

OF THE
AMERICAN

## MATHEMATICAL

## SOCIETY



OF THE

## AMERICAN MATHEMATICAL SOCIETY

Edited by John W. Green and Gordon L. Walker

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## MEETINGS

## Calendar of Meetings

NOTE: This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the $\mathcal{C}$ (otices 1 was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

| Meeting No. | Date | Place | Deadline for <br> Abstracts* |
| :---: | :---: | :---: | :---: |
| 621 | April 9-10, 1965 | Chicago, Illinois | Feb. 24 |
| 622 | April 12-15, 1965 | New York, New York | Feb. 24 |
| 623 | April 24, 1965 | Stanford, California | Feb. 24 |
| 624 | June 19, 1965 | Eugene, Oregon | May 5 |
| 625 | August 30 - September 3, 1965 (70th Summer Meeting) | Ithaca, New York | July 9 |
| 626 | October 30, 1965 | Cambridge, Massachusetts |  |
| 627 | November 12-13, 1965 | Lexington, Kentucky |  |
| 628 | November 26-27, 1965 | Iowa City, Iowa |  |
| 629 | December 29, 1965 | Berkeley, California |  |
| 630 | January 24-28, 1966 (72nd Annual Meeting) | Chicago, Illinois |  |
|  | August 29 - September 2, 1966 (71st Summer Meeting) | New Brunswick, New Jersey |  |
|  | January 24-28, 1967 <br> (73rd Annual Meeting) | Houston, Texas |  |
|  | August 28 - September 1, 1967 <br> (72nd Summer Meeting) | Toronto, Ontario, Canada |  |
|  | August 26-30, 1968 <br> (73rd Summer Meeting) | Madison, Wisconsin |  |

* The abstracts of papers to be presented in person at the meetings must be received in the Headquarters Offices of the Society in Providence, Rhode Island, on or before these deadlines. The deadlines also apply to news items. The next two deadline dates for the by title abstracts are April 28, and July 2, 1965.


The $\mathcal{C}$ (otices) of the American Mathematical Society is published by the Society in January, February, April, June, August, October and November. Price per annual volume is $\$ 7.00$. Price per copy $\$ 2.00$. Special price for copies sold at registration desks of meetings of the Society, $\$ 1.00$ per copy. Subscriptions, orders for back numbers (back issues of the last two years only are available) and inquiries should be addressed to the American Mathematical Society, 190 Hope Street, Providence, Rhode Island 02906.

Second-class postage paid at Providence, Rhode Island, and additional mailing offices. Authorization is granted under the authority of the act of August 24, 1912, as amended by the act of August 4, 1947 (Sec. 34, 21, P. L. and R.). Accepted for mailing at the special rate of Postage provided for in section 34,40, paragraph (d).

# Six Hundred Twentieth Meeting City College New York, New York February 27, 1965 

## $\overline{\overline{\text { PROGRAM }}}$

The six hundred twentieth meeting of the American Mathematical Society will be held at City College on Saturday, February 27 , 1965. All sessions will be in Shepard Hall.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings there will be an address in Room 306 of Shepard Hall at 2:00 P.M. Professor G. Washnitzer of Princeton University will speak on "Tempered Cohomology."

There will be sessions for contributed papers on Saturday, both morning and afternoon. There may be provision for a limited number of late papers.

The registration desk will be located in Shepard Hall at Convent Avenue and 139th Street. It will be open from 9:00 A.M. till 3:30 P.M.

Lunch will be available in a college cafeteria. This may be almost the only
place in the immediate vicinity for lunch.
Shepard Hall is two blocks east of the 137th Street Station of the IRT subway (the Broadway-7th Avenue Line, not the 7th Avenue Line). Also it is one block west and five blocks south of the 145th Street station of the IND subway (8th Avenue Line "A" train or 6th Avenue Line "D" train from mid-town New York).

Buses marked "Broadway-230th Street" or "Fort George" may be taken from the 125 th Street station of the New Haven or New York Central R. R. Riders should get off at l38th Street and walk east one block to Convent Avenue.

Persons who expect to travel by automobile may write to the Department of Mathematics, City College, New York 31, New York before the meeting date, requesting a one-day campus parking permit and directions to the parking area.

## PROGRAM OF THE SESSIONS

The time limits will be strictly enforced and the absolute times will be closely followed.

SATURDAY, 10:00 A.M.
First Session, Shepard Hall Room 306
10:00-10:10
(1) Bounds for moduli of rings in 3-space

Professor G. D. Anderson, University of Michigan and Eastern Michigan University (620-1)
10:15-10:25
(2) Differential-integral calculus for abstract algebraic-topological structures Mr. R. M. Sorensen, Trinity College (620-17)


