AMERICAN MATHEMATICAL SOCIETY

Notices

Edited by GORDON L. WALKER

VOLUME 6, NUMBER 7, PART I ISSUE NO. 42 DECEMBER 1959

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Please send in abstracts of papers to be presented in person well in advance of the deadline.

Published by the Society

ANN ARBOR, MICHIGAN and PROVIDENCE, RHODE ISLAND

Printed in the United States of America
NOTE: This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the NOTICES was sent to press. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

<table>
<thead>
<tr>
<th>Meeting No.</th>
<th>Date</th>
<th>Place</th>
<th>Deadline for Abstracts*</th>
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<tr>
<td>565</td>
<td>February 19-20, 1960</td>
<td>Tucson, Arizona</td>
<td>Jan. 6</td>
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<td>566</td>
<td>February 27, 1960</td>
<td>New York, New York</td>
<td>Jan. 6</td>
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<td>567</td>
<td>April 14-16, 1960</td>
<td>New York, New York</td>
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<td>568</td>
<td>April 22-23, 1960</td>
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<td>569</td>
<td>April 22-23, 1960</td>
<td>Berkeley, California</td>
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<td>570</td>
<td>June 18, 1960</td>
<td>Missoula, Montana</td>
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<td>571</td>
<td>August 29-September 3, 1960 (65th Summer Meeting)</td>
<td>East Lansing, Michigan</td>
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<td>572</td>
<td>October 22, 1960</td>
<td>Worcester, Massachusetts</td>
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<td>574</td>
<td>August, 1961 (66th Summer Meeting)</td>
<td>Stillwater, Oklahoma</td>
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<td>575</td>
<td>January, 1962 (68th Annual Meeting)</td>
<td>Kansas City, Missouri</td>
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<td>August, 1962 (67th Summer Meeting)</td>
<td>Vancouver, British Columbia</td>
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<td>577</td>
<td>August, 1963 (68th Summer Meeting)</td>
<td>Boulder, Colorado</td>
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*The abstracts of papers to be presented at the meetings must be received in the Headquarters Offices of the Society in Providence, R. I., on or before these deadlines. The deadlines also apply to news items.

The NOTICES of the American Mathematical Society is published seven times a year, in February, April, June, August, October, November, and December. Price per annual volume is $7.00. Price per copy, $2.00. Special price for copies sold at registration desks of meetings of the Society, $1.00 per copy. Subscriptions, orders for back numbers (none available before 1958), and inquiries should be addressed to the American Mathematical Society, Ann Arbor, Michigan, or 190 Hope Street, Providence 6, R. I.

Second-class postage paid at Ann Arbor, Michigan. Authorization is granted under the authority of the act of August 24, 1912, as amended by the act of August 4, 1947 (Sec. 3421, P. L. and R.). Accepted for mailing at the special rate of postage provided for in section 34-40, paragraph (d).
The sixty-sixth annual meeting of the American Mathematical Society will be held at the Conrad Hilton Hotel in Chicago, Illinois on January 27, 28, and 29, 1960. Due to the fact that a large convention will not leave the hotel until January 27, the scientific program of the meeting will not begin until the afternoon of the first day. Moreover, because of the prior convention, the Conrad Hilton will not be able to accommodate members of the Society for the night of January 26.

Several other mathematical organizations are also holding meetings during the period January 27-30. These are the Mathematical Association of America, the Society for Industrial and Applied Mathematics, and the National Council of Teachers of Mathematics. An outline of the program of these various organizations is found elsewhere within this program.

The thirty-third Josiah Willard Gibbs Lecture will be delivered by Professor Julian Schwinger of Harvard University. Professor Schwinger will speak at 8:00 P.M. on Wednesday, January 27, in the Grand Ball Room. The title of the lecture is "Quantum field theory".

The presidential address will be delivered by Professor Richard Brauer on Thursday, January 28, at 1:00 P.M. in the Boulevard Room. Professor Brauer's lecture is entitled "On finite groups and their characters". Immediately after the lecture, the Cole Memorial prize will be awarded, and following this the Business Meeting will be called to order.

By invitation of the Committee to Select Hour Speakers for Annual and Summer Meetings, there will be two addresses, Professor A. S. Besicovitch, of Cambridge University and the University of Pennsylvania, will speak on "Tangential properties of sets of arcs of infinite linear measure" at 2:00 P.M. on Wednesday, January 27. Professor Paul Rosenbloom, of the University of Minnesota, will speak on "Partial differential equations with a constant coefficient at 2:00 P.M. on Friday, January 29. Both lectures will be in the Boulevard Room.

Sessions for contributed papers, twenty-seven in all, will be held each day of the meeting, but due to the limitations of time, there will be no special sessions for papers which failed to meet the deadline. Abstracts of the papers to be presented at these sessions appear on pages 767-870 of these NOTICES.
The Council of the Society will meet in Parlor Dining Room 19 on Thursday, January 28, at 4:00 P.M.
There will be a meeting of the Conference Board of the Mathematical Sciences January 29, at 5:00 P.M., in Parlor Dining Room 19.
The Employment Register will function from 9:00 A.M. to 1:00 P.M. and 2:00 P.M. to 5:00 P.M. on Wednesday, Thursday, and Friday, January 27, 28, and 29. It will be located in Parlor Dining Room 19 from 9:00 A.M. to 1:00 P.M. on Wednesday, and thereafter in Parlor Dining Room 2.

ENTERTAINMENT AND RECREATION

It is necessary to plan evening sessions throughout the meeting, and hence no banquet will be held.
The entertainment possibilities offered by Chicago are so extensive as to make a tourist guide practically impossible within the confines of this program. There are several excellent restaurants within the hotel itself, and there is an ice show every evening in the Boulevard Room.
Among the many museums, the Chicago Natural History Museum, the Art Institute, the Adler Planetarium, and the Shedd Aquarium deserve special mention. There are two excellent libraries within a short distance of the hotel: the Chicago Public Library, and the John Crerar Library.

REGISTRATION AND ACCOMMODATIONS

Registration headquarters will be located in the third floor corridor of the Conrad Hilton. The office will be open on Wednesday, January 27, from noon on to 9:00 P.M., and on other days from 9:00 A.M. to 5:00 P.M. All members attending the meetings are requested to register at the headquarters on arrival. A directory of registration and an information service will be maintained at these headquarters.
The registration fee will be $1.00 for each member of any participating organization, and $.50 for each accompanying adult.

BOOK EXHIBITS

Various publishers will exhibit titles in Parlor Dining Rooms 12, 13, and 14 on the fourth floor.

TRAVEL INFORMATION

Chicago is said to be the world's railroad capital, and the world's busiest air center, with 22 and 19 lines respectively. The traveler should therefore have no difficulty in finding his way to the metropolis. It is even possible to arrive by boat. Any travel agent should have more than adequate information.
To provide information in the small, it is noted that the Conrad Hilton Hotel is located on Michigan Boulevard between 7th and 8th Streets.

RESERVATIONS

Members are urged to make reservations directly with the Conrad Hilton Hotel starting with the night of Wednesday, January 27. Due to the presence of a prior convention the hotel is sold out for the night of January 26. The reservation form at the back of the program should be employed, but one should not assume that the Conrad Hilton will be able to provide accommodations at the selected price. As is customary with all hotels, reservations will be made at the least expensive level at which rooms are available. Members are urged, whenever possible, to share double rooms, since the size of this meeting and their other commitments promises to strain the capabilities of the hotel.

COMMUNICATIONS

Mail and telegrams for those attending the meetings should be addressed in care of the American Mathematical Society, The Conrad Hilton Hotel, Chicago 5, Illinois.

Committee on Arrangements

H. L. Alder
H. J. Curtis
E. H. C. Hildebrandt

G. L. Walker
L. R. Wilcox, Chairman
J. W. T. Youngs
PROGRAM OF THE SESSIONS

The time limit for each contributed paper is ten minutes. The contributed papers are scheduled at 15 minute intervals so that listeners can circulate between the different sessions. To maintain this schedule, the time limit will be strictly enforced.

WEDNESDAY, 2:00 P.M.

Invited Address, Boulevard Room
Tangential properties of sets of arcs of infinite linear measure (One hour)
Professor A. S. Besicovitch, Cambridge University and the University of Pennsylvania

WEDNESDAY, 3:30 P.M.

Session on Topology, Waldorf Room
3:30 - 3:40
(1) On connectivity of graphs
   Dr. Andrew M. Duguid, Brown University (564-181)
3:45 - 3:55
(2) A special class of graphs
   Professor David M. Merriell, and Professor Paul J. Kelly, University of California, Goleta (564-167)
4:00 - 4:10
(3) Imbeddings of graphs
   Professor Louis Auslander and Professor J. W. T. Youngs, Indiana University (564-95)
4:15 - 4:25
(4) Extension of local and medial properties to compactifications
   Professor Raymond L. Wilder, University of Michigan (564-239)
4:30 - 4:40
(5) Spaces that admit a category measure
   Professor J. C. Oxtoby, Bryn Mawr College (564-128)
4:45 - 4:55
(6) On the homeomorphism group of a compact Hausdorff space
   Professor Johannes de Groot, Purdue University and University of Amsterdam
WEDNESDAY, 3:30 P.M.

Session on Logic and Foundations, Williford Room, Parlor A

3:30 - 3:40
(7) On the formal definition of categories
Professor Johann Sonner, University of South Carolina (564-14)

3:45 - 3:55
(8) The no counter-example interpretation
Professor J. R. Shoenfield, Duke University (564-84)

4:00 - 4:10
(9) A theorem on ordinal multiplication
Professor Leonard Gillman and Professor Anne C. Morel, Institute for Advanced Study (564-210)

4:15 - 4:25
(10) Extensions to isols. I
Professor Anil Nerode, Cornell University (564-140)

4:30 - 4:40
(11) Two forms of completeness
Dr. P. C. Gilmore, IBM Research Center, Yorktown, New York (564-76)

4:45 - 4:55
(12) A hierarchy of formulae of set theory
Professor Azriel Levy, University of California, Berkeley (564-138)

WEDNESDAY 3:30 P.M.

Session on Applied Mathematics, Williford Room, Parlor B

3:30 - 3:40
(13) A class of solutions of the membrane theory of shells
Dr. Stefan Drobot, University of Chicago (564-207)
(Introduced by Dr. Alvin N. Feldzamen)

3:45 - 3:55
(14) Neutron transport in systems with spherical symmetry
Dr. Emil Grosswald, Institute for Advanced Study (564-5)

4:00 - 4:10
(15) On some new continuation formulas and uniqueness theorems in the theory of elasticity
Dr. James H. Bramble, Naval Ordnance Laboratory, Silver Spring, Maryland, and Professor Lawrence E. Payne, University of Maryland (564-134)

4:15 - 4:25
(16) On the structure of the linearized equations governing a streaming electron-ion gas
Professor C. L. Dolph, University of Michigan (564-75)
4:30 - 4:40  (17) Scalar diffraction by elliptic cylinders or prolate spheroids whose eccentricities are almost one
Dr. R. F. Goodrich, University of Michigan, Professor N. D. Kazarinoff, Mathematics Research Center, U. S. Army, University of Michigan, and Professor R. K. Ritt, University of Michigan (564-113)

4:45 - 4:55  (18) Magnetohydrodynamic flow past a flat plate, in the presence of a transverse magnetic field
Dr. W. Ericson, Grumman Aircraft, and Professor J. Radlow, Adelphi College (564-107)
(Introduced by Professor H. C. Kranzer)

WEDNESDAY 3:30 P.M.

Session on Analysis, Williford Room, Parlor C
3:30 - 3:40  (19) Concerning the measurable boundaries of a real function
Professor Casper Goffman, and Professor Robert E. Zink, Purdue University (564-209)

3:45 - 3:55  (20) Köthe spaces and inverse limits
Mr. Robert Welland, Purdue University (564-234)

4:00 - 4:10  (21) Matrix representation of vector spaces of pseudo-regular functions
Professor W. V. Caldwell, University of Michigan and University of Delaware (564-98)

4:15 - 4:25  (22) The structure of certain measure algebras
Mr. K. A. Ross, University of Washington (564-101)

4:30 - 4:40  (23) Recurrent properties of conservative measurable transformations
Professor C. T. Taam, Georgetown University (564-105)

4:45 - 4:55  (24) Approximate similarity and almost periodic matrices
Dr. James C. Lillo, RIAS, Inc., Baltimore, Maryland (564-152)

WEDNESDAY, 8:00 P.M.
Josiah Willard Gibbs Lecture, Grand Ball Room
Quantum field theory (One hour)
Professor Julian Schwinger, Harvard University
THURSDAY, 9:00 A.M.

Session on Geometry, Waldorf Room
9:00 - 9:10
(25) Fixed ideals and tangent vectors in infinitely differentiable function rings
Professor Lyle E. Pursell, Grinnell College (564-151)

9:15 - 9:25
(26) On local and global existence of Killing vector fields
Professor Katsumi Nomizu, Catholic University of America (564-9)

9:30 - 9:40
(27) Invariance of tensor differential operations under mappings
Professor Albert Nijenhuis, University of Washington (564-227)

9:45 - 9:55
(28) Integration of subspaces derived from a linear transformation field
Mr. E. T. Kobayashi, University of Washington (564-43)

10:00 - 10:10
(29) Discrete uniform subgroups of a class of Lie groups
Professor Louis Auslander, Indiana University (564-96)

10:15 - 10:25
(30) Generalized Hilbert geometries, Preliminary report
Professor Leon W. Green, University of Minnesota (564-206)

10:30 - 10:40
(31) The existence of totally-geodesic submanifolds
Dr. Robert Hermann, Lincoln Laboratory, Massachusetts Institute of Technology (564-73)

10:45 - 10:55
(32) Some uniqueness theorems on Riemannian manifolds with boundary
Professor C. C. Hsiung, Mathematics Research Center, U. S. Army, University of Wisconsin (564-137)

THURSDAY, 9:00 A.M.

Session on Algebra, Williford Room, Parlor A
9:00 - 9:10
(33) Behavior of integral group representations under ground ring extension
Professor Irving Reiner, University of Illinois (564-61)

9:15 - 9:25
(34) Derived functors without projective and injective resolutions
Professor S. A. Amitsur, Hebrew University and Yale University (564-78)
9:30 - 9:40
(35) Automorphisms of split algebras  
Professor Alex Rosenberg and Professor Daniel Zelinsky, Northwestern University (564-112)

9:45 - 9:55
(36) On Amitsur's complex, II  
Professor Alex Rosenberg and Professor Daniel Zelinsky, Northwestern University (564-111)

10:00 - 10:10
(37) Lattice-ordered rings and function rings. I  
Professor Melvin Henriksen and Professor J. R. Isbell, Purdue University (564-222)

10:15 - 10:25
(38) Lattice-ordered rings and function rings. II  
Professor Melvin Henriksen and Professor J. R. Isbell, Purdue University (564-223)

10:30 - 10:40
(39) On p-groups of class three generated by three elements  
Professor James R. Boen, Massachusetts Institute of Technology (564-117)

10:45 - 10:55
(40) Characterization of certain finite linear groups  
Professor John H. Walter, University of Washington (564-149)

THURSDAY, 9:00 A.M.

Session on Applied Mathematics, Williford Room, Parlor B

9:00 - 9:10
(41) Separability conditions for partial differential equations. Preliminary report  
Professor Hubert L. Hunzeker, University of Nebraska (564-225)

9:15 - 9:25
(42) Stability and bound for nonlinear system of difference and differential equations. II  
Dr. J. B. Rosen, Shell Development Corporation, Emeryville, California (564-90)

9:30 - 9:40
(43) Error estimates for the numerical solutions of elliptic differential equations  
Professor Johannes C. C. Nitsche, University of Minnesota, and Dozent Dr. Joachim Nitsche, University Freiburg i, Br. (564-195)

9:45 - 9:55
(44) Asymptotic properties of a boson field with given source  
Dr. Joseph M. Cook, Argonne National Laboratories, Lemont, Illinois (564-132)
10:00 - 10:10
(45) Some inequalities for membrane eigenvalues and torsional rigidity
Professor L. E. Payne and Professor Hans F. Weinberger, University of Maryland (564-193)

10:15 - 10:25
(46) Asymptotic factorization for perturbed Wiener-Hopf problems
Professor H. C. Kranzer and Professor James Radlow, Adelphi College (564-201)

10:30 - 10:40
(47) A Jacobi-type method for the automatic calculation of eigenvalues of non-symmetric matrices. Preliminary report
Dr. Patricia J. Eberlein, University of Rochester (564-82)

10:45 - 10:55
(48) Gaussian quadrature of non-polynomial functions with indefinite or complex weight functions: multiple sums, integrals, and derivatives: functional transformations
Dr. Paul F. Wacker, Radio Standards Division, National Bureau of Standards, Boulder, Colorado (564-218)

THURSDAY, 9:00 A.M.

Session on Analysis, Williford Room, Parlor C

9:00 - 9:10
(49) Concerning twin convergence regions for continued fractions
Professor George Copp and Professor D. F. Dawson, North Texas State College (564-97)

9:15 - 9:25
(50) A simultaneous solution of an algebraic equation. Preliminary report
Dr. Michael I. Aissen, Radiation Laboratory, Johns Hopkins University (564-175)

9:30 - 9:40
(51) Structure theorems for certain scalar-product algebras
Professor Parfeny P. Saworotnow and Reverend James F. Smith, Catholic University of America (564-130)

9:45 - 9:55
(52) Intertwining number theorems for induced representations of Lie groups
Professor Robert J. Blattner, University of California, Los Angeles (564-121)
10:00 - 10:10
(53) Fatou's theorem for elliptic partial differential equations
Mr. S. Łojasiewicz, University of Chicago and University of Krakow, Poland (564-228)
(Introduced by Professor J. W. T. Youngs)

10:15 - 10:25
(54) Difference properties for polynomials and exponential polynomials on topological groups
Professor Francis W. Carroll, Jr., Purdue University (564-179)

10:30 - 10:40
(55) Construction of a class of modular functions
Dr. Marvin I. Knopp, Institute for Advanced Study (564-203)

10:45 - 10:55
(56) Locating the maximum point on a unimodal surface
Professor D. J. Newman, Brown University and Sylvania Electronics (564-72)
(Introduced by Professor Anatole Beck)

THURSDAY, 1:00 P.M.

Presidential Address, Boulevard Room
On finite groups and their characters
Professor Richard Brauer, Harvard University

THURSDAY, 2:00 P.M.

Cole Prize Award and Business Meeting, Boulevard Room

THURSDAY 4:00 P.M.

Council Meeting, Parlor Dining Room 19

THURSDAY 4:00 P.M.

General Session, Waldorf Room
4:00 - 4:10
(57) On a theorem of E. Sparre Andersen and its applications to tests against trend
Professor H. D. Brunk, University of Missouri (564-79)

4:15 - 4:25
(58) Another reformulation of the entropy concept
Professor Joshua Chover, University of Wisconsin (564-45)

4:30 - 4:40
(59) On last exit times
Professor K. L. Chung, Syracuse University (564-87)
4:45 - 4:55  (60) Commutator problem. Preliminary report
Mr. E. A. Fay, Dr. J. E. Maxfield, and Mr. T. L. Reynolds, U. S. Naval Ordnance Test Station, China Lake, California (564-27)

5:00 - 5:10  (61) Some remarks on Kron's method of tearing. Preliminary report
Professor John B. Giever, New Mexico State University (564-212)

5:15 - 5:25  (62) Scaled iterations and linear equations
Mr. R. B. Kellogg and Mr. L. C. Noderer, Combustion Engineering, Incorporated, Windsor, Connecticut (564-40)

5:30 - 5:40  (63) Analytic continuation and hypergeometric series
Dr. W. C. Sangren and Miss Margaret L. Johnson, General Dynamics Corporation, San Diego, California (564-12)

5:45 - 5:55  (64) Inequalities for hypergeometric functions
Professor Thomas Erber, Illinois Institute of Technology (564-159)

THURSDAY, 4:00 P.M.

Session on Algebra, Williford Room, Parlor A

4:00 - 4:10  (65) On the existence of real-closed eta-sub-alpha-fields of power aleph-sub-alpha. Preliminary report
Professor Norman L. Alling, Purdue University (564-131)

4:15 - 4:25  (66) A note on matrix commutators
Professor Donald W. Robinson, Brigham Young University (564-115)

4:30 - 4:40  (67) On the reduction of group representations. Preliminary report
Professor J. S. Frame, Michigan State University (564-215)

4:45 - 4:55  (68) The square of a tree
Professor Frank Harary, University of Michigan, and Dr. I. C. Ross, Bell Telephone Laboratories (564-25)

5:00 - 5:10  (69) On the representation of directed graphs by orderings
Professor Paul Erdős and Professor Leo Moser, University of Alberta (564-31)
Tactical configurations. I. The incomplete balanced block designs
Professor Haim Hanani, Mathematics Research Center, U. S. Army, University of Wisconsin, and Israel Institute of Technology (564-136)

Non-extendibility conditions on mutually orthogonal Latin squares
Dr. E. T. Parker, Remington Rand UNIVAC, St. Paul, Minnesota (564-77)

Bounds for determinants with positive diagonals
Dr. Emilie V. Haynsworth, National Bureau of Standards, Washington, D. C. (564-205)

Session on Topology, Williford Room, Parlor B

Extension of mappings to weight-preserving compactifications
Professor Robert H. McDowell, Purdue University (564-198)

Linearization of autohomeomorphisms
Professor Arthur H. Copeland, Jr., Purdue University, and Professor Johannes de Groot, Purdue University and University of Amsterdam (564-180)

The algebra of peak functions on a triangulated manifold.
Preliminary report
Professor Merrill E. Shanks, Purdue University (564-186)

Obstructions to smoothing, for manifolds with boundary
Dr. James R. Munkres, Princeton University (564-127)

Postnikov invariants and Whitehead products
Professor Franklin P. Peterson, Massachusetts Institute of Technology (564-116)

On fibre spaces. Preliminary report
Professor Avrum I. Weinzweig, University of California, Berkeley (564-241)

Compact 0-dimensional transformation groups
Dr. C. N. Lee, University of Michigan (564-170)
5:45 - 5:55  
(80) A necessary and sufficient condition that an lc space be a quasi-complex  
Professor James E. Keisler, Louisiana State University (564-154)

THURSDAY, 4:00 P.M.

Session on Analysis, Williford Room, Parlor C

4:00 - 4:10
(81) A reflection formula in neutron transport theory  
Mr. D. C. McGarvey, RAND Corporation, Santa Monica, California (564-88)

4:15 - 4:25
(82) The exponential representation of functions analytic in the upper half-plane with positive imaginary part  
Professor Nachman Aronszajn and Professor W. F. Donoghue, Jr., University of Kansas (564-219)

4:30 - 4:40
(83) Lower bounds for eigenvalues by explicitly solvable intermediate problems  
Professor Norman W. Bazley, University of Maryland and National Bureau of Standards, Washington, D. C., and Professor D. W. Fox, University of Maryland (564-168)

4:45 - 4:55
(84) On a class of incomplete gamma functions  
Mr. Murray S. Klamkin, AVCO Research and Advanced Development Division, Wilmington, Massachusetts

5:00 - 5:10
(85) Generalized integrals  
Professor Costas Kassimatis, Cornell University (564-145)

5:15 - 5:25
(86) The greatest common divisor property for exponential polynomials  
Mr. Ward Bouwsma, University of Michigan (564-3)

5:30 - 5:40
(87) Complements to the Karhunen representation  
Dr. Isidore Fleischer and Dr. Anthony Kooharian, Sylvania Applied Research Laboratory, Waltham, Massachusetts (564-123)

5:45 - 5:55
(88) A standardized set of tables listing 500 binomial coefficient summations. Preliminary report  
Mr. Henry W. Gould, West Virginia University (564-110)
Session on Geometry and Topology, Waldorf Room

7:30 - 7:40
(89) Symmetry
Dr. Sally R. R. Struik, Belmont, Massachusetts, and Professor D. J. Struik, Massachusetts Institute of Technology (564-13)

7:45 - 7:55
(90) A finite number of reflections render a nonconvex plane polygon convex
Professor Nicholas D. Kazarinoff, Mathematics Research Center, U. S. Army, University of Wisconsin, and Professor R. H. Bing, University of Wisconsin (564-155)

8:00 - 8:10
(91) An axiom system for the geometries of the Euclidean family
Professor C. C. Buck, University of Alabama (564-80)

8:15 - 8:25
(92) On the endomorphism ring of a translation group
Professor Arno Cronheim, Ohio State University (564-81)

8:30 - 8:40
(93) Configurations of Desarguesian planes associated with lines of Veblen-Wedderburn planes
Professor T. G. Ostrom, Montana State University (564-91)

8:45 - 8:55
(94) Coordinates in affine planes
Professor Basil A. Rattray and Mr. A. Goldrich, McGill University (564-114)

9:00 - 9:10
(95) The Euler composition. I. B-manifolds. Preliminary report
Professor H. W. Guggenheimer, Washington State University (564-56)

9:15 - 9:25
(96) On certain invariants in Kaehler manifolds
Professor S. C. Saxena, Atlanta University (Introduced by Professor J. W. T. Youngs)

9:30 - 9:40
(97) Recursion in topological transformation semi-groups
Mr. Arlington M. Fink, Iowa State University (564-216)

THURSDAY, 7:30 P.M.

Session on Algebra, Williford Room, Parlor A
7:30 - 7:40
(98) On the primes between $n^2$ and $(n + 1)^2$
Professor Charles N. Moore, University of Cincinnati (564-231)

7:45 - 7:55
(99) Diagonal forms of odd prime degree over finite fields
Dr. J. F. Gray, St. Mary's University (564-86)

8:00 - 8:10
(100) Absolute valued algebras
Professor Kazimierz Urbanik and Professor F. B. Wright, Tulane University (564-93)

8:15 - 8:25
(101) A representation theorem for a class of archimedean lattice-ordered algebras. Preliminary report
Professor Donald G. Johnson, Pennsylvania State University (564-156)

8:30 - 8:40
(102) Generalization of a theorem of Rees
Professor Dov Tamari, University of Rochester and Technion, Israel (564-104)

8:45 - 8:55
(103) The multiplicative semigroup of integers modulo $m$
Professor Edwin Hewitt and Professor H. S. Zuckerman, University of Washington (564-124)

9:00 - 9:10
(104) The Schur indices of groups with Abelian Sylow subgroups
Mr. Louis Soloman, Haverford College (564-182)

9:15 - 9:25
(105) Connected ordered topological groupoids with idempotent endpoints
Professor Ronson J. Warne, Louisiana State University (564-233)

9:30 - 9:40
(106) Some irreducibility theorems for Bernoulli polynomials of higher order
Professor P. J. McCarthy, Florida State University (564-83)

THURSDAY, 7:30 P.M.

Session on Analysis, Williford Room, Parlor B

7:30 - 7:40
(107) Lower bounds for free membrane eigenvalues
Professor L. E. Payne, University of Maryland (564-194)

7:45 - 7:55
(108) An initial value problem for hyperbolic linear partial differential equations
Professor A. K. Aziz, Georgetown University, and Professor J. B. Diaz, University of Maryland (564-108)
8:00 - 8:10
(109) On the initial value problem for parabolic systems of differential equations. II
Professor Donald G. Aronson, University of Minnesota (564-166)

8:15 - 8:25
(110) On a diffusion equation of order one half and its application
Professor Hidehiko Yamabe, University of Minnesota (564-146)

8:30 - 8:40
(111) Asymptotic behavior of solutions of differential equations
Professor Avner Friedman, University of Minnesota (564-23)
(Introduced by Professor S. E. Warschawski)

8:45 - 8:55
(112) On almost periodic differential systems containing a small parameter
Dr. Jack K. Hale, RIAS, Incorporated, Baltimore, Maryland (564-157)

9:00 - 9:10
(113) Behavior of solutions near integral manifolds
Dr. J. K. Hale and Dr. A. P. Stokes, RIAS, Incorporated, Baltimore, Maryland (564-99)

9:15 - 9:25
(114) On periodic integral surfaces for analytic periodic differential systems. Preliminary report
Dr. George Seifert, RIAS, Incorporated, Baltimore, Maryland (564-187)

9:30 - 9:40
(115) Disconjugacy of a self-adjoint differential equation of the fourth order
Professor J. H. Barrett, Mathematics Research Center, U. S. Army, University of Wisconsin and University of Utah (564-119)

THURSDAY, 7:30 P.M.

Session on Analysis, Williford Room, Parlor C
7:30 - 7:40
(116) Some examples in the theory of absolutely convergent Fourier series
Mr. Yitzhak Katznelson, University of California, Berkeley (564-148)

7:45 - 7:55
(117) Essential finiteness of the interaction hamiltonian of certain quantum fields
Professor Irving E. Segal, University of Chicago (564-188)
8:00 - 8:10
(118) Indecomposable maximal ideals of a partially ordered vector space
Professor Joseph E. Kist, Pennsylvania State University (564-153)

8:15 - 8:25
(119) On the space of subsets of a topological vector space
Dr. Alex P. Robertson and Dr. Wendy Robertson, University of Kansas (564-189)
(Introduced by Professor Nachman Aronszajn)

8:30 - 8:40
(120) On the algebra of boundary-value problems
Professor Oswald Wyler, University of New Mexico (564-147)

8:45 - 8:55
(121) The spectral theorem in the uniform operator-topology. Preliminary report
Professor Gregers L. Krabbe, Purdue University (564-202)

9:00 - 9:10
(122) Box topologies on vector spaces over topological fields. Preliminary report
Professor Gerald Gould, Syracuse University (564-220)
(Introduced by Professor J. W. T. Youngs)

9:15 - 9:25
(123) Neighborhood bases of translation semi-rings in linear spaces. Preliminary report
Mr. P. H. Maserick, University of Maryland (564-229)

9:30 - 9:40
(124) Approximation of analytic polyhedra
Professor Errett A. Bishop, University of California, Berkeley (564-133)

FRIDAY, 9:30 A.M.

Session on Topology, Waldorf Room
9:30 - 9:40
(125) Homogeneous countable connected Hausdorff spaces
Mr. J. M. Martin, State University of Iowa (564-59)
(Introduced by Professor Steve Armentrout)

9:45 - 9:55
(126) Homogeneous continua which are almost chainable
Professor C. E. Burgess, University of Utah (564-178)

10:00 - 10:10
(127) Pushing a 2-sphere into its complement
Professor R. H. Bing, University of Wisconsin (564-165)
10:15 - 10:25
(128) Three-manifolds with strong simple-connectivity properties
Mr. Daniel R. McMillan, Jr., University of Wisconsin (564-139)

10:30 - 10:40
(129) Regular mappings and the space of homeomorphisms on a 3-manifold
Professor Mary-Elizabeth Hamstrom, Goucher College (564-39)

10:45 - 10:55
(130) Open mappings and solenoids
Professor Robert F. Williams, Purdue University and Institute for Advanced Study (564-236)

11:00 - 11:10
(131) Continuous mappings of the pseudo-arc
Professor Lawrence Fearnley, Brigham Young University (564-171)

11:15 - 11:25
(132) Branch sets of light open maps on n-manifolds, I
Professor Philip T. Church and Professor Erik Hemmingsen, Syracuse University (564-161)

11:30 - 11:40
(133) Branch sets of light open maps on n-manifolds, II
Professor Philip T. Church and Professor Erik Hemmingsen, Syracuse University (564-162)

FRIDAY, 9:30 A.M.

Session on Analysis, Normandie Lounge, Second Floor

9:30 - 9:40
(134) Uniform summability of orthonormal expansions
Professor Syed A. Husain, University of Seattle, and Professor D. Waterman, University of Wisconsin, Milwaukee

9:45 - 9:55
(135) Mean summability of ultraspherical polynomials
Mr. Richard Askey and Professor I. I. Hirschman, Jr., Washington University (564-24)

10:00 - 10:10
(136) On quotients of exponential polynomials
Professor Allen L. Shields, University of Michigan and New York University (564-184)

10:15 - 10:25
(137) Expansion of the Gaussian hypergeometric function in series of functions of the same kind
Mr. Richard L. Coleman and Mr. Yudell L. Luke, Midwest Research Institute, Kansas City, Missouri (564-172)

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10:30 - 10:40
(138) On economic representations of transcendental functions
Mr. Yudell L. Luke, Midwest Research Institute,
Kansas City, Missouri (564-199)

10:45 - 10:55
(139) Properties of a family of elliptic integrals
Dr. H. H. Germond, Radio Corporation of America,
Missile Test Project, Patrick Air Force Base, Florida
(564-213)

11:00 - 11:10
(140) Complementary spaces of Fourier coefficients
Dr. Günther W. Goes, Northwestern University

11:15 - 11:25
(141) A continuous analogue (for integral equation) of a result
of G. Szegő
Dr. Lonnie Cross, Atlanta University

11:30 - 11:40
(142) A class of linear differential-difference equations
Dr. M. L. Slater, Nuclear Development Corporation of
America, White Plains, New York, and Professor Her-
bert S. Wilf, University of Illinois (564-150)

FRIDAY, 9:30 A.M.

Session on Analysis, Williford Room, Parlor B
9:30 - 9:40
(143) On exceptional values of entire and meromorphic func-
tions
Professor Swarupchand M. Shah, Northwestern Univer-
sity, and Mr. S. K. Singh, University of Kansas (564-141)

9:45 - 9:55
(144) On the genus of a meromorphic function
Professor S. M. Shah, Northwestern University (564-26)

10:00 - 10:10
(145) The representation of the exponential function by
Laguerre series in the complex domain
Professor N. P. Yeardley, Thiel College (564-102)

10:15 - 10:25
(146) The existence of types of ambiguous points for bounded
analytic functions
Professor Gail S. Young, Jr., Tulane University
(564-237)

10:30 - 10:40
(147) On the analytic continuation of functions mapping the
upper half plane into itself
Professor D. S. Greenstein, University of Michigan
(564-30)
10:45 - 10:55  
(148) Abel-Poisson summability on matrix spaces  
Professor Josephine M. Mitchell, Pennsylvania State University (564-126)

11:00 - 11:10  
(149) Extremal problems for p-valent starlike functions  
Professor James A. Hummel, University of Maryland (564-125)

11:15 - 11:25  
(150) On the definition of quasi-conformality  
Dr. Heinz J. Renggli, Rutgers, The State University (564-190)

11:30 - 11:40  
(151) On an example of Koebe's for slit mappings  
Professor Edgar Reich, University of Minnesota (564-191)

FRIDAY, 9:30 A.M.

Session on Algebra, Williford Room, Parlor C

9:30 - 9:40  
(152) Simple algebraic and transcendental extensions of completely primary rings  
Professor Edmund H. Feller, University of Wisconsin, Milwaukee, and Professor Earl W. Swokowski, Marquette University (564-217)

9:45 - 9:55  
(153) On locally compact halfrings  
Professor Samuel G. Bourne, University of California, Berkeley (564-176)

10:00 - 10:10  
(154) An analogue of the Hilbert theory  
Mr. Luther E. Claborn, University of Michigan (564-163)

10:15 - 10:25  
(155) Intrinsic functions of matrices  
Professor R. F. Rinehart, Duke University (564-51)

10:30 - 10:40  
(156) Countably generated ideals in rings of continuous functions  
Professor Leonard Gillman, Purdue University and Institute for Advanced Study (564-32)

10:45 - 10:55  
(157) The order of the automorphism groups of infinite torsion Abelian groups  
Professor Elbert A. Walker, New Mexico State University (564-235)
11:00 - 11:10
(158) Central intertwining numbers for representations of finite groups
Mr. John Ernest, University of Illinois (564-55)

11:15 - 11:25
(159) A generalization of the Cartan-Brauer-Hua theorem
Professor Eugene Schenkman, Louisiana State University, and Professor W. R. Scott, University of Kansas (564-42)

11:30 - 11:40
(160) Abelian groups with minimal systems of generators. Preliminary report
Mr. S. A. Khabbaz, University of Kansas (546-6)

FRIDAY, 2:00 P.M.

Invited Address, Boulevard Room
Partial differential equations with a constant coefficient (One hour)
Professor Paul Rosenbloom, University of Minnesota

FRIDAY, 3:30 P.M.

Session on Topology, Waldorf Room

3:30 - 3:40
(161) On the group of all homeomorphisms of a manifold
Dr. G. M. Fisher, Princeton University (564-94)

3:45 - 3:55
(162) On the proximal relation in topological dynamics
Professor Joseph Auslander, Carnegie Institute of Technology (564-74)

4:00 - 4:10
(163) Clusters of indecomposability. Part II
Mr. R. L. Kelley and Professor Paul M. Swingle, University of Miami (564-144)

4:15 - 4:25
(164) A proof of the generalized Shoenflies theorem
Mr. Morton Brown, University of Michigan (564-38)

4:30 - 4:40
(165) On convergence of sequences in C(X) and pseudo-compactness
Professor Robert W. Bagley, Mississippi Southern College (564-143)

4:45 - 4:55
(166) Bases and local finiteness. Preliminary report
Professor Harry H. Corson, Professor Trevor J. McMinn, Professor Ernest A. Michael, and Professor Jun-Ito Nagata, University of Washington (564-109)
5:00 - 5:10  (167) Fixed point theorem in and completion of developable spaces  
Mr. J. C. Mathews, Iowa State University (564-36)  
(Introduced by Dr. H. P. Thielman)

5:15 - 5:25  (168) Intersections of absolute retracts and the fixed point property  
Professor E. H. Connell, University of Miami (564-29)

5:30 - 5:40  (169) Inscribed squares in plane Jordan curves  
Professor R. P. Jerrard, University of Illinois (564-103)

FRIDAY, 3:30 P.M.

Session on Statistics and Probability, Normandie Lounge, Second Floor

3:30 - 3:40  (170) Inference in stochastic processes. II. Asymptotic properties of maximum likelihood estimators  
Dr. M. M. Rao, Carnegie Institute of Technology (564-192)  
(Introduced by Professor Victor J. Mizel)

3:45 - 3:55  (171) A Daniell approach to stochastic processes  
Mr. S. H. Coleman, University of Virginia (564-173)

4:00 - 4:10  (172) The red and black casino  
Professor Lester E. Dubins, University of California, Berkeley, and Professor Leonard J. Savage, University of Chicago (564-160)

Dr. Malcolm Goldman, University of Michigan (564-208)

4:30 - 4:40  (174) Sample functions of time-homogeneous differential processes. I  
Professor R. M. Blumenthal and Professor R. K. Getoor, University of Washington (564-52)

4:45 - 4:55  (175) Sample functions of time-homogeneous differential processes. II  
Professor R. M. Blumenthal and Professor R. K. Getoor, University of Washington (564-53)
5:00 - 5:10
(176) Regression with special orthogonal vectors
Professor Andrew Sobczyk, R. C. A. Missile Test
Project and University of Florida (564-183)

5:15 - 5:25
(177) Statistical metric spaces arising from sets of random
variables. II
Professor Berthold Schweizer, University of California,
Los Angeles, and Professor Abe Sklar, Illinois
Institute of Technology (564-129)

5:30 - 5:40
(178) On the iterations of two conditional expectation operators
Dr. Yuan S. Chow, IBM Research Laboratory, Yorktown
Heights, New York (564-135)

FRIDAY, 3:30 P.M.

Session on Analysis, Williford Room, Parlor B
3:30 - 3:40
(179) Coefficients of power series
Mr. George R. Blakley, University of Maryland
(564-120)

3:45 - 3:55
(180) Hölder continuity of n-dimensional quasi-conformal
mappings
Dr. E. D. Callender, Philco Corporation, Palo Alto,
California (564-46)

4:00 - 4:10
(181) The spherical summability of conjugate multiple
Fourier-Stieltjes series
Professor Victor L. Shapiro, Rutgers, The State
University (564-232)

4:15 - 4:25
(182) The Lebesgue constants for regular Hausdorff means
Professor Lee Lorch, University of Alberta, and Pro-
fessor D. J. Newman, Brown University (564-7)

4:30 - 4:40
(183) Tchebycheff approximations
Dr. J. R. Rice, National Bureau of Standards, Washing-
ton, D. C. (564-60)

4:45 - 4:55
(184) On the non-minimality of the Weierstrass product
Professor Paul Malliavin, University of Caen, France,
and Professor Lee A. Rubel, University of Illinois
(564-230)

5:00 - 5:10
(185) (C,1)-summability of orthogonal series
Mr. Dan J. Eustice, Purdue University (564-221)
5:15 - 5:25
(186) Chebyshev approximations to integral transforms
Mr. Jet Wimp, Midwest Research Institute (564-238)
(Introduced by Mr. Y. L. Luke)

5:30 - 5:40
(187) Generalized Fibonacci numbers and associated matrices
Professor E. P. Miles, Jr., Florida State University (564-49)

FRIDAY, 3:30 P.M.

Session on Analysis, Williford Room, Parlor C
3:30 - 3:40
(188) An extension of the Cesari-Cavalieri inequality
Professor R. E. Fullerton, University of Maryland (564-214)

3:45 - 3:55
(189) A characterization of AC maps
Professor C. J. Neugebauer, Purdue University (564-226)

4:00 - 4:10
(190) The Geôcze k-area
Professor Togo Nishiura, Purdue University (564-89)

4:15 - 4:25
(191) On the asymptotic behavior of certain integrals
Professor G. R. MacLane, Rice Institute (564-200)

4:30 - 4:40
(192) Functions with rectangular projections
Dr. W. M. Gilbert, Princeton University (564-211)

4:45 - 4:55
(193) The extinguishing of a class of functions
Dr. H. Fast and Professor Kazimierz Urbanik, Wroclaw University, Poland, and Tulane University (564-92)
(Introduced by Professor F. B. Wright)

5:00 - 5:10
(194) Lebesgue density as a set function
Dr. N. F. G. Martin, University of Virginia (564-100)

5:15 - 5:25
(195) Differentiation of set functions using Vitali coverings
Professor William E. Hartnett, College of the Holy Cross, and Dr. A. H. Kruse, University of Kansas (564-158)

5:30 - 5:40
(196) A functional inequality
Dr. G. U. Brauer, University of Minnesota (564-37)
FRIDAY, 7:30 P.M.

General Session, Williford Room, Parlor A
7:30 - 7:40
   (197) A decision procedure for transformations of trees
   Professor Trevor Evans, Emory University and University of Nebraska (564-106)

7:45 - 7:55
   (198) On uppersemicontinuous decompositions of $E^3$ into $E^3$
   Professor Louis F. McAuley, University of Wisconsin and Louisiana State University

8:00 - 8:10
   (199) The groups of all homeomorphisms of the solenoids
   Professor Richard D. Anderson, Louisiana State University (564-224)

8:15 - 8:25
   (200) Commutative multiplications on the plane
   Professor J. G. Horne, Jr., University of Georgia (564-204)

8:30 - 8:40
   (201) On the weak topology of a Banach space
   Professor Harry H. Corson, III, University of Washington (564-122)

8:45 - 8:55
   (202) Interpolation by sectionally linear functions
   Professor H. W. E. Schwerdtfeger, McGill University (564-41)

9:00 - 9:10
   (203) Matrix inversion and determinant evaluation by symmetric method
   Professor J. V. Talacko, Marquette University (564-28)

9:15 - 9:25
   (204) Necessary conditions for variational problems with side conditions
   Professor Philip Cooperman, University of Pittsburgh (564-118)

9:30 - 9:40
   (205) On differentiation of set functions
   Professor W. E. Hartnett, College of the Holy Cross, and Mr. A. H. Kruse, University of Kansas

FRIDAY, 7:30 P.M.

Session on Analysis, Williford Room, Parlor B
7:30 - 7:40
   (206) On a theorem of Maddaus
   Professor Jesus Gil de Lamadrid, University of Minnesota (564-48)
7:45 - 7:55
(207) A representation theorem for bounded convex sets
Dr. R. R. Phelps, Institute for Advanced Study (564-50)

8:00 - 8:10
(208) Quasi-barrelled locally convex spaces
Professor Mark Mahowald and Professor Gerald Gould,
Syracuse University (564-85)

8:15 - 8:25
(209) A generalization of the Banach closed graph theorem
Dr. W. Slowikowski, University of Wisconsin, Milwaukee
(Introduced by Professor J. W. T. Youngs)

8:30 - 8:40
(210) Means on the integers invariant under certain permuta-
tions
Dr. D. W. Dean, Yale University, and Professor R. A.
Raimi, University of Rochester (564-22)

8:45 - 8:55
(211) Equivalent permutations of the positive integers
Dr. D. W. Dean, Yale University, and Professor R. A.
Raimi, University of Rochester (564-47)

9:00 - 9:10
(212) Some examples of quasi-invariant distributions on Hil-
bert space
Professor Jacob Feldman, University of California,
Berkeley (564-18)

9:15 - 9:25
(213) The initial-boundary value problem for the wave equation
in an exterior domain with spherical boundary. Prelimin-
ary report
Professor Calvin H. Wilcox, Mathematics Research
Center, U. S. Army, University of Wisconsin and Cali-
forinia Institute of Technology (564-240)

FRIDAY, 7:30 P.M.

Session on Algebra, Williford Room, Parlor C
7:30 - 7:40
(214) A generalization to functions of density theorems for
sums of sequences of integers. Preliminary report
Professor William O. J. Moser, University of Manitoba
(564-196)

7:45 - 7:55
(215) An algebraic characterization of the polynomial solutions
of certain partial differential equations
Professor Harold S. Shapiro, New York University
(564-185)
8:00 - 8:10
(216) On the Mills-Seligman axioms for Lie algebras of classical type
Dr. Richard Block, California Institute of Technology (564-177)

8:15 - 8:25
(217) Moments and characteristic roots
Mr. F. L. Bauer, Institute of Applied Mathematics, Gutenberg University, and Dr. A. S. Householder, Oak Ridge National Laboratory (564-21)

8:30 - 8:40
(218) Projective and torsion free modules
Dr. Hyman Bass, Columbia University (564-164)

8:45 - 8:55
(219) On the theory of ring-logics. II. Preliminary report
Professor Adil M. Yaqub, Purdue University (564-142)

9:00 - 9:10
(220) Continued function expansions of real numbers
Mr. B. K. Swartz and Dr. Burton Wendroff, University of California, Los Alamos (564-16)

9:15 - 9:25
(221) Simple (-1,1) rings with an idempotent. Preliminary report
Mr. Carl C. Maneri, Ohio State University (564-197)

9:30 - 9:40
(222) Modulated and partition lattices
Mr. David Sachs, Illinois Institute of Technology (564-44)

SUPPLEMENTARY PROGRAM
(To be presented by title)

(223) The sound speeds of a diabatic fluid
Mr. Ralph Abraham, University of Michigan
(Introduced by Professor J. W. T. Youngs)

(224) The sound speeds of a charged fluid
Mr. Ralph Abraham, University of Michigan
(Introduced by Professor J. W. T. Youngs)

(225) Generalized link products
Dr. R. J. Aumann, The Hebrew University (564-1)

(226) An equicontinuity condition for transformation groups
Professor J. D. Baum, Oberlin College

(227) Polynomials defined by a difference system. I
Professor G. E. Baxter, University of Minnesota

(228) Polynomials defined by a difference system. II
Professor G. E. Baxter, University of Minnesota
(229) Probabilistic and Tauberian methods for stability analysis of certain integro-differential equations
   Dr. Václav E. Beneš, Bell Telephone Laboratories, Murray Hill, New Jersey, and Dartmouth College

(230) On mappings of an extended class in the theory of functions of two complex variables. I
   Professor Stefan Bergman, Stanford University (564-62)

(231) On mappings of an extended class in the theory of functions of two complex variables. II
   Professor Stefan Bergman, Stanford University (564-63)

(232) On random resolvent operators
   Professor A. T. Bharucha-Reid, University of Oregon

(233) On Reynolds operators in finite-dimensional algebras
   Mr. Martin Billik, Massachusetts Institute of Technology (564-2)
   (Introduced by Professor G.-C. Rota)

(234) A 3-cell is the only object whose cartesian product with an arc is a 4-cell
   Professor R. H. Bing, University of Wisconsin

(235) Sample functions of time-homogeneous differential processes. III
   Professor R. M. Blumenthal and Professor R. K. Getoor, University of Washington (564-54)

(236) On the factors of automorphy for the group of integral modular-substitutions of second degree
   Dr. U. H. R. O. Christian, Institute for Advanced Study

(237) A characterization of integer group and real group
   Dr. Hsin Chu, University of Michigan
   (Introduced by Dr. G. L. Walker)

(238) Some structure theorems in topological dynamics
   Dr. Hsin Chu, University of Michigan
   (Introduced by Dr. G. L. Walker)

(239) The Cauchy initial value problem for charged, compressible, relativistic fluids
   Professor Nathaniel Coburn, University of Michigan (564-35)

(240) A set of integers
   Professor Eckford Cohen, University of Tennessee

(241) Class number formula for relative-quadratic fields over 2 1/2 or 3 1/2. Preliminary report
   Professor Harvey Cohn, University of Arizona (564-34)

(242) Embedding incomplete Latin squares
   Professor Trevor Evans, Emory University and University of Nebraska

(243) The equivalence of fiber spaces and bundles
   Professor Edward R. Fadell, University of Wisconsin
(244) Some classes of equivalent Gaussian processes on the interval
Professor Jacob Feldman, University of California, Berkeley (564-20)

(245) The adjoint Weyr characteristic
Dr. A. N. Feldzamen, University of Wisconsin and University of Chicago (564-33)

(246) On the group of all homeomorphisms of a manifold
Dr. G. M. Fisher, Princeton University

(247) On the interpolation of $L^p$ functions by Jackson polynomials
Professor Richard P. Gosselin, University of Connecticut

(248) On the numerical solution of linear elliptic differential equations
Professor Donald Greenspan, Purdue University (564-4)

(249) The Euler composition. III. Antisymmetric linear connections. Preliminary report
Professor H. W. Guggenheimer, Washington State University (564-58)

(250) The Euler composition. II. Restricted and general tensors. Preliminary report
Professor H. W. Guggenheimer, Washington State University (564-57)

(251) Local triviality of certain restriction mappings
Dr. M. W. Hirsch and Dr. R. S. Palais, Institute for Advanced Study (564-64)

(252) On the topology of spaces of immersions of differentiable manifolds
Dr. M. W. Hirsch and Dr. R. S. Palais, Institute for Advanced Study (564-65)

(253) Results on arcs in semigroups
Professor R. P. Hunter, University of Georgia and Oxford University

(254) Direct decomposition of certain types of skew-lattice. Preliminary report
Dr. J. A. Kalman, University of Auckland, New Zealand (564-66)

(255) Isomorphism of ultraproducts
Mr. H. Jerome Keisler, University of California, Berkeley
(Introduced by Mr. J. A. Zilber)

(256) Properties preserved under reduced products
Mr. H. Jerome Keisler, University of California, Berkeley
(Introduced by Mr. J. A. Zilber)
(257) A nowhere euclidean cartesian factor of $E^4$
Professor K. W. Kwun, Seoul National University

(258) Maximal ideals
Professor W. G. Leavitt, University of Nebraska

(259) On the Hahn-Mazurkiewicz theorem in non-metric spaces
Dr. Sibe Mardešić, University of Zagreb (564-8)

(260) A general form of functional dependence. Preliminary report
Professor Gary H. Meisters, University of Nebraska

(261) Asymptotic properties of derivatives of stationary measures
Professor Shu-Teh C. Moy, Wayne State University

(262) A new parameter relating hyperbolic and circular functions
Dr. Charles A. Muses, Locarno, Switzerland

(263) Extensions to isols. II
Professor Anil Nerode, Cornell University

(264) Existence of slices for actions of non-compact Lie groups
Dr. R. S. Palais, Institute for Advanced Study (564-67)

(265) A computer search for latin squares orthogonal to latin squares of order ten. Preliminary report
Dr. E. T. Parker, Remington Rand UNIVAC, St. Paul, Minnesota (564-71)

(266) A generalization of Haar's theorem on unique best approximation
Dr. R. R. Phelps, Institute for Advanced Study (564-70)

(267) Integral closure of linear differential equations
Dr. E. C. Posner, University of Wisconsin

(268) Accumulability and preservation of sums
Dr. E. C. Posner, University of Wisconsin

(269) Canonical conformal maps onto a circular slit annulus
Professor Edgar Reich and Professor S. E. Warchawski, University of Minnesota

(270) Involutory matrices modulo a prime power
Professor Irma Reiner, University of Illinois
(Introduced by Professor Irving Reiner)

(271) Split Runge-Kutta for simultaneous equations
Dr. J. R. Rice, National Bureau of Standards, Washington, D. C. (564-68)

(272) A new representation of Gegenbauer's functions
Dr. J. R. Rice, National Bureau of Standards, Washington, D. C. (564-69)

(273) The undecidability of exponential Diophantine equations
Dr. Julia B. Robinson, Berkeley, California

(274) Duality in modules
Professor Alex Rosenberg and Professor Daniel Zelinsky, Northwestern University
Operator characterization of conditional expectation
Professor G.-C. Rota, Massachusetts Institute of Technology (564-10)

Alternative formulas for osculatory and hyperosculatory inverse interpolation
Dr. H. E. Salzer, CONVAIR, San Diego, California (564-11)

On uniformization of sets in topological spaces
Professor Maurice Sion, University of California, Berkeley (564-13)

Hereditarily compact spaces
Dr. A. H. Stone, Manchester University

N-universally complete sets of sentences of formalized theories. Preliminary report
Mr. Konrad Suprunowicz, University of Nebraska

"Near-groups" as generalized normal multiplication tables (g.n.m.t.)
Professor Dov Tamari, Technion and University of Rochester

Inversion of Toeplitz matrices. III
Professor Harold Widom, Cornell University and Institute for Advanced Study

Subgroups of compact connected Lie groups
Dr. J. A. Wolf, University of Chicago (564-17)

Concerning a question about Vitali coverings
Dr. John M. Worrell, Jr., University of Texas
(Introduced by Professor R. L. Moore)

A fixed-point theorem for arcwise connected continua
Professor Gail S. Young, Tulane University

Bloomington, Indiana
December 14, 1959

J. W. T. Youngs
Associate Secretary
The five hundred sixty-fifth meeting of the American Mathematical Society will be held on Thursday, Friday and Saturday, February 18-20, 1960 at the University of Arizona, Tucson, Arizona. This extraordinary meeting of the Far Western Section of the Society is planned as a part of the seventy-fifth anniversary celebration of the University of Arizona.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, and with the financial support of the National Science Foundation, a Symposium on "Differential Geometry" will be held on Thursday and Friday. The Program Committee for the Symposium consists of Professors C. B. Allendoerfer, chairman, Herbert Busemann, Hans Samelson, and D. C. Spencer.

By invitation of the same Committee, there will be an address on Saturday by Professor T. S. Motzkin of the University of California, Los Angeles on "Convex sets in analysis". There will also be sessions for contributed papers on Saturday.

Tucson is served by American, Continental, Frontier and Trans-World Airlines, by Southern Pacific Railroad, and by Greyhound Bus Line. The principal highway through Tucson is U. S. 80-89.

An extensive program of entertainment has been planned for members and their families at this meeting. The events include a Rodeo Parade, luncheon and fashion show, and tours of the campus, San Xavier Mission, the Desert Museum, and the Old Tucson Movie Set.

The Committee on Arrangements for this meeting would like to know as early as possible the approximate number of people to expect. Since February is the peak of the tourist season in Tucson, there will be great difficulty in securing accommodations unless reservations are made before the meeting. The Department of Mathematics at the University of Arizona will handle requests for hotel, motel, or dormitory rooms, provided they are received early enough. Additional information on this meeting will be given in the Program which will appear in the February issue of these NOTICES.

R. S. Pierce
Associate Secretary

Seattle, Washington
November 4, 1959
The five hundred sixty-sixth meeting of the American Mathematical Society will take place on Saturday, February 27, 1960 at the Washington Square Campus of New York University in New York City. All sessions will be in the Main Building.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, Professor Serge Lang of Columbia University will deliver an hour address at 2:00 P.M. in Room 703. The title of his address is "Some conjectures in diophantine equations".

There will be sessions for contributed papers at 10:00 A.M. and 3:15 P.M.

The registration desk will be near Room 703.

Further details of arrangements and the program of the meeting will appear in the February issue (no. 43) of these NOTICES. It is to be expected that the abstracts of most of the contributed papers to be presented in person will appear in the same issue of the NOTICES as the program.

The Main Building of New York University is at the northeast corner of Washington Square.

Washington Square may be reached by public transportation as follows:

Lexington Avenue (Interborough) Subway (IRT) -- Local to Astor Place Station. Walk west on Astor Place to Broadway, then south on Broadway to Waverly Place, and west on Waverly Place to Washington Square.

Seventh Avenue (Interborough) Subway (IRT) -- Local to Sheridan Square Station. Walk east on Waverly Place to Washington Square.

Broadway (Brooklyn-Manhattan) Subway (BMT) -- Brighton local or Fourth Avenue local to Eighth Street Station. Walk south on Broadway to Waverly Place, then west on Waverly Place to Washington Square.

Sixth or Eighth Avenue (Independent) Subway (IND) -- Express to West Fourth Street-Washington Square Station. Walk east on West Fourth Street or Waverly Place to Washington Square.

Fifth Avenue Bus -- Busses numbered 3, and some numbered 5, to last stop, which is Washington Square.

Everett Pitcher
Associate Secretary
ACTIVITIES OF OTHER ASSOCIATIONS

THE FORTY-THIRD ANNUAL MEETING OF THE MATHEMATICAL ASSOCIATION OF AMERICA will be held at the Conrad Hilton Hotel, Chicago, Illinois, from Thursday to Saturday, January 28 to 30, 1960, in conjunction with meetings of the American Mathematical Society, the National Council of Teachers of Mathematics, and the Society for Industrial and Applied Mathematics. The Saturday sessions of the Association will be joint sessions with the National Council of Teachers of Mathematics.

Sessions of the Mathematical Association of America will be held on Thursday at 9:15 a.m., on Friday at 9:15 a.m. in the Boulevard Room of the Conrad Hilton Hotel and on Saturday at 9:00 a.m. and 2:00 p.m. in the Eighth Street Theater, located near the corner of Eighth and Wabash on the southwest corner of the block containing the Conrad Hilton Hotel.

The only sessions of the National Council will be joint sessions with the Association on Saturday at 9:00 a.m. and 2:00 p.m. in the Eighth Street Theater.

Immediately following the annual business meeting of the Association, to be held on Friday at 10:15 a.m., the Chauvenet Prize will be awarded to Professor Cornelius Lanczos of the Dublin Institute for Advanced Studies, for his paper "Linear Systems in Self-Adjoint Form," American Mathematical Monthly, Vol. 65 (1958), pp. 665-679.

The Board of Governors of the Association will meet on Thursday evening at 7:30 p.m. in Private Dining Room 18.

FIRST SESSION, THURSDAY: 9:15 A.M.
Boulevard Room, Conrad Hilton Hotel
"Exterior Differential Forms in the Undergraduate Program"

Moderator: Harley Flanders, University of California, Berkeley

9:15 - 9:30 Introductory Remarks
Carl B. Allendoerfer, University of Washington

9:30 - 10:20 Exterior Differential Calculus
Lawrence Markus, University of Minnesota

10:20 - 10:30 Intermission

10:30 - 11:20 Applications to Geometry
Albert Nijenhuis, University of Washington

11:20 - 11:35 Concluding Remarks
Harley Flanders, University of California, Berkeley

SECOND SESSION, FRIDAY: 9:15 A.M.
Boulevard Room, Conrad Hilton Hotel

9:15 Lecture: The Four and Eight Square Problem and Division Algebras
Charles W. Curtis, University of Wisconsin

10:15 Annual Business Meeting of the Association. Award of the Chauvenet Prize

10:45 Lecture: Entropy in Ergodic Theory
Paul R. Halmos, University of Chicago
JOINT MEETING OF M.A.A. AND N.C.T.M.
SATURDAY: 9:00 A.M. - EIGHTH STREET THEATER

GENERAL SESSION, 9:00 - 10:15 A.M.

