AMERICAN MATHEMATICAL SOCIETY

Notices

Edited by J. H. CURTISS

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Contents

Please send in abstracts of papers to be presented in person well in advance of the deadline.

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MEETINGS

CALENDAR OF MEETINGS

NOTE: This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the NOTICES was sent to press. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

Meeting No.	Date	Place	Deadline for Abstracts*
549	October 25, 1958	Princeton, New Jersey	Sept. 11
550	November 21-22, 1958	Pomona, California	Oct. 8
551	November 28-29, 1958	Durham, North Carolina	Oct. 8
552	November 28-29, 1958	Evanston, Illinois	Oct. 8
553	January 20-22, 1959 (65th Annual Meeting)	Philadelphia, Pennsylvania	Dec. 5
554	February 28, 1959	New York, New York	Jan. 15
	April, 1959	Monterey, California	
	April, 1959	New York, New York	
	Summer Meeting, 1959	Salt Lake City, Utah	
	November, 1959	Detroit, Michigan	

*The abstracts of papers to be presented at the meetings must be received in the Headquarters Offices of the Society in Providence, R. I., on or before these deadlines. The deadlines also apply to news items.

The NOTICES of the American Mathematical Society is published seven times a year, in February, April, June, August, October, November, and December. Price per annual volume is $7.00. Price per copy, $2.00. Special price for copies sold at registration desks of meetings of the Society, $1.00 per copy. Subscriptions, orders for back numbers (none available before 1958), and inquiries should be addressed to the American Mathematical Society, Ann Arbor, Michigan, or 190 Hope Street, Providence 6, R. I.

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SIXTY-THIRD SUMMER MEETING

Massachusetts Institute of Technology

Cambridge, Massachusetts

August 26-29, 1958

PROGRAM

The sixty-third summer meeting of the American Mathematical Society will be held from Tuesday, August 26, through Friday, August 29, 1958, at the Massachusetts Institute of Technology, Cambridge, Massachusetts, in conjunction with meetings of other organizations as follows: Mathematical Association of America from Monday, August 25, through Thursday, August 28; Institute of Mathematical Statistics from Monday, August 25, through Thursday, August 28; Society for Industrial and Applied Mathematics from Monday, August 25, through Wednesday, August 27; the Econometric Society from Tuesday, August 26, through Thursday, August 28; and the Pi Mu Epsilon Fraternity on Wednesday, August 27. Some mention of the programs of these other meetings will be found in the Time Table for the Society meeting, and elsewhere in these NOTICES under "Activities of Other Associations", but for details one should consult he programs of the individual organizations. All times listed in this program are Eastern Daylight Saving Time.

By invitation of the Committee to Select Hour Speakers for Annual and Summer Meetings, five hour addresses will be given. Professor B. J. Pettis of the University of North Carolina will address the Society at 2:00 P.M. on Tuesday on "The existence and extension of measures". Professor Eldon Dyer of the University of Chicago will deliver an address entitled "The effect of mappings on dimension" at 9:00 A.M. on Wednesday. On Thursday there will be two invited addresses: Professor Walter Rudin of the University of Rochester will speak at 9:00 A.M. on "Measure algebras on abelian groups", and at 10:30 A.M. Professor O. M. Nikodym of Kenyon College will deliver an address entitled "Mathematically precise setting of the genuine Dirac's δ-function and proof of its basic properties". Professor Jürgen Moser of the Massachusetts Institute of Technology will address the Society at 9:00 A.M. on Friday on "The recent development in the theory of Hamiltonian systems". All invited addresses will be given in Kresge Auditorium.

Since this meeting will be almost concurrent with the International Congress of Mathematicians to be held in Edinburgh, Scotland, no Colloquium lectures are scheduled.

There will be a business meeting of the Society in Kresge Auditorium at 10:15 A.M. on Wednesday. The Council has recommended that at this meeting certain changes in the By-laws be considered. One of these is the repeal of Article IX, Section 10, which has to do with the allocation of portions of the dues to journals. This is brought about by the necessity of frequent revisions of these allocations to meet postal regulations. The others are amendments to Article XI, Sections 1 and 2, listing the NOTICES as one of the official publications of the Society, and placing its editorial management in the hands of the Executive Director.

The Council of the Society will meet at 5:00 P.M. on Tuesday in the Hayden Library Lounge, Room 14E-310, reconvening after dinner in the Faculty Club.

There will be a meeting of the Conference Board of the Mathematical Sciences from 5:30 to 7:30 P.M. on Wednesday in the Hayden Library Lounge, Room 14E-310.

Sessions for contributed papers will be held at 3:30 P.M. on Tuesday, at 1:00 P.M. on Wednesday, at 2:00 P.M. on Thursday, and at 10:30 A.M. on Friday. Abstracts of the papers to be presented in these sessions appear on pages 467-510 of these NOTICES. There are cross references to the abstracts in the program. Thus, for example, the title of paper (1) in the program is followed by (548-15), indicating that the abstract can be found under the designation 548-15 among the published abstracts. Sessions for late papers (received on or before August 4) will be held at 1:00 P.M. on Friday. Program for the sessions of late papers will be available in mimeographed form at the registration desk.

The Employment Register will be maintained near the registration desk. The Providence office of the Society is now handling all of the details for the Employment Register. Listings for this meeting should be sent to 190 Hope Street, Providence 6, Rhode Island, or brought to the meeting.

REGISTRATION

Registration headquarters will be in the lobby of Kresge Auditorium, and will be open Sunday afternoon and evening, August 24, from 2:00 to 10:00 P.M. and from 8:30 A.M. to 5:00 P.M., Monday through Friday. All those attending the meetings are requested to register as soon as possible. Those arriving after the registration desk is closed, and having reservations, may go directly to Baker House. A directory of all persons registered, and an information desk will be available in the registration area. The text book exhibits will be located in Baker House.

There will be a registration fee of $2.00 for each member of any participating organization and $1.00 for each accompanying adult. Tickets for the beer party will cost $1.00 per person.

ROOMS AND MEALS

Dormitory accommodations are available. Walker Memorial, near the dormitories and meeting places, will be open especially for these meetings for a light breakfast a la carte, from 7:30 to 9:30 A.M. and a set $ .99 lunch, from 11:30 to 1:30, from Monday breakfast through Friday noon. Tickets for lunch may be obtained at the registration desk and at the cafeteria. Supper may be had at one of several nearby restaurants or at the M. I. T. Graduate House Dining Room, from 5:00 to 7:00 P.M. Saturday breakfast may also be obtained at the Graduate House from 9:00 to 11:00 A.M. Dormitory rooms may be occupied from Sunday evening, August 24, until the following Saturday. A late-registration form will be found on page 517. Members who did not return the form sent with the June NOTICES are requested to return the late-registration form to Professor Philip Franklin, 2-163, M. I. T., Cambridge 39, Massachusetts, as soon as possible, regardless of whether or not they wish dormitory reservations.

Many double rooms and a limited number of single rooms in the M. I. T. dormitory system are available.

For single men or women, the housing rate will be $3.50 per person for each twenty-four hour unit beginning at 6:00 P.M.; for married couples $2.50 per person; and for children 16 and under $2.00 per person. [For children under 12 who wish to be in the same room as their parents, M. I. T. will make available cots at a rate of $1.00 per person.]

Some of the Boston and Cambridge hotels which are conveniently located with respect to M. I. T. are listed below. A few of the accommodations for motorists within commuting distance are also listed. Since the number of rooms in these motels is relatively small, it is suggested that those desiring such accommodations make their reservations well ahead of time.

HOTELS

Boston		Rates* per person
Kenmore Hotel KEnmore 6-2770	490 Commonwealth Ave. at the Kenmore Square	$6.00 up
Ritz-Carlton Hotel KEnmore 6-5700	Arlington and Newbury Sts.	9.00 up
Sheraton Plaza Hotel COpley 7-5300	138 St. James Ave. in Copley Square	6.85 up
Sherry Biltmore Hotel COpley 7-7700	150 Massachusetts Ave.	5.00 up
Somerset Hotel KEnmore 6-2700	400 Commonwealth Ave.	7.00 up

Statler Hotel HAncock 6-2000	Arlington Street at Park Square	$ 6.50 up

Cambridge (near Harvard Square)

Ambassador Hotel UNiversity 4-6200	1737 Cambridge Street	6.00 up
Commander Hotel KIrkland 7-4800	16 Garden Street	6.00 up
Continental Hotel KIrkland 7-6100	29 Garden Street	5.00 up

MOTELS AND MOTOR COURT HOTELS

Boston	Rates*	Single	Double
Hampton Court Hotel BEacon 2-7500	1223 Beacon Street Brookline, Mass. Kitchen-ettes, family rates, parking.	$9.50 up	$14.00 up
1200 Beacon Street	1200 Beacon Street Brookline, Mass. No charge for children under 14 in the same room.	10.50 up	15.00 up
Terrace Motel Apts. LOngwood 6-6260	1650 Commonwealth Ave. Boston 35, Mass. Airconditioned	7.00 up	10.00 up
Charterhouse Motel LAsell 7-9000	On State Rte. 9 at Chestnut Hill	8.80 up Family -- $16.60 No Kitchenettes	13.30 up
Bedford			
Bedford Motel CRestview 4-6300	South Edge of Bedford on State Rte. 4	7.00 up	9.00 up
Concord			
The Concordian Motel COlonial 3-7765	4 miles west of Concord on State Rte. 2 P. O. Acton	7.00 up	10.00 up
Colonial Inn EMerson 9-4600	Monument Square in Con-cord. On State Rte. 2A	7.00 up	10.00 up
Framingham Center			
Framingham Motor Inn TRinity 2-1206	1 1/2 miles west of Framingham on State Rte.9	8.50 up	
Lexington			
Lexington Inn VOlunteer 2-8700	Exit 38 on State Rte. 128, Junction State Rtes. 2A and 128	8.00 up	12.00 up

*Rates are subject to change.

ENTERTAINMENT AND RECREATION

Lounges - Rehearsal Room A on the lower floor of the Kresge Auditorium will be open daily including Sunday, from 8:00 A.M. to 11:00 P.M.; the Mathematics Common Room (Room 2-290) will be open from 9:00 A.M. to 5:00 P.M. Monday through Friday; the lounge in Baker House will be available at all times.

The Alumni Pool will be open for swimming Monday through Friday; on Wednesday from noon to 6:30 P.M. and for women only from 6:30 to 8:00 P.M.; and otherwise from noon to 8:00 P.M.; suits, towels and lockers will be provided. Women should provide their own caps. Those interested in sailing in the M. I. T. dinghies on the Charles River must first obtain from the swimming pool a certificate of swimming ability. The duPont tennis courts on Briggs Field are open from 8:00 A.M. to dusk and may be reserved by telephone or at the courts for not more than one hour a day. Badminton and volleyball courts in the Armory are open for play from 9:00 A.M. to 6:00 P.M.; lockers and the equipment required for softball, badminton and volleyball will be supplied on request. Registrants will be issued an Athletic Privilege Card which will provide admission without charge to any of these recreational facilities. Telephone: Tennis Courts -- Ext. 2912; Briggs Field -- Ext. 2913.

The annual SIAM beer party, open by ticket to all registrants, will be held in the Walker Memorial Building on the M. I. T. Campus on Tuesday evening at 9:00 P.M. Admission is $1.00 per person; tickets available at the registration desk in advance.

The M. I. T. Administration cordially invites all registrants to a tea and reception to be held Wednesday afternoon, 4:00 to 5:30 P.M., in the President's House, 111 Memorial Drive.

A list of baby sitters will be available at the registration desk.

Information with respect to special events and points of interest on the campus and in and about Greater Boston, and a list of restaurants, will be available at the registration desk.

TRANSPORTATION

M. I. T. is located on the Charles River opposite the Back Bay region of Boston. The main building is at 77 Massachusetts Avenue, Cambridge. Registration headquarters will be in the lobby of Kresge Auditorium, distinguished by its spherical triangular dome, and located directly across Massachusetts Avenue from the main building.

Boston and Cambridge are on U. S. Rtes. 1, 3, and 20; and Mass. State Rtes. 2, 9, 16, 28, 30, and 38. The East-West Mass. Turnpike ends at State Rtes. 128 and 30 on the outskirts of Boston. Follow Rte. 30 Eastbound along Commonwealth Avenue to the intersection with U. S. Rte. 1 at the Boston University Bridge; then U. S. Rte. 1 Northbound across the river and to M. I. T. at the next bridge which is on Massachusetts Avenue.

Boston is served by Greyhound, Vermont Transit, and Trailways Buses. The terminals are located in Park Square near the Arlington Street Subway entrance. From the Arlington Street subway station take Beacon Street or Commonwealth Avenue car to Massachusetts Station, then Harvard Square bus to M. I. T. Taxi fare to M. I. T. from Park Square is approximately $1.25.

Rail passengers arriving at South Station or North Station may take the subway to Park Street, thence southbound to Massachusetts Station, thence Harvard Square Bus to M. I. T. From Back Bay Railroad Station, taxi fare to M. I. T. is approximately $1.

Airline passengers arriving at the Logan Airport in Boston may use public bus and subway transportation (20 cents fare) as follows: MTA bus from airlines terminal to Airport subway station, subway to Scollay Square, then Beacon Street or Commonwealth Avenue car to Massachusetts Station, transfer there to Harvard Square Bus and get off at M. I. T.

MAIL AND TELEGRAMS

Correspondence for those attending the meetings should be addressed in care of the American Mathematical Society, Kresge Auditorium, M. I. T., Cambridge 39, Massachusetts. Mail may be obtained from the mail desk in the registration area.

Committee on Arrangements:

J. H. Curtiss	Norman Levinson
Philip Franklin	Hartley Rogers, Jr.
H. M. Gehman	R. D. Schafer
F. B. Hildebrand	G. B. Thomas, Jr., Chairman

TIME TABLE
(Eastern Daylight Saving Time)

SUNDAY, AUGUST 24

2:00 P.M. - 10:00 P.M.	Registration, Kresge Auditorium.
7:30 P.M.	MAA Executive and Finance Committees, Hayden Library, Room 14N-317.

MONDAY, AUGUST 25

8:30 A.M. - 5:00 P.M.	Registration, Kresge Auditorium.
9:00 A.M.	MAA Session, Kresge Auditorium. Hedrick Lecture I, A. S. Householder; T. C. Koopmans; Robert Solow; J. L. Snell.
9:00 A. M.	IMS Session, Room 10-250. A. W. Wortham; M. B. Wilk; Cuthbert Daniel.
11:15 A.M.	IMS Session, Room 10-250. Wald Lecture I, J. W. Tukey.

2:00 P.M. MAA Session, Kresge Auditorium. Hedrick Lecture II, A. S. Householder; H. J. Greenberg; S. P. Diliberto; J. B. Diaz.

2:00 P.M. IMS Session, Room 10-250. J. L. Hodges, Jr.; Milton Sobel; Charles Stein.

3:30 P.M. SIAM Session for contributed papers, Room 3-133.

4:00 P.M. IMS Session, Room 10-250. I. R. Savage; H. O. Hartley; L. A. Goodman.

7:30 P.M. SIAM Session, Kresge Auditorium. Invited Lecture, Harlan Mills.

7:30 P.M. MAA Board of Governors, Hayden Library Lounge, Room 14E-310.

8:00 P.M. IMS 1958 Council Meeting, Green Room, Kresge Auditorium.

Employment Register.

TUESDAY, AUGUST 26

8:30 A.M. 5:00 P.M. Registration, Kresge Auditorium.

9:00 A.M. MAA Session, Kresge Auditorium. Hedrick Lecture III, A. S. Householder; Business Meeting; Alfred Schild; K. O. Freidrichs; Richard Duffin.

9:00 A.M. IMS Sessions for contributed papers, Rooms 1-190, 1-390.

11:15 A.M. IMS Session, Room 10-250. Wald Lecture II, J. W. Tukey.

12:30 P.M. SIAM Council Meeting.

2:00 P.M. Invited Address, Kresge Auditorium, B. J. Pettis.

2:00 P.M. IMS Session, Room 10-250. C. L. Mallows; Michel Loève; Patrick Billingsley.

3:30 P.M. Sessions for contributed papers.
Analysis, Room 2-190.
Applied Mathematics, Room 6-120.
Algebra, Logic and Foundations, Room 4-270.

3:30 P.M. SIAM Session for contributed papers, Room 3-133.

4:00 P.M. IMS Session, Room 10-250. Special Invited Paper, Milton Sobel.

5:00 P.M. Council, Hayden Library Lounge, Room 14E-310.

7:30 P.M. SIAM Session, Kresge Auditorium. Invited Lecture, S. H. Crandall.

7:30 P.M. MAA Section Officers, Hayden Library Lounge, Room 14E-310.

9:00 P.M. SIAM Beer Party, Walker Memorial Building.

Employment Register.

WEDNESDAY, AUGUST 27

8:30 A.M. - 5:00 P.M. Registration, Kresge Auditorium.
7:45 A.M. Pi Mu Epsilon Breakfast. Graduate House, West Dining Room.
9:00 A.M. Invited Address, Kresge Auditorium, Eldon Dyer.
9:00 A.M. IMS Session, Room 10-250. T. W. Anderson; C. R. Blyth; M. H. DeGroot.
10:15 A.M. Business Meeting, Kresge Auditorium.
11:15 A.M. IMS Session, Room 10-250. Wald Lecture III, J. W. Tukey.
1:00 P.M. Sessions for contributed papers.
Analysis, Room 2-190.
Applied Mathematics, Room 6-120.
Algebra, Room 4-270.
Topology, Room 4-370.
2:00 P.M. MAA Session, Kresge Auditorium. G. E. Forsythe; John Todd; C. E. Shannon.
2:00 P.M. IMS Sessions, Rooms 1-190, 1-390. A. V. Balakrishnan; R. G. Laha; Ronald Pyke; F. J. Anscombe; E. M. L. Beale; C. L. Mallows; A. P. Dempster.
3:30 P.M. SIAM Session for contributed papers, Room 3-133.
4:00 P.M. Tea and Reception, President's House, 111 Memorial Drive.
4:00 P.M. IMS Session, Room 10-250. Wald Lecture IV, J. W. Tukey.
5:30 P.M. Conference Board of the Mathematical Sciences, Hayden Library Lounge, Room 14E-310.
5:30 P.M. IMS Business Meeting, Kresge Auditorium.
8:00 P.M. IMS 1959 Council, Hayden Library Lounge, Room 14E-310.

Employment Register

THURSDAY, AUGUST 28

8:30 A.M. - 5:00 P.M. Registration, Kresge Auditorium.
9:00 A.M. Invited Address, Kresge Auditorium, Walter Rudin.
9:00 A.M. IMS Sessions for contributed papers, Rooms 1-190, 1-390.
10:30 A.M. Invited Address, Kresge Auditorium, O. M. Nikodym.
11:15 A.M. IMS Session, Room 10-250. Special Invited Paper, E. J. Williams.
2:00 P.M. Sessions for contributed papers.
Analysis, Room 2-190.
Analysis, Room 6-120.
Algebra and Theory of Numbers, Room 4-270.
Geometry, Room 4-370.

2:00 P.M. MAA Session, Kresge Auditorium. G. B. Price; E. A. Cameron; H. M. Gehman; R. D. James; J. R. Mayor; A. E. Meder, Jr.; Rothwell Stephens.

2:00 P.M. IMS Session, Room 10-250. Special Invited Paper, W. E. Deming.

3:15 P.M. IMS Session, Room 10-250. H. L. Jones; Allan Birnbaum; S. W. Roberts.

Employment Register.

FRIDAY, AUGUST 29

8:30 A.M. - 5:00 P.M. Registration, Kresge Auditorium.

9:00 A.M. Invited Address, Kresge Auditorium, Jürgen Moser.

10:30 A.M. Sessions for contributed papers.
Analysis, Room 2-190.
Statistics and Probability, Room 6-120.
Topology, Room 4-370.

1:00 P.M. Sessions for late papers.

Employment Register.

PROGRAM OF THE SESSIONS

(Time limit for each contributed paper, 10 minutes)

TUESDAY, 2:00 P.M.

Invited Address, Kresge Auditorium

The existence and extension of measures (One hour)
Professor B. J. Pettis, University of North Carolina

TUESDAY, 3:30 P.M.

Session on Analysis, Building 2, Room 2-190

(1) New continued fraction expansions for the ratios of Heine functions
Professor Evelyn Frank, University of Illinois (548-15)

(2) On the number of distinct zeros of polynomials
Professor M. S. Klamkin, Avco Research and Development Division and Polytechnic Institute of Brooklyn, and Dr. D. J. Newman, Avco Research and Development Division and Massachusetts Institute of Technology (548-25)

(3) An asymptotic relation between Tschebycheff coefficients and the best polynomial approximation
Dr. E. K. Blum, Ramo-Wooldridge Corporation, and Professor P. C. Curtis, Jr., University of California, Los Angeles and Ramo-Wooldridge Corporation (548-88)

(4) n-Parameter families of functions and best approximation
Professor P. C. Curtis, Jr., University of California, Los Angeles (548-70)

(5) Sets of sequences on which certain generalized limits coincide
Mr. Lawrence Shepp, Avco Research and Development Division and Princeton University (548-54)
(Introduced by Professor M. S. Klamkin)

(6) An approach to certain reflection principles
Dr. E. P. Miles, Jr., University of Maryland and Alabama Polytechnic Institute (548-75)

(7) Cesàro partial sums of harmonic series expansions
Professor M. S. Robertson, Rutgers University (548-77)

Session on Applied Mathematics, Building 6, Room 6-120

(8) Application of generalized axially symmetric potential theory to Stokes flow problems
Dr. W. H. Pell, National Bureau of Standards, Washington, and Dr. L. E. Payne, University of Maryland and National Bureau of Standards, Washington (548-5)

(9) On electrodynamics in material media
Professor Domina E. Spencer, University of Connecticut (548-7)

(10) Heat flow in a composite solid
Professor W. P. Reid, Michigan State University (548-

(11) Concerning the time-independent multigroup diffusion equations
Dr. G. J. Habetler and Dr. M. A. Martino, General Electric Company, Schenectady, New York (548-62)

(12) Analysis of the nonlinear Stefan problem
Dr. W. L. Miranker and Mr. H. L. Frisch, Bell Telephone Laboratories, Murray Hill, New Jersey (548-96)

(13) Asymptotic radiance distributions in eventually separable media
Dr. R. W. Preisendorfer, University of California, La Jolla (548-98)

(14) Absorption of sound waves in a uniform stream
Mr. R. P. Kanwal, University of Wisconsin (548-18)

Session on Algebra, Logic and Foundations, Building 4, Room 4-270

(15) Translation lattices. I
Professor R. S. Pierce, University of Washington (548-11)

(16) Free products of alpha-distributive Boolean algebras
Professor R. S. Pierce, University of Washington, and Miss Dorothy J. Christensen, Wellesley College (548-46)

(17) A theorem on independence of universal algebras
Dr. E. S. O'Keefe, New York University (548-48)

(18) Abstract linear dependence relations
Dr. G. B. Preston and Mr. M. N. Bleicher, Tulane University (548-99)

(19) Combinatory formulation of standard theories
Mr. D. E. Schroer, University of Rochester (548-44)

(20) On certain aspects of relatable functions
Mr. L. W. Small, Yonkers, New York (548-55)

WEDNESDAY, 9:00 A.M.

Invited Address, Kresge Auditorium
The effect of mappings on dimension (One hour)
Professor Eldon Dyer, University of Chicago

WEDNESDAY, 10:15 A.M.

Business Meeting, Kresge Auditorium

WEDNESDAY, 1:00 P.M.

Session on Analysis, Building 2, Room 2-190

(21) Interpolation problems for functions of several complex variables
Professor George Springer, University of Kansas (548-10)

(22) Meromorphic maps into complex spaces
Dr. W. F. Stoll, Institute for Advanced Study and University of Tubingen (548-16)

(23) On the degree of convergence of sequences of extremal functions
Professor J. L. Walsh, Harvard University, and Professor Annette Sinclair, Purdue University (548-50)

(24) On meromorphic functions of order less than one
Professor Albert Edrei, Syracuse University, and Professor W. H. J. Fuchs, Cornell University (548-71)

(25) A property of entire functions with negative zeros
Professor Albert Edrei, Syracuse University, and Professor W. H. J. Fuchs, Cornell University (548-72)

(26) On a theorem of A. Huber
Professor Wilfred Kaplan, University of Michigan (548-74)

(27) On the bound of convexity of a class of star shaped mappings. Preliminary report
Professor Albert Schild, Temple University (548-79)

(28) Coefficients in certain asymptotic factorial expansions of the second kind
Dr. T. D. Riney, Bell Telephone Laboratories, Allentown, Pennsylvania (548-19)

Session on Applied Mathematics, Building 6, Room 6-120

(29) The theory of automata
Mr. Donald Bratton, Control Instrument Company, Incorporated, Brooklyn, New York (548-9)

(30) Solving linear programs in integers
Dr. R. E. Gomory, Princeton University (548-94)

(31) Programming with nonlinear constraints. Preliminary report
Dr. Philip Wolfe, RAND Corporation, Santa Monica, California (548-102)

(32) Solution of a multimove tactical game
Dr. L. D. Berkovitz and Dr. Melvin Dresher, RAND Corporation, Santa Monica, California (548-87)

(33) Simplified simplex method applied to the solution of simultaneous linear equations
Professor Valdemars Punga, Rensselaer Polytechnic Institute (548-39)

(34) Singular perturbations of polynomials in several variables. Preliminary report
Miss Czerna Flanagan and Dr. J. E. Maxfield, Naval Ordnance Test Station, China Lake, California (548-93)

(35) On the reversion of power series
Mr. P. L. Chessin, University of Maryland (548-35)

(36) On the convergence of sequences of singular integrals of Cauchy type. Part I, general
Mr. C. E. Stewart, Illinois Institute of Technology (548-83)

Session on Algebra, Building 4, Room 4-270

(37) On nilalgebras and linear varieties of nilpotent matrices. III
Professor Murray Gerstenhaber, University of Pennsylvania and Institute for Advanced Study (548-31)

(38) Cartan invariants of UMFR algebras
Professor D. W. Wall, University of North Carolina (548-67)

(39) On Abelian Galois groups
Professor C. C. Faith, Pennsylvania State University (548-73)

(40) Note on a theorem of Fuglede and Putnam
Professor S. K. Berberian, State University of Iowa (548-29)

(41) Various averaging operations onto subalgebras
Dr. Chandler Davis, Institute for Advanced Study (548-22)

(42) Prime ideals in rings of continuous functions
Professor Leonard Gillman and Professor Meyer Jerison, Purdue University (548-52)

(43) Characters of Cartesian products of rings
Professor S. L. Warner, Duke University (548-106)

(44) A derivative for Hausdorff-analytic functions
Professor W. O. Portmann, Case Institute of Technology (548-2)

Session on Topology, Building 4, Room 4-370

(45) A T-like plane continuum which is not a quasi-complex
Mr. J. E. Keisler, University of Michigan (548-38)

(46) Set-valued functions with and without fixed points. Preliminary report
Dr. R. L. Dunn, Ramo-Wooldridge Corporation, Los Angeles, California (548-20)

(47) Closed sets in Stone-Čech compactifications
Professor Leonard Gillman and Professor Meyer Jerison, Purdue University (548-51)

(48) FH spaces and intersection of FK spaces
Professor Albert Wilansky, Lehigh University, and Dr. Karl Zeller, Tübingen University (548-28)

(49) On the structure of threads. Preliminary report
Mr. C. R. Storey, Tulane University (548-100)

(50) Some semigroups on the two-cell
Miss Anne Lester, Tulane University (548-43)

(51) Widely connected algebras
Professor P. M. Swingle, University of Miami (548-84)

(52) A note on topological semirings
Mr. John Selden, Tulane University (548-80)

THURSDAY, 9:00 A.M.

Invited Address, Kresge Auditorium

Measure algebras on abelian groups (One hour)
Professor Walter Rudin, University of Rochester

THURSDAY, 10:30 A.M.

Invited Address, Kresge Auditorium

Mathematically precise setting of the genuine Dirac's δ-function and proof of its basic properties (One hour)
Professor O. M. Nikodym, Kenyon College

THURSDAY, 2:00 P.M.

Session on Analysis, Building 2, Room 2-190

(53) Tensor products of Banach algebras
Professor B. R. Gelbaum, University of Minnesota (548-13)

(54) On Banach algebras of infinite matrices
Mr. E. K. Dorff, Lehigh University (548-45)

(55) On the automorphism group of the bounded algebra of a real Hilbert space
Dr. Leonard Gross, Yale University (548-61)

(56) On the equivalence of weak and norm convergence in the space of functions of bounded variation
Professor Pasquale Porcelli, University of Wisconsin (548-34)
(57) A class of singular non-self-adjoint differential operators
Professor R. R. D. Kemp, Queen's University (548-95)
(58) Operators commuting with translation by one
Mr. D. C. McGarvey, RAND Corporation, Santa Monica, California (548-104)
(59) Semigroups and unbounded difference operators
Professor S. E. Puckette, University of the South (548-47)

Session on Analysis, Building 6, Room 6-120
(60) On discontinuous functions
Professor M. E. Mahowald, Xavier University (548-27)
(61) On the topological foundation of the theory of distributions
Professor I. S. Gál, Cornell University and Yale University (548-1)
(62) Locally o-convex spaces. III
Professor J. E. Kist, Wayne State University (548-3)
(63) Reversal of Peano continua by analytic maps
Professor G. S. Young, University of Michigan (548-41)
(64) Nonoscillation and disconjugacy in the complex plane. Preliminary report
Professor W. J. Coles, Professor J. H. Barrett, and Professor D. V. V. Wend, University of Utah (548-90)
(65) Periodic solutions of a perturbed autonomous system. II
Professor W. S. Loud, University of Minnesota (548-64)
(66) On the fundamental solution of a parabolic system of equations. Preliminary report
Dr. D. G. Aronson, University of Minnesota (548-68)
(67) Existence and convergence of formal solutions of certain systems of differential equations. Preliminary report
Dr. M. I. Aissen, Johns Hopkins University (548-86)

Session on Algebra and Theory of Numbers, Building 4, Room 4-270
(68) Blocks with normal defect group
Professor W. F. Reynolds, Tufts University (548-33)
(69) Modules over finite groups and cohomology
Dr. D. S. Rim, Columbia University (548-36)
(70) Groups which are automorphism groups of a simply ordered set. Preliminary report
Professor C. C. Chang, University of Southern California (548-59)
(71) Conjugate invariants. Preliminary report
Professor D. A. Norton, University of California, Davis (548-97)

(72) Existence theorems on a multiplicative system. II
Dr. Naoki Kimura, Tulane University and Tokyo Institute of Technology (548-63)

(73) On a problem of Mahler's
Professor A. C. Woods, Tulane University (548-107)
(Introduced by Professor G. O. Sabidussi)

(74) Further identities and congruences for the coefficients of modular forms
Dr. Morris Newman, National Bureau of Standards, Washington (548-21)

(75) Conclusive elementary proof of Fermat's last theorem for all odd $n \geqq 3$
Mr. Abraham Brind, Fairleigh Dickinson University (548-69)

Session on Geometry, Building 4, Room 4-370

(76) Multiples of divisors
Professor Ernst Snapper, Indiana University (548-24)

(77) Line element fields on the torus. Preliminary report
Dr. B. L. Reinhart, University of Michigan (548-23)

(78) Generalized exterior differentiation
Professor V. L. Shapiro, Rutgers University and Institute for Advanced Study (548-37)

(79) Limit sections and universal points of convex surfaces
Professor Z. A. Melzak, McGill University (548-42)

(80) Characterizations of Riemann n-spheres
Professor G. F. Feeman, Muhlenberg College, and Professor C. C. Hsiung, Lehigh University (548-26)

(81) A note on a surface of order seventeen
Professor W. R. Hutcherson, University of Florida (548-4)

(82) A pair of mutually independent dual theorems in projective geometry
Professor C. C. Buck, Army Ballistic Missile Agency, Redstone Arsenal, Alabama (548-89)

(83) Partitioning of areas by straight lines
Dr. D. J. Newman, Massachusetts Institute of Technology and Avco Research and Development Division (548-108)
(Introduced by Professor M. S. Klamkin)

FRIDAY, 9:00 A.M.

Invited Address, Kresge Auditorium
The recent development in the theory of Hamiltonian systems (One hour)
Professor Jürgen Moser, Massachusetts Institute of Technology

FRIDAY, 10:30 A.M.

