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REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1991 Mathematics Subject Classification can be found in the annual subject index of *Mathematical Reviews* starting with the December 1990 issue.

19[65–02, 65G05]—*Lectures on finite precision computations*, by Françoise Chaitin-Chatelin and Valérie Frayssé, Software, Environments and Tools, SIAM, Philadelphia, PA, 1996, ix + 235 pp., $25\frac{1}{2}$ cm, softcover, $44.50

Serious textbooks on Numerical Analysis have an initial part which considers finite precision computation. Standard topics are the generation and propagation of round-off, condition, forward and backward errors, and numerical stability. The extent to which this material is referenced later varies considerably; but whenever it occurs, a *regular* situation is generally assumed. Only recently, numerical analysis *research* has attempted to gain a more precise understanding of the phenomena governing finite precision computation: On the one hand, normwise perturbation bounds have been replaced by componentwise bounds, and "sufficiently small" perturbations by ones of realistic size; on the other hand, attention has been focused on computations which proceed near a singularity of the data-result mapping, a fact which may strongly alter the expected behavior of the computation.

The volume under review gives the first comprehensive presentation of this new research area to which the authors have heavily contributed themselves. It offers fascinating reading for numerical analysts of any flavor because the results concern fundamental aspects of their work; also the numerous examples throughout the book deal with linear algebra and univariate polynomial zeros which are common ground for everybody and from where one is able to connect to one's own area of interest.

Unavoidably, the treatise begins with the general concepts, formulated in the customary framework. But the second part of the introductory Chapter 2 points clearly beyond the classical syllabus: It introduces the influence of singularities, arithmetically robust convergence, finite precision computability for iterative and approximate algorithms, and a detailed discussion of the chaotic iteration $x_{n+1} := rx_n(1 - x_n)$. Chapter 3 on condition measures for regular problems gives the most comprehensive collection of expressions for normwise and componentwise condition ever published in one place, with many illuminating examples and remarks.

Chapter 4 deals with computation near a singularity. Since $Ax = \lambda x$ is singular at an eigenvalue, the condition of eigen*vectors* is treated here, in the convincing form developed by one of the authors. The reciprocal relation between the condition of a regular linear problem and its distance to the nearest singularity is established. The appropriate extension of this principle to nonlinear problems is discussed; the suggested formulation is later confirmed by computational experimentation.

Chapter 5 on backward errors emphasizes the dependence on the class of admissible perturbations; the definition via the *inf* of the feasible scalings within that class allows a great deal of flexibility. Expressions for many tasks are displayed and

interesting situations discussed including the role of iterative refinement. Chapter 6 on the finite precision behavior of iterative and approximate algorithms presents material which has not appeared in textbook form so far; it displays and explains a number of interesting phenomena.

After an account of various other approaches and tools for round-off error analysis in Chapter 7, the toolbox PRECISE for computer experimentation in round-off error analysis is introduced in Chapter 8. This toolbox is meant for a MATLAB environment and consists of two modules: The first one allows experimentation by random perturbation of selected data, with automatic sampling and plots; the other contains tools for sensitivity analysis, plots of pseudospectra, pseudozeros etc. The remainder of the book is largely devoted to a discussion and explanation of experimental results obtained with the help of PRECISE.

Studied are algorithms for the solution of $F(x) = y$ which yield a result $x_\theta = G_\theta(y)$ with $F(x_\theta) =: \eta_\theta$ upon execution with a finite precision θ. A perturbation Δz applied to specified data elements causes changes Δx_θ and $\Delta\eta_\theta := F(x+\Delta x_\theta)-\eta_\theta$, yielding a compound backward error $\Delta y_\theta := F(x + \Delta x_\theta) - y$. The following three *indicators* are fundamental for the assessment of an algorithm at precision θ (sup is over $\|\Delta z\| = \delta$):

the local sensitivity of G_θ	$L_{\theta\delta} := \sup \|\Delta x_\theta\|/\|\Delta z\|$;
the local sensitivity of F^{-1}	$K_{\theta\delta} := \sup \|\Delta x_\theta\|/\|\Delta\eta_\theta\|$;
the "reliability"	$I_{\theta\delta} := \sup \|\Delta y_\theta\|/\|\Delta z\|$.

