AMERICAN MATHEMATICAL SOCIETY

Notices

VOLUME 7, NUMBER 3 ISSUE NO. 46 JUNE 1960

AMERICAN MATHEMATICAL SOCIETY

Notices

Edited by GORDON L. WALKER

Contents

MEETINGS

CALENDAR OF MEETINGS

NOTE: This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the NOTICES was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

Meeting No.	Date	Place	Deadline for Abstracts*
571	August 29-September 3, 1960 (65th Summer Meeting)	East Lansing, Michigan	July 15
572	October 22, 1960	Worcester, Massachusetts	Sept. 8
573	November 18-19, 1960	Nashville, Tennessee	Oct. 5
574	November 19, 1960	Pasadena, California	Oct. 5
575	November 25-26, 1960	Evanston, Illinois	Oct. 5
576	January 24-27, 1961 (67th Annual Meeting)	Washington, D. C.	Dec. 9
	April, 1961	Stanford, California	
	August, 1961 (66th Summer Meeting)	Stillwater, Oklahoma	
	November 17-18, 1961	Milwaukee, Wisconsin	
	January, 1962 (68th Annual Meeting)	Cincinnati, Ohio	
	August, 1962 (67th Summer Meeting)	Vancouver, British Columbia	
	August, 1963 (68th Summer Meeting)	Boulder, Colorado	

*The abstracts of papers to be presented at the meetings must be received in the Headquarters Offices of the Society in Providence, R. I., on or before these deadlines. The deadlines also apply to news items.

The NOTICES of the American Mathematical Society is published by the Society seven times a year, in February, April, June, August, October, November, and December. Price per annual volume is $7.00. Price per copy, $2.00. Special price for copies sold at registration desks of meetings of the Society, $1.00 per copy. Subscriptions, orders for back numbers (none available before 1958), and inquiries should be addressed to the American Mathematical Society, Ann Arbor, Michigan, or 190 Hope Street, Providence 6, Rhode Island.

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FIVE HUNDRED SEVENTIETH MEETING

Montana State University
Missoula, Montana
June 18, 1960

PROGRAM

The five hundred seventieth meeting of the American Mathematical Society will be held on Saturday, June 18, 1960 at Montana State University, in Missoula, Montana. There will be a meeting of the Mathematical Association of America on Friday, June 17, and a meeting of the Society for Industrial and Applied Mathematics on Friday and Saturday.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, the Society will be addressed by Professor Samuel Karlin of Stanford University on Saturday afternoon at 2:00 P.M. in Room 109 in the Chemistry-Pharmacy Building. His talk is entitled "Total positivity and applications to probability".

A session for contributed papers will be held at 10:00 A.M. in Room 109 of the Chemistry-Pharmacy Building. Abstracts of papers to be presented at these sessions appear on pages 368-386 of these NOTICES. There are cross references to the abstracts in the program. For example the title of paper (1) in the program is followed by (570-38) indicating that the abstract can be found under the designation 570-38 among the published abstracts.

If necessary, there will be a session for late papers on Saturday morning in Room 108 of the Chemistry-Pharmacy Building. Information concerning late papers will be available at the registration desk which will be located in the lobby of Craig Hall.

The meeting of the MAA and SIAM on Friday will include an hour address by Professor Paul Rosenbloom, University of Minnesota on "Implication for colleges of the new school mathematics programs", and shorter addresses by Professor Arvid Lonseth, Oregon State College, on "The computer as an object of research", Professor J. E. Whitsitt, Montana State College, on "Topics in Boolean algebras", Professor R. A. Beaumont, University of Washington, on "Generalized rings", and Dr. J. D. Esary, Boeing Scientific Research Laboratories, Seattle, on "Some characteristics of the reliability of complex structures".

Dormitory accommodations will be available in Craig Hall. The rates are $3.00 per night for adults and $1.25 per night for children. The Mathematical Association of America is sponsoring a smorgasbord dinner on Friday, June 17. The price of this dinner is

$2.25 per person. Dormitory and banquet reservations can be obtained by writing to Professor W. M. Myers, Department of Mathematics, Montana State University, Missoula, Montana. The application for a reservation should include the applicant's name and address, his expected times of arrival and departure, the name and relationship of each member of his party, and the number of banquet reservations desired.

Cafeteria service will be provided at the Montana State University Lodge, about one-half block from Craig Hall dormitory. There will be a tea for wives on Friday afternoon, June 17. Baby sitting service for families attending the tea or banquet on Friday can be arranged.

There are numerous hotels and motels in Missoula. The prices of these range from $6.00 to $10.00 per night. The Florence Hotel, 111 North Higgins, is convenient. The Big Chief Motel, 744 East Broadway, the Bel Aire Motel, 300 East Broadway, and the City Center Motel, 338 East Broadway, are reasonably close to the University.

Missoula is served by Northwest Airlines, Northern Pacific Railroad, and by the Milwaukee Railroad. The town is situated at the intersection of U. S. highways 10 and 93. Members who drive to the meeting will find adequate parking near Craig Hall.

PROGRAM OF THE SESSIONS

The time limit for each contributed paper is ten minutes. The contributed papers are scheduled at 15 minute intervals so that listeners can circulate between the different sessions. To maintain this schedule, the time limit will be strictly enforced.

SATURDAY, 10:00 A.M.

General Session, Room 109, Chemistry-Pharmacy Building

10:00 - 10:10

(1) Geometric ABA-groups. II
Professor D. G. Higman and Professor J. E. McLaughlin, University of Michigan (570-38)

10:15 - 10:25

(2) Polar means of convex bodies
Professor W. J. Firey, Washington State University (570-15)

10:30 - 10:40

(3) An experiment by computing machine on the four color problem

Professor Hidehiko Yamabe and Professor David Pope, Institute of Technology, University of Minnesota (570-35)

10:45 - 10:55

(4) Three-dimensional lattice paths and the Ising problem

Professor H. S. Green, University of Adelaide, Dr. Roy Leipnik and Dr. J. E. Maxfield, U. S. Naval Ordnance Test Station, China Lake, California (570-11)

11:00 - 11:10

(5) Optimum interpolation of noisy signals from finite records

Dr. Roy Leipnik and Dr. J. E. Maxfield, U. S. Naval Ordnance Test Station, China Lake, California (570-12)

11:15 - 11:25

(6) First passage times for symmetric stable processes in space

Professor R. K. Getoor, University of Washington and Massachusetts Institute of Technology (570-16)

11:30 - 11:40

(7) A convergence theorem for sequences of polynomials

Mr. J. D. Buckholtz, University of Texas (570-33)

SATURDAY, 2:00 P.M.

Invited Address, Room 109, Chemistry-Pharmacy Building

Total positivity and applications to probability

Professor Samuel Karlin, Stanford University

SUPPLEMENTARY PROGRAM
(To be presented by title)

(8) A series considered by Ramanujan

Professor R. P. Boas, Jr., Northwestern University

(9) On a hierarchy of monadic predicate quantifiers. Preliminary report

Professor J. Richard Buchi, University of Michigan

(10) On a problem of Tarski. Preliminary report

Professor J. Richard Buchi, University of Michigan

(11) Some arithmetic properties of Bernoulli convolutions

Professor Adriano Garsia, University of Minnesota

(12) Expected first passage times for symmetric stable processes
Professor R. K. Getoor, University of Washington and Massachusetts Institute of Technology

(13) A property of an upper envelope of pluri-subharmonic functions
Dr. Jerzy Gorski, University of Krakow, Poland
(Introduced by Professor Stefan Bergman)

(14) Plemelj's formula for the functions of two complex variables
Dr. Jerzy Gorski, University of Krakow, Poland
(Introduced by Professor Stefan Bergman)

(15) Values of quadratic forms
Dr. Leon Greenberg, Brown University

(16) Arc-wise connectedness in semi-metric spaces
Dr. R. W. Heath, University of North Carolina
(Introduced by Professor F. Burton Jones)

(17) Geometric ABA-groups. I
Professor D. G. Higman and Professor J. E. McLaughlin, University of Michigan

(18) A closure and complement problem
Professor K. Jaakko J. Hintikka, University of Helsinki
(Introduced by Professor Hartley Rogers, Jr.)

(19) Algebraic structure of the group of recursive permutations
Mr. C. F. Kent, Massachusetts Institute of Technology

(20) Spectral representations for abstract functional equations
Mr. George Maltese, Yale University

(21) Convex ideals in ordered generalized convolution algebras
Mr. George Maltese, Yale University

(22) Stable periodic solutions of a non-linear difference-differential equation
Dr. W. L. Miranker, IBM Research Center, Yorktown Heights, New York

(23) A new class of stability theorems for differential equations
Dr. W. L. Miranker, IBM Research Center, Yorktown Heights, New York

(24) The concept and calculus of an operational continuum. Preliminary report
Dr. C. A. Muses, The Barth Foundation, Denver, Colorado

(25) On the unique ordering of the decimal forms
Dr. C. A. Muses, The Barth Foundation, Denver, Colorado

(26) The homeostasis of the Fibonacci series
Dr. C. A. Muses, The Barth Foundation, Denver, Colorado

(27) On a generalized Feld series
Sister Mary Redempta Nedumpilly and Professor Francis Regan, St. Louis University

(28) A weak form of the star conjecture for manifolds
Dr. R. H. Rosen, University of Michigan

(29) Some new forms of the axiom of choice
Professor Herman Rubin and Mrs. Jean E. Rubin, Michigan State University

(30) Two propositions equivalent to the axiom of choice only under both the axioms of extensionality and regularity
Professor Herman Rubin, Michigan State University

(31) A continuum of pairwise incomparable degrees. Preliminary report
Mr. Gerald E. Sacks, Cornell University

(32) Hypersurfaces with no negative sectional curvature
Dr. Richard Sacksteder, Yale University

(33) Produit direct de quasigroupes orthogonaux
Professor A. J. V. Sade, Lycee Perier Marseille, France

(34) The Dirichlet problem for degenerate elliptic equations
Professor Martin Schechter, New York University

(35) Too few and too many boundary conditions
Professor Martin Schechter, New York University

(36) The norm of the inverse of a certain tri-diagonal matrix
Dr. D. P. Squier, California Research Corporation, La Habra, California

(37) A maximal ergodic theorem in information theory
Dr. Alexandra Ionescu Tulcea, Yale University
(Introduced by Professor C. E. Rickart)

(38) The *-transpose of a matrix
Professor N. A. Wiegmann, Catholic University of America

(39) Two-point boundary problems involving a parameter linearly
Professor H. J. Zimmerberg, Rutgers, The State University

Seattle, Washington
May 9, 1960

R. S. Pierce
Associate Secretary

PRELIMINARY ANNOUNCEMENT OF MEETING

SIXTY-FIFTH SUMMER MEETING

AND

THIRTY-NINTH COLLOQUIUM

Michigan State University
East Lansing, Michigan
August 30-September 2, 1960

The Sixty-fifth summer meeting and the thirty-ninth Colloquium of the American Mathematical Society will be held at Michigan State University, East Lansing, Michigan, from Tuesday, August 30 to Friday, September 2, 1960. During the same week there will be meetings of the Mathematical Association of America and the Society for Industrial and Applied Mathematics.

Professor S. S. Chern of the University of California, Berkeley, will deliver the Colloquium Lectures: "Geometrical structures on manifolds." The first of these lectures will be held on Tuesday, August 30 at 1:30 P.M., and will be followed by three others on Wednesday, Thursday, and Friday at 9:00 A.M. All of the lectures will be delivered in the Anthony Hall Auditorium (C103).

The Committee to Select Hour Speakers for Summer and Annual Meetings has invited Professor Paul Halmos of the University of Chicago and Professor P. E. Conner of the University of Virginia to address the Society. Professor Halmos will speak at 1:30 P.M. on Thursday, September 1. His title is "Recent progress in ergodic theory." Professor Conner's lecture, "Involutions and equivariant maps," will be delivered at 1:30 P.M. on Friday, September 2. Both addresses will be delivered in the Anthony Hall Auditorium (C103).

There will be various sessions for contributed papers at times to be announced in the final Program. There will, however, not be any special sessions for papers which failed to meet the deadline.

The Council of the Society will meet at 4:00 P.M. on Tuesday, August 30, and there will be a Business Meeting of the Society at 10:15 A.M. on Wednesday, August 31 in Anthony Hall.

At the Business Meeting the Society will be asked to vote on certain changes in the By-Laws concerning the position of Executive Director. These suggested changes were prepared by a Committee appointed by the President and have been approved by the Council, and have been proposed in order to bring practice in the Society into line with that which has proved successful in other professional and learned societies.

It is proposed that the first sentence of Article II, Section 3 read as follows:

The Board of Trustees shall have the power to appoint such assistants and agents as may be necessary or convenient to facilitate the conduct of the affairs of the Society, and to fix the terms and conditions of their employment.

and Article VI, Sections 2 and 3, read as follows:

Section 2: The Executive Director shall be appointed by the Board of Trustees with the consent of the Council. The terms and conditions of his employment shall be fixed by the Board of Trustees.

Section 3: The Executive Director shall work under the immediate direction of a committee consisting of the President, the Secretary, and the Treasurer, of which the President shall be chairman ex-officio. The Executive Director shall attend meetings of the Board of Trustees, of the Council, and of the Executive Committee, but he shall not be a member of any of these bodies.

The complete By-Laws are to be found in the November, 1958 issue of the BULLETIN of the Society.

REGISTRATION, ROOMS, MEALS

Registration headquarters will be in the lobby of Snyder Hall, situated near the northeast corner of the campus on Bogue Street, approximately 300 yards south of East Grand River Avenue (U.S. Route 16). All persons attending the meetings are requested to register immediately upon arrival, whether or not they are staying in the dormitories. The registration fee for those attending the meetings is $2.00 for each member of any of the participating organizations and fifty cents for each accompanying adult. There is no registration fee for children. A directory of all persons attending the meetings and an information desk will be maintained at registration headquarters in Snyder Hall.

Snyder Hall and Phillips Hall (adjoining Snyder) dormitories will be available to all attending the meetings, and to their families, and will accommodate new arrivals at all hours of the day and night after 2:00 P.M. on SUNDAY, 28 August. The cost of housing is $3.00 a day per person in double rooms with separate beds or $18.00 per week. Single rooms when available will cost $4.50 per day or $27.00 per week. A state use tax of 4% is added to these room rates. Bedding, towels, and soap will be furnished and daily maid service with clean towel and clean linens each day will be provided. Children under 8 years of age will be charged half price for rooms, and there will be no charge for infants. Automatic washing and drying equip-

ment and electric irons are available in the dormitories. Rooms may be occupied from two o'clock Sunday afternoon, August 28, until Saturday noon, September 3.

Cafeteria service will be maintained in the dormitory dining rooms during the meetings, beginning Sunday, August 28, at 5:30 P.M. Meal hours will be: breakfast 7:45 to 8:45 A.M.; lunch 12:15 to 1:15 P.M., dinner 5:45 to 6:45 P.M.

RESERVATIONS

A reservation form will be found on the last page of these NOTICES. Persons desiring dormitory accommodations are requested to complete the form and mail it to Professor L. M. Kelly, Department of Mathematics, Michigan State University, East Lansing, Michigan, as soon as possible.

HOTELS AND MOTELS

Persons desiring hotel or motel accommodations should make their reservations directly with the hotel or motel management. Among Lansing hotels (about 4 or 5 miles from the meetings) are Hotel Olds ($6.50 single, $11.00 twin), Hotel Porter ($4.75 single, $8.50 twin), and Hotel Roosevelt ($5.50 single, $9.00 twin), quoted rates being the minima for rooms with single beds and twin beds respectively. Among East Lansing motels (about 2 miles from the meetings) are Amity Hall ($7.00 single, $10.00 twin, $15.00 suite), Holiday Inn ($7.50 single, $11.00 twin), Albert Pick Motor Hotel ($7.50 single, $11.00 twin, extra rollaway beds $2.50), and Poplars Inn ($7.50 single, $11.50 twin). The state use tax of 4% will be added to all of these.

ENTERTAINMENT AND RECREATION

The lounges and recreation rooms of Snyder-Phillips, the Conference Room of the Physics-Mathematics Building (Room 221), and the adjoining Mathematics Library will be open to the members and guests of the participating organizations.

Athletic facilities available to members and guests include table tennis in the dormitory recreation rooms, tennis courts near the stadium on campus, outdoor and indoor swimming pools (restricted to persons aged 14 years or over) near the stadium on campus, and an 18-hole golf course on the southwestern corner of the campus. Visitors should bring their own sports equipment.

On Monday evening there will be an informal coffee hour at 8:00 P.M. in the Phillips Hall, Lower Lounge.

An informal tea will be held Tuesday afternoon from 4:00 to 6:00 P.M. in Phillips Hall, Lower Lounge.

A chicken barbecue for members and guests of the participa-

ting organizations will be held Wednesday evening at 5:00 P.M. on the campus. The price will be $2.75 (plus tax) per person.

TRAVEL INFORMATION

East Lansing is four miles east of Lansing and eighty miles west of Detroit on U. S. Highway 16. Persons driving to Lansing from the South or Southwest will wish to use Highways 27 or 78.

Those driving from Wisconsin may cross Lake Michigan from Milwaukee or Muskegon by using the Wisconsin and Michigan Steamship Company Lines.

The Chesapeake and Ohio streamliner trains leave Detroit (Fort Street) and Grand Rapids twice daily, arriving at the Michigan Avenue Station in Lansing which is about a half mile east of the Capitol. The Grand Trunk Railway line from Chicago to Port Huron has service through Lansing and its Lansing station is about one mile south of the Capitol on Washington Avenue. The Hotel Olds facing the Capitol may be reached from the Grand Trunk Station by taxicab or Washington Avenue bus.

Buses marked College-Fisher leave the southeast corner of Washington and Michigan Avenues in Lansing (one block east of the Hotel Olds) about every 25 minutes, and pass the Chesapeake and Ohio Depot on the way to East Lansing. (Snyder Hall is near the northeast corner of the campus on Bogue Street approximately 300 yards south of the Bogue Street stop.) Greyhound buses come to East Lansing from Detroit (80 miles), Ann Arbor (60 miles), Battle Creek (55 miles), Chicago (220 miles) and other points.

Capitol Airlines has direct service to Lansing from New York, Cleveland, Willow Run (Detroit), Chicago, Milwaukee and Minneapolis, and connects with other airlines. The Lansing airport is 5 miles west of Lansing and 9 miles from East Lansing on U. S. Highway 16.

Those driving should note that there are no camping facilities within forty miles of the Lansing area.

J. W. T. Youngs
Associate Secretary

Bloomington, Indiana
May 5, 1960

ACTIVITIES OF OTHER ASSOCIATIONS

THE AUSTRIAN MATHEMATICAL SOCIETY will hold the Fifth Austrian Congress of Mathematicians from September 12 to 17, 1960, which again will be combined with an International Meeting of Mathematicians. This Congress will be held in Innsbruck and will follow the tradition of past meetings (1949 at Innsbruck, 1952 in Salzburg, 1956 in Vienna). Members of the American Mathematical Society are invited to participate in the Congress. The following is a translation of a portion of the letter of invitation, signed by Professor Dr. H. Hornich, Vorsitzender der Oesterr Mathematischen Gesellschaft:

"It is our chief concern to organize the Congress in such a manner that its scientific goals will be achieved. It seems essential to us to give mathematicians of different countries and languages a chance to meet and hold fruitful discussions. Since this meeting of mathematicians is not limited to a specific field of research, mathematics as such should be enhanced and better understanding among mathematicians should be served.

"You could help us considerably in our preparations if you would forward your reservation as early as possible. We should be pleased if you would care to give a lecture to add to the scientific program, but this is not a prerequisite to participation in the Congress. Please forward the title of your lecture (which should not exceed 20 minutes) as well as the section in which it should be delivered. The abstracts - in as few words and with as few formulas as possible because of printing difficulties - should be sent to the Committee of the Congress before June 30 in order to supply advance notice to the other participants. The abstracts will be assembled in one volume and distributed at the beginning of the Congress. Reservations for rooms should be made as soon as possible and in any event before August 21. Requests for reservations may be directed to the Staedtische Verkehrsbuero, Innsbruck, Burggraben 3, or enclosed when you send us notification of your participation. You will receive a reply within a few days, confirming your reservation and advising the name of the hotel in which a room has been reserved for you. The registration fee is S 150. per person.

"Please direct all questions relating to the organization of the Congress to:

Organisationsleitung des
V. Oesterreichischen Mathematikerkongresses
Mathematisches Institut der Universitaet
Innrain 52, Innsbruck

The following gentlemen are in charge: Professor F. Cap, Professor W. Groebner, Professor H. Schatz."

AN INTERNATIONAL SYMPOSIUM ON THE COORDINATION OF INSTRUCTION IN MATHEMATICS AND PHYSICS jointly organized by the Yugoslav Union of Societies of Mathematicians and the International Commission for Mathematical Instruction will be held in Belgrade on September 19-24, 1960.

The preliminary announcement of the Symposium states:

"In view of the increasingly close interconnections between mathematics and physics and in view of their growing importance in science, technology, and the arts, there is urgent need for a discussion of the ways in which the teaching of these subjects may be better coordinated in the interests of progress, both theoretical and practical. It is the aim of this Symposium to provide a convenient opportunity for such a discussion. All levels of instruction--primary, secondary, university, and post-graduate--will be examined by the Symposium, and all aspects of coordination both scientific and pedagogical will be studied. The discussion should also permit the comparison of measures currently applied in different countries to secure closer coordination of the teaching of mathematics and physics at different levels. For this reason in particular it is hoped that the different countries of various continents, especially those at no great distance from the host country, will be well represented at this Symposium."

In addition to papers arranged by the Organizing Committee through individual invitations, the program will include twenty-minute contributed papers. Those intending to contribute papers should provide the Committee with their titles and abstracts not later than June 15 next at the address indicated below, and should submit their papers not later than August 1 next at the same address. In order to make possible the prompt publication of the proceedings of the Symposium, it is essential that these dates be strictly observed.

More detailed information concerning the program and the practical arrangements for the Symposium will be communicated to prospective participants subsequently. The Symposium will be held at the same time and at the same place (Belgrade, September 19-24, 1960) as the Third Congress of Mathematicians and Physicists of Yugoslavia.

All correspondence and inquiries concerning the Symposium or the Congress should be addressed to:

SAVEZ DRUŠTAVA MATEMATIČARA I FIZIČARA
Pošt fah. 791
Belgrade, Yugoslavia

The invitation to the Symposium was signed by: Professor Dr. Marshall M. Stone, President of the International Commission for Mathematical Instruction; D'Voja Dajovic, Secretary of the Organizing Committee; Professor Dr. Djuro Kurepa, President of the Union of Societies of Mathematicians and Physicists of Yugoslavia, Vice President of the IOMI.

THE ASSOCIATION FOR COMPUTING MACHINERY will hold its fifteenth annual meeting at Marquette University, Milwaukee, Wisconsin, on August 23-25, 1960. Local arrangements will be under the direction of Professor Arthur Moeller, Marquette University.

Contributed papers concerned with all phases of analog and digital computation, business applications, and data processing are solicited. Each person wishing to contribute a paper to the program has been requested to send to the Program Committee by May 9, 1960, four copies of a 100-word abstract. The abstract should be suitable for inclusion in the printed Program of the meeting. The amount of time which can be allotted to each contributed paper is limited to 10 minutes followed by 5 minutes for discussion. The papers will not be refereed and comments may be expected from the audience. Abstracts should be sent to: J. H. Wegstein, Chairman, ACM Program Committee, National Bureau of Standards, Washington 25, D. C.

The program will include a substantial number of invited 35 minute papers, survey talks, round-table discussions, and a "Hall of Discussions". Suggestions for invited speakers and topics are solicited and should be sent to any member of the Program Committee. The members of the Program Committee are: Jack Belzer, Howard Bromberg, Jim Douglas, Jr., George Forsythe, Werner L. Frank, Jack Heller, Randall E. Porter, R. E. Utman.

This Calendar includes symposia, seminars, and institutes sponsored by the Society, but does not include regular meetings of the AMS or the MAA, which are listed elsewhere in the NOTICES.

June, 1960

Conference of Directors of University Computing Laboratories

Hotel: Shoreland Hotel
Location: Chicago, Illinois
Date: June 2-4, 1960
Secretary: Professor Walter Freiberger, Department of Applied Mathematics, Brown University, Providence 12, Rhode Island

2nd Conference of the Computing and Date Processing Society of Canada

Place: University of Toronto
Location: Toronto, Canada
Date: June 6-7, 1960

International Conference on Many-Body Problems

Location: Utrecht, Netherlands
Date: June 7-12, 1960
Contact: Professor L. C. P. van Hove, Department of Theoretical Physics, State University, Utrecht, Netherlands

Mathematics Research Center, U. S. Army; University of Wisconsin

"International Conference on Partial Differential Equations and Continuum Mechanics"
Place: University of Wisconsin, Madison, Wisconsin
Housing: Elizabeth Waters Hall - a university dormitory
Date: June 7-15, 1960
Contact: Professor R. E. Langer, Mathematics Research Center, U. S. Army, The University of Wisconsin, Madison 6, Wisconsin

International Union of Pure and Applied Physics Commission on Thermodynamics and Statistical Mechanics Meeting

Location: Utrecht, Netherlands
Date: June 12-16, 1960
Contact: Professor L. C. P. van Hove, Department of Theoretical Physics, State University, Utrecht, Netherlands

International Mathematical Union

International Colloquium on "Topology and Differential Geometry"
Location: Zürich, Switzerland
Date: June 20-26, 1960
Chairman: Professor H. Hopf

International Statistical Institute, Session

Location: Tokyo, Japan
Date: June, 1960
Contact: E. Lunenberg, Director of the Permanent Office
2 Oostduinlaan, The Hague, Netherlands

The Statistical Laboratory of the University of California

Fourth Berkeley Symposium on Mathematical Statistics and Probability
Date: June 20-July 30, 1960
Contact: Director, Statistical Laboratory, University of California, Berkeley 4, California

International Federation of Automatic Control
International Congress for Automatic Control

Location: Moscow, USSR
Date: June 25 - July 5, 1960
Contact: Secretariat of the Federation of Automatic Control, c/o Verein Deutsche Ingenieure, 79 Prinz-George-Strasse, Düsseldorf, Germany

July, 1960

International Mathematical Union

International Symposium on "Linear Spaces - Geometrical Aspects and Applications to Analysis"
Location: Jerusalem, Israel
Date: July, 1960
Chairman: Professor A. Dvoretzky

International Union of Theoretical and Applied Mechanics

"High Temperature Creep of Structures"
Place: Stanford University
Location: Stanford, California
Date: July, 1960

International Union of Theoretical and Applied Mechanics

"Aerodynamical Phenomena in Stellar Atmospheres"
Location: Varenna, Italy
Date: July, 1960

American Mathematical Society Summer Seminar in Applied Mathematics

"Modern Physical Theories and Associated Mathematical Developments", Sponsored by Atomic Energy Commission; National Science Foundation; Office of Naval Research; Office of Ordnance Research, U. S. Army
Place: University of Colorado
Location: Boulder, Colorado
Date: July 24-August 19, 1960
Chairman: Professor K. O. Friedrichs, New York University, 25 Waverly Place, New York 3, New York

August, 1960

American Mathematical Society Summer Institute on Finite Groups, Sponsored by the National Science Foundation

Place: California Institute of Technology
Location: Pasadena, California
Date: August 1-28, 1960
Chairman: Marshall Hall, Jr.