Session on Analysis, Building 2, Room 2-190

(84) Extension of methods used in solving Riccati's nonlinear differential equation
Mr. Iwao Sugai, IBM Research Center, Poughkeepsie, New York (548-14)

(85) On an Nth order system of differential equations
Mr. E. J. Putzer, CONVAIR, San Diego, California (548-53)

(86) Existence of singular points for a well known class of nonhomogeneous differential equations
Professor Diran Sarafyan, University of Florida (548-78)

(87) An extended ordering principle for generalized solutions of a quasi-linear partial differential equation
Professor Avron Douglis, University of Maryland (548-92)

(88) Linear and nonlinear boundary problems
Professor M. H. Martin, University of Maryland (548-32)

(89) An approximate solution of an improper boundary value problem
Professor Jim Douglas, Jr., Rice Institute, and Professor T. M. Gallie, Jr., Duke University (548-60)

(90) Asymptotic solutions of the equation $y^{IV} + 2x^{-1}y^{III} + y = 0$
Professor H. S. Heaps, Nova Scotia Technical College (548-85)

Session on Statistics and Probability, Building 6, Room 6-120

(91) On triangle inequalities for statistical metric spaces. II
Professor Berthold Schweizer, University of California, Los Angeles, and Professor Abe Sklar, Illinois Institute of Technology (548-81)

(92) Neighborhoods in statistical metric spaces
Professor Berthold Schweizer, University of California, Los Angeles, and Professor Abe Sklar, Illinois Institute Technology (548-82)

(93) A moment analogue to Spitzer's theorem
Mr. J. S. White, Minneapolis-Honeywell Regulator Company, Minneapolis, Minnesota (548-40)

(94) Some results in Markov chain theory
Professor D. G. Austin, University of Miami (548-57)

(95) Note on the convergence of sequences of stochastic processes
Dr. E. G. Kimme, Bell Telephone Laboratories, Murray Hill, New Jersey (548-30)

(96) A generalization of the Glivenko-Cantelli theorem
Professor H. G. Tucker, University of California, Riverside (548-17)

(97) The general queue with one server
Dr. V. E. Beneš, Bell Telephone Laboratories, Murray Hill, New Jersey (548-58)

(98) Abstract martingale convergence theorems
Dr. F. S. Scalora, University of Illinois and IBM Corporation, New York (548-66)
(Introduced by Professor Emanuel Parzen)

(99) A note on order statistics and stochastic independence
Professor G. S. Rogers, University of Arizona (548-49)

Session on Topology, Building 4, Room 4-370

(100) On Global cluster sets of meromorphic functions
Professor P. T. Church, Syracuse University (548-12)

(101) Fixed point theorems for connectivity maps
Mr. J. R. Stallings, Princeton University (548-56)

(102) Products of fixed point spaces
Mr. E. H. Connell, Lockheed Aircraft Corporation, Palo Alto, California, and Stanford University (548-91)

(103) Orbits in fixed sets
Professor W. L. Strother, Lockheed Aircraft Corporation and University of Miami (548-101)

(104) Stationary points for finite transformation groups
Dr. J. J. Greever, University of Virginia (548-6)

(105) p-adic transformation groups
Professor C. T. Yang, University of Pennsylvania (548-103)

(106) On a compact Lie group action on a sphere
Professor P. S. Mostert, Tulane University (548-76)

(107) Actions of the groups S0(3) and Sp(1) on the four-sphere
Mr. R. W. Richardson, Jr., University of Michigan (548-105)

FRIDAY, 1:00 P.M.

Sessions for Late Papers

SUPPLEMENTARY PROGRAM
(To be presented by title)

(108) On the solution of an implicit equation
Professor Smbat Abian and Professor A. B. Brown, Queens College

(109) On automorphic-inverse properties in loops
Dr. Rafael Artzy, Israel Institute of Technology and University of Wisconsin

(110) On higher order boundary value problem for hyperbolic partial differential equations
Professor A. K. Aziz, Georgetown University

(111) An identity for Jordan algebras. Preliminary report
Mr. J. W. Blattner and Professor L. J. Paige, University of California, Los Angeles

(112) Maps of spheres
Professor D. G. Bourgin, University of Illinois

(113) Some results on uniqueness and successive approximations
Dr. F. G. Brauer, University of British Columbia

(114) Typically-real functions
Professor R. K. Brown, Rutgers University

(115) A Riemann mapping theorem for quasi-conformal maps on certain domains in E^3
Professor P. T. Church, Syracuse University

(116) The cohomology groups of certain fibre spaces
Dr. W. H. Cockcroft, University of Southampton, Southampton, England

(117) The characteristic divisors of a polynomial function of a matrix
Professor Eckford Cohen, University of Tennessee

(118) A class of residue systems (mod r) and related arithmetical functions. II. Higher dimensional analogues
Professor Eckford Cohen, University of Tennessee

(119) The Brauer-Rademacher identity
Professor Eckford Cohen, University of Tennessee

(120) Convergence of Stieltjes type continued fractions
Professor D. F. Dawson, University of Missouri

(121) An extension of a theorem of Herstein
Professor W. E. Deskins, Michigan State University

(122) On idempotent stochastic matrices. Preliminary report
Mrs. Mary J. Etter, Tulane University

(123) Minimal coverings of pairs by triples
Professor M. K. Fort, Jr., University of Georgia, and Professor G. A. Hedlund, Yale University

(124) Distributions on measure spaces
Professor I. S. Gál, Cornell University and Yale University

(125) On the numerical solution of Dirichlet problems
Professor Donald Greenspan, Purdue University

(126) Hopf algebras of class 1
Dr. Edward Halpern, University of Michigan

(127) Unitary triangularization of a matrix
Dr. A. S. Householder, Oak Ridge National Laboratory

(128) On the quadratic detector. I
Professor Frank Kozin, Purdue University

(129) On the eigenvalues of Hermitian matrices
Professor E. O. A. Kreyszig, Ohio State University

(130) Characterization of the n-sphere
Dr. K. W. Kwun, University of Michigan and Tulane University

(131) A sufficient condition that an upper semicontinuous decomposition of S^3 yield a topological 3-sphere
Dr. K. W. Kwun, University of Michigan and Tulane University

(132) On modular forms of dimension 2
Professor Joseph Lehner, Michigan State University

(133) On the decompositions of a group algebra into a sum of a field and a nil ideal
Mr. G. O. Losey, University of Wisconsin

(134) Prime power representations of finite simple Lie groups
Professor J. E. McLaughlin, University of Michigan

(135) On the zeros of infrapolynomials for partly arbitrary point sets
Professor Morris Marden, University of Wisconsin

(136) Entire operators and functional equations
Professor Z. A. Melzak, McGill University

(137) On the inequality $\sum_{r=1}^{n}(x_r/(x_{r+1} + x_{r+2})) \geqq n/2$ and some others
Professor L. J. Mordell, St. Johns College, Cambridge University

(138) Differentiable isotopies on the two-sphere
Dr. J. R. Munkres, Princeton University

(139) Concerning boundary value problems
Dr. J. W. Neuberger, Illinois Institute of Technology

(140) An alternate characterization of the lower nil radical
Dr. E. C. Posner, University of Wisconsin

(141) A theory of incidence geometries
Professor Walter Prenowitz, Brooklyn College

(142) Dimension of closed subsets of homology manifolds
Mr. F. A. Raymond, University of Michigan

(143) On the class number of representations of an order
Professor Irving Reiner, University of Illinois

(144) Actions of the groups SO(3) and Sp(1) on the five-sphere
Mr. R. W. Richardson, Jr., University of Michigan

(145) Unknotted curves on 2-manifolds in 3-manifolds
Mr. J. R. Stallings, Princeton University

(146) Commutators in free products with amalgamation
Mr. J. R. Stallings, Princeton University

(147) On a class of quadratic transformations
Mr. Paul Stein, Los Alamos Scientific Laboratory
(Introduced by Dr. S. M. Ulam)

(148) On the convergence of sequences of singular integrals of Cauchy type. Part II, polygonal approximations
Mr. C. E. Stewart, Illinois Institute of Technology

(149) On the existence of singular integrals of Cauchy type
Mr. C. E. Stewart, Illinois Institute of Technology

(150) Lebesgue nonmeasurability and connected locally connected algebras
Professor P. M. Swingle, University of Miami

(151) Concerning local separability in locally peripherally separable spaces
Mr. L. B. Treybig, University of Texas

(152) On the law of the iterated logarithm
Professor Mary Weiss, De Paul University

(153) On a problem of Littlewood
Professor Mary Weiss, De Paul University

(154) On the law of the iterated logarithm for uniformly bounded orthonormal systems
Professor Mary Weiss, De Paul University

(155) On homogeneous algebras of functions
Professor A. B. Willcox, Amberst College

R. D. Schafer
Associate Secretary

Storrs, Connecticut
July 11, 1958

PRELIMINARY ANNOUNCEMENT OF MEETING

FIVE HUNDRED FORTY-NINTH MEETING

Princeton University
Princeton, New Jersey
October 25, 1958

The five hundred forty-ninth meeting of the American Mathematical Society will be held at Princeton University, Princeton, New Jersey, on Saturday, October 25, 1958.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, Professor Arnold Shapiro of Brandeis University will give an hour address entitled "Imbedding a complex in a cell" at 2:00 P.M. in Room 301 of the Palmer Physical Laboratory.

There will be sessions for contributed papers at 10:30 A.M. and 3:15 P.M.

All times given above are Eastern Daylight Saving Time (Princeton returning to Eastern Standard Time on Sunday, October 26).

The meeting will be held in Fine Hall and the Palmer Physical Laboratory, which are adjoining buildings on the Princeton University campus. Princeton can be reached by car using U. S. 1 or U. S. 206. One turns off U. S. 1 at the Penns Neck traffic circle on Washington Road. Palmer Laboratory is on the west side of Washington Road, near the corner of Prospect Street. A campus policeman will be on duty to direct people for parking. Railroad service to Princeton is via the Pennsylvania Railroad, stopping at Princeton Junction with a connecting shuttle train to Princeton. Bus service between Princeton and the Port of New York Authority, 41st Street and 8th Avenue, New York City is available via the Suburban Transit Corporation.

Further details of the meeting will appear in the next issue of the NOTICES. Abstracts of contributed papers should be sent to the American Mathematical Society, 190 Hope Street, Providence 6, Rhode Island, so as to arrive PRIOR TO THE DEADLINE, September 11. It is expected that abstracts of all papers to be presented in person will appear in the same issue of the NOTICES, but only abstracts which meet the specifications stated on the abstract blanks can be published.

R. D. Schafer
Associate Secretary

Storrs, Connecticut
July 11, 1958

ACTIVITIES OF OTHER ASSOCIATIONS

THE THIRTY-NINTH SUMMER MEETING OF THE MATHEMATICAL ASSOCIATION OF AMERICA will be held at Massachusetts Institute of Technology, Cambridge, Massachusetts, from Monday, August 25 to Thursday, August 28, 1958, in conjunction with summer meetings of the American Mathematical Society, the Econometric Society, the Institute of Mathematical Statistics, the Society for Industrial and Applied Mathematics, and the Pi Mu Epsilon Fraternity. Sessions of the Mathematical Association will be held on Monday at 9:00 a.m. and 2:00 p.m., on Tuesday at 9:00 a.m., and on Wednesday and Thursday at 2:00 p.m. All sessions will be held in Kresge Auditorium. The Board of Governors of the Association will meet on Monday evening at 7:30 in the Hayden Library Lounge, Room 14E-310. A meeting of officers of the Sections of the Association will be held on Tuesday evening at 7:30 in the Hayden Library Lounge, Room 14E-310.

MONDAY, 9:00 A.M.

First Session, Kresge Auditorium

The Earle Raymond Hedrick Lectures: Some Mathematical Problems Arising in Computations with Matrices, Lecture I
Dr. A. S. Householder, Oak Ridge National Laboratory

Decentralization of Optimizing Decisions Through a Price System
Professor T. C. Koopmans, Yale University

The Role of the Maximum in Economics
Professor Robert Solow, Massachusetts Institute of Technology

Markov Chains and Their Applications
Professor J. L. Snell, Dartmouth College

MONDAY, 2:00 P.M.

Second Session, Kresge Auditorium

The Earle Raymond Hedrick Lectures: Lecture II
Dr. A. S. Householder, Oak Ridge National Laboratory

Plasticity and the Mechanics of Structures
Professor H. J. Greenberg, Institute of Mathematical Sciences, New York University

Satellite Orbits
Professor S. P. Diliberto, University of California, Berkeley

On Mean Value Theorems in the Theory of Elasticity
Professor J. B. Diaz, University of Maryland

TUESDAY, 9:00 A.M.

Third Session, Kresge Auditorium

The Earle Raymond Hedrick Lectures: Lecture III

Dr. A. S. Householder, Oak Ridge National Laboratory

Business Meeting of the Association

The Clock Paradox in Relativity Theory

Professor Alfred Schild, University of Texas and Hughes Research Laboratory

Mathematical Problems of Boundary Layer Theory

Professor K. O. Friedrichs, Institute of Mathematical Sciences, New York University

Network Analysis of Thermal Conduction Problems

Professor Richard Duffin, Duke University and Office of Ordnance Research

WEDNESDAY, 2:00 P.M.

Fourth Session, Kresge Auditorium

The Role of Numerical Analysis in an Undergraduate Program

Professor G. E. Forsythe, Stanford University

The Introduction of Numerical Analysis

Professor John Todd, California Institute of Technology

The Leading Ideas of Information Theory

Professor C. E. Shannon, Massachusetts Institute of Technology

THURSDAY, 2:00 P.M.

Fifth Session, Kresge Auditorium

Report on the Washington Conference of the Mathematical Association of America

Panel: G. B. Price, Chairman; E. A. Cameron, H. M. Gehman, R. D. James, J. R. Mayor, A. E. Meder, Jr., Rothwell Stephens

THE ANNUAL MEETING OF THE INSTITUTE OF MATHEMATICAL STATISTICS will be held at the Massachusetts Institute of Technology, Cambridge, Massachusetts, from Monday, August 25 through Thursday, August 28. As of July 11 (the deadline for this issue of the NOTICES) the program stood as follows:

MONDAY, 9:00 A.M.

Invited Papers on Regression and Analysis of Variance

Variance Component Analysis in Models Where Effects are Time Variables

A. W. Wortham, Texas Instruments, Inc., Dallas. 30 min.

Confidence and Significance Procedures for Nonlinear Models
M. B. Wilk, Bell Telephone Laboratories, Murray Hill. 45 min.

Industrial Experience with 2^{p-q} Fractional Factorial Experiments
Cuthbert Daniel, New York City. 30 min.

MONDAY, 11:15 A.M.

Wald Lecture I.
The Mathematical Basis of Fiducial Inference
John W. Tukey, Princeton University.

MONDAY, 2:00 P.M.

Invited Papers on Estimation and Testing
Power of the Chi-square Test
J. L. Hodges, Jr., University of California, Berkeley. 30 min.

On Solutions of Dorfman's Mass-testing Problem
Milton Sobel, Bell Telephone Laboratories, Allentown. 20 min.

Estimation of a Covariance Matrix
Charles Stein, University of California, Berkeley. 30 min.

MONDAY, 4:00 P.M.

Invited Papers on Testing
Partial Orderings of Probabilities of Rank Orders, I.
Richard Savage, University of Minnesota. 30 min.

Maximum Likelihood Estimation for Incomplete Data
H. O. Hartley, Iowa State College. 45 min.

On Some Statistical Tests for Markov Chains
Leo A. Goodman, University of Chicago. 30 min.

MONDAY, 8:00 P.M.

1958 Council Meeting -- Green Room -- Kresge Auditorium

TUESDAY, 9:00 A.M.

Contributed Papers, I.

Contributed Papers, II.

TUESDAY, 11:15 A.M.

Wald Lecture II.
The Mathematical Basis of Fiducial Inference (continued)
John W. Tukey, Princeton University

TUESDAY, 2:00 P.M.

Invited Papers on Probability and Stochastic Processes, I.

A Moment-Problem with Restriction on Smoothness

C. L. Mallows, Princeton University. 20 min.

About the Central Limit Problem

Michel Loève, University of California, Berkeley. 45 min.

Hausdorff Dimension and Information Theory

Patrick Billingsley, Princeton University and University of Chicago. 30 min.

TUESDAY, 4:00 P.M.

Special Invited Paper

Multiple Decision Selection Procedures

Milton Sobel, Bell Telephone Laboratories, Allentown

WEDNESDAY, 9:00 A.M.

Invited Papers on Sequential Analysis

A Modification of Sequential Analysis to Reduce the Sample Size

T. W. Anderson, Center for Advanced Study in the Behavioral Sciences and Columbia University. 45 min.

Binomial Sequential Testing

Colin R. Blyth, Stanford University. 30 min.

Unbiased Sequential Estimation for Binomial Populations

Morris H. DeGroot, Carnegie Institute of Technology. 30 min.

WEDNESDAY 11:15 A.M.

Wald Lecture III

The Interpretation of Fiducial Inference

John W. Tukey, Princeton University

WEDNESDAY, 2:00 P.M.

Invited Papers on Probability and Stochastic Processes II

Semigroups of Operators and Stochastic Processes

A. V. Balakrishnan, University of California, Los Angeles. 30 min.

Independent Polynomials in Normal Variates

R. G. Laha, Catholic University of America. 45 min.

On Multi-event Renewal Processes

Ronald Pyke, Stanford University. 30 min.

Invited Papers on Random Balance

Introductory Remarks

Frank J. Anscombe, Princeton University

On the Analysis of Screening Experiments

E. M. L. Beale, Princeton University, and C. L. Mallows, Princeton University. 30 min.

Analysis Methods for Randomly Balanced Factorial Designs
A. P. Dempster, Bell Telephone Laboratories and Harvard University. 30 min.

Discussion. To be announced

WEDNESDAY, 4:00 P.M.

Wald Lecture IV

What Importance Should We Place on Fiducial Inference?
John W. Tukey, Princeton University

WEDNESDAY, 5:30 P.M.

Business Meeting. Kresge Auditorium

WEDNESDAY, 8:00 P.M.

1959 Council Meeting. Hayden Library Lounge, Room 14E - 310

THURSDAY, 9:00 A.M.

Contributed Papers III

Contributed Papers IV

THURSDAY, 11:15 A.M.

Special Invited Paper

Estimation Methods in Multivariate Analysis
Evan J. Williams, North Carolina State College

THURSDAY, 2:00 P.M.

Special Invited Paper

On a Formal Structure of Professional Practice in Sampling
W. Edwards Deming, New York University

THURSDAY, 3:15 P.M.

Invited Papers on Mixed Topics

The Number of Occupied Cells of a Particular Subclass (When Objects Are Assigned to Cells at Random)
Howard L. Jones, Illinois Bell Telephone Company, Chicago. 30 min.

Statistical Theory of Tests of an Ability
Allan Birmbaum, Columbia University. 45 min.

Properties of Some Control Chart Tests for Detecting Shifts in a Process Average
S. W. Roberts, Bell Telephone Laboratories, New York 45 min.

THE SOCIETY FOR INDUSTRIAL AND APPLIED MATHEMATICS will meet at the Massachusetts Institute of Technology from Monday, August 25 through Wednesday, August 27. The program is as follows:

MONDAY, 3:30 P.M.

Contributed Papers. Room 3-133

MONDAY, 7:30 P.M.

Invited Lecture (One hour), Kresge Auditorium
Mathematical Programming
Dr. Harlan Mills, Princeton University

TUESDAY, 12:30 P.M.

Open Council Meeting

TUESDAY, 3:30 P.M.

Contributed Papers, Room 3-133

TUESDAY, 7:30 P.M.

Invited Lecture (One hour), Kresge Auditorium
Boundary Conditions for Finite Difference Approximations to Differential Equations
Professor Stephen H. Crandall, Massachusetts Institute of Technology

TUESDAY, 9:00 P.M.

Beer Party, Walker Memorial Building

WEDNESDAY, 3:30 P.M.

Contributed Papers, Room 3-133

DELEGATION OF THE NATIONAL ACADEMY OF SCIENCES-NATIONAL RESEARCH COUNCIL TO THE INTERNATIONAL MATHEMATICAL UNION. The NAS-NRC has announced that it will be represented at the General Assembly of the International Mathematical Union to be held at St. Andrews, Scotland, August 1-13, and at the subsequent International Congress, by the following delegation: Marston Morse, Institute for Advanced Study, Princeton, New Jersey, Chairman; L. V. Ahlfors, Harvard University; A. A. Albert, University of Chicago; Richard Brauer, Harvard University; Richard Courant, New York University; J. L. Doob, University of Illinois; Nathan Jacobson, Yale University; Saunders MacLane, University of Chicago; Deane Montgomery, Institute for Advanced Study, Princeton, New Jersey; S. S. Wilks, Princeton University.

A QUARTERLY REVIEW TO BE PUBLISHED BY THE INTERNATIONAL ASSOCIATION FOR CYBERNETICS. It has been announc that as of May, 1958, the International Association for Cybernetics will be publishing a quarterly review called "Cybernetica" containing articles of a scientific nature on subjects relating to the various domains of cybernetics.

Through this publication the Association will be able to carry out the aims it has set itself and which are as follows:
1) To ensure a permanent and organized liaison between researchers whose work in various countries is related to different sectors of cybernetics.
2) To promote the development of this science and of its technical applications, as well as the propagation of results obtained in this field.

Each number will be 16 x 24 cms in size and will consist of about 70 pages. There will be an average of four articles on varied subjects. Whenever there is sufficient material there will be a special feature called "Recent Publications" dealing with articles and other works connected with cybernetics. One page will be devoted to news of the activities of the International Association for Cybernetics.

The columns of "Cybernetica" are open to all specialists in cybernetics and contributions from members of the Association, which at the moment group nearly 1,000 people from 33 countries, will be particularly welcome.

Articles will be published in French or in English, according to the wishes of the author. The terms of a subscription for 1958 are as follows: Members of the International Association for Cybernetics: 150 Belgian francs per year; Non-members: 300 Belgian francs per year. Separate numbers of the review may be obtained on the following terms: Members of the International Association for Cybernetics: 50 Belgian francs per number; Non-members: 100 Belgian francs per number.

The membership fee of the Association has been fixed for 1958 at 200 Belgian francs for individual members and at 1,000 Belgian francs for organizations.

Subscriptions and applications for membership should be sent to Association Internationale de Cybernetique, 13, Rue Basse-Marcelle, Namur, Belgique.

THE SECOND SYMPOSIUM ON NAVAL HYDRODYNAMICS, sponsored by the Office of Naval Research and the National Academy of Sciences-National Research Council, will be held in Washington, D. C., from August 25 through 29, 1958. This symposium, like the symposium held in 1956, is an outgrowth of a continually growing need for meetings devoted exclusively to developments in a rapidly advancing field of hydrodynamics, particularly in the areas basic to naval and marine applications. The intent of this symposium is to focus detailed attention on aero- and hydrodynamic noise and supercavitating and ventilated flows.

The program for the symposium includes the following speakers: A. Powell on theory and experiments in aero-dynamic research; R. H. Kraichnan on developments in turbulence theory; E. A. Muller on turbulence noise and scattering; E. Mollo-Christensen on boundary layer noise and pressure fluctuation measurement; U. Ingard on flow excited resonators; T. B. Benjamin on pressure waves from collapsing cavities; E. Meyer on experiments in ultrasonic cavitation noise; H. M. Fitzpatrick on cavitation noise in flowing liquids; E. E. Callaghan on noise in hypersonic boundary layers; M. P. Tulin on linearized theory of supercavitating flows; A. Acosta on fully developed cavity flow in pumps; W. Cornell on aerodynamic cavity flow in flight propulsion systems; H. Lerbs on supercavitating propellers; A. Tachmindji on the design and performance of supercavitating propellers; R. DiPrima on wall effects in cavity flow; T. Y. Wu on unsteady supercavitating flow; R. Timman on supercavitating hydrofoils; I. J. Campbell on air-filled cavities; and, G. Birkhoff on jets, wakes and cavities.

The meetings will be open to all those interested. The program will include tours of Navy Laboratories in the Washington area.

Formal programs and registration cards will be distributed in the near future. Further information may be obtained by contacting Phillip Eisenberg, Mechanics Branch, Office of Naval Research, Washington 25, D. C., or George W. Wood, National Academy of Sciences-National Research Council, Washington 25, D. C.

THE FIRST INTERNATIONAL CONGRESS IN THE AERONAUTICAL SCIENCES will be held under the auspices of the International Council of the Aeronautical Sciences in Madrid, Spain, from September 8 - 13, 1958. It will be held at the Instituto Nacional de Prevision, Alfonso XI, No. 1. The Registration Desk will be open Sunday, September 7, from 4 p.m. to 8 p.m., and on Monday, September 8, from 8:30 on. Dr. Theodore von Karman, First Honorary President of ICAS will preside. The program and arrangements for the Congress have been organized by committees chairmanned by Professor Maurice Roy, Director of the National Office for Aeronautical Study and Research, Paris, France; and Colonel Antonio Perez-Marin, General and Technical Secretary, Institute of Aeronautical Technology, Madrid, Spain.

Inquiries from those interested in attending the Congress should be made through Mr. S. Paul John, Director, The Institute of Aeronautical Sciences, 2 East 64th Street, New York 21, New York or Colonel Antonio Perez-Marin, Asociacion de Ingenieros Aeronauticos, Serrano 43, Madrid, Spain.

A detailed program is available, but it is too long to give here. The special sessions are listed on the program for Hypersonic Flow, Structures and Aeroelasticity, Heat Transfer and Heat Barrier, Jet Engines and Noise, Navigation and Guidance, Boundary Layer Control, VTOL/STOL, Heat Resistant Materials, Human Engineering, and Telecommand and Telemetering. Speakers will include: Theodore von Karman, Pedro Blanco, Robert T. Jones, William H. Stephens, M. P. Roy, W. D. Hayes, M. J. Lighthill, J. Lukasiewicz, A. van der Neut, Luigi Broglio, Robert Mazet, Marten T. Landahl, Ernst R. G. Eckert, R. J. Monaghan, R. Siestrunck, Jean-Joseph Bernard, E. Schmidt, J. M. de Sendagorta, F. B. Greatrex, D. M. Brown, K. K. Neely, H. S. Ribner, B. Etkin, William Littlewood, H. J. L. Le Boiteux, A. M. A. Majendie, Kurt Magnus, C. S. Draper, W. Wrigley, R. B. Woodbury, H. Schlichting, G. V. Lachmann, P. Carriere, P. Poisson-Quinton, E. A. Eichelbrenner, D. G. Hurley, G. H. Küssner, Antonio Ferri, Lucien C. Malavard, J. P. Campbell, D. Keith-Lucas, Gerhard Eggers, Günther Ernst, N. J. Hoff, Pol Duwez, D. A. Oliver, D. G. Simons, G. Melvill Jones, E. Evrard, R. A. Leslie, P. O. Gillard, J. T. Mengel, W. Moeckel, W. B. Thompson, V. H. Blackman.

NEWS ITEMS AND ANNOUNCEMENTS

EXPOSITORY REPORTS IN MATHEMATICS. The Society has entered into a contract with the Air Force Office of Scientific Research to provide for the preparation of expository mathematical books of high quality by leaders in modern mathematics. The exposition is to be aimed at a level accessible to first- or second-year graduate students. The authors will be encouraged to emphasize points of contact with other disciplines. Thus the books will presumably be both interesting and intelligible to professionals in other areas, such as physicists, chemists, and engineers. It is planned that each book be systematic and as far as possible self-contained.

To receive applications, select authors and supervise the project, a Committee on Expository Reports consisting of the following five members has been appointed: Professor E. J. McShane, Chairman, Professor Lipman Bers, Professor Salomon Bochner, Professor Andrew Gleason, and Professor Deane Montgomery.

Each prospective author should send to the Chairman of the Committee a detailed outline of what he proposes to write with a sample chapter or two. Six copies of this material should be submitted to the Chairman to facilitate distribution to the Committee. If it is not convenient for a prospective author to have so many copies made, the Headquarters Offices of the Society will upon request provide the necessary number of photo copies free of charge.

Negotiations will proceed along approximately the following lines: When an author has been selected by the Committee, he will obtain a leave of absence from his institution for a twelve to fifteen month period during which he will be employed full time under a contract with the Society. The author will not be required to remain at his university and it will be arranged to provide for customary services at the university of residence. In order to attract the best possible authors, the Society will be able to make arrangements to pay the author a stipend at a rate in excess of his normal academic rate.

Negotiations for the publication of a report in book form will lie between the author and a publisher of his choice. The usual royalty and copyright arrangements between an author and publisher will not be impaired by the fact that the book was written under this project except to permit the Government to reproduce the book in whole or in part if it chooses.

Proposals for preparation of such reports are currently being received and should be sent to Professor E. J. McShane, c/o Dr. J. H. Curtiss, Executive Director, American Mathematical Society, 190 Hope Street, Providence 6, Rhode Island.

NATIONAL BUREAU OF STANDARDS AND NATIONAL SCIENCE FOUNDATION TRAINING PROGRAM IN NUMERICAL ANALYSIS FOR SENIOR UNIVERSITY STAFF. The National Bureau of Standards is planning, conditional upon support by the National Science Foundation, to hold its second Training Program in Numerical Analysis for Senior University Staff during the spring term of 1959.

The purpose of this program is to give regular university staff a training in numerical analysis which will enable them to direct the operation of a university computing center, and to organize training and research in numerical analysis on their return to their own institutions. It will occupy the whole of the second semester of the academic year 1958-1959 (from February 9 to June 5) and has been arranged for that time so that participants may become familiar with the details of their own computing equipment during the following summer and be able to conduct courses in the academic year 1959-1960.

In order that individual attention can be given to each student, the number of participants will be limited to twelve. Selection will be made on the basis of qualifications of the individual and of the computational program of his institution. It will be made by a committee approved by the National Science Foundation. Payment to individuals will be arranged by the National Bureau of Standards. Candidates must be citizens of the U. S. A. Since it is anticipated that participants will require access to various National Bureau of Standards facilities, acceptance of participants as guest workers of the Department of Commerce will be essential and security assurance will be obtained for them. Appropriate forms will be submitted after the participants have been selected.

Applications must be received not later than October 15, 1958. They should be addressed to: Dr. Philip J. Davis, Chief, Numerical Analysis Section, Applied Mathematics Division, National Bureau of Standards, Washington 25, D. C., and should include:

1. The academic history of applicant and a list of his publications.
2. A statement about the computational program of the institution.

As soon as the selection is completed and the National Science Foundation has finally approved the program, the selected candidates will be informed.

Each selected candidate will receive from his institution (1) his usual salary for one semester, (2) travel not to exceed one first class round trip fare to Washington, D. C., and (3) subsistence (the sum of $850.00 will be allowed). Each institution sending a participant will be reimbursed for half the participant's salary for one semester and for his travel and subsistence. Upon the submission of vouchers to the National Bureau of Standards by the business office of the participant's institution, partial payments will be made on April 1, 1959 and June 1, 1959.

Assistance in obtaining accommodations in Washington will be provided by the National Bureau of Standards.

The program will be under the general direction of Dr. Philip J. Davis, Chief, Numerical Analysis Section, Applied Mathematics Division, National Bureau of Standards. He will be assisted by Dr. Morris Newman.

AWARDS UNDER THE FULBRIGHT ACT. A booklet describing the U. S. Government awards under the Fulbright and Smith-Mundt Acts has been prepared by the Conference Board of Associated Research Council, 2101 Constitution Avenue, Washington 25, D. C. The booklet, together with application forms and additional information are obtainable by writing to the above address. The closing date for applications for 1959-1960 is October 1, 1958.

APPLICATIONS FOR POSTDOCTORA WARDS INVITED BY NATIONAL SCIENCE FOUNDATION. Applications will be accepted through September 2, 1958, by the National Science Foundation for a second group of postdoctoral fellowships to be awarded during 1958 in the regular postdoctoral program. Fellowships will be awarded in the mathematical, physical, medical, biological, engineering, and other science fields, including anthropology, psychology (other than clinical), geography, certain interdisciplinary fields, and selected social science fields. Names of successful fellowship candidates will be announced on October 15, 1958. To be eligible for these awards, candidates must be citizens of the United States with special aptitude for advanced training and must hold the doctoral degree or have the equivalent in training or experience. Fellows will be selected on the basis of ability as evidenced by letters of recommendation and other evidence of attainment. Candidates' qualifications will be evaluated by carefully chosen panels of scientists under arrangements made by the National Academy of Sciences. Final selection of Fellows will be made by the National Science Foundation. A stipend of $3800 per year will be awarded to successful applicants in the regular postdoctoral program. Dependency allowances will be made to married Fellows. A limited allowance to aid in defraying a Fellow's cost of travel will be paid as well as tuition and fees. Applications for the regular postdoctoral fellowships may be obtained by writing to the National Academy of Sciences - National Research Council, 2101 Constitution Avenue, N. W., Washington 25, D. C.

1959 WESTERN JOINT COMPUTER CONFERENCE. Papers are being solicited for the 1959 Western Joint Computer Conference, to be held at the Fairmont Hotel, San Francisco, on March 3-5, 1959. The theme of this conference will be "New Horizons with Computer Technology."

In keeping with the theme, there is particular emphasis on factual papers dealing with the newer applications of computer techniques, such as Information Retrieval, Operation Control, Pattern Analysis, Decision Making, Computer Communications, Learning Concepts, ... and the like, as well as on papers dealing with advances in computer component and systems design.

It is also hoped that there will be two sessions of a speculative nature: A "Blue Sky Session," and one on "Philosophy and Responsibility of Computers in Society." Papers intended for the "Blue Sky Session" should deal with the extension of computer technology into areas not considered feasible at present. They should indicate the advantages of such extension, and also the area of research necessary to bring this application into the feasible range. Papers for the session on "Philosophy and Responsibility of Computers in Society," should deal with the philosophic and/or social implications of the widespread application of automatic computer techniques. The papers for these sessions should be of the type to invite serious discussion. These two sessions will be definitely scheduled only in the event that enough suitable papers are received.

