REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1980 Mathematics Subject Classification can be found in the December index volumes of Mathematical Reviews.

1[35Bxx, 49Gxx].—C. BANDLE, *Isoperimetric Inequalities and Applications*, Monographs and Studies in Mathematics 7, Pitman, Boston, 1980, vii + 228 pp., 24 cm. Price $51.00.

This well written and timely book by one of the leading researchers in the area of isoperimetric inequalities serves the useful purpose of organizing and unifying the large number of special techniques that have been developed in this area over the past 30 years. In this sense it may be regarded as a supplement to the treatise of Pólya and Szegö (Isoperimetric Inequalities in Mathematical Physics) which appeared in 1951. It is largely self contained with numerous examples, and an assortment of problems of various degrees of difficulty.

This book clearly displays the roles played by geometry and analysis in this diverse and rather complex area of study and shows how tools from both areas combine in applications.

The first chapter introduces the needed geometric tools and extends a number of classical results to domains and surfaces. The second chapter deals with various types of symmetrization techniques and applies them to a number of specific problems. Chapter 3 is devoted to linear eigenvalue problems, bringing together a large number of seemingly unrelated techniques and results, and Chapter 4 deals with boundary and initial value problems for linear and nonlinear elliptic and parabolic equations.

Although this book does not purport to give a complete up-to-date picture of the present state of the field of isoperimetric inequalities, it does contain an extensive bibliography which refers to most of the important advances made over the past 30 years. It would clearly be an excellent reference source for graduate students, applied mathematicians and researchers in differential geometry and partial differential equations.

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2[41A21].—J. GILEWICZ (Editor), *First French-Polish Meeting on Padé Approximation and Convergence Acceleration Techniques*, Proceedings of a Conference held in Warsaw, June 1–4, 1981, CPT, CNRS, Marseille, 1982, 94 pp., 21cm.

For several years close contacts between French and Polish mathematicians have been developed in the field of Padé approximation and convergence acceleration

0025-5718/84 $1.00 + $.25 per page

methods. Thus a meeting was organized by the computing center Cyfronet of the Institute of Nuclear Research in Swierk. Participants from England, France, Italy, The Netherlands, Poland and Spain were able to attend the conference.

The proceedings contains abstracts of some of the conferences and the complete text of some others.

The content is as follows:

C. BREZINSKI: "The state of the art in convergence acceleration methods and Padé-type approximation" (abstract). Review of recent results on the subjects.

M. G. DE BRUIN: "Generalized Padé tables and some algorithms therein". Regular algorithms for computing the generalized Padé tables introduced by Hermite, Padé and others are given. Some previous methods of the author are rediscovered.

L. CASASUS: "Nonlinear integration techniques. Application to nuclear reaction problems" (abstract). Construction of nonlinear integration methods by using Padé-type approximants.

F. CORDELLIER: "On the reliability and stability of the vector ε-algorithm" (abstract). It is shown how to modify the rules of the vector ε-algorithm to obtain a reliable and stable algorithm.

A. DRAUX: "Two point Padé approximants and general orthogonal polynomials" (abstract). The structure of the two point Padé table is studied and the approximants are related to the theory of formal orthogonal polynomials.

J. GILEWICZ: "Review of the best Padé approximation techniques in practical computation". Review on the problem of the choice of the "best" Padé approximant among the triangular part of the Padé table that can be constructed from a finite number of coefficients of the initial series.

P. R. GRAVES-MORRIS: "Toeplitz equations, Kronecker's algorithm and Polish polynomials". The construction of Padé approximants is related to the solution of Toeplitz systems of linear equations. An algorithm of the Kronecker and Euclid type is given for the normal case. The so-called Polish polynomials are introduced.

B. GERMAIN-BONNE: "Accelerable and non-accelerable sets of sequences" (abstract). Sufficient conditions for a set of sequences to be accelerable or to be non-accelerable are given. Examples of such sets are exhibited.

W. GUZINSKI: "Acceleration of convergence of power iterative process" (abstract). A variant of the vector ε-algorithm for accelerating sequences of vectors is presented. This algorithm is suitable for accelerating the vectors produced by the power method.

A. ISERLES: "Padé and rational approximations to the exponential and their applications to numerical analysis". Rational approximations to the exponential are very useful in constructing A-stable methods for integrating differential equations. A review of the subject is given.

