MEETINGS

Calendar of Meetings

NOTE: This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the Notices was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

<table>
<thead>
<tr>
<th>Meeting No.</th>
<th>Date</th>
<th>Place</th>
<th>Deadline for Abstracts*</th>
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<tr>
<td>626</td>
<td>October 30, 1965</td>
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<td>627</td>
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<td>628</td>
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<td>(72nd Annual Meeting)</td>
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<td>August 29 - September 2, 1966</td>
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<td>(71st Summer Meeting)</td>
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<td>January 24-28, 1967</td>
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<td>(73rd Annual Meeting)</td>
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<td>August 28 - September 1, 1967</td>
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<tr>
<td></td>
<td>(72nd Summer Meeting)</td>
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<td>January, 1968</td>
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<td>August 26-30, 1968</td>
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<td></td>
<td>(73rd Summer Meeting)</td>
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*The abstracts of papers to be presented in person at the meetings must be received in the Headquarters Offices of the Society in Providence, Rhode Island, on or before these deadlines. The deadlines also apply to news items. The next two deadline dates for the by title abstracts are September 3, 1965 and September 21, 1965.

The Notices of the American Mathematical Society is published by the Society in January, February, April, June, August, October, November, and December. Price per annual volume is $7.00. Price per copy $2.00. Special price for copies sold at registration desks of meetings of the Society, $1.00 per copy. Subscriptions, orders for back numbers (back issues of the last two years only are available) and inquiries should be addressed to the American Mathematical Society, Box 6248, Providence, Rhode Island 02904.

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500
Seventieth Summer Meeting
and
Forty-Third Colloquium
Cornell University
Ithaca, New York
August 31—September 3, 1965

PROGRAM

The American Mathematical Society will hold its seventieth summer meeting at Ithaca, New York from Tuesday through Friday, August 31-September 3, 1965.

All sessions will be held in lecture rooms and classrooms on the campus of Cornell University.

Professor A. P. Calderón of the University of Chicago will present the Forty-Third Colloquium in a set of four lectures with the title "Singular integrals." The first lecture will be given in the Alice Statler Auditorium of Statler Hall on Tuesday, August 31 at 2:00 P.M. The second will be in the same auditorium on Wednesday, September 1 at 9:00 A.M. The third and fourth will be on Thursday, September 2 at 2:00 P.M. and on Friday, September 3 at 9:00 A.M., respectively, but in Room B17 of Upson Hall.

By invitation of the Committee to Select Hour Speakers for Summer and Annual Meetings, Professor George Lorentz of Syracuse University will address the Society on Friday, September 3 at 2:00 P.M. in the Alice Statler Auditorium. He will speak on "Applications of entropy to approximation."

There will be sessions for contributed papers on Tuesday, August 31 in the afternoon; Wednesday, September 1 in the morning; Thursday, September 2 in the afternoon; and Friday, September 3 both morning and afternoon. All sessions for contributed papers are in rooms well supplied with blackboards.

There will be no provision for late papers.

Several organizations will cooperate in holding meetings or council meetings on the same campus as the Society and at approximately the same time. These include Pi Mu Epsilon, Mu Alpha Theta, and the Society for Industrial and Applied Mathematics. Some details of their programs are presented in the section titled Activities of Other Associations beginning on page 529. In particular SIAM will present the John von Neumann Lecture in the Alice Statler Auditorium on Thursday, September 2 at 8:00 P.M. The invited speaker is Professor Freeman J. Dyson of the Institute for Advanced Study, whose topic is "Applications of group theory in particle physics."

The Mathematical Association of America will hold their forty-sixth summer meeting from Monday, August 30 through Thursday, September 2. This meeting marks the fiftieth anniversary of the Association, an event which will be heralded in the special character of their program. In deference to the Association, the Society has yielded a half day of time normally devoted to Society sessions in order to permit the Association to present an expanded anniversary program.

In many of the meeting rooms there is a NO SMOKING rule which must be strictly enforced as a safety measure.

COUNCIL AND BUSINESS MEETINGS

The Council of the Society will meet at 5:00 P.M. on Tuesday, August 31 in Room 320 of the Industrial and Labor Relations (ILR) Conference Center. There
will be an intermission for dinner at 6:30 P.M.

The Business Meeting of the Society will be held in Room B17 of Upson Hall on Thursday, September 2 at 4:45 P.M.

On recommendation of the Council, there will be proposed changes in the by-laws to introduce Mathematics of Computation as a Society publication, and to make the members of the Editorial Committee members of the Council.

ADVANCE REGISTRATION

The advance registration procedure was used for this meeting, as explained in the preliminary announcement and on the back cover of the April issue and of the June issue of these Notices. Those who have registered in advance have the advantage of assured reservations at all events and the convenience of being able to pick up a completed information packet without delay.

REGISTRATION

The Registration Desk will be in the Memorial Lobby of Willard Straight Hall. This is at the north end of the main floor. It will be open on Sunday, August 29 from 2:00 to 8:00 P.M.; on Monday, August 30 from 8:00 A.M. to 5:00 P.M.; on Tuesday through Thursday, August 31 through September 2, from 9:00 A.M. till 5:00 P.M.; and on Friday, September 3 from 9:00 A.M. till 3:30 P.M.

The registration fees will be as follows:
- Member $2.00
- Member's family .50
- Student No fee
- Others $5.00

The preferred procedure was to register in advance. One then completes the process by picking up the badge and information packet at the registration desk. It is desirable to have one's local address already established when completing registration as this information will be recorded at the registration desk for the visual index. In particular, persons with dormitory reservations should go first to University Hall I before completing registration. See the section on DORMITORY HOUSING.

It is possible to register at the desk without advance registration. The number of available dormitory beds exceeds a generous estimate of attendance. However, one may find that some of the facilities for which attendance is estimated and guaranteed will be sold out.

EMPLOYMENT REGISTER

The Mathematical Sciences employment Register will be in Rooms 200 and 205 of the ILR Conference Center. It will be open Tuesday through Thursday, August 31 through September 2, from 9:00 A.M. to 5:00 P.M. on each of the three days. Attention is invited to the announcement of the Employment Register on page 528, in particular to the necessity for prompt registration at the Employment Register Desk by both applicants and employers.

EXHIBITS

Book exhibits and exhibits of educational media will be displayed in the Memorial Room and in the Music Room of Willard Straight Hall on Tuesday, Wednesday, and Thursday, August 31, September 1 and 2. The Memorial Room is in the northwest corner of the main floor, to the west of the lobby which contains the registration desk. The Music Room is in the southwest corner of the main floor.

BOOK SALE

Books published by the Society will be sold for cash prices somewhat below the usual prices when these same books are sold by mail on invoice.

DORMITORY HOUSING

Dormitory rooms will be available in University Halls and Baker Dormitories. The rates are $5.50 per day for a single room and $3.50 per day per person for a double room. Soap, towels, and maid service are provided.

Rooms will be available from Saturday, August 28 to Saturday, September 4. Rooms must be vacated by 9:00 A.M. on September 4.
Dormitory room reservations which were made on the advance registration form, accompanied with payment for one night have been confirmed. A local map accompanied the confirmation.

When people with dormitory reservations arrive, they should go directly to University Hall I, without going first to the meeting registration desk. The office in University Hall I will be open 24 hours a day to give room assignments and keys and to collect the remainder of the room rent. Persons arriving without dormitory reservations may be able to obtain dormitory accommodations through a dormitory representative at the registration desk during the hours that desk is open and at University Hall I during other hours.

During the day, bellhops will be available. They will accept tips. It is not necessary to use their services.

Dormitory rooms are not air-conditioned.

There are no special provisions for small children. For an older child (but not for an adult) a cot 30 inches by 6 feet can be put in a double room at a cost of $2.00 per night.

United Rent-Alls, 363 Elmira Road, Ithaca, New York (Phone Area 607, AR3-1807) has standard cribs at $3.50 per week and portacribs at $2.50 per week. Their local supply is limited.

There will be a short list of babysitters at the registration desk.

**MOTELS AND HOTELS**

There are a number of motels in the Ithaca area, including the following:

**Collegetown Motor Lodge** — 312 College Avenue — 41 rooms — singles $9, doubles $12—10 minute walk from campus — air-conditioned

**Howard Johnson Motor Lodge** — North Triphammer Road and Route 13 — 72 rooms — swimming pool — singles $10.50 to $16.50, doubles $14.50 to $18.50 and family units — 15-20 minute drive from campus — air-conditioned

**Meadow Court** — 529 South Meadow Street — 50 rooms — singles $8 and up, doubles $12 and $13, family units $20—15 minute drive from campus — air-conditioned.

**Plaza Motel** — Corner Meadow Street and Elmira Road — 84 rooms — swimming pool — singles $8, doubles $12 and $13, for three $15 to $17, for four $17—15 minute drive from campus — air-conditioned.

**Wonderland Motel** — 654 Elmira Road — 27 rooms — swimming pool — singles $8 to $13, doubles $10 to $14, family units $19 to $21—25 minute drive from campus — air-conditioned.

**Hillside Inn** — 518 Stewart Avenue — 41 rooms — singles $6, doubles $8, family units — 35 rooms have private baths and are air-conditioned — 10 minute walk from campus.

**Ithaca Hotel** — 219 East State Street — 73 rooms, of which about three-quarters are air-conditioned — singles $7.50 to $8.50, doubles $12 to $14 — on bus line which goes to campus.

Advance booking of motel and hotel accommodations has been handled by Mr. B. Anderson, Meadow Court, 529 South Meadow Street, Ithaca, New York 14850. It is not clear whether accommodations will be available at the time this program is delivered.

**CAMPING**

There are three state parks in the area with tent and trailer sites for camping. The location and the mail address for information and reservations follows:


**Taughannock Falls State Park** — 8 miles north of Ithaca on route 89. R. D. 3, Trumansburg, New York 14850.

**MEALS**

The Willard Straight cafeteria will be open all day for meals, beginning at 7:00 A.M., and through the evening for light refreshment. The Sage Hall cafeteria will be open all day for meals. Both of these
operate on a cash basis.

The Statler Hall dining room will be open for breakfast, lunch, and dinner. One should realize that it is more expensive than the cafeterias and that reservations may be necessary.

A list of local restaurants will be at the registration desk.

ENTERTAINMENT

The Society for Industrial and Applied Mathematics will sponsor their traditional Beer Party at Noyes Lodge on the evening of Monday, August 30 at 8:00 P.M. In addition to the tickets sold through the advance registration form, some tickets will be sold at the registration desk at $1.25 each.

There will be a chicken barbecue on Wednesday evening at 6:00 P.M., in Alumni Field, or in Barton Hall in case of rain. Tickets were sold through the advance registration form at $2.75 per person and for children up to the age of 12 at $1.50 for a half portion. Some additional tickets will be sold at the registration desk.

For non-mathematicians, there will be an excursion by bus on Thursday, September 2 to the Corning Glass factory. The round trip bus tickets were sold for $1.75 per person. There may be additional tickets for sale at the registration desk. Departure time will be immediately after lunch at 12:45 P.M. The trip takes about four hours but the time of return is not guaranteed.

Persons registered at the meeting will be able to use the Cornell Golf Course by showing their badges and paying a greens fee of $2.00 on weekdays and $3.00 on weekends. Golf clubs can be rented.

There is swimming and picnicking at the state parks and at other places. There are tennis courts adjacent to the dormitories and bowling alleys near the campus. Information about these diversions may be obtained at the registration desk.

TRAVEL

Ithaca is centrally located in the Finger Lakes region of New York State. Mohawk Airlines serves the region with connecting flights from the principal surrounding large cities—New York, Newark, Boston, Pittsburgh, Cleveland, Buffalo, Detroit. The Tompkins County Airport is two miles from the University with taxi, limousine, and car rental service available. American Airlines serves this area through Syracuse. Empire State Airlines connects Binghamton, Ithaca, Syracuse, Elmira, and New York, thus connecting Ithaca with major air carriers. Also, Commuter airlines provide service between Binghamton and Washington, D.C. The Airline Guide should be consulted for flight times.

There is no direct railroad service to Ithaca, but the New York Central goes to Syracuse and the Erie Lackawanna to Binghamton and Owego. Buses connect Syracuse and Binghamton with Ithaca, but there is no public transportation between Owego and Ithaca. Owego is approximately 30 miles from Ithaca.

The Greyhound Bus Company runs several buses daily that connect Ithaca with New York, Buffalo, Rochester, Syracuse, Binghamton, and Scranton.

Ithaca can be reached by private car by using an excellent system of connecting highways. Coming from the west one uses the New York Thruway to Waterloo and then connects with New York Route 89 to Ithaca; coming from the New England area one uses the New York Thruway to Syracuse where U.S. Route 11 and New York Route 13 lead to Ithaca; coming from New York City one uses New York Route 17 to Owego where New York Route 96 connects directly to Ithaca. From a southerly direction, one can take the northerly extension of the Pennsylvania Turnpike from Philadelphia to Scranton and then Interstate Highway 91 to Binghamton, and then proceed as if from New York City.

The city of Ithaca operates a bus line which provides infrequent local service within the city. Persons employed by certain universities and government agencies are entitled to car rental discounts. Participants should determine if they qualify for this discount. Car rental agencies exist both in Ithaca and in the surrounding cities of Syracuse, Binghamton, and Elmira.

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Labor Day week-end follows this meeting. Persons who need return travel reservations are advised to make them in advance even though there will be a travel desk near the Registration Desk.
PARKING

The Safety Division (campus police) will issue cards through the registration desk which will allow parking on the campus.

WEATHER

The mean temperature during the week of August 30 is 67°. During this period the average maximum is 79°, and the average minimum is 55°. However, maxima as high as 90° are not impossible. Minima as low as 40° have also been known to occur. The average rainfall is 69/100 inches for the week. The humidity is between 50 and 60 percent in the afternoon and rises to an average of 90 percent at night.

MEDICAL SERVICES

The staff of the Gannett Clinic will provide people attending the meeting with daytime out-patient medical care for acute illness and injury. Such medical care will be charged, at a reasonable fee, to the individual seeking medical care.

In general, those requiring hospitalization and night-time emergency care will rely on private medical facilities of the community. The Gannett Clinic physician who is on emergency call will assist in arranging such care if his help is needed, and can be contacted by calling Sage Hospital, AR2-6962, or Gannett Clinic, Ext. 3493.

ADDRESS FOR MAIL AND TELEGRAMS

The address for mail and telegrams is in care of Mathematics Meetings, Willard Straight Hall, Cornell University, Ithaca, New York 14850. Individuals should check for mail from time to time in the vicinity of the registration desk.

COMMITTEE

The committee on arrangements consists of

H. L. Alder
S. U. Chase
W. H. J. Fuchs
R. Greenblatt
May Kinsolving
E. Pitcher
G. S. Rinehart
A. Rosenberg, Chairman
G. Sacks
G. L. Walker
R. J. Walker
S. Wainger
H. Widom

PROGRAM OF THE SESSIONS

The time limit for each contributed paper is ten minutes. The papers are scheduled at 15 minute intervals in order that the listeners can circulate among sessions. To maintain the schedule, the time limit will be strictly enforced.

TUESDAY, 2:00 P.M.

Colloquium Lecture I, Alice Statler Auditorium

Singular integrals
Professor A. P. Calderón, University of Chicago

TUESDAY, 3:15 P.M.

First Session on Analysis, Olin Hall M

3:15 - 3:25
(1) Norm decreasing homomorphisms in group algebras
Mr. F. P. Greenleaf, University of California, Berkeley (625-53)

3:30 - 3:40
(2) An integral in topological spaces
Mr. V. F. Pfeffer, The George Washington University (625-37)
(Introduced by Professor David Nelson)
3:45 - 3:55
(3) Hp spaces derived from function algebras
Mr. Samuel Merrill, Yale University (625-93)

4:00 - 4:10
(4) Characterization of lattices of real-valued continuous functions
Professor G. A. Jensen, University of Florida (625-47)

4:15 - 4:25
(5) A 1-function double normalization problem
Professor R. D. Sinkhorn, University of Houston (625-94)

4:30 - 4:40
(6) The strict topology and compactness in the space of measure. I. Preliminary report
Professor J. B. Conway, Louisiana State University at Baton Rouge, (625-126)
(Introduced by Dr. S. H. Gould)

4:45 - 4:55
(7) The period of a positive operator
Professor S. -T. C. Moy, Syracuse University (625-106)

TUESDAY, 3:15 P.M.

Second Session on Analysis, Upson Hall B-17
3:15 - 3:25
(8) Differential-integral calculus for abstract algebraic-topological structures, III
Mr. R. M. Sorensen, Trinity College (625-176)

3:30 - 3:40
(9) Two examples on the Lane integral
Professor F. M. Wright, Iowa State University (625-172)

3:45 - 3:55
(10) A characterization of Baire's first class
Mr. C. S. Reed, The University of Texas (625-96)
(Introduced by Professor H. S. Wall)

4:00 - 4:10
(11) An affirmative answer to a problem of Zahorski and some consequences
Professor A. M. Bruckner, University of California, Santa Barbara (625-85)

4:15 - 4:25
(12) On fixed points of commuting continuous functions

4:30 - 4:40
(13) Smoothness of functions generated by Riesz products
Professor P. L. Duren, University of Michigan (625-25)

4:45 - 4:55
(14) Remarks on a paper of Fort and Schuster
Professor G. U. Brauer, University of Minnesota (625-2)

TUESDAY, 3:15 P.M.

Session on Number Theory, Phillips Hall 101
3:15 - 3:25
(15) Factorable polynomials in several indeterminates over a finite field
Mr. Andrew Long, Duke University (625-75)
(Introduced by Professor Leonard Carlitz)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.
3:30 - 3:40
(16) Arithmetic properties of the q-binomial coefficient
Professor R. D. Fray, Duke University (625-70)

3:45 - 3:55
(17) The enumeration of certain triangular arrays
Mr. D. P. Roselle, Duke University (625-66)

4:00 - 4:10
(18) Inhomogeneous minimum of indefinite quaternary quadratic forms with signature $\pm 2$.
   Preliminary report
   Mr. V. C. Dumir, Ohio State University (625-142)
   (Introduced by Professor R. P. Bambah)

4:15 - 4:25
(19) How not to prove Waring's Conjecture
   Mr. George Glauberman, University of Chicago and Mr. G. S. Stoller*,
   Harvard University (625-167)

4:30 - 4:40
(20) Some corollaries of the prime number theorem
   Professor D. R. Hayes, University of Tennessee (625-39)

4:45 - 4:55
(21) Inhomogeneous minima of indefinite binary quadratic, ternary cubic and quaternary quartic forms in fields of formal Laurent series.
   Preliminary report
   Mr. S. K. Aggarwal, Ohio State University (625-127)
   (Introduced by Professor R. P. Bambah)

TUESDAY, 3:15 P.M.

Session on Graph Theory, Hollister Hall B-14
3:15 - 3:25
(22) Some theorems about n-vertex connected graphs
   Professor M. E. Watkins* and Professor D. M. Mesner, University of North Carolina (625-31)

3:30 - 3:40
(23) Cycles and vertex connectivity
   Professor D. M. Mesner* and Professor M. E. Watkins, University of North Carolina (625-32)

3:45 - 3:55
(24) On cliques in graphs
   Mr. J. W. Moon* and Professor Leo Moser, University of Alberta (625-18)

4:00 - 4:10
(25) Cluster-star inversion
   Professor Seymour Sherman, Indiana University (625-52)

4:15 - 4:25
(26) The reconstruction of a tree from its maximal proper subtrees
   Professor Frank Harary* and Mr. Ed Palmer, University of Michigan (625-115)

4:30 - 4:40
(27) On the core of a graph
   Professor Frank Harary and Mr. M. D. Plummer*, University of Michigan (625-116)

4:45 - 4:55
(28) Binary matrices with equal determinant and permanent
   Dr. L. W. Beineke* and Professor Frank Harary, University of Michigan (625-125)
First Session on Applied Mathematics, Hollister Hall, 110

3:15 - 3:25
(29) Weighted information theory. Preliminary report
Dr. M. A. Hyman, IBM Corporation, Bethesda, Maryland (625-6)

3:30 - 3:40
(30) Drag simulation for trajectory optimization
Dr. Mark Lotkin, General Electric Company, Philadelphia, Pennsylvania (625-83)

3:45 - 3:55
(31) Error estimation for Runge-Kutta methods
Professor Diran Sarafyan, Louisiana State University in New Orleans (625-90)

4:00 - 4:10
(32) New formulas for numerical solution of ordinary differential equations
Mr. G. Landry, Louisiana State University in New Orleans (625-110)  
(Introduced by Professor James W. Ellis)

4:15 - 4:25
(33) Solution of transcendental equations by transformation to differential equations
Mr. H. E. Fettis, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio (625-119)

4:30 - 4:40
(34) A simplex sufficiency condition for quadrature formulas
Professor C. S. Duris, Michigan State University (625-120)

4:45 - 4:55
(35) General numerical continued fractions
Professor Evelyn Frank, University of Illinois (625-160)

WEDNESDAY, 9:00 A.M.

Colloquium Lecture II, Alice Statler Auditorium

Singular integrals
Professor A. P. Calderón, University of Chicago

WEDNESDAY, 10:15 A.M.

Third Session on Analysis, Olin Hall M

10:15 - 10:25
(36) On dilations of operators
Dr. J. E. Simpson, Marquette University (625-23)

10:30 - 10:40
(37) Sequential convergence in spaces of finitely additive measures
Professor C. A. Green, University of Maine (625-59)

10:45 - 10:55
(38) Integral representation of the linear continuous functionals in the space of vector-valued continuous functions
Professor W. M. Bogdanowicz, Catholic University of America (625-65)

11:00 - 11:10
(39) A representation theorem of set-valued additive functions
Professor D. R. Henney, The George Washington University (625-67)

11:15 - 11:25
(40) On discretization and differentiation of operators with application to Newton's method
Professor J. M. Ortega* and Professor W. C. Rheinboldt, University of Maryland (625-69)
11:30 - 11:40
(41) Commutation relations among unbounded operators and bounded semi-groups
Mr. R. T. Moore, University of California, Berkeley (625-109)

11:45 - 11:55
(42) Remarks on a theorem of Yuji Ito
Professor P. C. Shields, Wayne State University (625-134)

WEDNESDAY, 10:15 A.M.

Fourth Session on Analysis, Upson Hall B-17

10:15 - 10:25
(43) A Riccati matrix integral equation
Professor W. T. Reid, University of Oklahoma (625-171)

10:30 - 10:40
(44) A perturbation theorem for a linear differential system with general linear boundary conditions. Preliminary report
Mr. R. N. Bryan, University of Utah (625-179)

10:45 - 10:55
(45) Regular-singular systems of linear differential equations. Preliminary report
Mr. D. A. Lutz, Syracuse University (625-74)

11:00 - 11:10
(46) The problem of singular perturbations of linear ordinary differential equations at regular singular points
Dr. D. L. Russell*, Mathematics Research Center, U. S. Army and University of Wisconsin and Dr. Yasutaka Sibuya, University of Minnesota (625-56)

11:15 - 11:25
(47) Bounds for functions of eigenvalues of vibrating systems. Preliminary report
Professor D. O. Banks, University of California, Davis (625-71)

11:30 - 11:40
(48) On a class of nonlinear fourth order differential equations
Professor P. K. Wong, Michigan State University (625-92)

11:45 - 11:55
(49) Stability, periodicity and almost periodicity of the solutions of nonlinear differential equations in Banach spaces II, with an unbounded operator
Professor C. T. Taam, Georgetown University (625-102)

WEDNESDAY, 10:15 A.M.

Fifth Session on Analysis, Hollister Hall 110

10:15 - 10:25
(50) Analytic continuation for functions of several variables
Professor R. H. Cameron and Professor D. A. Storvick*, University of Minnesota (625-107)

10:30 - 10:40
(51) Generalization of the Runge-Behnke-Stein approximation theorem on open Riemann surfaces
Professor Annette Sinclair, Purdue University (625-161)

10:45 - 10:55
(52) A translation theorem for analytic Feynman integrals
Professor R. H. Cameron* and Professor D. A. Storvick, University of Minnesota (625-108)

11:00 - 11:10
(53) A class of singular integral operators
Mr. J. E. Lewis, Rice University (625-13)
11:15 - 11:25  
(54) A theorem of Littlewood and lacunary series for compact groups  
Professor Alessandro Figá-Talamanca* and Mr. D. Rider, Massachusetts  
Institute of Technology (625-135)

11:30 - 11:40  
(55) On a general estimation principle and a theory of comparison-factors  
Professor W. C. Rheinboldt, University of Maryland (625-42)

11:45 - 11:55  
(56) Closed form solutions to some boundary value problems  
Dr. T. C.-H. Li, North American Aviation, Incorporated, Downey, Cali-  
ifornia (625-129)

WEDNESDAY, 10:15 A.M.

First Session on Algebra, Phillips Hall 101

10:15 - 10:25  
(57) m-semigroups, semigroups, and function representations  
Professor J. D. Monk, University of Colorado and Professor F. M. Sioson*,  
University of Florida (625-21)

10:30 - 10:40  
(58) Congruences on some special completely simple semigroups  
Professor Mario Petrich, Pennsylvania State University (625-89)

10:45 - 10:55  
(59) On completely 0-simple semigroups with their graphs  
Mr. J. B. Kim, Michigan State University (625-153)  
(Introduced by Dr. S. H. Gould)

11:00 - 11:10  
(60) A theorem on generalized inverses of matrices  
Professor M. H. Pearl, University of Maryland (625-154)

First Session on Topology, Hollister Hall B-14

10:15 - 10:25  
(63) The core of a manifold. Preliminary report  
Mr. R. E. Chandler, Duke University (625-29)

10:30 - 10:40  
(64) A certain class of knots  
Professor J. P. Levine, University of California, Berkeley (625-36)

10:45 - 10:55  
(65) Inequivalent complexes in hyperplanes of $E^n$  
Professor David Gillman, University of California, Los Angeles (625-41)

11:00 - 11:10  
(66) Disproof of the Hauptvermutung for manifold-pairs  
Mr. J. D. Sondow, Princeton University (625-58)

11:15 - 11:25  
(67) On aspherical embeddings of 2-spheres in the 4-sphere  
Dr. C. H. Giffen, The Institute for Advanced Study (625-68)
11:30 - 11:40
(68) Cobordism operations
Professor P. S. Landweber, Harvard University (625-111)

11:45 - 11:55
(69) Some nice embeddings in the trivial range
Mr. Jerome Dancis, University of Wisconsin (625-177)

WEDNESDAY, 10:15 A.M.

Session on Logic and Foundations, Phillips Hall 219
10:15 - 10:25
(70) Diophantine correct nonstandard models in the isols
Professor Anil Nerode, Cornell University (625-77)

10:30 - 10:40
(71) Mathematics of incompleteness and undecidability
Professor Vladeta Vuckovic, University of Notre Dame (625-82)
(Introduced by Professor Cecil Mast)

10:45 - 10:55
(72) Some theorems on extensions of arithmetic
Professor R. A. DiPaola, University of California, Los Angeles (524-104)

11:00 - 11:10
(73) The iterated hyper-jump
Mr. S. K. Thomason, Cornell University (625-130)

11:15 - 11:25
(74) A normal form meta-theorem for proof in number theory
Professor C. F. Kent, Case Institute of Technology (625-156)

11:30 - 11:40
(75) Some set-theoretic problems related to closed embedding
Professor S. G. Mrowka, The Pennsylvania State University (625-159)

THURSDAY, 2:00 P.M.

Colloquium Lecture III, Upson Hall B-17

Singular integrals
Professor A. P. Calderón, University of Chicago

THURSDAY, 3:15 P.M.

General Session, Olin Hall M
3:15 - 3:25
(76) Arithmetic properties of the Bell polynomials
Professor Leonard Carlitz, Duke University (625-100)

3:30 - 3:40
(77) Conull and coregular FK spaces
Mr. A. K. Snyder, Lehigh University (625-54)

3:45 - 3:55
(78) On graphs with transitive automorphism groups
Professor M. H. McAndrew, University of Washington (625-103)

4:00 - 4:10
(79) Extremal problems in packings and coverings, Preliminary report
Mr. R. I. J. Hans, Ohio State University (625-132)
(Introduced by Professor R. P. Bambah)

4:15 - 4:25
(80) The range of Perron's modular function
Dr. J. R. Kinney* and Dr. T. S. Pitcher, Massachusetts Institute of Technology (625-51)
THURSDAY, 3:15 P.M.

Sixth Session on Analysis, Upson Hall B-17

3:15 - 3:25
(81) Some generalized Hardy spaces
Professor L. D. Meeker, Rutgers, The State University (625-8)

3:30 - 3:40
(82) Approximation of symmetric operations by operators of finite rank
Professor Shmuel Kaniel, Stanford University (625-178)

3:45 - 3:55
(83) A strange dilation theorem. Preliminary report
Professor C. A. Berger, New York University (625-152)
(Introduced by Dr. S. H. Gould)

4:00 - 4:10
(84) The existence of invariant subspaces
Professor Louis de Branges and Professor J. L. Rovnyak*, Purdue University (625-99)

4:15 - 4:25
(85) Jordan structures for bounded linear transformations in Hilbert space
Mr. L. E. Winslow, Duke University (625-11)

THURSDAY, 3:15 P.M.

Seventh Session on Analysis, Phillips Hall 101

3:15 - 3:25
(86) A necessary and sufficient condition that an $L_2$ kernel possess a finite number of characteristic values
Dr. D. W. Swann, Bell Telephone Laboratories, Incorporated, Whippany, New Jersey (625-137)

3:30 - 3:40
(87) Extensions of the Paley-Wiener theorem
Dr. Jacob Burlak, Duke University (625-163)

3:45 - 3:55
(88) Evaluation of some improper integrals containing Hermite polynomials
Professor M. O. Gonzalez, University of Alabama (625-98)

4:00 - 4:10
(89) A distributional Hankel transformation
Professor A. H. Zemanian, SUNY at Stony Brook (625-61)

4:15 - 4:25
(90) Inverse series relations and other expansions involving Humbert polynomials
Professor H. W. Gould, West Virginia University (625-46)

THURSDAY, 3:15 P.M.

Session on Geometry, Hollister Hall B-14

3:15 - 3:25
(91) Diagrams and Schlegel diagrams
Professor Branko Grünbaum, Hebrew University of Jerusalem and Michigan State University (625-112)

3:30 - 3:40
(92) Spherical linkages and framed structures. Preliminary report
Mr. Michael Goldberg, Bureau of Naval Weapons, Washington, D. C. (625-73)

3:45 - 3:55
(93) Two points on a certain quartic surface are both perfect and nonperfect
Professor W. R. Hutcherson*, University of Florida and Mr. Stanley Frank, Dynamics Research Corporation (625-34)
4:00 - 4:10
(94) On Witt's theorem in the countably infinite case
Professor Herbert Gross, Montana State College (625-24)

4:15 - 4:25
(95) A characterization of the m-sphere using topological geometries. Preliminary report
Professor M. C. Gemignani, SUNY at Buffalo (625-7)

THURSDAY, 3:15 P.M.

Session on Statistics and Probability, Hollister Hall 110
3:15 - 3:25
(96) Series expansions for order statistics
Dr. J. S. White, General Motors Corporation, Warren, Michigan (625-22)

3:30 - 3:40
(97) Incomplete coupon collector's test for random sampling digits
Professor R. E. Greenwood, The University of Texas (625-44)

3:45 - 3:55
(98) Ordered cycle lengths in a random permutation
Dr. L. A. Shepp and Dr. S. P. Lloyd*, Bell Telephone Laboratories, Murray Hill, New Jersey (625-45)

4:00 - 4:10
(99) Some random variables defined on permutations
Professor R. A. Scoville, Duke University (625-168)

4:15 - 4:25
(100) Invariance of probability measures on a circle under stochastic rotations. Preliminary report
Professor C. E. Langenhop, Southern Illinois University (625-147)

THURSDAY, 4:45 P.M.

Business Meeting, Upson Hall B-17

FRIDAY, 9:00 A.M.

Colloquium Lecture IV, Upson Hall B-17

Singular integrals
Professor A. P. Calderón, University of Chicago

FRIDAY, 10:15 A.M.

Eighth Session on Analysis, Olin Hall M
10:15 - 10:25
(101) Spectral manifolds for a class of operators
Dr. Daniel Kocan, Stevens Institute of Technology (625-146)

10:30 - 10:40
(102) A representation of linear operator-valued functions. Preliminary report
Professor J. A. Reneke, Newberry College (625-78)

10:45 - 10:55
(103) Differentiable functionals with bounded nonempty support on Banach spaces. Preliminary report
Mr. J. H. M. Whitfield, Case Institute of Technology (625-162)

1:00 - 11:10
(104) Differentiability of subconvex functionals on Banach spaces. Preliminary report
Professor E. B. Leach, Case Institute of Technology (625-164)
11:15 - 11:25
(105) Remark on fixed point theorems and their extensions
Professor W. V. Petryshyn, The University of Chicago (625-155)

11:30 - 11:40
(106) Fine structure of norm in B-convex spaces
Professor Daniel Giesy, University of Southern California (625-141)

11:45 - 11:55
(107) On the support of representing measures for the disc algebra
Professor J. V. Ryff, University of Washington (625-105)

FRIDAY, 10:15 A.M.

Ninth Session on Analysis, Upson Hall B-17
10:15 - 10:25
(108) Some notes on positive functions
Mr. W. J. Schneider, Syracuse University (625-138)

10:30 - 10:40
(109) Convolutions of convex functions
Mr. T. J. Suffridge, Kansas University (625-118)

10:45 - 10:55
(110) A series useful in the study of difference equations
Professor Tomlinson Fort, Emory University (625-87)

11:00 - 11:10
(111) The asymptotic distribution of harmonic interpolation of extremal points
Professor J. H. Curtiss, University of Miami (625-84)

11:15 - 11:25
(112) Interpolation in a Hilbert space of analytic functions
Dr. J. T. Rosenbaum, SUNY at Buffalo (625-49)

11:30 - 11:40
(113) Extremal problems for the class of typically real functions. Preliminary report
Professor W. E. Kirwan, University of Maryland (625-19)

11:45 - 11:55
(114) Some relations associated with Koshliakov's formula
Mrs. K. Soni, Endwell, New York (625-17)
(Introduced by Professor J. R. F. Kent)

FRIDAY, 10:15 A.M.

Second Session on Algebra, Phillips Hall 101
10:15 - 10:25
(115) Free vector lattices
Mr. K. A. Baker, Harvard University (625-101)
(Introduced by Professor Garrett Birkhoff)

10:30 - 10:40
(116) An abstract property $P$ for groupoids such that locally locally $P$ is weaker than locally $P$
Professor A. H. Kruse, New Mexico State University (625-131)

10:45 - 10:55
(117) A description by generators and relations of the derived functors of the n-fold tensor product
Professor T. W. Hungerford, University of Washington (625-20)

11:00 - 11:10
(118) On coverings of universal algebras
Professor G. A. Gratzer, Pennsylvania State University (625-95)
<table>
<thead>
<tr>
<th>Time</th>
<th>Session</th>
<th>Speaker</th>
<th>Institution</th>
<th>Room</th>
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<tbody>
<tr>
<td>11:15 - 11:25</td>
<td></td>
<td>Injective boolean $\sigma$-algebras</td>
<td>Mr. F. E. J. Linton, Wesleyan University</td>
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<td>625-166</td>
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<tr>
<td>11:30 - 1:40</td>
<td></td>
<td>Representations of graded Lie algebras</td>
<td>Professor L. E. Ross, University of California, Los Angeles</td>
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<td>625-48</td>
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<tr>
<td>11:45 - 11:55</td>
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<td>Some new relations in universal envelope of a semi-simple Lie algebra</td>
<td>Mr. D.-N. Verma, The University of Chicago</td>
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<td>625-140</td>
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**Third Session on Algebra, Hollister Hall B-14**

<table>
<thead>
<tr>
<th>Time</th>
<th>Session</th>
<th>Lecture Title</th>
<th>Speaker</th>
<th>Institution</th>
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<tr>
<td>10:15 - 10:25</td>
<td></td>
<td>Subnormally abelian bounded groups</td>
<td>Mr. R. E. Phillips, University of Kansas</td>
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<td>625-144</td>
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<tr>
<td>10:30 - 10:40</td>
<td></td>
<td>On the existence of Cartan subgroups of finite groups</td>
<td>Mr. S. K. Sehgal, The Ohio State University</td>
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<td>625-128</td>
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<td></td>
<td></td>
<td>(Introduced by Professor H. J. Zassenhaus)</td>
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<td>10:45 - 10:55</td>
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<td>On ascending chain condition of lattices in a Lie group</td>
<td>Professor H. C. Wang, Northwestern University</td>
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<td>625-79</td>
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<tr>
<td>11:00 - 11:10</td>
<td></td>
<td>On finite groups with a CCT-subgroup. Preliminary report</td>
<td>Dr. Marcel Herzog, Cornell University</td>
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<td>625-76</td>
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<tr>
<td>1:15 - 11:25</td>
<td></td>
<td>Generators for the alternating group over finite fields</td>
<td>Mr. C. F. Wells, Duke University</td>
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<td>625-62</td>
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<tr>
<td>11:30 - 11:40</td>
<td></td>
<td>Residual properties of generalized free products. Preliminary report</td>
<td>Professor R. J. Gregorac, Iowa State University</td>
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<td>625-28</td>
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<tr>
<td>11:45 - 11:55</td>
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<td>Groups with normal abelian, non-central Hall p'-subgroups</td>
<td>Professor D. S. Passman, University of California, Los Angeles</td>
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<td>625-5</td>
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**Second Session on Topology, Hollister Hall, 110**

<table>
<thead>
<tr>
<th>Time</th>
<th>Session</th>
<th>Lecture Title</th>
<th>Speaker</th>
<th>Institution</th>
<th>Room</th>
<th>Phone</th>
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<tbody>
<tr>
<td>10:15 - 10:25</td>
<td></td>
<td>Total normality and the hereditary property</td>
<td>Dr. R. E. Hodel, Duke University</td>
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<td>625-97</td>
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<tr>
<td>10:30 - 10:40</td>
<td></td>
<td>Between $T_1$ and $T_2$</td>
<td>Professor Albert Wilansky, Lehigh University</td>
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<td>625-4</td>
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<tr>
<td>10:45 - 10:55</td>
<td></td>
<td>Topologies and Borel structures associated with Banach algebras of functions. II. Preliminary report</td>
<td>Mrs. P. R. Strauss, Columbia University</td>
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<td>625-133</td>
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<tr>
<td>11:00 - 11:10</td>
<td></td>
<td>Absolute Borel sets in their Stone-Cech compactifications</td>
<td>Professor Stephen Willard, University of Rochester and Lehigh University</td>
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<td>625-63</td>
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<tr>
<td>11:15 - 11:25</td>
<td></td>
<td>Continuity of partial order and of lattice operations</td>
<td>Mr. Janardan Dehpandé, University of Florida</td>
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<td>625-12</td>
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<td></td>
<td></td>
<td>(Introduced by Professor A. D. Wallace)</td>
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</table>
11:30 - 11:40
(134) Sequences in topological spaces. II
Professor C. E. Aull, Kent State University (625-86)

11:45 - 11:55
(135) Statistical topological spaces
Professor F. J. Wagner, University of Cincinnati (625-145)

FRIDAY, 2:00 P.M.

Invited Address, Alice Statler Auditorium

Applications of entropy to approximation
Professor George Lorentz, Syracuse University

FRIDAY, 3:15 P.M.

Tenth Session on Analysis, Olin Hall M
3:15 - 3:25
(136) Infinite series to the power of infinite series
Mr. N. P. Salz, Cornell Aeronautical Laboratory, Buffalo, New York (625-10)

3:30 - 3:40
(137) Transformation of power series. Preliminary report
Mr. Al Berin, Hughes Aircraft Company, Culver City, California (625-148)

3:45 - 3:55
(138) A generalization of the \( f, d_n \) method of summability
Professor Gaston Smith, University of Southern Mississippi (625-180)

4:00 - 4:10
(139) Borel exceptional values of an entire function
Professor Alfred Gray, University of California, Berkeley and Professor S. M. Shah*, University of Kansas (625-117)

4:15 - 4:25
(140) Power series with periodic gaps
Dr. Basil Gordon, University of California, Los Angeles (625-121)

4:30 - 4:40
(141) Coefficient extremal problems for Schlicht functions
Professor J. T. Poole, Florida State University (625-139)

4:45 - 4:55
(142) Entire functions of bounded index
Dr. Benjamin Lepson, U. S. Naval Research Laboratory, Catholic University of America (625-151)

FRIDAY, 3:15 P.M.

Eleventh Session on Analysis, Upson Hall B-17
3:15 - 3:25
(143) Sufficiency in a discontinuous variational problem allowing for certain boundary subarcs
Professor G. T. McAllister, Lehigh University (625-27)

3:30 - 3:40
(144) Fundamental solutions for elliptic equations with several singular lines
Professor R. J. Weinacht, University of Delaware (625-114)

3:45 - 3:55
(145) Huygens' principle and Hadamard's conjecture
Professor J. E. Lagnese, Georgetown University (625-80)
4:00 - 4:10
(146) Series representation of solutions of the generalized heat equation in N-dimensions
Professor D. T. Haimo, Southern Illinois University and Harvard University (625-60)

4:15 - 4:25
(147) Maximizing elliptic operators
Professor Carlo Pucci, University of Genova, Italy and Louisiana State University at Baton Rouge (625-43)

4:30 - 4:40
(148) An isoperimetric inequality for multiply-connected minimal surfaces
Professor J. C. C. Nitsche, University of Minnesota (625-15)

4:45 - 4:55
(149) Removable singularities and polar sets
Professor Walter Littman, University of Minnesota (625-173)

FRIDAY, 3:15 P.M.

Fourth Session on Algebra, Phillips Hall 101
3:15 - 3:25
(150) m-rings
Professor R. S. Pierce, University of Washington (625-157)

3:30 - 3:40
(151) A generalization of quasi-Frobenius rings. Preliminary report
Professor B. L. Osofsky, Rutgers, The State University (625-143)

3:45 - 3:55
(152) On a class of right alternative rings without nilpotent ideals. Preliminary report
Dr. M. M. Humm, Syracuse University (625-123)

4:00 - 4:10
(153) Semi-prime modules
Professor E. H. Feller, University of Wisconsin, Milwaukee and Professor E. W. Swokowski*, Marquette University (625-64)

4:15 - 4:25
(154) D. C. C. rings with a cyclic group of units
Professor Irwin Fischer, University of Colorado and Mr. K. E. Eldridge*, University of Colorado and Ohio University (625-57)

4:30 - 4:40
(155) Orders on commutative rings. Preliminary report
Professor C. W. Kohls and Professor J. D. Reid, Syracuse University (625-55)

4:45 - 4:55
(156) Finitely generated ZG-modules, G cyclic of prime order
Professor L. S. Levy, University of Wisconsin and University of Chicago (625-26)

FRIDAY, 3:15 P.M.

Third Session on Topology, Hollister Hall B-14
3:15 - 3:25
(157) Sums of solid horned spheres
Mr. L. O. Cannon, Utah State University (625-122)

3:30 - 3:40
(158) A continuum which admits only the identity mapping onto a non-degenerate subcontinuum
Professor Howard Cook, University of North Carolina (625-3)
3:45 - 3:55
(159) On the Eilenberg-Moore spectral sequence
Mr. Larry Smith, Yale University (625-30)

4:00 - 4:10
(160) Fiber spaces with totally pathwise disconnected fibers. Preliminary report
Mr. G. S. Ungar, Rutgers, The State University (625-170)

4:15 - 4:25
(161) On subgroups of the homeomorphism group of the Cantor set
Professor A. R. Vobach, University of Georgia (625-165)

4:30 - 4:40
(162) Variants of Sperner's lemma
Dr. Phillip Bacon, Bangor, Maine (625-72)

4:45 - 4:55
(163) On the proximate fixed-point property
Professor A. L. Yandl, Western Washington State College (625-149)

FRIDAY, 3:15 P.M.
Third Session on Applied Mathematics, Hollister Hall 110

3:15 - 3:25
(164) On the spectrum of an operator arising in the theory of hydrodynamic stability
Professor S. I. Rosencrans*, Tulane University and Professor D. H. Sattinger, University of California, Los Angeles (625-175)

3:30 - 3:40
(165) Some remarks on the Stefan problem
Dr. A. D. Solomon, New York University (625-158)

3:45 - 3:55
(166) Alternating direction implicit methods with smooth initial error
Professor R. E. Lynch*, The University of Texas and Professor J. R. Rice, Purdue University (625-136)

4:00 - 4:10
(167) Estimates at infinity for stationary solutions of the Navier-Stokes equations in two dimensions
Dr. D. R. Smith, Stanford University (625-91)

4:15 - 4:25
(168) Pointwise bounds in the Cauchy problem for elliptic systems of partial differential equations
Dr. James Conlan, U. S. Naval Ordnance Laboratory, White Oak, Maryland and Professor G. N. Trytten*, University of Maryland (625-88)

4:30 - 4:40
(169) On the Cauchy problem for an elliptic system
Professor P. W. Schaefer, University of South Florida (625-81)

Bethlehem, Pennsylvania

Everett Pitcher
Associate Secretary
**TIME TABLE**
(Daylight Saving Time)

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<tr>
<th>SUNDAY</th>
<th>August 29</th>
<th>American Mathematical Society</th>
<th>Mathematical Association of America</th>
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<tbody>
<tr>
<td>10:00 A.M.</td>
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<td>Board of Governors</td>
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<td>Industrial and Labor Relations</td>
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<td>Conference Center, Room 105</td>
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<tr>
<td>2:00 P.M. - 8:00 P.M.</td>
<td>REGISTRATION - Willard Straight Hall, Memorial Lobby</td>
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<td>7:00 P.M. - 8:10 P.M.</td>
<td>FILM PROGRAM</td>
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<td>Alice Statler Auditorium, Statler Hall</td>
<td>WHAT IS MATHEMATICS AND HOW DO WE TEACH IT?, with Lipman Bers, Samuel Eilenberg, A. M. Gleason, Leo Zippin, and H. O. Pollak</td>
</tr>
<tr>
<td>8:10 P.M. - 9:10 P.M.</td>
<td>LET US TEACH GUESSING: A Demonstration with George Polya</td>
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<th>MONDAY</th>
<th>August 30</th>
<th>AMS</th>
<th>MAA</th>
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<tbody>
<tr>
<td>8:00 A.M. - 5:00 P.M.</td>
<td>REGISTRATION - Willard Straight Hall, Memorial Lobby</td>
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<tr>
<td>9:00 A.M.</td>
<td>Alice Statler Auditorium, Statler Hall</td>
<td>Earle Raymond Hedrick Lectures: Differential Topology, Lecture I</td>
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<td></td>
<td>J. W. Milnor</td>
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<td>10:10 A.M.</td>
<td>History of the MAA Before World War II</td>
<td>A. A. Bennett</td>
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<tr>
<td>11:10 A.M.</td>
<td>History of the MAA After World War II</td>
<td>R. A. Rosenbaum</td>
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<tr>
<td>2:00 P.M.</td>
<td>Hedrick Lecture II</td>
<td>J. W. Milnor</td>
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<tr>
<td>3:10 P.M.</td>
<td>History of Mathematical Education</td>
<td>P. S. Jones</td>
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<tr>
<td>4:20 P.M. - 5:20 P.M.</td>
<td>FILM PROGRAM</td>
<td>Alice Statler Auditorium, Statler Hall</td>
<td>THE KAKEYA PROBLEM, with A. S. Besicovitch</td>
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<tr>
<td>8:00 P.M.</td>
<td>SIAM - BEER PARTY - NOYES LODGE</td>
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<thead>
<tr>
<th>TUESDAY</th>
<th>August 31</th>
<th>American Mathematical Society</th>
<th>Mathematical Association of America</th>
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<tbody>
<tr>
<td>9:00 A.M. - 5:00 P.M.</td>
<td>REGISTRATION - Willard Straight Hall, Memorial Lobby</td>
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<td>EXHIBITS - Willard Straight Hall, Memorial Room and Music Room</td>
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<td>EMPLOYMENT REGISTER - Industrial and Labor Relations Conference Center, Rooms 200, 205</td>
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<tr>
<td>9:00 A.M.</td>
<td>Alice Statler Auditorium, Statler Hall</td>
<td>Hedrick Lecture III</td>
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<td></td>
<td>J. W. Milnor</td>
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<td>10:10 A.M.</td>
<td>Business Meeting, ceremonies honoring the charter members</td>
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<tr>
<td>11:10 A.M.</td>
<td>Retiring Presidential Address: Challenging Theorems</td>
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<td></td>
<td>R. H. Bing</td>
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<tr>
<td>2:00 P.M.</td>
<td>Alice Statler Auditorium, Statler Hall</td>
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<td></td>
<td>Colloquium Lecture I</td>
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<td></td>
<td>Singular Integrals</td>
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<tr>
<td></td>
<td>A. P. Calderón</td>
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519
### TIME TABLE
(Daylight Saving Time)

<table>
<thead>
<tr>
<th>TUESDAY August 31</th>
<th>American Mathematical Society</th>
<th>Mathematical Association of America</th>
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<tbody>
<tr>
<td>3:15 P.M.</td>
<td>Contributed Papers</td>
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<td></td>
<td>Session on Number Theory</td>
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<td></td>
<td>Phillips Hall 101</td>
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<td></td>
<td>First Session on Analysis</td>
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<td></td>
<td>Olin Hall M</td>
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<td></td>
<td>Session on Graph Theory</td>
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<td></td>
<td>Hollister Hall B-14</td>
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<tr>
<td></td>
<td>First Session on Applied Mathematics</td>
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<td>Hollister Hall 110</td>
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<td></td>
<td>Second Session on Analysis</td>
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<tr>
<td></td>
<td>Upson Hall B-17</td>
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<tr>
<td>3:30 P.M.</td>
<td>PI MU EPSILON FRATERNITY - Contributed Papers - Olin Hall, Room B</td>
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<tr>
<td>5:00 P.M.</td>
<td>Council Meeting</td>
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<tr>
<td></td>
<td>Industrial and Labor Relations</td>
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<tr>
<td></td>
<td>Conference Center, Room 320</td>
<td></td>
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<tr>
<td></td>
<td>(Dinner Intermission 6:30 P.M.)</td>
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<tr>
<td>6:30 P.M.</td>
<td>PI MU EPSILON BANQUET - Alt Heidelberg Restaurant</td>
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<tr>
<td></td>
<td>&quot;Should We Take Undergraduates Seriously?&quot;</td>
<td>K. O. May</td>
</tr>
<tr>
<td>7:00 P.M. - 7:30 P.M.</td>
<td>FILM PROGRAM - Alice Statler Auditorium, Statler Hall</td>
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<tr>
<td></td>
<td>TOPOLOGY, with Professors Marston Morse and Raoul H. Bott</td>
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</tr>
<tr>
<td>7:30 P.M. - 8:00 P.M.</td>
<td>CEM ANIMATED CALCULUS FILMS, including INFINITE ACRES by Melvin Henriksen, THE VOLUME OF A SOLID OF REVOLUTION by George Leger, Chandler Davis, et al</td>
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</tr>
<tr>
<td>8:10 P.M.</td>
<td>To be Announced</td>
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<tr>
<td>8:30 P.M.</td>
<td>PI MU EPSILON - Contributed Papers - Olin Hall, Room B</td>
<td></td>
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</tbody>
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<table>
<thead>
<tr>
<th>WEDNESDAY September 1</th>
<th>AMS</th>
<th>MAA</th>
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</thead>
<tbody>
<tr>
<td>8:00 A.M.</td>
<td>PI MU EPSILON - Breakfast Meeting - Willard Straight Cafeteria, Kimball A, B, C</td>
<td></td>
</tr>
<tr>
<td>9:00 A.M. - 5:00 P.M.</td>
<td>REGISTRATION - Willard Straight Hall, Memorial Lobby</td>
<td></td>
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<tr>
<td></td>
<td>EXHIBITS - Willard Straight Hall, Memorial Room and Music Room</td>
<td></td>
</tr>
<tr>
<td></td>
<td>EMPLOYMENT REGISTER - Industrial and Labor Relations Conference Center Rooms 200, 205</td>
<td></td>
</tr>
<tr>
<td>9:00 A.M.</td>
<td>MU ALPHA THETA - Governing Council - Industrial and Labor Relations Conference Center, Room 306</td>
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</tr>
<tr>
<td>9:00 A.M.</td>
<td>Alice Statler Auditorium, Statler Hall</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Colloquium Lecture II</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A. P. Calderón</td>
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<tr>
<td>9:15 A.M.</td>
<td>PI MU EPSILON - Contributed Papers - Olin Hall, Room B</td>
<td></td>
</tr>
<tr>
<td>10:00 A.M.</td>
<td>SIAM - Council Meeting - Industrial and Labor Relations Conference Center, Room 105</td>
<td></td>
</tr>
</tbody>
</table>

