

# Transactions of the Moscow Mathematical Society

---

2021

---

*A Translation of*

ТРУДЫ  
МОСКОВСКОГО  
МАТЕМАТИЧЕСКОГО  
ОБЩЕСТВА

ТОМ 82, выпуск 1 2021

ТОМ 82, выпуск 2 2021



AMERICAN  
MATHEMATICAL  
SOCIETY

Providence, Rhode Island USA

# TRANSACTIONS OF THE MOSCOW MATHEMATICAL SOCIETY

Translation edited by Frances H. Goldman  
with the assistance of AMS staff

*A translation of*

ТРУДЫ МОСКОВСКОГО МАТЕМАТИЧЕСКОГО ОБЩЕСТВА

## Editorial Board

A. G. Sergeev (Editor in Chief)  
V. M. Buchstaber V. L. Popov  
Yu. S. Il'yashenko A. A. Shkalikov  
I. D. Shkredov  
A. V. Domrin (Secretary of the Board)

Library of Congress Card Number 65-4713

**SUBSCRIPTION INFORMATION.** Transactions of the Moscow Mathematical Society for 2021 will consist of one issue. Beginning in 2004, *Transactions of the Moscow Mathematical Society* is accessible from [www.ams.org/journals/](http://www.ams.org/journals/). The subscription prices for 2021: **Paper format with free electronic access:** List US\$751; AMS Institutional Member US\$600.80; **Electronic only:** List US\$713; AMS Institutional Member US\$570.40. Subscription renewals are subject to late fees.

See [www.ams.org/journal-faq](http://www.ams.org/journal-faq) for more journal subscription information.

**BACK NUMBER INFORMATION.** For back issues see [www.ams.org/bookstore](http://www.ams.org/bookstore).

Subscriptions and orders should be addressed to American Mathematical Society, P.O. Box 845904, Boston, MA 02284-5904, USA. *All orders must be accompanied by payment.* Other correspondence should be addressed to 201 Charles St., Providence, RI 02904-2213, USA.

**COPYING AND REPRINTING.** Individual readers of this publication, and nonprofit libraries acting for them, are permitted to make fair use of the material, such as to copy an article for use in teaching or research. Permission is granted to quote brief passages from this publication in reviews, provided the customary acknowledgment of the source is given.

Republication, systematic copying, or multiple reproduction of any material in this publication is permitted only under license from the American Mathematical Society. Request for permission to reuse portions of AMS publication content are handled by the Copyright Clearance Center. For more information, please visit [www.ams.org/publications/pubpermissions](http://www.ams.org/publications/pubpermissions).

Excluded from these provisions is material in articles for which the author holds copyright. In such cases, requests for permission to use or reprint should be addressed directly to the author(s). (Copyright ownership is indicated in the notice in the lower right-hand corner of the first page of each article.)

**Transactions of the Moscow Mathematical Society** (ISSN 0077-1554 (print); ISSN 1547-738X (online)) is published by the American Mathematical Society, 201 Charles Street, Providence, RI 02904-2213, USA and is mailed from Providence, Rhode Island. Postmaster: Send address changes to Transactions of the Moscow Mathematical Society, American Mathematical Society, 201 Charles Street, Providence, RI 02904-2213, USA.

© 2021, American Mathematical Society. All rights reserved.

This journal is indexed in *Mathematical Reviews* and *Zentralblatt MATH*.

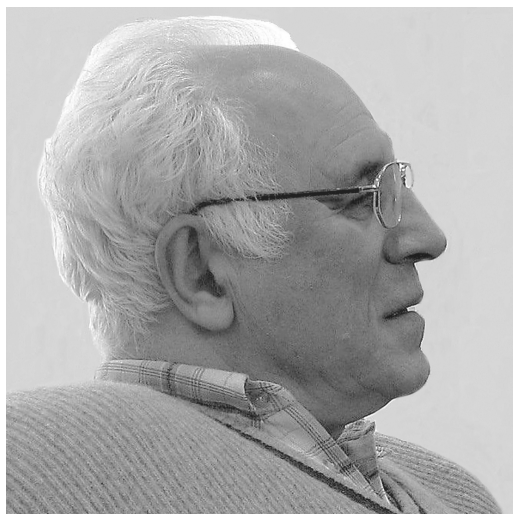
This journal is archived in *Portico*.

