MATHEMATICS OF COMPUTATION
Volume 76, Number 260, October 2007, Pages 2256–2258
S 0025-5718(07)02043-1
Article electronically published on May 14, 2007

7[68U10, 65F22, 94A08, 49N45]—*Deblurring images: Matrices, spectra, and filtering*, by Per Christian Hansen, James G. Nagy, and Dianne P. O'Leary, SIAM, Philadelphia, PA, 2006, xiv+130 pp., softcover, US$63.00; SIAM member US$44.00, ISBN 978-0-898716-18-4

If a senior asks for a summer project on imaging, or a beginning graduate student inquires about an inviting entrance into contemporary imaging science, I would most probably recommend the subject of *image deblurring*, and to my great satisfaction, I will hand out this remarkable introductory book by Hansen, Nagy, and O'Leary, which is well designed and orchestrated.

To absorb most of the materials, all one needs are (1) some innate curiosity and passion in imaging, (2) very primitive knowledge on how to *open* MATLAB (but not how to *program* in MATLAB), and (3) some basic linear algebra on eigenvalues and singular values, which is often covered in sophomore courses. The authors have promised in the Preface that "*The book is intended for beginners in the field of image restoration and regularization.*" Indeed, the promise has been faithfully kept throughout the manuscript.

While the cliche goes: "Never judge a book by its cover" (or *its physical layout*), I must confess that a professional layout design, *just like an elaborate stage setting in a Broadway show*, does help convey more effectively the intellectual messages of a book. This is especially true for an *imaging* book like the present in this *multimedia* era. *Four colors* have been well orchestrated throughout the book, in contrast with the prevailing practice of monochrome printing in black ink. Framed *red* texts are entitled "Pointers" by the authors, pointing towards extra technical, bibliographical, historic, or online information. Framed *green* texts are named "Challenges," which are carefully crafted tasks intended to help a reader get hands dirty and mind engaged, and to foster a *proactive* learning experience. Finally, highlighted or framed *purple-blue* texts are termed "Very Important Points" (VIPs), signifying either important concepts, programming tips, or key properties. In addition to these colored texts, the SIAM Publisher has also been generously offering color plates for most figures, which further enhances the physical and informative appeal of the book.

We now turn to the modeling, analytical, and computational aspects of the book.

Like denoising or dejittering [4] in image processing, the prefix "de-" invariably refers to an inverse action intended to reverse a *corruptive* process tagged by the name that follows it, e.g., noising or jittering. *Deblurring is to reverse a blurring process, so that a blurry image could regain its original sharpness and clearness.* It is crucial (and rewarding in return) to realize that deblurring is first of all an *inverse problem.*

The world appears blurry to deficient naked eyes. With a properly prescribed pair of glasses, one can still enjoy clear and sharp views. The distorted pupil lenses generate blurry images on the two retinae, while a good pair of glasses deblur them. This is probably the most tangible example of deblurring in daily life. One can almost breathe the importance of deblurring — a blurry world simply effaces too many details and easily frustrates an observer.

In the broader contexts of medical, astronomical, or surveillance imaging, blurring is also ubiquitous. But one spots a significant difference from glasses-based correcting. The latter deblurs images by interfering with the *image acquisition process*, while in the former contexts, most often a captured image is already blurry and final, and one does not have the luxury of going back to adjust the acquisition process. To be concrete, consider a camera for automatic traffic monitoring. Suppose that it captures the image of a car that violates a red-light signal. If the car moves fast enough, the image will be corrupted with severe motion blur. But the event has already occurred and the image is final. We are ridiculed if demanding that violating driver to slow down and pose well for a clear image.

To summarize, in most applications, one is prompted to study the following problem:

given a blurry image B, how to reconstruct its sharp and clear version X.

(*Here we have followed the notations in the book.*) Since this is an inverse problem, one has to first ask what the *forward problem* or *model* is, i.e., a model for blurring in deblurring problems. The most common model for a blurring process is given by

$$B = A[X] + N,$$

where A is a blurring process, be it linear or nonlinear, and N denotes some additive noising process. As a result, usually deblurring has to perform simultaneous denoising.

In the present book, A is assumed to be known and linear, and is discussed in full detail in **Chapter 3**. An unknown A would result in the so-called "blind"-deblurring problem, which is more challenging [2] and beyond the introductory level of the present book.

For digital images, both X and B are discrete vectors or arrays, and subsequently a linear blur A would correspond to a **matrix**. Then deblurring is very similar to the *linear regression* problem in statistical modeling. Furthermore, when the blur operator A is shift invariant, the corresponding blur matrix will be *well structured*, and could be Toeplitz, circulant, or Hankel, for example. There has been a wealth of numerical linear algebra on computing with structured matrices [3, 5, 6], which has been concisely reviewed in **Chapter 4**. In particular, for the structured blur matrices in the present book, fast Fourier transform (FFT) and discrete cosine transform (DCT) are indispensable tools.

The involvement of either FFT or DCT naturally points toward the spectral method. In the Fourier domain, or equivalently, the **spectral domain**, blurring amounts to suppressing high-spectra details, which is realized by

multiplying the spectrum indexed by i by a small weight σ_i.

In fact, this σ_i is precisely one of the singular values of A. Thus deblurring has to undo the distortion: multiplying the i-indexed spectrum (of the blurry image B) by $1/\sigma_i$. Here arises the central challenge for the deblurring problem: for high spectra, σ_i's are often close to zero, and how can one divide them cleverly so that numerical errors or noisy components in the data will not be amplified? For example, if $\sigma_i = \exp(-15)$ (which indeed surfaces out of Gaussian blurs), dividing σ_i could become very problematic even for moderately noisy data. The nature of this challenge has been friendly and is explored in **Chapter 5**.

By this point in the book, the reader should have been sufficiently motivated and eager to know how to fix this "multiplication" problem. Staged subsequently is the core of the entire book: the spectral **filtering** method in **Chapter 6**. The idea is to "regularize" the above naive multiplication by a tuning companion ϕ_i, so that in combination the multiplication of ϕ_i/σ_i yields controlled deblurring. It is clear that each ϕ_i has to depend upon its associated singular value σ_i, and also has to be sufficiently small when σ_i is, in order to avoid erroneous amplification of noises or errors. **Chapter 6** explores the theory behind this intuition, as well as develops effective numerical schemes and selecting strategies for parameters.

Therefore, the book title could never be more compact: "*Deblurring images: Matrices, spectra, and filtering.*"

Many useful MATLAB basics are explained in **Chapter 2**. Extension to color images and more regularizing techniques are discussed in **Chapter 7,** the last chapter. Then about 20-page MATLAB codes employed in the book are attached in the Appendix, which can also be downloaded online from its website:

www.siam.org/books/fa03

"fa03" indicates the third book in the SIAM series of "Fundamentals of Algorithms."

At a very affordable and introductory level, the book has successfully integrated an emerging important application, mathematical analysis, efficient algorithms, and practical software implementation into a coherent and very appealing subject. The book can be an ideal choice for undergraduate Math Clubs, senior projects, beginning graduate students potentially interested in a degree on imaging research, as well as a friendly reference tool for imaging researchers being perplexed by blurring issues.

The book has also paved much needed stepping stones for mathematically more sophisticated imaging monographs such as [1, 2, 7], just to name a few.

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