[^0]```
4:30-4:40
    (14) A sufficient condition for absolute continuity in the transformation theory for
        measure space
            Dr. R. W. Chaney, Western Washington State College (620-21)
                SATURDAY, 3:15 P.M.
Third Session, Shepard Hall Room 315
    3:15-3:25
            (15) Invariance of open sets in generalized manifolds
                Professor R. G. Lintz, University of Michigan (620-4)
3:30-3:40
            (16) Still on approximation theorems. Preliminary report
                Professor S. G. Mrowka, The Pennsylvania State University (620-12)
3:45-3:55
            (17) On Moessner's theorem on integral powers
                Professor C. T. Long, Washington State University (620-2)
4:00-4:10
            (18) The number of partitions of an integer
                    Mr. N. P. Salz, Cornell Aeronautical Laboratory, Buffalo, New York
                    (620-10)
4:15 - 4:25
            (19) Divisibility in semigroups of n }\timesn\mathrm{ unimodular matrices
                    Professor R. J. Wisner, New Mexico State University (620-5)
    4:30-4:40
            (20) A theorem on purely inseparable extensions
                    Dr. Morris Weisfeld, Aerospace Corporation, San Bernardino, California
                    (620-15)
    4:45 - 4:55
            (21) On the stability of difference schemes with singular coefficients
                    Mr. Dennis Eisen, Adelphi University (620-16)
                    Bethlehem, Pennsylvania
```


## PRELIMINARY ANNOUNCEMENTS OF MEETINGS

Six Hundred Twenty-First Meeting University of Chicago Chicago, Illinois April 9-10, 1965

The six hundred twenty-first meeting of the American Mathematical Society will be held at the University of Chicago on April 9 and 10, 1965.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, there will be addresses by Professor Cassius Ionescu Tulcea of the University of Illinois, by Professor James M. Kister of the University of Michigan and a special session of 20 minute papers on "Harmonic Analysis" under the chairmanship of Professor Ralph Boas of Northwestern University. There will probably be four to six sessions of contributed papers.

No papers will be presented at Eckhart Hall. All papers will be presented about a half mile to the southeast at the new University of Chicago Conference Center, officially entitled, "The Center
for Continuing Education," and located at 1307 East 60th Street. Get off the I. C. at 59th Street.

Rooms will be available at the Center. A reservation form is enclosed at the back of these $\mathcal{C}$ (otices). In the event of an overflow, the Center undertakes to place people at nearby hotels, and the hotel in question will, in that case, confirm the reservation. All meals will be served at the Center and there is a bar which opens daily at 11:30 A.M. The official at the Center in charge of the meeting is Mr. B. Berlin, and inquiries pertaining to the Center may be directed to him.

There will be a registration fee (no charge for students).

Seymour Sherman
Associate Secretary
Bloomington, Indiana

## Six Hundred Twenty-Second Meeting W aldorf-Astoria Hotel New York, New York April 12-15, 1965

The six hundred twenty-second meeting of the American Mathematical Society will be held at the Waldorf-Astoria in New York on April 12-15, 1965.

Contributed papers and invited addresses will be scheduled on Monday, April 12 and on the morning of Tuesday, April 13.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, Professor James Eells,

Jr. of Cornell University and Professor Robert D. M. Accola of Brown University each will address the Society. Professor Eells will speak on "A setting for global analysis."

Abstracts of contributed papers should be sent to the American Mathematical Society, 190 Hope Street, Providence, Rhode Island 02906, so as to arrive prior to the deadline of February 24.

There will be a Symposium on Magneto-fluid and Plasma Dynamics on the afternoon of Tuesday, April 13 and on Wednesday and Thursday, April 14 and 15.

The subject was chosen by the Committee on Applied Mathematics, which consisted of V. Bargmann, G. E. Forsythe, P. R. Garabedian, C. C. Lin, Alfred Schild, Chairman, and David Young.

Financial support comes from the Air Force Office of Scientific Research and the U. S. Army Research Office (Durham).

The Invitations Committee, responsible for the planning of the program and the choice of speakers, consists of

Professor Harold Grad, Chairman New York University
Dr. Andrew Lenard, Princeton Plasma Physics Laboratory
Professor Marshall N. Rosenbluth
University of California at La Jolla
Professor William R. Sears, Cornell University
Professor Harold Weitzner New York University

The principal topics to be discussed are the following:

Plasma equations
Stability
Propagation of waves and shocks Macroscopic flows

The speakers will include:
M. Trocheris, Association Euratom P. Germain, Office Nationale d'Etudes et de Recherche Aéronautique
E. A. Frieman, Princeton Plasma Physics Laboratory
John M. Greene, Princeton Plasma Physics Laboratory
C. K. Chu, Columbia University
G. S. S. Ludford, Cornell University
W. V. R. Malkus, Woods Hole Institute of Oceanography
N. Rostoker, University of California at La Jolla
as well as some of the members of the Invitations Committee.

The Waldorf-Astoria occupies an entire City block on the east side of New York City, from 49 th to 50th Street and from Lexington to Park Avenues.

Those arriving by train at Pennsylvania Station may take the Independent Subway System (E or F cars) to the 53rd Street and Lexington Avenue stop, a short walk from the hotel.

From Grand Central Station one may take the I.R.T. Lexington Avenue local subway to the 5 lst Street stop.

Those arriving by bus may take the Independent Subway System ( E or F cars) from the west side bus terminal. There is a shuttle bus service from LaGuardia and Kennedy Airports to The East Side Terminal with a transfer bus to Grand Central Station. (It is suggested that those arriving in a group of three or more may find it as economical to take a taxi directly to the hotel.)

Those arriving at Newark Airport can take a shuttle bus to the west side terminal and use the Independent Subway System (E or F cars) to the 53rd Street stop.

Those arriving by car will find many parking facilities in the neighborhood in addition to those at the hotel. Pickup and delivery service can be arranged through the hotel at a cost of $\$ 3.25$ for a 24 -hour period, plus $\$ 1.25$ for each pickupdelivery.

## RESERVATIONS

Persons intending to stay at the Waldorf-Astoria should make their own reservations with the hotel. Areservation blank and a listing of room rates are on the inside back cover of this issue of these $\mathcal{C}$ (otices).

Everett Pitcher Associate Secretary
Bethlehem, Pennsylvania

# Six Hundred Twenty Third Meeting Stanford University Stanford California April 24, 1965 

The six hundred twenty-third meeting of the American Mathematical Society will be held on Saturday, April 24, 1965 at Stanford University, Stanford, California.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, there will be addresses by Professor E. G. Straus of the University of California, Los Angeles, and by Professor Harvey Cohn of the University of Arizona. Professor Straus will speak at 11:00 A.M. in Room 100 of the Physics Lecture Hall on the "Arithmetic of analytic functions". The title of Professor Cohn's talk is "Some elementary aspects of modular functions in several variables". This address will be given at 2:00 P.M. in Room 100, Physics Lecture Hall.

There will be sessions for contributed papers at 9:30 A.M. and at 3:30 P.M. in the Alfred P. Sloan Mathematics Center. Registration for the meeting will take place in the lobby of the Sloan Mathematics Center, beginning at 9:00 A.M.

A tea for persons attending the meeting will be given in the Mathematics

Library on the fourth floor of the Sloan Mathematics Center at 4:30 P.M. Luncheon will be available at noon in the Old Union.

Stanford University is about thirty miles south of San Francisco, adjacent to the town of Palo Alto. The Southern Pacific Railroad stops at Palo Alto. Limousine service from the San Francisco International Airport to the Hotel President in Palo Alto is available. It is recommended that taxis be used to get from Palo Alto to the Stanford Campus.

There are two hotels in Palo Alto and numerous motels on the El Camino Real near the Stanford Campus. A complete list of hotels and motels within five miles of the University, giving location and rates, can be obtained from the Palo Alto Convention and Visitors Bureau, P.O. Box 1321, Palo Alto, California.

R. S. Pierce<br>Associate Secretary

Seattle, Washington

## MEMORANDA TO MEMBERS

## LIST OF <br> RETIRED MATHEMATICIANS

In February, the latest compilation of names and qualifications of retired mathematicians available for employment will be published by the Mathematical Sciences. Employment Register. Free copies of the list may be obtained by addressing a request to the Mathematical Sciences Employment Register, 190 Hope Street, Providence, Rhode Island 02906.

The Register is sponsored jointly
by the American Mathematical Society, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics.

## ERRATA

The title of the doctorate thesis by Spencer E. Dickson listed on page 507 of these CNotices) should read "Torsion theories for abelian categories."

# ACTIVITIES OF OTHER ASSOCIATIONS 

## CANADIAN MATHEMATICAL CONGRESS <br> 1965 SEMINAR AND CONGRESS

The Tenth Biennial Seminar and Seventh Congress of the Canadian Mathematical Congress will be held at Laval University, Quebec, during the period August 16 to September 8. The Seminar will be held from August 16 to September 3 with lectures beginning on August 16 and the Congress from September 5 to September 8 with registration on Sunday, September 5.

## General Program:

The theme of the Seminar will be Functional Analysis but the program will include other topics. A symposium on Numerical Analysis will be held during the Congress meetings and the four research lecturers will be invited to give lectures on specialized mathematical topics or more general topics. Various social events will be held during the Seminar and Congress.

## Research Lectures:

N. Aronszajn, University of Kansas

Spectral problems, finite dimensional perturbations and variational approximation methods.
J. Dixmier, Faculté des Sciences, Paris

Calcul fonctionnel dans les algèbres de Banach.
E. Hewitt, University of Washington, Seattle

The Measure Algebra of a locally compact Abelian Group.
F. Smithies, St. John's College, Cambridge

Some Problems in Banach Space Theory.
Professor I. M. Gelfand of Russia has also been invited to be a lecturer.

## Instructional Lectures:

C. Davis, University of Toronto

Analyse fonctionnelle et approximation des fonctions.
P. G. Rooney, University of Toronto

Fourier and Hilbert Transformations and Generalizations.
M. Sion, University of British Columbia

Measure Theory.
S. Zaidman, Université de Montréal

Introduction à la théorie des distributions d'après L. Hormander.

## Accommodation

Excellent residential accommodation and dining room service will be available at reasonable rates during the entire period.

Further information can be obtained from the Secretariat at the following address:

Canadian Mathematical Congress
985 Sherbrooke Street West, Montreal, Quebec, CANADA.

## LETTERS TO THE EDITOR

Editor, The $\mathcal{C}$ (otices)
In the CNotices) 11 (1964)639-640 an announcement was made concerning the activities of the Summer Research Institute on Algebraic Geometry which was held in July 1964 with financial support of the Air Force Office of Scientific Research, the Office of Naval Research and the U.S. Steel Foundation. One paragraph announced that "mimeographed copies of the fifteen formal lectures are collected in a paperbound volume of the proceedings of the Institute and in addition, brief summaries of the seminars are being collected in a second and smaller volume. Copies of the lecture notes have been sent to the supporting agencies, to the participants, and to about 90 other persons interested in algebraic geometry, but there are none available for further distribution. No formal publication of the proceedings of the institute is planned...". (Emphasis supplied.)

I wish to call attention to the underlined portions of this statement and to its consequences to the mathematical community. First, it should be said that this practice of making available only a limited number of copies, of proceedings at publicly supported conferences which are not generally open to the mathematical public, is not uncommon. What this means is that these notes are not generally available at libraries throughout the country nor are they available to faculty and students who have an interest in these matters unless they had friends among the participants who requested that copies be sent them. This has the consequence that the dissemination of this information is curtailed and delayed both to the disadvantage of those on the "outside" and to the progress of mathematics itself. Although it is true that the more significant results will be embodied in research papers published elsewhere, this does not detract from the argument that the time lapse may be significant. Nor does it meet the objection
that well developed expository talks may find no subsequent publication.

It would appear that the governmental agencies supporting such conferences have an obligation to encourage the most wide spread and the quickest dessemination of the notes of such institutes that they support. By this I do not mean that they need require the preparation of such notes; but once these notes have been prepared there is no excuse for a limited circulation. It must surely be possible for the AFOSR, ONR, NSF and others to arrange for an agency such as the Office of Technical Services to keep such notes available for a period of several years in the same way as it does for the reports of various government agencies and contractors.

My point is, simply, that the supporting agencies should adopt this as a normal operating procedure. Such a practice will enhance the benefits to be derived from public funds and hence is in the public interest.

## Lowell Schoenfeld

Editor, The $\mathcal{C}$ Notices)
We, the undersigned, would like to have the American Mathematical Society offer its services to the Library of Congress in the matter of classification of the new books which are being published.

It is quite apparent, from a casual glance at the geometry section of the classification used by Library of Congress, that this system is woefully out of date. As the system is presently used, all topology books are classified QA 611, while there are five categories for books in trigonometry. It would be nice to have a completely new system. However, until this system comes into being, we can help librarians classify our books as effectively as possible with the present system. We
all know examples where the librarians have made mistakes. We submit that with the system as it presently stands, mathematicians would not always find agreement among themselves as to the classification of books.

In order to understand the difficulties, we propose that the readers of this letter try and classify the following recent books:

Lectures on Lie group and Lie algebras, by G. D. Mostow
Representation theory of Lie algebras, by G. Hochschild
Discontinuous subgroups of classical groups, by A. Weil
Convex sets, by F.A. Valentine
Global theory of representations of locally compact groups, by V.S. Varadarajan
Sheaf theory, by C. H. Dowker
Foundations of fiber bundles, by $S$. Eilenberg

General topology, by J. L. Kelley Morse theory, by J. W. Milnor

Signed:

Richard Askey
M. N. Bleicher

Fred Brauer
R. C. Buck
S.H. Coleman
C. C. Conley
D. W. Crowe

Frank Forelli
S. Y. Husseini
H. Jerome Keisler
S. C. Kleene Frank Kosier Seymour Parter Walter Rudin
Robert D. Ryan
J. R. Smart
K. T. Smith
R.E.L. Turner

Michael Voichick
L. C. Young

Editorial Note: The Council has approved in principle the idea of participation by the Society in the revision of the Library of Congress classification system. Pressure of other business prevented the Secretary from implementing this participation in time for the most recent revision, but it is anticipated that the Society will be able to render assistance on the next revision.

# NEWS ITEMS AND ANNOUNCEMENTS 

## INSTITUTE IN HOMOLOGICAL ALGEBRA AND ITS APPLICATIONS

From June 22 to August 12, 1965, Bowdoin College will hold a summer institute combining a research program for postdoctoral mathematicians with advanced instruction for graduate students. This program is financed by the National Science Foundation. For the graduate students the central formal offering will be a course in Homological Algebra given by Professor Ernst Snapper of Dartmouth College. The research program will center on a Colloquium in Homological Algebra and Its Applications at which will appear a sequence of distinguished visiting speakers, including H. Bass, P. Freyd, G. Hochschild, D. S. Rim, and others.

Departments at Ph.D. granting institutions are invited to nominate appropriate graduate students by providing a letter of endorsement from the chairman
or from the candidate's expected dissertation advisor. When more than one nomination is made, the candidates should be ranked. Candidates will be selected on the basis of (1) endorsement by a graduate department, (2) proven ability as evidenced by previous record, and (3) a statement of reasons for entering this particular program. Postdoctoral candidates should similarly provide an academic vita, a list of publications or research in progress, an endorsement by an established mathematician, and a statement of purpose.

For further details concerning procedures, stipends, dependency and travel allowances, housing, academic facilities, etc., candidates should address: Professor Dan E. Christie, Chairman, Department of Mathematics, Bowdoin College, Brunswick, Maine 04011.

MEMORANDA TO MEMBERS

## INCREASE IN PAGE CHARGES

The change in individual membership dues proposed for 1966 represents the first dues increase in 15 years. It is essential, of course, to keep dues to a minimum, and so the Board of Trustees has found it necessary to re-examine all sources of income to the Society from publication charges, subscriptions, and other fees to insure that each form of income was adequate for its purposes. Thus, the Trustees have been led to a reassessment of publication charges. The practice of levying page charges is discussed in a report made by the Society committee consisting of L. J. Paige (Chairman), W. T. Martin, and A. Rosenberg, who were charged in January, 1962 with studying the means of financing mathematical journals. The report was not confined to Society journals. It was presented in the summer of 1964 to editors of mathematical journals, heads of mathematical departments, the Office of Scientific Information Service of the NSF, and members of the governing bodies of the Society. The following statement appears in the report:

The most obvious conclusion that we can deduce from an examination of the present financing of mathematical journals is that page charges, in their various forms and range of prices, are a fundamental part of the financial support. As we have mentioned earlier, we have considered alternatives to page charges and, except for the unpractical alternative of direct federal subsidies as a form of support of basic research, we have come to the conclusion that page charges when appropriately computed and assessed offer the most equitable means of distributing a portion of the financial support of mathematical publications to the sponsors of research. The remaining support must be supplied by various sub-
sidies and a reasonable subscription price.

In 1960, the Society began to levy page charges on institutions sponsoring research whose results are published in our journals. Since that time, publication costs have risen to such a height that the Board of Trustees reluctantly has found it necessary to raise the rate of assessing page charges to $\$ 30$ per page for all papers published in the BULLETIN, TRANSACTIONS, and MATHEMATICS OF COMPUTATION. The new rate will be effective for papers accepted on or after September 1, 1965.

## JOURNALS IN MICROF ORM

The Society Headquarters Staff has investigated the availability of mathematical journals on microfilm, micro-opaques, and microfiches, and has found that about seventy journals are available in such form from American and British microprint publishers. The survey was conducted by senđing a form letter to two hundred commercial microform companies, to large American and foreign universities having microcopying services, and to a number of public service microform information agencies. A list of the available publications is being kept on file at the Society office and will be published in the next (April 1965) issue of New Publications. Additions to the list will appear in subsequent issues.

New Publications is now a quarterly listing of new mathematical monographs and lecture notes. Members may receive free a subscription upon request, and non-members may subscribe for $\$ 1.00 \mathrm{a}$ year.

## Backlog of Mathematical Research Journals

Information on this important matter is being published twice a year, in the February and August issues of the NOTICES, with the kind cooperation of the respective editorial boards.

It is important that the reader should interpret the data with full allowance for the wide and sometimes meaningless fluctuations which are characteristic of them. Waiting times in particular are effected by many transient effects, which arise in part from the refereeing system. Extreme waiting times as observed from the published dates of receipt of manuscripts may be very misleading, and for that reason, no data on extremes are presented in the table at the bottom of this page.

Some of the columns in the table are not quite self-explanatory, and here are some further details on how the figures were computed.

Column 2. These numbers are rounded off to the nearest 50 .

Column 3. For each journal, this is the estimate as of the indicated dates, of the total number of printed pages which will have been accepted by the next time that manuscripts are to be sent to the printer, but which nevertheless will not be sent to the printer at that time. (Pages
received but not yet accepted are being ignored.)
Column 4. Estimated by the editors (or the Editorial Department of the American Mathematical Society in the case of the Society's journals) and based on thesefactors; manuscripts accepted, manuscripts received and under consideration, manuscripts in galley, and rate of publication. There is no fixed formula.

Column 5. The first quartile $\left(Q_{1}\right)$ and the third quartile $\left(Q_{3}\right)$ are presented to give a measure of the dispersion which will not be too much distorted by meaningless extreme values. The median (Med.) is used as the measure of location. The observations were made from the latest issue received in the Headquarters Offices before the deadline date for this issue of the NOTICES. The waiting times were measured by counting the months from receipt of manuscript in final revised form, to month in which the issue was received at the Headquarters Offices. It should be noted that when a paper is revised, the waiting time between receipt by editors of the final revision and its publication may be much shorter than is the case for a paper which is not revised, so these figures are to that extent distorted on the low side.

|  | 1 | 2 | 3 |  | 4 |  | 5 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| JOURNAL | No. issues per year | Approx. no pages per year | $$ |  | Est. time for paper submitted currently to be published (in months) | Observed waiting time in latest published issue (in months) |  |  |
|  |  |  | pages | pages |  | $\mathrm{Q}_{1}$ | Med. | $\mathrm{Q}_{3}$ |
| American J. Math. | 4 | NR* | NR* | NR* | NR* | 4 | 6 | 8 |
| Annals of Math. | 6 | 1200 | NR* | NR* | 10 | 12 | 12 | 14 |
| Annals of Math. Statist. | NR* | NR* | NR* | 0 | NR* | 6 | 7 | 9 |
| Arch. Hist. Exact Sciences | not fixed | 100 | 0 | 0 | 8 | 8 | 14 | 19 |
| Arch. Rational Mech. Anal. | not fixed | 1300 | 0 | 0 | 6 | 6 | 6 | 6 |
| Canad. J. Math. | 6 | 1200 | 490 | 400 | 12 | 14 | 14 | 16 |
| Duke Math. J. | 4 | 680-700 | 934 | 550 | 12-18 | 16 | 17 | 18 |
| Illinois J. Math. | 4 | 700 | 660 | 850 | 20 | 17 | 18 | 18 |
| J. Math. Analyses and Appl. | 6 | 1100 | NR* | NR* | 12 | ** | ** | ** |
| J. Math. and Mech. | 6 | 1000 | 1000 | 1000 | 12 | ** | ** | ** |
| J. Math. and Phys. | 4 | 350 | 100 | 0 | 6-9 | 8 | 9 | 11 |
| J. Mathematical Physics | NR* | NR* | NR* | NR* | NR* | 6 | 6 | 7 |
| J. Symbolic Logic | NR* | NR* | NR* | 0 | NR* | 23 | 27 | 27 |
| Math. Comp. | 4 | 700 | 75 | 50 | 9 | 7 | 8 | 10 |
| Michigan Math. J. | 4 | 400 | 50 | 25 | 9 | 8 | 9 | 10 |
| Pacific J. Math. | 4 | 1400 | NR* | 1200 | 16 | 13 | 14 | 15 |
| Proceedings of the AMS | 6 | 1000 | 650 | 800 | 17 | 16 | 17 | 18 |
| Quarterly of Appl. Math. | 4 | NR ${ }^{*}$ | NR* | NR* | NR* | 10 | 10 | 11 |
| SIAM Journal | 4 | 1000 | 125 | 100 | 9-12 | 10 | 12 | 13 |
| SIAM Journal on Control | 3 | 450 | 20 | 0 | 6-9 | 10 | 11 | 13 |
| SIAM J. on Numer. Anal. | 3 | 450 | 0 | - | 6-9 | - | - | - |
| SIAM Review | 4 | 550 | 25 | 0 | 6-9 | 8 | 8 | 8 |
| Transactions of the AMS | 12 | 2200\# | 630 | 1035 | 13 | 19 | 19 | 20 |

[^1]
## UNIVERSITY OF MANITOBA, WINN..PEG, MANITOBA, CANADA

N. S. Mendelsohn, Head, Department of Mathematics


Applications must be filed by March 15, 1965.

* Not restricted to mathematics students.
** An additional $\$ 1,000$ may be available for the summer months.
WESTERN RESERVE UNIVERSITY, CLEVELAND, OHIO 44106
W alter Leighton, Head, Department of Mathematics
Total university enrollment in 1965-1966

| Undergraduates |  |
| :---: | :---: |
| 3,800 | Graduates |
| 40 | 22 |

Number of mathematics majors enrolled in 1965-1966
4022
Number of mathematics majors subsidized in 1965-1966

| Type of | Number Anticipated | Stipend |  | Tuition |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| financial | in | 9 or |  | not included | required |
| assistance | 1965-1966 | 12 mo . | Amount | in stipend | (Hours/week) |
| Teaching Assistantship | 18 | 9 | \$2,200 |  | 3 |

Applications must be filed by March 1, 1965.

UNIVERSITY OF WISCONSIN, MIL WAUKEE, WISCONSIN
E. H. Feller, Head, Department of Mathematics

| Type of | Number <br> Anticipated | Stipend |  | Tuition payable, if not included in stipend | $\begin{gathered} \text { Service } \\ \text { required } \\ \text { (Hours/week) } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| financial assistance | $\begin{gathered} \text { in } \\ 1965-1966 \\ \hline \end{gathered}$ | $\begin{array}{r} 9 \mathrm{or} \\ 12 \mathrm{mo} \\ \hline \end{array}$ | Amount |  |  |
| Teaching Assistantships | 40 | 9 | \$2,790 | \$300/yr.* | 8 |
|  |  |  | -3,300 |  |  |
| Research Assistantships | 3 | 9 | 2,700 | 300/yr.* | No special |
|  |  |  | -3,000 |  | duties |
| Special Fellowships | 3 | 9 | 2,800 | 300/yr.* | No special |
|  |  |  | -3,000 |  | duties |

[^2]
# NEW AMS PUBLICATIONS 

## MEMOIRS

Number 52

## GALOIS THEORY AND COHOMOLOGY <br> of commutative Rings

By S. U. Chase, D. K. Harrison, and A. Rosenberg

84 pages. List Price $\$ 1.80$; Member Price \$1.35

This Memoir consists of three related papers: "Galois Theory and Galois Cohomology of Commutative Rings", "Amitsur Cohomology and the Brauer Group" and "Abelian Extensions of Commutative Rings".

In the first paper the main theorems of Galois theory for fields are generalized to commutative rings. Also, for general Galois extensions, a seven-term exact sequence is displayed which reduces in the field case to Hilbert's theorem 90 and to the cohomology characterizations of the Brauer group.

In the second paper, the Brauer group $B(S / R)$ of Azumaya $R$-algebras split by $S$, where $S$ is a finitely generated faithful projective commutative R -algebra, is described cohomologically. This paper also exhibits a seven-term exact sequence, of which the one mentioned above is a special case.

In the third paper the set of all Galois extensions of a commutative ring with a given Abelian Galois group is shown to be a group. Using this result, a compact Abelian group is associated to each commutative ring in a manner which generalizes the absolute Abelian Galois group of fields. This group is studied in a Kummer theory for rings.

## TRANSLATIONS OF MATHEMATICAL MONOGRAPHS

Volume 13

## ADDITIVE THEORY OF PRIME NUMBERS <br> By L. K. Hua

Approximately 200 pages. Prepublication prices valid until March 20, 1965: List Price $\$ 10.00$; Member Price $\$ 7.50$. After publication: List Price at least $\$ 11.00$; Member Price \$8.25.

A detailed exposition, from the beginning, of the Waring-Goldbach problem of representing positive integers as sums
of a given number of $k$ th powers of primes. The work of Vinogradov on trigonometric sums is supplemented by many original results of the author, including his improved form of the Vinogradov mean-value theorem and his extension of the Waring problem to the representation of integers as sums of polynomials with integral coefficients.

## PERSONAL ITEMS

Mr. D. R. BAKER of the International Telephone and Telegraph Company, Paramus, New Jersey has accepted a position as Senior Mathematician with the Advanced Computer Laboratories of Melpar Incorporated, Falls Church, Virginia.

Professor E. A. BISHOP of the University of California, Berkeley has been appointed a Research Professor in the Miller Institute for Basic Research, University of California for the academic year 1964-1965.

Dr. D. M. BLOOM of the University of Massachusetts has been appointed to an assistant professorship at Brooklyn College.

Mr. S. J. BRYANT of the University of California, Berkeley has been appointed to an assistant professorship at the San Diego State College.

Professor H. O. CORDES of the University of California, Berkeley will be on sabbatical leave for the academic year 1964-1965. He will be in residence in Berkeley.

Dr. A. S. COVER of Pennsylvania State University has been appointed to an assistant professorship at the University of Arizona.