The London Mathematical Society is incorporated under Royal Charter and is registered with the Charity Commissioners.

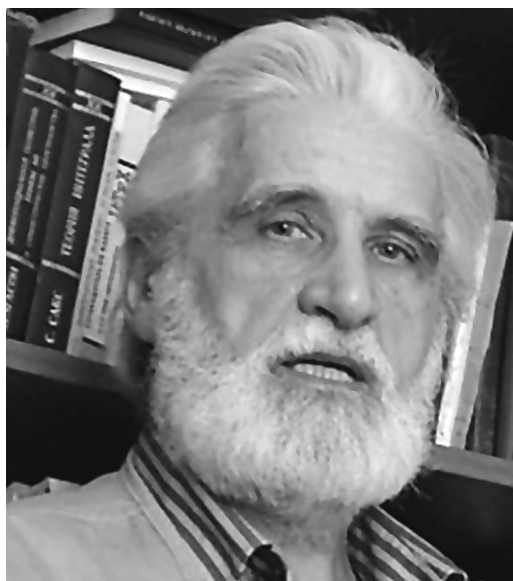
Printed in the United States of America.

⊗ The paper used in this journal is acid-free and falls within the guidelines established to ensure permanence and durability.

10 9 8 7 6 5 4 3 2 1      26 25 24 23 22 21



The first part of this volume is dedicated to the 80th birthday anniversary of Professor Valiriy Iustinovich Oseledetz (both May 25, 1940)



and the 80th birthday anniversary of Professor Anatoliy Mikhailovich Stepin (July 20, 1940–November 7, 2020).

This collection of articles on topics of current interest in the theory of dynamical systems are written by their friends, colleagues, and students. The collection was compiled by V. V. Ryzhikov.

# Transactions of the **Moscow** **Mathematical** **Society**

---

2021

---

*A Translation of*

ТРУДЫ  
МОСКОВСКОГО  
МАТЕМАТИЧЕСКОГО  
ОБЩЕСТВА

ТОМ 82, выпуск 1 2021

ТОМ 82, выпуск 2 2021

This translation was prepared jointly by  
the American Mathematical Society  
and the London Mathematical Society



---

AMERICAN  
MATHEMATICAL  
SOCIETY

Providence, Rhode Island USA

# TRANSACTIONS OF THE MOSCOW MATHEMATICAL SOCIETY

Translation edited by Frances H. Goldman  
with the assistance of AMS staff

*A translation of*

ТРУДЫ МОСКОВСКОГО МАТЕМАТИЧЕСКОГО ОБЩЕСТВА

## Editorial Board

A. G. Sergeev (Editor in Chief)  
V. M. Buchstaber V. L. Popov  
Yu. S. Il'yashenko A. A. Shkalikov  
I. D. Shkredov  
A. V. Domrin (Secretary of the Board)

Library of Congress Card Number 65-4713

**SUBSCRIPTION INFORMATION.** Transactions of the Moscow Mathematical Society for 2021 will consist of one issue. Beginning in 2004, *Transactions of the Moscow Mathematical Society* is accessible from [www.ams.org/journals/](http://www.ams.org/journals/). The subscription prices for 2021: **Paper format with free electronic access:** List US\$751; AMS Institutional Member US\$600.80; **Electronic only:** List US\$713; AMS Institutional Member US\$570.40. Subscription renewals are subject to late fees.

See [www.ams.org/journal-faq](http://www.ams.org/journal-faq) for more journal subscription information.

