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2[65G20, 65G30, 65G40, 65G50]—*Introduction to interval analysis*, Ramon E. Moore, R. Baker Kearfott, and Michael J. Cloud (Editors), SIAM, Philadelphia, PA, 2009, xi+223 pp., softcover, US$72.00, ISBN 978-0-898716-69-6

It is quite natural to ask, how far away lies a computed value from the genuine one? There are many possibilities to find an answer. The interval methods represent a unified approach to analyze such questions and to give the rigorous bounds on computational errors. These methods provide rigorous bounds on accumulated rounding errors, approximation errors, and propagated uncertainties in input data during the course of the computation. An another important field of applications of interval analysis is rigorous computer-assisted proofs of mathematical assertions.

The book opens with a brief introduction containing some simple examples which show that repeating a calculation (seeming to be convergent) with more precision does not necessarily provide a basis for determining the accuracy of the results. This should motivate the reader to get to know an alternative method for computations using the interval number system and interval arithmetic. The next chapter, "The interval number system", formally introduces this new number system, the interval arithmetic as well as the interval vectors and matrices. The chapter ends with some historical references concerning the pioneers of interval computations and the early works on the topic.

Chapter 3 introduces the first simple applications of interval arithmetic where the last one provides a natural way of incorporating measurement uncertainties of the variables into a calculation using a given formula. This chapter contains also a description of several software packages implementing the interval arithmetic. Particulary convenient is the free available INTLAB which provides an interactive environment within MATLAB.

In the next two chapters the authors describe some further properties of interval arithmetic and introduce the basics of interval-valued functions. It is shown that due to the lack of distributivity and of the additive and multiplicative inverses in interval arithmetic, two expressions defining a real-valued function of real arguments can be equivalent in real arithmetic but not equivalent in interval arithmetic. If F is an interval extension of f (i.e. $F = f$ for degenerate interval arguments consisting of one real number), then it is shown under which conditions it holds $f(X_1, ..., X_n) \subseteq F(X_1, ..., X_n)$ (the fundamental theorem).

The convergence and continuity are the two central concepts of analysis and in Chapter 6, "Sequences of intervals and interval functions", the authors generalize these concepts for the case of interval arithmetic. They introduce a suitable metric, convergence and continuity in interval mathematics, subdivisions and refinements and discuss the finite convergence and stopping criteria.

In Chapter 7, "Interval Matrices", the authors introduce interval computations in the context of numerical linear algebra. They discuss specific features of the Gauss elimination in the interval arithmetic and the interval versions of the fixed point iteration and of Gauss-Seidel iteration.

In Chapter 8, "Interval Newton Methods", the ideas of interval mathematics are used to develop methods for rigorous bounds on the solutions to nonlinear systems of equations. At the beginning the authors consider some interval generalizations of the fixed point iteration and of the Newton iteration first for a single nonlinear equation and then for a system of nonlinear equations.

Chapter 9, "Integration of Interval Functions", is devoted to the enclosures for the definite integrals of real-valued functions. The authors define the interval integral and discuss its application to obtain the interval enclosures for the definite integrals and the integration of power-type functions having interval coefficients.

In Chapter 10 the enclosures of the solution of operator equations of the form

$$y(t) = p(y)(t)$$

are discussed where the operator p may include integrals of the function $y(t)$. In this framework the integral equations and the initial as well as the boundary value problems for ordinary differential equations are considered.

Finally, in Chapter 11 the authors discuss the use of the interval arithmetic in the following fields: computer-assisted proofs, global optimization and constraint satisfaction, parameter estimation, robotics, chemical engineering and other applications.

The Appendices "Sets and Functions", "Formulary", "Hints for selected Exercises", "Internet Resources", and "INTLAB Commands and Functions" follow the last chapter.

The list of references including 257 titles and the Index round off the book.

The book contains many examples of demonstrated theoretical constructions as well as examples showing the computation with INTLAB.

This book is aimed primarily at those not yet familiar with interval computation and interval analysis. The book can serve as a support for an introduction to interval analysis for all who are studying or teaching on this topic. The reader will be able to follow the presentation with some previous knowledge in analysis, numerical methods and programming in MATLAB.

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