Mr. GEORGE DANELUK of Rutgers, The State University has been appointed to an assistant professorship at the Jersey City State College.

Dr. K. H. DANIEL of the Universität Münster, Germany has been appointed to an assistant professorship at the University of Maryland.

Professor D. G. De FIGUEIREDO of the University of Brasilia, Brasilia, Brazil has been appointed to an associate professorship at the Mathematics Research Center, University of Wisconsin.

Professor R. J. De VOGELAERE of the University of California, Berkeley will be on sabbatical leave for the academic year 1964-1965. He plans to remain in Berkeley.

Professor ISTVAN FARY of the University of California, Berkeley will continue on leave for the fall semester 1964.

He will carry on research in Berkeley.
Professor JACOB FELDMAN of the University of California, Berkeley will be on sabbatical leave for the academic year 1964-1965. He will be in residence in Berkeley.

Dr. J. L. FIELDS of Harvard University has been appointed an Associate Mathematician with the Midwest Research Institute.

Mr. W. E. FRANCK, JR. of the University of New Mexico has been appointed an Assistant Professor of Statistics at the University of Missouri.

Mr. M. A. GERAGHTY of the University of Alabama has been appointed to an assistant professorship at the University of Iowa.

Professor D. B. GILLIES of Stanford University has been appointed a Research Professor of Applied Mathematics at the University of Illinois.

Professor J. G. GLIMM of the Massachusetts Institute of Technology will be on leave during the academic year 19641965.

Mr. H. H. GLOVER of the University of Michigan has been appointed to an assistant professorship at the University of Minnesota.

Dr. J. B. GOEBEL of General Electric Company has accepted a position as Senior Mathematician with the Battelle Memorial Institute.

Dr. R. W. GOODMAN of Harvard University has been appointed a Lecturer at the Massachusetts Institute of Technology.

Mr. W. B. GRAGG, JR. of Belcomm has been appointed to the staff of the Oak Ridge National Laboratory.

Mr. EDMOND GRANIRER of the University of Illinois has been appointed to an assistant professorship at Cornell University.

Professor HELMUT GROEMER of Oregon State University has been appointed to a professorship at the University.

Professor OMA HAMARA of Pacific

Union College has been appointed to an assistant professorship at the University of Arizona.

Professor W. R. HARE of Duke University has been appointed to an associate professorship at Clemson University.

Professor O. G. HARROLD of the University of Tennessee has been appointed to a professorship at Florida State University.

Professor SIGURDUR HELGASON of the Massachusetts Institute of Technology will be on leave during the academic year 1964-1965.

Professor HENRY HELSON of the University of California, Berkeley has been appointed Research Professor in the Miller Institute for Basic Research, University of California, for the fall semester 1964.

Professor R.G. HE YNEMAN, on leave from Cornell Universitys has been appointed a visiting Lecturer at the Massachusetts Institute of Technology.

Professor M. W. HIRSCH of the University of California, Berkeley has been awarded a Sloan Fellowship for 19641966. He will be on leave from teaching duties for the fall semester 1964.

Professor G. P. HOCHSCHILD of the University of California, Berkeley will be on sabbatical leave for the academic year 1964-1965. He will be in residence in Berkeley.

Professor J. H. HODGES of the University of Colorado has been awarded the National Science Foundation Science Faculty Fellowship. He will spend the academic year 1964-1965 at the University of California, Berkeley.

Professor D. L. JAGERMAN of Fairleigh Dickinson University has accepted a position as a Member of the Technical Staff with the Bell Telephone Laboratories, Incorporated, Whippany, New Jersey.

Professor J. L. KELLEY of the University of California, Berkeley will be on leave for the academic year 1964-1965. He will spend the year in Kanpur, India, taking part in the Kanpur Indo-American Project, of which the University of California, Berkeley is a joint sponsor.

Dr. SHIN'ICHI KINOSHITA of Osaka University, Japan has been appointed to a visiting associate professorship at Florida State University.

Professor SHOSHICHI KOBA YASHI of the University of California, Berkeley has been awarded a Sloan Fellowship for 19641966. He will be on leave from teaching duties for the spring semester, 1965 and plans to visit Japan.

Professor E. E. KOHLBECKER of the University of Utah has been appointed to a professorship and Acting Head of the Department of Mathematics at MacMurray College.

Dr. TAKESHI KOTAKE of Kyoto University, Japan has been appointed to an assistant professorship at the Massachusetts Institute of Technology. He will spend the year on leave in Japan.

Dr. B. R. KRIPKE of the University of Texas has been appointed to an assistant professorship at the University of California, Berkeley.

Professor K. W. KWUN of Florida State University has been appointed a Member of the Institute for Advanced Study at Princeton for the period September 1964 to June 1965.

Captain PATRICK LEEHEY of the David Taylor Model Basin has been appointed to an associate professorship of Naval Architecture and Mechanical Engineering at the Massachusetts Institute of Technology.

Professor R. S. LEHMAN of the University of California, Berkeley will be on sabbatical leave for the academic year 1964-1965. He plans to carry on research at the Mathematical Institute, University of Göttingen, Germany.

Dr. J. P. LEVINE of Cambridge University has been appointed to an assistant professorship at the University of California, Berkeley.

Professor HANS LEWY of the University of California, Berkeley will be on sabbatical leave for the academic year 1964-1965. He will spend the year in Pisa, Italy.

Mr. H. T. McADAMS of the Cornell Aeronautical Laboratories has accepted a position as Engineer at the Homer Laboratory of the Bethlehem Steel Company, Bethlehem, Pennsylvania.

Mr. SCOTTY NEAL of the University of California, Riverside has accepted a position as Research Mathematician with the U. S. Naval Ordnance Test Station, China Lake, California.

Professor L. W. NEUSTADT of the University of Michigan and Aerospace Corporation has been appointed to a professorship in the Department of Electrical Engineering at the University of Southern California.

Professor R. H. OEHMKE of Michigan State University and the Institute for Defense Analyses has been appointed to a professorship at the University of Iowa.

Professor SIGERU OKAMOTO of Mie University, Tu-City, Japan has been appointed to an associate professorship at the Yamaguchi University, YamaguchiCity, Japan.

Professor G. P. PATIL of McGill University has been appointed to a professorship at Pennsylvania State University.

Dr. J. T. POOLE of the University of Maryland has been appointed to an assistant professorship at Florida State University.

Professor H. H. PRICE of Pasadena College has been appointed to a professorship at Texas Wesleyan College.

Mr. T. M. PRICE of the University of Wisconsin has been appointed to an assistant professorship at the University of Iowa.

Mr. E. N. REISER of the University College of Syracuse University has accepted a position as Mathematician with the U. S. Army Materiel Command, Gravelly Point, Virginia.

Mr. KENNETH ROGERS of the University of California, Los Angeles has been appointed to an associate professorship at the University of Hawaii.

Mr. D. M. SANDELIUS, Private Consultant, has been appointed a Lecturer at Skovde Handelsgymnasium, Skovde, Sweden.

Professor R. D. SCHAFER of the Massachusetts Institute of Technology will be on leave during the spring semester of 1965 :

Professor I. E. SEGAL of the Massachusetts Institute of Technology will be on leave during the academic year 19641965.

Mr. A. M. SETT of Patna University, Bihar, India has been appointed to an associate professorship at Southern University.

Dr. BERNARD SHERMAN of the Space Technology Laboratories, Redondo Beach,

California has accepted a position as Senior Technical Specialist with Rocketdyne, Canoga Park, California.

Professor D. R. SHREVE, Director of the Computing Center of North Carolina State University has been appointed Director of the Computing Center at the University of Mississippi Medical Center.

Dr. R. E. SLOVER of Georgetown College has been appointed to an assistant professorship at the Virginia Polytechnic Institute.

Professor STEPHEN SMALE of Columbia University has been appointed to a professorship at the University of California, Berkeley.

Dr. R. S. SPIRA of Duke University has been appointed to an assistant professorship at the University of Tennessee.

Sister HELEN SULLIVAN of Mount St. Scholastica College has been appointed to a visiting professorship at the University of Minnesota.

Professor P. E. THOMAS of the University of California, Berkeley will be on sabbatical leave for the spring semester 1965. He plans to carry on research at the Institute for Advanced Study, Princeton and at Oxford University, England.

Professor G. P. WEEG of Michigan State University has been appointed to a Professor and Director of the Computer Center at the University of Iowa.

Professor J. W. T. YOUNGS of the Indiana University has been appointed to a professorship at the University of California, Santa Cruz.

The following promotions are announced:
J. J. ANDREWS, Florida State University, to an associate professorship.
N. C. ANKENY, Massachusetts Institute of Technology, to a professorship.

STEVE ARMENTROUT, University of Iowa, to a professorship.

MICHAEL ARTIN, Massachusetts Institute of Technology, to an associate professorship.
S. K. BERBERIAN, University of Iowa, to a professorship.
R. K. BHATTACHARYA, University of Arizona, to an associate professorship.
M. S. CHEEMA, University of Arizona, to an associate professorship.
E. B. CURTIS, Massachusetts Insti-
tute of Technology, to an assistant professorship.
J. W. GIVENS, JR., Argonne National Laboratory, to Director of the Applied Mathematics Division.
K. M. HOFFMAN, Massachusetts Institute of Technology, to a professorship.
J. F. JAKOBSEN, University of Iowa, to an associate professorship.

MEYER JORDAN, Brooklyn College, to an associate professorship.
C. W. McARTHUR, Florida State University, to a professorship.
D. E. MYERS, University of Arizona, to an associate professorship.
R. S. B. ONG, University of Michigan, to a professorship.

HARTLEY ROGERS, JR., Massachusetts Institute of Technology, to a professorship.
W. G. STRANG, Massachusetts Institute of Technology, to an associate professorship.
D. W. WALL, University of Iowa, to a professorship.

The following appointments to Instructorships are announced:

Brooklyn College: H. L. ZUCKERBERG; University of California, Berkeley: W. W. ADAMS, F. P. GREENLEAF; Madison College: R. J. HURSEY, JR.; Massachusetts Institute of Technology:
A. L. FIGA-TALAMANCA, A. C. KERRLAWSON, E. G. K. LOPEZ-ESCOBAR, R. L. PENDLETON, D. G. QUILLEN, D. G. RIDER, S.I. ROSENCRANS, NORTON STARR, TZEE-CHAR KUO, J. J. UCCI; New York University: JOHN MINEKA; Princeton University: J. R. HATTEMER; Siena Heights College: Sister MARY CATHARINA BEREITER; SUNY at Buffalo: IRA GEDAN; Villanova University: R. A. DERRIG.

Deaths:

Dr. W. C. BRENKE of the University of Nebraska died on August 20, 1964 at the age of 90 . He was a member of the Society for 50 years.

Professor E. T. CARROLL-RUSK of Wells College died on December 5, 1964 at the age of 64. He was a member of the Society for 43 years.

Professor E. J. MILES of Yale University died on November 3, 1964 at the age of 78. He was member of the Society. for 56 years.

Professor C. H. SISAM of Colorado College died on December 4, 1964 at the age of 85 . He was a member of the Society for 55 years.

Mr. E. E. SLAUGHTER, JR. of the University of California, Berkeley died on September 23,1964 at the age of 27.

# MEMORANDA TO MEMBERS 

## CORPORATE MEMBERS

We are pleased to announce that, as of January 15, 1964, the following companies and corporations are supporting the Society through Corporate Membership.

Academic Press, Incorporated
Bell Telephonè Laboratories, Incorporated
The Boeing Company
Corning Glassworks Foundation
E. I. DuP ont de Nemours and Company, Incorporated
Eastman Kodak Company
Ford Motor Company
General Motors Corporation

Hughes Aircraft Company
International Business Machines Corporation
Lockheed Missiles and Space Company Marathon Oil Company
Radio Corporation of America
Remington-Rand UNIVAC
Shell Development Company
Socony Mobil Oil Company, Incorporated
Space Technology Laboratories, Incorporated
Standard Oil Company, Incorporated of New Jersey
United Gas Corporation

## NEWS ITEMS AND ANNOUNCEMENTS

## MATHEMATICS RESEARCH INSTITUTE, OBERWOLFACH

The following is a program of the meetings planned to date for 1965. The dates include arrival and departure days.

January 3-6 Work Session of the Frankfurt Seminar Chairman: R. Baer, Frankfurt a. M.

January 18-24 Work Session on Functional Analysis Chairman: H. Konig, Koln

March 1-7 Seminar on Questions on the Borderline between Differential Geometry and the Calculus of Variations Chairman: W. Barthel, Wurzburg

March 8-14 Partial Differential Equations Chairmen: W. Haack, Berlin; G. Hellwig, Berlin

March 22-28 Theory of Satellites and Spaceflight
Chairmen: K. Magnus, Stuttgart; P. Sagirow, Stuttgart
March 29- Work Session on Cybernetics
April 4 or
Chairman: B. Hassenstein, Freiburg i. Br.
April 12-15
April 5-11 Mathematical Logic and Research in the Foundations of Mathematics

Chairmen: H. Hermes, Munster; H. Arnold Schmidt, Marburg
April 19-25 Study Group on Representation Theory
Chairmen: W. Gaschutz, Kiel; W. Jehne, Heidelberg
April 26- The Geometry of Groups and the Groups of Geometry, with Special
May 2 Attention to Finite Structures

Chairmen: R. Baer, Frankfurt a. M.; J. Tits, Bonn
May 31 - History of Problems in Mathematics
June 5 Chairman: J. E. Hofmann, Ichenhausen
June 7-13 Foundations of Geometry
Chairmen: F. Bachmann, Kiel; E. Sperner, Hamburg; T. A. Springer, Utrecht
June 17-20 Work Session of the Frankfurt Seminar Chairman: R. Baer, Frankfurt a. M.

June 21-26 Numerical Problems in the Theory of Approximation Chairmen: L. Collatz, Hamburg; G. Meinhardus, ClausthalZellerfeld

June 27-30 Syntax of Natural Languages and Data Processing Chairman: H. Pilch, Freiburg i. Br.

July 5-11 Functional Equations
Chairmen: J. Aczél, Debrecen; O. Haupt, Erlangen; A. Ostrowski, Basel

July 12-18 Finite Groups and Lie Rings
Chairmen: H. Wieland, Tubingen; H. Zassenhaus, Columbus
July 27-
August 1 Ergodic Theory
Chairman: K. Jacobs, Gottingen
August 2-8 Harmonic Analysis and Integral Transformations Chairman: P. L. Butzer, Aachen

August 30 - Complex Analysis
Sept. 5 Chairmen: H. Grauert, Gottingen; R. Remmert, Gottingen; K. Stein, Munchen

Sept. 6-12 Number Theory, in particular Additive and Analytic Number Theory, Diophantine Approximations

Chairmen: G. Hoheisel, Koln; Th. Schneider, Freiburg i. Br.
Sept. 13-16 Reserved for a meeting of the Deutsche Mathematiker Vereinigung
Sept. 20-30 Topology
Chairmen: A. Dold, Heidelberg; D. Puppe, Saarbrucken; H. Schubert, Kiel

Oct. 4-10 Geometry
Chairman: K. H. Weise, Kiel
Oct. 11-17 Curriculum in Advanced Mathematics for Secondary School Teachers

Chairman: H. Kneser, Tubingen
Oct. 18-24 Study Group
Chairman: To be announced
Oct. 25-31 The Teaching of Mathematics in Secondary Schools (The Cooperation between the School and the University)

Chairmen: M. Barner, Freiburgi. Br.; K.Fladt, Freiburgi.Br./ Calw.

Nov. 15-21 Methods in Functional Analysis Applied to Numerical Mathematics Chairmen: L. Collatz, Hamburg; H. Unger

Personal invitations are sent separately for each meeting. As far as possible, all inquiries from interested persons will, of course, be taken into account.

M. Barner<br>Director of the Institute 78 Freiburg i. Br.<br>Hebelstrasse 29<br>Germany

## F OURTH BRAZILIAN COLLOQUIUM IN MATHEMATICS

The Fourth Brazilian Colloquium in Mathematics will be held in Pocos de Caldas, State of Minas Gerais, Brazil (July 5-24, 1965). It will consist of instructional courses, one-hour surveylectures, short communications and sessions
devoted to mathematics education. For information concerning membership, write to the organizing institution: Director, Instituto de Matemática Pura $e$ Aplicada, Rua Sao Clemente 265, Rio de Janeiro, GA, Brazil.

## SUPPLEMENTARY PROGRAM—Number 30

During the interval from November 26, 1964 through January 14, 1965 the papers listed below were accepted by the American Mathematical Society for presentation by title. After each title on this program there is an identifying number. The abstracts of the papers will be found following the same number in the section on Abstracts of Contributed Papers in this issue of these $\mathcal{C}$ (otices).

One abstract presented by title may be accepted per person per issue of the CNotices. Joint authors are treated as a separate category; thus in addition to abstracts from two authors individually, one joint abstract by them may be accepted for a particular issue.
(1) A remark on coercive operators Professor R. D. Adams, University of Kansas (65T-141)
(2) The six-sphere does not admit a complex structure. Preliminary report

Professor Alfred Adler, Purdue University (65T-113)
(3) Generalized splines and the best approximation of linear functionals

Dr. J. H. Ahlberg, United Aircraft Corporation, East Hartford, Connecticut; Dr. E. N. Nilson, Pratt and Whitney Aircraft Corporation, East Hartford, Connecticut and Professor J. L. Walsh, Harvard University (65T-125)
(4) Summability of nonnegative-valued set functions. Preliminary report Professor W. D. L. Appling, North Texas State University (65T-74)
(5) Analyticity in operator algebras. Preliminary report

Mr. W. B. Arveson, U. S. Naval Ordnance Test Station, Pasadena, California (65T-134)
(6) Sequences in topological spaces Professor C. E. Aull, Kent State University (65T-79)
(7) On a nonlinear elliptic boundary value problem with generalized Goursat data

Professor A. K. Aziz; Professor R. P. Gilbert and Professor H. C. Howard, University of Maryland (65T-124)
(8) On the differential-difference equation $\mathrm{dx} / \mathrm{dt}=\sum_{\mathrm{i}=0}^{\mathrm{n}} \mathrm{a}_{\mathrm{i}} \mathrm{x}(\mathrm{t}-\mathrm{i})$

Professor H. R. Bailey and Mr. M. Z. Williams, Colorado State University (65T-91)
(9) Normal extensions of formally normal ordinary differential operators

Mr. Richard Balsam, University of California, Los Angeles (65T-145)
(10) An asymptotic analogue of the Fuchs regularity theorem. Preliminary report

Dr. S. B. Bank, University of Illinois ( $65 \mathrm{~T}-111$ )
(11) An extension of a theorem of Bernstein to meromorphic functions

Mr. R. C. Basinger, University of Kansas (65T-114)
(12) Pseudo-Frattini subgroups and Lseries

Professor H. F. Bechtell, Bucknell University ( $65 \mathrm{~T}-68$ )
(13) On fixed-point properties of plane continua

Professor Harold Bell, State University of New York at Stony Brook (65T-107)
(14) A general ergodic theorem with weighted averages on continuous flows. Preliminary report

Mr. K. N. Berk, University of California, San Diego (65T-77)
(15) Lattice complements of certain topologies

Professor M. P. Berri, Tulane University (65T-94)
(16) Radial engulfing

Professor R. H. Bing, University of Wisconsin (65T-101)
(17) Extreme operators into spaces of continuous functions

Professor R. M. Blumenthal; Professor Joram Lindenstrauss and Professor R. R. Phelps, University of Washington (65T-147)
(18) On function spaces of finite-dimen-
sional compact Hausdorff spaces Professor C. J. R. Borges, University of Nevada (65T-140)
(19) Structure of commutators of operators

Professor Arlen Brown and Professor C. M. Pearcy, University of Michigan (65T-119)
(20) Extremal measures and loops. Preliminary report

Professor J.R. Brown, Oregon State University (65T-66)
(21) A close look at Quine's axioms Mr. K. R. Brown, IBM Corporation, Cambridge, Massachusetts (65T63)
(Introduced by Professor Hao Wang)
(22) Groups graphs and Fermat's last theorem

Professor S. J. Bryant, San Diego State College (65T-120)
(23) Stability by use of orthogonal trajectories

Professor T. A. Burton, University of Alberta (65T-109)
(24) Fibring within an oriented cobordism class

Professor P. E. Conner and Professor E. E. Floyd, University of Virginia ( $65 \mathrm{~T}-128$ )
(25) Construction of hyperbolic planes from inversive planes of odd order Professor D. W. Crowe, University of Wisconsin (65T-65)
(Introduced by Professor S. Y. Husseini)
(26) On a representation of solutions of linear partial differential equations Mr. E. B. Davis, Stanford University (65T-146)
(27) Representation theorems for operators into a locally convex function space

Mrs. D. A. Daybell, New Mexico State University (65T-103)
(28) The irreducible representations of the unimodular group. Preliminary report

Professor P. C. Deliyannis, Illinois
Institute of Technology (65T-89)
(29) Perturbation of spectra in SturmLiouville problems

Professor J. W. Dettman, Oakland University and Mr. R. L. Raymond, Case Institute of Technology (65T93)
(30) Some properties of a singular differential operator

Professor J. R. Dorroh, Louisiana State University, Baton Rouge (65T122)
(31) On a density theorem for finite differences and an application to Hausdorff means

Professor K. W. Endl, University of Utah (65T-96)
(32) Singular integrals and partial differential equations of parabolic type Mr . E. B. Fabes, University of Chicago (65T-69)
(33) Uniqueness of extremal doubly stochastic measures

Professor Jacob Feldman, University of California, Berkeley (65T126)
(34) Prime modules

Professor E. H. Feller, University of Wisconsin-Milwaukee and Professor E. W. Swokowski, Marquette University (65T-121)
(35) The asymptotic expansion of a ratio of gamma functions

Mr. Jerry Fields, Midwest Research Institute, Kansas City, Missouri (65T-98)
(36) Short proof of a theorem of Rado on graphs

Mr. B. L. Foster, Marathon Oil Company, Littleton, Colorado (65T86)
(37) Deterministic context-free languages. I. Preliminary report

Dr. Seymour Ginsburg, System Development Corporation, SantaMonica, California and Dr. Sheila Greibach, Harvard University (65T155)
(38) A statement equivalent to the axiom of choice

Professor G. A. Grätzer, Pennsylvania State University (65T-61)
(39) A formula connected with Hankel transforms

Professor J. L. Griffith, University of Kansas (65T-142)
(40) On Witt's theorem

Professor Herbert Gross, Montana State College ( $65 \mathrm{~T}-85$ )
(41) Series expansions for functions with the Huygens property

Professor D. T. Haimo, Southern Illinois University and Harvard

University (65T-156)
(42) Diffusion with "collisions" between particles

Dr. T. E. Harris, The RAND Corporation, Santa Monica, California (65T-83)
(43) Purity in lattices

Professor T. J. Head, Iowa State University (65T-116)
(44) Finite-dimensional approximations to White noise and Brownian motion. Preliminary report

Mr. T. Hida, Indiana University and
Nagoya University, Japan (65T-99) (Introduced by Professor P. R. Masani)
(45) Monogenic Post normal systems of arbitrary degree

Mr. P.K. Hooper, Harvard University (65T-82)
(Introduced by Professor Hao Wang)
(46) Doubly-transitive groups, nearfields and geometry

Professor W. J. Jonsson, University of Manitoba ( $65 \mathrm{~T}-106$ )
(47) Extending models of set theory. II. Preliminary report

Professor H. J. Keisler, University of Wisconsin (65T-151)
(48) On complete surfaces in 3 -space with constant nonzero mean curvature

Professor Tilla Klotz, New York University (65T-137)
(49) A note on the Hausdorff moment problem

Professor C.W.Leininger, Arlington State College (65T-153)
(50) Generalized functions and Cech homology theory

Mr. R. G. Lintz, University of Michigan (65T-76)
(51) An abstract characterization of the semigroup of all closed functions on a $\mathrm{T}_{1}$ space

Professor K. D. Magill, Jr., State University of New York at Buffalo (65T-75)
(52) A chainable continuum notrepresentable as an inverse limit on $[0,1]$ with only one bonding map

Professor W. S. Mahavier, Emory University (65T-71)
(53) The normality of time-invariant, subordinative operators in a Hilbert space

Professor P. R. Masani, Indiana

University (65T-100)
(54) Finding the points of attachment in a graph

Mr. B. H. Mayoh, Universitet i Oslo, Blindern, Norway (65T-73)
(55) Some properties of generalized multivariate t-distributions

Dr. K. S. Miller, Columbia University (65T-87)
(56) Quadratic forms positive for nonnegative variables not all zero

Professor T.S. Motzkin, University of California, Los Angeles (65T-84)
(57) Analogues of perfect numbers and amicable numbers

Mr. A. A. Mullin, University of California, Livermore (65T-64)
(58) Embedding properties of a class of 2-metrics

Mr. G. P. Murphy, St. Louis University (65T-117)
(59) Lattices of width 2 and subdirect products of finite lattices. Preliminary report

Mr. C. T. Nelson, Jr., Vanderbilt University ( $65 \mathrm{~T}-108$ )
(60) On finite rings. Preliminary report Professor R. E. Peinado, University of Iowa ( $65 \mathrm{~T}-150$ )
(61) Gödel's second theorem for elementary arithmetic

Mr. L. J. Pozsgay, University of Washington ( $65 \mathrm{~T}-133$ )
(62) On sets of functions that can be multiplicatively completed

Professor J. J. Price, Institute for Advanced Study and Professor R.E. Zink, Purdue University (65T-105)
(63) Summability in topological groups Professor D. L. Prullage, University of Kentucky (65T-78)
(64) Conditional expectations and closed projections

Professor M. M. Rao, Carnegie Institute of Technology ( $65 \mathrm{~T}-102$ )
(65) Extending homeomorphisms between certain subsets of $S^{3}$

Mr. D. S. Ray, Bucknell University (65T-80)
(66) Semi-multiplicative functions

Professor D. F. Rearick, University of Colorado ( $65 \mathrm{~T}-110$ )
(67) Recurrent inequalities

Professor R. M. Redheffer, University of California, Los Angeles (65T-144)
(68) Nilpotent element in rings of integral representations

Professor Irving Reiner, University of Illinois (65T-67)
(69) Primary groups as modules over their endomorphism rings. Preliminary report

Professor Fred Richman and Professor E. A. Walker, New Mexico State University (65T-92)
(70) Radii of star-likeness and close-toconvexity

Professor M. S. Robertson, Rutgers, The State University (65T154)
(71) The intersection of the free maximal ideals in a complete space

Professor S. M. Robinson, Union College (65T-112)
(72) Minimax theorems and conjugate saddle-functions

Professor R. T. Rockafellar, The University of Texas ( $65 \mathrm{~T}-136$ )
(73) On the Wedderburn principal theorem for nearly ( 1,1 ) algebra. Preliminary report

Professor D.J. Rodabaugh, Vanderbilt University (65T-157)
(74) The bounded holomorphic functions in the strict topology

Professor L. R. Rubel, University of Illinois and Professor A. L. Shields, University of Michigan (65T-138)
(75) On class preservation by parabolic singular integral operators

Miss Cora Sadosky, University of Chicago (65T-135)
(Introduced by Professor A. P. Calderón)
(76) Homology ring of a finitely generated abelian group

Mr. J. A. Schafer, University of Chicago (65T-148)
(77) Second separation property for existential second-order classes

Mr. Jack Silver, University of California, Berkeley (65T-139)
(78) Tensor products of complete commutative locally m-convex Q-algebras

Dr. H. A. Smith, University of Pennsylvania (65T-95)
(79) Some groups of type $\mathrm{D}_{4}$ defined by Jordan algebras. Preliminary report Mr. D. C. Soda, Washington Uni-
versity (65T-90)
(80) The measure problem. Preliminary report

Dr. Robert Solovay, Institute for Advanced Study (65T-62)
(81) Elimination of quantifiers and the length of formulae

Professor E. P. Specker, Eidg. Technische Hochschule, Zurich, Switzerland and Dr. Louis Hodges, IBM, Yorktown Heights, New York (65T-143)
(82) On the Riemann hypothesis

Professor R. S. Spira, University of Tennessee (65T-129)
(83) On the remainder term in renewal theory

Professor Charles Stone, University of California, Los Angeles ( $65 \mathrm{~T}-123$ )
(84) Convolutions of convex functions. Preliminary report

Mr. T. J. Suffridge, Kansas University (65T-149)
(85) A matrix proof for a quaternion identity leading to an eight square identity

Dr. Olga Taussky, California Institute of Technology ( $65 \mathrm{~T}-88$ )
(86) A generalization of Cauchy's inequality

Mr. S. S. Wagner, Clemson, South Carolina (65T-72)
(Introduced by Professor W. S. Mahavier)
(87) Endomorphism rings of generators Professor C. P. Walker and Professor E. A. Walker, New Mexico State University ( $65 \mathrm{~T}-130$ )
(88) On Bergman's kernel function for some uniformly elliptic partial differential equations

Dr. G. G. Weill, Yeshiva University (65T-118)
(89) Sentences preserved under direct products. Preliminary report

Mr. J. M. Weinstein, University of Wisconsin (65T-104)
(90) Characterizations and representations of semi-normed algebras. I. Preliminary report

Professor Chien Wenjen, California State College at Long Beach (65T-127)
(91) An integral transform pair involving Whittaker's function

Mr. Jet Wimp, Midwest Research Institute, Kansas City, Missouri (65T-97)
(92) On boundaries of elements of upper semicontinuous decompositions. I Dr. J. M. Worrell, Jr., Sandia Corporation, Albuquerque, New Mexico (65T-70)
(93) Diagonal cylindrifications and linear orderings under one-one reducibility

Professor P. R. Young, Reed College (65T-131)
(94) Asymptotic error estimates in solving the biharmonic equation by the method of finite differences Professor Milos Zlamal, University of Maryland (65T-81)
(Introduced by Professor L. E. Payne)

# NEWS ITEMS AND ANNOUNCEMENTS 

## FIRST WORLD CONGRESS OF THE ECONOMETRIC SOCIETY

The Econometric Society will hold its First World Congress in Rome on September 9-14, 1965. The program consists of about 35 sessions covering nearly all subjects of professional interest to econometricians, and includes a few sessions for contributed papers. An excursion and social events are also planned. The Congress will be open to non-members. The dates of the meeting were chosen in consultation with the International Statistical Institute, the International Association for Research in Income and Wealth, and The Institute of Management Sciences, all of which will also be meeting in Europe in September of 1965.

