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REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1991 Mathematics Subject Classification can be found in the annual subject index of *Mathematical Reviews* starting with the December 1990 issue.

1[65-00, 41-00, 41A05, 41A21, 41A55, 41A65, 65C05, 65H05, 65T20, 73C50, 73V20]—*Handbook of numerical analysis*, Vol. III, P. G. Ciarlet and J. L. Lions (Editors), *Techniques of scientific computing* (Part 1), *Numerical methods for solids* (Part 1), *Solution of equations in* $\mathbb{R}^n$ (Part 2), North-Holland, Amsterdam, 1994, x+778 pp., $24\frac{1}{2}$ cm, \$154.25/Dfl. 270.00

This book contains three parts: 1) Techniques of Scientific Computing; 2) Numerical Methods for Solids; 3) Solution of Equations in $\mathbf{R}^n$. Nevertheless, only the second part gets close to "numerical methods for large-scale computation". The aims of the book are rather "constructive methods", which are often the background to numerical methods.

The first part is concerned with three aspects of approximation theory. First, Claude Brezinski provides an interesting historical perspective on interpolation, approximation and numerical quadrature since the year 1611. His main contribution, however, is a survey on Padé approximation, written jointly with van Iseghen. The algebraic theory naturally elucidates the connections with orthogonal polynomials and continued fractions. The convergence theory contains, on the one hand, a complete theory for Stieltjes functions and, on the other hand, only convergence in capacity for the general case. A contribution of Sendov and Andreev includes such varied topics from approximation theory as the Abel-Gontcharov interpolation problem, the fast Fourier transform, the Kolmogorov criterion for uniform linear interpolation, Monte Carlo methods for numerical integration, and finally approximation in the Hausdorff metric.

The second part of the book with the subtitle "Numerical Methods for Nonlinear Three-Dimensional Elasticity" provides a survey on a topic which is rarely found in the literature in such a nice and condensed form. Le Tallec, on 200 pages, addresses people who know how to solve linear elliptic problems (by finite elements) but who may have little experience in elasticity theory. The appropriate setting for treating the nonlinear equations from elasticity by Newton's method in a robust way is a crucial theme. Specifically, algorithms for such problems, which are of general interest, have to cover the treatment of incompressible or almost incompressible materials; the restriction $\det(I + \operatorname{grad} u) = 0$ is genuinely nonlinear. Emphasis is on the augmented Lagrangian method, which is treated in the framework of dualization. The contribution concludes with an excursion into viscoelastic materials.

The last part by Sendov, Andreev and Kjurkchiev concentrates on polynomial equations in one variable. The survey starts with a wrong proof of the fundamental lemma of algebra. Much space is given to estimates of the number of roots in

special subsets of the complex plane, and to bounds on circles which contain all roots. The authors emphasize that the old methods of Bernoulli, Graeffe, Laguerre, and Lehmer-Schur are not only of historical interest. Weierstrass' iteration for the computation of all roots, which has been rediscovered several times, is treated with care. Bounds on the complexity conclude the text.

The most original part of the book is the contribution on solids, and even if the book were restricted to that alone, it would be worth having it.

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2[65-06, 65Nxx]—*Boundary value problems for partial differential equations and applications*, J.-L. Lions and C. Baiocchi (Editors), Research Notes in Applied Mathematics, vol. 23, Masson, Paris, 1993, xii+460 pp., 24 cm, softcover, F 390

This volume contains 44 papers with a total of 71 authors. The papers were all solicited as a tribute to Enrico Magenes on the occasion of his 70th birthday. Given the reputations of the contributing authors and the great esteem in which the honoree is held among workers in Numerical PDEs, it is no surprise that the papers are of high quality. It is likely that quite a number of these papers will be of interest to many readers of *Mathematics of Computation*.

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3[49-02, 70-08, 70Q05]—*Control and estimation in distributed parameter systems*, H. T. Banks (Editor), Frontiers in Applied Mathematics, vol. 11, Society for Industrial and Applied Mathematics, Philadelphia, PA, 1992, xii + 227 pp., 25 cm, softcover, $56.50

This book is volume 11 in the Frontiers in Applied Mathematics series published by SIAM. It consists of five, primarily review, contributions, each between 40 and 54 pages in length and each with extensive bibliographies.

The contributors and their topics, in order, are

1. J.-L. Lions on "Pointwise control for distributed systems",
2. M. C. Delfour and M. P. Polis on "Issues related to stabilization of large flexible structures",
3. J. S. Gibson and A. Adamian on "A comparison of three approximation schemes for optimal control of a large flexible structure",
4. D. L. Russell on "Mathematical models for the elastic beam with frequency proportional damping", and
5. R. F. Curtain on "A synthesis of time and frequency domain methods for the control of infinite-dimensional systems: a system theoretic approach".

