REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the revised indexing system printed in Volume 28, Number 128, October 1974, pages 1191–1194.

12 [4.05.3, 12.05.1].–L. F. SHAMPINE & M. K. GORDON, *Computer Solution of Ordinary Differential Equations*, Freeman, San Francisco, 1975, vi + 318 pp., 24 cm. Price $13.95.

This book concerns the computer solution of the initial value problem in ordinary differential equations. It differs from the numerous other texts available in this area in that the authors discuss only the mathematical and computational considerations relevant to one particular state-of-the-art nonstiff solver, rather than survey many particular methods or several classes of methods. The exposition is quite elementary and self-contained and avoids almost all mathematics not directly related to the given code. In particular, only the Adams-Bashforth-Moulton methods in PECE form are studied.

The centerpiece of the book is the suite of FORTRAN programs DE/STEP, INTRP. These form an exceptionally well written code which, while judged among the best available general purpose codes for the solution of nonstiff initial value problems, is also very readable. In addition to the mathematical theory of the Adams algorithms employed in the code, there is a full discussion of the variable-step implementation used (Krogh's modified divided differences), and of the step design and order selection procedures. Such discussions enable the reader to understand a significant code in full detail and appreciate the many decisions that are made in its development, and thereby furnish a perspective rather different from and complementary to that furnished by the asymptotic analysis of algorithms traditionally taught in numerical analysis courses. This makes *Computer Solution of Ordinary Differential Equations* a distinct and valuable addition to the numerical analysis literature.

Of course, one can question some of the choices made by the authors in the code and their heuristic justifications, and when the book is used as a text it is useful to supplement these by a discussion of their limitations and of other possibilities which can be pursued. For example, in STEP the user supplied error tolerance controls the local error *per step* in the *predictor* formula. The chief justification of this is that the error *per unit step* in the accepted *corrector* value is then roughly a constant multiple of the tolerance. This constant, however, depends on a ratio of higher derivatives of the solution and so is unavailable to the user even a posteriori, and, in fact, the "constant" changes with the order. It is possible, though more expensive, to directly control the local error-per-unit step in the accepted solution. This is preferable because this quantity relates directly to the solution furnished the user and under certain hypotheses may be viewed as an asymptotically correct estimate of its residual.

0025-5718/81/0000-0122/$02.50

One of the main uses of this work is as a textbook. Because of the authors' novel approach, however, there is some difficulty in incorporating it into a more or less standard curriculum. Most teachers of courses on the numerical solution of ordinary differential equations will not wish to use it as the primary textbook because it leaves vast areas untouched: single-step methods; Dahlquist's theory of multistep methods; stiffness, A-stability, and related concepts; to list a few. Fortunately, the elementary and lucid style of this exposition makes it quite accessible to the student who has already encountered the basic mathematical theory of multistep methods, and therefore it can be used as a supplementary text relatively easily. Worked exercises to test the reader's understanding are included. Through lectures and reading assignments the reviewer very successfully covered virtually the entire text in an introductory graduate level course in two and one half weeks. The authors also suggest that their book be used as text for a topics course or as a source of supplementary readings in a survey course on numerical analysis.

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13 [2.05.5].—CLAUDE BREZINSKI, *Padé-Type Approximation and General Orthogonal Polynomials*, Birkhäuser Verlag, Basel, Boston, 1980, 250 pp. Price $34.00.

In recent years, we have witnessed a surge of interest in the field of rational approximation with emphasis on computation and evaluation of functions. In particular, considerable emphasis has been placed on approximations of analytical type with remainder as opposed to those approximations which require tabular values of the functions being approximated, that is, approximations of the curve fitting variety.

Let $f(x)$ be a function which possesses the at least formal representation $f(x) = \sum_{k=0}^{\infty} c_k x^k$. Let $P_m(x)$ and $Q_n(x)$ be polynomials in x of degree m and n, respectively, such that $Q_n(x)f(x) - P_m(x) = O(x^{r+1})$ where r is some positive integer. Then $T_{mn}(x) = P_m(x)/Q_n(x)$ is a rational approximation to $f(x)$ with the property that its expansion in powers of x agrees with that of $f(x)$ through the first r powers. The fraction $T_{mn}(x)$ fills out a matrix $m, n = 0, 1, 2, \ldots$. If $m = n$, we have the main diagonal while if $m = n - 1$, we have the first subdiagonal etc. If $r = m + n$, the approximations go by the name Padé, for which there is considerable literature.

In previous studies the reviewer [1], [2], [3] has developed rational approximations principally for generalized hypergeometric series with $m = n - a$, $a = 0$ or $a = 1$ such that $r = n - a$ at least. Originally, they were developed on the basis of Lanczos' τ-method and later it was shown how they could be obtained by appropriate weighting of the partial sum of the series for $f(x)$. The latter weighting or summability procedure can be extended at least formally for any series. For the series ${}_2F_1(1, b; c; -z)$ and its confluent forms, a certain set of the above rational approximations are those of the Padé class. There are other methods for getting rational approximations. For example, appropriate quadrature of the Stieltjes integral $f(z) = \int dg(t)/(1 - zt)$ leads to rational approximations of the above kind and if the quadrature is Gaussian, then Padé approximations emerge.

