MEETINGS

Calendar of Meetings ................................................. 4
Program of February Meeting in New York ......................... 5
Abstracts for the Meeting, pages 49 - 52

PRELIMINARY ANNOUNCEMENTS OF MEETINGS ...................... 7

ACTIVITIES OF OTHER ASSOCIATIONS .............................. 11

ANNUAL MEETING IN WASHINGTON ................................. 15

NEWS AND COMMENT FROM THE CONFERENCE BOARD
OF THE MATHEMATICAL SCIENCES .................................. 17

MATHEMATICS IN TRANSLATION ...................................... 20

FROM THE AMS SECRETARY ........................................ 21

MATHEMATICAL LIFE IN THE USSR - by N. D. Kazarinoff ........... 22

REPORT ON SOVIET MATHEMATICS - by J. P. LaSalle ............... 25

NEWS ITEMS AND ANNOUNCEMENTS ................................ 30

PERSONAL ITEMS .................................................. 33

LETTERS TO THE EDITOR .......................................... 37

CORPORATE AND INSTITUTIONAL MEMBERS
OF THE AMERICAN MATHEMATICAL SOCIETY ...................... 38

MEMORANDA TO MEMBERS

An Announcement from the Proceedings Editorial Committee .... 41
Reciprocity Agreement with the Calcutta Mathematical Society .... 41
Backlog of Mathematical Research Journals ....................... 42

NEW PUBLICATIONS .................................................. 43

SUPPLEMENTARY PROGRAM - No. 2 .............................. 47

ABSTRACTS OF CONTRIBUTED PAPERS ............................. 49

RESERVATION FORM .................................................. 79
## MEETINGS

### CALENDAR OF MEETINGS

*Note:* This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the NOTICES was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

<table>
<thead>
<tr>
<th>Meeting No.</th>
<th>Date</th>
<th>Place</th>
<th>Deadline for Abstracts*</th>
</tr>
</thead>
<tbody>
<tr>
<td>578</td>
<td>April 5-8, 1961</td>
<td>New York, New York</td>
<td>Feb. 20</td>
</tr>
<tr>
<td>579</td>
<td>April 14-15, 1961</td>
<td>Chicago, Illinois</td>
<td>Feb. 20</td>
</tr>
<tr>
<td>580</td>
<td>April 22, 1961</td>
<td>Stanford, California</td>
<td>Feb. 20</td>
</tr>
<tr>
<td>581</td>
<td>June 13-16, 1961</td>
<td>Seattle, Washington</td>
<td>April 28</td>
</tr>
<tr>
<td>582</td>
<td>August 29 - September 1, 1961 (66th Summer Meeting)</td>
<td>Stillwater, Oklahoma</td>
<td>July 15</td>
</tr>
<tr>
<td>583</td>
<td>October 28, 1961</td>
<td>Cambridge, Massachusetts</td>
<td>Sept. 14</td>
</tr>
<tr>
<td>584</td>
<td>November 17-18, 1961</td>
<td>Milwaukee, Wisconsin</td>
<td>Oct. 4</td>
</tr>
<tr>
<td>585</td>
<td>November 17-18, 1961</td>
<td>Gainesville, Florida</td>
<td>Oct. 4</td>
</tr>
<tr>
<td>586</td>
<td>November 18, 1961</td>
<td>Santa Barbara, California</td>
<td>Oct. 4</td>
</tr>
<tr>
<td></td>
<td>January, 1962</td>
<td>Cincinnati, Ohio</td>
<td></td>
</tr>
<tr>
<td>587</td>
<td>(68th Annual Meeting)</td>
<td>August, 1962</td>
<td>Vancouver, British Columbia</td>
</tr>
<tr>
<td>588</td>
<td>January, 1963</td>
<td>Berkeley, California</td>
<td></td>
</tr>
<tr>
<td>589</td>
<td>(69th Annual Meeting)</td>
<td>August, 1963</td>
<td>Boulder, Colorado</td>
</tr>
<tr>
<td></td>
<td>(68th Summer Meeting)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* The abstracts of papers to be presented in person at the meetings must be received in the Headquarters Offices of the Society in Providence, Rhode Island, on or before these deadlines. The deadlines also apply to news items. The next two deadline dates for by title abstracts are February 13 and April 21.

The NOTICES of the American Mathematical Society is published by the Society seven times a year, in February, April, June, August, October, November, and December. Price per annual volume is $7.00, Price per copy, $2.00. Special price for copies sold at registration desks of meetings of the Society, $1.00 per copy. Subscriptions, orders for back numbers (none available before 1958), and inquiries should be addressed to the American Mathematical Society, 1350 Main Street, Ann Arbor, Michigan, or to 190 Hope Street, Providence 6, Rhode Island.

Second-class postage paid at Ann Arbor, Michigan. Authorization is granted under the authority of the act of August 24, 1912, as amended by the act of August 4, 1947 (Sec. 34.21, P. L. and R.). Accepted for mailing at the special rate of postage provided for in section 34.40, paragraph (d).

*Copyright © 1960 by the American Mathematical Society
Printed in the United States of America*
The five hundred seventy-seventh meeting of the American Mathematical Society will be held at Hunter College, New York, New York, on February 25, 1961.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, Professor John Wermer of Brown University will address the Society on "Uniform approximation and maximal ideal spaces" in Room 300 at 2:00 P.M.

The registration desk will be open from 9:00 A.M. till 3:30 P.M. and will be located on the third floor between the elevator and Room 300. Room 305 will serve as a coat room and Rooms 302 and 306 as conversation rooms.

Hunter College is on Park Avenue at 68th Street. It may be reached from the area of Grand Central Station by the Lexington Avenue subway, which has a 68th Street stop. Persons attending the meeting are requested to use the entrance on 69th Street. Elevator service will be provided.

PROGRAM OF THE SESSIONS

The time limit for each contributed paper is ten minutes. The papers are scheduled at fifteen minute intervals for the convenience of all. In order that each speaker and listener may know the precise time at which each scheduled paper will be heard, the time limit will be strictly enforced.

SATURDAY, 10:00 A.M.

Session on Analysis, Room 300
10:00 - 10:10
Professor J. L. Ullman, University of Michigan (577-2)

10:15 - 10:25
(2) A note on inverse function theorems
Professor E. B. Leach, Case Institute of Technology (577-1)

10:30 - 10:40
(3) Spectral theorems in Banach space
Professor G. L. Krabbe, Yale University and Purdue University (577-9)

10:45 - 10:55
(4) A note on logarithms of normal operators
Dr. Svetozar Kurepa, University of Maryland (577-12)

11:00 - 11:10
(5) Generalized functions of class \( C^p \)
Dr. T. P. G. Liverman, The George Washington University (577-13)

11:15 - 11:25
(6) Some properties of the canonical decomposition for representations of locally compact groups
Mr. J. A. Ernest, The Institute for Advanced Study (577-10)

11:30 - 11:40
(7) Boundary value problems for generalized analytic functions. I. Preliminary report
Professor Walter Koppelman, University of Pennsylvania (577-11)

11:45 - 11:55
(8) On the functional equation \( f(\lambda x) = \lambda^N f(x) \)
Mr. Louis V. Quintas, Columbia College and City College and Professor Fred Supnick, City College (577-8)
SATURDAY, 2:00 P.M.

Invited Address, Room 300
Uniform approximation and maximal ideal spaces (One hour)
Professor John Wermer, Brown University

SATURDAY, 3:15 P.M.

General Session, Room 300
3:15 - 3:25
(9) On Weyl's inequality and Waring's theorem problem for cubes
Professor S. Chowla, University of Notre Dame and Professor H. Davenport, Trinity College, Cambridge, England (577-7)

3:30 - 3:40
(10) On the existence of polynomials of arbitrary degree in many variables, which represent every natural number exactly once
Mr. Paromita Chowla, South Bend, Indiana (577-6)
(Introduced by S. Chowla)

3:45 - 3:55
(11) A nonlinear integral equation from the theory of servomechanisms
Dr. V. E. Beneš, Bell Telephone Laboratories, Murray Hill, New Jersey (577-4)

4:00 - 4:10
(12) Pointwise bounds for the solution of parabolic differential equations, Part I
Lieutenant Fred J. Bellar, Jr., University of Maryland and U. S. Naval Postgraduate School (577-5)

4:14 - 4:25
(13) Synthesis of logical systems with many outputs
Dr. J. P. Roth, IBM Research Center, Yorktown Heights, New York (577-3)

Bethlehem, Pennsylvania
January 12, 1961
The dates of the April Meeting in New York are April 5-8. This represents a change from the dates announced in the November and December issues of these NOTICES. Consequently it has been necessary to change the DEADLINE DATE for Abstracts. Please note that the DEADLINE DATE for the April Meetings is February 20.

FIVE HUNDRED SEVENTY-EIGHTH MEETING

Hotel New Yorker
New York, New York
April 5-8, 1961

The five hundred seventy-eighth meeting of the American Mathematical Society will be held on Wednesday through Saturday, April 5-8, 1961 at the Hotel New Yorker.

Two symposia will be held in conjunction with the meeting. A symposium on Mathematical Problems in the Biological Sciences will be held during the morning and afternoon on Wednesday, Thursday, and Friday. This is another in the series of Symposia in Applied Mathematics sponsored by the Office of Ordnance Research and is sponsored also by the National Science Foundation. The Invitations and Steering Committee includes Dr. S. M. Ulam, Chairman, Dr. R. E. Bellman, Secretary, Professor Anthony Bartholomay, Dr. John Jacquez, Professor Theodore Puck, and Professor C. E. Shannon.

A Symposium on Recursive Function Theory, with financial support from project FOCUS of the Institute for Defense Analyses, will be held on Thursday morning and afternoon and Friday morning. The Organizing Committee consists of Professor S. C. Kleene, Chairman, Professor J. C. E. Dekker, Professor John McCarthy, Professor J. B. Rosser, and Professor J. R. Shoenfield.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, Professor Jacob Wolfowitz of Cornell University will address the Society on "Coding theorems of information theory" at 2:00 P.M. on Friday.

By invitation of the same Committee, Professor Daniel M. Kan of the Massachusetts Institute of Technology will address the Society on Saturday afternoon at 2:00 P.M.

There will be sessions for contributed papers on Friday afternoon and on Saturday morning and afternoon. Abstracts of contributed papers should be sent to the American Mathematical Society, 190 Hope Street, Providence 6, Rhode Island, so as to arrive PRIOR TO THE DEADLINE of February 20, 1961.

The speakers and titles of some of the addresses at the Symposium on Mathematical Problems in the Biological Sciences are as follows:

- Machine Models of Self-Reproduction
  Dr. Edward F. Moore, Bell Telephone Laboratories

- Mathematical Problems in Heuristic Programming
  Professor Marvin L. Minsky, Massachusetts Institute of Technology

- Mathematical Models in the Behavior of the Central Nervous System
  Professor Herbert D. Landahl, Committee on Mathematical Biology, University of Chicago

- Protein Synthesis
  Professor Paul C. Zamecnik, Professor of Medicine, Harvard Medical School

- Enzymology and Kinetics
  Professor Irwin W. Sizer, Chairman of the Department of Biology, Massachusetts Institute of Technology

- Bio-chemistry: Sterile or Virgin for Mathematicians?
  Professor Arthur B. Pardee, Virus Laboratory, University of California, Berkeley

- Intracellular Information Transfer
  Professor Alexander Rich, Department of Biology, Massachusetts Institute of Technology

- Mathematical Problems in Modern Physiology
  Dr. John Jacquez, Sloan-Kettering Institute for Cancer Research
Mathematical Aspects of Adaptive Control  
Dr. Robert Kalaba, The RAND Corporation

Medical Diagnosis and Modern Decision Making  
Dr. Robert Ledley, National Biomedical Research Foundation

The full program of the Symposium will be included in the program of the meeting in the April issue of these NOTICES.

The tentative program of the Symposium on Recursive Function Theory follows.

THURSDAY, 9:30 A.M.

First Session  
Chairman: Professor S. C. Kleene, University of Wisconsin

- Infinite Series of Isols (25 minutes)  
  Professor J. C. E. Dekker, Rutgers, The State University

- \( \Omega \) (25 minutes)  
  Professor John Myhill, University of California, Berkeley

- Non-standard models and arithmetically isolated sets (25 minutes)  
  Professor Anil Nerode, Cornell University

- Representability of sets in formal sets (50 minutes)  
  Professor Andrzej Mostowski, University of Warsaw

THURSDAY, 1:45 P.M.

Second Session  
Chairman: Professor Frederick B. Fitch, Yale University

- Algebras of sets binumerable in complete extension of arithmetic (25 minutes)  
  Professor Dana S. Scott, University of California

- Some problems in hierarchy theory (25 minutes)  
  Professor John W. Addison, University of Michigan

- Provably recursive functionals in analysis (25 minutes)  
  Professor Clifford Spector, Ohio State University

- Recursive functions of higher types (25 minutes)  
  Professor Joseph R. Shoenfield, Duke University

FRIDAY, 9:15 A.M.

Third Session  
Chairman: Professor J. Barkley Rosser, Cornell University and Institute for Defense Analyses

- Applications of recursive function theory to number theory (25 minutes)  
  Professor Martin Davis, Yeshiva University

- Sequence generators and digital computers (25 minutes)  
  Professor Arthur Burks, University of Michigan

- The treatment of ambiguity and paradox in mechanical languages (25 minutes)  
  Professor Saul Gorn, University of Pennsylvania

- Checking proofs by computer (25 minutes)  
  Professor John McCarthy, Massachusetts Institute of Technology

- Solvable subclasses of the Suranyi reduction class (25 minutes)  
  Professor Burton Dreben, Harvard University

- Discussion of above papers led by  
  Professor Marvin Minsky, Massachusetts Institute of Technology

The Council of the Society will meet at 5:00 P.M. on Friday and will continue its meeting after an intermission for dinner.

The Hotel New Yorker is on the corner of 34th Street and 8th Avenue, one block west and one block north of the Pennsylvania Station, from which there is a tunnel direct. The hotel is on the 8th Avenue line of the Independent Subway, at the 34th Street or Pennsylvania Station stop. The hotel can be reached by walking underground from the 34th Street station of the IRT subway. Airline service into LaGuardia and Idlewild airports is served by the East Side Terminal, eight blocks north and two blocks west of the New Yorker, which is somewhat nearer to the hotel than the East Side Terminal is. The hotel has its own garage, which advertises reasonable rates.

A reservation blank will be found on page 79 of this issue of the NOTICES. Persons intending to stay at the Hotel New Yorker should use this blank and send it in as soon as possible.

Further details about the meeting will appear in the April issue of these NOTICES.

Everett Pitcher  
Associate Secretary
FIVE HUNDRED SEVENTY-NINTH MEETING

Chicago, Illinois
April 14-15, 1961

The five hundred seventy-ninth meeting of the American Mathematical Society will be held at the University of Chicago, Chicago, Illinois on Friday and Saturday, April 14-15, 1961. There will probably be four to six sessions for the presentation of contributed papers.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings Professor Israel N. Herstein of Cornell University and the University of Chicago, and Professor James A. Jenkins of Washington University will address the Society. Professor Herstein's title is "Lie and Jordan structures in simple, associative rings" and Professor Jenkins' title is "The general coefficient theorem and certain applications."

It should be noted that the deadline for the receipt of abstracts is March 1, 1961, and it may not be possible to accommodate those whose abstracts fail to meet the deadline by having special sessions for late papers.

Complete details of the program will appear in the next issue of the NOTICES.

J. W. T. Youngs
Associate Secretary

FIVE HUNDRED EIGHTIETH MEETING

Stanford University
Stanford, California
April 22, 1961

The five hundred eightieth meeting of the American Mathematical Society will be held on Saturday, April 22, 1961 at Stanford University, Stanford, California.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, there will be an address at 11:00 A.M. by Professor Hans Zassenhaus of McGill University and the University of Notre Dame on "Modern developments in number geometry."

Sessions for contributed papers will be held at 9:30 A.M. and 2:00 P.M. Registration and the morning sessions will be in the Little Theater, at the side of Memorial Auditorium. The afternoon sessions will be held in the Mathematics Building (Building 60).

A tea for persons attending the Meeting will be given at the Bowman Alumni House following the afternoon sessions.

Stanford University is about thirty miles south of San Francisco, adjacent to the town of Palo Alto. The Southern Pacific Railroad stops at Palo Alto. Limousine service from the San Francisco International Airport is available for about $2.50.

There are numerous hotels and motels within easy driving distance of the campus. A detailed list can be obtained from the Palo Alto Chamber of Commerce. The hotels closest to the campus are the Cardinal Hotel and the President Hotel

R. S. Pierce
Associate Secretary
With the support of the National Science Foundation, the Eighth American Mathematical Society Summer Institute will be held at Stanford University in August, 1961.

The purpose of the Institute is to present some recent applications of the methods and ideas of functional analysis to various problems in harmonic analysis, theory of differential and integral operators, operator theory, Banach algebras, and other topics, as well as the application of the methods of one and several complex variables to some problems of functional analysis.

Professors A. Beurling, A. Calderon, L. Ehrenpreis, L. Hormander, and I. Singer will each present a series of lectures on some of their recent researches and related matters. Other participants will have an opportunity to communicate their recent results in seminars.

The Program Committee consists of Professor Ralph Phillips of the University of California at Los Angeles; Professor Henry Helson of the University of California at Berkeley, and Professor Peter D. Lax of the Institute of Mathematical Sciences, New York University, Chairman. Those interested in participating in the Institute should contact members of the Committee before March first. Financial support is available to about 30 participants, some of which will be used to support younger people (recent Ph.D.'s).

Room and board at the dormitories of Stanford University will be available to participants and their families at a reasonable rate.

Peter D. Lax, Chairman
Organizing Committee

Recently published

International Series of Monographs on Pure and Applied Mathematics, Vol. 18
POLYNOMIALS ORTHOGONAL ON A CIRCLE AND INTERVAL
by Ya. L. Geronimus
One of the most important problems of the theory of orthogonal polynomials is that of the conditions of convergence of the expansion of a given function into a series of orthogonal polynomials. This work deals with the problem and will be invaluable to mathematicians in solving complex problems. $8.50

Vol. 11 AXIOMATICs OF CLASSICAL STATISTICAL MECHANICS
by Rudolf Kurth
This book constructs classical mechanics as a deductive system, founded only on the equations of motion, and a few well known postulates which formally describe the concept of probability. A chapter is included in which are surveyed the mathematical tools used, so that only a knowledge of the elements of calculus and analytical geometry need be known by the reader. $7.50

Vol. 14 ANALYTICAL QUADRICS
by Barry Spain
In this book the elementary theory of the plane, straight line, sphere, cone, central quadrics and paraboloids are developed 'ab initio'. The concept of the plane at infinity is introduced using homogenous cartesian coordinates and the properties of the quadric given by the general equation are studied. $5.50

In preparation

Vol. 13 INTRODUCTION TO SET THEORY AND TOPOLOGY
by K. Kuratowski
Vol. 15 THE THEORY OF MEASURE AND LEBESGUE INTEGRATION
by S. Hartman and J. Mikusiński

Pergamon Press
122 East 55th Street, New York 22, N.Y.
ACTIVITIES OF OTHER ASSOCIATIONS

CANADIAN MATHEMATICAL CONGRESS

A JOINT SEMINAR AND CONGRESS will be held by the Canadian Mathematical Congress and the Theoretical Physics Division of the Canadian Association of Physicists at the University of Montreal, P. Q., from August 14 to September 9, 1961. The first three weeks will be devoted to the Seminar and the final week to the Congress.

Further details of the program will be announced later. The list of lecturers and their subjects will be as follows:

CONGRESS RESEARCH LECTURES (Two lectures and one seminar each week)

Arthur Erdélyi California Institute of Technology Solutions of ordinary differential equations containing a large parameter
Leon Henkin University of California, Berkeley The Theory of cylindrical algebras
André Lichnerowicz College de France Transformations analytiques des variétés kahlériennes
Ian N. Sneddon The University, Glasgow Mixed boundary value problems in mathematical Physics

THEORETICAL PHYSICS RESEARCH LECTURES (Three lectures each week)

G. F. Chew University of California Strong coupling
S. R. de Groot University of Leyden Irreversible thermo-dynamics
M. H. L. Pryce University of Bristol Magnetic relaxations

A fourth lecturer will be announced later.

JOINT RESEARCH LECTURES

A joint research lecturer from Russia will contribute to the program.

CONGRESS INSTRUCTIONAL LECTURES (Five lectures each week)

Israel Halperin Queen's University Continuous geometry
Thomas E. Hull University of British Columbia Numerical analysis
Hale F. Trotter Queen's University Probability theory
J. M. A. Maranda Université de Montréal Algèbre

THEORETICAL PHYSICS INSTRUCTIONAL LECTURES (Three lectures each week)

A. B. Bhatia University of Alberta Solid state
F. A. Kaempffer University of British Columbia Elementary particles
H. Reeves University of Montreal Astrophysics

Financial aid will be available for members of the staff of, and graduate students in Canadian universities.

Reduced railway fares will be available for travel in Canada. For information concerning living accommodation and board, please write to: Professor R. Brossard, University of Montreal, Montreal, P. Q.

Other information can be obtained from the Secretariat, Canadian Mathematical Congress, Chemistry Building, McGill University, Montreal, P. Q., or from the Secretary-Treasurer of the Theoretical Physics Division, Dr. D. D. Betts, University of Alberta, Edmonton, Alberta.
A two-day meeting of SIAM will be held at the U. S. Naval Postgraduate School in Monterey, California on Thursday and Friday, April 20-21, 1961. It is timed to immediately precede the meeting of the American Mathematical Society at Stanford, California on Saturday, April 22, 1961.

The meeting will include a symposium on "Lunar and Planetary Probes" on Friday morning at 9:30 A.M.

Sessions for contributed papers will be held on Thursday afternoon at 3:30. Abstracts for contributed papers should be mailed to: Professor Robert E. Gaskell, Department of Mathematics, Oregon State College, Corvallis, Oregon, before February 28, 1961.

Information concerning hotel or motel accommodations can be obtained by writing: Monterey Peninsula Convention Bureau, P. O. Box 1571, Monterey, California. The registration fee is $2.00 for each member of SIAM, and $3.00 for a non-member.

The open mess facilities of the U. S. Naval Postgraduate School will be available for meal service.

Monterey is situated approximately 120 miles south of San Francisco on California Highway No. 1, and 320 miles north of Los Angeles. Monterey Airport is serviced by United Airlines and Pacific Airlines. Southern Pacific trains depart from San Francisco for Monterey once daily. Monterey can also be reached via Greyhound busses.

Further details about the meeting will be given in the April issue of the NOTICES.

FUTURE MEETINGS OF RELATED ORGANIZATIONS

This Calendar includes symposia, seminars, and institutes sponsored by the Society, but does not include regular meetings of the AMS or the MAA, which are listed elsewhere in the NOTICES.

March, 1961

The University of Arizona
"Conference on Data Processing Problems in Engineering and Scientific Research"
Date: March 16-17, 1961
Contact: Miss Betty L. Takvam, Conference Secretary, Numerical Analysis Laboratory, University of Arizona, Tucson, Arizona

American Mathematical Society, Office of Ordnance Research, U. S. Army, and the National Science Foundation
A Symposium on "Mathematical Problems in the Biological Sciences"
Hotel: Hotel New Yorker
Location: New York, New York
Date: April 5-7, 1961
Chairman: Dr. S. M. Ulam, Los Alamos Scientific Laboratories, University of California, Los Alamos, New Mexico

American Mathematical Society, Association for Computing Machinery, Association for Symbolic Logic, and The Institute for Defense Analyses
A Symposium on "Recursive Function Theory"
Hotel: Hotel New Yorker
Location: New York, New York
Date: April 6-7, 1961
Chairman: Stephen C. Kleene

April, 1961

A Symposium on "Electro-Magnetic Waves"
Place: Mathematics Research Center, University of Wisconsin
Location: Madison 6, Wisconsin
Date: April 10-12, 1961

A Symposium on "Information and Decision Processes"
Place: Purdue University
Location: Lafayette, Indiana
Date: April 12-13, 1961
Contact: Dr. Robert E. Machol, School of Mechanical Engineering, Purdue University, Lafayette, Indiana

Society for Industrial and Applied Mathematics Meeting
Place: U. S. Naval Postgraduate School
Location: Monterey, California
Date: April 20-21, 1961

A Symposium on "Lunar and Planetary Probes" will be held April 21, 1961.

May, 1961

Western Joint Computer Conference
Hotel: Ambassador Hotel
Location: Los Angeles, California
Date: May 9-11, 1961
Information Systems Branch, Office of Naval Research
A Symposium on "Large Capacity Memory Techniques for Computing Systems"
Place: Department of Interior Auditorium, C. Street between 18th and 19th Streets, N. W.
Location: Washington, D. C.
Date: May 23-25, 1961
Contact: Miss Josephine Leno, Code 430A, Office of Naval Research, Washington 25, D. C.

Operations Research Society of America
19th National (9th Annual)
Location: Chicago, Illinois
Date: May, 1961
Meeting Chairman: Mr. Donald H. Schiller, Caywood-Schiller Associates, 203 North Wabash Avenue, Chicago 1, Illinois

Spring, 1961
International Mathematical Union
International Symposium on "Fonctions de variables complexes et analyse fonctionnelle"
Location: Portugal
Date: Spring, 1961
Chairman: Professor J. Vicente Goncalves

June, 1961
American Mathematical Society and Air Force Office of Scientific Research
A Symposium on "Convexity"
Location: Seattle, Washington
Date: June, 1961
Chairman: Professor Victor Klee, Department of Mathematics, University of Washington, Seattle 5, Washington

July, 1961
Air Force Office of Scientific Research/Aeronautical Sciences Directorate and RIAS
A Symposium on "Differential Equations in Non-Linear Mechanics"
Location: Air Force Academy, Colorado
Date: July 31-August 4, 1961
Contact: Captain John Gilbert, Air Force Office of Scientific Research, Washington 25, D. C., or Dr. Joseph LaSalle, RIAS, Baltimore, Maryland

August, 1961
American Mathematical Society and the National Science Foundation
1961 Summer Research Institute on the topic "Applications of Functional Analysis"
Location: Stanford, California
Place: Stanford University
Date: August, 1961
Chairman: Peter D. Lax, Institute of Mathematical Sciences, New York University, New York, New York

September, 1961
International Association for Analog Computation
Location: Belgrade, Yougoslavia
Date: September 4-9, 1961
Contact: Dusan Strujic, President, Comité Yougoslave de l'Electronique, des Telecommunications, de l'Automatisme et de la Technique Nucléaire, Decanska 14/IV, Belgrade, Yougoslavia

7th Midwestern Conference
"Fluid Mechanics and Solid Mechanics"
Place: Michigan State University
Location: East Lansing, Michigan
Date: September 6-8, 1961
Contact: Professor J. E. Lay, Mechanical Engineering Department, Michigan State University, East Lansing, Michigan

Association for Computing Machinery - National Meeting
Hotel: Statler Hilton Hotel
Location: Los Angeles, California
Date: September 6-8, 1961
Chairman: Ben Handy, Litton Industries

International Association for Cybernetics
International Congress on Cybernetics, 3d
Location: Namur, Belgium
Date: September 11-15, 1961

November, 1961
Czechoslovak National Committee of Mathematicians (assisted by Polish mathematicians)
"Topology and its Methods in Other Mathematical Disciplines"
Location: Prague, Poland
Date: September, 1961
Contact: Professor Kazimierz Kuratowski, University of Warsaw, Warsaw, Poland

Operations Research Society of America - 20th National Meeting
Hotel: Jack Tar
Location: San Francisco, California
Date: November 9-10, 1961
Chairman: Paul Stillson
Fall, 1961
33rd Session of the International Statistical Institute
Location: Paris, France
Date: Fall, 1961

December, 1961
American Association for the Advancement of Science - 128th Annual Meeting
Location: Denver, Colorado
Date: December 26-31, 1961

The Econometric Society
Location: New York, New York
Date: December 27-29, 1961 with AEA and ASA

Eastern Joint Computer Conference
Location: Washington, D. C.
Date: December, 1961

Date Unknown
International Union of Theoretical and Applied Mechanics
Colloquium on "Non-Linear Vibrations"
Location: Moscow, USSR
Date: 1961
Contact: Maurice Roy, Membre, Académie des Sciences, 29, av, de la Division Leclerc Châillonsous-Bagneux (Seine), France

1962 Preview
The International Congress of Mathematicians will be held in Stockholm, August 15-22, 1962.

Announcing a new series of publications

SELECTED TRANSLATIONS IN MATHEMATICAL STATISTICS AND PROBABILITY

Volume I
This volume contains 25 papers. Published for the Institute of Mathematical Statistics by the American Mathematical Society

25% discount to members of IMS and AMS

306 pages $4.80

Orders for copies of Volume I and standing orders for this new series should be sent to the

AMERICAN MATHEMATICAL SOCIETY
190 Hope Street, Providence 6, R. I.
IN WASHINGTON

Left top: J. J. Stoker - Gibbs Lecture
Left bottom: Executive Committee of the Association: H. M. Gehman; C. B. Allendorfer; H. L. Alder; E. Johnson; Harley Flanders; R. D. James; A. S. Householder; A. W. Tucker.
Right top: W. H. Turner, Exec. Dir. of the United States Steel Foundation presenting a check to H. F. Bohnenblust for the Society.
Right center: At the Registration Desk
Right bottom: One of the Book exhibitors.
Washington is also the place where the day-by-day operation of many programs in support of science, mathematics, and education is conducted. The National Science Foundation has its headquarters at 1951 Constitution Avenue, and the Office of Naval Research is nearby. The Office of Scientific Research has its headquarters in Washington, and the National Institutes of Health are located in Bethesda, Maryland (a suburb of Washington). The U. S. Office of Education is located in the Health, Education, and Welfare Building. At least one agency with a program which supports mathematics is located elsewhere: the Office of Ordnance Research is located at Duke University. It may be remarked that no one of the well-known private foundations is located in Washington.