For the most part these articles are independent and treat the formulation of, and analytical questions about, specific classes of control problems. Only the third chapter emphasizes computational questions and methodology. The last four chapters each use various beam equations and models as examples for their analysis. The book provides a good snapshot of the state of the art of these topics at the

time the articles were written, together with useful bibliographies. In particular, the last chapter by Ruth Curtain provides an excellent synthesis and review of a large body of material.

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4[94-01, 94D05]—*The fuzzy systems handbook: A practitioner's guide to building, using, and maintaining fuzzy systems*, by Earl Cox, AP Professional, Boston, MA, 1994, xxxviii + 615 pp., $23\frac{1}{2}$ cm, softcover, $49.95

As a result of highly successful industrial applications of fuzzy systems in the 1990s, primarily in Japan, interest in this area has been rapidly growing during the last few years. This is currently reflected in the large increase of the literature dealing with fuzzy systems and related subjects, including scores of books.

While most books on fuzzy systems currently on the market are edited collections of papers or monographs on special topics, textbooks on fuzzy systems are still in short supply. Since Earl Cox's book is written as a self-contained introductory textbook on fuzzy systems, it fills an important need.

The book is in some sense unique. It is the only book currently on the market that introduces basic concepts of fuzzy set theory, fuzzy logic, and fuzzy systems from the standpoint of practical applications. As suggested by its subtitle, the book is oriented to practitioners. This orientation of the book is very clearly reflected in its style.

In general, the book shies away from mathematics. It teaches by examples and with the help of relevant computer software, which is included in the book on a diskette. Individual topics are introduced along with associated computer programs (properly explained) to allow the reader to develop his or her own hands-on experience with the topic. The book contains 80 code listings along with 401 figures (computer-generated graphs are, unfortunately, somewhat antiquated). In studying the various topics, the reader is constantly reminded of practical applications of the introduced concepts and supporting computer programs.

The book is perfectly suited for self study. When the study is completed, the reader is familiar not only with fundamentals of fuzzy set theory, fuzzy logic, and fuzzy systems, but also with relevant computer software. Moreover, by studying the book, he or she develops a good feeling for practical applicability of these novel theoretical tools.

The material covered in the book is organized into ten chapters. Six chapters cover basic concepts of fuzzy set theory and fuzzy logic, three chapters deal with various issues regarding the development of fuzzy models in industrial and business applications (including the six in-depth case studies), and one chapter is devoted to an overall description of the computer software designed for fuzzy systems modeling. The software is written in the programming language $C++$.

In general, the book is well conceived and well written. Although it is primarily oriented to practitioners, applied mathematicians and computer scientists will find

it an invaluable source of convincing industrial and business applications of fuzzy set theory.

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5[41-06, 65-06, 42-06, 46-06]—*Advances in computational mathematics: New Delhi, India*, H. P. Dikshit and C. A. Micchelli (Editors), Series in Approximations and Decompositions, Vol. 4, World Scientific, Singapore, 1994, xvi + 319 pp., $22\frac{1}{2}$ cm, $75.00

Twenty years ago, relatively little was known about the approximation, representation and analysis of functions of several variables. Since then the theoretical aspects, and more recently the computational development, have evolved quite nicely. In particular, the representation of curves, surfaces and functions has been enhanced by subdivision algorithms, neural network theory and radial basis functions, the analysis of functions by wavelet theory and the numerical solution of equations by multigrid techniques. Many of these computational developments are highlighted in the Proceedings volume under review. It includes 20 articles by many leaders of their fields. For the reader's convenience, the book is subdivided into four main areas: Finite element methods for PDE's, Geometric modeling for curves and surfaces, Wavelets, and Approximation.

The first section contains three papers which report on current numerical approaches to solving certain partial differential and integral equations. Both multigrid and multiscale techniques are utilized in these articles. The next section contains four papers. These articles illustrate the importance of rational splines for geometric modeling of curves and surfaces. Section 3 contains five papers related to wavelets and frames. The articles also include applications of wavelets to compressed representation and reconstruction of curves and images. The final section deals with the representation and approximation of functions of several variables with applications to Neural Networks.

In summary, I believe that the main contribution of this book is that it gives the reader a good feel for several directions of progress made in computational mathematics over the last 15 or 20 years.