Preparations for the Congress are being made by a committee under the chairmanship of L. R. Klein of the University of Pennsylvania; the SecretaryGeneral of the Congress is F. Archibugi, Director of the Centro di Studi e Piani Economici in Rome, who is in charge of local arrangements. The program committee consists of H. S. Houthakker, Harvard University (chairman); K. Arrow,

Stanford; R. Bentzel, Industriens Utredningsinstitut, Stockholm; M. Boiteux, Electricite de France, Paris; G. Debreu, University of California, Berkeley; S. Ichimura, Osaka University; M. Kemp, University of New South Wales; M. Kurz, Hebrew University; L. McKenzie, University of Rochester; A. L. Nagar, Delhi School of Economics; G. Orcutt, University of Wisconsin; J. D. Sargan, London School of Economics; and H. Theil, Netherlands School of Economics.

Those wishing to present a paper in English or French should submit an abstract as soon as possible to a member of the program committee located in or near their country of residence. This does not apply to members who have already been approached by the organizer of any session. All persons interested in attending should write to Dr. Franco Archibugi, Centro di Studi e Piani Economici, 26 Via Piemonte, Rome, Italy for further announcements about the program, hotel reservations, and other arrangements.

# ABSTRACTS OF CONTRIBUTED PAPERS <br> The February Meeting in New York, New York February 27, 1965 

620-1. G. D. ANDERSON, 1 Shuman Road, Marblehead, Massachusetts 01945. Bounds for moduli of rings in 3 -space.

We use the terminology of Gehring (Trans. Amer. Math. Soc. 101 (1961), 499-519; Michigan Math. J. 9 (1962), 137-150) and Väisälä (Ann. Acad. Sci. Fenn. A I 298 (1961)). First we obtain a pair of upper bounds for the moduli of rings in 3-space by means of axial and Steiner symmetrization. This leads naturally to the study of certain extremal rings. Among these are the plane ring $\mathrm{R}_{2}$ consisting of the unit disk minus the symmetric slit - $a \leqq x_{1} \leqq a, x_{2}=0$, where $0<a<1$, and the space rings $R_{3, s}, R_{3, d}$ obtained by rotating $R_{2}$ about its real and imaginary axes, respectively. The modulus of $R_{3, d}$ exceeds that of any other space ring $R_{3}$ consisting of the unit ball minus a continuum whose projection on some diametral plane is at least $\pi a^{2}$. The rings $R_{2}$ and $R_{3, s}$ have similar extremal properties. We show that $\bmod R_{3, d} \leqq \bmod R_{2} \leqq \bmod R_{3, s}$ and determine the asymptotic behavior of these moduli as a tends to 0 and to 1 . Both $\bmod R_{3, s}$ and $\bmod R_{3, d}$ behave like mod $R_{2}$ as a tends to 0 . But as a approaches 1 , $\bmod R_{2}$ behaves like $c_{1}\left(\bmod R_{3, ~}\right)^{2}$ and like $c_{2}\left(-\log \bmod R_{3, d}\right)^{-1}$, where $c_{1}$, $c_{2}$ are constants; thus $\bmod R_{3, d}$ goes to zero much faster than $\bmod R_{3, s}$. Finally, we show by means of an example that the extremal quasiconformal mapping of a prolate ellipsoid onto the unit ball is not the affine map as might be expected.

620-2. C. T. LONG, Washington State University, Pullman, Washington. On Moessner's theorem on integral powers.

Alfred Moessner [S.-B. Math.-Nat. K1. Bayer. Akad. Wiss. 1951, 29] conjectured that the natural kth powers can be generated in the following interesting way: From the series of natural numbers strike out every kth number. From the resulting series form the series of partial sums and from this series strike out every ( $\mathrm{k}-1$ ) st number. Again form the series of partial sums and then delete every ( $k-2$ ) nd number. Repeat the process $k-1$ times, striking out every $2 n d$ number the last time and forming the final series of partial sums. This final series of partial sums is the series of natural kth powers. Moessner's theorem was proved by Perron on pages 31-34 of the journal in which Moessner's article appeared. In this paper we show that if Moessner's process is applied to the arithmetic progression $a, a+d, a+2 d, \ldots$, the final series of partial sums is the series $a \cdot 1^{k-1},(a+d) \cdot 2^{k-1},(a+2 d) \cdot 3^{k-1}, \ldots$. The proof depends on a generalization of the Pascal triangle. (Received November 20, 1964.)

620-3. P. K. HOOPER, Computation Laboratory, Harvard University, Cambridge, Massachusetts. The undecidability of the unrestricted halting problem ( $\mathrm{T}_{2}$ ). Preliminary report.

The $T_{2}$ problem of J. R. Buchi (Abstract 61T-153, these CNoticis) 8 (1961), 354) is shown to be recursively undecidable by reducing to $\mathrm{T}_{2}$ the (undecidable) halting problem for initialized,

2-symbol Turing-machines operating on semi-infinite blank tape. For each such machine $\mathrm{M}_{\mathrm{i}}$, a machine $\bar{M}_{i}$ (a 2-symbol TM operating on a 2-way infinite tape) is constructed with the property that $\bar{M}_{i}$ has a total configuration (finite or infinite) that does not lead to a halt if and only if $M_{i}$ does not halt. The construction utilizes the two-tape, nonwriting machines of M. L. Minsky (Ann. of Math. (2) 74 (1961), 437) and relies heavily on the observation that, if such a machine is represented by three markers on a tape, all computation can be effected by moving only two of them and never even crossing the third. Hence one can nest instantaneous descriptions of these Minsky machines (to an arbitrary depth) by encoding the state as a block of markers "outside" the computation region. This permits replacing each unbounded search by a bounded search which, if it fails, causes the computation to begin anew, within but not interfering with the original computation. (Received December 11, 1964.)

620-4. R. G. LINTZ, University of Michigan, Ann Arbor, Michigan. Invariance of open sets in generalized manifolds.

In (R. G. Lintz, Bol. Soc. Mat. São Paulo 15 (1964), 67-75) has been introduced a more general concept of topological manifolds including the classical one. In a recent paper (to appear in Ann. Mat. Pura Appl. (Italy)) it has been shown that several important properties of classical manifolds can be extended to the new class of manifolds. In particular we have the invariance of open sets: Theorem. If $M_{m}$ and $M_{m}^{\prime}$ are locally compact $m-(A, B)$-manifolds where ( $A, B$ ) is an admissible pair and if $f$ is a homeomorphism $f: A \rightarrow M_{m}^{\prime}$, where $A$ is an open set in $M_{m}$, then $f(A)$ is an open set in $M_{m}^{\prime}$. (Received December 14, 1964.)

620-5. R. J. WISNER, New Mexico State University, University Park, New Mexico. Divisibility in semigroups of $n \times n$ unimodular matrices.

The notation and terminology are as in a previous paper (see Abstract 600-4, these CNotices) 10 (1963), 264). Given here is a new proof of the fact that $U_{2}^{0}$ is a free semigroup with two generators. It is then established that none of the semigroups $U_{n}^{0}$ for $n>2$ is finitely generated, and none is free. Some questions are raised about the primes in $U_{n}^{1}$ and a few are answered. (Received November 23, 1964.)

620-7. T. A. BUR TON, University of Alberta, Edmonton, Alberta, Canada. The Lienard equation.

Consider the system (1) $\{\dot{x}=y, \dot{y}=-f(x, y)-g(x)\}$, where $f(x, y) y>0$ if $y \neq 0$ and $x g(x)>0$ if $x \neq 0$ 。 Let $f$ and $g$ satisfy a Lipschitz condition with respect to $x$ and $y$. Theorem l. Let $f(x, y)=f(x) y$. The null solution to (l) is globally asymptotically stable if and only if both $\int_{0}^{\infty 0}[f(x)+g(x)] d x$ and $\int_{0}^{-\infty}[f(x)+|g(x)|] d x$ diverge. The following lemma was proved by E. Whitney for this paper. Lemma. $\operatorname{Inf}_{x}|f(x, y)|$ exists and is an upper semicontinuous function $h(y)$. Theorem 2. If $\int_{B}^{0}[y / h(y)] d y$ is finite for every finite $B$, then the null solution to (l) is globally asymptotically stable. Theorem 3 。 If there exists a continuous function $h(x)$ such that $|f(x, y) / y| \geqq h(x)$ and if $\int_{0}^{x} h(u) d u \rightarrow \pm \infty$ as $x \rightarrow \pm \infty$, then the null solution to (1) is globally asymptotically stable. Theorem 4. If there exists a continuous function $h(x)$ such that $|f(x, y) / y| \leqq h(x)$ and if either $\int_{0}^{\infty}[h(x)+g(x)] d x$ is finite or $\int_{0}^{-\infty}[h(x)+|g(x)|] d x$ is finite, then the null solution to (1) is not globally a symptotically stable. (Received December 14, 1964.)

620-8. D. T. HAIMO, 77 Snake Hill Road, Belmont, Massachusetts 02178. Integral representation of generalized temperature functions.

A generalized temperature function is a function $\in C^{2}$ which satisfies the equation $\Delta_{x} u(x, t)=$ $(\partial / \partial t) u(x, t)$, where $\Delta_{x} f(x)=f^{\prime \prime}(x)+(2 \nu / x) f^{\prime}(x), \nu$ a fixed positive number. H denotes the class of such functions. Let $G(x, y ; t)=(1 / 2 t)^{\nu+1 / 2} \exp \left[-\left(x^{2}+y^{2}\right) / 4 t\right] \mathscr{P}(x y / 2 t)$, where $\mathscr{\mathcal { I }}(\mathrm{z})=\mathrm{c}_{\nu} \mathrm{z}^{1 / 2-\nu} \mathrm{I}_{\nu-1 / 2}(\mathrm{z})$, $I_{a}(z)$ being the Bessel function of imaginary argument of order $a$, and $c_{\nu}=2^{\nu-1 / 2} \Gamma(\nu+1 / 2)$. $H^{*}$ denotes the class of functions $u(x, t) \in H$ such that $u(x, t)=\int_{0}^{a} G\left(x, y ; t-t^{\prime}\right) u\left(y, t^{\prime}\right) d \mu(y), d \mu(x)=$ $c^{-1} x^{2 \nu} d x$, the integral converging for every $t, t^{\prime}, a<t^{\prime}<t<b$. Theorem. If $\int_{0}^{\infty} G(s, y ; t) d a(y), s=$ $\sigma+\mathrm{i} \tau, \mathrm{t}>0$, converges for $\mathrm{s}=\sigma_{0} \geqq 0$, then the integral converges uniformly in $|\sigma|<\mathrm{R}$ for every positive real number $R$, and represents there an analytic function. Theorem. Let $\phi$ be integrable in every finite interval, and let $u(x, t)=\int_{0}^{\infty} G(x, y ; t) \phi(y) d \mu(y)$ converge absolutely at some point $\left(\mathrm{x}_{0}, \mathrm{t}_{0}\right), \mathrm{t}_{0}>0,0 \leq \mathrm{x}_{0}<\infty$. If $\lim _{\mathrm{h} \rightarrow 0}+\int_{\mathrm{x}}^{\mathrm{x}+\mathrm{h}}[\phi(\mathrm{y})-\phi(\mathrm{x})] \mathrm{d} \mu(\mathrm{y})=0$, then $\phi(\mathrm{x})=\int_{0}^{\infty} \mathrm{G}(\mathrm{ix}, \mathrm{y} ; \mathrm{t}) \mathrm{u}(\mathrm{iy}, \mathrm{t}) \mathrm{d} \mu(\mathrm{y})$, $0<t \leqq t_{0}, 0 \lesseqgtr \mathrm{x}<\infty$. Theorem. If $\mathrm{u}(\mathrm{x}, \mathrm{t})=\int_{0}^{\infty} \mathrm{G}(\mathrm{x}, \mathrm{y} ; \mathrm{t}) \mathrm{da}(\mathrm{y})$, the integral converging for $0<\mathrm{t}<\mathrm{c}$, then $u(x, t) \in H^{*}$ there, and the integral converges absolutely for $0<t^{\prime}<t<c$. Theorem. The conditions (i) $u(x, t) \in H$, (ii) $\int_{0}^{\infty}|u(x, t)| G(x ; c-t) d \mu(x)<M, 0<t<c$, are necessary and sufficient that $u(x, t)=\int_{0}^{\infty} G(x, y ; t) d a(y)$, where $\int_{0}^{\infty} G(x ; c)|d a(x)|<\infty$. (Received December 16, 1964.)

620-9. JACOB FELDMAN, University of California at Berkeley, Berkeley, California. Borel sets of states and of representations.

Let $\mathscr{A}$ be a separable c* algebra, and $\mathbb{R}$ the "concrete dual" of $\mathscr{A}$, i.e. the set of representations of $\mathscr{A}$ in certain fixed Hilbert spaces, with a certain natural Borel structure. Then the primary
representations in $\mathbb{R}$ form a Borel set, as do also the representations of type $\mathrm{II}_{1}$ and $\mathrm{I}_{\mathrm{n}}$, while those of type $\mathrm{II}_{\mathrm{o}}$ form at any rate an analytic set, and those of type III a co-analytic set. The same holds true for the sets of states giving rise to the aforementioned types of representations. (Received December 18, 1964.)

620-10. N. P. SALZ, Cornell Aeronautical Laboratory, 4455 Genessee Street, Buffalo, New York. The number of partitions of an integer.

An algorithm is first presented that constructively generates the partitions of an integer (following N. P. Salz, The solution of a partial differential equation of mathematical statistics, Abstract 612-17, these $\mathcal{C}$ (otices 11 (1964), 355). The following classic problem is then solved: Given the number of partitions of all positive integers less than a given integer. Determine without enumeration the number of partitions of the integer. (Received December 21, 1964.)

620-11. RAFAEL ARTZY, Rutgers, The State University, New Brunswick, New Jersey. Non-euclidean incidence planes. I.

In a projective plane a point set C satisfying certain incidence axioms serves as a generalized absolute conic. C -projectivities are defined as bijective mappings $\pi$ of C on itself such that for all point couples $P, Q \in C$, the intersections of lines $P^{\pi} Q$ and $P Q^{\pi}$ lie on a line, the axis of $\pi$. The validity of Pascal's theorem for $C$ and a given axis is equivalent to the existence of all C -projectivities with this axis. All C-projectivities with a fixed axis then form an abelian group. All C-projectivities with axes through a fixed point of $C$ form a group if certain Pascal conditions hold for $C$. Let the points of $C$ be labeled by a set $S, 3$ distinct of them named 0,1 and 00 . Then the $C$-projectivities with the axis that intersects $C$ only in $\infty$ provide an addition in $S$, and those with the axis through 0 and $\infty$ provide a multiplication in $S-\{0\}$. If $\infty$ is disregarded, ( $S,+$ ) and ( $S-\{0\}, \cdot$ ) are commutative loops and, if Pascal's theorem for $C$ and the two axes is assumed, abelian groups. With additional Pascal conditions for $C, S$ becomes a field, and all C-projectivities with axes through $\infty$ are exactly all mappings $x \rightarrow a x+b$ with $a, b \in S, a \neq 0$. Hilbert's field of "ends" is a specialization of ( $S,+, \cdot \cdot$ ). (Received December 22, 1964.)

620-12. S. G. MROWKA, The Pennsylvania State University, 227 McAllister Hall, University Park, Pennsylvania. Still on approximation theorems. Preliminary report.

Let $A$ be a subset of $X$ containing all constant functions. Consider four conditions for separation of $Z$-sets in $X$ by members of $A$. $\left(S_{1}\right):(G) f\left[Z_{1}\right]=0$ and $f\left[Z_{2}\right]=1$; ( $S_{2}$ ): (G) $\sup \left\{f(p): p \in Z_{1}\right\}$ $<\inf \left\{f(p): p \in Z_{2}\right\} ;\left(S_{3}\right):(G) \overline{f\left[Z_{1}\right]} \cap \overline{f\left[Z_{2}\right]}=\varnothing$; ( $S_{4}$ ): (G) $\overline{f\left[Z_{1}\right]} \cap f\left[Z_{2}\right]=\varnothing$, where (G) stands for the statement: "for every disjoint nonempty $Z$-sets $Z_{1}$ and $Z_{2}$ in $X$ there is an $f \in A$ with". Theorems: 1. If A is a uniformly closed (= closed with respect to uniform convergence) linear subspace of $C^{*}(X)$ satisfying $\left(S_{1}\right)$, then $A=C^{*}(X)$. 2. If $A$ is a uniformly closed subalgebra of $C^{*}(X)$ satisfying $\left(S_{3}\right)$, then $A=C^{*}(X)$. 3. If $A$ is a uniformly closed and inverse closed $(=$ if $f \in A$ and $1 / f$ is welldefined, then $1 / f \in A$ ) subalgebra of $C(X)$ satisfying $\left(S_{4}\right)$, then $A=C(X)$. In all theorems, $X$ is an arbitrary completely regular space. Theorem 2 is an improvement of Hewitt's theorem (Duke Math. J. 14 (1947), 419) concerning ( $\mathrm{S}_{2}$ ) separation, and Theorem 3 is an improvement of author's theorem
(these $\mathcal{C}$ Notices $11(1964), 666$ ) concerning $\left(S_{3}\right)$ separation. Theorem 2 is false if $\left(S_{3}\right)$ is satisfied by $\left(S_{4}\right)$. (Received November 30,1964 .)

620-13. E. F. WHITTLESEY, Trinity College, Hartford, Connecticut 06106. The Poincare conjecture.

Theorem: The only closed, simply-connected 3 -manifold is a 3 -sphere. If $M_{k}$ is the k -skeleton of a finite, CW-, 3-complex, formulas for attaching maps may be given explicitly for $M_{k}, k>0$; these afford presentations of $\pi_{1}$ and of $\pi_{2}\left(M_{3}, K\right), M_{1} \subset K \subset M_{2}$. If $M_{3}$ is a closed, simply-connected, 3-manifold, and is represented as a CW-complex with the fewest possible cells, it is enough to demonstrate that the group of integral 2 -chains, $C_{2}$, is trivial. Tietze transformations on the presentation of $\pi_{1}$ are effected, in part, by corresponding operations on the presentation of the relative $\pi_{2}$ carrying it and the absolute $\pi_{1}, \pi_{2}$ to isomorphs. After a finite number of steps, an empty set of edges is obtained, and the triviality of the absolute $\pi_{1}, \pi_{2}$ assures the same for $\pi_{2}\left(M_{2}, M_{1}\right)$ also, so $C_{2}$ is trivial, using results of Peiffer. (NSF 42005.) (Received January 5, 1965.)

620-14. B. V. O'NEILL, JR., Brown University, Providence, Rhode Island 02912. Parts and one-dimensional analytic spaces.
$X$ is a compact Hausdorff space, $C(X)$, resp. $C_{R}(X)$, the space of all continuous complex-, resp. real-, valued functions on $X$ Let $A$ be a closed subalgebra of $C(X)$ separating points and containing the constants. ReA $=\{\operatorname{ReF}: F$ in $A\} . A^{-1}=\left\{F\right.$ in $A: F^{-1}$ in $\left.A\right\}$. Theorem 1: Assume there exist $Z_{i}$ in $A, i=1, \ldots, N$, with $\operatorname{Re} A$ and $\log Z_{i}, i=1, \ldots, N$, spanning a dense subspace of $C_{R}(X)$. Then every Gleason part, $P$, of the maximal ideal space, $M(A)$, having more than one point can be given the structure of a one-dimensional analytic space such that every $F$ in $A$ is analytic on $P$. This generalizes a result of J. Wermer in Analytic disks in maximal ideal spaces, Amer. J. Math., Jan. 1964, 161-170. In the course of the proof it is shown that every two points $a, b$ in $P$ have mutually boundedly absolutely continuous Arens-Singer measures. Theorem 2: Assume instead that there exist $G_{i}$ in $C_{R}(X), i=1, \ldots, N$, such that $R e A$ and $G_{i}, i=1, \ldots, N$, span a dense subspace of $C_{R}(X)$. Then through every point a in $M(A)$ whose part $\neq\{a\}$ passes an analytic disk. The proof of Theorem 2 makes essential use of an unpublished result of E. Bishop. (Received January 11, 1965.)

620-15. MORRIS WEISFELD, Aerospace Corporation, San Bernardino Operations, San Bernardino, California. A theorem on purely inseparable extensions.

Let $F$ be a purely inseparable extension of $C$. A subset $B$ of $F$ is called a subbasis of $F$ over $C$ if and only if $B \cap C=\emptyset, F=C(B)$ and, for any finite subset $\left\{b_{1}, \ldots, b_{n}\right\}$ of $B$, the canonical homomorphism of the tensor product $C\left(b_{1}\right) \otimes C\left(b_{2}\right) \otimes \ldots \otimes C\left(b_{n}\right)$ into $F$ is a monomorphism. Theorem. Let $F$ be a purely inseparable extension having exponent e over $C$. Then, among the subfields of $F$ having a subbasis over C, there is a maximal subfield with respect to set inclusion. The proof is not trivial since the characterizing property is not directly amenable to the use of Zorn's lemma. (Received January 11, 1965.)

620-16. DENNIS EISEN, Adelphi University, Garden City, Long Island, New York. On the stability of difference schemes with singular coefficients.

Consider the spherically symmetric diffusion equation in m dimensions $u_{t}=u_{r r}+(m-1) r^{-1} u_{r}$ which is differenced by $u(r, t+\Delta t)=C(\Delta t) u(r, t)=u(r, t)+\lambda[u(r+\Delta r, t)-2 u(r, t)+u(r-\Delta r, t)]+$ $\sqrt{\lambda \Delta t}(m-1) r^{-1}[u(r+\Delta r, t)-u(r-\Delta r, t)] / 2$, where $\lambda=\Delta t /(\Delta r)^{2}$. Theorem: The difference operator $C(\Delta t)$, above, is unbounded with respect to the $L_{2}$ norm. If there is introduced a sequence of finitedimensional Banach spaces $V_{M}$ with the $M$ norm $\|u\|_{M}=\max \left\{\left|u_{j}\right|\right\}, l \leqq j \leqq M$, where $M=M(\Delta t) \rightarrow \infty$ as $\Delta t \rightarrow 0$, then the matrix representations of $C(\Delta t)$ with respect to $V_{M}$ have an invariant subspace of fixed dimension for any even value of m . This permits one to prove the Theorem: The family of operators $\left\{[\mathrm{C}(\Delta \mathrm{t})]^{\mathrm{n}}\right\}$ is uniformly bounded with respect to the M norm for all n and all $\Delta \mathrm{t}$ such that $0 \leqq n \Delta t \leqq T$ and for all values of $\lambda$ in the range $0 \leqq \lambda<1 / 2$ when $m$ is even. This establishes stability and therefore convergence of the difference solutions to those of the differential equation. (Received January 11, 1965.)

620-17. R. M. SORENSEN, 3400 Toledo Terrace, Apartment J-1, Hyattsville, Maryland. Differential-integral calculus for abstract algebraic-topological structures.

The computational formula for the derivative, limit of difference quotients, is discarded. The methods of Newton and Leibniz are reinstated, and new definitions are given which can be extended to abstract structures in such a manner that derivative and integral remain inverse to each other. Some generality is lost, for example, there is only a concept for differentiability on a set, not at a point, much as for analytic functions. Except for this, the theory reduces to the usual theory in the case of the real numbers. These ideas and methods show that our current idea of derivative is entirely algebraic and linear. They help to explain expansions in trigonometric series, orthogonal polynomials, and Chebyshev polynomials. Moreover, the way is opened for development of a nonlinear calculus. A development exactly parallel to that for the real numbers, for example with a topological group $G$, is possible. One can have a calculus for a $G$-valued function of a $G$-variable, instead of for a real-valued function of a real variable, except that "curve" is not well defined as in $R \times R$. An " n -dimensional calculus" can be defined for $\mathrm{G} \times \mathrm{G} \times \ldots \times \mathrm{G}$ (product of n factors). (Received January 12, 1965.)

620-18. MICHAEL BALCH, Courant Institute of Mathematical Sciences, 25 Waverly Place, New York, New York 10003. A generalized Laurent expansion.
$\widetilde{L}$ is a linear elliptic partial differential operator with coefficients that are analytic in a domain $D$ of $R_{n}$. For solutions to $\widetilde{L u}=0$ having in $D$ an isolated singularity of finite order, F. John has developed a generalization to the Laurent expansion in which the role of the Cauchy kernel $1 /(\xi-\eta)$ is taken over by a fundamental solution $K(x ; y)$ for the adjoint operator $L_{x}$ (Comm. Pure Appl. Math. 3 (1950), 273-304). M. Wachman extended this result to solutions with essential singularity in $D$, for the special case that $\widetilde{\mathrm{L}}$ has constant coefficients and is homogeneous (Proc. Amer. Math. Soc. 15 (1964), 101-108). Wachman's result is here obtained for operators $\tilde{L}$ named at the outset; viz., in terms of a "symmetric" fundamental solution $K(x ; y)$ for $L_{x}$ and $\widetilde{L}_{y}$, a solution $u(y)$ with isolated singularity at $z$ has, for $y \neq z$, a representation $u(y)=w(y)+\sum c_{k} D_{z}^{k} K(z$;y), where $w$
satisfies $\tilde{L} w=0$ and is regular at $z$. The development parallels the corresponding one of analytic function theory: the main tool is an addition formula for $K, K(x ; y)=\sum A_{k}(x ; z) D_{z}^{k} K(z ; y)$, where the key feature is that the $A_{k}$ satisfy $L_{x} A_{k}=0$ and are regular in a common neighborhood of $z$. This includes the familiar special case for Bessel functions: $Y^{(0)}\left(\sqrt{R^{2}+r^{2}-2 R r \cos \theta}\right)=$ $\sum_{-\infty 0}^{\infty} J_{k}(r) Y^{(k)}(R) e^{i k \theta}$. (Received January 13, 1965.)

620-19. GUIDO SANDRI and R. D. SULLIVAN, Aeronautical Research Associates of Princeton, Inc., 50 Washington Road, Princeton, New Jersey. On the classical N-body problem.

We have obtained the complete solution for the motion of three (classical) bodies of identical radii and masses (in $\boldsymbol{\nu}$-dimensional space). The complete classification of the three-body orbits is achieved by obtaining explicitly the domains $\Gamma_{n}$ of the phase space for which $n$ binary collisions occur. Our major result is that $\Gamma_{n}, n \geqq 5$, is empty. The calculation of $\Gamma_{n}$ consists in solving an appropriate set of nonlinear inequalities among the positions and momenta of the three bodies (Phys. Rev. Letters 13 (1964), 743). The extension of our classification of orbits to more than three bodies is formulated. (Received January 13, 1965.)

620-20. B. A. FUSARO, University of Maryland, College Park, Maryland. The permutability of an elliptic operator and a mean value operator in a harmonic space.

The elliptic operator $\Delta^{2}$ defined by $\sqrt{ } g \cdot \Delta^{2}=\partial / \partial x^{i}\left(g^{i j} \sqrt{ } g \partial / \partial x^{j}\right)$ will be said to be in a certain Riemannian m-space if $d s^{2}=g_{i j} d^{i}{ }^{i} x^{j}$ defines a (positive definite) metric for such a space. Here $g=\operatorname{det}\left(g_{i j}\right)$, and $\left(g^{i j}\right)=\left(g_{i j}\right)^{-1}$. Let $M(P, t ; f)$ denote the mean value of a function $f$ averaged over a geodesic hypersphere with radius $t$ and center at $P$. It is well known that in $E^{m}$ (where $\Delta^{2}=\Delta$ ) $\Delta M(t, P ; f)=M(t, P ; \Delta f)$. P. Günther (1957) proved via eigenvalue methods that this permutability holds in a harmonic space. Here this theorem is proved by a new, essentially geometric, method. Let $P$ be the origin of a Riemannian normal coordinate system in the space, let $Q$ be a neighboring point, and let r denote the geodesic distance between P and Q . Lichnerowicz (1944) characterized a harmonic space in these coordinates as one in which $g$ depends on $r$ alone, the variable point $Q$ not otherwise figuring in $g$. Moreover, he showed that the origin $P$ does not enter $g$ parametrically. From these results of Lichnerowicz follows the corollary that in a harmonic space the surface measure of a sphere with radius $t$ and center $P$ is independent of $P$. Using this corollary it can be proved, using methods quite similar to those used in $E^{m}$, that the permutability of the averaging and differential operators $\Delta^{2} M(t, P ; f)=M\left(t, P ; \Delta^{2} f\right)$ is valid in a harmonic space. (Received January 14, 1965.)

620-21. R. W. CHANEY, Western Washington State College, Bellingham, Washington. A sufficient condition for absolute continuity in the transformation theory for measure space.

Reichelderfer has developed a transformation theory for a function $T$ whose domain is a measure space ( $\underline{S}, \underline{M}, u$ ) and whose range is a measure space ( $\underline{S}^{\prime}, \underline{M}^{\prime}, u^{\prime}$ ) (see Abstract 61T-267, these $\mathcal{C}$ Notices) 8 (1961), 518). Relative to a certain collection $\underline{D}$ of subsets $D$ of $\underline{S}$, a weight function $W^{\prime}$ is a nonnegative extended real-valued function defined on $\underline{S}^{\prime} \times \underline{D}$ which is inner continuous and under additive, which vanishes when $s^{\prime}$ is not in TD, and for which each function $W^{\prime}(\cdot, D)$ is $\underline{M}^{\prime}-m e a s u r a b l e$.

The $u^{\prime}$ integral of $W^{\prime}(\cdot, D)$ is termed the weight $W D$ attached to $D$. $T$ is absolutely continuous with respect to $W$ ( $A C W$ ) if each $W D<\infty$ and if there is a nonnegative $u$-integrable function $f$ defined on $\underline{S}$ such that the u integral of $f$ equals WD for each D. Under certain assumptions it is shown that a sufficient condition for $T$ to be ACW is that T transform certain sets of measure zero into sets of $u^{\prime}$ measure zero; an example shows that this sufficient condition is not necessary. It is shown that, under standard hypotheses, certain weight functions $W^{\prime}$ have the property that the functions $W^{\prime}(\cdot, D)$ all vanish a.e. $u^{\prime}$ off the $T$ image of some set of $u$ measure zero. (Received January 14, 1965.)

620-22. F. M. WRIGHT, Iowa State University, Department of Mathematics, I.S.U. Ames, Iowa. The outer and inner area functions associated with a quasi-monotone nondecreasing function.

The concept of a quasi-monotone nondecreasing real-valued function $g$ on the entire real plane $E^{2}$ consisting of all ordered pairs ( $x, y$ ) of real numbers is described. With such a $g$ is associated an outer area function $A_{g}^{*}$ such that $A_{g}^{*}$ is a monotone, finitely sub-additive, non-negative real-valued function with domain the collection consisting of all bounded subsets of $E^{2}$. It is shown that for any two bounded subsets $S$ and $T$ of $E^{2}$ which are a positive distance apart it follows that $A_{g}^{*}(S+T)=A_{g}^{*}(S)+A_{g}^{*}(T)$. It is demonstrated that for $a, b, c, d$ real numbers such that $c>a$ and $d>b$, and for $S$ the set consisting of all ordered pairs ( $x, y$ ) of real numbers such that a $\leqq x \leqq c$ and $b \leqq y \leqq d$, it follows that $A_{g}^{*}(S)=\left[g\left(c^{+}, d^{+}\right)-g\left(c^{+}, b^{-}\right)+g\left(a^{-}, b^{-}\right)-g\left(a^{-}, d^{+}\right)\right]$. It is noted that, for example, one can establish by using a process similar to that used in establishing the preceding result the analogous result for $a, b, c, d$ real numbers such that $c>a$ and $d>b$, and for $S$ the set consisting of all ordered pairs ( $x, y$ ) of real numbers such that $a \leqq x<c$ and $b \leqq y \leqq d$. It is then shown how one can use the preceding results in what is believed to be a novel way to associate an inner area function $A_{*}$ g with g. (Received January 14, 1965.)

## ABSTRACTS PRESENTED BY TITLE

65T-61. G. A. GRÄTZER, Pennsylvania State University, McAllister Building, University Park, Pennsylvania. A statement equivalent to the Axiom of Choice.

Let $B$ be a Boolean algebra $H=\left\{h_{\lambda} ; \lambda \in \Lambda\right\}$ and $K=\left\{k_{\lambda} ; \lambda \in \Lambda\right\}$ be subsets of $B, 0 \notin H$. An ideal $I$ of $B$ is an implication-ideal if $h_{\lambda} \in I$ implies $k_{\lambda} \in I$ for all $\lambda \in \Lambda$. Consider the following statement: ( P ) There exists a maximal proper implication ideal. Theorem. ( P ) is equivalent to the Axiom of Choice. (Received September 24, 1964.)

65T-62. ROBERT SOLOVAY, Institute for Advanced Study, Princeton, New Jersey. The measure problem. Preliminary report.