Association for Computing Machinery

National Association for Computing Machinery Conference
Place: Marquette University
Location: Milwaukee, Wisconsin
Date: August 23-25, 1960

The Institute of Mathematical Statistics, American Statistical Association, and the Econometric Society

Place: Stanford University
Location: Stanford, California
Date: August 23-26, 1960
Associate Secretary: Professor Gerald Lieberman, Department of Statistics, Stanford University, Stanford, California

International Congress of Logic, Methodology and Philosophy Science, Sponsored by the International Union for History and Philosophy of Science

Place: Stanford University
Location: Stanford, California
Date: August 24 - September 2, 1960
Contact: Professor Patrick Suppes, Serra House, Stanford University, Stanford, California

Hungarian Mathematical Congress
2nd Hungarian Mathematical Congress, Organized by Hungarian Academy of Sciences and Janos Bolyai Mathematical Society

Place: Hungarian Academy of Sciences
Location: Budapest, Hungary
Date: August 24-31, 1960
Contact: MTA Matematikai Kutato Intezete (Mathematical Institute of the Hungarian Academy of Sciences) Budapest, V., Realtanoda u. 13-15, Hungary

4th London Symposium on Information Theory

Location: London, England
Date: August 29 - September 3, 1960
Contact: Professor E. C. Cherry, Imperial College, London, S. W. 7, England

10th International Congress of Applied Mechanics

Place: Stresa, Italy
Date: August 31 - September 7, 1960
Chairman: Professor Gustavo Colonnetti, Organizing Committee, c/o Consiglio Nazionale delle Ricerche, Piazzale delle Scienze 7, Rome, Italy

September, 1960 — 2nd International Conference on Operational Research

Location: Aix-en-Provence, France
Date: September 5-10, 1960
Contact: John B. Lathrop, ORSA Representative, IFORS, Operations Research Division, Lockheed Aircraft Corporation, Burbank, California

5th Austrian Congress of Mathematicians

Place: University of Innsbruck
Location: Tyrol, Innsbruck, Austria
Date: September 12-18, 1960
Chairman: Professor W. Grobner, Mathematical Institute of the University, Innrain, 52, Innsbruck, Austria

The Yugoslav Congress of Mathematicians and Physicists

Location: Belgrade, Yugoslavia
Date: September 19-24, 1960
Contact: Savez Drŭstava Matematičara i Fizičara, Pŏst fah. 791, Belgrade, Yugoslavia

October, 1960 — Operations Research Society of America

18th National Meeting
Hotel: Statler Hilton Hotel
Location: Detroit, Michigan
Date: October 10-12, 1960
Chairman: Mr. George O'Brien; Touche, Niven, Bailey and Smart; 1292 National Bank Building, Detroit 26, Michigan

December, 1960 — American Association for the Advancement of Science

127th Annual Meeting
Location: New York, New York
Date: December 26-31, 1960

The Econometric Society (in conjunction with the American Economic Association)

Location: St. Louis, Missouri
Date: December 28-30, 1960

Eastern Joint Computer Conference

Location: New York, New York
Date: December, 1960
Contact: Dr. Nathaniel Rochester, IBM, Yorktown Heights, New York

1960 (date not yet determined) — International Mathematical Union

International Symposium on "Fonctions de plusieurs variables complexes et analyse Fonctionnelle"
Location: Portugal
Date: 1960
Chairman: Professor Sebastiao e Silva

April, 1961

American Mathematical Society and the Office of Ordnance Research, U. S. Army

"Mathematical Problems in the Biological Sciences"
Location: New York, New York
Date: April, 1961
Chairman: Dr. S. M. Ulam, Los Alamos Scientific Laboratories, University of California, Los Alamos, New Mexico

American Mathematical Society and The Institute for Defense Analyses

"Recursive Function Theory"
Location: New York, New York
Date: April, 1961
Chairman: Stephen C. Kleene

May, 1961

Western Joint Computer Conference

Hotel: Ambassador Hotel
Location: Los Angeles, California
Date: May 9-11, 1961

Operations Research Society of America

19th National (9th Annual)
Location: Chicago, Illinois
Date: May, 1961
Meeting Chairman: Mr. Donald H. Schiller, Caywood-Schiller Associates, 203 North Wabash Avenue, Chicago 1, Illinois

NEWS ITEMS AND ANNOUNCEMENTS

FERRANTI FELLOWSHIP IN COMPUTING, tenable at the Oxford University Computing Laboratory for one year from 1 October 1960 is now accepting applications from candidates. The successful candidate, who should have at least two years research experience, will be expected to do original work in any field connected with computation or the application of digital computers, and will have access to a Ferranti MERCURY. The Fellowship is open to candidates of any nationality, has a stipend of £800, and is subject to the provision of F. S. S. U. (Federated Superannuation Scheme for Universities), where applicable. Applications, with details of academic and research experience, and the names of two references, should be sent before 31 July 1960 to the Director, University Computing Laboratory 9, South Parks Road, Oxford.

THE GENERAL CONFERENCE OF UNESCO at its tenth session authorized the establishment, under the 1959-1960 regular program, of six fellowships in Information Processing and Electronic Computation.

These fellowships are planned to enable highly qualified specialists to undertake research study abroad for a period of six months in one of the following fields: (1) Use of electronic computers for mechanical translation, (2) Theory of switching, (3) Use of computers for the reduction of geophysical data.

NSF POSTDOCTORAL FELLOWSHIP AWARDS. Award of 145 Postdoctoral Fellowships for advanced study and research in the sciences, mathematics, and engineering, was announced by Dr. Alan T. Waterman, Director of the National Science Foundation. The Fellowships awarded total approximately $800,000.

Fellows were selected on the basis of ability as evidenced by letters of recommendation, academic records, and other evidence of scientific attainment. Applications were evaluated by panels of outstanding scientists appointed by the National Academy of Sciences - National Research Council. Selection of awardees was made by the National Science Foundation.

National Science Foundation Postdoctoral Fellows receive an annual stipend of $4,500, an allowance for dependents, and a limited allowance to aid in defraying the cost of travel to the affiliated institution. These awards will enable the recipients to study or carry on their research at 26 institutions in the United States and in 46 foreign institutions for periods ranging from six to 24 months.

The National Science Foundation expects to reopen the Post-

doctoral Fellowship program in July for awards to be made in October 1960.

There were 20 awards in mathematics. The following is a list of individuals receiving awards in mathematics and the institutions with which they will be associated.

Name	Fellowship Institution
Abraham, Ralph H.	Princeton University
Bonic, Robert A.	Massachusetts Institute of Technology
Brown, Morton	Institute for Advanced Study
Cantor, David G.	Princeton University
Crawley, Peter L.	University of Minnesota
Dheedene, Robert N.	Harvard University
Faith, Carl C.	Institute for Advanced Study
Feldman, Jacob	Institute for Advanced Study
Greendlinger, Martin	New York University
Gross, Leonard	Institute for Advanced Study
Hajian, Arshag B.	Yale University
Harris, Bruno	Institute for Advanced Study
Jans, James P.	Institute for Advanced Study
Jones, Richard H.	University of Stockholm
Marshall, Albert W.	Princeton University
Stokes, Arnold P.	Johns Hopkins University
Thomas, Paul E.	Institute for Advanced Study
Tully, Edward J., Jr.	California Institute of Technology
Weiss, Guido L.	University of Paris
Williams, Robert F.	Institute for Advanced Study

NSF ANNOUNCES 2970 GRADUATE FELLOWSHIPS. Almost 3000 high-ability college and university students will be aided in their graduate study in the sciences, mathematics, and engineering through three National Science Foundation fellowship programs.

The Foundation awarded 1190 Cooperative Graduate Fellowships for the 1960-61 academic year, 580 Summer Fellowships for Graduate Teaching Assistants for study and research in the summer of 1960, and 1200 graduate Fellowships for 1960-61.

The approximately 150 cooperating colleges and universities that sponsor the programs originally evaluated applications for these prized awards. A second evaluation was made by 39 eminent scientist-scholars appointed by the National Academy of Sciences-National Research Council. Final selections were made by the Foundation solely on the basis of ability.

Of the Cooperative Graduate awards, 171 were made in mathe-

matics, 268 in engineering, 462 in the physical sciences, including a number in interdisciplinary fields, 254 in the life sciences, and 35 in the social sciences. Fellows were selected from 3091 applicants representing all 50 States, the District of Columbia, and Puerto Rico.

Cooperative Graduate Fellows will receive a basic, 12-month stipend of $2200 from funds provided by the National Science Foundation. At its own discretion each fellowship institution may augment the stipend by not more than $800 per year. In lieu of tuition and fees, institutions will receive a predetermined and standardized cost-of-education allowance for each Fellow.

Of the 580 Teaching Assistant awards, 99 were made in mathematics, 76 in engineering, 245 in the physical sciences, including a number in interdisciplinary fields, 143 in the life sciences, and 17 in the social sciences. Fellows were selected from 1362 applicants representing all 50 States and the District of Columbia.

Teaching Assistants will receive stipends ranging from $50 to $75 for each week of their Summer Fellowships, the exact amount to be determined by each fellowship institution according to local conditions. The Foundation will also pay the fellowship institutions the tuition and fees incurred by the Summer Fellows.

The 1200 individuals awarded graduate Fellowships were selected from 4696 applicants from all parts of the United States and its territories. The Foundation also released names of 2147 persons accorded Honorable Mention.

These awards were made in furtherance of the Foundation's policy of encouraging outstanding college graduates to obtain advanced training in the sciences on a full-time basis.

Of the awards, 233 were made in the life sciences, 946 in the physical sciences, including a number in interdisciplinary fields, and 21 awards were made in certain areas of the social sciences.

Graduate Fellowship applicants were required to take examinations for scientific aptitude and achievement. These tests were administered by the Educational Testing Service, Princeton, New Jersey. The test scores, academic record, and recommendations regarding each applicant's abilities were then considered by panels of outstanding scientists in the respective fields of the applicants; the panels were appointed by the National Academy of Sciences - National Research Council. Selection of awardees was made by the National Science Foundation.

National Science Foundation Fellows may attend any appropriate nonprofit American or nonprofit foreign institution.

Graduate Fellowships provide basic stipends (for 12 months) of $1,800 for the first year level of graduate study, $2,000 for intermediate years, and $2,200 for the terminal year. These fellowships include additional allowances for dependents and tuition, as well as a limited travel allowance.

An announcement of the National Science Foundation Graduate Fellowship program for 1961-1962 will be made in October 1960. Application forms will not be available until the program is announced.

All awards were approved by Dr. Alan T. Waterman, Director of the Foundation.

UNITED STATES STEEL FOUNDATION, INCORPORATED. A $2,677,000 program of aid to education, with grants to 710 liberal arts colleges, universities and institutes and to 27 organizations devoted to elevating educational quality in America, was announced by Roger M. Blough, Chairman of the Board of Trustees of United States Steel Foundation, Incorporated.

"As formulated by the Trustees of the Foundation," said Mr. Blough, "the program is directed toward specific efforts for the improvement of educational quality and toward encouraging all segments of the public to increase their support of the nation's educational facilities. Particular stress is laid upon stimulating colleges to attract their alumni into long-term support plans through an annual Alumni Incentive Awards Competition open to every institution of higher education in America. Continued support is provided for the state and regional liberal arts college associations which enlist business and other donors to aid the colleges financially. To provide additional opportunities for advanced academic development of competent students, a substantial expansion of graduate-study Fellowships is authorized."

The Foundation's program includes:

Operating Grants--$780,000. Operating aid in the amount of $462,000 is afforded to all 438 accredited four-year institutional members of all the 41 state and regional liberal arts college associations; additionally $43,000 of operating aid goes to Negro colleges, including all members of the United Negro College Fund.

Assistance channeled through the National Fund for Medical Education, at $80,000, is continued to the more than four-score private and public schools of medicine. In addition to support of group-affiliated institutions, the 1960 program provides $103,000 of operating assistance to non-group institutions, as well as $92,000 for the operating needs of universities, science and engineering institutes not otherwise assisted.

Major-Purpose or Capital Grants - $1,280,000. After designating $475,000 of the authorized total payments for installments on prior commitments, the Foundation is making new major-purpose of capital grants amounting to $805,000. Of this total, the sum of $670,000 for one-time grants -- ranging from $15,000 to $35,000 -- goes to 31 liberal arts and science colleges, women's colleges,

regional colleges, and universities.

The balance of $60,000 covers initial installments for commitments to three private universities as first-time participants under the Leadership Institution Aid Plan part of the overall program. Under this plan -- subject to periodic review and approval -- financial support for selected outstanding institutions is afforded for a term of years on a substantial basis. It replaces, with respect to the participating institutions, piecemeal consideration of the separate requirements of university departments and university needs for particular capital or other major-purpose program outlays. The sums becoming available are for unrestricted use.

For a five-year period, payable in installments or otherwise as determined, a total of $100,000 each is initially committed for Brown, Tulane, and Vanderbilt universities. These three institutions, together with 17 other private institutions, all of which have been designees of major grants in recent years, are among the influential leading private universities and institutes which, thus far, have been recognized for participation within the Leadership Institution Aid Plan.

Educational Association Support -- $120,000. Recognizing the continued need to broaden the base of participation in the financing of education, allotments are made to certain educational associations concerned with extending public acceptance of responsibility for adequate financing of education.

Advancement of Quality Standards -- $97,000. Support is again provided toward the attainment of improved library service and better facilities through a renewed grant to the Association of College and Research Libraries to assist many college libraries.

Aid Toward Teaching Excellence -- $69,500. The Associated Colleges of the Midwest--a group of ten colleges-- for the first time receives a grant for a project related to research and development in language-teaching techniques. Also, a first-time grant is made to Recording for the Blind to assist in teaching blind college students by use of tape and other recording devices. An initial operating grant also is made to the Center for Advanced Study in the Behavioral Sciences in California to forward interchange among scholars and to advance intellectual frontiers in the social sciences. In the area of assistance toward international educational problems, seminars and knowledge exchange, several renewed grants are made. Repeat grants are also made to the American Council of Learned Societies toward the cost of publishing scholarly works and to forward studies of the Humanities Center of the University of Massachusetts.

Fellowships -- $330,000. The Foundation's Graduate-Study Fellowship Plan is expanded with the installation of 23 new Fellowships at the doctoral level.

Research, Science and Training. Under consideration for later action by Trustees are grants in support of scientific research in the inter-related areas of polar regions, oceanography, and the earth's crust (included in the 1959 Aid-to-Education Program), and to major scientific organizations.

The United States Steel Corporation supports the activities of the American Mathematical Society through its Corporate Membership in the Society. (See Page 312).

THE MATHEMATICAL ASSOCIATION OF AMERICA has established an award in the amount of $500, to be given not more than once each year, for outstanding service to mathematics, other than mathematical research. The contribution should be such as to influence significantly the field of mathematics or mathematical education on a national scale. Nominations are invited by the Committee on the award. The members of the Committee are:

Professor Wallace Givens, Wayne State University
Professor John W. Green, University of California at Los Angeles
Professor R. J. Walker, Cornell University

THE CONFERENCE BOARD OF THE MATHEMATICAL SCIENCES was formally incorporated in the District of Columbia on February 29, 1960. The 18 members of the Board are:

American Mathematical Society

E. J. McShane
University of Virginia

J. W. Green
University of California, Los Angeles

Association for Symbolic Logic

H. B. Curry
Pennsylvania State University

P. R. Halmos
University of Chicago

Mathematical Association of America

C. B. Allendoerfer
University of Washington

H. L. Alder
University of California, Davis

National Council of Teachers of Mathematics

B. W. Jones
University of Colorado

J. R. Mayor
American Association for the Advancement of Science

Institute of Mathematical Statistics	Society of Industrial and Applied Mathematics
Z. W. Birnbaum University of Washington	Brockway McMillan Bell Telephone Laboratories, Murray Hill
Murray Rosenblatt Brown University	Gilbert Kaskey Remington-Rand, Philadelphia

Members-at-Large

H. Bode Bell Telephone Laboratories	Mina Rees Hunter College
A. E. Meder, Jr. Rutgers, The State University	Marie S. Wilcox Thomas Carr Howe High School, Indianapolis
G. B. Price University of Kansas	S. S. Wilks Princeton University

The officers of the Conference Board for 1960 are: Chairman, G. Baley Price; Secretary, J. R. Mayor; and Treasurer, A. E. Meder, Jr.

G. Baley Price has been appointed first Executive Secretary of the Conference Board and will open the Washington office on July 1. He will be in Washington on leave of absence from the University of Kansas.

THE FIRST NATIONAL CONFERENCE OF UNIVERSITY COMPUTING CENTER DIRECTORS was held June 2, 3, and 4 at the Shoreland Hotel in Chicago, with an attendance of approximately 120 invited participants. Arrangements for the Conference were made by the Headquarters Offices of the American Mathematical Society under financial sponsorship of the National Science Foundation. The following Committee planned the program:

Philip Morse, Massachusetts Institute of Technology, (Chairman)
Walter Feiberger, Brown University, (Secretary)
E. L. Buell, Worcester Polytechnic Institute
S. Gorn, University of Pennsylvania
T. A. Keenan, University of Rochester
W. B. Kehl, University of Pittsburgh
A. J. Perlis, Carnegie Institute of Technology
H. R. Rymer, Northwestern University
D. R. Shreve, Oklahoma State University
W. J. Viavant, University of Oklahoma
W. H. Wells, Georgia State College
A. W. Wymore, University of Arizona

The purpose of the Conference was to investigate problems peculiar to University Computing Centers and to publish a report embodying the findings. Five topics were selected for the agenda: (1) Budget and administration; (2) Computer-oriented research; (3) Curriculum and instruction; (4) Computing service to university departments; (5) Government relations.

Professor Morse opened the three-day meeting at a plenary session. His opening address was followed by brief talks by the Chairman of the five sections. At this general meeting every participant was given an opportunity to voice his opinion on matters pertaining to all subjects, giving to the individual sections which followed material to work on during their sessions the next day.

Highlights of the meeting were the technical lectures on each of the three days: Nicolas Metropolis, Director of the Institute of Computer Research at the University of Chicago, on Computer Logic; David Young, University of Texas, on Numerical Analysis; and Alan Perlis, Carnegie Institute of Technology, on Programming.

On the final day the Chairmen of the five sections presented their conclusions to the full meeting. The discussion of these conclusions led to firm recommendations for the conference report, which is at present in the process of being printed. It is planned to distribute one free copy of the Proceedings to each of the approximately 160 Institutional Members of the Society and to each of the participants in the Conference. Copies of the Proceedings are available, at no charge, upon request.

A SYMPOSIUM ON ENGINEERING APPLICATIONS OF PROBABILITY AND RANDOM FUNCTION THEORY sponsored by Purdue University will be held November 15-16, 1960, at Lafayette, Indiana. The symposium will stress applications of probability and random function theory to problems associated with factors of safety in structures, reliability of structures and systems, optimization of systems of all types which are subject to random disturbances, jet and rocket engine noise fields and traffic control. Professors M. Kac of Cornell, A. J. F. Siegert of Northwestern, E. W. Montroll of the University of Maryland have already agreed to participate. A detailed program will be released within the next two months.

Requests for further information should be addressed to either J. L. Bogdanoff or F. Kozin, co-chairmen of the Symposium, Division of Engineering Science, Purdue University, Lafayette, Indiana.

THE FOURTH SYMPOSIUM ON MATHEMATICAL STATISTICS AND PROBABILITY will be held at Berkeley on June 20 to July 30, 1960. The Symposium is organized by the Statistical Laboratory of the University of California with the financial support of NSF, ONR,

OOR, AFOSR, and NIH (National Institutes of Health). The Advisory Committee of the Symposium consists of Delegates of the American Mathematical Society: Professors J. H. Curtiss, J. L. Doob, and William Feller. Delegates of the Institute of Mathematical Statistics: Professors A. H. Bowker and H. E. Robbins. Editor of the Annals of Mathematical Statistics: Professor W. H. Kruskal. The Committee on Local Arrangements is: Professors M. W. Eudey, E. Fix, L. LeCam, J. Neyman, E. L. Scott, and H. G. Tucker.

The preliminary program lists approximately 120 lectures and it is planned to ordinarily schedule four lectures for each day of the symposium. It is expected that there will be approximately 50 foreign participants.

Symposium lectures will be approximately organized in four cycles: Theory of Probability and Theory of Statistics, conducted concurrently from June 20 to July 15; Problems of Health, July 18 to July 25; and Probability and Statistics in Physical Sciences, July 26 to July 29. Lectures will be interspersed with excursions to various points of interest. In response to the cordial invitation of Professor Herbert Solomon, Chairman of the Department of Statistics, Stanford University, the day of July 1 will be spent at Stanford. Also, at the kind invitation of Professor A. E. Whitford, Director of the Lick Observatory, there will be a visit to the Observatory on July 22.

THE TENTH INTERNATIONAL CONGRESS OF THE HISTORY OF SCIENCE will be held in the United States of America, 26 August - 2 September, 1962. Opening sessions of the Congress will be held at Cornell University, Ithaca, New York, and the concluding sessions will be held at the American Philosophical Society, Philadelphia, Pennsylvania. The President of the Congress is Professor Henry Guerlac of Cornell University. The Secretary of the Congress is Professor C. Doris Hellman.

All inquiries should be addressed to The Secretary, Xth International Congress of the History of Science, Cornell University, Ithaca, New York (U.S.A.). Those wishing to receive bulletins concerning the congress are requested to communicate with the Secretary.

ROMAN JAKOBSON, Chairman of the Organizing Committee of the AMS - Institute for Defense Analyses Symposium on the Structure of Language and its Mathematical Aspects, has been accorded a high honor by the American Council of Learned Societies. ACLS Prizes for Distinguished Scholarship in the Humanities, of $10,000 each, were awarded on January 20, 1960 to ten scholars, including Professor Jakobson, in recognition of their contributions to human-

istic learning.

At the award ceremony, the following citation was read:

"Roman Jakobson -- restless conquistador of the jungled frontiers of linguistic and literary scholarship; pioneer both as zealous organizer and as scholarly exemplar in the establishment of Slavic literary studies throughout the world; internationally honored critic of many literatures, in many works, who writes in many languages; historian and mythologist, prosodist and musicologist, grammarian and folklorist, whose multi-faceted scholarship is only now coming to its climax in his important contribution to linguistic science as a function of poetics, and who, in these studies, vivifies linguistics, sharpens poetics, and deepens our understanding of the literary imagination in some of its most technical and elusive manifestations"

AAGE BOHR, Professor of the Institute for Theoretical Physics of the University of Copenhagen, Denmark, has been named the second recipient of the Dannie Heineman Prize in Mathematical Physics.

The prize of $2,500 was awarded to Dr. Bohr at the dinner of the Spring meeting of the American Physical Society in Washington, D. C. on Wednesday, April 27, in the Sheraton-Park Hotel.

The prize is given by the American Physical Society and the American Institute of Physics in behalf of the Heineman Foundation. Dr. Victor F. Weisskopf, Professor of Physics at the Massachusetts Institute of Technology and President of the A. P. S. made the presentation.

Dr. Bohr, physicist son of Nobel Laureate Niels Bohr of Denmark, in 1952 advanced the theory of the collective model of the nucleus (a scientific explanation of the effect of the collective motion of particles in the nucleus).

Since 1946 he has been a member of the Institute for Theoretical Physics of the University of Copenhagen and has served as Professor of Physics since 1956.

His research is concerned with theoretical, atomic and nuclear physics. The principal topics he has investigated are: penetration of charged atomic particles through matter; atomic hyperfine structure; and, since 1949 especially, nuclear structure encompassing the theory of collective motion in the nucleus, nuclear vibrational and rotational spectra, nuclear moments, and the fission process.

HUGH C. WOLFE, Head of the Physics Department at Cooper Union School of Engineering, has been named Director of Publications of the American Institute of Physics.

Announcement of this major appointment to the executive staff

of the Institute was made by Dr. Elmer Hutchisson, Director, who said that Professor Wolfe will assume his new duties August 1.

The American Institute of Physics is a federation of the five principal scientific societies in the field of physics, and in cooperation with the societies publishes the leading journals of physics.

"Dr. Hugh Wolfe will become our first director of publications at a time when scientific journals in physics are expanding in number, size, and readers, and scientists as well as scientific societies are facing critical problems in keeping abreast of the rising tide of new research," said Dr. Hutchisson.

In this new position, Dr. Wolfe will supervise publication of the most significant basic and applied research reports which underlie advances in such fields as atomic energy and space physics.

STEPHEN I. JUHASZ has been named editor of Applied Mechanics Reviews, published by The American Society of Mechanical Engineers.

Dr. Juhasz, who was formally executive editor of the publication, is at the Southwest Research Institute of San Antonio, Texas, where the periodical is edited. He succeeds Martin Goland, President of Southwest Research Institute, who has edited the magazine for ten years. Mr. Goland remains as editorial adviser.

Honorary editors of the publication are: Hugh L. Dryden, Deputy Administrator, National Aeronautics and Space Administration; Theodore von Karman, Chairman, NATO, Advisory Group for Aeronautical Research and Development; and Stephen P. Timoshenko, Professor, Theoretical and Applied Mechanics, Stanford University.

Applied Mechanics Reviews is published monthly by The American Society of Mechanical Engineers. It contains critical reviews of articles in the field of applied mechanics, which originate in more than 40 countries. Over 7000 items, including authors' summaries and title listings are published annually. More than 1500 engineers and scientists in 30 countries serve as voluntary reviewers.

UNESCO SYMPOSIUM ON 'NONLINEAR PHYSICAL PROBLEMS' AND FIFTH CONGRESS ON THEORETICAL AND APPLIED MECHANICS. The symposium was held on the 21st and 22nd December at the University of Roorkee, Roorkee, which played host to over a hundred delegates and members who had assembled from all parts of India and abroad. Foreign participants included Folke K. G. Odqvist, President of the International Union of Theoretical and Applied Mechanics, E. Saibel and S. Kumar from U. S. A.

The symposium began with a welcome address by Dr. A. N. Khosla, Member, Planning Commission, who is also the President of the Indian Society of Theoretical and Applied Mechanics. He paid tri-

bute to the collaboration between UNESCO and the Society. The symposium was inaugurated by Sir Harrold Williams, Director of the Central Buildings Research Institute.

In his opening address, B. R. Seth dealt with 'Nonlinear Physical Problems: Survey and Prospects.' Linearized approximations for a physical problem do not explain satisfactorily a number of secondary effects, and the development of nonlinear continuum mechanics has led to substantial advance. Among the nonlinear problems that have recently come into prominence are: vibration problems, particularly in high speed phenomena; nonlinearities introduced by non-Newtonian and visco-elastic characteristics of certain motions; and the creep properties of materials subjected to fluctuating stresses at high temperatures. Success with such problems will continue to depend on advances in experimental techniques. Meanwhile, high speed computational methods have facilitated the solution of many practical problems.

The preliminary success in handling nonlinear problems has prompted the study of interaction with linear domains. Notable advances in this field relate to the pinch effect, force-free fields, etc, in magneto-hydrodynamics, and to the theory of lubrication and creep. On the other hand, the whole field or interaction of two or more nonlinear domains has remained essentially untouched.

Half-hour addresses were delivered by F. K. G. Odqvist, E. Saibel, and S. N. B. Murthy and A. K. Chaudhuri. Odqvist spoke on creep of plates and membranes, Saibel on general solutions for evaluating stresses and deformation in circular plates subject to ring loads, and Murthy and Chaudhuri on the behavior and optimum nonlinear control of turbo-jet engines.