Some may be surprised to learn that the Department of State operates extensive programs which are concerned with science, mathematics, and education. The Office of the Science Adviser to the Secretary of State endeavors to follow very closely new discoveries or proposals for scientific research or support that may affect the course of events in a particular country, a region, or an international organization. This office appoints scientific attachés who serve as advisers to ambassadors abroad. The Department of State contains also the Bureau of Educational and Cultural Affairs. It is responsible for educational and cultural exchange programs, and also for the Secretariat of the U. S. National Commission for UNESCO. The American Mathematical Society nominates a representative for membership on this Commission. The Department of State has many connections with activities throughout the world. Report (4) is one sample of planning and operations that are carried on through NATO, OEEC, and similar organizations.

A significant change has occurred recently in the operations of the National Science Foundation. The Act of Congress which established NSF in 1950 permitted it to support the social sciences, but did not require it to do so. In 1958 NSF organized its Office of Social Sciences and provided a small budget. This Office has now been changed into the Division of Social Sciences. (The other divisions of the foundation are the Division of Mathematical, Physical, and Engineering Sciences; the Division of Biological and Medical Sciences; and the Division of Scientific Personnel and Education.) The activities of each division of the NSF are guided by a divisional committee. Professor S. S. Wilks of Princeton University is a member of the divisional committee for the

Washington is also the place where the day-by-day operation of many programs in support of science, mathematics, and education is conducted. The National Science Foundation has its headquarters at 1951 Constitution Avenue, and the Office of Naval Research is nearby. The Office of Scientific Research has its headquarters in Washington, and the National Institutes of Health are located in Bethesda, Maryland (a suburb of Washington). The U. S. Office of Education is located in the Health, Education, and Welfare Building. At least one agency with a program which supports mathematics is located elsewhere: the Office of Ordnance Research is located at Duke University. It may be remarked that no one of the well-known private foundations is located in Washington.

Some may be surprised to learn that the Department of State operates extensive programs which are concerned with science, mathematics, and education. The Office of the Science Adviser to the Secretary of State endeavors to follow very closely new discoveries or proposals for scientific research or support that may affect the course of events in a particular country, a region, or an international organization. This office appoints scientific attachés who serve as advisers to ambassadors abroad. The Department of State contains also the Bureau of Educational and Cultural Affairs. It is responsible for educational and cultural exchange programs, and also for the Secretariat of the U. S. National Commission for UNESCO. The American Mathematical Society nominates a representative for membership on this Commission. The Department of State has many connections with activities throughout the world. Report (4) is one sample of planning and operations that are carried on through NATO, OEEC, and similar organizations.

A significant change has occurred recently in the operations of the National Science Foundation. The Act of Congress which established NSF in 1950 permitted it to support the social sciences, but did not require it to do so. In 1958 NSF organized its Office of Social Sciences and provided a small budget. This Office has now been changed into the Division of Social Sciences. (The other divisions of the foundation are the Division of Mathematical, Physical, and Engineering Sciences; the Division of Biological and Medical Sciences; and the Division of Scientific Personnel and Education.) The activities of each division of the NSF are guided by a divisional committee. Professor S. S. Wilks of Princeton University is a member of the divisional committee for the
Division of Social Sciences.

The new Division of Social Sciences will organize its support of basic research in the social sciences under four programs: (a) anthropological sciences, including ethnology, archeology, linguistics, and physical anthropology; (b) economic sciences, including econometrics, economic and social geography, the economics of research and innovation, and general mathematical economics; (c) sociological sciences, including demography, social psychology, psycho-linguistics, and the sociology of science; and (d) a program supporting basic research in the history and philosophy of science.

The National Science Foundation provides pamphlets (5) which describe its program activities, and it publishes an extensive annual report (6). Furthermore, it issues regularly press releases which describe important events and significant new developments in its operations. The other agencies of the Federal Government with programs which support science, mathematics, and education also supply information about their nature and operation, and about how to submit proposals.

The National Academy of Sciences-National Research Council is a unique organization which has its headquarters in its building at 2101 Constitution Avenue in Washington. The National Academy of Sciences was organized by President Lincoln in 1863 in connection with the Civil War; the National Research Council was organized by President Wilson in connection with World War I. The Academy-Council is described as a semi-governmental organization: it is a private organization, but it has responsibility for giving advice to the government on scientific matters. Many of its activities are carried on by committees whose members are drawn from throughout the United States; its extensive operations, however, have their headquarters in Washington.

The collection of information, much of it of a statistical nature, is a major activity in Washington. Reports (7) through (13) indicate the types of information collected and published by the National Science Foundation. Report (13) was prepared by the Office of Scientific Personnel of the National Academy of Sciences-National Research Council, under a grant from the National Science Foundation, The Office of Education collects and publishes information of many kinds; reports (14) through (17) are samples.

There are many private organizations in Washington that collect and publish information. The National Education Association and the American Council on Education both have their headquarters in Washington. Both are heavily involved in planning, and both collect and supply much information to their members. Reports (18) and (19) are samples of their publications.

Mathematics is in a transition period. Formerly, mathematical activity was restricted almost entirely to college and university professors who engaged in teaching and research. Now, mathematics is a profession, and many make a living from employment as mathematicians in business, industry, and government. The reports listed at the end of this article emphasize the change and also the problems that arise as a result. These reports show clearly the small (and inadequate) production of Ph. D. mathematicians, the increasing enrollments in mathematics, the increasing shortages of staff, and the increasing employment of mathematicians in business, industry, and government.

Mathematics is in the very center of the scientific and technological revolution in progress at the present time. It is important that mathematicians examine and evaluate the information and data that concern mathematics. Others may collect the information, but the mathematicians themselves must study and evaluate it and help to insure that it is properly used in planning for the future.

REFERENCES


4. Increasing the Effectiveness of Western Sci-
ence. Fondation Universitaire, Brussels, 1960. This report is an outgrowth of discussions at meetings of the Science Committee of the North Atlantic Treaty Organization.

5. Program Activities of the National Science Foundation, NSF 59-32, 1959.


9. Scientific Manpower Bulletin No. 12 (December 1960), National Science Foundation, NSF 60-78.

10. Funds for Research and Development in Industry 1957, National Science Foundation, NSF 60-49.

11. Scientific Manpower 1959, Papers of the Eighth Conference on Scientific Manpower, National Science Foundation, NSF 60-34.


VISITING FOREIGN MATHEMATICIANS. Here is a supplement to the list published in the December, 1960 NOTICES, pages 833-840.

<table>
<thead>
<tr>
<th>Name</th>
<th>Host Country</th>
<th>Host Institution</th>
<th>Period of visit From To</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freudenthal, H.</td>
<td>Netherlands</td>
<td>Yale University</td>
<td>September 1960 - September 1961</td>
</tr>
<tr>
<td>Mostowski, A.</td>
<td>Poland</td>
<td>AMS, Project FOCUS</td>
<td>April 5, 1960 - April 23, 1960</td>
</tr>
<tr>
<td>Siciak, J.</td>
<td>Poland</td>
<td>Stanford University</td>
<td>November 1960 - November 1961</td>
</tr>
</tbody>
</table>
In February, 1961, the first volume will appear in the new series of Selected Translations in Mathematical Statistics and Probability. This series, described in more detail in the New Publications section of this issue of the NOTICES, is published for the Institute for Mathematical Statistics by the American Mathematical Society.


To the list of sources from which the American Mathematical Society inquires about existing or intended translations the following two additions have been made: Consultants Bureau Enterprises, Incorporated, 227 West 17th Street, New York 11, New York; Morris D. Friedman, Incorporated, Foreign Technical Translations, 1383A Washington Street, - Mail: P. O. Box 35, West Newton 65, Massachusetts.

The Consultants Bureau is under the direction of Mr. and Mrs. Earl Coleman, who recently visited the USSR and had conversations with the directors of various presses there, in particular at the Academy of Sciences and at the Publishing House for the Literature of Mathematics and Physics: Izdat. Fiz.-Mat. Lit. The directors of the Russian presses agreed to furnish the Consultants Bureau with advance information about the appearance of significant books on Russian science.


N. N. Bogolyulov, B. V. Medvedev and M. K. Polivanov: Questions in the theory of dispersion relations. This book is in the same series as the above. It was published in Moscow in 1958 but the translation will be based upon a revised and supplementary text prepared by the authors.

The attention of mathematicians is also called to the existence of a translation published by the Consultants Bureau last year of the following book, in which mathematical methods play a considerable role. N. N. Bogolyulov, V. V. Tolmachev and D. V. Shirkov: A new method in the theory of superconductivity. The original Russian text was published by the Academy of Sciences in 1958 under the auspices of the V. A. Steklov Institute of Mathematics and the Joint Institute for Nuclear Research (Dubna).

The Friedman Corporation has just completed, among others, the following translations, which are available in multilithed form:


The Dutch publishers Erven P. Noordhoff, Ltd., Groningen, announce their intention of publishing a translation of the following Russian book, which was reviewed in Mathematical Reviews vol. 21 no. 958.

From the AMS Secretary
John W. Green, UCLA

In the last two decades the pace of mathematical activities in the United States has increased by a factor, a conservative estimate of which is two or three. In addition to the greatly increased number of those going into mathematics there is evidence that on the average the individual mathematician is more active in his use of publications, attendance at meetings, etc., than twenty years ago. One of the results of this is the large increase in the size of our national meetings, which has in turn made the scheduling of these meetings more of a problem.

Up until January 1958, when the Annual Meeting was held at the Sheraton-Gibson Hotel in Cincinnati, almost all of the Society meetings had been held at academic institutions. For the Summer and Annual meetings, where it is necessary to supply living accommodations, use had usually been made of university dormitories, which have the advantage of being both convenient and economical. Even with present day large meetings, it appears possible to arrange Summer Meetings at universities with living accommodations in dormitories, and Summer Meetings through 1966 are presently so scheduled.

However, the change from the Christmas vacation to the between semesters date for the Annual Meeting made the use of university dormitories impossible. Even during Christmas vacation, dormitory arrangements are more difficult to arrange than in late summer. For one thing, use frequently has to be made of rooms in which students' personal belongings have been left - an unsatisfactory arrangement, as I certainly would maintain if I were one of the students concerned. For another, it is desirable to hold winter meetings in locations easily and rapidly accessible by air, and this rules out many schools in rural locations with excellent facilities.

Once having given up the use of university dormitories, it appeared logical to make use of other hotel facilities and hold our meetings entirely in hotels. This has several advantages, of which the holding of all the activities of a meeting under one roof in winter weather is not the least. There are also disadvantages, such as the extreme dependence of mathematicians on excellent blackboard facilities, which are sometimes available in universities but almost never in hotels. An experiment was held at the recent Washington, D. C., meeting in large scale use of projecting equipment to overcome the blackboard difficulty in hotels. The results are not known at this time of writing.

Our experience with hotel meetings is not yet such as to allow us to anticipate all difficulties and get things running smoothly, or in fact to allow us to assert that they are better than meetings at universities with living accommodations in hotels. A meeting of this latter sort is to be held at the University of California, Berkeley, in January, 1963. At the 1960 Annual Meeting in Chicago, considerable feeling for this kind of meeting was expressed. However, a solid week of rain in Berkeley in 1963 would very likely result in a swing the other way.

One type of hotel with large and pleasant facilities for such meetings as ours, and of which we have not made use of in the past, is the resort hotel. Its disadvantages are in its close proximity to no university, its sometimes inconvenient location, and also sometimes its price (together with the fact that there are no less expensive hotels nearby). The only meeting of this sort I can recall was one held at Yosemite Lodge in Yosemite National Park in May, 1954. This was a small regional meeting, but was highly successful from the viewpoint of good facilities. We may wish to look further into the use of resort hotels.

WANTED:

Periodicals:
Mathematics, Physics, etc.

HIGH PRICES PAID

ABRAHAMS
MAGAZINE SERVICE
Dept. M 56 E. 13th St., N. Y. 3
MATHEMATICAL LIFE IN THE USSR

By N. D. Kazarinoff

The part of Moscow which lies southeast of the Moscow River is bisected by one of Moscow's busiest and most spacious boulevards, Leninskii Prospect. We live on this Prospect about three miles from the center of town. Our apartment, the interior of which has a 1920 look, is on the seventh floor of a new eight-story apartment building. This building "belongs" to the ANSSSR (Academy of Sciences of the USSR). In it live workers and intelligentsia. There are about 190 one-, two- and three-room apartments in the building, which has six entrances and six elevators. The hall, kitchen, washroom, and W. C. are not counted as rooms. Our three rooms have two exposures and 54 square meters. Even in this new building there is often one family to a room. A family of four with one son or daughter married may have a two-room flat. We are living in a nicer, more modern, more pleasantly situated building and are less crowded than most Moscow mathematicians. I hereby publicly thank the ANSSSR for delivering us from our initial desperate plight to our seventh heaven on Leninskii Prospect.

The housing situation is changing fast. Buildings such as ours have been put up by the hundreds in the last three years, and construction continues with no sign of abatement.

A quarter of a mile east of our apartment lies the Steklov Institute. I think of the east side of the Prospect as the quiet side (except for streetcars). The Institute livens up a bit at seminar time and when students from the special high school for mathematics arrive for a lecture or when high school classes, which have been singled out for training in programming, come in for instruction. There is an acute shortage of programmers in the USSR. Computing machines are being introduced in industry to make automation possible, but there are not enough engineers and programmers to run them. The Novikov-Markov logic seminar, Pontryagin's seminar on automatic control, and Nikol'skiđ's seminar on analysis (in particular on functions of several complex variables) seem to be the major seminars at the Institute. The seminars in mathematical physics (Bogolyubov's group) and in probability appear to be smaller. Seminars meet for two hours.

By contrast MSU (Moscow State University) is always crowded and bustling with activity. One needs a special permit, complete with photograph, to get in. It took me half a day to obtain my permit. Once at a Mathematical Society meeting at the University someone complained about the "propusk" or entry-card system at MSU and the inconveniences it caused him because he was not a faculty member. "Even I have difficulties," Alexandrov volunteered. "You are in good company. Be patient like the rest of us."

At popular hours all classrooms and lecture halls are in use. A partial list of the mathematics seminars is given at the end of this report. These are weekly seminars which run for two semesters. A review seminar surveys the current literature in the area of interest.

Gelfand's seminar is the most remarkable one I have ever attended. It begins at 7 p.m. and lasts three hours or more without interruption. Announcements of research completed are made; reports on papers are given; summaries of results presented at meetings are delivered (for example, the Warsaw Conference on Functional Analysis). Gelfand is continually interrupting speakers and redoing their exposition so as to make it clearer. Upon hearing a result new to him he specializes it, generalizes it, compares it with what seems similar, gives it a setting in a different theory, and in the end understands it. He asks questions and suggests problems. He criticizes and praises. He ignores the speaker and talks earnestly with Silov. He picks out a student who should be able to understand a discussion but does not, and he goes over everything again with him bit by bit. Attendance is large. The order of magnitude is $10^2$. People come from all over the city. Last month, Ladyzenskaya had a six-hour train layover in Moscow; she spent most of it at the seminar. In fact, out-of-town guests come often. V. I. Šik is a regular; the twin brothers Yaglom come; Naimark is a regular contributor. United States mathematics is carefully watched. Comparisons of different schools are made. Gelfand is interested in mathematicians as people. His questions run: "What does he like to do?" "How old is he?" "What are you going to be doing ten years hence?" "What do you want to be doing ten years hence?" Any mathematics is fair game for Gelfand's seminar, but the emphasis this year has been on differential equations (ordinary and par-

* This is the second in a series of articles by Professor Kazarinoff, the first "A Report from the USSR" appeared in the December, 1960 NOTICES, page
tial), and on the theory of functions of several complex variables. Many Russian mathematicians are now orienting themselves so as to participate in building this theory.

The Alexandrov-Keldysh (Ludmila Keldysh) seminar has been concerned with Brown's beautiful result* and its numerous consequences. Oleinik's seminar is giving special attention to the behavior of solutions of nonlinear systems, discontinuities, etc. The Novik-Lyusternik seminar is an attempt on the part of the directors to find out what the problems of diffraction theory are. They began with a review of Buchar's and Keller's recent paper and a review of Keller's geometric theory. In general, the attendance at seminars is impressive. The audiences are both friendly and critical, and the students "know their stuff."

The Moscow Mathematical Society meets on Tuesday evenings at the University. At a recent business meeting Alexandrov was re-elected president over his lively protestations, and Kuros and Kolmogorov were re-elected vice-presidents. The last meeting of the Society was devoted to three papers on the theory of programming. The most interesting paper was Etsov's. A month ago he established the equivalence between the four-color problem and one of programming theory.

Wednesday evenings the high school section of the Society meets at MSU. Again, attendance is impressive. The students solve problems, present solutions, and pose problems. People such as V. G. Boltyanskii, who have a special interest in elegant elementary mathematics, lecture. Boltyanskii's pamphlet on figures of equal area (or


PARTIAL LIST OF SEMINARS AT THE MOSCOW STATE UNIVERSITY

<table>
<thead>
<tr>
<th>Subject</th>
<th>Type (if special)</th>
<th>Who may attend (indicated by class year)</th>
<th>Directors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Selected questions of algebraic geometry</td>
<td>Review</td>
<td>Gel'fand, Postnikov, Safarevich</td>
<td>Aleksandrov</td>
</tr>
<tr>
<td>2. Topology</td>
<td>Study and investigatory</td>
<td>4</td>
<td>Aleksandrov, Keldysh, Smirnov</td>
</tr>
<tr>
<td>3. Set theoretic topology</td>
<td></td>
<td>3</td>
<td>Oleinik</td>
</tr>
<tr>
<td>4. Topology</td>
<td></td>
<td>2</td>
<td>Nemyckii</td>
</tr>
<tr>
<td>5. Partial differential equations</td>
<td></td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>6. Ordinary differential equations</td>
<td></td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Subject</td>
<td>Type (if special)</td>
<td>Who may attend (indicated by class year)</td>
<td>Directors</td>
</tr>
<tr>
<td>----------------------------------------------</td>
<td>-------------------</td>
<td>------------------------------------------</td>
<td>------------------------------------------------</td>
</tr>
<tr>
<td>7. Systems of ordinary differential equations</td>
<td></td>
<td>4,5</td>
<td>Nemyckii, Demidovlev, El'gin</td>
</tr>
<tr>
<td>8. Operator differential equations</td>
<td></td>
<td>3,4</td>
<td>Nemyckii, Gusarova, Wainberg</td>
</tr>
<tr>
<td>9. Operator equations</td>
<td></td>
<td>5</td>
<td>Nemyckii</td>
</tr>
<tr>
<td>10. Analysis IV</td>
<td></td>
<td>4</td>
<td>Šilov</td>
</tr>
<tr>
<td>11. Partial differential equations</td>
<td></td>
<td>4,5</td>
<td>Landis, Gal'pern, Borovikov</td>
</tr>
<tr>
<td>12. Theory of numbers</td>
<td></td>
<td>2,3</td>
<td>Šidlovskii</td>
</tr>
<tr>
<td>13. Selected problems</td>
<td></td>
<td>3,4,5</td>
<td>Dynkin</td>
</tr>
<tr>
<td>14. Functional Analysis</td>
<td>Universal</td>
<td>4</td>
<td>Gel'fand</td>
</tr>
<tr>
<td>15. History of mathematics</td>
<td></td>
<td>3</td>
<td>Rybnikov</td>
</tr>
<tr>
<td>16. History of mathematics</td>
<td>Study and investigatory</td>
<td>5</td>
<td>Rybnikov, Yanovskaya, Yuškevicĭ</td>
</tr>
<tr>
<td>17. Current problems in uniform topology</td>
<td>Review</td>
<td>3,4,5</td>
<td>Smirnov</td>
</tr>
<tr>
<td>18. Theory of orthogonal series</td>
<td></td>
<td>3,4,5</td>
<td>Men'sov, Bari</td>
</tr>
<tr>
<td>19. Functions of a real variable</td>
<td>Study and investigatory</td>
<td>4</td>
<td>Men'sov, Bari</td>
</tr>
<tr>
<td>20. Differential equations and generalized functions</td>
<td></td>
<td>5</td>
<td>Šilov</td>
</tr>
<tr>
<td>21. Theory of representations</td>
<td></td>
<td>4,5</td>
<td>Kirillov, Berezin</td>
</tr>
<tr>
<td>22. Theory of functions of a real variable</td>
<td></td>
<td>2</td>
<td>Men'sov, Vinogradov</td>
</tr>
<tr>
<td>23. Some mathematical problems in quantum mechanics</td>
<td></td>
<td>4</td>
<td>Milos, Berezin</td>
</tr>
<tr>
<td>24. Mathematical physics</td>
<td></td>
<td>2,3</td>
<td>Godunov</td>
</tr>
<tr>
<td>25. Functional methods of solution of boundary value problems for partial differential equations</td>
<td></td>
<td>4</td>
<td>Il'in, Kondrat'ev</td>
</tr>
<tr>
<td>26. Free sums and free derivatives</td>
<td></td>
<td>2,3,4,5</td>
<td>Skornyakov</td>
</tr>
<tr>
<td>27. Problems of mathematical logic and history of logic and semantics</td>
<td></td>
<td>-</td>
<td>Avkuznecov</td>
</tr>
<tr>
<td>28. Theory of group representations</td>
<td></td>
<td>2</td>
<td>Naǐmark</td>
</tr>
<tr>
<td>29. Elementary theory of generalized functions</td>
<td></td>
<td>2</td>
<td>Kreines, Vainstein, Aizenstat</td>
</tr>
<tr>
<td>30. Numerical methods for solution of equations of gas dynamics</td>
<td></td>
<td>4,5</td>
<td>Rusanov</td>
</tr>
<tr>
<td>31. Algebraic and differential topology</td>
<td></td>
<td>2</td>
<td>(Aspirant) Vinogradov</td>
</tr>
<tr>
<td>32. Introductory algebraic topology</td>
<td></td>
<td>1,2</td>
<td>Fuchs, Turina</td>
</tr>
<tr>
<td>33. Analysis III</td>
<td></td>
<td>3</td>
<td>Kostyučenko, (Aspirant) Mityatin</td>
</tr>
<tr>
<td>34. Particular problems in differential geometry</td>
<td></td>
<td>2</td>
<td>Efimov, Poznyak</td>
</tr>
<tr>
<td>35. Theory of associative rings</td>
<td></td>
<td>4</td>
<td>Kuroš, Andrunakievič</td>
</tr>
<tr>
<td>36. General algebra</td>
<td></td>
<td>2,3,4,5</td>
<td>Kuroš</td>
</tr>
<tr>
<td>37. Partial differential equations</td>
<td>Research</td>
<td>4</td>
<td>Petrovskii, Kreines</td>
</tr>
<tr>
<td>38. Partial differential equations</td>
<td></td>
<td>3,4,5</td>
<td>Landis, Oleinik, Gal'pern</td>
</tr>
<tr>
<td>39. Stochastic processes and differential equations</td>
<td></td>
<td>3,4,5</td>
<td>Dobrušin, Has'minskii, Girsanov, Blagoveščenskii, Višik, Lyusternik</td>
</tr>
<tr>
<td>40. Diffraction</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
A REPORT ON SOVIET MATHEMATICS
by J. P. LaSalle

After a year's study a panel organized by RIAS has completed a survey of recent Soviet contributions to mathematics. The report of this panel is to be published by The Macmillan Company and is due to appear May 1961. The report consists of some general sections, and extensive surveys of each of the mathematical fields covered. The fields covered were (1) algebra, (2) control and stability theory, (3) functional analysis, (4) numerical analysis, (5) partial differential equations, (6) probability and statistics, and (7) topology. Ordinary differential equations, where the Soviet Union clearly leads the rest of the world, was not included. It was the subject of a previous report. The panel members were: J. B. Diaz, University of Maryland; R. Fortet, Institute of Henri Poincare, Paris; J. H. Giese, Ballistic Research Laboratory, Aberdeen Proving Grounds; R. A. Good, University of Maryland; J. P. LaSalle, RIAS; S. Lefschetz, RIAS; T. Leser, Ballistic Research Laboratory, Aberdeen Proving Grounds; E. Lukacs, Catholic University; C. Masaitis, Ballistic Research Laboratory, Aberdeen Proving Grounds; Edith Moulier, University of Poitiers, France; J. Neveu, CNRS, Paris; L. Pukanszky, RIAS; H. A. Steeves, RIAS. A. P. Stokes, RIAS, served as secretary to the panel.

Major conclusions of the panel were:

1. In mathematics the Soviet Union and the United States lead the world and are about at the same level. Other than algebraic geometry there is no broad area of mathematics in which Soviet mathematicians are not working and in which they are not making significant contributions.

2. The Soviet Union will move at a faster rate than the United States in the practical application of mathematical theories. Further, there is a distinct possibility that the Soviet Union will gradually surpass us in certain areas more or less directly connected with applications: for example, control theory, numerical analysis, ordinary and partial differential equations.

Our main reason for believing this is that in Russia the application of mathematics by mathematicians is encouraged in many ways. In this country, however, there is a growing gap between pure mathematicians and those interested in applications. Consider the following:

a. Leading Soviet mathematicians have an interest in and contribute to both pure and applied mathematics. In the United States a pure mathematician rarely has an interest in applications.

b. Soviet scientists and engineers have a high regard for mathematics. There are a large number of Soviet mathematical-engineers who keep up with the latest mathematical developments and who exploit the deepest kind of mathematics. These mathematical-engineers have no counterpart in this country.

c. Soviet mathematicians make serious and successful efforts to communicate the latest theoretical advances to engineers and scientists. Long expository articles are encouraged and many fine books are written by Soviet mathematicians for

engineers and others. In the United States the space in mathematical journals is inadequate and conciseness in presentation is often enforced to the point of unintelligibility. Our specialists write for each other.

d. The above points indicate that applied mathematics is quite respectable in the USSR. In addition, certain areas (control theory, numerical analysis, differential equations) are being emphasized in Russia and this tends to attract good young people into these fields. In the United States, among pure mathematicians, there is an increasing tendency towards greater and greater abstraction, with a lessening regard for application, which tends to discourage good young people from ever approaching the applications. From this it is obvious that the quality of applied mathematics in Russia will tend to improve while in this country the quality of applied mathematics will tend to decrease.

3. An increasingly large number of Soviet mathematicians are working on control theory and numerical analysis. In quality they match the United States. In quantity of output and in numbers of people they lead the world.

4. Their effort in control theory is part of an all-out Soviet scientific effort to advance their knowledge and technical competence in the field of automatic control. Applications are to missiles and space vehicles and to the automation of industrial processes and industries.

5. For the most part the very breadth of Soviet mathematics conceals special objectives. As in the rest of the world, individual motivation is still more important in determining problems studied than direction from above. The prestige of mathematics and of individual mathematicians still protects Soviet mathematics in this regulated society. There is a strong "pressure to publish" and many articles which, by Western standards, are of poor quality do appear in their journals.

6. Many fine mathematical books are published in the USSR at fantastically low prices. In price our books cannot compete. This gives them a dangerous and potent weapon for enhancing their own scientific prestige and influence in the world.

The following are some comments on Soviet activities in each of the particular fields covered by the report:

1. Algebra.

Algebra is a strong point of Soviet mathematics, and they make a steady and relentless push toward a deeper understanding of algebra. Their research in this field is comparable to American research, being neither predominantly inferior nor disturbingly superior.

Judged by the publications available to us, more emphasis is placed on abstract algebra than on topics where the possibilities of application are more apparent. Their recent research on abstract group theory is certainly an example remote from applications. On the other hand, we did not see as much Soviet research on such topics as matrix theory and Latin squares (for which there are many practical applications) as was to be expected. This may indicate that such work is not openly published and may not be a true indication of Soviet lack of interest in matrix theory. An excellent book has been written by Gantmaher on matrix theory and applications and has been translated into English.

A half-dozen major centers for the study of algebra have flourished in the Soviet Union. Among their outstanding men are Kuroš, Mal'cev, Lyapin, Černíkov, Kantorovič, and Čunilin. Some of these leaders have been publishing on one general subject for many years, and they can be expected to continue to guide their colleagues in the same directions.

2. Control and stability.

Mathematical research in the theory of control is carried on in the USSR at two levels. There is a strong group of pure mathematicians developing a general theory of optimal control and doing research on problems in differential equations and stability that are of importance to control theory. At the same time there is an equally strong group of mathematical-engineers who are more directly concerned with applications. (Mathematicians in this country call them engineers and American engineers who know them call them mathematicians.)

The strongest purely mathematical group working on a control theory is that led by Academician L. S. Pontryagin at the Steklov Mathematical Institute of the Academy of Sciences USSR in Moscow. This group has developed a general mathematical theory of optimal control. Although they are not directly concerned with applications, they are certainly motivated by practical considerations. They work independently of other mathematicians in the Soviet Union (there is in fact a great deal of competition between mathematicians) but quite closely with the mathematical-engineers of the Institute of Automatics and Tele-mechanics (Automation and Remote Control) of the Academy of Sciences of the USSR. Their formula-
tion of the mathematical problem contains conditions which reflect constraints which occur in the practical design and operation of control systems.

As a group, the mathematical-engineers make important mathematical contributions, follow closely mathematical research, work on engineering problems, and serve to interpret and transmit mathematical results to engineers. Some of them undoubtedly advise mathematicians in the formulation of significant mathematical problems. (They are better informed of the work of American mathematicians on control theory than are American engineers.)

Soviet mathematicians appear to enjoy freedom of research. In this regulated society incentives are used to direct research towards fields such as control theory and numerical analysis, while at the same time, mathematics is supported for mathematics’ sake.

It is difficult to determine the extent to which mathematical control theory has been applied in the Soviet Union. There are a number of claims that applications have been made and that improvements in design have been achieved. They are certainly interested in applying it, and looking for applications, and are trying to use the theory to develop numerical and experimental methods of designing control systems. (The development of practical computational procedures for the design of control systems is a first step toward the development of control systems that are truly adaptive.)

The view we have of the Soviet research program leads to the conclusion that they are mounting a total scientific effort in automatic control. By contrast there is no such balanced program in the USA where developments in automatic control are primarily in the hands of engineers. (This is clearly illustrated by a study of the American “theoretical” papers presented at the First International IFAC Congress in Moscow, June 27th to July 7th, 1960.)