For fixed precision θ, the indicators are *estimated statistically* for a given δ through (small) samples of random perturbations Δz of norm δ.

$I_{\theta\delta}$ is an important new assessment tool introduced by the authors; it should be approximately *constant* and of order 1 as a function of δ. In the experiments, this holds only within a *reliability interval* $[s, r]$, where s is of the order of the backward error at x_θ. The measured estimate for the condition $K_{\theta\delta}$ should also be constant in δ. Near a singularity, this can only hold up to some δ_0 which is of the order of the distance to the singularity. If $[s, \min(r, \delta_0)]$ is not empty it indicates the perturbation range (within the chosen class) for which the computation will behave regularly and valid results may be expected from the experiments. Chapter 9 presents and discusses PRECISE plots for numerous interesting examples.

Chapter 10 discusses *nonnormality* in matrices and its effect on finite precision computations, a central research area of the authors. Chapter 11 introduces and discusses plots of pseudoeigenvalues and pseudozeros gained with PRECISE. The concept of *qualitative computing* is coined to denote situations where only partial information may be gained through finite precision computation; examples are given. An Annex lists samples of MATLAB codes which have been used to perform the PRECISE experiments in the volume.

The fascination of large parts of this treatise is slightly hampered by unmotivated changes and inconsistencies in the notation. Also, in spite of its essentially narrative style, the text is rather concise; at the first reading, one would welcome more explanations and perhaps repetitions of definitions or facts from previous chapters. The title "Lectures on ..." is definitely not appropriate from this point of view. Naturally, one also finds some loose formulations here and there. But these little flaws do not diminish the high value of this text which surveys a good deal of new

territory in a pioneering way. Also it makes one realize where one would like to know more; thus, further reaching investigations will certainly be initiated by it.

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20[65F05, 65G05, 65F10, 65F35]—*Accuracy and stability of numerical algorithms*, by Nicholas J. Higham, SIAM, Philadelphia, PA, 1995, xxv + 688 pp., $23\frac{1}{2}$ cm, $39.00

Nick Higham is well-known for his contributions to error analysis and for his ability to communicate results that are necessarily full of intricate details. Only someone with his renowned writing skills could organize a work of this magnitude and still make it appealing. The book is close to 700 pages in length and is chock full of results, problems, and references.

Higham begins with four chapters that describe what life is like in the presence of roundoff error. A later chapter deals with software issues in floating point arithmetic and completes what I think is one of the best portrayals of finite precision arithmetic in the literature.

Chapters on polynomials, norms, and linear system perturbation theory set the stage for the analysis of $Ax = b$ algorithms, in many ways the real business of the book. Chapters on the LU, block LU, Cholesky, and QR factorizations are complemented by chapters that deal with related issues such as triangular system solving, iterative improvement, condition estimation, and matrix inversion.

Underdetermined systems and full rank least squares problems are also covered. By sticking to the full rank case, the singular value decomposition (SVD) can be avoided. Indeed, the SVD is relegated to a brief appendix and a few problems that are concerned with the pseudo-inverse. As Higham states in the preface, the treatment of singular value and eigenvalue computations requires a book in itself. I appreciate this point but still feel that the SVD should have been introduced as an analytical tool early in the book. It is just too powerful a decomposition to ignore in a major text like this that deals with numerical stability in matrix computations.

Other portions of the book reflect Higham's penchant for answering important stability questions. The reader interested in stationary iterative methods, matrix powers, the Sylvester equation, Vandermonde systems, and fast matrix multiplication will appreciate the author's treatment of these topics. A brief chapter on the FFT includes a nice proof of stability. New results for Newton interpolating polynomial evaluation, iterative refinement, Gauss-Jordan elimination, and the QR factorization are also included.

The overall style is perfect for the specialist who needs detail and rigor. But Higham's legendary expository skills also take care of the casual reader who needs intuition and a passing appreciation of the issues. Especially helpful in this regard are the "Notes and References" that appear at the end of each chapter. Higham is an extraordinary bibliophile and for every topic covered in this book you get the feeling that no stone is left unturned. There are over 1100 entries in the master bibliography and Higham's chapter-ending pointers into the literature are informative and impart a real historical sense.

