AMERICAN MATHEMATICAL SOCIETY

Notices

VOLUME 9 NUMBER 5 ISSUE NO. 63 OCTOBER 1962

THE AMERICAN MATHEMATICAL SOCIETY

Notices

Edited by John W. Green and Gordon L. Walker

CONTENTS

MEETINGS

CALENDAR OF MEETINGS

NOTE: This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the NOTICES was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

Meeting No.	Date	Place	Deadline for Abstracts*
593	October 27, 1962	Hanover, New Hampshire	Sept. 12
594	November 16-17, 1962	Tallahassee, Florida	Oct. 2
595	November 17, 1962	Los Angeles, California	Oct. 2
596	November 23-24, 1962	Northwestern University	Oct. 2
597	January 24-28, 1963 (69th Annual Meeting)	Berkeley, California	Nov. 23
598	February 23, 1963	New York, New York	Jan. 9
599	April 19-20, 1963	Chicago, Illinois	Mar. 5
600	April 26-27, 1963	New Mexico State University	
601	April 29-May 3, 1963	New York, New York	
	August 26-30, 1963 (68th Summer Meeting)	Boulder, Colorado	
	January 20-24, 1964 (70th Annual Meeting)	Miami, Florida	
	August, 1964 (69th Summer Meeting)	Ann Arbor, Michigan	
	January, 1965 (71st Annual Meeting)	Denver, Colorado	
	August, 1965	Ithaca, New York	
	August, 1966	New Brunswick, New Jersey	

* The abstracts of papers to be presented in person at the meetings must be received in the Headquarters Offices of the Society in Providence, Rhode Island, on or before these deadlines. The deadlines also apply to news items. The next deadline date for by title abstracts is November 17.

The NOTICES of the American Mathematical Society is published by the Society in January, February, April, June, August, October and November. Price per annual volume is $7.00. Price per copy, $2.00. Special price for copies sold at registration desks of meetings of the Society, $1.00 per copy. Subscriptions, orders for back numbers (not available before 1958), and inquiries should be addressed to the American Mathematical Society, 190 Hope Street, Providence 6, Rhode Island.

Second-class postage paid at Providence, Rhode Island, and additional mailing offices. Authorization is granted under the authority of the act of August 24, 1912, as amended by the act of August 4, 1947 (Sec. 34, 21, P. L. and R.). Accepted for mailing at the special rate of postage provided for in section 34.40, paragraph (d).

Five Hundred Ninety-Third Meeting

Dartmouth College
Hanover, New Hampshire
October 27, 1962

PROGRAM

The five hundred ninety-third meeting of the American Mathematical Society will be held at Dartmouth College on Saturday, October 27, 1962. All sessions will be held in the Albert Bradley Center for Mathematics. All times given below are Daylight Saving Time.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, Dr. M. F. Atiyah will address the Society at 2:00 P.M. in the Lincoln Filene Auditorium, which is Room 101 Bradley. The title of his address is "Topology and linear algebra".

There will be a session for contributed papers at 10:00 A.M. and one at 3:15 P.M.

The registration desk will be on the first floor of Bradley. It will be open from 9:00 A.M. to 3:30 P.M. Coat and conversation space will be nearby on the same floor.

ROOMS

Each traveler should make his own reservation of rooms. A list of inns and motels follows.

Hanover Inn, Hanover. Center of town Owned and operated by Dartmouth College.
Phone 643-4300.

Hanover Inn Motor Lodge, Lebanon Street, Hanover. One block from center of town, three blocks from Bradley Center.
Phone 643-4400.

Green Lantern Inn, Main Street, Hanover. Two blocks from center of town, four blocks from Bradley Center.
Phone 643-3410.

Norwich Inn and Motel, Norwich, Vermont. One mile from Hanover.
Phone 649-1143.

Chieftan Motel, Lyme Road, Hanover. Two miles north of Hanover on Route 10.
Phone 643-2550.

Sunset Motel, Hanover Road, West Lebanon, New Hampshire.Three Miles south of Hanover on Route 10.
Phone 298-8721.

MEALS

The noon meal will be available at the college dining hall.

TRAVEL

Hanover is on New Hampshire Route 10. The natural highway approach from the south is U.S. Route 5. From Boston, the suggested approach is via Routes 3 and 4 through Concord, New Hampshire to Lebanon, New Hampshire and then Route 120 to Hanover. Hanover is 264 miles from New York by car and about 135 miles from Boston.

The Northeast Airlines give service from Boston and from New York to the Lebanon Airport about five miles from Hanover. There is taxi and bus service from the airport to Hanover.

There is limited rail service on the Boston and Maine Railroad from Boston to White River Junction, Vermont, which is about 5 miles from Hanover. There is bus service from Boston to White River Junction. There is local bus service from White River Junction to Hanover.

The Albert Bradley Center for

Mathematics is two blocks north of the center of Hanover, directly behind Baker Library on the north side of the campus. Baker Library is said to resemble a magnified Independence Hall.

PROGRAM OF THE SESSIONS

The time limit for each contributed paper is ten minutes. The contributed papers are scheduled at 15 minute intervals so that listeners can circulate between the different sessions. To maintain this schedule, the time limit will be strictly enforced.

SATURDAY, 10:00 A.M.

Session on Analysis, Algebra and Topology, Lincoln Filene Auditorium

10:00 - 10:10
(1) Homotopy commutators of finite order
Dr. M. A. Arkowitz, Princeton University and Professor C. R. Curjel*, Cornell University (593-6)

10:15 - 10:25
(2) Some noninvertible knots
Professor H. F. Trotter, Princeton University (593-10)

10:30 - 10:40
(3) The abelianized commutator subgroup of a knot theory
Professor R. H. Crowell, Dartmouth College (593-12)

10:45 - 10:55
(4) Boundary links
Professor R. H. Fox, Princeton University (593-14)

11:00 - 11:10
(5) Rotation-invariant algebras on the n-sphere
Professor Karel de Leeuw, Stanford University and Professor Hazelton Mirkil*, Dartmouth College (593-9)

11:15 - 11:25
(6) Doeblin's ratio limit theorem
Professor J. G. Kemeny, Dartmouth College (593-8)

11:30 - 11:40
(7) On monotonic solutions of certain linear integral equations
Professor John Lamperti, Rockefeller Institute (593-7)

11:45 - 11:55
(8) On the generalized inverse of a closed linear operator with an everywhere dense domain on a Hilbert space
Dr. Adi Ben-Israel*, Carnegie Institute of Technology, and Professor Abraham Charnes, Northwestern University (593-2)

SATURDAY, 2:00 P.M.

Invited Address, Lincoln Filene Auditorium
Topology and linear algebra (One hour)
Dr. M. F. Atiyah, Harvard University

* For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.

Session on Logic and Algebra, Lincoln Filene Auditorium

3:15 - 3:25

(9) Proof indices and recursion properties of provable recursive functions
Professor P. C. Fischer, Harvard University (593-11)

3:30 - 3:40

(10) Persistence and Herbrand expansions
Professor J. S. Wholey, Institute for Defense Analyses, Princeton, New Jersey (593-15)

3:45 - 3:55

(11) Some results in hyperarithmetic analysis
Mr. W. E. Ritter, Massachusetts Institute of Technology (593-1)

4:00 - 4:10

(12) An abstract form of Godel's incompleteness theorem
Professor Kurt Bing, Rensselaer Polytechnic Institute (593-4)

4:15 - 4:25

(13) Relative group extensions and simple algebras
Dr. D. R. Ostberg, Dartmouth College (593-13)

4:30 - 4:40

(14) On anti-commutative algebras with an invariant form
Dr. A. A. Sagle, Syracuse University (593-3)

4:45 - 4:55

(15) Single variable equations over a group
Professor Frank Levin, Rutgers, The State University (593-5)

Bethlehem, Pennsylvania

Everett Pitcher
Associate Secretary

MEMORANDA TO MEMBERS

RETIRED MATHEMATICIANS

The Mathematical Sciences Employment Register, 190 Hope Street, Providence 6, Rhode Island, announces that a small supplementary List of Retired Mathematicians Available for Employment has been prepared. The List is distributed by the Mathematical Sciences Employment Register sponsored by the American Mathematical Society, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics.

Copies of the supplementary list may be obtained free of charge by writing to the above address.

PRELIMINARY ANNOUNCEMENTS OF MEETINGS

FIVE HUNDRED NINETY-FOURTH MEETING
Florida Agricultural and Mechanical University
and
Florida State University
Tallahassee, Florida
November 16-17, 1962

The five hundred and ninety-fourth meeting of the American Mathematical Society will be held in Tallahassee, Florida on November 16 and 17, 1962. The sessions for contributed papers will be held in the Mathematics-Meteorology Building on the Florida State University Campus, and the invited address will be held in the Charles Winter Wood Theater on the Florida A and M Campus.

By invitation of the Committee to Select Hour Speakers for Southeastern Sectional Meetings, Professor J. S. MacNerney of the University of North Carolina will speak on "A linear initial-value problem", at 2:00 P.M., Friday, November 16.

There will be sessions for contributed papers at 10:00 A.M. and 4:00 P.M. on Friday, November 16 and at 10:00 A.M. on Saturday, November 17. Abstracts of papers should be sent to the American Mathematical Society, Providence 6, Rhode Island, prior to the deadline of October 2, 1962.

The registration desk will be in Room 303, Mathematics-Meteorology Building, F. S. U. This and the adjacent room will serve also as the coffee-and-conversation area.

ROOMS

Each traveler should make his own reservations of rooms. The following is a partial list of hotels and motels in Tallahassee.

Hotels

A and M Hotel
316 W. Virginia

Cherokee Hotel
Park Avenue and Calhoun

Duval Hotel
745 N. Monroe

Motels

Apalachee Motor Lodge
Apalachee Parkway

Driftwood Motel
1402 W. Tennessee

Holiday Inn Motel
1302 Apalachee Parkway

Howard Johnson Motel
738 Apalachee Parkway

Lake Ella Motor Court
1500 N. Monroe

Skyline Motor Lodge
2301 W. Tennessee

Southernaire Motel
W. Brevard and Tennessee

Tallahassee Travelodge
691 W. Tennessee

Williams Motel
1315 S. Boulevard

TRAVEL

Tallahassee is served by Eastern Air Lines (Atlanta-Miami or Tampa flights) and National Air Lines (Jacksonville-Pensacola or Mobile flights). U. S. Highways 90, 319 and 27 pass through Tallahassee. Florida State University is just west of town on U. S. 90, and the Mathematics-Meteorology Building is separated from the corner of U. S. 90 and Woodward Avenue by the Geological Survey Building. Maps will be available at the various entrances to the F.S.U. campus.

G. B. Huff
Associate Secretary

Athens, Georgia

FIVE HUNDRED NINETY-FIFTH MEETING
University of California
Los Angeles, California
November 17, 1962

The five hundred ninety-fifth meeting of the American Mathematical Society will be held on Saturday, November 17, 1962 at the University of California, Los Angeles.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, there will be hour addresses by Professor Dana Scott of the University of California at Berkeley, and by Professor Victor Shapiro of the University of Oregon. Professor Scott will speak at 11:00 A.M. on "The set of all sets". Professor Shapiro's talk on "Harmonic analysis and the theory of cochains" will be given at 2:00 P.M. Both of these lectures will be in Room 3400 of the Engineering Building (adjacent to the Mathematical Sciences Building on the UCLA campus).

Sessions for contributed papers will be held at 9:30 A.M. and 3:30 P.M. in the Mathematical Sciences Building. Registration will take place in the main departmental office, Room 6115 of the Mathematical Sciences Building, beginning at 9:00 A.M.

Luncheon on Saturday will be available in Dykstra Hall. The luncheon tickets will be sold at the Registration Desk.

The University of California campus is located in Westwood Village, about thirteen miles west of downtown Los Angeles. There are numerous motels within easy driving distance of the campus. The Dracker Hotel, 10824 Lindbrook Drive, Los Angeles 24, California is within walking distance of the campus.

R. S. Pierce
Associate Secretary

Seattle, Washington

FIVE HUNDRED NINETY-SIXTH MEETING
Northwestern University
Evanston, Illinois
November 23-24, 1962

The five hundred ninety-sixth meeting of the American Mathematical Society will be held at Northwestern University on Friday and Saturday, November 23-24, 1962. Meeting headquarters will be at the Pick-Georgian Hotel in Evanston, Illinois, though registration and all the scientific sessions of the meeting will be held at Northwestern University.

The rates of the Pick-Georgian for single rooms run from \$8.50 to \$14.50 and for double rooms from \$12.00 to \$14.50. However, the hotel has guaranteed single rooms to the Society at \$8.50 and doubles at \$12.00. Consequently, those who wish to stay where most of the members of the Society will be housed, and wish to avail themselves of these special rates, should use the reservation form at the back of these NOTICES so as to make themselves eligible for these special prices.

The Committee to Select Hour Speakers for Western Sectional Meetings has so far invited Albert E. Heins of the University of Michigan to address the Society.

Further details including travel information will appear in the final program.

J. W. T. Youngs
Associate Secretary

Bloomington, Indiana

ACTIVITIES OF OTHER ASSOCIATIONS

1962 FALL MEETING OF SIAM

A three-day meeting of the Society for Industrial and Applied Mathematics will be held at the Massachusetts Institute of Technology, Cambridge, Massachusetts on Thursday, November 1; Friday, November 2; and Saturday, November 3, 1962. The entire meeting will take place in rooms located in the Kresge Auditorium.

A Symposium on the Theory of Multivariable Control Systems will take place on Thursday afternoon, November 1, and on Friday morning and afternoon, November 2. Three sessions, devoted to the areas of theory, computational techniques, and applications, respectively, will be held. A total of eleven talks have been scheduled.

A Symposium on Continuum Mechanics will be held on Saturday morning. Dr. R. C. Toupin, IBM Research Center, Professor Fritz John, New York University, and Professor Eli Sternberg, Brown University have been invited to speak at this Symposium.

On Saturday afternoon, Professor Jean Leray of the College de France has been invited to deliver the Third John von Neumann Lecture, entitled "The Functional Transformations Required by the Theory of Partial Differential Equations." Following the von Neumann Lecture, sessions for contributed papers will be held, including a session for papers in the area of Continuum Mechanics.

A complete program for the meeting may be obtained by writing:

SIAM Programs,
Box 7541,
Philadelphia 1, Pennsylvania.

SUMMER RESEARCH INSTITUTE OF THE AUSTRALIAN MATHEMATICAL SOCIETY

The Third Summer Research Institute of the Australian Mathematical Society is to be held at the Australian National University, Canberra, between January 8th and February 15th, 1963. The Director of the Institute is B. H. Neumann (Institute of Advanced Studies, A.N.U.); he will be assisted by two Secretaries, G. Szekeres (University of Adelaide), G. E. Wall (University of Sydney) and a Treasurer, A. Brown (School of General Studies, A.N.U.).

R. H. Bruck of the University of Wisconsin, has accepted an invitation to attend the S.R.I., as has also K. Mahler of the University of Manchester. It is also hoped that H. Wielandt from Tubingen will be able to take part in the Institute.

THE ANNUAL SALARY SURVEY

The 1962 Salary Survey is the sixth in an annual series of surveys of academic institutions initiated in May 1957 by the Society's Committee on the Economic Status of Teachers. This year usable returns were received from 294 institutions reporting on 2,850 academic positions in 1961-1962 and 3,096 positions at the same institutions in 1962-1963. Thus there has been an 8.6% average increase in the staffs of the institutions reporting.

Tabulations of the salary surveys were originally based on a version of the classification of academic institutions introduced in the 1955-1956 Survey of Training and Research Potential in the Mathematical Sciences (the "Albert Survey"). In the interest of securing a classification which would reflect the mathematical activity of the institutions, a system based on institutional membership in the Society was introduced last year and is followed in this report. Institutional membership readily provides data on which to base the kind of classification desired, since the dues of a member institution are determined by the number of pages of research results sponsored by the institution and contained in the journals published or subsidized by the AMS. The dues paid by a member can therefore be taken as an indication of the amount of research activity in that institution.

This survey thus classifies institutions into two major types: Institutional Members and Non-Institutional Members. Members are in turn divided into two groups: I, those which in the three-year period 1959-1961 sponsored 37 1/2 or more pages in journals published or subsidized by the Society, and II, those which contributed less than 37 1/2 pages in the three-year period.

The salaries covered by the survey are based on an academic year (9 to 10 months). It was indicated in the questionnaire that by salary is meant the payment by the school to the individual for full-time work. Grants and contracts are included, but sabbatical payments and other part-time salaries are excluded. All salary figures are given in hundreds of dollars.

The salary information obtained from each institution is the minimum, medium, and maximum salary for each academic rank. In past surveys the report has given the ranges of data received from each group of institutions. In this report intervals are specified which contain the middle 50% of data received from each group. For example, among the 55 institutions in Group I, the following report states that the minimum salary of instructors with a Ph.D. in 1962-1963 is less than $6,000 at 25% of the institutions and greater than $7,000 at 25% of the institutions.

INSTITUTIONAL MEMBERS OF THE SOCIETY "I"

Number of usable returns: 55

Total number on the staffs working full time on the campus

RANK	1961-1962	1962-1963
Instructor (only those holding Ph.D.)	86	81
Assistant Professor	363	430
Associate Professor	330	355
Professor	416	457
TOTAL	1195	1323

Salary Survey

RANK	1961-1962			1962-1963		
	Minimum	Median	Maximum	Minimum	Median	Maximum
Instructor (only those holding Ph.D.)	59- 66	61- 68	64- 70	60- 70	65- 70	67- 70
Assistant Professor	66- 74	72- 80	78- 90	70- 78	77- 84	83- 94
Associate Professor	77- 89	86- 97	93-108	84- 94	91-104	100-117
Professor	100-120	114-145	133-177	104-120	125-150	143-190

INSTITUTIONAL MEMBERS OF THE SOCIETY "II"

Number of usable returns: 89

Total number on the staffs working full time on the campus

RANK	1961-1962	1962-1963
Instructor (only those holding Ph.D.)	24	29
Assistant Professor	299	304
Associate Professor	233	248
Professor	252	272
TOTAL	808	853

Salary Survey

RANK	1961-1962			1962-1963		
	Minimum	Median	Maximum	Minimum	Median	Maximum
Instructor (only those holding Ph.D.)	57- 66	57- 66	60- 68	58- 68	62- 69	65- 70
Assistant Professor	59- 70	65- 74	68- 83	63- 74	70- 79	73- 85
Associate Professor	72- 87	78- 90	83- 95	78- 90	83- 95	89-102
Professor	85-104	93-115	100-133	87-110	98-123	106-140

NON-INSTITUTIONAL MEMBERS

Number of usable returns: 150

Total number on the staffs working full time on the campus

RANK	1961-1962	1962-1963
Instructor (only those holding Ph.D.)	8	11
Assistant Professor	345	380
Associate Professor	257	271
Professor	237	258
TOTAL	847	920

Salary Survey

RANK	1961-1962			1962-1963		
	Minimum	Median	Maximum	Minimum	Median	Maximum
Instructor (only those holding Ph.D.)	55-65	58- 68	58- 68	54- 63	54- 63	54- 63
Assistant Professor	58-69	63- 73	65- 78	62- 71	66- 76	68- 82
Associate Professor	69-81	72- 86	75- 92	70- 85	72- 90	75- 97
Professor	79-99	83-105	85-114	85-105	87-113	90-121

SUMMARY OF ALL INSTITUTIONS SURVEYED

Number of usable returns: 294

Total number on the staffs working full time on the campus

RANK	1961-1962	1962-1963
Instructor (only those holding Ph.D.)	118	121
Assistant Professor	1007	1114
Associate Professor	820	874
Professor	905	987
TOTAL	2850	3096

Salary Survey

RANK	1961-1962			1962-1963		
	Minimum	Median	Maximum	Minimum	Median	Maximum
Instructor (only those holding Ph.D.)	56- 66	60- 68	62- 68	58- 70	60- 70	64- 70
Assistant Professor	60- 70	65- 75	68- 82	64- 75	68- 80	71- 87
Associate Professor	71- 86	77- 92	81-100	75- 90	79- 98	85-105
Professor	80-108	88-120	90-140	88-113	93-127	98-145

STARTING SALARIES FOR MATHEMATICIANS WITH A Ph.D

This is the third annual Survey of Starting Salaries of graduates with the Ph.D. degree in Mathematics. The figures are based on 228 usable replies to 308 questionnaires sent to individuals receiving the Ph.D. in mathematics during the past year. Last year 304 questionnaires were sent and 151 usable replies received.

Of the 228 Ph.D.'s reporting, 177 took employment in academic institutions: 154 of them in universities and colleges, 10 in technical institutes, and 13 in research institutes. Industry and government accounted for 45 and 6 respectively.

Only thirty of the Ph.D.'s reported less than 1/2 year previous professional work experience; 35 reported between 1/2 and 1 year, while 145 Ph.D.'s reported more than 1 year of work experience before receiving the Ph.D.

Geographically, the heaviest concentration of new appointments of mathematicians is again in the North East, with 45.3 percent; the Mid West has 24.1 percent; the South 15.8 percent; and the Far West 14.8 percent.

