THE AMERICAN MATHEMATICAL SOCIETY

Notices

Edited by John W. Green and Gordon L. Walker

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MEETINGS

CALENDAR OF MEETINGS

NOTE: This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the NOTICES was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

<table>
<thead>
<tr>
<th>Meeting No.</th>
<th>Date</th>
<th>Place</th>
<th>Deadline for Abstracts*</th>
</tr>
</thead>
<tbody>
<tr>
<td>592</td>
<td>August 28-31, 1962</td>
<td>Vancouver, British Columbia</td>
<td>July 6</td>
</tr>
<tr>
<td>593</td>
<td>October 27, 1962</td>
<td>Hanover, New Hampshire</td>
<td>Sept. 12</td>
</tr>
<tr>
<td>594</td>
<td>November 16-17, 1962</td>
<td>Tallahassee, Florida</td>
<td>Oct. 2</td>
</tr>
<tr>
<td>595</td>
<td>November 17, 1962</td>
<td>Los Angeles, California</td>
<td>Oct. 2</td>
</tr>
<tr>
<td>596</td>
<td>November 23, 24, 1962</td>
<td>Northwestern University</td>
<td>Oct. 2</td>
</tr>
<tr>
<td></td>
<td>January 24-28, 1963</td>
<td>Berkeley, California</td>
<td>Nov. 23</td>
</tr>
<tr>
<td></td>
<td>April 26-27, 1963</td>
<td>New Mexico State University</td>
<td></td>
</tr>
<tr>
<td></td>
<td>August 26-30, 1963</td>
<td>Boulder, Colorado</td>
<td></td>
</tr>
<tr>
<td></td>
<td>January 20-24, 1964</td>
<td>Miami, Florida</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(70th Annual Meeting)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>August, 1964</td>
<td>Ann Arbor, Michigan</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(69th Summer Meeting)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>January, 1965</td>
<td>Denver, Colorado</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(71st Annual Meeting)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>August, 1965</td>
<td>Ithaca, New York</td>
<td></td>
</tr>
<tr>
<td></td>
<td>August, 1966</td>
<td>New Brunswick, New Jersey</td>
<td></td>
</tr>
</tbody>
</table>

* The abstracts of papers to be presented in person at the meetings must be received in the Headquarters Offices of the Society in Providence, Rhode Island, on or before these deadlines. The deadlines also apply to news items. The next two deadline dates for by title abstracts are June 29 and September 5.

The NOTICES of the American Mathematical Society is published by the Society six times a year, in February, April, June, August, October and November. Price per annual volume is $7.00. Price per copy, $2.00. Special price for copies sold at registration desks of meetings of the Society, $1.00 per copy. Subscriptions, orders for back numbers (non available before 1958), and inquiries should be addressed to the American Mathematical Society, 190 Hope Street, Providence 6, Rhode Island. Second-class postage paid at Providence, Rhode Island, and additional mailing offices. Authorization is granted under the authority of the act of August 24, 1912, as amended by the act of August 4, 1947 (Sec. 34. 21, P. L. and R.). Accepted for mailing at the special rate of postage provided for in section 34.40, paragraph (d).

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The sixty-seventh summer meeting of the American Mathematical Society will be held at the University of British Columbia, Vancouver, B. C., from Tuesday, August 28, to Friday, August 31, 1962. During this week there will be meetings at the University of British Columbia of the Mathematical Association of America, and the Society for Industrial and Applied Mathematics. The Hedrick Lectures of the Mathematical Association of America will be given by Professor Andrew Gleason of Harvard University.

There will be no Colloquium Lectures at this meeting because of the occurrence in August 1962 of the International Congress of Mathematicians at Stockholm, Sweden.

The Committee to Select Hour Speakers for Summer and Annual Meetings has invited Professor C. C. Lin of the Massachusetts Institute of Technology, Professor Charles Loewner of Stanford University, Professor Albert Nijenhuis of the University of Washington, Professor R. S. Pierce of the University of Washington, and Professor R. L. Vaught of the University of California, Berkeley to address the Society. All of these lectures will be given in the Auditorium Building (about 200 yards west of the Buchanan Building). Professor Lin's address, entitled "Hydrodynamic stability -- a study in applied mathematics", will be given at 9:00 A.M. on Wednesday, August 29. Professor Loewner will speak at 2:00 P.M. on Thursday, August 30. The title of his talk is "On semigroups in analysis and geometry". Professor Nijenhuis's talk on "Derivations and structure" will be given at 2:00 P.M. on Tuesday, August 28. Professor Pierce will speak on "Representations of lattices" at 9:00 A.M. on Friday, August 31. Professor Vaught's address will be given at 9:00 A.M. on Thursday, August 30. The title of his talk is "Models of complete theories".

There will be sessions for contributed papers in Buchanan Building at times to be announced in the Program of the Meeting which will appear in the August NOTICES. No sessions will be held for papers which fail to meet the July 6 deadline.

The Council of the Society will meet at 4:00 P.M. on Thursday, August 30 in the Study Room of the International House. There will be a Business Meeting of the Society at 10:30 A.M. on Friday, August 31, in the Auditorium Building.

REGISTRATION

Registration headquarters will be in the main floor corridor of the Buchanan Building. All persons attending the meeting are requested to register immediately on arrival. The schedule of registration fees is as follows:

- Members of participating organizations (except students) $2.00
- First nonmember in member's family .50
- Other nonmembers in member's family free
- Students free
- Nonmembers not in any of the above categories $5.00

A directory of all persons attending the meetings will be located at registration headquarters. The employment register will be located in Room Bu-1221 of the Buchanan Building. There will be exhibits in the main hall on the second floor of the Buchanan Building.
ROOMS AND MEALS

Accommodations will be available in the University residences to all attending the meetings and to adult members of their families. Only a few units are available on campus which are suitable for families with children under twelve. Children twelve and over can be accommodated in the regular residences at the regular rates. Low cost dormitory space is available in converted huts in Fort Camp and Acadia Camp. The cost of dormitory housing will be $2.00 a night per person in a single room. In a double room with separate beds the cost is $1.50 a night per person. The university also has a number of new, modern, three-storey permanent residences containing mostly single rooms. The cost of permanent residence housing will be $4.00 a night per person. A small number of double rooms with separate beds are available at $3.00 a night per person. Towels and bedding will be provided. Automatic laundry facilities are available. Rooms in residences may be occupied from 2:00 P.M., Saturday, August 25 to 2:00 P.M., Saturday, September 1.

The nearest hotels or motels are located in downtown Vancouver (about 6 miles from the campus). The nearest public campsites are located at Alouette Lake, about 40 miles from the campus.

The main cafeteria located in the Auditorium Building will be open all day from 8:30 A.M. Cafeteria service will also be available in three dining rooms located in the three main residence areas. Meal hours are: breakfast 7:30 - 8:30 A.M.; lunch 11:45 - 1:00 P.M.; dinner 5:00 - 6:30 P.M. Prices of regular cafeteria meals are: breakfast 50 cents; lunch 65 cents; dinner 85 cents.

RESERVATIONS

Reservations for University residences and dormitories should be made by writing directly to the Conference Office, University of British Columbia, Vancouver 8, Canada at the earliest possible date and before August 9. A reservation form for this purpose can be found on the inside back cover of these NOTICES.

HOTELS AND MOTELS

Persons desiring hotel and motel accommodations should make their reservations directly with the appropriate manager, and under no circumstances write to the Conference Office which is in charge only of the University facilities.

It is advisable to make reservations early because the Seattle World’s Fair is expected to attract an unusually large number of tourists to the Pacific Northwest.

Suggested Hotels and Motels

<table>
<thead>
<tr>
<th>Name</th>
<th>Address</th>
<th>Phone</th>
<th>Units or Rooms</th>
<th>Minimum - Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Burrard Motel</td>
<td>1100 Burrard</td>
<td>MU 1-2331</td>
<td>60</td>
<td>$8.00 $11.00</td>
</tr>
<tr>
<td>Devonshire Hotel</td>
<td>894 W. Georgia</td>
<td>MU 1-5481</td>
<td>150</td>
<td>7.50 10.50</td>
</tr>
<tr>
<td>Doric Howe Motor Hotel</td>
<td>1061 Howe St.</td>
<td>MU 2-3171</td>
<td>103</td>
<td>8.00 10.00</td>
</tr>
<tr>
<td>Georgia Hotel</td>
<td>801 W. Georgia</td>
<td>MU 3-1182</td>
<td>314</td>
<td>9.00 12.50</td>
</tr>
<tr>
<td>Georgian Towers Motor</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kamlo Apt. Hotel</td>
<td>1120 Denman St.</td>
<td>MU 4-7474</td>
<td>47</td>
<td>7.00 10.00</td>
</tr>
<tr>
<td>Travelodge Motel</td>
<td>1304 Howe St.</td>
<td>MU 2-2767</td>
<td>74</td>
<td>7.00 9.00</td>
</tr>
<tr>
<td>Bayshore Inn</td>
<td>1601 Georgia St.</td>
<td>MU 2-3377</td>
<td>308</td>
<td>12.00 15.00</td>
</tr>
<tr>
<td>Sands Motor Hotel</td>
<td>1755 Davie St.</td>
<td>MU 2-1831</td>
<td>100</td>
<td>8.50 9.50</td>
</tr>
<tr>
<td>Sylvia Hotel</td>
<td>1154 Gilford St.</td>
<td>MU 1-9321</td>
<td>125</td>
<td>5.00 8.00</td>
</tr>
<tr>
<td>Vancouver Hotel</td>
<td>900 W. Georgia</td>
<td>MU 4-3131</td>
<td>561</td>
<td>9.00 11.50</td>
</tr>
</tbody>
</table>
ENTERTAINMENT AND RECREATION

There will be a barbecue on Wednesday, August 29 at 6:00 P.M. The cost will not exceed $2.50 per adult, with half price for children under twelve. A tea will be given between 4:00 P.M. and 6:00 P.M. on Tuesday, August 28. More complete information concerning the tea and barbecue will be given in the Program of the Meetings.

Some of the athletic facilities of the University will be available to members and their guests. The Seattle World's Fair and the Vancouver International Festival will be in operation during the meeting.

TRAVEL

Vancouver is approximately 140 miles north of Seattle, Washington on Highway 99. The best route to the University from the south is on Highway 99-B via Deas Island Tunnel (toll), Oak Street Bridge (toll), West 41st Avenue and Marine Drive.

Vancouver is served by Canadian Pacific, Qantas, Trans Canada, and United Airlines; the Canadian National, Canadian Pacific, and Great Northern Railroads; and the Greyhound Busline. Automobile ferry service is available from Vancouver Island on British Columbia Government ferries from Nanaimo and Swartz Bay, and by Canadian Pacific from Nanaimo.

Public transportation in Vancouver is provided by buses from the city center to the Blanca loop (15 cent fare), transferring to the U.B.C. bus running to the campus (10 cents).

Airport, bus, and limousine service. Bus and limousine service costs $1.25, cab from the airport to Vancouver about $4.50, cab from the airport to U.B.C. about $6.00, and cab from the city center to U.B.C. about $3.00. For one or two persons, the most economical way to travel is to take the bus from the airport to Granville and Broadway streets, and continue by taxi to U.B.C. (total cost about $4.00 for one person).

Steamship Lines. The only regularly scheduled ships between the California ports and Vancouver are operated by the Pacific and Orient Lines, but their sailing and arrival dates make them completely unsuitable for persons attending the summer meetings. There are other lines that operate freighters with excellent accommodations for a small number of passengers, but they do not keep very rigid schedules. The trip from Los Angeles to Vancouver takes between 5 and 8 days and the fares vary from about $130 to $180 on the Holland American Line. Less expensive accommodations are available on the American Mail and the Canadian Blue Star Lines. Reservations should be made in advance.

Persons who are planning to attend both the Vancouver meeting and the International Congress of Mathematicians in Stockholm may find the following flight schedules helpful.

<table>
<thead>
<tr>
<th>Flight</th>
<th>Destination</th>
<th>Time</th>
<th>Airline</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saturday Flight</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ar.</td>
<td>Amsterdam</td>
<td>11:05 A.M.</td>
<td></td>
</tr>
<tr>
<td>Lv.</td>
<td>Amsterdam</td>
<td>1:25 P.M. (Sat. or Sun.)</td>
<td>CPA Flight 301</td>
</tr>
<tr>
<td>Ar.</td>
<td>Vancouver</td>
<td>4:35 P.M.</td>
<td></td>
</tr>
<tr>
<td>Sunday Flight</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lv.</td>
<td>Stockholm</td>
<td>6:05 P.M. (Sun.)</td>
<td>SN Flight 174</td>
</tr>
<tr>
<td>Ar.</td>
<td>Amsterdam</td>
<td>7:20 P.M.</td>
<td></td>
</tr>
<tr>
<td>Lv.</td>
<td>Amsterdam</td>
<td>8:30 P.M. (Sun. only)</td>
<td>CPA Flight 363</td>
</tr>
<tr>
<td>Ar.</td>
<td>Vancouver</td>
<td>11:40 P.M.</td>
<td></td>
</tr>
</tbody>
</table>
### Saturday or Sunday Flight

<table>
<thead>
<tr>
<th>Lv.</th>
<th>Stockholm</th>
<th>8:30 A.M. (daily)</th>
<th>SAS Flight 551</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ar.</td>
<td>Copenhagen</td>
<td>9:45 A.M.</td>
<td></td>
</tr>
<tr>
<td>Lv.</td>
<td>Copenhagen</td>
<td>10:30 A.M.</td>
<td>SAS Flight 551</td>
</tr>
<tr>
<td>Ar.</td>
<td>Amsterdam</td>
<td>11:55 A.M.</td>
<td></td>
</tr>
<tr>
<td>Lv.</td>
<td>Amsterdam</td>
<td>1:25 P.M. (Sat. or Sun.)</td>
<td>CPA Flight 301</td>
</tr>
<tr>
<td>Ar.</td>
<td>Vancouver</td>
<td>4:35 P.M.</td>
<td></td>
</tr>
</tbody>
</table>

### Thursday, Friday, and Saturday Flights

<table>
<thead>
<tr>
<th>Lv.</th>
<th>Stockholm</th>
<th>9:45 P.M. (Thurs., Fri., or Sat.)</th>
<th>SAS Flight 935 (via Oslo, Copenhagen, and Greenland)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ar.</td>
<td>Los Angeles</td>
<td>5:20 A.M. (Fri., Sat., or Sun.)</td>
<td></td>
</tr>
<tr>
<td>Lv.</td>
<td>Los Angeles</td>
<td>7:30 A.M. (Fri., Sat., or Sun.)</td>
<td>There would be an added cost of about $10 on this flight due to surcharge on jet travel in the U.S.</td>
</tr>
<tr>
<td>Ar.</td>
<td>Vancouver</td>
<td>12:00 Noon</td>
<td></td>
</tr>
</tbody>
</table>

With a Vancouver-Stockholm ticket, stopovers are permitted in European cities anywhere west and south of Stockholm at no added cost. IT IS ADVISABLE TO MAKE FLIGHT RESERVATIONS AS EARLY AS POSSIBLE.

### CUSTOMS INFORMATION

Crossing the United States-Canadian border either way is made without difficulty or delay by citizens or permanent residents of the United States. Passports are not needed. However, U.S. citizens should carry some document establishing their citizenship. Alien permanent residents of the United States are advised to have their Alien Registration Receipt Card (U.S. Form 1-151). Visitors to the United States who have only a single entry visa to the U.S.A. should present their documents at an office of the United Immigration and Naturalization Service before entering Canada, to be sure that they have the necessary documentation for their re-entry to the United States. Further information regarding entry to Canada and what merchandise may be taken back into the United States may be obtained by writing to the Canadian Government Travel Bureau at one of the following addresses: Ottawa, Ontario, Canada; Canada House, 680 Fifth Avenue, New York 19, New York; 102 West Monroe Street, Chicago 3, Illinois; 1 Second Street, San Francisco 5, California.

The Canadian currency system is the same as that of the United States, but the rate of exchange between the American and Canadian dollar varies from time to time. U.S. currency is accepted by most merchants in Canada. However, United States visitors are advised to exchange their currency for Canadian funds at a bank in Canada to be assured of receiving the prevailing rate of exchange.

### MAIL AND TELEGRAM

Communications with members of the Society and their guests should be addressed to them in care of the American Mathematical Society, The University of British Columbia, Vancouver 8, Canada.

R. S. Pierce  
Associate Secretary

Berkeley, California
DOCTORATES CONFERRED IN 1961

The list of doctorates in the mathematical sciences and related subjects from universities in the United States and Canada has been published annually in the May issue of the BULLETIN. The Editors of the BULLETIN and NOTICES have decided to now publish the list in the June issue of the NOTICES. This list is based on information supplied by the National Research Council, National Academy of Sciences.

In 1961, 370 individuals were granted a doctorate in the mathematical sciences. Following are the names of the individuals and in each case when available, the university, the month in which the degree was conferred, minor subjects (other than mathematics) and the title of the dissertation.

Acheson, Willard Phillips
University of Pittsburgh, January
Minor - Physics
Dynamics of an elastic-Bingham body

Adler, Irving
Columbia University, November
Composition rings

Adler, Roy Lee
Yale University, June
Some algebraic aspects of measure preserving transformation

Albert, Eugene
University of Virginia, June
Markov chains and eigenvectors

Alberti, Furio
Illinois Institute of Technology, June
Ordinal invariants of quadratic forms over a quadratic field

Alexander, Forrest Doyle
Peabody College, May
Minor - Education Administration
An experiment in teaching mathematics at the college level by closed-circuit television

Ali, Mir Maswood
University of Toronto, June
Minor - Geophysics
Contributions to linear orderly estimates of location and scale parameters

Allaud, Guy Dante
University of Wisconsin, January
Fiber spaces and loop actions

Alperin, Jonathan Lazare
Princeton University, October
On a special class of regular p-groups

Andalafte, Edward Ziegler
University of Missouri, June
A metric characterization of Banach and Euclidean spaces

Anderson, Stuart Lee
University of Washington, December
Lebesgue constants in a certain class of groups

Andrew, David Robert
University of Pittsburgh, August
Weak and strong continuities in topological spaces

Andrews, Newton Steele
Auburn University, June
Orthogonal harmonic homogeneous polynomials in three variables

Antliff, William Bruce
Queen's University at Kingston, Ontario, Canada, May
The limit of \( (n)_{m}(E(n)x) \) as \( n \to \infty \)

Armendariz, Armando A.
Rice University, June
Cluster sets and quasiconformal mappings

Askey, Richard Allen
Princeton University, April
Mean convergence of orthogonal series and conjugate series

Aswad, Alfred George
University of California (Berkeley), June
Some statistical problems arising in a certification procedure, Part I: Estimation of the density of a material with irregular surfaces, Part II: Economic sample size

Avila, Geraldo Severo de Souza
New York University, June
The asymptotic field of A point sources in an inhomogeneous medium

Ax, James B.
University of California (Berkeley), September
The intersection of norm groups
Bailey, Duane W.  
University of Oregon, June  
Semi-normed algebras with involution

Bailey, Paul Bernard  
University of Washington, June  
Removal of the log factor in the asymptotic estimates of the membrane eigenvalues

Baldwin, George Lewis  
University of Oklahoma, June  
Minor - Chemistry  
Some questions on distributions, nets and duality

Banilower, Howard  
Purdue University, January  
Certain ideals in algebras of extended real-valued functions

Bardwell, George Eldred  
University of Colorado, June  
Certain discrete distributions

Bauserman, Thomas  
University of Pittsburgh, August  
Anti-involutions in the plane of two complex variables

Bednarek, Alexander Robert  
University of Buffalo, June  
On the Mickle-Rado theorems and binary relations

Bell, Howard Edwin  
University of Wisconsin, August  
Reduction of matrices to canonical form under generalized Lorentzian transformations

Bell, Lloyd Franklin  
Stanford University, April  
A mathematical theory of guarantee policies

Bellar, Fred James, Jr.  
University of Maryland, June  
Minor-Physics  
Pointwise bounds in parabolic and elliptic partial differential equations

Benton, Edward Rowell  
Harvard University, March  
Minor - Aeronautical Engineering  
Aerodynamic origin of the Magnus effect on a finned missile

Bereiter, Mary Catharine, Sister, O.P.  
Wayne State University, June  
Linear sequence spaces

Berman, Simeon M.  
Columbia University, June  
Limiting distribution of the maximum term in sequences of dependent random variables

Berri, Manuel Philip  
University of California (Los Angeles), August  
Minimal topological spaces

Bhat, Beliyar Ramdas  
University of California(Berkeley), September  
Bayes sequential tests for Markov chains

Bishir, John William  
North Carolina State College, May  
Two problems in the theory of stochastic branching processes

Bland, Richard Park  
University of North Carolina, June  
A minimum average risk solution for the problem of choosing the largest mean

Bleicher, Michael N.  
Tulane University, August  
Lattice coverings of n-dimensional Euclidean space by equal radius spheres

Bohachovsky, Ihor O.  
New York University, June  
Simple waves and shocks in a system of conservation laws

Bohn, S. Elwood  
University of Nebraska, June  
Minor - Physics  
A sub-function study of the Dirichlet problem for a quasi-linear differential equation

Bollinger, Richard Coleman  
University of Pittsburgh, August  
Summability of power series

Borelli, Mario  
Indiana University, June  
Residual varieties

Brande, Edward W., S. J.  
St. Louis University, June  
Minor - Philosophy  
The representations of binary quadratic forms by quinary quadratic forms
Brayton, Robert King  
Massachusetts Institute of Technology, September  
Minor - Physics  
On the asymptotic behavior of the number of trials necessary to complete a set with random selection

Brillinger, David Ross  
Princeton University, June  
Asymptotic means and variances in the \( k \)-dimensional case

Briney, Robert Edward  
Massachusetts Institute of Technology, June  
Minor - Languages  
Intersection theory on quotients of algebraic varieties

Browder, Andrew  
Massachusetts Institute of Technology, September  
Minor - Music  
Cohomology of maximal ideal spaces and some Dirichlet algebras

Buley, Ernest Robert  
University of California (Berkeley), January  
Differentiability of solutions of certain types of variational problems

Burdick, Donald Smiley  
Princeton University, October  
Stage by stage modification of polynomial estimators by the jackknife method

Bures, Donald John Charles  
Princeton University, June  
The type of certain factors constructed as infinite tensor products

Burke, George  
University of Missouri, August  
Some generalizations of the Glivenko-Cantelli theorem

Burton, David MacGregor  
University of Rochester, June  
The solution of hyperbolic systems of partial differential equations by double Laplace transforms

Buser, Mary Paul, Sister, C. S. J.  
St. Louis University, June  
Minor - Philosophy  
On the coefficients of the approximating polynomial for partitions into unit and prime summands.