Theorem I. We suppose that Zermelo-Fraenkel + "there exists an inaccessible cardinal" is consistent. Then Zermelo-Fraenkel remains consistent if the following four axioms are adjoined: (1) the principle of dependent choices (cf. Fund. Math. 16 (1948), 127); (2) every set of reals is Lebesgue measurable; (3) every set of reals has the property of Baire; (4) every uncountable set of reals contains a perfect subset. A set of reals is tame if it satisfies the conclusions of (2)-(4)-it is known that all Borel sets are tame. Theorem II. Suppose that Z-F + 'there exists an inaccessible cardinal" is consistent. Then Z-F remains consistent if the following axioms and axiom schema are adjointed: (1) the axiom of choice; (2) the generalized continuum hypothesis; (3) (schema) every set of reals definable from a fixed real and/or an ordinal is tame. The proofs use ideas of Cohen and Levy. (Received October 26, 1964.)

65T-63. K. R. BROWN, 28 Aberdeen Avenue, Cambridge 38, Massachusetts. A close look at Quine's axioms.

The axioms of Quine's Set theory and its logic are restated somewhat more simply and related to traditional axioms for set theory. The simplification consists of dispensing with ordered pairs in the formulation of the assumption of finite sets, thus making unnecessary a prior assumption of sets of up to two members. The relation to traditional axioms is surprising. Although Quine's axioms assure only the usual finite sets, they are not provable even in so powerful a theory of finite sets as what remains of the Zermelo-Fraenkel system, ZF, if the axiom of infinity and the schema of foundation are omitted. However, Quine's axioms are provable in ZF by appeal to either the axiom of infinity or the schema of foundation. For example, Quine's axioms are derivable from extensionality, adjunction, and foundation. (Received November 12, 1964.)

65T-64. A. A. MULLIN, University of California, Lawrence Radiation Laboratory, Box 808, Livermore, California 94551. Analogues of perfect numbers and amicable numbers.

As with the notions of perfect numbers and amicable numbers [L. E. Dickson, History of the theory of numbers, Vol. 1, Divisibility and primality, pp. 3-50, Chelsea, New York, 1952] define $\sigma^{*}$
as follows: $\sigma^{*}(n)$ is the sum of the natural number equivalents (under the inverse of the l-1 canonic map $\nu$ ) of the distinct submosaics of $n$. By a pluperfect number is meant a natural number n satisfying $\sigma^{*}(\mathrm{n})=2 \mathrm{n}$. By agapistic numbers is meant natural numbers m and n satisfying $\sigma^{*}(\mathrm{n})=$ $\mathrm{m}+\mathrm{n}=\sigma^{*}(\mathrm{~m})$. Lemma. There exist both pluperfect numbers and agapistic numbers. Comment. E. g., the infinitude of the cardinalities of the sets of such kinds of numbers remains an open question. (Received November 23, 1964.)

65T-65. D. W. CROWE, University of Wisconsin, Madison, Wisconsin. Construction of hyperbolic planes from inversive planes of odd order.

Dembowski (Bull. Amer. Math. Soc. 69 (1963), 850-854) has shown that a finite abstract inversive plane, $\mathrm{I}_{2 \mathrm{n}}$, of even order $2 \mathrm{n}\left(2 \mathrm{n}+1\right.$ points on each circle) in fact has order $2^{\mathrm{k}}$. This result suggests that a Poincare orthogonal-circles model of a finite hyperbolic plane (Crowe, Mathematika 11 (1964), 83-88) can always be constructed in $\mathrm{I}_{2 \mathrm{n}}$. The present paper proves that this is the case. Furthermore, it is shown that an inversive plane $I_{2 n+1}$ of odd order also admits such a model, providing $\mathrm{I}_{2 \mathrm{n}+\mathrm{l}}$ admits a strong enough orthogonality relation. The essential property of the orthogonality relation is that the circles orthogonal to a given circle C containing a given point $\mathrm{P} \notin \mathrm{C}$ must be exactly the $2 n+1$ circles through $P$ and some other fixed point $P^{*}$. (Received November 23, 1964.)

65T-66. J. R. BROWN, Oregon State University, Corvallis, Oregon. Extremal measures and loops. Preliminary report.

Let $\mu$ be a doubly stochastic measure on the unit square $I \times I$, i.e. a regular Borel measure such that $\mu(\mathrm{A} \times \mathrm{I})=\mu(\mathrm{I} \times \mathrm{A})=\mathrm{m}(\mathrm{A})$ (Lebesgue measure) for all Borel sets A. A measurable rectangle $\mathrm{A} \times \mathrm{B}$ is called full if $\mu(\mathrm{A} \times \mathrm{B})>0$ and the measures. $\mu_{1}(\mathrm{C})=\mu(\mathrm{C} \times \mathrm{B})$ and $\mu_{2}(\mathrm{D})=\mu(\mathrm{A} \times \mathrm{D})$, defined for Borel sets $C \subset A$ and $D \subset B$, are equivalent to Lebesgue measure. A loop for $\mu$ is a finite sequence $A_{1} \times B_{1}, A_{2} \times B_{2}, \ldots, A_{2 n} \times B_{2 n}, n \geqq 2$, of full rectangles such that $B_{1} \subset B_{2}, A_{2} \subset A_{3}, B_{3} \subset B_{4}$, $\ldots . . B_{2 n-1} \subset B_{2 n}, m\left(A_{2 n} \cap A_{1}\right)>0$ and $m\left(A_{i} \cap A_{j}\right)=m\left(B_{i} \cap B_{j}\right)=0(i \neq j)$ otherwise. It is shown that $\mu$ is loop-free iff for each full rectangle $A \times B$ and each $\epsilon>0$ there exist $f, g \in L_{1}(m)$ such that $\int\left|\chi_{A}(x) \chi_{B}(y)-f(x)-g(y)\right| d \mu<\epsilon$ and $f(x)=0$ for $x \in A$. In particular, if $\mu$ is loop-free, then $\mu$ is extremal in the convex set of doubly stochastic measures on $I \times I$. It is not known whether the converse is true. (Received November 23, 1964.)

65T-67. IR VING REINER, University of Illinois, Urbana, Illinois. Nilpotent elements in rings of integral representations.

Let $G$ be a finite group, $R$ a discrete valuation ring of characteristic zero with maximal ideal $P$, where $R / P$ has nonzero characteristic. Suppose that the Krull-Schmidt theorem holds for RG*-modules, for all subgroups $G^{*}$ of $G$. The integral representation ring $A(R G)$ is defined as the additive group generated by symbols $\{M\}$, one for each isomorphism class of $R$-torsionfree $R G-m o d u l e s$, with relations $\{M \oplus N\}=\{M\}+\{N\}$. Multiplication in $A(R G)$ is defined by taking tensor products of modules. The following result is proved, generalizing an earlier result by the author. Theorem. Assume that $G$ contains a cyclic subgroup $H$ of order $n$, where $n \in P^{2}$, and where
also $n \in 2 P$ if $2 \in P$. Then $A(R G)$ contains at least one nonzero nilpotent element. The proof uses induced representations, Mackey's subgroup theorem, and special properties of some RH-modules. (Received November 23, 1964.)

65T-68. H. F. BECHTELL, Bucknell University, Lewisburg, Pennsylvania 17837. PseudoFrattini subgroups and L-series.

The intersection of the normal maximal subgroups, $R$, and the intersection of the selfnormalizing maximal subgroups, L, are characteristic subgroups of a finite group G. L is nilpotent and contains the hypercenter of $G, Z^{*}$. An L-series is defined as $L_{0}=L, L_{l}=[L, G], \ldots, L_{j}$ $\left[L_{j-1}, G\right], \ldots$ its properties analogous to the upper central series. The terminal member, $L^{*}$, is contained in the Frattini subgroup of $G$ and $L^{*}=1$ implies that $L$ coincides with $Z^{*}$. Moreover $L^{*}=1$ also implies $L^{*}(N)=1$ for each normal subgroup $N$ of $G$, i.e. $L(N)$ coincides with the hypercenter of N. Also $L^{*}=1$ and each subgroup of $G / \Phi(G)$ being the direct product of elementary Abelian p-groups is equivalent to a group having each proper subgroup nilpotent. (Received November 20, 1964.)

65T-69. E. B. FABES, University of Chicago, Chicago 37, Illinois. Singular integrals and partial differential equations of parabolic type.

Let $\mathrm{x} \in \mathrm{E}^{\mathrm{n}}, \mathrm{y} \in \mathrm{E}^{\mathrm{n}}, 0<\mathrm{t}<\infty, 0<\mathrm{s}<\infty, \lambda>0, \mathrm{~m}$ any positive integer $\geqq 2$, and suppose that the kernel, $k(x, t ; y, s)$, satisfies the conditions (i) $k\left(x, y ; \lambda y, \lambda^{m} s\right)=\lambda^{-n-m} k(x, t ; y, s)$;
(ii) $\int_{E_{n}^{n}} k(x, t ; y, 1) d y=0$; (iii) Every derivative of $k(x, t ; y, 1)$ with respect to $y$ decreases at infinity like $e^{-A|y|} \mathbb{P}_{\text {for }}$ some $P>1, A>0$ depending only on the order of the derivative. Given $f(y, s) \in L^{p}\left(E^{n+1}\right)$, $\mathrm{f}=0$ for $\mathrm{s} \leqq 0$, and $\epsilon>0$, set $\widetilde{f}_{\epsilon}(\mathrm{x}, \mathrm{t})=\int_{0}^{\mathrm{t}-\epsilon} \int_{\mathrm{E}^{\mathrm{n}}} \mathrm{k}(\mathrm{x}, \mathrm{t} ; \mathrm{x}-\mathrm{y}, \mathrm{t}-\mathrm{s}) \mathrm{f}(\mathrm{y}, \mathrm{s})$ dyds. Theorem: For $\mathrm{l}<\mathrm{p} \leqq 2$ we have $\left\|\widetilde{f}_{\epsilon}\right\|_{p} \leqq A_{p}\|f\|_{p}, A_{p}$ depending only on $p$, and $\lim _{\epsilon \rightarrow 0} \widetilde{f}_{\epsilon}$ exists in the $L^{p}$-sense. Let $L u=$ $\sum_{|a|=m}{ }^{a}{ }_{a}(x, t)(\partial / \partial x)^{a} u(x, t)-(\partial / \partial t) u(x, t)$ be a differential operator parabolic in the sense of Petrovsky. Assume that the coefficients are bounded Hölder continuous functions of $(x, t)$. Let $L_{m, 1}^{p}\left(E_{n} \times(0, R)\right)$ be the set of all functions $u(x, t)$ on $E^{n} \times(0, R)$ such that $(\partial / \partial x){ }^{a} u(x, t),|a| \leqq m$, and $(\partial / \partial t) u$ exist in the sense of distributions and belong to $L^{p}\left(E_{n} \times(0, R)\right)$. Theorem: If $f \in L^{p}\left(E_{n} \times(0, R)\right)$, $1<p<\infty$, then there is a function $u(x, t) \in L_{m, l}^{p}\left(E_{n} \times(0, R)\right)$ with $u(x, 0)=0$ and such that $L u=f$ almost everywhere on $E_{n} \times(0, R)$. (Received November 27, 1964.)

65T-70. J. M. WORRELL, JR., Division 5421, Sandia Corporation, Sandia Base, Albuquerque, New Mexico 87115. On boundaries of elements of upper semicontinuous decompositions. I.

Theorem 1. If (1) $G$ is an upper semicontinuous decomposition of a $T_{1}$ developable topological space and (2) there exists an element $g$ of $G$ that is noncompact or has an interior point, then the collection of all g's as in (2) is the sum of ountably many sets that are closed and isolated with respect to the decomposition space. Theorem 2. If (1) the axiom that $\kappa_{1}=c$ holds true, (2) S is a first countable regular $T_{1}$ topological space, and (3) $\overline{\bar{S}} \leqq c$, then there exist a locally bicompact regular $T_{1}$ space $R$ having a base of countable order and an upper semicontinuous decomposition $G$ such that (a) the elements of $G$ have interior points and also have non-Lindelöf boundaries and (b) the decomposition space is homeomorphic with S. Remark. For relevant background material, the reader is referred to Abstracts $64 \mathrm{~T}-202$ (these $\mathcal{C}$ (otices) 11 (1964), 250), 64T-401 (ibid. 11 (1964), 595), and

64T-511 (ibid. 11 (1964), 773) and to work of K. Morita [Proc. Japan Acad. 32 (1956), 539-543]. (Received November 27, 1964.)

65T-71. W. S. MAHAVIER, Emory University, Atlanta, Georgia 30322. A chainable continuum not representable as an inverse limit on $[0,1]$ with only one bonding map.

The example given by J. J. Andrews in Proc. Amer. Math. Soc. 12 (1961), 333-334, is a nondegenerate, compact, chainable continuum as described in the title. This follows from the observation that if the continuum $M$ is representable as an inverse limit with only one bonding map from $[0,1]$ on $[0,1]$ then there is a nontrivial homeomorphism of $M$ onto $M$. It is also noted that (1) there are compact chainable continua which are not homeomorphic to inverse limits on $[0,1]$ with only one bonding map and on which there are nontrivial homeomorphisms, and (2) if $M$ is a compact chainable continuum, there is a map $f$ of $[0,1]$ on $[0,1]$ whose inverse limit contains a continuum homeomorphic to M. (Received November 27, 1964.)

65T-72. S. S. W AGNER, 340 Pendleton Road, Clemson, South Carolina 29631. A generalization of Cauchy's inequality.

Theorem: If $a_{1}, \ldots$ and $b_{1}, \ldots$ are number sequences such that $\sum a_{i}^{2}$ and $\sum b_{i}^{2}$ exist and $x$ is in the number interval $[0,1]$, then $\left[\left(\sum_{i} b_{i}\right)+x \cdot \sum_{i \neq j} a_{i} b_{j}\right]^{2} \leqq\left[\left(\sum a_{i}^{2}\right)+2 x \cdot \sum_{i \neq j} a_{i} a_{j}\right]$ $\cdot\left[\left(\sum b_{i}^{2}\right)+2 x \cdot \sum_{i \neq j} b_{i} b_{j}\right]$, and (for $\left.x \neq 1\right)$ equality holds only if the vectors ( $a_{1}, \ldots$ ) and ( $b_{1}, \ldots$ ) are linearly independent. Here are the main ideas of the proof. Writing $F(x)$ for the right side of the inequality minus the left side, and $F_{n}(x)$ for the same difference but with finite summations $\sum^{n}$, a limiting argument shows that if each of $F_{1}, \ldots$ is non-negative on $[0,1]$ then so is $F$. By partial differentiation, first w.r.t. $x$ and then w.r.t. $b_{n+1}, F_{n+1}$ is shown to be non-negative on $[0,1]$ provided a discriminant parabola $G\left(a_{n+1}\right)=A a_{n+1}^{2}+B a_{n+1}+C$ is negative nowhere (where $A$ is a sum of 5 summations, $B$ is a sum of 11 summations, and $C$ is a sum of 24 summations). A is shown to be non-negative by an inductive supposition, and $4 \mathrm{AC}-\mathrm{B}^{2}$ to be 0 by exhibiting the coefficients of all (317) possible terms in the products 4 AC and $\mathrm{B}^{2}$. (Received November 27, 1964.)

65T-73. B. H. MAYOH, Institutt i Matematiske Fag, Universitet i Oslo, Blindern, Norway. Finding the points of attachment in a graph.

A point of attachment in a graph is a vertex whose suppression disconnects the component in which it lies. An algorithm is described that not only determines all such vertices in a given graph but also defines the subgraphs into which the suppression of the point of attachment divides the component in which it lies. A matrix M is associated with a graph (directed or undirected) by the rule that $M(i, j)$ shall be the ith prime if $i=j$ or an edge joins the ith and jth vertices, and 0 otherwise. With an appropriate redefinition of + and $\times$, the $(i, j)$ th entry of a sufficiently high power of $M$ will be divisible by the kth prime if and only if every path joining the ith and jth vertices passes through the kth vertex (and $\mathrm{k} \neq \mathrm{j}$ ). For an undirected graph this ensures that the kth vertex is a point of attachment. The algorithm can be used to determine a strategy for solving a large set of simultaneous equations with many zero coefficients, for inverting matrices with many zero entries, and
indeed for any branching process that reduces a large problem to several smaller problems. (Received November 27, 1964.)

65T-74. W. D. L. APPLING, North Texas State University, Denton, Texas. Summability of nonnegative-valued set functions. Preliminary report.

Suppose $U$ is the set of all real nonnegative-valued functions on a field $S$ of subsets of a set $R$, and $U^{*}$ is the set of all finitely additive elements of $U$. All integrals discussed are Hellinger type limits of the appropriate sums. Suppose $m$ is in $U^{*}$. Suppose $W$ is the set of all $Q$ in $U$ such that for each $K \geqq 0, \int_{R} \min \{K, Q(I)\} \operatorname{m(I)}$ exists, and l.u.b. $\left\{z \mid z=\int_{R} \min \{K, Q(I)\} m(I), 0 \leqq K\right\}<\infty$. If $Q$ is in $W$, we let $s(Q)$ denote the function on $S$ such that for each $V$ in $S, s(Q)(V)=1$.u.b. $\{z \mid z=$ $\left.\int_{V} \min \{K, Q(I)\} m(I), 0 \leqq K\right\}$; and we see that $s(Q)$ is in $U^{*}$. Suppose each of $H$ and $L$ is in $U$. Theorem 1. If $L$ is in $W$, then $H$ is in $W$ iff $L+H$ is in $W$, in which case $s(L+H)=s(L)+s(H)$. Theorem 2. If $H$ is in $W$ and $L$ is bounded and $\int_{R} L(I) m(I)$ exists, then $L H$ is in $W$ and $\int_{R}\left|s(L H)(V)-\int_{V} L(I) s(H)(I)\right|=0$. Theorem 3. If $0<p<1$, and each of $H$ and $L$ is in $W$, then $H^{p_{L}}{ }^{1-p}$ is in $W$ and $\int_{R}\left|s\left(H^{p} L^{1-p}\right)(V)-\int_{V} s(H)(I)^{p} s_{(L)(I)}{ }^{1-p}\right|=0$. Theorem 4. If $1<p$ and $H$ is in $W$, then $H^{p}$ is in W iff $\int_{R} s(H)(I)^{P_{m}}{ }^{(I)}{ }^{1-p}$ exists, in which case $\int_{R}\left|s\left(H^{p}\right)(V)-\int_{V} s(H)(I)^{p_{m}(I)}{ }^{1-p}\right|=0$. Theorem 5. If $H$ is in $W$, and for some $c>0, H-c$ is in $U$, then $\int_{R}\left|H(V)^{-1} m(V)-\int_{V} s(H)(I) r^{-1} m(I)^{2}\right|$ $=0$. (Received November 22, 1964.)

65T-75. K. D. MAGILL, JR., Michael Hall, State Universitv of New York, Buffalo, New York 14214. An abstract characterization of the semigroup of all closed functions on a $\mathrm{T}_{1}$ space.

Let $Z(S)$ denote the ideal of left zeros of a semigroup $S$. If for $a, b \in S$, there is a $z \in Z(S)$ such that $a z=b$, we say $a x=b$ is $Z$-solvable. If $T \subset S$ and $a x=b$ is $Z$-solvable for each $a \in T$, we say $T x=b$ is $Z$-solvable. $T$ is a $Z$-set if $T \neq \emptyset$ and $T x=z$ is $Z$-solvable for some $z \in Z(S)$. Definition 1 . A semigroup $S$ is a $\Gamma$-semigroup if (i) $S$ has an identity, (ii) $a, b \in S$ and $a \neq b$ implies $a z \neq b z$ for some $z \in Z(S)$, (iii) for $a, b \in S$ there is a $c \in S$ such that for each $z \in Z(S), c x=a z$ and $c x=b z$ are $Z$-solvable and either $a x=c z$ or $b x=c z$ is $Z$-solvable, and (iv) for each $Z$-set $T$, there is $a b \in S$ such that for any $z \in Z(S), T x=b z$ is $Z$-solvable and if for $v \in Z(S), T x=v$ is $Z$-solvable, then $\mathrm{bx}=\mathrm{v}$ is Z -solvable. Definition 2. A $\Gamma$-semigroup T is a $\Gamma$-extension of a subsemigroup S if for each a $\in T$ there is $a b \in S$ such that for each $z \in Z(S)$, there are elements $v$ and $w$ in $Z(S)$ satisfying $\mathrm{av}=\mathrm{bz}$ and $\mathrm{bw}=\mathrm{az}$. Theorem. A semigroup S is isomorphic to the semigroup of all closed functions (under the binary operation of composition) on a $T_{1}$ space if and only if $S$ is a $\Gamma$-semigroup and for any proper $\Gamma$-extension $T$ of $S, Z(T) \neq Z(S)$. (Received November 27, 1964.)

65T-76. R. G. LINTZ, University of Michigan, Ann Arbor, Michigan. Generalized functions and Čech homology theory.

In (R. G. Lintz, A generalization of the concept of continuous function and homeomorphism, to appear in Ann. Mat. Pura Appl. (Italy)) we have introduced the concept of generalized continuous functions and modeling functions from a space $X$ with a family of open coverings $\mathscr{V}$ into another space $Y$ with a family of open coverings $\mathscr{V}^{\prime}$. It can be proved that if $\mathscr{V}$ and $\mathscr{V}^{\prime}$ above satisfy certain require-
ments, then $f$ is continuous if induces in a natural way homomorphism of the Cech homology groups of $X$ into that of $Y$. If $f$ is a modeling function this homeomorphism is an isomorphism. Also some results are obtained, in the category of compact metric spaces, on the problem of realization of homomorphisms. That is, if some homomorphism h from the Čech homology group of $X$ into that of $Y$ is given, under what conditions can it be proved that $h$ is induced by some generalized continuous function? (Received November 30, 1964.)

65T-77. K. N. BERK, University of California, San Diego, La Jolla, California. A general ergodic theorem with weighted averages on continuous flows. Preliminary report.

Let $(S, \Sigma, \mu)$ be a $\sigma$-finite measure space, $L_{1}(S)$ the integrable functions on $S,\left\{T^{a}, 0 \leqq a<\infty\right.$ a set of positive linear operators from $L_{1}(S)$ to $L_{1}(S)$ with $\left(T^{a+} \beta_{f}\right)(s)=\left(T^{a}\left(T \beta_{f}\right)\right.$ (s) almost everywhere, $\left|T^{a_{f}}\right|_{1} \leqq|f|_{l}$, and ( $\left.T^{a_{f}}\right)(\mathrm{s})$ measurable with respect to the product of Lebesgue measure and $(S, \Sigma, \mu)$, for each $f \in L_{1}(S), s \in S$. Let $v(a)$ be a solution of the renewal equation $v(a)=$ $1+\int_{0}^{a} v(a-\beta) d w(\beta), 0 \leqq a<\infty$, with $w$ a probability distribution on $(0, \infty)$. For $f \in L_{1}(S) l e t z(f ; c, s)=$ $\int_{0}^{c}\left(T^{a_{f}}\right)(s) d v(a)$. Then if $f \in L_{1}(S), p \in L_{1}(S), p \geqq 0$, there is a null set $N(f, p) \subseteq S$ such that, for $s \notin N(f, p) \cup\{s: p=0\}, z(f ; c, s)$ and $z(p ; c, s)$ exist for $0 \leqq c<\infty$ and $\lim _{c \rightarrow \infty} z(f ; c, s) / z(p ; c, s)$ exists. This result is an extension of work by G. E. Baxter (A general ergodic theorem with weighted averages, Abstract 64T-326, these $\mathcal{C}$ (otices) 11 (1964), 464). (Received November 30, 1964.)

65T-78. D. L. PRULLAGE, University of Kentucky, Lexington, Kentucky 40506. Summability in topological groups.

Let $G$ be a topological Hausdorff group which satisfies the first axiom of countability. A limitation method on $G$ is defined to be a sequence of algebraic homomorphisms $\{\mathrm{f}(\mathrm{m})\}$ such that for each $m$, the domain of $f(m)$ is some subset of $S(G)$, the class of all sequences of elements of $G$, and the range is contained in $G$. A limitation method is said to be triangular if for each $m$, the domain of $f(m)$ is all of $S(G)$ and if $X=\{x(n)\}$ and $Y=\{y(n)\}$ in $S(G)$ satisfy the condition that $x(n)=y(n)$ for $n=1,2, \ldots, m$, then $f(m: X)=f(m: Y)$. Regular methods are defined and characterized. Several results then follow which set restrictions on the convergence field of a regular triangular method. I then consider the cases where $G$ is, in turn, $R^{t}$, the unit circle, the p-adic numbers, and normed linear spaces. In all cases regular triangular methods are characterized. If $G$ is $R^{t}$ it is proven that for each regular triangular method, there exists a triangular matrix method equivalent to and consistent with it. Other results are obtained for $\mathrm{R}^{\mathrm{t}}$. (Received November 30, 1964.)

65T-79. C. E. AULL, Kent State University, Kent, Ohio. Sequences in topological spaces.
The following results are proved. A sequentially compact topological space is maximally sequentially compact iff every sequentially compact subset is closed. In such spaces, countably compact subsets are closed and the topology is sequential. [See R. M. Dudley, On sequential convergence, Trans. Amer. Math. Soc. 112 (1964), 483-507.] If a topological space is sequential, every sequentially compact subset is closed. These topological spaces are S spaces [espaces S of Frechet] iff for $x \in X, M \subset X$ such that $x \in M^{\prime}$, there exists $N \subset M$ such that $[x]=N^{\prime}$. (Received November 19, 1964.)

65T-80. D. S. RAY, Bucknell University, Lewisburg, Pennsylvania. Extending homeomorphisms between certain subsets of $S^{3}$.

Suppose the dendrite $D$ has a countable number of endpoints. Then $D$ can be expressed as the union of a countable number of nonoverlapping arcs. If each of these arcs is polyhedral, $D$ is called essentially polyhedral. Theorem. If $D$ and $D^{\prime}$ are essentially polyhedral dendrites in $E^{3}$ then a given homeomorphism from $D$ onto $D^{\prime}$ which is semi-linear on each polyhedron in $D$ can be extended to $E^{3}$. A cactoid $C$ in $S^{3}$ is called essentially polyhedral if it has properties (1)-(3): (1) Each 2-sphere of $C$ is polyhedral. (2) If $D$ is a dendrite in $C$ which has no more than one point in common with any 2-sphere of $C$, then $D$ is essentially polyhedral. (3) The collection of 2-spheres $S$ of $C$ for which $S$ contains more than one cut point $x$ of $C$ such that no component of $C-x$ is a dendrite is a finite collection. Theorem. Suppose $C$ and $C^{\prime}$ are essentially polyhedral cactoids in $S^{3}$ and $h$ is an orientation-preserving homeomorphism from $C$ onto $C^{\prime}$ which is semi-linear on each polyhedron in $C$. Then $h$ can be extended to $S^{3}$ iff $h$ preserves regions in $C$ and $h^{-1}$ preserves regions in $C^{\prime}$. To say that $h$ preserves regions in $C$ means that if two points of $C$ are not separated by a 2 -sphere in $C$, then their images are not separated by the image 2 -sphere. (Received November 23, 1964.)

65T-81. MILOS ZLAMAL, Institute for Fluid Dynamics and Applied Mathematics, University of Maryland, College Park, Maryland. Asymptotic error estimates in solving the biharmonic equation by the method of finite differences.

Let $R$ be a plane region which is bounded by a simple closed curve $S$ consisting of segments parallel to either the $x$ or $y$ axis. The biharmonic equation $\Delta^{2} u=\phi$ and its thirteen-point difference analogue are considered in $R$. Under the usual assumptions the asymptotic estimate $E=O\left(h^{2}\right)$ for the discretization error $E=u-U$ is proved in the case of the boundary conditions $u=f$ and $\Delta u+\lambda(\partial u / \partial \nu-g)=0$ on $S(\lambda \geqq 0)$. In the case of boundary conditions $u=f$ and $\partial u / \partial \nu=g$ on $S$ the estimate is weaker, namely, $\mathrm{E}=\mathrm{O}\left(\mathrm{h}^{3 / 2}\right.$ ). (Received November 23, 1964.)

65T-82. P. K. HOOPER, Harvard University, Computation Laboratory, Cambridge 38, Massachusetts. Monogenic Post normal systems of arbitrary degree.

It is a well-known result of combinatorial systems theory that the halting problem (HP) for a Turing machine (TM) can be reduced to that of a monogenic Post normal system (MPNS), implying that the HP for a MPNS can be of the complete degree. A two-way reduction is now constructed such that for any instantaneous description (ID) of the TM there is a corresponding word of the MPNS having the same halting problem; and, moreover, every word of the MPNS either has a trivial halting problem or is the concatenation of a finite number of representations of ID's of the TM. These representations of ID's operate independently in the MPNS, so the HP of a word of the MPNS can be determined from the solution of the HP for each ID of the TM represented in the word. Therefore, the TM and the MPNS which simulates it have halting problems of the same (bounded truth-table) degree, demonstrating the existence of MPNS's of arbitrary degree. (Received November 30, 1964.)

65T-83. T. E. HARRIS, The Rand Corporation, Santa Monica, California. Diffusion with "collisions" between particles.

Let $x_{1}(t)$ and $x_{2}(t), t \geqq 0$, be Wiener processes with mutually independent increments, with $x_{1}(0)<x_{2}(0)$. Put $y_{1}(t)=\min \left(x_{1}(t), x_{2}(t)\right), y_{2}(t)=\max \left(x_{1}(t), x_{2}(t)\right)$. There is some justification for regarding $y_{1}$ and $y_{2}$ as the respective sample functions of two particles diffusing on the same line, unable to pass one another. The definition is extended appropriately to the case of infinitely many particles on the same line, no two of which can pass one another. Suppose that at time 0 particles are situated on the whole $x$-axis in accordance with a Poisson process of rate 1 , and an extra particle is placed at position 0 initially. Let $y_{0}(t)$ be the position at time $t$ of the extra particle. It is shown that for large $t$ the distribution of $y_{0}(t)$ is approximately Gaussian with mean 0 and standard deviation $(2 t / \pi)^{1 / 4}$. A different definition of diffusion with collisions has been given by H. P. McKean, Jr., using nonlinear diffusion equations. (Received December 2, 1964.)

65T-84. T. S. MOTZKIN, University of California, Los Angeles, California 90024. Quadratic forms positive for non-negative variables not all zero.

For real $\mathrm{a}_{\mathrm{jk}}=\mathrm{a}_{\mathrm{kj}}, \mathrm{j}, \mathrm{k}=1, \ldots, \mathrm{n}, \mathrm{n} \geqq 2$, we have $\sum \mathrm{a}_{\mathrm{jk}} \mathrm{x}_{\mathrm{j}} \mathrm{x}_{\mathrm{k}}>0$ for all $\mathrm{x}_{1} \geqq 0, \ldots, \mathrm{x}_{\mathrm{n}} \geqq 0$ except $x_{1}=\ldots=x_{n}=0$ if and only if all $a_{j j}>0$ and each principal minor a of ( $a_{j k}$ ) for which the cofactors of its last row are all positive is itself positive. The set $P_{+}$of such ("copositive") matrices in ( $\left.\begin{array}{c}n+1 \\ 2\end{array}\right)$-space is bounded by $2^{n}-1$ hypersurfaces $a=0$, whereas the set $P$ of coefficient matrices of positive definite quadratic forms is bounded only by one hypersurface, $\operatorname{det}\left(\mathrm{a}_{\mathrm{jk}}\right)=0$. And while P is the solution set of a system of n polynomial inequalities, $\mathrm{P}_{+}$is not the solution set of any system of finitely many polynomial inequalities. (See also, and amend, NBS Report 1818 (1952), 11-12.) (Received December 2, 1964.)

65T-85. HERBERT GROSS, Montana State College, Bozeman, Montana. On Witt's theorem.
A Kneser field is a commutative, nonreal field with $\operatorname{char}(\mathrm{k}) \neq 2$ and finite group $\mathrm{k}^{*} /\left(\mathrm{k}^{*}\right)^{2}$ (multiplicative group $\mathrm{k}^{*}$ of the nonzero elements of k modulo squares). Theorem: Let E be a k -vector space of a denumerable (algebraic) dimension, k a Kneser field. Let furthermore $\phi$ : $E \times E \rightarrow K$ be a nondegenerate symmetric bilinear form. If $F_{1}$ and $F_{2}$ are subspaces of $E$ with $F_{1}^{\perp}=(0), F_{2}^{\perp}=(0)$ and $\operatorname{codim}\left(F_{1}\right)=\operatorname{codim}\left(F_{2}\right)$, then there exists an orthogonal automorphism of the space E which maps $\mathrm{F}_{1}$ onto $\mathrm{F}_{2}$. The theorem is a companion to the corresponding theorem by Kaplanski, in which $k$ is an arbitrary commutative field but $\phi$ alternate, i.e. $\phi(x, x)=\sigma$ for all $\mathrm{x} \in \mathrm{E}$. (Received November 23, 1964.)

65T-86. B. L. FOSTER, Marathon Oil Company, Denver Research Center, P. O. Box 269, Littleton, Colorado 80121. Short proof of a theorem of Rado on graphs.

In a note to appear in Proc. Amer. Math. Soc., a new proof is given of Rado's theorem, using König's lemma. Theorem. Given a locally finite graph, ( $G, \Gamma$ ), a finite set of integers $K$ and a mapping $T$ of subsets of $K$ to subsets of $K$; if each finite subgraph, $\left(A, \Gamma_{A}\right)$, admits a function $\Phi_{A}$ such that $\Phi_{A}(\mathrm{x}) \in \mathrm{T}\left\{\Phi_{\mathrm{A}}\left(\Gamma_{\mathrm{A}} \mathrm{x}\right)\right\}$, for all $\mathrm{x} \in \mathrm{A}$, then $(\mathrm{G}, \Gamma)$ admits a function $\Phi$ such that $\Phi(\mathrm{x}) \in \mathrm{T}\{\Phi(\Gamma \mathrm{x})\}$, for
al: $x \in G$. Lemma. If ( $A_{1}, A_{2}, \ldots$ ) is a sequence of nonempty, pairwise disjoint finite sets and $<$ is any relation between elements of consecutive sets such that for all $x_{n} \in A_{n}$, an element $x_{n-1} \in A_{n-1}$ exists with $x_{n-1}<x_{n}$, then a sequence ( $a_{1}, a_{2}, \ldots$ ) exists with $a_{n} \in A_{n}$, for all $n$, such that $a_{1}<a_{2}<\ldots<a_{n}<\ldots$. The idea of the proof is to let $a_{n}$ be a suitable function on a suitable finite subgraph and to let < be functional extension. Other proofs [R. Rado, Axiomatic treatment of rank in infinite sets, Canad. J. Math. l (1949), 337-343; and C. Berge, The theory of graphs and its applications]. (Received November 20, 1964.)

65T-87. K. S. MILLER, Columbia University, Electronics Research Laboratories, 632 West 125 Street, New York 27, New York. Some properties of generalized multivariate t-distributions.