Presiding: Carl B. Allendoerfer, University of Washington
Short and Long Range Improvement of the School Mathematics Curriculum
Paul C. Rosenbloom, University of Minnesota
Better Curriculum or Better Pedagogy?
Morris Kline, New York University

SESSION ON ADVANCED PLACEMENT, 10:30 - 11:45 A.M.

Moderator: Harold P. Fawcett, Ohio State University

Collegiate Views and Experiences:
E. P. Vance, Oberlin College
W. T. Scott, Northwestern University

High School Views and Experiences:
Edwin C. Douglas, The Taft School, Watertown, Connecticut
Henry Swain, New Trier Township High School, Winnetka, Illinois

SESSION ON THE NATURE AND ROLE OF GEOMETRY IN HIGH SCHOOL, 2:00 - 3:15 P.M.

Moderator: Max Beberman, University of Illinois

Collegiate Views:
Kenneth B. Leisenring, University of Michigan
Robert R. Christian, University of British Columbia and University of Illinois

High School Views:
Henry Syer, Kent School, Kent, Connecticut
Roderick McLennon, Arlington Heights High School, Arlington Heights, Illinois

SESSION ON CALCULUS IN HIGH SCHOOL, 3:15 - 4:30 P.M.

Moderator: Phillip S. Jones, University of Michigan

Collegiate Views:
Albert A. Blank, New York University
J. H. Neelley, Carnegie Institute of Technology

High School Views:
W. Eugene Ferguson, Newton High School, Newtonville, Massachusetts
Hubert Davis, Cranbrook School, Bloomfield Hills, Michigan
THE SINGLE SIAM SESSION FOR THE WINTER MEETINGS OF THE MATHEMATICAL SOCIETIES IN CHICAGO is as follows:

Date: Friday, January 29, 1960
Time: 8:00 P.M.
Place: Waldorf Room, Conrad Hilton Hotel

Speakers and Titles:
- On Prolate Spheroidal Wave Functions I: Fourier Analysis; Dr. David Slepian, Bell Telephone Laboratories, Inc.
- On Prolate Spheroidal Wave Functions II: Uncertainty; Dr. H. O. Pollak, Bell Telephone Laboratories, Inc.

EDITOR'S NOTE: In the past, this department has presented programs of future meetings of societies whose activities were closely related to our own. We shall continue this practice in the future, but to provide a broader coverage, we introduce below a Calendar of Future Meetings. Our purpose is to both extend the time scale for announcements and to provide more information on international meetings.

The Calendar includes symposia, seminars, and institutes sponsored by the AMS, but does not include regular meetings of the Society, which are listed on the inside front cover of each issue of the NOTICES.

FUTURE MEETINGS OF RELATED ORGANIZATIONS

**February, 1960**
- American Mathematical Society and the National Science Foundation
  - "Symposium on Differential Geometry"
  - in conjunction with 1960 meeting of the Society
  - Place - University of Arizona
  - Location - Tucson, Arizona
  - Date - February 19-20, 1960

**March, 1960**
- Society of Actuaries
  - Hotel - Mayflower Hotel
  - Location - Washington, D. C.
  - Date - March 24-25, 1960
  - Secretary - Dr. A. A. McKinnie, Society of Actuaries, 208 South La Salle Street, Chicago 4, Illinois

**April, 1960**
- The Institute of Mathematical Statistics
  - Place - Purdue University
  - Location - Lafayette, Indiana
  - Date - April 7-9, 1960
  - Secretary - Dr. George E. Nicholson, Jr., Department of Statistics, University of North Carolina, Chapel Hill, North Carolina
American Mathematical Society and The Institute for Defense Analyses

"Symposium on Mathematical Problems in the Structure of Language"
in conjunction with 1960 meeting of the Society
Hotel - Hotel New Yorker
Location - New York, New York
Date - April 14-16, 1960

American Mathematical Society and the Office of Ordnance Research, U. S. Army

"Symposium on Stability Problems in Hydrodynamics"
in conjunction with the 1960 meeting of the Society
Hotel - New Yorker Hotel
Location - New York, New York
Date - April 14-16, 1960

The Institute of Mathematical Statistics

Place - Columbia University
Location - New York, New York
Date - April 21-23, 1960
Secretary - Dr. George E. Nicholson, Jr., Department of Statistics, University of North Carolina, Chapel Hill, North Carolina

American Mathematical Society and the Air Force Office of Scientific Research

"Symposium on Differential Equations"
in conjunction with 1960 meeting of the Society
Place - University of California
Location - Berkeley, California
Date - April 22-23, 1960

Western Joint Computer Conference
Location - San Francisco, California
Date - May 2-6, 1960
Chairman - H. M. Zeigler, Technical Program Committee, WSJCC, Stanford Research Institute, Menlo Park, California

Society of Actuaries

Hotel - Roosevelt Hotel
Location - New Orleans, Louisiana
Date - May 5-6, 1960
Secretary - Dr. A. A. McKinnie, Society of Actuaries, 208 South La Salle Street, Chicago 4, Illinois
May, 1960

Operations Research Society of America
17th National (8th Annual Meeting)
Hotel - Statler Hilton Hotel
Location - New York, New York
Date - May 19-20, 1960
Chairman - Professor Max Woodbury, Department of Mathematics, New York University, New York 53, New York

June, 1960

The American Society of Mechanical Engineers
Semi-Annual Meeting and Aviation Conference
Hotel - Statler Hilton
Location - Dallas, Texas
Date - June 5-10, 1960
Director of Public Relations - L. S. Dennegar, The American Society of Mechanical Engineers, 29 West 39th Street, New York 18, New York

International Conference on Many-Body Problems
Location - Utrecht, NETHERLANDS
Date - June 7-12, 1960
Contact - Professor L. C. P. van Hove, Department of Theoretical Physics, State University, Utrecht, NETHERLANDS

Mathematics Research Center, U. S. Army; University of Wisconsin

"International Conference on Partial Differential Equations and Continuum Mechanics"
Place - University of Wisconsin, Madison, Wisconsin
Housing - Elizabeth Waters Hall - a university dormitory
Date - June 7-15, 1960
Contact - Professor R. E. Langer, Mathematics Research Center, U. S. Army, The University of Wisconsin, Madison 6, Wisconsin

International Union of Pure and Applied Physics Commission on Thermodynamics and Statistical Mechanics Meeting
Location - Utrecht, NETHERLANDS
Date - June 12-16, 1960
Contact - Professor L. C. P. van Hove, Department of Theoretical Physics, State University, Utrecht, NETHERLANDS

The American Society of Mechanical Engineers,
Applied Mechanics Conference
Place - Pennsylvania State University
Location - University Park, Pennsylvania
Date - June 20-22, 1960
Director of Public Relations - L. S. Dennegar, The American Society of Mechanical Engineers, 29 West 39th Street, New York 18, New York
International Mathematical Union

International Colloquium on "Topology and Differential Geometry"
Location - Zürich, SWITZERLAND
Date - June 20-26, 1960
Chairman - Professor H. Hopf.

International Statistical Institute, Session

Location - Tokyo, JAPAN
Date - June, 1960
Contact - E. Lunenberg, Director of the Permanent Office, 2 Oostduinlaan, The Hague, NETHERLANDS

International Federation of Automatic Control International Congress for Automatic Control

Location - Moscow, USSR
Date - June 25-July 5, 1960
Contact - Secretariat of the Federation of Automatic Control, c/o Verein Deutsche Ingenieurs, 79 Prinz-Georg-Stasse, Düsseldorf, GERMANY

July, 1960

International Mathematical Union

International Symposium on "Linear Spaces - Geometrical Aspects and Applications to Analysis"
Location - Jerusalem, ISRAEL
Date - July, 1960
Chairman - Professor A. Dvoretzky

American Mathematical Society Summer Seminar

"Modern Physical Theories and Associated Mathematical Developments"
Sponsored by Atomic Energy Commission; National Science Foundation; Office of Naval Research; Office of Ordnance Research, U. S. Army
Place - University of Colorado
Location - Boulder, Colorado
Date - July 24-August 19, 1960
Chairman - Professor K. O. Friedrichs, New York University, 25 Waverly Place, New York 3, New York
American Mathematical Society Summer Institute on Finite Groups

Sponsored by the National Science Foundation
Place - California Institute of Technology
Location - Pasadena, California
Date - August 1-28, 1960
Chairman - Marshall Hall, Jr.

Association for Computing Machinery

National Association for Computing Machinery Conference
Place - Marquette University
Location - Milwaukee, Wisconsin
Date - August 23-25, 1960

The Institute of Mathematical Statistics and American Statistical Association

Place - Stanford University
Location - Stanford, California
Date - August 23-26, 1960
Secretary - Dr. George E. Nicholson, Jr., Department of Statistics, University of North Carolina, Chapel Hill, North Carolina

International Congress for Logic, Methodology and Philosophy of Science

Sponsored by the International Union for History and Philosophy of Science
Place - Stanford University
Location - Stanford, California
Date - August 24-September 2, 1960
Contact - Professor Patrick Suppes, Serra House, Stanford University, Stanford, California

4th London Symposium on Information Theory

Location - London, ENGLAND
Date - August 29-September 3, 1960
Contact - Professor E. C. Cherry, Imperial College, London S.W. 7, ENGLAND

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10th International Congress of Applied Mechanics

Place - Stresa, ITALY
Date - August 31-September 7, 1960
Chairman - Professor Gustavo Colonnetti, Chairman, Organizing Committee, c/o Consiglio Nazionale delle Ricerche, Piazzale delle Scienze 7, Rome, ITALY

September, 1960

Second International Conference on Operational Research

Location - Aix-en-Provence, FRANCE
Date - September 5-10, 1960
Contact - John B. Lathrop, ORSA Representative, IFORS, Operations Research Division, Lockheed Aircraft Corporation, Burbank, California

5th Austrian Congress of Mathematicians

Place - University of Innsbruck
Location - Tyrol, Innsbruck, AUSTRIA
Date - September 12-18, 1960
Chairman - Professor W. Grubner, Mathematical Institute of the University, Innrain 52, Innsbruck, AUSTRIA

The Yugoslav Congress of Mathematicians and Physicists

Location - Belgrade, YUGOSLAVIA
Date - September 25-October 1, 1960

Society of Actuaries

Hotel - Edgewater Beach Hotel
Location - Chicago, Illinois
Date - September 28-30, 1960
Secretary - Dr. A. A. McKinnie, Society of Actuaries, 208 South La Salle Street, Chicago 4, Illinois

October, 1960

Operations Research Society of America

18th National Meeting
Hotel - Statler Hilton Hotel
Location - Detroit, Michigan
Date - October 10-12, 1960
Chairman - Mr. George O'Brien; Touche, Niven, Bailey & Smart; 1292 National Bank Building; Detroit 26, Michigan
November, 1960  The American Society of Mechanical Engineers

Annual Meeting
Hotel - Statler Hilton
Location - New York, New York
Date - November 27-December 2, 1960
Director of Public Relations - L. S. Dennegar, The American Society of Mechanical Engineers, 29 West 39th Street, New York 18, New York

December, 1960  American Association for the Advancement of Science

127th Annual Meeting
Location - Philadelphia, Pennsylvania
Date - December 26-31, 1960

Eastern Joint Computer Conference

Location - New York, New York
Date - December, 1960

1960 (date not yet determined)  International Mathematical Union

International Symposium on "Fonctions de plusieurs variables complexes et analyse fonctionnelle"
Location - Portugal
Date - 1960
Chairman - Professor Sebastiao e Silva

The Statistical Laboratory of the University of California

Fourth Berkeley Symposium on Mathematical Statistics and Probability
Date - summer, 1960
Contact - Director, Statistical Laboratory, University of California, Berkeley 4, California
Society of Actuaries

Hotel - Hotel Commodore
Location - New York, New York
Date - March 23-24, 1961
Secretary - Dr. A. A. McKinnie, Society of Actuaries, 208 South La Salle Street, Chicago 4, Illinois

Society of Actuaries

Hotel - Hotel Statler Hilton
Location - Dallas, Texas
Date - April 6-7, 1961
Secretary - Dr. A. A. McKinnie, Society of Actuaries, 208 South La Salle Street, Chicago 4, Illinois

Society of Actuaries

Hotel - Royal York Hotel
Location - Toronto, Ontario, CANADA
Date - June 1-2, 1961
Secretary - Dr. A. A. McKinnie, Society of Actuaries, 208 South La Salle Street, Chicago 4, Illinois

Society of Actuaries

Hotel - Ambassador Hotel
Location - Los Angeles, California
Date - June 8-9, 1961
Secretary - Dr. A. A. McKinnie, Society of Actuaries, 208 South La Salle Street, Chicago 4, Illinois

Association for Computing Machinery

National Meeting
Hotel - Statler Hilton Hotel
Location - Los Angeles, California
Date - September 6-8, 1961
Chairman - Ben Handy, Litton Industries
Society of Actuaries

Hotel - The Greenbrier
Location - White Sulphur Springs, West Virginia
Date - November 13-15, 1961
Secretary - Dr. A. A. McKinnie, Society of Actuaries,
208 South La Salle Street, Chicago 4, Illinois

American Association for the Advancement of Science

128th Annual Meeting
Location - Denver, Colorado
Date - December 26-31, 1961

Eastern Joint Computer Conference

Location - Washington, D. C.
Date - December, 1961

Operations Research Society of America

19th National Meeting (9th Annual)
Location - Chicago, Illinois
Date - Spring 1961
Chairman - Mr. Donald H. Schiller, Caywood-Schiller
Associates, 203 N. Wabash Avenue, Chicago 1, Illinois

Operations Research Society of America

20th National Meeting
Location - San Francisco, California
Date - Fall, 1961

International Union of Theoretical and Applied Mechanics

Colloquium on Non-Linear Vibrations
Location - Moscow, USSR
Date - 1961
Contact - Maurice Roy, Membre, Académie des Sciences, 29, av. de la Division Leclerc, Chatillon-sous-Bagneux, (Seine), FRANCE
1962 Society of Actuaries

Hotel - Hotel Sheraton Pennsylvania
Location - Philadelphia, Pennsylvania
Date - March 22-23, 1962
Secretary - Dr. A. A. McKinnie, Society of Actuaries,
208 South La Salle Street, Chicago 4, Illinois

Society of Actuaries

Hotel - Hotel Muehlebach
Location - Kansas City, Missouri
Date - April 5-6, 1962
Secretary - Dr. A. A. McKinnie, Society of Actuaries,
208 South La Salle Street, Chicago 4, Illinois

Society of Actuaries

Hotel - Hotel Meyer
Location - Jacksonville, Florida
Date - May 24-25, 1962
Secretary - Dr. A. A. McKinnie, Society of Actuaries,
208 South La Salle Street, Chicago 4, Illinois

Society of Actuaries

Hotel - Drake Hotel
Location - Chicago, Illinois
Date - June 7-8, 1962
Secretary - Dr. A. A. McKinnie, Society of Actuaries,
208 South La Salle Street, Chicago 4, Illinois

4th U. S. National Congress of Applied Mechanics

Location - Berkeley, California
Date - June 18-21, 1962
1962 cont.

Association for Computing Machinery

National Meeting
Location - Syracuse, New York
Date - August 29-31, 1962

Society of Actuaries

Hotel - Chateau Frontenac
Location - Quebec City, Quebec, CANADA
Date - October 15-17, 1962
Secretary - Dr. A. A. McKinnie, Society of Actuaries,
208 South La Salle Street, Chicago 4, Illinois

Eastern Joint Computer Conference

Hotel - Bellevue-Stratford Hotel
Location - Philadelphia, Pennsylvania
Date - December 4-7, 1962

Operations Research Society of America

21st National Meeting (10th Annual)
Location - Washington, D. C.
Date - Spring, 1962

Operations Research Society of America

22nd National Meeting
Location - Philadelphia, Pennsylvania
Date - Fall, 1962
NEWS ITEMS AND ANNOUNCEMENTS

BENJAMIN PEIRCE INSTRUCTORSHIP AT HARVARD. There will be one or more Benjamin Peirce Instructorships open next year at Harvard. The present base salary for this post-doctoral instructorship is $6,000 and may possibly be higher next year. It can be augmented by summer work and possibly on research contracts and teaching in summer school. The teaching commitment is six hours one semester and nine hours the other, and includes the option of an advanced half-course of the instructors own choosing. The appointments are annual, but carry a presumption of two renewals.

Interested persons should apply to the Harvard Mathematics Department before February 1, and should be seconded by supporting letters.

THE C. L. E. MOORE INSTRUCTORSHIPS AT M. I. T. The Department of Mathematics at the Massachusetts Institute of Technology wishes to announce the availability of C. L. E. Moore Instructorships in Mathematics for 1960-1961, open to young mathematicians with doctorates who show definite promise in research. The base salary for these instructorships is $6,700 and the teaching load will be six hours per week. The salary can be supplemented by summer work on research contracts or by teaching in the summer session. The appointments are annual but are renewable for one additional year.

Applications should be filed not later than January 25, 1960 on forms obtained from the Department.

THE MERSHON NATIONAL GRADUATE FELLOWSHIPS AND POSTDOCTORAL FELLOWSHIPS. The Ohio State University has established the Mershon Committee on Education in National Security to promote a variety of educational and research programs in the broad field of national security.

To encourage graduate and postgraduate study in national security, the Committee offers a number of graduate and postdoctoral fellowships for the academic year 1960-1961. Because of the scope of national security, no restrictions as to field of interest have been set.

Fellowships will be awarded on the basis of merit and, at the discretion of the Committee, may be renewed. Preference will be given to citizens of the United States.

GRADUATE FELLOWSHIPS $3,000

Mershon Graduate Fellows must devote their entire time to graduate study during the regular academic year September, 1960...
to June, 1961. They must attend the National Security Course (3 hours each quarter) and register for additional courses in their major field of interest. They must pay tuition of $270 per academic year and the usual matriculation and laboratory fees. First-year graduate students are especially invited to apply.

POSTDOCTORAL FELLOWSHIPS $7,500

Mershon Postdoctoral Fellows must possess the Ph.D. degree or its equivalent. They are expected to devote full time to research and writing in fields related to national security. They must be in residence during the academic year but will be permitted to travel in connection with their research. Limited travel and research expenses may be granted Fellows at the discretion of the Committee.

Applications must be filed before February 15, 1960. For applications and further details, write: The Dean of the Graduate School, The Ohio State University, Columbus 10, Ohio.

PREDOCTORAL FELLOWSHIPS IN THE ATMOSPHERIC SCIENCES AND OCEANOGRAPHY. M. I. T. announces a new program of predoctoral fellowships in the atmospheric sciences (meteorology) and oceanography. These fellowships are made possible by a grant from the Ford Foundation to M. I. T. Applicants need not have prior training in the atmospheric sciences or oceanography. Fellowships will be awarded on the basis of the extent and quality of the applicant's preparation in the physical sciences and mathematics, his overall academic record, the opinions of references and his scientific objectives. The results of the Graduate Record Examinations will also be considered, if available. The first awards will be made for the 1960-1961 academic year. Applications must be received by February 1, 1960, and awards will be announced not later than March 15, 1960. Fellowships may be renewed for a maximum of three years if satisfactory progress is made. The stipend is $3000 per year plus full tuition. Applications and further information may be obtained from Professor Henry G. Houghton, Room 24-516, Massachusetts Institute of Technology, Cambridge 39, Massachusetts.

POSTDOCTORAL RESIDENT RESEARCH ASSOCIATESHIPS.
The National Academy of Sciences - National Research Council announces the renewal of its program of postdoctoral research associateships, supported by several agencies of the Federal government and tenable at certain universities and government laboratories. Through these associateships, young investigators of promise are offered an exceptional opportunity to receive advanced training in well-equipped laboratories among highly qualified scientists dealing with various fields of fundamental and applied research.
Participating laboratories are the National Bureau of Standards, Boulder, Colorado, and Washington, D. C.; Naval Research Laboratory, Washington, D. C.; Naval Ordnance Laboratory, White Oak, Silver Spring, Maryland; Navy Electronics Laboratory, San Diego, California; Army Biological Warfare Laboratories, Fort Detrick, Maryland; Quartermaster Research and Engineering Center Laboratories, Natick, Massachusetts; Quartermaster Food and Container Institute, Chicago, Illinois; and three technical centers of the Air Research and Development Command -- Air Force Cambridge Research Center, Bedford, Massachusetts; Air Force Missile Development Center, Alamogordo, New Mexico; and Wright Air Development Center, Dayton, Ohio.

Other Research associateships in the program include those of the National Aeronautics and Space Administration, tenable at the Goddard Space Flight Center, near Washington, D. C., and the ARDC Postdoctoral University Research Associateships sponsored by the Air Force Office of Scientific Research and tenable at 26 universities throughout the United States.

Applicants will be required to produce evidence of training equivalent to that represented by the Ph.D. or Sc.D. degree and to demonstrate superior ability for creative research. Stipends for most of the program vary between $7030 and $8000; those for the Quartermaster research laboratories will be based, in part, on the candidate's normal salary.

Brochures describing these separate associateships in detail may be obtained by writing to the Fellowship Office, National Academy of Sciences -- National Research Council, 2101 Constitution Avenue, N. W., Washington 25, D. C. In order to be considered for 1960-1961 awards, applications must be filed with the Fellowship Office on or before February 1, 1960; applications for the ARDC Postdoctoral University Research Associateships must be received by January 4, 1960. Awards will be announced about April 1, by the participating laboratories.

EXCHANGE VISITS WITH USSR. The National Academy of Sciences (U. S. A.) and the Academy of Sciences of the U. S. S. R. in July, signed a two-year agreement providing for exchange visits by research scientists of each country for periods of up to one year. Each Academy designated 20 fields of specialized scientific inquiry in which its scientists desire to observe or conduct research within the host country.

American participants in this program will be designated by the Council of our National Academy. However, many exchanges have been and will continue to be arranged independent of the inter-Academy agreement. Scientists interested in professional visits to
the U. S. S. R. should give serious consideration to alternative possibilities.

An independently arranged exchange can be worked out between any two American and Soviet scientific institutions, the arrangements being made by direct correspondence, or through the intermediary of the National Academy of Sciences or the Department of State. A potential American sponsor should also ascertain whether any of the places to be visited by the American and Soviet scientists are in areas closed to travel under government regulations of the two countries. This can be done by letter to the East-West Contacts Staff, Department of State, Washington 25, D.C.; or the National Academy will request the Department of State to provide this information.

Inquiries to the National Academy may be addressed to Mr. Lawrence C. Mitchell, Professional Associate, East-West Exchange Program, National Academy of Sciences, 21 Constitution Avenue, Washington 25, D. C.

AMS SUMMER INSTITUTE ON FINITE GROUPS (preliminary announcement). With the support of the National Science Foundation, a Summer Institute on Finite Groups will be held at the California Institute of Technology, Pasadena, from August 1 to August 28, 1960 under the sponsorship of the American Mathematical Society.

Two distinguished group theorists have been invited from abroad and have accepted our invitation. These are Professor Graham Higman of Oxford University and Professor Helmut Wielandt of the University of Tübingen.

Participation in the Summer Institute will be by invitation, but the lectures and seminars will be open to interested mathematicians who happen to be in the area.

The Program Committee consists of Professors Richard Brauer, Richard Bruck, H. S. M. Coxeter, Robert Dilworth, Herbert Ryser, and Marshall Hall, Jr., Chairman. Professor Hall has prepared the following statement.

Recent Progress in the Theory of Finite Groups

The Theory of Groups is now a major area of research in mathematics. This now includes Topological Groups and various studies on infinite groups, mainly the study of infinite Abelian groups. The theory of finite groups, historically the first to be developed, was at a relatively inactive stage in the early 1930's when the problems which had not been solved by the classical school appeared to be too difficult to treat.

A new era was coming. A paper by Philip Hall on the theory of p-groups in 1933 and a paper by Grün in 1935 which introduced the transfer may be regarded as the harbingers of this new era. New
methods and concepts have now been found so that at the present re-
searches on finite groups are producing many new and exciting re-
sults. In a paper as yet unpublished, John Thompson has resolved a
conjecture of Frobenius more than fifty years old. His methods are
not restricted to the particular problem of Frobenius but have wide-
spread application. Thompson's work is based on general methods
developed in a long paper by Philip Hall and Graham Higman on the
idea of "p-length" of p-solvable groups. Recent progress on the
Burnside problem, proposed in 1902, has been impressive. Finitely
generated groups of exponent 2 and 3 were shown to be finite in Burn-
side's 1902 paper. The finiteness for exponent 4 was shown by Sanov
in 1940 and for exponent 6 by Marshall Hall in 1957. The restricted
Burnside problem for prime exponents was solved by Kostrikin in
1958, and now it is reported that Novikov has shown that the Burnside
groups with two generators and exponent 72 or greater are all infinite.

Further important results have been found recently by Wielandt
and Huppert, giving important properties of solvable and supersolv-
able groups. R. Brauer's characterization of characters of finite
groups is another milestone and valuable applications of this principle
have been made by Suzuki, Feit and Brauer himself.

THE AMS SUMMER SEMINAR ON MODERN PHYSICAL THEO-
RIES AND ASSOCIATED MATHEMATICAL DEVELOPMENTS will be
held in Boulder, Colorado, July 24 to August 19, 1960.

This will be the second in the series of Summer Seminars on
applied mathematics and mathematical physics which are being con-
ducted by the American Mathematical Society with the co-operation
of the University of Colorado, Boulder, Colorado. The first Summer
Seminar in the series was held in 1957. The Seminar will be spon-
sored by the Atomic Energy Commission, the National Science-Foun-
dation, the Office of Naval Research, and the Office of Ordnance Re-
search, U. S. Army.

The purpose of the Seminar is primarily instructional, with
emphasis on a number of carefully prepared basic courses. Lectures
on selected topics given by outstanding scientists are an added feature.
The Seminar is planned to give mature mathematicians the opportunity
to hear from leading physicists about physical theories developed
during recent years and to acquaint them with relevant mathematical
notions and methods.

While the lectures will be pitched at an advanced level of
general mathematical maturity (that indicated by the attainment of
Ph.D. degree, or its equivalent, will probably be necessary for maxi-
mum benefit from the Seminar), they will be integrated and self-
contained, and designed as expositions of their subjects. In order to
benefit significantly, the participants should possibly be familiar

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with the fundamentals of quantum mechanics, operator theory, and probability theory. The Seminar will provide an opportunity for pure mathematicians to familiarize themselves with major sectors of modern applied mathematics, and thereby respond to the increasing need for people versed in these fields. At the same time the Seminar will offer specialists in applied mathematics an opportunity to catch up on progress in related fields and to acquire new interests.

Three major courses will be given by Professors V. Bargmann, R. Jost, and G. E. Uhlenbeck on (1) classical quantum theory. (2) quantum theory of fields and elementary particles, and (3) statistical physics. These courses will meet three times each week. Two courses on related mathematical problems, meeting twice a week, will be given by Professors K. O. Friedrichs and I. E. Segal.

A number of special lectures will be held to supplement the basic courses. They will form an integral part of the Seminar for they have been worked out in close collaboration with the lecturers in the basic courses, to illuminate the various courses, and to deepen the understanding of them.

To date Professors Elliott Montroll, G. W. Mackey, and Arthur S. Wightman have agreed to give special lectures.

With few exceptions, no formal lectures are scheduled in the afternoons. The Program Committee has kept this time free purposely, believing that an opportunity for the participants to engage in free discussion in small informal groups among themselves and with the distinguished speakers on the program may prove to be one of the lasting benefits to be gained from the Seminar. No academic credit will be granted and there are no tuition charges.

The dormitories and dining facilities of the University of Colorado will be available to participants in the Seminar and their families.

Application for Admission - Financial Assistance

Application blanks for admission to the Seminar can be obtained from, and when completed should be directed to: Professor K. O. Friedrichs, Chairman of the Organizing Committee, New York University, 25 Waverly Place, New York 3, New York.

In view of limited accommodations, the committee requests that applications reach Professor Friedrichs by February 15, 1960. The applicant should indicate his scientific background and interest. Those who wish to apply for a grant in aid should indicate this. A limited amount of financial help will be available from funds supplied the Society.

A detailed announcement (covering registration, rooms and meals, entertainment and recreation, transportation, mail and telegrams) will appear later with the final program.
The Program Committee for the Summer Seminar is as follows:
Professors Mark Kac, M. M. Schiffer, G. E. Uhlenbeck, E. P. Wigner,
and K. O. Friedrichs, Chairman.

The 1960 Southern Regional Graduate Summer Session in Statistics will be held at the University of Florida at Gainesville from June 20 to July 29, 1960. The University of Florida, North Carolina State College, Virginia Polytechnic Institute and Oklahoma State University have agreed to operate a continuing program of graduate summer sessions in statistics to be held at each institution in rotation. The first such session was held at Virginia Polytechnic Institute in the summer of 1954.

It is the purpose of this program to serve: (1) teachers of introductory statistical courses and college teachers of mathematics who want formal training in modern statistics; (2) research and professional workers who want intensive instruction in basic statistical concepts and modern statistical methodology; (3) professional statisticians who wish to keep informed about advanced specialized theory and methods; (4) prospective candidates for graduate degrees in statistics; and (5) graduate students in other fields who desire supporting work in statistics.

The session will last six weeks and courses will carry three semester hours of credit. Not more than two courses may be taken for credit at any one session. The summer work in statistics may be applied as residence credit at any one of the cooperating institutions, as well as certain other universities, in partial fulfillment of the requirements for a graduate degree. The program may be entered at any session, and consecutive courses will follow in successive summers.

The National Science Foundation is making available to the University of Florida grants for college teachers of statistics and college teachers of mathematics who wish to attend the 1960 session. The stipend is $75 per week for the six weeks of the session plus additional amounts for dependents and travel allowances. Tuition will also be paid by the National Science Foundation. Participants must meet all the admission requirements of the Graduate School of the University of Florida and must be admitted thereto or must be a graduate student in good standing at one of the cooperating institutions. Applicants for these grants should be employed by an institution of higher learning as a teacher of mathematics or statistics; those from institutions wherein there is no opportunity for formal training in modern inferential statistics and probability will be given priority. No geographical or age limitations will be imposed, though preference will be given to the younger age group if other things are equal. The previous academic and professional record of appli-
cants will be considered. Those who received grants in a previous session who meet the other conditions of eligibility are encouraged to apply. Applications for grants should be postmarked not later than February 15, 1960 to be assured of full consideration.

Requests for application blanks for the summer session and for National Science Foundation grants should be addressed to Dr. Herbert A. Meyer, Statistical Laboratory, University of Florida, Box 3568, Gainesville, Florida.

NSF INSTITUTES FOR SCIENCE AND MATHEMATICS TEACHERS. The National Science Foundation has announced the award of grants totaling about $9,200,000 to 33 colleges and universities to support Academic Year Institutes for science and mathematics teachers. This will be the fifth year of this program, whose purpose is to help teachers improve their subject matter knowledge through a year's advanced study on a full-time basis.

Approximately 1,600 experienced teachers will be enrolled in the Institutes in the 1960-1961 academic year, of whom the great majority will be secondary school teachers. A limited number of openings for college teachers will also be available in some of the Institutes. Each teacher will pursue a program of study in science or mathematics planned especially for him and conducted by scientists noted both for competence in their fields and for skill in presentation.

The Foundation grants will provide stipends of $3,000 for each participant, with additional allowances for dependents, books, and travel. Some Institutes will provide an additional summer training program to enable teachers more easily to fulfill graduate degree requirements. Supplementary allowances will be provided for teachers participating in this extended program.

The list below shows the institutions receiving grants for institutes with exclusively mathematical programs and names the Institute directors. Information and application forms can be obtained from the directors of the individual Institutes, NOT from the National Science Foundation.

- **Boston College**
  - Rev. Stanley J. Bezuszka, S. J.
  - Chairman, Department of Mathematics

- **Illinois, University of**
  - Professor Joseph Landin
  - Department of Mathematics

- **Kansas, University of**
  - Professor Lee M. Sonneborn
  - Department of Mathematics

- **Louisiana State University**
  - Dr. Houston T. Karnes
  - Department of Mathematics

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Minnesota, University of
Professor Charles Hatfield
Department of Mathematics

Notre Dame, University of
Professor Arnold E. Ross, Head
Department of Mathematics

Puerto Rico, University of
Dean Mariano Garcia
Department of Mathematics

San Diego State College
Dr. John E. Eagle
Department of Mathematics

NSF ANNOUNCES 379 SUMMER INSTITUTES. Financial aid will be available in 1960 for about 18,000 high school and college teachers of science, mathematics, and engineering to participate in Summer Institutes sponsored by the National Science Foundation. Three hundred and seventy-nine Institutes will be supported by the Foundation next summer, in 265 educational institutions.

Awards of grants totaling more than $21,000,000 for the Summer Institutes were announced by Dr. Alan T. Waterman, Director of the National Science Foundation. Institutes will be held in all 50 states, the District of Columbia and Puerto Rico. Three hundred and sixteen of the Institutes will be open to high school teachers only, 37 will be for college teachers only, and 24 will be for both high school and college teachers, and two will be for technical institute personnel. Roughly 16,000 high school teachers and 2,000 college teachers will be enabled to participate through stipends provided by the National Science Foundation.

The number of teachers who will receive financial support in each of the 379 Institutes will average nearly 50, and will vary from 15 to more than 100. Tuition and fees will be paid for these teachers. They will also receive stipends of not more than $75 per week for the duration of the institute, plus allowances for travel and dependents. The Institutes will vary in length from four to twelve weeks.

Participants will be chosen by the Institutes themselves, NOT by the National Science Foundation.

DATA ON OOR PROPOSALS. The Office of Ordnance Research had received 2,598 proposals as of June 30, of this year. Of these 372 were active, 555 had been completed, 1204 had been declined, 336 had been withdrawn and 131 were pending. This represents an average of approximately 325 proposals received for each year since June 1951, when this office was first established.

THE FORD FOUNDATION has announced a grant of $900,000 to the College of Engineering of the University of Michigan. The pur-
The purpose of the grant is to introduce students to the use of computers at the sophomore level and to study problems at all undergraduate levels suitable for application of computer techniques.

The Air Force Office of Scientific Research awarded 67 contracts, for a total value of $1,794,115, during October, 1959, for basic research in science. These included the following contracts in mathematics:

<table>
<thead>
<tr>
<th>Contractor</th>
<th>Principal Investigator</th>
<th>Title</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ohio State University</td>
<td>E. Kreyszig</td>
<td>Differential Equations</td>
<td>$ 6,705</td>
</tr>
<tr>
<td>University of North Carolina</td>
<td>A. T. Brauer</td>
<td>Roots of Matrices</td>
<td>8,070</td>
</tr>
<tr>
<td>University of Illinois</td>
<td>D. G. Bourgin</td>
<td>Topological Invariants</td>
<td>5,751</td>
</tr>
<tr>
<td>American Mathematical Society</td>
<td>G. L. Walker</td>
<td>Conference on Differential Equations</td>
<td>6,958</td>
</tr>
<tr>
<td>Purdue Research Foundation</td>
<td>G. L. Krabbe</td>
<td>Study of Matrices</td>
<td>7,642</td>
</tr>
<tr>
<td>Wayne State University</td>
<td>S. I. Goldberg</td>
<td>Differential Geometry</td>
<td>7,419</td>
</tr>
<tr>
<td>Syracuse University</td>
<td>K. L. Chung</td>
<td>Markov Chains, etc.</td>
<td>21,939</td>
</tr>
<tr>
<td>Stanford University</td>
<td>K. de Leeuw</td>
<td>Functional Analysis</td>
<td>13,645</td>
</tr>
<tr>
<td>Institute of Aeronautical Sciences</td>
<td>S. P. Johnston</td>
<td>Aeronautical Abstracts</td>
<td>22,800</td>
</tr>
<tr>
<td>Syracuse University</td>
<td>G. G. Lorentz</td>
<td>Complex Variables</td>
<td>17,850</td>
</tr>
<tr>
<td>George Washington University</td>
<td>R. S. Ledley</td>
<td>Digital Computer Engineering</td>
<td>1,141</td>
</tr>
<tr>
<td>Northwestern University</td>
<td>R. R. Goldberg</td>
<td>Fourier Integrals</td>
<td>6,310</td>
</tr>
</tbody>
</table>

The November, 1959 contract awards of AFOSR consisted of 41 contracts amounting to $1,590,612 and contained the following in mathematics:
<table>
<thead>
<tr>
<th>Contractor</th>
<th>Principal Investigator</th>
<th>Title</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Harvard University</td>
<td>O. Zariski</td>
<td>Studies in Linear Systems</td>
<td>$26,394</td>
</tr>
<tr>
<td>Johns Hopkins University</td>
<td>P. Hartman</td>
<td>Partial Differential Equations</td>
<td>10,850</td>
</tr>
<tr>
<td>University of Maryland</td>
<td>A. Douglis</td>
<td>Partial Differential Equations</td>
<td>16,780</td>
</tr>
<tr>
<td>American Mathematical Society</td>
<td>G. L. Walker</td>
<td>Advisory and Evaluation Services</td>
<td>14,877</td>
</tr>
<tr>
<td>Yale University</td>
<td>N. Jacobson</td>
<td>Research on Modern Algebras</td>
<td>24,100</td>
</tr>
<tr>
<td>University of Miami</td>
<td>D. G. Austin</td>
<td>Analytical Properties of a Markoff Process</td>
<td>5,110</td>
</tr>
<tr>
<td>California Institute of Technology</td>
<td>P. Lazerstrom</td>
<td>Equations of motion for Viscous Fluids</td>
<td>25,000</td>
</tr>
<tr>
<td>Institute for Advanced</td>
<td>M. Morse</td>
<td>Advanced Mathematical Analysis</td>
<td>44,000</td>
</tr>
<tr>
<td>Princeton University</td>
<td>S. Bochner</td>
<td>Harmonic Analysis and Probability Theory</td>
<td>12,000</td>
</tr>
</tbody>
</table>

JOURNAL OF MATHEMATICAL PHYSICS, a new bimonthly to be published by the American Institute of Physics, 335 East 45th Street, New York 17, is to include new mathematical methods for the solution of physical problems as well as original research in physics furthered by such methods.

Covering mathematical aspects of quantum field theory and statistical mechanics of interacting particles, new approaches to eigenvalue and scattering problems, theory of stochastic processes, and novel variational methods, the journal will also report other recent developments in mathematical physics. Review papers on mathematical topics for Physicists will also appear occasionally.

The first issue is to appear in February, 1960.

PROCEEDINGS of the Sixth Midwestern Conference on Fluid Mechanics, and PROCEEDINGS of the Fourth Midwestern Conference
on Solid Mechanics, held jointly at the University of Texas on September 9-11, 1959, have now been published. These conferences are sponsored by a number of organizations including the American Mathematical Society.

Copies of the PROCEEDINGS may be obtained at a cost of $12.50 per volume by addressing orders to: Engineering Institutes, Division of Extension, The University of Texas, Austin, Texas.

THE CHAUVENET PRIZE of the Mathematical Association of America will be awarded to Dr. Cornelius Lanczos at the annual meeting in Chicago on January 29, 1960. Dr. Lanczos's paper on "Linear Systems in Self-Adjoint Form" was cited as the outstanding expository article published in the American Mathematical Monthly in three years. The last award of the Chauvenet Prize was in 1956.

Dr. Lanczos, of the Dublin Institute for Advanced Study, is spending the current academic year at the U. S. Army Mathematics Research Center of the University of Wisconsin.

STANFORD UNIVERSITY has announced plans for an expanded Computation Center.

Professor John G. Herriot, of the Department of Mathematics, is Director of the Center. Mr. Robert M. Gordon is Associate Director. Professor George E. Forsythe, of the Department of Mathematics, is Chairman of an ad hoc Committee for the Center.

The Center will be maintained as an educational, research, and service facility within the Applied Mathematics and Statistics Laboratory. However, the staff, which is presently being enlarged, will offer its services to every academic and administrative department of the University.

ABRAHAM FLEXNER died on September 21, 1959, after a lifetime of service to higher education.

His first book, a severe criticism of the American college, brought him to the attention of Henry S. Pritchett, head of the Carnegie Foundation for the Advancement of Teaching, who asked him to conduct an investigation of this country's medical schools.

Dr. Flexner found that the majority of the 150-odd medical institutions in the United States and Canada were no more than commercial enterprises devoid of qualified faculties, adequate facilities, or suitable entrance requirements. So scathing was his report that its publication in 1910 was enough to close nearly half of the medical schools in the country.

But he did not stop at mere criticism. Wherever he destroyed, he was the first to begin rebuilding. He proposed that at least thirty
top medical colleges be established, and he obtained the necessary funds for this purpose, reshaping the pattern of medical training in the nation.

To fulfill a dream he had of a school in which top students and teachers could work and study together, he raised six million dollars for the founding of the Institute for Advanced Study at Princeton and served as its first director.

A CORRECTION. On page 589 of the November issue of the NOTICES it is stated that out of 35 National Science Foundation post-doctoral fellowships announced on October 16, 1959, only one was granted in mathematics. This unfortunate ratio is fortunately incorrect. There were four awards in mathematics to John A. Ernest of the University of Illinois, Paul J. Koosis of New York University, Charles E. Watts of the University of Chicago, and Joseph A. Wolf of the University of Chicago.
NEW PUBLICATIONS

MEMOIR No. 34

SPACES WITH NON-SYMMETRIC DISTANCE

by

E. M. Zaustinsky

This Memoir treats Finsler spaces, without the assumption of a symmetric distance, by the geometric methods developed by Busemann. One would have expected that the suppression of the hypothesis that the distance be symmetric would entail the loss of most of the theory; we have the very surprising result, however, that nearly every theorem of the symmetric theory (so far attempted) which can be formulated at all without this assumption holds without this assumption. Among the topics treated are the one-dimensional spaces, the theory of perpendiculars and parallels, the determination of the two-dimensional spaces with transitive groups of motions, and spaces with non-positive curvature.

91 pages List Price: $2.00, Members and Agents: $1.50

MEMOIR No. 35

ASYMPTOTIC BEHAVIOR OF SOLUTIONS OF DIFFERENTIAL–DIFFERENCE EQUATIONS

by

Richard Bellman and K. L. Cooke

This paper provides the first detailed discussion of the determination of the asymptotic series for the solutions of linear differential–difference equations whose coefficients possess asymptotic series.

91 pages List Price: $2.00, Members and Agents: $1.50
VISITING FOREIGN MATHEMATICIANS. The following list of visiting foreign mathematicians has been prepared by the Division of Mathematics of the National Academy of Sciences - National Research Council. This list is dated October 12, 1959.

<table>
<thead>
<tr>
<th>Name</th>
<th>Home Country</th>
<th>Host Institution</th>
<th>Period of Visit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abuauad Abujatim, Cesar</td>
<td>Chile</td>
<td>University of Chicago</td>
<td>Sept. 1959-June 1960</td>
</tr>
<tr>
<td>Agmon, Shmuel</td>
<td>Israel</td>
<td>University of Maryland</td>
<td>Sept. 1959-Dec. 31, 1959</td>
</tr>
<tr>
<td>Altman, M.</td>
<td>Poland</td>
<td>California Institute of Technology</td>
<td>Oct. 1, 1958-Nov. 30, 1959</td>
</tr>
<tr>
<td>Alvarez de Araya, Jorge</td>
<td>Chile</td>
<td>University of Washington</td>
<td>Sept. 1958-Aug. 1960</td>
</tr>
<tr>
<td>Amitsur, Shimshon A.</td>
<td>Israel</td>
<td>Yale University</td>
<td>Fall Term 1959 Indefinite</td>
</tr>
<tr>
<td>Araki, Shôrô</td>
<td>Japan</td>
<td>Institute for Advanced Study</td>
<td>Sept. 1959-April 1960</td>
</tr>
<tr>
<td>Asplund, O. Edgar</td>
<td>Sweden</td>
<td>Institute for Advanced Study</td>
<td>Sept. 1958-April 1960</td>
</tr>
<tr>
<td>Atiyah, Michael F.</td>
<td>U. K.</td>
<td>Institute for Advanced Study</td>
<td>Sept. - Dec. 1959</td>
</tr>
<tr>
<td>Aubert, Karl E.</td>
<td>Norway</td>
<td>Institute for Advanced Study</td>
<td>Sept. 1958-April 1960</td>
</tr>
<tr>
<td>Banaschewski, Bernhard</td>
<td>Germany</td>
<td>Tulane University</td>
<td>Academic Year 1959-1960</td>
</tr>
<tr>
<td>Bernays, Paul</td>
<td>Switzerland</td>
<td>Institute for Advanced Study</td>
<td>Nov. 1959-April 1960</td>
</tr>
<tr>
<td>Bhattacharyya, B. B.</td>
<td>India</td>
<td>North Carolina State College, Raleigh</td>
<td>Oct. 1, 1959-Indefinite</td>
</tr>
<tr>
<td>Bolt, Bruce A.</td>
<td>Australia</td>
<td>Lamont Geological Observatory; Columbia University</td>
<td>Jan. 1960-Oct. 1960</td>
</tr>
<tr>
<td>Borsuk, Karol</td>
<td>Poland</td>
<td>University of California, Berkeley</td>
<td>Fall Term 1959</td>
</tr>
<tr>
<td>Name</td>
<td>Home Country</td>
<td>Host Institution</td>
<td>Period of Visit</td>
</tr>
<tr>
<td>------------------------</td>
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<td>--------------------------------------------------</td>
<td>-------------------------------</td>
</tr>
<tr>
<td>Cherry, T. M.</td>
<td>Australia</td>
<td>California Institute of Technology</td>
<td>Nov. 1, 1959 - Feb. 29, 1960</td>
</tr>
<tr>
<td>Chisnell, R. J.</td>
<td>England</td>
<td>Massachusetts Institute of Technology</td>
<td>Sept. 1, 1959- Dec. 31, 1959</td>
</tr>
<tr>
<td>Christian, Ulrich</td>
<td>Germany</td>
<td>Institute for Advanced Study</td>
<td>Sept. 1958- April 1960</td>
</tr>
<tr>
<td>Constantine, Alan</td>
<td>Australia</td>
<td>Yale University</td>
<td>Fall Term 1959- Indefinite</td>
</tr>
<tr>
<td>Demazure, Michel</td>
<td>France</td>
<td>Princeton University</td>
<td>Sept. 1959- June 1960</td>
</tr>
<tr>
<td>DeHeuvels, René</td>
<td>France</td>
<td>Yale University</td>
<td>Fall Term 1959- Indefinite</td>
</tr>
<tr>
<td>Deuring, Max F.</td>
<td>Germany</td>
<td>Institute for Advanced Study</td>
<td>Sept. 1959- April 1960</td>
</tr>
<tr>
<td>Dombrowski, Peter L.</td>
<td>Germany</td>
<td>Massachusetts Institute of Technology</td>
<td>Sept. 15, 1959- June 15, 1960</td>
</tr>
<tr>
<td>Dugué, Daniel</td>
<td>France</td>
<td>Catholic University</td>
<td>Mar. 1, 1960- July 1, 1960</td>
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<tr>
<td>Durbin, James</td>
<td>U. K.</td>
<td>University of North Carolina</td>
<td>July 1959- June 1960</td>
</tr>
<tr>
<td>Eicker, Frederick</td>
<td>Germany</td>
<td>University of North Carolina</td>
<td>March 1959- Indefinite</td>
</tr>
<tr>
<td>Elston, R. C.</td>
<td>U. K.</td>
<td>University of North Carolina</td>
<td>July 1959- June 1960</td>
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<tr>
<td>Engeler, Erwin</td>
<td>Switzerland</td>
<td>University of Minnesota</td>
<td>1958-1960</td>
</tr>
<tr>
<td>Fichera, G.</td>
<td>Italy</td>
<td>Mathematics Research Center (Army), University of Wisconsin</td>
<td>July 1959- Dec. 1959</td>
</tr>
<tr>
<td>Fixman, Uri</td>
<td>Israel</td>
<td>University of Kansas</td>
<td>Feb. 1, 1959- Jan. 31, 1960</td>
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<tr>
<td>Foguel, Shaul R.</td>
<td>Israel</td>
<td>University of California, Berkeley</td>
<td>Sept. 1958- June 1960</td>
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<tr>
<td>Froelicher, Alfred</td>
<td>Switzerland</td>
<td>University of Washington</td>
<td>July 16, 1959- Oct. 15, 1959</td>
</tr>
<tr>
<td>Fujisaki, G.</td>
<td>Japan</td>
<td>Massachusetts Institute of Technology</td>
<td>Sept. 1, 1959- Aug. 31, 1960</td>
</tr>
<tr>
<td>Name</td>
<td>Home Country</td>
<td>Country</td>
<td>Host Institution</td>
</tr>
<tr>
<td>-------------------------------</td>
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<tr>
<td>Fullerton, G. H.</td>
<td>N. Ireland</td>
<td>Ireland</td>
<td>Massachusetts Institute of Technology</td>
</tr>
<tr>
<td>Gårding, Lars</td>
<td>Sweden</td>
<td>Sweden</td>
<td>Institute for Advanced Study</td>
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<tr>
<td>Godement, Roger J.</td>
<td>France</td>
<td>France</td>
<td>University of California, Berkeley</td>
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<tr>
<td>Goes, Gunther</td>
<td>Germany</td>
<td>Germany</td>
<td>Northwestern University</td>
</tr>
<tr>
<td>Grünbaum, Branko</td>
<td>Israel</td>
<td>Israel</td>
<td>Institute for Advanced Study</td>
</tr>
<tr>
<td>Gutwirth, Azriel</td>
<td>Israel</td>
<td>Israel</td>
<td>University of California, Berkeley</td>
</tr>
<tr>
<td>Ha, Kwang Chul</td>
<td>Korea</td>
<td>Korea</td>
<td>University of North Carolina</td>
</tr>
<tr>
<td>Haefliger, André</td>
<td>Switzerland</td>
<td>Switzerland</td>
<td>Institute for Advanced Study</td>
</tr>
<tr>
<td>Hart, V.</td>
<td>Ireland</td>
<td>Ireland</td>
<td>Massachusetts Institute of Technology</td>
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<tr>
<td>Helgason, Sigurdur</td>
<td>Iceland</td>
<td>Iceland</td>
<td>Columbia University</td>
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<tr>
<td>Hirzebruch, Friedrich</td>
<td>Germany</td>
<td>Germany</td>
<td>Institute for Advanced Study</td>
</tr>
<tr>
<td>Hlawka, Edmund</td>
<td>Austria</td>
<td>Austria</td>
<td>Institute for Advanced Study</td>
</tr>
<tr>
<td>Igusa, Jun-Ichi</td>
<td>Japan</td>
<td>Japan</td>
<td>Institute for Advanced Study</td>
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<tr>
<td>Ionescu, Cassius I.</td>
<td>Rumania</td>
<td>Rumania</td>
<td>Yale University</td>
</tr>
<tr>
<td>Kampé de Fériet, Joseph</td>
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The September 26, 1959 issue of Kexue Tongbao (Scientia) was dedicated to "Greeting the Tenth Anniversary of our People's Republic" and contained eight articles on progress during the last ten years in mathematics, physics, chemistry, biology, geology, physical geography, geophysics, and engineering, respectively. A good deal of interest and importance is attached to these reports since, according to the most reliable information available, there are 450 scientific and technical publications (including 80 in the basic sciences) currently being published in Communist China and these journals, or abstract reports of their contents, are not adequately available in this country.

An English version of the report on mathematical research has been prepared for publication here under the direction of S. H. Gould. The advice and assistance of many colleagues, who desire to remain anonymous, is gratefully acknowledged.

In a few of the more specialized mathematical passages it has been found necessary, because of certain technical difficulties, to make a paraphrase rather than a translation. No other editorial changes of significance have been attempted.

Following the report is a short biography of the author, prepared by Lowell Schoenfeld.

Mathematical Research in China in the Last Ten Years
by Loo-keng Hua
Director of the Institute of Mathematics of Academia Sinica
Kexue Tongbao (Scientia), 18(1959), 565-567 (Chinese)

Ancient China had a long and brilliant history in mathematics, but as a result of the long stagnation produced by the feudal system in our society, mathematics dropped behind in recent centuries. Only within the last two or three decades of the present century has research in modern mathematics begun. Before the liberation, there were only a few universities and individuals in the whole of China who had done any mathematical research, most of which was merely a continuation of already existing mathematics with no new ideas and scarcely any new methods. Most of the mathematical articles were published in the journals of foreign capitalist nations or else in the combined reports of universities, which were not issued very frequently. The only publication specializing in creative mathematics before the liberation consisted of two volumes of the journal "Acta Mathematica Sinica", in which there were only 34 papers. Circumstances were unfavorable for the development of mathematics and the mere training of personnel capable of producing original mathematics was a difficult task.

Up to the time of the liberation the general situation was as follows. Creative work in modern mathematics had been started, but still had a semi-feudal and semi-colonial social character, since it had no connection with the practical development of our country and produced no fruitful results in this direction. Mathematics was restricted to a small range, and the more closely it was connected with practical applications, the less sound were its results.

We are liberated! Under the brilliant leadership of the Chinese Communist Party, mathematics has striven forward and is now on the right
and ideal road. It is no longer a branch cut from a capitalist tree but is a sapling, growing up on the fertile soil of our own country.

Ten years is a short time for the founding and development of any branch of science. Our work has just started. However, as we shall see by comparing our present achievements with the period before the liberation, we have now realized that science has found its true master and have gradually perceived its hitherto unrevealed power, since our people have come into control of it.

Let us make some comparisons.

Our group is now larger. Before the liberation, only 74 authors had published papers, while 342 new authors have appeared in the ten short years since the liberation. Before the liberation, almost no mathematical worker devoted his life to the practical problems of engineering science, but now numerous mathematicians are contributing their knowledge to the building up of our country. It is obvious that the number of these mathematicians cannot be stated as explicitly as the above number of authors who have published papers.

The results have been richer than before. There were 652 papers published before the liberation, but between 1950 and 1958 there have been 983 papers. The 652 papers mostly appeared in foreign journals, while the above figure 983 includes only papers published in our country. Also, papers relating to the history of mathematics are not included.

The range of our research is wider now. Even before the liberation, some workers were already enlarging their scope and introducing many new subjects for research; at present, some branches are increasing in quantity and some are improving in quality. In the following remarks I will give a description of the general status of each branch.

In numerical analysis, very little had been done and, although we realized its significance after the liberation, it was not until 1953 that we began to make energetic progress. Since that time we have received a great deal of assistance from the Soviet Union and our modern computation technique has developed swiftly. Our numerical computation group is growing rapidly. Its members make use of actual research problems in developing their computation methods for electronic computers, and thereby solve numerical problems occurring in engineering, industry, and the various branches of science.

In partial differential equations, we have studied equations of mixed type related to aerodynamics, non-linear equations of elliptic and parabolic type occurring in the study of capillarity, and partial differential equations related to the theory of thin shells. All these topics were studied under the direction of Soviet mathematicians. As for ordinary differential equations, we have studied stability in the neighborhood of a singular point, stability for equations with lagging time argument, and asymptotic stability, and in the qualitative branch of the theory, the distribution of limit cycles. During the great leap forward, researchers in differential equations have studied stress distribution and unstable flow in large hydraulic installations, long period data prediction in meteorology, vibration of machinery, aerodynamics, and thin shell problems in big buildings.

In probability and statistics, we have studied problems in statistical analysis, Markov processes, prediction theory for stationary processes, empirical distributions, etc. Many of these fields were undeveloped in our country before the liberation and most of our knowledge was obtained from the Soviet Union or else acquired by us directly under the guidance of
the Soviet Union and Poland. During the great leap these researchers applied their statistical methods to mining, manufacturing, etc. and thereby solved several important problems, for example, concerning the properties of cotton thread. They also studied prediction problems for peak construction needs in the region of the Three Gorges of the Yangtze River.

The three fields mentioned above have been in the foreground of the application of mathematics to practical problems. As a result of the achievements in these fields, we have paid particular attention to them in our science projects, and the resulting experience has shown us that the fields of operations analysis and formal logic should be developed as quickly as possible.

Operations analysis, especially in the field of linear programming, was a most active subject during the great leap. Since last October it has been applied everywhere in the country to the building up of the national economy, and many practical problems have been solved, with a great saving of national funds. We especially wish to mention the so-called graphical method, which was gradually developed through the experience of our comrades in the transportation section. During the great leap we gave a rigorous proof for this method and it is now widely employed in practice.

In logic, we have studied two-valued and multiple-valued logic, and the axioms of modal logic. We have also obtained some results in the theory of recursive functions, etc. During the great leap, researchers in this field have participated in the design of entire computer systems, including special machines for automatic maneuver on railroads, and in this way have further strengthened the bond between theory and practice.

The five fields mentioned above have had a more direct connection with the building up of the national economy, and also a weaker theoretical foundation. Because of the great concern of the party for these fields, our groups in them have grown from small to large, and although the results so far obtained are preliminary, further development is assured on the basis of the foundations already laid.

We have made studies in the field of integral equations, for example in eigenvalue problems.

In functional analysis, we started our work after the liberation on the basis of Russian experience and have acquired some concrete and stronger results connected with the concept of inner product space, approximate solutions of inconsistent equations, generalized inversion of linear operators, and classification of closed linear operators and bi-orthogonal systems. Some new treatments of generalized functions have been suggested and we have also studied generalized Mellin transforms. In linear topological spaces, we have introduced the concept of B.S. spaces, associated with the classical resonance theorem. With the help of Polish specialists, we have obtained some results regarding continuity and complete continuity, approximation theory for linear and non-linear equations, Orlicz spaces, imbedding theorems, and the like.

The theory of functions is one of the oldest branches of our research, so that there are comparatively more researchers working in this field than in others. Since 1950 great progress has been made on the basis of earlier work in such fields as the fundamental theory of the relationship between entire and meromorphic functions, prime numbers, normal families of analytic functions, the geometric theory of complex functions (particularly of univalent functions), the convergence of series of orthogonal polynomials and of trigonometric series, etc. These achievements are due to the continuous hard work of several diligent and persevering mathematicians and of a certain number of young comrades under their direction.
Following the lead of Russian mathematicians, we have carried on the study of the structure theory and the approximation theory of functions, and as for general analytic functions and conformal mapping, we have laid the groundwork for further progress. These latter studies are closely related both to practical applications and to other branches of mathematics.

The trend of research to the theory of functions of several complex variables began in 1952. We have our own methods and have obtained some systematic results based on extensive application of other branches of mathematics, e.g. matrix theory, group theory, Riemannian geometry, etc., which are also generally associated with such subjects as harmonic analysis on a compact group, partial differential equations of elliptic and mixed types, and generalized function theory. Up to the present, only a few people have devoted themselves to this subject and further developments are expected.

In number theory, we have paid special attention to the powerful method introduced by the Russian Academician Vinogradov and his school, namely the method of trigonometric sums. Since the liberation, we have obtained satisfactory results by applying this method to estimation of the major arcs in Waring's problem. Many researchers have done a great deal of work on the estimation of the remainder for number theoretic functions and have started studying the sieve method. A proof has been given for the theorem that a sufficiently large even number is the sum of a product of two primes and a product of three primes. Besides, we have determined the classification of positive definite quadratic forms, with unit determinant, in 12, 13, 14, 15 variables.

In algebra, more light has been thrown on the study of such topics as quasi-automorphisms of a field, the multiplicative group of a skew field, automorphisms of the classical groups over a field and matrix geometry over a field.

In Lie groups and Lie algebras, we have proved the algebraicity of certain Lie algebras, and have deepened our knowledge of some fundamental facts concerning the Lie algebras of algebraic Lie groups; and the diagrammatic method for systems of roots has been applied to the classification of real semi-simple Lie algebras and the determination of their automorphism group. Some problems have been solved concerning the existence of subalgebras of Lie algebras and of conjugation.

