**MEETINGS**

**Calendar of Meetings**

NOTE: This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the *Notices* was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

<table>
<thead>
<tr>
<th>Meeting No.</th>
<th>Date</th>
<th>Place</th>
<th>Deadline for Abstracts*</th>
</tr>
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<tbody>
<tr>
<td>631</td>
<td>February 26, 1966</td>
<td>New York City</td>
<td>Jan. 13</td>
</tr>
<tr>
<td>632</td>
<td>April 4-7, 1966</td>
<td>New York City</td>
<td>Feb. 18</td>
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<tr>
<td>633</td>
<td>April 9, 1966</td>
<td>Honolulu, Hawaii</td>
<td>Feb. 18</td>
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<tr>
<td>634</td>
<td>April 21-23, 1966</td>
<td>Chicago, Illinois</td>
<td>Feb. 18</td>
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<tr>
<td>635</td>
<td>June 18, 1966</td>
<td>Victoria, British Columbia</td>
<td>May 4</td>
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<tr>
<td></td>
<td>August 29 - September 2, 1966 (71st Summer Meeting)</td>
<td>New Brunswick, New Jersey</td>
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<td></td>
<td>January 24-28, 1967</td>
<td>Houston, Texas</td>
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<tr>
<td></td>
<td>August 28 - September 1, 1967 (72nd Summer Meeting)</td>
<td>Toronto, Ontario, Canada</td>
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<tr>
<td></td>
<td>January, 1968</td>
<td>San Francisco, California</td>
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</table>

*The abstracts of papers to be presented *in person* at the meetings must be received in the Headquarters Offices of the Society in Providence, Rhode Island, on or before these deadlines. The deadlines also apply to news items. The next two deadline dates for the by title abstracts are February 11, 1966 and April 27, 1966.*
Seventy-Second Annual Meeting
Sherman House
Chicago, Illinois
January 24-28, 1966

PROGRAM

The seventy-second Annual Meeting of the American Mathematical Society will be held at the Sherman House in Chicago, Illinois. This meeting will be held in conjunction with the Annual Meeting of the Mathematical Association of America. The Society will meet from Monday, January 24 through Thursday, January 27. The Mathematical Association of America will meet from Wednesday, January 26 through Friday, January 28.

The thirty-ninth Josiah Willard Gibbs Lecture, "Stellar evolution," will be delivered by Professor Martin Schwarzschild of Princeton University in the Grand Ballroom of the Sherman House at 8:00 P.M. on Monday, January 24, 1966.

As Past President of the Society, Professor J. L. Doob will give an address, "Application to analysis of a topological definition of smallness of a set," in the Grand Ballroom of the Sherman House at 3:00 P.M. on Wednesday, January 26.

By invitation of the Committee to Select Hour Speakers for the Annual and Summer Meetings, an address will be given by Professor Richard Askey of the University of Wisconsin on "Norm inequalities for some orthogonal expansions" and an address will be given by Professor George Glauberman of the University of Chicago on "Sylow 2-subgroups of finite groups." Professor Askey will speak at 2:00 P.M. on Wednesday, January 26, and Professor Glauberman's talk will be given at 2:00 P.M. on Thursday, January 27. Both of these addresses will be presented in the Grand Ballroom.

The Veblen Prize will be awarded in the Grand Ballroom at 2:00 P.M. on Monday, January 24.

As at the past three Annual Meetings of the Society, there will be sessions of selected twenty-minute papers. The first Special Session will be held on Monday, January 24, at 3:15 P.M. in the Grand Ballroom. This session on Partial Differential Equations was arranged by Professor Felix Browder; it will consist of papers by R. Seeley, J. Serrin, T. Kato, F. Browder, and E. Nelson. A Special Session on Markov Processes, with the program arranged by Professor Wendell Fleming, will be held on Tuesday, January 25, at 9:00 A.M. in the Grand Ballroom. The papers for this session are by F. B. Knight, J. Lamperti, S. Port, Y. S. Chow and H. E. Robbins, R. A. Gangolli, S. Orey and Benton Jamison. Another Special Session will be held on Tuesday afternoon at 3:15 P.M. in the Grand Ballroom. Professor Herbert Ryser has organized the program of this session on Combinatorial Mathematics, in which the speakers will be R. H. Bruck, D. R. Fulkerson, G. -C. Rota, V. Klee and D. W. Walkup, and M. Marcus. The fourth Special Session will be devoted to papers on Algebraic Groups. Professor Robert Steinberg has organized the program of this session which includes papers by J. -I. Igusa, G. D. Mostow, T. Ono, T. A. Springer, and E. J. Taft. This session on Algebraic Groups begins at 3:15 P.M. on Thursday, January 27 in the Grand Ballroom.

There will be sessions for contributed ten-minute papers at 9:00 A.M. and 3:15 P.M. on Monday, 9:00 A.M. and 2:00 P.M. on Tuesday, and 3:15 P.M. on Thursday. No sessions for late papers will be held.

The Business Meeting of the Society will be held at 11:00 A.M. on Monday, January 24 in the Grand Ballroom.

The Council of the Society will meet at 4:00 P.M. on Sunday, January 23 in the Ruby Room on the First Floor.
REGISTRATION

The Registration Desk for this meeting will be in the George Bernard Shaw Room on Sunday and in the foyer of the Grand Ballroom on the Mezzanine Floor of the Sherman House on Monday through Friday. The Registration Desk will be open from 2:00 P.M. to 8:00 P.M. on Sunday, January 23; from 8:00 A.M. to 5:00 P.M. Monday through Thursday; and from 9:00 A.M. to 3:00 P.M. on Friday. Those persons who have sent in their Advance Registration can pick up badges at the Registration Desk. Others must go through the usual registration process. It is requested that everyone attending the meeting register or pick up his badge; if registered in advance, as soon as possible after arrival. The fees for registration are as follows:

- Member: $2.00
- Member's family: .50
- Student: No fee
- Others: $5.00

for the first such registrant and no charge for additional registrants.

The Employment Register will be maintained from 9:00 A.M. to 5:00 P.M. Tuesday, Wednesday and Thursday on the Mezzanine Floor in Parlors J and K. Book and other exhibits will be in the Exhibit Hall on the Mezzanine Floor. The exhibits will be open from Tuesday through Thursday.

All mail and telegrams for those attending the meeting should be addressed in care of the American Mathematical Society, Sherman House, Clark & Randolph, Chicago, Illinois.

A Message Center will be maintained in the Registration Area providing a central location where members at the meeting may receive telephone calls. All calls will be taken and filed at this booth. These messages can be picked up from the operator at the booth. The number of the Message Center will be 726-9346. The Chicago Area Code is 312. The Center will be open from 1:30 to 8:00 P.M. on Sunday, January 23; from 8:00 A.M. to 5:00 P.M. on Monday through Thursday; and from 9:00 A.M. to noon on Friday.

Those wishing to make reservations after January 10 should write to the hotel of their choice. The accompanying map gives the location of the various hotels that have reserved rooms for the meeting, including the headquarters hotel, the Sherman House.

ACCOMMODATIONS

Room rates are listed below the map.

ENTERTAINMENT AND RECREATION

Illinois Institute of Technology and the University of Illinois, Chicago Circle, will be hosts at a tea to which all members of the American Mathematical Society and the Mathematical Association of America are cordially invited. The tea will take place on the Mezzanine Floor of the Sherman House on Wednesday afternoon, January 26, from 4:30 to 6:00 P.M.

Since the Sherman House is right in the heart of Chicago's busy Loop, it is convenient to many of the city's attractions and amenities. The Merchandise Mart, Marina City, theatres, fine restaurants, specialty shops, art galleries, book stores, as well as the celebrated department stores on State Street and the elegant shops that line Michigan Avenue's "Magnificent Mile," are all within easy walking distance.

The Art Institute of Chicago, with its collection of French impressionist paintings, considered the world's finest, is only a few blocks from the hotel; and such notable institutions as the Adler Planetarium, the Shedd Aquarium, and the Chicago Museum of Natural History are all readily accessible. There is an efficient transit system, radiating from the Loop, to take visitors to the many points of interest that lie outside the immediate area of downtown Chicago.

At the Registration Desk on the Mezzanine Floor of the Sherman House, brochures and folders can be obtained giving more detailed information on how members may add to the enjoyment of their stay in Chicago. Arrangements can be made for special trips to points of professional and educational interest as well as for general sight-seeing bus tours. Recommendations for restaurants, shops, and places of entertainment will also be available at the Registration Desk.
LOCAL AND TRAVEL INFORMATION

The normal temperature in Chicago during the month of January is 26°F., and the precipitation is normally 1.9 inches.

Chicago is easily reached from any part of North America. Its airport, O'Hare Field, is the world's busiest; and there are 22 railroad lines serving the city. Excellent expressways lead into Chicago for those who prefer to travel by car, and these expressways also permit frequent fast service by the bus lines.

The airport limousine service from O'Hare Field arrives and departs from the La Salle Street entrance of the Sherman House every half hour; the fare is $2.00. The railroad stations are well served by taxis, and the trip to the hotel from any of the stations should cost about $1.00. The Greyhound Bus Lines terminal is right across the street from the Clark Street entrance to the Sherman House.

ARRANGEMENTS

| TIME TABLE  
(Central Standard Time) |  
|--------------------------|  
| **SUNDAY**  
January 23 |  
| 1:30 P.M. - 8:00 P.M. | MESSAGE CENTER -- MEZZANINE  
| 2:00 P.M. - 8:00 P.M. | REGISTRATION -- GEORGE BERNARD SHAW ROOM  
| 4:00 P.M. | Council Meeting  
Ruby Room -- First Floor  
| **MONDAY**  
January 24 |  
| 8:00 A.M. - 5:00 P.M. | MESSAGE CENTER -- MEZZANINE  
| 8:00 A.M. - 5:00 P.M. | REGISTRATION -- MEZZANINE  
| 9:00 A.M. | Session on Analysis I  
Grand Ballroom, Mezzanine  
Session on Analysis II  
Assembly Room, Mezzanine  
Session on Logic and Foundations  
Gold Room, First Floor  
Session on Algebra and Theory of Numbers  
Louis XVI Room, First Floor  
Session on Algebra  
Crystal Room, First Floor  
11:00 A.M. | Business Meeting  
Grand Ballroom, Mezzanine  
2:00 P.M. | Veblen Prize Award and Lecture  
Grand Ballroom, Mezzanine  
3:15 P.M. | Special Session on Partial  
Differential Equations  
Felix Browder, Chairman  
Grand Ballroom, Mezzanine  
Session on Analysis I  
Assembly Room, Mezzanine  
Session on Topology  
Gold Room, First Floor  
Session on Algebra I  
Louis XVI Room, First Floor  
Session on Algebra II  
Crystal Room, First Floor  
8:00 P.M. | Gibbs Lecture  
Stellar evolution  
Martin Schwarzschild  
Grand Ballroom, Mezzanine  
| **TUESDAY**  
January 25 |  
| 8:00 A.M. - 5:00 P.M. | MESSAGE CENTER -- MEZZANINE  
| 8:00 A.M. - 5:00 P.M. | REGISTRATION -- MEZZANINE  
| EXHIBITS -- MAIN EXHIBITION HALL  
EMPLOYMENT REGISTER -- MEZZANINE PARLORS J & K  
<p>|</p>
<table>
<thead>
<tr>
<th>Time</th>
<th>AMERICAN MATHEMATICAL SOCIETY</th>
<th>MATHEMATICAL ASSOCIATION OF AMERICA</th>
</tr>
</thead>
</table>
| 9:00 A.M. | Special Session on Markov Processes  
W. Fleming, Chairman  
Grand Ballroom, Mezzanine  
Session on Analysis  
Assembly Room, Mezzanine  
Session on Algebra  
Louis XVI Room, First Floor  
Session on Topology  
Crystal Room, First Floor  
Session on Geometry  
Gold Room, First Floor | Board of Governors Meeting  
Ruby Room, First Floor |
| 10:00 A.M. |                                                                                           |                                                                                                |
| 2:00 P.M. | Special Session on Combinatorial Mathematics  
Herbert J. Ryser, Chairman  
Grand Ballroom, Mezzanine  
Session on Topology  
Assembly Room, Mezzanine  
Session on Analysis I  
Louis XVI Room, First Floor  
Session on Analysis II  
Crystal Room, First Floor  
Session on Applied Mathematics and Numerical Analysis  
Gold Room, First Floor |                                                                                                |
| 7:15 P.M. |                                                                                           | Film: "The Analog Computer and its Application to Ordinary Differential Equations"  
Grand Ballroom, Mezzanine |
| 7:50 P.M. |                                                                                           | Film: "Can you hear the Shape of a Drum?  
A Lecture by Mark Kac"  
Grand Ballroom, Mezzanine |
| 9:00 P.M. |                                                                                           | Film: "Challenge in the Classroom; The Methods of R. L. Moore"  
Grand Ballroom, Mezzanine |

### WEDNESDAY

<table>
<thead>
<tr>
<th>Time</th>
<th>AMS</th>
<th>MAA</th>
</tr>
</thead>
<tbody>
<tr>
<td>8:00 A.M. – 5:00 P.M.</td>
<td>MESSAGE CENTER--MEZZANINE</td>
<td></td>
</tr>
</tbody>
</table>
| 8:00 A.M. – 5:00 P.M. | REGISTRATION--MEZZANINE  
EXHIBITS--MAIN EXHIBITION HALL  
EMPLOYMENT REGISTER--MEZZANINE PARLORS J & K |       |
<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
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<tbody>
<tr>
<td>9:00 A.M.</td>
<td>Invited Address: Norm Inequalities for Some Orthogonal Expansions</td>
</tr>
<tr>
<td>2:00 P.M.</td>
<td>Presidential Address: Application to Analysis of the Topological</td>
</tr>
<tr>
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<td>Definition of Smallness of a Set</td>
</tr>
<tr>
<td>4:30 P.M.</td>
<td>Grand Ballroom, Mezzanine</td>
</tr>
<tr>
<td>7:15 P.M.</td>
<td>Grand Ballroom, Mezzanine</td>
</tr>
<tr>
<td>7:50 P.M.</td>
<td>Grand Ballroom, Mezzanine</td>
</tr>
<tr>
<td>9:00 P.M.</td>
<td>Grand Ballroom, Mezzanine</td>
</tr>
</tbody>
</table>

**THURSDAY January 27**

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
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<tbody>
<tr>
<td>8:00 A.M.</td>
<td>Invited Address Sylow 2-subgroups of finite groups</td>
</tr>
<tr>
<td>10:40 A.M.</td>
<td>General Discussion</td>
</tr>
<tr>
<td>11:20 A.M.</td>
<td>Business Meeting</td>
</tr>
<tr>
<td>2:00 P.M.</td>
<td>Film: &quot;Norbert Wiener--Scientist and Philosopher&quot;</td>
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<td>Film: &quot;The Search for Solid Ground, A Panel Discussion&quot;</td>
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<tr>
<td></td>
<td>Film: &quot;Mathematical Induction, Two Lectures by Leon A. Henkin&quot;</td>
</tr>
</tbody>
</table>
| **THURSDAY**  
*January 27* | **AMERICAN MATHEMATICAL SOCIETY** | **MATHEMATICAL ASSOCIATION OF AMERICA** |
|---|---|---|
| 3:15 P.M. | Special Session on Algebraic Groups  
Robert Steinberg, Chairman  
Grand Ballroom, Mezzanine  
General Session  
Assembly Room, Mezzanine  
Session on Analysis I  
Louis XVI Room, First Floor  
Session on Analysis II  
Crystal Room, First Floor | Grand Ballroom, Mezzanine  
Film: "Infinite Acres" by Melvin Henriksen  
Film: "Two Puppet Shows" by Charles and Ray Eames  
Film: "Mathematics of the Honeycomb"  
Film: "The Classical Groups as a Source of Algebraic Problems. Lecture by C, W. Curtis"  
Film: "Pits, Peaks, and Passes. A lecture on Critical Point Theory by Marston Morse" |
| 7:15 P.M. |  | |
| 7:23 P.M. |  | |
| 7:35 P.M. |  | |
| 8:00 P.M. |  | |
| 9:10 P.M. |  | |

| **FRIDAY**  
*January 28* | **AMS** | **MAA** |
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<tr>
<td>9:00 A.M.-12 noon</td>
<td>MESSAGE CENTER--MEZZANINE</td>
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<tr>
<td>9:00 A.M.-3:00 P.M.</td>
<td>REGISTRATION--MEZZANINE</td>
<td></td>
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</tbody>
</table>
| 9:00 A.M. |  | Grand Ballroom, Mezzanine  
Mathematical Theory of Optimal Stopping Rules  
H. E. Robbins  
Function Space Integrals and Non-linear Equations  
M. D. Donsker  
Who Killed Determinants?  
K. O. May |
| 10:00 A.M. |  |  |
| 11:00 A.M. |  |  |
| 2:00 P.M. |  | Grand Ballroom, Mezzanine  
Symposium on Recent Advances in Group Theory  
Recent Developments in the Theory of Group Characters  
P. Fong  
Classification Problems  
Daniel Gorenstein  
Recent Developments in Finite Groups  
Michio Suzuki |
| 3:00 P.M. |  |  |
| 4:00 P.M. |  |  |
PROGRAM OF THE SESSIONS

The time limit for each contributed paper is ten minutes. To maintain this schedule, the time limit will be strictly enforced.

MONDAY, 9:00 A.M.

Session on Analysis I, Grand Ballroom
9:00 - 9:10
(1) Solution of the Cauchy problem for a generalized EPD equation with polyharmonic data
   Dr. E. P. Miles, Jr.* and Dr. E. C. Young, Florida State University (630-10)
9:15 - 9:25
(2) Solution of a Cauchy problem for the nonhomogeneous EPD equation
   Dr. E. C. Young, Florida State University (630-12)
9:30 - 9:40
(3) A quasi-linear singular Cauchy problem
   Professor D. W. Lick, University of Tennessee (630-16)
9:45 - 9:55
(4) Quasilinear uniformly elliptic partial differential equations and difference equations
   Dr. G. T. McAllister, Lehigh University (630-117)
10:00 - 10:10
(5) Global solution of the Cauchy problem for quasi-linear first order equations in several space variables
   Professor E. D. Conway*, University of California, San Diego and Professor J. A. Smoller, University of Michigan (630-130)
10:15 - 10:25
(6) Diffraction by a smooth convex body
   Professor Donald Ludwig, New York University (630-131)

MONDAY, 9:00 A.M.

Session on Analysis II, Assembly Room
9:00 - 9:10
(7) Group algebra modules, I
   Mr. S. L. Gulick*, University of Maryland, Mr. T. S. Liu, University of Massachusetts and Mr. A. C. M. van Rooij, R. C. University, Nijmegen, The Netherlands (630-27)
9:15 - 9:25
(8) Group algebra modules, II
   Mr. S. L. Gulick, University of Maryland; Mr. T. S. Liu*, University of Massachusetts and Mr. A. C. M. van Rooij, R. C. University, Nijmegen, The Netherlands (630-28)
9:30 - 9:40
(9) Convolution measure algebras with group maximal ideal spaces. Preliminary report
   Professor J. L. Taylor, University of Utah (630-102)
   (Introduced by Dr. S. H. Gould)
9:45 - 9:55
(10) Some analysis on compact groups
    Dr. D. R. Beldin, Washington University (630-144)

* For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.
10:00 - 10:10
(11) Uniform approximation by Fourier-Stieltjes transforms. Preliminary report  
Mr. D. E. Ramirez, Tulane University (630-172)

10:15 - 10:25
(12) An operational calculus for locally compact Abelian groups  
Dr. Kenneth Whyburn, Cornell University (630-7)

10:30 - 10:40
(13) Multipliers on $D_a$  
Professor G. D. Taylor, University of Arizona (630-3)

MONDAY, 9:00 A.M.

Session on Logic and Foundations, Gold Room
9:00 - 9:10
(14) Non-standard analysis in topological algebra  
Mr. A. L. Stone, University of Oregon (630-39)

9:15 - 9:25
(15) Order preserving extensions to regressive isols  
Professor F. J. Sansone, Arizona State University (630-61)

9:30 - 9:40
(16) Spanning in model theory  
Mr. W. E. Marsh, Dartmouth College (630-91)

9:45 - 9:55
(17) The least upper bound principle in hyperarithmetic analysis  
Professor W. E. Ritter, University of Washington (630-174)

10:00 - 10:10
(18) Strong representability of partial recursive functions  
Professor R. W. Ritchie*, University of Washington and Professor P. R. Young, Stanford University (630-188)

10:15 - 10:25
(19) Operations in certain categories  
Dr. V. S. Krishnan, SUNY at Buffalo (630-201)

MONDAY, 9:00 A.M.

Session on Algebra and The Theory of Numbers, Louis XVI Room
9:00 - 9:10
(20) Zeros of approximate functional approximations  
Professor Robert Spira, University of Tennessee (630-5)

9:15 - 9:25
(21) 2nd term approximations of sieve generated sequences  
Professor R. G. Buschman and Professor M. C. Wunderlich*, SUNY at Buffalo (630-26)

9:30 - 9:40
(22) A combinatorial interpretation of generalized powers for finite linear spaces  
Professor Gloria Olive, Anderson College (630-38)

9:45 - 9:55
(23) A Minkowskian bound for a class of biquadratic fields arising from fixed points of the Hilbert modular group  
Mr. J. E. Nymann, University of Hawaii (630-50)

10:00 - 10:10
(24) Indecomposable modules over rings with minimum condition  
Mr. R. R. Colby, The Boeing Company, Seattle, Washington (630-51)

10:15 - 10:25
(25) A new proof of a theorem of Kummer  
Professor W. J. Leahey, University of Hawaii (630-66)
10:30 - 10:40
(26) Heights of subspaces and Diophantine approximations
Professor W. M. Schmidt, University of Colorado (630-133)

MONDAY, 9:00 A.M.

Session on Algebra, Crystal Room
9:00 - 9:10
(27) Matrix commutators
Professor R. C. Thompson, University of California, Santa Barbara (630-137)

9:15 - 9:25
(28) An inequality for the elementary symmetric functions of characteristic roots
Professor Marvin Marcus, and Professor Henryk Minc*, University of California, Santa Barbara (630-118)

9:30 - 9:40
(29) Implicative pairs in orthomodular lattices
Mr. D. E. Catlin, University of Massachusetts (630-35)

9:45 - 9:55
(30) A matrix reduction problem
Dr. J. W. Moon* and Professor Leo Moser, University of Alberta, Canada (630-9)

10:00 - 10:10
(31) Concerning nonnegative matrices and doubly stochastic matrices
Professor R. D. Sinkhorn* and Professor Paul Knopp, University of Houston (630-43)

10:15-10:25
(32) Functions on semi-simple algebra
Professor C. G. Cullen*, University of Pittsburgh and Dr. C. A. Hall, U.S. Army, White Sands Missile Range, New Mexico (630-2)

MONDAY, 11:00 A.M.

Business Meeting, Grand Ballroom

MONDAY, 2:00 P.M.

Veblen Prize Award and Lecture, Grand Ballroom

MONDAY, 3:15 P.M.

Special Session on Partial Differential Equations, Grand Ballroom
3:15 - 3:35
The powers $A^s$ of an elliptic operator $A$
Professor Robert Seeley, Brandeis University (630-212)

3:45 - 4:05
Removable singularities of solutions of elliptic equations
Professor James Serrin, University of Minnesota (630-213)

4:15 - 4:35
Wave operators and similarity for some nonselfadjoint operators
Professor Tosio Kato, University of California, Berkeley (630-214)

4:45 - 5:05
Infinite dimensional manifolds and nonlinear elliptic eigenvalue problems
Professor Felix Browder, University of Chicago (630-215)

5:15 - 5:35
Classical and quantized partial differential equations
Professor Edward Nelson, Princeton University (630-216)
Session on Analysis I, Assembly Room

3:15 - 3:25
(33) Translation properties of finite partitions of the positive integers
Professor R. A. Raimi, University of Rochester (630-31)

3:30 - 3:40
(34) Some topological and linear topological properties of separable function algebras. Preliminary report
Mr. A. Pełczynski, University of Warsaw and University of Washington (630-36)

(Introduced by Professor Victor Klee)

3:45 - 3:55
(35) Uniform approximation by polynomials with integral coefficients. Preliminary report
Professor L. O. Ferguson, University of California, Riverside (630-40)

4:00 - 4:10
(36) Denjoy integration in abstract spaces. I
Mr. D. W. Solomon, Wayne State University (630-45)

4:15 - 4:25
(37) Extreme positive operators on algebras of functions
Professor F. F. Bonsall, University of Edinburgh, Scotland; Professor Joram Lindenstrauss, Hebrew University, Israel and Mr. R. R. Phelps*, University of Washington (630-59)

4:30 - 4:40
(38) An isomorphism invariant for spin factors
Dr. D. M. Topping, University of Washington (630-60)

4:45 - 4:55
(39) A mixing condition for extreme left invariant means
Dr. S. P. Lloyd, Bell Telephone Laboratories, Murray Hill, New Jersey (630-69)

5:00 - 5:10
(40) The Stone Čech operator and its associated functionals. Preliminary report
Mr. J. B. Deeds, The University of Michigan (630-74)

5:15 - 5:25
(41) Approximations for the eigenvalues of certain nonself-adjoint operators
Professor J. E. Osborn, University of Maryland (630-77)

5:30 - 5:40
(42) Direct sums of Banach spaces with respect to a basis. I
Professor W. J. Davis* and Mr. D. W. Dean, The Ohio State University (630-87)

5:45 - 5:55
(43) Direct sums of Banach spaces with respect to a basis. II
Professor W. J. Davis and Mr. D. W. Dean*, The Ohio State University (630-88)

Session on Topology, Gold Room

3:15 - 3:25
(44) Equivalent decompositions of E
Professor Steve Armentrout, University of Iowa; Dr. L. L. Lininger*, University of Missouri and Professor D. V. Meyer, Central College (630-4)

3:30 - 3:40
(45) Concerning a wild 3-cell described by Bing
Professor Steve Armentrout, University of Iowa (630-168)
3:45 - 3:55
(46) Multi-valued functions and semi-continuous collections. Preliminary report
Mr. R. F. Jolly, University of California, Riverside (630-15)

4:00 - 4:10
(47) A mildly wild 2-cell in E^4
Mr. Ralph Tindell, Florida State University (630-78)
(Introduced by Professor O. G. Harrold)

4:15 - 4:25
(48) Consequences of the Jordan curve theorem
Mr. L. G. Gref, University of California, Riverside (630-107)

4:30 - 4:40
(49) Toroidal decompositions of E^3
Mr. R. B. Sher, University of Utah (630-143)

4:45 - 4:55
(50) Compact 0-dimensional decompositions of E^3
Mr. H. W. Lambert, University of Utah (630-164)
(Introduced by Professor C. E. Burgess)

5:00 - 5:10
(51) Cellular decompositions of E^3 which have only countably many nondegenerate
Professor C. E. Burgess, University of Utah (630-180)

5:15 - 5:25
(52) Trennungsaxioms in quasi-uniform spaces
Professor M. G. Murdeshwar* and Professor S. A. Naimpally, University
of Alberta, Canada (630-11)

5:30 - 5:40
(53) A closed set axiom of countability
Professor C. E. Aull, Virginia Polytechnic Institute (630-21)

MONDAY, 3:15 P.M.