Papers for the 1959 WJCC should be prepared based on a 20-minute delivery time. Selection of papers for presentation will be made from the complete text of the paper. There are no format requirements for these submission drafts. Three copies of each proposed paper should be submitted to the Technical Program Committee, IBM Research Laboratory, San Jose, California, by October 1st of this year. After review, final selection of papers will be made and the authors will be notified by December 1, 1958. Submission of final texts of the selected papers in the form required by the Publications Committee should be made by February 1, 1959.

CARLETON COLLEGE SUMMER INSTITUTE FOR HIGH SCHOOL TEACHERS OF MATHEMATICS. This institute was held in June and July and was featured by the presence of a number of well-known research mathematicians, according to information recently released by the Carleton College News Bureau. These included Dr. George Polya, Professor Emeritus of Mathematics at Stanford University, who spoke on Heuristic; Professor Preston C. Hammer, of the University of Wisconsin, who spoke on Numerical Analysis; Professor Dirk J. Struik, professor of mathematics at MIT, who talked on the history of mathematics; and Professor R. H. Bing of the University of Wisconsin, who spoke on Topology.

A DATA PROCESSING CENTER FOR BROWN UNIVERSITY. It has recently been announced that an elaborate data processing center will be presented to Brown University as a gift. The center will be established in memory of the late Thomas J. Watson, Sr. The sponsors are Mrs. Thomas J. Watson, Sr. and Mr. Thomas J. Watson, Jr., who is now President of the International Business Machines Corporation. Machines of various types and from various different manufacturers will be installed in the center, which will include a 2,000 square-foot computer room, adjoining lecture and conference rooms, offices and equipment space.

A NEW INSTITUTE FOR APPLIED MATHEMATICS has been established at the University of Mainz, Germany. The director is Professor Friedrich Baur, formerly at the Munich Computation Center. The computer laboratory of the new Institute will be equipped with a digital computer, Zuse Z No. 22, and an analog computer, Telefunken, both of which will be used primarily for teaching purposes. Research work on formula translation and in numerical analysis will be done in close collaboration with the computation centers of the Technische Hochschule in Munich and the ETH in Zurich.

THE PRESENTATION OF THE WILLIAM MARSHALL BULLITT MATHEMATICS LIBRARY TO THE UNIVERSITY OF LOUISVILLE. Mrs. William Marshall Bullitt is giving to the University of Louisville Library the unique collection of books, pamphlets, and other documents on mathematics and related subjects which her husband, the late William Marshall Bullitt, assembled. The mathematical collection was built up over a period of about twenty years, starting in 1936, and contains approximately 350 items, almost all of which are first published works of eminent and famous mathematicians. The list of authors includes Archimedes, Gauss, Newton, Abel, Cauchy, Euler, Fermat, Galois, Lagrange, Riemann, Einstein, Cantor, Eudoxus, Dirichlet, and Weierstrass. The gift was inaugurated on June 26 by the presentation by Mrs. Bullitt to the Library of the Halifax copy of Newton's "Principia"; a copy of the first edition of the "Mémoire sur les équations algébriques" by N. H. Abel, published in 1824, and five pamphlets from the "Annalen der Physik", 1905, Nos. 6-10, containing the first publications of writings by Albert Einstein. The original idea for the Bullitt collection was suggested to Mr. Bullitt by Professor Harlow Shapley of Harvard University and the late Professor G. H. Hardy of Cambridge, England. Professor E. T. Bell contributed to the compilation of the list for the collection.

APPOINTMENT OF ASSISTANT DIRECTOR, MATHEMATICAL, PHYSICAL AND ENGINEERING SCIENCES, OF THE NATIONAL SCIENCE FOUNDATION. The NSF has announced that Dr. Randal M. Robertson joined the staff of the Foundation as Assistant Director for Mathematical, Physical and Engineering Sciences on July 7. Dr. Robertson is a physicist. He received his M. A. degree in Mathematics and Natural Philosophy from the University of Glasgow and his Ph. D. in Physics from the Massachusetts Institute of Technology. He taught briefly at both institutions and served as a Research Assistant at Columbia University for a year. From 1937 to 1942 he was a Research Associate with the Norton Company in Worcester, Massachusetts. From 1942 to 1946 he was on the staff of the Radiation Laboratory at MIT and in 1946 came to Washington as Head of the Mechanics and Materials Branch of the Office of Naval Research. In 1948 he became Director of the Physical Sciences Division and has been Director of the Research Group since 1952.

THE ANNUAL REPORT OF THE TECHNICAL ADVISORY COMMITTEE TO THE APPLIED MATHEMATICS DIVISION OF THE NATIONAL BUREAU OF STANDARDS. For the period under review the committee consisted of David Blackwell, E. U. Condon, Mark Kac, Philip M. Morse, Mina Rees, and A. H. Taub, Chairman. The text of the report as submitted to the NOTICES by Professor Taub is as follows:

1. This committee is pleased to report that the program of the Applied Mathematics Division is progressing well and that this division continues to provide excellent mathematical, computational and statistical services to other agencies of the government, as well as other Bureau divisions. Members of the division are carrying out significant research and training programs in addition to their service duties.

The Applied Mathematics Division has a serious problem in recruiting personnel. There is a serious shortage of people trained in applied mathematics in this country, and the rules of the Department of Commerce concerning the hiring of non-citizens make it difficult to take advantage of the possibility of obtaining the services of foreign scientists if and when they become available.

2. The Numerical Analysis Section (11.01) has prepared a number of chapters for the Handbook of Tables being prepared under the direction of Dr. M. Abramowitz*, Head of Section 11.02. Members of this section have carried out research in various branches of mathematics and have conducted numerical experiments on the SEAC and IBM 704 computers.

One noteworthy experiment involved the evaluation of matrix inversion programs. The programs of a number of computing organizations were used to invert a set of matrices which were felt

*Editor's Note: since deceased.

to be a representative set of those that occur in practice. The methods were ordered in goodness in accordance with various measures of the "errors" involved. The error estimates given by von Neumann and Goldstine in their classic paper, "The Numerical Inverting of Matrices of High Order", were found to be accurate and to apply to those codes which used floating point arithmetic in spite of the fact that they were derived assuming fixed point arithmetic operations.

Section 11.01 is planning another training program so that regular university staff, not trained in numerical analysis, may become acquainted with that field and with the problems arising in the operation of a university computing center. A successful program of this type was conducted from February 11, 1957, to June 7, 1957. The new program is being planned to start in February, 1959, and end in June, 1959. Funds to support this program are being solicited from the National Science Foundation. It is hoped that it will be possible to make an early announcement of the training program and that applications for trainee positions can be solicited in good time.

3. The Computation Laboratory (Section 11.02) is fruitfully engaged in operating the IBM 704 installation for divisions of the Bureau and other Government agencies. In spite of the prevalence of computing services in and outside of Government, the Computation Laboratory's services are very much in demand and there seems to be no difficulty in selling machine time and coding services. This speaks very well for the quality of the services rendered by Section 11.02.

This section is making very good progress on the Handbook of Mathematical Tables. It is presently estimated that the Handbook will consist of about 1,000 pages, divided into 27 chapters. Each chapter will contain formulas of importance connected with the function being discussed and tabulated, as well as information as to how the tables can be used and extended.

4. The approximately 12 scientist years of work in the Statistical Engineering Laboratory (Section 11.03) during 1957 were divided about as follows:

Within NBS consulting	5.4
NBS supported research and professional activities	3.1
Non-NBS (contract) work	3.5

The non-NBS work is of a type which the SEL is uniquely (among Government agencies) qualified to perform, and the results have been excellent. The post office project has apparently gone well in spite of the press of other work this section had. About nine research papers were completed during 1957. Professional activities include Eisenhart's serving as an editor of JASA and Mrs. Rosenblatt's teaching a course at the Bureau.

Some of the members of SEL feel that they are undertaking too many activities. For example, while no consulting problem is ever refused, because of the lack of time, the members of SEL usually stop with answering the immediate question raised instead of pursuing the matter further as they would like to do in many cases.

Because the section is understaffed, it has not been able to seek out statistical problems in other divisions of the Bureau; it has had to divert attention from the reliability study; and has not been able to do consulting on problems in stochastic processes. Given the present staff, the division of activities seems reasonable and well carried out.

5. The Section of Mathematical Physics (Section 11.04) has continued the excellent work discussed in last year's Report; it has been able to start work in only a few new fields. The most important new field is that of magnetohydrodynamics, a good choice both because of its growing importance and also because its cultivation requires skill in analysis and access to high-speed computers, both of which the division has.

The work in hydraulics and fluid dynamics, in general, continues at about the same pace as last year; likewise with the useful and important work in theoretical elasticity. Detailed consulting work for other divisions of the Bureau has continued to about the same extent as before.

Some calculations of satellite problems have been carried out, for example, the effect of the earth's magnetic field on a spinning satellite. Calculations are nearly complete on the behavior or steam-filled bubbles in water and other two-phase hydrodynamic problems.

A NEW MEMOIR. Memoirs No. 29, "Twisted Polynomial Hyperalgebras" by Edward Halpern is now being printed and is expected to be available by September 1. The price is $1.50 list and $1.13 to members. The author has submitted the following description of this Memoir: "The notion of a hyperalgebra arises in considering H-spaces and includes Pontrjagin and Hopf algebras. A twisted polynomial hyperalgebra over an integral domain A is (algebraically) a tensor product of elementary types in which generators multiply $x_m x_n = a_{m,n} x_{m+n}$ and the coefficient $a_{m,n}$ divides $(m + n)!/m!n!$. In the paper these elementary types are classified in terms of sequences and it is shown that quite general ones may occur topologically. The main theorem gives sufficient conditions for a Hopf algebra to be a twisted polynomial hyperalgebra."

FURTHER DOCTORATES CONFERRED IN 1957. The following doctorates conferred in 1957 were omitted from the list published in the May 1958 issue of the Bulletin:

Kupperman, Morton, The George Washington University, February, 1957, Further applications of information theory to multivariate analysis and statistical inference.

Bartholomay, A. F., Harvard University, June, 1957, A stochastic approach to chemical reaction kinetics.

Goldberg, Karl, The American University, June, 1957, The formal power series for log $e^x e^y$, II.

BACKLOGS OF CERTAIN MATHEMATICAL RESEARCH JOURNALS. Information on this important matter is being published twice a year in the NOTICES, with the kind cooperation of the respective editorial boards.

It is important that the reader should interpret the data with full allowance for the wide and sometimes meaningless fluctuations which are characteristic of them. The waiting times in particular are affected by many transient effects, which arise in part from the refereeing system. Extreme waiting times as observed from the published dates of receipt of manuscripts may be very misleading, and for that reason, no data on extremes are presented in the table below.

	1	2	3		4	5		
Journal	No. Issues per year	Approx. no. pages published currently per year	Backlog 5/31/58 (Pages)	Backlog 11/30/57 (Pages)	Estimated Current Waiting Time (Months)	Observed Waiting time in latest issue Q (Mos.)	Med. (Mos.)	Q (Mos.)
American Jnl.	4	1,000	300	NR[a]	6	3	4	9
Annals of Math. Statistics	4	1,300	150	100	10	6	7	12
Annals of Math.	6	1,200	725	300	14	6	10	11
Canadian Jnl.	4	650	130	90	11	10[b]	12[b]	13[b]
Duke Journal	4	700	300	160	14	9	10	11
Illinois Jnl.	4	600	300	188	15	10	12	12
Jnl. of Math. and Mechanics	6	1,000	100	0	7	--[c]	--[c]	--[c]
Jnl. of the Soc. for Ind. and Appl. Math.	4	400-450	0	0	4-6	2	3	4
Michigan Jnl.	3	300	30	40	10	1[d]	6[d]	9[d]
Pacific Jnl.	4	900	0	0	6	5[e]	7[e]	8[e]
Proceedings	6	1,000	80	100	8	7	8	9
Transactions[f]	6	1,650	1,400	1,300	21	18	19	19

Footnotes: a. NR means either that no response was received to a request for information, or else that the response contained data in another form.

b. Issues are undated; these data based on the first number of the 1958 volume, which was arbitrarily counted as the March, 1958 issue.

c. Dates of receipt of mss. not indicated in this journal.

d. Issues are undated; these data based on the third number of the 1957 volume which was arbitrarily counted as the December, 1957 issue.

e. Issues are undated; these data based on the Spring 1958 issue, which was arbitrarily counted as the March, issue.

f. Beginning in 1959 there will be 12 issues per year, and approximately 2200 pages published per year thereafter.

Some of the columns in the table above are not quite self-explanatory, and here are some further details concerning how the figures were computed.

Column 2. These figures are rounded off to the nearest 50.

Column 3. For each journal, this is the estimate as of the indicated dates, of the total number of printed pages which will have been accepted by the next time that manuscripts are to be sent to the printer, but which nevertheless will not be sent to the printer at that time. (It should be noted that pages received but not yet accepted are being ignored.)

Column 4. Estimated by the editors (or the Editorial Department of the American Mathematical Society in the case of the Society's journals), and based on these factors: manuscripts accepted, manuscripts received and under consideration, manuscripts in galley, and rate of publication.

Column 5. The first quartile (Q_1) and the third quartile (Q_3) are presented, to give a measure of the dispersion which will not be too much distorted by meaningless extreme values. The median (Med.) is used as the measure of location. The observations were made from the latest issue received in the Headquarters Offices before the deadline date for this issue of the NOTICES. The waiting times were measured by counting the months from receipt of manuscript in final revised form, to nominal month of issue (not counting in month of receipt but counting in month of issue). It should be noted that when a paper is revised, the waiting time between receipt by editors of the final revision and its publication may be much shorter than is the case for a paper which is not revised, so these figures are to that extent distorted on the low side.

PERSONAL ITEMS

(This section is restricted to members of the Society)

Professor N. C. Ankeny has been awarded a Guggenheim Fellowship and will be on leave from Massachusetts Institute of Technology for the academic year 1958-1959 at Cambridge University, Cambridge, England.

Professor H. D. Brunk, on leave from the University of Missouri, will be a Fulbright lecturer at the University of Copenhagen, Copenhagen, Denmark from July 1958 until May 1959.

Mr. F. W. Carroll of Purdue University has received a Fulbright Fellowship to study at the University of Amsterdam, Amsterdam, Netherlands.

Assistant Professor Bernard Greenspan, on leave of absence for the 1958-59 term to accept a National Science Foundation Faculty Fellowship at the University of California, Berkeley, has been promoted to an associate professorship at Drew University.

Professor Jacob Korevaar of the University of Wisconsin was presented the Benjamin Smith Reynolds Award of $1,000 for excellence in teaching engineering students.

Assistant Professor L. H. Lange of Valparaiso University continues on leave and has been awarded a Science Faculty Fellowship at the University of Notre Dame by the National Science Foundation.

Professor M. H. Martin of the University of Maryland has been granted an honorary degree of Doctor of Sciences by Lebanon Valley College.

Associate Professor Alice T. Schafer of Connecticut College has been awarded a National Science Foundation Science Faculty fellowship for 1958-1959, and will spend the year on leave at the Institute for Advanced Study.

Professor R. D. Schafer of the University of Connecticut has been awarded a National Science Foundation Senior Postdoctoral fellowship for 1958-1959, and will spend the year on leave at the Institute for Advanced Study.

Assistant Professor G. B. Seligman, on leave from Yale University for the academic year 1958-1959, has received a Fulbright award to lecture at the University of Munster, Munster, Germany.

Dr. A. G. Anderson of the General Tire and Rubber Company, Akron, Ohio has been appointed to a professorship at the Western Kentucky State College.

Dr. J. F. Andrus of the University of Florida has accepted a position as senior mathematical engineer with the Lockheed Aircraft Corporation, Marietta, Georgia.

Professor Emil Artin of Princeton University will be on leave of absence in 1958-1959, and will be at the University of Hamburg, Hamburg, Germany.

Mr. A. E. Babbitt of Rutgers, The State University, has accepted a position as mathematician with the United States Signal Corps, Fort Monmouth, New Jersey.

Dr. J. L. Bailey of Michigan State University has been appointed to an assistant professorship at the Case Institute of Technology.

Dr. E. H. Batho will be on leave from the University of Rochester for the academic year 1958-1959 and will be at the Institute for Advanced Study.

Assistant Professor W. E. Baxter of the Ohio University has been appointed to an assistant professorship at the University of Delaware.

Dr. Anatole Beck of Tulane University of Louisiana has been appointed to an assistant professorship at the University of Wisconsin.

Mr. A. T. Beyer of the General Electric Company, Cincinnati, Ohio has accepted a position as scientist with the Litton Industries, Beverly Hills, California.

Dr. P. P. Billingsley of Princeton University has been appointed to an assistant professorship at the University of Chicago.

Dr. Richard Block of Indiana University has been appointed a research associate at Yale University.

Dr. R. K. Brown of Rutgers, The State University, has accepted a position as mathematician with the Evans Signal Corps Laboratories, Belmar, New Jersey.

Dr. J. R. Buchi of the University of Illinois has been appointed an associate research mathematician at the University of Michigan.

Mr. R. G. Busaker has accepted a position as operations analyst in the operations research office of Johns Hopkins University, Bethesda, Maryland.

Dr. L. L. Campbell of the Defense Research Board of Ottawa, Ontario, Canada has been appointed to an assistant professorship at the Assumption University of Windsor, Windsor, Ontario, Canada.

Dr. J. F. Carpenter of the Boeing Airplane Company has accepted a position as senior scientist with the Dalmo Victor Company, Belmont, California.

Assistant Professor R. V. Chacon of the University of Illinois has been appointed to an assistant professorship at the University of Wisconsin.

Mr. P. T. Church of the University of Michigan has been appointed to an assistant professorship at Syracuse University.

Professor Harvey Cohn of Washington University has been appointed to a professorship at the University of Arizona.

Mr. L. F. Cremona of the Combustion Engineering, Inc. has accepted a position as government contracts administrator with the ACF Industries, Inc., Paramus, New Jersey.

Assistant Professor C. H. Cunkle of Colorado State University has been appointed to an associate professorship at Dickinson College.

Dr. H. B. Curtis, Jr. of the Rice Institute has been appointed to an assistant professorship at the University of Texas.

Dr. J. M. Danskin of the Institute for Advanced Study has been appointed to an assistant professorship at Rutgers, The State University.

Dr. M. P. Drazin of Northwestern University has accepted a position as senior scientist with RIAS, Inc., Baltimore, Maryland.

Dr. R. L. Dunn of the Hoffman Laboratories, Los Angeles, California has accepted a position as member of the technical staff of the Space Technology Laboratories Division of the Ramo-Wooldridge Corporation, Los Angeles, California.

Mr. E. O. Elliott of the Operations Evaluation Group, Massachusetts Institute of Technology, Navy Department, Washington, D. C. has accepted a position as operations analyst with the Stanford Research Institute, Menlo Park, California.

Miss Jane L. Evans has accepted a position as teacher at the Saint Petersburg Junior College.

Assistant Professor W. H. Fleming of Purdue University has been appointed to an assistant professorship at Brown University.

Assistant Professor Ilse N. Gál of Cornell University will be on leave at Yale University for the academic year 1958-1959.

Assistant Professor I. S. Gál, on leave from Cornell University for the academic year 1958-1959, has been appointed a research associate at Yale University.

Professor R. W. Gardner of the Olivet Nazarene College has been appointed Dean of Students and a professor at the Eastern Nazarene College.

Mr. P. A. Gillis of Remington-Rand, Inc. has accepted a position as associate mathematician in the Atomic Energy Division of the Westinghouse Electric Corporation, Bettis, Pennsylvania.

Dr. P. C. Gilmore of Pennsylvania State University has accepted a position as mathematician with the International Business Machines Corporation, Yorktown, New York.

Dr. S. R. Goldner of New York University has been appointed to a professorship at Stellenbosch University, Stellenbosch, South Africa.

Dr. H. H. Goldstine of the Institute for Advanced Study has been appointed to the position of research advisor at the Research Center of the International Business Machines Corporation, Yorktown, New York.

Dr. R. L. Graves of the Standard Oil Company of Indiana has been appointed assistant professor of applied mathematics in the School of Business, and associate director of the Operations Analysis Laboratory at the University of Chicago.

Mr. J. R. Hatcher has accepted a position as research engineer with the North American Aviation, Inc., Los Angeles, California.

Professor M. H. Heins of Brown University has been appointed to a professorship at the University of Illinois.

Mr. Michael Held of New York University has accepted a position as mathematician in the research center of the International Business Machines Corporation, Yorktown, New York.

Dr. T. P. Higgins of the Boeing Airplane Company has accepted a position as senior mathematician with Dalmo-Victor, Belmont, California.

Dr. M. W. Johnson, Jr. of the Massachusetts Institute of Technology has been appointed to an assistant professorship in mechanics at the University of Wisconsin.

Mr. R. B. Kellogg of Northwestern University has accepted a position as mathematician with Combustion Engineering, Inc., Windsor, Connecticut.

Dr. R. R. D. Kemp of the Institute for Advanced Study has been appointed to an assistant professorship at Queen's University, Kingston, Ontario, Canada.

Associate Professor J. H. B. Kemperman of Purdue University will be on leave for the academic year 1958-1959.

Dr. C. W. Kohls of Columbia University has been appointed to an assistant professorship at the University of Illinois.

Assistant Professor L. A. Kokoris, on leave of absence for the 1958-1959 academic year to accept an associate professorship at the Illinois Institute of Technology, has been promoted to an associate professorship at Washington University.

Assistant Professor A. G. Kostenbauder of Syracuse University has been appointed to an associate professorship at Wilkes College.

Dr. L. D. Kovach will retain his position as design specialist at the Douglas Aircraft Company, El Segundo, California, while filling an appointment as professor of mathematics at the George Pepperdine College, Los Angeles, California.

Associate Professor Stephen Kulik of the University of South Carolina has been appointed to a professorship at the Utah State University.

Dr. K. W. Kwun of the University of Michigan has been appointed a research associate at the Tulane University of Louisiana.

Mr. W. J. Leinbach of Wake Forest College has accepted a position in the space technical laboratory of the Ramo-Wooldridge Corporation, Los Angeles, California.

Dr. R. B. Leipnik of the University of Washington has accepted a position as mathematician, consultant at the United States Naval Ordnance Testing Station, China Lake, California.

Professor Lee Lorch of Philander Smith College, on leave for the academic year 1958-1959, will serve as visiting lecturer in mathematics at Wesleyan University, Middletown, Connecticut.

Professor G. G. Lorentz of Wayne State University has been a visiting professor at the University of Tubingen, Germany, during the second part of the summer semester of 1958.

Mr. J. C. McCall of the University of Illinois has accepted a position as general manager of the Midwest Computer Service, Inc., Decatur, Illinois.

Dr. P. E. McDougle of the University of Virginia has been appointed to an assistant professorship at the University of Miami, Coral Gables, Florida.

Associate Professor E. K. McLachlan of Baylor University has been appointed to an associate professorship at the Oklahoma State University.

Professor H. B. Mann of the Ohio State University has been appointed a temporary member of the Mathematics Research Center of the University of Wisconsin for the summer of 1958.

Dr. A. I. Martin of the Institute for Advanced Study has accepted a position as scientist with the British Welding Research, Abington Hall, Cambridge, England.

Dr. H. D. Mills of the General Electric Company, New York, New York has accepted a position with the Market Research Corporation of America, New York, New York.

Professor L. M. Milne-Thomson of Brown University has been appointed a member of the Mathematics Research Center of the University of Wisconsin, Madison, Wisconsin.

Associate Professor Josephine M. Mitchell of the University of Pittsburgh has been appointed to an associate professorship at the Pennsylvania State University.

Mr. R. P. Mitchell of the Naval Ordnance Laboratory, Corona, California has accepted a position as scientist with the Lockheed Aircraft Corporation, Palo Alto, California.

Mr. J. H. Monahan of Boston College has accepted a position as staff member at the Lincoln Laboratory of Massachusetts Institute of Technology.

Mr. T. P. Mulhern of Brown University has been appointed to an assistant professorship at Fordham University.

Dr. A. E. Nussbaum of Rensselaer Polytechnic Institute has been appointed to an assistant professorship at Washington University.

Associate Professor Anne F. O'Neill of Wheaton College will be on sabbatical leave for the first semester of the academic year 1958-1959.

Dr. E. T. Parker of the University of Michigan has accepted a position as mathematician with the Remington Rand Corporation, St. Paul, Minnesota.

Dr. F. P. Pedersen, on leave from the University of Copenhagen, Denmark, has been appointed visiting assistant professor at the University of Southern California for the academic year 1958-1959.

Professor Pasquale Porcelli of the Illinois Institute of Technology is on leave until September 1959 and will spend the time in residence at the Mathematics Research Center, U. S. Army, University of Wisconsin.

Dr. Marian B. Pour-El of Berkeley, California has been appointed to an assistant professorship at the Pennsylvania State University.

Dr. J. R. M. Radok of the Shell Development Company, Emeryville, California has been appointed to an associate professorship at the Polytechnic Institute of Brooklyn.

Dr. H. J. Renggli of the Tulane University of Louisiana has been appointed to an assistant professorship at Rutgers, The State University.

Mr. J. A. Riley of Brandeis University has accepted a position as research associate with the Parke Mathematical Laboratories, Inc., Carlisle, Massachusetts.

Dr. J. D. Riley of the Iowa State College has accepted a position as mathematician with the Ramo-Wooldridge Corporation, Los Angeles, California.

Mr. W. W. Sawyer of the University of Illinois has been appointed to a professorship at Wesleyan University, Middletown, Connecticut.

Dr. E. C. Schlesinger of Yale University has been appointed to an assistant professorship at Wesleyan University, Middletown, Connecticut.

Dr. Lowell Schoenfeld of the Westinghouse Electric Corporation, Pittsburgh, Pennsylvania has been appointed to an associate professorship at the Pennsylvania State University.

Mr. S. J. Scott of the Vitro Corporation of America has accepted a position as operations analyst with the United States Air Force, Colorado Springs, Colorado.

Assistant Professor G. F. Simmons of the University of Rhode Island has been appointed to an assistant professorship at Williams College.

Dr. Stephen Smale, on leave from the University of Chicago, will be a member of the Institute for Advanced Study for the coming academic year.

Dr. W. A. Small of Grinnell College was appointed visiting professor of mathematics for the 1958 summer session at Nebraska State Teachers College, Chadron, Nebraska.

Associate Professor W. K. Smith of Antioch College has been appointed to an associate professorship at Bucknell University.

Professor Ernst Snapper of Miami University has been appointed to a professorship at Indiana University.

Dr. H. D. Sprinkle of the University of Arizona has accepted a position as research scientist with Litton Industries, Beverly Hills, California.

Associate Professor R. A. Struble of the Illinois Institute of Technology has been appointed to an associate professorship at the State College of Agriculture and Engineering, University of North Carolina, Raleigh, North Carolina.

Dr. W. C. Swift of the Bell Telephone Laboratories, Murray Hill, New Jersey has been appointed to an assistant professorship at Rutgers, The State University.

Dr. L. H. Turner of Purdue University has accepted a position as mathematician with the Ramo-Wooldridge Corporation, Los Angeles, California.

Associate Professor G. W. Whitehead, on leave from the Massachusetts Institute of Technology, has been appointed to a visiting professorship at Princeton University for the 1958-1959 term.

Professor E. H. Zarantonello has returned from his visiting position at the Mathematics Research Center, U. S. Army, at Madison, Wisconsin, to his post at the University of Cuyo, Mendoza, Argentina.

The following promotions are announced:

A. F. Bartholomay, Harvard University School of Public Health, to an assistant professorship.

C. S. Coleman, Wesleyan University, to an assistant professorship.

Trevor Evans, Emory University, to a professorship.

W. F. Freiberger, Brown University, to an associate professorship.

K. A. Hirsch, Queen Mary College, University of London, to a professorship.

Tosio Kato, University of Tokyo, to a professorship.

Solomon Leader, Rutgers, The State University, to an associate professorship.

F. I. Mautner, Johns Hopkins University, to a professorship.

Henry Sharp, Jr., Emory University, to an associate professorship.

K. G. Wolfson, Rutgers, The State University, to an associate professorship.

The following appointments to instructorships are announced:

University of British Columbia: Dr. J. V. Whittaker; Brown University: Dr. Leon Greenberg; Cornell University: Mr. P. E. Ney; Duke University: Mr. G. J. Minty; University of Florida: Mr. F. J. Lorenzen, Jr.; Harvard University: Dr. W. W. Comfort; Loyola College: Reverend F. A. Homann; Massachusetts Institute of Technology: Dr. D. W. Bressler, Dr. P. J. Cohen; Rutgers, The State University: Mr. M. J. Greenberg, Mr. P. E. Martin; West Virginia University: Mr. H. W. Gould.

LETTERS TO THE EDITOR

Editor, the NOTICES.

A total of $1,716.01 has been contributed toward the legal expenses of Professor Lee Lorch by 190 persons, nearly all mathematicians, according to the records of the Committee accepting such contributions. This includes funds sent either to the Committee or directly to Professor Lorch and reported by him to the Committee. In addition, he has received other contributions from various individuals totalling $837.50, making an overall total of $2,553.51. The total legal expenses have amounted to about $5,100 and so further contributions will be welcomed. They should be sent to the Secretary of the Committee, Professor David Blackwell, Statistics Department, University of California, Berkeley 4, California.

The legal expenses arose out of his prosecution on the charge of "contempt of Congress" following his testimony before the House Un-American Activities Committee. He was acquitted in Federal District Court in November, 1957.

David Blackwell

Editor, the NOTICES.

While I agree with R. P. Boas and G. Piranian (See their letter to the Editor of the NOTICES, June 1958) in deploring the idea of an author's submitting manuscripts to several journals simultaneously, I think that we should discourage this practice by other means than by insisting on original typewritten manuscripts. I feel that multilith copies should be acceptable to the editor of a journal because they are as legible as typewritten copies, they can be made on good quality paper, and they are much easier to prepare. Indeed, one can make corrections on a multilith master much more easily than one can on typewriter paper and carbons. In addition, the chance of error is reduced since formulas need be filled in only once rather than n + 1 times, where n is the number of carbons. Because extra copies are conveniently available, the author can circulate copies to several of his colleagues, who may be able to make use of the results in their own research and who may discover errors. Finally, some papers are written under government contracts which require extra copies to be presented for clearance and for other purposes prior to publication.

David M. Young, Jr.
The Ramo-Wooldridge Corporation

Editor, the NOTICES.

While one agrees with Messrs. Boas and Piranian that something should be done to prevent the practice of submitting the same article to several journals simultaneously, it isn't quite clear that their proposal to require all manuscripts to be typewritten originals would do this. Certainly any "varmint" who would do this in the first place should not find this requirement too great an inconvenience, especially since it could be done by any typist, and also since it would enhance acceptance possibilities. What to do? Why not bar the culprit from future publication in the journal in question, or, perhaps, in any of the journals of the Society?

Finally, won't anyone defend the practice of "ditto-ing" manuscripts on the basis of efficiency and increased (non-journal) circulation?

Carl C. Faith

Editor, the NOTICES.

Without disputing the legitimacy of Professors Boas' and Piranian's complaint in their recently published letter to these NOTICES, I should like to raise an objection to their suggestion that only typewritten manuscripts be accepted by editorial boards. But first, I must be excused for my attitude, for I write with limited personal experience as an author, referee, and editor.

To be sure typescripts are neat, legible, etc., as well as evidence (?) of the unicity of the author's presentation to journal editors. It is widely recognized, however, that many of the papers submitted for publication originated as government or industry sponsored research reports. Generally these reports are mimeographed, multilithed, dittoed, etc., and rarely are carbon copies. Moreover, as regards format, style, and rigor, these reports are acceptable manuscripts. I feel it is asking too much to insist that only original documents be submitted to editors. Moreover, papers purposely prepared for publication by authors employed in government bureaus and industry almost always are to be reviewed by several higher ranking officials. To this end the manuscripts are usually prepared for duplication. Again, it is asking too much to overrule this procedure (this letter requires ten copies for company inspection and approval purposes).

No, I can't agree with insisting on typescripts. This is not prima facie evidence of ethical behavior.

Paul L. Chessin

Editor, the NOTICES.

Several young mathematicians, postdoctoral fellows of the National Science Foundation, encountered an unexpected obstacle to their plans to attend the 1958 International Congress of Mathematicians in Edinburgh.

The Congress is in the middle of August; it lasts for eight days. The Fellows involved notified the Foundation of their wish to attend the Congress. The Foundation thereupon agreed to defray a part of the Fellows' travel expenses. By this action and by the explicit statements of some of its officials the Foundation indicated that it approved attendance at the Congress as proper activity for a Fellow. So far all had gone well.

In due course the Fellows notified the NSF of their proposed dates of leaving and returning. They wanted to take advantage of the opportunity to spend all or part of the summer in Europe. They were not asking for any additional money; they just notified the NSF of their proposed whereabouts as a matter of courtesy. At this point the unexpected obstacle arose. The Foundation informed the Fellows of its view that spending unplanned and unsupervised time in Europe is not proper activity during the tenure of an NSF fellowship. If a Fellow's time away from home came to more than eight days plus the minimum time required for a roundtrip to Edinburgh, the Foundation would withhold or diminish the Fellow's regular monthly checks so as to reduce the total of his stipend by an amount proportional to the time of his absence.

On learning these facts, I wrote to Dr. Bowen C. Dees of the NSF staff, asking him to explain to me the Foundation's policy. His reply was, in part, as follows.

