REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1980 Mathematics Subject Classification can be found in the December index volumes of Mathematical Reviews.

1[01A55, 01A70, 41-03, 41A21].—H. PADÉ, *Œuvres*, rassemblées et présentées par Claude Brezinski, Librairie A. Blanchard, Paris, 1984, 457 pp., 24 cm. Price 195ff.

When, in 1953, at the age of 90, Henri E. Padé died at Aix-en-Provence, little could he imagine the importance his name would achieve, when specialists decided to use it in connection with the rational approximations he had studied some 60 years before. He never pretended to have invented them, in fact these approximations already appear in their present form in the texts of Lagrange, Jacobi (1846) and many others. Then came his contemporaries: Poincaré, who said that the theory of continued fractions is "terra incognita", the omnipresent Hermite, whose classes Padé attended in Paris, Klein and Van Vleck, with whom he spent a year in Göttingen, Tchebycheff and finally Stieltjes and Frobenius, quoted for the first time by Padé in his lecture of 1900. When Padé began to study continued fractions, "Padé" approximants appear only as elements characterizing an isolated fraction. Classifying them all in a matrix table, Padé in fact identified them with the whole set of continued fractions associated with a series. It is this Padé table, whose structure he studied in detail, which gave, by extension, its name to the approximants.

After the death of Henri Padé, two new developments led scientists to bring Padé approximants to light. On the one hand, it was the advent of numerical calculation, which needed answers to the question: how can we decently replace infinite processes, such as the extrapolation of sequence limits or series summation, by objects manipulated by the finite automaton which is a computer? On the other hand, physicists seeking a concrete interpretation of asymptotic series, such as perturbation series, tried to sum them using a proven algorithmic method, and in particular, they tried to rediscover the singularities of a function starting from its formal series, which does not have any. The impulse for the first task was given by D. Shanks, the inventor of the covergence acceleration methods; the second task, tied up with the problem of analytic continuation of series, was initiated by G. A. Baker, Jr. There were few interested scientists at this time, a hundred in 1972 for the first Conferences on Padé approximants, held in Canterbury and Boulder, but there are thousands today, ranging from the ordinary user of the method to the pure mathematician.

These specialists wondered discreetly: "Who was Padé, and what did he do after his thesis, submitted in 1892 at the Sorbonne?"

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The aim of the enormous job of research and bibliography carried out by C. Brezinski throughout France, was to answer these questions. He had the good fortune to meet Padé's direct descendants, who were not all aware of the notoriety of their name. Thanks to this opportunity, Brezinski was able to draw the portrait of Henri Padé, an integral man, going to the very depth of the work he began, "enjoying" research as well as teaching, nature, music, poetry and maybe even administrative duties. There is here a mystery, which is the fact that Padé immersed himself in administration, never to come back to research. He published between 1888 and 1907, and had no research students, although he continued his professorial duties after his nomination as rector. Perhaps by courtesy, Brezinski gives no explanation of this change in Padé's life. It is however possible to imagine explanations with the help of a careful reading of Padé's work.

The Padé "Œuvres", collected by C. Brezinski, contain a detailed bibliographic note on the mathematician's life, some 400 pages covering 42 publications, with analyses done at the time of the publications. Thanks are due to the publisher Blanchard for having agreed to publish a forgotten mathematician's work.

Publication of a mathematician's collected works is of interest to historians of science, as well as to scientists who may find original ideas, left aside for different reasons, and currently exploitable. We consider the more important in today's terms. The term *continued fraction* is considered as an abbreviation for a sequence of approximants, whether convergent or not. The term "rediscovery" is not ironical.

Sur la convergence des fractions continues simples (1891)

The first ideas which attach the convergence of Padé approximants to the behavior of their denominators, ideas which lead to Padé's conjecture of 1907.

Sur la représentation approchée d'une fonction par des fractions rationnelles (90 pp., thesis, 1892)

Detailed analysis of blocks in a Padé table. One can recognize existing and nonexisting Padé approximants. All this was "rediscovered" by W. B. Gragg (1972), G. A. Baker, Jr. (1973) and by myself (1978).

Sur la généralisation des fractions continues algébriques (40 pp., 1894)

Follows an article with the same title by Hermite. The original idea for this generalization was stated by Hermite in a letter to Stieltjes on March 17, 1887. Without considering existence hypotheses, Padé creates general recurrence formulae to calculate these approximants. Following Paszkowski, the ideas given by Padé can still be used. Padé-Hermite approximants are today at the start of their development. They reappear in R. E. Shafer (1974, quadratic approximation), S. Paszkowski (1980, constructive algorithms), G. V. Chudnovsky (1980, about transcendence) and J. Della-Dora (1980, thesis; numerical computation).

Mémoire sur les développements en fractions continues de la fonction exponentielle pouvant servir d'introduction à la théorie des fractions continues algébriques (30 pp., 1899)

Elegant update which completes other short notes by Padé on the exponential function. Convergence theorem of Padé approximants for this function, rediscovered and generalized to functions of the class S (Pólya frequency series) by R. J. Arms and A. Edrei in 1970. Historical forms of continued fractions to e. At the same time, Padé published several notes on the continued fraction expansion of the function $(1 + x)^m$. It is here that he introduced for the first time the notion of "holoïd" continued fractions, a notion that he considered essential, but which nobody has taken up.

Sur la distribution des réduites anormales d'une fonction (1900)

In this short note the notion of equivalence transformation can already be found.