Bush, George Clark  
Queen's University at Kingston, Ontario, Canada, May  
On embedding a semi-group in a group

Cantrell, James Cecil  
University of Tennessee, December  
Separation of the \( n \)-sphere by an \( (n-1) \)-sphere

Cantrell, Robert Headden, Jr.  
Harvard University, June  
Gas film effects in the linear pyrolysis of solids

Carlyle, Jack Webster  
University of California (Berkeley), September  
Equivalent stochastic sequential machines

Carter, Frank Sydney  
University of Southern California, June  
Contributions to the theory of Hilbert space operators

Cayford, Afton Herbert  
University of California (Los Angeles), June  
A class of integer valued entire functions

Chamberlain, Erling William  
Columbia University, June  
Families of principal solutions of ordinary differential equations

Chambers, John Carlton  
Case Institute of Technology, June  
On the theory of minimum-cost information-collection systems

Channapragada, Rao S.  
University of Illinois, June  
Singular integral equations with an infinity of intersecting arcs

Chao, Chong-Yun  
University of Michigan, June  
Nilpotent Lie algebras

Cheema, Mohindar Singh  
University of California (Los Angeles), June  
Vector partitions and permutation vectors

Chicks, Barbara Jean Thomson  
University of Oregon, June  
k-sample rank order statistics
Chou, Tao-Hsiung
Iowa State University, Ames, July
Minor - Economic Statistics
Mixed integer programming with application of dynamic and non-linear models for agriculture

Chowla, Paromita
University of Colorado, August
The class-number of real quadratic fields and some diophantine equations in cyclotomic fields

Chwe, Byoung-Song
University of California (Berkeley), September
Relative homological algebra and homological division of Lie algebra

Clark, Allan Hersh
Princeton University, October
Some applications of Hopf algebras and cohomology operations

Clay, Jesse Paul
University of Pennsylvania, June
Proximity and equicontinuity in transformation groups

Clay, Robert Edward
Notre Dame University, August
Contributions to Meteorology

Clifton, Yeaton Hopley
Columbia University, June
Completely integrable systems of flat Pfaffian equations

Cohn, James Alan
Harvard University, June
Some results in the cohomology theory of finite groups

Cohn, Martin
Harvard University, June
Switching function canonical forms over integer fields

Coleman, Donald Brooks
Purdue University, June
On group rings

Conley, Charles Cameron
Massachusetts Institute of Technology, September
Minor - Electrical Engineering
On periodic solutions of long period for the moon problem

Conner, Howard Emmett
Massachusetts Institute of Technology, June
Minor - Physics
A limit theorem for a space-dependent branching process

Conway, Melvin Edward
Case Institute of Technology, June
A set-theoretic model for logic systems

Cooley, James William
Columbia University, June
Computational methods for the study of diatomic molecules

Cooper, Dale Edward
North Carolina State College, February
Minor - Soils
Available soil moisture as a stochastic process

Corneliussen, Arvid Helge, Jr.
Brown University, June
Finite deformation of elastic membranes with application to the stability of an inflated and extended circular tube

Crawley, Peter Linton
California Institute of Technology, June
A decomposition theory for lattices without chain conditions

Cross, Myrle Vivian
University of Michigan, June
Minor - Philosophy
On spaces approximated by open sets with compact closures

Cundiff, Joyce Coleman
University of Florida, June
Minor - Astronomy
Confluent cases of second order linear differential equations with four singular points

Daniel, Klaus H.
University of California (Berkeley), September
A delivery-lag inventory model with emergency

Das, Krishna Mohan
Carnegie Institute of Technology, June
Singularity and zero-free solutions for a class of complex non-linear differential equation
Davis, Edward D.  
University of Chicago, August  

Over-rings of commutative rings

Davis, Kenneth Samuel  
Stanford University, January  
The study of certain hyperbolic differential equations and their corresponding symmetric Hamburger moment problems

Davis, Morton David  
University of California (Berkeley), June  

On an infinite game of perfect information

DeMar, Richard Francis  
University of Wisconsin, January  
Existence of interpolating functions of exponential type

DeMarr, Ralph Algernon  
University of Illinois, October  
Minor - Statistics  
Point-to-set mappings and semi-group transformations, with applications to fixed set theory

Descloux, Alfred  
University of North Carolina, August  
On the covariance between the number of offered and the number of overflow requests in systems with limited capacity

Dinsmore, Robert Sunderland  
Stanford University, October  
Extremum problems for harmonic polynomials in higher Euclidean spaces

Dixon, John Douglas  
McGill University, May  
On general group extension

Drobnies, Saul Isaac  
University of Texas, June  
Minor - Business Administration  
Concerning the uniform polynomial approximation of a bounded function

Drufenbrock, Madeleine Sophie, Sister, O.S.F.  
University of Illinois, October  
Minor - Physics  
On some metabelian groups of order $p^10$

Duda, Edwin  
University of Virginia, June  
Brouwer property spaces

Eagon, John Alonzo  
University of Chicago, August  
Ideals generated by the subdeterminants of a matrix

Edwards, Harold Mortimer  
Harvard University, June  
Applications of intersection theory to differential equations

Eigel, Edwin George, Jr.  
St. Louis University, June  
Minor - Philosophy  
Paraphrases and divisor functions

Ellis, Homer Godsey  
University of Texas, June  
Solutions of analytic equations and solutions of first order, analytic differential equations

Engelhardt, Jacob  
New York University, February  
Solitary waves in two layer systems where the upper layer has variable density and the lower layer constant density

Falb, Peter Lawrence  
Harvard University, June  
On differentials in function fields

Faust, Claude Marie, Sister, C.C.V.I.  
University of Notre Dame, June  
On the boundary behavior of holomorphic functions in the unit disc

Feigl, Polly C. Bartholomew  
University of Minnesota, June  
Design considerations for the post-operative evaluation of analgesic drugs

Feinberg, Irwin  
New York University, June  
On the existence of two-dimensional subsonic compressible jets

Feldman, Dorian  
University of California (Berkeley), June  
Contributions to the two armed bandit problem

deFigueiredo, Djairo Guedes  
New York University, June  
The coerciveness problem for forms over vector-functions

Fishman, Robert Sumner  
Boston University, June  
The integral as an average on a group of translations in a subinvariant measure space

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Foland, Neal Eugene  
University of Missouri, June  
Continuous flows on arcs and 2-cells

Forelli, Frank John, Jr.  
University of California (Berkeley), June  
Marcel Riesz's theorem on conjugate functions

Fox, David Lawrence  
New York University, June  
Perturbation methods in a problem of waveguide theory

Frandsen, Henry  
University of Illinois, June  
Minor - Philosophy  
On metabelian groups of order $p^{10}$ with five generators

Frederick, Karen Norma  
New York University, October  
The Hopfian property for fundamental groups

Freese, Raymond William  
University of Missouri, June  
Ptolemaic metric spaces

Friedman, Manfred  
New York University, February  
Two problems in transonic fluid dynamics

Furman, Walter Laurie  
University of Florida, February  
Minor - Physics  
Mathematical research in Latin American universities

Galmarino, Alberto Raul  
Massachusetts Institute of Technology, September  
Minor - Economics  
Isotropic diffusions

Gangolli, Ramesh Anand  
Massachusetts Institute of Technology, June  
Minor - Economics  
Brownian motion and vector fields on a Riemannian manifold

Geiger, David Scott  
University of Illinois, October  
Metabelian groups of order $p^{10}$

Gershenson, Hillel Halkin  
University of Chicago, August  
Some relationships between the Adams spectral sequence and Toda's calculations of the stable homotopy groups of spheres

Gillilan, James Horace  
University of Illinois, June  
Minor - Astronomy  
Generalized set functions

Gilmer, Robert William, Jr.  
Louisiana State University, August  
Rings in which every ideal has a regular representation

Gindler, Herbert Aaron  
University of California (Los Angeles), August  
Some properties of operator spectra

Ginsburg, Michael  
Massachusetts Institute of Technology, June  
Minor - French Literature  
On the Lusternik-Schnirelman category of space

Giri, Narayan Chandra  
Stanford University, October  
On tests with likelihood ratio criteria in some problems of multivariate analysis

Glasgow, Mark Othello  
University of Texas, June  
Some marginal distributions associated with permutation cycles

Gleiberman, Leon  
University of Pittsburgh, August  
Representation of selected functions of a complex variable by surfaces in Argund four space and resulting classification of Riemann surface theory

Glick, Irving Isadore  
University of Maryland, June  
On an analogue of the Euler-Cauchy polygon method for the partial differential equation $u_{x_1\cdots x_n} = f$.

Gluck, Herman Randolph  
Princeton University, June  
The embedding of two-spheres in the four-sphere

Gold, Phillip John  
New York University, June  
The mapping class and symplectic modular groups

Goldman, Aaron Simpson  
Oklahoma State University, May  
On the determination of sample size
Goodman, Arnold Frank
Stanford University, October
The selection of an optimum sampling partition

Govindarajulu, Zakkula
University of Minnesota, March
Central limit theorems and asymptotic efficiency for one sample non-parametric procedures

Green, Paul Edgar
University of Pennsylvania, December
Minor - Economics
Some intra-firm applications of Bayesian decision theory to problems in business planning

Greenleaf, Newcomb
Princeton University, June
Local zeros of global forms

Guard, James Russell
Princeton University, June
The independence of transfinite induction up to 20-20 in recursive arithmetic

Gurtin, Morton Edward
Brown University, June
Some theorems in the linear theory of elasticity

Ha, Kwang Chul
University of North Carolina, August
The radical of topological abelian groups

Hachigian, Jack
Indiana University, June
Some further results on functions of Markov processes

Haddock, Aubura Glen
Oklahoma State University, August
Some theorems concerning compact continua in the plane

Hahn, Hwa Suk
University of Illinois, October
Minor - Astronomy
On the relative growth of differences of partition functions

Hanna, Samuel Charles
University of Pittsburgh, January
Metrization and uniform continuity

Hare, William Ray, Jr.
University of Florida, June
Minor - Philosophy
Projective convexity

Harvey, John Grover, II
Tulane University, June
Complete holomorphs and chains in partially ordered groups

Hayden, Thomas Lee
University of Texas, June
Minor - Physics
A convergence problem for continued fractions

Hellerstein, Simon
Syracuse University, September
Deficiencies of a meromorphic function and distribution of values in angles and annuli

Hershenov, Joseph
Massachusetts Institute of Technology, September
Numerical integration over hyper-shells

Hickman, James Charles
State University of Iowa, February
On random sets, derived from subsamples, for statistics based on the entire sample

Hill, Bruce Marvin
Stanford University, April
A test of linearity versus convexity of a median regression curve

Hills, Norman Leslie
New York University, October
Semi-infinite diffraction gratings

Hilt, Arthur Lincoln
Lehigh University, October
Vector fields and infinitesimal transformations on Almmost-Hermitian manifolds with boundary

Ho, Hung-Ta
Brown University, June
The compressible viscous layer in rarefied hypersonic flow

Ho, Yu-Chi
Harvard University, March
A study of the optimal control of dynamic systems
Holland, Samuel Shaheen, Jr.  
Harvard University, March  
The Radon-Nikodym theorem in dimension lattices

Holland, Wilbur Charles, Jr.  
Tulane University, June  
Extensions of ordered algebraic structures

Hooker, William Weston  
University of California (Berkeley), January  
Lower bounds for the first eigenvalue of elliptic equations of orders two and four

Hopper, Edgar Hugh  
Auburn University, August  
Orthogonal polynomials over discrete sets

Hudson, Anne Lester  
Tulane University, June  
On the structure of certain classes of topological semigroups

Hunt, Robert Weldon  
University of Utah, June  
The behavior of solutions of ordinary, self-adjoint differential equations of arbitrary even order

Iglehart, Donald Lee  
Stanford University, October  
Dynamic programming and stationary analyses of inventory problems

James, Donald D.  
Iowa State University, June  
Minor - Philosophy  
Periodic integral surfaces of periodic differential equations

Jarvis, Stephen, Jr.  
Columbia University, June  
Apparent wall shapes in viscous shear flows

John, Chelikuzhiel Thomas  
Boston University, June  
Spherical bearing under axial and radial loading

John, Floyd Idwal  
Purdue University, August  
On queueing, storage and related topics

Johnsen, Eugene Carlyle  
Ohio State University, December  
Matrix rational completions satisfying generalized incidence equations, and integral solutions to the incidence equation for finite projective plane cases of orders \( n \equiv 2 \) (mod 4)

Jones, Benjamin Franklin  
Rice University, June  
The determination of a coefficient in a parabolic differential equation

Jones, Richard Hunn  
Brown University, June  
Spectral estimates and their distributions

Joseph, Roger David  
Cornell University, September  
Contributions to perception theory

Kahn, Donald W.  
Yale University, June  
On the real cohomology of the fibre spaces

Kahn, Paul Markham  
University of Michigan, June  
Some applications of collective risk theory to reinsurance and group experience rating

Kapadia, Chandrakant Harilal  
Oklahoma State University, August  
Variance components in two way classification models with intersection

Kaplan, Stanley  
University of Pittsburgh, January  
Minor - Mechanical Engineering  
Extensions and applications of the method of finite transform

Karrass, Abraham  
Adelphi College, June  
Parallel geometries and finite non-Desarguian planes

Keisler, H. Jerome  
University of California (Berkeley), June  
Ultraproducts and elementary classes

Kelly, Edgar Preston, Jr.  
Oklahoma State University, May  
On abstract summability

Kenelly, John Willis, Jr.  
University of Florida, February  
Minor - Philosophy  
An involution of period seventeen
Kennedy, Hubert Collings  
St. Louis University, June  
Minor - Philosophy  
Group membership in rings

Killam, Eleanor  
Yale University, June  
Locally M-convex algebras

Kishore, Nand  
University of California (Berkeley), September  
Arithmetical properties of Bessel functions

Koch, Charles Frederick  
University of Illinois, June  
Minor - Physics  
Density theorems in the product space of two abstract measure spaces

Konsler, Thomas Rhinehart  
North Carolina State College, Raleigh, May  
Minor - Genetics  
A quantitative analysis of the growth and regrowth of a forage crop

Kristensen, Leif  
University of Chicago, August  
On the cohomology of two-stage Postnikov systems

Krueger, Charles Gordon  
University of Illinois, February  
Minor - Philosophy  
Baire classes in a Denjoy-Tvitzinsky measure space

Kullback, Joseph Henry  
Stanford University, January  
A quality control model for complex items

Lange, John Edward  
University of Wisconsin, June  
Entire functions as limits of zero-restricted polynomials

Larkin, William James, III  
University of Cincinnati, June  
On the first-order homogeneous bi-linear algebraic differential equation

La Rue, James Arthur  
University of Pittsburgh, August  
Comparison of relative strengths of regular matrix methods and their respective submethods

Lawson, Charles Lawrence  
University of California (Los Angeles), June  
Contributions to the theory of linear least maximum approximation

Leaf, Gary Kristian  
University of Illinois, October  
Minor - Physics  
A spectral theory for a class of linear operators

Lee, Shing-Meng  
Cornell University, June  
On a two-parameter family of summation methods

Leetch, James Frederick  
Ohio State University, August  
Functions whose values are sets in a vector space

Lehman, Eugene H., Jr.  
North Carolina State College, May  
Estimation of the scale parameter in the Weibull distribution

Leibowitz, Martin A.  
Harvard University, March  
Rigorous derivation of Fermi age theory

Levy, Lawrence Sherwin  
University of Illinois, February  
Minor - Music  
Unique subdirect sums of rings

Lick, Don Raymond  
Michigan State University, June  
Sets of points of non-uniform convergence of Taylor series and trigonometric series

Lincoln, Andrew James  
Harvard University, June  
Magnetic devices in sampled analog systems

Long, Paul Edward  
Oklahoma State University, May  
Properties of certain non-continuous transformations

Low, Leone Yarborough  
Oklahoma State University, May  
Estimation of variance components, Eisenhart's Model II, balanced on complete block and general two-way classification
Lowengrub, Morton  
Duke University, June  
Minor - Physics  
Stress in the vicinity of a crack in a thick elastic plate

Lubarsky, Bernard  
Case Institute of Technology, June  
Generalized functions and the fundamental solutions for polyharmonic difference operations

Lukaweccki, Stanley Michael  
Auburn University, August  
Diffraction of elastic waves by a semi-infinite plane

Lutts, John A., S. J.  
University of Pennsylvania, December  
Some imbedding properties of locally compact Hausdorff spaces which possess unisolvent systems

MacGregor, Thomas Harold  
University of Pennsylvania, June  
Topics in the theory of Schlicht functions

MacRae, Robert Eugene  
University of Chicago, June  
Homological dimension of ideals in Noetherian rings

Maeda, Fumiyuki  
Yale University, June  
Spectral theory on locally convex spaces

Majumdar, Kuliendra Narayan  
Purdue University, January  
An investigation of the properties of incomplete block designs

Marcia, John Glen  
University of California (Berkeley), January  
Almost periodic surfaces

Martin, Charles John  
Rensselaer Polytechnic Institute, June  
Vibrations of an elastic plate under uniform tension

Martino, Joseph Paul  
Ohio State University, December  
Minimum-expected-cost filters for arbitrary cost functions

Mayer, Alan Lionel  
Princeton University, October  
On the Jacobi inversion problem

McCarty, George Schultz, Jr.  
University of California (Los Angeles), June  
Homeotope groups

McCoart, Richard Felix, Jr.  
University of North Carolina, August  
Irreducibility of certain classes of Legendre polynomials

McCord, James Richard, III  
Massachusetts Institute of Technology, June  
Minor - Physical Chemistry  
Generalized multidimensional Poisson distributions for finite Markov chains

McCoy, Thomas La Rue  
University of Wisconsin, June  
Entire functions with restraints on the zeros of the partial sums

McCullough, Roger Stewart  
Iowa State University, February  
Testing equality of means under variance heterogeneity

McKinney, Earl Harry  
University of Pittsburgh, January  
Vibration analysis of continuous beam-columns with uniformly distributed axial load

McLean, Robert T.  
University of Pittsburgh, August  
Defining relations which characterize groups having elements of the form $A_1^{x_1}A_2^{x_2} \ldots A_n^{x_n}$

McQuillan, Donald Laurence  
Johns Hopkins University, June  
A generalization of a theorem of Hecke

Mertz, Robert Theodore  
Columbia University, June  
On the continuation of solutions of a linear ordinary differential equation with polynomial coefficients

Metcalf, Frederic Thomas  
University of Maryland, August  
Bounds on apsidal angles

Meyer, Donald Lee  
University of Minnesota, June  
Minor - Educational Psychology  
Response surface designs with integer valued factors
Mikhail, Wadie Faltas  
University of North Carolina, June  
On the monotonicity and admissibility of some tests in multivariate analysis

Mikulski, Piotr Witold  
University of California (Berkeley), September  
Some problems in the asymptotic theory of testing statistical hypotheses - efficiency of nonparametric procedures

Miller, Millage Clinton, III  
University of Oklahoma, August  
An investigation of the application of the method of steepest ascent in medical research

Miller, Robert Gerard  
Harvard University, June  
Minor - Meteorology  
An application of multiple discriminant analysis to the probabilistic prediction of meteorological conditions affecting operational decisions

Mitra, Shashanka Shekhar  
University of Washington, December  
Asymptotic rate of convergence for a process with stationary independent increments

Moeller, James Walter  
New York University, February  
On the spectra of some translation invariant spaces

Monahan, Irene Patricia  
North Carolina State College, June  
Incomplete variable designs in multivariate experiments

Monahan, Irene Patricia  
North Carolina State College, June  
Incomplete variable designs in multivariate experiments

Montzingo, Lloyd J., Jr.  
University of Buffalo, June  
Minor - Physics  
Convergence of distribution functions in a parameter space

Moroney, Richard Morgan, Jr.  
Massachusetts Institute of Technology, June  
A class of characteristic value problems

Morrison, David Dean  
University of California (Los Angeles), June  
Errors in the solution of eigenvalue problems by finite difference methods

Mundt, Marvin Glen  
Iowa State University, November  
Minor - History of Science  
Sufficient conditions for a summability method for improper Lebesgue-Stieltjes integrals involving an integral-to-function transformation to be regular

Murthy, Vrudhula Krishna  
University of North Carolina, June  
Minor - Mathematical Economics  
On the general renewal process

Muskat, Joseph Baruch  
Massachusetts Institute of Technology, February  
Minor - Electrical Engineering  
Criteria for prime power residuacity

Myers, Earle Frederick  
University of Pittsburgh, January  
Behavior of curves and surfaces through fundamental elements of birational transformations

Nanda, Jagdish Lal  
Indiana University, June  
Einstein's connections: degenerate cases of the first class

Nassar, Hanna I.  
Lehigh University, October  
Parallel and central transformations of Riemannian manifolds

Neumann, Peter Gabriel  
Harvard University, June  
Efficient error-limiting variable-length codes

Neuts, Marcel Fernand  
Stanford University, April  
Games on the unit-square with discrete payoff

Newman, Rogers Joseph  
University of Michigan, February  
Minor - Physics  
Capacity and Tchebycheff polynomials
Ney, Peter Ernest  
Columbia University, June  
Some contributions to the theory of cascades

Nieto-De-Pascual, Jose  
Iowa State University, Ames, February  
Theory of minimum variance estimation with applications

Odeh, Farouk Mohamad Saleh  
University of California (Berkeley), January  
Uniqueness theorems under the radiation condition

Ogawa, Hajimu  
University of California (Berkeley), January  
On difference methods for the solution of partial differential equations of mixed type

Ogg, Andrew Pollard  
Harvard University, June  
Cohomology of Abelian varieties over function fields

Oglesby, Francis Clyde  
Lehigh University, June  
An examination of a decision procedure

Onishi, Hironori  
Massachusetts Institute of Technology, June  
Minor - Philosophy  
The general sieve method

Ono, Tamio  
University of California (Berkeley), September  
Local theory in function analysis

Ossesia, Michel Germain  
University of Pittsburgh, May  
Some invariant hyper surfaces

Paolucci, Dominic Anthony  
Indiana University, June  
The effects of higher order viscosity terms on fluid flow

Patel, Mavibhai Sukhabhai  
University of North Carolina, June  
Investigations on factorial designs

Peach, Paul  
Case Institute of Technology, June  
Some results of queueing and vulnerability relevant to aircraft maintenance

Peressini, Anthony Louis  
Washington State University, February  
Topologies in ordered vector spaces

Perrin, Edward Burton  
Stanford University, January  
The estimation of parameters in systems related to the observations by an unknown monotone transformation

Petrich, Mario  
University of Washington, June  
Semicharacters of direct products and of certain other semigroups

Petridis, Nicholas Constantine  
University of Chicago, June  
Quasiconformal mappings and pseudomeromorphic curves

Petro, John William  
State University of Iowa, June  
Some results in the theory of pseudovaluations

Petryshyn, Walter Volodymyr  
Columbia University, June  
Direct and iterative methods for the solution of linear operator equations in Hilbert space

Pixley, Alden French  
University of California (Berkeley), June  
Clusters of algebras: identities and structure lattices

Pohl, William Francis  
University of California (Berkeley), September  
Differential geometry of higher order

Posten, Harry O.  
Virginia Polytechnic Institute, June  
Power of the likelihood-ratio test of the general linear hypothesis in multivariate analysis

Poxon, Nancy Jim Moody  
University of Illinois, June  
Minor - Physics  
Representations of functions in terms of logarithmic potentials

Quesenberry, Charles Price  
Virginia Polytechnic Institute, June  
Some tests for outliers

Rao, Jonnagadda Nalini Kanth  
Iowa State University, February  
Minor - Mathematical Economics  
Sampling procedures involving unequal probability selection
Ratliff, Louis Jackson, Jr.
State University of Iowa, June
Affine rings over rank two regular local rings

Reinitz, Rudolf N. C.
Stanford University, June
An integrated job shop sequencing problem

Reinke, William Andrew
Western Reserve University, June
Minor - Economics
Dynamic production and inventory control in the face of uncertainty

Rhodes, Benjamin Thomas, Jr.
Oklahoma State University, May
The use of combined information in interval estimation

Rich, Robert Norman
University of Pennsylvania, December
Simple and weakly almost periodic transformation groups

Riehm, Carl Richard
Princeton University, October
On the integral representations of quadratic forms over local fields

Ritchie, Robert Wells
Princeton University, January
Classes of recursive functions of predictable complexity

Rodin, Burton
University of California (Los Angeles), June
Reproducing formulas on surfaces

Romano, Albert
Virginia Polytechnic Institute, June
A trapezoidal approximation in spectral-density estimation

Rosenthal, Paul Lester
Oregon State University, June
On a generalization of Mahler’s inversion formula and some of its applications

Roy, Prabir
University of North Carolina, August
Separability of metric spaces

Rumsey, Howard Calvin, Jr.
California Institute of Technology, June
Sets of visible points

Rung, Donald Charles, Jr.
University of Notre Dame, August
Results on the order of functions defined in the unit disk, with applications to holomorphic functions

Rushforth, Norman Burleigh
Cornell University, February
Minor - Psychology
A comparison of sample correlation matrices and a multivariate analysis of job concepts of selected industrial executive groups

Rutenberg, Yechezkel Henryk
Case Institute of Technology, June
Sequential decision models

Saber, Nicholas John
University of Pittsburgh, January
On the product of real valued uniformly continuous functions in metric spaces

Sacks, Gerald Enoch
Cornell University, June
On suborderings of degrees of recursive unsolvability

Sandler, Reuben Irving
University of Chicago, August
Autotopism groups of some finite non-associative algebra

Sanwal, Jagdish C.
Indiana University, June
Minor - Physics
On locally affine spaces

Sarason, Leonard
New York University, February
On boundary value problems for hyperbolic equations

Sasieni, Maurice Wolf
Case Institute of Technology, June
Some problems in the analysis of queuing systems

Schrader, Walter Reginald
University of Wisconsin, January
The epistola de proportione et proportionalitate of Ametus Filius Iosephi

Seshadri, Vanamamalai
Oklahoma State University, May
Estimation in the balanced incomplete block design

Shepp, Lawrence Alan
Princeton University, April
Recurrent sums of random variables
Sherman, Gordon Rae  
Purdue University, January
Minor - Economics
Combinatorial scheduling: on finding a partition of a finite set which maximizes a set function

Sherry, Murray Elliot  
Harvard University, March
Minor - Applied Physics
Syntactic analysis in automatic translation

Shinbrot, Marvin  
Stanford University, April
Difference kernels

Shtern, Israel Hirsh  
McGill University, May
On some reducible diophantine equations

Simpson, James Edward  
Yale University, June
On spectral measures and spectral operators

Sinclair, Roy George  
Massachusetts Institute of Technology, February
Torsion and extension of helicoidal strips

Singh, Shobh Nath  
University of California (Berkeley), September
On a hypothetical chance mechanism for variation in the number of births per couple

Slagle, James Robert Paul  
Massachusetts Institute of Technology, June
Formal integration by a digital computer

Sledd, William Tazwell  
University of Kentucky, May
Minor - Physics
Some properties of Karamata summability matrices

Smart, John Roderick  
Michigan State University, June
Modular forms of dimension - 2 for subgroups of the modular group

Smith, Georgia Alberta Caldwell  
University of Pittsburgh, January
Some results on the anticenter of a group

Smith, Thomas Jefferson  
University of Wisconsin, August
Planar line families

Sparer, Gerson H.  
New York University, June
Units of algebraic number fields

Sparkman, Mary Clarice, Sister  
University of Texas, June
Minor - Education
Relationship and analogue between the finite calculus and the limit calculus

Sprague, Richard Howard  
University of Kentucky, January
Minor - Philosophy
Second variations of analytic functions

Sreedharan, V. P.  
Carnegie Institute of Technology, June
Function theoretic solutions of certain boundary value problems

Stanek, Peter F. G.  
University of Chicago, August
Two-element generation of the symplectic group

Starr, Selig  
George Washington University, June
Some algebraic aspects of the analysis of variance

Stasheff, James Dillon  
Princeton University, June
Homotopy associativity of H-spaces

Stearns, Richard Edwin  
Princeton University, October
Three person cooperative games without side payments

Steele, William Frank  
University of Pittsburgh, May
Summability of infinite sequences by submatrix methods

Steinberg, Leon  
University of Pennsylvania, June
Partitions into bounded summands from arbitrary sets

Stell, George Roger  
New York University, June
The solutions of the hierarchy equations of statistical mechanics for mixtures

Stengle, Gilbert Allan  
University of Wisconsin, June
A construction for solutions of an Nth order differential equation in the neighborhood of a turning point
Steward, Robert Franklin
Auburn University, June
Continuity and uncertainty of interpolating rational functions

Still, Harold Armstrong
Virginia Polytechnic Institute, June
Analysis of multiple covariance when the regression coefficients depend on the blocks

Stoddard, James Harwood
University of Michigan, June
Regular convergence in a paracompact space

Stolzenberg, Louis Gabriel
Massachusetts Institute of Technology, June
Minor - Linguistics
A maximal ideal space with no analytic structure

Stone, Charles Joel
Stanford University, October
Limit theorems for birth and death processes and diffusion processes

Stone, Jeremy Judah
Stanford University, January
Exponential polynomials on commutative semigroups

Struble, George Waring
University of Wisconsin, August
Orthogonal polynomials: variable-signed weight functions

Suh, Tae-il
Yale University, June
On isomorphisms of Zittle projective groups of Cayley planes

Summers, John Willis
University of California (Berkeley), June
On Kan fibre spaces

Summers, George William
Case Institute of Technology, June
Financing and initial operations of new firms

Suprunowicz, Konrad
University of Nebraska, February
Minor - Physics
Diagram normal forms and their applications to the theory of models

Suryanarayan, E. Ramnath
University of Michigan, June
The geometry of fluid flows in relativity

Tewarson, Reginald Prabhakar
Boston University, June
Solution of a generalized diffusion equation by difference methods

Thigpen, Charles Craig
Virginia Polytechnic Institute, June
Distribution of the largest observation in normal samples under nonstandard conditions

Throckmorton, Thomas Neil
Iowa State University, February
Structures of classification data

Thurber, James Kent
New York University, June
An asymptotic method for determining the lift distribution of a swept-back wing of finite span

Tilley, John Leonard
University of Florida, February
Minor - Education
Stress distribution of a rotating limacon

Trahan, Donald Herbert
University of Pittsburgh, August
Green’s theorem in the complex domain and related theorems

Trawinski, Benon John
Virginia Polytechnic Institute, June
Minor - Physics
Selection of the best treatment in a paired-comparison experiment

Troy, Allan
University of Illinois, October
Integral representations of cyclic groups of order p^2.