> " ... special travel (when it involves an extended period of time) is supposed to be an integral part of a Fellow's application and is subject to review by the panel and a determination made by the panel as to the consistency of such travel with the applicant's proposed program of activities. We feel an obligation to see that Fellows are given every possible opportunity for self improvement during their tenure, but I am sure you will agree that the staff of the Foundation should not as a rule consider it their prerogative to approve major changes of programs which modify the proposal as seen by the panels. ... I should like to point out that our operations and programs are subject to review each year within the Foundation, by the National Science Board, and the Bureau of the Budget; to careful scrutiny by the appropriations subcommittees of both Houses of the Congress; and to audit by the General Accounting Office. ...
>
> " ... If a Fellow requests permission to attend a meeting and we approve, we notify him and send only information necessary for essential travel, assuming that he plans to go to the meeting and return directly to his institution. We have not in the past had to consider the possibility of a Fellow planning to take an extended vacation of several months duration either before or after the meeting or both. If at any time it becomes evident that a Fellow is being unrealistic in his planning, we have no alternative but to advise him that the things he proposes to do are not consistent with his approved program of activities or the policies of the Foundation.

"Take Dr. ---'s case ... when his travel advance request was submitted, it showed dates of travel as June 3, 1958 to September 26, 1958. This was so unusual that Dr. Fontaine called him immediately to explain our position. The fact that our position came as quite a surprise to Dr. --- should not lead one to conclude that Dr. Fontaine's statements "involved some harsh and seemingly arbitrary decisions". What surprised Dr. Fontaine was Dr.---'s view that since the Institute for Advanced Study operated only six months each year, the remaining six months were his to do as he pleased and if he wished to use it all for vacation purposes while on tenure that should be all right. I am sure you will agree that this is not the proper view for our Fellows and is not consistent with a proper expenditure of Federal funds. It is difficult for me to believe that "responsible and serious research workers" might expect the National Science Foundation to enter into this type arrangement or expect it to condone such action. ...

" ... The National Science Foundation, being a federal establishment, can not operate like some academic institutions and, while fully informed on what is generally considered "normal academic procedure", must bear constantly in mind the boundary conditions which traditionally circumscribe the expenditure of Federal money.

"I am sure that you agree that few if any universities which make 12-month appointments to their junior staff members would adopt the attitude that, once the academic year is over, the staff member's time is his own, to do with as he likes, with no responsibility even for reporting to his department chairman or anyone else as to what his plans are for the summer. ..."

I should like to comment on some aspects of the entire situation.

The direct large-scale support of basic scientific activity by the federal government of the U. S. is a relatively new phenomenon. The government's previous experience in finance and administration is not necessarily pertinent to the problems of the NSF. The postmaster in Cochise, Arizona, operates under federal administration, but his activities seem to demand a type of regulation somewhat different from that needed for scientific research. It is arguable, I think, that "the boundary conditions which traditionally circumscribe the expenditure of Federal money" might be subject to reexamination when it comes to laying down the policies of the National Science Foundation. If it should turn out that the best way to spend money on science is indeed different from the traditional governmental procedure, then the task of explaining the difference to Congress and the General Accounting Office would fall on the shoulders of the NSF staff. In the long run this might well be less onerous than continually anticipating and avoiding trouble with the Bureau of the Budget. In the present relatively healthy climate of opinion about science, in the country as a whole and in Washington in particular, the traditionally circumscribed operations may turn out to be not the best guides to major scientific policy.

The two principles on which successful fellowships have always been based are, first, to award a stipend to a man who deserves it, and, second, to leave him alone.

The first principle implies, among other things, that fellowships should be given to deserving men regardless of any particular plan of work or any particular academic affiliation. A fellowship should be awarded to a man, not to a project.

The panel that evaluates fellowship applications has before it each applicant's "study program". Why? Not, I think, in order to select one program in preference to another, but just to get a feeling for how the applicant sees his field and how mature his views are. If a Fellow announces that he wants to study topology but finds that his interests start to lean toward algebra, no one criticizes him for his broadened point of view. At least I don't think any mathematician would be critical of such a "major change of program". Similarly, a geographic change might well be considered a private matter. It is important that a Fellow do the best he can; what he calls it and where he does it can probably best be left to him.

A point of semantic disagreement between the Fellows and the Foundation, one that has to do with the second of the principles mentioned before, concerns the word "vacation". The Fellows do not want a vacation, and it doesn't seem quite right to classify their proposed travels under that heading. Perhaps there is an easily rectified misunderstanding about how mathematicians operate. Many research workers are tied, by the nature of their subject, to a laboratory, an observatory, or an excavation. Others, working in mathematical branches of astronomy, economics, physics, statistics, and, of course, in pure mathematics itself, are tied to nothing less mobile than a clipboard and a pencil. Mathematics can be and has been done in strange places and at strange times. When a Fellow proposes to work in Denmark for a couple of months, it seems to him that he is just proposing a healthy change of pace. The Fellows did not ask for a vacation. They are dedicated mathematicians who can no more stop thinking about mathematics than they can stop breathing.

The Fellows are somewhat hurt by the apparent assumption that they are naughty. They are made to feel that they tried to take unfair advantage of the U. S. government and that they became Fellows for financial gain. A perhaps pertinent fact is that every one of the Fellows could earn three times his present income right now, and a correspondingly comfortable amount all his life, just by deciding to apply his training commercially. These Fellows are not expense account manipulators. It's not that they are above crass monetary considerations; the point is that the people in whose life such considerations play a large role do not wish to become Fellows.

To supervise the activities of a Fellow once the award has been made seems to be as insufficient as it is unnecessary. I think the U. S. is capable of producing all the dedicated scientists it needs. If, perish the thought, that is wrong, then supervision, light or severe, is not going to help. Not that disapproval of a proposed trip constitutes supervision in any effective sense; if a Fellow's aim in life were to avoid doing his work, he could probably do that at home as well as abroad.

Could a fellowship program be administered in accordance with the two principles above? I think so. One good reason for thinking so is the eminent success of the John Simon Guggenheim Memorial Foundation. Guggenheim fellowships have existed for over thirty years by now. Once a Guggenheim fellowship is awarded, all that the Fellow is required to do is to tell the Secretary General the address to which his check should be mailed. If the Fellow chooses to live in a bungalow on Catalina Island, that is his business. If after a few months he chooses to go to Europe, that is more of his business. There is no starting certificate to say that a Fellow has reported to some institution, there is no termination certificate, and not a word is said about vacations. This sensible policy has never yet been subject to any gross abuses, and, in fact, the luster and honor attached to the Guggenheim fellowships is growing every year.

It is regrettable that a few young men were subjected to petty annoyances and financial penalties. It is even more regrettable.that the chief governmental body charged with ministering to the needs of scientific research in the U. S. should show such early signs of bureaucratic petrification. This letter is being written in the hope that if responsible scientists learn the facts and the tendencies the facts point to, then those scientists can and will use their influence to rectify the policies that threaten to throttle the vitally necessary scientific renaissance of the past decade.

Paul R. Halmos
University of Chicago

MEMORANDA TO MEMBERS

DEADLINE FOR THE 1958-1959 COMBINED MEMBERSHIP LIST

Members are advised that the Headquarters Offices' deadline for changes of addresses which should appear in the forthcoming COMBINED MEMBERSHIP LIST is October 15. The Headquarters Offices will appreciate having any changes which must be incorporated in the CML sent in as soon as possible.

Please remember that if a new position goes with a new address, full information should be given for the CML. Please also advise as to when the new address is to be effective.

A change of address should always be submitted to the Society at least six weeks before it is to become effective. Otherwise one or more copies of the journals will be sent to the wrong address, and return or forwarding postage must be paid by the Society. Each of the journals of the Society is mailed by the printer from a strip list furnished by the Headquarters Offices, and as all readers of the NOTICES undoubtedly know from experience with other magazine subscriptions, it is not feasible to avoid a considerable time lag between receipt of and the carrying out of instructions from a subscriber.

LIST OF RETIRED MATHEMATICIANS AVAILABLE FOR EMPLOYMENT

This list is being maintained by the Headquarters Offices of the American Mathematical Society, 190 Hope Street, Providence 6, Rhode Island. Copies of the current issue may be obtained by writing to that address.

Mathematicians who are retiring this year and who are interested in being included in the list are asked to send the following information to the Headquarters Offices: name; date of birth; highest degree and where obtained; most recent employment; present address, date available; preferences, including preference for academic or industrial employment.

It should also be mentioned in this connection that in 1954 an Emeriti Employment Registry, Box 24451, Los Angeles 24, California, was established as a non-profit service to assist retired professors in the United States. Expenses are met by a $5 charge to individuals and $10 to institutions using this service. Persons interested may inquire at the Los Angeles address.

CORPORATE AND INSTITUTIONAL MEMBERS OF THE AMERICAN MATHEMATICAL SOCIETY

As of July 11, 1958, the following were supporting the Society through Corporate or Institutional memberships:

CORPORATE MEMBERS

Bell Telephone Laboratories, Incorporated
Ford Motor Company
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Shell Development Company
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INSTITUTIONAL MEMBERS

Acadia University, Wolfville, N. S., Canada
Alabama Polytechnic Institute, Auburn, Alabama
University of Alabama, University, Alabama
University of Alberta, Edmonton, Alberta, Canada
Amherst College, Amherst, Massachusetts
University of Arizona, Tucson, Arizona

Beloit College, Beloit, Wisconsin
Brandeis University, Waltham, Massachusetts
Brigham Young University, Provo, Utah
University of British Columbia, Vancouver, B. C., Canada
Brooklyn College, Brooklyn 10, New York
Brown University; Providence 12, Rhode Island
Bryn Mawr College, Bryn Mawr, Pennsylvania
University of Buffalo, Buffalo, New York

California Institute of Technology, Pasadena 4, California
University of California, Berkeley 4, California
University of California, Davis, California
University of California, Los Angeles 24, California
University of California, Riverside, California
Carnegie Institute of Technology, Pittsburgh 13, Pennsylvania
Case Institute of Technology, Cleveland 6, Ohio
Catholic University of America, Washington 17, D. C.
University of Chicago, Chicago 37, Illinois
University of Cincinnati, Cincinnati 21, Ohio
City College, New York 31, New York
University of Colorado, Boulder, Colorado
Columbia University, New York 27, New York
Connecticut College, New London, Connecticut
Cornell University, Ithaca, New York

Dartmouth College, Hanover, New Hampshire
University of Delaware, Newark, Delaware
De Paul University, Chicago 14, Illinois
University of Detroit, Detroit 1, Michigan
Duke University, Durham, North Carolina
Duquesne University, Pittsburgh 19, Pennsylvania

Florida State University, Tallahassee, Florida
University of Florida, Gainesville, Florida

Georgetown University, Washington 7, D. C.
University of Georgia, Athens, Georgia
Gettysburg College, Gettysburg, Pennsylvania
Goucher College, Towson, Maryland
Grinnell College, Grinnell, Iowa

Harpur College, Endicott, New York
Harvard University, Cambridge 38, Massachusetts
Haverford College, Haverford, Pennsylvania
College of the Holy Cross, Worcester 3, Massachusetts
University of Houston, Houston 4, Texas

Illinois Institute of Technology, Chicago 16, Illinois
University of Illinois, Urbana, Illinois
Immaculata College, Immaculata, Pennsylvania
Indiana University, Bloomington, Indiana
Institute for Advanced Study, Princeton, New Jersey
Iowa State College of Agriculture and Mechanic Arts, Ames, Iowa
State University of Iowa, Iowa City, Iowa

Johns Hopkins University, Baltimore 18, Maryland

University of Kansas, Lawrence, Kansas
University of Kentucky, Lexington 29, Kentucky
Kenyon College, Gambier, Ohio

Lehigh University, Bethlehem, Pennsylvania
Louisiana State University and Agricultural and Mechanical College, Baton Rouge 3, Louisiana

McGill University, Montreal, Quebec, Canada
University of Manitoba, Winnipeg, Manitoba, Canada
University of Maryland, College Park, Maryland
Massachusetts Institute of Technology, Cambridge 39, Massachusetts
University of Massachusetts, Amherst, Massachusetts
Mathematical Association of America, Buffalo 14, New York
University of Miami, Coral Gables, Florida
University of Michigan, Ann Arbor, Michigan

Michigan State University of Agriculture and Applied Science, East Lansing, Michigan
University of Minnesota, Minneapolis 14, Minnesota
Mississippi State College, State College, Mississippi
University of Mississippi, University, Mississippi
University of Missouri, Columbia, Missouri
Montana State College, Bozeman, Montana
Montana State University, Missoula, Montana
Mount Holyoke College, South Hadley, Massachusetts

University of Nebraska, Lincoln 8, Nebraska
University of New Hampshire, Durham, New Hampshire
New Mexico College of Agriculture and Mechanic Arts, State College, New Mexico
University of New Mexico, Albuquerque, New Mexico
New York University, New York 3, New York
University of North Carolina, Chapel Hill, North Carolina
North Texas State College, Denton, Texas
Northwestern University, Evanston, Illinois
University of Notre Dame, Notre Dame, Indiana

Oberlin College, Oberlin, Ohio
Ohio State University, Columbus 10, Ohio
Oklahoma State University of Agriculture and Applied Science, Stillwater, Oklahoma
University of Oklahoma, Norman, Oklahoma
Oregon State College, Corvallis, Oregon
University of Oregon, Eugene, Oregon

Pennsylvania State University, University Park, Pennsylvania
University of Pennsylvania, Philadelphia 4, Pennsylvania
University of Pittsburgh, Pittsburgh 13, Pennsylvania
Pomona College, Claremont, California
Princeton University, Princeton, New Jersey
Purdue University, Lafayette, Indiana

Queens College, Flushing 67, New York

Randolph-Macon Woman's College, Lynchburg, Virginia
Reed College, Portland, Oregon
University of Rhode Island, Kingston, Rhode Island
Rice Institute, Houston 1, Texas
University of Rochester, Rochester 3, New York
Rutgers, The State University, New Brunswick, New Jersey

College of Saint Thomas, St. Paul 5, Minnesota
University of Santa Clara, Santa Clara, California
University of Saskatchewan, Saskatoon, Saskatchewan, Canada
Seattle University, Seattle 22, Washington

Smith College, Northampton, Massachusetts
South Dakota State College of Agriculture and Mechanic Arts, Brookings, South Dakota
University of Southern California, Los Angeles 7, California
Southern Illinois University, Carbondale, Illinois
Southern Methodist University, Dallas 5, Texas
Stanford University, Stanford, California
Swarthmore College, Swarthmore, Pennsylvania
Sweet Briar College, Sweet Briar, Virginia
Syracuse University, Syracuse 10, New York

Temple University, Philadelphia 22, Pennsylvania
University of Tennessee, Knoxville 16, Tennessee
Texas Agricultural and Mechanical College, College Station, Texas
Texas Christian University, Fort Worth, Texas
Texas Technological College, Lubbock, Texas
University of Texas, Austin, Texas
University of Toronto, Toronto 5, Ontario, Canada
Tulane University of Louisiana, New Orleans 15, Louisiana

United States Air Force Academy, Denver 8, Colorado
United States Naval Postgraduate School, Monterey, California
University of Utah, Salt Lake City 1, Utah

Vanderbilt University, Nashville 4, Tennessee
Virginia Polytechnic Institute, Blacksburg, Virginia
University of Virginia, Charlottesville, Virginia

State College of Washington, Pullman, Washington
Washington University, St. Louis 5, Missouri
University of Washington, Seattle 5, Washington
Wayne State University, Detroit 2, Michigan
Wellesley College, Wellesley 81, Massachusetts
Wells College, Aurora, New York
Wesleyan University, Middletown, Connecticut
Western Reserve University, Cleveland 6, Ohio
Wheaton College, Norton, Massachusetts
Worcester Junior College, Worcester 8, Massachusetts
University of Wichita, Wichita, Kansas
Williams College, Williamstown, Massachusetts
University of Wisconsin, Madison 6, Wisconsin

Yale University, New Haven 11, Connecticut

CATALOGUE OF LECTURE NOTES

Supplement No. 2

HARVARD UNIVERSITY

The following item announced in the October and December 1957 NOTICES is no longer available.

P. C. ROSENBLOOM, "Linear partial differential equations".

The following items may be ordered from: Department of Mathematics, Harvard University, 2 Divinity Avenue, Cambridge 38, Massachusetts. (Advance payment only)

R. BRAUER, "Galois theory", 144 pp. $2.00

O. ZARISKI, "An introduction to the theory of algebraic surfaces", 100 pp. $2.00

NEW PUBLICATIONS

Adler, C. F. Modern geometry. An integrated first course. New York, McGraw-Hill, 1958. 14 + 215 pp. $6.00.

Anderson, T. W. An introduction to multivariate statistical analysis. New York, Wiley, 1958. 12 + 374 pp. $12.50.

Balatoni, J. See Chintschin, A. J.

Berge, C. Théorie générale des jeux à n personnes. (Mémor. Sci. Math., no. 138.) Paris, Gauthier-Villars, 1957. 114 pp.

Bianchi, L. Opere. Vol. VIII. Classi speciali di superficie. Ed. by the Unione Matematica Italiana with contributions by the Consiglio Nazionale delle Ricerche. Rome, Cremonese, 1958. 398 pp. 3500 Lire.

Bogolyubov, N. N., and Širkov, D. V. Vvedenie v teoriyu kvantovannyh poleĭ. Moscow, Gosudarstv. Izdat. Tehn.-Teor. Lit., 1957. 442 pp., 1 insert. 18.50 rubles.

Booth, A. D., Brandwood, L., and Cleave, J. P. Mechanical resolution of linguistic problems. New York, Academic Press, 1958. 7 + 306 pp. $9.80.

Booth, K. H. V. Programming for an automatic digital calculator. New York, Academic Press, 1958. 7 + 238 pp. $7.50.

Brandwood, L. See Booth, A. D.

Buckland, W. R. See Kendall, M. G.

Chintschin, A. J., Faddejew, D. K., Kolmogoroff, A. N., Rényi, A., and Balatoni, J. Arbeiten zur Informationstheorie, I. Mathematische Forschungsberichte, IV. Berlin, VEB Deutsche Verlag der Wissenschaften. 134 pp. 18.40 DM.

Cleave, J. P. See Booth, A. D.

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ABSTRACTS OF CONTRIBUTED PAPERS

THE JUNE MEETING IN CORVALLIS, OREGON

June 20, 1958

547-20. Elliott Mendelson: The Axiom of Fundierung and the axiom of choice.

The Axiom of Fundierung is taken in the form which asserts that there are no infinitely-descending ϵ-sequences. H_1 is the weak form of the axiom of choice according to which every denumerable set of unordered pairs has a choice set. It is shown that, in the system given by Gödel in The Consistency of the Continuum Hypothesis, neither H_1 nor Axiom D is provable from axioms A, B, C ("basic set theory") and the Axiom of Fundierung. The method of proof is an extension of that used by the author in a previous paper (J. Symb. Logic, vol. 21 (1956) pp. 350-366). (Received March 10, 1958.)

547-21. R. W. Carroll: On the EPD equation and subharmonic distributions. Preliminary report.

Let $\mu_x(R)$ and $A_x(R)$ be the surface and solid mean values respectively, considered as distribution-valued functions of R, i.e. functions $f(R):[0,\infty) \longrightarrow E'_x$ (Notations of L. Schwartz). They are then differentiable and it is shown that $\mu_x(R)*T$ satisfies an Euler-Poisson-Darboux (or EPD) equation of index m-1 for any $T \in D'_x$ where m is the dimension. If T is a subharmonic distribution and the measure ν, in the Riesz decomposition of T has compact support and is absolutely continuous then $\mu_x(R)*T$ is a solution almost everywhere of the Cauchy problem for this EPD equation in the usual sense. If moreover ν has a continuous density this holds everywhere. Weinstein's unified treatment of the family of EPD equations (Ann. Mat. Pura Appl. vol. 43 (1957)) is extended in the case of the wave equation to a solution of the Cauchy problem with initial data $T \in D'_x$. In the case of subharmonicity his results on the minimum and convexity of solutions are generalized. (Received March 13, 1958.)

547-22. G. T. Whyburn: On the invariance of openness.

A topological space X has the Brouwer Property provided any subset of X which is homeomorphic with an open subset of X is itself necessarily open. In this paper the following invariance theorem is proven: Let f(X) = Y be a mapping where X and Y are locally connected generalized continua. If there exists a closed set F in Y such that no locally compact set homeomorphic with a subset of F separates any region in Y and such that f is locally topological on $X - f^{-1}(F)$, then if X has the Brouwer Property, so also has Y. This yields not only the existence of spaces other than Euclidean manifolds having this property but also provides a method of generating such spaces in a variety of ways. (Received March 13, 1958.)

547-23. E. J. McShane: A canonical form for antiderivatives.

In anticipation of uses with "generalized functions", for each partial-differentiation operator D^p a kernel is defined which when applied to a continuous f yields an F such that $D^pF = 0$; it has the further property that if the functions $f_1, f_2, \ldots$ have any solutions $G_1, G_2, \ldots$ of $D^pG_n = f_n$ which converge uniformly on compact sets, the functions F_n obtained from the kernel representation also converge uniformly on compact sets. As applications, a rather general form of the fundamental lemma of the calculus of variations is obtained, and also a characterization of Bochner's "weak solutions" of the equation $D^pF = 0$. (Received March 13, 1958.)

547-24. O. G. Harrold, Jr.: A sufficient condition that a monotone image of the three-sphere be a topological three-sphere.

A recent example of R. H. Bing, Ann. of Math. vol. 65 (1957) pp. 484-500, gives a decomposition of the three-sphere, S^3, into points and tame arcs such that the decomposition space is topologically different from S^3. In this paper it is shown that if f is a continuous, monotone transformation of S^3 onto M such that (i) if $Y = \{y \in M \text{ such that } f^{-1}y \text{ is not a point}\}$, then given $y \in \bar{Y}$ and $\epsilon > 0$ there is a topological 2-sphere in the ϵ-neighborhood of y that separates y and the complement of the ϵ-neighborhood in M that is disjoint to $\bar{Y}$, then M is a topological S^3. (Received March 19, 1958.)

547-25. G. T. Whyburn: Compactness of certain mappings.

If X and Y are locally compact separable metric spaces and the metric in Y is such that all bounded sets are compact (or for some $r > 0$, all closed spheres of radius r are compact), the set of all compact mappings of X into Y is open in the mapping space Y^X provided the latter is topologized by the uniform convergence metric. Examples show this does not hold without the restriction on the metric in Y or when the mapping space is topologized by almost uniform convergence. Also if X and Y are homeomorphic locally compact separable metric spaces having the Brouwer property, any (1-1) mapping of X onto Y is a homeomorphism. From this we get that if $f(X) = Y$ is monotone, where Y is locally compact, has the Brouwer property and is homeomorphic with the natural decomposition space of f, then f is compact. In particular, any monotone mapping of a plane or line onto itself is compact. (Received March 13, 1958.)

547-26. J. W. Milnor: On the cobordism ring Ω^*.

It is proved that the cobordism groups Ω^n (defined by R. Thom, Comm. Math. Helv. vol. 28 (1954)) have no odd torsion. The quotient ring, Ω^* modulo torsion elements, is a polynomial ring on generators of dimension 4,8,12,... . A given manifold M^{4k} qualifies as a generator if and only if it satisfies the following condition. Let $s_k(\sigma_1,\ldots,\sigma_k)$ denote the polynomial which expresses the symmetric function $t_1^k + \ldots + t_m^k$ in terms of the elementary symmetric functions σ_i; and let $p_1,\ldots,p_k$ denote the Pontrjagin classes of M^{4k}. Then the characteristic number $s_k(p_1,\ldots,p_k)[M^{4k}]$ must be equal to $\pm q$ if $2k + 1$ is a power of the prime q; and must be equal to ± 1 if $2k + 1$ is not a prime power. The proof makes use of a spectral sequence due to F. Adams. (Received March 26, 1958.)

547-27. Donald Greenspan: On the numerical solution of Poisson's equation with given boundary values.

A numerical method is described for the solution of: $u_{xx} + u_{yy} = f(x,y)$, with given boundary values. Conditions are established under which the numerical solution converges to the analytical solution. (Received March 27, 1958.)

547-28. Herbert Federer: On sets with positive reach, applied to curvature theory. Preliminary report.

Say a closed subset A of Euclidean n space has positive reach if every point sufficiently close to A has a unique nearest point in A. This is proved equivalent to certain differentiability properties of the function associating with x the distance from x to A. Closed convex sets have positive reach, and so do compact sets locally definable by equations and inequalities involving continuously differentiable real functions with linearly independent Lipschitzian gradients, which includes class 2 manifolds with or without boundary. Steiner's formula holds whenever A has positive reach; for small $r > 0$ and each Borel set $S \subset A$, the n dimensional measure of the set of points within r of A and whose nearest point in A belongs to S is given by a polynomial of degree $\leq n$ in r; the coefficients define n signed Borel measures over A, called the curvatures of A. In this context the principal kinematic formula is proved; if A and B are compact sets with positive reach, then $A \cap m(B)$ has positive reach for almost all rigid motions m, and the integrals with respect to m of the curvatures of $A \cap m(B)$ are expressible bilinearly in terms of the curvatures of A and B. (Received March 28, 1958.)

547-29. R. K. Lashof and Stephen Smale: On the self-intersection of an immersed manifold.

An immersion $f:M^k \longrightarrow X^{k+r}$ of C^∞ manifolds is called n-normal if for each n-tuple of distinct points $p_1,\ldots,p_n$ of M with $f(p_1) = \ldots = f(p_n)$ the images $\overline{M}_{p_i}$ of the tangent spaces M_{p_i} have the minimum possible intersection in X_x, $x = f(p_i)$; explicitly, $\dim \bigcap_{i=1}^{n} \overline{M}_{p_i} = k - (n-1)r$. It is proved that every immersion of a closed manifold can be C^s-approximated (any s) by an n-normal immersion. If $f:M \rightarrow X$ is n-normal, M closed, and $(f)^n$ the n-fold product map, $(f)^n:(M)^n \longrightarrow (X)^n$, then the restriction F of $(f)^n$ to the subspace of distinct n-tuples of the n-fold product space $(M)^n$, is t-regular on the diagonal Δ of $(X)^n$. $F^{-1}(\Delta) = \Sigma_n$ is a closed manifold of dimension $k-(n-1)r$ called the n-self-intersection manifold of f. $\pi_1:(M)^n \longrightarrow M$ (projection onto first factor) restricted to Σ_n is an immersion and $\pi_1(\Sigma_n)$ is the set of points of M which are mapped n (or more) to one by f. Let ν be the fundamental class of M and $\mu = f^*\lambda f_* \nu$, where λ is Poincaré duality. Let $\overline{W}^r$ be the normal Stiefel-Whitney class of the immersion. Then if $r > 1$ and M and X are orientable $\lambda\Sigma_n = (\mu - \overline{W}^r)^{n-1}$ (cup product), with integer coefficients, where Σ_n is considered as a cycle in the natural fashion. If $r = 1$ or M or X is nonorientable then the result holds mod 2. For an early formulation of the case $n = 2$ see H. Whitney, Lectures in topology, Michigan, 1941, p. 131. (Received April 3, 1958.)

547-30. G. A. Heuer: Algebraic extensions of Banach algebras.

In a recent paper (Proc. Amer. Math. Soc. vol. 7 (1956) pp. 203-210), Arens and Hoffman norm the algebra B obtained by field extension methods from an algebra A and a polynomial $\alpha(x)$, and show that tractability of A is preseved under extension if the discriminant of α is not zero. (See above paper for notation and terminology.) The present author shows that if the discriminant of α is in no maximal ideal of A, then each component of the maximal ideal space H(B) is a covering space for the space H(A), and that several properties of A in addition to tractability may be inherited by B. Denote the statement "the discriminant of α is in no maximal ideal of A" by "condition (d)". Theorem: $\hat{A} = C(H(A))$ and condition (d) imply $\hat{B} = C(H(B))$. Theorem: $\hat{A}$ is dense in C(H(A)) and condition (d) imply $\hat{B}$ is dense in C(H(B)). Theorem: A is regular and self-adjoint, and condition (d), imply B is regular and self-adjoint. The second of these three theorems holds under a slight weakening of condition (d). The proofs utilize the fact that, while $\alpha(x) = 0$ has no solutions in A, the representation algebra A does contain local solutions. The last theorem also employs the construction technique for a partition of Dieudonné. (Received April 7, 1958.)

547-31. Eckford Cohen: Some remarks on Galois theory.

In the Artin approach to Galois theory, (Artin, Galois theory, 1946) a galoisian extension of a field F is characterized as a root field of a separable polynomial over F. The purpose of this note is to extend the Artin development so as to eliminate the condition of separability. This is accomplished by introducing the radical R of an extension E over F, defined to be the subfield of E consisting of all elements that are totally inseparable over F. Further, E is defined to be R-galoisian over F provided E is a finite extension of F with radical R and there exists a group of F-automorphisms of E with R as fixed field. On the basis of this definition and several simple lemmas, a theory of R-galoisian extensions is sketched along the pattern of Artin. The ordinary Galois theory results in the special case in which the radical R coincides with the base field F. (Received April 11, 1958.)

547-32. Eckford Cohen: Arithmetical inversion formulas.

The following general arithmetical inversion theorem is proved: $f(n,r) = \sum_{d|(n,r)} g(d,r/d) \rightleftarrows g(s,t) = \sum_{d|s} f(s/d,r)\mu(d)$, provided $r = st > 0$ and f(n,r) is an even function of n (mod r). A second inversion formula for a special class of even functions is also proved, and these two relations are shown to lead to simple proofs of several important properties of even functions, proved previously by other methods. The above inversion formula becomes equivalent to the Möbius inversion formula in case f(n,r) is restricted to the subclass of completely even functions (mod r), that is, functions of the form $f(n,r) = F((n,r))$,

$n \geqq 0$, $r > 0$. (Received April 11, 1958.)

547-33. D. G. Malm: On the cohomology ring of a sphere bundle.

Let $p: E \longrightarrow B$ be an orientable $(k - 1)$-sphere bundle, with B a finite polyhedron. For k odd, an invariant $P \in H^{2k-2}(B,Z)$ is defined which is closely related to the Pontrjagin class. Theorem 1. Suppose $H^{2k-2}(B,Z)$ has no two-torsion. Then P is the Pontrjagin class, and two $(k - 1)$-sphere bundles over B with the same Stiefel-Whitney classes W_k and W_{k-1} and the same invariant P have isomorphic integral cohomology rings. Theorem 2. For k even, if $H^{2k-2}(B,Z)$ has no two-torsion, then two $(k - 1)$-sphere bundles over B with the same Stiefel-Whitney classes W_k, W_{k-1}, and W_{k-2} have isomorphic integral cohomology rings. Using the real numbers or a cyclic group of odd order as coefficients for cohomology, the cohomology of the base space and the characteristic class W_k determine the cohomology ring of the total space E, for k even. This extends a theorem of R. Thom in the 1954-1955 H. Cartan Seminar Notes. The first results extend recent results of W. S. Massey in the J. Math. Mech. vol. 7. (Received April 11, 1958.)

547-34. D. E. Schroer: Unified interpretation of logical systems.

For terminology see Abstract 546-41 Notices Amer. Math. Soc. vol. 5 (1958) p. 251. A combinatory structure $\mathfrak{A}$ is defined to be a set A, called the set of $\mathfrak{A}$-objects, closed under a (usually nonassociative) binary operation called $\mathfrak{A}$-application, together with a subset T of A called the set of $\mathfrak{A}$-truths. A combinatory logic $\mathfrak{L}$ is interpreted in a combinatory structure $\mathfrak{A}$ by assigning $\mathfrak{A}$-objects to $\mathfrak{L}$-formulas so that combination of expressions denotes $\mathfrak{A}$-application of $\mathfrak{A}$-objects, denotations of all $\mathfrak{L}$-axioms are $\mathfrak{A}$-true, and T is closed under validity as determined by the set of all $\mathfrak{L}$-derived rules (of the same simple type as the underived $\mathfrak{L}$-rules). Every combinatory logic $\mathfrak{L}$ is then self-reflexive in the precise sense that $\mathfrak{L}$ can be naturally interpreted in the combinatory structure $\mathfrak{A}$ (called its principal structure) such that the $\mathfrak{A}$-objects are precisely the $\mathfrak{L}$-formulas, $\mathfrak{A}$-application is combination, and the $\mathfrak{A}$-truths are precisely the $\mathfrak{L}$-theorems. It follows that every combinatory logic $\mathfrak{L}$ has an interpretation in a countable structure, and that all $\mathfrak{L}$-formulas true in every interpretation of $\mathfrak{L}$ are $\mathfrak{L}$-theorems. For a strong natural kind of homomorphism, it is shown that the set of all combinatory structures in which a given combinatory logic can be interpreted is closed under the formation of homomorphic images. (Received April 14, 1958.)

547-35. William Browder: Loop spaces of suspensions. II.