Practitioners and experimentalists will enjoy several features of this book. There are many references to LAPACK, the software package of choice for solving most of

the matrix problems given in the text. How to get software over the Internet and the author's MATLAB toolkit for test matrices are also discussed in the appendices.

There are over 200 problems in the volume and they resonate with the text very well. Except for the "research problems," solutions to the exercises can be found in a 50-page appendix. The book could be used as a text for advanced graduate-level courses in matrix computations. However, the main role that the book will assume in the coming years will be as a reference and as a companion text in the classroom. Wilkinson's *Algebraic Eigenvalue Problem* played a similar role in the 1970s and 1980s and I bet Higham's book will prove to be equally valuable in the long run.

The volume is laced with great quotations and my favorite is due to Beresford Parlett:

> One of the major difficulties in a practical [error] analysis is that of description. An ounce of analysis follows a pound of preparation.

No numerical analyst can change that ratio. But what Higham has shown is that this quote "scales up." With tons of preparation Higham has given us a hundredweight of analysis—enough to keep the field on solid foundation for years to come.

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21[65N06, 65-04]—*Algorithms for elliptic problems: Efficient sequential and parallel solvers*, by Marián Vajteršic, Mathematics and Its Applications (East European Series), Vol. 58, Kluwer, Dordrecht, 1993, xviii + 292 pp., $24\frac{1}{2}$ cm, $152.00/Dfl.240

This book presents a survey of fast numerical methods for elliptic partial differential equations. Chapters 1 and 2, forming the first of the two parts of the book, examine methods designed for sequential computers. Chapters 3 through 6, forming the second part of the book, consider extensions to parallel computers. While the title of the book implies a somewhat general treatment of the subject, the author deals mainly with the case of finite difference discretizations of linear second- and fourth-order elliptic equations with Dirichlet boundary conditions on the unit square in two dimensions. However, the treatment of this restricted set of topics is quite thorough. Each chapter begins with a resonably complete review of the related literature, as well as an entertaining historical overview. An extensive bibliography follows each chapter.

In Chapters 1 and 2, fast sequential methods for the Poisson and biharmonic equations are examined. Chapter 1 begins with direct methods based on the FFT, cyclic reduction, and marching algorithms. Suitable modifications for handling various boundary conditions are then discussed. The survey then moves to iterative methods, such as classical relaxation and multigrid methods. While the unit square domain is considered for most of the book, the first chapter ends with a discussion of the treatment of L-shaped, octagonal, and circular domains, again remaining in the finite-difference setting. Chapter 2 begins with direct methods for the biharmonic equation and concludes with methods for the biharmonic eigenvalue problem. The presentation in Chapter 2 includes direct methods (the Buzbee-Dorr

algorithm, Bjorstad's algorithm, and Golub's algorithm), and methods based on operator splitting.

The focus of Chapters 3 through 5 is on the extension of the methods of Chapters 1 and 2 to parallel computers. A discussion of the basic ideas of solving elliptic equations in parallel is presented in Chapter 3. Parallel versions of marching algorithms, cyclic reduction, and domain decomposition algorithms for the Poisson equation are examined in detail, followed by a discussion of the parallel implementation of classical relaxation iterative methods. The biharmonic equation is the subject which concludes Chapter 3; the presentation includes the extension of parallel methods for the Poisson equation to the biharmonic case. In Chapter 4, the implementation of the methods of Chapter 3 on specific computers is examined, including implementation on the ICL DAP, the CDD Star-100, the Cray-1, the EGPA, the Connection Machine, and the MasPar computer. While Chapter 4 is the most practical chapter of the book, and gives the most meaningful performance indications for the various parallel algorithms considered, it also dates the book due to the rapid evolution of computer hardware. Chapter 5 examines the parallel implementation of a single algorithm, namely the multigrid method. Since this algorithm has such favorable complexity properties, and yet is somewhat complex to implement in parallel, it is no surprise that the author devotes more than forty pages to this topic. The chapter includes discussion of basic principles, SIMD versus MIMD algorithms, hypercube topologies, and complexity models for various hardware topologies. The chapter concludes with some numerical experiments on a Cray X-MP and other machines.