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6[01A70, 65-06, 65F10, 65T20, 76W05, 83-06, 83C05, 85-06]—*Proceedings of the Cornelius Lanczos international centenary conference*, J. David Brown, Moody T. Chu, Donald C. Ellison, and Robert J. Plemmons (Editors), SIAM Proceedings Series, Society for Industrial and Applied Mathematics, Philadelphia, PA, 1994, 1xvi + 644 pp., $25\frac{1}{2}$ cm, softcover, $78.50

The conference was held during December 12–17, 1993 at North Carolina State University and had about 600 attendees. This scholarly volume describes Lanczos'

life (1893-1974) and presents a clear development of the many fields he opened. The material is presented in five parts:

(1) The Life and Works of Cornelius Lanczos;
(2) Plenary Presentations: Computational Mathematics;
(3) Plenary Presentations: Theoretical Physics and Astrophysics;
(4) Mathematics Minisymposia;
(5) Physics Minisymposia.

Part (1) begins with a collection of seven photographs of Lanczos from 1910 to 1972. This is followed by a fascinating twenty-eight page annotated story, "Cornelius Lanczos: A Biographical Essay", by Barbara Gellai. The fortuitous inclusion of this excellent biography makes the Proceedings a more complete and desirable volume!

Peter D. Lax, mathematician, presents the first symposium essay, a four-page insightful and pithy description, "Cornelius Lanczos (1893-1974), and the Hungarian Phenomenon in Science and Mathematics". Lax describes how Lanczos developed therein, flourished thereafter, and ends with, "In the end there is no explanation for superior achievement. All we can do is accept gratefully the brilliant contributions of this remarkable man".

This is followed by an analogous four plus page study, "The Roots of Cornelius Lanczos", by George Marx, physicist, who describes the development and significance of Lanczos' early work in Physics and the remarkable way that Lanczos returned to the subject in his final years. Marx concludes with, "In the name of the Roland Eotvos Physical Society of Hungary, I express my deepest thanks to North Carolina State University and to the organizers and speakers of the Lanczos Centenary Symposium in Raleigh NC, for keeping the memory of the Hungarian-American-Irish-Jewish Cornelius Lanczos alive, and for transferring his human message to the incoming generations . . . ".

John Todd, mathematician, concludes the essays in Part (1) with "Reminiscences of Cornelius Lanczos". In a self-deprecating (now humorous) story, he tells of the first time they met. Todd then presents an interesting fantasy that describes how Bloom might have been replaced by Lanczos as the central character in Joyce's "Ulysses", since Joyce and Lanczos would have "surely met" in Dublin, had they been there at the same time!

Part (1) concludes with a six-page list: "Published Papers and Books of Cornelius Lanczos".

The 140-page Part (2) contains eleven papers that describe Lanczos' discoveries of numerical algorithms. The authors present careful analyses of these methods and their latest refinements, which are extremely useful in current computing practice even though they were developed by Lanczos when he worked with electric-mechanical desktop calculators. The reader of these articles will get a complete understanding of the history of the numerical methods and their underlying mathematical structure. Lanczos would have enjoyed seeing what he had spawned:

James W. Cooley, "Lanczos and the FFT: A Discovery Before its Time";

Jane K. Cullum, "Lanczos Algorithms for Large Scale Symmetric and Nonsymmetric Matrix Eigenvalue Problems";

Roland W. Freund, "The Look-Ahead Lanczos Process for Nonsymmetric Matrices and its Applications";

Anne Greenbaum, "The Lanczos and Conjugate Gradient Algorithms in Finite Precision Arithmetic";

Martin H. Gutknecht, "The Lanczos Process and Padé Approximation";

Eduardo L. Ortiz, "The Tau Method and the Numerical Solution of Differential Equations: Past and Recent Research";

C. C. Paige, "Krylov Subspace Processes, Krylov Subspace Methods, and Iteration Polynomials";

Beresford N. Parlett, "Do We Fully Understand the Symmetric Lanczos Algorithm Yet?";

Pál Rózsa, Francesco Romani, and Roberto Bevilacqua, "On Generalized Band Matrices and Their Inverses";

Yousef Saad, "Theoretical Error Bounds and General Analysis of a Few Lanczos-Type Algorithms";

G. W. Stewart, "Lanczos and Linear Systems".

The 98-page Part (3) contains seven papers which should be easily understood by physicists. This not so savvy reviewer did recognize that the technical articles were careful expositions, written mainly for physicists, concerning subjects that Lanczos thought and wrote about in his early career as a theoretical physicist! He was raised on the new Relativity Theory and Quantum Mechanics. By examining the list of Lanczos' publications, it is clear that he returned often to Einstein's quest for a unified field theory. So, I believe that Lanczos would have enjoyed learning about these current views of Physics, Astrophysics, and Cosmology:

Abbay Ashtekar, Donald Marolf, and José Mourão, "Integration on the Space of Connections Modulo Gauge Transformations";

James B. Hartle, "Quasiclassical Domains in a Quantum Universe";

D. Petiot and Y. Takahashi, "Gauge Invariant Energy-Momentum Tensor in Spinor Electrodynamics";

Tsvi Piran, "γ-Ray Bursts and Neutron Star Mergers";

John Stachel, "Lanczos's Early Contributions to Relativity and His Relationship with Einstein";

Claudio Teitelboim, "Topological Roots of Black Hole Entropy";

Robert M. Wald, "Variational Principles, Local Symmetries, and Black Hole Entropy".

