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A SEARCH FOR ALIQUOT CYCLES BELOW 10^{10}

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ABSTRACT. A search for aliquot k-cycles below 10^{10} with $k \geq 3$ is described. Two new 4-cycles are exhibited. Six new 4-cycles not below 10^{10} are also exhibited.

1. THE SEARCH PROCEDURE

For all natural numbers n, define $\sigma(n)$ to be the sum of the divisors of n, and define $s(n)$ to be the sum of the divisors of n exclusive of n, that is, $s(n) = \sigma(n) - n$. An *aliquot cycle* of length k is a finite sequence of distinct natural numbers $(a_1, \dots, a_k)$ such that $a_1 = s(a_k)$, and for each $i = 1, \dots, k-1$, $a_{i+1} = s(a_i)$. Aliquot cycles of length 1 correspond to perfect numbers, and cycles of length 2 are commonly referred to as amicable pairs. Several thousand amicable pairs have been discovered; te Riele [9] has conducted an exhaustive computer search which found 1427 amicable pairs with smaller member less than 10^{10}.

Cycles with length exceeding 2 have been called *sociable numbers*; only a few are known. Poulet [8] discovered two such cycles in 1918, one of length 5 and one of length 28. Borho derived forms which could be used to construct aliquot cycles with lengths exceeding 2. He was able to use one of these forms [1, Theorem 6] to construct a 4-cycle. Twenty-one more 4-cycles, two 8-cycles, and a 9-cycle were later found by computer searches ([2, 7, 10], cf. [4]). Little is known about sociable numbers. It has been conjectured [5] that for all k, infinitely many cycles of length k exist. Erdös [6] has proved that for each $k \geq 2$, the density of the members of aliquot cycles of length k is 0.

We conducted several brute-force computer searches for aliquot cycles in the range from 1 to 10^{10}. These searches discovered two new cycles of length 4. In addition, six cycles of length 4 outside this range were discovered of the form presented in [1, Theorem 6].

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To begin with, two searches were carried out by starting with the putative smallest member of an aliquot cycle, a_1, and repeatedly constructing $a_i = s(a_{i-1})$. The iteration was terminated when either a_i equalled a_1, in which case an aliquot cycle was found, or when a_i became less than a_1 or either a_i or i became too big, in which case an aliquot cycle was not found. One search was made with a_1 ranging between 1 and $5 \cdot 10^9$ and continuing iteration until a_i/a_1 exceeded or equalled 2 or i equalled 33, and one was made with a_1 ranging between $5 \cdot 10^9$ and 10^{10} and continuing iteration until a_i/a_1 exceeded or equalled $7/4$ or i equalled 5, except that in this second search, iteration was immediately terminated if $a_2/a_1 = s(a_1)/a_1$ exceeded or was equal to $7/5$. It will be seen that the first search was capable of finding cycles of length 32 or less, while the second search was only capable of finding cycles of length 4 or less.

Following the first version of this paper, a third search was carried out on the referee's advice. This search located all aliquot cycles with length 4 or less and largest member below 10^{10}. For all a_1 between 1 and 10^{10}, $a_2 = s(a_1)$ was constructed. The number a_2 was then the putative largest member of the aliquot cycle; hence if $a_2 > 10^{10}$ or $a_2 < a_1$, the search stopped immediately. Otherwise, $a_i = s(a_{i-1})$ was constructed for $i = 3, 4$, and so on. This process continued until some $a_i = a_1$, in which case an aliquot cycle was found, or until $i = 5$ or $a_i > a_2$, in which case it was not.