The symposium was followed by the Fifth Congress, Dec. 23-26. In his presidential address, Dr. A. N. Khosla described the projected 320 foot span shell-roof auditorium at Roorkee. The papers presented can be broadly classified under the heads: (i) Elasticity and plasticity, (ii) Fluid Mechanics, (iii) Vibration and lubrication studies, (iv) Thermodynamics and heat transfer, and (v) Statistics and computation.

S. C. Das, V. Lakshmikantan, D. N. Mitra and P. D. S. Verma presented Noll's general theory of continuity in solid and fluid states and dealt with boundary value problems; C. N. Lakshminarayana presented a frozen stress photoelastic model, and K. G. Chandiramani explained some measurements of the cutting forces on a lathe.

Starting with the general 'nonlinear shallow water theory' for variable water depth, C. N. Kaul showed an analogy in such a theory between the propagation of weak discontinuities and the gasdynamics. In describing 'Hydromagnetic Shocks,' O. P. Bhutani showed how the large compression caused by radiation escape amplifies the magnetic

field. G. L. Saini developed an analysis for the study of certain 'Singular Hypersurfaces' in general relativity. A. C. Srivastava dealt with the problem of 'Steady two dimensional hydromagnetic flow between non-parallel walls.' Ram Kumar discussed the unsteady motion of a viscous fluid in the presence of an infinite circular cylinder. P. V. Abdurahiman dealt with problems of boundary layer responses to free stream velocity and with drag coefficient for bodies started with supersonic velocity. G. A. Nariboli considered the steady flow of fluid through pipes of constant curvature. P. D. S. Verma, M. N. L. Narasimhan, Y. D. Wadhwa and R. S. Nanda presented some aspects of flow through channels with porous walls. S. K. Sharma discussed flow of a visco-elastic liquid near a stagnation point. G. Bandyopadhyay dealt with unsteady rectilinear flow of non-Newtonian fluids. Ram Ballabh discussed superposability and self-superposability. L. N. Nigam, H. C. Agarwal, N. R. Rajappa and A. C. Jain presented papers on flow past a wing of circular plane form, flow with heat transfer, compressible flow, and flow with porous walls.

S. N. Singh discussed the solution of a certain differential equation of second order of generalized parabolic type with non-homogeneous boundary conditions. B. Mohan Rao described secondary flow stresses in rotating fluid in a vertical shaft.

J. N. Kapur gave an extension of his earlier work on exact solution of the equations of internal ballistics for the pressure loads law of burning. R. G. Mokadam described some inconsistencies in the general thermodynamic analysis of the Darcy Law.

Sudhir Kumar gave an account of his recent work on scabbing, B. M. Belgaumkar described an equivalent shaft for transverse vibration of crank shafts, and J. V. Nagaraja dealt with electrical analogy methods for flexure of uniformly loaded plates.

THE BOARD OF DIRECTORS OF THE MATHEMATICAL SOCIETY OF JAPAN have the sad duty of announcing to its foreign colleagues the death of Teiji Takagi, Professor Emeritus of the University of Tokyo, Member of the Japan Academy, and Member of the National Committee of Mathematics of the Science Council of Japan.

He passed away at the age of eighty-five years on February 28, 1960 by the cerebral apoplexy.

Tokyo, March 1960

Zyoiti Suetuna
Shokichi Iyanaga
Kôsaku Yosida

EDWARD ČECH, the eminent Czech mathematician, died on March 15, 1960.

JOHN HENRY CONSTANTINE WHITEHEAD, F. R. S., distinguished British mathematician, Wayneflete Professor of Pure Mathematics, Magdalen College, Oxford University, died suddenly in Princeton on May 8, 1960, at the age of 55. This Spring he was a visiting member of the Institute for Advanced Study.

Henry Whitehead has been a leading figure in the world of mathematics. He specialized mainly in the relatively new field of topology, where his work was influential in inspiring some of the rapid developments of the last 25 years. He published numerous research papers, some jointly with American mathematicians. With Oswald Veblen, he wrote THE FOUNDATIONS OF DIFFERENTIAL GEOMETRY (Cambridge, 1932).

He was educated at Eton, Balliol College (Oxford University), and Princeton University, where he received his Ph.D. in 1930. He was Commonwealth Fund research fellow at Princeton, 1929-1932, lecturer and fellow at Balliol College, Oxford 1932-1946, visiting lecturer at Princeton, 1946-1947, and Wayneflete Professor of Pure Mathematics at Oxford since 1947. He was elected Fellow of the Royal Society in 1944 and was President of the London Mathematical Society, 1953-1955.

He was born in Madras, India, the son of the late Right Reverend Henry Whitehead, Bishop of Madras. He was a nephew of the late Alfred North Whitehead, mathematician and philosopher. He is survived by his wife, Barbara, and two sons. Their address is Manor Farm, Noke, Oxford, England.

A REPORT ON CORPORATE MEMBERSHIPS

The following article is being published in a booklet which the Society is sending to corporations and industrial firms, explaining the importance of mathematics to industrial research and the service of the AMS to mathematics, and encouraging these organizations to become corporate members of the Society. We are publishing the article here to inform our members of what has been and is being done in this area, and to enlist their support.

Corporate memberships were established in 1957, after several years consideration, largely as a result of a number of inquiries from industrial firms interested in keeping informed of our meetings and publications for the sake of scientists on their staff who need to keep up with current mathematics. The definition of the category of corporate members, and the first members in this category, were achieved largely due to the work of John H. Curtiss, former Executive Director of the Society, and the Committee on Corporate Memberships, consisting of M. M. Flood, B. P. Gill, and C. C. Hurd, Chairman. During 1959 the following organizations were corporate members of the Society:

Bell Telephone Laboratories, Incorporated
E. I. DuPont de Nemours and Company, Incorporated
Ford Motor Company
General Motors Corporation
Hughes Aircraft Company
International Business Machines Corporation
Procter and Gamble Company
Radio Corporation of America
RIAS, Incorporated
Shell Development Company
Space Technology Laboratories, Incorporated
Sun Oil Company
United Gas Corporation
United States Steel Corporation

Corporate memberships are very valuable to the Society for several reasons. They enable business organizations, especially those whose activities are affected, directly or indirectly, by research and education in higher mathematics, to cooperate with the Society in the development of the important area of human knowledge served by our program. Also, the dues paid by corporations enable us to keep the costs of our services and publications (which are unusually expensive to print) at a reasonable level, usually below cost, for the individual member.

Although industrial research, like all scientific research, depends upon mathematics, and is affected by new developments in

mathematics, still the connection between the final product and the mathematical foundations is often so attenuated that it is far from obvious to the producer, especially to the non-scientific businessman. For this reason the AMS often has some difficulty in recruiting corporate members. We have in the past used various documents to support our appeal for corporate members, but in view of the special position mathematics occupies with regard to industrial research, a more elaborate statement of the importance of mathematics in this research-conscious age seemed to be in order at this time.

Our individual members are often in a position to be very helpful in influencing the decisions of corporations to support our Society, and the Corporate Membership Committee would welcome your support in this effort to secure corporate members.

MATHEMATICS AND BASIC RESEARCH

> "Arts of public use--as fortification, making of engines, and other instruments of war--because they confer to defense and victory, are power, and though the true mother of them be science--namely the mathematics--yet, because they are brought into the light by the hand of the artificer, they be esteemed as his issue, the midwife passing with the vulgar for the mother."
>
> --Thomas Hobbes, Leviathan (1651).

Mathematics, often called the "language of science," is indeed, as Hobbes saw in that great age of discovery three centuries ago, of fundamental importance to all scientific and technological development. For mathematics, a non-empirical study, is not a science in the sense that physics, chemistry, geology, astronomy, etc., are sciences, with a definite, delineable chunk of the physical universe as its subject matter. Mathematics is basic to all these sciences, and as it has become more refined and complex, it has made possible more refined and complex observations of the world, which in turn illuminate and render explicable vast areas of empirical fact. Progress in application and development cannot proceed very far without advances in the basic body of knowledge.

The great men of the past, whom we recognize as having materially advanced our knowledge and control of the physical universe, are those who made not things, but theories, inventors not of machines but of concepts. Those great 17th century contemporaries of Hobbes whose discoveries and inventions ushered in a new age of science--Newton, Descartes, Leibnitz, Galileo--were not interested in making engines, or rockets, or satellites, or in getting anywhere faster, or blowing up more people into smaller pieces; they sought rather to achieve those ends that always have, and do now, and al-

ways will, spur on the finest minds--to reduce chaos to order, to fit the disparate parts into a coherent whole, to complete the design, to impose elegance and precision upon the apparent disorder and confusion of experience.

> "The Foundation's Program for Basic Research in the Physical Sciences is based on the philosophy that more new scientific ideas per dollar spent can be obtained by supporting 'creative people' rather than by supporting 'research projects.' The unique feature of our program is that the scholars selected are free to choose the scientific problems which they wish to investigate, and they are also free to change or modify their researches. We believe that anyone engaged in basic research needs such freedom because creative thinking cannot be charted in advance or put on a schedule."
>
> ---Alfred P. Sloan, Jr., President
> Alfred P. Sloan Foundation

The scientist engaged in basic research, in contrast to applied research and development, seeks only to know and understand the subject under study, adding his discoveries to the body of scientific knowledge by communicating them unambiguously to others. Basic research is usually the work of an individual or at most of a small group; it cannot, by the very nature of things, be mass-produced or put on a rigid schedule. Its results are not things but knowledge, usually made public in the form of papers so complex and difficult that only a few other scientists can recognize their significance. The man in the street would no doubt consider a great, intricate, multi-million dollar machine, like a jet plane or one of the large computers, as a greater achievement than a paper on non-linear differential equations, yet great technological achievements could never come about unless certain theoretical problems are solved. Still, because this very basic work is done out of sight, in the interior of a man's head, and the only external evidence of what has been accomplished may be no more than a short paper presented at a meeting or published in a journal, heard or read by a handful of people, it is easy to overlook its vital importance to the productions that engage the greater part of our attention, effort, and money.

Yet the basic research represented by these papers constitutes the new scientific knowledge from which spring advances in the national welfare, economy, and military strength--advances which generally require years to develop, since the opportunities presented by basic research must be followed up by applied research, development, and production before full fruition is realized. And, since it is a natural human disposition to attend to the immediate and concrete

rather than the remote and abstract, the basic research which made the entire advance possible tends to be forgotten. Even the men who did the groundwork may no longer be interested, having long since gone on to other problems.

> "The most impractical thing that can be done in designing and directing programs of scientific research is to worry about how 'practical' they are."
>
> ---President Eisenhower's Science Advisory Committee.

The time lapse between original discovery and application is nowhere greater than in mathematics, which often seems so "pure" as to have no possible connection with any "earthly" thing. Yet some of the most abstruse and apparently impractical of mathematical inventions have proved necessary to the revolutionary developments in physics and engineering of the 20th century.

Non-Euclidean geometry, invented about 1830, and the closely related tensor analysis, developed in 1895, remained unapplied until Einstein used them to formulate his General Theory of Relativity in 1915. Quantum field theory is based on the theory of Hilbert space, a branch of mathematics named for its prolific inventor, David Hilbert, whose "useless" inventions were among the most significant mathematical achievements of the 20th century. Mathematicians themselves often doubt that their work can be applied; for example, Arthur Cayley, in 1858, stated quite confidently that his newly developed algebra of matrices would never have any application, yet today matrices are one of the standard tools of physics and engineering.

Similar examples could be multiplied. Non-linear differential equations, which until recently were not seriously studied by American engineers, have assumed vital importance in these days of missiles and satellites. The advances of the Russians in this branch of mathematics are at the root of their success with space satellites and ballistic missiles.

The scientist, then, must not be expected to justify his work in terms of its ultimate utility, nor can the journals that publish it afford to make such judgments. If the research results in an enlargement of knowledge, uses for that knowledge will be found sooner or later. In these days of vast scientific knowledge and accelerated activity, the applications are likely to come sooner, development being very fast compared with an earlier time when science was young and scientific knowledge scanty.

SHOCK WAVE THEORY--A Case History

A dramatic example of the way in which a theoretical advance can illuminate a whole area of study, and lead to an entirely new field

of technology, is seen in the case of shock wave theory, which has led to modern developments in such diverse areas as underwater sound, solar physics, ballistic missiles, explosives, jet engines, wind tunnels, rockets, satellites, supersonic aircraft, and thermo-nuclear devices.

Development in this area can be traced back through the work of many scientists on the theory of sound, to Poisson, Laplace, Lagrange, d'Alembert, and even Newton. The field became an active subject of discussion in 1848 when Stokes, seeking a unique solution for the equation formulated by Poisson in 1808 to describe the flow of a gas, suggested that "a surface of discontinuity is formed, across which there is an abrupt change in velocity and density." Methods for actually calculating the position of this discontinuity, and the extent of the changes across it, were developed in 1860 by Riemann, who introduced some very abstruse mathematics for this purpose. The work of Rankine of Great Britain in 1870, and of Hugoniot of France in 1889, placed the subject on a firm theoretical basis.

The basic work up to that point was enough to set other scientists looking for compatible facts in many fields, guided by a theory which became more and more suggestive as it became more detailed. Discovery accelerated at a rapid rate in such diverse fields as physics, chemistry, mathematics, and aerodynamics, and is still going on today, as the ramifications of shock wave theory are being worked out. The development of the theory shows clearly how the work of one man stimulates another, how problems raised but left unsolved by one scientist engage the attention of many others, so that finally hundreds of men over long periods of time each make their unique contribution.

> "We might liken our pool of basic scientific knowledge to a savings account from which we make withdrawals as we convert that knowledge through applied research to new products and processes. As with all savings accounts, bankruptcy lies ahead when withdrawals exceed deposits. To keep our scientific balance in healthy condition, we must insure always that our deposits derived from basic research are never less than our technological withdrawals."
>
> --Crawford H. Greenewalt, President
> E. I. du Pont de Nemours and Company, Incorporated.

In the case of shock wave theory, as in so many cases, the original insights and discoveries, when compared with the results that follow from them, seem simple and small. They are the work of only

a few people in each generation, and the needs of these few, for laboratories, equipment, and channels of communication, are minimal compared with the expense of development and production. Yet failure to support the fundamental activity would inevitably have disastrous effects on the development which comes from it, a fact increasingly recognized among responsible people, not only in science but in industry and government.

Millions of dollars are spent by the Government on military research and development, and much of this money is wasted because engineering projects are undertaken without adequate preparation. "The trouble with engineers in the United States," according to Dr. Solomon Lefschetz, one of the leading mathematicians in the western world, and at present director of the Research Institute for Advanced Study in Baltimore, "is that they do too much 'hardware' and not enough thinking. They believe in empiricism--the old trial-and-error method." A more economical method would be to detect the error before the trial, to bank more on basic research than on specific projects, and to eliminate mistakes on the blackboard before moving to the drawing board.

At present only about six percent of the Government's total budget for research and development is earmarked for basic research, the remaining 94 percent going to pay for an almost endless list of "things"--jet planes, missiles, nuclear reactors, satellites, insecticides, vaccines, drugs, rocket fuels, computers, and space suits. According to Dr. James R. Killian, Jr., the President's former Special Assistant for Science and Technology, this distribution of effort is highly uneconomical, since applied research and development proceed more rapidly, and at a lower cost, when adequately backed by basic research.

> "The cost of development is far greater than the cost of research, and if a big development gets off on the wrong foot, the cost is terribly high."
>
> ---Frederick R. Kappel, President
> American Telephone and Telegraph Company

In industry, organized research is a fairly recent development, but is growing rapidly as the larger companies find that heavy concentration on research pays off. In the last forty years the number of industrial laboratories in this country has risen from 290 to more than 5,000. Yet even the largest company, no matter how heavily it invests in basic research, cannot hire every scientist whose work is valuable in keeping up the pace of modern progress. Fortunately it does not need to. The theoretical scientist or mathematician, whether in a university or in a laboratory, regards himself as part of the community of science, with an interest in communicating his findings to

his fellow researchers through the established journals of his profession. In this way scientists throughout the country are kept abreast of the newest developments in their fields, wasteful duplication of effort is avoided, and progress is made at a much faster rate than would be possible if each scientific department or laboratory were a self-contained unit, depending entirely upon its own resources.

Thus it can be seen that the media of communication, the life blood of the scientific enterprise, perform a vital service that lies at the very heart of scientific and technological development. The industrial firms whose scientific researchers have access to these media benefit immensely, and at very little cost to themselves. Since it would obviously be impossible for them to buy the personal services of every major scientist, it is clearly in their interest to support the general media of basic research, namely the journals and scientific meetings. This can be done no more effectively than through the established societies devoted to scientific activity and communication.

> The American Mathematical Society, organized in 1888, is one of the oldest societies in the world for the promotion of mathematical scholarship and research. The frequent scientific meetings of the Society, and the books and journals which it publishes or supports, are important media for communication of the basic research done in industrial and government laboratories, in universities, and by individual researchers.

Two national meetings and a number of widely distributed sectional meetings are held every year. These meetings enable mathematicians to keep abreast of current developments, provide occasions for announcement and discussion of new findings, and give opportunity for informal conversation on scientific topics. The regularly scheduled meetings are supplemented by various special meetings and symposia, some of them conducted by the Society acting on its own, and some co-sponsored with other organizations. Of particular note is a series of annual symposia in applied mathematics which have been scheduled in recent years. Besides these formal meetings, membership in the Society promotes informal communication between individuals, enabling mathematicians to meet and talk with people from different places and in different fields. This kind of informal contact is immensely stimulating to the individual, and its value can hardly be overestimated.

The publications of the American Mathematical Society are many and varied. In addition to publishing two research journals,

two organs containing reports on the activities of the mathematical community, and a comprehensive reviewing and abstracting journal, the Society helps to support a number of other research journals, and publishes three series of monographs, various memorial volumes, and proceedings of symposia. Furthermore, the Society, with Government support, publishes translations from languages unfamiliar to the majority of American mathematicians. The most important of these, the Russian translations, include the entire pure mathematics section of the REPORTS OF THE ACADEMY OF SCIENCES OF THE USSR, books on higher mathematics, and selected articles issued periodically in bound volumes.

> The objectives for which the Society was founded in 1888 have since become vitally important not only to mathematics and science, but also to the national welfare, and its original methods of achieving these objectives, through meetings and publications, are still the most effective for advancing mathematical research. But while the present vital significance of mathematics has increased the importance and expanded the activities of the American Mathematical Society, it has also placed on the Society a financial burden that it cannot carry unaided.

Mathematics differs from other sciences in being more basic and general in its application, a difference which works to the disadvantage of the American Mathematical Society in getting assistance from industrial firms. For although mathematics underlies all science, it has no such direct and apparent connection with particular industries as that which exists, for example, between the American Geological Institute and the oil industry, or between the American Chemical Society and the chemical industry. These societies, having specific ties with particular industries, are generously supported by them.

Membership dues and subscriptions do not begin to meet the Society's costs of operation; nor should they be raised to cover the costs, since this would mean that the people who are doing the mathematical research so basic to modern science would themselves be paying the costs of their own work. Although the Government helps to support the Society, it is of the utmost importance that mathematical research, like other such activities, should not become dependent upon the Government, which has not always made the wisest decisions about what kind of activity is or is not worthwhile. Yet this dependence seems inevitable unless corporations and individuals are willing to assist.

The American Mathematical Society, like every scientific society, needs and deserves such assistance. By bringing the results of mathematical research the world over to scientists in industrial and government laboratories and in institutions of higher learning, and by encouraging and supporting the work of the mathematician, the Society provides an indispensable service not duplicated by any other organization. It is a service, moreover, that is valuable to all sciences, and thus the American Mathematical Society indirectly but indispensably benefits every industry that engages in scientific research and development. For whatever products or processes are sought, "science--namely the mathematics--is their true mother."

PERSONAL ITEMS

(This section is reserved for members of the Society)

Dr. H. B. CURRY of Pennsylvania State University was honored by being named Evan Pugh Research Professor effective February 1, 1960.

Dr. L. A. AROIAN of Hughes Aircraft Company has accepted a position as member of the technical staff of Space Technology Laboratories, Incorporated, Los Angeles, California.

Assistant Professor J. L. BAGG of Florida State University has accepted a position as quality analysis statistician with RCA Service Company, Patrick Air Force Base, Florida.

Dr. ARCHIE BLAKE of Bendix Aviation Corporation has accepted a position as manager of the Analysis Section, Equipment Division, Raytheon Company, Sudbury, Massachusetts.

Dr. J. L. BOAL of Massachusetts Institute of Technology has been appointed to an assistant professorship at the University of South Carolina.

Mr. L. L. BUMGARNER of the University of North Carolina has been appointed to the staff of the Oak Ridge National Laboratory, Oak Ridge, Tennessee.

Professor LAMBERTO CESARI of Purdue University has been appointed to a professorship at the University of Michigan.

Dean HAROLD CHATLAND of Montana State University has accepted a position as engineering specialist in the Electronic Defense Laboratory of Sylvania Electric Products, Incorporated, Mountain View, California.

Dr. E. E. ENOCHS of the University of Chicago has been appointed to an assistant professorship at the University of South Carolina.

Dr. R. A. EUBANKS of Borg Warner Corporation has been appointed senior research scientist at the Armour Research Foundation, Illinois Institute of Technology.

Mr. R. O. EXCELL of the Pure Oil Company has accepted a position as senior operations research analyst with the Brunswick-Balke-Collender Company, Chicago, Illinois.

Dr. A. M. FINK of Iowa State University has accepted a position as assistant consultant with Mathematica, Princeton, New Jersey.

Miss J. B. FRIEDMAN of Technical Operations, Incorporated has accepted a position as member of the technical staff of the MITRE Corporation, Bedford, Massachusetts.

Dr. HANS GRAUERT of the University of Münster has been appointed an ordentlicher professor at the University of Göttingen, Germany.

Associate Professor SIMON GREEN of Essex College has been appointed to a professorship at Arizona State University.

Mr. ALEXANDER HACHIGIAN of the University of Illinois has accepted a position as staff member with Sandia Corporation, Albuquerque, New Mexico.

Assistant Professor JOHN HILZMAN of Harpur College has been appointed to an assistant professorship at Idaho State College.

Associate Professor MAURICE HOLT of Brown University has been appointed to a visiting professorship at the University of California, Berkeley.

Dr. NAI-CHAO HSU of Washington University has accepted a position as associate mathematician with International Business Machines Corporation, Yorktown Heights, New York.

Professor N. D. KAZARINOFF, on leave from the University of Michigan, will be at the Steklov Mathematical Institute, Moscow, U.S.S.R. for a period of six months beginning in August.

Mr. S. A. KHABBAZ of the University of Kansas has been appointed to an assistant professorship at the University of Massachusetts.

Mr. G. R. KUHN of Battelle Memorial Institute has accepted a position as mathematician with Bendix Aviation Corporation, Detroit, Michigan.

Assistant Professor H. T. LaBORDE of the University of Cincinnati has been appointed to an associate professorship at the University of South Carolina.

Mr. D. M. LONG of Barnard School for Boys has accepted a position as applied science representative with International Business Machines Corporation, Los Angeles, California.

Associate Professor R.C.MEACHAM of the University of Florida has been appointed to a professorship at the new Florida Presbyterian College, St. Petersburg, Florida.

Dr. G. J. MINTY of the University of Washington has been appointed to an assistant professorship at the University of Michigan.

Associate Professor KATSUMI NOMIZU of Catholic University of America has been appointed to an associate professorship at Brown University.

Professor E. P. NORTHROP, on leave from the University of Chicago for 18 months beginning April 1, 1960, will serve as the Ford Foundation's resident representative in Turkey and as consultant to the Turkish Ministry of Education.

Associate Professor INGRAM OLKIN of Michigan State University has been appointed to an associate professorship at the University of Minnesota.

Dr. L. L. PHILIPSON of Planning Research Corporation has accepted a position as director of advanced planning with Marc Shiowitz and Associates, Incorporated. In addition, he retains a position as consultant with Planning Research Corporation.

Assistant Professor H. J. RENGGLI of Rutgers, The State University, has been appointed to an assistant professorship at the University of New Mexico.

Dr. D. S. SCOTT of the University of Chicago has been appointed to an assistant professorship at the University of California, Berkeley.

Dr. T. I. SEIDMAN, while retaining his position as mathematician in the Radiation Laboratory of the University of California, Livermore, has been appointed a lecturer at the University of California, Berkeley, for Spring term 1960.

Dr. HAROLD SHULMAN of New York University has accepted a position as consultant with Service Bureau Corporation, New York, New York.

Dr. STEPHEN SMALE of the Institute for Advanced Study has been appointed to an associate professorship at the University of California, Berkeley.

Mr. C. J. SMITH of Washington University has accepted a position as methods technician in the Remington Rand UNIVAC Division of Sperry Rand Corporation, St. Louis, Missouri.

Dr. T. W. TING of Indiana University has accepted a position as senior mathematician in the Research Laboratories of General Motors Corporation, Warren, Michigan.

Dr. R. S. VARGA of Westinghouse Electric Corporation has been appointed to a professorship at Case Institute of Technology.

Assistant Professor DANIEL WATERMAN of Purdue University has been appointed to an assistant professorship at the University of Wisconsin, Milwaukee.

Associate Professor N. A. WIEGMANN of Catholic University of America has been appointed to a professorship at George Washington University.

Mr. TSE-CHIEN WOO of Brown University has accepted a position as senior researcher in the Research Center of Pittsburgh Plate Glass Company, Pittsburgh, Pennsylvania.

Mr. S. A. ZADOFF of Sperry Gyroscope Company has accepted a position as staff engineer with Radio Receptor Company, Westbury, New York.

The following promotions are announced:

H. A. ELIOPOULOS, Assumption University of Windsor, to an associate professorship.

MARIANO GARCIA, Jr., University of Puerto Rico, to Dean of Studies.

R. V. KADISON, Columbia University, to a professorship.

W. R. SLINKMAN, Bemidji State College, to an assistant professorship.

IRWIN STONER, Service Bureau Corporation, to senior analyst in the Special Projects Division, Poughkeepsie, New York.

J. L. WULFF, Sacramento State College, to an assistant professorship.

The following appointments to instructorships are announced:

Harvard University: Dr. DAVID LUBELL; University of Notre Dame: Mr. J. T. YAMADA; Princeton University: Dr. S. U. CHASE; University of San Francisco: Mr. T. E. FRAYNE; Yale University: Dr. J. G. STAMPFLI.

Deaths:

Miss RUTH B. EDDY of D. C. Heath and Company died on July 29, 1959 at the age of 47 years.

Professor C. G. LATIMER of Emory University died on February 3, 1960 at the age of 66 years. He had been a member of the Society for 35 years.

Professor Emeritus C. E. LOVE of the University of Michigan died on January 31, 1960 at the age of 77 years. He had been a member of the Society for 46 years.

Professor J. C. SMITH of High Point College died on October 27, 1959 at the age of 42 years. He had been a member of the Society for 18 years.

Mr. T. D. TRUAN of the Crucible Steel Company of America died on February 2, 1959 at the age of 37 years.

LETTERS TO THE EDITOR

Editor, the NOTICES

Members of the American Mathematical Society might be interested in the following editorial, which appeared in the St. Louis Post Dispatch of February 5, 1960.