There is reason to believe that the USSR can achieve a rapid acceleration in the rate of technological progress by an all-out scientific program in the field of automatic control. It seems clear that they intend to make the effort and it is unwise to assume that they will not be successful. They express the belief that through automation they can demonstrate the superiority of their system - that only a communist society can carry out automation of industry without a breakdown in its economy. They expect that we cannot in our system achieve automation as rapidly as they can. For them this may be sufficient reason for an expanded program in automatic control. There is no evidence to be found in Soviet mathematical literature that the USSR is ahead of us or is behind us in the field of control. Although there are far fewer mathematicians in the USA interested in control theory than in the USSR, our contributions are comparable in quality to theirs. At present we seem to be going in the same directions. We can expect, because of the great effort, that their progress will be faster than ours. Because they have a better balanced research program in control they may recognize long before we do the importance of a fundamental idea, if the theory is applicable, they are likely to exploit it before we do.

3. Functional analysis.

Soviet mathematicians have a strong tradition in functional analysis that goes back to the founding of the subject. This subject provides a general language for the formulation and study of problems in many fields of mathematics (ordinary and partial differential equations, integral equations, numerical analysis and analysis in general). It is these applications to other fields of mathematics that are of greatest interest to Soviet mathematicians. By contrast, American mathematicians are primarily concerned with the structure of functional analysis and the achievement of more abstractness and greater generality.

The most important Soviet leaders in this field are: I. M. Gel’fand, M. A. Naǐmark, D. A. Raǐkov, and M. G. Kreǐn.

Special mention should be made of the number of excellent textbooks on this subject that have recently been published (especially those of Naǐmark, Gel’fand and Sil’ov, and Livšic and Brodskii). In the presentation of results and the explanation of current research they are far ahead of anything the West can claim. They are written at a level that makes the subject widely accessible.


In the USSR, great emphasis is being placed on computers (and closely related data processing and automatic control equipment). As a result, a substantial portion of the world’s research in numerical analysis, of excellent quality, is produced in the USSR. Official encouragement of work in this field will probably swell the USSR’s proportionate share of contributions. An abundance of timely new books (very inexpensive, at least by Western standards) and survey articles synthesizing the latest and most modern achievements in mathematics (and naturally including topics in or closely related to numerical analysis) is being written (frequently in a style to assure accessibility to
large relatively unspecialized audiences) and published in mammoth editions. As an example, one may cite the book of Sobolev on applications of functional analysis in mathematical physics, or the exposition of Panov of numerical methods for solving hyperbolic differential equations, oriented towards engineering applications.

Also, it has been frequently remarked that the most outstanding Soviet mathematicians take a keener and more active interest in numerical analysis and computational mathematics than do their Western counterparts. This interest cannot fail to affect beneficially the growth and development of the field of numerical analysis, and may even influence the areas of computer design and development. That this last is possible, and may even be encouraged in the USSR, is evident upon comparing the section on computing machines in the Mathematical Reviews, in this country, with the corresponding section in the Referativnyi Zhurnal. In the Soviet publication, the computing machine coverage seems fuller than that in Mathematical Reviews, and even goes to the extent of showing circuit diagrams and pictures of equipment and components.

5. Partial differential equations.

The calibre of Soviet research in the field of partial differential equations is on a par with that in the United States. This is a vast field of research and there is no evidence of a concerted attack on any particular class of problems. Instead, their research appears to arise from the individual motivations of mathematicians. It is quite possible that some of their research is directed to specific problems but such direction is not apparent.

They are doing a large amount of research on problems related to fluid dynamics. There was a notable absence of research on the solution of boundary value problems which arise in such fields of research as plasma physics, high speed fluid dynamics, magnetohydrodynamics and neutron diffusion. No particular significance is attached by us to this remark. The literature surveyed bore the label "mathematics" and the examination of papers in engineering and physical papers would, we feel sure, give a different impression.


The quality of the research in the areas of probability theory, information theory and mathematical statistics in the USSR is on a level with that performed in the United States. There are some slight differences, not of any great significance, in the method of approach used in some fields. For instance, Linnik uses methods of classical analysis in his study of characteristic functions which are somewhat deeper in nature than any used here, and through their use, has produced some fundamental results. Again, in mathematical statistics, scientists in this country concentrated on the development of so-called parametric methods, based on the assumption of a normal distribution in the population at hand. In Russia, however, the main interest was directed towards statistical methods which avoided the assumption of normality and this resulted in the development of what is now known as nonparametric methods. After some time the results so obtained were also appreciated in the United States, and the study of this new branch of statistics was undertaken here. But the neglect of the parametric methods was only recently ended in Russia. Now, however, both parametric and nonparametric methods are taught in Russia.

The field of mathematical statistics is receiving added emphasis at the University of Moscow, as indicated by the shift of Gnedenko from Kiev to Moscow, where he will organize a program in mathematical statistics. This program was undertaken at the request of Kolmogorov, who apparently felt that this field was not receiving adequate attention at the University in Moscow. Whether this is an indication of the influence exerted by the National Academy on the direction of science in Russia is a matter of speculation.

As regards applications, an appreciable amount of work is being done in two areas of practical interest, queuing theory and statistical quality control. In 1955 an excellent monograph on queuing theory appeared by Hinčin, which has appeared in an English translation (Griffin, London, 1960). Gnedenko also has done some work in this area.

A large number of papers on applied statistics, primarily on quality control, have appeared in the Proceedings of the Academy in Uzbekistan. Because of the obvious practical importance of this area, a number of books have appeared concerning topics related to applied statistics. These range from a book on least squares by Linnik, written for mathematical statisticians, to books by Cebotarev and Lukomskii, written for workers in geodesy and for engineers and economists respectively. In this regard it is also worthy of note that a number of books on mathematical statistics designed for engineers have appeared in rather large editions.

7. Topology.
Because of its primitiveness, topology lies in the background of an enormous and ever growing body of mathematics. As a foundation element, one will not expect to find (and does not find) many practical applications of this science.

This field started some 30 years ago as much geometry and little algebra, and consists today of much algebra and little geometry. This is definitely the trend in the United States, but much less so in the Soviet Union. In that country there are comparatively few topologists and, if anything, their number tends to decrease. As a body they are, more or less, grouped around two outstanding leaders, both among the world's top mathematicians: P. S. Aleksandrov and L. S. Pontryagin. The first group deals mostly with the same type of problems that occupied Aleksandrov a generation ago, and which has ceased to occupy the attention of world topologists. The Pontryagin group, on the other hand, includes by far the best of the younger Soviet topologists: Postnikov, Boltyanski, Rohlin, Bokstein. This group is in close contact with the work of modern algebraic topologists outside the USSR, and has made, in recent years, highly significant contributions to topology.

8. Russian journals.

One or two general conclusions may be drawn from the pattern of Soviet publications. The older serials for mathematical research are generally those with the formal titles of Trudy, Doklady or Izvestiya which are the official organs of the scientific academies and institutions of learning. There are a few well-known exceptions to this such as the Uspehi and the Sbornik which actually contain the word mathematics in the title. The newer journals show a tendency to imitate the prevailing Western practice of describing in the title the range of content such as the Teoriya Veroyatnosti i Ee Primeneniya, and Problemy Kibernetiki.

In the West, good mathematical papers may be published in the proceedings of a learned society, but the preponderance of them find their way to a journal specializing in mathematics or a particular field of mathematics. In the Soviet Union, however, an important paper may turn up in the Uchenye Zapiski of a small pedagogical institute in Ulan-Ude or Irkutsk, buried among less noteworthy writings in the broad scientific field which may never be available outside the USSR. One can only hope that with the growing number of publication exchange agreements between libraries this difficulty of access will be obviated in the future.
NEWS ITEMS and ANNOUNCEMENTS

NSF SENIOR POSTDOCTORAL AND SCIENCE FACULTY FELLOWSHIP AWARDS. Awards of 376 fellowships in two programs were announced December 16 by NSF. There were 275 applicants for Senior Postdoctoral and 754 applicants for Science Faculty Fellowships. In mathematics, there were 8 Senior Postdoctoral and 56 Science Faculty Fellowships awarded.

SENIOR POSTDOCTORAL FELLOWSHIP AWARDS

<table>
<thead>
<tr>
<th>Name</th>
<th>Present Institution</th>
<th>Fellowship Institution</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUSLANDER, Maurice</td>
<td>Brandeis University</td>
<td>University of Paris (France)</td>
</tr>
<tr>
<td>BOOTHBY, William M.</td>
<td>Washington University</td>
<td>Institute for Advanced Study</td>
</tr>
<tr>
<td>CHERNOFF, Herman</td>
<td>Stanford University</td>
<td>London School of Economics (England)</td>
</tr>
<tr>
<td>ELLIOTT, Joanne</td>
<td>Barnard College</td>
<td>Institute for Advanced Study; University of Paris (France)</td>
</tr>
<tr>
<td>HAYS, William L.</td>
<td>University of Michigan</td>
<td>Center for Advanced Study in the Behavioral Sciences</td>
</tr>
<tr>
<td>OREY, Steven</td>
<td>University of Minnesota</td>
<td>University of Oxford (England)</td>
</tr>
<tr>
<td>PIERCE, Richard S.</td>
<td>University of Washington</td>
<td>University of California (Berkeley)</td>
</tr>
<tr>
<td>SAMPSON, Joseph H.</td>
<td>Johns Hopkins University</td>
<td>Institute for Advanced Study</td>
</tr>
</tbody>
</table>

SCIENCE FACULTY FELLOWSHIP AWARDS

<table>
<thead>
<tr>
<th>Name</th>
<th>Present Institution</th>
<th>Fellowship Institution</th>
</tr>
</thead>
<tbody>
<tr>
<td>BEIL, Robert J.</td>
<td>Vanderbilt University</td>
<td>Purdue University</td>
</tr>
<tr>
<td>BENSON, Russell V.</td>
<td>Long Beach</td>
<td>University of Southern California</td>
</tr>
<tr>
<td>BRANNEN, Joseph P.</td>
<td>University of Texas</td>
<td>University of Texas</td>
</tr>
<tr>
<td>BRIGGS, William E.</td>
<td>University of Colorado</td>
<td>University of London (England)</td>
</tr>
<tr>
<td>CARLBORG, Frank W.</td>
<td>Rockford College</td>
<td>University of Chicago</td>
</tr>
<tr>
<td>CHITTIM, Richard L.</td>
<td>Bowdoin College</td>
<td>Edinburgh University (Scotland)</td>
</tr>
<tr>
<td>CHRESTENSON, Hubert</td>
<td>Reed College</td>
<td>Yale University</td>
</tr>
<tr>
<td>CLARKE, Rev. Arthur A.</td>
<td>Fordham University</td>
<td>Yeshiva University</td>
</tr>
<tr>
<td>CLOCK, Daniel A.</td>
<td>Northern Michigan College</td>
<td>University of Wisconsin</td>
</tr>
<tr>
<td>COMSTOCK, Craig</td>
<td>U. S. Naval Postgraduate School</td>
<td>Princeton University</td>
</tr>
<tr>
<td>CRAIG, Cecil, Jr.</td>
<td>University of Cincinnati</td>
<td>University of Cincinnati</td>
</tr>
<tr>
<td>CULLEN, Charles G.</td>
<td>Case Institute</td>
<td>Case Institute</td>
</tr>
<tr>
<td>CURRAN, Peter M.</td>
<td>Fordham University</td>
<td>Columbia University</td>
</tr>
<tr>
<td>DAVIS, Alpheus G.</td>
<td>Clarkson College</td>
<td>Harvard University</td>
</tr>
<tr>
<td>DURFEE, William H.</td>
<td>Mt. Holyoke College</td>
<td>Cambridge University (England)</td>
</tr>
<tr>
<td>DUTCHER, Barry C.</td>
<td>University of Rochester</td>
<td>Brown University</td>
</tr>
<tr>
<td>ELICH, Joseph</td>
<td>Utah State University</td>
<td>University of California(Berkeley)</td>
</tr>
<tr>
<td>ELLIS, Wade</td>
<td>Oberlin College</td>
<td>University of Michigan</td>
</tr>
<tr>
<td>GOLDHABER, Jacob K.</td>
<td>University of Washington (Seattle)</td>
<td>University of London (England)</td>
</tr>
<tr>
<td>Name</td>
<td>Present Institution</td>
<td>Fellowship Institution</td>
</tr>
<tr>
<td>---------------------------</td>
<td>--------------------------------------------------------------</td>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td>GOTTER, Elroy E.</td>
<td>Wisconsin State College</td>
<td>University of Wisconsin</td>
</tr>
<tr>
<td>GRACE, Edward E.</td>
<td>Emory University</td>
<td>University of Wisconsin</td>
</tr>
<tr>
<td>GRUDIN, Arnold</td>
<td>Denison University</td>
<td>Massachusetts Institute of Technology</td>
</tr>
<tr>
<td>HAAG, Vincent H.</td>
<td>Franklin and Marshall College</td>
<td>University of Wisconsin</td>
</tr>
<tr>
<td>HALBERG, C. J. A., Jr.</td>
<td>University of California, (Riverside)</td>
<td>University of Wisconsin</td>
</tr>
<tr>
<td>HART, Mrs. Jean G.</td>
<td>University of Maine</td>
<td>University of Copenhagen (Denmark)</td>
</tr>
<tr>
<td>HAYDEN, Seymour</td>
<td>Clark University</td>
<td>Brown University</td>
</tr>
<tr>
<td>HOBLER, John H.</td>
<td>Ball State Teachers College</td>
<td>Harvard University</td>
</tr>
<tr>
<td>JOHNSON, David E.</td>
<td>Louisiana Polytechnic Institute</td>
<td>Washington University</td>
</tr>
<tr>
<td>JOHNSON, Wendell G.</td>
<td>Hiram College</td>
<td>Stanford University</td>
</tr>
<tr>
<td>LAWRENCE, Willard E.</td>
<td>Marquette University</td>
<td>University of California (Berkeley)</td>
</tr>
<tr>
<td>LAWS, Leonard S.</td>
<td>Southwestern College</td>
<td>University of Wisconsin</td>
</tr>
<tr>
<td>LEE, Joseph R.</td>
<td>College of William and Mary</td>
<td>Stanford University</td>
</tr>
<tr>
<td>LEWIS, Jesse C.</td>
<td>Jackson State College</td>
<td>University of California (Berkeley)</td>
</tr>
<tr>
<td>LOCKLEY, Jeanette E.</td>
<td>Oakland City College</td>
<td>Syracuse University</td>
</tr>
<tr>
<td>LORENZEN, Fred J. Jr.</td>
<td>University of Florida</td>
<td>University of California (Berkeley)</td>
</tr>
<tr>
<td>LYNCH, Roger V.</td>
<td>Baker University</td>
<td>University of Florida</td>
</tr>
<tr>
<td>MANHEIM, Jerome H.</td>
<td>Montclair State College</td>
<td>Oklahoma State University</td>
</tr>
<tr>
<td>McCULLY, Joseph C.</td>
<td>Western Michigan University</td>
<td>Columbia University</td>
</tr>
<tr>
<td>McLEAN, Leslie D.</td>
<td>Hofstra College</td>
<td>Harvard University</td>
</tr>
<tr>
<td>MEYER, Paul R.</td>
<td>Columbia University</td>
<td>University of Wisconsin</td>
</tr>
<tr>
<td>MOBLEY, Jean B.</td>
<td>Flora Macdonald College</td>
<td>Columbia University</td>
</tr>
<tr>
<td>NORTHEY, James H.</td>
<td>Eastern Michigan University</td>
<td>University of Wisconsin</td>
</tr>
<tr>
<td>O'NEILL, Anne F.</td>
<td>Wheaton College</td>
<td>University of North Carolina</td>
</tr>
<tr>
<td>PILGRIM, Donald H.</td>
<td>Luther College</td>
<td>University of Michigan</td>
</tr>
<tr>
<td>PINZKA, Charles F.</td>
<td>University of Cincinnati</td>
<td>University of California (Berkeley)</td>
</tr>
<tr>
<td>PRATHER, Ronald E.</td>
<td>San Jose State College</td>
<td>University of Wisconsin</td>
</tr>
<tr>
<td>REIFMAN, Lucille K.</td>
<td>American University</td>
<td>University of Cincinnati</td>
</tr>
<tr>
<td>RESCHOVSKY, Helene</td>
<td>University of Connecticut</td>
<td>Stanford University</td>
</tr>
<tr>
<td>ROSEN, David</td>
<td>Swarthmore College</td>
<td>University of Maryland</td>
</tr>
<tr>
<td>SKIBINSKY, Morris</td>
<td>Purdue University</td>
<td>University of California (Berkeley)</td>
</tr>
<tr>
<td>SMITH, Sigmund A.</td>
<td>Teachers College of Brockport</td>
<td>University of Glasgow (Scotland)</td>
</tr>
<tr>
<td>SPITAL, Sidney</td>
<td>University of Toledo</td>
<td>Yale University</td>
</tr>
<tr>
<td>STANAITIS, Otonas E.</td>
<td>St. Olaf College</td>
<td>University of Illinois</td>
</tr>
<tr>
<td>STENSTROM, Robert C.</td>
<td>Augsburg College</td>
<td>University of California (Los Angeles)</td>
</tr>
<tr>
<td>TURNER, Walter W.</td>
<td>General Motors Institute</td>
<td>Stanford University</td>
</tr>
<tr>
<td>VAN METER, Robert G.</td>
<td>Geneva College</td>
<td>University of Minnesota</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Michigan State University</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Duke University</td>
</tr>
</tbody>
</table>
THE UNIVERSITY OF NEBRASKA mathematics department announces the introduction of a curriculum leading to the M. A. or M. S. and Ph. D. degrees in Statistics. A bachelor's degree program in Statistics is also under consideration at present. Courses are offered in Statistical Methods, Statistics for Engineers, Theory of Probability, Information Theory, Methods of Experimental Design, Stochastic Processes, Theory of Games and Statistical Decision Theory, and Topics in Probability and Statistics. Additional courses will be added as needed. A number of assistantships and fellowships are available to qualified students. For information concerning degree requirements and for information concerning assistantships, direct inquiries to Professor Bernard Harris, Department of Mathematics, University of Nebraska, Lincoln 8, Nebraska.

THE HOLLOMAN SUMMER SCIENTIFIC SEMINARS will be held in the two week period June 19 - June 30, 1961, at Cloudcroft, New Mexico. The topic will be Astrophysics and the program will present Eberhard Hopf in five lectures on ergodic theory and R. S. Richardson in five lectures on celestial mechanics. Two lectures each will be given by ten other leading scientists on as many topics of astronomy and astrophysics. These speakers are Otto Struve, Donald H. Menzel, Gerard P. Kuiper, Carl Sagan, Seth B. Nicholson, John D. Strong, Herbert Friedman, N. U. Mayall, John D. Kraus, and George Gamow. A day-by-day calendar of the program is available and arrangements for attendance to all or any of these lectures may be made by writing to J. R. Foote, P. O. Box 1053, Holloman AFB, New Mexico.

THE MATHEMATICAL RESEARCH INSTITUTE AT OBERWOLFACH (MRI) was established in 1944 by Professor W. Suss. The objective of the organization is the advancement of mathematical science, mainly through regular scientific colloquia in the entire field of mathematics.

The MRI furthers mathematical research through a number of publications. In cooperation with distinguished mathematicians MRI takes part in editing the following: Archiv der Mathematik published by Birkhauser, Basel; Mathematisch-Physikalische Semesterberichte published by Vandenhoeck and Ruprecht, Gottingen; and the textbook series, Studia Mathematica, published by Vandenhoeck and Ruprecht, Gottingen. Soon after its establishment MRI published the first two volumes on pure mathematics in the Fiat Review of German Science. Single publications and several dissertations were also produced at Oberwolfach.

Since the death of W. Suss the Society for Mathematical Research has sponsored MRI. It will continue to be a center of intensive scientific collaboration for researchers of various countries. Professor H. Kneser, Tubingen is the head of MRI and Professor Th. Schneider, Freiburg i.B., is the Executive Director. A number of famous West German mathematicians serve as scientific council members.

In past years a great number of special sessions on mathematics were held at Oberwolfach for a wide circle of mathematicians from many countries. Some of the colloquia were on: Ordered Sets, Group Theory, Algebraic Groups, Number Theory, Partial Differential Equations, Special Functions, Functional Analysis, Geometry, Groups in Geometry, Mathematical Statistics, Probability Theory, System- and Information Theory applications in Biology, Operations Research, History of Mathematics, Didactic Questions in Mathematics, High-Energy Physics, Questions of Theoretical Physics.


The "Lorenzhof" at Oberwolfach, is the residence of MRI. It is a former hunting resort and is located four miles from the Wolfach Railroad Station in the center of the Black Forest about forty miles from Freiburg i.B. Accommodations for 30 to 40 house guests are available. The house has a lecture room and an extensive library. Sessions last, in general, over one week. Due to limited accommodations at the "Lorenzhof" participants should get in touch with Dr. Theodor Schneider, Freiburg, i.B. to make arrangements.
PERSONAL ITEMS

Dr. G. MALTESE of Yale University has been awarded a NATO postdoctoral Fellowship at the University of Gottingen, Germany.

Professor W. R. TRANSUE, on leave from Kenyon College, has received a National Science Foundation Faculty Fellowship for 1960-1961 at the University of Paris, France.

Associate Professor M. A. AL-BASSAM of the University of Baghdad has been appointed to an associate professorship at Texas Technological College.

Dr. M. A. ARKOWITZ of Cornell University has been appointed to a research associateship at Johns Hopkins University.

Professor R. G. AYOUB of Pennsylvania State University has returned after spending the academic year 1959-1960 as a research fellow at Harvard University.

Professor F. E. BAKER of Vassar College is on a sabbatical leave at the University of North Carolina.

Mr. A. F. BAUSCH has been appointed to an assistant professorship at Kalamazoo College.

Mr. M. BILLIK has accepted a position as mathematical engineer at Lockheed Aircraft Corporation, Sunnyvale, California.

Assistant Professor R. L. BLAIR of the University of Oregon has been appointed to an associate professorship at Purdue University.

Mr. K. R. BLAKE of United Aircraft Corporation, East Hartford, Connecticut has accepted a position as senior mathematician at International Business Machines Corporation, White Plains, New York.

Mr. M. N. BLEICHER, on leave from Tulane University will spend the academic year 1960-1961 at the University of Warsaw, Warsaw, Poland.

Dr. R. A. BONIC of the University of Southern California has been appointed to a National Science Foundation Post Doctoral Fellowship at the Institute for Advanced Study for the academic year 1960-1961.

Mr. K. C. BOUCHELLE of Lehigh University has accepted a position as a teacher of junior high school mathematics at Roslyn, New York.

Associate Professor J. W. BRACE, of the University of Maryland has returned after spending the academic year 1959-1960 at the University of California, Berkeley.

Professor C. M. BRADEN of Macalester College has returned after spending the academic year 1959-1960 at the University of California, Berkeley.

Mr. J. A. BROWN of Montana State College has accepted a position as operations analyst at Johns Hopkins University.

Assistant Professor M. BROWN, on leave from the University of Michigan, will spend the academic year 1960-1961 at the Institute for Advanced Study.

Assistant Professor L. C. BUTLER, on leave from Alfred University for the next two years, has accepted a position as director of the Central Institute of Mathematics at the University of Concepcion, Concepcion, Chile.

Mr. M. L. CANTOR of Brooklyn New York will spend the academic year 1960-1961 as a National Science Foundation participant at the University of Michigan.

Dr. S. U. CHASE of Princeton University has returned after spending the academic year 1959-1960 at Massachusetts Institute of Technology.

Dr. F. L. CLEAVER of Tulane University has been appointed to an assistant professorship at the University of South Florida.

Associate Professor H. D. COLSON of the Air Force Institute of Technology has been appointed to an associate professorship at Ohio State University.

Dr. R. J. CRITTENDEN of Wellesley College has been appointed to a visiting assistant professorship at Northwestern University.

Mr. D. B. DAVIS of Stanford University has accepted a position as mathematician at Philco Corporation, Palo Alto, California.

Associate Professor M. D. DAVIS of Rensselaer Polytechnic Institute has been appointed to an associate professorship at Yeshiva University.

Mr. C. B. de LYRA, on leave from the University of Sao Paulo, Brazil, will spend the academic year 1960-1961 at the Institute for Advanced Study.

Professor R. DUBISCH of Fresno State College has returned after spending the academic year 1959-1960 at the University of California, Berkeley.

Professor G. C. EVANS of the University of California, Berkeley, has retired with the title Professor Emeritus.

Mr. L. R. EVERINGHAM of Radiation, Incorporated, Orlando, Florida has accepted a position as general manager at the Aerolab Development Company division of Ryan Aeronautical
Company, San Diego, California.
Mr. R. O. EXCELL of Pure Oil Company, Crystal Lake, Illinois, has accepted a position as manager of operations research at the Brunswick-Balke-Collender Company, Chicago, Illinois.
Professor K. FAN of the University of Notre Dame has been appointed to a professorship at Wayne State University.
Assistant Professor A. N. FELDZAMEN of the University of Wisconsin has returned after spending the academic year 1959-1960 at the University of Chicago.
Assistant Professor T. S. FERGUSON of the University of California, Los Angeles has returned after spending the academic year 1959-1960 at Princeton University.
Mr. H. FISHMAN of the University of Wisconsin has been appointed a member of the project research staff at the Forrestal Research Center, Princeton University.
Professor O. FRINK of Pennsylvania State University will spend the academic year 1960-1961 at University College, Dublin, Ireland as a visiting lecturer.
Dr. G. H. FULLERTON of Massachusetts Institute of Technology has been appointed a lecturer at the Queen's University of Belfast, Belfast, Ireland.
Dr. C. W. GEAR of the University of Illinois has accepted a position as engineer at International Business Machines Corporation, Winchester, England.
Assistant Professor J. D. GILBERT of Louisiana Polytechnic Institute has returned after spending the academic year 1959-1960 at Auburn University.
Mr. P. A. GILLIS of Westinghouse Electric Corporation has accepted a position as advanced research engineer at Sylvania Electric Corporation, Needham, Massachusetts.
Mr. A. G. GLUCKMAN of Curtiss-Wright Corporation, Newark, New Jersey, has accepted a position as mathematician at the Goddard Space Flight Center, Greenbelt, Maryland.
Mr. D. S. GRANT of the University of Manitoba has accepted a position as technical programmer at Ontario Hydro Corporation, Toronto, Canada.
Mr. T. F. GREEN of CONVAIR has accepted a position as mathematician at General Electric Company, Philadelphia, Pennsylvania.
Professor Emeritus F. L. Griffin of Reed College has retired after spending the academic year 1960-1961 at Wesleyan University.
Dr. B. I. GROSS of the University of Pennsylvania has been appointed to an assistant professorship at Rutgers, The State University.
Dr. L. GROSS of Yale University has been appointed to an assistant professorship at Cornell University.
Professor J. GURLAND of Iowa State University has been appointed to a professorship at the University of Wisconsin Army Research Center for the academic year 1960-1961.
Mr. J. M. GWYNN of the University of North Carolina has been appointed an assistant research mathematician at Georgia Institute of Technology.
Dr. W. N. HALLETT of Sears Roebuck and Company has accepted a position as insurance counselor at the New York Life Insurance Company, Pittsburgh, Pennsylvania.
Dr. O. HAMARA of Washington Missionary College has been appointed a lecturer at Massachusetts Institute of Technology for the academic year 1960-1961.
Assistant Professor B. HARRIS of Northwestern University will spend the academic year 1960-1961 at the Institute for Advanced Study and Princeton University.
Assistant Professor F. C. HATFIELD of Mankato State College has returned after spending the academic year 1959-1960 at the University of Florida.
Assistant Professor M. HERSCHORN, on leave from McGill University, will spend the academic year 1960-1961 as a temporary member at the Institute of Mathematical Sciences, New York University.
Professor I. N. HERSTEIN of Cornell University has been appointed to a visiting professorship at the University of Chicago for the academic year 1960-1961.
Mr. T. Y. HICKS of Service Bureau Corporation, Dallas, Texas has been appointed district manager of Service Bureau Corporation, Los Angeles, California.
Mr. J. A. HIGGINS of Harpur College has been appointed a teaching fellow at Boston College.
Professor T. H. HILDEBRANDT of the University of Michigan has been appointed to a visiting professorship at Brown University for September 1960 - January 1961.
Mr. D. B. HOUGHTON of Franklin Institute has accepted a position as director of business systems equipment at Westinghouse Electric Corporation, Pittsburgh, Pennsylvania.
Assistant Professor R. A. HULTQUIST of DePauw University has been appointed to an assistant professorship at Oklahoma State University.
Professor D. H. HYERS of the University of
Southern California has been appointed a research mathematician at the University of California, Berkeley for the academic year 1960-1961.

Dr. G.S. JONES of General Electric Company, Cincinnati, Ohio, has accepted a position as mathematical applications specialist at General Electric Company, Evendale, Ohio.

Dr. K. S. KUNZ of Schlumberger Well Surveying Corporation has been appointed to a professorship at New Mexico State University.

Dr. R. A. KUNZE of Massachusetts Institute of Technology has been appointed to an assistant professorship at Brandeis University.

Professor M. KURANISHI of Nagoya University has been appointed to a visiting professorship at Princeton University.

Dr. D. LUBELL, on leave from Harvard University, will spend the academic year 1960-1961 at Massachusetts Institute of Technology as a visiting scientist.

Assistant Professor R. H. McDOWELL of Rutgers, The State University has been appointed to an assistant professorship at Washington University, St. Louis, Missouri.

Associate Professor J. E. McLAUGHLIN, on leave from the University of Michigan has been appointed a research fellow at Harvard University for the academic year 1960-1961.

Mr. S. W. MALINOWSKI of the University of Pennsylvania has accepted a position as mathematician at General Dynamics Corporation, Rochester, New York.

Professor L. J. MISHOE of Morgan State College has been appointed president of Delaware State College.

Professor Emeritus L. J. MORDELL, on leave from Cambridge University, England, has been appointed to a visiting professorship at the University of Notre Dame for the academic year 1960-1961.

Assistant Professor D. E. MOSER of the University of Massachusetts has been appointed to an associate professorship at the University of Vermont.

Assistant Professor J. K. MOSER of Massachusetts Institute of Technology has been appointed to a professorship at New York University.

Associate Professor S. MOWSHOWITZ of the University of Bridgeport has been appointed to an assistant professorship at Hofstra College.

Dr. T. NISHIURA of Purdue University has been appointed to an assistant professorship at the University of Wisconsin.