A naïve method of performing the computation required in the searches would be to factorize each a_i completely by trial division, construct $\sigma(a_i)$ from the prime factorization of a_i, and then set $a_{i+1} = \sigma(a_i) - a_i$. Three devices were used to speed up the computation. First, the factorization of a_i by trial division can often be cut short, as explained in [9]. Let p_j denote the jth prime, and suppose we know that we will terminate our search if $s(a_i)$ falls outside the range $[A, B]$. Let a_i be factored into $p_1^{e_1} p_2^{e_2} \cdots p_j^{e_j} R$, where R is divisible by no prime less than p_{j+1}. Then $\sigma(R)/R$ is bounded above by $\prod(1 - 1/p)^{-1}$, where the product is taken over all distinct primes p dividing R. Assuming then that $p_{j+1} \cdots p_k > R$ for some k, $\sigma(R)/R$ will be bounded above by $(1 - 1/p_{j+1})^{-1} \cdots (1 - 1/p_{k-1})^{-1}$, and it is trivially bounded below by 1. Since $\sigma(a_i) = \sigma(p_1^{e_1}) \cdots \sigma(p_j^{e_j})\sigma(R)$, our bounds for $\sigma(R)/R$ give us bounds for $\sigma(a_i)$. If these bounds prove that $s(a_i)$ is not in $[A, B]$, our search can be terminated before a_i is fully factored.

Second, the computation of a_2 from a_1 can be sped up by using the fact that in this step, we are taking $\sigma(r)$ for a series of r's that are consecutive integers. Suppose that we wish to compute $\sigma(M), \ldots, \sigma(M + N - 1)$. Two vectors of length N are used, q and r. The elements $q[j]$ are initialized to $M + j$ and $r[j]$ to 1, for $j = 0, \ldots, N - 1$. Then for each prime power p^e with $p \leq \sqrt{M + N - 1}$ and $p^e \leq M + N - 1$, and for each j with $M + j$ divisible by p^e and not by p^{e+1}, the $r[j]$-values are multiplied by $\sigma(p^e)$ and the $q[j]$-values are divided by p^e. At the end of this process the $q[j]$-values will be 1 unless $M + j$ had a prime factor p exceeding $\sqrt{M + N - 1}$, in which

case $q[j]$ will be p. Hence, if we make a final pass multiplying each $r[j]$ by $q[j]+1$ if $q[j] \neq 1$, we will have set each $r[j]$ equal to $\sigma(M+j)$. This sieving process produces the $\sigma(a_1)$-values very quickly, since the time spent dividing by the prime powers involved is amortized over the successive a_1-values. Our programs used a value of 10^5 for N. The sieve method only applies to the computation of a_2 from a_1, but it provides a considerable speedup nonetheless because most numbers n are deficient, that is, for most n, $s(n) < n$, and for deficient a_1's, our searches all stop after the computation of $s(a_1)$. In fact, we find that the fraction of deficient numbers in the interval between 10^9 and $10^9 + 10^6$ is 0.75241. Charles Wall states [11] that the density of the set of deficient numbers is between 0.75107 and 0.75250.

Finally, since these searches were conducted on a supercomputer, the trial divisions involved in factoring were sequenced in a way that permitted them to be vectorized.

2. Results

All searches were performed on an IBM 3090/180E. The first search consumed approximately 180 CPU hours, the second about 150 CPU hours, and the third about 175 CPU hours. 1427 amicable pairs were found in the first and second searches, reproducing the list in [8]. The third search found 1391 of these amicable pairs, which are all of the pairs found in [9] with larger member below 10^{10}. No 3-cycles were found by any search; hence, in particular, there are none with largest member below 10^{10}. The first search found the Poulet 5-cycle [8]. The Poulet 28-cycle was not found since the ratio of its largest member to its smallest member exceeds 2. Twenty-four 4-cycles were found by the first and second searches; these 4-cycles were also all found again by the third search. They are shown in entries 1–24 of Table 1. Cycle 8 was constructed by Borho in [1]. Cycles 1–7 and 9–14 were found by Cohen [2], David (cf. [4]) and Root [10]. Cycles 15–22 were unknown to us at the outset but were independently found by Achim Flammenkamp [7]. Cycle 17 was also independently found by Ren Yuanhua; see below. Cycles 23–24 are believed to be new. Our search also independently found the 8-cycle found by Flammenkamp [7] with smallest member 1095447416.