Using the operations in I, Abstract 546-61 Notices Amer. Math. Soc. vol. 5 (1958) p. 370, together with Hopf-algebra and spectral sequence arguments, one can calculate the homology ring $H_*(\Omega^n(s^nY);Z_p)$ where the coefficients Z_p are the integers modulo p, p an odd prime, $p > n/2$. For every graded Z_p-module M and any positive integer n, one defines a graded Z_p-module $\mathcal{P}(M,n)$. It is then proved that $H*(\Omega^n(s^nY);Z_p)$ is isomorphic to the free anti-commutative algebra generated by $\mathcal{P}(H(Y;Z_p),n)$, (for $n \geqq 2$). A free anti-commutative algebra is the tensor product of a polynomial ring on the even dimensional generators and an exterior algebra on the odd dimensional generators. This isomorphism leads to a definition of homology operations mod p, $\mathcal{P}_k$,$(0 \leqq k \leqq n)$, in the category of n-fold loop spaces of an H-space. $\mathcal{P}_0(x) = x^p$. These operations are in general only defined modulo an ideal J in the homology ring. If σ is the homology suspension (as in I) for the coefficient domain Z_p, we get the relation $\sigma\,\mathcal{P}_k(x) = \mathcal{P}_{k-1}(\sigma x)$, ($J \subseteq$ kernel of σ). It then follows that $\mathcal{P}_{k-1}(t)$ is transgressive if t is. $H_*(\Omega^m(s^nY);Z_p)$ is primitively generated for $0 < m < n$ and thus its cohomology mod p can be calculated using Hopf-algebra arguments. (Received April 17, 1958.)

547-36. G. C. K. Yeh and J. Martinek: Disturbance of a many-dimensional field satisfying the Helmholtz equation due to the presence of a hyperplane boundary.

Theorem: Let $\Phi_0(x_1) = \Phi_0(x_1,x_2,\dots,x_n)$ be a real single-valued, twice continuously differentiable solution of Helmholtz equation $\nabla_n^2\Phi_0 + \lambda\Phi_0 = 0$ with $\nabla_n^2 = \sum_{i=1}^n \partial^2/\partial x_i^2$ (and λ = a real constant) with singularities lying entirely in $x_1 > 0$ and $\Phi_1(x_1) = \Phi_1(x_1,x_2,\dots,x_n)$ be a real, single-valued, twice continuously differentiable solution of same Helmholtz equation $\nabla_n^2\Phi_1 + \lambda\Phi_1 = 0$ with singularities lying entirely in $x_1 < 0$. Then the functions $\Phi_+(x_1) = \Phi_+(x_1,x_2,\dots,x_n)$ and $\Phi_- = \Phi_-(x_1,x_2,\dots,x_n)$ defined by $\Phi_+(x_1) = \Phi_0(x_1) + B\Phi_0(-x_1) + \int_{-\infty}^0 F(s)\Phi_0(s - x_1)ds + C\,\Phi_1(x_1) + \int_0^\infty G(s)\,\Phi_1(s + x_1)ds$ (for $x_1 > 0$), $\Phi_-(x_1) = \Phi_1(x_1) + b\,\Phi_1(-x_1) + \int_0^\infty g(s)\,\Phi_1(s - x_1)ds + c\,\Phi_0(x_1) + \int_{-\infty}^0 f(s)\,\Phi_0(s + x_1)ds$ (for $x_1 < 0$) where $k = \nu_1/(\mu\nu_1 + \nu_2)$, $\alpha = (\nu_4 - \mu\nu_3)/(\mu\nu_1 + \nu_2)$, $b = -B = (1 - 2\mu k)$, $c = 2k$, $C = 2\mu(1 - \mu k)$, $\mu f(s) = F(s) = -2\mu k\alpha e^{\alpha s}$ and $\mu g(s) = G(s) = 2\mu(1 - \mu k)\alpha e^{-\alpha s}$ have the following properties (I) $\nabla_n^2(\Phi_+ - \Phi_0) + \lambda(\Phi_+ - \Phi_0) = 0$ for $x_1 > 0$,

(II) $\nabla_n^2(\Phi_- - \Phi_1) + \lambda(\Phi_- - \Phi_1) = 0$ for $x_1 < 0$, (III) $\Phi_+ = \mu\Phi_-$ and $\nu_1 \partial\Phi_+/\partial x_1 + \nu_3\Phi_+ = \nu_2\partial\Phi_-/\partial x_1 + \nu_4\Phi_-$ as $x_1 \to 0$, μ, ν_1, ν_2, ν_3 and ν_4 are constants. The theorem is proved by proving a corresponding lemma for harmonic field in n + 1 variables. (Received April 22, 1958.)

547-37. Burton Wendroff: On centered difference equations for hyperbolic systems.

For semi-linear symmetric hyperbolic systems of first order partial differential equations, with positive definite coefficients, it is shown how one may use centered difference equations to obtain approximate solutions. The method is most easily described for the special case $u_t + u_x = 0$, $u(x,0) = \phi(x)$, $u(0,t) = \psi(t)$. The corresponding difference equation is $u_{i-1/2}^{n+1} - u_{i-1/2}^{n} + \lambda(u_i^{n+1/2} - u_{i-1}^{n+1/2}) = 0$, where $\lambda = \Delta t/\Delta x$, to which is adjoined the consistency condition that $u_i^{n+1/2} + u_{i-1}^{n+1/2} = u_{i-1/2}^{n+1} + u_{i-1/2}^{n}$. Given $u_{i-1}^{n+1/2}$ and $u_{i-1/2}^{n}$, the difference equation and the consistency condition uniquely determine $u_i^{n+1/2}$ and $u_{i-1/2}^{n+1}$; therefore, the initial data $u_0^{n+1/2} = \psi(t^{n+1/2})$ and $u_{i-1/2}^{0} = \phi(x_{i-1/2})$ determine the mesh function in the first quadrant. An energy relation analagous to the one available for the differential equation is obtained by multiplying the difference equation by $u_{i-1/2}^{n+1} + u_{i-1/2}^{n}$ and summing over the mesh. The stability and convergence of the difference scheme are derived from the energy relation. The above analysis is carried out for the general case. (Received April 28, 1958.)

547-38. G. A. Baker, Jr. and T. A. Oliphant: An implicit, numerical method for solving the two-dimensional, heat equation.

A generalization of the one-dimensional, Peaceman and Rachford method is derived [Bruce, Peaceman, Rachford, and Rice, Trans. A. I. M. E. vol. 198 (1953) pp. 79-92]. In this generalization simultaneous equations are set up and solved once for all values of the temperature over the entire two-dimensional mesh. This method is extended to treat nonlinear heat flow and it is unconditionally stable, both for linear and nonlinear problems. In the nonlinear case an iterative scheme is employed to solve the simultaneous equations which provides second-order convergence. This method differs from the well-known, alternating-direction [Douglas and Peaceman, A. I. Ch. E. Jour. vol. 1 (1955) pp. 505-512] in that the alternating-direction method does not solve the complete set of simultaneous equations at each time step but only a one-dimensional facsimile of them, and its range of applicability is more restricted. Preliminary investigations indicate that this method can be readily generalized to n-dimensions. (Received April 28, 1958.)

547-39. E. S. Rapaport: Finite groups. Coset decompositions.

If A is a complex of k elements, C a complex of j elements of the group G of order g, kj $= g$, a_i an element in A, c_i an element of C, and $a_i c_h = a_r c_s$ if and only if $i = r$, $h = s$, then $G = a_1 C + a_2 C + \ldots + a_k C$ is a "coset decomposition" of G; two sets $a_i C$ and $a_h C$ are "congruent". Neither A nor C need be a subgroup; it can happen that both A and C generate G. Pairs (A,C) of such complexes are generated and investigated. (Received April 29, 1958.)

547-40. E. S. Rapaport: Finite groups. Graphs.

If a group G of order g is given as the factor group F_n/N of the free group $F_n = F(g_1,\ldots,g_n)$, and S is a Schreier system of coset representatives of N in F_n, $w_1, w_2, \ldots$, the set of free generators of N obtained from S, then the following two statements are equivalent: (1) S consists of the initial segments S_k of just one word W of F_n, so that $w_i = S_{k_i} g_{h_i} \bar{S}^1_{r_i}$ for every i; (2) the graph (Cayley color group) of G on the generators $g_1,\ldots,g_n$ has a Hamilton line (a closed connected path of edges without multiple points and containing every vertex). If the number of generators is large, a Hamilton line, H, always exists: G is "unicursally" generated. The question of the existence of H for a minimal set of generators under various restrictions is discussed. For nonoriented graphs (involutory generators) of the symmetric groups, S_n, certain minimal sets of generators are shown, by algebraic and geometric means, to generate S_n unicursally. (Received April 29, 1958.)

547-41. E. O. A. Kreyszig: Singularities of solutions of partial differential equations in three real variables.

Various theorems about analytic functions of a complex variable can be translated into theorems about elliptic partial differential equations in two variables with real analytic coefficients. For equations of the form (1) $v_{xx} + v_{yy} + v_{zz} + a(y,z)v = 0$ there exist similar possibilities; here a is assumed to be an entire function when continued to complex values of both variables. There are Bergman operators which transform a finite sequence of analytic functions $f_1, f_2, \ldots, f_N$ of the complex variable $Z = (z + iy)/2$ into a solution (2) $v(x,y,z) \equiv w(x,Z,Z^*) = \sum_{n=0}^{N} \sum_{m,p=0}^{\infty} k_{nmp} x^n Z^m Z^{*p}$, $Z^* = -(z - iy)/2$, of (1).

The relation between $f_1, \ldots, f_N$ and w is very simple. In this way properties of w can be derived from those of the coefficients k_{nm0}, $n = 0,1,\ldots,N$, $m = 0,1\ldots$, in (2). It is natural to ask to what extent similar information can be obtained from other subsequences of those coefficients. This basic problem can be solved by investigating a finite sequence of suitable auxiliary functions which are related by a system of linear second order partial differential equations. In this way the domain of regularity, the location and nature of singularities, and other important properties of w can be characterized in terms of arbitrary subsequences $\{k_{nmp}\}$, $p > 0$ and fixed, in the corresponding development (2). These relations are independent of the special form of a(y,z). (Received May 2, 1958.)

547-42. R. C. Lyndon: A semantic Herbrand-Gentzen lemma.

New proofs are given for Interpolation theorems of W. Craig (J. Symb. Logic vol. 22, p. 267) and of the author (Bull. Amer. Math. Soc. Abstract 63-4-553). The relation 'P implies Q' is interpreted in the sense of the theory of models. Under this interpretation, the argument, which utilizes familiar ideas of Skolem, Tarski, Henkin and others, is straightforward, self contained, and independent of any considerations from the theory of deductive proof. The central point of the proof is a semantic lemma which serves in this context as a substitute for results of Herbrand and of Gentzen. (Unpublished results similar to those announced here have been obtained independently by Grzegorczyk, Mostowski and Ryll-Nardzewski.) (Received May 7, 1958.)

547-43. K. L. Brinkman and L. I. Deverall: Application of group representation theory to determination of natural frequencies of vibrating plates.

If the differential equation governing the vibration of plates is replaced by an appropriate difference equation defined over a grid of points R_G which is an approximation to the plate region R, the values of the deflection at the grid points may be considered to be the components of an n-dimensional vector space S, where n is the number of interior points of R_G. Following the methods used by Stiefel (J. Res. Nat. Bur. Stands. vol. 48 (1952)) for membrane eigenvalue problems, the space S may be decomposed into subspaces appropriate to the symmetry classes of the group G of R_G. The boundary conditions are satisfied by augmenting R_G with fictitious exterior points and then using the conditions of Barton (Def. Res. Lab., Report 175, University of Texas). Determination of a particular eigenvalue involves working only within the corresponding subspace, and hence the amount of numerical work is greatly reduced. The first sixteen eigenvalues of a clamped square plate have been studied in detail for two difference operators corresponding to the biharmonic operator; the results are in good agreement with the eigenvalues determined by variational methods. The results indicate that good accuracy can be obtained for a small number of grid points provided that an improved difference operator (Collatz, Eigenwertaufgaben, p. 451) is used. (Received May 5, 1958.)

547-44. C. S. Coleman: Stable conal regions in the neighborhood of a singular point in three space.

Let (1) $\dot{x} = F_m(x) + G(x)$ be a vector differential equation in the three-vector x, where $F_m(x)$ is a homogeneous vector polynomial of degree m in the components of x and G(x) is analytic in x in some neighborhood of the origin. Suppose the components of F_m have no common nonconstant factors. Let y denote a vector on the two sphere. Singular points and periodic solutions of the equation (2) $\dot{y} = F_m(y) + (y, F_m(y))y$, where $(y, F_m(y))$ denotes the scalar product, generate "special directions" and "special cones" of (1). Suppose such a singular point or periodic solution is an asymptotically stable solution of (2). Then there is a conal neighborhood N of the corresponding "special direction" or "special cone" such that all solutions of (1) entering N close enough to the origin tend to the origin asymptotic to the "special direction" or "special cone" (Received May 6, 1958.)

547-45. Edmund Eisenberg and David Gale: Aggregation of utility functions.

An economy consists of m buyers $B_1, \ldots, B_m$. The i-th buyer has an income β_i which he may spend in purchasing various amounts of n different goods, $G_1, \ldots, G_n$. Such a purchase or bundle is represented by an n-vector $x_i = (\xi_{i1}, \ldots, \xi_{in})$ where ξ_{ij} is the amount of G_j purchased by B_i. It is assumed that B_i ranks all possible bundles by a utility function ϕ_i and that at a given set of prices $p = (\pi_1, \ldots, \pi_n)$ he makes a purchase x_i which maximizes ϕ_i among those bundles costing not more than β_i. If each buyer behaves in this manner then for each price vector p the community as a whole will purchase some bundle x_0. The question now arises as to whether the community is also acting so as to maximize some function ϕ_0. In general, this will not be the case. However, if each function ϕ_i is concave, continuous and homogeneous then there does exist such an aggregate utility function. This allows one to define the "standard of living" in a consistent manner. (Received May 8, 1958.)

547-46. J. E. Maxfield and Czerna Flanagan: Estimates of the roots of certain polynomials.

In the stability analysis of certain linear systems for servo-mechanisms, guided missiles, and general electronic circuits it is usually expedient to simplify the differential system by neglecting certain higher order derivatives. Frequently it is necessary, after studying the reduced system, to reintroduce the higher order derivatives and study the system as a whole. This paper gives a way of reintroducing these terms one at a time so that the analyst can observe their effects. Let $f(x)$ be a polynomial of degree n with real coefficients and known roots. Let $g(x) = \epsilon x^{n+1} + f(x)$, where ϵ is a small real number. Estimates for the roots of $g(x)$ are derived. One formula is given for the roots of $g(x)$ arising from the simple roots of $f(x)$, another for those arising from the multiple roots of $f(x)$, and a third estimating the new root introduced by raising the degree. (Received May 12, 1958.)

547-47. C. B. Morrey, Jr.: Elliptic differential equations with Holder-continuous coefficients.

Let L be a linear elliptic operator of order 2m with variable coefficients which merely satisfy uniform Hölder conditions with exponent μ, $0 < \mu < 1$, on the closure $D \cup \partial D$ of a domain D of class $C^{2m+\mu}$ having boundary ∂D. It is shown that if λ does not belong to a set $\mathcal{L}$ of characteristic values which has no finite limit point, then the Dirichlet problem for the equation $Lu + \lambda u = f$ has a unique solution u of class $C^{2m+\mu}$ on $D \cup \partial D$ for each $f \in C^{\mu}$ on $D \cup \partial D$; if $f \in \mathcal{L}_2$, this problem has a unique solution $u \in H_{2m}$ on D. The idea is to set $u = Ug$ where U is an integral operator involving a sort of parametrix and is so chosen that $Ug = 0$ only if $g = 0$, U is completely continuous, and $Lu = g - Tg$ where T is completely continuous. Then $Lu + \lambda u = f \leftrightarrow g - Tg + \lambda Ug = f$ and the only solutions u are of the form Ug. Corresponding results are obtained for the strongly elliptic systems of Nirenberg. (Received May 5, 1958.)

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August 26-29, 1958

548-1. I. S. Gál: On the topological foundation of the theory of distributions.

Let X be a unitary space with inner product (x,y) and let $\mathcal{A}$ be a family of linear operators A: $X \longrightarrow X$ with adjoints A^*. The space X need not be complete and the operators A need not be continuous with respect to $\|x\| = (x,x)^{1/2}$. The linear space X forms a topological vector space under the family of semi-norms $\|y\|_x = |(x,y)|$, where x varies over X. Let $\bar{X}$ denote the completion of X with respect to the uniform structure $\mathcal{U}$ induced on X by the semi-norms $\|y\|_x$. The operators A are all uniformly continuous with respect to $\mathcal{U}$ and so they can be extended to uniformly continuous operators A: $\bar{X} \longrightarrow \bar{X}$. The elements of $\bar{X}$ are called distributions or generalized functions. Let Y be a set with uniform structure $\mathcal{V}$ such that $X \subset Y$ and the trace of $\mathcal{V}$ on $X \times X$ is $\mathcal{U}$. If X is dense in Y the completions of X and Y coincide and so some of the elements of $\bar{X}$ can be identified with elements of Y. The elements of $\bar{X}$ - Y will be called ideal elements or proper distributions relative to Y. For each fixed $x \in X$ the inner product (x,y) is uniformly continuous in the variable $y \in X$ so that (x,y) can be extended to $X \times \bar{X}$. The extension is uniformly continuous in $y \in X$ but in general it is discontinuous in $x \in X$ for some proper distributions $y \in \bar{X} - X$. (Received June 9, 1958.)

548-2. W. O. Portmann: A derivative for Hausdorff-analytic functions.

The concept of an analytic function of a hypercomplex variable, proposed by F. Hausdorff (Leipziger Berichte, vol. 52 (1900) pp. 43-61), is extended to include a definition of a derivative. A study is then made of Hausdorff-analytic functions on the total matric algebra, showing that all matric functions with analytic components are Hausdorff-analytic and exhibiting a form for computing their derivatives; also, if a function of a matrix is Hausdorff-analytic then it has a derivative as defined by R. F. Rinehart (Proc. Amer. Math. Soc. vol. 8 (1957) pp. 329-335) and the two derivatives are equal. It is then shown that a matric function arising from a scalar function is Hausdorff-analytic and that the derivative, as here defined, of such a function is equal to the matric function arising from the derivative of the scalar function. (Received June 9, 1958.)

548-3. J. E. Kist: Locally o-convex spaces. III.

The following proposition generalizes a result recently reported in these abstracts (Bull. Amer. Math. Abstract 63-6-702). If a partially ordered vector space E is equipped with a sequentially complete locally o-convex topology T for which P, the positive cone of E, is a closed set, then the following statements are equivalent: (a) Every homomorphism of E with range in any locally o-convex space is continuous. (b) Every homomorphism of E into any locally o-convex space which takes P-bounded sets into bounded sets is continuous. (A P-bounded set is a bounded subset of P; all other notation as in Bull. Amer. Math. Soc. Abstracts 63-6-701 and 63-6-702.) (c) Every o-convex, symmetric, and absorbing subset of E is a neighborhood of the origin. (d) Every o-convex, symmetric, and absorbing set which absorbs all P-bounded sets is a neighborhood of the origin. (e) $T = \mathcal{T}_\eta(E,E')$, and every positive linear functional on E is continuous. (f) $T = \mathcal{T}_\eta(E, E')$, and every positive linear functional which takes P-bounded sets into bounded sets is continuous. (g) No strictly finer locally o-convex topology on E has the same P-bounded sets. (h) E is the o-inductive limit, with respect to the inclusion maps, of the normed locally o-convex spaces $\{E_a: a \in P\}$, where E_a, the linear space generated by the segment [-a,a], is normed by taking the latter set as unit ball. (Received June 4, 1958.)

548-4. W. R. Hutcherson: A note on a surface of order seventeen.

A surface of order 17, invariant under an involutory homography, $x_1':x_2':x_3':x_4' = x_1;\ \epsilon x_2:\ \epsilon^{226}x_3:\ \epsilon^{156}x_4$ where $\epsilon^{241} = 1$, is investigated by methods used by Lucien Godeaux [Sur quelques surfaces algebriques representant des involutions cycliques, Bulletin de l'Acad. Roy. de Belgique, 1951, pp. 1106-1119] and the writer [Su alcune involuzioni cicliche dotate periodo non inferiore a 157, Le Matematiche (Catania) 1955, pp. 15-17]. The coincidence point (1,0,0,0) on this surface was found to be a point of multiplicity fifteen, on a certain invariant curve in the tangent plane, by using either the Method of Godeaux or the classical method. (Received June 4, 1958.)

548-5. W. H. Pell and L. E. Payne: Application of generalized axially symmetric potential theory to Stokes flow problems.

If an axially symmetric body is placed in a uniform flow of speed U of an infinite, viscous, incompressible fluid, with the body axis parallel to the direction of the uniform flow, then the differential equation for the stream function

ψ is $L^2_{-1}\psi = 0$, where $L_{-1} = \partial^2(\ )/\partial x^2 + \partial^2(\ )/\partial r^2 - r^{-1}\partial(\ )/\partial r$, and x and r are axial and radial coordinates, respectively. On the body, ψ and its normal derivative must vanish, and $\psi \longrightarrow Ur^2/2$ infinitely far from the body. The solutions u of $L_k u = 0$, where $L_k = \partial^2(\ )/\partial x^2 + \partial^2(\ )/\partial r^2 + k/r\partial(\ )/\partial r$, k real, have been called generalized axially symmetric potentials, and have been studied by A. Weinstein [Bull. Amer. Math. Soc. vol. 59 (1953) pp. 20-38]. More recently, Weinstein [Ann. Mat. Pura Appl. vol. 39 (1955) pp. 245-254] and Payne [Tech. Note BN-122, Inst. fôr Fluid Dy. and Appl. Math., Univ. of Md., 1958] have discussed the representation of solutions of $L_\beta L_\alpha u = 0$. Using these results, the problem of Stokes flow about a lens-shaped body athwart a uniform stream has been solved. (Received June 2, 1958.)

548-6. J. J. Greever: Stationary points for finite transformation groups.

Let X be a finite-dimensional compact Hausdorff space, let G be a group of homeomorphisms of X into itself and let F_G be the set of points in X fixed under each element of G. Let p, q and r be primes, let α and β be non-negative integers and let R be the field of rationals. Homology groups are to be understood in the sense of Čech, the 0-dimensional groups being reduced. Suppose hat each integral Čech cohomology group of F_H is finitely generated whenever H is a subgroup of G. Theorem I: If each $H_i(X;R)$ is trivial and G is either of order pq or abelian and of order $p^\alpha q$, then F_G is nonempty. Theorem II: If each $H_i(X;Z_p)$ is trivial and ORD G $= p^\alpha q$, then F_G is nonempty. Theorem III: If each $H_i(X;Z_r)$ is trivial and G is either of order pqr where $p < q < r$ or abelian and of order $p^\alpha q r^\beta$, then F_G is nonempty. Thus, for example, one has Corollary: If each $H_i(X;Z)$ is trivial and either G is abelian and ORD G < 420 or G is non-abelian and $36 \neq$ ORD G < 60, then F_G is nonempty. (Received June 2, 1958.)

548-7. D. E. Spencer: On electrodynamics in material media.

An earlier postulational formulation (Moon and Spencer, A postulational approach to electromagnetism, J. Franklin Inst. vol. 259 (1955) p. 293) of Maxwellian theory is now extended to the general case of material media. The electromagnetic field vectors are defined in terms of scalar and vector potentials, ϕ and $\underset{\sim}{A}$, while material media are specified in terms of the polarization and magnetization vectors $\underset{\sim}{P}$ and $\underset{\sim}{M}$. The whole development is based on three physical postulates: conservation of charge, retardation, and the definition of orce in terms of the field vectors. A generalized set of partial differential equations results. These equations are valid for the general case of material media and reduce to Maxwell's equations in appropriate special cases. Maxwell's equations for div $\underset{\sim}{B}$ and curl $\underset{\sim}{E}$ are found to be of general validity, but his equations for div $\underset{\sim}{D}$ and curl $\underset{\sim}{H}$ must be generalized by the inclusion of additional terms when effects of free charge, bound charge, and spinning charge are included. The generalized formulation is invariant under transformation between observers whose motion is accelerated in an arbitrary fashion if the retardation terms are suitably interpreted. (Received May 21, 1958.)

548-8. D. F. Dawson: Convergence of Stieltjes type continued fractions.

If b is a complex number sequence, let g(b) denote the continued fraction $1/\overline{b_1 + 1}\,/\overline{b_2 + 1}\,/\overline{b_3 + \dots}$. Let $\{g_p\}$ denote the sequence of approximants of g(b). Theorem. If $\{g_{2p-1}\}$ ($\{g_{2p}\}$) converges absolutely and $\{g_{2p}\}$ ($\{g_{2p-1}\}$) converges, then g(b) converges if, and only if, b satisfies one of the following conditions: (1) the series $\sum|b_{2p-1}|$ (the series $\sum|b_{2p}|$) diverges, (2) the sequence $\{b_2 + b_4 + \dots + b_{2p}\}$ (the sequence $\{b_1 + b_3 + \dots + b_{2p-1}\}$) contains an unbounded subsequence. This theorem extends a previous result of the author (Notices Amer. Math. Soc. vol. 5, no. 1, p. 89). Let D_q denote the denominator of g_q. Theorem. If $\sum|b_{2p+1}||D_{2p}|^2$ converges and there exists a positive integer k such that $\{g_p\}_{p=k}^{\infty}$ is a bounded complex number sequence, then g(b) converges if, and only if, b satisfies condition (H) (H. S. Wall, Analytic theory of continued fractions, New York, 1948, p. 122). This theorem is used in constructing a simple proof of a theorem of Scott and Wall (Theorem D, p. 554, Amer. J. Math. vol. 46, July, 1947). (Received May 5, 1958.)

548-9. Donald Bratton: The theory of automata.

A theory of automata is constructed. A machine M: $A \longrightarrow B$ is the structure defined by giving a set S (called the set of states of M) and a function $f: S \times A \longrightarrow S \times B$ (called the transition function of M). When an initial state is prescribed, the structure is called a device. The input-output transformation effected by a device is embodied in a function $\alpha: \mathcal{L}(A) \longrightarrow \mathcal{L}(B)$, called the transfer function of D. ($\mathcal{L}(A)$ denotes the free semigroup generated by A). Two devices with the same transfer function are called equivalent. If there exists a homomorphism between two devices, they are equivalent. A universal machine U: $A \longrightarrow B$ is constructed. Each machine M: $A \longrightarrow B$ has a unique homomorphism ϕ: $M \longrightarrow U$. The reduced form of a device D is defined, with the help of U, gen alizing the notion and results of E. F. Moore [Automata studies, Princeton University Press, 1956]. A device D: $A \longrightarrow B$ is called an automaton when the set of states is finite. [A. J. Burks and H. Wang, J. Assoc. Comput. Mach. vol. 4 (1957) pp. 193-218, 279-297]. A necessary and sufficient condition that a function $\alpha: \mathcal{L}(A) \longrightarrow \mathcal{L}(B)$ be the transfer function of an automaton, is given. (Received April 23, 1958.)

548-10. George Springer: Interpolation problems for functions of several complex variables.

Let D be a bounded domain in the space C_n of n complex variables and let f be a function which is holomorphic in $\bar{D}$ and vanishes on a set M. Let $L^2(D)$ denote the class of functions g which are holomorphic in D such that $\|g\|^2 = \int_D |g|^2 dV < \infty$. The problem of finding the function in $L^2(D)$ which assumes on M the same values as a given function g and which has minimum norm is solved in terms of the weighted Bergman kernel function K_f with weight function $|f|^2$. If $L^2_M(D)$ represents the space of functions of the form $cg + f\phi$ where $\phi \in L^2(D)$, then $L^2_M(D)$ is a linear space for which the Bergman kernel K_M can be defined. The following relation exists between K_M and K_f for $T \in M$ and $\zeta \in D$: $[K_M(Z,\bar{T})/K_M(T,\bar{T})]g(T) = g(Z) - f(Z)\int_D g(W)\overline{f(W)}K_f(Z,\bar{W})dV_W$. Similar considerations are carried out for a class of meromorphic functions in a domain with distinguished boundary surface in C_2. (Received April 7, 1958.)

548-11. R. S. Pierce: Translation lattices. II.

A distributive translation lattice (or d.t.l.) of functions is a system of bounded, real valued functions which is closed under addition of constants (translation) and both lattice operations. Such a d.t.l. generates a unique minimal ℓ-algebra of functions. Theorem. Let T be a d.t.l. of functions which contains the zero function. Let A be the ℓ-algebra of functions generated by T. Let A_1 be an ℓ-algebra of bounded real valued functions, containing all constant functions. Suppose Φ is a (1-1) mapping of A into A_1, preserving translation, lattice operations and the zero function. Then Φ has a unique extension to an ℓ-algebra isomorphism of A into A_1. The proof of this theorem rests on the following fact: if T is a uniformly complete d.t.l. of functions and $\alpha \geqq 0$, $\beta \geqq 0$ are real numbers such that $\alpha + \beta = 1$, then for any $f, g \in T$, $\alpha f + \beta g = \text{l.u.b.} [(f - \lambda\beta) \wedge (g + \lambda\alpha) | \lambda \text{ real}] \in T$ (the bound exists in T). Corollary. Let Y be a closed subset of the compact Hausdorff space X. Suppose $T \subseteq C(Y)$ is closed under pointwise join, meet, translation, contains the zero function and separates points. Then the functions of T can be simultaneously extended to C(X), preserving translation, join, meet and zero only if Y is a retract of X. The case $T = C(Y)$ gives a theorem of Yoshizawa (Proc. Imp. Acad. Tokyo vol. 20 (1944)). (Received April 2, 1958.)

548-12. P. T. Church: On global cluster sets of meromorphic functions.

Given f mapping $|z| < 1$ into the sphere S, the global cluster set of f, denoted C(f), is the set of points y on S such that there exists a sequence of points z_n, $|z_n| < 1$ and $|z_n| \to 1$, with $f(z_n) \to y$. If f is continuous, then C(f) is a continuum. Collingwood and Cartwright (Acta Math., 1952) asked whether every continuum on S is C(f) for a function f meromorphic on $|z| < 1$. D. B. Potyagailo (Dokl. Akad. Nauk SSSR, 1952) and W. Rudin (J. London Math. Soc., 1955) independently gave as counter-example the same continuum (not a peano space). The author has a peano space which is not C(f) for any meromorphic f. He also has a topological sufficient condition, different from one of Potyagailo, for a continuum to be so representable. If f is meromorphic on $|z| < 1$ and continuous on $|z| \leqq 1$, the image of $|z| = 1$ is a peano space. G. MacLane (Proc. Amer. Math. Soc., 1955) asked whether every peano space is so representable. The author's example thus answers this question in the negative also. (Received June 6, 1958.)

548-13. B. R. Gelbaum: Tensor products of Banach algebras.

Let: A_1, A_2 be commutative Banach algebras; $A_T = A_1 \otimes A_2$ have a norm satisfying $\|a_1 \otimes a_2\|_T = \|a_1\|_{A_1} \cdot \|a_2\|_{A_2}$; A be the completion of A_T in this norm. Then A has a maximal ideal space $\mathfrak{M}$ homeomorphic with $\mathfrak{M}_1 \times \mathfrak{M}_2$, ($\mathfrak{M}_i$ the maximal ideal space of A_i, $i = 1, 2$). The 1-1 map between $\mathfrak{M}$ and $\mathfrak{M}_1 \times \mathfrak{M}_2$ can be constructed in more general situations. Compare A. Hausner [Bull. Amer. Math. Soc. Abstract 62-4-493, Proc. Amer. Math. Soc., April, 1957], G. P. Johnson [Bull. Amer. Math. Soc. Abstract 62-4-458, Trans. Amer. Math. Soc., to appear], for special cases. If A_1 is $L^1(G)$, G loc. cpct. abel., and if A_2 has an identity and an involution, there is a 1-1 correspondence between epimorphisms ρ: $A \to A_2$ and unitary representations of G into the multiplicative group of A_2. The compact-open and weak* topologies of $\tilde{G}$ (the group of such representations) are identical. There is a natural mapping ϕ: $\tilde{G} \to C(\hat{G}, \mathfrak{M}_2)$. If A_2 is semisimple ϕ is 1-1; not conversely. Under mild hypotheses, ϕ is an isomorphism iff. A_2 and $C(\mathfrak{M}_2)$ are equivalent. Partial sponsorship by Air Force Defense Command. (Received June 16, 1958.)

548-14. Iwao Sugai: Extension of methods used in solving Riccati's nonlinear differential equation.

A new transformation for reducing Riccati's nonlinear differential equation to a linear equation has been found. While the conventional transformation does not reduce a second degree first order inhomogeneous equation, this newly found transformation does. The homogeneous case of the Riccati's equation is generalized and two transformations are also extended to this equation. (Received June 18, 1958.)

548-15. Evelyn Frank: New continued fraction expansions for the ratios of Heine functions.

New continued fraction expansions for the ratios of two contiguous Heine functions are described here in detail. The characterizing feature of each of these expansions is the fact that it is equal throughout the finite z-plane to the ratio of the functions which generates it. Certain equal continued fractions are studied, and special continued fraction expansions are considered. (Received June 19, 1958.)

548-16. W. F. Stoll: Meromorphic maps into complex spaces.