Let $X, X_{1}, X_{2}, \ldots, X_{p}$ be Gaussian vectors of dimensions $p, n_{1}, n_{2}, \ldots, n_{p}$ respectively. Let $t_{k}=x_{k} / r_{k}, l \leqq k \leqq p$, where $X=\left\{x_{1}, x_{2}, \ldots, x_{p}\right\}$ and $r_{k}=\left|X_{k}\right|, l \leqq k \leqq p$, is the norm of $X_{k}$. Then $T=\left\{t_{1}, t_{2}, \ldots, t_{p}\right\}$ will be called a generalized $p$-dimensional $t$ random vector. Many "generalized t variates" that have been observed in the literature may be subsumed under the above definition. Under various assumptions regarding the correlation of the $X, X_{1}, X_{2}, \ldots, X_{p}$ vectors, density functions and moments of the $T$ vector have been calculated. In some cases it is possible to give closed form expressions in terms of higher transcendental functions. Some results on joint distributions of ratios of correlated Gaussian variates have also been obtained. The main techniques used are those of linear algebra and the theory of special functions. (Received December 3, 1964.)

65T-88. OLGA TAUSSKY, California Institute of Technology, Pasadena, California. A matrix proof for a quaternion identity leading to an eight square identity.

It is shown that det $\mathrm{X} \cdot \overline{\operatorname{det} \mathrm{X}}=\operatorname{det}\left(\mathrm{XX}^{*}\right)$ if $\mathrm{X}=\left(\begin{array}{ll}\xi_{1} & \xi_{2} \\ \eta_{1} & \eta_{2}\end{array}\right)$ where $\xi_{1}, \xi_{2}, \eta_{1}, \eta_{2}$ are quaternions with $\xi_{1} \neq 0$ and $\operatorname{det} \mathrm{X}=-\xi_{1} \eta_{1} \xi_{1}^{-1} \xi_{2}+\xi_{1} \eta_{2}$. This leads to the identities mentioned. (Received December 4, 1964.)

65T-89. P. C. DELIYANNIS, Illinois Institute of Technology, Chicago, Illinois 60616. The irreducible representations of the unimodular group. Preliminary report.

The purpose of this note is to show how Bargmann's analysis of the infinitesimal (irreducible unitary) representations of the $2 \times 2$ real unimodular group (the continuous series) can be used, together with results of the author obtained in his thesis, to find the concrete form of these representations. [A similar argument, yielding the form of the discrete series, will appear in Proc. Amer. Math. Soc.] The idea, in all cases, is to obtain a commutative algebra of operators in the representation space, realize this as a function algebra on a quotient space of the group, and then interpret the inner products as measures on this space. (Received December 3, 1964.)

65T-90. D. C. SODA, Washington University, St. Louis, Missouri 63130. Some groups of type $D_{4}$ defined by Jordan algebras. Preliminary report.

For definitions consult Jacobson, Groups of transformations. II, Crelle's Journal 201 (1960). Let $H$ be an exceptional central simple Jordan algebra over a field $F$ of characteristic $\neq 2,3$. H has
degree three and contains subfields which are cubic extensions of $F$. If $K$ is such a subfield, the group Aut $(H / K)$ of automorphisms of $H$ leaving $K$ elementwise fixed is a linear algebraic group of type $D_{4}$. If $H$ is split, Aut $(H / K)$ is a simple group. If $H$ is split and $K$ is a cyclic extension of $F$, this group is isomorphic to the group constructed by Tits (Sur la trialite et certains groupes qui s'en deduisent, Inst. Hautes Études Sci. Publ. Math. 2 (1959)) and the group of type $\mathrm{D}_{4}^{2}$ constructed by Steinberg (Variations on a theme of Chevalley, Pacific J. Math. 9 (1959), No. 2). If H is split and K is not a cyclic extension of $F$, then $\operatorname{Aut}(H / K)$ is isomorphic to the group ot type $D_{4}^{3}$ constructed by Steinberg (loc.cit.). The involutions of $\operatorname{Aut}(\mathrm{H} / \mathrm{K}), \mathrm{H}$ split, are reflections in 15 -dimensional central simple subalgebras which contain $K$. When $A$ is reduced but not split, Aut(H/K) has no unipotent elements. (Received December 3, 1964.)

65T-91. H. R. BAILEY and M. Z. WILLIAMS, Colorado State University, Fort Collins, Colorado 80521. On the differential-difference equation $d x / d t=\sum_{l=0}^{n} a_{i} x(t-i)$.

An explicit solution of the title equation is obtained for $t>0$ when the $a_{i}, i=0,1, \ldots, n$, are constants and $x(t)=0$ for $t<0, x(0)=1$. Sufficient conditions for stability of the zero solution are obtained by applying Liapounov's Second Method. Necessary and sufficient conditions for stability of the zero solution are given in the case $a_{0}=0 ; a_{i}=0, i=3,4, \ldots, n ; a_{1} \neq 0, a_{2} \neq 0$. This result is put in the form of a closed region in the $a_{1}, a_{2}$ plane within which the zero solution is stable. (Received November 20, 1964.)

65T-92. FRED RICHMAN and E. A. WALKER, New Mexico State University, University Park, New Mexico. Primary groups as modules over their endomorphism rings. Preliminary report.

Let $G$ be a $p$-group and $E$ its ring of endomorphisms. It is well known that $E$ determines $G$. A direct construction of $G$ from $E$ yields: Theorem 1. $G$ is a flat $E$-module iff $G$ is a direct limit of cyclic $E$-summands of $E$ iff $G$ is not bounded plus nonzero divisible. Criteria for $G$ to be a projective E-module are contained in: Theorem 2. The following conditions are equivalent: (1) G is a projective E-module, (2) $G$ is an $E$-summand of $E$, (3) $G$ is a submodule of $E$, (4) $G$ is a submodule of a free $E$-module, (5) $G$ is a cyclic $E$-module, (6) $G$ is bounded, (7) $G$ is reduced and every element of $E$ can be extended to an endomorphism of any group containing $G$ as its torsion subgroup. (Received November 23, 1964.)

65T-93. J. W. DETTMAN, Oakland University, Rochester, Michigan and R. L. RAYMOND, Case Institute of Technology, Cleveland, Ohio. Perturbation of spectra in Sturm-Liouville problems.

A class of spectral perturbation problems is considered for the Sturm-Liouville problem: $y^{\prime \prime}+[\lambda+q(x)] y=0, a \leqq x \leqq b, a_{1} y^{\prime}(a)+\beta_{1} y(a)=0, a_{2} y^{\prime}(b)+\beta_{2} y(b)=0 . q, a, b, a_{1}, a_{2}, \beta_{1}, \beta_{2}$ can be functions of a small parameter $\epsilon>0$, thus generating a large class of perturbation problems. The problem is formulated as a Fredholm integral equation involving a Green's function which is a function of $\epsilon$. It is proved that the Green's function is analytic in $\epsilon$ for small $\epsilon$ and, therefore, that the Fredholm operator can be expanded in a power series in $\epsilon$ and the methods of quantum mechanics can be used to find the perturbation of the spectrum as a function of $\epsilon$. The method also has applications in partial differential equations. (Received November 23, 1964.)

65T-94. M. P. BERRI, Tulane University, New Orleans, Louisiana. Lattice complements of certain topologies.

Hartmanis (Canad. J. Math. 10 (1958), 547-553) has shown that for any finite set the lattice of topologies on this set is always complemented. Gaifman (Abstract 61T-161, these CNotices) 8 (1961), 356) has asserted that the lattice of topologies on a countable set is complemented. In this abstract, we discuss the complements of certain topologies on sets of arbitrary infinite cardinality. Theorem 1. Let ( $\mathrm{X}, \mathscr{G}$ ) be a topological group and let H be a dense, non-open, countable subgroup of X . Then $\mathscr{T}$ is complemented in the lattice of topologies on X 。Corollary. If $(\mathrm{X}, \mathscr{T})$ is the space of the real numbers with the natural topology, then ( $\mathrm{X}, \mathscr{T}$ ) is complemented in the lattice of topologies on the set of real numbers. Theorem 2. The minimum Frechet topology on any set X is complemented. Furthermore, if X is infinite the minimum Frechet topology always has more than one complement. Hartmanis (ibid.) has shown that on any finite set containing three or more elements any topology which is neither the discrete topology nor the indiscrete topology has at least two complements. Corollary. If X is a set of three or more elements, then the lattice of topologies on X is not distributive. (Received November 25, 1964.)

65T-95. H. A. SMITH, 4625 Larchwood Avenue, Philadelphia, Pennsylvania 19143. Tensor products of complete commutative locally m-convex Q -algebras.

The problem of giving satisfactory conditions under which the complete projective tensor product of two complete locally m-convex algebras will be a Q-algebra was posed in Abstract 614-133 (these $\mathcal{C}$ (Notices) 11 (1964), 567). It is shown here that the complete projective tensor product of two complete commutative locally m-convex Q-algebras is again such an algebra. As a corollary, the space of nonzero multiplicative functionals (with relative weak topology induced by the algebra) on such a tensor product is characterized as the topological product of those of the two algebras. If the two algebras are also semi-simple, then the complete projective tensor product is semi-simple if and only if the tensor product of the dual spaces of the two algebras is dense in that of the complete projective tensor product (with the weak topology induced by the algebra). These results generalize theorems of Gelbaum and Tomiyama. The complete algebra of D-valued functions on a locally compact Abelian group is presented and studied as an example; D being the space of infinitely differentiable functions of compact support. (Received November 25, 1964.)

65T-96. K. W. ENDL, University of Utah, Salt Lake City, Utah. On a density theorem for finite differences and an application to Hausdorff means.

Let $N=\left\{n_{1}, n_{2}, \ldots\right\}$ be a finite or infinite set of natural numbers. We consider the problem under what assumptions on $N$ and a sequence $\left\{S_{n}\right\}_{0}^{\infty}$ the equations (I) $\Delta^{n_{i}} \mathrm{~S}_{0}=0$ imply (II) $S_{n}=P$ ( $n$ ) where $P(x)$ is a polynomial. The following is shown: If (1) $S_{n}=O\left(n^{k}\right)(k \geqq 0)$, (2) $\underline{D}(N)=$ $\lim _{\inf }^{x \rightarrow \infty}<(2 / \pi) \int_{0}^{\infty}(N(t) / t)\left(x /\left(x^{2}+t^{2}\right)\right) d t \geqq 1 / 2(\underline{D}(N)$ is the Poisson density, introduced by R. C. Buck) then (I) implies (II) with $P(x)$ of degree $\leqq[k]$. This result generalizes and extends results by R. P. Agnew, R. P. Boas, W. H. J. Fuchs and H. Pollard. The proof is based on Newtonian interpolation series. (Received November 25, 1964.)

65T-97. JET WIMP, Midwest Research Institute, Mathematical Analysis Section, 425 Volker Boulevard, Kansas City, Missouri 64110. An integral transform pair involving Whittaker's function.

In this paper we investigate an integral transform and derive an inversion formula for it. Both the kernel of the transform and of its single integral inversion formula contain a Whittaker function, and integration is performed with respect to a parameter of that function. Our main theorem bears a relationship to the numerous formulae, due to Buchholz, which express various types of waves as integrals of the functions of the paraboloid of revolution. (Received November 27, 1964.)

65T-98. JERRY FIELDS, Midwest Research Institute, Mathematical Analysis Section, 425 Volker Boulevard, Kansas City, Missouri 64110. The asymptotic expansion of a ratio of gamma functions.

A proof is given for the following result: Theorem. If a and $\beta$ are bounded quantities, $\mathrm{E}(\mathrm{v})$ is a function analytic in a neighborhood of $v=0, E(0)=1$, and $w$ is defined implicitly by $z+(a+\beta-1) / 2$ $=w E\left(w^{-2}\right)$, then there exist numbers $c_{j}$ such that $\Gamma(z+a) / \Gamma(z+\beta) \sim \sum_{j=0}^{\infty} c_{j} w^{\alpha-\beta-2 j}, z \rightarrow \infty$, $|\arg (z+a)|<\pi, c_{0}=1$. (Received November 27, 1964.)

65T-99. T. HIDA, Indiana University, Bloomington, Indiana 47405. Finite-dimensional approximations to White noise and Brownian motion. Preliminary report.

Let ( $\Omega, \mathrm{B}, \mathrm{P}$ ) be the Gaussian measure space obtained by taking the projective limit of the measure spaces ( $\Omega_{2 n}, B_{2 n}, P_{2 n}$ ), $n \geqq 1$, where $\Omega_{n}$ is obtained by deleting a certain ( $n-2$ )-dimensional space from the $n$-sphere $S^{n}$ in ( $n+1$ )-dimensional Euclidean space, $B_{n}$ is the family of Borel subsets of $\Omega_{n}$, and $P_{n}$ is the restriction to $B_{n}$ of the uniform probability measure over $S^{n}$ [cf. T. Hida and H. Nomoto, Proc. Japan Acad. 40 (1964)]. The measure P shares some of the important properties of $P_{n}$. We consider a group $\left\{\mathrm{T}_{\mathrm{t}}^{(\mathrm{n})}\right.$; t real $\}$ of transformations of $\Omega_{2 n}$ into itself, which are restrictions of certain rotations of $\mathrm{S}^{2 \mathrm{n}}$, and such that $\mathrm{T}_{\mathrm{t}+2 \pi}^{(\mathrm{n})}=\mathrm{T}_{\mathrm{t}}^{(\mathrm{n})}$. We let $\mathrm{T}_{\mathrm{t}}$ be the projective limit of $\mathrm{T}_{\mathrm{t}}^{(\mathrm{n})}$, i.e., $\mathrm{T}_{\mathrm{t}}\left(\omega_{2}, \omega_{4}, \ldots\right)=\left(\mathrm{T}_{\mathrm{t}}^{(1)}\left(\omega_{2}\right), \mathrm{T}_{\mathrm{t}}^{(2)}\left(\omega_{4}\right), \ldots\right), \omega_{2 \mathrm{n}} \in \Omega_{2 \mathrm{n}}$. We can then show that the flow $\left\{\mathrm{T}_{\mathrm{t}} ; \mathrm{t}\right.$ real $\}$ restricted to $[0,2 \pi$ ) is equivalent to the flow of Brownian motion on $(\Omega, \mathrm{B}, \mathrm{P}$ ) over the time interval $[0,2 \pi$ ). We can also express the Brownian motion process and White noise as limits of processes in $L^{2}\left(\Omega_{2 n}, B_{2 n}, P_{2 n}\right)$. Our expression for Brownian motion is analogous to the formula for Brownian motion as a random Fourier series due to Paley and Wiener. This work suggests a method for studying spherical harmonics and related questions in infinite-dimensional spaces. (Received November 27, 1964.)

65T-100. P. R. MASANI, Indiana University, Bloomington, Indiana 47405. The normality of time-invariant, subordinative operators in a Hilbert space.

Let ( $U_{t}, t$ real) be a strongly continuous group of unitary operators on a Hilbert space $\mathscr{H}_{\text {, }}$ E be its spectral measure on the family $B$ of Borel subsets of the reals $\mathscr{R}$, and $\mathscr{S}_{\mathrm{x}}$ be the cyclic subspace generated by $x$ under the action of $U_{t}, t \in \mathscr{R}$. Let $T$ be any (single-valued, unbounded) operator from $H$ to $H$ such that (i) $U_{t} T=T U_{t}, t \in \mathscr{R}$; (ii) $T(x) \in \mathscr{S}_{\mathrm{x}}$, for all $\mathrm{x} \in \mathscr{D}_{\mathrm{T}}$; (iii) T is closed and $\mathscr{H}_{0}=$ clos $\mathscr{D}_{\mathrm{T}}$ is separable. We show that there exists a B-measurable complex-valued
function $\phi$ on $\mathscr{R}$ such that $\mathrm{T}=\int_{-\infty}^{\infty} \phi(\lambda) \mathrm{E}_{0}(\mathrm{~d} \lambda)$, where $\mathrm{E}_{0}$ is the restriction of E to $\mathscr{H}_{0}$. This result provides a rigorous basis for the introduction of the frequency response function into the statistical theory of time-invariant, linear filters. The conditions (i)-(iii) are equivalent to the condition that the commutant of $\int_{-\infty}^{\infty} \lambda E(d \lambda)$ is included in that of $T$. But whereas the well-known theorem involving commutants follows easily from ours, the derivation of our result from the latter is nontrivial.
(Received November 27, 1964.)

65T-101. R. H. BING, University of Wisconsin, Madison, Wisconsin. Radial engulfing.
Stallings and Zeeman showed that if $U$ is an open subset of a piecewise-linear n-manifold $M^{n}, i \leqq n-3$, and ( $M^{n}, U$ ) is i-connected, then for each finite $i$-complex $K$ in $M^{n}$ there is an engulfing isotopy $h_{t}: M^{n} \rightarrow M^{n}(0 \leqq t \leqq 1)$ such that $h_{0}=I, h_{t}=I$ except on a compact set and $K \subset h_{1}(U)$. We strengthen the result in two directions. In one direction we show that if $G$ is an upper-semicontinuous decomposition of $M^{n}$ such that the homotopies showing that ( $M^{n}, U$ ) is i-connected can be chosen so that each point moves near some element of $G$, then the engulfing isotopy can be chosen so that each point flows near an element of $G$. In another direction we loosen the closed restriction on $k$. If $C$ is a closed set, $X$ is a compact set such that $X-C$ is locally polyhedral, dimension $(X-C) \leqq i$, and $X \cap C \subset \bar{U}$ then there is an engulfing isotopy $h_{t}$ such that $h_{t} / C=I, X-C \subset h_{1}(U)$ if ( $M^{n}-C, U-C$ ) is i-connected and locally i-connected at each point of $\bar{U}$. (Received November 30, 1964.)

65T-102. M. M. RAO, Carnegie Institute of Technology, Schenley Park, Pittsburgh, Pennsylvania 15213. Conditional expectations and closed projections.

Let $(\Omega, \Sigma, \mu)$ be a probability space, and $L^{\Phi}(\Sigma), L^{\Psi}(\Sigma)$ be (complementary) Orlicz spaces on it. Let $M^{\Phi} \subset L^{\Phi}(\Sigma)$ be the subspace spanned by simple functions, and $L^{\Phi}(B) \subset L^{\Phi}(\Sigma)$ for a $\sigma$-field $\mathfrak{B} \subset \Sigma$. Theorem 1. Let $M^{\Phi}=L^{\Phi}(\Sigma)$, and $P$ on $M^{\Phi}$ onto $L^{\Phi}(\mathfrak{B})$ be a bdd projection. If either (i) $\|\mathrm{P}\| \leqq 1$ and the $\Psi$ is continuous, or (ii) the bdd functions of $L^{\Phi}(\mathfrak{B})$ are fix points of the adjoint $P^{*}$ of $P$, then $\operatorname{Pf}=E^{\mathfrak{B}}(f), \forall f \in M^{\Phi}$. ( $E^{\mathfrak{B}}(\cdot)=$ conditional expectation.) Theorem 2. If $L^{\Phi}(\Sigma)$ is any Orlicz space and bdd $P$ on $L^{\Phi}$ is an averaging (i.e., $f, g \in L^{\Phi}(\Sigma)$, bdd $\Rightarrow P(f P g)=(P f)(P g)$ and $\left.P 1=1\right)$, then $\exists$ a $\sigma$-field $\mathfrak{B} \subset \Sigma$ with $P \chi_{A}=\chi_{A}$ for $A \in \mathfrak{B}$, and a $g \in L^{\Psi}(\Sigma), \ni\left({ }^{*}\right) \operatorname{Pf}=E^{\mathscr{B}}(\mathrm{fg}), \forall f \in L^{\Phi}(\Sigma)$. ( $\chi_{\mathrm{A}}=$ indicator of A.) Definition. Let $\mathrm{f} \in \mathrm{L}^{\Phi}(\Sigma)$ and $L^{\Phi}(\mathfrak{B}) \subset L^{\Phi}(\Sigma), L^{\Phi}$ separable. ( $\mu$ may be nonfinite.) Then a proj $P_{\mathfrak{B}}^{f}$ in $L^{\Phi}(\Sigma)$ onto $L^{\Phi}(\mathfrak{B})$ is called a closed conditional expectation w.r.t. $\mathfrak{B}$ and $f$ iff: $\exists g \in L^{\Phi}(\mathfrak{B}), \ni\|f-g\| \leqq\|f-h\|, \forall h \in L^{\Phi}(\mathfrak{B}) \Rightarrow f \in \operatorname{dom}\left(P_{\mathfrak{B}}^{f}\right)$ and $P_{\mathfrak{B}}^{f} f=g$, a.e. ( $P_{\mathfrak{B}}^{f}$ exists, has many properties of $E^{\mathfrak{B}}(\cdot)$, and is a closed projection, but does not preserve the "closeness" property under addition.) Theorem 3. Let $L^{\Phi}(\Sigma)$ be uniformly convex and the $\sigma$-fields $\mathfrak{B}_{\mathrm{n}} \subset \Sigma$ be $\nearrow$, and $\mathfrak{B}=\sigma\left(\bigcup_{1}^{\infty} \mathfrak{B}_{\mathrm{n}}\right)$. If $\mathrm{f} \in \mathrm{L}^{\Phi}(\Sigma)$ and $\mathrm{g}_{\mathrm{n}}=\mathrm{P}_{\mathfrak{B}_{\mathrm{n}}}^{\mathrm{f}}$ in $L^{\Phi}\left(\mathfrak{B}_{\mathrm{n}}\right)$, then $\left\{g_{\mathrm{n}}\right.$, $\left.\mathrm{n} \geqq 1\right\}$ is a (closed) martingale, $g_{n} \rightarrow g=P_{\mathfrak{B}}^{f} f$ in $L^{\Phi}$-norm and also a.e. (Received December 2, 1964.)

65T-103. D. A. DAYBELL, New Mexico State University, University Park, New Mexico. Representation theorems for operators into a locally convex function space.

Let S be a topological space, $\mathscr{A}$ a collection of subsets A of S , with $\mathrm{S} \subset \cup \mathscr{A}$. Let $\mathrm{BC}_{\mathscr{A}}$ (S) be the space of all continuous complex-valued functions on $S$ which are bounded on each member of $\mathscr{A}$, with the topology of uniform convergence on members of $\mathscr{A}$. Let E be a locally convex space,

W a convex, circled, $W^{*}$-closed, equicontinuous ( $\mathrm{ccw}^{*}$ e) subset of the dual $\mathrm{E}^{\prime}$. Let $\mathrm{E}_{\mathrm{W}}^{\prime}$ be the Banach space whose unit sphere is W . Theorem: If T is a continuous linear operator from E to $\mathrm{BC}_{\mathscr{A}}(\mathrm{S})$, then there is a unique map prom $S$ to $E^{\prime}$ which satisfies: (a) (Tx)(s) $=(p s)(x)$ for $x$ in $E, s$ in $S$; (b) for each A in $\mathscr{A}, \mathrm{p}[\mathrm{A}]$ is equicontinuous; (c) p is continuous with respect to the $\mathrm{w}^{*}$-topology on $\mathrm{E}^{\prime}$. Conversely, if $p$ is a map from $S$ to $E^{\prime}$ which satisfies (b) and (c), then (a) defines a continuous linear operator $T$ from $E$ to $\mathrm{BC}_{\mathscr{A}}(\mathrm{S})$. Theorem: If T is a w -compact [compact] operator from E to $\mathrm{BC}_{\mathscr{A}}(\mathrm{S})$, there are a ccw*e set $W$ in $E^{\prime}$ and a map $p$ from $S$ to $E_{W}^{\prime}$ which satisfies: (a) above; (b') for each $A$ in $\mathscr{A}, p[A]$ is relatively w-compact [rel. compact] in $E_{W}^{\prime}$; ( $c^{\prime}$ ) $p$ is continuous with respect to the $w$-topology on $E_{W}^{\prime}$. Conversely, if there are a ccw*e set $W$ in $E^{\prime}$ and a map prom $S$ to $E_{W}^{\prime}$ which satisfies ( $b^{\prime}$ ) and ( $c^{\prime}$ ), then (a) defines a w-compact [compact] operator $T$ from $E$ to $B C \mathscr{A}^{(S)}$. (Received December 7, 1964.)

65T-104. J. M. WEINSTEIN, University of Wisconsin, Madison, Wisconsin 53706. Sentences preserved under direct products. Preliminary report.

Let $s_{1}, s_{2}$, $s$ be formulas of a countable first-order language which has variables $v_{n}(n<\omega)$. Write " $s_{1} \times s_{2} \rightarrow s$ " if - treating free variables as constants $-s$ holds in every direct product $A_{1} \times A_{2}$ of models $A_{1}$ of $s_{1}$ and $A_{2}$ of $s_{2}$. In this paper are constructed sets $T_{n}(n<\omega)$ of formulas and a primitive recursive relation $R$ of triples of formulas such that: (1) To each finite set of formulas there is a $T_{n}$ containing logical equivalents of each of the formulas of the set; (2) If $R\left(s_{1}, s_{2}, s\right)$, then $s_{1} \times s_{2} \rightarrow s$; (3) If $s_{1}, s_{2}$ and $s$ are in some $T_{n}$ and $s_{1} \times s_{2} \rightarrow s$, then $R\left(s_{1}, s_{2}, s\right)$. From these properties follows: Theorem. A sentence $s$ is preserved under direct products iff $s$ is equivalent to a sentence $t$ satisfying the primitive recursive condition $R(t, t, t)$. This theorem yields a similar primitive recursive criterion for sentences preserved under direct powers, since such sentences can be shown to be equivalent to disjunctions of sentences preserved under direct products. (Received December 2, 1964.)

65T-105. J. J. PRICE, Institute for Advanced Study, Princeton, New Jersey and R. E. ZINK, Purdue University, Lafayette, Indiana. On sets of functions that can be multiplicatively completed.

Boas and Pollard have shown that if $\left\{\phi_{n}\right\}_{1}^{\infty}$ is a complete orthonormal set in $L^{2}(0,1)$, then given any positive integer $k$, there is a bounded measurable nonnegative function $m$ such that the set $\left\{\mathrm{m} \phi_{\mathrm{n}}\right\}_{\mathrm{k}}^{\infty}$ is complete. They exhibit an orthonormal set that cannot be completed by adjoining a finite number of elements but that can be multiplicatively completed in this way. Let $\Phi=\left\{\phi_{n}\right\}_{1}^{00}$ be a family of measurable real functions on a separable measure space $S$ of total measure 1 . We say that $\Phi$ has property BP if there is a bounded measurable nonnegative function $m$ such that $\left\{m \phi_{n}\right\}_{l}^{\infty}$ is a total set in $L^{2}(S)$. Property $B P$ is proved equivalent to each of the following two properties. (a) $\Phi$ is total in the sense of convergence in measure. (b) For each positive number $\epsilon$, there is a set $S_{\epsilon}$ of measure at least $1-\epsilon$, such that $\Phi$ is total in $L^{2}\left(S_{\epsilon}\right)$. It follows then by a result of Goffman and Waterman that from any set $\Phi$ having property BP an infinite number of deletions can be made so that the resulting set $\Phi^{\prime}$ will also have property BP. (Received November 24, 1964.)

65T-106. W. J. JONSSON, University of Manitoba, Winnipeg, Canada. Doubly-transitive groups, nearfields and geometry.

Let $G$ be doubly transitive on $\Sigma$. A well-known theorem of Jordan states that if $G$ is sharply doubly transitive, then the identity together with those elements fixing no symbol of $\Sigma$ form a normal subgroup regular on $\Sigma$. This theorem may be proven geometrically as follows: Construct a net $N$ whose points are elements of $\Sigma \times \Sigma$. The orbit of a point $(x, y)$ with $x \neq y$ under the stabilizer of $\mathrm{z} \in \Sigma$. Together with $(\mathrm{z}, \mathrm{z})$ is a line. Points $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right),\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right)$ are not joined if and only if either one of them is in the diagonal or the unique $\sigma \in G$, with $\mathrm{x}_{1}^{\sigma}=\mathrm{x}_{2}, \mathrm{y}_{1}^{\sigma}=\mathrm{y}_{2}$, fixes no points. The net can be completed in a unique manner to an affine plane by adjunction of one class of parallel lines. Jordan's theorem follows from the fact that two points (not in the diagonal) lie on a line of the new parallel class if and only if they are of the type described above. As a byproduct one finds that the plane can be co-ordinatized by a nearfield. (Received November 30, 1964.)

65T-107. HAROLD BELL, State University of New York at Stony Brook, Stony Brook, New York. On fixed-point properties of plane continua.

For a plane continuum $Q, T(Q)$ shall denote the complement of the unbounded component of the complement of $Q$. Let $f$ be a complex-valued continuous function defined on $T(Q)$, for some plane continuum $Q$. If $f(Q) \subset T(Q)$, there is an $x \in T(Q)$ such that $x=f(x)$, or there is an indecomposable continuum $M \subset Q$ such that $f(M)=M$. Furthermore, there is an indecomposable continuum $K \subset T(Q)$ such that $f(K)=K$ and if $N$ is a subcontinuum of $T(K)$ for which $f(N) \subset T(N), T(K)=T(N)$. (Received November 25, 1964.)

65T-108. O. T. NELSON, JR., Box 1645, Station B, Vanderbilt University, Nashville, Tennessee. Lattices of width 2 and subdirect products of finite lattices. Preliminary report.

Let $L$ be a lattice. Define distinct $x, y \in L$ to be finitely pointwise separable if and only if there exists a congruence relation $\theta$ over $L$ which determines a finite number of equivalence classes and such that $\mathrm{x} \not \equiv \mathrm{y}(\theta)$. Using a result of Birkhoff (Lattice theory) one has: Theorem 1 . L is a subdirect product of finite lattices if and only if for distinct $x, y \in L, x$ and $y$ are finitely pointwise separable. It is shown that the normal completion of a lattice of width 2 is of width 2 . From Theorem 1, the fact that the completion of a lattice of width 2 is of width 2, and Dilworth's decomposition of partially ordered sets (Ann. of Math. (2) 51 (1950), 161-166), it is shown that: Theorem 2. All lattices of width 2 are subdirect products of finite lattices. The proof involves construction of congruence relations satisfying Theorem 1. (Received November 27, 1964.)

65T-109. T. A. BURTON, University of Alberta, Edmonton, Alberta, Canada. Stability by use of orthogonal trajectories.

A system (1) $\left\{x^{\prime}=P(x, y), y^{\prime}=Q(x, y)\right\}$ is considered where $P$ and $Q$ are continuously differentiable and ( 0,0 ) is an isolated singular point. We form (2) $d y / d x=Q / P$ and the equation of the orthogonal trajectories (3) $d y / d x=-P / Q$. It is observed that solutions for (3) may be easily found in cases where solutions of (2) are not (e.g., the Lienard equation). We show that stability statements
about the null solution of (1) can be inferred from solutions of (3). Theorem 1. If ( 0,0 ) is a saddle point for (3), then ( 0,0 ) is a saddle point for (1). Theorem 2. If $P d x+Q d y=0$ is exact (or has an integrating factor $z(x, y)$ which is positive or negative definite) and has a solution $V(x, y)=c$ which is definite, then V is a Liapunov function for (1). This theorem is modified to make Cetaev's theorem apply. Theorem 3. Let $G$ be a periodic solution to (3) with inward normal (X,Y). G is the boundary of a positively invariant set for ( 1 ), if and only if $[(P, Q),(X, Y)]$ is positive on $G$. Other theorems regarding periodic solutions are obtained. Finally, if the solution $V(x, y)=c$ of (2) is a spiral in every sufficiently small neighborhood of ( 0,0 ), then conditions are given which reflect the asymptotic stability or instability of the null solution of (1). (Received November 30, 1964.)

65T-110. D. F. REARICK, University of Colorado, Boulder, Colorado 80304. Semi-multiplicative functions.

By arithmetical function we understand a complex-valued function of a real variable which is zero if the argument is not a positive integer. We introduce the following generalization of multiplicative arithmetical function: $f$ is semi-multiplicative if there exists a constant $c$, a positive integer a, and a multiplicative function $f^{\prime}$ such that $f(n)=c f^{\prime}(n / a)$ for all $n$. Theorem: $f$ is semi-multiplicative if and only if $f$ satisfies the functional equation $f(m) f(n)=f((m, n)) f(\langle m, n\rangle)$. Semi-multiplicative functions possess Euler products and thereby retain the essential advantage of multiplicativity. There are examples of familiar functions which are semi-multiplicative but not multiplicative. Furthermore, semi-multiplicativity is less fragile than multiplicativity and is preserved under many transformations which destroy the latter. (Received November 23, 1964.)

65T-111. S. B. BANK, University of Illinois, Urbana, Illinois. An asymptotic analogue of the Fuchs regularity theorem. Preliminary report.

Fuchs' theorem on nth order homogeneous linear ordinary differential equations states, in part, that if $\infty$ is a regular singularity of $\Omega(z, w)=\sum_{m=0}^{n} P_{n-m}(z) w(m)=0$, then this equation possesses a fundamental set of solutions, each asymptotically equivalent, as $z \rightarrow \infty$, to a function of form $c z^{a}(\log z)^{m}$. Here we consider a broader class of equations, namely those of the above form whose coefficients $P_{j}$ lie in a logarithmic domain of rank zero (see Strodt, Mem. Amer. Math. Soc. No. 13 (1954) and No. 26 (1957)). Roughly this means that in some sectorial region, the $P_{j}$ are analytic and have asymptotic expansions there ( $\mathrm{as} \mathrm{z} \rightarrow \infty$ ) in terms of decreasing real powers of $z$ and/or functions asymptotically smaller (in the sense of Memoir No. 13) than all powers of $z$. Our main result states that if such an equation $\Omega(\mathrm{z}, \mathrm{w})=0$ satisfies a condition analogous to regularity in the Fuchs theory, then in some sectorial region the equation has a fundamental set of solutions each asymptotically equivalent (in the sense of Memoir No. 13) to a function of form $\mathrm{cz}^{\mathrm{a}}(\log \mathrm{z})^{\mathrm{m}}$ as $\mathrm{z} \rightarrow \infty$. Sufficient conditions for existence of $\mathrm{q}(<\mathrm{n})$ such linearly independent solutions are also obtained. (Received December 11, 1964.)