An amicable pair is said to be of *Euler's first form* [3] if it is of the form (apq, ar), where p, q, and r are distinct primes not dividing a. If an amicable pair is of Euler's first form, p, q, and r can be solved for in terms of a and additional parameters c and e which satisfy $ce = a^2$. In [1], Borho generalizes this form to produce an analogous form for aliquot $2k$-cycles for any k. For 4-cycles, Borho states that if a_1, a_2, d_1, and d_2 are natural numbers, $a_1 a_2 = d_1 d_2$, and, for $i = 1, 2$,

$$p_i = (s(a_2)\sigma(a_1) + d_i\sigma(a_2))/(a_1 a_2 - s(a_1)s(a_2)),$$
$$q_i = (s(a_1)\sigma(a_2) + d_i\sigma(a_1))/(a_1 a_2 - s(a_1)s(a_2)),$$
$$r = s(a_1 p_1 p_2)/a_1, \quad \text{and} \quad t = s(a_2 q_1 q_2)/a_2$$

are all integral and prime, $p_1 \neq p_2$, $q_1 \neq q_2$, p_1, p_2, and r do not divide a_1, and q_1, q_2, and t do not divide a_2, then $(a_1p_1p_2, a_1r, a_2q_1q_2, a_2t)$ forms an aliquot cycle of length 4. There may, however, exist aliquot cycles of the form $(a_1p_1p_2, a_1r, a_2q_1q_2, a_2t)$ that are not given by the above formula.

Our brute-force searches discovered two 4-cycles fitting the above formula, numbers 8 and 17. This prompted us to look for more 4-cycles of this form. A check was made to see if aliquot 4-cycles given by this formula existed, where (a_1, a_2) was restricted to be of the form (ap, aq) with p and q distinct primes not dividing a. Searches were made for all $a \leq 10^5$ and all $p, q \leq 1233$, and all $a \leq 50000$ and all $p, q \leq 7919$. This search produced seven 4-cycles, shown in Table 1 as numbers 8, 17, and 25–29. After the first version of this paper was submitted, we searched all $a \leq 5.02 \cdot 10^6$ and all $p, q \leq 541$, and all $a \leq 100400$ and all $p, q \leq 7919$. This found one additional 4-cycle, number 30 in Table 1. Cycles number 25–30 were not found by our brute-force searches and were believed to be new. Since writing the first version of this paper, however, the referee has informed us that Ren Yuanhua had earlier found cycles numbers 17, 25, and 28.

It has been conjectured from numerical evidence [2] that there exists a constant $\beta > 0$ such that $\log b_j$ is asymptotically $\beta \log j$, where b_j is the smaller member of the amicable pair with the jth smallest smaller member. In fact it appears [9] that $\beta = 2$. Figure 1 is intended to provide evidence for or against a similar conjecture for 4-cycles. It shows $\log a_j$ plotted against $\log j$, where a_j is the jth smallest largest member of an aliquot 4-cycle, and j ranges from 1 to 24. The least squares best-fit line for this data is also shown. It has slope 3.23 and y-intercept 11.9.

TABLE 1
Aliquot 4-cycles

1	$(1264460 = 2^2 \cdot 5 \cdot 17 \cdot 3719$,	$1547860 = 2^2 \cdot 5 \cdot 193 \cdot 401$,
	$1727636 = 2^2 \cdot 521 \cdot 829$,	$1305184 = 2^5 \cdot 40787)$
2	$(2115324 = 2^2 \cdot 3^2 \cdot 67 \cdot 877$,	$3317740 = 2^2 \cdot 5 \cdot 165887$,
	$3649556 = 2^2 \cdot 107 \cdot 8527$,	$2797612 = 2^2 \cdot 331 \cdot 2113)$
3	$(2784580 = 2^2 \cdot 5 \cdot 29 \cdot 4801$,	$3265940 = 2^2 \cdot 5 \cdot 61 \cdot 2677$,
	$3707572 = 2^2 \cdot 11 \cdot 84263$,	$3370604 = 2^2 \cdot 23 \cdot 36637)$
4	$(4938136 = 2^3 \cdot 7 \cdot 109 \cdot 809$,	$5753864 = 2^3 \cdot 23 \cdot 31271$,
	$5504056 = 2^3 \cdot 17 \cdot 40471$,	$5423384 = 2^3 \cdot 53 \cdot 12791)$
5	$(7169104 = 2^4 \cdot 17 \cdot 26357$,	$7538660 = 2^2 \cdot 5 \cdot 376933$,
	$8292568 = 2^3 \cdot 59 \cdot 17569$,	$7520432 = 2^4 \cdot 127 \cdot 3701)$
6	$(18048976 = 2^4 \cdot 11 \cdot 102551$,	$20100368 = 2^4 \cdot 919 \cdot 1367$,
	$18914992 = 2^4 \cdot 37 \cdot 89 \cdot 359$,	$19252208 = 2^4 \cdot 1203263)$

