MATHEMATICS OF COMPUTATION
Volume 68, Number 225, January 1999, Pages 453–459
S 0025-5718(99)01069-8

REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1991 Mathematics Subject Classification can be found in the annual subject index of *Mathematical Reviews* starting with the December 1990 issue.

1[65-01, 65Fxx]—*Numerical linear algebra*, by Lloyd N. Trefethen and David Bau, III, SIAM, Philadelphia, PA, 1997, xii+361 pp., 25 cm, softcover, $34.50

For most people numerical linear algebra is part of the infrastructure of numerical computations—roadways and bridges that connect one part of a calculation to another. People assume that the road will be smooth, that the bridge will not fall down, and they give the matter little additional thought. For the most part their confidence is justified. The matrix computation community has taken pains to market its algorithms in attractive, reliable formats, whether in packages such as LAPACK or in interactive systems such as MATLAB.

But numerical linear algebra is also a fascinating subject, well worth studying in its own right. Moreover, packages cannot solve every problem, and at some point most people performing numerical computations will have to involve themselves with the details of a matrix algorithm. Once again these people have been well served by the community. The area of matrix computations is supplied with well-written books ranging from introductory textbooks to advanced monographs.

The book under review is a textbook and a most unusual one. Introductions to the field tend to come in two flavors. The first is the classroom textbook, of which Datta's *Numerical Linear Algebra and Applications* is a fine example. The second is the grand survey, of which Golub and Van Loan's *Matrix Computations* is definitive. The present book is unusual in its stress on the *ideas* of matrix computations. This is not to say that ideas are absent from other works. But the senior author, Nick Trefethen, is known for taking a broad view of things, and that view is what informs the entire book.

The coverage is what you would expect in an introductory text—fundamentals, the QR decomposition and least squares, condition and stability, Gaussian elimination, eigensystems, and iterative methods. It is not to be expected that the treatment of this wide range of topics in a book of 350 pages would be exhaustive. However, it is good enough to bring an intelligent student to the point where he or she can proceed alone. An unusual feature of the book is that it treats the QR decomposition before Gaussian elimination. The justification is that the QR decomposition is easier to understand than Gaussian elimination, which teeters on the brink of instability. Another reason, I suspect, is that the QR approach to least squares gives the authors more scope for geometrizing. In any event, the decision is defensible and the result is a refreshing change.

As hinted above, geometry plays an important role in this book. The singular value decomposition is related to the mapping by a matrix of the unit ball into a hyperellipsoid. I particularly liked the treatment of projectors, where algebra and geometry are played off against each other. Geometry of another sort plays a role

in the discussion of Arnoldi's eigenvalue method where the authors use "Arnoldi leminscates" to illustrate the convergence of the method.

The authors also stress the interrelation between algorithms. For example they use a four-way division of Krylov sequence methods (linear systems vs. eigenproblems and Hermitian vs. non-Hermitian) to guide their discussion. Again, the authors make an amusing distinction between "orthogonal structuring" and "structured orthogonalization" to illustrate the difference between algorithms based on Householder transformations and those based on orthogonalizing a sequence of vectors.

The book concludes with an essay by Trefethen on the definition of numerical analysis. One does not have to agree with the definition itself to appreciate the important issues Trefethen raises so entertainingly.

I have two reservations about the book—neither damning. First, there could be more stress on implementation issues (e.g., convergence criteria for the QR algorithm). It is natural that a book of this sort would not spend a great deal of time on the minutiae, but given the many ways you can shoot yourself in the foot while computing with matrices, a few more examples of the pitfalls would be helpful. Second, the material and presentation was developed for graduate students at two high-powered institutions (MIT and Cornell). I would certainly not say that the book is unsuitable for other schools, but the instructor who uses it should be prepared to field some difficult questions.

These reservations aside, I can strongly recommend this book. The authors are to be congratulated on producing a fresh and lively introduction to a fundamental area of numerical analysis.

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2[65-01, 65Lxx, 65Mxx]—*A first course in the numerical analysis of differential equations*, by Arieh Iserles, Cambridge Texts in Applied Mathematics, Cambridge University Press, New York, New York, 1996, xvi+378 pp., softcover, $27.95, hardcover, $74.95

This is a lively textbook that is suited for mathematics graduate students or for well-prepared (mathematically) engineering students. This text is, on the one hand, rigorous and concise in its presentation of mathematical ideas and, on the other hand, verbose in its discussion of the big picture, i.e., "the ways and means whereby computational algorithms are implemented" and developed. To quote Professor Iserles: "In this volume we strive to steer a middle course between the complementary vices of mathematical nitpicking and of hand-waving".

