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MEETINGS
CALENDAR OF MEETINGS

NOTE: This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the NOTICES was sent to press. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

<table>
<thead>
<tr>
<th>Meeting No</th>
<th>Date</th>
<th>Place</th>
<th>Deadline for Abstracts</th>
</tr>
</thead>
<tbody>
<tr>
<td>578</td>
<td>April 6-8, 1961</td>
<td>New York, New York</td>
<td>Feb. 22</td>
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<tr>
<td>580</td>
<td>April 22, 1961</td>
<td>Stanford, California</td>
<td>Feb. 22</td>
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<tr>
<td>581</td>
<td>June 13-16, 1961</td>
<td>Seattle, Washington</td>
<td>April 28</td>
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<tr>
<td>582</td>
<td>August 29-September 1, 1961</td>
<td>Stillwater, Oklahoma</td>
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<tr>
<td>583</td>
<td>October 28, 1961</td>
<td>Cambridge, Massachusetts</td>
<td>Sept. 14</td>
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<td>November 17-18, 1961</td>
<td>Milwaukee, Wisconsin</td>
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<td>November 18, 1961</td>
<td>Santa Barbara, California</td>
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<td></td>
<td>January, 1962</td>
<td>Cincinnati, Ohio</td>
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<td></td>
<td>(68th Annual Meeting)</td>
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<td></td>
<td>August, 1962</td>
<td>Vancouver, British Columbia</td>
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<td></td>
<td>(67th Summer Meeting)</td>
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<tr>
<td></td>
<td>January, 1963</td>
<td>Berkeley, California</td>
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<td></td>
<td>(69th Annual Meeting)</td>
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<td></td>
<td>August, 1963</td>
<td>Boulder, Colorado</td>
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<td></td>
<td>(68th Summer Meeting)</td>
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</tbody>
</table>

*The abstracts of papers to be presented at the meetings must be received in the Headquarters Offices of the Society in Providence, Rhode Island, on or before these deadlines. The deadlines also apply to news items.

The NOTICES of the American Mathematical Society is published by the Society seven times a year, in February, April, June, August, October, November, and December. Price per annual volume is $7.00. Price per copy, $2.00. Special price for copies sold at registration desks of meetings of the Society, $1.00 per copy. Subscriptions, orders for back numbers (none available before 1958), and inquiries should be addressed to the American Mathematical Society, 1350 Main Street, Ann Arbor, Michigan, or to 190 Hope Street, Providence 6, Rhode Island.

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772
The sixty-seventh Annual Meeting of the American Mathematical Society will be held at the Hotel Willard in Washington, D. C., from Monday, January 23 to Thursday, January 26, 1961. All sessions will be held in public rooms of the hotel. During the same week with headquarters at the same hotel there will be meetings of the Association for Symbolic Logic, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics.

The thirty-fourth Josiah Willard Gibbs Lecture will be delivered by Professor J. J. Stoker of New York University. Professor Stoker will speak at 8:00 P. M. on Tuesday, January 24, in the Grand Ballroom on the tenth floor on "Problems in nonlinear elasticity."

By invitation of the Committee to Select Hour Speakers for Summer and Annual Meetings, Professor Lars Hormander of the University of Stockholm and The Institute for Advanced Study will address the Society in the Grand Ballroom at 2:00 P. M. on Monday. The title of his address is "On the range of differential operators."

By invitation of the same committee, Professor Helmut Wielandt of the University of Tubingen and the California Institute of Technology will address the Society in the Grand Ballroom at 2:00 P. M. on Wednesday. The title of his address is "On the structure of finite groups."

Sessions for contributed papers will be held on the morning and afternoon of each day that the Society is meeting. Rooms used for this purpose will be the Congressional Room on the ground floor, the Cabinet and Presidential Rooms on the first floor and the Grand Ballroom and South Ballroom on the tenth floor. The projecting machine called Vu-Graph will be available. The sessions will not include any provision for late papers. There will be no supplementary program.

There will be a business meeting of the Society at 2:00 P.M. on Thursday, January 26, in the Grand Ballroom.

The programs of the mathematical organizations meeting jointly with this Society will be found in the section headed Activities of Other Associations, which begins on page 815 of this issue. There is a time-table of meetings open to members and guests and of committee meetings beginning on page 776. The listing of committee meetings is not guaranteed to be complete.
COUNCIL

The Council of the Society will meet at 5:00 P.M. on Wednesday, January 25, in the Executive Room on the first floor and will reconvene after an intermission for dinner.

EMPLOYMENT REGISTER

The Employment Register will function on Tuesday, Wednesday and Thursday, January 24, 25, and 26, from 9:00 A.M. till 5:00 P.M. with headquarters in Room 228 and additional space for interviews in Rooms 223 and 224.

REGISTRATION

Registration headquarters will be located in the Caucus Room on the first floor (up one flight) of the Hotel Willard. The Registration Desk will be open on Sunday, January 22, from 2:00 P.M. till 8:00 P.M., on Monday through Thursday, January 23-26, from 9:00 A.M. till 5:00 P.M., and on Friday, January 27, from 9:00 A.M. till 2:00 P.M. All persons attending the meetings are requested to register at the headquarters on arrival. The directory of registration and an information service will be maintained at these headquarters.

The registration fee will be $2.00 for each member of any participating organization and $.50 for each accompanying adult. The fee to non-members is $5.00, except that there is no fee for students.

EXHIBITS

Various publishers will have exhibits in the Jackson Room and the Caucus Room on the first floor on Tuesday through Thursday from 9:00 A.M. to 5:00 P.M.

There will be a demonstration of the Vu-Graph on Monday through Friday from 9:00 A.M. to 1:00 P.M. and from 2:00 P.M. to 5:00 P.M. in Room 212, which will include instruction of speakers who wish to use the machine.

The U. S. Naval Weapons Laboratory will conduct tours of their computation Center at Dahlgren at the time of the meeting. The equipment which can be seen will include the NORC, the IBM 7090, the Universal Data Transcriber, and the Space Surveillance Operations Center. The NWL will have its own desk in the registration area for people to sign up for a tour. The NWL will provide transportation to and from Dahlgren.

PRESS ROOM

Room 221 has been reserved as a press room. Typewriters will be available.
ACCOMMODATIONS

The Willard Hotel is the official hotel for this meeting. The Willard, the Washington Hotel (next door to the Willard), and the Raleigh Hotel (two blocks away) are cooperating in reserving blocks of rooms. There is a reservation blank in the back of this issue of the NOTICES: It should be mailed to the Hotel Willard for a reservation at any of the three cooperating hotels. It is important both to the Society and to the individual that the form be used, first because the hotel room rates have been negotiated and second because the potential of the Society to make suitable future arrangements with other hotels depends on the records of attendance and related hotel reservations.

The Y.M.C.A., 15th and G Streets, N. W., and the Y.W.C.A., 17th and K Streets, N. W., are not far from the Hotel Willard. The Marriott Motor Hotels, Twin Bridges on U. S. I, Washington 1, D.C. is also near the Willard and will have rooms available. For reservations with any of these three institutions, one should write directly.

TRAVEL INFORMATION

The Willard Hotel is located at 14th and Pennsylvania Avenue, N. W. Taxicab and limousine service is available from Washington National Airport directly to the hotel. The limousine fare is $1.20. Any street car serving northwest Pennsylvania Avenue from Union Station will stop opposite the hotel. It is best to take a taxicab from the bus terminal. The fare is $.50. Those arriving by auto can follow U. S. I until it joins 14th street near the Washington Monument, which is three blocks from the Willard Hotel.

The Washington Hotel is located at 15th and Pennsylvania Avenue, Northwest, adjoining the Willard. The Raleigh Hotel is located at 12th and Pennsylvania Avenue, Northwest, two blocks from the Willard.

COMMUNICATIONS

Mail and telegrams for those attending the meetings can be addressed in care of the American Mathematical Society, Hotel Willard, Washington 4, D.C.

COMMITTEE ON ARRANGEMENTS

The Committee on Arrangements for this meeting is a joint committee of the Mathematical Association of America and of this Society, and cooperates with the other associations making up the joint meeting. It consists of H. L. Alder, J. W. Brace, M. W. Oliphant, Everett Pitcher (Chairman), and G. L. Walker.
<table>
<thead>
<tr>
<th>Time</th>
<th>Event Description</th>
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<tbody>
<tr>
<td>SUNDAY, January 22</td>
<td><strong>American Mathematical Society</strong></td>
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<tr>
<td>2:00 p.m. - 8:00 p.m.</td>
<td>REGISTRATION - CAUCUS ROOM</td>
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<tr>
<td>MONDAY, January 23</td>
<td><strong>Mathematical Association of America</strong></td>
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<tr>
<td>9:00 a.m. - 5:00 p.m.</td>
<td>REGISTRATION - CAUCUS ROOM</td>
</tr>
<tr>
<td>9:00 a.m. - 1:00 p.m., 2:00-5:00 p.m.</td>
<td>VU-GRAPH DEMONSTRATION - ROOM 212</td>
</tr>
</tbody>
</table>
| 10:00 a.m. - 12:00 p.m. | Sessions for Contributed Papers  
Analysis, Presidential Parlors  
Algebra, Congressional Room  
Applied Mathematics, Cabinet Room  
Statistics and Probability, South Ballroom |
| 2:00 p.m. - 3:00 p.m. | Invited Address - Grand Ballroom  
Lars Hörmander: On the Range of Differential Operators |
| 3:15 p.m. - 5:00 p.m. | Sessions for Contributed Papers  
Analysis, Presidential Parlors  
Analysis, Cabinet Room  
Algebra and Theory of Numbers, Congressional Room  
Topology, South Ballroom |
**TIME TABLE**
(Eastern Standard Time)

<table>
<thead>
<tr>
<th>TUESDAY, January 24</th>
<th>American Mathematical Society</th>
<th>Mathematical Association of America</th>
<th>Society for Industrial and Applied Mathematics</th>
<th>Association for Symbolic Logic</th>
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<tr>
<td>9:00 a.m. - 5:00 p.m.</td>
<td><strong>REGISTRATION - CAUCUS ROOM</strong></td>
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<td>9:00 a.m. - 1:00 p.m., 2:00 p.m. - 5:00 p.m.</td>
<td><strong>VU - GRAPH DEMONSTRATION - ROOM 212</strong></td>
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<td>9:00 a.m. - 1:00 p.m., 2:00 p.m. - 5:00 p.m.</td>
<td><strong>EMPLOYMENT REGISTER - ROOM 228</strong></td>
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<td>9:00 a.m. - 1:00 p.m., 2:00 p.m. - 5:00 p.m.</td>
<td><strong>NATIONAL REGISTER FOR SCIENTIFIC AND TECHNICAL PERSONNEL - ROOM 228</strong></td>
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<td>9:00 a.m. - 5:00 p.m.</td>
<td><strong>BOOK EXHIBITS (JACKSON ROOM AND CAUCUS ROOM)</strong></td>
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<td>9:30 a.m.</td>
<td>Sessions for Contributed Papers</td>
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<td>10:00 a.m. - 12:00 m.</td>
<td>Analysis, Presidential Parlors</td>
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<td>Algebra, Congressional Room</td>
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<td></td>
<td>Applied Mathematics, South Ballroom</td>
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<td></td>
<td>Geometry, Cabinet Room</td>
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<td>11:00 a.m.</td>
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<td>2:00 p.m.</td>
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<td>3:00 p.m.</td>
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<td>*Executive and Finance Committee - Room 220</td>
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<td>*to be continued in evening</td>
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**GRAND BALLROOM**

Contributed Papers
R. C. Lawlor
C. J. Maloney
H. H. Schneider
M. Davis and H. Putnam

Invited Address
Abraham Robinson

Council - Monroe Room
Presidential Address - Grand Ballroom
S. C. Kleene
<table>
<thead>
<tr>
<th>Time</th>
<th>American Mathematical Society</th>
<th>Mathematical Association of America</th>
<th>Society for Industrial and Applied Mathematics</th>
<th>Association for Symbolic Logic</th>
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<tbody>
<tr>
<td>3:15 p.m. - 5:00 p.m.</td>
<td>Sessions for Contributed Papers Analysis, Presidential Parlors Algebra, Congressional Room Applied Mathematics, South Ballroom Topology, Cabinet Room Logic and Foundations, Grand Ballroom</td>
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<tr>
<td>4:30 p.m.</td>
<td>Gibbs Lecture - Grand Ballroom</td>
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<td>Council - Executive Room</td>
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<tr>
<td>8:00 p.m.</td>
<td>J. J. Stoker: Problems in nonlinear elasticity</td>
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<td>Time</td>
<td>American Mathematical Society</td>
<td>Mathematical Association of America</td>
<td>Society for Industrial and Applied Mathematics</td>
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<td>9:00 a.m. - 1:00 p.m., 2:00-5:00 p.m.</td>
<td>VU - GRAPH DEMONSTRATION - ROOM 212</td>
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<td>9:00 a.m. - 1:00 p.m., 2:00-5:00 p.m.</td>
<td>EMPLOYMENT REGISTER - ROOM 228</td>
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<td>9:00 a.m. - 5:00 p.m.</td>
<td>BOOK EXHIBIT (JACKSON ROOM AND CAUCUS ROOM)</td>
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<tr>
<td>9:15 a.m. - 12:05 p.m.</td>
<td>First Session - Grand Ballroom</td>
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<tr>
<td>10:00 a.m. - 12:00 m.</td>
<td>Sessions for Contributed Papers Analysis, Presidential Parlors Topology, South Ballroom Geometry, Cabinet Room</td>
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<td>12:00 m.</td>
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<tr>
<td>1:00 p.m.</td>
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<td>Invited Address - Grand Ballroom</td>
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<td>2:00 p.m. - 3:00 p.m.</td>
<td>Helmut Wielandt: On the Structure of Finite Groups</td>
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<tr>
<td>3:15 p.m. - 5:00 p.m.</td>
<td>Sessions for Contributed Papers Analysis, Presidential Parlors Analysis, Cabinet Room Algebra, Congressional Room Topology, South Ballroom</td>
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<tr>
<td>5:00 p.m.</td>
<td>Council - Executive Room (Recess for dinner)</td>
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<td>7:30</td>
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<td></td>
<td>Presidential Address - Grand Ballroom</td>
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<td>Brockway McMillan</td>
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<td>TIME TABLE</td>
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<tr>
<td><strong>THURSDAY, January 26</strong></td>
<td>American Mathematical Society</td>
<td>Mathematical Association of America</td>
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<td>9:00 a.m. - 5:00 p.m.</td>
<td>REGISTRATION - CAUCUS ROOM</td>
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<td>9:00 a.m. - 1:00 p.m., 2:00 - 5:00 p.m.</td>
<td>VU - GRAPH DEMONSTRATION - ROOM 212</td>
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<td>9:00 a.m. - 1:00 p.m., 2:00 - 5:00 p.m.</td>
<td>EMPLOYMENT REGISTER - ROOM 228</td>
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<td>9:00 a.m. - 5:00 p.m.</td>
<td>NATIONAL REGISTER FOR SCIENTIFIC AND TECHNICAL PERSONNEL - ROOM 228</td>
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<td>9:00 a.m.</td>
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<td>9:15 a.m. - 12:05 p.m.</td>
<td>Trustees - Executive Room</td>
<td>Second Session - Grand Ballroom</td>
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<tr>
<td>10:00 a.m. - 12:00 m.</td>
<td>Sessions for Contributed Papers</td>
<td>9:15 a.m. G. C. Lorentz</td>
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<td></td>
<td>Analysis, Presidential Parlors</td>
<td>10:15 a.m. Business Meeting</td>
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<td></td>
<td>Analysis and Applied Mathematics,</td>
<td>11:15 a.m. A. M. Gleason</td>
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<td>South Ballroom</td>
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<td>Algebra, Congressional Room</td>
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<td>Statistics and Probability, Cabinet Room</td>
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<td>2:00 p.m.</td>
<td>Business Meeting - Grand Ballroom</td>
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<td>3:00 p.m. (Or Earlier)</td>
<td>Conference Board - Executive Room</td>
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<td>3:15 p.m. - 5:00 p.m.</td>
<td>Sessions for Contributed Papers</td>
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<td>Analysis, Presidential Parlors</td>
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<td>Analysis, Cabinet Room</td>
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<td>Algebra and Theory of Numbers,</td>
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<td>Congressional Room</td>
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<td></td>
<td>Topology, South Ballroom</td>
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| TIME TABLE  
(Eastern Standard Time) |
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<tbody>
<tr>
<td>FRIDAY, January 27</td>
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<td>9:00 a.m. - 2:00 p.m.</td>
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<td>9:00 a.m. - 2:00 p.m.</td>
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<tr>
<td>9:15 a.m. - 12:05 p.m.</td>
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<td>2:00 p.m. - 4:50 p.m.</td>
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The time limit for each contributed paper is ten minutes, however, the papers are scheduled at 15 minute intervals. Thus the audience has an opportunity to circulate between the various sessions, and those who are interested in a particular paper are certain of the exact time at which it will be presented. To achieve these objectives, the time limit will be strictly enforced.

MONDAY, 10:00 A.M.

Session on Analysis, Presidential Parlors
10:00 - 10:10
(1) A note on the theory of analytic functions in Banach algebras
   Professor W. A. J. Luxemburg, California Institute of Technology (576-212)

10:15 - 10:25
(2) A representation theorem for a class of lattice ordered real Banach algebras
   Professor Ronald McHaffey, University of Massachusetts (576-150)

10:30 - 10:40
(3) Involutions on annihilator algebras. Preliminary report
   Mr. Neill McShane, Yale University (576-124)

10:45 - 10:55
(4) A locally convex algebra of analytic functions
   Dr. Louis Brickman, Yale University (576-91)

11:00 - 11:10
(5) A cosine functional equation in Banach algebras
   Mr. Svetozar Kurepa, University of Maryland (576-107)

11:15 - 11:25
(6) The second conjugate space of a Banach algebra as an algebra
   Professor Paul Civin, University of Florida and Professor Bertram Yood, University of Oregon (576-69)

11:30 - 11:40
(7) On the adjoint of the closed span of multiplicative functionals. Preliminary report
   Professor F. T. Birtel, The Ohio State University (576-11)

11:45 - 11:55
(8) Partial order and spectral theory. Preliminary report
   Professor H. H. Schaefer, University of Michigan (576-22)
Session on Algebra, Congressional Room
10:00 - 10:10
(9) Lattice isomorphisms of finite non-abelian groups of exponent p
   Professor R. F. Spring, Ohio University (576-132)
10:15 - 10:25
(10) Groups of associative ring multiplications
   Mr. F. Lane Hardy, Emory University (576-217)
10:30 - 10:40
(11) Kurosh radicals that are nil on rings with D. C. C.
   Professor N. Divinsky, University of British Columbia (576-215)
10:45 - 10:55
(12) Trace forms on Lie algebras
   Professor Richard Block, California Institute of Technology (576-207)
11:00 - 11:10
(13) Characterizations of generalized uniserial rings, III
   Professor Drury W. Wall, University of Iowa (576-176)
11:15 - 11:25
(14) Structure of prime rings. Preliminary report
   Professor Carl Faith, Institute of Advanced Study and Pennsylvania State University (576-190)
11:30 - 11:40
(15) Subrings of algebraic number fields
   Professor R. A. Beaumont and Professor R. S. Pierce, University of Washington (576-46)
11:45 - 11:55
(16) Rings of integer-valued continuous functions
   Professor R. S. Pierce, University of Washington (576-50)

Session on Applied Mathematics, Cabinet Room
10:00 - 10:10
(17) Some results on asymptotic expansions
   Dr. John T. Moore, University of Florida (576-173)
10:15 - 10:25
(18) An asymptotic expansion of the error of numerical quadrature schemes
   Dr. Seymour Haber, National Bureau of Standards, Washington, D. C. (576-182)
10:30 - 10:40
(19) On calculating solutions of quasi-linear, first order, partial differential equations
   Dr. Avron Douglis, University of Maryland (576-200)
10:45 - 10:55
(20) On multi-line iterative methods for elliptic difference equations
Professor Seymour V. Parter, Cornell University (576-122)

11:00 - 11:10
(21) Calculation of pressure pulse transmission in solids. Preliminary report
Dr. Mary F. Gourley, Austin College and C. R. Cassity, General Electric Company, Philadelphia, Pennsylvania (576-104)

11:15 - 11:25
(22) Analysis of the bi-stable oscillations in the parametron by Lyapunov's method
Dr. R. A. Willoughby, IBM Research Center, Yorktown Heights, New York (576-188)

11:30 - 11:40
(23) An existence theorem for an N-body problem of classical electrodynamics
Dr. R. D. Driver, RIAS, Baltimore, Maryland (576-203)

11:45 - 11:55
(24) A general method for the approximate solution of ordinary differential equations of first order
Professor Diran Sarafyan, Utah State University (576-224)

Session on Statistics and Probability, South Ballroom
10:00 - 10:10
Professor Bayard Rankin, Case Institute of Technology (576-218)

10:15 - 10:25
(26) A decomposition of a continuous parameter stochastic process. Preliminary report
Professor Guy Johnson, Jr., Rice University (576-202)

10:30 - 10:40
(27) Convergence in measure and convergence of distributions on topological spaces
Professor Herman Rubin, Michigan State University (576-163)

10:45 - 10:55
(28) Hölder conditions for realizations of Gaussian processes
Dr. Zbigniew Ciesielski, Cornell University (576-81) (Introduced by R. J. Walker)
11:00 - 11:10  
(29) The 2-sided absorption problem for asymmetric Cauchy processes  
Professor Harold Widom, Cornell University (576-63)

11:15 - 11:25  
(30) Markov chains and eigenvectors. Preliminary report  
Mr. Eugene Albert, University of Virginia (576-49)

11:30 - 11:40  
(31) Use of general regressors for smoothing and estimation of trend  
Professor Andrew Sobczyk, University of Miami (576-73)

11:45 - 11:55  
(32) A limit theorem for a function of the increments of a decomposable process  
Professor Robert Cogburn and Professor Howard G. Tucker, University of California, Berkeley (576-2)

MONDAY, 2:00 P.M.

Invited Address, Grand Ballroom  
On the range of differential operators (One hour)  
Professor Lars Hormander, University of Stockholm and The Institute for Advanced Study

MONDAY, 3:15 P.M.

Session on Analysis, Presidential Parlors  
3:15 - 3:25  
(33) Self-adjoint Wiener-Hopf operators. Preliminary report  
Professor M. Rosenblum, University of Virginia (576-165)

3:30 - 3:40  
(34) The Ilstow and Feynman integrals  
Professor R. H. Cameron, University of Minnesota (576-145)

3:45 - 3:55  
(35) A note on translation invariants  
Mr. R. L. Adler and Dr. A. G. Konheim, IBM Research Center, Yorktown Heights, New York

4:00 - 4:10  
(36) Trigonometrical series with quasi-monotone coefficients  
Professor S. M. Shah, University of Kansas (576-23)

4:15 - 4:25  
(37) Generalized Hermite polynomials  
Mr. H. W. Gould and Mr. A. T. Hopper, West Virginia University (576-89)
4:30 - 4:40
(38) Mean convergence of orthogonal series
Mr. R. A. Askey, Washington University (576-141)

Session on Analysis, Cabinet Room
3:15 - 3:25
(39) Nonsymmetric projections in Hilbert space
Professor Victor J. Mizel and Professor M. M. Rao,
Carnegie Institute of Technology (576-232)
3:30 - 3:40
(40) Algebras of operators and of continuous functions
Professor Leopoldo Nachbin, Brandeis University and
the Institute for Pure and Applied Mathematics, Rio de
Janeiro, Brazil (576-96)
3:45 - 3:55
(41) Extension of a theorem of Beurling
Professor H. S. Shapiro, New York University (576-191)
4:00 - 4:10
(42) A convexity condition in B-space and the strong law of large
numbers
Professor Anatole Beck, University of Wisconsin and
Cornell University (576-16)
4:15 - 4:25
(43) Extensions of linear functionals
Professor R. E. Fullerton, University of Maryland
(576-152)
4:30 - 4:40
(44) Absolutely continuous contractions
Professor Morris Schreiber, Cornell University
(576-230)
4:45 - 4:55
(45) Existence of compact linear maps between Banach spaces
Professor Seymour Goldberg, New Mexico State Univer-
sity and Professor A. H. Kruse, New Mexico State Uni-
versity and University of Kansas (576-17)

Session on Algebra and Theory of Numbers, Congressional Room
3:15 - 3:25
(46) Finite stability of a system of inequalities
Dr. Edmund Eisenberg, Hughes Aircraft Company, Los
Angeles, California (576-119)
3:30 - 3:40
(47) On a theorem on free Boolean algebras
Professor Philip Dwinger, Purdue University (576-92)
3:45 - 3:55
(48) The nonexistence of certain differences sets, Preliminary
report
Mr. Richard Turyn, Litton Systems, Incorporated
(576-121)
4:00 - 4:10
(49) Numbers which can be added to normal numbers yielding normal sums
Dr. Roy Leipnik, Naval Ordnance Test Station, China Lake, California and Dr. J. E. Maxfield, University of Florida (576-68)

4:15 - 4:25
(50) Generalization of two theorems of Romanov in additive number theory
Professor Georg J. Rieger, Purdue University (576-187)

4:30 - 4:40
(51) Congruence properties of $\sigma_T(n)$
Professor M. V. Subba Rao and Professor Vincent C. Harris, University of Missouri (576-161)

4:45 - 4:55
(52) Negative discriminants of binary quadratic forms with a single class in each genus
Professor Emil Grosswald, University of Pennsylvania (576-35)

Session on Topology, South Ballroom
3:15 - 3:25
(53) Separability of connected, locally connected, metric spaces
Mr. Prabir Roy, University of North Carolina (576-138)

3:30 - 3:40
(54) A question of Katětov concerning the Hilbert parallelootope
Professor V. L. Klee, Jr., University of Washington (576-156)

3:45 - 3:55
(55) Sheaves with prescribed local structure
Dr. Johann Sonner, University of South Carolina (576-24)

4:00 - 4:10
(56) Real commutative semigroups on the plane. II
Professor J. G. Horne, University of Georgia (576-194)

4:15 - 4:25
(57) On the topology of homeomorphism group of SLH space
Mr. Wu Ta-Sun, St. Mary's Dominican College (576-174)

4:30 - 4:40
(58) Fan-Gottesman compactifications and filter spaces. Preliminary report
Professor F. J. Wagner, Marquette University (576-195)

4:45 - 4:55
(59) Conditions for equality of the inductive dimensions
Professor Louis F. McAuley, University of Wisconsin (576-234)
TUESDAY, 10:00 A.M.

**Session on Analysis**, Presidential Parlors

10:00 - 10:10

(60) On an extension of Bernstein's theorem to meromorphic functions.

Professor A. J. Macintyre, University of Cincinnati and
Professor S. M. Shah, University of Kansas (576-168)

10:15 - 10:25

(61) Abelian and Tauberian theorems for orthogonal series on matrix spaces

Professor Josephine Mitchell, The Pennsylvania State University (576-222)

10:30 - 10:40

(62) Polynomial interpolation in points equi-distributed on the unit circle

Professor J. H. Curtiss, University of Miami (576-180)

10:45 - 10:55

(63) On the zeros of sections of the exponential series

Mr. J. D. Buckholtz, University of North Carolina (576-143)

11:00 - 11:10

(64) Meromorphic functions with sectors free of zeros and poles

Mr. Simon Hellerstein, Syracuse University (576-123)

11:15 - 11:25

(65) Meromorphic functions with small characteristic and no asymptotic values

Professor G. L. MacLane, Rice University (576-120)

11:30 - 11:40

(66) On the number of deficient values of entire functions of finite order

Professor Albert Edrei, Syracuse University and Professor W. H. J. Fuchs, Cornell University (576-102)

11:45 - 11:55

(67) Perron-Frobenius theory and the zeros of polynomials

Professor Herbert S. Wilf, University of Illinois (576-14)

**Session on Algebra**, Congressional Room

10:00 - 10:10

(68) A description of all globally idempotent threads with zero

Professor C. R. Storey, Florida State University (576-185)

10:15 - 10:25

(69) Conditions for the modularity of an orthomodular lattice

Professor D. J. Foulis, Wayne State University (576-59)
10:30 - 10:40
(70) On a class of nonflexible algebras
Dr. Frank Kosier, University of California, Berkeley
(576-37)

10:45 - 10:55
(71) Compact connected topological dissociative commutative loops
Dr. K. H. Hofmann, Tulane University
(Introduced by Professor A. D. Wallace)

11:00 - 11:10
(72) The group of commutative Schreier semigroup extensions of a group
Mr. V. Ray Hancock, Virginia Polytechnic Institute
(576-56)

11:15 - 11:25
(73) Concerning o-complete o-groups
Mr. J. G. Harvey, Tulane University (576-78)

11:30 - 11:40
(74) p-automorphisms of solvable groups
Mr. J. L. Alperin, University of Chicago (576-140)

11:45 - 11:55
(75) Primitive algebras with involution
Dr. W. S. Martindale, 3rd, Smith College (576-171)

Session on Applied Mathematics, South Ballroom

10:00 - 10:10
(76) Heat transfer coefficient of freezing metal
Professor Hans Bueckner, University of Wisconsin and
Dr. G. Horvay, General Electric Company, Schenectady, New York (576-9)
(Introduced by R. E. Langer)

10:15 - 10:25
(77) An explicit-implicit method for approximating the solution of the heat equation with radiation or conduction type boundary conditions
Dr. William C. Orthwein, International Business Machines Corporation, Owego, New York (576-19)

10:30 - 10:40
(78) Error estimates for equations of heat conduction
Professor Eugene Isaacson, New York University
(576-136)

10:45 - 10:55
(79) The error in a finite difference approximation
Dr. R. Bruce Kellogg, Combustion Engineering, Incorporated, Windsor, Connecticut (576-108)
11:00 - 11:10
(80) An interpolation formula for "nearly-odd" functions, with an application to the summation of even functions
Dr. Herbert E. Salzer, Convair-Astronautics, San Diego, California (576-237)

11:15 - 11:25
(81) Solutions of linear partial differential equations with variable coefficients by a substitution-separation procedure.
Preliminary report
Professor William F. Ames, University of Delaware (576-133)

11:30 - 11:40
(82) Physical systems Sk of curves in a central field of force
Mr. Adam Czarnecki, DePaul University (576-146)

11:45 - 11:55
(83) Determinant form of series solutions of Böcher equations
Professor Domina Eberle Spencer, University of Connecticut (576-116)

Session on Geometry, Cabinet Room
10:00 - 10:10
(84) Function-theoretic characterization of Einstein spaces and harmonic spaces
Professor Avner Friedman, University of Minnesota (576-28)

10:15 - 10:25
(85) Conformal transformations of compact Riemannian manifolds
Dr. Morio Obata, University of Illinois (576-114)

10:30 - 10:40
(86) Homogeneous manifolds of constant curvature
Mr. J. A. Wolf, Institute for Advanced Study, Princeton, New Jersey (576-44)

10:45 - 10:55
(87) Coboundary operators on the ring of alternating tensors on a manifold
Mr. P. J. Zwier, Calvin College (576-103)

11:00 - 11:10
(88) The parallelizability of solvmanifolds
Dr. R. H. Szczarba, Yale University (576-117)

11:15 - 11:25
(89) Second order tangent vectors and derivations, Preliminary report
Dr. B. L. Foster, Boeing Airplane Company, Renton, Washington (576-151)
11:30 - 11:40
(90) A characterization of the catenoid
Professor J. C. C. Nitsche, University of Minnesota and the University of Puerto Rico (576-175)

11:45 - 11:55
(91) The moduli of non-hyperelliptic curves of genus 3
Professor Irwin Fischer, University of Colorado and Professor Daniel Gorenstein, Clark University (576-178)

TUESDAY, 3:15 P.M.

Session on Analysis, Presidential Parlors
3:15 - 3:25
(92) Permutations of Fourier coefficients of continuous functions
Professor Walter Rudin, University of Wisconsin (576-172)

3:30 - 3:40
(93) Cauchy formulas for symmetric domains in several complex variables
Professor Raoul Bott, Harvard University and Dr. Adam Koranyi, University of California, Berkeley (576-109)

3:45 - 3:55
(94) On the zeros of functions with finite Dirichlet integral. Preliminary report
Professor H. S. Shapiro, New York University and Professor A. L. Shields, University of Michigan (576-216)

4:00 - 4:10
(95) Grunsky inequalities and coefficients of bounded schlicht functions
Dr. Vikramaditya Singh, Harvard University (576-169)

4:15 - 4:25
(96) A theorem on vanishing differences
Mr. R. F. DeMar, Miami University (576-88)

4:30 - 4:40
(97) Inequalities for functions having real part positive in the right half-plane
Dr. J. L. Goldberg, Bell Telephone Laboratories, Whippany, New Jersey (576-181)

4:45 - 4:55
(98) Families of binary relations and the theory of analytic functions of a complex variable. Preliminary report
Dr. Dov Tamari, Institute for Advanced Study and Israel Institute of Technology, Haifa (576-223)
Sessions on Algebra, Congressional Room
3:15 - 3:25
(99) The invariance of symmetric functions of singular values

3:30 - 3:40
(100) The permanent function
Professor Marvin Marcus, U. S. National Bureau of Standards, Washington, D. C. and Mr. Frank May, University of British Columbia (576-31)

3:45 - 3:55
(101) Note on a theorem by Marcus and Lopes
Dr. E. V. Haynsworth, Auburn University (576-214)

4:00 - 4:10
(102) Symmetric means and matrix inequalities
Professor Peter Bullen, University of British Columbia, and Professor Marvin Marcus, U. S. National Bureau of Standards, Washington, D.C. (576-65)

4:15 - 4:25
(103) Matrix commutators
Professor M. F. Smiley, University of California, Riverside (576-99)

4:30 - 4:40
(104) Determinants in projective modules
Professor Oscar Goldman, Institute for Advanced Study, and Brandeis University (576-134)

4:45 - 4:55
(105) Generalized Hadamard matrices, Preliminary report
Mr. A. T. Butson, Boeing Airplane Company, Seattle, Washington (576-139)

Session on Applied Mathematics, South Ballroom
3:15 - 3:25
(106) On the state assignment problem for sequential machines, I
Dr. Juris Hartmanis, General Electric Company, Schenectady, New York (576-53)

3:30 - 3:40
(107) On the state assignment problem for sequential machines, II
Mr. R. E. Stearns, Princeton University and Dr. Juris Hartmanis, General Electric Company, Schenectady, New York (576-54)

3:45 - 3:55
(108) A graph theoretic approach to matrix inversion by partitioning
Professor Frank Harary, University of Michigan (576-70)
4:00 - 4:10
(109) Almost strictly competitive games
Dr. R. J. Aumann, Princeton University (576-198)

4:15 - 4:25
(110) A simple signal identification model

4:30 - 4:40
(111) A quasi-finite algorithm for a problem in nonlinear programming. Preliminary report
Dr. A. A. Goldstein, Massachusetts Institute of Technology (576-10)

Session on Topology, Cabinet Room
3:15 - 3:25
(112) Locally flat embeddings of topological manifolds
Professor Morton Brown, Institute for Advanced Study (576-127)

3:30 - 3:40
(113) Collared subsets of metric spaces
Professor Morton Brown and Professor Ernest Michael, Institute for Advanced Study (576-128)

3:45 - 3:55
(114) On combinatorial manifolds and differentiable structures
Professor Stewart S. Cairns, University of Illinois (576-209)

4:00 - 4:10
(115) A proof and extension of Brouwer's fixed point theorem for closed n-cell
Professor Smbat Abian, University of Pennsylvania, and Professor Arthur B. Brown, Queens College (576-15)

4:15 - 4:25
(116) Locally unknotted combinatorial n-1 manifolds in euclidean n-space have shell neighborhoods
Professor O. G. Harrold, Jr., University of Tennessee (576-106)

4:30 - 4:40
(117) The separation of $S^3$ by a double torus
Mr. C. H. Edwards, Jr., University of Tennessee (576-43)

4:45 - 4:55
(118) Point-like decompositions of $E^3$
Professor R. H. Bing, University of Wisconsin (576-144)
Session on Logic and Foundations, Grand Ballroom
3:15 - 3:25
(119) On recursively enumerable degrees. Preliminary report
Mr. Gerald E. Sacks, Cornell University (576-21)
3:30 - 3:40
(120) Classes of recursive functions of predictable complexity
Mr. R. W. Ritchie, Dartmouth College (576-210)
3:45 - 3:55
(121) Augmenting axiomatic set theory. Preliminary report
Mr. A. H. Kruse, New Mexico State University and University of Kansas (576-225)
4:00 - 4:10
(122) Inductive limits and elementary classes
Dr. Isidore Fleischer, University of California, Berkeley, (576-51)
4:15 - 4:25
(123) The isomorphism of ultrapowers of elementarily equivalent systems
Professor Simon Kochen, Cornell University (576-199)

TUESDAY, 8:00 P.M.
The Josiah Willard Gibbs Lecture, Grand Ballroom
Problems in nonlinear elasticity (One hour)
Professor J. J. Stoker, New York University

WEDNESDAY, 10:00 A.M.
Session on Analysis, Presidential Parlors
10:00 - 10:10
(124) An existence theorem for an nth order partial differential equation
Mr. Irving I. Glick, U. S. Naval Ordnance Laboratory, Hyattsville, Maryland (576-186)
10:15 - 10:25
(125) An existence theorem for a 3rd order partial differential equation
Dr. James Conlan, U. S. Naval Ordnance Laboratory, Silver Spring, Maryland (576-193)
10:30 - 10:40
(126) A decomposition formula for differential equations
Professor Kuo-Tsai Chen, Institute for Advanced Study and Instituto Tecnologico de Aeronautica, Brazil (576-57)
10:45 - 10:55
(127) Two-point boundary-value problems for fourth-order differential equations with middle term
Professor John H. Barrett, University of Utah (576-204)
11:00 - 11:10
(128) An oscillation criterion for self-adjoint differential systems
Professor William T. Reid, State University of Iowa (576-197)

11:15 - 11:25
(129) Singular perturbations of eigenvalue problems
Dr. W. A. Harris, Jr., University of Wisconsin (576-105)

11:30 - 11:40
(130) The cross-ratio property for the matrix Riccati equation
Professor W. J. Coles, University of Utah (576-147)

11:45 - 11:55
(131) Periodic solutions of an autonomous perturbed second-order equation
Professor W. S. Loud, University of Minnesota (576-159)

Session on Topology, South Ballroom
10:00 - 10:10
(132) Local cohomology groups with closed supports
Professor Frank Raymond, University of Wisconsin (576-35)

10:15 - 10:25
(133) On the homology rings of certain Lie groups
Dr. William Browder, Cornell University (576-86)

10:30 - 10:40
(134) On generalized cohomotopy groups
Dr. J. W. Jaworowski, Institute for Advanced Study (576-183)
(Introduced by Dr. S. H. Gould)

10:45 - 10:55
(135) Jordan curves and free homotopy classes on compact surfaces
Professor Bruce L. Reinhart, University of Maryland and RIAS (576-71)

11:00 - 11:10
(136) Secondary operations
Professor E. H. Spanier, University of California, Berkeley (576-62)

11:15 - 11:25
(137) Homotopy retraction properties for compact Hausdorff spaces
Professor C. W. Saalfrank, Lafayette College (576-20)

11:30 - 11:40
(138) A note on periodic transformations
Mr. J. C. Su and Professor C. T. Yang, University of Pennsylvania (576-26)
11:45 - 11:55
(139) Nonorientable closed surfaces in euclidean spaces
Professor C. T. Yang, University of Pennsylvania
(576-25)

Session on Geometry, Cabinet Room
10:00 - 10:10
(140) An enumeration of the five parallelohedra
Professor William Moser, University of Manitoba
(576-113)

10:15 - 10:25
(141) Convex polyhedra with regular faces, Preliminary report
Mr. Norman W. Johnson, Carleton College (576-157)

10:30 - 10:40
(142) N-gon rotors making n + 1 contacts with a simple closed curve
Mr. Michael Goldberg, Bureau of Naval Weapons, Washington, D. C. (576-34)

10:45 - 10:55
(143) On a characterization of circles
Mr. M. S. Klamkin, Avco Research and Advanced Development Division, Wilmington, Massachusetts (576-76)

11:00 - 11:10
(144) A generalization of Helly's theorem
Dr. Branko Grünbaum, University of Washington
(576-154)

11:15 - 11:25
(145) The spin model of euclidean 3-space
Professor W. F. Eberlein, University of Rochester
(576-148)

11:30 - 11:40
(146) The tri-spherical intersection property and inner product spaces
Dr. W. W. Comfort and Dr. Hugh Gordon, Harvard University (576-1)

11:45 - 11:55
(147) Projective convexity in topological linear spaces and a certain connection with property $P_3$
Mr. W. R. Hare, Jr., and Dr. J. W. Gaddum, University of Florida (576-228)

WEDNESDAY, 2:00 P.M.

Invited Address, Grand Ballroom
On the structure of finite groups (One hour)
Professor Helmut Wielandt, University of Tubingen and the California Institute of Technology

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Session on Analysis, Presidential Parlors
3:15 - 3:25
(148) Some convergence regions for continued fractions. Preliminary report
Mr. T. L. Hayden, University of Texas (576-129)

3:30 - 3:40
(149) On the existence of power series. Preliminary report
Professor J. W. Neuberger, University of Tennessee (576-131)

3:45 - 3:55
(150) On modular forms of dimension -2 for subgroups of the modular group. Preliminary report
Mr. J. R. Smart, Michigan State University (576-32)

4:00 - 4:10
(151) Some sets of totally monotone sequences. Preliminary report
Dr. B. E. Rhoades, Lafayette College (576-47)

4:15 - 4:25
(152) On the use of parameters in S-fraction transformations. Preliminary report
Professor E. P. Merkes, Marquette University (576-208)

4:30 - 4:40
(153) Systems of linear inequalities and moment sequences
Professor Kurt W. Endl, The Ohio State University (576-220)

4:45 - 4:55
(154) On a Lagrange mean value theorem of the differential calculus of vector valued functions
Professor A. K. Aziz, Georgetown University and Professor J. B. Diaz, University of Maryland (576-227)

Sessions on Analysis, Cabinet Room
3:15 - 3:25
(155) Subalgebras of measure algebras
Professor Arthur B. Simon, Northwestern University (576-42)

3:30 - 3:40
(156) Idempotent measures on compact semigroups
Professor H. S. Collins, Louisiana State University (576-27)

3:45 - 3:55
(157) Minimal centers of attraction for non-compact transformation groups
Dr. Arthur Schwartz, Wayne State University (576-97)
4:00 - 4:10
(158) An isomorphism invariant of measure preserving transformations. Preliminary report
Dr. S. P. Lloyd, Bell Telephone Laboratories, Murray Hill, New Jersey (576-18)

4:15 - 4:25
(159) General local-integrability
Mr. R. R. Welland, Ohio State University (576-90)

4:30 - 4:40
(160) The Radon-Nikodym theorem for non-sigma-finite measures
Professor A. C. Zaanen, California Institute of Technology (576-231)

4:45 - 4:55
(161) On the structure of measure spaces
Professor Robert E. Zink, Purdue University (576-101)

Session of Algebra, Congressional Room
3:15 - 3:25
(162) Monotone behavior of cohomology groups under proper mappings
Professor Ernst Snapper, Indiana University (576-74)

3:30 - 3:40
(163) Dominance over the classical groups
Professor Murray Gerstenhaber, University of Pennsylvania (576-153)

3:45 - 3:55
(164) Lie admissible algebras. Preliminary report
Professor Philip J. Laufer, College Militaire Royal de St. Jean, Quebec and Professor M. L. Tomber, Michigan State University (576-160)

4:00 - 4:10
(165) On quasi-decompositions of torsion free Abelian groups
Professor J. D. Reid, Syracuse University (576-164)

4:15 - 4:25
(166) Rings of quotients of rings of continuous functions, I
Professor N. J. Fine, University of Pennsylvania, Professor Leonard Gillman, University of Rochester and Professor Joachim Lambek, McGill University (576-219)

4:30 - 4:40
(167) Groups with the property $T_2$
Professor H. W. E. Schwerdtfeger, McGill University (576-98)
Session on Topology, South Ballroom

3:15 - 3:25
(168) Longest simple paths in polyhedral graphs
   Dr. B. Grünbaum and Professor T. S. Motzkin, University of California, Los Angeles (576-6)

3:30 - 3:40
(169) On covers and independent sets in graphs
   Professor Frank Harary, University of Michigan, and Professor Robert Z. Norman, Dartmouth College (576-4)

3:45 - 3:55
(170) A fixed point theorem for the hyperspace of a snake-like continuum
   Dr. Jack Segal, University of Washington (576-61)

4:00 - 4:10
(171) Point-like, simplicial mappings of a three-sphere
   Mr. Ross Finney, Harvard University and University of Michigan (576-126)

4:15 - 4:25
(172) Light open maps on the n-sphere
   Mr. P. T. Church and Professor Erik Hemmingsen, Syracuse University (576-87)

4:30 - 4:40
(173) An example of an involution of $E^4$
   Dr. Ronald H. Rosen, University of Michigan (576-38)

4:45 - 4:55
(174) n-space modulo an arc
   Professor J. J. Andrews, University of Wisconsin and Professor M. L. Curtis, Florida State University (576-45)

THURSDAY, 10:00 A.M.

Session on Analysis, Presidential Parlors

10:00 - 10:10
(175) A characterization of spectral operators on locally convex spaces
   Mr. Fumiyuki Maeda, Yale University (576-60)

10:15 - 10:25
(176) Roots of scalar-type operators
   Mr. J. G. Stampfli, Yale University (576-137)

10:30 - 10:40
(177) Spectral decomposition for Pontriagin operators with application to differential eigenvalue problems
   Professor N. Aronszajn, University of Kansas (576-206)

10:45 - 10:55
(178) Hermitian moment sequences
   Professor J. S. MacNerney, University of North Carolina (576-95.)
11:00 - 11:10
(179) Semigroups of matrices defining linked operators with different spectra
Professor Charles J. A. Halberg, Jr., University of California, Riverside (576-75)

11:15 - 11:25
(180) On similarity invariants of certain operators
Professor G. K. Kalisch, University of Minnesota (576-184)

11:30 - 11:40
(181) A new formulation of the Hausdorff inclusion problem
Professor J. H. Wells, University of California, Berkeley and University of North Carolina (576-201)

11:45 - 11:55
(182) On linear operator equations
Professor Ralph E. Lane, University of Texas (576-158)

Session on Analysis and Applied Mathematics, South Ballroom
10:00 - 10:10
(183) On the solution of an elliptic differential equation over a rectangular domain
Mr. Tsuch Wu Ting, General Motors Research Laboratories, Warren, Michigan (576-72)

10:15 - 10:25
(184) Type-insensitive approximation methods. II, Galerkin's method. Preliminary report
Professor C. K. Chu, New York University (576-189)

10:30 - 10:40
(185) Some higher order integral identities with application to bounding techniques
Dr. J. H. Bramble, University of Maryland and Dr. B. E. Hubbard, U. S. Naval Ordnance Laboratory (576-155)

10:45 - 10:55
(186) Minimization of a convex function by iteration
Dr. Samuel Schechter, New York University (576-211)

11:00 - 11:10
(187) Asymptotic behavior of the spectral matrix of the operator of elasticity
Professor F. J. Bureau, University of Liége, Belgium and Professor E. J. Pellicciaro, University of Delaware (576-167)

11:15 - 11:25
(188) A theory in functional analysis concerning some iteration procedures and error estimates
Mr. Johann Schroder, University of Wisconsin and University of Hamburg (576-115)
11:30 - 11:40
Periodic response and superposition in on-off control system
Professor B. A. Fleishman, Rensselaer Polytechnic Institute (576-58)

11:45 - 11:55
On quasi-asymptotic stability
Professor K. H. Matthies, University of South Carolina (576-233)

Session on Algebra, Congressional Room
10:00 - 10:10
Some limiting examples in ordered groups
Mr. W. Charles Holland, Tulane University (576-79)

10:15 - 10:25
The number of basic subgroups of a p-group
Dr. S. A. Khabbaz, Lehigh University and Dr. E. A. Walker, New Mexico State University (576-5)

10:30 - 10:40
Triply transitive groups in which only the identity fixes four letters
Professor Daniel Gorenstein, University of Chicago and Professor D. R. Hughes, University of Michigan (576-29)

10:45 - 10:55
Finite groups of order $4g'$
Professor Daniel Gorenstein, University of Chicago and Clark University, Professor J. H. Walter, University of Chicago and University of Washington (576-205)

11:00 - 11:10
Abelian $\sum$-groups
Dr. John M. Irwin, and Dr. Elbert A. Walker, New Mexico State University (576-93)

11:15 - 11:25
Symmetric embeddings of complete graphs
Mr. J. R. Edmonds, U. S. National Bureau of Standards and University of Maryland (576-149)

11:30 - 11:40
Torsion endomorphic images of mixed Abelian groups
Dr. Elbert A. Walker, New Mexico State University (576-100)

11:45 - 11:55
Direct products of copies of the integers
Professor R. J. Nunke, University of Washington (576-130)
Session on Statistics and Probability, Cabinet Room

10:00 - 10:10
(199) Maximal correlation and uncertainty
Dr. H. P. Kramer, General Electric Company, Santa Barbara, California (576-213)

10:15 - 10:25
(200) Upper and lower bounds on the length of the longest code
Professor J. H. B. Kemperman, University of Wisconsin (576-94)

10:30 - 10:40
(201) A class of multivariate rank statistics
Mr. Thomas A. Willke, The Ohio State University (576-82)

10:45 - 10:55
(202) Fortune's formula: The game of blackjack
Dr. Edward O. Thorp, Massachusetts Institute of Technology (576-118)

11:00 - 11:10
(203) A Markovian walk on a plane grid. Preliminary report
Professor C. E. Langenhop, Mathematica Incorporated, Princeton, New Jersey (576-110)

11:15 - 11:25
(204) Generalizations of Shannon-McMillan theorem
Professor Shu-Teh C. Moy, Syracuse University (576-48)

11:30 - 11:40
(205) Branching processes and semigroups of operators
Professor A. T. Bharucha-Reid, University of Oregon (576-177)

11:45 - 11:55
(206) Frequency probability root relations
Dr. Clifford J. Maloney, U.S. Chemical Corps Biological Laboratories, Fort Detrick, Maryland (576-12)

THURSDAY, 2:00 P.M.

Business Meeting, Grand Ballroom

THURSDAY, 3:15 P.M.

Session on Analysis, Presidential Parlors
3:15 - 3:25
(207) The effect of a certain multiplicative relationship on the internal structure of a matrix
Professor D. G. Austin and Professor H. Meyer, University of Miami (576-179)
3:30 - 3:40
A theorem on strictly convex functions
Dr. Barry Bernstein and Mr. R. A. Toupin, U. S. Naval Research Laboratory, Washington, D. C. (576-7)

3:45 - 3:55
B(\mathcal{C})-property and the open mapping theorem
Dr. Tadqir Husain and Professor Mark Mahowald, Syracuse University (576-135)

4:00 - 4:10
On derivative functions. Preliminary report
Professor N. F. G. Martin, University of Virginia (576-112)

4:15 - 4:25
A generalized derivative. Preliminary report
Professor Edward D. Gaughan, New Mexico State University (576-55)

4:30 - 4:40
The convergence of measures on parametric surfaces
Mr. J. H. Michael, Purdue University (576-84)

4:45 - 4:55
Fourier transforms related to convex sets
Professor Carl S. Herz, Cornell University (576-196)

Session on Analysis, Cabinet Room
3:15 - 3:25
Symmetric exterior derivatives and flat forms
Professor Victor L. Shapiro, University of Oregon (576-192)

3:30 - 3:40
A characterization of the elliptic operators with variable coefficients
Dr. Joseph Nieto, University of Maryland (576-229)

3:45 - 3:55
On Harnack’s theorem
Professor J. Moser, New York University (576-221)

4:00 - 4:10
An inequality for uniformly elliptic operators
Professor Hans F. Weinberger, University of Minnesota (576-170)

4:15 - 4:25
Truncation errors for Cauchy problems
Professor Garrett Birkhoff, Harvard University and Dr. Richard S. Varga, Case Institute of Technology (576-8)
4:30 - 4:40
(219) Parabolic equations with nonsmooth coefficients
Professor D. G. Aronson, University of Minnesota (576-125)

4:45 - 4:55
(220) On a method of associating partial differential systems with the ordinary ones
Professor M. Z. Krzywoblocki, Michigan State University (576-41)

Session on Algebra and Theory of Numbers, Congressional Room
3:15 - 3:25
(221) "Color Group" graphs of binary systems
Dr. R. Artzy, University of North Carolina (576-142)

3:30 - 3:40
(222) On the algebra of functions
Professor B. Schweizer, University of Arizona and Professor A. Sklar, Illinois Institute of Technology (576-166)

3:45 - 3:55
(223) Abstract mean values, II
Professor Trevor Evans, Emory University (576-226)

4:00 - 4:10
(224) Multi-terminal network flows
Dr. R. E. Gomory and T. C. Hu, IBM Research Corporation, Yorktown Heights, New York (576-52)

4:15 - 4:25
(225) Bounds for certain sums; a remark on a conjecture of Mahler
Mr. Wolfgang Schmidt, University of Colorado (576-64)

4:30 - 4:40
(226) Partitions of vectors
Mr. G. R. Blakley, Cornell University (576-67)

4:45 - 4:55
(227) The maximum values of some trigonometric determinants
Professor Benjamin Lepson, U. S. Naval Research Laboratory and Catholic University of America (576-238)

Session on Topology, South Ballroom
3:15 - 3:25
(228) The Wallace functions for sets or ordered pairs of sets
Professor P. C. Hammer, University of Wisconsin (576-236)

3:30 - 3:40
(229) Uniform irreducible expansions
Professor J. R. Isbell, University of Washington (576-36)
An example in the theory of topological semigroups and homeomorphism groups
Professor Paul S. Mostert, Tulane University (576-66)

On a semigroup of right ideals of a semigroup
Miss Anne Lester, Tulane University (576-77)

Groups of homeomorphisms of uniform spaces
Mr. S. M. Hudson, Tulane University (576-80)
(Introduced by P. S. Mostert)

Test spaces for homological dimension
Mr. Yukihiro Kodama, Miami University (576-83)
(Introduced by W. L. Strother)

Finite orbit structure on locally compact manifolds, Preliminary report (576-111)
Dr. L. N. Mann, University of Virginia

Bethlehem Pennsylvania
December 3, 1960
J. J. STOKER
The subject matter of the lecture belongs in the field of mechanics of continuous media. This is a classical subject which has occupied the attention of many of the best mathematicians, notably Newton, Euler, Cauchy, Poisson, Kirchhoff, Kelvin, Poincaré. It is still a very alive subject, including, as it does, such fields as hydrodynamics, aerodynamics, elasticity, plasticity, acoustics, and electromagnetic wave propagation. The subject has been important not only for the applications but also of very great importance for the development of mathematics itself, above all mathematical analysis.

The present lecture is concerned with problems in elasticity and especially certain types of problems which are non-linear. Basically, all of continuous mechanics is non-linear, although the bulk of the literature concerns itself with approximate solutions obtained through linearizations. There are, however, many types of problems which are essentially non-linear in character. In elasticity, for example, problems concerned with the stability of equilibrium configurations are of necessity non-linear in character. These problems are quite important in practice since they involve the question of possible buckling of such structural elements as columns, plates, and shells. These buckling problems are concerned with thin-walled elastic solids. In recent years, there has been considerable interest in the stability of thick-walled solids, and this theory poses still more difficult problems than those attacked hitherto. A considerable portion of the Gibbs lecture will be devoted to them. The non-linear boundary value problems associated with partial differential equations to which such problems lead are intricate and it strains the resources of modern analysis to solve them.
FIVE HUNDRED SEVENTY-SEVENTH MEETING

Hunter College
New York, New York
February 25, 1961

The five hundred seventy-seventh meeting of the American Mathematical Society will be held at Hunter College, New York, New York, on February 25, 1961.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, Professor John Wermer of Brown University will address the Society on "Uniform approximation and maximal ideal spaces" in Room 300 at 2:00 P.M.

The registration desk will be open from 9:00 A.M. till 3:30 P.M. and will be located on the third floor between the elevator and Room 300. Room 305 will serve as a coat room and Rooms 302 and 306 as conversation rooms.

There will be sessions for contributed papers Saturday morning and afternoon. Abstracts of contributed papers should be sent to The American Mathematical Society, 190 Hope Street, Providence 6, Rhode Island in time to arrive prior to the deadline, January 12, 1961.

Hunter College is on Park Avenue at 68th Street. It may be reached from the area of Grand Central Station by the Lexington Avenue subway, which has a 68th street stop. Persons attending the meeting are requested to use the entrance on 69th street. Elevator service will be provided.

Bethlehem, Pennsylvania
December 9, 1960

Everett Pitcher
Associate Secretary

A JOINT PERIODICAL EXHIBIT has been arranged for the Annual Meeting in Washington. The scientific journals included in this exhibit will be displayed near the Registration Desk in the Caucus Room of the Hotel Willard. Upon conclusion of the meeting, the journals will be sent as a gift to the Department of Mathematics of the University of Gorakhpur in India.
News and comment from the

CONFERENCE BOARD OF THE

MATHEMATICAL SCIENCES

G. Baley Price  Washington, D. C.

CONFERENCE ON THE SUPPORT OF
HIGHER EDUCATION BY THE FEDERAL GOVERNMENT

Many agencies of the Federal Government administer programs of various kinds in support of higher education. The Federal Government and also various educational and scientific organizations are engaged in collecting information about these programs, their operation, and their influence on higher education, and in planning for the revision and expansion of these programs. Several opportunities have been offered to the Conference Board of the Mathematical Sciences to express its opinion on these programs and to make suggestions for their modification and further development in the future. As a result, the Conference Board called a special Conference on the Support of Higher Education by the Federal Government to consider all of these matters; it was held in the Allan Room of the NEA Building in Washington, D. C., on November 12-13, 1960.

About twenty persons, in addition to the speakers, were present at the Conference. Each member organization of the Conference Board was represented by its president or by his representative. Others were invited in an effort to obtain proper representation from all parts of the mathematical community.

On the first day of the Conference the following speakers provided information about the problem before the Conference and on representative programs of the Federal Government in support of higher education: Professor S. S. Wilks, Chairman, Conference Board of the Mathematical Sciences; Dean Mina S. Rees, Dean of the Faculty and Professor of Mathematics, Hunter College; Dr. J. Kenneth Little, Director, Survey of Federal Programs in Higher Education, U. S. Office of Education; Dr. Charles G. Dobbins, Staff Associate and Secretary, Committee on Relationships of Higher Education to the Federal Government, American Council on Education; Dr. Harold F. Dorn, Chief, Biometrics Branch, Division of Research Services, National Institutes of Health; Dr. C. Russell Phelps, Program Director for Academic Year Institutes, National Science Foundation; Dr. Philip W. Hemily, International Science Education Program Director, Office of Special International Programs, National Science Foundation; Dr. Gilbert Anderson, Office of Educational Exchange, Bureau of Educational and Cultural Affairs, Department of State; Dr. Arthur Grad,
Program Director for the Mathematical Sciences, Division of Mathematical, Physical, and Engineering Sciences, National Science Foundation; Dr. F. J. Weyl, Director of Research, Office of Naval Research.

At the end of the afternoon of the first day of the Conference Professor Wilks appointed the following committees to draft recommendations during the evening for consideration on the second day: (1) committee on support for teaching and mathematics education; (2) committee on fellowships, scholarships, and other support for students; (3) committee on support for research; (4) committee on buildings and facilities; and (5) committee on mathematical activities in the international field. These committees met during the evening of November 12, and they presented their recommendations on November 13. Twenty of their recommendations were adopted by the Conference.

A complete report of the Conference, including summaries of the talks and the complete text of the recommendations, is being prepared and will be available soon.

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The official textbook for

CONTINENTAL CLASSROOM

The official text for the Continental Classroom course to be taught by Professor Mosteller—begins Jan. 30 over the NBC Television Network

A thorough exposition of probability with statistical applications; numerous examples and problems. Explores the subject deeply, yet assumes only two years of high-school algebra

A Guide containing study aids, problem-solving hints, and answers to problems is available for the student taking course by correspondence or extension

READY JANUARY 15

PROBABILITY AND STATISTICS By Mosteller, Rourke, Thomas paperb. $4.00
GUIDE TO PROBABILITY AND STATISTICS By Noether paperbound $1.00

SEE IT AT THE AMS-MAA JANUARY MEETING

ADDISON-WESLEY PUBLISHING COMPANY, INC. Reading, Mass.

810
From

The AMS Secretary
John W. Green, UCLA

One aspect of meetings of the Society that has received considerable attention from the Council and Officers in the past few years is the question of ten-minute contributed papers. It is a long and cherished tradition that a member may present a ten-minute paper on a topic of his choice at any Society meeting, subject to acceptance of his abstract by the Associate Secretary in charge of the meeting. The custom has been for the Associate Secretary to be rather broadminded in his acceptance of abstracts - perhaps too broadminded on some occasions - so that there has been an essentially unrestricted privilege of contributed papers.

A result of this is that at our larger meetings, particularly the Annual Meeting now held in January, are offered such an extremely large number of ten-minute papers that scheduling them becomes difficult and results both in extending the meeting in number of days and number of hours per day, and in prohibiting the adding of other attractive features to the program, such as symposia on special topics, etc. For example, the January 1960 meeting in Chicago, had 222 contributed papers. This amounts to about 55 hours of sessions, enough to be difficult to schedule without suffocating the rest of the program.