UNIVERSITIES, COLLEGES AND TECHNICAL INSTITUTES (Nine Month Salary)

	TEACHING			RESEARCH		
Year	Minimum	Median	Maximum	Minimum	Median	Maximum
1958	$4,800	$5,500	$7,100	$5,000	$6,000	$7,000
1959	5,000	6,300	7,600	5,000	6,000	7,700
1960	4,900	6,500	8,000	5,200	6,500	8,000
1961	4,500	6,300	8,250	4,800	6,500	9,000
1962	4,320	7,000	9,250	4,500	6,500	9,000

INDUSTRY (Twelve Month Salary)

Year	Minimum Salary	Median Salary	Maximum Salary
1958	$8,600	$10,300	$12,700
1959	7,800	10,500	16,000
1960	7,800	11,000	15,000
1961	8,700	11,000	17,400
1962	9,000	11,500	16,200

RESEARCH INSTITUTES (Twelve Month Salary)

Year	Minimum Salary	Median Salary	Maximum Salary
1958	$4,000	$ 8,250	$11,000
1959	6,900	9,500	14,000
1960	9,700	10,500	14,000
1961	8,400	11,000	14,200
1962	6,000	10,000	14,500

GOVERNMENT (Twelve Month Salary)

Year	Minimum Salary	Median Salary	Maximum Salary
1958	$7,500	$ 9,750	$11,600
1959	8,800	10,200	13,000
1960	7,200	9,300	13,000
1961	7,780	8,900	16,000
1962	8,800	10,700	14,300

NEWS ITEMS AND ANNOUNCEMENTS

THE DIVISION OF MATHEMATICS of the NAS-NRC has issued its annual report. The introduction contains the following summary of the year's activities.

One of the most important functions of the Division is to render advice in the selection of candidates for fellowships and other awards and this year, as usual, committees of the Division made recommendations in such areas as the award of Fulbright grants, ONR research associateships, and NSF travel grants and fellowships.

The Committee on Travel Grants secured sufficient funds to support the travel of approximately 130 mathematicians to the Twelfth International Congress of Mathematicians to be held in Stockholm, Sweden, in August 1962. The Committee screened over 700 applications for travel support. Financial support came from the National Science Foundation and from the following industrial firms: Armco Foundation, Avco Research Laboratory, Ford Motor Company, General Dynamics Corporation, General Electric Company, Hughes Research Laboratories, International Business Machines Corporation, North American Aviation, Incorporated, and Socony Mobil Oil Company, Incorporated.

The program of summer Institutes for Advanced Graduate Students, initiated by the Committee on Regional Development, has proved very successful with three Institutes being held during the summer of 1961. In keeping with Academy-Research Council policy, the Committee on Regional Development is divesting itself of this activity, for the Committee on Graduate Instruction and Research of the American Mathematical Society has become interested in this activity and will henceforth assume responsibility for it.

The National Academy of Sciences has concluded an agreement with the Academy of Sciences of the USSR to continue the exchange of scientific personnel for two more years. Mathematicians interested in the exchange program may obtain additional information from the Executive Secretary of the Division of Mathematics, or from the Office of the Foreign Secretary of the National Academy of Sciences - National Research Council.

The plunge into space has brought into prominence many problems of a mathematical nature. So far, these have been receiving little attention from the mathematical community. Efforts are being made to remedy this situation. An ad hoc committee is recommending various measures to the Space Science Board. As a result of personal pressure by members of the committee, the American Mathematical Society will sponsor a Summer Symposium on Space Mathematics in 1963. This will be in cooperation with a summer study group in celestial mechanics which has been held for four summers now by a group of astronomers.

A grant of $4,000 from the National Science Foundation, coupled with $4,000 jointly from the Mathematical Association of America and the National Council of Teachers of Mathematics, enabled 200,000 additional copies of the pamphlet, "Careers in Mathematics," to be printed and distributed to the participants in the National High School Mathematics Contest. This brings the total number of copies printed to 700,000, all of which were distributed by the National Council of Teachers of Mathematics.

"Guide to Tables in Mathematical Statistics" by J. Arthur Greenwood and H. O. Hartley, sponsored by the Committee on Statistics of the Division, has been published by the Princeton University Press; and Derrick Henry Lehmer's "Guide to Tables in the Theory of Numbers," which has been out of print since 1954, has been reprinted by the Academy-Research Council.

This year the following committees are on stand-by status: Committee on Revolving Fund for Publication of Mathematical Books, Committee on Revolving Fund for Publication of Mathematical Tables, and Committee on Educational Policy. Two additional committees requested and were granted stand-by status, namely, Committee on Revision of Mathematical Tables and Committee on Mathematical Films and Television.

The undersigned concludes his term as Chairman this year and is pleased to announce that E. J. McShane, now Chairman Designate, will serve as Chairman of the Division for two years beginning July 1, 1962. During this period, G. A. Hedlund will be Chairman Designate and will assume the Chairmanship for two years beginning July 1, 1964. Dr. McShane, of the University of Virginia, has been a member of the National Academy of Sciences since 1948, and has served as a member of the Division and of several committees of the Division at various times since 1952.

J. Barkley Rosser, Chairman

A CONFERENCE ON ABELIAN GROUPS was held at New Mexico State University during the first week of June, 1962. The conference was supported by a National Science Foundation grant; it was directed by Elbert A. Walker and John M. Irwin, both of New Mexico State University. The proceedings of the conference will be published in the near future. Following is the list of major participants: James Armstrong, Ross Beaumont, Gilbert Baumslag, Delmar Boyer, Stephen Chase, Paul Conrad, Spencer Dickson, Donald Dubois, Lazslo Fuchs, David Harrison, Franklin Haimo, Thomas Head, Paul Hill, John Irwin, Samir Khabbaz, George Kolettis, Nancy Lackey, Horst Leptin, Ronald Nunke, Carol Peercy, Richard Pierce, James Reid, Joseph Rotman, William Scott, Ti Yen, Elbert Walker and Robert Wisner.

NASA'S $2 MILLION PREDOCTORAL RESEARCH TRAINING GRANT PROGRAM TO START IN TEN INSTITUTIONS. This fall will mark the beginning of a training program supported by the National Aeronautics and Space Administration at ten universities in an effort to increase the supply of scientists and engineers to meet the long-range objectives of the national space program. Under this predoctoral training program, block grants of money are made to the universities included in the program for: 1. Annual stipend to the student of $2,400. 2. An additional allowance to the student of up to $1,000 per year to be administered by the university in accordance with its established policy. 3. An allowance to the university to cover usual expenses attributed to training, such as tuition, laboratory fees, small items of equipment, costs incurred in course content improvement, an appropriate share of faculty augmentation, etc.

Grants are made to qualified universities, and graduate students of unusual promise with an interest in the space sciences are selected by the university to receive the traineeships. The student may continue for three years providing he maintains a satisfactory record at the university. The universities judge candidates on the basis of their academic record, personal qualifications, and plans for research and study. In all but exceptional cases approved by NASA, recipients will be citizens of the United States.

During the first experimental year, each of the ten universities selected, representing all parts of the nation, will train ten predoctoral graduate students. Universities selected for participation are: Rensselaer Polytechnic Institute, University of Maryland, Georgia Institute of Technology, University of Michigan, University of Chicago, University of Minnesota, State University of Iowa, Texas A and M College, Rice University, and University of California at Los Angeles.

The program's first-year cost is expected to be about $2,000,000. NASA has indicated that this figure represents a modest initial effort, and that the expenditure as well as the number of participating universities will increase in the future. In announcing the program, NASA's Administrator James E. Webb said, "We expect that this program will prove so valuable that we will want to increase it considerably in the years to come."

Additional information about the program or participation in it may be obtained from the Office of Grants and Research Contracts, Office of Space Sciences, Code SC, National Aeronautics and Space Administration, Washington 25, D. C.

REPORT OF THE AFFAIRS OF THE SOCIETY
Read by the Secretary at the Business Meeting in Vancouver

Before starting to talk about affairs of the Society in general, I would like to make a few remarks about one of its particular affairs -- namely the present meeting at the University of British Columbia. I am sure that every member of the Society in attendance here will be happy to join with me in expressing congratulations, appreciation, and warmest thanks to our hosts for the excellent facilities and arrangements and for the warm hospitality that we have been enjoying here. This will undoubtedly remain in the memories of all present as a most pleasant experience. In particular, I wish to express our appreciation to the Committee on Arrangements and its Chairman, Professor B. N. Moyls, and to its other local members, Professors N. J. Divinsky, R. D. James, R. A. Restrepo, and W. H. Simons. Perhaps we should also express our appreciation to the local meteorological bureau for the most excellent weather we have experienced here.

I might mention at this point that in five years we expect to have the pleasure of returning to Canada -- both the Society and Association have voted to accept the invitation of the University of Toronto to meet there in the summer of 1967. The meeting will help celebrate the 100th anniversary of the establishment of the Dominion of Canada.

I would like also to make another remark or two concerning the present meeting. It was anticipated that because of competition from the International Congress in Stockholm, this meeting would not be an extremely large one. Actually, it turned out to be a fairly sizeable one, with about 595 registrants, not counting families. However, the fact that there was serious competition from the International Congress is indicated by the fact that there were 615 American mathematicians registered at Stockholm -- almost exactly one-fourth of the total there. Perhaps the 1966 Congress, particularly if it is held in Novosibirsk, will not offer such heavy competition, and our meeting that year at Rutgers will be a well attended and representative one.

You will recall that next year's summer meeting will be at the University of Colorado at Boulder. It is practically certain that this will be a lively and enthusiastically attended meeting. The 1963 Colloquium lectures will be given by Professor Saunders MacLane. In a similar connection, I might mention that the Gibbs lecture next January at Berkeley will be given by Professor Claude Shannon.

In the Summer of 1963, the Society will conduct, with the financial support of the N.S.F., a month-long Summer Institute in Differential Topology, under the Chairmanship of Professor Norman Steenrod. The exact location has not yet been set. Also it will hold at Cornell University a six weeks Summer Seminar in Applied Mathematics on the subject of Space Mathematics with the sponsorship of several of the Federal Agencies. The Organizing Committee will be under the Chairmanship of Professor Barkley Rosser, and the Summer Seminar will incorporate the 1963 equivalent of the Summer Institutes in Dynamical Astronomy which have been held in recent years at Yale University under the direction of Professor Dirk Brouwer.

Now a few remarks on our publishing activities. The Society will translate in entirety the Russian journal, TRUDY MUSKOVKOGO MATEMATICESKOGO OBSCESTVA. This is a high quality research journal, containing papers of substantial nature, and it is planned eventually to translate it from its beginning, which was ten years ago, so as to make a complete set available.

A new book has been accepted for the Surveys series -- namely Linear Approximations by Arthur Sard.

Effective June 1, there was a major editorial change in Mathematical Reviews. Dr. A. J. Lohwater, formerly managing editor, became Executive Editor of Mathematical Reviews, replacing Dr. S. H. Gould. Dr. Gould, who had been Executive Editor for five years and had been supervising the extensive translation activities of the Society, is now devoting his full time to the latter.

I would like to mention in connection with Mathematical Reviews that a new system of compilation is expected to permit, beginning with the 1963 volumes, the index to come out within a month or two of the last issue of the volume. This will be a very welcome improvement over the approximately two year delay that has sometimes been the case.

One anticipated change involves the Bulletin and the Notices. You no doubt have observed that many things formerly in the Bulletin have been moved to the Notices, until the only nonscientific material left in the Bulletin are the reports of meetings and the Treasurer's report. It did not appear feasible to move these by administrative action, because of the explicit statement in the By-Laws that the Bulletin is the official organ of the Society -- By-Laws made, of course, before the Notices existed. It is planned to take the necessary steps to straighten the matter out, and leave the Bulletin for strictly scientific purposes.

NEWS ITEMS AND ANNOUNCEMENTS

CHINESE MATHEMATICS - ACTA. Two questions about Chinese Mathematics-Acta have been raised, one concerning the Chinese characters on the cover and one concerning the numbering of the volumes.

These are the characters:

数 学 学 报

and the standard Wade-Giles transliteration is:

SHU HSUEH HSUEH PAO

The following translations will explain why the second and third characters are identical:

SHU = number
HSUEH = science
SHUHSUEH = mathematics

HSUEH = science
PAO = newspaper (periodical)
HSUEHPAO = science journal

SHUHSUEH HSUEHPAO
= mathematics journal

As for the numbering, Volume 1, 1962, of the translation corresponds to Volume 10, 1960, of the original. The first nine volumes (1951-1959) of the original have not been translated. Volume 2, 1963, of the translation will correspond to Volume 11, 1961, of the original. It is planned to publish also Volume 3 of the translation (corresponding to Volume 12 of the original) during the year 1963. In each case the translation will have the same number of issues as the original.

The first two issues of Volume 1 of Chinese Mathematics-Acta have been published. The subscription prices per volume are $20 list price, $17.50 agents' price, and $12 members' price.

FELLOWSHIP AND RESEARCH OPPORTUNITIES IN MATHEMATICS. The Division of Mathematics, National Academy of Sciences -- National Research Council, calls attention to a variety of fellowship and other support for basic research in mathematics at both the predoctoral and postdoctoral levels to be awarded during the year 1962-1963. Copies of the complete announcements are available from the Division of Mathematics, National Academy of Sciences -- National Research Council, 2101 Constitution Avenue, Washington 25, D. C.

PERSONAL ITEMS

Assistant Professor R. W. ABEL of the University of Nebraska has been appointed to an associate professorship at Western Washington State College.

Mr. R. H. ABRAHAM of the University of California has been appointed a Research Associate at Columbia University.

Professor BRIAN ABRAHAMSON of Rhodes University, Grahamstown, South Africa has been appointed to an associate professorship at the University of Toronto.

Assistant Professor J. W. ADDISON, JR. of the University of Michigan has been appointed to an associate professorship at the University of California, Berkeley.

Mr. N. L. ALLING of Harvard University has been appointed a Lecturer at the Massachusetts Institute of Technology.

Professor R. D. ANDERSON of Louisiana State University will be on a sabbatical leave for the academic year 1962-1963 at the Mathematisch Centrum, Amsterdam, Netherlands.

Mr. G. H. ANDREWS of the University of Michigan has been appointed to an assistant professorship at Oberlin College.

Dr. J. F. ANDRUS of Lockheed Aircraft Corporation has accepted a position as Consulting Mathematician with General Electric Company, Huntsville, Alabama.

Mr. ANDREW ASTROMOFF of the University of California, Berkeley has been appointed to an assistant professorship at the San Francisco State College.

Mr. C. E. AULL of the University of Colorado has been appointed to an assistant professorship at Kent State University.

Assistant Professor M. C. AYER of the University of Missouri has been appointed to an associate professorship at the University of Oklahoma.

Dr. L. D. BERKOVITZ of the Rand Corporation has been appointed to a professorship at Purdue University.

Professor F. T. BIRTEL of Ohio State University has been appointed to an assistant professorship and research associate at Tulane University.

Professor B. H. BISSINGER on leave from Lebanon Valley College for the academic year 1962-1963 has been appointed to a visiting professorship at the University of Miami.

Dr. G. R. BLAKLEY of Harvard University has been appointed to an assistant professorship at the University of Illinois.

Dr. J. R. BLUM of the Sandia Corporation has been appointed to a professorship at the University of New Mexico.

Mr. J. R. BOEN of the University of Michigan has received a postdoctoral fellowship at Stanford University.

Mr. R. C. BOLLINGER of Westinghouse Electric Corporation has been appointed to an assistant professorship at Pennsylvania State University.

Professor R. D. BOSWELL, JR. of the State College, Mississippi has been appointed to a professorship at Monmouth College.

Dr. LOUIS de BRANGES of New York University has been appointed to an associate professorship at Purdue University.

Dr. D. G. BRENNAN of Massachusetts Institute of Technology has been elected President and Member of the Board of Trustees of the Hudson Institute.

Mr. R. E. BRINEY of the Massachusetts Institute of Technology has been appointed to an assistant professorship at Purdue University.

Assistant Professor J. R. BROWN of the University of Massachusetts has been appointed to an assistant professorship at Oregon State University.

Mr. MORTON BROWN of the Institute for Advanced Study has been appointed to an associate professorship at the University of Michigan.

Mr. R. J. BUMCROT of the University of Missouri has been appointed to an assistant professorship at Ohio State University.

Professor C. E. BURGESS on leave from the University of Utah will spend the academic year 1962-1963 at the Institute for Advanced Study.

Dr. D. G. CANTOR of Princeton University has been appointed to an assistant professorship at the University of Washington.

Dr. K. T. CHEN of the Institute for

Advanced Study has been appointed to an associate professorship at Rutgers, The State University.

Assistant Professor E. W. CHENEY of Iowa State University has been appointed to an assistant professorship at the University of California, Los Angeles.

Mr. R. K. CLARK of Matson Navigation Company, has accepted a position as Assistant Mathematician with the Argonne National Laboratories, Argonne, Illinois.

Dr. G. F. CLEMENTS of Syracuse University has been appointed to an assistant professorship at the University of Colorado.

Associate Professor P. E. CONNER, JR. of the University of Virginia has been appointed to an associate professorship at the Massachusetts Institute of Technology.

Dr. S. D. CONTE of Aerospace Corporation has been appointed to a professorship and Director of the Computation Center, Purdue University.

Assistant Professor P. L. CRAWLEY of the University of Minnesota has been appointed to a visiting assistant professorship at the University of California, Berkeley.

Dr. D. M. DAHM of Princeton University has accepted a position as Senior Applied Programmer with the Burroughs Corporation, Pasadena, California.

Mr. DAR-VEIG HO of Brown University has been appointed to an assistant professorship at the Georgia Institute of Technology.

Professor D. G. DE FIGUEIREDO of the Instituto de Mathematica Pura E Aplicada, Rio de Janeiro, Brasil has been appointed to an associate professorship at the Universidade de Brasília, Brasil.

Dr. R. P. DE FIGUEIREDO of the Portuguese Atomic Energy Center, Sacavem, Portugal has been appointed to a visiting associate professorship at Purdue University.

Dr. S. F. DICE of Bucknell University has been appointed to an assistant professorship at Carleton College.

Dr. M. P. DRAZIN of RIAS, Baltimore has been appointed to an associate professorship at Purdue University.

Dr. R. M. DUDLEY of Princeton University has been appointed Instructor and Junior Research Mathematician at the University of California, Berkeley.

Mr. J. E. DUEMMEL of Ohio State University has been appointed to an assistant professorship at the Air Force Institute of Technology.

Mr. W. S. EBERLY of Washington State University has been appointed to an assistant professorship at the University of New Mexico.

Mr. MICHAEL EDELSTEIN of Israel Institute of Technology, Haifa, Israel has been appointed to a visiting assistant professorship at Cornell University.

Assistant Professor H. G. ELLIS of the University of Utah has been appointed to an assistant professorship at the University of Washington.

Assistant Professor ERWIN ENGELER of the University of Minnesota has been appointed to a visiting assistant professorship and Assistant Research Mathematician at the University of California, Berkeley.

Mr. J. A. ENGLUND of Strategic Air Command has accepted a position as Military Systems Analyst with the U. S. Arms Control and Disarmament Agency, Washington, D. C.

Dr. J. A. ERNEST of the Institute for Advanced Study has been appointed to an assistant professorship at the University of Rochester.

Dr. R. E. ESCH of Harvard University has accepted a position as Head of the Applied Mechanics Department with the Sperry Rand Research Center, Sudbury, Massachusetts.

Reverend W. J. FEENEY on leave from Weston College has received a National Science Foundation Fellowship at the University of California, Berkeley for the academic year 1962-1963.

Professor Emeritus H. H. FERNS of the University of Saskatchewan, Canada has been appointed to a visiting professorship at Victoria College, Canada.

Mr. P. C. FISCHER of the Massachusetts Institute of Technology has been appointed to an assistant professorship at Harvard University.

Associate Professor W. H. FLEMING on leave from Brown University has been appointed to a visiting associate professorship at the University of Wisconsin.

Assistant Professor MARTIN FOX of Michigan State University will be on leave of absence in 1962-1963 and has received a Fulbright grant to lecture at Tel-Aviv University, Tel-Aviv, Israel.

Assistant Professor A. S. FRAENKEL of the University of Oregon has been appointed a Senior Scientist at the Weizmann Institute of Science, Rehovot, Israel.

Mr. J. B. FRALEIGH of Dartmouth College has been appointed to an assistant professorship at the University of Rhode Island.

Associate Professor AVNER FRIEDMAN of the University of Minnesota has been appointed to a professorship at Northwestern University.

Professor BERNARD FRIEDMAN of the University of California, Berkeley has been appointed Miller Research Professor for the academic year 1962-1963.

Mr. GEBHARD FUHRKEN of the University of California, Berkeley has been appointed to an assistant professorship at the University of Minnesota.

Dr. R. A. GANGOLLI of the Massachusetts Institute of Technology has been appointed to an assistant professorship at the University of Washington.

Assistant Professor A. M. GARSIA of the University of Minnesota has been appointed to an associate professorship at the California Institute of Technology.

Mr. I. L. GLICKSBERG of the Institute for Advanced Study has been appointed to a professorship at the University of Washington.

Mr. ROSS GOLDBERG of Weather System Center has accepted a position as Computer Systems Analyst with Computer Concepts, Incorporated, Washington, D. C.

Assistant Professor MALCOLM GOLDMAN of Reed College has been appointed to an assistant professorship at New York University.

Associate Professor D. M. GOLLMAR of the State University of New York has been appointed to an assistant professorship at Wisconsin State College.

Assistant Professor E. L. GRIFFIN, JR. of the University of Michigan has accepted a position as Associate Editor, Mathematical Reviews, American Mathematical Society, Providence, Rhode Island.

Professor Emeritus F. L. GRIFFIN of Reed College has been appointed to a visiting professorship at Portland State College.

Dr. WILLIAM GUSTIN of Indiana University has accepted a position as Mathematician in the Numerical Analysis Section, National Bureau of Standards, U. S. Department of Commerce, Washington 25, D. C.

Mr. JACK HACHIGIAN of Indiana University has been appointed to a visiting assistant professorship at Cornell University.

Dr. J. K. HARRIS of the University of Oregon has been appointed to an assistant professorship at Portland State College.

Assistant Professor R. T. HARRIS, JR. of Duke University has been appointed to a visiting assistant professorship at the University of California, Berkeley.

Mr. MORISUKE HASUMI of Ibaraki University, Japan has been appointed Instructor and Junior Research Mathematician at the University of California, Berkeley.

Dr. ALLAN HAYES of the Massachusetts Institute of Technology has been appointed to an assistant professorship at Purdue University.

