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Large-Scale Computations in Fluid Mechanics

Bjorn E. Engquist, Stanley Osher and Richard C. J. Somerville, Editors

This is the proceedings of an AMS-SIAM Summer Seminar on Applied Mathematics held at Scripps Institution of Oceanography in 1983, whose purpose was to bring scientists interested in computational fluid mechanics together with numerical analysts and mathematicians working in large-scale computations. The complexity of many contemporary problems of fluid mechanics is so great as to tax the capabilities of present-day computers. There is a real need and opportunity for numerical analysis to aid research on the physical problems of achieving optimal utilization of current computers.

Fifty lectures were given on subjects equally divided between mathematics and applications. The numerical modeling included geophysical problems of the atmosphere, ocean, and interior of the earth, and planetary, solar, and stellar atmospheres. Applications ranged from idealized turbulence in laboratory convection models to operational weather prediction. Engineering applications included aerodynamics, combustion, and flow in porous media. Recent advances in numerical analysis which have applications to these problems were stressed. These include shock capturing algorithms, spectral methods, boundary treatments, vortex methods, and parallel computing.

In addition to specialized research lectures, several speakers gave talks surveying important areas of numerical analysis and computational fluid dynamics.

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Computer Communications
B. Gopinath, Editor

This is the proceedings of the 1983 American Mathematical Society Short Course given at Denver, Colorado.

Computer communications is characterized by rapid technological advances presenting problems of a theoretical nature that are often very difficult to solve. They range from those that arise on a single chip, where communication among thousands of elements on a chip is influenced by electrical properties, to those that arise when human beings communicate with data bases where the logical aspects of communications play a more important role. A variety of mathematical methods is needed to attempt to solve such problems; they vary from partial differential equations to temporal or modal logic.

The article on "Diffusion Approximation" is probably the best review of this field, accessible to a mathematician, which is available today. The book is unique in having contributors from a variety of different fields of computer communications. All of the articles are of high research value and are self-contained.

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R. P. Kurshan, *Modelling concurrent processes*
T. Lengauer, *The communication complexity of VLSI circuits*
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Factorizations of $b^n \pm 1$, b = 2, 3, 5, 6, 7, 10, 11, 12 up to High Powers

John Brillhart, D. H. Lehmer, J. L. Selfridge, Bryant Tuckerman, and S. S. Wagstaff, Jr.

Every person interested in factorization has been waiting eagerly to see this book. Now and then during the past twenty or so years word has reached us about the so-called Cunningham Project, being carried out by a group of dedicated persons. No conceivable effort has been spared to make these factor tables as complete and as accurate as possible, even though their planned range surpasses the limits of what can be achieved by today's factorization algorithms and computers.

The book begins with some handy short tables for the factorizations of $2^n \pm 1$ and $10^n \pm 1$. Then follows a most readable historic account on the development of factorization methods and tools over the past years, covering 40 pages of text. The rest of the book is taken up by the Main Tables, giving all to date known factorizations of the numbers within the range set out for each of the tables.

And—best of all—update sheets containing brand new information found by the authors or communicated to the authors by other researchers are offered the owners of the book. One formal update, covering the period from autumn 1982, when the manuscript was sent to the printer, and to July 20, 1983, when the book was issued, is included in a pocket on the back inside cover. It contains such interesting factorizations as $2^{212}+1$, $2^{253}-1$, and $10^{64}+1$. After this, several "informal" sheets with further factorizations have been sent out, giving among others, the factors of $2^{211}-1$, $2^{251}-1$, and $10^{67}-1$.

– *Hans Riesel,*
Royal Institute & Technology, Stockholm

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