REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1980 Mathematics Subject Classification can be found in the December index volumes of Mathematical Reviews.

4[65-01].—R. L. Johnston, *Numerical Methods—A Software Approach*, Wiley, New York, 1982, ix + 276 pp., $23\frac{1}{2}$ cm. Price $24.95.

Several years ago, I proposed to some of my colleagues the following thesis on the subject of teaching numerical methods to scientists and engineers. Since it is difficult enough for most professionals to keep abreast of one's specialty, they will certainly not be up-to-date concerning the state-of-the-art in numerical algorithms and software. Hence they will continue to use the methods they learned in college throughout their entire career. This is borne out by the experience of many numerical analysts associated with scientific computing centers who discover in their conversations with customers that they have been using obsolete and inefficient methods to solve their problems. Therefore, I suggested that instead of teaching numerical methods, we teach how to use the mathematical software libraries such as IMSL and NAG. This is by no means a trivial task, and it requires much thought and effort to prepare such a course. The hope is that the libraries will be periodically updated and will incorporate the latest numerical analysis wisdom. In this way, the user will always have the best methods at his disposal. Thus, we should condition the user of numerical methods to turn to the latest edition of the mathematical software library, and we should teach how to read the manuals and use the software. This involves choosing the appropriate program, testing the programs before using them, understanding how to prepare the input and interpret the output, knowing what to do in case the software doesn't work, etc. Pressure should also be put on the writers of these manuals to make them easier to understand and on the writers of the programs to make them easier to use and more automatic and foolproof. One could develop this thesis at greater length but I believe the reader will get the flavor of the idea. Note, however, that the rapid spread of microcomputers may require a modification of this thesis; the problems are much more difficult!

Now when I received the book by Johnston, I hoped it would be a step in the direction of implementing this approach. And indeed, there are some indications that this book is going in the right direction in that it includes calling sequences to subroutines for executing many of the standard procedures in numerical analysis such as solving a system of linear algebraic equations, finding a least squares fit of data using cubic splines, evaluating a fast Fourier transform, finding the root of a nonlinear equation or the zeros of a polynomial, and integrating a system of ordinary differential equations. On the other hand, much more space is devoted to

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describing the algorithms for these tasks and others such as computing the eigenvalues and eigenvectors of a matrix, interpolation and approximation, quadrature, etc. The treatment of these matters is on a very elementary level and is to a great extent superfluous. And if we were to accept the assumption of the author that a good knowledge of the algorithms is necessary in order to use them, then there are some serious omissions. Thus, in the discussion of the solution of linear equations, there is no mention of scaling, while in the description of the QR algorithm, the crucial point of using shifts is ignored. One serious mistake occurs on p. 225, where the definition that is given for a (convergent) improper integral is actually the definition of a Cauchy principal value integral.

Finally, since this book also deals with the writing of mathematical software, it provides at least one example of our thesis. In the chapter on quadrature, adaptive quadrature is described using a local strategy. Current practice prefers the use of a global strategy. Hence, anyone writing an adaptive quadrature routine based on the material in this book is not using the best strategy, and twenty years from now, he will still be writing such programs, instead of referring to a mathematical software library for a program using the latest techniques.

P. R.

5[46-01, 65J10].—COLIN W. CRYER, *Numerical Functional Analysis*, Oxford University Press, New York, 1982, iv + 568 pp., 24 cm. Price $39.00.

This book is intended as the first volume of two. It is concerned with teaching the foundations of Functional Analysis and with its interplay with Numerical Analysis. The second volume will treat elliptic boundary value problems and nonlinear problems.

Special features of this text are as follows: In the systematic introduction of concepts of Functional Analysis frequent stops are made for applications relevant to Numerical Analysis. Thus the students immediately see the concepts in action. Many counterexamples are given to delineate the basic definitions and theory. There is plenty of exercises, and they come with solutions or references.

The choice of material is standard, as the following list of the first eight chapter headings indicates: Introduction, Topological vector spaces, Limits and convergence, Basic spaces and problems, The principle of uniform boundedness, Compactness, the Hahn-Banach theorem, Bases and projection. The ninth and last chapter covers approximate solution of linear operator equations (about a hundred pages) with emphasis on integral equations. This book also includes a list of notation, an excellent index and, still more excellent, a list of theorems, lemmas and corollaries. It ends with references and solutions, the latter covering a hundred and fifty pages.