Mr. T. J. HEAD of Kansas University has been appointed to an assistant professorship at Iowa State University.

Professor PETER HENRICI of the University of California, Los Angeles has been appointed to a professorship at the Eidgenossische Technische Hochschule, Zurich, Switzerland.

Dr. ROBERT HERMANN of the University of California, Berkeley has been appointed to an associate professorship at Northwestern University.

Professor I. N. HERSTEIN of Cornell University has been appointed to a professorship at the University of Chicago.

Mr. F. A. HIERSCH of Continental Aviation and Engineering Corporation has accepted a position as Senior Technical Specialist with Atomics International, Canoga Park, California.

Mr. W. A. HOLLEY of the University of Texas has accepted a position as Engineering Specialist with Ling-Tempco-Vought, Dallas, Texas.

Dr. R. C. N. HOURSTON of Cornell University has been appointed to an assistant professorship at Knox College.

Mr. H. F. HUNTER of Knolls Atomic Power Laboratories has been appointed to an assistant professorship at Rensselaer Polytechnic Institute.

Dr. L. C. HUTCHINSON of Marshfield, Massachusetts has been appointed to an associate professorship at Northeastern University.

Professor V. S. HUZURBAZAR of the University of Poona, Poona, India has been appointed to a visiting professorship for the academic year 1962-1963 at Iowa State University.

Dr. R. E. ISAAC of the Yeshiva University has been appointed to an assistant professorship at Hunter College.

Professor KENKICHI IWASAWA of the Massachusetts Institute of Technology will spend the academic year 1962-1963 at the University of Tokyo, Japan.

Associate Professor H. G. JACOB, JR. of Louisiana State University has been appointed to a professorship at the University of Massachusetts.

Associate Professor BERNARD JACOBSON of Franklin and Marshall College has accepted a position as Associate Director of the Committee on the Undergraduate Program in Mathematics for the period from September 1962 to August 1963.

Dr. RONALD JACOBOWITZ of the Massachusetts Institute of Technology has been appointed to an assistant professorship at the University of Arizona.

Assistant Professor A. A. JOHNSON of Ohio Wesleyan University has been appointed to an associate professorship at the University of Toledo.

Dr. B. F. JONES, JR. of New York University has been appointed to an assistant professorship at William Marsh Rice University.

Associate Professor COSTAS KASSIMATIS of North Carolina State College has been appointed to an associate professorship at the Assumption University of Windsor.

Professor I. W. KAY of New York University has accepted a position as Senior Research Mathematician with The Conductron Corporation, Ann Arbor, Michigan.

Professor J. L. KELLEY of the University of California, Berkeley will be on sabbatical leave during the fall at Stanford University.

Assistant Professor E. P. KELLY, JR. of Stephen F. Austin State College has been appointed to a professorship and Dean at the University of Southern Mississippi.

Assistant Professor R. N. KESARWANI of Washington University has been appointed to an associate professorship at Wayne State University.

Associate Professor J. E. KIST of Pennsylvania State University has been appointed to a visiting associate professorship at Purdue University for the academic year 1962-1963.

Mr. M. S. KLAMKIN of AVCO, Wilmington, Massachusetts has been appointed to a professorship at the University of Buffalo.

Mr. P. J. KNOPP of the University of Texas has been appointed to an assistant professorship at the University of Missouri.

Assistant Professor SHOSHICI KOBAYASHI of the University of British Columbia, Canada has been appointed to an assistant professorship at the University of California, Berkeley.

Dr. E. C. KOENIG of Allis-Chalmers Manufacturing Company has been appointed to an associate professorship at the University of Wisconsin.

Assistant Professor A. A. KOSINSKI on leave from the University of California, Berkeley has been appointed a Member of the Institute for Advanced Study for the academic year 1962-1963.

Professor BERTRAM KOSTANT of the University of California, Berkeley has been appointed to a professorship at the Massachusetts Institute of Technology.

Professor N. H. KUIPER of the University Duivendaal, Wageningen, Netherlands has been appointed to a professorship at the University of Amsterdam, Netherlands.

Professor MASATAKE KURANISHI of Princeton University has been appointed to a professorship at Columbia University.

Mr. G. R. LAWSON of the General Electric Company has accepted a position as Chief Engineer with the Leviton Manufacturing Company, Brooklyn, New York.

Professor D. H. LEHMER of the University of California, Berkeley has been appointed Miller Research Professor for the academic year 1962-1963.

Mr. C. W. LEMMON of the University of Virginia has been appointed to an assistant professorship at Queens College.

Mr. B. L. LERCHER of the University of Rochester has been appointed to an acting assistant professorship at Harpur College

Mr. R. J. LIBERA of Rutgers, The State University has been appointed to an assistant professorship at the University of Delaware.

Dr. J. A. LINDBERG, JR. of Yale University has been appointed to an assistant professorship at Syracuse University.

Professor J. L. LIONS of the University of Nancy has been appointed to a professorship at the University of Paris, France.

Dr. A. L. LIULEVICIUS of the Institute for Advanced Study has been appointed to an assistant professorship at the University of Chicago.

Dr. C. A. LOVELL of Bell Telephone Laboratories has accepted a position as Director, Launch Planning Division, Bellcomm, Incorporated, Washington, D. C.

Professor EUGENE LUKACS on leave from the Catholic University of America has returned after a leave at the University of Paris, France and at the Swiss Federal Institute of Technology in Zurich, Switzerland.

Assistant Professor S. M. LUKAWECKI of Auburn University has been appointed to an assistant professorship at the University of Southwestern Louisiana.

Dr. J. R. McCORD, III of Massachusetts Institute of Technology has been appointed to an assistant professorship in Chemical Engineering and has also received a postdoctoral fellowship under a Ford Foundation program at Massachusetts Institute of Technology for the academic year 1962-1963.

Mr. B. J. McDONALD of Eglin Air Force Base has been awarded a NDEA Fellowship at Florida State University.

Mr. H. V. McINTOSH of RIAS has been appointed Senior Research Scientist at the University of Florida.

Dr. C. J. MALONEY, formerly Chief, Biomathematics Division, Fort Detrick, has transferred to the Division of Biologics Standards, National Institutes of Health, Bethesda, Maryland, as a Mathematical Statistician.

Associate Professor MARY MARQUARDT of Quincy College has been appointed to an associate professorship at the Ferris Institute.

Mr. L. C. MARSHALL of Sperry Rand Corporation has accepted a position as Program Analyst with Computer Concepts, Washington, D. C.

Assistant Professor Z. A. MELZAK of the University of British Columbia has been appointed to an associate professorship at McGill University, Montreal, Canada.

Assistant Professor R. V. MENDENHALL of the University of Miami has been appointed to an associate professorship at Ohio Wesleyan University.

Mr. P. R. MEYER of Columbia University has been appointed to an assistant professorship at St. John's University.

Miss VALERIE MIKE on leave from Bell Telephone Laboratories, Incorporated has received a National Science Foundation Fellowship at New York University for the academic year 1962-1963.

Professor J. W. MILNOR has been appointed to the Henry Putnam University Professorship at Princeton University. The Chair is of unique distinction within the University and has been established "to provide recognition to a scholar of extraordinary ability in any discipline."

Mr. J. B. MINKUS of the University of Chicago has been appointed to Instructor and Junior Research Mathematician at the University of California, Berkeley.

Assistant Professor L. J. MONTZINGO, JR. of the University of Buffalo has been appointed to an associate professorship and Chairman of the Mathematics Department at the Seattle Pacific College.

Dr. M. D. MORELY of the University of Chicago has been appointed to Instructor and Junior Research Mathematician at the University of California, Berkeley.

Professor A. P. MORSE of the University of California, Berkeley will be on sabbatical leave for the academic year 1962-1963, remaining at Berkeley.

Professor MARSTON MORSE of the Institute for Advanced Study has been elected Foreign Member of the Academia Nazionale dei Lincei of Italy.

Dr. F. S. MULLA of the University of Kansas has been appointed to Instructor and Junior Research Mathematician at the University of California, Berkeley.

Professor Emeritus J. R. MUSSEL-

MAN of Western Reserve University has been appointed to a visiting professorship at Hiram College.

Dr. C. J. NEUGEBAUER of Purdue University will be on leave for the academic year 1962-1963 at the Institute for Advanced Study.

Associate Professor WALTER NOLL of the Carnegie Institute of Technology has been appointed to a visiting professorship at Johns Hopkins University.

Dr. A. P. OGG, a National Science Foundation Fellow at the University of Paris, has been appointed to an assistant professorship at the University of California, Berkeley.

Professor GORDON PALL of Illinois Institute of Technology has been appointed to a professorship at the University of Arizona.

Dr. LEIF-NORMAN PATTERSON of Suffolk University has been appointed to Instructor and Junior Research Mathematician at the University of California, Berkeley.

Dr. R. R. PHELPS of the University of California, Berkeley has been appointed to an assistant professorship at the University of Washington.

Mr. S. C. PORT of Northwestern University has accepted a position as Associate Mathematician with the Rand Corporation, Santa Monica, California.

Mr. M. R. PORTER of the University of California has been appointed to an assistant professorship at the Sacramento State College.

Dr. C. M. PRICE of Aerospace Corporation, Los Angeles, California has been appointed Associate Head of the Astrodynamics Department.

Dr. N. J. PULLMAN of Syracuse University has been appointed Lecturer at McGill University.

Associate Professor M. O. RABIN of the Hebrew University, Jerusalem, Israel has been appointed to a visiting associate professorship at the Massachusetts Institute of Technology.

Professor PHILBURN RATOOSH of the University of California, Berkeley has been appointed to a professorship at the San Francisco State College.

Associate Professor F. A. RAYMOND of the University of Michigan has been appointed to a visiting associate professorship at the University of California, Berkeley.

Professor C. B. READ of the University of Wichita has been appointed to a professorship at Central Michigan University.

Professor W. C. RHEINBOLDT of the University of Maryland has been appointed Director of the Computer Science Center and Research Associate Professor at the Institute for Fluid Dynamics and Applied Mathematics.

Dr. J. I. RICHARDS of Massachusetts Institute of Technology has been appointed to an assistant professorship at the University of Minnesota.

Dr. P. B. RICHARDS of General Electric Company has accepted a position as Principal Staff Scientist with General Precision, Incorporated, Little Falls, New Jersey.

Professor FAZLOLLAH REZA of Syracuse University has been appointed to a visiting professorship at the Swiss Federal Institute of Technology, Zurich, Switzerland.

Mr. C. E. ROBINSON of the University of Alabama has been appointed to an assistant professorship at Auburn University.

Professor D. W. ROBINSON on sabbatical leave from Brigham Young University has received a National Science Foundation Fellowship and has been appointed Senior Research Fellow at the California Institute of Technology.

Dr. J. B. ROSEN of Shell Development Company has been appointed to a visiting professorship at Stanford University.

Assistant Professor N. J. ROTHMAN of the University of Rochester has been appointed to an assistant professorship at the University of Illinois.

Mr. DAVID RYEBURN of Ohio State University has been appointed to an assistant professorship at Kenyon College.

Associate Professor P. T. RYGG of the State University of South Dakota has been appointed to an associate professorship at the Western Washington State College.

Professor H. J. RYSER of Ohio State University has been appointed to a professorship at Syracuse University.

Dr. G. E. SACKS of the Institute for Advanced Study has been appointed to an assistant professorship at Cornell University.

Associate Professor H. M. SCHAERF of Washington University has been appointed to a visiting professorship at the University of Wisconsin.

Professor P. A. SCHEINOK of Wayne State University has accepted a position as Engineer with the Radio Corporation of America, Moorestown, New Jersey.

Assistant Professor J. R. SCHUE of Oberlin College has been appointed to an associate professorship at Macalester College.

Dr. T. I. SEIDMAN of the University of Wisconsin has accepted a position as Mathematician with the Boeing Scientific Research Laboratories, Seattle, Washington.

Dr. E. P. SHELLY of the Knolls Atomic Power Laboratory has been appointed to an associate professorship at the University of Connecticut.

Dr. MARVIN SHINBROT of the University of Chicago has been appointed to an assistant professorship at the University of California, Berkeley.

Associate Professor R. J. SILVERMAN of Illinois Institute of Technology has been appointed to a professorship at the University of New Hampshire.

Mr. B. T. SIMS of Iowa State University has been appointed to an assistant professorship at San Jose State College.

Mr. F. W. SINN, JR. of American Airlines, New York has accepted a position as a member of the Technical Staff of Computronics, Incorporated, Fort Lee, New Jersey.

Professor W. A. SMALL of the Tennessee Polytechnic Institute has been appointed to a professorship and Coordinator at the State University College, Genesee, New York.

Dr. C. V. L. SMITH of the National Aeronautics and Space Administration has accepted a position as Head of the Mathematics and Computer Programs with the U. S. Atomic Energy Commission, Washington, D. C.

Professor R. C. T. SMITH on sabbatical leave from the University of New England, Armidale, Australia will spend the academic year 1962-1963 at the University of Maryland.

Assistant Professor W. F. STINESPRING of the University of Chicago has been appointed to a visiting assistant professorship at the Massachusetts Institute of Technology.

Mr. C. J. STONE of Princeton University has been appointed to an assistant professorship at Cornell University.

Dr. W. G. STRANG of the Mathematical Institute, Oxford, England has been appointed to an assistant professorship at the Massachusetts Institute of Technology.

Assistant Professor TSUNEO TAMAGAWA of Tokyo University, Tokyo, Japan has been appointed to a professorship at Northwestern University.

Mr. ALAN TROY of the University of Illinois has been appointed to an assistant professorship at the University of Washington.

Assistant Professor SAN FU TUAN of Brown University has been appointed to an assistant professorship at Purdue University.

Mr. F. S. VAN VLECK of the Massachusetts Institute of Technology has been appointed to an assistant professorship at the University of Kansas.

Professor B. W. VOLKMANN of the University of Mainz, Germany has been appointed to a visiting professorship at the University of Utah.

Associate Professor JU-KWEI WANG of the National Taiwan University, China, has been appointed Lecturer at the University of California, Berkeley, for the academic year 1962-1963.

Professor J. G. WENDEL of the University of Michigan has been appointed to a professorship at the Aarhus Universitet, Aarhus, Denmark.

Dr. J. T. WHITE of the University of Texas has been appointed to an assistant professorship at the University of Kansas.

Assistant Professor J. S. WHOLEY of Rutgers, The State University has accepted a position as Member, Technical Staff, Weapons Systems Evaluation Group, Institute for Defense Analyses, Pentagon, Washington, D. C.

Dr. L. H. WILLIAMS of Duke University has been appointed to an assistant professorship and consultant of the Computing Center, Florida State University.

Mr. PUI-KEI WONG of the Carnegie

Institute of Technology has been appointed to an assistant professorship at Lehigh University.

Dr. ARTHUR WOUK of Sylvania Electronic Systems has been appointed to a visiting professorship at the University of Wisconsin.

Assistant Professor F. M. YAQUB of Purdue University has been appointed to an assistant professorship at the University of California, Davis.

The following promotions are announced:

S. P. AVANN, University of Washington, to an associate professorship.

J. B. AX, Cornell University, to an assistant professorship.

W. E. BAXTER, University of Delaware, to an associate professorship.

KURT BING, Rensselaer Polytechnic Institute, to a professorship.

E. A. BISHOP, University of California, Berkeley, to a professorship.

D. W. BLACKETT, Boston University, to a professorship.

LOUIS BRICKMAN, Cornell University, to an assistant professorship.

ANDREW BROWDER, Brown University, to an assistant professorship.

F. E. BROWDER, Yale University, to a professorship.

WILLIAM BROWDER, Cornell University, to an associate professorship.

L. L. CAMPBELL, Assumption University of Windsor, to an associate professorship.

C. R. CASPAR, Cornell University, to an assistant professorship.

S. U. CHASE, Cornell University, to an assistant professorship.

H. H. CORSON, III, University of Washington, to an associate professorship.

CECIL CRAIG, JR., University of Cincinnati, to an assistant professorship.

THOMAS ERBER, Illinois Institute of Technology, to an associate professorship.

J. H. ENGEL, to Director, Operations Evaluation Group.

J. R. EWBANK, St. Benedicts College, to an assistant professorship.

ISTVAN FARY, University of California, Berkeley, to a professorship.

K. M. HOFFMAN, Massachusetts Institute of Technology, to an associate professorship.

J. R. ISBELL, University of Washington, to a professorship.

E. D. KANN, University of Southern California, to an assistant professorship.

HARRY KESTEN, Cornell University, to an associate professorship.

T. L. McCOY, Illinois Institute of Technology, to an assistant professorship.

ANIL NERODE, Cornell University, to an associate professorship.

D. S. NEWMAN, University of Washington, to an assistant professorship.

E. N. NILSON, Pratt and Whitney Aircraft Division of United Aircraft Corporation, Chief, Scientific Staff.

C. W. PATTY, University of North Carolina, to an assistant professorship.

C. F. PINZKA, University of Cincinnati, to an assistant professorship.

RONALD PYKE, University of Washington, to an associate professorship.

RUSSELL REMAGE, JR., University of Delaware, to a professorship.

THOMAS ROBERTSON, Occidental College, to an assistant professorship.

GIAN-CARLO ROTA, Massachusetts Institute of Technology, to an associate professorship.

D. S. SCOTT, University of California, Berkeley, to an associate professorship.

DAVID SHALE, University of California, Berkeley, to an assistant professorship.

E. M. STEIN, University of Chicago, to an associate professorship.

PAUL VERMES, Birkbeck College, London, England, to a Reader in mathematics.

C. R. YEAMAN, Hercules Powder Company, Magna, Utah, to a Technical Specialist.

G. C. WEBBER has been named H. Fletcher Brown Professor of Mathematics at the University of Delaware.

P. M. WEICHSEL, University of Illinois, to an assistant professorship.

The following appointments to instructorships are announced:

Allegheny College: R. F. McDERMOT; Brown University: Mr. HIDEKI OZEKI, M. I. ROSEN, YUJI ITO; University of California, Berkeley: R. M. FESQ, JR; Carleton College: R. B. KIRCHNER; University of Chicago: P. A. FILLMORE; Claremont Men's College: D. L. OUTCALT; Columbia University: D. I. KNEE; Cornell University: J. N. FRAMPTON, GUNJI HIROSHI, STEPHEN WAINGER; Dartmouth College: W. E. RITTER; University of Delaware: HOWARD WILSON; Foothill College: J. G. SOWUL; Grinnell College: A. M. ADELBERG; Illinois Institute of Technology: P. C. DELIYANNIS, D. J. RODABAUGH, E. F. STUEBEN; Kansas University: W. H. JOBE; Louisiana State University in New Orleans: B. L. BRECHNER; Massachusetts Institute of Technology: J. L. ALPERIN, D. R. ANDERSON, E. B. CURTIS, H. B. ENDERTON, A. B. RAMSAY; Michigan State University, Oakland: T. M. JENKINS; Northeastern University: H. H. CRAPO; Ohio Wesleyan University: L. E. DE NOYA; Pennsylvania State University: ANTON GLASER; Princeton University: SHMUEL KANTOROVITZ, B. S. RANDOL; St. John's University: C. H. GOLDBERG; St. Mary's College: J. E. JOHNSON; University of South Carolina: J. B. LANE; Stanford University: W. F. POHL, S. S. SHATZ; University of Tennessee: W. L. MORRIS; University of Washington: L. N. ARGABRIGHT, K. J. CRASWELL, A. J. FRODERBERG, M. L. WEISS; University of Wisconsin, Milwaukee: D. H. CARLSON; Wittenberg University: E. L. WILSON.

ERRATA

The following are corrections of announcements in the August issue of the NOTICES:

Mr. D. M. BLOOM of Harvard University has been appointed to an assistant professorship at the University of Massachusetts.

Dr. R. S. FREEMAN of the University of California, Livermore has been appointed to an assistant professorship at the University of Maryland.

Professor HOWARD LEVI of Columbia University has been appointed to a professorship at Hunter College.

NEW AMS PUBLICATIONS

SELECTED TRANSLATIONS SERIES II, VOLUME 21

Nine papers on Topology, Lie groups, and differential equations.

410 pages; List Price $6.00; 25% discount to members.

COLLOQUIUM PUBLICATIONS

VOLUME XIII

Completely Revised (First edition published in 1932)

FOUNDATIONS OF POINT SET THEORY
by R. L. Moore

420 pages; List Price $12.10; 25% discount to members.

In this revised edition there is again presented what may be roughly termed a largely self-contained treatment of the foundations of continuity, or point set-theoretic, analysis situs (topology). There is appended a bibliography which is much more extensive than the one in the first edition.

VOLUME XXIX

Revised and enlarged in 1962 (First edition published in 1944)

FOUNDATIONS OF ALGEBRAIC GEOMETRY
by Andre Weil

363 pages; List Price $12.20; 25% discount to members.

The main purpose of the book is to present a detailed and connected treatment of the properties of intersection-multiplicities, which includes all that is necessary and sufficient to legitimize the use made of these multiplicities in classical algebraic geometry, especially of the Italian school. In order to enable students of the modern literature in algebraic geometry to locate in this volume the references to the theorems of the first edition, a table of concordance is given, which indicates for each result in the first edition, the result in the present one which is equivalent to it or which contains it as a special case.

NEWS ITEMS AND ANNOUNCEMENTS

THE INSTITUTE FOR FLUID DYNAMICS AND APPLIED MATHEMATICS of the University of Maryland wishes to announce the availability for the year 1963-1964 of a limited number of post doctoral appointments for research in mathematics and its application to the physical or life sciences. Inquiries concerning appointments should be directed to the Committee on Fellowships of the Institute, and all applications should be returned prior to March 1.

ONR ESTABLISHES A LECTURE SERIES. The Department of Mathematics of the George Washington University, Washington, D. C., announces a series of thirty lectures in fifteen areas of mathematical research. The lectures are sponsored by the Office of Naval Research and will be given at the University during the academic years 1962-1963 and 1963-1964. The lectures will be published under the editorship of Dr. Thomas Saaty.