**BACK NUMBER INFORMATION.** For back issues see [www.ams.org/bookstore](http://www.ams.org/bookstore).

Subscriptions and orders should be addressed to American Mathematical Society, P.O. Box 845904, Boston, MA 02284-5904, USA. *All orders must be accompanied by payment.* Other correspondence should be addressed to 201 Charles St., Providence, RI 02904-2213, USA.

**COPYING AND REPRINTING.** Individual readers of this publication, and nonprofit libraries acting for them, are permitted to make fair use of the material, such as to copy an article for use in teaching or research. Permission is granted to quote brief passages from this publication in reviews, provided the customary acknowledgment of the source is given.

Republication, systematic copying, or multiple reproduction of any material in this publication is permitted only under license from the American Mathematical Society. Request for permission to reuse portions of AMS publication content are handled by the Copyright Clearance Center. For more information, please visit [www.ams.org/publications/pubpermissions](http://www.ams.org/publications/pubpermissions).

Excluded from these provisions is material in articles for which the author holds copyright. In such cases, requests for permission to use or reprint should be addressed directly to the author(s). (Copyright ownership is indicated in the notice in the lower right-hand corner of the first page of each article.)

**Transactions of the Moscow Mathematical Society** (ISSN 0077-1554 (print); ISSN 1547-738X (online)) is published by the American Mathematical Society, 201 Charles Street, Providence, RI 02904-2213, USA and is mailed from Providence, Rhode Island. Postmaster: Send address changes to Transactions of the Moscow Mathematical Society, American Mathematical Society, 201 Charles Street, Providence, RI 02904-2213, USA.

© 2021, American Mathematical Society. All rights reserved.

This journal is indexed in *Mathematical Reviews* and *Zentralblatt MATH*.

This journal is archived in *Portico*.

The London Mathematical Society is incorporated under Royal Charter and is registered with the Charity Commissioners.

Printed in the United States of America.

⊗ The paper used in this journal is acid-free and falls within the guidelines established to ensure permanence and durability.

10 9 8 7 6 5 4 3 2 1      26 25 24 23 22 21

## CONTENTS

<b>Wen Huang, Jian Li, and Xiangdong Ye, Positive entropy implies chaos along any infinite sequence</b> .....	1
<b>E. Glasner, J.-P. Thouvenot, and B. Weiss, On some generic classes of ergodic measure preserving transformations</b> .....	15
<b>V. A. Kibkalo, A. T. Fomenko, and I. S. Kharcheva, Realizing integrable Hamiltonian systems by means of billiard books</b> .....	37
<b>C. González-Tokman and A. Quas, Lyapunov exponents for transfer operator cocycles of metastable maps: A quarantine approach</b> .....	65
<b>Vassili N. Kolokoltsov, On a probabilistic derivation of the basic particle statistics (Bose–Einstein, Fermi–Dirac, canonical, grand-canonical, intermediate) and related distributions</b> .....	77
<b>R. Grigorchuk, R. Kogan, and Ya. Vorobets, On Mealy–Moore coding and images of Markov measures</b> .....	89
<b>V. V. Ryzhikov, Compact families and typical entropy invariants of measure-preserving actions</b> .....	117
<b>B. M. Gurevich, Conditions for the existence of a regular sequence of finite subgraphs of an infinite loaded linear graph</b> .....	125
<b>O. Paris-Romaskevich, Tiling billiards and Dynnikov’s helicoid</b> .....	133
<b>M. E. Lipatov, The asymptotic behaviour of cocycles over flows</b> .....	149
<b>M. Artigiani, C. Fougeron, P. Hubert, and A. Skripchenko, A note on double rotations of infinite type</b> .....	157
<b>S. V. Tikhonov, A violation of multiple mixing close to an extremal</b> .....	173
<b>A. Plakhov, On generalized Newton’s aerodynamic problem</b> .....	183
<b>A. V. Domrin, Holomorphic solutions of soliton equations</b> .....	193
<b>Kh. A. Khachatryan and A. S. Petrosyan, Alternating bounded solutions of a class of nonlinear two-dimensional convolution-type integral equations</b> ...	259
<b>G. G. Kazaryan, A. N. Karapetyants, V. N. Margaryan, G. A. Mkrtchyan, and A. G. Sergeev, New classes of function spaces and singular operators</b> .....	273
<b>M. A. Stepanova, CR-manifolds of infinite type in the sense of Bloom and Graham</b> .....	289
<b>A. I. Shafarevich and A. V. Tsvetkova, The wave equation with symmetric velocity on the hybrid manifold obtained by gluing a ray to a three-dimensional sphere</b> .....	305
<b>A. Yu. Pirkovskii, Letter to the Editors</b> .....	327