Session on Algebra I, Louis XVI Room

3:15 - 3:25
(54) On prime distributively generated near-rings
Professor J. C. Beidleman, University of Kentucky (630-46)

3:30 - 3:40
(55) The structure of certain rings
Professor Jiang Luh, Indiana State University (630-48)

3:45 - 3:55
(56) D. C. C. rings with finitely many zero divisors
Mr. K. E. Eldridge, Ohio University (630-73)

4:00 - 4:10
(57) Simple (n, n + 1)-alternative rings
Professor D. L. Outcalt, University of California, Santa Barbara (630-126)

4:15 - 4:25
(58) Coefficients of the Hilbert-Samuel polynomial for complete intersection
Mr. J. P. Fillmore, 100 Woodland Court, Laurel, Maryland (630-127)

4:30 - 4:40
(59) Semi-primal categorical independent algebras
Professor A. M. Yaqub, University of California, Santa Barbara (630-56)

4:45 - 4:55
(60) Projective (K,S)-configurations. Preliminary report
Mr. S. E. Payne* and Dr. M. F. Tinsley, The Florida State University
(630-75)

5:00 - 5:10
(61) Isomorphisms of simple Lie rings
Mr. E. A. Klotz, Swarthmore College (630-110)
MONDAY, 3:15 P.M.

Session on Algebra II, Crystal Room
3:15 - 3:25
(62) An embedding property of generalized Carter subgroups
Mr. Edward Cline, California Institute of Technology (630-181)

3:30 - 3:40
(63) Categories of extensions
Professor R. J. Nunke, University of Washington (630-125)

3:45 - 3:55
(64) Weakly semisimple finite dimensional algebras
Professor W. E. Clark, University of Florida (630-135)

4:00 - 4:10
(65) Characteristic roots of M-matrices
Professor D. E. Crabtree, University of Massachusetts (630-156)

4:15 - 4:25
(66) Results in Post algebras
Professor Philip Dwinger, University of Illinois (630-158)

4:30 - 4:40
(67) On pseudo-valued global fields
Sister Maris Stella Schrot, University of California, Los Angeles (630-167)

4:45 - 4:55
(68) Undirected graphs realizable as graphs of semi-modular lattices
Mr. L. R. Alvarez, The University of the South (630-193)

5:00 - 5:10
(69) Endomorphism rings of torsionless modules over semiprime rings
Mr. J. M. Zelmanowitz, University of Wisconsin (630-198)

5:15 - 5:25
(70) Intrinsic functions on semi-simple algebras
Lt. C. A. Hall, U. S. Army, White Sands Missile Range, New Mexico (630-112)

MONDAY, 8:00 P.M.

Gibbs Lecture, Grand Ballroom
Stellar evolution
Professor Martin Schwarzschild, Princeton University

TUESDAY, 9:00 A.M.

Special Session on Markov Processes, Grand Ballroom
9:00 - 9:20
A super-Markovian approach to $\Delta^2/2 + V$
Professor F. B. Knight, University of Illinois (630-206)

9:30 - 9:50
Semi-stable Markov processes. Preliminary report
Professor John Lamperti, Dartmouth College (630-207)

10:00 - 10:20
A system of denumerably many transient Markov chains
Professor S. C. Port, RAND Corporation, Santa Monica, California (630-208)

10:30 - 10:50
On optimal stopping for Markov processes
Professor Y. S. Chow*, Purdue University and Professor H. E. Robbins,
University of Minnesota (630-209)
11:00 - 11:20
On a class of random walks and certain function algebras associated with them.
Professor R. A. Gangolli, University of Washington (630-210)

11:30 - 11:50
Markov chains recurrent in the sense of Harris
Professor Steven Orey*, and Professor Benton Jamison, University of Minnesota (630-211)

TUESDAY, 9:00 A.M.

Session on Analysis, Assembly Room
9:00 - 9:10
(71) Singularity-free regions for solutions of some nth order nonlinear differential equations
Professor D. V. V. Wend, University of Utah (630-32)

9:15 - 9:25
(72) A characterization of differential operators
Mr. Martin Engert, University of North Carolina (630-42)

9:30 - 9:40
(73) General solutions of nonlinear difference equations
Professor L. J. Grimm*, University of Utah and Professor W. A. Harris, Jr., University of Minnesota (630-49)

9:45 - 9:55
(74) The asymptotic growth of solutions to a nonlinear system
Professor S. P. Hastings, Case Institute of Technology (630-57)

10:00 - 10:10
(75) A generalized method of averaging, with applications to slightly damped nonlinear oscillations
Dr. J. A. Morrison, Bell Telephone Laboratories, Murray Hill, New Jersey, (630-83)

10:15 - 10:25
(76) Lack of self-adjointness in three point boundary value problems
Professor Anton Zettl, Louisiana State University, Baton Rouge (630-94)

10:30 - 10:40
(77) Cyclicly related differential systems
Dr. G. J. Etgen, National Aeronautics Space Administration, Washington, D. C. (630-99)

10:45 - 10:55
(78) Existence criterion for almost periodic solutions of A. P. System
Professor R. K. Miller, University of Minnesota (630-109)

11:00 - 11:10
(79) A nonhomogeneous differential equation with a second order Turning point
Professor D. J. McGuinness, University of Maryland (630-129)

11:15 - 11:25
(80) A duality relation in differential equations and some associated functional equations
Mr. M. S. Klamkin, Ford Scientific Laboratory, Dearborn, Michigan (630-132)

11:30:11:40
(81) On the resolvent set and spectrum of homogeneous elliptic differential operators with constant coefficients
Professor R. S. Freeman, University of Maryland (630-152)

11:45 - 11:55
(82) Eigenfunction expansions and scattering theory for the wave equation
Mr. N. A. Shenk, II, Columbia University (630-155)
(Introduced by Dr. S. H. Gould)
TUESDAY, 9:00 A.M.

Session on Algebra, Louis XVI Room

9:00 - 9:10
(83) Artinian and noetherian hypercentral groups
Dr. Hermann Simon, University of Miami (630-23)

9:15 - 9:25
(84) Group rings, semigroup ring and their radicals
Professor Hans Schneider and Mr. Julian Weissglass*, University of Wisconsin (630-37)

9:30 - 9:40
(85) On the normal structure of the automorphism group and the ideal structure of the endomorphism ring of Abelian P-groups
Professor A. G. Mader, University of Hawaii (630-62)

9:45 - 9:55
(86) A purification problem in primary abelian groups
Professor Fred Richman* and Mrs. Carol Walker, New Mexico State University (630-84)

10:00 - 10:10
(87) Partial homomorphic images of certain groupoids
Professor Mario Petrich, The Pennsylvania State University (630-89)

10:15 - 10:25
(88) Special abelian group difference sets
Professor E. C. Johnsen, University of California, Santa Barbara (630-151)

10:30 - 10:40
(89) The near-ring generated by the inner automorphisms of a group
Dr. A. J. Chandy, Southeastern Massachusetts Technological Institute (630-171)

10:45 - 10:55
(90) Completeness and generalized primary groups
Mr. Ray Mines, New Mexico State University (630-176)

11:00 - 11:10
(91) On restricted roots of a semisimple algebraic group. Preliminary report
Miss Doris Schattschneider, University of Illinois at Chicago Circle (630-190)

11:15 - 11:25
(92) On mixed groups of torsion-free rank one
Dr. C. K. Megibben, University of Houston (630-192)

11:30 - 11:40
(93) Remarks on absolutely convex subgroups. Preliminary report
Professor A. C. Morel, University of Washington (630-142)

11:45 - 11:55
(94) On cancellative and abelian congruences in semigroups and groupoids
Mr. P. -A. Grillet, University of Florida (630-160)
(Introduced by Professor W. E. Clark)

TUESDAY, 9:00 A.M.

Session on Topology, Crystal Room

9:00 - 9:10
(95) Smoothing imbeddings of spheres
Mr. R. C. Kirby, University of California, Los Angeles (630-29)

9:15 - 9:25
(96) Relations between usual and generalized functions
Professor R. G. Lintz, University of Manitoba, Canada (630-44)

9:30 - 9:40
(97) Some theorems on retracts
Professor R. C. O'Neill, The University of Michigan (630-64)
9:45 - 9:55  
(98) Homotopy-commutativity in H-spaces  
Dr. R. R. Douglas, University of British Columbia, Canada (630-72)

10:00 - 10:10  
(99) Fixed points and fibre maps  
Professor R. F. Brown, University of California, Los Angeles (630-81)

10:15 - 10:25  
(100) An application of Morse theory to certain symmetric spaces  
Dr. S. Ramanujam, University of Washington (630-92)

10:30 - 10:40  
(101) Almost locally flat n-cells. Preliminary report  
Mr. R. C. Lacher, University of Georgia (630-103)

10:45 - 10:55  
(102) A necessary condition that an H-space be homotopy-Abelian  
Professor F. D. Williams, New Mexico State University (630-146)

11:00 - 11:10  
(103) On the enumeration of oriented vector bundles over a finite complex  
Professor L. L. Larmore, University of Illinois at Chicago Circle (630-186)

11:15 - 11:25  
(104) Isotopy classes of imbeddings  
Professor C. W. Patty, University of North Carolina (630-169)

11:30 - 11:40  
(105) A characterization of polyhedra embeddable in the plane  
Professor Sibe Mardešić, University of Zagreb and University of Washington and Professor Jack Segal*, University of Washington (630-53)

11:45 - 11:55  
(106) Orders of local knottedness. Preliminary report  
Professor S. J. Lomonaco, Jr., Florida State University (630-147)

TUESDAY, 9:00 A.M.

Session on Geometry, Gold Room

9:00 - 9:10  
(107) An axiomatization of N-partition geometries  
Dr. T. J. Brown, Loyola University, Chicago (630-1)  
(Introduced by Dr. S. H. Gould)

9:15 - 9:25  
(108) Some integral-geometric uniqueness theorem. Preliminary report  
Dr. Henryk Fast, University of Notre Dame (630-8)

9:30 - 9:40  
(109) Affine differential geometry of closed hypersurfaces  
Professor C. C. Hsiung and Professor J. K. Shahin*, Lehigh University (630-19)

9:45 - 9:55  
(110) The dual Lüneburg plane  
Professor T. G. Ostrom, Washington State University (630-30)

10:00 - 10:10  
(111) A theorem on local isometries  
Professor W. A. Kirk, University of California, Riverside (630-54)

10:15 - 10:25  
(112) Intersectional bases of convex cones  
Mr. E. P. Geyer*, University of Montana and Professor J. R. Reay, Michigan State University (630-63)

10:30 - 10:40  
(113) Noneuclidean incidence planes, II  
Professor Rafael Artzy, SUNY at Buffalo (630-68)
10:45 - 10:55  (114) Imbedding of convex metric spaces in $E^N$
   Mr. F. A. Toranzos, Consejo Nacional Investigaciones, Cientificas y Tecnicas, Argentina and University of Washington
   (Introduced by Professor Victor Klee)

11:00 - 11:10  (115) A polar Morse function for Riemannian manifolds of pinching greater than $1/4$
   Mr. Nathaniel Grossman, Institute for Advanced Study (630-120)

TUESDAY, 2:00 P.M.
Special Session on Combinatorial Mathematics, Grand Ballroom
2:00 - 2:20  Spreads of finite projective 3-space
   Professor R. H. Bruck, University of Wisconsin (630-202)

2:30 - 2:50  Incidence matrices and interval graphs
   Dr. D. R. Fulkerson* and Mr. O. A. Gross, RAND Corporation, Santa Monica, California (630-203)

3:00 - 3:20  A new look at the coloring problem
   Professor Gian-Carlo Rota, Rockefeller Institute (630-217)

3:30 - 3:50  The d-step conjecture for polyhedra of dimension $d < 6$
   Professor Victor Klee, University of Washington and Dr. D. W. Walkup*, Boeing Scientific Research Laboratories, Seattle, Washington (630-204)

4:00 - 4:20  Symmetry classes and combinatorial identities
   Professor Marvin Marcus, University of California, Santa Barbara (630-65)

TUESDAY, 2:00 P.M.
Session on Topology, Assembly Room
2:00 - 2:10  (116) Every nonnormable Fréchet space is homeomorphic with all of its closed convex bodies
   Professor Victor Klee*, University of Washington and Professor Czeslaw Bessaga, University of Warsaw, Poland (630-18)

2:15 - 2:25  (117) Absolute Baire sets
   Professor Stelios Negrepontis, Indiana University (630-187)

2:30 - 2:40  (118) On the permutability of certain operators in topological spaces
   Mr. L. E. De Noya, The Babcock & Wilcox Company, Alliance, Ohio (630-58)

   Professor R. W. Heath, Arizona State University (630-106)

3:00 - 3:10  (120) The edge-path group and Čech homology groups of a generalized uniform space
   Mr. R. W. Deming, Idaho State University (630-115)

3:15 - 3:25  (121) Selection theorems for multi-valued functions
   Professor R. E. Smithson, University of Florida (630-124)

3:30 - 3:40  (122) Strong proximate retracts
   Professor A. L. Yandl, Western Washington State College (630-134)
3:45 - 3:55
(123) Height of uniformities
Professor J. L. Hursch, Jr., The University of Vermont (630-139)

4:00 - 4:10
(124) Selfreproductive sets
Professor R. P. Bennett, Knox College (630-148)

4:15 - 4:25
(125) Projective topological groups
Dr. C. E. Hall, Virginia Polytechnic Institute (630-22)

4:30 - 4:40
(126) Homomorphic retracts in compact semigroups. Preliminary report
Mr. J. T. Borrego, University of Florida (630-85)

4:45 - 4:55
(127) On compactness of the order topology of a lattice. Preliminary report
Mr. A. J. Insel, University of California, Berkeley (630-86)

TUESDAY, 2:00 P.M.

Session on Analysis I, Louis XVI Room
2:00 - 2:10
(128) An approach to the theory of integration generated by positive linear functionals
Professor W. M. Bogdanowicz, Catholic University of America (630-92)

2:15 - 2:25
(129) Differentiable retracts in Banach spaces
Professor S. B. Nadler, Jr., Wayne State University (630-93)

2:30 - 2:40
(130) Tensor products over H*-algebras
Mr. L. C. Grove, Dartmouth College (630-121)

2:45 - 2:55
(131) Representing measures for function algebras
Mr. Patrick Ahern*, University of California, Los Angeles and Professor
Donald Sarason, University of California, Berkeley (630-128)

3:00 - 3:10
(132) The equivalence of Harnack's principle and Harnack's inequality in the axiom­
atic system of Brelot
Professor P. A. Loeb* and Professor Bertram Walsh, University of Cali­
fornia, Los Angeles (630-154)

3:15 - 3:25
(133) Partition-subspaces and norm-one bases in C(X)
Professor E. A. Michael* and Professor Aleksander Pełczynski, Univer­
sity of Washington (630-165)

3:30 - 3:40
(134) Algebraic theory of Fredholm operators
Professor L. A. Coburn*, Purdue University and Professor Arnold Lebow,
University of California, Irvine (630-173)

3:45 - 3:55
(135) On the state diagram of a linear operator and its adjoint in locally convex
spaces
Professor V. Krishnamurthy, Birla Institute of Technology and Science,
Pilani, India and Professor Joaquin Loustaunau*, New Mexico State Uni­
versity (630-177)

4:00 - 4:10
(136) On $t_m$-semireflexive locally convex spaces
Professor T. L. Hicks, Illinois State University (630-194)

4:15 - 4:25
(137) A structure theorem for n-parameter semigroups
Professor Dagmar Henney, The George Washington University (630-199)
4:30 - 4:40
(138) Some properties of a partial differential operator
Professor J. R. Dorroh, Louisiana State University (630-33)

4:45 - 4:55
(139) Atomicity of spectral measures on certain spaces
Professor B. J. Walsh, University of California, Los Angeles (630-200)

5:00 - 5:10
(140) Vectorial integration generated by finite additive vector measures
Professor W. M. Bogdanowicz and Mr. R. H. Sullivan*, Catholic University of America (630-108)

TUESDAY, 2:00 P.M.

Session on Analysis II, Crystal Room

2:00 - 2:10
(141) On the convexity of level curves of a polynomial
Professor A. W. Goodman, University of South Florida (630-13)

2:15 - 2:25
(142) Extremal SARIO linear operators
Professor G. G. Weill, Polytechnic Institute of Brooklyn (630-25)

2:30 - 2:40
(143) Analytic Schottky differentials
Professor Myron Goldstein, Arizona State University (630-70)

2:45 - 2:55
(144) Inequalities on holomorphic functions omitting one value
Professor D. C. Rung, Pennsylvania State University (630-166)

3:00 - 3:10
(145) Some theorems concerning the class of weight functions in the transformation theory for measure space
Dr. Robin Chaney, Western Washington State College (630-6)

3:15 - 3:25
(146) A distributional K transformation
Professor A. H. Zemanian, SUNY at Stony Brook (630-34)

3:30 - 3:40
(147) Holomorphic functions with gap power series. III
Professor Alfred Gray, University of California, Berkeley and Professor S. M. Shah*, University of Kansas (630-67)

3:45 - 3:55
(148) On high indices theorems
Mr. J. S. Ratti, Wayne State University (630-76)

4:00 - 4:10
(149) The L(r,t) summability transform
Mr. R. E. Powell, Lehigh University (630-95)

4:15 - 4:25
(150) Some properties of oricyclic cluster sets
Mr. T. A. Vessey, University of Minnesota (630-98)

4:30 - 4:40
(151) On total inclusion for Norlund methods of summability. II
Professor B. E. Rhoades, Indiana University (630-105)

4:45 - 4:55
(152) Some new identities for the modified Bessel function
Mr. J. A. Donaldson, Howard University (630-113)

5:00 - 5:10
(153) Laguerre functions with truncated domains
Professor Bayard Rankin, Case Institute of Technology (630-122)
Professor R. E. Moore, University of Wisconsin (6340-96)

2:15 - 2:25
Proof of global convergence of an iterative method for calculating complex zeros of a polynomial
Dr. J. F. Traub, Bell Telephone Laboratories, Inc., Murray Hill, New Jersey (630-196)

2:30 - 2:40
The vertical jet under gravity
Dr. W. E. Conway, West Texas State University (630-20)

2:45 - 2:55
Degenerate fields with twisting rays
Professor Ivan Robinson, Southwest Center for Advanced Studies, Dallas, Texas, Professor J. R. Zund and Professor J. D. Zund*, North Carolina State University, Raleigh (630-52)

3:00 - 3:10
On the convergence of discrete approximations to mixed boundary value problems
Professor N. M. Wigley, University of Arizona (630-80)

3:15 - 3:25
Approximate solution of the exterior Dirichlet problem and the calculation of electrostatic capacity
Professor Donald Greenspan*, Mathematical Research Center, University of Wisconsin and Professor Edward Silverman, Purdue University (630-55)

3:30 - 3:40
Quasi-linearization and a new monotone iteration
Dr. L. F. Shampine, Sandia Corporation, Albuquerque, New Mexico (630-104)

3:45 - 3:55
Elastic-plastic torsion of a square bar
Professor T. W. Ting, North Carolina State University (630-119)

4:00 - 4:10
Association of calculus of variations with dynamic programming
Dr. Abolghassem Ghaffari, NASA Goddard Space Flight Center, Greenbelt, Maryland (630-123)

4:15 - 4:25
Asymptotic error conjecture for retarded ordinary differential equations
Professor M. A. Feldstein, Brown University (630-178)

4:30 - 4:40
Some mathematical aspects of models of the special relativity
Professor M. Z. v. Krzywoblocki, Michigan State University (630-185)

WEDNESDAY, 2:00 P.M.

Invited Address, Grand Ballroom
Norm inequalities for some orthogonal expansions
Professor Richard Askey, University of Wisconsin

WEDNESDAY, 3:00 P.M.

Presidential Address, Grand Ballroom
Application to analysis of the topological definition of smallness of a set
Professor J. L. Doob, University of Illinois
**THURSDAY, 2:00 P.M.**

**Invited Address, Grand Ballroom**

Sylow $2$-subgroups of finite groups  
Professor George Glauberman, University of Chicago

**THURSDAY, 3:15 P.M.**

**Special Session on Algebraic Groups, Grand Ballroom**

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<thead>
<tr>
<th>Time</th>
<th>Topic</th>
<th>Speaker and Institution</th>
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<tbody>
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<td>3:15</td>
<td>Algebraic geometry and automorphic functions</td>
<td>Professor J. -I. Igusa, Johns Hopkins University (630-218)</td>
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<tr>
<td>3:45</td>
<td>On the conjugacy of subgroups of semi-simple groups</td>
<td>Professor G. D. Mostow, Yale University (630-219)</td>
</tr>
<tr>
<td>4:15</td>
<td>On the Tamagawa number of homogeneous spaces</td>
<td>Professor Takashi Ono, University of Pennsylvania (630-220)</td>
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<tr>
<td>4:45</td>
<td>Jordan decomposition in the Lie algebra of a linear group</td>
<td>Professor T.A. Springer, University of California, Los Angeles (630-221)</td>
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<tr>
<td>5:15</td>
<td>Cohomology of algebraic groups and invariant splitting of algebras</td>
<td>Professor E. J. Taft, Rutgers, The State University (630-222)</td>
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**THURSDAY, 3:15 P.M.**

**General Session, Assembly Room**

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<thead>
<tr>
<th>Time</th>
<th>Topic</th>
<th>Speaker and Institution</th>
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<tr>
<td>3:15</td>
<td>Connectivity, divisibility, and torsion</td>
<td>Mr. L. C. Robertson, University of Washington (630-101)</td>
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<td>3:30</td>
<td>Strictly convex metrics</td>
<td>Mr. W. A. Glynn, Western Illinois University (630-136)</td>
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<tr>
<td>3:45</td>
<td>Complete separation in the Stone topology</td>
<td>Professor G. A. Jensen, University of Florida (630-159)</td>
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<td>4:00</td>
<td>Spaces of continuous relations</td>
<td>Dr. J. M. Day, Carnegie Institute of Technology and Professor S. P. Franklin*, University of Florida (630-175)</td>
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<tr>
<td>4:15</td>
<td>On 1-factors in nonseparable graphs</td>
<td>Professor L.W. Beineke*, Purdue University, Fort Wayne, and Mr. M. D. Plummer, University of Michigan (630-195)</td>
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<tr>
<td>4:30</td>
<td>On processes equivalent to Feller processes</td>
<td>Professor T. I. Seidman* and Professor A. T. Bharucha-Reid, Wayne State University (630-47)</td>
</tr>
<tr>
<td>4:45</td>
<td>Expected number of real zeros of polynomials</td>
<td>Professor D. C. Stevens, New Mexico State University (630-79)</td>
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<td>5:00</td>
<td>Noncommutative Markov processes</td>
<td>Professor J. E. de Pillis, University of California, Riverside (630-183)</td>
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<tr>
<td>5:15</td>
<td>Stochastic point processes in $\mathbb{R}^n$: Limit theorems</td>
<td>Mr. J. R. Goldman, Harvard University (630-189)</td>
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</tbody>
</table>
Session on Analysis I, Louis XVI Room

3:15 - 3:25
(174) A theorem applicable to the Whittaker W problem
   Professor R. F. DeMar, University of California, Davis (630-97)

3:30 - 3:40
(175) On quasiconformal mappings with given boundary values
   Mr. T. J. Reed, University of Minnesota (630-145)

3:45 - 3:55
(176) The univalence of functions asymptotic to nonconstant logarithmic monomials
   Mr. E. W. Chamberlain, University of Vermont (630-149)

4:00 - 4:10
(177) On a min-max problem of Leo Moser
   Professor G. F. Clements, University of Colorado (630-150)

4:15 - 4:25
(178) Simultaneous approximation of a function and its derivatives
   Professor Amram Meir and Professor A. Sharma*, University of Alberta, Canada (630-157)

4:30 - 4:40
(179) Concerning local symmetry in the moment problem
   Dr. G. G. Johnson, University of Georgia (630-184)

4:45 - 4:55
(180) Extensibility of outer measures
   Professor Stephen Silverman, New York University (630-114)
   (Introduced by Dr. S. H. Gould)

5:00 - 5:10
(181) Quasi-normal currents having normal plane intersections
   Lt. J. E. Brothers, United States Army, Warrenton, Virginia (630-116)

5:15 - 5:25
(182) Uniformly non-$l^1_\infty$ Orlicz spaces
   Professor Kondagunta Sundaresan, Carnegie Institute of Technology (630-138)

5:30 - 5:40
(183) On Muirhead's theorem
   Professor J. V. Ryff, University of Washington (630-191)

5:45 - 5:55
(184) An extension of weighted average property
   Dr. A. K. Bose, University of Alabama (630-111)

6:00 - 6:10
(185) Finding the mean value. Preliminary report
   Professor Mason Henderson, University of Montana (630-141)

THURSDAY, 3:15 P.M.

Session on Analysis II, Crystal Room

3:15 - 3:25
(186) Concerning the general maximum principle
   Professor J. C. C. Nitsche, University of Minnesota (630-163)

3:30 - 3:40
(187) Uniqueness for the Dirichlet problem in an unbounded domain, and related problems
   Professor P. C. Fife, University of Minnesota (630-183)

3:45 - 3:55
(188) Solution of Burger's equation with Cauchy data
   Dr. E. Y. Rodin, Wyle Laboratories, Huntsville, Alabama (630-170)
4:00 - 4:10
(189) Normal derivatives of potentials. Preliminary report
Professor Josef Král, Brown University (630-197)
(Introduced by Professor W. H. Fleming)

4:15 - 4:25
(190) The radial heat equation and Laplace transforms
Professor L. R. Bragg, Case Institute of Technology (630-153)

4:30 - 4:40
(191) Asymptotically self-invariant sets and conditional stability. Preliminary report
Professor V. Lakshmikantham, University of Alberta, Canada and Miss S. Leelamma*, Marathwada University (630-161)

4:45 - 4:55
(192) Parabolic differential equations and conditional stability. Preliminary report
Professor V. Lakshmikantham*, University of Alberta, Canada and Miss S. Leelamma, Marathwada University (630-162)

5:00 - 5:10
(193) Compact solutions, and their approximation, of nonlinear differential equations in Banach spaces
Professor C. T. Taam and Mr. J. N. Welch*, Georgetown University (630-90)

5:15 - 5:25
(194) Asymptotically periodic and almost periodic solutions of nonlinear differential equations
Professor C. T. Taam, Georgetown University (630-179)

5:30 - 5:40
(195) Some applications of the theory of unbounded operators to ordinary differential equations
Professor Seymour Goldberg*, University of Maryland and Professor Cedric Schubert, University of California, Los Angeles (630-140)

5:45 - 5:55
(196) A Hilbert space generalization of Kantorovich's inequality
Dr. Bertram Mond, Wright-Patterson Air Force Base, Ohio (630-100)
The six hundred thirty first meeting of the American Mathematical Society will be held on the Washington Square Campus of New York University at the Courant Institute of Mathematical Sciences. All sessions will be in Warren Weaver Hall.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, Professor W. H. J. Fuchs of Cornell University will give an hour address in Room 109 of Warren Weaver Hall at 2:00 P.M. His title is "Developments in the classical Nevanlinna theory of meromorphic functions."

There will be sessions for contributed papers both morning and afternoon. There may be provision for a limited number of late papers.

The registration desk will be set up in the lobby at the entrance to Warren Weaver Hall and will be open at 9:00 A.M.

A list of restaurants near Washington Square will be available at the registration desk.

Warren Weaver Hall is at 251 Mercer Street, one block east of the southeast corner of Washington Square between Third and Fourth Streets. Fourth Street borders the south side of Washington Square but the street signs on the stretch along the square read "Washington Square South."

Subway and bus transportation may conveniently be used as follows: Lexington Avenue (Interborough) Subway (IRT)--Local to Astor Place Station. Walk west on Astor Place to Broadway, then south on Broadway to Fourth Street and west to Mercer Street.

Seventh Avenue (Interborough) Subway (IRT)--Local to Sheridan Square Station. Walk east on Waverly Place to Washington Square.

Broadway (Brooklyn-Manhattan) Subway (BMT)--Brighton local or Fourth Avenue local to Eighth Street Station. Walk south on Broadway to Fourth Street and west to Mercer Street.

Sixth or Eighth Avenue (Independent) Subway (IND)--Express to West Fourth Street - Washington Square Station. Walk east on West Fourth Street to Washington Square.

Fifth Avenue Bus--Busses numbered 3, and some numbered 5, to University Place. Walk south and cross the square to Washington Square South. Walk east to Mercer Street.