The theory of group representation has found applications in the theory of functions of several complex variables and the theory of inequalities. By variational methods we have proved a convergence theorem for Abelian summability of the Fourier expansion of a continuous function on a compact group.

In the theory of algebras and valuation theory, some corrected statements, proofs, and generalizations have been made for Gr"unwald's theorem, both in the general case and in the prime-power case. Meanwhile, we are proceeding in the preliminary study of the structure of the multiplicative group of simple algebras over an algebraic number field, the infinite valuation of a complete field and the structure of its multiplicative group.

Also, we have carried out some work on ring theory, general associative algebras, and non-associative algebras.

In differential geometry, we are continuing the study of metric and projective differential geometry, which had been relatively well developed before the liberation. Recently initiated research on the geometry of general differentiable manifolds, and in particular of k-spreads, is now under way. In this direction, we have systematically discussed the theory of projective motions, integrability conditions, plane theorems, and a new theory of representation, etc., and some important corrections have been made. A series
of imbedding problems has been undertaken; for example, to imbed a Riemannian space, or a space with a projective connection, or a Finsler space, in Euclidean space (or a space of constant curvature), in projective space or in Minkowski space.

The investigation of differentiable manifolds and the geometry of Lie groups was started after the liberation. We have studied Lie groups, homogeneous spaces, and such topological properties of Riemannian manifolds as, for instance, Betti numbers and the results of integration of Gauss-Bonnet form. We are also continuing the study of classification of homogeneous spaces. Some geometric problems arising from the developments of the theory of functions of several complex variables have also been undertaken.

In topology, on the basis of the proof of the topological invariance of the Stiefel-Whitney characteristic classes for a compact differential manifold, we have proved that the Pontryagin characteristic classes after reduction mod 3 or mod 4 are topological invariants. We have also introduced some non-homotopy type topological invariants by means of which we obtained a necessary condition that a topological space can be realized in an n-dimensional Euclidean space \( R^n \) and a sufficient condition that an n-dimensional complex (with \( n \neq 2 \)) can be realized in \( R^{2n} \); we have completely determined the dimension of a normal space of countable basis, and have proved that any two differentiable realizations of a compact manifold in \( R^{2n+1} \) are differentiably isotopic.

With respect to a space \( X \) with a group of transformations of prime period \( \varphi \), we have investigated the relationship of the homology mod \( \varphi \) of \( X \), the homology mod \( \varphi \) of the set of fixed points of \( X \), and the multiplicative structure of the cohomology of \( X \). We have also proved that in a 4-dimensional Euclidean space there always exists a fixed point for a periodic transformation.

We have introduced the method of symmetric products of fiber bundles for dealing with the problems of obstruction classes for bundles and have obtained the formulas for the second obstruction to extensions and homotopies of cross sections of sphere bundles.

We have discovered that there are seven fundamental types of \( A_n \)-polyhedra (for \( n > 2 \)) and have used them to construct the normal forms for the homotopy type of \( A_n \)-polyhedra; we have begun to establish the algebraic theory of the homotopy type of such polyhedra and have introduced new homotopy invariants and algebraic quantities, such as block invariants, \( E_2 \)-torsion and \( F_2 \)-torsion, characteristic polynomials and characteristic coefficients. Using these entities we have given a necessary condition for the existence of a cross-section of a fiber space, and using homotopy and cohomology operations we have calculated the homotopy group \( \pi_{n+2}(x) \) for an \( (n-1) \)-connected space (with \( n > 4 \)).

The above is a general survey of our research work in various branches in the past ten years.

During the great leap, the motivation and direction of our research has been entirely changed. Mathematical workers, almost without exception, have tried to do some work related to the practical purposes of production. Although this is only the first step and some of the methods are tentative, the range of our research extends to almost every department of production; its scope is so wide and its record so brilliant that any attempt to give a detailed report of it would be in vain. Consequently, I have been able to give only a brief description based on what I have seen and on the incomplete information I have obtained.

Some of the practical fields related to our research are: forestry, hydrology and irrigation, civil engineering and architecture, railroad-
ing, mechanical and electrical engineering, postoffice communication problems, mining, metallurgy, chemical engineering, petroleum industry, geological survey, textile industry, transportation, resource allocation, plant design, production management, industry allocations, planning of manpower and tools, meteorology, techniques of calculation, etc.

The most important mathematical methods that have been applied to these practical sciences are: linear programming, numerical analysis, mathematical statistics and differential equations. Some of the practical problems require only differential or integral calculus and some of them even only high school mathematics, but others have been solved using linear algebra, mathematical logic and number theory.

The essential achievement in this new direction of research is not the results already attained but rather its profound influence for the future. It leads mathematicians to venture out from their past limited environment and pay attention to their real responsibility for the building up of the fatherland. Essential progress is represented by the mere abandonment of the former unrealistic doctrine that mathematics should be studied for the sake of mathematics alone. As a result, many mathematicians are now investigating the relation between mathematics and the other sciences and are beginning to study other subjects. In order to apply their mathematical tools, some of them have even gone over to the domains of the other sciences, such as nuclear physics, structure of matter, automation, electronics, elasticity, fluid dynamics, space dynamics, meteorology, hydrology, etc. In the present short article we cannot list their achievements but can only say that certain concrete problems have been solved by mathematicians in collaboration with researchers in the other sciences.

Ten years is a short period and much time has been spent on restoration and rehabilitation, on reformation of our methods of thought, on correction of our teaching methods, and on the training of many staff members, so that our results to date can only be considered as preliminary. However, in comparison with the period before the liberation, our confidence has been doubled. Under the leadership of the Chinese Communist Party and with the help of Soviet and other brother countries, we, the Chinese mathematical workers, unanimously recognize that it is possible in China to make great progress at a rapid rate in the field of mathematics. In the future, it is certain that we will have more and more workers of a high quality, capable of advancing both our theoretical research and its practical applications.

A Biographical Note on Professor Loo-keng Hua
by Lowell Schoenfeld

Loo-keng Hua was born in October, 1909, in the province of Kiangsu in eastern China. He appears to have had little formal university education and to have taken no university degree. However, as a result of his independent studies on trigonometrical series and on the insolvability of algebraic equations by radicals, he was appointed an assistant at the National Tsing Hua University in Peking in 1931, and he received successive promotions during the next few years. After spending several years at Cambridge University, England, as a Research Fellow of the China Foundation for the Promotion of Education and Culture, he returned to Tsing Hua University in 1938 as Professor of Mathematics. He served in this capacity until 1946 when he became a member of the Institute for Advanced Study in Princeton, New Jersey. In 1948, he went to the University of Illinois in Urbana as a visiting professor, remaining there until his return to Tsing Hua University in February of 1950. Since 1951 he has been Professor of Mathematics at
Peking University. In addition, he has been the Director of the Institute of Mathematics of Academia Sinica since 1950.

Professor Hua is the author of several books and has written over 120 papers on various mathematical subjects. From 1934 to 1944 this work was almost exclusively in the field of the theory of numbers and centered around the Waring and Goldbach problems together with the associated subject of exponential sums. Since 1944, a large amount of his work has been in the geometry of matrices, several complex variables and various aspects of algebra although some work in analytic number theory has been continued. The work is of broad scope and is more than ample to qualify him as one of the foremost mathematicians in the world.
PERSONAL ITEMS
(This section is restricted to members of the Society)

Associate Professor R. G. Ayoub is on leave from Pennsylvania State University for the 1959-1960 academic year. He is a Research Fellow at Harvard University while writing an expository book under the Society's contract with AFOSR.

Professor Cristóbal de Losada y Puga has been incorporated as "miembro de número" in the Academia Peruana correspondent of the Real Academia Española. Besides, he has been elected Director of Publications in the Academia Nacional de Ciencias Exactas, Físicas y Naturales de Lima.

Assistant Professor L. E. Dubins of Carnegie Institute of Technology has been awarded a National Science Foundation Post Doctoral Fellowship and is spending the 1959-1960 academic year at the University of California, Berkeley.

Associate Professor Russell Remage, on leave from the University of Delaware, has been awarded a National Science Foundation Faculty Fellowship and will spend the 1959-1960 academic year in Cambridge, England.

The Institute of Mathematical Sciences at New York University has announced the following Temporary Members and Visiting Research Scientists: Assistant Professor Geraldine A. Coon of the University of Connecticut, Dr. J. M. Gonzáles-Fernández of the Mayo Clinic, Assistant Professor E. F. Low, Jr. of the University of Miami, and Professor I. M. Singer of Massachusetts Institute of Technology.

Assistant Professor J. J. Andrews of the University of Georgia has been appointed to an assistant professorship at the University of Wisconsin.

Mr. G. L. Baldwin of the University of Oklahoma has been appointed to an assistant professorship at New Mexico State University.

Professor Margaret M. Baskervill of Shorter College has been appointed to an assistant professorship at Alabama Polytechnic Institute.

Mr. M. L. Bender of Yale University has accepted a position as mathematics master at Adisadel College, Cape Coast, Ghana, Africa.

Mr. A. I. Benson of General Electric Company has accepted a position as Technical Assistant with System Development Corporation, Santa Monica, California.

Professor Dorothy L. Bernstein of the University of Rochester has been appointed to a professorship at Goucher College.

Dr. W. A. Beyer of Pennsylvania State University has accepted a position as staff member at the Los Alamos Scientific Laboratory of the University of California, Los Alamos, New Mexico.
Dr. J. W. Blattner of the University of California has been appointed to an assistant professorship at San Fernando Valley State College.

Dr. J. R. Boen of the University of Illinois has been appointed to an assistant professorship at Southern Illinois University.

Assistant Professor W. M. Boothby of Northwestern University has been appointed to an associate professorship at Washington University.

Professor Emeritus C. C. Camp of the University of Nebraska has been appointed a Lecturer at the Dearborn Center of the University of Michigan.

Visiting Associate Professor Buchanan Cargal of the University of Texas has accepted a position as Member of the Technical Staff of Land-Air, Incorporated, Point Mugu, California.

Dr. F. W. Carroll, Jr. has been appointed to an assistant professorship at Purdue University.

Dr. C. R. Cassity of New Mexico Institute of Mining and Technology has accepted a position as shock hydrodynamicist with General Electric Company, Philadelphia, Pennsylvania.

Dr. Ward Cheney of CONVAIR has accepted a position as member of the technical staff of Space Technology Laboratories, Incorporated, Los Angeles, California.

Professor S. S. Chern of the University of Chicago has been appointed to a professorship at the University of California, Berkeley.

Dr. W. R. Cowell of Montana State University has accepted a position as member of the technical staff of Bell Telephone Laboratories, Incorporated, Murray Hill, New Jersey.

Dr. G. E. Cross of Victoria College has been appointed to an assistant professorship at the University of Western Ontario.

Mr. I. T. Cundiff of St. Peter's College has accepted a position as mathematician with Vitro Laboratories, West Orange, New Jersey.

Dr. Aubert Daigneault of the Royal Military College has been appointed to an assistant professorship at the University of Ottawa.

Professor M. D. Donsker of the University of Minnesota is spending a sabbatical leave at the University of Aarhus in Denmark.

Dr. H. P. Edmundson of RAND Corporation has accepted a position as Senior Associate with Planning Research Corporation, Los Angeles, California.

Mr. G. W. Fairchild of Bendix Aviation Corporation has accepted a position as research scientist with Litton Industries, Beverly Hills, California.

Dr. Harry Ferguson of Wright-Patterson Air Force Base has been appointed to an associate professorship at the University of Cincinnati.

Assistant Professor Ben Fitzpatrick, Jr. of the University of Texas has been appointed to an assistant professorship at Alabama Polytechnic Institute.
Mr. H. P. Friedman of the Bulova Research and Development Laboratories has accepted a position as mathematician with the System Development Corporation, Lodi, New Jersey.

Dr. M. P. Gaffney of New York University has been appointed assistant program director at the National Science Foundation, Washington, D.C.

Assistant Professor S. G. Ghurye of the University of Chicago has been appointed to an associate professorship at Northwestern University.

Mr. Seymour Hayden of the Air Force Cambridge Research Center has been appointed to an assistant professorship at Clark University.

Mr. N. W. Hill, Jr. of the University of Miami has accepted a position as Data Reduction Analyst with RCA Service Company, a Division of Radio Corporation of America, Patrick Air Force Base, Florida.

Mr. R. R. Hornby of the University of Nebraska has accepted a position as Associate Research Engineer-Programmer with Boeing Airplane Company, Wichita, Kansas.

Dr. W. A. Howard of Bell Telephone Laboratories has been appointed to an assistant professorship at Pennsylvania State University.

Dr. R. A. Hultquist of Oklahoma State University has been appointed to an assistant professorship at DePauw University.

Dr. R. S. Johnson of the University of Pennsylvania has accepted a position as Engineer with Radio Corporation of America, Morningside, New Jersey.

Dr. Julius Kane of New York University has accepted a position as research physicist with Dorne and Margolin, Westbury, New York.

Dr. M. L. Katz, Jr. of the University of California has been appointed to an assistant professorship at the University of Chicago.

Dr. J. S. Klein of the University of Michigan has been appointed to an associate professorship at Lafayette College.

Dr. H. P. Kramer of Bell Telephone Laboratories, Incorporated has accepted a position as mathematician with General Electric Company, Santa Barbara, California.

Dr. H. C. Kranzer of New York University has been appointed to an associate professorship at Adelphi College.

Dr. Lester Kraus of CONVAIR has accepted a position as scientist with Republic Aviation Corporation, Farmingdale, New York.

Assistant Professor K. T. Leung of Miami University has been appointed to an assistant professorship at the University of Cincinnati.

Reverend C. J. Lewis of Fordham University is on leave for the 1959-1960 academic year to accept a National Science Foundation Faculty Fellowship at Harvard University.

Assistant Professor R. H. McDowell, on leave from Rutgers, The State University, has been appointed to an assistant professorship at Purdue University.
Mr. E. G. McNeil of DePaul University has accepted a position as Senior Mathematical Analyst with Lockheed Aircraft Corporation, Sunnyvale, California.

Dr. J. T. Mohat of Duke University has been appointed to an assistant professorship at North Texas State College.

Dr. E. O. Nelson of the University of Minnesota has been appointed to an associate professorship at the University of North Dakota.

Dr. Anil Nerode of the University of Chicago has been appointed to an assistant professorship at Cornell University.

Associate Professor Athanasios Papoulis, on leave from the Polytechnic Institute of Brooklyn, will be at the Institut fuer Elektrotechnik, Technische Hochschule, Darmstadt, Germany for the academic year 1959-1960.

Dr. S. T. Rio of Oregon State College has been appointed to an assistant professorship at Western Washington College of Education.

Dr. S. M. Robinson of Duke University has been appointed to an assistant professorship at the University of Rhode Island.

Dr. N. S. Rosenfeld of City College has been appointed an Assistant Research Scientist at New York University.

Dr. R. G. Segers of Bell Telephone Laboratories, Incorporated has accepted a position as senior mathematician with Vitro Laboratories, West Orange, New Jersey.

Assistant Professor R. J. Smith of Northern State Teachers College has been appointed a lecturer at the University of Sydney, Sydney, Australia.

Dr. D. P. Squier of San Diego State College has accepted a position as Mathematician with the California Research Corporation, La Habra, California.

Dr. W. F. Stinespring of the Institute for Advanced Study has been appointed to an assistant professorship at the University of Chicago.

Associate Professor G. M. Wing of the University of New Mexico has joined the staff of Sandia Corporation, Albuquerque, New Mexico.

The following promotions are announced:

Bess E. Allen, Wayne State University, to an associate professorship.

Rafael Artzy, Technion, Israel Institute of Technology, Haifa, Israel, to an associate professorship.

E. M. Beesley, University of Nevada, to a professorship.

Jerome Berkowitz, New York University, to an associate professorship.

G. H. Butcher, Howard University, to an associate professorship.

H. E. Campbell, Michigan State University, to an associate professorship.

C. L. Dolph, University of Michigan, to a professorship.

Harry Hochstadt, Polytechnic Institute of Brooklyn, to an associate professorship.

Solomon Hurwitz, City College, New York, New York, to an associate professorship.

Arno Jaeger, University of Cincinnati, to a professorship.

W. M. Laird, University of Pittsburgh, to an associate professorship.

Joseph Landin, University of Illinois, to a professorship.

D. B. Lowdenslager, Princeton University, to an assistant professorship.

R. H. McDowell, Rutgers, The State University, to an assistant professorship.

Nathaniel Macon, Alabama Polytechnic Institute, to Professor of Mathematics and Director of the Computer Laboratory.

Elisha Netanyahu, Israel Institute of Technology, Haifa, Israel, to a professorship.

Steven Orey, University of Minnesota, to an associate professorship.

R. H. Owens, University of New Hampshire, to an associate professorship.

R. A. Raimi, University of Rochester, to an associate professorship.

S. L. Ross, University of New Hampshire, to an associate professorship.

N. J. Rothman, University of Rochester, to an assistant professorship.

Martin Schechter, New York University, to an assistant professorship.

Oswald Wyler, University of New Mexico, to an associate professorship.

G. A. Yanosik, New York University, to a professorship.

The following appointments to instructorships are announced:

University of British Columbia: Dr. D. W. Bressler, Dr. M. W. Katz; University of California, Berkeley: Dr. C. S. Ballantine; Chapman College: Mr. M. C. Tews; Columbia University: Dr. Hyman Bass; Contra Costa College: Mr. W. C. Frey; Long Island University: Mr. Martin Orr; Louisiana State University: Mr. W. G. Franzzen; University of Nebraska: Mr. L. D. Fountain; Princeton University: Dr. M. L. Balinski, Dr. C. H. Sah; St. Joseph's College: Mr. J. J. Costello; Villanova University: Mr. Brindell Horelick; Univer-
Deaths:

Professor H. P. Evans of the University of Wisconsin died on June 25, 1959 at the age of 59 years. He had been a member of the Society for 33 years.

Assistant Professor Werner Gautschi of Ohio State University died on October 3, 1959 at the age of 31 years. He had been a member of the Society for 4 years.

Dr. Gertrude S. Ketchum of Urbana, Illinois died on September 27, 1958 at the age of 55 years. She had been a member of the Society for 23 years.

Professor Jakob Nielsen of the University of Copenhagen, Denmark, died on August 3, 1959 at the age of 68 years. He had been a member of the Society for 11 years.

Professor Emeritus Arthur Rosenthal of Purdue University died on September 8, 1959 at the age of 72 years. He had been a member of the Society for 18 years.

Mr. Balthasar van der Pol, Director Emeritus of the International Telecommunications Union, died on October 10, 1959 at the age of 70 years. He had been a member of the Society for 12 years.

CORRECTION:

On page 599 of the November issue of the NOTICES, it was incorrectly reported that Associate Professor J. W. Brace has received a National Science Foundation Fellowship. Instead, his work at the University of California will be supported in part by a grant from the National Science Foundation.
LETTERS TO THE EDITOR

The following are three short notes from Paul Erdős, containing a good deal of news concerning current European mathematical activity.

Editor, the NOTICES. 1959 X 10

Many thanks for your letter. I heard of Senator Humphrey's bill + the goings on at Salt Lake City. Perhaps I will see you in the autumn of 1960.

I leave for Poland in two weeks. Will visit also East + West Berlin and Halle. I start on my big trip in December. My probable route will be Moscow-Peking-Canton-Hong Kong-Singapore-Australia (not counting 2^0 other stops).

There was a three-day meeting on infinite series here (October 6, 7, and 8). Zamansky (France), Vermes (London) and several other foreigners were here. Next week there will be a meeting on graph theory. Tutte from Canada is expected. Linnik from Leningrad arrived yesterday. I wanted to meet him since a long time. His first question was: Is Erdős now in Hungary? He will preach on his recent results on number theory and probability. There was an Italian math meeting in Naples last month. Several mathematicians from here attended, but I was too busy here to go (I was twice in Naples already). Last month there was the British math Colloquium in Cardiff, but due to lack of time I could not go there either.

I just got a letter from Pierce with the copy of Humphrey's bill. Just saw Vermes and boss off to London, will have dinner tonight with Linnik. My mother is well and sends regards. Poor Fejér is very ill. He had a stroke and there is not much hope for his recovery.

1959 X 24

I was at the U. S. Consulate yesterday. There seems to be a good chance that I will get back to Samland next autumn. The consul said that it would help if American educational institutions would write him asking for my presence. I think Notre Dame will do this, but perhaps it will also help if the American Mathematical Society writes him. I believe I am still visiting lecturer of the Society and could give the lectures starting in the autumn of 1960 if I get the visa. I will be back from Australia in Europe not later than August, 1960. The name of the consul is Pratt Byrd, American Embassy, SZABADSÁG tér 12, Budapest.

There was a three-day long meeting on graph theory in Hungary October 20-22. Tutte (Toronto), A. Stone (Manchester), Smith (London), Berge (Paris), Mycielski (Wroclaw), G. Dirac (Hamburg), Reichardt, Sachs (East Germany) attended + many Hungarians.
I am expected to leave for Poland on October 27 (if I recover from my cold which the s.f. saw fit to send me). I am just trying to reach Kac or Professor Stark in Poland on the phone, so far no answers. Kac leaves Poland tomorrow or Monday so I will just miss him.

It now seems almost certain that I go to Australia via Moscow-Leningrad-Peking-Canton-Singapore. The consul said that my trip will in no way make it more difficult to return to the U.S.

1959 XI 18

Many thanks for your letter. Poor Fejér died late in October. On October 27 I went to Poland and stayed 12 days. I preached in Warsaw, Wroclaw + Posnan. I preached on set theory and number theory and probability. Just missed Kac who left on October 26 for Aarhus (Denmark). Sierpiński was in Paris, thus I did not see him. The interest in Poland for abstract set theory is small, a great change since 20 years. Logic and foundations, functional analysis and topology are most popular, among the young people several are interested in number theory. Sierpiński who started his career with number theory (he did some important work 55 years ago) now is most interested in number theory.

From Warsaw I went to Berlin. I preached in East + West Berlin, Rostock + Halle. Renyi was in Stockholm and Aarhus, met Kac there and we met in Berlin. In Aarhus there is a new math institute which is slowly becoming an international meeting place. Renyi + I visited there in 1957 when it was being born. I am invited to go there. Renyi + I work on probability and graph theory also we are supposed to write a book on applications of probability to number theory. I am going to start for Australia in about 5 weeks.

Kind regards to all,
E. P.

Editor, the NOTICES.

I do not agree with the suggestions made by Professor H. A. Pogorzelski (Notices Amer. Math. Soc. p. 496) about the printing of mathematics. As long as mathematics continues to be produced by hand and brains, not by machines, I do not see anything shocking in having it printed partly by hand.

As to speed, let us remember that a 20 page article is often the fruit of a year's work; what difference would it then make to have it printed in a few hours instead of a few days? Also the time of a mathematician is surely worth the time of a typesetter. An article that is hard to read for typographical reasons might save the time of one typesetter, but waste the time of perhaps more than 100 readers.
Indeed the suggestions of Professor H. A. Pogorzelski make the reading very hard. E. g. can one immediately say which one of the matrices

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 1 & 1
\end{bmatrix}
\]

is a unit matrix, or a permutation matrix? Computing the trace and the determinant of \(\begin{bmatrix} 1204 & 2401 \\ 1235 & 0106 \end{bmatrix}\) becomes a long job, in which simplifications may easily be overlooked. Also, with the suggested notation for diagrams, what would it mean to say that a diagram is commutative, or that its columns are exact? Except for describing the rows in a convenient manner, the suggested notation for diagrams would not be much easier to grasp than a clumsy sentence like: "Let \(A, B, C, A', B', C'\) be modules over a ring \(R\), and \(u, v, a, b, c, u', v'\) homomorphisms of \(A\) to \(B\), \(B\) to \(C\), \(A\) to \(A'\), \(B\) to \(B'\), \(C\) to \(C'\), \(A'\) to \(B'\) and \(B'\) to \(C'\) respectively; we assume that \(u'a = bu\), \(v'b = cv\), \(\text{Im}(u) = \text{Ker}(v)\) and \(\text{Im}(u') = \text{Ker}(v')\)" (describing a 3-row, 2-column commutative diagram with exact columns, which the writer was forced to write down in order to avoid mistakes!).

I also feel that the suggestion is somewhat insulting to the printers. Printing is not only a business, but also a kind of art. Many a printer feels, justly, proud of his books, and this makes his profession more rewarding than the selling of peanuts. Do we really want to strip the printing profession of all its artistic and rewarding aspects?

As to costs, finally, I think that a world that was able to afford the luxury of well printed books and journals and the cost of artistic and rewarding professions when it was poor, should also be able to afford them in the wealthy days of 1959.

P. Samuel
University of Illinois
Editor, the NOTICES.

It is not possible, as Professor Pierre Samuel found out, to give a large square or rectangular diagram a linear style without some surrender of notational eloquence. On the other hand, by using the contraction form, it is possible, for example, to give the currently not printable, cubic diagram consisting of 4 objects and 21 mappings a rectangular style which is printable at least. Professor Samuel's 3-row by 2-column commutative diagram with exact columns can be more clearly expressed and yet set by machine.
The basic issue here is not concerned with art, printers, pride, typesetters, hands, machines nor peanuts, but simply with the possibility of developing expeditious and modern methods of handling mathematical papers and reviews.

H. A. Pogorzelski
MEMORANDA TO MEMBERS

THE MATHEMATICAL SCIENCES EMPLOYMENT REGISTER

The following item is repeated from the November, 1959, issue of the NOTICES, and a more detailed time schedule given.

The Mathematical Sciences Employment Register, established by the American Mathematical Society, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics, will be maintained at the Winter Meeting in Chicago, Illinois, on January 27-29, 1960. The Register will be conducted from 9:00 A.M. to 5:00 P.M. on January 27, 28, and 29.

On Wednesday, January 27, from 9:00 A.M. to 1:00 P.M., the Placement Service Desk will be located in Room 19 on the fourth floor of the Conrad Hilton Hotel. This time will be devoted to the registration of both employers and applicants and to distribution of the listings.

Beginning at 2:00 P.M. on Wednesday, January 27, and from then on daily from 9:00 A.M. to 5:00 P.M., the Employment Register will be located in Private Dining Room No. 2 on the third floor of the Conrad Hilton Hotel. Interview appointments will start Wednesday at 1:00 P.M. There will be no charge for registering either to job applicants or to employers, except when the Late Registration Fee for employers is applicable. Provision will be made for anonymity of applicants upon request.

Job applicants and employers who wish to be listed will please write to the Employment Register, 190 Hope Street, Providence 6, Rhode Island, for application forms and for position description forms, which must be completed and returned to Providence not later than January 8, 1960, in order to be included free of charge in the listings at the meeting in Chicago. Forms which arrive after this closing date, but before January 26, will be included in the listings at the meeting for a Late Registration Fee of $3.00, and will also be included in the sold listings, but not until ten days after the meeting. Printed listings will be available for sale both during and after the meeting. The prices are as follows: Position descriptions, $2.00; listing of applicants, academic only, $5.00; comprehensive listing of applicants, academic, industrial, and government, $20.00.

It is essential that applicants and employers register at the Placement Service Desk promptly upon arrival at the meeting to facilitate the arrangement of appointments.

EMPLOYMENT OF RETIRED MATHEMATICIANS

The Headquarters Offices of the American Mathematical Society, 190 Hope Street, Providence 6, Rhode Island, again plans to
issue the List of Retired Mathematicians Available for Employment, which it has maintained yearly.

Mathematicians who are retiring this year and who are interested in being included in the list are asked to send the following information to the Headquarters Offices: name; date of birth; highest degree and where obtained; most recent employment; present address; date available; preferences, including preference for academic or industrial employment.

Data thus received will be incorporated into the next issue of the List, the issuance date of which will be announced in the February issue of the NOTICES.

RECIPROCITY AGREEMENT WITH THE AUSTRALIAN MATHEMATICAL SOCIETY

The American Mathematical Society has entered into a reciprocity agreement with the Australian Mathematical Society, by which members of each may become members of the other by paying half the regular dues. Half the regular dues of the Australian Mathematical Society are £2.15.0, amounting to about $7.50 a year. Members will receive the Journal of the Australian Mathematical Society. At present, two parts are published each year, and four parts form a volume. It is intended later to publish four parts each year. They shall receive also any other benefits of membership provided only that members under the reciprocity agreement spending time in Australia shall pay regular dues while they are resident there.

Those members of the American Mathematical Society wishing to join the Australian Mathematical Society, or presently being members of the latter and wishing to take advantage of the reciprocity agreement should write Professor H. O. Lancaster, General Secretary, Australian Mathematical Society, Department of Mathematical Statistics, University of Sidney, Sidney, Australia. Dues are payable to Professor C. S. Davis, University of Queensland, St. Lucia, Brisbane, Queensland, Australia.
CORPORATE AND INSTITUTIONAL MEMBERS OF THE AMERICAN MATHEMATICAL SOCIETY

As of December 5, 1959, the following were supporting the Society through Corporate or Institutional memberships:

CORPORATE MEMBERS

Bell Telephone Laboratories, Incorporated
E. I. Du Pont de Nemours and Company, Incorporated
Ford Motor Company
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INSTITUTIONAL MEMBERS

Acadia University, Wolfville, N. S., Canada
Alabama Polytechnic Institute, Auburn, Alabama
University of Alabama, University, Alabama
University of Alberta, Edmonton, Alberta, Canada
Amherst College, Amherst, Massachusetts
University of Arizona, Tucson, Arizona
Beloit College, Beloit, Wisconsin
Bowdoin College, Brunswick, Maine
Brandeis University, Waltham, Massachusetts
Brigham Young University, Provo, Utah
University of British Columbia, Vancouver, B. C., Canada
Brooklyn College, Brooklyn 10, New York
Brown University, Providence 12, Rhode Island
Bryn Mawr College, Bryn Mawr, Pennsylvania
University of Buffalo, Buffalo 14, New York
California Institute of Technology, Pasadena 4, California
University of California, Berkeley 4, California
University of California, Davis, California
University of California, Los Angeles 24, California
University of California, Riverside, California
University of California, Santa Barbara, California
Carnegie Institute of Technology, Pittsburgh 13, Pennsylvania
Case Institute of Technology, Cleveland 6, Ohio
Catholic University of America, Washington 17, D. C.
University of Chicago, Chicago 37, Illinois
University of Cincinnati, Cincinnati 21, Ohio
City College, New York 31, New York
University of Colorado, Boulder, Colorado
Columbia University, New York 27, New York
Connecticut College, New London, Connecticut
Carnegie Institute of Technology, Pittsburgh 13, Pennsylvania
Dartmouth College, Hanover, New Hampshire
University of Delaware, Newark, Delaware
De Paul University, Chicago 14, Illinois
University of Detroit, Detroit 1, Michigan
Duke University, Durham, North Carolina
Duquesne University, Pittsburgh 19, Pennsylvania
Emory University, Emory University, Georgia
Florida State University, Tallahassee, Florida
University of Florida, Gainesville, Florida
Fordham University, New York 58, New York
Georgetown University, Washington 7, D. C.
University of Georgia, Athens, Georgia
Gettysburg College, Gettysburg, Pennsylvania
Goucher College, Towson, Maryland
Grinnell College, Grinnell, Iowa
Harpur College, State University of New York, Endicott, New York
Harvard University, Cambridge 38, Massachusetts
Haverford College, Haverford, Pennsylvania
College of the Holy Cross, Worcester 3, Massachusetts
University of Houston, Houston 4, Texas
Illinois Institute of Technology, Chicago 16, Illinois
University of Illinois, Urbana, Illinois
Indiana University, Bloomington, Indiana
Institute for Advanced Study, Princeton, New Jersey
Iowa State University of Science and Technology, Ames, Iowa
The State University of Iowa, Iowa City, Iowa
The Johns Hopkins University, Baltimore 18, Maryland
University of Kansas, Lawrence, Kansas
University of Kentucky, Lexington 29, Kentucky
Kenyon College, Gambier, Ohio
Lafayette College, Easton, Pennsylvania
Lehigh University, Bethlehem, Pennsylvania
Louisiana State University and Agricultural and Mechanical College, Baton Rouge 3, Louisiana
McGill University, Montreal, Quebec, Canada
University of Manitoba, Winnipeg, Manitoba, Canada
Marquette University, Milwaukee, Wisconsin
University of Maryland, College Park, Maryland
Massachusetts Institute of Technology, Cambridge 39, Massachusetts
University of Massachusetts, Amherst, Massachusetts
Mathematical Association of America, Buffalo 14, New York
University of Miami, Coral Gables, Florida
University of Michigan, Ann Arbor, Michigan
Michigan State University of Agriculture and Applied Science, East Lansing, Michigan
University of Minnesota, Minneapolis 14, Minnesota
Mississippi State College, State College, Mississippi
University of Mississippi, University, Mississippi
University of Missouri, Columbia, Missouri
Montana State College, Bozeman, Montana
Montana State University, Missoula, Montana
Mount Holyoke College, South Hadley, Massachusetts
University of Nebraska, Lincoln 8, Nebraska
University of New Hampshire, Durham, New Hampshire
New Mexico State University of Agriculture, Engineering and Science, University Park, New Mexico
University of New Mexico, Albuquerque, New Mexico
New York University, New York 12, New York
University of North Carolina, Chapel Hill, North Carolina
North Texas State College, Denton, Texas
Northwestern University, Evanston, Illinois
University of Notre Dame, Notre Dame, Indiana
Oberlin College, Oberlin, Ohio
Ohio State University, Columbus 10, Ohio
Ohio Wesleyan University, Delaware, Ohio
Oklahoma State University, Stillwater, Oklahoma
University of Oklahoma, Norman, Oklahoma
Oregon State College, Corvallis, Oregon
University of Oregon, Eugene, Oregon
Pennsylvania State University, University Park, Pennsylvania
University of Pennsylvania, Philadelphia 4, Pennsylvania
University of Pittsburgh, Pittsburgh 13, Pennsylvania
Pomona College, Claremont, California
Portland State College, Portland, Oregon
Princeton University, Princeton, New Jersey
Purdue University, Lafayette, Indiana
Queens College, Flushing 67, New York
Randolph-Macon Woman's College, Lynchburg, Virginia
Reed College, Portland, Oregon
University of Rhode Island, Kingston, Rhode Island
Rice Institute, Houston 1, Texas
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BONN UNIVERSITY

The following items may be ordered from: Department of Mathematics, Bonn University, Wegelerstrasse 10, Bonn, Germany.

H. I. LEVINE, Singularities of Differentiable Mappings I, 69 pp., (in connection with a lecture by R. Thom) DM 3.00

R. BOTT, Lectures on Morse theory and its application to homotopy theory, (Notes by A. J. van de Ven) in preparation

THE UNIVERSITY OF BRITISH COLUMBIA

The following item may be ordered from: Department of Mathematics, University of British Columbia.

MARVIN MARCUS, Lectures on tensor and Grassmann products, 88 pp. $ 1.00

BROWN UNIVERSITY

The following items may be ordered from: Brown University Press, Providence 12, Rhode Island.

WILLIAM PRAGER, Theory of Structures, 66 pp. $ 3.00

HANS REISSNER, Hydrodynamical theory of propellers, 52 pp. $ 1.50

CALIFORNIA INSTITUTE OF TECHNOLOGY

The following items may be ordered from: Caltech Campus Bookstore, California Institute of Technology, Pasadena, California.

M. ALTMAN, Approximation methods in functional analysis, 130 pp. (Not available until February 1, 1960) $1.25


M. HALL, JR., Projective planes and related topics, 77 pp. $1.25
UNIVERSITY OF CHICAGO

The following items may be ordered from: Mathematics Lecture Notes, Eckhart Hall, University of Chicago, Chicago 37, Illinois. Please send payment in advance. Make checks payable to: Mathematics Lecture Notes.

YASUO AKIZUKI, Local Rings, 191 pp. $2.00
SAMUEL EILENBERG, Foundations of Fibre Bundles, 75 pp. 1.25
LARS J. GARDING, Cauchy's Problem for Hyperbolic Equations, 106 pp. 1.75
L. M. GRAVES, The Estimations of Schauder, and their Application to Existence Theorems for Elliptic Differential Equations, 67 pp. 1.00
PAUL R. HALMOS, Boolean Algebras, 175 pp. 3.00
Entropy in Ergodic Theory, 40 pp. .85
IRVING KAPLANSKY, Rings of Operators, 53 pp. 1.00
G. W. MACKEY, Theory of Group Representations, 182 pp. 3.00
Reports, Seminar in Topology, 120 pp. 2.00
ANDRE WEIL, Fibre Spaces in Algebraic Geometry, 48 pp. 1.00
Discontinuous Subgroups of Classical Groups - Notes by A. H. Wallace, 91 pp. 1.50

HARVARD UNIVERSITY

The following items may be ordered from: Department of Mathematics, Harvard University, 2 Divinity Avenue, Cambridge 38, Massachusetts.

P. C. ROSENBLOOM, Linear Partial Differential Equations, 237 pp. $2.50
O. ZARISKI, An Introduction to the Theory of Algebraic Surfaces, 100 pp. 2.00

The following item may be ordered from: Department of Mathematics, c/o Mrs. Laura Schlesinger, Harvard University, 2 Divinity Avenue, Cambridge 38, Massachusetts.

GARRETT BIRKHOFF and GIAN-CARLO ROTA, Ordinary Differential Equations, 155 pp. $2.00
STATE UNIVERSITY OF IOWA

The following item may be ordered from the: Campus Stores, State University of Iowa, Iowa City, Iowa.

S. BERBERIAN, A First Course in Hilbert Space, 87 + iii pp. (Bound) $1.64

THE JOHNS HOPKINS UNIVERSITY

The following item may be ordered from: The Johns Hopkins University Bookstore, The Johns Hopkins University, Baltimore 18, Maryland.

C. L. SIEGEL, Celestial Mechanics, 237 pp. $3.50

LEBANON VALLEY COLLEGE

The following items may be ordered from: Mathematics Department, Lebanon Valley College, Annville, Pennsylvania.

B. H. BISSINGER, and W. C. FITCH, Seminar in Group Physical Mortality, Part I, Techniques presently recognized by Commissions in light of modern statistical concepts, 210 pp. $4.00

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B. H. BISSINGER, Introduction to Operations Research, 151 pp. (Ready February 1960) $3.00

Logic, Sets, and the Number System - a first year graduate course for secondary teachers of mathematics, 175 pp. (Ready February 1960) $3.50

UNIVERSITÉ DE MONTRÉAL

The following item may be ordered from: Department of Mathematics, University of Montreal, P. O. Box 6128, Montreal, P. Q., Canada.

P. DEDECKER, Théorie des Structures, 50 pp. $1.00
The following item may be ordered from: Department of Mathematics, University of New Mexico, Albuquerque, New Mexico.

I. I. KOLODNER, Lectures on Principles of Applied Mathematics, 526 + 23 pp. $5.50
Notes contain a review of information student is supposed to know followed by chapters on linear algebra, ordinary differential equations, and integral equations.

NEW YORK UNIVERSITY

The following items may be ordered from: Institute of Mathematical Sciences, New York University, 25 Waverly Place, New York 3, New York. Please add $0.15 for first title and $0.10 for each additional title for postage within the United States. Do not enclose check when ordering from foreign countries (including Canada). Pro forma invoices will be sent for these orders.

E. ARTIN, Elements of Algebraic Geometry, 1955, 142 pp. $2.50
Modern Higher Algebra (Galois Theory), 1947, 198 pp. 3.25

L. BERS, Riemann Surfaces, 1957-1958, 259 pp. 4.00
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R. COURANT, Calculus of Variations, revision by J. Moser, 280 pp. 4.00
Theory of Functions of a Complex Variable, 226 pp. 3.50

K. O. FRIEDRICH, Advanced Ordinary Differential Equations, 281 pp. 3.25
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K. O. FRIEDRICH, SHAPIRO and others, Integration of Functionals, 213 pp. 3.50

E. ISAACSON, Introduction to Applied Mathematics and Numerical Methods, 1957, 233 pp. 4.00

F. JOHN, Advanced Numerical Analysis, 1957, 194 pp. 3.00
F. JOHN, Partial Differential Equations, 1953, 211 pp. $3.50

P. LAX, Partial Differential Equations (with Appendix by A. Douglis), 1952, 276 pp. 4.00
   Theory of Functions of a Real Variable, 1958-1959, 154 pp. 3.00

W. MAGNUS, Discrete Groups, 1953, 116 pp. 2.25

A. PETERS, Linear Algebra, 1948, 154 pp. 2.50

V. RAYUDU, (Venkatarayudu) Applications of Group Theory to Physical Problems, 1954, 208 pp. 3.50

F. RELLICH, Perturbation Theory of Eigenvalue Problems, 164 pp. 2.75

J. SCHWARTZ, Relativity in Pictures, 1958, 83 pp. 2.00
   Statistical Mechanics, 1957-1958, 103 pp. 1.50

SEMINAR: Recent Developments in the Theory of Wave Propagation, 1950, 219 pp. 2.50

J. J. STOKER, Differential Geometry, 1948, 157 pp. 2.50
   Introduction to the Geometry of Point Sets, 114 pp. 2.25

UNIVERSITY OF NORTH CAROLINA

The following items may be ordered from: Book Exchange, University of North Carolina, Chapel Hill, North Carolina.

J. S. MacNERNEY, Introduction to Analytic Functions $3.00
   (Self contained, no proofs; zerox copy, pressboard binding.) 100 pp.

J. W. CARR, III, Computer Programming and Artificial Intelligence (an intensive course for practicing scientists and engineers) Summer Session, University of Michigan, 518 pp. 5.87

IT MANUAL - Automatic Programming, using the IT Compilers for the Univac 1105 and IBM 650, 90 pp. Edited by Mrs. Sylvia M. Hubbard 1.00
OKLAHOMA STATE UNIVERSITY

The following item may be ordered from: Department of Mathematics, Oklahoma State University, Stillwater, Oklahoma.

Proceedings of the Symposium on Spectral Theory and Differential Problems (1950)  $3.50
(Prepaid if payment accompanies order)

QUEEN'S UNIVERSITY

The following item may be ordered from Professor I. Halperin, Queen's University, Kingston, Ontario, Canada.

I. HALPERIN, Lectures on Operator Theory (Elementary)  $1.00
65 pp.
Work-book type of exposition of spectral theorem for bounded Hermitian operators, for beginning graduate students.
560-64. **R. V. Chacon and D. S. Ornstein**: A general ergodic theorem.

We prove the following theorem which was conjectured by E. Hopf:

**Theorem.** Let \((S, \Sigma, \mu)\) be a positive measure space and let \(T\) be a linear operator on \(L_1(S, \Sigma, \mu)\) which is positive and whose \(L_1\) norm is less than or equal to one. Then if \(f\) and \(p\) are two functions in \(L_p\) and \(p \geq 0\), the limit \(\lim_{n \to \infty} \left( \sum_{k=0}^{n-1} T^k f / \sum_{k=0}^{n-1} T^k p \right)\) exists and is finite almost everywhere on the set \(A = \{s: T^n p > 0\text{ for some } n \geq 0\}\). (Received September 24, 1959.)

560-65. **J. de Groot**: Every continuous mapping is linear.

1. If \(\phi: M \to M\) is a continuous map of a metrizable \(M\) into itself, \(\phi\) is topologically linear, i.e. \(M\) can be embedded topologically in some real Hilbert space \(H\), such that the induced \(\phi\) equals a bounded linear operator \(\Phi\) of \(H\) onto itself, restricted to the embedded \(M\). \(\Phi\) may be chosen universally, i.e. independent of \(\phi\). 2. If \(\phi\) is a retraction, \(\Phi\) may be chosen as a projection of \(H\) into itself. 3. If \(\phi\) is an autohomeomorphism, \(\Phi\) may be chosen (universally) as a bounded linear autohomeomorphism of \(H\). 4. Analogous results hold if a locally compact group \(G\) of autohomeomorphisms with a countable base acts on \(M\). If \(G\) has no countable base, the corresponding statement is false, in general. If \(G\) is compact, \(M\) can be metrized in such a way that it spans up some real Hilbert space \(H\) over which \(G\) can be extended, such that \(G\) acts as a compact group of unitary linear homeomorphisms on \(H\). 5. If \(G\) is some set of homeomorphisms of a completely regular space, all homeomorphisms can be extended (topologically) to linear homeomorphisms of a real linear space. (Received September 8, 1959.)


The Green's function \(G(x; t)\) for \(L(D)y = \sum_{k=0}^{n} p_k(x)y^{(n-k)}(x) = 0\) and \(U_1(y) \equiv \sum_{j=1}^{n} a_{i,j-1} y^{(j-1)}(0) + b_{i,j-1} y^{(j-1)}(1)\) \((i \leq 1, 2, \ldots, n)\), is given by \(G(x; t) \equiv Z(x; t) / \Delta\) (B. Kamke, *Differentialgleichungen*, Leipzig, 1956), where \(\Delta \equiv \det U_1(y_j) \neq 0\), the \(y_j(x)\) are a fundamental set (of solutions) of \(L(D)y = 0\),
and \( g(x;t) \) is a fundamental solution of \( L(D)y = 0 \). Two fundamental sets \( y_j(x) \), \( y^j(x) \) are called complementary if \( \gamma_{ij} \) are constants such that \( \det \gamma_{ij} = 1 \), and \( \gamma_{ij} = \sum_{j=1}^{n} \gamma_{ij} y_j(x) \). Either set leads to the same \( \Delta \) and also to the same \( Z(x;t) \) independently of the \( g(x;t) \) used, since \( G(x;t) \) is unique. Each of the \( C_{2n,n} \) determinant minors of the matrix of coefficients in the \( U_j(y) \) appears as a coefficient of one term in the expansion of \( Z(x;t) \) (by minors of its first row) and one in the expansion of \( \Delta \), after collecting terms. Each such term is obtainable from either complementary set. If \( y_{(i-1)|}^{(i-1)}(0) = \delta_j, \) and \( W_b(x) \) is the Wronskian of \( y_{b|}(x) \), then \( (W_b(0))^{1/n} y_{aj}(x) \) and \( y_{aj}(x) \) are complementary. If the \( n \times n \) minor has more "a" columns than "b" columns, then one uses \( (W_b(0))^{1/n} y_{aj}(x) \) and \( g \) given by \( g = 0 \) when \( x < t \), \( g = y(x;t) \) when \( x > t \), where the solution \( y(x;t) \) satisfies \( (\partial y/\partial x)^{k-1} x = t \) \((k = 1, \ldots, n - 1)\), \( (\partial y/\partial x)^{n-1} x = t \) \( = 1/p(t); \) otherwise one uses \( y_{b|}(x) \) and \( g \) given by \( g = -y(x;t) \) when \( x < t \), \( g = 0 \) when \( x > t \). (Received September 16, 1959.)


Given a measure-preserving flow on a measure space \( X \) there are induced probability semi-groups on the Lebesgue spaces \( L^p \) with respect to the invariant measure. Let \( P\{t\}, t > 0 \) denote the semi-group and set \( g(x,t) = P(t)f(x) \) for \( f \in L^p \). Let \( \mathcal{O}(x) = \sup_{0 < t < \infty} \int_0^t |g(x,\tau)| d\tau \). The maximal ergodic theorem states that for \( 1 < p < \infty \), \( \|\mathcal{O}\|_p \leq A_p \|f\|_p \). Now suppose \( \{Q(s)\}, s > 0 \), is another semi-group formally related to \( P(t) \) by \( Q(s) = \int_0^s P(t)dt \). Set \( h(x,s) = Q(s)f(x) \) for \( f \in L^p \) and \( f^*(x) = \sup_{S > 0} |h(x,s)| \). If \( t\phi(s,t) \) is of bounded variation in \( t \) uniformly in \( S \) then it follows immediately that \( \|f^*\|_p \leq B_p \|f\|_p \) for \( 1 < p < \infty \), which is the Hardy-Littlewood Maximal Theorem. As an application of this remark, consider the Hardy-Littlewood theorem for functions harmonic in the unit ball in \( n \)-dimensions. The Laplacian may be written as \( \Delta = -r^{-n} \partial/r \partial r (r^2 \partial/r \partial r) + r^{-2} \Lambda \) where \( \Lambda \) is the Beltramic operator on the unit sphere. (Thus \( P(t) = \exp(-t\Lambda) \) is the semi-group associated with Brownian motion on the sphere.) If \( U(x,r) \) is harmonic in \( r < 1 \), i.e. \( \Delta U = 0 \), we have \( r\partial U/\partial r = [(\Lambda + k^2)^{1/2} - k] U \) with \( 2k + 1 = n \). Define \( \phi(s,t) \) by \( \exp\{-s[(\Lambda + k^2)^{1/2} - k]\} = \int_0^\infty \exp(-t\lambda)\phi(s,t)dt \). Then \( U(x,r) = h(x, -\log r) \) as above. A simple computation yields \( \phi(s,t) \) explicitly whereupon one easily establishes that \( t\phi(s,t) \) is of uniformly bounded variation. (Received September 21, 1959.)
A formalism for representing functions of symbolic expressions is described and the functions so representable are proved to be the computable partial functions. Because the representations are symbolic expressions themselves, the self-application of the system is particularly simple. Moreover, formalism has been made to serve as the basis of the LISP programming system for the IBM 704 electronic computer. Some examples of symbolic calculations are given. (Received September 18, 1959.)

Let \( a, b, c, u, r, s, w \) and \( z \) be linear operators on a Hilbert space \( H \), let \( r, s, a, b, l - wc, 1 - cw \) be nonsingular, let \( \| b(1 - wc)^{-1}w a \| < 1 \), and let \( \xi = u + rz(1 - wz)^{-1}s \). Then there are operators \( r_0, s_0, u_0 \), with \( r_0 \) and \( s_0 \) nonsingular, such that the circles \( \| a^{-1}(z - c)b^{-1} \| < 1, \| a^{-1}(z - c)b^{-1} \| = 1, \) \( a^{-1}(z - c)b^{-1} \text{ unitary} \), are mapped respectively onto the circles \( \| r_0^{-1}(\xi - u_0)s_0^{-1} \| < 1, \| r_0^{-1}(\xi - u_0)s_0^{-1} \| = 1, r_0^{-1}(\xi - u_0)s_0^{-1} \text{ unitary} \). If "circle" is defined as image of \( \| z \| < 1, \| z \| = 1, z \text{ unitary} \) under a function of form \( f(z) =AzB + C \), some of the conditions concerning nonsingularity are dropped in hypothesis and conclusion. (Received September 16, 1959.)

Let \( u, r, s, w \) and \( z \) be linear operators defined on a Hilbert space \( H \). We say \( (ursw) \) has property \( P \) if \( f(z) = u + rz(1 - wz)^{-1}s \) satisfies \( \| f(z) \| \leq 1 \) for all \( z \) such that \( \| z \| \leq 1 \) and such that \( (1 - wz)^{-1} \) exists. If also \( f(z) \) is unitary for all such unitary \( z \), \( (ursw) \) has property \( U \). The quadruple has property \( P_C \) if \( c \) is a positive scalar and the matrix with rows \( (cr, u) \) and \( (ws, u) \) has norm \( \leq 1 \) on \( H \times H \). It has property \( U_C \) if this matrix is unitary. \textbf{Theorems:}

Let \( r \neq 0, s \neq 0, \| u \| \leq 1, \| w \| \leq 1 \). Then \( (ursw) \) has property \( P \) if and only if \( (wsru) \) does. Property \( P_C \) implies property \( P \) always, and property \( P \) implies property \( P_C \) for some \( c \) if \( \| w \| \leq 1 \) and \( u, r, s, w \) are completely continuous. Property \( U \) is equivalent to "property \( U_C \) for some \( c \)" if \( \| w \| < 1 \). (As an application these results give the relation between passivity and stability for multiple transmission lines. The proof uses the fact that "property \( P \) when \( \| w \| \leq 1 \) and \( s \neq 0 \)" is equivalent to "\( |V_2| \leq |wV_2 + sV_1| \) implies \( |V_1| \leq |rV_2 + uV_1| \) for all \( V_i \) in \( H \)" together with a theorem of Hestenes, Reid, et al, on quadratic forms.) (Received September 16, 1959.)
560-71. John Wermer: **Dirichlet algebras.**

Let $X$ be a compact space, $C(X)$ the algebra of all continuous complex-valued functions on $X$ and let $A$ be a closed proper subalgebra of $C(X)$ with $1$ in $A$. Following Gleason we call $A$ a Dirichlet Algebra if the real parts of functions in $A$ are dense in the real parts of functions in $C(X)$. Let $M$ be the maximal ideal space of $A$. For $x, y$ in $M$ put $x \sim y$ if for some $k < 2$, $|f(x) - f(y)| < k$ for all $f$ in $A$ of norm $\leq 1$. Gleason showed that $\sim$ is an equivalence relation on $M$. **Theorem:** Let $A$ be a Dirichlet algebra and let $P$ be an equivalence class of $M$ under $\sim$. Then either $P$ consists of a single point or there exists a map $\phi$ of the open unit disk $|z| < 1$ onto $P$ such that $\phi$ is continuous and one-one and $f(\phi(z))$ is analytic in $|z| < 1$ for each $f$ in $A$. (Received October 2, 1959.)

560-72. R. S. Varga: **Continued fractions and stable implicit methods for parabolic difference equations.**

For various parabolic differential equations, such as $\partial u/\partial t = \partial^2 u/\partial x^2$, it can be shown that by discretizing only the space variables, one obtains the following ordinary matrix differential equation: (1) $\frac{d\mathbf{u}(t)}{dt} = -A \mathbf{u}(t) + \mathbf{g}$, where $\mathbf{u}(t)$ is a vector with $N$ components, and $A$ is a real $N \times N$ matrix all of whose eigenvalues have their real parts positive. The vector $\mathbf{g}$ is a constant vector, and the vector $\mathbf{u}(0)$ is a given vector initial condition. The solution of (1) is (2) $\mathbf{u}(t) = A^{-1} \mathbf{g} + \exp(-tA)\{\mathbf{u}(0) - A^{-1} \mathbf{g}\}$, $t > 0$. The well-known Crank-Nicolson implicit method which takes discrete time steps forward is shown to correspond to the following approximation (3) $\exp(-tA) \approx (I + (tA/2))(I - (tA/2))$. If one considers the function $f(z) = e^z$, then (3) corresponds to the rational approximation $f_{1,1}(z)$ of the Padé table for $e^z$. It is shown that the rational approximations $f_{p,q}(z)$ of the Padé table for $e^z$ give rise to matrix approximations as (3), which correspond to stable implicit difference methods for $p \geq q$. Finally, it is shown for the example $p = q = 2$, that the stable implicit method corresponding to the approximation (4) $\exp(-tA) \approx (1 + 6tA + (tA)^2)^{-1}(1 - 6tA + (tA)^2)$ can be efficiently carried out in the case of one space variable. (Received September 28, 1959.)

Let $P_n(x)$ be the $n$-th Legendre polynomial and let $L_n(x)$ be $P_n(x)$ or $x^{-1}P_n(x)$ for even or odd $n$, respectively. Nearly fifty years ago the irreducibility of $L_n(x)$ was proved for certain values of $n$. Later other values of $n$ were added, but still the problem is not completely solved. Let $p$ be a prime. J. B. Holt in two papers [Proc. London Math. Soc. vol. 11 (1912) pp. 351-356; vol. 12 (1913) pp. 126-132] proved the irreducibility for certain values of $n$, for instance, for $n = p, p + 1$, and $2p - 1$. It followed that $L_n(x)$ is irreducible for all $n < 200$ except possibly 122, 185, and 186. Other classes of irreducibility were obtained by H. Ilie [Jahrbuch der Dissertationen der Universität Berlin, 1924] and J. H. Wahab [Duke Math. J. vol. 19 (1952) pp. 165-176]. From these results it followed that $L_n(x)$ is irreducible for all $n < 500$ except possibly nine values, among them 326. In this paper new classes of irreducibility are obtained, and it follows, for instance, that $L_{326}(x)$ is irreducible. (Received October 13, 1959).

Let G be an Abelian torsion group. Call a subgroup B of G basic in G if every primary component of B is basic (in the sense of Kulikov) in the corresponding primary component of G. The following is obtained: Theorem: Let G be an Abelian torsion group. Then the following statements are equivalent: (I) G has a minimal system of generators. (II) G has the same power as a basic subgroup of itself. (III) G has a direct summand which is a direct sum of cyclic groups and has the same power as G. In his book Abelian groups, L. Fouchs asks: which are the cardinals m such that there exist no m-indecomposable reduced primary groups of power m. As an application of the equivalence of II and III of the above theorem we obtain: Theorem: There exist no infinite m-indecomposable reduced primary groups of power m if and only if m > ℵ₀ and for every cardinal n < m we have n^ℵ₀ < m. (Received September 21, 1959.)

562-14. V. L. Klee, Jr.: Polyhedral sections of convex bodies.

A finite-dimensional convex body K is said to be a-universal (resp. s-universal) for a family ℋ of convex bodies provided every member of ℋ is affinely equivalent (resp. similar) to some proper section of K. And K is centrally a-universal for ℋ provided K is centered and every centered member of ℋ is affinely equivalent to some central section of K. For 2 ≤ n ≤ r, $\xi_{a,f}(n,r)$ (resp. $\xi_{s,f}(n,r)$) is defined to be the smallest integer k such that some k-dimensional convex body is a-universal (resp. s-universal) for all n-dimensional convex polyhedra having r + 1 maximal faces. And $\gamma_{a,f}(n,r)$ is the smallest k such that some k-dimensional convex body is centrally a-universal for all n-dimensional centered convex polyhedra having 2r maximal faces. It is proved that $n(r + 1)/(n + 1) \leq \xi_{a,f}(n,r) \leq r$, $n(r + 2)/(n + 1) \leq \xi_{s,f}(n,r) < \infty$, and $\gamma_{a,f}(n,r) = r$. Sharper results are obtained for special values of n and r, and four conjectures of Melzak are validated (Canad. Bull. Math. vol. 2 (1959) pp. 31-32). The results are obtained by studying certain spaces of convex bodies.
under the Hausdorff metric, some spaces of equivalence-classes under Macbeath's metric (Canad. J. Math. vol. 3 (1951) pp. 54-61), and certain transformations involving these spaces. The principal task is to determine the Hausdorff dimensions of the spaces, prove that the transformations are Lipschitzian, and utilize the known behavior of Hausdorff measure under Lipschitzian transformations. (Received September 25, 1959.)

562-15. Jerzy Łoś: A theorem on the subgroups of a complete direct sum of abelian groups.

Let $S^* = \sum_{t \in T} G_t$ be a complete direct sum where $G_t = G$ for $t \in T$ and $G$ is an abelian group of the power $|G| = \mathcal{M}$. We say that a subgroup $H$ of $S^*$ lies in the axes of $T_0$ in the sum $\sum_{t \in T} G_t$, if $T_0$ is a subset of $T$ and for every $t \in T - T_0$ and every $x \in H$, we have $x_t = 0$. Theorem. Suppose $H \subseteq S^*$ be a subgroup of $S^*$ of the power $|H| = \mathcal{M}$. There exists an automorphism $\alpha$ of $S^*$ such that in the complete direct sum $\alpha S^* = \sum_{t \in T} \alpha G_t$, the subgroup $H$ in the axes of a set $T_0$ of power $|T_0| \leq \mathcal{M}^{\mathcal{M}}$. As an example it may be shown that the estimation $|T_0| \leq \mathcal{M}^{\mathcal{M}}$ can not be improved. (Received October 26, 1959.)


