CORRIGENDA


We are grateful to Carla Manni for pointing out an error in the proof of Lemma 1. This necessitates corrections in the statement of Lemma 1 and its consequences. In addition to the notations used in the paper, we set

\[ \beta_d' = \left[ \binom{d-r+1}{2} - 2 \binom{r+1}{2} + \binom{r-(d-2r)+1}{2} \right]_+, \]

\[ \gamma_d' = \left[ \binom{d+2}{2} - \binom{2r+2}{2} \right]_+, \]

\[ \sigma_d'(n) = \begin{cases} 
\sum_{j=r+1}^{d-r} [r+j+1-nj]_+ & \text{for } d \geq 3r+1, \\
\sum_{j=r+1}^{d-r} [r+j+1-n(j+d-3r-1)]_+ & \text{for } 2r+1 \leq d \leq 3r, \\
0 & \text{otherwise.}
\end{cases} \]

The correct formulation of Lemma 1 is then as follows.

Lemma 1. Let \( \Delta_0 \) be a rectilinear grid partition, as described. Then

\[ \dim S_d'(\Delta_0) = \left( \frac{d+2}{2} \right) + \left[ n_0 \beta_d' + \sigma_d'(N_0) - \gamma_d' \right]_+. \]

In particular, it is clear that for \( d \geq 4r+1 \) or \( d \leq 2r \) the formulation of this lemma reduces to the formulation of Lemma 1 in the original paper. For \( 2r+1 \leq d \leq 4r \), the mistake in the proof of Lemma 1 occurs in the formulation of equation (24). A correct statement is the following. For \( d \geq 3r+1 \), we have

\[ \rank H' = \min \left( \sum_{j=r+1}^{d-2r} \rank H_j + \sum_{j=d-2r+1}^{d-r} \rank [B_{r+1}^j \cdots B_{j-1}^j \overline{H}_j], n_0 \beta_d' \right) \]

\[ = \min \left( \sum_{j=r+1}^{d-r} (r+j+1-[r+j+1-N_0j]_+), n_0 \beta_d' \right) \]

\[ = n_0 \beta_d' - \left[ n_0 \beta_d' - \gamma_d' + \sigma_d'(N_0) \right]_+. \]
For $2r < d < 3r + 1$, we have
\[
\text{rank } H' = \min \left( \text{rank } H_{r+1} + \sum_{j=r+2}^{d-r} \text{rank} [D_{r+1}^j \cdots D_{j-1}^j H_j], n_0 \beta_d^r \right) = n_0 \beta_d^r - [n_0 \beta_d^r - \gamma_d^r + \sigma_d^r(N_0)]_+.
\]

The above reformulation of Lemma 1 entails changes in the statements of Lemma 2, Theorems 1, 2, 3, 4 and three of the corollaries. The correct statements of these results are given below.

**Lemma 2.** Let $\Delta_1$ be a rectilinear grid partition and $\epsilon = 0$ or 1 as described. Then
\[
\dim \tilde{S}_d^r(\Delta_1) = \binom{d+2}{2} + E_d \beta_d^r - \epsilon[\gamma_d^r - \sigma_d^r(N_0)] + \epsilon[\gamma_d^r - \sigma_d^r(N_0)] - n_0 \beta_d^r + 1.
\]

**Theorem 1.** Let
\[
D_d^r = \text{dim } \tilde{S}_d^r(\Delta) - \binom{d+2}{2} - E_d \beta_d^r + V_1 \gamma_d^r.
\]
Then
\[
\sum_{i=1}^{V_i} \sigma_i \leq D_d^r \leq \sum_{i=1}^{V_i} \tilde{\sigma}_i,
\]
where $\sigma_i = \sigma_d^r(e_i) + (\gamma_d^r - \sigma_d^r(e_i) - d \beta_d^r)$, $\tilde{\sigma}_i = \sigma_d^r(e_i) + (\gamma_d^r - \sigma_d^r(e_i) - d \beta_d^r)$, with $d_i$ and $e_i$ denoting the number of edges and the number of edges with different slopes attached to the vertex $A_i$, respectively, and $\tilde{d}_i$ and $\tilde{e}_i$ denoting the number of edges and the number of edges with different slopes attached to the vertex $A_i$, but not $A_j$, $j < i$, respectively, where $i = 1, \ldots, V_i$. In particular, for $d \geq 3r + 1$ and all $\tilde{e}_i \geq 2$,
\[
\dim \tilde{S}_d^r(\Delta) = \binom{d+2}{2} + E_d \beta_d^r - V_1 \gamma_d^r.
\]