Troy, Daniel Joseph
St. Louis University, June
Minor - Philosophy
Hadamard operators on Lambert series

Tucker, Patricia Anne
University of Wisconsin, August
On the reduction of induced representations of finite groups

Varley, Eric
Brown University, June
Flows of dilatant fluids

Von Wolff, Mary Robert, Sister
University of Notre Dame, August
Densest admissible point sets for two star domains
Wachman, Murray
New York University, February
Essential isolated singularities in solution of elliptic partial differential equations with analytic coefficients

Wagner, Norman Rayfield, Jr.
Massachusetts Institute of Technology, June
Minor - Electrical Engineering
Existence theorem for a nonlinear boundary value problem in ordinary differential equations

Wainger, Stephen
University of Chicago, August
Special trigonometric series in higher dimensions

Wall, Francis Joseph
University of Minnesota, June
Minor - Industrial Engineering
Biostatistical linear models in longitudinal medical research

Walston, Dale Edouard
University of Texas, August
An existence and uniqueness theorem for integro-partial differential equations

Walter, Everett Leon
New Mexico State University, August
Generalized conjugate spaces

Warten, Ralph Martin
Purdue University, August
On the approximate solution of axially symmetric problems by means of finite differences

Webster, Porter Grigsby
Auburn University, June
Determination of the convergence of Lagrange interpolation by Runge's method

Weidlich, John Edward, Jr.
University of California (Berkeley), January
The asymptotic behavior of the real solutions of certain first order nonlinear differential systems

Weinberg, Elliot Carl
Purdue University, January
Higher degrees of distributivity in lattices on continuous functions

Weinberg, Israel Jacob
Massachusetts Institute of Technology, February
Minor - Physics
Symmetric finite deflections of circular plates subjected to edge forces and edge moments

Weitkamp, Harvey Meredith
University of Connecticut, June
Minor - Education
The equation $X^n + Y^n + Z^n = 0$ in rational binary matrices

Wells, Mark Brimhall
University of California (Berkeley), June
Simplification of normal form expressions for Boolean functions of many variables

White, Alvin Murray
Stanford University, January
Singularities of harmonic functions of three variables generated by a Whittaker-Bergman operator

Whiteman, Russell A.
Illinois Institute of Technology, January
Concerning certain classes of analytic functions and the Hadamard product

Whitman, Andrew Peter
Catholic University of America, February
Minor - Philosophy
Invariant connections in principal fiber bundles over locally homogeneous spaces

Whitten, Wilbur Carrington, Jr.
University of Pittsburgh, August
Projective coordinates and rigid motions in the plane of two complex variables interpreted in argand four-space

Williams, James Stanley
Washington State College, May
An evaluation of the worth of some selection indices

Williams, Leland Hendry
Duke University, June
Minor - Physics
Extended algebra of polynomials in several variables for a digital computer

Williams, Vincent Cassell
Harvard University, March
On conformal maps of regions of infinite connectivity
Wilson, Atholl Livingstone  
University of California (Berkeley), June  
An approach to n-person games

Wiser, Horace Clare  
University of Utah, June  
Decomposition and homogeneity of continua on a 2-manifold

Woods, Walter Max  
Stanford University, January  
Variables sampling inspection procedures which guarantee acceptance of perfectly screened lots

Worrell, John Mays, Jr.  
University of Texas, August  
Minor - Zoology  
Concerning scattered point sets

Yaspan, Arthur Joel  
Case Institute of Technology, June  
The inventory problem with periodic reorder

Yearout, Paul Harmon  
University of Washington, December  
A study of (m,n) distributive rings

Young, Lael Marcus  
Stanford University, June  
On certain cyclic extensions of the field of rational numbers

Youngdahl, Carl Kerr  
Brown University, June  
Transient thermal stresses in a circular cylinder

Yozwiak, Bernard James  
University of Pittsburgh, May  
Relations between the Lototsky and certain other methods of summability

Zaki, Ramzi Mohamed  
North Carolina State College, May  
Applications of linear programming techniques to some problems of production planning over time

Zechmann, Albert William  
Iowa State University, November  
Minor - Physics  
Problems in viscoelasticity

Ziemer, William Paul  
Brown University, June  
Integral currents mod 2

Zimmerman, Jack Miles  
University of Southern California, June  
Band-limited functions and improper problems in partial differential equations

NEWS ITEMS AND ANNOUNCEMENTS

THE FOURTH SUMMER INSTITUTE IN DYNAMICAL ASTRONOMY, jointly sponsored by the Yale University Observatory and the National Science Foundation, will be held at the Yale University Observatory, New Haven, Connecticut, from June 25 to August 3, 1962. The Institute is open to astronomers, physicists, and mathematicians interested in the field of celestial mechanics. The Institute wishes particularly to call the attention of mathematicians to the last two weeks of the program, which will emphasize the mathematical aspects of celestial mechanics.

Academic applicants who are accepted will receive stipends to defray expenses for themselves and their families, plus a travel allowance. For further information write Professor Dirk Brouwer, Yale University Observatory, Box 2023, Yale Station, New Haven, Connecticut.

The Summer Institute in Dynamical Astronomy is considered to be of special interest to mathematicians at this time, in view of the plan to combine it with the American Mathematical Society's Summer Seminar in Applied Mathematics in 1963, to be held at Cornell University. The Applied Mathematics Committee of the AMS selected "Space Mathematics" as the topic for the 1963 Summer Seminar, and its recommendation for a joint program with the Yale Observatory was approved by the Council at the Atlantic City meeting of the Society. The Organizing Committee will consist of J. Barkley Rosser and Dirk Brouwer, and several others yet to be selected.

Plans now call for a program of six weeks duration, subdivided into sections devoted to narrower topics. Although the sections will be related in the same broad field, each may also be considered as an
independent unit, so that individuals may conveniently attend for shorter periods on just those topics of special interest to them.

Further announcements on the Summer Seminar will be made as plans progress.

A $10,000 CORNING SCIENCE PRIZE to be awarded to the best book manuscript in the field of the natural or physical sciences for the general reader has been announced by Little, Brown and Company, in co-operation with Corning Glass Works. For further information write to Little, Brown and Company, 34 Beacon Street, Boston 6, Massachusetts.

AUTOMORPHIC FUNCTIONS OF SEVERAL COMPLEX VARIABLES is the subject of a conference to be held at Stanford University on June 12-15, 1962. For further information write to Professor Stefan Bergmann at the Department of Mathematics at Stanford University, Stanford, California.

NSF POSTDOCTORAL FELLOWSHIP AWARDS for March were announced March 22. Of the 185 fellowships granted for advanced study and research in the sciences, thirteen were in mathematics.

<table>
<thead>
<tr>
<th>Name</th>
<th>Present Institution</th>
<th>Fellowship Institution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bass, Hyman</td>
<td>University of Chicago</td>
<td>College de France</td>
</tr>
<tr>
<td>Dudley, Richard M.</td>
<td>Princeton University</td>
<td>University of California</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(Berkeley)</td>
</tr>
<tr>
<td>Edwards, Harold M.</td>
<td>Harvard University</td>
<td>Institute for Advanced Study</td>
</tr>
<tr>
<td>Guilleming, Victor W.</td>
<td>Harvard University</td>
<td>University of Paris</td>
</tr>
<tr>
<td>Hales, Alfred W.</td>
<td>California Institute of Tech-</td>
<td></td>
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<tr>
<td></td>
<td>nology</td>
<td></td>
</tr>
<tr>
<td>Knill, Ronald J.</td>
<td>University of Notre Dame</td>
<td>Cambridge University</td>
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<tr>
<td>Martin, Joseph M.</td>
<td>State University of Iowa</td>
<td>Northwestern University</td>
</tr>
<tr>
<td>McMillan, Daniel R., Jr.</td>
<td>University of Wisconsin</td>
<td>Institute for Advanced Study</td>
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<tr>
<td>Resnikoff, Howard L.</td>
<td>University of California</td>
<td>University of Paris</td>
</tr>
<tr>
<td></td>
<td>(Berkeley)</td>
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<tr>
<td>Rhodes, John L.</td>
<td>Massachusetts Institute of</td>
<td>University of Münster</td>
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<tr>
<td></td>
<td>Technology</td>
<td>Institut des Hautes Etudes,</td>
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<tr>
<td></td>
<td></td>
<td>Paris</td>
</tr>
<tr>
<td>Topping, David M.</td>
<td>Tulane University</td>
<td>Massachusetts Institute of</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Technology</td>
</tr>
<tr>
<td>Vandeneeynden, Charles L.</td>
<td>University of Oregon</td>
<td>University of Michigan</td>
</tr>
<tr>
<td>Vasquez, Alphonse T.</td>
<td>University of California</td>
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<tr>
<td></td>
<td>(Berkeley)</td>
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</table>

FORTY-NINE NATO POSTDOCTORAL FELLOWSHIPS IN SCIENCE have been announced by the State Department and the National Science Foundation. Designed to encourage further study abroad, these awards will enable U.S. Fellows to study in ten foreign nations, while scientists from NATO countries will come to the United States for further training. Of the awards just announced, 15 are in the life sciences, one in the social sciences, and 33 in the physical sciences and mathematics. The four awards in mathematics were as follows:

<table>
<thead>
<tr>
<th>Name</th>
<th>Present Institution</th>
<th>Fellowship Institution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foulser, David A.</td>
<td>University of Michigan</td>
<td>Oxford University</td>
</tr>
<tr>
<td>Hudson, Anne L.</td>
<td>Tulane University</td>
<td>University of Tubingen</td>
</tr>
<tr>
<td>Mayer, Alan L.</td>
<td>Princeton University</td>
<td>University of Pisa</td>
</tr>
<tr>
<td>Topping, David M.</td>
<td>Tulane University</td>
<td>University of Paris</td>
</tr>
</tbody>
</table>
Mr. D. R. BAKER of System Development Corporation has accepted a position as Computer Programmer with International Telephone and Telegraph, Paramus, New Jersey.

Mrs. R. A. BATTERTON of Thompson Ramo-Wooldridge, Incorporated has accepted a position as an Associate with Planning Research Corporation, Los Angeles, California.

Dr. GERTRUDE BLANCH of Wright-Patterson Air Force Base has accepted a position as Senior Scientist in the Department of Defense, Washington, D. C.

Dr. JOSEPH BRAM of David Taylor Model Basin has accepted a position as a Mathematical Analyst with Operations Evaluation Group, Massachusetts Institute of Technology, Arlington, Virginia.

Professor J. R. BRITTON has been appointed Chairman of the Department of Applied Mathematics at the University of Colorado.

Mr. H. H. BROWN of North American Aviation Incorporated has accepted a position as Mathematician with National Aeronautics and Space Administration, Greenbelt, Maryland.

Mr. J. R. BRUGGENMAN of The Martin Company has accepted a position as Senior Engineer at Thiokol Chemical Corporation, Brigham City, Utah.

Dr. E. W. CANNON of the National Bureau of Standards, has been awarded the Department of Commerce Gold Medal for Exceptional Service.

Mr. E. D. CAREY of Federal Telecommunications Laboratories has accepted a position as Mathematician with the General Electric Company, MSVD, Philadelphia, Pennsylvania.

Dr. J. F. CARPENTER of Dalmo Victor Company has accepted a position as Section Head with Aerospace Corporation, El Segundo, California.

Mr. C. H. COOK of The Martin Company has been appointed to an assistant professorship at the University of Oklahoma.

Dr. CHANDLER DAVIS of New York University has been appointed to an associate professorship at the University of Toronto.

Dr. R. M. DAVIS of David Taylor Model Basin has accepted a position as Staff Assistant with the Department of Defense, Washington, D. C.

Mr. D. G. de FIGUEIREDO of Universidade do Ceara has been appointed to a research professorship at the Instituto de Matematica Pura e Aplicada, Rio de Janeiro, Brazil.

Captain R. L. EISENMAN of the Air Force Academy will be assigned for the academic year 1962-1963 to the new Research Department of the Air Force Academy, Colorado.

Mr. MAREK FISZ of Columbia University has been appointed to a professorship at New York University.

Dr. LEONARD GELLER of Combustion Engineering Company has accepted a position as Senior Associate with S. M. Stoller Associates, New York.

Mr. J. J. GILVARRY of Allis-Chalmers Manufacturing Company has accepted a position as Senior Staff Scientist at General Dynamics Astronautics, San Diego, California.

Associate Professor DONALD GREENSPAN of Purdue University has accepted a position as Staff Member at the Mathematics Research Center, United States Army, University of Wisconsin.

Mr. SIN HITOTUMATU of the University of Tokyo has been appointed to a professorship at St. Paul's University, Tokyo, Japan.

Mr. S. S. HOLLAND of the National Academy of Science has been appointed to an assistant professorship at Boston College.

Mr. R. E. HOLZMAN of Sylvania Electric Products Incorporated has accepted a position as Research Engineer with Jet Propulsion Laboratory of the University of California, Pasadena, California.

Assistant Professor W. B. HOUSTON, JR. of Carleton College has been appointed to an assistant professorship at Antioch College.

Mrs. A. IONESCU TULCEA of Yale
University has been appointed Research Associate at the University of Pennsylvania.

Mr. C. IONESCU TULCEA of Yale University has been appointed to an associate professorship at the University of Pennsylvania.

Dr. R. C. KAO of System Development Corporation has accepted a position as Research Economist with The Rand Corporation, Santa Monica, California.

Mr. MASUDA KATSUHIKO of Yamagata University, Japan has been appointed to a professorship at Okayama University, Japan.

Dr. R. B. KELMAN of Remington Rand UNIVAC Division of the Sperry Rand Corporation has been appointed a Lecturer at Howard University.

Dr. H. P. KERFOOT, JR. of the University of Southern California, has accepted a position as Director, Mechanical and Mathematical Sciences Laboratory of Lockheed Missile and Space Company, Palo Alto, California.

Mr. G. W. KIMBLE of the University of California has been appointed to an assistant professorship at Long Beach State College.

Mr. W. E. KOPKA of Service Bureau Corporation has accepted a position as Advisory Programmer, International Business Machines Corporation, Poughkeepsie, New York.

Professor TADAO KUBO of Kyoto Prefectural University, Japan has been appointed to a professorship at Kobe University, Japan.

Dr. H. E. KYBURG of Wesleyan University has been appointed to an associate professorship at the University of Denver.

Mr. L. L. LAYTON of David Taylor Model Basin, Washington, D. C. has accepted a position as Aerospace Technologist at the National Aeronautics and Space Administration, Greenbelt, Maryland.

Dr. J. H. LEVIN of Datamatic Corporation has accepted a position as Director, Programming Systems Division with Minneapolis Honeywell Regulator Company, Wellesley Hills, Massachusetts.

Dr. G. E. LEWIS of New York University has been appointed to an assistant professorship at the University of Colorado.

Mr. W. G. MADISON of National Company Incorporated has accepted a position as Senior Engineer with Edgerton, Gemeshhausen and Grier Incorporated, Boston, Massachusetts.

Mr. G. T. MALLISTER of the University of California has accepted a position as Research Mathematician with Sperry Rand Corporation, Sudbury, Massachusetts.

Dr. J. GAYLORD MAY of the U. S. Army Ordnance, Aberdeen Proving Ground has been appointed to an assistant professorship at Wake Forest College.

Mr. R. A. MICHELSON of the University of Washington has accepted a position as Associate Research Engineer with The Boeing Company, Renton, Washington.

Mr. DIETRICH MORGENSTERN of the University of Munster, Germany has been appointed to a professorship at the Universitat Freiburg, Freiburg, Germany.

Mr. D. E. MORRILL of Thiokol Chemical Corporation has accepted a position with International Business Machines Corporation, Poughkeepsie, New York.

Mr. P. D. OYER of Westinghouse Electric Company has accepted a position as Research Mathematician with Armour Research Foundation, Annapolis, Maryland.

Dr. L. G. PECK has been appointed Chairman of the Mathematics Department at the State University of New York.

Dr. A. J. PENICO of General Telephone and Electronics Laboratories Incorporated has been appointed Research Mathematician at Stanford University.

Dr. W. B. PENNINGTON of Westfield College, University of London has been appointed to a professorship at the University College of Wales.

Mr. H. K. PHILIPS of General Precision Incorporated has accepted a position as Programmer Analyst with National Computer Analysts, Incorporated, Princeton, New Jersey.

Dr. R. R. RAU of the University of Illinois has been appointed to an assistant professorship at Ohio State University.

Dr. R. W. RECTOR of Space Technology Laboratories Incorporated has accepted a position as Administrative Manager of the Aerospace Corporation, Los Angeles, California.

Mr. G. S. RINEHART of the Univer-
University of California, Berkeley, California has been awarded a National Academy of Sciences-National Research Council Postdoctoral Research Fellowship.

Professor HANNO RUND of the University of Natal, South Africa has been appointed to a professorship at the University of South Africa.

Mr. D. A. RUSSO of Grumman Aircraft Engineering Corporation has accepted a position as Project Engineer with the Defense Electronics Products, Morristown, New Jersey.

Dr. W. C. SANGREN of General Dynamics Corporation has accepted a position as Vice President and Director of West Coast Operations, Computer Applications Incorporated, San Diego, California.

Mr. H. E. SCHLOSS of the Hughes Aircraft Company has accepted a position as a Mathematician with the Burroughs Corporation, Pasadena, California.

Mr. J. D. SIDLEY of Georgetown University has accepted a position as Mathematician with the Bureau of Naval Weapons, Washington, D. C.

Mr. M. S. SKAFF of The University of Illinois has accepted a position as Computing Analyst with the Douglas Aircraft Company, Santa Monica, California.

Dr. C. V. L. SMITH of National Aeronautics and Space Administration has accepted a position as Head of the Mathematics and Computer Programs, Research Division, Atomic Energy Commission, Germantown, Maryland.

Mr. N. R. STANLEY of Sperry Rand Corporation has accepted a position as Senior Mathematical Physicist with The Perkin-Elmer Corporation, Norwalk, Connecticut.

Mr. H. R. STEVENS of Duke University has been appointed to an assistant professorship at Pennsylvania State University.

Dr. HARUO SUZUKI of Tohoku University, Sendai, Japan has been appointed to an assistant professorship at Kyushu University, Fukuoka, Japan.

Mr. J. T. WARFIELD of Johns Hopkins University has accepted a position as Senior Engineer with the Martin-Marietta Corporation, Baltimore, Maryland.

Mr. R. J. WERNICK of General Electric Company has accepted a position as Applied Mathematician with Mechanical Technology Incorporated, Schenectady, New York.

Dr. NEAL ZIERLER after a leave of absence has returned to Lincoln Laboratory, Massachusetts Institute of Technology.

The following promotions are announced:

C. B. BELL, San Diego State College, to an associate professorship.

J. L. BOAL, University of South Carolina, to an associate professorship.

O. S. BOWMAN, Bridgewater College, to an assistant professorship.

J. CHOVER, University of Wisconsin, to an associate professorship.

L. GARFIN Pacific Mutual Life Insurance Company, Los Angeles, California, to an Actuary.

C. A. HUTCHINSON, University of Colorado, to Associate Dean of the College of Engineering.

R. A. JORGENSEN, JR. Andrews University, to an assistant professorship.

F. B. KNIGHT, University of Minnesota, to a professorship.

P. J. LAUFER, College Militaire Royal de St. Jean, to a professorship.

J. R. McCORD, Massachusetts Institute of Technology, to a research associate.

E. E. POSEY, Virginia Polytechnic Institute, to a professorship.

Y. SIBUYA, Ochanomizu University, Tokyo, Japan, to an assistant professorship.

T. H. SLOOK, Temple University, to an assistant professorship.

J. SONNER, University of South Carolina, to a professorship.

L. ZEMMER, JR., University of Missouri, to a professorship.

The following appointments to instructorships are announced:

Cornell University: D. S. GILLMAN; Ivanovo State Furmanov Teachers' College, U.S.S.R.: M. GREENDLINGER; Keukuk University, Seoul, Korea: S. K. KIM; Montana State University: M. E. MANIS; Xavier University: W. A. HORN.
Deaths:

Professor ODDVAR BJORGUM of the University of Bergen, Norway died on December 22, 1961 at the age of 45.

Dr. H. T. ENGSTROM of Remington Rand Division of Sperry Rand Corporation died on March 8, 1962 at the age of 60.

Professor Emeritus M. G. GABA of The University of Nebraska died on February 11, 1962 at the age of 78.

Mrs. L. F. HUTCHINSON of Bryn Mawr College, died on January 29, 1962 at the age of 48.

Professor Emeritus F. S. NOWLAN of the University of British Columbia, Vancouver, Canada died on September 8, 1961 at the age of 75. He had been a member of the Society for 49 years.

Professor L. M. REAGAN of the University of Wichita died on December 29, 1961 at the age of 57. He had been a member of the Society for 22 years.

ERRATA

The announcement on page 91 of the April issue of the NOTICES concerning Dr. B. Bernholtz should read as follows:

Dr. B. BERNHOLTZ of the Hydro-Electric Power Commission of Ontario, Canada has been appointed to an Associate Professorship in the Department of Industrial Engineering at the University of Toronto, Toronto, Canada.

The announcement on page 549 of the December issue of the NOTICES concerning Dr. H. E. CONNER should read as follows:

Dr. H. E. CONNER on leave of absence from Lincoln Laboratory, Massachusetts Institute of Technology has been appointed a member of the Mathematics Research Center, United States Army, University of Wisconsin.

The announcement on page 92 of the April issue of the NOTICES concerning Mr. D. J. OSTROFF should read as follows:

Instructorship: San Fernando Valley State College: D. J. OSTROFF.

NEWS ITEMS

THE MATHEMATISCHE FOR-SCHUNGSINSTITUT OBERWOLFACH announces its schedule of meetings for 1962 as follows:

June 12-16. Foundation of Geometry
Chairmen: F. Bachmann, Kiel; E. Sperner, Hamburg; R. Baer, Frankfurt.

June 21-25. Workshop
Chairman: R. Baer, Frankfurt.

Chairman: L. Fejes Toth, Budapest August 6-10. Group Theory
Chairmen: R. Baer, Frankfurt; H. Wie-landt, Tübingen
August 27-31. Workshop
Chairman: H. König, Aachen.

September 3-7. Partial Differential Equations
Chairmen: W. Haack and G. Hellwig, Berlin

September 17-20. History of Mathematics
Chairman: J. E. Hofmann, Ichenhausen

September 24-29. Geometry
Chairman: K. H. Weise, Kiel

October 1-6. Functional Analysis
Chairman: G. Köthe, Heidelberg

Any additional meetings or changes will be announced.