Dr. J. W. ODLE of Avco Manufacturing Corporation, Cincinnati, has accepted a position as staff member at Arthur D. Little, Incorporated, Cambridge, Massachusetts.

Dr. C. L. PERRY, JR. of the Stanford Research Institute has accepted a position as director of computation facility at the University of California, La Jolla.

Professor H. RADEMACHER of the University of Pennsylvania will spend the academic year 1960-1961 at the Institute for Advanced Study.

Dr. J. D. REID of the University of Washington has been appointed to an assistant professorship at Syracuse University.

Dr. H. G. RICE of System Development Corporation, Santa Monica, California has accepted a position as consultant at the Computer Sciences Corporation, Santa Monica, California.

Professor D. C. ROSE of Transylvania College has been appointed to an associate professorship at the University of South Florida.

Associate Professor L. A. RUBEL, on leave from the University of Illinois, will spend the academic year 1960-1961 at Columbia University.

Mr. D. R. RYAN of Hughes Aircraft Company, Culver City, California, has accepted a position as member of the technical staff at American Systems Incorporated, Inglewood, California.

Assistant Professor J. SACKS of Columbia University has been appointed to an assistant professorship at Cornell University.

Assistant Professor D. SARAFLAN of the University of Florida has been appointed to an associate professorship at Utah State University.

Mr. N. S. SCARRITT, on leave from Purdue University, will spend the academic year 1960-1961 at the University of Wisconsin.

Dr. R. G. SINCLAIR of Massachusetts Institute of Technology has been appointed to an assistant professorship at the University of Alberta, Edmonton, Alberta, Canada.

Mr. L. B. SKLAR of Rutgers, The State University has been appointed to an associate professorship at Trenton State College.

Dr. W. F. STOLL of the University of Tubingen, Germany has been appointed to a professorship at the University of Notre Dame.

Professor S. R. SUKHON of the National School of Mathematical Sciences, Beirut, Lebanon, has been appointed to a professorship at the Moroccan Ministry of Education, Morocco.

Professor M. SUZUKI, on leave from the University of Illinois, will spend the academic year 1960-1961 at the University of Chicago as a visiting professor.

Assistant Professor G. TAKEUTI, on leave from Tokyo University, has returned after spend-
The following promotions are announced:

H. I. ANSOFF, Lockheed Electronics Company, to vice president of Plans and Programs.
K. E. AUBERT, University of Oslo, Norway, to an associate professorship.
R. G. AYOUB, Pennsylvania State University, to a professorship.
S. K. BERBERIAN, State University of Iowa, to an associate professorship.
R. BLOCK, California Institute of Technology, to an assistant professorship.
C. M. BRADEN, Macalester College, to a professorship.
R. A. GOOD, University of Maryland, to a professorship.
R. W. MCKELVEY, University of Colorado, to an associate professorship.
H. MINC, University of Florida, to an associate professorship.
H. MIRKIL, Dartmouth College, to an associate professorship.
R. B. RICE, Ohio Oil Company, of the physics department.

The following appointments to instructorships are announced:

Brown University: Mr. A. BROWDER; University of California, Berkeley: Dr. F. J. KOSIER; University of Chicago: Dr. C. C. MOORE; Lady-cliff College: Sister M. DENIS; Long Island University: Mr. N. SVILOKOS; Massachusetts Institute of Technology: Dr. S. Y. HUSSEINI, Dr. F. S. VAN VLECK; University of Michigan: Mr. J. A. COHN; Ohio State University: R. T. BARNES; Princeton University: Dr. D. G. CANTOR; Rutgers, The State University: J. H. OPPENHEIM; University of Southern California: Dr. E. D. KANN; Syracuse University: Dr. T. HUSAIN; University of Virginia: Dr. A. M. FINK; Yale University: Dr. D. B. LISSNER.

Deaths:

Professor B. L. BEEGLE of Bothell, Washington died on April 7, 1960. He has been a member of the Society for 25 years.

Dr. O. E. GLENN of Lansdowne, Pennsylvania died on November 9, 1960 at the age of 82. He had been a regular member of the Society for 28 years and an emeritus member for 13 years.

Mr. L. N. KING of Little Rock, Arkansas died on August 31, 1960 at the age of forty-four.

Associate Professor G. B. LANG of the University of Florida died on July 21, 1960 at the age of 54 years. He had been a member of the Society for thirty years.

Mr. C. E. VAN ORSTRAND of Manito, Illinois died on July 23, 1959 at the age of 89 years. He had been a member of the Society for 56 years.

Professor H. YAMABE of Northwestern University died on November 20, 1960 at the age of 37 years.

ERRATA.

The announcement on page 612 of the October, 1960 issue of the NOTICES concerning Professor S. A. AMITSUR of Yale University should read as follows: Professor S. A. AMITSUR of the Hebrew University of Jerusalem has returned after spending a leave of absence at Yale University and the University of Notre Dame.
LETTERS TO THE EDITOR

Editor, the NOTICES

At its inception in the June 1958 issue of the NOTICES, the purpose of the "Letters to the Editor" department was avowed to be the provision of "a forum for discussions of the programs of the Society" and "a method for communicating information of interest to the membership." I believe that the interest referred to may justifiably be presumed to be professional interest. However, several of the letters published in this department in the last year or so are completely devoid of material which may be construed as falling within the putative purview of the department. The letters of Lee Lorch appearing in the April 1960 and December 1960 issues of the NOTICES epitomize these gratuitous and irrelevant billets-doux: although written by a member of the Society and embellished with the terms "mathematics", "mathematicians" and "mathematical community", these screeds fail to evince one tittle of fact or opinion bearing on the programs of the Society or containing information of any conceivable professional interest to the membership.

With due regard for the delicate sensitivities of certain chimerical mathematicians residing behind the Iron Curtain who shudder when confronted with a "cold-war epithet", I believe there are many more members of the Society residing in the United States who object - as I do - to being made a captive audience for political and social diatribes and to giving financial support, in the form of dues, to such a gross misuse of an organ of the Society. I further believe that a continuation of a policy which admits to the pages of the NOTICES letters which properly belong, if anywhere, in the editorial pages of the morning papers can result only in a loss of prestige to the Society and, ultimately, in a substantial impairment of the effectiveness of the Society in areas which are of vital concern to its membership.

Whatever value the NOTICES may possess as a vehicle for improvement of international scientific cooperation is incidental to its value as a medium for dissemination of information of great professional interest to those members of the Society living in the United States and constituting the bulk of the Society's membership. It is distressing to contemplate the decline of this latter value as a result of a pernicious perversion of the function of the NOTICES to include the promulgation of propaganda. On page 854 of the December 1960 issue of the NOTICES appears a statement to the effect that the NOTICES is "one of the most widely read periodicals in the world dealing exclusively (emphasis mine) with the mathematical sciences." I hope that a return to former standards in future issues of the NOTICES will endow this statement with an accuracy it does not now possess.

T. F. Bridgland, Jr.
CORPORATE AND INSTITUTIONAL MEMBERS OF
THE AMERICAN MATHEMATICAL SOCIETY

As of January 23, 1961, the following were supporting the Society through Corporate or Institutional memberships:

CORPORATE MEMBERS
Bell Telephone Laboratories, Incorporated
E. I. du Pont de Nemours and Company, Incorporated
Eastman Kodak Company
Ford Motor Company
General Motors Corporation
Hughes Aircraft Company
International Business Machines Corporation
Procter and Gamble Company
Radio Corporation of America
Remington-Rand UNIVAC
RIAS
Shell Development Company
Space Technology Laboratories, Incorporated
United Gas Corporation
United States Steel Corporation

INSTITUTIONAL MEMBERS
Acadia University, Wolfville, N. S., Canada
Adelphi College, Garden City, Long Island, New York
University of Alabama, University, Alabama
University of Alberta, Edmonton, Alberta, Canada
Amherst College, Amherst, Massachusetts
Andrews University, Berrien Springs, Michigan
University of Arizona, Tucson, Arizona
Arizona State University, Tempe, Arizona
Atlanta University, Atlanta, Georgia
Auburn University, Auburn, Alabama
Beloit College, Beloit, Wisconsin
Bowdoin College, Brunswick, Maine
Brandeis University, Waltham, Massachusetts
Brigham Young University, Provo, Utah
University of British Columbia, Vancouver, B. C., Canada
Brooklyn College, Brooklyn, New York
Polytechnic Institute of Brooklyn, Brooklyn, New York
Brown University, Providence, Rhode Island
Bryn Mawr College, Bryn Mawr, Pennsylvania
Bucknell University, Lewisburg, Pennsylvania
University of Buffalo, Buffalo, New York
California Institute of Technology, Pasadena, California
University of California, Berkeley, California
University of California, Davis, California
University of California, Los Angeles, California
University of California, Riverside, California
University of California, Santa Barbara, California
Carleton College, Northfield, Minnesota
Carnegie Institute of Technology, Pittsburgh, Pennsylvania
Case Institute of Technology, Cleveland, Ohio
Catholic University of America, Washington, D.C.
University of Chicago, Chicago, Illinois
University of Cincinnati, Cincinnati, Ohio
City College, New York, New York
University of Colorado, Boulder, Colorado
Columbia University, New York, New York
Connecticut College, New London, Connecticut
Cornell University, Ithaca, New York
Dartmouth College, Hanover, New Hampshire
University of Delaware, Newark, Delaware
Denison University, Granville, Ohio
DePaul University, Chicago, Illinois
University of Detroit, Detroit, Michigan
Duke University, Durham, North Carolina
Duquesne University, Pittsburgh, Pennsylvania
Emory University, Emory University, Georgia
Florida Presbyterian College, St. Petersburg, Florida
Florida State University, Tallahassee, Florida
University of Florida, Gainesville, Florida
Fordham University, New York, New York
Georgetown University, Washington, D. C.
University of Georgia, Athens, Georgia
Gettysburg College, Gettysburg, Pennsylvania
Goucher College, Towson, Maryland
Grinnell College, Grinnell, Iowa
Hampshire College, State University of New York, Endicott, New York
Harvard University, Cambridge, Massachusetts
Haverford College, Haverford, Pennsylvania
College of the Holy Cross, Worcester, Massachusetts
University of Houston, Houston, Texas
University of Idaho, Moscow, Idaho
Illinois Institute of Technology, Chicago, Illinois
Illinois State Normal University, Normal, Illinois
University of Illinois, Urbana, Illinois
Indiana University, Bloomington, Indiana
Institute for Advanced Study, Princeton, New Jersey
Institute for Defense Analyses, Princeton, New Jersey
Iowa State University of Science and Technology, Ames, Iowa
The State University of Iowa, Iowa City, Iowa
The Johns Hopkins University, Baltimore, Maryland
University of Kansas, Lawrence, Kansas
Kent State University, Kent, Ohio
University of Kentucky, Lexington, Kentucky
Kenyon College, Gambier, Ohio
Lafayette College, Easton, Pennsylvania
Lehigh University, Bethlehem, Pennsylvania
Louisiana Polytechnic Institute, Ruston, Louisiana
Louisiana State University and Agricultural and Mechanical College, Baton Rouge, Louisiana
Loyola College Library, Baltimore, Maryland
McGill University, Montreal, Quebec, Canada
McMaster University, Hamilton College, Hamilton, Ontario, Canada
Macalister College, St. Paul, Minnesota
University of Maine, Orono, Maine
University of Manitoba, Winnipeg, Manitoba, Canada
Marquette University, Milwaukee, Wisconsin
University of Maryland, College Park, Maryland
Massachusetts Institute of Technology, Cambridge, Massachusetts
University of Massachusetts, Amherst, Massachusetts
Mathematical Association of America, Buffalo, New York
University of Miami, Coral Gables, Florida
University of Michigan, Ann Arbor, Michigan
Michigan State University of Agriculture and Applied Science, East Lansing, Michigan
Michigan State University - Oakland, Rochester, Michigan
Middlebury College, Middlebury, Vermont
University of Minnesota, Minneapolis, Minnesota
Mississippi State University, State College, Mississippi
University of Mississippi, University, Mississippi
University of Missouri, Columbia, Missouri
Montana State College, Bozeman, Montana
Montana State University, Missoula, Montana
Mount Holyoke College, South Hadley, Massachusetts
University of Nebraska, Lincoln, Nebraska
University of Nevada, Reno, Nevada
University of New Hampshire, Durham, New Hampshire
New Mexico State University of Agriculture, Engineering and Science, University Park, New Mexico
University of New Mexico, Albuquerque, New Mexico
New York University, New York, New York
University of North Carolina, Chapel Hill, North Carolina
North Carolina State College of Agriculture and Engineering, Raleigh, North Carolina
North Texas State College, Denton, Texas
Northwestern University, Evanston, Illinois
University of Notre Dame, Notre Dame, Indiana
Oberlin College, Oberlin, Ohio
Ohio State University, Columbus, Ohio
Ohio Wesleyan University, Delaware, Ohio
Oklahoma State University, Stillwater, Oklahoma
University of Oklahoma, Norman, Oklahoma
Oregon State College, Corvallis, Oregon
University of Oregon, Eugene, Oregon
Pennsylvania State University, University Park, Pennsylvania
University of Pennsylvania, Philadelphia, Pennsylvania
University of Pittsburgh, Pittsburgh, Pennsylvania
Pomona College, Claremont, California
Portland State College, Portland, Oregon
Princeton University, Princeton, New Jersey
Purdue University, Lafayette, Indiana
Queens College, Flushing, New York
Randolph-Macon Woman's College, Lynchburg, Virginia
Reed College, Portland, Oregon
Rensselaer Polytechnic Institute, Troy, New York
University of Rhode Island, Kingston, Rhode Island
University of Rochester, Rochester, New York
Rutgers, The State University, New Brunswick, New Jersey
Sacramento State College, Sacramento, California
St. Louis University, St. Louis, Missouri
The College of St. Thomas, St. Paul, Minnesota
San Fernando Valley State College, Northridge, California
University of Santa Clara, Santa Clara, California
University of Saskatchewan, Saskatoon, Saskatchewan, Canada
Seattle University, Seattle, Washington
Smith College, Northampton, Massachusetts
| University of the South, Sewanee, Tennessee                      |
| South Dakota School of Mines and Technology, Rapid City, South Dakota |
| South Dakota State College of Agriculture and Mechanic Arts, Brookings, South Dakota |
| University of Southern California, Los Angeles California |
| University of South Carolina, Columbia, South Carolina |
| Southern Illinois University, Carbondale, Illinois |
| Southern Methodist University, Dallas, Texas |
| University of Southwestern Louisiana, Lafayette, Louisiana |
| Stanford University, Stanford, California |
| Swarthmore College, Swarthmore, Pennsylvania |
| Sweet Briar College, Sweet Briar, Virginia |
| Syracuse University, Syracuse, New York |
| Temple University, Philadelphia, Pennsylvania |
| University of Tennessee, Knoxville, Tennessee |
| Agricultural and Mechanical College of Texas, College Station, Texas |
| Texas Christian University, Fort Worth, Texas |
| Texas Technological College, Lubbock, Texas |
| University of Texas, Austin, Texas |
| University of Toronto, Toronto, Ontario, Canada |
| Trinity College, Hartford, Connecticut |
| Tulane University of Louisiana, New Orleans, Louisiana |
| United States Air Force Academy, Denver, Colorado |
| United States Naval Postgraduate School, Monterey, California |
| University of Utah, Salt Lake City, Utah |
| Vanderbilt University, Nashville, Tennessee |
| Vassar College, Poughkeepsie, New York |
| Virginia Polytechnic Institute, Blacksburg, Virginia |
| University of Virginia, Charlottesville, Virginia |
| Washington State University, Pullman, Washington |
| Washington University, St. Louis, Missouri |
| University of Washington, Seattle, Washington |
| Wayne State University, Detroit, Michigan |
| Wellesley College, Wellesley, Massachusetts |
| Wells College, Aurora, New York |
| Wesleyan University, Middletown, Connecticut |
| Western Michigan University, Kalamazoo, Michigan |
| Western Reserve University, Cleveland, Ohio |
| Western Washington College of Education, Bellingham, Washington |
| Wheaton College, Norton, Massachusetts |
| University of Wichita, Wichita, Kansas |
| Wiley College, Marshall, Texas |
| College of William and Mary, Norfolk, Virginia |
| William Marsh Rice University, Houston, Texas |
| Williams College, Williamstown, Massachusetts |
| Worcester Junior College, Worcester, Massachusetts |
| Worcester Polytechnic Institute, Worcester, Massachusetts |
| Xavier University, Cincinnati, Ohio |
| Yale University, New Haven, Connecticut |
| Yeshiva University, New York, New York |

**THE CHELSEA PUBLISHING COMPANY**
calls attention to a discount of 15% from list price allowed to members of the AMS, MAA, IMS, and SIAM on orders of Chelsea books shipped within the United States. Orders are shipped anywhere in the United States without additional charge for postage, etc., if payment accompanies the order.

Members of one of the four above societies residing outside continental United States are not allowed a discount. But there is no charge for handling, export packing, or export postage. Foreign orders must be accompanied by payment.

**EMPLOYMENT OF RETIRED MATHEMATICIANS**

The Headquarters Offices of the American Mathematical Society, 190 Hope Street, Providence 6, Rhode Island, again plans to issue the list of Retired Mathematicians Available for Employment, which it has prepared yearly. The list will be compiled by February 10. Copies may be obtained free of charge by writing to the Headquarters Offices.
MEMORANDA to MEMBERS

AN ANNOUNCEMENT FROM THE PROCEEDINGS EDITORIAL COMMITTEE

We should like to call the attention of prospective authors of papers in the Proceedings to the following matters of editorial policy, some old, some new.

The Society never intended to establish a journal for good papers and another for bad ones; the intended distinction between the Transactions and the Proceedings is in the length, not the quality, of the papers they publish. The upper limit on the length of Proceedings papers varies from time to time; because of the size of the current backlog, it has recently been reduced to 8 printed pages. The following statistics may aid an author in estimating the length of the printed version of his manuscript. An average Proceedings page has 400 words; an average typewritten page has 300. A rough rule of thumb, accordingly, is that under 10 typewritten pages is within the allowed limit, but over 11 typewritten pages is not. A better approximation is this. A printed page of the Proceedings (with no displayed material) consists of about 43 lines, each with between 11 and 12 words. A space after a theorem or a section heading is half a line; a single-line display is two lines; displayed fractions, summations, integrals are three lines. In terms of printed lines, the maximum acceptable length is between 340 and 350.

We take this opportunity to announce the establishment of a new department in the Proceedings, on a temporary, experimental basis. The name of the department will be MATHEMATICAL PEARLS. The purpose of the new department is to publish very short papers (maximum length, 1 printed page) of an unusually elegant and polished character, for which there is normally no other outlet. The definitive new proof of an important and long-known theorem, an illuminating (but briefly statable) new fact, the central core of a long and computational argument, an interesting lemma used for the proof of a theorem in a different field - these are examples of subjects suitable for publication in the proposed new department. Many mathematicians encounter such pearls from time to time, and do not know what to do with them. The result is that more and more pearls pass into the folklore of mathematics - a phenomenon that makes it difficult for outsiders (e.g., the young mathematicians, or mathematicians in a country other than the one in which the pearl was discovered) to become up-to-date. The editors hereby invite contributions to the department of MATHEMATICAL PEARLS. Such contributions will be treated similarly to other Proceedings papers, and, in particular, they will be referred as usual.

The appropriate addresses to use for various kinds of Proceedings correspondence, as well as such policy matters as the current length limitation, appear on the inside of the cover of each issue of the Proceedings. Authors could save themselves time and trouble by reading those instructions. We call attention, in particular, to the address of the Society's Headquarters; the Editorial board is unable to help disgruntled authors and subscribers who complain about real or imaginary delays in the delivery of reprints or about damaged copies of the journal.

R. P. Boas
P. R. Halmos, Chairman

A. Rosenberg
E. H. Spanier

RECIPROCITY AGREEMENT WITH THE CALCUTTA MATHEMATICAL SOCIETY

The American Mathematical Society has entered into a reciprocity agreement with the Calcutta Mathematical Society by which members of each may become members of the other by paying half the regular dues. The regular dues of members of the Calcutta Mathematical Society are $3; therefore an American Mathematical Society member would pay $1.50 yearly dues and an admission fee of $2.50. Privileges of membership include all the usual privileges of membership in the Calcutta Mathematical Society such as receiving the notices of meetings, the BULLETIN, and presenting papers at the meetings of the Society. Those members of the American Mathematical Society wishing to take advantage of this arrangement should write to

Dr. P. P. Chattarji, Secretary
Calcutta Mathematical Society
92, Acharya Prafulla Chandra Road
Calcutta 9, India

It is understood that members under the reciprocity agreement spending time in the other country should pay the regular dues while they are there.
Information on this important matter is being published twice a year, in the February and the August issues of the NOTICES, with the kind cooperation of the respective editorial boards.

It is important that the reader should interpret the data with full allowance for the wide and sometimes meaningless fluctuations which are characteristic of them. Waiting times in particular are affected by many transient effects, which arise in part from the refereeing system. Extreme waiting times as observed from the published dates of receipt of manuscripts may be very misleading, and for that reason, no data on extremes are presented in the table at the bottom of this page.

Some of the columns in the table are not quite self-explanatory, and here are some further details on how the figures were computed.

Column 2. These numbers are rounded off to the nearest 50.

Column 3. For each journal, this is the estimate as of the indicated dates, of the total number of printed pages which will have been accepted by the next time that manuscripts are to be sent to the printer, but which nevertheless will not be sent to the printer at that time. (It should be noted that pages received but not yet accepted are being ignored.)

Column 4. Estimated by the editors (or the Editorial Department of the American Mathematical Society in the case of the Society's journals), and based on these factors: Manuscripts accepted, manuscripts received and under consideration, manuscripts in galley, and rate of publication. There is no fixed formula.

Column 5. The first quartile (Q₁) and the third quartile (Q₃) are presented to give a measure of the dispersion which will not be too much distorted by meaningless extreme values. The median (Med.) is used as the measure of location. The observations were made from the latest issue received in the Headquarters Offices before the deadline date for this issue of the NOTICES. The waiting times were measured by counting the months from receipt of manuscript in final revised form, to month in which the issue was received at the Headquarters Offices (not counting month of receipt of manuscript but counting month when issue was received at Headquarters Offices). It should be noted that when a paper is revised, the waiting time between receipt by editors of the final revision and its publication may be much shorter than is the case for a paper which is not revised, so these figures are to that extent distorted on the low side.

<table>
<thead>
<tr>
<th>Journal</th>
<th>1 No. issues per year</th>
<th>2 Approx. no. pages published currently/year</th>
<th>3 Backlog 11/30/60, 5/31/60 pages</th>
<th>4 Estimated current waiting time months</th>
<th>5 Observed waiting time in latest issue months</th>
<th>Q₁ Med. Q₃ months</th>
</tr>
</thead>
<tbody>
<tr>
<td>American Journal of Math.</td>
<td>4</td>
<td>NR (a)</td>
<td>NR (a) NR (a)</td>
<td>N/A (b)</td>
<td>10 (c)</td>
<td>14 (c) 12 (c)</td>
</tr>
<tr>
<td>Annals of Math.</td>
<td>6</td>
<td>1200-1400</td>
<td>N/A (b)</td>
<td>N/A (a)</td>
<td>12 (c)</td>
<td>9 (c) 10 (c)</td>
</tr>
<tr>
<td>Annals of Math. Statistics</td>
<td>4</td>
<td>1,300</td>
<td>0 0</td>
<td>10 (c)</td>
<td>7 (c)</td>
<td>9 (c) 12 (c)</td>
</tr>
<tr>
<td>Arch. Rat. Mech. Anal.</td>
<td>Not fixed</td>
<td>1,000</td>
<td>0 0</td>
<td>5 (c)</td>
<td>6 (c)</td>
<td>6 (c) 6 (c)</td>
</tr>
<tr>
<td>Canadian Journal of Math.</td>
<td>4</td>
<td>704</td>
<td>180 NR (a)</td>
<td>14 (c)</td>
<td>16 (c)</td>
<td>17 (c) 19 (c)</td>
</tr>
<tr>
<td>Duke Math. Journal</td>
<td>4</td>
<td>650</td>
<td>150 50</td>
<td>8 (c)</td>
<td>8 (c)</td>
<td>10 (c) 13 (c)</td>
</tr>
<tr>
<td>Illinois Journal of Math.</td>
<td>4</td>
<td>650</td>
<td>110 95</td>
<td>10-12 (c)</td>
<td>12 (c)</td>
<td>13 (c) 14 (c)</td>
</tr>
<tr>
<td>Journal Math. Analyses Appl.</td>
<td>4</td>
<td>NR (a)</td>
<td>NR (a) 150</td>
<td>NR (a)</td>
<td>(c) (c)</td>
<td>(c) (c) (c)</td>
</tr>
<tr>
<td>Journal of Math. and Mechanics</td>
<td>6</td>
<td>1,000</td>
<td>200 250</td>
<td>13 (c)</td>
<td>(c) (c)</td>
<td>(c) (c) (c)</td>
</tr>
<tr>
<td>Journal of Math. and Physics</td>
<td>4</td>
<td>350</td>
<td>100 NR (a)</td>
<td>9 (c)</td>
<td>8 (c)</td>
<td>10 (c) 11 (c)</td>
</tr>
<tr>
<td>Journal of Mathematical Physics</td>
<td>6</td>
<td>NR (a)</td>
<td>NR (a) 350</td>
<td>NR (a)</td>
<td>5 (c)</td>
<td>6 (c) 7 (c)</td>
</tr>
<tr>
<td>Michigan Math. Journal</td>
<td>Not fixed</td>
<td>400</td>
<td>40 40</td>
<td>9 (c)</td>
<td>11 (c)</td>
<td>12 (c) 13 (c)</td>
</tr>
<tr>
<td>Pacific Journal of Math.</td>
<td>4</td>
<td>1,500</td>
<td>900 800</td>
<td>11-13 (c)</td>
<td>12 (c)</td>
<td>12 (c) 15 (c)</td>
</tr>
<tr>
<td>Proceedings of the AMS</td>
<td>6</td>
<td>1,000</td>
<td>520 460</td>
<td>13 (c)</td>
<td>9 (c)</td>
<td>11 (c) 11 (c)</td>
</tr>
<tr>
<td>Quarterly of Applied Math.</td>
<td>4</td>
<td>450</td>
<td>200 200</td>
<td>10-12 (c)</td>
<td>11 (c)</td>
<td>14 (c) 16 (c)</td>
</tr>
<tr>
<td>SIAM Journal</td>
<td>4</td>
<td>750</td>
<td>30 150</td>
<td>12 (c)</td>
<td>7 (c)</td>
<td>11 (c) 15 (c)</td>
</tr>
<tr>
<td>SIAM Review</td>
<td>4</td>
<td>400</td>
<td>0 30</td>
<td>6-12 (c)</td>
<td>5 (c)</td>
<td>6 (c) 8 (c)</td>
</tr>
<tr>
<td>Transactions of the AMS</td>
<td>12</td>
<td>2,200</td>
<td>0 310</td>
<td>8 (c)</td>
<td>10 (c)</td>
<td>11 (c) 16 (c)</td>
</tr>
</tbody>
</table>

(a) NR means that no response was received to a request for information.
(b) NA means that the response requested was not available.
(c) Dates of receipt of manuscripts not indicated in this journal.
NEW PUBLICATIONS

SCIENCE AND MATH WEEKLY, a new weekly science newspaper for senior high school and junior college classes in biology, chemistry, physics, and mathematics has been announced by the publisher, Wesleyan University.

Scheduled publication is 32 weekly issues during the school year; the first issue is dated December 7, 1960. Subscriptions and inquiries are filled by SCIENCE AND MATH WEEKLY, Education Center, Columbus 16, Ohio.

The INSTITUTE FOR MATHEMATICAL STATISTICS and The AMERICAN MATHEMATICAL SOCIETY jointly announce the publication of Volume I of a new series of books SELECTED TRANSLATIONS IN MATHEMATICAL STATISTICS AND PROBABILITY - 306 pages, List price: $4.80. For IMS and AMS members: $3.60.

SELECTED TRANSLATIONS IN MATHEMATICAL STATISTICS AND PROBABILITY, Volume I, published for the Institute of Mathematical Statistics by the American Mathematical Society, will be published in February, 1961. These translations were made under a grant from the National Science Foundation.

The American Mathematical Society has been publishing mathematics in translation since 1948, and among its two-hundred-fifty-odd translated articles there have appeared several in Probability and a few in Statistics. But, with the great increase in the program in the last two years it became clear that Statistics and Probability should have a series on its own. In 1959, the American Mathematical Society Russian Translation Committee became a Joint Committee with the Institute of Mathematical Statistics, and the Institute appointed two members to work with the five members of the American Mathematical Society who were on the Committee. Translations in Statistics and Probability authorized by the Joint Committee, beginning in 1959, are to be published in this new series.

Volume I contains 25 papers (306 pages) authorized in 1959. The translation program for 1961 includes about 3000 pages from all branches of mathematics, of which about 1000 pages will be in Statistics and Probability.

Orders for copies of Volume I of SELECTED TRANSLATIONS IN MATHEMATICAL STATISTICS AND PROBABILITY and standing orders for this new series should be sent to the American Mathematical Society, 190 Hope Street, Providence 6, Rhode Island.

Adashko, J. G. See Stanyukovich, K. P.
Akilov, G. P. See Kantorovich, L. V.
Akulov, N. S. See Prigozin, I.
Bachmann, F. See Grundzüge der Mathematik.
Behnke, H. See Grundzüge der Mathematik.

Bityuckov, V. I. See Matematika v SSSR...

Bogolyubov, N. N. See Karleman, T.

Boltyanski, V. G. See Matematika v SSSR...

Boltyanskii, V. G. See Rassloennye prostranstva...

Boron, L. F. See Gelfond, A. O.


Bryan, J. G. See Wadsworth, G. P.

Budak, B. M. See Rihtmaier, R. D.


Carleman, T. See Karleman, T.

Cohn-Vossen, S. E. See Kon-Fossen, S. E.

Davis, P. See Krzywoblocki, M. Z.

Desirant, M. See Electromagnetic wave propagation.


Doroznov, N. I. See Anglo-russki'slovor' po radio­ elektronike.


Dynkin, E. B. See Matematika v SSSR...