S. PASZKOWSKI: "Generalized Padé approximation and hypergeometric series". Study of generalizations of Padé approximants which can be written as combinations of the values of an arbitrary function. Such generalizations are related to the hypergeometric series.

M. PINDOR: "Variational calculation of matrix elements of operator Padé approximants" (abstract). It is shown that a variational technique based on the Schwinger principle can be used for calculating matrix elements of operator Padé approximants.

G. SERVIZI, G. TURCHETTI: "On the perturbation expansion for classical Anharmonic oscillators". Determination of the stochastic transition by methods based on Padé approximation to locate the singularities of the perturbation series.

J. M. TROJAN: "An upper bound on the acceleration of convergence". The problem of the global estimation of the quality of the acceleration obtained with a given algorithm for a given class of sequences is studied. It is shown how much one can accelerate sequences obtained by iterative processes.

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3[65G10, 65L05].—P. EIJGENRAAM, *The Solution of Initial Value Problems Using Interval Arithmetic Formulation and Analysis of an Algorithm*, MC Tract 144, Mathematisch Centrum, Amsterdam, 1981, iii + 185 pp., 24 cm. Price Dfl. 24,15.

It has been well known that one should not use straightforward vectors of intervals in the stepwise attempt to bound the solution of an initial value problem for a system of first-order ordinary differential equations as their span may grow considerably faster than is appropriate for the analytic problem. Instead, as has been suggested by R. E. Moore, one should rather employ linear transforms of such interval vectors.

This idea has been elaborated in the tract under review. Eijgenraam constructs an algorithm which proceeds as follows:

Given an interval vector $\bar{y}_{n-1} = A_{n-1}\bar{x}_{n-1} \in \mathbf{IR}^M$, where A_{n-1} is an $M \times M$-matrix, the algorithm finds a step h_n, a matrix A_n and an interval vector $\bar{x}_n$ such that the interval $\bar{y}_n = A_n\bar{x}_n$ contains the values at $t_n = t_{n-1} + h_n$ of all solutions of $y'(t) = f(y(t))$ which have taken a value in $\bar{y}_{n-1}$ at t_{n-1}. For this purpose, it is necessary that the following functions may be formed for $0 \leqslant i \leqslant k - 2$ ($k \geqslant 2$) to be evaluated for $x \in \mathbf{R}^M$:

$$f_0(x) := f(x), \qquad f_i(x) = f'_{i-1}(x)f(x).$$

Furthermore, interval inclusions of f_{k-1} and of f'_i, $0 \leqslant i \leqslant k - 2$, are needed, for interval arguments from $\mathbf{IR}^M$.

The algorithm has been carefully motivated, introduced, analyzed, and discussed, with a great number of proven results and valuable observations about its performance. Thus the volume is a valuable contribution towards the goal of designing efficient software which generates realistic strict error bounds in conjunction with a stepwise approximate solution of a first-order system of ordinary differential equations.

H. J. S.

4[10A40, 10A25, 10–04].—H. J. J. TE RIELE, *Table of* 1870 *New Amicable Pairs Generated from* 1575 *Mother Pairs*, Report NN 27/82, Mathematical Centre, Amsterdam, Oct. 1982, 43 pages.

This is ref. [8] of te Riele's paper [1]. There he gives methods of deriving new amicable pairs ("daughters") from known pairs ("mothers"). Table 1 lists the "1575 mother amicable pairs" taken mostly from Lee and Madachy and from Woods (see the paper). L & M included *every* amicable whose smaller member is less than 10^8. The first mother listed here, not in L & M, is mother #266 with smaller member 176632390. Some of the daughters are known pairs but most of them, namely 1782, are new. And 88 more "granddaughters" are also listed. Since there are more daughters than mothers, this gives a heuristic argument for the existence of infinitely many amicable pairs.

Table 2a lists the number of new daughters for each of the corresponding mothers. Mother #1398 has 85 daughters! Table 2b lists the 1782 new daughters and Table 3b lists the 88 new granddaughters (that he computed).

The smallest new daughter is the pair 114944072, 125269528 arising from the proud mother #37. This daughter would come between pairs 243 and 244 in L & M.

D. S.

1. HERMAN J. J. TE RIELE, "On generating new amicable pairs from given amicable pairs," *Math. Comp.*, v. 42, 1984, pp. 219–223.