The 190-page Part (4) consists of 48 brief papers that cover the analysis and evaluation of algorithms studied in these 12 Lanczos inspired minisymposia: Eigenvalue Computations: Theory and Algorithms; Eigenvalue Computations: Applications; Moments in Numerical Analysis; Iterative Methods for Linear Systems; Least Squares; Software for Lanczos-based Algorithms; Tau Method; Chebyshev Polynomials; Lanczos Methods in Control and Signal Processing; Development of the FFT; The FFT in Signal Processing; Wavelets.

The 213-page Part (5) consists of 56 brief papers that cover numerical and theoretical investigations that were specially chosen to encourage an active exchange of ideas across these 13 minisymposia: Computational Magnetohydrodynamics in Astrophysics; Numerical Simulations of Collisionless Space Plasmas; Detection of Gravitational Radiation from Astrophysical Sources; Lanczos H-tensor; Cosmic Censorship; Cauchy Problem of General Relativity; Black Hole Evaporation and Thermodynamics; The Problem of Time in Quantum Gravity; New Variables and Loop Quantization; Decoherence and the Foundations of Quantum Mechanics;

Open Questions in Particle Theory; Supercollider Physics; Symplectic Methods in Physics.

The concluding Author Index lists the 169 writers of the articles in Parts (2)–(5).

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7[41-06, 41A15, 65-06, 65D07, 65D10]—*Curves and surfaces in geometric design*, Pierre-Jean Laurent, Alain le Méhauté, and Larry L. Schumaker (Editors), A K Peters, Wellesley, MA, 1994, xvi + 490 pp., $23\frac{1}{2}$ cm, $69.95

Curves and Surfaces in Geometric Design is one of two books resulting from the June 1993 Conference on Curves and Surfaces held in Chamonix-Mont-Blanc, France. This book contains 58 research papers relating to Computer Aided Geometric Design. The other book, *Wavelets, Images, and Surface Fitting* [1], contains an additional 48 papers.

Computer Aided Geometric Design (CAGD) is the computer-assisted representation and analysis of shape. It draws upon such areas as approximation theory, differential geometry, optimization, mechanical CAD, and computer science. The 58 papers in this collection do a nice job in representing a large number of currently vital research topics. These range from theoretical concerns to properties of spline curves and surfaces to interpolation and approximation schemes to data structures and software approaches to use of CAGD techniques in specific applications. Although the papers are generally scattered throughout the subareas of CAGD, topics of current concern are well represented. For example, the book contains a number of research papers arising from minisymposia on software infrastructure, spline conversion, rational approximation, and constrained approximation.

While a few of the articles in *Curves and Surfaces in Geometric Design* are survey articles, the vast majority are research articles describing recent work. Most of the articles are fairly short—8 pages—so this is not a book for readers anticipating a detailed or leisurely exposition. Nor is it a book for readers desiring an introduction to CAGD, as most articles assume familiarity with terminology, basic results, and issues. What the book is, is a good collection of recent work by many of the foremost researchers in this area. Its strengths are its breadth, the generally high quality of the articles, and the important topics addressed. I recommend the book to anyone who is familiar with the basic issues in CAGD and desires to read about recent work.

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REFERENCES

1. P.-J. Laurent, A. Le Méhauté and L. L. Schumaker (eds.), *Wavelets, images, and surface fitting*, A K Peters, Wellesley, MA, 1994. MR **95f:**65010

8[41-04, 41A30, 42A38, 65D99, 68U10]—*Adapted wavelet analysis from theory to software*, by Mladen Victor Wickerhauser, A K Peters, Wellesley, MA, 1994, xii + 486 pp., $23\frac{1}{2}$ cm, $59.95

Wavelet analysis is one of the richest subject areas in recent years to be included in the field of computational mathematics. Although there are already several popular monographs in the literature devoted to this subject, the book under review is the first one that goes beyond the mathematical treatment to aid those who write computer programs to analyze real data. It addresses the important properties of the wavelet transform so as to establish the criteria by which the proper analysis tool may be chosen, and it then details the software implementations for computational need. On the other hand, this book is rather self-contained, including even the necessary preliminary materials, such as mathematical analysis in Chapter 1, programming techniques in Chapter 2, and the discrete Fourier transform in Chapter 3. Chapters 4–10 are devoted to the algorithmic approach of wavelet analysis, and the final chapter includes applications to image compression, speech signal segmentation and scrambling, and signal denoising. In addition, an extensive appendix, giving solutions of selected problems in Chapters 2 and 4-9 as well as several tables of filter coefficients, is included.