Quasi-equ-i-nfinitesimal analysis (in re Equi-infinitesimal analysis, see abstracts of Meeting 561) combined with previous work in the geometry of the complex domain (lecture on The imaginary as a dimensionality operator, 1952, application-C file, U. S. Copyright Office and Library of Congress, Washington, D.C.) yielded the following applied results in the field of the physical theory of fundamental particles. Proton and electron appear to be respectively vertex-seeking and base-seeking conical vortical structures, the former much smaller and turning clockwise as viewed from base; the latter larger and with far less compactness and angular momentum, turning clockwise viewed from base. For anti-proton and anti-electron (positron) reverse the sense of rotation. The gross structure of the neutron then emerges as a pair of vertical-conical vortices, the two members being of different sizes, and one interpenetrating the other to some degree. The exact shape of the vortex profile is, from hydrodynamic considerations, probably more that of a horn-angular solid of revolution than a strictly conical form. Charge and Coulomb forces then relate to
angular momentum, and magnetic, to Coriolis-type effects. P and £ motions would be achieved via a jet effect. Gravitation appears to be related to the massed effect of the electron jet streams. (Received October 2, 1959.)

562-17. I. E. Segal: Generalized Heisenberg-Schroedinger systems and one-dimensional cohomology.

In reformulating the Heisenberg commutation relations and the Schroedinger representation thereof so as to be explicitly invariant under general coordinate transformations, and eventually so as to apply to an arbitrary orientable $C^\infty$ manifold $M$, it is found that although there is a unique situation locally, there may be a number of inequivalent such quantum-mechanical systems based on $M$ on the large. In purely mathematical terms any such system may be specified by a unitary representation of the group of all diffeomorphisms of $M$, in the space $L_2(M)$ of all square-integrable functions on $M$ relative to an arbitrary positive form of maximal dimension, which has the formal property that $U(a)f(a)^{-1} = f(a)$, where $f(a)$ is the operation of multiplication by the general bounded $C^\infty$ function $f$, and $f(a)(x) = f(a^{-1}(x))$, and the regularity property that $U(exp(tX))g(x)$ is a $C^\infty$ function of $(t,x)$, where $X$ is any generator of a one-parameter group of diffeomorphisms, and $g$ is any $C^\infty$ function in $L_2(M)$. It is shown that, within unitary equivalence, such representations are in canonical one-to-one correspondence with a substructure of the group of real closed one-forms on $M$ modulo the subgroup of those of the form $df/|f|$, where $f$ is a $C^\infty$ function of unit modulus on $M$. In particular, if $M$ is simply connected, there is uniqueness as in the euclidean case. (Received October 7, 1959.)


Let $X$ and $X'$ be Hausdorff spaces, $K$ and $K'$ the families of compact sets in $X$ and $X'$ respectively. The Souslin sets in $X$ are those obtained by applying the operation $\mathcal{A}$ to $K$; the Borelian sets in $X$ are those belonging to the smallest family containing $K$ and closed under countable unions and intersections. We have the following results: Two disjoint Souslin sets in $X$ can be separated by Borelian sets. If $X$ is a $K_\sigma$ then the continuous image of a $K'_\sigma\delta$ is Souslin in $X$. If $X$ is a $K_\sigma$ and the difference of two sets in $K'$ is a $K'_\sigma$ then the continuous, countable-to-one image of a $K'_\sigma\delta$ is Borelian in $X$. (Received September 29, 1959.)
This paper develops results analogous to the classical Picard Theorem for nonisolated singularities of functions meromorphic in the unit disk. Let $f(z)$ be meromorphic in $|z| < 1$. Let $C(f,P)$ and $C_\rho(f,P)$ be the cluster set and radial cluster set, respectively, of $f(z)$ at a point $P = e^{i\theta_0}$ on $|z| = 1$. Let $E$ be a set of capacity zero on $|z| = 1$. The radial boundary cluster set modulo $E$, $C_{R-E}(f,P)$, of $f(z)$ at $P = e^{i\theta_0}$ is defined as the intersection over all $\eta > 0$ of the closures of the unions $\bigcup C_\rho(f,e^{i\theta})$ for $0 < |\theta - \theta_0| < \eta$, $e^{i\theta} \notin E$. It is shown that in every neighborhood of $P$, $f(z)$ assumes all values save at most 2 in each component of $C(f,P) - C_{R-E}(f,P)$. Any such omitted values are asymptotic values of $f(z)$ at $P$ or at a sequence of boundary points having $P$ as limit point. (Received August 21, 1959.)


A plane domain $D$ will be called rigid if every analytic map of $D$ into itself is either one-to-one onto or else takes some nonzero homology class of curves into the zero homology class. Several criteria for rigidity are derived, including the following purely geometric one. The domain $D$ will be rigid if it satisfies the following conditions: (a) every boundary point of $D$ is contained in a non-degenerate boundary continuum; (b) whenever a boundary point $q$ is the limit of points on a sequence of disjoint boundary continua $b_n$, then the diameter $d_n$ of $b_n$ and the distance $r_n$ from $b_n$ to $q$ satisfy the relation $r_n/d_n \to 0$, where all distances are measured on the Riemann sphere. A similar result was proved by H. Huber for domains satisfying condition (a) and a stronger form of (b) which requires that $d_n \geq d > 0$. (Received November 16, 1959.)
563-18. I. L. Glicksberg: Some special transformation groups.

Let $H$ be a compact connected abelian group, $G$ an equicontinuous group of self-homeomorphisms of $H$ containing all translations. Then each $g$ in $G$ is of the form $h \mapsto h \sigma_g(h)$, where $\sigma_g$ is an automorphism of $H$. Further, if $H$ is finite dimensional, there is an integer $k$ for which $\sigma_g^k(h) = h$, all $g, h$. This result follows from standard vector valued integration, the existence of approximate identities consisting of trigonometric polynomials, and one conceivably new (but trivial) fact: on a compact connected abelian group a unimodular trigonometric polynomial is a monomial. As a special case, if $H$ is also metric, one obtains the form of all isometries of $H$ relative to any invariant metric.

(Received October 6, 1959.)


Let $G, H$ be locally compact abelian groups, with duals $G^*, H^*$. In 1952 Kaplansky proved the following result using structure (unpublished): (1) If $\tau: G \to H$ is an algebraic homomorphism with a dual $\tau^*: H^* \to G^*$ (so that $\langle \tau g, h^* \rangle = \langle g, \tau^* h^* \rangle$, all $g, h^*$) then $\tau$ is continuous. Here (1) is obtained from (2): If $K \subset G$ is compact in the topology of pointwise convergence on $G^*$, $K$ is compact. (1) bears a striking similarity to a well known fact about Banach spaces, a consequence of uniform boundedness; adopting compactness as the appropriate notion of boundedness for groups, (2) is the exact analogue of uniform boundedness for groups, since boundedness in the topology of pointwise convergence on the dual implies boundedness in the topology of uniform convergence on bounded subsets of the dual. Some consequences: (3) A measurable homomorphism of $G$ into $H$ is continuous; (4) Let $\text{Hom}(G, H)$ be the space of all homomorphisms of $G$ into $H$ in the compact-open topology. Then $T \subset \text{Hom}(G, H)$ has compact closure iff $Tg$ and $T^* h^*$ have, all $g \in G$, $h^* \in H^*$, where $T^*$ is the set of dual maps. The (nonstructural) proof of (2) is based on a result of Grothendieck [Amer. J. Math. vol. 74 (1952) pp. 168-182, Theorem 5].

(Received October 6, 1959.)

For $f(n)$ a real function defined for $n = 0, 1, \ldots$. $V[f]$ denotes the number of changes of sign of $f$. Let $\&$ denote the convolution operation for functions defined on $0, 1, \ldots$ associated with the ultraspherical polynomials of index $\nu$, as given for example by the author in the Cornell Symposium on Harmonic analysis and related integral transforms (1956). Let $W_\nu(n, x)$ be the ultraspherical polynomials of index $\nu$ normalized by the condition $W_\nu(n, 1) = 1$, and let $\omega_\nu(x) = a_\nu(1 - x^2)^{\nu - 1/2}dx$ where $a_\nu$ is a constant depending on $\nu$. Suppose that

$$
\int E(x)W_\nu(n, x)dx = H(n) \text{ where } E(x) = \exp Cx - \prod (1 + xA_k^{-1})^{-1} \prod (1 - xB_k^{-1})^{-1};
$$

where $C \geq 0, 1 \leq A_k, 1 < B_k, \sum_k A_k^{-1} + B_k^{-1} < \infty$. It is then proved that $H$ is variation diminishing in the sense that $V[H \& f] \leq V[f]$ for every bounded $f$. Presumably this condition is necessary as well as sufficient. (Received by October 7, 1959.)

563-21. J. M. Kister: Isotopies in 3-manifolds with boundaries. II.

(1) If $L$ is a compact tame 2-manifold in a 3-manifold $M$ and if $\varepsilon > 0$, there is a $\delta > 0$ so that if $h$ is any homeomorphism of $L$ into $M$ moving no point more than $\delta$ and $h(L)$ is tame there will be an extension of $h$ to $h^*$ in $H(M)$ and $h^*$ is $\varepsilon$-isotopic to the identity. (2) If $M$ is a 3-manifold having triangulation $\Sigma$ and $\varepsilon > 0$ there is a $\delta > 0$ so that if $h$ is a homeomorphism of the 2-skeleton $K$ of $\Sigma$ into $M$ moving no point more than $\delta$ (the metric is the barycentric one) and such that $h(K)$ is tame, then there is an extension of $h$ to $h^*$ in $H(M)$ and $h^*$ is $\varepsilon$-isotopic to the identity. (3) If $M$ is a compact 3-manifold with boundary then $h \in H(M)$ is isotopic to the identity if and only if $h = h_1 \cdots h_{k-1} h_k^* h_{k-1} \cdots h_1$ where each $h_i$ is the identity outside a polyhedral 3-cell. (Received October 1, 1959.)


A new realm of the figures formed by finite numbers of equal infinitesimals in various dimensions (D) is introduced. The infinitesimal units chosen for reasons both of convenience and generality are spheres of equal radii, including hyper- and hypo-spheres, depending on the manifold. In this realm the circle can be squared, but in just one way, producing the area of $169 = 13^2$, also a central hexagonal number, an infinitesimal disc of diameter 15 in the domain. Similarly, in this domain the sphere can be cubed, also uniquely, as $3^3 = 27$. 764
which is also a sphere of volume 27 and diameter 5 in the 3-D infinitesimal realm. It is also demonstrated that in the 3-D infinitesimal realm only 7 different cores, consisting respectively of 1 through 7 units, for any centrally symmetric structure are possible, a result immediately enlargeable to spherical geometry. These 7 cores can generate 7 kinds of sub-spaces (space lattices), 3 of which (1-, 2-, and 6-core) have strong inter-affiliations. 1-core in 2-D relates to the Fermat primes; 5-core in 3-D, to the Mersenne primes. The 5-core development of forms demonstrates the unique and striking fact of a developmentally necessary discontinuity, furnishing usable biomorphogenic correspondences (archemorphology). Usual length, area, and volume formulas are specifically altered in the equi-infinitesimal realm. (Received October 2, 1959.)

563-23. Dov Tamari: Nonassociative systems satisfying the Malcev conditions.

Let \( M \) be a (quasi-ordered, partial) multiplicative system \((m, xs.)\) with an identity or certain weaker conditions on the existence of binary factorizations. The Malcev conditions \((M)\) then imply cancellation and all associative \((asso.)\) laws \((A)\): to every \( A_n \in (A) \) for \((n + 1)\)-sequences there is a \( M_{2n} \in (M) \) such that \( M_{2n} \) implies \( A_n \) [Thesis, Paris, 1951; C. R. Acad. Sci Paris, vol. 232 (1951) p. 1332; Bull. Soc. Math. France, vol. 82 (1954) p. 53; P. Dubreil Algebre, 2nd ed., Paris, 1954, p. 268]. Effectively nonasso. m.s. satisfying \((M)\) are e.g., m.s. permitting at most one factorization of an el.; the only possible equations of form \( ax = by \) are even of form \( ax = ax \), i.e. identities; \((M)\) is satisfied because the hypotheses of the conditions can be satisfied only trivially, and then the corresponding conclusions are also trivially valid. But we can exhibit effectively nonasso. m.s. satisfying \((M)\) nontrivially: such are e.g. the quasi-groups \( Q_n \) \((n \geq 3)\), where multiplication is understood to be subtraction mod \( n \). The proof is similar to the proof of necessity of \((M)\) for the embeddability of m.s. into groups. (Received November 23, 1959.)

563-24. F. W. Carroll: Difference properties for continuity and Riemann integrability on topological groups.

Let \( G \) be a locally compact topological group. All functions considered are complex valued functions on \( G \). A function \( \Gamma(x) \) is additive if \( \Gamma(x + y) = \Gamma(x) + \Gamma(y) \) for all \( x, y \in G \). A function \( g(x) \) is Riemann integrable \((R.I.)\) on \( G \)
if, on each compact subset of $G$, $g(x)$ is bounded and continuous a.e. with respect to the Haar measure on $G$. **Theorem I:** Let $f(x)$ be a function on $G$. Suppose that, for each fixed $h$ in $G$, the functions $f(x + h) - f(x)$ and $f(h + x) - f(x)$ are continuous on $G$. If $G$ is compact and second countable, or if $G$ is abelian, then (*) $f(x) = g(x) + \Gamma(x)$, $\Gamma(x)$ additive, $g(x)$ continuous. **Theorem II:** Let $G$ be abelian and first countable, with $A$ an open subset of $G$. Let $f(x)$ be a function such that, for each $h$ in $G$, $f(x + h) - f(x)$ is continuous on $A \cap (A - h)$. Then (*) holds with $\Gamma(x)$ additive on $G$ and $g(x)$ continuous on $A$. **Theorem III:** If $G$ is abelian and second countable, and if $f(x)$ is a function such that, for each $h$ in $G$, $f(x + h) - f(x)$ is R.I. on $G$, then (*) holds with $\Gamma(x)$ additive and $g(x)$ R.I. For $G$ the reals and $G$ the reals (mod 1), these theorems are due to de Bruijn (Nieuw Archief voor Wiskunde (2) vol. 23 (1951) pp. 194-218), Nederl. Akad. Wetensch. Proc. Ser. A vol. 55 (1952) pp. 145-151). For $G$ the $n$-dimensional Euclidean space, Theorems I and II are due to Kemperman (Trans. Amer. Math. Soc. vol. 86 (1957) pp. 28-56). (Received October 8, 1959.)


Jacobson (Structures of rings, pp. 146-147) gives a sufficient condition (Nakayama) for a subring of a full ring of linear transformations to be Galois. A new sufficient condition is given here. The author extends the definition of homogeneous module to a class of noncompletely reducible modules and proves the **Theorem:** Let $R$ be a subring of $L_\Delta(M,M)$ ($M$ a finite dimensional vector space over the division ring $\Delta$, ($\Delta;\Gamma$) finite, $\Gamma$ the center of $\Delta$) such that $R$ contains $\Gamma$, $R$ is a principal ideal ring and $M$ is a homogeneous $R$-module, and let $\text{Hom}_R(M,M)$ be generated over $\Delta$ by $\Delta$-semi-linear transformations of $M$. Then $R$ is Galois in $L_\Delta(M,M)$. If $T$ is a linear transformation of $M$, $\Gamma[T]$ is a principal ideal subring of $L_\Delta(M,M)$ containing $\Gamma$. When the other hypotheses of the theorem hold, $\Gamma[T]$ is Galois in $L_\Delta(M,M)$. Several other results concerning the rings $R$ and $\Delta R$ (of special interest $\Gamma[T]$ and $\Delta[T]$) are obtained. (Received October 8, 1959.)

Let \( K = (K_1, \ldots, K_\mu) \) and \( L = (L_1, \ldots, L_\nu) \) be links, \( \mu, \nu \geq 2 \); set \( K^* = (K_1, \ldots, K_{\mu-1}) \) and \( L^* = (L_1, \ldots, L_{\nu-1}) \). Let \( U \) and \( V \) be closed tubular neighborhoods of \( K_\mu \) and \( L_\nu \) respectively, \( \partial U \) and \( \partial V \) their respective boundaries, and \( Q_K \) and \( Q_L \) the respective closures of \( S^3 - U \) and \( S^3 - V \). Form a compact 3-manifold \( Q_{KL} \) by "pasting together" \( Q_K \) and \( Q_L \) along their boundaries \( \partial U \) and \( \partial V \), in such a way that meridians of \( \partial U \) are identified with parallels of \( \partial V \), and parallels of \( \partial U \) are identified with meridians of \( \partial V \). If \( Q_{KL} \) is a 3-sphere, then \( KL \) is said to exist; in this case \( KL \) is defined to be the link \( K^* \cup L^* \), as imbedded in \( Q_{KL} \). Let \( t = (t_1, \ldots, t_{\mu-1}), s = (s_1, \ldots, s_{\nu-1}), t^k = t_1 \cdots t_{\mu-1}, \) and \( s^k = s_1 \cdots s_{\nu-1} \), where \( k_1 \) is the linking number of \( K_1 \) with \( K_{\mu-1} \), and \( k_1 \) is the linking number of \( L_1 \) with \( L_{\nu-1} \). Then \( \Delta_{KL}(t, s) = \Delta_K(t, t^k) \Delta_L(s, s^k) \), the \( \Delta \) being the appropriate Alexander polynomials. This generalizes previous results of Seifert and of Torres. If \( K_\mu \) is unknotted then \( KL \) is the link obtained by "tying \( K^* \) inside \( L^* \)"; and \( k_1 \) is the algebraic number of times that \( K_1 \) goes around the torus in which \( K^* \) is imbedded. (Received October 2, 1959.)


Let \( A \) be an algebra over a field of characteristic zero. A Reynolds operator in \( A \) is a linear operator of \( A \) into itself which satisfies the identity \( R(fg) = RfRg + R[(f - Rf)(g - Rg)] \) for \( f \) and \( g \) in \( A \). It is easily shown that an idempotent Reynolds operator satisfies the averaging identity \( R(fRg) = (Rf)(Rg) \). Counterexamples are known showing that not every Reynolds operator satisfies the averaging identity. A result of Molinaro (Alger Mathém., 1957, pp. 87-101) shows that if \( A \) is the algebra of all real functions on a finite set, and if \( R \) is an order-preserving Reynolds operator, then \( R \) satisfies the averaging identity. This result is now strengthened as follows: Theorem: Let \( A \) be the algebra of complex functions on a finite set, and let \( R \) be a Reynolds operator in \( A \). Then \( R \) satisfies the averaging identity. Thus every Reynolds operator is an averaging operator in the sense of Kelley (Pacific J. Math., 1958, pp. 214-223) and the
complete classification of Reynolds operators follows from Kelley's theory in the case of finite-dimensional function algebras. (Received October 15, 1959.)

564-3. Ward Bouwsma: The greatest common divisor property for exponential polynomials.

Exponential polynomials admit a unique factorization into irreducible and simple factors, as was shown by J. F. Ritt (Trans. Amer. Math. Soc., vol. 29 (1927) pp. 584-596). This factorization enables one to show that two exponential polynomials have a g.c.d. in the ring of exponential polynomials (which may not be a g.c.d. in the ring of entire functions). An exponential polynomial will be called simple in b if every exponent is an integer multiple of b. Hence it will also be simple in b/k, if k is an integer. Such a function can be considered as an ordinary polynomial in exp(bz/k). An enumeration of the factors of the corresponding ordinary polynomials for all k shows that all common factors of two given simple functions either belong to or are factors of elements belonging to a finite set of common factors; hence any two simple functions have a g.c.d. It then follows from the Ritt factorization that any two exponential polynomials have a g.c.d. (Received November 2, 1959.)


One considers in $E^n$ a Dirichlet problem with associated differential equation $\sum_{j=1}^{n} a_j(x_1,x_2,\ldots,x_n) \partial u/\partial x_j + b(x_1,x_2,\ldots,x_n) u = f(x_1,x_2,\ldots,x_n)$. It is assumed that (1) $b(x_1,x_2,\ldots,x_n)$ is continuous and nonpositive, and (2) each $a_j(x_1,x_2,\ldots,x_n)$, $j = 1,2,3,\ldots,n$ is itself either nonnegative or nonpositive. The analytical problem is reduced to a problem in finite differences by replacing the differential equation by the finite difference analogue $u(x_1,x_2,\ldots,x_n) = [1/(2B' - Hb_1)]\sum_{j=1}^{n} [u_{h_j} + B_j + (H_a(1)/2h_j)] + u_{h_j} - (H_a(1)/2h_j]) + Hf/(Hb_1 - 2B')$, and by using the method of Collatz at the boundary lattice points. The well known Gerschgorin results generalize readily and the error in the numerical solution is shown to be of order $O(h^2)$. (Received November 2, 1959.)


Let $\sigma(r,v)$ and $T(r,v,w)$ be given, nonnegative step functions in $r$, defined
for \( r_0 \leq r \leq R \leq \infty \), having salti at most at a finite set \( P = \{ r_j \}, r_0 < r_j < R \) \((j = 1, 2, \ldots, n)\). For \( r \notin P \), \( \sigma \) and \( \tau \) are continuous in \( v \) and \( w \). Let \(-1 \leq \mu \leq 1\), \( V > 0 \) and consider the equation \( \mu(\partial N/\partial r) + r^{-1}(1 - \mu^2)(\partial N/\partial \mu) + \sigma(r,v)N = 2^{-1} \int_0^\infty \int_{-1}^1 N(r,v,w)dw dy \tau(r,v,w)dw \equiv \Psi(r,v) \). Wanted is a solution \( N = N(r,\mu,v) \), continuous in all its arguments for \( r_0 \leq r \leq R \), \(-1 \leq \mu \leq 1\), \( 0 \leq v \leq V \), at least twice differentiable in \( r \) and \( \mu \) for \( r \notin P \) and satisfying \( N(R,\mu,v) = 0 \) for \( \mu < 0 \) and \( N(r_0,\mu,v) = \phi(\mu,v) \) for \( \mu > 0 \). Here \( \phi(\mu,v) \equiv 0 \) is a preassigned function, continuous in \( v \), differentiable in \( \mu \). \( N(r,\mu,v) \) represents the density of the neutron flux in a shield formed by homogeneous, concentric shells (isotropic scattering assumed), at distance \( r \) from the center, in the direction \( \theta = \arccos \mu \), of velocity \( v \) (variable). It is shown that \( \Psi(r,v) \) satisfies the integral equation \( \int_0^V \int_{-1}^1 \Psi(t,w)G(t,w;r,v)dt dw + F(r,v) = \Psi(r,v) \) with \( F \) and \( G \) known from the data of the problem. As stated, this problem has, in general, no solution \( N \). There exists a unique solution, whereby \( N \) is represented explicitly by integrals over \( \Psi \) as kernel, satisfying all conditions, except that \( N \) has, for each \( r \), a discontinuity at \( \mu = (1 - r_0^2/r^2)^{1/2} \). The corresponding saltius vanishes, however, if \( \phi \) satisfies an additional condition, which actually holds in the case of the neutron transport problem. (Received October 26, 1959.)


**Theorem:** Let \( G \) be a primary group. Then the following statements are equivalent: (I) \( G \) has a minimal system of generators, (II) \( G \) has the same power as a basic subgroup of itself, (III) \( G \) has a direct summand which is a direct sum of cyclic groups and has the same power as \( G \). In his book Abelian groups L. Fuchs asks: Which are the cardinals \( m \) for which there are no \( m \)-indecomposable reduced primary groups of power \( m \)? As a corollary of the equivalence of (II) and (III) of the above theorem we obtain: **Theorem:** There exists no infinite \( m \)-indecomposable reduced primary group of cardinal \( m \) if and only if either \( m = K_0 \) or for every cardinal \( n < m \) we have \( n^K_0 < m \). Also equivalent to the above statements is: (IV) \( G \) is finite or the power of the automorphism group of \( G \) is \( 2^{\text{power } G} \). For \( G \) infinite, this follows from (III) and the result proved by E. Walker (received through oral communication) that if the power of \( G \) exceeds that of a basic subgroup of itself then the power of the automorphism group is equal to that of \( G \). I, II, and III are equivalent for torsion groups. (Received August 26, 1959.)
Let $L(n;g)$ be the $n$th Lebesgue constant (norm) for the regular Hausdorff summation method with weight function $g$, applied to Fourier series. It is shown here that (i) $L(n;g) = C(g) \log n + O(\log n)$, where $C(g)$ is an explicitly determined constant depending only on $g$; (ii) $0 \leq C(g) \leq \frac{4}{\pi^2}$; (iii) $C(g) = 0$ if and only if $g$ is continuous; (iv) $C(g) = \frac{4}{\pi^2}$ if and only if the method is ordinary convergence; (v) if $g(1-) = g(1)$, then $C(g) \leq \frac{2}{\pi^2}$, with $C(g) = \frac{2}{\pi^2}$ if and only if the method is of Euler type. Part (iii) provides an alternative proof of a result due to Hille and Tamarkin (Math. Ann. vol. 108 (1933) Theorem 14,1) that, corresponding to any Hausdorff method with a discontinuous weight function, there exists a continuous function not summable by that method at a preassigned point. For the special cases of Cesàro and Hölder summation of order $k$, $0 < k < 1$, the limits of $L(n;g)$, $n \to \infty$, are found. For $(C,k)$ this provides an alternative derivation of a result obtained by Cramér (Arkiv f. Mat., Astr. o. Fys., vol. 13, 1918); for $(H,k)$ the result is new. The two limits differ. (Received October 28, 1959.)


In 1914 H. Hahn and S. Mazurkiewicz topologically characterized continuous curves as metric compact connected and locally connected spaces. This classical result was never extended to nonmetric spaces, although there existed a conjecture seemingly known to many topologists. The conjecture asserted that Hausdorff compact connected and locally connected spaces coincide with Hausdorff spaces which are continuous images of ordered continua. This paper rejects the conjecture by producing a counter-example, which is effectively constructed. More precisely, the following theorem is established: Theorem: There exists a Hausdorff compact connected and locally connected space $X$ and two points $x_0 \in X$ and $x_1 \in X$ such that, given any ordered continuum $C$, there is no continuous map $f: C \to X$ sending the two end-points of $C$ into $x_0$ and $x_1$ respectively. The construction of $X$ depends on defining a certain family of continuous curves $X_r$ and monotone maps $p_r: X_r \to I$ onto the real line segment $I = [0,1]$ in such a fashion that, given any ordered continuum $C$ and a map $g: C \to I$ sending the end-points of $C$ into 0 and 1 respectively, there exists an $r$ such that no continuous map $h_r: C \to X_r$ satisfies $p_r h_r = g$. (Received October 13, 1959.)

Let $M$ be a connected Riemannian manifold of class $C^\infty$. For each point $x$, one defines $k^*(x)$ to be the vector space of all germs of Killing vector fields defined around $x$ and $k(x)$ to be the vector space formed by all pairs $(X,A)$, where $X$ is a tangent vector at $x$ and $A$ is a skew-symmetric endomorphism of the tangent space which together satisfy certain algebraic conditions involving the curvature tensor and its covariant derivatives. A point $x$ is called $k^*$-regular (resp. $k$-regular) if $\dim k^*(y)$ (resp. $\dim k(y)$) is constant in a neighborhood of $x$. The main results are the following. A $k$-regular point is $k^*$-regular. If $M$ is analytic, every point is $k$-regular. If $M$ is simply connected and every point is $k^*$-regular, every element of $k^*(X)$ can be extended to a Killing vector field on $M$. If $x$ is a $k$-regular point, every element of $k(x)$ induces a Killing vector field in a neighborhood of $x$. The last theorem is a precise formulation of a theorem in classical Riemannian geometry. As one of the applications, a characterisation of Riemannian homogeneous spaces is obtained in terms of $k(x)$. (Received September 23, 1959.)


Let $(S, \Sigma, \mu)$ be a probability space (that is, $\mu(S) = 1$). Let $\Sigma'$ be a $\sigma$-subfield of the $\sigma$-field $\Sigma$. If $f$ is a real-valued integrable function on $S$, let $\mu_f(E) = \int_E f(s) \, \mu(ds)$. The restriction of the indefinite integral $\mu_f$ to the $\sigma$-subfield $\Sigma'$ is a $\mu$-continuous set-function; hence, by the Radon-Nikodym Theorem, there exists a $\Sigma'$-measurable function $f'$ such that $\int_E f'(s) \, \mu(ds) = \int_E f(s) \, \mu(ds)$ for every set $E$ in $\Sigma'$. The linear operator $A f = f'$ is called conditional expectation. Theorem: Let $A$ be a bounded everywhere defined linear operator in $L_p(S, \Sigma, \mu)$ (for fixed $p$, $1 \leq p < \infty$) satisfying the following conditions: (a) $A$ is a contraction operator, (b) if $I$ is the identity function on $S$, then $AI = I$, (c) if $f$ is of class $L_p$ and $g$ is essentially bounded, then the function $(Af)(s)(Ag)(s)$ is of class $L_p$, and the averaging identity $A(gAf) = (Ag)(Af)$ is satisfied. Then there exists a unique $\sigma$-subfield $\Sigma'$ of $\Sigma$ such that $A$ is the conditional expectation operator relative to $\Sigma'$. This strengthens some results of S. T. C. Moy (Pacific J. Math. (1954) pp. 47-63). A counterexample is given to show that the same result fails in $L_\infty(S, \Sigma, \mu)$. (Received October 15, 1959.)

For osculatory and hyperosculatory inverse interpolation for \( f(x) \), the Lagrange-Hermite formula is applied to the inverse function \( x(f) \). We seek \( x \) when given \( f(x) \), \( f(x_1) \) and \( x_1 \) with \( x_1' = 1/f'(x_1) \) or with \( x_1' \) and \( x_1'' = x''(f_1) = -f''(x_1)/[f'(x_1)]^3 \). Comparison is made with the writer's previously given formulas for osculatory and hyperosculatory inverse interpolation, all based upon the power series obtained by inversion of the Lagrange-Hermite direct interpolation polynomial. These alternative schemes, especially for higher accuracy, have less cumbersome formulas, are more truly interpolatory, converge better outside a middle interval and are no more complicated for \( x_1 \) irregularly spaced or complex. The formulas are arranged concisely by using a decomposability and uniqueness property of the general Lagrange-Hermite formula. A special application yields extremely accurate formulas for determining zeros of functions. These schemes suggest having with mathematical tables, several extra columns of auxiliary quantities for inverse interpolation to the fullest extent possible. Actual count of the number of operations for 10th degree accuracy by the older method and 11th degree accuracy (either 6-point osculatory or 4-point hyperosculatory) using the present scheme, showed the latter to involve only around one-fourth of the number of operations in the former. (Received October 23, 1959.)


A code called BEAUTY has been written in the FORTRAN language for evaluating the generalized hypergeometric series \( \mathbf{pFq}_j(\alpha_1, \alpha_2, ..., \alpha_p; \gamma_1, \gamma_2, ..., \gamma_q; z) \). \( p \) and \( q \) may be as large as 10 and all the parameters and the argument may be complex. Another FORTRAN code called BEAST permits the numerical analytical continuation of the ordinary hypergeometric series. (Received October 5, 1959.)


Let \( X \) and \( Y \) be Hausdorff spaces and suppose in addition that \( Y \) is regular, has a base whose power is at most that of the first noncountable cardinal and every family of open sets in \( Y \) has a countable subfamily with the same cover. Let \( Q \subset X \times Y \), \( P \) be the projection of \( Q \) onto \( X \), and \( K \) be the family of compact
sets in some auxiliary Hausdorff space. Then there exists a function \( f \) on \( P \) to \( Y \) such that, for every \( x \in P \), \( (x,f(x)) \in Q \) and: (1) if \( Q \) is compact then \( f \) is a Borel function; (2) if \( Q \) is the continuous image of a \( K_{\sigma\delta} \) then \( f \) is a \( \mu \)-measurable function for all Carathéodory outer measures \( \mu \) on \( X \) such that closed sets are \( \mu \)-measurable. (Received October 26, 1959.)


In order to provide for the existence of categories the theory of sets is generalized by introducing a relational sign \( \sim \) of scope two. It obeys an axiom of extensionality similar to that employed in set theory for the sign \( \in \).

The existence of categories is guaranteed by a scheme which tells us that the relation \( (\exists x)(\exists y)(z = (x,y) \land y \in T \land R) \) is categorizing in \( z \). By this we mean that \( (\exists P)(\forall z)(z \sim K \iff P) \) is a theorem where \( P \) stands for the above relation. The term \( \tau_K((\forall z)(z \sim K \iff P)) \) is called the category of the objects \( z \) such that \( P \), and is denoted by \( C_z(P) \). A category with morphisms is a category \( K \) endowed with two terms \( \sigma \) and \( \Gamma \), verifying the following axioms:

\[
(\text{MO}_I) \Gamma \{X,Y,Z\} \in F(\sigma \{Y,Z\}\sigma \{X,Y\},\sigma \{X,Z\})
\]

Denote \( \Gamma \{X,Y,Z\} \) by \( (g,f) \rightarrow g \circ f \).

\[
(\text{MO}_{II}) h^*(g \circ f) = (h \circ g)^*f
\]

\[
(\text{MO}_{III}) \text{There exists } e_E, \text{for each } E, \text{with } e_E \in \sigma \{E,E\} \land e_E \circ f = f \land e_E = g.
\]

By applying the existence scheme to appropriate terms and relations, one obtains for instance the category of sets, structural categories and categories of diagrams. In a straightforward manner, functors and quasi-morphisms can be defined. (Received November 2, 1959.)


A fundamental, fertile concept in affine axiomatics, the affine reflection, an obvious generalization, it being analogous to the reflection in a mirror in metric space, be the mirror a point (in one-space) or a line (in two-space) or a plane (in three-space) [resp. a line (in three-space)] or a plane (in four-space), etc. was evaluated by an "Abschätzung" by Georg Pick (Gerhard Kowalewski), to check up on the feasibility and usefulness of such a concept in higher dimensions, in the "Prager Mathematisches Kränzchen" in June 1917. Several recent publications, using the same idea analytically, yield results, not necessarily trivial. However, the heuristic value of the concept "affine reflection" emerges in an nonaffine side result, a fundamental theorem on metric areas (according to a letter of Paul Bernays, January 17, 1958). The metric reflection, as is common knowledge, is capable, by repeated
application, to produce all possible euclidean motions; so is the Affine reflection of course, capable of producing all affine transformations ("affine motions") purely synthetic, from one. To the naïve observer this seems "no news." He derives the classification, analytically, with a few strokes of the pen. (Received October 22, 1959.)

564-16. B. K. Swartz and Burton Wendroff: Continued function expansions of real numbers.

The ideas of Bissinger (Generalized continued fractions, Bull. Amer. Math. Soc. vol. 50 (1944) pp. 868-876) and Everett (Generalized decimal expansions, Bull. Amer. Math. Soc. vol. 52 (1946) pp. 861-869) are extended in an algorithm which associates with each real number \( x_0 \) in an interval \( R \) a unique finite or infinite sequence of integers \( \{c(0), c(1), \ldots\} \) as follows: Let there be the following: \( R \) partitioned into intervals, \( I_n; \) intervals \( M_n \subset I_n \) identical with \( I_n \) except perhaps for an endpoint; homeomorphisms \( h_n: M_n \leftrightarrow M \) where \( M \) is an interval and \( \bigcup M_n \subset M \subset \bigcup I_n. \) Since \( x_0 \) is in \( I_m, \) let \( c(0) = m. \) If \( x_{k-1} = h c(k-2)/(h c(k-3)/(\ldots h c(0)(x_0))) \notin M \) represent \( x_0 \) by \( \{c(0), \ldots, c(k - 1)\}. \) Otherwise \( c(k) \) is defined by \( x_k \in I_{c(k)}. \) The main theorem is: Let the sequences arising from the above algorithm using \( \{h_n\} \) be in 1-1 correspondence with the points that yield them. Then \( g_n \) have individually the same sense as the \( h_n \) and one obtains a 1-1 correspondence between points and sequences using \( \{g_n\} \) if and only if \( \exists \) an increasing homeomorphism \( F: R \leftrightarrow R, M_n \leftrightarrow M_n, \) such that \( g_n = F^{-1}(h_n(F)). \) For Bissinger and Everett, where \( I_n = [n, n + 1) \) and known \( \{h_n\} \) giving 1-1 correspondences are, \( h_n = l/(x - n) \) and \( h_n = p^{x-n}, \) respectively; the theorem states \( g_n(x - n) = g_0(x) \) and \( g_0 = F^{-1}(l/F(x)), \) \( g_0 = F^{-1}(p^{F}(x)), \) respectively. If \( \{h_n\} \) is 1-1 and \( x \in R \) yields \( \{c(0), c(1), \ldots\} \) under the algorithm, the expansion \( x = h c(0)/(h c(1)/(\ldots)) \) is defined. (Received October 9, 1959.)

564-17. J. A. Wolf: Subgroups of compact connected Lie groups.

Given a compact connected Lie group \( G \) of rank \( n, \) the problem of finding its closed connected subgroups of rank \( n - 1 \) can be reduced to the case where \( G \) is a torus or simple, or, if one is also interested in the imbedding, to the case where \( G \) is a torus or a product of isomorphic simple groups. Given a closed connected subgroup \( G' \) of rank \( n - 1 \) in \( G, \) techniques of J. de Siebenthal (Comment. Math. Helv. (1951)) give us a closed connected subgroup \( G'' \) of rank \( n \) in \( G, \) in which \( G' \) is a maximal closed connected subgroup. The classification
of A. Borel and J. de Siebenthal (Comment. Math. Helv. (1949)) tells us the possibilities for $G''$. The problem being local, write $G'' = G_1 \times \ldots \times G_s \times A$ with $A$ abelian and the $G_j$ simple, simply connected. Re-ordering the $G_j$: either $G' = G_1 \times \ldots \times G_s \times A'$, $A'$ a sub-torus of co-dimension 1 in $A$; or $G' = H \times G_{t+1} \times \ldots \times G_s \times A$, $H$ a semisimple maximal closed connected subgroup of rank 1 less in $G^\# = G_1 \times \ldots \times G_t$, $G_1$ through $G_t$ mutually isomorphic, and $H$ isomorphic to $H' \times G_2 \times \ldots \times G_t$ where $H'$ is a semisimple closed connected subgroup of rank 1 less in the compact connected simple group $G_1$.

(Received October 22, 1959.)


Previously all known examples of quasi-invariant ergodic distributions on Hilbert space were equivalent to Gaussian ones. A class of examples which are not equivalent to Gaussian ones, is given. (Received November 4, 1959.)

564-19.

WITHDRAWN.

564-20. Jacob Feldman: Some classes of equivalent Gaussian processes on the interval.

Let $\rho(s)$ be the Fourier transform of $1/(1 + x^2)^n$, and $\sigma(s)$ the Fourier transform of $(1 + \Psi(x))/(1 + x^2)^n$, $n \geq 1$, where $\Psi$ is any real integrable with $1 + \Psi(x) \geq \alpha$ for some $\alpha > 0$. $\rho$ and $\sigma$ can be considered as covariance functions of Gaussian stochastic processes with mean 0 and covariance $\rho$ and $\sigma$ respectively. Consider the processes on an interval. Let $\mu$ and $\nu$ be the measures they induce on the space of all functions on that interval. Then $\mu \sim \nu$. (Received November 4, 1959.)
Let the matrix $A = P A P^{-1}$, $A = \text{diag}(\lambda_1, \ldots, \lambda_n)$; $\kappa = \|P\| \|P^{-1}\|$, using the spectral norm. Then $K \geq 1$, with equality if and only if $A$ is normal. For any vector $x \neq 0$, and any polynomials $\alpha(\lambda)$ and $\beta(\lambda)$, the set $L_1 = [\lambda \left| \alpha(\lambda)/\beta(\lambda) \right| \leq \kappa \|\alpha(A)x\|/\|\beta(A)x\|]$ is an inclusion set for $A$, which is to say that at least one $\lambda_1 \in L_1$. Corollaries to this main theorem are that if $\alpha(A)x$ and $\beta(A)x$ are orthonormal vectors, then $L_2 = [\lambda \left| \alpha(\lambda) - \delta \beta(\lambda) \right|/|\delta\alpha(\lambda)| + \beta(\lambda)| \leq \kappa]$; and for any $\tau = \exp i\theta$, and any scalar $\delta$, $L_3 = [\lambda | \text{Re} \tau \alpha(\lambda)/\beta(\lambda) | \leq (\kappa - \kappa^{-1})/2]$ are inclusion sets. Theorem and corollaries imply separation theorems of Bueckner [Praktische Behandlung von Integralgleichungen] for hermitian matrices, separation and inclusion theorems stated by Wielandt in a series of papers, as well as more special results published by Temple, Weinstein, Collatz, Walker and Weston, Bartsch, Lehmann, Franklin, and others. Vector norms are Euclidean and expressible in terms of the moments $\mu_{ij} = x_i^H x_j$, $x_i = A^i x$. (Received November 6, 1959.)

Let $N$ be the positive integers, $E$ the Banach space of all bounded real functions on $N$, and $E'$ its conjugate space. Let $\tau$ be a 1:1 mapping of $N$ into $N$; there is a corresponding $T:E \rightarrow E$ defined by $(Tf)(n) = f(\tau(n))$. A $\tau$-mean is an element $\sigma \in E'$ satisfying (a) $\|\sigma\| = 1$, (b) $f \equiv 0$ implies $(\sigma, f) \equiv 0$, and (c) $(\sigma', f) = (\sigma', Tf)$ for all $f \in E$. Let $\sigma$ be another 1:1 mapping of $N$ into $N$, and $S$ its corresponding linear transformation of $E$; one may now speak of $\sigma$-means. Theorem. The following statements are equivalent: (1) Every $\sigma$-mean is a $\tau$-mean; (2) $\lim_n(S_n T - S_n) = 0$ in either the strong or uniform operator topology, where $S_n = (1/n)(S + S^2 + \ldots + S^n)$; (3) $\lim_n(1/n)\zeta[[\sigma(p), \sigma^2(p), \ldots, \sigma^n(p)]\Delta[\tau\sigma(p), \ldots, \tau\sigma^n(p)]] = 0$ uniformly in $p$, where $\zeta$ means 'cardinality of', and $\Delta$ denotes symmetric difference; (4) $\tau$ is $\sigma$-bounded except on a set of $\sigma$-density zero, which means there is some $k \in N$ such that $\tau(p) \in \{\sigma^{-k}(p), \sigma^{-k+1}(p), \ldots, \sigma^k(p)\}$ except for $p \in J$, where $J$ is a set on whose characteristic function all $\sigma$-means vanish. Work supported by National Science Foundation grant NSF G-9396. (Received November 9, 1959.)

Let $D$ be an $(n+1)$-dimensional domain, let $(x,t) = (x_1, \ldots, x_n, t)$ vary in $D$ and assume that for every $\sigma > 0, B_\sigma = D \cap \{t = \sigma\}$ is a bounded domain and, as $\sigma \to \infty$, $\lim B_\sigma$ exists and is bounded. Consider a solution $u$ in $D$ of a parabolic equation of any order $\partial u/\partial t = Lu + f(x,t)$ with smooth coefficients depending on $(x,t)$. If the Dirichlet data on the lateral boundary of $D$ tend to a limit as $t \to \infty$, and if $f$ and the coefficients of $L$ tend to limits as $t \to \infty$, then as $t \to \infty$, $u(x,t)$ tends in the $L^2(B_t)$-sense to a solution of the limit elliptic equation which satisfies the limit boundary conditions. For second order parabolic equations, a uniform convergence is established for both the first and the second boundary problems. This last result is also proved for second order elliptic equations. It is finally proved that fundamental solutions of elliptic equations of any order in $(x,t)$, tend "averagely" to fundamental solutions of elliptic equations in $x$. (Received November 9, 1959.)


The results of the present paper apply to ultraspherical polynomials. However for reasons of notational simplicity we confine ourselves here to the special case of Legendre polynomials. Let $P_n(x)$, $n = 0, \ldots$, be the Legendre polynomials normalized in the usual fashion. For $f(x) \in L^1(-1,1)$ we set $f^n(n) = \int_{-1}^{1} f(x)P_n(x)\,dx$. The formal development of $f(x)$ as a series of Legendre polynomials is then $\sum_{n=0}^{\infty} (n+1/2) f^n(n)P_n(x)$. Let $f(x) \in L^p(-1,1)$. Pollard has proved that if $4/3 < p < 4$ the series of partial sums converges in the mean of order $p$ to $f(x)$. In the present paper it is shown that the Cesàro sums of order $\alpha$ converge in the mean of order $p$ to $f(x)$ if $0 \leq \alpha \leq 1/2$ and if $4/(3 + 2\alpha) < p < 4/(1 - 2\alpha)$. The conclusion is false if $p \geq 4/(3 + 2\alpha)$ or $p \leq 4/(1 - 2\alpha)$. The case $\alpha = 0$ is Pollard's Theorem. (Received November 9, 1959.)

564-25. Frank Harary and I. C. Ross: The square of a tree.

The adjacency matrix of a graph of $n$ points is the square matrix of order $n$ in which the $i$, $j$ element is 1 if and only if the $i$'th point and $j$'th point are adjacent or $i = j$, and is 0 otherwise. Let $A$ be the adjacency matrix of graph $G$, considered as a boolean matrix, so that $1 + 1 = 1$. Then $G^2$, the square of $G$,
is the graph whose adjacency matrix is $A^2$. We obtain a necessary and sufficient condition for a graph to be the square of a tree by providing an algorithm for determining a tree which is the square root of any graph known to be the square of some tree. This algorithm cannot be carried through when a graph is not the square of a tree. It is shown that if a graph is the square of a tree, then it has a unique tree square root. The method utilizes a previous result for determining all the cliques in a given graph, where a clique is a maximal complete subgraph. This result was obtained while attempting the more general problem of characterizing boolean matrices having a square root or in general an n'th root. (Received November 9, 1959.)


Albert Edrei and S. M. Shah constructed an entire function $f(z)$, for which $g(f')$ (genus of $f'$) is less than $g(f)$ and $f$ has no finite defect value, to disprove a conjecture of R. Nevanlinna. In this paper a meromorphic function $f(z)$ of integer order $\rho$ is constructed for which $g(f') < g(f)$ and $f$ has no exceptional value $N$. Let $g(z) = g(z)/h(z)$, $g(z) = \prod_{1}^{\infty} (1 - z/a_n)$, $h(z) = \prod_{1}^{\infty} (1 + z/b_n)$ where $a_n > 0$, $b_n > 0$, $n = 1, 2, ...$, and $\sum a_n^{-1}, \sum b_n^{-1}$ convergent and so chosen that $\delta(0,0) = \delta(\infty,0) = 0$. Let $F(z) = g(z^a)$, $f(z) = F(z) + a$, $a \neq 0$. Then $f(z)$ is the required function of order $\rho$ for which $\sum \delta(\alpha, f) = 0$, and $g(f') < g(f)$. (Received November 12, 1959.)


Let there be given a commutator having $N$ assignable terminals equally spaced in time. A necessary and sufficient condition that these terminals can be assigned to $n_1$ channels with a sampling frequency $\nu_1$, $n_2$ channels with a sampling frequency $\nu_2$, etc., where $\sum n_i \nu_i = N$ and $\nu_i |N$ for all $i$, is proved. Further, under certain restrictions on the $\nu_i$, generating functions determining the locations of the channels are derived. (Received November 13, 1959.)


This algorithm is a by-product of the new general method of numerical solution of Linear programming and Matrix games problems. See Abstract
Matrix inversion algorithm: Let $A = [a_{ij}]$ be an $n \times n$ nonsingular matrix of real numbers. Let $(km)$ be the pivot field with $a_{km} = p$, other pivot-column values $a_{im} = c$, other pivot-row values $a_{kj} = r$, other elements of $A$, $a_{ij} = s$. First iteration matrix $A^{(1)}$ has these values: In the $(km)$ field $p^{-1}$, column fields $(im) = p^{-1}c - r$, row fields $(kj) = p^{-1}r$, other fields $(s - c \cdot p^{-1} \cdot r)$. The procedure terminates by $A^{(n)}$, which gives exact inverse $A^{-1}$. The system $Ax = b$ is solved in $n$ steps: $[Ab] \rightarrow \ldots \rightarrow [A^{-1}x]$. The algorithm holds for partitions. If $A \sim (P, R, S$ and $C$ clockwise), two iterations by $P^{-1}$, $P^{-1}R$, $(S - CP^{-1}R)$ and $-CP^{-1}$ give the $A^{-1}$. Similar algorithm gives a pivotal method of determinants evaluation by matrix manipulation. Connections with papers of Petrie (1951) and Gomory (1959) are indicated. Advantages and applications are demonstrated in this paper. (Received November 19, 1959.)


Suppose $X$ is a compact subset of $Q$, the Hilbert cube, and $X$ is the monotonic intersection of compact absolute retracts. If $M$ is an absolute neighborhood retract intersecting $X$ and $f$ maps $X$ into $X^0$, the interior of $X \cap M$ relative to $M$, then $f$ has a fixed point. It follows that if $X$ is a compact, connected subset of $E_2$ which does not separate $E_2$ and $f$ maps $X$ into the interior of $X$, then $f$ has a fixed point. Borsuk has given an example of a bounded, connected open set $0$ in $E_3$ such that $0$ is the monotonic intersection of closed 3-cells, yet $0$ does not have the fixed point property, even for homeomorphisms onto. By the theorem stated, if $f$ maps $0$ into $0$, then $f$ must have a fixed point. A variation of the theorem stated gives the corollary: If $X$ is a contractible, compact subset of $Q$ and $f$ maps $X$ into the interior of $X$, then $f$ has a fixed point. It is known that such $X$ need not have the fixed point property. (Received November 16, 1959.)

564-30. D. S. Greenstein: On the analytic continuation of functions mapping the upper half plane into itself.

If $f(z)$ is analytic in the upper half plane with $\text{Im} f(z) > 0$ there, then a result of Verblunsky says that $f(z) = Az + b + \int_{-\infty}^{\infty} (1 + tz)/(t - z) \, d\Psi(t)$ ($\text{Im} z > 0$), where $\Psi(t)$ is bounded and nondecreasing. We consider the problem of determining conditions on $\Psi(t)$ under which $f(z)$ may be continued across a
given interval \((a,b)\) of the real axis. A necessary and sufficient condition for such continuability is that \(\psi(t)\) be real analytic in \((a,b)\). The proof of the necessity depends on an inversion formula similar to the Stieltjes inversion formula for \(f(z)\) of the type \(\int_{-\infty}^{\infty} (t - z)^{-1} \psi(t) \, dt\). The inversion formula also makes it possible to give a continuation formula similar to the Plemelj formula. (Received November 16, 1959.)


An \(m \times n\) matrix each of whose rows consist of a permutation of \(1,2,\ldots,n\) is called an \(R\) matrix. Corresponding to every \(R\) matrix we define an associated oriented graph \(G = G(R)\) on the vertices \(1,2,\ldots,n\), in which \(i\) and \(j\) are joined by an edge oriented from \(i\) to \(j\) if \(i\) precedes \(j\) in a majority of the rows of \(R\). If \(i\) precedes \(j\) exactly as often as \(j\) precedes \(i\) then in \(G\) there is no edge joining \(i\) and \(j\). Let \(f(n)\) be the smallest number \(m\) such that the set of all \(m \times n\) matrices represent all oriented graphs \(G\) on \(n\) vertices. Main theorem: There exist positive constants \(C_1\) and \(C_2\) such that \(C_1 n / \log n < f(n) < C_2 n / \log n\). Let \(g(n)\) be the largest number such that every complete oriented graph \(G\) on \(n\) vertices contains a complete subgraph of \(g(n)\) vertices in which the orientation of \(G\) is transitive. Theorem: \(2 \log^2 n + 1 \leq g(n) \leq \log 2n + 1\). (Received November 17, 1959.)


The problem concerns ideals in \(C(X)\) (notation as in the forthcoming book by Jerison and the author). Previous results (see Kohls, Illinois J. Math. vol. 2 (1958) pp. 505-536): (a) \(O^P\) is \(CG\) (\(\sim\) countably generated) iff \(p \in X\) and \(p\) has a countable base; (b) \(O^P\) is prime and \(CG\) iff \(M^P\) is principal, and iff \(p \in X\) and \(p\) is isolated; (c) no lower prime ideal is \(CG\). Present results generalize these. Call an ideal \(ZCG\) if its \(z\)-filter is \(CG\). (1) A \(CG\) ideal is \(ZCG\), but not conversely (whether fixed or free). (2) A \(ZCG\) ideal contained in a hyper-real ideal is contained in \(2^C\) hyper-real ideals. (3) A free \(ZCG\) ideal is contained in \(2^C\) hyper-real ideals, but in no real ideal. (4) Hence no free \(O^P\), no free prime ideal, no free maximal ideal are \(ZCG\). (5) If \(K\) is the ideal of all functions vanishing on a compact set \(F\), if \(J\) is any free ideal or all of \(C(X)\), and if \(J \cap K\) is \(CG\), then \(F\) is open. (6) Hence if \(M^P\) is \(CG\) then \(p \in X\) and \(p\) is isolated. (7) A nonmaximal
prime ideal can be CG; any such is the union of a countable chain of upper ideals and hence cannot be a countable intersection of lower ideals. (8) Hence the intersection of a strictly decreasing sequence of prime ideals cannot be CG. (9) A nonmaximal prime \( z \)-ideal cannot be ZCG. (Received November 18, 1959.)

564-33. A. N. Feldzamen: The adjoint Weyr characteristic.

Let \( X \) be a Banach space of uniform finite multiplicity with respect to a complete spectral measure \( E \), let \( Q \) be a quasi-nilpotent commuting with \( E \), and \( \mathcal{W} \) the Weyr characteristic defined by \( E \) and \( Q \) (cf. Bull. Amer. Math. Soc. vol. 65 (1959) pp. 79-83). Then on the adjoint space \( X^* \), the Weyr characteristic defined by \( E^* \) and \( Q^* \) is also \( \mathcal{W} \). To apply this to operators on Hilbert space, for a regular, countably additive, totally finite measure \( \mu \) on the Borel subsets \( \mathcal{B} \) of the complex plane, define \( \overline{\mu} \) by \( \overline{\mu}(S) = \mu(\{x|x \in S\}) \), \( S \in \mathcal{B} \). Let \( T \) be an essentially finite spectral operator on Hilbert space with Weyr characteristic \( \mathcal{W} \). Then \( T^* \) is also essentially finite and has Weyr characteristic \( \mathcal{W}^* \) given by \( \mathcal{W}^*(\mu, k) = \mathcal{W}(\overline{\mu}, k) \), for every \( k \). (Received November 16, 1959.)

564-34. Harvey Cohn: Class number formula for relative-quadratic fields over \( 2^{1/2} \) or \( 3^{1/2} \). Preliminary report.

Given a square-free integer \( \mu = a + b\sqrt{2} \gg 0 \), then if \( H \) is the class number of \( R(2^{1/2}, (- \mu)^{1/2}) \), and if \( A \) is the number of representations of \( \mu \) as the sum of three integral squares in \( 2^{1/2} \), it follows that \( A = HG \) (ignoring simple cases where complex roots of unity occur where \( H = 1 \)). Here \( G = 48 \) if \( 2 = P^2Q^2 \), \( G = 96 \) if \( 2 = PQ \), and \( G = 24 \) if \( 2 = P^4 \). According to preliminary calculations, a similar result seems true for \( 3^{1/2} \). The method of proof uses theta-functions of two complex variables, like Gotzky's proof (Math. Ann., 1928). On the other hand, a similar proof along the Siegel-Maass line of approach (Hamb. Abh., 1941) seems invalid. In previous work (Amer. J. Math., to appear) the author showed the sufficiency of four squares in representing all such \( \mu \) for \( R(2^{1/2}) \), but could not quite show the result for \( R(3^{1/2}) \). It may well happen that three squares suffice in both cases! Experiments of the author on the GEORGE computer leave little doubt (Num. Math., 1959). Research supported by National Science Foundation Research Grant G-7412, computer time donated by the Argonne National Laboratory of the Atomic Energy Commission. (Received November 19, 1959.)

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The covariant derivatives of density, entropy, unit tangent world-line vector, electromagnetic field tensor, and current vector are decomposed into their tangential and normal derivatives with respect to an arbitrary hypersurface, $\phi =$ constant. The known tangential derivatives, of all quantities, and the quantities, except current, are assumed continuous along, $\phi =$ constant. It is shown that the following laws are insufficient to determine the normal derivatives of the above quantities: the first law of thermodynamics, conservation of entropy along a world-line, conservation of mass, current, and stress-energy, Maxwell relations. If current (and hence its tangential derivatives) is continuous along $\phi =$ constant, or if the covariant derivatives of the electromagnetic field tensor have a specific structure, (cases A and B), then the above laws suffice. In the other cases, Ohm's law, with undetermined conductivity and proper charge can be used to complete the system. It is shown that the discontinuity (characteristic) manifolds, when the system is complete (cases A and B), are identical with those of the noncharged, compressible, relativistic fluid. (Received November 23, 1959.)


Let $X$ be a topological space having a development $G_1, G_2, \ldots$. A $G$-chain is any sequence $\{U_j\}$ of open sets where $U_i \subseteq G_i$ and $U_i \cap U_{i+1} \neq \emptyset$. A $G$-chain $\{U_j\}$ is said to converge if and only if (i) there exists a nested $G$-chain $\{V_j\}$ such that each $V_i$ eventually contains the entries of $\{U_j\}$ and (ii) $\overline{V}_{i+1} \subseteq V_i$. $G$ is called a convergent development for $X$ if and only if (i) each $G$-chain converges and (ii) every nested $G$-chain $\{V_j\}$ is such that $\bigcap \{\overline{V}_j\}$ consists of at most one point. A $G$-Cauchy sequence $\{x_j\}$ in $X$ is any sequence such that there is a $G$-chain $\{U_j\}$ such that eventually $x_j$ and $x_{j+1}$ are in $U_m(j)$ for all $j$. It is shown that in a space $X$ having a convergent development one may establish a natural generalization of Kolmogorov and Fomin's (Functional analysis, vol. 1) contraction mapping theorem provided the space is $G$-complete (every $G$-Cauchy sequence converges). It is also shown that each space $X$ having a convergent development may be completed using a method resembling the classical method used to complete a metric space. (Received November 12, 1959.)
Let \( \phi(x) \) be a continuous strictly increasing function such that \( \lim_{x \to -\infty} \phi(x) = -\infty \), \( \lim_{x \to \infty} \phi(x) = \infty \). Conditions that a continuous function \( f(x) \) satisfy the inequality \( f^n(x) \geq \phi(x) \) are studied (here \( f^n \) denotes the \( n \)th iterate of \( f \), \( n \) is a given positive integer). Theorem 1. If \( f \) is a solution of (1) for some odd \( n \), then \( f(x) \leq \phi(x) \) whenever \( f(x) \leq x \). Theorem 2. If \( f \) is a solution of (1) for an odd value of \( n \) then \( (f(x) - x)/(\phi(x) - x) \) has positive infimum on the set where \( \phi(x) > x \). Theorem 3. If \( \phi(x) - x \) is nonnegative and decreasing on an interval \( a \leq x \leq b \), \( \phi(b) = b \), and \( f(x) \leq \phi(x) \) for \( a \leq x \leq b \), then \( f \) cannot be a solution for any odd \( n \). Theorem 4. If \( \phi(x) - x \) is increasing, \( f(x) - x \leq m \phi(x) - x \) for some positive constant \( m \) whenever \( \phi(x) \leq x \), and \( f(x) \leq \phi(\beta) \) when \( f(x) \leq x \) where \( \beta \) is the supremum of \( x \)-values such that \( f(x) < x \), then \( f \) is a solution for \( n \geq \lceil 1/m \rceil \). If \( f \) satisfies (1) for some even value of \( n \), then \( f^2 \) must satisfy the conditions stipulated for \( f \) in Theorems 1, 2, and 3. (Received November 20, 1959.)


