REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1980 Mathematics Subject Classification (1985 Revision) can be found in the December index volumes of Mathematical Reviews.

30[65N25, 76W05].—Ralf Gruber & Jacques Rappaz, *Finite Element Methods in Linear Ideal Magnetohydrodynamics*, Springer Series in Computational Physics (H. Cabannes et al., Editors), Springer-Verlag, Berlin, 1985, xi + 180 pp., $23\frac{1}{2}$ cm. Price $34.00.

This book is about finite elements for linearized spectral stability analysis of magnetically confined plasmas. The purpose of such confinement is to reach energy levels at which sustained, controlled thermonuclear fusion can take place in order to generate electric power. This has yet to be accomplished in a practical way, and the Introduction to this book explains why: There are only a few geometries in which confinement has been achieved, and the more efficient the process is energetically, it seems, the more susceptible it is to catastrophic instability which disrupts the confinement.

The magnetohydrodynamic (MHD) equations combine those of compressible, inviscid fluid flow (the plasma) with Maxwell's equations and Ohm's law (the electromagnetic fields). Since the plasma is to be confined, there are regions of magnetic field only (vacuum) and an interface with the confined plasma. Linearization about a stably confined plasma simplifies the problem greatly, but practical geometries are complex (such as helical or toroidal), and the linearized equations are formidable. This is a natural setting for numerical exploration—first to understand the nature of the instabilities, and then to use the understanding to guide the design process. When this reviewer conjures up a picture of the practical applications of his own research, the picture which most often comes to mind is one of some gooey fluid being dolloped out by a dingy industrial process. There is a real flair in being able to model cosmic forces on your computer and watch as the elusive ions escape on the wisps of seconds. This book conveys that romance well, but it is hard for the reader without training in high-energy physics to come away with a more quantitative impression.

So the dutiful reviewer tries to look at the book from the perspective of the finite element analyst, and that approach starts well. It is surprising to find out how easy it is to get into trouble with seemingly simple, symmetric eigenvalue problems in one space dimension! The problems are of the form $|A - \lambda B| = 0$, but the continuum problem can have an infinitely degenerate eigenvalue $\lambda = 0$, and a continuous spectrum. Going at this in a brute-force manner, with linear elements for all fields, leads to what the authors call "spectral pollution". Finite element methods are known to produce inaccurate high-spectrum eigenvalues. In the present problem,

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these eigenvalues are introduced with each mesh refinement, and though each new eigenvalue may actually converge to zero, new inaccurate ones continue to be introduced and cloud the *low* spectrum with nonsense.

A little more investigation shows that spectral pollution results from implied constraints in the multifield Lagrangian. The resolution to pollution is in balancing constrained and unconstrained degrees of freedom: an old finite element story. The problem looks at first like a Mindlin beam, and reduced integration might work. But, no, the derivative is on the wrong term in the constraint equation, and reduced integration may not work. The authors' solution is something that looks like a combination of upwind differencing and lumping—but which is achieved by selection of balanced interpolates.

The point here is that the numerical problems and their resolution seem closely related to, yet distinctly different from,those found in other areas of finite analysis, yet the authors make no connection. Later on in the book, there is another link to constrained media problems. Some solutions (the "Alfvén modes") are characterized by the vanishing of the divergence of one of the fields. The authors develop a rudimentary form of "constraint counting" familiar to Navier-Stokes modelers, but again the authors make no connection. They develop a method—the "finite-hybrid element approach"—argue that it works (but do not present error analysis in this book), and go on to talk about the physics, with its "kink" modes, "ballooning" modes and "Alfvén" modes.

If this book is intended to primarily reach high-energy physicists and tell them new things about the stability of these modes under various confinement conditions, it may well do so admirably. This reviewer is not competent to comment on that. But if that is the case, there seems to be more dwelling on numerical technique than one might expect. Yet, as we have observed, the numerical technique is not comprehensively linked to the numerical world outside of high-energy physics. What, then, *is* the purpose of this book? We do not find out until we reach far into the appendices. These appendices are long and contain a whole code, ERATO 3, with a very thorough write-up. If motivated, we all could compute kink modes. The reviewer is not making light of things here, because in Appendix C we find this revealing and admirable statement:

> "Publishing numerical results is similar to publishing experimental results. The reader of such a publication relies on the integrity of the authors, since it is *often not possible to check the given results*." (Reviewer's emphasis.)

Here, perhaps, we have found this book. It is the authors' consciences speaking, not the other things we thought it might be. This is their integrity on the line: a complete guide to reproducing their results, for those who may doubt them. Along the way, the reader may or may not like their mathematics or physics, but one imagines that the authors' consciences are clear.

David S. Malkus

Mathematics Research Center
and Engineering Mechanics Department
University of Wisconsin
Madison, Wisconsin 53705

31[65-01, 65R20].—L. M. DELVES & J. L. MOHAMED, *Computational Methods for Integral Equations*, Cambridge University Press, Cambridge, 1985, xii + 376 pp., $23\frac{1}{2}$ cm. Price \$69.50.

This welcome edition consists of one of the most complete elementary expositions of numerical methods for solving one-dimensional integral equations that have been published to date. It covers the classical types of equations: Fredholm, Volterra, first kind, second kind, and third kind. On the theoretical side, one finds discussions of convergence and error of methods confined more frequently to the practical $L^2(a, b)$, $L^\infty(a, b)$, and $C^m(a, b)$ spaces, than in previous texts. Much space is devoted to important discussions of different types of quadrature schemes that may be used for obtaining numerical solutions of integral equations. One finds a discussion of the treatment of singularities, by subtracting them out or by ignoring them. Another novelty of the text is a discussion of the numerical solution of integro-differential equations. There is a refreshingly frequent switching from theoretical to numerical, presumably for purposes of keeping the subject matter interesting to the reader. Finally, one finds many examples, and while most of these do not come from real-life situations, they do illustrate difficulties of the type that one may encounter in real-life problems.