In addition there has been the complaint, frequently justified, that these ten minute papers are of poor quality, not only in content, but also in presentation. This is no doubt related to the fact that presentation of a paper in person is often a prerequisite to getting one's transportation paid to the meeting, and many a pot boiler has been dashed off on deadline day and regretted on presentation day.

There have been several suggestions as to how to mitigate both of these unfortunate by-products of our relatively free-and-easy system. One is to eliminate contributed papers altogether, making our program entirely by invitation. The young person, trying to put his wares before the public and before potential employers, would lose by this, as well as the potential employer. Another is to referee abstracts and select a suitable number of the most significant. This is extremely difficult and would advance the deadline date a great deal. A third is to accept only abstracts of papers that have been submitted for publication in a recognized journal. No proposed solution has come up so far, however, that looks sufficiently appealing to adopt.

These matters are still under consideration and I would appreciate having expressions of opinion from those who would be interested in supplying them.
MATHEMATICS IN TRANSLATION

S. H. Gould

Translations of scientific material are being produced at an accelerated pace throughout the world. But mathematical translations often escape notice because they are included in much longer lists of translations from science in general. For example, the semi-monthly Technical Translations, issued by the Office of Technical Services in the Department of Commerce, Washington, D. C., has approximately twenty sections, ranging from the Behavioral Sciences to Nuclear Physics. In recent months a committee of the Conference Board of the Mathematical Sciences has been considering ways and means of selecting and publicizing current information about translations of interest to mathematicians. The following items are presented as the sort of information which it is hoped may later be incorporated in a regular series of articles in the Notices.

The Hindustan Publishing Corporation, 6-UB., Jawahar Nagar, Delhi 6, India, publishers for The National Institute of Sciences of India, announces regular publication of English translations of two Russian mathematical journals: Bulletin of the USSR Academy of Sciences, Mathematics Series (Izvestiya Akademii Nauk SSSR, Seriya Matematika); and Mathematical Symposium of the USSR Academy of Sciences and the Moscow Mathematical Society (Matematiceskii Sbornik). Translation of the Izvestiya begins with vol. 24, no. 1, January-February 1960, and will appear in six issues per year, each issue containing approximately 150 pages. Translation of the Sbornik begins with vol. 51 (93), no. 1, January 1961, and will appear in twelve issues per year, each issue containing approximately 125 pages. It is planned that in each case the English translation shall be available four months after the appearance of the Russian original. Subscription rates per annum are: for the Izvestiya, Rs. 100 domestic and Rs. 125 foreign; for the Sbornik, Rs. 130 domestic and Rs. 150 foreign.

The same corporation has also published English translations of six Russian mathematical monographs and announces twenty or more other titles in preparation. Four of the six books already published are:

Mikhlin, S. G.: Linear Integral Equations. $5.00
Korovkin, P. P.: Linear Operators and Approximation Theory. $5.00
Ventzel, E. S.: Elements of the Theory of Games. $3.00
Khinchin, A. I.: A Course of Mathematical Analysis. $10.00

In August 1960, the National Science Foundation, Washington 25, D. C., issued a pamphlet (NSF 60-46) entitled: A list of Russian scientific journals available in English. This list contains 85 Soviet
journals translated cover-to-cover and 6 journals translated on a selective basis. Under the specific heading Mathematics the following four items are listed: Doklady (American Mathematical Society), Prikladnaya Matematika (American Society of Mechanical Engineers), Selected Translations (AMS), Teoriya Veroyatnostei i ee primenenie (Society for Industrial and Applied Mathematics). Following the publication of the NSF list the London Mathematical Society announced plans to publish translations of all the survey articles from Uspekhi Matematischeskikh Nauk (see NOTICES, October 1960, p. 630).

In choosing its Selected Translations the American Mathematical Society inquires from the following sources whether they know of any existing or intended translation of the proposed article into English, French or German. The Society hopes to add other names to this list.

Miss Eva Skelley  
Collet's Scientific Bookshop  
23 Museum Street  
London W. 1, England

Captain I. R. Maxwell, Director  
Pergamon Institute  
4-5 Fitzroy Square  
London W. 1, England

Aslib  
Mr. Leslie Wilson, Director  
3 Belgrave Square  
London S. W. 1, England

Drs. Beranek and Ulrich  
VEB Deutscher Verlag der Wissenschaften  
Niederwallstrasse 39  
Berlin S. W. 1, Germany

VEB Verlag Technik  
Technisch-Wissenschaftl. Literatur  
Berlin C 2  
Oranienburger Str. 13/14

Mr. B. Cotton  
Department of Scientific and Industrial Research  
Lending Library Unit  
20 Chester Terrace, Regents Park  
London, N. W. 1, England
CALCULUS
by Wray G. Brady, Washington and Jefferson College,
and Maynard J. Mansfield, Washington and Jefferson College
A beginning calculus, rigorous and modern in treatment. In style it is formal — definitions and theorems are clearly stated and proofs are given where appropriate. This book deals with most of the topics treated in the traditional calculus course, plus a great deal of new material. The material has been class-tested for many years and the attitude has been to strive for rigor rather than for simplicity. The outstanding features of this book are the introduction and use of set theory, the Riemann-Stieltjes integral with its attendant applications to distributions, and, to a lesser degree, the circumvention of differentials.
472 Pages

A SURVEY OF BASIC MATHEMATICS
by H. G. Apostle, Grinnell College
A basic text for liberal arts and non-science students who desire an introduction to mathematics: its methods, techniques, and applications. Further satisfies all requirements for students planning to take additional work in science and mathematics. The subject matter is presented with clarity in familiar terms and with definitions to prevent confusion. Very little technical knowledge of mathematics is presupposed, and any student may take the course.
480 Pages

Little, Brown and Company
Boston 6, Mass.
ACTIVITIES OF OTHER ASSOCIATIONS

THE JANUARY 23-27, 1961, MEETING
WILLARD HOTEL, WASHINGTON, D.C.

The Association for Symbolic Logic, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics will meet in conjunction with the AMS at the Willard Hotel from Monday through Friday, January 23-27, 1961. The Association for Symbolic Logic will meet on Tuesday, January 24 and present the following program:

ASSOCIATION FOR SYMBOLIC LOGIC PROGRAM, TUESDAY

9:30 A.M. Contributed Papers (20 minutes each): Grand Ballroom

1. Logical Theory of United States Patent Claims
   Reed C. Lawlor

2. Contribution to the Foundation of Logic and Information Retrieval
   Clifford Joseph Maloney, U.S. Army Chemical Corps

3. A Syntactical Characterization of the Predicate Calculus with Identity and Universal Validity
   Hubert H. Schneider, the University of Nebraska

4. Diophantine Sets over Polynomial Rings
   Martin Davis and Hilary Putnam, Yeshiva University and Princeton University

11:00 A.M. Invited Address: Grand Ballroom

Non-Standard Arithmetics and Non-Standard Analysis
Abraham Robinson, Hebrew University and Princeton University

12:00 noon Luncheon meeting of the Council in the Monroe Room, to be reconvened at 5:00 P.M. if necessary.

2:00 P.M. Presidential Retiring Address: Grand Ballroom

Foundations of Intuitionistic Mathematics
S. C. Kleene, The University of Wisconsin

3:15 P.M. An AMS session of contributed papers on Logic and Foundations will be held in the Grand Ballroom following Professor Kleene's address. For a list of the papers consult the AMS program.
To be presented by title (ASL):

1. Completeness Theorems for Some Many Valued Functional Calculi
   Andrzej Mostowski, The University of Warsaw

2. Two Simple-minded Unsolvable Algebraic Problems
   A. A. Mullin, The University of Illinois.

The Mathematical Association of America will hold its forty-fourth annual meeting from Wednesday through Friday. The MAA Program Committee consisting of W. H. Durfee, Samuel Goldberg, S. A. Jennings, B. J. Pettis, and E. E. Moise, has prepared the following program:

FIRST MAA SESSION, WEDNESDAY
Grand Ballroom
Probability and Statistics

9:15 - 10:05 Finite Random Walks
   Professor Hale F. Trotter, Princeton University

10:15 - 11:05 Hitting Probabilities
   Professor Frank L. Spitzer, Princeton University

11:15 - 12:05 Applied Statistical Decision Theory
   Professor Howard Raiffa, Harvard University

SECOND MAA SESSION, THURSDAY
Grand Ballroom

9:15 - 10:05 Metric Entropy and Applications
   Professor George G. Lorentz, Syracuse University

10:15 - 11:00 Annual Business Meeting of the Association. Vote on Amendments to By-Laws. A Summary of the Status and Activities of the Association, presented by President Carl B. Allendoerfer

11:15 - 12:05 Undergraduate Preparation for Graduate Work
   Professor Andrew M. Gleason, Harvard University

THIRD MAA SESSION, FRIDAY
Grand Ballroom
Applied Mathematics

9:15 - 10:05 Quasi-conformal Mappings
   Professor Lipman Bers, New York University

10:15 - 11:05 Applied Mathematics as a Science
   Professor H. P. Greenspan, Massachusetts Institute Technology
11:15 - 12:05 Some Problems in Applied Mathematics
Professor J. B. Keller, New York University

FOURTH MAA SESSION, FRIDAY
Grand Ballroom
Topology

2:00 - 2:50 The Topology of Group-like Spaces
Professor Eldon Dyer, University of Chicago

3:00 - 3:50 Some Applications of Homotopy Theory to Geometric Problems
Professor Raoul Bott, Harvard University

4:00 - 4:50 Some Geometric Applications of Algebraic Topology
Professor John W. Milnor, Princeton University

At the annual business meeting of the Association on Thursday at 10:15 A.M., a motion will be voted upon to amend the By-Laws of the Association as described on page 951 of the November 1960 issue of the American Mathematical Monthly.

The Board of Governors of the Association will meet on Wednesday afternoon at 1:00 P.M. in Room 220 of the Willard Hotel.

The Society for Industrial and Applied Mathematics will meet on Tuesday, January 24 and Wednesday, January 25.

A retiring presidential address will be delivered by Dr. Brockway McMillan of Bell Telephone Laboratories, Inc. Dr. McMillan will speak at 7:30 P.M. on Wednesday, January 25, in the Grand Ballroom on "An Elementary Approach to the Theory of Information."

The council of SIAM will meet at 4:30 P.M. on Tuesday, January 24, in the Executive Room. In addition, arrangements have also been made for a luncheon meeting of the Section Chairmen, on Wednesday January 25 at 12:00 noon.

AUSTRALIAN MATHEMATICAL SOCIETY
SUMMER RESEARCH INSTITUTE

The Australian Mathematical Society will hold its first Summer Research Institute at the Australian National University, Canberra, January 3 - 31, 1961. Professor T.M. Cherry, F.R.S., University of Melbourne, is to be the first director of the Institute, and will be assisted by Drs. H. Levey and J. Gani, University of Western Australia, as secretaries. The Australian National University has agreed to provide working accommodations for the 14 Fellows of the Institute.

The Australian Summer Research Institute has been inspired by its Canadian counterpart, held yearly at Queen's College, Kingston,
Ontario. It is designed to resolve similar problems of communication between mathematical specialists in allied fields, working at widely distant universities.

Two groups, one in the Mechanics of Continua, and the other in Probability and Statistics, will be carrying out research at the Institute this summer; four survey lectures reviewing particular branches of these fields have been arranged, but the Institute will otherwise remain fairly informal.
NEWS ITEMS AND ANNOUNCEMENTS

NSF POSTDOCTORAL FELLOWSHIP AWARDS announced October 21 were made to 40 recipients, including three in mathematics to:

<table>
<thead>
<tr>
<th>Name</th>
<th>Present Institution</th>
<th>Fellowship Institution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blattner, Robert J.</td>
<td>University of California</td>
<td>Mass. Inst. of Tech.</td>
</tr>
<tr>
<td>Liulevicius, Arunas L.</td>
<td>University of Chicago</td>
<td>Inst, for Advanced Study</td>
</tr>
<tr>
<td>Ernest, John A.</td>
<td>Institute for Advanced Study</td>
<td>Inst, for Advanced Study</td>
</tr>
</tbody>
</table>

Each of the above awards were for a tenure of 12 months.

FELLOWSHIPS AND ASSOCIATESHIPS recently announced by the Fellowship Office, National Academy of Sciences-National Research Council, have included the following:


Postdoctoral Resident Research Associateships. Closing Date: February 1, 1961; Announcement of Awards: April 1, 1961. These one-year appointments, for the development of creative abilities and advanced education of young investigators of unusual promise, are open to United States citizens. The stipend, subject to income tax, will be $8,955. Appointments are offered at:

National Bureau of Standards
Washington, D. C.
Boulder, Colorado

Naval Research Laboratory
Washington, D. C.

Pure and Applied Mathematics; Operations Research; Physics; Electronics; Optics; Molecular Structure; Theoretical Astrophysics; Solid State Physics; Ultrasonics; Experimental Thermodynamics; Statistics.

Applied Mathematics; Chemistry; Electronics; Metallurgy; Nuclear Studies; Physics; Solid State; Engineering Psychology.
Naval Ordnance Laboratory  
White Oak, Silver Spring, Maryland  
Aeroballistics; Chemistry; Mathematics; Physics

Naval Weapons Laboratory  
Computation and Analysis Center, Dahlgren, Virginia  
Mathematics and Mathematical Physics; Mathematical Statistics; Digital Computer Systems; Missile and Satellite Technology

Navy Electronics Laboratory  
San Diego, California  
Applied Mathematics; Meteorology and Radio Physics; Oceanography; Physics; Engineering Psychology; Psychophysics.

Air Research and Development Command

Air Force Research Division  
Bedford, Massachusetts  
(Cambridge Research Labs.)  
Astronomy; Electronics; Mathematics; Physics

Dayton, Ohio  
(Aeronautical Research Labs.)  
Ceramics; Chemistry; Mathematics; Metallurgy; Physics

Air Force Missile Development Center  
Alamogordo, New Mexico  
Aeromedicine; Mathematics

U. S. Army Chemical Corps Biological Laboratories

Fort Detrick, Frederick, Maryland  
Aerobiology; Bacteriology; Biochemistry; Biomathematics; Biophysics; Immunology; Medical Entomology; Virology

RESEARCH INSTRUCTORSHIPS AT WISCONSIN. The University of Wisconsin invites applications for research instructorships or research assistant professorships from young mathematicians who show definite promise in research and teaching. The intent of the award is to allow the recipients to have substantial time for research. The teaching load is just half that of an ordinary staff member on a two-semester average. The number of awards depends on the number of highly qualified applicants. Salary is dependent on qualifications. Appointments are renewable. Application forms may be obtained from the Department of Mathematics.

TEMPORARY MEMBERSHIPS IN THE INSTITUTE OF MATHEMATICAL SCIENCES, NEW YORK UNIVERSITY. The Institute offers temporary memberships for the Academic Year 1961-1962 to mathematicians and other scientists holding the Ph.D. degree and who intend to study and do research in the fields in which the Institute is particularly active. These fields include FUNCTIONAL AN-
The temporary membership program is designed primarily to alleviate the present critical shortage of scientists trained in mathematical physics, applied mathematics, and mathematical analysis. The program is being supported by the National Science Foundation and also by funds contributed by industrial firms to New York University.

Temporary membership carries no specific obligation or duty. The entire program of the Institute is open to participation by temporary members. This includes research projects, the advanced graduate courses, and the research seminars of the Institute as well as the use of our computational facilities. The Computing Center now has an IBM 704 which will be replaced by a 7090 in the spring of 1961.

Grants to temporary members will be made in accordance with their professional status.

Temporary memberships are awarded for one year, but may be renewed in special cases. Appropriate arrangements can be made for applicants who expect to be on leave of absence from their institutions.

Requests for information should be addressed to the Membership Committee, Institute of Mathematical Sciences, 4 Washington Place, New York 3, New York. Applications should be submitted by February 15, if possible.

VISITING APPOINTMENTS TO RIAS for nine mathematicians were announced by Dr. Solomon Lefschetz, director of the Mathematics Group at RIAS, Ruxton, Maryland. The principal area of study at RIAS is ordinary nonlinear differential equations.

Appointed to the staff were Dr. Emilio Oscar Roxin, University of Buenos Aires; Dr. Czeslaw Olech, Mathematics Institute, Polish Academy of Sciences, Cracow; Dr. Masahiko Saito, University of Tokyo; Dr. Juan J. Shaffer, University of Uruguay, and Richard Bucy, of the Johns Hopkins Applied Physics Laboratory. Dr. Taro Yoshizawa, of the University of Nihon, Japan, was reappointed for one year to continue his studies in the stability of motion.

One-year grants from the Army Office of Ordnance Research for study at RIAS were awarded to Dr. Gary Meisters, University of Nebraska; Dr. Joseph Auslander, Carnegie Tech, and Dr. Rodney D. Driver, University of Minnesota.
AN INFORMAL CONFERENCE IN ALGEBRAIC TOPOLOGY will be held at the University of California, Berkeley, during the months of July and August, 1961. Foreign mathematicians in residence during this period will include: J. F. Adams, A. Haefliger, I. James, and R. Thom. Several other mathematicians have also indicated their intention to spend all or part of the summer months at Berkeley.

A SYMPOSIUM ON ELECTRO-MAGNETIC WAVES will be held April 10-12, 1961, at the Mathematics Research Center, University of Wisconsin. The topics to be considered are to be the propagation of waves in anisotropic media, diffraction, antenna theory, and numerical methods by which investigations of these can be submitted to electronic computation. The plan is to discuss the topics both from the practical (engineering) and the theoretical (mathematical) standpoint.

The program (when it has been fixed) may be had by addressing a request to the Mathematics Research Center, United States Army, The University of Wisconsin, Madison 6, Wisconsin.

A SYMPOSIUM ON INFORMATION AND DECISION PROCESSES will be held April 12-13, 1961, at Purdue University. The list of speakers and their subjects follows:


Tjalling C. Koopmans, Cowles Foundation for Research in Economics at Yale University, currently visiting at Harvard University - "Axioms for Persistent Preference"

Bradford Dunham, International Business Machines Company - "Exploratory Mathematics by Machine"

Norbert Wiener, Institute Professor and Professor of Mathematics, Emeritus, Massachusetts Institute of Technology - "Mathematics of Self-Organizing Systems"

Kai-Lai Chung, Professor of Mathematics, Syracuse University - "The Ergodic Theorem of Information Theory"

Sigeiti Moriguti, Professor, Faculty of Engineering, University of Tokyo, currently visiting at Columbia University - "Further Results in the Theory of Numerical Convergence"

L. J. Savage, Professor of Mathematics, University of Michigan - "Bayesian Statistics"
Howard Raiffa, Professor of Business Administration, Harvard University -
"Some Techniques for the Application of Bayes Decision Theory"
Paul F. Chenea, Head of the School of Mechanical Engineering, Purdue University, will address a banquet meeting.

Complete information on the symposium may be had by addressing Dr. Robert E. Machol, School of Electrical Engineering, Purdue University, Lafayette, Indiana.

A PROGRAM IN BIOMETRY has been established by the Statistical Laboratory of the Catholic University of America. This includes a training program and also a consultation service for biological scientists. Associated with the program as visiting professors for the academic year 1960-1961 are Professor Eduard Batschelet, on leave from the University of Basel, and Professor Harald Bergstrom of the Institute of Applied Mathematics of Chalmers University of Technology, Goeteborg, Sweden. Professor D. Duge of the Sorbonne and Dozent T. E. Dalenius of Stockholm University are expected to visit Catholic University during the spring term of 1961.

THE IBM JUNIOR SCIENCE SYMPOSIUM was held October 11-14 for 300 outstanding science and mathematics secondary school students in the metropolitan New York area. The symposium was co-sponsored by the Science Manpower Project of Columbia Teachers College and Columbia University. Lipman Bers of New York University spoke on "Infinite Cardinals and Transcendental Numbers" and Dean Mina S. Rees of Hunter College conducted a panel discussion on "Tomorrow's Opportunities in Science."

THE BOOKS FOR ASIAN STUDENTS PROGRAM of The Asia Foundation is now in its sixth year. Under this program over two million books and journals have been shipped to Asian educational institutions. The Asia Foundation will welcome donations of books suitable for this program. Suitable books are:

1. University, college, and secondary school books in good condition, published after 1945.
2. Scholarly, scientific, and technical journals in runs of five years or more.
3. Works by standard authors (e.g., Dickens, Hawthorne, Hemingway, Plato, W. James, T. Huxley, etc.)
The Asia Foundation will pay transportation costs from the donor to San Francisco and thence to Asia. Before shipping books to the Foundation, further information should be requested from: Books for Asia Students, 21 Drumm Street, San Francisco 11, California.

PROBABILITY AND STATISTICS, a nationally televised college-credit mathematics course, will be offered on the second semester of Continental Classroom beginning January 30, 1961.

Presented by Learning Resources Institute in cooperation with the Conference Board of the Mathematical Sciences, and telecast in color and black-and-white by the National Broadcasting Company, the course will be taught by Professor Frederick Mosteller, Chairman, Department of Statistics, Harvard University, and Professor Paul C. Clifford, Professor of Mathematics, Montclair (New Jersey) State College.

More than 300 colleges and universities are expected to offer the televised course for college credit. It will be carried by 170 stations throughout the nation, from 6:30 to 7:00 A.M. Monday through Friday in each time zone.

Dr. Frederick Mosteller, pictured on the set of Continental Classroom.
College and university students seeking undergraduate credit will view the lessons taught Monday, Wednesday and Friday by Professor Mosteller. Before joining the Harvard faculty, Dr. Mosteller taught at Carnegie Institute of Technology and Princeton University.

Teachers and others enrolled for graduate credit will view additional lessons on Tuesday and Thursday devoted to classroom teaching and problem-solving. These sessions will be conducted by Professor Clifford, who has lectured in statistics at New York University, Newark College of Engineering, Rutgers University and University of Michigan.

The first semester of Continental Classroom's course in Contemporary Mathematics featured Modern Algebra. It marked Continental Classroom's third consecutive year of education through network television.

Financial support for Contemporary Mathematics is provided by The Ford Foundation and the following industries: Bell Telephone System, E. I. du Pont de Nemours and Company, General Foods Fund, IBM Corporation, Radio Corporation of America, Union Carbide Corporation and United States Steel.

LAG IN BRITISH MATHEMATICS NOTED. In its annual report, recently issued, the Advisory Council on Scientific Policy warned that Britain is losing ground to the United States, the Soviet Union, and to smaller countries in the field of pure mathematics. The Council said this was because Britain had perhaps "persisted too long in the tradition of the lone worker."

"To play its proper part in the development of mathematics, as in most other subjects, a university must now provide itself with the facilities for cooperative work, and there should be provision for the organization of conferences, summer schools, and facilities for the reception of leading mathematicians from other countries."

With regard to scientific research, the Council took somewhat the same view. In the past, it said, new fields of scientific study had developed in the universities around brilliant individuals. The report continues:

"This procedure has been highly satisfactory in the past where scientific research was relatively inexpensive, but, when the total cost of activity on a worthwhile scale may run into millions of pounds, the situation is quite different."

"Our resources are limited and more concentration of our effort is required; attempts to pursue certain types of scientific activity in many different universities can result in a dispersal and dilution of effort such that in the end we achieve much less than could be reasonably expected from the total expenditure involved."

The Council found an encouraging rise in expenditure for scien-
tific and technological research and development in recent years and noted that there does not appear to be a great discrepancy between the proportion of national resources devoted to this effort in the United States and Britain.

The Council is a group appointed by the Government to advise the Minister of Science on the formulation and execution of Government scientific policy.

THE USE OF COMPUTERS IN UNDERGRADUATE ENGINEERING EDUCATION PROJECT, sponsored by the Ford Foundation, has now completed one full semester and one summer session, and the first report is available to those who are interested. The 600-page report may be obtained by writing to Professor Donald L. Katz, Department of Chemical Engineering, University of Michigan, Ann Arbor, Michigan. During the spring semester students in 49 different undergraduate courses were assigned homework problems to be written in MAD, the Michigan Algorithm Decoder and run on the 704. Descriptions of many of the problems which were assigned, together with the instructor's solution and a typical student's solution are included in the report. During the summer 18 visiting faculty members from other universities participated in a nine week program, and 75 others worked through a one week intensive course. The progress of the project was presented at a two-day conference early in September.

WOMEN RESUMING CAREERS IN MATHEMATICS after years of inactivity in that field will be retrained at Rutgers University, starting in February. Special mathematics courses, designed to bring professional knowledge up to date, will be offered at both the Newark and New Brunswick campuses of the state university, it was announced last week.

A grant from the Ford Foundation financed a study made last year by the university's women's college to determine how many women living in a ten-county New Jersey area wished retraining in mathematics. About 300 women who had studied the subject in college indicated a desire to take courses in order to qualify for scientific or teaching jobs. The survey further revealed that in the same area, 450 more mathematics teachers and 500 mathematically trained industrial personnel would be needed within five years.

As a result of these findings, Rutgers will begin an experimental retraining program in January, under a $155,000, two-year grant from the Ford Foundation.
On August 2, 1960 I arrived in Moscow to spend six months at the Steklov Mathematical Institute of the Academy of Sciences of the USSR (ANSSSR) whose Director is Academician I. M. Vinogradov. I came to Russia under the auspices of the National Science Foundation and the nebulous Cultural Exchange Agreement between the U.S.A. and USSR and under the sponsorship of Academician A. A. Dorodnitzyn, Director of the Computing Center of the Academy of Sciences of the USSR. My qualifications for writing this personal report are three: I am a mathematician; I have become fluent in the Russian language, albeit my grammar is atrocious; and for the most part I am treated as an ordinary citizen.

However, there are invisible barriers to understanding that are a constant source of frustration. The worlds in which we and the Russians live are as different as they are alike. Many words remain the same, although their meanings change. "Beefsteak" on a "stolovaya" menu may mean "hamburger." Observers of the same phenomenon react differently. For instance, I was recently asked for the 10^3 time since my arrival, "Why do you write with your left hand?" I replied politely, "Why do you write with your right hand?" "Everyone in the USSR is taught to write with his right hand", was the answer. "And in my country everyone is taught to write with his left hand", I ventured. My colleague believed me.

A Russian visited the U.S.A. and upon returning home wrote a book in which he reported that Americans eat grapefruit as an appetizer to each meal. "I did not really believe this" a supper guest told us recently, "but tonight I see it is really true." And so it goes.

The following paragraphs contain some of the facts I think I have learned and some of my reactions to them. Salaries mentioned are in rubles per month. The figures may not be exact but they are reasonable approximations in any case. I receive 10 rubles to the dollar. Overall, I feel that what you buy for a dollar costs about 20 rubles here. But it should be mentioned that the essentials of life are generally cheaper here than in the United States and the cultural amenities such as books, operas, ballets, etc., are considerably cheaper than in the United States. Also, both man and wife almost always work.

The scientific personnel of the institute are in the following categories: aspirant (780-1000 rubles per month, the latter if they have worked for two years after finishing a university), junior scientific worker without a degree, but generally a university graduate (1050-1350 rubles per month); junior scientific worker with a degree of "kandidat" (1850-2100 rubles per month); senior scientific worker
with degree of "kandidat" (2500-3000 rubles per month); senior scientific worker with degree of "doctor of physical and mathematical sciences" (3500-4500 rubles per month); assistant director (5000 rubles per month); director (6000 rubles per month). Among the academicians associated with the institute are Vinogradov, Novikov, Kolmogorov, P. S. Alexandrov, Pontryagin, Bogolubov and Sedov. Less exalted personages working here full or part time are: Keldyš (Novikov's wife), Safareivič, Markov, M. M. Postnikov, A. G. Postnikov, Boltyanskiĭ, Prokhorov, Sitnikov, Delone, Miščenko (Assistant Director and Director of the Party Organization -- the former post which has a new occupant each year, is not a requirement for the latter or vice versa); Nikolskiĭ (Assistant Director), and Gamkelidze (Editor of RZMat).

The Institute is divided into departments: differential equations, topology and geometry, theory of probability, mathematical physics, algebra, theory of numbers, mechanics, and theory of functions. Each department has a head. For example, P. S. Alexandrov and Pontryagin are the heads of the topology and differential equations departments, respectively. Department heads receive an additional 15% in salary. Humdrum administrative duties are handled by a "scholarly secretary" who, as the title implies, is a mathematician. This position rotates. The occupants are young men. Both Matematicheskiĭ Shornik and Izvestiya ANSSSR Ser. Mat. are edited at the Institute. The Institute has its own excellent library with a staff of about six and a cafeteria. There are also bookkeepers, a large secretarial-clerical staff, maintenance shops, and a large number of technicians who do numerical work, etc. These people have just a high school education and are paid a salary of about 700 rubles per month.

The Institute accepts about 15 aspirants each year. Entrance exams are held in September. The matriculants are generally university graduates. The entrance examinations consist of a comprehensive oral in mathematics, plus oral exams in philosophy (Marxism-Leninism) and a foreign language. This year's philosophy exam was rated as exceedingly difficult by the students with whom I talked. The mathematics exam resembles a Ph.D. oral prelim at Wisconsin. It lasts about an hour. There are three examiners. The object is to see which of the students has potentialities. Examinees pass if they show any spark of life. Upon passing they become "aspirants." If they are in need of housing, they can then stop living with their friends and move into the huge "dorm aspirantov" down the block from the Institute. Aspirants take two examinations and deliver three reports in their first two years. One of the reports must be on machine programming. The examinations may be on such frightening subjects as "Lie groups à la Cartan and dynamical systems à la Birkhoff". The reports are detailed oral expositions of the knowledge in such areas. They are
presented orally. Two hours a day for three days occurred in one case reported to me by M. M. Postnikov. There are additional examinations in a foreign language and philosophy. At the end of his second year, the aspirant begins on his dissertation if he has passed his examinations. This dissertation must now be published before it is defended. The thesis may consist of several published papers.

The program is impressive: a five year university program heavily weighted to mathematics, final examinations and a dissertation, then the aspirantship. These students are not narrowly educated. I find they are well-read. Many follow the theatre. Others play an instrument well. They are poor if they have only their stipends to live on, but they have spirit and faith in the future. They are a marvelous youth.

If an aspirant successfully defends his dissertation against two opponents and if a review committee of readers approves it, he receives the degree of "kandidat." The ceremony of defense of a thesis is an interesting one. A moderate company of inattentive mathematicians (and perhaps relatives of the candidate) convenes in the third floor auditorium. Vinogradov sits at a center table facing the audience and inanimately directs the proceedings. He calls the meeting to order and calls for the reading of the candidate's curriculum vitae. "Are there any questions?" Vinogradov asks. After questions the candidate speaks on his dissertation summarizing the main results. When he has finished, if there are no questions, his two opponents (Boltyanskiĭ is a vigorous one) read their statements. These contain an honest evaluation of the dissertation. Criticisms may be abundant. But in the end they recommend that the degree be granted. The candidate replies to the criticisms, usually agreeing to all suggested changes. The last part of the procedure is a vote which is by secret ballot. The candidate must receive a two-thirds vote of those present to win. In the "provinces" the electorate is not solely composed of mathematicians, and a two-thirds vote can at times be hard to muster, Nikolskiĭ tells me. An election commission counts the ballots, and the results are announced with ceremony. A. A. Markov is unrivalled in his ability to add interest and suspense to this final announcement.

My favorite quotation from life at the Institute is Pontryagin's remark to an aspirant giving a paper in his seminar. The aspirant had just finished writing down his last hypothesis when Pontryagin said, "We had a beautifully clear problem in front of us, but somehow it succeeded in running away."
Technical meetings, in general, and the presentation of technical papers, in particular, have been subjected to close scrutiny by a number of organizations in the last few years. This is a natural result of the growing dissatisfaction with the amount of information that is disseminated at any given meeting. While more papers than ever are being presented, the net information received by each listener seems to be approaching zero. That this approach appears to be more than asymptotic should be some cause for alarm.

The manner in which technical papers are presented has drawn fire from such organizations as the American Institute of Electrical Engineers, the American Society of Mechanical Engineers, the Institute of Radio Engineers, and the American Society of Chemical Engineers. One of these, the Institute of Radio Engineers, has organized a Professional Group on Engineering Writing and Speech which publishes a quarterly Transactions. Although many of the members of this group are technical writers, others look to the Transactions as a means of improving their technical papers.

Since the human ear, per se, is not a highly efficient receiver of technical information, the oral presentation of papers is a natural place to begin improvement. Some of the technical societies have published papers on how to present papers. (1,2)

Most frequent criticisms have been pointed at the speaker who seizes the opportunity to make an impression. It is a matter of professional pride for this individual to parade as much of his technical knowledge as he can in as brief a time as possible. He aims to impress. His ammunition is a series of complex equations, a galaxy of undefined terms, and lantern slides which contain ten times as much information as any human mind can absorb during the ten seconds each slide is flashed onto the screen. He is often characterized by illegible handwriting and acute eraserphobia. This last causes him to seek out unused portions of the blackboard so that before he is through, he has written equations between the equations between the equations.


Next, there is the speaker who is not a speaker at all because he reads his material—very rapidly and very evenly. Reading is a handy way to increase the snow factor to a dangerously high level. More bucketsful of technical information per minute can be slopped over a long-suffering audience in this manner than any other. In a desperate move to stop this torture, the WESCON (Western Electronic Show and Convention) flatly stated that no papers could be read during the August 23-26, 1960 conference in Los Angeles. The idea, of course, was that the speakers were less likely to become over-technical during their twenty-minute periods.

This problem does not really affect the American Mathematical Society because very few speakers read their ten-minute papers. This length of time is short enough so that even the most complex formulae and the most obtuse notation can be committed to memory. Hence, rules forbidding reading of papers are not required, nor would they be followed in any event.

The Society has a unique problem, one that is not shared by other technical societies. This is the problem of the speaker who is not a speaker at all, who isn’t even a reader, who isn’t even there at all. Mathematicians are very proud of their individuality and guard it jealously. This is natural and expected. But when an eager listener makes a mad dash from one technical session to another in order to hear a particular paper only to find that the author is a "no-show", it is time to examine this business of individuality.

It is more than high time to ask questions like the following. What is a technical paper intended to accomplish? Is it the speaker's objective to impress his audience or to teach his audience? Do mathematicians attend meetings in order to be impressed or to learn? Is it possible to justify technical meetings unless the papers are tutorial? Why should attendance at meetings be limited to those seeking to hire instructors, those seeking jobs, those seeking to renew old acquaintances, and those who wish to get away from the regular routine? If every speaker made a truly honest effort to present a tutorial paper, attendance at the technical sessions would increase, there would be more thought-provoking discussion and there would be a general feeling of satisfaction that some useful purpose has been accomplished.

To the question, "How can a highly technical paper be made tutorial?", the following answers can be given. Limit the number of new concepts that are introduced to one or two. It is not necessary to cover every detail of three years of research in a ten-minute paper. Point out the relationship between your highly specialized field and other branches of mathematics. Don't be afraid to say that your highly specialized field is merely a special case of some well-known branch of mathematics. Use historical facts freely but do not merely recite the names of those who have made contributions to your particular field. Always keep in mind that you are the teacher and that
your audience consists of students most of whom have not heard your definitions or seen your notation until this minute. Use analogies freely in comparing your topic with more familiar ones. Test your presentation occasionally by throwing out a question to see if you get a response. (This takes real courage!)

Every paper presented at a meeting should be tutorial. The time has come to widen the communication channels and increase the flow of information.

MODERN FACTOR ANALYSIS

By Harry H. Harman

A sophisticated, accurate, and up-to-date account of factor analysis from its basic foundations through the latest and most advanced techniques, including the use of high-speed electronic computers.

Designed for use as both a text and a reference, this book will ably serve the needs of graduate students and researchers using factor analysis as a statistical tool. 480 pages. 1960.

Harry H. Harman, co-author of an earlier book, Factor Analysis, is manager of the Systems Simulation Research Laboratory at the System Development Corporation in California. $10.00
VISITING FOREIGN MATHEMATICIANS. The following list of visiting foreign mathematicians has been prepared by the Division of Mathematics of the National Academy of Sciences - National Research Council. This list is dated October 12, 1960.

<table>
<thead>
<tr>
<th>Name</th>
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<th>Period of visit</th>
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<td>Germany</td>
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<td>Sept. 1960 - June 1961</td>
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<td>Switzerland</td>
<td>Switzerland</td>
<td>Catholic University</td>
<td>Sept. 1960 - June 1961</td>
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<td>Bhattachayya, P.R.</td>
<td>India</td>
<td>India</td>
<td>University of North Carolina</td>
<td>Sept. 1960 - June 1961</td>
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<td>Bose, A. K.</td>
<td>India</td>
<td>India</td>
<td>University of North Carolina</td>
<td>Sept. 1960 - June 1961</td>
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<tr>
<td>Cavailles, Paul L.</td>
<td>France</td>
<td>France</td>
<td>University of California, Berkeley</td>
<td>Sept. 1960 - June 1961</td>
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<td>Chakravarti, I. M.</td>
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<td>Feb. 1961 - May 1961</td>
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<td>Finland</td>
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<td>April 1960 - Nov. 1960</td>
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<td>Hayes, Allan</td>
<td>U. K.</td>
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</table>
Dr. ROBERT W. BASS was named Maryland's outstanding scientist of 1960 by the Maryland Academy of Sciences.

Dr. P. J. KOOSIS of New York University has been awarded a National Science Foundation Postdoctoral Research Fellowship and is at the Institut Henri Poincaré, Paris, France.

Assistant Professor F. P. PETERSON, on leave from Massachusetts Institute of Technology, has been appointed an Alfred P. Sloan Postdoctoral Research Fellowship at Oxford University for the academic year 1960-1961.

Associate Professor D. B. RAY, on leave from Massachusetts Institute of Technology, has been appointed an Alfred P. Sloan Postdoctoral Research Fellowship at the Institute for Advanced Study, Princeton, New Jersey.

Associate Professor H. ROGERS, on leave from Massachusetts Institute of Technology for the academic year 1960-1961, has been awarded a Guggenheim Fellowship.

Associate Professor F. L. SPITZER, on leave from the University of Minnesota, has been awarded a National Science Foundation Visiting Senior Postdoctoral Research Fellowship and will be at Princeton University for the 1960-1961 academic year.

Dr. D. W. SWANN of Stanford University has been awarded a NATO Post Doctoral Fellowship in Science at the University of Cambridge, England.

Professor E. P. VANCE, on leave from Oberlin College, has been awarded a National Science Foundation Faculty Fellowship and will be at Stanford University for the 1960-1961 academic year.

Dr. A. W. ADLER of Massachusetts Institute of Technology has been appointed to an assistant professorship at Rutgers, The State University.

Dr. A. R. AMIR-MOEZ of Queens College has been appointed to an assistant professorship at Purdue University.

Dr. W. C. BENNEWITZ of the University of Southern California has been appointed to an assistant professorship at Southern Illinois University.

Associate Professor M. CASTELLANI of the University of Kansas City has been appointed to a professorship at Fairleigh Dickinson University.
Dr. B. Chang of the University of British Columbia has been appointed to an assistant professorship at Ewha Women's University, Seoul, Korea.

Professor M. L. Curtis, on leave from Florida State University, will return after spending the academic year 1959-1960 at Cambridge University, England.

Dr. E. C. Dade of Princeton University has been appointed a Bateman Fellow at California Institute of Technology.

Mr. E. I. Deaton of the University of Texas has been appointed to an assistant professorship at San Diego State College.

Dr. P. DeDecker, on leave from the University of Liège, has been appointed to a professorship at the Universidad Central de Venezuela, Caracas, Venezuela.

Dr. W. W. Denton has been appointed to a professorship at Pikeville College.

Mr. E. A. Enrione has accepted a position as mathematician with Douglas Aircraft Company, Long Beach, California.

Associate Professor B. Epstein of the University of Pennsylvania has been appointed to a professorship at Yeshiva University.

Professor G. C. Evans, on leave from the University of California, is spending the winter in Rome, Italy.

Dr. G. M. Ewing of the University of Oklahoma has been appointed to a professorship at the University of Oklahoma.

Dr. C. C. Faith, on leave from Pennsylvania State University, will be at the Institute for Advanced Study for the 1960-1961 academic year.

Associate Professor H. C. Filgo, Jr., on leave from the University of Alabama, has been appointed to a professorship at the University of Georgia.

Associate Professor H. Flanders of the University of California, Berkeley, has been appointed to a professorship at Purdue University.

Mr. L. D. Fountain of the University of Nebraska has been appointed to an assistant professorship at San Diego State College.

Dr. R. A. Gambill of General Motors Corporation has been appointed to an associate professorship at Purdue University.

Associate Professor F. W. Gehring, on leave from the University of Michigan, has returned after spending the academic year 1959-1960 at the Swiss Federal Institute of Technology.

Mr. A. Gewirtz of Waldorf Industrial Controls, Huntington Station, New York has accepted a position as project engineer at Veeco Vacuum Corporation, Plainview, New York.

Professor Wallace Givens of Wayne State University has been appointed to a professorship at Northwestern University.
Assistant Professor S. I. GOLDBERG, on leave from Wayne State University, has been appointed to a visiting associate professorship at the University of Illinois for the academic year 1960-1961.

Professor J. R. HANNA of the University of Wichita has been appointed to an associate professorship at the University of Wyoming.

Assistant Professor C. HATFIELD, JR. of the University of Minnesota has been appointed to a professorship at the University of North Dakota.

Professor K. A. HIRSCH, on leave from the University of London, has been appointed to a visiting professorship at Washington University, St. Louis, Missouri.

Dr. B. E. HOWARD of the University of Chicago has been appointed to a professorship at the University of Miami.

Dr. H. C. HOWARD of the University of Wisconsin, Madison, has been appointed to an associate professorship at the University of Wisconsin, Milwaukee.

Professor C. C. HSIUNG, on leave from Lehigh University, has returned after spending the academic year 1959-1960 at the University of Wisconsin, Army Research Center.

Associate Professor C. W. HUFF of Auburn University has been appointed to a professorship at Winthrop College.

Dr. H. B. JENKINS of New York University has been appointed to a visiting assistant professorship at Stanford University.

Mrs. A. M. JONES, on leave from the University of Alabama, has been appointed an associate professorship at Millsaps College.

Dr. W. KAHAN has been appointed to an assistant professorship at the University of Toronto.

Dr. J. KANE, on leave from the University of Rhode Island, has accepted a position as staff consultant at Dorne and Margolin, Westbury, New York.

Associate Professor L. H. KANTER of Montclair State College has been appointed to an assistant professorship at Queens College.

Assistant Professor S. A. KHABBAZ of the University of Massachusetts has been appointed to an assistant professorship at Lehigh University.

Associate Professor E. E. KOHLBECKER, on leave from the University of Utah, has been appointed to a visiting assistant professorship at the University of Illinois.

Mr. R. R. KORFHAGE of the University of Michigan has been appointed to an assistant professorship at North Carolina State College.

Dr. S. KOTZ of Cornell University has been appointed a lecturer at Bar-Ilan University, Ramat Gan, Israel.

Dr. L. C. LAY of Pasadena City College has been appointed to a professorship at Orange County State College.
Dr. J. C. LILLO of RIAS has been appointed to an assistant professorship at Purdue University.

Mr. J. M. McLYNN of George Washington University has accepted a position as project director at Booz-Allen Applied Research Incorporated, Bethesda, Maryland.

Assistant Professor R. A. MACAULEY of the University of Washington has been appointed to an assistant professorship at the University of Nevada.

Dr. H. MacNEILLE of Washington University has been appointed to a professorship at Case Institute of Technology.

Assistant Professor J. G. MARICA of Fresno State College has been appointed to an assistant professorship at Sacramento State College.

Acting Assistant Professor A. W. MARSHALL of Stanford University has accepted a position as mathematician at the Institute for Defense Analyses, Princeton, New Jersey.

Dr. P. R. MASANI of Brown University has been appointed to a professorship at Indiana University.

Dr. H. F. MATHIS of Goodyear Aircraft Corporation has been appointed to a professorship at Ohio State University.

Associate Professorship W. H. MILLS, on leave from Yale University for the 1960-1961 academic year, has been appointed a visiting scholar at the University of California, Berkeley.

Dr. H. J. MISER of Research Triangle Institute has joined the staff of the Navy's Operations Evaluation Group as Director of its newly established Applied Science Division at the Massachusetts Institute of Technology, Cambridge, Massachusetts.

Professor G. D. MOCK of the State University of New York has been appointed to an associate professorship at Western Illinois University.

Dr. J. R. MUNKRES of Princeton University has been appointed to an assistant professorship at Massachusetts Institute of Technology.

Professor LEOPOLDO NACHBIN of the Institute for Pure and Applied Mathematics, Rio de Janeiro, Brazil, has been appointed to a visiting professorship at Brandeis University from December, 1960 through March, 1961.

Professor T. G. OSTROM of Montana State University has been appointed to a professorship at Washington State University.

Dr. F. P. PALERMO of the University of Michigan has accepted a position as staff mathematicians with International Business Machines Corporation, Yorktown Heights, New York.

Assistant Professor D. A. POPE of the University of Minnesota has been appointed to an associate professorship at the University of California, Davis.

Dr. W. E. PRUITT of Stanford University has been appointed to an assistant professorship at the University of Minnesota.
Assistant Professor RONALD PYKE of Columbia University has been appointed to an assistant professorship at the University of Washington.

Associate Professor L. B. RALL of Lamar State College of Technology has been appointed to an associate professorship at Virginia Polytechnic Institute.

Dr. F. A. RAYMOND of the Institute for Advanced Study has been appointed to an assistant professorship at the University of Wisconsin.

Dr. J. R. RICE of the General Motors Research Laboratories has taken a six month leave of absence for a tour of active duty in the Army.

Dr. R. W. RICHARDSON, Jr. of Princeton University has been appointed to an assistant professorship at the University of Washington.

Assistant Professor G. J. RIEGER of the University of Maryland has been appointed to an associate professorship at Purdue University.

Dr. T. D. RINEY of Bell Telephone Laboratories, Inc., has accepted a position as space mechanics mathematician with Space Science Laboratory, Philadelphia, Pennsylvania.

Assistant Professor E. A. ROBINSON, on leave from the University of Wisconsin, will spend the 1960-1961 academic year at the Statistiska Institutionen vid Uppsala Universitet, Uppsala, Sweden.

Assistant Professor J. I. ROSENBLATT of Purdue University has been appointed to an associate professorship at the University of New Mexico.

Dr. S. M. ROSENZWEIG of Massachusetts Institute of Technology has been appointed to an assistant professorship at Rutgers, The State University.

Mr. C. R. SELIGER of The Martin Company has accepted a position as senior research scientist in the Research Division of Aeronca Manufacturing Corporation, Baltimore, Maryland.

Visiting Professor S. M. SHAH of Northwestern University has been appointed to a visiting professorship at the University of Kansas.

Professor V. L. SHAPIRO of Rutgers, The State University, has been appointed to a professorship at the University of Oregon.

Mr. W. T. SHARP of Atomic Energy of Canada, Ltd., has been appointed to an associate professorship at the University of Alberta.

Dr. O. SHISHA of Harvard University has accepted a position as mathematician with the National Bureau of Standards, Commerce Department, Washington 25, D. C.

Assistant Professor SHU-TEH C. MOY of Wayne State University has been appointed to an associate professorship at Syracuse University.
Professor A. SOBCZYK of the University of Florida has been appointed to a professorship at the University of Miami.

Mr. A. S. STANKOVICH of the Air Force Department has accepted a position as operations research analyst with the System Development Corporation, Paramus, New Jersey.

Assistant Professor R. S. STEMMLER of the University of Illinois has been appointed to an assistant professorship at Purdue University.

Dr. K. R. STROMBERG of the University of Chicago has been appointed to an assistant professorship at the University of Oregon.

Dr. KONRAD SUPRUNOWICZ of the University of Nebraska has been appointed to an assistant professorship at the University of Idaho.

Mr. C. M. TERRY of the Defense Department, Washington, D. C. has accepted a position as mathematician at the Institute for Defense Analyses, Princeton, New Jersey.

Professor H. P. THIELMAN of Iowa State University has accepted a position as mathematician with Land-Air Incorporated, Point Mugu, California.

Associate Professor C. J. TITUS, on leave from the University of Michigan, has accepted a position as mathematician with the Institute for Defense Analyses, Princeton, New Jersey.

Professor C. B. TOMPKINS, on leave from the University of California, Los Angeles, has accepted a position as mathematician with the Institute for Defense Analyses for the academic year 1960-1961.

Dr. SAN FU TUAN of the University of Chicago has been appointed to an assistant professorship in the physics department of Brown University.

Dr. L. H. TURNER of Space Technology Laboratories, Incorporated has been appointed to an assistant professorship at the University of Minnesota.

Assistant Professor H. S. VALK of the University of Oregon has been appointed to an assistant professorship in the physics department of the University of Nebraska.

Dr. R. L. VAN de WETERING of Stanford University has been appointed to an assistant professorship at San Diego State College.

Assistant Professor M. J. WALSH of the University of Wyoming has accepted a position as design specialist with CONVAIR, San Diego, California.

Mr. C. M. WARDEN of California Institute of Technology has accepted a position as mathematical engineer in the Missiles and Space Division of Lockheed Aircraft Corporation, Sunnyvale, California.

Associate Professor J. WERMER of Brown University has returned after spending the academic year 1959-1960 at Harvard University.
Dr. W. L. WILSON, JR. of North American Aviation, Incorporated has been appointed to an assistant professorship at the University of Alabama.

Associate Professor YUE-KEI WONG of Adelphi College has been appointed to an associate professorship at the University of Toledo.

Associate Professor S. D. ZELDIN has retired from Massachusetts Institute of Technology with the title of associate professor emeritus.

Dr. P. J. ZWIER of Purdue University has been appointed to an assistant professorship at Calvin College.

The following promotions are announced:

I. K. BRAUER, Boston University, to an assistant professorship.

K. L. COOKE, Pomona College, to an associate professorship.

E. A. DAVIS, University of Utah, to an associate professorship.

L. K. DURST, William Marsh Rice University, to an associate professorship.

C. C. FAITH, Pennsylvania State University, to an associate professorship.

JACOB FELDMAN, University of California, Berkeley, to an associate professorship.

R. K. GETOOR, University of Washington, to an associate professorship.

C. C. HSIUNG, Lehigh University, to a professorship.

LAWRENCE GOLDMAN, Stevens Institute of Technology, to an associate professorship.

M. JERISON, Purdue University, to a professorship.

G. JOHNSON, JR., William Marsh Rice University, to an assistant professorship.

E. E. KOHLBECKER, University of Utah, to an associate professorship.

W. A. J. LUXEMBURG, California Institute of Technology, to an associate professorship.

H. P. McKEAN, JR., Massachusetts Institute of Technology, to an associate professorship.

H. P. MULHOLLAND, University of Exeter, England, to reader in analysis.

R. H. NIEMANN, Colorado State University, to an associate professorship.

K. B. O'KEEFE, University of Washington, to an assistant professorship.

R. S. PIERCE, University of Washington, to a professorship.

D. B. RAY, Massachusetts Institute of Technology, to an associate professorship.
P. D. RITGER, Stevens Institute of Technology, to an associate professorship.

N. J. ROSE, Stevens Institute of Technology, to a professorship.

P. W. SHAW, San Diego State College, to an associate professorship.

ABE SKLAR, Illinois Institute of Technology, to an associate professorship.

N. B. SMITH, San Diego State College, to an associate professorship.

G. B. THOMAS, Massachusetts Institute of Technology, to a professorship.

P. E. THOMAS, University of California, Berkeley, to an associate professorship.

R. L. VAUGHT, University of California, Berkeley, to an associate professorship.

D. V. V. WEND, University of Utah, to an associate professorship.

C. H. WILCOX, California Institute of Technology, to an associate professorship.

W. B. WOOLF, University of Washington, to an assistant professorship.

The following appointments to instructorships are announced:

Agnes Scott College: Miss MARY B. WILLIAMS; California Institute of Technology: Dr. C. R. B. WRIGHT; University of California, Los Angeles: Dr. E. R. BERKSON; Catholic University of America: Dr. INGE CHRISTENSEN, Mr. GUSTAV HENSEL; University of Chicago: Dr. L. EVENS, Dr. A. A. SAGLE; Drexel Institute of Technology: Mr. H. F. THORNTON; University of Illinois: Dr. DAVID SACHS, Mr. E. C. WEINBERG; Indiana State Teachers College: Mr. J. E. STROUT; Massachusetts Institute of Technology: Dr. R. JACOB WITZ, Mr. J. I. RICHARDS, Mr. F. S. VAN VLECK; North Dakota Agricultural College: Mr. G. V. ROWELL; Oregon State College, Corvallis: Mr. E. L. ROETMAN; Princeton University: Dr. D. A. LUDWIG; Syracuse University: Mr. J. D. REID; University of Washington: Mr. K. A. ROSS, Dr. JACK SEGAL; University of Wisconsin, Milwaukee: Mr. J. E. McADAM; Yale University: Mr. L. J. GERENDE, Dr. F. J. HAHN.

Deaths:

Mr. D. D. MANN of the University of California died on September 8, 1960 at the age of 27 years.

Professor A. L. NELSON of Wayne State University died on April 5, 1960 at the age of 69 years. He had been a member of the Society for 44 years.
Professor Emeritus OSWALD VEBLEN of the Institute for Advanced Study died on August 10, 1960 at the age of 81 years. He had been a member of the Society for 54 years.

Errata:

The announcement on page 451 of the 47th issue of the NOTICES concerning Dr. S. D. CHATTERJI of Michigan State University should read as follows:

Dr. S. D. CHATTERJI of Michigan State University has been appointed a lecturer at the School of Mathematics, University of New South Wales, Kensington, Australia.

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Editor, the NOTICES

Not versed in the psychology of Martians, I am unable to contribute to any discussion of the temptations that Professor A. V. Martin (NOTICES, August 1960) fears they may face on earth. However, I can find nothing in terrestrial logic to justify making the inference that he suggests can be drawn from my earlier letter (NOTICES, April 1960).

That portion of my letter in which I referred to a few of the many instances in which American mathematics and mathematicians have encountered violations of the principles of freedom cannot be rebutted by claims that things may be worse elsewhere. Presumably, American behavior should be judged by the standards America proclaims.

Accordingly, it is not incumbent upon me to enter into any discussion of life in the USSR. I do not even have to try to balance what Professor Martin asserts is the fear possessed by the government of the USSR of some scientists leaving their jobs against the fears possessed by a number of American scientists of their jobs leaving them. It would seem more appropriate in this situation for Americans seeking to dig up some liberty to follow the advice of Mr. Booker T. Washington who urged: "Put down your buckets where you are."

Professor Martin asserts that "...the United States government makes no effort to prevent citizens from leaving the country,..." This is not the case. For example, some American mathematicians did not get to the 1954 (Amsterdam) International Mathematical Congress because of the passport situation. Other Americans have been -- and still are -- forced to sue for their passports, having been deprived of them for years. Further, anyone who has visited China, e.g., in recent years is denied even now a passport unless he certifies in writing that he will not visit there again (except in certain special cases). The courts have sustained the state department in this. They have ruled against Mr. William Worthy, a reporter for the Afro-American newspaper chain who was denied a passport because of his intention to revisit China in his capacity as a journalist, and against Representative Charles O. Porter (Democrat, Oregon) who wanted to visit China in order to gain information and insights which he believed might assist him in forming judgments on matters on which he would vote in the Congress.

In the reverse direction as well, the United States government complicates many scientific gatherings. I doubt if it admits scientists from China or east Germany. Last summer, it required eastern
European statisticians attending the Berkeley Symposium to remain within a 25 mile radius of Berkeley. When the Symposium moved to Stanford, 35 miles away, special permission had to be obtained from Washington for eastern European participants to attend. Other ludicrous restrictions were imposed as well.

Nor are America's own citizens free of humiliating restrictions. Last April, conditions surrounding a meeting of the Mathematical Association held in Professor Martin's home state, South Carolina, were so inconsistent with, to use his phrase, America's "libertarian heritage and principles" that colored mathematicians walked out of the meeting in protest.

It would be well to remember that the systematic and deliberate violation of America's "libertarian heritage and principles" with respect to its colored citizenry has, among effects far more serious still, the consequence of depriving the mathematical community of a significant number of worthy members.

The heavy restrictions on Americans whose social views or skin pigmentation are of a "wrong" color are too well known to require elaboration here. But these repressions should never be forgotten, excused or tolerated.

Lee Lorch

Editor, the NOTICES

In the October 1960 issue of these NOTICES, toward the bottom of page 622, appears the following: "The future professional mathematician -- from computer programmer to the brilliant research man...". I decidedly object to the implication that computer programming is the lower bound on mathematical activity. Quite true, much programming is mechanical, demanding accuracy but little creativity. Will any mathematician claim that a large fraction of published research papers indicate unusual insight? In less than fifteen years, digital computers backed up by programmers have produced at least partial answers to a fair number of mathematical questions. Whether this trend will continue at an accelerated rate over the next few years depends primarily on the ingenuity and ambition of programmers. Authors and editors would do well to guard against printing such remarks in publications of the American Mathematical Society.

E. T. Parker
Much has been said in the mathematical literature concerning the updating of published works in mathematics. A particularly interesting article, On the Desirability of Publishing Classified Bibliographies of the Mathematics Literature, by H. S. Vandiver, was published in the January, 1960, issue of the American Mathematical Monthly. In his paper, Vandiver discussed the need for up to date reviews as an aid to the research mathematician, both in his research and as an aid in avoiding endless time spent on duplication of previous publications. In our opinion, publications of bibliographies classified according to topics of all branches in pure and applied mathematics as suggested by Vandiver, would be of outstanding value to both the research mathematician and teachers of mathematics.

Several of the journals publish, from time to time, statements of problems that individual mathematicians submit because they feel the problems are interesting and that their solution would contribute to the general process of mathematics. We believe further consideration should be given to the task of periodically publishing a work, classified according to topics in all branches of mathematics, which would deal specifically with the major unsolved problems in mathematics with references related to each individual problem whenever possible.

Whereas the magnitude of publishing classified bibliographies, as stated by Vandiver, is understood to demand considerable maturity, and, further require access to vast references obtainable only in the mathematical libraries, this would not be the case in a publication dealing with unsolved problems. Authorities in each branch of mathematics could be contacted annually and asked to submit clear and concise statements of existing unsolved problems known to them. Probably a capable graduate student, with both the desire and patience to undertake a task of this type, could do a satisfactory job; or, a more thorough work could be obtained by appointing a committee of graduate students, properly supervised by one or more professors, and designating specific responsibilities to each. Thus, many capable mathematicians who do not have access to the finer libraries nor have the chance to communicate with the leading authorities in their particular field would surely benefit from a work of this nature and, at the same time, create a more broadened interest in the field of mathematical research. Although we cannot expect to ascertain the exact amount of value such a work would yield, it would be a definite aid to the research mathematician and would probably, in time, lead to the solution of many previously unsolved problems.

William Edward Christilles
MEMORANDA TO MEMBERS

THE MATHEMATICAL SCIENCES EMPLOYMENT REGISTER

The following item is repeated from the November, 1960 issue of the NOTICES, and gives a more detailed time schedule.

The Mathematical Sciences Employment Register, established by the American Mathematical Society, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics, will be maintained at the Annual Meeting at the Willard Hotel in Washington, D. C., on January 24, 25, and 26, 1961. The Register will be conducted from 9:00 A.M. to 5:00 P.M. on each of these three days. The morning of Tuesday, January 24, will be devoted to the registration of both employers and applicants and to distribution of the listings.

The Employment Register Desk will be located in Room 228. There is no charge for registering, either to job applicants or to employers, except when the Late Registration Fee for employers is applicable. Provision will be made for anonymity of applicants upon request and upon payment of $1 to defray the cost involved in handling anonymous listings.

Job applicants and employers who wish to be listed will please write to the Employment Register, 190 Hope Street, Providence 6, Rhode Island, for application forms and for position description forms, which must be completed and returned to Providence not later than January 4, 1961, in order to be included free of charge in the listings at the meeting in Washington, D. C. Forms which arrive after this closing date, but before January 17, will be included in the listings at the meeting for a Late Registration Fee of $3.00, and will also be included in the printed listings, but not until ten days after the meeting. The printed listings will be available for distribution both during and after the meeting. The prices are as follows: Position descriptions, $2.00; listing of applicants, academic only, $5.00; comprehensive listing of applicants, academic, industrial, and government, $20.00.

It is essential that applicants and employers register at the Employment Register Desk promptly upon arrival at the meeting to facilitate the arrangement of appointments.

SUMMER EMPLOYMENT FOR MATHEMATICIANS AND COLLEGE MATHEMATICS STUDENTS

The Headquarters Office of the Society has compiled a list of opportunities for summer employment for mathematicians and college mathematics students. Members who are interested in summer em-
ployment and would like to obtain a copy of this list should please write to the Special Projects Department, American Mathematical Society, 190 Hope Street, Providence 6, Rhode Island. There is no charge to members for this list.

Copies will be available at the annual meeting in Room 228 of the Willard Hotel.

THE NATIONAL REGISTER OF
SCIENTIFIC AND TECHNICAL PERSONNEL

The Mathematical Sciences Section of the register will maintain a desk during the Annual Meeting in Washington on January 24, 25, and 26. The National Register desk will be located in Room 228. The attendants will be pleased to assist with registrations and to supply information. The National Register as a whole is a responsibility of the National Science Foundation. The Mathematical Sciences Section is operated by the American Mathematical Society under contract with the Foundation.

THE 1961 NOTICES

After seven years of steady growth, the NOTICES will be appreciably larger in 1961. Beginning with the February issue, the NOTICES will be published as a two-column journal, graduating from its present format to 8" × 10" page size. The subscription rate will not be increased for the 1961 volume. The list price for a yearly subscription remains $7.00 per volume; the NOTICES is distributed free to individual, institutional, and corporate members of the Society. We hope that the new format with its attractive cover will result in an even wider distribution of the NOTICES of the American Mathematical Society, one of the most widely read periodicals in the world dealing exclusively with the mathematical sciences.