TABLE 1 (*continued*)

7	$(18656380 = 2^2 \cdot 5 \cdot 932819$,	$20522060 = 2^2 \cdot 5 \cdot 13 \cdot 17 \cdot 4643$,
	$28630036 = 2^2 \cdot 19 \cdot 449 \cdot 839$,	$24289964 = 2^2 \cdot 97 \cdot 62603)$
8	$(28158165 = 3^3 \cdot 5 \cdot 7 \cdot 83 \cdot 359$,	$29902635 = 3^3 \cdot 5 \cdot 7 \cdot 31643$,
	$30853845 = 3^3 \cdot 5 \cdot 11 \cdot 79 \cdot 263$,	$29971755 = 3^3 \cdot 5 \cdot 11 \cdot 20183)$
9	$(46722700 = 2^2 \cdot 5^2 \cdot 47 \cdot 9941$,	$56833172 = 2^2 \cdot 11 \cdot 53 \cdot 24371$,
	$53718220 = 2^2 \cdot 5 \cdot 2685911$,	$59090084 = 2^2 \cdot 43 \cdot 343547)$
10	$(81128632 = 2^3 \cdot 13 \cdot 19 \cdot 41057$,	$91314968 = 2^3 \cdot 23 \cdot 29 \cdot 109 \cdot 157$,
	$96389032 = 2^3 \cdot 41 \cdot 71 \cdot 4139$,	$91401368 = 2^3 \cdot 149 \cdot 76679)$
11	$(174277820 = 2^2 \cdot 5 \cdot 29 \cdot 487 \cdot 617$,	$205718020 = 2^2 \cdot 5 \cdot 17 \cdot 43 \cdot 1407$,
	$262372988 = 2^2 \cdot 47 \cdot 107 \cdot 13043$,	$210967684 = 2^2 \cdot 23 \cdot 2293127)$
12	$(209524210 = 2 \cdot 5 \cdot 7 \cdot 19 \cdot 263 \cdot 599$,	$246667790 = 2 \cdot 5 \cdot 17 \cdot 59 \cdot 24593$,
	$231439570 = 2 \cdot 5 \cdot 19 \cdot 23 \cdot 211 \cdot 251$,	$230143790 = 2 \cdot 5 \cdot 17 \cdot 499 \cdot 2713)$
13	$(330003580 = 2^2 \cdot 5 \cdot 16500179$,	$363003980 = 2^2 \cdot 5 \cdot 18150199$,
	$399304420 = 2^2 \cdot 5 \cdot 1163 \cdot 17167$,	$440004764 = 2^2 \cdot 110001191)$
14	$(498215416 = 2^3 \cdot 19 \cdot 47 \cdot 69739$,	$506040584 = 2^3 \cdot 7 \cdot 233 \cdot 38783$,
	$583014136 = 2^3 \cdot 72876767$,	$510137384 = 2^3 \cdot 19 \cdot 797 \cdot 4211)$
15	$(1236402232 = 2^3 \cdot 13 \cdot 41 \cdot 53 \cdot 5471$,	$1369801928 = 2^3 \cdot 11 \cdot 17 \cdot 863 \cdot 1061$,
	$1603118392 = 2^3 \cdot 313 \cdot 640223$,	$1412336648 = 2^3 \cdot 35543 \cdot 4967)$
16	$(1799281330 = 2 \cdot 5 \cdot 7 \cdot 11 \cdot 139 \cdot 16811$,	$2267877710 = 2 \cdot 5 \cdot 7 \cdot 32398253$,
	$2397470866 = 2 \cdot 7 \cdot 17 \cdot 10073407$,	$1954241390 = 2 \cdot 5 \cdot 19 \cdot 73 \cdot 140897)$