## RUSSIAN CONTENTS\*

*Труды Московского математического общества*  
*Том 82 (2021), выпуск 1*

<i>W. Huang, J. Li, X. Ye</i> Positive entropy implies chaos along any infinite sequence	3
<i>E. Glasner, J.-P. Thouvenot, B. Weiss</i> On some generic classes of ergodic measure preserving transformations	19
<i>В. А. Кибкало, А. Т. Фоменко, И. С. Харчева</i> Реализация интегрируемых гамильтоновых систем бильярдами книжками	45
<i>C. González-Tokman, A. Quas</i> Lyapunov exponents for transfer operator cocycles of metastable maps: a quarantine approach	79
<i>V. N. Kolokoltsov</i> On a probabilistic derivation of the basic particle statistics (Bose–Einstein, Fermi–Dirac, canonical, grand-canonical, intermediate) and related distributions	93
<i>R. Grigorchuk, R. Kogan, Ya. Vorobets</i> On Mealy–Moore coding and images of Markov measures	105
<i>В. В. Рыжиков</i> Компактные семейства и типичные энтропийные инварианты сохраняющих меру действий	137
<i>Б. М. Гуревич</i> Об условиях существования правильной последовательности конечных подграфов бесконечного нагруженного линейного графа	147
<i>O. Paris-Romaskevich</i> Tiling billiards and Dynnikov’s helicoid	157
<i>М. Е. Лунатов</i> Об асимптотическом поведении коциклов над потоками	175
<i>M. Artigiani, C. Fougerson, P. Hubert, A. Skripchenko</i> A note on double rotations of infinite type	185
<i>С. В. Тихонов</i> О нарушении кратного перемешивания, близком к экстремальному	205
<i>A. Plakhov</i> On generalized Newton’s aerodynamic problem	217

---

\*The American Mathematical Society scheme for transliteration of Cyrillic may be found at the end of index issues of *Mathematical Reviews*.

*Труды Московского математического общества*  
*Том 82 (2021), выпуск 2*

<i>А. В. Домрин</i> Голоморфные решения солитонных уравнений	227
<i>Х. А. Хачатрян, А. С. Петросян</i> О знакопеременных и ограниченных решениях одного класса нелинейных двумерных интегральных уравнений типа свертки	313
<i>Г. Г. Казарян, А. Н. Карапетянц, В. Н. Маргарян, Г. А. Мкртчян, А. Г. Сергеев</i> Новые классы функциональных пространств и сингулярные операторы	329
<i>М. А. Степанова</i> О CR-многообразиях бесконечного типа по Блуму — Грэму	349
<i>А. В. Цветкова, А. И. Шафаревич</i> Волновое уравнение с симметричной скоростью на гибридном многообразии, образованном приклежкой луча к трехмерной сфере	369
<i>А. Ю. Пирковский</i> Письмо в редакцию	393



Correspondence between *Trudy Moskov. Mat. Obshch.* and *Trans. Moscow Math. Soc.*