Dynkin, E. B. See Rassloennye prostranstva...


Efimov, N. V. See Kon-Fossen, S. E.


Emde, F. See Jahnke, E.


Fedorova, R. M. See Burunova, N. M.


Fladt, K. See Grundzüge der Mathematik.


Gol'danskiǐ, V. I. See Baldin, A. M.


Goodman, R. See Annual review in automatic programming.

Gorbunov, A. D. See Rihtmaier, R. D.


Green, S. See Schwartz, M.

Grundzüge der Mathematik. Vol. II: Geometrie. Ed. by H. Behnke, F. Bachmann, K. Fladt, and

Rabinowitcz, P. See Krzywoblocki, M. Z.


Rozental', I. L. See Baldwin, A. M.

Rutkovskii, V. Yu. See Gorskaya, N. S.

Rutledge, W. A. See Schwartz, M.


Sevast'yanov, B. A. See Bartlett, M. S.

Silov, G. E. See Matematika v SSSR...


Stohr, A. See Gantmaher, F. R.


Surányi, J. See Erdős, P.

Suss, W. See Grundzüge der Mathematik.


Yaglom, I. M. See Yaglom, A. M.

Yuskevič, A. P. See Matematika v SSSR...

Zuckerman, H. S. See Niven, I.

THE UNIVERSITY OF MINNESOTA has established a Special Program in Applied Mathematics. In association with the program, several three-year Department of Health, Education, and Welfare Fellowships will be awarded to students beginning graduate work in the Department of Mathematics, Institute of Technology, of the University.

The program consists of nine independent one-quarter courses in graduate mathematics, extending over three years, designed to introduce doctoral candidates in mathematics to a diverse selection of topics and techniques in applied mathematics. A doctoral candidate in this program is expected to meet high standards in the traditional mathematical fields of analysis, algebra, and geometry. In addition he would be expected to take about one-third of his graduate mathematics from the courses in the Special Program.

For information and application forms, write to Professor S. E. Warschawski, Head, Department of Mathematics, Institute of Technology, University of Minnesota, Minneapolis 14, Minnesota.

Completed applications must be received by February 20, 1961.
SUPPLEMENTARY PROGRAM No. 2

During the interval from December 3, 1960 through January 3, 1961 the papers listed below were accepted by the American Mathematical Society for presentation by title. Readers may wish to refer to page 713 of the November, 1960 issue (No. 49) of these NOTICES where it is explained in detail that the presentation of papers by title is now dissociated from meetings of the Society. Supplementary program No. 3 will cover the interval from January 4, 1961 through February 13, 1961.

After each title on this program is an identifying number. The abstract of the paper will be found following the same number in the section on Abstracts of Contributed Papers in this issue of these NOTICES or in succeeding issues.

PAPERS PRESENTED BY TITLE

(1) Some identity and uniqueness theorems for normal meromorphic functions
   Mr. Frederick Bagemihl, Ann Arbor, Michigan (61T-8)

(2) A proposition of elementary plane geometry that implies the continuum hypothesis
   Mr. Frederick Bagemihl, Ann Arbor, Michigan (61T-9)

(3) Finite collections of 2-spheres in \( \mathbb{E}^3 \)
   Professor B. J. Ball, University of Georgia (61T-10)

(4) On the asymptotic behavior of Toeplitz determinants
   Professor Glen Baxter, Aarhus Universitet, Denmark (61T-11)

(5) Embedding circle like continua in the plane
   Professor R. H. Bing, University of Wisconsin (61T-12)

(6) Metric characterizations of Banach and euclidean spaces
   Professor L. M. Blumenthal and Mr. E. Z. Andalafte, University of Missouri (61T-13)

(7) Ptolemaic metric spaces
   Professor L. M. Blumenthal and Mr. R. W. Freese, University of Missouri (61T-14)

(8) Axioms that define semi-metric, Moore and metric spaces. Preliminary report
   Professor J. R. Boyd, Arlington State College (61T-15)

(9) The Schoenflies theorem for polyhedral spheres
   Professor S. S. Cairns, University of Illinois (61T-16)

(10) A stochastic treatment of some classical interpolation problems
    Professor J. H. Curtiss, University of Miami (61T-17)

(11) Theory of models of infinite valued logic. I
    Professor C. C. Chang, University of California, Berkeley (61T-49)

(12) Theory of models of infinite valued logic. II
    Professor C. C. Chang, University of California, Berkeley (61T-50)

(13) Theory of models of infinite valued logic. III
    Professor C. C. Chang, University of California, Berkeley (61T-51)

(14) Additive representations of pairs of integers with prime and square-free summands
    Professor Eckford Cohen, University of Tennessee (61T-52)

(15) Derivations of commutative Banach algebras. II
    Professor P. C. Curtis, Jr., University of California, Los Angeles (61T-18)

(16) Appell’s transformation in Riemannian space \( \mathbb{V}_n \)
    Professor John De Cicco, DePaul University (61T-19)

(17) Lagrangian and Hamiltonian equations of a natural family of a conservative field of force in Riemannian space
    Professor John De Cicco, DePaul University (61T-20)

(18) A partial converse of a theorem of N. Ito. Preliminary report
    Professor W. E. Deskins, Michigan State University (61T-21)

(19) The maximum principle and biharmonic functions
    Professor R. J. Duffin, Carnegie Institute of Technology (61T-22)

(20) An exponential extrapolator
    Professor R. J. Duffin and Mr. Phillips Whidden, Carnegie Institute of Technology (61T-23)

(21) On the braid groups of \( \mathbb{E}^2 \) and \( S^2 \)
    Professor Edward Fadell and Mr. James Van Buskirk, University of Wisconsin (61T-24)

(22) Rings with minimum condition on principal ideals. II
    Professor Carl Faith, Institute for Advanced Study and Pennsylvania State University (61T-25)

(23) Strongly regular extensions of rings
    Professor Carl Faith, Institute for Advanced Study and Pennsylvania State University (61T-26)
(24) Rings of quotients of rings of continuous functions. II
Professor N. J. Fine, University of Pennsylvania, Professor L. Gillman, University of Rochester and Professor J. Lambek, McGill University (61T-27)

(25) Rings of quotients of rings of continuous functions. III
Professor N. J. Fine, University of Pennsylvania, Professor L. Gillman, University of Rochester and Professor J. Lambek, McGill University (61T-28)

(26) On the singularities of harmonic functions in \((p + 2)\)-dimensions. I
Professor R. P. Gilbert, Michigan State University (61T-54)

(27) On the singularities of harmonic functions in \((p + 2)\)-dimensions. II
Professor R. P. Gilbert, Michigan State University (61T-55)

(28) On the singularities of harmonic functions in \((p + 2)\)-dimensions. III
Professor R. P. Gilbert, Michigan State University (61T-56)

(29) The coefficients of the inverse function
Dr. Karl Goldberg, National Bureau of Standards, Washington, D. C. (61T-29)

(30) On translation continuity and trigonometric polynomials
Professor R. P. Gosselin, University of Connecticut (61T-30)

(31) On the approximate solution of elliptic differential equations with mixed partial derivatives
Professor Donald Greenspan, Mathematics Research Center, U. S. Army, University of Wisconsin (61T-31)

(32) Analytic derivations on \(p\)-adic fields
Professor Nickolas Heerema, Florida State University (61T-32)

(33) Eigenvalues of the nonhomogeneous rectangular membrane by finite difference methods
Dr. B. E. Hubbard, U. S. Naval Ordnance Laboratory, Silver Spring, Maryland (61T-33)

(34) Replete relational systems
Mr. H. J. Keisler, University of California, Berkeley (61T-35)

(35) The repeletenness of ultraproducts
Mr. H. J. Keisler, University of California, Berkeley (61T-36)

(36) Isomorphism of ultraproducts. II
Mr. H. J. Keisler, University of California, Berkeley (61T-37)

(37) Properties preserved under reduced products. II
Mr. H. J. Keisler, University of California, Berkeley (61T-38)

(38) On radius of univalence of polynomials
Dr. Viktors Linis, University of Ottawa, (61T-34)

(39) Homeotopy groups. Preliminary report
Mr. G. S. McCarty, Jr., University of California, Los Angeles (61T-48)

(40) Some inequalities for the permanent. Preliminary report

(41) On a mixed initial-boundary value problem for the wave equation
Professor C. S. Morawetz, New York University (61T-40)

(42) Almost-symmetric solutions of some symmetric \(n\)-person games
Professor O. Morgenstern, Princeton University (61T-53)

(43) Error estimates for the solutions of first order ordinary differential equations (using computers)
Mr. Johann Schroder, Mathematics Research Center, U. S. Army, University of Wisconsin (61T-41)

(44) The behaviour of entire functions determined from their Taylor series and a conjecture of Erdos
Professor S. M. Shah, University of Kansas (61T-42)

(45) A structure theorem for infrapolynomials with prescribed coefficients
Dr. Oved Shisha, National Bureau of Standards, Washington, D. C. (61T-43)

(46) A note on the generation of integrable sets
Professor Choy-tak Taam, Georgetown University (61T-44)

(47) A fixed point theorem for monotone transformations on a bush
Professor L. E. Ward, Jr., University of Oregon (61T-45)

(48) Difference analogues for axially symmetric problems. Preliminary report
Mr. R. M. Warten, Mathematics Research Center, U. S. Army, University of Wisconsin and Purdue University (61T-47)

(49) Necessary and sufficient conditions for unitary similarity
Professor N. A. Wiegmann, George Washington University (61T-46)

(50) On Helly-Riesz-Hahn property with application to regular ring
Professor Y. K. Wong, University of Toledo (61T-57)
ABSTRACTS of CONTRIBUTED PAPERS

THE FEBRUARY MEETING IN NEW YORK, NEW YORK

February 25, 1961

577-1. E. B. Leach: A note on inverse function theorems.

Let $U$ and $V$ be Banach spaces and $f: U \rightarrow V$ a function. A strong differential of $f$ at a point $x_0 \in U$ is a bounded linear transformation $a: U \rightarrow V$ with the property that for every $\varepsilon > 0$, there is a $\delta > 0$ such that if $|x' - x_0| < \delta$ and $|x'' - x_0| < \delta$, then $|f(x'') - f(x') - a(x'' - x')| \leq \varepsilon |x'' - x'|$. If $a$ has a bounded left or right inverse $\beta$, then $f$ has a corresponding local left or right inverse $g$ with differential $\beta$ at $f(x_0)$. The proof uses successive approximations and assumes completeness, but not local compactness of $U$. (Received December 12, 1960.)


A matrix method (M method) of approximate integration with respect to a nondecreasing distribution function $\alpha(x)$ on $[-1, 1]$ consists of two infinite triangular arrays, $(x_{i,j})$, $(A_{i,j})$, $i \leq j$, $j = 0, 1, 2, \ldots$, with $-1 \leq x_{i,j} \leq 1$. An M method is said to be effective if $\lim L_n(f) = L(f)$, where $f$ is in $C[-1, 1]$, $L_n(f) = \sum_{i=1}^{n} A_{i,n}^T f(x_{i,n})$, $L(f) = \int_{-1}^{1} f(x) \alpha(x) \, dx$. An M method is said to be a Tchebycheff method (T method) if $A_{i,n} = 1/n$ and $L_n(p) = L(p)$ for $p$ a polynomial of degree $n$. T methods, when they exist, are unique and effective. In the following theorems we add the condition that the spectrum of $\alpha(x)$ contains -1 and 1.

Theorem I. There are T methods corresponding to infinitely many distributions of the type $\alpha(x)$. Theorem II. If a T method satisfies $L_n(x^{n+1}) = L(x^{n+1})$, $n \geq 2$, then it must correspond to $\alpha(x) = \int_{-1}^{x} (1 - z^2)^{1/2} \, dz$. Theorem I is proved by explicit construction. Theorem II yields a result of Posse on the Gaussian method of approximate integration, using simpler hypotheses. (Received December 21, 1960.)


A logical building block with $r$ inputs and $s$ outputs consists in a graph having $r$ input branches, directed toward an interior node and $s$ output branches directed away from this interior node. To each input branch $i$ is attached a binary variable $x_i$. To each output branch $q$ is attached a Boolean function $a_q = a_q(x_1, \ldots, x_r)$. A set of these logical building blocks are joined together to form a so-called Boolean graph. These graphs are formed by successively identifying the input branches of one such block with the output branches of others. Each such Boolean graph defines a set of Boolean functions, one for each output branch of the graph, each of these being functions of the variables attached to the input branches of the graph. Suppose now that a cost is associated with each of a prescribed set $B$ of logical building blocks. Let the cost of a Boolean graph be the sum of the cost of the building blocks of which it is composed. Problem: to find a Boolean graph constructed from the set $B$ realizing a given set of Boolean functions, of minimum cost. A special case is the classical problem of finding a functional expression for a Boolean function using AND's, OR's, and NOT's which has a minimum number of these primitive functions. An algorithm is given for this slight generalization of this problem, involving "don't-care" conditions. (Received December 12, 1960.)
577-4. V. E. Beneš: A nonlinear integral equation from the theory of servomechanisms.

The equation (1) \( x(t) = s(t) - \{k \ast F(x)\}(t) \), where \( s(\cdot) \) is a given function, \( F(\cdot) \) a nonlinear function, \( k(\cdot) \) the response of a linear system, and \( \ast = \text{convolution} \), describes a general class of servomechanisms. Specific properties of a solution \( x(\cdot) \) can be established (without actually solving the equation!) by finding a fixed point (corresponding to a solution) in a specific set of a function space, using Schauder's theorem. E.g., let \( B \) be the set of functions "band-limited" to \((-\Omega,\Omega)\), i.e., whose Fourier transforms vanish off \((-\Omega,\Omega)\). Let \( S \) be a subset of \( L_2 \cap B \) consisting of functions of uniformly integrable square, i.e., \( \int_{|u|>\epsilon}|x(u)|^2du < h(t) \) uniformly in \( x(\cdot) \). Then \( S \) is compact in \( L_2 \), and under suitable inequalities on \( k(\cdot) \) and \( s(\cdot) \), the operator of the equation (1) maps \( S \) into itself. Sample result: there exists a solution \( x(\cdot) \) of equation (1) such that \( x \in S \) and \( |x(t)| \leq \langle \Omega/\pi \rangle^{1/2} \|x\| \). (Received January 9, 1961.)


Pointwise bounds are obtained for a class of parabolic problems where theoretically the bounds may be made arbitrarily small if the solution is known to exist. Let \( V \) be a \((n + 1)\) dimensional domain bounded by two hyperplanes \( y = 0 \) and \( y = y_0 > 0 \) (denoted by \( D(0) \) and \( D(y_0) \) respectively) and a surface \( S \) lying between these planes. Let the unit normal \( (n_1, n_2, \ldots, n_N) = (n_1, \ldots, n_N, n_N) \) satisfy \( \min_n \sum |n_i|^2 = m_1 > 0 \). The problem considered is as follows: \( (a_{ij}w_{ij})_{ij} - Z w_{ij} = f(x,y,W,\nabla W) \) in \( V \cup D(y_0) \), \( W(x,0) = g(x) \) on \( \partial D(0) \), and \( a_{ij}w_{11}n_j = \ell(x,y,W) \) on \( S \) where \( W \) is twice piecewise continuously differentiable with respect to the \( x_i \) for \((x,y) \in V \) and piecewise continuously differentiably in \( \mathbb{V} \) with a continuous normal derivative across \( S \). The elliptic matrix \( a_{ij} \) is piecewise continuously differentiable while the piecewise continuous function \( Z \) has a piecewise continuous derivative in \( y \) where \( \min_y Z = m_2 > 0 \). The boundary functions \( f \) and \( \ell \) satisfy a Lipschitz condition in \( W \) and \( \mathbb{V} W \), as applicable. In subsequent parts of this paper problems shall be considered where the boundary data on \( S \) or the coefficients of the differential operator have been modified. (Received January 9, 1961.)

577-6. Paromita Chowla: On the existence of polynomials of arbitrary degree in many variables, which represent every natural number exactly once.

The author has proved the existence of a polynomial of degree \( m \) in \( m \) variables \((\text{where } m \text{ is any natural number})\) which represents every natural number exactly once \((\text{the range of each variable being the set of natural numbers})\). (Received January 9, 1961.)


We prove the Theorem. Let \( f(x,y) \) be any fixed binary cubic form with integral coefficients and non-zero discriminant, and let \( \phi(x,y) \) be any real polynomial of degree at most 2. Let \( (1) \ S = \sum_{x=1}^{P} \sum_{y=1}^{Q} e^{i(dx(y)+\phi(x,y))} \) where \( e(x) = e^{2\pi i x} \) and \( 1 \leq Q \leq P \). Then subject to \((2) \ (h,g) = 1, \ |h - h/g| \leq g^{-2} \) we have \( |S| \leq P^{3/2} + P^{1/2}g^{1/2} + P^{2}g^{1/2} \) for any fixed \( e > 0 \). This theorem enables us to generalize some of the results related to Waring's theorem for cubes. (Received December 15, 1960.)
L. V. Quintas and Fred Supnick: On the functional equation \( f(\lambda x) = \lambda f(x) \).

Let \( \Lambda \subset \mathbb{R} \) where \( \mathbb{R} \) is the set of all real numbers. A study is made of the dependence on \( \Lambda \) of the class \( C_{\Lambda} \) of all functions on \( \mathbb{R} \) to \( \mathbb{R} \) satisfying \( f(\lambda x) = \lambda f(x) (\lambda \in \Lambda) \). \( C_{\Lambda} \) depends on the multiplicative subgroup \( \langle \Lambda \rangle \) of \( \mathbb{R} \) generated by \( \Lambda - \{0\} \). Specifically, if \( \langle \Lambda \rangle = \langle \Lambda' \rangle \neq \{0\} \) then \( C_{\Lambda} = C_{\Lambda'} = C_{\langle \Lambda' \rangle} \). If \( B \) is a set of representatives for \( \mathbb{R}/\langle \Lambda \rangle \) then \( f \in C_{\Lambda} \) is completely determined when \( f \) is known on \( B \); conversely, any function defined on \( B \) to \( \mathbb{R} \) can be extended uniquely to a function \( f \in C_{\Lambda} \). Typical results are as follows: If \( \langle \Lambda \rangle \) is dense in \( \mathbb{R} \) with \( \mathbb{R}/\langle \Lambda \rangle \) infinite, then \( f \in C_{\Lambda} \) is continuous everywhere, continuous only at the origin, or totally discontinuous, and the closure of the cartesian graph of \( f \) is, in each case, a set of lines through the origin. If \( \langle \Lambda \rangle \) is infinite discrete in \( \mathbb{R} \) a decomposition of the graph of \( f \) is obtained such that each member \( \langle \lambda \rangle \) of the decomposition is the image of a dilation of the plane applied to the graph of \( f \) over the interval \( 1 \leq x < g \) (where \( g \) is the smallest member of \( \langle \Lambda \rangle \) greater than one) defined by \( D_{\lambda}(x,y) = (\lambda x, \lambda y) (\lambda \in \langle \Lambda \rangle) \). (Received January 10, 1961.)

G. L. Krabbe: Spectral theorems in Banach space.

Let \( X \) be a suitable Hausdorff space, and let \( B_1 \) and \( B_2 \) be Banach spaces that can be continuously injected into \( X \). Suppose further that \( B_1 \cap B_2 \) is dense in \( B \) (when \( n = 1 \) and \( n = 2 \)). Denote by \( [B_n] \) the space of bounded linear operators in \( B_n \). If \( T \in [B_2] \), denote by \( T^0 \) the restriction of \( T \) to the set \( B_1 \cap B_2 \), while \( \|T^0\|_1 = \sup \{\|T^0 x\|_1 : x \in B_1 \text{ and } |x|_1 \leq 1\} \) (here \( |x|_1 \) is the norm pertaining to \( B_1 \)). Suppose that \( E \) is a function of bounded variation on \( (-\infty,\infty) \) having compact support; the range \( \{E(\lambda) : \lambda \in (-\infty,\infty)\} \) is a subset of \( [B_2] \), and \( \|E(\lambda)^0\|_1 \) is uniformly bounded for \( \lambda \in (-\infty,\infty) \). Finally, suppose that \( H = \int_{-\infty}^{\infty} \lambda \cdot dE(\lambda) \) belongs to \( [B_2] \). Under these circumstances, there exists a member \( H_1 \) of \( [B_1] \) such that \( H_1 \supset H^0 \); to \( H_1 \) belongs an operational calculus \( f \rightarrow f(H_1) \) whose domain contains all functions of bounded variation on \( (-\infty,\infty) \); let \( E^1(\lambda) \) be the continuous extension of \( E(\lambda)^0 \) to the whole space \( B_1 \) (whence \( E^1(\lambda) \in [B_1] \)); we have \( f(H_1) = \int_{-\infty}^{\infty} f(\lambda) \cdot dE^1(\lambda) \) and \( H^1 = \int_{-\infty}^{\infty} \lambda \cdot dE^1(\lambda) \). The operator-valued function \( E^1 \) need not be of bounded variation (in particular, the semi-variation of \( E^1 \) may be infinite). (Received January 10, 1961.)

J. A. Ernest: Some properties of the canonical decomposition for representations of locally compact groups.

It is well known that every unitary representation \( L \) of a separable locally compact group \( G \) may be decomposed canonically into a direct integral \( L = \int_Z L^y d\mu(y) \) such that \( \mu \)-almost all of the components \( L^y \) are factor representations of \( G \). (See G. W. Mackey, Induced representations of locally compact groups, II, Ann. of Math., vol. 55 (1952) pp. 101-139.) This decomposition is shown to have the following additional properties. 1. There exists a Borel subset \( Z' \) of \( Z \) such that \( \mu(Z - Z') = 0 \) and \( L^y \) and \( L^z \) are disjoint whenever \( y \) and \( z \) are distinct points of \( Z' \). (Two representations are said to be disjoint if no subrepresentation of one is equivalent to a subrepresentation of the other.) 2. Two representations \( L \) and \( M \) are quasi-equivalent if and only if they have canonical decompositions over the same measure space, say \( L = \int_Z L^y d\mu(y) \) and \( M = \int_Z M^y d\mu(y) \), such that \( L^y \) is quasi-equivalent to \( M^y \) for \( \mu \)-almost all \( y \). (L is said to cover \( M \) if no subrepresentation of \( M \) is disjoint from \( L \). \( L \) and \( M \) are said to be quasi-equivalent if \( L \) covers \( M \) and \( M \) covers \( L \).) The proof of statement one is a perturbation of a proof of A. Guichardet (Sur un problème posé par...
G. W. Mackey, C. R. Acad. Sci. Paris vol. 250, pp. 962-963. The second statement justifies the introduction of the concept of direct integrals of quasi-equivalence classes of representations. (Received January 10, 1961.)


Preliminary report.

Let D be a finite Riemann surface, with boundary \( \partial D \), and suppose that \( \Delta \), \( \gamma \) are given functions on \( \partial D \), with \( |\Delta| = 1 \). Under the assumption that \( \Delta \) is Hölder continuously differentiable and \( \gamma \) is Hölder continuous, we treat the boundary value problem \( \text{Re}(\Delta w) = \gamma \) for solutions of the differential equation \( w_z = Aw + Bw \), where \( A \) and \( B \) are bounded measurable coefficients of a conjugate differential. The derivatives here are considered in the weak \( L^2 \) sense. The proof utilizes a suitable linear operator which maps solutions of the differential equation into analytic functions. Using certain extensions of a coercive inequality noted by M. Schechter (Abstract 538-19, Notices Amer. Math. Soc. vol. 5 (1958) p. 58) one may show that this operator is of Fredholm type. Known results for the same boundary value problem for analytic functions (Koppelman, Comm. Pure Appl. Math. vol. 12 (1959) pp. 13-35) can then be utilized to deduce corresponding results for solutions of the differential equation. (Received January 12, 1961.)


All operators considered in this note are bounded and defined on a fixed Hilbert space \( X \). If \( T: X \to X \) is a linear operator such that \( \exp T = N \) is a normal operator then \( T = N_0 + 2\pi i W \) where \( N_0 \) is a normal operator such that \( \exp N_0 = N \). The bounded operator \( W \) commutes with \( N_0 \) and there exists a bounded and regular selfadjoint \( Q \) such that \( W_0 = Q^{-1} WQ \) is a selfadjoint operator the spectrum of which belongs to the set of all integers. If the spectrum of \( N \) belongs to the set \( \Omega = \{ \text{re}^{i\theta} \mid -\pi \leq \theta \leq \pi, 0 \leq \alpha \leq 1/2, \tau \leq \varepsilon \geq 0 \} \) and \( \|T\| < (1-\alpha^2/4)^{1/2} \) then \( T \) is a normal operator. If \( \exp T = H \) is a regular positive definite selfadjoint operator then \( \|T\| < 2\pi \) implies that \( T \) is selfadjoint. This is a generalisation of the result obtained by C. R. Putnam; Proc. Glasgow Math. Assoc. vol. 4 (1958) pp. 1-2. If \( X \) is finite dimensional, \( H \) a positive definite selfadjoint operator then \( \exp T = H \) and \( \|T\| \leq 2\pi \) imply that \( T \) is normal. (Received January 12, 1961.)

577-13. T. P. G. Liverman: Generalized functions of class \( (\mathcal{E}^{P}_{\gamma}) \).

\( p, \gamma, \) constants \( p > 1, 0 \leq \gamma \leq p \), \( (t, x) \in R \times R^n \) (here below \( n = 1 \) for notational convenience), \( \phi(t,x) \in (\mathcal{E}^{P}_{\gamma}) \) iff \( |D^\alpha D_x^\gamma \phi(t,x)| \leq \psi(t) |x|^p |\alpha|^\gamma \), \( \phi(t) \) is nondecreasing; \( \phi(t) \) is carried on \([a, \infty) \times R^n\). A piecewise continuous \( f(t,x) \) is of class \( \mathcal{F}^{P}_{\gamma} \) if \( f \) is carried on \([a, \infty) \times R^n\) and to every \( \varepsilon > 0 \) corresponds \( K_{\varepsilon}(t) \) with \( |f(t,x)| \leq K_{\varepsilon}(t) \). If \( \phi(t) \) determines a functional on \( \mathcal{E}^{P}_{\gamma} \) with the value \( \langle f, \phi \rangle = \int_a^\infty \int_0^{\infty} f(t,x) \phi(t,x) dt dx \). \( (\mathcal{E}^{P}_{\gamma}) \) is defined as the set of equivalence classes of weakly convergent sequences of such functionals. Certain series of the form \( \sum_{j=0}^\infty \omega_j Q^{j} \) constitute a commutative ring of operators on \( \mathcal{E}^{P}_{\gamma} \) while submitting to the manipulations of ordinary power series. Theorem. Given the polynomial \( P(Q,D) \) polynomial in \( D_x \) of degree \( n \) such that \( P(Q,D) = P_{\gamma}(D_x) \), the series \( A(\varepsilon) \) is a linear operator on every class \( \mathcal{E}^{P}_{\gamma} \) which verifies the strict inequalities: \( 1 < p < 1/(1 - \mu) \). (Received January 5, 1961.)

52
61T-1. T. A. Brown: Maximal simple paths on convex polyhedra.

A graph G is said to be d-polyhedral if it can be represented as the vertices and edges of a convex polyhedron in \( \mathbb{E}^d \). Let \( p(G) \) denote the number of vertices contained in a maximal simple path on G, and set \( p_d(n) = \min \{ p(G) | G \text{ is d-polyhedral of order } n \} \). A subset \( W \) of the vertices of a graph G is said to be connectionless if no single edge of G connects two vertices in \( W \). Theorem 1: If \( W \) is connectionless in a 3-polyhedral graph of order \( n \), then \( \text{card } W \leq (2n - 4)/3 \). By constructing for each positive integer \( n \) of form \( n = 3k + 2 \) a 3-polyhedral graph \( H_K \) with a connectionless \( W \) for which the equality sign above holds, we deduce Theorem 2: \( p_3(n) \leq (2n + 13)/3 \). One can apply a method due to Grünbaum and Motzkin (see a forthcoming paper in the Pacific J. Math.) to extend the graphs \( H_K \) into a family of 3-polyhedral graphs whose maximal simple paths contain an arbitrarily small proportion of the vertices in the whole graph. Thus, we prove Theorem 3: If \( n = (49,13k - 3)/2 \), then \( p_3(n) < 1,851n^{8107} \). On the other hand, we have Theorem 4: If \( v(n) \) is the solution to \( Xx = n \), then \( v(n) < p_3(n) \). Thus \( p_3(n) \to \infty \) as \( n \to \infty \). Finally, we are able to construct a family of d-polyhedral graphs which prove Theorem 5: If \( n = d - d^k + 2 \sum_{i=1}^{k} d_i \), then \( p_d(n) \leq (2n^2 + 2(d + 1)(d - 2) + d)/d \). (Received November 30, 1960.)


Let A and B be Banach algebras with second conjugate spaces \( A^{**} \) and \( B^{**} \), respectively. Regard \( A^{**} \) and \( B^{**} \) as Banach algebras according to the procedure given by R. Arens (Monatsh. Math. vol. 55 (1951) pp. 1-19; Proc. Amer. Math. Soc. vol. 2 (1951) pp. 839-848). The possibility of extending an algebra homomorphism from A into \( B^{**} \) to an algebra homomorphism from \( A^{**} \) into \( B^{*} \) is considered, along with similar extensions involving certain quotient algebras of \( A^{**} \) and \( B^{**} \). If \( G \) and \( H \) are locally compact abelian groups, with \( H \) compact, a class of homomorphisms from \( L^1(G) \) into \( M(H) \) is shown to be obtainable from homomorphisms of \( L^1(G) \) into \( (L^1(H))^{**} \). (Received December 1, 1960.)