17	$(2387776550 = 2 \cdot 5^2 \cdot 19 \cdot 31 \cdot 89 \cdot 911$,	$2497625050 = 2 \cdot 5^2 \cdot 19 \cdot 31 \cdot 84809$,
	$2550266150 = 2 \cdot 5^2 \cdot 31 \cdot 59 \cdot 79 \cdot 353$,	$2506553050 = 2 \cdot 5^2 \cdot 31 \cdot 59 \cdot 27409)$
18	$(2717495235 = 3^2 \cdot 5 \cdot 7 \cdot 13 \cdot 19 \cdot 53 \cdot 659$,	$3509525565 = 3^2 \cdot 5 \cdot 7 \cdot 13 \cdot 857027$,
	$3977471043 = 3^2 \cdot 7 \cdot 13 \cdot 1451 \cdot 3347$,	$3100575933 = 3^2 \cdot 7 \cdot 13 \cdot 19^2 \cdot 10487)$
19	$(2879697304 = 2^3 \cdot 11 \cdot 19 \cdot 1722307$,	$3320611496 = 2^3 \cdot 17 \cdot 71 \cdot 343891$,
	$3364648984 = 2^3 \cdot 31 \cdot 13567133$,	$3147575336 = 2^3 \cdot 47 \cdot 8371211)$
20	$(3705771825 = 3^2 \cdot 5^2 \cdot 7 \cdot 1019 \cdot 2309$,	$3890616975 = 3^3 \cdot 5^2 \cdot 7 \cdot 503 \cdot 1637$,
	$4298858865 = 3^3 \cdot 5 \cdot 7 \cdot 79 \cdot 89 \cdot 647$,	$4659093135 = 3^3 \cdot 5 \cdot 101 \cdot 341701)$
21	$(4424606020 = 2^2 \cdot 5 \cdot 41 \cdot 103 \cdot 52387$,	$5186286908 = 2^2 \cdot 11 \cdot 1861 \cdot 63337$,
	$4720282996 = 2^2 \cdot 11 \cdot 13 \cdot 1301 \cdot 6343$,	$4993345292 = 2^2 \cdot 13 \cdot 1291 \cdot 74381)$
22	$(4823923384 = 2^3 \cdot 7^2 \cdot 1087 \cdot 11321$,	$5708253896 = 2^3 \cdot 23 \cdot 211 \cdot 147029$,
	$5513075704 = 2^3 \cdot 67 \cdot 97 \cdot 107 \cdot 991$,	$5196238856 = 2^3 \cdot 37 \cdot 743 \cdot 23627)$
23	$(5373457070 = 2 \cdot 5 \cdot 17 \cdot 19 \cdot 1663609$,	$5406735730 = 2 \cdot 5 \cdot 11 \cdot 31 \cdot 599 \cdot 2647$,
	$5575049870 = 2 \cdot 5 \cdot 13 \cdot 17^2 \cdot 179 \cdot 829$,	$5983131730 = 2 \cdot 5 \cdot 19 \cdot 569 \cdot 55343)$
24	$(8653956136 = 2^3 \cdot 7 \cdot 154534931$,	$9890235704 = 2^3 \cdot 23 \cdot 21859 \cdot 2459$,
	$9468980296 = 2^3 \cdot 19 \cdot 1051 \cdot 59273$,	$9237894104 = 2^3 \cdot 31 \cdot 1583 \cdot 23531)$
25	$(88585861815 = 3^4 \cdot 5 \cdot 11 \cdot 53 \cdot 59 \cdot 6359$,	$90937094985 = 3^4 \cdot 5 \cdot 11 \cdot 53 \cdot 385139$,
	$90251247735 = 3^4 \cdot 5 \cdot 11 \cdot 59 \cdot 107 \cdot 3209$,	$90965321865 = 3^4 \cdot 5 \cdot 11 \cdot 107 \cdot 190829)$

TABLE 1 (*continued*)