The presentation of wavelet analysis in Chapters 4-10 is different from those in the existing wavelet books. Since the discrete Fourier transform has already been reviewed in Chapter 3, the subject of localized trigonometric series (or local trigonometric transform) presented in Chapter 4 provides a continuous flow of ideas from global to local analyses. Also, since the main concern of this book is computer implementation, a thorough discussion of quadrature mirror filters in subband coding theory in Chapter 5 is probably the most natural approach for introducing the so-called discrete wavelet transform, DWT (or wavelet series), in Chapter 6. Beyond DWT, but still within the realm of discrete computational analysis, are wavelet packets, their corresponding best-basis algorithm, and multidimensional library trees, discussed in Chapters 7, 8, and 9, respectively. Of course, a chapter on time-frequency analysis must be included in any book on wavelet applications, and this is done in Chapter 10.

Each chapter discusses the technicalities of implementation, giving examples in pseudocode, which is backed up with machine-readable Standard C source code available on a diskette that can be purchased separately. This book, beautifully written by a master of the subject, should be a valuable addition to the personal collections of those who are interested in the subject of wavelet analysis and its applications to data analysis.

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9[42-06, 42C10, 86-06, 86-08]—*Wavelets in geophysics*, Efi Foufoula-Georgiou and Praveen Kumar (Editors), Wavelet Analysis and Its Applications, Vol. 4, Academic Press, San Diego, CA, 1994, xiv + 372 pp., $23\frac{1}{2}$ cm, $59.95

Seismic signals are dispersive, owing to attenuation and dispersion, and therefore, traditional methods that assume stationarity of data are not always useful in

spectral analysis. Because of this, the wavelet transform was developed in exploration geophysics in 1982 for the time-frequency analysis of seismic signals. Since then, wavelet transforms have developed into a new branch of mathematics and have been applied to a wide variety of problems, although interestingly their applications in geophysics have been rather limited. Only in the past couple of years have geophysicists started to reevaluate the use of wavelet transforms in their applications. Thus it is very encouraging to find a new publication detailing some recent developments and applications of the use of wavelet transforms in geophysics.

The book under review is of definite interest to all applied researchers in the field. While many geophysicists may be aware of the more fundamental work in the field of wavelet transforms, few real data examples that use the concepts of wavelet theory to solve practical problems in geophysics exist. Of particular value in this book is the introduction written by the editors, which explains the basic ideas of wavelet transform theory by analogy with Fourier transform and windowed Fourier transform methods. This chapter serves as the example from which all the other articles in the volume take advantage. Each of the other articles illustrates a practical example in the analysis of geophysical data using wavelet transforms and reiterates the basic ideas underlying wavelet transforms in general, and then their particular relevance to the problem being addressed. By the time the reader has read the introduction and a couple of the articles (any order is possible) he has already thought of several applications in his own research area where wavelet transforms are likely to be useful. As the reader continues, he is convinced that the problems where wavelets will prove useful are many, and that an explosion of successful practical applications is likely.

As mentioned in the preface, this book is a collection of a series of papers presented at the spring AGU meeting in 1993 from different fields of geophysics: meteorology (atmospheric turbulence), oceanography and marine geophysics, hydrology, and exploration geophysics. The first four chapters show results from the application of wavelet transform based techniques to atmospheric turbulence data with different objectives. The first paper outlines a technique to provide signal decomposition which preserves coherent structures. This technique will certainly find further application in other areas, including exploration geophysics, well log data analysis, ocean bottom scattering, etc. The other three applications in this category are also very interesting and use orthonormal wavelet expansions, piecewise constant Haar decomposition and various properties of wavelet transforms to analyze day-time and night-time turbulence data. The next two papers are based on applications in oceanography such as the analysis of wind-generated ocean waves and bathymetry data. Results from both these applications are quite encouraging and superior compared to standard Fourier transform based tchniques. It is rather amazing when the author S. Little shows how one can detect scarp and faulting in swath-mapped bathymetric data. Another contribution applies the wavelet transform to high-resolution seismic data for time-frequency analysis and obtains attenuation estimates. These were traditionally done by windowed Fourier transforms, and clearly the wavelet transform approach offers significant advantages. Another application is in the characterization of a hydraulic conductivity distribution in which a multi-scale reconstruction method was developed using forward transforms, and then an inverse transform was applied to fill in missing information around sparse data. This technique is also directly applicable to the problem of reservoir characterization performed by petroleum engineers. The last two papers are more general.