I found the treatment very well suited to what I perceive as the "typical student" in Applied Mathematics. Concepts are thoroughly motivated, and interesting and enlightening applications to Numerical Analysis (and related fields) are given. In the long list of counterexamples I missed one of a linear map defined on the whole of a Banach space but discontinuous (one occurs implicitly on p. 273). In my experience in teaching Functional Analysis this is one counterexample students will be asking for if you withhold it for too long. It should be pointed out that it is not assumed that the reader knows Measure Theory or Lebesgue Integration. These things are briefly summarized in an appendix to Chapter Four.

The only serious mistake I found is in connection with the Principle of Uniform Boundedness and its application to the Lax Equivalence Theorem. It is assumed, (5.47) and also p. 155, that the semigroup $E(t)$, the solution operator in L_2 to the homogeneous heat equation, is continuous with respect to t in the operator norm. This is false. However, for the Lax Equivalence Theorem it suffices that $E(t)v$ is continuous in t for each fixed v in L_2, also known as "strong convergence". The mistake is embarrassing since the proof of (5.47) is given as a problem, no. 5.29. The "solution" given to this problem ends by purporting to have shown a certain inequality. This inequality actually shows only "strong convergence". However, the proof of even this weaker result is completely wrong, although the result is true.

I also found two problems that I regard as "marginally misleading". Problem 8.27 might lead the unwary student to deduce a result which is true only in one space dimension. Interpreting the undefined and unlisted symbol $\mathscr{C}_0^2(\overline{\Omega})$ in the most charitable sense, one looks for solutions of a "smooth" second order elliptic problem $Lu = f \in \mathscr{C}(\overline{\Omega})$ with homogeneous Dirichlet conditions in $\mathscr{C}^2(\Omega) \cap \mathscr{C}^0(\overline{\Omega})$ which are zero on $\partial\Omega$ (classical solutions). In more than one space dimension the problem is correct only by virtue of its hypotheses always being false. (E.g., the problem $\Delta u = f$ in Ω a bounded domain in R^n, $n \geqslant 2$, $u = 0$ on $\partial\Omega$ nice, will *not* have a classical solution for every f in $\mathscr{C}(\overline{\Omega})$.) Problem 8.20 might lead the unsuspicious reader to believe that the Legendre series for a function in $\mathscr{C}^2([-1, 1])$ converges only to order $n^{-1/2}$ in the maximum norm; this is a full order n^{-1} off.

My impression is that in spite of minor flaws this is an excellent text for a stimulating one-year course in Functional Analysis with applications.

L. B. W.

6[65-01].—J. F. Botha & G. F. Pinder, *Fundamental Concepts in the Numerical Solution of Differential Equations*, Wiley, New York, 1983, xii + 202 pp., 24 cm. Price \$24.95.

This volume is intended for the novice practitioner who needs to read up quickly on basic practical numerical methods for (mainly) partial differential equations. It by and large bypasses theory, but some understanding and much practical advice is given. In the above respects it resembles von Rosenberg's brief volume, [2]. von Rosenberg's book treated finite difference methods, whereas the present exposition, written a decade and a half later, gives equal space to Galerkin Finite-Element methods, Collocation Finite-Element methods, and Boundary Element methods. It can be viewed as a pared-down version of Lapidus and Pinder [1]. I quote from the Preface: "This book is designed to provide an affordable reference on the methodology available for the solution of ordinary and partial differential equations in science and engineering."

It is hard to judge whether the authors have succeeded in their goal. Certainly the treatment is brisk and to the point and always elucidated with examples showing the "how-to" of the method. A basic tenet is said to be the following: "The various numerical methods are developed from one fundamental base—the theory of interpolation polynomials." Such a foundation is rather proper in one-dimensional situations, cf. the elegant little book by Wendroff, [3]. In more than one dimension the claim is somewhat overstated. You will need much more than interpolation polynomial theory to conquer a given partial differential equation.

The brisk treatment means that the authors often give their own opinions in practical matters without supporting evidence. For example, on page 36 the following statement appears: "It is enough to say here that the application of Galerkin's method to first order equations is never worth the effort."