SUPPLEMENTARY PROGRAM No. 13

During the interval from June 28, 1962 through September 5, 1962, the papers listed below were accepted by the American Mathematical Society for presentation by title. Readers may wish to refer to Page 713 of the November, 1960 issue (No. 49 of these NOTICES) where it is explained in detail that the presentation of papers by title is now dissociated from meetings of the Society.

After each title on this program is an identifying number. The abstract of the paper will be found following the same number in the section of Abstracts of Contributed papers in this issue of these NOTICES.

(1) A summation procedure for certain Feynman integrals
Professor D. G. Babbitt, University of California, Los Angeles (62T-298)

(2) Integral representations for harmonic functions in three real variables
Miss D. G. Becker, Bell Telephone Laboratories, Murray Hill, New Jersey (62T-280)
(Introduced by Professor J. Mitchell)

(3) On the representation of analytic functions as Laplace and Laplace-Stieltjes integrals
Mr. Hubert Berens and Professor P. L. Butzer, Technical University of Aachen, Aachen, Germany (62T-321)

(4) Column sequences in Hausdorff matrices.
Professor J. D. Buckholtz, University of North Carolina (62T-274)

(5) A method for establishing undecidability
Mr. Alan Cobham, International Business Machines Corporation, Yorktown Heights, New York (62T-310)

(6) Establishing the nonexistence of proof and disproof procedures
Mr. Alan Cobham, International Business Machines Corporation, Yorktown Heights, New York (62T-311)

(7) Undecidability in group theory
Mr. Alan Cobham, International Business Machines Corporation, Yorktown Heights, New York (62T-312)

(8) Simple waves in relaxation hydrodynamics. I
Professor Nathaniel Coburn, University of Michigan (62T-331)

(9) Three footnotes to two earlier papers
Professor Eckford Cohen, University of Tennessee (62T-317)

(10) On the distribution of the k-free integers in residue classes
Professor Eckford Cohen and Mr. R. L. Robinson, University of Tennessee (62T-327)

(11) The number of solutions of certain types of congruences in algebraic number fields
Professor J. T. Cross, University of the South (62T-294)

(12) Weak convergence of finitely additive measures on a sigma algebra
Dr. R. B. Darst, Purdue University (62T-324)

(13) Expansiveness and light open mappings
Dr. Edwin Duda, University of Miami (62T-289)

(14) Order ideals in C*-algebras and their duals. I
Mr. E. G. Effros, Columbia University (62T-271)

(15) Majorant for a modified Bessel function
Professor Thomas Erber, Illinois Institute of Technology (62T-304)

(16) Optimal steering programs for idealized missiles
Professor G. M. Ewing, University of Oklahoma (62T-305)

(17) Extension of Bernstein polynomials for the complex plane
Professor J. J. Gergen, Professor F. G. Dressel and Mr. W. H. Purcell, Jr., Duke University (62T-316)

(18) Convergent chain sequences
Mr. Hadi Haddad, University of Texas (62T-303)
(Introduced by Dr. H. S. Wall)

(19) On Riemannian manifolds of nonpositive curvature
Dr. Robert Herman, University of California, Berkeley (62T-283)

(20) On a theorem of Szegő
Professor I. I. Hirschman, Jr., Washington University (62T-308)

(21) Extreme eigenvalues
Professor I. I. Hirschman, Jr., Washington University (62T-309)

(22) On the characterization of spectral operators
Dr. Shmuel Kantorovitz, Princeton University (62T-332)

(23) Operational calculus in Banach algebras for algebra-valued functions
Dr. Shmuel Kantorovitz, Princeton University (62T-333)

(24) A note on Boolean algebraic representability
Dr. C. R. Karp, University of Maryland (62T-269)

(25) On the splitting problem in abelian groups. Preliminary report
Dr. S. A. Khabbaz, Lehigh University (62T-284)

(26) Associator dependent rings
Professor Erwin Kleinfeld, Ohio State University and Syracuse University (62T-295)

(27) Postscript to "The geometry of extremal quasiconformal mappings"
Dr. Tilla Klotz, University of California, Los Angeles (62T-270)

(28) The dimension of product spaces
Mr. Yukihiro Kodama, University of Michigan (62T-276)

(29) Cohomology dimension of fully normal spaces
Mr. Yukihiro Kodama, University of Michigan (62T-277)

(30) H. Hopf extension theorem
Mr. Yukihiro Kodama, University of Michigan (62T-278)

(31) The dominators of a semigroup
Mr. J. F. Latimer, U. S. Naval Ordnance Test Station, China Lake, California (62T-279)

(32) Affine geometry via parallel projections
Professor Howard Levi, Hunter College (62T-328)

(33) Interpolation theorem for an infinitary language
Mr. E. G. K. Lopez-Escobar, University of California, Berkeley (62T-323)

(34) An extension of a result of Friedberg
Mr. T. G. McLaughlin, University of California, Los Angeles (62T-325)

(35) On covering systems
Mr. D. J. Mallory and Professor Maurice Sion, University of British Columbia (62T-318)

(36) Nonlinear programming problems with stochastic objective functions
Dr. O. L. Mangasarian, Shell Development Company, Emeryville, California (62T-290)

(37) E^3 modulo a 3-cell
Mr. D. V. Meyer, State University of Iowa (62T-302)

(38) Topics on mutation. I, direct products with mutant factors
Mr. A. A. Mullin, University of Illinois (62T-296)

(39) Topics on mutation. II, logical algebra
Mr. A. A. Mullin, University of Illinois (62T-299)

(40) Topics of mutation. III, relational abstractions
Mr. A. A. Mullin, University of Illinois (62T-300)

(41) A lattice connected with relative interpretability of theories
Professor Jan Mycielski, University of California, Berkeley and Polish Academy of Sciences (62T-315)

(42) A remark on the curvature tensor field
Professor Katsumi Nomizu and Dr. Hideki Ozeki, Brown University (62T-275)

(43) Some generalisations of the notion of well ordering. Preliminary report
Mr. R. J. Parikh, Stanford University (62T-330)

(44) 1-dimensional cell complexes with homeotopy group equal to zero
Mr. L. V. Quintas, Columbia University (62T-297)

(45) On the prime ideal theorem with remainder term
Professor G.J.Rieger, Purdue University and University of Munich, Germany (62T-320)

(46) A minimax theory for overdamped systems
Mr. E. H. Rogers, The Royal College of Science and Technology,

Glasgow, Scotland (62T-291)

(47) Neighborhoods of one-simplexes in triangulated manifolds
Professor R. H. Rosen, University of Michigan (62T-322)

(48) An interpolation theorem for recursively enumerable degrees
Professor G. E. Sacks, Cornell University (62T-326)

(49) Paratopies et auto-paratopies des quasigroupes
Professor A. J. V. Sade, Lycée Perier Marcelle, Marseille, France (62T-306)

(50) Tables of coefficients for obtaining central differences from the derivatives
Dr. H. E. Salzer, General Dynamics/Astronautics, San Diego, California (62T-314)

(51) Nevanlinna's second main theorem without exceptional intervals
Professor Leo Sario, University of California, Los Angeles (62T-285)

(52) Meromorphic functions and conformal metrics on Riemann surfaces
Professor Leo Sario, University of California, Los Angeles (62T-286)

(53) Islands and peninsulas on arbitrary Riemann surfaces
Professor Leo Sario, University of California, Los Angeles (62T-287)

(54) Functions with bounded characteristics on Riemann surfaces
Professor Leo Sario, University of California, Los Angeles (62T-288)

(55) Extension of measures and simplification of integration on a ring of sets
Professor H. M. Schaerf, Washington University and the University of Wisconsin (62T-281)

(56) On L^p estimates and boundary problems. III. Preliminary report
Professor Martin Schechter, New York University (62T-273)

(57) Approximation properties of measures generated by continuous set functions
Professor Maurice Sion and Mr. D. K. Sjerve, The University of British Columbia (62T-319)

(58) Extension of groupoids with operators
Professor Takayuki Tamura and Mr. D. G. Burnell, University of California, Davis (62T-282)

(59) Tag systems and lag systems
Professor Hao Wang, Harvard University (62T-313)

(60) The space of real parts
Professor John Wermer, Brown University (62T-329)

(61) Asymptotic distribution of the eigenvalues of certain integral operators. Preliminary report
Professor Harold Widom, Cornell University (62T-307)

(62) Homogeneity and bounded isometries in manifolds of negative curvature
Professor J. A. Wolf, University of California, Berkeley (62T-301)

NEWS ITEM

RESEARCH SUPPORT FOR MATHEMATICAL METHODS IN LOGISTICS. The third topic in the monograph series and research support program of the Office of Naval Research, selected in cooperation with the Institute of Management Sciences, is "Capital Budgeting of Interrelated Projects."

Manuscripts, which must provide reports of research yielding either increased knowledge or improved ability to deal with important classes of problems, will be accepted until December 31, 1963. Preference will be given to manuscripts in which the underlying research leans heavily on mathematics, statistics or computer simulation.

The author of the manuscript judged best, upon recommendation of the Institute of Management Sciences, will be offered the opportunity to spend a year continuing his research efforts in logistics or related areas on a full time basis at a salary of $11,000.

The papers judged best will be included in the third volume of the Office of Naval Research monograph series on Mathematical Methods in Logistics.

Any interested person who has received his last academic degree within the last seven years, or who is still a student, may write to the Logistics and Mathematical Statistics Branch, Office of Naval Research, Washington 25, D.C., concerning detailed requirements for submitting a manuscript.

ABSTRACTS OF CONTRIBUTED PAPERS

THE OCTOBER MEETING IN HANOVER, NEW HAMPSHIRE
October 27, 1962

593-1. W. E. RITTER, Massachusetts Institute of Technology, Cambridge, Massachusetts. Some results in hyperarithmetic analysis.

A branch of "constructive analysis" (of the type considered by Moschovakis and Hodes) over the hyperarithmetic reals is given an abstract but natural formulation in terms of notation systems. The isomorphism lemma of Rogers (J. Symb. Logic 23, 331-341) for Gödel numberings of partial recursive functions is extended to general notation systems enjoying an appropriate "effective fixed point property" and applied to show that there is exactly one h-complete h-real field up to recursive isomorphism. Kreisel's result (ibid., 460-461; Bull. Acad. Polon. Sci. 8, 621-626) on the Cantor-Bendixson Theorem is obtained in the present context: There is an h-closed set of h-reals whose scattered part is not hyperarithmetic. Finally it is shown that there are sets of h-reals which are (h-) measurable but not in the (constructive) Borel hierarchy (the hyperarithmetic σ-ring generated by the open sets) and nonmeasurable but "nameable" sets (such that they and their complements in the h-reals are determined by Π_1^1 sets of integers). Useful tools are the Single-valuedness Theorem of Rogers for Π_1^1 relations and the techniques and results of Spector (J. Symb. Logic 20, 151-163), particularly in the construction and application of an appropriate Π_1^1 well-ordering of the h-reals. (Received May 28, 1962.)

593-2. ADI BEN-ISRAEL, Carnegie Institute of Technology, Pittsburgh 13, Pennsylvania and ABRAHAM CHARNES, The Engineering Sciences Department, The Technological Institute, Northwestern University, Evanston, Illinois. On the generalized inverse of a closed linear operator with an everywhere dense domain on a Hilbert space.

Let A: $\mathcal{H} \to \mathcal{H}$ be a bounded symmetric linear operator on a Hilbert space $\mathcal{H}$, D an A-admissible domain with $\sigma(A) \subset D$ and $C = \partial D$. Theorem. The sequence of operators $e^{-A^2 n} = (2\pi i)^{-1} \int_C e^{-\lambda^2 n} R_\lambda(A) d\lambda$ converges uniformly to $P_{N(A)}$. (Follows from Theorem 3.6 in Dunford: Trans. Amer. Math. Soc. 54 (1943), 185-217). Definition: For $g_n(\lambda) = \int_0^n \lambda e^{-\lambda^2 x} dx$ let $A_n^+ = (2\pi i)^{-1} \int_C g_n(\lambda) R_\lambda(A) d\lambda$. The generalized inverse (g.i.), of A is $A^+ = \lim_{n\to\infty} A_n^+$. Theorem: (a) $N(A^+) = N(A_n^+) = N(A)$, (b) $R(A) \subseteqq D(A^+)$, (c) $AA^+ = P_{R(A)}$, (d) $\overline{D(A^+)} = \mathcal{H}$. (e) A^+ is closed and self adjoint, (f) $A^+A = P_{\overline{R(A^+)}}$. These results are extended to unbounded self adjoint linear operators T: $\mathcal{H} \to \mathcal{H}$ by using the Riesz-Lorch representation, and to closed linear operators S: $\mathcal{H} \to \mathcal{H}$ with $\overline{D(S)} = \mathcal{H}$ by using the Von Neumann canonical decomposition (Theorem 7, Annals of Math. 33 (1932), 294-310). The g.i. of S, S^+ is defined and satisfies Theorem: (a) $N(S^+) = N(S^*)$, (b) $R(S) \subseteqq D(S^+)$, (c) $\overline{D(S^+)} = \mathcal{H}$. (d) $\overline{R(S^+)} = \overline{R(S^*)}$, (e) S^+ is closed, (f) $SS^+ = P_{\overline{R(S)}}$, (g) $S^+S = P_{\overline{R(S^+)}}$. S^+ coincides with the maximal g.i. of Tseng: Dokl. Akad. Nauk. SSSR. (N.S.) 67 (1949), 431-434. (Received June 28, 1962.)

593-3. A. A. SAGLE, Syracuse University, Syracuse 10, New York. On anti-commutative algebras with an invariant form.

An anti-commutative algebra A with an invariant form satisfies (1) xy = - yx for all x,y in A; (2) A possesses a symmetric bilinear form f(x,y) such that f(xy,z) = f(x,yz) for all x,y,z in A. Lie and Malcev algebras are examples of such algebras and generalizations of these algebras are obtained by introducing commutation [x,y] = xy - yx as a new multiplication in the noncommutative Jordan algebras considered by L. J. Paige, A note on noncommutative Jordan algebras, Portugal. Math. (1957), 15-18. Thus if $\mathcal{A}$ is such a Jordan algebra, form the anti-commutative algebra $\mathcal{A}^{(-)}$ which is the same vector space $\mathcal{A}$ but with commutation [x,y] as multiplication. If $\mathcal{C} = \{x \text{ in } \mathcal{A}^{(-)}: [x,y] = 0$, all y in $\mathcal{A}^{(-)}\}$, then the anti-commutative algebra $\mathcal{A}^{\circ} = \mathcal{A}^{(-)}/\mathcal{C}$ can be used to construct a family of simple Jordan and anti-commutative algebras which generalize the split Cayley-Dickson and Malcev algebras. Theorem. If A is a finite dimensional anti-commutative algebra with an invariant form f(x,y) over a field F of characteristic not 2, then there exists a noncommutative Jordan algebra $B = 1F \oplus \overline{B}$ with identity element 1 such that $B^{(-)}/1F$ is isomorphic to A. Furthermore if f(x,y) is nondegenerate and the mapping $x \longrightarrow R_x$, where R_x is right multiplication by x, is injective, then B is simple. (Received September 4, 1962.)

593-4. KURT BING, Rensselaer Polytechnic Institute, Troy, New York. An abstract form of Gödel's incompleteness theorem.

Terms and notation are those of Smullyan, Theory of formal systems, Princeton, 1961. Theorem 1 below is thought to be an accurate abstract version of Gödel's actual construction in his incompleteness theorem of 1931. The remaining statements are generalizations. H consistently represents A iff for every number n: $Hn \in T \longrightarrow n \in A$ and $Hn \in R \longrightarrow n \in \tilde{A}$. H consistently separates A from B iff H consistently represents a superset K of A disjoint from B. Theorem 1. If H consistently represents $\tilde{T}^*$ (R^*) in Z, then Hh is undecidable in Z. (In Gödel's 1931 proof, the formula with Gödel number p consistently represents $\tilde{T}^*$ if the system is ω-consistent. See also Kleene, Introduction to metamathematics, New York, 1952, Theorem XIII.) Diagonalization lemma: If H consistently separates W^* from T^* in Z and if Hh is decidable, then Hh is outside both W and T. Theorem 2. If H consistently separates R^* from T^* in Z, then Hh is undecidable in Z. (Analogously, in Chapter III of Smullyan's work, Theorem 3 also holds of $\tilde{T}^*$; also, consistency is dispensable for Theorem 6.) (Received July 13, 1962.)

593-5. FRANK LEVIN, Rutgers, The State University, New Brunswick, New Jersey. Single variable equations over a group.

To any nontrivial group G there exist elements h_i, i = 1,2,..., in the free product $P = G * C$, where C is an infinite cyclic group, such that $H = gp(G,h_1,h_2,...) \subsetneq P$ has the properties: (i) $H = G * gp(h_i)$; (ii) let N be a normal subgroup of H, N^P its normal closure in P, $N \cap G = 1$. Then, $N^P \cap H = N$. This result is applied as follows: A system of equations over G is compatible over G if G can be embedded in a group which contains a solution of the equations. Theorem. To any compatible system of m equations in n variables over G (m,n countable) there exists a group G' and an element t in G' such that G is embedded in G', G' = gp(G,t), and the equations have a solution in G'. In particular, G is (m,n)-algebraically closed, i.e., contains a solution of every compatible system of m equations in n variables over G, if it is (m,1)-algebraically closed. (Received September 5, 1962.)

593-6. M. A. ARKOWITZ, Fine Hall, Box 708, Princeton University, Princeton, New Jersey and C. R. CURJEL, Cornell University, Ithaca, New York. Homotopy commutators of finite order.

Let $\pi(\Sigma X,Y)$ denote the group of homotopy classes of maps of ΣX, the reduced suspension of the polyhedron X, into the polyhedron Y. Let ${}^n\Sigma X = \Sigma X \vee \dots \vee \Sigma X$ be the wedge of n copies of ΣX. The generic n-commutator $\psi_n(\Sigma X) \in \pi(\Sigma X, {}^n\Sigma X)$ is defined as the n-fold (simple) commutator of the n injections of ΣX into ${}^n\Sigma X$ (see Berstein and Ganea, Illinois J. Math. 5 (1961), 102). Theorem. $\psi_n(\Sigma X) \in \pi(\Sigma X, {}^n\Sigma X)$ has finite order if and only if all cup products of any n positive dimensional elements in $H^*(X;Q)$ vanish (Q = the field of rationals). In the proof the following theorem is applied to the adjoint of $\psi_n(\Sigma X)$ in $\pi(X,\Omega({}^n\Sigma X))$. Theorem. Let $f:X \longrightarrow \Omega Y$ be any map. The element in $\pi(X,\Omega Y)$ determined by f has finite order if and only if the homomorphism $f^*: H^*(\Omega Y;Q) \longrightarrow H^*(X;Q)$ is zero. (Received September 6, 1962.)

593-7. JOHN LAMPERTI, The Rockefeller Institute, New York 21, New York. On monotonic solutions of certain linear integral equations.

Consider the integral equation (*) $G(y) = \int_0^\infty K(x,y)dG(x)$, where the solution G is to be non-decreasing, right continuous, and $G(0) = 0$. For each x, $K(x,\cdot)$ is a probability distribution on $[0,\infty)$, and K is continuous in x in the weak * sense. Assume there exists $\epsilon > 0$: $K(x,x+\epsilon) < 1 - \epsilon$ and $K(x,(x-\epsilon)^+) > \epsilon$ for all x. Finally define, for $k \leqq 4$, $\mu_k(x) = \int_0^\infty (y-x)^k K(x,dy)$, and assume that as $x \longrightarrow \infty$, $x\mu_1(x) \longrightarrow \alpha$, $\mu_2(x) \longrightarrow \beta$ and $\mu_4(x) = O(1)$. Theorem 1. If $\alpha < \beta/2$, (*) has a unique solution G having a unit discontinuity at 0; this solution is bounded if $\alpha < -\beta/2$ but unbounded if $\alpha > -\beta/2$. Theorem 2. $-\beta/2 < \alpha < \beta/2$, then $G(y) = y^{1+2\alpha/\beta}L(y)$, where L is a slowly varying function. The cases $\alpha \pm \beta/2$ can be included under additional hypotheses. When $1 - K(x,y) = F(x-y)$, where F is an absolutely continuous probability distribution function, (*) is equivalent to a Wiener-Hopf equation; our results in that case are weaker than those of F. Spitzer. The proof uses probabilistic results due to T. E. Harris and the author. (Received September 7, 1962.)

593-8. J. G. KEMENY, Dartmouth College, Hanover, New Hampshire. Doeblin's ratio limit theorem.

For a denumerable Markov chain, whose states communicate, Doeblin showed that the ratios $N_{ij}^{(n)}/N_{km}^{(n)}$ always converge. [$N_{ij}^{(n)}$ is the mean number of visits to j in n steps, starting at i.] For a transient chain the numerator and denominator converge separately, hence the result is trivial. For a recurrent chain the terms tend to infinity, and the ratio tends to α_j/α_m. [α is the invariant measure.] The following is a surprisingly elementary proof of this result. The inequalities, $0 \leqq N_{jj}^{(n)} - N_{ij}^{(n)} \leqq {}^iN_{jj}$, are probabilistically obvious. [${}^iN_{jj}$ is the mean number of visits to j before hitting i, starting at j.] Dividing through by $N_{jj}^{(n)}$, and letting n tend to infinity, yields that

$N_{ji}^{(n)}/N_{jj}^{(n)} \longrightarrow \alpha_i/\alpha_j$. By computing the limit of the product of four such ratios Doeblin's theorem is obtained. (Received September 7, 1962.)

593-9. KAREL de LEEUW, Stanford University, Palo Alto, California, and HAZELTON MIRKIL, Dartmouth College, Hanover, New Hampshire. Rotation-invariant algebras on the n-sphere.