Chapter 6, the final chapter, is in some ways the most interesting chapter of the book. The topic is the implementation, in hardware, of some of the parallel algorithms (e.g., cyclic reduction and multigrid) presented earlier in the book using VLSI technology. The presentation is complete with VLSI schematics for various core computational kernels. Algorithms are constructed by piecing the schematics together, much as they would be laid out in silicon. The chapter concludes with a discussion of the biharmonic operator, complete with the VLSI schematic layout of a capacitance matrix solver and a multigrid-based solver.

While the first two chapters contain complete complexity statements for all of the methods presented, notable omissions throughout the rest of the book are theoretical performance models appropriate for parallel computers, which would provide a framework for meaningful performance analysis and comparisons of the various methods. (An exception is the excellent discussion in Chapter 5.) Rather than presenting these performance models, the author gives timings on various computers (many of which are now extinct), which unfortunately dates the book somewhat. However, the author makes it clear in the Introduction that the omission of complex theoretical models was intentional for readability. The book is otherwise thorough, well-written and contains few errors. The notation is simple and easy to follow; a list of the symbols used appears at the beginning of the book. This book could be used in a course on parallel numerical methods for model elliptic equations in two dimensions.

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22[65–00, 65–01]—*Integral equations*: *Theory and numerical treatment*, by Wolfgang Hackbusch, International Series of Numerical Mathematics, Vol 120, Birkhäuser, Basel, 1995, xiv + 359 pp., 24 cm, $79.50

This is an excellent reference text in the subject of integral equations, their analysis, and their numerical treatment for both the student as well as for the solver of integral equations. The type of integral equations considered include linear and nonlinear, as well as first and second kind Fredholm and Volterra integral equations, and singular integral equations. Integral equation methods can be used for solving differential equations. They are especially important when they enable a considerable saving of computation time resulting from the fact that the solution of the integral equation can be achieved by solving a lower dimensional problem than that for the case of the solution of the differential equation.

The text is organized into the following chapters:

1. *Introduction.* Tools of real and functional analysis are a convenient way to study integral equations, and the author discusses these tools in this introductory chapter. Spaces of continuous (including Hölder continuous) and differentiable functions that house solutions of integral equations are presented via real analysis methods. Also covered in this chapter are some theoretical aspects of general, polynomial, and spline interpolation, quadrature, the convergence of these processes, and the role of the condition number of a system of algebraic equations.
2. *Volterra Integral Equations.* In this chapter we encounter a theoretical treatment of these equations including existence, uniqueness, and regularity properties of solutions of Volterra integral equations of the first and second kind, as well as a discussion of equations of convolution type. Numerical solution methods presented are based on one and multistep methods of solving differential equations, and on the use of simple piecewise linear splines.
3. *Theory of Fredholm Integral Equations of the Second Kind.* Here we find a discussion of compactness of the integral equation operator and its consequences, with application to spaces of continuous functions, to square integrable functions, and to the case of unbounded intervals. Approximability and convergence, as well as properties of the map via the integral equation operator are discussed in general and for several specific types of integral equation operators.
4. *Numerical Treatment of Fredholm Integral Equations of the Second Kind.* The following topics are presented on the subject of replacing a Fredholm integral equation by a system of algebraic equations:
   (a) Stability, consistency, error of approximation, and condition numbers;
   (b) Replacement of the equation by a discrete system;
   (c) Projection methods of approximation;
   (d) Collocation method of approximation;
   (e) Galerkin's method;
   (f) Nyström's method; and
   (g) Supplements to the above, including solving eigenvalue problems, use of extrapolation, defect corrections, and the Fredholm alternative.
5. *Multigrid Methods for Solving Systems Arising from Integral Equations of the Second Kind.* This, too, is a lengthy chapter, which discusses the details of

multigrid implementation, including direct solution, Picard iteration, and the conjugate gradient method. We find here, discussions of the interpolation or projection error, levels of discretizations, two-grid, and more general multigrid iteration, and nested iteration.