RECPROCITY AGREEMENT WITH THE
OSTERREICHISCHE MATHEMATISCHE GESELLSCHAFT

The American Mathematical Society has entered into a reciprocity agreement with the Österreichische Mathematische Gesellschaft by which members of each may become members of the other by paying half the regular dues. Dues of members of the American Mathematical Society wishing to join the Österreichische Mathematische Gesellschaft are one dollar. Privileges of membership include free attendance at meetings and receiving the Internationalen Mathematischen Nachrichten. Those members of the American Mathematical Society wishing to take advantage of this arrangement
should write to

Professor Dr. Hans Hornich
Oesterreichische Mathematische Gesellschaft
Karlsplatz 13
Vienna IV, Austria

it is understood that members under the reciprocity agreement spending time in the other country should pay the regular dues while they are there.

RECIPIROCITY AGREEMENT WITH THE GESELLSCHAFT FUER ANGEWANDE MATHEMATIK UND MECHANIK

The American Mathematical Society has entered into a reciprocity agreement with the Gesellschaft fuer Angewandte Mathematik und Mechanik by which members of each may become members of the other by paying half the regular dues. Dues of members of the American Mathematical Society wishing to join the Gesellschaft fuer Angewandte Mathematik und Mechanik are one dollar. Privileges of membership include receiving the regular publications of the Gesellschaft fuer Angewandte Mathematik und Mechanik and certain books published in association with commercial publishers at reduced rates.

Those members of the American Mathematical Society wishing to take advantage of this arrangement should write:

Professor Henry Gortler
Mathematisches Institut der Universitat Freiburg
Hebelstrasse 60
Freiburg i. Br., Germany

it is understood that members under the reciprocity agreement spending time in the other country should pay the regular dues while they are there.

RECIPIROCITY AGREEMENT WITH THE SOCIEDAD MATEMATICA MEXICANA

The American Mathematical Society has entered into a reciprocity agreement with the Sociedad Matematica Mexicana by which members of each may become members of the other by paying half the regular dues. Members of the American Mathematical Society may then become members of the Sociedad Matematica Mexicana by paying $2.00. Privileges of membership include the presentation of papers at Society meetings organized by the Sociedad Matematica Mexicana; receiving free the Revista Matematica (2 numbers a year
and the Boletín de la Sociedad Matemática Mexicana (2 numbers a year); purchasing the publications of the Sociedad at reduced rates (about 50% discount). Those members of the American Mathematical Society wishing to take advantage of this arrangement should write:

Dr. Alfonso Napoles Gandara, President
Sociedad Matemática Mexicana
Tacuba No. 5
Mexico 1, D. F., Mexico

It is understood that members under the reciprocity agreements spending time in the other country should pay the regular dues while they are there.

RECIPROCITY AGREEMENT WITH THE UNION MATEMATICA ARGENTINA

The American Mathematical Society has entered into a reciprocity agreement with the Union Matematica Argentina by which members of each may become members of the other by paying half the regular dues. Dues of members of the American Mathematical Society wishing to join the Union Matematica Argentina are $1.25. Privileges of membership include free attendance at meetings of the Union Matematica Argentina and receiving the Review of the Union Matematica Argentina and the Asociacion Fisica Argentina.

Those members of the American Mathematical Society wishing to take advantage of this arrangement should write:

Secretary of the Union Matematica Argentina
Casilla de Correo 3588
Buenos Aires, Argentina

It is understood that members under the reciprocity agreement spending time in the other country should pay the regular dues while they are there.

OREGON STATE COLLEGE has five National Defense Education Act fellowships for first year Ph.D. students beginning in the fall of 1961. There will be three fellowships in Applied Analysis and two in an interdepartmental program (with Electrical Engineering) in Computer Science and Technology. A grant of $200,000 has been received from the National Science Foundation to begin construction of a high speed computer. Present plans call for building a machine similar to the Maniac 3 now being completed at the University of Chicago.

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NEW PUBLICATIONS

Adler, R. B., Chu, L. J., and Fano, R. M. Electromagnetic energy
$14.50.

1960. 9 + 485 pp. $7.50.

Akivis, M. A. See Blyaške, V.

Alexits, G. Konvergenzprobleme der Orthogonalreihen. Budapest,
Verlag der Ungarischen Akademie der Wissenschaften, 1960.
307 pp.


Bartlett, M. S. An introduction to stochastic processes, with special
reference to methods and applications. New York, Cambridge

Beyer, R. T. See Brekhovskikh, L. M.

Library of Psychology, Philosophy and Scientific Method.) Paterson,
219 pp. $1.50.

Blyaške, V. Vvedenie v geometriyu tkanei. [Einführung in die Geo-
metrie der Waben.] Trans. by M. A. Akivis; ed. by I. M. Yaglom.

Boas, R. P., Jr. A primer of real functions. (The Carus Mathematical
Monographs, No. 13) Published by the Mathematical Association
$4.00.

Borevič, Z. I. See Ševalle, K.

Boron, L. F. See Naïmark, M. A.

(Proceedings of a symposium conducted by the Mathematics Re-
search Center at the University of Wisconsin, Madison, April
10 + 324 pp. $4.00.

Brekhovskikh, L. M. Waves in layered media. Trans. by D. Lieberman;
translation ed. by R. T. Beyer. (Applied Mathematics and
pp. $16.00.

Brelot, M. Lectures on potential theory. Notes by K. N. Gowrisan-
karan and M. K. V. Murthy. (Lectures on Mathematics, 19.) Bom-
bay, Tata Institute of Fundamental Research, 1960. 2 + 155 pp.

Brunk, H. D. An introduction to mathematical statistics. Boston,
Ginn, 1960. 11 + 403 pp. $7.00.

Burbaki, N. Obščaya topologiya: Čisla i svyazannye s nimi gruppy i
prostranstva. [General topology: Numbers and the groups and
spaces related to them.] Trans. by C. N. Kračkovskij; ed. by
Burgers, J. M. See Goldstein, S.
Chan, K.-K. Summation of the Fourier series of orthogonal functions. Peking, Science Press, 1957. 4 + 174 pp. 17s. 6d.
Chu, L. J. See Adler, R. B.
Debrunner, H. See Hadwiger, H.
Drabkin, I. E. See Galilei, G.
Drake, S. See Galilei, G.
Fano, R. M. See Adler, R. B.

Flügge, S. See Handbuch der Physik.


Gerretsen, J. See Sansone, G.


Gowrisankaran, K. N. See Brelot, M.


Halmos, P. R. Lectures on ergodic theory. New York, Chelsea, 1960. 7 + 101 pp. $2.95.


Hill, R. See Progress in solid mechanics.

Hirsch, K. A. See Kurosh, A. G.

Hoffmann, B. See Kron, G.

Huschke, R. E. See Glossary of meteorology.


Kleinerman, G. I. See Kaplanski, I.


Kračkovskii, C. N. See Burbaki, N.


Langer, R. E. See Boundary problems in differential equations.

Langer, R. E. See Frontiers of numerical mathematics.

Levin, L. M. See Gantmaher, F. R.

Lieberman, D. See Brekhovskikh, L. M.


Manuel of the dozen system. New York, Duodecimal Society of America, 1960. 5 + 33 pp. $1.00.


May, D. C. Jr. See Burington, R. S.

Milloux, H. See Valiron, G.

Mitrinovič, D. S. See Fempl, S.


Murthy, M. K. V. See Brelot, M.

Naǐmark, M. A. Normed rings. Trans. by L. F. Boron. Groningen, Noordhoff, 1959. 16 + 560 pp. Paperbound, Dfl. 45.00, $12.00; cloth, Dfl. 48.75, $13.00.

Nelson, W. C. See Selected topics on ballistics.


Novikov, P. S. Élementy matematicheskoj logiki. [Elements of mathematical logic.] (Matematičeskaya Logika i Osnovaniya Matematiki.) Moscow, Gosudarstv. Izdat. Fiz.-Mat. Lit. 1959. 400 pp. 11.05 r.


Postnikov, M. M. See Kaplanskiǐ, I.


Raǐkov, D. A. See Burbaki, N.


Roslyakov, G. S. See Žogolev, E. A.


Selected topics on ballistics. Ed. by W. C. Nelson. (Cranz Centenary Colloquium, University of Freiburg, Freiburg, West Germany. AGARDograph No. 32.) New York, Pergamon, 1959. 7 + 280 pp. (1 plate) $9.00.


Sneddon, I. N. See Progress in solid mechanics.

Stojaković, M. D. See Dirac, G. A.
Șura-Bura, M. R. See Žogolev, E. A.
Tables of Whittaker functions (Wave functions in Coulomb field). II. (Report No. 11 of Numerical Computation Bureau.) Tokyo, 1959. 52 pp. $3.00.
Talbot, L. See Schaaf, S. A.
Trifonov, N. P. See Žogolev, E. A.
Vajda, S. An introduction to linear programming and the theory of games. London, Methuen; New York, Wiley; 1960. 76 pp. 9s. 6d.
Valiron, G. Fonctions entières d'ordre fini et fonctions méromorphes. From a manuscript rev. and prefaced by H. Miloux. (Monographies de "L'Enseignement Mathématique", No. 8.) Genève, Institut de Mathématiques, Université, 1960. 150 pp. 20 fr. s.
Ver Ecke, P. See Caravelli, Vito.
Wilson, J. G. See Progress in elementary particle and cosmic ray physics.
Wouthuysen, S. A. See Progress in elementary particle and cosmic ray physics.
Yaglom, I. M. See Blyaške, V.
CATALOGUE OF LECTURE NOTES

BRANDEIS UNIVERSITY

The following item may be ordered from: Mathematics Department, Brandeis University, Waltham 54, Massachusetts.

M. AUSLANDER, Rings, Modules and Homology, Chapters I and II, 131 pp. $1.00

UNIVERSITY OF CHICAGO

The following items may be ordered from: Mathematics Lecture Notes, Eckhart Hall, University of Chicago, Chicago 37, Illinois.

REPORTS, Seminar in Topology, 120 pp. $2.00
Y. AKIZUKI, Local Rings, 191 pp. $2.00
S. S. CHERN, Differentiable Manifolds (Revised Edition), 132 pp. $2.00
S. EILENBERG, Foundations of Fibre Bundles, 75 pp. $1.25
L. J. GARDING, Cauchy's Problem for Hyperbolic Equations, 106 pp. $1.75
L. M. GRAVES, The Estimates of Schauder and their Application to Existence Theorems for Elliptic Differential Equations, 67 pp. $1.00
P. R. HALMOS, Boolean Algebras, 175 pp. $3.00
P. R. HALMOS, Entropy in Ergodic Theory, 40 pp. $0.85
I. KAPLANSKY, Rings of Operators, 53 pp. $1.00
G. W. MACKEY, Theory of Group Representations, 182 pp. $3.00
A. WEIL, Fibre Spaces in Algebraic Geometry, 48 pp. $1.00
A. WEIL, Discontinuous Subgroups of the Classical Groups (Notes by A. H. Wallace), 91 pp. $1.50

COLUMBIA UNIVERSITY

The following item may be ordered from: Department of Mathematics, Columbia University, New York 27, New York

JAMES EELLS, JR., Singularities of Smooth Maps, 134 pp.
The following items may be ordered from: Department of Mathematics, Harvard University, 2 Divinity Avenue, Cambridge 38, Massachusetts.

OSCAR ZARISKI, An Introduction to the Theory of Algebraic Surfaces, 100 pp. $2.50

GEORGE W. MACKEY, Lecture Notes on Quantum Mechanics, 220 pp. $3.50

JOHN TATE, Class Field Theory, approx. 250 pp. (To be available approximately March 15, 1961) $3.00

RICHARD BRAUER, Non-Commutative Rings (To be available approximately April 15, 1961) in preparation

The following items may be ordered from: Department of Mathematics, c/o Mrs. Laura Schlesinger, Harvard University, 2 Divinity Avenue, Cambridge 38, Massachusetts

GARRETT BIRKHOFF, Lecture Notes on Lattice Theory, approx. 300 pp., (Prepared by students in Mathematics 252 at Harvard; to be available January, 1961)(Without covers) $4.00

GARRETT BIRKHOFF AND GIAN-CARLO ROTA, Lectures on Ordinary Differential Equations, revised. Part A (Chapters I - VI), approx. 250 photo-offset pp. (With covers) $3.50

Part B (Chapters VII - X), approx. 200 photo-offset pp. (With covers) $3.50

HAVERFORD COLLEGE

The following item may be ordered from: Department of Mathematics, Haverford College, Haverford, Pennsylvania.

DEANE MONTGOMERY, Topological Groups, 56 pp. (Lectures delivered at Haverford College under the Philips Program) Free

STATE UNIVERSITY OF IOWA

The following item may be ordered from: Campus Stores, State University of Iowa, Iowa City, Iowa.

S. BERBERIAN, A First Course in Hilbert Space, 3 + 87 pp., bound $1.64

JOHNS HOPKINS UNIVERSITY

The following item may be ordered from: The Johns Hopkins University Bookstore, Baltimore 18, Maryland.
C. L. SIEGEL, Celestial Mechanics, 237 pp. (Limited number still available) $3.50

UNIVERSITY OF KANSAS

The following items may be ordered from: Student Union Book Store, University of Kansas, Lawrence, Kansas.

J. L. LIONS, Boundary Value Problems, 101 pp. $2.00

W. R. SCOTT, CALVIN V. HOLMES, and ELBERT A. WALKER, Contributions to the Theory of Groups, 129 pp. $2.00

ARTHUR H. KRUSE, Introduction to the Theory of Block Assemblies and Related Topics in Topology, 306 pp. $3.00

ARTHUR H. KRUSE, Ergodic Estimates in the Theory of Periodic Functions by Use of Simple Continued Fractions and Related Topics, 247 pp. $3.00


EBERHARD HOPF, PETER D. LAX, and M. M. SCHIFFER, Lecture Series of the Symposium on Partial Differential Equations, 149 pp. (Berkeley, California; Summer, 1955) $1.50

CONFERENCE on Partial Differential Equations, 148 pp. (University of Kansas; Summer, 1954) $2.50

N. ARONSZAJN and W. F. DONOGHUE, Variational Approximation Methods Applied to Eigenvalues of a Clamped Rectangular Plate. Part I, Auxiliary Problems, 76 pp. $1.00

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During the interval from September 20, 1960 through December 2, 1960 the papers listed below were accepted by the American Mathematical Society for presentation by title. Readers may wish to refer to page 713 of the November issue (no. 49) of these NOTICES where it is explained in detail that the presentation of papers by title is now dissociated from meetings of the Society. Supplementary Program Number Two will cover the interval from December 3, 1960 through January 3, 1961.

After each title on this program is an identifying number. The abstract of the paper will be found following the same number in the section on Abstracts of Contributed Papers in this issue of these NOTICES or in succeeding issues.

PAPERS PRESENTED BY TITLE

(1) Nestinvertible matrices
   Mr. Edgar Asplund, University of Stockholm (60T-1)

(2) Choleski decomposition of 2k-difference matrices
   Mr. Edgar Asplund, University of Stockholm (60T-32)

(3) Pathological extensions of difference fields
   Mr. A. E. Babbitt, Jr., U. S. Army Signal Research and Development Laboratory, Newshrewsbury, New Jersey (60T-35)

(4) Koebe arcs and Fatou points of normal functions
   Mr. F. Bagemihl and Professor W. Seidel, University of Notre Dame (60T-27)

(5) On lie algebras of derivations. II. Preliminary report
   Mr. Richard T. Barnes, Ohio State University (60T-20)

(6) Consistency and solvability of a system of Boolean equations
   Mr. Michael Barr, University of Pennsylvania (60T-28)

(7) A fixed point method for studying the stability of a class of integrodifferential equations
   Dr. V. E. Beneš, Bell Telephone Laboratories, Incorporated, Murray Hill, New Jersey (60T-2)

(8) Univalence and disconjugacy of the complex plane
   Dr. R. K. Brown, U. S. Army Signal Corp Research and Development Laboratories, Fort Monmouth, New Jersey (61T-5)

(9) Maximal simple paths on convex polyhedra
   Mr. T. A. Brown, Harvard University (61T-1)

(10) A note on perturbed simultaneous Wiener-Hopf equations
    Professor John B. Butler, University of Arizona (60T-29)

(11) Extensions of homomorphisms
    Professor Paul Civin, University of Florida (61T-2)
(12) Unitary functions (mod r). II  
Professor Eckford Cohen, University of Tennessee (60T-4)

(13) Simultaneous binary compositions involving pairs of relatively k-free integers  
Professor Eckford Cohen, University of Tennessee (61T-6)

(14) Cusp functions involved in decomposition into the sum of four squares in R(3^{1/2}). Preliminary report  
Professor Harvey Cohn, University of Arizona (61T-7)

(15) Vitali's theorem for invariant measures  
Professor W. W. Comfort and Dr. Hugh Gordon, Harvard University (61T-3)

16) Cobordism classes of bundles  
Professor P. E. Conner and Professor E. E. Floyd, University of Virginia (60T-23)

(17) Cobordism for differentiable involutions  
Professor P. E. Conner and Professor E. E. Floyd, University of Virginia (60T-30)

(18) Banach spaces of Lipschitz functions  
Professor Karel deLeeuw, Stanford University and Institute for Advanced Study (60T-31)

(19) Postscript to "Maximality and ultracompleteness in normed modules"  
Professor Isidore Fleischer, University of California, Berkeley (60T-21)

(20) Singularities of three-dimensional harmonic functions. I  
Professor Robert P. Gilbert, Michigan State University (60T-5)

(21) Singularities of three-dimensional harmonic functions. II  
Professor Robert P. Gilbert, Michigan State University (60T-6)

(22) Singularities of harmonic functions in four variables  
Professor Robert P. Gilbert, Michigan State University (60T-7)

(23) Binary collisions in the n-body problem  
Dr. A. A. Goldstein, Massachusetts Institute of Technology (60T-8)

(24) Conjugate pairs of set-valued set-functions  
Professor P. C. Hammer, University of Wisconsin (60T-9)

(25) Enlarging functions on partially ordered systems  
Professor P. C. Hammer, University of Wisconsin (60T-10)

(26) On a theorem of I. Schur concerning matrix transformations  
Professor Marvin Marcus, U. S. National Bureau of Standards, Washington, D. C. and Mr. Frank May, University of British Columbia (60T-19)

(27) Almost complex structures on 8-dimensional manifolds. I  
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(28) On the linear independence of Laplace integral solutions of certain differential equations
   Professor K. S. Miller, New York University and Professor H. S. Shapiro, University Heights (60T-12)

(29) Representation theorems for cylindric algebras
   Mr. Donald Monk, University of California, Berkeley (60T-22)

(30) Plane semigroups
   Professor P. S. Mostert, Tulane University (576-13)

(31) A new type of product space
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(32) A double generalization of the concept of a mutant. Preliminary report
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   Professor Katsumi Nomizu, Brown University (60T-25)

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   Professor S. V. Parter, Cornell University (60T-26)

(35) Stability of differential equations and equivalent nonlinear programming problems
   Dr. Judah B. Rosen, Shell Development Company, Emeryville, California (60T-13)

(36) The universality of the recursively enumerable degrees. Preliminary report
   Mr. Gerald E. Sacks, Cornell University (60T-14)

(37) Suborderings of degrees that have cardinality of the continuum
   Mr. Gerald E. Sacks, Cornell University (60T-36)

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(41) The Wiener process with moving absorbing barriers. Preliminary report
   Professor Harold Widom, Cornell University (60T-33)

Bethlehem, Pennsylvania
December 2, 1960

Everett Pitcher
Associate Secretary

In a previous paper: Criteria for Kummer's congruences (to appear in Acta Arithmetica) necessary and sufficient conditions are obtained that a sequence \(\{a_n\}\) satisfy
\[
\sum_{s=0}^{r} (-1)^s \binom{r}{s} a_{n+s(p-1)} \equiv 0 \pmod{p^r},
\]
\((n \geq r)\). In the present paper these criteria are extended to congruences of the type
\[
\sum_{s=0}^{r} (-1)^s \binom{r}{s} a_{n+s(p^r-1)} \equiv 0 \pmod{p^r}.
\]
However it is now necessary to assume that \(r < p\). (Received October 4, 1960.)


Bell (Ann. of Math. vol. 35 (1934) pp. 258-277) showed that the Bell polynomial \(\phi_n(a_1, a_2, \ldots)\) satisfies the congruence \(\phi_p \equiv a_1^p + a_p \pmod{p}\). In the present paper it is shown that \(\phi_{p^r} \equiv a_1^{p^r} + a_1^{p^{r-1}} + \ldots + a_p^r \pmod{p}\), \(\phi_{p^n}(a_1, a_2, \ldots) \equiv \phi_p(\phi_p, a_2p, a_3p, \ldots) \pmod{p}\). It is also shown that the polynomial \(C_n(a_1, a_2, \ldots)\), the cycle indicator of the symmetric group, satisfies \(C_{n+p} \equiv (a_1^p - a_p)C_n \pmod{p}\). (Received October 4, 1960.)


The writer has proved (Proc. Amer. Math. Soc. vol. 11 (1960) pp. 456-459), that if a polynomial \(f(x)\) with coefficients in GF(q), where \(q = p^n\) is odd, satisfies \(f(0) = 0\), \(f(1) = 1\) and \(\psi(f(x) - f(y)) = \psi(x - y)\) for all \(x, y \in F\), where \(\psi(x) = x(q-1)/2\), then \(f(x) = x^p^j\) for some \(j\). The present paper contains the following generalization. Let \(\lambda_1 = \pm 1, \ldots, \lambda_k = \pm 1\) be assigned and let \(\psi(f(x_1, \ldots, x_r, \ldots, x_k) - f(x_1, \ldots, y_r, \ldots, x_k)) = \lambda_r \psi(x_r - y_r)\) for all \(r = 1, \ldots, k\) and all \(x_j, y_j \in F\). Then \(f(x_1, \ldots, x_k) = c_1 x_1^{p_1^r} + \ldots + c_k x_k^{p_k^r} + d\), where \(\psi(c_j) = \lambda_j\) and \(0 \leq r_j < n\). (Received October 4, 1960.)

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Let \( L = \partial^2/\partial t^2 + [\alpha(t) + \beta(t)] \partial/\partial t + \gamma(t) \) where \( \gamma \in C^0[0,b]; \beta \in C^0[0,b]; \alpha \in C^1(0,b); |\alpha| \leq c \text{ Re } \alpha \text{ on } [0,T_0] \) for some \( T_0 \leq b < \infty; \int_0^b \text{Re } \alpha \gamma \to \infty \text{ as } \xi \to 0; \phi = a_1/a_2 \in C^0[0,T_0]; \text{ and } |\alpha(t)/\alpha(\xi)| \leq N \) for \( 0 \leq \xi \leq t \leq T_0. \) (In many cases the latter hypothesis on \( \alpha \) is a consequence of the others.) Let \( Q \in C^1[0,b] \) with \( 0 < q \leq Q(t) \) where \( q \) is constant and \( Q(t) \) is real. Let \( \mathcal{H} \) be a separable Hilbert space and \( \Lambda \) a self-adjoint operator \( \mathcal{H} \to \mathcal{H} \) with domain \( D(\Lambda); \) assume \( \Lambda \) is semi-bounded below. Let \( T \in D(\Lambda), \tau \geq 0, \) and consider the Cauchy problem \( (L + Q(t)\Lambda)\omega(t) = 0, \omega(\tau) = T, \) \( (\partial/\partial t)\omega(\tau) = 0, \) with \( t \to \omega(t) \in C^0(D(\Lambda)) \) and \( t \to \omega(t) \in C^2(\mathcal{H}). \) There exists a unique solution to this problem. (Received October 4, 1960.)


If \( B_1 \) and \( B_2 \) are two solid tori (of genus 1) with \( B_2 \) interior to \( B_1, \) then \( B_1 \) and \( B_2 \) are said to be concentric if and only if \( C^1(B_1 - B_2) \) is the topological product of a torus and an interval. The simple closed curve \( J \) in the 3-sphere \( S^3 \) is said to have the concentreal enclosure property if and only if there exists a sequence \( \{B_n\}_{n=1}^\infty \) of polyhedral solid tori in \( S^3 \) such that (1) \( B_{n+1} \subseteq \text{Int } B_n \) for \( n \geq 1, \) (2) \( B_i \) and \( B_j \) are concentric if \( i \neq j, \) and (3) \( \bigcap_{n=1}^\infty B_n = J. \) The curve \( J \) is said to pierce a disk at the point \( x \in J \) if and only if there exists a disk \( D \) such that (a) \( D \) is locally polyhedral mod \( J, \) (b) \( D \cap J = x, \) and (c) \( \text{Bd } D \) links \( J. \) It is shown that the simple closed curve \( J \) in \( S^3 \) is locally unknotted and locally peripherally unknotted if and only if \( J \) has the concentreal enclosure property and pierces a disk at each of its points. By the characterization theorem of Harrold, Griffith, and Posey, this proves that the simple closed curve \( J \) is tamely imbedded in \( S^3 \) if and only if it has the concentreal enclosure property and pierces a disk at each point. (Received October 3, 1960.)

In an Abelian group the sum of two endomorphisms is an endomorphism although this is implied also by the weaker identity \((x + y) + (z + w) = (x + z) + (y + w)\) in a groupoid. In this note conditions for an abstract algebra to have an algebra of endomorphisms, that is, for the set of endomorphisms to be closed under the operations of the algebra are investigated. It is shown that if, for every pair of operations \(f, g\) where \(f\) is \(m\)-ary and \(g\) is \(n\)-ary, the identity \(f\{g(R_1), g(R_2), \ldots, g(R_n)\} = g\{f(C_1), f(C_2), \ldots, f(C_m)\}\) is satisfied where \(R_1, R_2, \ldots, C_1, C_2, \ldots, C_m\) are the row and column sequences of an arbitrary \(m \times n\) array of elements, then an algebra of endomorphisms can be constructed. In a sense, these conditions are the weakest which imply this endomorphism closure property. (Received October 4, 1960.)

573-34. R. P. Hunter: Certain two dimensional semigroups.

If \(S\) is a two dimensional compact connected semigroup with identity \(1\), such that \(H(1)\) is not totally disconnected, then there is a local one parameter semigroup \(A\) such that \(A \cap H(1) = 1\). (Received November 3, 1960.)


1. The number of copies sold, reported as 555, is not at the present accurate.

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The self-adjoint fourth-order equation (1) \((r(x)y'')'' - p(x)y = 0\), where \(r(x)\) and \(p(x)\) are both continuous on \([a, \infty)\), is investigated with respect to a second-order vector-matrix equation (2) \(a'' = A(x)a\) with \(a = (y_1)\) and \(y_1 = ry''\). Paralleling previous work of the author [Abstract 564-119, Notices Amer. Math. Soc. vol. 6 (1959) p. 818] two sets of two-point boundary conditions: (3) \(y(a) = y_1(a) = 0 = y(b) = y_1(b)\) and (4) \(y(a) = y_1(a) = 0 = y'(b) = y_1'(b) = 0\) are compared and related to those in the above-mentioned work. Particular significance is attached to the equation (5) \((y''/q(x))'' - q(x)y = 0\), which is studied in detail and its oscillatory properties are found to imply similar properties for certain other equations of the form (1). (Received October 6, 1960.)

574-32. Harvey Cohn: Proof that Weber's normal unit is no perfect power.

The author shows that the unit \(\mathcal{U} = \tan \pi/m\) for the field \(\mathbb{R}(\cos(2\pi/m))\) of degree \(m/4\) is no perfect \(q\)-power. Here \(m = 2^t \geq 8\) and the even powers \(q\) were disposed of by Weber in his famous work on abelian fields. The proof consists mainly of noticing first that any integer has a trace which is a multiple of \(m/4\), that \(\mathcal{U}\) has trace exactly \(-m/4\) and that the "trace-function" of \(\mathcal{U}^{1/q}\) \((q \text{ odd})\) is monotonic in \(q\). The significance of the result is that in conjunction with the work A numerical study of Weber's real class number calculation, to appear in Numerische Mathematik, we conclude that the fields \(\mathbb{R}(\cos(2\pi/m))\) all have either class number 1 or else the class number has only prime factors that are at least as large as 257. No factors are actually known! Research supported in part by NSF grant G-7412. Supporting computer time was donated by the Argonne National Laboratory. (Received October 3, 1960.)
574-33. Gunter Ewald: **Imbedding of projective planes into non-desarguean spaces.**

A class of generalized projective 3-spaces is defined, points and planes being undefined elements. These spaces can be coordinatized by ternary rings as defined by M. Hall, (Trans. Amer. Math. Soc. vol. 54, pp. 229-277). Any projective plane may be considered as a plane in such a space. Additional incidence properties lead to new geometrical characterizations of some special ternary rings like quasifields, or cartesian number systems. Furthermore, characterizations of projective 3-spaces by weakened incidence axioms are obtained. (Received October 7, 1960.)

574-34. G. S. Jones: **On the existence of periodic solutions of**

\[ f^*(x) = -af(x - 1) \{1 + f(x)\}. \]

This paper develops an existence theorem for periodic solutions of

\[ f'(x) = -a f(x - 1) \{1 + f(x)\}, \quad a > \pi/2, \]

where \( f \) is specified on a real unit interval \([x_0 - 1, x_0]\) and satisfies (1) for all \( x > x_0 \). This equation and closely related ones occur widely in diverse applications and have been previously shown to have bounded undamped oscillatory solutions about \( \theta(x) \equiv 0 \), for \( a > \pi/2 \). Let \( b_1, b_2, \) and \( \varepsilon \) be positive constants and define \( S(\varepsilon) \) to be the set of continuously differentiable functions defined on \([x_0 - 1, x_0]\) such that for every \( \Phi \) in \( S(\varepsilon) \) we have \( \Phi(x_0 - 1) = 0, \varepsilon \leq d(\Phi, \theta) \leq b_1, \) and \( 0 \leq d(\Phi', \theta) \leq b_2, \) where \( d(u,v) = \max \{|u(x) - v(x)|: x \in [x_0 - 1, x_0]\} \). For each \( \Phi \) in \( S(\varepsilon) \) let \( \{z_n(\Phi)\} \) denote the set of zeros of the corresponding solution \( f \) of (1), where \( z_{n+1} > z_n \) for all \( n \geq 1 \). Let \( T \) denote the transformation defined on \( S(\varepsilon) \) by

\[ T \Phi(x) = f(x + z_2(\Phi) - x_0 + 1) \text{ for } x \in [x_0 - 1, x_0]. \]

For \( \varepsilon \) sufficiently small, \( T^n \) for some finite \( n \) maps \( S(\varepsilon) \) topologically into itself. The Schander-Tychonoff Fixed Point Theorem then establishes the existence of periodic solutions of (1). (Received October 13, 1960.)

574-35. H. J. Keisler: **On the class of limit ultrapowers of a relational system.**

For the definition of ultrapower see abstract 550-7. Let \( I \) be a set, \( S \) an ideal in \( 2^I \), \( D \) a prime ideal in \( 2^I \), and \( R \) a relational system on the set \( A \). In abstract 559-137, the limit ultra power \( R_S^I/D \) was defined as the restriction
of the ultrapower $\mathcal{U}/D$ to the set $A^I/D = \{ f/D \in A^I/D \mid \{i,j\} \in I \times I \land f(i) \neq f(j) \} \in S_I^J$. Theorem 1. There exists $\mathcal{U}$ and a strong limit ultrapower $\mathcal{U}^S/D$ which is not isomorphic to any ultrapower of $\mathcal{U}$. $\mathcal{U}$ is complete (cf. Rabin, Nederl. Akad. vol. 62, p. 439) if every relation on $A$ is specified in $\mathcal{U}$. Theorem 2. If $\mathcal{U}$ is complete, $\mathcal{L}$ is isomorphic to a limit ultra power of $\mathcal{U}$ iff $\mathcal{L}$ is isomorphic to an elementary extension of $\mathcal{U}$. An isomorphism $\phi: \mathcal{U}^I/D \to \mathcal{U}^J/E$ is canonical if there exists $\forall J \to I$ such that $D = \{ H \subseteq I \mid |\psi^{-1}(H)| \in E \}$, and for each $f/D \in A^I/D$, $\phi(f/D) = g/E$ where $g(j) = f(\psi(j))$ for $j \in J$. A canonical direct system of ultrapowers of $\mathcal{U}$ is a system $\{ A^I_{X/D; x} \mid x \in X \}, \langle X, \leq \rangle, \{ \phi_{xy} \mid x, y \in X, x < y \}$ such that (a) for all $x, y \in X$ there exists $z \in X$ with $x < z$ and $y < z$, (b) whenever $x < y$, $\phi_{xy}$ is a canonical isomorphism of $A^I_{x/D; x}$ into $A^I_{y/D; y}$, (c) whenever $x < y < z$, $\phi_{xz} = \phi_{yz} \phi_{xy}$. Theorem 3. $\mathcal{L}$ is isomorphic to a limit ultrapower of $\mathcal{U}$ iff $\mathcal{L}$ is isomorphic to the direct limit of a canonical direct system of ultrapowers of $\mathcal{U}$. (Received October 3, 1960.)


For notation see the preceding abstract. Theorem 1. For any infinite set $A$ and index set $I$, there is a prime ideal $D$ in $2^I$ such that $A^I/D$ is of greater cardinality than $I$. This improves a theorem of Frayne, Morel, and Scott, abstract 550-8. Theorem 2. Suppose $D$ is a prime ideal in $2^I$ which is not countably complete, and the power of $A$ is cofinal with $\omega$. Then $A^I/D$ is of greater cardinality than $A$. Combining this with results of the preceding abstract, we obtain Theorem 3. Suppose $\mathcal{X}$ is a complete relational system, every countably complete prime ideal in $2^A$ is principal, and the power of $A$ is cofinal with $\omega$. Then any proper elementary extension of $\mathcal{X}$ has greater cardinality than $A$. This improves a theorem of Rabin (Nederl. Akad. vol. 62, p. 439) which gives the same conclusion but with the additional assumptions of the generalized continuum hypothesis and that the power of $A$ is accessible from $\omega$. Using Theorem 2 of the preceding abstract, we obtain Theorem 4. If $\mathcal{X}, \mathcal{L}$ have isomorphic limit ultrapowers (i.e. they are elementarily equivalent, according to abstracts 559-138, 559-97), then they have isomorphic limit ultrapowers of cardinality at most that of $2^A \cup 2^B$. (Received October 3, 1960.)
Consider the problem of obtaining a path on a rectangular board of \( m \) by \( n \) squares with both terminals at the edges of the board. A square is said to be covered when the path enters one edge of the square and leaves an adjacent edge. All other squares are said to be missed. Paths are found which cover a maximum number of squares (maximal paths). For \( m = n \), \( m - 2 \) squares are missed when \( m \) is even, and \( m - 1 \) squares are missed when \( m \) is odd. For \( m < n \), \( m - 2 \) squares are missed if \( m \) is even, and \( n - 2 \) squares are missed if \( m \) is odd. The proofs (that the paths exhibited are indeed maximal) involve elementary combinatorial arguments. The method of proof for \( m \) and \( n \) even is quite different from that for \( m \) or \( n \) odd. Certain properties of terminal positions, path length, types of missed squares, and unique paths are also investigated. The dependence of the results on the parity of \( m \) and \( n \) is again very striking. (Received October 6, 1960.)


In a previous paper (Ann. of Math., vol. 67 (1958) pp. 590-606) we defined a homomorphism of the group of divisors of an algebraic function field \( K \) into the group of divisors of an algebraic function field \( \bar{K} \). In one of the steps of this mapping an ideal \( \mathcal{U} \) of the ring of integers \( \mathcal{H}_K \) is mapped onto an ideal \( \mathcal{U} \) of the ring \( \mathcal{H}_{\bar{K}} \). In general, \( \mathcal{H}_K \) is not the entire ring of integers \( \mathcal{H}_{\bar{K}} \) and \( \mathcal{V} \) is not in general an ideal of \( \mathcal{H}_{\bar{K}} \). It was shown that \( \mathcal{V} \subset \mathcal{V} \) and that \( (\mathcal{V} \mathcal{H}_{\bar{K}})^{-1}(\mathcal{V} \mathcal{H}_{\bar{K}}) = (\mathcal{V} \mathcal{H}_{\bar{K}}) \). We now show that the modules \( \mathcal{V} \) are invertible and that \( \mathcal{V} \mathcal{V} = \mathcal{V} \). (Received October 6, 1960.)


(See abstract 571-49, Notices Amer. Math. Soc. vol. 7 (1960)). The results therein about strictly sublinear cases are extended to the asymptotically linear case \( a = 1 \). Put \( \lim_{|u| \to \infty} u^{-1} f(x,u) = A(x) \geq 0 \). \( f(x,u) \) as restricted in above reference.) Let \( \{\gamma_n^2\} \) be the eigenvalue sequence for the "linearized problem at infinity" with DE \( (p(x)h_x) + \lambda^{-1} A(x) h = 0 \) just as \( \{\mu_n^2\} \) was the sequence for the "linearization at the origin." \( 0 < \gamma_n < \mu_n \), \( n = 1, 2, \ldots \), and \( \lim_{n \to \infty} \mu_n = 0 \). The
spectrum for the main problem \( p(x)u'_x + \lambda^{-1}t(x,u) = 0, \ p(x) > 0, \ u(0) - ap(0)u_x(0) = 0, \ u(1) + bp(1)u_x(1) = 0, \ a,b > 0, \) consists of the intervals \( [\gamma_n, \mu_n], \ n = 1,2, \ldots. \) Associated with each interval is a branch of eigenfunctions \( u_n(x,\lambda). \) A "characteristic value function," \( r_n(\lambda) = \|u_n(x,\lambda)\| \) is defined on each interval such that \( \lim_{\lambda \to \mu_n} r_n(\lambda) = 0, \lim_{\lambda \to \gamma_n} r_n(\lambda) = \infty. \) \( \| \cdot \| \) denotes the \( L_2(0,1) \) norm. The branch \( u_n(x,\lambda) \) bifurcates at \( \lambda = \mu_n. \) Consider a sequence of problems approximating a linear problem. Then for \( n = 1,2, \ldots, \lim \gamma_n = \mu_n. \) The curves \( r_n(\lambda) \) approach perpendicular lines; the branches \( u_n(x,\lambda) \) of eigenfunctions become linear eigenspaces. For the strictly sublinear problems discussed in 571-49, \( \gamma_n = 0 \) for each \( n; \) the spectral intervals are the nested sets \( [0, \mu_n]. \) (Received October 3, 1960.)


As to the construction of a finite semigroup whose greatest c-homomorphic image is a group, the author obtained the result last year. The purpose of this note is to give a fundamental theorem in the theory of the structure of a finite semigroup whose greatest c-homomorphic image is a unipotent semigroup without zero. **Theorem.** Let \( S \) be a finite semigroup whose greatest c-homomorphic image \( U \) is a unipotent semigroup without zero, and let \( I \) be the minimal ideal of \( S. \) Then \( I \) is mapped to the kernel \( G \) of \( U \) under the homomorphism \( S \to U, \) so that the homomorphism \( I \to G \) gives the greatest c-decomposition of \( I. \) In order to prove the theorem we use three lemmas. Consider the difference semigroup of \( S \) modulo \( I \) and denote it by \( Z = (S:I). \) Then \( Z \) is clearly a finite unipotent semigroup with zero. In order to construct the required finite semigroup whose greatest c-homomorphic image is a unipotent semigroup without zero, for given \( I \) and \( Z \) we may construct \( S \) such that \( Z = (S:I) \) where \( I \) is a finite simple semigroup whose greatest c-homomorphic image is a group. (Received October 18, 1960.)


It is shown that the essential arithmetical properties of second order linear recurrences discovered by Lucas (Amer. J. Math. vol. 1 (1878) pp. 184-240; 289-321) may be extended to linear recurrences of any order by introducing a suitably defined vector recurrence which satisfies a linear difference equation of order two with matrix coefficients. (Received October 13, 1960.)
574-42. V. L. Klee: Topological equivalence of a Banach space with its unit cell.

In an earlier paper (Trans. Amer. Math. Soc. vol. 74 (1953) pp. 10-43), the author proved that Hilbert space $H$ is homeomorphic with its unit cell $\{x \in H: \|x\| \leq 1\}$. That result is now extended by means of the following Theorem: If a Banach space $E$ has a closed linear proper subspace which is $h$-compressible, then $E$ is homeomorphic with its unit cell. (A normed linear space $J$ is said to be $h$-compressible provided it is homeomorphic with a compressible normed linear space, and to be compressible provided the space $J \times [0,1]$ is homeomorphic with $J \times [0,1] \cup W \times \{0\}$ for some closed linear subspace $W$ of infinite deficiency in $J$.) From the theorem it follows, in particular, that the infinite-dimensional Banach space $E$ is homeomorphic with its unit cell provided any of the following supplementary conditions is satisfied: (1) $E$ is a separable conjugate space; (2) $E$ admits an unconditional basis; (3) $E$ is reflexive; (4) $E$ is the space $CM$ of all bounded continuous real-valued functions on a metric space $M$. (Received October 27, 1960.)
575-31. R. V. Chacon: On the ergodic theorem without the assumption of positivity.

Dunford and Schwartz have obtained a generalization of an ergodic theorem of E. Hopf to include nonpositive operators in spaces of complex-valued functions. We give a proof of a maximal ergodic lemma for operators which are not necessarily positive. This lemma is used to further generalize Hopf's theorem so that it applies to operators in spaces of functions which take their values in reflexive Banach spaces. The method of proof we have used appears to be shorter and simpler than those used previously. Further, our result has the additional advantage that it is sufficiently general to give as a direct corollary a theorem of Kakutani and of Beck and Schwartz. (A vector-valued random ergodic theorem, Proc. Amer. Math. Soc. vol. 8 (1957) pp. 1049-1059). Our result is the following: Theorem. Let \( \mathcal{X} \) be a reflexive Banach space and let \( (S, \Sigma, \mu) \) be a \( \sigma \)-finite measure space. Let \( T \) be a mapping of \( L_1(S, \Sigma, \mu, \mathcal{X}) \) into itself, with \( \|T\|_1 \leq 1, \|T\|_\infty \leq 1 \). Then for every \( p \) with \( 1 \leq p < +\infty \) and \( f \in L_p(S, \Sigma, \mu, \mathcal{X}) \), the limit \( \lim \frac{1}{n \sum_{m=0}^{n-1} T^m(s)} \) exists strongly for almost all \( s \in S \). Furthermore, if \( f \in L_p, p > 1 \) then there exists a function \( f^* \in L_p \) such that \( f^* \) dominates \( n^{-1} \sum_{m=0}^{n-1} T^m f(s) \) for every \( n \). (Received October 10, 1960.)


Let \( M \) be a connected, paracompact \( C^\infty \) manifold. Let \( V(M) \) be the set of \( C^\infty \) vector fields on \( M \), considered both as a real Lie algebra under the Jacobi bracket operation and as a module over the ring \( C^\infty(M) \) of real, \( C^\infty \) functions on \( M \). A foliation with singularities is defined by a subspace \( F \subset V(M) \) that is both a submodule and a subalgebra. We must suppose further that \( F \) is locally finitely generated in the sense that every \( x \in M \) has a neighborhood \( U \) and a finite dimensional real subspace \( S_U \subset V(U) \) such that: \( F \) restricted to \( U \) is just the submodule of \( V(U) \) generated by \( S_U \). Theorem: There is a unique maximal connected integral submanifold of \( F \) through each point of \( M \). This is
a generalization of a well-known theorem of Chevalley for the nonsingular case ("involutive distributions"). The new fact is that $\dim(F(x))$ is constant along the integral curves of $F$, although it is of course not necessarily constant on all of $M$. (Received October 10, 1960.)

575-33. C. A. Muses: An explicit formulation for the value of a fractional factorial.

Where $x$, $q$ and $p$ are positive and real, and $(q/p) < 1$, and $x \geq 1$,
\begin{enumerate}
\item $(x + q/p)! = (q/p - 1)! (q/p)(1 + q/p)(2 + q/p)\ldots(x + q/p)$,
\item $(q/p)!/(q/p - 1)! = q/p$,
\item $(-\infty)! = 0$,
\item $(-(x + q/p))! = (q/p - 1)!/(q/p - 1)(q/p - 2)\ldots(q/p - (x - 1))$.
\end{enumerate}

Thus $(-1.6)! = (-3/5)!(-5/3) \approx (-0.27049)^{-1}$. There exist similar expressions, immediately derivable from the above, for $(x + q/p)!$ and $(x - q/p)!$. Previously known were only the general formulas for such factorials where $q/p = 1/2$; but these expressions are simply one isolated example of the much more general expressions given above. The derivation of the above formulations depends upon a more general definition of the factorial than here-tofore recognized; namely, that where $r$ is real, $(r)! = (r)(r - 1)(r - 2)(r - 3)\ldots(r - 4)\ldots(-\infty)(-\infty)!$. It can be proved from an analysis of the factorials of negative fractions that this definition is demanded rather than the usual and incomplete one which stops at $(r - m) = 1$. The gamma function is similarly redefined. From these new enlarged definitions a straightforward elementary proof follows that $(-1/2)! = \pi^{1/2}$. (Received October 4, 1960.)

575-34. Eckford Cohen: Arithmetical Notes, VI. Simultaneous binary compositions involving coprime pairs of integers.

Let $m, n$ denote positive integers, $n \geq m$, and let $Q(m, n)$ be the number of solutions in positive integral $x_1, x_2, y_1, y_2$ of $m = x_1 + y_1, n = x_2 + y_2$, under the restriction, $(x_1, x_2) = (y_1, y_2) = 1$. It is shown in this note that $Q(m, n)$
\begin{equation}
\sim mn \prod (1 - 1/p^2) \prod (1 - 2/q^2)
\end{equation}
as $m \to \infty$, $p$ ranging over the primes such that $p | (m, n)$ and $q$ over the primes such that $q \not| (m, n)$. As a corollary, it follows that $Q(m, n) > 0$ for $m$ sufficiently large. A sharper estimate for $Q(m, n)$ is actually proved. The method employed is elementary. (Received October 27, 1960.)

A real normed linear space $E$ is said to have the tri-spherical intersection property if, whenever three open (equivalently, closed) spheres have a point in common, they have a point in common which lies in the convex hull of their centres. **Theorem:** Let $\dim E > 2$. Then (a) $E$ is an inner product space if and only if (b) $E$ has the tri-spherical intersection property. To prove that (a) $\Rightarrow$ (b), project. The proof that (b) $\Rightarrow$ (a) uses Helly's theorem and the following theorem of James (Inner products in normed linear spaces, Bull. Amer. Math. Soc. vol. 53 (1947) pp. 559-566): (a) holds if and only if for each hyperplane $H$ through $\phi$ there exists $x \in E$ with $x \neq \phi$ and $H \perp x$. (By $H \perp x$, we mean $\|y + ax\| \geq \|y\|$ for each $y \in H$ and each real $a$). **Definition:** A subset $T$ of $\mathbb{R}^n$ is a homothetic copy of $S \subset \mathbb{R}^n$ if $T = \alpha S + x$ for some $x$ in $\mathbb{R}^n$ and some $\alpha > 0$.

**Corollary:** Let $S$ be a compact symmetric convex body in $\mathbb{R}^n$. If $S$ is an ellipsoid, then every three homothetic copies of $S$ which intersect must have a point in common which lies in the convex hull of their centres; and conversely. (Received May 27, 1960.)


Let $T = [a,b]$ be a bounded interval of real numbers and $\{\mathcal{O}_n(T)\}$ be a sequence of subdivisions of $T$ simply ordered by refinement and such that the length of the longest subinterval of $\mathcal{O}_n(T)$ converges to 0 as $n \to \infty$, and let $X_{nk}$ denote the increment of $X_T$ over the $k$th subinterval of $\mathcal{O}_n(T)$. **Theorem.** Let $X_T$ be a separable, centered, decomposable stochastic process whose trend term is a function of bounded variation over $T$, and let $g$ be a continuous function on the real line having a second derivative at 0 and with $g(0) = 0$. Then

$$\int_a^b g(dX_t) = \lim_{n \to \infty} \sum_k g(X_{nk}) = g'(0) (X_b - X_a) + (g''(0)/2) \sigma^2 + \sum g(J_t)$$

almost surely, where $\sigma^2$ is the variance of the normal part of $X_b - X_a$ and the final sum is over the jumps, $J_t$, of the process $X_T$. In particular, when $g(x) = x^2$, the theorem is a generalization of one due to Paul Lévy for Brownian motion.
processes. The methods of symmetrization and truncation are applied repeatedly, leading to the use of the martingale theorems. (Received June 30, 1960.)

576-3. WITHDRAWN


Two point (lines) of a graph are adjacent if they have a common incident line (point). A line-cover (point cover) of a graph is a set of lines (points) such that each point (line) is incident to a line (point) of this set. A set of points (lines) is independent if no two are adjacent. A minimum cover has the fewest possible points or lines; a maximum independent set the most possible. (Cf. Berge Proc. Nat. Acad. Sci. vol. 43 (1957); pp. 842-844, Norman-Rabin Proc. Amer. Math. Soc. vol. 10 (1959) pp. 315-319.) The line-coverance, lc(G) (point-coverance, pc(G)) of a graph G is the number of lines (points) in a minimum cover. The number of lines (points) in a maximum independent set is its line-independence, li(G) (point-independence, pi(G)). Let G have p points. Theorem. lc(G) + li(G) = pc(G) + pi(G) = p. Proof. For a minimum line-cover C let m_i be the C-degree of a point a_i. Clearly lc(G) = \( \sum_{i=1}^{p} (m_i - 1) \). It follows from Section 5 of Norman-Rabin that li(G) = lc(G) - \( \sum_{i=1}^{p} (m_i - 1) \). Thus
lc(G) + li(G) = p. To show that pc(G) + pi(G) = p, observe that the complement of a minimum point-cover is a maximum independent set of points and vice versa. (Received July 20, 1960.)

576-5. S. A. Khabbaz and E. A. Walker: The number of basic subgroups of a p-group.

In his book Abelian groups, page 110, L. Fuchs asks: Determine the cardinality of the set of different basic subgroups in a p-group G. The following is obtained: (1) If G is of bounded order or divisible this number is one.

(2) If \( G = C_1(p^\alpha) + ... + C_n(p^\alpha) + D_1 + ... + D_m \) where \( C_1 \) is quasicyclic and \( D_j \) is cyclic of order \( p^{k_j} \), then the number of basic subgroups is \( p^{\sum_{j=1}^{m} k_j} \).

(3) In all other cases the number is \( |G/B| \), where B is a basic subgroup of G and \( |L| \) denotes the cardinal of L. In this case if G is reduced then of course \( |G/B| = 2^{|B|} \). (Received August 17, 1960.)


A graph G is d-polyhedral if its vertices and edges may be identified with those of a d-dimensional convex polyhedron. Let \( n(G) \), \( v(G) \), \( c(G) \), \( p(G) \) be the number of vertices and the valence of G and the maximal number of vertices in a simple circuit or path contained in G. In 1956, W. T. Tutte (J. London Math. Soc. vol. 21, pp. 98-101) found a 3-polyhedral graph with \( c < n \) (\( c = 45 \), \( n = 46 \), \( v = 3 \)). A 3-polyhedral graph with \( p < n \) (\( p = 153 \), \( n = 154 \), \( v = 3 \)) is constructed, and letting \( p(n;d) = \min\{p(G): G \text{ is } d\text{-polyhedral and } n(G) = n\} \), \( p^*(n;d) = \min\{p(G): G \text{ is } d\text{-polyhedral, } v(G) = d \text{ and } n(G) = n\} \), it is shown that \( p(n;d) < 2(d - 2)n^a \) for \( d \leq 3 \), and \( p^*(n;3) < 2n^a \), with \( a < 1 \), e.g. \( a = 1 - 2^{-18} \). (Received August 18, 1960.)


The following theorem is proven: If W is a strictly convex function defined in some domain D in n-dimensional real affine space, then the matrix of the second derivatives of W must be positive semidefinite in D and positive.
definite except on a nowhere dense subset of $D$. A counterexample shows that the converse of this theorem is false. (Received October 3, 1960.)


A study is made of the convergence of difference approximations to the exact solution of Cauchy problems, for systems of homogeneous linear partial differential equations with constant coefficients. For any regular such system (in the sense of G. Birkhoff, J. Soc. Indust. Appl. Math. vol. 2 (1954) pp. 57-67), and any sequence of homogeneous meshes $h_n$ whose norms approach zero, it is shown that one can find a sequence of time-steps $\Delta t_n$ for which convergence is sure (in the norm of G. Birkhoff and T. W. Mullikin, Proc. Amer. Math. Soc. vol. 9 (1958) pp. 18-25). The preceding norm refers to convergence in the mean, averaged over the set of all mesh-origins, as in the Lax-Richtmyer Theorem. It is then shown that convergence can also be proved for every individual sample mesh, for sufficiently regular initial values. (Received September 13, 1960.)

576-9. Hans Bueckner and Gabriel Horvay: **Heat transfer coefficient of freezing metal.**

When a cold slab travels at speed $U$ through a liquid metal bath, it freezes out metal at the rate of $V$. It is shown that the problem of determining the heat transfer coefficient $h$ at the interface of the liquid and solid phases is equivalent - under certain simplifying assumptions - to solving the (time independent) wave equation in a sector. For the case of $V/U = \tan \alpha < 1$ this problem is reduced, through suitable changes of variables and a Fourier sine transform, to the solution of Dirichlet's problem for the Laplace equation. The temperature field $T(X,Y)$ is expressed as an inverse sine transform, involving a single integration (in the transform variable). One finds that at the entrance cross-section to the bath, $X = 0$, the heat transfer coefficient is zero, then it rapidly approaches the asymptotic ("fully developed") value $h = \nu V \cos \alpha$. The heat transfer coefficient is determined in closed form for $\alpha = \pi/6$, and asymptotic expressions of it are derived for very small and very large distances from the origin when $\alpha$ is arbitrary ($0 < \alpha < \pi/4$). It is furthermore shown that for certain angles $\alpha \approx \pi/4$ (e.g., $\alpha = \pi/4, \pi/3$) the problem may be solved in terms of simple exponential functions. (Received October 5, 1960.)

An algorithm is given for minimizing the function \( F(x) = \max_{1 \leq i \leq k} \frac{[(A^i, X) + c]/(1 + (B^i, X)) - b_i]}{} \) on any subset \( S \) of \( E_n \) which consists of the intersection of a finite number of closed half-spaces and which has the additional property that \( X \in S \) implies that \( 1 + (B^i, X) \) is positive and bounded away from zero. Here \( A^i, B^i \) and \( X \) are points of \( E_n \), \( c \) and \( b_i \) are real numbers, \( k > n \) and \( (Y, X) \) denotes the inner product of \( X \) and \( Y \). The algorithm is of the type analogous to the "simplex method" in that it requires only a finite number of cycles. Each cycle, however, requires the solution of a certain characteristic value problem. As a special case, one has the Tchebycheff problem for approximation by rational functions on a discrete point-set. (Received September 20, 1960.)

On the adjoint of the closed span of multiplicative functionals. Preliminary report.

Let \( \Delta \) be the set of multiplicative functionals on a commutative semi-simple Banach algebra \( A \), and let \( B^* \) be the Banach space adjoint of the uniform operator closed span of \( \Delta \). \( B^* \) is a commutative Banach algebra with multiplication defined as in R. Arens' *Operators induced on function classes*, Monatsh. Math. vol. 55 (1951) pp. 1-19. \( A \) can be embedded in \( B^* \) and, if \( A \) has a weak approximate identity, the algebra \( A^m \) of multipliers of \( A \) (see Abstract 571-69, Notices Amer. Math. Soc. vol. 7 (1960) p. 505) is embeddable in \( B^* \). For tauberian \( A \) embedded topologically in \( B^* \), \( A^m = B^* \), when \( A \) has discrete maximal regular ideal space and an approximate identity. *Main Theorem.* For regular \( A \) with an approximate identity, \( F \subseteq B^* \) is a multiplier of \( A \) if and only if \( F \) belongs locally to \( \hat{\Delta} \) at each \( \rho \in \Delta \). See Abstract 572-15, Notices Amer. Math. Soc. vol. 7 (1960) p. 716, and compare this condition with Eberlein's n.a.s.c. for a function to be a Fourier-Stieltjes transform of a measure (Duke Math. J. vol. 22 (1955) pp. 465-468). Using Eberlein's characterization and the \( B^* \) construction described above, multipliers of \( L_1(G_+) \), with \( G_+ \) certain semigroups of a locally compact abelian group \( G \), are identified as those measures on \( G \) with supports contained in \( G_+ \). (Received November 17, 1960.)

Let \( f(x) \) and \( g(x) \) be two continuous functions, nonnegative in some domain \((A, B)\) and yielding the same integral over it. Then \( f(x) = g(x) \) for at least one point \( x \), not \( A \) or \( B \). If \( f(x) \) and \( g(x) \) are unimodal with continuous derivatives, and \( f(x) \neq g(x) \), they have at most two distinct points, \( x \) and \( y \), such that \( f(x) = g(x) \), \( f(y) = g(y) \). If the domain is the entire real axis, and \( f(x) \) and \( g(x) \) have a single intersection, then they are infinitesimals of the same order at plus and minus infinity, yielding further restrictions on the probability functions.

(Received October 18, 1960.)


Let \( S \) be a topological semigroup with identity which is topologically the plane, and \( G \) the maximal connected subgroup containing the identity. It is known (Mostert and Shields, Trans. Amer. Math. Soc., 1960) that \( G \) is an open subgroup. Let \( L = G^* \setminus G \). It is proved that \( L \) is either a (closed) line, ray, or circle, and the possible multiplications on \( L \) are determined completely, answering a question of the above paper. (Received October 6, 1960.)


The principal results are (I) if \( f(z) = a_0 + a_1 z + \ldots + a_n z^n \) is a polynomial of degree \( n \) with complex coefficients, then all the zeros of \( f(z) \) lie in the circle 
\[
|z| \leq \max_{1 \leq i \leq n} \left\{ \frac{|a_{n-1}|}{|a_n|} \left| \frac{x_i}{x_1} + \frac{x_i}{x_1} \right| \right\}
\]
where the \( x_i \) are arbitrary positive numbers, and further, there exists a choice of the \( x_i \) such that the above bound is the positive root of the associated Cauchy polynomial \( f_c(z) = |a_0| + |a_1|z + \ldots + |a_{n-1}|z^{n-1} - |a_n|z^n \). By various special choices of the \( x_i \) one obtains well known theorems of Cauchy, Kojima and Fujiwara. (II) If \( \{ \phi_n(x) \} \) is a sequence of orthogonal polynomials satisfying the recurrence 
\[
x_n \phi_n(x) = \lambda_n \phi_{n+1}(x) + \lambda_{n-1} \phi_{n-1}(x) + A_n \phi_n(x) \text{ and } x_{nn} \text{ is the largest zero of } \phi_n(x) \text{ then } \min_{1 \leq i \leq n} \{ \lambda_i x_i + A_i + \lambda_{i-1} x_{i-1} \} \]
\[
\leq x_{nn} \leq \max_{1 \leq i \leq n} \{ \lambda_i x_i + A_i + \lambda_{i-1} x_{i-1} \},
\]
there being a choice of the \( x_i \) for which simultaneous equality holds. The proofs are both by application of the Perron-Frobenius theorem and a lemma of Wielandt, in the first case to the companion matrix of \( f(z) \) and in the second case to the Jacobi matrix implied by the recurrence relation. (Received September 29, 1960.)

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Generalizing the results of an earlier paper, the authors prove the following theorem. Let $E^n$ be a closed solid n-sphere with the boundary $S^{n-1}$ in a Euclidean n-space $R^n$, and $f$ a continuous map of $E^n$ into $R^n$ which leaves no point of $S^{n-1}$ fixed. If there exist an inner point $z$ of $E^n$ and a constant angle $\alpha$ with $0 \leq \alpha \leq \pi$, such that for no point $s \in S^{n-1}$ is $\alpha$ an angle from the vector $s$, $f(s)$ to the vector $z,s$, then $f$ leaves at least one point fixed. In particular, $f$ has a fixed point if it maps $S^{n-1}$ into $E^n$. This result is also extended to:

Theorem: If a continuous map $f$ of a closed n-cell $\sum$ into $R^n \supset \sum$ maps the boundary of $\sum$ into $\sum$, then $f$ leaves at least one point fixed. This is a stronger theorem than that of Brouwer's inasmuch as the latter requires that $\sum$ be mapped into $\sum$. The principal tool in the proofs is consideration of the degree of mappings. The results of this paper will be extended and generalized in various ways later. (Received October 28, 1960.)


A B-space $X$ will be defined to have property (A) if for some integer $k > 0$ and some $\varepsilon > 0$, every choice of $k$ elements $(a_1, ..., a_k)$ from the unit ball of $X$ can be written with an appropriate collection of $+$ and $-$ signs so that $\|a_1 \pm a_2 \pm ... \pm a_k\| < k(1 - \varepsilon)$. A B-space $X$ is said to have property (B) if, for every sequence $\{X_i\}$ of independent random $X$-variables with $E(X_i) = 0$ and $\text{Var}(X_i) = E(\|X_i\|^2) \leq M$, we have $\|n^{-1}\sum_{i=1}^{n}X_i\| \rightarrow 0$ almost surely. Our theorem states that property (A) is necessary and sufficient for property (B). (Received October 21, 1960.)