Let X be the n-dimensional sphere, $n \geqq 2$. Let C(X) be the pointwise algebra of complex continuous functions on X. Then C(X) contains only one nontrivial closed subalgebra invariant under rotations. Proof. (1) The invariant subspaces of C(X) are in one-one correspondence with the invariant subspaces of P(X) = the spherical harmonics. (2) The invariant symmetric subalgebras of C(X) are in one-one correspondence with the invariant fiberings of X. (3) All invariant subspaces of P(X) are symmetric. (4) There is exactly one nontrivial invariant fibering of X. (Received September 10, 1962.)

593-10. H. F. TROTTER, Princeton University, Fine Hall, Box 708, Princeton, New Jersey. Some non-invertible knots.

A knot K in the 3-sphere S is invertible iff there exists an autohomeomorphism of the pair (S,K) which preserves the orientation of S and reverses that of K. Suppose K is a pretzel-knot with a_1, a_2, a_3 crossings in its three braids, as in Reidemeister, Knotentheorie (Erg. d. Math. 1 (1932), 65). Then if the a_i are odd and $a_1 > a_2 > a_3 \geq 3$, K is not invertible. The proof depends on showing that $G = \pi_1(S - K)$ admits no automorphism which reverses meridians and longitudes, and involves mapping G onto a group of isometries of the hyperbolic plane. (The same representation of G was used by Reidemeister (loc. cit.) in classifying the alternating pretzel knots.) (Received September 10, 1962.)

593-11. P. C. FISCHER, 15 Robinhood Road, Winchester, Massachusetts. Proof indices and recursion properties of provable recursive functions.

Reference is made to abstract 590-33 these Notices 9 (1962), 128. Let ϕ_e denote the partial recursive function with Gödel number e. A natural number i will be called a proof index of a p-function f if there is an e such that $f = \phi_e$ and i is a Gödel number of a proof that ϕ_e is totally defined. We will let Φ_i denote the p-function with proof index i. p-functions may be indexed either by their Gödel numbers (since they are partial recursive functions) or they may be indexed by their proof indices. The set of Gödel numbers of p-functions is known to be recursively enumerable; the set of proof indices of p-functions is recursive. Theorem 1. There exists a p-1-1 p-function n(e) such that for all e, $\phi_{\phi_e(n(e))} = \phi_{n(e)}$. Theorem 2. If S is sound, there exists a p-function f such that, for every x, $\Phi_{f(x)} \neq \Phi_x$. Theorem 1 is a stronger version of the recursion theorem of Kleene. Theorem 2 shows that there is no analogous theorem for p-functions indexed by proof indices. (Received September 10, 1962.)

593-12. R. H. CROWELL, Dartmouth College, Hanover, New Hampshire. The abelianized commutator subgroup of a knot theory.

Let H be an infinite cyclic multiplicative group generated by t. Consider a ZH-module A which has an $n \times n$ relation matrix with determinant $\Delta(t) = c_0 + \dots + c_d t^d$, $c_0 c_d \neq 0$, where g.c.d.$(c_0, \dots, c_d) = 1$. It is known that if K is a tame knot and $G = \pi_1(S^3 - K)$, then the abelianized commutator subgroup G'/G'' is such a module. We prove that A is a torsion free Z-module of rank d. Let R be any subring of the rationals that contains the integers. Then $R \otimes_Z A$ is a finitely generated R-module iff c_0 and c_d are units of R; in either case $R \otimes_Z A = \oplus_{i=1}^{d} R$. Let R_Δ be the ring of all rational numbers $i/c_0^j c_d^k$, where $i,j,k \in Z$. If $p_0, \dots, p_s$ are the positive primes that divide $c_0 c_d$, then $A = \oplus_{i=1}^{d} R_\Delta$ iff each p_i divides all the coefficients of $\Delta(t)$ except one (which may depend on i). It follows that G'/G'' is characterized for many knots. (Received September 10, 1962.)

593-13. D. R. OSTBERG, Dartmouth College, Hanover, New Hampshire. Relative group extensions and simple algebras.

Let L be a finite separable extension of a field F, N the normal field associated with L, N* the multiplicative group of N, G the Galois group of N/F, and G_L the subgroup of G associated with L. Then the Brauer group of central simple F-algebra classes split by L is naturally isomorphic to the group of (G, G_L)-extensions of N*. (For the definition of group extensions of this type see Trans. Amer. Math. Soc. 82 (1956), 263.) The proof of this result is accomplished by means of the following theorem which is also of independent interest. Theorem. Let (E, ϕ) be an extension of an abelian group M by a group G, and let H be a subgroup of G. Let E_H be the complete inverse image of H under ϕ, and suppose that the sequence $(1) \to M \to E_H \to H \to (1)$ is split exact. Then if the one-dimensional cohomology groups $H^1(H \cap gHg^{-1}, M)$ are trivial for every g in G, (E, ϕ) is a (G,H)-extension of M. (Received September 7, 1962.)

593-14. R. H. FOX, Fine Hall, Princeton University, Princeton, New Jersey. Boundary links.

Call a link L of μ components $L_1, \dots, L_\mu$ a boundary if there exist disjoint orientable surfaces $F_1, \dots, F_\mu$ whose respective boundaries are $L_1, \dots, L_\mu$. It is well known that every knot is a boundary; for $\mu \geqq 2$ there exist links that are not boundaries, even though all the linking numbers are equal to zero. If L is a boundary then (i) its elementary ideals $E_0, E_1, \dots, E_{\mu-1}$ are all equal to (0), (ii) the μ-th elementary ideal E_μ is a principal ideal $(\Delta_\mu(x_1, \dots, x_\mu))$, (iii) $\Delta_\mu(t, \dots, t)$ is the Alexander polynomial of a knot. Thus to a boundary link is uniquely associated a class of knots. (Received September 12, 1962.)

593-15. J. S. WHOLEY, 1803 N. Edgewood, Arlington, Virginia. Persistence and Herbrand expansions.

A sentence S is said to be persistent with respect to a set of sentences K if the following holds: for every model M of K, if S is true in M, then S is true in every extension M' of M such that M' is a model of K. The Herbrand expansion of a set of first-order sentences K, which we denote by H(K), is a set of quantifier-free formulas $Z'_1, \dots, Z'_n, \dots$ in the notation of K, generated from K in a uniform manner, and such that K is consistent if and only if every conjunction $Z'_1 \& \ldots \& Z'_n$ of members of H(K) is consistent in the propositional calculus with identity. Theorem. Let K be a set of first-order sentences, and let S be a sentence in the notation of K. Then S is persistent with respect to K if and only if, for some n, the sentence $S \longleftrightarrow \mathcal{E}(Z'_1 \& \ldots \& Z'_n)$ is deducible from K, where $\mathcal{E}(Z'_1 \& \ldots \& Z'_n)$ is the existential closure of a conjunction of the first n members of $H(K \cup \{S\})$. This theorem improves on a result of A. Robinson (J. Symb. Logic 21 (1956), 33-35). An easy extension of the argument yields some related results on model-complete sets of sentences. (Received September 12, 1962.)

ABSTRACTS PRESENTED BY TITLE

62T-269. C. R. KARP, 7421 Patrician Road, Norfolk 18, Virginia. A note on Boolean algebraic representability.

Let α be a regular infinite cardinal. A Boolean algebra is α-representable if it is an α-homomorphic image of an α-complete field of sets. Examples are given of complete α-representable Boolean algebras not α^+-representable making no use of the continuum hypothesis. The examples in the literature have depended on the assumption $\alpha^+ = 2 \exp \alpha$. The algebras are also (β, ∞)-distributive for all $\beta < \alpha$ and not (α, α)-distributive. (Received July 3, 1962.)

62T-270. TILLA KLOTZ, University of California, Los Angeles 24, California. Postscript to "The geometry of extremal quasiconformal mappings".

The paper (*) named above appears in the Michigan Math. J. 9, 129-136. Theorem 1 of (*) and its corollary are strengthened as follows. Also two elementary remarks are made, each stronger than the original corollary. Let σ and $\hat{\sigma}$ immerse the oriented surfaces S and $\hat{S}$ smoothly in E^3. Theorem 1'. If the standard map $f: S \longrightarrow \hat{S}$ between distinct parallel surfaces is a Teichmüller mapping, then S and $\hat{S}$ are isothermal and satisfy the Weingarten relations $(1 - tk_1) = K(1 - tk_2)$ and $(1 + t\hat{k}_2) = K(1 + t\hat{k}_1)$ respectively. Moreover S and $\hat{S}$ are umbilic free if f is nonconformal. Corollary'. The standard map between compact parallel surfaces of genus $g \neq 1$ is never a nonconformal Teichmüller mapping. Remark 1. If S and $\hat{S}$ are compact of genus $g \neq 1$, and $f: S \longrightarrow \hat{S}$ is a nonconformal Teichmüller mapping, then neither $\hat{\sigma} \circ f$ nor $\sigma \circ f^{-1}$ is an immersion. Remark 2. If a nonconformal Teichmüller mapping $f: S \longrightarrow \hat{S}$ preserves a net of lines of curvature with at least one irremovable umbilic, then neither $\hat{\sigma} \circ f$ nor $\sigma \circ f^{-1}$ is an immersion. Corrections. On page 135 of (*) insert "corresponding" between "of" and "nonplanar" in line 8, and replace "removable" by "nonplanar" in third line past (7). (Received July 2, 1962.)

62T-271. E. G. EFFROS, 100 Maple Drive, Great Neck, New York. Order ideals in C*-algebras and their duals. I.

Let X be a partially ordered vector space. A subset N of X is an order ideal if for all x and y in N and scalars $r \geqq 0$, $x + y$ and rx are in N. Let A be a C*-algebra, A* its dual, with their natural orderings. The map $L \longrightarrow L^+$ is a 1-1 correspondence between the norm-closed left ideals L and the norm-closed order ideals in A. A operates to the left on A* by $(af)(b) = f(ab)$. $L \longrightarrow L^+$ is a 1-1 correspondence between the w*-closed left invariant subspaces L and the w*-closed order ideals in A*. $N \longrightarrow N^{\perp}$ (the positive annihilator of N) defines a 1-1 correspondence between the norm-closed order ideals in A and the w*-closed order ideals in A*. The norm-closure of an order ideal in A or A* is an order ideal. The w*-closure of an order ideal in A* need not be an order ideal. A necessary and sufficient condition for it to be an order ideal may be formulated using Tomita's concept of "regular" projections (cf. Tomita, Spectral theory of operator algebras. I, Math. J. Okayama Univ. 9 (1959), 63-98.) (Received July 2, 1962.)

62T-272. WITHDRAWN.

62T-273. MARTIN SCHECHTER, New York University, 25 Waverly Place, New York 3, New York. On L^p estimates and boundary problems, III. Preliminary report.

In the notation of Abstract 62T-20, Notices Amer. Math. Soc. 9 (1962), 40, let A_i be a differential operator of order $m_i \leq m$ (the order of A) with coefficient in $C^\infty(\bar{G})$, $1 \leqq i \leqq \ell$, where $\ell \geqq 1$. If A is elliptic on U, then for every real s satisfying $0 \leqq s \leqq m - \max m_i$ there is a constant C_s such that $\|u\|_{s,p} \leqq C_s(\sum_{i=1}^{\ell} \|f_i\|_{s+m_i-m,p} + \|u\|_{s-m,p})$ for all u and f_i in $C^\infty(\bar{G})$ satisfying $(u, A'v) = \sum_{i=1}^{\ell}(f_i, A_i'v)$ whenever $v \in V$. In particular this holds if $u \in U$, $Au = \sum_{i=1}^{\ell} A_i f_i$ and $\sum_{i=1}^{\ell} [(f_i, A_i'v) - (A_i f_i, v)] = 0$ for $v \in V$. (Received July 3, 1962.)

62T-274. J. D. BUCKHOLTZ, University of North Carolina, 378 Phillips Hall, Chapel Hill, North Carolina. Column sequences in Hausdorff matrices.

For a Hausdorff matrix H, it is well known that if the condition (*) $\sum_{k=0}^{n} |H_{nk}| \leqq M$ (M independent of n) is satisfied, then the sequence $\{H_{n0}\}_{n=0}^{\infty}$ converges, and, for every positive integer k, the sequence $\{H_{nk}\}_{n=0}^{\infty}$ converges to zero. Even without (*), however, one can prove a weaker form of the above in which convergence is replaced by Cesàro summability. Theorem. If H is a Hausdorff matrix and $\{H_{n0}\}_{n=0}^{\infty}$ converges, then for every positive integer k, $\{H_{nk}\}_{n=0}^{\infty}$ is (C,k) summable to zero. (Received July 5, 1962.)

62T-275. KATSUMI NOMIZU and HIDEKI OZEKI, Brown University, Providence 12, Rhode Island. A remark on the curvature tensor field.

The main result of the paper, A theorem on curvature tensor fields, Proc. Nat. Acad. Sci. 48 (1962), 206-207, is now established without assuming the completeness of the space: If the mth covariant differential $\nabla^m R$ of the curvature tensor field R of a Riemannian manifold is zero for some $m \geqq 1$, then ∇R is zero. (Received July 5, 1962.)

62T-276. YUKIHIRO KODAMA, The University of Michigan, Ann Arbor, Michigan. The dimension of product spaces.

Let E be an n-simplex and S the boundary of E. Theorem. Let X and Y be fully normal spaces. Suppose that there exists a map f from E into Y such that, for every point y of f(S), $f^{-1}(y) \cap S$ is contained in an open star of S. Then $\dim(X \times Y) \geqq \dim X + n$. The following corollary which is a generalization of K. Morita's theorem is a consequence of the theorem. Corollary. Let X be a fully normal space and Y a locally compact 2-dimensional ANR. Then $\dim (X \times Y) = \dim X + \dim Y$. (Received July 5, 1962.)

62T-277. YUKIHIRO KODAMA, The University of Michigan, Ann Arbor, Michigan. Cohomology dimension of fully normal spaces.

Denote by $\mathcal{Q}$ the rational field, by R_p the subgroup of $\mathcal{Q}$ consisting of rationals whose denominators are coprime with a prime p, by Z_p a cyclic group with order p and by $\mathcal{Q}_p$ all rationals mod 1 whose denominators are powers of p. Theorem. There exist Cantor manifolds M_0, M_p, P_p and N_p such that (i) $D(X:\mathcal{Q}) = \dim X$ if and only if $\dim (X \times M_0) = \dim X + 2$, (ii) $D(X:R_p) = \dim X$ if and only if $\dim (X \times M_p) = \dim X + 2$, (iii) $D(X:Z_p) = \dim X$ if and only if $\dim (X \times P_p) = \dim X + 2$, and (iv) $D(X:\mathcal{Q}_p) = \dim X$ if and only if $\dim (X + N_p) = \dim X + 2$, where D is the cohomological dimension and X is a finite dimensional fully normal space. Since the universal coefficient theorem does not hold for nonlocally compact spaces, the theorem does not follow from Bockstein's theorem. (Received July 5, 1962.)

62T-278. YUKIHIRO KODAMA, The University of Michigan, Ann Arbor, Michigan. H. Hopf extension theorem.

There exists a pair (K, L) of countable CW complexes satisfying the following properties: (i) $\dim K = n + 1$ and L is a one point union $\bigvee_i S_i^n$ of n-spheres S_i^n; (ii) A map f of L into an n-sphere S^n is extendable over K if and only if the degree of the restricted map $F|S_i^n$ is 0 for each i; (iii) A map f of L into S^n is extendable over every compact subset of K and not extendable over K if and only if the degree of the restricted map $f|S_i^n$ is a nonzero d for each i. Moreover, for a map f whose restriction to S_i^n has the same degree, the induced homomorphism f_* maps the subgroup $\partial H_{n+1}(K,L:G)$ into zero, where H_n is the homology group and G is an abelian group. The property (iii) shows that the H. Hopf extension theorem in terms of homology does not hold for infinite complexes. (Received July 5, 1962.)

62T-279. J. F. LATIMER, c/o Professor T. Tamura, University of California, Davis, California. The dominators of a semigroup.

An element x of a semigroup S is called a dominator of S if xax = x for all a in S. The set D of all dominators of S, if any exist, is called the dominator of S. Right-zero (xy = y for all x,y), left-zero (xy = x for all x,y), and rectangular semigroups (xyx = x for all x,y) are examples of semigroups for which every element is a dominator. If a semigroup S has a dominator D, then D is a two-sided idempotent ideal of S. It is also a rectangular subsemigroup of S. A semigroup has a unique dominator if and only if it has a zero element, namely the zero element itself. Using the lemma - If D is any rectangular semigroup and for a, b $\in$ D, there exists exactly one element c $\in$ D such that xc = xa and cx = bx for every x $\in$ D - then the following theorem can be proved: If D is any rectangular semigroup and Z any semigroup with a zero element, there exists at least one semigroup S such that D is the dominator of S, and the Rees factor semigroup of S modulo D is isomorphic to Z. (Received July 5, 1962.)

62T-280. D. G. BECKER, 871 Midwood Drive, Rahway, New Jersey. Integral representations for harmonic functions in three real variables.

Let $X = (x,y,z)$ be a point in three-dimensional euclidean space and $f(u,\zeta)$ an analytic function of two complex variables u and ζ; u, ζ, and X being connected by the relation $u = x + (1/2)(iy + z)\zeta + (1/2)(iy - z)\zeta^{-1}$. The Whittaker-Bergman operator $(1/2\pi i)\int_{|\zeta|=1} f(u,\zeta)d\zeta/\zeta$ is known to represent a complex harmonic function H in a neighborhood of some fixed point X_0 [See, Bergman, Ergebnisse der Math. Wissen., N. F., 23 (1961)]. Bergman has described the function and its singularities in the case $f = \zeta^k/(u - iA)$, A a real constant. For $k > 0$ H is a two-valued harmonic function which for $A = 0$ is singular along the negative x-axis in the first sheet and for $A \neq 0$ branches along the circle $x = 0$, $y^2 + z^2 = A^2$. The traces of the level surfaces Re $H = C$ in the plane $x = 0$ are studied. For $A = 0$ such traces are rose curves of k leaves for k odd and 2k leaves for k even. For $A = 1$ the traces are obtained for $C = \pm 1/2, \pm 3, \pm 3\ 1/2$, for points inside and outside the circle $y^2 + z^2 = 1$. (Received July 6, 1962.)

62T-281. H. M. SCHAERF, University of Wisconsin, Madison 6, Wisconsin. Extension of measures and simplification of integration on a ring of sets.

Let m be an extended real valued set function on a class R of sets such that $\theta \in R$ and $m(\theta) = 0$. Assume that R contains the union of any two of its disjoint members and let $\tilde{R}$ be the class of all sets whose intersection with every element of R is in R. Theorem. The set function $\tilde{m}$ defined for all $A \in \tilde{R}$ as the least upper bound of the values of m on those members of R which are subsets of A is finitely or countably additive if so is m, respectively. If R is a ring or σ-ring of sets, then $\tilde{R}$ contains R and is an algebra or σ-algebra of sets, respectively. If R is a ring of sets, then each R-measurable function is $\tilde{R}$-measurable and its integrals with respect to m and $\tilde{m}$ coincide if m is nonnegative. Remarks: 1. If R contains all closed compact sets of a topological space, then $\tilde{R}$ contains all closed sets, hence the theorem extends Haar's measure correspondingly. 2. $\tilde{m}$ inherits most of the properties of m. 3. The above theorem was first introduced by the author in his lectures at Washington University in 1958-1959. Since it considerably simplifies integration on rings of sets, it is here communicated to a wider public. (Received July 5, 1962.)

62T-282. TAKAYUKI TAMURA and D. G. BURNELL, University of California, Davis, California. Extension of groupoids with operators.

By a mono-endomorphism of a groupoid is meant a one-to-one endomorphism. Let G be a groupoid, an operation of which is denoted by +, and suppose that Γ is a commutative semigroup of some mono-endomorphisms α of G and αx denotes the image of x under α, in other words, (1.1) $\alpha(x + y) = \alpha x + \alpha y$. (1.2) $\alpha(\beta x) = \beta(\alpha x)$. (1.3) $\alpha x = \alpha y$ implies $x = y$. Denote such a G by (G, Γ). Theorem. For (G, Γ) there exists $(\bar{G}, \bar{\Gamma})$ such that (2.1) G is embedded into $\bar{G}$. (2.2) Γ and $\bar{\Gamma}$ are isomorphic. (2.3) Each $\alpha \in \bar{\Gamma}$ is an extension of $\alpha \in \Gamma$ to $\bar{G}$, and $\bar{\alpha}$ is an automorphism of $\bar{G}$. (2.4) $(\bar{G}, \bar{\Gamma})$ is the smallest extension of (G, Γ). This is a generalization of the earlier abstract 591-9 [Notices Amer. Math. Soc. 9 (1962), 133] and also is applied to some extension theory of semirings. (Received July 5, 1962.)

62T-283. ROBERT HERMAN, University of California, Berkeley, California. On Riemannian manifolds of non-positive curvature.

Theorem. Let M be a complete Riemannian manifold. Suppose that N is a closed totally geodesic submanifold such that the sectional curvature is nonpositive for all tangent planes containing a tangent vector to a geodesic that is perpendicular to N. Suppose further that the inclusion map of N into M sends the fundamental group of N onto the fundamental group of M. Let $N^{\perp}$ be the normal bundle of N. Then, the exponential map is a diffeomorphism of $N^{\perp}$ onto M. (Received July 9, 1962.)

62T-284. S. A. KHABBAZ, Lehigh University, Bethlehem, Pennsylvania. On the splitting problem in abelian groups. Preliminary report.