THIRTY-SIX FELLOWSHIPS FOR AMERICAN WOMEN will be awarded by the American Association of University Woman Educational Foundation for the period July 1, 1963, to June 30, 1964. These awards, in varying amounts, are unrestricted as to age or field and may be used in the United States or abroad.

Application forms are available from Fellowships Office, AAUW Educational Foundation, 2401 Virginia Avenue, N. W., Washington 7, D. C. Requests for applications must indicate present academic status. The deadline for filing applications is December 1, 1962, and notification of awards will be made on March 1, 1963.

For information about fellowships offered for women of other countries, write to the above address.
NEW AMS PUBLICATIONS

PROCEEDINGS OF SYMPOSIA IN APPLIED MATHEMATICS

Volume 13

HYDRODYNAMIC INSTABILITY
Edited by G. Birkhoff, R. Bellman, and C. C. Lin

319 pages; $7.90 List Price; 25% discount to members.

Proceedings of the Symposium in Applied Mathematics of the American Mathematical Society, held in New York City, April 1960. Papers by: C. C. Lin and D. J. Benney; K. M. Case; Marshall N. Rosenbluth; R. J. Donnelly; Garrett Birkhoff; Russell E. Duff; John W. Miles; O. M. Phillips; M. S. Longuet-Higgins; M. St. Denis; Eberhard Hopf; J. Kampé de Fériet; Robert H. Kraichnan; Joseph B. Keller; S. Ulam; Brockway McMillan; Richard Bellman; R. E. Kalman; J. P. LaSalle.

MEMOIR 42
THE FACTORIZATION OF CYCLIC REDUCED POWERS
BY SECONDARY COHOMOLOGY OPERATIONS

by Arunas Liulevicius

113 pages; $1.90 List Price; 25% discount to members.

J. F. Adams has proved that the Steenrod squares $Sq^{2r+4}$ can be factored into stable secondary cohomology operations. The analogous theorem for cyclic reduced powers $p^n$ is proved here. One consequence of the result is the triviality of the mod $p$ Hopf invariant in most dimensions.

NEWS ITEMS AND ANNOUNCEMENTS

THE COMMITTEE ON INTERNATIONAL EXCHANGE OF PERSONS of the Conference Board of Associated Research Councils has for distribution a list of foreign scholars available for remunerative positions in American universities and colleges for the Summer Term 1962 and Academic Year 1962-1963. This list is compiled annually by the Committee and includes information about scholars recommended by the United States Educational Commissions and Foundations abroad. Each scholar is eligible for a Government travel grant covering costs of round-trip transportation to the United States if arrangements are completed for a lecturing or research appointment at an American university or college.

Mathematicians on this year’s list are Godfrey L. Isaacs, Head of the Department of Mathematics at the University of Witwatersrand, South Africa, and Vasanti S. Huzurbazar, Head of the Department of Mathematics and Statistics at the University of Poona, India. For details write Mrs. John D. Leary, Program Officer, Conference Board of Associated Research Councils, 2101 Constitution Avenue, N.W., Washington 25, D. C.
Editor, the NOTICES

Several years ago J. P. Phillips, in an article "The individual in chemical research" (Science, vol. 121 (1955), pp. 311-312), gave a rough measure of the decrease in individual research in contrast to team research in chemistry as measured by the number of papers published each year by a single author compared with the number of papers published and having two, or more, authors. This study was based on a single journal, the Journal of the American Chemical Society, with partial results for the Journal of Chemical Physics. This was not a deep and exhaustive study of the increase in team research but it was, as the author claimed, interesting to learn that between 1918 and 1950 the number of papers published by a single author in the Journal of the American Chemical Society decreased monotonically from 45% to 14% of the total papers published in that journal.

The last paragraph of the article of Phillips follows. "The real frontier for the individual in science, however, would appear to be in mathematics. In 1950, 98 percent of the papers in the American Mathematical Monthly were written by individuals."

Evidence from a large sample of papers is provided in the following paragraphs. This census, like that of Phillips, is informal and is not intended to have breadth or depth. Also, I join Phillips in not drawing conclusions, moral or otherwise.

Table 1 is based on titles in Mathematical Reviews Vol. 1 (1940), Vol. 11 (1950) and Vol. 21 (1960). These are "titles" in contrast to "research papers" to emphasize that I have counted the titles without making judgments as to which papers represent research and which do not represent research except in the opinion of their authors. Books containing new results as well as some advanced textbooks are reviewed in this journal and their titles were counted.

Mathematical Reviews has always reviewed some of the papers in some fields closely related to mathematics, such as astrophysics, mechanics, etc. There would be common agreement to exclude certain of these fields as not being mathematics but there are others about which there would be some debate. I've arbitrarily settled this by excluding those fields now reviewed in Part B (the green-stripe part) of Mathematical Reviews even though this excludes probability and statistics, both of which are usually considered as genuine mathematics.

TABLE 1. MATHEMATICAL REVIEWS

<table>
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<th>Year</th>
<th>Number of titles</th>
<th>Number jointly authored</th>
<th>Percent jointly authored</th>
</tr>
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<tbody>
<tr>
<td>1940</td>
<td>1579</td>
<td>92</td>
<td>5.8</td>
</tr>
<tr>
<td>1950</td>
<td>3298</td>
<td>214</td>
<td>6.5</td>
</tr>
<tr>
<td>1960</td>
<td>4393</td>
<td>473</td>
<td>10.8</td>
</tr>
</tbody>
</table>

Mathematical Reviews is an international publication in that it reviews essentially all of the mathematics published throughout the world. Since it only began publishing in 1940, it is necessary to go to other sources for evidence prior to 1940. Rather than examine an older review journal, I've selected, arbitrarily, three of the older research journals of the United States in forming Table 2. The journals are the American Journal of Mathematics, the Annals of Mathematics and the Transactions of the American Mathematical Society. In Table 2 the numbers following the year are the totals for all three of these journals. Since Vol. 21 of the Annals of Mathematics is dated 1919-1920 and Vol. 22 is dated 1920-1921, I've included both volumes in the tabulation for the year 1920.

TABLE 2. THREE U.S. JOURNALS

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of papers published</th>
<th>Number jointly authored</th>
<th>Percent jointly authored</th>
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<tr>
<td>1920</td>
<td>89</td>
<td>2</td>
<td>2.2</td>
</tr>
<tr>
<td>1930</td>
<td>170</td>
<td>7</td>
<td>4.1</td>
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<tr>
<td>1940</td>
<td>170</td>
<td>31</td>
<td>18.2</td>
</tr>
<tr>
<td>1950</td>
<td>214</td>
<td>39</td>
<td>18.2</td>
</tr>
<tr>
<td>1960</td>
<td>228</td>
<td>29</td>
<td>12.7</td>
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</tbody>
</table>
The percentages in Table 2 are conspicuously not monotone. In each of the three years 1940, 1950, 1960 the American Journal of Mathematics had a greater percent of jointly authored papers than either of the other two journals. Even if this journal is excluded and one makes the table for the other two, the percentages are still not monotone. In fact, the percentage column will not be monotone if formed for any combination of two of these journals for the years considered in Table 2.

W. R. Utz

Editor, the NOTICES

The number of those institutions whose Departments of Mathematics have recently begun, or are in the process of starting doctoral programs is, to say the least, surprisingly large. Whether the magnitude is to be regarded with alarm or with satisfaction is not the present concern, which is rather that they should be good doctoral programs.

Institutions inaugurate doctoral programs for, no doubt, a variety of reasons. Among these are their desires to meet the repeated pleas of the smaller colleges for mathematical doctors, their desire to meet their own staffing problems (by way of teaching assistants), their normal aspirations for bettering themselves and, possibly, their hope of financial assistance. In some instances the pressure for a doctoral program comes from within the Department of Mathematics and in other instances it is from the administration.

The real question here is not whether there should be more doctoral programs (there are more and there are more coming into existence) but whether that body of mathematicians who have, in recent months, been so seriously concerned with maintaining standards in this context, can and will play the role of mathematical midwives. That this sort of obstetrical assistance is needed, seems to me beyond question.

A few years ago, when this matter had not reached its present proportions, I suggested to the Council that it should appoint a visiting committee to give aid and assistance to those Departments of Mathematics who desired it and who might profit from it. Recently, I have again transmitted this proposal, in the present instance to Professor Bing's committee, but with what results I do not know.

Relative to this proposal let me make clear that it is in no sense whatsoever an accrediting device.

It would be a sort of "consultant's bureau" comprised of mathematicians of repute, not necessarily always those of great distinction, but mathematicians who (i) are held in some esteem for their research, (ii) have had experience in directing dissertations, (iii) are now and have been concerned with formulating and maintaining a reputable graduate program, (iv) have some sense of the financial difficulties involved and (v) are cognizant of the present national state of our art in most of its ramifications - teaching loads, availability of mathematicians with certain training and talent, and so on. Of course they should be wise beyond measure and tactful beyond reproach. (Needless to say, in a mathematical sense.)

The first duty of such a group would be to prepare a prospectus for a sound doctoral program - necessary library facilities, suitable courses (and texts!), suggestions as to seminars, teaching loads, salaries, and other pertinent items. They should then - in the most discreet fashion possible - advertise their existence and their availability.

Should it be so that a Department of Mathematics and the administration of an institution made clear the desire and necessity for a consultant, one would be assigned by the committee. Upon visiting the institution he would consult with all parties, give whatever advice tact and the situation called for and report to the institution and to the committee upon his visit. Such reports would, of course, be confidential.

It is not my thought, even in this best possible of worlds, that such a bureau would solve all problems inherent in our present position. Nothing will. It could, I think, do much good over the next ten years. Reputable doctoral programs are not brought into existence with pen and paper, even if some of the latter is of the appropriate green. It is not a short-term program and I contemplate the notion that a given institution might be visited several times over a period of years.

Alexander Doniphan Wallace
MEMORANDA TO MEMBERS

THE EMPLOYMENT REGISTER

The Mathematical Sciences Employment Register, established by the American Mathematical Society, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics, will be maintained at the Summer Meeting at the University of British Columbia at Vancouver, British Columbia, Canada, on August 28, 29, and 30, 1962. The Register will be conducted from 9:00 A.M., to 5:00 P.M., on each of these three days in Room 1221 of the Buchanan Building.

There is no charge for registration, either to job applicants or to employers, except when the late registration fee for employers is applicable. Provision will be made for anonymity of applicants upon request and upon payment of $3.00 to defray the cost involved in handling anonymous listings.

Job applicants and employers who wish to be listed will please write to the Employment Register, 190 Hope Street, Providence 6, Rhode Island, for application forms or for position description forms. These forms must be completed and returned to Providence not later than August 3, 1962, in order to be included free of charge in the listings at the Summer Meeting in Vancouver. Forms which arrive after this closing date, but before August 10, will be included in the register at the meeting for a late registration fee of $3.00, and will also be included in the printed listings, but not until ten days after the meeting. The printed listings will be available for distribution both during and after the meeting.

It is essential that applicants and employers register at the Employment Register Desk promptly upon arrival at the meeting to facilitate the arrangement of appointments.

DUES OF RECIPROCITY MEMBERS
OF THE
MATHEMATICAL SOCIETY OF JAPAN

Owing to increasing printing and other costs, the Mathematical Society of Japan has found it necessary to raise members' dues for 1962. At the same time it has been decided to give reciprocity members a choice of journal privileges, with the dues scaled accordingly. Reciprocity members in the United States will belong to either of two categories, according to whether they are (A) interested only in the Journal of the Mathematical Society of Japan (in European languages), or (B) interested also in receiving Sugaku (in Japanese). The dues for these two groups are now as follows:

<table>
<thead>
<tr>
<th></th>
<th>1961</th>
<th>1962</th>
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<tbody>
<tr>
<td>Group A</td>
<td>$2.00</td>
<td>$2.50</td>
</tr>
<tr>
<td>Group B</td>
<td>$3.50</td>
<td>$4.00</td>
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The NOTICES of the Society (in Japanese) are sent to all members in both groups.

Most reciprocity members paid $2.50 in dues for 1961, and have been sent issues No. 1 and No. 2 of Volume 13 of Sugaku, issued in fiscal year 1961, which ended March 31, 1962. The new policy change was only recently agreed upon, and it was decided to put it into effect immediately. Therefore the remaining issues of Volume 13 of Sugaku will be sent only to those members who specifically request it, and make an additional payment of $1.00 to dues for 1961.

New applicants for membership are requested to indicate whether they wish to belong in Group A or Group B.
During the interval from February 21, 1962 through April 27, 1962, the papers listed below were accepted by the American Mathematical Society for presentation by title. Readers may wish to refer to Page 713 of the November, 1960 issue (No. 49 of these NOTICES) where it is explained in detail that the presentation of papers by title is now dissociated from meetings of the Society.

After each title on this program is an identifying number. The abstract of the paper will be found following the same number in the section on Abstracts of Contributed papers in this issue of these NOTICES.

(1) Concerning Abelian differentials on parabolic Riemann surfaces. Preliminary report
Professor R. D. M. Accola, Brown University (62T-151)

(2) On generalized harmonic measures of Riemann surfaces. I. Preliminary report
Professor R. D. M. Accola, Brown University (62T-152)

(3) On generalized harmonic measures on Riemann surfaces. II. Preliminary report
Professor R. D. M. Accola, Brown University (62T-153)

(4) On $T^n$-functions
Professor M. A. Al-Bassam, Texas Technological College (62T-176)

(5) Some existence theorems of differential equations of noninteger order
Professor M. A. Al-Bassam, Texas Technological College (62T-177)

(6) Solution of loop equations by adjunction
Professor Rafael Artzy, Rutgers, The State University (62T-163)

(7) Enumeration and the Grzegorczyk hierarchy
Professor Paul Axt, Michigan State University (62T-124)

(8) On the construction of models of analytic polyhedra
Professor Stefan Bergman, Stanford University (62T-120)

(9) Some Abelian results for Dirichlet series
Professor W. E. Briggs, University College, London, England (62T-130)

(10) Formal power series
Dr. E. D. Cashwell and Dr. C. J. Everett, Los Alamos Scientific Laboratory, Los Alamos, New Mexico (62T-131)

(11) An elementary estimate for the k-free integers
Professor Eckford Cohen, University of Tennessee (62T-125)

(12) On cartesian products which are homotopy manifolds
Professor M. L. Curtis and Professor D. R. McMillan, Florida State University (62T-127)

(13) On certain sequence to sequence transformations which preserve convergence
Professor D. F. Dawson, North Texas State University (62T-158)

(14) Concerning a theorem of Hadamard
Professor D. F. Dawson, North Texas State University (62T-159)

(15) Convergence and divergence of continued fractions
Professor D. F. Dawson, North Texas State University (62T-159)

(16) A solvable infinite class of AEA formulas with axioms of infinity
Professor Burton Dreben, Mr. A. S. Kahr and Professor Hao Wang, Harvard University (62T-106)

(17) Invariant subspaces of tridiagonal operators
Dr. P. L. Duren, Stanford University (62T-114)

(18) Open 3-manifolds which are simply connected at infinity
Professor C. H. Edwards, Jr., University of Wisconsin (62T-128)

(19) On the rate of convergence of a symmetric successive overrelaxation iterative method. Preliminary report
Mr. L. W. Ehrlich, The University of Texas (62T-162)

(20) Hierarchies in recursive function theory
Mr. H. B. Enderton, Harvard University (62T-148)
(21) Directed distance geometry
Professor C. M. Fulton, University of California, Davis (62T-171)

(22) Partial recursive functionals of finite type
Dr. R. O. Gandy, The University, Manchester, England (62T-149)

(23) Complete predicates for the analytic hierarchy
Dr. R. O. Gandy, The University, Manchester, England (62T-150)

(24) Brownian motion and harmonic forms on a compact manifold. Preliminary report
Dr. R. A. Gangolli, Massachusetts Institute of Technology (62T-123)

(25) A deformation theory for algebras
Professor Murray Gerstenhaber, University of Pennsylvania and Institute for defense Analyses, Princeton, New Jersey (62T-122)

(26) Poisson's equation and generalized axially symmetric potential theory
Professor R. P. Gilbert, Institute for Fluid Dynamics and Applied Mathematics, University of Maryland (62T-129)

(27) Note on difference approximations with non-negative coefficients for partial differential equations
Dr. Donald Greenspan, Mathematics Research Center, University of Wisconsin (62T-165)

(28) On a problem of Erdös and Tarski
Mr. W. P. Hanf, IBM Corporation and University of California, Berkeley (62T-172)

(29) A two dimensional analogue to boolean algebras
Mr. C. M. Howard, University of California, Berkeley (62T-140) (Introduced by William Craig)

(30) Indeterminate forms on $E^n$
Mr. Seymour Kass and Mr. Arthur Fine, Illinois Institute of Technology (62T-156)

(31) A new potential operator for recurrent Markov chains
Professor J. G. Kemeny and Professor J. L. Snell, Dartmouth College (62T-170)

(32) On metric compactifications, Preliminary report
Mr. L. R. King, Duke University (62T-154)

(33) Some uses of the second conformal structure on strictly convex surfaces
Professor T. S. Klotz, University of California, Los Angeles (62T-146)

(34) Non-Desarguesian planes of order $2^{2m+1}$, Preliminary report
Mr. D. E. Knuth, California Institute of Technology (62T-137)

(35) New characterizations of the class of hyperarithmetic sets of natural numbers.
Professor Georg Kreisel, Institut Henri Poincaré, Paris, France (62T-174) (Introduced by Dr. S. H. Gould)

(36) Formalizing predicative analysis
Professor Georg Kreisel, Institut Henri Poincaré, Paris, France (62T-175) (Introduced by Dr. S. H. Gould)

(37) Traffic in a polar coordinate net of streets
Professor E. O. A. Kreyszig, Ohio State University (62T-116)

(38) Traffic in a city having radial and circular streets
Professor E. O. A. Kreyszig, Ohio State University (62T-117)

(39) On a class of integral operators
Professor E. O. A. Kreyszig, Ohio State University (62T-118)

(40) Integral operators for generating harmonic functions of four variables
Professor E. O. A. Kreyszig, Ohio State University (62T-119)

(41) Imbedding and immersion of projective spaces
Mr. J. P. Levine, Massachusetts Institute of Technology (62T-142)

(42) Imbedding and immersion of manifolds
Mr. J. P. Levine, Massachusetts Institute of Technology (62T-143)

(43) Convergence of series of iterates
Professor M. A. McKiernan, University of Waterloo, Ontario, Canada (62T-173) (Introduced by Professor Gerald Berman)

(44) Two theorems on contraproducive centers
Mr. T. G. McLaughlin, University of California, Los Angeles (62T-126)

(45) A note on pseudo D. C. pairs
Mr. T. G. McLaughlin, University of California, Los Angeles (62T-144)
(46) Stiefel-Whitney classes of a nonorientable manifold
Professor W. S. Massey, Yale University (62T-141)

(47) Categoricity in power
Mr. M. D. Morley, Berkeley, California (62T-136)

(48) On some algebraico-topological analogs
Mr. A. A. Mullin, University of Illinois (62T-103)

(49) On the mutant double-cosets of any group
Mr. A. A. Mullin, University of Illinois (62T-138)

(50) On differences of algebraic structures
Mr. A. A. Mullin, University of Illinois (62T-168)

(51) On complete sets of mutually orthogonal latin squares of Euler conjecture orders. Preliminary report
Dr. E. T. Parker, Remington Rand UNIVAC Division, St. Paul, Minnesota and University of Minnesota (62T-133)

(52) Failure of the Krull-Schmidt theorem for integral representations
Professor Irving Reiner, University of Illinois (62T-111)

(53) On the representation of algebraic integers as the sum of four squares
Professor G. J. Rieger, Purdue University and University of Munich, W. Germany (62T-108)

(54) 0-sequences and 0-nets
Mr. L. C. Robertson and Mr. S. P. Franklin, University of California, Los Angeles (62T-139)

(55) A generalization of the flexible law
Mr. D. J. Rodabaugh, Illinois Institute of Technology (62T-121)

(56) The dual space of $H^p$, Preliminary report
Dr. B. W. Romberg, Arthur D. Little, Incorporated, Cambridge, Massachusetts (62T-112)

(57) Coerciveness in $L^p$ for bilinear forms. Preliminary report
Professor Martin Schechter, New York University (62T-115)

(58) Nonexistence of some unsymmetrical partially balanced incomplete block designs
Professor S. S. Shrikhande, Banaras Hindu University, Mr. D. Raghav Rao and Mr. S. K. Tharthare, Bombay University, India (62T-109)

(59) Existence and uniqueness of solutions of the Poisson interface problem
Dr. D. P. Squier, California Research Corporation, La Habra, California (62T-160)

(60) On the existence and analyticity of solutions of regular variational problems
Professor Guido Stampacchia, Courant Institute of Mathematical Sciences, New York University and University of Pisa, Italy (62T-104)

(61) Kernel constructions and Borel sets
Professor A. H. Stone, University of Rochester (62T-134)

(62) Borel isomorphism and $o$-discreteness
Professor A. H. Stone, University of Rochester (62T-135)

(63) An ergodic application of Banach limits. II
Professor Louis Sucheston, University of Wisconsin, Milwaukee (62T-107)

(64) Extreme points in order intervals of self-adjoint operators
Mr. D. M. Topping, Tulane University (62T-113)

(65) On the truncation error associated with a class of elliptic equations
Dr. P. M. Treuenfels, Minneapolis-Honeywell Regulator Company, Minneapolis, Minnesota (62T-147)

(66) Concerning homogeneity in totally ordered, connected topological spaces
Professor L. B. Treybig, Tulane University (62T-102)

(67) A sequence of rational functions with application to approximation by bounded analytic functions
Professor J. L. Walsh, Harvard University (62T-110)

(68) A direct characterization of recursive predicates
Professor Hao Wang, Harvard University (62T-164)

(69) An interface problem in the theory of acoustic wave fields
Dr. Peter Werner, Mathematics Research Center, University of Wisconsin (62T-105)
(Introduced by Professor R. E. Langer)

(70) A class of nilstable algebras
Mr. W. G. Witthoft, Illinois Institute
AN INSTRUCTIONAL CONFERENCE ON MATHEMATICAL PROBABILITY is to be held during the fortnight March 28 to April 11, 1963, in the Department of Mathematics, Durham Colleges, University of Durham, under the auspices of The London Mathematical Society.

This will be the second in the London Mathematical Society's series of Instructional Conferences; the first, on Functional Analysis and some of its applications, was held in April, 1961. On this occasion the subject will be Mathematical Probability, and while it is hoped that there will be much in the program to interest specialists the main purpose of the Conference is to meet the needs of mathematicians and statisticians who would like a thorough introduction to this field.

The Conference will begin with two series of introductory lectures delivered in parallel on the first day; the first of these, on the applications of probability theory, will consist of informal accounts of some practical problems which have motivated much recent theoretical work, while the second will survey some of the mathematical prerequisites for the subsequent courses.

The introductory lectures will be followed by a course of basic lectures on the fundamentals of the subject; these will be supplemented by classes for informal discussion.

The second part of the Conference will be devoted to three series of advanced lectures. Each of these, starting where the basic course leaves off, will attempt to give a thorough review of some branch of the subject in which recent research has been most active.

Arrangements are being made for accommodations and full board to be provided at the Durham Colleges at a charge of thirty shillings per day per person, inclusive of gratuities. Full details of the organisation and program of the Conference will be advertised in due course, but those wishing to be kept informed of the arrangements are encouraged to register now. The registration fee of two guineas (which covers participation in all the scientific activities of the Conference) should be sent to the Secretary, Science Laboratories, South Road, Durham, England, and the envelope should be marked 'Probability Conference;' remittances should be made payable to 'The London Mathematical Society.'
ABSTRACTS OF CONTRIBUTED PAPERS

The April Meeting in Atlantic City
April 16-19, 1962

590-40. MELVIN DRESHER, RAND Corporation, 1700 Main Street, Santa Monica, California. On an infinite game with a discontinuous game value.

Suppose the payoff \( M(x, y) \) in a game over the unit square takes on the three values \( b \), \( a \), or \( 1 \) depending upon whether \( y \geq x, x < y \geq x + t, \) or \( x + t < y \), respectively \( (a < b \leq 1, t < 1) \). It is shown that this game has optimal mixed strategies and a game value which is discontinuous. (Received February 28, 1962.)

590-41. S. P. LLOYD, Bell Telephone Laboratories, Murray Hill, New Jersey. On positive projections of unit norm.

With \( X \) compact Hausdorff, let \( T \) be a positive projection in \( C(X) \) of unit norm. There is an upper semicontinuous decomposition of \( X \) into closed equivalence classes, such that all functions \( Tf \) are constant on equivalence classes. It is shown that an equivalence class is either ergodic or dissipative, as follows. The value of \( T \) on an ergodic class is the average of \( f \) with respect to a regular Borel probability measure living on the ergodic class, The value of \( T \) on a dissipative class is a subaverage of \( f \) over ergodic classes. An application to Markov chains is given. (Received February 28, 1962.)