J. D. Weston [J. London Math. Soc. vol. 32 (1957) pp. 186-187] showed that for every separable Banach space $Y$ there exist a Banach space $X$ and a compact one-one linear map $T:X \rightarrow Y$ with range dense in $Y$. This theorem is generalized as follows: Let $X$ and $Y$ be infinite-dimensional Banach spaces. Let $K(X,Y)$ be the set of all linear $T:X \rightarrow Y$ such that (i) $T$ is compact, (ii) $TX$ is
dense in \( Y \), (iii) \( T \) is the limit (in norm) of a sequence of bounded linear operators with finite-dimensional range, and (iv) \( T \) is one-one. Let \( K^{-}(X,Y) \) be the set of all linear \( T:X \rightarrow Y \) such that (i), (ii), and (iii) hold. \textbf{Theorem.} (A) \( K(X,Y) \) is nonempty if and only if \( Y \) is separable and \( X' \) has a countable total subset. (B) \( K^{-}(X,Y) \) is nonempty if and only if \( Y \) is separable. \textbf{Corollary.} If \( X \) and \( Y \) are separable, then \( K(X,Y) \) and \( K(X',Y) \) are nonempty. (Received October 3, 1960.)


In \( (X,\mathfrak{S},p,T) \), a measure space with measure preserving transformation \( T \), let \( \mathcal{F} \) denote an arbitrary finite subfield of \( \mathfrak{S} \) with atoms \( A_i, i = 1,\ldots,\mu \). Define \( \phi_n(t,T,\mathcal{F}) = (n+1)^{-1} \log \sum i_0 \cdots \sum i_n \sum_i [P(A_i_0 \cap T^{-1} A_i_1 \cap \cdots \cap T^{-n} A_{i_n})]^{1-t} \), \( 0 \leq t < 1 \), and then \( \phi(t,T,\mathcal{F}) = \lim_n \phi_n(t,T,\mathcal{F}) \) and \( \phi(t,T) = \sup_{\mathcal{F}} \phi(t,T,\mathcal{F}) \). The function \( \phi(t,T), 0 \leq t < 1 \), is an isomorphism invariant, by construction; the invariant entropy [Ya.G. Sinai, Dokl. Akad. Nauk vol. 124 (1959) pp. 768-771] is given by \( h(T) = \phi'(0,T) \) (when finite). Several properties of \( h(T,\mathcal{F}) \) (and \( h(T) \)) generalize to \( \phi(t,T,\mathcal{F}) \) (and \( \phi(t,T) \)). Strong monotonicity (i.e., for invertible \( T \), if \( \mathcal{F} \subset \mathcal{V} \) then \( \phi(t,T,\mathcal{F}) \leq \phi(t,T,\mathcal{V}) \)) would be most desirable for computational purposes, but proof appears to be difficult. (Received October 27, 1960.)

576-19. W. C. Orthwein: \textit{An explicit-implicit method for approximating the solution of the heat equation with radiation or conduction type boundary conditions}.

Analysis of the error propagation in the numerical solution of the heat equation in three spatial dimensions subject to either radiation or conduction type boundary conditions shows that for the explicit method (see W. E. Milne, \textit{Numerical solution of differential equations}, New York, 1953, p. 120) the departures (see G. E. Forsythe and W. R. Wasow, \textit{Finite-difference methods for partial differential equations}, New York, 1960, p. 29) due to the introduction of an error at a previous time step are greater than in the corresponding solution to the problem wherein the boundary temperatures are fixed. In the present paper it is shown that for radiation and conduction type boundary conditions the departures may be reduced by employing a combination of explicit and implicit methods. The implicit method discussed here is essentially an extension of the
Crank-Nicolson procedure to three spatial dimensions. Incidental to this it is formally shown that the optimization of the parameter $\theta$ (see S. H. Crandall, Quart. Appl. Math. vol. 13 (1955) pp. 320-322) to obtain a formal truncation error of the order of $h^6$ is not possible for higher dimensions. (Received October 20, 1960.)


Using a modified form of the definition of homotopy retraction given by A. F. Bartholomay (Portugal. Math. vol. 13 (1954) pp. 105-110), the author develops a homotopy retraction theory for compact Hausdorff spaces. Absolute homotopy retracts (AHR) and absolute neighborhood homotopy retracts (ANHR) are defined and are characterized as homeomorphs of closed homotopy retracts and closed neighborhood homotopy retracts, respectively, of Tychonoff cubes. It is shown that the topological product of any set of AHR or any finite set of ANHR is an AHR or an ANHR, respectively. It is shown that a necessary and sufficient condition for a set $A$ to be an AHR (ANHR) is that $A$ be a compact Hausdorff space, and that any continuous mapping $f$ defined on a closed subset $P$ of a compact Hausdorff space $P_1$ such that $f: P \rightarrow A$, admits a homotopy extension over $P_1$ relative to $A$ (over some open subset $V$ of $P_1$ relative to $A$). (Received October 20, 1960.)


A degree $c$ is said to be recursively enumerable in a degree $b$ if there is a set of degree $c$ which is recursively enumerable in a set of degree $b$. Theorem 1: if $b$ and $d$ are degrees such that $b$ is less than $d$ and $d$ is recursively enumerable in $b$, then there is a degree $c$ such that $b$ is less than $c$ and $c$ is less than $d$ and $c$ is recursively enumerable in $b$. Theorem 2: if $b$, $c$ and $d$ are degrees such that $b$ is less than $c$ and $c$ is less than $d$ and both $c$ and $d$ are recursively enumerable in $b$, then there is a degree $g$ which is greater than $b$, incomparable with $c$ and less than $d$ and which is recursively enumerable in $b$. Theorem 3: if $A$ and $B$ are sets such that $B$ is recursively enumerable in $A$ and the degree of $A$ is less than the degree of $B$, then $B$ is the union of two disjoint sets whose degrees are incomparable and are recursively enumerable in the degree of $A$. 893
Corollary: if $b$ and $c$ are degrees such that $b$ is less than $c$ and $c$ is recursively enumerable in $b$, then the set of degrees less than $c$ has $c$ as its least upper bound. (Received October 24, 1960.)


An ordered $B$-algebra $A$ is a (real or complex) Banach algebra with unit $e$, whose underlying $B$-space is (partially) ordered with closed, normal positive cone $K$, such that $e \in K$, and $a, b \in K$ provided $a, b \in K$, $ab = ba$. ($K$ is normal if $\forall \gamma > 0 \|x + y\| \leq \gamma \|y\|$ for all $x, y \in K$). Let $[0, e] = \{z \in A: 0 \leq z \leq e\}$. Theorem: Let $A$ be an ordered $B$-algebra such that $[0, e]$ is weakly semi-complete. Suppose $a \in A$ is in the real linear hull of $[0, e]$: $c_1 e \leq a \leq c_2 e$ $(c_1, c_2 \in \mathbb{R})$. Then $\sigma(a) \subset [c_1, c_2]$, and there exists a spectral measure $\mu$, i.e., a $\sigma$-additive homomorphism of the Boolean $\sigma$-algebra of Borel subsets of $\sigma(a)$ onto a Boolean $\sigma$-algebra of idempotents in $[0, e]$, with these properties: $\int 1 d\mu \in A$ for every bounded Borel function on $\sigma(a)$, and $f \rightarrow \int f 1 d\mu$ is an order preserving homomorphism of the algebra of bounded Borel functions on $\sigma(a)$, into $A$ such that $\int d\mu = e$, $\int \lambda d\mu = a$. If $A$ is an ordered $B$-algebra of operators with $[0, e]$ weakly semi-complete (such as an algebra of Hermitean operators in their usual order, or the algebra of operators on a weakly semi-complete Banach lattice), then each $a \in \bigcup_{1}^{\infty} [\text{ne, ne}]$ is a scalar type spectral operator in the sense of Dunford (Bull. Amer. Math. Soc. vol. 64 (1958) pp. 217-274). Thus there is a general spectral theorem independent of the notion of Hilbert space, and there exist nontrivial spectral operators on every weakly semi-complete $B$-space. The proof is based on the solution of a generalized moment problem. (Received October 24, 1960.)


In this note we extend a theorem of Chaundy and Jolliffe on the uniform convergence of $S(x) = \sum b_n \sin nx$ (i) and two theorems of Boas (Quart. J. Math. Oxford (2) vol. 3 (1952) pp. 217-221). Let $0 < b_{n+1} \leq b_n(1 + \alpha/n), n > n_0$ $b_n = o(1)$. (ii). It is proved that a necessary and sufficient condition that the series (i) should be uniformly convergent throughout any interval is that $n b_n = o(1)$. Let $(b_n)$ satisfy (ii) and $C(x) = \sum b_n \cos nx$ (iii) and let $P(k)$ denote the number
of terms $b_n$ such that $n \leq k$, $b_{n-1} < b_n$. If either $P(n) = O(n^\beta)$, $0 < \beta < 1$ (iv) or $\sum b_n/n$ is convergent, then the series (i) and (iii) are convergent for every $x$ save possibly $x = 0$ in the case of (iii). If (iv) holds and $S(x) \in L$, then $\sum b_n/n$ is convergent. If $\sum b_n < \infty$ (v) and $0 < \gamma \leq 1$ then $x^{-\gamma}S(x) \in L$. If $0 < \gamma < 1$ then $x^{-\gamma}C(x) \in L$. If $\sum |b_n - b_{n+1}| \log n < \infty$ then $C(x)$ and $S(x) \in L$. (Received October 26, 1961.)


Let $\mathcal{H}$ be a sheaf of groups on the topological space $X$. (See R. Godement, Théorie des Faisceaux, for notations.) We are going to interpret the elements of $H^1(X;\mathcal{H})$ by sheaves $\mathcal{G}$ with prescribed local structure, called principal Bundle-Spaces and satisfying: (BP) $\mathcal{H}$ operates in $\mathcal{G}$, (BP). Every point of $X$ possesses an open neighbourhood $U$ such that $\mathcal{G}|U$ and $\mathcal{H}|U$ (with $\mathcal{H}|U$ as Group of Operators) are isomorphic. By a slight modification of the standard procedure, one can construct a principal Bundle-Space $\mathcal{W}(g, M)$ associated with a given element $(g, M)$ of the sum of the family $(\mathcal{Z}(M;\mathcal{H}))_{M \in R(X)}$ using the lemma: Let $M \in R(X)$, and, for each $L \in X$, let $\mathcal{H}_L$ be a pre-sheaf on $M_L$. Assume, that $\mathcal{F}_L | M_{Lk} = \mathcal{F}_K | M_{Lk}$. Then there exists a sheaf $\mathcal{F}$ and morphisms $\mathcal{H}_L$ of $\mathcal{F}_L$ into $\mathcal{F} | M_L$, solving a certain universal problem. Proposition 1. Every principal Bundle-Space on $X$ with structural Group $\mathcal{G}$ is isomorphic to a $\mathcal{W}(g, M)$. Proposition 2. There exists a bijective mapping of the set $\mathcal{G}(X;\mathcal{H})$ of the types of principal $\mathcal{G}$-Bundle-Spaces on $X$ into $H^1(X;\mathcal{H})$. (Received October 26, 1960.)


Gleason's conjecture: Any compact $k$-independent subset of the euclidean $n$-space (Borsuk, On the $k$-independent subset of the euclidean space and of the Hilbert space, Bull. Acad. Polon. Sci. Cl. III, vol. 5 (1957) pp. 351-356, XXIX) can be topologically imbedded into an $(n-k)$-sphere. As a special case of the conjecture, it is proved that given any 2-dimensional nonorientable closed surface topologically imbedded into the euclidean $n$-space, there exists an $(n-3)$-dimensional hyperplane which intersects the surface at more than $n-2$ points. To prove the result, we first assume the assertion false and then obtain a contradiction by computing certain linking numbers of integral cycles. (Received October 28, 1960.)

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The dimension parity theorem of Floyd (Trans. Amer. Math. Soc. vol. 72 (1952) pp. 138-147) and that of Liao (Ann. of Math. vol. 56 (1952) pp. 68-83) are strengthened. Let $X$ be a compact Hausdorff space having the mod $p$ cohomology group of an $n$-sphere and let $T$ be a periodic transformation of $X$ of period $p$, where $p$ is a prime. It is known that the fixed point set of $T$ has the mod $p$ cohomology group of an $r$-sphere, where $-1 \leq r \leq n$ and $(-1)$-sphere means the null set. In this paper it is proved that if $p$ is odd, then $n - r$ is even, and that if $p = 2$, then $X$ can be naturally oriented such that $n - r$ is even or odd according as $T$ preserves or reverses the orientation of $X$. Not as in the theorems of Floyd and Liao, $X$ is neither assumed to be finite-dimensional (Larry Mann, Dissertation, University of Pennsylvania, 1959) nor assumed to have a finitely generated integral cohomology group. (Received October 28, 1960.)

H. S. Collins: Idempotent measures on compact semigroups.

Let $S$, $\overline{S}$, $C(S)$, $C_R(S)$, and $H$ (respectively) denote a compact topological semigroup, the convolution semigroup of (non-negative normalized regular Borel) measures on $S$, all continuous complex functions on $S$, all real continuous functions on $S$, and the carrier of a $\mu \in \overline{S}$. If $\mu$ is a fixed element of $\overline{S}$ and $f \in C(S)$, let $f'(x) = \mu(t^x)$, where $t^x(y) = f(yx)$. **Theorem 1.** $H$ is a simple semigroup ($K = H$, with $K$ the kernel of $H$) iff $H^2 = H$ and max $f'(H) = \max f'(K)$, for all $f \in C_R(S)$. **Theorem 2.** Suppose $H^2 = H$. Then $\mu^2(t^x) = \mu(t^x)$ for all $f \in C(S)$ and $x \in H$ iff (i) $H$ is simple and (ii) $f \in C(S)$ implies $f'$ is constant on each minimal left ideal of $H$. **Theorem 3.** If $\mu^2 = \mu$, then (i) $H$ is simple, (ii) $f \in C(S)$ implies $f'$ is constant on each minimal left ideal of $H$, and (iii) $\mu^3 = \mu$; the converse also holds. **Theorem 4.** If $H$ is simple then $\mu^2 = \mu$ iff $\mu$ is a $\overline{\mu}$ measure (see J. Rosenblatt, Math. Mech. vol. 9 (1960) pp. 293-305). **Theorem 5.** Let $\Lambda$ be a compact semigroup of $S$. Then there is a $\nu^2 = \nu \in \overline{S}$ with carrier $\nu = \Lambda$ iff (i) $\Lambda$ is simple and (ii) there exist regular Borel measures $\alpha$ and $\gamma$ on (respectively) $L$ and $R$ (the minimal left and minimal right ideal spaces of $A$) such that carrier $\alpha = L$, carrier $\gamma = R$. (Received November 7, 1960.)

Let $R_n$ be an $n$-dimensional Riemannian manifold with positive metric, and denote by $M(u,x^0,R)$ the spherical mean value of a function $u$ over the geodesic sphere with center $x^0$ and radius $R$. It is proved that $R_n$ is Einsteinian at $x^0$ if and only if every solution $u(x)$ of Laplace-Beltrami equation $Δu = 0$ in some neighborhood $B(x^0,u)$ of $x^0$ satisfies:

$M(u,x^0,R) = u(x^0) + O(R^3)$, $ΔM(u,x^0,R)/ΔR = u(x^0) + O(R^3)$.

Next it is proved that if $u ≡ 0$ or $u ≡ \lambda$, $Δu = 0$ and $AΔu = 0$ in some neighborhood of $x^0$, then $M(u,x^0,R) = u(x^0) + O(R^3) + Δu(x^0) \left\{ (R^2/2n) + ρR^4/[12n^2(n+2)] + O(R^5) \right\}$ where $ρ$ is the scalar curvature at $x^0$. For $R_n$ harmonic at $x^0$, the first result with $O(R^3) ≡ 0$ was proved by Willmore. The second result is improved, in the harmonic case, and yields instead of $ρ$ the fundamental solution $φ = φ(R)$ of $Δφ = 0$. (Received November 7, 1960.)

576-29. Daniel Gorenstein and D. R. Hughes: Triply transitive groups in which only the identity fixes four letters.

Let $G$ be a finite triply transitive group in which only the identity fixes four letters. Then it is shown that $G$ is one of the known groups with this property. Such a group is: (1) sharply four-fold transitive; (2) sharply triply transitive; or (3) a "full semi-linear fractional group" over a field $GF(2^p)$, where $p$ is a prime. The proof depends heavily on recent results due to Feit and Suzuki, as well as the earlier work of Zassenhaus. Trivially, this classifies $t$-fold transitive groups in which only the identity fixes $t + 1$ letters, for $t ≡ 3$; in fact, for $t ≡ 4$, such a classification is easily carried out directly. (Received November 10, 1960.)


Let $M_{m,n}$ be the vector space of $m × n$ rectangular matrices over the complex numbers. Let $E_r(Λ)$ denote the $r$th elementary symmetric function of the squares of the singular values of $Λ$ (eigenvalues of $Λ^*Λ$). Let $T$ be a linear transformation of $M_{m,n}$ into itself. Theorem: If $r ≡ 2$ then $E_r(T(Λ)) = E_r(Λ)$ for all $Λ ∈ M_{m,n}$ if and only if $T$ has the form (1) $T(Λ) = UAV$ for all $Λ ∈ M_{m,n}$.
in case \( m \neq n \); or \( T \) has either the form (1) or (2) \( T(A) = UA'V \) all \( A \) in case \( m = n \), where \( U \) and \( V \) are unitary of appropriate size. This result is similar to one of Morita (Jap. J. Math, vol. 19 (1944) pp. 45-56) in which the linear operators on the space of skew-Hermitian matrices holding the maximum singular value (the Hilbert norm) fixed are determined. (Received November 4, 1960.)


Let \( F \) be a field of at least \( r \) elements not of characteristic 2. Let \( M_n \) be the vector space of \( n \)-square matrices over \( F \) and let \( T \) be a linear transformation of \( M_n \) into itself. Let \( p(X) = \sum_{\sigma} \prod_{i=1}^{n} x_{i\sigma(i)} \) denote the permanent of \( X \). The summation extends over all permutations \( \sigma \) of \( 1, \ldots, n \). Theorem. If \( n > 2 \) then \( p(T(X)) = p(X) \) for all \( X \in M_n \) if and only if \( T(X) = DPXQL \) for all \( X \in M_n \) or \( T(X) = DPX'QL \) for all \( X \in M_n \) where \( D \) and \( L \) are diagonal matrices satisfying \( p(DL) = 1 \) and \( P \) and \( Q \) are permutation matrices. This result extends to the permanent function the classical theorems on determinants of Frobenius and Schur. A theory of permanental compounds is developed that parallels the determinantal compounds. (Received November 4, 1960.)


Lehner has given a method for defining Poincaré series of dimension -2 on \( \Gamma(1) \) (the modular group) which does not rely on the Hecke idea of introducing a convergence factor. He has shown that, although these Poincaré series are not absolutely convergent, when summed in the proper order they converge uniformly on compact subsets of the upper half plane and represent modular forms of dimension -2. In this paper these results are extended to those subgroups of finite index of \( \Gamma(1) \) for which the Kloosterman sums which arise can be estimated as \( O(c^{1/2+\epsilon}) \). In this direction, Petersson has proved that the required estimate holds in the case of congruence subgroups \( \Gamma \) for which the multiplier system \( v(\Gamma) \) is identically 1 on one of the levels \( \Gamma(N) \) of \( \Gamma \). The proof that the Poincaré series \( G(z,v,A,\Gamma,\mu) \) satisfy the proper functional equation is established by proving a generalization of a lemma originally given by Rademacher concerning the rearrangement of conditionally convergent double series. Finally, the Fourier series expansion for \( G(z,v,A,\Gamma,\mu) \) is obtained. The coefficients of this expansion agree with those obtained by Petersson using the Hecke method. (Received November 14, 1960.)
576-34. Michael Goldberg: N-gon rotors making \( n + 1 \) contacts with a simple closed curve.

There are polygons of \( n \) nearly-rectilinear sides which can be rotated through all orientations while remaining tangent to \( n + 1 \) fixed circles (M. Goldberg, Rotors tangent to \( n \) fixed circles, J. Math. Phys. vol. 37 (1958) pp. 69-74). These polygons can be replaced by polygons of perfectly rectilinear sides if the circles are replaced by special curves. A kinematic method of generating these curves is shown for polygons of an odd number of sides. The method is generalized to include regular rotors of equal symmetric arcs and some classes of rotors of an even number of sides. (Received November 14, 1960.)

576-35. Emil Grosswald: Negative discriminants of binary quadratic forms with a single class in each genus.

Let \( \chi \) be a character \((\text{mod } k)\), denote by \( K \) the set of integers \( \Delta \) such that \( -\Delta \) is a discriminant with exactly one class per genus and consider the following hypotheses: \( H_1: L(53/54, \chi) \geq 0 \) for \( k > 10^{14} \); \( H_2: L(53/54, \chi) \geq 0 \)
for \( k > 10^7 \); \( H_3 \): \( L(s, \chi) \neq 0 \) for \( s > 1/2 \) and \( k > 10^7 \); \( H_4 \) and \( H_5 \) are \( H_2 \) and \( H_3 \) respectively, stated only for \( k = 4n > 10^7, k \equiv 0 \mod p \), for every prime \( p \equiv 3 \mod 4 \). Chowla and Briggs proved (Canad J. Math. vol. 6 (1954) pp. 463-470): \( H_1 \) implies that \( \Delta \notin K \) for \( \Delta > 10^{14} \). Swift had shown (Bull. Amer. Math. Soc. vol. 54 (1948) pp. 560-561) that \( \Delta \notin K \) if 7392 < \( \Delta < 10^7 \). If \( H_2 \) replaces \( H_1 \), Chowla and Briggs method permits to reduce the gap 10\(^7\) < \( \Delta < 10^{14} \).

Using Littlewood's approach (J. London Math. Soc. vol. 27 (1928) pp. 358-372) and a method of descent, \( H_2 \) permits to close the gap, except for some integers, that can be handled individually by known criteria (see Swift, loc. cit.) The stronger \( H_3 \) reduces computations considerably. The general case can be reduced to that of a fundamental discriminant. Conclusions: \( H_3 \) (or \( H_2 \) upon finite checking) implies that \( K \) consists of the 110 known integers (\( \Delta \leq 7392 \)).

Corollary (see Grosswald and Calloway, Proc. Amer. Math. Soc. vol. 10 (1959) pp. 451-455): \( H_5 \) (or \( H_4 \) upon finite checking) implies that every natural integer \( n = 4^m n_1 \) (integral \( m \geq 0, n_1 \neq 0 \mod 4 \)) is the sum of three positive squares, unless \( n_1 \equiv 7 \mod 8 \), or \( n_1 \) is one of the integers 1, 2, 5, 10, 13, 25, 37, 58, 85, 130.

(Received November 14, 1960.)

576-36. J. R. Isbell: **Uniform irreducible expansions.**

Freudenthal's Exchange Theorem says that any two irreducible expansions of a compact metric space as an inverse limit of finite complexes can--by selecting subsequences and applying small "zulässige Abänderungen"--be interlaced into a single inverse mapping system with the same limit. The theorem generalizes to complete metric spaces and finite-dimensional uniform complexes. The complete metric spaces which admit such an expansion are those which have bases of finite-dimensional uniform coverings and have bases of uniform coverings which are essential, that is, not able to be shrunk to a uniform covering with a smaller nerve. Every complete separable metric space is homeomorphic with a space admitting an irreducible expansion. (Received November 14, 1960.)

576-37. Frank Kosier: **On a class of nonflexible algebras.**

In (Notices Amer. Math. Soc. vol. 7 (1960) p. 250, Abstract 568-21) the author defined a class of algebras and it was shown that relative to quasi-equivalence there are four residual types of such algebras; those that satisfy
some one of the following four identities; (i) \((xy)z - x(yz) = (zy)x - z(yx)\);
(ii) \(x(xy) + (yx)x = 2(xy)x\); (iii) \((xy) + (yx)x = x(xy) + (xy)x\); (iv) \(x(yz + zy) + (yz + zy)x = (xy + yx)z + z(xy + yx)\). For any \(n \geq 1\) it is possible to construct a simple power-associative algebra \(A\) of degree \(n\) such that \(A\) satisfies (i) but not the flexible identity \(x(yx) = (xy)x\). Employing the methods of Oehmke (On flexible algebras, Ann. of Math. vol. 68 (1948) pp. 221-230) it is shown that any semi-simple strictly power-associative algebra \(A\) over a field \(F\) of characteristic \(p \neq 2, 3\) satisfying (ii) has a unity element and is the direct sum of simple algebras. If, in addition, \(A\) is simple, then \(A\) is one of the following: (a) a commutative Jordan algebra; (b) a quasi-associative algebra; (c) an algebra of degree 1 or 2. (Received November 14, 1960.)

576-38. R. H. Rosen: An example of an involution of \(E^4\).

Recently the author has given an example of a 3-dimensional space which is totally non-Euclidean (that is, it contains no topological 3-cell) but whose Cartesian product with a line is homeomorphic to \(E^4\) (see Notices Amer. Math. Soc. vol. 6 (1959) p. 641, Abstract 563-1). By modifying his previous construction and applying some results of R. H. Bing (Ann. of Math. vol. 70 (1959) pp. 399-412), the author has proved the following: Theorem. There is a period 2 homeomorphism \(\theta\) of \(E^4\) with totally non-Euclidean fixed point set \(F\) so that the restriction of \(\theta\) to \(E^4 - F\) is piece-wise linear. This result was conjectured by Professor Bing. Similar theorems are proved for \(I^4, S^4, S^3 \times S^1\).

(Received November 14, 1960.)

576-39. WITHDRAWN

576-40. WITHDRAWN


In some cases and under the suitable structure of a partial differential system, this system can be associated with an ordinary differential system. The fundamentals of this procedure based upon the transformations in the Banach space were laid down by the late Professor Michael. In the present paper, the author develops a systematic and methodical approach of associating a partial differential system with an ordinary one. The technique is illustrated by means of several examples. (Received November 17, 1960.)
576-42. A. B. Simon: Subalgebras of measure algebras.

Let G be a locally compact Abelian group and let M(G) be the algebra of all bounded regular Borel measures on G. Suppose P is an independent subset of G. Let $M_0$ be the algebra generated by $M_G(P \cup -P)$ (= the continuous measures concentrated on $(P \cup -P)$) and all discrete measures; then set $M_1$ equal to the set of all measures absolutely continuous w.r.t. some element of $M_0$. The closure $M$ of $M_1$ is a closed subalgebra of $M(G)$. Result: The ideal space of $M$ is homeomorphic to $S \times \Gamma$ where $S$ is the closed unit ball of $[M_G(P \cup -P)]^*$ (in the $w^*$-topology) and $\Gamma$ is the character group of G considered as a discrete group. (Received November 16, 1960.)


By an equator of a double torus $T$ (closed orientable surface of genus 2) is meant a simple closed polygon $e$ on $T$ such that $T - e$ consists of two components, the closure of each of which is a torus with one hole. A solid torus $B$ of genus 2 in the 3-sphere $S^3$ is said to be unknotted if and only if $B$ can be expressed as the union of two unknotted solid tori of genus 1, meeting in a disk common to their boundaries. The following two theorems are proved: Theorem 1. Let $T$ be a polyhedral double torus in $S^3$. Suppose that $e$ is an equator of $T$, and that $m_1$ and $m_2$ are non-nullhomologous simple closed polygons on $T$ which are separated on $T$ by $e$. If $A$ is a component of $S^3 - T$ such that each of the polygons $e$, $m_1$, $m_2$ bounds a disk in $Cl A$, then $Cl A$ is a solid torus of genus 2. Theorem 2. Suppose that $T$ is a polyhedral double torus separating $S^3$ into two components $A$ and $B$. Let $e$ be an equator on $T$, and let $m_1$, $m_2$, $n_1$, $n_2$ be non-nullhomologous simple closed polygons on $T$ such that $e$ separates $m_1 \cup n_1$ and $m_2 \cup n_2$ on $T$. If $e, m_1, m_2$ bound disks in $Cl A$, and $e, n_1, n_2$ bound disks in $Cl B$, then both $Cl A$ and $Cl B$ are unknotted solid tori of genus 2. (Received November 28, 1960.)
Let $M$ be a connected homogeneous pseudo-Riemannian manifold (non-degenerate, but possibly indefinite, metric $g_{ij}$) of constant curvature $k$ $(R_{ijk}l = k\left[\delta_{jk}g_{il} - g_{ik}g_{jl}\right])$, with $g_{ij}$ of signature $(h, n - h)$ $(h < n - h < 1)$ if $k \neq 0$, signature $(n - h, h)$ if $k < 0$. If $k = 0$, $M$ is isometric to a manifold $R^n_h/D$ where $R^n_h$ is Euclidean $n$-space with metric $\sum_{1}^{h} x_j^2 + \sum_{h+1}^{n} y_j^2$ and $D$ is a discrete vector subgroup. If $k \neq 0$, $(h, n)$ $\neq (1, 2)$ and either $2h \neq n$ or $n + 1 - h$ is odd, then $M$ is isometric to a manifold $Q^v_u(F)/D$ where $F$ is one of the fields $R$ (real), $C$ (complex) or $K$ (quaternion), $D$ is a finite subgroup of the multiplicative group of unimodular elements of $F$, $f = \dim R F$, $fu = h$, $fv = n + 1$, $Q^v_u(F)$ is the quadric $-\sum_{1}^{h} x_j^2 + \sum_{h+1}^{n} x_j^2 = 1$ in a left $v$-dimensional vector space over $F$, and $D$ acts on $Q^v_u(F)$ by scalar multiplication. Structure theorems for $\tau_f(M)$ are found for the other cases. Drop the homogeneity assumption on $M$. If $M$ is isotropic, it is symmetric; if $M$ is symmetric, it is homogeneous; if $M$ is homogeneous and $k = 0$, it is symmetric; if $M$ is symmetric and $k \neq 0$, it is isotropic; if $M$ is isotropic and $k = 0$, $M$ is isometric to $R^n_h$; if $M$ is symmetric and $k \neq 0$, then $M$ is a (metric) covering manifold of $Q^{n+1}_h(R)/\{\pm 1\}$. (Received November 21, 1960.)

Let $a$ be an arc in euclidean $n$-space $E^n$ and let $E^n/a$ be the decomposition space. That is, the points of $E^n/a$ are the points of $E^n - a$ and the arc $a$. The topology of $E^n/a$ is the topology induced by the natural map of $E^n$ onto $E^n/a$. In general $E^n/a$ is not topologically equivalent to $E^n$, but the space $E^n/a \times E$ is topologically equivalent to $E^{n+1}$. (Received November 21, 1960.)

If $A$ is a subring of an algebraic number field $L$, then $A$ is a subring of a field $K$ such that (1) $L \supseteq K \supseteq A$, and (2) for any $x \in K$, there is a nonzero integer $n$ such that $nx \in A$. $A$ is said to be full in $K$. This paper is devoted to the classification of full subrings of an algebraic number field $K$. If $A$ and $B$ are full subrings of $K$, then $A$ is quasi-equal to $B$ if there exists a nonzero integer $n$ such that $nA \subseteq B$ and $nB \subseteq A$. The rings are first classified up to quasi-
equality. In each quasi-equality class of full subrings of K, there is one and only one integrally closed ring \( J_\pi = \bigcap_{P \in \pi} J_P \), where \( \pi \) is a set of prime ideals in the ring of integers J of K and \( J_P \) is the valuation ring corresponding to the prime ideal P. Each set \( \pi \) of prime ideals gives rise to a quasi-equality class. The ring \( J_\pi \) is the integral closure of every ring in the class. The classification of the rings in a given quasi-equality class is equivalent to the classification of the subrings of finite index in \( J_\pi \). There is a (1-1) correspondence between the subrings of finite index in \( J_\pi \) and the open subrings of \( J(\pi) \), where \( J(\pi) \) is the complete direct sum of the rings \( J(P), P \in \pi \), and \( J(P) \) is the \( P \)-adic completion of J, and \( J(\pi) \) has the product topology. (Received November 21, 1960.)


A sequence \( \{s_k\} \) is said to be totally monotone if \( \Delta^ns_k \geq 0 \), \( k,n = 0,1,2,\ldots \), where \( \Delta s_k = s_k - s_{k+1} \), \( \Delta^ns_k = \Delta(\Delta^{n-1}s_k) \). Let T denote the set of totally monotone sequences, \( L = \{s|s \in T \{s \in T and \Delta^ns_k \geq 0 \text{ for } n = 1,2,\ldots; k = 0,1,2,\ldots\} \), and \( Q = \{s|s \in T and \{s_k/s_{k+1}\} \subseteq T \). Results. (i) \( L \cap Q \) is nonempty. (ii) With respect to sequence multiplication, L and Q are multiplicative semigroups containing the identity. (iii) L and Q are not closed with respect to sequence addition. (iv) \( L \subseteq Q \). Let \( \{s_k\} \in T \). Define a family of quotient operations on \( \{s_k\} \) by \( q_1(s_k) = s_k/s_{k+1} \), \( q_n(s_k) = q_1(q_{n-1}(s_k)) \). Let \( Q_n \) denote the set of totally monotone sequences whose first n "quotients" are not less than one. Then, from the definition of \( Q_n \), \( Q_1 \subset Q_2 \subset \ldots \). Let \( Q_{\infty} = \bigcap_{n=1}^{\infty} Q_n \). Results. (v) \( L = Q_\infty \). (vi) Q is a proper subset of \( Q_2 \). (vii) \( Q_2 \) is a proper subset of \( Q_1 \). (viii) \( Q_1 = T \). Conjectures. (i) \( Q_{n+1} \) is a proper subset of \( Q_n \) for each n. (ii) \( L = Q \). (Received November 23, 1960.)


Let X be a nonempty set and \( \mathcal{F} \) be a \( \sigma \)-algebra of subsets of X. Consider the infinite product space \( \Omega = \prod_{n=0}^{\infty}X_n \) where \( X_n = X \) for \( n = 0,1,2,\ldots \), and the infinite product \( \sigma \)-algebra \( \mathcal{F} = \prod_{n=0}^{\infty}\mathcal{F}_n \) where \( \mathcal{F}_n = \mathcal{F} \) for \( n = 0,1,2,\ldots \). Let \( \mathcal{F}_n \) be the sub-algebra of \( \mathcal{F} \) which consists of all sets in \( \mathcal{F} \) determined by the first n coordinates. Let \( \gamma \) be a stationary probability measure on \( \mathcal{F} \) and \( \mu \), a Markovian probability measure on \( \mathcal{F} \) which has stationary transition probabilities. Let \( \gamma_n, \mu_n \) be the contractions of \( \gamma, \mu \) respectively to \( \mathcal{F}_n \). Let \( \gamma_n \) be
absolutely continuous with respect to $\mu_n$ and $f_n$ be the Radon-Nikodym derivative. **Theorem:** If \( (1) \int (\log f_n - \log f_{n-1}) \, d\nu \leq M \) for \( n = 1,2,3,\ldots \), then \( \{n^{-1}(\log f_n - \log f_0)\} \) converges in $L_1(\nu)$. If (1) is replaced by a stronger condition: \( \int (f_n/f_{n-1}) \, d\nu \leq L \) for \( n = 1,2,3,\ldots \), then \( \{n^{-1}f_n\} \) converges with $\nu$-probability one. (Received November 23, 1960.)


Let $P = (p_{ij})$, $i,j = 0,1,2,\ldots$, be an ergodic aperiodic stochastic matrix with a finite number of nonzero elements in any column. Denote the elements of $P^n$ by $p_{ij}^{(n)}$. Suppose $\lim p_{0j}^{(n)}/p_{00}^{(n)} = u_j$ and $\lim p_{00}^{(n+1)}/p_{00}^{(n)} = c$ exist, as $n \to \infty$, for all $j$. **Theorem:** $(u_j)$ is an eigenvector of the adjoint $P^*$, with eigenvalue $c \leq 1$. The theorem can be extended to the periodic case. It is further proven that, under some additional assumptions, $c$ is the minimum eigenvalue for all non-negative eigenvectors of $P^*$. It may be remarked that if $P$ is the matrix of a recurrent chain, $c = 1$ (see Derman, Proc. Amer. Math. Soc. vol. 5 (1954) p. 332); if $c < 1$, the chain will have "drift". (Received November 25, 1960.)


Let $C(X,\mathbb{Z})$ be the ring of all integer-valued, continuous functions on a topological space $X$. Let $\mathcal{P}$ be all rational primes. Define $D(f)$

$$D(f) = \{(x,p) \in X \times \mathcal{P} | f(x) \equiv 0 \mod p\},$$

for $f \in C(X,\mathbb{Z})$, and $\mathcal{D} = D(C(X,\mathbb{Z}))$. If $J$ is an ideal in $C(X,\mathbb{Z})$, $D(J)$ is a filter in $\mathcal{D}$ and if $\mathcal{J}$ is a filter in $D$, then $D^{-1}(\mathcal{J})$ is an ideal in $C(X,\mathbb{Z})$. Moreover, $D(D^{-1}(\mathcal{J})) = \mathcal{J}$ for all $\mathcal{J}$, while $D^{-1}(D(J)) = J$ if $J$ is an intersection of maximal ideals. Thus, $D$ defines a (1-1) correspondence between maximal ideals of $C(X,\mathbb{Z})$ and ultrafilters of $\mathcal{D}$. $C(X,\mathbb{Z})$ has maximal ideals $M$ such that $C(X,\mathbb{Z})/M$ has zero characteristic iff $C(X,\mathbb{Z})$ contains unbounded functions. If $C(X,\mathbb{Z})/M$ has characteristic $p$, it is isomorphic to $\mathbb{Z}_p$. Otherwise, it is uncountable and, for suitable $X$ and $M$, it can have arbitrarily large cardinality. The field $C(X,\mathbb{Z})/M$ is always quasi-algebraically closed, but never algebraically closed. The subfield $\mathcal{L}$ of all elements in $C(X,\mathbb{Z})/M$ which are algebraic over the prime field is uniquely determined by $\mathcal{P} = \{A_x | x \in \mathbb{Z}_p\}$, where $A_x = \{p \in \mathcal{P} | f(x) \equiv 0 \mod p, \text{some } x \in X\}$. If $C(X,\mathbb{Z})$ contains unbounded functions, then $\mathcal{P}$ is an ultrafilter in the subsets of $\mathcal{P}$. In
the characteristic zero case, \( \mathcal{A} \) has infinite degree over its prime field \( \mathbb{Q} \) and it can be as large as the algebraic closure of \( \mathbb{Q} \). (Received November 25, 1960.)


Only finitary algebras and the lower predicate calculus are considered. If the mappings in an inductive system of algebras of some type are all homomorphisms, the limit is given the weakest structure of this type making all limit mappings homomorphisms. Theorem: An open sentence (quantifier-free propositional form) satisfied by a sequence of the system as well as by its images cofinally is satisfied by its image in the limit. Corollary: A closed sentence true cofinally for which the Skolem functions may be chosen to commute with the system mappings is true in the limit. (More generally, it suffices for each sequence picked out by the functions to have cofinal images satisfying the matrix.) For example, any closed sentence cofinally true in an injective system is true in the limit. "Reduced products" being special cases of inductive limits, the above essentially contains the results announced in Abstract 550-9, Notices Amer. Math. Soc. vol. 5 (1958) p. 674; indeed, the latter even hold for subdirect products whenever the Skolem functions can be chosen in the factors so as to map the product into itself. (Received November 25, 1960.)


We consider a network as nodes connected by links. A link which connects two nodes \( N_i \) and \( N_j \) directly has a given capacity of transferring flow between them. This capacity is denoted by \( b_{ij} \). If all \( b_{ij} \) in a network are given, there is a maximum flow possible between \( N_i \) and \( N_j \) using all possible links in the network. This is denoted by \( f_{ij} \). Both \( b_{ij} \) and \( f_{ij} \) are symmetrical matrices. The following three problems are solved. 1. What is a condition that a given symmetrical matrix with entries \( r_{ij} \) will be an \( f_{ij} \) matrix for some network? A necessary and sufficient condition is shown to be \( r_{ik} \geq \text{Min}(r_{ij}, r_{jk}) \) for all \( i,j,k \). 2. Given \( b_{ij} \), find \( f_{ij} \) between all pairs of nodes in a \( n \)-node network. This is shown here that all \( (n(n - 1)/2) f_{ij} \) can be found by solving only \( n - 1 \) flow problems. 3. Given a set of \( n(n - 1)/2 \) flow requirements, find \( b_{ij} \) such that all \( f_{ij} \) equal or exceed the corresponding flow requirements with minimum \( \sum b_{ij} \). (Received November 16, 1960.)

A finite state sequential machine is a system consisting of three finite sets $X$ (input alphabet), $S$ (set of internal states), $Z$ (output alphabet) and two single valued functions $f: X \times S \rightarrow S$ and $g: S \rightarrow Z$. Sequential machines are models of digital computers; the machine is in one of the states of $S$, $f$ determines the new state of the machine for any input of $X$, and $g$ yields the output for that state. **Problem:** study the structure of sequential machines and assign (binary) codes to the states of the machine so that the logical equations representing the machine are "simple". This is achieved by selecting assignments in which each (binary) variable describing the new state depends on the input and only a small subset of the variables of the old state. The basic tool is the partition on $S$ with Substitution Property (homomorphism). The existence of partitions with S.P. is closely related to the existence of assignments with subsets of state variables which can be computed independently. Decomposition of machines is discussed and algebraic properties of partitions with S.P. are derived to aid their application. (Received November 25, 1960.)

576-54. R. E. Stearns and Juris Hartmanis: On the state assignment problem for sequential machines, II.

Same problem and approach as in part I, but it deals with the cases which cannot be studied by "homomorphisms". The main tool is the partition pair on the states of a sequential machine, which is a generalization of the partition with S.P. **Definition:** A partition pair $(\pi, \pi')$ on $S$ (of a machine) is an ordered pair such that if the states $s_i$ and $s_j$ are in the same block of $\pi$, then for every input $x$ the new states $f(x,s_i)$ and $f(x,s_j)$ are in the same block of $\pi'$. Algebraic properties of these pairs are derived. **Definition:** A pair $(\pi, \pi')$ is an Mm pair if $\pi' = \max \{ \pi'_i | (\pi'_i, \pi') \text{ is a partition pair} \}$ and the dual condition on $\pi'$ holds. It is shown that the set of all Mm pairs of a machine form a lattice and that all partition pairs can be generated from this lattice. The relation between state assignments with reduced dependence and partition pairs is discussed. A necessary and sufficient condition is given in terms of these pairs for the existence of assignments with reduced dependence. Also machines with "don't care" conditions are discussed. (Received November 25, 1960.)
Let $X$ be a topological space, $R$ a nondiscrete topological field and $F = (f_1, \ldots, f_n)$ functions from $X$ into $R^n$. $J \subset X^n$ is a $G$-set at $x$ if $(x, \ldots, x) \in \overline{J} \setminus J$ and $\det \{g_i(x_j) - g_i(x)\} \neq 0$ for each $(x_1, \ldots, x_n) \in J$. For $A \subset X$ let $K = \{J_x \mid x \in A\}$ be such that $J_x$ is a $G$-set at $x$. Then $F$ is $K$-differentiable with respect to $G$ on $A$ if for each $x \in A$, $\lim \det \{f_i(x_j) - f_i(x)\} [\det \{g_i(x_j) - g_i(x)\}]^{-1}$ exist as $(x_1, \ldots, x_n) \in J_x$ tends to $(x, \ldots, x)$, call this limit $\nabla_G f_i(x)$. Define $f$ to be $K$-differentiable with respect to $g_1$ on $A$ if $\nabla_G g_1(x_1, \ldots, g_1(x_n), \ldots, g_n)$ exists on $A$. A $G$-set $J$ is regular at $x$ if for each neighborhood $B$ of $x$ there is a $j$ such that the $j$th projection of $J \cap \bigcap_{i=1}^n B$ contains a deleted open neighborhood of $x$. **Theorem.** Suppose $X$ is a uniform space, multiplication is locally uniformly continuous in $R$, $K$ is uniformly regular on an open set $A$, and $f_i$ is uniformly continuously $K$-differentiable with respect to $g_j$ on $A$ for all $i$ and $j$. Suppose $\nabla_G F(x) \neq 0$ for some $x \in A$. Then there exists a neighborhood $C$ of $x$ such that for $y, z \in A$, $G(y) = G(z)$ if and only if $F(y) = F(z)$, and $g_j$ is uniformly continuously $K$-differentiable with respect to $f_i$ on $C$ for all $i$ and $j$. (Received November 14, 1960.)

576-56. V. R. Hancock: *The group of commutative Schreier semigroup extensions of a group.*

If $F(S, Q)$ is the semigroup of all factor-systems for commutative Schreier extensions of a semigroup $S$ by a semigroup $Q$ and if $E(S, Q)$ is the subgroup of all elements of $F(S, Q)$ which are associate to $0$, then two elements of $F(S, Q)$ are equivalent (i.e., are factor-systems of equivalent extensions) if and only if they are in the same coset of $E(S, Q)$ in $F(S, Q)$. Equivalence is a congruence relation on $F(S, Q)$ and the quotient-semigroup is denoted by $H(S, Q) = F(S, Q)/E(S, Q)$.

**Theorem:** If $S$ is a group and if $Q^*$ is the difference-group of the maximal cancellative homomorphic image of a semigroup $Q$ then the natural homomorphism of $Q$ into $Q^*$ induces an isomorphism of $H(S, Q^*)$ onto $H(S, Q)$. **Corollary 1:** A commutative (resp. commutative and cancellative) semigroup $S$ is a direct summand of every commutative (resp. commutative and cancellative) semigroup extension of itself if and only if it is a divisible abelian group. **Corollary 2:** A commutative semigroup $Q$ is such that every Schreier extension of a group $S$ by $Q$ is a splitting extension if and only if the difference-group of the maximal cancellative homomorphic image of $Q$ is a free abelian group. (Received November 25, 1960.)

Let $A(t)$ and $B(t)$ be two infinitesimal transformations on a differentiable manifold $M$, both of which depend continuously on time $t$. Consider the differential equation $D_u = (A(t) + B(t))_u$, where $u = u(t)$ is a path on $M$, $D_u$ represents the tangent vector of the path, and $(A(t) + B(t))_u$ is the vector given by the infinitesimal transformation $A(t) + B(t)$ at the point $u(t)$. The solution of the differential equation with the initial condition $u(t_0) = p_0 \in M$ is denoted by $u = T(A + B; t, t_0)p_0$. We obtain the formula $T(A + B; t, t_0)p_0 = U(t)T(B^*; t, t_0)p_0^{-1}$ for $t$ near $t_0$, where $U(t) = T(A; t, t_0)$ and $B^*(t) = Ad_{U(t)}B(t)$, i.e., for any function $f$ defined about a point $p$ of $M$, $B^*(t)_pf = B(t)U(t)_pfU(t)^{-1}$. It is further proved that, for any infinitesimal transformation $X$ on $M$, $d(Ad_{U(t)}X)_pf/dt = [Ad_{U(t)}X, A(t)]_p f$. The results are then applied to study decompositions of differential equations in the sense of (1). (Received November 28, 1960.)


A simple on-off control system (governed by a piecewise-linear differential equation of second order) is subjected to rather arbitrary periodic input signals, and various periodic responses are studied. Two types of results are obtained. First of all, exact analytic expressions for periodic responses, of harmonic and subharmonic type, are derived. Secondly, using these explicit representations of periodic solutions, it is shown that for certain classes of periodic inputs various superposition properties hold. For example, the average of the harmonic responses to a set of inputs is just the response to the average input. It is thus seen that on-off control systems may display both nonlinear features (subharmonic oscillations) and linear features (superposition).

(Received November 28, 1960.)


An orthomodular lattice is a lattice $L$ with 0 and 1 which is equipped with an orthocomplementation $' : L \rightarrow L$ and which satisfies the orthomodular identity $e \leq f \Rightarrow f = e \lor (f \land e')$. A Baer *-semigroup is a semigroup $S$ with a zero element 0 which is equipped with an anti-automorphic involution $*: S \rightarrow S$ and with
a mapping $' : S \rightarrow S$ such that for $x \in S$, $x' = (x')^2 = (x')^*$ and \{ $y \in S | xy = 0$ \} = xS. $P'(S)$ denotes the image of the mapping $'$, and is an orthomodular lattice (in a natural way). $S$ coordinatizes $L$ in case $L$ is isomorphic to $P'(S)$. $S$ is range-closed in case $x \in S$, $e \in P'(S)$, $e \leq x'$ and $(ex^*)'' = (x^*)'' \Rightarrow e = x''$. $S$ is *-regular in case $x \in S \Rightarrow \exists e \in P'(S)$ such that $Sx = Se$. **Theorem.** The following conditions are mutually equivalent: (i) $L$ is modular, (ii) $L$ can be coordinatized by a range-closed $S$, (iii) $L$ can be coordinatized by a $*$-regular $S$. (Received November 28, 1960.)

576-60. Fumiyuki Maeda: A characterization of spectral operators on locally convex spaces.

Let $E$ be a separated locally convex space which is quasi-complete and barreled and let $T$ be a continuous linear operator of $E$ into itself. An $E$-valued measure $m$ on the complex plane $C$ is called a weak $T$-measure (resp. a $T$-measure) for $x \in E$, if $\lim_{n \rightarrow \infty} \{ \text{Var}((T - S_z)^R m(\cdot \cap \sigma), x)^{1/n} = 0 \}$. Here, $S_z$ is the transformation on measures with compact support defined by the equation: $S_z\gamma(\sigma) = \int_{[\sigma]} zd\gamma(z)$. For a continuous linear operator $T'$ on $E'$, a weak $T'$-measure (resp. a $T'$-measure) for $x' \in E'$ is similarly defined. Then the following theorem, which was proved for reflexive Banach spaces by E. Bishop (Trans. Amer. Math. Soc. vol. 86 (1957) pp. 414-445), is valid for any separated locally convex space $E$ which is quasi-complete and barreled: **Theorem:** $T$ is spectral (resp. scalar) if and only if every $x \in E$ has a weak $T$-measure (resp. a $T$-measure) and every $x' \in E'$ has a weak $tT$-measure (resp. a $tT$-measure). (Received November 28, 1960.)

576-61. Jack Segal: A fixed point theorem for the hyperspace of a snake-like continuum.

If $X$ is a metric continuum, $C(X)$ denotes the space of subcontinua of $X$ with the finite topology. As a partial answer to question 186 (due to B. Knaster 4/29/52) of the New Scottish Book it is shown that $C(X)$ has the fixed point property if $X$ is a snake-like continuum. This is done by showing that $C(X)$ is a zero-cyclic quasi-complex and then applying the Lefschetz Fixed Point Theorem. (Received November 28, 1960.)

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A general construction of secondary operations is given in terms of homotopy classes of mappings. Given a sequence of continuous maps
\[ A \xrightarrow{f} B \xrightarrow{g} C \xrightarrow{h} D \]
such that the composites \( gf \) and \( hg \) are both null homotopic there is constructed a homotopy class of maps of the suspension \( SA \) into \( D \).

The construction uses choices of the null homotopies, and the resulting homotopy class is defined modulo two types of indeterminacies, each due to one of the choices involved. Specializing the spaces in question yields several known examples of secondary operations including secondary cohomology operations, functional cohomology operations, and the toral construction of Toda. General properties of this construction are established and include most of the known properties of the special cases listed above. (Received November 28, 1960.)


Let \( X(t) \) denote a possibly asymmetric Cauchy process starting at
\[ x: E\{e^{itX(t)}\} = \exp(\xi x - t|\xi| - ia\xi t), \quad -\infty < a < \infty. \]
Set \( k(x,y,t) \)
\[ = \frac{d}{dy} \Pr\{\max_{\tau \leq t} |X(\tau)| < 1, \quad X(t) \equiv y\}, \quad K(x,y) = \int_0^\infty k(x,y,t)dt. \]
An explicit formula is found for \( K(x,y) \).

If \( T \) is the first passage time to the exterior of the unit interval then \( E\{T^n\} = n! \int_1^\infty K_n(x,y)dy \). Let \( 2\pi\beta \) denote that argument of \( (a - i)/(a + i) \) which lies between 0 and \( 2\pi \), \( \gamma^*(u) = |1 + u|^\beta |1 - u|^{1-\beta} \). Then \( E\{T^n\} = \gamma(x)/(a^2 + 1)^{1/2} \). For the distribution of the place of first passage, \( X(T) \), we have \( d/du \Pr\{X(T) \equiv u\} = \frac{\pi(a^2 + 1)^{1/2} \gamma(u)|x - u|^{-1} \gamma(x)}{x} \). These results were obtained by solving a singular integral equation of Cauchy type. (Received November 28, 1960.)

576-64. Wolfgang Schmidt: Bounds for certain sums; a remark on a conjecture of Mahler.

If \( P(x,y) = P_n(y)x^n + \ldots + P_1(y)x + P_0(y) \) is a polynomial with integral coefficients where \( P_n(y) \) has degree \( m \) and \( n \geq 1, \ 3m \geq n + 3 \), and if \( P(x,y) - k \) has no linear factor if \( k \neq 0 \), then we show \( \sum |P(x,y)|^{-1/2} = O(H^{2/3}) \), where the sum is taken over all integral pairs \( x, y \) in \(|x| \leq H, \ |y| \leq H \) having \( P(x,y) \neq 0 \).

This result enables us to prove \( \gamma_n(\xi) \leq 2 - \gamma/3_n \) for almost all \( \xi \) if \( n > 2 \), where \( \gamma_n(\xi) \) is a certain measure for the transcendency of \( \xi \). Mahler conjec-
tured $\mathcal{G}_n(\xi) \leq 1$ almost everywhere, the best result so far for general $n$, obtained by Kasch-Volkmann, was $\mathcal{G}_n(\xi) \leq 2 - 2/n$ a.e. (Received November 28, 1960.)

576-65. Peter Bullen and Marvin Marcus: **Symmetric means and matrix inequalities.**

If $(a)$ is an $n$-tuple of positive numbers $(a) = (a_1, \ldots, a_n)$ let $(\bar{a}) = (a_1, \ldots, a_n, a_{n+1})$ denote the $(n + 1)$-tuple obtained by adjoining $a_{n+1}$ to $(a)$. Following Hardy-Littlewood-Polya (Inequalities) let $p^r_\bar{a} = p^r(a)$ denote the mean of the $r$th elementary symmetric function of the numbers $(a)$ (i.e.)

$$
\sum_{(i_1, \ldots, i_r) \text{ increasing}} a_{i_1} \cdots a_{i_r} 
$$

where the summation extends over all $C(n, r)$ increasing sequences of integers $(i_1, \ldots, i_r)$ of length $r$ chosen from $1, \ldots, n)$. Let $p^r_{\bar{a}} = p^r(\bar{a})$. **Theorem:**

$$
p^r_{\bar{a}} \leq p^r_{\bar{a}_{k+1}} \text{ with equality if and only if } a_1 = \ldots = a_{n+1}.
$$

**Corollary:** If $A_n$ and $G_n$ are the arithmetic and geometric means of $n$ positive numbers respectively then $(A_n/G_n)^n$ strictly increases with $n$ unless all the numbers are equal. (This is an analogue of an inequality of Rado: Hardy-Littlewood-Polya, p. 61).

**Corollary:** If $H$ is positive definite Hermitian and $t_k$ and $d_k$ denote the trace and determinant respectively of the leading $k$-square principal submatrix of $H$ then $(t_k/k)^k d_{k+1} = (t_{k+1}/k+1)^{k+1} d_k$. These results and others will appear in the *Proc. Amer. Math. Soc.* (Received November 29, 1960.)

576-66. P. S. Mostert: **An example in the theory of topological semigroups and homeomorphism groups.**

The following two questions are equivalent: (1) if $S$ is a locally compact topological semigroup, and $G$ is a subgroup, is $G$ a topological group (i.e., is inversion continuous)? (2) if $X$ is a locally compact topological space, and $G$ a group of homeomorphisms of $X$ onto itself such that the closure $G^*$ of $G$ in $C(X,X)$ under the compact-open topology is locally compact, is $G$ a topological group under the compact-open topology? An example is constructed using the product defined in the preceding abstract to show that the answer to these questions is not yes as was generally conjectured. (Received December 1, 1960.)
576-67. G. R. Blakley: **Partitions of vectors.**

All components of any vector appearing below are non-negative integers. None of these vectors is the zero vector. The number \( p_r(\vec{a}) \) of partitions of the vector (multipartite number) \( \vec{a} = (a_1, a_2, \ldots, a_n) \) into \( r \) parts is the number of distinct solutions of the equation \( b_1^r + b_2^r + \ldots + b_r^r = \vec{a} \). A recursion formula is derived for \( p_r(\vec{a}) \). From this formula it follows that \( p_r(\vec{a}) \) is, essentially, a polynomial in the variables \( a_1, a_2, \ldots, a_n \). Let \( c = r! \cdot \binom{r}{r-1} \). Then the leading term of this polynomial is \( \frac{1}{c} \cdot a_1^{r-1} \cdot a_2^{r-1} \cdot \ldots \cdot a_n^{r-1} \). It is apparent that this is a direct generalization of an already known result in the theory of partitions of an integer. (Received December 1, 1960.)

576-68. Roy Leipnik and J. E. Maxfield: **Numbers which can be added to normal numbers yielding normal sums.**

It is known that the sum of two numbers that are normal to the scale \( r \) need not be normal, however if \( a \) is normal and \( a \) is rational then \( a + a \) and \( a + a \) are normal to scale \( r \). This determines a countable set, members of which can be added to \( a \) and the sum remain normal to scale \( r \). In this paper two distinct methods are used to define uncountable sets each of whose members when added to a normal number yields a normal sum. The sets which are defined contain the class of Liouville numbers defined in terms of series with increasing gaps. (Received December 1, 1960.)

576-69. Paul Civin and Bertram Yood: **The second conjugate space of a Banach algebra as an algebra.**

Let \( B \) be a Banach algebra with \( B^{**} \) its second conjugate space. A procedure is given by R. Arens (Monatsh. Math. vol. 55 (1951) pp. 1-19; Proc. Amer. Math. Soc. vol. 2 (1951) pp. 839-848) for defining a multiplication in \( B^{**} \) so that \( B^{**} \) becomes a Banach algebra. A systematic study is made of the algebra \( B^{**} \) so obtained, with particular emphasis on the case in which \( B \) is the group algebra of a locally compact group \( G \). In the latter case if \( G \) is infinite, it is shown that \( B^{**} \) is not commutative and not semi-simple. (Received December 1, 1960.)

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Frank Harary: A graph theoretic approach to matrix inversion by partitioning.

Let $M$ be a square matrix whose entries are in some field. Our object is to find a permutation matrix $P$ such that $PMP^{-1}$ is partitionable so that all submatrices below the diagonal of this partitioned matrix consist entirely of zeros. Let $A$ be the matrix obtained from $M$ by defining $a_{ij} = 0$ whenever $i = j$ or $m_{ij} = 0$, and $a_{ij} = 1$ otherwise. Then $A$ may be regarded as the adjacency matrix of a directed graph $D$. Call $D$ strongly connected or strong if any two points of $D$ are mutually reachable by directed paths. A strong component of $D$ is a maximal strong subgraph. The condensation $D^*$ of $D$ is that digraph whose points are the strong components of $D$ and whose lines are induced by those of $D$. By known methods (J. Math. Phys. vol. 38 (1959) pp. 104-111), we construct $D^*$ from the digraph $D$ whose adjacency matrix $A$ was obtained from the original matrix $M$. Let $A^*$ be the adjacency matrix of $D^*$. It is easy to show that there exists a permutation matrix $Q$ such that $QA^*Q^{-1}$ is an upper triangular matrix. The determination of a desired permutation matrix $P$ from this matrix $Q$ is straightforward. (Received December 1, 1960.)

B. L. Reinhart: Jordan curves and free homotopy classes on compact surfaces.

Let $M$ be a compact surface of constant negative curvature, bounded by geodesics, possibly one sided. It is a known fact that each free homotopy class (that is, conjugacy class in the fundamental group) admits a unique geodesic which is without multiple points if and only if the free homotopy class admits a Jordan curve (there is one minor exception). By use of this fact, an algorithm is given for determining whether the free homotopy class admits a simple closed curve. There are two reasons for a class's having essential multiple points: its being a power, and its winding around several handles in incompatible ways. (See also Proc. Nat. Acad. Sci. U. S. A., Sept. 1960.) (Received December 1, 1960.)

T. W. Ting: On the solution of an elliptic differential equation over a rectangular domain.

Let $G$ be a rectangle with boundary $\Gamma$. The boundary value problem is
to find the solution \( u \) for the equation \( \Delta u - q(x)u = f(x,y) \) in \( G \) with \( u = 0 \) on \( \Gamma \), \( u \in C_2 \) in \( G \), where the functions \( f(x,y) \), \( q(x) \) and their first partial derivatives are Hölder-continuous in \( G + \Gamma \), \( q(x) \geq 0 \) in \( G + \Gamma \). By means of the Dirichlet principle, this problem is reduced to a variational problem. With the expansion of the admissible functions into double sine series, the variational problem is further reduced to a problem of solving an infinite system of linear algebraic equations which determine the coefficients of the double sine series in the solution of the differential equation. It has been shown that (1) the bounded solution of the infinite linear algebraic system exists and is unique; (2) every finite algebraic system truncated from the infinite system always has a unique solution and this solution converges to the solution of the infinite system as the number of the unknowns increases; (3) the finite double sine series with the coefficients determined by the truncated finite system form a minimizing sequence for the variational problem. (Received December 1, 1960.)