6. *Abel's Integral Equation.* Special consideration is given to the solution of this class of Volterra integral equations, which are considered to be difficult to solve numerically. The chapter ends with a discussion of the solution of an Abel equation, for which the singular denominator is the function $(x-y)^{1/2}$; the equation is then solved via piecewise linear approximation of the unknown function.
7. *Singular Integral Equations.* This chapter is concerned mainly with the solution of Cauchy singular integral equations (CSIE). It starts with a careful examination of Hilbert and related transforms, and then leads to methods of approximation of solutions of CSIE, including Fourier series and multigrid methods. Applications are given for solving the interior and exterior Dirichlet problems, and illustrations are made of the use of single and double layer potentials for solving problems over planar regions. At the end of the chapter we encounter a refreshing discussion of hypersingular (Hadamard-type) integrals.
8. *The Integral Equation Method.* This chapter is concerned mainly with the reduction of differential equations to integral equations. Especially important are those cases when the solution of the equivalent integral equation formulation of the differential equation (over the boundary of the region) represents a problem of smaller dimensionality than that for the direct solution of the differential equation. In such cases the Green's function kernels have singularities, and we thus find a relevant study of continuity properties of the mappings via singular integral equation operators. In addition to Green's function kernels for solution of potential problems, which are discussed in detail, one also finds Green's function kernels for the *Helmholtz equation*, for the *biharmonic equation*, for the *Lamé equations*, and for the system of *Stokes equations.*
9. *The Boundary Element Method* (BEM). In this culminating chapter of the text, we encounter various methods of approximating the solution to the integral equation formulations (of differential equations) over the boundary of the region. Topics discussed include collocation, finite element, and multigrid methods. After obtaining a solution (i.e., the boundary "density") of the integral equation, we must integrate the product of this density with the Green's function over the boundary in order to find the corresponding solution of the differential equation in the interior of the region. Special care must be taken to achieve sufficiently accurate approximations of the singular integrals over parts of the boundary, both in the process of setting up the system of algebraic equations that approximate the solution to the boundary integral equation as well as to approximate the solution to the differential equation within the region. Estimates on the error are given for each type of approximation. The chapter ends with a discussion of the *panel clustering algorithm*, a research topic of the author. This algorithm enables an efficient evaluation of the matrix-vector multiplications in the system of algebraic equations whose solution approximates the solution to the integral equation.

There is an unusual inclusion in the text: following the table of contents, we find

a list of notations, as well as an explanation of numbers and formulae, the author's method of numbering theorems, and an explanation of the author's use of generic constants. These additions are worthwhile to both reader and student.

At the end of the text we find several pages devoted to a bibliography as well as to an index.

FRANK STENGER

23[41A05, 41A10, 42A15, 65D05, 65M70, 65T10]—*A practical guide to pseudospectral methods*, by Bengt Fornberg, Cambridge Monographs on Applied and Computational Mathematics, Cambridge Univ. Press, New York, NY, 1996, x + 231 pp., $23\frac{1}{2}$ cm, hardcover, $54.95

This is the first book in the series "Cambridge Monographs on Applied and Computational Mathematics". The stated goal of this series is to publish expositions on all aspects of applicable and numerical mathematics, with an emphasis on new developments in this fast-moving area of research. On the whole, this first book in the series is well written and is clearly in line with the stated goal of the series.

Spectral methods have been under rapid development in the last 20 years. There are many books written in this period, most notably the pioneer book by Gottlieb and Orszag in 1977 [1] and the comprehensive book by Canuto, Hussaini, Quarteroni and Zang in 1988 [2]. The book under review is different from all others in the following aspects. It is not a comprehensive book about spectral methods. The content is restricted to the subject of pseudospectral (PS) methods, which are equivalent mathematically to the interpolation, or collocation, methods. Galerkin methods are thus not covered in the book. Also, the author puts his own research experience into the book, notably the relationship between the finite difference (FD) and the PS methods. The approach of using the limit of FD when stencil is widened to define PS methods is advocated by the author. This book is perhaps the best resource for the readers to fully understand this approach.