> Three men, largely unknown to the general public, are now in prison in the United States for asserting their supposed rights under the First Amendment. It is strange indeed that so little attention has been paid to these cases, for all of us are directly concerned with freedom of speech, press, assembly, and religion.
>
> On February 2, H. Chandler Davis, 33-year-old former instructor in mathematics at the University of Michigan, went to federal prison for refusing to answer some of the questions of the House Un-American Activities Committee, put to him in 1954, as to the authorship of a pamphlet critizing the committee and published by two groups at the University of Michigan campus.
>
> Last November 10, Lloyd Barenblatt, former psychology instructor at Vassar College, entered federal prison to serve six months because he would not discuss his political affiliation, if any, with the House Un-American Activities Committee.
>
> Last December 14, Willard Uphaus, religious educator and director of World Fellowship, Incorporated, went to jail because he would not give up the guest list at a summer camp in Conway, New Hampshire, as demanded by the Attorney General of the State.
>
> After his session with the House committee, Dr. Davis was dismissed by the University of Michigan. The dismissal was condemned by the American Association of University Professors, the American Civil Liberties Union, and faculty and student groups. He then taught at Columbia University and was offered and accepted a fellowship at the Institute for Advanced Study in Princeton, New Jersey.
>
> On surrendering to the Federal Marshal in Grand Rapids, Michigan, Dr. Davis said
>
> > "There is no such thing as orthodoxy and heresy in political thought in this country - all opinion must be permitted. They must be permitted not only to be spoken but also to be heard with attention.
> >
> > It is wrong to take away a man's freedom of speech, not only because it is unfair to him, but because it hurts

the country by frustrating the circulation of relevant opinions from which the sovereign people can make unforced judgments.

We have to restore the reputability of dissent. I think such an effort is sufficiently important to justify extreme measures. It was an extreme measure for me to risk jail. I won't enjoy it. But it will be a satisfaction to affirm by this action that I value free speech this highly.

There are 36 other defendents awaiting final disposition of cases involving similar charges. Can it be that all will go to prison? If so the First Amendment does not mean what it says".

Karel deLeeuw

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1960

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K. O. Friedrichs, Chairman
Mark Kac — G. E. Uhlenbeck
M. M. Schiffer — E. P. Wigner

Invitations Committee for the Summer Institute on Finite Groups, 1960

Richard Brauer — R. P. Dilworth
R. H. Bruck — Marshall Hall, Chairman
H. S. M. Coxeter — H. J. Ryser

Invitations Committee for a Symposium on Stability Problems in Hydrodynamics, April, 1960

Richard Bellman — C. C. Lin
Garrett Birkhoff, Chairman
E. Hopf — Brockway McMillan

Committee to Nominate Officers and Members of the Council for 1961

A. A. Albert, Chairman — E. E. Floyd
R. Bott — D. C. Spencer
A. E. Taylor

Committee to Consider By-Laws Concerning Election and Term of Executive Director

S. Bochner — A. H. Taub
J. H. Curtiss — J. W. T. Youngs, Chairman

JOINT COMMITTEES

Committee of the Society and Association on Places of Meetings

R. H. Bruck 1961
Wallace Givens 1962
G. R. MacLane, Chairman 1960

Arrangements Committee for the Summer Meeting at Michigan State University in 1960

H. L. Alder — L. M. Kelly
W. E. Deskins — M. L. Tomber
J. S. Frame — Gordon L. Walker
F. Herzog — C. P. Wells
J. W. T. Youngs

Committee on Employment Opportunities

R. F. Rinehart — A. E. Taylor, Chairman
R. M. Thrall

Joint Committee to Consider the Setting of Winter Meetings

R. H. Bing — G. A. Hedlund, Chairman
J. W. T. Youngs

Term Expires
June 30

Committee on Translations from Russian and other Foreign Languages

H. A. Antosiewicz 1962
Felix Browder 1962
K. L. Chung 1962
Edwin Hewitt, Chairman 1961
E. R. Kolchin 1961
Eugene Lukacs* . . . . . .
Ingram Olkin* . . . . . .

*Appointed by the Institute of Mathematical Statistics

Term Expires
December 31

APPOINTED REPRESENTATIVES OF THE SOCIETY

To the U. S. National Committee on Theoretical and Applied Mechanics

William Prager 1960

On the Council of the American Association for the Advancement of Science

R. L. Wilder 1960
G. B. Price 1960

On the Editorial Board of the Duke Mathematical Journal

A. T. Brauer 1961
A. D. Wallace 1961

On the Board of Editors of the Annals of Mathematics

A. M. Gleason 1960
J. L. Kelley 1960
L. V. Ahlfors 1960

NATIONAL RESEARCH COUNCIL
Division of Mathematics

Officers

S. S. Wilks, Chairman
Harold W. Kuhn, Executive Secretary
J. B. Rosser, Vice Chairman
Paul A. Smith, Past Chairman

Executive Committee

M. R. Hestenes
Deane Montgomery
R. F. Rinehart
J. B. Rosser
J. L. Walsh
S. S. Wilks

Term Expires
June 30

Members of the Division Representing the Society

H. Busemann* 1963
William Feller 1962
K. O. Friedrichs 1960
P. R. Halmos 1961
B. W. Jones* 1963
Irving Kaplansky 1962
N. H. McCoy 1960
C. B. Morrey 1961

* Term starts July, 1960.

THE EMPLOYMENT REGISTER

The Mathematical Sciences Employment Register, established by the American Mathematical Society, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics, will be maintained at the Summer Meeting in East Lansing, Michigan, on August 29 to September 3, 1960. The Register will be conducted from 9:00 A.M. to 5:00 P.M. on August 29 to September 2 and from 9:00 A.M. to 12:00 noon on September 3.

The Employment Register Desk will be located in the Snyder Dormitory, the same building in which the Registration Desk will be maintained. There is no charge for registering to either job applicants or to employers, except when the Late Registration Fee for employers is applicable. Provision will be made for anonymity of applicants upon request.

Job applicants and employers who wish to be listed will please write to the Employment Register, 190 Hope Street, Providence 6, Rhode Island, for application forms and for position description forms, which must be completed and returned to Providence not later than August 5, 1960, in order to be included free of charge in the listings at the meeting in East Lansing. Forms which arrive after this closing date, but before August 22, will be included in the listings at the meeting for a Late Registration Fee of $3.00, and will also be included in the printed listings, but not until ten days after the meeting. The printed listings will be available for distribution both during and after the meeting. The prices are as follows: Position descriptions, $2.00; listing of applicants, academic only, $5.00; comprehensive listing of applicants, academic, industrial, and government, $20.00.

It is essential that applicants and employers register at the Employment Register Desk promptly upon arrival at the meeting to facilitate the arrangement of appointments.

NEW PUBLICATIONS

Ahiezer, A. I., and Beresteckiĭ, V. B. Kvantovaya èlektrodinamika. 2d ed., rev. Moscow, Gosudarstv. Izdat. Fiz.-Mat. Lit., 1959. 656 pp. 26.40 rubles.

Artin, E. Galoissche Theorie. (Mathematisch-Naturwissenschaftliche Bibliothek, 28.) Leipzig, Teubner, 1959. 4 + 86 pp. 5.30 DM.

Asser, G. Einführung in die mathematische Logik. I. Aussagenkalkül. (Mathematisch-Naturwissenschaftliche Bibliothek, 18.) Leipzig, Teubner, 1959. 6 + 184 pp. 11.25 DM.

The axiomatic method. With special reference to geometry and physics. Proceedings of an International Symposium held at the University of California, Berkeley, December 26, 1957 - January 4, 1958. Ed. by L. Henkin, P. Suppes, and A. Tarski. (Studies in Logic and the Foundations of Mathematics.) Amsterdam, North-Holland, 1959. 11 + 488 pp. $12.00.

Bachet, C.-G, Problèmes plaisants et délectables, que se font par les nombres. 5th ed. Rev., simplified and enlarged by A. Labosne. Paris, Blanchard, 1959. 8 + 243 pp. Paperbound: 9 NF.

Bellman, R. Introduction to matrix analysis. New York, McGraw-Hill, 1960. 20 + 328 pp. $10.00.

Beresteckiĭ, V. B. See Ahiezer, A. I.

Bol, G. See Finikow, S. P.

Boole, G. A treatise on differential equations. 5th ed. New York, Chelsea, 1959. 24 + 735 pp.

Brillouin, L. Wave propagation and group velocity. (Pure and Applied Physics, vol. 8.) New York, Academic Press, 1960. 11 + 154 pp. $6.00.

Brittin, W. E. See Lectures in theoretical physics.

Cassels, J. W. S. An introduction to the geometry of numbers. (Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen mit besonderer Berücksichtigung der Anwendungsgebiete, Vol. 99.) Berlin, Springer, 1959. 8 + 344 pp. 64.50 DM; Clothbound, 69 DM.

de Castro Brzezicki, A. Mecanica no-lineal. (Publicaciones del Instituto de Calculo del Consejo Superior de Investigaciones Cientificas. Madrid, Dossat, 1959. 106 pp. 76 pesetas.

The correspondence of Isaac Newton. Vol. I, 1661-1675. Published for the Royal Society. Ed. by H. W. Turnbull. New York, Cambridge University Press, 1959. 38 + 468 pp., 7 plates. $25.00.

Cot, D. Eléments des calculs d'interpolation. (Publ. Sci. Tech. Ministère d l'Air, Bull. Serv. Tech. no. 123.) Paris, 1959. 11 + 140 pp. 4.435 fr.

Debreu, G. Theory of value: an axiomatic analysis of economic equilibrium. (Cowles Foundation for Research in Economics at Yale University, Monograph 17.) New York, Wiley, 1959. 12 + 114 pp. $4.75.

Dedekind, R. Was sind und was sollen die Zahlen? 8th ed. Braunschweig, Vieweg, 1960. 12 + 47 pp. Paperbound, 3.80 DM.

Demidovič, B. P. See Zadači i upražneniya po matematičeskomu analizu dlya vtuzov.

Dunham, L. G. See Lectures in theoretical physics.

Finikow, S. P. Theorie der Kongruenzen. German edition ed. by G. Bol. (Mathematische Lehrbücher und Monographien, II Abt., Bd. X.) Berlin, Akademie-Verlag, 1959. 16 + 491 pp. 56.00 DM.

Flügge, S. See Handbuch der Physik.

Forsyth, A. R. Theory of differential equations. Six volumes bound as three. 1. Exact equations and Pfaff's problem. 2,3. Ordinary equations, not linear; 4. Ordinary linear equations; 5,6. Partial differential equations. New York, Dover, 1959. 13 + 340 pp.; 11 + 344 pp.; 10 + 391 pp.; 16 + 534 pp.; 20 + 478 pp.; 13 + 596 pp. $15.00 the set.

Gnedenko, B. W., and Kolmogorov, A. N. Grenzverteilungen von Summen unabhängiger Zufallsgrössen. German edition ed. by J. Heinhold. (Mathematische Lehrbücher and Monographien, II. Abt., Bd. IX.) Berlin, Akademie-Verlag, 1959. 8 + 279 pp. 44 DM; $10.48.

Golubev, I. F. Vyazkost' gazov i gazovyh smesei (Spravočnoe rukovodstvo). Moscow, Gosudarstv. Izdat. Fiz.-Mat. Lit., 1959. 375 pp. 12.40 rubles.

Grenander, U. See Probability and statistics.

Hadamard, Jacqueline. See Hadamard, Jacques.

Hadamard, Jacques. Essai sur la psychologie de l'invention dans le domaine mathématique. Trans. by Jacqueline Hademard. First French ed., rev. and enlarged by the author. Paris, Blanchard, 1959. 135 pp. Paperbound, 8 NF.

Handbuch der Physik. Bd. 41/1. Kernreaktionen II: Theorie. Ed. by S. Flügge. Berlin, Springer, 1959. 7 + 580 pp. 145.00 DM.

Hardy, G. H. Ramanujan: twelve lectures on subjects suggested by his life and work. New York, Chelsea, 1959. 3 + 236 pp., 1 plate.

Heinhold, J. See Gnedenko, B. W.

Henkin, L. See The axiomatic method.

Hess, S. L. Introduction to theoretical meteorology. New York, Holt, 1959. 14 + 362 pp. $8.50.

Hille, E. Analytic function theory. Vol. 1. Introductions to Higher Mathematics. Boston, Ginn, 1959. 11 + 308 pp. $6.50.

Holbrook, J. G. Laplace transforms for electronic engineers. (International Series of Monographs on Electronics and Instrumentation, Vol. 10.) New York, Pergamon, 1959. 13 + 259 pp. $10.00.

Kac, M. Statistical independence in probability, analysis and number theory. (The Carus Mathematical Monographs, No. 12.) Published by the Mathematical Association of America. New York, Wiley, 1959. 14 + 93 pp. $3.00.

Kamke, E. Differentialgleichungen. Lösungsmethoden und Lösungen. Teil I. Gewöhnliche Differentialgleichungen. 6. Aufl.; Teil II: Partielle Differentialgleichungen erster Ordnung für eine gesuchte Funktion. 4. aufl. (Mathematik und ihre Anwendungen in Physik und Technik, Reihe A. Bd. 18.) Leipzig, Geest and Portig, 1959. I, 26 + 666 pp., 36.80 DM.; II, 15 + 243 pp., 16.00 DM.

Karlin, S. Mathematical methods and theory in games, programming and economics. Vol. I: Matrix games, programming, and mathematical economics. Vol. II: The theory of infinite games. Reading, Addison-Wesley, 1959. Vol. I, 10 + 433 pp., $12.50. Vol. II, 11 + 386 pp. $12.50.

Kazinskiĭ, V. A. Matematičeskie tablicy dlya approksimacii geofizičeskih anomaliĭ i redukciĭ interpolyacionnymi mnogočlenami.(Akademiya Nauk SSSR, Institut Fiziki Zemli.) Moscow, Izdat. Akad. Nauk SSSR, 1959. 90 pp. 5.75 rubles.

Kemeny, J. G., and Snell, J. L. Finite Markov chains. (The University Series in Undergraduate Mathematics.) Princeton, Van Nostrand, 1960. 8 + 210 pp. $5.00.

Kneebone, G. T. See Semple, J. G.

Kolmogorov, A. N. See Gnedenko, B. W.

Kopal, Z. Figures of equilibrium of celestial bodies: with emphasis on problems of motion of artificial satellites. (Publ. No. 3, Mathematics Research Center, United States Army, University of Wisconsin.) Madison, University of Wisconsin Press, 1960. 6 + 135 pp. $3.00.

Kurth, R. Introduction to the mechanics of the solar system. New York, Pergamon, 1959. 9 + 177 pp. $6.50.

Labosne, A. See Bachet, C.-G.

Landau, L. D., and Lifshitz, E. M. Theory of elasticity. (Course of Theoretical Physics, Vol. 7.) Trans. by J. B. Sykes and W. H. Reid. Reading, Addison-Wesley, 1959. 8 + 134 pp. $6.50.

Lectures in theoretical physics, Vol. I. Lectures delivered at the Summer Institute for Theoretical Physics, University of Colorado, Boulder, 1958. Ed. by W. E. Brittin and L. G. Dunham. New York, Interscience, 1959. 7 + 414 pp. $6.00.

Lehmann, E. L. Testing statistical hypotheses. New York, Wiley, 1959. 13 + 369 pp. $11.00.

Levens, A. S. Nomography. 2nd ed. New York, Wiley, 1959. 8 + 296 pp., 2 unbound inserts.

Lifschitz, E. M. See Landau, L. D.

Lowe, P. G. See Novozhilov, V. V.

Lyapunov, A. M. Akademik A. M. Lyapunov; sobranie sočineniĭ. Tom IV. Moscow, Izdat. Akad. Nauk SSSR, 1959. 645 pp., 2 plates. 29.60 rubles.

Menninger, K. Mathematik und Kunst. Göttingen, Vandenhoeck and Ruprecht, 1959. 44 pp., 11 plates. Paperbound, 2.40 DM.

Minrath, W. R. Handbook of business mathematics. Princeton, Van Nostrand, 1959. 12 + 658 pp. $9.85.

Moisil, G. C. Scheme cu comandă directă cu contacte şi relee. [Schemes with direct command with contact relays.] (Comisia de Automatizări, Institutul de Matematici, Monografii Asupra Teoriei Algebrice a Mecanismelor Automate.) Editura Academiei Republicii Populare Romine, 1959. 205 pp., Mimeographed. 9.65 Lei.

Mordell, L. J. Reflections of a mathematician. Montreal, Canadian Mathematical Congress, 1959. 8 + 50 pp., 1 plate.

Neumark, M. A. Normierte Algebren. (Hochschulbücher für Mathematik, Bd. 45.) Berlin, VEB Deutscher Verlag der Wissenschaften, 1959. 572 pp. 48 DM.

Nevanlinna, F., and Nevanlinna, R. Absolute analysis. (Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen mit besonderer Berücksichtigung der Anwendungsgebiete, Bd. 102) Berlin, Springer, 1959. 7 + 259 pp. 36 DM.; clothbound, 39 DM.

Nevanlinna, R. See Nevanlinna, F.

Non-homogeneity in elasticity and plasticity. Symposium, International Union of Theoretical and Applied Mechanics, Warsaw, September 2-9, 1958. Ed. by W. Olszak. New York, Pergamon, 1959. 24 + 529 pp. $15.00.

Novozhilov, V. V. The theory of thin shells. Trans. by P. G. Lowe. Ed. by J. R. M. Radok. Groningen, Noordhoff, 1959. 16 + 376 pp. Paperbound, $9.50; 36.00 Dfl.

Olszak, W. See Non-homogeneity in elasticity and plasticity.

Polak, L. S. See Variacionnye principy mehaniki.

Probability and statistics: The Harald Cramér volume. Ed by U. Grenander. New York, Wiley, 1959. 434 pp. $12.50.

Radok, J. R. M. See Novozhilov, V. V.

Reid, W. H. See Landau, L. D.

Richards, P. I. Manual of mathematical physics. New York, Pergamon, 1959. 11 + 486 pp. $17.50.

Richter, W. Flugmechanik. (Mathematisch-Naturwissenschaftliche Bibliothek, 25.) Leipzig, Teubner, 1959. 6 + 296 pp. 15.60 DM.

Schwartz, L. Théorie des distributions à valeurs vectorielles. Grenoble, Annales de l'Institut Fourier, 1959. 204 pp. 30 Nfr.

Séminaire de mécanique analytique et de mécanique céleste dirigé par Maurice Janet. 2e année: 1958/59. (Faculté des Sciences de Paris.) Paris, Secrétariat mathématique, 1959. 173 pp., mimeographed.

Séminaire de topologie et de géométrie différentielle dirigé par Charles Ehresmann. 1re année: 1957/58. (Faculté des Sciences de Paris.) Paris, Secrétariat mathématique, 1959. 155 pp., mimeographed.

Semple, J. G., and Kneebone, G. T. Algebraic curves. London, Oxford University Press, 1959. 13 + 361 pp. 45 s.; $7.20.

Sideriades, L. Méthodes topologiques appliquées à l'électronique. (Publ. Sci. Tech. Ministère de l'Air, Notes Tech. no. 84.) Paris, 1959. 10 + 136 pp. 3.190 fr.

Širokov, A. P. See Širokov, P. A.

Širokov, P. A., and Širokov, A. P. Affinnaya differencial'naya geometriya. Moscow, Gosudarstv. Izdat. Fiz.-Mat. Lit., 1959. 319 pp. 9.85 rubles.

Smirnov, V. I. Kurs vysšeĭ matematiki. Vol. 5. Moscow, Gosudarstv. Izdat. Fiz.-Mat.-Lit., 1959. 655 pp. 11.10 rubles.

Snell, J. L. See Kemeny, J. G.

Stigant, S. A. The elements of determinants, matrices and tensors for engineers. London, Macdonald, 1959. 11 + 433 pp. 60 s.

Suppes, P. See The axiomatic method.

Swainger, K. Analysis of deformation. Vol. 4. Waves and vibrations. New York, Macmillan, 1959. 27 + 370 pp. $15.00.

Sykes, J. B. See Landau, L. D.

Symposium on the numerical treatment of partial differential equations with real characteristics: Proceedings of the Rome Symposium (28-29-30 January 1959) organized by the Provisional International Computation Centre. Rome, Veschi, 1959. 12 + 158 pp.

Tables numériques des fonctions associées de Legendre: Fonction associée de première espèce $P_n^m(\cos \theta)$. 2d ed. (Collection Technique et Scientifique du Centre National d'Études des Télécommunications.) Paris, Revue d'Optique, 1959. 12 + 639 pp., 3 plates. 5600 fr.

Tables of Whittaker functions (wave functions in Coulomb field), Part 2. (Report No. 11.) Tokyo, Numerical Computation Bureau, 1959. 52 pp. $3.00.

Tarski, A. See The axiomatic method.

Tatarskiĭ, V. I. Teoriya fluktuacionnyh yavleniĭ pri rasprostranenii voln v turbulentnoĭ atmosfere. (Akademiya Nauk SSSR, Institut Fiziki Atmosfery.) Moscow, Izdat. Akad. Nauk SSSR, 1959. 232 pp. 9.30 rubles.

Tietze, H. Gelöste und ungelöste mathematische probleme aus alter und neuer Zeit. 2d rev. ed. Munich, Beck, 1959. Vol. 1, 20 + 256 pp., 10 plates. Vol. 2, 4 + 298 pp., 8 plates. 28.00 DM; clothbound, 36.00 DM.

Tong, K. N. Theory of mechanical vibration. New York, Wiley, 1960. 12 + 348 pp. $9.75.

Turing, S. Alan M. Turing. Cambridge, Heffer, 1959. 14 + 157 pp., 7 plates. 21 s.

Turnbull, H. W. See The correspondence of Isaac Newton.

Vallentine, H. R. Applied hydrodynamics. Toronto, Butterworth, 1959. 8 + 272 pp. $10.00.

Variacionnye principy mehaniki. Ed. by L. S. Polak. Moscow, Gosudarstv. Izdat. Fiz.-Mat. Lit., 1959. 932 pp. 37.25 rubles.

Yakovkin, M. V. Čislennaya teoriya privodimosti mnogočlenov. (Akademiya Nauk SSSR, Vyčislitel'nyĭ Centr.) Moscow, Izdat. Akad. Nauk SSSR, 1959. 135 pp. 5 rubles.

Zadači i upražneniya po matematičeskomu analizu dlya vtuzov. Ed. by B. P. Demidovič. Moscow, Gosudarstv. Izdat. Fiz.-Mat. Lit., 1959. 472 pp. 10.30 rubles.

TWO NEW AMS PUBLICATIONS:

Proceedings of Symposia in Applied Mathematics, Vol. 10
COMBINATORIAL ANALYSIS

311 pages, $7.70 Richard Bellman and Marshall Hall, Jr., Editors

Problems involving the choice of a specified set of objects from a finite set of objects are usually called "combinatorial." Although there is seldom any theoretical difficulty concerning the existence of the desired subset, there is almost always an enormous amount of difficulty connected with the actual determination of this subset.

Questions of this type arise in all parts of mathematics, in algebra, geometry and analysis. In general, any advance in feasible procedures requires a major theoretical advance. Quite often, the physical background of the mathematical problem suggests a new mathematical approach to a longstanding problem.

This volume contains a variety of papers devoted to combinatorial problems arising in the mathematical areas cited above, and in the new fields of scheduling, transportation, communication and de-

sign of experiments. A number of papers are devoted to the use of digital computers to attack the fantastically complicated problems which arise from apparently simple physical problems.

Not only will this volume furnish a valuable survey of the current state of the art, but it will also provide a source for research in this vast and growing field.

Memoir 36
THE CLASSIFICATION OF G-SPACES

iv + 72 pp. $1.65 by R. S. Palais

Let G be a compact Lie group. A G-space is a completely regular space X together with a fixed action of G as a group of homeomorphisms of X. There has been a fruitful tendency in recent years to regard G-space as a generalization of principal G-bundles and to adapt concepts and techniques that have been successful in studying principal bundles. For example, the concept of a local cross-section has been directly generalized to the motion of a slice. The first part of this Memoir is an expository account of the theory of G-spaces with this point of view in mind. In the second part of the Memoir the well-known classification theorem for principal G-bundles with base space Y is generalized to give a classification of G-spaces with a given orbit structure Y. As in the principal case the classification is by homotopy classes of maps of Y into a classifying space and the latter is the orbit structure of a "universal" G-space, examples of which are constructed explicitly.

RECENT REPRINTS OF COLLOQUIUM PUBLICATIONS

Volume 20
INTERPOLATION AND APPROXIMATION BY RATIONAL FUNCTIONS IN THE COMPLEX DOMAIN

398 pp. $9.90 by J. L. Walsh

Originally published in 1935, some corrections were made and an appendix added in 1952. The present edition contains added references and additional corrections.

Volume 25
LATTICE THEORY

283 pp. $6.30 by Garrett Birkhoff

Published in 1940; enlarged and completely revised in 1948.

At present the author is preparing another revision of this book which will be published early in 1963.

"Lattice Theory has now become recognized as a substantial branch of modern algebra, and this revised edition of what was, we believe, the first volume devoted to the subject, will be of first importance to all students and mathematicians everywhere."

from BOOK EXCHANGE, August, 1949, p. 22

Volume 5, Part 2
ANALYSIS SITUS

194 pp. $5.30 - Published in 1922 by Oswald Veblen

An elegant and rigorous account of n-dimensional manifolds, originally presented as the Cambridge Colloquium Lectures of 1916, printed in Colloquium Series in 1922, revised in 1931, and frequently reprinted. It remains in constant demand as the first systematic treatment, still of great value, of the theory of polyhedra, the basis for a definition of a manifold.

Volume 9
DYNAMICAL SYSTEMS

295 pp. $6.00 - Published in 1927 by G. D. Birkhoff

ACTIVITIES OF OTHER ASSOCIATIONS

THE APRIL MEETING IN NEW YORK, NEW YORK

April 14-16, 1960

567-80. A. A. Albert and D. R. Hughes: On the André Veblen-Wedderburn systems.

The class of Veblen-Wedderburn (V-W) systems known as André V-W systems give finite non-Desarguesian projective planes, and the finite members of this class have been the only finite planes with undetermined collineation groups. It was known that all but finitely many finite near-fields are André V-W systems, and it is shown here that all finite Hall V-W systems give planes which are also André V-W planes (only one Hall V-W system is also an André V-W system, however). Furthermore, we determine the complete collineation group of any finite André V-W plane and show that the group is always solvable, excepting exactly for the André V-W planes which are also Hall V-W planes. (Received March 7, 1960.)

567-81. R. C. Bose and Sarvadaman Chowla: Theorems in the additive theory of numbers.

This paper generalizes earlier results by Erdös and Turan (J. London Math. Soc. (1941) pp. 212-215) and S. Chowla (Proc. Nat. Acad. Sci. Ind. Sec. A (1944) pp. 1-2). The following theorems are proved. Theorem 1. If m is a prime-power we can find m nonzero integers (less than m^r), $d_1 = 1, d_2, \ldots, d_m$ such that the sums $d_{i_1} + d_{i_2} + \ldots + d_{i_r}$, $1 \leqq i_1 \leqq i_2 \ldots \leqq i_r \leqq m$ are all different (mod $m^r - 1$). Theorem 2. If m is a prime-power we can find m + 1 integers (less than q) $d_0 = 0$, $d_1 = 1, d_2, \ldots, d_m$ such that the sums $d_{i_1} + d_{i_2} + \ldots + d_{i_r}$, $0 \leqq i_1 \leqq i_2 \leqq \ldots \leqq i_r \leqq m$ are all different (mod q), where $q = (m^{r+1} - 1)/(m-1)$. A B_r set can be defined as a set of non-negative distinct integers, $d_1, d_2, \ldots$ such that the sums $d_{i_1} + d_{i_2} + \ldots + d_{i_r}$ $(i_1 \leqq i_2 \leqq \ldots \leqq i_r)$ are all different. If $F_r(x)$ denotes the maximum number of members that a B_r set can have we prove: Theorem 3. If m is a prime power and r is a positive integer, $F_r(m^r) \geqq m + 1$; $F_r(1 + q) \geqq m + 2$, where $q = (m^{r+1} - 1)/(m - 1)$. Theorem 4. $\liminf F_r(y)/y^{1/r} \geqq 1$ as $y \longrightarrow \infty$. We may conjecture that $\limsup F_r(y)/y^{1/r} \leqq 1$, $y \longrightarrow \infty$, for any positive integer r. This was proved by Erdös and Turan for $r = 2$ in the paper referred to. (Received March 2, 1960.)