520
### TIME TABLE
(Daylight Saving Time)

#### WEDNESDAY
**September 1**

<table>
<thead>
<tr>
<th>Time</th>
<th>American Mathematical Society</th>
<th>Mathematical Association of America</th>
</tr>
</thead>
<tbody>
<tr>
<td>10:15 A.M.</td>
<td>Contributed Papers</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Third Session on Analysis</td>
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<tr>
<td></td>
<td>Olin Hall M</td>
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<tr>
<td></td>
<td>Fourth Session on Analysis</td>
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<tr>
<td></td>
<td>Upson Hall, B-17</td>
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</tr>
<tr>
<td></td>
<td>Fifth Session on Analysis</td>
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<tr>
<td></td>
<td>Hollister Hall 110</td>
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<tr>
<td></td>
<td>First Session on Algebra</td>
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<tr>
<td></td>
<td>Phillips Hall 101</td>
<td></td>
</tr>
<tr>
<td></td>
<td>First Session on Topology</td>
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</tr>
<tr>
<td></td>
<td>Hollister Hall B-14</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Session on Logic and Foundations</td>
<td></td>
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<tr>
<td></td>
<td>Phillips Hall 219</td>
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</tr>
<tr>
<td>12:15 P.M.</td>
<td>PI MU EPSILON COUNCIL LUNCHEON - Willard Straight Cafeteria</td>
<td></td>
</tr>
<tr>
<td>2:00 P.M.</td>
<td>Alice Statler Auditorium, Statler Hall</td>
<td>Trends in Algebra</td>
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<tr>
<td></td>
<td>C. W. Curtis</td>
<td></td>
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<tr>
<td>3:10 P.M.</td>
<td>Trends in Analysis</td>
<td></td>
</tr>
<tr>
<td></td>
<td>E. J. McShane</td>
<td></td>
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<tr>
<td>6:00 P.M.</td>
<td>CHICKEN BARBECUE - Alumni Field (in case of rain, Barton Hall)</td>
<td></td>
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</table>

#### THURSDAY
**September 2**

<table>
<thead>
<tr>
<th>Time</th>
<th>AMS</th>
<th>MAA</th>
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<tbody>
<tr>
<td>9:00 A.M.-5:00 P.M.</td>
<td>REGISTRATION - Willard Straight Hall, Memorial Lobby</td>
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<tr>
<td></td>
<td>EXHIBITS - Willard Straight Hall, Memorial Room and Music Room</td>
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<tr>
<td></td>
<td>EMPLOYMENT REGISTER - Industrial and Labor Relations Conference Center, Rooms 200, 205</td>
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<tr>
<td>9:00 A.M.</td>
<td>Alice Statler Auditorium, Statler Hall</td>
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<tr>
<td></td>
<td>CUPM, The History of an Idea</td>
<td>W. L. Duren, Jr.</td>
</tr>
<tr>
<td>10:00 A.M.</td>
<td>How Mathematicians Can Serve the Federal Government</td>
<td>J. W. Tukey</td>
</tr>
<tr>
<td>10:50 A.M.</td>
<td>The Future Role of the Federal Government in Mathematics</td>
<td>Saunders Mac Lane</td>
</tr>
<tr>
<td>11:30 A.M.</td>
<td>Discussion from the Floor</td>
<td></td>
</tr>
<tr>
<td>12:45 P.M.</td>
<td>CORNING GLASS WORKS - Bus Excursion</td>
<td></td>
</tr>
<tr>
<td>1:00 P.M.</td>
<td>CONFERENCE BOARD OF MATHEMATICAL SCIENCES - Council Meeting</td>
<td>Industrial and Labor Relations Conference Center, Room 105</td>
</tr>
<tr>
<td>2:00 P.M.</td>
<td>Upson Hall, B-17</td>
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<tr>
<td></td>
<td>Colloquium Lecture III</td>
<td>A. P. Calderón</td>
</tr>
<tr>
<td>3:15 P.M.</td>
<td>Contributed Papers</td>
<td>General Session</td>
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<tr>
<td></td>
<td>Olin Hall M</td>
<td>Olin Hall M</td>
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<tr>
<td></td>
<td>Sixth Session on Analysis</td>
<td>Upson Hall B-17</td>
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</tbody>
</table>
# TIME TABLE
(Daylight Saving Time)

<table>
<thead>
<tr>
<th>THURSDAY</th>
<th>American Mathematical Society</th>
<th>Mathematical Association of America</th>
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<tbody>
<tr>
<td>September 2</td>
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<tr>
<td>3:15 P.M.</td>
<td><strong>Contributed Papers</strong></td>
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<tr>
<td></td>
<td>Seventh Session on Analysis</td>
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<tr>
<td></td>
<td>Phillips Hall 101</td>
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<tr>
<td></td>
<td>Session on Geometry</td>
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<td></td>
<td>Hollister Hall B-14</td>
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<tr>
<td></td>
<td>Session on Statistics and Probability</td>
<td>Hollister Hall 110</td>
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<tr>
<td>4:45 P.M.</td>
<td><strong>Business Meeting</strong></td>
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<td></td>
<td>Upson Hall B-17</td>
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<tr>
<td>8:00 P.M.</td>
<td><strong>SIAM - John von Neumann Lecture</strong></td>
<td>Alice Statler Auditorium, Statler Hall</td>
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<td></td>
<td>&quot;Applications of Group Theory in Particle Physics&quot;</td>
<td>F. J. Dyson</td>
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<thead>
<tr>
<th>FRIDAY</th>
<th>AMS</th>
<th>MAA</th>
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<tbody>
<tr>
<td>September 3</td>
<td></td>
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<tr>
<td>9:00 A.M.-3:30 P.M.</td>
<td>REGISTRATION - Willard Straight Hall, Memorial Lobby</td>
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<tr>
<td>9:00 A.M.</td>
<td>Upson Hall, B-17</td>
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<tr>
<td></td>
<td>Colloquium Lecture IV</td>
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<tr>
<td></td>
<td>A. P. Calderón</td>
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<tr>
<td>10:15 A.M.</td>
<td><strong>Contributed Papers</strong></td>
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<td></td>
<td>Eighth Session on Analysis</td>
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<td></td>
<td>Olin Hall M</td>
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<td></td>
<td>Ninth Session on Analysis</td>
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<td>Upson Hall B-17</td>
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<td></td>
<td>Second Session on Algebra</td>
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<td></td>
<td>Phillips Hall 101</td>
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<td></td>
<td>Third Session on Algebra</td>
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<td>Hollister Hall B-14</td>
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<td></td>
<td>Second Session on Topology</td>
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<td></td>
<td>Hollister Hall 110</td>
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<tr>
<td>2:00 P.M.</td>
<td>Alice Statler Auditorium, Statler Hall</td>
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<tr>
<td></td>
<td>Invited Address: Applications of Entropy to Approximation</td>
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<td></td>
<td>G. Lorentz</td>
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<tr>
<td>3:15 P.M.</td>
<td><strong>Contributed Papers</strong></td>
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<td>Tenth Session on Analysis</td>
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<td></td>
<td>Olin Hall M</td>
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<td></td>
<td>Eleventh Session on Analysis</td>
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<td>Upson Hall B-17</td>
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<td>Fourth Session on Algebra</td>
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<td>Phillips Hall 101</td>
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<td>Third Session on Topology</td>
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<td>Hollister Hall B-14</td>
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<tr>
<td></td>
<td>Third Session on Applied Mathematics</td>
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<td>Hollister Hall 110</td>
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PRELIMINARY ANNOUNCEMENT OF MEETING

Six Hundred Twenty-Sixth Meeting
Massachusetts Institute of Technology
Cambridge, Massachusetts
October 30, 1965

The six hundred twenty-sixth meeting of the American Mathematical Society will be held at the Massachusetts Institute of Technology on Saturday, October 30, 1965.

There will be an invited address in the afternoon and sessions for contributed papers both morning and afternoon.

The registration desk will be set up on the first floor of Building 2.

The Massachusetts Institute of Technology is located on the Cambridge side of the Charles River approximately one to two miles from the various railway stations in Boston. It is easily accessible by automobile, subway, trackless trolley, or taxi-cab.

Parking space will be available; its precise location will be described in the program.

M.I.T. is a seven to ten minute walk from the Kendall Square station of the Cambridge-Dorchester subway. This subway may be boarded at various points, including South Station, Boston, and Harvard Square, Cambridge. The most convenient entrance for those coming by subway is at the Northwest corner of the Hayden Memorial Library.

Those coming by taxicab or trackless trolley will find it convenient to use the main entrance, 77 Massachusetts Avenue. Most of the entrances except the two mentioned above are closed on Saturday.

Lunch will be served in an M.I.T. cafeteria, and a list of nearby restaurants in Boston and Cambridge will be available.

Everett Pitcher
Associate Secretary
Bethlehem, Pennsylvania
NEWS ITEMS AND ANNOUNCEMENTS

NEW YORK UNIVERSITY OFFERS MASTER’S PROGRAM FOR HIGH SCHOOL MATHEMATICS TEACHERS

A new master's degree program for high school teachers of mathematics has been established by New York University. Courses leading to the master of arts degree will stress mathematical content that will aid instructors in their classroom work.

This degree can be earned through NYU's In-Service Institute for High School Teachers of Mathematics. Supported by a grant from the National Science Foundation, the Institute provides free tuition for students and an allowance for books and travel expenses. The ninth In-Service Institute will be conducted by the University's Courant Institute of Mathematical Sciences from September 23, 1965, through May 13, 1966.

Three one-year courses will be offered this year: "Programming for Digital Computers," "Modern Algebra," and "Aspects of Geometry."

All high school mathematics teachers are eligible to apply. Applications can be obtained from Dr. Melvin Hausner, Director of the Institute, New York University, 251 Mercer Street, New York, New York 10012.

TRAVEL GRANTS FOR INTERNATIONAL CONGRESS IN MOSCOW

Travel grants will be made to a number of qualified mathematicians for attendance at the International Congress of Mathematicians, to be held in Moscow, August 16-26, 1966.

Selection of the grantees will be made by the Committee on Travel Grants of the Division of Mathematics of the National Academy of Sciences—National Research Council. This committee will also include representatives of all mathematical societies affiliated with the Division and representatives of various governmental agencies concerned.

Applications can be obtained from the Division of Mathematics, National Academy of Sciences, National Research Council, 2101 Constitution Avenue, N. W., Washington, D. C. 20418, and must be received by the committee on or before November 1, 1965.

For information on plans for a charter flight to the Congress, see the inside back cover of this issue of the Notices.

MATSCIENCE SUMMER SCHOOL

The second MATSCIENCE Summer School will be held in Bangalore (South India) for three weeks from August 17 to September 4, 1965, on "Recent Trends in Theoretical Physics."

The following persons from outside India who will be resident at the Institute as visiting scientists will take part in the summer school: R. J. Oakes and A. Grossman from the United States, V. L. Teplitz from Switzerland, and J. P. Vigier from France.

About twenty research workers from various institutions in India including MATSCIENCE will be participants of the school and will deliver lectures and conduct seminars summarizing their recent research or the latest developments in their respective fields. Among these participants are: M. G. K. Menon, B. Misra, S. Kichenasamy, V. Devanathan, and S. K. Srinivasan. Also included are Alladi Ramakrishnan, T. K. Radha, N. R. Ranganathan, and K. Venkatesan, permanent members of the MATSCIENCE staff.
THIRD SOUTHEASTERN CONFERENCE ON THEORETICAL AND APPLIED MECHANICS

This Conference will be held on March 31, and April 1, 1966, at the University of South Carolina. The deadline for submission of papers has been tentatively set as November 15, 1965. Prospective authors should contact the Executive Chairman, Professor J. D. Waugh, University of South Carolina, Columbia, South Carolina, or the editorial chairman, Professor W. H. Shaw, Auburn University, Auburn, Alabama. All papers accepted will be presented at the conference and will be published in full in the proceedings volume.

UNIVERSITY OF HOUSTON INITIATES Ph.D. PROGRAM IN MATHEMATICS

The University of Houston has initiated a Ph. D. program in mathematics. Work toward the doctorate is available in analysis, algebra, and topology. Further details can be obtained by writing Dr. D. R. Traylor, Department of Mathematics, University of Houston, Houston, Texas 77004.

LECTURE SERIES IN DIFFERENTIAL EQUATIONS Washington, D.C., 1965-1967

The Air Force Office of Scientific Research, The Joint Graduate Consortium of American University, Catholic University, Georgetown University, George Washington University and Howard University and the University of Maryland will sponsor eight sessions of three lectures each on the first Saturday of October, December, March and May of the academic years 1965-1966 and 1966-1967. Each session will be devoted to a basic area of differential equations which is fundamental in applications. Each lecture will offer a survey and critical review of a particular aspect of that area, with emphasis on modern methods and new techniques.

Subject areas for each session will be control theory, boundary value problems, dynamical systems, differential operators, differential equations in Banach space, stochastic differential equations, numerical solutions, and differential equations of mathematical physics.

Interested persons are invited to write to Professor M. W. Oliphant, Department of Mathematics, Georgetown University, Washington, D. C. 20007, for the specific program and list of distinguished scholars who will speak.

PENNSYLVANIA STATE UNIVERSITY ESTABLISHES A COMPUTER SCIENCE DEPARTMENT

As of July 1965, The Pennsylvania State University has a separate Computer Science Department within the College of Science. There is a faculty of seven persons, Professors P. C. Hammer, and J. Selfridge, Associate Professors D. T. Laird, M. McCammon, Assistant Professors B. Barnes, F. R. Deutsch and P. Wegner. The fields now represented include numerical analysis, programming languages and systems, non-numeric programming, automata theory and extended topology.

The new department is authorized to grant the M. S. degree in Computer Science and a proposal has been submitted to the administration to authorize a Ph.D. program. P. C. Hammer is the Head of the Computer Science Department.

The Computer Science Department has both teaching and research assistantships, for which applications are invited. Further information can be obtained by writing to Professor P. C. Hammer, Computer Science Department, 426 McAllister Building, The Pennsylvania State University, University Park, Pennsylvania 16802.

ANNUAL REPORT TO BE ISSUED

In September, 1965, an Annual report will be published which will contain the annual Treasurer's report as well as reports on activities of individual committees within the Society. This report is available to all members on request.
MEMORANDA TO MEMBERS

SUBSCRIPTION PRICE CHANGES FOR 1966

Members are aware that the Council, at the business meeting held at the January meeting in Denver, presented a change in the Society's by-laws. Effective in 1966, dues will increase to $20 and membership privileges will include free subscriptions to the NOTICES and the BULLETIN. This is the first dues increase in sixteen years.

In past years, publications have increased in size and, necessarily, in cost. Notably, TRANSACTIONS has increased from 1,460 pages in 1949, to 3,850 pages at present. Because of these increases and because the Society wishes to keep the membership dues as low as possible, the following schedule of subscription prices, to be effective in 1966, was found to be necessary and was approved by the Board of Trustees.

In addition to this, the following statement on members' prices was adopted jointly by the Board of Trustees and the Executive Committee of the Council:

"Journals and books purchased at the members' prices are for the personal use of individual members only and are not intended for the use of libraries or other organizations. Members are entitled to purchase one subscription to each journal at members' price, all other subscriptions will be serviced at list price."

"The Society bears heavy losses to be able to offer reduced prices to members; for example, the production cost of a 1966 subscription to MATHEMATICAL REVIEWS will be approximately $125 but the members' price will be only $40.00. The losses are made up by dues, grants, gifts, and the sale of publications at regular prices. When libraries, business organizations, and other institutions able to pay the regular prices take advantage of the members' prices, the whole mathematical community is forced to suffer heavier costs."

<table>
<thead>
<tr>
<th>JOURNAL</th>
<th>ANNUAL LIST PRICE</th>
<th>MEMBERS' PRICE</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOTICES</td>
<td>$12.00</td>
<td>$ 6.00</td>
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<tr>
<td>BULLETIN</td>
<td>12.00</td>
<td>6.00</td>
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<tr>
<td>PROCEEDINGS</td>
<td>12.00</td>
<td>6.00</td>
</tr>
<tr>
<td>TRANSACTIONS (5 volumes)</td>
<td>50.00</td>
<td>25.00</td>
</tr>
<tr>
<td>MATHEMATICAL REVIEWS (2 volumes)</td>
<td>180.00</td>
<td>40.00</td>
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<tr>
<td>(Reviewers $20.00)</td>
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<tr>
<td>MATHEMATICS OF COMPUTATION</td>
<td>16.00</td>
<td>8.00</td>
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<td>ACTA</td>
<td>40.00</td>
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</tr>
<tr>
<td>DOKLADY</td>
<td>50.00</td>
<td>25.00</td>
</tr>
</tbody>
</table>

MAILING OF THE NOTICES

Because the Notices is sent by second class mail there is often a delay in receiving the issue in enough time to see the program of meetings and make the necessary arrangements to attend them. Therefore, this is to remind members that the chairmen of mathematics departments of institutional members always receive a first-class copy of the Notices which is available for reference.

An option is offered, however, to those members who wish to obtain their own first-class copy of Notices at the additional postage charge of $3.00 per year which is the difference in postal rate between first and second class mail. There will be a provision on the dues bill in September to take care of this charge for the 1966 issues.
Backlog of Mathematical Research Journals

Information on this important matter is being published twice a year, in the February and August issues of the Notices, with the kind cooperation of the respective editorial boards. It is important that the reader should interpret the data with full allowance for the wide and sometimes meaningless fluctuations which are characteristic of them. Waiting times in particular are affected by many transient effects, which arise in part from the refereeing system. Extreme waiting times as observed from the published dates of receipt of manuscripts may be very misleading, and for that reason, no data on extremes are presented in the table at the bottom of this page.

Some of the columns in the table are not quite self-explanatory, and here are some further details on how the figures were computed.

Column 2. These numbers are rounded off to the nearest 50.

Column 3. For each journal, this is the estimate as of the indicated dates, of the total number of printed pages which will have been accepted by the next time that manuscripts are to be sent to the printer, but which nevertheless will not be sent to the printer at that time. (Pages received but not yet accepted are being ignored.)

Column 4. Estimated by the editors (or the Editorial Department of the American Mathematical Society in the case of the Society's journals) and based on these factors; manuscripts accepted, manuscripts received and under consideration, manuscripts in galley, and rate of publication. There is no fixed formula.

Column 5. The first quartile (\(Q_1\)) and the third quartile (\(Q_3\)) are presented to give a measure of the dispersion which will not be too much distorted by meaningless extreme values. The median (Med.) is used as the measure of location. The observations were made from the latest issue received in the Headquarters Offices before the deadline date for this issue of the Notices. The waiting times were measured by counting the months from receipt of manuscript in final revised form, to month in which the issue was received at the Headquarters Offices. It should be noted that when a paper is revised, the waiting time between receipt by editors of the final revision and its publication may be much shorter than is the case for a paper which is not revised, so these figures are to that extent distorted on the low side.

<table>
<thead>
<tr>
<th>JOURNAL</th>
<th>No. issues per year</th>
<th>Approx. no. pages per year</th>
<th>BACKLOG 6/30/65</th>
<th>Est. time for paper submitted currently</th>
<th>Observed waiting time in latest published issue (in months)</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>12/31/64 pages</td>
<td>to be published (in months)</td>
<td>Q_1 Med. Q_3</td>
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<tr>
<td>American J. Math,</td>
<td>4</td>
<td>NR*</td>
<td>NR*</td>
<td>11</td>
<td>12 14</td>
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<tr>
<td>Annals of Math,</td>
<td>6</td>
<td>1200</td>
<td>1217 NR*</td>
<td>12</td>
<td>13 15 18</td>
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<tr>
<td>Annals of Math, Statist,</td>
<td>6</td>
<td>2000</td>
<td>0 NR*</td>
<td>5-8</td>
<td>6 10 14</td>
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<tr>
<td>Arch, Hist, Exact Sciences</td>
<td>not fixed</td>
<td>NR*</td>
<td>0 NR*</td>
<td>7</td>
<td>10 10 10</td>
</tr>
<tr>
<td>Arch Rational Mech, Anal,</td>
<td>not fixed</td>
<td>NR*</td>
<td>0 NR*</td>
<td>7</td>
<td>8 8 8</td>
</tr>
<tr>
<td>Canad. J. Math,</td>
<td>6</td>
<td>1000</td>
<td>500 490</td>
<td>14</td>
<td>15 18 18</td>
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<td>Duke Math, J.</td>
<td>4</td>
<td>680-700</td>
<td>661 934</td>
<td>15-18</td>
<td>16 17 18</td>
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<tr>
<td>Illinois J. Math,</td>
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<td>700</td>
<td>510 660</td>
<td>22</td>
<td>18 19 20</td>
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<tr>
<td>J. Math, Analyzes and Appl.</td>
<td>6</td>
<td>1300</td>
<td>NR*</td>
<td>12</td>
<td>** ** **</td>
</tr>
<tr>
<td>J. Math, and Mech,</td>
<td>6</td>
<td>1000</td>
<td>1000 NR*</td>
<td>12</td>
<td>** ** **</td>
</tr>
<tr>
<td>J. Math, and Phys.</td>
<td>4</td>
<td>350</td>
<td>400 100</td>
<td>12</td>
<td>5 8 10</td>
</tr>
<tr>
<td>J. Mathematical Physics</td>
<td>12</td>
<td>2016</td>
<td>25 NR*</td>
<td>6</td>
<td>7 8 12</td>
</tr>
<tr>
<td>J, Symbolic Logic</td>
<td>NR*</td>
<td>NR*</td>
<td>NR*</td>
<td>NR*</td>
<td>8 12 18</td>
</tr>
<tr>
<td>Math, Comp.</td>
<td>4</td>
<td>700</td>
<td>22 75</td>
<td>7</td>
<td>7 9 13</td>
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<tr>
<td>Michigan Math, J.</td>
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<td>50 50</td>
<td>9</td>
<td>7 8 9</td>
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<tr>
<td>Pacific J. Math,</td>
<td>4</td>
<td>1500</td>
<td>1050 NR*</td>
<td>138 or 18</td>
<td>17 18 20</td>
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<tr>
<td>Proceedings of the AMS</td>
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<td>18 19 20</td>
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<tr>
<td>Quarterly of Appl. Math.</td>
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<td>NR*</td>
<td>NR*</td>
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<tr>
<td>SIAM Journal</td>
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<td>SIAM Journal on Control</td>
<td>3</td>
<td>450</td>
<td>0 10</td>
<td>6-9</td>
<td>9 12 14</td>
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<tr>
<td>SIAM J. on Numer. Anal.</td>
<td>3</td>
<td>600</td>
<td>0 0</td>
<td>6-9</td>
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<tr>
<td>SIAM Review</td>
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<td>700</td>
<td>0 0</td>
<td>6-9</td>
<td>10 14 15</td>
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<tr>
<td>Transactions of the AMS</td>
<td>12</td>
<td>2200##</td>
<td>500 630</td>
<td>12</td>
<td>18 20 23</td>
</tr>
</tbody>
</table>

* NR means that no response was received to a request for information.

** Dates of receipt of manuscript not indicated in this journal.

# If we decide to publish 150 pages per month in 1966.

## #H 1500 in 1965 and 1966.
ESTABLISHMENT OF THE
JOSEPH FELS RITT MEMORIAL FUND

The American Mathematical Society is privileged to announce the receipt of a bequest from the estate of the late Estelle Ritt, widow of Joseph Fels Ritt. A Provision was made by Mrs. Ritt for the establishment of the Joseph Fels Ritt Memorial Fund for the purpose of publishing new mathematical literature. The Society extends its sincere appreciation for this generous gift and deems it a privilege to honor the memory of a distinguished mathematician.

Joseph Fels Ritt was an outstanding and active member of the Society from 1915 until his death in January, 1951. He was Vice-President of the American Mathematical Society from 1938-1940; Trustee, 1923-1924; member of the Council, 1926-1928; representative on the Board of Editors of the American Journal of Mathematics, 1936-1940; and member of the Colloquium Editorial Committee, 1943-1948. In 1932, he was Colloquium Lecturer of the Society and in subsequent years gave invited addresses to the International Congress in Zurich and the International Congress in Cambridge. Dr. Ritt received the unusual honor of having two of his books appear in the Colloquium Publications and the posthumous distinction of having his biography, written by E. R. Lorch, appear in the Bulletin of the American Mathematical Society.

MATHEMATICAL SCIENCES
EMPLOYMENT REGISTER

The Mathematical Sciences Employment Register, conducted by the American Mathematical Society, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics will be maintained at the Summer Meeting at Cornell University, Ithaca, New York on August 31 through September 2. The Register will be conducted from 9:00 A.M. to 5:00 P.M. in Rooms 200 and 205 of the Industrial and Labor Relations (ILR) Conference Center.

It is essential that applicants and employers register at the Employment Register Desk promptly to facilitate the arrangement of appointments.

The Register operates by arranging interviews between employers and applicants and by publishing listings of available positions and applicants. There is no charge for registration, either to job applicants or to employers, except when the late registration fee of $5.00 for employers is applicable. Provision will be made for anonymity of applicants upon request and upon payment of $5.00 to defray the cost involved in handling anonymous listings.

The latest compilations of available positions and of applicants for positions in the mathematical sciences, published August 15, 1965, may be purchased at the Employment Register Desk. The List of Applicants is available for $7.50; the List of Positions costs $3.00. These may also be purchased by writing to the Employment Register, P. O. Box 6248, Providence, Rhode Island 02904.

Persons who want to be included in the next listings, those to be published for the Winter Meeting, 1966, should write to the Employment Register at the above address.
ACTIVITIES OF OTHER ASSOCIATIONS

THE MATHEMATICAL ASSOCIATION OF AMERICA
Ithaca, New York--August 30--September 2, 1965

The fiftieth anniversary of the Mathematical Association of America will be celebrated at its forty-sixth summer meeting which will be held at Cornell University, Ithaca, New York from Monday, August 30, to Thursday, September 2, 1965. There will be a ceremony honoring the charter members of the Association at the business meeting of the Association, to be held on Tuesday, August 31 at 10:10 A.M. in the Alice Statler Auditorium, Statler Hall.

Professor J. W. Milnor of Princeton University will deliver the Hedrick Lectures. A complete program of the meeting is included in the time table in this issue of the Notices.

SOCIETY FOR INDUSTRIAL AND APPLIED MATHEMATICS
Seattle, Washington
November 12-13, 1965

The Society for Industrial and Applied Mathematics will hold a two-day Western Regional Meeting in Seattle, Washington, on Friday and Saturday, November 12-13, 1965. The meeting, with the Boeing Scientific Research Laboratories as host, will be held at the Seattle Center, the former site of the Seattle World's Fair, in downtown Seattle.

The program for the meeting will include sessions for contributed papers and will feature a Symposium on Reliability and Life Testing. The program for this symposium will include invited addresses by: Richard E. Barlow, Z. W. Birnbaum, Shanti S. Gupta, M. V. Johns, Ingram Olkin, Frank Proschan, Ernest M. Scheuer and Walter L. Smith.

As has been customary in the past, there will be sessions for contributed papers. Abstracts should be prepared with author's name and institution at top of first page, should be typewritten (double spaced) and should be 200 words or less in length. If an author is not a member of SIAM, a letter of introduction from a member should be included. Any special requirements such as projectors, size of slides, and so forth should be noted separately by the author. Abstracts should be received no later than September 15, 1965 and should be sent to: Dr. Frank Proschan, "Contributed Papers" —1965 Seattle Meeting, SIAM, 33 South 17th Street, Philadelphia, Pennsylvania, 19103.

For further information on the SIAM Western Regional Meeting, address Dr. B. H. Colvin, General Chairman, Boeing Scientific Research Laboratories, P. O. Box 3981, Seattle, Washington.

SOCIETY FOR INDUSTRIAL AND APPLIED MATHEMATICS
Ithaca, New York--September 2, 1965

The Society for Industrial and Applied Mathematics is pleased to announce that the Sixth John von Neumann Lecture will be delivered at the summer meeting by Professor Freeman J. Dyson of the Institute for Advanced Study. Professor Dyson will speak on "Applications of group theory in particle physics" on Thursday, September 2, in the Alice Statler Auditorium, Statler Hall.

SIAM will also conduct a beer party on Monday evening, August 30 at 8:00 P.M. at Noyes Lodge. In addition to the tickets sold through the advance registration form, some tickets will be sold at the registration desk at $1.25 each.
ASSOCIATION FOR SYMBOLIC LOGIC

There will be a meeting of the Association for Symbolic Logic at the Statler-Hilton Hotel in New York on Tuesday, December 28, 1965, in conjunction with a meeting of the American Philosophical Association. Professor Leon Henkin will give his Presidential address. Members wishing to contribute papers will please submit abstracts of not over 300 words in duplicate before October 15, 1965, sending them to either of the co-chairmen of the Program Committee: Professor Sidney Morgenbesser, Department of Philosophy, Columbia University, New York 27, New York, or Professor Elliott Mendelson, Department of Mathematics, Queens College, Flushing, New York 11367. A joint session with the APA on December 27, 1965 will feature a symposium on the Current Status of Set Theory.

PI MU EPSILON FRATERNITY
Ithaca, New York--August 31--September 1, 1965

The Pi Mu Epsilon Fraternity will hold a banquet in the Alt Heidelberg Restaurant on Tuesday evening, August 31, at 6:30 P.M. Professor K. O. May will speak at this banquet on "Should We Take Undergraduates Seriously?" Sessions for contributed papers will be held on Tuesday, at 3:30 P.M. and 8:30 P.M. in Olin Hall, Room B, and on Wednesday at 9:15 P.M. in Olin Hall, Room B. A Dutch treat breakfast meeting for members will be held on Wednesday, September 1, in Willard Straight Cafeteria, Kimball Rooms A, B, and C at 8:00 A.M.

OPERATIONS RESEARCH SOCIETY OF AMERICA
May 18-20, 1966

The 29th National Meeting of the Operations Research Society of America will be held at Los Angeles during May 18-20, 1966. The theme is "Expanding Horizons of Operations Research." Dr. John E. Walsh, System Development Corporation, Santa Monica, California is Chairman. January 1, 1966 is the deadline for contributed papers (from members and nonmembers).

NEWS ITEM

THE JOURNAL OF THE FRANKLIN INSTITUTE

The Journal of The Franklin Institute, the oldest continuing scientific publication in the United States, announces the founding of a special JOURNAL PREMIUM of $1000 to be awarded annually to the author of the outstanding paper published in the Journal during the preceding year. The award will be presented, in general, to the recipient of the Louis E. Levy Gold Medal for "...a paper of special merit...describing the author's experimental and theoretical researches in a subject of fundamental importance."
The following list contains the names and addresses of foreign mathematicians who will be visiting in the United States in 1965-1966. It has been compiled from the response received to the request for names which appeared in the Notices for June, 1965. These names will also be included in the more complete annual list to be published in the Notices for November, 1965.

<table>
<thead>
<tr>
<th>Name and Home Country</th>
<th>Host Institution</th>
<th>Period of Visit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alexits, George D. (Hungary)</td>
<td>University of Utah</td>
<td>Sept. 1965-June 1966</td>
</tr>
<tr>
<td>Anderson, C. T. (Canada)</td>
<td>The University of Iowa</td>
<td>Sept. 1965-June 1966</td>
</tr>
<tr>
<td>Aref, Cahit (Turkey)</td>
<td>The Institute for Advanced Study</td>
<td>Sept. 1965-June 1966</td>
</tr>
<tr>
<td>Cartier, Pierre (France)</td>
<td>The Institute for Advanced Study</td>
<td>Sept. 1965-June 1966</td>
</tr>
<tr>
<td>Freud, Geza (Hungary)</td>
<td>Oak Ridge National Laboratory</td>
<td>Sept. 1965-Sept. 1966</td>
</tr>
<tr>
<td>Garwick, Jan V. (Norway)</td>
<td>The Institute for Advanced Study</td>
<td>Sept. 1965-June 1966</td>
</tr>
<tr>
<td>Gėba, Kazimierz (Poland)</td>
<td>The Institute for Advanced Study</td>
<td>Sept. 1965-Jan. 1966</td>
</tr>
<tr>
<td>Gherardelli, Francesco (Italy)</td>
<td>University of Notre Dame</td>
<td>Sept. 1965-April 1966</td>
</tr>
<tr>
<td>Grauert, Hans (Germany)</td>
<td>The Institute for Advanced Study</td>
<td>Sept. 1965-June 1966</td>
</tr>
<tr>
<td>Herrera, M. E. M. (Argentina)</td>
<td>The Institute for Advanced Study</td>
<td>Sept. 1965-June 1966</td>
</tr>
<tr>
<td>Ihara, Yasutaka (Japan)</td>
<td>Cornell University</td>
<td>Aug. 1965-July 1966</td>
</tr>
<tr>
<td>Janich, Kalus Werner (Germany)</td>
<td>The Institute for Advanced Study</td>
<td>Sept. 1965-June 1966</td>
</tr>
<tr>
<td>Jeanquartier, Pierre (Switzerland)</td>
<td>Hughes Aircraft Company</td>
<td>Indefinitely</td>
</tr>
<tr>
<td>Kino, Dr. Akiko (Japan)</td>
<td>The Institute for Advanced Study</td>
<td>Sept. 1965-June 1966</td>
</tr>
<tr>
<td>Kiselman, Christier O. (Sweden)</td>
<td>The Institute for Advanced Study</td>
<td>Sept. 1965-June 1966</td>
</tr>
<tr>
<td>Klingens, Helmut (Germany)</td>
<td>The Institute for Advanced Study</td>
<td>Sept. 1965-June 1966</td>
</tr>
<tr>
<td>Mordell, L. J. (England)</td>
<td>Catholic University of America</td>
<td>Feb. 1966-June 1966</td>
</tr>
<tr>
<td>Murakami, Shingo (Japan)</td>
<td>The Institute for Advanced Study</td>
<td>Sept. 1965-June 1966</td>
</tr>
<tr>
<td>Raghavan, S. (India)</td>
<td>The Institute for Advanced Study</td>
<td>Sept. 1965-June 1966</td>
</tr>
<tr>
<td>Rangachari, S. (India)</td>
<td>The Institute for Advanced Study</td>
<td>Sept. 1965-June 1966</td>
</tr>
<tr>
<td>Remmert, Reinhold (Germany)</td>
<td>University of Notre Dame</td>
<td>Sept. 1965-Nov. 1965</td>
</tr>
<tr>
<td>Roth, K. F. (England)</td>
<td>Massachusetts Institute of Technology</td>
<td>Sept. 1965 ---</td>
</tr>
<tr>
<td>Shimogaki, Tetsuya (Japan)</td>
<td>Syracuse University</td>
<td>Sept. 1965-June 1966</td>
</tr>
<tr>
<td>Teleman, Costake (Rumania)</td>
<td>The Institute for Advanced Study</td>
<td>Sept. 1965-June 1966</td>
</tr>
<tr>
<td>Walter, Wolfgang (Germany)</td>
<td>University of Notre Dame</td>
<td>Sept. 1965-June 1966</td>
</tr>
<tr>
<td>Watson, John (England)</td>
<td>Rensselaer Polytechnic Institute</td>
<td>Sept. 1965 ---</td>
</tr>
</tbody>
</table>
PERSONAL ITEMS

Associate Professor BRIAN ABRAHAMSON of the University of Toronto has been appointed to a professorship at the School of Physical Sciences at the University of Adelaide at Bedford Park, Adelaide, South Australia.

Dr. N. L. ALLING of the Massachusetts Institute of Technology on leave from Purdue University has been appointed to an associate professorship at the University of Rochester.

Professor RAFAEL ARTZY of Rutgers, The State University has been appointed to a professorship at the State University of New York at Buffalo.

Mr. D. R. BAKER of Melpar Incorporated has accepted a position as a Technical Staff Member with the Wolf Research and Development Corporation, College Park, Maryland.

Professor HEINZ BAUER of the University of Hamburg, Hamburg, Germany has been appointed to a professorship at the University of Erlangen, Nurnberg, Germany.

Professor G. E. BAXTER of the University of California at San Diego has been appointed to a professorship at Purdue University.

Professor E. A. BISHOP of the University of California, Berkeley has been appointed to a professorship at the University of California, San Diego.

Dr. B. F. BRYANT of Vanderbilt University has won the 1964-1965 Madison Sarratt Prize for excellence in under graduate teaching.

Sir EDWARD COLLINGWOOD of Alnwick, Northumberland, England has been elected a Fellow of the Royal Society.

Dr. E. D. CONWAY III of the Courant Institute, New York University has been appointed to an assistant professorship at the University of California, San Diego.

Associate Professor FREDERIC CUNNINGHAM, JR. of Bryn Mawr College will spend the academic year 1965-1966 in France.

Professor T. T. FRANKEL of Brown University has been appointed to a professorship at the University of California, San Diego.

Assistant Professor J. B. FREIER of Rensselaer Polytechnic Institute has been appointed to an associate professorship at the Southeastern Massachusetts Technical Institute.

Professor R. H. GEESLIN of the International Christian University, Tokyo, Japan has been appointed to an associate professorship at the University of Louisville.

Associate Professor R. P. GILBERT of the Institute for Fluid Dynamics and Applied Mathematics at the University of Maryland has been appointed to a professorship at the Georgetown University.

Assistant Professor C. A. GREEN of Ohio University has been appointed to an assistant professorship at the University of Maine.

Dr. E. J. GUMBEL of the School of Engineering at Columbia University has been appointed a Visiting Professor of Mathematical Statistics at the University of Hamburg, Hamburg, Germany for the Summer Term 1965.

Dr. H. R. HALKIN of the Bell Research Laboratories, Whippany, New Jersey has been appointed to an associate professorship at the University of California, San Diego.

Professor F. S. HARPER of Georgia State College has been appointed a Professor of Actuarial Sciences at Drake University.

Associate Professor P. D. HILL of Emory University has been appointed to a professorship at the University of Houston.

Assistant Professor J. E. HOMER, JR. of Wisconsin State University has been appointed to an assistant professorship at Saint Procopius College.

Professor MELVIN HENRIKSEN of Purdue University has been appointed to a professorship at the Case Institute of Technology.

Assistant Professor H. L. JOHNSON of Purdue University has been appointed to an assistant professorship at the Virginia Polytechnic Institute.

Professor TATSUJI KAMBAYASHI of
Indiana University will be on leave for the academic year 1965-1966. He will spend the year at the University of Pisa, Italy.

Visiting Professor M. S. KLAMKIN of the University of Minnesota will be with the School Mathematics Study Group at Stanford University for the Summer. In September he will join the Ford Scientific Laboratory, Dearborn, Michigan as a Principal Research Scientist.

Mr. J. L. KLEMM of Purdue University has been appointed to an assistant professorship at the Indiana State College.

Assistant Professor F. J. KOSIER of the University of Wisconsin has been appointed to an associate professorship at the University of Iowa.

Dr. W. J. KOTZE of the University of Toronto has been appointed Senior Lecturer at the University of Cape Town, Cape Town, Africa.

Dr. Lester Kraus of the Republic Aviation Corporation, Farmingdale, New York has been appointed to a professorship in the Electrical Engineering Department at Drexel Institute of Technology. He will also Head the Electrophysics Program.

Assistant Professor V. Krishna-Murthy of the University of Illinois has been appointed to an associate professorship at Birla Institute of Technology and Science, Pilani, India.

Assistant Professor W. W. Leonard of Susquehanna University has been appointed to an assistant professorship at the Georgia State College.

Mr. E. E. McGehee Jr. of the University of South Carolina has been appointed to an assistant professorship at the Arkansas State Teachers College.

Professor Saunders MacLane of the University of Chicago has been awarded the degree of Doctor of Science from Purdue University.

Professor H. B. Mann of the Ohio State University has been appointed to a professorship at the University of Wisconsin and is on loan to the Mathematics Research Center, University of Wisconsin.

Mr. D. A. Mattson of the University of Wisconsin has been appointed to an assistant professorship at Trinity College.

Dr. K. O. May of the University of California at Berkeley has been awarded a grant by the Society of The Sigma XI to assist in his study of History of Linear Algebra.

Professor J. W. Milnor of Princeton University has been awarded the degree of Doctor of Science, honoris causa, by Syracuse University.

Mr. J. B. Moore of the University of Texas has accepted a position as a Research Engineer with the Boeing Company, Huntsville, Alabama.

Professor Leopoldo Nachbin of the University of Rochester will be a member of the Instituto de Matematica Pura e Aplicada, Rio de Janeiro, Brazil for the academic year 1965-1966.

Professor J. A. Nobel of the University of Wisconsin has been awarded a grant by the Air Force Office of Scientific Research. He will spend the academic year at the University of Paris, Paris, France.

Sir Alexander Oppenheim, presently Vice-Chancellor of the University of Malaya, has been appointed to a visiting professorship at the University of Reading, England for three years starting October 1, 1965.

Dr. G. M. Petersen of the University College of Swansea, Swansea, Great Britain has been appointed a Professor of Pure Mathematics at the University of Canterbury, Christchurch, New Zealand.

Dr. Stanley Preiser of the United Nuclear Corporation, White Plains, New York has been appointed to an assistant professorship at the Polytechnic Institute of Brooklyn.

Dr. T. J. Robertson of the University of Missouri has been appointed to an assistant professorship at the University of Iowa.

Mr. S. I. Rosenkrans of the Massachusetts Institute of Technology has been appointed to an assistant professorship at Tulane University.

Associate Professor Gian-Carlo Rota of the Massachusetts Institute of Technology has been appointed to a professorship at the Rockefeller Institute.

Professor D. A. Sanchez of the University of California at Los Angeles has been appointed a visiting Lecturer at Manchester University, Manchester, England for the academic year 1965-1966.

Dr. G. E. Schober of the University of Minnesota has been appointed to an acting assistant professorship for 1965-1966 at the University of California, San Diego.
Dr. GEORGE SENGE of the University of California, Los Angeles has been appointed to an acting assistant professorship for 1965-1966 at the University of California, San Diego.

Dr. H. A. SIMMONS a visiting Professor at Wartburg College has been appointed to a professorship at Indiana State College.

Assistant Professor W. R. SLINKMAN of Bemidji State College has received a National Science Foundation Science Faculty Fellowship award and will be on leave for the academic year 1965-1966 at Syracuse University.

Mr. D. J. SMITH of Goodyear Aerospace Corporation has accepted a position as Research Specialist Engineer with the Aerospace Division of Boeing, Seattle, Washington.

Mr. R. M. SOLOVAY of the Institute of Advanced Study has been appointed to an assistant professorship at the University of California at Berkeley.

Mr. J. D. TARWATER of the University of New Mexico has been appointed to an assistant professorship at Western Michigan University.

Professor S. J. TAYLOR of Westfield College, London, England has been appointed to a visiting professorship at the University of Michigan for the academic year 1965-1966.

Professor J. W. TUKEY of Princeton University has been awarded the degree of Doctor of Science from Brown University.

Mr. V. R. UPPULURI, formerly associated with Michigan State University has recently been appointed to the staff of the Oak Ridge National Laboratory, Oak Ridge, Tennessee.

Dr. GILBERT WALTER University of Wisconsin, Milwaukee has been appointed to a visiting assistant professorship for 1965-1966 at the University of California, San Diego.

Dr. P. E. WALTMAN of the Sandia Corporation has been appointed to an assistant professorship at the University of Iowa.

Dr. WARREN WEaver, consultant and former Vice President of the Alfred P. Sloan Foundation has been chosen as the 13th winner of the Kalinga Prize, awarded each year by UNESCO for distinguished contributions to public understanding of Science.

Mr. R. O. WELLS JR. of New York University has been appointed to an assistant professorship at Rice University.

Dr. S. G. WILLIAMSON of the University of California, Santa Barbara has been appointed to an assistant professorship at the University of California, San Diego.

Associate Professor OSWALD WYLER of the University of New Mexico has been appointed to a professorship at the Carnegie Institute of Technology.

The following promotions are announced:

L. R. BRAGG, Case Institute of Technology, to an associate professorship.

GEORGE BURKE, University of Iowa, to an associate professorship.

D. B. COLEMAN, Vanderbilt University, to an associate professorship.

JOHN DYER-BENNET, Carleton College, to a professorship.

M. A. GERAGHTY, University of Iowa, to an associate professorship.

H. W. GOULD, West Virginia University, to an associate professorship.

S. L. GULDEN, Lehigh University, to an associate professorship.

J. P. KING, Lehigh University, to an associate professorship.

M. O. MARCHAND, Bemidji State College, to an associate professorship.

G. A. STENGLE, Lehigh University, to an associate professorship.

FREDERICK WAY III, Case Institute of Technology has been appointed an Associate Professor of Computer Technology in the Engineering Division.

The following appointments to Instructorships are announced:

Morris Harvey College: E. N. BLACKWOOD JR.; Columbia University: S. L. KLEIMAN.

Deaths:

Dr. JOSE GALLEGOS-DIAZ of Caracas, Venezuela died February 1965 at the age of 52.

Dr. J. H. MACKAY of the Georgia Institute of Technology died on May 24, 1965 at the age of 42.

Mr. A. F. PAYTON of South Orange, New Jersey died on April 21, 1965 at the
age of 39. He was a member of the Society for 11 years.

Mr. J. R. PENN of Tucson, Arizona died on January 24, 1965 at the age of 45. He was a member of the Society for 13 years.

Professor LORRAINE SCHWARTZ of the University of British Columbia died on March 8, 1965 at the age of 36.

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**NEWS ITEM**

**PANEL OF VOLUNTEERS FOR CAREER INFORMATION**

The American Mathematical Society and many students who ask for career information from the Society are indebted to the volunteers listed below, who have, with impressive care and thoughtfulness, encouraged students in mathematics by answering their letters. During the past year, 167 letters were turned over to these volunteers:

R. V. Andree, University of Oklahoma; E. G. Begle, Stanford University; B. H. Bissinger, Lebanon Valley College; H. Brownson, National Science Foundation; R. C. Carson, University of Akron; D. Clock, Northern Michigan College; L. J. Cohen, Applied Data Research, Inc.; J. B. Diaz, University of Maryland; H. A. Gehman, Mathematical Association of America; H. A. Gindler, University of Pittsburgh; W. L. Grace, Massachusetts Mutual Life Insurance Company; F. Haimo, Washington University; R. G. Helsey, Ohio State University; H. L. Hunzeker, University of Omaha; L. D. Kovach, George Pepperdine College; D. M. Krabill, Bowling Green State University; G. R. Kuhn, Northwestern Michigan College; R. S. Ledley, National Biomedical Research Foundation; W. J. LeVeque, University of Michigan; M. Levine, National Science Foundation; C. B. Lindquist, Department of Health, Education and Welfare; W. F. Lucas, Princeton University; H. M. McNicoll, Case Institute of Technology; M. E. Mahowald, Northwestern University; B. McGovern, Great Valley Laboratory; R. A. Melter, University of Massachusetts; W. F. Miller, Stanford University; L. W. Neustadt, University of Southern California; M. W. Oliphant, Georgetown University; O. Pardee, Syracuse University; George Piranian, University of Michigan, L. E. Pursell, Grinnell College; P. Rotter, Mutual Benefit Life Insurance Company; J. P. Russell, Polytechnic Institute of Brooklyn; A. J. V. Sade, Pertuis, France; I. R. Savage, Florida State University; H. Sigal, Suffern, New York; L. W. Small, University of Chicago; R. M. Smith, Huntington, Tennessee; T. H. Southard, California State College at Hayward; R. A. Spong, General Dynamics; D. J. Struik, Massachusetts Institute of Technology; N. Tapper, Ithaca, New York; C. J. Thorne, Pacific Missile Range; J. W. Tukey, Princeton University; F. J. Van Antwerpen, American Institute of Chemical Engineers; W. Waterfall, American Institute of Physics; M. E. White, Stevens Institute of Technology.

An average of 240 requests for career information are received by the Society every month. Most of them are routine in nature and are answered with a form letter and pamphlets available for free distribution by the Society. Some of the letters, however, show a real interest in mathematics; it is these that are sent to the panel of volunteers to be answered.

Anyone who would be willing to be a part of this service is invited to send his name, address, and field of interest to the Providence Office.
LETTERS TO THE EDITOR

Editor, the Notices

Would anyone having information on the topic of the solubility of the diophantine equation $y^2 = x^3 + k$ please get in touch with me at Pennsylvania State University, McAllister Building, University Park, Pennsylvania 16802.

Hymie London

Editor, the Notices

We can observe that the restrictions written in books after "Copyright" become more and more rigorous. I found recently the following one: "No part of this book may be used or reproduced in any manner whatsoever without written permission except in the case of brief quotations embodied in critical articles and reviews." This "used in any manner whatsoever" is really annoying. I wonder what is your opinion: If I read the book without written permission, do I violate the law?

Zbigniew Semadeni

NEWS ITEMS AND ANNOUNCEMENTS

EASTERN EUROPEAN TRAVEL GRANTS

The Inter-University Committee on Travel Grants announces opportunities for advanced graduate students and scholars to engage in study and research in the Soviet Union, Bulgaria, Czechoslovakia and Hungary during the academic year, 1966-1967. This exchange, presently in its eighth year, is made possible by the intergovernmental agreements on exchanges with the USSR and agreements with the educational ministries of Bulgaria, Czechoslovakia, and Hungary.

All participants are chosen in national competition through application and interview. Participants must have proficiency in the language of the country commensurate with the needs of their programs. Applicants must be either American citizens or permanent residents of the United States.

For additional information write: Dr. Howard Mehlinger, Inter-University Committee on Travel Grants, 021 Lindley Hall, Indiana University, Bloomington, Indiana.

75th ANNIVERSARY OF THE GERMAN MATHEMATICAL SOCIETY

The 75th Anniversary of the German Mathematical Society will be celebrated at its annual meeting, September 12-18, 1965, in Freiburg in Breigau. All mathematicians, whether members of the German Mathematical Society or not, are cordially invited to take part in the meeting.