There are many voids in our understanding of rational approximations. Claude Brezinski's research on rational approximations and summability is brought to focus in the present volume which contains a number of novel and ingenious ideas on the subject. We present an illustration which is at the basis of his work.

Let c be a linear functional which acts on some normed linear space of functions whose domain includes polynomials in t and $(1 - xt)^{-1}$ where x lies in some region of the complex plane. Let $c(x^i) = c_i$, $i = 0, 1, 2, \ldots$. c_i is called the moment of order i of the functional c. Then $f(t) = c((1 - xt)^{-1})$. Let $v(x)$ be a polynomial in x of degree k. Then $w(t) = c((v(x) - v(t))/(x - t))$ is a polynomial of degree $k - 1$. Let $\bar{v}(t) = t^k v(1/t)$, $\bar{w}(t) = t^{k-1}w(1/t)$. Then $f(t) - T_{k-1,k}(t) = R(t)$, where $T_{k-1,k}(t) - \bar{w}(t)/\bar{v}(t)$ is a rational approximation to $f(t)$ and $R(t)$ is the remainder, $R(t) = \{t^k/\bar{v}(t)\}c[v(x)/(1 - xt)]$. The approximation $T_{k-1,k}(t)$ is called a Padé-type approximation for $f(t)$. It is readily shown how the analysis can be extended to give generally rational approximants of the form $T_{mn}(t)$ as discussed above.

The volume under review is devoted to the study of these Padé-type approximants. Basic properties are taken up in Chapter 1. They are divided into three classes. The first deals with algebraic properties; the second concerns the error; and the third explains the connection with polynomial interpolation and summation methods.

Suppose one desires to construct a Padé-type approximation whose order exceeds the degree of the numerator polynomial. This can be accomplished say for the first subdiagonal approximant by putting additional constraints on the polynomial $v(x)$. For instance, we can write $v(x) = u(x)S_m(x)$, $m < k - 1$, and determine the polynomial $S_m(x)$ by the conditions $c(x^i v(x)) = 0$, $i = 0, 1, \ldots, m - 1$. Define the functional c^* by $c^*(x^i) = c(x^i u(x))$. Then the added conditions are $c^*(x^i S_m(x)) = 0$. So $S_m(x)$ is a polynomial of degree m belonging to a family of orthogonal polynomials with respect to c^*. If $m = k - 1$, then we get the Padé approximation which occupies the $(k - 1, k)$ position in the matrix T. Many results on Padé-type approximation can be gotten by means of the theory of general orthogonal polynomials. For this reason the latter theory is extensively studied in Chapter 2.

In Chapter 3, the theory of general orthogonal polynomials is used to derive old and new results for Padé approximations. Some related items such as continued fractions and the ε-algorithm are treated. Two generalizations of Padé-type approximations are taken up in Chapter 4. The first is the case of series with coefficients in a topological vector space. Its application to sequences produces an ε-algorithm which generalizes the scalar ε-algorithm due to Wynn. The second generalization is concerned with Padé-type approximants for double power series.

An appendix gives a 'conversational program' in FORTRAN which computes recursively sequences of Padé approximants. There is a list of 149 references which is admittedly far from complete. The author has prepared a more extensive bibliography of some 2,000 items which has been published in the form of internal reports and is available on request. The shortened reference list aside, the reviewer noticed very few blemishes. One such is on page 30 and has to do with the Laplace transform $f(t) = \int_0^\infty e^{-xt}g(x)\,dx$. Suppose one approximates $f(t)$ by a Padé-type approximation where the degree of the denominator polynomial exceeds that of the numerator polynomial. Then an approximation for $g(x)$ as a series of exponentials follows by inversion of the approximation for $f(z)$. It is stated that this idea is due to Longman in a paper published in 1972. Actually an application of this idea was used by Longman in 1966. I recognize, there is some danger in saying who expressed an idea first. However, the reviewer gave several applications of this idea in papers published in 1957, 1962, 1963, and 1964. See [2, Volume 2, Section 16.4] for details.

The volume is well written and organized. It is valuable both for the information it provides and for pointing out directions for future research. For anyone interested in rational approximations, the book is a must on one's bookshelf.

Y. L. L.

1. Y. L. LUKE, *The Special Functions and Their Approximations*, Vols. 1, 2, Academic Press, New York, 1969.

2. Y. L. LUKE, *Mathematical Functions and Their Approximations*, Academic Press, New York, 1975. Also in Russian, Izdat. "Mir", Moscow, 1980.

3. Y. L. LUKE, *Algorithms for the Computation of Mathematical Functions*, Academic Press, New York, 1977.

14 [8.00, 2.00, 3.00].–W. J. KENNEDY & J. E. GENTLE, *Statistical Computing*, Marcel Dekker, New York, 1980, xi + 591 pp., $23\frac{1}{2}$ cm. Price $26.50.