Let G be a group which acts transitively on the Hausdorff space X, and let \( \mu \) be a regular Borel measure on X which is left-invariant under G. Let \( \mathcal{B} \) be a point of X, arbitrary but fixed. If \( B, C \subset X \), we define \( B^{-1}C = \bigcup \{ gC | g \in G, g \notin gB \} \). A sequence \( \{ S_n \} \) of subsets of X is called a sequence of quasispheres if there exists an \( e > 0 \) such that \( S_{n+1} \subseteq S_n \) and \( \mu^*S_{n+1} > e \mu^*S_n \) for each integer \( n > 0 \). Definition: A family \( \mathcal{U} \) of closed subsets of X, each with positive measure, is called a regular Vitali cover for the subset \( A \) of X if there is a sequence \( \{ S_n \} \) of quasispheres and a real-valued function M on A such that whenever \( V \) is a neighborhood of \( x \in A \) there exist \( U \in \mathcal{U} \), \( g \in G \) and an integer \( n \) for which \( x \in U \subset V \cap gS_n \) and \( \mu^*S_n \leq M(x)\mu U \). Theorem: If \( \mathcal{U} \) is a regular Vitali cover for \( A \), and if \( A \) is contained in some \( \sigma \)-finite subset of X, then there is a (possibly finite) sequence \( \{ U_k \} \) of pairwise disjoint elements of \( \mathcal{U} \) for which \( \mu^*(A \setminus \bigcup U_k) = 0 \). Application: Let \( \mu \) be left Haar measure on the Lie group X = G acting by left-translation.
Let $\rho > 2$ and, relative to a right-regular local coordinate system at $\phi = \text{identity}$, let $S_n$ be the sphere centered at $\phi$ of radius $\rho^{-m-n}$ (m sufficiently large). Then $\{S_n\}$ is a sequence of quasispheres with respect to which each subset of $X$ admits numerous regular Vitali covers. (Received December 1, 1960.)


Let $(E_i, F_i, I)$ be a collection where $I$ is an infinite set, for each $i \in I$, $E_i$ is a topological space, and $F_i$ a subspace of $E_i$. Define $S$ to be the collection of all functions $f: I \to \bigcup_i E_i$ such that $f(i) \in E_i$, and $f(i) \in F_i$ except for finitely many values of $i$. Take for a subbasis for a topology on $S$ the following sets: for each $i \in I$, and open set $U_i \subset E_i$, let $U = \{f \in S: f(j) \subset F_j, j \neq i, f(i) \in U_i\}$. Thus $S = \prod_i F_i$ is an open subset of $S$ on which our topology agrees with the Tychonoff topology. Among the interesting properties of $S$ is that it is locally compact if $E_i$ is locally compact for all $i$, and $F_i$ is compact for all but at most a finite number of $i$. This product generalizes both the Tychonoff product and the weak direct product for groups with the discrete topology. (Received December 1, 1960.)


The author considers the differential equations (a) $W'' + pW = 0$ and (b) $W'' + p^*W = 0$ and their respective solutions $W_\beta(z) = z^\beta \sum_{n=0}^{\infty} b_n z^n, (b_0 = 1)$, and $W_\beta^*(z) = z^{\beta*} \sum_{n=0}^{\infty} b_n^{*} z^n, (b_0^{*} = 1)$, corresponding to the smaller roots $\beta$ and $\beta^*$ of their respective indicial equations. It is required that (i) $0 < \beta < 1/2$, (ii) $z^2 p(z)$ and $z^2 p^*(z)$ are regular in some neighborhood of the origin, and (iii) $z^2 p^*(z)$ is real on the real axis. Using a modification of the methods of M. Robertson and P. Beesack the author derives conditions on $p(z)$ sufficient to ensure the univalence of $[W_\beta(z)]^{1/\beta}$ in $|z| < r_{\beta^*}$, where $r_{\beta^*}$ is the smallest positive zero of the function $W_\beta^*(z)$. Sufficient conditions for the disconjugacy of (a) in $|z| < r_{\beta^*}$ are also obtained. Applications of the results yield regions of univalence of the normalized Bessel functions $z^{1-\nu} J_{\nu}(z),(\nu > -1/2)$, and $J_\nu(z)J_{1-\nu}(z),(\nu > 1/2 < \nu < 3/2)$. (Received December 2, 1960.)


Let $m, n, k$ denote positive integers, and let $(m, n)_k$ represent the greatest common $k$th power divisor of $m$ and $n$, $(m, n) \equiv (m, n)_1$. It is the purpose of this paper to investigate the asymptotic behavior of $Q_k(m,n)$, defined to be the number of sets of positive integers, $x_1, x_2, y_1, y_2$, such that $m = x_1 + y_1, n = x_2 + y_2, (x_1, x_2)_k = (y_1, y_2)_k = 1$. The main result proved for $Q_k(m,n)$, stated here without remainder, is $Q_k(m,n) \sim mn \prod (1 - 1/p^{2k}) \prod (1 - 2/q^{2k})$ as $m \to \infty (n \geq m)$, where $p$ ranges over the primes such that $p^k|m,n$ and $q$ over the primes such that $q^k|m,n$. The special case $k = 1$ was treated in an earlier note (Arithmetical notes, VI). The method used in the present paper, while elementary, yields much sharper estimates than that used in the original note. For example, the result obtained for $Q_1(n,n^2)$ has a remainder term of order $O(n^2 \log^3 n)$ as compared with the earlier estimate, $O(n^3/\log n)^{1/4}$. (Received December 2, 1960.)
61T-7. Harvey Cohn: Cusp functions involved in decomposition into the sum of four squares in $R(3^{1/2})$. Preliminary report.

The author has previously developed an "approximate" formula for the number $A(p)$ of decomposition of totally positive integers $p = a + 2b 3^{1/2} = \sum (x_i + y_i 3^{1/2})$, $1 \leq i \leq 4$, by taking the easily calculated coefficient $Z(p)$ of the corresponding singular series of $\theta^4$ where $\theta = \sum \lambda \exp(\lambda^2/12)/12^{1/2}(\lambda^2 - 1^2/\lambda^2)$. [See current work in the Amer. J. Math. 1960, 1961]. Now unlike the case of $2^{1/2}$ and $5^{1/2}$, we find $A(p)$ is not equal to $Z(p)$ (which would have given an easy proof that $A(p) > 0$); but $A(p) - Z(p) = 4T(p)$ defines a function $T(p)$ with multiplicative properties like those of Ramanujan's function (and provable by an extension of Mordell's method). Here there is the additional advantage that $\mu$ might have an odd primary component $\mu_i$ not of the form $r + s 3^{1/2}$, for $s$ even then easily $T(\mu) = 0$. This extends the cases for which $\mu$ permits a decomposition as the sum of four squares, in fact far enough to show totally positive multiples of 2 permit such a decomposition. (Received December 2, 1976.)


Let $C$ be the unit circle and $D$ be the open unit disk in the complex plane, $\omega$ be a complex number or $\infty$, $M$ be a finite positive constant, and $\{z_n\}$ be a sequence of points in $D$ that has at least two limit points on $C$ and is such that $|z_n| \to 1$. Denote the hyperbolic non-Euclidean distance between $z_n$ and $z_{n+1}$ by $p_n(n = 1,2,3,\ldots)$. Let $f(z)$ be a normal meromorphic function in $D$ that omits the value $\omega$, $g(z)$ be a normal meromorphic function in $D$, and $h(z)$ be a meromorphic function in $D$ that is bounded away from $\omega$. (I) If $p_n < M$ for every $n$, and $f(z_n) \to c$, then $c \neq \omega$, and $f(z) \equiv c$. (II) If $p_n \to 0$ and $g(z_n) \to c$ (c finite or infinite), then $g(z) \equiv c$. (III) If $p_n < M$ for every $n$, and $f(z_n) - h(z_n) \to 0$, then $f(z) \equiv h(z)$. (IV) If $p_n \to 0$ and $g(z_n) - h(z_n) \to 0$, then $g(z) \equiv h(z)$. (Received December 5, 1960.)

61T-9. Frederick Bagemihl: A proposition of elementary plane geometry that implies the continuum hypothesis.

It is not known whether the following proposition (see Sierpiński, Cardinal and ordinal numbers, Warsaw, 1958, p. 399) is true or false, even if the continuum hypothesis ($H$) $2^\alpha = \aleph_1$ is assumed to be true: (P) The Euclidean plane is the union of three sets $E_i$ ($i = 1,2,3$) such that, for some three straight lines $D_i$ ($i = 1,2,3$) in the plane, for $i = 1,2,3$ the set $E_i$ intersects every straight line parallel to $D_i$ in only a finite number of points. We prove that (P) implies (H). It is shown that (H) is implied by even the following proposition: (Q) The Euclidean plane is the union of three sets $E_i$ ($i = 1,2,3$) such that, for some three straight lines $D_i$ ($i = 1,2,3$) in the plane, $E_i$ intersects every straight line parallel to $D_i$ in only a finite number of points, $E_2$ intersects every straight line parallel to $D_2$ in at most $\aleph_0$ points, and $E_3$ intersects every straight line parallel to $D_3$ in at most $\aleph_1$ points. (Received December 7, 1960.)


If $M$ is a compact subset of a plane and $\varepsilon$ is a positive number, then $M$ can be covered by the interiors of a finite number of mutually exclusive 2-cells, each lying in the $\varepsilon$-neighborhood of $M$. This theorem does
not generalize to $E^3$, as shown by Antoine's necklace and many other examples. However, the difficulty is always due to the impossibility of covering $M$ by small 3-cells whose boundaries do not intersect $M$; specifically, it is shown in this paper that if $M$ can be covered by a finite number of 3-cells, each lying in the $\varepsilon$-neighborhood of $M$, whose boundaries do not intersect $M$, then $M$ can be covered by the interiors of a finite number of mutually exclusive 3-cells, each lying in the $\varepsilon$-neighborhood of $M$. (Received December 29, 1960.)


Let $f(\theta) > 0$ be an integrable function on $-\pi \leq \theta \leq \pi$ with Fourier coefficients $c_k$, i.e. $f(\theta) \sim \sum_{-\infty}^{\infty} c_k \exp (ik\theta)$, and let $D_n(f) = \det(c_j-k), j,k = 0,1,\ldots,n$. Moreover, let $\log f(\theta) \sim \sum_{-\infty}^{\infty} k \exp (ik\theta)$, and set $G(f) = \exp(h_0)$. Theorem: If $\sum_{-\infty}^{\infty} k^{1/3} |c_k| < \infty$, then there exists a finite limit $\lim_{n \to \infty} D_n(f)/G(f)^{n+1} = \exp(\sum_{-\infty}^{\infty} k |h_k|^2)$. This is a sharpening of a result of Szegő (see Toeplitz forms and their application, Calif. Press 1958, Grenander and Szegö, page 76) and a result of Kac (Toeplitz matrices, translation kernels and a related problem in probability theory, Duke J. Math. vol. 21 (1954) pp. 501-510). The theorem is sharp in the sense that if $a < 1/2$, then there exist functions $f(\theta) > 0$ such that $\sum_{-\infty}^{\infty} k a |c_k| < \infty$ but $D_n(f)/G(f)^{n+1}$ does not have a finite limit. (Received December 27, 1960.)

61T-12. R. H. Bing: Embedding circle like continua in the plane.

A compact continuum is circle like if for each positive number $\varepsilon$, the continuum can be irreducibly covered by a circular $\varepsilon$-chain of open sets. It is shown that each circle like continuum that lies in a 2-manifold $M$ lies in a subset of $M$ which is either an annulus ring or a subset of a Moebius band. Although not each planar circle like continuum can be covered by an $\varepsilon$-chain with connected links, it is shown each is homeomorphic to a planar continuum whose links are small open circular disks. Furthermore, if $U$ is an open disk in the plane and the set $X$ of planar continua in $E^2$ which separate $U$ from infinity is metrized by the Hausdorff distance, then the set of elements of $X$ which cannot be covered by circular chains whose links are small open circular disks is of the first category. (Received December 8, 1960.)


A metric space $M$ has the Young property provided $p,q,r,q',r' \in M, pq' = qq' = (1/2)pq, pr' = rr' = (1/2)pr$ imply $q'r' = (1/2)qr$. Banach spaces with unique lines are characterized among the class of metric spaces by being complete, convex, externally convex, and possessing the two-triple and Young properties. (A metric space has the two-triple property if all triples of a quadruple are linear whenever two of them are.) If $p \in M$ and $\lambda$ denotes a metric line of $M, p \not\in \lambda$, let $f_p$ denote a foot of $p$ on $\lambda$. The space $M$ has the Ficken property provided $q, r \in \lambda, qf_p = rf_p$ imply $sq = sr$ for each point $s$ of line $(p,f_p)$. It is proved that a Banach space (as characterized above) is euclidean (not necessarily of finite dimension) if and only if it has the Ficken property. This provides a new completely metric proof of Ficken's theorem characterizing inner product spaces among normed linear spaces. (Received December 12, 1960.)

A metric space $M$ is ptolemaic if $p, q, r, s \in M$ implies that the three products of opposite distances $pq \cdot rs, pr \cdot qs, ps \cdot qr$ satisfy the triangle inequality. Variations of this condition yield spaces called weak strict, feeble strict, isosceles weak strict, isosceles feeble strict ptolemaic. This paper investigates metric properties of such spaces. Among the results established are (1) surfaces of nonpositive curvature are ptolemaic, (2) hyperbolic $n$-space is ptolemaic, (3) a normed linear space is feeble strict ptolemaic if and only if it has unique straight lines, (4) a complete convex metric space that is locally (irreducibly and homogeneously) euclidean $n$-dimensional is euclidean $n$-dimensional if and only if it is ptolemaic. A new class of ptolemaic straight line spaces, called hybrid, are defined. Parts of these spaces are euclidean and parts are hyperbolic. (Received December 12, 1960.)


L. F. McAuley (Pacific J. Math. vol. 6, p. 325) asked: is it possible to partition Moore's metrization theorem into three or more parts which begins with a condition for a topological space and which ends with a condition for a metrizable space, but with necessary and sufficient conditions somewhere between these extremes for semi-metric and Moore spaces? This paper answers the question in the affirmative. (Received December 6, 1960.)


**Theorem.** Let $P^{n-1}$ be a combinatorial $(n-1)$-sphere in a Euclidean $n$-space $E^n$, and let $N$ be an arbitrary neighborhood of $P^{n-1}$. Then there exists a homeomorphism of $E^n$ onto itself which maps $P^{n-1}$ onto the unit $(n-1)$-sphere $S^{n-1}$ and which is a $C^\infty$-diffeomorphism on $E^n-N$. This is proved with the aid of results due to H. Noguchi, The smoothing of combinatorial $n$-manifolds in $(n+1)$-space, Ann. of Math. vol. 77 (1960) pp. 201-215, and to M. Morse, Differentiable mappings in the Schoenflies theorem, Compositio Math. vol. 14 (1959) pp. 83-151. A construction by Noguchi can be modified to obtain an arbitrarily small isotopic deformation of $E^n$ taking $P^{n-1}$ into a position $Q^{n-1}$ admitting a transverse vector field. A neighborhood of $Q^{n-1}$ is fibred by $C^\infty$-$(n-1)$-spheres, Morse's work yields a $C^\infty$-diffeomorphism taking the interior of one of these spheres onto the interior of $S^{n-1}$, with an exceptional point which can be required to be in $N$.

**Corollary.** Given an $\varepsilon > 0$, $E^n$ admits an isotopic deformation $h_t$, $0 \leq t \leq 1$, such that $h_t(P^{n-1}) \subset B^i(t > 0)$ and $h_t(P^{n-1})$ is a $C^\infty$-$(n-1)$-sphere ($t > \varepsilon$). Noguchi assumes the theorem in the next lower dimension, so the above proof is inductive. (Received December 19, 1960.)

Let $T$ be a Jordan curve in the complex $z$-plane, let $f$ be a function given on $T$, and let $L_n(f;z)$ be the polynomial of degree at most $n-1$ found by interpolation to $f$ in the points $S_n: z_1, z_2, ..., z_n$. Let $w = \psi(z)$ give a schlicht mapping of the exterior of $T$ onto $|w| > 1$. Except in the classical case in which $f$ is analytic on and within $T$, convergence of the sequence $\{L_n\}$ has been proved only when the points $w_k$ are the $n$th roots of unity; and known necessary conditions are unsatisfactory. In the classical case, the necessary and sufficient condition is that the points $w_k = \psi(z_k)$ shall be equidistributed on $|w| = 1$. To study the role of equidistribution in the nonanalytic case, a probabilistic model is set up in which with probability one a sample sequence of points has the equidistribution property. It is found that for a function $f$ with a pole inside $T$ but otherwise analytic, the value of $L_n(f,z)$ (now a random variable) for $z$ inside $T$ will almost certainly be near the value to which convergence takes place in deterministic roots-of-unity interpolation. Various asymptotic expected value results are also established. (Received December 9, 1960.)


The following are extensions of results announced earlier in Abstract 574-18, Notices Amer. Math. Soc., vol. 7 (1960) pp. 740-741. Let $A$ be a regular semi-simple commutative Banach algebra, with maximal ideal space $S$. Let $C(S)$ ($B(S)$) be the algebra of complex continuous (bounded) functions on $S$. Assume for convenience that $A$ has a unit. Then any derivation $D$ of $A$ into $C(S)$ is necessarily a bounded operator. As a corollary it follows that $D \equiv 0$ if $D$ maps $A$ into $A$. If $D$ maps $A$ into $B(S)$, then $D$ may be unbounded. However, in this case, there exists a bounded derivation $D_1$ of $A$ into $B(S)$ and a finite subset $F$ of $S$ such that if $D_2 = D - D_1$, then $D_2x(s) = 0$, $x \in A$, $s \in S - F$. If $s \in F$, then $f_0(x) = D_2x(s)$ is an unbounded point derivation of $A$. (Received December 9, 1960.)

61T-19. John DeCicco: Appell's transformation in Riemannian space $V_n$.

Appell's transformation $T$ between two Riemannian spaces $V_n$ and $\overline{V}_n$, is a correspondence which converts every complete dynamical system in $V_n$ into a complete dynamical system in $\overline{V}_n$. A necessary and sufficient condition for this is that $T$ be a projective map between $V_n$ and $\overline{V}_n$ such that the related affine connections obey certain compatibility conditions of the first order. This is equivalent to saying that the contravariant form of the acceleration vector obeys the law of transformation $a_s^i = \mu^2a^i_s$, where $\mu$ is a positive scalar point function. If $V_n$ and $\overline{V}_n$ are each projectively equivalent to a Euclidean space $E_n$, then any projective collineation $T$ between $V_n$ and $\overline{V}_n$, can be considered to be an Appell correspondence $T$. (Received December 8, 1960.)

61T-20. John DeCicco: The Lagrangian and Hamiltonian equations of a natural family of a conservative field of force in Riemannian space.

A natural family $F$ is composed of the $e^{2n-2}$ extremals $C$ of the variation problem: $\int_1^2 \mu ds = \text{minimum}$, where $\mu$ is a positive scalar point function. If a particle due to the influence of a conservative field of force in a Riemannian space $V_n$, is constrained to move along a curve $C$ of this natural family $F$, then the Lagrangian function is $L = (\mu/v)(T - V + E) - E$, and the Hamiltonian function is $H = (\mu/v)(T + V - E) + E$, 58
where \( q_i = m(\mu/v)g_i \frac{dx^j}{dt} \). From these, the Lagrangian and Hamiltonian equations of a natural family \( F \), are obtained. Applications are made to a conservative physical system \( S_k \) where \( k \neq 1 \), but \( k \) may be infinite, of a Riemannian space \( V_n \). (Received December 8, 1960.)


N. Ito has proved (Nagoya Math. J. vol. 3 (1951) pp. 5-6) that the degree of an absolutely irreducible representation of a finite group \( G \) is a divisor of the index of a maximal normal abelian subgroup of \( G \). Here we prove the following partial converse. Theorem 1. If \( G \) is a solvable finite group, if \( H \) is a maximal normal abelian subgroup of \( G \), and if \( G/H \) contains an element of period \( p^n \), \( p \) a prime, then \( G \) has an absolutely irreducible representation whose degree is divisible by \( p^n \). We then prove the next result by applying this. Theorem 2. If each absolutely irreducible representation of the finite group \( G \) is of degree one or a power of the fixed prime \( p \), then \( G \) is an extension of an abelian group by a \( p \)-group. (Received December 12, 1960.)


This concerns the maximum principle which applies to solutions of partial differential equations of elliptic type. This principle asserts that the maximum of a solution occurs on the boundary of a region. Consideration of the ratio of solutions of an elliptic equation shows that the ratio satisfies the same maximum principle. This result is then used to obtain a maximum principle relating to biharmonic functions. These maximum principles give inequalities which biharmonic functions must satisfy. The relations and concepts developed have application in elasticity and in hydrodynamics. (Received December 9, 1960.)


A linear combination of \( m \) exponential functions (exponential) is fitted to a time series, such as daily observations. The fitting is carried out over all past times by weighted least squares with an exponential weight factor. The resulting minimizing function could be continued into the future. In particular tomorrow's predicted value is so defined. To solve this problem a formula is constructed which gives the predicted value as a linear combination of the last \( m \) observed values and the last \( m \) predicted values. The \( 2m \) coefficients of this formula are expressed as explicit rational functions of the \( m \) exponential bases. The extrapolating functions available with this method include polynomials, trigonometric polynomials, damped waves, etc. The prediction formula for the case of polynomial functions was given by R. J. Duffin and T. W. Schmidt [J. Math. Anal. Appl. vol. 1 (1960) pp. 215-227]. (Received December 9, 1960.)

61T-24. Edward Fadell and James Van Buskirk: On the Braid groups of \( E^2 \) and \( S^2 \).

Let \( M \) denote a manifold and \( F_n \) the configuration space of \( n \)-tuples \((x_1, \ldots, x_n)\) such that \( x_i \in M \) and \( x_i \neq x_j, \ i \neq j \). Then the symmetric group \( \Sigma^n \) on \( n \) letters acts freely on \( F_n \) and we set \( B_n = F_n/\Sigma^n \). Following R. H. Fox, we define the geometric braid group (\( n \) strings) on \( M \), \( G_n(M) \), by \( G_n(M) = \Xi(B_n) \). The purpose of this paper is to provide a new proof that \( G_n(E^2) \), where \( E^2 \) is the plane, is the classical algebraic braid group on \( E^2 \), and also to compute \( G_n(S^2) \), where \( S^2 \) is the 2-sphere. It turns out that \( G_n(S^2) \) is just \( G_n(E^2) \) with
one additional relation, namely \( \sigma_1 \sigma_2 \cdots \sigma_{n-2} \sigma_{n-1} \sigma_{n-2} \cdots \sigma_2 \sigma_1 = 1 \). The basic topological tool employed is the vanishing of certain second homotopy groups of configuration spaces. (Received December 7, 1960.)

61T-25. Carl Faith: Rings with minimum condition on principal ideals, II.

An (assoc.) ring \( A \) is bound if for each \( a \in A \), the equation \( aJ = Ja = 0 \) implies \( a \in J \), where \( J \) denotes the Jacobson radical of \( A \) \([M. \text{ Hall}, \text{ Jr.}, \text{ Trans.} \text{ Amer. Math. Soc.} \text{ vol. 48 (1940) pp. 394-404; Brown and McCoy, Proc. Amer. Math. Soc.} \text{ vol. 1 (1950) pp. 165-171.}] \). It is known (Hall, Brown and McCoy) that every \( m \)-ring \( A \) (a ring with minimum condition on left ideals) has the structure: \( A = R \oplus R^* \), where \( R, R^* \) are ideals, \( R^* \) a bound ring, and \( R \) is either 0, or a direct sum of simple \( m \)-rings. The result of the present paper is: If \( A \) is a \( MP \)-ring (= a ring with minimum condition on principal left ideals), then \( A = R \oplus R^* \), where \( R^* \) is bound, and \( R \) is either 0, or a direct sum of simple \( MP \)-rings. (cf. \[ \text{Arch. Math. vol. 10 (1959) pp. 327-330} \].)


An (assoc.) ring \( A \) is a strongly regular (s.r.) extension of a subring \( B \) if to each \( a \in A \) there exists \( x \in A \) such that \( a^2 x - a \in B \). (a) A directly irreducible s.r. extension of a division ring (resp. integral domain) is division (resp. integral); (b) If \( A \) is a primitive ring, not division, and if \( A/B \) is s.r., then \( B \) is dense in the finite topology in \( A \). Immediate corollaries: (1) a s.s. ring \( A \) which is s.r. over a commutative subring \( B \) is a sdsdr (= subdirect sum of division rings); (2) any s.r. extension of a division ring is a sdsdr. Applications of (1) and (2) produce extensions of commutativity theorems of Nakayama \[ \text{Nagoya Math. J. vol. 12 (1959) pp. 39-44} \]. In these new results commutative subrings, or division subrings, play the role which the center of the ring played in his theory. The paper concludes with a study of the position in a semi-simple ring \( A \) of the subrings \( E(A) \) (resp. \( T(A) \)) generated by the idempotents (resp. nilpotents) of \( A \): If \( A \) is primitive but not division, and if \( A \) is an algebraic algebra, or has nonzero socle, then \( E(A) \) and \( T(A) \) are dense in the finite topology in \( A \); If \( A \) is algebraic and semisimple, so is \( E(A) \). (Gen. refs: Arens and Kaplansky, Trans. Amer. Math. Soc. vol. 63 (1948) pp. 457-481; Martindale, Proc. Amer. Math. Soc. vol. 9 (1958) pp. 714-721; Utumi, Proc. Japan Acad. vol. 33 (1957) pp. 63-66.) (Received December 9, 1960.)

61T-27. N. J. Fine, L. Gillman and J. Lambek: Rings of quotients of continuous functions, II.

Notation is as in Abstract 576-219, Notices Amer. Math. Soc. vol. 7 (1960) p. 980. Let \( A \) be a subring of \( C(X) \) containing the reals. The following two conditions are equivalent: (1) Every maximal ideal of \( A \) is real, and there is a homeomorphism \( x \to M_x \) of \( X \) onto the maximal ideal space of \( A \) such that \( M_x(a) = a(x) \) for all \( x \in X \) and \( a \in A \). (2) Any function in \( A \) that vanishes nowhere on \( X \) is a unit of \( A \), and the zero-sets of \( A \) form a base for the closed sets in \( X \). Note that the second part of (2) holds in case \( A \) is a lattice and separates points of \( X \). If \( (X_i)_{i \in I} \) is an inverse system of compact spaces with limit \( X_\infty \), then the rings \( C(X_i) \)
form a direct system with limit $A$ canonically embeddable into $C(X_0)$, and $X_0$ is homeomorphic with the maximal ideal space of $A$. Application: For any $X$, $\mathcal{Q}(X)$ is isomorphic to $C(Y)$, where $Y$ is the inverse limit of the Stone–Čech compactifications of the dense open sets in $X$; furthermore, $Y$ is extremally disconnected. Using this, one can show that for any $X$, the lattice $\mathcal{Q}(X)$ (as well as $\mathcal{Q}^*(X)$) is conditionally complete. (Received December 9, 1960.)


Notation is as in Abstract 576-219, Notices Amer. Math. Soc. vol. 7 (1960) p. 980. A commutative ordered ring with 1 will be called real if a sum of squares of elements cannot be zero unless each of these elements is zero. Let $A$ be a real ring with absolute value: for each element $a$, there exists an element $|a|$ such that $|a| \geq 0$ and $|a|^2 = a^2$; if $A^*$ (the subring of all bounded elements) is semi-simple and all elements $\geq 1$ are units, then there is a canonical embedding of $A^*$ into $C(X)$, where $X$ is the maximal ideal space of $A^*$, and this may be extended to an embedding of the maximal ring of quotients of $A^*$ into the ring $Q(X)$. If $A$ is any regular real ring with absolute value, then $A$ is a ring of quotients of $A^*$, the Jacobson radical of $A^*$ consists of all infinitesimal elements of $A$, and $A$ and $A^*$ have homeomorphic ideal spaces. Any regular self-injective real ring ordered in such a way that the ordering cannot be extended is a ring with absolute value. (Received December 9, 1960.)


Let $f(z) = \sum_{m=0}^{\infty} a_m z^m$ and $g(w) = \sum_{m=0}^{\infty} b_m w^m$ with $a_m = b_m$ for all $m$; $f(g(w)) = w$ near 0. From the fact that $a_m$ and $b_m$ are polynomials in $m$ of degree $n$ we derive the formula $b_m = n(n + 1)(n + 2) \ldots (n + m - 1)\binom{n}{m}$ for all $m \geq 1$. This reduces to E. Jabotinsky's result: $b_m = n(n + 1)(n + 2) \ldots (n + m - 1)\binom{n}{m}$ for all $m \geq 1$. The resulting relation yields a relation between the positive indexed coefficients. For example, if $f(z) = e^z - 1$, then $b_m = m! S_n(m)/n!$ (the new symbols are the Stirling numbers of the second and first kind respectively, and the resulting relation was derived independently by H. W. Gould), and $\pi^{-1}(1) = B_n$, the nth Bernoulli number. (Received December 7, 1960.)


Let $x$ and $u$ be two reals not differing by a rational multiple of $\pi$. For a positive integer $n$, let the integer $r = r(x, u, n)$ be defined by $|x - u - 2\pi n| < |\pi/2|$. We say the measurable function $f$ is translation continuous at $x$ if $\lim_{u \to x} f(u) = f(x)$ for almost every $u$. A function $f$ satisfies condition $C_{a, p}$ if $\int_{-\pi}^{\pi} f(x + u) - f(x) |^p |u|^{-1-a} |du < \infty$, $a > 0$. Theorem (i). If $f$ satisfies $C_{a, p}$ for some $p \geq 1$ and some $a \geq (1/2 - 1/p)$, then $f$ is translation continuous almost everywhere. (ii) For every $a < 1/2$, $p \geq 1$, there is a function $f$ satisfying $C_{a, p}$ but such that $f$ is translation continuous almost nowhere. Applications to trigonometric interpolating polynomials are given. (Received December 5, 1960.)

A constructive, finite difference method is described for the approximate solution of Dirichlet problems associated with elliptic differential equation $a(x,y)u_{xx} + 2b(x,y)u_{xy} + c(x,y)u_{yy} + d(x,y)u_x + e(x,y)u_y + f(x,y)u = g(x,y); f \leq 0, a > 0, c > 0$. Two difference analogues of the differential equation are required and are used as follows: at grid points where $b \geq 0$, apply $(-2a - 2c + 2b + h^2f)u_0 + (a - b + dh/2)u_1 + (c - b + eh/2)u_2 + (a - b - dh/2)u_3 + (c - b - eh/2)u_4 + bu_5 + bu_7 = gh^2$; at grid points where $b < 0$, apply $(-2a - 2c - 2b + h^2f)u_0 + (a + b + dh/2)u_1 + (c + b + eh/2)u_2 + (a + b - dh/2)u_3 + (c + b - eh/2)u_4 - bu_6 - bu_8 = gh^2$. Under the assumption that $|b| < a, |b| < c$ and for appropriately selected $h$, uniqueness and convergence follow in the usual fashion. The method extends to mixed-type problems, n-dimensional problems, and mildly elliptic (in the sense of Bers) problems. (Received December 5, 1960.)