There will also be lectures on topology, algebra, theory of numbers, mathematical logic, numerical mathematics, and the history of the German Mathematical Society.

In addition, an invitation is extended to all interested persons to give a short address of not more than twenty minutes.

Further information may be obtained by writing to Professor Dr. Martin Barner (DMV-Tagung), Postscheckkonto Karlsruhe 1277, 78 Freiburg i. Br.
APPLICATIONS OF NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS IN MATHEMATICAL PHYSICS
Edited by Robert Finn

This volume contains the papers that were presented at a symposium held in New York City on April 20-23, 1964. There are seventeen articles arranged in four sections. The six articles on "General Nonlinear Theory" deal with such questions, concerning the solutions of nonlinear partial differential problems, as existence "in the large", behavior near isolated singularities or for unbounded values of the time, and application of the recently developed theory of monotone nonlinear operators from a Banach space to its dual. The three articles on "Finite Elasticity, Compressible Fluids" discuss finite elasticity, the theory of thin shells, and the important special problems that arise, in magnetohydrodynamics and elsewhere, for a flow near the point where a free boundary is met by a fixed one. The five articles in the section "Viscous Fluids, Magnetohydrodynamics" are chiefly devoted to the Navier-Stokes equations, and the three in the last section "General Relativity, Quantum Field Theory" deal with local existence and uniqueness theorems, and algebraically degenerate solutions, for Einstein's gravitational field equations, and with quasi-linear equations of evolution in quantum field theory.

THE H^p SPACES OF AN ANNULUS
By D. E. Sarason

This Memoir consists of two parts. In the first, the author derives, largely by standard and elementary techniques, the basic properties of Hardy classes of functions in a circular annulus. The second part is concerned with annular analogues of two famous disk theorems, the invariant subspace theorem of Beurling and the theorem of Szegö on mean square approximation by polynomials.

A HIERARCHY OF FORMULAS IN SET THEORY
By Azriel Levy

In this memoir a natural alternation-of-quantifiers hierarchy for the formulas of set theories of the Zermelo-Fraenkel type is introduced. An extensive investigation of the properties of this hierarchy is carried out. The known results concerning non-finitizability of the Zermelo-Fraenkel set theory are strengthened. Several statements of set theory are located in the hierarchy; in particular the axiom of choice, the simple and generalized continuum hypotheses and the axiom of constructibility. This location answers, at least from one point of view, the question as to whether these axioms are of an existential or a universal character.
SPECIAL CHAPTERS IN THE THEORY OF ANALYTIC FUNCTIONS OF SEVERAL COMPLEX VARIABLES
By B. A. Fuks

Approximately 400 pages; List Price $15.90, Member Price $11.93.

The five "special chapters" deal respectively with: approximation of functions and domains, the fundamental problems of Cousin and Poincaré, domains which are convex in the sense of Hartogs, holomorphic extension of domains, and biholomorphic mappings. Emphasis is placed on the discoveries of the last two decades, and in particular on the methods of holomorphic extension of domains which have recently become important in quantum field theory.

RECENT REPRINTS

The Calculus of Variations in the Large
By Marston Morse

378 pages; List Price $9.00, Member Price $6.75.

This volume is a reprinting of the standard, classical work, which has dominated the subject since its first publication in 1934. Subsequent advances in variational theory in the large, as made by the author and others and reported, for example, in the successive International Congresses since 1936, have been chiefly concerned with greater abstractness and generality for the Morse theory. As noted by A. S. Svarc in 1957 the index of multiply-covered closed extremals, as presented in the final chapter, requires correction. This evaluation leaves the main theory unaffected, in particular the passage from the critical point theory to the variational theory in the large, Morse's formulation of n-dimensional analogues of the Sturm-separation and comparison theorems, and the variational basis for differential topology, so that the 1934 volume is still an indispensable reference.

TRANSLATIONS OF MATHEMATICAL MONOGRAPHS

The Theory of Functions of Several Complex Variables
By B. A. Fuks

398 pages; List Price $13.00, Member Price $9.75.

This is the first part of the second edition of the famous Russian text on functions of several complex variables. The second part, which is also being translated in the present series, is concerned with special questions. The first part, dealing with the general theory, was extensively revised by the author, who contributed a foreword especially for this translation. The present volume is a reprint of the 1963 edition, with a greatly expanded subject index, prepared by Raymond H. Roper and an errata and addendum prepared by Professor Fuks.
MEMOIRS

Number 28

COHOMOLOGY GROUPS AND GENERA OF HIGHER-DIMENSIONAL FIELDS
By Ernst Snapper

100 pages; List Price $1.90, Member Price $1.43.

The Memoir is in four parts. Part I, which is purely topological in character and can be read without a knowledge of algebraic geometry, discusses the properties of the limit of an inductive system of sheaves and of the cohomology groups $H^n(E/k)$, where the extension $E$ of the field $k$ is generated by a finite number of elements. Part II develops purely field-theoretic definitions of the geometric genus, the irregularity and the arithmetic genus of $E/k$. In Part III it is shown that, if $r$ is the degree of transcendency of $E/k$, then the dimension of $H^q(E/k)$ is zero for $q > r$ and finite for $q = r$, while the principal unsolved problem of the Memoir is to show that this dimension is also finite for $0 < q < r$. Part IV is a study of the relationship between the cohomology groups of $E/k$ and $G/k$, where $G$ is the image of $E$ under some place of $E/k$.

This volume is reprinted without changes from the first edition of 1957.

SELECTED TRANSLATIONS SERIES II

Volume 6

FIVE PAPERS ON ALGEBRA AND GROUP THEORY
487 pages; List Price $14.70, Member Price $11.03
By Dynkin, Naǐmark, Raševskii, Vilenkin.

MEMO TO MEMBERS

THE COMBINED MEMBERSHIP LIST
1965-1966

The deadline for changes of listings in the next issue of the COMBINED MEMBERSHIP LIST is September 15, 1965. If your listing was incorrect in the 1964-65 CML, or if any part of your listing has changed since October 1, 1964, please send the information requested below to the Society before the deadline date.

1. Name in full and highest earned degree.
2. Job title and name of place of employment. Please include your full business address; it is needed for our files even though it will not be a part of the listing.
3. Job title and name and address of place of employment for a secondary permanent job or for a temporary position. Please indicate which is the secondary position.
4. Mailing address, including ZIP code number.

Please note that in the CML listings, students are listed at colleges and universities without job titles, secondary and temporary positions are listed after primary positions, "Mathematics Department" is understood to be a part of all university addresses, and only one address, the mailing address, will appear.
SUPPLEMENTARY PROGRAM—Number 33

During the interval from April 29, 1965 through July 2, 1965 the papers listed below were accepted by the American Mathematical Society for presentation by title. After each title on this program there is an identifying number. The abstracts of the papers will be found following the same number in the section on Abstracts of Contributed Papers in this issue of these Notices.

One abstract presented by title may be accepted per person per issue of the Notices. Joint authors are treated as a separate category; thus in addition to abstracts from two authors individually one joint abstract by them may be accepted for a particular issue.

(1) On representation of partially ordered sets
   Professor Alexander Abian and Mr. D. L. Deever, The Ohio State University (65T-365)

(2) Analogue of a theorem of Khintchine in a field of formal Laurent series.
   Preliminary report
   Mr. S. K. Aggarwal, Ohio State University (65T-357)
   (Introduced by Professor R. P. Bambah)

(3) The geometry of a third order differential element
   Professor Rodney Angotti, SUNY at Buffalo and Professor Mario Benedicty, University of Pittsburgh (65T-347)

(4) A uniqueness characterization of absolute continuity
   Professor W. D. L. Appling, North Texas State University (65T-320)

(5) The classification of recursive sets of number-theoretic functions
   Mr. R. F. Barnes, Jr., University of California, Berkeley (65T-363)

(6) On automorphic forms and Poincaré series. Preliminary report
   Mr. David Bell, Brown University (65T-314)

(7) Oscillating polynomials and approximation to \(|x|\)
   Mr. R. A. Bell, Gustavus Adolphus College, and Professor S. M. Shah, University of Kansas (65T-348)

(8) Prime rings with a one-sided ideal having a polynomial identity
   Professor L. P. Belluce, and Mr. S. K. Jain, University of California, Riverside (65T-358)

(9) Fixed-point theorems for families of contraction mappings
   Professor L. P. Belluce and Professor W. A. Kirk, University of California, Riverside (65T-333)

(10) Regular extensions of vector-valued, sigma-additive set-functions to measures
    Professor W. M. Bogdanowicz, Catholic University of America (65T-321)

(11) On metrizability of topological spaces
    Professor C. J. R. Borges, University of Nevada (65T-306)

(12) Multiplicity of solutions in frame mappings
    Professor D. G. Bourgin, University of Illinois (65T-302)

(13) Taming polyhedra in the trivial range
    Mr. J. L. Bryant, University of Georgia (65T-332)

(14) Triality in plane geometry
    Professor J. M. Cardoso, Universidade do Paraná, Brazil (65T-299)

(15) Bisected chords of a convex body
    Dr. G. D. Chakerian and Professor S. K. Stein, University of California, Davis (65T-315)

(16) An application of the Bott suspension map to the topology of EIV
    Professor L. W. Conlon, St. Mary's College (65T-339)

(17) Non-monotone operations in lattices
    Professor N. C. A. da Costa, Universidade do Paraná, Brazil (65T-297)

(18) Transformation theory of isothermal families in Riemannian space \(V_n\)
    Professor John De Cicco and Mr. R. V. Anderson, Illinois Institute of Technology (65T-293)

(19) One sided inequalities for inhomogeneous indefinite quadratic forms. Pre-
liminary report
Mr. V. C. Dumir, The Ohio State University (65T-362)
(Introduced by Professor R. P. Bambah)
(20) Transfer kernels in finite groups
Professor J. R. Durbin, University of Texas (65T-351)
(21) On some possibilities of generalizing the Lorentz group in the special relativity theory
Dr. C. J. Everett, University of California, Los Alamos and Dr. S. M. Ulam, Los Alamos Scientific Laboratory, Los Alamos, California (65T-335)
(22) Parabolic partial differential equations with uniformly continuous coefficients
Mr. E. B. Fabes and Mr. N. M. Rivière, University of Chicago (65T-338)
(23) Chronological bibliography of the Cauchy integral theorem. Preliminary report
Mr. H. K. Fallin, Jr. Ballistic Laboratory, Aberdeen, Maryland and Professor H. W. Gould, West Virginia University (65T-330)
(24) Inner c* subalgebras
Professor Jacob Feldman, University of California, Berkeley (65T-296)
(25) Multipliers from L^p to L^q
Professor Alessandro Figà-Talamanca, Massachusetts Institute of Technology and Mr. Garth Gaudry, Australian National University (65T-305)
(26) Pairwise uniform spaces. Preliminary report
Mr. Peter Fletcher, The University of North Carolina (65T-328)
(Introduced by Professor C. W. Patty)
(27) On the strong integral closure of an integral domain
Professor R. W. Gilmer, Jr., Florida State University (65T-344)
(28) On the existence of exceptional field extensions
Professor R. W. Gilmer, Jr., Mr. W. J. Heinzer and Professor H. F. Kreimer, Florida State University (65T-313)
(29) Research bibliography of two special number sequences. Preliminary report
Professor H. W. Gould, West Virginia University (65T-329)
(30) On equational classes of lattices
Professor G. A. Grätzer, Pennsylvania State University (65T-340)
(31) Covering constants of some non-convex domains. Preliminary report
Mr. Raj I. J. Hans, Ohio State University (65T-356)
(Introduced by Professor R. P. Bambah)
(32) Further results on O^*
Mr. Joseph Harrison, Stanford University (65T-304)
(Introduced by Professor Dana S. Scott)
(33) The structure of contours for non-degenerate surfaces defined on a two-dimensional manifold
Professor D. R. Henney, The George Washington University (65T-326)
(34) Two new classes of locally convex spaces
Professor Taqdir Husain, McMaster University (65T-334)
(35) Absolute Hardy-Bohr factor
Mr. R. L. Irwin, University of Utah (65T-325)
(Introduced by Professor Alexander Peyerimboff)
(36) General mixed problems on a half-space for certain higher order parabolic equations in one space variable
Professor R. K. Juberg, University of Minnesota (65T-308)
(37) Homogeneous theories. Preliminary report
Professor H. J. Keisler, University of Wisconsin (65T-292)
(38) Rings in which certain subsets satisfy polynomial identities
Professor T. P. Kezlan, University of Texas (65T-350)
(39) A fixed point theorem for weakly contractive sequences of contraction mappings
Dr. G. F. Kohlmayr, Pratt and Whitney Aircraft, East Hartford, Connecticut (65T-337)
(40) Local neighborhood strong deformation retraction
Professor A. H. Kruse, New Mexico State University (65T-355)
(41) Elementary characterization of prime polynomial ideal
Professor W. M. Lambert, Jr., Loyola University of Los Angeles (65T-352)
(42) On sum of respective intersections of cone and its dual with linear subspaces and its orthogonal complement
Professor Norman Levinson, Massachusetts Institute of Technology and Dr. T. O. Sherman, Institute for Advanced Study (65T-324)

(43) On defining well-orderings
Mr. E. G. K. Lopez-Escobar, Massachusetts Institute of Technology (65T-295)

(44) Singular homology groups and homotopy groups of finite spaces
Professor M. C. McCord, University of Georgia (65T-361)

(45) Some multivariate density functions of products of Gaussian variates
Dr. K. S. Miller, Columbia University (65T-309)

(46) A combinatorial result on maximally convex sets
Professor T. S. Motzkin, University of California, Los Angeles (65T-303)

(47) On the unique decomposition of natural numbers into finitary spreads of Mycielski numbers
Mr. A. A. Mullin, University of California, Livermore (65T-307)

(48) A characterization of differentiable submanifolds of $\mathbb{R}^n$
Mr. S. B. Nadler, Jr., University of Georgia (65T-346)

(49) On strongly continuous functions
Professor S. A. Naimpally, Iowa State University (65T-298)

(50) The synthesis of a certain subclass of tri-diagonal matrices with prescribed eigenvalues
Dr. Israel Navot, Israel Institute of Technology, Haifa, Israel (65T-359)

(51) Lower bounds for solutions of hyperbolic inequalities
Professor Hajimu Ogawa, University of California, Riverside (65T-331)

(52) Hypersurface in a manifold with an almost complex structure admitting contact transformations
Professor Tanjiro Okubo, Montana State College (65T-323)

(53) Characterization of the nested n-fold recursive functions via computational complexity
Mr. J. W. Robbin, Princeton University (65T-301)

(54) On the dual Pontrjagin classes of a manifold. II
Professor H. M. Roberts, University of Connecticut (65T-318)

(55) Prenex normal form in predicate calculi which are classical in implication and minimal in negation
Dr. T. T. Robinson, University of Illinois (65T-360)

(56) Some general Wedderburn theorems and their applications
Professor D. J. Rodabaugh, Vanderbilt University (65T-300)

(57) Rings of quotients as primitive classes of universal algebras
Mr. Martin Rutsch, University of Cincinnati (65T-317)

(58) Quasigroupes demi-symétriques. I
Professor A. J. Sade, Pertuis, Vaucluse, France (65T-319)

(59) On simple algebras obtained from reductive Lie algebras
Professor A. A. Sagle, University of California, Los Angeles (65T-364)

(60) The class number problem
Mr. K. Savithri, Defence Science Laboratory, Delhi, India (65T-367)

(61) Actions of cyclic groups on spheres
Mr. J. D. Sondow, Princeton University (65T-349)

(62) Differential-integral calculus for abstract algebraic-topological structures. II
Mr. R. M. Sorensen, Trinity College (65T-366)

(63) Zeros of sections of the Riemann Zeta function
Professor R. S. Spira, University of Tennessee (65T-345)

(64) Two step difference schemes. Preliminary report
Professor D. P. Squier, Colorado State University (65T-310)

(65) Topologies and Borel structures associated with Banach algebras of functions. I. Preliminary report
Mrs. P. R. Strauss, Columbia University (65T-353)

(66) On the asymptotic theory of ordinary differential equations of first order and second degree. Preliminary report
Professor W. C. Strodt, Columbia University (65T-311)

(67) On operators with completely continuous imaginary part
Professor Noboru Suzuki, Kanazawa University and University of Minnesota (65T-343)

(68) A generalized function calculus based on the Laplace transform
Mr. Charles Swartz, University of Arizona (65T-341)

(69) The ridge of a Jordan domain
Professor T. W. Ting, North Carolina State University at Raleigh (63T-316)

(70) Certain classes of bisimple inverse semigroups. Preliminary report
Professor R. J. Warne, West Virginia University (65T-336)

(71) On a linear operator connected with almost periodic solutions of linear differential equations in Banach spaces
Mr. J. N. Welch, Georgetown University and Dr. W. M. Bogdanowicz, Catholic University of America (65T-312)

(72) Space forms of irreducible Riemannian symmetric manifolds
Professor J. A. Wolf, University of California, Berkeley (65T-294)

(73) Completion of a separated uniform convergence space
Professor Oswald Wyler, University of New Mexico (65T-322)

(74) Completeness and compactness of a language with the equi-cardinality quantifier (but without equality)
Mr. Mitsuru Yasuhara, University of Montreal (65T-327)

(75) On the convergence of nth order spline functions
Mr. Zvi Ziegler, University of California, Los Angeles (65T-354)

NEWS ITEM

1966 INTERNATIONAL CONGRESS OF MATHEMATICIANS

The 1966 International Congress of Mathematicians will be held in Moscow from August 16-26, 1966 under the auspices of the Academy of Sciences of the USSR.

The Congress' scientific program will consist of daily sessions devoted to short communications by Congress members as well as one-hour and one half-hour addresses by invited lecturers.

There will be 15 sections of sessions on: mathematical logic and foundations of mathematics, algebra, theory of numbers, classical analysis, functional analysis, ordinary differential equations, partial differential equations, topology, geometry, algebraic geometry and complex manifolds, probability theory and statistics, applied mathematics and mathematical physics, mathematical problems of control systems, numerical mathematics, history and pedagogical questions.

The official languages of the Congress are English, French, German, and Russian.

Daily rates for "Intourist" service in Moscow are as follows:

\begin{tabular}{|c|c|}
\hline
     & \textbf{Russian} \\
\hline
De Luxe & 35,00 \\
Class A & 12,50 \\
Class B & 8,00 \\
University Student Hostel & 5,00 no more than \\
\hline
\end{tabular}

An application form is available by writing to the American Mathematical Society, Box 6248, Providence, Rhode Island, 02904, or the Division of Mathematics, National Academy of Sciences--National Research Council, 2101 Constitution Avenue, Washington, D. C. 20418. This form is not considered a registration form, but is only a request for additional information; the second communication form for registration will be sent to all those filing the first application form.

For further information write to the Secretariat of the International Congress of Mathematicians, Moscow University, Moscow V-234, USSR.
ABSTRACTS OF CONTRIBUTED PAPERS

The June Meeting in Eugene, Oregon
June 19, 1965

624-24. TAKURA TAMURA, University of California, Davis, California. Attainability of system of identities on all rings.

The author discussed the attainability of system of identities on all semigroups [Bull. Amer. Math. Soc. 71 (1965) to appear; Abstract 623-31, these Notices 12 (1965), 359]. In the present paper the same problem is considered in the case of rings. Let $\mathcal{R}$ be a class of rings and $\mathcal{I}$ be a system of identities. $\mathcal{I}$ is called attainable on all rings belonging to $\mathcal{R}$ if, for any $R \in \mathcal{R}$, the ideal of $R$ induced by the smallest $\mathcal{I}$-congruence on $R$ is $\mathcal{I}$-indecomposable. Does there exist an attainable, non-trivial system of identities on all rings? This paper gives a negative answer to this question. The author will also refer to the same problem on other algebraic systems, and will give some unsolved problems. (Received May 6, 1965.)


In studying the real group algebra $L^1_R(G)$ of a locally compact Abelian group $G$, one encounters two general problems which are closely connected with each other: (i) the study of $L^1_R(G)$ as an ordered ring under the usual pointwise ordering a.e., in particular the problem of determining its closed order-convex ideals; (ii) the study of the Fourier transforms of functions in $L^1_R(G)$, in particular the problem of determining whether there exist such real-valued functions (with possibly extra properties, such as positivity or compact support) having a Fourier-transform of a prescribed type. The purpose of the present paper is to present some results concerning these two problems. (Received May 6, 1965.)
625-1. WITHDRAWN.


Let $f(x)$ be a real-valued continuous function defined on an interval $I_b = [-b, b]$ satisfying

(i) $f(0) = 0$, $|f(x)| < |x|$, $x \neq 0$. For a point $x$ in $I_b$ let $u_0 = x$, $u_{n+1} = f(u_n)$, if $\sum_{n=1}^{\infty} u_n$ converges, $x$ is said to lie in the set of convergence for $f$, otherwise $x$ is said to lie in the set of divergence.

**Theorem 1.** If there are points in the set of convergence for $f$ immediately to the left of 0 and immediately to the right of 0 then each interval $[x, f(x)]$ contains points in the set of convergence.

**Theorem 2.** If $x_0$ lies in the set of divergence, then the series $\sum f^n(x)$ does not converge uniformly in any interval about $x_0$. For functions $f$ satisfying conditions similar to those imposed by Fort and Schuster (Abstract 618-19, these Notices, 11 (1964), 764), the set of convergence must be closed, and each closed set with 0 as an isolated point is a set of convergence for a function satisfying such conditions. (Received January 18, 1965.)

625-3. HOWARD COOK, University of North Carolina, Chapel Hill, North Carolina 27515. A continuum which admits only the identity mapping onto a non-degenerate subcontinuum.

**Theorem.** There exist compact, metric, one-dimensional, indecomposable continua $M_1$ and $M_2$ such that (1) no non-degenerate subcontinuum of either is topologically equivalent to a plane continuum, (2) if $H$ and $K$ are two non-degenerate subcontinua of $M_1$ and $f$ is a mapping of $H$ onto $K$, then $K$ is a subcontinuum of $H$ and $f$ is a retraction, (3) the identity is the only mapping of $M_1$ onto a non-degenerate subcontinuum of $M_1$, and (4) each mapping of $M_2$ onto a non-degenerate subcontinuum of $M_2$ is a homeomorphism of $M_2$ onto $M_2$ and the space of all such homeomorphisms is topologically equivalent to the Cantor set. (Received January 21, 1965.)

625-4. ALBERT WILANSKY, Lehigh University, Bethlehem, Pennsylvania. Between $T_1$ and $T_2$.

A KC topological space is one in which all compact sets are closed. A US space is one in which convergent sequences have unique limits. Then $T_2 \Rightarrow KC \Rightarrow US \Rightarrow T_1$ but no converse holds even in the presence of compactness. Locally compact KC or first countable US $\Rightarrow T_2$. A compact KC space may be of first category. A compact $G_\delta$ space is first countable if $T_2$ but need not be if KC.

**Theorem 1.** There exists a non-Hausdorff maximal compact topology. The space of rationals has no weaker compact Hausdorff topology but has a weaker compact KC topology. Let $Y$ be the one point compactification of $X$. **Theorem 2.** $Y$ is a KC space iff $X$ is a k space. If $X$ is US so is $Y$. Let $X$ be not locally compact; then $X$ may be hemicompact (i.e., $Y$ first countable at $\infty$) but if it is, it must not be first countable. **Theorem 1** has recently been published by N. Levine. (Received January 25, 1965.)
625-5. D. S. PASSMAN, University of California, Los Angeles, California. Groups with normal, abelian, non-central Hall $p'^{t}$-subgroups.

Let $G$ be a finite group and $p$ a fixed prime. If the Hall $p'^{t}$-subgroup of $G$ is normal and abelian, then the degrees of all the irreducible complex characters of $G$ are powers of $p$. We let $e(G)$ denote the exponent of $p$ of the biggest such degree. The functions $f$ and $g$ were defined for such groups in A characterization of groups in terms of the degrees of their characters [to appear in Pacific J. Math.] by I. M. Isaacs and this author. We introduce the function $f_p$ here by the following properties: If $G$ is a $p'$-group with $e(G) \leq e$ then $G$ has an abelian subgroup $A$ with $|G:A| \leq p^{f_p(e)}$. Moreover $f_p$ is the smallest such function. Theorem 1. $f(e) = f_p(e)$. Theorem 2. If either (i) $p \neq 2$ and $p$ is not a Mersenne prime; or (ii) $p > e$ then we have $g(e,s) = e + f_p(s) - s$. One interesting preliminary result needed here is the following. Theorem 3. Let $p'$-group $P$ act faithfully on abelian $p'$-group $H$. If $p \neq 2$ and $p$ is not a Mersenne prime then there exists $h \in H$ with $C_p(h) = 1$. In any case there always exists elements $h_1, h_2 \in H$ with $C_p(h_1) \cap C_p(h_2) = 1$. (Received April 19, 1965.)

625-6. M. A. HYMAN, 7220 Wisconsin Avenue, Bethesda, Maryland 20014. Weighted information theory. Preliminary report.

Classical information theory supposes that every character transmitted has the same expected "significance" (that is, utility at the receiving end). In actual practice, however, certain "words" may be more significant to the user than others. Likewise certain positions within a word may be more significant than others; consider numbers written using positional notation. This paper proposes a measure for "weighted information" (measured in "sigs") and demonstrates how one can often greatly reduce the "technical" information transmitted (measured in "bits") while transmitting almost all of the "weighted" or "semantic" information. Among the topics re-examined are coding, channel capacity, sampling of a continuous function, and quantization. (Received March 22, 1965.)


The terminology used is that introduced in the author's paper Topological geometries and a new characterization of $\mathbb{R}^m$ (to appear in the Notre Dame J. Formal Logic). A complete set of topological invariants for $S^m$ is given by Theorem 1: A space with geometry $G$ of length $m - 1$ is said to form a spherical $m$-arrangement if (i) each 0-flat consists of precisely two points; (ii) every linearly independent subset of $X$ has a convex hull; (iii) if $W$ is any convex subset of $X$, then $W$ with geometry $G_W$ forms a $(\delta(W) + 1)$-arrangement; (iv) if $f$ is a k-flat contained in a $k + 1$-flat $g$, then $f$ disconnects $g$ into two convex components; and (v) $G$ is semi-projective. $S^m$ is then the only second countable space $X$ which admits a geometry $G$ such that $X$ and $G$ form a spherical $m$-arrangement. The proof involves a sort of generalized triangulation together with Theorem 2: Let $X$ be a second countable space with geometry $G$ such that $X$ and $G$ form an $m$-arrangement. Then if $S = \{x_0, \ldots, x_k\}$ is any linearly independent subset of $X$ and $T = \{p_0, \ldots, p_k\}$ is a linearly independent subset of $R^m$ (with the usual Euclidean geometry $\Gamma$), there is a homeomorphism $u$ which maps $C(S)$ onto $C(T)$ and $P^+C(S)$ onto $P^+C(T)$ such that $u(G_{C(S)}) = u(G_{C(T)})$. (Received March 22, 1965.)

In this paper generalizations of the classical Hardy \( H^p \) spaces are discussed. Classes of analytic and quasi-analytic functions are defined on the "Big Disk" of Arens-Singer and Hoffman. Hardy spaces are defined in this context and the question of boundary values of such functions considered. In certain cases the Hardy spaces are shown to be isometrically isomorphic to closed subspaces of the associated \( L^p \) spaces on the boundary group. (Received March 24, 1965.)

625-9. WITHDRAWN.

625-10. N. P. SALZ, P. O. Box 235, Buffalo, New York 14221. *Infinite series to the power of infinite series.*

Let \( A \) be the series resulting from exponentiating \( B \) with \( C \), where \( A, B, \) and \( C \) are infinite power series with respective coefficients \( a_i, b_i, \) and \( c_i \), where \( a_0 = b_0 = 1 \) and \( c_0 = 0 \) (N. P. Salz, *The solution of exact first order differential forms*, Abstract 614-39, *these Notices*, 11 (1964), 538). The general solutions are presented for each series when the other two series are known. (Received March 29, 1965.)


Let \( T \) be a closed, bounded, domain dense transformation defined on the Hilbert space \( \mathcal{H} \). Let \( \mathcal{M} \) be the set of finite linear combinations of elements in eigenmanifolds of \( T \). With the assumption that a Jordan box can have infinite length, it is shown that \( T \) restricted to the closure of \( \mathcal{M} \) is similar, by an "into" similarity, to a transformation which is the sum of Jordan boxes whose domains are pairwise orthogonal. (An into similarity is a closed, bounded, domain dense, linear transformation \( S \) such that \( S^{-1} \) exists but need not be bounded.) When \( \mathcal{H} \) is finite dimensional this reduces to the usual Jordan normal form result. To treat elements which are not in the closure of \( \mathcal{M} \), one introduces the concept of a chained CGA (Countably generated, additive) set which corresponds to the domain of a Jordan box which is infinite in both directions. One also assumes, without loss of generality, \( T^{-1} \) exists and is bounded. Then, it is shown that there exists a double sequence of chained CGA sets \( \{\{\mathcal{M}_{ij}\}_{j=1}^{n_1}\}_{i=1}^{n_2} \) such that (i) for all \( i \), \( \{\mathcal{M}_{ij}\}_{j=1}^{n_1} \) are linearly independent and (ii) \( \bigcup_{n_2}^{\infty} \bigoplus_{n_1}^{\infty} \{\mathcal{M}_{ij}\}_{i=1}^{n_2} \) is dense in \( \mathcal{H} \). (Received April 19, 1965.)


A partial order \( \preceq \) on a topological space \( X \) is *continuous* if for \( p, q, \in X; p \preceq q \) implies the existence of neighborhoods \( G_p, G_q \) of \( p \) and \( q \) respectively such that every \( x \in G_p \) is strictly less than every \( y \in G_q \); while \( p, q \) incomparable implies the existence of \( G_p, G_q \) such that every \( x \in G_p \),
Theorem 1: The partial order \( \preceq \) is continuous iff \( P = \text{cl} P \) and \( F(P) \subset \Delta \). Theorem 2: Let \( X \) be a \( T_1 \)-space with a continuous semi-lattice operation \( \wedge \). Then the associated partial order is continuous. Conversely let \( X \) be a compact space with continuous partial order. If it is also equipped with lattice operations \( \wedge \) and \( \vee \), then they are continuous. Corollary: For a compact \( T_1 \)-space \( X \) with partial order \( \preceq \), the associated lattice operations \( \wedge \) and \( \vee \) are continuous iff \( P = \text{cl} P \) and \( F(P) \subset \Delta \).

(Received April 5, 1965.)


For \( 0 < a < \infty \), \( U_a = \{(x,t) \in \mathbb{R}^{n+1}: |x| \leq a, |t| \leq a^2\} \), where \( a \equiv 1 \). Let \( k(x,t) \) be a locally integrable function in \( \mathbb{R}^{n+1} - \{0\} \). Let there exist an increasing family of sets \( \text{Ma} \) for \( 0 < a < \infty \), where \( U_a \subset \text{Ma} \subset U_{2a} \) and \( \int \int_{M_{\lambda} - M_\epsilon} k(x,t)dxdt = 0 \) for \( 0 < \epsilon < \lambda < \infty \). Let \( \int \int_C (U_{2a}) |k(x - y, t - \tau) - k(x,t)|dxdt \leq C \) for \((y,\tau) \in U_a\). Suppose \( \int \int_{U_{2a} - U_a} |k(x,t)|dxdt \leq C \). Theorem 1. If \( k_\epsilon(x,t) = k \) in \( M_{\lambda} - M_\epsilon \) and 0 otherwise, then the Fourier transform of \( k_\epsilon \lambda \) on \( \mathbb{R}^{n+1} \) satisfies \( \|k_\epsilon \lambda\| \leq KC \), where \( K \) depends only on \( a,n \).

Theorem 2. \( k = \lim k_\epsilon \lambda \) exists in \( S' \) as \( \epsilon \to 0, \lambda \to \infty \). Hence convolution with the kernel \( k \) represents a bounded operator in \( L^p \), \( 1 < p < \infty \). This generalizes a recent theorem of Jones. The results are applied to the heat equation. (Received April 5, 1965.)

625-14. WITHDRAWN.

625-15. J. C. C. NITSCH, Institute of Technology, University of Minnesota, Minneapolis, Minnesota 55455. An isoperimetric inequality for multiply-connected minimal surfaces.

Proof of the Theorem. Let \( S \) be a minimal surface of the type of the circular annulus of area \( A \) (finite or infinite), bounded by two distinct Jordan curves \( \Gamma_1 \) and \( \Gamma_2 \) of lengths \( L_1 \) and \( L_2 \), respectively, (finite or infinite). If these curves are rectifiable, then the area of \( S \) is finite, and the inequality \( (L_1 + L_2)^2 - 4A > 0 \) is satisfied.—Further remarks. Illustrations. (Received April 9, 1965.)

625-16. WITHDRAWN.


Voronoi's sum formula and the functional equation for \( \phi^2(z) \) are obtained from Koshliakov's formula and, in the process, we find two more equivalent relations. However, the problem is concerned with a generalization of Koshliakov's formula and we obtain the above results by specializing parameters. (Received April 12, 1965.)
A complete subgraph of a graph is called a clique if it is not contained in any other complete subgraph of the graph. Erdős and Moser asked the following questions: What is the maximum number of cliques possible in a graph with $n$ nodes and which graphs have this many cliques? These questions are answered. It is also shown that the maximum number of different sizes of cliques that can occur in a graph with $n$ nodes is $n - \log_2 n + O(\log_2 \log_2 n)$. (Received April 12, 1965.)

Extremal problems for the class of typically real functions. Preliminary report.

Let $T$ denote the class of functions $f(z) = z + a_2 z^2 + \ldots$ which are regular in the unit circle and satisfy $\text{Im} f(z) \cdot \text{Im} z > 0$ for $\text{Im} z \neq 0$. $T$ is called the class of typically real functions. The following theorem is proved for the class $T$. Theorem 1. Let $|c| < 1$ and let $F(w_0, \ldots, w_{n+1})$ be analytic on $\bigcup_{t \in T} \{ f(c), \ldots, f^{(n)}(c), c \}$. Then $\max_{t \in T} \text{Re } F(f(c), \ldots, f^{(n)}(c), c)$ is attained for a function of the form $f(z) = z \sum_{k=0}^{n+1} s_k (1 - t_k z + z^2)^{-1}$ where $-1 \leq t_k \leq 1$, $s_k \geq 0$, and $\sum_{k=0}^{n+1} s_k = 1$. Using Theorem 1 the following two results are obtained. Theorem 2. Let $f(z) \in T$. Then $f(z)$ maps $|z| < \sqrt{2} - 1$ onto a domain starlike with respect to the origin. Theorem 3. Let $f(z) \in T$. Then $f(z)$ is univalent in $|z| < \sqrt{2} - 1$. (Received April 14, 1965.)

Description by generators and relations of the derived functors of the $n$-fold tensor product. If $R$ is a ring (with unit) and $A_1, A_2, \ldots, A_n$ are $R$-(bi)modules, then $\text{Mult}_{n}^R(A_1, \ldots, A_n)$ is defined to be the $i$th left derived functor of the $n$-fold tensor product $A_1 \otimes \ldots \otimes A_n$ ($\otimes = \otimes_R$); i.e., $\text{Mult}_{n}^R(A_1, \ldots, A_n) = H_i(K^1 \otimes \ldots \otimes K^n)$ where each $K^i$ is a projective resolution of $A_i$. A description of $\text{Mult}_{n}^R(A_1, \ldots, A_n)$ is given in terms of generators and relations, analogous to that given in MacLane: Homology for the case $n = 2$ [and $\text{Mult}_1 = \text{Tor}_1^R(A_1, A_2)$]. (Received April 15, 1965.)

$m$-semigroups, semigroups, and function representations.

An $m$-semigroup $\langle S, (\cdot) \rangle$ is an algebraic structure with one $m$-ary operation satisfying the $m$-associative law $((x_1 x_2 \ldots x_m) x_{m+1} \ldots x_{2m-1}) = (x_1 x_2 \ldots x_i (x_{i+1} x_{i+2} \ldots x_{i+m}) x_{i+m+1} \ldots x_{2m-1})$ for all $i < m$ and $x_1, x_2, \ldots, x_{2m-1} \in S$. An $m$-semigroup $\langle S, (\cdot) \rangle$ is a subreduct of an ordinary or $2$-semigroup $\langle A, (\cdot) \rangle$ iff $A$ contains a subset $S'$ such that $\langle S', (\cdot) \rangle$ is an $m$-semigroup isomorphic to $\langle S, (\cdot) \rangle$ under the operation defined by $x_1 x_2 \ldots x_m = x_1 \cdot x_2 \cdot \ldots \cdot x_m$ for all $x_1, x_2, \ldots, x_m \in S'$. A disjoint $m$-semigroup of $m$-adic transformations is an $m$-semigroup of functions $f: \bigcup_{i=1}^{m-1} X_i \to \bigcup_{i=1}^{m-1} X_i$ such that $f(X_i) \subseteq X_{\sigma(i)}$ ($\sigma$ being the cyclic permutation $(1 \ 2 \ \ldots \ m - 1)$) under the operation of composition of any $m$ such functions, and where $X_1, X_2, \ldots, X_{m-1}$ are any $m - 1$ pairwise disjoint sets. Theorem 1. Any $m$-semigroup $\langle S, (\cdot) \rangle$ is a subreduct of an ordinary semigroup $\langle A, (\cdot) \rangle$ such that $S$ generate $A$ and if $\langle S, (\cdot) \rangle$ is a subreduct of any other semigroup $\langle B, (\cdot) \rangle$, then a
homomorphism exists from A into B which is the identity on S. Theorem 2. Every m-semigroup is isomorphic to a disjoint m-semigroup of \( m \)-adic transformations. Theorem 3. An m-semigroup is isomorphic to a disjoint m-semigroup of one-to-one \( m \)-adic transformations iff it is left-cancellative. The Post Coset Theorem is a corollary of the above result. (Received April 15, 1965.)


Let \( X_1 \leq \ldots \leq X_n \) be the order statistics of a random sample of size \( n \) from a population with continuous distribution function \( F(x) \); then \( U_i = F(X_i) \) is distributed as the \( i \)th order statistic from a uniform \((0,1)\) distribution. Let \( G(u) \) be the inverse of \( F(x) \) (i.e., \( u = F(x) \) implies \( G(u) = x \)); then the Taylor expansion of \( G(u) \) about \( u = p \) is \( x = G(u) = \sum (u - p)^k G'(p)/k! \). If we set \( X_i = x_i p = i/(n + 1) = E(U_i) \), we obtain an expansion for \( X_i \) in terms of \((U_i - p)^k\) and hence an expansion for \( E(X_i) \) in terms of the central moments of \( U_i \). Expansions for higher moments of \( X_i \) and cross moments \( E(X_i X_j) \) may be obtained by multiplying appropriate series. These series expansions are generally asymptotic expansions and may not give sufficient accuracy for small \( n \). Examples for the normal, half normal, and log Weibull distributions are given. (Received April 22, 1965.)

625-23. J. E. SIMPSON, Marquette University, Milwaukee Wisconsin 53233. On dilations of operators.

Let \( 1 \) and \( Z \) be the functions \( 1(z) = 1 \) and \( Z(z) = z \) for all \( z \in \mathbb{C} \), \( B^{00}(\mathbb{C}) \) the Banach algebra of complex-valued bounded Borel functions on \( \mathbb{C} \), \( E \) a separated complete locally convex topological vector space over \( \mathbb{C} \) with strong dual \( E' \), and \( L(E,E) \) the algebra of continuous linear mappings of \( E \) into itself with the topology of uniform convergence on bounded subsets of \( E \). A family \( \mathcal{F} = (m_{x,x'})_{x \in E, x' \in E} \) of bounded Radon measures on \( E \) will be called spectral if there is a mapping \( f \rightarrow \mathcal{U}_f \) of \( B^{00}(\mathbb{C}) \) into \( L(E,E) \) which is a continuous algebra representation satisfying \( \langle \mathcal{U}_f x, x' \rangle = \int_x f \, dm_{x,x'} \) for all \( f \in B^{00}(\mathbb{C}), x \in E, x' \in E' \), and \( \mathcal{U}_1 = I \). An operator \( T \in L(E,E) \) is scalar if there is a spectral family such that \( Z \) is \( m_{x,x'} \)-integrable and \( \langle Tx, x' \rangle = \int_x Z \, dm_{x,x'} \) for all \( x \in E, x' \in E' \). Theorem. Let \( (T_j)_{j=1,2,\ldots,n} \) be a finite subset of \( L(E,E) \). Then there is a separated complete space \( F \) which contains \( E \) and an idempotent element \( P \) of \( L(F,F) \) with range \( E \), and \( n \) regular (i.e., with compact spectrum) commuting scalar operators \( T_j \) in \( L(F,F) \) such that \( T_j x = P T_j x \) for all \( x \in E, j = 1,2,\ldots,n \). Conditions will also be given under which an element \( T \) of \( L(E,E) \) is the restriction of a scalar operator and not merely its projection. (Received April 22, 1965.)


Theorem. Let \( \phi \) be a non-degenerate, alternate form on a \( k \)-vectorspace \( E \) of denumerable dimension, \( k \) an arbitrary commutative field (of any characteristic). Let \( H \) and \( \overline{H} \) be subspaces of \( E \) with \( \text{rad}(H) = \text{rad}(\overline{H}) = \text{rad}(H^\perp) \) satisfying the following conditions: (i) \( \dim H = \dim \overline{H}, \dim(H/\text{rad} H) = \dim(\overline{H}/\text{rad} \overline{H}), \dim(H + \overline{H}/H) = \dim(\overline{H} + H/\overline{H}), \dim((\text{rad} H)/H + \overline{H}/H) = \dim((\text{rad} \overline{H})/(\overline{H}^\perp + H^\perp) + \overline{H}/H) = \dim((\text{rad} \overline{H})/(\overline{H}^\perp + H^\perp) + \overline{H}/H). \) Then there exists a metric automorphism of \( E \) which maps \( H \) onto \( \overline{H} \). The theorem also holds in the case of a symmetric form.
provided that the underlying field is a Kneser field and condition (i) be replaced by the (necessary) conditions $H \cong H$ and $H^\perp \cong H^\perp$. The class of subspaces covered contains thus in particular all closed subspaces $H(H^\perp = H)$ and all dense subspaces $H, H^\perp = E$ (i.e., $H^\perp = \{0\}$). (Received April 26, 1965.)


Let $A^*$ denote the class of continuous functions $\phi$ such that $|\phi(t + h) - 2\phi(t) + \phi(t - h)| \leq A|h|$ for some constant $A$. It is known that any function of class $A^*$ must have modulus of continuity $\omega(t) = O(t\log 1/t)$. Let $n_1, n_2, \ldots$ be positive integers increasing so rapidly that $n_{j+1}/n_j \geq q > 3$; and let $a_1, a_2, \ldots$ be real numbers with $0 < |a_j| \leq 1$. Then the Riesz product with these parameters generates a continuous non-decreasing function (see Zygmund, *Trigonometric Series*, vol. I, p. 208):

$$F(x) = \lim_{k \to \infty} \int_0^1 \prod_{j=1}^k (1 + a_j \cos n_j t) dt.$$ Results: If $\prod_{j=1}^k (1 + |a_j|) = O(n_k^{-1/3})$, then $F$ is of class Lipschitz $\alpha$ ($0 < \alpha \leq 1$). The converse is true if $\alpha < 1$ and all $a_j > 0$. If $\prod_{j=1}^k (1 + |a_j|) = O(\log n_k)$, then $F$ has modulus of continuity $\omega(t) = O(t\log 1/t)$. In particular, $F$ can be singular and yet have $\omega(t) = O(t\log 1/t)$. However, $F$ cannot be both singular and of class $A^*$. (Received April 26, 1965.)


Let $Z = R_1$ be the ring of integers, $G$ a cyclic group of prime order $p$, $\theta$ a primitive $p$th root of 1, and $R_2 = Z[\theta]$ ($R_2$ is a Dedekind domain). There exist ring homomorphisms $f_i: R_1 \to \mathbb{Z}/p\mathbb{Z}$. Proposition. The ring $R = \{(x_1, x_2): f_1(x_1) = f_2(x_2)\}$ is isomorphic to $ZG$. Let $P_i = \ker f_i$. A $ZG$-module (hence also an $R$-module) is said to have a separated representation if there exist epimorphisms $g_i: M_i \to \overline{M}(M_i$ an $R_i$-module, $\overline{M}$ a vector space over $\mathbb{Z}/p\mathbb{Z}$ with $\ker g_i = P_i M_i$ and monomorphisms $h_i: \overline{M} \to P_i M_i (\overline{M}$ a vector space over $\mathbb{Z}/p\mathbb{Z}$) such that $M \cong \{m_1, m_2\}: g_1(m_1) = g_2(m_2)/\{h_1 m, h_2 m\}$). (To be more complete one should also specify a homomorphism or the numerator of the preceding fraction onto $M$.) Theorem. Every finitely generated $ZG$-module $M$ has a separated representation; and the representation is unique in the sense that given another one, there exist isomorphisms: $M_i$ onto $\overline{M}_i$, $\overline{M}$ onto $\overline{M}'$, and $\overline{M}$ onto $\overline{M}'$ such that the diagram containing these maps and the ones previously mentioned is commutative and induces the identity map on $M$.

Corollary 1. Every finitely generated $ZG$-module without elements of order $p$ is the direct sum of ideals of $ZG$ and cyclic modules. Corollary 2. Given $n(>0)$, there are only a finite number of isomorphism classes of finitely generated $ZG$-modules whose torsion subgroup has order $n$ and (if $n \neq 1$) els of order $p$. (Received April 26, 1965.)


Let $\phi(x, y)$ be a smooth function with $\phi_y \neq 0$. Let $R^+, R^-$ be regions for which $\phi > 0, \phi < 0$. Let $f = f^+ + f^-$ in $R^+, = f^-$ in $R^-$ with $f^+, f^- \in C^2$. The problem of minimizing $J(y) = \int_1^2 f(x, y, y') dx$, over $(C^1, D^0), (x_1, y_1) \in R^+$ and $(x_2, y_2) \in R^-$, has been treated by Bliss and Mason with the condition that $y$ intersects $\phi = 0$ in only one point. Those methods are used for extremals with a subarc on $\phi = 0$. On $\phi = 0$ the value of $f$, for a minimizing curve, is min $[f^+, f^-]$. For subarcs belonging to $f^+(\cdot)$ we
must have $\lambda \leq 0 (\lambda \geq 0)$. We assume $f^+_{y'y'}$ and $f^-_{y'y'}$ are not zero for $(x,y,p)$ with $\phi(x,y) = 0$ and $p$ arbitrary. **Theorem.** If the above conditions hold together with (II) and (IV), then there exists a simple covering in a neighborhood of $y(x)$. Moreover, if the E-functions are positive, then $y(x)$ furnishes a weak minimum. The construction of the simple cover follows a method of Bliss. If $y(x)$ enters $\phi = 0$ (at $x = \xi_1$) on a tangent and exits (at $x = \xi_2$) on a tangent the covering is as follows: Construct tangents $f^+$-extremals from $\phi = 0$ into $R^+$ for $\xi_1 \leq x \leq \eta$ where $x = \eta$ is a point of transition—before $\eta, y$ belongs to $f^+$ and after $\eta, y$ belongs to $f^-$. For $\eta \leq x \leq \xi_2$, construct $f^+$-extremals into $R^+$ with corners of $\phi = 0$. For $\xi_1 \leq x \leq \eta$ construct $f^-$-extremals into $R^-$, etc. (Received April 26, 1965.)


Let $A \star B | U$ be the generalized free product of $A$ and $B$ with $U$ amalgamated. For certain classes of groups necessary and sufficient conditions are given for $A \star B | U$ to be residually a $P$-group, where $P$ is one of the properties, finite, finite $p$-group, or solvable. For example, let $A$ and $B$ be finitely generated torsion free nilpotent groups. **Theorem.** $A \star B | U$ is residually a finite $p$-group if and only if (1) $A$ and $B$ have normal series $A = A_0 \supseteq A_1 \supseteq A_2 \supseteq \ldots$, $B = B_0 \supseteq B_1 \supseteq \ldots$, such that $[A_i:A_{i+1}] = p = [B_i:B_{i+1}]$, $i = 0,1,2,\ldots$; (2) $\cap_i A_i = 1 = \cap_i B_i$; (3) $U \cap \{A_i\} = U \cap \{B_i\}$ (where $U \cap \{A_i\}$ means the distinct terms $U \cap A_i = G_i$) and there are not more than a finite number of $A_i$ and $B_i$ such that $U \cap A_i = G_i = U \cap B_i$, $i = 0,1,2,\ldots$; (4) $\cap_1 U A_i = U = \cap_1 U B_i$. An example shows the amalgam $A \cup B | U$ can be embedded in a group $C$ which is residually a finite $p$-group even when $A \star B | U$ is not residually a finite $p$-group. Properties 1,2,3, are necessary, but not sufficient, and 4 is not necessary for embedding $A \cup B | U$ in $C$. The above extends results of G. Higman (Amalgams of $p$-groups, Journal of Algebra, vol. 1, no. 3, 1964). (Received April 21, 1965.)


Let $M^n$ be a compact $n$-manifold with boundary $\partial M^n$. A closed subset $X$ of $M^n$ will be said to have property $C$ provided there is a homeomorphism $h: M^n - X \rightarrow \partial M^n \times [0,1)$ such that $h(x) = (x, 0)$ for all $x \in \partial M^n$. If $X$ has property $C$ and no proper subset of $X$ has property $C$, then $X$ is a core of $M^n$. Suppose now that $M^n$ is a contractible $n$-manifold with $\pi_1(\partial M^n) = 1$. For each positive integer $p$ let $X_p$ denote a core of $M^n \times [0,1)^p$. Let Core $(n)$ denote the following conjecture: There is a core $Y$ of $M^n$, a point $x_p \in [0,1)^p$, and an integer $N$ such that $Y \times x_p \subset X_p$ for all $p \geq N$. **Theorem 1.** Core $(n)$ is equivalent to the generalized $n$-dimensional Poincaré conjecture. **Theorem 2.** If we do not require $\pi_1(\partial M^n) = 1$ above, then Core $(n)$ is false. (Received April 29, 1965.)

625-30. LARRY SMITH, Box 2155 Yale Station, New Haven, Connecticut. **On the Eilenberg-Moore spectral sequence.**

In an unpublished manuscript of Eilenberg and Moore a spectral sequence of great use in algebraic topology is developed. Very briefly the situation is as follows. Suppose $F \rightarrow E \rightarrow B$ and $F_0 \rightarrow E_0 \rightarrow B_0$, are fibrations, $B$ and $B_0$ simply connected. Let $f:B \rightarrow B_0$ and $\tilde{f}:E \rightarrow E_0$ be a map of fibre spaces (draw a diagram), then for any field $k$ there exists a spectral sequence of algebras.
\[ \{ E_r, d_r \} \text{ with } E_r \Rightarrow H^*(E, k) \text{ and } E_2 = \text{Tor} H^*(B_0, k)(H^*(B, k), H^*(E_0, k)). \] As usual, conditions under which the spectral sequence collapses are of great use. Definition. If \( A \) is a graded algebra, a sequence of homogenous elements \( a_1, a_2, \ldots \) of \( A \) is called an ESP-sequence iff \( a_1 \) is not a zero divisor in the algebra \( A/(a_1, \ldots, a_{i-1}) \). An ideal in \( A \) is called a Borel ideal iff it is generated by an ESP-sequence. Assume that the cohomology of all spaces in sight is of finite type. Theorem 1. (The Little Collapse Theorem). Let \( B_0 = \text{pt.}, E_0 = F \) and suppose that \( p_0 \) is onto and \( \ker p_0 \) is a Borel ideal. Then \( E_2 = E_\infty \). Theorem 2. (The Big Collapse Theorem). Let \( p_0 \) be onto with \( \ker p_0 \) a Borel ideal and suppose that the ideal in \( H^*(B, k) \) generated by \( \ker p_0 \) under \( f^* \) is a Borel ideal. Then \( E_2 = E_\infty \). (Received April 29, 1965.)