Historically, statistical practice was limited by what was computationally feasible. Modern computing capability has expanded the horizons on computational feasibility to where what might have been the toys of mathematical statisticians in the past have become part of today's common statistical practice. This success and aim have been the fruits and the goal of the field of statistical computing.

The technique of nonlinear regression epitomizes the toy that is now common practice. It also illustrates the fragmentation that statistical computing suffered from. That is, contributors have come from many disciplines, from those in applications like chemical engineers, to those in operations research and numerical analysis, mathematicians, economists, and, finally, statisticians themselves. These advances have appeared in a great variety of publications. Many wheels have been reinvented many times.

The publication of this book, I believe, is a milestone, although it is not the first in its field (Hemmerle, [1]). Kennedy and Gentle have done an outstanding job of assembling the best techniques from a great variety of sources, establishing a benchmark for the field of statistical computing. Its imperfections will be discussed later.

Disregarding four preparatory chapters on the fundamentals of computing, the authors treat four main topics in depth:

1. Computing probabilities and percentile points.
2. Random numbers and simulation.
3. Linear models computations.
4. Nonlinear regression and optimization.

The final two chapters deal with robust methods and techniques for multivariate statistical analysis.

In the discussion of the first topic, methods are presented to satisfy most of the needs of statisticians. Some topics in numerical analysis are discussed in context: root finding, series approximations, continued fractions and quadrature.

For the second main topic, the authors begin with a discussion of generating the uniform distribution and the testing of random number generators. Following an explanation of general techniques, algorithms for generating from many nonuniform distributions are given. The list is reasonably up to date: most algorithms are the state of the art to 1977, some to 1978 and 1979. Here the authors have done a creditable job with a difficult topic.

The material on linear models computations is split into three chapters. The first chapter introduces Cholesky, Gram-Schmidt, Gaussian elimination/sweeps, and Householder transformations. Application of these techniques to the regression problem forms the middle chapter. The final chapter deals exclusively with classification models. The more conventional material has been treated elsewhere by Lawson and Hanson [2] and especially well by Stewart [4]. However, the last chapter on this topic has not and a great wealth of work is summarized. Of the 114 references for this chapter, 108 are to journal articles, conference proceedings, theses and reports; four of the remainder are statistics texts.

The last topic, covered in a single chapter, begins with the fundamentals of minima, saddle points and gradients. The discussion then moves to the elements of iterative search algorithms: direction, step size and convergence. A number of algorithms for unconstrained minimization are explained in detail. The important special case of nonlinear least squares is then covered extensively. Eighteen test problems are included, and the references for this chapter number 167.

This book is also intended to be a textbook for a graduate level course. The authors clearly have tried to write to such an audience (or lower). But so much of the material is at such an advanced level that it is better described as a reference work. I find it to be an adequate text, partly for personal reasons, but unquestionably superior to the alternative: no text. The many problems given at the end of each chapter are an obvious aid.

My major criticism of this book is that it is not very statistical. Computational techniques can be found in texts on mathematical statistics (Rao [3, p. 302]); the authors, statisticians, appear to have avoided any statistics beyond the bare necessities. In the long discussion of unconstrained minimization, maximum likelihood, the main interest of statisticians, is barely mentioned. The standard asymptotic theory of maximum likelihood (as well as exceptions) and their relationship to the computational problem would be both valuable and instructive here. Also overlooked is the relationship between conditioning and the associated statistical problem. In least squares, for example, solving the normal equations is ill conditioned when X^TX is nearly deficient in rank, hence an accurate solution may be unattainable. But a large condition number, measured by, say $\|X^TX\| \cdot \|(X^TX)^{-1}\|$, indicates that the entries of $(X^TX)^{-1}$ are unexpectedly large, hence the variances of the least squares estimates are likely to be large, and, hence, any computationally accurate solution would still be statistically crude. This relationship is stronger in maximum likelihood estimation.

Secondly, the distinct value of books written by those distinguished in a given field are their opinions and experiences. The authors have chosen to be diplomatic. They include some inferior techniques for completeness. Comparisons of techniques, their requirements and tradeoffs appear unemotional and neutral (a plus for the left-brained) but also indecisive and not compelling (and sometimes incomplete). They avoid the fray regarding regression computations on p. 325; their opinions and experiences are missed. For the most part, I miss the comments based on experience, which I compare to Stewart [4, p. 93 and pp. 152–154]. Notable exceptions are the discussion of checking regression computations on p. 329 and the advantages of GFSR on p. 162.

To summarize, *Statistical Computing* by Kennedy and Gentle is comprehensive: both broad and deep. It can replace for me the small library of books and articles I have heretofore needed. I bought a copy of this book before the editor sent me a second copy for review. That purchase has not been regretted.

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1. W. H. HEMMERLE, *Statistical Computations on a Digital Computer*, Blaisdell, Waltham, Mass., 1967.
2. C. L. LAWSON & R. J. HANSON, *Solving Least Squares Problems*, Prentice-Hall, Englewood Cliffs, N. J., 1974.
3. C. R. RAO, *Linear Statistical Inference and Its Applications*, Wiley, New York, 1965.
4. G. W. STEWART, *Introduction to Matrix Computations*, Academic Press, New York, 1973.

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