65T-112. S. M. ROBINSON, Union College, Schenectady, New York. The intersection of the free maximal ideals in a complete space.

Theorem. If X admits a complete uniform structure, then the intersection of all free maximal ideals in $C(X)$ is $C_{K}(X)$, the ring of functions with compact support. This result extends a theorem of Gillman and Jerison (Rings of continuous functions, p. 123). In particular, our theorem shows that the result holds for all paracompact and metric spaces, even assuming the existence of measurable cardinals. If all cardinals are nonmeasurable, our theorem is equivalent to that of Gillman and Jerison. (Received December 11, 1964.)

65T-113. ALFRED ADLER, Purdue University, Lafayette, Indiana. The six-sphere does not admit a complex structure. Preliminary report.

If J is a complex structure on $\mathrm{S}^{6}$ and h a compatible hermitian metric on $\mathrm{S}^{6}$, then the first Chern form $c_{1}$ of $h$ defines a compatible symmetric bilinear form on $S^{6}:(X, Y)=c_{1}(X, J Y)$. The fundamental form of this bilinear form is $c_{1}$, and hence is closed. An integrability condition for almost-comp.lex structures is found (use is made at this point of the Nash imbedding theorem and of the associated mappings into Grassmannians), a condition which in the case of $S^{6}$ shows that the form ( $\mathrm{X}, \mathrm{Y}$ ) is positive-definite. Thus this form defines a Kähler metric on $\mathrm{S}^{6}$, which is too ridiculous for words in view of the fact that $\mathrm{H}^{2}\left(\mathrm{~S}^{6}\right)=0$. (Received November 23, 1964.)

65T-114. R. C. BASINGER, 1716 Tennessee Street, Lawrence, Kansas. An extension of a theorem of Bernstein to meromorphic functions.

The following theorem is proved by the method of A. J. Macintyre and S. M. Shah (J. Math. Anal. Appl. 3 (1961), 351-354) and extends the result given there. Theorem. Let $f(z)$ be a meromorphic function which satisfies: (l) $\lim _{\sup } \mathrm{ram}_{\rightarrow \infty} \mathrm{r}^{-1} \mathrm{~T}(\mathrm{r}, \mathrm{f})=\mathrm{t}<\infty$, (2) $|\mathrm{f}(\mathrm{x})| \leqq \mathrm{A} \exp \left(\mathrm{B}|\mathrm{x}|^{\mathrm{p}}\right)$ for $-\infty<x<\infty(A, B>0 ; 0 \leqq p<1)$, (3) $f(z)$ has no poles in $|\operatorname{Im}(z)|<h$, and (4) $\sum_{1}^{\infty}\left|\operatorname{Im}\left(b_{n}\right)\right|\left|x-b_{n}\right|^{-2}$ $\leqq C<\infty$ for $-\infty<x<\infty$, where $b_{1}, b_{2}, \ldots$ are the poles of $f(z)$. Then $\left|f^{\prime}(x)\right|<K \exp ((3+2 / \pi) B|x| p)$ for $-\infty<x<\infty$, where $K=K(A, B, C, t, p, h)$. (Received December 2, 1964.)

65T-115. WITHDRAWN.

65T-116. T. J. HEAD, Iowa State University, Ames, Iowa. Purity in lattices.
The concept of purity in Abelian group theory can be carried to the theory of compactly generated lattices $L$ by: p pure in $L$ if for each compact element $c$ in $L$ there is an element $x$ in $L$ for
which $p \cup x=p \cup c$ and $p \cap_{x}=0$. The assumption of modularity on $L$ is sufficient to give: $x$ pure in $L$ and $y$ pure in $x / 0 \Rightarrow y$ pure in $L$; $x$ pure in $L$ and $y$ pure in $1 / x \Rightarrow y$ pure in $L$. The union of a chain of pure elements is pure only under a further hypothesis, such as: the compact elements of L form an ideal. Call lattices satisfying all conditions mentioned Noetherian. The notion of purity in Noetherian lattices can be used to clarify the lattice-theoretic character of propositions in Abelian groups. Example: The discussion of almost local purity by D. L. Boyer and E. A. Walker [Pacific J. Math. 9 (1959), 409-413] can be carried out in this context. Our techniques are elementary. (Received November 27, 1964.)

65T-117. G. P. MURPHY, St. Louis University, St. Louis, Missouri 63103. Embedding properties of a class of 2 -metrics.
A. Froda (Espaces p-metriques et leur topologie, C. R. Acad. Sci. Paris 247 (1958), 849-852) generalizes the concept of a metric to that of a p-metric. In the euclidean plane if $p$ is taken to be 2 , one p -metric is the area determined by a triple of points. For a class of 2 -metrics we have the following Theorem: If every seven points of a 2-metric space $S$ containing more than seven points are "area" embeddable in $\mathrm{E}_{2}$, then S is "area" embeddable in $\mathrm{E}_{2}$ (i.e., there exists an area-preserving homeomorphism of $S$ onto a subset of $E_{2}$ ). Under certain conditions the theorem is true if "seven points" is replaced by "six points". An example is given of a space containing six points every five of which are "area" embeddable in $E_{2}$ but the six are not. (Received November 23, 1964.)

65T-118. G. G. WEILL, Belfer Graduate School of Science, Yeshiva University, Amsterdam Avenue and 186th Street, New York, New York. On Bergman's kernel function for some uniformly elliptic partial differential equations.

A generalization of the theory or Bergman's kernel function is presented for uniformly elliptic partial differential equations of the second order, of divergence type $D_{k}\left(a_{i k}(x) D_{i} u\right)=0$ where $\mathrm{x} \in \Omega, \Omega$ a regular open set in $\mathrm{R}^{\mathrm{n}}$. Let $\mathrm{E} \subset \Omega$ be compact, and consider the family of regular solutions of $D_{k}\left(a_{i k}(x) D_{i} u\right)=0$ vanishing at $x_{0} \in \Omega$. The existence of the kernel follows from an estimate $|u(x)|^{2} \leqq C(E) \int_{\Omega} a_{i k}(x) D_{i} u D_{k} u d X$ for $x \in E$ where $C(E)$ is a constant whose value depends only on $E$. (Received November 30, 1964.)

65T-119. ARLEN BROWN and C. M. PEARCY, The University of Michigan, 347 West Engineering, Ann Arbor, Michigan. Structure of commutators of operators.

Let $H$ be a separable Hilbert space. A bounded, linear operator $C$ on $H$ is an additive commutator if there are (bounded, linear) operators $A$ and $B$ on $H$ such that $C=A B-B A$. Theorem. An operator C on H is an additive commutator if and only if C is not of the form $\lambda+\mathrm{C}$ where $\lambda$ is a nonzero scalar and C is a compact operator. An invertible operator C on H is a multiplicative commutator if there exist invertible operators $A$ and $B$ on $H$ such that $C=A B A^{-1} B^{-1}$. Theorem. A normal invertible operator $C$ is a multiplicative commutator if and only if $C$ is not of the form $\lambda+C$ where $\lambda$ is a scalar of modulus different from one and $C$ is a compact operator. (Received November 19, 1964.)

65T-120. S. J. BRYANT, San Diego State College, San Diego, California. Groups graphs and Fermat's last theorem.

Call p a bad prime if $2^{\mathrm{p}-1} \equiv 1 \bmod \mathrm{p}^{2}$. The prime 1093 is the first bad prime. Such primes are related to Fermat's theorem, i.e., if no one of $x, y, z$ are divisible by $p$ and $2^{p-1} \equiv 1$ mod $p^{2}$ then $x^{p}+y^{p} \neq z^{p}$. The following theorem holds: Theorem. If $G$ and $H$ are abelian groups of order $n$ and n is not divisible by the square of a bad prime and there is a square-preserving function on $G$ onto $H$, then $G$ and $H$ are isomorphic. Furthermore, the isomorphism type of $G$ can be computed if the square of every element of $G$ is given. (Received November 27, 1964.)

65T-121. E. H. FELLER, University of Wisconsin-Milwaukee, Milwaukee, Wisconsin and E. W. SWOKOWSKI, Marquette University, Milwaukee, Wisconsin. Prime modules.

In the following, R is a ring satisfying Goldie's right quotient conditions [Proc. London Math. Soc. 8 (1958), 589-608] and "R-module" always means "right R-module". An R-module M is prime if $(0: N)=0$ for every nonzero submodule $N$ of $M$ and the singular submodule $M$ is zero. If $M$ is a prime $R$-module, then $R$ is a prime ring. Theorem 1 . An $R$-module $M$ is prime if and only if $M$ is contained in a completely reducible unitary right $S$-module, where $S$ is a simple ring with minimum condition on right ideals and a right quotient ring for $R$. Theorem 2. An $R$-module $M$ is Noetherian and prime if and only if $M$ is a finite subdirect sum of uniform Noetherian prime R-modules. Theorem 3. Let $M$ be a subdirect sum of uniform Noetherian prime $R$-modules $M_{1}, \ldots, M_{t}$. Then for each i there exists $x_{i} \in M_{i}$ such that $D_{i}=\operatorname{Hom}_{R}\left(x_{i} J, x_{i} J\right)$ is a right Ore domain, where $J$ is a uniform right ideal in $R$. Moreover if $S$ is the quotient ring of $R$ then $S$ is isomorphic to a total matrix ring $Q_{n}$, where $Q$ is the quotient ring of $D_{i}$. (Received December 2, 1964.)

65T-122. J. R. DORROH, Louisiana State University, Baton Rouge, Louisiana. Some properties of a singular differential operator.

Let $X$ denote the B-space of all bounded and uniformly continuous complex-valued functions defined on the real numbers, and let $p$ denote a real-valued function in $X$ which has uniformly bounded difference quotients. Let $D$ denote the set of all functions x in X which are differentiable at every nonzero of $p$ and for which the function $A x$ defined by $\left([A x](s)=p(s) x^{\prime}(s)\right.$ for $p(s) \neq 0,[A x](s)=0$ for $p(s)=0$ ) is again a function in $X$. The operator $A$ with domain $D$ generates a strongly continuous group [ $T(t)$ ] of operators in $X, A^{2}$ generates a strongly continuous semi-group [V(t)] of operators in $X$, and simple formulas are given for $T(t)$ and $V(t)$. One easy corollary of the results is the following. If $x$ and $q$ are continuous complex-valued functions defined on $[-1,1], \operatorname{Re}\{q(s)\}<0$ for $-1 \leqq s \leqq 1$, $\left(1-s^{2}\right)(d / d s)\left[\left(1-s^{2}\right) x^{\prime}(s)\right]+q(s) x(s)=0$ for $-1<s<1$, and $\left(1-s^{2}\right) x^{\prime}(s) \rightarrow 0$ as $s \rightarrow \pm 1$, then $x(s)=$ 0 for $-1 \leqq s \leqq 1$. (Received November 27, 1964.)

65T-123. CHARLES STONE, University of California, Los Angeles, California 90024. On the remainder term in renewal theory.

Let F be a right-continuous distribution function having finite moment of integer order $\mathrm{m} \geqq 2$ and first moment $\mu>0$. Let $f$ denote the characteristic function of $F$ and $F^{(n)}$ the $n$-fold convolution
of $F$ with itself. Set $U(x, h)=\sum_{n \geqq 0}\left(F^{(n)}(x+h)-F^{(n)}(x)\right), \epsilon(x, h)=U(x, h)-\mu^{-1} h-\mu^{-2} h \int_{x}^{\infty}(1-F(y)) d y$ for $x \geqq 0$ and $\epsilon(x, h)=U(x, h)-\mu^{-2} h \int_{-\infty}^{x} F(y) d y$ for $x<0$. F is lattice with lattice constant $d>0$ if $f$ is periodic with period $2 \pi d$ and $f(\theta) \neq 1$ for $0<|\theta| \leqq \pi d$. F is strongly non-lattice if
$\lim \inf |\theta| \rightarrow \infty$ |l-f( $\theta) \mid>0$. Theorem. If $F$ is lattice with lattice constant $d$, then $\epsilon(x, d)=$ $o\left(x^{-m}\right)$ as $|x| \rightarrow \infty$. If $F$ is strongly non-lattice, then $\epsilon(x, h)=o\left(x^{-m} \log |x|\right)$ as $|x| \rightarrow \infty$ uniformly for $h$ in bounded sets. This extends a result of A. O. Gelfond (Teor. Verojatnost. i Primenen. 9 (1964), 327-331) that if $F$ is lattice with lattice constant $d$ and $F(x)=0$ for $x<0$, then $\epsilon(x, d)=$ $\mathrm{O}\left(\mathrm{x}^{-\mathrm{m}} \log \mathrm{x}\right)$ as $\mathrm{x} \rightarrow \infty$. (Received December 14, 1964.)

65T-124. A. K. AZIZ, R. P. GILBERT, and H. C. HOWARD, Fluid Dynamics Institute, University of Maryland, College Park, Maryland. On a nonlinear elliptic boundary value problem with generalized Goursat data.

The following elliptic boundary value problem is considered: $\Delta u=\widetilde{f}\left(x, y, u, u_{x}, u_{y}\right)$ with $u_{x}(x, y)=$ $\widetilde{a}_{0}(x) u(x, y)+\widetilde{a}_{1}(x) u_{y}(x, y)+\widetilde{g}(x)$ on the curve $\Gamma_{1} \equiv\left\{(x, y) \mid y=f_{1}(x)\right\}, u_{y}(x, y)=\widetilde{\beta_{0}}(y) u(x, y)+\widetilde{\beta_{1}}(y) u_{x}(x, y)$ $+\widetilde{h}(y)$ on the curve $\Gamma_{2} \equiv\left\{(x, y) \mid x=f_{2}(y)\right\}$, and $u(0,0)=\gamma,(0,0) \in \Gamma_{1}$ and $\Gamma_{2}$. Replacing $x$ and $y$ by (1) $x=\left(z+z^{*}\right) / 2, y=\left(z-z^{*}\right) / 2 i$ with $z$ and $z^{*}$ new independent complex variables (and assuming (hypothesis A) that $f$, the coefficients in the boundary data, and the curves $\Gamma_{1}$ and $\Gamma_{2}$ possess suitable continuations if their arguments are taken to be complex-valued) one obtains an equivalent boundary value problem, $P$ say. By introduction of appropriate Banach spaces one shows that the existence of a unique solution to problem $P$ in the small is equivalent to the existence of a fixed point of the operator $T s\left(z, z^{*}\right)=f\left(z, z^{*}, B_{1} s, B_{2} s, B_{3} s\right)$ where the $B_{i} s$ depend upon the solutions of an auxiliary operator equation. With the notation $S=$ bounded domain in the $z$ plane, $\bar{S}=\left\{z^{*} \mid z^{*}=\bar{z}, z \in S\right\}$ we have the following theorem: If (1) hypothesis A holds in the bicylinder ( $\mathrm{S}, \overline{\mathrm{S}})$, (2) $\mathrm{S} \supset \Delta \rho=\{\mathrm{z} \mid(\mathrm{z}) \leqq \rho\}$, $\rho=\mathrm{const}, \overline{\mathrm{S}} \supset \Delta^{*} \rho=\left\{\mathrm{z}^{*}| | \mathrm{z}^{*} \mid \leqq \rho\right\}$, (3) $\mathrm{F}_{1}: \Delta \rho \rightarrow \Delta^{*} \rho, \mathrm{~F}_{2}: \Delta^{*} \rho \rightarrow \Delta \rho$, (4) $\left|\mathrm{a}_{1}(0) \beta_{1}(0)\right|<1$ ( $\mathrm{F}_{\mathrm{i}}, \mathrm{a}_{1}, \beta_{1}$ the continuations of $f_{i}, \widetilde{a}_{1}, \widetilde{\beta}_{1}$ under (1)), (5) either $y=f_{1}(x) \neq A x$ or $x=f_{2}(y) \equiv B y$ ( $A, B$ real constants), then problem P has a unique solution in $\left(\Delta \rho, \Delta^{*} \rho\right.$ ). (Received December 14, 1964.)

65T-125. J. H. AHLBERG, United Aircraft Corporation, East Hartford, Connecticut, E. No NILSON, Pratt and Whitney Aircraft, East Hartford, Connecticut, and J. L. WALSH, Harvard University, Cambridge 38, Massachusetts. Generalized splines and the best approximation of linear functionals.

Let $f^{(n-1)}$ be absolutely continuous on $[0,1], f^{(n)} \in L^{2}(0,1), \mathscr{L}_{f}=\sum_{j<\eta} \int_{0}^{1} f^{(j)}(t) d \mu_{j}(t)$ $(\eta \leqq n), L f=a_{n}(x) f^{(n)}+\ldots+a_{0}(x)$, each $\mu_{j} \in V_{0}^{1}$, each $a_{i}(x) \in C^{n}(0,1)$, and $a_{n}(x) \neq 0$ on $[0,1]$. Let $\Delta: 0=x_{0}<\ldots<x_{N}=1$ and $B f=\sum_{i=0}^{N} \sum_{j=0}^{n_{i j}} b_{i f} f^{(j)}\left(x_{i}\right)$ be given with each $n_{i}<\eta$ and certain $b_{i j}=0$ prescribed. If $\mathrm{Rf} \equiv \mathscr{L} \mathrm{f}-\mathrm{Bf}=0$ when $\mathrm{Lf}=0$, then $\mathrm{Rf}=\int_{0}^{1} \mathrm{~K}_{\mathrm{B}}(\mathrm{t}) \mathrm{Lfdt}$ where $\mathrm{K}_{\mathrm{B}}(\mathrm{t}) \in \mathrm{L}(0,1)$ and is independent of $f$. If $K_{B} \in L^{2}(0,1)$ for $B \equiv 0$, this is true for every $B$ and $\int_{0}^{1} K_{B}^{2} d t$ is minimized when $\mathrm{Bf} \equiv \mathscr{L}_{S_{\Delta, f}}$, where $\mathrm{S}_{\Delta, \mathrm{f}}$ exists for N sufficiently large as the unique generalized spline function having $2 n-2-n_{i}$ continuous derivatives at $x_{i},(L f)^{(j)}\left(x_{i}\right)=0\left(j=n+n_{i}, \ldots, 2 n-2 ; i=0, N\right)$, $f^{(j)}\left(x_{i}\right)=S_{\Delta, f}^{(j)}\left(x_{i}\right)\left(b_{i j} \neq 0\right), \lim _{t \rightarrow x_{i}+}\left(L S_{\Delta, f}\right)^{(2 n-j-1)}(x)=\lim _{t \rightarrow x_{i}}\left(L S_{\Delta, f}\right)^{(2 n-j-1)}(x) \quad\left(b_{i j}=0\right.$, $i \neq 0$, $\left.i \neq N),\left(L S_{\Delta, f}\right)(2 n-j-1)_{x_{0}}\right)=0\left(b_{0 j}=0\right)$, and $\left(L S_{\Delta, f}\right)^{(2 n-j-1)}\left(x_{N}\right)=0\left(b_{N j}=0\right)$. This extends Schoenberg's results (1964) for polynomial and trigonometric splines and allows additional functionals important to numerical integration quadrature and interpolation (c.f. Sard. Linear approximation).

For a suitable sequence of orthonormal splines $\left\{S_{i}(x)\right\}, K_{B}(t)=\sum_{i=1}^{\infty} L S_{i}(t) \int_{0}^{1} \sum_{j<\eta} S_{i}^{(j)}(t) d \mu_{j}(t)$ and, for $\eta<n, K_{B}(t) \in L^{2}(0,1)$. Analogous results hold for multi-dimensional splines. (Received December 8, 1964.)

65T-126. JACOB FELDMAN, University of California, Berkeley, California. Uniqueness of extremal doubly stochastic measures.

Let $\lambda$ be Lebesgue measure on $I=[0,1]$. A Borel probability measure $\mu$ on $I$ will be called doubly stochastic if $\mu(\mathrm{A} \times \mathrm{I})=\mu(\mathrm{I} \times \mathrm{A})=\lambda(\mathrm{A})$ for all Borel sets $\mathrm{A} \subset \mathrm{I}$. J. Lindenstrauss has shown that $\mu$ is extreme in the set $\Sigma(\lambda)$ of doubly stochastic measures if and only if the functions on $\mathrm{I} \times \mathrm{I}$ of the form $(\mathrm{x}, \mathrm{y}) \rightarrow \mathrm{f}(\mathrm{x})+\mathrm{g}(\mathrm{y})$, where $\mathrm{f}, \mathrm{g}$ are in $\mathscr{L}_{1}(\lambda)$, form a dense set in $\mathscr{L}_{1}(\mu)$. We show as a consequence of this that if $\mu, \nu$ are in $\Sigma(\lambda), \mu$ extremal, and $\nu<\mu$, then $\nu=\mu$. Consequently, there is at most one extreme point of $\Sigma(\lambda)$ in any equivalence class under mutual absolute continuity. (Received December 18, 1964.)

65T-127. CHIEN WENJEN, California State College at Long Beach, Long Beach 4, California. Characterizations and representations of semi-normed algebras. I. Preliminary report.

Let $A$ be a commutative complete semi-normed algebra with unity over complex numbers and with a set $\mathscr{V}$ of semi-norms. (See Arens, Pacific J. Math. 2 (1952), 455-471.) Theorem 1. A is equivalent to the algebra $C(T, K)$, with compact-open topology, of all continuous complex-valued functions on a locally compact completely regular space T if and only if the following condition is satisfied: (I) For each $V \in \mathscr{V}$, there exists an element $x \in A$ such that $\bar{V}=\sup \{V ; V(x) \leqq 1, V \in \mathscr{V}\}$ is a semi-norm belonging to $\mathscr{V}$. This is the second solution of the question raised by M. A. Nairmark in his treatise Normed rings, on p. 237. (For the first solution, see Wenjen, Pacific J. Math. 8 (1958), 177-186.) Theorem 2. A satisfying the condition (I) is equivalent to $C(T, K)$, with compact-open topology, where $T$ is a locally compact realcompact space if and only if every convex set $F$ in $A$ containing 0 and absorbing bounded sets, contains a multiplicatively convex and idempotent subset $U$ such that $0 \in U$ and $\lambda U \subset F$ with $0 \leqq \lambda<1+h$. A subset $G$ of $A$ is said to be bounded if the set $\{V(x) ; x \in G\}$ is bounded for each $V \in \mathscr{V}$. (See Michael, Mem. Amer. Math. Soc., no. 11 (1952), and Nachbin, Proc. Nat. Acad. Sci. U.S.A. 40 (1954), 471-474.) (Received December 1, 1964.)

65T-128. P. E. CONNER and E. E. FLOYD, University of Virginia, Charlottesville, Virginia. Fibring within an oriented cobordism class.

We show that if $M^{4 n}$ is a closed oriented manifold with index 0 , then modulo an element of order 2 the oriented cobordism class $\left[\mathrm{M}^{4 n}\right]$ is represented by the total space of a differentiable bundle over the 2 -sphere with structure group $U(1)$ and fibre a closed connected oriented manifold. It is clear that if $\left[M^{4 n}\right]$ can be so represented, then the index of $M^{4 n}$ vanishes. (Received December 2l, 1964.)

65T-129. R. S. SPIRA, University of Tennessee, Knoxville, Tennessee. On the Riemann hypothesis.

Let $\mathrm{g}_{\mathrm{N}}(\mathrm{s})=\sum_{\mathrm{n}=1}^{\mathrm{N}}\left[\mathrm{n}^{-\mathrm{s}}+\chi(\mathrm{s}) \mathrm{n}^{\mathrm{s}-\mathrm{l}}\right]$ be an approximation to the Riemann zeta function arising from the approximate functional equation. $g_{N}(s)$ satisfies the same functional equation as the zeta function. Let $\mathrm{s}=\sigma+\mathrm{it}$. It is shown for $\mathrm{N}=1$ and 2 that for sufficiently large t all the complex zeros of $\mathrm{g}_{\mathrm{N}}(\mathrm{s})$ lie on the line $\sigma=1 / 2$. The case $\mathrm{N}=3$ appears much more difficult. (Received December 21, 1964.)

65T-130. C. P. WALKER and E. A. WALKER, New Mexico State University, University Park, New Mexico. Endomorphism rings of generators.

Let $U$ be a generator in the category $\mathscr{M}_{R}$ of right $R$-modules, $E=\operatorname{Hom}_{R}(U, U)$, and $\mathrm{T}_{\mathrm{U}}: \mathscr{M}_{\mathrm{E}} \rightarrow \mathscr{M}_{\mathrm{R}}: \mathrm{A} \rightarrow \mathrm{A} \otimes_{\mathrm{E}} \mathrm{U}$. It is known that $\mathrm{T}_{\mathrm{U}}$ induces an equivalence between $\mathscr{M}_{\mathrm{E}} / \mathrm{Ker}^{\mathrm{T}} \mathrm{T}_{\mathrm{U}}$ and $\mathscr{M}_{R}$. Theorem. Let $\overline{\mathrm{E}}$ be an injective envelope of E in $\mathscr{M}_{\mathrm{E}}$. Then $\overline{\mathrm{E}} \otimes_{\mathrm{E}} \mathrm{U}$ is an injective envelope of $U$ in $\mathscr{M}_{R}$. Theorem. $E$ is a self-injective ring if and only if $U$ is an injective $R-m o d u l e$. Let $I=\operatorname{Im}\left(U \otimes_{R} \operatorname{Hom}_{E}(U, E) \rightarrow E: u \otimes f \rightarrow f(u)\right)$. $\quad I$ is a two-sided ideal of $E$. Lemma. $I^{2}=I$ and if $J$ is a right ideal of $E, J U=J$ if and only if $I \subseteq J$. Theorem. If $A$ is a right $E$-module, $A \in K e r T_{U}$ if and only if $A I=0$. Theorem. If $U$ is projective in $\mathscr{M}_{R}$ then $I=\{f \in E \mid f(U) \subseteq$ a finitely generated submodule of $U\}$. Theorem. If $U$ is a co-generator in $\mathscr{M}_{R}$ then $I=\bigcap\{J \mid J$ is a right ideal of $E$ and $\left.\operatorname{Hom}_{E}(\mathrm{E} / \mathrm{J}, \mathrm{E})=0\right\}$, so that $\operatorname{Ker} \mathrm{T}_{\mathrm{U}}$, and hence the category $\mathscr{M}_{R}$, is completely determined by the ring E. There are generalizations to Abelian categories with exact inductive limits and generators. (Received November 24, 1964.)

65T-131. P. R. YOUNG, Reed College, Portland, Oregon. Diagonal cylindrifications and linear orderings under one-one reducibility.

Let N be the set of nonnegative integers. For infinite $\mathrm{A} \subset \mathrm{N}$ define $\Phi(\mathrm{A})$ (the diagonal cylindrification of $A$ ) to be $A \times N \cup \cup_{k=0}^{00}(\{k\} \times\{k+2, k+3, \ldots\})$. If $A$ is not immune, then $A$ is a cylinder iff $\Phi(\mathrm{A}) \leqq_{1}$ A iff $\Phi(\mathrm{A})$ is a cylinder. Let $\nu$ be the order type of the rationals. Modifying our construction of $\Phi$ we obtain: Every nonrecursive m-1 degree consists either of a single 1-1 degree or contains a sequence of sets which has, under $\leqq_{1}$, order type $\nu$. Let $D_{i}$ be any of the standard classes of r.e. sets listed in Abstract 63T-368 (these CNotices) $10(1963), 586) ; \Phi\left(D_{i}\right) \subset D_{i}$, and this may be used to locate l-l orderings of r.e. sets of order type $\nu$. Combining the above methods with methods due to Dekker, we obtain: The m-l degree of every simple set contains a sequence of simple sets which has, under $\leqq_{1}$, order type $\left(\omega^{*}+\omega\right) \cdot \nu$. If $M$ is hyperhypersimple, $\Phi(\mathrm{M})$ is a nonhyperhypersimple, hypersimple set whose complement is not point-decomposable (Abstract 612-19, these $\mathcal{C}$ Notices) 11 (1964), 355). (Received November 23, 1964.)

65T-132. WITHDRAWN

65T-133. L. J. POZSGAY, Rockhurst High School, 9301 State Line Road, Kansas City, Missouri, 64114. Gödel's second theorem for elementary arithmetic.

Elementary arithmetic (EA) is a quantifier-free formal system which formalizes the elementary functions of Kalmar in a way similar to that in which primitive recursive arithmetic (PRA) formalizes the primitive recursive functions (see Abstract 578-5, these CNotices) 8 (1961), 140). Since the elementary functions form a small subclass of the primitive recursive functions (see Grzegorczyk, Some classes of recursive functions, Rozprawy Mat. 4 (1953)), EA may be regarded as a weak subsystem of PRA. Formalization of intuitive results of Grzegorczyk provides formal counterparts of theorems of number theory up to and including the prime decomposition theorem. By means of a special arithmetization technique which assigns to each functor, term, formula, and proof of EA a natural number which in its prime decomposition mirrors the tree structure of the given object, basic metamathematical facts about EA are rendered expressible as formal theorems of EA. Theorem. The formula $\left.\sim \underline{\operatorname{PRF}( }{ }^{\top} 0=l^{\urcorner}, x\right)$ - which says intuitively that $x$ is not the number of a proof of $0=1$ - is valid but unprovable in EA. (Received December 23, 1964.)

65T-134. W. B. ARVESON, 380 No. Madison, Apartment 7, Pasadena, California 91106. Analyticity in operator algebras. Preliminary report.

The author's intent is to exploit the attitudes and ideas of function theory in the study of certain algebras of operators on Hilbert space. In the terminology of the text of Dixmier, let $\Phi$ be a positive normal mapping of the von Neumann algebra $\mathfrak{B}$ into itself such that $\Phi^{\circ} \Phi=\Phi$, and $\Phi(H)=0$ implies $H=0$, for every positive $H \in \mathfrak{B}$. A subalgebra $\mathfrak{A}$ of $\mathfrak{B}$ is called superdiagonal (with respect to $\Phi$ ) if $\mathfrak{N}+\mathscr{U}^{*}$ contains a weakly dense *-subalgebra of $\mathfrak{B}, \Phi(A B)=\Phi(A) \Phi(B)$ for all $A, B \in \mathfrak{A}$, $\Phi(\mathfrak{H}) \subseteq \mathscr{2} \cap \mathfrak{A}^{*}$, and $\left(\mathfrak{H} \cap \mathfrak{N}^{*}\right)^{2}$ has trivial nullspace. The main result is this. Let $\mathfrak{A}$ be a maximal (by Zorn) superdiagonal subalgebra of $\mathfrak{B}$, and suppose there is a distinguished faithful normal finite trace $\phi$ on $\mathfrak{B}$ such that $\phi \circ \Phi=\phi$. Then every regular operator X in $\mathfrak{B}$ admits a factorization $\mathrm{X}=\mathrm{UA}$, where $U$ is a unitary operator in $\mathfrak{A}$ and $A \in \mathfrak{A} \cap \mathfrak{U}^{-1}$. It follows that $\mathfrak{A}$ is logmodular in the sense that every self adjoint-element of $\mathfrak{B}$ has the form $\log |A|$, with $A \in \mathfrak{A} \cap \mathfrak{Y}^{-1}$. Analogs of Jensen's inequality, Jensen's formula, and Szegö's theorem (for analytic functions in the unit disc) are cast in this setting. Utilizing the factorization theorem, the author shows that these three propositions are equivalent. (Received November 24, 1964.)

65T-135. CORA SADOSKY, University of Chicago, Chicago, Illinois 60637. On class preservation by parabolic singular integral operators.

Consider an $f(x, t) \in L^{p}\left(E_{n} \times(0, \infty)\right), 1 \leqq p \leqq \infty$. Write $[x, t]=\left(|x|^{2 m}+t^{2}\right)^{1 / 2 m}, m$ positive integer. We say that $f \in T_{u}^{p}\left(x_{0}, t_{0}\right), u \geqq-(n+m) / m p$ if there exists a polynomial $P\left(x_{0}, t_{0}\right)=$ $\sum_{|a|+m \beta<\mathrm{mu}^{\mathrm{a}}{ }_{\mathrm{a} \beta}\left(\mathrm{x}-\mathrm{x}_{0}\right)^{\mathrm{a}}\left(\mathrm{t}-\mathrm{t}_{0}\right)^{\beta}(\mathrm{P} \equiv 0 \text { if } \mathrm{u} \leqq 0) \text { such that }\left(\rho^{-\mathrm{n}-\mathrm{m}} \int\left[\mathrm{x}-\mathrm{x}_{0}, \mathrm{t}-\mathrm{t}_{0}\right] \leqq \rho\right.} \mid \mathrm{f}(\mathrm{x}, \mathrm{t})-\mathrm{P}\left(\mathrm{x}-\mathrm{x}_{0}\right.$, $\left.\mathrm{t}-\mathrm{t}_{0}\right)\left.\right|^{\left.\mathrm{P}_{\mathrm{dxdt}}\right)^{1 / \mathrm{p}} \leqq \mathrm{A} \rho^{\mathrm{mu}}, 0<\rho<\infty \text {. This is a Banach space with the norm } \mathrm{T}_{\mathrm{u}}^{\mathrm{p}}\left(\mathrm{x}_{0}, \mathrm{t}_{0} ; f\right)=}$
$\|f\|_{p}+\inf A+\sum\left|a_{a \beta}\right|$ (see A. P. Calderon and A. Zygmund, Local properties of solutions of elliptic partial differential equations, Studia Math. 20 (1961), 171-225). Let $\widetilde{f}(x, t)$
 be the parabolic singular integral, where (i) $\int_{E_{n}} k(x, 1) d x=0$; (ii) $k\left(\lambda x, \lambda^{m_{t}}\right)=\lambda^{-n-m_{k}(x, t),} \lambda>0$. Theorem: If $\left|(\partial / \partial x)^{a}(\partial / \partial t) \beta_{k(x, t) \mid} \leqq m[x, t]^{-n-m-|a|-m \beta}, 0 \leqq|a|+m \beta \leqq m u+1\right.$ if $u>0$, $|a|+m \beta=0$ if $u \leqq 0$ and if $f \in T_{u}^{p}\left(x_{0}, t_{0}\right)$, then (I) If $m u \neq 0,1,2, \ldots$, then $\widetilde{f} \in T_{u}^{p}\left(x_{0}, t_{0}\right)$ and $T_{u}^{p}\left(x_{0}, t_{0} ; \widetilde{f}\right)$ $\leqq \mathrm{C}_{\text {upk }} \mathrm{T}_{\mathrm{u}}^{\mathrm{p}}\left(\mathrm{x}_{0}, \mathrm{t}_{0} ; \mathrm{f}\right)$; (II) If $\mathrm{mu}=1,2, \ldots$, then the result (I) holds with $\mathrm{T}_{\mathrm{u}}^{\mathrm{p}}\left(\mathrm{x}_{0}, \mathrm{t}_{0} ; \widetilde{f}\right) \leqq \mathrm{C}_{\text {puk }} \mathrm{T}_{\mathrm{u}}^{\mathrm{p}}\left(\mathrm{x}_{0}, \mathrm{t}_{0} ; \mathrm{f}\right)+\mathrm{CM}$ provided that $f(x, t)\left[x-x_{0}, t-t_{0}\right]^{-n-m-m u} \in L\left(E_{n} \times(0, \infty)\right)$; (III) If $m u=0$, let $\widetilde{f_{*}}(x, t)=\sup \boldsymbol{f}_{\epsilon}(x, t)$ and suppose that $\widetilde{\mathrm{f}_{*}}\left(\mathrm{x}_{0}, \mathrm{t}_{0}\right)<\infty$, then the result (I) holds with $\mathrm{T}_{0}^{\mathrm{p}}\left(\mathrm{x}_{0}, \mathrm{t}_{0} ; \widetilde{\mathrm{f}}\right) \leqq \mathrm{C}_{\mathrm{pk}} \mathrm{T}_{0}^{\mathrm{p}}\left(\mathrm{x}_{0}, \mathrm{t}_{0} ; f\right)+\widetilde{\mathrm{Cf}}_{*}\left(\mathrm{x}_{0}, \mathrm{t}_{0}\right)$. (Received December 24, 1964.)

65T-136. R. T. ROCKAFELLAR, Computation Center, The University of Texas, Austin, Texas 78712. Minimax theorems and conjugate saddle-functions.