One deals with multi-fractal analysis of nonstationary and intermittent geophysical analysis. One area where this will find immediate application is in the stochastic description of sea floor and other geologic structures. Traditional methods based on the assumption of stationarity can be avoided by using wavelet based techniques. The last paper considers noise suppression and signal compression and is the only paper that shows an example from exploration seismology. For the migrated seismic section example, a compression ratio of 20.34 was obtained, which resulted in 93.24% of the original data being discarded. This result is of particular importance for data transmittal and archiving, as a modern seismic surveying for exploration continues to demand an increase in the information required to accurately image subsurface structures in detail.

Thus, this book is very timely; the papers cover a wide range of applications and the results are well presented. We recommend this book without hesitation to any researcher, practitioner or student in geophysics.

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10[41-02, 41A17, 26C05, 26C10, 26Dxx]—*Topics in polynomials: extremal problems, inequalities, zeros*, by G. V. Milovanović, D. S. Mitrinović, and Th. M. Rassias, World Scientific, Singapore, 1994, xiv + 821 pp., $22\frac{1}{2}$ cm, $115.00

This is a remarkable book, offering a cornucopia of results, all connected by their involvement with polynomials. The scope of the volume can be conveyed by citing some statistics: there are 821 pages, 7 chapters, 20 sections, 108 subsections, 95 pages of references (distributed throughout the book), a name index of 16 pages, and a subject index of 19 pages. A brief description of each chapter follows.

Chapter 1 concerns generalities and discusses algebraic polynomials in one or several variables, as well as trigonometric polynomials. The Fejér-Riesz representation of nonnegative trigonometric polynomials is proved, as are representations for nonnegative algebraic polynomials on the real line or the half-line. Orthogonal systems are briefly dealt with, but the authors wisely do not attempt to include that vast subject in their book. Multivariable polynomials of various types are considered, such as symmetric polynomials and homogeneous polynomials. Resultants and discriminants are discussed.

Chapter 2 addresses polynomial inequalities for algebraic and trignonometric polynomials. Here we find inequalities satisfied by the zeros, by the moments, by the coefficients, by derivatives, and so on.

Chapter 3 is on the zeros of polynomials, and includes classical results such as the Gauss-Lucas Theorem and much very recent work (109 pages of text and 15 pages of references).

Chapter 4 is on inequalities involving trigonometric sums. Classical work by many persons is presented in a section of 33 pages. Another section of 40 pages emphasizes positivity results, mostly of recent origin.

Chapter 5 concerns extremum problems, particularly minimum norm problems. Incomplete polynomials receive the attention of one section, and inequalities involving trigonometric polynomials with different norms (inequalities of the Nikolskii type) are the subject of another section.

Chapter 6, having 200 pages, is on the extremal problems exemplified by the Markoff and Bernstein inequalities. The inequalities are given for various domains, various norms and for various subclasses of polynomials, both algebraic and trigonometric.

Chapter 7 sets forth some interesting applications: least squares approximation with constraints, simultaneous approximation, the Bernstein conjecture, and computer-aided design.

The book is written in a gentle style: one can open it anywhere and begin to understand, without encountering unfamiliar notation and terminology. It is strongly recommended to individuals and to libraries.

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11[49-02, 49J10, 65K10]—*Optimization and nonstandard analysis*, by J. E. Rubio, Monographs and Textbooks in Pure and Applied Mathematics, Vol. 184, Dekker, New York, 1994, xii + 356 pp., $23\frac{1}{2}$ cm, $135.00

Nonstandard analysis not only provides powerful tools for simplifying standard proofs and for proving or refuting new conjectures; it also gives precise meaning to many informal and intuitive notions. For example, in nonstandard analysis, each real-valued objective function with a lower bound has near-minimizers, even if—like the exponential function $x \mapsto e^x$ whose near-minimizers are large and negative—it has no minimizer. These near-minimizers provide a theoretical counterpart to the approximations to minimizers obtained by optimization algorithms in finitely many iterations.

While Rubio introduces near-minimizers on page 15 of his text, more than a hundred pages pass before they are mentioned again. In the interim, Rubio succinctly summarizes a considerable part of set theory, topology, model theory, and measure theory, both standard and nonstandard. He has organized this text into six chapters:

1. Optimization and Nonstandard Analysis, with a now typical ultrapower approach to nonstandard analysis and a proof of the transfer theorem;

2. Further Concepts and Applications, defining internal sets, enlargements of superstructures, and saturation;

3. Measure Theory and Infinite-Dimensional Linear Programming, presenting the theory of Loeb measure and a nonstandard characterization of realcompact spaces;

4. Linear Spaces, A Variational Principle and Penalties, with applications to optimal control and simple variational problems;

5. The Control of Homogenized Systems, deriving the homogenized nonlinear diffusion equation; and

6. Distributions, using certain optimal control problems to illustrate the nonstandard theory.

In addition to the notes and references at the end of each chapter, Rubio provides exercises for each chapter in one appendix, and an 85-page bibliography in another.