Assume that discrete data \( \{ y_t \} \), \( t = \ldots , -1,0,1,2, \ldots \) consists of a true trend \( \{ u_t \} \) plus error, \( y_t = u_t + x_t \), where the trend \( u_t \) satisfies some difference equation with constant coefficients, for example \( k_3 u_{t-3} + k_2 u_{t-2} + k_1 u_{t-1} + u_t = 0 \), and \( \{ x_t \} \) is a sequence from the ensemble of a stationary time series. The difference equation determines regressors \( \phi^{(0)}(t) \), \( \phi^{(1)}(t) \), \( \phi^{(2)}(t) \) such that \( \{ u_t \} \) is a linear combination \( c_0 \phi^{(0)} + c_1 \phi^{(1)} + c_2 \phi^{(2)} \). It is shown that because of the constancy of coefficients, the regressors have the property that in any panel of \( n \) data values, \( n > 3 \), they span the same linear subspace of the linear space \( E_n \) of all functions on the \( n \) values of \( t \); and conversely. Accordingly the least squares best estimate of trend value at say the midpoint of each panel is given by \( y_h = \sum_{t=0}^{n-1} a_{t+h} y_{t+h} \), where the weights \( a_{t+h} = a_t \) do not change from panel to panel. Procedures are discussed for estimation of \( k_1, k_2, k_3 \) in case they are not known in advance, and for pre-whitening of \( \{ e_t \} = \{ k_3 x_{t-3} + k_2 x_{t-2} + k_1 x_{t-1} + x_t \} \) in case \( \{ e \} \) is not white error, to make appropriate the least squares estimation of the coefficients \( c_0, c_1, c_2 \) for the regression on \( \phi^{(0)}, \phi^{(1)}, \phi^{(2)} \). (Received December 2, 1960.)

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576-74. Ernst Snapper. Monotone behavior of cohomology groups under proper mappings.

Let \( f : X \rightarrow Y \) be a proper mapping from an irreducible, quasi-projective variety \( X \) onto an algebraic variety \( Y \) of the same dimension \( r \). Let \( F \) be a coherent sheaf over \( X \) and \( f(F) \) its direct image over \( Y \). We prove that the natural homomorphism \( g : \text{H}^r(Y, f(F)) \rightarrow \text{H}^r(X, F) \) is an epimorphism. Several conclusions can be drawn from this. For example, if \( f \) is birational, \( X \) is projective, \( Y \) is normal and \( r = 2 \), it follows immediately that \( \chi (Y, 0_Y) \cong \chi (X, 0_X) \); here, \( \chi \) denotes the Euler-Poincaré characteristic and \( 0_X \), respectively \( 0_Y \), sheaves of local rings. This behavior of \( \chi \) was established by Muhly and Zariski [Trans. Amer. Math. Soc. vol. 69 (1950) pp. 78-88]. The proof that \( g \) is an epimorphism uses spectral sequences and the following three lemmas: We drop the assumption that \( X \) is irreducible and that dimension \( (X) = \text{dimension} (Y) \). We denote \( s = \text{Minimum} (\text{dimension} (t^{-1}(y))) \) where \( y \) runs through \( Y \); and \( Y_q = \{ y \mid y \in Y \text{ and dimension} (t^{-1}(y)) \leq q \} \) for \( q \leq 0 \). Then, (1) \( Y_q \) is closed in \( Y \) for \( q \geq 0 \); (2) \( \text{Supp} (R^q f(F)) \subseteq Y_q \) for \( q \geq 0 \); (3) dimension \( (X) - \text{dimension} (Y) \geq s \). (Received December 2, 1960.)

576-75. C. J. A. Halberg, Jr.: Semigroups of matrices defining linked operators with different spectra.

Let \( A \) denote the infinite matrix \( (a_{ij}) \), \( A^t \) its transpose matrix, and \( \bar{A} = (\bar{a}_{ij}) \), where \( \bar{a} \) is the complex conjugate of \( a \). Also let \( A_p \) denote the operator defined on \( \mathcal{L}_p \) by \( A, |\sigma(A_p)| \) the spectral radius of \( A_p \), and \( \left[ \mathcal{L}_p \right] \) the algebra of bounded linear operators on \( \mathcal{L}_p \). Using a generalization of Schur's inequality and the Toeplitz theory of summability the following theorems are obtained:

**Theorem I.** Suppose \( T \) and \( T^t \) belong to \( \left[ \mathcal{L}_p \right] \), \( T^t/\|T_1\| \) is regular, and \( \|T^t\| < \|T_1\| \). Then \( |\sigma(T^t)| > |\sigma(T_1)|, 1 < p \leq \infty \). **Theorem II.** Suppose \( T/\|T_1\| \) and \( T^t/\|T_1\| \) are regular and \( \|T^t\| < \|T_1\| \). Then \( A = T^t + T \) is a hermitian symmetric operator such that \( |\sigma(A_2)| < |\sigma(A_1)| \). **Theorem III.** Suppose \( T \) and \( T^t \) belong to \( \left[ \mathcal{L}_p \right], t_{ij} \) is real, and the infimum of the column sums is greater than \( \|T_1\| \). Then \( |\sigma(T^t)| < |\sigma(T_1)| \). Operators on different \( \mathcal{L}_p \) spaces defined by the same infinite matrix are special cases of "linked operators" introduced by A. E. Taylor and the author (Duke Math. J. vol. 6 (1956) pp. 283-290). The set of operators satisfying the hypotheses of any one of the above theorems forms a semigroup. (Received December 7, 1960.)
576-76. M. S. Klamkin: On a characterization of circles.

It is an elementary theorem that if the circumference of a circle be divided arbitrarily into any four arcs, and these arcs be bisected, then the chords joining the opposite points of bisection are orthogonal. It is shown that this latter property characterizes circles only. This will still be valid even if we put restrictions on combinations of the four points coinciding or not. However, the class of curves considered will be different in some of the cases. The five cases considered are (1) All four points coincide and the class of curves are ovals, i.e., plane, convex, twice differentiable curves, (2) Three points coincide and the class of curves are simple, closed, and rectifiable, (3) Two sets of two points coincide, (4) One pair of points coincide, (5) All points are distinct (the class of curves for the latter three cases are simple, closed, and differentiable). (Received December 2, 1960.)

576-77. Anne Lester: On a semigroup of right ideals of a semigroup.

Let $S$ be a compact connected semigroup with zero, $0$, and such that $S = SE$, where $E$ denotes the set of idempotents of $S$. Let $\mathcal{S}$ denote the space of nonempty closed subsets of $S$ with the finite topology, and let $\mathcal{R}$ be the subset of $\mathcal{S}$ consisting of all closed right ideals of $S$. The following statements are proved: (a) If multiplication in $\mathcal{S}$ is defined by $m(A, B) = AB$ for $A$ and $B$ in $\mathcal{S}$, then $\mathcal{S}$ is a compact connected semigroup with zero, $\{0\}$, and $\mathcal{R}$ is a closed left ideal of $\mathcal{S}$; (b) $\mathcal{R}$ is a compact arc-wise connected subsemigroup of $\mathcal{S}$ with zero, $\{0\}$, and right identity, $S$. (Received December 2, 1960.)

576-78. J. G. Harvey: Concerning o-complete o-groups.

An o-group is said to be o-complete if it has a trivial center and all of its o-automorphisms are inner. A subgroup $H$ of an o-group $G$ is said to be o-characteristic in $G$ if every o-automorphism of $G$ maps $H$ into $H$. If $G$ is an o-group for which the group of o-automorphisms $A_0(G)$ can be ordered, then the subgroup $K_0(G) = A_0(G) \times G$ of the holomorph of $G$, ordered lexicographically from the left, is called the o-holomorph of $G$. The author proves the following theorems. Theorem 1. If $x \rightarrow 2x$ is an automorphism of an abelian o-group $G$ and if $A_0(G)$ can be ordered, then $K_0(G)$ is o-complete if and only if $G' = \{1\} \times G$ is o-characteristic in $K_0(G)$. From this it is not too hard to prove that under
the same hypotheses and if \( G \) has well-ordered rank, then \( K_0(G) \) is \( o \)-complete.

Theorem 2. Any divisible, torsion-free, abelian group \( G \) can be ordered so that (i) \( A_0(G) \) can be ordered and (ii) \( K_0(G) \) is \( o \)-complete. Hence, every torsion-free abelian group can be embedded in an \( o \)-complete \( o \)-group. Theorem 3. Suppose that \( D \) and \( N \) are \( o \)-groups, that \( G = D \oplus N \), and that \( G \) is ordered lexicographically from the left. Then \( G \) is \( o \)-complete if and only if \( D \) and \( N \) are \( o \)-complete and \( N^1 = \{ \emptyset \} \times N \) is \( o \)-characteristic in \( G \). (Received December 2, 1960.)


The author gives examples of: (1) an \( R \)-group which cannot be ordered; (2) an ordered group of rank three which is not embeddable in a divisible ordered group of rank three; (3) an abelian ordered group of rank two whose group of order-preserving automorphisms cannot be ordered; (4) an abelian ordered group in which each component is rational, whose group of order-preserving automorphisms cannot be ordered. With regard to (1), it is known that every \( o \)-group is an \( R \)-group. As for (2) - (4), Conrad (Proc. Amer. Math. Soc. vol. 6 (1955) pp. 516-528, and Proc. Amer. Math. Soc. vol. 9 (1958) pp. 382-389) has shown that every \( o \)-group of well-ordered rank with rational components has an orderable order-automorphism group, and that every \( o \)-group of rank two is embeddable in a divisible such. In example (3), the largest component is not rational, and in example (4), the rank is not well-ordered. (Received December 2, 1960.)

576-80. S. N. Hudson: Groups of homeomorphisms of uniform spaces.

Let \((X, \mathcal{U})\) be a separated uniform space such that for each entourage \( A \) of the uniformity there exists an entourage \( B \) of the uniformity such that \( B \subseteq A \) and \( B(x) \) is connected for all \( x \in X \). A uniform space with this property is called a uniformly connected space. Let \( G \) be a transitive group of homeomorphisms of \( X \) such that for each \( x \) of \( X \) and open neighborhood \( V \) of \( x \), there exists an open neighborhood \( U \) of \( x \) contained in \( V \) such that \( y \) an element of \( U \) implies there exists a homeomorphism \( g \) in \( G \) with \( g(x) = y \) and \( g(z) = z \) for all \( z \) not in \( U \). A space with this property is called a strongly locally homogeneous space. If \( G \) is given the topology of uniform convergence, then it is shown that
G induces on X another uniformity $\mathcal{U}'$ in a natural way; that is, if $p$ is fixed in $X$, the function $\pi_p^*: G \to X$ defined by $\pi_p^*(g) = g(p)$ induces a uniformity on $X$. It is shown that $\mathcal{U}$ and $\mathcal{U}'$ are the same uniformity on $X$. This implies that $X$ is homeomorphic to $G/G_p$, the coset space of $G$ with respect to $G_p$, $(G_p = \{g \in G: g(p) = p\})$ with the quotient topology. (Received December 2, 1960.)


Let $\{x(t), 0 \leq t \leq 1\}$ denote the real Gaussian process with the following properties: $x(0) = 0$, $E\{x(t)\} = 0$ and $E\{x(t) x(s)\} = \rho(t, s)$. Theorem. Let the correlation function $\rho(t,s)$ of the real Gaussian process $\{x(t), 0 \leq t \leq 1\}$ satisfy the following Hölder condition: $|\rho(t_1, s) - \rho(t_2, s)| \leq K|t_1 - t_2|^a$ for $t_1, t_2$, $s \in \langle 0, 1 \rangle$, where $K$ and $a (0 < a < 1)$ are some fixed numbers. There exists a positive absolute constant $c$ such that $\lim_{\delta \to 0} \sup_{|t_1 - t_2| \leq \delta} |((x(t_1) - x(t_2))|^{1/2} - \left(\frac{2}{\log |t_1 - t_2|}\right)^{-1} < (C/a)^{1/2}$ holds with probability one.

(Received December 5, 1960.)


Each $r$-dimensional continuous random variable, $X = (X_1, ..., X_r)$, has $2^r - 1$ distribution functions of the form $F_{a_1 a_2 ... a_p}(x_{a_1}, ..., x_{a_p}) = \text{Prob}(X_{a_1} \leq x_{a_1}, 1 = 1, ..., p)$ for $a_1 < a_2 < ... < a_p$ and $p = 1, ..., r$. Let $\mathcal{F}$ be the $2^r - 1$ dimensional vector of these functions arranged in some fixed order. For a sample of size $n$ from $X$ let $\mathcal{F}(n)$ be the corresponding vector of sample distribution functions. A sample statistic, $S_n$, is a member of the class $\mathcal{J}$ if it is of the form, $S_n = \int Q(\mathcal{F}(n))d\mathcal{F}(n)_{12...r}$, where the integral is over $r$-dimensional real space and $Q(t_1, ..., t_{2^r-1})$ is defined $0 \leq t_k \leq 1, k = 1, ..., 2^r - 1$. The class $\mathcal{J}$ contains, for example, for $r = 2$, Spearman's $\rho$ and Kendall's $\tau$, and for $r > 2$, natural extensions of these. If $Q$ is a polynomial it is shown that $\lim_n S_n = \text{Expectation}\ (Q(\mathcal{F}))$. Conditions for consistency of a sequence of tests of an hypothesis based on $S_n$ follow. The well-known conditions for the consistency of $\rho$ and $\tau$ in the usual tests for independence, as well as similar conditions for their extensions to $r \geq 2$, are special cases. A parallel development of a class of two-sample statistics is possible. (Received December 5, 1960.)
A topological space \( M \) is called a test space with respect to an abelian group \( G \) if, in case \( X \) is compact, the homological dimension of \( X \) with respect to \( G = \dim X \) if and only if \( \dim (X \times M) = \dim X + \dim M \). Let \( R = \) the additive group of all rationals, \( Q_p = \) the additive group of all rationals reduced mod 1 whose denominators are powers of a prime \( p \) and \( Z(\mathcal{H}) = \) the limit group for a \( k \)-sequence \( \mathcal{H} \). Then we can construct test spaces \( M_0, M_p \) and \( Q(\mathcal{H}) \) with respect to \( R, Q_p \) and \( Z(\mathcal{H}) \) respectively. Moreover, we can prove: In order that \( \dim (X \times Y) = \dim X + \dim Y \) for compact spaces \( X \) and \( Y \) it is necessary and sufficient that at least one of the following four conditions hold; (i) \( \dim (X \times M_0) = \dim X + \dim M_0 \) and \( \dim (Y \times M_0) = \dim Y + \dim M_0 \), (ii) \( \dim (X \times M_p) = \dim X + \dim M_p \) and \( \dim (Y \times Q(\mathcal{H})) = \dim Y + \dim Q(\mathcal{H}) \) for some prime \( p \), (iii) \( \dim (X \times Q(\mathcal{H}^p)) = \dim X + \dim Q(\mathcal{H}^p) \) and \( \dim (Y \times M_p) = \dim Y + \dim M_p \) for some prime \( p \), (iv) \( \dim (X \times Q(\mathcal{H})) = \dim X + \dim Q(\mathcal{H}) \) and \( \dim (Y \times Q(\mathcal{H})) = \dim Y + \dim Q(\mathcal{H}) \) for some prime \( p \), where \( \mathcal{H}^p = (p, p^2, p^3, \ldots) \) and \( \mathcal{H} = (p, p, p, \ldots) \). (Received December 5, 1960.)
transformation invariants \( \rho = \{ \rho_\omega(\cdot) : \omega \in \Omega \} \) is complete if \( \rho_\omega(f) = \rho_\omega(g) \) for all \( \omega \in \Omega \) implies \( g \in \mathcal{J}[f] \). The kth order autocorrelation function of \( f \in L_{1,r}(G) \) is the real-valued function \( \rho_k(f) = \rho_k(f)(x_1, x_2, \ldots, x_k) \) with domain \( G^{(k)} = G \times G \times \ldots \times G \) (k copies) defined by \( \int_G f(\xi) f(\xi x_1) f(\xi x_2) \ldots f(\xi x_k) d\mu(\xi) \).

Theorem: \( \{ \rho_k(f) : k = 1, 2, \ldots \} \) is a complete set of translation invariants for \( L_{1,r}(G) \). (Received December 5, 1960.)

576-86. William Browder: On the homology rings of certain Lie groups.

If \( G \) is a group, \( Z(G) \) the center of \( G \), \( PG = G/Z(G) \). **Theorem 1.**

\( H_*(PSU(n^r); Z_p) \) is not commutative if \( r > 1, n > 0, \) and \( H_*(PSO(4n + 2); Z_2) \) is not commutative if \( n > 0 \). The proof demonstrates the noncommutativity without actually computing the ring structure. It uses the Bockstein spectral sequence and various technical results concerning it. (Received December 5, 1960.)

576-87. P. T. Church and Erik Hemmingsen: Light open maps on the n-sphere.

This abstract is a continuation of No. 571-107 (Notices Amer. Math. Soc. vol. 7 (1960) pp. 521-522). Let \( f : M \to N \) be a map, where \( M \) and \( N \) are n-manifolds; its branch set \( B_f \) is the set of points \( p \) at which \( f \) is not a local homeomorphism; \( f \) is said to be light, if, for each \( p \) in \( f(M) \), each component of \( f^{-1}(p) \) is a single point. If \( f \) is locally at most \( k \)-to-1 at \( p \), then \( f \) is said to have exceptionality \( e(p) = k - 1 \). I. For each \( n \geq 4 \), there exists a simplicial open map \( f : S^n \to S^n \) of degree 3 for which \( B_f \) contains no \((n-2)\)-manifold (C. J. Titus had conjectured in the Problem Book at the University of Michigan that for any map \( f : S^n \to S^n \) of degree greater than one, \( B_f \) contains an \((n-2)\)-sphere. The cases \( n = 1 \) and \( 2 \) are trivial; \( n = 3 \) is still open). II. For each \( n \geq 3 \), there exists a finite-to-one open map \( f : S^n \to S^n \) such that \( \dim(B_f) = \dim f(B_f) = n - 2 \), but, for each \( k \leq n - 2 \), there exist infinitely many components of \( B_f \) having dimension \( k \). Thus, \( B_f \) need not be locally connected. III. On the other hand, if \( f : M \to N \) is open, \( M \) and \( N \) are 3-manifolds, and each branch point of \( M \) has exceptionality one, then \( B_f \) is the union of homeomorphs of circles and open intervals, and locally there are only a finite number of them. (Received December 5, 1960.)
A theorem on vanishing differences.

Let \( \{a_n\}_{n=0}^{\infty} \) be an even sequence of complex numbers such that \( a_n = O(|n|^k) \) for some non-negative integer \( k \). Let \( k \) be the smallest such integer.

Let the differences \( \Delta a_n = 0 \) for all \( n \) belonging to a set \( A \) of non-negative integers of density \( d \geq 1/2 \). Then there exists a polynomial \( p \) of degree \( k \) such that \( p(n) = a_n \) for \( n = 0,1,2,\ldots \). In particular, if \( \{a_n\} \) is bounded, then it is a constant sequence. This theorem is analogous to theorems concerning the vanishing of differences \( \Delta^n a_0 \) proved by Agnew (On sequences with vanishing even or odd differences, Amer. J. Math. vol. 66 (1944) pp. 339-340), Pollard (Sequences with vanishing even differences, Duke Math. J. vol. 12 (1945) pp. 303-304), and Fuchs (A theorem on finite differences with application to the theory of Hausdorff summability, Proc. Cambridge Philos. Soc. vol. 40 (1944) pp. 188-198). However, in the above theorem, the condition that the set \( A \) be of density \( \geq 1/2 \) is weaker than the corresponding assumptions in those theorems. (Received December 5, 1960.)

Generalized Hermite polynomials.

Explicit formulae involving the polynomials \( H_n^r(x,a,p) = (-1)^n x^a \exp(px^r) \sum_{k=0}^n \frac{D_x^k}{k!} \frac{x^k}{k!} \exp(-px^r) \) and \( g_n^r(x) = \exp(-D_x^r) x^n \) are developed. The ordinary Hermite polynomials occur when \( r = 2 \). The polynomials do not in general possess orthogonality properties. Expansion formulae for operators such as \( (D_x - prx^{r-1} + a/x)^n \) are found. Put \( H_n^r(x) = H_n^r(x,0,1) \); we note that \( H_n^{r+1}(x) = \sum_k (-1)^k \binom{n}{k} (r-k)x^{r-k}1H_{n-k}^r(x) \). These polynomials are motivated by extensive attempts to find closed summation formulae for binomial coefficient series of the type \( \Delta n^a \binom{a+bk}{j} \). (Received December 5, 1960.)

General local-integrability.

A cover of a locally compact space \( X \) with a Radon measure \( \mu \) is a set \( C \) of measurable sets such that every measurable set is contained in some countable union of members of \( C \). The space of \( c \)-locally integrable functions \( \Omega_c = \{ f : X^E f \in L^1(x,\mu) \text{ for all } E \in \mathcal{C} \} \). A function \( f \) on \( X \) is unbounded at \( x \) if \( \mu(\{ y : |f(y)| > n^j \cap V \} > 0 \) for every neighborhood \( V \) of \( x \), and \( n = 1,2,\ldots \); it is strictly unbounded at \( x \) if \( \chi_V \cdot |f| \notin L^1(x,\mu) \) for every neighborhood \( V \) of \( x \),

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\( \mu(V) \neq 0 \). Let \( F \) be a set of functions \( f \) on \( X \) with \( X \in C \); denote by \( S(F) \) the set of points where elements of \( F \) are unbounded and denote by \( T(F) \) the points where elements of \( F \) are strictly unbounded. **Theorem.**

If \( \mu(S(F)) = 0 \) there exists a cover \( C \) of \( X \) such that \( F \subset \Omega_C \). When \( X \) is a locally compact group, it suffices to assume that \( \mu(S(F) \cap E) = 0 \) for every \( E \) in \( C \). If \( X = [0,1] \) and \( \mu \) is Lebesgue measure, there exists a countable set \( F \) and a cover \( C \) such that \( F \subset \Omega_C \) and \( \mu(T(F)) = 1 \). (Received December 5, 1960.)

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Let \( \mathcal{A}_R \), \( 0 < R \leq 1 \), denote the set of functions analytic in the disk \( \mathcal{D}_R \) \( \{ z \mid |z| < R \} \) and vanishing at 0. Define a multiplication on \( \mathcal{A}_R \) by \( \{fg\}(z) = \sum_{n=1}^{\infty} a_n^* b_n \), where \( f(z) = \sum_{n=1}^{\infty} a_n z^n \), \( g(z) = \sum_{n=1}^{\infty} b_n z^n \). The following results are obtained. (A) With the above multiplication, the usual addition and scalar multiplication, and the topology of uniform convergence on compact subsets of \( \mathcal{D}_R \), \( \mathcal{A}_R \) is a locally convex topological algebra with jointly continuous multiplication and with identity e. (B) \( f \in \mathcal{A}_R \) is invertible if and only if \( f'(0) \neq 0 \). **Corollary.** Let \( g, f \in \mathcal{A}_R \) with \( g'(0) \neq 0 \). Then there is a unique sequence \( \{c_n\} \) such that \( f(z) = \sum_{n=1}^{\infty} c_n g(z^n) \), \( z \in \mathcal{D}_R \). (This result is due to E. Hille, Duke Math. J. vol. 3 (1937).) (C) Let the series \( \sum_{q=0}^{\infty} a^* q^q \) have radius of convergence \( \rho \). Then \( |f'(0)| < \rho \Rightarrow \sum_{q=0}^{\infty} a^* q^q \) converges, \( |f'(0)| > \rho \Rightarrow \sum_{q=1}^{\infty} a^* q^q \) diverges.

(D) The exponential function \( \exp f = \sum_{q=0}^{\infty} (1/q!) f^q \) maps \( \mathcal{A}_R \) onto the subset of invertible elements. \( \exp f = \exp g \) if and only if \( f - g = 2n\pi i e \) for some integer \( n \). (Received December 6, 1960.)

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**Theorem.** Let \( a \) be an infinite cardinal and let \( A_2^a \) be the free Boolean algebra on \( 2^a \) generators. Then \( A_2^a \) is a subalgebra of the Boolean algebra of all subsets of a set of \( a \) elements. This theorem follows from a theorem proved by Hausdorff (Studia Math. vol. 6 (1936) pp. 18-19). (For \( a = \aleph_0 \) this was proved earlier by Fichtenholz and Kantorowitsch.) In the present paper the following theorem is proved. **Theorem.** Let \( a \) be an infinite cardinal and let \( C_2^a \) be the topological product of \( 2^a \) copies of the two-elements discrete space. Then \( C_2^a \) has a dense subset of \( a \) points. The above theorem follows as a corollary.
(using Stone's representation theory). It also generalizes an argument of Mrowka (Colloq. Math. vol. VII. 1 (1959) pp. 530-5) showing that $\mathcal{X}$ has $2^C$ points, where $X$ is the discrete space of $\mathcal{X}_0$ points, to the case $|X| \geq \mathcal{X}_0$. (Received December 6, 1960.)

576-93. J. M. Irwin and E. A. Walker: Abelian $\Sigma$-groups.

Let $G^1$ be the subgroup of elements of infinite height of an Abelian group $G$. A subgroup $H$ of $G$ is called a high subgroup of $G$ iff $H$ is maximal with respect to $H \cap G^1 = 0$. We have shown that if $H$ is a high subgroup of any Abelian group $G$, then $H$ is pure and $G/H$ is divisible. A theorem concerning high subgroups is: Theorem 1. Let $H$ and $K$ be high subgroups of a torsion group $G$. Then if $H$ is a direct sum of cyclic groups, so is $K$. Moreover, $H \cong K$. This theorem gives rise to the following notion. A $\Sigma$-group is any Abelian group $G$ all of whose high subgroups are direct sums of cyclic groups. This means that in a torsion $\Sigma$-group every high subgroup is basic. Furthermore in a torsion $\Sigma$-group every high subgroup is an endomorphic image. Examples of $\Sigma$-groups are very easy to find. For instance direct sums of countable groups turn out to be $\Sigma$-groups. Also, any Abelian group $G$ such that $G/G^1$ is a direct sum of cyclic groups is a $\Sigma$-group. Other results concerning these groups are: Theorem 2. For Abelian torsion groups it is true that a direct sum of $\Sigma$-groups is a $\Sigma$-group. Theorem 3. An Abelian torsion group $G$ is a $\Sigma$-group iff $G$ contains a maximal basic subgroup. (Received December 6, 1960.)


By a channel $S$ we mean a collection $\mu(\cdot | x), x \in \mathcal{A}$, of probability measures on a fixed measurable space $(\mathcal{B}, \mathcal{B}')$. It is called semi-continuous if the input alphabet $\mathcal{A}$ is finite. If $x_i \in \mathcal{A}, B_i \in \mathcal{B}', \mu(B_i | x_i) \geq 1 - \lambda_i$, $(i = 1, \ldots, N)$, with the $B_i$ disjoint then the sequence $(x_i, B_i)$ is called a $\lambda$-code for $S$. The largest possible integer $N$ will be denoted as $N(S, \lambda)$, $(0 < \lambda < 1)$. Let $S_1, \ldots, S_n$ be given channels and let $S^{(n)}$ denote their direct product, (a so-called memoryless channel of length $n$). Given suitable conditions, (e.g. if all the $S_i$ are semi-continuous), rather close upper and lower bounds for $N(S^{(n)}, \lambda)$ are found. As an illustration, let $S_1 = \{\mu_1(\cdot | x); 0 \leq x \leq \frac{1}{12}\}$ with
u_i(\cdot|x) as the normal \((x,\sigma_i)\) distribution on the reals, \(\sigma_i \equiv \varepsilon > 0\), \((i = 1, ..., n)\). Then 
\[ \log N(S^{(n)}, \lambda) - \sum C_i \leq K(n \log n)^{1/2}, \]
where \(K\) depends on \(\varepsilon\) and \(\lambda\) only; here, \(C_i\) denotes the Shannon capacity of \(S_i\). These bounds hold equally well if one allows a "noisy feedback channel". Further, bounds on the maximal length of a code which is to be a \(\lambda\)-code for a whole class of memoryless channels \(S^{(n)}\) of equal components. Finally, a result on a finite memory channel having a denumerable output alphabet. (Received December 6, 1960.)

576-95. J. S. MacNerney: **Hermitian moment sequences.**

If \(\{S, Q\}\) is a complete inner product space and \(H\) is the set of all functions from \(S\) into \(S\) which are Hermitian with respect to \(Q\) and \(C\) is an infinite sequence with values in \(H\), the following are equivalent: (1) There exists a nondecreasing function \(\phi\) from the real line into \(H\) such that 
\[ \int T^0 d\phi = C_n \]
\((n = 0, 1, ...). (2) There is a member \(A\) of \(H\) and an infinite symmetric matrix \(M\) with values in \(H\) such that \(M(i, j) = 0\) for \(|i - j| > 1\) \((i, j = 0, 1, 2, ...)\) and \(A^2 = C_0\) and \(A \cdot M^p(0, 0) \cdot A = C_p\) for each positive integer \(p\). (3) There is, for each positive integer \(k\), a number interval \([a_k, b_k]\) such that 
\[ a_k \sum_{j=0}^{k} Q(x, C_{i+j}x_j) \leq \sum_{j=0}^{k} Q(x, C_{i+j}x_j) \leq b_k \sum_{j=0}^{k} Q(x, C_{i+j}x_j) \]
for each sequence \(x, i, k\) with values in \(S\). The \(\phi\) is nondecreasing in the sense that \(Q(x, \phi(u)x) \leq Q(x, \phi(v)x)\) for each point \(x\) in \(S\) and each number interval \([u, v]\); \(I\) denotes the identity function on the real line; indicated Stieltjes integrals (over the real line) are strong limits with respect to the norm corresponding to the inner product \(Q\). There is in (1) an odd \(\phi\), \(\phi[-1] = -\phi\), only in case (2) holds with the proviso \(M(n, n) = 0\) \((n = 0, 1, ...),\) and also only in case \(C\) has zero odd part and (3) holds for the even part of \(C\) with the proviso \(a_k \geq 0\) \((k = 1, 2, ...). Related continued fraction expansions provide refinements of all these results. (Received December 6, 1960.)

576-96. Leopoldo Nachbin: **Algebras of operators and of continuous functions.**

Let \(E\) be a real topological vector space and \(A\) a commutative algebra of operators in \(E\) containing the unit. Assume \(A\) and \(E\) to satisfy the following conditions: (1) The neighborhoods \(V\) of 0 in \(E\) such that \(V = \bigcap V + IE\), where \(I\) is an ideal in \(A\) of codimension 1 and \(IE\) is the vector space spanned by all \(T(x), T \in I, x \in E\), form a basis; (2) The neighborhoods \(V\) of 0 in \(E\) for each of
which there is $\lambda = \lambda(V, T)$ such that $T(V) \subseteq \lambda V$ for all $T \in A$, form a basis. If $F$ is a vector subspace of $E$ invariant under $A$, let $E' = E/F$ and $A'$ be the algebra induced by $A$ on $E'$. Then $E'$ and $A'$ satisfy (1) and (2), with $E$ and $A$ replaced by $E'$ and $A'$. The proof is an application of the Weierstrass-Stone theorem. If $E$ and $A$ satisfy only (1), then $E'$ and $A'$ need not satisfy (1), this being related to the Bernstein approximation problem. (1) and (2) are necessary and sufficient for $E$ and $A$ to be representable by continuous functions.

(Received December 6, 1960.)


Consider a transformation group $[G, M]$ where $G$ is a noncompact, locally compact topological group and $M$ is a metric space. Let $\int f(g)dg$, $m(K) = \int \chi_K(g)dg$ be the Haar integral and corresponding Haar measure in $G$. Let $E$ and $E'$ be the ends (in the sense of Freudenthal) of $G$. If $E$ and $E'$ are distinct, choose neighborhoods $N'(E)$ and $N(E')$ such that $N' \cap N = \emptyset$. If $G$ has one end, set $N(E) = \emptyset$. For $x \in M$ and $B$ a Borel set in $M$ and $U$ an arbitrary neighborhood of $E$, set $\tau(x, B, U) = m(G - N \cup U)^{-1} \int \chi_B(gx)dg$. One may order the neighborhoods of $E$ by inclusion, then $[\tau(x, B, U): U \text{ a neighborhood of } E]$ becomes a net and it follows that $0 = \lim_{U \to E} \inf [\tau(x, B, U)] = \lim_{U \to E} \sup [\tau(x, B, U)] = 1$. One then defines $\bar{P}(gx \in B, E) = \lim_{U \to E} \sup [\tau(x, B, U)]$ and $P(gx \in B, E) = \lim_{U \to E} \inf [\tau(x, B, U)]$. If $P(gx \in B, E) = P(gx \in B, E)$, define $P(gx \in B, E) = \bar{P}$. Definition: A compact, invariant set $C \subseteq M$ is called a center of attraction for $x$ as $g \to E$ if for any open set $V \supset C$, $P(gx \in V, E) = 1$. Such a set is called a minimal center of attraction for $x$ as $g \to E$ if there exists no proper subset having the above properties. Theorem: If $Gx$ is compact, there exists a minimal center of attraction for $x$ as $g \to E$. (Received December 6, 1960.)

576-98. H. W. E. Schwerdtfeger: Groups with the property $T_2$.

A group $G$ is said to have the property $T_2$ with respect to a subgroup $\gamma$ if for every noninvolutory $A \in \gamma$, $A \not\in \gamma$, there is a unique element $H \in \gamma$ such that $H A H^{-1} = T A T^{-1}$ for every $T \in G$, and exactly two different such elements $H \in \gamma$, if $A^2 = I$, $A \not= I$. By a geometrical consideration it has been found that the group of all Moebius transformations $(M, T)$

$z \to (az + \beta)/(\gamma z + \delta) = A(z)$ (a, $\beta, \gamma, \delta$ complex, $a \delta - \beta \gamma \not= 0$, $z$ in the completed
complex plane) has this property with respect to the subgroup of all integral M. T. $z \to az + b$. The proof is based on the fact that every M.T. A defines, and is defined by, its characteristic parallelogram whose vertices are the fixed points and the poles $\delta/\nu$ and $\alpha/\rho$ of A and $A^{-1}$ respectively. Also the group of all real M.T. has the property $T_2$, as well as the group of all M.T. over a finite field. (Received December 6, 1960.)


Let $K_n$ be the set of all $n$ by $n$ matrices with elements in an algebraically closed field $K$ of characteristic zero. Define $\Delta_X Y = XY - YX$ for $X$, $Y$ in $K_n$. Let $A$, $B$ be in $K_n$ and let $s$ be a positive integer. This note proves briefly that if $\Delta^s_X B = 0$ for all $X$ in $K_n$ such that $\Delta^s_A X = 0$, then $B$ is a polynomial in $A$ with coefficients in $K$. The case $s = 1$ is classical. Marcus and Khan have recently established the result for $s = 2$ (Canad. J. Math. vol. 12 (1960) pp. 269-277). The proof rests on the observation that for nonderogatory $X$, $\Delta^s_X B = 0$ only if $\Delta_X B = 0$. (Received December 6, 1960.)

576-100. E. A. Walker: Torsion endomorphic images of mixed Abelian groups.

L. Fuchs, in his book *Abelian groups*, asks the following questions.

1) Which are the torsion Abelian groups $T$ that are endomorphic images of all Abelian groups containing them as maximal torsion subgroups? (2) Which are the torsion Abelian groups $T$ such that a basic subgroup of $T$ is an endomorphic image of any Abelian group $G$ containing $T$ as its maximal torsion subgroup? It is proved by homological methods that the answers to both questions are the same, and that $T$ is such a group if and only if $T = D \oplus B$, with $D$ divisible and $B$ of bounded order. Equivalently, $T$ is such a group if and only if $T$ is a direct summand of every Abelian group $G$ containing $T$ as its maximal torsion subgroup. (Received December 6, 1960.)


Let $(X, \mathcal{S}, \mu)$ be a totally-finite, normalized, nonatomic and homogeneous measure space, and let $(I, \mathcal{I}, m)$ be the Lebesgue measure space associated with $I = [0,1]$. If $(X, \mathcal{S}, \mu)$ is separable, then there exists a mapping $T$ of $X$ into $I$ such
that $X_r = T^{-1}(\{r\})$ is a null set for each $r$ in $I$, and for each $E$ in $\mathcal{F}$ there is an $E_0$ in $\mathcal{L}$ such that $\mu(E) = m(E_0)$, and $\bigcup \{X_r : r \in E_0\}$ differs from $E$ by a set of measure zero. In the general case, let $\beta$ be the least ordinal corresponding to the character of $(X, \mathcal{F}, \mu)$. There then exists a family $\{T_a : a < \beta\}$ of mappings of $X$ into $I$ such that $X_{ar} = T_a^{-1}(\{r\})$ is a null set for each $a < \beta$ and each $r$ in $I$, the $\sigma$-algebra generated by the class of sets $\{\bigcup \{X_{ar} : r \in E_0\} : a < \beta, E_0 \in \mathcal{L}\}$ is equivalent to $\mathcal{F}$, and $\mu(\bigcup \{X_{ar} : r \in E_0, E_0 \in \mathcal{L}\}) = m(E_0)$, for all $a < \beta$. The proofs depend on the fundamental isomorphism theorems of Halmos - von Neumann and Maharam and a theorem concerning the relation between the lattices of measurable functions modulo null functions defined on isomorphic measure spaces. (Received December 6, 1960.)


It is an open problem whether the number of deficient values of an entire function of finite order has an upper bound depending on the order only. The authors prove: Theorem. Hypothesis. Given $\varepsilon > 0$ there is a positive integer $m = m(\varepsilon)$ and a positive number $r_0 = r_0(\varepsilon)$ such that all zeros of the entire function $f(z)$ of finite order $\lambda$ in $|z| > r_0$ die in $m$ sectors $|z| > r_0$, $a_j < \arg z < \beta_j$ ($j = 1, 2, \ldots, m$) with $\sum_{j=1}^{m}(a_j - \beta_j) < \varepsilon$. Conclusion. Then $f(z)$ has at most $[2\lambda]$ deficient values other than 0 and $\infty$. The hypothesis can be relaxed considerably. The same conclusion holds if the sectors containing the zeros are curvilinear, provided they do not spiral too rapidly. There may also be zeros outside the sectors, provided the number of these zeros in $r_0 < |r| \leq r$ is $o(T(r,f))$. (Received December 6, 1960.)

576-103. P. J. Zwier: Coboundary operators on the ring of alternating tensors on a manifold.

Let $S = \sum S^p$ be the graded ring of alternating contravariant tensors on an infinitely differentiable manifold $\mathcal{M}$ of dimension $n$. Let $x \in \mathcal{M}$ and for each integer $p$, $0 \leq p \leq n$, let $S^p_x$ be the set of all elements $L^p$ of $S^p$ such that $x$ support $L^p$. The question arises as to what coboundary operators can be defined on $S$. An answer is given by the following theorem. Let $d^*$ be an operator on $S$ satisfying the following conditions: (i) For each $a, \beta \in S$, $d^*(a + \beta) = d^*(a) + d^*(\beta)$. (ii) For each $a \in S^p, \beta \in S$, $d^*(a @ \beta) = d^*a @ \beta + (-1)^p a @ d^*\beta$. 

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(iii) \(d^*f \in S^1\) and \(d^*d^*f = 0\) for all \(f \in S^0\). (iv) For each \(x \in \mathcal{M}\), \(d^*f = 0 \mod S^1_x\) if and only if \(f = c \mod S^0_x\). Then there exists a second order contravariant tensor \(g^{ij}\) on \(\mathcal{M}\) such that, except for sets of empty interior, \(g = |g^{ij}| \neq 0\), and 
\[d^* = \sigma_{p+1} d\sigma_p\] where \(d\) is the classical exterior derivative operator on the differential forms and \(\sigma_p\) is the ring isomorphism defined by 
\[(\sigma_p L^p) = g_{j_1k_1}g_{j_2k_2}\cdots g_{j_pk_p}(L^p)_{i_1j_2\cdots i_p}\] and \(g_{mt} \equiv g_{ms} = \delta t\). The tensor \(g^{ij}\) is not necessarily symmetric and its existence follows from conditions (i) - (iii) and the "if" part of (iv). The nonvanishing of \(g\) on open sets follows from (iv). (Received December 5, 1960.)


The von Neumann spring and mass point model is used to develop a set of finite difference equations suitable for machine calculation. A component of pressure having the form of the time derivative of compression is used to smooth out shocks and to decrease the oscillations behind shocks. The equations have been used to study the effects of different equations of state, reflections from fixed and free surfaces, and numerical effects of the number of iterations in the system of implicit equations. Results are presented in the form of graphs of pressure profiles at fixed time intervals. (Received December 5, 1960.)

576-105, W. A. Harris, Jr.: Singular perturbations of eigenvalue problems.

Consider the eigenvalue problem (1) \(L [y,a] = dy/dt - \{aA(t) + B(t,a)\} y = \lambda R(t)y\), (2) \(s[y,a] = M(a)y(a,a) + N(a)y(b,a) = 0\), where \(A,B,R,M,\) and \(N\) are square matrices of order \(n + m\), \(y\) is a vector, and \(\alpha\) is a large parameter. The problem is to investigate the spectrum of \(L[y,a]\), \(s[y,a]\) in its dependence on \(\alpha\). The character of this problem as \(\alpha \to \infty\) depends on the matrix \(A\) which we assume has its first \(n\) rows identically zero. When \(\alpha \to \infty\), formally (1) becomes a differential system essentially of order \(n\). With appropriate boundary conditions related to \(s[y,\alpha]\) a degenerate eigenvalue problem of order \(n\) can be defined and the problem (1), (2) considered as a perturbation of this degenerate problem. Under suitable restrictions corresponding to each simple eigenvalue \(\lambda_0\) of the degenerates there exists a unique eigenvalue of the problem (1), (2)
which has an asymptotic expansion $\lambda (a) \approx \lambda_0 + (1/a) \lambda_1 + \ldots$. An eigenvector $y(t,a)$ corresponding to $\lambda (a)$ may be determined so that $y(t,a) \approx y_0(t) + (1/a)y_1(t) + \ldots$, for $a < t < b$ and uniformly on any closed sub-interval, where $y_0(t)$ is an eigenvector for the degenerate problem corresponding to $\lambda_0$. For a class of self-adjoint problems this procedure characterizes the asymptotic behavior of all the eigenvalues of the problem (1), (2). (Received December 5, 1960.)

576-106. O. G. Harrold, Jr.: Locally unknotted combinatorial $n - 1$ manifolds in euclidean $n$-space have shell neighborhoods.

A topological $k$-manifold $M$ is called locally unknotted at $x$ in euclidean $n$-space, $E^n$, if there is a closed $k + 1$ cell $D$ such that the common part of $M$ and $D$ is a neighborhood of $x$ in $M$ that lies on a parameter $k$-cell of $D$ spanning the boundary of $D$ (Bull, Amer. Math. Soc. vol. 63 (1957) p. 298). If $M$ has this property at each point we say $M$ is locally unknotted in $E^n$. Let $M$ be a topological image of a closed, combinatorial manifold that is locally unknotted in $E^n$. Then there is a homeomorphism of $M \times [0,1]$ into $E^n$ such that $M \times (1/2)$ is the identity map. Strong use is made of the M. Brown version of the generalized Schoenflies theorem. Corollary: If $M$ is also an $n - 1$ sphere, the closure of the bounded component of the complement of $M$ is a closed $n$-cell. The theorem is a partial generalization of the useful result in 3-dimensional topology that "locally tame sets are tame." (Received December 5, 1960.)


We prove the following theorems: Theorem 1. Let $R$ be the set of all real numbers, $B$ a Banach algebra with unit $e$, $f: R \rightarrow B$ a function such that

\begin{itemize}
  \item[(1)] $f(t + s) + f(t - s) = 2f(t)f(s)$, $f(0) = e$ holds for all $t, s \in R$. If $f$ is measurable on $R$ then an element $a \in B$ exists such that (2) $f(t) = \sum_0^\infty a^n t^{2n}/(2n)!$ Banach algebra $B$ can be imbedded in a Banach algebra $B'$ of all $2 \times 2$ matrices, matrix elements of which belong to $B$ in such a way that
  \begin{align*}
    \left( \begin{array}{c}
      f(t) \\
      0 \\
    \end{array} \right)
    = \sum_0^\infty (a')^{2n} t^{2n}/(2n)!
  \end{align*}

  with $a' \in B'$. If the element $a$ in (2) possesses a regular square root in $B$, then (1) can be reduced to the functional equation $g(t + s) = g(t)g(s)$, $g(0) = e$.

  Theorem 2. Let $X$ be a Banach space, $F: X \rightarrow B$ a function such that $F(x + y) + F(x - y) = 2F(x)F(y)$, $F(0) = e$ for all $x, y \in X$. If $F$ is measurable on every ray then $F(x) = \sum_0^\infty [A(x)]^2/(2n)!$, where $A: X \rightarrow B$. The function $A$ is
continuous if and only if \( \lim F(tx) = e \) (\( t \to 0, t \in R \)) uniformly with respect to \( x \) in some sphere. If \( \|e - A(x)\| < 1 \) for some \( x \) then \( A(x) = [L(x)]^2 \) where \( L: x \to B \) is an additive function which is continuous if \( A \) is continuous. (Received December 5, 1960.)


Let \( x = (x_1, \ldots, x_n) \in E_n \), real \( n \)-space, let \( c_{ij} \), \( 1 \leq i \leq n \), be a set of real numbers, and consider in \( E_n \) the rectangular array \( M \) of mesh points defined by the intersections of the hyperplanes \( x_i = c_{ij} \). Let \( R \) be a bounded region of \( E_n \) whose boundary consists of portions of the above hyperplanes. Let \( a \) be be the diffusion operator \( au = - \nabla \cdot (D\nabla u) + cu \), and consider the equation \( au = f \) in \( R \) with \( u = 0 \) on the boundary of \( R \). Suppose that all the derivatives of \( u \) (resp. \( D \)) of orders up to and including the fourth (resp. third) exist and are continuous in \( \mathbb{R} \). Consider the usual \( 2n + 1 \) difference approximation \( A u = F \) on \( M \) such that \( A \) is a symmetric matrix. Let there be \( N \) mesh points in \( R \) and let \( h \) be the maximum mesh spacing in \( R \). Then \( \{N^{-1}\sum_{P \in M \cap R} [u(P) - U(P)]^2\}^{1/2} \leq ch^2 \) where the sum ranges over \( P \in M \cap R \) and where \( c \) is independent of the mesh except for dependence on quantities like the ratio of the largest to smallest mesh spacing in \( R \). The proof uses a finite difference analogue of the dual energy inequality for elliptic operators. The proof also holds if \( D \) is discontinuous at some mesh hyperplanes provided \( u \) and \( D \partial u/\partial n \) are continuous, \( n \) being the normal vector to the hyperplane. (Received December 5, 1960.)

576-109. Raoul Bott and Adam Korányi: Cauchy formulas for symmetric domains in several complex variables.

According to a theorem of E. Cartan and Harish-Chandra the irreducible bounded symmetric domains in complex \( n \)-space are exactly the homogeneous spaces \( D = G/K \) where \( G \) is simple noncompact, \( K \) is maximal compact in \( G \) and has a nondiscrete center. Our purpose is to study some elementary questions of function theory on symmetric domains using this algebraic characterization rather than the explicit representations of their various types. \( D \) has a natural imbedding into \( GU/K \) (\( GU \) a compact form of \( G \)); let \( 0 \leq t_1 \leq \ldots \leq t_r \leq 2 \) be the fundamental simplex of the symmetric space \( GU/K \), then the simplex \( S: 0 \leq t_1 \leq \ldots \leq t_r < 1 \) will be fundamental for \( D \) under \( K \). The boundary of \( D \) decomposes into \( r \) \( G \)-orbits; the orbits of the \( r \) vertices different from the
origin of $S$. Exactly one of these, the orbit of $t_1 = \ldots = t_r = 1$, is also a $K$-orbit; this is the Šilov-boundary of $D$. An analogue of the Cauchy formula can be established for the Šilov-boundary, or just as well for any $G$-orbit on the boundary of $D$. Restricted to $S$, the kernels for these formulas, and similarly the Bergman kernel function of $D$, are of the form $c \prod_{i=1}^{r} (1 - t_i)^{-\rho(H)}$ where $H$ is a certain element of the Lie algebra $\mathfrak{g}$ of $G$, $\rho$ is a sum of certain positive roots of $\mathfrak{g}$, and $c$ is a constant which can be computed explicitly; only $\rho$ and $c$ depend on the particular kernel we consider. In the classical case these expressions reduce to the formulae of Hua. (Received December 5, 1960.)

576-110. C. E. Langenhop: **A Markovian walk on a plane grid.**

Preliminary report.

A particle, starting from the origin, moves on a rectangular plane grid, each step depending on the previous step so that at the grid points the probability of a turn through 90$^\circ$ degrees is $p_k$, $k = 0,1,2,3$. Among others, formulas for the means and variances of the coordinates after $s$ steps are obtained, e.g. if the first step is in the positive $x$ direction, then $\bar{x}(s) = \sum_{n=0}^{s-1} \rho^n \cos n\theta$, where $\bar{x}(s)$ is the mean of the $x$ coordinate after $s$ steps, $\rho \cos \theta = p_0 - p_2$ and $\rho \sin \theta = p_1 - p_3$. The method involves the sum of a sequence of dependent random vectors, $V_n$, related to a Markov chain with transition matrix $P = (p_{ij})$, where $p_{ij} = p_{j-i}$, $i,j = 1,2,3,4$ and $j - i = k$, mod 4. (Received December 5, 1960.)

576-111. L. N. Mann: **Finite orbit structure on locally compact manifolds.** Preliminary report.

Montgomery raised the following conjecture. If a compact Lie group $G$ operates on a compact manifold $X$, then there are only a finite number of conjugate classes of isotropy subgroups $G_x$, $x \in X$. This conjecture was answered in the affirmative through the joint efforts of Floyd (Ann. of Math. vol. 65) and Mostow (Ann. of Math. vol. 65). A counterexample due to Montgomery exists for the conjecture when $X$ is not compact. In this paper the author shows, however, that the conjecture still holds when $X$ is an orientable cohomology manifold with finitely generated cohomology over the integers, $X$ not necessarily compact. The techniques used are based upon those of Floyd's paper (Ann. of Math. vol. 65) and the following useful proposition due to Frank Raymond (Pacific J. Math. vol. 10). Let $X$ be a locally compact Hausdorff space and $L$
a principal ideal domain. Then if \( X \) is of finite cohomology dimension over \( L \) and is clc over \( L \), the following two conditions are equivalent: (i) \( H^*_c(X;L) \) is finitely generated, (ii) \( \hat{X} \), the one-point compactification of \( X \), is clc over \( L \) at the point at infinity. Alexander-Spanier cohomology with compact supports is used in the above. (Received December 5, 1960.)


Suppose \( f \) is a function on a nondiscrete space \( X \) to a space \( Y \). Then \( f \) is continuous if and only if for every open set \( W \) in \( Y \), \( f^{-1}(W) \subset \left[ f^{-1}(Y - W) \right]' \), where prime denotes the derived set. A function \( f \) will be called pseudo-continuous if for every open set \( W \), \( f^{-1}(W) \subset \left[ f^{-1}(W) \right]' \). Let \( (X, \mathcal{U}) \) be the reals with the usual topology or an interval with the relative topology. Denote by \( \mathcal{F} \) the family of all subsets of \( X \) which have positive outer Lebesgue measure. Then \( (X, \mathcal{U}) \) and \( \mathcal{F} \) define a topology \( \mathcal{V} \) for \( X \) which is finer than \( \mathcal{U} \). [See Abstract 573-17, Notices Amer. Math. Soc. vol. 7 (1960) p. 728]. Theorem. Every derivative function is \( \mathcal{V} \)-pseudo-continuous. Let \( \mathcal{W} = \{ W : \text{for all } x \in W, D^*(x, X - W) = 0 \} \), where \( D^*(x, X - W) \) denotes the outer metric density of \( X - W \) at \( x \). Then \( \mathcal{W} \) is a topology for \( X \) which is finer than \( \mathcal{V} \). Theorem. A necessary and sufficient condition that \( f \) be \( \mathcal{W} \)-continuous is that \( f \) be approximately continuous. Corollary. Every bounded \( \mathcal{W} \) continuous function is a derivative function. Corollary. A function is measurable if and only if it is \( \mathcal{W} \) continuous except for a \( \mathcal{W} \)-discrete set. Corollary. Every derivative function is \( \mathcal{W} \) continuous except for a \( \mathcal{W} \)-discrete set. (Received December 5, 1960.)


A parallelohedron is a convex polyhedron in real affine 3-space which can be repeated by translation to fill the whole space without interstices. It has centrally symmetrical faces (Minkowski, Ges. Math. Abt. vol. 2) and hence is centrally symmetrical. (This theorem is due to Alexandroff; see Burckhardt, Vierteljschr. Naturf. Ges. Zurich vol. 85 (1940).) Furthermore it has at most 14 faces (Voronoi, J. Reine Angew. Math. vol. 134 (1908) p. 278). Let \( F_i \) denote the number of faces each having exactly \( i \) edges, \( V_i \) the number of vertices each incident with \( i \) edges, \( E \) the number of edges, \( n \) the number of sets of parallel edges, \( P = \sum F_i \), \( V = \sum V_i \). Then \( \sum iV_i = \sum iF_i = 2E \), \( V - E + F = 2 \)
and $\sum k(k - 1)F_{2k} = 2n(n - 1)$ (Coxeter, *Regular polytopes*, p. 29). These relations are used to determine the five possible parallelohedra, namely the parallelepiped, hexagonal prism, rhombic dodecahedron, elongated dodecahedron and truncated octahedron (cf. Federov, *Mineralogicheskoе Obshchestvo*, Leningrad, vol. 214 (1885)). (Received December 5, 1960.)

576-114. Morio Obata: *Conformal transformations of compact Riemannian manifolds.*

Let $(M, g)$ and $(M', g')$ be connected compact Riemannian manifolds with scalar curvatures $K_g$ and $K_{g'}$ respectively. We assume that $K_g$ and $K_{g'}$ are everywhere nonpositive. Let $f$ be a conformal transformation of $(M, g)$ onto $(M', g')$ with $f^* g' = e^{2\phi} g$, where $\phi$ is a scalar on $M$. Then we prove the following: (1) $f$ is homothetic if and only if there exists a constant $k$ such that $f^* K_{g'} = e^{-k} K_g$. (2) $f$ is an isometry if and only if $f^* K_{g'} = K_g$. (3) If $(M, g)$ has a nonpositive constant scalar curvature $K_g$, every conformal transformation of $(M, g)$ onto itself is an isometry. (Received December 5, 1960.)


A theory is developed which covers several known results about iteration procedures in partially ordered spaces as well as in (generalized) metric spaces. Moreover the theory yields a generalization of error estimates (proved by using fixed point theorems) for equations $(* ) Au = Bu$. Let $N_1, N_2$ be partially ordered spaces and suppose that each element $u$ of a set $R$ is minorized majorized in some sense by an element $\alpha_1(u) \in N_1 \{f_2(u) \in N_2\}$. Suppose further that $A$ and $B$ are minorized and majorized by operators in the $N_1$ and $N_1 \times N_1$ respectively. Using suitable assumptions one gets error estimates for a given approximate solution $u_0$ of $(* )$ in terms of elements $\in N_1$ starting with the residual $-Au_0 + Bu_0$. Moreover the solution is enclosed between the approximations of an iteration procedure. A special case is $N_1 = R$, $\alpha_1(u) = u$. If the $N_1$ are linear and a "lower \{upper\} distance" $\delta_1(u, v) \in N_1 \{\delta_2(u, v) \in N_2\}$ is associated to each pair $u, v \in R$, one can use $\rho_1(u) = \delta_1(u, u_0)$. Starting with more restrictive assumptions including bounds of the $\delta_1(Bu, Bv)$ one obtains existence and uniqueness theorems concerning the special equation $u = Bu$ as well as error estimates and convergence theorems for the iteration procedure $u_{n+1} = Bu_n$. (Received December 5, 1960.)

The separation equations of classical field theory in 40 coordinate systems can be expressed as nine special cases of Bocher equations [Moon and Spencer, Field theory for engineers, Princeton, New Jersey, D. Van Nostrand Company, 1961]. Solutions of three of these equations are well known. The other six are the Bessel wave equation, the Baer wave equation, the Legendre wave equation, the Lamé wave equation, the Wangerin equation, and the Heine equation. Methods of solving differential equations in terms of convergent infinite series have been developed by Frobenius [J. Reine Angew. Math. vol. 76 (1873) p. 214] and Fuchs [J. Reine Angew. Math. vol. 68 (1868) p. 354] and others. The present paper presents a systematic way in which the general term of the series can be written directly in determinant form and applies the method to obtaining the series solutions of the six unfamiliar Bocher equations. (Received December 5, 1960.)


A nilmanifold (solvmanifold) is the quotient of a nilpotent (solvable) Lie group by a closed subgroup. A theorem of A. I. Malcev (On a class of homogeneous spaces, Izvestiya Akad. Nauk SSSR vol. 13 (1949) pp. 9-32; Amer. Math. Soc. Translation No. 39 (1951)) states that any nilmanifold can be expressed as the quotient of a nilpotent Lie group by a discrete subgroup. From this it follows easily that any nilmanifold is parallelizable. H. C. Wang (Discrete subgroups of solvable Lie groups. I, Ann. of Math. vol. 64 (1956) pp. 1-19) has shown that the analogue of Malcev's theorem for solvmanifolds is false (even for compact solvmanifolds). The purpose of this paper is to prove that, in spite of this difficulty, compact solvmanifolds are parallelizable. (Received December 5, 1960.)


It has long been an open question as to whether those standard gambling games which are not repeated independent trials admit strategies favorable to the player. We present a strategy for the widely played casino game of blackjack, or twenty-one, which gives the player an appreciable positive expectation
The situation, using our strategy, is closely approximated by a favorable coin-toss game (see Feller) with \( p = 0.515, \) \( q = 0.485, \) and an effective betting rate of 5-15 bets/hr. In such a game, a player with 80 units of capital can, on the average, double his fortune in about 300 hours, with a ruin probability of less than 1 percent. The trials in blackjack are dependent on the outcome of previous trials. The strategy involves making large bets in favorable situations and "waiting" by making small bets otherwise. If \( M \) is the size of a large bet and \( m \) is the size of a minimum bet, the player's expectation \( E, \) approaches \( 0.0329 \) as \( M/m \rightarrow \infty. \) However the approach is rapid: \( M/m = 25 \) gives \( E = 0.030. \) Thus, with a ruin probability of less than 1 percent, \$3200 \) (unit size \$40) will provide a "living wage" of \$10/hr. (Received December 5, 1960.)


The following has applications in economics and statistics. Given positive integers \( m \) and \( n \) and non-negative numbers \( b_i, u_{ij}, i = 1,...,m, \) \( j = 1,...,n, \) such that \( b_i > 0 \) all \( i, \) \( \sum_i b_i = 1, \) each row and each column of the matrix \( [u_{ij}] \) has at least one positive element. One wishes to find non-negative numbers \( \pi_j, \beta_{ij} \) such that \( \sum_i \beta_{ij} = \pi_j \) all \( j, \) \( \sum_j \beta_{ij} = b_i \) all \( i, \beta_{ij}u_{ik}\pi_k \geq \beta_{ij}u_{ik}\pi_j \) all \( i, j, k. \) It is known that a solution exists with the \( \pi_j \)'s being unique. We give a method whereby a solution is obtained by means of a finite sequence of iterations each iterative step being equivalent to solving a special linear programming problem. The iteration is "globally stable" in the sense that the unique equilibrium \( \pi_j \)'s will be obtained from any initial \( (\pi_1,...,\pi_n) \) in the \( (n+1) \)-dimensional unit simplex. (Received December 7, 1960.)

576-120. G. L. MacLane: Meromorphic functions with small characteristic and no asymptotic values.

By the results of Fatou and Nevanlinna any function \( F(z), \) meromorphic in \(|z| < 1, \) with bounded characteristic has radial limits almost everywhere. Lohwater and Piranian (Ann. Acad. Sci. Fenn. A I 239, 1957) constructed an \( F(z) \) without radial limits and such that \( T(r) = 0(- \log(1 - r)). \) The object of the present note is to prove: let \( p(r) \) be a given function in \([0,1), 0 < p(r) \uparrow \infty. \) Then there exists a function \( F(z), \) meromorphic in \(|z| < 1, \) such that \( T(r,F) \leq p(r) \) and such that if \( C \) is any continuous curve in \(|z| < 1, \) tending to \(|z| = 1, \) then
the image of \( C \) by \( F \) is dense on the sphere. In particular then, \( F(z) \) has no radial limits. The construction is of the form \( F(z) = \phi(f(z)) \) where: (1) \( f(z) \) is holomorphic in \( |z| < 1 \), of slow growth, and assumes arbitrarily large values on any curve tending to \( |z| = 1 \). \( f \) is constructed as a gap Taylor series, similar to the example of Lusin and Priwaloff (Ann, École Norm. Sup. (1925) pp. 147-150), (2) \( \psi(z) \) is meromorphic in \( |z| \to \infty \), of known growth, and such that the image by \( \psi \) of any unbounded curve in \( |z| < \infty \) is dense on the sphere. \( \psi \) is constructed by specifying the Riemann surface of its inverse as a covering of the sphere. (Received December 7, 1960.)


Marshall Hall (Proc. Amer. Math. Soc. vol. 7 (1956) pp. 975-986) has listed 12 sets of parameters \( v, k, \lambda, n \) which satisfy \( k(k - 1) = \lambda(v - 1) \), \( n = k - \lambda \) for which the question of the existence of a difference set remains open. Three of these sets belong to the class \( v = 4n \), which is the class of cyclic Hadamard matrices. Of the 12 sets of parameters, 11 have \( (v,n) > 1 \). It is shown that for 9 of the latter no (cyclic) difference sets exist. Also, for \( n \)-prime power (and others) cyclic Hadamard matrices do not exist. (Received December 7, 1960.)