The authors admit a lack of completeness of the text. For example, one does not find a treatment of nonlinear Fredholm equations, of multidimensional Fredholm equations, or of Cauchy singular equations.

The book is recommended for advanced undergraduates or beginning graduate students.

The following is a listing of the chapter-by-chapter layout of the text, along with some brief comments.

Ch. 0: Introduction and preliminaries.

This chapter consists mainly of a discussion of the aim of the book, a listing of other books in the field, and a classification of the different types of integral equations.

Ch. 1: The space $L^2(a, b)$.

Here one finds a complete and self-contained discussion of the theory of $L^2(a, b)$, which is relevant to the contents of the text.

Ch. 2: Numerical quadrature.

Standard classical methods of quadrature and errors of quadrature are discussed here.

Ch. 3: Introduction to the theory of linear integral equations of the second kind.

Here one finds a discussion of the classical theory of Fredholm and Volterra integral equations, in an $L^2(a, b)$ setting. The important Fredholm alternative is not discussed until Ch. 6.

Ch. 4: The Nyström (quadrature) method for Fredholm equations of the second kind.

The Nyström method is very powerful for solving Fredholm equations that have continuous kernels. A complete discussion is given in this chapter of the important Nyström method, along with an error analysis.

Ch. 5: Quadrature methods for Volterra equations of the second kind.

Here one finds a discussion of the implementation of classical numerical methods for solving ordinary differential equations to the numerical solution of one, and systems of Volterra integral equations.

Ch. 6: Eigenvalue problems and the Fredholm alternative.

The discussion of eigenvalues is related to arbitrary Fredholm equations in terms of degenerate systems. No error analysis is given.

Ch. 7: Expansion methods for Fredholm equations of the second kind.

The Galerkin and Ritz-Galerkin methods are discussed for reducing a linear Fredholm integral equation problem to a problem involving a system of linear algebraic equations.

Ch. 8: Numerical techniques for expansion methods.

Explicit methods are discussed for actually carrying out the details of the Galerkin methods of the previous section.

Ch. 9: Analysis of Galerkin method with orthogonal basis.

Truncation and roundoff errors of the Galerkin method with orthogonal basis are discussed.

Ch. 10: Numerical performance of algorithms for Fredholm equations of the second kind.

This chapter compares the time required of different existing computer algorithms for solving Fredholm integral equations.

Ch. 11: Singular integral equations.

The type of singularities discussed are: (i) an infinite, or semi-infinite range of integration in the integral operator; (ii) a discontinuous derivative in the kernel or in the nonhomogeneous term; and (iii) an infinite or nonexisting derivative of some finite order.

Ch. 12: Integral equations of the first kind.

Eigenfunction expansions, regularization, and computational methods are discussed.

Ch. 13: Integro-differential equations.

Several methods are discussed for solving Volterra and Fredholm-type integro-differential equations.

Appendix: Singular expansions.

Explicit formulas are tabulated, for the coefficients in the expansions of the solution in terms of Chebyshev polynomials, for the case when the kernel has various types of special singular forms.

F. S.

32[41-01, 34A40, 34A50].—Richard E. Bellman & Robert S. Roth, *Methods in Approximation—Techniques for Mathematical Modelling*, Reidel, Dordrecht, 1986, xv + 224 pp., 23 cm. Price $49.00/Dfl. 120.00.

According to the editor of the series in which this book appears, it is a survey of the thoughts of R. Bellman on the how and why of approximation over the past twenty-five years. This seems an accurate enough description, provided that one takes the right definition of the word approximation.

The book consists of ten chapters. Aside from the first chapter, which serves to introduce some basic ideas, the book essentially divides into two parts: three chapters which deal with topics from classical Approximation Theory, and six chapters which deal with finding approximate solutions to several types of problems involving differential equations.

The chapters on classical approximation include material on polynomials, piecewise linear functions, splines, exponential sums, and even a special finite element. The treatment is fragmentary at best, and the reader who wants to know something more about these subjects (beyond their connection with dynamic programming) will have to look elsewhere. (The references at the ends of these chapters will not be of much help.) It is indeed the case that these approximation tools play a central role in the numerical solution of many problems in applied mathematics, but no systematic development of their use is given here. In fact, I could not find much of a connection between the three approximation chapters and the rest of the book.

The other six chapters of the book deal with quasilinearization, differential approximation, differential quadrature, and differential inequalities. These methods are applied to solve some problems in parameter identification, finding unknown initial conditions, to replace systems of nonlinear partial differential equations by systems of nonlinear ODE's, and to replace systems of nonlinear ODE's by linear ones. Several specific types of equations are considered, including the renewal equation and an equation arising in pharmacokinetics, as well as the Riccati, van der Pol, and Maxwell equations.

Each of the chapters in the book includes a bibliography, with almost all references being to papers and books of Bellman and his coauthors, mostly published in the 60's and 70's. The quality of exposition varies, and there are a fair number of problems with mangled sentences, missing or extra words, misspelled words, and notation. Some of these could cause some confusion for a novice reader, such as the use of the notation $(0, 1)$ for the closed unit interval, or the use of (f, g) for the norm of $f - g$. Some of the sections read as if they were typed directly from lecture notes with no further editing. It is hard for me to see how this book will be of much use to people in classical approximation theory. Other potential users such as numerical analysts, applied mathematicians, and engineers should form their own opinions.

L. L. S.