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REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme accessible from `www.ams.org/msc/`.

2[65N38, 35J08, 31A10]—*Fast multipole boundary element method*, by Yijun Liu, Cambridge University Press, Cambridge, 2009, xviii+235 pp., hardcover, US$85.00, ISBN 978-0521-11659-6

This work presents an introduction to the fast multipole boundary element method (BEM) which is a powerful numerical tool for solving large-scale mathematical models based on the boundary integral equation (BIE). Clarity of exposition, a lot of illustrating examples and programms are the distinguished features of this book. Within each major topic the exposition is, as a rule, inductive, nearly always proceeding from the statement and the roots of a problem through a theoretical analysis toward fundamental ideas of numerical approximations and numerical examples. The close relationship of the mathematical models for different applications from different application fields in various chapters is emphasised. Each chapter is rounded off by a short summary and a collection of exercises to review the materials covered in the chapter.

The book consists of six chapters and two appendices.

Chapter 1 "Introduction" provides a general introduction of the boundary integral equations (BIE) and of the boundary element methods (BEM). A comparison of the BEM and the finite element method (FEM) is discussed. As a test example a simple beam problem is used. Some mathematical results that are needed in the development of the BIE and the BEM are reviewed.

Chapter two is devoted to the conventional BEM for potential problems and can be considered as the basis for the following chapters. It is shown, how a boundary value problem (BVP) for a partial differential equation can be transformed into BIE with the help of the fundamental solution or of the Green's function. The handling of the singularities at the BIE is discussed. The discretisation procedures with constant, linear and quadratic boundary elements are discussed. Programming for the BEM using the conventional approach is addressed and some numerical examples are given to demonstrate the accuracy of the BEM for solving potential problems. The following chapters use the same presentation scheme beginning with the basic ideas through BIE formulation, BEM discretisation procedures and programming towards the numerical results for practical problems.

The third Chapter "Fast Multipole Boundary Element Method for Potential Problems" introduces the main topic of the book for 2D and 3D potential problems. The main idea of the fast multipole BEM is to use the multipole expansions of kernels such as

$$G(x,y) = \sum_i G_i^x(x,y_c)G_i^y(y,y_c)$$

and then to write

$$\int_{S_c} G(x,y)q(y)dS(y) = \sum_i G_i^x(x,y_c) \int_{S_c} G_i^y(y,y_c)dS(y)$$

where S_c is a subset of the boundary S away from x and then to replace the "element-to-element" interactions with "cell-to-cell" interactions. Some additional expansions and translations, as well as the hierarchical tree structure of the elements, are introduced to further reduce the computational costs. Using this idea one can reduce the solution time and the memory requirements of the corresponding BEM system with N boundary elements from $\mathcal{O}(N^2)$ (with iterative solvers) to $\mathcal{O}(N)$. Complete formulations, implementation details as well as Fortran code provided in Appendix B are also discussed in this chapter. The efficiencies of the fast multipole BEM are demonstrated by solving large-scale 2D and 3D discretizations of several mathematical models in new technologies such as image-based modelling and simulations in reverse engineering and biochemical engineering.

Chapter 4 deals with the mathematical modelling of elastostatic problems using fast multipole BEM. The fundamental solutions of the governing equations and the generalized Green's identity are introduced to formulate the corresponding BIE. The fast multipole BEM is described in detail for 2D problems and the results for 3D problems are presented. Numerical examples demonstrate the accuracy and efficiencies of the fast multipole BEM.

In Chapter 5 the BIE formulations for 2D and 3D Stokes flow problems are presented. The fast multipole BEM is discussed for 2D problems and its formulation for 3D problems is presented. The 2D examples of the flow in an infinite medium that is due to a rotating circular cylinder, the flow between two parallel plates, the Stokes flows through a channel placed with one or multiple cylinders as well as the 3D examples of a translating sphere in an infinite fluid and of multipole particles that move through an infinite fluid are the practical engineering problems discussed in this chapter.

Formulations of the fast multipole BEM for solving Helmholtz equations describing various acoustic wave problems in both two and three dimensions are presented in Chapter 6. Scattering from cylinders in a 2D medium, radiation from a pulsating sphere, scattering from multiple scatterers, an engine-block model, a submarine model, an Airbus A320, a model of the impact of noise on human hearing, acoustic problems in a half-space such as an airport or other traffic noise control problems are the numerical examples of this chapter demonstrating the accuracy and efficiency of the fast multipole BEM for solving large-scale acoustic wave problems in both two and three dimensions.

Analytical integration of the kernels for 2D potential, elasticity, and Stokes flow cases and the samples of computer programs for both the 2D potential conventional BEM and the fast multipole BEM are provided in Appendices A and B.

The literature list consisting of 145 titles and the subject index round up the book.

The book under consideration is aimed primarily at graduate students (as a text book for a graduate course in engineering) and at researchers in the field of applied mechanics, and engineers from industries who would like to apply or to further develop the fast multipole BEM for solving large-scale problems in their own field.

The reader will be able to follow the presentation with some previous knowledge from senior-level courses in linear algebra, the theory and numerical analysis of partial differential equations and programming.

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