567-82. Sarvadaman Chowla: A formula for the fundamental unit of a real quadratic field whose class number is 1.

With the usual notation (Ankeny, Artin, Chowla in Ann. of Math., 1952) it is proved that if $p \equiv 1 \pmod 4$, then I. $(t/2)(-1)^{(h+1)/2} \equiv ((p-1)/2)! \pmod p$. II. If $h = 1$, "u" and "t" are given as the unique solutions of a linear congruence $(\mathrm{mod}\ 2^p - 1)$: $uA \equiv +(A^2 + B^2)/AB \pmod{2^p - 1}$. Here (r and n run resp. over quadratic residues and non-res. between 0 and p) $A = \sum_r 2^r - \sum_n 2^n$, $B = \prod_n (2^n - 1)$. Since A and B are prime to $2^p - 1$ and $0 < u < 2^p - 1$, our "u" is determined uniquely. For the primes $p < 10{,}009$ which are $\equiv 1 \pmod 4$, Schaffstein [Math. Ann. vol. 97 (1928)] showed that $h = 1$ in more than half the cases. Kronecker found a formula analogous to I for primes $p \equiv 3 \pmod 4$. (Received March 2, 1960.)

567-83. Sarvadaman Chowla and F. B. Correia: The falsity of a certain asymptotic relation for the product of the first n primes.

In the booklet, Primzahlen, Band II, by Ernst Trost, published by Verlag Birkhauser, Basel-Stuttgart, 1953, on page 56 is a statement concerning the asymptotic relation of the product of the first n primes. The statement was suspect because its acceptance together with some newly derived results by F. B. Correia implied the truth of Cramer's conjecture concerning the difference of consecutive primes. The statement by Ernst Trost was then proved to be false by an indirect proof. The assumption of the statement by Ernst Trost led to the result that the limit of the quotient $d_n/\log p_n$ is equal to one, where d_n is the difference of consecutive primes and p_n is the nth prime. A contradiction is thus established since it is known that the limit superior of the same quotient is unbounded. For a reference see Karl Prachar; Primzahlverteilung; Springer-Verlag, Berlin-Gottingen-Heidelberg, 1957, page 157. (Received February 29, 1960.)

567-84. G. A. Coon and D. L. Bernstein: A class of boundary value problems for hyperbolic partial differential equations with constant coefficients.

Given an equation $P(D)U = F(x,y)$, where P(D) is a homogeneous differential operator with constant coefficients, the problem considered is whether a solution can be found, in the entire first quadrant, which assumes, together with certain of its partial derivatives, given values on the coordinate

axes. Using double Laplace transforms, it is shown that the boundary values must satisfy certain compatibility conditions, when the characteristic equation has real roots (either simple or multiple), so that only certain boundary value problems can be solved in general. Existence and uniqueness theorems are proved under fairly general hypotheses on the given functions, and explicit formulas are obtained for the solutions. (Received March 2, 1960.)

567-85. Eckford Cohen: On the inversion of even functions of finite abelian groups (mod H).

Let X denote the semigroup of finite abelian groups and let H be a group of X. A function f(G,H), defined for all $G \in X$, is called even (mod H) if $f(G,H) = f(D,H)$, where D denotes the greatest common direct factor of G and H in X. In this paper, inversion theories of both Fourier and Möbius type are developed for this class of functions. The results obtained are analogous to those proved previously for even functions of integers; however, the methods are quite different. In the case of the integers, the Fourier theory was developed on the basis of trigonometric sums, while the Möbius theory was treated independently along arithmetical lines. A unified theory is developed in this paper. First the Fourier theory is treated by purely arithmetical methods, and then the Möbius theory is shown to result as a natural consequence. The proof of one of the main results is based on an analogue of the Brauer-Rademacher identity; a new proof of the latter identity is included. This paper is preliminary to a study of a much wider class of functions defined in X. (Received March 16, 1960).

567-86. L. M. Court: Certain generalizations of Schwartz's inequality, etc.

Several generalizations of Schwartz's and kindred inequalities, potentially useful in analysis, are made both for integrals and the case of the analogous finite sums. E.g. the formula $\{\sum_{i=1}^{n} y_i\}^2 \leq n \sum_{i=1}^{n} y_i^2$ is obtained (note that the inequality would not be valid if the n factor were dropped). The integral analogue of this inequality often enables us to set upper or lower bounds on integrals that are formally recalcitrant: thus, $\int_1^{\alpha} x^{-2}(\log x)^2 dx \geq (\alpha - 1)^{-1}\{\int_1^{\alpha} x^{-1}\log x\, dx\}^2 = (\log \alpha)^4/4(\alpha - 1)$; $\int_{\alpha}^{\beta} dx\, (h + x^r)^{1/2} \leq \{h(\beta - \alpha)^2 + (\beta - \alpha)(\beta^{r+1} - \alpha^{r+1})/(r+1)\}^{1/2}$; etc. Other generalizations are developed. (Received March 16, 1960.)

567-87. Jane Cronin-Scanlon: An upper bound for the number of periodic solutions of a perturbed system.

Consider the n-dimensional system (1) $\dot{x} = Ax + \mu f(x,t,\mu)$ where A is a constant matrix; the components of f have period 2π; and μ is a small parameter. An upper bound for the number of distinct periodic solutions (of period 2π) of (1) is obtained for the degenerate case. If the components of f are polynomials in the components of x; if m is the maximum degree of these polynomials; if q is the degree of degeneracy of (1) (i.e., q is the number of linearly independent periodic solutions of $\dot{x} = Ax$); and if a function h(t), arbitrarily small, is added to $f(x,t,\mu)$; then the upper bound is m^q. (Received February 24, 1960.)

567-88. Herbert Federer: Approximation of integral currents by cycles.

If A is a compact Lipschitz neighborhood retract in R^n, then there exists a number q with the following property: For each k + 1 dimensional integral current T of A there is an integral cycle S of A such that $M(T - S)^{k/(k+1)} \leqq qM(\partial T)$; here "M" means "mass". A more general theorem involving pairs and relative cycles is also proved. (Received March 1, 1960.)

567-89. Günther Goes: Characterization of the space of bounded generalized Fourier coefficients.

If $\overset{\circ}{f} = \sum_{k=-\infty}^{\infty} c_k e^{ikx}$, $F \sim \overset{\circ}{F} = \sum_{k\neq 0;k=-\infty}^{\infty} (ik)^{-1} c_k e^{ikx} \in L_2$, $\sigma_n(\overset{\circ}{f}) = \sum_{k=-n}^{n} (1 - |k|/(n+1))\, c_k e^{ikx}$, $g \sim \overset{\circ}{g} = \sum_{k=-\infty}^{\infty} d_k e^{ikx}$, $g * \overset{\circ}{f} = \sum_{k=-\infty}^{\infty} c_k d_k e^{ikx}$ and E, E' are spaces between the space C (coefficients of continuous functions) and the space dV (Fourier-Stieltjes coefficients), then the space of $\overset{\circ}{f}$ with $\sup_k |c_k| < \infty$ (bounded generalized Fourier coefficients) can be defined by any one of the following five equivalent properties: (1) For any system I_n of disjoint intervals $[a_i^n, b_i^n]$ $(i = 1,2,\ldots,m_n;\ n = 1,2,\ldots)$ contained in $[0,2\pi]$, $\sup_{I_n} \int_0^{2\pi} |\sum_{i=1}^{m_n} \{F(t + b_i^n) - F(t + a_i^n)\}|^2\, dt < \infty$. (2) If E' is a Banach space, $L_2 \subseteq E' \subseteq dV$ and for $g \in L_2$ the norm $\|g\|_2 = (\int_0^{2\pi} |g|^2 dt)^{1/2}$, then $\sup_n \sup_{\|g\|_2 \leqq 1} \|g * \sigma_n(\overset{\circ}{f})\|_{E'} < \infty$. (3) If E is a Banach space, $C \subseteq E \subseteq L_2$, then $\sup_n \sup_{\|g\|_E \leqq 1} \|g * \sigma_n(\overset{\circ}{f})\|_2 < \infty$. (4) If $L_2 \subseteq E' \subseteq dV$, then $g * \overset{\circ}{f} \in E'$ for every $g \in L_2$. (5) If $C \subseteq E \subseteq L_2$, then $g * \overset{\circ}{f} \in L_2$ for every $g \in E$. (Received February 26, 1960.)

567-90. Karl Goldberg: Hadamard matrices of order cube plus one.

Let H be an Hadamard matrix of order h ($H = (\pm 1)$, $HH^T = hI$), satisfying $H + H^T = 2I$. Then there exist Hadamard matrices of orders 2h, h(h - 1), $(q^a + 1)h$ (where q is a prime of the form 4n - 1 and a is odd), and $(h - 1)^3 + 1$, with the same property. This last result is new. (Received February 26, 1960.)

567-91. G. H. Golub and R. S. Varga: Chebyshev semi-iterative methods, successive overrelaxation iterative methods, and second order Richardson iterative methods.

The three iterative methods of the title are applied to the numerical solution of the matrix equation $\vec{x} = B\vec{x} + \vec{g}$, where B is an n X n symmetric and convergent matrix. For the case where B is primitive, the three iterative methods are formulated in a way which makes them very similar, and it is shown, using the spectral norm of the various iteration matrices as a basis for comparison, that the Chebyshev semi-iterative method gives rise to the smallest spectral norm. For the case where B is cyclic (property (A) is the sense of Young), a new iterative method, the modified Chebyshev semi-iterative method, is formulated, and it is shown, again using spectral norms as a basis of comparison, that this new iterative method gives rise to a smaller spectral norm than do the successive over-relaxation iterative method and its recent modification by Sheldon [J. A. C. M. vol. 6 (1959) pp. 494-505]. (Received February 26, 1960.)

567-92. Bruno Harris: Cohomology of Lie triple systems.

A cohomology theory is developed for Lie triple systems: These are subspaces T of a Lie algebra closed under the triple product $[[xy]z]$ (and are definable by identities); equivalently, T is the subspace of those elements t of a Lie algebra L with automorphism A of period 2 satisfying $A(t) = - t$. A T-module M is defined by a composition $[t_1[t_2m]]$ for t_i in T, m in M. We define the cohomology groups of T with coefficients in M as the cohomology groups of a certain supplemented associative algebra E (universal enveloping algebra of T) with coefficients in an E-module N(M) (enveloping module of N). The first and second cohomology groups are isomorphic to the group of derivations modulo inner ones, and of factor sets modulo trivial ones, respectively;

for finite-dimensional semi-simple T of characteristic zero these two groups are zero — a result of Lister (A structure theory of Lie triple systems Trans. Amer. Math. Soc. vol. 72 (1952) pp. 217-242), the analogue for Lie triple systems of the first and second Whitehead lemmas for Lie algebras. These cohomology groups are distinct from the relative cohomology groups of the Lie algebra L relative to the subalgebra of fixed points of the automorphism A. (Received March 2, 1960.)

567-93. D. G. Johnson: A structure theory for a class of archimedean lattice-ordered rings. Preliminary report.

For definitions and previous results, see Abstracts 553-64, 559-91, and 564-156, Notices Amer. Math. Soc., vols. 5,6. Let A denote an archimedean f-ring containing no nonzero nilpotent elements, $\mathfrak{X}_A$ the totality of maximal ℓ-ideals of A such that, for some $\bar{a} \in A/M$ and some positive integer n, $(n+1)\bar{b} \leqq \bar{a}\bar{b} \leqq (n+2)\bar{b}$ for each positive $\bar{b} \in A/M$. Then $\bigcap\{M:M \in \mathfrak{X}_A\} = \{0\}$, and the hull-kernel topology for $\mathfrak{X}_A$ is locally compact and Hausdorff. We let $\mathfrak{X}_A$ denote this topological space. If A* denotes the bounded elements of A, then $\mathfrak{X}_A$ and $\mathfrak{X}_{A^*}$ are homeomorphic. Theorem 1: A is isomorphic to an f-ring $\bar{A}$ of continuous functions f on $\mathfrak{X}_A$ into the two-point compactification of R such that the set on which f is real-valued is everywhere dense. Moreover, $\bar{A}^*$ separates points and closed sets. This representation yields internal characterizations of the f-algebras $C_s(\mathfrak{X})$ [resp., $C_\infty(\mathfrak{X})$] of all continuous real-valued functions with compact supports [resp., "vanishing at ∞"] on a locally compact Hausdorff space $\mathfrak{X}$. Moreover, the representation of Theorem 1 yields, in a natural way, Theorem 2: An archimedean f-ring A is isomorphic to a sub-f-ring of an archimedean f-ring with ring unit element if (and only if) A contains no nonzero nilpotent elements. (Received February 29, 1960.)

567-94. Erwin Kreyszig: Bergman operators and coefficient problems for harmonic functions in three variables.

Bergman's operator method permits the translation of results about analytic functions of a complex variable into theorems on solutions of linear partial differential equations; concerning equations in two independent variables see e.g. Bergman (Mat. Sb. vol. 2 (1937) p. 1169) and Kreyszig (J.Math. Mech. vol. 6 (1957) p. 361). The present paper deals with relations between coefficients of developments of harmonic functions H of three real variables in

terms of spherical harmonics and singularities of the functions H. Bergman (J. Rat. Mech. Anal. vol. 3 (1954) p. 539) considered subsequences $(A_{a_n b(a_n)})$ of coefficients in the development $H = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} A_{nm} \Gamma_{nm}$ where the Γ_{nm} are essentially spherical harmonics. In certain cases he establishes conditions in order that $H_b = \sum_{n=0}^{\infty} A_{a_n b(a_n)} \Gamma_{a_n b(a_n)}$ is singular along certain curves. In the present paper it is shown that this approach can be generalized in various directions. A systematic study is made of functions a_n, $b(a_n)$ for which the resulting function H_b is the sum of an entire function H_e and a singular function H_s which can be represented in a closed form. In this way the geometric shape of the curves of singularities of H_b and the behavior of H_b in a neighborhood of these curves can be determined. (Received February 25, 1960.)

567-95. Erwin Kreyszig: Meromorphic associated functions for Bergman operators generating harmonic functions of three variables.

Using the methods described in the previous abstract, conditions are obtained under which the associated function of H in the Bergman integral operator is of the form (1) $B_{mk} \xi^m (u\xi^k - a)^{-m} (k = -1, 0, 1)$ or a sum of such functions. It is shown that in this case the resulting harmonic functions are algebraic functions which can be written in explicit form, and the curves of singularities of these functions H_s are determined. Furthermore certain general properties of the level surfaces of H_s are obtained. This includes certain results by Bergman (J. Rat. Mech. Anal. vol. 3 (1954) p. 539) as special cases. The further consideration is concerned with harmonic functions H_b having infinitely many algebraic curves of singularities and corresponding to an associated function which is an infinite series of terms of the form (1) multiplied by suitable convergence-generating factors. Using Bergman's operator method conditions for the coefficients B_{mk} in order that the family of those curves have a limit curve, are obtained, and upper bounds for the harmonic function in the neighborhood of the limit curve are determined. (Received February 25, 1960.)

567-96. Sibe Mardešić: Locally connected, ordered and chainable continua.

By a continuum we mean a Hausdorff compact and connected space; an ordered continuum is a set C provided with a total ordering such that C is

a continuum under the induced order topology; a continuum is chainable (stronglychainable) provided that each open covering admits a chain refinement (having connected links). Theorem 1. Each locally connected continuum X is the inverse limit of an inverse system $\{X_\alpha; \pi_{\alpha\alpha'}\}$, where X_α are Peano continua and $\pi_{\alpha\alpha'}$ are monotone maps; also the converse is true. Corollary. Each ordered continuum X is the inverse limit of an inverse system $\{I_\alpha; \pi_{\alpha\alpha'}\}$, where all I_α are arcs and $\pi_{\alpha\alpha'}$ are monotone maps; also the converse is true. Theorem 1 follows from Theorem 2. Let X and Y be two continua and f: $X \longrightarrow Y$ a light mapping onto Y. If X is locally connected then X and Y have equal weights. A consequence of Theorem 1 is Theorem 3. These three classes of spaces coincide: (a) ordered continua, (b) locally connected chainable continua, (c) strongly chainable continua. (Received February 24, 1960.)

567-97. Martin Schechter: Negative norms and boundary problems. I

Elliptic boundary problems are defined and solved for generalized functions (distributions). This is made possible by inequalities involving negative norms. Let A be an elliptic partial differential operator of order 2r in a bounded domain G in E^n. Let $\{B_j\}_{j=1}^r$ be a normal set of differential boundary operators which cover A (for definitions cf. Comm. Pure Appl. Math. vol. 12 (1959) pp. 457-486). For any function $v \in C^\infty(\bar{G})$ and integer $t > 0$ let $\|v\|_t$ be the sum of the $L^2(G)$ norms of all derivatives of v of orders $\leqq t$ and set $\|v\|_{-t} = \text{lub}|(v,w)|\cdot\|w\|_t^{-1}$, where $(v,w) = \int_G v\bar{w}dx$ and the lub is taken over all $w \in C^\infty(\bar{G})$. For functions g defined on the boundary ∂G of G set $\langle g\rangle_{-t} = \text{lub}\int_{\partial G} g\bar{w}d\sigma\|w\|_{t+1}^{-1}$. Then under suitable regularity assumptions, for every integer $s > 0$ there is a constant C_s such that $\|u\|_{-s} \leqq C_s(\|Au\|_{-2r-s} + \sum_{j=1}^r \langle B_j u\rangle_{-m_j-s} + \|u\|_{-2r-s})$ for all $u \in C^\infty(\bar{G})$. Stronger inequalities can also be proved. (Received February 25, 1960.)

567-98. Martin Schechter: Negative norms and boundary problems. II.

Employing the notation of the preceding abstract, let $H^t(G)$(resp. $H^{-t}(G)$) be the completion of $C^\infty(\bar{G})$ with respect to the $\|\ \|_t$ (resp. $\|\ \|_{-t}$) norm. (u,v) can be defined for $u \in H^{-t}(G)$, $v \in H^t(G)$ and satisfies $|(u,v)| \leqq \|u\|_{-t}\|v\|_t$. For $f \in H^{-2r-s}(G)$, an element $u \in H^{-s}(G)$ is a solution of (1) $Au = f$ in G, $B_j u = 0$ on ∂G, $1 \leqq j \leqq r$, if there is a sequence $\{u_k\} \subseteq C^\infty(\bar{G})$ such that $\|u_k - u\|_{-s} \longrightarrow 0$, $\|Au_k - f\|_{-2r-s} \longrightarrow 0$. Under the assumptions made in the preceding abstract,

it is proved that (1) has a solution if, and only if, $(f,w) = 0$ for all solutions $w \in C^{\infty}(\bar{G})$ of a null adjoint problem of the form $A^*w = 0$ in G, $B_j^!w = 0$ on ∂G, $1 \leqq j \leqq r$, where A^* is the formal adjoint of A. Moreover, if $f \in H^{-2r-s}(G)$, $u \in H^{-s-t}(G)$, and $(u,A^*(v)) = (f,v)$ for all $v \in C^{\infty}(\bar{G})$ satisfying $B_j^!v = 0$ on ∂G, $1 \leqq j \leqq r$, then actually $u \in H^{-s}(G)$ and is a solution of (1). If $f \in H^p(G)$, then $u \in H^{2r+p}(G)$, and if p is large enough u is a classical solution of (1). Other interesting results and strengthened versions of these are also obtained. (Received February 25, 1960.)

567-99. P. M. Swingle and R. L. Kelley: Connected semigroups with n-fold set of indecomposability.

The following is proved: Let S be a connected semigroup with n-fold set of indecomposability; let $\bar{S}$ be compact and the multiplication operation m be extendable to $\bar{S}$. Let C and Y be connected subsets of S. Then, for $y \in Y$, either every $y\bar{C}$ is region-containing (i.e. its interior with respect to S is non-null) or every $y\bar{C}$ is not region-containing; thus, for $s \in S$, either every $s\bar{S}$ is region-containing or every $s\bar{S}$ is not; hence if S has a zero no $s\bar{S}$ is region-containing and, if S has a unit, every $s\bar{S}$ is region-containing. Also, if $\bar{S}$ is a continuum with set Z of indecomposability, then $\bar{S}$ is a topological group. (Received February 25, 1960.)

567-100. John Wermer: Runge domains.

A domain D in the space C^n is called a Runge domain if each function analytic in D can be approximated by polynomials uniformly on all compact subsets of D. We construct in C^3 a domain R with the following properties: (i) R is a bounded domain of holomorphy; (ii) R is analytically equivalent to the polycylinder $|z_i| < 1$, $i = 1,2,3$, i.e. there exists an analytic homeomorphism mapping the polycylinder on R; (iii) R is not a Runge domain. The situation in C^3 thus differs from the corresponding situation in C^1 where every domain analytically equivalent to the monocylinder $|z| < 1$ is a Runge domain. (Received February 26, 1960.)

567-101. D. J. Hillman: On inflection functions.

Proto-syntactic analysis of English sentences, which recognizes the verbal or nominal character of word-sequences and imposes stations around them, is here distinguished from micro-syntactic analysis, which describes

the composition of phrases within these stations. This paper is concerned with the micro-syntax of the English verb-phrase. The analysis is undertaken to examine the behavior of transformations that affect the composition of the verb-phrase. It is shown that certain well-known transformations have restricted applicability and that new transformations are constructible in terms of micro-syntactic analysis. A technique is proposed whereby the verb-phrase is regarded as the result of a series of inflection-functions acting upon a main verb and its auxiliaries. The possibilities of transformation affecting the verb-phrase now become the possibilities of permuting and combining functions and arguments in the verb-phrase, preserving well-formedness. A scheme is described for representing the most basic of these inflection-functions, viz., the tense-function. Starting from two auxiliaries of conjugation and four primitives of tense-formation, rules are set up for producing all tenses of a given verb in terms of concatenated sequences of functions and their arguments. (Received March 25, 1960.)

567-102. D. J. Hillman: On defining linguistic modalities.

Some brief comments are first made concerning logical modalities and their treatment in Transformational Analysis. The nature of linguistic modalities is then described. Two problems are selected for discussion. The first concerns the analysis of modal auxiliaries in sentence-centers. The modally qualified verb-phrase is represented as a sequence of functions and their arguments containing a modal inflection-function. A technique of factorization is then described, whereby any element, or combination of elements, that can be extracted from a given sentence-center and relocated in the introductory clause of a factorization of that sentence-center may be regarded as a modality. Some conditions governing the admissibility of this procedure are formulated, and questions concerning the redistribution of inflection-functions under this factorization are discussed. A general rule is proposed, making the modal function of an element correspond to its extractability under factorization. The second problem concerns the description of a special class of modal auxiliaries. The members of this class may be enumerated by applying a special type of transformation, known as a lengthening transformation, to a set of kernel sentences. The theory of lengthening transformations is related to the theory of n-termed predicates in symbolic logic. (Received March 25, 1960.)

567-103. Iwao Sugai: On Sturm-Liouville equations.

Recently a second order nonlinear ordinary differential equation was solved (to be published in Proceedings of the IRE, Correspondence Section). For an unknown function y(x), $(r/y')' = -p/y$ was solved. Primes are used for ordinary derivatives with respect to x. The general Sturm-Liouville equation for W(x) is expressed by $(RW')' = -PW$. Equating both sides of these two equations separately, variable coefficients r, p, R and P are properly related. Therefore, W(x) is expressed in terms of y(x) and these inter-related variable coefficients whose arguments may include parameter λ, (eigen value) besides x. (Received April 5, 1960.)

567-104. M. J. Greenberg: Schemata over local rings.

Every Artin local ring R with algebraically closed residue field k can be made into an algebraic ring variety over k in a natural way. Reasons: In characteristic 0, R becomes a finite dimensional k-algebra. If k has characteristic $p > 0$, R is canonically a finitely generated module over a ring of Witt vectors. Then every schema of finite type over R (in the sense of Grothendieck) can be "realized" by an algebraic variety over k. As application, the rational points of any schema of finite type over a complete local ring with residue field k form the projective limit of algebraic varieties over k. (Received April 8, 1960.)

567-105. A. C. Morel: An algebra isomorphic to the ordinals under addition.

A relation R is homogeneous if given any two elements of the field of R there is an automorphism taking one to the other. Let HS be the class of homogeneous scattered order types. For ϕ an ordinal, $(\omega^* + 1 + \omega)_0^{\phi}$ is the type, defined by Hausdorff, of the anti-lexicographically ordered relation whose field consists of all sequences $\langle n_0,\ldots,n_\iota,\ldots\rangle$, with $\iota < \phi$, the n_ι's being integers and all but finitely many being 0. Theorem 1. For every type α, $\alpha \in$ HS iff there is a (unique) ordinal ϕ such that $\alpha = (\omega^* + 1 + \omega)_0^{\phi}$. Now let GS be the class of types of all scattered relations R such that, for some simply ordered group $\mathcal{G}$, R is the ordering of $\mathcal{G}$. Theorem 2. GS = HS. Finally, let $\cdot$ and $+$ denote ordinal multiplication and ordinal addition resp. Theorem 3. The algebras $\langle$HS, $\cdot\rangle$ and $\langle$Ordinals, $+\rangle$ are isomorphic. (Received February 29, 1960.)

567-106. E. T. Parker: Nonexistence of a type of complete sets of mutually orthogonal latin squares. Preliminary report.

In Abstract 567-4, these NOTICES, vol. 7 (1960) p. 208, Construction of mutually orthogonal latin squares, Mendelsohn, Dulmage, Johnson, and the author report construction of sets of 5 mutually orthogonal latin squares (m.o.l.s.) of order 12, all squares being the noncyclic abelian group with rows permuted. It is currently an open question whether there exists a complete set of m.o.l.s. of order 12 of this type. However, for infinitely many orders, not among those for which planes are excluded by the Bruck-Ryser theorem, complete sets of m.o.l.s., all an abelian group with rows permuted, can be shown nonexistent. (1) For all orders $4t + 2$, it is well known that a group table has no transversal — relevant except for order 2. (2) Existence of such a complete set of m.o.l.s. of order n implies (but not conversely) existence of an $n \times n$ matrix A of pth roots of unity (p a prime divisor of n) such that $AA' = nI$ (prime denoting complex conjugate transpose). Thus for odd n there must exist (fractional) x in the field of pth roots of unity such that $x\bar{x} = n$. The first order excluded by this criterion is 15. (Received February 29, 1960.)

567-107. Robert Hermann: Totally geodesic orbits of the isotropy group of an irreducible compact symmetric space.

Let $\underline{G}$ be a compact, simple Lie algebra $\sigma: \underline{G} \to \underline{G}$ on involutive automorphism, $\underline{K}$ the subalgebra of fixed elements of $\underline{G}$. Let G be the group of inner automorphisms of $\underline{G}$, $\sigma: \underline{G} \to G$ the corresponding automorphism, K' the subgroup of elements of G fixed under σ, K the connected component of the identity of K'. Let $M = G/K'$ be the symmetric Riemannian space of right cosets of K'. Theorem: The orbits of K on M that are totally geodesic in M are in one-one correspondence with the elements of order 2 of K', two such elements considered the same if they are conjugate under Ad K. (Received April 14, 1960.)