Russian	English	Russian	English
<b>12</b> (1963)	<b>1963</b> (1965)	<b>48</b> (1985)	<b>1986</b>
<b>13</b> (1965)	<b>1965</b> (1967)	<b>49</b> (1986)	<b>1987</b>
<b>14</b> (1965)	<b>1965</b> (1967)	<b>50</b> (1987)	<b>1988</b>
<b>15</b> (1966)	<b>1966</b> (1967)	<b>51</b> (1988)	<b>1989</b>
<b>16</b> (1967)	<b>1967</b> (1968)	<b>52</b> (1989)	<b>1990</b>
<b>17</b> (1967)	<b>17</b> (1969)	<b>53</b> (1990)	<b>1991</b>
<b>18</b> (1968)	<b>18</b> (1969)	<b>54</b> (1992)	<b>1993</b>
<b>19</b> (1968)	<b>19</b> (1969)	<b>55</b> (1994)	<b>1994</b>
<b>20</b> (1968)	<b>20</b> (1971)	<b>56</b> (1995)	<b>1995</b>
<b>21</b> (1970)	<b>21</b> (1971)	<b>57</b> (1996)	<b>1996</b>
<b>22</b> (1970)	<b>22</b> (1972)	<b>58</b> (1997)	<b>1997</b>
<b>23</b> (1970)	<b>23</b> (1972)	<b>59</b> (1998)	<b>1998</b>
<b>24</b> (1971)	<b>24</b> (1974)	<b>60</b> (1999)	<b>1999</b>
<b>25</b> (1971)	<b>25</b> (1973)	<b>61</b> (2000)	<b>2000</b>
<b>26</b> (1972)	<b>26</b> (1974)	<b>62</b> (2001)	<b>2001</b>
<b>27</b> (1972)	<b>27</b> (1975)	<b>63</b> (2002)	<b>2002</b>
<b>28</b> (1973)	<b>28</b> (1975)	<b>64</b> (2003)	<b>2003</b>
<b>29</b> (1973)	<b>29</b> (1976)	<b>65</b> (2004)	<b>2004</b>
<b>30</b> (1974)	<b>30</b> (1976)	<b>66</b> (2005)	<b>2005</b>
<b>31</b> (1974)	<b>31</b> (1976)	<b>67</b> (2006)	<b>2006</b>
<b>32</b> (1975)	<b>32</b> (1977)	<b>68</b> (2007)	<b>2007</b>
<b>33</b> (1976)	<b>1978</b> , issue 1	<b>69</b> (2008)	<b>2008</b>
<b>34</b> (1977)	<b>1978</b> , issue 2	<b>70</b> (2009)	<b>2009</b>
<b>35</b> (1976)	<b>1979</b> , issue 1	<b>71</b> (2010)	<b>2010</b>
<b>36</b> (1978)	<b>1979</b> , issue 2	<b>72</b> (2011)	<b>2011</b>
<b>37</b> (1978)	<b>1980</b> , issue 1	<b>73</b> (2012)	<b>2012</b>
<b>38</b> (1979)	<b>1980</b> , issue 2	<b>74</b> (2013)	<b>2013</b>
<b>39</b> (1979)	<b>1981</b> , issue 1	<b>75</b> (2014)	<b>2014</b>
<b>40</b> (1979)	<b>1981</b> , issue 2	<b>76</b> (2015)	<b>2015</b>
<b>41</b> (1980)	<b>1982</b> , issue 1	<b>77</b> (2016)	<b>2016</b>
<b>42</b> (1981)	<b>1982</b> , issue 2	<b>78</b> (2017)	<b>2017</b>
<b>43</b> (1981)	<b>1983</b> , issue 1	<b>79</b> (2018)	<b>2018</b>
<b>44</b> (1982)	<b>1983</b> , issue 2	<b>80</b> (2019)	<b>2019</b>
<b>45</b> (1982)	<b>1984</b> , issue 1	<b>81</b> (2020)	<b>2020</b>
<b>46</b> (1983)	<b>1984</b> , issue 2	<b>82</b> (2021)	<b>2021</b>
<b>47</b> (1984)	<b>1985</b>		