G will denote an undirected graph with no loops or multiple edges. \( V \) will denote its vertex set. A family of arcs is said to be openly disjoint if the arcs are pairwise disjoint except perhaps at common endpoints. Theorem 1. Let \( n \) be a positive integer. If \( |V| \geq n + 1 \), then the following \( 2n \) statements are equivalent: \( X: G \) is \( n \)-vertex connected; \( Y_m (m = 1, \ldots, n): \) Given any \( m + 1 \) vertices \( a, c_1, \ldots, c_m \in V \), there exists an openly disjoint family of \( n \) arcs \( P_i[a, c_i] \) (\( i = 1, \ldots, n, m + 1 \)) and \( S_i[a, c_i] \) (\( i = 2, \ldots, m \)); \( Z_m (m = 1, \ldots, n - 1): \) Given any \( m + 2 \) vertices \( a, b, c_1, \ldots, c_m \in V \), there exists an openly disjoint family of \( n - m \) arcs joining \( a \) and \( b \), each of which excludes \( c_1, \ldots, c_m \). This is an extension of a theorem by H. Whitney (Amer. J. Math. 54 (1932)). Consider the statement \( W(k_1, \ldots, k_m): \) Given any \( m + 1 \) vertices \( a, c_1, \ldots, c_m \in V \), there exists a cycle \( K \) containing \( a, c_1, \ldots, c_m \) and \( c \). Theorem 2. If \( G \) is \( n \)-vertex connected, then \( G \) satisfies \( W(k_1, \ldots, k_m) \) for any \((k_1, \ldots, k_m)\) such that \( \sum_{i=1}^{m} k_i = n \). Various necessary conditions for \( W(k_1, \ldots, k_m) \) are obtained, and the structure of the counter-examples to the converse of Theorem 2 is characterized. (Received April 29, 1965.)


Continue the notation of the preceding abstract. Define \( \gamma \) and \( \pi \) as the largest integers such that given a set \( \gamma \) (resp., \( \pi \)) vertices of \( G \), there exists a cycle (resp., an arc) in \( G \). Theorem 1: If \( G \) is \( n \)-vertex connected and \( n \geq 3 \), then \( \gamma \geq n \). Theorem 2: If \( G \) is \( n \)-vertex connected, then \( \pi \geq n + 1 \). The converses are false; however, the following holds.

Theorem 3: If \( G \) is \( n \)-vertex connected and \( n \geq 3 \), then \( \gamma = n \) if and only if there exists a set \( A \subset V \) with \( |A| = n \) such that \( G(V - A) \) has at least \( n + 1 \) components. Indication of proofs. Let \( c_1, \ldots, c_m \) \( (1 \leq m \leq n) \) be vertices of a cycle \( K \subset G \), and let \( c^* \) be any other vertex. By statement \( Y_n \) (previous abstract), \( G \) contains an openly disjoint family of arcs \( R_i[c^*, c_i] \) (\( i = 1, \ldots, m \)). If \( m < n \), a cycle \( K^* \) containing \( c_1, \ldots, c_m, c^* \) is constructed using the arcs \( R_i \), whence an inductive proof of Theorem 1. If \( m = n \), \( K^* \) can again be constructed unless some set \( A \) as described in Theorem 3 exists such that \( c_1, \ldots, c_n, c^* \) lie in distinct components of \( G(V - A) \). (Received April 29, 1965.)
Two points on a certain quartic surface are both perfect and nonperfect.

The quartic surface \( axyz^2 + bx + cx^2 + dx^3 + e = 0 \) is invariant under the cyclic collineation \( x' : y' : z' = x : Ey : E^5z \) where \( E^{11} = 1 \). [W. R. Hutcherson, A cubic surface as a carrier of an involution of period thirteen, Revista Matematica y Fisica Teorica (Tucuman) Serie A, Vol. XV (1964), 39-42].

Of the three points of coincidence on the surface, \( P_1 \) is simple, \( P_2 \) is triple, and \( P_3 \) is double. The triple point \( P_2 \) is shown to be perfect on the double tangent plane and imperfect on the single tangent plane. The double point \( P_3 \) is imperfect on one of its tangent planes, yet it is perfect on the other one.

(Received April 30, 1965.)

Some partial results are given on the well-known conjecture that two functions, which are continuous self-mappings of the interval \([0,1]\) into itself and which commute on that interval, possess a common fixed point. Without imposing any further requirements on these functions, it is shown that there exists a fixed point of one of the functions which is a limit point of the set of all fixed points of the compositional iterates of the other. This result is then applied to find under what conditions one of the two functions has a fixed point in common with an iterate of the other.

(Received May 3, 1965.)

A knot is a differentiably imbedded \( n \)-sphere in the \((n + 2)\)-sphere \( S^{n+2} \). A knot \( K \) is simple if its complement \( C \) fibers over a circle in such a way that the closure, in \( S^{n+2} \), of each fiber is bounded by \( K \). Simple knots are widespread and can be classified by considering certain pairs \((M,T)\) consisting of a manifold \( M \) and diffeomorphism \( T: M \rightarrow M \). This classification has some interesting consequences. (Received May 3, 1965.)
An integral in topological spaces.

Let $P$ be a locally compact Hausdorff space and let $\sigma$ be some ring of its subsets on which is defined a non-negative additive function $\mu$. With every $x \in P \cup \{\infty\}$ (one-point compactification of $P$) we shall associate a family $\kappa_x$ of nets $\{B_a\}_{a \in D} \subset \sigma$. If $M$ is a function on $\sigma$, $\mathcal{M}$ and $\mathcal{M}$ are functions defined by the rule $\mathcal{M}(x) = \inf \liminf M(B_a) / \mu(B_a)$ and $\mathcal{M}(x) = \inf \liminf M(B_a)$ for all $x \in P$ and $x \in P \cup \{\infty\}$ respectively. Here the lower bound is taken for all nets $\{B_a\}_{a \in D} \in \kappa_x$ such that $x \in B_a$ and $x \notin B_a$ for all $a \in D$ respectively. An additive function $M$ on $\sigma$ is termed a major function of a function $f$ on $P$ if and only if $\mathcal{M}(x) = \inf \liminf M(B_a)$ and $\mathcal{M}(x) = \inf \liminf M(B_a)$ for all $x \in P$ and $x \in P \cup \{\infty\}$ respectively. Using major functions we shall define the integral $I(f)$ as usual (see e.g., Saks: Theory of the Integral, Chapt. VI, 6). Under certain assumptions about $\sigma$, $\mu$ and $\kappa_x$, I will have all properties which an integral is usually required to have. In particular, if $\mu$ is a regular measure and a function $f$ has a Lebesgue integral $\int_P f \, d\mu$, then $I(f)$ exists and $I(f) = \int_P f \, d\mu$. Applying the theory to the discrete space of all positive integers, we obtain that a function $f$ is $I$-integrable if and only if the series $\sum_{n=1}^{\infty} f(n)$ is conditionally convergent, and $I(f) = \sum_{n=1}^{\infty} f(n)$. There are also several applications to the Euclidean spaces. (Received May 3, 1965.)

WITHDRAWN.

Some corollaries of the prime number theorem.

Suppose $h(n)$ and $f(n)$ are arithmetic functions for which $h(n) = \sum d \lfloor nf(n) \rfloor$. Theorem.

If $\sum_{n=1}^{\infty} h(n)/n$ is absolutely convergent, then the mean value of $f$ is 0. This theorem is shown to be an elementary equivalent of the prime number theorem. It is used to derive in a systematic and elementary way several corollaries of the prime number theorem, e.g., $\sum_{n=1}^{\infty} \mu(n) \theta(n) = 0$. Most of these corollaries are well known, and none would offer any difficulties to one trained in analytic methods. A more general theorem is derived using a strong form of the prime number theorem and a theorem of Landau [Rendiconti di Palermo, 24 (1907)] on the multiplication of Dirichlet series. (Received May 5, 1965.)
625-41. DAVID GILLMAN, University of California, Los Angeles, California 90024.
Inequivalent complexes in hyperplanes of $E^n$.

Bing and Kister have shown (Duke Math. J., 31, (1964), 491-511) that any two embeddings of a $k$-complex in an $m$-hyperplane of $E^n$ (Euclidean $n$-dimensional space) are equivalent if $k < m - 1$ and $n \geq m + k$. That is, one embedding of the complex may be carried onto the other by a homeomorphism of $E^n$ onto itself. **Theorem.** For $m = 3, 7, 15, k < m, n > m + k - 1$, there exist two embeddings of a $k$-complex in a $m$-hyperplane of $E^n$ which are not equivalent. The examples are, roughly speaking, obtained by lifting an $m - 1/2$ dimensional theta-curve into $E^n$ via the Hopf fibration. Thus, the lowest dimensional example obtained by this method is a 2-complex in a 3-hyperplane of $E^4$; the highest is a 14-complex in a 15-hyperplane of $E^{28}$. (Received May 7, 1965.)

625-42. W. C. RHEINBOLDT, University of Maryland, College Park, Maryland. **On a general estimation principle and a theory of comparison-factors.**

The idea of the comparison-factor for linear approximation problems—as introduced by G. Aumann [Numer. Math. (5) 68 (1963)]—is generalized as follows: Let $r, p$ be real-valued, non-negative, and non-negatively homogeneous functions of degree $\mu > 0$ on a cone $C$ in the real (complex) linear space $X$. If $r(x) = 0$ whenever $p(x) = 0$, then $r(x) \leq \gamma p(x), (x \in C)$, and $\gamma = \sup \left\{ \frac{r(y)}{p(y)} \right\} = 1$ is called the comparison-factor of $r$ with respect to $p$ on $C$. A number of results about the finiteness of $\gamma$ are proved under the assumption that $X$ is a topological linear space and that $r$ and $p$ satisfy various conditions. One useful condition involves the following concept: A function $p$ of the above type is called quasi-convex of degree $\alpha \geq 1$ if $p(\lambda x + (1 - \lambda)y) \leq \alpha^\mu \max(p(x), p(y))$ for $x, y \in C$ (C convex) and $0 \leq \lambda \leq 1$. Then transformations of the underlying space are considered which preserve finiteness of $\gamma$, and, in addition, the theory is applied to the special case of finite dimensional $X$. An important application of the theory involves a factorization theorem for linear operators which—when combined with one of the finiteness results—provides in particular the quotient theorem of A. Sard, and relates the theory also to other results in the theory of linear approximation. (Received May 7, 1965.)
Maximizing elliptic operators.

Let $\Omega$ be the sphere $\{x: |x| < R\}$ in $\mathbb{R}^m$, let $a$ be a constant, $0 < a \leq 1$, and $\mathcal{L}_a$ the class of differential operators $L$ defined in $\Omega$, $L = \sum a_{ij}(x)(\partial^2 / \partial x_i \partial x_j)$ with $a_{ij}$ measurable functions in $\Omega$, verifying in $\Omega$ the inequalities $a |\lambda|^2 \leq \sum a_{ij}(x)\lambda_i \lambda_j \leq |\lambda|^2$. For any function $u$ of class $C^2(\Omega)$ we define $M_a[u(x)] = \sup_{L \in \mathcal{L}_a} L u(x)$. Theorem I. For any function $u, \phi \in C^2(\Omega)$, there is an $L, \phi \in \mathcal{L}_a$, such that $L u = M_a[u]$. Theorem II. Let $\phi \in C^0(\partial \Omega)$ and $f \in C^0(\overline{\Omega})$. There is at most one function $\overline{u}$ such that $\overline{u} \in C^2(\Omega) \cap C^0(\overline{\Omega})$, $M_a[\overline{u}] = f$ in $\Omega$, $\overline{u} = \phi$ on $\partial \Omega$. If such function $\overline{u}$ exists, for any function $u$ with $u \in C^2(\Omega) \cap C^0(\overline{\Omega})$ and $L u = f$ in $\Omega$, $L \in \mathcal{L}_a$, $\overline{u} = \phi$ on $\partial \Omega$, we have $u(x) \leq \overline{u}(x)$ in $\Omega$. Special solutions of $M_a[u] = f$ are constructed and applied to questions of removable singularities and bounds on the solutions. (Received May 10, 1965.)

Incomplete coupon collector's test for random sampling digits.

From a collection of random sampling digits, digits are observed in sequence until a set of seven different digits has been obtained. Let $W_n$ be the probability that the number of observations required is exactly $n$; i.e., that the last of the seven different digits is observed for the first time on the $n$th observation. Values of $W_n$ are tabulated for $n = 7, 8, \ldots, 51$; the assumption is made that each digit is equally likely to be observed on any trial. This test requires smaller samples than the complete coupon collector's test (where observations continue until all ten digits 0 to 9 inclusive are observed). (Received May 10, 1965.)

Ordered cycle lengths in a random permutation.

Let the elements of the symmetric group $\mathcal{S}_n$ have probability $1/n!$ each. Let $L_r(\pi), S_r(\pi)$ be the length of the $r$th longest, $r$th shortest, cycle in $\pi \in \mathcal{S}_n$, with $L_r(\pi) = S_r(\pi) = 0$ when $\pi$ has fewer than $r$ cycles. For each $m, r = 1, 2, \ldots$ and as $n \to \infty$ we find $E_n \{[L_r(\pi)]^m\} \sim n^m G_{r,m}$, while $E_n \{[S_r(\pi)]^m\} \sim e^{-\gamma} (\log n)^r / r!$ when $m = 1$ and is $\sim n^{m-1} (\log n)^{-1} H_{r,m}$ when $m > 1$. The constants $G_{r,m}, H_{r,m}$ are given as integrals involving the exponential integral. Our methods are straightforward: we set up generating functions, get leading terms in closed form, and use Tauberian methods to recover the asymptotic dependence on $n$. The Tauberian side conditions are established by combinatorial arguments. (Received May 11, 1965.)

Inverse series relations and other expansions involving Humbert polynomials.

Put $H(t;m,x,y,p,C) = (C - mxt + yt^m)P = \sum_{n=0}^{\infty} t^n P_n(m,x,y,p,C)$ with $m$ = non-negative integer. This defines the $P_n$ as generalized Humbert polynomials. For suitable parameters we have a simple set of polynomials. Special cases include the polynomials of Legendre, Gegenbauer, Kinney [Amer. Math. Monthly, 70 (1963), 693], and Humbert [Proc. Edinburgh Math. Soc., 39 (1921), 21]. Elementary properties are found in the usual way. An expansion for $D_n^k \chi_{P_n+k}$ includes a result of Djokovic.
Functions $Q_j^r(x,t,m)$ are found so that $(t \Delta f \cdot f = \sum_{j=1}^{r} Q_j \Delta x f, r \geq 1,$ subject to $(t \Delta f \cdot f = (x - t^{m-1})\Delta x f$. The new pair of inverse series relations $(m \geq 1) F(n) = \sum_{0 \leq k \leq n} \frac{A_k(p - n,m)}{k!} (p - n,1)^{k} F(n - mk),$ with $(a + bk)A_k(a,b) = aC_{a+bk,k}$ is found. This result is related to previous theorems of the author [Duke Math. J., 29 (1962), 393]. Finally, it is shown that the inverse series relations allow an arbitrary polynomial to be expanded in a linear series of Humbert polynomials, although we do not have an orthogonal set of polynomials for general $m \geq 3$. (Received May 11, 1965.)


Let $R$ be the reals and $X$ a topological space. A sublattice $L$ of $C(X,R)$ is determining in case
(i) $R$ is a sublattice of $L$, (ii) if $A$ and $B$ are completely separated subsets of $X$ and if $r, s \in R$, then there exists $f \in L$ such that $f(x) \leq r$ for all $x \in A$ and $f(x) \geq s$ for all $x \in B$, and (iii) if $(f_n)_{n \in N} \subset L$ and the family of cozero sets of $(f_n)_{n \in N}$ form a star finite cover of $X$, then $\forall f_n \in L$. It was reported earlier (Abstract 618-18, these Notices, 11 (1964), 764) that determining sublattices of $C(X,R)$ characterize $\mathcal{U}$ and $\mathcal{V}$. Necessary and sufficient conditions are now obtained for a lattice $L$ to be $R$-isomorphic to a determining sublattice of $C(X,R)$ for some unique realcompact space $X$. The conditions use those of Anderson and Blair (Pacific J. Math. 9 (1959), 335-364). A Ky Fan (Ann. of Math. 51 (1950), 409-427) type extension is used to determine when determining sublattices are all of $C(X,R)$. (Received May 12, 1965.)


Let $L$ be a graded Lie algebra over a commutative ring $R$ with unit. We shall deal with graded representations of $L$. We define the universal enveloping algebra $U$ of $L$ and prove a Poincaré-Birkhoff-Witt theorem for $U$. We prove that if $L$ is a finite dimensional graded Lie algebra over a field of characteristic $\neq 2$, then $L$ has a faithful finite dimensional representation. A key lemma for this proof is a result on (non-graded) Lie algebras which states that if $L$ is a finite dimensional Lie algebra over a field of characteristic 0 then $L$ has a faithful, finite dimensional representation $f$ such that $f(x)$ is nilpotent whenever $ad x$ is nilpotent. (Received May 12, 1965.)


Let $D$ be the space of functions $f$, analytic in the unit disc, for which $f(0) = 0$ and $\int |f'|^2 dA < \infty$. Consider sequences of complex numbers $\{z_n\}$ satisfying $0 < |z_n| < 1$ and $z_m \neq z_n (m \neq n)$. Such a sequence is called a weighted interpolation sequence if, for every $w \in L^2$, we find an $f \in D$ such that $f(z_n) = w_n \sqrt{-\log(1 - |z_n|^2)}$, $(n = 1,2,\ldots)$. (The radical is the norm of the functional "evaluation at the point $z_n".) We show that if $\lim \sup \left[ \log(1 + |z_n|) \right]^{-1} < 1$ and if $\{z_n\}$ is a weighted interpolation sequence, then $f$ can be chosen such that its first $N$ derivatives (any natural number) will simultaneously interpolate any $N + 1$ appropriately weighted $L^2$ sequences. We further
show that if the lim inf above is less than 1/9, "\( \{ z_n \} \) is a weighted interpolation sequence" may be dropped from the hypothesis. (Received May 12, 1965.)

625-50. WITHDRAWN.

625-51. J. R. KINNEY and T. S. PITCHER, Massachusetts Institute of Technology, Lincoln Laboratory, P. O. Box 73, Lexington, Massachusetts 02173. The range of Perron's modular function.

Perron's modular function \( M(a) \) is the value for which \( |a - P/q| < (1 + d)/M(a)q^2 \) is satisfied infinitely often for all positive but only finitely often for negative \( d \). Perron has shown that the values of \( M(a) \) can take on, less than 3, are a countable set whose only limit point is 3. The values between 3 and \( 3\sqrt{2} \) have limits at 3 and at \( 3\sqrt{2} \). The only value between \( 3\sqrt{2} \) and \( (\sqrt{243} + 65)/22 \) is \( \sqrt{13} \). We show that \( M(a) \) takes on all values greater than \( 5 + \sqrt{2} \), and that the set of values taken in \( [3,3\sqrt{2}] \) is not dense. (Received May 12, 1965.)

625-52. SEYMOUR SHERMAN, Indiana University, Bloomington, Indiana. Cluster-star inversion.

Let \( \mathcal{S} \) be the union of the set of complete-star trees (for graph notation see [Uhlenbeck, G. W. and Ford, G. W. Theory of linear graphs, Studies in statistical mechanics 1 (1962), 123-211]) with points labeled 1, 2, ..., \( n \) and the set consisting of the single graph 0, with points labeled 1, 2, ..., \( n \) and no lines. If \( x, y \in \mathcal{S} \), then \( x \preceq y \) means each line of \( x \) is a line of \( y \). Thus \( \mathcal{S} \) is a finite partially ordered set with a minimum element 0 and a maximum element, the complete star \( K_n \) on the points 1, 2, ..., \( n \). An explicit formula is given for the Mobius function \( \mu(y,K_n) \) (for the concept see Rota, G-C., On the foundations of combinatorial theory I, Zeit, fur Wahr. 2 (1964), 340-368) in terms of the number of articulation points of \( y \), the number of maximal complete stars containing each articulation point and, in one formulation, the number of maximal complete stars in \( y \). In their monograph Uhlenbeck and Ford pointed out that the problem of inverting the relationship \( \mathcal{B}_n(\mathcal{V}_n) \) that defines the cluster or Ursell functions \( \mathcal{U}_n(\mathcal{V}_1, ..., \mathcal{V}_m) \) in terms of the star or Husimi functions \( V_n(\mathcal{V}_1, ..., \mathcal{V}_n) \) was open. By means of the Mobius function mentioned above an explicit cluster-star inversion is presented. This paper will appear in the Journal of Mathematical Physics. (Received May 17, 1965.)

625-53. F. P. GREENLEAF, University of California, Berkeley, California. Norm decreasing homomorphisms in group algebras.

The structure of norm decreasing homomorphisms \( T : L^1(F) \to M(G) \) is determined for all
locally, compact groups $F,G$. It is first shown that $T$ has a unique extension $\overline{T}: M(F) \rightarrow M(G)$ which is continuous on norm bounded sets when $M(G)$ has the weak-*-topology and $M(F)$ the strong operator topology obtained when it acts on $L^1(F)$ by left convolution (this strong operator topology is important in the whole theory). Many problems are concerned with norm decreasing monomorphisms, which have an elegant structure theory (arbitrary homomorphisms are much less pleasant). From this theory one proves: Theorem 1. A closed subalgebra in $L^1(G)$ is isometric and isomorphic to a group $L^1(F) \hookrightarrow H/K$ where $H \subset G$ is an open/closed subgroup and $K \subset H$ a compact subgroup, normal in $H$. Theorem 2. If $T: L^1(F) \rightarrow L^1(G)$ is a norm decreasing epimorphism, there is a multiplicative character $p$ on $F$, a closed normal subgroup $F_0 \subseteq F$ (with canonical map $1_\mu: F \rightarrow F/F_0$), and an isometric isomorphism $U: L^1(F/F_0) \rightarrow L^1(G)$ such that $T = U \circ \tau^* \circ A_p$, where (i) $A_p(\ell) = pf$ and (ii) $\langle \tau^* \mu, \psi \rangle = \langle \mu, \psi \circ \tau \rangle$ for $\mu \in M(F), \psi \in C_0(F/F_0)$. (Here recall Wendel's structure theory for isometric isomorphisms of group algebras.) (Received May 17, 1965.)


An FK space $X$ (see Wilansky, Functional analysis, Blaisdell, New York (1964), 202) is called conull if $\mathcal{V}_1 = 1 - \sum_{k=1}^{n} \delta^k = \{0,0,...,0,1,1,1,...\} \rightarrow 0$ weakly in $X$; otherwise coregular. (See Abstract 598-6, these Notices 10 (1963), 183.) Let $r = [r_n]$ be a sequence of positive integers, $r_\infty \uparrow \infty$. Let $\Omega(r)$ be the set of sequences $x$ such that $\max \{x_u - x_v; r_n \leq u < v \leq r_{n+1}\} \rightarrow 0$. Agnew proved that if the matrix $A$ is multiplicative - 0, then $\Omega(r) \cap m \subseteq c_A$ for some $r$. Zeller improved the inclusion to $\Omega(r) \subseteq c_A$ for conull matrices. The principal result of the present paper is the following generalization. Let $X$ be an FK space which includes $c$, the space of convergent sequences. The following conditions on $X$ are equivalent: (i) $\Omega(r) \subseteq X$ for some $r$; (ii) $X$ is conull; (iii) $1 \in \gamma$-linear closure of $\{l_n\}$ in $c$ considered as a two-norm space under its own norm and the metric of $X$. (Received May 17, 1965.)

625-55. C. W. KOHLS and J. D. REID, 15 Smith Hall, Syracuse University, Syracuse, New York 13210. Orders on commutative rings. Preliminary report.

Let $A$ be a commutative ring with identity. A subset $S$ of $A$ is a symmetric positive cone (SPC) if it satisfies: (1) $S + S \subseteq S$, and $a + b \in S$ implies $a \in S$ or $b \in S$; (2) $a \in S$, $ab \in S$ iff $b \in S$; (3) $0 \notin S$; (4) $S - S = A$. If $Q$ is a prime ideal and $A/Q$ is totally ordered, then $\{a \in A; Q(a) > 0\}$ is an SPC; with any SPC is associated a prime ideal $Q$ such that $A/Q$ is totally ordered. An ideal $I$ in an ordered ring $A$ is convex (absolutely convex) if $0 \leq a \leq b$ and $b \in I$ implies $a \in I$. Theorem 1. If $F$ is a positive cone containing an SPC, then the ideal $Q$ associated with the SPC is $P$-convex absolutely convex, and is comparable with all $P$-absolutely convex ideals. If $I$ is a $P$-convex ideal, either $I \subseteq Q$ or $P \cap Q \subseteq I$. Let $F$ be a totally ordered field. An $N$-set in $F$ is one of the form $\{f \in F; p|f| \geq 0, p \in F[x]\}$. A prime $N$-filter on $F$ is a proper prime dual ideal in the lattice generated by the $N$-sets in $F$. A gap $\ell^+$ in $F$ is algebraic if some $p \in F[x]$ changes sign at $\ell^+$. The prime $N$-filters on $F$ converging to gaps generate total orders on $F[x]$, and at algebraic gaps, they also generate other SPC. Theorem 2. If $\ell^+$ is an algebraic gap, there are at least two total orders on $F[x]$ such that $F \cup \{x\}$ has the same order as $F \cup \{\ell^+\}$. (Received May 17, 1965.)
The problem of singular perturbations of linear ordinary differential equations at regular singular points.

Consider a system of $n$ linear ordinary differential equations,

$$\epsilon^\sigma \frac{dy}{dx} = A(x,\epsilon)y,$$

where $\sigma$ is a positive integer, $\mu$ is a non-negative integer and $A(x,\epsilon)$ is an $n \times n$ matrix holomorphic for $|x| \leq x_0, |\epsilon| \leq \epsilon_0$. When $\mu = 0$ and $x = 0$ is not a transition point, a theory has been developed by M. Hukuhara and H. L. Turrittin. Theorem. Let $\arg \epsilon = \rho_0$ be a direction in the complex $\epsilon$-plane. Assume $x = 0$ is not a transition point for $tux (dy/dx) = A(x,\epsilon)y$. Then there are real numbers $\rho_1, \rho_2$ with $0 < \rho_1 \approx x_0, 0 < \rho_2 \approx t_0$, $\rho_1 \leq \arg \epsilon \leq \rho_2$ such that in the resulting system $\epsilon^\sigma (dz/dx) = B(x,\epsilon)z$, the matrix $B(x,\epsilon)$ is triangular. This theorem is the analogue of the Hukuhara-Turrittin theory. Further, let

$$\lambda_1, \lambda_2, \ldots, \lambda_m, (m \equiv n),$$

be the distinct eigenvalues of $A(0,0)$. A direction: $\arg \epsilon = \rho_0$, in the $\epsilon$ plane is a singular direction of the second kind if $\sigma \rho_0 = \arg (\lambda_j - \lambda_k), \mod \pi, j \neq k$. In a sector $\rho_1 \leq \arg \epsilon \leq \rho_2$ which does not contain such a singular direction one obtains a block-diagonalization corresponding to the distinct eigenvalues $\lambda_j$. Otherwise one obtains only a block-triangularization. (Received May 17, 1965.)

By a ring $R$ we shall mean an associative ring with identity, possibly non-commutative, and denote its group of units by $G$. Theorem 1. If $R$ satisfies the descending chain condition for either left or right ideals and if $G$ is cyclic, then $R$ is finite. Theorem 2. Let $R$ be a finite ring with characteristic a power of the prime $p$ and with $G$ cyclic. If $p$ is odd, then $R$ is commutative; if $p = 2$, then $R$ is either commutative or is the ideal direct sum $R = A \oplus B$, where the ideal $B$ is a commutative semi-simple ring and the ideal $A$ is isomorphic or anti-isomorphic to the matrix ring

$$\{I_{a,b} : a, b, c \in GF(2)\}. $$

Hence we obtain Theorem 3. A finite non-commutative ring $R$ with cyclic $G$ is the direct sum of a commutative ring and a ring $A$ described in Theorem 2. These theorems extend the work of R. W. Gilmer, Jr., Finite rings having a cyclic multiplicative group of units, Amer. J. Math. 85 (1963) 447-452, to non-commutative rings with d.c.c. (Received May 21, 1965.)

A smooth [piecewise linear (PL)] $n$-knot is a pair $K^n = (S^{n+2}, S^n)$ of standard spheres, with $S^n$ smoothly [PL] imbedded in $S^{n+2}$. If $f: S^n \times [0,1] \rightarrow S^{n+2} \times [0,1]$ is a smooth imbedding such that for $i = 0$ and $i = 1$, $f(S^n \times [0,1])$ intersects $S^{n+2} \times \{i\}$ in $f(S^n \times \{i\})$, transversely, and $S^{n+2} \times \{i\} - f(S^n \times \{i\})$ is a deformation retract of $S^{n+2} \times [0,1] - f(S^n \times [0,1])$, then the smooth $n$-knots

$$K_i^n = (S^{n+2} \times \{i\}, f(S^n \times \{i\}))$$

are strongly $h$-cobordant. Theorem 1. For every odd $n \geq 3$ there exist infinitely many smooth $n$-knots $K_i^n = (S^{n+2}, S^n)$, $j = 1, 2, \ldots$, which are pairwise strongly $h$-cobordant but distinct: the complements $S^{n+2} - T(S_i^n)$ of tubular neighborhoods $T(S_i^n)$ of the $S_i^n$ can be distinguished.
by their real representation torsion invariants. \textbf{Theorem 2.} $C^1$-triangulate the $K^n_{f_1}$. Then the PL$(n + k)$-knots $\sum_{j=0}^{k}K^n_{f_j}$, obtained by suspending the $K^n_{f_j}$ times, $k \geq 1$, are homeomorphic but PL distinct. \textbf{Corollary.} If $M^m$ and $W^{m+2}$, $m \geq 4$, are PL manifolds, where $M$ is compact (or $= \mathbb{R}^m$) and is PL imbeddable in $W$, then there exist infinitely many PL imbeddings $h_j: M \rightarrow W$ such that the manifold-pairs $(W, h_j(M))$ are homeomorphic but PL distinct. The proofs use methods of Milnor and Stallings. (Received May 19, 1965.)
of testing functions for which the \( m \)-th order Hankel transformation is a topological automorphism.

The dual space \( H_\mu^* \) consists of the \( m \)-th order Hankel-transformable distributions. If \( f \in H_\mu^* \), \( \theta \in H_\mu \), and \( \Phi = T_\mu \theta \), then \( F = T_\mu f \) is defined by generalizing Parseval's formula: \( \langle F, \Phi \rangle = \langle f, \theta \rangle \). If the support of \( f \in H_\mu^* \) is a compact subset of \((0,\infty)\), then \( (T_\mu f)(y) = \langle f(x), \gamma \gamma \gamma J_\mu(xy) \rangle \). The distributional transformation \( T_\mu \) is useful in solving distributional differential equations containing the Bessel operator. (Received May 20, 1965.)


In the following let \( q = p^n \), where \( p \) is a prime, and let \( \rho \) be a primitive root of \( GF(q) \).

**Theorem 1.** The alternating group \( A_q \) of permutations on the elements of \( GF(q) \) is generated by \( \rho^2 x, x + 1, \) and \( (x^{q-2} + 1)^{q-2} \). If \( q = 1 \pmod{4} \), \( A_q \) is generated by \( \rho^2 x, x + 1, \) and \( \rho^{q-2} \). If \( q = 3 \pmod{4} \), it is generated by \( \rho^2 x, x + 1, \) and \( x^{q-2} \). Now let \( GF(q) \) denote the extended domain obtained from \( GF(q) \) by adding an element \( \omega \) to \( GF(q) \) with rules of calculation \( a + \omega = a, \omega \omega = \omega, 1/0 = \omega, 1/\omega = 0 \). This enables us to consider \( x^{-1} \) and so all rational functions over \( GF(q) \). **Theorem 2.** The symmetric group \( S_{p+1} \) of permutations over \( GF(p) \) is generated by \( 1/x^{p-2} \) and \( x + 1 \). The symmetric group \( S_{q+1} \) over \( GF(q) \) is generated by \( 1/x^{q-2} \) and \( x + 1 \). The alternating group \( A_{q+1} \) is generated by the following functions: For \( q = 0 \) or \( 1 \pmod{4} \): \( 1/x, (x^{q-2} + 1)^{q-2}, x + 1, \rho^2 x, \) or \( 1/x, x + a, (x^{q-2} + \beta)^{q-2} \), where \( a, \beta \) run through \( GF(q) \). For \( q = 3 \pmod{4} \): \( x^{q-2}, (x^{-1} + 1)^{-1}, x + 1, \rho^2 x \) or \( 1/x, x + a, (x^{-1} + \beta)^{-1}, a, \beta \in GF(q) \). The proof is an extension of methods used by Carlitz (Proc. Amer. Math. Soc. 4 (1953), 538, and Duke Math. J. 29 (1962), 325-332) and Fryer (Proc. Amer. Math. Soc. 6 (1955), 1). (Received May 24, 1965.)

625-63. STEPHEN WILLARD, University of Rochester, Rochester, New York. Absolute Borel sets in their Stone-Cech compactifications.

\( X \) is a metrizable topological space. A \( G_\alpha \) set in a space \( Y \) is the countable union or intersection of \( G_\beta \) sets for \( \beta < \alpha \), when \( \alpha \) is an ordinal less than the first uncountable ordinal, with a \( G_0 \) set being open. An absolute \( G_\alpha \) means a metrizable space which is a \( G_\alpha \) set in every metric space in which it is embedded. In 1937, E. Cech proved the following result about \( G_1 \) sets: \( X \) is an absolute \( G_\delta \) iff \( X \) is a \( G_\delta \) in \( \beta X \). **Theorem.** \( X \) is an absolute \( G_\alpha \) iff \( X \) is a \( G_\alpha \) set in \( \beta X \). This yields as a corollary that every (metric) absolute Borel set is a Borel set in its Stone-Cech compactification and gives a start towards the reverse implication, the completion of which depends on two problems: (i) How far can the theorem above be carried (e.g. will it work for \( F_\alpha \) sets)? and (ii) Is there a convenient classification of the possible Borel types of \( X \) as a subset of \( \beta X \)? (Received May 24, 1965.)


Let \( M \) be a right module over a ring \( A \). A submodule \( N \) of \( M \) is **factor-prime** (f-prime) if (i) whenever \( a \in (N:N) \) for a submodule \( N \) \( \supseteq \) \( N \) then \( a \in (N:M) \) and (ii) \( cl(N) = N \). The module \( M \) is termed **annihilator-prime** if \( (0) \) is an f-prime submodule of \( M \). A **prime module** is a faithful annihilator prime module. The module \( M \) is called **semi-prime** if \( \bigcap P_i = 0 \) where \( \{P_i\} \) is the
collection of \( f\)-prime submodules of \( M \). Theorem. If \( M \) is annihilator-prime then \( M \) is prime as a module over \( A/(0:M) \). Theorem. If \( N \) is an \( f\)-prime submodule of \( M \) then \( M/N \) is a prime module over the prime ring \( A/(N:M) \). A module \( M \) is semi-prime if and only if \( M \) is a subdirect sum of annihilator-prime modules. If \( M \) is a faithful semi-prime module then \( A \) is a semi-prime ring. (Received May 24, 1965.)


Let \( X, \mathcal{B}, (Y, \mathcal{I}) \) be a compact Hausdorff space, the sigma-ring of Borel sets of \( X \), and a fixed complex or real Banach space respectively. Let \( C \) be the space of all continuous functions \( f \) from the space \( X \) into the Banach space \( Y \). The norm in the space \( C \) is defined by \( \|f\| = \sup \{|f(x)| : x \in X\} \). Let \( M \) be the space of all sigma-additive measures \( \mu \) from the ring \( B \) into the conjugate space \( Y^* \) such that \( \sup \{\sum_{i \in I} \|\mu(A_i)\| \} = \|\mu\| < \infty \), where the supremum is taken over all finite families \( A_i \in B(t \in T) \) of disjoint sets. The space \( M \) with the norm \( \|\mu\| \) is a Banach space. Define a bilinear functional \( u \) by the formula \( u(y, y^*) = y^*(y) \) for \( y \in Y \) and \( y^* \in Y^* \). Let \( \int f(t, d\mu) \) denote the trilinear integral defined in the paper: W. M. Bogdanowicz, A generalization of the Lebesgue-Bochner-Stieltjes Integral and a new approach to the theory of integration, Proc. Nat. Acad. Sci. U.S.A. 53 (1965), 492-498. Theorem. To every linear continuous functional \( \phi \) on the space \( C \) corresponds a unique vector-valued measure \( \mu \in M \) such that \( \phi(f) = \int f(t, d\mu) \) for all \( f \in C \). This correspondence establishes an isomorphism and an isometry of the conjugate space \( C^* \) with the space \( M \). (Received May 24, 1965.)


Let \( k \) be a positive integer. The enumeration of certain triangular matrices of order \( k \) leads to the numbers \( F_n(k) \) and \( N_n(k) \) defined by the recurrences \( F_n(k) = F_{n-1}(k) + F_{n-k}(k) \); \( N_n(k) = F_n(k)(N(k)-1)^k \), with the initial conditions \( F_1(k) = 1; N_0(k) = C_{n+k,n} \) for \( 0 \leq n \leq k \). Note that \( F_n(1) = N_n(1) = 2^n \), \( F_n(2) = F_{n+2} \), and \( N_n(2) = 3 \cdot 2^{n-1} \), where \( F_k \) denotes the usual Fibonacci number \( (F_0 = 0, F_1 = 1) \). For the numbers \( N_n(k) \), we obtain \( kN_n(\rho) = \sum_{j=0}^{k-1} \rho^j (\rho^j - 1)^k \) \( \rho = \text{a primitive } k \)-th root of unity. (Received May 24, 1965.)


Let \( Y \) denote a reflexive Banach space and let \( C(Y) \) be the space of all non-empty, closed, convex and bounded sets of the space \( Y \). The space \( C(Y) \) forms a semi-linear space under the operation of algebraic addition of sets and algebraic multiplication of a set by a scalar. Let \( A(t) \) be an additive function defined on the set \( S \) of all positive real numbers and let \( A(t) \) have values in \( C(Y) \). Consider a linear continuous functional \( y' \) on the space \( Y \). Define the following functional \( B(t, y') = \sup y'(A(t)) \) for \( t \in S \), \( y' \in Y' \). Theorem. If for every \( y' \) there exists an open interval \( I \) such that
the function $B(t,y')$ as a function of the variable $t$ is bounded either from above or from below on the interval $I$, then the function $A(t)$ is of the form $A(t) = t^A(1)$ for all $t > 0$. (Received May 26, 1965.)

625-68. C. H. GIFFEN, The Institute for Advanced Study, Princeton, New Jersey 08540. **On aspherical embeddings of 2-spheres in the 4-sphere.**

Let $(S,K)$ be a semilinear (not necessarily locally flat) $(4,2)$-sphere pair, $Q$ the complement of an open regular neighborhood of $K$ in $S$, and $B = Q$. Set $G = \pi_1(B)$, $H = \pi_1(Q)$, and $J = H/\langle i_* [G,G] \rangle$, where $i_*$ is the inclusion induced homomorphism. **Theorem.** $\pi_2(Q)$ is a free abelian group with basis the set of symbols $[a_c]_1 \neq c \in [J,J]$, and the $\pi_1$-action is that induced by the basis transformations $a_c \rightarrow a_c'$, where $h \in H$, $c' = h_* ch_*^{-1}$, and $h_*$ is the image of $h$ in $J$ under the natural projection, $J \rightarrow H$. **Corollary 1.** $Q$ is aspherical if and only if $i_*$ is onto. **Corollary 2.** $Q$ is aspherical if and only if $H = \mathbb{Z}$. **Corollary 3.** For $(S,K)$ locally flat, $Q$ is aspherical if and only if it has the homotopy type of a circle. Examples of non-locally flat $(S,K)$ with $H = \mathbb{Z}$ are given to show the independence of Corollaries 2, 3; a characterization of such examples appears difficult. The proof of the theorem splits into three parts: geometric, algebraic topological, and group theoretic. Geometric constructions and procedures provide the basis for an algebraic topological expression for $\pi_2(Q)$, and group theoretic methods translate this result into the simple form given above. (Received May 28, 1965.)

625-69. J. M. ORTEGA and W. C. RHEINBOLDT, University of Maryland, Computer Science Center, College Park, Maryland 20742. **On discretization and differentiation of operators with application to Newton's method.**

In the numerical solution of operator equations $Fx = 0$, discretization of the equation and then application of Newton's method often results in the same linear algebraic system of equations as application of Newton's method followed by discretization. To investigate this commutativity question, let $W, X, Y, \overline{X}, \overline{Y}$ be Banach spaces and $\Phi: X \rightarrow \overline{X}$, $\psi: Y \rightarrow \overline{Y}$ bounded, linear and onto mappings. For the class $D(\Phi_1)$ of operators $F$ such that $\Phi x_1 = \Phi x_2, x_1, x_2 \in \text{Domain}(F)$ implies $\psi(Fx_1) = \psi(Fx_2)$, define the mapping $\Phi_1 F = \overline{F}, \overline{F}(\phi x) = \psi(Fx)$ and similarly the mapping $\Phi_2$ for operators on $X \times X$. The mappings $\Phi_1$ and $\Phi_2$ describe that part of the discretization process induced by the space discretizations $\Phi$ and $\psi$. To specify a complete discretization process for, e.g., a class of nonlinear integral operators, let $K$ and $K_d$ be a fixed pair of bounded linear operators from $W$ to $Y$ and let $D(\psi)$ be the collection of all operators $F = KG$, where $G$ is a mapping from a domain in $X$ into $W$, such that $K_d G \in D(\Phi_3)$. Define the complete discretization mapping $\Psi_1(KG) = \Phi_1(K_d G)$ and similarly a mapping $\Psi_2$ for functions of two variables. **Theorem.** If $F = KG \in D(\Phi_3)$ is differentiable on $D'(F)$, then $(\Psi_1 F)' = \Psi_2 F'$ on $\phi(D'(F)) \times \overline{X}$. Similar commutativity results are obtained for operators of the form $GK$. (Received June 1, 1965.)

625-70. R. D. FRAY, Duke University, Durham, North Carolina. **Arithmetic properties of the q-binomial coefficient.**

For a non-negative integer $n$ the $q$-number $[n]$ is defined by $[n] = (q^n - 1)/(q - 1)$. If $r$ is a non-negative integer the $q$-binomial coefficient $[n,r]$ is defined to be $[n]_r /[r]_1$, where $[n]_r = [n][n - 1] \ldots [n - r + 1]$, $[r]_1 = [r]_r$ and $[0]_r = 1$. Let $q$ be a rational number which is integral
(mod p), then the highest power of p dividing \([n]\), the q-binomial coefficient and the q-multinomial coefficient \([m_1 + m_2 + \ldots + m_n]/[m_1][m_2] \ldots [m_n]\) is obtained. If \(q \neq 0 \pmod{p}\) let \(e\) be the exponent to which \(q\) belongs \((\pmod{p})\). **Theorem.** Let \(n = a_0 + ea\) and \(r = b_0 + eb\) where \(0 \leq a_0 < e\) and \(0 \leq b_0 < e\). Then \([n, r] = [a_0, b_0] C_{a, b} \pmod{p}\). The number of q-binomial coefficients \([n, r]\) prime to \(p\) then follows. In addition, the period of the q-binomial coefficient and the generating function for the number of coefficients \([n - kr, r]\) that are prime to \(p\), where \(k\) is a fixed positive integer, are determined. (Received June 2, 1965.)

625-71. D. O. BANKS, University of California, Davis, California. **Bounds for functions of eigenvalues of vibrating systems.** Preliminary report.

The eigenvalues \(0 < \lambda_1 < \lambda_2 < \lambda_3 < \ldots\) of the differential system \(u'' + A(p(x))u = 0, u(a) = u(b) = 0\) associated with a vibrating string are considered. Let \(F\) be a differentiable function of a finite number \(K\) of these eigenvalues. Then for certain classes of functions \(p\), there exists a function \(p_0\) in this class such that \(F\) attains its extreme values. By considering the variation of \(F\) as \(p\) varies within the given class, properties of \(p_0\) can be derived. These results are applied to the class of functions with the property that \(p(x) \leq H < \infty\) and \(\int_a^b p(x)dx = M\). (Received June 3, 1965.)

625-72. PHILIP BACON, 53 Court Street, Apartment 4, Bangor, Maine 04401. **Variants of Sperner's Lemma.**

**Definition.** A **Q-space** is an ordered triple \((X, A, T)\) such that \(X\) is a normal topological space, \(A\) is a closed subset of \(X\) and \(T\) is a homeomorphism from \(A\) onto \(A\) such that, if \(p \in A\), \(TTp = p\).

**Example.** \((D^n, S^{n-1}, T)\) is a Q-space, where \(D^n\) is the \(n\)-disk, \(S^{n-1}\) the \((n-1)\)-sphere that is the boundary of \(D^n\), and \(T\) is the antipodal map on \(S^{n-1}\). — For each positive integer \(n\) and each Q-space \((X, A, T)\) let \(QL_n(X, A, T)\) denote the proposition: "Suppose each of \(F_1, \ldots, F_{n+1}\) is a closed subset of \(A\), \(\bigcup_{i=1}^{n+1} F_i = A\), \(\bigcup_{i=1}^{n+1} (F_i \cap T F_i) = \emptyset\), \([C_1, \ldots, C_{n+1}]\) is a closed cover of \(X\) and, if \(M\) and \(N\) are disjoint nonempty sets whose union is \([1, \ldots, n + 1]\) then \(\bigcap_{i \in M} F_i \subset \bigcup_{i \in N} C_i\). Then \(\bigcap_{i=1}^{n+1} C_i \neq \emptyset\)." **Theorem.** For each positive integer \(n\), \(QL_n(D^n, S^{n-1}, T)\) holds. — For each positive integer \(n\) and each Q-space \((X, A, T)\) let \(QM_n(X, A, T)\) denote the proposition: "Suppose each of \(F_1, F_{-1}, \ldots, F_n, F_{-n}\) is a closed subset of \(A\), \(\bigcup_{i=1}^n (F_i \cup F_{-i}) = A\), and, if \(i \in \{\pm 1, \ldots, \pm n\}\), \(F_i \cap T F_i = \emptyset\), \(F_i \cap F_{-i} = B_i\) is a closed subset of \(X\) containing \(F_i\) and \(B_i \cup B_{-i} = X\). Then \(\bigcap_{i=1}^n (B_i \cap B_{-i}) \neq \emptyset\)." **Theorem.** \(QL_n(X, A, T)\) is logically equivalent to \(QM_n(X, A, T)\), if \((X, A, T)\) is a Q-space and \(n\) is a positive integer. — The paper contains additional theorems of a similar character. (Received June 3, 1965.)


General rules and formulas have been derived for the rigidity of framed structures and the mobility of linkages. Special boundary cases, which fall between rigid frames and movable linkages, have been derived in the plane and in three-space. However, corresponding boundary cases for structures and linkages on the surface of a sphere have received very little attention. Several examples, involving combinations of four-bar linkages, will be illustrated. (Received June 7, 1965.)
Let $A(z)$ denote an $n \times n$ matrix of meromorphic functions having a pole of the order $q$ at a point $z_0$. Regular-singular is defined in the sense of Coddington and Levinson [Ordinary Differential Equations, McGraw-Hill, p. 111]. Theorem 1. If $X' = AX$ is regular-singular, then the invariant of $A(z)$ of order $v$ can have a pole at $z_0$ whose order is at most $(v - 1)q$, for all $v$, $2 \leq v \leq n$, and these bounds are exact. Theorem 2. If $(A)_m$ denotes the truncation of the Laurent expansion for $A$ after $m$ terms, then $X' = AX$ is regular-singular iff $X' = (A)_m X$ is regular-singular in case $m \geq n(q - 1)$. This bound on $m$ is exact. Let $[A_v]$ denote the sequences of matrices inductively defined by $A_{v+1} = A_v^i + A_v A_0$; $A_0 = I$. Let $p[A_v]$ denote the order of the pole of $A_v$ at $z_0$. W. B. Jurkat proved that $X' = AX$ is regular-singular at $z_0$ iff $p[A_v] \leq v + (n - 1)(q - 1)$ for all $v = n, n + 1, \ldots$. (Received June 7, 1965.)

Factorable polynomials in several indeterminates over a finite field.

Dickson (Linear groups, §48) has proved that an irreducible ordinary polynomial of degree $s$ in $GF[q, x]$, $q = p^r$, is a product of $d$ irreducible factors of degree $s/d$ in $GF[q^r, x]$, $d$ being the greatest common divisor of $r$ and $s$. The present paper extends this theorem to irreducible factorable polynomials of an arbitrary number of variables. The proof uses a result of Carlitz (On factorable polynomials in several indeterminates, Duke Math. J. (4) Vol. 2.) Dickson's classification of irreducible ordinary polynomials of degree $p^s$ in $GF[q]$ is extended to irreducible factorable polynomials in several indeterminates of degree $p^k s$ in $GF[q]$. For a given irreducible factorable polynomial $P(x_1, \ldots, x_t)$ of degree $s$, the factorization of $P(x_1^{q^s} - x_1, \ldots, x_t^{q^s} - x_t)$ is obtained. The homogeneous and non-homogeneous cases require separate treatment. (Received June 8, 1965.)

On finite groups with a CCT-subgroup. Preliminary report.

Let $G$ be a finite group. A proper nontrivial subgroup $M$ of $G$ is a CCT-subgroup if the centralizer in $G$ of each nonunit element of $M$ is contained in $M$ and $M$ is a TI-subset of $G$. Groups $G$ with a CCT-subgroup $M$ were studied by W. Feit (Illinois J. Math. 4 (1960), 170-186), and important information was furnished there about the characters of $G$, under the assumptions: (i) $N_G(M) \neq M$; (ii) $|N_G(M)| \neq (|M| - 1)|M|$; (iii) $M$ is not a non-abelian $p$-group with $[M : M'] < 4|N_G(M) : M|^2$. In our study two assumptions are added: (iv) $M$ is non-cyclic; (v) $3$ divides $|M|$. In this case Theorem 1 of W. Feit-J. G. Thompson (Nagoya J. Math. 21 (1962), 185-197) is of fundamental importance. In an attempt to prove a more general conjecture of W. Feit the following results are obtained: let $|M| = m$, and let $N_G(M)$ be a semi-direct product $QM$ with $|Q| = q$. Theorem. Let $G$ be a finite group containing a non-normal CCT-subgroup $M$ satisfying conditions (i)-(v). Then: (a) $M$ is abelian; (b) if $Q$ is an abelian group of odd order then: $q = 1/2(m - 1)$, $m = 3^n$ and $G = PSL(2, m)$. (Received June 7, 1965.)
A diophantine correct nonstandard model in the isols.