Let A and B be any abelian groups whose torsion subgroups C and D are primary belonging to the prime p. Theorem. Suppose that $p(A/C) \neq A/C$ and that $p(B/D) \neq B/D$. Then the tensor product $A \otimes B$ splits (i.e. is the direct sum of a torsion subgroup and a torsion free subgroup) if and only if both A and B split. The theorem is valid for more than two variables, and is proved by showing that a certain subgroup of $A \otimes (B/D)$ containing the torsion subgroup of $A \otimes (B/D)$ and isomorphic to a direct sum of copies of A splits. Theorem. If $p(A/C) = A/C$, and if $f: A \longrightarrow A/C$ is the natural homomorphism, then A splits if and only if $f^{-1}(K)$ splits whenever K is a pure subgroup of A/C having rank one. Also determine the conditions (for the case $p(A/C) = A/C$) under which $A \otimes A$ splits without A splitting. (Received July 9, 1962.)

62T-285. LEO SARIO, 521 Georgina Avenue, Santa Monica, California. Nevanlinna's second main theorem without exceptional intervals.

Let Ω_0 and Ω be bordered compact subregions with borders β_0, β_Ω on an open Riemann surface W. Let v be harmonic in $\overline{\Omega} - \Omega_0$ with $v = 0$ on β_0, $v = k$ (const.) on β_Ω, $\int_{\beta_0} dv^* = 1$. For a meromorphic w on W and for $0 < h \leqq k$ let $\beta_h = \{z \mid v(z) = h\}$ and set $m(h,w) = \int_{\beta_h} \log^+ |w| dv^*$, $m(h,a) = \int_{\beta_h} \log^+ (1/|w - a|)\, dv^*$, $N(h,a) = \int_0^h n(h,a) dh$, $N(h,w) = N(h, \infty)$, and $T(h) = m(h,w) + N(h,w)$. For any function $F(h)$ in $[0,k]$ let $F_2(k) = \int_0^k \int_0^h F(t) dt\, dh$. Then $\sum_1^q m_2(k, a_i) < 2T_2(k) - M_2(k) + E_2(k) + O(k^3 + \log T_2(k))$, where M is the counting function N for multiple points of w and E is the (once) integrated Euler characteristic of $\Omega - \Omega_0$. For w with $\lim_{\Omega \to W} (k/T(k)) = 0$ the remainder is negligible. As a special case a second main theorem without exceptional intervals is obtained in the classical case of the plane. (Received July 9, 1962.)

62T-286. LEO SARIO, 521 Georgina Avenue, Santa Monica, California. Meromorphic functions and conformal metrics on Riemann surfaces.

On an open Riemann surface W let $ds = \lambda(z)|dz|$ be a conformal metric satisfying certain regularity conditions. Let W_0 be a compact bordered subsurface and let $W_\sigma \supset W_0$ be the region bounded by the compact "level line" $\beta_\sigma = \{z \mid d(z, W_0) = \sigma\}$ at distance σ from W_0 and of unit length. Given a meromorphic w on W set $m(\sigma,a) = 1/2\pi \int_{\beta_\sigma - \beta_0} \log(1/[w,a])\, ds$, $N(\sigma,a) = \int_0^\sigma n(\sigma,a) d\sigma$, and $T(\sigma) = n(\sigma,\infty) + N(\sigma,\infty) + n_0(\infty)\sigma$, with σ referring everywhere to $W_\sigma - W_0$. Then $m(\sigma,a) + N(\sigma,a)$

$+ n_0(a)\sigma = T(\sigma)$ for all a and $\sum_1^q m(\sigma, a_i) < 2T(\sigma) - N_1(\sigma) + N(\sigma, \lambda^{-1}) + 0(\sigma + \log T(\sigma))$ except in Δ with $\int_\Delta e^{a\sigma} d\sigma < \infty$ for $a \geqq 0$ and for $\sigma_\beta = \sup \sigma = \infty$. If $\sigma_\beta < \infty$, then $0(\sigma + \log T(\sigma))$ is replaced by $0(1/(\sigma_\beta - \sigma) + \log T(\sigma))$. The capacity function p_β on W gives the special metric $ds = |\text{grad } p_\beta||dz|$ and $N(\sigma, \lambda^{-1})$ becomes the integrated Euler characteristic $E(\sigma) = \int_0^\sigma e(\sigma)d\sigma$ of $W_\sigma - W_0$. A generalization of Picard's theorem is obtained: $P \leqq 2 + \eta$, where P is the number of Picard values and $\eta = \lim_{\sigma \to \sigma_\beta} E(\sigma)/T(\sigma)$. This bound is sharp. (Received July 9, 1962.)

62T-287. LEO SARIO, 521 Georgina Avenue, Santa Monica, California. Islands and peninsulas on arbitrary Riemann surfaces.

Let W be a covering surface of the complex plane W_0 and consider a compact bordered subsurface $\Omega \subset W$. The Ahlfors theory can be generalized from simply connected to arbitrary W as follows: Given $q \geqq 3$ disjoint simply connected regions Δ_j with boundaries a_j on W_0, cut Ω along cross-cuts ϑ above the a_j and remove the peninsulas $D_{p\vartheta}$ thus separated. Cut each component Ω_ϑ of the remainder of Ω along cycles σ on the boundaries of the remaining peninsulas $D_{p\sigma}$ and above the a_j, and remove these $D_{p\sigma}$. From each component Ω' of the remainder of Ω remove the islands D_i, with components $\bar{\Omega}$ remaining. The Euler characteristics e and the numbers N_1 of simply connected regions satisfy $\sum e(D_i) = N_1(\bar{\Omega}) + \sum e(\Omega') - \sum e^+(\bar{\Omega})$ where $N_1(\bar{\Omega}) \leqq N_1(\Omega_\vartheta)$, $\sum e(\Omega') - \sum e(\Omega_\vartheta)$, $\sum e^+(\bar{\Omega}) \geqq (q - 2)S + 0(L)$, and $\sum e(D_i) = \sum \mu(\Delta_j) - qS + \sum b(\Delta_j) + 0(L)$. A generalized nonintegrated second main theorem results: $\sum \mu(\Delta_j) < 2S - \sum b(\Delta_j) + e^+ + 0(L)$. If W is the image under a meromorphic w of an arbitrary Riemann surface F, let $\beta_\rho = \{z | d(z, \zeta) = \rho\}$ be the "level line" with distance ρ from a fixed $\zeta \in F$ in a suitable conformal metric $d\rho$ on F. Then $\overline{\lim}_{\rho \to R}[S(\rho)\int_\rho^R (d\rho/l(\rho))] = \infty$ with $R = \sup_F \rho$ is sufficient for regular exhaustability $\underline{\lim}_{\rho \to R}(L(\rho)/S(\rho)) = 0$. (Received July 9, 1962.)

62T-288. LEO SARIO, 521 Georgina Avenue, Santa Monica, California. Functions with bounded characteristics on Riemann surfaces.

Let W be an open Riemann surface and Ω a compact bordered subregion. The characteristic $T(\Omega)$ of a meromorphic w on W is defined in terms of the capacity function p_Ω of Ω. The class MB of meromorphic functions with bounded characteristics can thus be introduced on an arbitrarily given W. In case $w \neq 0, \infty$ at the singularity ζ of p_Ω, Nevanlinna's classical decomposition theorem carries over to W without difficulty. If, however, $\log|w|$ has a singularity at ζ, then it is necessary to extend MB to the class M_eB of functions e^{u+iu^*} with single-valued moduli, u being in the class L of functions harmonic on W except for logarithmic singularities with integral coefficients. It is shown that $w \in M_eB$ if and only if $\log|w|$ is the difference of positive functions in L. An explicit construction is given of the smallest functions in such decompositions. Another necessary and sufficient condition for $w \in M_eB$ is that $\int_{\beta_\Omega} \log^+|w| dp_\Omega^*$ is bounded and $\sum g(z, b_\nu)$ converges, with b_ν the poles of w. A regular harmonic function h on W is the difference of two positive harmonic functions if and only if $\int_{\beta_\Omega} h \, dp_\Omega^*$ is bounded. Theorems on MB are included as corollaries. The classical theory in the plane is contained as a special case. (Received July 9, 1962.)

62T-289. EDWIN DUDA, University of Miami, Coral Gables, Florida. *Expansiveness and light open mappings.*

For a light open mapping $f(X) = Y$, where X and Y are locally connected generalized continua, it is well known (G. T. Whyburn, Mem. Amer. Math. Soc. No. 1) that each point x of X is interior to a conditionally compact region R with f/R an expansive mapping of R onto $f(R)$ and $f(FrR) = Frf(R)$. It is shown that x is interior to a maximal region R on which f is expansive and that $f(FrR) \cdot f(R) = \emptyset$. It is not always true that $f(FrR) = Frf(R)$ for such a maximal region. Further restrictions are necessary in order to obtain the equality. (Received July 11, 1962.)

62T-290. O. L. MANGASARIAN, Shell Development Company, Emeryville, California. *Nonlinear programming problems with stochastic objective functions.*

Let $\phi(c,x)$ be a scalar function of the k-dimensional vector c and the n-dimensional vector x, and let $g(x)$ be a vector function of x. Assume further that c is a random vector with a known distribution, and let E denote the expectation with respect to this distribution. *Theorem. Let* $x = \bar{x}(Ec)$ *be the solution of* $\max_x \{\phi(Ec,x) \mid G(x) \geqq 0\}$. *Then* $E \max_x \{\phi(c,x) \mid g(x) \geqq 0\} \geqq \max_x \{E\phi(c,x) \mid g(x) \geqq 0\} \geqq E\phi(c,\bar{x}(Ec)) \geqq \max_x \{\phi(Ec,x) \mid g(x) \geqq 0\}$ *where it has been assumed for the last inequality alone, that* c *takes on values only in some (finite or infinite) convex region* R *of the k-dimensional Euclidean space and that* $\phi(c,x)$ *is a convex function of* c *in* R *for every* x *satisfying* $g(x) \geqq 0$. For the case when the c's are independent and defined over a finite k-dimensional rectangle, an upper bound to $E \max_x \{\phi(c,x) \mid g(x) \geqq 0\}$ has also been found. (Received July 12, 1962.)

62T-291. E. H. ROGERS, The Royal College of Science and Technology, Glasgow C. 1, Scotland. *A minimax theory for overdamped systems.*

A minimax theory for eigenvalues occurring nonlinearly in a vector relationship is developed. Of first concern are Lagrange's equations of small motion as modified by Rayleigh to cover dissipative systems. The classical minimax theory applies to nondissipative systems in which the eigenvalues occur linearly. The dissipation complicates this eigenvalue problem. Duffin (Arch. Rational Mech. Anal. 4 (1955), 221) has described a minimax theory for the real eigenvalues of a class of such motions. The present theory is a generalization of his work. First, a generalization of the Rayleigh quotient $p(u)$ is introduced, representing the zeros x of the quadratic form $(T_x u,u)$, where T_x is a symmetric linear transformation of R^n into itself. Values k of p are sought so that, for some nonzero vector v, $T_k v = 0$. The equivalence between such "eigenvalues" and "minimax" values defined with p is shown. Linear constraints are seen to have the same effect as in the classical case; that is, they tend to increase the eigenvalues. This theory can be extended to Hermitian operators and infinite dimensional spaces. (Received July 16, 1962.)

62T-292. WITHDRAWN.

62T-293. WITHDRAWN.

62T-294. J. T. CROSS, The University of the South, Sewanee, Tennessee. The number of solutions of certain types of congruences in algebraic number fields.

Let K denote a finite extension of the rational number field and D the domain of algebraic integers of K. Let P denote an odd prime ideal of D with norm $N(P) = p^h = q$, where p is an odd rational prime number and h is a positive integer. Formulas are obtained for the number of solutions of the congruence, $X_0^k + \alpha_1 X_1^2 + \dots + \alpha_s X_s^2 \equiv \rho \pmod{P^r}$, where ρ is an arbitrary integer of K, r and k are positive integers, each of $\alpha_1, \dots, \alpha_s$ is an integer of K, and the following conditions are satisfied: $(k,p) = 1$, $d = (k, q-1) > 1$, $(\alpha_1, \dots, \alpha_s, P) = 1$. Let C denote an ideal of D prime to P such that $PC = (\theta)$ is principal. Let b be the greatest integer n such that $0 \leqq n \leqq r$ and $P^n | \rho$. Then one can put $\rho \equiv \theta^b \eta \pmod{P^r}$, $(\eta, P) = 1$, where η is uniquely determined $\pmod{P^{r-b}}$ if $b < r$. Three cases are considered in the paper: (1) s even, (2) s odd and k even, and (3) s odd and k odd. In cases (2) and (3) the formulas obtained involve a special type of Jacobsthal sum. This sum occurs, however, only if b is a multiple of k, $\rho \not\equiv 0 \pmod{P^r}$, and bk is even. All other formulas are explicit. (Received July 16, 1962.)

62T-295. ERWIN KLEINFELD, Syracuse University, Syracuse 10, New York. Associator dependent rings.

In a paper entitled Associator dependent rings about to appear in Archiv der Mathematik rings are discussed which satisfy a nontrivial linear dependence relation among the six associators involving elements x,y,z, over some field of scalars of characteristic $\neq 2$ and 3, in addition to third power associativity. The discussion of such rings was incomplete and one can now assert that the only new classes are included among rings R that satisfy an identity of the form $(x,y,z) = a(x,z,y) + b(y,x,z) + c(z,y,x)$, where a,b,c are scalars, in addition to third power associativity. One need only consider the case where $a + b + c = -1$, and $a^2 + b^2 + c^2 = 1$. For certain values of a,b,c one obtains rings satisfying (1) $(x,x,y) = (x,y,x)$, or (2) $(x,x,y) = (y,x,x)$ or (3) $(x,y,x) = (y,x,x)$. For all others the result is obtained that if R is simple and contains an idempotent $e \neq 1$, then R must be alternative, hence either a Cayley-matrix algebra of dimension eight over its center or else associative. The comparable situation for (1), (2) and (3) has been studied by Frank Kosier. (Received July 17, 1962.)

62T-296. A. A. MULLIN, University of Illinois, Urbana, Illinois. Topics on mutation. I. Direct products with mutant factors.

Let $\{(A_i, *_i): i \in I\}$ be a nonempty family of semigroups. Let (B,0) be their ordinary (direct) product, chez e.g., C. Chevalley, Fundamental concepts of algebra. E.g., Lemma. If all of the factors of a Cartesian product are maximal mutants of their respective semigroups then their Cartesian product is a maximal mutant of the direct product (B,0). If a Cartesian product is a maximal mutant of the direct product then a mutant factor is a maximal one over its semigroup. Then, for simplicity, mapping properties of direct products of monoids, rather than semigroups, are treated. E.g., Theorem. Let there be given, for each $i \in I$, a homomorphism ϕ_i of a monoid (N,0) into a monoid $(A_i, *_i)$. Then every Cartesian product with a mutant factor of subsets from A_i induces a mutant set of (N,0) uniformly under one and the same mapping from (N,0) into $\prod_{j \in I}(A_j, *_j)$. Theorem. Let (A,0) and (B,0) be normal subgroups of a group (G,0) for which $A \cap B = 1$ and $A \cup B = G$. Let M and N be maximal mutants of (A,0) and (B,0), respectively. Then there exists a mapping from AXB into G such that the image of MXN is a maximal mutant of (G,0). (Received July 17, 1962.)

62T-297. L. V. QUINTAS, 915 West End Avenue, New York 25, New York. 1-dimensional cell complexes with homeotopy group equal to zero.

The homotopy groups of the homeomorphism group G(K) ~~of a~~ topological space K are called the homeotopy groups of K and are denoted $\mathcal{H}_k(K) \equiv \pi_k[G(K)]$. $\mathcal{H}_0(K) = 0$ is equivalent to every homeomorphism if K is isotopic to the identity. Let K denote a connected finite 1-dimensional cell complex, $a_0(K)$ and $a_1(K)$ its number of 0-cells and 1-cells respectively, and let $N(K) \equiv a_1(K) - a_0(K) + 1$ denote the nullity of K. Theorem 1. If $\mathcal{H}_0(K) = 0$, then $a_0(K) \geqq 7$, $a_1(K) \geqq 10$, and $N(K) \geqq 2$. Furthermore, there exist complexes K such that $\mathcal{H}_0(K) = 0$ and (i) K has a_0 0-cells for all $a_0 \geqq 7$, (ii) K has a_1 1-cells for all $a_1 \geqq 10$, and (iii) K has nullity N for all $N \geqq 2$. ((i), (ii), and (iii) are not necessarily satisfied simultaneously.) Let $G_0(I)$ denote the identity component of the homeomorphism group of the unit interval I. Theorem 2. If $\mathcal{H}_0(K) = 0$, then G(K) is isomorphic to the direct product of $a_1(K) - b_2$ homeomorphic copies of $G_0(I)$, where b_2 is the number of 0-cells in K of degree 2. Corollary. $\mathcal{H}_k(K) = 0$ $(k \geqq 0)$. (Received July 19, 1962.)

62T-298. D. G. BABBITT, University of California, Los Angeles 24, California. A summation procedure for certain Feynman integrals.

Theorem. Let V(x) be a continuous, real-valued function on E^m, bounded on bounded sets and bounded below everywhere. Let $(T_n(z)\phi)(x) \equiv (2\pi z/n)^{-mn/2} \int_{E^m} \cdots \int_{E^m} \phi(x_0) \cdot \prod_1^n \exp[-|x_j - x_{j-1}|^2/2 \cdot z/n - z/n \cdot V(x_{j-1})]\, dx_0 \cdots dx_{n-1}$ where $\phi(x) \in L_2(E^m)$, $Re(z) > 0$, $X_n \equiv X$. Then $T_n(z)\phi$ is in $L_2(E^m)$ and for $Re(\sigma) > 0$, $t > 0$, the L_2 double-limit $\text{l.i.m.}_{\sigma \to i}[\text{l.i.m.}_{n\to\infty} T_n(\sigma t)\phi]$ exists and equals $U_t\phi \equiv \phi(x,t)$, where U_t, $-\infty < t < \infty$ is a continuous unitary group of operators on $L_2(E^m)$. Denote by iA the infinitesimal generator of U_t, then A is the infinitesimal generator of the semi-group T(t) where $T(t)\phi(x) \equiv E\{\exp[-\int_0^t V(x(\tau) + x)d\tau]\phi(x(t) + x)\}$, $t > 0$, $\phi \in L_2(E^m)$. $E\{.\}$ denotes Wiener integration. Remark. By a theorem of Ray (Trans. Amer. Math. Soc. 77 (1954), 299-321) A is the Schrödinger operator when V(x) satisfies mild smoothness conditions. In this case $\phi(x,t)$ is a solution of Schrödinger's equation when $\phi \in \mathcal{D}(A)$. Theorem. If $[\text{l.i.m.}_{n\to\sigma} \text{l.i.m.}_{\sigma\to i} T_n(\sigma t)\phi]$ exists and is weakly continuous in t, it equals $U_t\phi$. Proof. By using the theory of holomorphic semi-groups, holomorphic vector-valued functions and Wiener integration the theorem is proved. Thus the double limit of the first theorem is used as a definition for certain Feynman integrals. (Received July 19, 1962.)

62T-299. A. A. MULLIN, University of Illinois, Urbana, Illinois. Topics on mutation, II. Logical algebra.

The results of an earlier paper (On the doubly mutant cosets generated by a skew field, Notices Amer. Math. Soc. 8 (1961), 627) are generalized to Stone rings so as to permit the investigation of consistent, but incomplete, logical algebras in the sense of, e.g., P. R. Halmos, Algebraic Logic, Chelsea Publishing Company, New York, 1962, especially pp. 18-19. Various interpretations are given to models of the composite form (i) a non-maximal ideal of refutable propositions, (ii) a singly mutant coset of provable propositions and (iii) singly mutant cosets of undecidable propositions. Other interpretations are given to models of the composite (i), (ii) and (iii)' maximal singly mutant cosets of undecidable propositions. E. g., a Boolean algebra can have no nonempty doubly mutant sets, although there is an abundance of nonempty singly mutant sets, viz., almost all cosets. Lattice-theoretical results are forthcoming. (Received July 23, 1962.)

62T-300. A. A. MULLIN, University of Illinois, Urbana, Illinois. Topics on mutation, III, Relational abstractions.

This paper abstracts from an earlier definition of a (λ, T, μ)-mutant set of a general algebraic system (see: Notices Amer. Math. Soc. 9 (1962), 37) to provide a concrete exemplar of a set relation that has many descriptive interpretations within, e.g., group theory, ring theory, elementary number theory, naive algebraic number theory, elementary mathematical biophysics and philosophical analysis. Specifically the set-theoretical inclusion relation of the definition of a (λ, T, μ)-mutant set is replaced by any element from a certain class of dyadic transitive relations R_i defined on relational algebraic systems, thereby yielding what are called R_i-mutant sets of relational systems. Various properties of these general R_i-mutant sets are investigated, e.g., regarding the existences of ultra- and infra-hereditariness and of extremal properties, relative to relational systems. In addition conditions are studied for which R_i-mutant sets are bidirectionally preserved under homomorphic mappings between relational systems. (Received July 24, 1962.)

62T-301. J. A. WOLF, University of California, Berkeley 4, California. Homogeneity and bounded isometries in manifolds of negative curvature.

The main result is Theorem. If N is a connected Riemannian homogeneous manifold of non-positive sectional curvature, then (1) N is isometric to the product of a flat torus with a simply connected Riemannian homogeneous manifold of nonpositive sectional curvature, (2) N is diffeomorphic to the product of a torus and a Euclidean space, (3) N admits a transitive solvable group of isometries, and (4) N is Riemannian symmetric iff it admits a transitive reductive group of isometries. This follows from Theorem. If Γ is a properly discontinuous group of isometries of a simply connected Riemannian homogeneous manifold M of nonpositive sectional curvature, then M/Γ is isometric to the product of a flat torus with a simply connected manifold, iff M/Γ is Riemannian homogeneous, iff every element of Γ is a Clifford translation (= constant displacement) of M, iff every element of Γ is a bounded isometry of M. The latter result is an easy consequence of Theorem. If M is a simply connected Riemannian manifold of nonpositive sectional curvature and γ is an isometry of M, then γ is a bounded isometry of M iff γ is an ordinary translation along the Euclidean factor of the de Rham decomposition of M. This last result is proved by geometric arguments. (Received July 25, 1962.)