Students can test their understanding of this material not only by working through the suggested exercises but also by correcting quite frequent typographical errors; e.g., replacing "$\subseteq$" by "$\supseteq$" near the bottom of page 18, "$\bigcup_{i\in\mathbb{N}}$" by "$\bigcup_{n\in\mathbb{N}}$" in Eq. 1.25, and "$\exists x\psi$" by "$\exists z\psi$" in Definition 1.7d.

Much of this book develops sophisticated mathematical concepts needed for advanced applications of nonstandard analysis, rather than concentrating on just the more familiar concepts needed for certain simpler applications. For example, properties of κ-saturated superstructures are developed for arbitrary infinite cardinals κ, before discussing the embedding of the ordered field $\mathbb{R}$ of reals into the ordered field ${}^*\mathbb{R}$ of hyperreals. Consequently, this book is more for the mathematician seeking powerful new ways to study advanced optimization theory, rather than for the numerical analyst with only a casual interest in nonstandard analysis.

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12[65-06, 65Y05]—*Parallel processing for scientific computing*, David H. Bailey, Petter E. Bjørstad, John R. Gilbert, Michael V. Mascagni, Robert S. Schreiber, Horst D. Simon, Virginia J. Torczon, and Layne T. Watson (Editors), SIAM Proceedings Series, Society for Industrial and Applied Mathematics, Philadelphia, PA, 1995, xviii + 875 pp., $25\frac{1}{2}$ cm, softcover, $105.00

These are the proceedings of the Seventh SIAM Conference on the topic of the title, held February 15–17, 1995, in San Francisco. Included are minisymposia papers, contributed papers, and short summaries of poster presentations. The nearly 200 papers, organized in three parts, each further subdivided into four chapters, give an impressive account of the current use of parallelism in a vast variety of application areas. Specifically, Part I entitled "Applications", contains chapters on image, signal, and information processing; optimization and control; computational physics; and mathematical applications. Part II, entitled "Algorithms", has chapters on n-body simulation; partial differential equations; sparse linear systems; and eigenvalues. Part III, entitled "Systems", finally concludes with chapters on mesh partitioning and load balancing; languages and compliers; libraries and runtime systems; and visualization and performance. There is a final chapter containing position papers from a Panel Discussion on the question "Is scalable parallel computing a myth?". An author index concludes the volume.

Walter Gautschi

13[11-01, 11Yxx]—*A course in computational algebraic number theory*, by Henri Cohen, Graduate Texts in Mathematics, Vol. 138, Springer, Berlin, 1993, xxii + 534 pp., 24 cm, $49.00

The present book is one of the most popular texts on computational number theory. Its attitude is *practical*. For instance, in the first chapter, among some gen-

eral remarks about multiprecision arithmetic, we find: "...Since we will be working mostly with numbers of up to roughly 100 decimal digits..."

The author, Henri Cohen from Bordeaux, is a mathematician of a rare wide culture. He can tell you the details of the special merits of the machine instructions of the Motorola 68040 micro-processor, but also explain the intricacies of modular forms of half-integral weight. He easily manipulates commutative diagrams and discusses the numerical evaluation of several special functions on the same page. Cohen's personal style of writing is quite amusing. The preface to this book reads like a "Who's Who" in computational number theory, and the bibliography comes with a three-line evaluation of the merits of each text. The comments are always friendly and to the point. Often the last line of the evaluation says: "A must on the subject". A judgement that definitely applies to Cohen's own book as well.

The first chapter of the book contains a very readable account of the fundamental number-theoretical algorithms: Euclid's gcd algorithm, computing in $\mathbf{Z}/n\mathbf{Z}$, quadratic residue computations, square roots modulo a prime and a few words about computations in the polynomial ring $\mathbf{F}_p[T]$.

The most important ingredients for many of the more sophisticated algorithms are algorithms for lattice reduction. These are algorithms that compute a basis of rather short and orthogonal vectors of a given integral lattice in Euclidean space. For one-dimensional lattices the usual Euclidean gcd algorithm does the job. In general, the so-called "LLL"-algorithm is used. This algorithm is due to H. W. Lenstra, A. K. Lenstra and L. Lovász. In their paper [2] it was used to construct the first polynomial-time algorithm to factor polynomials with coefficients in $\mathbf{Q}$. This algorithm has very, very many applications and has quickly become the principal building block of many modern, practical number-theoretical algorithms.