Define a saddle on $R^{m} \times R^{n}$ to be a triple $\{K, A, B\}$, where $A \subseteq R^{m}$ and $B \subseteq R^{n}$ are nonempty convex sets and $K(x, y)$ is a real function on $A \times B$, concave in $x$ and convex in $y$. As in Fenchel's theory of conjugate convex functions, a regularizing procedure turns each saddle into a "closed" one, unique up to a mild equivalence: $\left\{K^{\prime}, A^{\prime}, B^{\prime}\right\} \sim\{K, A, B\}$ when $A^{\prime}=A, B^{\prime}=B$, and $K^{\prime}(x, y)=K(x, y)$ unless $x \notin$ ri $A$ and $y \notin$ ri $B$ (ri $=$ rel. interior). Given a closed $\{K, A, B\}$, define $L_{1}$ and $L_{2}$ on $R^{m} \times R^{n}$ by $L_{1}(u, v)=\inf _{A} \sup _{B}\{u x+v y-K(x, y)\}$ and $L_{2}(u, v)=\sup _{B} \inf _{A}$. Theorem: The set of ( $\left.u, v\right)$, where $L_{1}$ and $L_{2}$ are both finite, is of form $C \times D$, and $\left\{L_{1}, C, D\right\}$ and $\left\{L_{2}, C, D\right\}$ are equivalent closed saddles depending only on the class of $\{K, A, B\}$. Thus for each equivalence class of closed saddles on $R^{m} \times R^{n}$ there is a conjugate class. Theorem: The conjugate of the conjugate class is the original class. This paper will appear shortly in Math. Scand. (Received November 30, 1964.)

65T-137. TILLA KLOTZ, Courant Institute of Mathematical Sciences, New York University, 4 Washington Place, New York, New York. On complete surfaces in 3-space with constant nonzero mean curvature.

Theorem. If $S \subset E^{3}$ is complete, with $H \equiv c \neq 0$ and $K \leqq 0$, then $S$ is a full right circular cylinder. Assume (with no loss) that $\mathrm{H}=1 / 2$. Then the proof involves choosing $\mathrm{x}, \mathrm{y}$ locally so that $I=\lambda\left(d x^{2}+d y^{2}\right)$ and $I I=L d x{ }^{2}+(L-1) d y^{2}$, where $L$ is a function on $S$ and $\lambda=2 L-1$. Since $K \leqq 0$, $\log \lambda \leqq 0$ is subharmonic on $S$. But $I / \lambda$ is a flat complete metric on $S$, giving $S$ a parabolic universal covering surface. Thus $\lambda$ is constant, implying the theorem. Similar reasoning yields the following related result. Theorem. If $S \subset E^{3}$ is complete with $H \equiv c \neq 0$ and $K \geqq 0$ bounded away from $c^{2}$ then $S$ is a full right circular cylinder. (Received January 4, 1965.)

65T-138. L. R. RUBEL, University of Illinois, Urbana, Illinois and A. L. SHIELDS, University of Michigan, Ann Arbor, Michigan. The bounded holomorphic functions in the strict topology.

Let $B$ denote the bounded analytic functions in the unit disc in the topology given by the family of seminorms: $\|f\|_{k}=\sup |f(z) k(z)|$, where $k(z)$ is any non-negative continuous function in the closed disc that vanishes on the boundary. R. C. Buck studied B as a topological algebra (Seminars on
analytic functions, Princeton, 1957). We show that every closed ideal is principal, that a principal ideal is closed if and only if the generating function is an inner function multiplied by a unit, and that a principal ideal is dense if and only if the generating function is an outer function. This leads to an extension of Beurling's theorem on invariant subspaces to any topological vector space of analytic functions in the disc that satisfies certain axioms. Buck has shown that the closed maximal ideals of $B$ correspond to points in the open disc. We extend this result to any region whose boundary is the union of nondegenerate continua. Rudin has shown (Bull. Amer. Math. Soc 70 (1964), 321-324) that this result need not be true when the boundary contains nowhere dense perfect sets (we assume that none of the boundary points represent removable singularities). Finally, for general regions we identify the conjugate space of the bounded analytic functions in the strict topology and we show that in the norm topology the bounded analytic functions are always a conjugate Banach space. (Received November 24, 1964.)

65T-139. JACK SILVER, University of California at Berkeley, 869 International House, Berkeley 4, California. Second separation property for existential second-order classes.

All relational structures are assumed to be of a fixed similarity type, having at least two unary relations. A class $K$ of relational structures is said to be a $V_{1}^{1}$-class (an existential secondorder class) if it is the class of models of some second-order sentence in prenex form whose only second-order quantifiers are existential; the definition of a $\Lambda_{1}^{1}$-class (a universal second-order class) is analogous. Theorem. The second separation property fails for $V_{1}^{1}$-classes, i.e. there are $V_{1}^{1}$-classes $K, L$, such that for no $V_{1}^{l}$-classes $K^{\prime}, L^{\prime}$, where $K^{\prime} \cup L^{\prime}$ contains all structures of the similarity type, do we have $K^{\prime} \cap(K \sim L)=0$ and $L^{\prime} \cap(L \sim K)=0$. The proof uses Corollary 4.7 of Vaught, Proceedings of the International Symposium on the Theory of Models (forthcoming). Corollary. The reduction property fails for $\Lambda_{l}^{1}$-classes. This answers a question of Addison in the Proceedings of the International Congress of Logic, Methodology, and Philosophy of Science (at Stanford, 1960). (Received January 5, 1965.)

65T-140. C. J. R. BORGES, University of Nevada, Reno, Nevada 89507. On function spaces of finite-dimensional compact Hausdorff spaces.
A. H. Stone [Proc. Amer. Math. Soc. 14 (1963), 81-83] asked whether ( ${ }^{I}$, co. topology) is normal whenever $I$ is the closed unit interval and $Y$ is a finite-dimensional (in the covering sense) compact Hausdorff space. We will answer this question negatively, as follows: Theorem l. There exists a 2 -dimensional (in the covering sense) compact Hausdorff space $Y$ such that ( $\mathrm{Y}^{\mathrm{I}}$, co. topology) is not a normal space. (Received January 6, 1965.)

65T-141. R. D. ADAMS, University of Kansas, Lawrence, Kansas. A remark on coercive operators.

A self adjoint operator $H$ on the Hilbert space $\mathscr{\mathscr { K }}$ is coercive if there is a completely continuous symmetric operator $B_{0}$ and a constant $c_{0}$ such that $c_{0} H+B_{0} \geqq I$. We prove the simple but interesting Theorem. Suppose $H$ is coercive and $B \geqq 0$ is completely continuous. Then there are
constants $c_{1}$ and $c_{2}$ such that $c_{1} H+c_{2} B \geqq I$ if and only if there is a constant $c>0$ such that $(H x, x) \geqq c\|x\|^{2}$ for all $x \in \mathfrak{N}(B)=\{x \mid B x=0\}$. (Received January 6, 1965.)

65T-142. J. L. GRIFFITH, University of Kansas, Lawrence, Kansas. A formula connected with Hankel transforms.

Suppose that the Hankel transform is defined as in Sneddon, Fourier transforms. Write this as $G(y)=T(f(x))$. It is easy to find a function the transform of which is $-y^{2} G(y)$. The author shows that, by using two different fractional derivatives, a formula involving real variables only may be obtained for a function whose transform is $\mathrm{y}^{2 \mathrm{n}} \mathrm{G}(\mathrm{y})$ for all $\mathrm{n}>0$. (Received November 27, 1964.)

65T-143. E. P. SPECKER, Eidg. Technische Hochschule, Zurich, Switzerland and LOUIS HODES, IBM, Research Division, Thomas J. Watson Research Center, P.O. Box 218, Yorktown Heights, New York 10598. Elimination of quantifiers and the length of formulae.

Let T be one of the following theories: (1) the theory of identity on an infinite set, (2) the theory of densely ordered sets, (3) the theory of ordered dense abelian groups. It is known that elimination of quantifiers holds in T, i.e. for every open formula $\phi$ in the free variables $\vec{x}=\left(x_{1}, \ldots, x_{k}\right), \vec{y}=\left(y_{1}, \ldots, y_{n}\right)$ and every prefix $Q=Q_{1} \ldots Q_{k}$ there exists an open formula $\psi$ in $\vec{y}$ such that $(Q \vec{x}) \phi$ is equivalent to $\psi$ in $T$. Let the length of a formula be the number of symbols providing in (3) that the length of the expression $n x(x+\ldots+x, n$ times ) be proportional to logn. Theorem. There exist constants $c_{k}(k=1,2, \ldots)$ such that, for every open formula $\phi$ in $\vec{x}, \vec{y}$ of length $t$ and for every $k$-place prefix $Q$, there exists an open formula $\psi$ of length less than $c_{k} t^{k+1}$ such that ( $\left.Q \overrightarrow{\mathrm{x}}\right) \phi$ is equivalent to $\psi$ in T. (Received January 7, 1965.)

65T-144. R. M. REDHEFFER, University of California, Los Angeles, California 90024. Recurrent inequalities.

Let $f_{k} \equiv f_{k}\left(a_{1}, a_{2}, \ldots, a_{k}\right)$ and $g_{k} \equiv g_{k}\left(a_{1}, a_{2}, \ldots, a_{k}\right)$ be real-valued functions defined for $a_{i} \in D_{i}$ where each $D_{i}$ is a given set. The inequality $\mu_{1} f_{1}+\mu_{2} f_{2}+\ldots+\mu_{n} f_{n} \leqq g_{1}+g_{2}+\ldots+g_{n}$ is recurrent if sup $\left[\mu f_{k}-g_{k}\right]=F_{k}(\mu) f_{k-1}$ for $k \geqq 1$, under the convention that $f_{0}=1$. The sup is over $a_{k} \in D_{k}$. By induction the recurrent inequality holds for all $a_{i} \in D_{i}$ if and only if $\mu_{k}=F_{k}^{-1}\left(\delta_{k}\right)-\delta_{k+1}$ for some sequence $\left\{\delta_{\mathrm{k}}\right\}$ with $\delta_{1} \leqq 0, \delta_{\mathrm{n}+1}=0$. This parametric solution gives an extremely simple proof of the inequalities of Carleman-Pólya-Kaluza-Szegö, and Hardy-Knopp. The forms obtained are both more general and sharper than those in the literature. Elementary inequalities are not used; on the contrary, the generalization of Carleman's theorem contains the inequality $G \leqq A$, and the generalization of Hardy-Knopp's theorem contains the Hölder inequality. About a dozen new inequalities, apparently unrelated to those cited above, also follow from the general theorem. (Received January 7, 1965.)

65T-145. RICHARD BALSAM, University of California, Los Angeles, California 90024.
Normal extensions of formally normal ordinary differential operators.
Let $L y=p_{0} y^{\prime \prime}+p_{1} y^{\prime}+p_{2} y, p_{0}, p_{1}$, and $p_{2} \in C^{\infty}(a, b)$. Assume $L^{+} L=L L^{+}$and let $N$ be the
minimal formally normal operator in $\mathscr{L}^{2}(a, b)$, induced by $L$. Theorem. N has a normal extension $\mathrm{N}_{1}$ in $\mathscr{L}^{2}(\mathrm{a}, \mathrm{b})$ if and only if $\mathrm{L}=\mathrm{c}_{0}\left(\mathrm{~L}_{\mathrm{s}}\right)^{2}+\mathrm{c}_{1} \mathrm{~L}_{\mathrm{s}}+\mathrm{c}_{2}$ for some $\mathrm{L}_{\mathrm{s}}=\mathrm{L}_{\mathrm{s}}^{+}\left(\mathrm{c}_{0}, \mathrm{c}_{1}\right.$, and $\mathrm{c}_{2}$ constants any of which may be 0 ), and $N_{1}=c_{0} A^{2}+c_{1} A+c_{2}$ I for some $A=A * \supseteq S$, $S$ being the minimal symmetric operator in $\mathscr{L}^{2}(a, b)$ induced by $\mathrm{L}_{\mathrm{s}}$. It is also shown, that constant coefficient ordinary linear differential operators with suitable boundary conditions provide examples of: (1) A formally normal operator N in Hilbert space $\mathfrak{F}$, such that N has no normal extension in $\mathfrak{F}$ or in any larger space. N may even be chosen so that $\mathscr{D}\left(\bar{N}^{*}\right)=\mathscr{D}\left(\mathrm{N}^{*}\right)$. (2) A formally normal N in $\mathscr{S}$, with $\mathfrak{M} \neq \overline{\mathfrak{M}}$, such that N does possess a normal extension in 5. (Notation same as Biriuk and Coddington, J. Math. Mech. 13 (1964), 617-638.) (Received January 7, 1965.)

65T-146. E. B. DAVIS, Stanford University, Box 2966, Stanford, California 94305. On a representation of solutions of linear partial differential equations.

The Bergman integral operator $C(f)$ transforms analytic functions $f(Z)$ of a complex variable Z into solutions of $\mathrm{L}(\phi)=\phi_{z z^{*}}+\mathrm{A} \phi_{z}+\overline{\mathrm{A}} \phi_{\mathrm{z}^{*}}+\mathrm{C}=0$, where $\mathrm{z}=\mathrm{x}+\mathrm{iy}, \mathrm{z}^{*}=\mathrm{x}$ - iy (see Bergman, Integral operators in the theory of linear partial differential equations, Ergebnisse der Mathematik und ihrer Grenzgebiete, Vol. 23, p. 10, Springer, Berlin, 1961). For x and y real, $\phi$ is generally complex. One can verify that if, for $k=0,1,2, \ldots, \phi_{2 k}=\operatorname{Re}\left(C\left(Z^{k}\right)\right)$ and $\phi_{2 k+1}=\operatorname{Im}\left(C\left(Z^{k}\right)\right)$, and if $A \neq 0, \phi$ can be expanded as $\sum_{j=0}^{\infty}\left(a_{j} \phi_{j} J+i a_{j} \phi_{j+J}\right)$, where $J=(-1)^{j}$ and the $a_{j}$ are real constants. If for some $\mathrm{r}, 0<\mathrm{r}<1, \phi$ is a real solution of $\mathrm{L}(\phi)=0$ in $|\mathrm{z}|<r$ which has there a representation $\sum \mathrm{Ja} \mathrm{j}_{\mathrm{j}} \phi_{\mathrm{j}}=\operatorname{Re}\left(\sum\left(\mathrm{J} \mathrm{a}_{\mathrm{j}} \phi_{\mathrm{j}}+\mathrm{ia} \mathrm{j}_{\mathrm{j}} \phi_{\mathrm{j}+\mathrm{J}}\right)\right.$ ), where $\mathrm{J}=(-1)^{\mathrm{j}}$, and if $\left\{\mathrm{b}_{\mathrm{k}}: \mathrm{b}_{\mathrm{k}}=\mathrm{a}_{2 \mathrm{k}}+\mathrm{ia} 2 \mathrm{k}+\mathrm{l}, \mathrm{k}=0,1,2, \ldots\right\}$ is a sequence of bounded variation which tends to zero, then the series $\sum \mathrm{Ja}{ }_{\mathrm{j}} \phi_{\mathrm{j}}$ converges absolutely in every compact subset of $\{\mathrm{z}:|\mathrm{z}| \leqq 1, \mathrm{z} \neq 1\}$. (Received January 7, 1965.)

65T-147. R. M. BLUMENTHAL, JORAM LINDENSTRAUSS and R. R. PHELPS, University of Washington, Seattle, Washington. Extreme operators into spaces of continuous functions.

Let $X, Y$ be compact Hausdorff spaces, and $C(X), C(Y)$ the spaces of real-valued continuous functions on $X, Y$, respectively, with the supremum norm. If $\phi: Y \rightarrow X$ is continuous and $\lambda \in C(Y)$, $|\lambda(y)|=1$ for all $y$, then it is easily verified that the operator $T$ defined by $\left(^{*}\right)(T f)(y)=\lambda(y) f(\phi y)$ is an extreme point of the unit ball $B$ of the space of all bounded operators from $C(X)$ to $C(Y)$, with operator norm. Theorem. If $X$ is metrizable, then every extreme point $T$ of $B$ is of the form (*)。 The proof depends on one of $E$. Michael's selection theorems. Similar questions are considered for operators from an arbitrary Banach space into C(Y). (Received November 30, 1964.)

65T-148. J. A. SCHAFER, University of Chicago, Chicago, Illinois. Homology ring of a finitely generated abelian group.

This note presents by means of generators and relations a complete and explicit description of the homology ring of a finitely generated abelian group in terms of the Betti numbers and invariant factors of the group. The indecomposable generators of positive degree in the homology ring of $\pi=Z_{\mathrm{h}_{1}} \oplus \ldots \oplus \mathrm{Z}_{\mathrm{h}_{\mathrm{n}}}$ where $\mathrm{h}_{\mathrm{n}} \mid \ldots \ldots, \mathrm{h}_{\mathrm{l}}$ are in a one-to-one correspondence with points of $\left(\mathrm{Z}^{+}\right)^{\mathrm{n}}-[0]$ 。 More explicitly, the homology ring is generated by the unit class and all symbols $\zeta(a)$ where $a=\left(a_{1}, \ldots, a_{n}\right)$ is in $\left(Z^{+}\right)^{n}-[0]$ and the degree of $\zeta(a)$ is equal to $2\left(\sum a_{1}\right)-1$. Moreover, if we define
$|a|=k$ if $a_{k} \neq 0$ and $a_{j}=0$ for all $j>k$ ，then $h_{|a|} \zeta(a)=0$ ．This is the only additive relation．The multiplicative relations are more complicated but still basically simple．This description is accom－ plished by means of the differential graded algebra resolution of certain Noetherian rings as developed by Tate［Illinois J．Math． 1 （1957），14－27］．Also used extensively is the notion of cross caps of cycles ［Eilenberg and Mac Lane，Ann．of Math。（2） 60 （1954），49－139］．Relations are obtained for cross caps when all complexes involved are differential graded rings．These relations lead directly to the relations among generators in the homology ring itself．（Received November 23，1964．）

65T－149．T．J．SUFFRIDGE，University of Kansas，Lawrence，Kansas 66045．Convolutions of convex functions．Preliminary report．

Let $f(z)=\sum_{1}^{\infty} a_{n} z^{n}$ be regular for $|z|<1$ ．We say $f$ is convex if $f(z)$ maps $|z|<1$ one－to－one onto a convex domain．Also，$f$ is called close－to－convex if there exists $g(z)$ convex such that $\operatorname{Re}\left[\mathrm{f}^{\prime}(\mathrm{z}) / \mathrm{g}^{\prime}(\mathrm{z})\right] \geqq 0$ for $|\mathrm{z}|<1$ 。In this paper the following theorems are proved．Theorem 1 。Suppose $h(t)$ is periodic with period $2 \pi$ and suppose $h(t)$ is nondecreasing for $t_{0}<t<t_{1}$ and $h(t)$ is nonincreas－ ing for $t_{1}<t<t_{0}+2 \pi$ 。Further，suppose $f(z)$ is convex．Then $\int_{-\pi} \pi_{f}\left(z e^{-i t}\right) h(t) d t$ is close－to－convex． Theorem 2．Let $f(z)=\sum_{1}^{\infty} a_{n} z^{n}$ and $g(z)=\sum_{l}^{\infty} b_{n} z^{n}$ be convex。 Then $F(z)=\sum_{l}^{\infty} a_{n} b_{n} z^{n}$ is close－to－ convex．（Received November 27，1964．）

65T－150．R．E．PEINADO，University of Iowa，Iowa City，Iowa 52240．On finite rings． Preliminary report．

Let $n$ be a non－negative integer．$n=\prod_{p_{i}} e_{i}, i=1, \ldots, r, p$ a prime，$e_{i} \geqq 0$ ，an integer。 $\tau(n)$ is the number of positive integer divisors of $n 。 \phi(n)$ the number of positive integers less than $n$ and relatively prime to $n$ ．Let $Z_{n}$ be the ring of integers modulo $n$ ．An element $e$ in $Z_{n}$ is idempotent if and only if $e=a p$ or $e=b q$ where $n=p q$ ，with $(p, q)=1$ and $l=a p+b q$ ．There are $2^{r}$ idempotent elements in $Z_{n}$ ．An element $b$ in $Z$ is nilpotent if and only if $b=\prod_{i}{ }^{t_{i}}, i=1, \ldots, r, t_{i}>0$ ．There are $\Pi p_{i}^{t_{i}-1}, i=1, \ldots, r$ ，nilpotent elements in $Z_{n}$ ．Theorem 1 ．There are $\tau(n)$ nonisomorphic finite rings $R$ with $n$ elements，whose additive structure is a cyclic group of order $n$ ．Let a be the generator of $\langle R,+\rangle$ ，then the structure of $R$ is determined by the value assigned to $a^{2}$ ．Let $R_{p}$ be the ring where $a^{2}=\mathrm{pa}$ ，and let $\mathscr{R}_{\mathrm{p}}$ be the isomorphism class of rings isomorphic to $\mathrm{R}_{\mathrm{p}}$ ．If $\mathrm{R}_{\mathrm{q}} \in \mathscr{R}_{\mathrm{p}}$ ，the isomorphism is given by a $\rightarrow$ xa where $(x, n)=1$ 。Let $\left(\mathscr{R}_{\mathrm{p}}\right)$ be the number of elements in the isomor－ phism class $\mathscr{R}_{\mathrm{p}}$ ．Then $\eta\left(\mathscr{R}_{1}\right)=\phi(\mathrm{n}), \eta\left(\mathscr{R}_{0}\right)=1$ and，if n is an even integer，$\eta\left(\mathscr{R}_{\mathrm{n} / 2}\right)=1$ ．Using the above－mentioned result on idempotent and nilpotent elements in $Z_{n}$ ，the idempotent and nilpotent elements of $R$ can be determined．Using the fundamental theorem of finite abelian groups，arbitrary finite rings of order $n$ can be constructed．Theorem 2 。For $n=\prod_{p_{i}}{ }^{e_{i}}, i=1, \ldots, r$ ，there are at least $\tau(\mathrm{n})=\prod\left(p_{i}+1\right), i=1, \ldots, r$ ，nonisomorphic rings of order $n$ ，having a fixed abelian group．（Received December 14，1964．）

65T－151．H．J．KEISLER，University of Wisconsin，Madison，Wisconsin 53706．Extending models of set theory．II．Preliminary report．

Let $Z$ be Zermelo－Fraenkel set theory with the axiom of choice．If $E$ is a binary relation， let $x_{E}=y: y E x$ ．Theorem 1．Suppose $\langle A, E\rangle$ is a countable model of $Z$ and $x$ is an infinite
regular cardinal in $\langle A, E\rangle$ ．Then there exists an elementary extension $\langle B, F\rangle$ of $\langle A, E\rangle$（in symbols， $\langle B, F\rangle\rangle\langle A, E\rangle$ ）such that $x_{F}$ has power $\kappa_{l}$ ，but $y_{F}=y_{E}$ for all yEx．Theorem 2．Assume there are $\kappa_{1}$ constructible sets of integers．Then for every countable well－founded model $\langle A, E\rangle$ of $Z$ there exists $\langle B, F\rangle\rangle\langle A, E\rangle$ such that $\omega$ is standard in $\langle B, F\rangle$ but $\langle B, F\rangle$ is not well－founded． Theorem 3．Suppose $\langle A, E\rangle$ is a natural model of $Z,\langle B, F\rangle\rangle\langle A, E\rangle$ ，and $\omega_{F}=\omega$ ．Then $x_{F}=x$ for every $x \in A$ of nonmeasurable power．Theorem 4．Let $x, y$ by two infinite cardinals of unequal confinalities in the sense of a model $\langle A, E\rangle$ of $Z$ ．Let $a, \beta$ be infinite regular cardinals such that $\max (a, \beta) \geqq$ power of $A$ ．Then there exists $\langle B, F\rangle\rangle\langle A, E\rangle$ such that the sets $x_{F}, y_{F}$ have confinali－ ties $a, \beta$ relative to the orderings induced by $F$ 。 Note：In Theorems $1,2,4$ ，we may make $B=$ $\bigcup\left\{x_{F}: x \in A\right\}$ ．We use the results of our abstract Extending models of set theory，to appear in J．Symbolic Logic．（Received January 11，1965．）

## 65T－152．WITHDRAWN

65T－153．C．W．LEININGER，Arlington State College，Arlington，Texas 76010．A note on the Hausdorff moment problem．

The solution of the Hausdorff moment problem given by J．H．Wells（Concerning the Hausdorff inclusion problem，Duke Math．J． 26 （1959），629－645）is extended to include Riemann－integrable mass functions．If $\left\{d_{n}\right\}$ is a number sequence，let $A_{n p}=C_{n, p} \Delta^{n-p_{d}}, n \geqq p, p=0,1,2, \ldots$ ．Theorem．If $\left\{d_{n}\right\}$ is a number sequence，the following two statements are equivalent：（i）There is a function $g$ Riemann－ integrable on $[0,1]$ such that $d_{n}=\int[0,1]^{I^{n}} d g, n=0,1,2, \ldots$ ；（ii）（a）there is a number $M$ such that $\left|\sum_{p=0}^{k} A_{n p}\right|<M, 0 \leqq k \leqq n, n=0,1,2, \ldots$ ，and（b）if $\epsilon>0$ and $0<\delta<1$ ，there is a finite collection $C$ of nonoverlapping subsegments（ $u, v$ ）of the segment $(0,1)$ such that $\sum_{C}(v-u)>1-\delta$ and if $u<y<z<v$ ，then there is a positive integer $N$ such that if $n>N,\left|\sum_{n y<p \leqq n z} A_{n p}+\sum_{n y \leqq p<n z} A_{n p}\right|$ $<\epsilon$ 。（Received January 13，1965．）

65T－154．M．S．ROBERTSON，Rutgers，The State University，New Brunswick，New Jersey． Radii of star－likeness and close－to－convexity．

Let $S_{a}$ denote the class of spiral－like univalent functions $f(z)$ ，regular in $E\{z:|z|<1\}$ and normalized so that $f(0)=0, f^{\prime}(0)=1$ ，with the property that in $E, \mathscr{R}\left\{e^{i a}\left(z f^{\prime}(z) / f(z)\right)\right\} \geqq 0$ for some real $a,|a| \leqq \pi / 2$ ．The radius $\rho_{a}$ of star－likeness for the class $S_{a}$ is defined as $\rho_{a}=\lim \inf f \in S_{a} \rho_{a}(f)$ ， where $f(z) \in S_{a}$ is star－like and univalent for $|z|<\rho_{a}(f)$ and in no larger circle。 If $f(z) \in S_{a}$ ，it is shown that，for $|z|=r<1, \mathscr{R}\left\{z f^{\prime}(z) / f(z)\right\} \geqq\left(1-2 r \cos a+r^{2} \cos 2 a\right) /\left(1-r^{2}\right)$ and that $\rho_{\alpha}=$ $[|\sin a|+\cos a]^{-1} \geqq 2^{-1 / 2}=0.707 \ldots \ldots,|a|<\pi / 2$ 。Similarly，every function $F(z)$ ，analytic and close－ to－convex in E relative to a non－normalized convex function $\phi(z)$ ，where $\phi^{\prime}(0)=e^{i a},|a|<\pi / 2$ ，is close－to－convex in $|z|<\rho_{a}$ relative to the normalized convex function $e^{-i a} \phi(z)$ ．These results are all sharp．（Received January 14，1965．）

65T－155．SEYMOUR GINSBURG，System Development Corporation， 2500 Colorado，Santa Monica，California and SHEILA GREIBACH，Harvard University，Cambridge，Massachusetts． Deterministic context－free languages．I．Preliminary report．
（ $\mathrm{K}, \Sigma, \Gamma, \delta, \mathrm{Z}_{0}, \mathrm{q}_{0}, \mathrm{~F}$ ）is a pushdown automaton（pda）if $\mathrm{K}, \Sigma, \Gamma$ are finite sets；$\delta$ is a mapping from $K \times(\Sigma \cup \epsilon) \times \Gamma$ to the finite subsets of $K \times \Gamma^{*} ; Z_{0}$ is in $\Gamma$ ；$q_{0}$ in $K$ ；and $F \subseteq K$ ．Let $\vdash$ be the following relation on $K \times \Sigma^{*} \times \Gamma^{*}$ 。For x in $\Sigma \cup \epsilon$ ，let $(\mathrm{p}, \mathrm{xw}, \mathrm{aZ}) \vdash(\mathrm{q}, \mathrm{w}, \mathrm{a} \gamma)$ if $\delta(\mathrm{p}, \mathrm{x}, \mathrm{Z})$ contains（ $\left.\mathrm{q}, \gamma\right)$ 。 Let $\vdash^{*}$ be the transitive extension of $\vdash$ ．A pda is deterministic if for each（ $q, Z$ ）in $K \times \Gamma$ ，（a）either $\delta(q, a, Z)$ contains exactly one element for all a in $\Sigma$ and $\delta(q, \epsilon, Z)=\phi$ ，or $\delta(q, \epsilon, Z)$ contains exactly one element and $\delta(\mathrm{q}, \mathrm{a}, \mathrm{Z})=\phi$ for all a in $\Sigma$ ；（b）if $\delta\left(\mathrm{q}, \mathrm{a}, \mathrm{Z}_{0}\right) \neq \phi$ ，a in $\Sigma \cup \epsilon$ ，then $\delta\left(\mathrm{q}, \mathrm{a}, \mathrm{Z}_{0}\right)=\left(\mathrm{p}, \mathrm{Z}_{0}, \mathrm{w}\right)$ for some（ $p, w$ ）in $K \times \Delta^{*}$ 。 Theorem．For $M$ a deterministic pda，let $T(M)=\left\{w \mid\left(q_{0}, w, Z_{0}\right) \vdash^{*}\left(q, \epsilon, Z_{0} \gamma\right)\right.$ ， $q$ in $F$ \}. Then $T(M)$ and $\Sigma^{*}-T(M)$ are unambiguous context－free languages．（ $T(M)$ is called a deterministic language．）（Received January 14，1965．）

65T－156．D．T．HAIMO， 77 Snake Hill Road，Belmont，Massachusetts．Series expansions for functions with the Huygens property．
$H^{*}$ denotes the $C^{2}$ class of functions $u(x, t)$ which，for $a<t<b$ ，satisfy the equation $\Delta_{\mathrm{x}} \mathrm{u}(\mathrm{x}, \mathrm{t})=(\partial / \partial \mathrm{t}) \mathrm{u}(\mathrm{x}, \mathrm{t})$ ，where $\Delta_{\mathrm{x}} \mathrm{f}(\mathrm{x})=\mathrm{f}^{\prime \prime}(\mathrm{x})+(2 \nu / \mathrm{x}) \mathrm{f}^{\prime}(\mathrm{x})$ ，and which are such that $\mathrm{u}(\mathrm{x}, \mathrm{t})=$ $\int_{0}^{\infty}{ }_{G}\left(\mathrm{x}, \mathrm{y} ; \mathrm{t}-\mathrm{t}^{\prime}\right) \mathrm{u}\left(\mathrm{y}, \mathrm{t}^{\prime}\right) \mathrm{d} \mu(\mathrm{y}), \mathrm{d} \mu(\mathrm{x})=2^{1 / 2-\nu}[\Gamma(\nu+1 / 2)]^{-1} \mathrm{x}^{2 \nu}{ }^{\mathrm{d}} \mathrm{x}$ ，for all $\mathrm{t}, \mathrm{t}^{\prime}, \mathrm{a}<\mathrm{t}^{\prime}<\mathrm{t}<\mathrm{b}$, the integral converging absolutely，where $G(x, y ; t)=(1 / 2 t)^{\nu+1 / 2} \exp \left[-\left(x^{2}+y^{2}\right) / 4 t\right] \mathscr{I}(x y / 2 t), \mathscr{I}(z)=$ $2^{\nu-1 / 2} \Gamma(\nu+1 / 2) \mathrm{I}_{\nu-1 / 2}(\mathrm{z}), \nu$ a fixed positive number。 Let $\mathrm{P}_{\mathrm{n}, \nu}(\mathrm{x}, \mathrm{t})=$ $\sum_{\mathrm{k}=0}^{\mathrm{n}} 2^{2 \mathrm{k}_{\mathrm{C}}} \mathrm{C}_{\mathrm{k}}[\Gamma(\nu+1 / 2+\mathrm{n}) / \Gamma(\nu+1 / 2+\mathrm{n}-\mathrm{k})] \mathrm{x}^{2 \mathrm{n}-2 \mathrm{k}_{\mathrm{t}} \mathrm{k}}$ and $\mathrm{W}_{\mathrm{n}, \nu}(\mathrm{x}, \mathrm{t})=\mathrm{G}(\mathrm{x} ; \mathrm{t}) \mathrm{P}_{\mathrm{n}, \nu}(\mathrm{x} / \mathrm{t},-1 / \mathrm{t})$ ，where $G(x ; t)=G(x, 0 ; t)$ ．Theorem．A necessary and sufficient condition that $u(x, t)=\sum_{n=0}^{\infty} a_{n} P_{n, \nu}(x, t)$ ，the series converging for $|t|<\sigma$ ，is that $u(x, t) \in H^{*}$ there．The coefficients $a_{n}$ have either of the determinations $a_{n}=u^{(2 n)}(0,0) /(2 n)!$ ，or $a_{n}=k_{n} \int_{0}^{\infty} u(y,-t) W_{n, \nu}(y, t) d \mu(y)$ ，where $k_{n}=$ $\Gamma(\nu+1 / 2) / 2^{4 n_{n}!} \Gamma(\nu+1 / 2+n)$ ．Theorem．If $u(x, t)=\sum_{n=0}^{\infty} a_{n} P_{n, \nu}(x, t)$ ，the series converging for $|t|<\sigma$ ，then $\mathrm{a}_{\mathrm{n}}=(-1)^{\mathrm{n}_{\mathrm{n}}} \int_{0}^{\infty} \mathrm{u}(\mathrm{ix}, \mathrm{t}) \mathrm{W}_{\mathrm{n}, \nu}(\mathrm{x}, \mathrm{t}) \mathrm{d} \mu(\mathrm{x})$ 。 Theorem．A necessary and sufficient condition that $u(x, t)=\sum_{n=0}^{\infty} b_{n} W_{n, \nu}(x, t)$ ，the series converging for $t>\sigma \geqq 0$ ，is that $u(x, t) \in H^{*}$ there and that $\int_{0}^{\infty}|\mathrm{u}(\mathrm{x}, \mathrm{t})| \mathrm{e}^{\mathrm{x}^{2} / 8 \mathrm{t}} \mathrm{d} \mu(\mathrm{x})<\infty, \sigma \leqq \mathrm{t}<\infty$ ．The coefficients $\mathrm{b}_{\mathrm{n}}$ have the determination $\mathrm{b}_{\mathrm{n}}=$ $\mathrm{k}_{\mathrm{n}} \int_{0}^{\infty} \mathrm{u}(\mathrm{y}, \mathrm{t}) \mathrm{P}_{\mathrm{n}, \nu}(\mathrm{y},-\mathrm{t}) \mathrm{d} \mu(\mathrm{y})$ ．（Received January 14，1965．）

65T－157．D．J．RODABAUGH，Vanderbilt University，Box 1631，Station B，Nashville，Tennessee． On the Wedderburn principal theorem for nearly（ 1,1 ）algebras．Preliminary report．

An algebra $A$ is said to be nearly（ 1,1 ）if $(x, y, x)=(x, x, y)$ and $(x, x, x)=0$ ．It is proved that， if $A-N$ is a simple associative algebra over an algebraically closed field，then $A=S+N$ where $S \cong A-N$ ．A consequence of this is that a $(1,1)$ algebra over an algebraically closed field has a Wedderburn decomposition．The principal theorem for an arbitrary nearly（ 1,1 ）algebra over an algebraically closed field is reduced to the case where $A$ has a unity element and $A-N$ is simple． If $\mathrm{A}-\mathrm{N}$ is a Cayley－Dickson algebra with divisors of zero，a subalgebra $\mathrm{A}^{\prime}$ of A can be constructed where $A^{\prime}=M_{2}+w M_{2}+M$ for $M$ a subideal of $N$ 。 It is further proved that $M \subseteq A_{11}^{\prime}\left(e_{11}\right)+A_{11}^{\prime}\left(e_{22}\right)$ and $\mathrm{M}^{2}=0$ ．（Received January 14，1965．）

## ERRATA <br> Volume 11

K. D. MAGILL, JR., Some homomorphism theorems for a class of semigroups. Page 662, Abstract 615-2.

At the end of the Abstract, after the word "isomorphism", insert the phrase "if $Y$ has more than one point'.
E. J. BELTRAMI and M. R. WOHLERS, On a converse to a theorem of Fatou, Page 663, Abstract 615-4.

Line 3. $" \mathrm{Df}=(1 / \pi \mathrm{i}) \mathrm{f} * \mathrm{pv}(1 / \mathrm{w})$ " should read $" \mathrm{Df}=(1 / \pi \mathrm{i}) \mathrm{f} * \operatorname{Dpv}(1 / \mathrm{w})$ "。
B. S. RANDOL, A number-theoretic estimate. Page 768, Abstract 64T-497.

The estimate in the last line should read " $\mathrm{N}_{\mathrm{kn}}(\mathrm{x})=\mathrm{Vx}^{\mathrm{n} / 2 \mathrm{k}}+\mathrm{O}\left(\mathrm{x}^{\mathrm{p}}\right)$, where $\mathrm{P}=$ $\mathrm{n}(\mathrm{n}-1)(2 \mathrm{k}-1)\left(4 \mathrm{nk}^{2}-2 \mathrm{nk}+2 \mathrm{k}\right)^{-1_{11}}$.
(Note: In the definition of $\mathrm{r}_{\mathrm{kn}}(\mathrm{m})$ (line 1), we distinguish between sums in which the summands occur in different order.)

## Volume 12

J. W. MOELLER, On the extrapolation of lacunary Fourier series. Page 89, Abstract 619-112.

| Line | $\underline{\text { For }}$ | Substitute |
| :---: | :---: | :---: |
| 1 | close subspace | closed subspace <br> 2 |
| 4 | $\left\{e^{\mathrm{u} \lambda \mathrm{x}}\right\}_{\lambda \in \Lambda}$ | $\left\{\mathrm{e}^{\mathrm{i} \lambda \mathrm{x}}\right\}_{\lambda \in \Lambda}$ |
| bouned | bounded |  |

JOHN de PILLIS, A curious result in linear algebra. Page 115, Abstract 619-194. Substitute the following for the text of the abstract:
Let $\mathfrak{A}$ be the algebra of $n \times n$ matrices and let $\mathfrak{B}$ represent the algebra of $m \times m$ matrices over the complex field. $\quad \mathfrak{Z}(\mathfrak{A}, \mathfrak{B})$ will denote the algebra of linear operators from $\mathfrak{A}$ to $\mathfrak{B}$, and $\mathfrak{R}$ denotes the cone of operators in $\mathfrak{P}(\mathfrak{A}, \mathfrak{B}$ ) that preserve positive semi-definite matrices (i.e. $T \in \mathbb{R}$ if and only if $T(A)$ is positive semi-definite in $\mathfrak{B}$ whenever $A$ is positive semi-definite in $\mathfrak{A}$ ). Theorem. Any operator $U \in \mathscr{R}(\mathfrak{A}, \mathfrak{B})$ can be written $U=\left(T_{1}-T_{2}\right)+i\left(T_{3}-T_{4}\right)$ where $T_{k} \in \Re$, $\overline{\mathrm{k}=1,2,3,4}$, and $\mathrm{i}^{2}=-1$.

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