Everett Pitcher
Associate Secretary

Bethlehem, Pennsylvania
The six hundred thirty-second meeting of the American Mathematical Society will be held at the Waldorf-Astoria Hotel in New York on April 4-7, 1966.

On Monday, April 4, and the morning of Tuesday, April 5, there will be a program of invited addresses and ten minute contributed papers. The deadline for receipt of the abstracts of contributed papers is February 18. Abstracts should be submitted to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904 so as to arrive prior to the deadline.

The Association for Symbolic Logic will meet in the same hotel, also on April 4 and the morning of April 5. Their call for papers will be issued in the near future. The program chairman is Professor Martin D. Davis, Courant Institute of Mathematical Sciences, 251 Mercer Street, New York, New York 10012.

SYMPOSIUM ON MATHEMATICAL ASPECTS OF COMPUTER SCIENCE

The Symposium will be scheduled in four sessions on the afternoon of Tuesday, April 5, on Wednesday, April 6, and on the morning of Thursday, April 7, as follows:

Session I. Computation with symbolic and algebraic data

Session II. Numerical methods for computers

Session III. Software systems; mechanical linguistics; computer analysis of language

Session IV. Theory of automata

The subject of the Symposium was chosen by the Committee on Applied Mathematics which consisted of A. H. Taub (Chairman), V. Bargmann, G. E. Forsythe, C. C. Lin, Alfred Schild, and H. S. Wilf. Financial support comes from the Air Force Office of Scientific Research; the Institute for Defense Analyses; and the U. S. Army Research Office -- Durham. The Association for Computing Machinery and the Association for Symbolic Logic are co-sponsoring the Symposium.

The Invitations Committee responsible for the planning of the program and the choice of speakers consists of: Jack Schwartz (Chairman), Courant Institute of Mathematical Sciences; Martin Davis, Courant Institute of Mathematical Sciences; H. H. Goldstine, IBM Research Center; D. H. Lehmer, University of California; John Todd, California Institute of Technology; H. S. Wilf, University of Pennsylvania; Calvin C. Elgot, IBM Research Center; Saul Gorn, University of Pennsylvania; Harry Huskey, University of California; and Anthony G. Oettinger, Harvard University.

ROOM RESERVATIONS

Persons intending to stay at the Waldorf-Astoria should make their own reservations with the hotel. A reservation blank and a listing of room rates are found on the last page of this issue of the Notices. A reservation blank will also appear in the February issue.

Everett Pitcher
Associate Secretary
Bethlehem, Pennsylvania
Six Hundred Thirty-Fourth Meeting
University of Chicago
Center for Continuing Education
Chicago, Illinois
April 21-23, 1966

The six hundred thirty-fourth meeting of the American Mathematical Society will be held at the University of Chicago Center for Continuing Education in Chicago, Illinois on April 21-23, 1966.

Contributed papers and invited addresses will be scheduled on Friday, April 22, and Saturday, April 23. There will be a Symposium on Singular Integrals on Thursday, April 21, and Friday morning, April 22. The invited addresses include: Glen Baxter, Purdue University, Some aspects of the Ising model; Hans Grauert, Göttingen and Notre Dame, Nonarchimedean analysis; and R. G. Swan, University of Chicago, Modules over finite groups.

The subject of the Symposium was chosen by the Committee to Select Hour Speakers for Western Sectional Meetings. The Committee consisted of Seymour Sherman (Chairman), Felix Browder, and Irving Reiner. Financial support comes from the National Science Foundation and the University of Chicago.

The Invitations Committee responsible for the planning of the program and the choice of speakers consists of: Alberto P. Calderón (Chairman), University of Chicago; K. O. Friedrichs, Courant Institute of Mathematical Sciences; Robert T. Seeley, Brandeis University; and Antoni Zygmund, University of Chicago.

The principal topics to be discussed are:
- Singular Integrals and Real Variables
- Singular Integrals and Partial Differential Equations
- Singular Integrals on Manifolds
- New and Miscellaneous Aspects of the Theory of Singular Integrals

Approximately 17 persons will be invited to participate as speakers or discussion leaders. There will be four hour lectures and a series of shorter papers presenting individual attainments in the field of Singular Integrals.

The deadline for receipt of abstracts for the sessions of ten minute contributed papers on Friday afternoon is February 18, 1966. Abstracts should be sent as usual to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904 so as to arrive prior to the deadline.

A reservation blank is on the last page of this issue of the Notices. A reservation blank will also appear in the February issue.

S. Sherman
Associate Secretary
Bloomington, Indiana
The thirteenth American Mathematical Society Summer Institute will be held from June 27 to July 22, 1966 in California, the exact location to be announced later. The topic of the Summer Institute will be Entire Functions and Related Parts of Analysis. The Institute will receive financial support from the National Science Foundation.

A large number of mathematicians have been working in the field of entire functions, and there has been a recent influx of new ideas and methods, principally from functional analysis and modern algebra. It is hoped that the lack of communication that has developed between the "classical" people and the "modern" can be substantially diminished by a successful Institute. To this end, seminars on current developments and lectures designed to give the present state of knowledge will be held to allow participants to become familiar with the different points of view of their colleagues.

Of the approximately 60 to 70 participants who are being invited, 15 are from Western Europe, 1 is from Hungary, and 6 are from Russia. A limited amount of financial support is available for a few qualified individuals other than those invited. Interested persons should write to the Chairman of the Committee. Graduate students should ask their references to send letters of recommendation.

The Invitations and Organizing Committee for the 1966 Summer Institute is composed of R. P. Boas of Northwestern University (Chairman), A. Beurling of the Institute for Advanced Study, L. Ehrenpreis of New York University, W. Fuchs of Cornell University, J. Korevaar of the University of California at La Jolla, and L. A. Rubel of the University of Illinois.

INTERNATIONAL CONFERENCE HELD IN AUSTRALIA

The International Conference on the Theory of Groups, sponsored by the International Mathematical Union, the Australian Academy of Science, and the Australian National University, was conducted August 10-20, 1965, at the Australian National University of Canberra, Australia. This was the first international conference on a specific mathematical topic to be held in Australia.

The scientific program of the conference concentrated mainly on non-abelian abstract groups; it consisted of 16 formal lectures, mostly surveys, and 10 informal study groups on special topics.

One-term visiting lecturerships in Australian universities were arranged for about 12 participants in order to ensure the participation of some of the younger workers in the field who might not receive support from industry, government or their own universities. These participants were given leaves of absence from their Australian host universities in order to attend the conference.

NEWS ITEMS AND ANNOUNCEMENTS

POSTDOCTORAL VISITING MEMBERSHIPS FOR THE ACADEMIC YEAR 1966-1967
NEW YORK UNIVERSITY
COURANT INSTITUTE OF MATHEMATICAL SCIENCES

Each year the Courant Institute of Mathematical Sciences awards a number of visiting memberships to mathematicians, scientists and engineers who hold the doctorate.

A visiting member has no duties other than to be in residence at the Institute and to participate in research activities. The stipend is determined on an individual basis depending on the visitor's professional status. Normally the award is for one year, but in special cases it may be renewed. A visiting member is invited to participate in the scientific life of the Courant Institute and in particular to take part in a variety of advanced seminars. He is given complete access to the facilities of the Courant Institute, including a CDC 6600 computer installation.

Some of the fields in which there is current research activity at the Courant Institute are: number theory, group theory, group representations, topology of manifolds, differential geometry, differential and integral equations, analytic function theory, probability, functional analysis, harmonic analysis, computer science, numerical analysis, mechanics, elasticity, fluid dynamics, electromagnetic theory, magneto fluid dynamics, quantum field theory, statistical inference, and dynamic meteorology.

Support for the visiting membership program is provided by the National Science Foundation, other government agencies, and industrial organizations.

For application forms and further information, please write to the Visiting Membership Committee
Courant Institute of Mathematical Sciences
New York University
251 Mercer Street
New York, New York 10012

Applications for the academic year 1966-1967 should be submitted before February 1, 1966.

FOURTH ANNUAL SYMPOSIUM ON BIOMATHEMATICS AND COMPUTER SCIENCE IN THE LIFE SCIENCES

The Division of Continuing Education of the University of Texas Graduate School of Biomedical Sciences at Houston announces its sponsorship of the Fourth Annual Symposium on Biomathematics and Computer Science in the Life Sciences. The symposium will be held at the Shamrock Hilton Hotel in Houston, Texas, on Thursday, Friday, and Saturday, March 24-26, 1966. The program will be centered around the following general topics: Stimulation and Modeling of Biological Processes, Training for Bio-engineering and Biomathematics, Analysis of Biological Processes with Time Sharing Computation, Records and Library Information Systems for a Medical Center, and Biostatistical Techniques and Biologically Oriented Computer Languages.

Further information may be obtained by writing to: Office of the Dean, Division of Continuing Education, The University of Texas Graduate School of Biomedical Sciences at Houston, 102 Jesse Jones Library Building, Texas Medical Center, Houston, Texas 77025.

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[Ed. Note: Because of printing requirements, it is often necessary to edit the original material which we receive for publication in the Notices. This editing is done with the intention of preserving the import of the original item. Upon occasion, however, the editing process may inadvertently impinge upon some facet of the material which, the author feels, should be printed as it stands. The editors of the Notices respect such opinions and are certainly willing to amend articles appearing in this journal.]

The October Notices published a partially edited version of the report of A. Borel and G. D. Mostow on the 1965 Summer Institute. The authors have requested us to reproduce in full their own version of the following items.

(1) The statement prefacing the complete program should read:

An effort was made to strike a balance between too many lectures on the one hand and too scanty an account of the principal developments on the other. Lectures were scheduled for each of five mornings from 9:30 to 10:30 and 11:00 to 12:00 and for three afternoons from approximately 3:15 to 4:15 and 4:30 to 5:30.

The first two weeks were devoted largely to survey lectures and these in turn merged increasingly into seminar type presentations.

(2) The section under acknowledgements should read:

The Summer Institute was very fortunate to have had as local administrator Mr. Robert Ellingwood of the University of Colorado whose intelligent and diligent planning contributed enormously to the smooth running of both the class room and dormitory arrangements. Mr. Ellingwood and the AMS secretary, Mrs. Lawton Smith, deserve special commendation for the efficient and speedy reproduction of the lecture notes. Finally, we acknowledge with pleasure the planning of Dr. Gordon L. Walker, Executive Director of the American Mathematical Society, and his office staff.

ADVANCED SUMMER SEMINAR IN ALGEBRAIC NUMBER THEORY AND CLASS FIELD THEORY
(Tentative Announcement)

In 1966 Bowdoin College plans to hold a second eight-week advanced summer seminar in algebra combining a research program for postdoctoral mathematicians with advanced instruction for graduate students. For the graduate students the central formal offering will be a course in Algebraic Number Theory and Class Field Theory based on lectures given by Professor George Whaples of Indiana University. The lectures will be backed up by work sessions throughout the program. Supplementary lectures on Galois Theory will be given by Professor Jonathan D. Lubin, Associate Director, during the first week. The research program will center on a Colloquium at which will appear a sequence of distinguished speakers.

The realization of these plans is contingent upon a grant by the National Science Foundation. If a grant is made, announcements will appear concerning the selection of members, stipends, exact dates, and the Colloquium speakers. Departments at Ph.D. granting institutions will be invited to endorse appropriate graduate students. Colloquium speakers will be asked to nominate postdoctoral students. Individual queries will be welcome.

Tentative plans are being formulated for a third advanced summer seminar in algebra for 1967. The proposed topic is Algebraic Foundations of Algebraic Geometry.

Questions should be addressed to: Professor Dan E. Christie, Chairman, Department of Mathematics, Bowdoin College, Brunswick, Maine 04011.
YESHIVA UNIVERSITY BREAKS GROUND FOR NEW SCIENCE CENTER

Yeshiva University held ground-breaking ceremonies on October 31, 1965, for a new 15-story, $15 million Science Center. In addition to Abe Gelbart, Dean of the Graduate School, and many prominent Governmental leaders, A. Adrian Albert, President of the American Mathematical Society, participated in the ceremonies. The Science Center ground-breaking inaugurated the sixth major unit in the University's "Blueprint for the Sixties" expansion program, initiated in 1961.

The new Science Center, which will house the University's Belfer Graduate School of Science, will be devoted primarily to physics, computing, mathematics, nuclear research, chemistry, biophysics and astrophysics.

PROFESSOR ANDRE WEIL RECEIVES SCIENCE AWARD

Professor Andre Weil, of the Institute for Advanced Study at Princeton, was the recipient of the fourth annual award for distinguished service to science, presented at the Science Conference sponsored by the Belfer Graduate School of Science. The award was presented Sunday, November 14, 1965, at the Hotel Astor in New York City and initiated the Conference which was held on November 15th and 16th at the hotel.

SEMINAR ON MATHEMATICAL LOGIC
Université de Montréal
June 27 to July 29, 1966

Under the sponsorship of the North Atlantic Treaty Organization (NATO) and the Canadian Mathematical Congress, the fifth session of the University of Montreal international "Séminaire de Mathématiques Supérieures" will be held next summer, from June 27 to July 29. The subject of this Seminar will be mathematical logic. The program will consist of six main courses given by invited professors and of a certain number of lectures given by guest speakers. Registrants may make application for financial assistance to cover travelling and living expenses. To obtain further information concerning the program and the invited lecturers, and to obtain registration forms, please write to: Séminaire de mathématiques supérieures, Université de Montréal, Case postale 6128, Montréal 3, Québec (CANADA).

UNIVERSITY OF CHICAGO FORMS INFORMATION SCIENCE COMMITTEE

A new academic committee offering a graduate program leading to advanced degrees in information sciences has been formed at the University of Chicago. Richard H. Miller, Director of the University's Institute for Computer Research, has been named as acting chairman. Additional members of the Committee include Robert Ashenhurst, Robert Graves, Alex Orden, Clemens Roothaan, Victor Yngve, Jay Goldberg, and Glenn Manacher.

The Committee on Information Sciences sponsors research and instruction, primarily on the graduate level. The program of the Committee is based on close cooperation with the Institute for Computer Research and the Computation Center, whose primary interests are in the computer area, as well as with supporting groups in the application areas such as the Biological Sciences Computation Center, the Center for Mathematical Studies in Business and Economics, and the information science program of the Graduate Library School.

The Committee will accept its first students this autumn to work for the degrees of Master of Science and Doctor of Philosophy. Students entering the program will be expected to have a background of courses in advanced calculus, linear algebra, numerical methods, and probability statistics.

MATHEMATICIAN RECEIVES NATIONAL MEDAL OF SCIENCE

It was recently announced that Professor Oscar Zariski of Harvard University was among recipients of the National Medal of Science. The announcement was made by President Johnson on Saturday December 11. Eleven scientists received the awards.
REPORT ON 1965 ADVANCED SUMMER SEMINAR IN HOMOLOGICAL ALGEBRA

The first NSF supported Advanced Seminar in Algebra at Bowdoin College brought together 60 predoctoral and numerous postdoctoral mathematicians for an eight-week program. The main formal offerings were a Course in Homological Algebra based on lectures by Professor Ernst Snapper of Dartmouth and a Colloquium on Homological Algebra and its Applications. Colloquium speakers included (in chronological order): Hyman Bass (Columbia), Michael Artin (M.I.T.), Daniel Zelinsky (Northwestern), Gerhard Hochschild (Berkeley), Alex Rosenberg (Cornell), Peter Freyd (Penn), Stephen S. Shatz (Penn), David A. Buchsbaum (Brandeis), Fred E. J. Linton (Wesleyan), Goro Azumaya (Massachusetts and Indiana), Paul Dedecker (Lille), John W. Gray (Illinois), Peter J. Hilton (Cornell), Ernst Snapper, D. S. Rim (Penn), and, courtesy of the MAA Cooperative Seminar, Israel N. Herstein (Chicago), and Nathan Jacobson (Yale). Other postdoctoral participants were Joseph T. Buckley, Mrs. Verena H. Dyson, Kenneth Grant, Morton E. Harris, Roger Hou, Ian Hughes, R. Wells Johnson, Tzee-Char Kuo, Jonathan D. Lubin, Alan McConnell, James N. McNamara, Horace Y. Mochizuki, E. James Peake, Jr., Leonard E. Ross, Edgar A. Rutter, John H. Smith, Daniel J. Sterling, Earl Willard. The formal program was reinforced by regular work-sessions and by at least 14 seminars.

Dan E. Christie, Director
Department of Mathematics
Bowdoin College

CONFERENCE ON HARMONIC ANALYSIS AND INTEGRAL TRANSFORMS

On August 2 to 10, 1965, a Conference on Harmonic Analysis and Integral Transforms was held at the MATHEMATICAL RESEARCH INSTITUTE, OBERWOLFACH. The Conference, which was by invitation and attended by thirty-two mathematicians from eleven countries, was directed by P. L. Butzer of the Technical University of Aachen, sponsored by the Oberwolfach Institute and supported by Philips (Hamburg), IBM (Sindelfingen) and Telefunken (Ulm).

The program centered around twenty one-hour addresses presented by specialists in the field: E. J. Akutowicz (Bologna), R. A. Askey (Madison, Wisconsin), F. Sunyer i Balaguer (Barcelona), H. Berens (Aachen), A. Dinghas (Berlin), A. Erdélyi (Edinburgh), H. Gunzler (Göttingen), St. Hartmann (Breslau), A. O. Huber (Zurich), S. Igarí (Sendai), J. P. Kahane (Paris), W. A. J. Luxemburg (Pasadena), Y. Meyer (Strassburg), B. Sz. Nagy (Szeged), R. J. Nessels (Aachen), A. Pflüger (Zurich), G. Weiss (St Louis, Missouri), J. D. Weston (Swansea), D. V. Widder (Cambridge, Massachusetts), A. C. Zaanen (Leiden).

The program included panel discussions and two special sessions on new and unsolved problems. The conference was dedicated to the memory of Professor Jean Favard, whose untimely death on January 21, 1965 kept him from delivering one of the invited lectures.

STIPEND INCREASED FOR PEIRCE INSTRUCTORSHIPS IN 1966-1967

The stipends offered in conjunction with the Benjamin Peirce Instructorships at Harvard University have been increased from the amounts noted in the November and December issues of the *Notices*. The salary has been increased from $7200 to $7800 with $400 rather than $300 annual increments. Word of this increase was received too late for inclusion in the special issue on grants and fellowships.
An International School of Nonlinear Mathematics and Physics will be held in Europe during June 27 - August 5, 1966. The school will be sponsored by NATO as part of their Advanced Study Institute Program. Directed by M. D. Kruskal and N. J. Zabusky, the courses offered at the School are designed to present a uniform view of nonlinear phenomena which have recently become more important because of the availability of intense coherent and incoherent sources, satellite data, and sophisticated computers.

The Physics session (three weeks) will cover: classical field theories, gravitation, statistical mechanics, optics, and turbulence, and the faculty includes: N. Bloembergen, I. Prigogine, C. Truesdell, A. A. Vedenov, and J. A. Wheeler.


For more information and application material write to Dr. N. J. Zabusky, Nonlinear School, Bell Telephone Laboratories, Incorporated, Whippany, New Jersey 07981, U.S.A.

MAA TO HOLD COOPERATIVE SUMMER SEMINAR

The third Cooperative Summer Seminar for Postdoctoral College mathematics teachers, sponsored by the MAA, will be held June 20 through August 12, 1966, at Bowdoin College, Brunswick, Maine. This program has a two-fold objective in trying to increase the mathematical competence of college teachers and attempting to improve the quality of mathematical instruction in the home institutions by encouraging participants to conduct seminars for their colleagues upon their return.

The program of activities includes daily lectures on Topics in Applied Mathematics, conducted by Professor E. J. McShane, University of Virginia, and Topics in Analysis, conducted by Professor G. F. Carrier, Harvard University. The Seminar will also feature occasional special lectures by eminent specialists.

Each participant will receive a stipend of $800 plus an extension grant of $700 to cover expenses incurred in conjunction with the home seminar the participant is expected to conduct. Travel allowances and additional allowances for dependents are also available.

The deadline for applications is January 5, 1966. Requests for further information and/or application forms should be sent to:

Professor Vincent O. McBrien, Director
MAA Cooperative Summer Seminar
Department of Mathematics
College of the Holy Cross
Worcester, Massachusetts 01610
The Joint Committee on the Graduate Program requested that the Association and the Society make known, through its publications, the following suggestions for the consideration of Departments of Mathematics—

(i) That an external minor not be required for the doctoral program, and
(ii) That a written magistral essay not be required in those departments having a fairly large doctoral program.

This was done at the Annual Winter Meeting in Miami, and thus far neither the Board nor the Council seem to have taken action on the Committee's request. These august bodies seem to have said neither yea nor nay.

Does the mathematical fraternity really care very much about what goes into the making of a Ph.D. at the approximately 90 universities which grant this degree? Most of these departments are rather small and isolated, many are just emerging. They are subjected to all sorts of pressures—lack of staff, poorly prepared students overloaded with teaching, demands that they produce mathematical doctors willy-nilly in a rather short period.

Quite obviously AMS-MAA cannot and should not get into the business of accreditation. On the other hand, its governing bodies have been adamant that the doctorate should not be lowered in quality, while simultaneously there is an enormous demand for mathematical doctors.

It seems to be that there is a problem here, though, having been wrong once before, it is possible that I err again.

Alexander Doniphan Wallace

Is it the opinion of a majority of the members of the American Mathematical Society that, in the twentieth century United States of America, ballots must require a validating signature of the voter?

Benjamin Volk
ACTIVITIES OF OTHER ASSOCIATIONS

ACM SYMPOSIUM ON SYMBOLIC AND ALGEBRAIC MANIPULATION

An ACM Symposium on Symbolic and Algebraic Manipulation will be held on March 29-31, 1966, at the Sheraton-Park Hotel, in Washington, D. C. The symposium is being sponsored by the Special Interest Committee on Symbolic and Algebraic Manipulation (SIC-SAM) of the Association for Computing Machinery.

The objectives of the symposium are: (1) To report on work done in the areas of list processing, string and character handling, and nonnumerical algebraic manipulation to the wider computing community by means of carefully selected expository papers; both manipulation systems and languages, and their applications will be covered; (2) To help promote new developments by making system builders aware, both of problems encountered by users, and of recently developed implementation techniques; (3) To encourage an exchange of new ideas among specialists in this area.

Further information about the symposium may be obtained by writing:
Miss Jean E. Sammet, Chairman
SIC-SAM
IBM Corporation
545 Technology Square
Cambridge, Massachusetts 02139

THE MATHEMATICAL ASSOCIATION OF AMERICA

The forty-ninth Annual Meeting of the Mathematical Association of America will be held at the Sherman House in Chicago, Illinois from Wednesday to Friday, January 26-28, 1966. A complete program of the meeting is included in the time table in this issue of the Notices.

1966 NATIONAL MEETING OF THE SOCIETY FOR INDUSTRIAL AND APPLIED MATHEMATICS
Iowa City, May 11-14, 1966

The 1966 SIAM National meeting will be held at the University of Iowa in Iowa City on May 11-14, 1966. The theme of the meeting will be numerical analysis with four sessions of invited papers on the topics of numerical analysis of non-linear differential equations, matrices in numerical analysis, numerical analysis of integral and integro-differential equations, and numerical methods in engineering and other physical sciences. The program will also include a panel discussion on education in applied mathematics as well as several sessions of contributed papers.

The session on differential equations will be held in cooperation with the Mathematics Research Center’s 1966 Symposium on the same subject which will be held in Madison, Wisconsin, on May 9-10, 1966, and on May 11, in Iowa City. Abstracts of contributed papers for both meetings should be sent to: R. J. Lambert or G. Seifert, "Contributed Papers"--1966 National Meeting, Department of Mathematics, Iowa State University, Ames, Iowa, before March 5, 1966. Contributed papers in the area of numerical analysis of non-linear differential equations will be included in a special joint SIAM-MRC session on May 11th.

For further information write to: W. J. Jameson, Jr., Collins Radio Company, 120-111, Cedar Rapids, Iowa 52406.
Flight Arrangements for the International Congress

As of December 15 the status of travel arrangements being made by the Society for the International Congress is as follows. So far there have been 319 reservations made, a majority of which state a preference for the direct New York to Moscow and return charter plans as originally announced in the August issue of the Notices. Accordingly, it has been necessary to secure two charter planes for this plan. Arrangements are now being concluded for a 138 Passenger Scandinavian Airlines jet leaving New York on August 14 for Moscow and returning on August 29, and for a 154 passenger Pan American Airlines jet leaving New York on August 14 and returning on August 30. Although there are now sufficient reservations to insure the 120 passenger minimum necessary to make each of these flights feasible, a few additional reservations can be accepted.

In the November issue of the Notices the Society announced plans for a charter from New York to Paris and return with arrangements for European travel left up to the individual. Since that time the International Air Transport Association (IATA) has adopted a new schedule of fares for group travel to Western Europe that makes it more practical to arrange for group travel to London and Paris rather than for charter flights. Group travel rates apply to a minimum of 25 passengers, whereas the minimum required for a charter is 120. Making arrangements for 25 passengers rather than 120 will allow for much more flexibility in schedules. Accordingly, arrangements are now being made for approximately 10 group flights from New York to Paris or London leaving between June 1 and August 4 and returning from Paris or London between September 6 and 21. Additional arrangements are being made for travel from Western Europe to Moscow immediately prior to the Congress and to return to Western Europe at the conclusion of the Congress. The fares established by IATA, which are still subject to ratification by the governments of the participating nations, will be for round-trip from New York to London $300, to Paris $331, and to Moscow $545; as compared to economy class fares of $484.50, $526.30, and $815.50, respectively. There will be an additional charge of not more than $10 to participants in travel groups to obtain the insurance protection described in the August Notices, page 655.

In addition, arrangements are being made for a flight from New York to Moscow via Amsterdam leaving New York on August 13 and returning from Moscow to Amsterdam about August 31. Individuals will be able to return from Amsterdam to New York at their convenience within a year. The fare will be approximately $545.

Since there will be a large number of people leaving New York on August 14 by charter, it may be possible for domestic travel groups to be arranged from Chicago, New Orleans, Seattle, San Francisco, and Los Angeles to New York. Domestic group travel is on regular commercial flights at a discount of 20% on economy fares for the round-trip. A group must travel together to New York, but individuals may schedule their return at any time within one year. The Society will arrange for domestic travel when it is practical.

Applications for charter or group flights will be accepted as long as arrangements can be made. The applicant should include his name, both home and business addresses, names and ages (if children) of any dependents who will be accompanying him. A deposit of fifty dollars must be included for each passenger. Applicants should also list their flight preferences in order, and if these cannot be met or an acceptable substitute found in the arrangements now being made, the application may be withdrawn and the deposit refunded. Preferred travel arrangements will be given to those applications received at the earliest date.
NEW AMS PUBLICATIONS

SELECTED TRANSLATIONS, Series II

Volume 49
276 pages; List Price $8.40, Member Price $6.30.

Ten papers on Functional Analysis and Measure Theory, by P. E. Sobolevskii,
Ju. V. Egorov, M. A. Naïmark, V. P. Palamodov, I. C. Gohberg, V. A. Rohlin,
L. M. Abramov, A. M. Il'in.

LECTURES IN APPLIED MATHEMATICS

Volumes 5, 6, and 7

SPACE MATHEMATICS
Edited by J. Barkley Rosser

These volumes consist, for the most part, of the lectures presented at the 1963 Summer Seminar in Applied Mathematics sponsored by the American Mathematical Society. A few of the notes from the 1961 and 1962 Institutes in Dynamical Astronomy are also included. The articles deal with the current state of research on the behavior of nonpropulsive space vehicles, indicate the more pressing unsolved problems, and furnish examples of mathematical techniques which are currently useful. Besides presenting much new and advanced material, an effort is made in these volumes to give readers basic information, in fields other than their own which they need in order to have a full understanding of space problems in their own fields.