Consider the Laplace difference equation in a rectangle with \( q - 1 = m \cdot k \) horizontal lines and \( p - 1 \) points on a line. We consider a \( k \)-line iterative scheme. Let \( \lambda_R(k), \lambda_L(k) \) and \( \lambda_E(k) \) be the dominant eigenvalues of the simultaneous-displacement (Richardson) method, successive-displacement (Liebmann) method and over-relaxation, successive-displacement (Extrapolated Liebmann) method respectively. Since \( \lambda_L = \lambda_R^2 \) and \( (\lambda_E + \omega - 1)^2 = \omega^2 \lambda_R^2 \lambda_E \), we need only estimate \( \lambda_R(k) \). One finds \( \lambda_R \gtrsim 1 - k/2y \Theta \) as an asymptotic lower bound (for notation see Parter: On two-line iterative methods for the Laplace and biharmonic difference equations, Numerische Mathematik 1, 240-252 (1959)). This estimate is exact for \( k = 1,2,3 \). This estimate for \( k = 3 \) is new. This result generalizes to more general elliptic difference equations in general domains. (Received December 7, 1960.)
576-123. Simon Hellerstein: **Meromorphic functions with sectors free of zeros and poles.**

Let \( f(z) \) be a meromorphic function which is not a polynomial. Assume that all the zeros and poles of \( f \) lie on the real axis. Let \( \epsilon > 0 \) be given and denote by \( n_\epsilon(r,k) \) the number of zeros of \( f^{(k)}(z) \) (taking multiplicities into account) which lie in the disk \( |z| \leq r \) and outside the angles \( -\epsilon < \arg z < \epsilon \); \( \pi - \epsilon < \arg z < \pi + \epsilon \). Then for functions of finite order and \( r \) sufficiently large, \( n_\epsilon(r,k) < (K/\epsilon)r \), where \( K \) depends only on \( f \) and \( k \). For functions of infinite order \( n_\epsilon(r,k) < (K/\epsilon)r^2 \log r \log T(r,f) \), provided \( r \) avoids the values of an exceptional set of finite measure, \( T(r,f) \) is the Nevanlinna characteristic of \( f \). The theorem is a consequence of a more general one for meromorphic functions with one sector \( S \) free of zeros and poles. Here again it is possible to prove that a sector interior to \( S \) contains few zeros of the successive derivatives of \( f \). (Received December 7, 1960.)

576-124. Neill McShane: **Involutions on annihilator algebras.**

Preliminary report.

Let \( A \) be a simple \( H^* \)-algebra or a two-sided ideal in the algebra of continuous linear operators over a Hilbert space. In the second case let \( A \) be a Banach algebra under some norm. Examples are the compact operators, the Schmidt class, and the trace class. For any involution \( ^t \) on \( A \) the following statements are equivalent. (1) \( ^t \) is proper. (2) \( ^t \) is symmetric. (3) \( ^t \) is Hermitian real. (4) \( ^t \) is regular. (5) If \( ^* \) is the usual operator adjoint over Hilbert space and if \( A \) is an ideal in the algebra of bounded operators over Hilbert space and a Banach algebra then there exists a positive self-adjoint operator \( U \) both continuous and linear such that \( x^t = Ux^*U^{-1} \). Statement (5) can be used to show that if \( ^t \) is an involution on such an ideal then \( ^t \) can be extended to the algebra of all bounded linear operators on the Hilbert space if it satisfies any of them on the ideal. Let \( C \) be a semi-simple complex Banach algebra with a continuous Hermitian bilinear form \( (, ) \) and an involution \( * \) such that \( (xy,z) = (y,x^*z) = (x,zy*) \). Assume every continuous linear functional on \( C \) is of the form \( f(x) = (x,y) \) for some \( y \in C \). Then \( C \) is the topological direct sum of simple \( H^* \)-algebras. Positive definiteness of \( (, ) \) is not used. Continuity of the involution is proved. (Received December 7, 1960.)
A function $f(x,y)$ defined on $E^n \times [0,Y] = E^n \times T$ is said to belong to $L^p,\kappa(E^n \times T)$ if $\int \int |f(x,y)|^p dx dy \leq \infty$, where $\varphi(t) = \int \int |f(x,y)|^p dx dy$. With the norm $\|f\|_{L^p,\kappa} = \left(\int \int |f|^p dx dy\right)^{1/\kappa}$, $L^p,\kappa$ is known to be a Banach space. We consider the initial value problem (*) $\Delta u = \Delta u + \sum a_i(x,y)u_{x_i} + a(x,y)u - u = 0$ in $E^n \times (0,Y]$, $u(x,0) = u_0(x)$ in $E^n$, where $a_i \in L^p,\kappa$, $a \in L^q,\lambda$, and $u_0 \in L^p(E^n) \cap L^q(T)$. $u = u(x,y)$ is said to be a generalized solution of (*) if $u \in L^q,\lambda$, and there exists a sequence $u_N(x,y) \in C(E^n \times T) \cap \{C^2(E^n) \times C^1(T)\}$ such that (i) $\|u_N - u\|_{q,\lambda} \to 0$, (ii) $\|u_N \|_{L^p,\kappa} \to 0$ as $M,N \to \infty$, (ii) $\|u_N(x,0) - u_0(x)\|_{q,\lambda} \to 0$, (iv) $\|\Delta u_N\|_{1,1} \to 0$. The following results are proved. Theorem 1. If $n/2p + 1/\kappa < 1/2$ and $n/2q + 1/\lambda < 1$, then there exists a unique generalized solution of (*). Theorem 2. If, in addition, $\kappa = \lambda$ and $u_0 \in L^\infty(E^n)$, then the generalized solution of (*) has first partial derivatives with respect to $x$ in $E^n \times (0,Y]$. These results can be extended to parabolic equations of arbitrary order and to systems, in case the coefficients of the highest order $x$-derivatives are constant or sufficiently smooth. (Received December 7, 1960.)

Ross Finney: Point-like, simplicial mappings of a threesphere.

A subset of a three-sphere is point-like if its complement in the sphere is homeomorphic to Euclidean three-space. A mapping of the three-sphere onto a topological space, $T$, is point-like if the inverse-image of each point of $T$ is a point-like set. Theorem. Let $M$ be a triangulated three-sphere, and let $T$ be a triangulated topological space. If there exists a point-like, simplicial mapping of $M$ onto $T$, then $T$ is a three-sphere. (Received December 7, 1960.)

Morton Brown: Locally flat embeddings of topological manifolds.

A compact $n-1$ manifold $M^{n-1}$ (without boundary) embedded in an $n$ manifold $N^n$ is locally flat if for each point $p \in M^{n-1}$ there is an open neighborhood (relative to $N^n$) $U_p$ and a homeomorphism $h_p: U_p \to E^n$ such that $h_p(U_p \cap M^{n-1})$ lies in the hyperplane $E^{n-1} \subset E^n$. Theorem 1: Let $M^{n-1}$ be two sided and locally flat in $N^n$. Then there is a homeomorphism $h: M^{n-1} \times [-1,1] \to N^n$. Theorem 2: If $M^{n-1}$ is locally flat in $N^n$ and $u_0 \in C^\infty(N^n)$, then there exists a unique locally flat embedding of $M^{n-1}$ in $N^n$ that satisfies $u(x,0,y) = u_0(x)$.
Theorem 2: Suppose the topological $n-1$ sphere $S^{n-1}$ is locally flat in $E^n$. Then there is a homeomorphism of $E^n$ upon itself carrying $S^{n-1}$ onto the unit $n-1$ sphere of $E^n$. Theorem 3: (The formulation of this theorem was suggested to me by E. A. Michael.) Let $M^{n-1}$ be the boundary of a compact manifold (with boundary) $N^n$. Then there is a homeomorphism $h: M^{n-1} \times [0,1] \to N^n$ such that $h|M^{n-1} \times 0 = 1$. Remarks: The proof of Theorem 1 and Theorem 3 are similar. Actually, Theorem 1 can be derived from Theorem 3. Theorem 2 is a consequence of Theorem 1 and the Generalized Schoenflies Theorem (M. Brown, Bull. Amer. Math. Soc. vol. 66 (1960) pp. 74-76) and considerably strengthens the latter theorem. (Received December 7, 1960.)


Theorems 3 and 1 of the preceding abstract will be generalized. Call a subset $A$ of a metric space $X$ **collared in $X$** if there exists a homeomorphism $h$ from $A \times [0,1]$ onto a neighborhood of $A$ in $X$ with $h(a,0) = a$ for $a$ in $A$; call $A$ **locally collared in $X$** if each $a$ in $A$ has a collared $A$-neighborhood. (Example: The boundary of a topological manifold with boundary is, by definition, locally collared.) **Theorem 1.** $A$ locally collared subset of a metric space is collared.

Now define (locally) bi-collared by replacing $[0,1]$ in the definition of (locally) collared by $[-1,1]$. (Example: If $A$ is an $(n-1)$-manifold in a topological $n$-manifold $X$, then $A$ is locally bi-collared in $X$ iff it is locally flat in the sense of the preceding abstract). **Theorem 2.** If $A$ is a locally bi-collared subset of a metric space $X$, then there exists a double covering $f: \bar{A} \to A$, and a homeomorphism $h$ from the mapping cylinder $M_f$ of $f$ onto a neighborhood of $A$ in $X$, such that (considering $A \subseteq M_f$) $h(a) = a$ for $a$ in $A$. If moreover $A$ is a two-sided $(n-1)$-manifold in a topological $n$-manifold $X$, then $f$ is the trivial double cover and $(M_f,A) \simeq (A \times [-1,1], A \times \{0\})$, so that $A$ is bi-collared in $X$. (Received December 7, 1960.)

Suppose $S$ is a sequence such that, $S_n = (1 - g_{n-1})g_n$, $0 < g_{n-1} < 1$, and $P$ an increasing sequence of positive integers such that $P_1 > 1$ and $P_{n+1} - P_n \geq 2$. **Theorem 1.** The continued fraction $1 + a_1/1 + a_2/1 + \ldots$ converges if (1) $|a_p| \leq S_p$, for $p$ not in $P$, (2) either $|a_p| \leq S_p$ or $|a_p| \leq (1 + g_{p-1})(2 - g_p)$, for $p$ in $P$ and (3) either $P$ is finite or, there exists an infinite subsequence $Q$ of $P$ such that, if $Q_m$ is $P_{n+1}$, then $P_{n+1} - P_n \geq 3$ and $\prod (k$ in $Q) |a_{k-1}/S_{k-1}| = 0$. **Theorem 2.** If $u_p = g_p/(1 - g_p)$ for $p$ not in $P$ and $u_p = (2 - g_p)/(l - g_p)$ for $p$ in $P$, then the continued fraction converges absolutely if (1) and (2) of Theorem 1 hold and, for each positive integer $n$, $u_n + u_nu_{n+1} + u_nu_{n+1}u_{n+2} + \ldots$ converges. (Received December 7, 1960.)

576-130. R. J. Nunke: Direct products of copies of the integers.

Let $P$ be the direct product of countably many copies of $Z$ (the additive group of the integers) and let $S$ be the direct sum. (1) If $f$ is an endomorphism of $P$, then the kernel and image of $f$ are direct products of copies of $Z$ and the kernel of $f$ is a direct summand of $P$. (2) If $P$ is given the cartesian product topology with $Z$ discrete and if $A$ is a closed subgroup of $P$, then $A$ is a product of copies of $Z$. (3) If $A$ is a subgroup of $P$ of infinite rank then $A$ is isomorphic to a subgroup of $P$ containing $S$. (4) If $G$ is a direct product of copies of $Z$ with the number of factors less than the first Ulam measurable cardinal and if $f$ is an endomorphism of $G$, then the kernel of $f$ is a direct summand of $G$ if and only if it is a product of copies of $Z$. (5) The group $S$ can be imbedded in the direct product $G$ of continuum many copies of $Z$ so as to be the kernel of an endomorphism of $G$. (Received December 7, 1960.)


Some considerations concerning finite differences yield the following theorem: If $f$ is a continuous function from $[0,1)$ to a number set, then the following two statements are equivalent: (A) If $x$ is in $[0,1)$, there is a number $M$ such that $\sum_{i=0}^{n} (-1)^{i} f(x/n) \leq M$ if $n = 2^k$ for some non-negative integer $k$, and (B) there is a number sequence $c_0, c_1, c_2, \ldots$, such that if $x$ is in $[0,1)$, $f(x) = c_0 + c_1 x + c_2 x^2 + \ldots$. (Received December 8, 1960.)

Denote the $\phi$-subgroup of a group $G$ by $\phi(G)$, and the lattice of subgroups of $G$ by $L(G)$. Group shall mean finite group in all that follows. Theorem 1. $G$ is a non-abelian group of exponent $p$ iff $L(G)$ satisfies the following: (i) $L(G)$ is lower semi-modular; (ii) all intervals of $L(G)$ are irreducible; (iii) an ideal of dimension 2 exists in $L(G)$ and every ideal of dimension 2 has $p+1$ atoms; and (iv) $L(G)$ is nonmodular. (i) and (ii) can be replaced by the condition, (v) $L(G/\phi(G))$ is a finite projective geometry over $GF(p)$. (iv) can be replaced by the condition, (iv)' $\phi(G) \neq I$. Theorem 2. If $L(G) \not\simeq L(H)$ and $G$ is a class $c$ ($c \geq 2$), $k$-step metabelian group of exponent $p$, then $H$ is also $k$-step metabelian (with the same value for $k$) of exponent $p$ and of the same class $c$; and the center and commutator subgroup of every subgroup of $G$ is mapped onto the center and commutator subgroup respectively of the corresponding subgroup of $H$. Theorem 3. If $G$ is a class 2 group of exponent $p$ with fewer than five independent generators, then $L(G) \not\simeq L(H)$ iff $G \not\simeq H$. (Received December 7, 1960.)


The equations of the title are the "energy" type equations of Fluid Mechanics $\nu_r \partial t/\partial r + (1/r)\nu_\theta \partial t/\partial \theta = K \nabla^2 t$ where $\nu_r$, $\nu_\theta$ are velocity components. Results have been obtained in several coordinate systems. In polar coordinates the following typical theorem has been proved and applied to several problems: Let $\nu_r$, $\nu_\theta$ belong to the admissable family "B" characterized by the properties (a) "Exactness" $\partial \nu_r/\partial \theta = \partial (\nu_\theta)/\partial r$ and (b) $r^2[\nu_r^2 + \nu_\theta^2] = p(\theta) + g(r)$. Then the "energy" equation under the transformation $t = Cr^{-1/2}\{\exp \{-(1/2K)\phi(r,\theta)\} \}$ reduces to the separable equation in $S$, $S[r^2/4K)(\nu_r^2 + \nu_\theta^2) - K/4] = K [r^2 \partial^2 S/\partial r^2 + \partial^2 S/\partial \theta^2]$ where $\phi(r,\theta)$ is the velocity potential. Application is made to several problems of heat transfer in fluids. (Received December 7, 1960.)
576-134. Oscar Goldman: Determinants in projective modules.

Let \( R \) be a commutative ring (with unit element), \( E \) a finitely generated projective \( R \)-module and \( \alpha \) an endomorphism of \( E \). By imbedding \( E \) as a direct summand of a finitely generated free module \( F \) and extending \( \alpha \) suitably to an endomorphism \( \alpha' \) of \( F \), the determinant of \( \alpha' \) does not depend on \( F \), and its value is defined to be \( \det \alpha \). The map \( \det: \text{Hom}_R(E,E) \to R \) has most of the usual properties of determinants in free modules. Extend \( R \) to \( R[x] \) and \( E \) to \( E \otimes R[x] \); then \( \phi(x) = \det(x - \alpha) \in R[x] \) is defined to be the characteristic polynomial of \( \alpha \). In general, \( \phi(\alpha,x) \) is not monic, but its leading coefficient and its degree depend only on \( E \) and not on \( \alpha \). The polynomial \( \phi(0,x) \) has several interesting properties: \( \phi(0,x) = \sum e_i x_i \) with \( e_i e_j = \delta_{ij} e_i \) and \( \sum e_i = 1 \). (Every polynomial of this type can occur.) If \( \mathfrak{p} \) is a prime ideal of \( R \), then exactly one of the coefficients of \( \phi(0,x) \) can fail to be in \( \mathfrak{p} \), and if \( e_i \in \mathfrak{p} \) then \( E \otimes R_{\mathfrak{p}} \) is a free \( R \)-module of rank exactly \( i \). Finally the annihilator of \( \bigwedge^{i+1} E \) is generated by \( e_0 + e_1 + \ldots + e_i \) (Received December 7, 1960.)

576-135. Taqdir Husain and Mark Mahowald: \( B(\mathcal{C}) \)-property and the open mapping theorem.

Let \( \mathcal{C} \) be a class of locally convex topological vector (l.c.) spaces. An l.c. space \( E \) is said to have the \( B(\mathcal{C}) \)-property if a linear, continuous, one-to-one, and almost open mapping \( f \) of \( E \) onto \( F \), for each l.c. space \( F \in \mathcal{C} \), implies \( f \) is open. In case \( \mathcal{C} \) represents the class \( \mathcal{F} \) (or \( \mathcal{J} \)) of all Fréchet spaces (or \( t \)-spaces), the \( B(\mathcal{C}) \)-property can be denoted by \( B(\mathcal{F}) \) (or \( B(\mathcal{J}) \)). It is known that for an l.c. space \( E \) to have the \( B(\mathcal{F})(\text{or } B(\mathcal{J})) \)-property it is sufficient that \( E \) be a Fréchet (or B-complete) space e.g. see Théorie des opérations linéaires by Banach, Chelsea Publishing Company, page 41 and On the closed graph theorem by Robertsons, Proc. Glasgow Math. Assoc. vol. 3 (1956) pp. 9-12. It is shown here that for \( E \) to have the \( B(\mathcal{F}) \) or \( B(\mathcal{J}) \)-property it is not necessary that \( E \) be a Fréchet or B-complete space respectively. Further, let \( \mathcal{C} \) denote the class of all strict inductive limits of l.c. spaces \( F_n \) (\( n \geq 1 \)) such that \( F_n \) is closed in \( F_{n+1} \) and \( F_n \in \mathcal{C} \) for each \( n \). It is shown that an l.c. space \( E \) has the \( B(\mathcal{C}) \)-property whenever each closed subspace of \( E \) has the \( B(\mathcal{C}) \)-property. (Received December 7, 1960.)

In a rod of two different materials the following equations may hold:

\[ u_t = f_1(x, t, u, u_x, u_{xx}) \text{ for } 0 < x < e, \ t > 0; \]
\[ u_t = f_2(x, t, u, u_x, u_{xx}) \text{ for } e < x < 1, \ t > 0; \]
\[ u(x, 0) = g(x) \text{ for } 0 \leq x \leq 1; \]
\[ u_x = a + b \text{ for } x = 0, \ t > 0; \]
\[ u_x = -c + d \text{ for } x = 1, \ t > 0; \]
\[ u(e, t) = u(e', t) \text{ for } t \geq 0; \]
\[ u_x(e, t) = u_x(e', t) \text{ for } t \geq 0; \]

The derivatives of \( f_1 \) and \( f_2 \) with respect to \( u_{xx} \) and the coefficients \( a, c, k_1 \) and \( k_2 \) are positive and bounded away from zero; the conditions match smoothly; the functions and coefficients which appear have enough continuous derivatives so that a unique solution \( u(x, t) \) with derivatives of "high enough" order exist.

The conventional explicit and fully implicit finite difference schemes have a local truncation error of order \( \Delta x^4 \) for \( x \neq 0, e, 1 \) while for \( x = 0, e, \) and 1 the error is of order \( \Delta x^3 \). Nevertheless, by means of a maximum principle argument the overall error for \( 0 \leq t \leq T \) is shown to be of order \( \Delta x^2 \).

(Received December 7, 1960.)

576-137. J. G. Stampfli: Roots of scalar-type operators.

Let \( T \) be a bounded linear operator on a Hilbert space \( H \), \( S \) a scalar-type operator on \( H \) with \( T^n = S; n \) a positive integer. Then (1) if \( S \) is invertible, \( T \) is a scalar-type operator, (2) if zero is an isolated point of the spectrum of \( S \), \( T \) is a spectral operator. (Received December 7, 1960.)


Alexandroff showed that if a connected metric space \( S \) is locally separable, then \( S \) is separable. Jones obtained the result that if a connected, locally connected, metric space \( S \) is locally peripherally separable, then \( S \) is separable. Treybig constructed an example of a connected, semi-locally-connected, metric space, which is locally peripherally separable, but which is not separable. This example affirms the distinction between the locally connected situation and the situation where local-connectedness is omitted. Recently Treybig and Roberts have gotten some results which complement Alexandroff's theorem. This paper develops an analogous idea in the direction of Jones' theorem in the locally connected situation. In this case it is shown that if no...
point separates a connected, locally connected, metric space $S$, and each pair of points in $S$ is separated by a separable closed set, then $S$ is separable.

(Received December 7, 1960.)


A square matrix $H$ of order $h$ is called a generalized Hadamard matrix if all of its elements are $p$th roots of unity and if $HH^T = hhI$. When $p = 2$, $H$ is an ordinary Hadamard matrix about which much is known, including the coexistence of certain Hadamard matrices, certain difference sets, and certain symmetric balanced incomplete block designs. In this paper many of these results are extended to the case where $p$ is an odd prime. For example, a method of constructing a generalized Hadamard matrix is given for $h = p^n$; and the resulting matrix is shown to induce a decomposition of the additive group $\mathbb{Z}_r$ of integers modulo $r = p^n - 1$ into $p$ sets, each of which is "almost" a difference set. (Received December 7, 1960.)

576-140. J. L. Alperin: $p$-automorphisms of solvable groups.

Theorem. If a finite solvable group $G$ possesses an automorphism of prime order $p$, leaving fixed exactly $p^n$ elements, then there exists a bound for the derived length of $G$ determined by $p$ and $n$. This is proven first for the case of $G$ a $p$-group by a combination of G. Higman's Lie ring methods and the Lemma. If a finite $p$-group $G$ possesses an automorphism of order $p$ having at most $p^n$ fixed elements, then every normal subgroup of $G$ can be generated by $np$ elements. The theorem is then proven in the general case by use of the upper $p$-series of $G$. As a consequence of the theorem one may obtain the Corollary. If $G$ is a $p$-group of maximal class then there is a bound for the derived length of $G$ determined solely by $p$. (Received December 8, 1960.)

576-141. R. A. Askey: Mean convergence of orthogonal series.

The results of the present paper hold for a class of orthogonal polynomials that generalize the Jacobi polynomials. However, for simplicity we state the results for Legendre polynomials. Let $P_n(x)$, $n = 0, 1, \ldots$, be the Legendre polynomials normalized in the usual fashion. For $f \in L^1 (-1,1)$ we
set \( f^\wedge(n) = \int_{-1}^{1} f(x) P_n(x) \, dx \). The formal development of \( f(x) \) as a series of Legendre polynomials is then \( \sum_{n=0}^{\infty} \left( (n + 1/2) f^\wedge(n) \right) P_n(x) \). Let \( \int_{-1}^{1} \left| f(x) \right|^p (1 - x^2)^{\frac{p}{2}} \, dx \) be finite. For \( a = 0 \) and \( 4/3 < p \leq 4 \) Pollard has shown that the series of partial sums converges in the mean of order \( p \) to \( f(x) \). We show that if \( 1 < p \leq 4/3 \), \( a < (3p - 4)/4p \), then the partial sums converge in the mean of order \( p \) to \( f(x) \). The conclusion is false if \( a = (3p - 4)/4p \). (Received December 8, 1960.)

576-142. R. Artzy: "Color group" graphs of binary systems.

Cayley diagrams (= color groups, Gruppenbilder) of general binary systems can be defined in the same way as for groups. Necessary and sufficient conditions are found for graphs to represent groupoids, semigroups, cancellative semigroups, quasigroups, loops and groups. These Cayley diagrams are shown to be useful in providing lucid alternative proofs for a number of facts about principal isotopes of commutative loops, inverse-property loops and Moufang loops. (Received December 8, 1960.)

576-143. J. D. Buckholtz: On the zeros of sections of the exponential series.

Theorem 1. Suppose \( \sum_{p=0}^{\infty} a_p z^p \) is a power series (not necessarily convergent if \( z \neq 0 \)). The following two statements are equivalent: (i) there is a positive number \( d \) such that for each \( n \), the disk \( |z| \leq nd \) contains no zero of \( \sum_{p=0}^{n} a_p z^p \); and (ii) \( \sum_{p=0}^{\infty} a_p z^p \) is the power series for \( a_0 \exp(a_1 z/a_0) \). As a result, the exponential series is completely determined by (i) and the condition that \( a_0 = a_1 = 1 \). Theorem 2. The largest positive number \( d \) such that for each \( n \), \( \sum_{p=0}^{n} z^p / p! \) has no zero in the disk \( |z| \leq nd \) has the property that \( de^d = 1/e \) and is the sum of the series \( \sum_{p=1}^{\infty} (-p)^p e^{-p} / p! \). (Received December 8, 1960.)

576-144. R. H. Bing: Point-like decompositions of \( E^3 \).

An upper semicontinuous decomposition \( G \) of \( E^n \) is point-like if for each element \( g \) of \( G \), \( E^n - g \) is topologically equivalent to the complement of a point. An example is given of a point-like decomposition \( G \) of \( E^3 \) such that \( G \) has only a countable number of nondegenerate elements but the decomposition space is
topologically different from $\mathbb{E}^3$. A property that distinguishes the decomposition space $G$ from $\mathbb{E}^3$ is that certain points do not have arbitrarily small neighborhoods whose boundaries are 2-spheres. Another point-like decomposition $H$ of $\mathbb{E}^3$ is described such that $H$ has a Cantor set of nondegenerate elements. Each element of $H$ is a straight segment. In fact, there are two parallel planes such that each element of $H$ has its ends in the two planes. Although it is not proved that the decomposition space $H$ is topologically different from $H$, various properties of $H$ are developed which cast doubt in this direction. (Received December 8, 1960.)


The Feynman integral is presented as the Fourier-Stieltjes transform of a new integral, the "Ilstow Integral", which is an abbreviation for "Inverse Laplace Stieltjes transform of Wiener's integral." The transformation is performed with respect to the reciprocal of the variance parameter which occurs in the definition of the Wiener integral. The Ilstow and Feynman integrability of certain types of functionals are established, and it is shown that under certain circumstances a solution to the Schroedinger equation can be expressed in terms of the Feynman integral. (Received December 8, 1960.)

576-146. Adam Czarnecki: Physical systems $S_k$ of curves in a central field of force.

This is an extension to central fields of force of some of the results contained in the paper by Kasner and De Cicco, Physical families in the gravitational field of force, Amer. Math. Monthly, 1951. Every trajectory of a physical system $S_k$ of a central field of force, is a plane curve whose plane passes through the center 0 of the field of force. The converse is that such a field of force is either central or parallel. A generalization is obtained of Kepler's second law of planetary motion, for a central physical system $S_k$ and for a central velocity system $S_\infty$. This result characterizes a central field of force, conservative or not. For the conservative case, the explicit equation of a central physical system $S_k$ including the case $k = \infty$, is found in terms of two quadratures. If a potential function $V$ of a conservative central field of force, is known, the explicit equation of such an $S_k$, is obtained in terms of a single
quadrature. The case where the potential \( V \) varies inversely as the \((n - 1)\) power of the radius vector \( r > 0 \), from the center \( 0 \), where \( n \neq 1 \), is studied in detail. Further amplifications of these results will be given in succeeding papers. (Received December 8, 1960.)

5 6-147. W. J. Coles: The cross-ratio property for the matrix Riccati equation.

Let \( A(t) \) be an \( n \times n \) matrix; let \( n_1 + \ldots + n_k = n \), and let \( A \) be partitioned into \( n_i \times n_j \) submatrices \( A_{ij} \) \((i,j = 1,\ldots,k)\). The system \( Y' = -YA_mY + AY \) is considered. A form for the general solution is obtained in terms of particular solutions and those of a certain linear system. The property reduces to the ordinary scalar cross-ratio property for \( k = 2, n_1 = n_2 = 1 \). (Received December 8, 1960.)


Let \( E_n(H_n) \) denote abstract Euclidean (unitary) \( n \)-space. The spin model of Euclidean 3-space consists of all self-adjoint linear transformations of \( H_2 \) into itself of trace 0 with the inner product \( (A,B)I = (AB + BA)/2 \). Then 2-component spinor analysis takes a particularly simple and transparent form. It turns out that, contrary to the usual statement in the physical literature, electron spin then appears as a nonrelativistic effect. (Received December 8, 1960.)

576-149. J. R. Edmonds: Symmetric embeddings of complete graphs.

A regular map, the topological analog of a regular polyhedron, as a surface has symmetrically embedded in it a symmetric graph consisting of the edges and vertices of the faces. The graph-embedding theorem (Abstract 572-1, Notices Amer. Math. Soc. vol. 7 (1960) p. 646) and its generalization to unoriented surfaces are used in constructing an efficient scheme to exhaust the symmetric embedding possibilities for each complete \( n + 1 \) graph, the graph of an \( n \) dimensional simplex. Among these maps are counter-examples to the conjecture of Coxeter and Moser (p. 102, Ergebnisse, 1957) that every regular map, except certain tori, has an automorphism which reflects the map about an edge. The simplest, a genus 7 embedding for \( n = 7 \), is given by \( \Lambda(0,1,2,3,4,5,6) \).
k(A,5 + k,3 + k, 2 + k,6 + k,1 + k, 4 + k) where addition is mod 7; each of the symbols, A and 0 through 6, corresponds to a vertex; \(X_0(X_1,\ldots,X_n)\) indicates how the edges \((X_0,X_i)\) are arranged cyclically around the vertex \(X_0\). The group of automorphisms, generated by \(R = (0123456)\) and \(S = (A042561)\), is defined by \(R^7 = S^7 = (RS)^2 = R^2S^{-1}RS^{-3} = 1\); whereas \(R^{-2}SR^{-1}S^3 \neq 1\). The complete 10 graph is one of the simplest symmetric graphs with no symmetric embedding. (Received December 8, 1960.)


If \(A\) is a lattice-ordered real Banach algebra with involution satisfying Bohnenblust's condition \(P\) relating norm and lattice properties (Duke Math. J. vol. 6 (1940) p. 627) and certain further conditions on regular maximal right ideals and their annihilators, then \(A\) is isomorphic and isometric, preserving the lattice structure and involution, to a certain type of closure of the direct sum of a class of algebras of all real, possibly infinite, matrices satisfying that \(|A_{i,j}|Pd_{ij}P < \infty\) for a value of \(1 \leq p \leq \infty\). The \(d_{ij}\) being a set of positive constants and \(p\) the same for each matrix algebra. A nearly complete converse to the representation is given and examples covering the remaining cases. The closure of the direct sum of matrix algebras consists of all sequences of elements chosen from the component matrix algebras, with at most a countable number of nonzero values satisfying the condition \((\sum_\alpha \|A(\alpha)\|P)^1/P < \infty\). This last serving as the norm for the sequence. The norm for the matrix coordinate \(\|A(\alpha)\| = (\sum_{i,j} |A_{i,j}(\alpha)|Pd_{ij})^{1/P}\). (Received December 8, 1960.)


Let \(A\) be the algebra of allowable functions defined in some neighborhood of a point, \(p\), of a \(C^r\) manifold \((0 \leq r \leq \omega; \text{or}\ r = \omega, \text{in case manifold is real analytic})\). Call a function in \(A\) stationary of the second order if all its first and second derivatives vanish at \(p\). A second order tangent vector is an element of the dual space, \((A/A_2)^*\), where \(A_2\) is the vector subspace of second order stationary functions. (Analogous to the definition of Newns and Walker, J. London Math. Soc. vol. 31 (1956) pp. 400-407.) Generalizing Chevalley, a second order derivation is a real linear map, \(L\), on \(A\) such that \(L(fgh) = f(p)L(gh) + g(p)L(fh)\)
+ h(p)L(fg) - g(p)h(p)L(f) - h(p)f(p)L(g) - f(p)g(p)L(h), for any \( f, g, h \) in \( A \). Then one readily verifies the following. (1) Any second order derivation on a \( C^0 \) manifold vanishes everywhere, but a \( C^1 \) manifold has nontrivial second order derivations. (2) On a \( C^{\infty} \) or \( C^{\omega} \) manifold, the notions of second order derivation and second order tangent vector coincide. (3) On a \( C^r \) manifold, \( 1 < r < \infty \), the space of second order tangent vectors is a proper subspace of the vector space of second order derivations. (4) The Lie bracket of a second order derivation with a first order derivation is a second order derivation. (Received December 6, 1960.)


Let \( X \) be a Banach space with real scalars. It is shown that a necessary and sufficient condition that a set \( Q^* \) of linear functionals of unit norm over \( X \) be the set of all norm preserving extensions of a functional \( f \) defined on a linear subspace \( M \subset X \) is that \( Q^* \) be a maximal convex subset of the surface of the unit sphere in \( X^* \) which lies in a weak * closed hyperplane variety in \( X^* \). In particular spaces, general conditions on linear subspaces are investigated which insure the uniqueness of norm preserving extensions of linear functionals defined over the subspaces. Theorems of Taylor [Duke Math. J. vol. 5 (1939)] Foguel [Proc. Amer. Math. Soc. vol. 9 (1958)] and Phelps [Trans. Amer. Math. Soc. vol. 95 (1960)] are shown to be special cases of the theorems developed here. (Received December 8, 1960.)

576-153. Murray Gerstenhaber: Dominance over the classical groups.

If \( G \) is a group variety operating on a variety \( V \), all defined over a field \( K \), and if \( P, Q \in V \), then we say that \( P \) dominates \( Q \) over \( G \) relative to \( K \) if there exists an \( x \in G \) such that \( Q \) is a specialization of \( Px \) relative to \( K \). It is shown that if \( G \) is the group of orthogonal or symplectic \( n \times n \) matrices (of arbitrary characteristic \( \neq 2 \)), \( V \) the corresponding Lie algebra and \( K \) arbitrary, then \( P \) dominates \( Q \) over \( G \) if and only if \( P \) dominates \( Q \) over the group of unimodular \( n \times n \) matrices. Simultaneously, certain nilpotent subalgebras (called "triangular") of the classical Lie algebras are studied and the similarity classes of their generic elements are determined as a step toward their classification. (Received December 8, 1960.)

For integers $k$ and $n$ satisfying $n \geq k \geq 0$, let $H(n,k)$ be the smallest integer with the following property: If $F$ is a finite family of convex sets in $E^R$, consisting of at least $H(n,k)$ members and such that the intersection of every $H(n,k)$ members of $F$ has dimension $k$ at least, then the intersection of all members of $F$ has dimension at least $k$. With this notation, Helly's theorem is $H(n,0) = n + 1$; a result of Klee (Amer. J. Math. vol. 75 (1953) pp. 178-188) implies $H(n,n) = n + 1$. From a theorem of Steinitz (see, e.g. Rademacher-Schoenberg, Canad. J. Math. vol. 2 (1950) pp. 245-256) it is easy to derive $H(n,1) = 2n$. Theorem. $H(n,k) = 2n - k$ for $n > k > 1$. In the proof use is made of a topological theorem due to Helly (Monatsh. Math. vol. 37 (1930) pp. 281-302). (Received December 8, 1960.)


Let $R$ be a simply connected region in $E_N$ with smooth bounding surface $S$. For a sufficiently differentiable set of functions we derive a class of quadratic integral identities relating surface integrals of derivatives to integrals over $R$. These identities are a generalization of a first order identity given by L. Hörmander (C. R. Douzième Congrès des Mathématiciens Scandinaves Tenu à Lund, 1953, pp. 105-115) and L. E. Payne and H. F. Weinberger (Pacific J. Math. (1958) pp. 551-573). As an example of an application of these identities we consider a solution $u$ of the boundary value problem $\Delta u - pu = F$ in $R$ and $u = f$ on $S$. Here $\Delta$ denotes the Laplace operator and $0 \leq p(x)$. We obtain pointwise a priori bounds for the derivatives of $u$ in $R$ in terms of a quadratic functional of an arbitrary function. Hence the Rayleigh-Ritz procedure can be used to make the error arbitrarily small. (Received December 8, 1960.)

576-156. V. L. Klee, Jr.: A question of Katětov concerning the Hilbert parallelootope.

At a recent conference in Warsaw, Professor Katětov asked whether one can define a notion of "dimensional deficiency" in purely topological terms, at least for the closed subsets of a compact metric space. Such a notion would be useful in applying dimension theory to infinite-dimensional compacta. He
suggested the following test-problem: Does the Hilbert parallelotope
\[ P = \{ x = (x_1, x_2, \ldots) \in \mathbb{R}^2 : \text{always } x^n \in [0, 1/n] \} \]
admit a self-homeomorphism which carries \( A \) onto \( B \), where \( A = \{ x \in P : x^1 = 0 \} \) and \( B = \{ x \in P : x^1 = x^2 = 0 \} \)?
Since the sets \( A \) and \( B \) are intuitively of different dimensional deficiency in \( P \),
the existence of such a homeomorphism would suggest nonexistence of the
notion referred to above. In the present note, such a homeomorphism is con­
structed by means of some mapping techniques originated by O. H. Keller and
developed further by the author. The following more general theorem is proved:
Suppose that (for \( i = a, b \)) \( K_i \) is a compact convex subset of a normed linear
space, \( H_i \) is a proper supporting hyperplane of \( K_i \), and \( C_i \) is an infinite-dimen­
sional closed convex subset of \( K_i \cap H_i \). Then there is a homeomorphism of \( K_a 
onto K_b \) which carries \( C_a \) onto \( C_b \). (Received December 8, 1960.)

Preliminary report.

The five Platonic solids, the thirteen Archimedean polyhedra, and the
infinite families of regular prisms and antiprisms are uniform in the sense
that they have regular faces and a symmetry group which is transitive on the
vertices. In addition to these there are numerous other convex polyhedra all
of whose faces are regular polygons. It is proved in this paper that, apart from
the prisms and antiprisms, there can be only a finite number of such solids.
The existence of nearly a hundred non-uniform convex polyhedra with regular
faces is known. It is interesting that, while many of these figures are quite
irregular, none have been found which are completely asymmetrical, i.e., whose
symmetry group consists of the identity alone. (Received December 8, 1960.)

576-158. R. E. Lane: On linear operator equations.

\[ QC_{0L} \]
is the set of quasi-continuous functions with value zero to the left
of the origin and everywhere continuous from the left; \( BV_{0L} \)
is the subset of
\[ QC_{0L} \]
whose members are of bounded variation in every interval. \( T_{0L} \)
is the
set of linear operators such that \( T \) is in \( T_{0L} \) if and only if there is in \( BV_{0L} \)
a function \( u \) such that \( Tf(s) = \int_0^Co f(s - t) du(t) \)
for each function \( f \) in \( QC_{0L} \) and
each real number \( s \). If \( f \) is in \( QC_{0L} \) and \( T \) is in \( T_{0L} \), then \( Tf \) is in \( QC_{0L} \).
There is in \( T_{0L} \) an identity transformation \( I \) such that \( If = f \) if \( f \) is in \( QC_{0L} \).
For a member \( T \) of \( T_{0L} \) to have an inverse it is necessary and sufficient that
\( T_{JL}(0^+) \neq 0 \), where \( J_L(t) = 0 \) if \( t \leq 0 \) and \( J_L(t) = 1 \) if \( t > 0 \). Hence if \( T_{JL}(0^+) \neq 1 \), then the operator \( I - T \) has an inverse \( V \); and if \( x \) is in \( QC_{0L} \) and \( y = Vx \), then \( y \) is a solution of the linear operator equation \( y = x + Ty \). Such an equation arises, for example, if \( x \) is the input to a physical system consisting of a linear device with feedback and \( y \) is the output. Numerical approximations for the inverse of a member of \( T_{0L} \) can be obtained by methods like those used for differential equations. (Received December 8, 1960.)

576-159. W. S. Loud: **Periodic solutions of an autonomous perturbed second-order equation.**

In the real autonomous differential equation (1) \( x'' + g(x,x') = ef(x,x',e) \) let \( g \in C^3, f \in C^2 \). Let (2) \( x'' + g(x,x') = 0 \) have the nonconstant periodic solution \( x = x_0(t) \), of least period \( L_0 \) with \( x_0(0) = 0 \). Let both characteristic multipliers of the related variation equation be equal to \( 1 \). Under certain conditions, a necessary condition being that \( \int_{0}^{L_0} x_0(t)f(x_0(t),x_0'(t),0) \exp \left[ \int_{0}^{s} g_x(x_0(s),x_0'(s)) \right] ds \) \( dt = 0 \). (1) has one or more periodic solutions of period \( L_0 + \tau(e) \) of the form \( x = x_0(s) + e x_1(s) + O(e) \) with \( x'(0) = 0 \) where \( s = tL_0 + \tau(e) \). Explicit formulas are given for \( \tau'(0) \) and \( x_1(0) \). In the simplest case \( x_1(0) \) is determined as a simple root of a certain algebraic equation. The existence of a simple root is, together with the above-mentioned conditions, sufficient for existence of the periodic solution. Cases are considered in which this equation has multiple roots, so that the results go beyond those of Malkin, *Some problems in the theory of nonlinear oscillations*, 1956. Stability of the periodic solutions found is determined. The methods depend on singular cases of the implicit function theorem. This work was done in part while the author was at the Mathematics Research Center, U. S. Army. (Received December 8, 1960.)

576-160. P. J. Laufer and M. L. Tomber: **Lie admissible algebras.**

Preliminary report.

Let \( A \) be an algebra and \( A^{(\cdot)} \) the algebra which is the same vector space as \( A \), but with multiplication given by \( [x,y] = xy - yx \), where \( xy \) denotes the product in \( A \). Following Albert (Trans. Amer. Math. Soc. vol. 64 (1948) pp. 552-593) we say that \( A \) is Lie admissible if \( A^{(\cdot)} \) is a Lie algebra, flexible if \( (xy)x = x(yx) \) and power associative if \( x^m x^n = x^{m+n} \). Theorem. Over an algebraically closed field \( \mathbb{F} \) of characteristic zero, if \( A \) is a flexible, power associa-
tive algebra and such that \( A(\cdot) \) is a simple Lie algebra, then \( A \) is a simple Lie algebra isomorphic to \( A(\cdot) \). The proof makes use of the "root system" technique due to Cartan. It is shown that if the "constants of structure" satisfy a certain equation, then \( A \) will be a simple Lie algebra. By extending the ground field \( \Phi \) to its algebraic closure \( \Omega \) and using the result that if \( A(\cdot) \) is a semi-simple Lie algebra then \( A \) is the direct sum of simple Lie algebras, we obtain the following: Corollary. Let \( A \) be a flexible, power associative algebra over \( \Phi \) and such that \( A(\cdot) \) is a semi-simple Lie algebra, then \( A \) is a Lie algebra.

(Received December 8, 1960.)

576-161. M. V. Subba Rao and V. C. Harris: Congruence properties of \( \sigma_r(n) \).

\( \sigma_r(n) \), the sum of the \( r \)th powers of the divisors of \( n \) can be split up into its "components", where a component of \( \sigma_r(m) \) is defined as \( d^r + d' \) with \( d \neq n \), \( 1 \leq d \leq n \), and \( d' \) if \( d = n \) is an integer. If every component of \( \sigma_r(n) \) is congruent to a \( (mod \ K) \), \( \sigma_r(n) \) is said to be componentwise congruent to a \( (mod \ K) \) and written \( \sigma_r(n) \equiv a (mod \ K) \). The following results are obtained:

I. If \( R, K, L \) are fixed positive integers with \( K \geq 3 \) and \( (L,K) = 1 \) and if \( a \) is a non-negative integer, the necessary and sufficient condition that \( \sigma_r(nK + L) \equiv a (mod \ K) \) for \( n = 0,1,2,\ldots \) is that \( a \) is a quadratic nonresidue of \( K \), \( b \) \( 1 + L \equiv a \ (mod \ K) \), \( c \) \( (w^r - 1)(w^r + 1 - a) \equiv 0 \ (mod \ K) \) for all \( w \) such that \( (w,K) = 1 \).

II. \( \sigma_1(nK + L) \equiv a (mod \ K) \) holds for suitable \( L \) and \( a \) and all \( n, n = 0,1,2,\ldots \)

if and only if \( K \) is one of the numbers \( 3,4,6,8,12,24 \).

III. \( \sigma_r(nK + L) \equiv a (mod \ K) \) and \( \sigma_r(nK + L) \equiv a (mod \ K) \) imply each other if and only if \( a = 0, n \) taking all non-negative integral values. (Received December 8, 1960.)


A chance device with probability distribution \( (q_0,\ldots,q_m) \) emits an integer \( S \) between 0 and \( m \). Player II, ignorant of \( S \), selects a real number \( y \) between 0 and \( n \). Player I, knowing only \( S + y \), wishes to identify \( S \); his payoff is 1 if successful, 0 otherwise. This zero-sum game is solved in the following cases:

1. \( q_i \)'s in geometric progression,
2. \( q_i \)'s in supergeometric progression and \( m = n \),
3. \( m = n = 2 \). (Received December 8, 1960.)
576-163. Herman Rubin: Convergence in measure and convergence of distributions on topological spaces.

Let \( X_\alpha \) be a net of random variables with values in a topological space and \( P_\alpha \) the induced distributions. Several definitions of convergence in measure and convergence of distributions are given and for some of these \( X_\alpha \rightarrow Y \) implies \( P_\alpha \rightarrow Q \), where \( Q \) is the distribution of \( Y \). Conversely, under suitable restrictions if \( R_\beta \rightarrow Q \) there is a net of random variables \( X_\alpha \rightarrow Y \) with \( R_\beta \) being a subnet of \( P_\alpha \) defined as above. If \( Q \) is real in the sense of A. D. Alexandroff, all the conditions are equivalent. In fact, if \( P_\alpha \rightarrow Q \) on Euclidean space, there exists, on a suitable probability space, \( X_\alpha \rightarrow Y \) with relations as above, where the convergence is almost everywhere. (Received December 8, 1960.)

576-164. J. D. Reid: On quasi-decompositions of torsion free abelian groups.

Notation and terminology are as in B. Jónsson, On direct decompositions of torsion free abelian groups, Math. Scand. vol. 7 (1959) pp. 361-371. Let \( V \) be a vector space over the rational number field \( \mathbb{Q} \), \( \mathcal{L}(V) \) the ring of linear transformations of \( V \) and \( G \) a subgroup of \( V \) such that \( V/G \) is a torsion group. Then \( E(G) = \{ \lambda \in \mathcal{L}(V) | n\lambda(G) \leq G \text{ for some } n \neq 0 \} \) is a ring and its units are called quasi-automorphisms of \( G \). It is assumed that \( E(G) \) has D.C.C. on left ideals. Theorem 1. Any decomposition \( G = \bigoplus \lambda A_\lambda \) with the \( A_\lambda \neq 0 \) has finitely many summands. There is a one-to-one correspondence between such decompositions of \( G \) and decompositions \( E(G) = \bigoplus \lambda L_\lambda \) of \( E(G) \) into nonzero left ideals. Corresponding decompositions have the same number of summands and, modulo notation, \( A_\lambda \) is strongly indecomposable if and only if \( L_\lambda \) is an indecomposable \( E(G) \)-module, Theorem 2. (Jónsson). Let \( \sum_{i=1}^{t} \lambda A_i \leq G \leq \sum_{j=1}^{s} B_j \) with the \( A_i \) and \( B_j \) strongly indecomposable. Then \( s = t \) and, modulo notation, \( A_i \cong B_i \) for all \( i \). Theorem 3. For any \( \lambda \in E(G) \) there is a decomposition \( G = A \oplus B \) such that \( \lambda \) induces a quasi-automorphism on \( A \) and \( \lambda \) is nilpotent on \( B \). (Received December 8, 1960.)
576-165. Marvin Rosenblum: **Self-adjoint Wiener-Hopf operators.**

Preliminary report.

Let P and K be the multiplication operators that multiply $L^2(-\infty, \infty)$ functions by the indicator function of $(0, \infty)$ and the essentially bounded function $k$ respectively. Suppose $F$ is the Fourier-Plancherel operator on $L^2(-\infty, \infty)$. Then $W = PF*KFP$, when considered as an operator $L^2(0, \infty)$, is the Wiener-Hopf operator associated with $k$. **Theorem 1.** $W$ is self-adjoint. Its spectral measure is weakly absolutely continuous with respect to Lebesgue measure.

**Theorem 2.** Suppose $k$ is even and strictly monotone on $(0, \infty)$. Then $W$ is unitarily equivalent to the multiplication operator $PKP$ on $L^2(0, \infty)$. (Received December 8, 1960.)

576-166. Berthold Schweizer and Abe Sklar: **On the algebra of functions.**

In a recent paper ([The algebra of functions; Math. Ann. vol. 139 (1960) pp. 366-382;](#) the notation used in the present abstract is that of this paper) we have studied an axiomatic system designed to describe, in an abstract setting, the properties of ordinary functions under their natural ordering by set inclusion and under any one of the three operations: addition, multiplication, composition. More recent studies have shown that it is desirable to add to this system a new axiom, comprising the following identities: $R(a \circ b) = R(Ra \circ b)$, $L(a \circ b) = L(a \circ Lb)$. These identities reflect certain "domain-range" properties of ordinary functions, and lead, among other things, to the following **Representation Theorem:** If $S$ is a system satisfying Axioms 1 - 4 of the paper mentioned above, plus the new axiom of this abstract, then there exists an order-preserving isomorphism between $S$ and a system $F$, whose elements are mappings from subsets of $S$ to $S$, partially ordered by set inclusion and combined by ordinary composition. This representation differs fundamentally from previous representations of semigroups as systems of mappings, by taking the ordering of the elements into account. (Received December 8, 1960.)

576-167. F. J. Bureau and E. J. Pellicciaro: **Asymptotic behavior of the spectral matrix of the operator of elasticity.**

Let $D$ be a bounded, open, and connected domain in $\mathbb{R}^3$ with boundary $D'$; set $L\vec{\nabla} = b^2 \text{grad} \cdot \vec{\nabla} - a^2 \text{rot} \cdot \vec{\nabla}$ with $a, b$ constant. The object of this paper
is to find the asymptotic representation of the spectral matrix of the problem
\[ L\vec{v} + \lambda \vec{v} = 0, \quad x \in D, \quad \vec{v} = 0, \quad x \in D', \lambda \text{ a parameter.} \] If \( D' \) is sufficiently smooth, this problem determines a sequence \( \{\lambda_k\} \) of positive eigenvalues such that \( \lambda_k \to \infty \) when \( k \to \infty \); each \( \lambda_k \) is associated with an eigenfunction \( \vec{v}_k \) with components \( v_{kj} \). Connected with the above is the Cauchy problem \( \vec{u}_{tt} - Lu = 0, \vec{u}(x;0) = \vec{f}(x), \vec{u}_t(x;0) = 0 \) whose solution \( \vec{u}_t(x;t) \) depends on an elementary solution and is written in terms of logarithmic parts of certain divergent integrals. Set
\[ (\lambda)^{1/2} = u, \quad (\lambda_k)^{1/2} = \mu_k, \quad \text{and} \quad \theta^+(x;\mu) = \sum_{i,j=1}^3 \Theta_{ij}(x,x;\mu), \text{ where } \mu \geq \mu_1, \theta_{ij}(x,y;\mu) = 0 \text{ when } 0 \leq \mu < \mu_1, \theta_{ij}(x,y;\mu) = -\delta_{ij}(x,y;\mu) \text{ when } \mu < 0. \] With the aid of a Tauberian theorem (see F. J. Bureau, Asymptotic representation of the spectral function of self-adjoint elliptic operators of the second order with variable coefficients. AFOSR-TN-59-1182, Technical Note No. 2; to appear in the Journal of Mathematical Analysis and Applications), one arrives at the result \( \theta^+(x;\lambda) \sim (1/6\pi^2)(2/a^3 + 1/b^3)\lambda^{3/2} \). (Received December 8, 1960.)

576-168. A. J. Macintyre and S. M. Shah: On an extension of Bernstein's theorem to meromorphic functions. If \( f(z) \) is an entire function of exponential type \( \tau \) and \( |f(x)| \lesssim M \) for \( -\infty < x < \infty \), then \( |f'(x)| \lesssim M \tau \). This theorem cannot be extended to meromorphic functions unless we impose some conditions on the poles of \( f(z) \). It is proved in this note that if \( f(z) \) be a meromorphic function with poles \( b_{\mu} \), satisfying (i) \( \lim_{r \to \infty} \sup |T(x,f)/r = \tau | < \infty \), (ii) \( |f(x)| \lesssim M \) for \( -\infty < x < \infty \), (iii) \( f(z) \) has no poles in the strip \( |\text{Im } z| < \rho \), (iv) \( \sum_{1}^{\infty} \text{Im } b_{\mu} |x - b_{\mu}|^2 \lesssim A \) for \( -\infty < x < \infty \), then \( |f'(x)| \lesssim K \) for \( -\infty < x < \infty \). Here \( K = K(\tau,M,\rho,A) \). (Received December 8, 1960.)


Let \( D \) be a bounded plane, finitely connected schlicht domain which contains the origin and whose boundary \( C \) consists of piecewise analytic curves. Let \( f(z) \) be single-valued regular in \( D \) and let \( K(z,\zeta) \) be the Bergman kernel function of \( D \) and \( \ell(z,\zeta) \) the single-valued regular function in \( D \), symmetric in \( z \) and \( \zeta \), defined by the boundary relation (1) \( K(z,\zeta) - 1/\pi (z - \zeta)^2 - \ell(z,\zeta), z \in C, \zeta = dz/ds, |dz| = ds \). Let \( k_{\mu\nu}, \ell_{\mu\nu}, c_{\mu\nu} \) and \( d_{\mu\nu} \) denote the
coefficients of $z^\mu \zeta^\nu$ in the power series expansion about the origin respectively of the functions $K(z, \bar{z})$, $\ell(z, \zeta)$, $(1/\pi)\left[(f'(z)f'(\zeta))/f(z)f(\zeta) - 1/(z - \zeta)^2\right]$ and $1/(f(z)f(\zeta))^2$ in $D$. Theorem 1. In order that $f(z)$ be schlicht, and $|f(z)| < 1$, in $D$ it is necessary and sufficient that for every complex vector $a_0, a_1, \ldots, a_N$ of the form (6)
\[ \sum_{\mu, \nu=0}^N (C_{\mu \nu} + \ell_{\mu \nu})a_\mu a_\nu \left| \leq \sum_{\mu, \nu=0}^N (k_{\mu \nu} - d_{\mu \nu})a_\mu a_\nu, \quad N = 0, 1, 2, \ldots, \]
where the coefficients $C_{\mu \nu}$, $\ell_{\mu \nu}$, $k_{\mu \nu}$, and $d_{\mu \nu}$ have been defined above. This is a generalization of a result due to Grunsky (Math. Z. vol. 45 (1936) pp. 27-61). Using the above theorem the following has been demonstrated: Using Theorem 1 it is possible to give sharp bounds on the fourth coefficient for the class of bounded schlicht functions in the unit disc with usual normalization. (Received December 8, 1960.)


Let $Lu = \sum_{i,j=1}^n a_{ij}u_{ij}$ be uniformly elliptic in the sense that $\sum_{i,j=1}^n a_{ij}x_i x_j \geq \sum_{i=1}^n x_i^2$ throughout a bounded $n$-dimensional domain $D$. The $a_{ij}$ need not be differentiable. It is shown by a symmetrization argument that if $u$ vanishes on $D$, then for any $p > n/2$ the inequality $|u(x)| \leq KV^{2/n-1/p}||Lu||_p$ holds throughout $D$. Here $V$ is the volume of $D$, $|| \cdot ||_p$ the $L_p$ norm, and $K$ a known constant independent of $D, u$, and $L$. Equality is attained when $L$ is the Laplace operator, and $D$ is a circle centered at $x$. (Received December 8, 1960.)


Theorem. If $A$ is a primitive algebraic algebra with involution, then either (1) $A$ is a division algebra or (2) $A$ is the ring of all $2 \times 2$ matrices over a division algebra or (3) $A$ contains a nontrivial symmetric idempotent.

Theorem. If $A$ is the ring of all $n \times n$ matrices over a division algebra and if $A$ has an involution, then $A$ contains a set of at least $k$ orthogonal non-nilpotent symmetric elements, where $2k \geq n$. Theorem. If $A$ is a primitive algebraic algebra with involution whose symmetric elements satisfy a polynomial identity, then $A$ is finite dimensional over its center. (Received December 8, 1960.)
Let $C(T)$ be the set of all Fourier series of continuous complex functions on the circle $T$, and let $Z$ be the set of all integers. Suppose $N \subseteq Z$ and $\alpha$ maps $N$ into $Z$. The following two conditions are shown to be equivalent: (I) The series $\sum_{n \in N} c(n) e^{inx}$ belongs to $C(T)$ whenever $\sum_{n \in Z} c(n) e^{inx}$ belongs to $C(T)$. (II) There are finitely many disjoint arithmetic progressions $S_j$ (infinite in both directions) whose union differs from $N$ by finitely many elements, and there are affine maps $\beta_j$ of $S_j$ into $Z$ (i.e., $\beta_j(n) = A_j n + B_j$, $A_j \neq 0$) such that $\alpha(n) = \beta_j(n)$ for all but finitely many $n \in S_j$. The proof that (I) implies (II) (the nontrivial part of the theorem) depends on the construction of a measure on the torus whose Fourier-Stieltjes transform is the characteristic functions of the graph of $\alpha$. The known structure of idempotent measures (P. J. Cohen, Amer. J. Math. vol. 82 (1960) pp. 191-212) then yields (II). If $\Gamma_1$ and $\Gamma_2$ are the dual groups of the compact abelian groups $G_1$ and $G_2$, the same procedure yields a complete description of those maps of $\Gamma_2$ into $\Gamma_1$ which carry $C(G_1)$ into $C(G_2)$. (Received December 8, 1960.)


In SIAM, December, 1960, the author extended an early result of Stieltjes to facilitate the computation of Bessel functions of orders zero and one. The present paper is a further extension of the Stieltjes method to include the modified Bessel functions $K_0, K_1, I_0, I_1$. A report is also made on the accuracy obtained by the earlier result of the author. (Received December 8, 1960.)

576-174. Wu Ta-Sun: On the topology of homeomorphism group of SLH space.

Let $X$ be a SLH completely regular space, $U$ be any uniformity of $X$ compatible with $X$. Let $G$ be a transitive group of homeomorphisms of $X$ to $X$ topologized by the uniform convergence under $U$. $\theta_z : G \to X$, where $z$ is some fixed point in $X$, and $\theta_z(g) = g(z)$. If $\theta_z$ is open, we say $G$ is quasi reasonable. (If $\theta_z$ is open and $G$ is a topological group, then $G$ is reasonable.) We can prove that $G$ is quasi reasonable under any uniformity $U$ of $X$; and there exists examples which show that $G$ is not reasonable under certain uniformity, but
if $X$ is locally compact connected, then there exists a transitive group which is reasonable under any uniformity of $X$ (cf. L. R. Ford, Jr., Homeomorphism groups and coset spaces, Trans. Amer. Math. Soc. vol. 77 (1954) pp. 490-497). Also one knows that if $X$ is compact and $n$-homogeneous (without assuming $X$ be an SLH space), then the full group of homeomorphisms of $X$ to $X$ is reasonable. Some other discussions of the topology of $G$ will be given. (Received December 8, 1960.)

576-175. J. C. C. Nitsche: A characterization of the catenoid.

The paper deals with doubly-connected minimal surfaces $S$ which have the property that their intersection $S_C$ with every plane $z = c$, where $c_1 < c < c_2$ ($-\infty \leq c_1 < c_2 \leq \infty$), is a Jordan curve. The following theorem is proved: "If $S$ is complete and if all curves $S_C$ are starshaped (with respect to some interior point) then $c_1 = -\infty$, $c_2 = \infty$, and $S$ must be a catenoid." This is a generalization of previous results; see Journal of Rational Mechanics and Analysis vol. 6 (1957) and Math. Z. vol. 74 (1960). (Received December 8, 1960.)


III.

Let $A$ be an Artinian ring with identity element. A left ideal is dominant if it is primitive and is dual to a primitive left ideal. A ring is dominant if it contains dominant ideals. Theorem. If, for every two-sided ideal $Z$ of $A$, $A/Z$ is dominant then $A$ is generalized uniserial. Proof. Construct an ascending sequence of two-sided ideals $Z_i$ of $A$ by making use of the socle of the dominant part of $A$ and of certain residue class rings of $A$. Show that if each $A/Z_i$ is dominant then $A$ is generalized uniserial. This theorem generalizes earlier results [I., Trans. Amer. Math. Soc. vol. 90 (1959) pp. 161-170; II., Proc. Amer. Math. Soc. vol. 9 (1958) pp. 915-919] in which stronger assumptions were placed upon the residue class rings $A/Z$ of $A$. Also, it is shown that in order that $A$ be generalized uniserial it is not sufficient that each $A/N^1$ be dominant, where $N = \text{Rad } A$. (Received December 8, 1960.)
This paper is concerned with the further development of the operator-theoretic approach to branching processes presented in an earlier paper (Bharucha-Reid and Rubin, Proc. Nat. Acad. Sci. U.S.A. vol. 44 (1958)). Let \( \mathcal{B} = \{X(t), t \geq 0\} \) be a continuous time parameter branching process; and let \( P_X(t) = \Pr(X(t) = x), \ x = 0,1, \ldots, \) and let \( F(s,t) = \sum_{x=0}^{\infty} P_X(t)s^x, \ |s| \leq 1, \) denote the generating function of the probabilities \( P_X(t). \) The generating function analogue of the backward Kolmogorov equations is \( \frac{\partial F}{\partial t} = \phi(s) \frac{\partial F}{\partial s}. \) Let \( F(s,t) = T_t g(s), \) where \( g(s) \) is an element of the Hilbert space \( H^2 \) (i.e. the Hardy class of functions); then \( F(s,t) \) is, for every fixed \( t \geq 0, \) an element of \( H^2. \) It is shown that \( \{T_t, \ t \geq 0\} \) is a semigroup of contraction operators on \( H^2 \) with infinitesimal generator \( \phi(s) \frac{\partial}{\partial s}. \) Using a theorem of Sz.-Nagy on the unitary dilation of contraction operators on Hilbert space an integral representation of \( F(s,t) \) is obtained, leading to a representation of the probabilities \( P_X(t) \) associated with \( \mathcal{B}. \) Various properties of the semigroups associated with \( \mathcal{B} \) are considered, and some properties of particular processes are examined.