Let G and H be complex spaces, $A \subset G$, and $M = G - A$ closed and thin. A holomorphic map τ: $A \longrightarrow H$ is called (weakly) meromorphic if $\sum(P,L) = \{Q | \lim_{\nu \to \infty}(P^\nu, \tau(P^\nu)) = (P,Q)$ and $P^\nu \in L \cap A$ consists of one (at most one) point for every $P \in M$ and every one-dimensional complex submanifold L of G when $L \cap M = \bar{L} \cap M = \{P\}$. According to R. Remmert τ is called R-meromorphic if $\overline{\{(P, \tau(P)) | P \in A\}}$ is analytic in $G \times H$. R-meromorphic maps are weakly meromorphic. The singularities of a meromorphic map are contained in a countable union of locally analytic sets of codimension 2. When H is a complex projective space, then τ is meromorphic (resp. R-meromorphic) if and only if τ is given by meromorphic functions. τ is called gapless if a subsequence of $\tau(P^\nu)$ converges for every convergent sequence $P^\nu \in A$ with $\lim_{\nu \to \infty} P^\nu \in M$. The space H is called M-complete if for every $Q_0 \in H$ functions $f_1, \ldots, f_k$ exist which are meromorphic in H and holomorphic in a neighborhood U of Q_0, such that $Q \in U$ and $f_\nu(Q) = f_\nu(Q_0)$ for $\nu = 1, \ldots, k$ implies

$Q = Q_0$. If H is M-complete and τ is gapless, then the following conditions are equivalent: (i) τ is meromorphic; (ii) τ is R-meromorphic; (iii) $f\tau$ is meromorphic on G if f is meromorphic on H and the inverse image $\tau^{-1}(N_f)$ of the set N_f of poles and points of indeterminations of f is thin. (Received June 24, 1958.)

548-17. H. G. Tucker: A generalization of the Glivenko-Cantelli theorem.

Theorem: Let $X_1, X_2, \ldots, X_n, \ldots$ be any (strictly) stationary sequence of random variables. Let $\mathcal{J}$ denote the invariant sigma-field of events determined by this sequence, let $\{F_n(x)\}$ denote the sequence of empirical distribution functions, and let $F(x|\mathcal{J})$ denote the conditional distribution function of X_1 given $\mathcal{J}$. Then $F_n(x) \longrightarrow F(x|\mathcal{J})$ as $n \longrightarrow \infty$ uniformly in x with probability one. (Received June 25, 1958.)

548-18. R. P. Kanwal: Absorption of sound waves in a uniform stream.

The object of this paper is to discuss the absorption of sound waves in a stream moving with a uniform velocity U, in the O'seens approximation. Both the vorticity waves and expansion waves are considered. Since the propagation of linearized vorticity waves is independent of bulk viscosity and heat conduction, Lin's analysis [Studies in Mathematical Mechanics, presented to Richard Von Mises, 1954, Academic Press, New York,] for the vorticity waves in an incompressible perfect fluid for cylindrical waves is shown to describe the absorption of these waves in the general class of fluids considered in this paper. These results are then extended to three-dimensional flows. As regards the expansion waves it is found that the absorption and dispersion of these waves is again governed by Langevin's generalization of Kirchhoff's biquadratic equation which has been analyzed by Truesdell [J. Rat. Mech. Anal., vol. 2, 1953]. The absorption coefficient for plane waves in this case is less than that found by Truesdell in still medium. In our extension of Truesdell's results to curved waves [J. Acoust. Soc. Am., vol. 29, 1957] we had shown that the absorption coefficient for cylindrical and spherical waves is that for plane waves. In the present case, however, there is a small region behind the curved oscillator, the source of disturbances, where the disturbances remain finite even at large distances. For the rest of the region their behavior is the same as that for the plane waves. (Received June 24, 1958.)

548-19. T. D. Riney: Coefficients in certain asymptotic factorial expansions of the second kind.

Let p and q denote integers $\geqq 0$ with $p \leqq q$, then J. G. van der Corput (Nederl. Akad. Wetensch Proc. Ser. A. vol. 60 (1957) pp. 337-351) has shown that the function $(g(w))^{-1} = (\Gamma(w + \rho_0)\Gamma(w + \rho_1) \dots \Gamma(w + \rho_q)) \cdot (\Gamma(w + \sigma_1)\Gamma(w + \sigma_2) \dots \Gamma(w + \sigma_p))^{-1}$ possesses for large $|w|$ in a right half-plane an asymptotic expansion of the form $(g(w))^{-1} = \alpha^{-\alpha w} \cdot \{\sum_{m=0}^{M-1}\gamma_m \Gamma(\alpha w + \beta - m) + O(\Gamma(\alpha w + \beta - M))\}$. Moreover, van der Corput gives an inductive formula for γ_m depending on all the previously calculated coefficients $\gamma_0, \dots, \gamma_{m-1}$. In this paper a simpler formula for these coefficients is derived by the method used previously for determining the coefficients in the asymptotic representation of g(w) (to be published in the Trans. Amer. Math. Soc.). A function $\phi(t)$ defined as an inverse transform of $(g(w))^{-1}$ is introduced and shown to satisfy a differential equation of Fuchsian type. The expansion of $\phi(t)$ about one of the regular singular points of the differential equation is shown to generate the γ_m. By this procedure a finite recursion formula of fixed length q is obtained for the coefficients γ_m. (Received June 27, 1958.)

548-20. R. L. Dunn: Set-valued functions with and without fixed points. Preliminary report.

The notation $\mu(F(x)) = a$ indicates that F(x) consists of a points. The function F is of multiplicity $(a_1, \dots, a_n) = S$ if for any $x \in X, \mu(F(x)) \in S$ and if for any $a_i \in S$, there is an x such that $\mu(F(x)) = a_i$. The following result is obtained: If X has a 2-cell as a retract, then for any set of positive integers S except $\{n\}$, $\{1,n\}$, $\{2,3n + 1\}$, and $\{2,3n + 2\}$ $(n \geqq 1)$, there is a continuous function $F: X \to X$ without fixed points such that $\mu(F) = S$. The existent knowledge may be summarized as follows: Consider any continuous self-map F of the p-cell $(p > 1)$ such that $\mu(F) = S$. (1) If $S = \{n\}$ or $\{1,n\}$ then F has a fixed point. (2) If $S = \{2,3n + 1\}$ or $\{2,3n + 2\}$ the question remains open. (3) If S is any other set of positive integers, then F does not necessarily have a fixed point. Hence, for cases other than (1) (and perhaps (2)) additional restrictions must be put on F to ensure the existence of fixed points. Some theorems and conjectures of this nature are presented. For example, if $\mu(F) = (m,n)$ $(0 < m \leqq n)$ and the set of all x such that $F(x) = n$ is connected then F has a fixed point. (Received June 27, 1958.)

548-21. Morris Newman: Further identities and congruences for the coefficients of modular forms.

A class of two-term recurrence formulas depending on a prime parameter p are derived for the coefficients $p_r(n)$ defined by $\sum p_r(n)x^n = \prod(1 - x^n)^r$, where the summation extends from 0 to ∞, the product from 1 to ∞, and r is odd, $1 \leqq r \leqq 23$. These are used to determine congruence properties for the coefficients of various modular forms. For example, if p(n) is the number of partitions of n, it is shown that $p(84n^2 - (n^2 - 1)/24) \equiv 0 \pmod{13}$, provided $(n,6) = 1$. (Received June 30, 1958.)

548-22. Chandler Davis: Various averaging operations onto subalgebras.

$\mathcal{A}$: finite-dimensional complex *-algebra. Given any orthogonal projections P_i adding to 1, the function taking each A to $\sum P_i A P_i$ is called a pinching. An averaging is a function $\mathcal{C}$ on $\mathcal{A}$ onto *-subalgebra $\mathcal{R}$ which is positive linear idempotent and satisfies $\mathcal{C}(A\,\mathcal{C}B) = (\mathcal{C}A)\mathcal{C}B$. Prime denotes commutor relative to $\mathcal{A}$. Main theorems: 1. If $\mathcal{R} \supset \mathcal{R}'$, there is just one averaging onto $\mathcal{R}$, a pinching. 2. If $\mathcal{R}' = \mathcal{A}'$, the homomorphisms included in the set of averagings onto $\mathcal{R}$ are exactly the set's extreme points. 3. Every averaging is obtainable as a pinching followed by a convex combination of homomorphisms. (Received June 30, 1958.)

548-23. B. L. Reinhart: Line element fields on the torus. Preliminary report.

A line element field on the torus is a C^1 cross-section of the projective tangent bundle of the torus, or equivalently, a map F: $T^2 \longrightarrow P^1 = S^1$. Such maps are classified up to differentiable homotopy by the induced map $F_{\#}$ on the fundamental groups, hence by a pair of integers (i,j). A line element field also defines a family of integral curves. Theorem. The homotopy class of a closed integral curve belongs to the kernel of $F_{\#}$; if $F_{\#} \not\equiv 0$, it is the generator of the kernel. To prove this, it is shown that such a curve is regularly homotopic to a curve with constant tangent line element; to show this, a winding number technique is used to make explicit the results of Smale (Trans. Amer. Math. Soc. vol. 87 (1958) pp. 492-512). The notion of cycle without contact may be generalized to cycle of minimal contact, defined as a cycle belonging to some fixed homotopy class not containing a closed integral curve; let μ count the (signed) number of points of contact. Let λ be the rotation number. Theorem. With a

suitable choice of basis for $\pi_1(T^2)$, $\mu = i$; if $\mu \neq 0$, then $\lambda = -j/i$. Thus, f $\mu \neq 0$, μ and λ form another complete set of homotopy invariants for F.
(Received June 30, 1958.)

548-24. Ernst Snapper: Multiples of divisors.

Let X be an irreducible, normal, projective variety, defined over an arbitrary groundfield, and with sheaf of local rings L. Let F be an algebraic coherent sheaf of X. If D is a divisor of X, we denote the sheaf $L(D) \otimes_L F$ by F(D), where the sheaf L(D) is defined as usual. The theorem we proved is: If $D_1,\ldots,D_n$ are divisors of X which are locally linearly equivalent to zero, the Euler-Poincaré characteristic $\chi(X, F(m_1 D_1 + \ldots + m_n D_n))$ is a polynomial in $m_1,\ldots,m_n$ with rational coefficients, for all rational integers $m_1,\ldots,m_n$. The degree of this polynomial does not exceed the dimension d of the support of F. Examples show that this degree may be less than d, and that the theorem is false for divisors which are not locally linearly equivalent to zero. The theorem is formulated in terms of line bundles, which causes it to hold also for projective varieties which are neither irreducible nor normal. The conjecture that the theorem holds for all complete algebraic varieties is amply supported by the paper. The coefficients of the above polynomials are investigated in the paper, Polynomials associated with divisors. (Received June 30, 1958.)

548-25. M. S. Klamkin and D. J. Newman: On the number of distinct zeros of polynomials.

Although a polynomial of degree N must have N zeros, we are well aware of the fact that some of them or all of them may coincide. Consequently, nothing much can be said about the number of distinct zeros of an arbitrary polynomial. If, however, we consider related pairs of polynomials, certain sharp estimates of this number can be obtained. Furthermore, from these very simply proven estimates, corollaries on a class of polynomial-Diophantine equations can be extracted which seem quite difficult to prove otherwise. (Received July 1, 1958.)

548-26. G. F. Feeman and C. C. Hsiung: Characterizations of Riemann n-spheres.

Let V^n be an orientable n-dimensional ($n \geqq 2$) Riemannian manifold of class C^3, with an (n-1)-dimensional boundary V^{n-1}, imbedded in a contractible region of an (n + 1)-dimensional Riemannian manifold V^{n+1}. The authors first derive some integral formulas for V^n, and then deduce some simple conditions on the mean curvatures of a closed V^n for V^n to be a Riemann n-sphere, that is, for every point of V^n to be umbilical with respect to V^{n+1}. By using different methods, one of the authors has obtained [C. C. Hsiung, Math. Scand. vol. 2 (1954) pp. 286-294; Pacific J. Math. vol. 6 (1956) pp. 291-299] all of these formulas and theorems in the case where V^{n+1} is Euclidean and V^n is of class C^2 instead of C^3, and some of these formulas and theorems in the case where V^{n+1} is of constant Riemannian curvature. (Received July 2, 1958.)

548-27. M. E. Mahowald: On discontinuous functions.

In Variational measures, (Trans. Amer. Math. Soc. vol. 83 (1956) pp. 205-221), Maurice Sion constructs several classes of measures on the real line for which open sets are measurable and obtains some theorems concerning sets which are v^*-measurable for all v in a class. In particular, he shows that, for his class M_2 (Def. 8.5 of his paper) all projective sets are measurable. Therefore, according to a theorem of Gödel, Lebesgue measure is not in M_2. From this conclusion it can be shown that there exists a function which is discontinuous on every set having positive outer measure without the hypothesis of the continuum. Also, Sion remarks that it would be interesting to prove his result directly and this is done under an additional hypothesis. These two results follow from the Theorem: These two statements are equivalent: (i) A measure, μ, is not in M_2. (ii) There exists a real-valued function of a real-variable which is discontinuous on every set, E, such that $\mu^*(E) > 0$. (Received July 3, 1958.)

548-28. Albert Wilansky and Karl Zeller: FH spaces and intersection of FK spaces.

Assume a fixed Hausdorff space H. An FH space is a subset of H with operations which make it into an (F) space with topology stronger than that of H. THEOREMS: 1. Given the linear operations, the topology is unique. 2. If $A \subset B$, the topology of A is stronger. 3. Of countably many FH spaces the intersection is, and the union (assuming no largest space) is not, an FH space (assuming compatibility). 4. A continuous linear functional on the intersection of a nested sequence of spaces is continuous on one of them, and a basis for each space is a basis for the intersection. With H = s (space of all sequences) we have the known FK spaces whose theory can be further developed in the FH setting. An example is given of nested domains of perfect (i. e. $\{\delta^n\}$ is fundamental) summability matrices whose intersection is the domain of a (necessarily row infinite) matrix. For domains with $\{\delta^n\}$ as basis this is impossible. (Received July 3, 1958.)

548-29. S. K. Berberian: Note on a theorem of Fuglede and Putnam.

Denote by (FT) the following theorem of B. Fuglede: If A, N are operators in a Hilbert space, N is normal, A is bounded, and $AN \subset NA$, then $AN^* \subset N^*A$. Denote by (PT) the generalization by C. R. Putnam: if M, N are normal, A is bounded, and $AM \subset NA$, then $AM^* \subset N^*A$ (see Putnam's paper in the Amer. J. Math. vol. 73, pp. 357-362). A simple 2×2 matrix argument is given for deducing (PT) from (FT), thus avoiding the repetition of Fuglede's argument employing the spectral projections. The (PT) result is established for the regular ring of a finite AW*-algebra of Type I, with all three of A, M, N allowed to be unbounded (see the author's paper in Annals of Mathematics, vol. 65, pp. 224-240); in this argument the classical (FT) is replaced by an algebraic lemma on matrices. (Received July 3, 1958.)

548-30. E. G. Kimme: Note on the convergence of sequences of stochastic processes.

Let $\{X_n(t,w), 0 \leqq t \leqq 1\}$ be a sequence of stochastic processes with independent increments. Using the definition of "uniform convergence in distribution" constructed by the author in a paper in the January 1957 Transactions, the following two conditions are shown to be equivalent: (1) The sequence of stochastic processes converges uniformly in distribution; (2) for fixed N, the random variables $\{\sup\{X_n(S): t_{j-1} < S \leqq t_j\}, \inf\{X_n(S): t_{j-1} < S \leqq t_j\}, 1 \leqq j \leqq N\}$ converge in distribution uniformly with respect to $0 \leqq t_1 \leqq t_2 \leqq \dots \leqq t_N$. In case $\{X_n(t,w), 0 \leqq t \leqq 1\}$ is of the form $X_n(t) = \sum\{X_{nk}: 1 \leqq k \leqq [tk_n]\}$, $\{X_{nk}, 1 \leqq k \leqq k_n\}$ being a row-independent array of infinitesimal random variables, conditions (1) and (2) are equivalent to: (3) Gnedenko's general convergence theorem for such arrays, applied to the sequence $\{X_n(t,w), n \geqq 1\}$ for fixed t, shall hold uniformly with respect to $t \in [0,1]$. These are a sharpening of the author's results in the paper referred to above. (Received July 3, 1958.)

548-31. Murray Gerstenhaber: On nilalgebras and linear varieties of nilpotent matrices. III.

In a previous paper the author proved that if V is a linear space of nilpotent $n \times n$ matrices over a field of at least n elements then $\dim V \leqq n(n-1)/2$, and if $\dim V = n(n-1)/2$ then V is similar to the set T_n of all $n \times n$ triangular matrices, i.e., matrices with zeros on and below the diagonal. Here these results are extended as follows: Let V be a linear space of nilpotent $n \times n$ matrices over a field of at least n elements and suppose $A \in V$ implies rank $A \leqq \rho$. Then $\dim V \leqq [n(n-1) - (n-\rho)(n-\rho-1)]/2 = n\rho$. If dim V = n then there is a set of $\rho + 1$ algebras $\mathfrak{A}_{\rho\mu}$, $\mu = 1, \ldots, \rho + 1$ of triangular matrices such that V is similar to one of these, except for $\rho = n - 1$ when there is only T_n, and the algebras $\mathfrak{A}_{\rho\mu}$ are not similar nor even isomorphic except $\mathfrak{A}_{11} \cong \mathfrak{A}_{12}$, these being zero algebras of the same dimension. As a trivial corollary it is shown that the algebra T_n / T_n^{n-1} is not representable by an algebra of $n \times n$ matrices if $n \geqq 4$. (Received July 7, 1958.)

548-32. M. H. Martin: Linear and nonlinear boundary problems.

The uniqueness of solutions to boundary problems: $\Delta u = 0$ in S, $u_n = h(s)f(u)$ on C is studied. C is the boundary of S, s is the arc length of C, and u_n denotes the external normal derivative of u. Carleman [Math. Z. (1921) p. 37] demonstrated uniqueness if $hf' < 0$. When this condition is not met, uniqueness no longer prevails, but the following results pertain. If u_2 is a harmonic solution of the boundary problem $u_n = h(s)u$ on C, any other such solution u_1 for which the ratio $\lambda = u_1/u_2$ is regular in S + C, is linearly dependent on u_2. If u_2 is a harmonic solution of the boundary problem $u_n = h(s)u^{1+p}$ on C $(p = 2, 4, \ldots)$, no other such solution u_1 exists for which the ratio λ is regular and $|\lambda| < 1$. (Received July 7, 1958.)

548-33. W. F. Reynolds: Blocks with normal defect group.

Let G be a group of finite order $p^a h$, p prime, $h \not\equiv 0 \pmod p$. Let B be a p-block of G which has a defect group D, of order p^d, normal in G [Brauer, Math. Z. vol. 63 (1956) pp. 406-444]. Let χ, of degree x, be an ordinary character contained in B; let p^e be the degree of an irreducible constituent of the restriction of χ to D. Then $x = p^{a-d+e}y$, $y \not\equiv 0 \pmod p$. Hence a necessary and sufficient condition for B to contain an ordinary character of degree divisible by p^{a-d+1} is that D be non-abelian. If G = DT, where T is the centralizer of D, then there is a natural one-to-one correspondence between the ordinary characers of B and the irreducible characters of D, and B contains only one modular character. (Received July 7, 1958.)

548-34. Pasquale Porcelli: On the equivalence of weak and norm convergence in the space of function of bounded variation.

Let B(a,b) denote the collection of all functions f of bounded variation on the close interval (a,b). If for each f in B(a,b), N(f) denotes the total variation of f on (a,b), then N(f) is a norm and B(a,b) is a Banach space under this norm. Theorem: If $f_n \in B(a,b)$, $n = 1,2,\ldots$, and $f_n \longrightarrow 0$ (weakly), then $N(f) \longrightarrow 0$: i. e. weak and norm convergence are equivalent in B(a,b). (Received July 7, 1958.)

548-35. P. L. Chessin: On the reversion of power series.

Formulas for reversing power series as given by van Orstrand (Phil. Mag. vol. 19, 1910) display coefficients for the first thirteen powers in the reversed series, but are not suitable, in general, for computation either by hand or by modern high speed electronic data processing machines. Since the reversion of power series is a useful device in the analyses of current astronautical problems as well as in the study of order-disorder phenomena in cooperative assemblies (which gave rise to the author's interest in this area) and since at the same time no machine program seems to have been published, the author by modifying van Orstrand's results describes a procedure for the ready generation of the coefficients in the reversed series which is especially suitable for computational purposes. (Received July 7, 1958.)

548-36. D. S. Rim: Modules over finite groups and cohomology.

Let π be a finite group. For a π-module A the following properties may be considered: π-free, π-projective (or π-injective), weakly π-projective, cohomologically trivial (i.e. $H(\pi',A) = 0$ for all subgroups $\pi' \subset \pi$). Among others the key results are the following: Theorem I. A is π-projective (or π-injective) if and only if A is Z-free (or Z-divisible) and cohomologically trivial, where Z= rational integers. Theorem II. The following statements are equivalent: (a) A is cohomologically trivial; (b) For each p-Sylow subgroup π', there exists an integer i_p such that $H^{i_p}(\pi',A) = H^{i_p+1}(\pi',A) = 0$; (c) A has finite π-projective (or π-injective) dimension. Theorem III. If A or B is cohomologically trivial, then $H^n(\pi',A \otimes_Z B) = H^{n+2}(\pi',\mathrm{Tor}_1^Z(A,B))$, and $H^n(\pi'\mathrm{Hom}_Z(A,B))$

$= H^{n-2}(\pi', \mathrm{Ext}^1_Z(A,B))$ for all n and all $\pi' \subset \pi$. Now let $\mathcal{P}_\pi$(or $\mathcal{F}_\pi$) be the class of all finitely generated and π-projective (or π-free) modules. Then $\mathcal{P}_\pi$ mod $\mathcal{F}_\pi$ suitably treated forms a group denoted by $\Gamma(\pi)$. The following result is deduced: Theorem IV. Let π be a cyclic group of prime order p. Then $\Gamma(\pi)$ is isomorphic to the ideal class group of cyclotomic extension of degree p over the rational number field. (Thus, in particular, there exist finitely generated π-projective modules which are not π-free. This settles a problem of Cartan-Eilenberg.) (Received July 7, 1958.)

548-37. V. L. Shapiro: Generalized exterior differentiation.

Let R be an open set in n-dimensional Euclidean space with the usual Cartesian coordinate system $x = (x_1,\dots,x_n)$. With $\lambda = \lambda_1 \cdots {}_r$, let $\omega = \sum_{\lambda_1 < \cdots < \lambda_r} \omega_\lambda(x) dx^{\lambda_1} \vee \dots \vee dx^{\lambda_r}$ be a continuous differential r-form defined in R with $r < n$. Let $C_{\lambda_1 \dots \lambda_{r+1}}(x,t)$ be the r-sphere with center x and radius t lying in the r + 1-plane parallel to the $x_{\lambda_1} \dots x_{\lambda_{r+1}}$-plane, and let S_{r+1} be the r + 1-dimensional volume bounded by $C_{\lambda_1 \dots \lambda_{r+1}}(x,1)$. Furthermore let $C_{\lambda_1 \dots \lambda_{r+1}}(x,t)$ be oriented as usual with respect to the outward normal. Then the upper generalized exterior derivative of ω at the point x, $D^*\omega$, is defined as follows: $D^*\omega = \sum_{\lambda_1 < \cdots < \lambda_{r+1}} B^*_{\lambda_1 \dots \lambda_{r+1}}(x) dx^{\lambda_1} \vee \dots \vee dx^{\lambda_{r+1}}$ where $B^*_{\lambda_1 \dots \lambda_{r+1}}(x) = \limsup_{t \to 0} (t^{r+1} S_{r+1})^{-1} \int_{C_{\lambda_1 \dots \lambda_{r+1}}(x,t)} \omega$. The lower generalized exterior derivative of ω, $D_*\omega$, is defined in a similar manner using lim inf. The following theorem then holds: Suppose that (i) $D^*\omega$ and $D_*\omega$ are finite - valued in R, and (ii) there exists a continuous r + 1 - form ξ in R such that $\xi = D^*\omega = D_*\omega$ almost everywhere in R. Then $D^*\omega$ is continuous in R, and Stokes' Theorem with respect to ω and $D^*\omega$ holds for every oriented standard r + 1-manifold contained in R. In particular, for every r + 1-simplex σ contained in R, $\int_{\partial\sigma} \omega = \int_\sigma D^*\omega$. The essential idea in the proof of the theorem is to use the theory of multiple trigonometric series. (Received July 7, 1958.)

548-38. J. E. Keisler: A T-like plane continuum which is not a quasi-complex.

The author proves that the T-like plane continuum, X, formed by a ray spiraling down on a "T", is not a quasi-complex. For if X were a quasi-complex, one could find T-like covers a,g,h as in the definition of a quasi-complex [Lefschetz, Algebraic topology, Amer. Math. Soc. Colloquium Publications, vol. 27, 1942, p. 323], but if h is sufficiently finer than g, this is shown to be impossible. The work for this paper has been sponsored in part by AF-49 (638)-104. Received July 7, 1958.)

548-39. Valdemars Punga: Simplified simplex method applied to the solution of simultaneous linear equations.

Given a linear system (1) $AX = A_0$, where A is a $n \times n$ matrix and X and A_0 are n-dimensional column vectors. Designating the i-th column of matrix A by A_i, (1) can be written as (2) $A_1x_1 + A_2x_2 + ,\ldots, + A_nx_n = A_0$. Introducing the n artificial variables $x_{n+1},\ldots, x_{2n}$, (2) can be rewritten as (3) $A_1x_1 + ,\ldots,+ A_nx_n + A_{n+1}x_{n+1} +,\ldots,+ A_{2n}x_{2n} = A_0$, where $A_{n+1},\ldots, A_{2n}$ are the columns of the identity matrix I_n. Forming the simplified simplex tableau of linear programming, we have to replace the unit vectors $A_{n+1},\ldots, A_{2n}$ in basis successively by the structural vectors $A_1,\ldots, A_n$ using usual formulas for the change of basis (4) $x'_{kj} = x_{rj}/x_{rk}$, $x'_{ij} = x_{ij} - (x_{rj}/x_{rk})x_{ik}$, where subscript k stands for "coming in" and r for "going out". The elements of the column vector A_0 in the final tableau will give the solution of the system (1). Here we do not have a linear form which needed to be optimized, nor the restrictions $x_i \geqq 0$ and from the simplex tableau we can omit the columns of artificial unit victors. The method yields nicely to the programming on the digital computer. Received July 7, 1958.)

548-40. J. S. White: A moment analogue to Spitzer's theorem.

Let $X_1, X_2,\ldots,X_n$ be a set of symmetrically dependent random variables. Let $a^+ = \max(0,a)$, $S_k = \sum_{i=1}^{k} X_i$, $\bar{S}_j = \max_{1 \leq k \leq j} S_k^+$, $T_{n-k} = (S_n - S_k)^+$. Using combinatorial techniques similar to those used by Spitzer (Trans. Amer. Math. Soc. vol. 82 (1956) pp. 323-339), the following recursion relation for the moments of $\bar{S}_n$ is obtained: $E(\bar{S}_n^{r+1}) = \sum_{k=0}^{n-1} E(T_{n-k}(T_{n-k} + \bar{S}_k)^r)/(n - k)$. Methods are given for the explicit evaluation of these moments in special cases. In particular if $X_1,\ldots,X_n$ are independent and identically distributed the recursion relation may be stated as $E(\bar{S}_n^{r+1}) = \sum_{k=1}^{K} \sum_{i=0}^{j} ((C_{j,i}) E(S_k^+)^{i+1}/k)\ E(\bar{S}_{n-k})^{j-i}$. (Received July 7, 1958.)

548-41. G. S. Young: Reversal of Peano continua by analytic maps.

Let D be the interior of the unit circle, and let P be a Peano continuum in the extended plane, that is, a locally connected, closed, connected set. It is proved that if f(Z) is a map of D into the extended plane, of a class that includes meromorphic functions of bounded characteristic, and C is a component of $f^{-1}(P)$, then the closure of C is itself a Peano continuum. This generalizes results of Lohwater, Ohtsuka, and Whyburn. (Received July 7, 1958.)

548-42. Z. A. Melzak: Limit sections and universal points of convex surfaces.

The following problem is due to S. Mazur: does there exist a closed convex surface whose plane sections give, up to affinities, all plane closed convex curves? We consider an extension based on a generalization of a plane section. Let S be a closed convex surface, s a point on S, π the (unique) supporting plane to S at s, $\{\pi_n\}$ a sequence of planes converging to π, C_n the section of S by π_n, $\{\lambda_n\}$ a sequence of constants tending to infinity, $\lambda_n C_n$ the section C_n scaled up in ratio λ_n: 1; then $C = \lim \{\lambda_n C_n\}$, if it exists, is called a limit section of S at s. A point s of S is called universal if the set of all limit sections of S at s contains the set of all plane closed convex curves. Here we prove several theorems about limit sections and universal points. Sample: there exists a closed convex surface every point of which is universal. Proofs make use of: (1) the existence of a basis for all plane closed convex curves, consisting of a countable set of analytic ovals, and (2) a method of condensation of singularities, based on successive approximations. The research reported upon has been sponsored by the O. N. R. (Received July 7, 1958.)

548-43. Anne Lester: Some semigroups on the two-cell.

If a semigroup, S, is defined on the two-cell so that $B^2 = B$, where B is the boundary of S relative to the plane, then, from a result of A. D. Wallace and R. J. Koch, either B is a group or the multiplication for x and y in B is of the trivial kind, $xy = x$ or $xy = y$. A semigroup satisfying the former case has been described by P. S. Mostert and A. L. Shields. This paper gives a description of a semigroup satisfying the latter condition, with the additional hypothesis that S has a zero but no other idempotents in the interior. It is shown that there exist one-parameter semigroups in a compact semigroup which satisfies the condition that there exist a set A such that $A \subset f(A)$, where f is the square function. Using a one-parameter semigroup, an I-semigroup, T, is constructed in S so that $S = BT$. This describes the multiplication in S for if $f, h \in B$, $t, s \in T$ then $(ft)(hs) = f(ts)$. It is also shown that if $ft = hs$ then $t = s$, but it is not necessarily the case that $f = h$, as there are examples to the contrary.
(Received July 7, 1958.)

548-44. David Schroer: Combinatory formulation of standard theories.

A standard theory (applied LPC) is called completely finite iff it has but finitely-many individual constant, operation, and relation symbols, and but finitely-many nonlogical axioms; an effective construction is given which assigns to each completely finite standard theory a unique combinatory logic (see Abstract 546-41) called its canonical combinatory formulation. This construction involves formalization of syntax rather than arithmetization, and commutes with all operations of theory extension achieved by adjoining finitely-many new individual constant, operation, and relation symbols and finitely-many new axioms. A (classical) structure (in the usual model-theoretic sense) is called completely finite iff its universe is a set and it has but finitely-many distinguished elements, operations, and relations; a set-theoretic construction is given which assigns to each completely finite classical structure a unique combinatory structure (see Abstract 547-34) called its canonical combinatory diagram. It is shown that a consistent completely finite standard theory $\mathcal{L}$ can be classically interpreted in a completely finite classical structure $\mathfrak{A}$ iff the canonical combinatory formulation of $\mathcal{L}$ can be interpreted (in the sense of Abstract 547-34) in the canonical combinatory diagram of $\mathfrak{A}$. (Received July 7, 1958.)

548-45. E. K. Dorff: On Banach algebras of infinite matrices.

For any matrix A, define $\|A\| = \sup_n \sum_{k=1}^{\infty} |a_{nk}|$. Let Φ be the set of all A with $\|A\| < \infty$. Φ is a Banach algebra; a closed subalgebra of the algebra of endomorphisms of m. The set Γ of all conservative A, and the set Δ of all conservative triangular A, are closed sub-Banach algebras of Φ. For any sequence x, let (x) denote the set of all $A \in \Phi$ such that $Ax \in c$. Then $x \notin m$ implies (x) is dense in Φ. Similarly $x \notin m$ implies $(x) \cap \Gamma$ is dense in Γ. However, this is not true in Δ. These lead to several corollaries which show for what x, (x) is a left, right or closed ideal. $A \in \Gamma$ is said to be coercive if $Ax \in c$ for all $x \in m$. If we assume A is not coercive, we can construct a bounded set $\mathcal{B} \subset m$ such that $A\mathcal{B}$ is not compact. Hence A is completely continuous iff A is coercive. As a corollary, every A in the radical of Δ is completely continuous, and conversely for matrices with zero diagonal. (Received July 7, 1958.)

548-46. R. S. Pierce and D. J. Christensen: Free products of alpha-distributive Boolean algebras.