This monograph is devoted to the numerical analysis of both ordinary and partial differential equations but, as needed, many other traditional topics are introduced and studied. These include interpolatory quadrature, Newton's method (and its variants) in $\mathbf{R}^d$, Gaussian elimination, iterative methods for sparse linear systems, and the FFT. There are several appendices, called "*Bluffer's guide to useful mathematics*", wherein important definitions and theorems from linear algebra, approximation theory, and ordinary differential equations are presented. Each chapter concludes with a short but challenging list of exercises and a Comments and Bibliography section. The text is organized into three parts. Part I consists of

six chapters devoted to numerical methods for ODE's. Part II deals with elliptic PDE's and Part III with evolution equations.

The material in Part I is as follows.

1. *Euler's method and beyond.* This introductory chapter is concerned with simple single step methods for first-order nonlinear systems of standard form, $y'(t) = f(t, y)$, with Lipschitz right-hand side. The presentation is limited to Euler's method, the implicit Trapezoidal rule, and the so-called θ-method that includes each of the aforementioned schemes as special cases. As an indication of the level of exposition, we mention that the implicit function theorem in $\mathbf{R}^d$ is invoked on p. 14 in the error analysis of the θ-method. 2. *Multistep methods.* This chapter is devoted to the Adams family of multistep methods, both explicit and implicit, and to backward difference formulae. There is a careful analysis of order and stability via the root condition. The classical Dahlquist convergence theorem is given and discussed. 3. *Runge-Kutta methods.* The chapter begins with a discussion of interpolatory quadrature with emphasis on Gaussian rules. Then explicit (ERK) and implicit (IRK) methods of Runge-Kutta type are developed. The chapter concludes with a derivation of collocation schemes that result in IRK methods. In the Comments and Bibliography section there is an introduction to the graph theoretic derivation of RK methods (due to Butcher). 4. *Stiff equations.* To begin the chapter, Professor Iserles has given an extensive and revealing discussion of a simple stiff system having one stable and one unstable eigenvector. In discussing the temptation to increase stepsize after the unstable eigenvector component has decayed, he admonishes the reader with the delightful analogy: "like a malign version of the Chesire cat, the rogue eigenvector might seem to have disappeared, but its hideous grin stays and is bound to thwart our endeavors". The bulk of the chapter is devoted to issues of A-stability for Runge-Kutta and multistep methods. 5. *Error control.* Up to this point in the text the author has not been concerned with the practical implementation of the many numerical methods derived and analyzed in previous chapters. The estimation of local errors (using another method in tandem) and controlling the error with stepsize changes is the theme of this chapter. A particularly nice aspect of the chapter is Iserles' device of applying each of the error-control devices to three specific simple systems of ODE's, one of which is moderately stiff. The presentation is limited to halving or doubling of the stepsize; however, both multistep and single step methods are considered. 6. *Nonlinear algebraic systems.* This final chapter of Part I is concerned with Newton's method (and variants) as applied to the solution of those nonlinear systems that arise in implicit methods (both RK and multistep) for ODE's. The Banach fixed point theorem is proved and utilized to establish convergence of the methods considered. The issue of *starting* the iteration is addressed in the last section wherein predictor-corrector schemes (PECE) are discussed and the equally important issue of *stopping* the iteration is also presented.

There are six chapters devoted to elliptic PDE's and related matters in Part II.

7. *Finite difference schemes.* The famous 5-point difference approximation to the Laplacian is used to solve the Poisson problem on a rectangle. The eigenvalues of this discrete Laplacian are determined and shown to approximate the eigenvalues of the Laplace operator. The 9-point operator and a powerful modification thereof are derived and utilized on a model problem. 8. *The finite element method.* Much of this chapter is introductory in nature and intended to give the reader some feeling for the main ideas involved in the FEM. A two-point boundary problem is utilized as

a vehicle for the description of the FEM that is presented as a Galerkin method for the differential equation as well as a Ritz method for the minimization of the appropriate functional. More general self-adjoint elliptic problems and the corresponding FEM are described later in the chapter with careful statements of important tools (for existence of the FEM solution and error analysis) such as Cea's lemma and the Lax-Milgram theorem. 9. *Gaussian elimination for sparse linear equations.* This brief chapter examines the issue of "fill in" in Cholesky factorization and the use of graphs to investigate the sparsity structure and factorization of matrices. 10. *Iterative methods for sparse linear equations.* The classical Jacobi, Gauss-Seidel, and SOR methods are analyzed with emphasis on SOR. Unfortunately, the powerful conjugate gradient method is relegated to the Comments and Bibliography sections at the end of the chapter. 11. *Multigrid techniques.* The author motivates the multigrid technique by demonstrating the smoothing property of Gauss-Seidel thereby revealing how one may accelerate via a hierarchy of grids. Then the basic ideas of the V-cycle and full multigrid iteration are discussed. No error analysis is presented. 12. *Fast Poisson solvers.* This chapter is concerned with the use of FFT techniques to efficiently solve block Toeplitz, symmetric tridiagonal systems that arise in certain finite difference (element) approximations.