Theorem: Let \( h: S^{n-1} \times I \to S^n \). Then \( h(S^{n-1} \times 1/2) \) bounds an \( n \)-cell on either side. Definition. A set \( M \) is cellular in an \( n \)-dimensional space if it is the intersection of a sequence of \( n \)-cells each lying in the interior of the previous one. Lemma 1: Let \( Q \) be an \( n \)-cell and \( M \) a cellular subset of \( \bar{Q} \). Then \( Q/M \) is an \( n \)-cell. Lemma 2. Let \( S^{n-1} \) be an \( n-1 \) sphere in \( S^n \) and \( D \) one of its complementary domains. Suppose \( f \) maps \( D \) onto an \( n \)-cell \( R \) such that the only nondegenerate inverse under \( f \) is a cellular subset \( M \) of \( D \). Then \( D \) is an \( n \)-cell. Lemma 3. Let \( f \) map an \( n \)-cell \( Q \) into \( S^n \). Suppose \( M \subset \bar{Q} \) is the only nondegenerate inverse under \( f \). Then \( M \) is cellular in \( Q \). Lemma 4. Let \( f \) map \( S^n \) onto itself have only two nondegenerate inverses \( A, B \). Then both \( A \) and \( B \) are cellular in \( S^n \). Proof of Theorem. Let \( A \) be the closure of the complementary domain of \( h(S^n \times 0) \) not containing \( h(S^n \times 1) \), and let \( B \) be the closure of the complementary domain of \( h(S^n \times 1) \) not containing \( h(S^n \times 0) \). Now apply Lemma 4 (which follows from Lemma 3) and then Lemma 1. (Received November 20, 1959.)


Suppose that \( M \) is a compact 3-manifold with boundary and that each compact, simply connected 3-manifold in \( M \) with boundary a 2-sphere is a 3-cell. It is first proved that if \( f \) is a homotopy 2-regular mapping of a metric space \( X \)
into a metric space \( Y \) such that each inverse under \( f \) is homeomorphic to \( M \), then \( f \) is completely regular. This result and techniques developed by the author and E. Dyer (Completely regular mappings, Fund. Math. vol. 45 (1957) pp. 103-118) are used to prove that the space of homeomorphisms of a compact 3-manifold with boundary onto itself is \( LC^n \) for each \( n \). (This duplicates in part results obtained independently and previously by J. M. Kister and G. M. Fisher.) It follows that if \( X \) is complete and \( Y \) is finite dimensional, then \((X,f,Y)\) is a locally trivial fibre space. (A proper open mapping \( f \) of a metric space \( X \) into a metric space \( Y \) is homotopy \( n \)-regular if for \( x \in X, \varepsilon > 0 \), there is a \( \delta > 0 \) such that every mapping of a \( k \)-sphere, \( k \leq n \), into \( S(x,\delta) \cap f^{-1}(y), y \in Y, \) is homotopic to 0 in \( S(x,\varepsilon) \cap f^{-1}(y) \) and is completely regular if for each \( \varepsilon > 0, y \in Y, \) there is a \( \delta > 0 \) such that \( d(y,y') < \delta, y' \in Y, \) implies the existence of an \( \varepsilon \)-homeomorphism of \( f^{-1}(y) \) onto \( f^{-1}(y') \).) (This research was supported by The National Science Foundation.) (Received November 20, 1959.)


Let \( A \) be an \( n \times n \) matrix and \( L \) a linear functional on \( n \)-space. The solution \( x \) of the system of linear equations \( x = Ax + b \) lies in the hyperplane \( H = z: L(I - A)z = Lb \). In the iterative scheme \( (1) \quad y_k = Ax_k + b, \quad x_{k+1} = (Lb/L(I - A)y_k)y_k, \) each iterate \( x_k \) also lies in \( H \). Let \( P \) be the projection operator \( Pz = z - (Lz/Lb)b \). Theorem. If \( Lx \neq 0, Lb \neq 0, \) \( (1) \) converges to \( x \) for any \( x_0 \) with \( Lx_0 \neq 0 \) if and only if \( (PA)^kz \) converges to 0, and the two convergence rates are the same. Corollary. For almost every \( b \) there is an \( L \) such that \( (1) \) is a conjugate gradient method. Corollary. If \( A \) is the overrelaxation matrix of a matrix \( T + B \) with overrelaxation parameter \( \omega \), and \( T(B) \) is an upper (lower) triangular matrix, the convergence rate of \( (1) \) is given by the solutions \( \lambda \) of \( (PT + \lambda PB)z = \omega^{-1}(\lambda + \omega - 1)z \). The optimal \( \omega \) is not known, even when \( PT + PB \) has Young's property (A) (Trans. Amer. Math. Soc. vol. 76 (1954) pp. 92-111). A continuous version of the theorem is given for operators on a Banach space. Some numerical examples are given. (Received November 20, 1959.)


Functions employed for the purpose of interpolation of real functions of one real variable are mostly polynomials. It is proposed to use sectionally
linear \((s, \mathcal{L})\) functions which share with polynomials the essential property that their values can be computed immediately. Every \(s, \mathcal{L}\) function over a partition \(x_0 < x_1 < \ldots < x_n\) of an interval \([x_0, x_n]\) can be written in the form 
\[
f(x) = \sum_{\nu=0}^{n} c_{\nu}(x) \phi_{\nu}(x)\]
where \(\phi_{\nu}(x) = \phi(x - x_{\nu - 1})\), \(\phi_0(x) = 1\), \(\phi(t) = (|t| + t)/2\).
The coefficients \(c_{\nu}\) are readily found in terms of divided differences, thus involving tedious computations. Therefore the functions \(\phi_0(x), \ldots, \phi_n(x)\) are orthogonalized into \(\psi_0(x), \ldots, \psi_n(x)\) so that 
\[
\sum_{\nu=0}^{n} \psi_{i}(x) \psi_{j}(x) = 0\text{ if } i \neq j.
\]
Then \(f(x) = \sum_{\nu=0}^{n} a_{\nu} \psi_{\nu}(x)\); the \(a_{\nu}\) are obtained as "Fourier coefficients". The method can be applied mutatis mutandis to the data fitting problem. Explicit computation of the orthogonal functions \(\psi_{\nu}(x)\) is simplified in the case of a partition by equidistant points; in this case \(\psi_{n-k}(x) = \alpha_{k} \phi_{n-k-2}(x) + \beta_{k} \phi_{n-k-1}(x) + \phi_{n-k}(x)\) where \(\alpha_{k} = (k + 1)(k + 2)/(k + 3)(k + 4)\), \(\beta_{k} = -2(k + 1)/(k + 3)\), for \(k = 0, 1, \ldots, n - 3\); \(\psi_{0}(x) = 1\), \(\psi_{1}(x) = -1/2 + x\), and \(\psi_{2}(x) = (n - 1)/(n + 1)(n + 2) - x(n - 1)(n + 4)/(n + 1)(n + 2) + \phi_{2}(x)\). (Received November 20, 1959.)


The Cartan-Brauer-Hua theorem says that the only invariant subdivision rings of a division ring \(K\) are \(K\) itself and subfields of the center of \(K\). In the present paper this is improved to the following theorem: The only subinvariant subdivision rings of a division ring \(K\) are \(K\) itself and subfields of the center of \(K\). This completes the proof in Schenkman, [Proc. Amer. Math. Soc. vol. 9 (1958) pp. 231-238]. (Received November 23, 1959.)

564-43. E. T. Kobayashi: Integration of subspaces derived from a linear transformation field.

Let \(\chi\) be the characteristic polynomial of a vector 1-form \(h\) on a \(\mathcal{C} \infty\)-manifold \(M\). If at a point \(p\) of \(M\), \(\chi_p\) has a factorization over the reals \(R\), 
\[
\chi_p = K_1^{m_1} \cdots K_g^{m_g},
\]
where \(g > 1\), \(K_i \in R[\lambda]\), with leading coefficients 1, and \(K_i\) are all distinct and irreducible over \(R\), then there is a neighborhood \(U\) of \(p\), where \(\chi\) has a unique factorization \(\chi = \chi_1 \cdots \chi_g\) on \(U\), satisfying 
(i) \(\chi_i \in R_U[\lambda]\), where \(R_U\) is the ring of \(\mathcal{C} \infty\) functions on \(U\); (ii) \(\chi_i\) has leading coefficient 1; (iii) \((\chi_i)_p = K_i^{m_i}\); (iv) \((\chi_i)_q\) and \((\chi_j)_q\) are relatively prime for \(q \in U, i \neq j\). Corresponding to each \(\chi_i\) there is a projection operator \(e_i(h)\) on \(U\) such that the image space of \((e_i(h))_q\) is equal to the zero space of \((\chi_i(h))_q\) for \(q \in U\). We obtain \(\mathcal{C} \infty\)-distributions \(\theta_{i_1} \cdots i_k\) on \(U\) defined by
\[ q \rightarrow (e_{i_1}(h) + \ldots + e_{i_k}(h))q, \text{ where } T_q \text{ is the tangent space to } M \text{ at } q \in U. \] All \( \theta_{i_1} \ldots i_k, 1 \leq k \leq g - 1 \) are integrable in a neighborhood of \( p \), if and only if \([e_i(h), e_i(h)] = 0\), for all \( i \), in a neighborhood of \( p \), where the bracket \([k,k]\) for a vector 1-form \( k \) denotes a vector 2-form defined by Nijenhuis. If \( e \) is a projection operator on \( U \) such that \( e = e(h) \) for some \( e \in \mathfrak{g} \), then on \( U \) we have \( e = e(h) \) \( = \sum_{i=1}^{g} \delta_i(h) \), where \( \delta_i = 0 \) or 1.

564-44. David Sachs: **Modulated and partition lattices.**

Let \( L \) be a left-complemented lattice (L. R. Wilcox, Bull. Amer. Math. Soc. vol. 48 (1942) pp. 453-457). We write \( bM \) if \((b,c)M\) for every \( c \), where \((b,c)M\) means \((d + b)c = d + bc\) for every \( d \leq c \). A left-complemented lattice with greatest element \( I \) is modulated if for every \( yM \) and for every \( z \geq y \), there exists \( xM \) such that \( x + z = 1, \ xz = y \). If \( L \) is a matroid lattice, then the set \( M \) of \( M \)-elements forms a lattice whose dual \( \overline{M} \) is left-complemented. Theorem: A lattice \( L \) of length \( \geq 5 \) is isomorphic to a lattice of partitions on a set \( \text{iff} \) \( L \) is an irreducible modulated matroid lattice having a pair of elements \((a,b)\) such that \( ab \neq 0 \), \((a,b)M'\). Theorem: A lattice \( L \) of infinite length is isomorphic to a lattice of partitions on a set \( \text{iff} \) \( L \) is a simple modulated matroid lattice.

(Received November 23, 1959.)

564-45. Joshua Chover: **Another reformulation of the entropy concept.**

Let \( X \) be a space, \( F \) a field of subsets of \( X \), and \( e \) an equivalence relation in \( F \). A notion of normalized entropy, \( K(m) \), for a measure \( m \) on \( F \) (with \( m(X) = 1 \)) may be given as follows. Every finite partition of \( X \) into \( F \)-sets splits into equivalence classes: form the usual conditional entropy of an equivalence class, normalize it by subtracting the logarithm of the number of non-null \( m \)-sets in the class; average over the equivalence classes; take the infimum over all such partitions. It can be shown that \( K(m) \) has properties analogous to the familiar ones. Because of normalization \( K(m) \) is always \( \geq 0 \). Continuity is replaced by upper-semi-continuity of \( K(m) \) as a function of \( m \).

Apart from finite normalization constants, \( K(m) \) coincides with the usual entropy for finite spaces, with entropy defined for one measure given another base measure (and using the Radon-Nikodym derivative), in particular with Shannon's entropy for continuous distributions (on bounded sets). Under suitable definition of \( e \), \( K(m) \) gives the entropy-per-step of a stationary process; and may be applied to nonstationary processes also. (Received November 23, 1959.)
564-46. E. D. Callender: Holder continuity of n-dimensional quasi-conformal mappings.

This paper is an extension of previous work on the Hölder continuity of two-dimensional quasi-conformal mappings. We shall use the approach of Finn and Serrin and prove analogous results in n dimensions. A two-dimensional quasi-conformal mapping is one which carries infinitesimal circles into infinitesimal ellipses of bounded eccentricity. An n-dimensional quasi-conformal mapping carries infinitesimal spheres into infinitesimal ellipsoids of bounded eccentricity. Finn and Serrin gave an elementary proof that a quasi-conformal mapping is uniformly Hölder continuous in compact subdomains. Their proof makes extensive use of the Dirichlet integral. We obtain similar results in n dimensions using a modified Dirichlet integral. (Received November 23, 1959.)

564-47. D. W. Dean and R. A. Raimi: Equivalent permutations of the positive integers.

Let N be the set of positive integers and let E be the Banach space of bounded real-valued sequences with its conjugate space denoted by E'. Let μ: N —> N be 1:1 and define U: E —> E by letting Uf(n) = f(μ(n)) for all f in E and n in N. Let M^μ be the set of those f' in E' for which (f', Uf) = (f', f), ||f'|| = 1, and f'(n) = 1 if f(n) = 1 for all n in N. Let σ: N —> N be 1:1. Say that μ and σ are equivalent if M^μ = M^σ. If the equation μ^i(n) = n has a solution for no i in N nor n in N, then there is a permutation σ of N which is equivalent to μ. Moreover σ can be chosen so that it has exactly one cycle; that is, for every n, m in N, there is an integer i for which σ^i(n) = m. Such a σ, and hence such a μ, is equivalent to a τ for which ∃ n in N such that given m in N, ∃ i in N such that τ^i(n) = m. Work supported by National Science Foundation Contract NSF-G 9396. (Received November 23, 1959.)


A theorem of Maddaus on the uniform approximability of a compact linear transformation T from F into E (two Banach spaces) by finite dimensional transformations is generalized. Maddaus assumes E is of Type A. (The identity I of E is the pointwise limit of a sequence {P^i} of finite dimensional linear transformations of E into E.) Type A is generalized (Type A') in two ways: (1) The topology τ, in which x = lim_i P^i x is meant, is weakened, (2) {P^i} is no longer
countable. These two changes lead to a stronger theory. For instance, we prove
that approximability of $T$ when its domain or range or the dual of one of them is
of Type $A'$. This is a considerable strengthening of Maddaus's result. Several
classes of spaces of Type $A'$, not known to be of type Type $A$, are discussed.
They are among the most useful spaces of analysis. The paper ends with a
characterization of inner product spaces, based on that of Kakutani (Jap. J. Math,
vol. 16 (1939) pp. 93-97) in terms of the systems $\{P_i\}$. (Received November 23,
1959.)

564-49. E. P. Miles, Jr.: Generalized Fibonacci numbers and associated
matrices.

Let $f_{j,k} = 0$ for $0 \leq j \leq k - 2$, $f_{k-1,k} = 1$ and $f_{j,k} = \sum_{n=1}^{k} f_{j-n,k}$ for $j \geq k$.
The sequence $f_{j,k}, k > 2$, is called a $k$-generalized Fibonacci sequence, because
$f_{j,2}$ is the ordinary Fibonacci sequence $f_j$ with each term after the second the
sum of its two immediate predecessors. The classical (Bernoulli) expansion
is obtained for $f_{j,k}$ as a linear combination of the $j$th powers of the roots of
$E_k: x^k - x^{k-1} - \ldots - x - 1 = 0$. $E_k$ is shown to have exactly $k - 1$ distinct roots
interior to the unit circle, while the remaining root $r_k, 2 - 2/(k - 1) < r_k < 2$, is
such that $\lim_{j \to \infty} f_{j+1,k}/f_{j,k} = r_k$. The $k$ by $k$ symmetric matrices $A_{j,k}$
having elements $a_{mn} = (\frac{1}{j-m-n-2,k},$ are studied. It is shown that $|A_{j,k}|$
$= (-1)^{(2j-k)(k-1)/2}$, a generalization of $f_{j,1}^2 - f_{j-1,1} f_{j+1,1} = (-1)^j$. These Fibonacci
matrices, which become arbitrarily ill-conditioned for sufficiently large $j$ and
fixed $k$, are used to discuss systems of linear equations with exact integral
solutions which become highly unstable when their coefficients are only approxi-
mately known. A generalization of the identity $f_n = \sum_{m=0}^{\infty} (n-1)/2 \left( \frac{n-1-m}{m} \right)$ is ob-
tained for the $f_{n,k}$. (Received November 23, 1959.)

564-50. R. R. Phelps: A representation theorem for bounded convex
sets.

A well-known theorem states that every closed convex subset of a normed
linear space can be represented as the intersection of all the closed half-spaces
which contain it. A study is made of spaces $E$ having property (I): Every
bounded closed convex subset $C$ of $E$ is the intersection of the cells containing
it, i.e. if $x \notin C$ there exist $y \in E$ and $r > 0$ such that $x \notin \{z: \|y - z\| \leq r\} \supset C$.
Theorem: A finite dimensional $E$ has property (I) if and only if the extreme
points of $U^*$ (the unit cell of $E^*$) are dense in the boundary of $U^*$. This condition
is seen to be a necessary one in infinite dimensional spaces. A sufficient

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condition, generalizing a result of S. Mazur (Studia Math., vol. 4 (1933) pp. 70-84) is obtained. An example, obtained by renorming $l_1$, shows that the above necessary condition does not imply property (I). The same example answers a question of Mazur (loc. cit.) by exhibiting a space in which the norm is weakly differentiable everywhere but strongly differentiable nowhere.

(Received November 25, 1959.)


Let $\mathcal{A}$ be a linear associative algebra with identity over a field $F$, and let $f$ be a single valued function with domain, $D$, and range in $\mathcal{A}$. Let $G$ be the group of all automorphisms and anti-automorphisms of $\mathcal{A}$ which leave $F$ element-wise invariant. Then the function $f$ is called intrinsic, if it admits $G$ as operator domain, i.e., if for every $\Omega \in G$ $(1) \Omega \in D$ implies $\Omega \in D$, and $(2) x \in D$ implies $f(\Omega x) = \Omega f(x)$. An earlier paper (these Notices, April, 1959, p. 147) characterized intrinsic functions on the algebra of real quaternions. The present paper characterizes intrinsic functions on total matrix algebras over the complex and real fields, as those functions $g(Z; \sigma_1, ..., \sigma_{n-1})$, $Z$ a matrix of order $n$, arising from the extension of scalar functions $g(x; \sigma_1, ..., \sigma_{n-1})$ of the complex variable $x$ and the $n - 1$ complex or real parameters $\sigma_1, ..., \sigma_{n-1}$, the extension being made in a manner entirely analogous to the classical case of a function of a single variable $g(x)$, with $\sigma_1, ..., \sigma_{n-1}$ assuming the values of the elementary symmetric functions of the eigenvalues of $Z$. Elemental results concerning the Hausdorff derivative of such functions are obtained. (Received November 25, 1959.)


Let $X(t)$ be such a (separable) process in $\mathbb{R}^N$, with $X(0) = 0$. Then $e^{t\psi(x, X(t))} = \exp(- t\psi(x))$ where $\psi(x) = i(a, x) + (1/2)x \sigma x' + \int(1 - e^{i(x, y)} + i(x, y)/(1 + |y|^2)) \nu(dy)$, $a$ in $\mathbb{R}^N$, $\sigma$ a non-negative definite symmetric matrix, $\nu$ a measure on $\mathbb{R}^N$ with $\int |y|^2/(1 + |y|^2) \nu(dy) < \infty$. Assume $\sigma = 0$ and define $\beta_1(x) = \beta_1(X) = \inf \{ \alpha > 0: \int |x|^2 \nu(dx) < \infty \}$. If $\beta_1 < 1$ we assume that $a$ is chosen so that $\psi(x) = \int(1 - e^{i(x, y)}) \nu(dy)$. Then the following results hold:

(a) If $\alpha > \beta_1$ then $\lim_{t \to 0} t^{-1/\alpha} X(t) = 0$ with prob. 1. (b) If $\alpha < \beta_1$ then $\lim \sup_{t \to 0} t^{-1/\alpha} |X(t)| = \infty$ with prob. 1. (c) $\beta_1 = \inf \{ \alpha > 0: |y|^{-\alpha} |\psi(y)| \to 0 \}$ as $|y| \to \infty$. (Received November 27, 1959.)

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The same notation is used as in the preceding abstract. Define
\[ \beta_2 = \sup \{ \lambda : \int |x|^{-\lambda} \chi_{N}(1 - e^{-Re\Psi(x)/Re\Psi(x)}\,dx < \infty \} \] and
\[ \beta'_2 = \sup \{ \lambda : |y|^{-\lambda} Re\Psi(y) \to \infty \text{ as } |y| \to \infty \}. \]
Let \( \dim E \) denote the Hausdorff-Besicovitch dimension of a subset \( E \) of \( \mathbb{R}^1 \) or \( \mathbb{R}^N \). The following results hold:

(a) \( 0 \leq \beta'_2 \leq \beta_2 \leq \beta_1 \leq 2 \).

(b) If \( E \) is a Borel subset of \([0,1]\) and \( \dim E = \gamma \), and if \( \beta'_2 \leq N \) then
\[ P\{ \dim X(E) \geq \beta_2 \gamma \} = 1. \]

(c) If \( \beta_1 < 1 \) then \( P\{ \dim X(E) \leq \beta_1 \gamma \} = 1. \)

(d) If for a fixed \( \sigma \) and for every \( \epsilon > 0 \)
\[ |x|^{-\sigma+\epsilon} Re\Psi(x) \to \infty \text{ as } |x| \to \infty \]
then \( \beta'_2 = \beta_1 = \sigma \). (e) If \( \alpha < N \) and \( \alpha < \beta_2 \), or in any case if \( \alpha < \beta'_2 \) then
\[ t^{-1/\alpha} |X(t)| \to \infty \text{ in prob. as } t \to 0. \]

The notation is that of the preceding abstracts. If \( X \) is a subordinator
(that is, \( N = 1 \) and \( X(*) \) is monotone nondecreasing with prob. 1) then
\[ \beta_1(X) \leq 1, \] and \( \gamma \) is concentrated on the half-line \( y \geq 0 \) and for \( x \geq 0, \]
\[ \mathcal{E}e^{-xX(t)} = \exp(-tg(x)) \]
where \( g(x) = \int (1 - e^{-xy}) \gamma(dy) \). For a subordinator define
\[ s_2 = \sup \{ \lambda : \int_{0}^{\infty} [(1 - e^{-x})/g(x)]\,dx < \infty \}. \]
The following results hold: (a) if \( X \) is a subordinator
then \( \beta'_1 = \inf \{ \sigma > 0 : x^{-\sigma} g(x) \to 0 \text{ as } x \to \infty \} \) and
\[ s_2 = \sup \{ \lambda : x^{-\lambda} g(x) \to \infty \text{ as } x \to \infty \}. \]
In addition \( \beta_2 \leq s_2 \leq \beta_1 \), and there is a subordinator for which the
strict inequality \( s_2 < \beta_1 \) holds. (b) If \( X \) is a subordinator and \( \dim E = \gamma \) then
\[ P\{ \gamma s_2 \leq \dim X(E) \leq \gamma \beta'_1 \} = 1. \]
(c) If \( X \) is any time-homogeneous differential process in \( \mathbb{R}^N \) and \( V_\alpha(t) = \alpha \)-variation of \( X \) over \([0,t]\) is finite with prob. 1 for
some (and hence for all) \( t > 0 \) then \( V_\alpha \) is a subordinator, and
\[ \beta_1(V_\alpha) = \beta_1(X)/\alpha. \]
If \( X \) is the symmetric stable process of index \( \alpha \) then \( V_\alpha \) is the stable subordinator
of index \( \alpha/\alpha \). Our definition of \( \alpha \)-variation is that of N. Wiener, The
vol. 3 (1924). (Received November 27, 1959.)

The central intertwining number of a representation \( L \) of a finite group \( G \)
is defined to be the vector space dimension \( CI(L) \) of the center \( CR(L) \) of the
intertwining algebra of \( L \). If \( CI(L) = 1 \), \( L \) is called a factor representation.
CR(L) is characterized as follows. Let $C_1, \ldots, C_m$ denote the distinct conjugacy classes of $G$ and let $S_1 = \sum_{s \in C_1} L_s$, $i = 1, \ldots, m$. Then $CR(L)$ is the vector space generated by $\{S_1, \ldots, S_m\}$. Now let $H$ be a subgroup of $G$ and let $L$ be a representation of $H$. Using the above characterization, a method for calculating $CI(U^L)$ is developed, where $U^L$ is the representation of $G$ induced by $L$. This method is used to derive the following result. **Theorem:** Let $H$ be a subgroup of a finite group $G$ and let $L$ be a factor representation of $H$. Then the induced representation $U^L$ is a factor representation of $G$ if and only if, for all conjugacy classes $D$ of $G$, $\sum_{x \in D} H^L_x = 0$, for all $r \in G$ such that $r \notin H$. (Received November 27, 1959.)


A functional defined on the automorphisms of a Banach space is said to transform by the Euler composition if $F[f \circ g] = g^{-1} \circ F[f] \circ g + F[g]$. Example: $\Phi[A] = A^{-1}dA$, a matrix function, appearing in the theory of fibre spaces, Frenet formulae, Coriolis force. The Euler formulae for the rotation of a rigid body result as $\Phi$ of the product of 3 rotations. Definitions: a B-manifold is a paracompact space covered by neighbourhoods homeomorphic to the unit ball of a real Banach space, with differentiable maps induced by intersections. A B-fibre space $(E, B, F)$ is a fibre space in which $B$ and $F$ are B-manifolds, and the fibre maps $f_b$: $F_b \rightarrow F$ are Fréchet differentiable in $b$. For bounded integrable $\Phi[f_b]$, the $f_b$'s are defined by $\Phi$ up to a constant automorphism of $F$. This approach includes, for suitable $F$, Finsler and Higher Order Geometries, eliminating tensor analysis where it is inadequate. If $F$ is the tangent space of $B$, parallelism is defined by $dv = -\Phi(v), v \in F_b$, curvature by $\Omega = d\Phi - \Phi \wedge \Phi$ with an appropriate definition of $\wedge$. The automorphism induced by parallelism along a closed curve may then be computed as $\int \Omega(v)$ over an extension of the parallel field with a singularity only at the starting point. (Received November 27, 1959.)


Two differential geometries of same type defined by B-fibre spaces $(E,F,B),(\tilde{E},B,F)$ with fibre maps $f_b, \tilde{f}_b$ may be transformed one into another by a differentiable family $g_b$ of automorphisms of $F$: $\tilde{f}_b = g_b \circ f_b$. A functional
defined on $F_B$ will be a general tensor, if it is mapped on the corresponding functional on $F_B$ by composition with $g_b$'s and $g_b^{-1}$'s. It will be a restricted tensor if it behaves tensorially only under maps $g_b$ deriving locally from local automorphisms of $B$ (in the finite dimensional case: $g_b$ is a Jacobian matrix).

The covariant differential, $d\nu_b + \phi(\nu_b)$ is a general covariant. The curvature form is only a restricted tensor, it transforms generally under the Euler composition: $\Omega[g \circ f] = g^{-1} \Omega[f] \circ g + \Omega[g]$. This yields a simple proof of the characterization of isometric Riemann spaces by the tensor character of the curvature, and corresponding theorems for all other geometric structures represented by $B$-fibre spaces. If a family of automorphisms $g_{b,t}, t \in [0,1]$, $g_{b,0} = \text{identity}$ is given, a Lie derivative may be defined. The Lie derivative of a quantity transformed by Euler composition is always a general tensor.

(Received November 27, 1959.)


Take a finite dimensional manifold. Its tangent space may be mapped into a standard Euclidean space by an affine mapping $A$. Skew $\phi[A]$ characterizes orthogonal matrices $A$. Writing $\Phi = \Gamma^i_{jk} dx^k$, one obtains by the integration theorem of abstract 564-56 a refinement of H. Weyl's integration theorem on linear connections: The symmetric part of the connection derives from a uniquely defined Riemann metric, whose matrix $G_\ast = (g_{ik}) = \Lambda^t \Lambda$ derives from a mapping of the tangent space into a standard Euclidean space. The torsion tensor derives as $\Phi(0)$ from a field of orthogonal matrices $O(c)$, being functionals of rectifiable curves on the manifold, acting by left translations on $A$, therefore leaving $G_\ast$ invariant. Hence, the antisymmetric connections on a manifold $M^n$ are in 1-1 correspondence with the principal $O(n,R)$ bundles on $M^n$. So the linear space of the global connections is $H^1(M^n, O)$, $O$ being the sheaf of germs of sections of $O(n)$ over $M^n$. There is an extension of the theorem to structures defined over a Hilbert space. (Received November 27, 1959.)


It is shown that there exists a countable connected Hausdorff space which is bihomogeneous, that is, if $x$ and $y$ are points of the space then there exists a homeomorphism $f$ of the space onto itself such that $f(x) = y$ and $f(y) = x$. This
space has the additional property that it has no cut point. If $Z$ is a connected topological space and $z \in Z$, then the statement that $z$ has component number $n$ means that $n$ is a positive integer and $Z - \{z\}$ has exactly $n$ components. It is shown that there exists a countable connected Hausdorff space $Z$ with the following properties: (1) $Z$ has no cut point and (2) distinct points of $Z$ have distinct component numbers. It follows that the only homeomorphism of $Z$ onto itself is the identity. (Received November 27, 1959.)

564-60. J. R. Rice: Tchebycheff approximations.

In this paper the theory of Tchebycheff approximations by $ab^x + c$ and $ab^x\cos(\theta_0 + \phi x) + c$ is studied where $a, b, c, \theta_0$ and $\phi$ are parameters to be varied. A complete development of the theory is made for $ab^x + c$. The topics considered are existence theorem, characterization of best approximations, approximations on subsets, uniqueness theorem, approximation on finite point sets, limiting relations between approximations on $[0,1]$ and finite point sets and computational procedures for finite point sets. A really surprising phenomenon of this theory is that best approximations do not necessarily exist on finite subset of $[0,1]$, although best approximations on $[0,1]$ exist for every continuous function. The results for $ab^x\cos(\theta_0 + \phi x) + c$ are very incomplete and only the rudiments of a theory are established. This is due mainly to the intractibility of the transcendental equations arising in the analysis. On the other hand some useful formulas for applications are obtained. (Received November 27, 1959.)

564-61. Irving Reiner: Behavior of integral group representations under ground ring extension.

For $i = 1, 2$ let $\mathfrak{p}_i$ be a discrete valuation of an algebraic number field $K_i$ with valuation ring $R_i$, where $K_2 \supset K_1$ and $R_2 \supset R_1$. Given a finite group $G$, an $R_iG$-module shall mean a left unital $G$-module with finite $R_i$-basis. Let $M, N$ be $R_iG$-modules. Theorem: $R_2M \cong R_2N$ (as $R_2G$-modules) implies $M \cong N$, if either (i) $\mathfrak{p}_1, \mathfrak{p}_2$ have the same residue class fields, or (ii) the irreducible constituents of $K_iM$ are distinct and absolutely irreducible. The theorem also holds for the special case where $G$ is a $p$-group ($p$ odd), and where $\mathfrak{p}_1$ is the $p$-adic valuation of the rational field $K_1$, assuming only that $K_1M$ is irreducible. Finally, let $\mathcal{O}_1$ be the ring of all algebraic integers in $K_1$, and $G$ any group. For $M, N$ a pair of $\mathcal{O}_1G$-modules, it is shown that $\mathcal{O}_2M \cong \mathcal{O}_2N$ implies $M \cong N$, provided that $K_1M$ is absolutely irreducible and $\mathcal{O}_1$ a principal ideal ring. This latter hypothesis cannot be dropped. (Received November 30, 1959.)
On mappings of an extended class in the theory of functions of two complex variables, I.

The class of analytic functions of two complex variables is too small for us to be able to extend various methods used in the case of one variable. When considering special domains, namely, those with a distinguished boundary surface, it is useful to introduce (real) functions of the "extended class." These functions play a role similar to that of potential functions in the case of one variable ([1] Bergman, Compositio Math. vol. 6 (1939) p. 305). In addition to functions one can consider mappings of an extended class ([2] Bergman, J. Math. Mech. vol. 7 (1958) p. 937, and [3] Hitotumatu, J. Math. Mech. vol. 8 (1959) p. 77).

Let $P^4$ be a domain bounded by two segments $g^3_k$, $k = 1, 2$, of analytic hypersurfaces. Here $g^3_1$ is a segment of the hypersurface $[h(z_1, z_2) = \exp(i\lambda_1)]$, $0 \leq \lambda_1 \leq 2\pi$, where $h$ is an analytic function of two complex variables, regular in a sufficiently large domain. ($g^3_1$ will be described later.)

By the mapping $z_1 = \zeta_1, z_2 = h(z_1, z_2)$, $P^4$ is transformed into $M^4$ bounded by $m^3 = \overline{z_1} + \overline{z_2}$, where $\overline{z_2}$ is a segment of $[z_2 = \exp(i\lambda_2)]$ and $\overline{z_1} = \overline{z_2} = \overline{z_2} = \overline{z_2} = \overline{z_2} = \overline{z_2}$.

564-63. Stefan Bergman: On mappings of an extended class in the theory of functions of two complex variables, II.

Assume that every $J^2(\lambda_1), 0 \leq \lambda_1 \leq 2\pi$, is a one-to-one image $z_1 = \phi_1(Z, \lambda_1), \nu = 1, 2$, of $|Z| \leq 1, |\phi_1(\exp(i\theta), \lambda_1)| = 1$. The $\phi_1$ are continuously differentiable functions of $\lambda_1$. Every $J^2(\lambda_2)$ is a simply connected domain lying in $z_2 = \exp(i\lambda_2)$. Let $T^2_1 = [z_1 = f_1(z_2)] \cap M^4$ be a segment of an analytic surface; $f_1(z_2)$ is regular and single valued in a sufficiently large domain. We assume that $T^2_1 \cap m^3$ is the boundary of $T^2_1$ and it is a simple, closed curve which lies inside of $z_2$. $G = - (\log |z_1 - f_1(z_2)| + g_1(z_1, z_2))$ is the Green's function of the extended class of $M^4$. (See [4] Bergman, Trans. Amer. Math. Soc. vol. 63 (1948) p. 523.) Let $H(z_1, z_2)$ be the conjugate of $G$ for every fixed value $z_2 = z_2, |z_2| \leq 1$. Then the pair $w_1 = \exp[G + iH], w_2 = z_2$ is a mapping of an extended class transforming $M^4$ into $B^4 = [|w_k| \leq 1, k = 1, 2]$. $M^4 \cap [z_1 = f(z_2)]$ goes into $[w_1 = 0] \cap B^4$. In analogy to considerations in [2], bounds for the ratio $ds/dS$ are given. Here $dS$ is the length of the line element of the invariant metric of $M^4$, $ds$ that of $B^4$. (Received November 27, 1959.)
564-64. M. W. Hirsch and R. S. Palais: Local triviality of certain restriction mappings.

Let $W$ be a compact differentiable (= $C^\infty$) manifold (possibly with boundary), $V$ a closed differentiable submanifold, and $M$ a differentiable manifold without boundary. Let $B$ be a differentiable fiber bundle over $W$ with fiber $F$ a manifold without boundary. Let $\mathcal{E}(W,M)$ and $\mathcal{E}(V,M)$ be the spaces of differentiable imbeddings of $W$ and $V$ respectively in $M$ in the $C^r$-topology $1 \leq r \leq \infty$ and let $\pi_1: \mathcal{E}(W,M) \to \mathcal{E}(V,M)$ be the restriction map: $\pi_1(f) = f|V$. Let $\mathcal{X}_d(W,B)$ and $\mathcal{X}_d(V,B|V)$ be the space of differentiable cross sections of the bundle $B$ over $W$ and $V$ respectively in the $C^r$-topology and let $\pi_2: \mathcal{X}_d(W,B) \to \mathcal{X}_d(V,B|V)$ be the restriction map. Let $\mathcal{X}(W,B)$ and $\mathcal{X}(V,B|V)$ be the spaces of continuous cross sections of $B$ over $W$ and $V$ respectively in the compact-open topology and let $\pi_3: \mathcal{X}(W,B) \to \mathcal{X}(V,B|V)$ be the restriction map. The following is proved: Theorem. The mappings $\pi_1$, $\pi_2$, and $\pi_3$ are locally trivial. If for $B$ we take $W \times M$, then an element of $\mathcal{X}(W,B)$ ($\mathcal{X}(W,B)$) is just a differentiable (continuous) map of $W$ into $M$ so restriction to $V$ on these spaces is again locally trivial. (Received November 25, 1959.)


Let $W$ be a compact $k$-manifold, $V$ a closed submanifold, and $M$ a manifold without boundary of dimension $> k$. Let $I(W)$ be the space of $C^\infty$ immersions of $W$ in $M$ in the $C^r$ topology, $1 \leq r \leq \infty$, and similarly for $I(V)$. Let $\pi: I(W) \to I(V)$ be the restriction map $\pi(f) = f|V$. Theorem 1. $(\pi, I(W), I(V))$ is a fiber space in the sense of HU (Proc. Amer. Math. Soc. vol. 1(1950) pp. 756-762). Corollary. $\pi$ has the covering homotopy property for arbitrary spaces. Let $Q$ be the space of $GL(k)$-equivariant maps $T_k(W) \to T_k(M)$ in the compact-open topology, where $T_k$ means the bundle of $k$-frames. Let $\phi: I(W) \to Q$ be defined by $\phi(f) (X_1, \ldots, X_k) = (df(X_1), \ldots, df(X_k))$. Theorem 2. $\phi$ is a homotopy equivalence. These are generalizations of results of S. Smale (Ann. of Math. vol. 69 (1959) pp. 327-344) and M. Hirsch (Immersions of manifolds, to appear in Trans. Amer. Math. Soc). (Received November 25, 1959.)

This paper is concerned with algebraic systems having two everywhere-defined binary operations $\Lambda$ and $\vee$. Such a system is called a "nest" (P. Jordan, Die Theorie der Schrägverbände, Abh. Math. Sem. Univ. Hamburg, vol. 21 (1957) p. 129) if it obeys the laws $x \Lambda y = x$, $x \vee y = y$. It is shown that a system which obeys the associative laws, the absorption laws $x \Lambda (x \vee y) = x$, $(y \Lambda x) \vee x = x$, $x \vee (x \Lambda y) = x$, and the pseudo-commutative laws $x \Lambda (y \Lambda z) = x \Lambda (z \Lambda y)$, $(x \vee y) \Lambda z = (y \vee x) \Lambda z$, is isomorphic to the direct union of a nest and a lattice. A system which obeys one associative law, one of the absorption laws $x \Lambda (y \Lambda x) = x$, $(x \Lambda y) \Lambda x = x$, the absorption laws $x \Lambda (x \vee y) = x$, $(y \Lambda x) \vee x = x$, and the distributive laws $(x \vee y) \Lambda z = (x \Lambda z) \vee (y \Lambda z)$, $x \vee (y \Lambda z) = (x \Lambda y) \Lambda (x \vee z)$, is isomorphic to the direct union of a nest and a distributive lattice. In each case the laws indicated are independent axioms for the family of all systems which obey them. Other sets of independent axioms for these families are given. (Research assisted by the University of New Zealand Research Fund.) (Received November 30, 1959.)


Let $G$ be a Lie group. A $G$-space is a completely regular space together with an action of $G$ on $X$. A $G$-space $X$ is said to admit slices along one of its orbits $\Omega$ if there is an equivariant retraction of an invariant neighborhood of $\Omega$ onto $\Omega$. Mostow has shown (Theorem 3.1, Equivariant imbeddings in Euclidean spaces, Ann. of Math., vol. 65 (1957)) that if $G$ is compact then every $G$-space admits slices along each of its orbits. We prove the following generalization: Theorem. Let $X$ be a $G$-space and $x \in X$. The following are equivalent: (1) The isotropy group at $x$ is compact and $X$ admits slices along the orbit of $x$. (2) There exists a neighborhood $V$ of $x$ in $X$ such that $\{g \in G | gV \cap V \neq \emptyset\}$ has compact closure in $G$. As a consequence we can prove the following generalization of Mostow's equivariant imbedding theorem.

Theorem. Let $G$ be a matrix group and let $X$ be a separable metric $G$-space of finite dimension. Suppose that the isotropy groups at points of $X$ fall into a finite number of conjugate classes. Then $X$ admits an equivariant imbedding in a linear $G$-space provided given $x$ and $y$ in $X$ there are neighborhoods $U$ and $V$ of $x$ and $y$ respectively such that $\{g \in G | gU \cap V \neq \emptyset\}$ has compact closure.

Consider two simultaneous first order differential equations \( x'(t) = F(x, y, t), \ y'(t) = G(x, y, t) \). Runge-Kutta type integration methods are developed which allow different integration steps to be used for these equations. These methods retain the desirable properties of Runge-Kutta methods, namely the self starting property and ease of change of integration step. Two different approaches are considered and extensive experimental work is reported upon. Experiments are done both in situations where these methods are advantageous and where they are not. It is seen that these methods are more efficient than the normal Runge-Kutta methods if they are at all applicable and in ideal situations they give the same accuracy with 90% less computation. These methods are applicable to six degree of freedom missile simulations, for which they were developed. (Received November 27, 1959.)


The function \( R(a, b, x) = [D - a][D - (a - 2)]...[D + (a - 2)][D + a] \sinh^b x \) is introduced and studied. \( D \) is the differentiation operator and \( a \) is an integer. \( R \) is shown to satisfy \([\sinh xD^2 + (a - b)\cosh xD - ab \sinh x] R = 0\). By means of the differential equation it is shown that \( R \) includes some classical Gegenbauer functions as special cases. Let \( C_n^\lambda(x) \) denote Gegenbauer's derived polynomial and \( C_n^{\lambda \alpha}(x) \) denote Gegenbauer's associated function. Typical relations are \( C_n^{\lambda \alpha}(\cosh x) = kR(\alpha + n + 2\lambda, n - \alpha, x) \) for \( \alpha > 0 \) and \( 2\lambda \) an odd integer and \( C_n^{\lambda \alpha}(\cosh x) = kR(n - \alpha - 1, n + \alpha + 2\lambda - 1, x) \sinh^{-\alpha} x \) when \( \lambda + \alpha > 0 \). A function analogous to \( R \) for circular arguments is defined and similar results are obtained. (Received November 27, 1959.)


Say that a subspace \( M \) of a (real) normed linear space \( E \) has the Haar property if to each \( x \) in \( E \) there corresponds a unique nearest point in \( M \). For \( E = C[0, 1], \) and \( M \) of dimension \( n \), Haar (Math. Ann. vol. 78 (1918) pp. 294-311) proved that \( M \) fails to have the Haar property if and only if there exists a function \( x \) in \( M \) which is not identically zero and which has at least \( n \) distinct zeros in \([0, 1]\). Recalling the characterization, by means of evaluation functionals, of
the extreme points of the unit sphere $S^*$ of $C[0,1]$; we may formulate Haar's theorem as follows: An $n$-dimensional subspace $M$ of $E$ fails to have the Haar property if and only if there exists an $x$ in $M$, $x \not= \emptyset$, and a linearly independent set $\{f_i: i = 1,2,\ldots,n\}$ of extreme points of $S^*$ such that $f_i(x) = 0$, $i = 1,2,\ldots,n$. The "only if" part of this theorem is shown to hold for arbitrary normed spaces $E$; simple examples show that the "if" part need not be true in general. The "only if" part is then used to prove the corresponding portion of the obvious analogue to Haar's theorem for $C_0(X)$ (the space of all continuous functions vanishing at infinity on the locally compact Hausdorff space $X$); the converse is proved directly. Open question: For what locally compact $X$, other than subsets of the line, does $C_0(X)$ possess Haar subspaces of dimension greater than 1? (Received November 25, 1959.)


The author has written a program for the UNIVAC M-460 Computer to find all latin squares orthogonal to a fixed latin square of order 10. The running time was about one hour per initial square in four cases where orthogonal mates were known in advance to exist. The speed of the search was due primarily to having storage sufficient to generate and store all ca. 800 transversals of the given latin square, then generate all possible latin squares from these transversals. The quantity of output is large and will require considerable processing for reduction of isomorphic examples. One result has been obtained: There exist pairs of orthogonal latin squares of order 10 with no pair of orthogonal latin subsquares of order 3. Bose, Shrikhande, and the author had previously constructed a fair variety of pairs of orthogonal latin squares of order 10 (mostly unpublished), in every instance with a pair of orthogonal subsquares of order 3. The author proved (these Notices, Abstract 558-54 vol. 6 (1959) p. 390) that no such pair of orthogonal latin squares of order 10 can have a third latin square orthogonal to both. Attempts to extend from two to three mutually orthogonal latin squares of order 10 have been unsuccessful to date. (Received December 2, 1959.)

If a curve is unimodal, i.e., increasing up to a point and then decreasing, then the optimal procedure for locating its maximum point to a given accuracy is well known. It is our purpose to extend this result about curves to higher dimensions. It is found that the most obvious analogue to the 1-dimensional problem is unsolvable and offers the following formulation: To find, with a minimum number, $N$, of evaluations of $f(x,y)$, a point $(\xi,\eta)$ such that $f(\xi,\eta) \geq \max_{0 \leq a \leq n; 0 \leq b \leq n} f(a/n, b/n)$, $a$ and $b$ integers, it is found that $N$ is of the order of $\log n$, in fact $\log n < N < 90 \log n + 5$. By restricting $f$ to be $C^1$ in addition to unimodal a procedure is given with $N < 12.5 \log n + 5$. Extensions to dimensions 3 and higher are also given. (Received November 30, 1959.)


Let $M$ be a complete Riemannian manifold, $x_0 \in M$. Let $N$ be a subspace of $M_{x_0}$, the tangent space to $M$ at $x_0$. A once-broken geodesic $g$, starting at $x_0$, formed of two geodesic segments $g_1$ and $g_2$, is called admissible if (1) the tangent vector to $g_1$ at $x_0$ belongs to $N$, as does the tangent vector to $g_2$ parallel-translated back to $x_0$ and (2) $g_2$ is contained in a geodesically-convex neighborhood of the end-point of $g_1$. For each point $x$ of such an admissible $g$, let $N_{g,x} \subset M_x$ be the parallel-translation of $N$ along $g$ to $x$. Suppose that, for $v_1 \in N_{g,x}$ and $v_2 \in N_{g,x}$, $R_x(v_1, v_2)$ (the infinitesimal rotation provided by the curvature tensor) leaves $N_{g,x}$ invariant. **Theorem:** With these assumptions, there is a complete, immersed totally-geodesic submanifold of $M$ whose tangent space at $x_0$ is $N$. The method of proof is similar to that used by Ambrose (Ann. of Math. (1956) pp. 337-363), and indeed his theorem on isometry is a consequence of this result. (Received December 1, 1959.)


Let $X$ be a compact Hausdorff space and let $T$ be a group of self homeomorphisms of $X$. The points $x$ and $y$ of $X$ are called proximal if, for any member $\beta$ of the uniformity of $X$, there exists $t \in T$ such that $(xt, yt) \in \beta$. Let $P$ denote the proximal relation in $X$. Let $E$ denote the closure of $T$ in $X^X$; $E$ is a compact semigroup. **Theorem 1.** Suppose $P$ is an equivalence relation in $X$. Then $X = \bigcup N_x$, where $N_x T \subset N_x$, each $N_x$ contains precisely one minimal set.
and, if \( x \in N_x - M_x \), then \( x \) is proximal to some \( x' \in M_x \). **Theorem 2.** If \( P \) is a closed relation, \( P \) is an equivalence relation. **Theorem 3.** If \( I \) is a minimal right ideal in \( E \), let \( J(I) \) denote the set of idempotents in \( I \). Let \( J = \bigcup [J(I)/I \) a minimal right ideal]. Suppose \( T \) is pointwise almost periodic on \( X \). Then

(i) If \( x \in X \), \( xJ = P(x) \). (ii) If \( I \) is a minimal right ideal in \( E \), then \( xJ(I) \) consists of mutually proximal points. Moreover, if \( y \) is proximal to all \( x' \in xJ(I) \), then \( y \in xJ(I) \). (iii) If \( p \in I \) is such that \((x,xp) \in P \) for all \( x \in X \), then \( p \in J(I) \).

(Received December 2, 1959.)

564-75. C. L. Dolph: *On the structure of the linearized equations governing a streaming electron-ion gas.*

The problem of a point source moving through a fully ionized gas has been treated by Kraus and Watson (Phys. Fluids, vol.1 (1958) p. 480) and by P. Greifinger (Rand Report R339, paper No. 19, June, 1959). In addition R. Pappert (Convair Report ZPH-026, December 1958) has demonstrated the equivalence of the approach used by Kraus and Watson to that of the Bohm-Pines theory of random phase approximation (Phys. Rev. vol. 85 (1952) p. 338) and to linearized fluid dynamics for an assumed isothermal expansion. In this paper the nine equations governing the flow in the above fluid dynamical situation are linearized about the streaming gas velocity and uncoupled by the introduction of two scalar fields in the spirit of the method of Oseen. This method is more general in that it permits the inclusion of a coupling between the electrons and ions proportional to their relative velocities (Ohm's law) and separate scalar viscosities. The resulting equations include those of Kraus and Watson and admit a class of separated solutions in all cylindrical coordinate systems. Finally, this method clearly demonstrates the irrotationality of the flows so obtained. (Received December 2, 1959.)

564-76. P. C. Gilmore: *Two forms of completeness.*

A theory \( F \) is any applied first order predicate calculus with at least one predicate symbol, at least one individual constant, perhaps some function symbols, and a set of closed well-formed formulas (sentences) specified as axioms. A sentence is provable in \( F \) if it is derivable from the set of axioms in the first order predicate calculus. An eligible semi-model for \( F \) is any consistent set of atomic sentences and negated atomic sentences such that each atomic sentence of \( F \) or its negation is a member of the set. An admissible
semi-model of F is an eligible semi-model that is consistent with respect to the set of sentences provable in F. A theory is said to be U-complete if and only if whenever Q(t) is provable for every constant term t, (x)Q(x) is also provable. A theory is said to be E-complete if and only if whenever a sentence (Ex)Q(x) is provable, Q(t_1)v...v Q(t_k) is provable for some k and some constant terms t_1,...,t_k. U-completeness is thus related to \( \Gamma \)-completeness as introduced by Henkin as an extension of \( \omega \)-completeness. We prove that every E-complete theory is U-complete and that every U-complete theory with only finitely many admissible semi-models is E-complete. (Received December 2, 1959.)

564-77. E. T. Parker: Nonextendibility conditions on mutually orthogonal latin squares.

Theorem 1. If a set of t mutually orthogonal latin squares of order n has a set of t mutually orthogonal latin subsquares of order r, with \( r < n \), then \( n \geq (t+1)r \). Theorem 2. If a set of t mutually orthogonal latin squares of order n has a set of t mutually orthogonal latin subsquares of order r, with \( r < n \), and there exists a latin square of order n orthogonal to all t, then \( r(n - r) \leq n \lfloor (n - r)/(t + 1) \rfloor \). (The bracket denotes greatest integer.) Both theorems are proved by counting cells on transversals. Note that the conclusion of Theorem 2 is stronger than that of Theorem 1 precisely when the number in brackets is not an integer. These results are generalizations of the author's in these Notices, Abstract 558-54, (vol. 6) p. 390. (Received December 3, 1959.)

564-78. S. A. Amitsur: Derived functors without projective and injective resolutions.

Let C be an abelian category with finite direct sums and let F be a covariant functor defined on C with values in a category C' of a similar type. Generalizing the notion of direct limits, we define the nth derived functor \( F^{(n)}(A) \) to be the generalized direct limit of the nth homology object (n \( \geq 0 \)) \( H^n[F(X)] \) of the complex \( F(X) \) where X ranges over all acyclic cochain complexes in C augmented by A. If F is half exact then \( F^{(n)} \) is a covariant half exact functor, not necessarily defined for all \( A \in C \). Nevertheless, if for a short exact sequence \( 0 \rightarrow A' \rightarrow A \rightarrow A'' \rightarrow 0 \), \( F^{(n)}(A') \), \( F^{(n)}(A) \) and \( F^{(n)}(A'') \) exist then we have the connecting exact sequence: \( \ldots \rightarrow F^{(n)}(A') \rightarrow F^{(n)}(A) \rightarrow F^{(n)}(A'') \rightarrow F^{(n+1)}(A') \rightarrow \ldots \) with the standard properties. The main idea
in this representation of derived functors is replacing injective resolutions by
generalized direct limits of complexes with a dual procedure for replacement
of projectives. (Received December 3, 1959.)

564-79. H. D. Brunk: On a theorem of E. Sparre Andersen and its
applications to tests against trend.

Let \( X_1, X_2, \ldots, X_n \) be symmetrically dependent random variables. Set
\[ S_0 = 0, \quad S_r = \sum_{i=1}^{r} X_i, \quad r = 1, 2, \ldots, n. \]
E. Sparre Andersen (Math. Scand. vol. 2 (1954) pp. 195-223) found the distribution of the number of sides of the greatest
convex minorant of the set of points \((0,S_0), (1,S_1), \ldots, (n,S_n)\). Andersen observed
that it is the same distribution as that of the number of cycles in a randomly
chosen permutation of the integers \(1, 2, \ldots, n\). (Cf. Feller, An introduction to
probability theory and its applications, vol. 1, 2d. ed., John Wiley and Sons Inc.,
1957, p. 242.) It is possible to exhibit both as special instances of a class of
random variables sharing this distribution. Theorems giving this result have
application also to the problem of determining the distribution of the maximum
likelihood statistic in tests against trend, when the populations belong to an
exponential family (a Koopman-Pitman class). These results specialize in the
case of normally distributed random variables to results of Bartholomew
(Biometrika vol. 46 (1959) pp. 37-48). (Received December 3, 1959.)

564-80. C. C. Buck: An axiom system for the geometries of the
Euclidean family.

The geometries of the Euclidean family, hyperbolic, Euclidean, and
elliptic, have often been treated informally as if they differed only in an
assumption regarding the existence of parallels. A corresponding formal treat-
ment appeared in an article by O. Wyler [Compositio Mathematica vol. 11
(1953) pp. 60-70], who handled the main problem, the order of points on a line,
by referring it back to the order of lines in a plane pencil. This notion is
sophisticated and does not provide a mathematical basis for the mentioned
informal treatments. Such a basis is provided in this paper by axiomatizing
the order of points in a closed segment and incorporating this in a formal sys-
tem more accessible to the intuition. (Received December 4, 1959.)

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Let $T$ be the translation group of a translation plane $A$ of char $\neq 2$, (see Pickert, Projekttive Ebenen). If $T = X \oplus Y$, a congruence preserving automorphism $r$ of $T$ is a reflection with axis $X$ and center $Y$ if $xr = x$ for $x$ in $X$ and $yr = -y$ for $y$ in $Y$. Theorem: $A$ is a translation plane over a quadratic alternative division algebra $F/F_0$ of char $\neq 2$ if and only if the translation group $T$ of $A$ permits a reflection $r$ and an additive group $H$ of congruence preserving automorphisms $a$ of $T$ such that (i) $Q(a) = a^2$ is in the kernel $K(A)$ of $A$; (ii) $a \pm r$ is congruence preserving; (iii) $ra + ar = 0$; (iv) given $x \neq 0$ in the axis of $r$ and $y \neq 0$ in the center of $r$, there is $a$ in $H$ such that $xa = y$; (v) there is $e$ in $H$ such that $Q(e)$ in $H$ for all $a$ in $H$. For the $F_0$-vector-space $V = F_0 x + F_0 H$, $Q(v) = v^2(v$ in $V)$ is a quadratic $F_0$-form. Being "axis and center of a reflection in $V"$ defines an orthogonality in $T$ (and in $A$), which can be described by a Hermitian form. Let $U$ be the unitary group generated by all nonisotropic $v$ in $V$. For $u$ in $U$ define $\bar{u}$ by $v \bar{u} = u^{-1}v$ for all $v$ in $V$. $u \rightarrow \bar{u}$ is a homomorphism from $U$ onto $0^+(F_0, Q)$. (Received December 4, 1959.)


Two dimensional transformations $P_i$ are chosen to minimize $N^2(P_i^{-1}AP_i)$, (since $\inf P N^2(P^{-1}AP) = \sum_{i=1}^{n} |\lambda_i|^2$) where $A$ is a real matrix with eigenvalues $\lambda_i$ and $N^2(A) = $ trace $AA^*$. Letting $P_m = T_{\gamma=1}^{m} P_i$, where the $P_i$ range over all two dimensional subspaces, it is shown that in general $N^2(P_m^{-1}AP_m)$ $\rightarrow m \sum_{i=1}^{n} |\lambda_i|^2$ and $(A^* A - AA^*) \rightarrow 0$. The limiting matrix $B = (b_{ij})$ is a normal matrix of the form: $b_{ii} = \lambda_i$ for $\lambda_i$ real; $b_{ii} = b_{jj}$ and $b_{ij} = -b_{ji}$ for $\lambda_{ij} = b_{ii} + b_{jj}$ and all other elements arbitrarily small. In the defective case a canonical form is attained from which the multiple roots may be read directly. Each $P_i$ in the $k$-$m$ plane is chosen as a product of a rotation ($r_{kk} = r_{mm}$ $= \cos \theta$; $r_{km} = -r_{mk} = \sin \theta$; and $r_{ij} = \delta_{ij}$ for $i,j \neq k,m$) and a shear ($s_{kk} = s_{mm} = \cosh x$; $s_{km} = s_{mk} = \sinh x$; and $s_{ij} = \delta_{ij}$ for $i,j \neq k,m$) where $\theta$ and $x$ are chosen to yield the maximum decrease in $N^2(P_i^{-1}AP_i)$. This choice also annihilates the kmth element of $A^* A - AA^*$. An experimental program has been written and tested successfully for a number of different types of matrices on the IBM 650. (Received December 4, 1959.)
564-83. P. J. McCarthy: Some irreducibility theorems for Bernoulli polynomials of higher order.

Let \( B_n^{(k)}(x) \) be the Bernoulli polynomial of order \( k \) and degree \( n \). By making use of the Eisenstein irreducibility criterion and some results concerning the Bernoulli numbers of higher order due to Carlitz (Proc. Amer. Math. Soc. vol. 3 (1952) pp. 608-613), the following results are obtained: (1) if \( p \) is an odd prime, \( k < p \) and \( 1 \leq m < p - k + 1 \), then \( B_m^{(k)}(p-1)(x) \) is irreducible (over the rational field); (2) if \( p \) is an odd prime, \( k \neq p \), \( t > 0 \) and \( 1 \leq m \neq p \), then \( B_m^{(k)}(p-1)p^t(x) \) is irreducible; (3) if \( p \) is an odd prime, \( k < p \), \( 1 \leq n \leq p - k + 1 \) and \( 2m = n(p - 1) \), then \( B_{2m+1}^{(k)}(x)/(x - k/2) \) has an irreducible factor of degree \( \geq 2m + 1 - p \); (4) for every integer \( k \geq 1 \) there is an integer \( T = T(k) \) such that for all \( t \geq T \), \( B_{2t}^{(k)}(x) \) is irreducible. (Received December 4, 1959.)

564-84. J. R. Shoenfield: The no counter-example interpretation.

New proofs are given for results of Kreisel (J. Symbolic Logic vol. 16, pp. 241-267 and vol. 17, pp. 43-58). The consistency proof of Ackermann, used in Kreisel's proof, is replaced by a consistency proof due to Schütte, after replacing the \( \omega \)-rule in Schütte's system by an effective \( \omega \)-rule.

(Received December 4, 1959.)