26 $(1092162882824 = 2^3 \cdot 13 \cdot 37 \cdot 443 \cdot 640691,$
$1177885756216 = 2^3 \cdot 13 \cdot 37 \cdot 306103367,$
$1264819120424 = 2^3 \cdot 37 \cdot 443 \cdot 1553 \cdot 6211,$
$1178275499416 = 2^3 \cdot 37 \cdot 1553 \cdot 2563207)$
27 $(1755676229313195 = 3^2 \cdot 5 \cdot 7 \cdot 53 \cdot 16694999 \cdot 6299,$
$1788418506686805 = 3^2 \cdot 5 \cdot 7 \cdot 53 \cdot 107123001299,$
$1821198145117995 = 3^2 \cdot 5 \cdot 7 \cdot 340073 \cdot 2699 \cdot 6299,$
$1788428908642005 = 3^2 \cdot 5 \cdot 7 \cdot 2103576173 \cdot 2699)$
28 $(114588454336625295 = 3^2 \cdot 5 \cdot 13 \cdot 17 \cdot 71 \cdot 4079659 \cdot 39779,$
$115087954524328305 = 3^2 \cdot 5 \cdot 13 \cdot 17 \cdot 71 \cdot 162992167519,$
$115583776699336335 = 3^2 \cdot 5 \cdot 13 \cdot 17 \cdot 103 \cdot 39779 \cdot 2836619,$
$115087979545630065 = 3^2 \cdot 5 \cdot 13 \cdot 17 \cdot 103 \cdot 112353848639)$
29 $(1277351117703084961561 05 = 3^3 \cdot 5 \cdot 13 \cdot 23 \cdot 607 \cdot 8476649 \cdot 615024539,$
$127870741499225281763895 = 3^3 \cdot 5 \cdot 13 \cdot 23 \cdot 607 \cdot 5218882699454189,$
$128006484638238134248905 = 3^3 \cdot 5 \cdot 13 \cdot 23 \cdot 8285887 \cdot 223947359 \cdot 1709,$
$127870742226200145943095 = 3^3 \cdot 5 \cdot 13 \cdot 23 \cdot 1853634766868767 \cdot 1709)$
30 $(455449879323655623656384 = 2^6 \cdot 79 \cdot 226691 \cdot 207722852483 \cdot 1913,$
$460256233251615186934336 = 2^6 \cdot 79 \cdot 47585829431911487 \cdot 1913,$
$465109226480399267470784 = 2^6 \cdot 479 \cdot 34620666713 \cdot 1913 \cdot 229081,$
$460256233273935581206336 = 2^6 \cdot 479 \cdot 7848184812741787 \cdot 1913)$

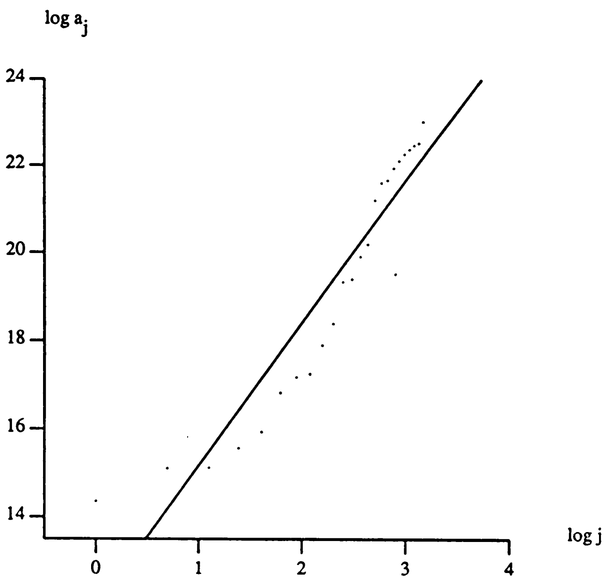

FIGURE 1

The distribution of 4*-cycles below* 10^{10}. *Slope of the best-fit line is* 3.23, *y-intercept is* 11.9

Added in proof. We have since extended our third search by searching for aliquot cycles of arbitrary length with the elements of the cycles prior to the largest elements not exceeding $1.7 \cdot 10^{10}$. We found no cycles of length exceeding 2 not already given in this paper, except for an 8-cycle with smallest element 1276254780 and a 9-cycle with smallest element 805984760, both previously found by A. Flammenkamp [7].

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