The final two chapters constitute Part III, namely partial differential equations of evolution.

13. *The diffusion equation.* The analysis, stability and convergence, of semidiscrete and fully discrete schemes for parabolic initial-boundary value problems is presented. The discussion is, by and large, limited to Euler and Crank-Nicolson time discretizations. 14. *Hyperbolic equations.* Professor Iserles motivates the development of numerical schemes for hyperbolic problems by considering the advection equation $\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0$ and its numerical solution by Euler and Crank-Nicolson with particular attention to stability. The remainder of the chapter deals with the wave equation and Burgers equation. (On p. 308, Burgers is incorrectly stated; the expression $\frac{\partial u^2}{\partial x^2}$ should read $\frac{\partial u^2}{\partial x}$.)

This is a well-written, challenging introductory text that addresses the essential issues in the development of effective numerical schemes for the solution of differential equations: stability, convergence, and efficiency. The softcover edition is a terrific buy—I highly recommend it.

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3[65M06, 65M12, 65M20]—*Numerical methods for the three dimensional shallow water equations on supercomputers*, by E. D. de Goede, CWI Tract, Vol. 88, Stichting Mathematisch Centrum, Amsterdam, 1993, x+124 pp., 24 cm, softcover, Dfl. 40.00

This book is a collection of articles on the development of a numerical method for the three dimensional shallow water equations. They are obtained by simplifying the Navier-Stokes model: the unknowns are the horizontal velocity and the water elevation as for the two dimensional model, but the velocity may depend on the vertical coordinate. The pressure gradient is directly linked to the water elevation,

and the vertical velocity can be deduced from the incompressibility condition. The equation for the water elevation is nonlocal since it involves the integrals in the vertical direction of the velocity components.

After presenting very clearly the physical model and the spatial discretization, the author focuses on the development of good time-stepping schemes for the three dimensional shallow water equations. He mainly proposes two schemes, which are first studied for a simplified model:

- A semi-implicit scheme (the vertical diffusion is treated implicitly and the scheme is fully explicit for the water elevation) leading to a block triangular linear system which can be solved in parallel by direct methods. For stabilization, smoothing techniques are applied and may deteriorate the accuracy.
- A time splitting scheme where the diffusion terms are decoupled from the terms responsible for the water waves. The linear systems are solved iteratively and preconditioners are used. The scheme is unconditionally stable and seems more robust and accurate than the previous one.

Finally, the parallel implementation is thoroughly discussed.

The theoretical part of the book is written in a very interesting way, and numerous and systematic tests are given. In addition to being of evident interest for the field specialists, this book can be seen as an example of how to tackle a complex numerical problem, and proves if necessary that numerical analysis can be very helpful for concrete engineering problems. I have enjoyed reading this book, although, since it is a collection of articles, it contains repetitions and redundancies.

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4[65-00, 65-01]—*Boundary element method, fundamentals and applications*, by Frederico Paris and Jose Canas, Oxford University Press, New York, NY, 1997, xv+392 pp., 24 cm, hardcover, $65.00

It is frequently possible to express the solution of a partial differential equation (PDE) in a region $\mathcal{B}$ in terms of an integral over the boundary of the region of some *Green's* function times a *potential density*. In such cases, the problem of solving the PDE at a discrete set of interior points in $\mathcal{B}$ can often be reduced to solving an integral equation for the *potential density* on the boundary of $\mathcal{B}$, and then using the integral expression for the solution to compute the solution in the interior of $\mathcal{B}$. This is the *boundary element method* (BEM). In the case when the BEM can be applied, it usually enables considerable saving of effort compared with direct solution of the PDE via classical methods. Other difficulties ensue, due to the presence of corners on the boundary of $\mathcal{B}$ which give rise to singularities of the *potential density*, and due to the singularities in the *Green's function*.