The main object of this paper is to answer some problems posed by J. Dieudonné in his paper Denumerability conditions in locally convex vector spaces, Proc. Amer. Math. Soc. vol. 8 (1957) pp. 367-372. His main two results are as follows: Proposition 1. If \( E \) is a barrelled space on which there is a countable fundamental system of convex compact subsets, then it is the strong dual of a Fréchet Montel Space. Proposition 2. If \( E \) is either bornological or barrelled, and if there is a countable fundamental system of compact subsets, then \( E \) is dense in the strong dual of a Fréchet Montel Space. Two questions raised by Dieudonné in connection with these results are: (a) If \( E \) is either bornological or barrelled then it certainly is quasi-barrelled. Can one substitute this weaker condition on \( E \) in Proposition 2? (b) Is there an example of a quasi-barrelled space which is neither barrelled nor bornological? The answer to both questions is yes. (Received December 4, 1959.)
564-86. J. F. Gray: Diagonal forms of odd prime degree over finite fields.

If \( p_0 \) is a fixed odd prime, a diagonal form \( a_1x_1^p + a_2x_2^p + \ldots + a_px_p^p \)
\((a \in k, p \geq p_0)\) of odd prime degree \( p \) over a finite field \( k \) has a nontrivial zero
in \( k \) for \( t = \left\lfloor \frac{(p_0 + 1)/2N} + 2N - 4 \right\rfloor \), where \( N = \left\lfloor (1 + (p_0 + 2)^{1/2})/2 \right\rfloor \) and all
square brackets indicate the greatest integer function. The problem is reduced
to that in which the coefficients are nonzero, lie in distinct cosets of \( k^* \), modulo
\( k^p \), and in which \( p | q - 1 \), where \( q \) is the number of elements in \( k \). This is then
handled by considering a form derived from \( lx_0^p + dx_1^p + \ldots + d^{p-1}x_{p-1}^p \) (d a sp-
ecly chosen generator of \( k^* \)) by replacing \( t \) of these coefficients by zeros.

The approach is constructive: \( 1 + d = da^p \), for some \( a \in k^* \) and some \( i \),
\( 2 \leq i \leq p - 1 \); from \( i \) is determined an integer \( j \) such that \( x_j, x_{j+1}, \) and \( x_{i+j} \)
are among the effectively appearing variables of the derived form, and hence
\(-E_j - E_{j+1} + aE_{i+j} \) is the desired solution, where \( E_x = (x_0, x_1, \ldots, x_{p-1}) \) and
\( x_r = 1, x_s = 0, s \neq r \). (Received December 4, 1959.)

564-87. K. L. Chung: On last exit times.

Let \( \{x_t\} \) be a Markov chain with stationary transition probabilities, which
is separable and measurable. For a given state \( i \) let \( S_i(w) = \{t: x(t,w) = i\} \),
\( S_i(w) = \) the closure of \( S_i(w) \). We define \( \gamma_i(t,w) = \sup \{S_i(w) \cap [0,t]\} \) to be the last
exit time from \( i \) before time \( t \). Properties of this random variable are studied
leading to results which are dual to those concerning first entrance times. The
duality is further shown up by considering the reversed Markov chain which
does not necessarily have stationary transition probabilities. (Received December 7, 1959.)

564-88. D. C. McGarvey: A reflection formula in neutron transport
theory.

For an arbitrary oriented rod \( B \) let \( T(\mathcal{L},B), T(r,B), R(\mathcal{L},B) \) and \( R(r,B) \)
be the left and right transmission and reflection operators of \( B \); for example,
if \( f \) represents an arbitrary initial flux of neutrons inserted at the left end of \( B \)
then \( T(\mathcal{L},B)f \) is the consequent expected flux emitted from the right hand end
over all time. If \( B \) is formed from \( B_1 \) and \( B_2 \) by joining the right end of \( B_1 \) to
the left end of \( B_2 \) then \( T(\mathcal{L},B) = T(\mathcal{L},B_2)[ \sum_{n=0}^{\infty} R(\mathcal{L},B_2)R(r,B_1) ] \) and
\( R(\mathcal{L},B) = R(\mathcal{L},B_1) + T(r,B_1)R(\mathcal{L},B_2) [ \sum_{n=0}^{\infty} R(\mathcal{L},B_2)R(r,B_1) ] \). Applications
to the determination of criticality by estimating the largest eigenvalue of

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(Received December 7, 1959.)

564-89. Togo Nishiura: The Geöcze k-area.

By suitable extension of the definition of the Geöcze area given in [L. Cesari, Surface area, Princeton, 1959] extensions of the theory of Geöcze k-area for mappings from admissible subsets of Euclidean k-space $E_k$ into $E_n$ ($n \geq k$) is made. The extension is complete for flat mappings ($k = n$). For mappings with $k = 2 < n$ a number of results can be extended in all generality. Other results, in particular the limit theorems for $2 < k < n$, extend under the additional hypothesis that the point set covered by the mapping has Hausdorff $(k + 1)$ - measure zero in Euclidean n-space. There is an example of a mapping $q = T(p)$ of finite Lebesgue 3-area, $p = (u,v,w) \in \Lambda \subset E_3$, $\Lambda$ a cube $q = (x,y,z,t) \in E_4$, such that, if $\tau$ is the projection of $E_4$ onto the $(x,y,z)$ - space $E_3$ and $T_4 = \tau T$, $Q = T_4(\Lambda)$, then for every point $q$ of the set $Q$ of positive 3-measure in $E_3$, $T_4^{-1}(q)$ is a continuum and $T$ is not constant on $T_4^{-1}(q)$.

(Received December 7, 1959.)

564-90. J. B. Rosen: Stability and bound for nonlinear system of difference and differential equations. II.

The system of n nonlinear differential equations (*) $\frac{dy}{dt} = - f(y,t)$, $y \equiv \{x_1, x_2, \ldots, x_n\}$, $f \equiv \{f_1, f_2, \ldots, f_n\}$, is considered, with $y(0) = y_0$. Denote by $B(y,t) = (\partial f_i/\partial x_j)$ the n X n first derivative matrix of $f(y,t)$. Let $\lambda(u,t) = \min.$ eigenvalue of $[B(u,t) + B^T(u,t)]/2$, and $\lambda$ be a constant such that $A(u,t) \geq \lambda$ for $\|u\| \leq \max_{0 \leq t \leq T} \sqrt{\int_0^t e^{-\lambda(t-\tau)}\|f(0,\tau)\|d\tau}$ and $0 \leq t \leq T$. Let $y_j$ be the solution of the implicit finite difference approximation to (*), $y_{j+1} - y_j = - (At/2)[e^{-\lambda t} \|y_0\| + \int_0^t e^{-\lambda(t-\tau)}\|f(0,\tau)\|d\tau]$ and $0 \leq t \leq T$. Then a bound on the rate of decay or growth of the solution $y_j$ is given by $\|y_j\| \leq e^{-\lambda(1/2)At} \|y_0\| + \sum_{j=0}^{1/2} e^{-\lambda(j-1/2)At} \|y_j(0)\|$, $j = 1, 2, \ldots, T/At$, for $1/2 \leq \theta \leq 1$ and every $\Delta t > 0$. An essential part of the proof consists of showing that $\| [1 - \varepsilon B] [1 + \beta B]^{-1} \| \leq 1$ for $B + B^T$ positive semi-definite and every $\beta \geq \beta > 0$. The corresponding bound for the solution $y(t)$ to (*) is $\|y(t)\| \leq e^{-\lambda t} \|y_0\| + \int_0^t e^{-\lambda(t-\tau)}\|f(0,\tau)\|d\tau$ for $0 \leq t \leq T$. This can be shown by a convergence proof as $\Delta t \to 0$, or directly from (*). The above is a generali-
zation of the previously reported result (Abstract 560-57, Notices Amer. Math. Soc. vol. 6 (1959) p. 625) which is obtained for \( \lambda = 0, f(0,t) = 0 \). It also follows that if \( \lambda > 0 \), and \( \|f(0,t)\| \) is bounded for \( 0 \leq t \), then both \( \|y_j\|, j = t/\Delta t \), and \( \|y(t)\| \) are bounded as \( t \to \infty \). (Received December 7, 1959.)

564-91. T. G. Ostrom: Configurations of Desarguesian planes associated with lines of Veblen-Wedderburn planes.

Every finite Veblen-Wedderburn plane \( T \) is of prime power order, and its points can be identified with those of a Desarguesian affine plane \( P \) of the same order. The set of points on a line of \( T \) then generates a configuration in \( P \). Such configurations resemble affine sub-spaces of \( P \). The study of these configurations suggests generalizations of some known methods for constructing Veblen-Wedderburn planes. (Received December 7, 1959.)


Let \( E \) be a set of real positive numbers and let \( L(E) \) be the family of all intervals of the form \( I = \{ ax + y = t, x \geq 0, y \geq 0 \} \), where \( a \in E \) and \( 0 < t < \infty \). A complex-valued continuous function \( F \) of two variables defined on the first quadrant is said to be extinguished by \( E \) if \( \int F(x,y)ds = 0 \) for any \( I \in L(E) \). Let \( A_n \) denote the class of all functions \( F \) of two variables having the representation \( F(x,y) = \sum_{j=1}^{n} f_j(x)g_j(y) \), where all functions \( f_1, ..., f_n, g_1, ..., g_n \) are continuous on the right half-line. \( B_n \) will denote the class of all sets \( E \) such that a unique function belonging to \( A_n \) and extinguished by \( E \) is the function identically equal to 0. Let \( P_n \) be the least power of sets belonging to \( B_n \). From Titchmarsh's Theorem on convolution it follows that \( P_1 = 1 \). For \( n \geq 2 \) the inequality \( n \leq P_n \leq (n^2 - n + 4)/2 \) is proved. (Received December 7, 1959.)


Let \( A \) be a (not-necessarily associative) linear algebra over the reals, and let \( A \) be absolute valued; i.e., there is a norm \(|x|\) satisfying \(|xy| = |x||y|\) for each \( x,y \) in \( A \). It is shown here that if \( A \) has an identity element, then \( A \) is isomorphic to one of the following: the real field \( R \), the complex field \( C \), the quaternion algebra \( Q \), the Cayley-Dickson algebra \( D \). A. A. Albert had previously established this result under the restriction that \( A \) be algebraic, in the sense that every element generates a finite-dimensional sub-algebra, and F. B. Wright
had shown that an absolute valued division algebra is algebraic. A simple example of an infinite dimensional absolute valued algebra shows that the existence of the identity is essential. The example also shows that the existence of an identity is essential to Kaplansky's theory of quadratic forms admitting composition. It is also shown in this note that a commutative absolute valued algebra has dimension \( \leq 2 \), and hence, by a result of Albert, is isomorphic to one of the algebras \( R, C, C^* \), where \( C^* \) is the vector space of complex numbers with the product of \( x \) and \( y \) defined to be \( \overline{xy} \). (Received December 7, 1959.)


Let \( G(M) \) be the group of all homeomorphisms of a closed \( n \)-manifold \( M \), \( n \leq 3 \), with the Frechet metric topology. The identity component of \( G(M) \) equals the group of \( h \) in \( G(M) \) isotopic to the identity and the group \( G^0(M) \) of \( h \) in \( G(M) \) such that \( h = \prod_{i=1}^{k} h_i \), \( h_1 \) the identity outside some internal polyhedral \( n \)-cell in \( M \) (cf. these Notices, Abstract 560-9, vol. 6 (1959) p. 543, first result). The proof uses a result on "small isotopies" obtained independently and almost simultaneously by Kister, Hamstrom, and this author (see Kister's announcement, Bull. Amer. Math. Soc. vol. 65 (1959) pp. 371-373). Let \( G^1(M) \) be the group of \( h \) in \( G(M) \) such that \( h = \prod_{i=1}^{k} h_i \), \( h_1 \) the identity inside some polyhedral \( n \)-cell in \( M \). Then \( G^1(M) \) is either of index 1 or 2 in \( G(M) \) (this implies the second result of Abstract 560-9). For any \( n, G^0(S_n) = G^1(S_n) \). Combining the two results above, and using a theorem of Brouwer, we conclude that two homeomorphisms of \( S_n \), \( n \leq 3 \), are isotopic if and only if they are homotopic. (Received December 7, 1959.)


A finite, connected graph is minimally imbedded in a (closed, orientable) 2-manifold if it cannot be imbedded in a 2-manifold of lower genus. The genus of the manifold is called the genus of the graph, and it is a classical problem to determine this number. In this paper minimal imbeddings are characterized. An imbedding of a graph \( G \) in a 2-manifold \( M \) is said to be a 2-cell imbedding if each component of \( M - G \) is an open 2-cell. A 2-cell imbedding of \( G \) in \( M \) is said to be maximal if for any imbedding of \( G \) in a 2-manifold \( N \) the number of components in \( M - G \) is no less than the number of components in \( N - G \). It is shown that an imbedding is minimal if and only if it is a maximal 2-cell
Embedding. Though the classical problem is restricted to orientable 2-manifolds, the concept of minimal embedding can be extended to all (closed) 2-manifolds by using the Euler characteristic instead of genus; the same theorem holds. In regard to applications, one obtains a simple proof of the not unexpected result that the 1-dimensional skeleton of a triangulation of a 2-manifold is minimally imbedded. In addition, it is shown that for each positive integer \( n \) there is a graph \( G_n \) of genus \( n \) which can be imbedded in the projective plane.

(Received December 7, 1959.)

564-96. Louis Auslander: Discrete uniform subgroups of a class of Lie groups.

Let \( N \) be a connected, simply connected nilpotent Lie group and let \( C \) be a compact group of continuous automorphisms of \( N \). If \( G \) denotes the semi-direct product of \( N \) and \( C \) and if \( \Gamma \) is a discrete uniform subgroup of \( G \), then \( \Gamma \cap N \) is a discrete uniform subgroup of \( N \) and of finite index in \( \Gamma \). Further, if \( \Gamma_1 \) and \( \Gamma_2 \) are isomorphic and discrete uniform subgroups of \( G \), then \( G/\Gamma_1 \) is homeomorphic to \( G/\Gamma_2 \). (Received December 7, 1959.)

564-97. George Copp and D. F. Dawson: Concerning twin convergence regions for continued fractions.

Let \( f(a) \) denote the continued fraction \( a_1/1 + a_2/1 + a_3/1 + \ldots \), where \( a_p \) is a complex number. \textbf{Theorem.} If \( k > 1 \), \( e > 0 \), and for each positive integer \( p \), \( a_p = r_p \exp i \theta p \), \( 0 \leq r_{2p-1} \leq k^2 \), and \( r_{2p} \leq 2(k^2 - \cos \theta_{2p}) + e \), then \( f(a) \) converges. \textbf{Corollary.} If \( k > 1 \), \( e > 0 \), and for each positive integer \( p \), \( |a_{2p-1}| \leq k^2 \) and \( |a_{2p}| \geq (1 + xk)^2 + e \), then \( f(a) \) converges, where \( x = -(1/k) \cdot \sqrt{1 - (2k^2 + 1)^2} \). \textbf{Theorem.} If \( k > 0 \) and for each positive integer \( p \), \( |a_{2p-1}| \leq k^2 \) and \( |a_{2p}| \geq (1 + yk)^2 \), then \( f(a) \) converges absolutely, where \( y = 1 - (1/k) \cdot \sqrt{1 - (2k^2 + 1)^2} \). Singh and Thron (Proc. Amer. Math. Soc. vol. 7 (1956) pp. 277-282) showed that if \( 0 < k < 1 \) and for each positive integer \( p \), \( |a_{2p-1}| \leq k^2 \), \( a_{2p} = r_{2p} \exp i \theta_{2p} \), and \( r_{2p}^2 + 2r_{2p}(\cos \theta_{2p} - k^2) + (1 - k^2) \geq 0 \), then \( f(a) \) converges. Copp (Dissertation, The University of Texas, 1950) and Cowling, Leighton, and Thron (Bull. Amer. Math. Soc. vol. 50 (1944) pp. 351-357) showed that if \( k = 1 \), \( e > 0 \), and for each positive integer \( p \), \( |a_{2p-1}| \leq k^2 \), \( a_{2p} = r_{2p} \exp i \theta_{2p} \), and \( r_{2p} \geq 2(1 - \cos \theta_{2p}) + e \), then \( f(a) \) converges. Hence the first theorem in this abstract can be regarded as an extension of known results for \( 0 < k \leq 1 \). (Received December 7, 1959.)

Let \( \mathcal{D} \) be a domain in \( \mathbb{E}^2 \) and let \( \mathcal{W} \) be a maximal real linear vector space of pseudo-regular functions (in the sense of Kakutani) defined on \( \mathcal{D} \). It is known that if \( \mathcal{W} \) contains two elements, \( f = u + iv \) and \( g = p + iq \), such that \( v_x q_y - v_y q_x \neq 0 \) on \( \mathcal{D} \), then \( \mathcal{W} \) consists of solutions to a uniquely determined elliptic system of first order partial differential equations. In this paper it is shown that if \( \mathcal{W} \) is the set of Jacobian matrices of elements of a vector space \( \mathcal{W} \) of pseudo-regular functions such that \( \mathcal{W} \) contains two everywhere linearly independent elements, there exist uniquely determined matrices \( S \) and \( T \) with continuous functions as entries, such that if \( J \in \mathcal{W} \), \( J = SCT \) where \( C \) is a matrix depending on \( J \). Further, if \( S = I \), the elements of \( \mathcal{W} \) satisfy a Bers system of equations and if \( T = I \), the elements of \( \mathcal{W} \) satisfy a Beltrami system. Finally, it is shown that if \( \mathcal{W}_1 \) and \( \mathcal{W}_2 \) are two such vector spaces and \( S_1, T_1, S_2, \) and \( T_2 \) are the associated matrices, \( \mathcal{W}_1 \cap \mathcal{W}_2 \neq 0 \) implies \( \| S_2^{-1} S_1 \| = \| T_1 T_2^{-1} \| \). (Received December 7, 1959.)


Assume that the equation (1) \( \dot{x} = X(x) \), where \( x,X \) are \( n \)-vectors, possess a \((k + 1)\)-parameter family of periodic solutions \( x_0(w(b)t + \phi, b) \), \( \phi \in \mathbb{R} \), \( b = (b_1,...,b_k) \in V \subset \mathbb{R}^k \), \( V \) open in \( \mathbb{R}^k \), \( x_0(s + 2\pi,b) = x_0(s,b) \). In the \((x,t)\) space, \( x_0 \) describes a manifold \( M \) as \( \phi \) varies in \( \mathbb{R} \), and \( b \) varies over \( V \). If (A) for each \( b \in V \), \( n -(k + 1) \) of the characteristic exponents of the linear variational equation of (1) have negative real parts, and (B) the matrix \( [\partial x_0(s,b)/\partial s, \partial x_0(s,b)/\partial b] \) has rank \((k + 1)\), then \( M \) is asymptotically stable in the strong sense that solutions near \( M \) tend to a particular solution in \( M \) as \( t \to \infty \). The proof proceeds by introducing local coordinates around \( M \), and (B) is a sufficient condition for the existence of such coordinates. Next the authors consider (2) \( \dot{x} = X(x) + X^*(t,x) \), where \( X^* \to 0 \) as \( t \to \infty \), \( \| x \| \) bounded. The rate at which \( X^* \to 0 \) determines the stability properties of \( M \). The method of proof, and some of the results relate to product space equations considered by Persidskii, Malkin, Dyhman; see Lefschetz, Differential equations-geometric theory, p. 117. (Received December 7, 1959.)

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It has been shown (A note on metric density of sets of real numbers, these Notices, Abstract 555-6, vol. 6, p. 149) that for every $F$ subset $Z$ of the real line $R$ whose Lebesgue measure is zero, and for every real number $d$ such that $0 < d < 1$ there exists a measurable set $E$ such that the Lebesgue density, $D_x(E)$, of $E$ exists at each point $x$ of $Z$ and has the value $d$. Thus if $x$ represents a given point on $R$ and if $\mathcal{D}(x)$ represents the class of all measurable subsets of $R$ whose density exists at $x$, $D_x$ may be thought of as a set function which maps $\mathcal{D}(x)$ onto the closed unit interval. This function fails to be a finitely additive measure in the respect that $\mathcal{D}(x)$ is not a ring. The upper density, $\overline{D}_x$, at $x$ is a finitely subadditive outer measure. It is shown that the class of $\overline{D}_x$-measurable sets coincides with the class of all sets whose density exists at $x$ and has the value zero or one. The same type of result holds for a Lebesgue density in which the single point $x$ is replaced by an $F_\sigma$ set of measure zero. (Received December 7, 1959.)

Let $G$ be a linearly ordered set; let $G$ have the order topology and suppose that $G$ is then locally compact. Define $xy = \max(x,y)$; then $G$ is a topological semigroup. Let $M(G)$ be the algebra of complex-valued countably additive regular Borel measures on $G$ such that $|\lambda| = |\lambda|_G < \infty$; define convolution as usual. Then $M(G)$ is a self-adjoint commutative Banach algebra and $M(G)$ is semi-simple. Also every non-negative measure $\lambda$ has a square root (with respect to convolution). However, $M(G)$ is not isomorphic to any $L_0(X)$, $X$ locally compact Hausdorff. The main result is that spectral synthesis holds for $M(G)$. (Received December 7, 1959.)

Let $\beta, t, \lambda, k$ be complex numbers. Let $\{L_n^{(\alpha)}(z)\} (n = 0, 1, 2, \ldots; z = x + iy; \alpha > -1)$ be the set of orthogonal polynomials, the Laguerre polynomials, let $a_n$ be the Fourier-Laguerre coefficients generated from the function, $\exp(\frac{k}{2} - 1 - (\lambda/2 \beta)^2 x)$. Corresponding to the exponential function $e^{-\lambda t}$ we develop a Laguerre series (1) $e^{-\lambda t} \sim \sum_{n=0}^{\infty} e^{-k \beta t} a_n L_n^{(\alpha)}(2 \beta t)$. Let the region, $R$, in the complex $\beta$-plane be defined as follows: (a) The region, exterior to a
certain circle $C_1$ through the origin ($R(k) < 1$). (b) The region in a half plane bounded by the line $1$ through the origin and tangent to $C_1 (R(k) = 1)$. (c) The region interior to a circle $C_2$, the reflection of $C_1$ in line $1$ ($R(k) > 1$). Let an arbitrary complex $\lambda$ be given and $\beta$ be a fixed point in region $R$, (which is defined for all complex $k$) then in any finite region of the $t$-plane the series (1) is an absolutely convergent series which converges uniformly to $e^{-\lambda t}$ ($\alpha > -1$).

(Received December 7, 1959.)


A square is inscribed in $C$ if its four corners lie on $C$. It is established that a square can be inscribed in any plane Jordan curve. Theorems similar to this have been proved for $E^2$, but with a restriction to convex sets. The proof is carried out by the following method. It is shown that every analytic simple closed curve $K$ contains a continuous family of squares with three corners on $K$, and that there are two members of this family which have their remaining corners on opposite sides of $K$. A limiting process extends the result to Jordan curves. It is also shown that as a Jordan curve undergoes a continuous deformation, there is a continuously varying square inscribed in it.

(Received December 7, 1959.)

564-104. Dov Tamari: Generalization of a theorem of Rees.

Given a family of binary 1-1 relations $R = \{R_i\}$ over an arbitrary set $N$ ($R_i \subset N \times N$, 1-1 mappings of parts of $N$ into $N$) satisfying (0) $\emptyset \notin R$; (1) $R_i \cap R_j \subseteq \exists R_k \in R (R_k \subset R_i \cap R_j)$; (2) $R_i \cap R_j \subseteq \exists R_j \in R (R_j \subseteq R_i \cap R_j)$. Lemma 1: The binary r. $\mathcal{E}$ over $\mathcal{R}$ defined by $R_i \in \mathcal{R} \iff \exists R_j \in \mathcal{R} (R_j \subseteq R_i \cap R_j)$ is an equivalence r. Theorem 1. $\mathcal{E}$ is a group. Lemma 2: If $R_m \in \mathcal{R}$ is a minimal r., $R_m$ is a permutation. Lemma 3: Two minimal r. are permutations of the same subset $N_m \subset N$. Theorem 2: The subfamily $\mathcal{R}_m$ of minimal r. in $\mathcal{R}$ forms a permutation group, subgroup of $\mathcal{R}/\mathcal{E}$. If $\mathcal{R}$ satisfies (3) "Every $r.R \in \mathcal{R}$ contains a minimal $r.$ $R_m \in \mathcal{R}$," then $\mathcal{R}_m = \mathcal{R}/\mathcal{E}$. Theorem 1 generalizes a theorem of Rees [J. London Math. Soc. vol. 22 (1947) p. 281, Theorem 2] work to be published in a joint paper of Abraham Ginzburg and the author on "multiplicative systems as homomorphic images of square sets." (Received December 14, 1959.)
564-105. C. T. Taam: Recurrent properties of conservative measurable transformations.

Let T be a conservative measurable transformation of a measure space into itself. In this note it shows that $T^n$ is also conservative, n being any positive integer. This result implies that T is strongly recurrent in the sense that if E is a measurable set, then almost every point of E returns to E infinitely often under the action of T. When T is also invertible, the above result is known (see Halmos: Invariant measure, Ann. of Math. (1947) p. 738).

(Received December 14, 1959.)

564-106. Trevor Evans: A decision procedure for transformations of trees.

Consider the following transformations on finite rooted trees; (i) replacing a subtree of a tree by another subtree, the allowable substitutions being given as a finite set of pairs of interchangeable trees, (ii) permuting the order of the segments from a point, the allowable permutations being given as subgroups of the symmetric groups of degree 2,3,4,... . A decision method is given here for the problem of deciding when one tree can be transformed into another by a finite sequence of such transformations. It is shown that this decision problem for trees is equivalent to the word problem for the variety of algebras having an n-ary operation for $n = 1,2,3,...$ and such that these operations satisfy generalised commutative laws. A solution of the word problem is given for this variety of algebras. (Received December 14, 1959.)

564-107. W. Ericson and James Radlow: Magnetohydrodynamic flow past a flat plate, in the presence of a transverse magnetic field.

The problem considered is that of determining the steady state flow of a viscous incompressible conducting fluid past a semi-infinite flat plate, in the presence of a magnetic field perpendicular to the plate. An electric field parallel to the plate is imposed to eliminate electromagnetic interaction between the free stream flow and the applied magnetic field, and an Oseen-type linearization then reduces the problem to a mixed boundary-value problem with data given on a half-plane $x \geq 0, y = 0$. A Wiener-Hopf argument yields an explicit solution. In particular, it is found that the velocity gradient $f(x)$ at the edge of the plate has an expansion (valid for small $x$) of the form $f(x) \sim c_0 x^{-1/2} [1 + c_1 e^x + c_2 (e^x)^2 + ...]$, where $e$ is the product of the conductivity...
magnetic permeability and viscosity of the fluid, and $\beta$ is the Alfvén Mach number. The result holds, and the steady flow exists, for all positive $\epsilon, \beta$. Numerical comparisons are made with the results of Rossow (Z. Angew. Math. Phys. vol. 9b (1958) pp. 519-527), who treated the problem from a boundary-layer point of view. (Received December 14, 1959.)


Here is described a method by which a large number of boundary value problems for $L(u) \equiv u_{xy} + au_x + bu_y + cu = d(x,y)$, including the classical boundary value problems (Cauchy, Goursat and Hadamard problems), can be reduced to an equivalent characteristic boundary value problem. To this end the equation $L(u) = d(x,y)$, satisfying the conditions $A(x)u(x,\alpha(x)) + B(x)u_x(x,\alpha(x)) = \phi(x)$ and $C(y)u(\beta(y),y) + D(y)u_y(\beta(y),y) = \psi(y)$, is considered. It is shown that the above problem has a unique solution in a rectangle $R$ (with sides parallel to the coordinate axes and lying entirely within the domain of continuity of the coefficients of $L(u)$), if $A(x)\cdot C(y) \neq 0$ in $R$, and if the functions $a, b, c,$ and $d$ are differentiable and $\alpha(x), \beta(y), A(x), B(x), C(y)$ and $D(y)$ are any prescribed continuous functions. In case $A(x)\cdot C(y) = 0$ in $R$, it is required in addition that $\alpha(x)$ and $\beta(y)$ be monotone increasing (decreasing) in $R$. The method of proof consists of reducing the given boundary value problem to an equivalent system of two integral equations, involving the value of the solution $u(x,y)$ on the characteristics, which has a unique solution. (Received December 14, 1959.)


Call a base $B$ of a metric space $X$ fine if every open covering $U$ of $X$ has a locally finite refinement $V \subset B$; call $B$ coarse if no locally finite $V \subset B$ covers $X$. The following results were obtained. (1) Every separable $X$ has a countable, fine base. (2) In a $\sigma$-compact $X$, every base is fine. (3) In a reflexive, infinite-dimensional Banach space, the base consisting of all bounded, open, convex subsets is coarse. (4) The irrationals have a coarse base. (5) A closed subset of a space possessing a coarse base has a coarse base. (6) Every non-$\sigma$-compact, complete, separable metric space has a closed subset homeomorphic to the irrationals; without completeness, this becomes false even for subsets of the line. (Received December 14, 1959.)

Binomial coefficient summation formulas, though of wide use in statistics, combinatorial analysis, and special function theory, are very scattered in the literature. Few existing tables of such formulas seem to be complete enough to be of much use. In the present paper 500 formulas have been selected and indexed in a special fashion to allow ease of use and flexibility in adding new formulas. New as well as old results are given, but it was found impossible to include detailed proofs in this first paper. It is hoped to extend the tables to include approximately 2000 formulas and develop a separate text giving complete proofs. This work has been supported by the Department of Defense through the agency of Science Service, Washington, D. C. The work is based on the author's study of the literature on this subject over the past 14 years. (Received December 14, 1959.)

564-111. Alex Rosenberg and Daniel Zelinsky: On Amitsur's complex, II.

In [Trans. Amer. Math. Soc. vol. 90 (1959) pp. 73-112] Amitsur proved that if $F$ is a finite dimensional extension field of $C$ then the Brauer group $B(F/C)$ of central simple $C$-algebras split by $F$ is isomorphic to the second cohomology group $H^2(F)$ of a certain complex. We extend this result to the cases (i) where $C$ is semi-local and $F$ a finitely generated projective $C$-module containing $C$ as a direct summand, and (ii) where $C = K[x]$, $F = L[x]$ with $L$ a finite dimensional commutative algebra over the field $K$. Under weaker hypotheses on $C$ and $F$ there is a homomorphism and even an epimorphism $B(F/C) \rightarrow H^2(F)$. This gives a new proof of the theorem of Auslander-Goldman which asserts that the natural mapping $B(L[x]/K[x]) \rightarrow B(L/K)$ is an isomorphism if $L$ is separable over $K$ and is not an isomorphism for some inseparable $L$. (Received December 14, 1959.)

564-112. Alex Rosenberg and Daniel Zelinsky: Automorphisms of split algebras.

Let $C$ be an integral domain, $V$ a finitely generated projective $C$-module and $A \text{ Hom}_C(V,V)$. The group of $C$-algebra automorphisms of $A$ modulo inner ones is isomorphic to the following subgroup of the group of projective fractional $C$-ideals modulo principal ones: $\{I \mid IA$ is a principal right $A$ ideal\}, or,
equivalently \( \{ 1 | IV \cong V \} \). If \( C \) is Dedekind this subgroup is also the subgroup of ideal classes of order dividing the rank of \( V \). (Received December 14, 1959.)

564-113. R. F. Goodrich, N. D. Kazarinoff and R. K. Ritt: Scalar diffract by elliptic cylinders or prolate spheroids whose eccentricities are almost one.

If the largest (smallest) radius of curvature of a scattering body is much greater (smaller) than the wave length \( \lambda \) of an incident wave, then generally neither geometrical optics nor Rayleigh-theory gives a satisfactory description of the total field. In this paper, the leading terms in the asymptotic expansions with respect to \( 2\pi a/\lambda \) of the surface distribution and the total field in the shadow zone are found for the exterior Dirichlet and Neumann problems for an elliptic cylinder (prolate spheroid) with semi-axes \( a \) and \( b \) (\( a > b \)) when \( [2\pi b^2/(\lambda a)] < 1 \) and \( 2\pi a/\lambda > 1 \), and where the incident wave is cylindrical (spherical) with axis parallel to that (center on the axis) of the cylinder (spheroid). Portions of the turning-point theory are used which are distinct from those employed by Kazarinoff and Ritt in [Annals of Phys. vol. 6 (1959) pp. 277-299; Proc. of the URSI Symp. on Electromagnetic Theory., Toronto, June, 1959 (to appear)]. The results used are McKelvey's [Trans. Amer. Math. Soc. vol. 79 (1955) pp. 103-123; same journal, vol. 91 (1959) pp. 410-424]. The solution for a strip agrees with Millar's [Proc. of the Cambridge Philos. Soc. (I,II) vol. 54 (1958) pp. 479-511]. However, his work yields asymptotic series. (Received December 14, 1959.)

564-114. B. A. Rattray and A. Goldrich: Coordinates in affine planes.

A new method is given for introducing coordinates in a Desarguesian affine plane. A vector is defined as an equivalence class of ordered point pairs and the set of vectors made into an abelian group \( V \). Each line determines a line subgroup of \( V \). A scalar is defined as an endomorphism of \( V \) which carries each line subgroup into itself. Every nonzero scalar is an automorphism and the scalars form a skew field \( F \). \( V \) is a two dimensional vector space over \( F \) and the line subgroups of \( V \) are the one-dimensional subspaces. (Received December 8, 1959.)
Motivated by a recent paper of I. N. Herstein [On a theorem of Putman and Wintner, Proc. Amer. Math. Soc. vol. 9 (1958) pp. 363-364], this note provides necessary and sufficient conditions for representing a given nonsingular matrix as a multiplicative commutator \( ABA^{-1}B^{-1} \) such that the additional condition \( A(AB - BA) = (AB - BA)A \) is satisfied. **Theorem 1.** Let \( D \) be a nonsingular \( n \)-by-\( n \) matrix over a field \( F \) of characteristic zero or prime \( p > n \). Then there are nonsingular matrices \( A \) and \( B \) over \( F \) such that \( D = ABA^{-1}B^{-1} \) with \( A(AB - BA) = (AB - BA)A \) if and only if \( D - I \) is nilpotent. **Theorem 2.** Let \( D \) be a nonsingular \( n \)-by-\( n \) matrix over a field \( F \neq \) the field of two elements. Then there are nonsingular matrices \( A \) and \( B \) over \( F \) such that \( D = ABA^{-1}B^{-1} \) with \( A(AB - BA) = (AB - BA)A \) if and only if \( D - I \) is similar to \( I - D^{-1} \).

(Received December 8, 1959.)

Let \( (E,p,B) \) be a principal fibre space with fibre \( F \). Let \( \mu: F \times E \rightarrow E \) be the operation of the fibre on the total space. When \( E \) is part of the Postnikov system of a space \( X \), the knowledge of \( \mu^* \) (the induced homomorphism on cohomology) gives one the Whitehead products in \( X \). The following theorem computes \( \mu^* \) in many cases. Let \( F \) be \((n - 1)\)-connected, \( \mathfrak{l} \in H^n(F;\pi) \) the canonical element, and let \( u = \tau(\mathfrak{l}) \in H^{n+1}(B;\pi) \) where \( \pi = \pi_n(F) \). Let \( x \in H^q(B) \) and \( \theta \) be a cohomology operation such that \( \theta(u) + x \cup u = 0 \). Let \( k \in H^{n+q}(E) \) be a representative of \((\theta + x \cup )_p(u)\). Then \( \mu^*(k) = 1 \otimes k + \sum \raisebox{0pt}{\(\mu^*(x) = 1 \otimes \theta(\mathfrak{l}), \quad \theta \otimes k \in H^{n+q}(F \times E), \text{where} \quad \theta(y) = 1 \otimes \theta(\mathfrak{l}), \quad \theta \text{ denoting the suspension of} \theta. \) This theorem can be used to compute Whitehead products in say complex Grassmannians, projective spaces, and to give necessary conditions that algebraic systems of Whitehead products be those of some space.

(Received December 8, 1959.)

Consider a \( p \)-group \((p > 3)\) of class three, generated by three elements, which is not the direct product of two of its proper subgroups, in which every element except the identity is of order \( p \). There is one such abstract group of order \( p^{15} \), and none of greater order. It is shown that there are as many such groups of order \( p^{14} \) as equivalence classes of 3-square matrices over GF(p),
where the matrices $A$ and $B$ are in the same class if and only if $A$ is similar to a scalar multiple of $B$. An extension concerning the number of such groups of order $n$ ($n < 14$) is conjectured. The method used is similar to that of H. R. Brahana (H. R. Brahana, Amer. J. Math. vol. 52 (1940) pp. 365-379). (Received December 8, 1959.)

564-118. Philip Cooperman: Necessary conditions for variational problems with side conditions.

The problem to be treated is that of finding explicitly certain necessary conditions on the solutions of variational problems in two functions $u(x,y)$ and $v(x,y)$ where $u$ and $v$ are connected by a general first order partial differential equation. The method used is based on a remark by R. Courant in his derivation of the Friedrichs transformation (Courant-Hilbert, vol. I, Chap. 4). This method makes use of an auxiliary multiplier which, in general, will be identical with the corresponding Lagrangian multiplier, but which is determined differently. The multiplier is determined explicitly in terms of the functions $u(x,y)$ and $v(x,y)$ and conditions for the uniqueness and existence of the multiplier are given. (Received December 8, 1959.)


This paper supplements the oscillation theory of Leighton and Nehari (Trans. Amer. Math. Soc. vol. 89 (1958) pp. 325-377, Part I) for fourth-order differential equations \((1) (r(x)y')' - p(x)y = 0\), where $r(x)$ and $p(x)$ are positive and continuous on $[a, \infty)$. A "focal-point" concept defined by the two-point boundary conditions \((2) y(a) = y'(a) = y_1(b) = y_1'(b) = 0\), where $y_1(x) = r(x)y''(x)$, is introduced and utilized to determine criteria for the existence (or nonexistence) of the first conjugate point of Leighton and Nehari. Also, it is noted that for certain solutions, $u$ and $v$, of (1) the quantities $\sigma = uv' - vu'$, $\tau = u'u_1 - v'u_1$, $\rho = u_1v'_1 - v'u'_1$ all satisfy second-order linear self-adjoint differential equations of the type \((3) (r(x)y')' + p(x)y = 0\) with positive coefficients to which recent results of the author [Proc. Amer Math. Soc. vol. 10 (1959) pp. 552-561] are applied. It is found that certain focal-conjugate point theorems for second-order equations of type (3) are true (almost verbatim) for fourth-order equations (1), with the interpretation (2) for focal points and the conjugate points of Leighton and Nehari. A by-product is a "free-end" Wirtinger-type inequality,
which was suggested by recent work of Coles [Duke Math. J., to appear].

(Received December 9, 1959.)

564-120. G. R. Blakley: Coefficients of power series.

The coefficients of certain functions of a power series have been determined, but usually only recursively, or implicitly by means of determinants as e.g., the coefficients of \(1/f\) where \(f(z) = \sum a_n z^n\). In this paper the coefficients \(c_n\) of \(f^k\) (\(k\) an arbitrary complex number), \(f(g)\) (\(g(z) = \sum b_n z^n\)), \(f^{-1}\), \(f'/f\) (and, hence, of \(\log f\)) are determined explicitly using a certain number theoretic function of \(n\). The solutions of some function theoretic problems thus yield number theoretic results and vice versa. If \(k\) is a fixed positive integer the coefficients of \(f^k\) depend only on the values of this number theoretic function for the first \(k\) positive integers and consequently these powers of \(f\) can be expressed in a particularly simple manner. The representation of the general term \(a_n^{(3)}\) of \(f^3\) is given as an example. (Received December 9, 1959.)

564-121. R. J. Blattner: Intertwining number theorems for induced representations of Lie groups.

Let \(G, \Gamma_1, \Gamma_2\) be Lie groups with modular functions \(\Delta, \delta_1, \delta_2\) resp. such that \(\Gamma_1, \Gamma_2\) are closed subgroups of \(G\). Let \(L, M\) be unitary representations of \(\Gamma_1, \Gamma_2\) resp. on Hilbert spaces \(V, W\) resp., \(W\) finite dimensional. Let \(n = \dim (G/\Gamma_2) > 0\). Let \(r\) be an integer \(> n/4\). Let \(S = 1 - \sum x_i^2\), the \(\{x_i\}\) being a basis for the (left-invariant) Lie algebra of \(G\). If \(K\) is compact in \(G\) and \(f \in C^\infty_0(K)\), set \(\|f\|_K = \|S^r f\|_\infty\). The \(C^\infty_0(K)\) being so topologized, give \(C^\infty_0(G)\) the inductive limit topology. Let \(A\) be the space of all \(A \in C^\infty_0(G), \delta(V,W)\) \(\exists: \lambda(t_{\xi^2}) = \delta_1(\xi_1)^{-1/2} \delta_2(\xi_2)^{1/2} \Delta(\xi_1^{1/2} \delta_2^{-1/2} M_{\xi_2} A(t)L_{\xi_1}^{-1}\) for all \(\xi \in \Gamma_1, f \in C^\infty_0(G)\), where \(t_{\xi^2}(x) = f(\delta_1^{-1} x \delta_2)\). Theorem 1: The intertwining number of the induced representations \(G L_{\xi}^U\) and \(G M_{\xi}^U\) is \(\leq \dim(\Delta)\). This is a sharpening of a result of Bruhat [Bull. Soc. Math. France vol. 84 (1956) pp. 97-205]. Let \(\Gamma_1, \Gamma_2\) be compact, \(V, W\) finite dimensional. Allow \(n \geq 0\). For each \(\Gamma_1, \Gamma_2\) double coset \(D\), let \(I(D)\) be the intertwining number of \(\xi \to L_{\xi}^D\) and \(\xi \to M_{\xi^{-1} x\xi}^D\), representations of \(\Gamma_1 \cap (x_\Gamma_2 x^{-1})\), \(x \in D\). \(I(D)\) depends only on \(D\). Theorem 2: Suppose \(G\) contains a Haar null set \(N\), union of double cosets \(\exists: I(D) = 0\) for \(D \notin N\). Then the induced representations \(G L_{\xi}^U\) and \(G M_{\xi}^U\) are disjoint. (Received December 9, 1959.)
A topological space is Lindelöf if each open cover has a countable sub-cover. The weak topology $\mathcal{T}$ of a Banach space $B$ is such that one might not expect $\mathcal{T}$ to be Lindelöf unless $B$ were separable or reflexive. However, if $X$ is locally compact and $C_0(X)$ is the space of continuous, complex valued functions on $X$ which vanish at infinity, then the following is true. Theorem. If $X$ is metric then the weak topology $\mathcal{T}$ of $C_0(X)$ is Lindelöf. The converse is not true, but it is if $X$ is a locally compact group. In contrast it can be shown that the space of all bounded functions on the integers, under its weak topology, is not even a normal space. (Received December 9, 1959.)

Let $x(t)$ be a family of random variables on a probability space $\Omega$ with zero mean and finite variance. Karhunen (Ann. Acad. Sci. Fenn. Ser. A, I. vol. 37 Helsinki (1947) pp. 1-79) has shown that if there exists a family $f(t,a)$ of square integrable functions on a finite measure space $(\mathbb{R}, \mu)$ such that $r(s,t) = E(x(s)x(t)) = \int_{\mathbb{R}} f(s,a)f(t,a)d\mu(a)$, then there exists an "orthogonal increments" process $Z$ on $\Omega$ indexed by the finite measurable subsets of $\mathbb{R}$ in terms of which $x(t) = \int_{\mathbb{R}} f(t,a)dz(a)$, the convergence being in the mean on $\Omega$. It is pointed out that if $t$ ranges over the infinitely differentiable functions with compact support and if $r$ is stationary in a suitable sense and continuous, then the hypothesis is satisfied in virtue of the distribution-theoretic version of Bochner's theorem; while if $t$ ranges over the functions square integrable on yet a third measure space and $r$ arises from a function square integrable on the corresponding product space, then the hypothesis holds with a discrete $\mathbb{R}$, although the fact that the convergence here is valid in the mean can be used to strengthen the conclusion. This last, along with applications, will be presented subsequently. (Received December 9, 1959.)

Let $S_m$ be the semigroup of integers modulo $m$ under multiplication, where $m$ is an integer with prime factorization $p_1^{\alpha_1}p_2^{\alpha_2}...p_r^{\alpha_r}$ ($\alpha_1, ..., \alpha_r$ are positive integers). The algebraic structure of $S_m$ is completely determined: all idempotents, subgroups, and ideals are identified. The semicharacters of
S_m (complex-valued functions \( \chi \) on \( S_m \) such that \( \chi \neq 0 \) and \( \chi(xy) = \chi(x)\chi(y) \) for all \( x, y \in S_m \)) are computed explicitly. **Theorem.** If \( \chi \) is a semicharacter of \( S_m \) assuming some value different from 0 and 1, then \( \sum_{\chi \in S_m} \chi(x) = 0 \).

**Theorem.** The semigroup under pointwise multiplication consisting of all semicharacters of \( S_m \) is isomorphic to the subsemigroup of \( S_m \) consisting of all integers of the form \( p_{11}^{\beta_1}p_{22}^{\beta_2}...p_{rr}^{\beta_r}x \) modulo \( m \), where \( \beta_j \) is 0 or \( \alpha_j \) \((j = 1, 2, ..., r) \) and \( x \) is prime to \( m \). (Received December 9, 1959.)

564-125. J. A. Hummel: **Extremal problems for p-valent starlike functions.**

A function \( f(z) \) is called a p-valent starlike function if \( f(0) = 0 \), \( f(z) \) is regular in \( |z| < 1 \), takes on each value at most \( p \) times, has \( p \) zeros in \( |z| < 1 \), and \( z f'(z)/f(z) \) has a positive real part in some annulus \( r < |z| < 1 \). Even under suitable normalizations, many extremal problems in this class do not have solutions. However, with the use of the variational method for starlike functions previously given by the author (Proc. Amer. Math. Soc. vol. 9 (1958) pp. 82-87) it can be shown that under suitable restrictions, if an extremal problem has a solution, it must be of the form \( f(z) = z^q \prod_{\gamma=1}^{m} (1 - \gamma z)^{-\alpha_\gamma} \prod_{\mu=1}^{p} (1 - z/\mu)^{1 - b^*z} \) where \( 1 < q < p \), \( |\gamma_\gamma| = 1 \), each \( \alpha_\gamma > 0 \), \( \sum \alpha_\gamma = 2p_0|b_i| < 1 \), and \( b^* \) is the complex conjugate of \( b^\mu \). This maps \( |z| < 1 \) onto a p-sheeted surface whose boundary consists of \( m \) radial slits. Bounds on \( m \) can be determined from the type of extremal problem. Some questions about the existence of the extremal function are investigated. (Received December 9, 1959.)

564-126. J. M. Mitchell: **Abel-Poisson summability on matrix spaces.**

Let \( D \) be the domain defined by \( 1 - zz^* > 0 \) where \( z \) is an \( n \) by \( n \) matrix of complex numbers, \( z^* \) its conjugate transpose and \( I \) the identity matrix.

\( D \) possesses a distinguished boundary \( B \), given by \( uu^* = I \), which is a proper part of its boundary. Let \( f \) be a real integrable function on \( B \) and consider the integral operator \( I(f,z) = \int_B P(z,u)f(u)dV \), where \( dV \) is Euclidean volume element and \( P(z,u) \) is the Poisson kernel of the Laplace equation which corresponds to the metric of \( D \), invariant with respect to the group of one to one analytic transformations mapping \( D \) onto itself. Hua and Lowdenslager have proved that given a real continuous function on \( B \), \( I(f,z) \) is harmonic on \( D \) and \( I(f,z) \) \( \to f(u_0') \) as \( z \to u_0 \) on \( B \) radially, that is, along the set \( \rho u_0, 0 \leq \rho < 1 \). However for the unit circle if \( f \) is merely assumed to be integrable, then \( I(f,\rho u_0) \to f(u_0) \).
as \( \rho \to 1 \) (Abel-Poisson summability). We prove this result for the matrix spaces \( D \) under radial approach to the boundary \( B \) in case \( f \) is integrable and satisfies certain other integrability conditions. (Received December 9, 1959.)

564-127. J. R. Munkres: Obstructions to smoothing, for manifolds with boundary.

Let \( M \) and \( N \) be \( C^2 \) \( n \)-manifolds; they are combinatorially equivalent if they have isomorphic \( C^2 \) triangulations. In such a case, one attempts to construct a diffeomorphism between them; obstructions arise, results concerning which were announced in Bull. Amer. Math. Soc. vol. 65 (1959) pp. 332-334. Here the theory is extended to the case in which \( M \) and \( N \) are manifolds with boundary. The main result is that the obstruction class lies in \( H_m(M, \partial M; \Gamma^{n-m}) \), which denotes homology based on infinite chains, having twisted coefficients if \( M \) is nonorientable. \( \Gamma^n \) is the group of orientation-preserving diffeomorphism of \( S^{n-1} \) modulo those extendable to diffeomorphisms of the closed unit ball \( B^n \). Corollary (Thom): Two differentiable structures on a manifold homeomorphic to \( B^n \) are diffeomorphic if they are combinatorially equivalent. (Received December 9, 1959.)

564-128. J. C. Oxtoby: Spaces that admit a category measure.

A category measure is a finite measure defined for sets having the property of Baire that vanishes for and only for sets of first category. A topological space is called quasi-regular (q.r.) if every nonempty open set contains the closure of some nonempty open set. It is said to have property (P) if every set of first category is nowhere dense. \( R(X) \) denotes the algebra of regular open subsets of \( X \). A Boolean algebra is called weakly distributive (w.d.) if the intersection of every sequence of dual ideals each having g.l.b. zero has g.l.b. zero. If \( X \) is q.r. and \( R(X) \) is w.d. then \( X \) has property (P). A Boolean space has property (P) iff it corresponds to a w.d. algebra. A Baire space \( X \) admits a category measure iff \( R(X) \) is measurable (and therefore w.d.). The measure is regular iff \( X \) is q.r. In that case \( X \) has property (P). By means of Stone's theory of maps in Boolean spaces all regularly measurable Baire spaces are constructed, and all spaces such that \( R(X) \) is isomorphic to a given complete Boolean algebra. In metrizable spaces category measures are necessarily trivial — confined to a countable dense set of isolated points. (Received December 9, 1959.)
564-129. Berthold Schweizer and Abe Sklar: Statistical metric spaces arising from sets of random variables. II.

In a previous abstract (these Notices, April 1958, pp. 219-220), several theorems on statistical metric spaces whose 'points' are mutually independent, spherically-symmetric Gaussian random vectors were announced. These theorems remain true if the qualification "Gaussian" is replaced by the considerably weaker requirement, "having unimodal densities", provided that certain not-too-severe restrictions are imposed on the convolutions of two such densities. (Received December 9, 1959.)


Let A be a complex associative algebra dense in some Hilbert space H. A will be called a scalar-product algebra (SP-algebra) if the following postulates hold: (1) The operators \( L_a^b : b \rightarrow ab \) and \( R_a^b : b \rightarrow ba \) are continuous for all \( a \) and \( b \) in A. Denote their extensions to H by \( L_a \) and \( R_a \). (2) Each operator \( L_x^b \rightarrow R_b^x \) and \( R_x^b \rightarrow L_b^x \), where \( b \in A \), \( x \in H \), has a closed linear extension \( L_x(R_x) \) which is the closure of the graph of \( L_x(R_x) \). (3) For each \( x \) in \( H \), \( L_x \) and \( R_x \) are both bounded, or both unbounded. (4) If \( L_x \) (or \( R_x \)) is bounded, then \( x \in A \). (5) If \( x \) in H is such that \( L_xa = 0 \) or \( R_xa = 0 \) for all \( a \) in A, then \( x = 0 \). The authors deal with semi-simple SP-algebras in which the orthogonal complement of a right ideal is a right ideal, where all these concepts are suitably defined for the system at hand. If for \( e \) in A, \( L_e \) is a projection that is minimal in the usual projection ordering, then \( eAe \) is isomorphic to the complex number field; moreover, if every projection \( L_f, f \in A \), is minorized by such a minimal projection, then \( A \) is a direct sum of simple SP-algebras that are two-sided ideals of A. Under an additional condition on the annihilators of ideals it is shown that \( A \) contains a dense left ideal isomorphic to a matrix algebra with a special type of trace scalar product. (Received December 9, 1959.)


Let \( \alpha \) be an ordinal number, \( \alpha > 0 \). Assume that \( K_\alpha \) is regular and that \( \sum_{\beta<\alpha} 2^{\alpha_\beta} \leq K_\alpha \). It has been shown that an \( \eta_\alpha \)-set E of power \( K_\alpha \) exists. Let G be the set of all elements of the Hahn group \( R[\{E\}] \) whose support is of power less than \( K_\alpha \). Then G, a totally ordered Abelian divisible group, is an \( \eta_\alpha \)-set
of power $K_{\alpha}$. Let $F$ be the set of all formal power series whose coefficients are in the reals, whose "exponents" are in $G$ and whose support is of power less than $K_{\alpha}$. It is easily seen that $F$ is a real-closed field. Further, $F$ is an $\eta_{\alpha}$-set of power $K_{\alpha}$. (Received December 9, 1959.)


Let $H$ be a Hilbert space, and $\mathcal{D} = \sum_{n=0}^{\infty} \otimes \mathcal{D}^{(n)}$ the corresponding Fock space of symmetric tensors of all orders $n$. (For notation, see Trans. Amer. Math. Soc. vol. 74 (1953) pp. 222-245.) If $H$ is a self-adjoint operator on $\mathcal{H}$, and $\phi \in \mathcal{D}_H$, define the self-adjoint operators $H_{\text{free}} = \mathcal{Q}(H)$ and $H_{\text{interaction}} = q(H\phi)$ on $\mathcal{D}$. Then $H_{\text{free}} + H_{\text{interaction}} + z^{-1}(H\phi,\phi)$ is essentially self-adjoint, so its closure, $H_{\text{total}}$, exists and is self-adjoint. If $e^{iHt}\phi \to 0$ (weak convergence) as $t \to +\infty$, then $\mathcal{U}(0,t) = \exp (iH_{\text{total}}t)$.

564-133. E. A. Bishop: Approximation of analytic polyhedra.

Let $M$ be an $n$-dimensional complex manifold. Let $A$ be an algebra of holomorphic functions on $M$. Let $P$ be an analytic polyhedron in $M$ defined by functions in $A$. That is, let $P$ be a relatively compact subset of $M$ such that there exist functions $f_1, ..., f_k$ in $A$ with the property that $P$ is a component of the open set $P_0 = \{z \in M, |f_j(z)| < 1 \text{ for } 1 \leq j \leq k\}$. The analytic polyhedron $P$ will be called elementary if $k = n$. Our main result states that any analytic polyhedron can be approximated arbitrarily closely by elementary analytic polyhedra defined by functions in the same algebra $A$. This result can be used to obtain an extremely simple higher-dimensional version of the Cauchy integral formula, the integration being done over the distinguished boundary of an elementary analytic polyhedron. The integral can be applied in the usual way to obtain theorems of the Runge type. (Received December 10, 1959.)
Let \( u_1 \) (the components of the displacement vector) satisfy the equations of elasticity in a region \( D \), a portion \( Q \) of whose boundary is spherical. On \( Q \) we prescribe data for one of the following sets of boundary operators:
(a) the normal component of the surface traction, the normal displacement and the normal component of the rotation vector; (b) the tangential components of surface traction and the tangential displacements. If we prescribe zero data on \( Q \) we obtain in each case formulas for the analytic continuation of \( u_1 \) across \( Q \). If \( Q \) is the entire surface of a sphere and \( D \) its interior we show that there are an infinite number of discrete values of Poisson's ratio \( \sigma \) for which the boundary value problems \( (a) \) and \( (b) \) do not have unique solutions. For all other values of \( \sigma \) the solution to \( (a) \) is unique and the solution to \( (b) \) is unique up to an arbitrary hydrostatic pressure. If the region \( D \) is the exterior of the spherical surface and suitable conditions are imposed at infinity then the solutions to \( (a) \) and \( (b) \) are unique for all values of \( \sigma \). (Received December 10, 1959.)

564-135. Y. S. Chow: On the iterations of two conditional expectation operators.

Let \((W,F,P)\) be a probability space, \( x \in L_1 \) and \( F_1 \) and \( F_2 \) two sub-\(\sigma\)-algebras of \( F \). Define \( x_0 = x \) and \( x_n = E(x_{n-1}|F_n) \) for \( n \geq 1 \), where \( F_{2n} = F_2 \) and \( F_{2n-1} = F_1 \). If \( x \in L_2 \), then \[ \lim_{n \to \infty} E |x_n - x_\infty|^2 = 0, \] where \( x_\infty = E(x|F_1 \cap F_2) \). The author obtains the following results: (1) \( \lim E |x_n - x_\infty|^2 = 0 \) for \( x \in L_1 \), (2) \( \lim x_n = x_\infty \) \( p. 1 \) for every \( x \in L_1 \), if and only if \( \sup x_n < \infty \) \( p. 1 \) for every \( x \in L_1 \), (3) if \( F_1 \) and \( F_2 \) are generated by atoms, then \( \lim x_n = x_\infty \) \( p. 1 \) for \( x \in L_1 \), (4) \( x_{2k} = x_\infty \) \( p. 1 \) for \( x \in L_2 \), (5) \( \lim (1/n)(x_1 + \ldots + x_n) = x_\infty \) \( p. 1 \) for \( x \in L_1 \) and \( \lim (1/n) \sum_{i=1}^n |x_i - x_\infty| = 0 \) \( p. 1 \) for \( x \in L_2 \). (Received December 10, 1959.)


An incomplete balanced block design is an arrangement of \( v \) elements in blocks of \( k \) distinct elements each in such a way that each unordered pair of elements occurs \( \lambda \) times. A necessary condition for the existence of such a design is, trivially \( \lambda (v - 1) \equiv 0 \pmod{(k - 1)} \), \( \lambda v(v - 1) \equiv 0 \pmod{k(k - 1)} \) and
v \geq k. It is proved that this condition is also sufficient for \( k = 3 \) and \( k = 4 \) (and for every \( \lambda \)). The proof is given by induction on \( v \) for any pair of fixed values of \( k \) and \( \lambda \). The induction works also for larger values of \( k \) but in this case the existence of designs for initial values of \( v \) remains undetermined.

(Received December 10, 1959.)

564-137. C. C. Hsiung: Some uniqueness theorems on Riemannian manifolds with boundary.

By deriving some new integral formulas the author obtains uniqueness theorems under the group of translations in a Euclidean space for \( n \)-dimensional Riemannian manifolds immersed in a Euclidean space of dimension \( n + m \) for any \( m > 0 \) with a boundary satisfying a natural boundary condition, as well as for hypersurfaces immersed in a Euclidean space with a boundary which can never be empty. The theorem in the first case contains the well-known Christoffel's uniqueness theorem as a special case, and in the second case the well-known Aleksandrov's condition is extended. As corollaries there are deduced conditions for an \( n \)-dimensional immersed manifold with boundary to be a compact subset of an \( n \)-sphere. (Received December 10, 1959.)