Approximately 200 pages per volume.

SPACE MATHEMATICS, Part I

Volume 5

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"The Lagrange-Hamilton-Jacobi Mechanics" by Boris Garfinkel
"Stability and Small Oscillations about Equilibrium and Periodic Motions" by P. J. Message
"Lectures on Regularization" by Paul B. Richards
"The Spheroidal Method in Satellite Astronomy" by John P. Vinti
"Precession and Nutation" by Alan Fletcher

"Notes on a Two-Degree-of-Freedom Irreversible Dynamical System: the Restricted Problem of Three Bodies" by Victor Szebehely
"Problems of Stellar Dynamics" by George Contopoulos
"Qualitative Methods in the n-Body Problem" by Harry Pollard

Until April 1, 1966 prepublication List Price: $8.91; Member's Price $6.68. After that date at least $9.90 List Price; $7.43 Member's Price.
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PERSONAL ITEMS

Dean A. A. ALBERT of the University of Chicago has been elected an honorary member of the Union Matematica Argentina and the Sociedad Matematica Mexicana.

Mr. R. M. ANDERSON of the Collins Radio Company, Cedar Rapids, Iowa has been appointed to an assistant professorship at the Texas Technological College.

Professor MASAHIKO ATSUJI of the Senshu University, Tokyo, Japan has been appointed to a professorship at the Josai University, Tokyo, Japan.

Professor GORO AZUMAYA of the University of Massachusetts has been appointed to a visiting professorship at Indiana University.

Professor L. C. BAGBY of the Detroit Institute of Technology has accepted a position as Vice President with the International Casting Corporation, New Baltimore, Michigan.

Mr. R. D. BARNARD of the Bell Telephone Laboratories, Murray Hill, New Jersey has been appointed to a professorship in the College of Engineering at Wayne State University.

Mr. R. F. BARNES, JR. of the University of California, Berkeley has been appointed to an associate professorship in the Department of Philosophy at Lehigh University.

Dr. L. C. BARRETT of the South Dakota School of Mines and Technology has been appointed as Professor and Chairman at Clarkson College of Technology.

Professor JOSE BARROS-NETO of the University of Montreal has been appointed to a visiting professorship at the University of Rochester.

Dr. J. Y. BARRY of the Institute for Defense Analyses, Washington, D. C., is on leave as Senior Research Fellow in the Department of Operational Research at the University of Lancaster, Lancaster, England.

Professor GILBERT BAUMSLAG of the Graduate Center of the City University of New York has been awarded a Sloan Foundation Fellowship and will be on a leave of absence from January to June 1966.

Mr. R. W. BEALS of Yale University has been appointed to a visiting assistant professorship at the University of Chicago.

Dr. P. O. BELL of Bellcomm Incorporated has been appointed to a professorship at the George Washington University.

Dr. RICHARD BELLMAN of the Rand Corporation, Santa Monica, California has been appointed Professor of Mathematics, Medicine and Engineering at the University of Southern California.

Dr. J. P. BENZECRI of the Rennes Faculté des Sciences, Rennes, France has been appointed to a professorship at the Faculté des Sciences, Département de statistique, University of Paris, Paris, France.

Mr. MICHAEL BERNKOPF of Hunter College has been appointed to an assistant professorship at Fairleigh Dickinson University.

Dr. BARRY BERNSTEIN of the National Bureau of Standards, Washington, D. C. will be on a leave of absence and has been appointed to a visiting associate professorship in the School of Aeronautics, Astronautics and Engineering Sciences at Purdue University.

Dr. DOROTHY BOLLMAN formerly of Michigan State University has been appointed to a visiting associate professorship at the University of Puerto Rico, Mayaguez, Puerto Rico for 1965-1966.

Professor R. A. BONIC of Cornell University has been appointed to an associate professorship at Northeastern University.

Dr. J. R. BOWER of the University of Michigan has been appointed as assistant Professor and Coordinator of Measurement Services at the Testing and Counseling Center of the University of Texas.

Mr. R. E. BRADFORD of the University of California, Berkeley has been appointed to an assistant professorship at the University of Southern California.

Mr. D. M. BROWN of the Department of Defense at the Computer Institute, Washington, D. C. has been appointed to a professorship at the Allegany Community Col-
Mr. B. W. BUGDEN of Queen Mary College, University of London, London, England has been appointed an Assistant Lecturer at the University of Birmingham, Birmingham, England.

Professor E. A. CAMERON of the University of North Carolina has been awarded a National Science Foundation Science Faculty Fellowship and will spend the academic year 1965-1966 at Harvard University.

Dr. B. A. CHARTRES of Brown University has been appointed an Associate Professor of Computer Science and Associate Director of the Computer Science Center at the University of Virginia.

Mr. C. L. CHRISTMAS of the University of Georgia has been appointed to an assistant professorship at Eastern Illinois University.

Professor D. S. COHEN of Rensselaer Polytechnic Institute has been appointed to an assistant professorship at the California Institute of Technology.

Professor J. S. CRONIN of the Polytechnic Institute of Brooklyn has been appointed to a professorship at Rutgers, The State University.

Dr. E. C. CURTIS of Dartmouth College has accepted a position as an Associate with Daniel H. Wagner, Associates, Paoli, Pennsylvania.

Professor K. L. DE BOUVERE of the University of Amsterdam, Amsterdam, Netherlands has been appointed to an associate professorship at the University of Santa Clara.

Mr. J. L. DENNY, JR. of Indiana University has been appointed to an assistant professorship at the University of California, Riverside.

Professor J. E. DE PILLIS of San Francisco State College has been appointed to an assistant professorship at the University of California, Riverside.

Mr. L. O. FERGUSON of the University of Washington has been appointed to an assistant professorship at the University of California, Riverside.

Mr. G. H. GLEISSNER of the U. S. Naval Weapons Laboratory, Dahlgren, Virginia has accepted a position as Associate Technical Director and Head of the Applied Mathematics Laboratory at the David Taylor Model Basin, Washington, D. C.

Dr. K. C. HA of the Seoul National University, Seoul, Korea has been appointed to an assistant professorship at the University of South Florida.

Professor PETER HENRICI has returned to the Eidgenoessische Technische Hochschule, Zurich, Switzerland from Harvard University where he was a Visiting Professor of Applied Mathematics during the spring of 1965.

Dr. P. N. HU of the Davidson Laboratory at the Stevens Institute of Technology has accepted a position as a Senior Scientist with the Space Sciences Incorporated, Wal­tham, Massachusetts.

Professor P. C. JAIN of the U. S. Army, Mathematical Research Center at the University of Wisconsin, has been appointed to a professorship at the Birla Institute of Technology and Science, Pilani (Rajasthan), India.

Professor G. P. JOHNSON of the University of the South has been appointed Professor and Chairman at Oakland University.

Dr. C. W. KIM of the University of Washington has been appointed to an assistant professorship at the University of Idaho.

Professor K. L. KIMURA of the University of Arkansas has been appointed to a professorship at the University of Arkansas.

Professor P. C. JAIN of the University of Washington has been appointed to a professorship at Stanford University.

Dr. L. G. KOVACS of the Research School of Physical Sciences, The Australian National University, Canberra, Australia has been appointed to a visiting professorship at the Graduate Center of the City University of New York from January to June 1966.

Professor H. W. KUHN of Princeton University has been awarded the National Science Foundation Senior Postdoctoral Fellowship and will be on leave for the academic year 1965-1966 at the University of Rome, Rome, Italy.

Dr. P. E. LONG of the Oklahoma State University has been appointed to an assistant professorship at the University of Arkansas.

Mr. G. D. LUDDEN of the University of Notre Dame has been appointed a Lecturer at Indiana University.

Mr. J. P. MALONEY of the Georgetown University has been appointed to an assis-
Dr. J. G. MARICA of the University of Chile, Santiago, Chile has been appointed to a visiting associate professorship at the University of Idaho.

Professor P. E. MARTIN of Rutgers, The State University has been appointed to an associate professorship at Dickinson College.

Dr. BROCKWAY MCWILLIAM, Under Secretary of the U. S. Air Force, Washington, D. C. has accepted a position as Executive Director of Military Research with the Bell Telephone Laboratories, Incorporated, Whippany, New Jersey.

Dr. M. V. K. MENON of the International Business Machines Corporation, San Jose, California has been appointed a Member of the Research Staff of the U. S. Army, Mathematical Research Center at the University of Wisconsin.

Dr. W. A. MICHAEL, Jr. of the International Business Machines Corporation, San Jose, California has accepted a position as a Member of the Research Staff of the Ampex Corporation, Redwood City, California.

Mr. WILLARD MILLER, JR. of the Courant Institute of Mathematical Sciences at New York University has been appointed to an assistant professorship at the University of Minnesota.

Professor G. J. MINTY of the University of Michigan has been appointed to a professorship at Indiana University.

Dr. H. G. von MISSEs of Harvard University has been appointed a Visiting Professor of Applied Mathematics for the spring semester 1966 at the University of Virginia.

Dr. B. M. MITCHELL of Columbia University has been appointed to an assistant professorship at Bowdoin College.

Dr. A. L. MULLIKIN of the University of Wisconsin has been appointed to an assistant professorship at the Georgia Institute of Technology.

Mr. STELIOs NEGREPONTIS of the University of Rochester has been appointed to a visiting assistant professorship at Indiana University.

Professor RUFUS OLDENBURGER, Director of the Automatic Control Center at Purdue University, was elected a Fellow of the American Society of Mechanical Engineers at the November 1965 winter annual meeting.

Professor WILLIAM PRAGER of the University of California, San Diego has been elected to the National Academy of Engineering.

Professor G. E. H. Reuter of the University of Durham, Durham, England has been appointed to a professorship at the Imperial College of Science and Technology, London, England.

Dr. FAZLOLLAH REZA of Syracuse University will be on a leave of absence and has been appointed to a visiting professorship at the Institut Henri Poincaré, Université de Paris, Paris, France for the fall term 1965-1966.

Associate Director B. E. RHODES of the Committee on the Undergraduate Program in Mathematics at the University of California, Berkeley has been appointed to an associate professorship at Indiana University.

Dr. HABIB SALEHI of Indiana University has been appointed to an assistant professorship in the Department of Statistics at Michigan State University.

Dr. G. L. SEEVER of the University of California, Los Angeles has been appointed to an assistant professorship at the California Institute of Technology.

Dr. J. G. SIMMONDS of the Massachusetts Institute of Technology has been appointed an Assistant Professor of Applied Mathematics at the University of Virginia.

Professor A. E. TAYLOR of the University of California, Los Angeles has been appointed Vice President for Academic Affairs of the statewide University. He will leave the Los Angeles mathematics department at the end of the 1965 fall semester.

Mr. P. C. TONNE of the University of North Carolina has been appointed to an assistant professorship at Emory University.

Dr. H. J. TRAMER of Johns Hopkins University has been appointed to an assistant professorship at the State University of New York at Stony Brook.

Dr. J. F. TRAUB of the Bell Telephone Laboratories Incorporated, Murray Hill, New Jersey has been appointed to a visiting associate professorship in the Computer Science Department at Stanford University for the spring of 1966.
The following promotions are announced.

To Head Professor:
Auburn University, L. P. BURTON.

To Research Professor:
Auburn University: E. V. HAYNSWORTH.

To Professor:
California Institute of Technology: A. M. GARSIA; University of Illinois: L. A. RUBEL; Luther College: H. J. REBASSOO; Sacramento State College: G. R. GLABE, CHING-HWA MENG; Trinity College: E. F. WHITTLESEY; Wayne State University: A. T. BHARUCHA-REID.

To Associate Professor:
American University: I. J. KATZ; Auburn University: M. M. BASKERVILLE, C. E. ROBINSON; California Institute of Technology: P. L. CRAWLEY; University of California, Santa Barbara: J. C. CEDAR, R. C. THOMPSON; University of Colorado: J. D. MONK; Emory University: B. K. YOUSE; Georgia Institute of Technology: M. Z. NASHED; Harvey Mudd College: JOHN GREEVER; Humboldt State College: R. W. TUCKER; Loretto Heights College: SISTER MARGARET GRACE; Oakland University: D. G. MALM; Southern Illinois University: D. T. HAIMO; Tulane University: F. T. BIRTEL; Wayne State University: CHARLES BRIGGS.

To Assistant Professor:
California Institute of Technology: H. A. KRIEGER; Delaware State College: HARRY WASHINGTON; Georgia Institute of Technology: G. L. CAIN; Humboldt State College: V. K. T. TANG; Pepperdine College: DENNIS DEPASSE.

To Deputy Director:
Communications Research Division of the Institute for Defense Analyses at Princeton: L. P. NEUWIRTH.

The following appointments are announced:

To Instructor:
Eastern Illinois University: J. W. LEDUC; Fordham University: E. W. BRANDE; Oakland University: V. M. SEHGL; Rosary College: P. A. OSTRAND; University of Santa Clara: GEORGE LANGBERG; Simmons College: J. L. FISHER; Stetson University: SHERWOOD HOADLEY; Trinity College: R. D. POLLACK.

Deaths:
Professor Emeritus C. R. ADAMS of Brown University died on October 15, 1965 at the age of 67. He was a member of the Society for 44 years.
Professor JESSE DOUGLAS of the City University of New York died on October 7, 1965 at the age of 68. He was a member of the Society for 45 years.
Professor Emeritus L. P. EISENHART of Princeton University died on October 28, 1965 at the age of 89. He was a member of the Society for 65 years.
Professor Emeritus R. K. MORLEY of Worcester Polytechnic Institute died on August 3, 1965 at the age of 83. He was a member of the Society for 44 years.
Professor Emeritus M. E. WELLS of Vassar College died on October 7, 1965 at the age of 84. She was a member of the Society for 52 years.

Errata
The following are corrections of announcements in the October issue of the Notice.

Dr. LOUISE HAY has been appointed to an assistant professorship at Mount Holyoke College.
Professor J. A. WOLF of the University of California, Berkeley is at the Institute for Advanced Study for the academic year 1965-1966.
SUPPLEMENTARY PROGRAM-Number 36

During the interval from September 22, 1965 through November 26, 1965 the papers listed below were accepted by the American Mathematical Society for presentation by title. After each title on this program there is an identifying number. The abstracts of the papers will be found following the same number in the section on Abstracts of Contributed Papers in this issue of these Notices.

One abstract presented by title may be accepted per person per issue of the Notices. Joint authors are treated as a separate category; thus in addition to abstracts from two authors individually one joint abstract by them may be accepted for a particular issue.

(1) On representation of partially ordered sets
   Professor Alexander Abian and Mr. David Deever, Ohio State University (66T-44)

(2) Convergence properties of cubic splines
   Professor J. H. Ahlberg, United Aircraft Research Laboratories, East Hartford, Connecticut, Dr. E. N. Nilson, Pratt and Whitney Aircraft, East Hartford, Connecticut and Professor J. L. Walsh, University of Maryland (66T-47)

(3) Concerning some characterizations of absolute continuity
   Professor W. D. L. Appling, North Texas State University (66T-62)

(4) Extension of an Euler transformation process. Preliminary report
   Mr. Al Berin, Hughes Aircraft Company, Culver City, California (66T-71)

(5) Linear positive machines
   Professor R. S. Bucy, University of Colorado (66T-41)

(6) On the inverse problem corresponding to certain ordinary even order differential operators. Preliminary report
   Professor J. B. Butler, Jr., Portland State College and Robert College, Turkey (66T-52)

(7) Some growth and convexity theorems for second order equations
   Professor Robert Carroll, University of Illinois (66T-43)

(8) The univalence of an integral
   Mr. W. M. Causey, University of Kansas (66T-7)

(9) On the length of programs for computing finite binary sequences by bounded-transfer Turing machines. Preliminary report
   Mr. G. J. Chaitin, The City College, CUNY (66T-26)
   (Introduced by Dr. Sondra Jaffe)

(10) Finitely generated N-semigroups
    Mr. J. L. Chrislock, University of California, Davis (66T-74)

(11) On the boundary behavior of functions meromorphic in the unit disk
    Professor Peter Colwell, Iowa State University (66T-46)

(12) Bordism and complex K-theory
    Professor P. E. Conner and Professor E. E. Floyd, University of Virginia (66T-63)

(13) The Tutte polynomial
    Professor H. H. Crapo, University of Waterloo, Canada (66T-8)

(14) Analogue of a theorem of Khintchine in a field of formal Laurent series
    Mr. T. W. Cusick, Churchill College, England (66T-34)

(15) Linear methods which sum sequences of bounded variation
    Professor D. F. Dawson, North Texas State University (66T-58)

(16) Some properties of a generalized Schwarzian derivative of a polygenic function
    Professor John DeCicco and Mr. Stavros Busenberg, Illinois Institute of Technology (66T-27)

(17) Orthogonal group matrices of hyperoctahedral groups
    Professor J. S. Frame, Michigan State University (66T-67)
(18) Some basic properties of polynomials over a commutative ring. Preliminary report
Professor R. W. Gilmer, Jr., Florida State University (66T-24)

(19) Infinitely divisible point processes in $\mathbb{R}^n$
Dr. J. R. Goldman, Harvard University (66T-61)

(20) Equality of minimal and maximal extensions of a partial differential operator in $L_p(\mathbb{R}^n)$
Mr. R. A. Goldstein, Courant Institute of Mathematical Sciences, New York University (66T-53)

(21) Congruence relations on partial algebras
Professor George Grätzer, Pennsylvania State University (66T-66)

(22) On Witt's Theorem in the denumerable case
Professor Herbert Gross, Montana State University (66T-3)

(23) Bilinear forms in the denumerable case and characteristic 2
Professor Herbert Gross and Professor R. D. Engle, Montana State University (66T-4)

(24) Perturbation of contraction semigroups
Dr. K. E. Gustafson, Battelle Institute, Switzerland (66T-33)

(25) Isotonic spaces in convexity. II
Professor P. C. Hammer, Pennsylvania State University (66T-65)

(26) Isomorphism types of index sets of partial recursive functions
Professor Louise Hay, Mount Holyoke College (66T-59)

(27) A Harnack-type inequality
Professor L. L. Helms, University of Illinois (66T-22)

(28) Representations by bilinear forms (mod $p^a$)
Professor J. H. Hodges, University of Colorado (66T-15)

(29) Seminormed Riesz spaces and Egoroff property
Mr. J. A. R. Holbrook, California Institute of Technology (66T-36)

(30) On the homotopy groups of an affine algebraic hypersurface
Mr. Alan Howard, Stanford University (66T-14)

(31) Absolute Hardy-Bohr factors
Mr. R. L. Irwin, University of Utah (66T-1)
(Introduced by Professor C. R. Wylie)

(32) Concerning local bounded variation in the moment problem
Dr. G. G. Johnson, University of Georgia (66T-13)

(33) Normed linear spaces equivalent to inner product spaces
Professor J. T. Joichi, University of Minnesota (66T-54)

(34) On the generalized inverse of products of matrices
Professor I. J. Katz, American University (66T-69)

(35) Locally flat strings and half-strings. Preliminary report
Mr. R. C. Lacher, University of Georgia (66T-51)

(36) A characterization of hh-simple sets
Mr. A. H. Lachlan, Simon Fraser University, British Columbia (66T-11)

(37) On functions which preserve Cauchy nets
Professor Yu-Lee Lee, University of Florida (66T-30)

(38) Decomposition of functions and the classification of spaces in axiomatic potential theory. Preliminary report
Professor P. A. Loeb and Professor Bertram Walsh, University of California, Los Angeles (66T-64)

(39) Completely regular mappings and the weak bundle properties
Professor L. F. McAuley, Rutgers University (66T-21)

(40) Another S-admissible class of spaces
Professor K. D. Magill, Jr., SUNY at Buffalo (66T-56)

(41) Universal sentences in languages with countable conjunction
Mr. Jerome Malitz, University of California, Berkeley (66T-10)

(42) A representation theorem for positive functionals on involution algebras
Professor G. J. Maltese, University of Maryland and Professor R. S. Bucy, University of Colorado (66T-57)

(43) On a generalisation of a lemma of Chow
Dr. S. M. Mazhar, Aligarh Muslim University, India (66T-25)

(44) Group-structural criteria for group orderings. II. Preliminary report
Professor A. C. Morel, University of Washington (66T-48)

(45) On fixed-point results for some computable functionals
(46) A generalisation of the Bolzano-Weierstrass theorem
Dr. Jan Mycielski, Polish Academy of Sciences, Poland (66T-16)
(47) On nonarchimedian Banach algebras
Mr. Lawrence Narici, Polytechnic Institute of Brooklyn (66T-40)
(48) Symmetry in non self-adjoint Sturm-Liouville systems
Professor J. W. Neuberger, Emory University (66T-6)
(49) Finite bases for identities in commutative semigroups
Mr. Peter Perkins, University of California, Berkeley (66T-37)
(50) m-frames in tame n-books in $E^3$
Lt. C. A. Persinger, Air Force Institute of Technology, Wright-Patterson Air Force Base, Ohio (66T-17)
(51) Formal contraction of the n-simplex
Mr. B. B. Peterson, Middlebury College (66T-32)
(52) A class of bounded analytic functions whose derivatives are not of bounded type
Mr. K. V. R. Rao, Purdue University (66T-76)
(53) Certain products of tori manifolds admit expansive homeomorphisms
Dr. W. L. Reddy, Webster College (66T-18)
(54) Homotopy functors determined by set-valued maps
Professor C. J. Rhee, Randolph-Macon Woman's College (66T-12)
(55) Some unsolvable problems involving functions of a real variable
Mr. Daniel Richardson, The University of Bristol, England (66T-31)
(56) The evaluation of a character sum
Mr. S. F. Robinson, University of Southern California (66T-28)
(57) The automorphism group of nets
Professor G. O. Sabidussi, McMaster University, Canada (66T-68)
(58) Quasigroupes demi-symetriques II, Autotopies gauches
Professor Albert Sade, 86, Cours de la Republique, Pertuis, Vaucluse, France (66T-39)
(59) On reductive Lie admissible algebras. Preliminary report
Mr. A. A. Sagle, Yale University (66T-55)
(60) Polynomial approximation of functions analytic in a disk
Mr. J. T. Scheick, Syracuse University (66T-19)
(61) Differential inequalities of the fourth order
Professor Johann Schroder, University of Cologne, Germany (66T-70)
(62) On the automorphisms of the unitary group over a field of characteristic 2
Mr. Eugene Spiegel, California Institute of Technology (66T-38)
(63) Locally cyclic semigroups
Professor Takayuki Tamura and Mr. R. Levin, University of California, Davis (66T-75)
(64) Construction of commutative Archimedean semigroups without idempotent
Professor Takayuki Tamura, University of California, Davis (66T-72)
(65) The Feller and Silov boundaries of a vector lattice
Professor J. C. Taylor, McGill University, Canada (66T-60)
(66) Existence of optimal stopping rules for linear and quadratic rewards
Professor Henry Teicher, Purdue University and Professor Jacob Wolfowitz, Cornell University (66T-45)
(67) Bernay's set theory and the continuum hypothesis
Mr. L. H. Tharp, Massachusetts Institute of Technology (66T-42)
(68) Lattice-embeddings in the degrees and hyperdegrees
Mr. S. K. Thomason, Cornell University (66T-35)
(69) A version of Riesz's Theorem for convex metric spaces
Mr. F. A. Toranzos, University of Washington and Consejo Nacional Investigaciones Cientificas y Tecnicas, Argentina (66T-29)
(70) Differences, convolutions, primes. II
Mr. Benjamin VolK, Yeshiva University (66T-5)
(71) The convergence of sequences of ra-
MEMORANDA TO MEMBERS

MATHEMATICAL SOCIETY OF FRANCE INCREASES DUES

Members of AMS who are members of the Mathematical Society of France under the terms of a reciprocity agreement should note that the dues in the French Society are $4, rather than $3 as listed in the "Report on Reciprocity Agreements." Apparently, the dues were increased from $3 to $4 for the year 1965. However, AMS was not notified of this change, thus the former figure appeared in our revised edition of "Report on Reciprocity Agreements." Fees are to be paid by sending a check for $4 to the AMS, with the notation, "reduced membership fee for the Mathematical Society of France 1966."

This fee, granted under the terms of reciprocity, applies only to membership for the year 1966. Payment of dues for former years, if they have not yet been paid, must be paid at the full rate, either (1) directly to France at the rate of 30 francs for each of the unpaid years for membership in arrears; or 50 francs for members in arrears who desire to receive the BULLETIN for the corresponding years; or (2) directly to AMS at the rate of $6 for each of the unpaid years for membership in arrears; or $10 for members in arrears who desire to receive the BULLETIN for the corresponding years.

AMS members who mistakenly paid only $3 for their 1965 membership are not required to pay the full rate as listed in the paragraph above provided that the extra dollar is paid to AMS before January 31, 1966.

In the future, the Mathematical Society of France would appreciate members paying their annual dues each year in the month of January.
ABSTRACTS OF CONTRIBUTED PAPERS

The October Meeting in Cambridge, Massachusetts
October 30, 1965


Let $A$ be a complex Banach algebra with identity and an involution $*$ s.t. $(za + b)^* = za^* + b^*$, $(ab)^* = b^*a^*$ and $a^{**} = a$ whenever $a, b \in A$ and $z$ is complex. $A$ is a B*-algebra iff $\|aa^*\| = \|a\| \|a^*\|$ when $a \in A$. $A$ is locally B*-algebra iff $\|aa^*\| = \|a\| \|a^*\|$ when $a \in A$ and $aa^* = a^*a$. It is well-known that each B*-algebra is completely isomorphic to a C*-algebra, i.e. a uniformly closed and *-closed algebra of operators on a Hilbert space. Theorem. The following 4 conditions on $A$ are equivalent:

(i) $A$ is a B*-algebra.
(ii) $A$ is a locally B*-algebra.
(iii) $\|\exp ih\| = 1$ for each Hermitian element $h$ of $A$.
(iv) There is a neighborhood $N$ of the identity in $A$, and a function $G: N \to \text{reals}$ s.t. $G(1) = 1, G$ is continuous at 1, and $\|a\| \|a^*\| \leq G(aa^*)$ whenever $a, a^*$, and $aa^*$ all lie in $N$. The proof is obtained by piecing together the work of Vidav (Math. Z. 66 (1956), 121); the author (Abstract 604-22, these Notices 11 (1964), 51), and Russo and Dye, (a forthcoming paper titled A note on unitary operators in C*-algebras). (Received October 22, 1965.)

The November Meeting in Lexington, Kentucky
November 12-13, 1965


Bing and Kirkor defined a concept called "strongly cellular" in their paper An arc is tame in 3-space if and only if it is strongly cellular (Fund. Math. 55 (1964), 175-180). The main theorem of the paper is stated by the title. There is a flaw in the early part of the argument that leads to an erroneous conclusion. In particular, the theorem is false under the definition of strong cellularity adopted in that paper. The purpose of the present paper is to modify the concept of strong cellularity so that the theorem referred to above will be true under the modified definition, and to show that the result can be extended to 3-cells. An immediate corollary is a criterion for tameness of 2-spheres. (Received September 29, 1965.)

627-53. E. L. BETHEL, Clemson University, Clemson, South Carolina. On reducibility of continua.

A continuum is said to be irreducible if there exists two points about which it is irreducible. It is shown in this paper that if $M$ is a compact metric continuum and $G$ is a nondegenerate monotone continuous decomposition of $M$ each of whose elements is nondegenerate and either snakelike or indecomposable, and $M/G$ has a dense set of separating points, then it is not the case that $M$ is irreducible. (Received September 29, 1965.)
627-54. T. G. PROCTOR, Clemson University, Clemson, South Carolina. An invariant curve theorem for analytic mappings of an annulus.