Let \( F \) be a finite \( CW \)-complex. Let \( H \) be the topological monoid of all homotopy equivalence of \( F \) onto itself, Dold and Lashof have constructed a space \( BH \) such that \( H \cong \Omega BH \). Theorem. Fibre homotopy equivalence classes of fibrings \( p: E \to X \) with fibres of the homotopy type of \( F \) are in 1-1 correspondence with homotopy classes of maps of \( X \) into \( BH \) (at least for reasonable \( X \)). The crucial step is the construction of fibrings by a generalization of the Hopf construction. (Received February 28, 1962.)


Let \( M \) be a second countable orientable Riemannian manifold. By dissecting the sample paths of skew Brownian motions, one in each tangent space, and injecting these into \( M \) via the exponential map, an almost Markovian path is constructed on \( M \). It is shown that as the mesh of the dissection approaches zero, these paths converge uniformly on compacts of \( [0, \infty) \) to the paths of a diffusion on \( M \) whose generator is identified. This leads to geometrical descriptions of the Laplace-Beltrami Brownian motion on \( M \). When \( M \) is homogeneous = \( G/H \), \( G, H \) being the Isometry and Isotropy groups of \( M \), a Brownian motion on \( M \) is defined as a diffusion whose law is invariant under \( G \). Under various
hypotheses on $M$, such motions are described. Sample results: Let $M$ be symmetric. Let $g, h$ be the Lie algebras of $G, H$, $g = h + m$, $\text{Ad}(H)m \subset m$; (i) Every probability measure $dp$ with global support on the space of orbits of points of the unit sphere of $m$ under the action of $\text{Ad}(H)$ gives rise to a Brownian motion on $M$ in a natural way. (ii) Every decent Brownian motion on $M$ arises in this way from such a measure $dp$. (Received March 1, 1962.)

590-44. J. M. WORRELL, JR., 1509 Windsor Road, Austin 3, Texas. Concerning upper semi-continuous collections of mutually exclusive closed and compact point sets.

Axiom 13. There exists a sequence $G_1, G_2, G_3, \ldots$, satisfying the conditions of R. L. Moore's Axiom 13 (Foundations of point set theory, Amer. Math. Soc. Colloquium Publications, Vol. 13) such that if $R_1, R_2, R_3, \ldots$, is a sequence such that for each $n$, $R_n$ belongs to $G_n$ and $R_n$ contains $R_{n+1}$, then there is a point $P$ which belongs to $R_n$ for each $n$. Suppose that $\Sigma$ is a space satisfying Axioms 0 and 13 of R. L. Moore and $G$ is an upper semi-continuous collection of mutually exclusive closed and compact point sets covering the set $S$ of all points. Let $\Sigma'$ denote the space obtained by interpreting "point" to mean "element of $G" and "region" to mean "subcollection $G'$ of $G$ such that $G'$ is a domain in $\Sigma'." The author shows that Axiom 13 holds for $\Sigma'$, thus settling a question raised by R. L. Moore, and that if Axiom 13 holds for $\Sigma$, then Axiom 13 holds for $\Sigma'$. R. L. Moore has settled an analogous question with respect to a considerably stronger axiom called Axiom 11 in Foundations of point set theory. (Received February 28, 1962.)

The April Meeting in Monterey
April 28, 1962


The authors propose an extension of the usual "predict-correct" procedure for the numerical solution of ordinary differential equations. The purpose of the extension is two fold: (1) to increase the accuracy without increase in the number of values of the dependent variable and its derivatives that must be carried along; and (2) to insure a high degree of stability in the computed values. The process consists in predicting the value of $y_{n+1}$ by an open type quadrature formula, computing $dy/dx = y'_{n+1}$, and correcting $y_{n+1}$ by a closed type formula, recomputing $y'_{n+1}$, and finally (the new feature) recomputing $y_n$ by a stable formula of higher accuracy than is available for $y'_{n+1}$. (Received February 28, 1962.)


Every $f$ whose $p$th power is locally integrable on the half-line $t \geq 0$ can be expressed as $f(t) = \int g(t-u)da(u)$ (a.e. $t \geq 0$) where $g$ is a function whose $p$th power is locally integrable, and $a$ is locally of bounded variation and $a(0^+) = 0$. A necessary and sufficient condition that a continuous
function $c$ can be expressed as $\int_0^t c_2(t-u) \, da(u)$ with $c_2$ continuous and $a$ as above is that $c_1(0) = 0$. There are continuous $c$, which cannot be so expressed if it is required that $a$ be absolutely continuous and zero at the origin. (Received February 28, 1962.)


[For terminology and notation see our papers in Pacific J. Math., 10 (1960), 313-334; Publ. Math., Debrecen 8 (1961), 169-186]. Let $(S,d)$ be a metric space, $a \neq 0$, and $G$ a distribution function which is strictly increasing on $(0,\infty)$, continuous, and such that $G(0) = 0$. For every pair of points $p, q$ in $S$ define a distribution function $F(p,q) = F_{pq}$ via $F_{pq}(x) = G(x/\theta(p,q))$. Then $(S,F)$ is a statistical metric space - which we call the $\alpha$-simple SM space generated by $G$ and $d$. For $0 \leq \alpha \leq 1$, this SM space is a Menger space under the $t$-norm $\min$, independently of $G$; for $\alpha > 1$, it is a Menger space under the $t$-norm generated by $(G^*)^{1/1-\alpha}$, where $G^*$ is the inverse of $G$. This result is best-possible. (For related results cf. E. O. Thorp, Proc. Amer. Math. Soc. 11 (1960), 734-740.) (Received February 28, 1962.)

591-29. ALFRED TARSKI, 462 Michigan Avenue, Berkeley 7, California. Solution of the decision problem for the elementary theory of commutative semigroups.

A commutative semigroup $\mathcal{G} = \langle S, \cdot \rangle$ is an algebraic structure with a binary operation $\cdot$ satisfying the commutative, associative, and cancellation laws, and having a unit 1; an element $x \in S$, $x \neq 1$, is indecomposable if $x = y \cdot z$, $y \neq 1$, always implies $z = 1$. If, in particular, $S$ is a set of positive integers closed under the ordinary multiplication $\cdot$, $\mathcal{G} = \langle S, \cdot \rangle$ is called an $M$-semigroup. Let $p_0, p_1, \ldots$ be the increasing sequence of all primes. For any natural $m,n$ let $m \cdot n = 2^{m+1}(2^{n+1} + 1)$. Let $m \cdot n$ mean that, for some odd number $k$, $2^m \cdot k \leq n < 2^{m+1}(k + 1)$; the relation $R$ is isomorphic to the membership relation between sets of finite rank. Let $x R^* y$ mean that $x = P_{2m+1} \cdot y = P_{2n+1}$, and $m \cdot n$ for some $m,n$. Let $\mathcal{G}$ be the $M$-semigroup generated by all the primes $P_{2m+1}$, $m = 0,1,\ldots$, and all the numbers $P_{2m+1} \cdot P_{2n+1}$ for which $m \cdot n$. $R^*$ is elementarily definable in $\mathcal{G}$; in fact, $x R^* y$ iff $x,y$ are indecomposable (in $\mathcal{G}$) and $x^3 \cdot y = y^3 \cdot x$ for some indecomposable $u \cdot y$. Hence, using terminology and results of Tarski-Mostowski-Robinson's Undecidable theories, one proves:

Theorem 1. The elementary theory of the $M$-semigroup $\mathcal{G}$ is hereditarily undecidable. Corollary 2. The elementary theories of the classes $K_1$ of all $M$-semigroups and $K_2$ of all commutative semigroups are undecidable. (Received February 28, 1962.)

591-30. E. J. TULLY, JR., 534 Midvale Avenue, Los Angeles, California. A class of commutative semigroups having a group-like property.

Let $S$ be a commutative semigroup with maximum condition on principal ideals. Let $b \in S$ be such that $\{x: x \rho b\} \neq \{x: x \sigma b\}$ whenever $\rho$ and $\sigma \neq \rho$ are congruence relations on $S$. Then $S$ must be of one of the following types. 1. Let $b$ be the identity of any group $G$, $n_1$ positive integers indexed by
a (possibly uncountable or empty) set \( I = \{0, \ldots \} \), \( S = G \cup \{(i,n): i \in I, n \text{ an integer}, 0 \leq n < n_i, (i,n) \neq (0,0) \} \) with an optional zero element adjoined. Define multiplication by \((i,n)g = g (g \in G), (i,n)(j,m) = (i,n + m) \) if \( i = j \) and \( n + m < n_i, (i,n)(j,m) = b \) otherwise. II. Let \( b \) be any element of a group \( G, S = G \cup \{c\} \), with an optional zero element. Define \( cg = bg (g \in G), c^2 = b^2 \) III. \( S = \{0,1, \ldots \} \) or \( \{1, \ldots \} \) if \( m \neq c \neq n \). Corollary. If \( S \) is as in the first sentence, and all the congruence relations on \( S \) commute, then \( S \) is of one of the following types. V. Let \( G_0, G_1, \ldots, G_n \) be groups, \( \pi_i \) (for \( 1 \neq i \neq n \)) a homomorphism of \( G_{i-1} \) onto \( G_i, S = G \cup \{c\} \) with a zero element adjoined, if \( x \in G_i \) and \( y \in G_j \), let \( x \circ y = (x\pi_{i+1}\pi_{i+2} \ldots \pi_{i+j}) (y\pi_{j+1}\pi_{j+2} \ldots \pi_{j+i}) \) if \( i + j \neq n, x \circ y = 0 \) otherwise. VI. \( S = \{1,2, \ldots, n\} \), \( ij = \min(i+j, n) \). VII. Let \( S \) be any group. (Received February 28, 1962.)

591-31. STEFAN BERGMAN, Stanford University, Stanford, California, On singularities of harmonic functions of three variables.

Let \( \mathcal{H} \) be a linear space of harmonic functions regular at the origin with a basis \( \{P_{n,k}(X), n = 0,1, \ldots, \kappa = 1, \ldots, 2n\} \). Here: \( X = (R, \theta, \phi); P_{n,k}(X) = C_{n,k} R^n P_{n,|\kappa-n|} (\cos \theta \exp(i\phi (\psi - n)); R, \theta, \phi \) are polar coordinates; \( P_{n,\mu} \) — the associated Legendre functions; \( C_{n,k} \) — numerical constants. \( H \) is an algebra with respect to the addition and the composition defined in (1) Math. Zeit. 24, p. 647 (see also (2) Arch. Rational Mech. Anal. 8, p. 216 and (3) Ergebnisse der Math. NF 23). Let \( T_{11,\mu} \) be the sub-space of \( \mathcal{H} \) with the basis \( \{C_{n,k} R^n P_{n,\mu} (\cos \theta \exp(i\phi), n = |\mu|, |\mu| + 1, \ldots\} \). For \( R > 0 \), sufficiently small, \( T_{11,\mu} \) is assumed to be isomorphic to the algebra of meromorphic functions (regular at the origin) of a complex variable. Let \( S_\mu(X) = \sum_{n=0}^{\infty} |\mu|^{n+\mu} a_{n,\mu+\mu} P_{n,\mu}(X) \in T_{11,\mu} \) and let \( a_n, n = 1, \ldots, N, \) be arbitrary complex constants. The function \( G_\mu(X) \) reciprocal to \( S_\mu(X) - \sum_{n=0}^{N} |\mu| a_n P_{n,\mu}(X) \) has branchlines \( b^{\mu}_{\lambda}(a) = [R \cos \theta = \text{Re} A_\phi(a), R \sin \theta = \text{Im} A_\phi(a)], \gamma = 1,2, \ldots, \) see (3), pp. 45-47. Further, if \( \lim_{n \to \infty} [n \log(-n \log|a_n,\mu+\mu|)]^{-1} \geq \rho \), then \( \sum_{n=1}^{\infty} |A_\phi(a)|^{-\lambda} < \infty \) for \( \lambda > \rho \). (Received April 2, 1962.)

591-32. MARTIN BLUMBERG, Department 58-22, Building 204, Lockheed Missiles and Space Company, 3251 Hanover Street, Palo Alto, California, Some formulas involving the products of trigonometric functions spaced at regular intervals.

In the study of conformal maps of infinite arrays of line charges there arise functions involving the products of sines and of cosines spaced at regular intervals. A typical formula is

\[ (-1/2)^{n-1} \sin nz = \sum_{m=1}^{n-1} \sin(z - m \pi/n). \]

The value of the constant, \( (-1/2)^{n-1} \), is obtained by several methods, one utilizing a formula due to Euler. Many other relations are derived, some involving sums of tangents and cotangents spaced at regular intervals. The manipulations leading to these are also of interest. (Received April 12, 1962.)
Abstracts Presented by Title


Let $F^k$ denote a positive definite left hermitian vectorspace over $F = \text{reals, complex numbers, or quaternions}$; if $0 < n < k$, let $G_{n,k}(F)$ be the Grassmann manifold of $n$-dimensional subspaces of $F^k$, with its usual structures as a Riemannian symmetric space. If $B$ is a collection of pairwise isoclinic subspaces of $F^k$, then it is shown that there is an orthogonal decomposition $F^k = V_1 \oplus \ldots \oplus V_m$, and that there are connected totally geodesic submanifolds $B^i$ of $G_{n,k}(F)$, such that (1) every element of $B^i$ lies in $V_i$, (2) $B \subset \bigcup B^i$, and (3) $\bigcup B^i$ is a collection of pairwise isoclinic subspaces of $F^k$. Results of an earlier paper (Geodesic spheres in Grassmann manifolds. See Abstract 62T-65, Notices Amer. Soc. 9 (1962), 143) show that each $B^i$ is a Riemannian symmetric space of rank one, and give the classification of those $B^i$ which are spheres. Here, it is shown that $B^i$ is a real, complex or quaternion projective space if it is not a sphere, and those $B^i$ are classified. (Received February 21, 1962.)


Suppose $L$ is a totally ordered, connected topological space such that (1) the topology of $L$ is the order topology, and (2) $L$ has no first or last point. Theorem 1. If $L$ is homogeneous, then $L$ satisfies the first axiom of countability. Theorem 2. $L$ is homogeneous if and only if each pair of closed intervals lying in $L$ are homeomorphic. Theorem 3. Suppose $L$ is homogeneous and $I$ is a closed interval lying in $L$. Then, if for each positive integer $p$, $I^p$ denotes a copy of $I$, then the space $X = L \times I^1 \times I^2 \times I^3 \times \ldots$ with the topology induced by the lexicographic order is also homogeneous. (Received February 23, 1962.)


Theorem 1: Let $T$ be a transformation from a topological space $T_1$ into a topological space $T_2$. Let $S$ be a homomorphism from an algebraic system $S_1$ into an algebraic system $S_2$. If there exists a mapping $a$ from the mutant sets of $S_1$ into the open (closed) sets of $T_1$ and if there exists a mapping $\beta$ from the open (closed) sets of $T_2$ into the mutant sets of $S_2$ such that $T^{-1}(P) = a(S^{-1}(\beta(P)))$, where $P$ is any open (closed) set of $T_2$, then $T$ is continuous. Definition. A homomorphism $\phi$ from an algebraic system $A_1$ into an algebraic system $A_2$ is said to be stationary provided $\phi(\bar{A}) \subseteq \phi(A)$ for every mutant set $A$ of $A_1$. Theorem 2. A stationary homomorphism is mutant preserving. Meta-theorem 1. If one sets up the appropriate correspondences between the symbols of the two syntax languages of Theorem 1 then "continuous transformations of the topological spaces" corresponds to "stationary homomorphisms of the algebraic systems," and vice versa. The theory of abelian U-diagrams can be used to treat the more special cases of isomorphic algebraic systems and homomorphic topological spaces. (Received February 26, 1962.)

Let \( f(p) \) be a convex function \( (p = (p_1, p_2, \ldots, p_n)) \), with the quadratic form: 
\[
\sum_{i,j} f(p_j)(p_i^2 - p_i p_j).
\]
positive definite at every point \( p \) (not necessarily uniformly). Consider the regular variational problem:

\[
I(u) = \int_{\Omega} f(\nabla u) \, dx = \min.
\]

It is supposed that \( \Omega \) is a bounded domain of \( E^n \) and that \( u \) equals a given function \( \phi(x) \) on the boundary \( \partial \Omega \) of \( \Omega \). Let \( \Gamma \) denote the \( (n - 1) \)-dimensional manifold of \( E_{n+1} \) \{ \( x \in \partial \Omega, u = \phi(x) \) \}. Let the boundary values \( \phi(x) \) satisfy the bounded slope condition \( K \) if for every point \( P_0 \) of \( \Gamma \) there exist two planes \( \pi_- \) and \( \pi_+ \) of slope \( \alpha \leq K \) passing through \( P_0 \) such that \( \Gamma \) is between \( \pi_- \) and \( \pi_+ \). Consider the variational problem (1) for all Lipschitz functions in \( \Omega \) having boundary values \( \phi \) satisfying the bounded slope condition \( K \). The existence and smoothness of the solution is shown, and in particular, if the data of the problem are analytic also the solution is analytic in the closure of \( \Omega \). The result, which is related, for \( n = 2 \), to a classic theorem by Haar, requires no conditions on the growth of the function \( f \) at infinity. The proof is based on De Giorgi's result and its extension at the boundary and uses an a priori bound for the maximum of the first derivatives of the solution. (Received February 26, 1962.)


Stationary acoustic wave fields in nonhomogeneous media are described by potentials \( U(x) \) satisfying \( \rho \nabla \cdot (\rho^{-1} \nabla U) + \kappa^2 U = f \) with given variable coefficients \( \rho, \kappa, f \). Study the case in which \( \rho, \kappa, f \) are discontinuous on a closed surface \( S \). The physical experience that pressure and normal velocity are continuous across \( S \) leads to two interface conditions (2), (3) linking the interior and the exterior field. Under certain assumptions on \( \rho, c, f \) at infinity, and additional regularity requirements for these coefficients and \( S \), it can be shown that a uniquely determined \( U \) exists satisfying (1), (2), (3) and Sommerfeld's radiation condition. This continues investigations of Sternberg, Freudenthal, Kupradze, Müller and Barrar-Dolph concerning interface problems for acoustic, electro-magnetic and elastic fields in homogeneous media. (Received February 26, 1962.)


Let \( J \) be the class of all AEA formulas \( AxEuAyMxuy \) with only dyadic predicate letters \( G_1, G_2, \) etc., and monadic ones such that, for every \( i \), \( G_{i}xy, G_{iy}u \) never occur in the matrix \( Mxuy \). It is proved by a new type of argument that \( J \) is solvable. A similar procedure also solves the class \( J^* \) in which \( G_{1}xy, G_{i}uy \) never occur. In both cases, it is further shown that (1) the formulas containing atomic formulas only of the forms \( G_{i}xy, G_{iy}u \) or only of the forms \( G_{1}yx, G_{iy}u \) do not include any axiom of infinity; (2) there are axioms of infinity in any class of all formulas containing three (or more) forms of atomic formulas which include either the two forms \( G_{1}xy \) or \( G_{iy}u \), or the two forms \( G_{1}yx, G_{iy}u \). For example, \( \neg G_{1}xx \& (G_{1}xy \rightarrow G_{1}uy) \& (G_{2}xy \equiv G_{1}uy) \& G_{2}xx \) is an axiom of infinity. It is
worth noting that the class in which $G_{1}uy$, $G_{1}yu$ never occur and the class in which $G_{1}xy$, $G_{1}yx$ never occur are solvable and indeed contain no axioms of infinity. (Received February 27, 1962.)

62T-107. LOUIS SUCHESTON, 3702 N. Murray, Shorewood 11, Wisconsin. An ergodic application of Banach limits. II.

Let $T$ be a measurable one-to-one transformation on a probability space $(\Omega, \mathcal{A}, P)$ such that if $P(A) = 0$ then $P(TA) = P(T^{-1}A) = 0$. A necessary and sufficient condition that there exist a finite invariant measure equivalent with $P$ is that for each measurable set $A$ all Banach limits on the sequence $\{P(T^nA)\}$ coincide (cf. Abstract 584-2, Notices Amer. Math. Soc. 8 (1961), 496). (Received February 26, 1962.)

62T-108. G. J. RIEGER, Dahlienstrasse 24, Ottobrunn near Munich, West Germany. On the representation of algebraic integers as the sum of four squares.

Let $K$ be an arbitrary algebraic number field. Theorem. The 4-fold Schnirelmann sum of the sequence of all squares of a sequence of totally positive integers of $K$ with positive Schnirelmann density has positive Schnirelmann density. The proof is elementary and uses ideas of Linnik. (Received February 26, 1962.)


In the case of a partially balanced incomplete block (PBIB) design with two associate classes, let $v, b, r, k$ have their usual meanings and let $N$ be the incidence matrix of the design. Then the characteristic roots of $NN'$ can be found in terms of the parameters of the design by the methods of W. S. Connor and W. H. Clatworthy (Ann. Math. Statist. 25 (1954), 100-112). One considers the case when $b = v - a$ and zero is a characteristic root of $NN'$ of multiplicity $a$, while $\rho_0 = rk$ is a root of multiplicity 1 and the remaining root is a rational number $\rho_1$ with multiplicity $a_1 = b - 1$. Let $Q_1$ and $Q$ be the gramians of the rational characteristic vectors corresponding to the roots $\rho_1$ and zero respectively. Then it can be proved that necessary conditions for the existence of the design are

\begin{align*}
(1') & \rho_0 \rho_1^{a_1} |Q| = \text{a perfect square and if (i) satisfied, then (ii) } (\rho_0 - v \rho_1^{a_1+1})_p (\rho_1, v)_p \\
& \times (-1, \rho_1)^{a_1(a_1+3)/2} c_p(Q) = 1, \text{ where } p \text{ is any odd prime and } (a, b)_p \text{ is the Hilbert norm residue symbol and } c_p(Q) \text{ is the Hasse-Minkowski invariant of } Q. \text{ This result is applied to group divisible, triangular and L_1 designs and is also extended to PBIB designs with more than two associate classes. (Received February 26, 1962.)}
\end{align*}


Let region $D$ be bounded by mutually disjoint Jordan curves $B_1, B_2, ..., B_{\mu}, C_1, C_2, ..., C_{\nu}$, the $B_j$ having smoothness properties. Let $u(z)$ be harmonic in $D$, continuous in its closure, zero on $B = \Sigma B_j$.
and unity on \( C = \Sigma C_j \). By a conformal map \([H. J. Landau]\) we choose \( D \) interior to \( C_1 \), and interior to each \( C_j \) \((j > 1)\) choose \( u(z) \) harmonic with a single singularity, of form \(-n_j \log|z - a_j|\); \( u(z) \) has a corresponding singularity at \( a_1 = \infty \). Points \( a_1, a_2, \ldots \), each some \( a_j \), exist [Ann. Mat. Pura. Appl., 39 (1955), 267-777, Lemma 1] distributed among the \( a_j \) asymptotically in the proportions \( n_j \). Points \( \beta_{nk}(k = 1, 2, \ldots, n + 1; n = 1, 2, \ldots) \) exist [same Lemma] equally spaced on each \( B_j \) with respect to the conjugate \( v(z) \) of \( u(z) \), distributed among the \( B_j \) asymptotically in the proportions \( \int_{B_j} dv \). Rational functions with poles in \( a_1, a_2, \ldots, a_n \) interpolating in the points \( \beta_{nk} \) to an arbitrary \( f(z) \) analytic on and within the \( B_j \) can be used to generalize previous results on approximation; the curves \( B_j \) need not be analytic. In particular \( p > 1 \) is a new treatment of Walsh and Sewell, Duke Math. J. 6 (1940), 658-705. (Received February 28, 1962.)

62T-111. IRVING REINER, University of Illinois, Urbana, Illinois, Failure of the Krull-Schmidt theorem for integral representations.

Let \( RG \) denote the group ring of a finite group \( G \) over the ring \( R \) of all algebraic integers in an algebraic number field \( K \). Consider \( RG \)-modules which are finitely-generated and \( R \)-torsion-free. It is shown that if \( G \) is not a \( p \)-group, and if \( G \) contains a normal subgroup of prime index, then the Krull-Schmidt theorem cannot be valid for \( RG \)-modules. Indeed, in a direct sum of indecomposable \( RG \)-modules, the \( R \)-ranks of the summands need not be uniquely determined by the direct sum. The proof depends on showing that if \( G \) satisfies the above conditions, then there exist \( RG \)-modules \( A, B \) and \( C \) such that \( KA, KB \) and \( KC \) are irreducible \( KG \)-modules no two of which are isomorphic, and such that there exist nonzero elements \( F \in \text{Ext}^1_{RG}(B, A) \) and \( F' \in \text{Ext}^1_{RG}(C, A) \) whose orders are relatively prime. (Received March 5, 1962.)