Let \( P \) be a set theory with set variables only, which contains all the axioms of Zermelo-Fraenkel including the axiom of foundation. A quantifier is called restricted if it is of the type \( (\exists x)(x \in y \cdot \phi) \) or \( (x)(x \in y \supset \phi) \). A hierarchy is defined by \( \sum_n = \{ \phi; P \vdash \phi \equiv (\exists x_1)(x_2)(\exists x_3)\ldots(Q x_n)\psi, \) where all the quantifiers in \( \psi \) are restricted\}, \( \prod_n = \{ \sim \psi; \psi \in \sum_n \}. \) This hierarchy has the usual hierarchy structure. One can construct formulae in \( \sum_n - \prod_n, \prod_n = \sum_n, \) \( (\prod_n \cap \sum_n) - (\prod_{n-1} \cup \sum_{n-1}) \) (for \( n > 0 \)) which stay there also in every consistent extension of \( P \). Using a theorem which is a common generalization of the Goedel-Rosser unprovability theorem and Tarski's truth definitions theorem one can, if \( P \) is axiomatizable, construct sentences in \( \sum_n - \prod_n, \prod_n - \sum_n, \) \( (\prod_n \cap \sum_n) - (\prod_{n-1} \cup \sum_{n-1}) \). In \( P \) one can prove the consistency of the set of Zermelo together with all the instances of the axiom of replacement which contain at most \( n \) unrestricted quantifiers, where \( n \) is any fixed finite number. If \( \phi(x) \in \sum_1 \) one can prove in \( P \) that \( (\exists x)\phi(x) \) implies the existence of an \( x \) such that \( \phi(x) \), \( x \) is constructible and the order of \( x \) is less than a certain fixed countable ordinal \( \Lambda \). (Received December 10, 1959.)
One of the outstanding problems in topology is whether every compact, connected, simply-connected three-manifold M is homeomorphic to the 3-sphere. In 1958, R. H. Bing showed that such an M is a 3-sphere if each simple closed curve in M lies in a topological cube in M. He also raised the question of whether this conclusion would hold if one were to replace the last condition with the condition that each simple closed curve in M can be shrunk to a point inside a solid torus (of genus one) in M. The author answers this question in the affirmative, using some of the techniques of the original paper, applied to a suitable cellular decomposition of M. This seems to have added interest in light of the fact that the former theorem extends to open subsets of E³ in a reasonable way, while the present theorem does not. (Received December 10, 1959.)

For theorems being improved see Nerode [Notices Amer. Math. Soc. (1958) p. 677], Myhill [Bull. Amer. Math. Soc. (1958) pp. 373-376, Notices Amer. Math. Soc. (1959) p. 526]. Let E = {0,1,2,...}, Λ = isols, Ω = recursive equivalence types. Let ≤, Λ be infimum and order for the power X kP(E) of the Boolean algebra P(E) of subsets of E. Let $F \subseteq X^kP(E)$ be all k-tuples of finite sets, Gödel numbered. A frame is an $F \subseteq \mathcal{F}$ closed under $\Lambda$; put $F^* = \{x \in \mathcal{F} | \exists y \in F, x \leq y \}$, $A(F) = \{y \in X^kP(E) | (\forall x \leq y)(\exists z \in F)(x \leq z \leq y)\}$; if $y \in X^kP(E)$ has upper bounds in $A(F)$, $C_Fy$ is the least. Call $F$ recursive if $F^*$ is recursively enumerable (re) and the map $x \in F^* \rightarrow C_Fx \in F$ is partial recursive. For a k-ary relation $R \subseteq X^kE$, call $F$ an R-frame if $\alpha \in F$, $\alpha_i$ of cardinality $x_i$, implies $x \in R$. Extend R to an $R_\Omega \subseteq X^k\Omega$ by requiring $x \in R_\Omega$ if a recursive R-frame $F$ and $\alpha_i \in x_i$ exist with $\alpha \in A(F)$. Let $R_\Lambda = R_\Omega \cap X^k\Lambda$. Call R totally unbounded (tu) if for all $n \in \mathbb{N}$ there exists $x \in R$ with $x_1, \ldots, x_k \leq n$: then $R = R_\Omega \iff R$ isolated, otherwise $R_\Lambda$ has power $c$: $(X^k\Omega) - R_\Omega = (X^kE) - R \iff (X^kE) - R$ finite, otherwise $(X^k\Lambda) - R_\Lambda$ has power $c$: $R_\Omega \cap X^k(\Lambda - E)$ empty $\iff R$ has no tu re subset, otherwise $R_\Lambda \cap X^k(\Lambda - E)$ has power $c$: $X^k(\Omega - E) \subseteq R_\Omega \iff (X^k\Omega) - R$ not tu; otherwise $(X^k(\Delta - E)) - R_\Lambda$ has power $c$. (Received December 10, 1959.)
Let \( f(z) \) be an entire function and let \( u(r,f) = \min|z| = r|f(z)|. \) It is proved in this paper that \( u(r,f) \to o \) as \( r \to \infty \) provided 'o' is an e.v.N. (exceptional value in the sense of Nevanlinna). In particular it is shown that \( u(r,f) \) tending to zero for \( f(z) \) does not imply that 'o' is an asymptotic value for \( f(z) \). Further for a meromorphic function \( F(z) \) it is proved here that \( \lim \sup r \to \infty \sum_{j=1}^{3} n(r,a_j) \cdot (\rho(r) \geq \rho)^{1/2} \) where \( \rho \) is the order of \( F(z) \), it being assumed that \( o < \rho < \infty \); the \( a_j \) being different from each other \( (o \leq |a_j| \leq \infty) \); and \( \rho(r) \) in the above denotes proximate order relative to Nevanlinna characteristic function \( T(r,F) \). Finally it is known that for a class of entire functions if \( n(r,a)/\rho(r) \to o \) as \( r \to \infty \), then \( o < n(r,x)/\rho(r) \to \infty \) for all \( x \neq a \). Here \( a \) will be called an e.v.L. if \( n(r,a)/\rho(r) \to o \) as \( r \to \infty \). In this paper various relations between e.v.P., e.v.B., e.v.E. e.v.N. and e.v.L. have been studied. For the definition of these exceptional values see S. M. Shah (Compositio Math. vol. 9 (1951) pp. 227-238). (Received December 10, 1959.)

Boolean rings and Boolean logics (= Boolean algebras) are equationally interdefinable in a familiar way (Stone, Trans. Amer. Math. Soc. vol. 40 (1936) pp. 37-111). The theory of ring-logics (Foster, University of California Publications vol. 1 (1951) pp. 385-396) raises this interdefinability to a more general level. Indeed, a ring \( R \) is studied modulo \( K \), where \( K \) is an arbitrary transformation group in \( R \). The Boolean theory results on choosing \( K = C = \text{Boolean group}, \) generated by \( x^* = 1 - x \) (order 2, \( x^{**} = x \)). More generally in a ring with identity 1 the natural group \( N \), generated by \( x^\wedge = 1 + x \) (with \( x^\vee = x - 1 \) as inverse) is of particular interest. Thus specialized to \( N \), a ring \( (R, x, +) \) with identity 1 is essentially a ring-logic, mod \( N \), if the + of \( R \) is equationally definable in terms of its \( N \)-logic \( (R, x, ^\wedge, ^\vee) \). The following theorem is proved: suppose \( R \) is a ring with identity 1 and suppose that, for every \( x \) in \( R \), there exists an integer \( n(x) > 1 \) such that \( x^n(x) = x \), then \( R \) is a ring-logic, mod \( N \). This result generalizes some known theorems on Boolean-rings, p-rings and \( p^k \)-rings. It is further shown that any finite commutative ring with zero radical is a ring-logic, mod. \( N \). (Received December 10, 1959.)
564-143. R. W. Bagley: **On convergence of sequences in \( C(X) \) and pseudo-compactness.**

In any topological space \( X \) the following are equivalent: 1. \( X \) is pseudo-compact. 2. Every sequence of continuous real-valued functions which converges strictly continuously to a continuous function converges uniformly. 3. Every sequence of continuous real-valued functions which converges jointly (in some admissible topology on \( C(X) \)) to a continuous function converges uniformly. 4. Every equicontinuous sequence of real-valued functions which converges pointwise converges uniformly. If \( X \) satisfies the first axiom of countability then the following are equivalent: 1. \( X \) is pseudo-compact, 2. Every sequence of continuous real-valued functions which converges continuously to a continuous function converges uniformly. 3. Every sequence of continuous real-valued functions which converges in the compact-open topology on \( C(X) \) to a continuous function converges uniformly. (Received December 10, 1959.)

564-144. R. L. Kelley and P. M. Swingle: **Clusters of indecomposability.** Part II.

Let \( W \) be a connected set, \( \bigcup Z_i \subset W \ (i = 1, 2, \ldots, n) \). Let \( p \in W, z_i \in Z_i - p \); let \( \{R_{ij}\} \) and \( \{R_{ij}'\} \) be classes of regions which close down on \( z_i \) and \( p \). Then we say \( W \) is shielded at \( p \) out from \( (z_1, z_2, \ldots, z_n) \) iff for each \( R_i, W - R_i = H \cup K \), where \( H \) and \( K \) are mutually separate and for all \( i \) and \( j \), \( R_{ij} \cap H \neq \emptyset \neq R_{ij}' \cap K \). We say \( W \) has \( n \)-fold set \( Z_i \) of indecomposability if for every region-containing connected subset \( V \) of \( W \), \( V \supset \) some \( Z_i \). We show that: In a compact metric space \( W \) is a continuum with \( n \)-fold set \( Z_i \) of indecomposability iff for every \( p \in W \) and every \( z_i \in Z_i - p \ (i = 1, 2, \ldots, n) \), \( W \) is shielded at \( p \) out from \( (z_1, z_2, \ldots, z_n) \). This is false if \( W \) is not closed. We show that continua with \( n \)-fold sets of indecomposability exist in \( E_n \) (\( n > 1 \)) by a modification of the network construction used by R. L. Wilder for Theorem 8 [Math. Ann., vol. 109 (1933) pp. 273-306]. These usually contain indecomposable subcontinua. (Received December 10, 1959.)

564-145. Costas Kassimatis: **Generalized integrals.**

The definitions of the operator \( H_n \) and of the generalized Riemann derivative can be found in: C. Kassimatis, Functions which have generalized Riemann derivatives, Canad. J. Math. vol. 10 (1958) pp. 413-420. In the present paper, the generalized integral \( I_n \) of a function \( f(x) \) is defined and necessary and suffi-
cient conditions for $I_n = 0$ are obtained. Moreover, an additivity law is established for $I_n$ when $n = 3$. The application of the integral $I_n$ to trigonometric series is based on the theorem: If a trigonometric series and its conjugate series are both summable $(C,k)$ for all $x$, and the first series integrated $n$ times, $n > k + 1$, converges to $F(x)$, then the $n$th order de la Vallée Poussin derivative of $F(x)$ exists and is equal to the sum of the first series. (Received December 10, 1959.)

564-146. Hidehiko Yamabe: On a diffusion equation of order one half and its application.

Notations. $D$: A bounded domain in a $C^k$ Riemannian manifold, $k \leq \infty$, or $\omega$, "dV": the volume element, $\Delta$ the corresponding Laplace-Beltrami operator, "$-\lambda_i$": eigenvalue of $\Delta$, $i = 1,2,\ldots$, $\theta_i$ eigenfunction of $\Delta$ corresponding to $-\lambda_i$. Consider an equation $\partial U/\partial t = -(\Delta)^{1/2}U$ with 0 boundary condition. Then the formal Green's function $L(x,y,t) = \sum \exp \left( -\frac{\lambda_i^{1/2}}{2} t \right) \theta_i(x)\theta_i(y)$ is square-integrable over $D \times D \times (E,\infty)$, satisfies $\partial^2 L/\partial t^2 + 1/2(\Delta_x + \Delta_y)L = 0$, is $C^k$ and complex analytic in $z$ with $\Re z > 0$ when the variable $t$ is extended to $t + (-1)^{1/4}$ and converges to $\delta_x(y)$ as $t$ tends to zero. $L$, considered as a function of $s$, satisfies $\partial^2 L/\partial s^2 = \Delta_x L$. Suppose that a solution $\partial u/\partial s^2 = \Delta u$ is given. Set $U = U(s) = (-\Delta)^{1/2}u + (-1)^{1/2} \partial u/\partial s$, $U$ considered as a complex valued function. Approximating $u$ by $u_t = \int L(x,y,t)u(y)dy$, $\partial u/\partial s_t$ satisfies the above equation converging to $U$ in the topology of the same function space. $\|U\|^2 = \int (\Delta u)udV + \int (\partial u/\partial s)^2dV = \|U(s)\|^2$ for each $s$. (Received December 11, 1959.)


Let $E,F$ be two locally convex linear spaces, and $E^*$ and $F^*$ their dual spaces. If $L$ is a closed linear transformation which maps a weakly dense subspace $D(L)$ of $E$ onto a subspace $R(L)$ of $F$, then a weakly continuous linear transformation $T$ from $F$ into $E$ is called a proper Green operator of $L$ if (a) $T$ maps $F$ into $D(L)$, and $T^*$ maps $E^*$ into $D(L^*)$, and (b) $T^*LT = T$ and $LTL = L$. Necessary and sufficient conditions for the existence of proper Green operators are given, and the properties of such operators discussed. (Received December 11, 1959.)

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Denote by A the Banach algebra of absolutely convergent Fourier Series. We shall say that a perfect totally disconnected subset of \([0,2\pi]\) has the Stone-Weierstrass property (the S-W property) if the set \(A_0(P)\) of twice differentiable functions the first derivative of which vanish on \(P\), is dense in \(A\). The closure \(A(P)\) of \(A_0(P)\) is always a separating self-adjoint subalgebra of \(A\). It is clear that if \(P\) is of measure zero one has \(A(P) = A\). We construct sets \(P\) for which \(A(P) \neq A\). This shows that the Stone-Weierstrass theorem cannot be extended to \(A\). We then construct a set \(P\) of positive measure which has the S-W property. The last construction shows that for every function \(g \in A\) one can find a sequence of differentiable functions \(g_n(x)\) such that \(\|g - g_n\| \to 0\) and that \(g'(x)\) tends to zero almost everywhere. (Received December 11, 1959.)

J. H. Walter: Characterization of certain finite linear groups.

Designate by \(Z(G)\), \(D(G)\), and \(T(G)\), respectively, the center, commutator subgroup, and subgroup generated by the involutions of a group \(G\). If \(H \subseteq G\) is a subgroup, let \(C(H)\) be the centralizer of \(H\) in \(G\). Denote by \(P(G)\) the factor group \(P(G)/Z(G)\). A linear group of rank \(n\) is a subgroup of \(GL(n,q)\) containing \(TL_n = T(GL(n,q))\). A projective linear group of rank \(n\) is a subgroup of \(PGL(n,q)\) containing \(PTL_n = P(TL(n,q))\). Let \(q\) be odd. Define inductively a quasilinear group \(G\) of rank \(n \geq 2\) as follows. If \(n = 2\) or \(n = 3\), let \(G\) be a linear group of rank \(n\). For \(n \geq 4\), \(G\) is to satisfy the following conditions:

A. There are exactly \(n + 1\) classes \(K_0, K_1, \ldots, K_n\) of conjugate involutions such that if \(U \in K_p\), \(T(C(U))\) is the direct product of quasilinear groups of rank \(p\) and \(n - p\).

B. \(D(G)\) is perfect and \(Z(D(G))\) is cyclic of order \((q - 1,n)\). There is an involution in \(G\) but not in \(D(G)\) and \(G/D(G)\) is cyclic.

C. \(Z(T(G)) = Z(G)\).

Theorem: If \(G\) is a quasilinear group of rank \(n\), then \(P(G)\) is a projective linear group of rank \(n\). If \(G\) is a direct factor in the centralizer of an involution in a quasilinear group, then \(G\) is a linear group of rank \(n\). This result is to be used in the direct characterization of projective linear groups to follow. (Received December 11, 1959.)


Consider the equation (1) \(\phi(x) = \int_x^{x+1} K(y)\phi(y)dy\) for \(-\infty < x < \infty\) with the
boundary condition (2) \( \lim_{x \to \infty} \phi(x) = 1 \). The main results are: 1° \( K(x) \) is measurable, 2° \( 0 < K(x) \leq 1 \) for almost all \( x \), 3° \( K(x) \) is increasing for \( x \gtrsim \) some number \( M \), and 4° \( \lim_{x \to \infty} \phi(x) = 1 \), then a solution \( \phi(x) \) of (1) and (2) exists if and only if \( \int_{-\infty}^{\infty} (1 - K(y)) dy < \infty \); II if 1°, 2°, 3°, and 4° hold as above, and \( \int_{-\infty}^{\infty} (1 - K(y)) dy < \infty \), then \( \lim_{x \to \infty} \phi(x) \) exists and \( \int_{-\infty}^{\infty} (1 - K(y)) \phi(y) dy = \frac{1}{2} \). The proof of I is made by constructing a minorant for \( \phi \), and of II by application of the Euler-MacLaurin sum formula. (Received December 11, 1959.)

564-151. L. E. Pur sell: Fixed ideals and tangent vectors in infinitely differentiable function rings.

An infinitely differentiable manifold \( P \) and its ring \( C^\infty(P) \) of infinitely differentiable functions are defined as in K. Nomizu, Lie groups and differentiable geometry (Math. Soc. of Japan, 1956). Tangent vectors are defined as in C. Chevalley, Theory of Lie groups, I (Princeton, 1946). Theorem 1. A maximal ideal in \( C^\infty(P) \) is fixed if and only if it has a finite basis. Definition: The maximal fixed ideal \( I(p) = \{ f \in C^\infty(P) | f(p) = 0 \} \). The annihilator of a set \( N \) of tangent vectors at \( p \) is \( \{ f \in I(p) | Lf = 0 \text{ for all } L \in N \} \). Theorem 2. If \( J \) is a subset of \( C^\infty(P) \) and \( I^2(p) \subset J \subset I(p) \), then \( J \) is an ideal in \( C^\infty(P) \) if and only if \( J \) is the annihilator of some linear subspace of the tangent space at \( p \). Theorem 3. The residue class ring \( C^\infty(P)/I^m(p) \) is isomorphic to \( R[x_1, \ldots, x_n]/[x_1, \ldots, x_n]^m \) where \( R[x_1, \ldots, x_n] \) is the ring of all polynomials in \( n \) variables over the real field and \( [x_1, \ldots, x_n]^m \) is the ideal generated by the set of all monomials of degree \( m \) in these variables. Theorem 4. \( C^\infty(P)/ \bigcap_{m=1}^{\infty} I^m(p) \) is isomorphic to the ring of all formal power series in \( n \) variables with real coefficients. (Received December 11, 1959.)

564-152. J. C. Lillo: Approximate similarity and almost periodic matrices.

Denote by \( F^n \) the space of all \( n \times n \) real valued continuous matrices with almost periodic entries and the topology defined by the uniform norm. Denote by \( G^n \) the set of all \( B(t) \in F^n \) such that \( B(t) \) is approximately similar (see author's article to appear in the Proceedings of the National Academy of Sciences) to some matrix \( A = \text{diag}(\lambda_1, \ldots, \lambda_n) \) where \( \lambda_j \) are real constants and \( |\lambda_i - \lambda_j| \neq 0 \) for \( i \neq j \). Here \( A \) depends on \( B(t) \). Then the set \( G^n \) is open in the
space $F^n$ and for every $B(t) \in G^n$ the associated system $\dot{x} = B(t)x$ possesses a fundamental solution whose associated Perron transformation to an upper triangular system is, along with its derivative, almost periodic. If $A(t) \in F^n$ is symmetric and if the constant terms of $A(t)$ form a matrix $A_0$ with $n$ distinct real roots then $A(t) \in G^n$. There exist $B(t) \in F^n$ such that for the system $\dot{x} = B(t)x$ none of the Perron transformations to an upper triangular system are almost periodic. (Received December 11, 1959.)


Let $E$ be a partially ordered vector space whose cone of positive elements is generating. If $a$ is a positive element of $E$, and $V$ a subspace of $E$, then $H_a(V)$ denotes the set of all $x$ in $E$ such that for each positive real number $e$, there exists an element $v_e$ in $V$ for which $-(ea + v_e) \leq x \leq ea + v_e$. The subspace $V$ is called perfect if $V \subseteq H_a(V)$ for some element $a$. A positive linear functional (p.l.f.) $f$ defined on $E$ is called indecomposable if the interval $[0,f]$ consists only of scalar multiples of $f$. In case $E$ has an order unit $e$, it is easy to see that a p.l.f. $f$ is indecomposable if, and only if, $f/f(e)$ is an extreme point of the set of all normalized p.l.f.'s. A maximal ideal of $E$ is called indecomposable if it is the kernel of an indecomposable p.l.f. It is shown that several results of Bonsall [Proc. Amer. Math. Soc. vol. 7 (1956); and Proc. London Math. Soc. vol. 4 (1954)] can be extended to spaces without order unit. For instance, (1) a maximal ideal of $E$ is indecomposable if, and only if, it is perfect; and (2) an indecomposable maximal ideal of $E$ admits every bounded linear operator. (Received December 11, 1959.)

564-154. J. E. Keisler: A necessary and sufficient condition that an lc space be a quasi-complex.

It is shown that a compact, Hausdorff, homology lc space, $X$, is a quasi-complex if and only if $X$ possesses a cofinal family of finite open coverings, $\{U_a\}$, such that each of the canonical homomorphisms $H[N(U_a),G]$ into $H[N(U_b),G]$, where $U_a$ refines $U_b$, is an isomorphism onto. Thus, an ANR is a quasi-complex if and only if it possesses such a cofinal family of finite open coverings. (Received December 11, 1959.)
Let $P$ be a nonconvex $n$-gon. Let $ab$ be a segment of the convex hull $H$ of $P$ such that $P \cap (ab) = a \cup b$. A reflection operation $r$ on $P$ reflects that piece of $P$ - $(a \cup b)$ on the interior of $H$ in $ab$ as a mirror. $r(P)$ is a $k$-gon ($n \leq k \leq n$) with sides congruent to corresponding sides of $P$. Consider a sequence $\{r_m(P)\}$ where $r_0(P) = P$, $r_m(P) = r_{m-1}(r_{m-1}(P))$ ($m > 0$) and the $r_m$'s are arbitrarily chosen reflection operations. **Theorem:** The sequence $\{r_m(P)\}$ is finite, and the last member is a convex $k$-gon ($k \leq n$). **Proof:** Let $v_m$ be corresponding vertices of the polygons $r_m(P)$. (1) Then, as $m \to \infty$, $v_m \to v$; hence, $r_m(P)$ converges pointwise to $r(P)$, a $k$-gon ($k \leq n$). To prove (1), observe that if $a_1, a_2, a_3$ are noncolinear points interior to $H$, then $\{v_j\}$ ($j = 1, 2, 3$) are bounded monotone-increasing sequences of positive numbers. Let $R_j = \lim a_jv_m$. The circles with radii $R_j$ and centers $a_j$ have only one point, $v$, in common; and $v_m \to v$. This implies (1). Now let $v$ be a vertex of the convex hull $Q$ of $r(P)$. (2) $v$ has moved only a finite number, $N_v$, of times. This follows from the fact that the angle at a vertex of $r(P)$, which is also a vertex of $Q$, is less than $\pi$. Finally, let $N = \max N_v$ for $v$ in $Q$. (3) $r_N(P) = Q = r(P)$. This is true since, otherwise, a portion of $r_{N+1}(P)$ must lie outside of the convex hull of $r_N(P)$, that is $Q$, which is impossible. **Conjecture:** For fixed $n$, $N$ is bounded by at least $2n$ for all $P$ and all choices of $r_m$'s. (Received December 11, 1959.)

564-156. D. G. Johnson: **A representation theorem for a class of archimedean lattice-ordered algebras.** Preliminary report.

A lattice-ordered algebra $A$ over the real field $R$ is called a (real) $f$-algebra if $a, b, c \in A$, $a \wedge b = 0$, and $c \geq 0$ imply $ac \wedge b = 0 = ca \wedge b$. A continuous function $f$ on a topological space $X$ into the two-point compactification of $R$ is called an extended (real-valued) function if the subset $\mathcal{F}(f)$ of $X$ on which $f$ is real-valued is everywhere dense. **Theorem:** If $A$ is an archimedean real $f$-algebra containing no nonzero nilpotent elements, then $A$ is isomorphic to an algebra of extended functions on a completely regular Hausdorff space. (This extends to algebras without ring unit element the representation theorem of Henriksen and the author; see Abstract 559-91 Notices Amer. Math. Soc. vol. 6 (1959) p. 434.) (Received December 11; 1959.)

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Consider the system (1) \( x' = Ax + \varepsilon B(t)x \), \( x = (x_1, ..., x_n) \), where \( \varepsilon > 0 \); 
\( C = \text{diag}(\rho_1, ..., \rho_n), \rho_{2j-1} = i\sigma_j, \rho_{2j} = -i\sigma_j, \sigma_j > 0, j = 1, 2, ..., m; \sigma_j = \alpha_j + i/\beta_j \), \( \alpha_j < 0, j = 2m + 1, ..., n \), \( B(t) \) is almost periodic in \( t \) in the sense of Bohr and let \( m[B] = \lim_{T \to \infty} T^{-1} \int_0^T B(t)dt \). Let \( P(t) = \text{diag}(\rho_1^t, ..., \rho_{2m}^t, 1, ..., 1) \) and \( B^*(t) \) be the am \( 2m \) matrix consisting of the first \( 2m \) rows and \( 2m \) columns of the matrix \( P^{-1}(t)B(t)P(t) \). 

**Theorem.** If the eigenvalues of the matrix \( m[B^*] \) have negative real parts, then, for any constant vector \( x_0 \), there exist constants \( \varepsilon_0 > 0, b > 0, \) independent of \( x_0 \), and a \( K(x_0) > 0 \), such that, for \( 0 < \varepsilon \leq \varepsilon_0 \), every solution \( x(t, t_0), x(t_0, t_0) = x_0, \) of (1) satisfies (2) \( \|x(t, t_0)\| \leq K(x_0) \exp[-eb(t-t_0)], t \geq t_0. \) It follows that the zero solution of (1) is asymptotically stable even under sufficiently small nonlinear perturbations \( f(x, t, \varepsilon) \). The "method of averaging" of Krylov-Bogoliubov-Mitropolski is used in the proof. If \( B(t) \) is periodic in \( t \), this theorem coincides with some previous results of the author [Proc. Symp. Ecua. Dif. Ord., Mexico City, 1959] which were obtained by the method developed by Cesari and the author [see L. Cesari, Ergebnisse 16, 1959]. (Received December 11, 1959.)


Suppose \( X \) is a separable metric space, \( \mu \) is a sigma-finite measure on \( \mathcal{B} \) the Borel sets of \( X \), and \( \mu \) has a regular completion with domain \( \mathcal{B}^* \). 

A \( \mu \)-Vitali covering of \( X \) is a subclass \( \mathcal{C} \) of \( \mathcal{B}^* \) which satisfies the conclusion of the Vitali Covering Theorem among other requirements. If \( \lambda \) is an extended-real-valued function whose domain includes \( \mathcal{C} \), then \( \lambda \) and \( \mathcal{C} \) may be used to define a lower measure \( \underline{m} \) and an upper measure \( \overline{m} \). Under certain conditions, the \( \mu \)-nonsingular component of \( \underline{m} = \int \underline{D}(\lambda; \mu, \mathcal{C}) \) and \( \overline{m} = \int \overline{D}(\lambda; \mu, \mathcal{C}) \) where \( \underline{D} \) and \( \overline{D} \) are the lower and upper derivative functions of \( \lambda \) with respect to \( \mu \) and \( \mathcal{C} \). Conditions may be stated in terms of \( \underline{m} \) and \( \overline{m} \) which insure the differentiability of \( \lambda \) with respect to \( \mu \) and \( \mathcal{C} \). (Detailed results may be found in a paper of the same title to appear in Trans. Amer. Math. Soc.) These results may be applied to the study of surface area; for the class \( \mathcal{F}_0 \) (see, Radó, Length and area, p. 439) this measure treatment gives the usual Lebesgue area. The details will appear in a later paper. (Received December 11, 1959.)
Thomas Erber: Inequalities for hypergeometric functions.

Matrix elements of first order "continuum-continuum" transitions in the Coulomb field (Bremsstrahlung, pair production, etc.) may be expressed in terms of Appell's generalized hypergeometric functions. However the explicit numerical evaluation of cross sections often involves further operations (e.g. summation) on these functions. A number of majorants which facilitate these operations have been derived for the Appell functions. Numerous inequalities for other functions are contained in the present results as special cases.

(Received December 11, 1959.)

L. E. Dubins and L. J. Savage: The red and black casino.

A gambler whose initial fortune is positive can make a sequence of bets on any one of which he can stake any amount then in his possession. His fortune increases or decreases after each bet by the amount staked with probabilities w and 1 - w respectively. Theorem 1. If w > 1/2, there exists k = k(w), 0 < k < 1, such that a gambler who adopts the strategy of staking c · y whenever his fortune is y will have his fortune converge almost certainly to +∞ or 0 according as 0 < c < k or c > k. If c = k, the lim and lim of his fortunes will almost certainly be +∞ and 0 respectively. Let G be a gambler's goal. Roughly speaking, a betting strategy is optimal if it maximizes the probability that the gambler's fortune ever attains the goal. A gambler plays boldly if he always stakes as much as possible consistent with the condition that his fortune can never exceed G. Theorem 2. If w < 1/2 bold play is optimal. Let Q(x) be the distribution function of Σ 1/n Y_n where Y_n is a sequence of independent random variables, Y_n = 0 and 1 with probabilities w and 1 - w respectively. Theorem 3. For 0 ≤ f ≤ 1 and 0 ≤ h ≤ g ≤ 1, Q(fg + (1 - f)h) ≤ Q(f)Q(g) + (1 - Q(f))Q(h) provided w ≤ 1/2. The inequality is reversed for w ≥ 1/2.

(Received December 11, 1959.)

P. T. Church and Erik Hemmingsen: Branch sets of light open maps on n-manifolds. I.

Throughout this abstract f will denote a continuous function of an n-manifold M into an n-manifold N. The branch set B_f is the set in M at which f fails to be a local homeomorphism. If dim f⁻¹(p) = 0, for p in N, then f is light. For n = 2 light open maps are topologically analytic (Stoilow, Principes topologiques de la theorie des fonctions analytiques, 1938, p. 121). The following theorems
are representative of results the authors have obtained about such maps for arbitrary $n$: I. If (1) $f$ is an open map of $E^n$ onto $E^n$, $n \neq 2$, (2) $\dim f(B_f) \leq n - 2$, (3) the restriction of $f$ to $E^n - f^{-1}(f(B_f))$ is a covering map, and (4) $B_f$ is compact, then $f$ is a homeomorphism. (The significance of this result is that it is false for $n = 2$ - $f(z) = z^2$ is a counterexample.) II. If $f$ is a light open $C'$ map of $U$, open in $E^n$, into $E^n$, and $\dim B_f \neq n - 1$, then, for all $p \in E^n$, $f^{-1}(p)$ consists of isolated points. (This settles a special case of an old conjecture.) III. If $f$ is light open, then there is no set $U$, open in $N$, such that $U \cap f(B_f)$ is a tamely embedded $(n - 1)$-manifold. (Received December 11, 1959.)

564-162. P. T. Church and Erik Hemmingsen: Branch sets of light open maps on $n$-manifolds, II.

The nature of $B_f$ (for terminology see the previous abstract) can be further restricted. IV. If $M$ is compact, then there is no closed subset $A$ of $N$ such that (1) $f(B_f) \subseteq A$, (2) $N - A$ is connected, (3) $\pi_1(N - A) = 0$, (4) there exists a neighborhood $V$ of a point $p$ of $f(B_f)$ such that $\dim (A \cap V) \leq n - 2$, (5) the restriction of $f$ to $f^{-1}(f(V))$ is light. Corollary: If $f$ is light, $M$ compact, and $\dim f(B_f) \leq n - 2$, then either $f$ is a homeomorphism or $\pi_1(N - f(B_f)) \neq 0$. Almost a corollary is V. If $n \geq 3$, then $B_f$ is perfect. Corollary: If $f$ is $C'$, mapping an open subset $U$ of $E^n$ into $E^n$, $n \geq 3$, and the Jacobian is zero only at isolated points, then $f$ is a local homeomorphism. VI. The authors conjecture that for $n \geq 3$ there is no light map for which $\dim f(B_f) = 0$ (equivalently, no open map for which $\dim B_f = 0$ and $\dim f(B_f) < n$). If such a map exists, then, for each "small" neighborhood $V$ of a point of $f(B_f)$, $\pi_1(V \cap (N - f(B_f))) \neq 0$ (thus $B_f$ must be locally something like Antoine's necklace, which as a particular example the authors can rule out); moreover, there would exist another such map for which $M \subseteq E^n$, $N = E^n$, $\dim B_f = \dim f(B_f) = 0$, and the restriction of $f$ to $M - f^{-1}(f(B_f))$ is a finite-to-one covering map onto $N - f(B_f)$. (Received December 11, 1959.)

564-163. L. E. Claborn: An analogue of the Hilbert theory.

Let $A$ be a Dedekind domain, $K$ the quotient field of $A$, $L$ a Galois extension of $K$, and $B$ the integral closure of $A$ in $L$. If $P$ is a prime ideal of $A$, and $Q$ a prime ideal of $B$ lying over $P$, then $B/Q$ is normal over $A/P$, and if $G'$ is the subgroup of $G$ (the Galois group of $L$ over $K$) leaving $Q$ setwise fixed, then $G'$ acts on $B/Q$ over $A/P$ and it is known that $G'$ gives all automorphisms of
B/Q over A/P. The following analogue of this theorem is proved for the Jacobson theory if L/K is a pure inseparable extension of exponent one: If B is finitely generated as an A module, then the ring of derivations of L/K mapping B into B and Q into Q (which then acts on B/Q over A/P) gives the full Lie ring of derivations of B/Q over A/P. This result is also valid if A is an arbitrary valuation ring and B is finitely generated as an A module. (Received December 11, 1959.)

564-164. Hyman Bass: Projective and torsion free modules.

Let R be a Noetherian integral domain. A module M over R[X] will be called extended if M \cong R[X] \otimes_R M_0 for some R-module M_0. C. S. Seshadri [Proc. Nat. Acad. Sci. U. S. A. vol. 44 (1958) pp. 456-458 and Proc. Amer. Math. Soc. vol. 10 (1959) pp. 670-673] has shown that for certain Dedekind rings R, every finitely generated projective R[X]-module is extended. An elaboration of his argument yields: Theorem: Let R be integrally closed, P a finitely generated projective R[X]-module. Then if either Krull dim R \leq 1 (R is Dedekind) or rank P \leq 1, P is extended. An example of S. Schanuel shows the failure of this result without integral closure. Theorem: Suppose R is finite under its integral closure. Then every finitely generated torsion free R-module is a direct sum of modules of rank 1 if and only if every ideal in R can be generated by two elements. (Received December 11, 1959.)

564-165. R. H. Bing: Pushing a 2-sphere into its complement.

Suppose S is a 2-sphere in E^3, U is one component of E^3 - S, and \varepsilon > 0. It is shown that S contains a Cantor set C such that there is a map f of S into U + C such that f moves no point more than \varepsilon and f is a homeomorphism on S - C. This result may be used to show that for each simple closed curve J in U there is a Cantor set C in S such that J can be shrunk to a point in U + C. It also shows that S contains a 0 dimensional F_\sigma set X such that U + X is uniformly locally simply connected. If there is a homotopy H_t(s) (0 \leq t \leq 1, s \in S) of S into S + U such that H_0 is the identity and H_t(S) \subseteq U for t > 0, then U is uniformly locally simply connected and S is tame from the U side. (Received December 11, 1959.)
We consider the \( N \times N \) matrix linear differential operator \( L = P(x,y;D) - E(\partial/\partial y) \) defined for \( (x,y) \in \mathbb{R}^n \times [y',y''] \). We assume that \( L \) is uniformly parabolic in \( \mathbb{R} \). Theorem: Assume \( A^{(k)}(x,y) \in \mathcal{C}^{r(k)+\infty}[\mathbb{R}^n \times C^0[y',y''] \), where \( r(k) \geq 0 \), and let \( s = \max |k| \leq 2b \). If \( u \) is a solution of the initial value problem: \( Lu = 0 \) for \( (x,y) \in \mathbb{R}^n \times (y',y'') \), then \( u \equiv 0 \) in \( \mathbb{R} \). If \( s = 0 \) the Hölder continuity of \( u \) can be dropped. This result contains and interpolates between the results of Efim'ev (Dokl. Akad. Nauk. vol. 120 (1958) pp. 980-983) for the case \( s = 0 \) and the author (Bull. Amer. Math. Soc. vol. 65 (1959) pp. 310-318) for the case \( s = 2b \). The proof is based on certain a priori estimates for \( \sum_k N_k(u; \gamma, y')d\gamma \) and is essentially a generalization of the method used for the case \( s = 2b \). (Received December 11, 1959.)
with inner product $[u,v] = (A_2u,v)$. Let $Q^1$ be the projection in $L^2$ on the span of the arbitrary elements $\{q_1,q_2,\ldots,q_j\}$, and let $P^k$ be the projection in $L^2$ on the span of $\{u^1_1,u^1_2,\ldots,u^1_k\}$. Define the operators $A_{1,k} = A_1P^k + A_1^{k+1}(I - P^k)$ and $A_{1,k}^1 = A_1,k + A_2Q^1$. The eigenvalues of $A_{1,k}^1$ are lower bounds determined from the zeroes of $\det(R_{1,k}A_2q_j + q_j^1q_j^1)$ where $R_{1,k}$ is the resolvent of $A_{1,k}$ given in closed form by $R_{1,k} = \sum_{i=1}^{k} (\lambda_i^{1,k} - \lambda_i^{-1})^{-1}(f,u_i^1)u_i^1 + (\lambda_i^{1,k} - \lambda_i^{-1})^{-1}[f - \sum_{i=1}^{k} (f,u_i^1)u_i^1]$. Applications will be given elsewhere. (Received December 11, 1959.)

564-169. M. S. Klamkin: On a class of incomplete gamma functions.

A finite set of functions $\{F_r(x)\} (r = 1,2,\ldots,n)$ is said to be "completely known" if all their derivatives and integrals can be expressed as elementary functions (polynomials, trigonometric exponentials and their inverses) of itself, elementary functions and other "completely known" functions. Since $\text{erfc } x = -\frac{1}{\sqrt{\pi}} e^{-x^2}$, $\int_{-\infty}^{x} \text{erfc } x \, dx = \frac{1}{\sqrt{\pi}} e^{-x^2}$, it follows that $\text{erfc } x$ is "completely known". The complete set (in $n$) of gamma functions $\Gamma(x) = \int_{0}^{\infty} e^{-x^m} \, dx, r = 0,1,2,\ldots,m - 2$ is shown to be "completely known" and some properties of this class are derived. (Received December 11, 1959.)

564-170. C. N. Lee: Compact 0-dimensional transformation groups.

Let $(G,X)$ denote a compact transformation group acting freely on a locally compact Hausdorff space $X$. Cartan-Leray-Borel spectral sequence for $(G,X)$ is established where $G$ is not required to be a Lie group. Using this, $H^*_c(X/G)$ is computed over various coefficients where $G$ is a p-adic group with the trivial induced action on $H^*_c(X)$. For instance, $H^*_c(X/G,Z_p) \cong H^*_c(S^1 \times X,Z_p)$ as a ring where $S^1$ is a circle. $X$ is said to have Property $R_n$ iff the integral cohomology dimension of $X$ is $n$ and $X$ has locally constant local $n$-cohomology $H^*_c(X,Z)$ isomorphic with $Z$. Theorem 1: Let $(G,X)$ be a $\{p_j\}$-adic transformation group acting freely on $X$ with Property $R_n$. Then $G$ is either finite or contains a $p$-adic subgroup for almost all $p \in \{p_j\}$. Theorem 2: Using the same notations as in Theorem 1, let $X$ be orientable, that is, $H^n_c(X,G,Z_p) \cong H_c^n(S^1 \times X,Z_p)$ as a ring where $S^1$ is a circle. $X$ is said to have Property $R_n$ iff the integral cohomology dimension of $X$ is $n$ and $X$ has locally constant local $n$-cohomology iff $G$ is finite. Theorem 3. Let $(G,X)$ be a compact 0-dimensional transformation group acting freely on an n-gm $X$ over the rational field $K$. Then $X/G$ is also an n-gm over $K$. Corollary. Let $(G,X)$ be any compact connected transformation.
group acting freely on a 3-gm over K. Then G is a Lie group. This is known for topological 3-manifold. (Received December 11, 1959.)

564-171. Lawrence Fearnley: Continuous mappings of the pseudo-arc.

The question "What characterizes all continuous images of the pseudo-arc?" has been raised by R. H. Bing (Summary of Lectures and Seminars, The Summer Institute on set theoretic topology, Madison, Wisconsin, 1955, p. 73). In this paper a generalization of chainability, named p-chainability, is defined and established as a characterization of the continuous images of the pseudo-arc. Using this characterization it is shown that continua which are finite unions or countable topological products of continuous images of the pseudo-arc are themselves continuous images of the pseudo-arc. This characterization is also used in showing that there exist compact metric continua which are not continuous images of the pseudo-arc. Specific examples of continua shown to be of this type include a tree-like plane continuum, an arc-wise connected continuum, and an aposyndetic continuum. (Received December 11, 1959.)


An expansion of the Gaussian hypergeometric function in series of functions of the same kind but with changed argument is derived. For special values of the parameters the expansion simplifies considerably and is especially advantageous since the change in argument yields more rapidly converging series. Known Chebyshev type expansions for the logarithm and arc tangent functions are derived. The Gauss transformation for the complete elliptic integral of the first kind is a special case. Finally, a rapidly converging expansion of the complete elliptic integral of the first kind is derived for modulus near 1. (Received December 14, 1959.)


Some of the difficulties encountered in the theory of stochastic processes with an uncountable parameter set have been attacked by Doob (Stochastic processes, Wiley, 1953) by modifying the process to obtain separability, and recently by Nelson (Ann. of Math. vol. 69 (1959) pp. 630-643) by considering the properties of regular Borel measures on function space. This paper applies
the McShane-Bourbaki generalization of the Daniell integral (the generalization uses limits of uncountable collections of functions) to families of consistent integrals on the class of those continuous functions which depend on only a finite number of coordinates to define an integral on the product of compact Hausdorff spaces. A regular Borel measure is obtained in a constructive way which yields readily theorems on the approximation of measurable sets which imply those of Nelson. The resulting stochastic process is separable, as follows from a significantly more general theorem. In addition, it is shown that the Baire functions on the space actually depend on only a countable number of coordinates. (Received December 14, 1959.)

564-174.

WITHDRAWN


An algorithm for determining all the zeros of a polynomial of degree \( N \) is developed. The algorithm is motivated by the relations between the zeros and coefficients and furnishes a sequence \( u(n) \). Conditions are derived under which \( \lim u(Nm + k) \) exists for \( k = 1, 2, \ldots, N \). The various limiting values are the zeros of the polynomial. (Received December 14, 1959.)


A halfring \( H \) is a semiring which can be embedded in a ring. Let \( R \) be the ring which is generated by \( H \). A topological halfring is a halfring \( H \) together with a Hausdorff topology on \( H \) under which the halfring operations are continuous. Following Rothman [Embedding of topological semigroups, Math. Ann. (to appear)], a halfring \( H \) has property \( F \) if \( i, x \in H \) and an open subset \( V_i \exists i \) imply that there exists an open subset \( U_x \exists x \) such that \( x + i \)
The following theorems are proved: Theorem 1. A locally compact topological halfring with property \( F \) is embeddable in a locally compact topological ring. A semisimple halfring \( H \) is said to be strongly semisimple, if the ring \( R \) generated by \( H \) is also semisimple. Theorem 2. A locally compact bounded strongly semisimple halfring \( H \) with property \( F \) is embeddable in a locally compact bounded semisimple ring. (Received December 14, 1959.)


Mills and Seligman [J. Math. Mech. (1957)] considered the following set of axioms for a (finite dimensional) Lie algebra \( L \) with Cartan subalgebra \( H \):

1. \( LL = L \);
2. \( L \) is centerless;
3. \( H \) acts diagonally on \( L \);
4. If \( \alpha \neq 0 \) is a root then \( L_{\alpha} \) is 1-dimensional;
5. No root string is infinitely long (i.e., circular).

They show, over a field \( F \) of characteristic \( \not\in \{2,3\} \), that these axioms imply that \( L \) is a direct sum of simple algebras of classical type, i.e., analogues over \( F \) of the simple Lie algebras of characteristic 0. A system of axioms is considered here from which the rather unnatural axiom (5) above is deleted, (3) and (4) are sharpened to:

3. Each root space is 1-dimensional;
4. If \( \alpha \neq 0 \) is a root then \( \alpha(L_{\alpha} \alpha) \neq 0 \). It is proved, over an algebraically closed field of characteristic \( \geq 5 \), that if \( L \) satisfies (1), (2), (3') and (4') then \( L \) is a direct sum of simple algebras which are either of classical type or of rank one and hence [Block, these Notices, Abstract 549-25, vol. 5 (1958) p. 605] either of classical type or Albert-Zassenhaus algebras. Conversely, any of these latter algebras has an \( H \) such that (1) - (4') hold. (Received December 14, 1959.)

564-178. C. E. Burgess: Homogeneous continua which are almost chainable.

A continuum \( M \) is said to be almost chainable if, for every positive number \( \epsilon \), there exist an \( \epsilon \)-covering \( G \) of \( M \) and a linear chain \( C(g_{1}, g_{2}, \ldots, g_{n}) \) of elements of \( G \) such that no \( g_{i} \) (\( 1 \leq i < n \)) intersects an element of \( G - C \) and every point of \( G \) is within a distance \( \epsilon \) of some element of \( C \). The following theorems are proved. An indecomposable plane continuum \( M \) is almost chainable if, for each positive number \( \epsilon \), \( M \) can be covered by a circular \( \epsilon \)-chain of open disks. A tree-like continuum is almost chainable if it is nearly homogeneous and has only a finite number of branches. If a nondegenerate homogeneous continuum \( M \) is almost chainable, then every proper subcontinuum of \( M \) is a
pseudo-arc. This last theorem and Bing's result that every homogeneous
chainable continuum is a pseudo-arc [Each homogeneous nondegenerate
chainable continuum is a pseudo-arc, Proc. Amer. Math. Soc. vol. 10 (1959)
pp. 345-346] hold for a weaker type of homogeneity where for each point p of M
and each nondegenerate subcontinuum K of M there is a homeomorphism of M
onto itself that carries p into a point of K. (Received December 14, 1959.)

564-179. F. W. Carroll: Difference properties for polynomials and
exponential polynomials on topological groups.

Let G be a locally compact abelian group, and let B be the closed sub-
group consisting of all compact elements of G, so that G/B = R^n + G_1, where
G_1 is discrete and torsion free. (Cf. Pontrjagin, Topologische Gruppen, vol. 2,
pp. 44-47.) A continuous complex-valued function P on G is a polynomial of
degree n if, for each (x_1, x_2) in G X G, the values P(x_1 + kx_2) (k = 0, ± 1, ...)
are the values assumed on {k} by some polynomial on R of degree ≤ n, and the
degree n is attained for some choice of (x_1, x_2). An exponential polynomial on
G is a finite sum \sum P j z_j, where each P_j is a polynomial and each z_j is a homo-
morphism of G into the multiplicative group of nonzero complex numbers.
Theorem I: Let f be a complex-valued function such that, \forall h \in G, \Delta_h f:
\Delta_h f(x) = f(x + h) - f(x), is an exponential polynomial. Then a N & S condition
for (*)f = g + T, where g is an exponential polynomial and T is additive (i.e., a
solution of the functional equation T(x + y) = T(x) + T(y)) is that G be compactly
generated. Theorem II: Let f be a c - v function such that, \forall h \in G, \Delta_h f is
a polynomial. Then a N & S condition for (*), with g a polynomial and T
additive, is that G_1 have a finite "Hamel basis". (Received December 14, 1959.)

564-180. A. H. Copeland, Jr., and Johannes de Groot: Linearization of
autohomeomorphisms.

An autohomeomorphism \phi of a completely regular space X may be regarded
as a linear map. That is, there exist a topological linear space H, a con-
tinuous linear transformation \eta on H and an imbedding \zeta: X \rightarrow H such that
\zeta \phi = \eta \zeta. If X is metrizable, H may be chosen to be a real Hilbert space and
\eta a bounded linear homeomorphism of H on itself. If, in addition, X has finite
dimension n and \phi has finite period k, then H may be chosen to be a finite
dimensional Euclidean space, and \eta to be an orthogonal transformation. If k is
prime, dim H = 2n + 2m + 1, where 2m is the smallest even integer larger than

Consider the following three properties of a graph G: I. G has at least \( n + 1 \) vertices, and it is impossible to drop out \( n - 1 \) or fewer vertices and the arcs upon them in such a manner that the resulting graph is not connected. II. Any two vertices \( a \) and \( b \) of G may be joined by \( n \) chains, any two of which have only the vertices \( a \) and \( b \) in common. III. Any two disjoint sets \( X = \{x_1, ..., x_n\} \) and \( Y = \{y_1, ..., y_n\} \), each of \( n \) distinct vertices of G, may be joined by \( n \) chains no two of which have a common vertex. H. Whitney (Amer. J. Math., vol. 54 (1932) pp. 150-168) has called a graph that satisfies I \( n \)-tuply connected, and has proved that for graphs with no 1-circuits and no 2-circuits I and II are equivalent. In the present paper a new and simpler proof of Whitney's theorem is given, using the max flow-min cut theorem of flows in networks (G. B. Dantzig and D. R. Fulkerson in Ann. of Math. Studies, No. 38, pp. 215-221; D. Gale, Pacific J. Math. vol. 7 (1957) pp. 1073-1082). The absence of 1-circuits and 2-circuits is not assumed, but G is assumed to have at least \( n + 1 \) vertices. It is also shown that if G has at least \( 2n \) vertices, then the properties I, II, and III, are all equivalent. (Received December 14, 1959.)

564-182. Louis Solomon: The Schur indices of groups with abelian Sylow subgroups.

Let G be a finite group and let \( \chi \) be an absolutely irreducible character of G. Schur has shown that the index \( m \) of \( \chi \) divides the degree \( f \) of \( \chi \) and that if \( m = f \) then the Sylow subgroups are either cyclic groups or generalized quaternion groups. This suggests a connection between the index and the manner of generation of the Sylow subgroups. Theorem: If the \( p \)-Sylow subgroup is abelian of order \( p^n \) and has \( d \) generators, then the highest power of \( p \) which divides \( m \) is at most \( p^{n-d} \). In particular, if the Sylow subgroups are all elementary abelian, then all the indices are one. The main tool in the proof is a reduction theorem of R. Brauer. (Received December 14, 1959.)
564-183. Andrew Sobczyk: Regression with special orthogonal vectors.

The special vectors \( \mathbf{s}^{(1)}, \ldots, \mathbf{s}^{(N)} \), for \( N = 2^p \), \( p \) an integer, are the vectors whose coordinates are the rows of the system of symmetric matrices which is defined inductively as follows: \( B_1 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \), \( B_p \) equals \( B_1 \) with \( \pm B_{p-1} \) substituted in place of \( \pm 1 \). It is shown that the \( \mathbf{s}^{(k)} \)'s may be indexed as follows: for each \( k = 1, \ldots, N \), \( k \) is one more than the number of changes of sign of the coordinates of \( \mathbf{s}^{(k)} \). Any function or empirical data \( (y_1, \ldots, y_N) = \{ y_t \} = Y \), \( t = 1, \ldots, N \), may be exactly represented in the form \( y_t = \sum_{k=1}^{N} b_k \mathbf{s}^{(k)} \), where the \( b_k \) are "Fourier" coefficients. For regression, an appropriate subset of \( p \) of the \( N \) special vectors is chosen, and the least squares or Markoffian best estimates (errors purely random or successively correlated) is calculated using Parseval's formula. A "finite harmonic analysis" is developed to a considerable extent, including a convolution theorem and an analogue of the random process called the "model of random phases" in terms of Fourier series. "Fourier" coefficients for polynomials are determined by recursion. A correspondence with the Rademacher orthogonal functions is indicated and exploited to obtain asymptotic results showing the smoothness of fit obtainable by relaxing the constraint to polynomials in the subspaces spanned by polynomials. (Received December 14, 1959.)


An exponential polynomial with polynomial coefficients is a finite sum \( f(z) = \sum P_n(z) \exp(a_nz) \), where the \( P_n \) are polynomials in \( z \). Let \( E_p \) denote the set of all such functions. In case the \( P_n \) are all constant, \( f \) is called an exponential polynomial with constant coefficients. Let \( E \) denote the set of such functions. J. F. Ritt proved that if \( f = gh \) where \( f, g \in E \) and \( h \) is entire, then \( h \in E \). P. Lax gave another proof of this result. We give a different proof, and prove also that if \( f \in E_p, g \in E \) and \( h \) is entire, then \( h \in E_p \). If \( f \in E \) and \( g \in E_p \) then \( h \) need not be in \( E_p \) (for example, \( f(z) = \sin z, g(z) = z \)). (Received December 14, 1959.)


Let \( K \) be any field, and \( P \) the polynomial ring in \( k \) indeterminates \( x_1, \ldots, x_k \) over \( K \). Let \( P' \) denote the (isomorphic) ring of differential operators generated over \( K \) by \( \partial/\partial x_1, \ldots, \partial/\partial x_k \). If \( p \in P \) we denote by \( D_p \) the corres-
ponding operator in $P'$, $D_p = p(\partial/\partial x_1, \ldots, \partial/\partial x_k)$. Let $p_1, \ldots, p_r \in P$ generate the ideal $I$, and denote by $S$ the set of $s \in P$ such that $D_{p_i} s = 0$ ($i = 1, \ldots, r$).

**Theorem.** If no sum of nonzero squares of elements of $K$ equals zero, and the $p_i$ are all homogeneous, then $P = I \oplus S$. For example, taking $K$ to be the real field, and a single homogeneous polynomial $q$, the polynomial solutions of $D_q u = 0$ are a maximal set of polynomials linearly independent (mod $q$), which implies that for $x = (x_1, \ldots, x_n)$ restricted to the algebraic manifold $q(x) = \text{const.}$, every polynomial $p(x)$ of degree $n$ admits the decomposition $p(x) = \sum s_j(x)$ ($j = 0, \ldots, n$), where $s_j$ has degree $\leq j$ and $D_q s_j = 0$.

**Remarks.** 1. The restriction on $K$ in the above theorem is necessary. 2. If one or more of the $p_i$ is not homogeneous, it is still true that the polynomials annihilated by all the $p_i$ are mutually incongruent (mod $I$). (Received December 14, 1959.)


Let $K(X)$ be the complex of a triangulated $n$-manifold $X$ and let $A(X)$ be the algebra of real functions generated by the constant functions and the "peak" functions $f_i$ which are 1 at the $i$th vertex $v_i$, 0 on the complement of the star of $v_i$ and linear on simplexes. The peak functions and the fixed maximal ideals of $A$ are characterized algebraically, and the set of fixed maximal ideals topologized to recover $X$. Simplicial mappings of $K(X)$ into $K(Y)$ generate and are generated by homomorphisms of $A(Y)$ into $A(X)$. The algebra $A$ provides a simple setting for much of differential geometry. Thus, there is a natural "Riemannian" metric and contravariant tensor fields are multilinear derivations on $A(X)$. Components of a tensor are defined in terms of action on peak functions. The algebra $A(X)$ and its tensor fields seem fruitful for relating concepts of differential geometry to those of combinatorial topology. For example, there is a natural correspondence between alternating contravariant tensor fields of order $p$ and certain $(n - p)$-cocycles whose coefficients are functions. (Received December 14, 1959.)


The integral surface, in fact, consists of almost periodic solution curves. We show, by a two dimensional example, that this result may be used to obtain such a periodic integral surface from a periodic perturbation of an autonomous system having an orbitally asymptotically stable limit cycle for which more than one of the characteristic exponents of the associated variational equations are zero, a case to which the Diliberto-Hufford perturbation theory (cf. Contributions to the theory of nonlinear oscillations, III) is not directly applicable. It is also shown that a 3-dimensional autonomous system of the type considered by L. L. Rauch, (Contributions to the theory of nonlinear oscillations, I) but somewhat less restricted, possesses such a periodic integral surface, in this case, since the system is autonomous, a cylinder. (Received December 14, 1959.)

564-188. I. E. Segal: Essential finiteness of the interaction hamiltonian of certain quantum fields.

Let $A_1, A_2, \ldots$, and $B_1, B_2, \ldots$, be two sequences of self-adjoint operators in a Hilbert space $K$, all of which commute. Then there exists a sequence of canonical variables $P_1, Q_1, P_2, Q_2, \ldots$, on a Hilbert space $M(\exp(isQ_j)\exp(itP_k) = \exp(is\delta_{jk})\exp(itP_k)\exp(isQ_j), \text{etc.})$ such that $M = K \times L$ for some Hilbert space $L$, and $\sum_{n=1}^{\infty} [P_n (A_n X 1) + Q_n (B_n X 1)]$ (I = identity on $L$) is essentially self-adjoint. (Received December 14, 1959.)

564-189. A. P. Robertson and Wendy Robertson: On the space of subsets of a topological vector space.

The uniform structure on a locally convex topological vector space $E$ gives rise to a natural uniform structure on the set $\mathcal{G}$ of closed subsets of $E$, analogous to the Hausdorff metric on the set of closed subsets of a metric space. The main result of this paper is that, if $E$ is complete, the set of weakly compact subsets of $E$ forms a closed subset of $\mathcal{G}$; this is a generalisation of the theorem that a uniform limit of weakly compact linear mappings is weakly compact. (Received December 14, 1959.)


Let $D$ be a region in the complex plane $E$, and let $f$ be a topological mapping of $D$ onto $D^*, D^* \subseteq E$. Let $V_i$ be an infinite set of quadrangles $V_i, V_i \subseteq D$, which are pairwise disjoint. Let $m_1$ be one of the modules of $V_i$, and $m_1^*$ be the
Theorem: A given $f$ is quasiconformal in the geometrical sense (Ahlfors, J. d'Analyse vol. 3 (1953-1954)), iff for any $V$ the series $\sum_{i=1}^{\infty} m_i$ and $\sum_{i=1}^{\infty} m^*$ are either both divergent or both convergent. The nontrivial part of the theorem is proved by showing that the so-called dilatation function $\delta(p)$ is bounded in $D$. Let $U, U \subset D$, be a neighborhood of a point $p, p \in D$. Then $\delta(U) = \sup_Q \left[ m^* / m, m / m^* \right]$ where $Q$ is the set of the quadrangles contained in $U$. Definition: $\delta(p) = \inf _U \delta(U)$ where $U$ is the filter of the $U$ of $p$. The proof of the boundedness of $\delta(p)$ relies upon the Lemma: Let the sequence $p_\nu$ have the properties that each $p_\nu$ is isolated and $\delta(p_\nu) \to \infty$. Then there exists a set $V$ such that $\sum_{i=1}^{\infty} m_i$ diverges but $\sum_{i=1}^{\infty} m^*$ converges.

(Received December 14, 1959.)

564-191. Edgar Reich: On an example of Koebe's for slit mappings.

In 1917, Koebe [Goettinger Nachrichten (1918) pp. 60-71] outlined the construction of conformally equivalent regions, $\Omega_1, \Omega_2$, of the $z$-plane with the following properties which have enabled Koebe and later writers to establish a variety of counterexamples in complex function theory: The regions $\Omega_k$ are such that the components of $E_1 = \text{bdry } \Omega_1$ are points or parallel slits (not all of zero length), $\text{Area } (E_1) = 0$, while the components of $E_2 = \text{bdry } \Omega_2$ are merely points on a fixed straight line. As Koebe showed, the existence of $\Omega_k$ with the above properties is guaranteed if it is possible to construct the slits of $E_1$ (say, parallel to the $y$-axis) such that $E_1$ is symmetric with respect to the $x$-axis, and such that each point on a nonpoint component of $E_1$ is the limit of points of $E_1$ both from the left and right. It does not, however, appear to have been previously noted that Koebe committed an oversight; his set $E_1$ is not closed. In the present work the gap in Koebe's reasoning is filled by constructing an $E_1$ by a technique differing from Koebe's. (Received December 14, 1959.)