In this paper a theorem is proved establishing the existence of curves with analytic representation which are invariant under a certain analytic area preserving mapping. The mapping is a perturbation of a rotation or twist mapping and arises naturally in some undamped physical motions. This theorem modifies an invariant curve theorem proved by J. Moser to guarantee that there are "analytic" curves invariant under the mapping and is an application of a technique for overcoming small divisors which was introduced by Kolmogorov in the study of perturbations of integrable Hamiltonian systems. In the proof of the analytic case to avoid requiring additional and impractical hypothesis it is convenient to introduce a modified mapping which also possesses invariant sets. The theorem is then obtained by showing the modification vanishes for real mappings with a curve intersection property. (Received September 29, 1965.)

627-55. B. A. FUSARO, University of South Florida, Tampa, Florida 33620. A nonuniqueness result for a generalization of the Euler-Poisson-Darboux (EPD) problem.

A generalized EPD problem in \( m + 1 \) variables with index \( k \) will be defined as \( u_{tt} + kb(t)u_t = \Delta_2 u \left( 0 < t \right), u(x,0) = f(x), u_x(x,0) = 0. \) The space operator \( \Delta_2 \) is the positive definite, class \( C^2 \) Beltrami second differential parameter of a harmonic space \( H^m, \) and the coefficient \( b(t) \) is defined by

\[
(1 - m) b(t) = \frac{\phi''(t)}{\phi'(t)};
\]

where \( \phi(t) \) is an elementary solution of \( \Delta_2 \phi = 0 \) depending on the geodesic distance \( r \) alone, the existence of \( \phi \) being a characterizing property of \( H^m. \) For negative \( k, \) functions \( w = w(k) \) are sought for the corresponding completely homogeneous problem so that if \( u = u(k) \) is a solution of the original EPD problem, then \( u + w \) is also a solution. In \( E^m, \Delta_2 = \Delta \) and \( b(t) = t^{-1}, \) so the equation reduces to that of the classical EPD problem \( u_{tt} + kt^{-1}u_t = \Delta u, \) where \( w = t^{1-k} (A. \text{ Weinstein, On the wave equation and the equation of Euler-Poisson, 1952}). \) It can be verified for the EPD problem in \( H^m, \) that a function \( w \) is given by \( w = \phi'(t)^k/(m-1), \) and hence this EPD problem in \( H^m \) cannot have a unique solution for \( k < 0. \) (Received September 30, 1965.)

Two functions of primes $p$, $f_3(p)$ and $f_5(p)$, are established so that each class of cubic nonresidues modulo $p$ contains a positive integer smaller than $f_3(p)$ and each class of quintic nonresidues modulo $p$ contains a positive integer smaller than $f_5(p)$. The function $f_3(p) = \exp[(n + \epsilon)\ln p]$, where $n = .191$ and $f_5(p) = [(\beta + \epsilon)\ln p]$, where $.27 < \beta < .2725$. The function $f_3(p)$ is consistent with Burgess' improvement of the Davenport and Erdős function but is obtained by different methods. The function $f_5(p)$ is better than the previous known bounds. (Received September 30, 1965.)

629-28. PETER GABRIEL, Université de Strasbourg, Strasbourg, France, and ULRICH OBERST, University of California, San Diego, La Jolla, California 92038. Spectral categories and V. Neumann regular rings.

All considered rings are unitary and all modules are unitary right modules. The spectrum of a ring $R$ is the "set" of all isomorphism classes of direct-indecomposable, injective $R$-modules. If $R$ is right-Noetherian the spectrum completely describes all injective $R$-modules. In this paper it is shown for arbitrary $R$ (even for every Grothendieck category) that this spectrum can be replaced by the so called spectral category. This spectral category is obtained by making formally invertible all essential monomorphisms. The spectral category of a Grothendieck category is a Grothendieck category in which every morphism splits. It is equivalent to a quotient category of the category of modules over a certain regular ring. As an application one obtains a system of invariants for every module over an arbitrary ring. These invariants (ranks) have formerly been defined by L. Fuchs (Ann. Univ. Sci. Budapest 6 (1963)) under very strong assumptions. (Received October 1, 1965.)

629-29. K. G. WHYBURN, 124 Triphammer Road, Ithaca, New York. On a different multiplication for functions on certain locally compact groups.

In a previous paper it has been shown that the $L^1$ functions on a compactly generated locally compact abelian group the compact part of which has a countable dual or the continuous functions on such a group without the countability condition may be imbedded 1-1 in a commutative ring with no divisors of zero. This ring has essentially ordinary addition and a multiplication very similar to convolution. In this paper it is shown that if $f$ and $g$ are two elements of this ring corresponding to functions on the group then it is possible to reconstruct their convolution as elements of $L^1$ from their product in the ring. This allows the transformation of the problem of solving convolution equations on the group into the problem of classifying elements of the field of quotients of the ring as the convolution equation may now be lifted into this field and solved there. This leaves the problem of interpreting the solution in the field in terms of the group. (Received October 1, 1965.)
Let $M$ be a bounded open analytic polyhedron in $\mathbb{C}^2$ and let $F$ be its Bergman-Šilov boundary. See Bergman, *Über eine integral darstellung ... Mat. Sb. 1 (43) (1936), 851-861* for a description of such domains. According to hypotheses and theorems presented there, every point of $\partial M - F$ belongs to a "lamina" which is a two dimensional image of the unit disk in the complex plane. Using this fact one can extend a function which is defined on and satisfies certain hypotheses on $F$ to all of $\partial M$ in such a way that it attains its maximum on $F$. Then using the solution of the Dirichlet problem for the present circumstances one obtains a function that is harmonic in $M$ and assumes its maximum on $F$, where it takes on the prescribed values at points where they are continuous. In particular, if $F = \bigcup_{k=1}^{m} F_k$ is a sufficiently regular decomposition of $F$, then there exist functions $H(z;M,F_k)$, $z = (z_1, z_2)$, $k = 1, \ldots, m$, such that $H(z;M,F_k)$ is harmonic in $M$, defined in $\overline{M} = 1$ on $F_k$, and $= 0$ on $F - F_k$. If $f(z)$ is a function holomorphic in $M$ and continuous in $\overline{M}$, and if $|f(z)| \leq a_k$ for $z \in F_k$, $k = 1, \ldots, m$, then $|f(z)| \leq \exp \sum_{k=1}^{m} H(z;M,F_k) \cdot \ln(a_k)$ for all $z \in M$. The proof proceeds by first showing that the desired inequality holds on all of $\partial M$, and then by following the proof of an analogous theorem for $\mathbb{C}$. (Received October 21, 1965.)
630-1. T. J. BROWN, Loyola University, Chicago, Illinois. An axiomatization of N-partition geometries.

Two points of a geometry are related iff the line they define is nontrivial. A chain is a finite sequence $C = (a_1, ..., a_m)$, $m \geq 0$, with $a_j$ related to $a_{j+1}$ for $0 \leq j < m$. The length, $|C|$, of $C$ is $m$ in this case. Two points are connected iff there exists a chain connecting them. Suppose that the point $a$ is connected to the point $b$; $d(a,b) = \min \{|C| \mid C$ is a chain connecting $a$ to $b\}$. Distances between subspaces are defined in the natural way. Theorem. Let $N \geq 1$ and let $g$ be a geometry on $W$, $|W| \cong N + 2$. The lattice of subspaces of $g$ is isomorphic to the lattice of all $N$-partitions on some set iff $g$ satisfies the following axioms: (1) There is a line with exactly $N + 2$ points on it. (2) If a point $a$, not on a nontrivial line $L$, is related to a point on $L$ then $a$ is related to precisely two points on $L$. (3) If $(a, b, c, a)$ and $(d, b, c, d)$ are chains with $a, d \not\in g(b,c)$ then $d(a,d) \leq 1$. (4) Any two points are connected. (5) If $L_1$ and $L_2$ are nontrivial lines with $d(L_1, L_2) = 1$, then there are exactly two points of $L_1$ which are solutions of $d(x, L_2) = 1$. Ore (Duke Math J., 9 (1942), 573-576) axiomatized 1-partition geometries using essentially axioms 1-4. If $N = 2$, Axiom 5 is independent of the preceding axioms. Hartmanis (Canad. J., Math., 11 (1959), 97-106) has obtained different axioms for 2-partition geometries. (Received August 16, 1965.)


Let $A$ be a finite dimensional linear associative algebra with identity over the real field $R$ or complex field $C$. Further, let $A = A_1 \oplus ... \oplus A_k$ be semisimple with the $A_i$ as simple direct summands. If $F_i$ is a function on $A_i$ ($i = 1, 2, ..., k$), then define $F = F_1 \oplus ... \oplus F_k$ on $A$ by $F(a) = F_1(a_1) + ... + F_k(a_k)$. The notions of primary, intrinsic and H - intrinsic functions are defined in Elements of a theory of intrinsic functions on algebras by R. F. Rinehart [Duke Math. J. 27 (1960), 1-20]. Let $H$ be the group of all automorphisms and anti-automorphisms of $A$ which leave the $A_i$ invariant. We prove that if the $F_i$ are "intrinsic" (primary with stem function $f$) on the $A_i$ then $F$ is "H-intrinsic" (primary with stem function $f$) on $A$, and conversely. "H-intrinsic" is replaced by the stronger "intrinsic" if in addition we know that $A_i$ is not isomorphic to $A_j$ ($i \neq j$), or if certain restrictions are placed on the functions $F_i$. As a preliminary result we also prove that in the Sylvester-Buchheime definition of primary function, the minimum polynomial may be replaced by any polynomial divisible by the minimum polynomial whose roots satisfy the necessary conditions. (Received August 16, 1965.)
Let $H$ be a given Hilbert space. Fix $a$ a real number, and set $D_a = \{ f(z) = \sum_{n=0}^{\infty} a_n z^n | a_n \in H \text{ for } n = 0, 1, 2, \ldots, \text{ and } \sum_{n=0}^{\infty} (n+1)^a \| a_n \|_H^2 < \infty \}$. Let $h(z)$ be an operator valued function mapping the open unit disc into the algebra of all bounded linear transformations of $H$ into $H$. Then $h(z)$ is a multiplier from $D_a$ to $D_\beta$ if $h \cdot f \in D_\beta$ for each $f \in D_a$, where $h \cdot f$ denotes pointwise multiplication. Let $M(D_a, D_\beta)$ denote the set of all multipliers from $D_a$ to $D_\beta$. The purpose of the paper is to characterize the sets $M(D_a, D_\beta)$ for different choices of $a$ and $\beta$. For example, we show that $M(D_a, D_\beta) = \{ h(z) | h(z) = 0 \}$ for $a < \beta$. Where a complete characterization has not been achieved, both a necessary condition and a sufficient condition for a function to be a multiplier are given. Further, the paper gives some examples and shows that some of the characterizations for the case when $H$ is finite dimensional fail to carry over to the case when $H$ is infinite dimensional. (Received August 9, 1965.)

If $G$ is any monotone decomposition of $E^3$, let $H_G$ denote the union of all the nondegenerate elements of $G$, and let $P_G$ denote the projection map from $E^3$ onto the decomposition space $E^3/G$ associated with $G$. Suppose that $F$ and $G$ are monotone decompositions of $E^3$ such that each of $\text{Cl}(P_F[H_F])$ and $\text{Cl}(P_G[H_G])$ is compact and 0-dimensional. Then $F$ and $G$ are equivalent decompositions of $E^3$ if and only if there is a homeomorphism $h$ from $E^3/F$ onto $E^3/G$ such that $h[\text{Cl}(P_F[H_F])] = \text{Cl}(P_G[H_G])$. A necessary and sufficient condition for two decompositions to be equivalent is given. It is shown that there is a decomposition with only a countable number of nondegenerate elements which is equivalent to the dogbone decomposition. There is a decomposition with uncountably many nondegenerate elements equivalent to the decomposition of section 3 of Point-like decompositions of $E^3$, Fund. Math. 50 (1962), 431-453; a number of additional properties of this decomposition are established. (Received August 13, 1965.)

For previous results see Abstract 65T-129, these Notices 12 (1965), 238. Let $s = \sigma + it$, $\tau = t/2\pi$. Zeros of $g_m(s) = \sum_{n=1}^{m} [e^{-s} + x(s)n^{s-1}]$ are calculated for $1/2 < \sigma \leq 5, 1 \leq t \leq 200$, $m \geq (1)100$. Zeros off the critical line are found for $m \geq 3$ outside the interval $\sqrt{m} \leq \tau \leq m$. (Received August 9, 1965.)

Reichelderfer has developed a transformation theory for a function $T$ whose domain is a measure space $(\mathcal{S}, \mathcal{M}, \mu)$ and whose range is a measure space $(\mathcal{S}', \mathcal{M}', \mu')$ (see Abstract 61T-267, these Notices 8 (1961), 518). The results in the present paper are established in a setting first described.
in that earlier paper. The definition of the term "quasi-weight function" is obtained from the definition of the term "weight function" by systematically replacing the expression "for every s' in" by "a.e. u' on" throughout the latter definition. In this paper it is shown that under certain standard conditions every quasi weight function gives rise to a weight function which agrees with it a.e. u' uniformly. It is then shown that an arbitrary real valued, non-negative, u-integrable function f with domain $S$ is a g.l.b. W for some weight function W' if and only if f vanishes a.e. u on the inverses of sets of u' measure zero. Finally, this theorem is applied to give several results, one of which is a uniqueness theorem for Lebesgue-type decompositions (see Abstract 619-90, these Notices 12 (1965), 82). (Received July 12, 1965.)

630-7. KENNETH WHYBURN, 124 Triphammer Road, Ithaca, New York. An operational calculus for locally compact abelian groups.

In this paper it is shown that given any compactly generated locally compact abelian group $G$ there is associated with it a field $\mathbb{F}(G)$ in which $\mathbb{F}(G,C)$ may be imbedded isomorphically using pointwise addition and a multiplication which is in some sense a convolution. This is done by first showing it for compact groups which may be imbedded in finite products of circle groups. Next a slight digression shows that if $G$ is compact and has a countable dual then we may embed $L^1(G)$ in such a field. Then it is demonstrated how to obtain $\mathbb{F}(G)$ for a compactly generated locally compact abelian group and also how to deal with the $L^1$ case when the compact part of $G$ has a countable dual. (Received August 31, 1965.)

630-8. HENRYK FAST, 812 Forest Avenue, South Bend, Indiana. Some integral-geometric uniqueness theorem. Preliminary report.

Let $Q$ be the class of all measure-preserving homeomorphisms from compacts in $R^{n-1}$ into $R^n$, i.e. homeomorphisms having the property of transforming Borelian subsets $R^{n-1}$ with given value of $L_{n-1}(n-1 \text{ dimensional Lebesgue measure})$ in sets in $R^n$ with equal value of $H_{n-1}(n-1 \text{ dimensional Hausdorff measure})$. $S$ denotes the unit sphere in $R^n$, $m$th rotation-invariant normalized Borelian measure on it. For $p \in R^n, s \in S, q \in Q$ define $N(p, s, q) = \text{the number of points in the set } \{ t: p + st \in q(E), t \in R \} \text{ in case this set is finite and } \infty \text{ otherwise. Here } E = \text{domain } (q)$. Put $\phi(p, q) = \int_S N(p, s, q)m(ds)$. The main result: supposing the set $\{ p: \phi(p, q_1) = \phi(p, q_2) \}, q_1 \in Q$ is dense in $R^n$, we have $q_1(E_1) = q_2(E_2)$, where $E_1 = \text{domain } (q_1), (i = 1, 2)$. A slight generalization of this result: Let the family of sets domain $(q), q \in Q$ consist of all compacts in $R^{n-1}$ modulo a set of $L_{n-1}$ measure zero. Then under the same assumption $q_1(E_i) (i = 1, 2)$ are equal modulo a set of $H_{n-1}$ measure zero. (Received July 19, 1965.)


An operation on an $n$ by $n$ matrix $A_n$ of 0's and 1's consists of adding one row to another row modulo two. Let $f(A_n)$ denote the least number of operations necessary to transform $A_n$ into the identity matrix, if this is possible. (We permit ourselves to interchange rows whenever necessary.)

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Then, there exist absolute constants $c_1$ and $c_2$ such that $c_1 n^2 / \log n < f(A_n) < c_2 n^2 / \log n$ for almost all matrices $A_n$, i.e., for all but a fraction that tends to zero as $n$ tends to infinity. (Received September 23, 1965.)


Let $P(x) = P(x_1, \ldots, x_m)$ be polyharmonic of order $p$. Let $L(u) = u_{tt} + (k/t)u_t - \Delta u - cu = 0$ where $\Delta$ is the Laplace operator in the variables $x_1, \ldots, x_m$, and $k$ and $c$ are real parameters, $k \neq -1, -3, \ldots$. The Cauchy problem $L(u) = 0$, $u(x,0) = P(x)$, $u_t(x,0) = 0$ is shown to have solution $u(x,t) = \sum_{n=0}^{P-1} (\Delta^n P) u_n(t)$ where $u_n(t) = (\Gamma[(k+1)/2]/\sqrt{\pi n!}) (\sqrt{\pi c t})^{(2n+1-k)/2} (k+2n-1)/2 (\sqrt{\pi c t})^{(2n+1-k)/2}$. From this result one can construct a solution of the Cauchy problem $u(x,0) = P(x)$, $u_t(x,0) = 0$ for the equation $L'(u) = u_{tt} + (k/t)u_t - \Delta u = R(x,t)$ where $R(x,t) = \sum_{s=0}^{P} Q_s t^s$, each $Q_s$ being polyharmonic of order $r_s$. (Received September 9, 1965.)


In this paper, simple characterizations of $T_i$ ($i = 0, 1, 2$) and $R_0$ spaces are given in terms of quasi-uniformities (without explicit reference to the induced topology) which shows that they are quasi-uniform invariants. A quasi-uniform space $(X, \mathcal{U})$ is (i) $T_0$ iff $\cap U$ is anti-symmetric, (ii) $T_1$ iff $\cap U = \Delta$, (iii) $T_2$ iff $\Delta = \cap U^{-1} \circ U$, (iv) $R_0$ iff $\cap U$ is symmetric. If $\mathcal{U}$ is a quasi-uniformity, then so is $\mathcal{U}^{-1} = \{ \mathcal{U}^{-1} \mid \mathcal{U} \in \mathcal{U} \}$. A property $P$ is conjugate-invariant iff whenever $P$ holds in $\mathcal{U}$, $P$ also holds in $\mathcal{U}^{-1}$. $T_0$, $T_1$ and $R_0$ are conjugate-invariant. An example is given to show that $T_2$ is not conjugate-invariant. However, if $\mathcal{U}$ is regular and $T_0$, then both $\mathcal{U}$ and $\mathcal{U}^{-1}$ are Hausdorff. Some related results are also proved. (Received September 10, 1965.)


A solution of the Cauchy problem $u(x,0) = 0$, $u_t(x,0) = 0$ for the $m$ dimensional Euler-Poisson-Darboux equation $u_{tt} + (k/t)u_t - \Delta u = f(x,t)$ where $\Delta$ is the Laplace operator in the variables $x_1, \ldots, x_m$ and $(x,t) = (x_1, \ldots, x_{m-1}, t)$ is found for all $-\infty < k < \infty$ using Riesz' method of analytic continuation. It is shown that when $k > m - 2$ the solution of the corresponding regular Cauchy problem with data on $t = t_0 > 0$ converges to the solution of the problem in question as $t_0 \rightarrow 0$. For other values of $k$ a method is used to reduce the problem to one in which $f(x,t)$ is of the form $t^p \phi(x,t)$ where $p$ is a positive integer depending on $m$ and $k$. It is noted that when $f(x,t)$ is an even function of $t$ the solution may exhibit a logarithmic singularity at $t = 0$ in certain of its derivatives for odd negative integers $k$. When $f(x,t)$ is an odd function of $t$, the same situation may occur for even negative integral values of $k$. (Received September 9, 1965.)
Let \( P(z) = \prod_{\alpha=1}^{m} (z - z_\alpha)^{k_\alpha} \) where \( \{z_1, z_2, \ldots, z_m\} \) are \( m \) distinct points in the complex plane and \( k_\alpha \) is a positive integer. Let \( E(c) \) be the set of points \( z \) for which \( |P(z)| < c \). Grunsky conjectured that if the constant \( c \) is such that the set \( E(c) \) has \( m \) components, then each component is a convex set. Pommerenke showed by an example that the conjecture is false. However his example polynomial has a multiple root of very high order. This paper gives an example polynomial of only fourth degree, in which all roots are simple, and the Grunsky conjecture is still false. (Received September 10, 1965.)

630-14. WITHDRAWN.


The author gives a somewhat complicated definition of both upper and lower semi-continuous collections (more general than definitions of either R. L. Moore or L. F. McAuley) which allows the elements to intersect and reduces to the usual definition when the elements are mutually exclusive. When restricted to the case of a collection \( C \) of bicompact point sets in a semi-metric space, this definition of upper semi-continuous collection is equivalent to saying that if \( \{P_n\} \) is a sequence of points and \( \{M_n\} \) is a sequence of elements of \( C \) such that for each \( n \), \( P_n \in M_n \) and \( \{P_n\} \) has a sequential limit point \( P \) belonging to some element of \( C \), then (i) some subsequence \( \{M_{n(k)}\} \) of \( \{M_n\} \) has a sequential limiting set \( M \) and (ii) there is a minimal element \( K \) of \( C \) (called the sequential limit point in the decomposition space \( S/C \)) which contains \( M \). On a basis of this definition several interesting theorems can be proved, for example, if \( S \) is either a metric, semi-metric or regular developable space and \( C \) (as above) fills up \( S \), then the decomposition space is respectively either a metric, semi-metric or regular developable space. (Received September 20, 1965.)


Let \( m \) be any positive real number and let \( I \) be any finite interval of the \( x \)-axis. The singular Cauchy problem for the second order quasi-linear hyperbolic partial differential equation \( u^{2m}u_{xx} - u_{yy} + f(x, y, u, u_x, u_y) = 0 \), with initial conditions \( u(x,0) = 0 \), \( u_y(x,0) = \phi(x) \), \( x \in I \), is considered. It is shown that under the appropriate conditions on \( f \) and \( \phi \), this problem has a unique solution in a neighborhood \( (y > 0) \) of \( I \). This is done by application of Schauder's Fixed Point Theorem. (Received September 23, 1965.)

630-17. WITHDRAWN
630-18. VICTOR KLEE, University of Washington, Seattle, Washington and CZESLAW BESSA,
University of Warsaw, Warsaw, Poland. Every non-normable Frechet space is homeomorphic with all
of its closed convex bodies.

It has been conjectured that every infinite-dimensional Frechet space is homeomorphic with
all of its closed convex bodies, and this has been proved for a class of spaces which is believed to
include all infinite-dimensional normable F-spaces (Banach spaces). However, the normable case
has not been fully settled and thus it is surprising that the non-normable case is tractable, as indicated
in the title. The key difference between the two cases is that every non-normable F-space has the
space $s(= R^0)$ as a topological factor, while no analogous result is known in the normable case. In
addition to proving the theorem stated in the title, the paper contains a complete topological classifi-
cation of the products $[0,1]^a \times [0,1]^\beta \times [0,1]^\gamma$ for arbitrary cardinals $\alpha, \beta$ and $\gamma$. (Received September
27, 1965.)

Affine differential geometry of closed hypersurfaces.

E. Cartan's method of moving frames and exterior differential calculus is used to establish
the affine differential geometry of closed hypersurfaces $M^n$ in an affine space $A^{n+1}$ of odd dimension
$n + 1 \geq 3$. In an invariant way we determine the Frenet affine frames and therefrom obtain the
fundamental Frenet formulas in the theory of the hypersurfaces $M^n$. By using these formulas, some
integral formulas are derived for a closed orientable hyper-surface in the space $A^{n+1}$, and various
characterizations of even-dimensional affine hyperspheres are deduced. (Received September 27,
1965.)

630-20. W. E. CONWAY, West Texas State University, Canyon, Texas. The vertical jet under
gravity.

The properties of the flow through a vertical jet in the lower wall of a channel are determinate
when the solution of an exact nonlinear integral equation has been found. The integral equation is
obtained by a conformal mapping onto a semicircle together with the condition of constant pressure
on the free streamlines. (Received September 27, 1965.)

630-21. C. E. AULL, Virginia Polytechnic Institute, Blacksburg, Virginia. A closed set
axiom of countability.

Definition. A topological space $(X, \mathcal{T})$ satisfies $C_2$, if for closed $F \subset X$, there is a countable
family $\{U_n\}$ of open sets, $F \subset U_n$ for all $n$, such that for open $V \supset F$, there exists $n$ such that $U_n \subset V$. 
The following are proved. Regular \( C^3 \) spaces are perfectly normal; perfectly normal, countably compact spaces satisfy \( C^3 \); \( T^3 \), \( C^3 \) spaces with a finite number of isolated points are countably compact; \( T^3 \), \( C^3 \) spaces are metrizable iff the topology has a \( \sigma \)-point finite base. A similar theorem holds for \( T^2 \) countably compact spaces. (Received September 27, 1965.)

630-22. C. E. HALL, Virginia Polytechnic Institute, Blacksburg, Virginia. Projective topological groups.

All topological groups discussed here are Abelian and Hausdorff and all maps are continuous homomorphisms. Let \( A \) and \( B \) denote topological groups. A topological group \( G \) is projective relative to a family \( \mathcal{F} \) of exact sequence of the form \( A \xrightarrow{f} B \xrightarrow{g} 0 \) iff for each \( (A \xrightarrow{f} B \xrightarrow{g} 0) \in \mathcal{F} \) and each map \( g \) from \( G \) to \( B \) there is a map \( h \) from \( G \) to \( A \) such that \( fh = g \). Let \( \mathcal{F} \) be the family of all exact sequences of the form \( A \xrightarrow{f} B \xrightarrow{g} 0 \) such that all free topological groups are projective relative to \( \mathcal{F} \). A topological group is projective iff it is projective to \( \mathcal{F} \). \textbf{Theorem 1.} A topological group is projective iff it is a summand of a free topological group. \textbf{Theorem 2.} If \( P \) is the free group on a finite set and is a projective topological group then \( P \) is a free topological group. \textbf{Theorem 3.} The class of free topological groups is a proper subclass of the class of projective topological groups. (Received September 27, 1965.)

630-23. HERMANN SIMON, University of Miami, Coral Gables 46, Florida. Artinian and noetherian hypercentral groups.

\textbf{Theorem 1.} An artinian group \( G \) is hypercentral iff each nonabelian subgroup of \( G \) has a noncyclic commutator factorgroup. This theorem contains as a special case the following theorem of Kemhadze: \textbf{Corollary.} A finite group \( G \) is nilpotent iff each nonabelian subgroup of \( G \) has a noncyclic commutator factorgroup. \textbf{Theorem 2.} A group \( G \) is noetherian and nilpotent iff (a) \( G \) is finitely generated, (b) each nonabelian factor of \( G \) has a noncyclic commutator factorgroup, and (c) there exists an integer \( n \geq 0 \) such that the nil-class of any finite nilpotent factorgroup of \( G \) does not exceed \( n \). \textbf{Remarks.} Artinian = minimum condition for subgroups. Noetherian = maximum condition for subgroups. Factor of \( G \) = epimorphic image of a subgroup of \( G \). \( Z(G) = \) center of \( G \). \( Z(0) = 1 \), \( Z(1) = Z(G) \), \( Z(n + 1)/Z(n) = Z(G/Z(n)) \), \( Z(\lambda) = \bigcup_{n<\lambda} Z(n) \) if \( \lambda \) is a limit ordinal. \( G = Z(\omega) \). Nilpotent i.e., \( G = Z(n) \), \( n \) integer. If \( G = Z(n) \), \( n \) integer and \( Z(n-1) < G \) then \( n \) is the nil-class of \( G \). (Received October 6, 1965.)