568-32. S. D. Chatterji: Almost everywhere convergence of Banach-valued strong martingales.

In continuation of the study of Banach-valued martingales, the mean convergence theorems which were presented at the December meetings of the Institute of Mathematical Statistics held in Washington, D. C. it is reported that several almost everywhere convergence theorems have been proved by using a generalization of a theorem of Banach and quite independently of the classical theory of martingales. Amongst these theorems is the following analogue of a classical theorem: Any uniformly integrable Banach-valued martingale converges almost everywhere and in the mean, if the Banach-space considered is reflexive. An example has been constructed to show that no kind of convergence is to be expected unless the Banach-space in which the r.v.'s take values is suitably restricted. (Received February 29, 1960.)

568-33. P. A. Clavier: Toward a product for distributions

It is shown that the product of distributions proposed by Guttinger and Gross does not possess any of the common properties of the product and leads to impossibilities when applied in theoretical physics. After imposing a restriction on the definition of the derivative of a distribution and introducing distributions multiplied by infinite weights, one can define a product for distributions which does possess the common properties of the product and also does not contradict available results in theoretical physics. Conclusions reached by König are circumvented by the restriction on the derivative. (Received February 26, 1960.)

568-34. Erwin Kreyszig: Operator methods for a class of linear partial differential equations.

There exists a Bergman integral operator $\phi(X) = p_3(H) \equiv H(X) + \int_0^1 B(r^2,t)H(t^2X)dt$, $X = (x,y,z)$, which transforms harmonic functions H of three variables into solutions ϕ of the equation (1) $\phi_{xx} + \phi_{yy} + \phi_{zz} + F(r^2)\phi = 0$ where F is an entire function of $r^2 = x^2 + y^2 + z^2$; cf. Bergman (Trans. Amer.

Math. Soc. 68 (1950) p. 461) and (Duke Math. J. vol. 13 (1946) p. 419). This operator has the property that the associated function of the solution ϕ in the characteristic space $x = 2(Z Z^*)^{1/2}$, $Z = (z + iy)/2$, $Z^* = (z - iy)/2$ coincides with the associated function of H, namely $H(2(ZZ^*)^{1/2},Z,Z^*) = \phi(2(ZZ^*)^{1/2},Z,Z^*)$. Due to this fact theorems about solutions of (1) can be obtained from results about harmonic functions of three variables, and methods used for harmonic functions in connection with integral operators can be extended to solutions of (1). For example, let ϕ be given by a development of the form (2) $\phi = \sum\sum\sum A_{nmp} x^n Z^m Z^{*p}$. Then, by introducing the particular solutions $\phi_{mn} = p_3(H_{mn})$ where $H_{mn} = (2\pi i)^{-1} \int_{|\zeta|=1} (Z\zeta + x + Z^*\zeta^{-1})^m \zeta^{n-1} d\zeta$ the series (2) may be written $\phi = \sum\sum B_{mn}\phi_{mn}(x,Z,Z^*)$, and, in analogy to the methods used in the case of harmonic functions, solutions of the form (3) $\phi^* = \sum_{n=0}^{\infty} B_{a_n b(a_n)} \phi_{a_n b(a_n)}$ may be considered. In this way it is possible to determine the growth and other properties of $|\phi^*|$ in the neighborhood of points at which the corresponding harmonic function is singular. (Received February 25, 1960.)

568-35. A. J. Lohwater: Meromorphic functions with discontinuous sets of singularities.

Let f(z) be single-valued meromorphic in $|z| < \infty$ except on a set E of essential singularities of logarithmic capacity zero. It is shown that, for each $P \in E$, f(z) assumes infinitely often every complex value--with two possible exceptions--in any neighbourhood of P. A similar result is applicable to functions which are locally quasi-conformal. (Received February 26, 1960.)

568-36. Sibe Mardešić: Mapping ordered continua onto product spaces.

A continuum is a Hausdorff compact and connected space; an ordered continuum is a set C provided with a total ordering such that C is a continuum under the induced order topology. Theorem 1. Let X and Y be nondegenerate continua and C an ordered continuum. If there exists a mapping $f: C \longrightarrow X \times Y$ onto $X \times Y$, then X and Y possess the Suslin property, i.e. each family of disjoint open sets of X (of Y) is at most countable. Corollary. Let X and Y be two nondegenerate locally connected continua at least one of which fails to possess the Suslin property. Then $X \times Y$ is a locally connected continuum having the property that there is no ordered continuum C which admits a mapping $f: C \longrightarrow X \times Y$ onto $X \times Y$. This provides numerous examples contradicting the

conjecture that each locally connected continuum is the continuous image of an ordered continuum. Specifically, take for X and Y the "long line". Theorem 2. The only ordered continuum C which admits a mapping onto $C \times C$ (the Peano phenomenon) is the real line segment I. This follows from Theorem 1 and a result of Dj. Kurepa (Publ. Inst. Math. Acad. Serbe Sc. vol. 4 (1952) pp. 97-108). (Received February 24, 1960.)

568-37. H. E. Rauch: Periods of Abelian integrals and the Douglas functional.

A simple relationship between the first variations of the quantities in question is exhibited. (Received March 10, 1960.)

568-38. W. Slowikowski and W. Zawadowski: Some correspondence between sets of ideals of a semiring.

Consider semiring $\mathcal{A}$ (cf. Slowikowski and Zawadowski, A generalization of maximal ideals method ..., Fund. Math. vol. 42 (1955) p. 215) without the assumption about existence of 0 and 1 in $\mathcal{A}$. Dual ideal D is said to be strong if ab $\in$ D implies a,b $\in$ D. We set $\mathrm{Rad}_D \mathcal{A} = \bigcap_{y \bar{\in} D} \{x | x + y \bar{\in} D\} = \bigcap_{M \in \mathfrak{M}_D} M$, where $\mathfrak{M}_D$ denote the set of all maximal ideals disjoint with D. D is said to be normal if for each x,y $\in \mathcal{A}$ with x + y $\in$ D there are u,v $\in \mathcal{A}$ with x + u, y + v $\in$ D and uv $\in \mathrm{Rad}_D \mathcal{A}$. Let $D' \supset D''$ be two strong normal dual ideals and let $xy \in \mathrm{Rad}_{D''} \mathcal{A}$ and $x \in D'$ imply $y \in \mathrm{Rad}_{D''} \mathcal{A}$. Theorem. Each $M' \in \mathfrak{M}_{D'}$ can be extended to exactly one $M'' \in \mathfrak{M}_{D''}$ and the mapping $\mathfrak{M}_{D'} \to \mathfrak{M}_{D''}$ is onto. If $\mathcal{A}$ is the semiring of open sets of a compact T_2 space X, then we can put for example D'' = (X) and $D' = \{A \in \mathcal{A} | \bar{A} = X\}$. (Received February 29, 1960.)

568-39. Peter Treuenfels: Remark on a paper by L. Collatz on bounds for eigenvalues of differential equations.

For the eigenvalue problem $-(pu')' + qu = \lambda ru$, $u(0) = u(b) = 0$, $p(x) > 0$, $r(x) > 0$, L. Collatz proved [Deutsche Mathematik vol. 2 (1937) pp. 189-215] that the lowest eigenvalue λ_h of a corresponding finite-difference equation satisfies $|\lambda - \lambda_h| \leqq C h^2$, where λ is the lowest eigenvalue of the differential equation, h is the mesh size, and C is a certain constant. In this note an upper bound for C is given in terms of b, p(x), q(x), r(x). Thus, given any $\epsilon > 0$, the mesh size h necessary to calculate bounds for λ differing by at most ϵ can be determined a priori. (Received February 24, 1960.)

568-40. Hidehiko Yamabe: On a deformation of Riemannian structures on compact manifolds.

Let S be a C^∞ Riemannian d-dimensional compact manifold with its fundamental tensor g_{ij}, $d \geqq 3$. The function v(q) minimizing the integral $\int_S ((4(d-1)/(d-2))|\nabla u|^2 + Ru^2)dV/\|u\|_q^2$ to $\mu(q)$ with $q \leqq 2d/(d-2)$ exists, is C^∞ and positive, and satisfies the equation $(4(d-1)/(d-2))\Delta v(q) - Rv(q) = -\mu(q)(v(q))^{q-1}$. Here Δ denotes the corresponding Laplace-Beltrami operator. Consider a new structure $\bar{g}_{ij} = (\bar{u})^{4/(d-2)}g_{ij}$ where $\bar{u} = v(p)$, $p = 2d/(d-2)$. The scalar curvature of the new structure is equal to the constant $\mu(p)$. Among lemmas used in this paper are Sobolev's lemma and its generalization; $\inf \|\psi - c\|_q \leqq \delta \|\nabla\psi\|_2$ where c is an arbitrary constant. This δ can be made small if ψ is weakly close to zero. Another lemma used here is that if $\|v(q)\|_q = 1$, for $q \leqq 2d/(d-2)$, and satisfies the elliptic equation weakly, then it is a genuine solution. (Received February 18, 1960.)

568-41. J. C. C. Nitsche: A uniqueness theorem for doubly connected minimal surfaces.

The following theorem is proved: Let S be a doubly connected minimal surface with the representation $x = r(z,\phi)\cos\phi$, $y = r(z,\phi)\sin\phi$. The positive distance function $r(z,\phi)$ is assumed to be of class C^2 and periodic: $r(z,\phi + 2\pi) = r(z,\phi)$, for all values of z and ϕ. Then S must be a catenoid whose axis is parallel to the z-axis. In a former paper (J. Rat. Mech. Anal. vol. 6 (1957)) this result had been ascertained under the additional assumption that all curves S_c of intersection of S with planes $z = c$ are convex. Thus the purpose of the present investigation consists in showing that, under the assumptions formulated above, the curves S_z are convex. (Received April 18, 1960.)

569-38. Zygmunt Charzynski: Bounds for analytic functions of two complex variables. I.

Let there be given a function $f(z_1,z_2)$ of two complex variables which is analytic in a closed domain $\overline{\mathcal{M}}$, with a distinguished boundary surface (see Bergman, Math. Z. vol. 63 (1955) pp. 173-194). Let $\dot{\nu}$ denote a part of the boundary of $\mathcal{M}$, given in a parametric form: $z_1 = h_1(Z,\lambda)$, $z_2 = h_2(Z,\lambda)$; where $|Z| \leqq 1$, $0 \leqq \lambda \leqq 2\pi$ are the complex and real parameters, respectively; $h_1(Z,\lambda)$, $h_2(Z,\lambda)$ are continuously differentiable with respect to Z and λ. Suppose that for every fixed λ, the composite function of one complex variable $f[h_1(Z,\lambda),h_2(Z,\lambda)]$ belongs to a normal family Φ of functions of one variable in $|Z| \leqq 1$. Let $M(R) = \sup_{\phi(Z)\in\Phi} \sup_{|Z|\leqq R} |\phi(Z)|$ for $0 < R < 1$. Denote also by ν_R the part of ν, which corresponds to the values $|Z| < R$ of the first parameter and all values of λ. Finally let there be an arbitrary analytic surface $\mathcal{S}_1$ given parametrically by the equations $z_1 = g_1(\zeta)$, $z_2 = g_2(\zeta)$, $|\zeta| \leqq 1$, lying in the set $\mathcal{M} + \nu_R + C\mathcal{M}$ whose boundary $\mathcal{S}_1$, corresponding to the values $|\zeta| = 1$, lies in $C\mathcal{M}$. Then we have: Lemma. In the set $\mathcal{S} \cap \mathcal{M}$ the following estimation holds: $|f(z_1,z_2)| < M(R)$. The lemma represents a generalization of analogous results of Bergman in the paper mentioned above. (Received March 14, 1960.)

569-39. Zygmunt Charzynski: Bounds for analytic functions of two complex variables. II.

The notions, notations and the result of the first part are used. Then one defines a sequence of sets $\{\mathcal{R}_m\}$ in the following way: 1°. $\mathcal{R}_1 = \sum_{\mathcal{S}_1\in B_1} \mathcal{S}_1 \cap \mathcal{M}$, where B_1 is the class of all surfaces $\mathcal{S}_1$ described in the lemma. 2°. $\mathcal{R}_{m+1} = \sum_{\mathcal{S}_m\in B_{m+1}} \mathcal{S}_{m+1} \cap \mathcal{M}$, where B_{m+1} is the class of all surfaces $\mathcal{S}_m$, lying in $\mathcal{M} + \mathcal{R}_1 + \ldots + \mathcal{R}_m$ and having its boundary in $\mathcal{R}_1 + \ldots + \mathcal{R}_m$. The sum $\mathcal{R}_R = \sum_{m=1}^{\infty} \mathcal{R}_m$ will be called the associated domain with respect to $\dot{\nu}$ and R. The set $\mathcal{R}_R$ has the following properties: (1) It is either empty or open, (2) it cannot be extended by the method of analytic surfaces used before, (3) it is pseudo-convex with respect to its exterior

at every point of its boundary which lies in $\mathfrak{M}$, (4) the inequality $|f(z_1,z_2)| \leqq M(R)$ holds in $\mathfrak{R}_R$. This complements Bergman's results (Math. Z. vol. 63 (1955) pp. 173-194). (Received March 14, 1960.)

569-40. J. J. Mehlberg: The representation and optimization of a class of linear filters.

The subject of this paper is the formulation of a set of conditions entailing the existence of weighting functions for a class of linear filters (time-invariant, stable and causal) whose input functions belong to certain function spaces. In particular, the following spaces of input functions are shown to have weighting functions for the aforementioned class of filters: certain Banach spaces, Saks spaces, and some special spaces of interest from a physical point of view (e.g. the space of continuous functions on $(-\infty,\infty)$ of polynomial growth i.e. such that for some $n > 0$, constants a,T, and $t \geqq T$ $|f(t)| \leqq a|t^n|$). The space of the weighting functions is determined by the space of the inputs. Furthermore, a method is indicated for optimizing this class of filters by minimizing the mean squared difference between actual and desired output, on the assumption that the signal and noise do not have the same spectral densities. (Received March 11, 1960.)

569-41. A. T. Bharucha-Reid: Approximate solutions of random operator equations. Preliminary report.

Let $x(\omega)$ be a generalized random variable (g.r.v.) with values in a Banach space $\mathfrak{X}$, and let $T(\omega,x(\omega)) = 0$ be a random operator equation in $\mathfrak{X}$. In this paper we consider the stochastic analogue of the Newton-Kantorovič process for the solution of operator equations in Banach spaces. In particular, we study the measurability and convergence properties of the process $\{x_n(\omega), n \geqq 0\}$, where $x_{n+1}(\omega) = x_n(\omega) - [T'(\omega,x_n(\omega))]T(\omega,x_n(\omega))$ and $x_0(\omega)$ is an arbitrary g.r.v. with values in $\mathfrak{X}$. Several applications and related statistical problems are also discussed. (Received March 2, 1960.)

569-42. R. A. Beaumont and R. S. Pierce: Torsion free abelian groups of rank two.

Let $(G; x_1,x_2)$ be a torsion free abelian group of rank two containing the independent pair of elements x_1,x_2 (x_1,x_2 is called a rational basis). A system of invariants for $(G; x_1,x_2)$ is obtained, and the effect on the invariants of a

change of rational basis is determined. Conditions are given in terms of these invariants for rank two groups to be quasi-equal and quasi-isomorphic. Conditions on the invariants in order that such a group be quasi-decomposable are determined. (Received March 3, 1960.)

569-43. C. B. Bell: On the cardinality of independent stochastic processes.

What is the maximum cardinality $M_C(\Omega)$ of an independent stochastic process on a given space Ω? Consider a process $\mathcal{F}_T = \{X_t, t \in T\}$ of cardinality $\overline{\overline{T}}$, such that $A_t = \{X_t = 1\}$; $A_t^0 = \Omega - A_t = \{X_t = 0\}$; $p_t = p_t^{(1)} = P(A_t)$ and $p_t^{(0)} = P(A_t^0)$. $\mathcal{F}_T$ is (C_T): totally indep. if $\bigcap_{t\in T} A_t^{i_t} \neq \emptyset$ for all T-sequences $\{i_t\}$ of 0's and 1's; (C_2): σ-indep. if $\bigcap_{n=1}^{\infty} A_{t_n}^{i_n} \neq \emptyset$ for all σ-sequences $\{t_n\} \subset T$ and all $\{i_n\}$; (C_1): Stochastically indep. w.r.t. $\{p_t\}$ if there exists P on $\mathscr{B}(\{A_t\})$ such that $P(\bigcap_{n=1}^{m} A_{t_n}^{i_n}) = \prod_{n=1}^{m} p_{t_n}^{(i_n)}$. It is known that $C_T \longrightarrow C_2 \longrightarrow C_1$ w.r.t. arbitrary $\{p_t\}$. Hence, $M_{C_1}(\Omega) \geqq M_{C_2}(\Omega) \geqq M_{C_T}(\Omega)$. It is proved that (1) C_T iff $2^{\overline{\overline{T}}} \leqq \overline{\overline{\Omega}}$; (2) C_2 if $\overline{\overline{T}}^{\aleph_0} \leqq \overline{\overline{\Omega}}$; and not if $\overline{\overline{T}}^{\aleph_0} > 2^{\overline{\overline{\Omega}}}$. Consequently, (3) if $\overline{\overline{\Omega}} < \aleph_0$, $M_{C_T}(\Omega) = M_{C_1}(\Omega) = [\log_2 \overline{\overline{\Omega}}]$; and (4) if $\overline{\overline{\Omega}}^{\aleph_0} = \overline{\overline{\Omega}}$, then $M_{C_1}(\Omega) \geqq M_{C_2}(\Omega) \geqq \overline{\overline{\Omega}}$. Finally, if $\overline{\overline{\Omega}} = \aleph_0$, there exist countable $\mathcal{F}_T$ and $\{p_t\}$ such that $M_{C_T}(\Omega) = M_{C_2}(\Omega) < M_{C_1}(\Omega) = \aleph_0$. (Received March 1, 1960.)

569-44. F. H. Brownell: The transition operator and a Lamb shift model. Preliminary report.

Consider self-adjoint H_0 in Hilbert X, consider H_1 symmetric with larger domain, $H = H_0 + H_1$, and suppose $\overline{H} = H^{**}$ self-adjoint. Then unitary $U_t = \exp(-it\overline{H})$ and $W_t = \exp(itH_0)\exp(-it\overline{H})$ exist for all real t, W_t being the transition operator. For given $u_0 \in X$, $U_t u_0$ represents a flow in X, and we study $\|PU_t u_0\|^2$ as a function of t for P an orthogonal projection (see Pacific J. Math. vol. 5 (1955) pp. 1-16). If P commutes with the spectral measure of H_0, then $\|PU_t u_0\|^2 = \|PW_t u_0\|^2$, which enables us to compute using the well known expansion of W_t in powers of H_1. Using some of our earlier ideas (Bull. Amer. Math. Soc. vol. 62 (1956) p. 587), we construct a simplified model by taking X a direct product of C_3 onto a Hilbert space X_4 representing 1-dimensional "photons", $H_0 = H_3 + H_4$, $H_3 = \begin{pmatrix} 0 & & \\ & 1 & \\ & & 1 \end{pmatrix}$ acting on C_3 only, and H_4 the pure "photon" field Hamiltonian is diagonal with eigenvalues $\rho(\sum m_n|bn|)$, with integer $m_n \geqq 0$ and summation over all integers n having $0 < |bn| \leqq R$, b being the

"photon" oscillator mesh and R the cut-off radius. H_1 is suitably chosen to represent an interaction Hamiltonian. By computing with suitable P and u_0 and then taking $b \to 0^+$ and $R \to +\infty$, we exhibit phenomena analogous to the Lamb shift, with the band width dependent upon the distribution of 1/b modulo the integers as $b \to 0^+$. (Received February 29, 1960.)

569-45. M. A. Geraghty: Continuous functions on spheres.

The development of Smith homology theory in terms of singular cycles is sketched and the Smith index of a T-space, or topological space with periodic homeomorphism T, is defined in terms of singular cycles. The index of a space with fixed-point-free involution, whose first n - 1 reduced singular homology groups are zero, is shown to be n, and for such spaces three theorems, including the Borsuk theorem that a mapping of such a space to Euclidean n-space must take some point and its involute to the same value, are established. If, moreover, the spaces are normal, four theorems, including the Lusternik-Schnirelman covering theorem, are established. The indices of the Stiefel manifolds $V_{2p,n}$ with the natural antipodal involutions are shown to be between 2p - 1 and s - 1, where s is the largest power of two dividing 2p. Covering and mapping theorems similar to those given above are then established for these manifolds, and one of the covering theorems is used to obtain some frame results, including the result that if f is an odd function from S^{2^k-1} to Euclidean n-space and if $2^k \quad mn - m + 1$, then given any n-frame on S^{2^k-1}, there exists a congruent frame whose points are mapped to the same value by f. (Received March 6, 1960.)

569-46. H. J. Keisler: The completeness of a certain class of first order logics with infinitary relations.

The system F_1 in Church, Introduction to mathematical logic, extends naturally to the system L whose symbols are: an infinite set V of individual variables; a set C of individual constants; sets $\mathfrak{R}_\alpha$ of relation symbols for each ordinal $\alpha < \overline{\overline{V}}$ (= cardinality of V); and $\rightarrow, \sim, \forall, (,)$. The set $\mathfrak{A}$ of formulas of L is defined as follows. If $R \in \mathfrak{R}_\alpha$ and $x \in (V \cup C)^\alpha$ then $Rx \in \mathfrak{A}$. If $A, B \in \mathfrak{A}$, $v \in V^\beta$, and the ordinal $\beta < m$, for a fixed cardinal $m \leqq \overline{\overline{V}}$, then $\sim A$, $A \rightarrow B$, and $\forall vA \in \mathfrak{A}$. The rules of inference are modus ponens and generalization (i.e. infer $\forall vA$ from A). The axioms are: all tautologies; $\forall v(A \rightarrow B) \rightarrow (A \rightarrow \forall vB)$, where no v_β is free in A; and $\forall vA \rightarrow B$,

where $\tau \in (V \cup C)^V$ and B arises from A by replacing each v_β free in A by $\tau(v_\beta)$, and with no free occurrence becoming bound. Conventional notions of (finitely long) proof, consistency, model, and satisfiability for L are defined. Theorem: (Extending Leon Henkin's proof of the Lowenheim-Skolem-Gödel Theorem). Let $\Gamma \subseteq \mathcal{A}$ be consistent, $\mathfrak{K} \geqq \overline{\overline{\mathcal{A}}}$, and $\mathfrak{K} = \mathfrak{K}^p$ for each cardinal $p \prec V$. Then Γ is satisfiable in a model of cardinality $\mathfrak{K}$. Examples show that in most cases L with identity is highly nonaxiomatizable, and does not satisfy this theorem. The consistency, converse of completeness, deduction theorem, and semantical compactness theorem for L are easily proved. (Received February 29, 1960.)

569-47. H. J. Keisler: The representation of polyadic algebras of infinite degree.

Consider the system L' defined in the same manner as the system L of the preceding abstract except that L' contains sets $\mathcal{Q}_\alpha$ of relation symbols for each $\alpha \leqq \overline{\overline{V}}$, and m is the least cardinal which is greater than $\overline{\overline{V}}$. Theorem 1 (in the notation of the preceding abstract): Let $\Gamma \subseteq \mathcal{A}$ be consistent. Let $\mathfrak{K} \geqq \overline{\overline{\mathcal{A}}}$ and $\mathfrak{K} = \mathfrak{K}^{\overline{\overline{V}}}$. Then Γ is satisfiable in a model of cardinality $\mathfrak{K}$. The polyadic algebras of P. Halmos (Fund. Math. vol. 43, pp. 255-325) are essentially algebraic analogues of L'. Theorem 1 has the following algebraic version (in the notation of Halmos). Representation theorem: Let A be a simple polyadic algebra of infinite degree d. If $\mathfrak{K} \geqq A$ and $\mathfrak{K} = \mathfrak{K}^d$, then A is isomorphic to a model whose domain has cardinality $\mathfrak{K}$. In the special case that A is locally finite, this becomes a theorem of Halmos. Corollary: Every polyadic algebra of infinite degree is isomorphic to a functional polyadic algebra (in fact, a subdirect product of models). (Received February 29, 1960.)

569-48. H. J. Keisler: A type of product more general than reduced products, and elementary classes closed under them.

For notation see the author's paper, J. Symb. Logic, June 1959, pp. 1-27 and these NOTICES, Abstract 564-265, vol. 7 (1960) p. 70. Let F be a GA set of formulas, and $\mathcal{O}\!\!\mathcal{L}$, $\mathcal{O}\!\!\mathcal{L}_i$, $i \in I$, be a relational systems. $\mathcal{O}\!\!\mathcal{L}$ is an F-product of $\langle \mathcal{O}\!\!\mathcal{L}_i | i \in I \rangle$ by the map ϕ of $\prod_{i \in I} \mathcal{O}\!\!\mathcal{L}_i$ onto $\mathcal{O}\!\!\mathcal{L}$ iff for any $a \in (\prod_{i \in I} \mathcal{O}\!\!\mathcal{L}_i)^\omega$ and any $f \in F$ such that for each $i \in I$ the ith projection of a satisfies f in $\mathcal{O}\!\!\mathcal{L}_i$, $\phi(a)$ satisfies f in $\mathcal{O}\!\!\mathcal{L}$. If $F = \{\text{atomic formulas}\}$, the notions of "F-product" and "homomorphic image of direct product" coincide. Theorem 1: Suppose

$2^{\aleph_0} = \aleph_1$, $K \subseteq H$, and $\mathfrak{A} \in (K^* \cap \{\forall, \exists\} \wedge F)^* \cap H$. Then for some countable set I, some $D \in P(I)$, and some $\{\mathfrak{A}_i | i \in I\} \subseteq K$, $\mathfrak{A}^I/D$ is an F-product of $\langle \mathfrak{A}_i | i \in I \rangle$. Corollary: If $2^{\aleph_0} = \aleph_1$, and Φ is a sentence, then (i), (ii), and (iii) below are equivalent. (i) Φ is preserved under countably indexed F-products. (ii) Φ is preserved under arbitrary F-products. (iii) $\Phi \in \{\forall, \exists\} \wedge F$. If D is an ideal in 2^I, let θ_D be the quotient map of $\prod_{i \in I} \mathfrak{A}_i$ onto $\prod_{i \in I} \mathfrak{A}_i/D$. Theorem 2: If F is the set of all formulas (or Horn formulas), then $\mathfrak{A}$ is an F-product of $\langle \mathfrak{A}_i | i \in I \rangle$ by ϕ iff for some prime ideal (or ideal) D in 2^I there exists ψ: $\prod_{i \in I} \mathfrak{A}_i/D \cong \mathfrak{A}$ such that $\phi = \psi\theta_D$. This shows Theorem 1 above generalizes Theorem 2 of Abstract 564-266. (Received February 29, 1960.)

569-49. H. J. Keisler: Conditions for inclusion between certain classes of algebraic operations sending members of one elementary class into members of another.