There are a few questionable statements in the book. The worst one appears in Section 1.4, pp. 5–6, on why and how boundary conditions must be specified. For example, for $\Delta u = 0$ in a two-dimensional region, since there are 4 derivatives involved, formal integration gives 4 unknown functions! Happily, if the region is rectangular, there are 4 sides and so we can specify the correct number of conditions! Woe if you were to solve the problem in a triangle or a pentagon or a disc! For the heat equation it is only by convention that one specifies initial time conditions (in addition to spatial boundary conditions). Final time conditions would be equally appropriate, it appears!

For the interested reader, I will record that I also found statements that I quarrel with on the following pages: 30, 49, 52, 55, 77, 97, 100–107, 117. Happily, most of these statements can be taken as starting-points for constructive discussions. Most likely, the authors have evidence which they do not present in this brief volume.

The practical hints given seem mostly to pertain to problems which require only low-accuracy solutions, say 5–10 percent relative error in multidimensional situations. A scientist who desires to illustrate her/his theory by compelling numerical examples might well heed different advice.

Having thus said that the book contains statements that merit reflection, for novice and expert alike, I point out that it is indeed a brisk and to the point introduction to numerical methods in partial differential equations. Most major classes of methods are treated in some detail. The "how-to" is explained with detailed examples, and the authors share their wealth of knowledge in practical evaluation of the methods. This volume should serve at least as an introduction to its stated purpose, "...to supply the inexperienced scientist or engineer with the fundamental concepts required to achieve this objective (to obtain a relevant numerical solution efficiently and accurately)".

L. B. W.

1. L. Lapidus & G. F. Pinder, *Numerical Solution of Partial Differential Equations in Science and Engineering*, Wiley, New York, 1982.

2. D. V. von Rosenberg, *Methods for the Numerical Solution of Partial Differential Equations*, Gerald L. Farrar and Associates, Inc., Tulsa, 1969.

3. B. Wendroff, *First Principles of Numerical Analysis*, Addison-Wesley, Reading, Mass., 1969.

7[65R20, 76B05].—H. Schippers, *Multiple Grid Methods for Equations of the Second Kind with Applications in Fluid Mechanics*, Mathematical Centre Tracts 163, Mathematisch Centrum, Amsterdam, 1983. iii + 133 pp., 24 cm. Price $6.00.

This Mathematical Centre Tract has been based on the author's Ph.D. Thesis at Delft University of Technology (with Professor Wesseling). It provides a well-written introduction into theoretical and practical aspects of the use of multigrid methods for the numerical solution of Fredholm integral equations of the second kind. Furthermore, it presents interesting applications to problems in fluid dynamics: the computation of the double layer distribution representing a potential flow around an airfoil, and the computation of the periodic flow generated by an infinite disk performing rotational oscillations.

After introductory remarks and a survey of the compactness results on sequences of approximations to Fredholm operators, the multigrid approach is introduced and asymptotic results about contraction rates, convergence and number of operations are derived and experimentally verified. Then a code is presented (in Algol 68) for the solution of Fredholm integral equations of the second kind. The approximations may be piecewise linear and trapezoidal rule, or cubic spline and Simpson's rule. A tolerance for the remaining discretization error may be specified and the code attempts to choose the finest grid accordingly. In numerical examples the code proves superior to Atkinson's IESIMP.

The second half of the monograph deals with two applications in fluid dynamics. First, the multigrid approach is applied to the numerical solution of the customary collocation system for a piecewise constant doublet distribution along the boundary. A clever analysis of many theoretical and algorithmic details leads to convergence results for the doublet distribution and its derivative and to contraction rates of various multigrid processes. The application to both noncirculatory and circulatory potential flows around Karman-Trefftz airfoils is demonstrated; the circulatory flows require a special type of smoothing and convergence cannot be established in a vicinity of the trailing edge.

A particularly interesting application to the computation of the periodic solution of a parabolic equation with periodic initial conditions concludes the treatise, which constitutes a welcome contribution to the multigrid literature.