Let α be an arbitrary infinite cardinal. Let $\mathcal{C}$ be a category of α-representable Boolean algebras and their α-homomorphisms and let $\{B_\omega | \omega \in \Omega\}$ be a set of algebras of $\mathcal{C}$. An α-representable Boolean algebra B is called a $\mathcal{C}$ product of the B_ω's if there exist α-isomorphisms $i_\omega \in \mathcal{C}$, i_ω: $B_\omega \rightarrow B$ for all $\omega \in \Omega$ such that for any set $\{h_\omega | \omega \in \Omega\}$ of α-homomorphisms in $\mathcal{C}$, h_ω: $B_\omega \longrightarrow \bar{B} \in \mathcal{C}$, there exists a unique α-homomorphism h: $B \longrightarrow \bar{B}$ which satisfies $h \circ i_\omega = h_\omega$ for all $\omega \in \Omega$. If $B \in \mathcal{C}$, B is called a free $\mathcal{C}$-product. It is shown that the free $\mathcal{C}$-product exists if $\mathcal{C}$ is the category of all α-representable Boolean algebras and their α-homomorphisms and if $\mathcal{C}$ is the category of all α-distributive Boolean algebras and their α-homomorphisms into α-distributive Boolean algebras. The free $\mathcal{C}$-product is unique (to isomorphism) when it exists. A characterization of the free α-distributive product is given in terms of a family of α-independent subalgebras whose union α-generates the product. It is shown that the free α-representable and free α-distributive products are not in general isomorphic. (Received July 7, 1958.)

548-47. S. E. Puckette: Semigroups and unbounded difference operator

The question "Can an unbounded difference operator generate a semigroup of operators?" is answered in the affirmative. In the complex Banach space $C_0[0,\infty]$, a necessary and sufficient condition that the unbounded multiplier operator $Uu(x) = a(x)u(x)$ generate a semigroup is that the range of a(x) be contained within a left half-plane. The general difference operator $Uu(x) = \sum_{k=0}^{n} a_k(x)u(x + k)$ is attacked by perturbation methods. For such an unbounded operator to generate a semigroup, it is necessary that its multiplier part be unbounded. If it is not, the spectrum of the operator is the entire complex plane Sufficient conditions are found for the nth-order operator to generate a semigroup. They are roughly that the coefficient functions be decreasing in size with increasing k. The concrete representation of such operator questions is a Cauchy-type problem involving mixed difference-differential equations. (Received July 7, 1958.)

548-48. E. S. O'Keefe: A theorem on independence of universal algebras.

An expression $\phi(\xi,...)$ of species Sp is a finite set of one or more operation symbols ξ, η etc., composed by means of the primitive operation symbols of

A set $\{\mathfrak{A}_i\}$ of n universal algebras of given species Sp is said to be independent for any set $\{\psi_i\}$ of n expressions of species Sp, there is a single expression K that represents each ψ_i in its corresponding algebra $\mathfrak{A}_i$; that is, $K = \phi_i(\mathfrak{A}_i)$. (See A. L. Foster, Math. Z. vol. 62 (1955) pp. 171-188 and vol. 65 (1956) pp. 70-75.) The ϕ-condition is said to hold in an algebra $\mathfrak{A}$ if, for each operation O of $\mathfrak{A}$, there are expressions $\phi_1(\xi),\ldots,\phi_r(\xi)$ so that $O(\phi_1(\xi),\ldots,\phi_r(\xi)) = \xi(\mathfrak{A})$. Theorem: Every set of algebras of the same species in which the ϕ-condition holds is independent if each subset of two algebras is independent. The relation does not necessarily hold without the ϕ-condition. (Received July 7, 1958.)

548-49. G. S. Rogers: A note on order statistics and stochastic independence.

The following theorem is proved. Let x be a continuous or discrete type real random variable. Let $x_1 \leqq \ldots \leqq x_n$ be the order statistics based on a random sample of size n from this x distribution. Let $z = z(x_1,\ldots,x_j)$ be a statistic based on the first $j < n$ items only. If z is stochastically independent of x_j, then z is stochastically independent of all x_k, $j < k \leqq n$; if z is stochastically independent of some x_k, $j < k \leqq n$, then z is stochastically independent of $_j$ and hence of all x_k, $j \leqq k \leqq n$. The first result is direct, since in terms of the conditional probability density functions, $g(z|x_j) = g(z|x_j,\ldots,x_n)$. For the second part, in $g(x_1,\ldots,x_{k-1}|x_k)$, let x_k be considered as a "parameter". Then $(x_{k-1}|x_k)$ is a "complete single sufficient statistic" for x_k; also, the distribution of $(z|x_k)$ is free of the "parameter x_k". By a well known theorem, (Basu, Sankhyā vol. 15 (1955) pp. 377-380), $(z|x_k)$ and $(x_{k-1}|x_k)$ are stochastically independent. It follows that z and x_{k-1} are stochastically independent; similarly, with an induction, z and x_k, $j \leqq k \leqq n$, are stochastically independent. (Received July 3, 1958.)

548-50. J. L. Walsh and Annette Sinclair: On the degree of convergence of sequences of extremal functions.

Let F(z) be of class L^p on an analytic Jordan curve γ in the z-plane and let $p_n(z)$ be the polynomial of degree n of best approximation to F(z) on γ in the sense of minimizing $\int_\gamma |F(z) - p_n(z)|^p |dz|$, $p \geqq 2$. The $p_n(z)$, $n = 1,2,3,\ldots$ may also be subjected to assigned interpolation conditions $p_n(\alpha_k)$ at a finite number of points α_k interior to γ. The object is to study the degree of convergence of $\{p_n(z)\}$ to a possible limit function. In the subclass of functions of H_p which satisfy the same interpolation conditions at the α_k there is a best approximating function f(z) to F(z) in the above sense. It is proved that if F(z) and f(z) are analytic on C, the closed Jordan region bounded by γ, then $\{p_n(z)\}$ converges maximally to f(z) on C. The analogous problem for the surface integral is treated. (Received July 3, 1958.)

548-51. Leonard Gillman and Meyer Jerison: Closed sets in Stone-Čech compactifications.

Let X be an arbitrary completely regular space, and N the countable discrete space. Theorem 1. Every nondiscrete closed set in $\beta X - \upsilon X$ contains a copy of βN (and hence has cardinal at least 2^c). Theorem 2. If X is extremally disconnected, then every infinite closed set in βX contains a copy of βN. These generalize results of Čech (Ann. of Math. vol. 38 (1937) pp. 823-844) and J. Novák (Fund. Math. vol. 40 (1953) pp. 106-112). The proofs are based on the following lemmas. A set S is said to be C-embedded [resp. C*-embedded] in X if every [bounded] continuous, real-valued function on S has a continuous extension to all of X. Lemma 1. If there exists a continuous function on X sending S homeomorphically onto a closed set of real numbers, then S is C-embedded in X. Lemma 2. Every nondiscrete closed set in $\beta X - \upsilon X$ has a countable di crete subset D with a limit point p in $\beta X - \upsilon X$. Lemma 3. There exists a continuous function on βX that vanishes at p but nowhere on $X \cup D$. Lemma 4. If X is extremally disconnected, then every countable discrete subset is C*-embedded. (Received July 8, 1958.)

548-52. Leonard Gillman and Meyer Jerison: Prime ideals in rings of continuous functions.

Let C denote the ring of all real-valued continuous functions on a space X. An ideal I is a Z-ideal if $f \in I$ whenever f has the same zeros as some member of I. A prime ideal is called an upper ideal if it has an immediate predecessor in some maximal chain of prime ideals; similarly for lower ideal. (1) The sum of two Z-ideals is a Z-ideal. (2) (C. W. Kohls, Illinois J. Math., to appear.) The set of all prime ideals containing a given prime ideal is a chain. (3) The intersection of all ideals common to two maximal chains of prime ideals is a prime Z-ideal. (4) The set of all prime ideals contained properly in an upper ideal has a largest member. (The corresponding result for lower ideals follows from (2).) (5) If Q is any nonmaximal prime ideal, then the set $\mathcal{U}$ of all uppe ideals between any two containing Q (and there exist two) is an η_1-set (this exends a result of Kohls, loc. cit.); hence the cardinal of the Dedekind-complete set $\mathcal{P}$ of all prime ideals therein is at least $2^{\aleph_1}$. Also, the continuum hypothesis implies that the sets of the form $\mathcal{U}$ of cardinal c (e.g., for X = integers, reals, et al.) are all similar, whence the corresponding sets $\mathcal{P}$ are also similar. (Received July 8, 1958.)

548-53. E. J. Putzer: On an Nth order system of differential equations.

Under the usual continuity hypotheses, together with the assumption that all the characteristic roots of A have negative real part, it is proved, for the equation $\dot{\vec{x}} = A\vec{x} + \mu\vec{f}(\vec{x},t,\vec{\alpha}) + \vec{\phi}(t,\vec{\beta})$; $\vec{x}(0) = \vec{C}$ that for a certain range of μ and $\vec{C}$ the solution exists on $[0,\infty)$, is unique, bounded, and asymptotically stable, and is a continuous function of the parameters $\vec{\alpha},\vec{\beta},\mu$, the continuity being uniform on $[0,\infty)$. A successive approximations method is given which is uniformly convergent on $[0,\infty)$ to the solution. The ranges of μ and $\vec{C}$, and the bound for the solutions are explicitly obtained. (Received June 30, 1958.)

548-54. Lawrence Shepp: Sets of sequences on which certain generalized limits coincide.

The convergence of sequences may be defined in various ways (viz. Cesaro, Abel). Herein is considered not a single definition of convergence, but rather a class U of such methods, each member of which is: linear, translational, consistent with convergence and non-negative definite. The central problem has been to ascertain the set of sequences upon which all the methods of the class U, coincide. Banach (see Operationes lineares pp. 29-34) has proven the existence of a nonempty subset V (of U), with the propèrty that each method in V assigns a limit to every bounded sequence. The set of sequences upon which all the methods of V coincide has been found by this author. Lastly, a simplified proof of Banach's above theorem is given. (Received July 1, 1958.)

548-55. L. W. Small: On certain aspects of relatable functions.

Two numbers, functions, or sets are said to be related if they can be constructed from $N \geqq 1$ of the following operations: $\otimes$, $\oplus$, $\ominus$,... . The numbers B and C are relatable if they both belong to the set of numbers formed by the abstract operation O; in symbols, $B_{B,C} \in {}_O RC$. This notation is more explicit than the conventional. Two sets which are not the sub-sets of a particular set

S are relatable by $D_{\subsetneq S}\mathcal{RE}$. It is shown that in a finite field F that there can exist a class of unrelatable functions. However, these are related to each oth That is, $x_1 Я y_1, x_2 Я y_2, \ldots, x_N Я y_N$, where $Я$ shows unrelatedness. Certain geometrical applications are given. Particularly, the $\mathcal{R}$-function is applied to the volume concept. (Received July 8, 1958.)

548-56. J. R. Stallings: Fixed point theorems for connectivity maps.

A connectivity map f: $X \rightarrow Y$ is a function such that its graph over every connected subset of X is connected. It is known that every connectivity map of an n-cell into itself has a fixed point [O. H. Hamilton, Proc. Amer. Math. Soc. vol. 8 (1957) p. 750]. This is generalized thus: If the polyhedron P admits a fixed point for every continuous function $P \rightarrow P$, then it has the fixed point property for connectivity maps also. This follows easily from the fact that a connectivity map f: $P \rightarrow Q$ of one polyhedron into another is polyhedrally almost continuous; that is, if Z is any closed subpolyhedron of $P \times Q$ which is disjoint from the graph of f, then there is a continuous function $P \rightarrow Q$ whose graph is disjoint from Z. If P is of dimension $\geqq 2$ at each point, it can be proved (using Hamilton's method) that connectivity maps f: $P \rightarrow Q$ are almost continuous; that is, Z can be any closed (not necessarily polyhedral) subset of $P \times Q$ whic is disjoint from the graph of f; but it is not known whether this last statement is true when P is one-dimensional. (Received July 8, 1958.)

548-57. D. G. Austin: Some results in Markov chain theory.

Let P(t) be a matrix of non-negative functions on $t \geqq 0$, satisfying (i)$\sum_j p_{ij}(t) = 1$, (ii) $P(t+s) = P(t) \cdot P(s)$ and (iii) $\lim_{t \rightarrow 0} P(t) = I$. It is shown that either $p_{ij}(t) \equiv 0$ for all t or $p_{ij}(t) > 0$ for all t. This result under the additional assumption that the diagonal elements have a finite derivative at 0 was obtained by P. Levy - his proof in the general case appears to be incomplete. Weak form theorems (fixed t) on continuity of the sample functions are obtained for stochastic processes with discrete sample space and stationary conditional probabilities satisfying (i) and (iii) above. It is shown that with the imposition of the additional assumption that the process is Markovian the fixed t hypothesis may be eliminated. In particular it is shown that there is a standard modification of any such Markov process with approximately right continuous sample functions. (Received July 9, 1958.)

548-58. V. E. Beneš: The general queue with one server.

Consider a queue with one server, and let the service-times and the interarrival times of customers be, within the limits of physical meaningfulness, quite arbitrary stochastic processes, hereafter called the input processes. Let service be in order of arrival, and define W(t) as the virtual waiting-time, that is, the time a customer would have to wait if he arrived at t. The following results are proven: (1) If $\mathcal{F}_t$ is the field corresponding to knowledge of the input processes up to t, then $\Pr\{W(t) \leqq w \mid \mathcal{F}_t\}$ satisfies an integrated, stochastic version of the integrodifferential (Kolmogorov) equation of Takács, which holds for the special case of Poisson arrivals and independent service-times; (2) There exist functions f,g, and a kernel K, connected with W(t) only via the event $\{W(t) = 0\}$, and otherwise dependent only on the input processes, such that the distribution of W(t) is determined by f,g, and $P = \Pr\{W(t) = 0\}$, with P a solution of the Volterra equation of the first kind, f = KP; (3) Under weak stationarity assumptions on the input processes, the equation f = KP is of convolution type and is solvable by transforms. (Received July 2, 1958.)

548-59. C. C. Chang: Groups which are automorphism groups of a simply rdered set. Preliminary report.

An automorphism of a simply ordered set is a one-to-one mapping of S onto S which preserves the ordering relation. An ordered group G is complete if every nonempty set of elements of G having an upper bound in G has a l.u.b. in G. Theorem: Every proper subgroup of a complete ordered group is the group of all automorphisms of some simply ordered set. The proof uses essentially the completeness and the ordering properties. Some applications: Every proper subgroup of the additive group R of the reals has the property indicated in the title. Since the reals R is group isomorphic to a proper subgroup of itself, R also has the property. Hence, every archimedean ordered abelian group has the property. Open problems: Is every ordered group an automorphism group of some simply ordered set? Also, is every torsion-free group? (Received July 9, 1958.)

548-60. Jim Douglas, Jr. and T. M. Gallie, Jr.: An approximate solution of an improper boundary value problem.

Let u be a function about which nothing is known except that on the half lane $y \geqq 0$ it is harmonic and bounded and that on a line $y = c > 0$ an arbitrarily

good approximation to the values of u can be obtained. It is desired to compute from these approximate boundary data the values of u in any half plane $y \geqq b$ For $b > c$ this is trivial, but for $b < c$ the problem is of the type called "improperly posed" since two harmonic approximations to u which agree arbitrarily well in $y \geqq c$ may bear no resemblance to one another (or to u) in $y < c$. In this paper a sequence of functions is given which is shown to converge (if u vanishes sufficiently rapidly at infinity) to u as the approximate boundary values converge to the true values. The computational technique is nonlinear programming; the proof uses the methods of normal families of functions.
(Received July 9, 1958.)

548-61. Leonard Gross: On the automorphism group of the bounded algebra of a real Hilbert space.

For terminology see I. E. Segal, Tensor algebras over Hilbert spaces, Trans. Amer. Math. Soc. vol. 81 (1956) pp. 106-134. Consider the normal distribution with parameter c over a real Hilbert space, H. The group of automorphisms of the algebra of bounded measurable functionals on H mod null functionals is algebraically isomorphic to a group, G, of unitary transformations of $L_2(H)$. If K is a closed subspace of H then $L_2(H)$ is naturally isomorphic tc $L_2(K) \otimes L_2(K^{\perp})$. If in this decomposition an operator A on $L_2(H)$ is of the form $A = B \otimes I$ then A is said to be based on K. Theorem. The subgroup G_0 of G consisting of those unitaries which are based on finite dimensional subspaces of H is dense in G in the strong operator topology. If H is separable and $\{x_k\}$ is an orthonormal basis of H and if U is in G then there exists a sequence, $\{U_n\}$, of unitaries in G_0 such that U_n converges to U strongly and U_n is based on span $(x_1,\ldots,x_n)$. (Received July 9, 1958.)

548-62. G. J. Habetler and M. A. Martino: Concerning the time-independent multigroup diffusion equations.

The time-independent eigenvalue problem for the n-group diffusion model of reactor theory (cf. these Notices, April 1958, p. 215, Abstract 544-28) can be expressed as follows: Let R be a finite, connected region in E_3 which is the union of a finite number of disjoint convex sets and take n to be a positive integer. A Hilbert space of functions $\Phi(r) = (\phi_1(r), \phi_2(r),\ldots,\phi_n(r))$ is defined in a natural way, depending on certain boundary conditions. Then (1) $D_j\phi_j + \sum_{k=1;k\neq j}^{n} a_{jk}\phi_k = 1/\lambda \sum_{k=1}^{n} b_{jk}\phi_k$, $1 \leqq j \leqq n$, where the D_j are second-order self-adjoint partial differential operators, the coefficients $a_{jk}(r)$, $b_{jk}(r)$ re everywhere non-negative and the matrix $C = (a_{jk}) + (b_{jk})$ satisfies a certain transitivity property. The authors have shown, for the general three-dimensional model with or without up-scattering, that (1) possesses a unique normalized positive eigenfunction Φ_0 corresponding to a positive eigenvalue λ_0 which is less than the modulus of any other eigenvalue of (1). An iterative procedure for the construction of Φ_0, λ_0 is shown. (Received July 9, 1958.)

548-63. Naoki Kimura: Existence theorems on a multiplicative system. II.

There exists a tensor algebra unique up to isomorphism over a given module. We shall generalize this to a very general case. (Received July 9, 1958.)

548-64. W. S. Loud: Periodic solutions of a perturbed autonomous system. II.

This paper is a sequel to Abstract 542-87, Notices Amer. Math. Soc. April 1958, p. 186, and the same notations are used. In (1) $x' = g(x) + \epsilon f(t,x,\epsilon)$ ($' = d/dt$) let x be an n-vector (real), and let $g(x)$ and $f(t,x,\epsilon)$ be in C^3 for appropriate values of (t,x,ϵ). Let $x' = g(x)$ have a periodic solution $x_0(t)$ of least period L_0, and let $f(t,x,\epsilon)$ be periodic in t of least period T, a rational multiple of L_0. Let L be the least common multiple of T and L_0. Let the variation equation (2) $y' = g'(x_0(t))y$ have no solution of period L linearly independent of $x_0'(t)$. Let $z_0(t)$ be the unique solution of period L of the system adjoint to (2) with $z_0^*(t)x_0'(t) = 1$. In the present paper (1) is studied for the case that $\int_0^L z_0^*(t)f(t,x_0(t),0)dt$ and $\int_0^L z_0^*(t)f_t(t,x_0(t),0)dt$ both vanish, in which the theory of Coddington and Levinson does not apply. Conditions are derived by implicit function methods that (1) have a solution of period L of the form $x = x_0(t) + \epsilon_1(t) + o(\epsilon)$. Stability is analyzed by studying the behavior near $\epsilon = 0$ of that characteristic multiplier of the variational system $y' = (g'(x(t)) + \epsilon f_x(t,x(t),\epsilon))y$ which is +1 for $\epsilon = 0$ and $x = x_0(t)$. The research for this paper was supported in part by the Office of Ordnance Research. (Received July 9, 1958.)

548-65. W. P. Reid: Heat flow in a composite solid.

Given heat flow just in the x direction in a composite solid. Medium 1, with initial temperature $F(x)$, extends from $x = -b$ to $x = 0$. At $x = -b$ it is exchanging heat by radiation with surroundings at temperature $\phi(t)$. Medium 2, with initial temperature $G(x)$, extends from $x = 0$ to $x = a$. At $x = a$ it is exchanging heat by radiation with surroundings at temperature $\psi(t)$. At $x = 0$ there is contact resistance. The radiative heat transfer, and also that at $x = 0$, is assumed to be proportional to the difference in temperatures of the surfaces. The temperature in each medium is determined by a modification of the separation of variables procedure. (Received July 9, 1958.)

548-66. F. S. Scalora: Abstract martingale convergence theorems:

Let $(\Omega, M, P,)$ be a probability space, and X a Banach space. An X-martingale is defined as a collection of X-valued random variables on Ω, together with corresponding sub Borel fields of $M, \{x_t, F_t, t \in T\}$ such that x_t is integrable in the sense of Bochner and strongly measurable relative to F_t or equal almost everywhere to such a function, $F_s \subset F_t$ for $s < t$, and $E\{x_t | F_s\} = x_s$ with probability 1 for $s < t$, where $E\{x_t | F_s\}$ is the strong conditional expectation of x_t given F_s. If X is a reflexive Banach space and $\{x_n, F_n, n \geqq 1\}$ is an X-martingale such that $E\{\|x_n\|\} \leqq K < \infty$ for every n, then there exists a strongly measurable random variable x_∞ such that $x_n \to x_\infty$ weakly with probability 1: that is, $f(x_n(\omega)) \to f(x_\infty(\omega))$ as $n \to \infty$ with probability 1 for every f in X^*. If instead $\{\|x_n\|, n \geqq 1\}$ is a uniformly integrable class of functions, then $\|x_n(\omega) - x_\infty(\omega)\| \to 0$ as $n \to \infty$ with probability 1, and $\{x_n, F_n, 1 \leqq n \leqq \infty\}$ is an X-martingale, where F_∞ is the smallest Borel field containing $\bigcup_n F_n$.
(Received July 9, 1958.)

548-67. D. W. Wall: Cartan invariants of UMFR algebras.

Let $\mathfrak{A}$ be a finite dimensional algebra with unit over a field, and $\{e_i\}_{i=1}^n$ be a maximal set of nonisomorphic primitive idempotents of $\mathfrak{A}$. Let c_{ij} be the number of irreducible constituents of $\mathfrak{A}e_i$ which are $\mathfrak{A}$-isomorphic to $\mathfrak{A}e_j/\mathfrak{N}e_j$. The c_{ij} are the Cartan invariants of $\mathfrak{A}$. Let $\mathcal{X}(\mathfrak{A}e_k) = \sum_{i=1}^n \sum_{j=1}^{g_{ki}} L_{ij}(\mathcal{X}(e_k\mathfrak{A}) = \sum_{i=1}^n \sum_{j=1}^{h_{ki}} \mathcal{R}_{ij})$ be a decomposition of the socle of $\mathfrak{A}e_k$ $(e_k\mathfrak{A})$ such that $\mathcal{L}_{ij}$ $(\mathcal{R}_{ij})$ is $\mathfrak{A}$-isomorphic to $\mathfrak{A}e_i/\mathfrak{N}e_i$ $(e_i\mathfrak{A}/e_i\mathfrak{N})$. Let $\Sigma_k = \{i \mid g_{ki} \neq 0\}$ and $\Pi_k = \{i \mid h_{ki} \neq 0\}$. If Σ_k (Π_k) has a single element denote it by $\sigma(k)$ $(\Pi(k))$. By using earlier results [Duke Math. J. vol. 25 (1958) pp. 321-329], inequalities for the Cartan invariants of UMFR algebras (algebras with unique minimal faithful representations) are obtained which generalize results obtained by R. M. Thrall [Trans. Amer. Math. Soc. vol. 64 (1948) pp. 173-183] for type ABC algebras. Theorem. If $\mathfrak{A}$ is a UMFR algebra then for every i and j, $c_{ij} \leqq \sum_{k=1}^{n} g_{ik} c_{jk}$ with equality holding for all j if and only if $\mathfrak{A} e_i$ is dominant; and $c_{ij} \leqq \sum_{k=1}^{n} h_{jk} c_{ki}$ with equality holding for all i if and only if $e_j \mathfrak{A}$ is dominant. For various subclasses of the UMFR algebras similar inequalities hold: Type AC: $c_{ij} \leqq \sum_{k \in \Sigma_i} c_{jk}$ and $c_{ij} \leqq \sum_{k \in \Pi_j} c_{ki}$; Type B: $c_{ij} \geqq g_{i\sigma(i)} c_{j\sigma(i)}$ and $c_{ij} \geqq h_{j\pi(j)} c_{\pi(j)i}$; and Type ABC: $c_{ij} \leqq c_{j\sigma(i)}$ and $c_{ij} \geqq c_{\pi(j)i}$. (Received July 9, 1958.)

548-68. D. G. Aronson: On the fundamental solution of a parabolic system of equations. Preliminary report.

We consider the parabolic system of equations (*) $\partial U/\partial y = P(x,y; \partial/\partial x)U$, where U is an N-vector and $P(x,y; \partial/\partial x)$ is a linear differential operator with coefficients depending on $x = (x_1,\ldots,x_n)$ and y. We assume that P is uniformly elliptic for $x \in E^n$ and $y \in I$, where I is some closed interval. Theorem: If the coefficients of P are uniformly continuous in $E^n \times I$ and satisfy a uniform Hölder condition with respect to x, then the fundamental solution matrix $\Gamma(x,y;\xi,\eta)$ of (*) exists for all (x,y), (ξ,η) in $E^n \times I$ such that $y > \eta$. Moreover $\Gamma(x,y;\xi,\eta) = G(x,y;\xi,\eta) + \int_\eta^y d\tau \int G(x,y;s,\tau)\psi(s,\tau;\xi,\eta)ds$, where G is a known parametrix and ψ is the solution of a certain integral equation. This result is a generaliation of S. D. Eĭdel'man's existence theorem (Mat. Sbornik vol. 33: 51-92) since it does not require any differentiability of the coefficients. The proof is an extension of the author's proof of the corresponding result for a single 2nd order equation (Bull. Amer. Math. Soc. Abstract 62-2-192). The parametrix G is the fundamental solution of $\partial V/\partial y = P(z,y; \partial/\partial x)V$, where z a parameter. An essential point in the proof is the Hölder continuity of G with respect to z. The existence of the solution of the initial value problem for (*) with unbounded data is also considered. (Received July 10, 1958.)

548-69. Abraham Brind: Conclusive elementary proof of Fermat's last theorem for all odd $n \geqq 3$.

To prove that $x^n + y^n = z^n$, n odd $\geqq 3$, cannot be satisfied by any integer triple (a,b,c) it is sufficient to prove that it cannot be satisfied by any positive integer triple. However, this being what Fermat called a "negative" theorem, the sufficient condition is really a necessary and sufficient one. For, it is clear that the theorem must be proved for at least positive integer triples. After transforming the additive Fermat equation into the multiplicative one $a(ka^{n-2} - k') = kk'$, a an arbitrarily fixed positive integer (a notion replacing the infinite descent argument which Fermat himself could not apply to the odd cases, as he expressly stated), k and k' positive integers, it is shown that in order to prove the theorem it is necessary and sufficient to prove that this latter equation is impossible. This, in turn, is proved by very elementary considcrations of products of primes. (Received July 10, 1958.)

548-70. P. C. Curtis, Jr.: n-parameter families of functions and best approximation.

Let F be an n-parameter family of real valued continuous functions, in the sense of Tornheim (Trans. Amer. Math. Soc. vol. 69, p. 457), defined on a finite interval [a,b]. Let $a \leqq x_1 < x_2 < \dots < x_{n+1} \leqq b$, and let h be an arbitrary real valued continuous function defined on [a,b]. Let $\delta(x_1,\dots,x_{n+1}) = \inf_{f \in F} \sup_{i=1,\dots,n+1} |f(x_i) - h(x_i)|$. It is shown that the problem of determining the best approximation to h by elements of F in the sense of the norm $\|f\| = \sup_{x \in [a,b]} |f(x)|$ is equivalent to determining points $\{x_1,\dots,x_{n+1}\}$ for which $\delta(x_1,\dots,x_{n+1})$ is an absolute maximum. Two iterative methods for determining the maximum values of $\delta(x_1,\dots,x_{n+1})$ are discussed, and convergence of both methods is proved. It is also shown that if f, g $\in$ F, then f - g can have at most n - 1 zeros counting multiplicity. If [a,b] is replaced by the finite point set $x_1,\dots,x_N$ and $\|f\|_{p,N} = \sum_{i=1}^{N} |f(x_i)|^p$ $p > 1$, then the function from F approximating h best in the above $L_{p,N}$ norm is unique if and only if F is convex. Further F is convex if and only if F is the translate of a linear n-parameter family. Lastly under additional hypotheses it is shown that if F is an n-parameter family defined on a connected, compact Hausdorff space S, then S must be homeomorphic to a compact subset of the circumference of the unit circle. If n is even this subset must be proper. (Received July 10, 1958.)

548-71. Albert Edrei and W. H. J. Fuchs: On meromorphic functions of order less than one.

Let f(z) be a meromorphic function of order $\lambda < 1$ and write $\delta(a)$ for the Nevanlinna deficiency of the value a with respect to f(z). 1. If a and b are any two distinct values, $u = 1 - \delta(a)$, $v = 1 - \delta(b)$, then u and v are subject to the following (best-possible) inequalities: $0 \leqq u \leqq 1$, $0 \leqq v \leqq 1$, $u^2 + v^2 - 2uv \cdot \cos \pi\lambda \geqq \sin^2 \pi\lambda$; if $u \geqq \cos \pi\lambda$, then $v = 1$; if $v \geqq \cos \pi\lambda$, then $u = 1$. 2. With the usual notations of the Nevanlinna theory $k = \lim \sup[N(r,f) + N(r,1/f)]/T(r) \geqq 1 (\lambda \leqq 1/2)$; $k \geqq \sin \pi\lambda (1/2 < \lambda \leqq 1)$. 3. If $1/2 < \lambda \leqq 1$ and $k = \sin \pi\lambda$, then f(z) is of regular growth (i.e. lim log T(r)/log r $= \lambda$). (Received July 10, 1958.)

548-72. Albert Edrei and W. H. J. Fuchs: A property of entire functions with negative zeros.

Let f(z) be an entire function vanishing at all points of the sequence $\{a_n\}_{n=1}^{\infty}$ and nowhere else. Assume that (i) $a_n < 0$ ($n = 1,2,3,\ldots$); (ii) $\sum |a_n|^{-\rho}$ converges for some value of ρ ($< +\infty$); (iii) $\sum |a_n|^{-1}$ diverges. Then $\delta(0,f) > A/1 + \log q$ ($q \geqq 1$), where $\delta(0,f)$ is the deficiency (in the sense of Nevanlinna) of the zeros of f, A (> 0) is an absolute constant and q is the genus of the canonical product formed with the zeros a_n. (Received July 10, 1958.)

548-73. C. C. Faith: On Abelian Galois groups.

(1) A is an algebra, with 1, over a field C such that $\bar{A} = A - R$ has minimum condition, where R is the radical. If S is a subring of A, $1 \in S$, S^* denotes the group of units of S. (2) THEOREM. A^*/C^* is Abelian if and only if A^* is. (This is well-known when A is simple.) Results of Shoda [Math. Ann. vol. 107 (1932-3) pp. 252-258] are applied in noting conditions (e.g. $C \neq GF(2)$) under which A^* generates A. Other conditions are found (e.g. $R = 0$) under which commutativity of A^*/C^* implies that of A. Below, K is a ring, with 1, whose center C is a field, and F is a Galois subring whose centralizer A in K obeys (1). (2) is applied in determining necessary conditions for the commutativity of the Galois group G (also the inner group A^*/C^*) of K/F. KEY LEMMA. If G is Abelian, and if $x, x-1 \in A^*$, $x \notin C$, then $x \in F$. (3) THEOREM. If A is simple, or if $C \neq GF(2)$ when $\bar{A}$ is simple but $R \neq 0$, or if $C \nsupseteq GF(k)$, $k = 3,4$, when A is nonsimple, then G Abelian implies $A \subseteqq F$ or $A = C$. (4) THEOREM. If K is simple of finite dimension over C, $C \neq GF(4)$ when $\bar{A}$ is nonsimple, then G Abelian implies that G is inner or outer. The proofs are highly computational. Examples illustrating various exceptional cases are given. (Received July 10, 1958.)

548-74. Wilfred Kaplan: On a theorem of A. Huber.

The following theorem is proved: If f(z) is an entire function, not a polynomial, then for each fixed λ, $\lambda \leqq 1/2$, there exists a corresponding path C leading to infinity in the z-plane such that $\int_C |f(z)|^{-\lambda} ds < \infty$. A. Huber proved this for all $\lambda > 0$ (Comment. Math. Helv. vol. 32 (1957) pp. 13-72, especially p. 52). The new proof for the special case is offered because of its greater simplicity. (Received July 10, 1958.)

548-75. E. P. Miles, Jr.: An approach to certain reflection principles.

Huber (Comm. Pure App. Math. vol. 9 (1956) pp. 471-478) established a reflection principle for functions polyharmonic of order p applicable when * the function and its first p - 1 derivatives with respect to x_1 vanish on the hyperplane of reflection, $x_1 = 0$. His theorem includes as special cases the classical principle of H. A. Schwarz for harmonic functions and corresponding results of H. Poritsky and R. J. Duffin for biharmonics. Assuming that a polyharmonic function of order p, meeting condition *, has an analytic continuation across this hyperplane, the author obtains a locally valid expansion for the function in terms of its Cauchy data on the hyperplane. From this expansion an alternate reflection principle of limited range is obtained which is shown to be reducible to the Schwarz, Duffin, Huber principles for p = 1,2,3, respectively, within their common range of validity. Results for the iterated Laplace Equation are used which closely parallel those for the iterated wave equation which the author discussed recently in Abstract 546-26, these NOTICES, April 1958, p. 245. (Received July 7, 1958.)