The book contains eight chapters and eight appendices. After a brief introduction in Chapter 1, the author introduces spectral methods as expansions in orthogonal functions in Chapter 2. Different ways of determining the expansion coefficients are briefly mentioned, and the goal of the book, namely the discussion of the PS method, is stated. Difficulties of using the spectral method to approximate discontinuous functions, namely the Gibbs phenomenon, is also mentioned early in this chapter. Chapter 3 begins the introduction to PS methods via finite differences. The mechanism of finding the interpolation or differentiation matrices is discussed, and examples given for these matrices for different node distributions. The need to choose special node distributions to avoid divergence (Runge phenomenon) is discussed. Chapter 4 is perhaps the main chapter about the methodology. Several important properties of the PS approximations are discussed. This includes discussion about approximations to both smooth and nonsmooth functions. However, the discussion about approximations to nonsmooth functions seems not comprehensive. In practice there are examples requiring more sophisticated strategies than the ones advocated here. See, e.g. [3]. Chapter 5 discusses PS variations and enhancements in implementations. Several useful tricks in applications are discussed

here. Chapter 6 describes PS methods in polar and spherical geometries. Chapter 7 compares the costs of FD and PS methods. Finally, in Chapter 8, applications of the PS method to turbulence modeling, nonlinear wave simulation, weather prediction, seismic exploration, and elastic wave solution, are discussed. Certain technical details are left to the appendices.

This is a useful book for practitioners who use PS methods to solve practical problems.

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Chi-Wang Shu

24[65Lxx, 65L06]—*Numerical solution of initial-value problems in differential-algebraic equations*, by K. E. Brenan, S. L. Campbell, and L. R. Petzold, Classics in Applied Mathematics, Vol. 14, SIAM, Philadelphia, PA, 1996, x + 256 pp., 23 cm, softcover, $29.50

This monograph is an updated reprint of a book that appeared with North-Holland earlier in 1989. Although DAE have been studied for some time now their discovery by numerical analysts is rather recent; Gear [1] was one of the first to study their numerical solution. In 1986 Griepentrog and März [2] published a first monograph on numerical treatment of DAE. The latter book together with the (second part) of Hairer and Wanner's ODE book [3] and the present monograph constitute the main textbook resource for the interested researcher.

DAE naturally appear in electric networks (from Kirchhoff's laws) and in multibody mechanics (where the restricted number of degrees of freedom provides for fewer state variables than needed to describe Newton's law of motion, cf. (robotic) arms), etc. In control theory they are indispensable. Interestingly some DAE appear naturally as a limiting case of a singularly perturbed ODE, i.e. the reduced equation. This then explains the strong relationship to numerical methods for stiff ODE (cf. [3]). In particular the work of Gear and also Petzold has been inspired strongly by the celebrated BDF methods, a particular class of multistep methods used for such stiff problems.

The book at hand follows a logical line in treating these DAE. It starts off with an overview of DAE arising in practical situations as mentioned above. In chapter 2 the index notion, related to a matrix pencil, is introduced. Since practical problems usually do not involve constant system matrices and are often even nonlinear in nature this index concept needs improvement. Therefore the solvability notion is first introduced here. In chapter 3 this is compared to an alternative found in [2], viz. transferability. The latter notion is somewhat more technical but appears to be geometrically fairly natural. Then the celebrated notion of (differential) index is given. There are various other index concepts; however, they appear to boil down

to the same, except for in some academic cases. The main emphasis throughout the book is on index 1 and index 2 systems. Often DAE appear in semi-explicit form, i.e. where an algebraic set of equations is appearing separately from the differential part (involving some or all the state variables). Higher index problems may be reduced to lower index ones, although it has a (numerical) price. In chapter 3 multistep methods are treated, with some particular emphasis on BDF methods. The code DASSL of the third author is given a lot of attention in chapter 5, which is anticipated here. Also some of the interesting work of März [2] c.s. is reviewed here. The important class of Runge-Kutta methods is considered in chapter 4, both for index 1 and index 2 systems. Here the material in [3] will be helpful as a more recent update of the state of affairs, as various articles by Petzold, Ascher, Lubich etc. on e.g. projected Runge-Kutta methods will be.