Let $\{X_n, n \geq 1\}$ be a process whose finite dimensional densities (Leb.meas.) $p_n(x_1, \ldots, x_n; \theta)$, or $p_n$, for each $n$ satisfy the conditions: (limits as $n \to \infty$)

(a) $d^i p_n / d\theta^i$ (i = 1, 2) exist and are continuous for $\theta$ in $\bar{A}$, $A$ being a finite non-degenerate interval. These are dominated by $G_i(x)$, where $E_\theta(G_i(X))$ are uniformly convergent for all $\theta$ in $\bar{A}$, (b) $C_n(\theta) = E_\theta(d\ln p_n / d\theta)^2 = E_\theta(\theta_n^2(\theta)) \to \infty$, $\bar{A}$.
all $\theta$ in $A$ (c) $E_0(\phi_n'(\theta)/C_n(\theta)) \leq M < \infty$, all $n$ and $\theta$ in $A$, where $\phi_n'(\theta) = d\phi_n/d\theta$,

(d) for $0 < \delta < 1$, $\exists e_\delta > 0$, $\forall$ for all $\theta$ in $A$ and all $n$, $Pr\{|\phi_n'(\theta)|/C_n(\theta) \geq e_\delta\} > 1 - \delta$. Theorem 1: If conditions (a) - (d) hold, and $p_n(x, \theta_1) \neq p_n(x, \theta_2)$ a.e.

for $\theta_1 \neq \theta_2$, then the maximum likelihood (m.l.) equation, $\phi_n'(\theta) = 0$, has a root $\theta_n$ which is a consistent estimator of $\theta$. ($\theta_n$ is consistent for $\theta$ if $(\theta_n - \theta) \to 0$ in prob.) Theorem 2: Every consistent m.l. estimator $\theta_n$ of $\theta$ is asymptotically efficient in the weak sense [i.e. $\exists$ random sequences $\{W_n\}$ and $\{V_n\}$ (i) $E_\theta(W_n) \to 0$, $E_\theta(W_n^2) \to 1$, (ii) $E_\theta(V_n) \to 1$, $Pr\{\lim V_n = 0\} = 0$ and (iii) $(C_n(\theta))^{1/2} \cdot (\theta_n - \theta) - (W_n/V_n) \to 0$ in prob.]. If $\theta$ is a k-vector a (somewhat weaker)

extension of the above results is obtained. These generalize the results of Wald (Ann. Math. Stat. vol. 19 (1949) pp. 40-46). (Received December 14, 1959.)


Let $D$ be a simply connected domain with area $A$ and perimeter $L$. Let $\lambda(k)$ be the fundamental frequency of the membrane problem $\Delta u + \lambda u = 0$ in $D$, $\partial u/\partial n + ku = 0$ on the boundary. Let $D'$ be the domain of area $A$ lying between two concentric circles, the outer of which has radius $L/2\pi$. Let $\lambda'(k)$ be the fundamental frequency of the membrane problem of $D'$ with $\partial u/\partial n + ku = 0$ on the outer circle and $\partial u/\partial n = 0$ on the inner circle. It is shown that $\lambda(k) \leq \lambda'(k)$. By the same method it is shown that the torsional rigidity $P$ of $D$ has the lower bound $2\pi PA^{-2} \leq 1 - 2d(1 - d)^{-2}(1 - d - d \ln d)$, where $d = 1 - 4\pi AL^{-2}$. (The inequality $2\pi PA^{-2} \leq 1$ was proved by G. Polya and G. Szegö). Equality in the bounds for $\lambda(k)$ and $P$ is attained when $D$ is a circle, and approached as $D$ tends to an infinite strip. These results extend and improve recent results of E. Makai [Acta Sci. Math. Szeged vol. 20 (1959) pp. 33-35] and G. Polya [J. Indian Math. Soc.]. (Received December 14, 1959.)


A lower bound for the lowest nonzero eigenvalue $\mu_2$ of the problem $\Delta u + \mu u = 0$ in an $N$-dimensional domain $D$ with $\partial u/\partial n = 0$ on the boundary is useful in approximating the solution of Neumann's problem in $D$. Let $\lambda(k)$ be the lowest eigenvalue of the problem $\Delta u + \lambda u = 0$ in $D$ with $\partial u/\partial n + ku = 0$ on the boundary. Simple lower bounds for $\lambda(k)$ have been given by the authors [J. Soc. Indust. Appl. Math. vol. 5 (1957) pp. 171-182]. Let $p_2$ be the first nonzero eigenvalue of the Stekloff problem $\Delta u = 0$ in $D$ with $\partial u/\partial n = pu$ on the
boundary. A lower bound for $p_2$ in terms of the diameter of $D$ and the curvature of its boundary is derived. The inequality $\mu_2(1 + kp_2^{-1}) \geq A(k)$ is found, and gives a lower bound for $\mu_2$. If $D$ is convex and its diameter is $d$, the simple lower bound $\mu_2 \geq \pi^2/(4d^2)$ is obtained. (Received December 14, 1959.)


Authors consider the boundary value problem of the linear elliptic differential equation $L(u) = au_{xx} + 2bu_{xy} + cu_{yy} = f(x,y)$ for the square $S: \{0 < x, y < 1\}$ with zero Dirichlet data. The coefficients are assumed to be continuous in $S$ and satisfy the uniform ellipticity condition. $f$ is continuous, or at least square integrable. The differential equation is replaced by a difference equation $L_{ik}(u^h) = F_{ik}$ for a grid function $u^h = \{U_{ik}\}$, defined in the grid point $P_{ik}$ of a square mesh of width $h = 1/(N + 1)$. For $F_{ik}$ the values $f(P_{ik})$ are taken, if $f$ is continuous, or else the values in the grid points of a double mean $\omega^2(f)$ over squares of side $\rho \geq h$. The following error estimate is derived: (*$\max_{1 \leq i, k \leq N} |u(ih, kh) - U_{ik}| \leq C[\epsilon^H(f) + \|f\|_2(a, b, c) + H^2\rho^{-4} + \rho^4].$ Here $\|f\|_2$ is the $L^2$-norm of $f$, and the $\epsilon^H$ denote the moduli of continuity. If $f$ is merely square integrable, then the term $\epsilon^H(f)$ has to be replaced by the grid norm $\|\omega^2(f) - F\|_g/4$ where $\|V\|_g = [\sum_{1 \leq i, k \leq N} V_{ik}^2]^{1/2}$. $C$ is a numerical constant, only depending on the constant of ellipticity of the operator $L(u)$.

With $\rho = h^{2/5}$ the last two terms in (*) contract to $2h^{2/5}$, and the bracket tends to zero with $h \to 0$. (Received December 14, 1959.)


The results of Brauer, Erdős, Selberg, Kasch, and others (see Kasch, Math. Z. vol. 62 (1955) pp. 368-387 for references) regarding the density of the sum of two sequences of integers is generalized as follows. Let $f(x)$ be defined, integrable and $0 \leq f(x) \leq 1$ on $[0,1]$; $g(x) \equiv 1 - f(x)$ on $[0,1]$, $f(x) \equiv g(x) \equiv 0$ outside $[0,1]$. Furthermore, let $\int_0^1 f(x)dx \geq \alpha(t)$ ($0 \leq t \leq 1, \alpha$ a fixed real number), $\alpha_0 = \int_0^1 f(x)dx$, $D(t) = \int_0^1 f(x)g(x + t)dx$, $C(t) = \alpha_0 + D(t)$, $D = \inf_{0 \leq t \leq 1} D(t)$, $C = \inf_{0 \leq t \leq 1} C(t)$. Then $C \geq \alpha + (3/4)\alpha(1 - \alpha)$. Also, if $\alpha_0 = 1/2$ and $M = \inf_{0 \leq t \leq 1} D(t)$, then $1.7677 < 2^{1/2}/8 \leq M \leq 1.9328$ (cf. L. Moser, Acta Arithmetica (1959) pp. 117-119). (Received December 14, 1959.)
Albert defines a finite dimensional algebra to be of type \((\gamma, \delta)\) if the identities, \((x, y, z) + (y, z, x) + (z, x, y) = 0\) and \((z, x, y) + \gamma(x, z, y) + \delta(y, z, x) = 0\), hold where \((x, y, z) = (xy)z - x(yz)\) and \(\gamma^2 - \delta^2 + \delta = 1\) (Portugal. Math. vol. 8 (1949) pp. 23-26). A ring satisfying these identities with \(\gamma = -1\) and \(\delta = 1\) is called a \((-1,1)\) ring. Let \(e\) be an idempotent of \(R\) where \(R\) is a \((-1,1)\) ring of characteristic not 2 or 3. Then the associator \((e, e, x)\) commutes with every element of \(R\). The identity \((e, e, (e, x))w = (e, e, w(e, x))\) is also valid in \(R\). If \(R\) is assumed to be simple and not associative this identity can be used to show that \((e, e, x) = 0\). This then gives a Pierce decomposition from which it is concluded that \(e\) is the unity of \(R\). This result along with those of Kokoris and Kleinfeld in (Proc. Amer. Math. Soc. vol. 6 (1955) pp. 291-296) enables us to conclude that an idempotent \(e\), of a simple \((\gamma, \delta)\) ring \(R\) of characteristic not 2 or 3, is the unity of \(R\) if \(R\) is not associative. An example given by Albert (Ann. of Math. vol. 50 (1949) pp. 318-328) is a \((-1,1)\) ring which is not simple and where the identity \((e, e, x) = 0\) fails. (Received December 14, 1959.)

R. H. McDowell: Extension of mappings to weight-preserving compactifications.

If \(X\) is a completely regular Hausdorff space, and \(\Phi\) a set of continuous mappings from \(X\) into \(X\), we define an \(m\)-\(\Phi\)-compactification of \(X\) to be a compact space of weight \(m\) containing \(X\) densely, over which all the functions in \(\Phi\) can be extended continuously. We show that if \(X\) has weight \(m\), and \(\Phi\) has cardinal \(m\), \(X\) always possesses an \(m\)-\(\Phi\)-compactification, while if the cardinal of \(\Phi\) exceeds \(m\), such a compactification can be found only when \(\Phi\) acts in a rather special way on \(X\). The constructions involve the systematic use of uniform structure theory, together with some results of Tychonoff. The results generalize theorems proved for the separable metric case by J. de Groot and the author (Extension of mappings on metric spaces, Fund. Math., to appear). (Received December 14, 1959.)


In previous studies, we considered the development of economic representations for the evaluation of certain transcendental functions. Computational
advantages of these algorithms are well known. The present paper shows how to generate rational approximations for a wider class of transcendents, those with a Laplace transform representation whose formal series expansion is divergent. It is shown how one may formally derive rational approximations which are a weighted sum of the truncated divergent series. If the function being approximated is a generalized hypergeometric series, it is shown that the weights can be chosen to yield convergent rational approximations which coalesce with those derived from the \( \tau \)-method. Numerous examples including approximations for Whittaker functions and Struve functions are presented.

(Received December 14, 1959.)

564-200. G. R. MacLane: On the asymptotic behavior of certain integrals.

Let \( \phi(t) \in C^1, \phi(t) > 0, \phi'(t) \rightarrow 0 \), for \( t \rightarrow 0 \). Let \( \psi(u) = \log \phi(e^u) \) and let \( \psi \) satisfy: \( 0 \leq \psi'(u) \psi(u) \leq u \log u \psi(u) \) for some positive \( \mu \) and \( u > u_0 \).

Let \( z \) be the function inverse to \( \psi \) and let \( \psi(0) = c \). Set \( q(x) = \int_1^{\infty} x^{-1} \phi(z^2) \, dz \). Then \( \log q(x) = \int_1^{\infty} x^{-1} \phi(z^2) \, dz \) for \( x \rightarrow \infty \). This result is weaker than that obtainable by the Laplace method in particular cases, for it operates on less information and the maximum of the integrand cannot be pinned down precisely. The proof is elementary. The result has applications to the growth of entire functions. (Received December 14, 1959.)


Let a factorization \( K(z) = P_+(z)P_-(z) \), with nonvanishing factors \( P_+, P_- \) regular and of algebraic growth in overlapping right and left half-planes, be given. Consider \( K^*(z) = K(z) + \epsilon M(z) \), where \( \epsilon \) is a small parameter, \( K^*(z) \) is regular in a vertical strip, and \( M/K \sim c|z|^{\alpha} \) near \( z = \pm \infty \). It is shown that \( K^* \) has a factorization \( K^* = P^*_+P^*_- \) for which \( P^*_+ - P^*_- = o(\epsilon) \) when \( 0 < \alpha < 1 \), \( O(\epsilon \log \epsilon) \) when \( \alpha = 1 \), \( O(\epsilon^{1/\alpha}) \) when \( \alpha > 1 \). Explicit formulas for the leading terms are derived in the cases \( \alpha \approx 1 \); these have been verified in problems where the \( P^*_+ \) are known, e.g., \( K^* = (1 - z^2)^{-1/2} + \epsilon \). The techniques developed yield approximate solutions of Wiener-Hopf integral equations with perturbed kernels. (Received December 14, 1959.)
Let $\mathcal{L}_p$ be the Banach algebra of all endomorphisms of $L_p$. Suppose $q \geq 1$ and let $W_q$ denote the Banach algebra of all complex-valued functions on $[0,1]$ whose $q$-variation is finite ($W_q$ contains all functions of bounded variation). Suppose $2q/(q+1) < p < 2q/(q-1)$. If $f \in W_q$, then $\mathcal{L}_p$ contains the multiplier transformation $T(f)$ determined by $f$; this has recently been proved by I. I. Hirschman, Jr. [Duke Math. J. vol. 26 (1959) pp. 221-242]. In this note, we show that the mapping $f \rightarrow T(f)$ is a continuous homomorphism of $W_q$ into $\mathcal{L}_p$; moreover, we construct a spectral resolution $E_p$ in $\mathcal{L}_p$, such that the relation $T(f) = \int f(\lambda) \cdot E_p(\lambda) d\lambda$ holds for all $f \in W_q$. This relation involves a Riemann-Stieltjes integral which converges in the strong operator-topology of $\mathcal{L}_p$. Corollary. There exists a class $\mathcal{S}_p \subseteq \mathcal{L}_p$ such that any $T$ in $\mathcal{S}_p$ gives rise to a spectral resolution $E_p^T$ and a relation $T = \int f \cdot E_p^T(\lambda) d\lambda$; this integral exists in the uniform operator-topology of $\mathcal{L}_p$. The class $\mathcal{S}_p$ contains the unitary shift operator and the Hilbert transformation; if $p \neq 2$, these are not spectral operators in the sense of Dunford (see [Krabbe, Math. Z. vol. 70 (1959) pp. 446-462]: $\mathcal{L}_p$ contains the class $\mathcal{L}_s$ described therein). (Received December 14, 1959.)

564-203. M. I. Knopp: Construction of a class of modular functions.

The principal consequence subgroup, $G(j)$, of level $j$ of the modular group is the set of transformations of the complex plane given by $T \rightarrow (aT + b)/(cT + d)$, where $a,b,c,d$ are integers, $ad - bc = 1$, and $a \equiv d \equiv 1$ (mod $j$), $b \equiv c \equiv 0$ (mod $j$). In this paper we construct functions, $\lambda^{(j)}_v(\tau)$ which are invariant under $G(j)$, for each integer $j \geq 2$. The $\lambda^{(j)}_v(\tau)$ are Fourier series which generalize the Fourier series expansion of $\lambda(\tau)$ ($\lambda^{(2)}_1(\tau)$ in our notation), the well-known invariant for G(2); this expansion was obtained by Simons [Canad. J. Math. vol. 4 (1952) pp. 67-80]. We show directly from the Fourier series definition of the $\lambda^{(j)}_v(\tau)$ that they satisfy the required functional equations. The method used is an extension of that introduced by Rademacher in treating $J(\tau)$, the invariant for the full modular group [Amer. J. Math. vol. 61 (1939) pp. 237-248]. The author previously used the same general methods in discussing $\lambda(\tau)$. An appropriate generalization of our method makes it possible to construct forms of all positive even integral dimensions for the groups $G(j)$. (Received December 14, 1959.)
564-204. J. G. Horne, Jr.: Commutative multiplications on the plane.

In effect, the main result of this note is that complex and coordinate-wise multiplication represent the only ways that the plane $E_2$ can be made into a commutative topological semigroup in such a way as to extend ordinary multiplication on a line (retaining the usual roles for 0 and 1). Thus, coordinate-wise multiplication is distinguished algebraically by the presence of at least three idempotents, and topologically by the fact that the component of the maximal subgroup which contains the identity has at least two boundary points. The proof depends on some results of Mostert and Shields in *Semigroups with an identity on a manifold* (Trans. Amer. Math. Soc. vol. 91 (1959) pp. 380-389) as well as a series of lemmas which make fairly standard use of connectedness and separation properties of the plane. The extent to which commutativity can be dropped as a hypothesis has not yet been investigated. (Received December 14, 1959.)

564-205. E. V. Haynsworth: Bounds for determinants with positive diagonals.

Lower bounds are found for the determinant of a real, $n \times n$ matrix $A = (a_{ij})$ with positive diagonal elements satisfying either (1) $a_{ij} \equiv \sum_{j \neq i} |a_{ij}| (i = 1,2,\ldots,n)$, or (2) $a_{ii} \geq nA_i^+ - \sum_{j \neq i} a_{ij} (i = 1,2,\ldots,n)$, where $A_i^+ = (\max_{j \neq i} a_{ij} + |\max_{j \neq i} a_{ij}|)/2$. For matrices satisfying (1), $\det A \geq \sum_{k=0}^n (\prod_{i=1}^k L_i)^n (\prod_{i=k+1}^n R_i)$, where $L_i + R_i = a_{ii}$, $L_i \geq \sum_{j<i} |a_{ij}|$, $R_i \geq \sum_{j>i} |a_{ij}|$, $(i = 1,2,\ldots,n)$. For those satisfying (2), $\det A \equiv \prod_{i=1}^n (\sum_{j=1}^n a_{ij} - nA_i^+)$. (Received December 14, 1959.)


Although most Riemannian manifolds admit no projective transformations, it is possible to obtain other kinds of geometries with the same geodesics. A particular local construction which yields straight $G$-spaces should be the analogue of Hilbert's construction applied to geodesically convex domains in Riemannian (or Finsler) manifolds: the "Hilbert-Riemann" distance between two points in such a set is defined to be the logarithm of the absolute value of the cross-ratio in which the points divide the segment of geodesic in the domain. However, only in special cases have we been able to prove the triangle
inequality for such a "metric". Some of these proof are easier than the corresponding result in the plane. (Received December 14, 1959.)


The paper gives an explicit solution of the equilibrium equations of the membrane theory of shells for all ruled surfaces, i.e., consisting of a family of straight lines, real or complex. The arbitrary functions occurring in these general solutions can be determined by some natural requirements concerning the compatibility of the stresses on the boundary of the shell. (Received December 14, 1959.)


Let $X(t)$ be the Wiener process with $X(0) = 0$. Let $Y(t) = \int_0^t x(s)ds$ and $M(t) = \max_{0 \leq s \leq t} Y(s)$. The joint c.f. of $(X(t), Y(t), M(t) - Y(t))$ is characterized as the unique function of a certain type which is the Fourier transform of a finite mass whose support is the half space $\{(x, y, z) | z \geq 0\}$ at all instants of time. It can be found, in principle, by inverting the transform and subjecting the resulting complicated integral expression to the condition of zero mass at all points where $z < 0$. The method should be applicable to other processes e.g. the integral of the Ornstein-Uhlenbeck process. The method is a continuous time version of that found in Wendel (Proc. Amer. Math. Soc. vol. 9 (1958) pp. 905-908). (Received December 14, 1959.)

564-209. Casper Goffman and R. E. Zink: Concerning the measurable boundaries of a real function.

While it is known that the equivalence classes of extended real-valued measurable functions on certain types of measure spaces form a complete lattice, this basic fact has found little application to problems concerning real functions. In this paper, the complete lattice property is used to simplify the theory of approximation of arbitrary real functions by means of measurable functions. The upper measurable boundary of an arbitrary real function $f$ on a totally $\sigma$-finite measure space is defined to be the equivalence class which is the infimum of the set of all measurable majorants of $f$, and the lower measurable boundary is defined similarly. For the case of Lebesgue measure, these
definitions agree with those given by Blumber (Acta Math. vol. 65 (1935) pp. 263-282) using metric density notions. Two applications are given: (a) for a bounded f, the upper integral of f is equal to the integral of the upper measurable boundary of f, (b) the Saks-Sierpinski theorem (Fund. Math. vol. 11 (1928) pp. 105-112) is extended to totally \( \sigma \)-finite measure spaces. (Received December 14, 1959.)


For each \( n < \omega \), let \( R_n \) be a reflexive binary relation, and let \( \phi \) be a function on \( \omega \) with \( \phi_n \in F(R_n) \) (the field of \( R_n \)). Next, let \( T = \prod (R_n, \phi_n) \) be the anti-lexicographically ordered relation whose field consists of all sequences \( \langle r_0, \ldots, r_n, \ldots \rangle \), where \( r_n \in F(R_n) \) for all \( n \), and \( r_n = \phi_n \) for almost all \( n \). (Cf. Hausdorff, Grundzüge der Mengenlehre, Chapter VI.) Put \( \tau(R_n) = \beta_n \), \( \pi(T) = \prod (\beta_n, \phi_n) \). Theorem I. If \( \alpha_n = \beta_n \cdot \alpha_{n+1} \) for \( n < \omega \), then \( \alpha_0 = \sum i \in F(R) \prod (\beta_n, \phi_n) \) for some reflexive relation \( R \) and functions \( \phi_i \). (This representation need not be unique.) If \( \pi(T) \) is independent of \( \phi \), we write \( \pi(T) \) as \( \prod (\beta_n) \). Corollary II. If all \( \alpha_n, \beta_n \) are simply ordered, and \( \prod (\beta_n) \) exists, then \( \alpha_0 = \prod (\beta_n) \cdot \gamma \) for some \( \gamma \). Corollary III. If \( \alpha_n = \beta \cdot \alpha_{n+1} \) for \( n < \omega \), \( \prod (\beta) \) exists, and \( \beta \) is left cancelling, then \( \alpha_0 = \alpha_1 = \ldots \). IV. There exist order types \( \alpha_n, \beta \) such that \( \alpha_n = \beta \cdot \alpha_{n+1} \) but, for all \( \gamma \) and \( \phi \), \( \alpha_0 \neq \prod (\beta, \phi_n) \cdot \gamma \). V. If each \( \beta_n \) is homogeneous, then \( \prod (\beta_n) \) exists and is homogeneous (i.e., for any two elements of \( F(T) \), there is an automorphism carrying one to the other hand); the conclusion can hold even when none of the \( \beta_n \) is homogeneous. (Received December 14, 1959.)

564-211. W. M. Gilbert: Functions with rectangular projections.

Suppose that \( f(x_1, x_2) \in L_1(E_2) \), that \( S \) is the set on which \( f(x_1, x_2) \neq 0 \), and that the integral of \( f(x_1, x_2) \) along any line intersecting \( S \) is equal to one. If the projection of \( S \) on any line is connected, then \( S \) is a disk, and, with a suitable choice of origin \( f(x_1, x_2) = k [1 - a^2(x_1^2 + x_2^2)]^{-1/2} \). This may be proved by elementary considerations and a theorem of Cramer and Wold. The proof seems simpler than that given for a slightly less general result by J. W. Green, Proc. Amer. Math. Soc. vol. 9 (1958) pp. 758-762. (Received December 14, 1959.)

Using Roth's formulation of the network problem (Proc. Nat. Acad. Sci. vol. 41 (1955) pp. 599-600) one notes that Kron's method of tearing is primarily a technique for manipulating matrices and bases of vector spaces and that a choice of bases is an intimate part of the technique. In particular, the choice of a "tree" base reduces tearing to an application of Woodbury's formula for inverting modified matrices. Kron's method in the case of complete tearing expresses the inverse of the node matrix in terms of the inverse of the loop matrix of the network (and is of dubious value to the electrical engineer). The possible applicability of tearing technique to abstract matrix inversions is considered with the result that Woodbury's formula can be considered as a matrix form of the dual loop-node approach to the solution of network problems due to Kirchoff and Maxwell (rather than Kron's method of tearing in the useful sense). (Received December 14, 1959.)


The function \( a_n = 4 \int_0^{\pi/2} \cos n\theta \arcsin(k \cos \theta) d\theta \), arising in certain Chebyshev expansions, can be expressed in terms of the complete elliptic integrals E and K. Evaluation as a hypergeometric series in \( k^2 = (\sin \theta)^2 \), or as the difference of two such series in \( m^4 = (\tan \theta/2)^4 \), is feasible in those cases where adequate precision cannot be obtained by use of E and K. This function satisfies the differential equation: \( k^2(1 - k^2) d^2a_n/dk^2 + k(1 - k^2) da_n/dk - n^2 a_n = 4k \cos n\pi/2 \), and various recurrence relations. The \( a_n \) of even order can be expressed in closed form involving \( \ln(1 + k)/(1 - k) \). If \( a_n \) be redefined as a solution of the homogenous differential equation, then \( a_n = k^n \cdot f(n/2, n/2, n + 1, k^2) \) for all positive \( n \). Those of even order reduce to closed form involving \( \ln(1 - k^2) \). The function \( b_n = 2 \int_0^{\pi/2} \cos n\theta (1 - k^2 \cos \theta)^{-1/2} d\theta \) is related to \( a_n \) in various ways such as: \( b_n = k(b_{n-1} - b_{n+1}) \), \( da_n/dk = b_{n-1} + b_{n+1}, d(k^n a_n)/dk = k^n b_{n-1} \), as well as exhibiting many recurrence relations within its own family. For positive, even orders, \( b_n = k^n f[(n + 1)/2, (n + 1)/2, n + 1, k^2] \). This expression applies for all positive values of \( n \) if \( b_n \) is redefined as a solution of \( k^2(1 - k^2) d^2b_n/dk^2 + k(1 - 3k^2) db_n/dk - (n^2 + k^2)b_n = 0 \). (Received December 14, 1959.)

Let $T: M \to \mathbb{E}^N$ be a continuous mapping from a two-manifold $M$ (with or without boundary) into euclidean $N$ space which defines a surface $S$. Let $[S]$ be the set of points occupied by $S$. Let $\mathcal{F}_K[S]$ be the class of all real valued functions on $[S]$ having the following property. For every $\varepsilon > 0$ there exists $\delta > 0$ such that if $p_1, p_2 \in [S]$, $d(p_1, p_2) < \delta$, then $|f(p_1) - f(p_2)| < (K + \varepsilon)d(p_1, p_2)$. Then if $L(S)$ is the Lebesgue area of $S$ and $\gamma_f(t)$ is the generalized length of the image of the contour defined by $f$ corresponding to the value $t$,

$$KL(S) \geq \int_{-\infty}^{\infty} \gamma_f(t) \, dt.$$ 

This extends an inequality of Cesari [Surface area, Princeton, 1956, p. 328] to a larger class of functions. (Received December 14, 1959.)


A method is described for extracting common constituents of two matrix representations $M: x \to M_x$ and $N: x \to N_x$ of a finite group $G$ with elements $x, y, \ldots$, assuming that $M_x$ is $m \times m$ and $N_x$ is $n \times n$ over a field $F$ whose characteristic does not divide the group order. Consider the "inducing" matrices $\mathcal{M} = \sum x^{-1} M_x$ and $\mathcal{N} = \sum y^{-1} N_y$ (over the group ring $FG$). Since $M_x^m = M_x$, $x \mathcal{N} = \mathcal{N} N_x$, we find $M_x (\mathcal{M} \mathcal{N}) = (\mathcal{M} \mathcal{N}) N_x$, where $V = \sum v_{ij} E_{ij}$ is any $m \times n$ matrix. If $M E_{ij} \mathcal{N} = \sum x^{-1} V_{ij,x}$, where $V_{ij,x}$ are $m \times n$ matrices over $F$, then the $V_{ij,1}$ span the vector space of intertwining matrices. If $M$ and $N$ are monomial representations $a_r \to \alpha_r, b_s \to \beta_s$ of subgroups $A, B$, of indices $m, n$, with coset representatives $\rho_1$ and $\sigma_j$ respectively, then $\alpha_r^{-1}(V_{ij,x}) \beta_s = V_{11,y}$ for $y = (a_r \rho_1)^{-1} x(b_s \sigma_j)$; and $V_{11,x} = 0$ unless the elements $b = x^{-1} a p x$ in $B \cap x^{-1} A x$ are represented by $\beta_q = \alpha_p$. Here the matrices $V_{11,y}$ for representatives of certain double cosets $A y B$ span the intertwining space. If matrices $S$ and $T$ reduce any intertwining matrix of rank $r$ to an $m \times n$ "diagonal" matrix $S^{-1} V T$, then the representations $S^{-1} M S$ and $T^{-1} N T$ display a common $r$-dimensional constituent as direct summand. (Received December 14, 1959.)

564-216. A. M. Fink: Recursion in topological transformation semigroups.

Let the topological transformation semigroup $(X, T, \pi)$ be defined analogously to the topological transformation group in Gottschalk and Hedlund, Topological dynamics. $\pi^f$ is not in general a homeomorphism as it is in the
case of the group. A condition which gives an approximate inverse of \( \pi^t \) is used to prove theorems about the recursive properties at \( x \) carrying over to points of the orbit of \( x \). This condition is called the friendly condition. \((X,T,\pi)\) is friendly at \( x \) if and only if given \( t \) and an open set \( U \) containing \( x \) in its interior it is true that there exists an \( s \) in \( T \) such that \( xts \) is in \( U \). (Received December 14, 1959.)


Let \( R \) be a completely primary ring satisfying the A,C,C. for right ideals. The structure of \( R(\pi) \) is given, where \( \pi r = r \pi \) for all \( r \in R \), in the cases where \( \pi \) is algebraic or transcendental over \( R \). In addition it is shown that \( R \) is properly contained in just such a ring. This is accomplished by using the quotient ring \( Q(R[x]) \) where \( x \) is an indeterminate. (Received December 14, 1959.)


To compute a "product" \( \langle f,g \rangle \) which is additive and homogeneous in the first function \( f \), consider a function \( L \) such that \( \langle f - L, g \rangle \) is small. Evaluation of \( f \) at zeros of \( f - L \) will often determine the parameters of \( L \), therefore \( \langle L, g \rangle \) and, at least approximately, \( \langle f, g \rangle \). If \( L = \sum s v_s(X,Y), \langle L, g \rangle = \sum s t_u(Y), \) the components of the vector \( Y \) being the variables of \( \langle L, g \rangle \) and those of \( X \) being the additional variables in \( L \). If the matrix \( \lambda_s(X,Y) \) is nonsingular for the evaluation points \( X, Y \), \( e_t = \sum \lambda_t(X,Y) \lambda_s \) where \( \lambda_s = \sum a_s a_{st} \langle v_s(X,Y) \rangle^{-1} \) and \( a_s \) satisfies \( u_s(Y) = \sum a_s \langle v_s, g \rangle \). Practical Gaussian integration formulas have been derived in which (1) the weight function changes sign an infinite number of times, assumes all complex values, or has a divergent integral and (2) \( f \) has an infinite number of step discontinuities or singularities. Further examples include multiple summation and integration, evaluation of a derivative at a point and the Cauchy principal part of a divergent integral, (vector) integration of the cross product of two vectors, and a functional transformation. Auxiliary information may give bounds for the error due to a small number of evaluation points. (Received December 14, 1959.)
564-219. Nachman Aronszajn and W. F. Donoghue, Jr.: The exponential representation of functions analytic in the upper half-plane with positive imaginary part.

Some earlier theorems of the authors (Journal d'Analyse vol. 5 (1957) pp. 321-388) are substantially generalized. For the sake of brevity we report only a part of the results. \( P \) is the class of functions of the title; the authors once found that a function \( \phi(\zeta) \) in \( P \) admits a representation \( \phi(\zeta) = \beta_{\infty} \int_{-\infty}^{\infty} d\mu(\lambda)/(\lambda - \zeta) \) where \( \beta_{\infty} > 0 \) and \( \mu \) is a positive measure for which \( (|\lambda| + 1)^{-1} \) is integrable if and only if it also admits the representation \( \exp \int_{-\infty}^{\infty} f(x)dx/(x - \zeta) \) where the positive, bounded and measurable \( f(x) \) is such that \( (|x| + 1)^{-1}f(x) \) is integrable. A positive, monotone and Lipschitzian function \( \eta(\lambda) \) defined for \( \lambda \geq 0 \) is called a comparison function. We sharpen the assertion above and show that the finiteness of \( \int_{-\infty}^{0} \eta^{-1}(-x)f(x)dx + \int_{0}^{\infty} \eta_{+}^{-1}(x)f(x)dx \) is equivalent to that of \( \beta_{\infty} \int_{-\infty}^{0} \eta^{-1}(-\lambda)d\mu(\lambda) + \int_{0}^{\infty} \eta_{+}^{-1}(\lambda)d\mu(\lambda) \) when \( \eta_{-}(\lambda) \) and \( \eta_{+}(\lambda) \) are any pair of comparison functions. Similar results are obtained for the cases \( \beta_{\infty} = 0 \) and \( \beta_{\infty} < 0 \), as well as when \( \phi(\zeta) \) involves a term of the form \( \zeta^\alpha \) with \( \alpha > 0 \). Finally, it is shown that if the requirement that the comparison functions be Lipschitzian is omitted, the theorem reported above is false; for every pair of non-Lipschitzian comparison functions there exists a \( \phi(\zeta) \) in \( P \) such that one and only one of the expressions above is finite. (Received December 14, 1959.)


If \( E_{i} \) is an infinite family of T.V.S.'s having cardinality \( \mathcal{M} \) over a non-discrete Hausdorff Field \( K \), and \( F = \bigcup_{i} E_{i} \) is given the box topology, then this topology is compatible with vector space structure iff axioms A and B below are both satisfied for the field \( K \). On the other hand if \( E \) is the direct sum of the \( E_{i} \), then as a sub-space of \( F \), the topology induced on \( E \) by the box topology is compatible with vector space structure iff axiom A below is satisfied. Each of these statements show that Exercise 7 of section 1 of Bourbaki, Espaces vectoriels topologiques, Chapter 1 states two theorems which are false in general. The axioms are as follows: Axiom A. There is a neighborhood \( V_{0} \) of zero in \( K \) such that for any neighborhood \( V \) of zero, there is \( \lambda \neq 0 \) such that \( \lambda V_{0} \subset V \). Axiom B. The intersection of any \( \mathcal{M} \) neighborhoods of zero in \( K \) is a neighborhood of zero. This is a non-Archimedian type property, examples of
564-221. D. J. Eustice: (C,1)-summability of orthogonal series.

Let \( \sum a_k g_k \) be an orthogonal series with coefficient series square convergent. Denote by \( S_n \) the n-th partial sum of the orthogonal series. Suppose \( \{n_k\} \) is a sequence of integers such that \( \Delta n_k \to \infty \) and \( \{S_{n_k}\} \) is (C,1)-summable a.e.; then it is proven that there exists a sequence \( \{m_j\} \) such that \( \{S_{m_j}\} \) is (C,1)-summable a.e., and such that the number of terms of \( \{m_j\} \) in the interval \( (n_k, n_{k+1}) \) tends to \( \infty \) as \( k \to \infty \). An investigation of the order of the number of terms of \( \{m_j\} \) in \( (n_k, n_{k+1}) \) is based on a generalization of a theorem of R. P. Gosselin (Proc. Amer. Math. Soc. vol. 7 (1956) pp. 392-397) and results of the author on Riesz summability. (Received December 14, 1959.)

564-222. Melvin Henriksen and J. R. Isbell: Lattice-ordered rings and function rings. I.

An f-ring \( A \) is a lattice-ordered ring that is a subdirect sum of totally ordered rings. It is unitable if it is a sub-f-ring of an f-ring with a multiplicative identity. If there is an \( e \in A \) such that \( ea \geq a \) and \( ae \geq a \) for all \( a \in A \), then \( A \) is said to have a superunit \( e \). If \( a^2 \leq a \) for all \( a \in A \), then \( A \) is called infinitesimal. Theorem 1. An f-ring \( A \) is unitable iff \( \sum_f (A(y - xy) + (x^2 - x) \wedge (y - xy)) \leq A(0) \) for all \( x, y \in A \). Theorem 2. An f-ring is unitable iff it is a subdirect sum of totally ordered rings that are either infinitesimal or have a superunit. Further results: An f-ring without nonzero left or right annihilators is unitable. An archimedean f-ring is unitable. An f-ring without proper \( f \)-ideals is unitable. All idempotents of a unitable f-ring are central. These results improve and generalize those of D. G. Johnson (Notices Amer. Math. Soc. vol. 6 (1959) p. 155). This research was sponsored by the Office of Naval Research. (Received December 14, 1959.)

564-223. Melvin Henriksen and J. R. Isbell: Lattice-ordered rings and function rings. II.

An f-ring that is a homomorphic image of an f-ring of real-valued functions is called a formally real f-ring. Theorem 1: Every commutative f-ring without nonzero nilpotent elements, every zero f-ring, and every archimedean f-ring is formally real. Theorem 2. Every formally real f-ring can be embedded in a commutative f-ring in which whenever \( |x| \leq |y| \), then \( x \) is a multiple.
of $y$. It is not known if the formally real $f$-rings can be characterized by means of a finite set of identities, but there is a 79-dimensional nonformally real totally ordered algebra in which any 8 elements generate a formally real subalgebra. This research was sponsored by the Office of Naval Research. (Received December 14, 1959.)

564-224. R. D. Anderson: The groups of all homeomorphisms of the solenoids.

The algebraic structures of the groups of all homeomorphisms of the various solenoids are studied. For any solenoid (not a circle) the subgroup of all elements isotopic to the identity is algebraically simple and is the group of all homeomorphisms which are composant-wise fixed. Various other normal subgroups are identified and the algebraic structures of some of the quotient groups are determined. For example, let $G$ be the group of all homeomorphisms of the dyadic solenoid. Let $G_I$ be its isotopic subgroup. Let $G_P$ be the group of all orientation-preserving homeomorphisms. Let $G_{LP}$ be the subgroup of $G_P$ consisting of all "level-preserving" homeomorphisms. Then $G \supset G_P \supset G_{LP} \supset G_I$ and each indicated group is normal in $G$. Further $G/G_P$ is of order two; $G_P/G_{LP}$ is infinite cyclic; and $G_{LP}/G_I$ can be considered as the dyadic integers modulo the group of ordinary integers as a subgroup. (Received December 14, 1959.)


A partial differential equation $L[U] = 0$ is said to be separable if a solution may be obtained by assuming it to be of the form $U(x) = F(x)\prod_{i=1}^{n} X_i(x_i)$ where each $X_i$ is a function of $x_i$ only. A sufficient condition for separability is: $L$ be factorable of the form $L[U] \equiv \sum_{i=1}^{n} M_i N_i [X_i]$ where the $N_i$ are ordinary differential operators. Applications to the Biharmonic Equation and Laplace's Equation are given. (Received December 14, 1959.)


For notations and definitions in this abstract see (1) L. Cesari: Surface area, Princeton University Press, 1956. Let $(T,Q)$ be a continuous mapping from the unit square $Q$ in $E_2$ into $E_3$ with area $V(T,Q) < \infty$. In (1) absolute continuity of $(T,Q)$ is defined by two conditions: (i) $e > 0$ implies there exists
64 0 such that for any finite system of nonoverlapping simple polygonal regions $\tau_1, \ldots, \tau_k \subset Q$ with $\sum_{i=1}^{k} |\tau_i| < \delta$, the sum $\sum_{i=1}^{k} V(T, \tau_i) < \varepsilon$. (ii) If $\Delta$ is a subdivision of a polygonal region $F \subset Q$ into polygonal regions $\tau$ then $\sum_{\tau \in \Delta} V(T, \tau) = V(T, F)$. This paper shows that a slightly stronger additivity property than (ii) already characterizes AC maps. In fact, $(T, Q)$ is AC if and only if $V(T, Q - E) = V(T, Q)$ for every compact set $E$ with $|E| = 0$. (Received December 14, 1959.)

564-227. Albert Nijenhuis: Invariance of tensor differential operations under mappings.

Every $C^{\infty}$ map $F: X \to Y$ of $C^{\infty}$ manifolds has induced maps $F_*, F^*$ on the contra- and covariant tensors (co- resp. contravariant functors). For mixed tensors (and objects like affine connections) there only is $F$-relatedness; e.g. vector 1-forms (fields of linear transformations) $h, h'$ over $X, Y$ are $F$-related if $F_*(h(u)) = h'(F_*(u))$ for all tangent vectors $u$ to $X$. Operations invariant under $F$-relatedness include $[P, Q]$, if (1) $P$ and $Q$ are contravariant tensors; (2) $P$ and $Q$ are vector forms; (3) $P$ is a vector field and $Q$ a tensor field. Also the formation of curvature of an affine connection. Not invariant: the trace of a vector 1-form; the derivation of Christoffel symbols from Riemann metric. The invariance of the bracket of vector 1-forms gives important applications to complex structures; e.g. (1) homogeneous complex structures, (2) holomorphic mappings, (3) families of complex structures on complex manifolds. In cases (1) and (3) substantial improvements over present methods have been obtained. [Detailed publications in co-operation with A. Frolicher are in preparation.] (Received December 14, 1959.)


Consider the equation \( \sum a_{ij}(x)u_{x_i} x_j + \sum b_1(x)u_{x_i} + c(x)u = f(x) \) with the $a_{ij}$ continuously differentiable and $b_1, c, f$ bounded on the closure of the cylinder $D$: $0 < x_1 < d, x_2^2 + \ldots + x_m^2 < R^2$; assume that there exists $h > 0$ such that $a_{ij}(x) y_i y_j \geq h \sum y_i^2$ for any real $y_i$ and $x \in \overline{D}$. If $u(x)$ is a solution bounded on $D$, then, for almost every point $(x_2, \ldots, x_m)$ of the base of the cylinder, the nontangential limit exists, i.e. the limit as $x \to (0, x_2, \ldots, x_m)$ so that $(x_2 - x_2^2)^2 + \ldots + (x_m - x_m^2)^2 = O(x_1^2)$ exists. (Received December 14, 1959.)

If X is a linear space then a semi-ring \( \mathcal{R} \) over X (Zaanen, Linear analysis) will be said to be a translation semi-ring (t.s.r) provided \( R^* \) is a translate of \( R \in \mathcal{R} \) implies \( R^* \in \mathcal{R} \). Let \( \Gamma \) be a set of linear functionals on X. It is shown that a t.s.r \( \mathcal{R}_\Gamma \) of convex polyhedra is an equivalent neighborhood base for the weak topology on X with respect to \( \Gamma \). The main result is the converse of this theorem, namely that if \( \mathcal{R} \) is a t.s.r. of convex sets with nonvoid interiors over a separable Banach space then \( \mathcal{R} \) is a neighborhood base for a weak topology with respect to a collection \( \Gamma \) of continuous linear functionals. Moreover it is proved that \( \mathcal{R} \) has a sub-t.s.r. \( \mathcal{R}' \) of convex polyhedra such that \( \mathcal{R}' \) is also a neighborhood base for this weak topology. (Received December 14, 1959.)


The Weierstrass product, \( W(Z) = \prod (1 - Z^2/\lambda_n^2) \), over a given nondecreasing sequence, \( \Lambda = \{\lambda_n\} \), of positive real numbers \( \lambda_n \), is the most convenient function in the class \( (\mathcal{F}:\Lambda) \) of entire functions of exponential type, not identically zero, that vanish at least on \( \Lambda \). Perhaps because of this, \( W(Z) \) has been universally used in the literature as a comparison function, and one might be led to the plausible guess that \( W(Z) \) is minimal in the appropriate sense. However, it follows from earlier work of the authors (Abstract 556-28) that \( W(Z) \) need not minimize the type on the imaginary axis for functions in \( (\mathcal{F}:\Lambda) \). It is shown here that \( W(Z) \) need not even have minimum overall type among all functions in \( (\mathcal{F}:\Lambda) \). Further, necessary and sufficient conditions are found on \( \Lambda \) that \( W(Z) \) does minimize the overall type. One form of these conditions is

\[
\bar{D}_p(\Lambda) = \bar{D}_L(\Lambda),
\]

where \( \bar{D}_p(\Lambda) = \pi^{-1} \lim_{y \to \infty} y^{-1} \int_0^\infty \log (1 + y^2 t^2) t d\Lambda(t) \) and

\[
\bar{D}_L(\Lambda) = \lim_{a \to \infty} (\log a)^{-1} \limsup_{x \to \infty} (\Lambda(ax) - \Lambda(x)),
\]

where \( \Lambda(t) = \sum_{n \leq t-1} \lambda_n^{-1} \). (Received December 14, 1959.)

564-231. C. N. Moore: On the primes between \( n^2 \) and \( (n + 1)^2 \).

The main objective of this paper is to show that there is always at least one prime between \( n^2 \) and \( (n + 1)^2 \). A further conclusion may be drawn from the discussion, namely, that the number of such primes increases with \( n \). The
The method used is of the sieve variety. The sieve used has been fully defined in an abstract of a paper on the Goldbach conjecture. (Cf. Abstract 557-61 in the June, 1959 issue of these Notices.) It should be mentioned that there is an error in the final formula of this abstract. The term \(3n/\log n + O(n/\log^2 n)\) should be replaced by \(9n/\log^2 6n + O(n/\log^3 6n)\). (Received December 14, 1959.)


Operating in \(k\)-dimensional Euclidean space, \(k \geq 2\), let \(K(x)\) be a Calderon-Zygmund kernel in class \(C^{(k+5/2)+\epsilon}\) if \(k\) is even and in class \(C^{(k+3)+\epsilon}\) if \(k\) is odd, \(\epsilon > 0\), and let \(\hat{K}(y)\) designate the principal-valued Fourier transform of \(K\). Also let \(S[\mu] = \sum a_m e^{im\cdot x}\) be a multiple Fourier-Stieltjes series, and call \(S_K = \sum a_m \hat{K}(m)e^{im\cdot x}\) the conjugate of \(S\) with respect to the kernel \(K\). Set \(\sigma_R^{\alpha}(x) = \sum |m| R^m \hat{K}(m)e^{im\cdot x}(1 - |m|^2/R^2)^{\alpha}\), and \(\overline{\mu}(x) = (2\pi)^{-k} \lim_{t \to 0} \lim_{\lambda \to \infty} \int D(x,t)-D(x,t) K(x-y) \, d\mu(y)\), wherever this limit exists, \(D(x,t)\) being the solid sphere with center \(x\) and radius \(t\). (It is known that this limit exists almost everywhere.) The following result is then obtained: For \(\alpha > (k - 1)/2\), \(\sigma_R^{\alpha}(x) \to \overline{\mu}(x)\) almost everywhere. This result is parallel to that of Bochner's for multiple Fourier-Stieltjes series. (Received December 14, 1959.)


The groupoids described in this title will be called \(g\)-threads. More specifically, a \(g\)-thread is a system \(G(o, <)\) with the following properties:

1. \(G\) is a groupoid with respect to \(o\), i.e. \(o\) is a single valued binary operation defined in \(G\) for every pair of elements \(a, b \in G\).
2. \(<\) is a total (i.e. linear or simple) order relation in \(G\).
3. The mapping \(p(x,y) = xoy\) of \(G \times G\) into \(G\) is continuous in the order topology.
4. \(G\) is connected in the order topology.
5. \(G\) has a zero, 0, and a unit, u. Zero is the least element, and u is the greatest element of \(G\) with respect to \(<\).

Our main result states: If \(G\) is a \(g\)-thread whose power associating elements form a dense subset and if \(G\) satisfies the cancellation law then there exists a function \(F\) from \(G\) onto the unit interval \([0,1]\) of real numbers under the usual topology and the usual multiplication, which is an isomorphism as well as order preserving homeomorphism. In particular any continuous multiplication on the interval \([0,1]\) with the usual topology is the usual multiplication if it satisfies the above conditions. (Received December 14, 1959.)
An ideal \( I \) in the algebra \( \mathcal{M} \) of measurable subsets of a space \( M \) with \( \sigma \)-finite measure \( \mu \) is called a cover of \( M \), if its \( \sigma \)-completion is \( \mathcal{M} \). The space \( \Omega_I \) of \( I \)-locally integrable functions is, \( \{ f \in R^M : \int f \cdot \chi_A d\mu < \infty \text{ for all } A \in I \} \). We say that \( (M, \mathcal{M}, \mu) \) has the \( I \)-piecing property, if for each subset \( (f_A)_{A \in I} \) of \( \Omega_I \), satisfying \( \chi_A \cdot f_B = \chi_B \cdot f_A \) for all \( A, B \in I \), there exists \( f \in \Omega_I \) such that \( \chi_A \cdot f = f_A \) for all \( A \in I \). For each \( A \in I \) let \( L_A = \{ f \in \Omega_I : \chi_A \cdot f = f \} \) and \( \mathcal{L}_I \) the inverse limit of the \( L_A \). Relating the spaces \( \mathcal{L}_I \) and \( \Omega_I \) is the mapping \( \varphi \) which takes \( f \in \Omega_I \) to \( F \in \mathcal{L}_I \), where \( F \) satisfies \( \sigma^A(F) = \chi_A \cdot f \) for all \( A \in I \), \( (\sigma^A) \) is the projection from \( \mathcal{L}_I \) onto \( L_A \). Giving \( \Omega_I \) the topology determined by the semi-norms \( N^A(f) = \int f \cdot \chi_A d\mu \), we have: Theorem 1. \( \varphi \) is a uniformity equivalence and an isomorphism into \( \mathcal{L}_I \). It is onto if and only if \( (M, \mathcal{M}, \mu) \) has the piecing property. Theorem 2. If \( \mathcal{V} \) is a set function absolutely continuous with respect to \( \mu \), such that \( |\mathcal{V}(A)| < \infty \) for all \( A \in I \), then there exists an \( F \in \mathcal{L}_I \) such that \( \mathcal{V}(A) = \int_{\mathcal{A}} \sigma^A(F) d\mu \) for all \( A \in I \). Pairs of subspaces \( (\Lambda, \Lambda^*) \) of \( \mathcal{L}_I \) analogous to Dieudonné's Köthe spaces may be defined, and a theory developed which is free from the restrictions on \( M \) of local compactness and \( \sigma \)-compactness. (Received December 14, 1959.)

564-235. E. A. Walker: The order of the automorphism groups of infinite torsion Abelian groups.

Let \( G \) be a primary reduced infinite Abelian group. Let \( A(G) \) be the automorphism group of \( G \), \( E(G) \) endomorphism ring of \( G \), and \( B \) be a basic subgroup of \( G \). Let \( |S| \) denote the cardinal of the set \( S \). If \( |G| \) is not the sum of a countable number of smaller cardinals, then it is proved that \( |A(G)| = |E(G)| = 2|B| \). In any case, \( |G| \leq |A(G)| \leq |E(G)| \), and \( |E(G)| = 2|B| \). If \( b \) is a cardinal with an immediate predecessor \( a \), if \( a \) is the sum of a countable number of smaller cardinals, then there exists a \( G \) such that \( |G| = b \), and \( |A(G)| = |E(G)| = |G| \). In fact, \( G \) can be chosen so as to have no elements of infinite height. The proofs are effected by standard Abelian group theory techniques. (Received December 14, 1959.)

564-236. R. F. Williams: Open mappings and solenoids.

It is shown that there is no open map \( f \) of a 3-manifold \( M \) onto a space \( Y \) such that \( f^{-1}(y) \) is a solenoid, for all \( y \in Y \). This follows from our principal result: Suppose \( f : X \rightarrow [0,1] = A \cup B \) is an open map such that \( f^{-1}(y) \) is a
Then $X$ cannot be embedded in any 3-manifold. Though condition (2) cannot be dropped, it can be weakened yielding a generalization of the theorem of Montgomery-Zippin which states that no solenoid can act effectively on a 3-manifold. The proof uses a generalization of Moore's triod theorem and similar lemmas to reduce the problem to the trivial case in which $X = [0,1] \times (a$ solenoid). (Received December 14, 1959.)

564-237. G. S. Young, Jr.: The existence of types of ambiguous points for bounded analytic functions.

Let $f(z)$ be bounded and analytic in the open unit disk $D$, and let $z_0$ be a point of the unit circle $C$. For $n = 2, 3, 4, \ldots$, $z_0$ has property $P_n$ if there exist $n$ arcs $A_1, A_2, \ldots, A_n$ terminating at $z_0$ and otherwise lying in $D$, such that each $n - 1$ of the cluster sets of $f(z)$ along the arcs $A_j$ meet in at least one point, but such that no point belongs to all $n$ arc cluster sets. **Theorem 1.** If the boundary cluster set $C_b(f, z_0)$ is not connected, or if there is a point in $C(f, z_0) - C_b(f, z_0)$ that is not in the range of $f(z)$ at $z_0$, then $z_0$ has property $P_n$ for each $n \geq 1$. **Theorem 2.** If the radial limit of $f(z)$ has modulus 1 almost everywhere on an open segment of $C$ containing $z_0$ and $C(f, z_0) - C_b(f, z_0)$ is not empty, then $z_0$ has property $P_n$ for all $n > 2$. **Theorem 3.** There is a function $f(z)$, analytic in $D$ and continuous in the closed disk except at $z = 1$, with $|f(z)| \leq 1$, such that for every arc in $D$ approaching $z = 1$, the arc cluster set is the entire closed unit disk. **Theorem 4.** There is a function $f(z)$, analytic in $D$ and continuous in the closed disk except at the points of a Cantor set on $C$, with $|f(z)| \leq 1$, such that each arc cluster set at a point of the Cantor set is the entire closed disk. (Received December 14, 1959.)

564-238. Jet Wimp: Chebyshev approximations to integral transforms.

Interpolatory properties of Chebyshev polynomials of the first kind are well known. However, the integral which defines the coefficients for the expansion of a function $g(x)$ in these polynomials usually is quite difficult to evaluate in simple form. In numerous cases it is shown that this difficulty can be overcome if $g(x)$ is replaced by an integral transform representation (e.g., Fourier, Laplace). If the order of integration can be interchanged, the inner integral can be evaluated by contour integration and the outer integral, in the case of the transform types noted above, becomes a Hankel transform for which an
excellent glossary is available. Some examples given include expansions for the logarithm of the gamma function and its derivative, the sine and cosine integrals and the Gaussian error function. (Received December 14, 1959.)

564-239. R. L. Wilder: Extension of local and medial properties to compactifications.

Let $X$ be a locally compact Hausdorff space, and let $A_r^F$ denote that the cohomology group $H^r_C(X)$ is finitely generated. A necessary and sufficient condition that the one point compactification of a connected and lc$^n$ space $X$ be lc$^n$ is that $A_r^F$ hold for $r = 1, \ldots, n$. If $X'$ is a compactification of $X$ such that $T = X' - X$ is closed and totally disconnected and $X$ has property $(P, Q, \sim)_n$ intrinsically, then $X'$ has $(P, Q, \sim)_n$ and $X$ has $(P, Q, \sim)_n$ extrinsically in $X'$. (A similar theorem holds for cohomology.) Consequently if $X$ has $(P, Q, \sim)_n$ and is $(n + 1)$-lc, then a sufficient condition that $X'$ be $(n + 1)$-lc is that $A_n^{n+1}$ hold; and if either (1) $n > 0$ or (2) $T$ is finite, then this condition is necessary. Conditions for extensions of lc$^n$ and lc$^n_k$ follow as corollaries. If $n \geq 1$, $X$ has $(P, Q, \sim)^{n+1}$ and $p^n(x) \leq \omega$ for all $x \in X$, then a sufficient condition that $p^n(x) \leq \omega$ for all $x \in X'$ is that $A_n^n$ hold; and this condition is necessary if $n > 1$ or $T$ is finite. Conditions for extensions of both medial and local connectedness properties follow as corollaries. (Received December 14, 1959.)


The problem can be stated as follows. If $x = (x_1, x_2, x_3) = ru$, $r = (x_1^2 + x_2^2 + x_3^2)^{1/2}$, find $y(x, t)$ in $r > 1, t > 0$ if $\psi_{tt} - \Delta \psi = \rho(x, t)$ in $r > 1, t > 0$; $\lim_{r \to 1^+} \psi(x, t) = f(u, t), t > 0; \psi(x, 0 +) = F(x), \psi_t(x, 0 +) = G(x)$ in $r > 1$. By adding the solution of a Cauchy problem $\phi, F$, and $G$ can be reduced to zero. For this case it is shown that if $f(u, t) = \sum_{n=0}^{\infty} f_n(u, t)S_n(u)$, $S_n$ a spherical harmonic of degree $n$, then $\psi(x, t) = 1/r f(u, t - r + 1) + \sum_{n=1}^{\infty} \sqrt{\frac{n}{(n+1)!}} r^{n+1} \exp\{\lambda_n^S(t - r)\} \cdot \cdot \cdot f_n(r)k_n(r\lambda_n^S/k_n^S) S_n(u) in t - r + 1 \geq 0$, where $k_n(z) = (\pi/2z)^{1/2}K_{n+1/2}(Z), K_{\nu}(z)$ is the modified Hankel function (Watson, Bessel functions, p. 78), and $\lambda_1^S, \ldots, \lambda_n^S$ are the roots of $k_n(\lambda) = 0$. Results of F. W. J. Olver (Philos. Trans. Roy. Soc. London Ser. A, vol. 247 (1954)) on Hankel functions of large order are used to derive estimates $Re \lambda_n^S \leq An^{1/3}, n \geq n_0; Re \lambda_n^S \leq - \mu < 0, all n and s; k_n(r\lambda_n^S/k_n^S) = O(n^{1/2}), n \to \infty$. These are used to prove that if
f(u,t) is square-integrable on compact subsets of $U \times (0,\infty)$, where $U$ is the unit sphere, then (1) the series for $\Psi$ converges absolutely and uniformly on compact sets, (2) $|\Psi(x,t) - 1/r \int f(u,t - r + 1)| \leq M \left( \int_0^{t-r+1} \int_U |f(u',\tau)|^2 \, du' \, d\tau \right)^{1/2}$, $du' = \text{element of area on } U$, and (3) if $f(u,t) = 0$ for $t \geq T$ then for fixed $x$ in $r > 1$, $\Psi(x,t) = O(\exp[-\mu t])$, $t \to \infty$. (Received December 14, 1959.)


Let $p$ be a map of the space $E$ onto the space $B$, $B\{\}$ the space of paths in $B$ (c-o topology) and $E \times B\{\}$ the subspace of $E \times B\{\}$ consisting of those $(e, \omega)$ with $p(e) = \omega(0)$. Identifying $e$ with $(e,p(e)\{\})$, $E$ can be regarded as a subspace of $E \times B\{\}$. A connection for the triple $\langle E, P, p \rangle$ is a fibre map $\tau: E \times B\{\} \to E$, that is, $p \tau(e, \omega) = \omega(1)$, such that $\tau|E$ is fibre homotopic to the identity of $E$. If $\tau(e,p(e)) = e$ for all $e \in E$ then the connection $\tau$ is regular. A (regular) fibre space in the sense of Hurewicz (Proc. Nat. Acad. Sci. vol. 41 (1955) pp. 956-961) always admits a (regular) connection. However, the existence of a lifting function is not invariant under a fibre homotopy equivalence whereas the existence of a connection is. The existence of a connection is equivalent to a slightly weakened form of the covering homotopy theorem, which, nevertheless implies the exactness of the homotopy sequence. If $B$ is wlc (Fadell, Trans. Amer. Math. Soc. vol. 90 (1959) pp. 1-14) then a triple with connection is a fibre space in the sense of Fadell. Conversely, a Fadell-fibre space with paracompact base space admits a connection whence a Fadell-fibre space satisfies the gCHT for paracompact spaces. (Received December 14, 1959.)
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