A model of formal (provable) arithmetic is diophantine correct if every diophantine equation without solutions in the natural numbers is also without solutions in the model. Theorem: A countable model of formal arithmetic is imbeddable in the isols if and only if it is diophantine correct.

(N. B. Of course the existence of diophantine incorrect models is unknown. Obviously their nonexistence would imply the recursive solvability of Hilbert's tenth problem.) (Received June 7, 1965.)


Suppose that \([S, Q]\) is a complete inner product space with norm \(N\), \(E\) is a set, and \(S'\) is a collection of functions from \(E\) into the linear operators of \(S\) which is a right module over \(T\), the ring of continuous linear operators of \([S, Q]\). Suppose that \(Q'\) is a function from \(S' \times S'\) into \(T\) such that if each of \(F, G,\) and \(H\) is in \(S', \lambda\) is in \(T\), and \(x\) is in \(S\) then \(Q'(FA, G) = Q'(F, G)A\), \(Q'(F + G, H) = Q'(F, H) + Q'(G, H)\), \(Q'(G, F) = Q'(F, G)\), \(Q'(F, F)\) is non-negative Hermitian, and \(Q'(F, F)x = 0\) only in case \(F(t)x = 0\) for each \(t\) in \(E\). A net \([D, F]\) in \(S'\) is said to be a Cauchy net provided that for each \(x\) in \(S\), \([D, F]\) is a Cauchy net with respect to the pseudo-norm \(N(Q'( , , )^{1/2}x)\). The following are equivalent: (1) \(S'\) is complete. (2) There is a complete inner product space \([S'', Q'']\) of functions from \(E\) into \(S\) such that a function \(F\) from \(E\) into the linear operators of \(S\) is in \(S'\) only in case \(Fx\) is in \(S''\) for each \(x\) in \(S\) and there is a number \(b\) such that \(Q''((Fx, Fx)^{1/2} \leq bN(x)\) for each \(x\) in \(S\), and \(Q(Q'(F, G)x, y) = Q''(Fx, Gy)\) for each \(F\) and \(G\) in \(S'\) and \(x\) and \(y\) in \(S\). If \(S'\) is complete, \(L\) is a linear function from \(S'\) into \(T\), \(L(FA) = L(F)A\) for each \(F\) in \(S'\) and \(A\) in \(T\), and \(b\) is a number such that \(N(L(F)x) \leq bN(Q'(F, F)^{1/2}x)\) for each \(F\) in \(S'\) and \(x\) in \(S\), then \(L = Q'( , , G)\) for some \(G\) in \(S'\). (Received June 7, 1965.)

On ascending chain condition of lattices in a Lie group.

Let \(G\) be a Lie group. By a lattice in \(G\), we mean a discrete subgroup \(D\) of \(G\) such that \(G/D\) is compact. \(D\) is said to satisfy the ascending chain condition (in short, a.c.c.) if, for every increasing sequence \(D \subset D_1 \subset D_2 \subset D_3 \subset \ldots\) of discrete subgroups of \(G\), there exists an integer \(n\) such that \(D_n = D_1\) for all \(i > n\). The following are proved: (A) If \(G\) is not semi-simple, each lattice is properly contained in another lattice, and hence no lattice in \(G\) satisfies a.c.c. (B) Suppose that \(G\) is semi-simple, and let \(K\) denote the maximal connected compact normal subgroup of \(G\). Then a lattice \(D\) in \(G\) satisfies a.c.c. if and only if \(Ad, D\) does not leave invariant any toral subgroup of \(K\) of positive dimension. As immediate consequence of (B), we have (C) If \(G\) is semi-simple and has no compact component, then each lattice in \(G\) satisfies a.c.c., and hence is contained in a maximal lattice. (Received June 4, 1965.)
A 2nd order linear partial differential operator \( L \) is said to satisfy Huygens' principle if the fundamental solution of \( Lu = \delta \) has support on the surface of the characteristic conoid associated with \( L \). We consider those operators of the form \( L_m u = \nabla_m u + c(t)u \) (with \( c(t) = \partial^2 / \partial t^2 - \sum_{i=1}^{m-1} \partial^2 / \partial x_i^2 \)) with \( c(t) \) analytic on some open, connected set. A result due jointly to K. L. Stellmacher and the author (Abstract 608-174, these Notices) 11 (1964), 107) states that with any such operator there is associated a family \( F \) of linear 1st order differential operators having the properties (i) for each \( \Lambda \in F \) there exists an operator \( L^*_m U = \nabla^*_m U + c^*(t)U \) for which \( \Lambda L^*_m = L^*_m \Lambda \), and (ii) \( L^*_m \) satisfies Huygens' principle if \( L^*_m \) has this property. In this paper is proved the following Theorem: If \( L^*_m \) satisfies Huygens' principle, there is a finite sequence of 1st order differential operators \( \Lambda_1, \Lambda_2, ..., \Lambda_k \), such that \( \Omega L^*_m = \nabla^*_m \Omega \), where \( \Omega \) denotes the product \( \Lambda_1 \Lambda_2 ... \Lambda_k \). This theorem shows that "Hadamard's conjecture" is essentially correct for the class of operators under consideration.

(Received June 4, 1965.)

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Consider the system of nonlinear elliptic equations in a domain \( \mathcal{D} \),
\[
\Delta u = f(x, u, u_x) \quad \text{and} \quad \Delta v = g(x, v, v_x) + c(x)u,
\]
where \( \Delta \) is the Laplacian, comma \( i \) denotes first order differentiation, \( f \) and \( g \) are Lipschitz continuous in their last two arguments, and \( c(x) \) is a nonzero \( C^2 \) function. Let \( W = v - \psi \) and \( w = u - \phi \), where \( \psi \) and \( \phi \) are sufficiently smooth approximating functions of \( v \) and \( u \) respectively. \( W \) is assumed to be uniformly bounded in \( \mathcal{D} \). By means of two suitably defined convex functionals and an auxiliary inequality which bounds certain integrals of \( w^2 \) in terms of the uniform bound for \( W \), uniqueness and stability results are obtained in the Cauchy problem for the above system. When specialized to the linear system for the deflection of elastic plates, this technique demonstrates that the assumption the Laplacian of the solution to the inhomogeneous biharmonic equation be uniformly bounded in \( \mathcal{D} \) can be removed [see Payne, J. Res. Nat. Bur. Standards, 65B (1961), 157-163]. (Received June 9, 1965.)

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Author presents in a general and pure mathematical form the arguments in the investigations of Gödel, Tarski and Rosser. To eliminate the distinguished role of true sentences the author considers a representation k-system \( Z \) which has \( k \) distinguished sets \( T_1, ..., T_k \) of true sentences. Relativizing then the notions of Gödel sentence, definability and representability to any of the sets \( T_i \), he proves counterparts of Gödel's theorem on undecidability, Tarski's theorems on representability and definability and Rosser's theorem on weak separability and undecidability in a general form, which, as he believes, points better the mathematical arguments involved in those theorems. Introducing then the notions of recursive and effective inseparability of \( k \)-tuples and of productive and creative \( k \)-tuple the author proves various theorems on those \( k \)-tuples and applies them to formal representation \( k \)-systems. (Received June 8, 1965.)

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625-81. P. W. SCHEIFER, Natural Science Division, University of South Florida, Tampa, Florida. **On the Cauchy problem for an elliptic system.**

625-82. VLADETA VUCKOVIC, University of Notre Dame, Notre Dame, Indiana. **Mathematics of incompleteness and undecidability.**

The accurate simulation of the trajectories for a specified ballistic body capable of reentering the atmosphere at prescribed flight conditions in general requires a six degrees of freedom computer representation. However, the relatively large computer times involved in the determination of these trajectories make the utilization of the somewhat less accurate but considerably more efficient three degrees of freedom computer simulation a practical necessity. The use of the latter, less complex program requires the construction of a sufficiently representative ballistic parameter $W/C_D A$ of the ballistic body. The discussion will be concerned with some of the techniques that have been developed and used with considerable success. (Received June 9, 1965.)

625-84. J. H. CURTISS, University of Miami, Coral Gables, Florida 33124. The asymptotic distribution of harmonic interpolation extremal points.

Recently the author [Bull. Amer. Math. Soc. 68 (1962), 333-337 and J. Soc. Indust. Appl. Math. 10 (1962), 709-736] introduced a new type of Fekete extremal points as modes for interpolation with harmonic polynomials. The $n$th set for any compact set $E$ is a set $\{z_{nk}\} k = 0, \ldots, 2n \subset E$ which maximizes $|\det A_n|$, where $A_n$ is them matrix with kth row $(1, z_{nk}, z_{nk}^2, \ldots, z_{nk}^n, z_{nk}^{*n})$, and $*$ denote complex conjugate. In the J. Analyse Math, 14 (1965), 396, Josef Siciak opens the problem of the convergence of the sequence $\{\max |\det A_n|/n(n+1)\}$ and proves that the limit is the transfinite diameter $d(E)$ when $E$ is an ellipse, as would be the case for general $E$ for the original Fekete points. He points out that it is 0, not $d(E)$, when $E$ is a line segment. In the present paper the limit is shown to be $d(E)$ for a wide class of starlike analytic curves. The method consists in first proving the convergence when $A_n$ is formed for the Fejér points by methods developed by the author involving the Faber polynomials and set forth in Math. Z. 86 (1964), 75-92. An extremal argument is then used for $\max |\det A_n|$. (Received June 9, 1965.)

625-85. A. M. BRUCKNER, University of California, Santa Barbara, California. An affirmative answer to a problem of Zahorski and some consequences.

Theorem. Let $P$ be a function-theoretic property (for real valued functions defined on an interval). Suppose $P$ is sufficiently strong to imply (i) that any Darboux Baire 1 function possessing property $P$ on an interval $I$ is VEG on $I$, and (ii) that any continuous function of bounded variation which possesses property $P$ on $I$ is nondecreasing. Then, any Darboux Baire 1 function possessing property $P$ on $I$, is continuous and nondecreasing on $I$. In particular, let $P$ be the property of having, with the possible exception of a denumerable set, a finite or infinite approximate derivative which is nonnegative a.e. Then $P$ satisfies the hypothesis of the theorem. This choice of $P$ answers in the affirmative a question raised by Zahorski (Trans. Amer. Math. Soc. 69 (1950), 1-54). The requirement that $f$ be a Baire 1 function cannot be weakened to the requirement that $f$ be a function in Baire class at most 2. Other theorems concerning Darboux Baire 1 functions are given, and some consequences of these theorems and the stated theorem are provided. (Received June 10, 1965.)
625-86. C. E. AULL, Kent State University, Kent, Ohio 44240. Sequences in topological
spaces, II.

Abstract 65T-227, these Notices 12 (1965), 463, with the following stronger theorem.

Theorem. If every sequence in a topological space has a subsequence without sidepoints, then the
countably compact subsets and the sequentially compact subsets are identical. (Received June 10, 1965.)

625-87. TOMLINSON FORT, 4 Heathwood Circle, Columbia, South Carolina. A series useful
in the study of difference equations.

In this paper series of the following type are considered \( \sum_{n=1}^{\infty} C_n [(z + h_1(z))(z + h_2(z))\ldots (z + h_n(z))]^{-1} \). Here \( 0 < \epsilon \leq h_n(z) \leq E_n \) and \( h_n(z) \to \infty \) when \( n \to \infty \) and \( x + h_n(z) \) increases with \( x \),
\( (z = x + yi) \). Although \( C_n \) is usually independent of \( z \), it is pointed out that in some of the theorems
\( C_n \) may depend upon \( z \). Convergence theorems are proved, also a uniqueness theorem for representa­
tion of functions. It is shown that the product of two functions represented by series of the type
treated is representable by a series of the same type where, however, the \( C_n \)'s depend upon \( z \)
through \( h_1, \ldots, h_n \). If \( h_n \) is independent of \( z \) we have an interesting case. If \( h_n = n \) we have ordinary
factorial series for which the theorems are already known. (Received June 11, 1965.)

625-88. JAMES CONLAN, U. S. Naval Ordnance Laboratory, White Oak, Maryland and
G. N. TRYTTEN, Institute of Fluid Mechanics and Applied Mathematics, University of Maryland,
College Park, Maryland. Pointwise bounds in the Cauchy problem for elliptic systems of partial
differential equations.

and P. W. Schaeffer (Ph. D. Thesis, University of Maryland (1964)) for obtaining pointwise bounds
for solutions to the Cauchy problem for elliptic partial differential equations and elliptic systems
are extended to systems of the form \( \mathfrak{A}_1 (u^i) = h_1 (x, u, u, u, i) \ldots \mathfrak{A}_m (u^m) = h_m (x, u, u, u, i) \),
\( u = (u^1, \ldots, u^m) \) where the \( \mathfrak{A}_a \) are uniformly elliptic operators. (Received June 14, 1965.)

625-89. MARIO PETRICH, 230 McAllister Building, University Park, Pennsylvania.
Congruences on some special completely simple semigroups.

Let \( E \) be a completely \( 0 \)-simple semigroup over the one element group, let \( G \) be a group, and
let \( S = E \times G \) (the Rees factor semigroup of \( E \times G \) modulo the ideal \( \{0\} \times G \). Theorem 1. If \( \eta \)
is a proper congruence on \( E \) and \( \gamma \) is a congruence on \( G \), then the equivalence on \( S \) whose classes are:
\( \{0\} \) and the sets \( \Lambda \times B \), where \( \Lambda \) is a class of \( \eta \) not containing \( 0 \) and \( B \) is a class of \( \gamma \), is a proper
congruence on \( S \); conversely all proper congruences on \( S \) are of this form. This establishes a 1-1
correspondence between the set of all pairs \( (\epsilon, N) \) where \( \epsilon \) is a contraction of the matrix of \( E \) and \( N \)
is a normal subgroup of \( G \), and the set of all proper congruences on \( S \). Let \( L, R, \) and \( G \) be a left zero
semigroup, a right zero semigroup, and a group, resp., and let \( K = L \times R \times G \). Theorem 2. Let \( \lambda \)
and \( \rho \) be equivalences on \( L \) and \( R \), resp., \( \gamma \) a congruence on \( G \). Then the equivalence on \( K \) whose
classes are the sets \( \Lambda \times \Lambda \times C \) where \( \Lambda, B, \) and \( C \) are classes of \( \lambda, \rho \), and \( \gamma \), resp., is a congruence on
\( K \); conversely every congruence on \( K \) is of this form. (Received June 14, 1965.)

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Error estimation for Runge-Kutta methods.

There are no satisfactory methods for the estimation of the errors committed in the numerical solution of differential equations with Runge-Kutta type formulas. The aim of this paper is to establish an error estimation method which not only will provide better results than the other known processes but will enable us also to determine an optimal or near optimal step-length \( h \) which may vary as we progress. This method is based upon the transformation of the Runge-Kutta method into a kind of iterative process. Specific formulas and easy-to-apply instructions will also be given to make the method practical. In order to make a careful comparison between the present method and other known ones the reader may refer advantageously to A survey of numerical analysis, by J. Todd, pp. 325-326. (Received June 14, 1965.)

Estimates at infinity for stationary solutions of the Navier-Stokes equations in two dimensions.

Let \( \Omega \) be a neighborhood of infinity in \( \mathbb{R}^2 \), \( x = (x_1, x_2) \in \Omega \) and \( w = (w_1(x), w_2(x)) : \Omega \rightarrow \mathbb{R}^2 \) a solution of the time-independent Navier-Stokes nonlinear partial differential equations in \( \Omega \). Assume \( |w(x) - w_0| = O(|x|^{-1/4 - \epsilon}) \) as \( x \rightarrow \infty \), for some non-zero constant vector \( w_0 = (w_0, 0) \) and some \( \epsilon > 0 \). Then \( w(x) \) exhibits a parabolic "wake" region behind the complement of \( \Omega \) (i.e., behind the "obstacle" in the flow), interior to which \( |w_1(x) - w_0| = O(|x|^{-1/2}) \) as \( x \rightarrow \infty \) and completely exterior to which \( |w_1(x) - w_0| = O(|x|^{-1}) \) as \( x \rightarrow \infty \). The component of \( w \) orthogonal to \( w_0 \) satisfies the faster decay \( |w_2(x)| = o(|x|^{-1} \log|\xi|) \) interior to the wake and \( |w_2(x)| = O(|x|^{-1}) \) exterior to the wake. Also, any such solution \( w(x) \) which satisfies the physical boundary condition \( w = 0 \) on the inner boundary of \( \Omega \) must necessarily satisfy \( |w_1(x) - w_0| \neq o(|x|^{-1/2}) \) as \( x \rightarrow \infty \). (Received June 14, 1965.)

On a class of nonlinear fourth order differential equations.

Let \( F(u,x) \) be continuous for \( 0 \leq x, u < \infty \) such that \( F(u,x) > 0 \) for \( u > 0 \). Suppose further that there is a positive constant \( \epsilon \) such that \( u^{-2 - \epsilon} F(u,x) \) is an increasing function of \( u \) for each \( x \). Necessary and sufficient conditions are found for the equations \( u^{iv} \pm F(u,x)u = 0 \) to have various types of positive solutions of class \( C^4[a, \infty) \), \( a \geq 0 \). Typical is the following result: Theorem. The equation \( u^{iv} = F(u,x)u \) will have positive \( C^4 \) solutions such that \( (\pm 1)^k u(k)(x) \geq 0 \) and which decrease monotonically to positive constants if, and only if \( \int_0^\infty x^3 F(B,x)dx < \infty \) for some constant \( B > 0 \). Results on two point boundary values problems for these equations are also given. (Received June 14, 1965.)

\( \text{H}^p \) spaces derived from function algebras.

Let \( \mathcal{A} = \mathcal{A}(X) \) be a Dirichlet algebra of functions on a compact Hausdorff space \( X \). Suppose that \( \phi \) is a member of a Gleason part of the carrier space of \( \mathcal{A} \) containing more than one element and that \( \phi \) is represented by the measure \( m \) on \( X \). For \( 1 \leq p < \infty \), define \( \text{H}^p(dm) \) as the closure of \( \mathcal{A} \) in \( \text{L}^p(dm) \), and \( \text{H}^\infty(dm) \) as \( \text{H}^2(dm) \cap \text{L}^\infty(dm) \). Let \( Z = Z_m^2 \) be the inner function in \( \text{H}^2(dm) \) from
which was constructed an embedding mapping of the unit disc onto the part containing $\phi$ by J. Wermer in Dirichlet Algebras, Duke Math. J. 27 (1960), 373-382. **Theorem.** The following is a necessary and sufficient condition that the $\text{sp}(Z_n^r)$, $n \geq 0$, be dense in $H^p(dm)$, i.e., that the natural isomorphic image of $H^p$ (of the disc algebra) fill $H^p(dm)$; $H^{\infty}(dm)$ is a maximal $w^*$ closed subalgebra of $L^{\infty}(dm)$. The theorem is also valid for algebras having the unique representing measures property. In this context an explicit determination of Wermer's embedding function is obtained in terms of a certain Radon-Nikodym derivative and used to generalize a theorem of Szegö on mean square approximation relating two different homomorphisms of the same Gleason part. (Received June 15, 1965.)


**Theorem.** Let $M$ be a compact topological space and $\mu$ be a regular Borel measure on $M$ such that $\mu(M)$ is finite and $\mu(K)$ is positive for each nonvoid open subset $K$ of $M$. If $F(y)$ and $h(x,y)$ are positive and continuous on $M$ and $M \times M$ respectively, there exists a unique positive continuous function $f(x)$ on $M$ such that $\int_M f(x)h(x,y)f(y)d\mu(x) = F(y)$ for all $y \in M$. The function $f(x)$ can be obtained via a certain iterative scheme. (Received June 15, 1965.)


Let $K$ be a class of universal algebras. Let $S(K)$, $H(K)$, $P(K)$, $P_s(K)$ denote the subalgebras, homomorphic images, direct products and subdirect products of algebras in $K$. B. H. Neumann defined the operator $C(K)$: an algebra is in $C(K)$ if it is a union of subalgebras each of which is isomorphic to some algebra in $K$. **Theorem 1.** $SC(K) = CS(K)$, $HC(K) = CH(K)$, $PC(K) \subseteq CP(K)$ for every class $K$. **Theorem 2.** There exist classes $K_1$, $K_2$ and $K_3$ such that $PC(K_1) \not\supseteq CP(K_2)$, $CPS(K_2) \not\supseteq P_s C(K_2)$ and $CP_s(K_3) \not\supseteq P_s C(K_3)$. **Theorem 3.** If $K$ is an equational class then so is $C(K)$. **Theorem 4.** For an equational class $K$, $C(K) = K$ if and only if $K$ is the class of all algebras satisfying a system of identities in one variable. Theorems 1 and 2 solve a problem of B. H. Neumann (Universal algebra, Lecture notes, 1962, pp. 48-49). (Received June 16, 1965.)

625-96. C. S. REED, The University of Texas, Austin, Texas 78712. A characterization of Baire's first class.

Suppose $G$ is the graph of the real valued function $f$ over the interval $[a,b]$. The following two statements are equivalent: (1) $f$ is the pointwise limit of a sequence of continuous functions and (2) if $p$ is a point of $G$, there exist two vertical lines $h$ and $k$ with $p$ between them such that if $u$ and $v$ are two horizontal lines and $T$ is some subset of $[a,b]$ having the property that each point of $G$ with abscissa in $T$ is between $h$ and $k$ and above $u$ and $v$ and $B$ is some subset of $[a,b]$ having the property that each point of $G$ with abscissa in $B$ is between $h$ and $k$ and below $u$ and $v$, then there is some point of $T$ which is not a limit point of $B$ or there is some point of $B$ which is not a limit point of $T$. (Received June 16, 1965.)

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A normal space $X$ is said to be totally normal if every open subset $U$ of $X$ can be written as a locally finite (in $U$) collection of open $F_{\sigma}$ subsets of $X$. The main theorem is as follows: if $X$ is a totally normal space having a topological property $FP$ satisfying certain axioms, then every subset of $X$ has property $P$. Two consequences of this theorem are: (1) every totally normal paracompact space is hereditarily paracompact; (2) every totally normal collectionwise normal space is hereditarily collectionwise normal. (Received June 16, 1965.)

625-98. M. O. GONZALEZ, University of Alabama, Box 1056, University, Alabama 35486. Evaluation of some improper integrals containing Hermite polynomials.

Let $\mathcal{L}(F(t)) = f(s)$, and $s^{1/2}f(s) = \sum_{n=0}^{\infty} b_n u^n$, where $u = (s - a)/s$. It follows $F(t) = \sum_{n=0}^{\infty} (-1)^n b_n 2^{n+1/2} n! t^{-1/2} H_{2n}(\sqrt{2at}/(2n)!)$, and $\int_{-\infty}^{\infty} e^{-t^{1/2}} H_{2n}(\sqrt{at})F(t)dt = (-1)^n a^{-1/2} 2^{n+1} n! b_n$. Similarly, if $s^{3/2}f(s) = \sum_{n=0}^{\infty} c_n u^n$, one finds $\int_{-\infty}^{\infty} e^{-t^{1/2}} H_{2n+1}(\sqrt{at})F(t)dt = (-1)^n a^{-1/2} 2^{n+1} n! c_n$. These general formulas are applied to the evaluation of a number of integrals involving Hermite polynomials and certain higher transcendental functions. (Received June 16, 1965.)

625-99. LOUIS de BRANGES and J. L. ROVNYAK, Purdue University, Lafayette, Indiana. The existence of invariant subspaces.

We previously announced (Bull. Amer. Math. Soc, 70 (1964), 718-721) that a bounded linear transformation in a Hilbert space always has invariant subspaces. This claim was withdrawn (Bull. Amer. Math. Soc, 71 (1965), 396) when a gap in the argument was discovered by Dr. P. A. Fillmore. We have filled the gap by using a new method of obtaining invariant subspaces from a factorization of the characteristic operator function. The result which we originally announced is true. (Received June 17, 1965.)

625-100. LEONARD CARLITZ, Duke University, Durham, North Carolina. Arithmetic properties of the Bell polynomials.

Put $e^{Ax} = \sum_{0}^{\infty} \phi_n(x)t^n/n!$, where $A = \sum_{1}^{\infty} a_n t^n/n!$ and the $a_j$ are indeterminates. Let $p$ be a fixed prime. For fixed $n$, $k$ let $\theta(n,k)$ denote the number of coefficients of $x^k$ in $\phi_n(x)$ that are not divisible by $p$. Also put $\Theta(n,x) = \sum_{0}^{\infty} \theta(n,k)x^k$. Explicit formulas are obtained for $\Theta(n,x)$. When $n = p^r_1 + \ldots + p^r_m$, where the $r_j$ are distinct, $\Theta(n,x)$ is exhibited in terms of the single variable Bell polynomial $B_n(x)$ defined by $B_0(x) = 1$ and $B_{n+1}(x) = x \sum_{n}^{C_n,s} B_s(x)$. In the general case $\Theta(n,x)$ depends upon $P_k(a_1,\ldots,a_m)$, the number of partitions of the vector $(a_1,\ldots,a_m)$ into exactly $k$ parts. For the special case $x = 1$ see Bull. Amer. Math. Soc., 71 (1965), 143-144. (Received June 17, 1965.)
Every vector-lattice identity which is true for the real numbers $\mathbb{R}$ is true in every vector lattice. It is shown that the free vector lattice $F_n$ on $n$ generators is therefore isomorphic to a vector lattice of piecewise linear functionals on $\mathbb{R}^n$, namely the vector lattice of continuous functions on $\mathbb{R}^n$ generated by the coordinate projections. Consequences for $n < \infty$: (1) The structure lattice of $F_n$ is isomorphic to the lattice of dual ideals of the lattice of conical polyhedra in $\mathbb{R}^n$. (2) $F_n$ cannot be embedded in $F_m$ for $m < n$. (3) An archimedean vector lattice with $n$ generators is isomorphic to a vector lattice of piecewise linear functionals on a suitable closed cone in $\mathbb{R}^n$. (4) Any identical implication true in a given non-trivial vector lattice is true in every vector lattice. (Received June 21, 1965.)


In this paper this equation $dX/dt + UX = f(t,X,A)$ is considered in a Banach space, $t$ being real and $\lambda$ a parameter. The linear operator $U$ is unbounded, but closed with a dense domain, and generates a semi-group of bound linear transformations. And $f$ is strongly measurable in $t$, continuous in $X$ and satisfying a Lipschitz condition in $X$ with a variable coefficient. Sufficient conditions for the existence, uniqueness and stability of bounded solutions periodic solutions and almost periodic solutions are obtained. The continuity of these solutions with respect to $\lambda$ is also studied. An associated differential equation with a bounded linear operator is constructed whose periodic or almost periodic solution approximates the solution of the same kind of the above equations. These results extend those obtained by the author in a recent note where the linear operator is bounded. (Research supported by the U. S. Army Research Office-Durham.) (Received June 21, 1965.)

On graphs with transitive automorphism groups.

If $\Gamma$ is a finite undirected graph whose group $G$ of automorphisms acts transitively on the vertices of $\Gamma$ then for some subgroup $H$ of $G$, with $\bigcap_{\gamma \in G} \gamma H \gamma^{-1} = \{1\}$, and for some subset $S$ of $G$, where $S$ is a union of double cosets $\bigcup_{\gamma \in G} \gamma H \gamma^{-1} = S$, there is a 1-1 correspondence $\pi$ between vertices of $\Gamma$ and right cosets of $H$ in $G$ such that $(\pi(x_1 H), \pi(x_2 H))$ is an edge of $\Gamma$ if and only if $x_1^{-1} x_2 \in S$. This generalizes a result of Sabidussi (Proc. Amer. Math. Soc. 9 (1958), 800-804). Conversely given $G,H,S$ as above then there is a graph $\Gamma(G,H,S)$ on which $G$ acts as a group of automorphisms; however $G$ may not be the complete automorphism group. A necessary condition for the latter is the following Theorem. If $\sigma$ is an automorphism of $G$ for which $H$ and $S$ are invariant then every double coset $HxH$ is invariant for $\sigma$. The following completes an investigation of Chao (Proc. Amer. Math. Soc. 15 (1964), 291-292). Theorem. An abelian group $G$ is the transitive automorphism group of a finite graph if and only if $G = C_2 \times C_2 \times \ldots \times C_2$, $r$ copies, where $r = 0, 1$ or $r \geq 5$. (Received June 21, 1965.)
The notation is that of Abstract 611-5, these Notices 11 (1964), 317. The following are established: Theorem 1. If $T$ is any $\omega$-consistent extension of Peano arithmetic $P$, there are $\omega$-consistent extensions $T_1$, $T_2$ of $T$ with the following property: if $(A,B)$ is any pair of re sets and $A \cap B$ is recursive, there is a $\Pi_1^0$ formula $F$ which represents $A$ in $T_1$ and $B$ in $T_2$. In terms of the cited abstract: if $N_{a_1}$, $N_{a_2}$ are pseudo-complement functions of $T_1$, $T_2$ respectively, there is a number $n$ with $\{N_{a_1}(n)\} = A$, $\{N_{a_2}(n)\} = B$. Theorem 2. There is a general method such that if $T$ is any $\omega$-consistent extension of $P$ and $\mathsf{Con}_n$ is a legitimate consistency statement for $T$, there can be constructed a strongly non-invariant transfinite recursive progression of theories having $T$ as base system such that the given consistency statement is unprovable throughout the progression.

Corollary. If $N_a$ is a fixed pseudo-complement function of $T$ and $N_B$ is the pseudo-complement function associated with any system of the progression, then for all $n$, $\{N_B(N_a(n))\} = \emptyset$. Theorem 3. If $T$ is as in Theorem 2 and $T'$ is $T$ with $\mathsf{Con}_n$ adjoined, then if $(A,B)$ is a pair of re sets such that $A \subseteq R \subseteq B$, with $R$ recursive, there is a $\Pi_1^0$ formula $F$ which represents $A$ in $T$ and $B$ in $T'$.

(Received June 21, 1965.)


Suppose that $A_0$ is the algebra of continuous complex-valued functions defined on the closed unit disc $D = \{ |z| \leq 1 \}$ which are analytic in the interior $\{ |z| < 1 \}$. Each multiplicative linear functional on $A_0$ is given by evaluation at a point of the disc: $f \mapsto f(0)$ and may be represented by $\int f d\mu$, where $\mu$ is a probability measure with support in the disc. If $\xi$ is a boundary point, $\mu$ is the point mass at $\xi$. Otherwise there are many such measures. Fixing $\xi = 0$, we show that to each such $\mu$ (distinct from the point mass at 0) which represents the origin there corresponds a certain simply connected domain $\Delta$ containing 0 such that $\partial \Delta \subseteq \text{supp} \mu \subseteq \overline{\Delta}$ where $\text{supp} \mu$ is the closed support of $\mu$ and $\partial \Delta$ the boundary of $\Delta$. The fact that $C - \Delta$ need not be connected leads to some interesting examples. Several conjectures in this area are still not settled. (Received June 21, 1965.)

625-106. S.-T. C. MOY, 950 Larchwood Avenue, Detroit, Michigan 48203. The period of a positive operator.

Let $(X, \mathcal{F}, \mu)$ be a $\sigma$-finite measure space and $M$, a linear operator on $L^\infty(\mu)$ to $L^\infty(\mu)$ satisfying the following conditions: (1) $Mf \geq 0$ a.e. $(\mu)$ if $f \geq 0$ a.e. $(\mu)$, (2) $Mf_n \downarrow 0$ a.e. $(\mu)$ if $f_n \uparrow 0$ a.e. $(\mu)$. Under the assumption that the measure algebra $\mathcal{F}$ is nontrivial and $M$ is irreducible, a period $\delta$ is defined for $M$ which may be $+\infty$ or a positive integer. $M$ is said to be $\lambda$-continuous if $M$ has the representation: $Mf(x) = \int m(x,y)f(y)\lambda(dy)$. It is shown that if $M$ is $\lambda$-continuous, the period $\delta$ of $M$ is a finite integer and $X$ is partitioned into $\delta$ cyclically moving sets $C_1, C_2, \ldots, C_\delta$ such that each $C_1$ is irreducibly $M^{\alpha\delta} -$ closed for $\alpha = 1, 2, \ldots$. (Received June 22, 1965.)

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625-107. R. H. CAMERON and D. A. STORVICK, University of Minnesota, Minneapolis, Minnesota. Analytic continuation for functions of several variables.

Let $\Omega_1$ be a simply connected domain in the $z_1$-plane and $\Omega_j$ denote the $z_j$-plane, $|z_j| < \infty$, for $j = 2, \ldots, n$. Let $L_1$ be a line segment, $L_1 \subset \Omega_1$ and $L_j$ be a complete line contained in $\Omega_j$ for $j = 2, \ldots, n$. Let $(z_1, \ldots, z_n)$ be defined and continuous on $S = (L_1 \times L_2 \times \cdots \times L_n) \cup \cdots \cup (L_1 \times \cdots \times L_{n-1} \times L_n)$. It is proved that if $f(z_1, \ldots, z_n)$ is analytic in $\Omega_1$ for $(z_2, \ldots, z_n) \in (L_2 \times \cdots \times L_n)$ and if $f(z_1, \ldots, z_n)$ is analytic in $\Omega_2$ for $(z_1, z_3, \ldots, z_n)$ $\in (L_1 \times L_3 \times \cdots \times L_n)$ and if $f(z_1, \ldots, z_n)$ is analytic in $\Omega_n$ for $(z_1, \ldots, z_{n-1}) \in (L_1 \times \cdots \times L_{n-1})$ then there exists a function $g(z_1, \ldots, z_n)$ analytic in $(L_1 \times \cdots \times L_n)$ such that $g(z_1, \ldots, z_n) = f(z_1, \ldots, z_n)$ on $S$.

(Received June 23, 1965.)


Conditions on the functional $F$ are given under which the following translation formula for analytic Feynman integrals (see J. Analyse Math, X, pp. 287-361) holds. It is shown that the analytic Feynman integral of $F(x + x_0)$ is equal to the product of $\exp \{i/2 \int_0^1 [x_0(t)]^2 dt\}$ with the analytic Feynman integral of $F(x) \exp \{i\int_0^1 x(t)dx(t)\}$.

(Received June 23, 1965.)

625-109. R. T. MOORE, University of California, Berkeley, California. Commutation relations among unbounded operators and bounded semi-groups.

Suppose a commutation relation such as (1) $[P,Q]f = PQf - Q Pf = cf$, $c$ a complex number, holds for all $f$ in a dense linear subset $D$ of a Banach space $X$, where $P$ and $Q$ are the (generally unbounded) generators of strongly continuous one parameter semi-groups, $\exp(sP)$ and $\exp(tQ)$, and both $PQ$ and $QP$ are defined on $D$. It is of interest in physics and group representations to know when (1) implies more tractable "bounded commutation relations" such as (2) $\exp(sP) \exp(tQ) = \exp(\lambda tQ) \exp(\lambda sP)$. Theorem 1. Suppose (1) holds as above, and in addition (a) $\exp(tQ)$ is a group (defined for all $t \in \mathbb{R}$) and leaves $D$ invariant, (b) $P \exp(tQ)f$ is weakly locally bounded at $t = 0$ for all $f \in D$, and (c) $P$ is the closure of its restriction to $D$. Then (2) holds. Theorem 2. Suppose (1) holds, and $P$ is the closure of its restriction to $(\lambda - Q)D$ for all sufficiently large $\lambda$. Then (2) holds. Theorems 1 and 2 are extended to more general "unbounded commutation relations" than (1), and the "smoothness condition" (b) in Theorem 1 is removed in special cases. Mixed relations between generators and semigroups are discussed. Applications to quantum mechanics and group representations are outlined. (Received June 23, 1965.)


Using a method originated by Sarafyan which makes possible the transformation of Runge-Kutta formulas into a finite iterative process, the author has established a set of formulas that give results as accurately as Nystrom's formula, but not as accurately as Sarafyan's fifth order formulas. However, the simplicity of the structure of the author's formulas makes them easier to program and their use requires less computer time than Nystrom's formula. Another advantage is that
they furnish a constant check on the committed error and, accordingly, enable one to increase or
decrease the steplength "h" and/or the number of significant figures used in the computations.
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625-111. P. S. LANDWEBER, Harvard University, Cambridge 38, Massachusetts.
Cobordism operations.

P. Conner and E. Floyd have shown [Cobordism theories, Seattle Topology Conference, 1963]
how one may define cobordism characteristic classes of vector bundles. These give rise to certain
operations of cobordism theories, by a well-known method of Thom and Wu. For the case of un-
oriented cobordism, one obtains a Hopf algebra \( A^* = \sum_{i=0}^{\infty} A_i \) of stable operations, \( A^* \) contains a Hopf
subalgebra isomorphic to the mod 2 Steenrod algebra. As a coalgebra, \( A^* \cong H^*(MO; \mathbb{Z}_2) \) with the
usual coproduct in \( A^* \) is complicated, but computable. A minimal set of generators of \( A^* \) is obtained,
and other algebraic problems are investigated. (Received June 24, 1965.)

625-112. BRANKO GRÜNBAUM, Michigan State University, East Lansing, Michigan.
Diagrams and Schlegel diagrams.

A d-diagram \( \mathcal{D} \) consists of a simplicial complex \( \mathcal{L} \) and a d-simplex \( D \) such that \( D \) is the
union of all members of \( \mathcal{L} \), the intersection of every member of \( \mathcal{L} \) with \( \partial D \) being a member of \( \mathcal{L} \).
(Simplices are understood as geometric simplices.) A special case of Steinitz's "Fundamentalsatz"
(E. Steinitz-H. Rademacher, Theorie der Polyeder, Springer 1934) asserts that every 2-diagram is
combinatorially equivalent to a Schlegel-diagram of some 3-dimensional convex polytope. In contrast
to this it is shown that there exist 3-diagrams which are not combinatorially equivalent to Schlegel-
diagrams of 4-dimensional convex polytopes. (Received June 25, 1965.)

625-113. WITHDRAWN.

625-114. R. J. WEINACHT, University of Delaware, Newark, Delaware 19711. Fundamental
solutions for elliptic equations with several singular lines.

Let \( B \Phi[u] = u_{x_1 x_1} + a_{x_1} u_{x_2} \) where \( a \) is a real constant. Various equations involving this
operator have been studied by A. Weinstein and others since 1948 (Weinstein, Trans. Amer. Math.
Soc. 63 (1948), 342-358). In the present paper fundamental solutions in the large are given for the
operator \( L = \sum_{j=1}^{m} B \Phi + \lambda^2 \) and iterates thereof. These results generalize previous work of the
author (Contributions to Differential Equations 3 (1964), 43-55). In the case \( m = 2, a_1 = a > 0, a_2 =
\beta > 0 \), the formula is \( C(\alpha, \beta) \int_{0}^{\pi} \int_{0}^{\pi} (\lambda \sigma) \sigma^{-1} \sin^{a-1} \theta \sin^{\beta-1} \phi \, d\theta \, d\phi \), where \( \sigma^2 = x_1^2 + a^2 - 2ax_1 \cos \theta + \)

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\[ x^2_1 + b^2 - 2bx_2 \cos \phi, \quad 2p = a + \beta \text{ and } C(a, \beta) \text{ is a "normalization constant".} \] The pole is at the point \((a, b)\) and \(N_p\) denotes the Neumann function of order \(p\). (Received June 25, 1965.)

625-115. FRANK HARARY and ED PALMER, University of Michigan, Ann Arbor, Michigan 48104. The reconstruction of a tree from its maximal proper subtrees.

Paul Kelly [Pacific J. Math. 7 (1957), 961-968] proved that if \(G\) and \(H\) are two trees with the same number \(p\) of points, for which the two subgraphs obtained by removing the \(i\)th point of each are isomorphic for all \(i = 1\) to \(p\), then \(G\) and \(H\) are themselves isomorphic. We now prove a stronger theorem with the same conclusion but a weaker hypothesis, namely that the subtrees obtained from \(G\) and \(H\) by removing the endpoints are pairwise isomorphic. The general problem for graphs, posed by S. Ulam [A collection of mathematical problems, Interscience, New York, 1960, p. 29], remains unsolved. (Received June 25, 1965.)

625-116. FRANK HARARY and M. D. PLUMMER, University of Michigan, Ann Arbor, Michigan 48104. On the core of a graph.

Dulmage and Mendelsohn (Trans. Roy. Soc. Canada, Sect. III., 53 (1959), 1-13) introduced the core of a bipartite graph and used it extensively in their canonical decomposition for such graphs. In this paper the core of any (not necessarily bipartite) graph is investigated. The core of an arbitrary graph \(G\) is the collection of all lines \(x\) such that \(x\) is a member of a set \(S\) of independent lines, where \(|S|\) equals the minimum number of points necessary to cover all the lines of \(G\). Although a bipartite graph always has a core, this is not true for graphs in general. Various properties of the core are investigated. In particular, characterizations are obtained for (1) graphs with an empty core and (2) graphs which are equal to their core. (Received June 25, 1965.)

625-117. ALFRED GRAY, University of California, Berkeley, California, and S. M. SHAH, University of Kansas, Lawrence, Kansas. Borel exceptional values of an entire function.

Let \(f(z) = \sum_{n=0}^{\infty} A_n z^n\), where the coefficients \(A_n\) are ultimately non negative, be an entire function having a Borel exceptional value. When \(f\) is of infinite order, it will be assumed that for some \(a\), the exponent of convergence of \(a\)-points of \(f\) is finite. Then it is shown that \(\limsup_{r \to \infty} \mu(r)/M(r) \leq 1/2\), where \(\mu(r)\) and \(M(r)\) denote respectively the maximum term and the maximum modulus of \(f\). (Received June 28, 1965.)

625-118. T. J. SUFFRIDGE, University of Kansas, Lawrence, Kansas 66045. Convolutions of convex functions.

Let \(K\) be the class of functions \(f\) which are regular in the \(E = \{z : |z| < 1\}\) and which satisfy the conditions \(f(0) = 0\) and \(f(E)\) is schlicht and convex. **Theorem.** \(f \in K\) and \(f\) is bounded if and only if there exist \(u(t)\) and \(v(t)\) satisfying the following conditions. (i) \(u\) and \(v\) are real valued, nonconstant, continuous and periodic with period \(2\pi\). (ii) There exist \(t_0(\phi)\) and \(t_1(\phi)\) such that \(u(t) \cos \phi - v(t) \sin \phi = p(t, \phi)\) is nondecreasing when \(t_0(\phi) < t < t_1(\phi)\) and nonincreasing when \(t_1(\phi) < t < t_0(\phi) + 2\pi\) for each real \(\phi\). (iii) \(\int_{-\pi}^{\pi} p(t, \phi)dt = 0\) for all real \(\phi\). (iv) \(f(z) = e^{i\phi} \int_{-\pi}^{\pi} p(t, \phi) dt\) for all
real $\phi$. **Theorem.** If $f \in K$, then for each $r$, $0 \leq r \leq 1$, and real $\theta$, the function $g(z;r,\theta) = \left[ f(z) - f(re^{i\theta}z) \right] / (1 - re^{i\theta})$ is starlike (where $g(z;1,2k\pi) = zf'(z)$ when $k$ is an integer). (Received June 28, 1965.)


Transcendental equations of the type $x = a + t\phi(x)$ can be always solved for $x$ as a power series in $t$ by applying Lagrange's theorem. In certain cases, simpler expressions for the coefficients of this series can be derived as recurrence relations by writing a differential equation for $x(t)$ and solving this equation by series substitution. (Received June 28, 1965.)

625-120. C. S. DURIS, Michigan State University, East Lansing, Michigan. A simplex sufficiency condition for quadrature formulas.

In this paper a procedure is described for deriving interpolatory type quadrature formulas by inverting linear systems of differentiation formulas. Because interpolatory type quadrature formulas are uniquely given in terms of the interval of integration and the interpolating points, no new formulas result. By carrying along the remainder terms for the differentiation formulas, remainder terms for the quadrature formulas can be obtained. Using this procedure a sufficient condition is given for a quadrature formula to have a remainder of the form $M_f(n; \alpha)$, where $f$ is the function being integrated and $M$ is a constant. This is equivalent to the statement that the quadrature formula is a simplex quadrature formula. Applications of this sufficiency condition are made, and in particular a fairly thorough investigation of the formulas having the form $\int_{-1}^{1} f(t)dt = C_0f(-1) + C_1f(\alpha) + C_2f(\beta) + C_3f(1)$ is made for $-1 < \alpha < \beta < 1$. (Received June 29, 1965.)

625-121. BASIL GORDON, University of California, Los Angeles, California 90024. Power series with periodic gaps.

The distinct residues $r_1, \ldots, r_g (\mod m)$ are called strongly distinct if for any divisor $d$ of $m$, the number of incongruent $r_i (\mod d)$ is $\leq \min (g,d)$. Suppose $f(z) = \sum_{n=0}^{\infty} a_n z^n$ has radius of convergence 1, and $a_n = 0$ for all $n = r_1, r_2, \ldots, r_g (\mod m)$ where $r_1, \ldots, r_g$ are strongly distinct residues. Then $f(z)$ has at least $\min (4g+1)$ singularities on $|z| = 1$, and this is the best possible result. This answers a question posed by Motzkin, who settled the case $g \leq 3$. (Received June 29, 1965.)

625-122. L. O. CANNON, Utah State University, Logan, Utah 84321. Sums of solid horned spheres.

A general definition is given for horned spheres and, in particular, for 2-horned spheres and 3-horned spheres. If $M$ is a horned sphere in $E^3$ and $U$ is the bounded component of $E^3 - M$, then $M^* = M \cup U$ is a solid horned sphere. Alexander's solid horned sphere as described by Bing (Ann. of Math., 56 (1952), 354-362) and Ball's solid horned sphere (Ann. of Math. 69 (1959), 253-257) are both examples of solid 2-horned spheres. A continuum is the sum of two solid horned spheres $M^*$ if it is the sum of three mutually exclusive sets $M, U^1, U^2$, such that there is a homeomorphism of
M ∪ U^i (i = 1, 2) onto M, taking M onto M. If, in addition, there is a homeomorphism of M ∪ U^1 onto M ∪ U^2 which is the identity on M, we say that M ∪ U^1 ∪ U^2 is the sum of two solid horned spheres sewn together on their boundaries with the identity homeomorphism and we use the notation \( \Sigma (M^*, M^*) \). We show that for any solid 2-horned sphere M^*, \( \Sigma (M^*, M^*) \) is homeomorphic with \( S^3 \).

One solid 3-horned sphere M^† is shown to have the property that \( \Sigma (M^†, M^†) \) is homeomorphic with \( S^3 \). We also describe a solid 3-horned sphere M^‡, such that the sum of two, even sewn together with the identity homeomorphism is not homeomorphic with \( S^3 \). (Received June 28, 1965.)


Let R be a right alternative ring of characteristic not 2 or 3 and such that there exists an idempotent e in R for which there are no nilpotent elements in either \( R_1 (e) \) or \( R_0 (e) \) as defined by A. A. Albert (see A. A. Albert, Power associative rings, Trans. Amer. Math. Soc. 64 (1948), 552-593). The following theorems are proved. **Theorem 1.** If R has no nilpotent ideals then R is alternative.

**Theorem 2.** If R is prime and not associative then R may be embedded in a Cayley-Dickson algebra over the quotient field of the center of R (see M. B. Slater, Weakly prime alternative rings, Abstract 65T-166, these Notices), 12 (1965), 367-368). **Theorem 3.** A simple right alternative ring R of characteristic not 2 or 3 which is not associative is alternative if and only if there exists an idempotent e in R such that there are no nilpotent elements in either \( R_1 (e) \) or \( R_0 (e) \) (see E. Kleinfeld, Simple alternative rings, Ann. of Math. 58 (1953), 544-547). (Received June 28, 1965.)

625-124. WITHDRAWN.

625-125. L. W. BEINEKE and FRANK HARARY, University of Michigan, Ann Arbor, Michigan 48104. Binary matrices with equal determinant and permanent.

Paul Turán [Ann. Polon. Math. 12 (1962), 50-53] asked for a necessary and sufficient condition that a binary matrix have equal determinant and permanent. We have obtained a structural characterization of such a matrix A by associating a directed graph D with A in the usual way, i.e., so that A is the adjacency matrix of D. For terminology, see Harary, Norman, and Cartwright [Structural models: an introduction to the theory of directed graphs, New York, 1965]. **Theorem.** A binary matrix A has equal determinant and permanent if and only if (1) all permutations in each strong component of D have the same parity, and (2) the number of strong components in which all permutations are odd is even. (Received June 29, 1965.)
Let S be locally compact and H ⊆ M(S), the bounded Radon measures. The strict topology $\beta$ was introduced by R. C. Buck who proved that $(C(S),\beta)^* = M(S)$. **Theorem 1.** H is $\beta$-equicontinuous iff (a) H is bounded, (b) $\epsilon > 0$ implies there is a compact $K \subseteq S$ with $|\mu|(S \setminus K) < \epsilon$ for all $\mu \in H$.

**Theorem 2.** S paracompact and H $\beta$-weak* countably compact implies H is $\beta$-equicontinuous. Consequently, $(C(S),\beta)$ is a Mackey space (this partially answers a question posed by Buck).

**Theorem 3.** If $\Omega_0$ is the space of ordinals $< \text{the first uncountable}$ then $(C(\Omega_0),\beta)$ is not a Mackey space.

**Theorem 4.** If S is metrizable these are equivalent: H is $\beta$-equicontinuous; H is $\beta$-weak* sequentially compact; H is $\beta$-weak* countably compact; H is $\beta$-weak* conditionally compact. If D is the open unit disk in the complex plane and $H_0^0$ the bounded analytic functions on D, then $H_0^0$ is closed in $(C(D),\beta)$. **Theorem 5.** $(H_0^0,\beta)$ is not a Mackey space (answering a question posed by A. Shields and L. Rubel). Finally, Theorem 2 can be applied to yield simple proofs of generalizations of some theorems of Dieudonné concerning various modes of convergence in M(S). (Some of these results will appear in Bull. Amer. Math. Soc.) (Received June 23, 1965.)

Let K be a field, $K[t]$ the ring of polynomials integrated over K and $K(t)$ the field of rational functions. Let $x = a_1 t^1 + \ldots + a_0 / b_1 t^1 + \ldots + b_0$ where $a_1 / b_0$ be a typical element of $K(t)$. Define a valuation in $K(t)$ by $|x| = e^{m-n}$, $|0| = 0$, $e > 1$. Let $K[t]$ be the completion of K(t) with respect to this valuation. Then we prove the following results: **Theorem.** Let K be a finite field in which $x^2 + 1 = 0$ is not solvable. Let $L_1$, $L_2$, $L_3$, $L_4$ be linear forms in variables $x, y, z, s$ with coefficients from $K[t]$ and determinant $\Delta \neq 0$. Then there exist $x_0, y_0, z_0, s_0 \in K[t]$ with $\min |\Delta| / e \leq \min |L_1^2 + L_2^2| + L_3 + L_4^2 : (x, y, z, s) = x_0 y_0^2 z_0^2 s_0 (mod K[t])$. We can also prove the corresponding theorem for $L_1 (L_2^2 + L_3^2)$ with $|\Delta| / e^3$ instead of $|\Delta| / e^4$. Also a similar result can be proved for $f(x, y) = a(x - \theta y)(x - \phi y)$ in an arbitrary finite field with $|a| |\theta - \phi| / e^2$ instead of $|\Delta| / e^4$. All these results are best possible. Similar results were proved by Armitage (by a different method) for $f(x, y)$ and $L_1 (L_2^2 + L_3^2)$ where $f(x, y) \neq 0$ for all $(x, y) \neq (0, 0)$ $(x, y \in K[t])$ and $L_2^2 + L_3^2 \neq 0$ for all $(x, y, z) \neq (0, 0, 0)$, $x, y, z \in K[t]$. (Received June 29, 1965.)