H. J. S.

8[10A25, 10-04].—JOHN BRILLHART, D. H. LEHMER, J. L. SELFRIDGE, BRYANT TUCKERMAN & S. S. WAGSTAFF, JR., *Factorizations of* $b^n \pm 1$, $b = 2, 3, 5, 6, 7, 10, 11, 12$ *up to high powers*, Contemporary Mathematics, Vol. 22, Amer. Math. Soc., Providence, R.I., 1983, lxvii + 178 pp., 25 cm. Price $22.00.

The purpose of this volume is to present several tables of factorizations of integers of the form $b^n \pm 1$. The first set of four tables gives the complete (with a few exceptions) factorization of all integers of the form $2^n - 1$, $2^n + 1$, $10^n - 1$, $10^n + 1$ for values of n up to 250, 238, 82, and 72, respectively. A second, larger set of tables presents factorizations of $b^n \pm 1$ for all $n \leqslant m$, where the values of b and m are set out in the table below.

N	m	N	m
$2^n \pm 1$	1200	$7^n \pm 1$	180
$3^n \pm 1$	330	$10^n \pm 1$	150
$5^n \pm 1$	210	$11^n \pm 1$	135
$6^n \pm 1$	195	$12^n \pm 1$	135

These main tables employ a more concise format than the earlier tables; however, the manner in which they are to be used is carefully explained.

There are also three appendices to the main tables. The first contains the actual decimal digits of all prime and probable prime factors in the main table that exceed 25 digits. The second supplies a short summary of the proof of primality of each prime between 25 and 72 digits. As the authors were not able to completely factor several of the numbers in their tables, they present, in the third appendix, the decimal digits of each composite cofactor which is no more than 64 digits.

The authors also provide a chapter on the developments in technology that have permitted them to complete this work. This is a fascinating blend of history, computing and the theory of factorization and primality testing. Unfortunately, the book was sent to press previous to two exciting new developments in this subject. The first of these is the primality test of Cohen and Lenstra (developed from the important work of Adelman, Pomerance, and Rumely), which will undoubtedly provide primality proofs for all the probable primes in the first appendix. The second is the implementation of the Quadratic Sieve Factoring technique by Davis and Holdridge. Indeed, activity in this subject is so great that the 10 "most wanted" factorizations listed by the authors have already been achieved, most of them by Davis and Holdridge.

A very useful feature of this work, especially in view of the intense activity mentioned above, is the provision of periodic updates, which will fit in a pocket in the back cover. One of these appears with the book now and another is to come very soon; still others are expected to follow.

This remarkable book, the product of decades of work, is indispensable to anyone who is interested in the problems of factoring and primality testing. It should be especially enlightening to those individuals who believe that nobody did any "serious factoring" previous to the last ten or fifteen years.

H. C. W.

9[65-06].—DAVID F. GRIFFITHS (Editor), *Numerical Analysis*, Lecture Notes in Math., Vol. 1066, Springer-Verlag, Berlin, 1984, ix + 275 pp., 24 cm. Price $14.00.

These are the proceedings of the 10th biennial conference on Numerical Analysis, held at Dundee June 28–July 1, 1983, containing 15 of the invited lectures. Among the subject areas represented are spline approximation, nonlinear equations and optimization, ordinary and partial differential equations, and weakly singular integral equations.

W. G.

10[41-02].—C. K. CHUI, L. L. SCHUMAKER & J. D. WARD (Editors), *Approximation Theory IV*, Academic Press, New York, 1983, xvii + 785 pp., $23\frac{1}{2}$ cm. Price $50.00.

This volume contains seven survey papers (289 pages) and 74 short research papers (447 pages) given at an international symposium on Approximation Theory held on the campus of Texas A & M University at College Station January 10–14,

1983. The survey papers are: "The best approximation of multivariate functions by combinations of univariate ones" by E. W. Cheney; "Recent progress in multivariate splines" by W. Dahmen & C. A. Micchelli; "A survey of some recent developments of approximation theory in China" by L. C. Hsu; "*n*-widths in approximation theory: A survey" by A. Pinkus; "General algorithms for discrete nonlinear approximation calculations" by M. J. D. Powell; "Incomplete and orthogonal polynomials" by E. B. Saff; "Truncation and factorization of biinfinite matrices" by P. W. Smith. In addition, there is a bibliography including 771 items on the theory and application of Bernstein-type operators compiled by H. H. Gonska & J. Meier.

W. G.