Sur l'extension des propriétés des réduites d'une fonction aux fractions d'interpolation de Cauchy (1900)

First study of Padé approximants of interpolation (N-point Padé approximants), generalized in the Padé-type approximants of Brezinski (1980).

Recherches nouvelles sur la distribution des fractions rationnelles approchées d'une fonction (36 pp., 1902); *Recherches sur la convergence des développements en fractions continues d'une certaine catégorie de fonctions* (60 pp., first prize at the Academy competition of 1907).

Fine studies on the algebraic theory of continued fractions at that time, but forgetting Frobenius, and with only a simple mention of Stieltjes. Holoïd fractions are still present, as in other notes, but with no success.

In his complimentary thesis report, Charles Hermite, promotor of the work, wrote: "Mr. Padé... has completely exhausted the question he set himself", that is, the problem of approximants. This was probably a concession to the style of the times, since it does not appear likely that Hermite could really think so, particularly when one reads his correspondence with Stieltjes between 1882 and 1894. In fact, Padé seems to be victim of his stubborn requirement for the clarity he brought to the classification of continued fractions. He thought that his theory of holoïd fractions should explain all, and his stubborness was the cause of increasing isolation, the new generation of specialists ignoring his work. The idea of holoïd fractions is purely algebraic, but Padé connects it, with a few hypotheses, to the theory of convergence, which led him to announce in 1907, in his next to last publication (*Sur la généralisation des formules de Sylvester...*) a "theorem" known today, in a form "slightly" changed by Baker, Gammel and Wills, under the name "Padé conjecture". This can be considered as an exceptional intuition of Padé, or as the ignorance of the author of this review, who had not understood the master's thoughts.

G. A. Baker, Jr., in "*Essentials of Padé Approximants*", points out that, in his 1892 thesis, Padé did not quote, and probably did not know about, the work of Jacobi and of Frobenius. In fact, he referred to Jacobi, but, curiously, he mentioned his two contemporaries, Stieltjes (deceased in 1894) and Frobenius, for the first time in 1900, at the Second Mathematicians Congress in Paris. He wrote on two occasions that the results of Stieltjes could be interpreted and generalized by his theory of holoïd fractions, but stopped this line of research following the publication of the "beautiful results" obtained by Van Vleck in 1903, as he himself said in the competition entry which won him the Grand Prix of Mathematics at the Academy in 1906. The work of Stieltjes, typically analytical, could not be interpreted by purely algebraic theory. Did Hermite, friend and admirer of Stieltjes, brutally point this out to Padé? Padé either chose, or was forced, to live in the country, and this cost him the loss of information from, and contacts with, Paris. He certainly carried out continuous correspondence with several mathematicians, and surely with Hermite. The reading of this correspondence could have explained a lot of things, but unfortunately Brezinski could not find it. Finally, Padé, solitary researcher, ended up in a cul de sac. Did he suffer some problem which left him without motivation for research, or did he simply find satisfaction in academic administration? Brezinski does not reply to these questions. As for the scientific work of Padé, it has a magnificent flair of combat, partially lost at first, and won 70 years later. From reading his papers, one can also profit from an interesting lesson on how to avoid scientific dead ends.

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2[65-02, 65F05, 65F10, 65L05, 65N10, 65N30, 65N50].—JOHN NOYE (Editor), *Computational Techniques for Differential Equations*, North-Holland, Amsterdam, 1984, vii + 679 pp., 24 cm. Price $74.50, Dfl. 175.00.

These are expanded versions of tutorial papers presented at the 1981 Conference on Numerical Solutions of Partial Differential Equations held at the University of Melbourne, Australia. The authors and their titles are: JOHN NOYE, "Finite Difference Techniques for Partial Differential Equations" (260 pages), CLIVE FLETCHER, "The Galerkin Method and Burgers' Equation" (121 pages), JOSEF TOMAS, "The Finite Element Method in Engineering Application" (47 pages), LEIGH WARDLE, "An Introduction to the Boundary Element Method" (27 pages), KEN MANN, "Direct Solution and Storage of Large Sparse Linear Systems" (69 pages), LEONARD COLGAN, "Iterative Methods for Solving Large Sparse Systems of Linear Algebraic Equations" (57 pages). In addition, a survey paper by ROBERT MAY on "The Numerical Solution of Ordinary Differential Equations: Initial Value Problems" (94 pages) has been included. The treatment is on a thoroughly practical level, but includes ample reference to theoretical results and to current activities in the field.

W. G.

3[65F15, 65F25, 82-02].—D. G. PETTIFOR & D. L. WEAIRE (Editors), *The Recursion Method and its Applications*, Springer Series in Solid-State Sciences 58, Springer-Verlag, Berlin, 1985, viii + 179 pp., $23\frac{1}{2}$ cm. Price $29.00.

By "Recursion Method" physicists refer to Lanczos-type methods for tridiagonalizing the Hamiltonian operator in some matrix form and for computing its spectrum, or part of it, and the corresponding eigenstates. This is closely tied up with continued fraction theory, orthogonal polynomials and the moment problem. The book under review contains the invited papers of a conference on the subject, held at Imperial College, London, September 13–14, 1984. Although written largely by physicists, in their own terminology, this collection of essays should be of interest to those who want to learn about current applications of Lanczos-type algorithms in solid-state theory, nuclear theory and lattice gauge theories.

W. G.