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CORRIGENDUM TO "SPHERICAL MARCINKIEWICZ-ZYGMUND INEQUALITIES AND POSITIVE QUADRATURE"

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Professor Dr. Joachim Stöckler has kindly pointed out to us that our proofs [2] of the estimates (4.5) and (4.9) are valid only in the case when we do not require the quadrature weights a_ξ to be nonnegative, since the Krein-Rutman extensions of nonnegative functionals are not guaranteed to be norm-preserving. The purpose of this note is to point out that the existence of nonnegative weights a_ξ satisfying (4.4) necessarily implies the following analogue (0.1) of (4.5). The existence of such weights was proved in the paper. (Here, and in the sequel, we use all the notation as in the paper.)

$$\left\| \frac{a_\xi}{\mu_q(R_\xi)} \right\|_{\mathcal{C},p'} \le c, \qquad 1 \le p' \le \infty. \tag{0.1}$$

The estimate (4.9) (being a special case of (4.5)) should also be modified accordingly in the case of nonnegative weights. The estimates are correct as stated in the case when the quadrature weights are not required to be nonnegative.

First, we prove (0.1) in the case when $p' = \infty$. Let N be an even integer with $\delta_{\mathcal{C}_0} \sim 1/N$, and such that the quadrature formula (4.4) is exact for Π_N^q. Now fix ξ, and let S be the spherical cap circumscribing R_ξ. Then the proof of Proposition 3.2 shows that $\mu_q(R_\xi) \sim \mu_q(S)$. Moreover, one can write $S := \{\mathbf{x} : \mathbf{x} \cdot \zeta \ge \cos\theta_\xi\}$ with $\theta_\xi \sim N^{-1}$. Now, let $y := \cos\theta_\xi$ and χ be the univariate function defined by

$$\chi(t) = \begin{cases} 1 & \text{if } y < t \le 1, \\ 0 & \text{if } -1 \le t \le y. \end{cases}$$

Then there exist ([1, Section I.5]) polynomials ϕ and Φ in Π_N^1 such that

$$\phi(t) \le \chi(t) \le \Phi(t), \qquad t \in [-1, 1], \tag{0.2}$$

and

$$\int_{-1}^{1} (\Phi(t) - \phi(t)) w_q(t) dt \le \lambda_{N/2}(y), \tag{0.3}$$

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where $\lambda_{N/2}$ denotes the Christoffel function for the weight w_q. In view of the quadrature formula (4.4), one obtains from (0.2) that

$$\int_{\mathbb{S}^q} \phi(\mathbf{x}\cdot\zeta)d\mu_q(\mathbf{x}) = \sum_{\mathbf{z}\in\mathcal{C}_0} a_{\mathbf{z}}\phi(\mathbf{z}\cdot\zeta) \le \sum_{\mathbf{z}\in\mathcal{C}_0} a_{\mathbf{z}}\chi(\mathbf{z}\cdot\zeta)$$
$$\le \sum_{\mathbf{z}\in\mathcal{C}_0} a_{\mathbf{z}}\Phi(\mathbf{z}\cdot\zeta) = \int_{\mathbb{S}^q} \Phi(\mathbf{x}\cdot\zeta)d\mu_q(\mathbf{x}).$$

Further, we obtain from (0.2) also that

$$\int_{\mathbb{S}^q} \phi(\mathbf{x}\cdot\zeta)d\mu_q(\mathbf{x}) \le \int_{\mathbb{S}^q} \chi(\mathbf{x}\cdot\zeta)d\mu_q(\mathbf{x}) = \mu_q(S) \le \int_{\mathbb{S}^q} \Phi(\mathbf{x}\cdot\zeta)d\mu_q(\mathbf{x}).$$

Therefore, in view of (0.3), we obtain

$$\begin{aligned}\left|\sum_{\mathbf{z}\in\mathcal{C}_0} a_{\mathbf{z}}\chi(\mathbf{z}\cdot\zeta) - \mu_q(S)\right| &\le \int_{\mathbb{S}^q} (\Phi(\mathbf{x}\cdot\zeta) - \phi(\mathbf{x}\cdot\zeta))d\mu_q(\mathbf{x}) \\ &= c\int_{-1}^{1} (\Phi(t) - \phi(t))w_q(t)dt \le c\lambda_{N/2}(y).\end{aligned} \tag{0.4}$$

Now, using [3, Lemma 5, p. 108] with $a = b = q/2 - 1$ and taking into account the fact that $y = \cos\theta_\xi$ with $\theta_\xi \sim 1/N$, we deduce that

$$\lambda_{N/2}(y) \le \frac{c}{N}\left(\sqrt{1-y} + 2/N\right)^{2a+1} \le cN^{-q} \le c\mu_q(S).$$

Therefore, from (0.4),

$$a_\xi \le \sum_{\mathbf{z}\in\mathcal{C}_0} a_{\mathbf{z}}\chi(\mathbf{z}\cdot\zeta) \le c\mu_q(S) \le c\mu_q(R_\xi).$$

This proves the estimate (0.1) in the case when $p' = \infty$. The case when $p' = 1$ follows by taking P to be the polynomial identically equal to 1 in (4.4). In the case when $1 < p' < \infty$, we use these cases to conclude that

$$\left\|\frac{a_\xi}{\mu_q(R_\xi)}\right\|_{\mathcal{C},p'}^{p'} = \sum_{\xi\in\mathcal{C}_0} a_\xi \left\{\frac{a_\xi}{\mu_q(R_\xi)}\right\}^{p'-1} \le c.$$

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