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REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 2000 Mathematics Subject Classification can be found in print starting with the 1999 annual index of *Mathematical Reviews*. The classifications are also accessible from `www.ams.org/msc/`.

1[65N55, 65F10, 65Y20]—*Practical Fourier analysis for multigrid methods*, by Roman Wienands and Wolfgang Joppich, Chapman & Hall / CRC, Boca Raton, FL, 2005, 217 pp + vii pp., hardcover, UK£49.99, ISBN 1-58488-492-4

Multigrid methods have been shown to be powerful techniques for solving a wide variety of partial differential equations. Multigrid consists of various components such as a coarse grid sequence, smoothing, interpolation, restriction, and coarse grid operators. The first thing a multigrid user faces is the correct choice of these components. The options are vast, but two slightly different combinations of multigrid components can lead to convergence or divergence. The authors are advocating the use of theoretically based techniques to provide practical guidance for multigrid. In particular, the tool considered is a Fourier-based method, or more precisely, the local Fourier analysis (LFA), which is able to provide quantitative estimates of multigrid performance. Specifically, the effectiveness of smoothers for reducing high frequency components and the rate of convergence of a two-grid/three-grid method are given by the values of the smoothing factor and the two-grid/three-grid factor, respectively.

The term "Local Fourier analysis" seems to suggest this book is technical, and full of tedious algebra and complex formulae. On the contrary, the authors intend to present the material from a practical point of view. In fact, a rather unique distinguished feature is the accompanying LFA software which is not often found in other multigrid books. Most of the commonly used techniques and multigrid components are implemented in the software. It has also provided a graphical user interface that makes the use quite easy. (I managed to become familiar with the software in less than an hour by just playing around with the buttons plus a brief reading of the documentation.)

Accordingly, the book is structured into Part I, Chapters 1-4 (Practical application of LFA) and Part II, Chapters 5-7 (Theory of LFA). Part I provides the basic knowledge of multigrid, an introduction to local Fourier analysis and a review of the LFA software. Part II provides the technical details of LFA for smoothing analysis, two-grid/three-grid analysis, and their results on a variety of problems.

Using multigrid terminology, Chapter 1 serves as "the coarse grid"; it provides a quick overview of the entire book. It first sets up notation, then provides a brief introduction of Fourier analysis and finally explains the two basic multigrid principles: smoothing and coarse grid correction. The software is introduced at this point as well, to prepare readers who are already familiar with basic multigrid and do not want to deal with the fuss of mathematical details.

Chapter 2 can be viewed as the "interpolation component"; it is a transition from coarse to fine details. This short chapter does not talk much about the details of LFA as yet, but rather engages the reader as to what LFA can and cannot do by making and explaining the necessary assumptions.

Chapter 3 provides a finer description of various choices of multigrid components that are often used in practice and are implemented in the LFA software. Unlike the usual treatment of multigrid components, the presentation is accompanied by the software dialogue windows. At first glance, while not knowing what those buttons and menu bars were, I found those windows somewhat distracting. However, after getting used to the visual layout, I actually appreciate the nice, concise, and succinct presentation of all kinds of variations within each component. Moreover, the multigrid options are quite extensive which include most of the choices that are used in practice.

Finally, it comes to the fine details. The power of LFA is the quantitative measure of multigrid effectiveness. However, these estimates depend heavily on the particular choice of multigrid components as well as the particular problem being solved. Chapter 4 provides results of LFA for an extensive list of problems. There are altogether 20 case studies which include 2D/3D anisotropic diffusion equation using second/fourth order discretization, Helmholtz equation, biharmonic equation, convection-diffusion equation, Stokes equations, Oseen equations, elasticity system and a linear shell problem. Each case study is accompanied by the numerical results given by the software. The effect of changing multigrid components for some case studies is also discussed.

Chapter 5 and 6 provide the fine details of theoretical background of LFA. Chapter 5 focuses on one grid, i.e. smoothing analysis. The smoothers considered are primarily relaxation types: Jacobi and Gauss-Seidel and their variants for scalar equations; collective, decoupled and distributive relaxations for systems. It should also be noted that it contains a rather thorough discussion on the red-black or the so-called pattern relaxation smoothing analysis. Chapter 6 deals with two and three grid analysis. To my pleasant surprise, there is no messy algebra nor long strings of formula. But rather, the authors focus on the main ideas. I particularly like the tables of Fourier symbols for various restriction and prolongation operators; they are not often found in other multigrid books. Furthermore, the LFA analysis for semi-coarsening, 3D cases, and systems are also included.

The last chapter serves as "the postsmoothing" phase that wraps up the book by further discussion of LFA on the issues of polynomial order versus high- and low-frequency order, simplified LFA, cell-centered multigrid, and multigrid as a preconditioner.

This book presents a thorough and systematic description of local Fourier analysis in the context of multigrid for general systems of partial differential equations. Two main features of this book are an extensive selection of problems of different kinds and an accompanying user-friendly software that can perform rather complex local Fourier analysis by just a few mouse clicks. It provides the classical local Fourier analysis results of multigrid as well as the state-of-the-art of the field. The numerical results given by the software further illustrate the flexibility and applicability of this analysis approach.

Overall, this book is not exactly an introductory text for multigrid. It is not a pure theory book for experts either. It is rather designed as a companion to any multigrid tutorial or book with a strong flavour on quantitative analysis and as a handy guide to the subject. It is aimed at researchers who are both practically and theoretically oriented, at graduate students who are interested in multigrid algorithms and theoretical justification, and at practitioners who develop efficient multigrid code for computer simulation. It was a joy reading this book, and I am happy to have it as a valuable addition to my multigrid book collection.

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