For notation see the preceding abstract. Theorem 1: Let K, M $\in EC_\Delta$, F, G be GA sets. Then (i) and (ii) below are equivalent. (i) If $\mathfrak{A} \in K$, $\mathfrak{B} \in M$, and ϕ: $\mathfrak{A} \to \mathfrak{B}$ is an F-homomorphism, then ϕ is a G-homomorphism. (ii) For every $g \in G$, there exists $f \in \{\forall, \exists, \wedge, \vee\} F$ such that $K^* \vdash g \supset f$ and $M^* \vdash f \supset g$. Special case: every homomorphism from $\mathfrak{A} \in K$ onto $\mathfrak{B} \in M$ is an isomorphism iff for every atomic formula g there is a positive formula f such that $K^* \vdash \sim g \supset f$ and $M^* \vdash f \supset \sim g$. Theorem 1a: Replace "homomorphism" by "abridgement" and "$\{\forall, \exists, \wedge, \vee\}$" by "$\{\forall, \wedge, \vee\}$" in Theorem 1. Theorem 1b: Replace "homomorphism" by "expansion" and "$\{\forall, \exists, \wedge, \vee\}$" by "$\{\exists, \wedge, \vee\}$" in Theorem 1. Special case: a consistent set Σ of sentences is model-complete iff for every formula g there is an existential formula f such that $\Sigma \vdash f \equiv g$ (Abraham Robinson). Theorem 2: If $2^{\aleph_0} = \aleph_1$, K, $M \in EC_\Delta$ and F, G are GA sets, then (i) and (ii) below are equivalent. (i) If $\{\mathfrak{A}_i | i \in I\} \subseteq K$, $\mathfrak{B} \in M$, and $\mathfrak{B}$ is an F-product of $\langle \mathfrak{A}_i | i \in I \rangle$ by ϕ, then $\mathfrak{B}$ is a G-product of $\langle \mathfrak{A}_i | i \in I \rangle$ by ϕ. (ii) For every $g \in G$, there exists $f \in \{\forall, \exists\} \wedge F$ such that $K^* \vdash g \supset f$ and $M^* \vdash f \supset g$. In view of the preceding abstract, Theorem 2 applies to reduced products and ultraproducts. (Received February 29, 1960.)

569-50. D. H. Lehmer: Mahler's matrices.

Recently Mahler called attention to a remarkable set of $\phi(2n)$ matrices of n rows and columns which form under multiplication the abelian group of the residue classes prime to 2n (modulo 2n) [Proc. Amer. Math. Soc. vol. 8

(1957) pp. 525-531]. It is shown that these matrices, whose elements are 0, 1 or - 1, have latent roots and determinants which can be given explicitly. The roots depend on Ramanujan's sum while the determinant is Jacobi's symbol. The case in which n is odd is more interesting and simpler than the even case. (Received March 2, 1960.)

569-51. Henryk Minc: Mutability of bifurcating root-trees.

A bifurcating root-tree can be regarded as the sum of its two sub-trees of the first order (Proc. Roy. Soc. Edinburgh vol. 64, p. 322). A knot of a tree P is said to be unbalanced if the subtree of P of which the knot is the root has nonconformal (i.e. not congruent when addition of trees is commutative) subtrees of the first order. The mutability of P is equal to the number of unbalanced knots in P. Thus the number of trees conformal to a given tree of mutability μ is 2^{μ}. The altitude of a tree is the distance from its root to its furthest knot; the potency is the number of free knots in the tree. If α, δ, μ are the altitude, potency and mutability of a bifurcating root-tree then $\alpha - \mu \leqq \log_2(\delta - \mu)$ and $\mu \geqq h - 1$, where h is the sum of digits in δ written in the binary scale of notation. These bounds are the best possible since, given any non-negative integers α, δ satisfying $\alpha + 1 \leqq \delta \leqq 2^{\alpha}$, one can construct a tree of altitude α, potency δ and mutability equal to the least integer satisfying $\alpha - \mu \leqq \log_2(\delta - \mu)$ and $\mu \geqq h - 1$. (Received February 25, 1960.)

569-52. T. S. Motzkin: Power series with gaps.

It is known (Mandelbrojt, Motzkin, Ostrowski) that $\sum a_n z^n$ with $a_n = 0$ for $n \equiv r_1, \ldots, r_k \pmod{q}$, q prime, has at least k + 1 singularities on its circle of convergence, and that a similar statement holds for general q and $k \leqq 3$. It is shown that this statement does not hold for $k = 4$ and $k \geqq 6$. (Received February 29, 1960.)

569-53. Morgan Ward: The vanishing of the homogeneous product sum on three letters.

If H_n denotes the homogeneous symmetric product sum on three letters x,y and z, it is proved that the diophantine equation $H_n = 0$ has only trivial integral solutions if n + 2 is a prime number. (Received March 7, 1960.)

569-54. H. J. Weinitschke: On asymptotic solutions in the theory of shallow shells. Preliminary report.

Consider axisymmetric deformations of a shallow elastic shell of revolution under rotationally symmetric axial load. It is known that the problem can be formulated in terms of two simultaneous nonlinear differential equations with appropriate boundary conditions. When the load parameter γ tends to infinity, the order of the system of equations is decreased and the solution is found to be of the boundary layer type. An asymptotic expansion for large loads is given, extending earlier work for flat plates under uniform pressure. It is shown how appropriate boundary conditions for the solution of the limit problem ($\gamma^{-1} = 0$) can be obtained by assuming the validity of the asymptotic expansion both in the interior and in the boundary layer of the shell. The existence and uniqueness of the solution of the limit problem is established. (Received March 2, 1960.)

569-55. R. R. Phelps: Extreme points of polar convex sets.

Let F and G be real linear spaces in duality under $\langle , \rangle$, let C be a convex subset of F containing the origin ϕ, and let p be the gauge functional defined by C. For $y \in C^0$ (the polar of C in G), let $F_y = \{x \in F: p(x) - \langle x,y \rangle \leqq 1\}$. Let M be a subspace of F and $M^\perp$ its annihilator in G. Theorem 1. A point $y \in C^0 \cap M^\perp$ is an extreme point of $C^0 \cap M^\perp$ if and only if $F_y - F_y + M$ is $\sigma(F,G)$-dense in F. Theorem 2. Suppose that F is a topological vector space, $G = F^*$ and $\phi \in$ int C. Then $F_y - F_y + M = F$ if (and only if) y is an extreme point of $C^0 \cap M^\perp$. The above theorems hold for complex spaces if $\langle , \rangle$ is replaced by $\mathrm{Re}\langle , \rangle$ in the definition of F_y. These generalize Buck's results (Bull. Amer. Math. Soc. vol. 65 (1959) pp. 130-133); Theorem 1 also yields dual versions of Buck's theorems, e.g. x is an extreme point of the unit ball of the normed space E if and only if $E_x^* - E_x^*$ is weak*-dense in E^*. Open question: Does there exist a Banach space E whose unit ball has an extreme point, the image of which (under the usual embedding) is not an extreme point of the unit ball of E^*? (Received March 17, 1960.)

570-1. R. K. Getoor: Expected first passage times for symmetric stable processes.

Let $\{X_\alpha(t): t \geqq 0\}$ be the symmetric stable process of index α, $0 < \alpha \leqq 2$, in Euclidean N-space, R^N. If $a > 0$ define $T_\alpha(a) = \inf\{t: |X_\alpha(t)| > a\}$. Thus $T_\alpha(a)$ is the first passage time of the process $X_\alpha(t)$ to the exterior of the solid sphere $\{x: |x| < a\}$. Theorem. $E_x\{T_\alpha(a)\} = [2^\alpha \Gamma(1 + \alpha/2)\Gamma((N + \alpha)/2)]^{-1} \cdot \Gamma(N/2)(a^2 - |x|^2)^{\alpha/2}$ provided $|x| \leqq a$, where $E_x\{\cdot\}$ denotes the expectation under the condition $X_\alpha(0) = x$. If $\alpha = 2$ this result is a standard fact in the theory of Brownian motion. If $N = 1$ and $\alpha = 1$ the above formula was obtained by Kac and Pollard (Partial sums of independent random variables, Canad. J. Math. vol. 11 (1950) pp. 375-384). More recently this result was obtained in the case $N = 1$ and $0 < \alpha < 1$ by J. Elliot (Absorbing barrier processes connected with the symmetric stable densities, Illinois J. Math. vol. 3 (1959) pp. 200-216). (Received March 25, 1960.)

570-2. R. W. Heath: Arc-wise connectedness in semi-metric spaces.

It is well known that a connected, locally connected, complete, metric space is arc-wise connected. In his book R. L. Moore has generalized this theorem. The author shows that the theorem cannot be generalized to Cauchy complete, regular semi-metric spaces. Some examples show rather clearly the limited nature of the progress that can be made in trying to extend the Arc Theorem to semi-metric spaces. Several topological conditions and completeness axioms are investigated to determine which of them might be used to generalize the Arc Theorem. It is established that a regular semi-metric space is a Moore space provided that it satisfies a certain completeness axiom which is slightly stronger than Cauchy completeness but weaker than the completeness of Moore's Axiom 1. Finally, the author obtains a sufficient condition for arc-wise connectedness in semi-metric spaces which is somewhat more general than Moore's theorem. (Received March 16, 1960.)

570-3. K. J. J. Hintikka: A closure and complement problem.

Suppose we have, for the same set S of points, two topologies, one of which is stronger (coarser) than the other. Given an arbitrary subset of S, how many different sets can be constructed from it by complementation and closure (with respect to either topology)? Answer: at most 126. (Received April 7, 1960.)

570-4. W. L. Miranker: Stable periodic solutions of a nonlinear difference-differential equation.

In this paper we study the periodic solutions of the nonlinear difference-differential equation $c(v)v_t(t)|_{t-\tau}^{t} = -v(t) + \phi(t - \tau/2)$. There are three main points to our investigation. First, when the period T of ϕ and the solution v is commensurable with the lag τ, we solve the nonlinear equation by solving successively two simple equations. These are a linear difference equation and a transcendental equation. Second, when we obtain the response curves for our equation in the general case by perturbations, the results of the above analysis demonstrate the necessity of developing v, T, and τ simultaneously in power series. This extends the method of perturbations for the case of nonlinear differential equations where only v and T are developed. Third, stability of the periodic solutions requires the proof that solutions of a linear difference-differential equation with variable coefficients vanish at infinity. The existing theorems in this field require that the coefficients converge to constants suitably fast. Our equation does not have this property. Nevertheless, we obtain the asymptotic vanishing by a method which employs a Phragmèn-Lindelöf theorem for Fourier transforms. We point out that this method is applicable to wide classes of differential and difference-differential equations. (Received April 11, 1960.)

570-5. W. L. Miranker: A new class of stability theorems for differential equations.

The existing theorems which guarantee the vanishing of solutions as x tends to infinity of linear differential equations with variable coefficients, require that the coefficients of the equation converge to constants at infinity. If the rate of convergence is suitably rapid, then the solutions of the variable coefficient equation behave at infinity like those of the constant coefficient equation. In this paper we eliminate the requirement of convergence of co-

efficients and produce new requirements for the asymptotic vanishing of solutions. We also extend our method to difference-differential equations. Our conditions are roughly that the variable coefficients need only be close enough to constants in a neighborhood of infinity (not necessarily converge to them). If these constants are such that the solutions, of the corresponding differential equation with their constants for coefficients, vanish at infinity, then the same is true for the variable coefficient equation. Closeness is measured by the minimum distance to the imaginary axis of the roots of the characteristic equation corresponding to the differential equation with constant coefficients. Our method of proof uses a Phragmèn-Lindelöf theorem for Fourier transforms. Specifically, one of the equations we consider is $y^{(n)}(x) + (a_1(x)y(x))^{(n-1)} + \ldots + a_n(x) = 0$. (Received April 11, 1960.)

570-6. G. E. Sacks: A continuum of pairwise incomparable degrees. Preliminary report.

J. R. Shoenfield (Proc. Amer. Math. Soc. vol. 11 (1960) pp. 61-62) has shown by means of Zorn's lemma that there is an uncountable set of pairwise incomparable degrees of recursive unsolvability. The theorem below provides a set of pairwise incomparable degrees whose power is that of the continuum without any use of the axiom of choice. Lemma 1: Let A be a countable set of nonrecursive functions; then there is a countably infinite set S of functions such that if f is a member of A or S, then f is not recursive in the members of any finite subset of S - ($\{f\}$ - A). Lemma 2. Let A and B be countable sets of functions, then (1) and (2) are equivalent: (1) There is a function f such that f is not recursive in any member of A, no member of A is recursive in f, and each member of B is recursive in f. (2) No member of A is recursive in any member of B. Theorem: Let A be a countable set of nonrecursive functions; then there is a set B of functions such that B has power of the continuum, no member of B is recursive in any other member of B or in any member of A, and no member of A is recursive in any member of B. The proof of the theorem requires only the two lemmas and one elementary fact of set theory: there is a set D, having power of the continuum, of sets of natural numbers such that no member of D is a subset of any other. (Received March 31, 1960.)

570-7. Richard Sacksteder: Hypersurfaces with no negative sectional curvatures.

Let M be a complete Riemannian (n - 1) manifold ($n \geq 3$) and let $F:M \longrightarrow E^n$ be an immersion of class C^n. Suppose that all of the sectional curvatures of M are non-negative and that at least one is positive. Theorem: F is an imbedding and F(M) is the boundary of a convex body. Supplement: E^n can be decomposed as a product $E^n = E^m \times E^{n-m}$ in such a way that $F(M) = P_1F(M) \times P_2F(M)$, $P_1F(M) = E^m$, and $P_2F(M)$ is the boundary of a convex body in E^{n-m}, which contains no complete line. Here P_1 and P_2 denote respectively the natural projections from E^n to E^m and E^{n-m}. The integer m is determined intrinsically, and satisfies $0 \leq m \leq n - 3$. The maximum rank of the second fundamental form of F(M) is n - m - 1. For smooth (C^n) hypersurfaces this result generalizes theorems of Hadamard, Stoker, Van Heijenoort, Chern and Lashof, and the author. It also complements a theorem of Hartman and Nirenberg who proved that if all of the sectional curvatures of M are zero, F(M) is a hypercylinder. (Received March 17, 1960.)

570-8. Martin Schechter: Too few and too many boundary conditions.

Let A be a linear elliptic partial differential operator of order 2r in a bounded domain $G \subset E^n$. Let $\{B_j\}_{j=1}^{2r}$ be a Dirichlet system of boundary operators such that $\{B_j\}_{j=1}^{r}$ covers A (for definitions cf. Comm. Pure Appl. Math. vol. 12 (1959) pp. 457-486). Let $\partial_1 G$ be a portion of the boundary ∂G of G which is open in ∂G, and set $\partial_2 G = \partial G - \overline{\partial_1 G}$. Under suitable regularity assumptions on G and the coefficients of A and the $\{B_j\}_{j=1}^{2r}$, there is an adjoint Dirichlet system $\{B_j'\}_{j=1}^{2r}$ such that the following statement holds: Given integers s,t satisfying either $0 \leq s \leq t \leq r$ or $r \leq s \leq t \leq 2r$, there is a solution $u \in C^\infty(\bar{G})$ of $Au = f$ in G, $B_j u = 0$ on $\partial_1 G$, $1 \leq j \leq s$, $B_j u = 0$ on $\partial_2 G$, $1 \leq j \leq t$, if, and only if, $\int_G f\bar{v}\,dx = 0$ for all $v \in C^\infty(\bar{G})$ satisfying $A^*v = 0$ in G, $B_j' v = 0$ on $\partial_1 G$, $1 \leq j \leq 2r - s$, $B_j' v = 0$ on $\partial_2 G$, $1 \leq j \leq 2r - t$, where A^* is the formal adjoint of A. This theorem has many interesting consequences. (Received April 5, 1960.)

570-9. Martin Schechter: The Dirichlet problem for degenerate elliptic equations.

The Dirichlet problem is considered for the equation $\sum_{i,j=1}^{n} a_{ij}\partial^2 u/\partial x_i \partial x_j + \sum_{i=1}^{n} a_i \partial u/\partial x_i + au = f$, $a_{nn} = 1$, $a \leq 0$, in which the

coefficients and f are permitted to become infinite near a portion of the boundary contained in the hyperplane $x_n = 0$. If the domain G is contained in the strip $0 \leqq x_n \leqq t_0$, it is assumed that there are functions $q(t) \geqq 0$, $p(t)$ continuous in $0 < t \leqq t_0$, such that $|a_{ij}|, |a_i|, |a|, |f| \leqq q(x_n)$, $i, j \neq n$, $a_n \leqq p(x_n)$ and $\int_0^{t_0} \exp[P(t)] \int_t^{t_0} \exp[-P(s)] q(s) ds dt < \infty$, where $P(t) = \int_t^{t_0} p(s) ds$. In closed subdomains of $\bar{G}$ (the closure of G) which do not touch $x_n = 0$ the equation is assumed regular and elliptic. It is shown that the Dirichlet problem can be solved in G for any continuous boundary values. An example shows that the hypotheses are essentially necessary. (Received April 5, 1960.)

570-10. H. J. Zimmerberg: Two-point boundary problems involving a parameter linearly.

The concepts of normality, symmetrizability and definiteness of Reid [Illinois J. Math. vol. 2 (1958) pp. 434-453] are extended to a system of n first order ordinary differential equations and two-point boundary conditions in vector form, $y' - A(x)y = \lambda B(x)y$, $(M_0 + \lambda M_1)y(a) + (N_0 + \lambda N_1)y(b) = 0$, where the $2n \times 2n$ matrix $\left\| \begin{matrix} M_0 & N_0 \\ M_1 & N_1 \end{matrix} \right\|$ has rank n plus the rank of $\| M_1 \; N_1 \|$. A boundary problem is termed normal if $y \equiv 0$ is the only vector satisfying $y' - A(x)y \equiv 0$ and $B(x)y \equiv 0$ on ab, $M_0 y(a) + N_0 y(b) = 0$ and $M_1 y(a) + N_1 y(b) = 0$. An analysis of the various conditions imposed on normal definite problems is made. In particular, the results of Reid are extended to show that for a definite boundary problem not satisfying the normality condition there exists an equivalent definite problem satisfying the normality condition whose boundary conditions also satisfy the corresponding matrix requirement above. Results for non-normal definite problems then follow from the application of previous results to the associated normal definite problem. (Received March 29, 1960.)

570-11. H. S. Green, Roy Leipnik and J. E. Maxfield: Three-dimensional lattice paths and the Ising problem.

A spinor algebra is developed for the paths in a three-dimensional crystal lattice. A pair of scalar invariants is obtained in terms of which closed paths can be described. A path weight function is recursively defined so as to vanish for paths with nodes. This apparatus is employed in conjunction with a general approach to order-disorder problems developed by Green and Leipnik

(Rev. Mod. Phys. vol. 32 (1960) p. 129) to obtain a formal recursive solution to the three-dimensional Ising problem for nearest neighbors. For the cubic lattice, with hypertorus identification, the first step is carried through explicitly, and the associated high temperature expansion is obtained for comparison with the known high temperature expansion. Agreement is found through the sixth power terms. The critical point is calculated and also the approximate shape of the critical region of the lattice for this first step in the exact solution. (Received April 14, 1960.)

570-12. Roy Leipnik and J. E. Maxfield: Optimum interpolation of noisy signals from finite records.

The Wiener theory of optimum interpolation is modified to consider finite records of noisy signals. Analysis in the time domain results in a matrix-inversion problem. This is solved explicitly in terms of the parameters in the autocovariances of several simple signals and noises. Comparisons are made with the limiting results for infinite records, and the effects of the finite record are noted. (Received April 14, 1960.)

570-13. D. P. Squier: The norm of the inverse of a certain tri-diagonal matrix.

Let $-A_j x_{j-1} + B_j x_j - C_j x_{j+1} = d_j$, $j = 1$ to n, be a set of linear equations, with $A_1 = C_n = 0$; $A_j, C_j \geqq 0$, and $B_j \geqq A_j + C_j + 1$. Then the inverse of the coefficient matrix has a norm not greater than one, the norm being defined as the maximum over rows of the sum of the absolute values of the row elements. The proof requires only elementary algebra. (Received April 18, 1960.)

570-14. R. P. Boas, Jr.: A series considered by Ramanujan.

The function f defined by $xf(x) = \sum_{n=1}^{\infty}(n^{n-1}/n!)(xe^{-x})^n$, $x > 1$, has been conjectured to be completely monotonic for $x > 1$, and is known to have $(-1)^k f^{(k)}(x) \geqq 0$ for $k = 0,1,2,3,4$. It is shown, however, that f is not completely monotonic in any interval (c,∞). (Received March 14, 1960.)

570-15. W. J. Firey: Polar means of convex bodies.

In Euclidean n-space, consider convex bodies K_i, $(i = 0,1)$, having a common interior point Q. If $F_i(x)$ is the distance function of K_i, then $M_p(F_0,F_1) = [(1-\vartheta)F_0^p + \vartheta F_1^p]^{1/p}$, $(1 \leqq p \leqq \infty, 0 \leqq \vartheta \leqq 1)$, is, (using

Minkowski's inequality), the distance function of a convex body $K_{\vartheta}^{(p)}$ containing Q. $\dot{K}_{\vartheta}^{(\infty)} = K_0 \cap K_1$ for $0 < \vartheta < 1$. Similarly $S_p(F_0,F_1) = [F_0^p + F_1^p]^{1/p}$ is the support function of a convex body $\dot{S}_p(K_0,K_1)$. If $\hat{K}$ signifies the polar reciprocal of K with respect to the unit sphere centered at Q, then $K_{\vartheta}^{(1)} = [(1 - \vartheta)\hat{K}_0 + \vartheta\hat{K}_1]\hat{}$ where $[(1 - \vartheta)\hat{K}_0 + \vartheta\hat{K}_1]$ is the weighted Minkowski sum of $\hat{K}_i$. A deviation $\dot{\delta}^{(p)}(K_0,K_1)$ can be defined as the g.l.b. of numbers $1/\lambda > 0$ such that $\dot{S}^{(p)}(K_0, \lambda E) \subseteq K_1$ and $\dot{S}^{(p)}(K_1,\lambda E) \subseteq K_0$ where λE is the sphere of radius λ centered at Q. This metric in the space of nondegenerate convex bodies is topologically equivalent to the usual deviation for $1 \leqq p < \infty$. The family $\{K_{\vartheta}^{(p)}\}$ is convex in ϑ monotonic decreasing (to $K_0 \cap K_1$) in p and continuous in K_i, p, ϑ. As an analogue of the Brunn-Minkowski theorem we have: $V_{\vartheta}^{1/n} \leqq 1/M_p(1/V_0^{1/n}, 1/V_1^{1/n})$ where V_{ϑ} is the volume of $K_{\vartheta}^{(p)}$, V_i that of K_i. For $1 \leqq p < \infty$, there is equality if and only if K_i are homothetic with center of homothety at Q. For $p = \infty$, $0 < \vartheta < 1$, the theorem is $V^{1/n}(K_0 \cap K_1) \leqq \min(V_0^{1/n}, V_1^{1/n})$. (Received April 25, 1960.)

570-16. R. K. Getoor: First passage times for symmetric stable processes in space.

Let $\{X(t); t \geqq 0\}$ be the symmetric stable process of index α in R^N. Define $T = \inf\{t:|X(t)| > 1\}$. Thus T is the first passage time of X(t) to the exterior of the unit ball in R^N. Recently (Abstract 567-37, Notices Amer. Math. Soc. vol. 7 (1960) p. 223) a formula for $E_x(T)$ was announced. The formula given there for the higher moments $E_x(T^k)$ is incorrect if $k > 1$. Here $E_x(\cdot)$ denotes the expectation operator under the condition $X(0) = x$. In the present paper it is proved $E_x(T^2) = \alpha\Gamma(N/2)^2[2^{\alpha}\Gamma(1 + \alpha/2)\Gamma((N + \alpha)/2)]^{-2} \cdot \int_{|x|^2}^1 (t - |x|^2)^{\alpha/2-1} F(-\alpha/2, N/2; (N + \alpha)/2; t)dt$, provided $|x| \leqq 1$. Here F is the usual hypergeometric function. (Received April 28, 1960.)

570-17. C. F. Kent: Algebraic structure of the group of recursive permutations.

Let G denote the group of permutations of the natural numbers, N; A, the group of arithmetical permutations (a permutation, f, is arithmetical if there exists an arithmetical set, M, such that f is recursive in M); R, the group of recursive permutations; F, the group of permutations which move only a finite number of numbers; A^+, the subgroup of F consisting of the even permu-

tations; and e the identity of G. Then $G \supset A \supset R \supset F \supset A^+ \supset \{e\}$. Onofri, Ann. di Mat. Ser. 4, 7, (103-130, and Schrier-Ulam, Studia, 4, (134-141), have independently shown that $G \supset F \supset A^+ \supset \{e\}$ is the unique composition series for G. Permutations on N can be factored into products of disjoint cycles. It is shown that (as in G) identical cycle-structure is a n.a.s.c. for conjugacy in A. The unique composition series for A is shown to be $A \supset F \supset A^+ \supset \{e\}$. Identical cycle-structure does not suffice for conjugacy in R. Nevertheless, it is shown that $R \supset F \supset A^+ \supset \{e\}$ is the unique composition series for R. (Received April 28, 1960.)

570-18. A. J. V. Sade: Produit direct de quasigroupes orthogonaux.

Dans, A. Sade, Quasigroupes, Marseille, 1950, puis C. R. Acad. Sci. Paris, vol. 237 (1953) pp. 372-374, a été défini un produit direct singulier pour deux quasigroupes. Ce concept, étendu aux quasigroupes orthogonaux ($xy = zt$, $x \times y = z \times t \Rightarrow x = z$, $y = t$) permet de construire des séries orthogonales d'ordre n, dans le cas où cette construction n'est pas réalisable au moyen du produit direct usuel, en particulier si $n = 4k + 2$. Ce produit peut être obtenu si, (i) M et M' sont deux quasigroupes orthogonaux admettent un sous-ensemble commun, qui soit un sous-groupe à la fois dans M et dans M', (ii) H et H' sont deux quasigroupes orthogonaux idempotents. Les produits directs singuliers MH et M'H' sont alors orthogonaux; si M et H appartiennent à des séries orthogonales, MH appartient à une série orthogonale. Si h,m,p sont les ordres respectifs de H, M et son sous-quasigroupe, l'ordre du produit MH est p+(m - p)h. La conjecture d'Euler-MacNeish, déjà infirmée par Parker, Bose and Shrikhande, reçoit ainsi une nouvelle réfutation. (Received April 20, 1960.)

570-19. N. A. Wiegmann: The *-transpose of a matrix.

If $A = A_1 + jA_2$ is a matrix with real quaternion elements (where A_1 and A_2 are complex matrices), for the transpose of A, A^T (which equals $A_1^T + jA_2^T$), many properties which hold in the complex case do not carry over to this non-commutative case. It is shown that a more suitable definition, in this sense, is that $A^* = A_1^T + A_2^T j$. First, a result is obtained for the complex case. If two complex matrices, A and B, are such that a nonsingular matrix P exists so that both $P^{-1}AP = B$ and $P^{-1}A^TP = B^T$ hold, then A and B are orthogonally similar, and conversely. Then definitions and basic properties of the *-transpose are

given, a diagonal form for *-symmetric matrices under *-congruence is obtained as well as necessary and sufficient conditions for *-orthogonal similarity, it is shown that a *-symmetric quaternion matrix is *-orthogonally similar to a known complex symmetric normal form, and, as a result, it is seen that if any two *-symmetric quaternion matrices are similar, they are *-orthogonally similar. (Received April 22, 1960.)

570-20. Leon Greenberg: Values of quadratic forms.