This text is about the BEM. It stems from a Ph.D. course taught by the authors in Blacksburg, USA, in 1991–92. It covers, in essence, the use of the BEM for solving first *Laplace-type* and then *elastic-type* problems. Initially, *Green's* theorem is used to transform the PDE to an integral equation, and the integral equation is then solved for the two dimensional case via *finite element method* (FEM). Piecewise

linear and piecewise quadratic elements are used for regions where the boundary is smooth, and the authors derive singular elements in the cases when the nature of the singularity is known. Such "singular elements" are not yet known for every type of corner and edge in three dimensions. Many simple examples are presented, enabling a student to get a good understanding of the BEM. In addition, at the end of each chapter one finds a set of problems which further facilitate the understanding of the BEM, as well as a bibliography on the subject matter of the chapter.

The text is organized into the following chapters.

1. *Preliminary concepts.* This chapter presents some basic mathematical concepts that are required for the conversion of a PDE to a boundary integral equation, and for replacing the boundary integral equation by a system of algebraic equations. Included are Gauss' divergence theorem, the concept of a delta function, the concept of a singular integral, including the Cauchy principal value. A brief description of approximate methods of solving boundary integral equations consists of the Galerkin scheme based on the use of basis functions, such as polynomials, trigonometric polynomials, finite elements, and delta functions. Also included is a discussion of the merits of the BEM versus the FEM for solving PDE's. The chapter then concludes with a listing of some problems that can be effectively dealt with by the BEM.
2. *Integral formulation of the Laplace equation.* In this chapter one encounters a derivation of the fundamental solution in two and three dimensions, a detailed derivation of the integral expression for the solution in terms of the fundamental solution by means of Green's theorem, and a derivation of the boundary integral equation from this integral representation.
3. *BEM applied to the Laplace equation.* Finite element approximations are extensively discussed for the two dimensional case using piecewise constant, piecewise linear, and piecewise quadratic elements. The use of singular elements is also discussed. Also discussed are the approximations to the solution in the interior of the region, once the solution to the BEM has been computed.
4. *A computer program to solve the* $2D$ *Laplace equation based on BEM.* The authors present a FORTRAN program for approximation of a solution to a Poisson problem based on the BEM under the assumption that the boundary is given in terms of piecewise linear elements. The resulting system of linear equations is then solved via Gaussian elimination. Some examples are given illustrating the use of the program. These include a heat conduction problem, a torsion problem, a Motz problem, and a ground water flow problem.
5. *Integral formulation of the elastic problem.* After deriving in detail the PDE of the elastic problem, the authors express in detail the solution to the PDE in the interior to the region as integrals over the boundary of the region in terms of Green's functions, and they then use these integral expressions to derive in detail the boundary integral equations. This is done in both two and three dimensions.
6. *BEM applied to elastic theory.* Just as for the case of Laplace's equation, the authors illustrate in detail for the two dimensional case the setting of the system of algebraic equations based on piecewise constant, piecewise linear, and piecewise linear discontinuous boundary elements. In addition, explicit evaluation of boundary integrals are also presented for the case of isoparametric elements, circular elements, and singular elements. Also discussed are

the approximation to the solution in the interior of the region via use of the integral expressions for the solution.

7. *A computer program to solve* $2D$ *elastic problems based on BEM.* This chapter discusses the numerical solution to the integral equations developed in the previous chapter using a FORTRAN computer program, which they present, the use of which is described in detail. They illustrate the use of their program for obtaining the stresses for the cases of: (i) a square plate in traction; (ii) the semi-infinite half space uniformly loaded along half the length; (iii) a plate with central hole in traction; and (iv) a plate with central crack in traction.

There are also the following appendices.

1. *Bringing the integral equation to the boundary without deforming it.* This appendix presents an alternative procedure for taking the integral expressions of the solution to the boundary.
2. *Numerical integration by the Gauss method.* Appendix 2 presents methods of numerical integration via: (i) the trapezoidal formula; (ii) Simpson's rule; and (iii) Gauss-Legendre quadrature. In addition, a FORTRAN program is given to evaluate the zeros and weights for Gauss-Legendre quadrature.
3. *Definition of a stress vector in terms of the principal directions obtained from the displacement field.* This appendix expresses the stress vector in the neighborhood of a corner, based on a piecewise linear finite element approximation.
4. *Analytic expressions of integration constants for parabolic elements in elasticity when the collocation point belongs to the element.*

Finally, the authors include a description of a diskette which is attached to the book inside the back cover. This is for *Mac* and *PC* use, and contains the files of the FORTRAN programs given in Chapters 4 and 7 of the text, including programs for the examples which they have presented.

The final pages of the text consists of a four page index, starting on page 289.

Frank Stenger