630-24. WITHDRAWN.
630-25. G. G. WEILL, 333 Jay Street, Brooklyn, New York 11201. **Extremal Sario linear operators.**

The abstract linear operators introduced by L. Sario to construct harmonic functions on Riemann surfaces, with given singularities and given boundary behavior form a convex set whose extremal points are characterized. They can be represented as isometries of suitable $L^p$ spaces. It is shown in particular that any multiplicative Sario operator is extremal. The relationship between the "principal" operators and the extremal ones is examined in the special case of an annulus. (Received October 1, 1965.)

630-26. R. G. BUSCHMAN and M. C. WUNDERLICH, State University of New York at Buffalo, Buffalo, New York. **2nd term approximations of sieve generated sequences.**

In a previous paper, Wunderlich gave a necessary and sufficient condition for a sieve generated sequence $\{a_n\}$ of the type described in Abstract 65T-192, these Notices 12 (1965), 375 to have the property $a_n \sim n \log n$. By similar methods, it can be further shown that a sieve generated sequence $\{a_n\}$ can be constructed such that $a_n - n \log n$ is asymptotic to any given function lying between $n(\log \log n)^2$ and $n \log n$. It is not possible, however, to construct a sequence using this sieve method for which $a_n - n \log n \sim p_n - n \log n = n \log \log n$. The authors have studied a sieve method which more closely resembles the prime sieve. At the nth execution of the sieve one begins sieving in the usual manner at the element $a_{n+a(n)}$ where $a(n)$ is a specified non-negative function. This corresponds to the old method when $a(n) = 0$ and resembles the prime number sieve when $a(n) \sim (1/2)n^2 \log n$. The authors show that for a large class of functions $a(n)$, $a_n \sim n \log n$. There are also some partial results about the asymptotic behavior of $a_n - n \log n$. (Received October 4, 1965.)

630-27. S. L. GULICK, University of Maryland, College Park, Maryland, T. S. LIU, University of Massachusetts, Amherst, Massachusetts and A. C. M. van ROOIJ, R. C. University, Nijmegen, the Netherlands. **Group algebra modules, I.**

Let $K$ be a Banach module over $L_1(G)$, where $G$ is a locally compact group. Consider the operators from $L_1(G)$ into $K$ and from $K$ into $L_\infty(G)$, which are linear, continuous, and which commute with the module composition $\circ$. We are interested in describing the space of such operators, which we call $\mathcal{D}(L_1(G), K)$ and $\mathcal{D}(K, L_\infty(G))$ respectively. To this end, let $K$ be a left Banach module over $L_1(G)$, and $K^*$ its conjugate space. For $k^* \in K^*$ and $f \in L_1(G)$, let $(k^* \circ f)k = k^*(f \circ k)$, $k \in K$, define $k^* \circ f \in K^* \circ L_1(G)$, and for $k^{**} \in K^{**}$ and $f \in L_1(G)$, let $(f \circ k^{**})k^* = k^{**}(f^* \circ f)$, $k^* \in K^*$, define
Let $f \circ k^{**} \in L_1(G) \circ K^{**}$. Finally let $\pi$ be the injection map of a space into its bidual. **Theorem 1.** $\mathscr{K}(L_1(G), K)$ is isometrically isomorphic to a subspace of $(K^* \circ L_1(G))^*$, and this subspace is indeed $(K^* \circ L_1(G))^*$ if and only if $L_1(G) \circ K^{**} \subseteq \pi(K)$. **Theorem 2.** $\mathscr{K}(K, L_\infty(G))$ is isometrically isomorphic to $(L_1(G) \circ K)^*$.

(Received October 4, 1965.)

630-28. S. L. GULICK, University of Maryland, College Park, Maryland, T. S. LIU, University of Massachusetts, Amherst, Massachusetts and A. C. M. van Rooij, R. C. University, Nijmegen, the Netherlands. **Group algebra modules, II.**

Let $\mathcal{P}$ be a locally compact group and let $\mathcal{X}$ be a locally compact Hausdorff space with a non-zero Radon measure $m_X$, quasi-invariant with respect to $\mathcal{X}$. We define on $L_p(\mathcal{X}), p \in [1, \infty), M(\mathcal{X}),$ and $C_c(\mathcal{X})$, a generalized convolution $\ast$ which renders each as a left $L_1(\mathcal{P})$-module. Then we prove **Theorem 1.** $L_p(\mathcal{X}), p \in [1, \infty), \text{ and } C_c(\mathcal{X})$ are factorable with respect to the module composition. **Theorem 2.** The space of linear, continuous operators from $L_1(\mathcal{P})$ to $L_p(\mathcal{X}), p \in (1, \infty)$, which commute with convolution, corresponds isometrically and isomorphically to $L_p(\mathcal{X})$. **Theorem 3.** The space of linear, continuous operators from $L_1(\mathcal{P})$ to $L_1(\mathcal{X})$ which commute with convolution corresponds isometrically and isomorphically to the subspace of $M(\mathcal{X})$ consisting of all those measures $\mu$ which have the property that $f \ast \mu \ll m_X$, for all $f \in L_1(\mathcal{P})$. (Received October 4, 1965.)

630-29. R. C. KIRBY, University of California at Los Angeles, Los Angeles, California. **Smoothing imbeddings of spheres.**

Let $f : S^k \rightarrow S^n, n \geq 5, k \neq n - 2$, be a locally flat imbedding. Suppose there exists a point $p \in S^k$ and a neighborhood $U$ of $f(p)$ in $S^n$ such that $(U, U \cap f(S^k))$ is stably homeomorphic to $(R^n, R^k)$. Then there exists an $\epsilon$-isotopy $F_t : S^n \rightarrow S^n, t \in [0, 1]$ such that $F_0 = \text{identity}$ and $fF_1$ is a differentiable imbedding. If $S^n$ is replaced by a combinatorial manifold $m^n, k < n - 2$, then there exists an $\epsilon$-isotopy $F_t : m^n \rightarrow m^n, t \in [0, 1]$ such that $F_0 = \text{identity}$ and $fF_1$ is a combinatorial imbedding. (Received October 4, 1965.)

630-30. T. G. OSTROM, Washington State University, Pullman, Washington 99163. **The dual Luneburg plane.**

Lüneburg has constructed a translation plane which admits the Suzuki group as a group of collineations. The dual of this plane and the one derived from it admit a group of collineations transitive on affine points. The other known finite planes with this property are translation planes. Both planes are semi-translation planes which are neither full translation planes nor strict semi-translation planes. (Received October 4, 1965.)

630-31. R. A. RAimi, University of Rochester, Rochester, New York 14627. **Translation properties of finite partitions of the positive integers.**

Let $N$ denote the positive integers, and let $s : N \rightarrow N$ be any 1:1 mapping without finite orbits (e.g. $s(n) = n + 1$). Let $\mathcal{I}$ be any nontrivial proper ideal in the Boolean algebra of all subsets of $N$, and let $\mathcal{I}$ be $s$-invariant, i.e. $F \in \mathcal{I} \Rightarrow s(F) \in \mathcal{I}$. (E.g. $\mathcal{I}$ may be the family of all finite subsets of 64
If $\mathcal{A}$ and $\mathcal{B}$ are two families of subsets of $\mathbb{N}$, ‘$\mathcal{A}$ refines $\mathcal{B} \pmod{r}$’ means that each member $A \in \mathcal{A}$ is contained in some set $B \cup F$, with $B \in \mathcal{B}$ and $F \in \mathcal{F}$ depending on $A$. We denote by $s^k\mathcal{B}$ the family $\{s^k(B) : B \in \mathcal{B}\}$. **Theorem.** There exists a finite partition $\mathcal{A}$ of $\mathbb{N}$ such that if $\mathcal{B}$ is any other finite partition of $\mathbb{N}$, some ‘translate’ $s^k\mathcal{B}$ fails to refine $\mathcal{A} \pmod{r}$. The proof depends on measure theoretical and topological properties of the Stone-Cech compactification $\beta\mathbb{N}$. (Received October 4, 1965.)

630-32. D. V. V. WEND, University of Utah, Salt Lake City, Utah 84112. **Singularity-free regions for solutions of some nth order nonlinear differential equations.**

In this paper conditions are obtained under which $(*)y^{(n)} + f(y,z) = 0$ has solutions satisfying given initial conditions which are regular in a disk and $y^{(m)} + p(z)y^{(n)} = 0$, $m > 1$, $n > 1$, has solutions regular in the unit disk and in a sector. These results generalize some theorems obtained by K. M. Das (J. Math. Mech. 13 (1964), 73-84). Let $G(s,t)$ be real-valued, continuous and nondecreasing in $s$, $0 \leq s < \infty$, and continuous in $t$, $0 \leq t < R$. Let the maximal integral of $(**)$ $\int y^{(n)} = G(y,x)$, $y^{(k)}(0) = a_k \geq 0$, $k = 0, \ldots, n - 1$, $a_0 + \ldots + a_{n-1} > 0$, be defined on $[0,r_0)$, where $r_0 \in \mathbb{R}$ is maximal. **Theorem.** Suppose $F(y,z)$ in $(*)$ is entire in $y$ and regular in $z$, $0 \leq |z| < R$, and satisfies $|F(y,z)| \leq G(|y|, |z|)$ for all $y$ and for $|z| < R$. Then any solution of $(*)$ for which $\left| y^{(k)}(0) \right| = a_k$, $k = 0, \ldots, n - 1$, is regular for $|z| < r_0$. **Corollary.** Suppose in addition solutions of $(**) y^{(n)} = G(y,x), y^{(k)}(0) = a_k, k = 0, \ldots, n - 1$, are unique and there exists a function $u(x)$ such that $u^{(k)}(0) = a_k$, $k = 0, \ldots, n - 1$, and $u^{(n)}(x) \geq G(u(x),x)$ on $[0,r)$, $r \in \mathbb{R}$. Then any solution of $(*)$ for which $\left| y^{(k)}(0) \right| = a_k, k = 0, \ldots, n - 1$, is regular for $|z| < r$. An example is given showing the necessity of the uniqueness hypothesis in the corollary. (Received October 4, 1965.)

630-33. J. R. DORROH, Louisiana State University, Baton Rouge, Louisiana. **Some properties of a partial differential operator.**

Suppose $S$ is a real or complex Banach space, that $p$ is a bounded, continuously Fréchet differentiable function from $S$ into $S$, and that $p'$, the Fréchet derivative of $p$, is a bounded function. Let $X$ denote the complex Banach space of all bounded and uniformly continuous complex valued functions on $S$. Let $A$ denote the operator in $X$ defined by $Ax = x'p$ for all Fréchet differentiable $x$ in $X$ such that $x'p$ is in $X$. If $S$ is real or complex Euclidean $n$-space, then $Ax = \sum p_i x_i$, where $p_i$ denotes the $i$th component of $p$, and $D_i x$ denotes the $i$th place partial derivative of $x$. There is a closed extension of $A$ which is the infinitesimal generator of a strongly continuous group of contraction operators in $X$, and a simple formula is given for this group. The group property in turn yields other properties of $A$ and other partial differential operators. (Received October 5, 1965.)

630-34. A. H. ZEMANIAN, State University of New York at Stony Brook, Stony Brook, New York. **A distributional K transformation.**

The $K$ transformation, $(T_\mu f)(s) = \int_0^\infty f(t)\sqrt{\mu(t)}K_\mu(st)dt$, where $K_\mu$ is the modified Bessel function of third kind and order $\mu (-1/2 \leq \text{Re} \mu \leq 1/2)$, is generalized to distributions as follows. A certain space $K_{\mu,a}$ of infinitely differentiable functions on $0 < t < \infty$ is constructed. $K_{\mu,a}$ contains $\sqrt{\mu(t)}K_\mu(st)$ for $Re s > a$. $K_{\mu,a}$ is also closed under a certain Bessel-type differentiation operator. If $f$ is a distribution in the dual space $K'_{\mu,a}$ of $K_{\mu,a}$, its $K$ transform is defined by $(T_\mu f)(s) = \left(f(t), \sqrt{\mu(t)}K_\mu(st)\right)$.
Theorems on analyticity, uniqueness, inversion, and continuity for $T_f$ are established. Also, $F(s) = T_f d$, where $d \in K'_\mu$, if and only if $F$ is analytic and bounded on $\text{Re } s \geq b > a$ by $|F(s)| \leq P_b(|s|)$, $P_b$ being some polynomial. (Received October 5, 1965.)

630-35. D. E. CATLIN, University of Massachusetts, Amherst, Massachusetts. Implicative pairs in orthomodular lattices.

Elements $e$ and $f$ in an orthomodular lattice $L$ are said to form an implicative pair, written $I(e,f)$, if and only if there exists $g \in L$ such that, (1) $eAg \leq f$ and, (2) if $h \in L$, then $eAh \leq f$ implies $h \leq g$, both hold. It is easily shown that $g$ is unique, in fact $g = e'vf$. If $I(a,b)$ and $I(a,c)$ both hold then $I(a,bv.c)$ and $I(a,bAC)$ both hold. Similarly if $I(a,c)$ and $I(b,c)$ both hold then $I(avb,c)$ and $I(aAb,c)$ both hold. It follows from this that if we define $A(a) = \{x| x \in L$ and $I(a,x)\}$ and $B(a) = \{x| x \in L$ and $I(x,a)\}$, then $A(a)$ and $B(a)$ are both sublattices of $L$. $A(a)$ turns out to be a sub-orthomodular lattice of $L$ exactly when $a \in C(L)$ = the center of $L$, and in this case $A(a) = L$. The formula $C(L) = \cap \{B(x)| x \in L\}$ can be shown. As a corollary we obtain that the statements $I(e,f)$ and $I(f',e')$ both hold if and only if $e'vf \in C(L)$. (Received October 6, 1965.)


Theorem. Let $A$ be a uniformly closed subalgebra of $C(S)$, $S$ compact metric. Let $A$ separate points of $S$ and contain constant functions. Then the following conditions are equivalent: (1) $S$ is uncountable, (2) the Šilov boundary of $A$ is uncountable, (3) the minimal boundary of $A$ is uncountable, (4) the space $A^*$, dual to $A$, is unseparable, (5) there exist an uncountable closed subset $S_0$ of $S$ and a linear operator $L : C(S_0) \to A$ with $\|L\| = 1$ and $L(f) = f(s)$ for $s \in S_0$ and for $f \in C(S_0)$; (6) each separable Banach space is isometrically isomorphic with a subspace of $A$. Corollary 1. Each infinite dimensional separable uniformly closed subalgebra of $C(S)$, $S$ compact Hausdorff, is homeomorphic with Hilbert space. Corollary 2. If $A$ is a separable uniformly closed subalgebra of $C(S)$, $S$ compact Hausdorff, which is complemented in $C(S)$ (i.e. there is a bounded linear idempotent operator from $C(S)$ onto $A$), then $A$ is linearly homeomorphic with $C(S_1)$, where $S_1$ is a compact metric space. (Received October 6, 1965.)

630-37. HANS SCHNEIDER and JULIAN WEISSGLASS, 213 Van Vleck, University of Wisconsin, Madison, Wisconsin. Group rings, semigroup rings and their radicals.

Let $R$ be an associative ring. Denote the upper nil radical of $R$ by $\mathcal{N}(R)$ and the Jacobson radical by $\mathcal{J}(R)$. A class of semigroups $\mathcal{L}$ is defined by means of combinatorial properties such that every strict fully ordered semigroup, every right ordered group, and every SN group with a normal system whose factors are torsion-free Abelian are in $\mathcal{L}$. Further $\mathcal{L}$ is closed with respect to direct and free products. It is conjectured that all torsion-free groups are in $\mathcal{L}$. If $D$ is a semigroup, let $RD$ denote the semigroup ring of $R$ over $D$. Theorem 1. If $\mathcal{N}(R) = \{0\}$ then $\mathcal{J}(RD) = \{0\}$.

Theorem 2. If $\mathcal{N}(R) = \{0\}$ and $G$ is a group such that the order of every element of finite order in $G$ is cancellable in $R$ then $\mathcal{N}(RG) = \{0\}$. Corollary. Let $\mathcal{N}(R) = \{0\}$ and suppose $G$ is an Abelian group with at least one element of infinite order such that the order of every element of finite order is can-
cellular in R. Then $\mathcal{I}(RG) = \{0\}$. Theorem 1 generalizes results of Amitsur and Bovdi. Theorem 2 generalizes results of Passman. (Received October 8, 1965.)


It has been shown (Amer. Math. Monthly 72 (1965), 624) that the generalized power $2^n(b)$ represents the number of subgroups in an elementary Abelian group of order $b^n$--and therefore $2^n(b)$ has a concrete interpretation when $b$ is a prime. In this paper $2^n(b)$ is shown to represent the number of subspaces in an $n$-dimensional linear space over a Galois field with $b$ elements--and therefore $2^n(b)$ is now seen to have a concrete interpretation when $b$ is a power of a prime. (Received October 8, 1965.)


Call an ultrafilter $\Delta$ on $I$ adequate for set $K$ if every ultrafilter on $K$ has a base of the form $t(\Delta)$ for some function $t : I \to K$. Adequate ultrafilters always exist if $\text{Card}(I) \geq 2^{\text{Card}(K)}$. Where $\Delta$ is adequate for $K$, let $\sigma$ be the natural map from the ultrapower $K^I_\Delta$ to $\beta K_d$ (the set of all ultrafilters on $K$). By the quasi-discrete topology on $K^I_\Delta$ we mean the $\sigma$-inverse image of the Stone-Cech topology on $\beta K_d$. Let $K$ be a group (ring, field). Theorem. The Hausdorff group (ring, field) topologies on $K$ are in natural 1-1 correspondence with the maximal partial group homomorphisms (ring homomorphisms, places) $\rho$ from $K^I_\Delta$ to $K$ such that $\rho(k) = k \forall k \in K$, whose kernels are quasi-discrete closed. (If we factor $\rho = \sigma t$ where $t$ is a partial function from $\beta K_d$ to $K$, then in the topology corresponding to $\rho$ an ultrafilter $D$ on $K$ converges to element $k$ iff $t(D) = k$.) Note that $\rho = \sigma t$ is the "standard part" function of Abraham Robinson's nonstandard analysis. One application of this approach: a ramification theory for field topologies that parallels that of valuation theory. (Received October 8, 1965.)


Let $A$ be a discrete subring of the complex numbers $\mathbb{C}$ with dimension 2 (that is, $A$ spans all of $\mathbb{C}$ as a real linear space). Let $X$ be a compact subspace of $\mathbb{C}^n$, $n$ any positive integer, such that $\mathbb{C}[z_1, \ldots, z_n]$ is uniformly dense in the space of all continuous complex valued functions defined on $X$ which are holomorphic on $X$ interior. (The existence of many such sets is demonstrated.) Furthermore, suppose that for some $z' \in A^n \cap X^0$, $X$ is contained in the open polycylinder centered at $z'$ with multiradii all equal to 1. Then a complex valued function on $X$ is in $A[z_1, \ldots, z_n]$ (where the bar denotes uniform closure) if and only if $f$ is continuous on $X$, holomorphic on $X$ interior, and the coefficients of its power series expansion about $z'$ lie in $A$. This result adds to those announced by Fekete concerning uniform approximation over certain compact subsets of the plane by polynomials whose coefficients lie in the ring of integers of an imaginary quadratic field. The latter results are also proved because, as far as is known, the proofs of them have never been published. (Received October 7, 1965.)
A characterization of differential operators.

Let $D$ be a continuous linear operator from $\mathcal{D}(\mathbb{R}^n)$ into $L^2(\mathbb{R}^n)$. Define $T(D)$ to be the complex linear space of operators spanned by $\{e^{-i(a,x)}De^{i(a,x)} : a \in \mathbb{R}^n\}$ where $e^{i(a,x)}$ and $e^{-i(a,x)}$ denote the operators of multiplication by the bounded exponentials $e^{i(a,x)}$ and $e^{-i(a,x)}$ respectively.

**Theorem.** $D$ is a differential difference operator if and only if $T(D)$ is finite dimensional. (Received October 11, 1965.)

Concerning non-negative matrices and doubly stochastic matrices.

**Theorem.** Let $A \neq 0$ be a non-negative square matrix. A necessary and sufficient condition that there exists a unique doubly stochastic matrix of the form $D_1AD_2$ where $D_1$ and $D_2$ are diagonal matrices with positive main diagonals is that each positive element of $A$ lie on at least one positive diagonal. The matrix $D_1AD_2$ is the limit of the sequence of matrices generated by alternately normalizing the rows and columns of $A$. (Received October 11, 1965.)

Relations between usual and generalized functions.

The author has improved former results on realization of homomorphism in Čech homology by generalized functions as stated by the following theorems: **Theorem 1.** If a usual continuous function is generated by a generalized function, they induce both the same homomorphism in the corresponding Čech homology groups. **Theorem 2.** If $f$ is a cofinal generalized continuous function, $f: (X, \mathcal{V}) \to (Y, \mathcal{V}')$ and if $X$ and $Y$ are compact metric spaces and $\mathcal{V}$ and $\mathcal{V}'$ are cofinal families of open coverings, then $f$ generates a well defined continuous function $\phi: X \to Y$, in the usual sense. From these theorems one concludes: **Theorem 3.** If $h: H_n(X, G) \to H_n(Y, G)$ is a homomorphism of Čech groups with a continuous spectral realization, and $X$ and $Y$ are compact metric spaces, there exists a continuous function $\phi: X \to Y$, such that $\phi^* = h$. (Received October 11, 1965.)
Let $X$ be a second countable, locally compact metric space which has a base, $\mathcal{B}$, satisfying Romanovski's axioms $I$-$X$ [Math. Sbornik 9 (51) (1941)], and $Y$ be a separable Banach space. This paper presents a descriptive and constructive definition of an integral of point functions, $f$, defined on a member of $\mathcal{B}$, and with range contained in $Y$, and studies some of the properties of this integral. The integral generalizes Romanovski's first integral [Math. Sbornik 9 (51) (1941)] and Bochner's integral [Fund. Math. 20 (1933)]. The constructive process given can be compared with that presented for the Denjoy-Perron integral in Saks's Theory of the integral. As in the classical case, any integral can be attained in at most countably many applications of the construction process. Since this integral reduces to Romanovski's in case $Y$ is the space of real numbers, in particular, a constructive definition is now available for Romanovski's integral. In the case of real-valued functions of a real variable, the descriptive definition of this integral reduces to the classical descriptive definition of the Denjoy-Perron integral. (Received October 11, 1965.)

R. R. Laxton (Prime ideals and the ideal-radical of a distributively generated near-ring, Math. Z. 83, 8-17 (1964)) introduced the concept of prime distributively generated (d.g.) near-ring. He gave an example of a finite d.g. near-ring that is prime but not primitive. We determine several necessary and sufficient conditions for a prime d.g. near-ring to be primitive. Now let $R$ be a d.g. near-ring (with identity). A nonzero $R$-module is said to be prime if $CB = 0$, $C$ an $R$-subgroup of $M$ and $B$ a right ideal of $R$, implies $C = 0$ or $B = 0$. A submodule $M'$ of an $R$-module $M$ is called a prime submodule if $NB \subseteq M'$ implies $N \subseteq M'$, where $N$ is any $R$-subgroup of $M$ and $B$ is any nonzero right ideal of $R$. A submodule of an $R$-module $M$ is called large if it has nonzero intersection with every nonzero submodule of $M$. A submodule of $M$ is called modular if it is maximal as an $R$-subgroup of $M$. The following theorems are proved: Theorem 1. If $M$ is a prime $R$-module, then $R$ is a prime d.g. near-ring. Theorem 2. If a prime $R$-module $M$ contains a modular submodule that is not large, then $R$ is a primitive d.g. near-ring. Theorem 3. Let $R$ be a primitive d.g. near-ring and let $M$ be a faithful minimal $R$-module. Then $M$ is a prime $R$-module and for each nonzero element $m \in M$ the right ideal $(0 : m) = \{ r | rm = 0 \}$ is a modular submodule of the prime $R$-module $R^+$. Theorem 4. If an $R$-module $M$ contains a modular submodule that is prime, then $R$ is a primitive d.g. near-ring. Theorem 5. If $R$ is a primitive d.g. near-ring, then the $R$-module $R^+$ contains a modular submodule that is prime. (Received October 11, 1965.)
Feller function if the semigroup operator $T(t)f(x) = \int_{E} P(t,x,dy)f(y)$ is an endomorphism of the Banach space $C(E,T,\Sigma)$ of continuous functions. Here we consider a related definition of equivalence, restricting $H$ in (*), such that the Feller property is preserved. **Theorem.** If (*) holds where the restriction of $H$ to $C_1$ is a homeomorphism of $C_1$ onto $C_2$ then either both $T_1$ and $T_2$ or neither are Feller. Several examples of operators $H$ which preserve the Feller property are given, for example (i) $Hf(x) = Gx$, where $G$ is a homeomorphism of $(E_1, T_1)$ onto $(E_2, T_2)$, (ii) $H$ is a convolution transform with appropriate kernel; when the $E_i$ are compact Hausdorff, $Hf(x) = \int_{E_1} f(y)K(dy, x)$, $f \in C_1$, $x \in E_2$, $dy \in \Sigma$ (cf. Bartle, et al., Canad. J. Math 7 (1955), 287-305). (Received October 13, 1965.)

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**630-48. JIANG LUH,** Indiana State University, Terre Haute, Indiana. **The structure of certain rings.**

A ring $R$ is called a J-ring if there exists an integer $n > 1$ such that $x^n = x$ for every $x$ in $R$, and is called a $p^k$-ring if there exist a prime $p$ and a positive integer $k$ such that $x^{p^k} = x$ and $px = 0$ for every $x$ in $R$. Using the Dirichlet's Theorem in number theory, one can prove: **Theorem 1.** A ring $R$ is a J-ring iff there exists a prime $p$ such that $x^p = x$ for every $x$ in $R$. **Theorem 2.** A ring $R$ is a J-ring iff $R$ is a direct sum of finitely many $p^k$-rings. (Received October 14, 1965.)

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**630-49. L. J. GRIMM,** University of Utah, Salt Lake City, Utah 84112 and W. A. HARRIS, JR., University of Minnesota, Minneapolis, Minnesota 55455. **General solutions of nonlinear difference equations.**

Consider the nonlinear $n$-dimensional system (1) $y(x + 1) = f(x,y(x))$ with components holomorphic in a region $R = \mathcal{S} \times U$, where $\mathcal{S}$ is a sector in the complex $x$-plane and $U$ is a neighborhood of $y = 0$. Conditions are given under which system (1) can be reduced in a region $R' \subseteq R$ by a uniformly and absolutely convergent transformation to the system (2) $u(x + 1) = A(x)u(x) + g(x,u(x))$, where the components of $g$ are polynomials in the components of $u$. Further, it is possible to solve the system (2) by recursively solving $r$ systems of linear equations. (Received October 11, 1965.)

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**630-50. J. E. NYMANN,** University of Hawaii, Honolulu, Hawaii. **A Minkowskian bound for a class of biquadratic fields arising from fixed points of the Hilbert modular group.**

Let $K_1$ denote a real quadratic extension of the rationals with class number 1 and let $\mu_0 = t^2 - 4\epsilon$ be totally negative where $t$ and $\epsilon$ (unit) are integers in $K_1$. Then $K_2 = K_1(\sqrt{\mu_0})$ will be called a fixed point field. (Such values of $\mu_0$ arise from fixed points of the Hilbert modular group for $K_1$.) A Minkowskian bound for fixed point fields is given by the following. **Theorem.** Every ideal class of a fixed point field contains an ideal of norm less than or equal to $N_{K_1}^{1/2}(\mu_0)/4H_1$ where $H_1 = \inf \{X : X = \text{Im } z \cdot \text{Im } z' \text{ for } (z, z') \text{ a fixed point in the fundamental domain of the Hilbert modular group for } K_1\}$. This result is obtained by using a generalization, due to H. Cohn, of the classical theorem which states that the solvability of $x^2 + 1 = 0 \pmod{m}$ implies the solvability of $x^2 + y^2 = m$. (Received October 11, 1965.)