**Theorem 2.** Let
\[
F_d^r(n) := (n \beta_d^r + \sigma_d^r(n) - \gamma_d^r).
\]
Then
\[
\dim \tilde{S}_d^r(\Delta_e) = \binom{d+2}{2} + L \beta_d^r + \sum_{i=1}^{V_i} F_d^r(l_i).
\]

**Corollary 1.** Let $d \geq 3r + 1$. Then
\[
\dim \tilde{S}_d^r(\Delta_{mn}^{(2)}) = mn \left[ 6 \binom{d-r+1}{2} - 12 \binom{r+1}{2} - 2 \binom{d+2}{2} + 2 \binom{2r+2}{2} \right] + (m+n) \left[ 5 \binom{d-r+1}{2} - 10 \binom{r+1}{2} - \binom{d+2}{2} + \binom{2r+2}{2} \right] + \binom{2r+2}{2} + 4 \binom{d-r+1}{2} - 8 \binom{r+1}{2}.
\]
Corollary 2. Let $\Delta_{mn}^{(1)}$ be a uniform type-1 triangulation of $\Omega_R$. Then
\[
\dim S_d^{(1)}(\Delta_{mn}^{(1)}) = \left( \frac{d}{2} + 2 \right) + mn[3\beta_d^r - \gamma_d^r + \sigma_d^r(3)] + (2m + 2n + 1)\beta_d^r.
\]

Corollary 3. Let $\Delta_{mn}^{(2)}$ be a uniform type-2 triangulation of $\Omega_R$. Then
\[
\dim S_d^{(2)}(\Delta_{mn}^{(2)}) = \left( \frac{d}{2} + 2 \right) + mn[6\beta_d^r - 2\gamma_d^r + \sigma_d^r(2) + \sigma_d^r(4)] + (m + n)[5\beta_d^r - \gamma_d^r + \sigma_d^r(2)] + 4\beta_d^r - \gamma_d^r + \sigma_d^r(2).
\]

In the following, we need the notations:
\[
\beta_d^{r, \rho} = \left( \left( \frac{d - r + 1}{2} \right) - 2\left( \frac{\rho - r + 1}{2} \right) + \left( \frac{\rho - r - (d - \rho) + 1}{2} \right) \right)_+,
\]
\[
\gamma_d^{r, \rho} = \left( \left( \frac{d}{2} + 2 \right) - \left( \frac{\rho + 2}{2} \right) \right)_+,
\]
\[
\sigma_d^{r, \rho}(n) = \begin{cases} 
\sum_{j=\rho-r+1}^{d-r} (r + j + 1 - n j)_+ & \text{for } d > 2 \rho - r, \\
\sum_{j=\rho-r+1}^{d-r} [r + j + 1 - n(j + d - 2 \rho + r - 1)]_+ & \text{for } \rho < d \leq 2 \rho - r, \\
0 & \text{otherwise.}
\end{cases}
\]

Theorem 3. Let
\[
D_{d}^{r, \rho} = \dim S_d^{r, \rho}(\Delta) - \left( \frac{d}{2} + 2 \right) + V_i \gamma_d^{r, \rho} - E_i \beta_d^{r, \rho}.
\]
Then
\[
\sum_{i=1}^{V_i} \sigma_i^{r, \rho} \leq D_d^{r, \rho} \leq \sum_{i=1}^{V_i} \tilde{\sigma}_i^{r, \rho},
\]
where $\sigma_i^{r, \rho} = \sigma_d^{r, \rho}(e_i) + (\gamma_d^{r, \rho} - \sigma_d^{r, \rho}(e_i) - d_i \beta_d^{r, \rho})_+$, $\tilde{\sigma}_i^{r, \rho} = \sigma_d^{r, \rho}(\tilde{e}_i) + (\gamma_d^{r, \rho} - \sigma_d^{r, \rho}(\tilde{e}_i) - \tilde{d}_i \beta_d^{r, \rho})_+$, and $e_i, d_i, \tilde{e}_i$ and $\tilde{d}_i$ have been defined in Theorem 1.

Theorem 4. Let $\rho \geq r$ and
\[
F_{d}^{r, \rho}(n) := (n \beta_d^{r, \rho} + \sigma_d^{r, \rho}(n) - \gamma_d^{r, \rho})_+.
\]
Then
\[
\dim S_d^{r, \rho}(\Delta_c) = \left( \frac{d}{2} + 2 \right) + L \beta_d^{r, \rho} + \sum_{i=1}^{V_i} F_d^{r, \rho}(l_i).
\]