Let $\Lambda(G)$ be the first term of the descending Loewy series (see H. Zassenhaus's book, The theory of finite groups). **Definitions.** Let G be a finite group. A subgroup U of G is called a Cartan subgroup of G if it satisfies the following conditions. (a) U is nilpotent, (b) $\exists$ a solvable subgroup L of G satisfying $U \subseteq L$; $\langle L^{N_G(U)} \rangle \Lambda(G) = G$, (c) U is maximal with respect to conditions (a) and (b), (d) if $\varphi$ is any automorphism of G, then $U^{\varphi}$ is a conjugate of U, (e) If N is any normal subgroup of G, then $UN/N$ satisfies the conditions (a), (b), (c) and (d) in G/N. **Theorem 1.** Cartan subgroups of finite solvable groups are the same as Carter subgroups [For the definition of Carter
subgroups, see R. W. Carter, Math. Z. 75-77 (1961), 136-139]. By making some assumptions on the structure of the non-abelian composition factors of the given groups we prove Theorem 2. Every finite group G has Cartan subgroups. Corollary, Cartan subgroups form a characteristic class of conjugate nilpotent subgroups such that images of Cartan subgroups under a group homomorphism are Cartan subgroups of the homomorphic images. (Received June 25, 1965.)


Consider the Neumann problem consisting of the Laplace equation (1) \( \nabla^2 \phi = 0 \) in three-space with the boundary condition (2) \( \partial \phi / \partial n = 0 \) at the surface (3) \( f_1(x_1,x_2,x_3) = C_1 \). The problem is "definitized" by considering \( C_1 \) as an arbitrary constant and by determining two families of surfaces (4) \( f_i(x_1,x_2,x_3) = C_i, \ i = 2,3 \), such that (5) \( \nabla^2 f_i = 0, \ i \neq j, j = 1,2,3 \), where \( C_2 \) and \( C_3 \) are arbitrary constants. Letting (6) \( f_i = \xi_i \), the Laplace equation becomes (1*) \( (\partial^2 \phi / \partial \xi_i^2)(\partial f_i / \partial \xi_1)^2 + (\partial \phi / \partial \xi_1)\partial^2 f_i = 0 \) while (2) and (3) reduce to (2*) \( (\partial \phi / \partial \xi_1) = 0 \) for (3*) \( \xi_1 = C_1 \). The definitized problem is much easier to solve, since the new boundary condition (2*) does not contain the derivatives of \( f_1 \). Closed form solutions are given for spheres, circular cylinders with flat ends, and for cone-cylinder-hemisphere combinations. (Received June 29, 1965.)


N is the set of non-negative integers. Let \( \alpha \) be any countable ordinal. The following refers to a certain class of natural definitions of \( A(\beta) \), for \( \beta \leq \alpha \), as the \( \beta \)-times-iterated hyper-jump of \( A \). Theorem 1. There exist sets \( A,B \leq \beta N(\alpha) \) such that, for every \( \beta < \alpha, A \not\leq h B(\beta) \) and \( B \not\leq h A(\beta) \). In fact, \( A \not\leq_T B(\alpha) \) and \( B \not\leq_T A(\alpha) \). Theorem 2. For any set \( B \), and for \( 1 \leq \beta \leq \alpha : \) there is a set \( A \) such that \( B = h A(\beta) \) if and only if \( N(\beta) \leq h B \). The present paper uses generalizations of the techniques mentioned in Abstract 624-3, these Notices 12 (1965), 449. For \( \alpha = 1 \), Theorem 1 becomes Theorem 1 of that abstract. Theorem 2, for \( \alpha = 1 \), is an unpublished result of Gandy. (Received June 29, 1965.)

625-131. A. H. KRUSE, New Mexico State University, Box Y, University Park, New Mexico. An abstract property \( P \) for groupoids such that locally locally \( P \) is weaker than locally \( P \).

P. M. Cohn, in his Universal algebra (Harper and Row, 1965), defines a property \( P \) pertaining to \( \Omega \)-algebras to be abstract iff \( P \) is invariant under isomorphism (p. 101) and to be local iff for each \( \Omega \)-algebra \( A \), if \( A \) is the union of a collection of subalgebras having property \( P \) with the collection directed by inclusion, then \( A \) has property \( P \) (p. 101). Cohn indicates (p. 106) that it is not known whether for each abstract property \( P \), locally locally \( P \) implies locally \( P \). In the following theorem, the set \( R \) of all real numbers is regarded as a groupoid (in fact, as a semigroup) with groupoid operation * given by \( x*y = \min(x,y) \) \((x, y \in R)\). Theorem. There is an abstract property \( P \) pertaining to groupoids such that each groupoid with property \( P \) is isomorphic to a subset of \( R \) and such that some subset of \( R \) has property locally locally \( P \) and does not have property locally \( P \). The proof expands a method of Ben Dushnik and E. W. Miller, Bull. Amer. Math. Soc. 46 (1940), 322-326. (Received June 30, 1965.)

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Let \( \Delta(S) \), \( c(S) \), denote the critical determinant and covering constant respectively of an \( n \)-dimensional set \( S \). Let \( \mathcal{L} \) be a class of sets in \( n \)-dimensional space. A set \( S \) is said to be \( \mathcal{L} \)-minimal (irreducible) for packings (or coverings) if \( S \in \mathcal{L} \) and for all \( T \in \mathcal{L} \), \( T \not\subseteq S \) we have \( \Delta(T) < \Delta(S) \) (or \( c(T) < c(S) \)). Similarly \( S \) is \( \mathcal{L} \)-maximal for packings (coverings) if \( T \in \mathcal{L} \), \( T \not\supseteq S \) implies that \( \Delta(T) > \Delta(S) \) (or \( c(T) > c(S) \)). A proof based on Zorn's lemma is given of the theorem of C. A. Rogers that if \( \mathcal{L} \) is the class of bounded closed symmetric star sets and \( S \in \mathcal{L} \), then \( S \) contains a set \( T \) such that \( \Delta(T) = \Delta(S) \) and \( T \) is \( \mathcal{L} \)-minimal for packings. The method is used to get similar results for minimal and maximal packing and covering sets contained in or containing a given set for various classes. If \( \mathcal{L} \) is the class of symmetrical convex bodies, it is shown that in \( \mathbb{R}^3 \) there exist sets which are \( \mathcal{L} \)-maximal for packings, but are not space filling. This shows that a straightforward generalization to higher dimensions of a famous theorem of Reinhardt and Mahler is not possible. Criteria are obtained for the \( \mathcal{L} \)-maximality for coverings of sets in \( \mathcal{L} \), where \( \mathcal{L} \) is the class of closed symmetrical star bodies (or all star bodies). These are applied to discuss the maximality of various sets. (Received June 30, 1965.)

625-133. P. R. STRAUSS, 157 South Harrison Street, East Orange, New Jersey 07018. Topologies and Borel structures associated with Banach algebras of functions, II. Preliminary report.

Conditions for completely regular spaces to be homeomorphic are developed. With terminology as in I, Abstract 65T-353, these \( \text{Notices} \) 12 (1965), 619, and the paper of E. R. Lorch cited there, \( B \) is admissible on \( E \), \( \mathcal{N}_a \) is now assumed regular. Restriction induces epimorphisms of bound 1 from \( I^a(B^\alpha) \) and \( A^a(B^\alpha) \) onto \( I^a(B) \) and \( A^a(B) \), respectively. The kernels consist of the functions vanishing on \( K_a(B) \), the \( \iota_{a} \)-closure of \( E \) in \( E^\alpha \). (\( K_1(B) = B ; K_a(B) = E \) for a large enough.) Bijections continuous to \( K_a(B) \) from \( K_a(I^a(B)) \) and \( K_a(A^a(B)) \) are defined and the m.p.c. in either of the latter are characterized among all m.p.c. in \( I^a(B) \) or \( A^a(B) \) by simple algebraically invariant properties. Given \( B_i \) admissible on \( E_i \), \( i = 1 \) or 2, an isomorphism \( \phi \) between \( B_1 \) and \( B_2 \) corresponds to a homeomorphism between the spaces \( K_a(B_1) \) (\( \beta \)-topologies) iff \( \phi \) can be extended to the algebras \( A^a(B_1) \) or \( I^1(\Lambda^a(B_1)) \) or \( I^a(B_1) \). Thus \( E \)' is determined in \( E^\alpha \) by the Baire functions and \( E \) by the Borel functions. If \( X \subseteq B \), \( B_X \) is the algebra (\( \chi \)-sup norm) of restrictions to \( X \) of the functions in \( B \), then \( X = K_a(B_X) \) iff \( X \) is \( \iota_a \)-closed in \( K_a(B) \). The theory includes Q-spaces. If \( C^*(E) \) is the algebra of all bounded \( \beta \)-continuous functions on \( E \) then Hewitt's realcompactification of \( E \) (\( \beta \)-topology) is \( K_1(C^*(E)) \).

(Received June 30, 1965.)


Let \( L^1(m) \) denote the Banach space of absolutely integrable functions with respect to the \( \sigma \)-finite measure \( m \). Suppose \( T \) is a positive linear operator of norm one on \( L^1(m) \) and that \( A_n = (1/n) \sum_{h=0}^{n-1} f_k \). Theorem. Suppose \( \rho \) is an a.e. positive function such that \( \{ A_n \rho \} \) converges in norm. Put \( h = \lim A_n \rho \) and let \( A = \{ x | h(x) = 0 \} \). Then for any \( f \) in \( L^1 \) the pointwise limit of \( A_n f \) is zero almost everywhere on \( A \). The theorem and its proof are generalizations of Lemma 5 of Y. Ito.
This shows that the mean ergodic theorem implies the pointwise ergodic theorem on $L^1(m)$, when $m$ is $\sigma$-finite. (Received June 30, 1965.)

ALESSANDRO FIGA-TALAMANCA and D. RIDER, Massachusetts Institute of Technology, Cambridge, Massachusetts. A theorem of Littlewood and lacunary series for compact groups.

Let $G$ be a compact group and $f \in L^2(G)$. It is proved that given $p < \infty$ there exists a unitary transformation $U$ of $L^2(G)$ into $L^2(G)$, which commutes with left translations and such that $Uf \in L^p$. The proof is based on techniques developed by S. Helgason for a similar question. The result stated above, which is an extension of a theorem of Littlewood for the unit circle is then applied to the study of lacunary Fourier series. (Received June 30, 1965.)

R. E. LYNCH, University of Texas, Austin, Texas and J. R. RICE, Purdue University, Lafayette, Indiana. Alternating direction implicit methods with smooth initial error.

The choice of parameters for use in alternating direction implicit methods for solving separable elliptic partial differential equations in two independent variables is related to the approximation problem: Find $\rho_1, \ldots, \rho_m$ which minimize the maximum of $|w(x) \prod_{j=1}^{m} (x - \rho_j)/(x + \rho_j)|$ on $0 < x \leq 1$ where $w$ is a positive weight function associated with the smoothness of the initial error. Existence, uniqueness, and characterization of the solution of this problem is established. For the case of the five-point approximation to Poisson's equation with mesh size $h$ precise asymptotic results, as $h \to 0$, are obtained in the case of $m = 1$ for initial condition chosen from functions in various smoothness classes, i.e., $C$, $C^1$, etc. Optimal parameters were computed and numerical experiments were performed. Larger error reductions were observed when smoothness was taken into account. It was also found that the optimal parameters as defined above can be considerably different than the parameters obtained by maximizing on the restriction of $a \leq x \leq 1$ to the set of eigenvalues of the iteration matrix, e.g., for $h = 1/4$, $m = 8$, the error curve (with optimal parameters) evaluated only at the eigenvalues has only four sign changes. (Received June 29, 1965.)

D. W. SWANN, Bell Telephone Laboratories, Incorporated, Whippany, New Jersey 07981. A necessary and sufficient condition that an $L^2$ kernel possess a finite number of characteristic values.

Let $K(s,t)$ be a complex-valued $L^2$ kernel in the sense of Smithies (Integral Equations, Cambridge Tracts No. 49) on the square $a \leq s, t \leq b$. Then the iterated kernels $K^{(v)}(s,t) = \int_a^b K(s,u)K^{(v-1)}(u,t)du$ are well-defined, as are the higher order traces $k_\nu = \int_a^b K^{(v)}(s,s)ds = \sum_{m=1}^\infty (1/\lambda_m^v)$ for $\nu \geq 2$, where $\{\lambda_m\}$ is the set of characteristic values, perhaps empty, belonging to $K(s,t)$. Unless $K(s,t)$ falls into certain specialized classes (e.g., normal, degenerate, positive definite), the number of characteristic values possessed by the kernel is generally unknown. This question is partially resolved by the following. Theorem. A necessary and sufficient condition that the $L^2$ kernel $K(s,t)$ possess a finite, nonzero number $n$ of characteristic values is that the higher order traces $k_\nu$ (which obviously must not all vanish) satisfy a linear, homogeneous recurrence relation of the form $k_{n+\nu} + a_1 k_{n+\nu-1} + a_2 k_{n+\nu-2} + \ldots + a_{n-1} k_{n+\nu-1} + a_n k_\nu = 0$, for every $\nu \geq 2$. The $a_1, a_2, \ldots, a_n$ are constants not all zero, and in particular $a_n \neq 0$. (Received June 30, 1965.)
A holomorphic function \( f(z) \) in the right half plane is said to be a positive function (PF) if the real part of \( f(z) \) is positive. A positive function \( f \) is said to be a positive real function (PRF) if it is real on the real axis and a positive imaginary function (PIF) if the real part of \( f(z) \) goes to zero as \( z \) approaches the imaginary axis. **Theorem 1.** If \( f \) is PF and if \( \theta = \sup_{x > 0} \arg f(x) \) and if \( \theta = \arg f(x) \), then \( \theta - |\arg z| + (1 - 2\theta/\pi) \leq \arg f(x) \leq \theta + |\arg z| + (1 - 2\theta/\pi) \). This is a generalization of the standard theorem for PRF (where \( \theta = 0 \)). **Corollary** (to the proof of the theorem). Let \( -1 \leq g_1(x) < g_2(x) \leq 1 \) with \( \lim_{x \to -\infty} g_k(x) = (-1)^k(k = 1,2) \) and let \( D = \{ z = x + iy \mid -\infty < x < \infty, g_1(x) \leq y \leq g_2(x) \} \). Consider \( D \) now as a channel for a plane steady irrotational flow of an ideal fluid starting from \( -\infty \) with uniform velocity in the \( x \)-direction. If the streamline, which approaches the \( x \)-axis as \( x \) goes to \( -\infty \), remains between the lines \( y = a \) and \( y = \beta (\beta < a) \), then the streamline which approaches the line \( y = c \) remains between the lines \( y = a + |c|(1 - a) \) and \( y = \beta - |c|(1 - \beta) \).

**Theorem 2.** Let \( f \) be PIF with \( H_R = \{ z = x + iy \mid x^2 + y^2 < R^2, x > 0 \} \) and \( a(R) \) equal to the area (counting multiplicity) of the image of \( H_R \) under \( f \), then \( R^2 - a(1) \leq a(R) \). (Received July 1, 1965.)

**625-139. J. T. POOLE,** Florida State University, Tallahassee, Florida 32306. **Coefficient extremal problems for schlicht functions.**

Let \( S \) be the class of functions \( f(z) = z + a_2z^2 + \ldots \), regular and schlicht in \( |z| < 1 \); let \( \sum \) be the class of functions \( \tilde{f}(z) = z + \tilde{a}_0 + (\tilde{a}_1/z) + \ldots \), regular and schlicht in \( |z| > 1 \) except for the simple pole at \( z = \infty \). Let \( S^{-1} \) and \( \sum^{-1} \) be the respective inverse classes. Finally, let \( S^*, \sum^*, \sum^*-1 \) and \( \sum^*-1 \) be the corresponding classes of starlike functions. According to a result due to E. Jubotinsky (Proc. Amer. Math. Soc. 4 (1953), 546-553) if \( \tilde{f}(z) = \sum \sum^{-1} \equiv \sum^* \equiv \sum^*-1 \equiv \sum^*-1 \) is a positive function in \( \sum \equiv \sum^* \equiv \sum^*-1 \equiv \sum^*-1 \), we have obtained the results:

1. \( \max_{f \in \mathcal{S}^*-1} |b_n| \leq (1/n) \max_{f \in \mathcal{S}^*} |a_n| \).
2. \( \max_{f \in \mathcal{S}^*-1} |b_n| \geq (1/n) \max_{f \in \mathcal{S}^*} |a_n| \).

Applying a generalization of the method of proof to J. Clunie (J. London Math. Soc. 34 (1958-1959), 215-216) we show that for \( f \in \mathcal{S}^* \), \( |a_{-t+n}| \leq C_{2t,n} \), \( n = 0,1, \ldots, t + 1 \). Using variational methods the same result is obtained for \( f \in \mathcal{S} \). Thus, applying (1) and (2) we have obtained the results: If \( \phi \in \mathcal{S}^{-1} \) then

\[ |b_n| \leq (1/n) C_{2n,n-1} \quad \text{and} \quad |\tilde{b}_n| \leq (1/n) C_{2n,n+1} \quad \text{if } n = 1,2, \ldots . \]

(Received July 1, 1965.)

**625-140. D-N, VERMA,** Andrews Hall, University of Colorado, Boulder, Colorado. **Some new relations in universal envelope of a semi-simple Lie algebra.**

Let \( \mathcal{L} \) be a finite-dimensional split semi-simple Lie algebra of characteristic 0 having root-space decomposition \( \mathcal{L} = \sum_{a \in \Delta} \mathcal{L}_a \) relative to Cartan subalgebra \( \mathcal{H} \). In \( \Delta \) take a simple system of roots \( \alpha_1, \ldots, \alpha_k \) and define basic weights \( \lambda_1, \ldots, \lambda_k \) by \( \langle a_1, \alpha_q \rangle = (1/2) \delta_{1,1} \). Denote Weyl reflection for \( a_1 \) by \( R_1 \) and Weyl group by \( W \). Following result amounts to certain relations satisfied by any fixed \( f_1 \in \mathcal{L}_{-a_1} \). **Theorem.** Let \( \lambda \) be an integral linear combination of \( \lambda_1 \)'s and \( \sigma = R_{11} R_{12} \ldots R_{1k} \in W \); for \( q = 1,2, \ldots, k \) let \( n_q \) be the coefficient of \( \lambda_1 \) in \( \lambda \). Then the monomial \( f_1 f_2 \ldots f_k \) in the division ring of quotients of the universal envelope \( U \) of \( \mathcal{L} \) depends only on \( \lambda \) and \( \sigma \)(not on the way \( \sigma \) is expressed as product of \( R_i \)'s).
Conjecture. This monomial is in \( U \) if \( \lambda - \lambda \sigma \) is a non-negative linear combination of \( a_i \)'s. This has deep consequences in study of infinite-dimensional representations and has been verified for \( \mathfrak{g} \), simple of Type \( A \). For \( \Lambda \), theorem asserts \( f_1^{m_1} f_2^{m_2} = f_1^{m_2} f_2^{m_1} \) and conjecture asserts left-divisibility of \( f_2 f_1^{m_1 + m_2} \) by \( f_1^{m_1}, m_1, m_2 \) any positive integers. (Received July 1, 1965.)

625-141. DANIEL GIESY, University of Southern California, Los Angeles, California 90007. Fine structure of norm in \( B \)-convex spaces.

A normed linear space (NLS) \( X \) is \( B \)-convex if there exist \( k \geq 2 \) and \( \epsilon > 0 \) such that for any choice of \( x_1, \ldots, x_k \) from the unit ball of \( X \), \( \| x_1 \pm x_2 \pm \cdots \pm x_k \| \leq (1 - \epsilon) \) for at least one choice of the + and - signs. It is known that a NLS \( X \) is \( B \)-convex iff the following strong law of large numbers is valid: If \( \{X_n\} \) is an independent sequence of \( X \)-valued random variables with 0 mean and uniformly bounded variances, then \( \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} X_i = 0 \) in the strong topology of \( X \) almost surely. I find classes of sequences \( \{X_n\} \) such that the \( n^{-1} \sum_{i=1}^{n} X_i \to 0 \) uniformly over the whole class in some sense and apply this result to derive the following: If \( X \) is \( B \)-convex, then for every \( \delta > 0 \) and \( r < 1 \) there exists \( N \) such that if \( n \geq N \) and \( x_1, \ldots, x_n \) are in the unit ball of \( X \), \( n^{-1} \| x_1 \pm x_2 \pm \cdots \pm x_n \| \leq \delta \) for at least \( r^n \) of the \( 2^n \) choices of + and - signs. (Received July 1, 1965.)

625-142. V. C. DUMIR, Ohio State University, Columbus, Ohio 43210. Inhomogeneous minimum of indefinite quaternary quadratic forms with signature 2, Preliminary report.

Theorem. Let \( \Phi(x,y,z,t) \) be a real indefinite quaternary quadratic form with signature \( \pm 2 \) and determinant \( D \). Then given any real numbers \( x_0, y_0, z_0, t_0 \) we can find integers \( x, y, z, t \) such that
\[
\| \Phi(x + x_0, y + y_0, z + z_0, t + t_0) \| \leq (|D|/3)^{1/4}.
\]
Equality is needed if and only if \( \Phi(x,y,z,t) \sim \rho \Phi_1 \) where \( \rho \neq 0 \) and \( \Phi_1 = x^2 + y^2 + 3z^2 - t^2 \). For \( \Phi_1 \) equality occurs if and only if \( (x_0, y_0, z_0, t_0) = (1/2, 1/2, 1/2, 1/2) \) (mod 1). The corresponding result for signature 0 is a consequence of Birch's Theorem for forms with \( 2n \) variables and signature zero. The analogous result for two variables was proved by Minkowski and that for three by H. Davenport. (Received July 1, 1965.)

625-143. B. L. OSOSKY, Rutgers, The State University, New Brunswick, New Jersey 08903. A generalization of quasi-Frobenius rings, Preliminary report.

A quasi-Frobenius ring \( R \) is a ring with minimum condition on right ideals and the properties (i) \( R \) is right self injective and (ii) every simple right \( R \)-module is isomorphic to a right ideal of \( R \). Theorem. Let \( R \) be a ring with properties (i) and (ii). Then \( R = \sum_{i=1}^{n} e_i R \) where \( e_i \in e_i^2 \subseteq R \), and \( e_i R/e_i J \) is a simple \( R \)-module for all \( i, 1 \leq i \leq n \), \( J \) the Jacobson radical of \( R \). Hence \( R \) must have maximum condition and minimum condition on direct summands. An example is constructed to show that (i) and (ii) imply no other chain conditions. Another example shows that even if the injective hull of every simple \( R \)-module is isomorphic to a right ideal of \( R \), if \( R \) is not self injective, \( R/J \) need not be semi-simple Artin. (Received July 1, 1965.)
Let $W$ be a class of groups. Groups with a $W$-category are defined by B. I. Plotkin (Generalized soluble and nilpotent groups, Amer. Math. Soc., Transl., Series 2, Vol. 17, p. 64). Let $\pi$ be an infinite cardinal number. Define $\pi^* = \pi[1]$, $\pi[2] = 2^\pi$ and inductively $\pi[n] = 2^{\pi[\pi^{n-1}]}$. Let $G$ be an infinite group, and $A$ be a subgroup of $G$. If $o(G) \leq o(A)[n]$ for some positive integer $n$, $G$ is said to be bounded by $A$. Classes of groups are determined that are bounded by an abelian subnormal subgroup. Among these classes are (1) metabelian groups; (2) $ZA$-groups; (3) $SI^*$-groups; (4) $SJ^*$-groups with the maximal condition for subnormal abelian subgroups; (5) Baer Nilgroups with the maximal condition for subnormal abelian subgroups; (6) groups with a finite solvable category; (7) groups with a finite $ZA$-category; (8) certain classes of $ZD$-groups; and (9) groups that possess a subnormal $SI^*$-subgroup whose centralizer is subnormal and $SI^*$. In some of the above cases, upper bounds are determined for $n$. (Received July 2, 1965.)


Statistical metric spaces (initiated by Karl Menger, Proc. Nat. Acad. Sci., USA, 28 (1942), 535-537) are generalized to statistical topological spaces and statistical uniform spaces by means of the concept $\lambda^A = \{x \in X | P(x \in \lambda) \geq \lambda\} (0 \leq \lambda \leq 1, A \subset X)$ and related concepts. Statistical metric spaces are characterized in terms of existence of "statistical distance functions". (Received July 2, 1965.)

625-146. DANIEL KOCAN, 1344 Taft Road, Teaneck, New Jersey. Spectral manifolds for a class of operators.

Let $T$ be a closed linear transformation defined on a dense subset of a Banach space $X$ which has a pure continuous spectrum on the real axis and whose resolvent operator $R(z) = (zI - T)^{-1}$ satisfies a growth condition $|\text{Im } z|^n \|R(z)\| \leq K$ for $0 < |\text{Im } z| < 1$; $|\text{Im } z| \|R(z)\| \leq K$ for $|\text{Im } z| \geq 1$ for positive constants $K$ and $n$, $n$ an integer. For each closed subset $F$ of the real line let $M(F)$ denote the set of all vectors $x$ in $X$ whose spectra are contained in $F$. It is shown that the vectors with compact spectra are dense in $X$ and that $T$ is bounded if and only if every vector has compact spectrum. $T$ has nontrivial invariant subspaces of the form $M(J)$ where $J$ is a compact interval, if $J = [a,b]$ then there exists a closed projection, unbounded in general, which has $M(J)$ as its range. For each real $t$ there is a closed projection $E(t)$ which has $M((-\infty,t])$ as range and $M([t,\infty))$ as null-space. The family $\{E(t): -\infty < t < \infty\}$ has the property that $E(s) \leq E(t)$ if $s < t$ and that $\lim_{t\to\infty} E(t)x = x$ and $\lim_{t\to-\infty} E(t)x = 0$ for a dense set of vectors. (Received July 6, 1965.)


Let $F$ and $G$ be real-valued monotonic nondecreasing functions on the real line, continuous from the left and such that $F(x + 1) - F(x) = G(x + 1) - G(x) = 1$ and $F(0) = G(0) = 0$. Such $F$ and $G$
define probability measures $P_F$ and $P_G$ on a circle of circumference 1. Accordingly, $P_F$ will be said to be rotationally invariant with respect to $P_G$ if $\int_{[0,1]} [F(x + t) - F(y + t)]dG(t) = F(x) - F(y)$ for all $x$ and $y$ in $[0,1)$. \textbf{Theorem.} A necessary and sufficient condition that $P_F$ be rotationally invariant with respect to $P_G$ is that $P_F$ be uniform on the circle ($F(x) = x$), or $P_G(\{0\}) = 1$, or $F(x) - x$ be periodic with period $1/q$ for some integer $q > 1$ and $P_G(\{k/q | k = 0, 1, \ldots, q-1\}) = 1$.

The sufficiency of the condition is readily demonstrated. The necessity follows from an analysis of the defining relation when $x$ and $y$ are chosen so as to characterize certain sets having distinctive $P_F$ measures in the event that $P_F$ is not uniform. (Received July 6, 1965.)


\textbf{Theorem.} Convergent power series and (the convergent portions of) asymptotic power series can be transformed, by use of the geometric series identity (*) $D/X^r - 2 (DX^2 + C) = \sum_{0}^{\infty} (-1)^n C^n / D^n X^{r + 2 n} \quad (C/DX^2 < 1; \ r \ fixed)$, to new series which converge within a few terms to highly accurate approximations. \textbf{Application.} Use of certain (*) identity equations will transform Stirling's formula to the highly accurate gamma function approximation $\Gamma(y + 1) = y! \sim X^{y-1/2} \sqrt{2\pi} \exp[X - D_1 X/C_1 (D_1 X^2 + C_1) - D_2/C_2 X^3 (D_2 X^2 + C_2)]$, where: $X = y + 1$; $+ 1 \leq y < + \infty$; $C_1 = 2/B_1$ ($B_r$ are Bernoulli numbers); $D_1 = 12/B_2$; $C_2 = 1/[B_3/30 - 2(B_2)^2/144B_1]$; and $D_2 = 1/[B_4/56 - 4(B_3)^3/(12)^3(B_2)^2]$. This new formula gives result data which is accurate, at $y = + 1$, to six places. Proof exists that result data accuracy improves continuously as the variable increases in magnitude. In conclusion, this transformation process works equally well with other asymptotic, as well as fully convergent, power series. (Received July 6, 1965.)


A mapping $f$ of a topological space $X$ into a metric space $M$ is said to be $\delta$-continuous provided each point of $X$ admits a neighborhood whose image is of diameter less than $\delta$. A metric space $M$ is said to have the proximate fixed-point property (p.f.p.p.) provided for each $\epsilon > 0$ there exists a $\delta > 0$ such that every $\delta$-continuous mapping of $M$ into itself has an $\epsilon$-invariant point. For compact metric spaces, the p.f.p.p. implies the f.p.p. but not conversely. A continuum is a compact connected metric space. An $\epsilon$-chain is a finite ordered collection $d_1, d_2, \ldots, d_n$ of open sets, each of diameter less than $\epsilon$, such that $d_i$ intersects $d_j$ if and only if $|i - j| \leq 1$. A continuum is said to be chainable if for each $\epsilon > 0$, $M$ can be covered by an $\epsilon$-chain. \textbf{Theorem 1.} If $\{M_i\}_{i=1}^{\infty}$ is a countable collection of metric spaces, then its product has the p.f.p.p. if and only if for each positive integer $n$, the product $X_{i=1}^{n} M_i$ has the p.f.p.p. \textbf{Theorem 2.} If $M_1, M_2, \ldots, M_n$ are chainable continua, then $X_{i=1}^{n} M_i$ has the p.f.p.p. \textbf{Corollary 3.} A countable product of chainable continua has the p.f.p.p. The notions above can be carried over to uniform spaces and Theorem 1 can be generalized to an arbitrary infinite collection of compact Hausdorff spaces. Corollary 3 is then a sharpening of a result of Dyer (A fixed point theorem, Proc. Amer. Math. Soc. 7 (1956), 662-672.) (Received July 6, 1965.)
Let \( R \) be a relation on a set \( X \). A set \( C \subseteq X \) is called an \( R \)-chain (complete subgraph in the graph-theoretic case) if \( (C \times C) \subseteq R \cup R^{-1} \cup \Delta \). Now let \( R^* \) be a reflexive-symmetric relation on a set \( Y \), let \( z \) be a fixed element of \( Y \) and let \( R \) be the restriction of \( R^* \) to \( X = Y \setminus z \). Denote by \( R^*z \) the set \( \{ x \mid x \in X \text{ and } (x,z) \in R^* \} \). Furthermore, let \( \mathcal{L}^* \) and \( \mathcal{L} \) denote the families of maximal \( R^* \)-chains and maximal \( R \)-chains respectively. **Theorem 1.** If \( M^* \in \mathcal{L}^* \), then \( M^* \in \mathcal{L} \) or \( M^* = z \cup (R^*z \cap M) \) for some \( M \in \mathcal{L} \). **Theorem 2.** For each \( M \in \mathcal{L} \), (i) if \( R^*z \supseteq M \), then \( M \cup z \in \mathcal{L} \); (ii) if \( R^*z \nsubseteq M \), then \( M \in \mathcal{L}^* \) and \( z \cup (R^*z \cap M) \in \mathcal{L}^* \) iff \( R^*z \cap M \) is a maximal (relative to set inclusion) element of \( \mathcal{F} = \{ S \mid S = R^*z \cap M', M' \in \mathcal{L} \} \). On the basis of these two propositions we are able to give a new inductive procedure (which offers the advantage of being easy to program for machine computation) for the identification of the maximal \( R \)-chains when \( X \) is finite. (Received July 6, 1965.)

**Entire functions of bounded index.**

Let \( \{a_n(w)\} \) be the sequence of coefficients in the Taylor series expansion of the entire function \( f(z) \neq 0 \) about the point \( w \). Let \( C(w) \) be the maximum of their moduli, and let \( I(w) \) be the largest value of \( n \) for which \( |a_n(w)| = C(w) \). Then \( C(w) \) is continuous for all finite \( w \) and \( I(w) \) is bounded on bounded sets. If \( I(w) \) remains bounded over the entire complex plane, then \( f(z) \) is said to be of bounded index. Some examples of entire functions of bounded index are polynomials, and the sine, cosine, and exponential functions. However, there exist functions of all orders, including zero and infinity, which are not of bounded index. Examples of such are constructed with the aid of the Weierstrass factorization theorem, as it can be seen that any entire function having zeros of arbitrarily large order is not of bounded index. The concept of entire functions of bounded index is due to the Rev. John J. MacDonnell, S. J., and the author, and was first investigated and used by the former in his dissertation entitled "Some convergence theorems for Dirichlet-type series whose coefficients are entire functions of bounded index" [Catholic University of America Press, Washington, D. C., 1957]. (Received July 6, 1965.)

**A strange dilation theorem.** Preliminary report.

Let \( T \) be a bounded linear operator on the complex Hilbert space \( H \). The numerical range of \( T \) is the set \( W(T) = \{ (Tx, x) : x \in H, \|x\| = 1 \} \). The numerical radius of \( T \) is the quantity \( w(T) = \sup \{ |z| : z \in W(T) \} \). **Theorem.** \( w(T) \leq 1 \) if and only if there is a Hilbert space \( K \supset H \) and a unitary operator \( U \) on \( K \) such that \( (1/2)(T^n)P = PU^nP n = 1,2, \ldots \) where \( P \) is the projection on \( K \) with range \( H \). If such a unitary operator does exist, then a minimal one exists, and all such minimal operators \( U \) are unitarily equivalent, **Corollary.** For any \( T \), \( w(T^n) \leq w(T)^n n = 1,2, \ldots \). Equivalently, if \( w(T) \leq 1 \), then \( w(T^n) \leq 1, n = 1,2, \ldots \). (Received July 6, 1965.)
Let $S$ be a semigroup with $0$. Let $a \in S \setminus 0$. Denote by $V(a)$ the set of all inverses of $a$ in $S$. A semigroup $S$ with $0$ is said to be homogeneous if the cardinal number of the set $V(a)$ of all inverses of $a$ is $n$ for every nonzero element $a$ in $S$, where $n$ is a fixed positive integer. Let $P = (p_{ij})$ be any $m \times m$ matrix over a group with zero $G^0$, and consider any $m$ distinct point $A_1, A_2, \ldots, A_m$ in the plane, which we call vertices. For every nonzero entry $p_{ij} \neq 0$ of the matrix $P$, we connect the vertex $A_i$ to the vertex $A_j$ by means of a path $A_iA_j$, which we shall call an edge directed from $A_i$ to $A_j$. In this way, with every $m \times m$ matrix $P$ can be associated a finite directed graph $G(P)$ of the matrix $P$. Let $S = M^0(G;m,m;P)$ be a Rees matrix semigroup. Then the graph $G(P)$ of the sandwich matrix $P$ is called the graph of $S$. **Theorem.** A Rees matrix semigroup $S = M^0(G;m,m;P)$ is homogeneous regular if the directed graph $G(P)$ of the semigroup $S$ is regular of degree $n$. (Received July 6, 1965.)

625-154. M. H. PEARL, University of Maryland, College Park, Maryland. **A theorem on generalized inverses of matrices.**

For each rectangular matrix $A$ with entries from the complex field there exists a unique Moore-Penrose generalized inverse (Penrose, R., Proc. Cambridge Philo. Soc. 51 (1955), 406-413) denoted by $A^+$. The following conditions are shown to be equivalent: (1) $A$ commutes with its generalized inverse, (2) $A$ is an $EPr$ matrix (See Schwerdtfeger, Introduction to linear algebra), (3) $A^+$ can be expressed as a polynomial in $A$. (Received July 7, 1965.)

625-155. W. V. PETRYSHYN, The University of Chicago, Chicago, Illinois 60637. **Remark on fixed point theorems and their extensions.**

Let $X$ be a complex reflexive Banach space and let $X^*$ be its conjugate space. For $u$ in $X^*$ and $x$ in $X$ we denote the value of $u$ at $x$ by $(u,x)$. Let $T$ be a nonlinear mapping from $X$ to $X^*$. We say that $T$ is demicontinuous (resp. strongly continuous) if $x_n \rightharpoonup x$ strongly in $X$ implies $Tx_n \rightharpoonup Tx$ weakly (resp. strongly) in $X^*$; $T$ is complex monotone if $|(Tx - Ty, x - y)| \leq \beta \|x - y\|^2$ for all $x, y$ in $X$ and some constant $\beta > 0$; $T$ satisfies the $k$-condition if $T$ has a demi-continuous inverse $T^{-1}$ mapping all of $X^*$ into $X$ and $|(Tx - T(0), x)| \leq k \|x\|^2$ for some constant $k > 0$ and all $x$ in $X$. We generalize in this paper both of Schauder's fixed point principles to yield the existence of solutions of nonlinear equations of the form $T(x) = S(x)$. **Theorem 1.** If $T$ is a complex monotone demicontinuous map of $X$ to $X^*$ and if $S$ is a completely continuous map such that $(1/\beta) \{S(x) - S(0)\}$ maps the ball $B_r = \{x \in X; \|x\| \leq r\}$ into the ball $B_{r^*} = \{u \in X^*; \|u\| \leq r^*\}$ with $r^* \leq r$, then there is at least one $x$ in $B_r$ such that $T(x) = S(x)$. **Theorem 2.** If $T$ satisfies the $k$-condition and $S$ is a strongly continuous operator from the ball $B_r \subset X$ to $X^*$ such that $(1/k) \{S(x) - T(0)\}$ maps $B_r$ into $B_{r^*}$ with $r^* \leq r$, then the equation $T(x) = S(x)$ has at least one solution $x$ in $B_r$. (Received July 7, 1965.)
625-156. C. F. KENT, Case Institute of Technology, Cleveland, Ohio 44106. A normal form meta-theorem for proof in number theory.

A normal form meta-theorem for proofs in number theory is proved for a system very similar to that of Schütte, Math. Ann. 122, 166-186. The meta-theorem is too involved to state in this abstract but its effect is to restrict the application of existential quantifiers which later vanish from proofs by cut. The proof uses reduction techniques similar to those of Schütte's papers and Beweistheorie, Springer 1960, and is by transfinite induction over the ordinals less than $\varepsilon_0$. For example, the normal form proof for a closed, quantifier-free, theorem is quantifier free. Immediate consequences are consistency, external and 1*-consistency of the system used, and related results. (Received July 7, 1965.)


Let $m \geq 2$. An m-ring is an associative (and necessarily commutative) ring with identity satisfying $x^m = x$ identically. Using previous results (Abstract 614-94, this Notices 11 (1964), 556) it is shown that every m-ring is isomorphic to the ring of global sections of a sheaf of finite fields over a Boolean space. This leads to a direct sum decomposition such that each component is a sheaf of subfields of some $GF(p^r)$. If $\mathcal{R}$ is a sheaf of subfields of $GF(p^r)$ over the Boolean space $X$, define for each $x \in X$, $\delta_{\mathcal{R}}(x) = r$ iff $\mathcal{R}_x = GF(p^r)$. Main Theorem: If $X$ is completely normal, and every open subset of $X$ is a disjoint union of clopen sets, then $\delta_{\mathcal{R}}$ is a complete invariant for $\mathcal{R}$. (Received July 8, 1965.)


Following a procedure indicated by M. Rose (Math. Comp., 14 (1960), 249-256) we seek to characterize the solution to the Stefan problem for the melting of a semi-infinite medium $x > 0$, with constant boundary and initial conditions, as a limit of smooth solutions of the equations $e_t = (k(e)e)_x$, where $e$ denotes the specific internal energy of the medium, and $k$ tends to zero in an interval of $e$ corresponding to the phase transition values of the medium. This indicates that numerical solutions to the Stefan problem, particularly in two and three dimensions, can be obtained without special consideration of the location of the interface. (Received July 8, 1965.)

625-159. S. G. MROWKA, Pennsylvania State University, 227 McAllister Hall, University Park, Pennsylvania 16801. Some set-theoretic problems related to closed embedding.

Let $E$ be a Hausdorff space and let $X$ be an $E$-compact space (R. Engelking and S. Mrowka, Bull. Acad. Polon. Sci., VI, No. 7 (1958), 429-436). The $E$-defect of $X$ is the smallest cardinal $m$ such that for every cardinal $n$ and for every embedding $X'$ of $X$ into $E^n$ there exists a closed embedding $X''$ of $X$ into $E^n \times E^m$ such that the projection of $X''$ on $E^n$ coincides with $X'$. It is shown that the statement $S(m)$: "$N$-defect of $X_m \leq m$" (where $X_m$ is the discrete space of cardinality $m$ and $N = X_{\aleph_0}$) holds for at least all constructible cardinals (in Gödel sense) as well as for some cardinals beyond the first inaccessible. This is a generalization of related results of Erdős and
Hajnal (concerning cardinals $\leq \aleph_\omega$) and of Mycielski (concerning cardinals that are weakly accessible from $\aleph_0$). It is not known if $S(m)$ implies $S(n)$ for $n < m$. $S(m)$ has various set-theoretical, topological and algebraic interpretations; for instance, (a) $S(m)$ iff $X_m$ is $Q$-closed in some compactification $cX$ with weight $cX \leq m$, (b) for $m$ with $\aleph_0 = m$, $S(m)$ iff $C(X_m)$ has a uniformly closed, inverse closed subalgebra $A$ such that card $A = m$, $A$ determines the topology, and every real homomorphism of $A$ is an evaluation. (Received July 8, 1965.)

625-160. EVELYN FRANK, P. O. Box 361, Evanston, Illinois. General numerical continued fractions.

Conditions for the convergence of general numerical continued fractions are given. Generalized regular continued fractions are defined, as well as a class of such continued fractions that converge to the generating function. (Received July 8, 1965.)

625-161. ANNETTE SINCLAIR, Purdue University, Lafayette, Indiana. Generalization of the Runge-Behnke-Stein approximation theorem on open Riemann surfaces.

On an open Riemann surface $S$, let $S$ be a set with compact components $S_j$ having no sequential limit point (limit point of points from different components of $S$) on $S$. Suppose each $S_j$ is the closure of a smooth region. Let $B^*$ consist of one point from the interior of each compact component of $R - S^0$. Then, corresponding to any function $F$ analytic on $S$ and to any sequence $\{f_j\}_j$, there exists $f$ meromorphic on $R$, analytic on $R - B^*$, such that $|F(Q) - f(Q)| < \epsilon_j$ when $Q \in S_j$. This is proved by applying the Behnke-Stein generalization of the Runge approximation theorem to open Riemann surfaces and an abstract approximation theorem [Pacific J. Math., 8 (1958), Theorem 1] previously proved by the writer. (Received July 8, 1965.)

625-162. J. H. M. WHITFIELD, Case Institute of Technology, 10900 Euclid Avenue, Cleveland, Ohio 44106. Differentiable functionals with bounded nonempty support on Banach spaces. Preliminary report.

Let $X$ be a Banach space with norm $\rho$ with the following property: $\exists K > 0$ such that for each $x \in X$, $\rho x = 1$, and every neighborhood $N$ of $x$, $x_1 x_2 \in N$, $\rho x_1 = \rho x_2 = 1$ and also $x_1 x_2 = X, |x_1^*| = |x_2^*| = 1, x_1^* x_1 = x_2^* x_2 = 1$ and $|x_1^* - x_2^*| \leq 3K$. Theorem 1. Let $X$ have the above property and let $f$: $X \to \mathbb{R}, f_0 = 0$ be continuously Fréchet differentiable on $U_\xi = \{x \in X: \rho x < 1 + \xi\}$, $\xi > 0$. Then $\exists x \in X$, $1 < \rho x < 1 + \xi$ such that $f x \leq \rho x$. The techniques of proof are similar to those used by J. Kurzweil (Studia Math. 14 (1954), 213-231). In the proof use is made of the Lemma. Let $X$ have the above property, then for each $x \in X$, $\rho x = 1$ and $\eta > 0 \exists u \in X, \rho u \leq 2$ such that, for $t$ real, $\rho(x + tu) > 1 + |t||K| - \eta$. The following result is obtained as a Corollary. If $X$ is any Banach space which admits an equivalent norm $\widetilde{\rho}$ such that the differential $\widetilde{\rho}'$ (which certainly exists as a sort of one-sided Gateaux differential at every point $x$) is uniformly discontinuous on the unit sphere, then there are no differentiable functionals with bounded nonempty support on $X$. (Received July 8, 1965.)
The Paley-Wiener Theorem asserts that the set of entire functions \( f(z) \) of exponential type \( \tau \) which lie in \( L^2(\mathbb{R}) \) when \( z \) is real is exactly the set of Fourier (Plancherel) transforms of functions in \( L^2(-\infty, \infty) \) which vanish outside \( (-\tau, \tau) \). A similar result, with some modifications, holds for \( L^p(-\infty, \infty) \), \( 1 < p \leq 2 \). One can extend this result in two different ways: one is to use more general transforms, e.g. the Hankel transform, the other is to use entire functions of other types. The extension to entire functions with polygonal indicator diagrams is straightforward but some new features appear in the following generalization of a result of Liht: Theorem. Let \( f(z) \) be an entire function of exponential type having a circle as its indicator diagram, let \( 1 < p \leq 2 \) and let \( 1/p + 1/q = 1 \). If \( f(z) \) is expressible in the form \( f(z) = \int e^{\lambda z} \phi(\lambda) d\lambda \), where \( \Gamma \) is the conjugate diagram and \( \phi(\lambda) \in L^p(\Gamma) \), then \( N_q(f) = \int_0^\infty e^{\lambda z} \phi(\lambda) d\lambda \), where \( \phi(\lambda) \in L^q(\Gamma) \). Conversely, if \( N_q(f) < \infty \) then \( f(z) = \int e^{\lambda z} \phi(\lambda) d\lambda \), where \( \phi(\lambda) \in L^q(\Gamma) \).

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A subconvex functional is a real-valued function whose domain is a convex open subset of a Banach space \( X \), satisfying the condition \( f((1 - t)x + ty) \leq (1 - t)f(x) + tf(y) \), for all \( x, y \) in the domain, and \( t \in [0, 1] \), the unit interval of real numbers. For any \( x \) in the domain and any vector \( u \) in \( X \), a one-sided directional derivative is defined by: \( f'(x)(u) = \lim_{t \to 0^+} f(x + tu) - f(x)/t \). When \( f \) is subconvex, \( f' \) exists at every point of its domain. For any \( x \), the set of vectors \( u \) such that \( f'(x)(u) = - f'(x)(-u) \) is a subspace, and the restriction of \( f'(x) \) to that subspace is linear. Finally, if \( f'(x) \) is a Fréchet differential at any point \( x \), then \( f' \) is continuous at \( x \), and so \( f'(x) \) is a strong differential at \( x \), in a sense that has been defined by the author, (Proc. Amer. Math. Soc. 12 (1961), 694-697). (Received July 8, 1965.)

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Let \( H(C) \) be the homeomorphism group of the Cantor set \( C \). Let \( M \) be a compact metric space and \( p \) a map of \( C \) onto \( M \). Let \( G(p, M) = \{ h \in H(C) \mid \forall x \in C, p(x) = ph(x) \} \). \( G(p, M) \) is a subgroup of \( H(C) \). The map \( p \) is a standard map if, for each pair of points \( x \) and \( y \) such that \( p(x) = p(y) \), there is a sequence \( \{ h_n \}_{n=1}^{\infty} \subset C \) and a sequence \( \{ x_n \}_{n=1}^{\infty} \subset C \) such that \( x_n \to x \) and \( h_n(x_n) \to y \). Standard maps are very naturally obtained. Theorem. Let \( p \) be a standard map of \( C \) onto \( M \). The compact metric space \( N \) is homeomorphic to \( M \) if and only if \( G(p, M) = G(q, N) \) for some standard map \( q \) of \( C \) onto \( N \). (Received July 8, 1965.)

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It remains unknown whether there is an injective in the category of boolean \( \sigma \)-algebras (other than \( \{0\} \)). This brief paper lays forth virtually all known properties of such injectives. The
The most startling result is that the mere existence of a non-trivial injective guarantees that every epimorphism of \( \sigma \)-algebras must be set-theoretically onto. Epimorphism of course means map \( f \) with the right cancellation rule \( xf = yf \Rightarrow x = y \); needless to say, it is an open question whether epimorphisms of \( \sigma \)-algebras need be onto. The proof of the result mentioned uses a general position argument and the fact that each non-trivial \( \sigma \)-algebra injective must be a cogenerator. (Received July 8, 1965.)

625-167. GEORGE GLAUBERMAN, University of Chicago, Chicago, Illinois and G. S. STOLLER, c/o Morton, 5 Bryant Street, Cambridge, Massachusetts 02138. How not to prove Waring's conjecture.

It was hoped that statement (S) were true, whence Waring's Conjecture (and a generalization of it to polynomials) would follow from Hilbert's Lemma 1 [F. Hausdorff, Math. Ann. 67 (1909), 301].

**Definition.** Let \( A = \{a_0, a_1, a_2, \ldots \} \) and \( B = \{b_0, b_1, b_2, \ldots \} \) be two sets of nonnegative integers where \( 0 \leq a_0 < a_1 < a_2 < \ldots \) and \( 0 \leq b_0 < b_1 < b_2 < \ldots \). \( B \) is more dense than \( A \) iff \( a_{n-1} < b_n \leq a_n \) for all \( n > 0 \). **Definition.** \( A \) spans the set of nonnegative integers iff there exists a positive integer \( k \) such that every nonnegative integer can be expressed as a sum of fewer than \( k \) elements from the set \( A \). **(S)** If \( A \) spans the set of nonnegative integers and \( B \) is more dense than \( A \), then \( B \) spans the set of nonnegative integers. This statement is false. **Theorem.** If \( B \) is a set of nonnegative integers and \( \overline{B} \) (topological closure in \( \mathbb{Z}_p \), the \( p \)-adic integers) is countable, then \( B \) does not span the nonnegative integers. **Theorem.** Let \( A = \{a_0, a_1, a_2, \ldots \} \) be a set of integers such that \( 0 = a_0 < a_1 < a_2 < \ldots \) and \( \lim_{n \to \infty} (a_{n+1} - a_n) = \infty \) in \( \mathbb{R} \). Pick any prime \( p \) and any \( c \in \mathbb{Z}_p \). There exists a set \( B = \{b_0, b_1, b_2, \ldots \} \) such that \( b_0 = 0 \) and \( a_{n-1} < b_n \leq a_n \) for all \( n > 0 \) and \( \lim_{n \to \infty} b_n = c \) in \( \mathbb{Z}_p \). (Received July 8, 1965.)