(Received December 8, 1960.)

Let \( k \) be an algebraically closed field. We represent the nonhyperelliptic functions fields over \( k \) of genus 3 by nonsingular plane quartics. Two such quartics are birationally equivalent if and only if they are projectively equivalent. Using some results and methods of Van der Waerden (Math. Ann. vol. 114 (1937) pp. 683-699) we study projective equivalence of nonsingular plane curves. Let \( C \) be a nonsingular plane curve of order \( n. \) Then the curves improperly projectively equivalent to \( C \) either: (1) have a point of multiplicity \( n, \) or (2) have a point \( P \) of multiplicity \( n - 1, \) and have only one tangent line at \( P. \) The variety of curves of these types has dimension \( n + 4. \) For the quartic case, this is 8. This, together with the fact that the plane projective group is of dimension 8, enables us to obtain a moduli-variety for the non-hyperelliptic curves of genus 3. (Received December 8, 1960.)

In Section 1 of this paper we study the analytic structure of matrices of functions from $[0, \infty]$ into $[0, \infty]$ which satisfy a functional relation but with no semi group property assumed. A matrix will be said to have property $P$ if its elements have property $P$. Let $F_t$ and $G_t$, $t \in [0, \infty]$ be two such matrices satisfying $F_{t_1 + t_2} = F_{t_1} G_{t_2}$ then: (1) If the diagonal elements $g_{11}(t)$, $i = 1, 2, \ldots$, of $G_t$ satisfy (*) $\lim_{t \to 0} g_{11}(t) = 1$ then $F_t$ is continuous. (2) If (*) holds and $F_t$ is monotone then $F_t$ has a right continuous right derivative and left continuous left derivative. (3) If (*) holds and both $F_t$ and $G_t$ are monotone then $F_t$ is analytic.

In Section 2 we generalize Ornstein's differentiation theorem (Bull. Amer. Math. Soc., vol. 66 (1960) pp. 36-39) to the abstract space case previously considered by Austin (Illinois J. Math. vol. 3 (1959) pp. 532-537). (Received December 9, 1960.)


Let $L_n(f;z)$ be the polynomial of degree at most $n - 1$ found by interpolation in the distinct points $z_{kn} = e^{i \theta kn}$, $k = 1, \ldots, n$, to a function $f$ given on $|z| = 1$. It is known that a necessary and sufficient condition that $\lim_{n \to \infty} L_n(f;z) = f(z)$, $|z| \leq 1$, for all $f$ analytic on $|z| \leq 1$, is that $\{\theta_{kn}\}$ be equidistributed on $[0, 2\pi]$. In nonanalytic cases, convergence has been established when $z_{kn}$ is an $n$th root of unity, but the behavior of $\{L_n\}$ with other spacings of the interpolation points is not clear. It is here proved that if $\theta_{kn}^*$, $k = 1, \ldots, n$, are independent random variables each with a uniform probability distribution and if $f$ satisfies certain mild smoothness restrictions on $|z| = 1$, then $E[L_n^*(f;z)] = (1/2\pi i) \int \left[ f(t)/(t - z) \right] \left[ 1 - (z^n/t^n) \right] dt$, where $L_n^*$ is found by interpolation to $f$ in the random points $z_k^* = e^{i \theta kn}$. The limit of the expected value is the same as that obtained for interpolation in the roots of unity. An example is constructed involving an equidistributed sample sequence $\{\theta_{kn}\}$ for which $L_n(f;z)$ diverges to infinity at a point $z$, $|z| < 1$, for at least one function $f$ continuous on $|z| = 1$. Generalizations to Jordan configurations other than the unit circle are indicated. (Received December 9, 1960.)
576-181. J. L. Goldberg: Inequalities for functions having real part positive in the right half-plane.

This paper contains several inequalities believed to be new, simplified proofs of two known inequalities, and various related results for the class of functions sometimes called, "Positive real." A function, \( f \), belongs to this class (we write \( f \) is PR) if for \( \text{Re} \ z > 0 \), (i) \( f \) is single-valued and analytic, (ii) \( \text{Re} \ f(z) > 0 \), (iii) \( f(\bar{z}) = \bar{f}(z) \). (See, P. Richards, A special class of functions with positive real part in a half-plane, Duke Math. J. vol. 14 (1947) pp. 777-788).

Some typical results. **Theorem.** If \( f \) is PR and \( |z - z_0| \leq k\text{Re} \ z_0 \) \((0 \leq k < 1)\) then \( |f(z) - f(z_0)| \leq k(1 - k)^{-1}\text{Re} f(z_0) \). **Theorem.** If \( f \) is PR and \( \text{Re} \ z = x > 0 \) then \( Ax \leq |f(z)| \leq A'/x \), where \( \lim_{x \to \infty} f(x)/x = A \) and \( \lim_{x \to \infty} xf(x) = A' \).

**Theorem.** If \( \{f_n(z)\} \) is a sequence of PR functions with \( f_n(1) = 1 \), for all \( n \) and \( \lim_{n \to \infty} f_n(z_0) = z_0 \), \( \text{Re} z_0 > 0 \) and \( z_0 \neq 1 \), then \( \lim_{n \to \infty} f_n(z) = z \) uniformly in any closed bounded subset of the open right half-plane. (Received December 9, 1960.)

576-182. Seymour Haber: An asymptotic expansion of the error of numerical quadrature schemes.

If \( R \) is a numerical quadrature rule, the composite rule \( n \times R \) is that in which the interval of integration is divided into \( n \) equal subintervals and the rule \( R \) applied in each. Automatic quadrature schemes used in computation generally consist of using successively the rules \( R, 2 \times R, 4 \times R, \ldots \) until some convergence criterion is satisfied. The Euler-Maclaurin sum formula may be regarded as an asymptotic expansion of the error of \( n \times R \) as a function of \( n \), where \( R \) is the trapezoid rule \( \int_a^b f(x)dx \approx \frac{f(a) + f(b)}{2} \). A similar expansion is given for the error of \( n \times R \) where \( R \) is any rule using only values of the function in the interval of integration. (Received December 9, 1960.)


K. Borsuk defined the generalized cohomotopy groups as follows: If \( A \) is closed in the space \( X \) with \( \dim A \leq k < 2n - 1 \), then \( \pi^n(A \subset X) \) denotes the subgroup of the cohomotopy group \( \pi^n(A) \) generated by the mappings of \( A \) into \( S^n \) extendable over \( X \). A closed subset \( A \) of \( X \) is called a \( k \)-skeleton, if \( \dim A \leq k \) and if every at most \( k \)-dimensional closed subset of \( X \) is deformable.
into A. Borsuk showed that if X is an ANR with property ($\Delta$), then for each k there exists a k-skeleton in X and, if A and B are two k-skeletons of X, then the groups $\pi_k^0(A \subset X)$ and $\pi_k^0(B \subset X)$ are isomorphic. Therefore they determine an abstract group $\pi_k^0(X)$ for each ($\Delta$)-ANR-space X and $k < 2n - 1$. Another definition of the groups $\pi_k^0(X)$ can be given, as direct limits of the corresponding groups of polyhedra. This definition is valid for all compact spaces and, if X is a ($\Delta$)-ANR-space, it coincides with the Borsuk's definition. The groups $\pi_k^0(X)$ can be expressed in some simple cases in terms of the cohomology groups by using the Hopf and Steenrod extension theorems. (Received December 8, 1960.)

576-184. G. K. Kalisch: **On similarity invariants of certain operators.**

Continuing earlier work of the author's on similarity invariants of various analytically given operators on $L = L_p[0,1]$ (see Ann. of Math. vol. 66 (1957); and two papers to appear in the Pacific J. Math, and the Proc. Amer. Math. Soc., resp.), this paper investigates similarity invariants of operators of the form $A_c = M + cT$ and $B_c = M + cT^*$ where for $f \in L$, $(Mf)(x) = sf(x)$, $(Tf)(x) = \int_x^1 f(y) dy$ and c is a constant. A detailed investigation of the spectrum, together with the application of certain similarity transformations (such as "fractional integration of pure imaginary order"), shows that under certain circumstances $A_c$ and $B_c$ are similar to M, and under certain circumstances $A_c$ (and $B_c$) is similar only to itself in the class of operators here considered. (Received December 9, 1960.)

576-185. C. R. Storey: **A description of all globally idempotent threads with zero.**

In The structure of threads (to appear in the Pacific J. Math.), the author shows that if S is a thread with a zero in which $S^2 = S$, then the set of elements equal to or greater than the zero forms a subthread having a quite manageable structure. The problem raised by this result of describing the multiplication on the remainder of the thread has been solved by devising several constructive processes by which all such threads can be obtained. (Received December 9, 1960.)
576-186. I. I. Glick: An existence theorem for an nth order partial
differential equation.

The existence of a solution of the classical initial value problem
\[ y' = f(x,y), \quad y(0) = y_0, \]
can be proved by the Euler-Cauchy polygon method.

J. B. Diaz (On an analogue of the Euler-Cauchy polygon method for the numeri-
cal solution of \( u_{xy} = f(x,y,u_x,u_y) \), Arch. Rational Mech. Anal., vol. 1 (1958))
devised a two dimensional generalization of this method, using a finite-sum
analogue of an integral inequality of Gronwall, and obtained an existence theo-
rem for the problem \( u_{xy} = f(x,y,u_x,u_y) \), \( u(x,y_0) = g(x), u(x_0,y) = h(y), g(x_0) = h(y_0) \),
where \( f \) is required to be continuous, bounded and Lipschitzian only in
\( u_x \) and \( u_y \). James Conlan has extended these considerations to the corresponding
partial differential equation \( u_{xyz} = f \) (NAVORD REPORT 6921). By means of a
generalization of Diaz's inequality to \( n \), dimensions, a corresponding existence
theorem is obtained for \( u_{x_1x_2...x_n} = f \), where \( f \) is continuous, bounded, and
Lipschitzian in the pure mixed derivatives \( u_{x_1},...,u_{x_n}u_{x_1,x_2},... \) up to order
\( n - 1 \). In the process, properties of certain multilinear interpolatory functions
are obtained. These functions, together with their pure-mixed one-sided
derivatives, converge to the approximated function and its pure-mixed deriva-
tives respectively. (Received December 9, 1960.)

576-187. G. J. Rieger: Generalization of two theorems of Romanov in
additive number theory.

It has been proven by Romanov (Math. Ann. vol. 109 (1934)) that the
sequences \( F_a = \{ x: x = p + a^m, \, p \text{ prime, } m = 1,2,... \} \) (a integer, \( >1 \)) and
\( F_m = \{ x: x = p + a^m, \, p \text{ prime, } a = 1,2,... \} \) (m integer, \( >0 \)) have positive lower
asymptotic density. This is generalized to arbitrary algebraic number fields.
(Received December 9, 1960.)

576-188. R. A. Willoughby: Analysis of the bi-stable oscillations in the
parametron by Lyapunov's method.

The parametron is a nonlinear and time varying electrical circuit which
can oscillate in two distinct steady states. The circuit is described by a
system of nonautonomous differential equations. The given system can be
approximated by a time averaged autonomous system of the form \( dw/dt \)
Lyapunov's method is applied to this equation. A function $S(w)$ is constructed which is distance in an appropriate coordinate system. Since $S$ has the property that $-dS/dt \geq 0$ it is a Lyapunov function and its critical points completely characterize the solution field of the differential equation. In this manner the bi-stable nature of the original electrical circuit is characterized. (Received December 9, 1960.)


This paper studies the application of Galerkin's method to fine approximate solutions to symmetric positive equations, which were introduced by Friedrichs (Comm. Pure Appl. Math. (1958)) to study partial differential equations of different types in a unified way. Let $Ku = f$ be the differential equation defined in a region $R$, and $Mu = 0$ the boundary condition defined on the boundary $\partial R$. The solution $u$ is assumed to exist and be in $H_1(R)$. Let $\{\phi_i\}$ be a sequence of functions complete in $H_1(R)$ and individually satisfying the boundary condition $M\phi_i = 0$. Let $u_n = \sum c_i\phi_i$ be the approximate solution, and $S_n$ be the subspace spanned by $\phi_1, \ldots, \phi_n$. The coefficients $c_i$ are to be determined by the condition that $Ku_n - f$ be orthogonal to $S_n$, which is a system of $n$ linear equations in $n$ unknowns. It is shown that the solution of these equations is unique (therefore, exists), and that $u_n$ converges to $u$ strongly in the $H_0$ norm. A special example is given for which the sequence $\{\phi_i\}$ is selected. (Received December 9, 1960.)


Theorem. If $R$ is a prime (assoc.) ring with ascending chain condition on left and right annihilator and complement ideals, and if $U$ (resp. $V$) is any minimal right (resp. left) annihilator ideal, such that $VU \neq 0$, then $K = U \cap V$ is a nonzero integral domain. Consider $U$ as a left $K$-module. Then, if $x_1, \ldots, x_n \in U$ are left linearly independent over $K$, and if $y_1, \ldots, y_n \in V$ are arbitrary, then there exist elements $a \in R$ and $0 \neq k \in K$ such that $x_ia = ky_i$, $i = 1, \ldots, n$. This result may be obtained as an application of a theorem of R. E. Johnson [Trans. Amer. Math. Soc. vol. 72 (1953) pp. 351-357] by verifying Johnson's axiom $P(5)$ when (in his notation) $P^* = \mathbb{P}^R \ell$ (resp. $P^* = \mathbb{P}^L \ell$) for any right (resp. left) ideal $P$, where $\mathbb{P}^R$ (resp. $\mathbb{P}^L$) denotes the right (resp. left)
annihilator of the ideal P. The axiom P(5) holds more generally in any ring in
which right and left complements are annihilators. That the rings of the theo-
rem have this latter property is A. W. Goldie's [Proc. London Math. Soc. vol. 10
(1958) p. 602, Theorem 9]. The representation theorem above also holds under
weaker conditions. (Received December 9, 1960.)


Let $H_2$ denote the Hilbert space of functions $f(z) = \sum_{n=0}^{\infty} a_n z^n$, with
$\|f\|^2 = \sum_{n=0}^{\infty} |a_n|^2$. Beurling (Acta Math. vol. 81) has shown that if $X$ is a
closed subspace of $H_2$, and $f \in X$ implies $zf \in X$, then $X$ consists of all functions
$f = f_0 g$, where $g \in H_2$ and $f_0$ is a certain bounded function in $H_2$. Lax (Acta
Math. vol. 101) proved an analogous theorem for functions in a half-plane. We
prove a more general theorem, valid for any domain in which the function class
$H_2$ can be introduced. Our proof does not, as previous proofs, use harmonic
analysis, but general Hilbert space methods, notably the concept of orthogonal
projection on a subspace and its realization by means of a reproducing kernel.
(Received December 9, 1960.)


Let $E^n$ be Euclidean n-space, and let $\omega$ be a continuous differential
$r$-form defined in bounded domain $R$ of $E^n$. With $E^n$ endowed with the usual
Cartesian coordinate system $x = (x_1, ..., x^n)$ and with $\mu$ the ordered $(r + 1)$-tuple
$\mu = (\mu_1, ..., \mu_{r+1})$, $\mu_1 < ... < \mu_{r+1}$, define the upper symmetric exterior deriva-
tive of $\omega$, $\bar{D} \omega(x)$, at the point $x$ in $R$ as follows: First define $\bar{D} \omega(x)$
by
$$\bar{D} \omega(x) = \lim_{t \to 0} \sup_{t \leq 1} |B_{\mu}(x,t)|^{-1} \int_{S_{\mu}(x,t)} \omega,$$
where $S_{\mu}(x,t)$ is the $r$-sphere with center $x$ and radius $t$ lying in the $(r + 1)$-plane parallel to the $x_1 \ldots x_{r+1}$-plane, $B_{\mu}(x,t)$
is the closed $(r + 1)$-ball bounded by $S_{\mu}(x,t)$, $|B_{\mu}(x,t)|$ is the volume of $B_{\mu}(x,t)$,
and $S_{\mu}(x,t)$ is oriented as usual with respect to the outer normal. Then define
$$D \omega(x) = \sum_{\mu_1 < ... < \mu_{r+1}} \bar{D} \omega(x) dx_{\mu_1} \ldots dx_{\mu_{r+1}}.$$ In a similar manner
define the lower symmetric exterior derivative. The following theorem is then
proved: A necessary and sufficient condition that $\omega$ be flat in $R$ is that its
upper and lower symmetric exterior derivatives be bounded in $R$. (Received
December 9, 1960.)
James Conlan: An existence theorem for a 3rd order partial differential equation.

The existence of solutions of the classical initial value problem, \( y' = f(x,y), y(0) = y_0 \), can be proved by (1) Picard's iteration method, (2) the Euler-Cauchy polygon method. In method (2) \( f \) is only assumed to be continuous in \((x,y)\), but not necessarily Lipschitzian in \( y \). J. B. Diaz (On an analogue of the Euler-Cauchy polygon method for the numerical solution of \( u_{xy} = f(x,y,u,u_x,u_y) \), Arch. Rat. Mech. Anal. (1958) pp. 357-390, extended method (2) to obtain an existence theorem for \( u_{xy} = f(x,y,u,u_x,u_y) \), \( u(x,0) = g(x) \), \( u(0,y) = h(y) \), \( g(0) = h(0) \), where \( f \) is continuous in all its arguments, and need be Lipschitzian only in \( u_x, u_y \). The method of Diaz has here been extended to obtain an existence theorem for the corresponding problem for \( u_{xyz} = f(x,y,z,u,u_x,u_y,u_z,u_{xy},u_{xz},u_{yz}) \), where \( f \) is continuous in all its arguments and need be Lipschitzian only in the derivatives of \( u \). (Received December 9, 1960.)

J. G. Horne: Real commutative semigroups on the plane, (II).

In an earlier note (Abstract 573-5, Notices Amer. Math. Soc. vol. 7 (1960) p. 722) a study was begun of commutative topological semigroups on the plane \( E \) which contains a copy of the multiplicative semigroup of real numbers \( R \) embedded so that 0 is a zero and 1 an identity. This note is chiefly concerned with strengthening the results of that study by finding conditions under which \( E \) is a real semigroup. We continue to assume that \( E \) is commutative and has a zero 0 and identity 1. As usual, \( H(1) \) denotes the maximal subgroup of \( E \). If \( G \) is a vector group in \( E \) and \( 0 \in G^- \) then there is a copy \( P \) of the group of positive real numbers contained in \( G \) such that \( 0 \in P^- \). A corollary of this result is that \( P^- \times P^- \) is the only commutative semigroup with identity on a half-plane having no nilpotent elements. A second corollary is that if \( 1 \) has a nontrivial square root then \( E \) is a real semigroup. Hence \( E \) is isomorphic to \( R \times R \) iff (1) \( E \) has no nilpotent elements, (2) \( E \) has exactly four idempotents and (3) \( E \) has at least two nontrivial square roots of 1. (Received December 9, 1960.)

Fan and Gottesman (Nederl. Akad. Wetensch. Proc. Ser. A vol. 14 (1952) pp. 504-510) have generalized a method of Freudenthal (loc. cit. vol. 54 (1951) pp. 184-192) for rim-compact spaces of compactifying a space X with a normal basis \( \mathcal{B} \) of open sets. The author (loc. cit. vol. 60 (1957) pp. 171-176) has shown that every compactification of a completely regular space X is homeomorphic to a filter space over X. The problem is to identify the Fan-Gottesman compactifications by means of filter spaces. A filter \( \mathcal{F} \) on X is said to be a \( \mathcal{B} \)-filter if \( \mathcal{F} \) is an open filter and, for every \( F \in \mathcal{F} \), there exists \( B \in \mathcal{B} \) such that \( \overline{B} \subseteq F \) and \( \overline{B} \cap \mathcal{A} \in \mathcal{B} \) implies \( \mathcal{A} \in \mathcal{F} \). Then the Fan-Gottesman \( \mathcal{B} \)-compactification of X is homeomorphic to the filter space of all maximal \( \mathcal{B} \)-filters on X. (Received December 9, 1960.)


Let \( C \) be a convex body in Euclidean space \( \mathbb{E}^{n+1} \) with boundary \( \partial C \). We consider \( \hat{\lambda}(C,t) \), the Fourier transform of the volume distribution on \( C \), i.e., the integral of \( \exp(2\pi it \cdot x) \) over \( C \), and \( \hat{s}(C,t) \), the Fourier transform of the surface area distribution on \( C \). The two are related in the following way. Let \( C + hB \) be the vector sum of \( C \) with a ball of radius \( h \) centered at the origin.

**Theorem 1.** \( \hat{s}(C + hB,t) \) is the derivative with respect to \( h \) of \( \hat{\lambda}(C + hB,t) \). The average asymptotic behavior of \( \hat{\lambda} \) is easily determined from **Theorem 2.**

\[
\int_{|t| \leq T} |t|^2 |\hat{\lambda}(C,t)|^2 dt \leq A_n S(C)T \quad \text{where} \quad A_n \quad \text{is a dimensional constant and} \quad S(C) \quad \text{is the surface area.} \]

The deepest result, and the one which is of interest in lattice-point problems, is a statement about the asymptotic behavior of \( \hat{\lambda} \) and \( \hat{s} \).

For this we have to assume that \( C \) is sufficiently smooth and has strictly positive Gaussian curvature at every point. Write \( t = r\theta \) where \( r > 0 \) and \( \theta \) is a unit vector; \( K(\theta) \) denotes the Gaussian curvature of the surface at the point where the outer normal is \( \theta \) and \( P(t) \) the support function of the convex body.

With the notation \( f(C,t) = r^{-m}K^{-1/2}(\theta) \exp(2\pi iP(t) - \pi i/4) \) where \( n = 2m \) we have **Theorem 3.** \( 2\pi ir\hat{\lambda}(C,t) = f(C,t) - \overline{f}(C,-t) + O(r^{-m-1}) \) and \( \hat{s}(C,t) = f(C,t) + \overline{f}(C,-t) + O(r^{-m-1}) \). (Received December 9, 1960.)
W. T. Reid: An oscillation criterion for self-adjoint differential systems.

For a differential system with complex coefficients that is of the general form of the accessory differential equations for a calculus of variations problem of Bolza type that is identically normal, variational methods are employed to prove an oscillation theorem which includes as a very special instance the recently established result of H. Howard, [Trans. Amer. Math. Soc. vol. 96 (1960) pp. 296-311], to the effect that if \( r(x) \) and \( p(x) \) are real-valued positive functions such that \( r(x) \in C'(0,\infty) \), \( p(x) \in C(0,\infty) \), and for \( a > 0 \) we have \( \int_a^\infty \left| r(x) \right|^{-1} dx = \infty \), then the fourth order linear differential equation \( (r(x)u'')'' - p(x)u = 0 \) is nonoscillatory on \((a,\infty)\) if and only if for arbitrary \( b > a \) the condition \( \lambda_b > 1 \) is satisfied by the smallest proper value \( \lambda_b \) of the boundary problem \( (r(x)u'')'' - \lambda p(x)u = 0, u = u' = 0 \) at \( x = a \), \( ru'' = (ru'')' = 0 \) at \( x = b \).

The criterion here derived also includes as special cases some of the oscillation theorems for more general fourth order differential equations presented by J. H. Barrett in a recent MRC Technical Report No. 150, April 1960, entitled Two-point boundary value problems and comparison theorems for fourth-order self-adjoint differential equations and second-order matrix differential systems. (Received December 9, 1960.)


In a 2-person game, a mixed strategy pair is an inverse equilibrium point if neither player can decrease the other player's payoff by a unilateral change in strategy. In strictly competitive games, this is the same as an equilibrium point, but there are other games as well for which the two concepts coincide (e.g., the prisoner's dilemma); such games will be called almost strictly competitive (a.s.c.). Assume that an a.s.c. game is given; a solution is an equilibrium point, and a good strategy is a component of a solution. Theorems. The equilibrium payoff is unique (it is called the value); any pair of good strategies constitutes a solution; a strategy is good if and only if it guarantees a player his component of the value, also if and only if it guarantees a player that the other player will not get more than his component of the value; a strategy that weakly dominates a good strategy is good; the value is maximin and minimax. Suppose that an extensive game \( \Gamma \) decomposes at a move \( X \), that the subgame \( \Gamma_X \) is a.s.c., and that the difference game \( \Gamma_D \) whose payoff
at $X$ is the value of $\Gamma_X$ is also a.s.c.; then $\Gamma$ itself is a.s.c., and a strategy is good in $\Gamma$ if it decomposes into good strategies in $\Gamma_X$ and $\Gamma_D$. As the prisoner's dilemma shows, the value need not be pareto optimal. (Received December 9, 1960.)

576-199. Simon Kochen: The isomorphism of ultrapowers of elementarily equivalent systems.

Assume the generalized continuum hypothesis. Theorem 1. Two systems $\mathfrak{A}$ and $\mathfrak{F}$ are elementarily equivalent iff there is a set $I$ and an ultra-filter $D$ over $I$ such that the ultrapowers $\mathfrak{A}^I/D$ and $\mathfrak{F}^I/D$ are isomorphic. Corollary 1. A class $K$ of systems is elementary (i.e. $K \in AC$) iff $K$ and its complement $K'$ are closed under isomorphism and ultraproducts. Corollary 2. $K \in AC_\delta$ iff $K$ is closed under isomorphism and ultraproducts and $K'$ is closed under ultrapowers. Theorem 1 follows from Theorem 2: Suppose $I$, $\mathfrak{A}$, and $\mathfrak{F}$ have the same infinite cardinality, $S$ is one-one map of $I$ onto the family of all finite subsets of $I$, and $D$ and $E$ are ultrafilters over $I$ having the sets $\{i|j \in I, j \in S(i)\}$ as elements, for all $j \in I$. Then $\mathfrak{A} \cong \mathfrak{F}$ iff $\mathfrak{A}^I/D \cong \mathfrak{F}^I/E$. Further theorems of this type are also proved for the case where $I$, $\mathfrak{A}$, and $\mathfrak{F}$ have unequal cardinalities. For the case where $I$ is denumerable the theorem takes the stronger form: Theorem 3: The continuum hypothesis implies that if $D$ and $E$ are any nonprincipal ultrafilters over $I$ and $\mathfrak{A}$ and $\mathfrak{F}$ have cardinalities $\aleph_0$ or $\aleph_1$ then $\mathfrak{A} \cong \mathfrak{F}$ iff $\mathfrak{A}^I/D \cong \mathfrak{F}^I/E$. Theorem 3 coincides with a result announced in Keisler, Abstract 564-265, Notices Amer. Math. Soc. vol. 7 (1960) pp. 70-71. (Received December 9, 1960.)


Economical ways to calculate the action and interaction of shocks in the solution of nonlinear hyperbolic equations are well tested empirically but have not been completely founded in theory. Here, theoretical justifications are offered of a family of explicit schemes for first order equations of the form $u_t + (F(x,t,u))_x + G(x,t,u) = 0$, where $F_{uu} \geq 0$. These explicit schemes include versions of the von Neumann-Richtmyer and the Lax - Wendroff methods; they are related to implicit schemes previously discussed. (Received December 9, 1960.)
Let B denote the functions of bounded variation on the unit interval \( I = [0,1] \) with convolution defined by \((a \ast \beta)(x) = \int_0^1 a(xy^{-1})d\beta(y)\) and let C denote the continuous functions on I. For \( a \in \beta \) denote by R(a) the range of the endomorphism of C defined by \( a(f) = f^*a \). It is shown that if \( a, \beta \in B \), then \( R(a) \supset R(\beta) \) if and only if the Hausdorff method \( |H;\beta| \) includes the Hausdorff method \( |H;a| \). From this equivalent formulation of the inclusion problem it follows that \( |H;a| \) is a prime Hausdorff method if \( R(a) \) is a maximal closed subspace of C. This fact provides an easy method of constructing all known primes. (See E. Hille and R. S. Phillips, *Functional analysis and semi-groups*, rev. ed., p. 617.) (Received December 9, 1960.)

Let \( \{x_t, \mathcal{F}_t, 0 \leq t\} \) be a stochastic process where \( E\{|x_t|\} < \infty \) and \( x_t \) is measurable with respect to the Borel field \( \mathcal{F}_t \) for each \( t \), the fields satisfying \( \mathcal{F}_s \subset \mathcal{F}_t \) for \( s < t \). Under certain conditions on the conditional expectation \( E\{x_t \| \mathcal{F}_s\} \) there exists a process \( \{y_t, \mathcal{F}_t, 0 \leq t\} \) such that \( x_t = w_t + \int_0^t y_s ds \) where the process \( \{w_t, \mathcal{F}_t, 0 \leq t\} \) is a martingale. If the \( x_t \)-process is a semimartingale then the \( \int_0^t y_s ds \)-process is a monotonic semimartingale. The corresponding result in the discrete parameter case was proved by Doob (*Stochastic processes*, Wiley, 1953, p. 296). As an example, if \( \{z_t, 0 \leq t\} \) is Brownian motion in the complex plane and \( u(z) \) has second derivatives which satisfy a Holder condition, then for the process \( \{u(z_t), 0 \leq t\}, y_t = \Delta u(z_t) \). (Received December 9, 1960.)

The relativistic equations of motion for N point charges can be transformed into a delay-differential system of the form \( y'(t) = f(t,y(t),y(u_1(t,y(t))),..., y(u_m(t,y(t)))) \) and the given functions \( u_j(t,y) \) are strictly retarded, i.e., \( u_j(t,y) < t \) (\( j = 1,...,m \)). Assume, as is true in electrodynamics, that for any \( y(t) \) satisfying these equations each
\( u_j(t, y(t)) \) is a strictly increasing function of \( t \). Specify \( y(t) \) for an appropriate interval \([a, t_0]\). Let each \( f_i, u_j, y_i(t) \), and \( y'_i(t) \) be Lipschitz continuous in an appropriate region. Then there is a unique extension of \( y(t) \), continuous in \([t_0, \beta)\), which satisfies the delay-differential system in \((t_0, \beta)\) except at points \( t_1 < t_2 < ... \) where \( u_j(t_k, y(t_k)) = t_0, t_1, ..., or t_{k-1} \) for some \( j \). For the \( N \)-body problem (neglecting radiation reaction) the following holds: One specifies trajectories of the \( N \) charges such that the speeds are less than the speed of light in \([a, t_0]\), the accelerations are Lipschitz continuous in \([a, t_0]\), no charges collide at \( t_0 \), and a set of functional equations involving the trajectories has a solution at \( t_0 \). Then there is a unique extension of the trajectories with continuous positions and velocities for \([t_0, \beta)\) and either \( \beta = +\infty \) or else a collision occurs at \( t = \beta \). (Received December 9, 1960.)


Most of the known results about the distribution of zeros of fourth-order equations of the form (1) \( [(r(x)y'')' + q(x)y']' - p(x)y = 0, \) \( r(x) \) positive and all coefficients continuous on \([a, \infty)\), require that \( p(x) \) does not change sign and either \( q(x) \) is zero or can be removed without changing signs of \( r(x) \) and \( p(x) \) (e.g., results of Leighton and Nehari, H. C. Howard and the author) or where \( q(x) \) and \( p(x) \) are both non-negative (e.g., H. M. and R. L. Sternberg and the author). This paper is primarily concerned with the case: \( p(x) \) non-negative and the second-order equation (2) \( (r(x)y')' + q(x)y = 0 \) is not disconjugate. Using the notation: \( y_1(x) = ry'' \), \( y_2(x) = y_1' + qy' \), conditions on the coefficients of (1) are found which ensure that there exists a solution having a double zero at \( x = a \) and which satisfies at some \( x = b > a \), the various end conditions:

\[
\begin{align*}
y(b) &= y'(b) = 0, \\
y_1(b) &= y_2(b) = 0, \\
y'(b) &= y_1(b) = 0 \text{ or } y'(b) = y_2(b) = 0.
\end{align*}
\]

For each case, attention is on the minimum \( b \) and the ordering of these minima. Also, included in the ordering are the corresponding numbers for the second-order equation (2). (Received December 9, 1960.)

576-205. Daniel Gorenstein and J. H. Walter: Finite groups of order \( 4g' \).

Theorem. Let \( G \) be a finite group of order \( 4g' \) where \( g' \) is odd. Assume that \( G \) possesses no normal subgroup of index 2 and has a Sylow 2-subgroup which is its own centralizer. Then \( G \) possesses a normal subgroup \( H \) of odd...
order such that $G/H$ is isomorphic to $\text{PSL}(2,q)$ where $q \equiv 3$ or $5$ modulo $8$. This theorem was conjectured by R. Brauer (Proc. Int. Cong., Amsterdam).

(Received December 9, 1960.)


A "Pontriagin" operator is an operator in a Hilbert space $\mathcal{H}$ of the form $UA$ where $A$ is a bounded self-adjoint operator and $U$ is a self-adjoint operator with a finite dimensional eigenspace corresponding to the eigenvalue $-1$. The normal case, i.e., where every algebraic finite dimensional spectral subspace of the operator is at the same time a topological one, is considered. There exists a decomposition into a direct sum, $\mathcal{H} = \mathcal{H}_k \oplus \mathcal{H}_+$ where $\mathcal{H}_k$ form a finite dimensional subspace composed of elementary divisors of $UA$ and the $\mathcal{H}_k$'s form minimal reducing subspaces for the quadratic form $Q(v) = (Uv,v) + i(Av,v)$. $\mathcal{H}_+$ is a reducing subspace for $Q(v)$ on which the form $(Uv,v)$ is positive definite and gives a norm equivalent to the norm of $\mathcal{H}$. Hence, on $\mathcal{H}_+$ the operator $UA$ has a usual spectral decomposition as a self-adjoint operator relative to the metric $(Uv,v)$. The case when $A$ is a completely continuous operator without null space is a normal case. This case arises in differential elliptic eigenvalue problems $Au = \lambda Bu$ with suitable boundary conditions when $A$ and $B$ are without null space under the boundary conditions and $B$ is of lower order than $A$. (Received December 9, 1960.)


Let $F$ be an algebraically closed field of characteristic $p > 5$, and $L$ a Lie algebra over $F$. A trace form $(t.f)$ on $L$ is a bilinear form $f$ on $L$ for which there is a representation $\Delta$ of $L$ such that $f(a,b) = tr((a\Delta)(b\Delta))$ ($a, b$ in $L$). $L$ is said to have a quotient trace form (q.t.f.) if there is a Lie algebra $K$ with a t.f. $f$ such that $L \cong K/K^\perp$, where $K^\perp$ is the radical of $f$. Zassenhaus (Proc. Glasgow Math. Assoc. (1959)) determined the structure of Lie algebras with q.t.f. in terms of simple algebras with q.t.f. Henceforth let $L$ be simple.

Theorem 1. A n.a.s.c. for $L$ to have a q.t.f. is that it be of classical type (in the sense of Mills and Seligman, J. Math. Mech. (1957)). Theorem 2. If an irreducible representation $\Delta$ of $L$ has nondegenerate t.f., then $\Delta$ is restricted. $L$ is said to be of type PA if it is isomorphic to a Lie algebra of $n \times n$ matrices

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(over F) of trace 0 modulo scalars, where $p | n$. **Theorem 3.** Algebras of type $PA$ have no nonzero t.f. Corollary 1. $L$ has a nondegenerate t.f. if and only if it is of classical type other than $PA$. **Corollary 2.** If $L$ has a nondegenerate t.f. then all derivations of $L$ are inner. (Received December 9, 1960.)


Let $M = (\hat{y}_{ij})$ be a $2 \times 2$ matrix with entries that are 0 or 1. The S-fraction (*) $1 + a_1 z/1 + \ldots + a_n z/1 + \ldots$ is said to be parametrically represented relative to $M$, written $p.r.M$, if there exists a sequence $\{g_n\}$ such that $g_0 = 1 - \hat{y}_{11}$, $a_{2n-1} = (g_{2n-2} + \hat{y}_{11} g_{2n-1} + \hat{y}_{12})$, and $a_{2n} = (g_{2n} + \hat{y}_{21})$ for $n = 1, 2, \ldots$. When equivalent types are identified, there are four distinct parametric representations which correspond to the matrices $M_0$ where all $\hat{y} = 0$, $M_1$ where only $\hat{y}_{11} \neq 0$, $M_2$ where only $\hat{y}_{12} \neq 0$, and $M_3$ where only $\hat{y}_{11} = \hat{y}_{22} 
eq 0$. Any S-fraction is $p.r.M_0$. If $f(z)$ denotes the formal power series corresponding to (*), a necessary and sufficient condition for (*) to be $p.r.M_i$ is that the S-fraction of the type (*) corresponding to $f(z) - z$ be $p.r.M_2$. Similar conditions for S-fractions $p.r.M_1$ and $p.r.M_3$ are contained in the results of Scott, Garabedian and Wall. Interrelations among the various classes of parameters are found and an application to moment problems is considered. In addition, necessary and sufficient conditions for the backward extension of (*) to be $p.r.M_i$ ($i = 1, 2, 3$) are derived. (Received December 9, 1960.)

576-209. S. S. Cairns: *On combinatorial manifolds and differentiable structures.*

**Theorem.** A combinatorial $n$-manifold $K^n$ can carry a compatible differentiable structure if and only if it admits a piecewise differentiable homeomorphic mapping $h$ into a differentiable $(n + 1)$-manifold $M^{n+1}$. The necessity of the condition is easily established. Its sufficiency is demonstrated with the aid of (1) special restrictions on $h$ ensuring a field of transverse geodesic arcs to $h(K^n)$ on $M^{n+1}$ and (2) "equipotential" manifolds on $M^{n+1}$ of a uniform distribution on $h(K^n)$. It follows that the $K^8$ of Milnor admitting no compatible differentiable structure and the $K^10$ of Kervaire admitting no differentiable structure at all, are not homeomorphic to $C^1$-complexes, in the sense of Whitehead, on differentiable 9-manifolds and 11-manifolds respectively. It also
follows that if $K^n$ admits no compatible differentiable structure, neither does $K^n \times S^1$, where $S^1$ is the circle, so that each such $K^n$ implies the existence of others, of the form $K^n \times S^1 \times \ldots \times S^1$, in all higher dimensions. (Received December 9, 1960.)


A sequence of classes of recursive functions of increasing computational difficulty is defined. First the class of functions computable by finite automata is studied and taken as the class of functions for which computation is least difficult. Each subsequent class is defined as the class of functions computable by Turing machines for which the amount of tape used in the computation is a function of the arguments which lies in the preceding class. These classes are studied and characterized independently of the Turing machine approach. It is shown that their union is precisely the class of elementary functions as defined by Kalmar and that each class properly contains the class $E^2$ introduced by Grzegorczyk (Some classes of recursive functions, Rozprawy Mat. (1953)). The characterizations are established by arithmetization of these classes of machines within subclasses of the elementary functions, using techniques derived from those introduced by Smullyan (Theory of formal systems, forthcoming, Annals of Mathematics Study) in connection with his class of rudimentary relations. (Received December 9, 1960.)

576-211. Samuel Schechter: Minimization of a convex function by iteration.

Let $E$ be a convex domain in Euclidean $n$-space and let $G(u)$ be a real valued function, bounded from below and in class $C^2$ for all $u \in E$. Let $r = \text{grad } G$ and assume that the matrix $A = (a_{ij})$, $a_{ij} = \frac{\partial^2 G}{\partial u_i \partial u_j}$ is positive definite for $u \in E$. Let $Z = (1,2,\ldots,n)$ and let $D$ be a bounded convex set such that (1) $\overline{D} \subset E$ (2) for any $u^0 \in D$ and any fixed $i \in Z$ there exists a $u' \in D$ such that $u_k^i = u_k^0$, $k \neq i$ and $r_i(u') = 0$. Let $\{i_k\}_{k=0}^{\infty}$ be a sequence of integers $i_k \in Z$ which exhaust $Z$ infinitely often, then (2) will define a "relaxation" process $\{u^k\}$ for this sequence. It is shown that $\{u^k\}$ converges to a unique solution $u^*$ of $r(u) = 0$ for every $u^0 \in D$. This extends known theorems on relaxation methods for linear equations. A partial converse is also obtained thus extending Reich's
theorem. Applications are made to nonlinear elliptic difference equations including an approximation to the minimal surface equation. The feasibility of an approximate relaxation method given by \( u_i^1 = u_i^0 - \omega_1 r_i(u^0)/a_{ii}(u^0) \) is also discussed. (Received December 9, 1960.)


Let \( B \) be a complex Banach algebra with a unit element and let \( \mathcal{L}(B) \) denote the Banach algebra of all bounded linear transformations of \( B \) into \( B \). By \( \mathcal{L}_x(B) \) (resp. \( \mathcal{L}_r(B) \)) we denote the set of all elements \( T \) of \( \mathcal{L}(B) \) of the form \( x \to ax \) (resp. \( x \to xa \)), where \( a \in B \). The sets \( \mathcal{L}_x(B) \) and \( \mathcal{L}_r(B) \) are closed linear subalgebras of \( \mathcal{L}(B) \). The smallest closed linear subalgebra of \( \mathcal{L}(B) \) which contains both \( \mathcal{L}_x(B) \) and \( \mathcal{L}_r(B) \) is denoted by \( \mathcal{H}(B) \) and is called the derived algebra. For hypercomplex systems \( \mathcal{H}(B) \) was first introduced by Haussdorff in 1900. Generalizing a definition of Haussdorff we say that a mapping \( f \) of \( B \) into \( B \) defined on an open subset \( D \) of \( B \) is \( H \)-analytic at a point \( x \in D \) if there exists an element \( T \in \mathcal{H}(B) \) such that \( f(x + h) - f(x) = Th + \|h\|e(x,h) \), where \( \varepsilon \) tends to zero as \( h \) tends to zero. The theory of \( H \)-analytic functions is discussed and it is shown among other things that Dunford analytic functions are \( H \)-analytic. (Received December 9, 1960.)


The notion of maximal correlation between two random variables (H. Gablelein, Das Statistische Problem, Z. Angew. Math, Mech, vol. 21 (1941) pp. 364-379) is extended to that of maximal correlation \( S(A,B) \) between two subfields \( A \) and \( B \) of the Borel field of a probability space \((\Omega, \Sigma, P)\). Let \( E_A \) and \( E_B \) be the projections on the manifolds of random variables measurable on \( A \) and on \( B \). \textbf{Theorem 1.} \( S(A,B) = \|E_B E_A \| \). \textbf{Theorem 2.} If \( Q = E_A E_B E_A \) is completely continuous, then \( S(A,B) = \Lambda^{1/2} \) where \( \Lambda \) is the largest eigenvalue of the positive definite operator \( Q \). \textbf{Theorem 3.} If \( X \in L_2(\Omega) \) and \( a(X) = \|E_A X\|/\|X\| \) and \( b = \|E_B X\|/\|X\| \), then \( \cos^{-1} a + \cos^{-1} b \geq \cos^{-1} S(A,B) \). Theorem 3 implies that if \( A \) and \( B \) are independent, then \( 0 \leq a(X) \leq 1, 0 \leq b(X) \leq 1 \) without additional restriction. However, if \( A \) and \( B \) are dependent, then \( a = b \). Cases that are intermediate have restrictions of intermediate severity. If \( a \) and \( b \) are taken as measures of uncertainty or information,
Theorem 3 relates the information gained from one measurement of a random variable to the information gained from a different measurement as a function of the interdependence of the two measurements. (Received December 9, 1960.)


A result of M. Marcus and L. Lopes (Inequalities for symmetric functions and Hermitian matrices, Canad. J. Math, vol. 9 (1957) pp. 305-312) is extended to give a new inequality for the elementary symmetric functions of the roots of the non-negative Hermitian matrices, A, B and A + B. If \( E_r(A) \) represents the \( r \)th elementary symmetric function of the roots of A, \( E_r(A + B) \)
\[
\geq \prod_{j=1}^{r} \left[ \frac{E_j(A)}{E_{j-1}(A)} + \frac{E_j(B)}{E_{j-1}(B)} \right] \text{ where } E_0(A) \text{ is defined to be 1.}
\]
This inequality is used to find a positive lower bound for the determinant of a positive definite symmetric matrix A, with positive elements. If \( \text{Det } A \) represents the determinant of A, \( T_rA \) represents the trace of A and \( M = \text{diag } (m_1, m_2, \ldots, m_n) \) where \( m_i \) is the minimum element in the \( i \)th row, then \( \text{Det } A/T_r A \geq 2(n - 2)^{n-1} \text{ Det } M/T_r M \). (Received December 9, 1960.)

576-215. N. J. Divinsky: Kurosh radicals that are nil on rings with D.C.C.

The general theory of radicals a la Kurosh is considered. Let \( N \) be the nil radical property. It is shown that a radical property \( Q \) coincides with \( N \) on rings with D.C.C. if and only if \( Q \) is between \( U \), the upper radical property determined by all finite dimensional total matric rings over division rings and \( D \), the lower radical property determined by all nilpotent rings which are nil radicals of rings with D.C.C. This sharpens Kurosh's statement that \( Q \) could be as small as \( L \), the lower radical property determined by all the zero simple rings. An example is given for which \( L \not\supset D \). Another example shows that \( D \not\supset B \), the Baer lower radical. The final conclusion then is that in general, \( L \not\supset D \not\supset B \not\supset N \not\supset U \) and \( Q \) coincides with \( N \) if and only if \( D \leq Q \leq U \). (Received December 9, 1960.)

Let $D$ denote the Hilbert space of functions $f(z)$ analytic for $|z| < 1$ with $f(0) = 0$ and $\int |f'(z)|^2 dx dy < \infty$ (or in terms of Taylor series, with $\sum n|a_n|^2 < \infty$). If $\{z_n\}$ is a sequence of points in the unit circle with no interior limit point, we wish to know when there will exist an $f \in D$ vanishing at all these points but not vanishing identically. Since $D$ is a subset of the Hardy space $H_2$, a necessary condition for the existence of such an $f$ is that $\sum (1 - |z_n|) < \infty$. Olli Lokki stated in his thesis (Acad. Fenn. No. 39 (1947)) that this condition is also sufficient, however his proof contains an error. In fact we show that there is a sequence $\{z_n\}$ on which no $f \in D$ ($f \not= 0$) can vanish, and for which $\sum (1 - |z_n|)^\varepsilon < \infty$ for every $\varepsilon > 0$. On the other hand we show that if the $z_n$ all lie on a radius (or all lie inside some polygon inscribed in the unit circle) and if $\sum (1 - |z_n|) < \infty$, then there is an $f \in D$ having these zeros and no others in the unit circle. (Received December 9, 1960.)

576-217. F. L. Hardy: Groups of associative ring multiplications.

If $(A,+)$ is an Abelian group, the set $\mathcal{D}$ of all distributive binary compositions on $A$ form a group with the operation $\circ$ defined by: $x[\theta_1 \circ \theta_2]y = x\theta_1 y + x\theta_2 y$ for $\theta_1, \theta_2 \in \mathcal{D}$ and all $x, y \in A$. $\mathcal{D}$ is then the set of ring multiplications on $A$. In general the set $\mathcal{A} \subset \mathcal{D}$ of associative ring multiplications on $A$ do not form a group with respect to this operation. Problem 62 of Fuchs' Abelian groups asks for a characterization of those groups on which the associative ring multiplications do form a group. The following theorems are proved. **Theorem 1.** The set $\mathcal{A}$ of associative ring multiplications on a torsion-free Abelian group $(A,+)$ form a group if and only if $(A,+)$ is the infinite cyclic group. **Theorem 2.** The set $\mathcal{A}$ of associative ring multiplications on an Abelian mixed group $(A,+)$ do not form a group. **Theorem 3.** Let $(A,+)$ be an Abelian torsion group with at least two generators $\{e_i\}$ and $e_i$ has order $n_i$. Then the set $\mathcal{A}$ of associative ring multiplications on $(A,+)$ form a group if and only if $(n_i,n_j) = 1$ for all $i \neq j$. (Received December 9, 1960.)

The postulates for a probability functional space as stated in the study of computable probability spaces (Acta Math. vol. 103 (1960) pp. 89-122) are augmented to include the assumption of weak monotone convergence (Abstract 558-57, Notices Amer. Math. Soc. vol. 6 (1959) p. 392). A function, \( F_X(t) \), for any random variable, \( X \), is then defined (for probability functional \( E \)) by

\[
F_X(t) = \lim_k E c_k(X)
\]

where \( c_1, c_2, \ldots \) is a sequence of real continuous functions defined on the real line and converging monotonely to the indicator function of the set \((-\infty, t)\). It is proved that \( F_X(t) \) is a distribution function and satisfies the requirements for a distribution function of \( X \). It is also shown that the assumption of weak monotone convergence and, thus, the ensuing analysis is true in the special case of computable probability spaces where monotone convergence, in general, is invalid. (Received December 9, 1960.)


Let \( X \) be a completely regular space; \( \tilde{X} \) its Stone-Čech compactification; \( C(X) \) the ring of all crvf's (continuous real-valued functions) on \( X \); \( Q(X) \) the ring of all crvf's on dense open subsets of \( X \), with two such f's identified when they coincide on the intersection of their domains. Put \( A^* = \) bounded elements of \( A \) and \( \tilde{S} = \) completion of a metric space \( S \). Then \( C(X) \) is an rq (ring of quotients) of \( C^*(X) \) [\( \cong C(\beta X) \)], and \( Q(X) \) is the maximal rq (in the sense of R. E. Johnson and Y. Utumi) of \( C(X) \) and \( C^*(X) \), so that \( Q(X) \cong Q(\beta X) \); indeed, every crvf on a dense open set in \( X \) has an extension to a crvf on a dense open set in \( \beta X \). One can put a natural metric on \( Q \) (though in general, \( Q \) will not become a topological ring). Then \( \overline{Q} \) is the (Dedekind) completion of \( Q \) and is isomorphic to the ring of all crvf's on dense \( G_\delta \)'s in \( \beta X \), identified as before. The D completion of \( Q(X) \) consists of those elements of \( \overline{Q} \) that are bounded by members of \( C \). Next, \( \overline{Q} \) is the maximal rq of \( \overline{Q}^* \), and \( \overline{Q}^* \) coincides with \( \overline{Q}^* \) and is both the D completion and the largest normed rq of \( C^* \). In special cases, such as when \( X \) is metric, \( Q \) is the classical rq of \( C \); also, \( \overline{Q} \) is invariant for all complete separable metric \( X \) without isolated points. For any \( X \), the maximal ideal spaces of \( Q, Q^*, \overline{Q}, \) and \( \overline{Q}^* \) are all homeomorphic. (Received December 9, 1960.)

If the sequence \( \{\lambda_n\}_0^\infty \) with \( 0 < \lambda_0 < \lambda_1 < \ldots < \lambda_n \to \infty \), and
\[
\sum_{n=1}^\infty \lambda_n^{-1} = \infty,
\]
is fixed, the set of solutions of the system \((-1)^{m-n} \left( \frac{\mu_n}{\lambda_n} \ldots \frac{\mu_m}{\lambda_m} \right) \geq 0\) is identical with the set of moment sequences \( \mu_0 = \bar{\mu} + \int_{\Delta_0} \lambda_0 \, d\chi(x), \mu_n = \int_{\Delta_0} \lambda^n d\chi(x) \) (\( n \geq 1 \)), where \( \bar{\mu} \geq 1 \) and \( \int_{\Delta_0} \lambda^n d\chi(x) < \infty \) (theorem of I. J. Schoenberg). An open problem has been the characterisation of those moment sequences for which \( \bar{\mu} = 0 \) and \( \chi(x) \) is bounded for \( x \to 0 \), i.e. the determination of the system of inequalities whose set of solutions is identical with the set of moment sequences \( \{\mu_n\}_0^\infty \). It is shown that for the system of inequalities is \((-1)^{m-n} \left( \frac{\mu_n}{\lambda_n} \ldots \frac{\mu_m}{\lambda_m} \right) \geq 0\), \((-1)^m \left( \frac{\mu_0}{\lambda_0} \ldots \frac{\mu_m}{\lambda_m} \right) \leq 1\). The method is based on the investigation of regularity problems of generalized Hausdorff means as recently introduced by A. Jakimovski. In the general case the problem is reduced to an interpolation problem at infinitely many points. (Received December 9, 1960.)


Consider an elliptic differential equation of the form \( \sum_{k,l=1}^n \left( \frac{\partial}{\partial x_k} \right)^2 \cdot (a_{kl}(x) \frac{\partial u}{\partial x_l}) = 0 \) where the \( a_{kl}(x) \) are measurable and the eigenvalues of the symmetric matrix \( (a_{kl}) \) are assumed to lie between the constants \( \lambda^{-1} \) and \( \lambda > 1 \). A solution is considered in the weak sense and is assumed to have a finite Dirichlet integral. For such differential equations a generalization of Harnack's theorem can be proven: If \( u > 0 \) is a weak solution of the above equation in \( |x| < 2 \) then there exists a constant \( c \) depending on \( n, \lambda \) only such that \( u(x) \leq cu(y) \) in \( |x|, |y| < 1 \) almost everywhere. This theorem implies, in particular, the Hölder continuity of the solution in the interior which was proven by de Giorgi in 1956. For \( n = 2 \) such a Harnack theorem had been proven by Bers, Nirenberg, Serrin. These proofs, however, are not generalizable to \( n > 2 \). Such a theorem was announced by J. Nash without a detailed proof. (Received December 9, 1960.)
Josephine Mitchell: Abelian and Tauberian theorems for orthogonal series on matrix spaces.

Let \( D \) be the domain defined by \( I - z z^* > 0 \) where \( z \) is an \( n \) by \( n \) matrix of complex numbers, \( z^* \) its conjugate transpose and \( I \) the identity matrix. It is known that there exists an orthonormal system \( \{ \phi_n \} \) of homogeneous polynomials on \( D \), complete with respect to functions of class \( L^2 \), that is, analytic functions with finite norm: \( \int_D |f|^2 dV \) (\( dV \) Euclidean volume element). For the orthogonal series \( S(z) = \sum_{n=0}^{\infty} a_n \phi_n(z) \) the following Abelian and Tauberian theorems hold: If \( S(I) \) exists and the series \( S(r) \) has bounded partial sums where \( r \) is a diagonal matrix in \( D \), then \( S(r) \to S(I) \) as \( r \to I \). If the series \( S(r) \) converges for \( r \) in \( D \), \( S(r) = 0(1) \), and the coefficients \( a_n \) satisfy a certain boundedness condition, then \( \lim_{r \to I} S(r) = S \) implies \( S(I) \) converges to \( S \). If \( [a_n] \) are the Fourier coefficients of a function of class \( L^2 \), they satisfy a certain Cauchy's inequality; also two mean value theorems may be proved which generalize analogous theorems for the unit disk. (Received December 9, 1960.)


For notations see Notices Abstract 564-104 vol. 6 p. 812). Assume now that the \( R_i \) are not necessarily \( 1 - 1 \): Replace \( N \) by \( N' = N/\Delta \), where \( \Delta \) is the equivalence generated by the \( T \) with \( T \subseteq RS^{-1} \) or \( R^{-1}S \), \( R' \subseteq S \), and \( R, S, T \in R' \). All results transfer to the induced \( R' = \{ 1 \} \), \( C' \) and \( R'/C' \). But if one suppresses axiom (1), the definition of \( C' \) must be supplemented by transitive closure, and \( R'/C' \) will be (in general) a hypermonoid (partial and/or many valued nonassociative multiplicative system), even if all individual \( R_i \) are \( 1 - 1 \).

Concepts as analytic continuation, monodromy, branching, carry over in a natural way from the theory of functions. As a particular case one interpretes the Malcev-conditions (n.a.s.c. for the imbeddability of a semigroup into a group) as conditions for "analytic" continuability, monodromy, uniqueness and associativity of composition (by substitution) of certain functions (quotients or binary \( r_\_ \)). This raises the problem of representation of nonassociative systems (e.g. loops) by families of analytic functions. (Received December 9, 1960.)

In higher order approximation methods for the solutions of the differential equation \( y' = f(x,y) \) such as the widely used Runge-Kutta method, it is assumed that \( f(x,y) \) must have a certain number of continuous partial derivatives. For instance in the Runge-Kutta method the continuity of nine partial derivatives: \( f_x, f_y, f_{xx}, f_{xy}, f_{yy}, f_{xxx}, f_{xxy}, f_{xyy}, f_{yyy} \) is assumed. Numerous examples can be given where Runge-Kutta method is applicable in spite of the fact that all these nine partial derivatives are discontinuous at the given initial point \( P_0(x_0,y_0) \). A method of solution is established, which requires only the continuity of \( f(x,y) \), based on a general formula. This method includes the higher order approximation methods and related formulas as special cases. In particular, when there is more than one integral curve passing through \( P_0 \) \( (f_y \) discontinuous), and if one of these curves is a straight line, the Runge-Kutta method will give this line. In order to obtain the other integral curves one should vary continuously the parameters which are involved in the above mentioned general formula. (Received December 9, 1960.)


By simple axiomatic set theory will be meant a theory whose primitive notions are element, class, =, and \( \in \) such that (1) a set is an element which is a class, (2) an atom is an element which is not a class, and (3) modifications of the usual axioms of (e.g.) Bernays hold except that instead of a universal class there is for each class \( A \) of atoms a "subuniversal" class built on \( A \) in an obvious way. Given a simple axiomatic set theory \( \Sigma_0 \), by an augmentation of \( \Sigma_0 \) will be meant an axiomatic set theory \( \Sigma_1 \) (whose primitive notions should be distinguished from those of \( \Sigma_0 \) whose atoms are the objects (i.e., elements and classes) of \( \Sigma_0 \), Theorem. Every model of simple theory contains inner models \( M_0 \) and \( M_1 \) such that \( M_0 \) is isomorphic with \( M \) and \( M_1 \) is an augmentation of \( M_0 \). Corollary 1. Every statement of simple theory which is a theorem of augmented theory is a theorem of simple theory. Corollary 2. If simple theory is consistent, so is augmented theory. These results may be sharpened by requiring axioms linking a simple theory more strongly with its augmentation. (Received December 9, 1960.)
576-226. Trevor Evans: Abstract mean values. II.

Let \( M \) be an \( n \)-ary operation on a set \( S \) satisfying the identity
\[
M(M(r_1, M(r_2), \ldots) = M(M(c_1, M(c_2), \ldots) \text{ where } r_1, r_2, \ldots, c_1, c_2, \ldots \text{ are the rows and columns of an arbitrary } n \times n \text{ array of elements of } S. \text{ This is one of the conditions assumed by Aczel and other authors for continuous real functions in characterizing mean values. It is shown in this paper under the condition only that } S \text{ contains one regular element with respect to the operation } M, \text{ that there is a commutative semigroup with zero having } S \text{ as its set of elements such that }

M(x_1, x_2, \ldots) = x_1a_1 + x_2a_2 + \ldots + x_na_n + d, \ a_1, a_2, \ldots, a_n \text{ being fixed pairwise commuting automorphisms of the semigroup and } d \text{ being a fixed element of it. From this follows characterizations of mean values essentially the same as those obtained for the real continuous case but without these assumptions. In addition, this result describes completely the structure of quasigroups and groupoids with a regular element which satisfy the multiplicative law } (xy)(zw) = (xz)(yw), \text{ the properties of which have been studied by Murdoch, Etherington, Stein and others.} (Received December 9, 1960.)


Suppose that: (1) the vector valued function \( x(t) \) is defined for all real \( t \) such that \( a \leq t \leq b \), where \( a < b \), and that its values are in a linear normed vector space \( B \) (the norm in \( B \) will be denoted by \( \| \| \) ); (2) \( \lim_{t \to a} x(t) = x(a) \) and \( \lim_{t \to b} x(t) = x(b) \) (i.e., for example \( \lim_{t \to a} \| x(t) - x(a) \| = 0 \) ); (3) the derivative \( x'(t) \) exists, and is finite, whenever \( a < t < b \) (i.e., there is a vector \( x'(t) \) in the space \( B \) such that \( \lim_{s \to t} \| k(s) - x(t)(s - t) - x'(t) \| = 0 \). Then there is a number \( \tau \), with \( a < \tau < b \), such that \( \| k(b) - x(a)/(b - a) \| \leq \| x'(\tau) \| \). (Received December 9, 1960.)


Let \( x \) and \( y \) be distinct points in the real topological linear space \( X \). The internal segment \( \bar{x}y \) is the set \( \{ z : z = \lambda x + (1 - \lambda)y, \ 0 \leq \lambda \leq 1 \} \) and the external segment \( \overline{xy} \) is the set \( \{ z : z = \lambda x + (1 - \lambda)y, \ \lambda \leq 0 \text{ or } 1 \leq \lambda \} \). A subset of the real topological linear space \( X \) is said to be projectively convex (here-
after, p.c.) if and only if, for two points \( x \) and \( y \) of \( S \), either the internal or the external segment is contained in \( S \). It is immediate that a convex set is p.c., so this is a "natural" generalization of convexity. Several general results for p.c. sets are obtained. Two of these are: (1) The interior and closure of a p.c. set are p.c.; and (2) if a closed or open p.c. set lies on one side of a hyperplane, then the set is either convex or is contained in a hyperplane. A theorem is proved which relates projective convexity with property \( P_3 \) in two-dimensional spaces. (See, F. A. Valentine, A three point convexity property, Pacific J. Math. vol. 7 (1957)). A set \( S \) has property \( P_3 \) if and only if, for points \( x,y,z \) of \( S \), one of the internal segments \( \hat{xy}, \hat{xz}, \) or \( \hat{yz} \) is contained in \( S \).) This theorem is:

Let \( S \) be a p.c. set in a two-dimensional topological linear space, and suppose that \( S \) does not have property \( P_3 \). Then (1) \( S' \) has property \( P_3 \), (2) \( S \) has at most two components, and (3) if \( S \) is open or closed, it is connected. (Received December 7, 1960.)


