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REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 2000 Mathematics Subject Classification can be found in print starting with the 1999 annual index of *Mathematical Reviews*. The classifications are also accessible from `www.ams.org/msc/`.

1[90C25, 90C51]—*A mathematical view of interior-point methods in convex optimization*, by James Renegar, SIAM, Philadelphia, PA, 2001, viii+177 pp., softcover, \$39.50, SIAM member price \$27.65, ISBN 0-89871-502-4

This book develops a comprehensive yet elegantly compact (116 pages) presentation of the modern mathematical theory of interior-point methods (IPMs) for convex constrained optimization. Through the author's insightful reinterpretation of this theory and his lucid writing style, he successfully develops the theory of IPMs in a manner that is (relatively) simple, genuinely accessible, and that conveys some of the deep intuition behind the theory. The book is designed to be used by research academicians, doctoral students, and scholars who want to invest in understanding and developing intuition into this important theory.

To place IPMs for convex optimization in context, recall that the "interior-point revolution" in optimization began in 1984 with Karmarkar's IPM for linear optimization. Thereafter, many researchers began to explore the extension of IPMs from linear optimization to nonlinear constrained convex optimization problems. In an incredible *tour de force*, Yurii Nesterov and Arkadii Nemirovskii [1] presented a deep and unified theory of IPMs for all of convex programming based on the notion of *self-concordant functions*. This theory and the resulting computational algorithms can be outlined in broad strokes as follows. After a straightforward transformation, any convex optimization problem can be put into the form (P:) $\min\{c^t x | x \in D\}$, for a suitable linear objective function $c^t x$ and feasible domain D. Supposing that there is an available *barrier function* $f_{D^o}(\cdot)$ on the relative interior D^o of D, one can construct the parametric family of barrier problems: (P_μ): $\min\{c^t x + \mu f_{D^o}(x) | x \in D^o\}$ for $\mu \in (0, \infty)$ and then approximately trace the path of solution $x(\mu)$ of (P_μ) using Newton's method for a sequence of parameter values $\mu_k \to 0$. This naturally leads to a list of questions concerning the theoretical efficiency of this scheme, the class or properties of barrier functions $f_{D^o}(\cdot)$ for which Newton's method will result in theoretically efficient methods, the class of feasible domains which easily give rise to "good" (i.e., theoretically efficiently computable) barrier functions, etc. These questions and their theoretical and practical answers are the core subject of IPMs for convex optimization.

The intellectual contributions of Nesterov and Nemirovskii's theory (and extensions by Nesterov and Michael Todd in [2], [3]) cannot be overstated, whether for its mathematical depth, its implications for the theory of convex optimization and computational complexity, or for its implications for computational practice. However, as is often the case in seminal technical literature, the presentation and the development of the theory in [1] is complicated, awkward in places, and in general not very accessible, especially with regard to developing intuition and understanding root themes. This is probably due to a variety of factors, including notational conventions, the generality of the set-up, cultural differences in mathematical writing, and the fact that the theory was so novel and did not have the benefit of hindsight and analysis by others.

Renegar's book is designed to present the theory of IPMs for convex optimization in a way that is as simple and lucid as possible, based only on first principles of multi-variable calculus, and in a way that conveys intuition as to what is really driving the theory. What makes the book more than a mere re-presentation of [1] is Renegar's reinterpretive perspective of the theory using his own concepts of local inner products and local norms. Beyond shortening many proofs and making for a more elegant presentation, the local inner product interpretation yields a different and more accessible intuition as to what drives the underlying theory of IPMs. It is this interpretation that is the book's intellectual contribution, quite substantial by itself, and which adds to Renegar's lucid style by which the book accomplishes its pedagogical goals.

The book is organized into three parts: preliminaries, basic theory, and cone-based duality theory. The preliminaries section is devoted to a review of the necessary elements of multi-variable calculus, notably gradients and Hessians, and their properties under changes in inner products. The basic theory of IPMs is presented next. This includes the theory of self-concordant functions, self-concordant barriers and their complexity values, and primal interior-point algorithms and their complexity. The third section of the book is devoted to conic duality, conjugate barrier functions, and the theory and algorithms based on self-scaled cones. The first two parts of the book truly shine for their motivating themes and the lucidity of presentation.

The book is an excellent resource for the serious researcher in optimization who wants to rapidly learn modern IPM theory and gain access to the research frontier. This reviewer has also used the book for a guided informal course of study with doctoral students, to help them learn and get up to speed with the latest developments in IPM theory. As this reviewer can attest, if one meets with one to three students once a week for two hours to go over approximately ten pages per week, the entire book can easily be covered in one semester.

References

[1] Y. Nesterov and A. Nemirovskii, *Interior Point Polynomial Algorithms in Convex Programming*, SIAM, Philadelphia, 1994. MR **94m:**90005

[2] Y. Nesterov and M. Todd, "Self-scaled barriers and interior-point methods for convex programming", *Mathematics of Operations Research* 22 (1997), pp. 1-42. MR **98c:**90093

[3] Y. Nesterov and M. Todd, "Primal-dual interior-point methods for self-scaled cones", *SIAM Journal on Optimization* 8 (1998), pp. 324-364. MR **99f:**90056

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