548-76. P. S. Mostert: On a compact Lie group acting on a sphere.

Let S^k denote the k-sphere. If G is a connected compact Lie group acti effectively on S^{n+1} in such a way that there is an n-dimensional orbit, then the author has proved (Ann. of Math. (1957) pp. 447-455) that the space of orbits S^{n+1}/G must be homeomorphic to one of the four 1-dimensional manifolds with regular boundary, which in our case is the arc, and that there are two singular orbits corresponding to the end points of the arc. If N denotes a nonsingular orbit, and N_1 and N_2 are the singular orbits, it is proved that for some positive integer r, N is an r-sphere bundle over N_1 and also N_2. Moreover, using singular homology groups over the integers J,$H_p(N_1) \approx H_p(N_2) \approx J$ if p is a multiple of r or zero, $p < n - r$, and $H_p(N_1) \approx H_p(N_2) \approx 0$ otherwise. Also, $H_p(N) \approx H_p(N_1) + H_p(N_2)$ if $0 < p < n$, $H_n(N) = J$. As a consequence, if n + 1 is even, r = n, N is an n-sphere, G = SO(n + 1) or $S_p(1)$, and S^{n+1} is fibred (singularly) by its polar decomposition. We conjecture from this that if n + 1 is odd, only the two standard decompositions of S^{n+1} occur: i.e., the one above and the case where $N \cong N_1 \times N_2$, and N_1 and N_2 are r spheres, $r = n/2$. (Received July 10, 1958.)

548-77. M. S. Robertson: Cesàro partial sums of harmonic series expansions.

Let the harmonic function $v(r,\theta)$ have the sine series expansion $v(r,\theta) = \sum_1^\infty A_v r^v \sin v\theta$, convergent for $0 \leqq r < 1$, and suppose that $v(r,\theta) \geqq 0$ for $0 < \theta < \pi$. Let $S_n^{(k)}(r,\theta) = \sum_1^n C_k^{n+k-v} A_v r^v \sin v\theta$ denote the nth Cesàro partial sum of order k, $k = 1,2,\ldots$. There exists an $R_n^{(k)}$, depending upon n and k only, so that $S_n^{(k)}(r,\theta) \geqq 0$ for $0 < r \leqq R_n^{(k)}$, $0 \leqq \theta \leqq \pi$, but not always for $r > R_n^{(k)}$. Asymptotic developments for $R_n^{(k)}$ are obtained for $k = 1,2,3$ as $n \to \infty$, correct to within $o(1/n)$. Applications are made to the partial sums of power series of schlicht functions convex in one direction. (Received July 10, 1958.)

548-78. Diran Sarafyan: Existence of singular points for a well known class of nonhomogeneous differential equations.

In a previous paper (Notices, vol. 6, No. 3, June 1958) the existence and the determination of singular points for differential equations of the form $dy/dx = f((a_1x + b_1y + c_1)/(a_2x + b_2y + c_2))$ was discussed. However, this discussion was based on the assumption that $a_1b_2 - a_2b_1 \neq 0$. The present paper deals with this special case $a_1b_2 - a_2b_1 = 0$, or more generally $c_1/c_2 \neq a_1/a_2$ $b_1/b_2 = K$, where K is a constant of proportionality. The author shows that if the equation $f((kr + c_1)/(r + c_2)) = -a_2/b_2$ does not have real valued roots r then the considered differential equation has no singular points. However, if it has real valued roots r_i, $(i = 1,2,\ldots)$ then there exist singular points and they are located on the straight lines $a_2x + b_2y = r_i$. In particular the differential equation $dy/dx = (a_1x + b_1y + c_1)/(a_2x + b_2y + c_2)$ has no singular points if $a_2 = b_1$. If $a_2 \neq b_1$ then the singular points are located on the straight line $a_2x + b_2y = (b_2c_1 - a_2c_2)/(a_2 - b_1)$. (Received July 10, 1958.)

548-79. Albert Schild: On the bound of convexity of a class of star shaped mappings. Preliminary report.

A. Marx in his paper: Untersuchungen über schlichte abbildungen, Math. Ann. vol. 107 (1932-1933) pp. 40-67 has shown that any univalent function $f(z) = z + \sum_{n=2}^\infty a_n z^n$ which maps $|z| < 1$ into a convex region satisfies $\operatorname{Re}\{zf'(z)/f(z)\} > 1/2$ for $|z| < 1$. In this paper the following problem is considered: What is the bound of convexity of the class of functions $f(z) = z + \sum_{n=2}^\infty a_n z^n$ for which $\operatorname{Re}\{zf'(z)/f(z)\} > 1/2$? (Received July 10, 1958.)

548-80. John Selden: A note on topological semirings.

By a semiring is meant a Hausdorff space together with two continuous associative operations, $\cdot$ and $+$, such that multiplication distributes over addition Using the (I)-semigroups and a slight generalization of the (L)-semigroups of Mostert and Shields (On the structure of semigroups on a compact manifold with boundary, Ann. of Math. vol. 65 (1957) pp. 117-143) the author proves: Theorem 1. In a semiring which is multiplicatively an (L)-semigroup each additive subgroup is a single point. The addition in a semiring which is multiplicatively an (I)-semigroup is discussed in Theorem 2. Theorem 3 characterizes the additio of a semiring which is compact, additively commutative, and multiplicatively a group with zero. The semiring of continuous functions from a semiring, S, into itself under the compact-open topology is represented by M(S). A semigroup is said to be compact if the closure of the set of multiples of each element is compact. In Theorem 4 it is shown that if the semiring, S, is additively Γ-compact with multiplicative unit then M(S) is Γ-compact. This leads to: Theorem 5. Let S be a compact semiring with multiplicative unit. If S is divisible then S is additively idempotent. (Received July 10, 1958.)

548-81. Berthold Schweizer and Abe Sklar: On triangle inequalities fo statistical metric spaces II.

For definitions and notation, see Abstract 545-20, these NOTICES, April, 1958, pp. 229-230. Condition 4 in the abstract referred to expresses a generalized triangle inequality. A generalized triangle inequality of a different type, due to A. Wald (Proc. Nat. Acad. Sci. vol. 29 (1943) pp. 196-197) is the following $F_{pr}(x) \geqq [F_{pq}*F_{qr}](x) = \int_{-\infty}^{\infty} F_{pq}(x - y)dF_{qr}(y)$. Theorem: If, in a statistical metric space, the Wald triangle inequality holds, then Condition 4 holds in the same space under the choice T = Product. The converse of this theorem is false, since there exist spaces in which Condition 4 holds with T = Product, but the Wald inequality does not. (Received July 10, 1958.)

548-82. Berthold Schweizer and Abe Sklar: Neighborhoods in statistical metric spaces.

For definitions and notation, see Abstract 545-20, these NOTICES, April, 1958, pp. 229-230. One introduces a concept of neighborhood into the theory of statistical metric spaces by defining, as an ϵ,λ-neighborhood of a point p, the set of all points q for which $F_{pq}(\epsilon) > 1 - \lambda$. With this definition of neighborh the statistical metric space becomes a Hausdorff space, provided that the func- n T appearing in Condition 4 of the abovementioned abstract (the triangle inequality) is chosen properly - e.g., T = Min. In certain special cases, such as, for example, when $F_{pq}(x) = H[x/d(p,q)]$, where H is a distribution function satisfying $H(0) = 0$ and d is an ordinary metric, the ϵ, λ-neighborhoods of p become spherical neighborhoods of p in the ordinary sense. (Received July 10, 1958.)

548-83. C. E. Stewart: On the convergence of sequences of singular integrals of Cauchy type. Part I, general.

An example is given of a sequence of Hölder functions uniformly convergent to 0 on [a,b], for which the corresponding sequence of Cauchy integrals fails to converge at an interior point of [a,b]. DEFINITION: A sequence $\{f_n\}_{n=1}^{\infty}$, is S-convergent (secant convergent) on (a,b) if the corresponding sequence of difference quotients is uniformly convergent in the open, diagonaless rectangle, $\{(x,y): a < x < b, a < y < b, x \neq y\}$. THEOREM I: If $\{f_n\}_{n=1}^{\infty}$ is S-convergent on (a,b) and if the Cauchy principal value of (i) $f_n^{(1)}(y) = \int_a^b (x - y)^{-1} ((b - x)(x - a))^{-1/2} f_n(x)dx$ exists for each y in (a,b), then the sequence $\{f_n^{(1)}\}_{n=1}^{\infty}$ converges uniformly on (a,b). LEMMA 1: If the sequence of derivatives $\{Df_n\}_{n=1}^{\infty}$ is uniformly convergent on (a,b), then $\{f_n\}_{n=1}^{\infty}$ is S-convergent on (a,b). THEOREM II: If $\{Df_n\}_{n=1}^{\infty}$ is uniformly conve... on (a,b) then $\{f_n^{(1)}\}_{n=1}^{\infty}$ (defined by (i)) is uniformly convergent on (a,b). DEFINITION: A sequence $\{f_n\}_{n=1}^{\infty}$ is S*-convergent on [a,b] if the corresponding sequence of difference quotients is uniformly convergent in the closed, diagonaless rectangle $\{(x,y): a \leqq x \leqq b, a \leqq y \leqq b, x \neq y\}$. THEOREM III. If $\{f_n\}_{n=1}^{\infty}$ is S*-convergent on [a,b] and if the Cauchy principal value of (ii) $f_n^{(2)}(y) = \int_a^b (x - y)^{-1} ((b - x)(x - a))^{-1/2} \cdot f_n^{(1)}(x)dx$ exists for each y in (a,b), where $f_n^{(1)}$ is defined by (i), then $\{f_n^{(2)}\}_{n=1}^{\infty}$ is uniformly convergent on (a,b). (Received June 19, 1958.)

548-84. P. M. Swingle: Widely connected algebras.

Let S be a Hausdorff semigroup whose continuous multiplication is extendable to $\bar{S}$; let further $\bar{S}$ be compact and perfectly separable. Let $A = [a|aS = aa$, i.e. is a point, $a \in S]$, $B = [b|Sb = bb, b \in S]$, $C = [c|c\bar{S} \supset S, c \in S]$ and $D = [d|\bar{S}d \supset S, d \in S]$. If S is also widely connected (Bull. Amer. Math. Soc. (1931) pp. 254-255) we must have one of the following: (1) $S = A \cap B$; (2) $S = A \cap D$; (3) $S = B \cap C$; (4) $S = A \cup (C \cap D)$; (5) $S = B \cup (C \cap D$; or (6) $S = C \cap D$. Thus S has a minimal ideal K and either $K = SS$ is a point or

$\overline{K} = \overline{S}$. In cases (1) - (3) SS is contained in the set of idempotents of S; in cases (4) - (6) $\overline{S}$ contains a unit for $C \cap D$ which is itself a semigroup closed in S. is known that a metric compact indecomposable continuum I can be a semigroup (Koch and Wallace, Duke Math. J. vol. 21 (1954) p. 683) and can be even a ring (R. Arens, 1955, Madison Set Theoretic Topology Institute, pp. 140-141). Under the hypothesis of the continuum and further algebraic or topological conditions, for example each proper subcontinuum of I is an arc, one can show that I contains densely a widely connected subset which inherits the algebra of I, i.e. for example if I is a ring it is also under the same algebraic operations. (Received July 10, 1958.)

548-85. H. S. Heaps: Asymptotic solutions of the equation $y^{IV} + 2x^{-1}y^{III} + y = 0$.

Series solutions of the differential equation $y^{IV} + 2x^{-1}y^{III} + y = 0$ have been described by Nevel [Conference on the Bearing Capacity of Ice, National Research Council of Canada, Ottawa, April, 1958]. The series are slowly convergent for large values of x and become infinite at $x = \infty$, which is a non-regular singularity of the differential equation. The present paper expresses each series in the form of an asymptotic expansion valid for large values of x (Received July 3, 1958.)

548-86. M. I. Aissen: Existence and convergence of formal solutions of certain systems of differential equations. Preliminary report.

The equation $t_N\, df/dt + af + b = 0$, is studied, where N is a nonnegative integer and a and b are formal power series in t. Necessary and sufficient conditions for the existence of a formal power series solution are given. A proof of convergence of the series is given in the case $N = 0, 1$ using a new method. Systems of such equations are also studied with corresponding results. (Receive July 11, 1958.)

548-87. L. D. Berkovitz and Melvin Dresher: Solution of a multimove tactical game.

The following multimove zero-sum two-person game is analyzed. At the n-th move or stage of the game the Blue player has resources given by the state variable p_n and assigns a value to each of two tactical variables under his control, x_n and u_n, subject to the constraints $x_n \geqq 0$, $u_n \geqq 0$, $x_n + u_n \leqq p_n$. At the same time Red has resources given by the state variable q_n and controls the lues of the tactical variables y_n and w_n subject to the constraints $y_n \geqq 0$, $w_n \geqq 0$, $y_n + w_n \leqq q_n$. The state variables at the (n + 1) st move are defined by $p_{n+1} = \max[0, p_n - \max(0, y_n - cu_n)]$, and $q_{n+1} = \max[0, q_n - \max(0, x_n - cw_n)]$. The payoff to Blue is given by $\sum_{n+1}^{N}[(p_n - x_n - u_n) - (q_n - y_n - w_n)]$, where N is the number of moves in the game. The authors analyze the optimal strategies as functions of the parameter c for $0 < c < 1$. The cases $c = 0$ and $c = 1$ have been previously reported. (Received July 11, 1958.)

548-88. E. K. Blum and P. C. Curtis, Jr.: An asymptotic relation between Tschebycheff coefficients and the best polynomial approximation.

Let f(x) be a real-valued function defined on [-1,1], and let its Tschebycheff series be $(a_0/2) + \sum_{k=1}^{\infty} a_k T_k(x)$, where $T_k(x) = \cos(k \arccos x)$ and $a_k = (2/\pi)\int_0^{\pi} f(\cos\theta)\cos k\theta d\theta$. Let P_n be the class of polynomials, p(x), of degree not greater than n. Define $\|f - p\| = \sup_{-1 \leqq x \leqq 1}|f(x) - p(x)|$, and let $M_n = \inf_{p \in P_n}\|f - p\|$. Let $\{a_{k_j}\}$ be the subsequence of the $\{a_k\}$ consisting of all the nonzero coefficients. The following result is proven. Theorem: If $|a_{k_{j+1}}/a_{k_j}| \leqq r < 1$ for all $j \geqq j_0$, then for all $j \geqq j_0$, $\max(1/2^{1/2}, (1 - 2r)/(1 - r))$ $M_{k_j - 1}/|a_{k_j}| \leqq 1/(1 - r)$ and $1/2^{1/2} \leqq M_{k_j}/|a_{k_{j+1}}| \leqq 1/(1 - r)$. Corollary: If $m_{j \to \infty}|a_{k_{j+1}}/a_{k_j}| = 0$, then $\lim_{j \to \infty}|M_{k_j - 1}/a_{k_j}| = 1$. (Received July 11, 1958.)

548-89. C. C. Buck: A pair of mutually independent dual theorems in projective geometry.

The theorem on the uniqueness of the projective construction of a fourth harmonic point implies (assuming the usual incidence axioms for the plane) a special case of Desargues' triangle theorem in which there are two extra incidences, hence it implies the dual of this theorem. On the other hand, this pair of dual theorems implies the uniqueness theorem. The dual theorems are mutually independent. (This research was supported by the Army Ballistic Missile Agency, Army Ordnance Missile Command.) (Received July 11, 1958.)

548-90. W. J. Coles, J. H. Barrett, and D. V. V. Wend: Nonoscillation and disconjugacy in the complex plane. Preliminary report.

This paper is concerned primarily with refinements and extensions of the work of Beesack (Trans. Amer. Math. Soc., 1956) and Schwarz (Trans. Amer. Math. Soc., 1955). The following are typical results concerning the complex differential equation $(P(z)W')' + F(z)W = 0$: Beesack's comparison theorems are extended to the case where the leading coefficient $P(z)$ is not identically one. When considered along a smooth path in the complex plane, some problem arise concerning complex equations (with a real independent variable) having a real nontrivial solution. Secondly, for $P(z) \equiv 1$, a circle of disconjugacy is obtained by applying a theorem of Schwarz on the non-Euclidean distance between zeros of a solution. Finally, Beesack's methods for finding domains of nonoscilation are defined to obtain other such domains. (Received July 11, 1958.)

548-91. E. H. Connell: Products of fixed point spaces.

In this note it is shown that if a metric space has the fixed point property it does not contain an infinite chain of arcs that is locally finite. Using this result, an example is given of a metric space X such that X has the fixed point property and $X \times X$ does not. (Received July 11, 1958.)

548-92. Avron Douglis: An extended ordering principle for generalized solutions of a quasi-linear partial differential equation.

Let $F(u)$ have a bounded, integrable second derivative $F''(u)$ such that $F''(u) \geqq a > 0$ (a = const.). Any function $u(x,t)$ defined and bounded in the upr half plane P: $t \geqq 0$ is called a "weak solution" of the partial differential equatio (*) $u_t + (F(u))_x = 0$, if u is integrable along every smooth path in P, and if $\oint -udx + F(u)dt = 0$ for every piecewise smooth, closed path of integration in P A weak solution $u(x,t)$ for which $u(x + h,t) - u(x,t) \leqq h/at$ for $h > 0$, $t > 0$, is called a "generalized solution" of (*), and it is said to be "normalized" in addition, if $u(x,t) = u(x - 0,t)$ for $t > 0$. Generalized solutions of (*) exist with arbitrary, bounded, measurable initial data. All normalized, generalized solutions of (*) are governed by the following ordering principle: If $u_1(x,t)$ and $u_2(x,t)$ ar any two such solutions, and if $u_1(x,0) \leqq u_2(x,0)$ for almost every value of x, thei $u_1(x,t) \leqq u_2(x,t)$ for every $t > 0$ and every x. This ordering principle extends that announced by the author in Bull. Amer. Math. Soc. vol. 63 (1957) p. 130. (Received July 11, 1958.)

548-93. Czerna Flanagan and J. E. Maxfield: Singular perturbations of polynomials in several variables. Preliminary report.

Certain approximate zeros of a class of polynomials in several variable are given. (Received July 11, 1958.)

548-94. R. E. Gomory: Solving linear programs in integers.

Let A be an $m \times n$ matrix, C an n-vector, C' its transpose, B and X n-vectors. The linear programming problem is the problem of obtaining an X of nonnegative components satisfying $AX = B$ and maximizing $C'X$. The integer programming problem differs only in that X is also required to have all components integers. A finite algorithm is described for obtaining a solution to the integer programming problem. The algorithm is based on Dantzig's simplex method and proceeds roughly as follows. Successive trial solutions to the linear programming problem are found by the simplex method. If any variables in the trial solution are not integers, an automatic procedure uses elements of A to produce a set of additional equations which are satisfied by the still unknown integer solution, but not by the current trial solution. One or more of these equations are adjoined to the original set and a new trial solution produced. If this solution is not in integers the process is repeated. If the additional equation is chosen by certain simple rules, the process is shown to give the integer solution in a finite number of steps. Equations can be dropped as well as added so the problem size never exceeds $m + n + 1$ equations. (Received July 11, 1958.)

548-95. R. R. D. Kemp. A class of singular non-self-adjoint differential operators.

Differential operators $L = L_0 + L_1$ on $L^p(-\infty, \infty)$ $1 \leqq p < \infty$ are discussed, where L_0 is of the nth order and has constant coefficients and L_1 is of order $n - 2$ with complex coefficients $b_k(x)$ satisfying $(x^2 + 1)^r |b_k(x)| \in L^1$ for a suitable r. Under these conditions one can construct solutions of $Ly - \lambda y = 0$ having prescribed asymptotic behaviour and use these to form the Green's function $G(x, \xi, \lambda)$ for λ not in the spectrum. If $L_0 y = \sum_{i=0}^{n} a_{n-j}(-i)^j d^j y/dx^j$ where a_j is a complex number and $a_0 = 1$ limit points of the spectrum are determined by properties of the polynomial $p(z) = \sum_0^n a_{n-j} z^j$. In particular the curve $\lambda = p(t)$ for real t contains the continuous and residual spectrum of L. The point spectrum is a bounded set with limit points at points $\lambda_0 = p(t)$ where $p(z) = \lambda_0$ has two or more real roots. If $G(x, \xi, p(t))$ is sufficiently well-behaved then one can also obtain an expansion in eigenfunctions for a sufficiently restricted class of functions by using the Cauchy Integral Technique. (Received July 11, 1958.)

548-96. W. L. Miranker and H. L. Frisch: Analysis of the nonlinear Stefan problem.

In this paper it is shown that the nonlinear Stefan problem possesses a $t^{1/2}$ time dependent solution. A method for approximating the solution for a given diffusion function is also given. The techniques for doing these things make use respectively of the Schauder fixed point theorem and the theory of asymptotic expansions. (Received July 11, 1958.)

548-97. D. A. Norton: Conjugate invariants. Preliminary report.

In a quasigroup with binary operation "$\circ$" if $a \circ b = c$, a permutation of the elements a,b and c yields a conjugate quasigroup. S. Stein has shown that mediality is an invariant identity under all possible conjugations. However, certain other properties such as the existence of sub-quasigroups and normal sub-quasigroups (with the definition of normality sufficiently generalized), and the lengths of cycles are conjugate invariants. Such conjugate invariants are studied in this paper. (Received July 11, 1958.)

548-98. R. W. Preisendorfer: Asymptotic radiance distributions in eventually separable media.

In radiative transfer theory we say that a plane parallel medium $Z \times \Xi$ (Z a copy of the non negative real line, Ξ is the unit sphere in E_3) is eventually separable if the phase function p on $Z \times \Xi \times \Xi$ is representable as: $p(\tau;\xi;\xi') = p_0(\tau;\xi;\xi') + \phi(\tau;\xi;\xi')$, where p_0 is independent of τ and not identically zero, and $\phi \to 0$ uniformly with respect to (ξ,ξ') as $\tau \to \infty$. If N is a radiance function (specific intensity) on $Z \times \Xi$, define $K(\tau,\xi) = (-1/N(\tau,\xi)) \cdot dN(\tau,\xi)/d\tau$. A similar definition, for K_q, holds using the equilibrium radiance $N_q(\tau,\xi) = (1/4\pi)\int_\Xi p(\tau;\xi;\xi')N(\tau,\xi')d\Omega(\xi')$. An asymptotic radiance distribution is said to exist if $K_\infty(\xi) = \lim_{\tau\to\infty}K(\tau,\xi)$ exists and K_∞ is independent of ξ. The main result is: An asymptotic radiance distribution exists in an arbitrary plane parallel medium if and only if the medium is eventually separable. In this case $K_\infty(\xi) = \lim_{\tau\to\infty}K_q(\tau,\xi) = \lim_{\tau\to\infty}k(\tau) = k_\infty$, for all $\xi \in \Xi$, $0 < k_\infty < \infty$; where $k(\tau) = (-1/h(\tau))dh(\tau)/d\tau$, $h(\tau) = \int_\Xi N(\tau,\xi)d\Omega(\xi)$ The proof makes use of the Riccati-like equation governing K: $dK(\tau,\xi)/d\tau = [K(\tau,\xi) - K_q(\tau,\xi)]\,[K(\tau,\xi) + 1/(\xi\cdot n)]$, (where n is the unit outward normal to $Z \times \Xi$), and the principles of invariance for non separable optical media (Proc. Nat. Acad. Sci. U. S. A. vol. 44 (1958) pp. 323-327). (Received July 11, 1958.)

548-99. G. B. Preston and M. N. Bleicher: Abstract linear dependence lations.

A relation $\prec$ on the subsets of a set is a dependence relation if (1) it is transitive, (2) $A \subset B$ implies $A \prec B$, (3) for elements x,y and any set $X, x \prec X$ and $x \not\prec X\backslash y$ implies $y \prec X \backslash x$, (4) $X_t \prec Y$ $(t \in T)$ implies $\bigcup_{t \in T} X_t \prec Y$. A set X is independent if $x \not\prec X\backslash x$ for any x in X. The relation $\prec$ is proper if the property of independence is of finite character. A dependence relation defined only on the finite subsets of a set can be equivalently described in terms of either R. Rado's I-functions [Quart. J. Math. vol. 13 (1942) pp. 83-89] or H. Whitney's rank functions [Amer. J. Math. vol. 57 (1935) pp. 509-533]. It is shown that with any set on which a proper dependence relation is defined is associated a unique rank which is the cardinal of any maximal independent subset. The corresponding results for groups, vector spaces etc. follow as corollaries. These results are obtained more easily than in Rado's treatment [Canad. J. Math. vol. 1 (1949) pp. 337-343]. Generalized vector spaces are defined in terms of dependence relations. A treatment of such spaces is begun. For example on the analogue of an important lemma of Banaschewski is obtained [Arch. Math. vol. 7 (1956) pp. 430-440, Lemma 4]. (Received July 11, 1958.)

548-100. C. R. Storey: On the structure of threads. Preliminary report.

A thread is a connected topological semigroup where the topology is the interval topology induced by a total ordering. It is shown that the closed interval between any two idempotents in a thread is a subthread, and thus the results of A. H. Clifford (Trans. Amer. Math. Soc. (to appear)) on bounded threads with idempotent endpoints may be applied to describe the multiplication in these intervals. Also, a characterization of all threads without idempotents is given which has the following corollary: If S is a thread, if S has no idempotents and if $S^2 = S$ then S is iseomorphic with the open unit interval under ordinary multiplication. (Received July 11, 1958.)

548-101. W. L. Strother: Orbits in fixed sets.

J. L. Kelley proved that a continuous function on a compact Hausdorff space X to itself has a fixed set A. Here some properties of the orbits of points in A are established. (Received July 11, 1958.)

548-102. Philip Wolfe: Programming with nonlinear constraints. Preliminary report.

A computational procedure is presented for minimizing the linear form $\sum_{j=1}^{n} c_j x_j$ under constraints $g_i(x) \leqq 0$ $(i = 1,\ldots,m)$, g_i convex and differentiable. Start: Given any $\bar{x}$, let x^0 yield the minimum of $\sum c_j x_j$ subject to (A): $g_i(\bar{x}) + \nabla g_i(\bar{x})(x - \bar{x}) \leqq 0$. Recursion: Given x^k, let $g_p(x^k) = \mathrm{Min}\{g_i(x^k): i = 1,\ldots,m\}$, and replace the pth constraint of (A) by $g_p(x^k) + \nabla g_p(x^k)(x - x^k) \leqq 0$; obtain the new minimum from the old employing the dual simplex method. Sufficient conditions for convergence of x^k to a solution of the problem are give (Received July 11, 1958.)

548-103. C. T. Yang: p-adic transformation groups.

In this paper it is proved that if X is a locally compact Hausdorff space of homological dimension $\leqq n$ (with respect to reals mod 1) and G is a p-adic group acting as a topological transformation group on X, then the orbit space X/G is of homological dimension $\leqq n + 2$. If, moreover, X is an n-dimensional manifold and G acts effectively on X, the homological dimension of X/G is actually equal to $n + 2$ and hence the dimension of S/G is equal to either $n + 2$ or ∞. The proof is based on a generalized special homology theory with rea mod 1 as coefficients. (Received July 11, 1958.)

548-104. D. C. McGarvey: Operators commuting with translation by one

The unit translation operator on $L_2(-\infty, \infty)$ is defined by $(Sf)(t) = f(t + 1)$. Let T_j, j integer, be defined by $(T_j f)(t) = f(t + j)$, $t \in [0,1)$, $(T_j f)(t) = 0$, $t \notin [0,1)$ Let X be the space of weakly measurable $L_2(0,1)$ operator valued functions $\mathcal{A}(\theta)$, $\theta \in (0,2\pi)$, for which $|\mathcal{A}|_\infty = \text{ess sup}\|\mathcal{A}(\theta)\| < \infty$. There is a 1-1 corres pondence between bounded operators A on $L_2(-\infty, \infty)$ for which $AS = SA$ and elements $\mathcal{A}(\theta)$ in X, denoted by $A \sim \mathcal{A}(\theta)$, such that $Af = \mathrm{LIM} \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} (2\pi i)^{-1} \int_0^{2\pi} T_j^* \exp(i\theta(j - k)) \mathcal{A}(\theta) T_k f d\theta$ and $\|A\| = |\mathcal{A}|_\infty$. If $A \sim \mathcal{A}(\theta)$, $B \sim \mathcal{B}(\theta)$ then $AB \sim \mathcal{A}(\theta)\mathcal{B}(\theta)$. Necessary and sufficient conditior that A be spectral in the sense of Dunford (Pacific J. Math. vol. 4 (1954) pp. 321-354) are given in terms of $\mathcal{A}(\theta)$. If A is the resolvent of the maximal (= minimal) operator T defined by a differential operator τ with coefficients of period 1 then $\mathcal{A}(\theta)$ is the resolvent of the operator $\mathcal{J}(\theta)$ on (0,1) defined by τ and the boundary conditions $f^{(j)}(1) = e^{i\theta} f^{(j)}(0)$, $0 \leqq j \leqq n - 1$. Thus perturbation techniques of Schwartz (Pacific J. Math. vol. 4 (1954) pp. 415-457) and Krame

(Pacific J. Math. vol. 7 (1957) pp. 1405-1435) applicable to the compact operators $\mathcal{A}(\theta)$ yield conditions on $\mathcal{T}$ that T be spectral. (Received July 11, 1958.)

548-105. R. W. Richardson, Jr.: Actions of the groups SO(3) and Sp(1) on the four-sphere.

A study is made of the actions of SO(3), the group of proper rotations of E^3, and Sp(1), the symplectic group of order one, on the four-sphere, S^4. It is shown that, to within topological equivalence, there are exactly two actions of SO(3) on S^4 and there is exactly one (effective) action of Sp(1) on S^4. These actions are equivalent to linear actions. No assumptions of differentiability are made. The method is briefly as follows: It is shown that the orbit space under such an action is homeomorphic to either an arc or a closed two-cell. It is then shown that for every such action the orbit space admits a cross section with prescribed stability groups. Such a cross section completely characterizes the action to within equivalence. (Received July 11, 1958.)

548-106. S. L. Warner: Characters of Cartesian products of rings.

This abstract continues work of Bialynicki-Birula and Zelazko [Bull. Acad. Polon. Sci. Cl. III vol. 5 (1957) pp. 589-593] and the author [Bull. Amer. Math. Soc. Abstract 660]. Let R be a commutative ring with identity, R(A) the Cartesian product of replicas of R, indexed by set A. A character of R(A), regarded as an R-algebra, is a homomorphism from R(A) onto R. 1. If R is finite and has n minimal idempotents, there is a canonical correspondence between the class of characters of R(A) and Φ^n, where Φ is the class of all ultrafilters on A. 2. Let R be an integral domain, K its field of quotients. Every character of R(A) is a projection if and only if: (a) every character of K-algebra K(A) is a projection; and (b) for any character u of R(A) and any f $\in$ R(A) such that $f(\alpha) \neq 0$ for all $\alpha \in A$, $u(f) \neq 0$. 3. If R is a principal ideal domain having at least two nonassociated primes, every character of R(A) is a projection provided A either admits no Ulam measure or has cardinality not greater than that of A.
(Received July 10, 1958.)

548-107. A. C. Woods: On a problem of Mahler's.

Let $x_1, x_2, \ldots, x_n$ be cartesian coordinates of a point X in euclidean n-space E_n. Further let K and C be convex bodies, symmetric in the origin, of n and m dimensions respectively where $m \leqq n$ such that $X \in C$ implies $x_{m+1} = x_{m+2}$

$= \cdots = x_n = 0$. For an arbitrary real and positive t define $K(C,t)$ to be the set of all points X such that $X \in K$ and $(x_1, x_2, \ldots, x_m, 0, 0, \ldots, 0) \in tC$. By using th symmetrization theorem of Brunn and Minkowski as presented in Bonnesen and Fenchel's <u>Theorie der Konvexen Korper</u> the following theorems are proved: <u>Theorem</u> 1: If $V(K)$ is the Jordan content of K then $V(K(C,t))/t^m$ is a decreasin function of t, $t > 0$. <u>Theorem</u> 2: If $\Delta(K)$ is the critical determinant and $c(K)$ th covering constant of K then $\Delta(K(C,t))/t^m$ and $c(K(C,t))/t^m$ are decreasing func tions of t, $t > 0$, provided that $m = n - 1$. <u>Theorem</u> 3: $\Delta(K(C,t))/t^m$ is a de- creasing function of t, $t > 0$, provided only that $n \leqq 3$. (Received July 10, 1958.

548-108. D. J. Newman: <u>Partitioning of areas by straight lines.</u>

The following two theorems are proven: If R is any open set in the plane then a point in the plane can be chosen such that every line through this point cuts R into two areas each of which is $\leqq$ twice the other. If R is a convex open set in the plane then a point in the plane can be chosen such that every line through this point cuts R into two areas each of which is $\leqq$ 5/4 of the other. Both of these results are best possible. One of the lemmas used is also quite interesting and seems not to have been previously noticed, namely; Given an open set R there exist three concurrent lines such that the six wedges thus formed each contain 1/6 the area of R. (Received July 1, 1958.)

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MODERN GEOMETRICAL OPTICS
By MAX HERZBERGER, *Kodak Research Laboratories, Eastman Kodak Company, Rochester, N. Y.*
1958. 516 pages, 104 illus., 11 tables. $15.00

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