Chapter 2 contains a discussion of algorithms involving linear algebra and lattices: how to compute determinants, characteristic polynomials, Gaussian elimination, the Hermite and Smith normal forms, the LLL-algorithm. The author restricts himself mainly to integral lattices and linear algebra over $\mathbf{Z}$ or a finite field. He does not discuss the usual problems of numerical linear algebra. In Chapter 3 the author discusses computational aspects involving polynomials: how to evaluate, how to compute disciminants. He discusses the Berlekamp and Cantor-Zassenhaus factorization algorithms for polynomials over finite fields. Finally he explains the Lenstra-Lenstra-Lovász algorithm to factor polynomials over $\mathbf{Q}$.

The next three chapters contain a description of the basic algorithms to do computations in algebraic number fields. Chapter 4 is of a somewhat auxiliary nature. In it the author discusses certain general problems: the subfield problem, how to represent an ideal, how to test membership. He introduces, without any proofs, the basic concepts of algebraic number theory.

In Chapter 5 the author focuses his attention to quadratic fields. For these fields the computational theory is furthest developed. He explains the close connections between binary quadratic integral forms and ideal classes of the rings of integers of these fields. He explains Shanks's baby-step-giant-step algorithm and Buchmann's subexponentional algorithm to compute the class group. The chapter is concluded with a brief discussion of the so-called Cohen-Lenstra heuristics on the statisical distribution of the class groups.

Chapter 6 contains a description of the fundamental algorithms in algebraic number theory: algorithms to compute the ring of integers of a given number field,

to compute the Galois group of a normal closure of a number field, algorithms to compute the unit group and the ideal class group of the ring of integers. The author is currently developing a program that routinely computes all these things for number fields which are given by a polynomial which is not "too large". At present, the program can handle number fields of degree at most 24 with moderate root discriminant.

Chapter 7 contains a description of several algorithms related to elliptic curves. The author gives an algorithm to transform a cubic curve in $\mathbf{P}^2$ with a point into Weierstrass form; he gives Tate's algorithm to determine the fibers in the Néron model of an elliptic curve over a discrete valuation ring. He gives the doubly exponentially convergent algorithms to compute the period lattice of an elliptic curve over $\mathbf{C}$. The remaining algorithms apply to elliptic curves over number fields: computation of the canonical height of a point, evaluation of the Hasse-Weil L-series and its derivatives, etc.

The 85 pages of the last three chapters contain a description of the currently most popular algorithms for factoring integers and for primality testing. Chapter 8 contains a description of some of the older practical algorithms: the $N-1$ test for proving primality and the Pollard ρ-method, the $p-1$ method and Shanks's class group method for factoring. In Chapter 9 the author discusses the two most efficient practical primality proving methods that are currently known: the "Jacobi sum test" and Atkin's algorithm. The Jacobi sum test was invented by Adleman, Pomerance and Rumely. The author discusses the efficient simplified version due to Hendrik Lenstra and Henri Cohen in detail. It is based on computations with Jacobi sums in cyclotomic fields. Atkin's algorithm, which is based on certain computations with elliptic curves with complex multiplication, is only discussed briefly. The reader is referred to the paper by Atkin and Morain [1] for the practical details. The current implementations of both algorithms routinely handle 1000-digit primes.

In Chapter 10 the modern factoring algorithms are discussed: first the classical continued fraction method, then an algorithm due to Lenstra and Schnorr that exploits class groups of complex quadratic number fields. Lenstra's celebrated elliptic curve method is explained and, finally, Pollard's "Number Field Sieve" is discussed in some detail.

I like this book very much. Its attitude is practical and it is written in plain English: the description of the algorithms is not obscured by the jargon that is often used in theoretical computer science. Therefore, this book is very useful for anyone who actually wishes to use the algorithms to do computations on a computer. It is also very useful for anyone who wishes to use one of the recent powerful computer algebra packages. The book provides a good theoretical background for this purpose.

The author discusses the computational aspects of $\mathbf{Z}/n\mathbf{Z}$, of rings of integers of number fields and of elliptic curves, but some other "natural" rings, such as polynomial rings in several variables over a finite field, or over $\mathbf{Q}$, are not discussed in great detail. For instance, all recent developments involving computational algebraic geometry, Gröbner bases, etc. are never mentioned.

Much of the contents of the book are very fundamental and probably will, like Knuth's 1973 textbooks, be of long-lasting value. But, given the philosophy of the book, some other parts have a somewhat more temporary character. One may

wonder what value the more practical sections of the book and the information on computer algebra packages will have in, say, 20 years from now.

René Schoof

References

1. A. O. L. Atkin and F. Morain, *Elliptic curves and primality proving*, Math. Comp. **61** (1993), 29–68. MR **93m:**11136
2. A. K. Lenstra, H. W. Lenstra and L. Lovász, *Factoring polynomials with rational coefficients*, Math. Ann. **261** (1982), 515–534. MR **84a:**12002