The software in chapter 5 comes in very handy as a reference source for users of the DASSL code. It contains both principles behind the implementation and hints how to use DASSL in various situations.

An application chapter concluded the first version of the book. In the present SIAM edition a new chapter has been added to fill some gaps and make at least the references more up to date. Some of them are really important ones and it is a pity that the authors have not seized the opportunity to rewrite the book for a second edition more thoroughly. As remarked in the beginning there are not many books on numerical ODE and this "SIAM classic" is therefore still a useful introduction and source of references on the subject.

References

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25[65F10, 65K10]—*Linear and nonlinear conjugate gradient-related methods*, Loyce Adams and J. L. Nazareth (Editors), SIAM, Philadelphia, PA, 1996, xvi + 164 pp., $25\frac{1}{2}$ cm, softcover, $32.00

This book contains a collection of papers presented at an AMS-IMS-SIAM Summer Research Conference held in July, 1995. Most of the contributions are short notes or surveys containing observations about the conjugate gradient method and its place in optimization and numerical analysis. There are also a few research papers. Most of the authors are from either a sparse linear algebra or an optimization background, and the articles successfully elucidate the many connections between these two areas.

The basic conjugate gradient (CG) algorithm is a method for solving a linear system of equations $Ax = b$, where the coefficient matrix A is symmetric and positive

definite. In exact arithmetic, the algorithm is known to converge to the solution in at most n iterations, where n is the dimension of A. In fact, at each iterate x_k, the remaining error $r_k = b - Ax_k$ is projected into a smaller and smaller subspace. The algorithm has maintained its popularity because of its simplicity and economy of implementation and its widespread applicability in sparse matrix computations, numerical analysis (in particular, numerical PDEs) and optimization.

O'Leary's paper (Chapter 1) gives a brief overview of the origins and development of CG and related algorithms, while Nazareth (Chapter 13) surveys the history of nonlinear extensions of the CG algorithm. Nocedal (Chapter 2) also discusses nonlinear variants of CG and their relationship and combination with Newton and quasi-Newton methods for unconstrained optimization. A longer paper by Conn et al. (Chapter 5) describes the "iterated-subspace minimization" technique for unconstrained minimization. In this method, iterates are updated not by searching along a single direction but rather by searching in a low dimensional subspace constructed from the steepest descent direction, the truncated Newton direction, and some iterates generated by the CG algorithm in its search for the Newton direction. Battery testing of their approach is not conclusive, but it shows this to be a promising direction of research.

In Chapter 8, Saunders describes regularization of linear least squares problems, with applications to the linear systems that arise in interior-point methods for linear programming. This contribution has little to do with CG, but it does raise interesting questions about the connections between the regularization strategy and proximal point methods. Mehrotra and Wang (Chapter 11) describe a dual interior-point algorithm for network linear programming, in which the linear system at each iteration is solved with a preconditioned CG method. The preconditioner is based on a minimum spanning tree for the underlying network. Numerical comparisons show the method to be slower than, but within reach of, a state-of-the-art network simplex code.

Davidon (Chapter 6) generalizes the notion of conjugacy to conic functions, while Dixon (Chapter 9) presents some results on two dimensional searches in modified CG algorithms that incorporate a steepest descent direction.

Greenbaum (Chapter 7) discusses the behavior of CG and Krylov methods in the presence of finite precision arithmetic. Barth and Manteuffel (Chapter 10) discuss classes of nonsymmetric matrices for which CG-like algorithms which do not require storage of all preceding iterates can be applied. DeLong and Ortega (Chapter 12) propose the use of SOR as a preconditioner for the nonsymmetric iterative method GMRES on a parallel architecture.

Edelman and Smith (Chapter 3) describe the role of CG-like algorithms in eigenvalue computations. They include a history of this applications class (for which the literature is widely spread) and make the connection between CG and the Lanczos procedure, Rayleigh quotient iterations, and so on. In Chapter 4, a Boeing group describes some of the difficult optimization problems that arise in aircraft design. They show how blind application of standard optimization paradigms yields unsatisfactory results, while smarter, customized strategies (such as merging the optimization and simulation procedures suitably) can be used to advantage.