Let \( X_n \) consist of all \( n \)-tuples \( x = (x_1, x_2, \ldots, x_n) \) with the \( x_i \)'s integers between 1 and \( n \). Certain random variables \( F_n \) defined on \( X_n \) are given. Let the random variable \( G_n = F_n \) restricted to the permutations of \( X_n \). Sufficient conditions on the functions \( F_n \) are given to insure convergence of \( F_n \) and \( G_n \) to the same limit distribution \( D(x) \). The characteristic function of \( D \) is given explicitly. (Received July 8, 1965.)

625-169. WITHDRAWN.
Let \( p \) be a mapping of a topological space \( T \) onto a topological space \( B \). It will be said that \( p \) is \( \alpha \)-light if \( p^{-1}(b) \) contains no arcs for every \( b \) in \( B \). Some theorems typical of those in this paper are as follows: Theorem 1. The following conditions are equivalent: A. If \( \sigma \) is a loop in \( B \) which is homotopic to a constant and \( y \in p^{-1}\sigma(0) \) then there exists a unique loop \( \tau \) which covers \( \sigma \) such that \( \tau(0) = y \). B. \( p \) is \( \alpha \)-light and is a fibering in the sense of Serre. Theorem 2. If \( p \) is an \( \alpha \)-light fibering and \( p \) has the covering homotopy property for maps of fibers and \( B \) is an arcwise connected, locally compact, uniformly locally arcwise connected, semi-locally simply connected metric space and \( p^{-1}(b) \) is compact then \( p \) is a locally trivial fibration. (Received July 9, 1965.)

The matrix integral equation considered is (I) \( W(t) + \int_a^t [W(s)A(s) + D(s)W(s) + W(s)B(s)W(s) - C(s)] ds = M_0(t), \quad a \leq t \leq b, \) where \( A(t), B(t), C(t), D(t), M_0(t) \) are \( n \times n \) matrices with \( A(t), B(t), C(t), D(t) \) (Lebesgue) integrable, and \( M_0(t) \) of bounded variation on the compact real interval \([a,b] \). Extending earlier results of the author, (Generalized linear differential systems, J. Math. Mech. 8 (1959), 705-726, and Principal solutions of non-oscillatory linear differential systems, J. Math. Anal. Appl. 9 (1964), 397-423), the basic relations between (I) and associated generalized vector differential systems (II) \( u' = A(t)u + B(t)v, \quad dv = [C(t)u - D(t)v]dt + [dM(t)]v \) are derived. In particular, if (II) is self-adjoint, (i.e., \( D(t) = A^*(t), B(t), C(t), M(t) \) hermitian), and \( B(t) \) non-negative definite on \([a,b] \), then (II) is nonoscillatory on \([a,b] \) if and only if there exists an hermitian nonincreasing \( n \times n \) matrix \( \Phi(t), \) (i.e., \( \Phi(t_1) - \Phi(t_2) \) non-negative definite for \( a \leq t_1 \leq t_2 \leq b \)), such that for \( M_0(t) = M(t) + \Phi(t) \) the integral equation (I) has an hermitian solution on \([a,b] \). (Received July 9, 1965.)

Let \( f \) and \( g \) be bounded real-valued functions on a nondegenerate finite closed interval \([a,b] \) of the real axis. It is shown that there are disjoint countable subsets \( S_1, S_2 \) of \((a,b) \) which are both everywhere dense on \((a,b) \) with the property that if \( k \) and \( \ell \) are bounded real-valued functions on \( S = S_1 + S_2 \) for which there is a positive real number \( M \) such that every non-degenerate open sub interval \((c,d) \) of \([a,b] \) has at least one point \( \xi \) of \( S_1 \) and at least one point \( \xi' \) of \( S_2 \) satisfying \( |k(\xi)\ell(\xi') - k(\xi')\ell(\xi)| \geq M, \) \( f^*(x) = f(x) + k(x) \) or \( f(x) \) for every \( x \) in \([a,b] \) according as \( x \) is in \( S \) or not, and \( g^*(x) = g(x) + \ell(x) \) or \( g(x) \) for every \( x \) in \([a,b] \) according as \( x \) is in \( S \) or not, then the Lane integral \( \int_a^b f^*(x)g^*(x) \) does not exist. It is further shown that if every nondegenerate open subinterval \((c,d) \) of \([a,b] \) has a point \( \xi \) such that at least one of \( f(\xi^+), f(\xi^-) \) exists and a point \( x^* \) such that at least one of \( g(x^+), g(x^-) \) exists, then every two disjoint countable subsets \( S_1, S_2 \) of \((a,b) \) which are both everywhere dense on \((a,b) \) have the above property. This last result shows that Example 1,1 given by R. E. Lane (Proc. Amer. Math. Soc. 6 (1955), 393) is incorrect. (Received July 9, 1965.)

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A compact set $S \subseteq \mathbb{R}^n$ is called $m$-p polar if the only element in $H^m_{mp}(\mathbb{R}^n)'$ with support in $S$ is the zero element. Let $L$ be a linear partial differential operator of order $m$ with smooth coefficients defined in a bounded domain $D$ having $S$ in its interior. We say $S$ is removable with respect to $(L_p, L, D)$ if every generalized solution of $Lu = 0$ in $D - S$ can be extended to be a generalized solution in $D$. Various extensions of the following fact are derived: If $S$ is $m$-p polar then it is removable with respect to $(L_p, L, D)$. If $L$ is elliptic and has the unique continuation property, the converse is true. (Received July 9, 1965.)

Most of the theory of Jacobi (tri-diagonal) matrices is built around a three term recurrence relation satisfied by the determinants of successive principal submatrices. We introduce and study a class of square matrices of order $n$ satisfying a recurrence relation enjoying many of the same properties as that for the Jacobi matrices. The resulting class of matrices includes the Jacobi matrices and finds interesting applications in the recent theory of qualitative economics and in the use of finite difference methods for ordinary and partial differential equations. The most important type of quasi-Jacobi matrices are those with the nonzero elements symmetrically placed. Such a matrix is quasi-Jacobi if and only if for each $r = 2, \ldots, n$, every principal submatrix of order $r$ has at most $r - 1$ nonzero elements above the main diagonal. If a quasi-Jacobi matrix $A$ is (i) non-singular, (ii) has symmetrically placed non-zero elements, (iii) has non-positive elements on the main diagonal, and (iv) if the product of each pair of symmetrically placed non-zero elements is negative, then every eigenvalue of $A$ has negative real part. This is a qualitative result, independent of the magnitudes of the elements of $A$. (Received July 9, 1965.)

The problem considered is that of hydrodynamic stability for two dimensional parallel flow of an inviscid incompressible fluid bounded by rigid walls. The problem is formulated as a Cauchy problem $u_t = Bu$ for a bounded operator $B$ on the Hilbert space $L^2(0,1)$. The operator $B$ has a point spectrum inside the unit disc; its continuous spectrum is $[-1,1]$. We show that the point spectrum cannot cluster except possibly at certain points associated with turning points of the basic flow. Further, if the basic flow has at most two turning points, then there are no cluster points, so that the number of unstable modes is finite. Assuming no turning points we prove that the continuous spectrum is stable (see Case, Phys. Fluids 3 (1960), 143). (Received July 8, 1965.)
Differential-integral calculus for abstract algebraic-topological structures. III.

The development previously outlined (Abstract 620-17, these Notices 12 (1965), 214) is continued. Let \( Q \) be the set of all closed squares \( S = \{(x,y) : a \leq x \leq b, c \leq y \leq d\} \) in \( \mathbb{R} \times \mathbb{R} \), and let \( P, f \) be mappings of \( \mathbb{R} \times \mathbb{R} \) into \( \mathbb{R} \). **Definition.** \( F \) has derivative \( f \) on \( Q \) iff given \( B \) in \( Q \) such that \( B \subset S \), and any two distinct points \((x_1,x_2), (y_1,y_2) \) in \( B \); there is a third point \((z_1,z_2) \) in \( B \) such that \( F(x_1,x_2) - F(y_1,y_2) = f(z_1,z_2) \prod_{i=1}^{2} \ast (x_i - y_i) \), in which \( \ast \) indicates the product is taken only over the non-zero factors. The partial derivatives \( F_1, F_2 \) are defined by holding one variable fixed and using the one-dimensional analog of the definition above. **Theorem 1.** \( F \) has derivative \( f \) on \( Q \) iff \( F \) has partial derivatives \( F_1, F_2 \), and if given \((x_1,x_2), (y_1,y_2) \) in \( S \) then there are \((q,r), (s,t) \) in \( S \) such that \( F(x_1,x_2) - F(y_1,y_2) = F_1(q,r)(x_1 - y_1) + F_2(s,t)(x_2 - y_2) \). The chain rule for derivatives is proved in two forms. One form corresponds to the equation given in the definition, the other corresponds to the equation involving partial derivatives which appears in Theorem 1. A theorem on implicit functions is proved, and the formula for the derivative as a ratio of the partial derivatives is derived. (Received July 9, 1965.)

**625-177. JEROME DANCIS, University of Wisconsin, Madison 6, Wisconsin. Some nice embeddings in the trivial range.**

Let \( P \) be a \( k \)-dimensional polyhedron and let \( M \) be an \( n \)-manifold, \( n \geq 2k + 2 \). A map \( f : P \rightarrow M \) is called countably p.w.l. if \( P \) may be written as the countable union of subsets \( X_i, i = 1,2,\ldots, \) such that for each \( i : f|X_i \) can be extended to a p.w.l. map on a subpolyhedron \( P_i \subset P \). **Theorem.** Suppose that \( f : P \rightarrow M \) where \( f \) is an embedding which is countably p.w.l. \( \ast \). Then \( f \) is a stably tame embedding of \( P \) into \( M \). Also if \( \epsilon > 0 \) is given, then there exists an \( \epsilon \)-push of \((M,f(P)) \) which sends \( f(P) \) onto a subpolyhedron of \( M \). **Corollary.** Let \( P \) be embedded in \( M \) such that the embedding is locally tame except at a countable number of points. Then this is a tame embedding. (Received July 9, 1965.)

**625-178. SHMUEL KANIEL, Stanford University, Stanford, California. Approximation of symmetric operations by operators of finite rank.**

The following is established: **Theorem.** Let \( A \) be a bounded symmetric operator in Hilbert space \( H \). Let \( f \in H \) be chosen and let \( H_1 \) denote the span of \( f, Af,\ldots,A^k f \). Let \( P \) be the orthogonal projection of \( H_1 \) and let \( B \) denote the restriction of \( PA \) to \( H_1 \). Then there exists an absolute constant \( c(k) \) tending to zero as \( k \) tends to infinity which does not depend on \( A \) nor on \( f \) so that for some \( \lambda \in \sigma(A) \) \(|A - \mu| \leq c(k)\|A\| \leq 2\sqrt{1/(k + 1)}\|A\| \). The proof involves some lemmas based on the elementary properties of symmetric operators and approximation by polynomials. (Received July 9, 1965.)

**625-179. R. N. BRYAN, University of Utah, Salt Lake City, Utah 84112. A perturbation theorem for a linear differential system with general linear boundary conditions. Preliminary report.**

Suppose \( U \) is a bounded linear transformation from \( C \), the space of \( n \times n \) continuous matrices on \([a,b] \) with uniform norm, to the space of matrix constants, such that for some nonsingular continuously differentiable \( Y \in C \), \( U(Y) \) is nonsingular. If, for \( A \in C \), the system \( Y' = AY; U(Y) = 0 \)
has a nontrivial solution and \( \epsilon > 0 \), then there is a \( B \in \mathbb{C} \) such that \( \| A - B \| < \epsilon \) and \( Y' = BY; U(Y) = 0 \) has only the trivial solution. (Received July 9, 1965.)

625-180, GASTON SMITH, University of Southern Mississippi, Hattiesburg, Mississippi.

A generalization of the \((f, d_n)\) method of summability.

Let \( f \) be a nonconstant function from a subset of the complex plane to the complex plane which is analytic at the origin. Let \( \{d_n\} \) be a sequence of complex numbers. There is a neighborhood \( N \) of the origin such that for each \( n \), an expansion of the form 
\[
\prod_{j=1}^{n} [f(z) + d_j] = \sum_{k=0}^{\infty} b_{nk} z^k
\]
is valid for each \( z \in N \). A sequence \( \{t_k\} \), or a series whose \( k \)-th partial sum is \( t_k \), is \((f, d_n, z)\)-summable to \( t \) if and only if 
\[
\lim_{n \to \infty} \sum_{k=0}^{\infty} a_{nk}^{(z)} = t,
\]
where \( a_{00} = 1 \), \( a_{0k} = 0 \) if \( k \neq 0 \), \( a_{nk} = \prod_{j=1}^{n} [f(z) + d_j]^{-1} b_{nk} z^k \) for each \( n > 0 \), and \( d_n \neq f(z) \) for each \( n > 0 \).

Theorem 1. If \( d_n \neq f(0) \) a necessary condition in order that the \((f, d_n, z)\) method be regular is that 
\[
\sum_{n=1}^{\infty} [f(0) + d_n]^{-1} = \infty.
\]

Theorem 2. If \( z > 0 \), \( d_n \geq 0 \), and the Taylor expansion of \( f \) about the origin has non-negative coefficients then the \((f, d_n, z)\) method is regular if and only if 
\[
\sum_{n=1}^{\infty} [f(z) + d_n]^{-1} = \infty.
\]
(Received July 9, 1965.)
ABSTRACTS PRESENTED BY TITLE


We follow the notation of Abstract 65T-291 on p. 482. B \leq A means that B is an elementary submodel of A. Theorem 1. If \omega \leq |g(A)| < |f(A)|, then there exist B, C such that B \leq A, |g(B)| = \omega, B \leq C, g(B) = g(C), and |f(C)| = \omega_1. This improves a theorem of Vaught (cf. Morley and Vaught, Math. Scand. 11, 37-57). Now let G be a set \{g_0, g_1, \ldots\} of formulas and let G(A) = \cap\{g_n(A): n < \omega\}. Theorem 2. Theorem 1 holds with G in place of g. A model A is homogeneous if whenever B \leq A and |B| < |A|, every isomorphism on B into A can be extended to an automorphism of A (cf. Morley and Vaught op. cit.). Assume hereafter that T is a theory in L such that every model of T of power \omega_1 is homogeneous. Theorem 3. Every uncountable model of T is homogeneous. Let \alpha(T) be the number of nonisomorphic models of T of power \alpha. Theorem 4. If \omega_1(T) < \omega, then \alpha(T) \leq \omega_1(T) for all uncountable \alpha. In the case \omega_1(T) = 1, this was proved by Morley in another way (Proc. Nat. Acad. Sci 49, 213-216). Theorem 5. If \omega_1(T) = \omega, then \alpha(T) < \alpha for all regular \alpha > \omega. Theorems 3-5 also hold for theories T which have existential second order quantifiers and arbitrary weak second order quantifiers (or quantifiers over natural numbers). Theorems 2-5 follow from Theorem 1. (Received April 28, 1965.)


For a conservative field of force in a Riemannian space V_n, there is a scalar function V = V(x), such that the covariant force vector is \phi_i = V_{,ij}. It is supposed that |\phi| > 0. The Lamé differential parameters of orders one and two of V = V(x), are \Delta_1(V) = g^{ij}V_{,ij} \neq 0, and \Delta_2(V) = \{\epsilon g\}^{\epsilon/2} \partial / \partial \varepsilon \{[\epsilon g]^{\epsilon/2}g^{ij}[\partial V / \partial x^j]\}, where g = |g_{ij}| \neq 0, and \epsilon = \pm 1. Then |\phi| = \{[\Delta_1(V)]^{\epsilon/2} > 0. The equipotential family V(x) = constant, is isothermal if and only if \Delta_2(V)/\Delta_1(V) = F(V), where F depends on V(x), alone. Under these circumstances, the isothermal parameter \mu can be found by two quadratures. If every isothermal family corresponds to an isothermal family under a cartogram T between two Riemannian spaces V_n and \overline{V}_n, then T is necessarily conformal. When n = 2, this condition is sufficient. However, for n \geq 3, the necessary and sufficient condition is that T be homothetic. For n = 2, the expression \Delta_2(V)/\Delta_1(V) is an absolute conformal invariant. However, for n \geq 3, this is an absolute conformal invariant if and only if the equipotential family V = V(x), constant, is orthogonal to the scale family e = e(x) = constant. (Received February 22, 1965.)
Let \( M = G/K \) be a compact simply connected irreducible symmetric space, \( p = \text{rank } G - \text{rank } K \). The space form problem for \( M \) (find all Riemannian manifolds covered by \( M \)) is feasible only when \( p ;; 1 \) \[ \text{(Amer. Math. 82 (1960), 661-688).} \] If \( p = 0 \) then the space form problem for \( M \) was settled \[ \text{(Canad. J. Math. 15 (1963), 193-205; Comment Math. Helv. 37 (1963), 266-295).} \] If \( p = 1 \) and \( M \) is a Grassmann manifold, then the problem was reduced \[ \text{(Canad. Math. 15 (1963), 193-205) to the author's solution} \] \[ \text{(Abstract 619-67, these Notices 12 (1965), 75) to the Clifford-Klein spherical space form problem.} \] The only remaining case with \( p = 1 \) was \( M = \text{SU}(3)/\text{SO}(3) \). For this case it is shown that the space forms are the quotients \( M/\Gamma_{n;k_1 k_2 k_3} \) where \( n > 0 \) is odd, \( k_i \) are prime to \( n \), and \( k_1 + k_2 + k_3 = 0 \) (mod \( n \)); \( \Gamma_{n;k_1 k_2 k_3} \) is the cyclic group of order \( n \) in \( \text{SU}(3) \) generated by the matrix \( \{e^{2\pi i k_1/n}, e^{2\pi i k_2/n}, e^{2\pi i k_3/n}\} \). Groups \( \Gamma_{n;k_1 k_2 k_3} \) and \( \Gamma_{m;l_1 l_2 l_3} \) give isometric quotient manifolds if and only if \( m = n \) and there is a permutation \( q \) of \( \{1, 2, 3\} \) and an integer \( b \) prime to \( n \) such that \( k_{q(i)} = b_{j} (\text{mod } n) \). (Received March 11, 1965.)

La.\( \omega \) is the extension of the first-order predicate logic obtained by allowing conjunctions and disjunctions of \( \mu \)-sequences of formulas \( (\mu < \alpha) \) in the formation rules of the formulas. \( \mathcal{B} \) is the class of all non-empty well-orderings. If \( \mathfrak{A} = (\Lambda, R\lambda, \lambda < \beta) \) and \( \mathfrak{B} = (\Lambda, R\gamma, \gamma < \delta) \) then \( \mathfrak{B} \) is the reduct of \( \mathfrak{A} \). Theorem. There does not exist a cardinal \( \alpha \) such that for some set \( T \) of sentences of \( La.\omega \) \( \mathcal{B} \) is the class of reducts of models of \( T \). The proof of the theorem makes use of an upper bound for the Hanf-number for \( La.\omega \). (Received March 18, 1965.)

Call a c* subalgebra \( A \) of the c* algebra \( M \) \textit{inner} if \( uAu^* = A \) for all unitaries \( u \) in \( M \). This is a stringent condition if \( M \) has an identity. Two classes of such subalgebras are (a) c* subalgebras of the center of \( M \), and (b) uniformly closed two-sided ideals of \( M \). Conjecture: Any inner c* subalgebra of a \( \text{w*} \) algebra is generated by one of type (a) and one of type (b). We have proven this for the case when \( M \) is a factor. (Received March 19, 1965.)

In (C. R. Acad. Sci. Paris 257 (1963), 3790-3792) we have constructed a hierarchy of propositional calculi, \( C_1, C_2, \ldots, C_n, \ldots, C_\omega \) for the inconsistent mathematical systems. These calculi are developed and applied in (Newton C. A. da Costa, C. R. Acad. Sci. Paris 258 (1964), 27-29, 1111-1113, 1366-1368 and 3144-3147) and in (Newton C. A. da Costa and Marcel Guillaume, Sur les calculs \( C_{n'} \) to appear in An. Acad. Brasil. Cl., and \textit{Négations composées et loi de Peirce dans les systèmes} \( C_n \), to appear in Portugal. Math.). The calculi \( C_n, 1 \leq n \leq \omega \), can be put into a lattice form, and the
resulting lattices are absolute implicative lattices in the sense of Curry (Foundations of Mathematical Logic, 1963), in which are defined unary non-monotone operations, corresponding to negations (that is, \(a = b\) does not necessarily imply \(a' = b'\); \(\equiv\) denotes an equivalence relation in Curry's treatment). These lattices are called \(C_n\)-algebras. To each \(C_n\)-algebra, \(1 \leq n < \omega\), it is associated, in a natural way, a Boolean algebra; as a corollary of this fact, a \(C_n\)-algebra, \(1 \leq n < \omega\), is also a classical implicative lattice. There are simple set theoretic and topological representation theorems for the \(C_n\)-algebras. (Received March 26, 1965.)


On strongly continuous functions.

Let X and Y be topological spaces. A function \(f: X \rightarrow Y\) is called strongly continuous iff for each subset \(K\) of \(X\), \(f(K) \subseteq f(K)\) [see Levine, Amer. Math. Monthly 67 (1960) 269]. Let \(S\) denote the set of all strongly continuous functions on \(X\) to \(Y\). **Theorem 1.** If \(Y\) is Hausdorff and \(S\) is given the pointwise convergence topology then \(f \in S\) implies that for each connected set \(K\) of \(X\), \(f(K)\) is a single point. **Theorem 2.** If \(X\) is locally connected and \(Y\) is Hausdorff then \(S\) is closed in \(Y^X\) in the pointwise convergence topology. [Compare this result with the corresponding function space of continuous functions.] Examples are constructed to show that the hypotheses of the above theorems cannot be weakened. **Theorem 3.** If \(X\) is locally connected and \(Y\) compact Hausdorff then \(S\) is compact in the pointwise convergence topology. **Theorem 4.** If \(X\) is locally connected and \(Y\) a uniform space then \(S\) is equicontinuous. **Theorem 5.** If \(X\) is locally connected and \(Y\) a uniform space then the pointwise convergence topology for \(S\) is equivalent to the topology of uniform convergence on compacts. Consequently, \(S\) is evenly continuous. (Received March 29, 1965.)


Triality in plane geometry.

The author presents a plane geometrical example of triality in the sense of E. Cartan (Bull. Sci. Math. 49 (1925), 361-374). Three classes of elements are considered in a plane: points, lines and circles. Some results are derived from the corresponding ("trial") propositions: (1) Two distinct points determine a circle of which they are diametral opposites; (2) Two distinct circles determine a line—their radical axis; (3) Two distinct lines determine a point—their intersection point. (Received April 2, 1965.)

65T-300. D. J. RODABAUGH, Box 1631, Station B, Vanderbilt University, Nashville, Tennessee.

Some general Wedderburn theorems and their applications.

A decomposable class \(P\) of algebras is a class of strictly power-associative algebras over fields of char. \(\neq 2, 3\) closed under homomorphic images and the formation of subalgebras with the conditions that a semisimple member is the direct sum of simple members, each with unity elements, and \((A_e(t))^2 \subseteq A_e(t)\) for \(t \neq 1/2\) and \(e\) an arbitrary idempotent in a member of \(P\). The center \(C(P)\) of \(P\) is the set of \(A\) in \(P\) such that \(A - N\) is simple and \(A\) has a unity element. Every member of \(P\) has a Wedderburn decomposition if and only if every member of \(C(P)\) does. Seven equivalent
conditions are given which imply that a member of $C(P)$ has a Wedderburn decomposition. From now on, assume that $A$ is power-associative over an algebraically closed field of char. $\neq 2,3$ and that $A - N$ contains no simple nodal subalgebras. As a consequence of the above-mentioned results, we prove that the following algebras possess a Wedderburn decomposition: (1) a static algebra; (2) an algebra satisfying $(x,y,z) + a(y,z,x) + a^2(z,x,y) = 0$ for $a^3 = 1, a \neq 1$; (3) an algebra satisfying $(x^2,y,z) = 2x^*(x,y,z)$ and $(z,y,x^2) = 2x^*(z,y,x)$ with the condition that $A - N$ contains no ideals isomorphic to $M_2$.  (Received April 16, 1965.)


For a Turing machine $Z$ define the functions $Z^*$ and $\overline{Z}$ as follows: $Z^*(x_1,...,x_r)$ is the number of operations performed by $Z$ until it halts when its input is $(x_1,...,x_r)$; $\overline{Z}(x_1,...,x_r)$ is the amount of tape used by $Z$. (These definitions depend on the notation used for numbers; the theorems below are, to an extent independent of the notation.) For a singulary function $T$ define $S^*(T)$ as the set of all functions $f$ computed by some Turing machine $Z$ which satisfies $Z^*(x_1,...,x_r) \leq T(x_1 + ... + x_r)$ for all but finitely many r-tuples $(x_1,...,x_r)$ (where $f$ is r-ary). $\overline{S}(T)$ is similarly defined. Let $N_n$ be the nested n-fold recursive functions. Theorem 1. $N_n = \bigcup S^*(W_a) = \bigcup S(W_a)$ where in both unions $a$ ranges over the ordinals $< \omega_1$. The definition of $W_a$ was given in my previous abstract; Theorem 1 is a corollary of the theorem (Abstract 65T-217, these Notices 12 (1965), 460) announced there. Now define $V_0(x) = x$ and $V_{n+1}(x) = 2^V$ where $V = V_n(x)$. Let $E$ be the elementary recursive functions. Theorem 2. $E = \bigcup S^*(V_n) = \bigcup \overline{S}(V_n)$. (Received April 9, 1965.)


If the integer part $[n - 1]/(k - 1)$ is $s$ where $k$ is prime and $n$ is odd, then for an arbitrary continuous mapping $S^{n-1} \rightarrow R^m$ the set of orthogonal k-tuples on $S^{n-1}$ each of which maps into a single point has at least $(k - 1)(2s - m) - 1$ nontrivial homology groups. (Received April 27, 1965.)

65T-303. T. S. MOTZKIN, University of California, Los Angeles, California 90024. A combinatorial result on maximally convex sets.

A point set $S$ that spans $d$-space is $k$-convex ($k$-neighborly), if for each $k$-tuple $s \subset S$ the convex hull conv $s$ of s lies on the boundary of conv $S$; if conv $s = S \cap H$ where $H$ is a support hyperplane of $S$ then $S$ is strictly $k$-convex. A set $S$ is $k$-convex iff every $(d + 2)$-tuple in $S$ is $k$-convex. Only for $k = [d/2]$ is it true that for every $m$ there is an $n$ such that every set $S$ of $n$ or more points contains an $m$-tuple that is $k$-convex and not $(k + 1)$-convex. The proof uses Ramsey's theorem and transposed complementarity (cf. Abstract 65T-285, these Notices 12 (1965), 481). (Received April 27, 1965.)
65T-304. JOSEPH HARRISON, Stanford University, Stanford, California. Further results on $O^*$. For notation and definitions see Feferman and Spector, Incompleteness along paths in progressions of theories, J. of Symb. Logic, 27 (1962), 383. Theorem 1. Let $a \in O^*$, $X \subseteq \{y : y \leq a\}$, then if $X \not= \emptyset$, $X$ has a least element with respect to $<_0$. Let $\omega_1$ be Church-Kleene $\omega_1$ and let $\eta$ be the order type of the rationals in the half open interval $[0,1)$. Let $^+'$ and $^*'$ denote sum and product of order types. Theorem 2. Suppose $a \in O^* - 0$. Then $\exists a < \omega_1$ such that $\{y : y \leq a\}$ is order isomorphic to $\omega_1 \cdot \eta + a$. If $a,b, \in O^*$, say $a$ and $b$ are hyperarithmetically comparable if there exists a hyperarithmetic function which maps $\{y : y \leq a\}$ onto an initial segment of $\{y : y \leq b\}$ isomorphically or vice versa. Theorem 3. Given any $a \in O^* - 0$ there exists $b \in O^*$ such that $a$ and $b$ are not hyperarithmetically comparable. Theorem 4. There exists a dense set of hyper-degrees less than the hyper-degree of 0. (Received May 17, 1965.)

65T-305. ALESSANDRO FIGA-TALAMANCA, Massachusetts Institute of Technology, Cambridge, Massachusetts and GARTH GAUDRY, Australian National University, Canberra, Australia. Multipliers from $L^p$ to $L^q$.

Let $G$ be an LCA Hausdorff group. If $L^p(G)$ and $L^q(G)$ with $p, q \in [1, \infty]$ are the usual Lebesgue spaces of index $p$ and $q$ respectively with respect to Haar measure on $G$, then a multiplier from $L^p(G)$ to $L^q(G)$ is a continuous linear map from $L^p(G)$ into $L^q(G)$ which commutes with the translation operators. If $1 \leq p \leq q \leq \infty$, define $1/r = 1/p + 1/q - 1$, and $A^q_p = \{u \in L^r(G) : u = \sum f_i * g_i \text{ a.e.,} \}$, with $f_i, g_i \in C_c(G)$, and $\sum \|f_i\|_p \|g_i\|_q < \infty$. (Here $r^*$ denotes the index conjugate to $r$.) Norm $A^q_p$ as follows: $\|u\| = \inf \{\sum \|f_i\|_p \|g_i\|_q : u = \sum f_i * g_i \text{ a.e.} \}$. The authors prove the Theorem: If $1 \leq p \leq q < \infty$ then the space of multipliers from $L^p(G)$ to $L^q(G)$ is isometrically isomorphic to the dual of $A^q_p$. This generalises Theorem 1 of an earlier announcement by the first-named author (Bull. Amer. Math. Soc., (5), 70 (1964), 666-669). (Received May 5, 1965.)


We prove some new and interesting metrization theorems, as well as improve some well-known results of Bing, Hanai, Nagata and Smirnov. As examples of our results we state: Theorem 1. A regular space is metrizable if and only if there exists a family $\{f_a\}_{a \in L}$ of continuous functions on $X$ to the closed unit interval satisfying (a) $\{f_a^{-1}(0, 1]\}_{a \in L}$ is a base for $X$, (b) $L = \bigcup_{n=1}^{\infty} L_n$ such that $\lim_{k \to \infty} x_k = x$ and $\{a(k)\}_{k=1}^{\infty} \subseteq L_j$ (some $j$) implies $\lim_{k \to \infty} [f_{a(k)}(x_k) - f_{a(k)}(x)] = 0$. Theorem 2. Let $f : X \to Y$ be a monotone ($f^{-1}(y)$ is connected for $y \in Y$) quotient map from a metrizable locally separable $X$ onto a regular first countable $Y$ then $Y$ is metrizable and locally separable. Theorem 3. $X$ is metrizable if and only if to each $x \in X$ one can assign neighborhood bases $\{S_{nx} \}_{n=1}^{\infty}$ such that (a) $S_{nx} \cap S_{ny} = \emptyset$ implies $x \in U_{ky}$ and $y \in U_{nx}$, (b) $x \in S_{nx}$ implies $S_{nx} \subseteq U_{nx}$. Theorem 4. A regular, pointwise metacompact (open covers have point-countable open refinements), $M$-space $X$ (i.e., there exists a dense metacompact space which is the image of $X$ under a perfect (proper)map) is metrizable if and only if its diagonal is a $G_\delta$-set in $X \times X$. (Received May 5, 1965.)
Recall that Euclid's Algorithm is obtained from the Division Algorithm by finite reflexive induction (Proc, Nat. Acad. Sci, U.S.A. 50 (1963), 604-606) upon the Division Algorithm, and by analogy the author's "mosaic model" of elementary unique factorization follows from the prime-power factorization by use of the method upon resulting exponents. This note applies the same method to the unique purely exponential decomposition of natural numbers into Mycielski numbers (i.e., natural numbers which are not powers of lesser natural numbers, as with, e.g., the primes). Thus, represent subsequences with repeated entries, if any, of the standard exponential sequences by "powers" of the repeated entries, and repeat this (effective) process upon the new "exponents" until, for any given natural number, termination results with a unique "pattern" of Mycielski numbers alone. Implicit in the technique is a concrete alternative to the standard prime-factorization scheme of Godel numbering the symbols, formulas, proofs, and sequences of proofs in any formal system. (Received May 6, 1965.)

General mixed problems on a half-space for certain higher order parabolic equations in one space variable.

Suppose u = u(x,t) solves (1) \(D_x^{2m}u + (-1)^m Du = 0\) for \((x,t) \in \mathbb{R}^2 \times (0,T),\mathbb{R}^+\) the positive half line, and satisfies also the following: (2) \(u(x,0^+) = 0, 0 < x < \infty\). Let \(B(D_x)\) be an operator with components \(B_j(D_x), 1 \leq j \leq m\), linear differential operators with constant coefficients. Let \(r\) be the maximum order of the \(B_j(D_x), 1 \leq j \leq m\), and set \(p = \max((r - 1)/m,0)\). We define \(x \rightarrow u(x,·)\) to be continuous at \(x = 0\) in the \(L^1\)-sense locally, if for each \(t \in (0,T) \lim_{x \rightarrow 0} \int_0^t |u(x,y) - u(0,y)|^r dy = 0\). Under the assumption \(\det(B_j(\omega_k z)) \neq 0, \omega_k (1 \leq k \leq m)\) the 2mth roots of \((-1)^{m+1}\) with negative real parts, we prove the uniqueness theorems. Theorem 1. Assume \(x \rightarrow u(x,·)\) is continuous at \(x = 0\) in the \(L^1\)-sense locally. If for each \(t \in (0,T) \lim_{x \rightarrow 0} \int_0^t (t - y)^r B(D_x)u(x,y)dy = 0\), then \(u = 0\) in \(R\). Theorem 2. Assume \(x \rightarrow u(x,·)\) is continuous at \(x = 0\) in the \(L^{2m+1}\)-sense locally where \(2m + = 2m + \epsilon\) for some \(\epsilon > 0\). If for every \(\phi \in \mathcal{C}^0\) with support compact in \((0,T) \lim_{x \rightarrow 0} \int_0^t \phi(t)(t')^{r-1} \int_0^t (t - y)^{r-1} B(D_x)u(x,y)dy dt = 0\), where the parenthetical factor in the integrand is to be taken as \(B(D_x)u(x,t)\) when \(p = 0\), then \(u = 0\) in \(R\). We prove also an existence theorem providing a framework for essentially best possible results in this context. (Received May 10, 1965.)

Some multivariate density functions of products of Gaussian variates.

 Certain results in the theory of multivariate t-distributions have suggested the following problems: (i) Let \(X = \{x_1, x_2, ..., x_p\}\) be a p-dimensional Gaussian vector with mean zero and positive definite covariance matrix. Let \(Y\) be an n-dimensional Gaussian vector independent of \(X\), with mean zero and covariance matrix the identity matrix. Let \(Z = \{z_1, z_2, ..., z_p\}\) where \(z_k = x_k Y\), \(1 \leq k \leq p\), and \(|Y|\) is the norm of \(Y\). Find the frequency function of \(Z\). (ii) Let \(X\) be as above. Let \(V = \{v_1, v_2, ..., v_{p-1}\}\) where \(v_k = x_k x_p\), \(1 \leq k \leq p - 1\). Find the frequency function of \(V\). It is shown that both these multidimensional distributions may be expressed in closed form in terms of a modified Bessel function of the second kind. (Received May 12, 1965.)

The difference analog \( *u_{n+1} = F(u_{n+1}, u_n, nh, h) \) of the differential equation system \( **dy/dt = A(t, y) \) (\( y \) a finite-dimensional vector) is called strongly consistent with \( ** \) if

\[
|y_{n+1} - F(y_{n+1}, y_n, nh, h)| \leq h[L_1 W(y', h) + L_2 W(y, h)]
\]

when and only when \( y \) is a solution of \( ** \), where \( W(y', h) \) and \( W(y, h) \) are the moduli of continuity of \( dy/dt \) and \( y \), respectively. Under standard assumptions on \( A \) and \( F \) of Lipschitz continuity, the following theorem holds: The two-step scheme \( * \) is convergent to the solution of the initial value problem \( ** \) with \( y(t_0) = y_0 \) if \( * \) is strongly consistent. This theorem generalizes the result of the two-step \( (\rho, \sigma) \) methods found in Henrici where strong consistency implies the root condition which implies stability which implies convergence. (Received May 12, 1965.)


Let \( P(x, y, Dy) = (Dy)^2 + a(x)y Dy + b(x)Dy + \sum_{j=0}^{n} c_j(x)y_j \), where \( a(x), b(x), \) and the \( c_j(x) \) are rational functions of \( x \), and \( c_0(x) \neq 0 \). Then there exists a sector \( S \) of the complex plane, and functions \( y_0(x) \) and \( cx^{r}(\log x)^{s} \) analytic in \( S \) (with \( r \) and \( s \) rational numbers), such that \( y_0(x) \) satisfies \( P = 0 \), and \( y_0(x) \sim cx^{r}(\log x)^{s} \) as \( x \to \infty \) in \( S \). An analogous result is obtained when the rationality of the coefficients is replaced by a much weaker condition. The proofs use methods and results of the author's On the Briot and Bouquet theory of singular points of ordinary differential equations, Math, Research Center, U. S. Army, University of Wisconsin, Tech. Summary Rep. 508 (1964), 103 pp. (Received May 10, 1965.)

65T-312. J. N. WELCH, Georgetown University, Washington, D. C. and W. M. BOGDANOWICZ, Catholic University, Washington, D. C. On a linear operator connected with almost periodic solutions of linear differential equations in Banach spaces. The following is a generalization of a previous result (Abstract 619-5, these Notices 12 (1965), 55-56). Let \( (X, | |) \) be a complex Banach space and let \( A \) be a linear continuous operator from \( X \) into \( X \) such that its spectrum \( \sigma A \) is in the right-half plane. Denote by \( P(X) \) the Banach space of all Bohr almost periodic functions from the additive group of reals \( R \) into the space \( X \). The norm in the space \( P(X) \) is defined by \( ||f|| = \sup(|f(x)| : x \in R) \). The operator \( g = Kf \) defined by \( g(t) = \int_{-\infty}^{t} e^{A(t-s)}f(s + t)ds \) (\( t \in R \)) for \( f \in P(X) \) is linear and continuous from \( P(X) \) into itself, and maps \( f \) into a solution of the equation \( g' + Ag = f \) (the proof is as in the paper of Bogdanowicz in Arch. Rational Mech. Anal. 13 (1963), 364-370). Theorem. The spectrum of the operator \( K \) is the set \( S = \{ u : |u - 2^{-1}\Re(\lambda)| = 2^{-1}\Re(\lambda) \text{ for some } \lambda \in \sigma A \} \). For \( u \notin \sigma K \) the resolvent of the operator \( K \) is given by the formula \( (uI - K)^{-1} = u^{-1}I + u^{-2}(K_1 - K_2) \). Where \( K_1f(t) = \int_{-\infty}^{0} e^{s(A-u^{-1})}P_1f(t + s)ds \) and \( K_2f(t) = \int_{0}^{\infty} e^{s(A-u^{-1})}P_2f(t + s)ds \) for any \( f \in P(X) \). \( P_1 \) and \( P_2 \) are some projection operators in \( X \) which depend on \( u \). (Received May 10, 1965.)
Let $F$ be a field of characteristic $p \neq 0$, and let $K$ be an inseparable algebraic field extension of $F$ which is not purely inseparable over $F$. $K$ is an exceptional extension of $F$ if $K$ contains no purely inseparable, nontrivial extension of $F$. $K$ splits over $F$ if $K$ can be obtained by a purely inseparable extension of $F$ followed by a separable extension. This paper considers the problem of determining necessary and sufficient conditions in order that $K$ should split over $F$ and such conditions in order that $K$ be an exceptional extension of $F$. This question can be reduced to considering the case when $K$ is a simple extension of $F$. Theorem 1. If $K = F(t)$ is a simple extension of $F$ of exponent $e$ and if $\sum_{i=0}^{r} a_i t^{ip^e}$ is the minimal polynomial for $t$ over $F$, then $K$ splits over $F$ if and only if $a_1^{1/p^e} \in K$ for $0 \leq i \leq r$—hence if and only if the normal closure of $K$ over $F$ is separable over $K$. $K$ is an exceptional extension of $F$ if and only if $F(a_0^{1/p}, ..., a_r^{1/p})$ is not a simple extension of $F$. 

Theorem 2. If $F$ is not separably algebraically closed, the following statements are equivalent: (i) $F^{1/p}$ is a simple extension of $F$, (ii) there are no exceptional extensions of $F$, (iii) each algebraic extension of $F$ splits over $F$, (iv) each finite dimensional extension of $F$ is simple over $F$. (Received May 10, 1965.)
Let $G$ be a Jordan domain with piecewise twice continuously differentiable boundary, $\partial G$. Let $\rho$ denote ordinary distance between two sets. We say $\Gamma$ is the ridge of $G$ if it consists of those points $\gamma \in G$ with the following properties: if $\gamma \in \Gamma$, $s \in \partial G$ are such that $\rho(\gamma, s) = \rho(\gamma, \partial G)$ then for every point $q \in G$, $\rho(\gamma, \partial G) \geq \rho(q, \partial G)$, provided the points $\gamma$, $q$ and $s$ are collinear. **Theorem 1.** By distinguishing the two sides of a curve, the ridge $\Gamma$ of the Jordan domain $G$ can be represented as a joint union of the continuous images of the subarcs of $\partial G$ between two consecutive corners and the continuous images of the interior angles of the reentrant corners of $G$. **Theorem 2.** If the curvature of $\partial G$ is in general monotone between consecutive corners, then the ridge $\Gamma$ of $G$ has continuously turning tangents except at a finite number of points in $\Gamma$. (Received May 10, 1965.)
Un groupoïde est demi-symétrique s'il coïncide avec ses transposes: \( G = GP^{123} = GP^{132} \); \( xy = z \Leftrightarrow zx = y \Leftrightarrow yz = x \). Définitions équivalentes: \( (xy)x = y \), ou \( x(yx) = y \); ou \( \Delta a \Gamma a = 1 \). Tout domaine multiplicatif demi-symétrique est un groupoïde ou un quasigroupe. Tout quasigroupe demi-symétrique entièrement gauche et idempotent définit un BIBD \([n, n(n-1)/3, n-1, 3, 2]\); exemple: sur l'anneau \( \mathbb{Z}/n(\cdot) \), \( x\cdot y = y \cdot 2 \cdot x + By \); \( \theta^3 = 1 \); \( \theta(\theta + 1) \) premier avec n). Tout idempotent satisfaisant \( xy = z \) et \( zx = y \) et \( x = y \) représente un BIBD avec \( k = 3, \lambda = 3 \) s'il est abélien et \( k = 3, \lambda = 6 \) s'il est complètement gauche. Ces conditions sont conservées par le produit direct.

Pour qu'un quasigroupe, Q, soit isotope d'un quasigroupe demi-symétrique il faut et il suffit que, parmi les isotopies qui appliquent Q sur son transposé il y en ait au moins une (a,b,c) qui satisfasse \( cba = 1 \).

Ce théorème permet de construire tous les quasigroupes non isomorphes, demi-symétriques, d'ordre donné, au moyen des tables de loops du même ordre. Jusqu'à \( n = 7 \), toutes les \((a, b, c)\) satisfont à la condition \( cba = 1 \).

(Received May 24, 1965.)
proof is based on the results of the paper: W. Bogdanowicz, *A generalization of the Lebesgue-
U.S.A., 53 (1963), 492-498. (Received May 24, 1965.)

65T-322, OSWALD WYLER, University of New Mexico, Albuquerque, New Mexico 87106. *Completion of a separated uniform convergence space.*

For definitions not given in this abstract, see Abstract 619-83, these *Notices* 12 (1965), 80.
A separated uniform convergence space, or su.c.s., A is called complete if every Cauchy filter on
A converges in A. A completion of a su.c.s., A is defined by an injective uniformly continuous, or
u.c., map j: A → A such that (i) A is a complete su.c.s., and (ii) for any u.c. map f from A to a com-
plete su.c.s. B, there is one, and only one, u.c. map f_1: A → B such that f = f_1 j. **Theorem.** Every
su.c.s. A has a completion A. **Construction of** A: The underlying set is the set of all equivalence
classes of Cauchy filters on A, and j(x) = q(x) for x ∈ A, where q is the filter on A generated by {x},
and q(F) the equivalence class of a Cauchy filter F. The uniform convergence structure on A is the
finest structure for which j is u.c. and j(F) converges to q(F) for every Cauchy filter F on A.
(Received May 24, 1965.)

65T-323. TANJIRO OKUBO, Montana State College, Bozeman, Montana 59715. *Hypersurface
in a manifold with an almost complex structure admitting contact transformations.*

Let M_{2n} (X^A) be a manifold of C^r, r ≥ 4, with an almost complex structure F^A_B;
F^A_B = - δ^A_C and M_{2n-1} (u^1) be its hypersurface whose tangent vectors Bf^A = (∂x^A/∂u^1) and the
pseudo-normal N^A the structure F^A_B acts in the most general ways that F^A_B|f^A = f^A_j + ξ^A_F^A,
F^A_B = η^A_B^j + φ^A_F^A. Then we have the relation η^j_i ξ^i_j = φ^j_i + 1 and find that f satisfies
ξ^j_i φ^j_i + η^j_i φ^i_j + ξ^j_i φ^i_j = 0. If we take du^1 and p_i for η^i_j and ξ^i_j, respectively, and impose the
condition on p_i that it be transformed to p_f by the coordinate transformation of U(u^1) in M_{2n-1} (u^1) in
such a way that p_f is homogeneous degree zero in the p's, then we have the **Theorem:** there exists a
hypersurface in a manifold with almost complex structure such that it admits a tensor field f of
type (1,1) satisfying f^3 + Φ^2 + f + Φ = 0 and also an extended homogeneous point transformation
u^1 = u^1 (u), p_i = p_i (Φ, u) with p_i du^1 = Ψ (u, p). Especially if Φ = 0, the hypersurface has a
well known property called a contact structure with f^3 = 0, admitting a restricted contact trans-
formation qdu^1 = 1. (Received May 25, 1965.)

65T-324. NORMAN LEVINSON, Massachusetts Institute of Technology, Cambridge, Massachu-
setts 02139 and T. O. SHERMAN, Institute for Advanced Study, Princeton, New Jersey. *On sum of
respective intersections of cone and its dual with linear subspace and its orthogonal complement.*

A wedge W = C + K, C ⊆ K, where C is a cone, not necessarily polyhedral, and K a closed linear
subspace in R^n, W* is the dual to W in R^n. **Theorem.** Let L be a closed linear subspace of R^n
and L_⊥ its orthogonal complement. Then there exists a ∈ W ∩ L and b ∈ W* ∩ L_⊥ such that a + b ∈ in-
terior (W + W*). In the special case where W is the self-dual polyhedral cone x ≥ 0 in R^n, (the non-
negative orthant), the theorem follows from a result of Ben-Israel. An example shows int(W + W*)
cannot be replaced by int W ∪ int W*. A corollary is the extension of Tucker's Key Theorem from

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x ≥ 0 to W: Let A be a k × n matrix with transpose A^T. W and W* are as above. Then there exist x ∈ W and z ∈ R^k such that Ax = 0, A^T z ∈ W* and x + A^T z ∈ int(W + W*). (Received May 24, 1965.)

65T-325. R. L. IRWIN, University of Utah, Salt Lake City, Utah. Absolute Hardy-Bohr factor.

Definition. If \( \sum \alpha_i \epsilon_i \in |A| \) whenever \( \sum \alpha_i \epsilon_i \in |A| \) then we write \( \epsilon_i \epsilon_i \in |A| \).

Theorem. Let A = BP^k, P a weighted mean and k ≥ 0 (integer). Also assume A is absolutely regular, \( |BP| ≥ |B| \) and \( \epsilon_i (B, C) \in |B| \epsilon_i \epsilon_i = 0(1) \). If \( \epsilon_i (B, C) \in |A| \) then \( \epsilon_i (A, C') \in |A| \).

(Received March 29, 1965.)


The main objective of this paper is to show that for a non-degenerate surface defined on a two-dimensional manifold, there exists a representation for which almost all contours are arcs, points or simple closed curves. The definitions and notations used are either the same or equivalent to those used by Professor R. E. Fullerton and Professor L. Cesari and this paper is a generalization of one of their results. Let M be a 2-dimensional manifold and let T be a continuous mapping from M into n-dimensional Euclidean space \( E^n \). The mapping T defines a Frechet surface S. Let \( \{S\} \) denote the set of points in \( E^n \) occupied by the surface. On \( \{S\} \) define a real-valued function f with upper and lower bounds \( t_1 \) and \( t_2 \), respectively. The contour \( C(t) \) associated with f,T,t is the set of all points p ∈ M such that \( f[T(p)] = t \) for t ∈ \([t_1, t_2]\). Finally let Q denote the unit square in \( E^2 \).

Theorem. Let S = (T,M) be a Frechet surface defined by a light mapping T on a 2-dimensional manifold M into n-dimensional Euclidean space \( E^n \). Let f be a real-valued continuous function on \( \{S\} \). Let \( \gamma \) be the set of all components of contours corresponding to f in Q and whose images are of finite length in the sense of Cesari. Then all components, except a countable number, will either be a point, a simple arc, or a simple closed curve. (Received May 26, 1965.)

65T-327. MITSURU YASUHARA, Université de Montréal, Montréal, Quebec, Canada. Completeness and compactness of a language with the equi-cardinality quantifier (but without equality).

The language used is the usual first-order language without equality but with an additional quantifier (Cx). (Cx)φ(x) "says" that the set \{x; φ(x)\} is equi-cardinal to the universe. Let \( \mathcal{G} \) be the Gentzen sequent calculus (c.f., Math. Z., vol. 39) augmented by the rule: from a sequent \( Θ(a), Γ \to Δ, A(a), ..., B(a) \) one may infer (Cx)Θ(x), Γ \to Δ, (Cy)Δ(y), ..., (Cx)B(z), provided the free variable a does not occur in the lower sequent and Θ(a) consists of at most one formula. Theorem. A set of formulae either has a finite subset Γ such that Γ \to Δ is deducible in \( \mathcal{G} \) or is satisfiable in any domain of cardinality > \( k_0 \) and cofinal with \( ω \). Corollary. If a set of formulae is satisfiable in an infinite domain then it is satisfiable in any of the domains mentioned above. The proof consists in forming a deduction-tree and making copies of elements in a special way. The absence of the equality sign is essential for this theorem, in view of a result of Fuhrken: (4) p. 291, Fund. Math. LIV. (Due to (1,4) p. 293, his \( V_0 \) is the same as the set of valid sentences with equality in the domains of cardinality \( k_0 \).) In \( \mathcal{G} \), the rule 'cut' is eliminable, and hence Craig's Interpolation Theorem and
Beth's Theorem hold. (Received May 27, 1965.)

65T-328. PETER FLETCHER, The University of North Carolina, Chapel Hill, North Carolina, 
Pairwise uniform spaces. Preliminary report.