Let $K$ be a p-adic field with residue field $k$ and place $H$. A derivation $D$ on $K$ into $K$ is analytic if it is continuous in the valuation topology. A derivation on $k$ into $k$ is said to be induced by $D$ if it is obtained from $D$ in the usual way by means of the homomorphism $H$. The following results are obtained. Theorem 1. Every derivation on $k$ into $k$ is induced by a (necessarily) analytic derivation on $K$. Corollary. $K$ possesses no non-trivial analytic derivation iff $k$ is perfect. The following application of Theorem 1 is made. Theorem 2. A p-adic field $K$ is uniquely embedded in any p-adic field $K'$ with the same residue field $k$ iff $k$ is perfect. The "if" part of Theorem 2 is, of course, well known. (Received December 12, 1960.)

61T-33. B. E. Hubbard: Eigenvalues of the nonhomogeneous rectangular membrane by finite difference methods.

The eigenvalue problem for the equation $A u(x,y) + \lambda k(x,y)u(x,y) = 0$ in a rectangle $R$ with boundary $C$ is studied. The function $k(x,y)$ is assumed to be positive and, together with its gradient, bounded in $R + C$. Finite difference problems are posed for both the fixed membrane, $u = 0$ on $C$, and the free membrane where $\partial u/\partial n = 0$ on $C$. In each case let $\lambda_n$ and $\lambda_n^*$ represent the $n$th eigenvalue of the membrane and its finite difference analogue respectively. Bounds on the truncation error $|\lambda_n - \lambda_n^*|$ are obtained in terms of properties of $k$ and its first derivatives. If $h$ is the mesh size then the bounds are $O(h^2)$. These results can be applied to obtain eigenvalues for certain regions which can be mapped conformally onto the rectangle. (Received December 8, 1960.)

61T-34. Viktors Linis: On radius of univalence of polynomials.

The radius of univalence of a polynomial $A_n(z)$ of degree $n > 1$ has been given by S. Kakeya [Tohoku Math. J, vol. 11 (1917) pp. 5-16] as $r \sin (\pi/n)$ where $r$ is the smallest modulus of the zeros of $A_n'(z)$. Taking into account the number $m < n/2$ zeros of $A_n'(z)$ in the annulus $1 \leq |z| \leq R$ the radius of univalence can be increased by a factor $a = \csc \left[\pi(n - 2m)/2n(n - m - 1)\right]$. (Received December 12, 1960.)
61T-35. H. J. Keisler: Replete relational systems. 

For terminology see Tarski-Vaught, Compositio Math., (1957). We consider relational systems 
\[ \mathcal{H} = (A, R_t)_{t \in T}, \mathcal{L} = (B, S_t)_{t \in T}. \] 
A subset \( X \subseteq A \) is \( \mathcal{H} \)-definable if there is a formula \( \theta(v) \) with the single 
free variable \( v \) such that \( X = \{ a \in A \mid \theta(a) \text{ holds in } \mathcal{H} \} \). Let \( \alpha, \gamma, \delta, \eta \) denote ordinals. \( \mathcal{H} \) is weakly 
\( \omega \)-replete if for any set \( S \) of \( \mathcal{H} \)-definable subsets of \( A \) such that \( S \) has power \( S < \omega \) and \( X, Y \subseteq S \) implies 
\( 0 \neq X \cap Y \subseteq S \), \( |S| \neq 0 \), \( \mathcal{H} \) is \( \omega \)-replete if \( (A, R_t)_{t \in T}, a \in A \) is weakly \( \omega \)-replete. Thus \( \mathcal{H} \) is always 
\( \omega_0 \)-replete. \( \mathcal{H} \equiv \mathcal{L} \) means \( \mathcal{H}, \mathcal{L} \) are elementarily equivalent, and \( \mathcal{H} \approx \mathcal{L} \) means \( \mathcal{H}, \mathcal{L} \) are isomorphic. 

Theorem 1. Suppose \( T \) is a \( \omega \)-complete ultrafilter on the set \( A \) and \( \omega \nmid A \). Then \( \mathcal{H} \) has an \( \omega_{a+1} \)-replete extension \( \mathcal{H}^a \). 
Theorem 2. Suppose \( \mathcal{H} \equiv \omega \), \( \omega \nmid \mathcal{K} \), and \( \omega \) is strongly inaccessible. Then \( \mathcal{H} \) has an \( \omega \)-replete 
elementary extension of power \( \omega_\alpha. \) 
Theorem 3. Suppose \( \mathcal{H} \equiv \mathcal{L}, \mathcal{L} = \mathcal{K} = \mathcal{L} = \omega \), \( \mathcal{H} \approx \mathcal{L} \), and \( \mathcal{H} \), \( \mathcal{L} \) are \( \omega \)-replete, then 
\( \mathcal{H} \approx \mathcal{L} \). 

Corollary 4. If \( \mathcal{H} \equiv \omega \) and \( \omega \)-replete \( \mathcal{H} \), then there is, up to isomorphism, exactly one 
\( \omega_{a+1} \)-replete system \( \mathcal{K} \) such that \( \mathcal{H} \equiv \mathcal{K} \) and \( \omega \). 

In fact, \( \mathcal{L} \) is exactly the system characterized by 
\( K \) be a set of \( \omega \)-replete systems such that every set of \( \omega \)-elements of \( K \) has a common extension 
in \( K \). Then \( \bigcup K \) is \( \omega \)-replete. (Received December 23, 1960.)


For notation see the preceding abstract and Abstract 550-7, Notices Amer. Math. Soc. vol. 5 (1958) 
p. 673. For any set \( Y \), let \( S(Y) = \{ Z \mid Z \subseteq Y \text{ and } \mathcal{Z} \subseteq \omega \} \). Let \( D \) be a prime ideal in the algebra \( S(X) \) of all subsets of \( X \). Let \( C(\omega, D) \) denote the condition that for every \( \beta < \alpha \) and every monotonic function \( F: S(\omega, D) \to D \) there is a finitely additive function \( G: S(\omega, D) \to D \) such that \( F(s) \subseteq G(s) \) for every \( s \in S(\omega, D) \). Theorem 1. If \( \alpha = 1 \), then \( C(\omega, D) \). 
Theorem 2. If \( D \) is \( \omega \)-complete then \( C(\omega, D) \). 
Theorem 3. If \( C(\omega, D) \) and \( D \) is not countable, then \( C(\omega, D) \). 
Theorem 4. If \( \omega \equiv \omega + 1 \) \( \text{iff } \omega \). 
Theorem 5. Let \( D \) be a prime ideal in \( S(I) \) such that \( C(\omega, D) \) and \( D \) is not countable. Then for any family \( \{ \mathcal{H}_i \} \subseteq I \), the ultraproduct 
\( \prod_{i \in I} \mathcal{H}_i / D \) is \( \omega \)-replete, \( \text{Corollary 6. If } \omega \equiv \omega + 1 \text{ and } \omega \equiv \omega \), then \( \mathcal{H} \) has an \( \omega \)-replete 
ultrafilter \( \mathcal{H}_\alpha^\omega / D \), and thus \( \omega \)-replete \( \mathcal{H}_\alpha^\omega / D \). \( \text{Theorem 7. If } \omega \equiv \omega + 1 \text{ and } \omega \equiv \omega + 1 \text{, then there exists a system } \mathcal{H} \text{ of power } \omega \). 

Corollary 8. \( \omega \equiv \omega + 1 \text{ iff } \omega \). \( \text{Corollary 9. If } \omega \equiv \omega + 1 \text{ and } \omega \equiv \omega + 1 \text{, } D \) is not countable complete. \( \text{Theorem 5. Let } D \) be a prime ideal in \( S(I) \) such that \( C(\omega, D) \) and \( D \) is not countable complete. Then for any family \( \{ \mathcal{H}_i \} \subseteq I \), the ultraproduct 
\( \prod_{i \in I} \mathcal{H}_i / D \) is \( \omega \)-replete, \( \text{Corollary 6. If } \omega \equiv \omega + 1 \text{ and } \omega \equiv \omega \), then \( \mathcal{H} \) has an \( \omega \)-replete 
ultrafilter \( \mathcal{H}_\alpha^\omega / D \), and thus \( \omega \)-replete \( \mathcal{H}_\alpha^\omega / D \). \( \text{Theorem 7. If } \omega \equiv \omega + 1 \text{ and } \omega \equiv \omega + 1 \text{, then there exists a system } \mathcal{H} \text{ of power } \omega \). 

61T-37. H. J. Keisler: Isomorphism of ultraproducts, II. 

For notation see the preceding two abstracts. Let \( P(I) \) be the set of all prime ideals which are not 
countably complete in \( S(I) \). \( K, L \) denote arbitrary classes of similar relational systems. \( \text{Theorem 1. Suppose } D \subseteq P(I), B \subseteq P(J), C(\omega, D), C(\omega, E), \mathcal{H} = \prod_{i \in I} \mathcal{K}_i / D, \mathcal{L} = \prod_{i \in J} \mathcal{L}_i / E, \text{ and } \mathcal{T} \subseteq \omega. \text{ If } \mathcal{X}, \mathcal{Y} \subseteq \omega \text{ and } \mathcal{H} = \mathcal{L} \text{, then } \mathcal{H} = \mathcal{L} \). \( \text{Corollary 2. If } \omega \equiv \omega + 1 \text{ and } \omega \equiv \omega + 1 \), then there exists \( D \subseteq P(\omega) \) such that whenever \( \mathcal{X}, \mathcal{Y} \subseteq \omega \text{ and } \mathcal{H} = \mathcal{L} \text{, then there exists } \mathcal{T} \subseteq \omega \text{ such that } \prod_{i \in I} \mathcal{K}_i / D \text{ is a model of } \mathcal{T}. \text{ Hereafter assume the generalized continuum hypothesis. } \text{Theorem 4. \( } \omega \text{.} \)
The elementary theory of $K$ is consistent with that of $L$ iff for some $I$ and $D \in \mathcal{P}(I)$, $\mathcal{G}'_{1} \subseteq L$, we have $\prod_{i \in I} \mathcal{G}'_{i}/D \cong \prod_{i \in I} \mathcal{G}'_{i}/D$. Theorem 5. $\mathcal{C} = \mathcal{G}'_{1}/D \cong \mathcal{G}'_{2}/D$ for some $I$ and $D \in \mathcal{P}(I)$. Theorem 6. $K \in E_{C}$ iff $K$ is closed under ultraproducts and isomorphisms and $K$ is closed under ultrapowers. Theorem 7. $K \in E_{C}$ iff $K$ and $\overline{K}$ are closed under ultraproducts and isomorphisms. Theorem 8. (Generalization of Craig's Interpolation Theorem). If $K \cap L = 0 \text{ and } K$, $L$ are closed under ultraproducts and isomorphisms, then there exists $M \in E_{C}$ such that $K \subseteq M$ and $L \cap M = 0$. (Received December 23, 1960.)

61T-38. H. J. Keisler: Properties preserved under reduced products. II.

For notation see the preceding three abstracts. Theorem 1. Suppose $\mathcal{A}$, $\mathcal{B}$ are $\omega_{\alpha}$-replete, every positive sentence holding in $\mathcal{A}$ holds in $\mathcal{A}$, $\mathcal{B}$ is $\omega_{\alpha}$, and $\mathcal{B} \subseteq \omega_{\alpha}$. Then $\mathcal{H}$ is a homomorphic image of $\mathcal{A}$. Theorem 2. Suppose $\mathcal{A}^{\omega}_{1} = \omega_{\alpha+1}$, $\mathcal{A}^{\omega}_{2} = \omega_{\alpha}$, $\mathcal{A}^{\omega}_{1}$ are similar systems for each $i \in I$. $\mathcal{A}$ is $\omega_{\alpha+1}$-replete, $\mathcal{A} = \omega_{\alpha+1}$, $\mathcal{B} \subseteq \omega_{\alpha+1}$ for each $i \in I$, and $\mathcal{F} \subseteq \omega_{\alpha}$. Then (i) and (ii) below are equivalent. (i) $\mathcal{H} \subseteq K$ for every Horn class (or elementary class) $K$ such that $\mathcal{H} \in \omega_{\alpha+1}$ and $\mathcal{S} \in \omega_{\alpha+1}$. (ii) For some ideal (or some prime ideal) $D \subseteq \mathcal{P}(I)$, $\mathcal{A}^{\omega}_{1} \subseteq \mathcal{A}^{\omega}_{2}/D$ and $\mathcal{S} \subseteq \mathcal{P}(I)$, $\mathcal{F} \subseteq \mathcal{D}$. Hereafter assume the generalized continuum hypothesis. Theorem 1'. Every positive sentence holding in $\mathcal{A}$ holds in $\mathcal{H}$ iff for some $I$ and $D \in \mathcal{P}(I)$, $\mathcal{A}^{\omega}_{1}/D$ is a homomorphic image of $\mathcal{A}^{\omega}_{2}/D$. Theorem 2'. For any $K$, $\prod_{i \in I} \mathcal{A}$ is a Horn class (or elementary class) and $K$ is closed under reduced products and isomorphisms, and $K$ is closed under ultraproducts (or ultrapowers). Theorem 4. If $K \in E_{C}$ and $K$ is closed under reduced powers and under direct products of two systems, then $K$ is closed under arbitrary reduced products. (Received December 23, 1960.)


The permanent of an $n$-square matrix $A$ is defined as $\sum_{\sigma(i)} \prod_{i=1}^{n} a_{i\sigma(i)}$ where the summation extends over all $n!$ permutations $\sigma$ of $1, \ldots, n$. Let $x_{1}, \ldots, x_{k}$ and $y_{1}, \ldots, y_{k}$ be two sets of $k$ vectors each in an $n$-dimensional unitary space with inner product $(x, y)$. Theorem. (*) $\text{per } (x_{1}, y_{1})^{2} \leq \text{per } (x_{1}, x_{1}) \text{ per } (y_{1}, y_{1})$. If the zero vector occurs in neither set then equality in (*) implies that the subspaces spanned by $x_{1}, \ldots, x_{k}$ and $y_{1}, \ldots, y_{k}$ are the same. If the two sets are linearly independent then equality in (*) implies that except for order and scalar factors the sets are termwise identical. From this a partial answer to a conjecture of van d. Waerden is obtained: if $A$ is an $n$-square doubly stochastic positive semi-definite symmetric matrix then per $(A) \equiv n!/n^{n}$ with equality if and only if $A$ is the matrix all of whose entries are $1/n$. Many other results are obtained, of which the following is typical: if $A$ is an $n$-square positive semi-definite Hermitian matrix then per $(A) \equiv \det (A)$ with equality if and only if $A$ has a zero row or $A$ is a diagonal matrix. Thus $\sum_{\sigma(i)} \prod_{i=1}^{n} a_{i\sigma(i)} \geq 0$ where $\sum'$ denotes the sum over all odd permutations $\sigma$ and equality holds if and only if $A$ has a zero row or $A$ is diagonal. (Received December 5, 1960.)
On a mixed initial-boundary value problem for the wave equation.

The following theorem concerning a classical problem for the wave equation is proved: Let $u$ be a solution of $\Delta u - u_{tt} = 0$ outside a three-dimensional star-shaped body, on which $u = 0$. Suppose $u$ satisfies Cauchy initial data with compact support. Then at a fixed point $(X,Y,Z)$ in space, $u$ decays in time at least as fast as $t^{-1/2}$. The proof consists of first showing that $u^2(X,Y,Z,t)$ is bounded by an energy integral $E_R(t - \tau(X,Y,Z)) = \int_0^t \int (u_x^2 + u_y^2 + u_z^2 + u_t^2) \, dx \, dy \, dz$ and similar $x^2 + y^2 + z^2 \leq R^2$ expressions for the time derivatives of $u$. Here $R$ depends only on $x^2 + y^2 + z^2$.

Using the observation, due to Protter (Journal of Rational Mechanics and Analysis vol. 3 (1954)) that $\int (u_xu_t + u_yu_t + u_zu_t + u_t^2) \, dx \, dy \, dz \, dt$ can be written as a surface integral, one shows secondly that $t \, E_R(t)$, and the corresponding expressions for the time derivatives of $u$, are bounded. Hence $E_R(t)$ decays at least like $t^{-1}$ and $u$ like $t^{-1/2}$.

Error estimates for the solutions of first order ordinary differential equations (using computers).

Let $u(t)$ be the exact solution of the problem $y' = f(t,y), y(0) = u_0$ and suppose that $y_k = u(kh)$ are approximate values. A systematical procedure for computers is developed to estimate the errors of the $y_k$. Using this method in a concrete problem one has to find three functions which describe the behavior of $f(t,y)$ in a certain sense and to write sub-programs for these functions. Then the computer delivers error bounds which take also in consideration the rounding off errors. No knowledge about $u(t)$ is needed. The results are correct for any kind of $y_k$, but good estimates can be expected for approximation methods of order $\leq 4$.

Four examples were computed using a second order method and the Runge-Kutta method of order 4. For the first method (100 steps) the bounds were almost equal to the exact errors; in case of the Runge-Kutta method (200, 30, 80, 100 steps) they exceeded the errors by a factor of 1.5 - 4 depending on the properties of $f(t,y)$.

For a problem with periodic character the bounds also behaved periodically. The error estimates are derived using theorems gained with methods of functional analysis. The numerical results show that these theorems have more than only theoretical interest. (Received December 6, 1960.)

The behaviour of entire functions determined from their Taylor series and a conjecture of Erdos.

Let $f(z) = \sum_0^\infty A_n z^n$ be an entire function, $\mu(r) = \max_n |A_n|^r n$, $U(f) = \limsup_\rho_\to_0 \mu(r,f)/M(r,f)$, $u(f) = \liminf_\rho_\to_\infty \rho^{\mu(r,f)/M(r,f)}$. $L(f) = L = \limsup_\int_0^n |A_n^2/A_{n-1}A_{n+1}|$, $l(f) = 1 = \liminf_\int_0^n |A_n^2/A_{n-1}A_{n+1}|$. P. Erdős conjectured that if $U(f) = u(f)$, then $U(f) = 0$. The following results are proved here. Let $|A_n/A_{n+1}| > \beta$:

(i) If $L = 1$, then $U(f) = 0$. (ii) If $L > 1$, then $U(f) \geq (L - 1)^{1/2}/(L + 1)^{1/2}$. (iii) If $L > 1$, then $u(f) \geq \{2 + 3L^{-1} + (2/(L^3 - 1))\}^{-1}$. (iv) If $n = \infty$, then $u(f) = 1/2$. If $L = \infty$, then $U(f) = 1$. If $f = FG$ and $F,G$ are entire functions satisfying (i) $M(r,F)M(r,G) = O(M(r,f))$, (ii) $M(ar,G)/M(r,G) = O(r^a) \ a > 0$, $A > 1$, (iii) $\mu(r,F)/M(r,F) = O(r^{-3})$ then $U(f) = 0$. If any one of the three conditions holds for a sequence of values $r \to_\infty$, and the remaining two conditions for $r \to_\infty$, then $u(f) = 0$. (Received December 7, 1960.)

Let \( n \) and \( s \) be integers, \( 1 \leq s \leq n \), let \( S \) be a sequence having one of the forms \( (0,1,\ldots, s-1), (n-s+1, n-s,\ldots,n), (0,1,\ldots,k,n-k-1,\ldots,n) \) \([k \geq 0, \ell \geq 0, k + \ell + z = s]\). Let \( S \) be a closed and bounded set in the \( z \)-plane \((0 \not\in S \text{ if } 0 \in \delta)\). Let \( A(z) = \sum_{n=0}^{N} a_n z^n \) be a polynomial having the property: there does not exist a polynomial \( B(z) = \sum_{n=0}^{N} b_n z^n \) with \( b_n = a_n \) for every \( \gamma \in \delta \), such that \(|B(z)| < |A(z)|\) whenever \( z \in S \) and \( A(z) \neq 0 \), \( B(z) = 0 \) whenever \( z \in S \) and \( A(z) = 0 \). Let \( C(z) = \sum_{n=0}^{N} c_n z^n \) \((\forall \gamma \text{ throughout } S, c_{\gamma} \neq 0, r \geq s)\) be a divisor of \( A(z) \). Then \( C(z) \) is a divisor of some \( Q(z) \equiv P(z)g(z) + zK + N - 1 \), the \( z_j \) are (distinct) points of \( S \), \( g(z) = \prod_{\gamma=1}^{N} (z - z_\gamma) \), \( \lambda_\gamma \) are positive reals with \( \sum_{\gamma=1}^{N} \lambda_\gamma = 1 \), \( P(z) \) is a polynomial of degree \( \leq s - 1 \) such that \( P(z)g(z) + zK + N - 1 \) is of degree \( \leq N + s - 2 \), and \( K = \min \left[ \gamma, \nu \notin \delta, \nu = 0,1,2,\ldots \right] \). Applications of this theorem to the study of the geometric location of the zeros of infrapolynomials with some prescribed coefficients will be found in a forthcoming joint paper of Professor J. L. Walsh and the author (The zeros of infrapolynomials with some prescribed coefficients, to appear in J. Analyse Math.) The above result generalizes a previous theorem of M. Fekete [Proc. Nat. Acad. Sci. vol. 37, pp. 95-103, Theorem 1]. (Received December 27, 1960.)


Let \( S \) be a given set and let \( L \) be a given class of real-valued functions on \( S \). It is assumed that \( L \) forms a real vector space and is also closed under the lattice operations \( \max(f,g), \min(f,g) \) and \( \min(f,1) \). A set \( A \) in \( S \) is called integrable if its characteristic function belongs to the Baire functions generated by \( L \). It is shown in this note that the smallest monotone family of subsets of \( S \) which contains all these subsets \( A = \{x: f(x) > \alpha\} \), for every choice \( f \in L \) and for every choice of the positive real constant \( \alpha \), is the family \( \{x: f(x) \geq \alpha\} \). This result makes it easy to deduce some of the basic properties of integrable sets. (Received December 7, 1960.)

61T-45. L. E. Ward, Jr.: A fixed point theorem for monotone transformations on a bush.

Call an hereditarily unicoherent, hereditarily decomposable continuum a bush. It is a theorem of O. H. Hamilton that each homeomorphism of a bush into itself has a fixed point. We prove: Each monotone transformation of a bush into itself has a fixed point. Corollary. Each monotone transformation of (a) an hereditarily decomposable tree-like continuum, or (b) a continuum each of whose subcontinua has a cutpoint, into itself has a fixed point. Now (a) is a special case of a much more difficult problem posed by Bing and (b) has been proposed by G. S. Young for continuous functions. The proof depends on the existence of an upper semi-continuous decomposition of a bush into subcontinua such that the resulting hyperspace is arcwise connected. Then the hyperspace has the fixed point property for upper semi-continuous continuum-valued mappings (see my Abstract 574-12, Notices Amer. Math. Soc. vol. 7 (1960) pp. 737-738). In addition, the partial order structure of the hyperspace induces a quasi order-theoretic characterization of bushes. (Received December 19, 1960.)
61T-46. N. A. Wiegmann: Necessary and sufficient conditions for unitary similarity.

Necessary and sufficient conditions that two complex matrices be unitarily similar have been developed from several points of view. It is shown that the unitary similarity of matrices with real quaternion elements with \( n \) rows and columns depends on the unitary similarity of matrices with complex elements of \( 2n \) rows and columns. Some known results for the complex case are then applied to this result and to the quaternion case. In addition the unitary similarity of sets of complex matrices and sets of quaternion matrices is considered where the matrices are general matrices as above and the sets are not necessarily finite. (Received December 29, 1960.)


The method of finite differences is applied to obtain a numerical solution to the Dirichlet problem for the second order elliptic partial differential equation \( u_{xx} + u_{yy} + (K/y)u_y = 0 \) in a bounded, simply connected region \( R \). \( R \) and its boundary \( B \) satisfy \( B = S_1 \cup S_2 \) and \( R \subset S_1 \times \{ y; y > 0 \} \), where \( S_1 \) is an interval on the \( x \)-axis and \( S_2 \) is piecewise analytic. The difference analogues used are of the form: \( \Delta^4 [u](P_i) \sum_{i=1}^{4} a_{ij} u(P_{ij}) \) where the \( P_{ij} \) are the four neighbors of \( P_i \). The \( a_{ij} \) are determined as follows. Let \( n_0 = \) integral part of max. \( [K/2,1] \), \( P_i = (mh, nh) \) \( n \equiv 1 \). Then for \( n > n_0 \) and all \( K \): \( a_{11} = a_{13} = 1, a_{12} = 1 + K/2n, a_{14} = 1 - K/2n \), \( n \equiv n_0 \) and \( K \equiv -1 \): \( a_{11} = a_{13} = 6/K + 4, a_{12} = [2(K+1)/(K+4)](1 + 1/n), a_{14} = [2(K+1)/(K+4)](1 - 1/n) \), \( n \equiv n_0 \) and \( K < -1 \): \( a_{11} = a_{13} = 1/K, a_{12} = (2n - 1)(2K + 1) + K/(2nK), a_{14} = 2(n + 1)(2K + 1) - K/(2nK) \).

Uniqueness of the numerical solution and convergence to an analytical solution of class \( C^2(R) \) (provided one exists) is proved. Moreover, consequences of the special difference methods described above are: Theorem: For \( K = -1 \), if \( u \) vanishes on \( S_2 \), is a solution of the differential equation on \( R \) and belongs to \( C^2(R) \), then \( u = 0 \) on \( R \); Corollary: For \( K = -1 \) one cannot solve the Dirichlet problem in general with arbitrary continuous and bounded boundary values if the solution is to be in \( C^2(R) \). (Received December 30, 1960.)


Let \( X \) be a locally compact, locally connected, Hausdorff space; let \( G(X) \) be its group of homeomorphisms, provided with the compact-open topology, and let \( G_0(X) \) be the arc-component of 1 \( \in G(X) \). The homeotopy group of \( X \) is defined to be \( \pi(X) = G(X)/G_0(X) \); it is shown to be a topological invariant which is not isotopy invariant, and that it is independent of the dimension and the homotopy groups. If \( A \subset X \) let \( G(X,A) = \{ g \subset G(X) : g|A \subset G(A) \} \) and let \( G_0(X,A) \) be the arc-component of 1 \( \in G(X,A) \). The homeotopy group of \( (X,A) \) is defined to be \( \pi(X,A) = G(X,A)/G_0(X,A) \). Fiber bundle methods are used to prove theorems relating the homeotopy groups of manifolds got by deletion of finite point sets from compact manifolds. The following are particularizations of the main results: Theorem 1: Let \( X \) be a compact, connected, simply connected, triangulable manifold of dimension at least 3; let \( A \) be a finite set of \( n \) points of \( X \). Then \( \pi(X,A) \cong \pi(X - A) \cong S_n \times \pi(X) \) where \( S_n \) denotes the symmetric group on \( n \) marks. Theorem 2: If \( X \) is a manifold and \( x \in X \), then there exists a central subgroup \( P \) of \( \pi_1(X, x) \) and appropriate homomorphisms such that the following sequence is exact: \( 0 \rightarrow P \rightarrow \pi_1(X, x) \rightarrow \pi(X, x) \rightarrow \pi(X) \rightarrow 0 \). (Received December 30, 1960.)
We consider similar systems $\mathcal{H} = \langle \Lambda , R_\Lambda , \equiv \rangle$ where $\Lambda \neq \emptyset$, $= (x,y) = \delta_{xy}$, $R_\Lambda \subseteq \{0,1\}^\Lambda$ $\forall t \in T$. $\mathcal{L} = \langle B, S_\mathcal{L}, \equiv \rangle$ is a subsystem of $\mathcal{H}$ if $B \subseteq A$, $S_\mathcal{L} = R_\Lambda \cap B^{\mathcal{L}(t)}$ for $t \in T$. $\mathcal{H}, \mathcal{L}$ are isomorphic, $\mathcal{H} \cong \mathcal{L}$, if there is a 1-1 function $f$ on $A$ onto $B$ such that for $t \in T$, $x \in A^{\mathcal{L}(t)}$, $R_\Lambda (x) = S_\mathcal{L}(f(x))$. $\mathcal{H}, \mathcal{L}$ are homomorphic, $\mathcal{H} \supseteq \mathcal{L}$, if there is a function $f$ on $A$ onto $B$ such that for $t \in T$, $x \in A^{\mathcal{L}(t)}$, $R_\Lambda (x) \subseteq S_\mathcal{L}(f(x))$. For a system $\mathcal{H}$, a sentence $\phi$, and a set of sentences $\Sigma$, let $V(\phi, \mathcal{H})$ be the value of $\phi$ on $\mathcal{H}$, $\mathcal{H} = \{ \phi; V(\phi, \mathcal{H}) = 1 \}$, $\phi^* = \{ \phi; V(\phi, \mathcal{H}) = 1 \}$ for each $\phi \in \Sigma$. $K \subseteq \Sigma$ if $K = \sum^*$ for some $\Sigma$. The class of similar 2-valued systems is in $EC_A$. A set $\mathcal{H}$ is weakly $\omega_1$-compact (replete) if $\mathcal{H}$ is a subsystem of $\mathcal{H}$ and for each formula $\phi(x_1, \ldots, x_k)$, elements $a_1, \ldots, a_k \in A$, $V(\phi(a_1, \ldots, a_k), \mathcal{H}) = 1$ iff $V(\phi(a_1, \ldots, a_k), \mathcal{L}) = 1$. A system $\mathcal{H}$ is closed if for each formula $\phi(x_1, \ldots, x_k, y)$, elements $a_1, \ldots, a_k \in A$, there is $b \in A$ such that $V(\exists y \phi(a_1, \ldots, a_k, y), \mathcal{H}) = V(\phi(a_1, \ldots, a_k, b), \mathcal{L})$. A set $X \subseteq A$ is a formula $\phi$ with one free variable $x$ such that $X = \{ a; \phi \in \mathcal{L} \}$ and $V(\phi(a), \mathcal{H}) = 1$. $\mathcal{H}$ is weakly $\omega_1$-compact if the system $\langle A, R_\Lambda, \equiv, a \rangle \subseteq T, a \notin A$ is weakly $\omega_1$-compact. (Received January 3, 1961.)