I found this collection of well written and (for the most part) brief articles by well known researchers to be a refreshing crash course on both the history of CG and related methods and the current state of the art.

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26[65K10]—*Numerical methods for least squares problems*, by Åke Björck, SIAM, Philadelphia, PA, 1996, xiii + 408 pp., $25\frac{1}{2}$ cm, softcover, $47.50

Conceptually, the easiest multi-dimensional optimization problem of all is the linear least squares problem. The problem is based on a linear model and the (unique) solution is easy to "write down" in the full rank case: it is the solution of the corresponding normal system of equations. Even in the rank-deficient case the solution set is well-understood and can be easily expressed in the language of the singular value decomposition. Therefore it might be surprising to learn that linear least squares problems (and many closely related problems) have considerable computational complexity. The business of numerically solving linear least squares problems is a serious one, requiring knowledge of a wide range of computational techniques.

"Numerical Methods for Least Squares Problems" by Åke Björck presents a modern "full-service" treatment of numerical methods for the linear least squares problem. This book is very well-written, comprehensive, and up-to-date. It is suitable for use as a textbook for a course on numerical methods for linear least squares problems as well as a reference for both practitioners and researchers.

The first two chapters cover the basic properties and factorizations: singular-value and QR decompositions, rounding error analysis, rank deficiency and ill-conditioning, and condition number estimation. A thorough treatment is given, though much of this material is covered in other modern texts on matrix computations, with minor variations in emphasis.

Efficiency in numerical methods is often obtained by exploiting the proximity of one problem instance to another. With cleverness much of the computational work in solving one instance can be amortized over several. In this setting "proximity" often means the defining matrices differ by a few rows or columns or, more generally, by a low-rank matrix. Chapter 3 discusses how to modify existing factorizations, in stable and efficient ways, to enable the efficient solution of proximal linear least squares problems.

The author does a wonderful service by including Chapter 4, a discussion of optimization problems closely related to least squares. In particular, I am very happy to see sections on l_1, l_p, l_∞, and robust linear regression (amongst others). Several modern numerical techniques for solving these less popular (but important!) norm-minimization problems are very similar indeed to the least squares machinery—this appealing aspect is recognized in this book.

Many practical least squares problems come with additional constraints on the variables. Chapter 5 discusses several numerical techniques for dealing with linear constraints, both equality and inequality. The chapter concludes with a short discussion of quadratically constrained least squares problems.

Chapters 6 and 7, concerned with large-scale problems, constitute the most important contribution of this book, in my opinion. Least squares problems that arise in serious applications are often very large systems, typically sparse. The robust and efficient methods discussed in the first five chapters become impractical, with respect to both space and time, in the large-scale setting. Chapters 6 and 7 discuss methods and techniques, direct and iterative, that are tailored to large (and sparse) problems. There is good detail here, especially with respect to sparse direct methods, and yet the big picture is never far from the surface. Many recent developments are discussed, briefly but informatively.

Finally, in chapters 8 and 9 the basic material is extended to problems with special bases (e.g., Toeplitz systems, Vandermonde matrices) and nonlinear problems.

In summary, this is a wonderful book, suitable as a text as well as a research reference. What is missing? The important bases are all touched upon, though two timely topics are given rather brief treatment: parallel methods and the surprising effectiveness of the (modified) normal system approach in interior point methods for linear programming. There is some discussion of the latter, but it is brief and (already) a bit out of date.

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27[11–00]—*Number-theoretic function products*, by R. G. Buschman, Buschman, Langlois, OR, 1996, vi +69 pp., 28 cm, vinyl cover, spiral bound, $12.50

In this work, the author has collected, for number-theoretic functions, properties for various products [Dirichlet, integer, lcm (Lehmer), Max, unitary, exponential, integral convolution]. Included are lists of specific products, multiple factor products, alternative factorizations (summation identities), and various inversion formulas.

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