62T-302. D. V. MEYER, State University of Iowa, Iowa City, Iowa. E^3 modulo a 3-cell.

If A is a compact continuum in E^3, let E^n/A be the decomposition of E^n whose only nondegenerate element is A. Andrews and Curtis [Ann. of Math. 75 (1962), 1-7] have shown that if A is an arc in E^n, then $E^n/A \times E^1$ is homeomorphic to E^{n+1}. Let C be a 3-cell in E^3 and let K be the set of points on BdC at which BdC is not locally polyhedral. Theorem. If K is a 0-dimensional set, then there exists an arc A on BdC such that E^3/C is homeomorphic to E^3/A. Corollary. $E^3/C \times E^1$ is homeomorphic to E^4. (Received July 26, 1962.)

62T-303. HADI HADDAD, The University of Texas, Austin 12, Texas. Convergent chain sequences.

Theorem 1. If $\{a^p\}_{p=1}^{\infty}$ is a sequence of chain sequences which converges weakly to a, then a is a chain sequence. Corollary. If a is a chain sequence which converges to k, then $0 \leq k \leq 1/4$. Theorem 2. Suppose that g is a link sequence of the chain sequence a. The following two statements are equivalent: (1) a converges to a limit distinct from 1/4. (2) g converges to a limit distinct from 1/2. Theorem 3. If g is a link sequence of the chain sequence 1/4, 1/4, 1/4,..., then g converges to 1/2. Theorem 4. Suppose that a and b are chain sequences, t is a number in [0,1] and C is the sequence such that for each positive integer p, $c_p = [ta_p + (1 - t)b_p]/2$. Then c is a chain sequence. 1/2 is the largest factor for which any such combination of chain sequences a and b is a chain sequence. (Received July 26, 1962.)

62T-304. THOMAS ERBER, Illinois Institute of Technology, Chicago 16, Illinois. Majorant for a modified Bessel function.

The usual series representation for $I_\nu(z)$ may be converted into an integral representation through the substitution of Laplace's Gamma function integral. The resulting expression may be bounded from above through the use of elementary inequalities. The free parameter of Laplace's integral can be eliminated through minimization. The final result is $I_\nu(z) < \pi^{-1/2}\{z/(\nu+\sqrt{\nu^2+z^2})\}^{\nu} \cdot \exp\sqrt{\nu^2+z^2}\} \times \Gamma(\nu/2)/\Gamma([\nu+1]/2)$; $\nu > 0$, $z > 0$. (Received July 27, 1962.)

62T-305. G. M. EWING, University of Oklahoma, Norman, Oklahoma. Optimal steering programs for idealized missiles.

Given positive nonincreasing functions m (a thrust program) and S (a staging program) together with any function θ (a steering program) which is measurable in terms of the measure μ generated by -c ln m, a planar trajectory is determined by the equations $x(t) = \int u\,d\tau$, $y(t) = \int v\,d\tau$, $ue^{I(t)} = \int (m/M)e^I \cos\theta\, d\mu$, $ve^{I(t)} = \int (m/M)e^I \sin\theta\, d\mu - g\int e^I d\tau$, where $M = m + S$ and $I(t) = \int (k/M)d\tau$. All integrals are over the closed interval [0,t]. With m and S fixed a necessary condition on θ which maximizes the range (loosely x(T) such that y(T) = 0) is that 0(t) be constant aeμ if $T > t_1$ = burnout or aeμ on [0,T) if $T \leq t_1$. This extends to arbitrary thrust and staging together with linear drag a result of Fried and Richardson, J. Appl. Phys. 27 (1956), 955-961, for the single-stage no-drag case with an AC function m and $T > t_1$. It is shown that such a function θ furnishes the absolute maximum range in comparison with all μ-measurable θ. (Received July 27, 1962.)

62T-306. A. J. V. SADE, 14 boulevard du Jardin Zoologique, Marseille 4, France. Paratopies et auto-paratopies des quasigroupes.

Une isotopie I d'un quasigroupe Q = E(.), suivie d'une parastrophie P, regardée comme une locomotion, est une paratopie. Le produit PI peut être exprimé comme une paratopie JP; cela permet de définir la transformée d'une isotopie par une parastrophie et de montrer que l'ensemble des paratopies ($I \in \mathcal{T} = \mathcal{S}_E^3$, $P \in \mathcal{S}_{123}$) est un groupe $\mathcal{G}_p$. $\mathcal{S}_{123}$ est homomorphe à $\mathcal{G}_p$. La transformée d'une parastrophie P' par une paratopie IP est une parastrophophie si I est invariante par P'. Si $\sigma \subseteq \mathcal{S}_{123}$, (i) les isotopies de Q invariantes par toute $P \in \sigma$ forment un groupe $\mathcal{T}_1 \subseteq \mathcal{T}$ (ii) $\sigma \lhd \mathcal{T}_1\sigma$ (iii) $\sigma \lhd \mathcal{S}_{123} \Rightarrow \sigma \lhd \mathcal{T}_1 \mathcal{S}_{123}$. Le groupe d'autotopie $\mathfrak{A}$ d'un quasigroupe Q est sous-groupe normal d'indice 6, 3, 2 ou 1 de son groupe d'auto-paratopie $\mathfrak{A}_p$ suivant qu'il existe ou non des isotopies projetant Q sur QP^i et seulement sur QP^i, $\forall P^i \in \mathcal{S}_{123}$, A_{123}, C_2 ou P_0 resp. L'ensemble des auto-paratopies IP où P décrit un sous-groupe $\sigma \subseteq \mathcal{S}_{123}$ est un groupe M. $M \lhd \mathfrak{A}_p \rightleftarrows \sigma \lhd \mathcal{S}_{123}$. (Received July 30, 1962.)

62T-307. HAROLD WIDOM, Cornell University, Ithaca, New York. Asymptotic distribution of the eigenvalues of certain integral operators. Preliminary report.

In Euclidean N-space E_N let V(x) be a non-negative function and k(x) an integrable function whose Fourier transform $K(\xi) = \int e^{i\xi \cdot x} k(x)\, dx$ is non-negative. Define $2\pi\sigma(\epsilon)$ to be the measure in $E_N \times E_N$ of the set of pairs (x,ξ) for which $V(x) \cdot K(\xi) > \epsilon$. Then under certain conditions, too complicated to state here, the integral operator with kernel $V(x)^{1/2}\, k(x - y)\, V(y)^{1/2}$ is positive semi-definite and completely continuous and, if its non-negative eigenvalues be $\lambda_1 \geqq \lambda_2 \geqq \ldots$, one has $\lambda_n \sim \sigma^{-1}(n)$ as $n \rightarrow \infty$. This holds in particular under the following hypotheses: V(x) is properly Riemann integrable; $K(\xi)$ is asymptotic (as $|\xi| \rightarrow \infty$) to $\phi(|\xi|)$ where ϕ is a completely monotonic function of one variable satisfying $\phi(r)/\phi(2r) = O(1)$. (Received July 30, 1962.)

62T-308. I. I. HIRSCHMAN, Jr., Washington University, St. Louis 30, Missouri. On a theorem of Szegö.

Szegö has shown that if $f(\theta)$ is periodic with period 2π, satisfies a Lipschitz condition of order greater than 1, and is positive, then the determinant of the section of index N of the Toeplitz matrix associated with $f(\theta)$ divided by the N + 1st power of the geometric mean of $f(\theta)$ approaches an explicitly evaluated limit. This result is extended to sections of matrices constructed using ultra-spherical polynomials, and is similar to Szegö's except that the N + 1st power of the geometric mean must be replaced by the N + 1st power of a weighted geometric mean--the weighing function depending upon N. The proof follows the ideas of Kac, Duke Math. J. 21 (1954), 501-509. (Received August 6, 1962.)

62T-309. I. I. HIRSCHMAN, Jr., Washington University, St. Louis 30, Missouri. Extreme eigenvalues.

H. Widom, Trans. Amer. Math. Soc. 88 (1958), 491-522, has obtained very precise asymptotic estimates for the largest eigenvalues of sections of order N of the Toeplitz matrix associated with a suitable restricted function f(θ). It is shown that Widom's ideas can be extended to the generalizations of Toeplitz matrices associated with a variety of Sturm-Liouville systems and their difference analogues. (Received August 6, 1962.)

62T-310. ALAN COBHAM, Thomas J. Watson Research Center, P. O. Box 218, Yorktown Heights, New York. A method for establishing undecidability.

Let J_1x, J_2x ... be a sequence of formulas of a first order theory T such that for all m, $\vdash_T \bigvee x J_m x$. A function f on the positive integers is strongly definable in T if there is a formula Fxy such that (i) $\vdash_T J_m x \wedge J_{f(m)} y \longrightarrow Fxy$ for all m, (ii) $\vdash J_m x \wedge Fxy \longrightarrow J_{f(m)} y$ for all m, (iii) $\vdash_T J_m x \wedge J_n y \longrightarrow \sim Fxy$ for all m, n with $n \neq f(m)$. If in (ii) "$\vdash$" is replaced by "$\vdash_T$", f is definable in T, the sense being essentially that of Tarski, Mostowski and Robinson, Undecidable Theories ("UT"). Theorem. If all recursive functions are strongly definable in T, and if T is weakly interpretable in S then S is undecidable. (Compare with Corollary 2, p. 49 of UT.) Theorem. If there is a formula Lxy such that $\vdash_T J_m y \longrightarrow (Lxy \longrightarrow \bigvee_{i=1}^{m} J_i x)$ for all m, and if all primitive recursive functions are definable in T then all recursive functions are strongly definable in T. A consequence is that if R (UT, p. 52; axiom scheme Ω_5 is unnecessary) is weakly interpretable in a theory S, S is undecidable. Thus R can replace Q (UT, p. 51) in arguments establishing undecidability. (Received July 16, 1962.)

62T-311. ALAN COBHAM, Thomas J. Watson Research Center, P.O. Box 218, Yorktown Heights, New York. Establishing the nonexistence of proof and disproof procedures.

In some circumstances extensions of the methods of the preceding abstract can be used to obtain a more explicit characterization of the set of valid sentences of an undecidable theory. Theorem. If T has a consistent, finite extension with the same constants in which all recursive functions are strongly definable then T has no disproof procedure, i.e., the set of its nonvalid sentences is not recursively enumerable. Theorem. Let $T_1 \supset T \supset T_2$ where T_1, T, T_2 are consistent and have the same constants and all recursive functions are strongly definable in T_1. Suppose there is a formula Nx such that every consistent, finite extension of T_2 with the same constants has a model in which $\hat{x}Nx = \bigcup_{n=1}^{\infty} \hat{x} J_n x$. Then T has no proof procedure; i.e., T is unaxiomatizable. If, in addition, $T_1 = T = T_2$ then T is nonarithmetic; i.e., the set of its valid sentences is not definable in elementary number theory. (Received July 16, 1962.)

62T-312. ALAN COBHAM, Thomas J. Watson Research Center, P.O. Box 218, Yorktown Heights, New York. Undecidability in group theory.

There is a sequence of formulas, J_1x, J_2x,... of elementary group theory such that in each symmetric group not of degree 6, $\hat{x}J_n x$ is the set of cyclic elements of order n. Relative to this sequence all recursive functions are strongly definable in every extension of the theory of symmetric groups which has only infinite models. Hence, if P is a property of one infinite or of arbitrarily large finite symmetric groups the theory of groups with property P is undecidable. The methods of the preceding abstract provide further classification. Examples: The theories of finitely generated groups and of complete groups have no disproof procedures; the theories of finite groups and of finitely generated periodic groups have no proof procedures; the theories of periodic groups and of locally finite groups are nonarithmetic. (Received July 16, 1962.)

62T-313. HAO WANG, 33 Oxford Street, Cambridge 38, Massachusetts. Tag systems and lag systems.

A tag system is a finite set of rules $s_i \longrightarrow E_i$ such that the first β symbols of a string are deleted and E_i is added at the end if the first deleted symbol is s_i. A lag system looks at the first β symbols but deletes only the first symbol. Both are assumed to be monogenic. Let ϵ and ϵ^- be the maximum and minimum of the length of the E_i's. Theorem 1. Every tag system with $\beta \geqq \epsilon$ or $\beta \leqq \epsilon^-$ is decidable, i.e., whether one string is derivable from another is decidable. Theorem 2. Every lag system with $\epsilon^- \geqq 1$ or $\epsilon \leqq 1$ is decidable. Theorem 3. Every tag or lag system with $\beta = 1$ is decidable. Theorem 4. There is a lag system with $\beta = \epsilon = 2$, whose halting problem (a special case of the derivability problem) is unsolvable. Theorem 5. There is a tag system with $\beta = 2$, $\epsilon = 3$, $\epsilon^- = 1$, whose halting problem is unsolvable. The proof of the last theorem uses methods developed by Cocke and Minsky in giving an undecidable tag system with $\beta = 2$, $\epsilon = 4$. (Received August 8, 1962.)

62T-314. H. E. SALZER, General Dynamics/Astronautics, P. O. Box 1128, San Diego, California. Tables of coefficients for obtaining central differences from the derivatives.

Central differences are expressible in terms of derivatives by (1) $\delta^k f(x_0) = \sum_{q=k,k+2,\ldots}^{n} M_k^q h^q f^{(q)}(x_0) + R_n$ and (2) $\mu\delta^k f(x_0) = \sum_{q=k,k+2,\ldots}^{n} M'^q_k h^q f^{(q)}(x_0) + R'_n$, where M_k^q and M'^q_k are given by the symbolic equations (3) $\delta^k = [2 \sinh(hD/2)]^k = \sum_{q=k,k+2,\ldots} M_k^q (hD)^q$ and (4) $\mu\delta^k = [\cosh(hD/2)]\,[2 \sinh(hD/2)]^k = \sum_{q=k,k+2,\ldots} M'^q_k (hD)^q$. In terms of general Bernoulli polynomials, (5) $M_k^q = B_{q-k}^{(-k)}(-(1/2)k)/(q-k)!$. In terms of central differences of zero, (6) $M_k^q = \delta^k 0^q/q!$. The coefficients M_k^q and M'^q_k, $k = 1(2)29$, $q = k(2)29$ and M_k^q, $k = 2(2)30$, $q = k(2)30$ were calculated exactly, and give 30th (31st) degree accuracy for k odd (even). The calculation, starting from $M_k^k = 1$, $M_1^{2q+1} = 1/2^q(2q+1)!$, $M_2^{2q} = 2/(2q)!$, was based upon a simplified arrangement of (7) $q(q-1) M_k^q = (k^2/4) M_k^{q-2} + k(k-1) M_{k-2}^{q-2}$, and (8) $M'^q_k = [(q+1)/(k+1)]\, M_{k+1}^{q+1}$. Precomputed central differences δ^k, k odd, (otherwise requiring f(x) at mid-interval arguments) would be especially useful in numerical integration based upon (9) $D^{-m} = h^m/[2 \sinh^{-1}(\delta/2)]^m$, m odd, or interpolation by (10) $f(x_0 + ph) = \sum_{k=0}^{n} p^{[k]}\delta^k f(x_0)/k! + E_n$, which employs δ^k for odd and even k. (Received August 8, 1962.)

62T-315. JAN MYCIELSKI, University of California, Berkeley 4, California. A lattice connected with relative interpretability of theories.

Let $\mathcal{T}$ be the class of all consistent theories formalized in the first order predicate calculus. For S, T $\in \mathcal{T}$ put $S \prec T$ if every sentence $\sigma \in S$ has a relative interpretation σ_I in the language of T, in which the formulas interpreting the predicates and the formula $\delta(x_0)$ to which the quantifiers are relativised may contain additional (free variables) $x_1, x_2, \ldots$, such that $\bigvee x_0 x_1 \ldots [\delta(x_0) \wedge \sigma_I] \in T$.

Let $\langle A, \leqq \rangle$ be the partial order $\langle \mathcal{T}, \prec \rangle / \equiv$, where $S \equiv T$ means $S \prec T \wedge T \prec S$, and $|T|$ denotes $T/\equiv$. The following facts hold: 1° each $a \in A$ contains a theory formalized by means of a denumerable language; 2° each subset of A totally ordered by $\leqq$ is at most denumerable and (Montague, J. Symb. Logic 23 (1958), 460) A has the potency of continuum; 3° $\langle A, \leqq \rangle$ is a complete lattice, i.e., the l.u.b. ($\cup$) and g.l.b. ($\cap$) of every subset of A exists; 4° If S and T do not have any common predicates then $|S| \cup |T| = |\{\sigma \wedge \tau : \sigma \in S, \tau \in T\}|$ and $|S| \cap |T| = |\{\sigma \vee \tau : \sigma \in S, \tau \in T\}|$; 5° The infinite distributivity law $a \cap \bigcup_t b_t = \bigcup_t (a \cap b_t)$ holds. Problem: does $a \cup \bigcap_t b_t = \bigcap_t (a \cup b_t)$ hold? (Received August 10, 1962.)

62T-316. J. J. GERGEN, F. G. DRESSEL and W. H. PURCELL, JR., Duke University, Durham, North Carolina. Extension of Bernstein polynomials for the complex plane.

For a function f(x) defined on $0 \leqq x < \infty$ the series $P_k(x;f) = \exp(-kx)\sum_{j=0}^{\infty} (xk)^j f(j/k)/k!$, $0 < k$, form a natural extension of the Bernstein polynomials corresponding to a function defined on [0,1]. The series $P_k(x;f)$ have been considered by Favard [J. Math. Pures Appl. 23], Szász [J. Res. Nat. Bur. Standards 45], and Butzer [Proc. Amer. Math. Soc. 5] for the real case. In this paper the series $P_k(x,f)$ are studied in the complex plane. Results obtained include the following generalization of the well-known theorem of Kantorovitch on the convergence of Bernstein polynomials in an ellipse: If the analytic function f(z), $z = x + iy$, can be represented in the parabolic domain $p(d) = \{z \mid |z| \leqq x + 2d^2\}$, $0 < d$, by a convergent Laguerre series, then $P_k(z;f) \to f(z)$ as $k \to \infty$ in p(d) this convergence being uniform on each compact subset of p(d). The condition on representation by a Laguerre series can be replaced by an order condition on f. Use is made of the work of Szász and Yeardley on Laguerre series [Pacific J. Math. 8]. (Received August 10, 1962.)

62T-317. ECKFORD COHEN, University of Tennessee, Knoxville, Tennessee. Three footnotes to two earlier papers.

In a paper to appear in the American Mathematical Monthly (Notices Amer. Math. Soc. 8 (1961), Abstract 61T-144, p. 277), the author considers two sequences of integers n, defined in terms of the multiplicities of the prime divisors of n. It is shown in the present note that the first of these sequences, $S_{a,b}$, may be characterized as the set of those integers representable in the form $d^a e^b$ A similar result is proved for the second sequence considered and also for their unitary analogues (Acta Arith. 7 (1961), 29-38). It is also shown how the basic identities of the first paper can be obtained without the use of generating functions. (Received August 13, 1962.)

62T-318. D. J. MALLORY and MAURICE SION, University of British Columbia, Vancouver, Canada. On covering systems.

Let X be a topological space; μ be a finite Carathéodory outer measure on X; for each $x \in X$, N(x) be a family of subsets of X; for each $A \subset X$, $\overline{N}(A)$ be the collection of all families F such that for every $x \in A$ and neighborhood U of x there exists $\alpha \in F \cap N(x)$ with $\alpha \subset U$. Suppose $\overline{N}(X)$ is not empty and for every $A \subset X$ and $F \in \overline{N}(A)$, there exists a countable $F' \subset F$ which covers μ-almost all of A. N is a V-system iff F' can be chosen to consist of disjoint μ-measurable sets; N is an S-system with factor λ iff $1 \leqq \lambda < \infty$ and F' can be such that for every B covered by F', $\sum_{a \in F'} \mu(a \cap B) \quad \lambda \cdot \mu B$; N is a T-system iff for every $\epsilon > 0$, F' can be chosen so that $\sum_{a \in F'} \mu a \leqq \mu A + \epsilon$. Theorems: V-systems are always S-systems; with mild conditions on μ, V-systems are T-systems; with stronger conditions, S-systems are T-systems. The converses in general do not hold. If N is a T-system and μ satisfies some regularity conditions, every μ measurable function is approximately continuous w.r.t. (μ, N) μ-almost everywhere and hence a density theorem holds. (Received August 13, 1962.)

62T-319. MAURICE SION and D. K. SJERVE, University of British Columbia, Vancouver 8, B. C., Canada. Approximation properties of measures generated by continuous set functions.

Let X be a compact metric space and τ a monotone increasing non-negative function on the subsets of X, continuous in the standard metric induced by the metric on X. Let $\mu_\delta^{(\tau)}$ for $\delta > 0$ and $\mu^{(\tau)} = \lim_{\delta \to 0} \mu_\delta^{(\tau)}$ be the outer measures generated by the Carathéodory process. The limits of $\mu_\delta^{(\tau)} A$ as any one of the variables A, δ, τ runs over a monotone sequence are studied and a general approximation theorem of an analytic set by a descending sequence of compact sets is established. These are used to prove that, given an analytic set E and a descending sequence τ, if E is not of the form $\bigcup_n A_n$ where A_n has σ-finite $\mu^{(\tau_n)}$-measure then there exist a compact $C \subset E$ and a τ' such that C has non-σ-finite $\mu^{(\tau)}$-measure and $\tau' A/\tau_n A \longrightarrow 0$ as diam $A \longrightarrow 0$ for every n. This yields as corollaries extensions of several known results. (Received August 13, 1962.)