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Let $A$ be a ring with identity and left minimum condition with the property that the center of the endomorphism ring of each simple $A$-module is infinite. **Definition.** $A$ is of **strongly unbounded module type** if for each integer $n$, there exists $m \geq n$ such that there are infinitely many nonisomorphic $A$-modules with composition length $m$. **Theorem.** If the ideal lattice of $A$ is infinite, then $A$ is of strongly unbounded module type. **Theorem.** If the ideal lattice of $A$ contains a vertex $V$ of order greater than three such that, for some primitive idempotent $e \in A$, the image $Ve$ of $V$ is a vertex of order greater than three in the submodule lattice of $Ae$, then $A$ is of strongly unbounded module type. (Received October 11, 1965.)

**Degenerate fields with twisting rays.**

A new class of exact solutions of Einstein's vacuum field equations which contain a twisting ray congruence are exhibited. This family of solutions contains as special cases (1) the Kerr solution for a spinning particle (2) the Newman, Tamburino and Unti generalization of the Schwarzschild solution and (3) a linear superposition of these two solutions. (Received October 11, 1965.)

**A characterization of polyhedra embeddable in the plane.**

Kuratowski's classical result [Fund. Math. 15 (1930), 271-283] on embedding linear graphs in $S^2$ is purely polyhedral in nature. On the other hand, Claytor's result [Annals of Math. 38 (1937), 631-646] on embedding Peano continua in $S^2$ makes use of nonpolyhedral skew curves. We obtain the following polyhedral extension of Kuratowski's theorem in which we avoid the nonpolyhedral skew curves by using the spiked disc (i.e., the union of a disc $D$ and an arc $A$ such that $D \cap A$ is an end point of $A$ and an interior point of $D$). **Theorem.** A connected polyhedron $P$ is embeddable in $S^2$ iff $P$ does not contain one of the two primitive skew curves of Kuratowski or a spiked disc. We give two proofs of this theorem. One is elementary and uses only techniques of plane topology and Kuratowski's theorem. The other makes use of Claytor's result by first showing that if a polyhedron contains a nonpolyhedral skew curve, then it also contains a spiked disc. (Received October 14, 1965.)

**A theorem on local isometries.**

For a fixed point $p$ in a $G$-space $R$ (Busemann, The geometry of geodesics, p. 37), consider the collection $G(p)$ of all nonoverlapping geodesic curves which begin and end at $p$. Define the subcollection $Q(p)$ of $G(p)$ as follows: A geodesic curve $h$ is in $Q(p)$ if $h \in G(p)$ and if there exists a point $z \in R$ such that $h = T_1(p,z) \cup T_2(p,z)$ where $T_1(p,z)$ and $T_2(p,z)$ denote metric segments with endpoints $p$ and $z$. For $h \in Q(p)$ let $\ell(h)$ denote the length of $h$. Let $\ell_1(p) = \inf \{ \ell(h) \}$ and $\ell_2(p) = \sup \{ \ell(h) \}$ for $h \in Q(p)$. If $Q(p) = \emptyset$ take $\ell_1(p) = \infty$ and $\ell_2(p) = 0$. Then let $\ell_1 = \inf \{ \ell_1(p) \}$ and $\ell_2 = \sup \{ \ell_2(p) \}$.
Theorem. If $l_1 > 0$ and $l_0 < \infty$, then every locally isometric mapping of $R$ on itself is an isometry. This theorem generalizes a result of Szente (Acta Math. Acad. Sci. Hung. 13 (1962), 433-441). (Received October 15, 1965.)

630-55. DONALD GREENSPAN, Mathematics Research Center, University of Wisconsin, Madison, Wisconsin and EDWARD SILVERMAN, Purdue University, Lafayette, Indiana. Approximate solution of the exterior Dirichlet problem and the calculation of electrostatic capacity.

A digital computer technique is described for approximating the solution of the exterior Dirichlet problem. The essence of the method is the combination of geometric inversion with a method for approximating the solution of an interior Dirichlet problem. Two and three dimensional problems are discussed separately because each requires a different treatment of boundary data. The technique for three dimensions extends to higher dimensional problems. Application is made to the calculation of electrostatic capacity by reducing the usual formula for capacity to one which requires only the evaluation at the origin of the solution of a particularly simple interior Dirichlet problem. Finally, the capacity of a unit cube is estimated to be 0.661. (Received October 15, 1965.)

630-56. A. M. YAQUB, University of California, Santa Barbara, California 93106. Semi-primal categorical independent algebras.

Let $U$ be an algebra of species $S$ and with more than one element. An $S$-expression is a finite composition of indeterminate symbols via the primitive operations of $U$. A map $f$ from $U^k$ to $U$ is expressible if there exists an $S$-expression $\phi$ which yields $f$ in $U$. A map $f(\xi_1, \xi_2, \ldots)$ is conservative if for each subalgebra $U_i$ of $U$, $a, \beta, \ldots \in U_i$ implies $f(a, \beta, \ldots) \in U_i$. The algebra $U$ is semi-primal if it is finite, and if every conservative map in $U$ is expressible. A $U$-algebra is an algebra (of same species as $U$) which satisfies all identities of $U$. $U$ is categorical if it is finite, and if every $U$-algebra is a subdirect power of $U$. A $U$-algebra is an algebra (of same species as $U$) which satisfies all identities of $U$. $U$ is independent if for each set $\Phi_1, \ldots, \Phi_n$ of $S$-expressions, there exists an $S$-expression $\psi$ such that $\psi = \Phi_i$ holds in each $U_i$. Theorem. Let $(R, X, +)$ be a finite commutative ring with unit. Then there exists a permutation of $R$ such that the algebra $(R, X, +, \cdot)$ is both semi-primal and categorical. Moreover, any finite subset of such algebras (with isomorphic copies deleted) is independent. (Received October 15, 1965.)

630-57. S. P. HASTINGS, 10900 Euclid Avenue, Cleveland, Ohio 44106. The asymptotic growth of solutions to a nonlinear system.

The system (1) $x(t) = y(t) + h(t), \ y(t) = -g(t, x(t))$ is considered, where $g(t, x)$ and $(\partial g/\partial t)(t, x)$ are continuous for $t \geq 0$, all $x$ and where $h(t)$ is continuous for $t \geq 0$. This system is equivalent to the equation $x(t) - x(0) + \int_0^t g(s, x(s))ds = h(t)$, and the theorem below extends results on a special case of this equation which were announced in Abstract 65T-208, these Notices 12 (1965), 157. Theorem. Assume that $xg(t, x) > 0$ for $t \geq 0$, $x \neq 0$. Let $G(t, x) = \int_0^t g(s, x(s))ds$ and assume that there exist positive constants $k_1$ and $k_2$ such that $xg(t, x) \leq k_1 G(t, x) + k_2$ for $t \geq 0$, all $x$. Further, suppose that for some positive numbers $a$, $k_3$, and $k_4$, $|x| \geq k_3 |y| + k_4 \Rightarrow |g(t, x)| \geq a|g(s, y)|$ if $0 \leq s \leq t$. Finally, suppose that there is a positive integrable function $a(t)$ with $(\partial g/\partial t)(t, x)/g(t, x) \leq a(t)$ for $x \neq 0$, $t \geq 0$, and $\int_0^t a(t)dt < \infty$. Let $(x(\cdot), y(\cdot))$ be any solution to (1) defined at $t = 0$. Then $(x(\cdot), y(\cdot))$ can be extended
to \([0, \infty)\) and \(|x(t)| = 0(1 + \int_0^t |h(s)| ds)\) as \(t\) approaches infinity. Other conditions on \(g(t, x)\) are found which also lead to this conclusion. (Received October 18, 1965.)

630-58. L. E. DeNOYA, 1562 Beeson Street, Alliance, Ohio 44601. On the permutability of certain operators in topological spaces.

The following theorem is established and its implications are discussed: Theorem. Let \(E\) be a subset of a topological space \(S\) and suppose that \(E\) is both open and closed. Then there can be constructed at most eight distinct sets by repeated application of the interior, the closure and the complement operators to \(E\). (Received October 18, 1965.)


Let \(A\) and \(B\) be algebras of real valued functions on the sets \(X\) and \(Y\), respectively. Assume that \(1 \in A\) and let \(K(A, B)\) denote the convex cone of all linear \(T: A \to B\) such that \(Tf \geq 0\) whenever \(f \geq 0\). Let \(K_1(A, B)\) be those \(T\) in \(K(A, B)\) such that \(T1 = 1\). Any multiplicative operator in \(K_1(A, B)\) is an extreme point of \(K_1(A, B)\). If \(A = B =\) polynomials on \(X = Y = \mathbb{R}\), then there exist extreme points of \(K_1(A, B)\) which are not multiplicative. If the functions in \(A\) are bounded, however, or if \(T^{-1} \in A\) whenever \(f \in A\) and \(f \geq \delta > 0\), then every extreme point of \(K_1(A, B)\) is multiplicative. Under these same hypotheses on \(A\), if \(T\) is on an extreme ray of \(K(A, B)\), then \(Tf = hMf\) (\(f \in A\)), where \(M\) is a positive multiplicative linear functional on \(A\) and \(h\) lies on an extreme cone of \(B\). No special hypotheses on \(A\) are needed to prove this last result when \(B = \mathbb{R}\), or to show that any operator of the above form lies on an extreme ray of \(K(A, B)\). For algebras which do not contain the constant functions it is possible to replace \(K_1(A, B)\) by the set \(K_0(A, B)\) (of those \(T \geq 0\) for which \(Tf \leq 1\) whenever \(0 \leq f \leq 1\)) and obtain analogous results. (Received October 18, 1965.)


Previous studies (Mem. Amer. Math. Soc. No. 53 (1965), 48 pp.) yielded infinite dimensional type I Jordan factors of finite class (called spin factors, loc. cit.). An abstract space-free characterization is given, using Segal’s construction of the Clifford algebra over a real Hilbert space. Results of E. Størmer and the author are obtained by elementary methods. A new construction of concrete spin factors is given, and shows that the uniformly closed linear span of an infinite system of Pauli spin relations \((s_is_k + s_k s_i = 2\delta_{ik}, s_i^2 = s_i)\) is weakly closed. Consequently, all operator topologies coincide on a spin factor. Under the norm \(\|a\|_2 = \text{trace}(a^2)^{1/2}\), a spin factor is a real Hilbert space (Størmer). Theorem 1. Two spin factors are isomorphic if and only if their real Hilbert space dimensions are equal. For any \(m \geq 3\), there is, up to isomorphism, exactly one spin factor of dimension \(m\). Corollary. The automorphism group of a spin factor is the stability subgroup of the full orthogonal group fixing the identity. Theorem 2. Every state \(\omega\) on a spin factor \(A\) has the form \(\omega(a) = \text{trace}(hah)\), all \(a \in A\), where \(h \in A^*\) and \(\|h\|_2 = 1\). The state is pure if and only if \(h^2\) is twice a pro-
jection and mixed if and only if h is invertible. Pure states are vector states and each mixed state is the sum of two vector states and is strictly positive. (Received October 18, 1965.)

630-61. F. J. SANSONE, Arizona State University, Tempe, Arizona 85281. Order preserving extensions to regressive isols.

We call the extension, $f_A$, of a recursive, eventually increasing function $f$ to the isols, ultimately order preserving in $A_R$ if there is a natural number $K$ such that $f_A$ preserves the order $\leq$ in the class of regressive isols which are greater than or equal to $K$. The principal result states that among the recursive, eventually increasing functions, those whose extensions are ultimately order preserving are exactly the functions whose first difference is eventually increasing. (Received October 18, 1965.)

630-62. A. G. MADER, 2565 The Mall, University of Hawaii, Honolulu, Hawaii 96822. On the normal structure of the automorphism group and the ideal structure of the endomorphism ring of abelian P-groups.

Let $G$ be an abelian $p$-group. For $H \leq G$ define $\text{Aut}(H)G = \{a \in \text{Aut} G : a[H = 1]\}$ and $\text{End}(H)G = \{a \in \text{End} G : Ha = 0\}$. The chain $... \geq p^nG \geq p^{n+1}G \geq ...$ of fully invariant subgroups gives rise to normal respectively ideal chains $... \leq \text{Aut}(p^nG)G \leq \text{Aut}(p^{n+1}G)G \leq ...$ and $... \leq \text{End}(p^nG)G \leq \text{End}(p^{n+1}G)G \leq ...$. The main theorem. Let $B(n) = \oplus \{B_1^{(n)} : 1 \leq 1\}$ be a basic subgroup of $p^nG$. For either positive integers $n$ and arbitrary $p$-groups $G$ or arbitrary ordinals $n$ and countable $p$-groups $G$, $\text{Ant}(p^{n+1})G/\text{Ant}(p^nG)G$ is isomorphic with the semidirect product of groups $A_1^{(n)}$, $A_2^{(n)}$, $A_3^{(n)}$, where $A_1^{(n)} \approx \text{Hom}(p^nG/p^{n+1}G, p^{n+1}G)$, $A_2^{(n)} \approx \text{Hom}(p^nG/(B_1^{(n)}} \oplus p^{n+1}G), B_1^{(n)}$, and $A_3^{(n)} \approx \text{Aut} B_1^{(n)}$. This result improves earlier results by Fuchs and Freedman. There is a corresponding theorem on $\text{End}(p^{n+1})G/\text{End}(p^nG)G$. (Received October 18, 1965.)


In the euclidean plane $E^2$, define an $\alpha$-cone to be any pointed cone subtending an angle of $\alpha$ radians, $0 \leq \alpha \leq \pi$. The set $S \subset E^2$ is said to be unbounded in the direction $\beta$ (where $\beta$ is a unit vector) if there is a half-line of the form $[x, x + \beta \ell]$ contained in $\text{conv} S$, for some $x \in S$. Let $A \subset E^2$ and $a \in [0, 2\pi]$ be given. Using the $\alpha$-cones which contain $A$ as an intersectional basis for the $\alpha$-convex hull of $A$, leads one to the following definition. For any $X \subset A$, define $\text{conv}_a X$ to be the intersection of all $\alpha$-cones $C$ such that $X \subset C$ and $C$ is unbounded in every direction in which $A$ is unbounded.

Theorem. If $A \subset E^2$ is unbounded wdf $a \in \text{int conv}_a A$, where $a \in (0, 2\pi]$, then there is a subset $F \subset A$ of at most 2 points such that $p \in \text{int conv}_a F$. A similar Caratheodory-type statement fails. In particular, $A \subset E^2$ and $p \in \text{conv}_a A$, does not even imply that there exists a finite subset $F$ of $A$ such that $p \in \text{conv}_a F$. (Received October 18, 1965.)
Three generalizations of the classical result \("S^n \) is not a retract of \(I^{n+1}\) are studied; the spaces concerned are CW-complexes. \textbf{Definition:} a subcomplex A of a CW-complex K is retractile in K provided SA is a retract of SK. \textbf{Theorem 1.} Let \((K, A)\) be a CW-pair. Suppose there is a connected H-space X and a map \(f: A \rightarrow X\) such that \(f_*: \\pi_k(A) \cong \pi_k(X)\) for all \(k \leq n\) and suppose \(H^{k+1}(K,A;\pi_k(A)) = 0\) for \(k \geq n + 1\). Then A is a retract of K iff A is retractile in K. \textbf{Theorem 2.} Let \(H^k(L) \cong G_k, \ k = 0, 1, \ldots\), and let \(i: A \subset K\). If \(i_*: H^k(K;G_k) \rightarrow H^k(A;G_k)\) is not epimorphic for some pair \((n, k)\) of integers, then \(A \ast L\) is not retractile in \(K \ast L\). \textbf{Theorem 3.} If \(A^*(L/B)\) is not retractile in \(K^*(L/B)\), then \(K \times B \cup A \times L\) is not retractile in \(K \times L\). Examples show these theorems to be "best possible." (Received December 2, 1965.)

\textbf{WITHDRAWN.}

\textbf{A new proof of a theorem of Kummer.}

Let p be an odd prime and denote by K the field obtained by adjoining a primitive pth root of unity \(\xi\) to the rational numbers. The methods of homological algebra are applied to prove the following theorem due to Kummer. \textbf{Theorem.} Let \(\epsilon\) be a unit in K and suppose that \(\epsilon = a \ (mod\ (1 - \xi^p))\), where \(a\) is a rational integer. Then if p is regular (i.e., p does not divide the class number of K) there exists \(\epsilon_1\) in K such that \(\epsilon_1^p = \epsilon\). (Received October 18, 1965.)

\textbf{Holomorphic functions with gap power series, III.}

Let \(t(r)\) be a counting function of a sequence \(\{r_n\}\) where \(0 < r_1 \leq r_\delta \ldots < r_n \rightarrow \infty\), and let \(x(r)\) be a counting function of \(\{(x_n)\}\). Then \(x(r)\) is a \(C^\infty\)-function.
be the range of \( t(r) \). Write 
\[
\ell = \liminf_{k \to \infty} \frac{\tau_{n_k+1}}{\tau_{n_k}}, \quad \rho = \limsup_{r \to \infty} \frac{\log t(r)}{\log r},
\]
\[
\limsup_{r \to \infty} \frac{\tau_{n_k+1}}{\tau_{n_k}} = \left( \frac{\rho}{b - 1} \right) \log b.
\]

Similar inequalities are proved for \( \liminf_{r \to \infty} \frac{t(r)}{I(r)} \). These inequalities are sharp. When \( \rho = 0 \) and \( t(r) = O((\log r)^\Delta) \), inequalities for the limiting values of \( I(r)/t(r) \log r \) are obtained. These results have applications to value distribution theory and to the theory of the maximum term of a holomorphic function. (Received October 19, 1965.)

630-68. RAFAEL ARTZY, State University of New York, Buffalo, New York 14214.
Noneuclidean incidence planes, II.

In a projective plane \( \pi \) a point set \( C \) ("oval") satisfies the following two axioms and their duals:
(1) No 3 points of \( C \) collinear, (2) Each point of \( C \) is on just one line ("tangent") which contains no other points of \( C \). In \( \pi \) the results of part I of this paper Abstract 620-11, (these Notices 12 (1965), 212) are valid and can be extended so as to provide line coordinates for all lines of \( \pi \). All points of \( \pi \) will then be characterized by equations in terms of a ternary ring \( T \). With minor modifications, \( T \) is a Hall ternary ring meeting the following additional requirements: (1) Both the additive and multiplicative loops are commutative, (2) \( x + x = a \) has at most one solution for \( x \), (3) \( T(a, x^{-1}, x) = a + a \) has just one solution for \( x \), (4) Each of the equations \( T(a, x^{-1}, x) = b, bx = a, \) and \( T(x, b, c) = \sqrt{x} + \sqrt{x} \) has at most 2 solutions for \( x \). Nondistributive examples of \( T \) exist. In contrast to R. Baer's planes (Studies and essays presented to R. Courant, New York, 1948, pp. 21-27), \( \pi \) may be finite. (Received October 19, 1965.)


Let \( LM \) be the set of left invariant means on the left amenable discrete semigroup \( \Sigma \). We call a bounded function \( u \) on \( \Sigma \) left almost convergent to value \( k \) provided \( (u, \lambda) = k \) for every \( \lambda \in LM \).
Several properties are discussed. Let \( V(\sigma), \sigma \in \Sigma \), be an antirepresentation of \( \Sigma \) by positive operators on \( C(X) \), with \( V(\sigma) = 1 \) and \( X \) compact Hausdorff. Let \( LM(X) \) be the means in \( C^*(X) \) fixed under all \( V(\sigma) \). We show that for \( \mu \in LM(X) \) to be extreme it is necessary and sufficient that \( (f \cdot V(\sigma)g(\mu), \sigma \in \Sigma, be left almost convergent to \( (\mu)(\mu)(g, \mu) \) for all \( f, g \in C(X) \). This is a mixing condition which generalizes the result of Sucheston for the case \( \Sigma = \) integers under addition (Duke Math. J. 30 (1963), 417 ).
(Received October 19, 1965.)

630-70. MYRON GOLDSTEIN, Arizona State University, Tempe, Arizona. Analytic Schottky differentials.

The problem has been raised by Rodin in his doctoral dissertation to characterize the period reproducer \( \psi(c) \) of the space of analytic Schottky differentials \( \Gamma_{as} \) on an arbitrary open Riemann surface in terms of principal functions. The author has shown that \( \psi(c) = \frac{1}{2}(\partial(\partial z) - (1)_{11}s - 1)_{11}dz \) where \( (1)_{11}s \) and \( 11r \) are certain principal functions. Furthermore, it is shown that as \( c \) runs through all cycles, the differentials \( \psi(c) \) span \( \Gamma_{as} \). (Received October 19, 1965.)
Using the terminology and results of A version of Riesz's Theorem for convex metric spaces (Abstract 66T-29, these Notices) it is shown: (1) A convex, complete, Blumenthal space of type \( n \) is congruently imbeddable in \( \mathbb{E}^n \). The following corollaries and refinements are obtained easily: (2) (Blumenthal) A convex, complete, externally convex, Blumenthal space of type \( n \) is congruent with \( \mathbb{E}^n \). (A metric space \( E \) is externally convex provided that \( \forall x, y \in E \exists z \) such that \( xyz \).) (3) If \( E \) is a convex, locally compact and thick Blumenthal space there exists a positive integer \( n \) such that \( E \) is congruently imbeddable in \( \mathbb{E}^n \). (4) A densely convex, Blumenthal space of type \( n \) is congruently imbeddable in \( \mathbb{E}^n \). (5) A convex, complete, and fe4p space of type \( n \) is congruently imbeddable in \( \mathbb{E}^n \). (fe4p stands for "feeble euclidean four point property". This concept has been defined in Pacific J. Math. 5 (1955), 161-167.) (Received October 20, 1965.)

Let \( G \) be a simply-connected, finite CW-complex. W. Browder has shown that if \( G \) is a homotopy-commutative, homotopy-associative H-space, then \( H^*(G) \) is torsion-free. Moreover, it is well known that under this hypothesis, \( H^*(G) \) is a primitively generated, exterior algebra on odd dimensional generators. The only known H-spaces \( G \) for which \( H^*(G) \) are primatively generated, torsion-free, exterior algebras on odd dimensional generators, are the Special Unitary groups \( SU(n) \), the Symplectic groups \( Sp(n) \) and the seven-dimensional sphere. It is well known that the 7-sphere has no homotopy-commutative multiplications. Let \( \Lambda(n) \) be the number of ones in the binary expansion of integer \( n \). Theorem. If \( n \) is odd and \( \Lambda(n) \leq 3 \), or if \( n = 8 \), then \( SU(n) \) can not support a homotopy-commutative, homotopy-associative H-space multiplication. Theorem. If \( n \) is odd and \( \Lambda(n) \leq 3 \), or if \( n \equiv 2 \pmod{4} \) and \( \Lambda(n) = 2 \), or if \( n < 8 \), then \( Sp(n) \) does not possess an H-space multiplication which is both homotopy-commutative and homotopy-associative. (Received October 21, 1965.)

In the following we assume that \( R \) is an associative ring satisfying the descending chain condition for either left or right ideals, and that \( J \) is the Jacobson radical of \( R \). Any \( b \in R \) is a zero divisor if for some nonzero \( c \in R \), \( cb \) or \( bc \) is zero. Proposition 1. For any \( b \in R \), either \( b \) is a zero divisor or \( R \) has an identity and \( b \) is a unit. Further assume that \( R \) has a finite number (\( \geq 1 \)) of nonzero zero divisors. Lemma 1. Either \( R \) is finite or \( R \) has an identity. In either case \( J \) is finite. Lemma 2. If \( J = (0) \), then \( R \) is finite. Proposition 2. If \( R/J \) is a division ring, then \( R \) is finite. Theorem. \( R \) is finite. This modifies the work of N. Ganesan, Properties of rings with a finite number of zero divisors, Math. Ann. 157 (1964), 215-218, from commutative rings to d.c.c. rings. (Received October 21, 1965.)

Let $H_L^*$ be the set of all bounded sequences $\{x_n\}$ taking values in a separable Hilbert space $H$. Define operations pointwise so $H_L^*$ is a vector space. Let $\langle x_n, y_n \rangle = L(x_n, y_n)$ where $L$ is a "generalized limit", and in the usual fashion construct a Hilbert space which is denoted by $H_L$. Every sequence $x_n$ determines a weakly continuous vector valued extension function $\hat{x}: \beta N \to H$ where $\beta N$ is the Stone-Čech compactification of the integers. Let $L_2^2(\mu, \beta N, H)$ be the space of norm-square $\mu$-integrable vector functions from $\beta N \to H$, where $\mu$ is the measure on $\beta N$ determined by $L$. Then $x_n \to \hat{x}$ is a partial isometry from $H_L \to L_2^2(\mu, \beta N, H)$. It is isometric on $U^1$. For any operator $A$ on $H$, $\lambda^0 \{x_n\}^i = \{Ax_n\}^i$ defines a representation $A \to A^0$ on $H_L$. If $Z \in U^1$ then $\langle A^0 Z, Z \rangle = \phi(A)$ is a linear functional which annihilates all compact operators. (Received October 21, 1965.)


A $(K,S)$-configuration $M$ of order $v$ is a system of $v$ points and lines, each line (point) containing $S$ points (lines) such that $K$ is the smallest integer $n$ ($n > 1$) for which there are $n$ distinct points $x_i$ and lines $L_i$, $1 \leq i \leq n$, with $L_i \cap L_{i+1} = \{x_{i+1}\}$, $1 \leq i \leq n - 1$, and $L_n \cap L_1 = \{x_1\}$. Then $v \geq \sum_{i=0}^{K-1}(S - 1)^i$, and in case of equality $M$ is a projective $(K,S)$-configuration. The existence of a projective $(K,S)$-configuration is shown to imply the existence of a $(v,k,\lambda)$-configuration, and in the case $K = 2j + 1$, the existence of $S - 2$ orthogonal Latin squares of order $(S - 1)^j$. A class of projective $(4,S)$-configurations is constructed from the projective geometries $PG(3,S - 1)$ in the case $S - 1$ is a prime power, and their collineations (permutations of points taking lines to lines) are completely determined and are induced by collineations of the corresponding $PG(3,S - 1)$. The inducing projective collineations of $PG(3,S - 1)$ are precisely those given by Dickson's general linear Abelian group, with a unique one taking an ordered pentagon into any other. It is shown that special $(K,S)$-configurations do not exist, but that there is no upper bound on those $K$ for which a $(K,S)$-configuration of some order exists for fixed $S \geq 3$. (Received October 21, 1965.)

630-76. J. S. RATTI, Wayne State University, Detroit, Michigan 48202. On high indices theorems.

The series $\sum a_n$ is said to be summable $[R, \lambda, k, \gamma]_m$ if $\int_1^\infty x^\gamma + m - 1 \left| (d/dx)x^{-k} A_\lambda^k (x) \right|^m dx < \infty$ for $m \geq 1$, $k > 0$, $km^1 > 1$, $k > \gamma - 1$, where $A_\lambda^k (x) = \sum_{\lambda_n < x} (x - \lambda_n)^k a_n$, and $\{\lambda_n\}$ is a sequence of positive numbers such that $1 \leq \lambda_1 < \lambda_2 < \cdots$. Theorem. If $\sum a_n$ is summable $[R, \lambda, k, \gamma]_m$, $0 < \gamma \leq 1 - (1/m)$, and $(\lambda_{n+1}/\lambda_n) \geq q > 1$, then $\sum |a_n|^m \lambda_n^\gamma < \infty$. The proof of the theorem is based upon theorem 3 of Waterman, Trans. Amer. Math. Soc. 69 (1950), 468-478. (Received October 21, 1965.)
Let \( W \) be a linear manifold in a Hilbert space \( H \). Let \( L : W \rightarrow H \) be a self-adjoint linear operator having a countable set of eigenvalues, each of which is positive. Let \( \{ \lambda_i \} \) be the the eigenvalues of \( L \), arranged in increasing order, and let \( \{ x_i \} \) be the corresponding eigenvectors. Let \( A : W \rightarrow H \) be a not necessarily self-adjoint operator such that there exists a positive sequence \( \{ \tau_i \} \) such that \( \sum \tau_i^{-1} |(Ay, x_i)|^2 \leq \|y\|^2 \). Then if the circles \( C_j = \{ z \mid |z - \lambda_j| \leq \tau_j \} \) are all disjoint, the operator \( \tilde{L} = L + A \) has a countable set of eigenvalues, one in each \( C_j \). The Rayleigh-Ritz method is applied to \( \tilde{L} \), using the eigenvectors of \( L \) as a basis, to obtain approximations to the eigenvalues of \( \tilde{L} \) and estimates are derived for the errors which arise. More precisely we have the following situation. Let \( \mu_p \) be the eigenvalue of \( \tilde{L} \) in \( C_p \). For \( n \geq p \) let \( \eta_1^n, \ldots, \eta_n^n \) be the eigenvalues of \( \tilde{L}_n = (P_n \tilde{L})|_{V_n} \), where \( P_n \) is the projection onto \( V_n \) the space spanned by the first \( n \) eigenvectors of \( L \). Then under certain other conditions we obtain a bound for \( |\mu_p - \eta_p^n| \). This bound is computable. Compare Abstract 621-12, these Notices 12 (1965), 306. (Received October 22, 1965.)