The following conjectures of K. Mahler are proved. 1. Let $F = \{s_k | s_k(z) = (\alpha_k z + \beta_k)/(\gamma_k z + \delta_k), \alpha_k, \beta_k, \gamma_k, \delta_k \text{ real}, \alpha_k\delta_k - \beta_k\gamma_k = 1\}$ be a Fuchsian group with compact fundamental region. Let f(u,v) be an indefinite real quadratic form. Then the values $f(\alpha_k, \gamma_k)$ are everywhere dense in the real numbers. (The case of definite forms was treated by Mahler.) 2. Let $F = \{s_k | s_k(z) = (\alpha_k z + \beta_k)/(\gamma_k z + \delta_k), \alpha_k\beta_k - \gamma_k\delta_k = 1\}$ be a Kleinian polyhedral group with compact fundamental region. Let f(u,v) be a Hermitian form. Then if f is indefinite (positive definite) the values $f(\alpha_k, \gamma_k)$ are everywhere dense in the (positive) real numbers. (Received April 29, 1960.)

570-21. Alexandra Ionescu Tulcea: A maximal ergodic theorem in information theory.

Let $(X, \mathcal{B}, P)$ be a probability space. For any $E \in \mathcal{B}$ and σ-algebra $\mathcal{C} \subset \mathcal{B}$ let $P(E|\mathcal{C})$ be the conditional probability of E relative to $\mathcal{C}$. For any finite σ-algebra $\mathcal{A} \subset \mathcal{B}$, let $\pi(\mathcal{A})$ be the finite partition of the space X induced by $\mathcal{A}$, and $\mathfrak{c}(\mathcal{A})$ the number of elements of $\pi(\mathcal{A})$. If $\mathcal{A} \subset \mathcal{B}$ is a finite σ-algebra and $\mathcal{C} \subset \mathcal{B}$ is an arbitrary σ-algebra, the conditional information of $\mathcal{A}$ relative to $\mathcal{C}$ is $I(\mathcal{A}|\mathcal{C})(x) = -\sum_{A \in \pi(\mathcal{A})} \phi_A(x) \log P(A|\mathcal{C})(x)$ (ϕ_A is the characteristic function of A). In the present note we prove the following theorem: Let $\mathcal{A} \subset \mathcal{B}$ be a finite σ-algebra, $(\mathcal{C}_n)_{0 \leq n < \infty}$ a sequence of σ-algebras such that $\mathcal{C}_n \subset \mathcal{C}_{n+1} \subset \mathcal{B}$ for every $n \geq 0$. Then, for every $t > 0$ we have $P(\{x | \sup_{0 \leq n < \infty} I(\mathcal{A}|\mathcal{C}_n)(x) > t\}) \leq \mathfrak{c}(\mathcal{A})e^{-t}$. As an immediate corollary, we deduce McMillan's theorem in information theory (see P. R. Halmos, Entropy in ergodic theory, Lecture notes, University of Chicago, 1959), with mean convergence in every L^p ($1 \leq p < \infty$), as well as the assertion that almost everywhere convergence holds in McMillan's theorem. (Received May 2, 1960.)

570-22. M. R. Nedumpilly and Francis Regan: On a generalized Feld series.

The generalized Feld series discussed in the paper is designated as the Q-series. The Q-series is defined as $Q(z) = \sum_{n=1}^{\infty} g_n(z)/(1 - f_n(z))$ where the sequences $\{f_n(z)\}$ and $\{g_n(z)\}$ are analytic functions in the neighborhood of the origin having value zero at the origin. The regions of ordinary and uniform convergence of the Q-series are determined. In the region of convergence the Q-series defines an analytic function. In the region of uniform convergence the Q-series may be expanded as a power series $\sum_{m=1}^{\infty}(\sum_{n=1}^{\infty} C_{nm})z^m$. Suppose the power series representations of $f_n(z)$ and $g_n(z)$ are $f_n(z) = \sum_{m=1}^{\infty} A_{nm}z^m$, $g_n(z) = \sum_{m=1}^{\infty} B_{nm}z^m$. Then the coefficients C_{nm} can be determined from the coefficients A_{nm} and B_{nm}. The formula for C_{nm} in terms of A_{nm}'s and B_{nm}'s has been proven by induction. The conditions for inverting power series defining analytic functions in the neighborhood of the origin into Q-series of special types are given. (Received May 2, 1960.)

570-23. Jerzy Gorski: A property of an upper envelope of pluri-subharmonic functions.

Let D be a bounded domain in the space of two complex variables and (I) $f(p) = -\log d_D(p)$, where $d_D(p)$ is the Euclidean distance of the point $p \in D$ from the boundary of D. Let h(p,q) be a function which satisfies the conditions: (1) h(p,q) is continuous with respect to the points p,q in a domain $\Delta \supset \overline{D}$, (2) $h(p,p) = 0$, $|h(p,q)| = |h(q,p)|$, (3) for fixed q, h(p,q) is analytic with respect to p. Applying the method of extremal points (Arch. Rat. Mech. Anal. vol. 4 (1960)) in the case of $E = \overline{D}$ and the function (I), we obtain the pluri-subharmonic function $u_h(p) = \int_D \log|h(p,q)|d\mu_h(q)$, which possesses the properties (1) $u_h(p) \leqq f(p) + \text{const}$ in D and (2) $u_h(p) = f(p) + \text{const}$ at every point p, which belongs to the kernel D_{μ_h} of the mass distribution μ_h. Let $\Phi(p)$ be the upper envelope of all pluri-subharmonic functions in D, which are $\leqq f(p)$ and let w(p) be the following function $w(p) = \lim \sup_{q\to p}\{\sup_h[u_h(q) - \text{const}]\}$. It is proved that $\lim \sup_{p\to p_0 \in \Sigma_h D_{\mu_h}} w(p) = \lim \sup_{p\to p_0 \in \Sigma_h D_{\mu_h}} \Phi(p) = f(p_0)$. The introduced function w(p) is a function of an extended class [Bergman, Trans. Amer. Math. Soc. vol. 63 (1948) p. 523] and has various applications in the theory of functions. (Received May 2, 1960.)

570-24. Jerzy Gorski: Plemelj's formula for the functions of two complex variables.

Let D be a bounded domain with a distinguished boundary surface in the space of two complex variables, i.e., D is bounded by $m > 2$ analytic hypersurfaces $\Phi_j(z_1,z_2,\lambda_j) = 0$, $j = 1,2,\ldots,m$ [Bergman, Mat. Sb. vol. 1 (1936) pp. 851-862]. Let z_1^0,z_2^0 be an arbitrary fixed point which lies on the intersection of no more than two boundary hypersurfaces. Suppose $f(\zeta_1,\zeta_2)$ is a continuous function defined on the distinguished boundary surface of D. Consider the function $F(z_1,z_2)$ which is defined by Bergman's integral formula $F(z_1,z_2) = -(1/4\pi^2)\sum_{j,k}\int\int_{d_{jk}} (f(\zeta_1,\zeta_2)/(\zeta_1 - z_1)(\zeta_2 - z_2))\{\Phi_j(z_1,\zeta_2,\lambda_j)/j^{(z_1,z_2,\lambda_j)} - \Phi_k(z_1,\zeta_2,\lambda_k)/k^{(z_1,z_2,\lambda_k)}\}\,d\zeta_1 d\zeta_2$, where d_{jk} is the intersection of $\Phi_j = 0$ with $\Phi_k = 0$. If the hypersurfaces, on whose intersection lies the point z_1^0,z_2^0, satisfy the Hölder condition, then there exists the interior and the exterior limit of the function $F(z_1,z_2)$, where the point z_1,z_2 tends to z_1^0,z_2^0 along a path, which is nontangiential to the boundary hypersurfaces. The difference between these limits equals to $f(z_1^0,z_2^0)$. In the case where $\Phi_j = 0$, $j = 1,2,\ldots$, satisfies more general conditions (for instance, in the case of a bicylinder) there exist three different exterior and an interior limit, depending on the path along which the point z_1,z_2 approaches to z_1^0,z_2^0. (Received May 2, 1960.)

570-25. C. A. Muses: The concept and calculus of an operational continuum. Preliminary report.

Any discrete series whatsoever may be treated as a sequence of ordinal integers. This raises the question of the analytic meaning of nonintegral, negative, imaginary and complex ordinal values of terms of such a sequence, since all integer sequences presuppose the number continuum. Thus we may validly ask for and seek the meaning of "the possible combinations of 3 1/2 things taken 1/4 at a time" or of "the number of combinations of -5 things taken 3i at a time." Such questions in their most generalized form lead to the concept of an operational continuum, taking us behind the scenes of the usual integer or "output" terms of a series, to the inner workings of the operation under consideration. Applying this approach to the particular examples given, we soon learn that "combining" or taking things n at a time implies in some fundamental sense a rotational process, as angle is explicitly involved in the nonintegral steps of the operation, as Gauss' factorial function speedily proves. Further

investigation shows that this entry of angle into the operational continuum is quite general, and that the most discrete terms (i.e. operationally most completed) of any operation representable by the general term of a series correspond to the five angles with integral sine or cosine. The concept of an integer denoting the completing of some recurrent cycle of operations emerges, as well as the interesting question of the operational continuum of the series of dimensions. The hypersphere was found specially useful in investigating the geometry of negative and imaginary dimensions. (Received May 2, 1960.)

570-26. C. A. Muses: On the unique ordering of the decimal forms.

The problem is actually one of coding. The basic difficulty in any ordinary attempt to order uniquely the decimal forms lies in the fact that an indeterminate number of zeros may preface the same decimal sequence of digits, making the simple linear sequence required for countability impossible. However, instead of saying 0.000123, for instance, let us agree to code this decimal uniquely as the integer 321,000 by a device of notational reversal. Now the unique linear sequence for all decimal forms becomes at once possible, in which 0.01 is the 10th form; 0.071, the 170th; and in general 0.abc...x, the (x...cba)th. Another confusion in this question lay in vague conceptions of "decimal form." The exact value of π thus can never be a decimal form, for a decimal form is not an infinite decimal expansion. This fact excludes any infinite decimal expansion from the decimal forms. However, we can approximate any infinite decimal expansion by a decimal form (a finite decimal expansion) as closely as we please. Thus Cantor's 1873 dictum did not prove the uncountability of decimal forms, but simply the uncountability of irrational numbers. (All infinite decimal expansions which do not represent irrational numbers may be counted by reference to their corresponding rational fractions.) All decimal forms may thus be enumerated by the simple coding method of notational reversal herein indicated. (Received May 2, 1960.)

570-27. C. A. Muses: The homeostasis of the Fibonacci series.

This homeostasis closely reproduces the original Fibonacci terms when the higher terms are divided by certain powers of 10. There is a similar phenomenon for the Auxiliary F-series (Notices Amer. Math. Soc. vol. 6 (1959) p. 558) given by $F'_n = F_{n+1} + F_{n-1}$. The deviations from the exact reproduction of either the F_n or F'_n are also expressible as other F-terms. The sets of

deviations alternate in being less and more than the terms to which these remarkable approximations tend, with the oscillation (F_n vs. F'_n) characteristic of all homeostasis. Thus $F_n/10^5$ for n = 32,...,41, are given by 21 +1, 34 + 1, 55 + 2, 89 + 3, 144 + 5, 233 + 8, 377 + 13 + 1, 610 + 21 + 1, 987 + 34 + 2, 1597 + 55 + 3. This example already shows a tertiary F-series. Indeed the Fibonacci series is thus homeostatic to the nth such order, even if n tends to infinity. Similar deviations in the F'_n series are negative, and likewise expressible in F_n terms. Explicit formulae, for which there is no space here, have been derived; as also the fact that $F_n \equiv 0 \pmod{n}$ if $n = 12k$ or 5^k, k = 0, 1,2,3, etc., and that $F_p \equiv \pm 1 \pmod{p}$ if p is prime: +1 if p ends in 1 or 9, and -1 if p ends in 2,3 or 7. Moreover, if $n = 5^k$ composed of d digits in base 10, the last d digits of F_n yield n. This behavior stems from groups of 5 (hence 10) Fibonacci terms possessing deep-seated correspondences, 5 being the key structural number in the series. Both negative F_n and F'_n terms alternate in sign. (Received May 2, 1960.)

570-28. R. H. Rosen: A weak form of the star conjecture for manifolds.

A simple proof is given of the following Theorem. Let v be a vertex of a triangulated n-manifold M; the open star of v in M is homeomorphic to E^n. The proof proceeds from Lemma 1. If X is a compact space which is the union of two open subsets each homeomorphic to E^n, then X is homeomorphic to S^n. This may be proved from Brown, Bull. Amer. Math. Soc. vol. 66 (1960) p. 76, in a manner analogous to Moise, Ann. of Math. vol. 58 (1953) p. 107. Lemma 1 in turn implies, Lemma 2. If the cone over X is locally euclidean of dimension n at the vertex, then the suspension of X is homeomorphic to S^n. This is similar to a conjecture mentioned by Mazur, Bull. Amer. Math. Soc. vol. 65 (1959) p. 65. Lemma 2 implies the theorem and also that if B is the boundary of the closed star of v, the suspension of B is topologically S^n. (Received May 2, 1960.)

570-29. Herman Rubin and J. E. Rubin: Some new forms of the axiom of choice.

The notation is as in D. Kurepa, Sur la relation d'inclusion et l'axiome de choix de Zermelo, Bull. Soc. Math. France vol. 80 (1952) pp. 225-232. Consider the thirty propositions $F \longrightarrow G$, where F and G are distinct elements of $D,\bar{D},I,\bar{I},K,\bar{K}$. If every set can be ordered, all thirty are equivalent to the

axiom of choice. Otherwise, we have not been able to show that the five with consequent $\overline{K}$ and $\overline{I} \rightarrow D$ and $\overline{D} \rightarrow I$ imply the axiom of choice; the other twenty three are equivalent to it. (Received May 2, 1960.)

570-30. Herman Rubin: Two propositions equivalent to the axiom of choice only under both the axioms of extensionality and regularity.

In all proofs of consistency of the negation of the axiom of choice of which the author is aware, either the axiom of extensionality or the axiom of regularity is weakened. We here give two forms which, with an obvious exception of one model for the second, are valid in the models constructed in all of these papers. The forms are (1) The power set of any well-ordered set can be well-ordered, and (2) Any ordered set can be well-ordered. As (2) is an immediate consequence of (1), it is sufficient to establish (1). This can be done by the use of rank and Hartogs' Theorem on the existence of ordinals. (Received May 2, 1960.)

570-31. J. R. Buchi: On a hierarchy of monadic predicate quantifiers. Preliminary report.

Let $x, x_1, x_2, \ldots$ be variables ranging over natural numbers, and over monadic predicates (sets) of natural numbers, let $\underline{i}, \underline{j}$ denote k-tuples of i's. The hierarchy $[\Sigma_m, \Pi_m]$ of predicates $P(\underline{i})$ on predicates is defined thus: Σ_0 = all predicates $P(\underline{i})$ definable by formulas $K[\underline{i}(0)] \vee (\exists x)H[\underline{i}(x), \underline{i}(x+1)] \vee (\forall x)U[\underline{i}(x)]$, whereby K, H, U are truth functions, $\Pi_m = \{\sim P \mid P \in \Sigma_m\}$, $\Sigma_{m+1} = \{(\exists \underline{j})P \mid P \in \Pi_m\}$. A k-tuple $\underline{i}$ of monadic predicates may be considered to be an infinite sequence $\underline{i}(0)\underline{i}(1)\underline{i}(2),\ldots$, whose elements are drawn from the finite alphabet consisting of all k-tuples of truth-values. Let $\underline{u}, \underline{v}, \ldots$ denote finite words on this alphabet, let $\underline{u}^\frown\underline{v}$ denote concatenation. Def: $P(\underline{i})$ is of finite rank if there is a $^\frown$-congruence relation $\underline{u} \backsim \underline{v}$ on words, with finite partition, and such that $\underline{u}_q \backsim \underline{v}_q$ implies $P(\underline{u}_1{}^\frown\underline{u}_2{}^\frown \ldots) \equiv P(\underline{v}_1{}^\frown\underline{v}_2{}^\frown\ldots)$. Theorem: The following are equivalent conditions on $P(\underline{i})$: (1) $P \in \Sigma_2$, (2) P is of finite rank, (3) $P = P_1 \cup \ldots \cup P_a$ whereby each P_c is of the form $R^\frown S^\frown S^\frown S^\frown \ldots$, R and S being regular sets of finite words. This is established by using basic facts from automata-theory (regularity), and Ramsey's theorem A (Proc. London Math. Soc. (2) vol. 30 (1930) pp. 264-286). Corollary: $\Sigma_2 = \Pi_2$ and therefore $\Sigma_n = \Pi_n = \Sigma_2$ for $n \geq 2$. (Received April 18, 1960.)

570-32. J. R. Buchi: On a problem of Tarski. Preliminary report.

Let SC be the interpreted system containing variables ranging over natural numbers, and over monadic predicates on natural numbers, the succesor function, propositional connectives, and quantifiers for both types of variables. Using theorem and definition of Abstract 570-31 one obtains: Theorem 1: The predicates $P(i)$ definable in SC are exactly those belonging to Σ_2. Theorem 2: Truth of sentences in SC is decidable. This answers a question of Tarski (See R. M. Robinson, Proc. Amer. Math. Soc. vol. 9 (1958) pp. 238-242). Note that strong assertions about infinity can be stated in SC (parts of the fan-theorem and Ramsey's theorem). Theorem 2 may be stated thus: Theorem 2': The first order theory of [Re, +, Nn, Pw] is decidable. Here Re is the set of positive reals, Nn is the set of natural numbers and Pw is the set of integral powers of 2. Let SC_{per} be like SC, except that the variables i range over ultimately periodic predicates. Theorem 3: A sentence is true in SC_{per} if and only if it is true in SC. i.e., [Re, +, Nn, Pw] and [R, +, Nn, Pw] are arithmetically equivalent. Here R stands for the set of rational numbers. (Received April 18, 1960.)

570-33. J. D. Buckholtz: A convergence theorem for sequences of polynomials.

Let $P_n(z) = \prod_{p=1}^{m_n}(1 + a_{np}z)$ be a sequence of polynomials such that, for some positive integer k, $\lim_{n\to\infty}\sum_{p=1}^{m_n}|a_{np}|^{k+1} = 0$. Let M be the set of complex numbers z for which $\lim_{n\to\infty}P_n(z)$ exists and is not zero. Then each of the following three statements implies the other two: (i) There is a complex number sequence $\{s_q\}_{q=1}^{k}$ such that $\lim_{n\to\infty}\sum_{p=1}^{m_n}(a_{np})^q = s_q$, $q = 1,2,\ldots,k$; (ii) M is not mapped into a subset of the real numbers by a nonconstant polynomial transformation of degree k or less; (iii) The sequence $\{P_n(z)\}$ converges uniformly in every bounded region to the function $\exp[\sum_{q=1}^{k}(-1)^{q-1}s_q z^q/q]$. Note: Statement (ii) is equivalent to the statement that M contains $2k + 1$ points $z = re^{i\theta}$ which are not contained by an algebraic curve of the type $\sum_{q=1}^{k} r^q(A_q\cos q\theta + B_q\sin q\theta) = 0$. In particular, if M contains $2k + 1$ points equidistant from the origin, then (ii) is true. (Received May 4, 1960.)

570-34. Adriano Garsia: Some arithmetic properties of Bernoulli convolutions.

Let $F(x,r)$ denote the infinite convolution $\ast_{n=1}^{\infty} F(x/r_n)$ where $F(x)$ represents the distribution function of a random variable which assumes the values ± 1 with equal probability and $r = (r_1, r_2, \ldots, r_n, \ldots)$ denotes a sequence of real numbers such that $\sum r_n^2 < \infty$. It is well known that $F(x,r)$ is either purely singular or absolutely continuous. Here some sufficient conditions for the singularity and some sufficient conditions for the absolute continuity are given. These conditions are of arithmetical character but they are by no means necessary. However, they yield some interesting implications in some special cases. For instance if $r(\beta) = (\beta, \beta^1, \ldots, \beta^n, \ldots)$ and $0 < \beta < 1$ then $F^p(x, r(\beta))$ (the pth convolution of F with itself) is singular for β in a small interval on the right of $1/p$. (If $0 < \beta \leqq 1/p$ the singularity of $F^p(x, r(\beta))$ is quite trivial.) If $1/\beta$ is an algebraic integer such that the product of $1/\beta$ and all of its conjugates that are in absolute value greater than 1 is in absolute value equal to 2 then $F(x, r(\beta))$ is absolutely continuous with a bounded derivative. Also a new measure theoretical proof is given of the well known result that $F(x, r(\beta))$ is singular when $1/\beta$ is a Pisot-Vijaiaraghavan number. (Received May 4, 1960.)

570-35. Hidehiko Yamabe and David Pope: An experiment by computing machine on the four color problem.

The four color problem can easily be reduced to the case when at each vertex three boundaries meet. If this is the case the problem can be further reduced to a three 'color' problem for the 1-dimensional complex consisting of boundaries and vertices such that at each vertex three different colors meet. This is equivalent to a two 'color' problem to assign either +1 or -1 such that the sum of such values of vertices on the boundary of each country is congruent to zero modulo 3. A searching routine, using the Univac 1103 computer, has been coded to solve this assignment problem. All relevant permutations of ± 1 are generated and tested, and the assignments satisfying the congruence property are printed out. The searching routine takes about 40 minutes for a map of 36 vertices and 20 countries. In this experiment not more than 73 inequivalent solutions were found. This exhausts all possible ways of coloring the above map. (Received May 4, 1960.)

570-36. George Maltese: Spectral representations for abstract functional equations.

Let Z be a locally compact Abelian group and let $L^1(Z,m)$ denote the corresponding convolution algebra of Z. Let H be a Hilbert space and B(H) the set of bounded linear transformations of H into H. Let $z \longrightarrow U_z$ be a mapping of Z into B(H) such that (for all $s,t \in Z$) (S) $aU_{s+\alpha(t)} + bU_{s+\beta(t)} = g(t)U_sU_{\phi(t)}$ where a,b are complex numbers, α,β,ϕ are m-measure preserving homeomorphisms of Z onto Z, and g is a continuous (nonzero) function on Z. A continuous ($\not\equiv 0$) complex-valued function χ is called a character of (S) if (for all $s,t \in Z$) $a\chi(s + \alpha(t)) + b\chi(s + \beta(t)) = g(t)\chi(s)\chi(\phi(t))$. Under certain weak continuity conditions the following spectral representation is valid (for all $z \in Z, x,y \in H$): $(U_zx|y) = \int_{E(r)} \chi(z)d\mu_{x,y}(\chi)$ where E(r) is a locally compact set of characters and $\mathcal{F} = (\mu_{x,y})_{x,y \in H}$ is a spectral family on E(r). The recent results of Kurepa (Canadian J. Math. vol. 12 (1960) pp. 45-50) are obtained as particular cases of the spectral representation theorem. Results concerning measurability and continuity of the solutions of (S) are obtained. Spectral representations for systems of functional equations are also presented. (The above results may be proved in the setting of generalized convolution algebras.) (Received May 5, 1960.)

570-37. George Maltese: Convex ideals in ordered generalized convolution algebras.

If $\{Z, m_z \text{ (real)}, m, (f,g) \longrightarrow f * g\}$ is a generalized convolution algebra (see for instance C. Ionescu Tulcea and A. Simon, Proc. Nat. Acad. Sci. U.S.A. vol. 45 (1959) pp. 1765-1767), the subset of all real-valued m-integrable functions form an ordered ring L^1_R with respect to the convolution product and pointwise addition and ordering. An ideal $m_R \subset L^1_R$ is said to be a convex ideal if $f,g \in m_R$, $h \in L^1_R$ and $f \leqq h \leqq g$ implies that $h \in m_R$. An ideal $m_R \subset L^1_R$ is said to be absolutely convex if $f \in m_R$, $g \in L^1_R$ and $|g| \leqq |f|$ implies that $g \in m_R$. Generalizing a result of Aubert (Math. Scand. vol. 6 (1958) pp. 181-188) the following is proved: Theorem (A) Every (proper) regular maximal convex ideal $m_R \subset L^1_R$ is the kernel of a positive multiplicative linear functional defined on L^1_R. (B) The algebra L^1_R does not contain any (proper) absolutely convex ideal which is either closed or regular. (The method of proof does not depend on a duality theory which is essentially used in Aubert's proof.) A (new)

proof is given for the existence of a positive multiplicative measure on certain generalized convolution algebras. In fact a more general existence theorem for multiplicative linear functionals on ordered algebras is proved. (Received May 5, 1960.)

570-38. D. G. Higman and J. E. McLaughlin: Geometric ABA-groups. I.

A group of collineations of an incidence system will be called sharply transitive if it is transitive on the configurations consisting of an incident point and line. By a sharply transitive representation of a group G on an incidence system Σ will be meant a homomorphism of G onto a sharply transitive group of collineations of Σ. A finite incidence system will be called a 2-design of type (h,k) if each point lies on $h \geq 2$ lines, each line contains $k \geq 2$ points, and each pair of points lies on exactly one line. It is shown that a finite group G admits a sharply transitive representation on a 2-design if and only if it possesses subgroups A and B such that (1) $G = ABA$, (2) $AB \cap BA = A + B$, and (3) $A \not\subseteq B$ and $B \not\subseteq A$. Such a group we call a geometric ABA-group. A finite group G admits a sharply transitive representation θ on a projective plane π if and only if it is a geometric ABA-group satisfying (3') $A:A \cap B = B:A \cap B \geq 3$; and then π is Desarguesian and $\theta(G)$ contains the Little Desargues group if and only if $G = A + A \times A$. As an application it is shown that a finite simple group satisfying Steinberg's axioms [Canad. J. Math. vol. 9 (1957) p. 347], with the symmetric group of degree 3 as Weyl group, is a Little Desargues group. (Received May 2, 1960.)

570-39. D. G. Higman and J. E. McLaughlin: Geometric ABA-groups. II.

A necessary and sufficient condition that a finite group G be a sharply regular group of collineations of a 2-design, in the sense that it be a sharply transitive group in which no element $\neq 1$ fixes an incident point and line, is that G be a geometric ABA-group such that $A \cap B = 1$ [see Geometric ABA-groups I, Abstract 570-38, for definitions]. This means precisely that G is an independent ABA-group in the sense of Gorenstein [Canad. J. Math. vol. 11 (1959) pp. 39-42]. It is shown that the only Desarguesian projective planes admitting sharply regular groups of collineations are those of orders 2 and 8. It follows that these are the only Desarguesian cyclic planes generated by perfect residue difference sets as defined by E. Lehmer [Canad. J. Math. vol. 5 (1953) pp. 425-432]. It is shown moreover that a sharply transitive group on a

Desarguesian projective plane contains the Little Desargues group, if it is not sharply regular. Some analogous results are obtained for affine planes.

(Received May 2, 1960.)

ERRATA, Volume 6

Seymour Sherman: Combinatorial aspects of the Ising model for ferromagnetism. I. A conjecture of Feynman on paths and graphs.

Page 554, Abstract 560-34. At the end of line 3 change "D_{j_i}" to "$D_{j_i}^{\mu_i}$".

Line 4. Change "of $D_{j_{i+1}}$" to "of $D_{j_{i+1}}^{\mu_{i+1}}$".

Line 9. Change "(D_{j_i})" to "$\phi(D_{j_i})$".

C. A. Muses: The geometry of equi-infinitesimals.

Page 764, Abstract 563-22. Line 7 on page 765. Interchange "Fermat" and "Mersenne".

Line 9 on page 765. Insert "at 47 units" after "discontinuity".

ERRATUM, Volume 7

B. E. Rhoades: Totally conservative matrices.

Page 238. Abstract 567-91. Replace line 8 by "a monotone increasing unbounded sequence. There A is not".

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