J. C. Kelly [Proc. London Math. Soc. (3) 13 (1963) 71-89] defined bitopological space and 
proved several theorems including a "metrization" theorem. We define pairwise completely regular 
and show that it lies between Kelly's pairwise normal and pairwise regular. **Definition,** $(X,P,Q)$ is 
pairwise completely regular provided that if $C$ is a $P$-closed set and $x \in X - C$ there is a function 
$f : X \rightarrow [0,1]$ such that $f(x) = 0$, $f(C) = 1$ and $f$ is $P$-upper-semi-continuous and $Q$-lower-semi-continuous;
and the above conditions hold with $P$ and $Q$ interchanged. **Definition,** $(X,P,Q)$ is pairwise uniform 
provided there is a quasi-uniformity $\mathcal{U}$ for $X$ such that $P = T_{\mathcal{U}}$ and $Q = T_{\mathcal{U}}^{-1}$. **Theorem,** Every 
pairwise completely regular space is pairwise uniform and conversely. **Theorem,** If $(X,P',Q')$ is a pairwise uniform 
and $\mathcal{U}'$ has a countable base, then $(X,P'',Q'')$ is quasi-pseudo metrizable. If $(X,P,Q)$ is a quasi-pseudo metrizable space, there is a quasi-uniformity 
$\mathcal{U}$ for $X$ such that $P = T_{\mathcal{U}}$ and $Q = T_{\mathcal{U}}^{-1}$ and $\mathcal{U}$ has a countable base. (Received May 28, 1965.)

65T-329. H. W. GOULD, West Virginia University, Morgantown, West Virginia 26506, 
Research bibliography of two special number sequences. Preliminary report.

The sequence of numbers $1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, ...$ was first studied by Euler, 
Fuss, Von Segner, Catalan, and others in connection with the combinatorial problem of dissecting a 
convex polygon into triangles by diagonals. The sequence of numbers arises in studies of non-
associative algebras in regard to ways of interpretation of an indicated product. The sequence is 
generated by $c_{2n,n} = n/(n + 1)$ for $n = 0, 1, 2, ...$. It is shown how this sequence and its associated con-
volution are related to work of the present author on binomial addition theorems. Over 100 pertinent 
items in the literature are collected here. The sequence of numbers $1, 1, 2, 5, 15, 52, 203, 877, 4140, 
21147, 115975, ...$ is also related to combinatorial studies, and over 100 bibliographical references are 
presented. This sequence appears to have first been studied by Christian Kramp and others in con-
nection with the expansion of the function $f(x) = \exp(e^x - 1)$ in a power series. Rota (Amer. Math, 
Monthly, 71 (1964), 498-504) has given an interesting account of this sequence. W. G. Brown (to be 
published) has given an account of the former sequence. Because of the current research involving 
these two rather different sequences the present bibliographies are offered as a research aid.
(Received June 4, 1965.)

65T-330. H. K. FALLIN, JR., Ballistics Research Laboratory, Aberdeen Maryland and 
H. W. GOULD, West Virginia University, Morgantown, West Virginia 26506. **Chronological bibli-
ography of the Cauchy Integral Theorem.** Preliminary report.

Vandiver (Amer. Math. Monthly, 67 (1960), 47-50) has ably pointed out the value of classified 
research bibliographies for mathematics. The present paper presents a chronological history of the 
Cauchy Integral Theorem, starting with the letter of Gauss to Bessel in 1811 when the theorem was 
first intimated. Critical remarks are appended to many of the items and in all around 150 significant 
references are given, together with citations in the Fortschritte and Math. Reviews. The present
report is the outgrowth of a master's thesis by one of the authors (Fallin), and the authors solicit suggestions as to any omissions dealing with proofs of the integral theorem or its generalizations. (Received June 4, 1965.)

65T-331. HAJIMU OGAWA, University of California, Riverside, California. **Lower bounds for solutions of hyperbolic inequalities.**

Let \( u \in C^2_{2}(\mathbb{R}) \), \( R = D \times I \), \( D \) is a bounded domain in \( \mathbb{R}^n \), with boundary \( \Gamma \), and \( I = [0, \infty) \). Define \( Lu = u_{tt} - Au \), where \( A \) is the elliptic operator defined by \( Au = \sum_{i,j=1}^{n} (a_{ij}u_{ij}) \) (subscripts \( i \) and \( j \) denote differentiation by \( x_i \) and \( x_j \)). Assume \( a_{ij} = a_{ji} \in C^1(\mathbb{R}) \) and define \( \|u(t)\|_0 = \int_D u^2 \, dx \) and \( \|u(t)\|_2 = \int_D (u_t^2 + \sum a_{ij}u_{ij}^2) \, dx \). Suppose \( \Phi \) is a function satisfying \( \int_D \sum a_{ij}u_{ij} \, dx \leq 2\Phi(t)\|u(t)\|^2 \), and let \( K \) be a positive, differentiable, non-decreasing function such that \( \lim_{t \to \infty} K(t) = \infty \). Theorem. Let \( u \) be a solution of \( \|Lu(t)\|_0 / \Phi(t) \|u(t)\|_0 < \ell(t) \|u(t)\|_2 \) such that \( u = 0 \) on \( \Gamma \times I \). Set \( f = \Phi + \Phi \). If \( \|u(t_0)\|_0 < 0 \), \( t_0 \geq 1 \), and either (i) \( (K'/K)^{1/p-1} \ell \in L_p(1, \infty) \) for some \( p, 1 \leq p < \infty \), or (ii) \( Kf/K' \in L_0(1, \infty) \) and \( \|Kf/K'\|_\infty \leq 1 \), then there exists a positive constant \( C \) such that \( \|u(t)\|_0 \geq C\|u(t_0)\|_0 K(t)^{-1} \), \( t \geq t_0 \). This is a generalization of some results of M. H. Protter. (Received June 7, 1965.)

65T-332. J. L. BRYANT, University of Georgia, Athens, Georgia. **Taming polyhedra in the trivial range.**

Theorem. Suppose \( X \) is a polyhedron of dimension \( k \), \( P \) is a polyhedron of dimension \( \ell \leq k \) tamely embedded in \( \mathbb{R}^n \), with \( n \geq 2k + 2 \), and \( f \) is an embedding of \( X \) into \( \mathbb{R}^n \) such that \( f|X - C \), is locally tame, where \( C = r^{-1}(P \cap t(X)) \). Then, given \( \epsilon > 0 \), there is an isotopy \( h_t : X \to \mathbb{R}^n \) piecewise linear, and \( h_t|X - U_\epsilon(f(X)) \) is the identity for each \( t \in I \). This is a generalization of the theorem stated in Abstract 65T-233 (these Notices 12 (1965), 415), and the method of proof is an extension of the method indicated there. John Cobb previously obtained this result, except for \( n \geq 3k + 1 \), Abstract 618-11 (these Notices 12 (1964), 762). As an example of how the theorem can be applied, one has the following generalization of a result obtained by Harvey Durham Abstract 65T-42 (these Notices 12 (1965), 137). Corollary. A \( k \)-dimensional polyhedron in \( \mathbb{R}^n \) (\( n \geq 2k + 2 \)) which is locally tame modulo a tame Cantor set is tame. (Received May 3, 1965.)

65T-333. L. P. BELLUCE and W. A. KIRK, University of California, Riverside, California. **Fixed-point theorems for families of contraction mappings.**

Let \( X \) be a nonempty, bounded, closed and convex subset of a Banach space \( B \). A mapping \( f : X \to X \) is called a contraction mapping if \( \|f(x) - f(y)\| \leq \|x - y\| \) for all \( x, y \in X \). Let \( \mathcal{F} \) be a nonempty commutative family of contraction mappings of \( X \) into itself. Theorem 1. If there is a compact subset \( M \) of \( X \) and a mapping \( f_1 \in \mathcal{F} \) such that for each \( x \in X \) the set \( \{f_1^n(x)\} \) has an accumulation element in \( M \), then there is a point \( x \in M \) such that \( f(x) = x \) for each \( f \in \mathcal{F} \). Theorem 2. If \( X \) is weakly compact and the norm of \( B \) is strictly convex, and if for each \( f \in \mathcal{F} \) the \( f \)-closure of \( X \) [Edelstein, Proc. Cambridge Philos. Soc. 60 (1964), 439-447] is nonempty, then there is an \( x \in X \) such that \( f(x) = x \) for each \( f \in \mathcal{F} \). (Received June 10, 1965.)

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Let $E$ be a Hausdorff locally convex (abbreviated to l.c.) space and $E'$ its dual. $E$ is said to be a countably barrelled (countably quasibarrelled) space if each weak*- or $\sigma(E', E)$- (strongly) bounded subset of $E'$ which is the countable union of equicontinuous subsets of $E'$ is itself equicontinuous. Countably barrelled and countably quasibarrelled spaces, respectively, generalize barrelled and quasibarrelled spaces. Two of the main results are the following: Theorem. Let $E$ be a countably barrelled space. If a $\sigma(E', E)$-bounded sequence $\{f_n\}$ in $E'$ converges to a linear functional $f_0$ in the topology $\sigma(E', E)$, then $f_0 \in E'$ and $\{f_n\}$ converges to $f_0$ uniformly on each precompact subset of $E$.

This is the celebrated Banach-Steinhaus theorem which is known to be true for barrelled spaces. Theorem. Let $E$ be a metrizable countably barrelled l.c. space, $F$ a metrizable topological space and $G$ an Lc. space. A family $H$ of mappings of the product space $E \times F$ into $G$ is equicontinuous if the following conditions hold: (i) for each fixed $y \in F$, $\{f(\cdot, y)\}_{f \in H}$ is an equicontinuous family of linear mappings of $E$ into $G$; (ii) for each fixed $x \in E$, $\{f(x, \cdot)\}_{f \in H}$ is an equicontinuous family of mappings of $F$ into $G$. (Received June 10, 1965.)

The Lorentz group is considered as the group of linear transformations of space $E \times T$, where $E$ is the geometrical space and $T$ is the time space, which preserve the class of sets of the form $\|x\| = \|t\|$ where $\| \|$ stands for the norm. The problem is: what is this group when $E$ and $T$ are structures more general than the euclidean vector spaces? In case $E$ is the $k$-dimensional $p$-adic vector space and $T$ the $p$-adic number system, this group of transformations is explicitly determined. (Received June 14, 1965.)

Let $S$ be a semigroup and $E_S$, the collection of idempotents of $S$. If $E_S = I^0 \times I^0$, where $I^0$ is the non-negative integers and the natural order on $E_S$ is $(m, n) < (s, t)$ if $m < s$ or $m = s$ and $n < t$, we say $E_S$ is lexicographically ordered. If $C$ is a bicyclic semigroup, define the following multiplication on $C \times C$, $((i, j), (k, l))((m, n), (s, t)) = ((i, j)(m, n), (k, l))$ if $j > m$, $((i, j)(m, n), (s, t))$ if $m > j$, and $((i, j)(m, n), (k, 1)(s, t))$ if $j = m$, juxtaposition denotes product in $C$. We call $C \times C$, with this multiplication a Bruck product and denote it by $C \circ C$. Theorem. $S$ is a bisimple inverse semigroup with $E_S$ lexicographically ordered and trivial unit group iff $S = C \circ C$. The definitions and theorem may be extended to arbitrary finite dimensions. (Received June 14, 1965.)
A fixed point theorem for weakly contractive sequences of contraction mappings.

Let \( X \) be a B-space. A sequence \( \{T_i\} \) of continuous mappings \( T_i : X \to X \) is said to be weakly \( \beta \)-contractive if there is a number \( \beta, 0 \leq \beta < 1 \), such that \( \|T_{i+2}x - T_{i+1}x\| \leq \beta \|T_{i+1}x - T_ix\| \) for all \( x \in X \) and all nonnegative integers \( i \). \( \{T_i\} \) is said to be \( \alpha \)-contractive if there is a number \( \alpha, 0 < \alpha < 1 \), such that \( \|T_ix - T_iy\| \leq \alpha \|x - y\| \) for all \( x, y \in X \). \( T_i \) is said to be weakly \( \gamma \)-bounded on \( S \), if there is a number \( \gamma \) such that \( \|T_ix\| \leq \gamma \) for all \( x \in S \). Let \( S \) be a closed sphere of radius \( \gamma + \|a\|, a \in X \). We prove the following Theorem. Assume \( \beta \leq \alpha \) and let \( \{T_i\} \) be a weakly \( \beta \)-contractive sequence of \( \alpha \)-contractive mappings \( T_i \) which are weakly \( \gamma \)-bounded on \( S \). Then \( \{A_i\} \), defined by \( A_i(\cdot) = T_i(\cdot) + a \) converges weakly to \( A(\cdot) = T(\cdot) + a \) for all \( x \in S \) and the sequence \( \{x_i\} \) of successive iterates, inductively defined by \( x_{i+1} = A_1x_i \), where \( x_0 \in S \), converges in norm to the unique fixed point of \( Ax = x \). Furthermore, an upper error bound is given by:

\[
\|x - x_i\| \leq a^i \|x_1 - x_0\|/(1 - a) + 2\gamma[a^i/(1 - a) - \beta^i/(1 - \beta)]/a(1 - \beta/a) \text{ for } \beta < a \text{ and by } \|x - x_i\| \leq a^i \|x_1 - x_0\|/(1 - a) + 2\gamma[a^i/a(1 - a)]a^{i-1}/(1 - a) \text{ for } \beta = a.
\]

(Received June 15, 1965.)

Parabolic partial differential equations with uniformly continuous coefficients.

Consider the following system of equations: (1) \( \sum_{j=1}^M \sum_{a|m} a^{k_j}(x,t)(\partial/\partial x)u_j \delta_{kj} \partial/\partial t u_k = f_k, k = 1, \ldots, M \). Here \( x \) denotes a point in \( E_n \) and \( t \in (0, R), R < \infty \). Assume the system (1) is parabolic in the sense of I. G. Petrovsky i.e. the roots \( \lambda(x,t;z) \) of the equation:

\[
det(L^{a|m}_{a|m} l_{a|m} \sum_{a|m} a^{k_j}(x,t)(iz)^a \delta_{kj} \partial/\partial t u_k)
\]

satisfy \( \text{Re} \lambda(x,t;z) < -\delta < 0 \), for \( \|z\| = 1 \), independent of \( (x,t) \). Define \( L^{0|m}_{0|m} l_{0|m} (E_n x(0,R)) \) to be the closure in the class of distributions of \( E_{n+1} \) of the functions \( u \in C_0^\infty(E_n x(0,\infty)) \) with respect to the norm:

\[
\|u\|_{m,1} = \sum_{a|m} a^{k_j}(x,t)(R \int_E |x_0|^m |x|^a dV dt)^{1/p} + (R \int_E |x_0|^m |x|^a dV dt)^{1/p}.
\]

Define \( L^{0|m}_{0|m} l_{0|m} (E_n x(0,R)) \) to be all vectors \( u = (u_1, \ldots, u_M) \) with \( u_k \in L^{0|m}_{0|m} l_{0|m} (E_n x(0,R)) \). Concerning the coefficients of (1) assume that: (i) \( a^{k_j}(x,t) \) are bounded measurable over \( E_n x(0,R) \) for all \( a, k, j \). (ii) For \( |a| = m, a^{k_j}(x,t) \) are uniformly continuous in \( E_n x(0,R) \), for all \( k, j \). Theorem. Given any vector valued \( f = (f_1, \ldots, f_M) \) where \( f_k \in L^p(E_n x(0,R)) \) there exists a unique \( u \in (L^{0|m}_{0|m} l_{0|m} (E_n x(0,R)))^M \) satisfying system (1). (Received June 14, 1965.)

An application of the Bott suspension map to the topology of EIV.

Let \( (E_6, F_4) \) be the compact simply connected symmetric pair whose local structure is usually so designated. The corresponding symmetric space is denoted EIV (by a slight abuse of E. Cartan's notation). Let \( W \) be the Cayley projective plane, \( \Sigma(W) \) the nonreduced suspension of \( W \), and \( E : \Sigma(W) \to \text{EIV} \) the Bott suspension map defined by means of the shortest geodesics joining the two nontrivial points of the "center" of EIV. Theorem. For suitable \( q : S^{25} \to \Sigma(W) \), \( E \) extends to a homeomorphism \( E^* : \Sigma(W) \cup q_* e_{26} \to \text{EIV} \). Corollary. \( E^* : \pi_7(\Sigma(W)) = \pi_7(\text{EIV}), 0 \leq j \leq 24, \) and the sequence \( 0 \to \pi_25(S^{25}) \to Q_* \pi_25(\Sigma(W)) \to E_* \pi_25(\text{EIV}) \to 0 \) is exact and canonically split. Using this corollary, we compute \( \pi_7(\text{EIV}) = \pi_7(\Sigma(W)), 16 \leq j \leq 24, \) to be (in order) as follows: \( 0, Z + (Z_2)^2, (Z_2)^3, Z_6, Z_{1512} + Z_2, 0, Z_3, Z_4, Z_{225} + (\text{finite 2-primary group}). \) Also, \( \pi_25(\Sigma(W)) = Z + (\text{finite 25-primary group}) \).

(Received June 14, 1965.)

65T-338. E. B. FABES and N. M. RIVIERE, University of Chicago, Chicago 37, Illinois.

65T-339. L. W. CONLON, St. Mary's College, St. Mary's, Kansas 66536.
The equalities $\pi_j(EIV) = \pi_j(S^9)$, $0 \leq j \leq 15$, also follow, but are already well known. (Received June 10, 1965.)


B. Jonsson raised the following problem (Algebras whose congruence lattices are distributive, Prepublication copy, 1963): what is the number of equational classes $K$ of modular lattices, which cover the class $M_5$, generated by the five element modular and non-distributive lattice? It is easy to show that such a class $K$ is always generated by a single subdirectly-irreducible lattice. Theorem. The number of such subdirectly-irreducible and finite lattices is two. Problem. Is there any infinite lattice satisfying these properties? (Received June 17, 1965.)


By using a construction due to J. Mikusinski (Fund. Math. 35 (1948)), the classical one-sided Laplace transform is extended to a generalized function space. The usual formulas for translation, differentiation, etc., are extended to the generalized functions. It is shown that the generalized Laplace transform maps the generalized functions onto the class of all functions which are analytic in some half plane. Several characterizations of the generalized functions of finite order are given; in particular, it is shown that a generalized function is of finite order iff its Laplace transform is analytic in some half plane and of polynomial growth in the half plane. (Received June 21, 1965.)

65T-342. WITHDRAWN.

65T-343. NOBORU SUZUKI, University of Minnesota, Minneapolis, Minnesota. On operators with completely continuous imaginary part.

The algebraic structure of an operator $A$ on a Hilbert space is closely related to the type of the von Neumann algebra $R(A)$ generated by $A$. An operator $A$ on a Hilbert space is said to be of type I(II, III) if the von Neumann algebra $R(A)$ is of type I(II, III). Which non-normal operators are of type I? A few answers are known. We will answer this question: Theorem. An operator $A$
whose imaginary part \((A - A^*)/2i\) is completely continuous is of type I. **Corollary.** A completely continuous operator is of type I. (Received June 23, 1965.)

65T-344. R. W. GILMER, JR., Florida State University, Tallahassee, Florida. **On the strong integral closure of an integral domain.**

If \(R\) is a subring of the commutative ring \(S\), an element \(s\) of \(S\) is **strongly integral** over \(R\) if all powers of \(s\) belong to a finite \(R\)-submodule of \(S\). The set \(R^*\) of elements of \(S\) strongly integral over \(R\) is a subring of \(S\) containing \(R\); \(R^*\) is the **strong integral closure** of \(R\) in \(S\) and if \(R = R^*\), \(R\) is strongly integrally closed in \(S\). **Theorem 1.** If \(t\) is an indeterminate over \(S\), \(R^*[t]\) is the strong integral closure of \(R[t]\) in \(S[t]\). **Theorem 2.** If \(R\) is a domain and if \(S\) is the quotient field of \(R\), then if \(R^*\) is strongly integrally closed in \(S\), the strong integral closure of \(R\) in any domain \(D\) containing \(S\) is strongly integrally closed in \(D\). **Theorem 3.** If \(R\) is a domain with quotient field \(S\), then \(R^*\) is strongly integrally closed in \(S\) if either (a) \(R\) is Noetherian, (b) \(R\) is Prüfer and each nonunit of \(R\) belongs to only finitely many maximal ideals, (c) the conductor of \(R\) in \(R^*\) is nonzero, or (d) \(R^*\) is a finite ring extension of \(R\). (Received June 23, 1965.)

65T-345. R. S. SPIRA, University of Tennessee, Knoxville, Tennessee 37916. **Zeros of sections of the Riemann zeta function.**

Let \(s = \sigma + it\). Zeros of \(\sum_{n=1}^{N} n^{-s}\) are calculated for \(N = 3(1)12, 10^k; k = 2(1)5,10, t \leq 100\). The calculations support Turán's conjecture that all zeros have \(\sigma \leq 1\). The known proofs of the conjecture for \(N \leq 5\) have been extended to a few more \(N\). (Received June 24, 1965.)

65T-346. S. B. NADLER, JR., University of Georgia, Athens, Georgia 30601. **A characterization of differentiable submanifolds of \(\mathbb{R}^n\).**

It is known that any differentiable submanifold of \(\mathbb{R}^n\) is a class \(C^1\) neighborhood retract. The following theorem shows that the subsets of \(\mathbb{R}^n\) which are class \(C^1\) neighborhood retracts are precisely the differentiable submanifolds of \(\mathbb{R}^n\). **Theorem.** Let \(U\) be a connected open subset of \(\mathbb{R}^n\) and let \(f: U \rightarrow U\) be a differentiable (class \(C^1\)) retraction of \(U\). If \(r = \text{rank } (f'(x))\), some \(x \in f(U)\), then \(f(U)\) is a class \(C^1\) differentiable submanifold of \(\mathbb{R}^n\) of dimension \(r\). **Corollary.** Every \(m\)-dimensional differentiable retract of \(\mathbb{R}^n\), \(n \geq 2\), is homeomorphic with \(\mathbb{R}^m\), where \(m = 1,2\). (Received June 24, 1965.)

65T-347. RODNEY ANGOTTI, State University of New York at Buffalo, Buffalo, New York and MARIE BENEDICTY, University of Pittsburgh, Pittsburgh, Pennsylvania. **The geometry of a third order differential element.**

In a paper of E. Bompiani, *Configurations associées à un élément curviligne du troisième ordre dans l'espace projectif* (J. Math. Pures Appl., 41 (1962), 193-200), it is shown that the configuration of a point, a line and a third order curvilinear differential element \(E_3\) in the ordinary projective three space \(P_3\) has a projective invariant. The purpose of this note is to show that this invariant completely characterizes the subgroup of the projective transformations which leave an
element $E_3$ invariant. This subgroup depends on six parameters and, in this respect, is similar to the group of transformations of the classical non-euclidean geometry in $P_3$. The subgeometry of $P_3$ with this fundamental group (in the sense of Klein) is an example in the ordinary space of what Bompiani calls a pseudo-euclidean geometry. (Received June 28, 1965.)

65T-348. R. A. BELL, Gustavus Adolphys College, St. Peter, Minnesota and S. M. SHAH, University of Kansas, Lawrence, Kansas. Oscillating polynomials and approximation to $|x|$.

Let $0 \leq a_0 < a_1 < ... < a_n$ be a given set of integers and let $c_i$ (and $a_i$) be real for all $i$. Then $p(x) = C_0 x^{a_0} + C_1 x^{a_1} + ... + C_n x^{a_n}, c_i \neq 0$ for all $i$, is an oscillating polynomial in $[0,1]$ if and only if $\max_{0 \leq x \leq 1} |p(x)|$ is attained for $n+1$ values of $x$ in $[0,1]$. Oscillating polynomials with three exponents $0, h, k$ where $1 \leq h < k$, and four exponents $0, h, 2h, 4h$ have been obtained. With the help of these results, the values of $d_2$ and $d_4$ have obtained where $d_2 = \min_{n+1} \max_{1 < i < n} x_i = \{x_0, x_1, ..., x_n\}$ and $d_4 = \min_{n+1} \max_{1 < i < n}$ $|x| - (a_0 + a_1 x^2 + ... + a_n x^{2n})|$. (Received June 28, 1965.)

65T-349. J. D. SONDOW, Fine Hall, Princeton University, Princeton, New Jersey. Actions of cyclic groups on spheres.

Let $(S^n, S^k)$ be the standard smooth or piecewise linear (PL) sphere-pair, where $S^{-1} = \emptyset$. Let $Z_q$ be the cyclic group of order $q$. A relatively free action of $Z_q$ on $(S^n, S^k)$ is an action of $Z_q$ on $S^n$ which is free off the fixed-point set $S^k$ (hence $n-k$ must be even if $q > 2$). Two smooth actions $\phi_1, \phi_2$ of $Z_q$ on $S^n$ are $C^1$-PL equivalent if there is a PL action $\phi$ of $Z_q$ on $S^n$ and $C^1$-triangulations $\psi_i: S^n \rightarrow S^n$ equivariant with respect to $\phi$ and $\phi_i, i = 1,2$. Theorem 1. If $n \geq 6, k \geq 0$, and either $n$ is even or $n - k = 1$ or $2$ or $q \leq 4$ or $q = 6$, then any relatively free smooth action of $Z_q$ on $(S^n, S^k)$ is $C^1$-PL equivalent to an orthogonal action. Theorem 2. If $n$ and $k$ are odd, $n \geq 5, k \geq -1, n - k \geq 4$, and $q \geq 5, q \neq 6$, then there exist infinitely many relatively free smooth actions of $Z_q$ on $(S^n, S^k)$ which are not smoothly or $C^1$-PL equivalent to orthogonal actions or to each other. Milnor proved this for $k = -1$. Theorem 3. If $n - k \geq 4$ is even, $n \geq 5, k \geq 0$, and $q \geq 5, q \neq 6$, then there exist infinitely many relatively free PL actions of $Z_q$ on $(S^n, S^k)$ which are topologically equivalent but PL distinct. In fact, the orbit spaces $S^n/Z_q$ are counterexamples to the Hauptvermutung for complexes. The proofs use the theory of Whitehead torsion. (Received June 29, 1965.)

65T-350. T. P. KEZLAN, University of Texas, Austin, Texas 78712. Rings in which certain subsets satisfy polynomial identities.

Let $R$ be a ring and $\mathcal{S}$ a family of polynomials (each in some finite number of non-commuting indeterminates) with integer coefficients. $R$ is called an $\mathcal{S}$-ring (resp., $\mathcal{S}^+$-ring) if and only if for every finite subset (resp., finitely generated additive subgroup) $S$ of $R$, there is an $f \in \mathcal{S}$ which vanishes identically on $S$. Conditions on $\mathcal{S}$ are considered which will imply that every $\mathcal{S}$-ring (resp., $\mathcal{S}$-$^+\text{-ring}$) has nil commutator ideal. Using the results obtained, several theorems concerning $K$-rings (defined by Drazin, Amer. J. Math. 77 (1955), 895-913) are obtained. Some other theorems proved are the following. Theorem. Suppose $\text{char}(R) = 0$. If for every finitely generated additive subgroup $S$ of $R$ there exists $n \geq 1$ such that $(xy - yx)^n = 0$ for all $x, y$ in $S$, then the commutator ideal
of $R$ is nil. **Theorem.** If the commutators of $R$ commute, then the commutator ideal of $R$ is nil.

(Received June 29, 1965.)

65T-351. J. R. DURBIN, University of Texas, Austin, Texas 78712. **Transfer kernels in finite groups.**

If $G' \subseteq A \subseteq G$, where $G$ is a finite group, then $A$ is called a transfer kernel of $G$ if there is a subgroup $H$ of $G$ such that $A = \text{Ker } T(H)$, where $T(H) : G \rightarrow H/H'$ is the transfer of $G$ into $H$. Although for subgroups $H$ and $K$ of $G$, $\text{Ker } T(H) \cup \text{Ker } T(K) \subseteq \text{Ker } T(H \cap K)$ and $\text{Ker } T(H) \cap \text{Ker } T(K) \supseteq \text{Ker } T(H \cup K)$, the containments are, in general, strict. However, in the case of Abelian groups $G$, at least, the transfer kernels do form a sublattice of the subgroup lattice of $G$ (even though the above containments are still usually strict). Other questions concerning the transfer kernels of a group are also considered. (Received June 29, 1965.)

65T-352. W. M. LAMBERT, JR., 7101 West 80th Street, Los Angeles, California 90045. **Elementary characterization of "prime polynomial ideal".**

There is a system of predicates in the elementary language of fields which represents the notion of prime polynomial ideal, in the following sense. Let $L$ be the lower predicate calculus with identity and with operation symbols for (field) addition and multiplication; for each field $A$ let $L(A)$ be $L$ augmented by an individual constant for each member of $A$. There is a triple sequence $P$ of $L$-predicates such that, for each field $A$ and each system $\{\phi_1, \ldots, \phi_n\}$ of members of $A[x_1, \ldots, x_n]$, each $\phi_i$ of degree $\leq q : P_{\text{eqn}}$ is satisfied in $A$ by the $L(A)$-constants for the coefficients of the $\phi_i$ (under a conventional ordering) if and only if the ideal $P_{\text{eqn}}$ is prime. This solves a problem raised by Abraham Robinson (Fund. Math. 14 (1957), 309-329). The sequence $P$ corresponds to a procedure which is "effective" with respect to polynomial factorization, in the sense of a natural extension of Herbrand-Gödel-Kleene recursion to arbitrary structures. Our technique follows that of G. Herrmann (Math. Ann. 95 (1926), 736-788). (Received June 29, 1965.)

65T-353. P. R. STRAUSS, 157 South Harrison Street, East Orange, New Jersey 07018. **Topologies and Borel structures associated with Banach algebras of functions, I. Preliminary report.**

An admissible Banach algebra $B$ of bounded functions on a set $E$ is the kind of E. R. Lorch, Acta Sci. Math. (Szeged) XXIV (1963), 204-218. For each infinite cardinal $\kappa$, $\alpha$, $\beta$, $\delta$, $\lambda$, there are associated to such $B$ completely regular topologies, $\tau_\alpha$, and related classes of Borel sets, semi-continuous functions, Daniell integrals, weak m.p.c. in $B$, and $\beta$-pseudocompact spaces as well as related generalizations of Baire sets, Baire functions, and $\beta$-realcompact spaces. The $\tau_1$-topology coincides with the $\tau$ and those $E$ which are $\tau_\alpha$-dense in $E^\alpha$ are described by equivalent statements corresponding to those in Lorch's Theorem 11. These spaces are $\beta$-second (Baire) category and Baire spaces in their $\tau$-topologies, $\tau$-meager subsets being $\tau$-nowhere dense. Classical Baire-measurable subsets are thus Baire spaces in their $\tau$-topologies. The generalized Baire functions of type $\alpha$, denoted $1^\alpha(B)$, coincide with the classical ones when $\alpha = 1$. For $\kappa$ regular the weak $1^\alpha(B)$-topology is $\tau_{\kappa}$. The subalgebra $\Lambda^\alpha(B)$ of $1^\alpha(B)$ is admissible, also induces the $\tau_{\alpha}$-topology, and for a large enough the determining sets of $\tau(\Lambda^\alpha(B))$ are the Borel sets. (Received June 30, 1965.)
On the convergence of nth order spline functions.

Let \( \{X\}_r = \{x_{r,0}, x_{r,1}, \ldots, x_{r,n}\} \), \( r = 1, 2, \ldots, n \), \( n = n(r) \) be a sequence of subdivisions of \([0,1]\) such that the norm of the subdivision tends to 0. Let \( x_{r,kn+v} = x_{r,v} + k, k = \pm 1, \pm 2, \ldots; \) \( v = 0, 1, 2, \ldots, n - 1 \). Theorem. Let \( f(x) \) be of class \( C^{2m-2}(-\infty, \infty) \) and let it be periodic with period 1. Let \( \phi_r(x) \) be the periodic spline function of order \( 2m \) interpolating to \( f(x) \) at the points \( \{x_{r,i}\}_{i=0}^n \). Then \( \phi_r^{(k)}(x) \rightarrow f^{(k)}(x) \) uniformly for \( x \in [0,1] \) \( (k = 0, 1, \ldots, 2m - 2) \). Theorem. Let \( f(x) \) be of class \( C^{2m-1}(-\infty, \infty) \) with period 1. Let \( \phi_r(x) \) be the interpolatory spline as above. If there exists a universal constant \( K \) (independent of \( r \)) such that for every set of \( 2m - 1 \) consecutive points \( (y_1, \ldots, y_{2m-1}) \) from \( \{x_{r,i}\}_{i=-\infty}^{\infty} \) we have \( \max_{2 \leq i \leq 2m-1} (y_i - y_{i-1}) \leq K \min_{2 \leq i \leq 2m-1} (y_i - y_{i-1}) \), then \( \phi_r^{(k)}(x) \rightarrow f^{(k)}(x) \) uniformly for \( x \in [0,1] \) \( (k = 0, 1, \ldots, 2m - 1) \). These results represent an extension to higher order splines of results obtained by Sharma and Meir (Abstract 64T-496, these Notices 11 (1964), 768). (Received June 30, 1965.)

Local neighborhood strong deformation retraction.

Theorem. Suppose that \( X \) is a collectionwise normal space and \( A \) is a closed subset of \( X \) which is fully normal. Then \( A \) is a neighborhood strong deformation retract of \( X \) iff for each \( a \in A \) there is a neighborhood \( N \) of \( a \) in \( X \) such that \( A \) is a strong deformation retract in \( X \) of \( A \cup N \). \( \) That the set \( N \) may be taken to be a neighborhood of \( A \) in \( X \) is the defining condition for neighborhood strong deformation retract.\( \) Remark 1. A similar theorem holds for neighborhood retracts, but the localizing condition is not the obvious one. \( \) Remark 2. The theorem and embedding in convex sets may be used to prove Hanner's theorem that ANR (metrizable) is a local property. (Received November 23, 1964.)

Covering constants of some non-convex domains. Preliminary report.

Let \( S \) be an \( n \)-dimensional set and \( A \) an \( n \)-dimensional lattice. \( (S,A) \) is said to be a lattice covering of the \( n \)-dimensional space \( R^n \) if \( R^n \subseteq \bigcup_{A} c(A) + A \). \( c(S) = \sup d(A) \) for all lattice coverings \( (S,A) \) is said to be the covering constant for \( S \). It is known that if \( S \) is a symmetrical convex domain in \( R_2 \), then \( c(S) = H(S) \) where \( H(S) \) denote the area of the largest hexagon inscribed in \( S \). The only other sets for which \( c(S) \) is known are \( n \)-dimensional space filling bodies, cylinders with a two dimensional symmetric convex base and three dimensional spheres. In this paper a method is developed to determine \( c(S) \) for a number of non-convex domains in \( R_2 \). (Received June 30, 1965.)

Analogue of a theorem of Khintchine in a field of formal Laurent series. Preliminary report.

Let \( K \) be a field, \( K[t] \) the ring of polynomials and \( K(t) \) be the field of rational functions and \( K[t] \) be the field of formal Laurent series. Then elements of \( K[t] \) are of the form \( x = a_n t^n + a_{n-1} t^{n-1} + \ldots \) (up to \( -\infty \)), \( a_i \in K \). If \( a_n \neq 0 \), define a valuation in \( K[t] \) by \( |x| = e^n (e > 1) \)
and \(|0| = 0\). Also define \(\|x\| = |x|\) if \(n < 0\); \(\|x\| = |a_1 t^{-1} + a_2 t^{-2} + \ldots|\) if \(n \geq 0\). Then we prove the following analogue of a theorem of Khintchine. \textbf{Theorem.} Let \(K\) be a finite field. Let \(m, n\) be positive integers. Let \(L_t(U) = L_t(u_1, \ldots, u_n)(1 \leq i \leq m)\) be a system of linear form over \(K[t]\). Then there exist \(C = (C_1, \ldots, C_m)\), \(C_i \in K[t]\) \((1 \leq i \leq m)\) such that \((\max_{1 \leq i \leq m} \|L_t(U) + C_i\|)^n \cdot (\max_{j \geq 1} |u_j|)^m \geq e^{-m^{-2n}}\) for all \(0 \neq (u_1, \ldots, u_n) \in K^n\) \((1 \leq j \leq n)\). However, for \(m = n = 1\), we can prove that \(|u|\cdot \|\theta u + c\| \geq e^{-2}\) for all non-zero \(u \in K\).

65T-358. L. P. BELLUCE and S. K. JAIN, University of California, Riverside, California. \textbf{Prime rings with a one-sided ideal having a polynomial identity.}

Let \(R\) be a prime ring and \(I\) a non-zero right ideal satisfying a polynomial identity. The following results are obtained. \textbf{Theorem 1.} If \(I \neq 0\) then \(R\) also satisfies a polynomial identity. \textbf{Theorem 2.} If \(R\) is right-quotient simple then \(R\) satisfies a polynomial identity. \textbf{Theorem 3.} \(R\) satisfies a polynomial identity iff \(\hat{R}\), the right singular ideal is 0, and \(\hat{R}\), the right quotient ring has at most finitely many orthogonal idempotents. These results extend similar results of the authors announced in Abstract 614-89, these \textit{Notices} 11 (1964), 554. (Received June 29, 1965.)

65T-359. ISRAEL NAVOT, Technion, Israel Institute of Technology, Haifa, Israel. \textbf{The synthesis of a certain subclass of tri-diagonal matrices with prescribed eigenvalues.}

A well known technique for the calculation of eigenvalues and eigenvectors of real matrices is their initial reduction to a tri-diagonal form by orthogonal transformations (for references see, for example, La Budde, Math. Comp. 17 (1963), 433-437). We consider the inverse problem of finding a certain subclass of tri-diagonal matrices with prescribed eigenvalues, i.e., given \(D = \text{diag.} (\lambda_1, \lambda_2, \ldots, \lambda_n)\); Real \(\lambda_i > 0\) complex \(\lambda_i\) appear in conjugate pairs, we have to determine all tri-diagonal matrices \(K = \|k_{ij}\|\) with \(k_{11}, k_{nn}\) and \(k_{i,i+1}\) positive, \(k_{i+1,i}\) negative \((i = 1, 2, \ldots, n; n = 1, 2, \ldots)\) and all other \(k_{ij}\) zero such that \(|K - \lambda I| = |D - \lambda I|\). In network theory we associate with \(K + sI\) a reactive ladder network terminated in resistances and express the network functions of interest in terms of continuants obtained from \(K + sI\). From this aspect it is easy to show that there are \(2[(n-1)/2]\) basically different ladders and therefore the same number of essentially different matrices \(K\). An algorithm for calculating these matrices is described. (Received June 30, 1965.)

65T-360. T. T. ROBINSON, 2107 Grange Drive, Urbana, Illinois 61801. \textbf{Prenex normal form in predicate calculi which are classical in implication and minimal in negation.}

In Abstracts 64T-381, 64T-414 of these \textit{Notices} 11 (1964), 592, 668, the author gave a sequenzen-type propositional calculus \(PS_\sim\) which is classical in \(\lor\) and minimal in \(\sim\). Let \(LS_\sim\) be the predicate calculus obtained by adjoining Kleene's \(V\) - \(3\) -rules (\textit{Intro}, Metamathematics, pp. 442-443) to \(PS_\sim\). We then obtain a non-intuitionistic fragment of the classical predicate calculus which Curry (\textit{Foundations of Math}, Logic, p. 261) calls classical refutability. For example, 

\[
\sim \sim A \lor A \Rightarrow B \text{ is not a theorem schema of } LS_\sim. 
\]

This Abstract is to report that every wff of \(LS_\sim\) is provably equivalent to its prenex normal form in \(LS_\sim\). We show that quantifiers can be moved outside \(\sim\) despite the special restriction on \((\sim \rightarrow)\) in \(LS_\sim\) (omitting some structural steps):

\[
\begin{align*}
\Lambda &\rightarrow \Lambda, \rightarrow \Lambda, \sim \Lambda (\sim \rightarrow); \rightarrow \Lambda (\exists a) \sim \Lambda (\rightarrow \exists); \rightarrow (a) \Lambda, (\exists a) \sim \Lambda (\rightarrow \forall); \rightarrow (\exists a) \sim \Lambda, \sim \Lambda, (a) \Lambda (\rightarrow P, W);
\end{align*}
\]
65T-361. M. C. McCORD, University of Georgia, Athens, Georgia. Singular homology groups and homotopy groups of finite spaces.

**Theorem.** Given any finite simplicial complex $K$, there is a finite $T_0$ topological space $X_K$ having the same singular homology groups and homotopy groups as $|K|$. This is a consequence of the stronger Theorem 1 below. Let $K$ be an arbitrary (not necessarily finite) simplicial complex, with the weak topology on the associated polyhedron $|K|$. For each simplex $s$ of $K$, let $s^*$ denote the set of all faces of $s$. A space $X_K$ is constructed as follows. The points of $X_K$ are the simplexes of $K$. As a basis for the open sets of $X_K$ take the collection $\{s^* : s \in K\}$. It is possible to define, using the first barycentric subdivision of $K$, a continuous map $f_K$ of $|K|$ onto $X_K$ satisfying Theorem 1 below.

Let $H_q$ denote singular homology. Theorem 1. For each $q$, the induced homomorphisms $(f_K)^*: H_q(|K|) \rightarrow H_q(X_K)$ and $(\pi_q)^*: \pi_q(|K|) \rightarrow \pi_q(X_K)$ (for any base point) are isomorphisms. For each simplicial map $g: K \rightarrow L$ there is associated a continuous map $g': X_K \rightarrow X_L$ such that $g'f_K = f_Lg$.

Theorem 2. If $X$ is a finite space with $n$ points, then $H_q(X)$ is finitely generated for all $q$, and $H_q(X) = 0$ for $q > \max(0, (n - 2)/2)$. The latter inequality is the "best possible." Theorem 3. Every finite space has the same homotopy type as a finite $T_0$ space. (Received July 1, 1965.)

65T-362. V. C. DUMIR, The Ohio State University, Columbus, Ohio 43210. One sided inequalities for inhomogeneous indefinite quadratic forms. Preliminary report. Let $Q(x_1, x_2, \ldots, x_n)$ be a real indefinite quadratic form in $n$ variables with determinant $D \neq 0$ and signature $s$. Then it is known that there exists a constant $\gamma_{n,s}$ depending only on $n$ and $s$ such that given any real numbers $a_1, \ldots, a_n$ we can find integers $x_1, x_2, \ldots, x_n$ satisfying $0 < Q(x_1 + a_1, x_2 + a_2, \ldots, x_n + a_n) \leq (\gamma_{n,s} |D|)^{1/n}$. Denoting by $\gamma_{n,s}$ the least possible value of $\gamma_{n,s}$ the following results are obtained. $\gamma_{3,-1} = 8, \gamma_{4,0} = 16, \gamma_{4,2} = 16/3$ and the critical forms in each case are described. The value $\gamma_{2,0} = 4$ is due to H. Davenport and H. Heilbronn and $\gamma_{3,1} = 4$ is due to E. S. Barnes. (Received July 1, 1965.)

65T-363. R. F. BARNES, JR., 1127D Ninth Street, Albany, California. The classification of recursive sets of number-theoretic functions. The results of Abstract 622-67, these Notices 12 (1965), 346, are affectivized to yield a classification of the $\Delta^0_1$ (i.e., $\sum^0_1 \cap \Pi^0_1$, or recursive) subsets of $\mathbb{N}^N$; special notation is as there defined. Let $\{\varphi^n_e : e \in \mathbb{N}\}$ be a standard enumeration of the partial recursive functions. Let $K^\#_0 = \{\langle 0, \varphi^0_e \rangle, \langle 1, \mathbb{N}^N \rangle\}$ and for any positive ordinal $\nu$ let $K^\#_\nu = \{\langle e, X \rangle : \forall n (\varphi^n_e(n), X(n)) \in \bigcup \{K^\#_\mu : \mu < \nu\}\}$; for any ordinal $\nu$ let $K^\#_\nu$ be the range of $K^\#_\nu$. Then $K^\#_\nu|_{\omega_1}$ is a hierarchy invariant under recursive isometry and $K^\#_\nu|_{\omega_1} = \bigcup \{K^\#_\nu : \nu < \omega_1\}$. Strong separation principle. For any disjoint $\prod^0_1$ subsets.
X, Y of \( N^N \); X, Y are Ka\( \omega \), -separable. Corollary (Construction principle). For any subset X of \( N^N \), 
\[ X \in \Delta^0 \iff X \in Ka\omega. \]

For any subset X of \( N^N \) let 
\[ C(X) = \{ S: S \subseteq \Delta^0 \land (\forall u \in S) \exists v \in S \implies u \in S \}. \]

and let 
\[ B(X) = \{ S \cap T: S \in C(X) \land T \in C(\sim X) \}. \]

Separability test. For any disjoint subsets X, Y of \( N^N \) and any ordinal \( \nu \), X, Y are Ka\( \omega \), -separable \( \iff \) for some \( S \in B(X) \) and \( T \in B(Y) \), 
\[ \Delta^0, \nu \vdash \text{Dv}(S \cap T) = \emptyset. \]

Corollary (Classification test). For any subset X of \( N^N \) and any ordinal \( \nu \), 
\[ X \in \text{Ka}_\omega^\nu \iff \exists S \in B(X), \text{Dv}(S) = \emptyset. \]

Theorem. For any ordinal \( \nu \), 
\[ \text{Ka}_\omega^\nu \subseteq \text{Ka}_\omega^\nu \cap \Delta^0 \nu, \quad \nu < \omega \implies \text{Ka}_\omega^\nu \cap \Delta^0 \nu = \text{Ka}_\omega^\nu \cap \Delta^0 \nu. \]

(Received July 1, 1965.)

65T-364. A. A. SAGLE, University of California, Los Angeles, California 90024. On simple algebras obtained from reductive Lie algebras.

A Lie subalgebra \( h \) of a Lie algebra \( g \) is reductive in \( g \) if there exists a complementry subspace \( m \) of \( h \) such that \( g = m + h \) (vector space direct sum) and \([m, h] \subseteq m\), where \([a, b]\) denotes multiplication in \( g \). For \( x, y \in m \) an anticommutative multiplication \( xy \) is introduced in \( m \) by \([x, y] = xy + h(x, y)\), where \( xy \) (resp. \( h(x, y) \)) is the component of \([x, y] \in g\) in \( m \) (resp. \( h \)) relative to a fixed decomposition \( g = m + h \). A general structure theory is developed concerning the semi-simplicity of \( g \) and \( h \), the \( h \)-complete reducibility of \( m \), and the "semi-simplicity" of the algebra \( m \) over fields of characteristic zero. Some results are: (1) if \( m \) is \( h \)-irreducible and \( \text{m}^2 \neq 0 \), then \( m \) is simple; (2) if \( g \) and \( h \) are simple and \( \text{m}^2 \neq 0 \), then \( m \) is simple. Derivations of the algebra \( m \) are also discussed. (Received July 2, 1965.)

65T-365. ALEXANDER ABIAN and D. L. DEEVER, The Ohio State University, 231 West 18th Avenue, Columbus, Ohio 43210. On representation of partially ordered sets.

Definition. Let \( (\pi_k) \) and \( (\lambda_j) \) be two sequences (of the same finite or transfinite type) made up of the numbers 0, 1 and the letter \( u \). The sequence \( (\pi_k) \) is said to be less than or equal to the sequence \( (\lambda_j) \) according to the principle of first numerical difference if \( (\pi_k) \) is equal (identical) to \( (\lambda_j) \), or, if \( (\pi_k = 1) \) \( \implies (\lambda_k = 1) \) for every index \( k \) and \( \pi_j = 0 \) and \( \lambda_j = 1 \) for some index \( j \). Treating a cardinal number as an initial ordinal number the following is established. Theorem. Every partially ordered set \( P \) is isomorphic to a set of sequences of type \( P \) made up of 0, 1, \( u \) and ordered according to the principle of first numerical difference. (Received July 2, 1965.)

65T-366. R. M. SORENSEN, 3400 Toledo Terrace, Apartment J-1, Hyattsville, Maryland. Differential-integral calculus for abstract algebraic-topological structures. II.

The development previously outlined (Abstract 620-17, these Notices 12 (1965), 214) is continued. Let \( B \) be the collection of all closed intervals \( I \) in the real numbers \( R \), and let \( F, f \) be two mappings of \( R \) into \( R \). Definition. \( f \) is the derivative of \( F \) on \( I \) in \( B \) iff given any \( I_0 \) in \( B \) such that \( I_0 \subseteq I \), and any two distinct points \( x, y \) in \( I_0 \), there exists a third point \( z \) in \( I_0 \) such that \( F(x) - F(y) = f(z)(x - y) \). Rolle's Theorem is immediate as are: Theorem 1. If \( f \) is the derivative of \( F \) on \( I \) in \( B \), then \( f \) is the derivative of \( F + C \) on \( I \), in which \( C \) is any constant; furthermore, \( cf \) is the derivative of \( cf \) on \( I \); \( c \) any constant, Theorem 2. If \( f \) is the derivative of \( F \) on \( I \) in \( B \), and if \( H \) is defined on \( I \) by \( H(x) = F(x)G(x) \), then \( H \) has a derivative \( h \) on \( I \) with \( h(z) = f(z_1) + g(z_2) \) in which \( z \) corresponds to \( (z_1, z_2) \) in a one to one correspondence of \( I \) to \( I \times I \). The intermediate value

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Theorem for derivatives is proved as well as: **Theorem 3.** If \( f \) is the derivative of \( F \) on \( I \subseteq B \), and if \( F \) has a maximum at \( c \in I \), then either \( f(c) = 0 \) or \( c \) is a limit point of zeros of \( f \). (Received July 6, 1965.)

65T-367. **K. SAVITHRI**, Defence Science Laboratory, Meh Calf House, Delhi 6, India.

**The class number problem.**

The author wishes to announce that, using summability arguments, it is possible to give a (rather long) proof to obtain the results of Siegel (Acta Arith., 1931) and Chowla (Math. Z., 1933) on the class number problem of imaginary quadratic fields. A more detailed paper will be communicated to the next conference of the Indian Mathematical Society. Publication is delayed because some improvement of the techniques is likely to give some results for algebraic number fields. It will be interesting to pursue by these methods the importance of Siegel's zeros. (Received June 11, 1965.)
ERRATA

Volume 11

GEORGE GLAUBERMAN. Fixed point subgroups which contain centralizers of involution. Preliminary report. Page 760, Abstract 618-5

Line 5. Omit the condition "if 3 does not divide |G|".

Volume 12


Line 6. "X closed in X" should read "S closed in X".

Line 5, page 86. "∂X" should read "vX" (where vX is the Hewitt real-compactification of X).


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Line 5. "\subseteq" should read "\rightarrow".

Line 6. "R \subseteq P" should read "R \subseteq P".

Line 1, page 117. After "yield different y's" insert "if sup_{(0, x]} |u| > 0 whenever x > 0".


Line 1. "such that \Sigma a_i^2 and \Sigma b_i^2 exist" should read "such that \Sigma |a_i| and \Sigma |b_i| exist".


The proof of the asserted theorem has turned out to be incorrect.


Lines 3-4.

For \"K_{v-1/2}(z \cosh \tau), z > 0, I_m \nu = 0; \int_0^\infty \cos x \kappa x_{2/3} (y/z \cosh x) + \cosh x \sinh x \kappa x_{5/3} (y/z \cosh x), y > 0,\"  

Read \"K_{v-1/2}(z \cosh \tau), z > 0, I_m \nu = 0; \int_0^\infty \cos x \kappa x_{2/3} (y/z \cosh x) + \cosh x \sinh x \kappa x_{5/3} (y/z \cosh x), y > 0,\".
On the occasion of the Fiftieth Anniversary Meeting of The Mathematical Association of America, D. Van Nostrand would take this opportunity to congratulate the members and staff of the Association and to thank them, as individuals, for their contribution to the success of the Van Nostrand publishing program in mathematics. This program is embodied chiefly in the three series listed below in which we take special note of recent and forthcoming volumes.

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