The space \( H_p \) is defined to be the class of all functions \( f(z) = \sum a_n z^n \), analytic for \(|z| < 1\) and such that \( \sup_{r \in [0, 1]} |f(re^{it})| \) exists for almost all \( t \). The following theorems give a complete characterization of \( H^*_p \) for the case \( 1/p \) not an integer, and a partial characterization when \( 1/p \) is an integer. Define \( k \) as the greatest integer less than \( 1/p - 1 \) and \( \ell = \text{lip}(1/p - 1) \) as the class of functions \( g(z) \), analytic for \(|z| < 1\) and such that \( \lim_{r \to 1} g^{(k)}(re^{it}) \) exists for almost all \( t \) and satisfies a Lipschitz condition of order \( 1/p - (k - 1) \). Then we have: Theorem I. If \( 1/p \) is not an integer, then \( g \) is the kernel of some \( T \in H_p^* \) if and only if \( g \in \ell = \text{lip}(1/p - 1) \). Theorem II. Suppose \( 1/p - 1 = j \) for some integer \( j \). If \( g(z) = \sum b_n z^n \) is analytic for \(|z| < 1\) and there exists a function \( F(x) = \sum c_n e^{inx} \), bounded in the unit circle and such that \( c_n (\Gamma(n + 1 - j)/\Gamma(n + 1)) b_n \), then \( g \) is the kernel of some \( T \in H_p^* \). Conversely, if \( \ell \) then \( T \) has a kernel \( g(z) \) such that \( \lim_{r \to 1} g^{(k)}(re^{it}) \) exists and has the modulus of continuity \( w(t) = O[(\ln 1/t)^{1/p}] \). (Received March 5, 1962.)

Let $e \equiv 0$ be a s.a. operator and let $[0,e] = \{a: a \text{ is s.a. and } 0 \leq a \leq e\}$. Clearly $[0,e]$ is convex and is easily seen to be compact in the weak operator topology; extreme points abound. **Theorem.** For $v \in [0,e]$ the following are equivalent: (1) $v$ is an extreme point of $[0,e]$ with $ve = ev$; (2) $ve = v^2 (= ev)$; (3) $ve = ev$ and for each $\phi \in N(v)^t$, $v\phi = e\phi$ (here $N(v)$ denotes the null space of $v$). Moreover, if $v\phi = e\phi$, for each $\phi \notin N(v)^t$, then $v$ is extreme. If $e$ is a projection, the extreme points of $[0,e]$ are just the sub-projections of $e$ and, as is well-known, $e$ commutes with each $a \in [0,e]$. The extreme points need not commute among themselves (as the case $e = 1$ clearly shows), nor need they even commute with $e$. The proof employs simple lattice techniques locally, where commutativity and lattice structure are available. (Received March 8, 1962.)


Let $T x = y$ be a bounded operator defined on the complex Hilbert sequence space $l_2$ by

$$y_n = \gamma_n x_{n-1} + a_n x_n + \beta_n x_{n+1}, \quad n \geq 0, \quad x_{-1} = 0,$$

where $\gamma_n, a_n, \beta_n$ are complex numbers and $\beta_n \neq 0$.

For complex $\lambda$, the vector equation $(T - \lambda I)x = 0, x_0 = 1$, recursively generates a sequence $x_n = p_n(\lambda)$. Suppose the polynomials $p_n(\lambda)$ are orthogonal, with respect to a positive continuous weight function, over some rectifiable Jordan curve bounding a Smirnov domain, and suppose the norms $\|p_n\|$ are bounded away from 0 and $\infty$. Then the invariant subspaces of $T$ have the same lattice structure as those of the shift operator $(\gamma_n \equiv 0, a_n \equiv 0, \beta_n \equiv 1)$. Several specific applications can be made. (Received March 8, 1962.)


Let $G$ be a bounded domain in $\mathbb{R}^n$ and $[u,v] = \int_G |\mu|, |\tau|^m \mu_{ij} D^m u D^i v \ dx$ a bilinear integro-differential form of order $m$. Let $B_1, \ldots, B_r$ be boundary operators which are nowhere characteristic to the boundary $\partial G$ of $G$ and have distinct orders. Let $V$ be the set of those $u \in C^\infty(G)$ which satisfy $B_j u = 0$ on $\partial G, 1 \leq j \leq r$. The form $[u,v]$ is said to be coercive over $V$ if $\|u\|^2_{m,2} \leq \text{const.}$ \(\text{Re } [u,u] + \|u\|^2_{0,2}\) for all $u \in V$, where for any real $p > 1$ and $s$ the norm $\|\cdot\|_{s,p}$ is defined as in Abstract 62T-20, Notices Amer. Math. Soc. 9 (1962), 40. Necessary and sufficient conditions for $[u,v]$ to be coercive over $V$ have been given by Agmon (J. Analyse Math. 6 (1958), 216). Set $[u]_{s,p} = \sup_{v \in V} \|u\|_{m-s,q}$, where $q = p/(p - 1)$. **Theorem.** Under suitable regularity conditions, if $[u,v]$ is coercive over $V$, then for each real $s \leq m, \|u\|_{s,p} \leq \text{const.}$ for all $u \in V$. For each integer $s, \|u\|_{s,p} \leq \text{const.}$ for all $u \in V$. For each integer $s, \|u\|_{s,p} \leq \text{const.}$ for all $u \in C^\infty(G)$, where the $\langle \cdot \rangle_{t_j}$ are appropriate boundary norms. (Received March 1, 1962.)
It is shown that a previously developed theory for estimating the density of traffic in a net of radial and corresponding orthogonal circular streets can be generalized to include the case where the distribution of the population is completely arbitrary. The application to freeway systems is illustrated by an example. The situation is treated as a discrete problem using a suitable subdivision of the city area into small portions. (Received March 9, 1962.)

Consider a city whose main traffic lines are concentric circles and straight streets originating from the center. Subdividing the city into "cells" (small portions around the points of intersection of the streets) and assuming that the traffic flows along the lines of minimum distance in the net of streets, simple formulas for computing the number of vehicles passing through the various portions of the net of streets are derived. Here it is assumed that the density of the population depends only on the distance from the center. By considering particular densities the influence of the geometrical shape of the shortest paths in the net can be estimated. (Received March 9, 1962.)

The present paper is concerned with a class of integral operators for generating harmonic functions of three variables from analytic functions. The class includes the well-known Bergman-Whittaker operator and a similar operator used by A. Erdelyi. Each operator of the class generates infinitely many sets of homogeneous harmonic polynomials of degree m, where m = 0, 1, ... . Each set is orthogonal on the unit sphere. The totality of these polynomials constitutes a base for the harmonic functions H of three variables in the sense that any such function H can be represented by a series of the polynomials. This representation converges absolutely and uniformly in a sufficiently small neighborhood of a point at which H is regular. The polynomials can be represented in terms of spherical harmonics. Each operator has an inverse in a restricted sense. This inverse can be written in the form of a single definite integral. The operators can be used for investigating the coefficient problem, in particular with respect to the position and nature of curves of singularities. Some of the results can be translated into results about solutions of more general partial differential equations by the use of Bergman's theory. (Received March 9, 1962.)

If $\mathbf{a}$ is an isotropic vector, $\mathbf{x} = (x_1, ..., x_n)$, $u = \mathbf{a} \cdot \mathbf{x}$, and $f(u)$ is analytic then $f(u)$ is a harmonic function of $n$ variables. The operators under consideration involve two integrations in the complex plane, the integrands being analytic functions of $u$, depending on two parameters, where $n = 4$. These
analytic functions are called associated functions of the corresponding harmonic functions with respect to the operator. The operators generate harmonic polynomials which are homogeneous and can be represented in terms of Jacobian polynomials, multiplied by some simple functions. It follows that these polynomials are orthogonal on the surface of the hypersphere of radius 1 in four-space. Choosing a fixed operator and introducing a certain class of associated functions the correspondence between these functions and the harmonic functions generated by that operator becomes one-to-one. Then the inverse of that operator can be defined. It is represented by a relatively simple double integral. The class of operators under consideration includes an operator for generating solutions of the three-dimensional wave equation introduced by R. P. Gilbert. Any harmonic function of four variables which is regular at the origin can be represented by a series of those polynomials. The proof is similar to that in the three-dimensional case. (Received March 9, 1962.)

62T-120. STEFAN BERGMAN, Stanford University, Stanford, California. On the construction of models of analytic polyhedra.

The author describes a combined camera- and computing machine for the construction of models of analytic polyhedra (domains with a distinguished boundary surface), as described in (1) Proc. of Intern. Congress of Math. I (1950), 363 ff. An analytic polyhedron $A^4$ can be described analytically by several equations (see (2.1a) - (2.1d) of (1)). Let $x_1, y_1, x_2, y_2$ be the coordinates of the four-dimensional space. For a number of discrete values $x_2^{(v)}, y_2^{(v)}, v = 1, 2, ..., n$, the machine computes the boundary curve of the intersection of $A^4$ with $x_2 = x_2^{(v)}, y_2 = y_2^{(v)}$. Then it cuts out from cardboard the intersections $S_2 = A^4 \cap (x_2 = x_2^{(v)}, y_2 = y_2^{(v)})$ and places $S_2$ on the height $x_2 = x_2^{(v)}$. The still obtained in this way is photographed by the camera. This is repeated for a number of values of $y_2$ to obtain stills indicated in Fig. 1.1 - 1.4 of (1). These stills are used to prepare a film. Projecting this film, we will obtain the movie model. It is useful to make pictures which correspond to the image of the body using different coordinates $x_1^{(n)}, y_1^{(n)}, x_2^{(n)}, y_2^{(n)}$ which one obtains by orthogonal linear transformations. In this way is "shown" the four-dimensional body from different angles. Compare p. 365 and p. 367 of (1) where the same body is shown from different angles. (Received March 12, 1962.)


Algebras satisfying $(x, x, x) = 0$ and $(x_1, x_2, x_3) = e(x_{\tau(1)} x_{\tau(2)} x_{\tau(3)})$ with $e$ in $F$, the base field and $\tau$ in $S_3$ satisfy (for char. $\neq 2, 3$): (1) $(x, y, z) = (z, y, x)$ and $(x, x, x) = 0$, (2) $(x, y, z) = (y, z, x)$ or are well known. Those satisfying (2) are power associative for char. $\neq 2, 3, 5$ and have an associative type decomposition. Such a semisimple algebra is the direct sum of associative or nodal algebras if $F$ is algebraically closed. Algebras satisfying (1) (studied by F. Kosier, Trans. Amer. Math. Soc. 102, No. 2, 299-319) are power associative for char. $\neq 2$. For such a semisimple algebra $A$, a necessary and sufficient condition is given so that $A$ has an identity and is the direct sum of simple algebras, which are nodal or associative. In a semisimple algebra with identity satisfying (1) either $A_{10}(e) + A_{01}(e) = 0$ or $A_{10}(e)A_{01}(e) = A_{11}(e)$ where $e$ is primitive and always $A_{11}(e)^2 \cap A_{11}(e) / \subseteq Z = \{x: (x, y) = 0\}$. If $F$ is algebraically closed, altering products by elements of $Z$ changes a semi-
simple algebra into a direct sum of nil, matrix and degree 1 algebras and a simple algebra is changed into a direct sum of degree 1 algebras. Partial converses are proved. A simple non-nil algebra has an identity. For a nodal algebra $A$ with $A^+$ associative, alteration of multiplication produces $A$ from $A^+$ and conversely. (Received March 12, 1962.)


Defining a deformation of a Lie or associative algebra $A$ to be a multiplication of the same type in the tensor product of $A$ with a power series ring in $t$ specializing at $t = 0$ to the original multiplication, $H^2(A, A)$ becomes naturally the space of infinitesimal deformations. These have successive obstructions in $H^3$. If $H^3 = 0$, all elements of $H^2$ are "integrable" to deformations. A deformation has an algebraic variety of moduli (analogous statements hold for module structures over $A$). Deformation of an algebra of characteristic $p$ to characteristic zero may be defined, following Goldman, using rings complete in a valuation; here there are further obstructions to integrating an infinitesimal deformation. (Received March 12, 1962.)


Let $M$ be a compact Riemannian manifold, $\alpha$ be a 1-form, $\Delta$ the Laplace-Beltrami operator. The solution $\beta(t) = T_t \alpha$ of $\Delta \beta = \partial \beta / \partial t$, the heat equation for 1-forms can be written explicitly in terms of the Brownian motion on $M$ as follows. $(T_t \alpha)(x) = E_x [B(t) \alpha(x_t)]$, where $x_t$ is the Brownian motion on $M$, $E_x$ is the expectation operator for this motion starting at $x$ and $B(t)$ is a multiplicative matrix-valued functional of the Brownian path in the sense of Dynkin. In fact if $R = R_{ij}$ is the mixed Ricci tensor of $M$, $B(t) = \lim_{k \to \infty} \prod_{m=0}^{2k-1} \exp [- (x_{m^2-k_i})^{2-k}]$, where $\exp$ is matrix exponentiation and $\prod$ is matrix multiplication, the limit existing uniformly with Probability 1. From this and similar formulas, it is possible to present simple proof of various known theorems of Hodge, Bochner and Yano as ergodic theorems. (Received March 13, 1962.)


The method of extending classes of functions used by Kleene (Extension of an effectively generated class of functions by enumeration, Colloq. Math. 6 (1958), 67-78) is applied to give a hierarchy having as its initial class the class $\mathcal{F}_4$ of the Grzegorczyk hierarchy (Some classes of recursive functions, Rozprawy Mat. Warsaw (1953)), the latter having been established by the use of a scheme of bounded primitive recursion and a version of the Ackermann majorizing function. The segment of the resulting hierarchy corresponding to the finite ordinals is shown to be identical to the Grzegorczyk hierarchy beginning with class $\mathcal{F}_4$. (Received March 5, 1962.)
An elementary estimate for the k-free integers.

Let $k$ denote an integer $> 1$ and let $Q_k(x)$ represent the number of $k$-free integers $\le x$. It is proved in this note, using no analytical fact beyond the mere convergence of $\sum_{n=1}^{\infty} 1/n^k$ ($s > 1$), that $Q_k(x) = \zeta^{-1}(k)x + O(x^{1/k+\varepsilon})$ for all $\varepsilon > 0$, where $\zeta(k) = \sum_{n=1}^{\infty} 1/n^k$. The proof is based on an expression for the characteristic function of the $k$-free integers, which involves the trigonometric sum $c_k(n, r)$ of the author. (Received March 14, 1962.)

Two theorems on contraproductive centers.

It has been shown, by Carol Karp, that any productive set $a$ has $k_0$ mutually disjoint productive centers. A relatively simple proof is given (using the result of Myhill that any productive set is $P$-productive) that every productive set admits $k_0$ productive functions with mutually disjoint ranges (which of course implies the above result). Further, in the presence of a simple lemma on existence of 1-1 contraproductive functions, the same argument yields Theorem 1. Every $P$-contraproductive set admits $k_0$ contraproductive functions with mutually disjoint ranges. Theorem 2. If $a$ is a set productive w.r.t. the partial function $p(x)$, and if range $(p) \cap a$ is r.e., then $a$ has a contraproductive center lying inside range $(p) \cap a$. The hypotheses of Theorem 2 can be satisfied in such a way that $\overline{a}$-(range$(p) \cap a$) is r.e., but $\overline{\left(\left(\overline{\text{range}}(p) \cap a\right) \subseteq \overline{\text{range}}(p) \subseteq a\right)}$. (Received March 29, 1962.)

On cartesian products which are homotopy manifolds.

Let $\pi$ be an arc (preferably wild) in $\mathbb{R}^3$, and let $X$ denote $\mathbb{R}^3$ modulo $\pi$ (i.e., the quotient space obtained by collapsing $\pi$ to a point). Then $X \times X$ is a homotopy manifold. It is known that $X$ may fail to be a homotopy manifold. It is not known if $X \times X$ is always a manifold, and, indeed, there is no known example of a nonmanifold $X$ such that $X \times X$ is a manifold. The theorem is proved by using a lemma of Penrose-Whitehead-Zeeman (Ann. of Math., 1960), M. Brown's theorem on cellular sets (Bull. Amer. Math. Soc., 1960), and Kwon's theorem on decompositions of $\mathbb{S}^3$ which give homotopy manifolds (Proc. Amer. Math. Soc., 1961). More general results are also obtained by using these same techniques. (Received March 19, 1962.)

Open 3-manifolds which are simply connected at infinity.

Let a triangulated open manifold $M$ be called simply connected at infinity if each compact subset $C$ of $M$ is contained in a compact polyhedron $P$ in $M$ such that each component of $M - P$ is simply connected. Now suppose that $U$ is an open 3-manifold which is simply connected at infinity, and such that each compact subset of $U$ can be imbedded in $E^3$. It is then shown that $U$ is the union of a sequence $\{C_i\}^\infty_{i=1}$ of punctured cubes, with $C_i \subseteq \text{Int } C_{i+1}$ for each $i$. This implies that there is a totally disconnected subset $Y$ of $E^3$ such that $U$ and $E^3 - Y$ are homeomorphic. If in addition $U$ is
contractible, it follows that $U$ is homeomorphic to $E^3$. In order to prove the 3-dimensional Poincaré conjecture, it would suffice to obtain this last result without the hypothesis that each compact subset of $U$ can be imbedded in $E^3$. (Received March 19, 1962.)

62T-129. R. P. GILBERT, Institute for Fluid Dynamics and Applied Mathematics, University of Maryland, College Park, Maryland. Poisson's equation and generalized axially symmetric potential theory.

In this paper a method is developed by which one may obtain solutions to the nonhomogeneous equation of generalized axially symmetric potential theory (GASPT). Representation formulae are obtained and the method is shown to be extendable under certain symmetry conditions to the Poisson equation in three variables. (Received March 19, 1962.)


If $f(s)$ is the analytic function defined by $\sum_{n=1}^{\infty} a_n n^{-s}$, certain properties of $f(s)$ can be deduced from information about $A(x) = \sum_{n=1}^{\infty} x^n a_n$, (an Abelian theorem). In this paper it is shown that if $A(x) = Cx \log^k x + O(x^b)$, where $k$ is a positive integer and $0 < b < 1$, then the coefficients in the Laurent series for $f(s)$ at $s = 1$ are related to constants which appear in the estimates for $\sum_{n=1}^{\infty} x^n a_n \log^n x$. If $A(x) = Cx + O(x/\log^t x)$ for arbitrarily large $t$, a similar result will follow under the assumption that $f(s)$ has a pole at $s = 1$. This theorem includes the important special case of $f(s) = \zeta'(s) / \zeta(s)$ which arises when $a_n = \Lambda_n$. (Received March 22, 1962.)


Generalizing our previous result (Pacific J. Math. 9 (1959), 975-985) we prove that the ring $P$ of formal power series, over an arbitrary domain of integrity $C$, in an arbitrary (cardinal) number of variables, is factorial (unique factorization domain) if and only if the rings $C[[x_1,\ldots,x_n]]$ of such series in every finite number $n$ of variables are factorial. From a recent theorem of D. Buchsbaum (J. Math. Mech. 10 (1961), 749-753) if follows that $P$ is factorial whenever the coefficient domain $C$ is a principal ideal ring. In the countable case it appears that the ring of all integer-valued arithmetic functions is a unique factorization domain. These statements appear as corollaries of a general theorem on semigroups. (Received March 26, 1962.)


Let $A$ be a simple algebra over an algebraically closed field of characteristic $\neq 2,3,5$ satisfying $x(ax) + (ax)x = 2(ax)x$ and $x^2 x^2 = x^3 x$. Then $A$ is power associative, has a unit, $1$, and has been shown by Kosier (Trans. Amer. Math. Soc. 102 (1962), 299-318) to be Jordan or quasiassociative whenever degree $A \neq 3$. Suppose $A$ is of degree 2; then $1 = e + f$ with $e$ and $f$ orthogonal primitive idempotents,
A is the direct sum of subspaces $A_e(\lambda)$ where $(ex + xe)/2 = \lambda x$ for all $x \in A_e(\lambda)$ and $\lambda = 1, 1/2, \text{ or } 0$, $A_e(1) = eF + N_1$, and $A_e(0) = fF + N_0$ with $N_1$ and $N_0$ nilsubalgebras of $A_e(1)$ and $A_e(0)$, respectively. Suppose $A$ is nilstable, i.e., that $A_u(\lambda)A_u(1/2)$ and $A_u(1/2)A_u(\lambda) \subseteq A_u(1/2) + N_{1-\lambda}$ for $\lambda = 1, 0$ and for every idempotent, $u$. Then, following Oehmke (Proc. Amer. Math. Soc. 12 (1961), 151-158) and Kokoris in an article on nilstable flexible algebras to appear in the Proceedings of the Amer. Math. Soc., $A$ is shown to be a noncommutative Jordan algebra. In case $ex = x/2$ for all $x \in A_e(1/2)$, $A$ is commutative Jordan. Should $A$ fail to be nilstable, $A$ might be of a class of commutative power associative algebras of degree two that was constructed by Kokoris (Proc. Nat. Acad. Sci. 38 (1952), 534-537) and shown by him not to satisfy the Jordan identity. (Received March 26, 1962.)


Let $L$ be a latin square of order $4t + 2$, $t$ a positive integer. Let $B$ be a $(2t + 1) \times (2t + 1)$ block of $L$. Designate as a special cell any cell of $B$ containing a digit in a fixed subset of $2t + 1$ digits. In Bull. Amer. Math. Soc. 50 (1944), 249-257, Mann proves: If $L$ has an orthogonal mate, then $B$ contains at least $t + 1$ special cells. Under a more stringent hypothesis, the author has sharpened this inequality slightly. Theorem. If $L$ is contained in a complete set of mutually orthogonal latin squares (equivalent to an affine plane of an Euler conjecture order), then $B$ contains at least $t + 1 + f(t)$ special cells; $f(t)$ grows as $ct^{1/2}$, and is positive if $t \equiv 3$. For the most interesting order $4t + 2 = 10$, the author's techniques give some information, but show no prospect of improving the inequality. (Received April 11, 1962.)


Let $P$ be a class of spaces such that, whenever $B$ is a closed subspace of $A \in P$, then $B \in P$. The "non-locally-P kernel" $K = K(P,X)$ of a space $X$ is the largest closed subset of $X$ in which no neighborhood is in $P$. Suppose for simplicity that all the spaces mentioned are metrizable. Generalizing the Cantor-Bendixson theorem, it is shown that $X - K$ is the union of a sequence of closed locally $P$ sets. Generalizations of a theorem of Banach are deduced, including: if $X \subset Y$ and every non-empty closed subset of $X$ is somewhere of the first category in $Y$, then $X$ is of the first category in $Y$. Similar generalizations are given of a theorem of Montgomery (roughly that locally Borel sets are Borel). The spaces which are absolutely both $F_\sigma$ and $G_\delta$ are shown to be those for which the non-locally-compact kernel is empty, and the spaces all of whose subsets are absolutely both $F_\sigma$ and $G_\delta$ are shown to coincide with the scattered spaces, as do those all of whose nonempty subsets are of the second category (in themselves). (Received March 27, 1962.)


All spaces considered are metrizable and absolutely Borel. A Borel isomorphism is a 1-1
correspondence \( f \) such that both \( f \) and \( f^{-1} \) take Borel sets into Borel sets. Theorem. The property of being \( \sigma \)-discrete is invariant under Borel isomorphism. The proof uses the methods and results of earlier work of the author (Rozprawy Mat, 28). It follows that every (Borel) space \( X \) is either \( \sigma \)-discrete or contains a homeomorph of the Cantor set. The Borel classification of the spaces of weight \( \chi_1 \) is completed, by means of a result of independent interest: If to each countable limit ordinal \( \alpha \) we assign a sequence \( x = (x_1, x_2, \ldots) \) of ordinals \( x_n < \alpha \) converging to \( \alpha \), then the set \( X \) of all the sequences \( x \) is never a Borel set (in either of two reasonable topologies) in the space of all sequences of countable ordinals. (Received March 27, 1962.)


Theorem. If a first order theory is categorical in one uncountable power, then it is categorical in every uncountable power. This completely solves the problem raised by Löb in On the categoricity in power of elementary deductive systems, Colloq. Math, 3 (1954), 58-62. (Received March 28, 1962.)

62T-137. D. E. KNUTH, California Institute of Technology, Pasadena, California. Non-Desarguesian planes of order \( 2^{2m+1} \). Preliminary report.

Let \( x \) be a primitive element of \( GF(2^{2m+1}) \), \( m > 1 \). A new multiplication is defined on the elements of this field by letting \( x^a \cdot x^b = x^{a+b} \), \( x^{2m} \cdot x^a = x^a \cdot x^{2m} = x^{(x^{2m} + x^a + 1)} \), and \( x^{2m} \cdot x^{2m} = (x^{2m} + 1)^2 \), where \( 0 \leq a, b < 2m \). The resulting system is a commutative non-associative division algebra. For if \( s \) and \( t \) are arbitrary nonzero polynomials in \( x \) of degree less than \( 2m \), \( s \cdot t = st \neq 0 \), and \( (x^{2m} + s) \cdot t = (x^{2m} + s + t + 1)t \neq 0 \), and \( (x^{2m} + s) \cdot (x^{2m} + t) = (s + t)^2 + (s + t)y + y^2 \neq 0 \) where \( y = x^{2m} + s + 1 \). Furthermore, \( (x^m \cdot x^m) \cdot x \neq x^m \cdot x(x^m \cdot x) \), so this system coordinatizes a non-Desarguesian projective plane. (Received March 29, 1962.)


Theorem: Let \( (G, \cdot) \) be any group. Then every double-coset (modulo the normal subgroups \( (H, \cdot) \) and \( (K, \cdot) \)), with the exceptions of \( H \cdot K \) (clearly \( (H \cdot K, \cdot) \) is a subgroup) and \( (\lambda, H \cdot K) \)-mutant double-cosets, is a \( (\lambda + 1, G) \)-mutant set. No systematic method has appeared to the author for determining which, if any, of the abundance of mutant double-cosets are nontrivial maximal ones over some set \( N \subseteq G \). (Received March 30, 1962.)

