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REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 2000 Mathematics Subject Classification can be found in print starting with the 1999 annual index of *Mathematical Reviews*. The classifications are also accessible from `www.ams.org/msc/`.

1[65F10]—*An introduction to iterative Toeplitz solvers*, Raymond Hon-Fu Chan and Xiao-Qing Jin (Editors), Fundamentals of Algorithms, Vol. 5, SIAM, Philadelphia, 2007, xii + 108 pp., softcover, US$67.00, member US$46.90, ISBN 978-0-898716-36-8

This book is the fifth in the series "Fundamentals of Algorithms" that SIAM has been publishing since 2003. The books in the series, including the one under review, are slim monographs presenting a topic at an introductory level, so that practitioners can get information in a self-contained manner. This volume is possibly the one which is most focused and least broad in the series so far. Within this context, the authors have succeeded in developing the material in an accessible manner, and the book can indeed serve as a guide for the uninitiated. Furthermore, relevant m-files for matlab codes are provided (but more on this later).

The area described in the book is the solution of algebraic linear systems of equations governed by symmetric Toeplitz matrices. More specifically, the solution method discussed is preconditioned Conjugate Gradients (PCG). The essence of the book is the description and analysis of a handful of preconditioners for these matrices. Block versions are also briefly discussed (block Toeplitz solvers are well developed in the earlier tome [1] by the second author).

Toeplitz matrices are matrices with a very special structure, namely, constant along diagonals. The entries of the matrix are often obtained as the Fourier coefficients of a *generating function*. These matrices appear in many applications, in particular, in some discretizations of partial and ordinary differential equations (ODEs) and differential algebraic equations, and in image restoration problems in image processing. In this book only the symmetric positive definite case is considered, and this implies that the spectrum of the matrix is real and positive.

As is well known, the Conjugate Gradient iterative method exhibits, in most cases, superlinear convergence; see, e.g., [2]. One such case is when the spectrum of the matrix is clustered around 1, and with only a small number of eigenvalues outside a small interval around 1. This property forms the basis of most of the analysis of the preconditioners studied in the book: it is shown that the preconditioned matrix has such a spectrum.

The preconditioners studied are named after the author or authors who first proposed them: Strang, T. Chan, R. Chan, Huckle, and Ku and Kuo. A few others are also considered. These preconditioners are introduced, their spectra analyzed (showing that they form a cluster around one), and numerical experiments with simple examples are presented, with the number of PCG iterations reported for each of them. All the preconditioners are presented in a unified manner as convolutions of the generating function with specific kernels. There is also a brief chapter with considerations for ill-conditioned problems.

While I am aware that the series imposes space constraints, I would have liked to have seen some examples coming from real applications, such as from ODEs. Similarly, there is no motivation for the chapter on ill-conditioned systems, which are ubiquitous in image processing.

The appendix with the matlab code is very useful, but to my taste its level of documentation is deficient. One such example is the PCG code. If one copies the m-file, and provides the input as specified in the headings, a flag tells you that there are undefined functions. Indeed, one needs a matrix-vector product function, but this is not specified in the header. Furthermore, in this day and age one would expect that the fifteen pages of code be available in electronic form somewhere, at the publisher's web site, at the authors' web site, or in an accompanying CD. Unfortunately, they are not available.

In spite of these shortcomings, the volume is a very good introduction to modern preconditioners for the iterative solution of symmetric Toeplitz linear systems.

References

[1] Xiao-Qing Jin, *Developments and Applications of Block Toeplitz Iterative Solvers*, Science Press, Beijing/New York, and Kluwer Academic Publishers, Dordrecht/Boston/London, 2002. MR1980639 (2004d:65041)

[2] Ragnar Winther, Some Superlinear Convergence Results for the Conjugate Gradient Method, *SIAM Journal on Numerical Analysis*, vol. 17 (1980) 14–17. MR559456 (81k:65060)

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