
If a senior asks for a summer project on imaging, or a beginning graduate student inquires about an inviting entrance into contemporary imaging science, I would most probably recommend the subject of image deblurring, and to my great satisfaction, I will hand out this remarkable introductory book by Hansen, Nagy, and O'Leary, which is well designed and orchestrated.

To absorb most of the materials, all one needs are (1) some innate curiosity and passion in imaging, (2) very primitive knowledge on how to open MATLAB (but not how to program in MATLAB), and (3) some basic linear algebra on eigenvalues and singular values, which is often covered in sophomore courses. The authors have promised in the Preface that "The book is intended for beginners in the field of image restoration and regularization." Indeed, the promise has been faithfully kept throughout the manuscript.

We now turn to the modeling, analytical, and computational aspects of the book. Like denoising or dejittering in image processing, the prefix "de-" invariably refers to an inverse action intended to reverse a corruptive process tagged by the name that follows it, e.g., noising or jittering. Deblurring is to reverse a blurring process, so that a blurry image could regain its original sharpness and clearness. It is crucial (and rewarding in return) to realize that deblurring is first of all an inverse problem.

The world appears blurry to deficient naked eyes. With a properly prescribed pair of glasses, one can still enjoy clear and sharp views. The distorted pupil lenses generate blurry images on the two retinas, while a good pair of glasses deblur them. This is probably the most tangible example of deblurring in daily life. One can almost breathe the importance of deblurring — a blurry world simply effaces too many details and easily frustrates an observer.

In the broader contexts of medical, astronomical, or surveillance imaging, blurring is also ubiquitous. But one spots a significant difference from glasses-based
correcting. The latter deblurs images by interfering with the image acquisition process, while in the former contexts, most often a captured image is already blurry and final, and one does not have the luxury of going back to adjust the acquisition process. To be concrete, consider a camera for automatic traffic monitoring. Suppose that it captures the image of a car that violates a red-light signal. If the car moves fast enough, the image will be corrupted with severe motion blur. But the event has already occurred and the image is final. We are ridiculed if demanding that violating driver to slow down and pose well for a clear image.

To summarize, in most applications, one is prompted to study the following problem:

\[ \text{given a blurry image } B, \text{ how to reconstruct its sharp and clear version } X. \]

(Here we have followed the notations in the book.) Since this is an inverse problem, one has to first ask what the forward problem or model is, i.e., a model for blurring in deblurring problems. The most common model for a blurring process is given by

\[ B = A[X] + N, \]

where \( A \) is a blurring process, be it linear or nonlinear, and \( N \) denotes some additive noising process. As a result, usually deblurring has to perform simultaneous denoising.

In the present book, \( A \) is assumed to be known and linear, and is discussed in full detail in Chapter 3. An unknown \( A \) would result in the so-called “blind”-deblurring problem, which is more challenging [2] and beyond the introductory level of the present book.

For digital images, both \( X \) and \( B \) are discrete vectors or arrays, and subsequently a linear blur \( A \) would correspond to a matrix. Then deblurring is very similar to the linear regression problem in statistical modeling. Furthermore, when the blur operator \( A \) is shift invariant, the corresponding blur matrix will be well structured, and could be Toeplitz, circulant, or Hankel, for example. There has been a wealth of numerical linear algebra on computing with structured matrices [3, 5, 6], which has been concisely reviewed in Chapter 4. In particular, for the structured blur matrices in the present book, fast Fourier transform (FFT) and discrete cosine transform (DCT) are indispensable tools.

The involvement of either FFT or DCT naturally points toward the spectral method. In the Fourier domain, or equivalently, the spectral domain, blurring amounts to suppressing high-spectra details, which is realized by

\[
\text{multiplying the spectrum indexed by } i \text{ by a small weight } \sigma_i.
\]

In fact, this \( \sigma_i \) is precisely one of the singular values of \( A \). Thus deblurring has to undo the distortion: multiplying the \( i \)-indexed spectrum (of the blurry image \( B \)) by \( 1/\sigma_i \). Here arises the central challenge for the deblurring problem: for high spectra, \( \sigma_i \)'s are often close to zero, and how can one divide them cleverly so that numerical errors or noisy components in the data will not be amplified? For example, if \( \sigma_i = \exp(-15) \) (which indeed surfaces out of Gaussian blurs), dividing \( \sigma_i \) could become very problematic even for moderately noisy data. The nature of this challenge has been friendly and is explored in Chapter 5.

By this point in the book, the reader should have been sufficiently motivated and eager to know how to fix this “multiplication” problem. Staged subsequently is the core of the entire book: the spectral filtering method in Chapter 6. The
idea is to “regularize” the above naive multiplication by a tuning companion $\phi_i$, so that in combination the multiplication of $\phi_i/\sigma_i$ yields controlled deblurring. It is clear that each $\phi_i$ has to depend upon its associated singular value $\sigma_i$, and also has to be sufficiently small when $\sigma_i$ is, in order to avoid erroneous amplification of noises or errors. Chapter 6 explores the theory behind this intuition, as well as develops effective numerical schemes and selecting strategies for parameters.

Therefore, the book title could never be more compact: “Deblurring images: Matrices, spectra, and filtering.”

Many useful MATLAB basics are explained in Chapter 2. Extension to color images and more regularizing techniques are discussed in Chapter 7, the last chapter. Then about 20-page MATLAB codes employed in the book are attached in the Appendix, which can also be downloaded online from its website:

www.siam.org/books/fa03

“fa03” indicates the third book in the SIAM series of “Fundamentals of Algorithms.”

At a very affordable and introductory level, the book has successfully integrated an emerging important application, mathematical analysis, efficient algorithms, and practical software implementation into a coherent and very appealing subject. The book can be an ideal choice for undergraduate Math Clubs, senior projects, beginning graduate students potentially interested in a degree in imaging research, as well as a friendly reference tool for imaging researchers being perplexed by blurring issues.

The book has also paved much needed stepping stones for mathematically more sophisticated imaging monographs such as [1] [2] [7], just to name a few.

References


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