62T-139. L. C. ROBERTSON and S. P. FRANKLIN, University of California, Los Angeles 24, California. \( \alpha \)-sequences and \( \alpha \)-nets.

An \( \alpha \)-net (\( \alpha \)-sequence) is a net (sequence) which is either eventually inside or eventually outside any given open set. (See Levine, Amer. Math. Monthly 67 (1959), 667-668). The relationship between \( \alpha \)-nets and universal nets shows that convergence of every \( \alpha \)-net is equivalent to compactness (not necessarily \( T_2 \)). Tychonoff's theorem is an easy corollary, using familiar techniques. Convergence of every \( \alpha \)-sequence is equivalent to nonvoid intersection of every countable nest of point 218
closures. This result can be formulated in terms of Davis' characterization of topological spaces (Amer. Math. Monthly 68 (1961), 886-894). Davis' condition $R_0$ (point closures form a partition) is related in that an O-sequence converges to almost all points of its range in $R_0$ spaces. Spaces in which O-sequences have finite range or are eventually constant are characterized in terms of the possibility of separating countably infinite sets into two infinite subsets by intersection with an open set and its complement. It sometimes suffices to require separation of compact countable subsets.

Relationship to separation axioms $T_0$, $T_1$, and $T_2$ is displayed. (Received April 2, 1962).

62T-140. C. M. HOWARD, University of California, Berkeley, California.  A two dimensional analogue to boolean algebras.

A concrete $E_2$-algebra is an algebra $\langle A, +, \cdot, -, S, D, d \rangle$ such that for some set $B$, $+,$ $\cdot,$ $-$ are union, intersection, and complement with respect to $B \times B$, $0 \neq A \subseteq P(B \times B)$, and for each $a \in A$, $S(a) = \{ \langle u, v \rangle | \langle v, u \rangle \in a \}, D(a) = \{ \langle u, v \rangle | \langle u, u \rangle \in a, v \in B \}, d = \{ \langle u, u \rangle | u \in B \}$. (1) The concrete $E_2$-algebras are those classes $A$ of binary relations on a set $B$, such that $A$ is closed under all the operations expressible without quantifiers in the first order binary predicate calculus with identity. (2) A natural, finite, set of identities $I$ together with two universal sentences can be shown to characterize all concrete $E_2$-algebras. (3) The finitely generated algebras satisfying $I$ are finite. (4) For the algebras satisfying $I$, the notions of being (a) simple, (b) subdirectly irreducible, (c) weakly subdirectly irreducible, (d) concrete, (e) directly irreducible can be expressed by a universal sentence. (5) Each property (a) - (e) implies the next with only (b) and (c) coinciding. (Received April 2, 1962).


The following theorem is proved: Let $V^n$ be a compact connected non-orientable $n$-dimensional manifold with $n$ even and not a power of 2. Then the dual Stiefel-Whitney class of $V^n$ in dimension $n - 1$ (with twisted integer coefficients) vanishes. The restriction that $n$ is not a power of 2 is essential here, as the example of real projective space of dimension $2^k$ shows. In view of recent results of M. Hirsch and A. Haefliger (as yet unpublished), this theorem has implications regarding the possible imbeddings of such a $V^n$ in Euclidean space. (Received April 2, 1962.)


The following results on imbedding and immersing projective spaces in Euclidean space are obtained: (A) If $n - 1$ is a power of two, $P^n$ is not imbeddable in $E^{2n-2}$, $C^n$ not in $E^{3n-5}$, $Q^n$ not in $E^{8n-11}$; (B) If $n > 7$, $P^n$ is not immersible in $E^{n+2}$. (A) is proved by examining the cohomology of the normal bundle, à la Massey (Pacific J. Math. 9 (1959)); (B) is proved by a study of 2-plane bundles over $P^n$. (Received April 4, 1962.)
It has been proved by Haefliger (see Comment. Math. Helv. 36 (1961), 47) that any closed q-connected n-manifold can be differentiably imbedded in $E^{2n-q}$ if $n \geq 2q + 3$. This result is improved as follows. Let M be an orientable $(q-1)$-connected n-manifold with normal Stiefel-Whitney class $w_{n-q}$ equal to zero. Then, for $n \geq 2q + 3$, M can be imbedded in $E^{2n-q}$. Furthermore, if the homomorphism $\pi_{n-1}(S^{n-q-1}) \to \pi_{n-1}(W_{2n-q,n+1})$ induced by the natural inclusion is zero, M can be immersed in $E^{2n-q-1}$ (e.g., if $(n,q) = (4k + 1,1), (4k + 1,2), (4k + 2,2), (8k + 3,3), (n,4)$ or $(n,5)$). Some corollaries are: (1) Any orientable closed n-manifold can be imbedded in $E^{2n-1}$. (2) If n is not a power of two, $\mathbb{C}P^n$ is imbeddable in $E^{4n-2}$, $\mathbb{Q}P^n$ in $E^{8n-4}$ and $\mathbb{Q}P^n$ is immersible in $E^{8n-5}$. (Received April 4, 1962.)

R. M. Smullyan has suggested the name "pseudo doubly productive" for the pairs defined as doubly productive in his Theory of formal systems (Annals of Mathematics Studies 47, Princeton, University Press, Princeton, 1960). Define accordingly the notions "pseudo doubly creative" and "pseudo D. C. +" (cf. Smullyan, Theory of formal systems, pp. 109, 120), and verify the existence of pseudo D. C. + pairs $(a, \beta)$ such that both of $\alpha - \beta, \beta - \alpha$ (and hence also the symmetric difference of $\alpha, \beta$) are: (i) recursive; (ii) r.e. but not recursive; (iii) not r.e. (for the reason for the introduction of the adjective "pseudo," see Abstract 588-28, Notices Amer. Math. Soc., 9 (1962), 34). (Received April 9, 1962.)

We consider equations of the form \( (1) \ a(y)u_{tt} = b(y)u_{xx} + u_{yy} \), $a \in C^2$, $\alpha > 0$, $b \in C^0$, and seek $C^2$ solutions in a region $R = \{ (t,x,y), -\infty < t, x < \infty, 0 \leq y \leq h < \infty \}$, given initial data (2) $u(t,x,0) = f(t,x)$, $u_y(t,x,0) = g(t,x)$ on the timelike manifold $y = 0$. When $b(y) > 0$ for at least some subinterval of $[0,h]$, such problems are improper in the sense of Hadamard. We have previously shown [J. Math. Mech., 11, no. 2 (1962)] that if $f(t,x), g(t,x) \in L_2$ with respect to $t$ and $x$, and their Fourier transforms $F(\tau, \omega), G(\tau, \omega)$ have compact support in both variables, then there exists a unique $L_2$ solution which depends Lipschitz continuously (in the $L_2$ norm) on the initial data $f, g$. In this paper we show that for the problem (1), (2) it is sufficient to assume only that $F(\tau, \omega), G(\tau, \omega)$ have compact supports in $\omega$ in order to obtain the same results. Our methods generalize to the initial value problem for the ultrahyperbolic equation \( \sum_{i=1}^m a_i(y)u_{i|t|} = \sum_{j=1}^n b_j(y)u_{x_jx_j} + u_{yy} \). In the case $m = n = 1$ a more refined result is obtained relaxing the assumption that $f, g \in L_2$ with respect to $t$, and using the norm $\|u(y)\| = \sup_{t \in [0,\infty]} \int_0^\infty |u(t,x,y)|^2 dx^{1/2}$. In this case the coefficients $a, b$ may depend on $t$ and $y$. (Received April 10, 1962.)
Some uses of the second conformal structure on strictly convex surfaces.

$S$ is immersed $C^3$ in $E^3$ with $K > 0$, oriented so that $H > 0$. Form a Riemann surface $R$ by using conformal parameters $z = x + iy$ on $S$, where $x,y$ are bisothermal coordinates making $\Pi = \mu(x,y) \cdot (dz^2 + dy^2)$. Then $\Omega = \phi dz^2$ with $\phi = (E - G - iF)/2$ is a quadratic differential on $R$. **Lemma.** $\Omega$ is holomorphic on $R$ iff $K \equiv$ constant on $S$. **Corollary.** Liebmann's Theorem. **Corollary.** If $K \equiv$ constant, then $S$ is bisothermal, i.e., bisothermal lines-of-curvature coordinates exist in the neighborhood of all non-umbilics on $S$. **Lemma.** The index of an isolated umbilic $U$ is $j = (-1/2)((1/2\cdot \Delta \arg X_x \cdot X_y)$ where $X$ describes the immersion of $S$ in $E^3$. (Note that only first derivatives of $X$ appear above.) **Corollary.** Carathéodory's conjecture (that $j \equiv 1$) holds if $K \equiv$ constant near $U$. Analogous facts have long been known involving ordinary conformal structure on $S$, isothermal for bisothermal co-ordinates, $H$ for $K$, and $(L - N - iM)/2$ for $\phi$. (See Heinz Hopf, Lectures on differential geometry in the large, mimeographed notes, Stanford University, 1956.) (Received April 13, 1962.)

**62T-148.** H. B. ENDERTON, 18 Prentiss Street, Watertown, Massachusetts. Hierarchies in recursive function theory.

Let $\prec_1$ be a partial well-ordering. For each $b$ in the field $D$ of $\prec_1$ the ordinal $|b|$ can be defined inductively to be the least ordinal $\sigma$ such that $(\forall y \prec_1 b) |y| < \sigma$. Consider only orderings which are neat and reasonable at successors, but allow them to be completely wild at limit points. Given either the jump or the hyperjump operation, a hierarchy of sets $H_b$ indexed by the members of $D$ can be constructed. This is done by following the example of the hyperarithmetic hierarchy, except that, when $|b|$ is a limit ordinal, let $H_b = \{x \prec_1 y : y <_1 b \land x \in H_y\}$. **Theorem.** If the set of predecessors of each member of $D$ is uniformly recursive in an earlier $H$-set, then the Turing degree of $H_b$ depends only on the ordinal $|b|$, and not on the choice of $b$ or even on the choice of the particular ordering $\prec_1$. This theorem allows construction of a hierarchy of Turing degrees indexed by a segment of the ordinals, without using a particular ordering of integers. In the case of the jump operation just the hyperarithmetic hierarchy is attained. With the hyperjump a hierarchy of $\sum_2^1 \cap \Pi_2^1$ degrees is obtained. (Received April 16, 1962.)


For notations see Kleene, Trans. Amer. Math. Soc. 91 (1959), 1-52. **Theorem.** There are $\varphi$-operators, recursive in $E^2$ (the representing functional for $\lambda x_1 (E(x))(a_1(x) = 0)$ for partial recursive
functionals ('p.r.f.') whose arguments are of type not greater than 2. E.g. there is a p.r.f. 
\( \psi(z, a^1, a^2, \rho^2) \) such that \( \{z\} (\psi(z, a^1, a^2, E^2), a^1, a^2) = 0 \) whenever \( (E \in \{z\}) (a^1, a^2) = 0 \). The theorem has a number of obvious but interesting corollaries. E.g. (1) Let \( F^2 \) be any type 2 functional in which \( E^2 \) is recursive, then \( (\kappa)(Ea^1 \in [E^2]) R (\kappa, a^1, F^2) \supset (Ea^1 \in [E^2])(\kappa) R (\kappa, a^1(\kappa^{a^1}), F^2) \) where \( [E^2] \) is the class of functions recursive in \( F^2 \). (2) Let \( E_1^2 \) be the representing functional for \( \lambda a^1(\rho^1) (Ea^1) \cdot (a^1(\rho^{-1}(\kappa)) = 0) \); if \( \mathcal{H}(a) = 0 \equiv (a^1 \in [E^2])(E \rho^1 \in [E^2]) a^1(\rho^1, a, a) \equiv (Ea^1 \in [E^2])(\rho^1 \in [E^2]) B(a^1, \rho^1, a) \) with arithmetic \( A, B \), then \( \psi \in [E^2] \). The proof of the theorem depends on the fact that the significant positions on the Kleene tree for a (defined or undefined) computation are well-ordered lexicographically. The theorem can be extended to p.r.f.'s with arguments of type \( \leq j \), but the \( \psi \)-operators then require a well-ordering of the type \( \leq 2 \) as an argument as well as the representing functional for \( \lambda a^1(\rho^1-2)(a^1(\rho^{-2}) = 0) \). (Received April 16, 1962.)


Let \( \Delta^3 \) be the representing functional for \( \lambda a^2(Ea^1 \in [E^2])(a^2(a^1) = 0) \), and note that \( E^2, E_1^2 \), are recursive in \( \Delta^3 \); (notation as in previous abstract). Theorem 1. There is a p.r. function \( \psi(z) \) such that if \( \{z\} (a, \Delta^3) \) is defined for all \( a \), then \( \psi(z) \) is an index number for the expression of \( \{z\} (a, \Delta^3) = 0 \) in both 2-function-quantifier forms. (But there are predicates of \( \Sigma^1 \cap \Pi^1 \) which are not recursive in \( \Delta^3 \).) Let functionals \( D_n^2 \) and predicates \( F_{n+2} \) be defined as follows: \( D_0^2 = \lambda a^1 0; D_1^2 \) is the representing functional for \( \lambda a^1 (\{a^1\}(t \lambda a^1(t+1), D_n^2, \Delta^3) \) is defined) and \( F_{n+2} = \lambda z (\{z\}(D_n^2, \Delta^3) \) is defined). Theorem 2. \( F_{n+2} \) is a complete predicate of the form \( \Pi^1_{n+2} (m = 0, 1, ...), \) Thus every analytic predicate is recursive in \( D_n^2 \) for some \( n \). The proof of Theorem 1 starts from the observation that given both 2-function-quantifier forms for the graph of \( a^2 \) we can, using Kleene's XXVI and XXVII, write down the 2-function-quantifier forms for \( \Delta^3(a^2) = 0 \), and then proceeds by induction on \( \{z\}(\cdot) \). Theorem 2 follows fairly directly, using Kleene's definition of 'is defined'. (Received April 16, 1962.)


Let \( \Gamma^a \) stand for the Hilbert space of square integrable abelian differentials on a parabolic Riemann surface \( W \) of infinite genus. Let \( \Gamma^a \) be the subset of \( \Gamma^a \) consisting of differentials which are exact in a neighborhood of the ideal boundary. Lemma: If \( a \) and \( \beta \in \Gamma^a \) then the function \( a/\beta \) is a meromorphic function of finite valence. Theorem: If \( \dim \Gamma^a = 2 \) then there exists a closed surface \( W' \) and an analytic map \( f \) of finite valence from \( W \) into \( W' \) such that every differential in \( \Gamma^a \) is an abelian differential on \( W' \) raised via \( f \). Corollary: \( \Gamma^a \) is a finite dimensional subspace. Proofs: The lemma follows in the case where the ideal boundary of \( W \) has a finite number of points from a theorem of M. Heins (Riemann surfaces of infinite genus, Ann. of Math. 55 (1952), Theorem 5.1.) In the general situation it follows from a theorem of H. Royden (unpublished). The theorem follows from another theorem of M. Heins since \( W \) admits a nonconstant meromorphic function of finite valence. (On certain meromorphic function of bounded valence, Rev. Math. Pures Appl., II (1957). Theorem 1.) (Received April 16, 1962.)
Definitions. Let $\Gamma_h(W)$ be the Hilbert space of square integrable harmonic differentials on a Riemann surface $W$. Let $\Gamma_{he}$ be the exact forms in $\Gamma_h$ and let $\Gamma_{h0} = \Gamma_{he}^{*A}$. A harmonic function, $u$, will be called a generalized harmonic measure (g.h.m.) if the greatest harmonic minorant of $u$ and $1 - u$ is zero. **Theorem.** Let $u$ be a g.h.m. on $W$ such that $du \in \Gamma_{he}^*$. Then: (1) The double of the surface $\{ \varepsilon \leq u \leq 1 - \varepsilon \}$ is parabolic for all $\varepsilon > 0$. (2) The set of $t$, such that the level curves $\{ u = t \}$ contain a noncompact component, has measure zero. (3) For $\sigma \in \Gamma_h^*, \int_{t=0}^{t=\sigma} \sigma$ converges absolutely to $(\sigma, du^*)$ for almost all $t$. (4) $\int_{t=0}^{t=\sigma} du^* = \| du \|^2$ for all $t$. (5) $du \in \Gamma_{h0}^*$. (6) There exists an exhaustion $\{ \Omega_n \}$ and $du_n \in \Gamma_{h0}^* \cap \Gamma_{he}^*(\Omega_n)$ such that $\| du_n - du \| \to 0$ as $n \to \infty$. (7) Let $A$ be a nonbranched component of the set of level curves $\{ u = t_0 \}$. Parametrize $A$ by $f: [0, a] \to A$ such that $s = \int_{t(0)}^{t(s)} du^*$ and $a = \int_{A} du^*$. Let $l(s)$ be the trajectory through $f(s)$ orthogonal to the level curves of $u$. Then the set of $s$, such that the variation of $u$ on $l(s)$ is less than one, has measure zero. **Proof.** Part (1) follows by an extention of the methods of Kuramochi (Capacity of subsets of the ideal boundary, Proc. Japan Acad, 32 (1956), 111-116). The remaining parts follow from part (1) via theorems of the author as reported in the abstract Semi-parabolic Riemann surfaces, Notices Amer. Math. Soc. 7 (1960), 650. (Received April 16, 1962.)

Continue the notation of the preceding abstract. **Theorem 2:** Suppose $\Gamma_{he}^*(W)$ is finite dimensional. If $du \in \Gamma_{he}^*$ and $a \leq u \leq b$, then there exists a partition of $[a, b]$, $a = t_0 < t_1 < \ldots < t_n = b$ such that $(u - t_{k-1})/(t_k - t_{k-1})$ is a g.h.m. on the surface $\{ t_{k-1} < u < t_k \}$. **Corollary.** Suppose $W$ can be embedded in $W'$ such that $\partial W$ is piecewise analytic and $\Gamma_{he}^*(W')$ is finite dimensional. If $du \in \Gamma_{he}^*(W)$ then the set of $t$, such that the level curves $\{ u = t \}$ contain a noncompact component, has measure zero. **Remark.** There exists a surface $W$ with $du \in \Gamma_{h0}^*(W)$ such that the level curves of $u$ do not satisfy the condition of the corollary. **Theorem 3.** Let $u$ be a singular harmonic function on $W$ such that for some $a > 0$, $du$ has finite norm on $[0 < u < a]$. Then $(u - a)/(b - a)$ is a g.h.m. with finite norm on the surface $\{ a < u < b \}$ for any pair of nonnegative numbers, $a, b$. (Received April 16, 1962.)

J. deGroot and R. H. McDowell have raised the following question (Fund, Math 48 (1959/60), 256): Let $X$ be a metric space, dim $X = 0$, $\Phi \subseteq C(X, X)$, (not necessarily countable), and suppose there is a metric compactification, $X^*$, of $X$ allowing continuous extension to $X^*$ of each $\varphi \in \Phi$ (i.e., $X^*$ is a $\Phi$-compactification of $X$). Then, does there exist a $\Phi$-compactification of dimension zero? Engelking asks (Fund, Math, 48 (1959/60), 324): If $\Phi = \{ \varphi_k : k \leq 1 \} \subseteq C(X, X)$, $X$ separable metric, dim $X \leq n$, what is the category of the set of homeomorphisms $h \in C(X, I^{2n+1})$ such that $h(X)$ is a $\Phi$-compactification of $X$, in the space $C(X, I^{2n+1})$? The first question is answered in the negative, and $X = \mathbb{R}$.
numbers in the unit interval I, \( \Phi = \text{all continuous functions } \not\equiv X \to X \) with an extension to I is a counterexample. The second question is answered by the following theorem: Let X be a separable metric space, \( \dim X \equiv n \), \( \Phi = \{ f_k \in C(X;X) : k \equiv 1 \} \). Then the set of homeomorphisms \( h \in C(X, I^{2n+1}) \) satisfying \( \dim h(X) \equiv n \) and \( h(X) \) is a \( \Phi \)-compactification of X contains a dense G-delta set in \( C(X, I^{2n+1}) \), for any positive integer n. (Received April 19, 1962.)

62T-155. WITHDRAWN.

62T-156. SEYMOUR KASS and ARTHUR FINE, Illinois Institute of Technology, Technology Center, Chicago 16, Illinois. Indeterminate forms on \( E^2 \).

The following is a generalization of the classical theorems of L'Hospital and Stolz for indeterminate forms of 1-place functions. For simplicity, the theorem is stated for 2-place functions and a straightforward extension to n-place functions is indicated in the proof. Theorem. Let \( \lambda:(a,b) \in \mathbb{R} \), where \( R \subseteq E^2 \) is star-shaped with respect to \( \lambda \). Let f and g be functions defined on \( R \) which, for \( P \in R \), are both differentiable on the segment \( \overline{PA} \), except possibly at \( \lambda \). Moreover, let

\[ (x - x')g_1(P) + (y - y')g_2(P) \neq 0 \]

on every segment of the form \( \overline{QPQ'} \), where \( Q:(x,y), Q':(x',y') \) and \( P \) are in \( R \) (perhaps \( Q' = \lambda \)). With the understanding that all limits are taken from within \( R \), there are two cases. (1) \( f, g \to 0 \) at \((a,b)\), (2) \( |g| \to \infty \) at \((a,b)\). In either case, we have that if \( \lim_{(a,b)} Df/Dg = L \) then \( \lim_{(a,b)} f/g = L \), where D is a derivation on the ring of all (infinitely) differentiable functions, defined by \( (Df)(x,y) = (x - a)f_1(x,y) + (y - b)f_2(x,y) \). It should be noted that the nonvanishing of \( (x - x')g_1(P) + (y - y')g_2(P) \) in the hypothesis of the theorem is a simple matter to investigate if one takes into account that the points involved lie on radial segments. Moreover, Euler's theorem on homogeneous functions makes it possible to develop a calculus for the derivation D which makes the theorem easy to apply. (Received April 19, 1962.)

62T-157. D. F. DAWSON, North Texas State University, Denton, Texas. On certain sequence to sequence transformations which preserve convergence.

The following question is considered. For what subsets U of the set S of all complex sequences is it true that any triangular matrix A (\( A_{pq} = 0 \) if \( q > p \)) which is convergence preserving over U is convergence preserving? Several subsets of S having this property are given here. The methods of proof generally do not make use of the three well-known conditions of Silverman and Toeplitz which are necessary and sufficient for such a matrix to be convergence preserving. Typical results are as follows. (1) Suppose X is a real sequence which converges to \( k \neq 0 \). If A is convergence preserving over the set of all real sequences which converge to k slower than X, then A is convergence
preserving. (Here \([\{a_p\}\] converges slower than \([\{b_p\}\]) means \(\lim a_p = a, \lim b_p = b,\) and \(\lim (b_p - a)/(a_p - a) = 0.\) (2) If \(A\) is convergence preserving over the set of all sequences whose terms are elements of the Cantor set, then \(A\) is convergence preserving. (Received April 23, 1962.))


The following theorem essentially combines two results of Hadamard (Acta Math., 27 (1903), 177-183), and includes theorems of Abel, Dedekind, and others. **Theorem.** In order that the complex number sequence \(\{a_i\}\) be such that the series \(\sum a_i\) and \(\lim \sum E_i a_i\) converge or diverge together for every choice of \(\sum a_i\), it is necessary and sufficient that \(\sum_{N}^{\infty} |1 - p_{i+1}/p_i| < \infty\) for some \(N\). The proof is based on the following property of complex number sequences. If \(z_i \to z\) as \(i \to \infty\), then \(\sum_{N}^{\infty}|z_i - z_{i+1}/(z_i - z)|\) converges for no \(N\). This result, which includes several theorems of Dini, Stipanic, and others on positive term series, is equivalent to the fact that a product \(\prod(1 + a_i)\) converges absolutely if and only if \(\sum a_i\) converges absolutely. The theorem above is equivalent to a result of Agnew (Pacific J. Math., 1 (1951), 1-3): Suppose \(c_i \neq -1, i = 1, 2, 3, \ldots\). In order that \(\{c_p\}\) be such that \(\sum a_p\) and \(\sum b_p\) converge or diverge together whenever \(a_{n+1}/a_n = (b_{n+1}/b_n)(1 + c_n)\), \(n = 1, 2, 3, \ldots\) it is necessary and sufficient that \(\sum |c_p| < \infty\). (Received April 23, 1962.)

62T-159. D. F. DAWSON, North Texas State University, Denton, Texas. Convergence and divergence of continued fractions.

Let \(g(b)\) denote the continued fraction \(1/1 + 1/b_2 + 1/b_3 + \ldots\) with \(p\)th approximant \(g_p = C_p/D_p\). Lane and Wall (Trans. Amer. Math. Soc., 77 (1949), 368-380) studied the convergence and divergence of \(g(b)\) in terms of properties of \(\{h_p\}\) defined as follows: \(h_1 = -b_2, h_{p+1} = (1 - h_p)b_{p+1}/h_p, p = 1, 2, 3, \ldots\) this sequence having interesting properties with respect to the linear fractional transformations which generate the continued fraction equivalent to \(g(b)\) which has partial denominators \(1\). In this paper the following theorem is proved and is used to obtain several theorems on the convergence and divergence of \(g(b)\). **Theorem.** Suppose \(\{z_p\}\) is a complex number sequence whose terms are distinct from \(0\) and 1. Let \(B\) denote the set of all continued fractions \(g(b)\) such that \(h_{2p} = z_p, p = 1, 2, 3, \ldots\). Then the following two statements are equivalent: (1) If \(g(b) \in B\), then \(\sum_{2p} b_{2p}\) both converge or both diverge, and \(2) \sum |z_p| < \infty\). A similar result holds if the roles of even and odd indices are interchanged. (Received April 24, 1962.)