62T-320. G. J. RIEGER, Dahlienstrasse 24, 8012 Ottobrunn, Germany. On the prime ideal theorem with remainder term.

For an arbitrary algebraic number field K, let $\pi_K(x)$ denote the number of all prime ideals of K with norm less than x. Generalizing the Selberg-Erdös approach to the prime number theorem, Shapiro (1949) gave an elementary proof of the prime ideal theorem: $\pi_K(x) = (x/\log x)(1 + o(1))$ $(x \longrightarrow \infty)$. In the present paper, an elementary proof of $\pi_K(x) = (x/\log x)(1 + O(\log^{-c} x))$ $(c > 0, x \longrightarrow \infty)$ is given. A corresponding refinement of the prime number theorem was obtained by several authors (van der Corput (1955), Kuhn (1955), and Breusch (1960)). (Received August 15, 1962.)

62T-321. HUBERT BERENS and P. L. BUTZER, Technical University of Aachen, Aachen, Germany. On the representation of analytic functions as Laplace and Laplace-Stieltjes integrals.

D. V. Widder considered representation theorems expressed in terms of the real Post-Widder inversion operator. The authors establish corresponding results in terms of the complex inversion operator: $F_T(t) = (1/2\pi i)\int_{c-iT}^{c+iT}(1 - |\tau|/T)e^{st}f(s)ds$ $(s = c + i\tau,\ c > 0)$. Let f(s) be analytic in Re $s > 0$. Theorem 1. A necessary and sufficient condition that f(s) can be represented in the form $f(s) = \int_0^\infty e^{-st}dh(t)$ (Re $s > 0$), h(t) being of bounded variation in every neighbourhood of t $(t \geqq 0)$ with $\int_0^\infty e^{-ct}|d\,h(t)| < +\infty$, $c > 0$ arbitrary fixed, is that $\|e^{-ct}F_T(t)\|_1 = O(1)$, all $T \geqq 0$. Theorem 2. One has $f(s) = \int_0^\infty e^{-st}F(t)dt$ (Re $s > 0$), where $e^{-ct}\,F(t) \in L_p(0,\infty)$, $1 < p \leqq \infty$, $c > 0$ arbitrary fixed, iff $\|e^{-ct}F_T(t)\|_p = O(1)$, all $T \geqq 0$. Corresponding results are shown for L_1-space. (Received August 15, 1962.)

62T-322. R. H. ROSEN, University of Michigan, Ann Arbor, Michigan. Neighborhoods of one-simplexes in triangulated manifolds.

Let M be a triangulated n-manifold and e, a one-simplex of M. Let V be the open stellar neighborhood of e in M; V is, of course, an open mapping cylinder over e. Let N be the closed first barycentric neighborhood of e in M; N is a closed submapping cylinder of V. The double of N consists of two copies of N with boundaries identified in a canonic manner. Theorem. The double of N is homeomorphic to S^n. (Received August 20, 1962.)

62T-323. E. G. K. LOPEZ-ESCOBAR, 1337 Shattuck Avenue, Berkeley 9, California. Interpolation theorem for an infinitary language.

$L_{\omega_1,\omega}$ is the first order infinitary language whose formulas are obtained from simpler ones by means of forming (i) the negation $\neg\phi$ of a formula ϕ, (ii) the existential quantification $\bigvee x\phi$ of a formula ϕ with respect to an individual variable x, (iii) the disjunction $\sum_{\xi<\delta}\phi_\xi$ of a countable sequence of formulas $\langle\phi_\xi\rangle_{\xi<\delta}$. If A, B are countable sequences of formulas from $L_{\omega_1,\omega}$, then the formal expression $A \Longrightarrow B$ is called a sequent. Using a natural extension of the postulates for Gentzen's formal system (see S. C. Kleene, Introduction to Metamathematics, North-Holland Publishing Co.) it can be shown that, Theorem I. If a sequent is valid, then it is provable without cut. Theorem II. If the sequent $\langle\phi\rangle \Longrightarrow \langle\eta\rangle$ is provable and the formulas ϕ and η have at least one predicate symbol in common, then there is a formula π whose predicate symbols (free individual variables) are those that occur (occur free) in both ϕ and η. Corollary. If the sequent $\langle\phi\rangle \Longrightarrow \langle\eta\rangle$ is provable and the formulas ϕ and η have no predicate symbol in common, then either $\langle\phi\rangle \Longrightarrow$ or $\Longrightarrow \langle\eta\rangle$ is provable. (Received August 20, 1962.)

62T-324. R. B. DARST, Purdue University, West Lafayette, Indiana. Weak convergence of finitely additive measures on a sigma algebra.

The purpose of this paper is to list three corollaries of a recent result of T. Andô's (Pacific J. Math. 11 (1961), 395-404). These corollaries extend well known results on weak convergence of sequences of countably additive measures on a sigma algebra B of subsets of a set I to the case of a sequence $\{u_k\}_{k=0}^{\infty}$ of bounded and finitely additive measures on B. (1,2) $\{u_k\}$ is weakly convergent $\Longleftrightarrow$ $\lim u_k(E)$ exists (and is finite) for every $E \in B \Longleftrightarrow \lim \int_I f du_k$ exists for every bounded function f on I that is measurable with respect to B. Moreover (3), if there exists a sequence $\{f_k\}_{k=1}^{\infty}$ of bounded B-measurable functions on I such that $u_k = \int f_k du_0$ for $k \geq 1$, then $\{u_k\}$ converges in norm $\Longleftrightarrow$, (i) $\{f_k\}$ converges in u_0-measure and (ii) $\lim \int_E f_k du_0$ exists for every $E \in B$. (Apply Theorems IV. 9. 12 and III. 3. 6 of Linear operators, Vol. 1, by Dunford and Schwartz, to Andô's result.) (Received August 27, 1962.)

62T-325. T. G. McLAUGHLIN, University of California, Los Angeles 24, California. An extension of a result of Friedberg.

The theorem of R. Friedberg, that any essentially r.e. ("e.r.e.") set is the disjoint sum of two e.r.e. sets, extends, by a straightforward modification of Friedberg's argument, to the result that an e.r.e. set is a decidably indexed infinite sum of pairwise-disjoint e.r.e. sets. (A proof using neither Friedberg's approach nor the Myhill isomorphism theorem is available for creative sets, and gives some added properties for that case.) One quickly gets some facts (involving uniform creativity in the creative case) about r.e. chains of e.r.e. sets, whose adjacent terms are (i) separated by infinite relative differences, and (ii) in certain relationships of "effective covering." (Reference to "decidable indexing" is simply to existence of a recursive set of indices.) We take the present opportunity to remark on an oversight: the second theorem of abstract 62T-126 (these Notices, June, 1962) already follows, without assuming range (f) $\cap \tilde{\alpha}$ r.e., from proofs (in the literature) of certain theorems of Myhill. If the conclusion is strengthened to read "all of range (f) $\cap \tilde{\alpha}$ is a contraproductive center for α", then that assumption becomes operative again, and our proof is more elementary than that which follows from the aforementioned proofs. (Received August 27, 1962.)

62T-326. G. E. SACKS, Cornell University, Ithaca, New York. An interpolation theorem for recursively enumerable degrees.

By degree is meant degree of recursive unsolvability. A degree is called recursively enumerable if it is the degree of a recursively enumerable set. Theorem. If b and c are recursively enumerable degrees such that $b < c$, then there exists a recursively enumerable degree d such that $b \leqq d < c$ and $d' = c'$. Corollary. If b and c are recursively enumerable degrees such that $b < c$ and $b' < c'$, then there exists a recursively enumerable degree d such that $b < d < c$. (Received August 31 1962.)

62T-327. ECKFORD COHEN and R. L. ROBINSON, University of Tennessee, Knoxville, Tennessee. On the distribution of the k-free integers in residue classes.

Let k, r, h denote non-negative integers, $h \geqq 1$, $k > 1$, and let $Q_k(x; h,r)$ denote the number of k-free integers $\leqq x$ which are $\equiv r \pmod{h}$. This paper is concerned with generalizations and refinements, valid for arbitrary k, of Landau's asymptotic estimates for $Q_2(x; h,r)$ (Handbuch der Primzahlen, II, pp. 633-636). As an application, it is proved that the k-free integers are equidistributed (mod h) if and only if h is k-full, that is, if each prime divisor of h has multiplicity $\geqq k$. Application is also made to the problem of the number of representations of an integer as a sum of a j-free integer and a k-free integer. The results of this paper reveal that the formulas for $Q_k(x; h,r)$ as stated by Ostmann are erroneous (Additive Zahlentheorie, II, p. 23). Furthermore, the paper shows that the role played by the Euler totient $\phi(n)$ in the Landau formulas is special to the case $k = 2$; in the general case, it is the unitary analogue of $\phi(n)$ that is fundamental. (Received August 27, 1962.)

62T-328. HOWARD LEVI, 784 Columbus Avenue, New York 25, New York. Affine geometry via parallel projections.

Plane affine geometry is developed from four axioms. The first three are also the first three given in Artin (Geometric algebra, Interscience, New York, 1957, Chapter II.) They make it possible to define parallel projection from a line to a line, and to prove that if A and B are distinct points and if A' and B' are distinct points then there is a map from line AB to line A'B' under which A goes to A' and B goes to B' and which is the composition of parallel projections. Our fourth axiom asserts that this map is unique. We show that these axioms characterize the usual affine number plane relative to a (not necessarily commutative) field. A fifth axiom, a variant of Pappus' theorem, guarantees the commutativity of the field. It is: If f,g and h are parallel projections from a line l to a line l' then $fg^{-1}h = hg^{-1}f$. (Received August 27, 1962.)

62T-329. JOHN WERMER, Brown University, Providence, Rhode Island. The space of real parts.

Let X be a compact Hausdorff space. Denote by C(X) the class of all complex-valued continuous functions on X. Let A be any closed subalgebra of C(X), separating points on X and containing the constants. Put Re A = $\{\mathrm{Re}\, f \mid f \in A\}$. Theorem. If Re A is a ring, A = C(X). This result was conjectured by K. Hoffman, and its proof uses earlier work by K. Hoffman and the author. Corollary. There exists a continuous real u on $|z| = 1$ having a continuous conjugate, but such that u^2 fails to have a continuous conjugate. This had been proved last year by J. P. Kahane (unpublished). (Received August 28, 1962.)

62T-330. R. J. PARIKH, Stanford University, Stanford, California. Some generalisations of the notion of well ordering. Preliminary report.

Let $\alpha(a,b) \equiv a \leq b$ (in α) be a linear ordering over some subset of the integers. If A is a class of number theoretic functions, then we will say that α is A-founded if α has no infinite descending sequences belonging to the class A. Let P(R) denote the class of primitive (general) recursive functions. We shall consider recursive linear orderings α, β, for which operations $\alpha + \beta$, $\alpha \cdot \beta$ (α taken β times) α^β can be defined uniformly. (i) α is R-founded iff every recursive nonempty subset of the field of α has a least element. (ii) If α, β are R-founded and γ is a well ordering, then $\alpha + \beta$, $\alpha \cdot \beta$ and α^γ are R-founded. (iii) There exists an R-founded α such that 2^α is not R-founded. (iv) If α, β are P-founded and γ is a well ordering, then $\alpha + \beta$, $\alpha \cdot \gamma$ are P-founded. (v) There exist primitive recursive β and γ such that β is P-founded and γ is a well ordering of type ω such that β^2 and $\gamma \cdot \beta$ are not P-founded. (Received August 31, 1962.)

62T-331. NATHANIEL COBURN, University of Michigan, Ann Arbor, Michigan. Simple waves in relaxation hydrodynamics. I

In this paper, the general nonlinear theory of simple waves in relaxation hydrodynamics is formulated. The existence of such waves for the linearized, steady, nonmagnetic flow past a wedge has been shown, (Stakhanov, I. P. and Stupochenko, E. V., Dokl. Akad. Nauk SSSR, 134 (1960) No. 5). By use of the equation for characteristic manifolds (Coburn, N., The limiting speeds of characteristics in relaxation hydrodynamics, J. Math. Anal. Appl., to appear) the equations for the bicharacteristics are obtained. It is shown that when the local sound speed and other scalars are replaced by new invariants, then most of the general theory of relaxation hydrodynamics is similar to that of equilibrium hydrodynamics. Thus, in equilibrium hydrodynamics, simple waves do not exist for non-isentropic, one-dimensional, non-steady flows (Germain, P. and Gundersen, R., C. R. Acad. Sci.

Paris, 241, 925, 1955). However, in this type of flow, simple waves always exist in the nonmagnetic case. Further, if energy is withdrawn from the system then: (1) in the supersonic case, the particle velocity decreases but density increases; (2) in the subsonic case, the particle velocity increases but density decreases. (Received September 4, 1962.)

62T-332. SHMUEL KANTOROVITZ, Princeton University, Princeton, New Jersey. On the characterization of spectral operators.

Let B(X) be the Banach algebra of all bounded linear operators on the reflexive Banach space X into itself; $S \in B(X)$ is called pseudo-hermitian (p.h.) if it is spectral of scalar type with real spectrum (Cf. Dunford, Bull. Amer. Math. Soc. 64 (1958), 217). We characterize p.h. operators by certain properties of the group $\{e^{-2\pi it\,S};\ t \text{ real}\}$. Let C(R) be the space of all bounded continuous complex-valued functions on the real line R with sup norm. For $N > 0$, $u \geqq 0$, $s \in R$ and $S \in B(X)$, we write $G_N(s,u) = \int_R \exp\pi[-(t/N)^2 - 2u|t| + 2its] \cdot e^{-2\pi itS}dt$. Theorem. S is p.h. if and only if (a) $\|e^{-2\pi itS}\| \leqq M < \infty$ for all $t \in R$; (b) for every $f \in C(R)$ and $u \geqq 0$, $\lim_{N\uparrow\infty}\int_R f(s)G_N(s,u)ds = B(f;u)$ exists in the weak operator topology, is in B(X) and $\|B(f;u)\| \leqq K\|f\|$ $(K = K(u))$; (c) for $f(s) = e^{-2\pi ist}$ $(t \in R)$, $B(f;\ )$ is right continuous at $u=0$. Unlike the general theorems of Dunford about the spectrality of an operator, the theorem above is easily applied to analytically given operators on concrete spaces. (Received September 4, 1962.)

62T-333. SHMUEL KANTOROVITZ, Princeton University, Princeton, New Jersey. Operational calculus in Banach algebras for algebra-valued functions.

Let A be a Banach algebra with unit. For x in A, let F_x be the algebra of all A-valued functions of a complex variable locally holomorphic in an open neighborhood of the spectrum $\sigma(x)$ of x. The map $f \to f(x)$ of F_x into A is defined by $f(x) = (2\pi i)^{-1}\int_C f(\lambda)x(\lambda)d\lambda$, where C is suitably chosen and $x(\cdot)$ is the resolvent of x. Theorems. (1) the map of $f \to f(x)$ is multiplicative if and only if $x \in$ center (A); (2) if $f \in F_x$ is such that x and f(x) commute, then $\sigma(f(x)) \subset \bigcup\{\sigma(f(\lambda));\ \lambda \in \sigma(x)\}$. Let g be an A-valued function locally holomorphic in an open neighborhood of the closure of the latter set, and let $h(\lambda) = g(f(\lambda))$. Then $h(x) = g(f(x))$. (3) for $x, y \in A$, we have $(x + y)(\lambda) = (2\pi i)^{-1}\int_C x(\lambda - \mu)y(\mu)d\mu$ for all $\lambda \notin \sigma(x) + \sigma(y)$ and suitable C if and only if x and y commute. Similarly $(xy)(\lambda) = (2\pi i)^{-1}\int_C x(\lambda\mu^{-1})y(\mu)d\mu/\mu$ for $\lambda \notin \sigma(x)\sigma(y)$. Using (3), a theorem of Foguel (Ark. Math. 3 (1957), 449-461, Theorem 7) is improved. (Received September 4, 1962.)

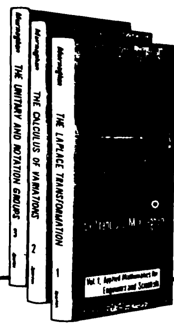
A DEFINITIVE WORK . . .

APPLIED MATHEMATICS

FOR ENGINEERS AND SCIENTISTS

By FRANCIS D. MURNAGHAN
Consultant, Applied Mathematics Lab.
The David Taylor Model Basin

Completely self-contained with detailed proof of all basic mathematical theorems



IN THREE VOLUMES:

Volume I — The Laplace Transformation
Volume II — The Calculus of Variations
Volume III — The Unitary and Rotation Groups

The three volumes comprising this outstanding treatise are based on lectures given by the author at the Applied Mathematics Laboratory of the U. S. Navy Department's David Taylor Model Basin, Washington, D. C.

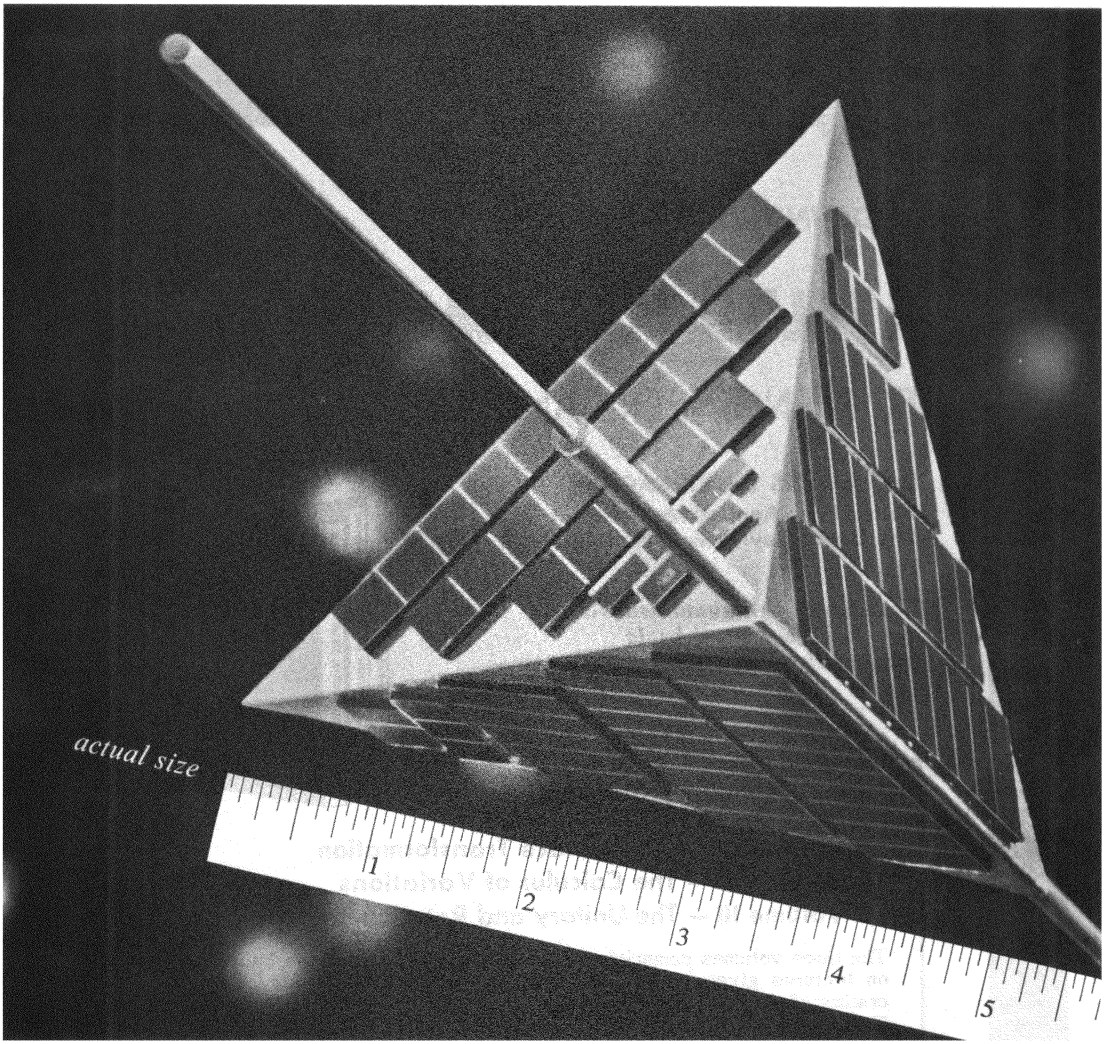
This definitive work—for workers in the fields of hydrodynamics, aerodynamics, structural mechanics, electronic engineering and computer technology as well as graduate and undergraduate students—has been carefully re-edited by the author, and is presented by Spartan Books in an attractive, durable format.

In all three volumes—each of which is also available separately—care has been taken to make the treatment completely self-contained by giving detailed proofs of the basic mathematical theorems. Examples and exercises are given in nearly every chapter, and the sequence of chapters has been planned to make the volumes suitable for seminar use, classroom lectures, refresher courses, or home study.

1962 ● Cloth, three volumes, ... $16.00
Each volume also available separately at . . $5.75

SPARTAN BOOKS

6411 CHILLUM PLACE, N.W. ● WASHINGTON 12, D.C.


actual size
1
2
3
4
5

INDEX TO ADVERTISERS

RESERVATION FORM

Evanston Meeting

November 23 and 24, 1962

Send this form directly to

Hotel Pick-Georgian
422 Davis Street
Evanston, Illinois

Please reserve accommodations for

Name ____
Please print

Institution ______________________ Address ______________________

Arrival date __________ Time __________ : Departure date __________ Time __________

Check room desired

Single room __________ Special rate $8.50

Double room __________ Special rate $12.00

AMERICAN MATHEMATICAL SOCIETY
190 Hope Street
Providence 6, Rhode Island

SECOND-CLASS POSTAGE
PAID AT
PROVIDENCE, RHODE ISLAND
AND
ANN ARBOR, MICHIGAN

Return Requested