The following theorem characterizes the linear elliptic differential operators with variable coefficients: A differential operator of order \( m \):

\[
P(x,D) = \sum_{|\alpha| \leq m} a_\alpha(x)D^\alpha \quad (a_\alpha(x) \in \mathbb{E})
\]

elliptic if its principal part \( P_0(x,D) = \sum_{|\alpha| \leq m} a_\alpha(x)D^\alpha \) has the property, that for any open subset \( \Omega \subset \mathbb{R}^n \), \( P_0(x,D)\phi \in L^0_\Omega \) implies \( \phi \in L^m_\Omega \). Here \( \mathbb{R}^n \) denotes the \( n \)-dimensional Euclidean space; \( L^0_\Omega \) is the space of the locally square summable functions in \( \Omega \), and \( L^m_\Omega \) is the space of the functions \( f \), such that for each \(|p| \leq m\), \( D^p f \) is locally square summable in \( \Omega \). (Received December 9, 1960.)


Let \( A \) be a contraction (\( \|A\| \leq 1 \)) on complex Hilbert space \( H \), for which there exists an integrable function \( K(t,x,y) \) defined for each \( x,y \in H \) for a.e.\( t \),

\[
0 \leq t \leq 2 \pi,
\]

such that (1/2 \( n \)) \( \int_0^{2\pi} e^{i\tau \phi} K(t,x,y)dt = (A^nx,y) \) for \( x \not= 0 \) and

\[
(A^nx,y) \text{ for } n \leq 0.
\]

Such \( A \) we call absolutely continuous (abbr. (ac)). The unilateral and bilateral shifts are (ac), as are multiplications on \( L^1_2 \) by sequences \( [a_n] \) with \(|a_n| < 1 \) (which includes convolution by suitable \( L^1_1 \) functions), and in particular proper contractions \( A (\|A\| < 1) \) (ac), with the additional property (which is characteristic of proper contractions) that \( \|K(\cdot,x,y)\|_\infty \leq c \|x\| \|y\|, c \)
a constant. For (ac) operators earlier results [Trans. Amer. Math. Soc. vol. 87 (1958) pp. 108-118] are extended as follows. The map \( f \rightarrow f(A) \) is a norm-decreasing algebra homomorphism on \( H^\infty \), the space of functions bounded and analytic for \(|z| < 1\), into the strong closure of the algebra generated by \( A \) and \( I \) (the identity operator); if the approximate point spectrum \( S_{ap} \) is dense on \(|z| = r\) for some \( 0 < r < 1 \) then the homomorphism is an isomorphism; and if \( S_{ap} \) is dense in \(|z| \leq 1\) then it is an isometry (onto a norm-closed subring). The isometry holds also for functions of \( A^* \) when it does for functions of \( A \). It follows that these particular "fullness of spectrum" conditions are not necessary for isometry. In weaker form (restriction to subsets of \( H^\infty \)) these results persist for arbitrary contractions. (Received December 9, 1960.)


The validity of the Radon-Nikodym theorem with regard to a non-\( \sigma \)-finite measure \( \mu \) in the point set \( X \) was investigated by I. E. Segal (Am. J. Math. vol. 73 (1951)) under the additional hypothesis that \( \mu \) has the finite subset property (i.e. any \( E \) with \( \mu(E) > 0 \) contains a subset \( F \) with \( 0 < \mu(F) < \infty \)), and one of Segal's main results is that the R.-N. theorem holds if and only if \( \mu \) is localizable. The proof, in his paper, is imbedded in the proof of a more extensive theorem connecting localizability also with some further properties, and it seemed worthwhile to indicate a somewhat more direct way from localizability to the R.-N. theorem and backwards, and to pay attention also to the case when \( \mu \) does not have the finite subset property. (Received December 9, 1960.)


Theorem 1. On a Hilbert space \( \mathcal{H} \), \( P \) is a (not necessarily s.a.) projection iff there exist (I) a bounded s.a. operator \( S \), \( \exists S^2 - S \geq 0 \) (II) a unitary operator \( U \) on \( \mathcal{H} \) (i) \( U^2 = -I \), (ii) \( SU = U(I - S) \), in terms of which we have (*) \( P = S + U(S^2 - S)^{1/2} = \lambda^{1+a} \lambda \ dE + \lambda^{1+a} (-\lambda)^{1/2} \ dE_\lambda \), for some \( a \geq 0 \). Here \( \{E_\lambda\} \) is the spectral family of \( S \), the s.a. part of \( P \). Theorem 2. On \( \mathcal{H} \) a bounded s.a. operator \( S \) is the s.a. part of a projection iff (I) \( S^2 - S \geq 0 \), and (II) \( S \mid \mathcal{H} \) and \((I - S)|\mathcal{H}\) are unitarily equivalent. Def. If \( \mathcal{H} \) is real
P_1 \succ P_2$ signifies that $(x,Ax) \leq 0$ for all $x$ ($A = P_1 - P_2$ not necessarily s.a.). For complex $\mathcal{X}$ it signifies $\text{Re} \{ (x,Ax) \} \geq 0$. $\mathcal{C}$ denotes the class of projections $\{ P \}$ whose s.a. parts $\{ S \}$ possess compact negative part. **Theorem 3:** If $P_1,P_2 \in \mathcal{C}$, then the following conditions are equivalent: (I) $P_1 \succ P_2$, (II) $S_1^2 - S_1 = S_2^2 - S_2 = Q$ with (i) $S_1 = S_2$ on $\mathcal{R}_Q$, (ii) $S_1(\mathcal{N}_Q) \subset S_2(\mathcal{N}_Q)$, (III) $E_\lambda^{(1)} = E_\lambda^{(2)}$, $\lambda \notin [0,1)$ and $\mathcal{K}_{E_\lambda^{(1)}} \subset \mathcal{K}_{E_\lambda^{(2)}}$, where $\{ E_\lambda^{(j)} \}$ is the spectral family of $S_j$. A counterexample shows that some restriction such as $P_i \in \mathcal{C}$ is necessary for the theorem. (Received December 9, 1960.)


Let $I$ denote the interval $[0,\infty]$ and $\mathcal{R}$ a normed finite dimensional real or complex vector space. Let $C$ denote the set of continuous mappings from $I$ into $\mathcal{R}$, $B$ the subset of bounded and continuous mappings. $B_0$ denotes the set bounded and continuous mappings from $\mathcal{R} \times I$ into $\mathcal{R}$, such that for $Q \in B_0$ the mapping $t \mapsto Q(0,t)$, $t \in I$ is the zero-mapping in $C$. With $Y \in B_0$ and $K$ a suitable matrix, define the mapping $T \mapsto T(f,X,Q) (t) = Y(X,t) + \int_0^t K(t,s)Q(s(s),s)ds$ from $I$ into $C$ where $(s,X,Q) \in C \times \mathcal{R} \times B_0$. Define finally the mapping $d: X \mapsto \text{Max}(\lim_{t=\infty} \| Y(t,X) \|, \lim_{t=\infty} \sqrt{\int_0^t \| K(t,s) \| ds})$. Then the following statement holds: $d = 0$ is a necessary and sufficient condition that for every fixed point, $f^+$, of every mapping $f \mapsto T(f,X,Q)$, $f \in C$, with $(X,Q) \in \mathcal{R} \times B_0$ holds $\lim_{t=\infty} \| f^+ (t) \| = 0$. Since the zero mapping is a fixed point if $x = 0$ for every $Q \in B_0$ the latter condition is necessary and sufficient for quasi-asymptotic stability of the zero mapping. (Received December 9, 1960.)


Suppose that a metric space $X$ which has inductive dimensions $\leq n$ ($\text{ind} n \leq n$) also has property $P$, that is, there exists a sequence $\{ B_i \}$ such that (1) $B_i$ is a discrete collection of open sets, (2) $\text{ind} (B \cap g)$ less than or equal to $n - 1$ for each $g$ in $B_i$, and (3) $\sum B_i$ is a basis for $X$. One of the main results is the following. **Theorem.** If $X$ is a metric space, $\text{ind} X \leq n$ and $X$ has property $P$, then $\text{ind} X = \text{Ind} X$. Smirnov-Morita has shown that $\text{ind} X = \text{Ind} X$ for metric spaces with the star finite property. It is observed that a connected metric space with the star finite property is separable. One condition that a
metric space X has property P is that there exists $\varepsilon > 0$ such that for each subset A in X with diameter $< \varepsilon$ it is true that ind $A = \text{Ind} A \leqslant n$. The main result holds for collectionwise normal spaces. (Received December 9, 1960.)

576-235. Frank Raymond: **Local cohomology groups with closed supports.**

Let X be a locally compact cohomology locally connected Hausdorff space. Then it is shown that for each $x \in X$ the sequence, $0 \rightarrow \text{Ext} \left( H^C_p(X, X - x; \mathbb{K}), B \right) \rightarrow I^C_p(X; B) \rightarrow \text{Hom} \left( H^C_{p+1}(X, X - x; \mathbb{K}), B \right) \rightarrow 0$, is exact. In this universal coefficient theorem for local groups $\mathbb{K}$ is a principal ideal domain, $B$ is a $\mathbb{K}$-module, $H^C_p$ denotes the Borel-Moore homology theory with compact supports, $I^C_p(X; B)$ is the direct limit $\bigoplus \{ H^p(U, X - x; B) \}$ where $\{ U \}$ forms a neighborhood system of $x$ and $H^p$ denotes the augmented Čech cohomology with closed supports. Such a formula, as pointed out by A. Borel, would be very useful in passing automatically from global situations to local situations in transformation groups. It is shown, e.g., that the Smith theorems for manifolds follow automatically from the Smith theorems for homology spheres. For compact generalized n-manifold X it is shown, as a consequence of the methods used, that $H^C_p(X, X - Y; \mathbb{K}) \cong H^{n-p}(Y; J)$, where $Y$ is an arbitrary subset of X and $J$ is the orientation sheaf. This gives a new proof and extension of K. Sitnikov's duality theorem. (Received December 9, 1960.)

576-236. P. C. Hammer: **The Wallace functions for sets of ordered pairs of sets.**

Let $M$ be the space and $N$ its null set. Let $\overline{M}$ be the class of all subsets of $M$. Let $\overline{T} \subseteq \overline{M}^2$, the Cartesian square of $\overline{M}$. For weak separation sets $T$, A. D. Wallace (Ann. of Math., vol. 42, pp. 687-697) defined a function $w$. I extend this definition for arbitrary $T$ and also define the conjugate Wallace function $w^*$ and an alternate pair $w_1, w_1^*$ of Wallace functions. Let $cX = M - X$, i.e. $c$ is the complement function. $wX = \bigcap \{ cY: (Y, X) \subseteq T \}$, $w^*X = \bigcap \{ cwY: (X, Y) \subseteq T \}$, $w_1X = \bigcap \{ cY: (X, Y) \subseteq T \}$, $w_1^*X = \bigcap \{ cw_1Y: (Y, X) \subseteq T \}$. Then $(w^*)^* = w$, $(w_1^*)^* = w_1^*$, $w^*X \supseteq X$, $w_1^*X \supseteq X$ and $T \subseteq \{(X, Y): [w^*X \cap wY] \cup [w_1X \cap w_1^*Y] \}$ = $N_j$. Restrictions on $T$ are introduced and the effects on the Wallace functions studied. (Received September 29, 1960.)
576-237. H. E. Salzer: **An interpolation formula for "nearly-odd" functions, with an application to the summation of even functions.**

In an earlier work, it was shown how replacement of the convergent sequence $S_j = \sum_{k=1}^{j} f(k)$ by $S'_j = \sum_{k=1}^{j-1} f(k) + f(j)/2$, for many odd functions $f(x)$, led to a formal expansion in $1/j^2$ for $S'_j$, by application of the Euler-Maclaurin formula. For $n$ points, Lagrangian extrapolation for variable $1/j^2$ instead of $1/j$ gave much greater accuracy in determining $S'_j$ for $j = m$ or $j = \infty$. However, for $f(x)$ even, we need a new basic $n$-point interpolation polynomial, $P_{2n-3}(x) = a_0 + a_1 x + a_2 x^3 + \ldots + a_n x^{2n-3}$, which is equal to $\phi(x_i)$ at $x = x_i$, $i = 1, 2, \ldots, n$, and applicable to $S'_j = b_0 + b_1/j + b_2/j^3 + \ldots$ (i.e., $S'_j$ is "nearly-odd"). The condition for uniqueness, $\det (x_1 x_2 \ldots x_{2n-3}) \neq 0$, is not always satisfied. The additional assumption of every $x_i \neq 0$ precludes both immediate expression of $P_{2n-3}(x)$ in terms of a usual $(n-1)$-point Lagrangian polynomial in $X = x^2$, as well as initial knowledge of $\lim_{j \to \infty} S'_j$. Two proofs are given that $P_{2n-3}(x) = \sum_{l=1}^{n} \phi_l(x)/\phi'_l(x_j) \phi(x_i)$, where $\phi_l(x) \equiv 1 - \sum_{l=1, l \neq 1}^{n} (x/x_j) \prod_{k=1, k \neq i, k \neq j}^{n} [(X - X_k)/(X_j - X_k)]^l$. From $P_{2n-3}(x)$, setting $n = 7$, $x_i = 1/(i + 3), \phi(x_i) = S'_{i+3}$, $x=0$, we obtain an extrapolation formula for $\lim_{j \to \infty} S'_j$. Tests upon $S'_j = \sum_{l=1}^{j-1} 1/[l^2 + 1/2j^2 + 1/2j^2] + S'_j = 1 - 2 \sum_{l=1}^{j-1} [1/(16x^2 - 1)] - 1/(16j^2 - 1)$ gave the answers $\pi^2/6$ and $\pi/4$ to 3 more places than by a previous formula. (Received Dec. 5, 1960.)

576-238. Benjamin Lepson: **The maximum values of some trigonometric determinants.**

The following occur as system determinants of linear systems arising in the theory of electrical networks: Case I with $a_{ij} = \cos (j - 1) x_i$ for $i, j = 1, \ldots, n$; Case II with $a_{ij} = \sin j x_i$ for $i, j = 1, \ldots, n$; Case III with $a_{ij} = \cos (j - 1) x_k$ or $\sin (j - 1) x_k$ if $i$ is odd or even, respectively, where $k$ is whichever of the two numbers $i/2$ and $(i + 1)/2$ is integral, for $i, j = 1, \ldots, 2n$. They can be evaluated to yield the following: Case $I = 2(n-2)(n-1)/2 \prod_{i,j=1, i \neq j}^{n} (\cos x_i - \cos x_j) = D$; Case $II = Dz^{n-1} \prod_{i=1}^{n} \sin x_i = E$; Case $III = Dz^{n-1} E$; [see Muir, History of determinants (1930), p. 200, for these results in Cases I and II]. The maxima for cases I and II, in algebraic rather than trigonometric form, were obtained by Stieltjes [C. R. Acad. Sci. Paris, vol. 100 (1885), pp. 439-440]. These occur when the $x_i$ are the roots of the integrals of the Legendre polynomials and of the Legendre polynomials themselves, respectively, of degree $n$. A proof for
case I is given by Schur [Math. Zeit. vol. 1 (1918) pp. 377-402], which involves the differential equations for these polynomials. This paper gives a similar result and proof for Case III, which has apparently not been considered previously, which shows that the maximum value is \(n^n\) and that this value is attained when the \(x_i\) are the roots of the Tschebycheff polynomial of the first kind of degree \(n\). (Received December 9, 1960.)

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Robert Hermann: Totally geodesic \textit{orbits of the isotropy group of an irreducible compact symmetric space.}

Page 355, Abstract 567-107. In line 7 replace "with the elements of order 2 of \(K\)" by "with the elements of order 2 of \(\text{Exp} M\), where \(M\) is the orthogonal complement of \(K\) in \(G\)."

G. E. Sacks: \textit{A continuum of pairwise incomparable degrees.}

Page 370, Abstract 570-6. In lines 11-12 replace "any member" by "the members of any finite subset".
60T-1. Edgar Asplund: Nestinvertible matrices.

A nonsingular matrix A = (a_{ij}), 1 \leq i \leq n, 1 \leq j \leq n with inverse A^{-1} = B = (b_{ij}), 1 \leq i \leq n, 1 \leq j \leq n is called nestinvertible if for each integer k such that 1 \leq k \leq n the submatrix (b_{ij}), n - k + 1 \leq i \leq n, n - k + 1 \leq j \leq n of B is the inverse of the submatrix (a_{ij}), 1 \leq i \leq k, 1 \leq j \leq k of A, and moreover a_{11} = b_{nn} = 1. **Theorem 1.** A nestinvertible matrix is unimodular. **Theorem 2.** A matrix A = (a_{ij}) is nestinvertible if and only if its elements are of the form a_{ij} = \sum_{k=0}^{1+\text{Min}(i,j)} c_{i-k}d_{j-k} with c_1 = d_1 = 1. The theorems are valid for matrices over arbitrary associative rings with a unit except that for Theorem 1 one has to have some kind of determinant defined. (Received October 5, 1960.)

60T-2. V. E. Beneš: A fixed point method for studying the stability of a class of integrodifferential equations.

The integrodifferential equation \( \dot{x} = \omega - k * F(x) \), \( t \geq 0 \), with \( \omega \) a constant, \( F(\cdot) \) a nonlinear function, \( k(\cdot) \) an impulse response, and * = convolution, arises in the theory of various synchronization phenomena. The behavior of \( \dot{x} \) as \( t \to \infty \) is studied by finding a fixed point (corresponding to a solution of the equation) in a specific set of a function space. The method used depends on Schauder's theorem, and requires only that \( F(\cdot) \) be bounded and Lipschitzian, that \( k(\cdot) \) be bounded and integrable, and that two integral inequalities involving the rate of convergence of \( \int_0^t k(u)du \) to its limit have positive integrable solutions vanishing at \( \infty \). (Received October 27, 1960.)


Let \( \Omega_r \) be the determinant of order \( k \) with \( j \)th column 1, \( x_j',...,x_j^{k-2}, x_j^r \) \( \partial/\partial x_j \) and let \( T \) be the difference product of \( x_1,...,x_k \). The paper is concerned with equations of the type (*) \( T^{-1}\Omega_r F = S \), where \( S \) is symmetric. It is shown first that any \( F \) satisfying (*) is necessarily symmetric. For \( 0 \leq r \leq k \), the equation is always solvable. For \( r = k \), the operator \( T^{-1}\Omega_r \) induces a nonsingular linear transformation on the space of symmetric polynomials of weight
N. If \( a_1, ..., a_k \) denote the elementary symmetric polynomials in \( x_1, ..., x_k \), then relative to the basis \( a_1, ..., a_k \) (\( n_1 = 2n_2 + ... + kn_k = N \)) the matrix of the linear transformation is in diagonal form. The characteristic values are the numbers \( \lambda = n_1 + n_2 + ... + n_k \). (Received October 4, 1960.)

60T-4. Eckford Cohen: Unitary functions (mod r), II.

In Part I (Notices Amer. Math. Soc. vol. 7 (1960) p. 491 Abstract 571-37) the class of unitary functions (mod r) was shown to be characterized by representations of the form (i) \( \sum d |r,d| \mu(d,r/d) \). In the present paper this class of functions is also shown to be characterized by representations of the type (ii) \( \sum d |r,(n,d) = 1 h(d,r/d) \). Moreover, a complete inversion theory of the representations (ii) is developed, corresponding to that developed in I for the representations (i). The trigonometric inversion theory of I is also reformulated and given a new treatment. Application is made to the evaluation of the function \( \omega_S(n,r) \), defined to be the number of integers \( x \) (mod r) such that \((x,r)=1\) and \((n-x,r) \in S\), where \( S \) denotes an arbitrary set of positive integers. Special cases of this combinatorial problem are considered in detail. Connections with the class of primitive functions (mod r) are noted, and a group-theoretical characterization of the latter class of functions is given. (Received October 10, 1960.)

60T-5. R. P. Gilbert: Singularities of three-dimensional harmonic functions I.

The Whittaker-Bergman operator, \( B_3(f,\mathcal{L}X^0) \) transforms functions of two complex variables, \( f(t,u) \), into harmonic functions in three variables, \( H(X) = B_3(f,\mathcal{L}X^0) \equiv (1/2\pi i)\int f(t,u)du/u \), where \( t = [(x_1 - ix_2)(u/2) + x_3 + (x_1 + ix_2)/2u] \), \( |X - X^0| < \epsilon \), \( X \equiv (x_1,x_2,x_3) \), \( \epsilon > 0 \) is sufficiently small, and \( \mathcal{L} \) is a closed differentiable arc. In this paper the singularities of \( H(X) = B_3(f,\mathcal{L}X^0) \) are investigated by using an extension of an idea originally employed by Hadamard in the proof of his multiplication of singularities theorem. We prove the following result, Theorem. If \( Z^3 \equiv B\{S(X;u) = 0\} \) is the singularity manifold of \( u^{-1}f(t,u) \), then \( H(X) = B_3(f,\mathcal{L}X^0) \) is regular at \( X \) providing this point does not lie simultaneously on the two surfaces \( S(X;u) = 0 \), and \( (\partial/\partial u)S(X;u) = 0 \). (Parts I and II are to appear in the Pacific J. Math.) (Received October 21, 1960.)
By continuing the arguments $x_1, x_2, x_3$, of $H(X) = B_3(f, \mathcal{L}^0)$ to the complex spherical polar coordinates, $r = \sqrt{(x_1^2 + x_2^2 + x_3^2)/2}$, $\zeta = ((x_1 + i x_2)/(x_1 - i x_2))^{1/2}$, $\xi = x_3/r$, one may obtain an inverse Whittaker-Bergman operator. In this paper the following lemma and theorem are proved: **Lemma.** Let

$$V(r, \cos \theta, e^{i \phi}) = H^{00}(X) = B_3(f, \mathcal{L}^0; x_0) = \left(\frac{1}{2\pi r} \right)^{1/2} g(t, u) du/u$$

be a harmonic function regular at infinity ($g(t, u) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} a_{nm} t^{-n} u^{-m}$). $g(s, u)$ may then be generated by the following integral operator $g(s, u) = B_4(V; C, \Gamma; s_0, u_0)^{-1} (1/4\pi i)^{-1/2} \int_{C} (r(s + t)/(s - t)^2) V(r, \xi, \zeta) d\xi/d\zeta$. The integrand is a single-valued analytic function of $\xi, \zeta$. The integrations' paths are the real axis connecting $-1$ to $+1$, and the unit circle in the $\xi, \zeta$-planes respectively. **Theorem.** Let $Z \equiv E^5 \{ x = \Phi(\xi, \zeta) \}$ be the singularity manifold of $V(r, \xi, \zeta) = H^{00}(X)(X \in C^3)$, where $C^3$ is a region in the three dimensional space of complex points, then the function $g(s, u) = B_4(V; C, \Gamma; s_0, u_0)$ is regular at $(s, u)$ providing this point does not lie simultaneously on the hypersurfaces $\Psi(s, u|\xi, \zeta) = \Phi(\xi, \zeta)[\xi + (i/2) \cdot (1 - \xi^2) \cdot (u/\xi) + (\xi^2/\zeta)] - s = 0$, and $\partial \Psi/\partial \xi + (\partial \Psi/\partial \zeta) \nabla(\xi) = 0$, where $\nabla = \nabla(\xi)$ is an arbitrary relationship between $\xi$ and $\zeta$. (Received October 21, 1960.)
concerning singularities of certain solutions to the wave equation, such as those obtained by Stieltjes' method, and axially symmetric wave functions, will appear in Crelle's Journal under the title Singularities of solutions to the wave equation in three-dimensions. An inverse for the operator $B_4(F; \mathcal{L}_u, \mathcal{L}_v, X^0)$ is also developed in this paper. (Received October 21, 1960.)

60T-8. A. A. Goldstein: Binary collisions in the n-body problem.

The present paper applies Sundman's methods for the regularization of binary collisions in the 3-body problem to the regularization of binary collisions in the n-body problem. Recurrence formulae for the coefficients of the Taylor series for the regularized differential equations are given. (Received September 20, 1960.)


Let $\mathcal{M}$ be the class of all subsets of a set $M$ with null set $N$. A pair $\{u,v\}$ of functions mapping $M$ into itself is called conjugate provided the following equalities hold for all $X$: $u_X \cap v_Y = N$, $v_X = \cap \{c_Y: u_Y \cap v_X = N\}$. Here $c$ is the complement function: $c_X = M - X$. Let $\{f,g\}$ be any pair of functions mapping $\mathcal{M}$ into itself. Then there exist two conjugate pairs of functions $\{f_1, g_1\}$ and $\{f_2, g_2\}$ such that the solutions $(X,Y)$ of each of the following equations comprise the same subset of $\mathcal{M}^2$: $f_X \cap g_Y = N$, $f_1 X \cap g_1 Y = N$, $f_2 X \cap g_2 Y = N$. The effects of restrictions on $f, g$ are studied. The definitions and procedures extend to the case in which $f$ and $g$ map an arbitrary set $D$ into $\mathcal{M}$. The Wallace functions of the preceding paper form conjugate pairs by the definition given here. However, the Wallace functions $w^*, w$ of $T = \{(X,Y): u_X \cap v_Y = N\}$ does not generally give $w^* = u, w = v$ when $\{u,v\}$ is a conjugate pair of functions. (Received September 29, 1960.)

60T-10. P. C. Hammer: Enlarging functions on partially ordered systems.

Theorem. Let $M$ be a set partially ordered by $(p \leq q, q \leq p$ implies $p = q)$ and suppose further that every simply ordered subset of $M$ has a supremum (unique) in $M$. Let $f: M \rightarrow M$ be an enlarging function, i.e. $f p \leq p$ for all $p \in M$. Then there exists an idempotent enlarging function $u$ mapping $M$ into itself.
such that \( f(up) = up \) for all \( p \) and \( \{up\} \) is the set of fixed elements of \( f \). In particular, \( f \) has at least one fixed point. **Note.** This theorem extends Theorem 5, p. 5 of Dunford and Schwartz, *Linear operators*, New York, 1958. An idempotent function \( u \) has the property \( u(up) = up \) for all \( p \). (Received September 29, 1960.)

60T-11. K. H. Hofmann: **Compact connected topological diassociative commutative loops.**

**Theorem.** Every compact connected topological loop, in which each pair of elements lies in a commutative subsemigroup is a group. **Remark.** There are compact connected topological diassociative loops, which are not associative (e.g. the Cayley numbers of norm 1) and locally compact connected topological diassociative and commutative loops, which are not associative. The theorem is proved by showing, that the set of all elements lying in some connected subgroup is a closed connected normal subgroup contained in the center, and that every loop as stated in the theorem contains at least one nontrivial connected subgroup. This suffices to show, that the factorloop modulo the maximal connected subgroup must be trivial. By the same methods the structure of all compact topological diassociative and commutative loops can be almost completely determined. (Received October 10, 1960.)

60T-12. K. S. Miller and H. S. Shapiro: **On the linear independence of Laplace integral solutions of certain differential equations.**

Let \( S \) be the class of ordinary linear differential operators of the \( n \)th order whose coefficients are linear functions of the independent variable. It is shown that \( n \) linearly independent solutions in the form of (Laplace) contour integrals may be constructed for the equation \( Lw = 0 \) where \( L \in S \). In order to establish the linear independence certain tools from the modern theory of functions are employed; notably Mergelyan's theorem on polynomial approximation and results on approximation in the space \( H_1 \). The techniques may also be applied to a certain class of differential operators which is larger than \( S \). (Received October 7, 1960.)
The system of \( n \) linear or nonlinear equations (*) \( \frac{dx_i}{dt} = f_i(x,\omega,t) \), \( i = 1,\ldots,n \) is considered, with \( x = \{x_1,\ldots,x_n\} \) and \( \omega = \{\omega_1,\ldots,\omega_k\} \). The parameters \( \omega_i \) are held fixed for any particular solution. Continuous \( f_i \) are assumed. Let 

\[ R(\rho) = \{ x \mid \|x\| \leq \rho \} , \quad S = \{ \omega \omega_i \leq \omega_1 \leq \omega_i \} , \quad i = 1,\ldots,n \} \quad \text{and} \quad I = \{ t \mid 0 \leq t < T \} \].

For a specified \( \rho > 0 \), the system (*) is \( \rho \)-stable if for every \( (t',\omega) \in I x S \), \( x(t') \in R(\rho) \) implies \( x(t) \in R(\rho) \) for \( t' \leq t < T \). That is every solution which starts in the cylinder \( I x R(\rho) \), remains in it. For the Euclidean norm let \( \|x\| = \|x\|_2 \) and consider the nonlinear programming problem 

\[ \Lambda_2(\rho) = \max \{ \sum_{i=1}^n x_i f_i(x,\omega,t) / \rho^2 | \|x\| = \rho , (t,\omega) \in I x S \} . \]

For the maximum norm let \( \|x\| = \|x\|_\infty = \max |x_i| \), and consider 

\[ \Lambda_\infty(\rho) = \max \{ \sum_{i=1}^n x_i f_i(x,\omega,t) / \rho^2 | \|x\| = \rho , x \in R(\rho) , (t,\omega) \in I x S , i = 1,\ldots,n \} \].

It is shown that the necessary and sufficient condition for \( \rho \)-stability is 

\[ \Lambda_2(\rho) \leq 0 \] for the Euclidean norm, and 

\[ \Lambda_\infty(\rho) \leq 0 \] for the maximum norm. In addition, a bound on the rate of decay or growth of the norms is given by an appropriate gauge function for systems which are not necessarily \( \rho \)-stable.

Thus, \( \|x(t)\| \leq u(t) \), \( (t,\omega) \in I x S \), where \( u(t) \) is the gauge function determined by the single equation \( du/dt = u(\Lambda(u,t)) \), with \( u(0) = \|x(0)\| \). \( \Lambda(u,t) \) is the solution of a programming problem similar to that for \( \Lambda_2(\rho) \) or \( \Lambda_\infty(\rho) \). (Received October 19, 1960.)
Friedberg's solution to Post's problem, as well as some new theorems of the author, are easy consequences of the theorem below. A requirement is a sequence of ordered pairs of finite sets of natural numbers. A set \( T \) meets requirement \( R = \{(F_k, H_k) : k = 0,1,2,\ldots\} \) if there is a \( k \) such that \( F_k \subseteq T \) and \( H_k \cap T = \emptyset \). \( W = (t,g) \) is an enumeration of a sequence \( Q = \{R_i : i = 0,1,2,\ldots\} \) if
\[
(1) \quad (i) (R_i = \{(F^i_k, H^i_k) : k = 0,1,2,\ldots\});
(2) \quad (s) (E_k)(t(s) = (F^g(s), H^g(s)));
(3) \quad (i)(k)(E_s)(g(s) = i \text{ and } t(s) = (F^i_k, H^i_k)).
\]
The value of \( t \) for argument \( s \) will be denoted by \( (F^t(s), H^t(s)) \). A set \( T \) meets requirement \( R \) by virtue of \( s' \) if \( g(s') = g(s), \quad F^t(s') \subseteq T \) and \( H^t(s') \cap T = \emptyset \). The \( W \)-generated priority sequence of \( Q \) is defined by induction:
\[
T^0_{Q,W} = \emptyset; \quad T^s_{Q,W} = T^{s-1}_{Q,W} \text{ if there is an } s' < s \text{ such that } T^{s-1}_{Q,W} \text{ meets } R \text{ by virtue of } s' \text{ or if there is an } s' < s \text{ such that } g(s') < g(s) \text{ and } T^{s-1}_{Q,W} \text{ meets } R \text{ by virtue of } s' \text{ but } T^{s-1}_{Q,W} \cup F^t(s) \text{ does not or if } H^t(s) \cap (F^t(s) \cup T^{s-1}_{Q,W}) \neq \emptyset; \quad T^s_{Q,W} = T^{s-1}_{Q,W} \cup F^t(s) \text{ otherwise.}
\]
\( R \) is \( W \)-dense in \( Q \) if for each \( s \) such that \( g(s) = i, \quad H^t(s) \cap (F^t(s) \cup T^{s-1}_{Q,W}) = \emptyset \), and if for each finite set \( L \) and each \( k \), there is an \( m \) such that \( m > k \) and \( L \cap F^1_{m,W} = \emptyset \). Theorem. If \( W \) is an enumeration of a sequence \( Q \) of requirements, then \( \bigcup_{s=1}^{\infty} T^s_{Q,W} \) is recursively enumerable in \( t,g \) and meets every requirement of \( Q \) which is \( W \)-dense in \( Q \). (Received November 14, 1960.)

A topological (resp. locally convex) vector lattice is a vector lattice and a topological (resp. locally convex) Hausdorff vector space with a 0-basis of solid neighborhoods. \( A \subset L \) is order complete if \( X \subset A \) implies \( \sup X \in A \) for every directed (\( \subseteq \)) set majorized in \( L \). Nakano (J. Fac. Sci. Hokkaido University, Ser. I vol. 12 (1953) pp. 87-104) has asserted that a topological vector lattice is topologically complete, provided that it has a 0-basis of order complete solid neighborhoods, and that every directed (\( \subseteq \)) topologically bounded subset has a sup. However, his proof is incomplete in the nonmetrizable case. It can be shown that the theorem holds, and as a consequence it follows that the strong dual of a locally convex nonarch vector lattice is topologically complete. Thus, in particular, every locally convex vector lattice which is a reflexive space, is order complete and topologically complete. (Received October 24, 1960.)
60T-17. L. M. Sonneborn: A generalization of Abian and Brown's extension of the Brouwer fixed point theorem.

Let $\mathbb{R}^{n+1}$ be Euclidean $n + 1$-dimensional space, $S^n = \{x|x \in \mathbb{R}^{n+1}, |x| = 1\}$ and $E^{n+1} = \{x|x \in \mathbb{R}^{n+1}, |x| \leq 1\}$. Abian and Brown (Notices Amer. Math. Soc. vol. 7 (1960) p. 649 Abstract 572-9) have announced the following result for $n = 1$. Theorem: If $f$ is a continuous function from $E^{n+1}$ into $\mathbb{R}^{n+1}$ satisfying $f(S^n) \subset E^{n+1}$, then $f$ has a fixed point. This theorem is true for all $n \geq 0$, and can be proved using the Brouwer fixed point theorem and some elementary homotopy theory. Suppose $f$ has no fixed point, and consider $g(x) = f(x) - x/|f(x) - x|$. $g$ is then a function from $E^{n+1}$ to $S^n$ whose restriction to $S^n$ is homotopic to the antipodal map. This leads to a contradiction. (Received October 20, 1960.)


Let $0 < b_{n+1} \leq b_n(1 + c/n)$, $n > n_0$ (*), $b_n = o(1)$. Let $P(k)$ denote the number of $b_n$ such that $n \leq k$, $b_{n-1} < b_n$. Then it is proved that the series $\sum b_n z^n$ is convergent at every point $z = e^{i\theta}$, $z \neq 1$ if either $\sum b_n/n$ is convergent or $P(k) = O(k^\beta)$, $\beta < 1$. Let $\Phi(x) \uparrow$, $\lim f(n)/n^\beta = d < \infty$, and $\sum 1/n^\beta(n) = \infty$. Then it is proved that there exists a power series $S(z) = \sum_{n=1}^{\infty} b_n z^n$ such that $(b_n)$ satisfy (*), $\phi(n)b_n = o(1)$, and the series $S(z)$ is divergent for $z = 1$ and $-1$.

(Received October 21, 1960.)


Let $M_{m,n}$ be the vector space of $m \times n$ rectangular matrices over the complex numbers. The theorem in question (I. Schur, Preuss. Akad. Wiss. Sitzungsber. Berlin (1925) pp. 454-463) is the following: Let $T$ be a linear map of $M_{m,n}$ into itself. If $2 \leq r \leq \min(m,n)$ and the $r$th order subdeterminants of $Y = T(X)$ are fixed linearly independent linear homogeneous functions of the $r$th order subdeterminants of $X$ then there exist fixed nonsingular $A$ and $B$ in $M_{m,m}$ and $M_{n,n}$ respectively, such that for $m \neq n$ (1) $T(X) = AXB$. If $m = n$ then $T$ has the form (1) or (2) $T(X) = AX'B$ where $X'$ denotes the transpose of $X$. A new and succinct proof of this is given by showing that the hypotheses on $T$ imply that if $X \in M_{m,n}$ is of rank 1 then $T(X)$ is of rank 1. The result on the structure of
T then follows immediately from (M. Marcus and B. N. Moyls, Pacific J. Math. vol. 9 (1959) pp. 1215-1221, Theorem 1). This paper will appear shortly in the Arch. Math. (Received November 4, 1960.)


Let $A$ be a f.d. algebra with 1 over $\mathbb{F}$ and $\mathcal{O}(\mathbb{F})$ the class of algebras $B$ over $\mathbb{F}$ such that $B_\Omega = A_\Omega$, $\Omega$ the algebraic closure of $\mathbb{F}$. For any algebra $C$, let $\mathcal{O}(C)$ be the derivation Lie algebra and $G(C)$ the group of automorphisms of $C$. For $\alpha \in G(C)$, let $\alpha$ denote the automorphism $d \mapsto \alpha^{-1}d\alpha$ of $\mathcal{O}(C)$. Assume that for every $B \in \mathcal{O}(\mathbb{F})$ there exists (1) a linear functional $\phi_B$ such that $B = \Phi B', B' = \{b|\phi_B(b) = 0\}$, (2) a pair of elements $b_1', b_2' \in B'$, with $b_1'b_2' \neq 0$ in $B'$, (3) an isomorphism $\gamma_B$ of $B_\Omega$ onto $A_\Omega$ such that $\gamma_B^{\alpha}_A = \phi_B$ where the functionals are extended in the natural way. Further assume (4) $\alpha \mapsto \alpha$ maps $G(A_\Omega)$ onto $G(\mathcal{O}(A_\Omega))$ and (5) $\mathcal{O}(A_\Omega)$ sends $A_\Omega$ onto $A'_\Omega$ and acts irreducibly in $A'_\Omega$. Under these assumptions one proves: If $B_1$ and $B_2$ are in $\mathcal{O}(\mathbb{F})$ and $\mathcal{O}(B_1) \cong \mathcal{O}(B_2)$, then $B_1 \cong B_2$ and any isomorphism of $\mathcal{O}(B_1)$ onto $\mathcal{O}(B_2)$ has the form $d \mapsto \beta^{-1}d\beta$, $\beta$ an isomorphism of $B_1$ onto $B_2$. Except for a few low characteristics the above can be applied to extend the known results for characteristic 0 when $A$ is split Cayley or split exceptional simple Jordan algebra, and thus to classify central simple Lie algebras of types $G_2$ and $F_4$. (Received November 7, 1960.)

60T-21. Isidore Fleischer: Postscript to maximality and ultracompleteness in normed modules.

Theorem 3 of the paper cited (Proc. Amer. Math. Soc. vol. 9 (1958) pp. 151-157) can be extended and sharpened as follows: Every maximal module is ultracomplete if and only if every ideal in the coefficient ring is projective, Sufficiency is shown as before; indeed arbitrary extensions of the residue modules can even be accommodated. The requisite extension lemma is proved as follows: Let $F$ be a free module mapping on $\hat{M}$; by projectivity the restriction of this map to the inverse image of $\phi(M)$ is a multiple of $\phi$; $\hat{M}$ is obtained as $F \oplus M$ modulo the graph of this multiple. To see necessity, let $\phi$ be a non-splitting homomorphism of a module $M$ on an ideal conceived as a submodule of the free module with generator $x$. Take the direct sum of both $M$ and the
latter with a free module on countably many generators \( \{ x_n \} \) extending \( \phi \) by the identity on the common summand. The submodules \( M_n \) generated by all \( x - x_k \) for \( k \geq n \) induce a separated topology in which \( x_n \) converges to \( x \); their inverse images under \( \phi \), along with its kernel, define a norm on its domain. If the latter admitted an ultracomplete normed extension, \( \phi \) would be extendable to a map onto \( x \), contrary to construction. (Received November 7, 1960.)

60T-22. Donald Monk: **Representation theorems for cylindric algebras.**

The notation of Henkin and Tarski, *Cylindric algebras*, Proc. of Symposia in Pure Mathematics, vol. 3, Lattice Theory, 1960 is used. Let \( a \) be an infinite ordinal, and let \( \mathcal{O} = \langle A, +, -, c, d_{K^A} \rangle \) be a \( CA_a \). **Theorem 1.** If \( \mathcal{O} \) has a Halmos-type substitution \( S \) (i.e., if \( \mathcal{O} \) is a polyadic equality algebra except for having quantifications only on finite sets), then \( \mathcal{O} \) is representable. **Theorem 2.** If for all nonzero \( x \in A \) and for every finite subset \( F \) of \( a \) there are distinct \( K, \lambda \in a \sim F \) such that \( x \cdot d_{K^A} = 0 \), then \( \mathcal{O} \) is representable. **Corollary 1.** If \( \mathcal{O} \) is simple, then \( \mathcal{O} \) is representable. **Corollary 2.** If for all \( x \in A \), \( a \sim \Delta x \) is infinite, then \( \mathcal{O} \) is representable. **Corollary 3.** If there is a finite irreflexive subset \( F \) of \( a \times a \) such that \( \prod_{(K, \lambda) \in F} d_{K^A} = 0 \), then \( \mathcal{O} \) is representable. **Theorem 1** was independently obtained by Alfred Tarski, and subsequently generalized by him. **Theorem 1** cannot be strengthened by replacing "substitution \( S \)" by "substitution \( S \) acting only on finite transformations". From **Corollary 1** it follows that for no \( \beta \) with \( 2 \leq \beta < \omega \) is it true that every \( CA_a \) can be embedded in a \( CA_{\omega^\beta} \). **Corollary 2** answers a question of Henkin, and shows that **Theorem 2** generalizes the representation theorem for \( DCA_a \)'s. (Received November 7, 1960.)

60T-23. P. E. Conner and E. E. Floyd: **Cobordism classes of bundles.**

For a space \( X \), consider pairs \( (M^n, f) \) of compact differentiable oriented manifolds \( M^n \) and maps \( f: M^n \to X \). Two such pairs \( (M_1^n, f_1) \) are cobordant if there exists an oriented \((n + 1)\)-manifold \( V^{n+1} \), with boundary the disjoint union \( M_1^n - M_2^n \), and a map \( F: V^{n+1} \to X \) with \( F|_{M_1^n} = f_1 \). The equivalence classes form a group \( \Omega_n(X) \). The definition can be extended to pairs, and one obtains a homology theory satisfying the Eilenberg-Steenrod axioms, with the exception of the dimensional axiom. For a finite simplicial complex \( K \) there is a spectral sequence for which \( E^2_{p,q} = H_p(K; \Omega_q) \), \( \Omega_q \) the Thom group, and whose \( E^\infty \)-term is
associated with a filtration of $\Omega_*(K)$. For a classifying space $B_G, \Omega_n(B_G)$ represents the cobordism classes of $G$-bundles over oriented differentiable $n$-manifolds. In case all torsion of $H_*(B_G;\mathbb{Z})$ consists of elements of order 2, we show that $\Omega_n(B_G) \approx \sum_{p+q=n} H_p(B_G;\Omega_q)$. Corresponding to the Whitney numbers and Pontryagin numbers of ordinary cobordism, there are here defined mixed Whitney numbers and Pontryagin numbers, using combinations of degree $n$ of tangential classes of the base $M^n$ and characteristic classes of the bundle.

For $G$ as above, two $G$-bundles are cobordant if and only if corresponding mixed Whitney numbers and Pontryagin numbers are all equal. (Received November 14, 1960.)

60T-24. W. S. Massey: Almost complex structures on 8-dimensional manifolds, I.

Let $M$ be a compact, connected, orientable, differentiable 8-dimensional manifold whose second Stiefel-Whitney class vanishes. Theorem I. If there exists a continuous field of tangent 2-frames on $M$, then $M$ admits an almost-complex structure. Theorem II. If $H^2(M,\mathbb{Z}) = 0$ and $M$ admits an almost-complex structure, then there exists a continuous field of tangent 2-frames on $M$; as a consequence, the Euler characteristic of $M$ vanishes. It follows from Theorem II that the quaternionic projective plane does not admit an almost complex structure; this answers a question raised by Hirzebruch (Ann. of Math. vol. 60 (1954) p. 224). The example $M = S^2 \times S^2 \times S^2 \times S^2$ shows that the hypothesis $H^2(M,\mathbb{Z}) = 0$ is essential in Theorem II. The proof of Theorems I and II makes use of the facts that the group of the tangent bundle of $M$ can be "lifted" to the group Spin (8), and that Spin (8) has interesting outer automorphisms. (Received November 14, 1960.)


Some years ago H. Ozeki proved that any differentiable manifold $M$ satisfying the second axiom of countability admits a complete Riemannian metric. It is now shown that if every Riemannian metric which $M$ admits is complete, then $M$ is compact. This follows from the following result. A Riemannian metric $g$ on $M$ is called bounded if $M$ is bounded with respect to the distance $d(x,y)$ defined naturally by $g$ as the infimum of all rectifiable curves joining
x and y. **Theorem.** For any Riemannian metric \( g \) on \( M \), there exists a bounded Riemannian metric which is conformal to \( g \). (Received November 14, 1960.)

60T-26. S. V. Parter: **Extreme eigenvalues of Toeplitz forms and applications to elliptic difference equations.**

The results of Kac, Murdoch and Szego *(On the eigenvalues of certain hermitian forms, Journal of Rational Mechanics and Analysis vol. 2 (1953) pp. 767-800)* are extended as follows. Let \( f(\theta) \) be a real continuous function, and periodic with period \( 2\pi \). Let \( \min, f(\theta) = f(0) \), and let \( \theta = 0 \) be the only value of \( \theta \) (mod \( 2\pi \)) for which this minimum is attained. Finally, let \( f(\theta) \) have \( 2a \) continuous derivatives in some neighborhood of \( \theta = 0 \) and \( f^{2a} (0) \neq 0 \) is the first nonvanishing derivative of \( f(\theta) \) at \( \theta = 0 \). Let \( T_n[\{f\}] \) be the Toeplitz matrix associated with \( f(\theta) \). Let \( \lambda_{\nu,n} \) be the eigenvalues of \( T_n[\{f\}] \) ordered in nondecreasing order. **Theorem.** \( \lambda_{1,\nu} \sim f(0) + O(1/n)^{2a} \) where \( O \) cannot be replaced by \( o \).

**Theorem.** Let \( a = 2 \), then \( \lambda_{\nu,n} \sim f((2\nu + 1 + E\nu)/2(n + 3) + o(1/n)) \) where tan \( ((2\nu + 1 + E\nu)/4) = (-1)^\nu \tanh ((2\nu + 1 + E\nu)/4) \). Additional results are obtained for even functions which attain their minimum only at \( \pm 8 \odot 0 \) (mod \( 2\pi \)). These results are used to estimate \( \lambda_{R,1,n} \) for certain cases where \( f(\theta) \) is a matrix-valued function. Finally, these results are applied to problems in numerical analysis. In particular, we obtain an estimate for the Richardson eigenvalue \( \lambda_{R} \) of the "two-line" iterative methods for the biharmonic difference equation. (Received November 14, 1960.)

60T-27. Frederick Bagemihl and Wladimir Seidel: **Koebe arcs and Fatou points of normal functions.**

Let \( C \) be the unit circle and \( D \) be the open unit disk. A strong Koebe sequence of arcs is defined to be a Koebe sequence of arcs in \( D \) such that for every \( \zeta \in C \) there is a rectilinear segment extending from \( \zeta \) to a point of \( D \) and intersecting infinitely many of the arcs. (1) If \( f(z) \) is a normal meromorphic function in \( D \) that tends to a constant \( b \) along a Koebe sequence of arcs, then \( f(z) \equiv b \). (2) If \( f(z) \) is a normal holomorphic function in \( D \) that is bounded by the constant \( M \) on a strong Koebe sequence of arcs, then \( |f(z)| \leq M \) in \( D \). (3) If \( f(z) \) is a normal holomorphic function in \( D \), and if the set of Fatou points of \( f(z) \) on an open subarc \( A \) of \( C \) is of Lebesgue measure zero, then \( A \) contains a Fatou point of \( f(z) \) at which the corresponding Fatou value is \( \infty \). **Corollary.**
The set of Fatou points of a normal holomorphic function in $D$ is everywhere dense on $C$. Given $\varepsilon > 0$, there exists a normal holomorphic function in $D$ whose set of Fatou points on $C$ is of measure less than $\varepsilon$, but for which $\infty$ is not a Fatou value. This paper will appear in the Commentarii Mathematici Helvetici. (Received November 17, 1960.)


Let $B$ be a Boolean ring and $I$, $J$ two index sets. A system of equations
\[(1) \{ p_i(\ldots x_j \ldots) = c_i \}_i \in \{ I \},_j \in \{ J \} \]
where $p_i(\ldots x_j \ldots)$ is a Boolean polynomial in the variables $x_j$ over $B$ having no terms of zero degree and $c_i \in B$, is called consistent if $F(\ldots p_i \ldots) \equiv 0$ implies $F(\ldots c_i \ldots) = 0$ for any Boolean polynomial $F$ in $p_i(\ldots x_j \ldots)$ over $B$. System (1) is called solvable if there exists a set $\{ b_j \}_j \in \{ J \}$ of elements of $B$ such that $\{ p_i(\ldots b_j \ldots) = c_i \}_i \in \{ I \},_j \in \{ J \}$. In this paper it is proved: Theorem. A necessary and sufficient condition for any consistent system of Boolean equations over a Boolean ring $B$ be solvable is that $B$ be complete. The principal tool in the proof is the consideration of the notion of the extension of algebra homomorphisms and the extensive use of properties of Boolean rings. (Received November 17, 1960.)


Let (i) $K(a)\Psi_+(a) = \Psi_-(a) + F(a)$ be a Wiener-Hopf system of equations where $K$ is an $(n,n)$ matrix and $\Psi_+, \Psi_-$, $F$ are $n$-vectors. $K(a)$, $F(a)$ are analytic in the strip $\tau_1 < \mathcal{C}(a) < \tau_2$. Define a norm $\|u\| = (u,u)^{1/2}$ where $(u,v)$
\[= \int_{-\infty}^{\infty} e^{\tau z} \sum_{i,j=1}^{n} u_i \bar{v}_j d\tau, \mathcal{C}(a) = \tau_1 < \tau < \tau_2. \]
It is assumed: (ii) $K(a) = K_0(a) + eq(a)$; (iii) $K_0(a) = \lim_{\epsilon \to 0} K_0(a)K_0^\dagger(a)$ where $K_0(a)$ is analytic $\mathcal{C}(a) > \tau_1$, $K_0^\dagger(a)$ is analytic $\mathcal{C}(a) < \tau_2$; (iv) $(K_0^\dagger)^{-1}$ exists $\mathcal{C}(a) < \tau_2$, and the elements of $(K_0^{-1})^{-1}$ have bounded modulus $\mathcal{C}(a) < \tau_2$; (v) $q(K_0^{-1})^{-1}$ exists except at $a = \beta_i$, $i = 1,\ldots,N$, $\mathcal{C}(\beta_i) < \tau_1$, and the elements of $q(K_0^{-1})^{-1}$ have order $p_i(a - \beta_i)^{k_1}$, $-1 < k_1 < 0$, at $a = \beta_i$ and are bounded in modulus $|a| \to \infty$; (vi) the elements of $F$ are analytic $\mathcal{C}(a) < \tau_2$ except for simple poles at $a = a_i$, $i = 1,\ldots,M$, $a_i \neq \beta_j$ and are bounded in modulus $|a| \to \infty$; $\mathcal{C}(a) < \tau_2$. Systems (i) satisfying (ii), (iii) have been considered by B. Noble, H. C. Kranzer and J. Radlow.
If the matrix $K$ and the vector $F$ satisfy (ii),(iii),(iv),(v),(vi) then for sufficiently small $e$ solutions $\phi'_+(a)$, $\psi'_-(a)$ of (i) exist for $T = \partial(a)$ which are regular in $e$, bounded in norm, and $\phi'_+(a)$ is analytic $\forall T < \partial(a) < T_2$, $\psi'_-(a)$ is analytic $\forall T_1 < \partial(a) < T$. (Received November 16, 1960.)


Without considering orientation the groups $\nabla_n(X)$ are defined for a space $X$. For a finite simplicial complex $\nabla_n(K) \cong \sum_{p+q=n} H_p(K;Z_2) \otimes \nabla_q$. There is a homomorphism $k_*: \nabla_f(B(O(n - r))) \to \nabla_{n-1}(B(O(1)))$ defined by assigning to each differentiable $(n - r - 1)$-sphere bundle over a closed $r$-manifold the mod 2 cobordism class of the fixed point free involution on the bundle space which, on each fibre, reduces to the antipodal involution. Two differentiable involutions $(T,M_i)$ are cobordant mod 2 if there is an involution $(\tau,B^{n+1})$ on a manifold with boundary $M^n_1 \cup M^n_2$ such that $\tau|_{M^i} = (T,M^n_i)$. By means of disjoint union an abelian group structure is imposed on $\nabla_n$, the collection of mod 2 cobordism classes of involutions. Let $F^X$ denote the union of the $r$-dimensional components of the fixed point set of $(T,M^n)$. For $0 \leq r < n$, let $a_r \in \nabla_f(O(n - r))$ be the mod 2 cobordism class of the normal bundle to $F^X$ in $M^n$. Let $a_n = [F^n]_2 \in \nabla_n$. We define $i_*: \theta_n \to \nabla_n + \sum_{r=0}^{n-1} \nabla_r(B(O(n - r)))$ by $i_*([T,M^n]_2) = (a_n,a_{n-1},...,a_0)$. We define $j_*: \nabla_n + \sum_{r=0}^{n-1} \nabla_r(B(O(n - r))) \to \nabla_{n-1}(B(O(1)))$ by $j_* (a_n,a_{n-1},...,a_0) = \sum_{r=0}^{n-1} k_* (a_r)$. The sequence $0 \to \theta_n \to \nabla_n + \sum_{r=0}^{n-1} \nabla_r(B(O(n - r))) \to \nabla_{n-1}(B(O(1))) \to 0$ is exact. (Received November 17, 1960.)


If $0 < a < 1$, Lip $a$ is the space of all functions on the real line of period 1 that satisfy $|f(\sigma + T) - f(\sigma)| = O(|T|^a)$ uniformly in $\sigma$. It is a Banach space under the norm $\|f\|$ defined by $\|f\| = \sup \{|f(\sigma)|, |T|^{-a}[f(\sigma + T) - f(\sigma)]\}$. Lip $a$ is the closed linear subspace of Lip $a$ consisting of those $f$ with $|f(\sigma + T) - f(\sigma)| = o(|T|^a)$ uniformly in $\sigma$. It is shown that Lip $a$ is canonically isomorphic and isometric to the second dual space of the Banach space Lip $a$. The extreme points of the unit sphere of the dual of lip $a$ are found to be of the form $f \to \lambda f(\sigma)$ and $f \to \lambda|\sigma - T|^{-a}(f(\sigma) - f(\tau))$. As a consequence one obtains the fact that lip $a$ has no isometries besides the expected ones. (Received November 17, 1960.)

A matrix $A = (a_{ij})$, $1 \leq i, j \leq n$, whose elements are given by $a_{ij} = (-1)^{i+j} \binom{2k}{k-j+1}$ for $|j - i| \leq k$ and by $a_{ij} = 0$ for $|j - i| > k$ may be called a 2k-difference matrix. The matrix $A$ is positive definite and has a Choleski decomposition $A = LL^T$ where $L$ is a lower triangular matrix and $L^T$ its transpose. **Theorem:** The elements of $L = (l_{ij})$ are given explicitly by

$$l_{ij} = (-1)^{i+j} \binom{k}{i-j} \left[ (j + i) \ldots (j + 2k - i) \right]^{1/2} \left[ (i + 1) \ldots (i + k - 1) \right]^{-1}$$

for $0 \leq i - j \leq k$ and by $l_{ij} = 0$ otherwise. (Received November 28, 1960.)


Let $X(t)$ denote the Wiener process starting at zero, $f(t)$ a continuous function. Denote by $p(t_0 , t)$ the probability that $X(t) = f(t)$ for $t_0 \leq t \leq t$. Then

$$\int_{t_0}^{\infty} \exp \left\{ - \frac{s^2 t}{2} + sf(t) \right\} dp(t_0 , t) = - (2\pi t_0)^{-1/2} \int_{-\infty}^{\infty} \exp \left\{ - x^2 / 2t_0 \right\} dx.$$  

The transform can be inverted if $f$ is a quadratic in $t$ or $(t + a)^{1/2}$. If $f(t) \equiv 0$ denote by $q(t_0 , t)$ the probability that $|X(t)| \leq f(t)$ for $t_0 \leq t \leq t$. Then

$$\int_{t_0}^{\infty} \exp \left\{ - \frac{s^2 t}{2} \right\} \cosh sf(t) dq(t_0 , t) = - (2\pi t_0)^{-1/2} \int_{-1}^{1} \exp \left\{ - x^2 / 2t_0 \right\} dx.$$  

This transform can be inverted if $f$ is a constant or a constant multiple of $(t + a)^{1/2}$. The formulas were obtained by applying Fourier transforms to the diffusion equation with the appropriate boundary conditions. (Received November 28, 1960.)

60T-34. A. A. Mullin: A double generalization of the concept of a mutant. Preliminary report.

This abstract gives two generalizations to the concept of a mutant as given and developed elsewhere (see: Abstract 571-10 Notices Amer. Math. Soc. vol. 7 (1960) p. 480. Erratum: in the middle of line 4 read "$\bar{M}$" for "$M$"; Some logico-philosophical comments on self-organizing systems, Technical report no. 5 for contract NONR 1834(21), Electrical Engineering Department, University of Illinois, Urbana, Illinois, U.S.A.). By a $(P,T)$-mutant of an algebraic system $(S, *)$ is meant a subset $M$ of $S$ that satisfies the condition

$$\mathcal{X}_P M \subseteq M(T),$$

for some positive integer $P \geq 2$ and some closed (relative to $*$) subset $T$ of $S$, where $\mathcal{X}_P M = \{ a_1^* a_2^* \ldots a_P : a_1 \in M, a_2 \in M, \ldots, a_P \in M \}$ and $M(T) = T \cap \bar{M}$, i.e., the set of all of the elements of $T$ not in $M$. Then there is an abundance of more general theorems which are analogous to the ones in the above abstract, e.g., concerning subsets, homomorphic mappings, isomorphic mappings, a maximal property and preservation of the maximal property under
homomorphic mappings. However the generalization seems to lack, for the author, in richness of concrete examples when $P > 2$ and $T$ is a proper subset of $S$. (Received November 21, 1960.)

60T-35. A. E. Babbitt, Jr.: Pathological extensions of difference fields.

A difference field $G$ is said to be a pathological extension of a difference field $F$ if either $G$ is a monadic extension of $F$, or $G$ is incompatible with some other extension of $F$. Pathological extensions of difference fields have been studied by R. M. Cohn (Amer. J. Math. vol. 74 (1952) pp. 507-530) who showed that a difference field admits finitely generated pathological extensions only if it admits such extensions of order zero. In this paper a structure theorem is developed that leads to two principal theorems. Theorem 1. If a difference field admits a finitely generated pathological extension of order zero, then it admits a finitely generated pathological extension of finite degree. Theorem 2. Let $F$ represent the difference field of rational functions of $x$ with complex coefficients and transforming defined by $f(x)$ maps into $f(x + 1)$. Then $F$ admits no finitely generated pathological extensions. (Received November 25, 1960.)

60T-36. G. E. Sacks: Suborderings of degrees that have cardinality
of the continuum.

Call a partially ordered set $P$ completely normal if there is an ordinal $\alpha$ and a collection \{ $P_{\gamma}\mid \gamma < \alpha$ \} of subsets of $P$ such that $P = \bigcup_{\gamma < \alpha} P_{\gamma}$ and for each ordinal $\gamma < \alpha$: $\bigcup_{\delta < \gamma} P_{\delta}$ has cardinality less than that of the continuum; $P_{\gamma}$ is countable; no member of $P_{\gamma}$ is $\subseteq$ any member of $\bigcup_{\delta < \gamma} P_{\delta}$; for each $n \in P_{\gamma}$, the set $L_n' = \{ m \mid m \in \bigcup_{\delta < n} P_{\delta}, m < n \}$ is countable and any two members of it have an upper bound in it. Call a partially ordered normal if it is imbeddable in some completely normal partially ordered set. Theorem 1: let $A$ and $B$ be sets of degrees such that $A$ is countable and $B$ has cardinality less than that of the continuum; let $D$ be the set of all degrees greater than every member of $A$ and incomparable with every member of $B$; then a nonempty, normal partially ordered set $T$ is imbeddable in $D$ if and only if no member of $B$ is $\subseteq$ any finite union of members of $A$. Note: a partially ordered set of cardinality aleph-one is normal if and only if each member of it has at most a countable number of predecessors. The following is proved without the axiom of choice. Theorem 2: If $T$ is a partially ordered set of cardinality of the continuum and no member of $T$ has infinitely many predecessors, then $T$ is imbeddable in the upper semi-lattice of degrees. (Received November 14, 1960.)
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