The Poisson interface problem asks for a function \(u\) satisfying Poisson's equation in a simply connected bounded domain \(D\), taking prescribed values on the boundary \(C\), and satisfying \(K\partial u^+ / \partial n = \partial u^- / \partial n\) on an arc \(F\) (the interface) in \(D\) which terminates on \(C\). Here \(K\) is a positive constant. Existence and uniqueness of the solution are proved under suitable regularity conditions on the given data when \(F\) is an analytic arc. The method may also be used for the condition \(Ku^+ = u^-\) on \(F\). Existence and uniqueness are shown also when there are several nonintersecting analytic
of times that $x$ appears in $w$. Form $f(x)$ from $w$ by inserting parentheses between its letters so as to make it into a well-defined product if juxtaposition means loop-multiplication. If $r \in L$, $f(x) = r$ is said to be an integral equation over $L$, $n$ its degree. Every integral equation has a solution in a suitably constructed extension loop $(C_n, L)$, $C_n$ being the cyclic group of order $n$. The number of solutions is studied, and additional results pertaining to loops with identities are obtained for the special case $n = 2$. (Received April 24, 1962.)


There are infinitely many individual variables and predicate letters. In addition, there are the individual constant 0, the equality predicate, the successor function, the truth functions, all with the intended interpretation. Quantifiers are not permitted. An individual variable or 0 is a term, applying the successor operation on a term gives again a term. A predicate followed by a suitable number of terms is a proposition, applying truth functions to propositions again give a proposition. A proposition is a sequential definition if it has a unique model for the occurring predicates, relative to zero, equality, and successor. Theorem 1. A predicate is recursive in the usual sense if and only if it is definable by a sequential definition. By using predicates instead of functions, one can apply here the infinity lemma, so that the Herbrand type of definition in terms of unique models determines the same set of predicates as the Gödel type of definition in terms of derivability. Theorem 2. A proposition $p$ is a sequential definition if and only if, for every occurring predicate $R$ and every set of constant arguments $a, \ldots, b$, either $Ra \ldots b$ or $\neg Ra \ldots b$ is derivable from $p$ but not both. (Received April 24, 1962.)

62T-165. DONALD GREENSPAN, Mathematics Research Center, United States Army, University of Wisconsin, Madison, Wisconsin. Note on difference approximations with non-negative coefficients for partial differential equations.

On a region $R$, the linear operator $L[u(x,y)] = u_{xx} + 2b(x,y)u_{xy} + c(x,y)u_{yy}$, $c(x,y) > 0$, is considered. Conditions are sought under which at $(x,y) \in R$, $L$ has a difference approximation $L_h[u] \equiv \sum_{i=0}^{8} a_i u_i$ with non-negative coefficients. If $b$ and $c$ are continuous and $u \in C^2(R)$; or if $b$ and $c$ are analytic, $u \in C^2(R)$, and $u$ is a solution of $L[u] = 0$, then it is proved that a necessary and sufficient condition that $L_h$ exist is that $|b(x,y)| \leq \min[1, c(x,y)]$. When the latter inequality is valid, $L_h$ is given constructively. Application is made to an extension of a numerical method of Bers for mildly nonlinear elliptic differential equations. (Received April 24, 1962.)


Let $N$ be a complete connected locally symmetric Riemannian manifold with every sectional curvature \( \geq 0 \). Then there is a finite covering $N' \to N$ and a deformation retraction of $N$ onto a compact totally geodesic submanifold, such that $N' = E \times T \times M'$ where $E$ is a Euclidean space, $T$ is a torus, $M'$ is a compact simply connected Riemannian symmetric space, and the retraction of $N$ lifts
interfaces. The method of proof uses the Schwarz iteration process and conformal mapping to extend the domain of existence from symmetric domains of existence. (Received by April 23, 1962.)

62T-161. Y. C. WONG, University of Hong Kong, Hong Kong. Geometry of n-planes in a real or complex Euclidean space. Preliminary report.

The objects of study are the n-planes (i.e., n-dimensional subspaces) in a real or complex inner-product vector space of dimension \( n > 1 \). Given any two n-planes A and B, we define in a natural way the (real) angle \( \theta(u) \) between a direction \( u \) in A and its orthogonal projection in B. The \( n \) extreme values of \( \theta(u) \) as \( u \) varies in A are called the angles between A and B, and the directions in A giving these extreme values the angle directions of A with respect to B. **Theorem 1.** Semi-perpendicular to both A and B, there are \( n \) mutually orthogonal 2-planes (some of which may degenerate into 1-planes) having the property that each of them cuts A in an angle direction of A with respect to B and cuts B in an angle direction of B with respect to A. **Theorem 2.** The \( n \) angles between A and B completely determine the relative position of A and B. These results and others will form the basis of a study of the differential geometry of the Grassmann manifolds. As in Memoirs Amer. Math. Soc., 41: Isoclinic n-planes in Euclidean 2n-space, etc., the method of matrices is used. (Received April 23, 1962.)


Sheldon [Math. Tables Aids Comput. 9 (1955), 101-112] suggested the use of a (point) symmetric successive overrelaxation (SSOR) iterative method with a semi-iteration acceleration procedure [Varga, J. Soc. Indust. Appl. Math. 5 (1957), 39-46] for the solution of a linear system of algebraic equations. Let \( R_1(\omega) \) denote the average rate of convergence [Young, J. Math. Physics 32 (1954), 243-255] of the SSOR method with semi-iteration and \( R_2(\omega) \) denote the rate of convergence [Young, Trans. Amer. Math. Soc. 76 (1954), 92-111] of the successive overrelaxation (SOR) method [Young, Trans. Amer. Math. Soc., op. cit.]. It is shown that for a symmetric and positive definite matrix with property \( \lambda' \) [Young, Trans. Amer. Math. Soc., op. cit.; Arms, Gates and Zondek, J. Soc. Indust. Appl. Math. 4 (1956), 220-229] using an \( \omega \) as per Habtler and Wachspress [Math, Comput. 15 (1961), 356-362] one has \( R_1(\omega) \geq k R_2(\omega) \) for some constant \( 3^{-1/2} < k < 1 \), asymptotically as \( n \to 1 \). Here \( \omega \) is the "best" \( \omega \) [Young, Trans. Amer. Math. Soc., op. cit.]. In particular, for the Dirichlet problem in a square or a subregion of a square, for point SSOR, \( R_1(\omega) \geq 2(\pi h)^{1/2} \) for sufficiently small mesh size \( h \). For block SSOR in a square, if \( \omega_1 = (\beta/2)(\beta - (\beta^2 - 4)^{1/2}) \), where \( \beta = 4 - 2 \cos \pi h \), then the largest eigenvalue of the method is \( \omega_1 - 1 \), and \( R_1(\omega_1) \sim 2(2\pi h)^{1/2} \) for small \( h \). (Received April 24, 1962.)

62T-163. RAFAEL ARTZY, Rutgers, The State University, New Brunswick, New Jersey. Solution of loop equations by adjunction.

Let \( w \) be a word whose letters are a symbol \( x \) and elements of a loop \( L \). Let \( n \) be the number
to a deformation retraction of $N'$ onto $T \times M'$. In particular, the betti numbers (singular theory) of $N$ are finite and the Euler-Poincare characteristic $\chi(N)$ is defined. Furthermore, $\chi(N) \equiv 0$. Finally, if $\chi(N) \neq 0$ then the fundamental group $\pi_1(N)$ is a finite 2-group. The rest of the paper is devoted to classification of the manifolds $N$ with $\chi(N) \neq 0$ in a special case which includes the case where the non-Euclidean part of the universal Riemannian covering manifold of $N$ is irreducible, and to proving some of the results on the structure of $N$ under weakened hypotheses. (Received April 25, 1962.)


A Riemannian nilmanifold is a Riemannian manifold which admits a transitive nilpotent group of isometries. Let $N$ be a connected Riemannian nilmanifold. It is shown that $N$ is isometric to a connected nilpotent Lie group in a left invariant Riemannian metric, that the nilradical of the full group of isometries of $N$ is the only nilpotent connected transitive group of isometries of $N$, and that the full group of isometries of $N$ is the semidirect product of this nilradical with any isotropy subgroup. If $N$ is simply connected and $D$ is the group of deck transformations of a Riemannian covering $N \rightarrow M$, then these are equivalent: (1) $M$ is a Riemannian nilmanifold, (2) $M$ is Riemannian homogeneous, (3) $D$ consists of Clifford translations (isometries of constant displacement), and (4) $D$ consists of isometries of bounded displacement. Other results are obtained on Riemannian coverings involving Riemannian nilmanifolds. (Received April 25, 1962.)


The theory of mutant sets investigates relations between similarities of algebraic structures and differences of algebraic structures, permitting one to focus on the differences, if one wishes to do so. Although the property of being a mutant set is an algebraic property one can show that, e.g., under a very weak hypothesis an anti-homomorphism between two algebraic systems is bidirectionally mutant preserving. Under the same hypothesis the image of a maximal mutant under an anti-homomorphism is a maximal mutant. Those two lemmas provide the tools for establishing nontrivial conditions under which the image of a maximal multiply mutant set under an anti-homomorphism is a maximal multiply mutant set. The case of anti-isomorphism is considered implicitly, but not explicitly. (Received April 25, 1962.)

62T-169. WITHDRAWN


An analogue of the theory of logarithmic potentials for recurrent Markov chains was worked out by the authors [J. Math. Anal. Appl., October, 1961]. This theory was based on a pair of dual potential operators $C$ and $G$. Potential measures of the form $\psi = -\mu C$ and functions of the form $g = -Gf$ where the charges $\mu$ and $f$ were of total charge 0 were considered. A new self-dual operator $K$ defined by $K_{ij} = C_{ij} + (G_{0j} - C_{0j}) = G_{ij} + (C_{i0} - G_{i0})a_j/a_0$, where $0$ is any fixed state, was introduced.
K exists if and only if the chain is normal (C and G both exist), and it serves as a potential operator in place of C and G for both measures and functions. Extend the class of potentials considered to include functions $g = -Kf$ and measures $\nu = -\mu K$ where $f$ and $\mu$ have finite support. It is shown that an equilibrium potential exists for every finite set and the probabilistic interpretation for its charge is given. The capacity of a set is the value of its equilibrium charge on the set. This capacity satisfies the capacity inequalities. Versions of classical principles, e.g., balayage and domination, are established. (Received April 25, 1962.)

62T-171. C. M. FULTON, University of California, Davis, California. Directed distance geometry.

Letting AB denote the distance from point A to point B, the geometry in question is based on the following two axioms. Axiom (I). $AB \cdot CD + BC \cdot AD + CA \cdot BD = 0$. Axiom (II). If $AB \neq 0$, $AC = AD$, $BC = BD$, then C and D are the same point. Using these axioms, definitions of straight lines and angles are given and the basic trigonometry is developed. Coordinates and their transformations are also dealt with. (Received February 9, 1962.)


An $\alpha$-complete Boolean algebra $\mathcal{A} = \langle A, \mathcal{I} \rangle$ is said to have an ordered $\alpha$-basis if there exists a set $B \subseteq A$ such that $B$ is simply ordered by $\subseteq$ and every element of $A$ is a sum of fewer than $\alpha$ differences of elements of $B$. Theorem 1. If $\alpha$ is a strong limit number, then every $\alpha$-complete and $\alpha$-distributive Boolean algebra of power at most $\alpha$ has an ordered $\alpha$-basis. Theorem 2. Suppose there exists an $\alpha$-complete and $\alpha$-distributive Boolean algebra $\mathcal{A}$ of power $\alpha$ such that every $\alpha$-complete prime ideal of $\mathcal{A}$ is principal. Then there exists a simply ordered set of power $\alpha$ which has no well ordered or inversely well ordered subset of power $\alpha$. Combining Theorem 2 with known results (see P. Erdős and A. Tarski, Essays on the foundations of mathematics, Jerusalem, 1961, 50-82), one sees that conditions $P_1$, $P_2$, $R$, and $Q$ of Erdős-Tarski are all equivalent for inaccessible cardinals. Thus $P_1$ (i.e, the conclusion of Theorem 2 above) and $P_2$ apply to a comprehensive class of inaccessible cardinals as well as to all accessible cardinals. Abstract 61T-240 shows that $P_3$ does not imply $P_2$ (unless all cardinals $\alpha > \omega$ satisfy $P_2$); the problem remains, does $P_4$ imply $P_3$? (Received April 27, 1962.)


Let $f(x)$ be analytic about $x = 0$, $f(0) = 0$ and $f'(0) = a$ for $0 < |a| < 1$. Define $f^{[n]}(x)$ recursively by $f^{[0]}(x) = x$ and $f^{[n+1]}(x) = f\{f^{[n]}(x)\}$. Form the generating function $\phi(x;z) = \sum_{n=0}^{\infty} f^{[n]}(x) z^{-n+1}$. The singularities in $z$ of $\phi(x;z)$ are among the points $z = a^n$ for $n = 1,2,\ldots$. Let $z = 0$ be a limit point of these singularities, and let $h(z) = \sum_{n=0}^{\infty} a_n (z - \beta)^n$ converge for $|z - \beta| < |\beta|$. Then Theorem. A sufficient condition for these series $\sum_{n=0}^{\infty} a_n \sum_{r=0}^{n} R_{\nu}^{(n)} (-\beta)^{n-R_{\nu}^{(n)}} f^{[n]}(x)$ to converge uniformly for all $x$ in some $e$ neighborhood of $x = 0$ is that there exist a $\nu > 0$ such that $N^{-\nu} |\sum_{n=0}^{N} a_n \beta^n|^{1/\nu}$ is bounded in $N$, and that the singularities in $z$ of $\phi(x;z)$, with the exception of $z = 0$, lie within the intersection of
the regions \(|z - \beta| < |\beta|\) and the angular sector \(\arg \beta - \pi/2 + \epsilon \leq \arg z \leq \arg \beta + \pi/2 - \epsilon\) for some \(\epsilon > 0\). Corollary. If \(\alpha\) is real, \(0 < \alpha < 1\), then for any complex \(s\), the Cayley-Schroder series

\[
\sum_{n=0}^{\infty} s^n \sum_{r=0}^{n-1} \binom{n}{r} (-1)^{n-r} f_r^n(x)
\]

converges uniformly in some \(\epsilon\) neighbourhood of \(x = 0\) to a generalized iterate \(f^\infty(x)\). (Received April 26, 1962.)


Rules (i) and (ii) of FPA(I) are suggested by the following theorems, dealing with hyperarithmetic sets, functions, resp.: (1) HYP is the smallest class \(C\) of sets closed under arithmetic operations and such that, with \(R\) and \(S\) arithmetic, \(
\bigwedge n \left[ (\forall x \in C) \land mR(m,n,x) \iff (\forall x \in C) VmS(m,n,x) \right]
\)

\(\iff \left\{ n : (\forall x \in C) \land mR(m,n,x) \right\} \in \mathcal{C}. \) (2) HYP is the smallest class \(C\) of functions closed under recursive operations and such that \(\bigwedge m (\forall a \in C) \land nR(m,n,a) \iff (\forall a_1 \in C) \land m \land nR'(m,n,a_1)\). Direct predicative justification of the rules (i), (ii) is also given. Much of classical analysis, including the theory of Lebesgue measure, is readily developed in \(EPA\) and \(EPA^+\). (Received January 2, 1962.)


The traditional way of making the notion of predicative definition and proof explicit is by use of ramified hierarchies and theories. These have the defect that the relation between the contents of statements in predicative and impredicative (ordinary) analysis is not at all clear; for a universal (existential) quantifier over sets of level \(\sigma\) is weaker (stronger) than the use of the corresponding quantifier over arbitrary sets. The following alternative formalization of predicative analysis within the same language as impredicative analysis \(\mathcal{A}\) is proposed. For simplicity, consider only variables \(n, m, \ldots\), ranging over the set \(\mathbb{N}\) of natural numbers and \(a, \beta, \ldots\), ranging over functions in \(\mathbb{N}\), \(a_1, \beta_1, \ldots\), over \(\mathbb{N} \times \mathbb{N}\), etc. Besides the usual logical and number-theoretical axioms (with induction applied to arbitrary formulas) consider the following schemes of inference (\(R, S\), any primitive recursive relations): (i) \(EPA\) (Elementary predicative analysis). From \(\bigwedge m \left[ (\forall \beta \land n R(m,n,\beta,\gamma) \iff \bigwedge \beta V_n S(m,n,\beta,\gamma) \right] \) infer \(\forall a \land m \left[ a(m) = 0 \iff \forall \beta \land n R(m,n,\beta,\gamma) \right] \). (ii) \(EPA^+\) (Constructive axiom of choice). From \(\bigwedge m \forall a \land n R(m,n,a,\gamma) \) infer \(\forall a_1 \land m \land nR'(m,n,a_1,\gamma)\), where \(R'(m,n,a_1,\gamma)\) is obtained from \(R(m,n,a,\gamma)\) by replacing \(\lambda t a(t)\) by \(\lambda t a_1(m,t)\). It is shown that \(EPA \subseteq EPA^+ \subseteq \mathcal{A}\). (Received January 2, 1962.)

62T-176. M. A. AL-BASSAM, Texas Technological College, P.O.Box 4346 Tech. Station, Lubbock, Texas. On \(T^a\)-functions.

Definition. \(f(x) \in T^a \iff (S): f(x) = \sum_{n=0}^{\infty} a_n \cdot (x - a)^n, a > 0, \text{ on } L: 0 \leq a \leq x \leq b < \rho \) where \(\rho\) is the radius of convergence of \((S)\). It is shown that: (1) If \(g(x) = \sum_{n=0}^{\infty} b_n \cdot (x - a)^n\) such that \(\lim_{n \to \infty} |b_n|^1/|a^n| = 1/r, \) or \(\lim_{n \to \infty} |b_{n+1}|^1/b_n = 1/r^n, r > 0, \) then \(g(x) \in T^a\). (2) If \(f_1(x), f_2(x) \in T^a, \) then \(f_1 + f_2 \in T^a\), and \((f_1 \cdot f_2) \in T^a\). (3) If \(f \in T^a, \) then (i) \(f\) is continuous at \(x = a, \) and at every interior point of \(L.\) (ii) \(f^n_0(x) = -\) the \(n\)th derivative of \(f\) with respect to \((x - a)^n, \) and
\[ f(t(x))(na - 1) = I_a x^1 \cdot an t', \text{ (for details see the author's work: Ann. Sci. Norm. Sup. Pisa, XV, Serie III (1961), 1-24.) are continuous on L for every positive integer n.} \]

(iii) \( a_n = f_n(a)/(n + 1), \) or \( [f(t(a))](na - 1)/\Gamma(na + 1), \) where \( [f(t(a))]^{-1} = f(a). \)

(4) If \( f \in T^a, \) then \( I_a x^p a f \in T^a, \) \( (p = 1,2,...), \) and if \( \{b_n\} \) is a sequence of numbers such that \( |b_{n+1}/b_n| \) is bounded, then \( F(x) = \sum_{q=0}^{\infty} b_q I_a x^q a f, \) and if \( F(x) \in T^a. \)

(5) Let \( m \) be a number then \( E_n = \sum_{p=0}^{\infty} a_m p - 1 x^{pa - 1}/\Gamma(p\alpha), \) \( S_n = \sum_{p=1}^{\infty} (-1)^p \alpha x^{2pa - 1}/\Gamma(2\alpha), \) and \( C_n = \sum_{p=0}^{\infty} (-1)^p b^p x^{pa - 1}/\Gamma(2\alpha), \) may be regarded as generalized exponential and circular functions. Some properties of these functions have been obtained. (Received May 4, 1962.)

62T-177. M. A. AL-BASSAM, Texas Technological College, P. O. Box 4346 Tech. Station, Lubbock, Texas. Some existence theorems of differential equations of noninteger order.

Theorem A. If \( p(x) \in T^a \) on \( L, \) and \( a \) is an ordinary point, then there exists a solution \( y = Y(x) \in T^a \) which satisfies the differential equation of order \( a = 1: y(a) + p(x)y = 0, \) and is such that \( Y = Y(x; a_0). \)

Example. By assuming \( y = \sum_{n=0}^{\infty} a_n (x - a)^{n + c}, \) \( c > 0, n < a < n + 1, \) it is found that the equation \( y(a) + my = 0, \) is satisfied by \( y = \sum_{k=1}^{\infty} C_k y_k, \) where \( C_k \) are arbitrary constants and \( y_k = E^P_k(-m; x). \) A similar result was obtained by J. H. Barrett (Canad. J. Math. (1954) 535-538) by solving the equivalent integral equation by the method of successive substitution. Theorem B. If \( P(x) \) and \( R(x) \in C \) on the interval \( (\lambda, \mu), \) then the differential equation \( y(a) + P(x)y = R(x), \) \( n < a < n + 1, \) admits of a unique solution which together with \( y(a-k)(x), \) \( (k = 1,2,...,n), \) is continuous on \( (\lambda, \mu) \) and satisfies the conditions \( y(a-k)(a) = \gamma_k, \) where \( a \) is a point of \( (\lambda, \mu). \) Clearly for \( a = n, \) this represents the well-known existence theorem of the nth order equation of this type. The unique solution satisfying the equation of the above mentioned example and these initial conditions is found to be the one in which \( C_k = \gamma_k. \) (Received April 18, 1962.)

ERRATA - Volume 9

D. C. RUNG. Results on cluster sets of normal functions defined in the unit disk.

Page 43, Abstract 62T-33.

Line 6. For "\( \Delta_1 \) and \( \Delta_2 \)" read "\( \Delta_1 \) and \( \Delta_2 \) with \( \Delta_2 \subseteq \Delta_1. \)"

Line 6. For "\( R \Delta_1 (t, e^{i\theta}) \)" read "\( R \Delta_2 (t, e^{i\theta}) \)."

Line 7. For "\( R \Delta_1 (t, e^{i\theta}) - \bigcap \Delta_2 (t, e^{i\theta}) \) is an open set" read "\( R \Delta_1 (t, e^{i\theta}) \) - \( \bigcap \Delta_2 (t, e^{i\theta}) \) is contained in \( R \Delta_2 (t, e^{i\theta}) \)."

T. G. McLAUGHLIN. Some remarks about productive and contraproductive centers.


This abstract was published in error.

WILLIAM CRAIG and WILLIAM HANF. On relative characterizability in a language.

Page 152, Abstract 62T-95.

Page 153, Line 3. For "has no nontrivial automorphism" read "is not isomorphic to any of its proper subsystems."
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May 1962 certainly stood out as “Operations Research Month”—in fact, if not by official proclamation. May saw the 10th anniversary meeting in Washington of the Operations Research Society of America (ORSA) as well as OEG’s 20th Anniversary Conference on Operations Research, also in Washington.

OEG, created in May 1942, is the oldest continuing military operations research organization in the country. The 20th Anniversary Conference—of international scope—reviewed applications of OR in NATO, in industry and public health, and in fiscal planning for defense, as well as considering OR education.

OEG acts as civilian scientific advisor to the Chief of Naval Operations and the Commandant, U. S. Marine Corps, functioning in diverse problem areas. Typical problems OEG has been called upon to solve include:

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2. Conduct studies of other Army problems that will respond to War Gaming techniques, and maintain liaison with other Army and Army-supported groups. One such program, nearing completion at STAG is a computer-programmed mathematical model of a Field Army Ballistic Missile Defense System (FABMDS).

3. Provide Army participation in joint exercises and technological advice and assistance to other war games groups. For example, STAG is sponsoring a War Gaming Symposium at its Bethesda, Maryland headquarters in September, 1962.

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RESERVATION FORM

UNIVERSITY OF BRITISH COLUMBIA MEETING, Vancouver, Canada
August 26 - 31, 1962

UNIVERSITY HOUSING RESERVATION FORM

Please make reservations by using the form below. It should be mailed to Conference Office, University of British Columbia, Vancouver 8, Canada, at the earliest possible date.

Those who wish to stay at a hotel or motel should make their reservations directly with the hotel or motel.

American Mathematical Society
Housing Reservation Form - Vancouver Meeting

Name ___________________________ (first) ___________________________ (last)

Institution ___________________________ Address ___________________________

Arrival Date _______________________ Time _______: Departure Date _______________________ Time _______

Housing available from 2:00 p.m. Saturday, August 25th, to 2:00 p.m. Saturday, September 1st.

Number of persons requiring accommodation (including children 12 years and over):

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<th>Male</th>
<th>Female</th>
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<th>Husband + Wife</th>
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<td>in double rooms at $1.50 per person</td>
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Only a few housing units suitable for families with young children are available on campus. Please apply as early as possible. You will receive an immediate reply.

Total number in family: _______; number of adults _______; ages of children ____________________________;
Can you provide your own crib/camp bed and sleeping bag/bedding for children: Yes _____ No _____;
Number expecting to attend barbecue _______.

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HYDRODYNAMIC INSTABILITY
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