Let $P^*$ be a maximal ideal in the MV-algebra $[0,1]^I$. We identify $[0,1]^I$ with $[0,1]$. Let $P^* = \{ \{X; \forall f \in P, X = \{ i; f(i) = 0 \}\} \}$. $P^*$ is a maximal ideal in $[0,1]^I$ iff $P$ is a maximal ideal in $[0,1]$. $P$ is countably incomplete if $P^*$ is countably incomplete. Following Keisler, we say the condition $C(\alpha, P)$ holds if for each $\beta < \alpha$ and each monotonic function $F: S_\alpha(\alpha_\beta) \rightarrow P^*$ there is a finitely additive function $G: S_\alpha(\alpha_\beta) \rightarrow P^*$ such that $F(s) \subset G(s)$ for each $s \in D(F)$. The ultraproduct $\prod_{i \in I} A_i / P = \langle A, R_A, \equiv \rangle \iota \in T$ of a family of systems $A_i, i \in I$, is defined as follows: for $f, g \in \prod_{i \in I} A_i$, $i \equiv \langle i; f(i), g(i) \rangle / P$; $f / \equiv = \{ g; i \equiv \langle i; f(i), g(i) \rangle / P \}$; $A = \{ f / \equiv; f \in \prod_{i \in I} A_i \}$; for $t \in T$, $R_A(f_1 / \equiv, \ldots, f_k / \equiv) = \{ i; R_{A_i}(f_1(i), \ldots, f_k(i)) / P \}$. We define the ultra-power $A^P / P = \prod_{i \in I} A_i / P$ if $A = A_i, i \in I$. A formula (formula of propositions) is positive if it is built up from the atomic formulas (propositional variables) with $+, \cdot, \wedge, \vee, \exists, \forall$ (with $+, \cdot, \vee, \Lambda$). Theorem 1. If $\mathcal{A} = B$, $V(\phi, \mathcal{A}) \leq r \leq V(\phi, \mathcal{B})$, then there is a $C$ such that $\mathcal{A} = V(\phi, \mathcal{C}) = r$. Theorem 2. For each rational $r$, $0 < r < 1$, there are positive formulas $\xi_1(p)$ and $\xi_2(p)$ with one propositional variable $p$ such that for each system $\mathcal{A}$, formula $\phi(x_1, \ldots, x_k, y)$, elements $a_1, \ldots, a_k \in A$, $V(\xi_1(\phi(a_1, \ldots, a_k), \mathcal{A}) \equiv r$ and $V(\xi_2(\phi(a_1, \ldots, a_k), \mathcal{A}) \equiv 0$ iff $V(\phi(a_1, \ldots, a_k), \mathcal{A}) \equiv r$. Theorem 3. $A^* = B^*$ iff for each sentence $\phi, V(\phi, \mathcal{A}) = V(\phi, \mathcal{B})$. (Received January 3, 1961.)

Theorem 4. Let $X$ be a closed subset of $[0,1]$, $\Sigma$ a set of sentences, $K = \{ \mathcal{H}; V(\phi, \mathcal{H}) \subseteq X \}$ for each $\phi \in \Sigma$. Then $K \subseteq EC_A$. Theorem 5. $\mathcal{K} \subseteq EC$ if there is a $\phi$ assuming only the values 0 and 1, $K = \phi^*$, $\mathcal{K} = (\neg \phi)^*$. Theorem 6. $\mathcal{H} \subseteq \mathcal{H}/P$. Theorem 7. For each $\mathcal{H}$, there is a closed system $\mathcal{L}$ such that $\mathcal{H}/P \subseteq \mathcal{L}$. Theorem 8. (Lowenheim-Skolem) Let $\max(\omega_1) \leq \beta < \alpha$, $C \subseteq A$, $C \subseteq \subseteq$. There is a system $\mathcal{A}$, $C \subseteq B$, $\mathcal{A} \subseteq \mathcal{L}$. Theorem 9. (Compactness) Let $\Sigma$ be a set of sentences. If for each $\sigma \in S_\alpha(\Sigma)$, $\mathcal{A}^* \subseteq \sigma^*$, then there is an ultra-product $\prod_{\sigma \in S_\alpha(\Sigma)} \mathcal{A}^*/P \subset \sigma^*$. Theorem 10. Let $P$ be a countably incomplete...
plete maximal ideal in $[0,1]^T$ and $C([0,1]^T, P)$. Then for any family $\mathcal{C}_i$, $i \in I$, the ultraproduct $\prod_{i \in I} \mathcal{C}_i / P$ is $\omega_a$-compact. Theorem 11. If $\omega_{a+1} = \omega_a$, then any two equivalent $\omega_a$-compact systems of power $\omega_a$ are isomorphic.

Theorem 12. If $\omega_{a+1} = \omega_a$, then any two equivalent $\omega_a$-compact systems of power $\omega_a$ are isomorphic. Theorem 13. If $\mathcal{C}_1 \subseteq \omega_{a+1}$, then (i) $\mathcal{C}_1^* = \mathcal{C}_2^* \iff \mathcal{C}_1^{-a}/P \sim \mathcal{C}_2^{-a}/P$ for some $P$, (ii) every positive sentence having value 1 on $\mathcal{C}_1$ has value 1 on $\mathcal{C}_2^{-a}/P \mathcal{C}_2^{-a}/P$ for some $P$. Hereafter assume the generalized continuum hypothesis. Theorem 14. $K \subseteq EC_a$ iff $K$ closed under isomorphisms, ultraproducts, $\mathcal{C}_1$ closed under ultrapowers. Theorem 15. If $K \subseteq EC_a$ iff $K$ closed under isomorphisms, ultraproducts. Theorem 16. $K = \Sigma^*$, $\Sigma$ a set of positive sentences, iff $K$ closed under homomorphisms, ultraproducts, $\mathcal{C}_1$ closed under ultrapowers. (Received January 3, 1961.)


For positive integers $m, n (m \leq n)$ let $E_i(m, n), i = 1, 2, 3$, denote the number of positive integral solutions of $m = x_1 + y_1, n = x_2 + y_2$, with $(x_1, x_2) = 1$, corresponding, respectively, to the following sets of conditions: (1) $y_1$ prime, $y_2$ square-free, (2) $y_1, y_2$ square-free, (3) $y_1, y_2$ prime. Asymptotic formulas for $E_i(m, n)$ are obtained in each of the three cases. As a corollary, it is shown that $E_1(m, n) > 0$, provided that $m$ is sufficiently large. A similar result is proved for $E_3(m, n)$, subject to certain conditions relating $m$ and $n$. The proofs are based on known results concerning the distribution of primes and square-free integers in arithmetical progressions. (Received January 3, 1961.)


This abstract summarizes incomplete joint work by the author and the late J. von Neumann. Let $G$ be a symmetric simple $n$-person game in which the minimal winning coalitions are those containing precisely $n - 1$ players. Let $V$ be a solution to $G$ which is symmetric in all the players except one, say player $n$ (such solutions exist). Let $A$ be the projection of $V$ on the $x_n$-axis, i.e., the set of all payoffs to the exceptional player for imputations in $V$; $A$ is compact, so its complement is open. We use the $(-1, 0)$ normalization. Theorem 1. The minimum of $A$ is $< 1/(n - 1)$. Theorem 2. If $(b, a)$ is a bounded component of the complement of $A$, then $a \leq (b + 1)(n - 2)/(n - 1)$. Furthermore, the estimates in these theorems are optimum. Proofs of the theorems will be given in RAND paper P-2169. (Received January 3, 1961.)

61T-54. R. P. Gilbert: On the singularities of harmonic functions in $(p + 2)$ dimensions. I.

The author develops two integral operators, which map functions of $(p + 1)$ complex variables onto the harmonic functions in $(p + 2)$-variables. The first operator $B_N(F; \mathcal{L}_1^p, \mathcal{L}_2^p, \ldots, \mathcal{L}_p^p; X^0)$ is a direct extension of the Whittaker-Bergman operator. This operator generates $(2n + 1)^p$ harmonic polynomials of degree $n$ (whereas the number of linearly independent harmonic polynomials of degree $n$ of $(p + 2)$-variables is only $(2n + p)(n + p - 1)!/p!n!$), hence if one attempted to invert this operator one would have to choose a linearly independent set of the harmonic polynomials of degree $n$. The operator $B_N(F; \mathcal{L}_1^p, \mathcal{L}_2^p, \ldots, \mathcal{L}_p^p; X^0)$, which is constructed so as to map functions of $(p + 1)$ complex variables onto homogeneous, linearly independent, harmonic polynomials does not have this difficulty and we are able to obtain an inverse integral operator. (Received December 30, 1960.)
In this paper the author extends his method of the envelope for obtaining singularity manifolds of harmonic functions etc. in three variables, (R. Gilbert, Pacific J. Math., Crelle's J. and Notices Amer. Math. Soc., about January 1961) to the case of harmonic functions in \( n \)-variables. Theorems are stated for both the operators \( B_N(F; \omega; X_0) \) and \( B_N^*(F; \omega; X_0) \). For instance, we prove the Theorem: If \( Z(x; u_1, u_2, \ldots, u_p) = 0 \) is the singularity manifold of \((1/u_1 u_2, \ldots, u_p) F(r; u_1, \ldots, u_p)\), then the \( N \)-dimensional harmonic function \( H(x) \equiv B_N(F; \omega; X_0) \) is regular at the point \( X \) providing \( X \) does not lie simultaneously on the surfaces

\[
S(x; u_1, u_2, \ldots, u_p) = 0 \quad \text{and} \quad \frac{\partial S}{\partial u_1} + \frac{\partial S}{\partial u_2} \frac{\partial \omega}{\partial u_1} + \frac{}{} + \frac{\partial S}{\partial u_p} \frac{\partial \omega}{\partial u_p} = 0,
\]

where the \( u_k = \phi_k(u_1), (k = 2, 3, \ldots, p) \) are \((k - 1)\) arbitrary functions of \( u_1 \). Several corollaries to this theorem are immediate. (Received December 30, 1960.)

If one considers just those solutions of the \( n \)-dimensional Laplace equation, which depend on the variables \( z = x_1, \rho = (x_2^2 + x_3^2 + \ldots + x_n^2)^{1/2} \), certain interesting results may be obtained connecting their singularities with those of analytic functions. For instance, we may prove the Theorem: Let \( u(r, \theta) = A_n (r, \omega; P_0) \equiv \int_0^{r+1} f(\sigma)(u - 1/\sigma)^{-1} d\sigma/u_1 \), (where \( \sigma = z + i/2 \rho(1 + ru), \| P - P_0 \| \in \epsilon, P \equiv (r, \theta), \rho^0 \equiv (r^0, \theta^0), (r^2 + \rho^2), \theta = \cos^{-1} z/r, \epsilon \in 0 \) is sufficiently small, and the integration path is the upper semi-circular arc) be a generalized axially symmetric potential. Furthermore, let us continue the arguments of \( u(r, \theta) \) to complex values, such that \( r = +z^2 + \rho^2)^{1/2}, \xi = z/r, \and \, u(r, \theta) = w(r, \xi) \) when \( z, \rho \) are real. Then, \( W(r, \xi) \) is singular if and only if the point \((r, \xi) \) lies on the locus \( r^2 - 2ar\xi + a^2 = 0 \) (here \( \sigma = a \) is a singularity of \( f(\sigma) \)), the one exception being the points on the \( z \)-axis \((r, \pm 1)\), which may be singular points of \( W(r, \xi) \) without corresponding to singularities of \( f(\sigma) \). These results will appear under the title Singularities of GASP (Arch. Rat. Mech. Anal., vol. 6, pp. 171-176). (Received December 30, 1960.)

Consider the collection of elements \( [x_a; a \in A] \) in a normed linear space \( X \), and a collection of numbers \( [\xi_a; a \in A] \). If \( \sum_{\sigma} \tau_a x_a \| \leq M \| \sum_{\sigma} \tau_a x_a \| \) holds for every finite subset \( \sigma \) of \( A \) and every choice of numbers \( [\xi_a] \), then we say that Helly-Riesz-Hahn property holds in \( [x_a] \) and \( [\xi_a] \). By considering the Hilbert space over a division ring (of complex numbers), Helly-Riesz-Hahn property assumes a special form. John von Neumann proved that a ring of bounded operators in Hilbert space is regular if and only if it possesses a finite basis. From our special form of Helly-Riesz-Hahn property, we establish a new equivalent condition for a ring of bounded operators in Hilbert space to be regular. (Received December 12, 1960.)
MATHEMATICIANS
PROGRAMMERS

Inquire about Opportunities with

LOCKHEED
in
GEORGIA
located near beautiful Atlanta

Enjoy pleasant year-round climate and the highest professional environment

B. Sc. degree and experience programming scientific and engineering problems on 7090, 709 or 704 computer required. Intermediate and senior level openings available. Also, excellent opportunity for Programmer experienced with APT System for numerically controlled milling machines.

Write now to: Dr. E. K. Ritter, Manager
Mathematical Analysis Dept.

LOCKHEED AIRCRAFT CORP.
834 WEST PEACHTREE ST. • ATLANTA, 8 GA.
planned through advanced communication techniques at Amherst Laboratories

Signal transmission can be protected and hidden from interception by using natural or man-made disturbances. Locating such a signal in intelligible form at a designated point has been accomplished by scientists at Amherst Laboratories.

These and other accomplishments are a result of long-range experimental investigation currently in progress. We are concerned with wave propagation and properties of the propagation medium, as applied to advanced Ground, Air and Space Communications.

PROFESSIONAL STAFF AND MANAGEMENT OPPORTUNITIES are unlimited for Physicists, Mathematicians and Electronic Engineers with advanced degrees and creative desire.

You are invited to direct inquiries in confidence to Dr. R. Malm, Amherst Laboratories... or call NF 3-8315 for information. All inquiries will be acknowledged promptly.
Proceedings of Symposia in Pure Mathematics — Volume 3

DIFFERENTIAL GEOMETRY

The Symposium on Differential Geometry was organized as a focal point for the discussion of new trends in research. Modern differential geometry has become to a large degree differential topology, and the methods employed are a far cry from the tensor analysis of the differential geometry of the 1930's.

The papers in this volume give a cross-section of many of the types of differential geometry of major current interest: differential topology, Lie groups, complex manifolds, fiber bundles, and differential geometry in the large.

Table of Contents

A Report on the Unitary Group ... By Raoul Bott
Vector Bundles and Homogeneous Spaces .......... By M. F. Atiyah and F. Hirzebruch
A Procedure for Killing Homotopy Groups of Differentiable Manifolds .......... By John Milnor
Some Remarks on Homological Analysis and Structures .. . . . By D. C. Spencer
Vector Form Methods and Deformations of Complex Structures .......... By Albert Nijenhuis
Almost-Product Structures .. . . . . . . . . . . . . . By A. G. Walker
Homology of Principal Bundles ................ By Eldon Dyer and R. K. Lashof
Alexander-Pontryagin Duality in Function Spaces .... By James Eells, Jr.
The Cohomology of Lie Rings .......... By Richard S. Palais
On the Theory of Solvmanifolds and Generalization with Applications to Differential Geometry . . . . . . By Louis Auslander
Homogeneous Complex Contact Manifolds .. . . . . . By William M. Boothby
On Compact, Riemannian Manifolds with Constant Curvature. I . . By Eugenio Calabi
Elementary Remarks on Surfaces with Curvature of Fixed Sign . . . By L. Nirenberg
Canonical Forms on Frame Bundles of Higher Order Contact .. By Shosichi Kobayashi
On Immersion of Manifolds . . By Hans Samelson

Edited by Carl B. Allendoerfer

25% discount to members

200 Pages $7.50 approx.

American Mathematical Society
190 Hope Street
Providence 6, Rhode Island

OUTSTANDING ASSIGNMENTS for OUTSTANDING SCIENTISTS at the OPERATIONS EVALUATION GROUP of M. I. T.

Seeking scientists who have the ability and imagination to apply their broad knowledge with originality in the field of research, the Operations Evaluation Group of the Massachusetts Institute of Technology offers stimulating career opportunities to scientists with advanced degrees in mathematics and the physical sciences.

For over 18 years, OEG, has served as advisor to the Office of Chief of Naval Operations and the operating fleet. Engaged in both conventional operations research and in the solution of complex problems far out of the realm of the ordinary, OEG has the responsibility for conducting research that cuts laterally across many scientific disciplines.

If you have the interest and the creative ability to apply your basic research findings to the solution of problems that are vital to the Navy and the national security, you are invited to write to OEG. Working in a professional atmosphere and exchanging stimulating ideas with colleagues of the same discipline, you will find a rewarding opportunity for increased scientific stature and personal growth.

OPERATIONS EVALUATION GROUP

An Activity of the Massachusetts Institute of Technology
Department E
Washington 25, D. C.

Physicists • Physical Chemists
Mathematicians • Economists
Electronics Engineers
for the Mathematician

SPACE TECHNOLOGY
OFFERS WORK OF THE FUTURE...TODAY

Applied Mathematicians, Numerical Analysts

and Scientific Programmers

Challenging new problems in the areas of aerodynamics, supersonic flow, shock theory, elasticity, and information theory require individuals with superior qualifications in the fields of Applied Mathematics, Numerical Analysis, and Scientific Programming.

The STL Computation and Data Reduction Center is one of the largest and most advanced computation facilities in the nation. CDRC personnel daily utilize two IBM 7090’s and sophisticated data reduction systems in support of the Air Force Ballistic Missile Program and space flight studies. Resumes are invited.

SPACE TECHNOLOGY LABORATORIES, INC.
P. O. BOX 95005, LOS ANGELES 45, CALIFORNIA
EXPERIENCED

Applied Mathematicians

You are invited to consider positions now available in a new applied mathematics group being formed within General Electric's Heavy Military Electronics Department. The Department's activities encompass design and manufacture of land-based and seaborne military electronics equipment including: radar, sonar, data processors, communication systems and guidance equipment.

Areas for mathematical investigation include:

- Antennae
- Boolean Algebra
- Probability
- General Numerical Analysis
- Management Sciences

An IBM 7090 and programming services are available for problems requiring machine solution.

Write in full confidence to
Mr. W. J. Eschenfelder,
Dept. 123-MB.

HEAVY MILITARY ELECTRONICS DEPT.
GENERAL ELECTRIC
Court Street, Syracuse, New York

PSAM Vol. XII

PROCEEDINGS OF THE SYMPOSIUM
ON THE STRUCTURE OF LANGUAGE
AND ITS MATHEMATICAL ASPECTS

The twenty articles in this book are texts of addresses which were delivered at the symposium held in April, 1960.

The authors contributing papers to this book are: W. V. Quine; Noam Chomsky; Hilary Putnam; H. Hiz; Nelson Goodman; Haskell B. Curry; Yuen Ren Chao; Murray Eden; Morris Halle; Robert Abernathy; Hans G. Herzberger; Anthony G. Oettinger; Victor H. Yngve; Gordon E. Peterson and Frank Harary; Joachim Lambek; H. A. Gleason, Jr.; Benoit Mandelbrot; Charles F. Hockett; Rulon Wells; Roman Jakobson.

285 pp. approx. $6.60 approx.
25% discount to members

AMERICAN MATHEMATICAL SOCIETY
190 Hope Street, Providence 6, Rhode Island

RUSSIAN TRANSLATION PROJECT

SERIES II VOLUME XVI

35 Scientific communications and 1 article from the All-Union Conference on Functional Analysis and its Applications (Jan. 17-24, 1956) and 4 papers on Analysis, 1960, ii.

25% Discount to members

484 pages $5.40

American Mathematical Society
190 Hope Street
Providence 6, Rhode Island
BARNES & NOBLE COLLEGE OUTLINES and EVERYDAY HANDBOOKS remain landmarks in educational paperback publishing. For thirty years these well-organized teaching and study aids have been recommended by educators for use in college and in advanced high school classes, as well as in industrial training courses. The following titles are highly valued in the field of mathematics. ■ ALGEBRA, by PROF. GERALD E. MOORE, University of Illinois, offers all topics studied in a first college course; special chapters on problem solving plus exams and answers. ($1.50)

■ ALGEBRA PROBLEMS, by DR. DONALD S. RUSSELL, Ventura College, is in large format, and includes 800 solved problems covering the normal topics found in a course in Intermediate Algebra. Considerable theory is included with introduction of new topics, and, as need arises during the explanation of steps in the problems. ($1.75)

■ THE CALCULUS, by C. O. OAKLEY, Head of Mathematics Dept. Haverford College, summarizes fundamentals; exercises, exams, answers, tables of integrals. ($1.50)

■ ANALYTIC GEOMETRY, by C. O. OAKLEY, gives the student a brief but comprehensive introduction and review. ($1.50)

■ ANALYTIC GEOMETRY PROBLEMS, by C. O. OAKLEY, presents a review of basic principles combined with 341 problems with solutions; 345 problems with answers. ($1.95)

■ COLLEGE GEOMETRY, by NATHAN ALTSCHILLLER COURT, Prof. Emeritus of Math, University of Oklahoma, has long been a classic in its subject—an introduction to modern geometry of the triangle and the circle. Mathematics Teacher said of this book that it “will be a prized possession of the serious teacher of mathematics.” ($1.95)

■ GEOMETRY THROUGH PRACTICAL APPLICATION, by PROF. JULIO A. MIRA, Manhattanville College, will give a popular presentation of Euclidean geometry, both plane and solid, with many practical applications, problems, and solutions. Ready summer 1961 ($1.50)

■ PLANE GEOMETRY PROBLEMS WITH SOLUTIONS, by MARCUS HORBLIT and PROF. KAJ L. NIELSEN. An excellent compilation of source material for students and teachers; useful as a “brush-up” by draftsmen and designers. ($1.25)

■ LOGARITHMIC AND TRIGONOMETRIC TABLES, by KAJ L. NIELSEN, Missiles Research Analyst, Allison Division, General Motors. This convenient set of tables, to five places, also provides adequate instructions and numerous illustrative examples for their use. ($1.00)

■ COLLEGE MATHEMATICS, by KAJ L. NIELSEN. Basic Course in essentials of algebra, trigonometry, analytic geometry, calculus, with exercises, examinations, answers. ($1.75)

■ SLIDE RULE AND HOW TO USE IT, by CALVIN C. BISHOP, Retired Senior Designing Engineer, California Institute of Technology. This practical guide with complete instructions, examples, problems, and answers “contains a thoroughness which is rare for a book of its size.”—The Mathematics Teacher. ($1.25)

■ PLANE AND SPHERICAL TRIGONOMETRY, by KAJ L. NIELSEN and JOHN H. VAN LONKHUYZEN. Two practising technicians have collaborated to give a brief but comprehensive review of the subject with examinations and answers. Five-place tables. ($1.75)

We would be pleased to send free examination copies of books on the subjects which you teach. Please give the names of the courses for which you may be considering the books you request.

BARNES & NOBLE
105 FIFTH AVENUE, NEW YORK 3, N.Y.
PROCEEDINGS OF SYMPOSIA IN
APPLIED MATHEMATICS

These symposia were held under the auspices of the American Mathematical Society and other interested organizations. The Society itself published the first two volumes. The McGraw-Hill Book Company, Inc., published and sold Numbers 3 through 8. These six volumes have now been transferred to the American Mathematical Society by special arrangement with the McGraw-Hill Book Company, Inc. Orders should be placed through the Society.

Members of the Society are entitled to the usual 25% discount on all the volumes.

**Volume 1**
NON-LINEAR PROBLEMS IN MECHANICS OF CONTINUA, 1949. vii + 219 pp. $5.25

**Volume 2**
ELECTROMAGNETIC THEORY, 1950. iii + 91 pp. $3.60

**Volume 3**
ELASTICITY, 1950. vi + 233 pp. $6.00

**Volume 4**
FLUID DYNAMICS, 1953. vi + 186 pp. $7.00

**Volume 5**
WAVE MOTION AND VIBRATION THEORY, 1954. vi + 169 pp. $7.00

**Volume 6**
NUMERICAL ANALYSIS, 1956. vi + 303 pp. $9.75

**Volume 7**
APPLIED PROBABILITY, 1957. v + 104 pp. $5.00

**Volume 8**
CALCULUS OF VARIATIONS AND ITS APPLICATIONS, 1958. v + 153 pp. $7.50

**Volume 9**
ORBIT THEORY, 1959. v + 195 pp. $7.20

**Volume 10**
COMBINATORIAL ANALYSIS, 1960. vii + 311 pp. $7.70

**Volume 11**
NUCLEAR REACTOR THEORY (In Preparation)

**Volume 12**
THE STRUCTURE OF LANGUAGE AND ITS MATHEMATICAL ASPECTS (In Preparation)

Order from

AMERICAN MATHEMATICAL SOCIETY
190 Hope Street, Providence 6, Rhode Island
new textbooks

THEORY OF FUNCTIONS OF A REAL VARIABLE, PRELIMINARY ED.
Edwin Hewitt, University of Washington
A first year, graduate level, real variables text which offers a thorough introduction to concrete functional analysis. A knowledge of advanced calculus is presupposed. Professor Hewitt deals with Lebesgue measure and integrals, other measures on the line, Hilbert space, and a variety of function spaces. This text is intended for courses in mathematics and statistics; theory of functions of a real variable; measure theory; advanced probability theory. (1960, 334 pp., $4.00 paper)

Ready in Spring 1961

BASIC ANALYSIS
Stephen P. Hoffman, Jr., Trinity College, Conn.
This two- or three-semester calculus with analytics is intended primarily for use by liberal arts students. The calculus of polynomial functions is done without using limit or derivative theorems for anything other than sums or powers. Professor Hoffman’s intention is to show the structure of theory. Functions are discussed in terms of ordered pairs, variables, and domain-range rule. (1961, 604 pp., $8.50 tentative)

CONVEX FIGURES
I. M. Yaglom and V. G. Boltyanskiĭ
Translated by Paul J. Kelly and Lewis F. Walton, both of the University of California, Santa Barbara
This collection of geometry problems about plane convex figures is now available in English for the first time. The material is highly intuitive. Though the concepts and methods used are elementary, the material is not now covered in the standard curriculum of either high schools or colleges, and many of the results are of mathematical depth. (1961, 320 pp., $6.00 tentative)

CURRENT ISSUES IN THE PHILOSOPHY OF SCIENCE
Edited by Herbert Feigl and Grover Maxwell, University of Minnesota
Covers the proceedings of the section on History and Philosophy of Science of the 1959 AAAS national meeting. (1961, 512 pp., $6.50 tentative)

Holt, Rinehart and Winston, Inc.
383 Madison Avenue, New York 17, New York
Announcing a new publication

PROCEEDINGS OF SYMPOSIA IN PURE MATHEMATICS, VOLUME II

LATTICE THEORY

The eighteen articles in this volume are the papers presented at the Symposium on Partially Ordered Sets and Lattice Theory held in April, 1959.

The authors contributing papers to this book are:

R. P. Dilworth
P. M. Whitman
Juris Hartmanis
R. A. Dean
C. C. Chang and Alfred Horn
Israel Halperin
B. Jonsson
K. D. Fryer
J. E. McLaughlin
Leon Henkin and Alfred Tarski
P. R. Halmos
C. C. Chang
R. S. Pierce
Philip Dwinger
Garrett Birkhoff
Marshall Hall, Jr.
L. W. Anderson
F. W. Anderson

202 pages

25% discount to members approx. $6.00

AMERICAN MATHEMATICAL SOCIETY

190 Hope Street, Providence 6, Rhode Island

NEW YORK, NEW YORK MEETING AT THE HOTEL NEW YORKER
April 5-8, 1961

RESERVATION FORM

The NEW YORKER is the official headquarters hotel. Your room reservations should be sent directly to Hotel New Yorker, Reservation Department, 34th Street and 8th Avenue, New York 1, New York. PLEASE TEAR OFF THE ATTACHED COUPON AND USE AS YOUR ROOM RESERVATION BLANK. Be sure to specify the names of persons for whom reservations are being made. BE SURE TO BRING THE CONFIRMATION SLIP WITH YOU AS PROOF OF YOUR RESERVATION. Unless otherwise requested, the hotel will hold reservations only to 6:00 P.M. of the day or your arrival. Check-out time is 3:00 P.M.

HOTEL NEW YORKER - RATES

<table>
<thead>
<tr>
<th>Room and Bath for one per day</th>
<th>Double bed Room with bath for two per day</th>
<th>Twin bed Room with bath for two per day</th>
<th>Suites Living Room Bedroom and bath for 1 or 2 per day</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ 8.00</td>
<td>$ 9.50</td>
<td>$ 11.50</td>
<td>$ 15.50</td>
</tr>
<tr>
<td>$10.00</td>
<td>$12.50</td>
<td>$13.50</td>
<td>$15.50</td>
</tr>
<tr>
<td>$13.00</td>
<td>$14.50</td>
<td>$15.50</td>
<td>$18.50</td>
</tr>
</tbody>
</table>

Note: All rates subject to New York City 5 per cent hotel tax.

To: Hotel New Yorker
Reservation Department
34th Street and 8th Avenue
New York 1, New York

Please enter my reservation for

<table>
<thead>
<tr>
<th>Single Room</th>
<th>Twin Bed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Double Bed</td>
<td>Suite</td>
</tr>
</tbody>
</table>

Please print below additional names of persons in your group:

Rate, Approximately $______ to $______
(I) (We) plan to arrive
and remain until

Signature

Address

City and State

25% discount to members approx. $6.00
SOVIET MATHEMATICS

DOKLADY

A Translation of all the Pure Mathematics Sections of Doklady Akademii Nauk SSSR

The total number of pages of the Russian journal to be translated in 1961 will be about 1600. All branches of Pure Mathematics are covered in the DOKLADY in short articles which provide a comprehensive, up-to-date report of what is going on in Soviet mathematics.

Six Issues a year

Domestic Subscriptions ........................................ $17.50
Foreign Subscriptions ........................................... 20.00
Single Issues ....................................................... 5.00

Send Orders to

American Mathematical Society
190 Hope Street
Providence 6, Rhode Island