630-78. RALPH TINDELL, Florida State University, Tallahassee, Florida. A mildly wild 2-cell in \( E^4 \).

An example is given of a wild 2-cell \( D \) in 4-space with a point \( p \) on its interior such that any 2-cell subset of \( D \) with \( p \) on its boundary is flat. The particular cell is the one suggested by Fox in problem 21 of "Some problems in knot theory", Topology of 3-manifolds, Prentice-Hall, 1962. The proof depends on the following Theorem. Suppose \( D \) is a 2-cell in \( E^4 \), \( p \in D \), and \( D_i \) is a combinatorial 2-cell satisfying (i) \( D - p = \bigcup_{i=1}^{\infty} D_i \), (ii) \( D_i \cap D_{i+1} \) is a common face and \( D_i \cap D_j = \emptyset \) if \( i \neq j \) if \( i, j + 1 \). (iii) \( D_m^1 = \bigcup_{i=1}^{m} D_i \) is (piece-wise linearly) flat. Then \( D \) is flat. The above theorem is also true for \( k \)-cells \( D \) in \( E^n \) for \( n \geq 5 \) and \( k \neq n - 2 \). The author has an example of a 2-cell in \( E^4 \) satisfying hypotheses (i) and (ii) which is wild. (Received October 25, 1965.)

630-79. D. C. STEVENS, New Mexico State University, Box AM, University Park, New Mexico. Expected number of real zeros of polynomials.

It is known that the expected number of real zeros of the random polynomial \( \sum_{i=0}^{n} X_i x_i \) is asymptotic to \( (2/\pi) \log n \) if the \( X_i \) are independent, identically distributed (either Gaussian or uniform) random variables, and \( E(X_i) = 0 \); M. Kac has established these results. The following theorem will be presented: If the \( X_i \) are independent random variables satisfying \( E(X_i) = 0 \), \( E(X_i^2) = 1 \), \( E(X_i^4) < B \), and for \( i = 0 \), and \( i = n \) \( (d/dy)P(X_i < y) \leq B/(1 + y^1+B) \) for some \( B < \infty \), then the expected number of real zeros of \( \sum_{i=0}^{n} X_i x_i \) is asymptotic to \( (2/\pi) \log n \). The proof is made by showing that if \( x_0 \) is sufficiently near \( \pm 1 \) and \( \Delta x \) is sufficiently small, then the expected number of zeros in \( (x_0, x_0 + \Delta x) \) is sufficiently near the expected number of zeros in \( (x_0, x_0 + \Delta x) \) for the Gaussian case. Bounds are established for the expected number of zeros in the remainder of \( (-\infty, 0) \). The case of purely discrete random variables remains open. (Received October 25, 1965.)
Let \( D \) be a bounded, simply connected domain in the plane with piecewise smooth boundary \( \Gamma \). Let \( \Gamma \) be composed of two parts \( \Gamma_1 \) and \( \Gamma_2 \). The mixed boundary value problem is that of finding a function harmonic in \( D \) with specified values on \( \Gamma_1 \) and specified normal derivative on \( \Gamma_2 \). Let a mesh of width \( h \) be drawn and set up difference equations in the fashion of Batschelet, Über die Numerische Auflosung von Randwertproblemen bei Elliptischen partiellen Differentialgleichungen, ZAMP 3 (1952), 165-193. Let the "corners" of \( D \) be the points of \( \Gamma \) where the smoothness is violated, together with the points common to both \( \Gamma_1 \) and \( \Gamma_2 \). Let \( \delta_h \) be the error committed in the approximation and let \( D' \) be any subdomain of \( D \) which excludes the corners. Then \( \delta_h = O(h^\gamma) \) where \( \gamma \) depends on the size of the angles at the corners. If all the corners are acute one can take \( \gamma = 1 \). The derivatives of the harmonic function are allowed to have singularities at the corners. The proof is based on methods of Laasonen, On the degree of convergence of discrete approximations for the solution of the Dirichlet problem, Ann. Acad. Sci. Fenn. Ser. AI, 246 (1957). (Received October 25, 1965.)

Let \( \tilde{\mathcal{F}} = (E,p,B) \) be a (Hurewicz) fibre space with lifting function \( \lambda \). A fibre (-preserving) map \( f: E \to E \) induces \( \bar{f}: B \to B \) such that \( \bar{f}p = pf \). If \( [W \cup \tilde{f}(W)] \subseteq V \subseteq B \) where \( V \) is pathwise connected, define \( \tilde{f}_b^V: p'^{-1}(b) \to p'^{-1}(b) \), for \( b \in W \), by \( \tilde{f}_b^V(e) = \lambda(f(e), \omega)(1) \), where \( \omega \) is a path in \( V \) from \( \tilde{f}(b) \) to \( b \). Let \( i \) be a fixed point index for compact ANR's. Let \( Q \) denote the rationals. Write \( \text{bd} \) for boundary and \( \text{cl} \) for closure. Theorem. Let \( \tilde{\mathcal{F}} = (E,p,B) \) be a fibre space such that \( E,B \) and all fibres are compact ANR's. Let \( f: E \to E \) be a fibre map. If \( U \subseteq B \) is open with \( \tilde{f}(b) \neq b \) for all \( b \in \text{bd}(U) \) and \( \text{cl}(U \cup \tilde{f}(U)) \subseteq V \subseteq B \) where \( V \) is open and connected and \( \tilde{\mathcal{F}}|V = (p'^{-1}(V),p,V) \) is \( Q \)-orientable, then \( i(f,p'^{-1}(U)) = i(\tilde{f},U) \cdot L(\tilde{f}_b^V) \) for all \( b \in U \), where \( L \) denotes the Lefschetz number. In particular, if \( B \) is connected and \( \tilde{\mathcal{F}} \) is \( Q \)-orientable, then \( L(f) = L(\tilde{f}) \cdot L(\tilde{f}_b^V) \) for any \( b \in B \), which extends the well-known formula \( X(E) = X(B) \cdot X(F) \) (\( X = \text{Euler characteristic} \)). The theorem has application to the computation of the index of a fixed point class, to the construction of fixed point free maps, and to the study of the fixed point property. (Received October 25, 1965.)

By using methods similar to those used by T. T. Frankel [Differential and combinatorial topology (a symposium in honor of Marston Morse), Princeton Univ. Press, Princeton, N. J., 1965, 37-53], Morse decomposition of the classical structures of Riemannian globally symmetric spaces (such as \( U(n)/O(n), Sp(n)/U(n), U(2n)/Sp(n), SO(2n)/U(n) \) and real, complex and quaternionic Grassmannians) are obtained. The Morse-Bott inequalities obtained are shown to be equalities by using a result of E. E. Floyd. To illustrate the results obtained we give an example: \( Sp(n)/U(n) = pt \cup \bigcup_{k=1}^{n-1} (W(n,1) \cup W(n,2) \cup \cdots \cup W(n,k)) \) where \( W(n,k) \) is a \((k+1)\)-dimensional plane bundle over the complex Grassmannian \( W(n,k) = U(n)/U(k) \times U(n-k) \) (\( = \) means up to homotopy type, \( \cup \) stands for attaching bundles). The Poincaré polynomial of \( Sp(n)/U(n) \) is related to that of the
complex Grassmannian as follows: \( P(\text{Sp}(n)/U(n);t) = \sum_{k=0}^{n} k(k+1) P(W_n,k;t) \). (Received September 29, 1965.)


The application of a generalized method of averaging to a perturbed vector system of equations containing a "rapidly rotating" phase, the rate of change of which depends on the phase even for vanishing perturbation parameter, is considered. This generalizes a method given by Bogoliubov and Mitropolsky (Asymptotic methods in the theory of nonlinear oscillations, New York, 1961).

Application of the method is made to the problem of the damping of oscillations corresponding to a nonlinear (odd) restoring force, the equation of motion being first transformed by introducing an amplitude and a phase. In the particular case of linear damping the rate of decay of the amplitude is exhibited. An extension of the results is made to nonlinear oscillating systems which contain a slowly varying parameter. A second application considers systems with a periodic restoring force (with zero mean) and deals with the case of revolutions, rather than oscillations. In the particular case of linear damping plus constant torque (e.g., the synchronous motor), it is found that the revolutions persist if the torque algebraically exceeds a critical value, but otherwise the revolutions will cease after a finite time and damped oscillations will follow. (Received October 27, 1965.)

630-84. FRED RICHMAN and CAROL WALKER, New Mexico State University, University Park, New Mexico. A purification problem in primary abelian groups.

Theorem. If \( G \) is an abelian p-group and \( K \) is a subgroup of \( G^1 \) then there exists a pure subgroup \( P \) of \( G \) with \( P \cap G^1 = K \) precisely when \( [(pG^1 \cap K)/pK] \) is no greater than the final rank of a high subgroup of \( G \). (Received October 27, 1965.)


Let \( f \) be a function from a compact semigroup, \( S \), into itself. \( f \) is called a homomorphic retraction iff \( f \) is a retraction and a homomorphism. **Theorem 1.** If \( A \) is a subsemigroup of \( S \), then every continuous homomorphism defined on \( A \) has continuous homomorphic extension to \( S \) iff \( A \) is a homomorphic retract of \( S \). **Theorem 2.** The minimal ideal, \( K \) of \( S \) is a homomorphic retract of \( S \) iff there exists a function \( f: S \to K \cap E \) (\( E \) denotes the set of idempotents) with the following properties (i) If \( y \in K \cap E \), then \( y = f(x) \) if and only if \( xy = xy \) (ii) \( f(xy) = f(x)f(y) = f(xy)f(xy) \) for all \( x \) and \( y \) in \( S \). **Remark.** If such an \( f \) exists and if \( x \in K \) then \( f(x) \) is the unit of the maximal subgroup to which \( x \) belongs. **Theorem 3.** \( K \) is a homomorphic retract iff there exist functions \( g \) and \( g^t: S \to K \) such that \( xK = g(x)S \) and \( Kx = Sg^t(x) \) for all \( x \in S \). (Received October 27, 1965.)
In the paper (Insel, Proc. Amer. Math. Soc. 15 (1964), 847-50) a sufficient condition was obtained for the order topology of a complete lattice to be compact--namely that its complete topology is Hausdorff. We assert that the converse is true under the condition that the complete topology of the lattice satisfies the first axiom of countability. Let L be a complete lattice. Then (1) If \( \{ s_n \} \) is a sequence in L with no order convergent subset, the range of \( \{ s_n \} \) is closed. (2) If \( S \subseteq L \) is compact with respect to the order topology for L, then every sequence on S has an order convergent subset. Lemma (1) implies Lemma (2) and (2) implies the result. E. E. Floyd has shown (Pacific J. Math., 5 (1955), 687-689) that there exists a complete lattice which is not Hausdorff in its order topology. Lemma (2) may be used to obtain: If L is complete and first countable in its order topology, L is Hausdorff in its order topology. (Received October 27, 1965.)

Pelczynski (Studia Math. 19 (1958)) studied complemented subspaces of \( l_p, c_0, m \) and \( s \) by considering countable direct sums of these spaces with respect to their generalized bases. Using his definition of the direct sum, let \( (e_n) \) be a basis for the Banach space \( E \), and let \( (e_{ij}) \) be the corresponding basis for \( \sum E = (E \oplus E \oplus \ldots) \). Let \( N \) denote the positive integers, and let \( \tau : N \rightarrow N \times N \) be any one to one, onto map. If there exists an isomorphism (isometry) \( T : E \rightarrow \sum E \) with \( T e_n = e_{\tau(n)} \), \( \| T \| \leq K \), and \( \| T^{-1} \| \leq C \) (independent of \( \tau \)), then we say that \( (e_n) \) is almost dispersed (dispersed). Proposition. An orthogonal basis (Gelfand, Ucn. Zap, MGU, 148 (1951)) \( (e_n) \) is almost dispersed if and only if \( E \sim \sum E \) under some \( \tau \) induced \( T \) as above, and if \( (e_n) \) is symmetric (Singer, Rev. Math. Pures Appl. (Acad. R.P.R.) 6 (1961)). One obtains complementation theorems such as Pelczynski's Proposition 4, and Proposition. Let \( E \) and \( F \) have orthogonal, dispersed bases \( (e_n) \) and \( (f_n) \) respectively. Then \( E \) can be embedded in \( F \) if and only if \( E \sim F \). Concerning the conjugate spaces, one derives: Proposition. Let \( E \) have an orthogonal, dispersed basis \( (e_n) \). Then \( E^* \sim (\sum E^*)_E^* \). (Received October 27, 1965.)

Let \( E \) have the orthogonal basis \( (e_n) \) and define \( \sum E \) to be all sequences \( (x_n) \) with values in \( E \) such that \( \sum \| x_n \| e_n \) is in \( E \). Let \( \| (x_n) \| = \| \sum x_n \| e_n \| \). Let \( e_{jk} \) stand for the kth basis element in the jth copy of \( E \) in the sum. Say that \( (e_n) \) is almost dispersed if for each one to one and onto \( \tau \) from the positive integers \( N \) to \( N \times N \) there is an isomorphism \( T : E \rightarrow \sum E \) such that \( Te_n = e_{\tau(n)} \) if \( \tau(n) = (j,k) \) and \( \| T \| \) is bounded independently of \( \tau \). Lemma: If \( (e_n) \) is almost dispersed then every one to one \( \tau \) from \( N \) to \( N \times N \) induces an isomorphism \( T \) of \( E \) into \( \sum E \) with \( Te_n = e_{jk} \) if \( \tau(n) = (j,k) \). Let the support of \( e_n \) be \( \{ n : a_n \neq 0 \} \). Theorem. \( (e_n) \) is almost dispersed if and only if there are constants \( C, K > 0 \) such that if \( x_1, \ldots, x_n \) have mutually disjoint supports then \( C \| \sum x_n \| \leq \| \sum x_n \| e_n \| \leq K \| \sum x_n \| \). Example. An almost dispersed basis is symmetric but a symmetric basis is not necessarily almost dispersed. The space of sequences \( (a_n) \) such that \( \sup_p \sum (\| a_n \| /p(n)) = \| (a_n) \| \)
where \( p \) is a permutation of \( N \), has a symmetric basis which does not satisfy the geometric condition in the theorem above. This space is a conjugate space not isomorphic to \( \ell_p \) (or \( c_0 \)). \textbf{Theorem.} If \( (e_n) \) is almost dispersed and if \( (x_n) \) is a sequence of nonzero elements with mutually disjoint supports then \( [x_n] \) is isomorphic to \( E \) and is a direct factor of \( E \). (Received October 27, 1965.)

630-89. MARIO PETRICH, Pennsylvania State University, McAllister Building, University Park, Pennsylvania. \textit{Partial homomorphic images of certain groupoids.}

Let \( E \) be a completely \( 0 \)-simple semigroup over a one-element group, \( G \) a group, \( E \times G/N \) the Cartesian product of \( E \) and \( G \) modulo the ideal \( N = \{0\} \times G \). Call the groupoid \((E \times G/N)^*\) obtained from \( E \times G/N \) by removing the zero a \textit{rectangular groupoid} (\( \mathcal{L} \) and \( \mathcal{R} \)-classes as well as the structure group of \((E \times G/N)^*\) have the meaning inherited from \( E \times G/N \) which is completely \( 0 \)-simple semigroup). \textbf{Theorem 1.} Every regular \( D \)-class of any semigroup is a partial isomorphic image of a rectangular groupoid having the same structure group, the same number of \( \mathcal{R} \)-classes, and the same number of \( \mathcal{L} \)-classes as \( D \). \textbf{Theorem 2.} Let \( G \) be a group, \( S \) a semigroup, and \( D \) a regular \( D \)-class of \( S \). Then \( D \) is a partial homomorphic image of a Brandt groupoid with the structure group \( G \). (For the terminology and related results see: A. H. Clifford, Proc. Amer. Math. Soc. 16 (1965), 538-544.) (Received October 27, 1965.)


Let \( F(t,y) \) be a sufficiently regular operator from \( R \times Y \) into \( Y \), where \( R, Y \) are the spaces of reals and a complex Banach space respectively. Let \( F \) satisfy a Lipschitz condition in \( y \) with a coefficient depending on \( t \). Let \( U \) be the generator of a strongly continuous semigroup \( E(t) \) on \((-\infty,0]\). Let \( Z \) be the space of locally Bochner summable functions from \( R \) into \( Y \) such that \( \|f\| = \sup(\int_0^{t+1} |f(s)|ds : t \in R) < \infty \). Assume \( UF(\cdot,y) \in Z \) for \( y \in Y \). For the equation \( y' + Uy = F(\cdot,y) \) sufficient conditions are given for the existence and uniqueness of a solution \( y \) such that \( y(R) \) is precompact in \( Y \), \( y \) is locally absolutely continuous, \( E(t-s)y(t) \) for each fixed \( s \) is locally absolutely continuous in \( t \), and \( y' \in Z \). If \( Y \) has a Schauder basis \( \{x_i\} \), set \( P_n x = \sum_{i=1}^n a_i x_i \) for \( x = \sum_{i=1}^\infty a_i x_i \). If \( \|P_n\| = 1 \) for all \( n \) then it is proven that the solution can be uniformly approximated by fixed points \( y_n \) of some contractive operators defined in closed subsets of the Banach space of bounded continuous functions from \( R \) into \( Y \). When \( U \) is linear and bounded we also have that \( y_n' \) approximates \( y' \) in the norm of the space \( Z \). When \( \|P_n\| \geq 1 \) it is shown that the solution can be uniformly approximated by continuous bounded functions from \( R \) into \( Y \). (Research supported by the U. S. Army Research Office - Durham). (Received October 27, 1965.)


For notation see [Morley, Proc. Nat. Acad. Sci. (2) 49 (1963), 213-216]. \textbf{The subspace} \( S_X \) \textit{spanned by} \( X \) \textit{is the union of all finite subsets of} \( |A| \) \textit{which are definable by members of} \( F(A,X) \). \textbf{The dimension} of \( S_X \) \textit{is the smallest} \( \nu \) \textit{such that there is a} \( Y \subseteq |A| \) \textit{with} \( S_X = S_Y \) \textit{and} \( \nu \) \textit{is the cardinality of} \( Y \). \textbf{Proposition.} If \( S_X = S_Y \) \textit{are isomorphic, then they are the same dimension.} \textbf{Theorem.} If \( \sum \)}
is categorical in an uncountable power but not in \( \mathbb{N}_0 \), then there is an \( n < \mathbb{N}_0 \) such that the denumerable models of \( \sum \) are (up to isomorphism) the subspaces of dimension \( m, n \leq m \leq \mathbb{N}_0 \), and there is a unique subspace for each such \( m \). Corollary. For \( \sum \) as above, there are \( \mathbb{N}_0 \) denumerable isomorphism types. This answers a question mentioned by Vaught [Proc. Sympos. Foundations of Math.: Infinitistic Methods, Warsaw, 1959, p. 320.] (Received October 28, 1965.)


Let \( C_0 \) be a linear lattice of real-valued functions on a set \( X \). A linear positive functional \( J \) on \( C_0 \) is called a Daniell functional if \( f_n \to 0 \) on \( X \) implies \( Jf_n \to 0 \). Let \( C = \{ f : \text{there exist } f_n \in C_0 \text{ such that } f_n / f \text{ on } X \text{ and } \sup \{ |f_n| \} < \infty \} \). Let \( D = \{ f : \text{there exist finite-valued } g_1, g_2 \in C \text{ such that } f = g_1 - g_2 \} \). Denote by \( J_0 \) the natural extension of \( J \) onto \( D \). Let \( V \) be the family of all sets of the form \( \Lambda = Q_1 \setminus Q_2 \) where \( Q_1 \subseteq C \) and \( Q_1 < f \) on \( X \) for some \( f \in C_0 \) depending on \( A \), \( i = 1, 2 \). Put \( v(A) = J_0 X_\Lambda \). Theorem 1. The family \( V \) is a prerger of sets, that is if \( A_1, A_2 \in V \) then \( A_1 \cap A_2 \in V \) and there exist disjoint \( B_1, \ldots, B_k \in V \) such that \( A_1 \setminus A_2 = B_1 \cup \ldots \cup B_k \). The set function \( v \) is a volume, that is, it is countably additive on \( V \). Using the results of the paper: W. Bogdanowicz, A generalization of the Lebesgue-Bochner-Stieltjes integral and a new approach to the theory of integration, Proc. Nat. Acad. Sci. USA 53 (1965), 492 -498; in which has been presented a direct approach to the theory of the space \( L(v,Y) \) of Lebesgue-Bochner summable functions generated by any volume \( v \), we get:

Theorem 2. If \( (X, V, v) \) is the volume space generated by a Daniell functional \( J \) and \( f \cap 1 \subseteq C_0 \) for every \( f \in C_0 \), then \( C_0 \subseteq L(v,R) \) and \( \int f \, dv = Jf \) for all \( f \in C_0 \), \( R \) being the space of reals. This result will appear in Math. Ann. (Received October 28, 1965.)

630-93. S. B. NADLER, Jr., Wayne State University, Detroit, Michigan 48202. Differentiable retractions in Banach spaces.

Let \( K \) be a subset of a Banach space \( A \) and let \( f : K \to K \). Then \( f \) is a differentiable retraction of \( K \) iff \( f \) is a continuous retraction (\( f \circ f = f \)) of \( K \) which is class \( C^1 \) on \( \text{int}(K) \). If \( f \) is not the identity map, then \( f \) is called a proper differentiable retraction, Theorem. Let \( A \) be a Banach space and let \( K \) be a subset of \( A \) such that \( \text{int}(K) \) is connected and \( \text{int}(K) \supset K \). If \( f : K \to K \) is a proper differentiable retraction of \( K \), then \( \text{int}(f(K)) \) is empty. It is shown that the unit sphere in \( L^2 \) is a differentiable retract of the unit ball. Theorem. Let \( U \) be a connected open subset of \( E^n \) and let \( f : U \to U \) be a differentiable retraction of \( U \). If \( r = \text{rank}(f'(x)) \), for some \( x \in f(U) \), then \( f(U) \) is a class \( C^1 \) differentiable submanifold of \( E^n \) of dimension \( r \). This Theorem, together with the Tubular Neighborhood Theorem (see J. Milnor, Topology from the differentiable viewpoint), characterizes the class \( C^1 \) differentiable submanifolds of \( E^n \) as class \( C^1 \) neighborhood retracts. (Received October 28, 1965.)

630-94. ANTON ZETTL, Louisiana State University, Baton Rouge, Louisiana 70803. Lack of self-adjointness in three point boundary value problems.

Denote by \( a_i, i = 1, 2, 3 \) real numbers such that \( a_1 < a_2 < a_3 \) and by \( A_i, B_i, i = 1, 2, 3 \) complex \( k \times k \) matrices. Let \( P = (f_{ij}) \) be a \( k \times k \) matrix of continuous complex valued functions such that \( f_{ij} = 0 \) if \( i + j \) is even or \( j > i + 1 \) and \( f_{i, i+1}(t) \neq 0 \), \( t \in [a_i, a_{i+1}] \), \( i = 1, \ldots, k - 1 \). Let \( H = (\bar{e}_{k+1-j, k+1-i} \cdots e_{k+1-i, k+1-j}) \).
Consider the boundary value problems (i) $Y' = FY$ with
$\sum_{i=1}^{3} B_i X(a_i) = 0$,
and (ii) $X' = HX$ with
$\sum_{i=1}^{3} B_i X(a_i) = 0$. Suppose (i) and (ii) have only the trivial solution. Let $K = (K_{ij})$ and $G = (G_{ij})$ denote Green's matrices for problems (i) and (ii), respectively. Theorem. If either $A_2 \neq 0$ or $B_2 \neq 0$ then there exist $t,u$ in $[a_1,a_3]$ such that $K_{1k}(t,u) \neq -1)G_{1k}(u,t)$. The generalization of this theorem to n point problems for $n > 3$ is straightforward. (Received October 28, 1965.)

630-95. R. E. POWELL, Lehigh University, Bethlehem, Pennsylvania 18015. The $L(r,t)$ summability transform.

Define the $L(r,t)$ summability transform $(b_{nk})$ by $b_{nk} = 0$ if $k < n$ and $b_{nk} = (1 - z)^{n+1}$ $\exp(1/(1 - z))^{(n)}(z)k^{-n}$ if $k \geq n$ where $L_j^{(n)}(t)$ denotes the Laguerre polynomial of degree $j$. The special case, $L(r,0)$, is the well-known Taylor matrix $T(r)$. The following are established: Theorem 1. $L(r,t)$ is regular for each $t$ if and only if $\text{Im}(z) = 0$ and $0 \leq \text{Re}(z) < 1$. Theorem 2. Let $0 \leq r < 1$. For each $t$, the $L(r,t)$ transform continues the geometric series analytically into the region $\{z: \{|1 - rz/(1 - rz)| < 1\} \cap \{|z| < 1\}\}$. Theorem 3. A sequence $(x_{n})$ which is $T(r,t)$-summable to $x$ is $L(r,t)$-summable to $x$ provided (i) $|x_1| < 1$, $r_2 \neq 1$. (ii) $|x_1| < |r_2|$, (iii) $|x_1| + |x_2| < |1 - r_1|$, and (iv) $|1 - r_2| + |x_1 - r_2| = |1 - r_1|$. (Received October 29, 1965.)

630-96. R. E. MOORE, University of Wisconsin, Computer Sciences Department, Madison, Wisconsin. Interval analysis: an approach to constructive two-sided approximation for nonlinear problems.

The concepts of interval arithmetic and interval valued rational functions of several interval variables form the basis for a set of techniques which can be programmed for digital computers to obtain intervals of arbitrarily small width containing exact values of solutions to nonlinear problems. Such results have been described for the initial value problem for systems of nonlinear ordinary differential equations (R. E. Moore, "The automatic analysis and control of error in digital computation based on the use of interval numbers", Error in digital computation, Vol. I, Ed. by L. B. Rall, Wiley, 1965, pp. 61-130). In this paper similar results are presented for solutions of nonlinear integral equations. The two point boundary value problem for systems of nonlinear ordinary differential equations is included as a special case. Under hypotheses which can be tested by the computer, a nested, contracting sequence of interval valued step functions converges to the solution. The result is a constructive version of a method of Rall (Numerical integration and the solution of integral equations by the use of Riemann sums, SIAM Review, (1) 7 (1965), 55-64). (Received October 29, 1965.)

630-97. R. F. DeMAR, University of California, Davis, California 95616. A theorem applicable to the Whittaker $W$ problem.

The following theorem is proved: Let $(a_n)$ be a periodic sequence of complex numbers of period $p$, i.e., $a_{n+p} = a_n$; $k = 0, 1, \ldots, p - 1$. Let $w$ be a primitive $p$th root of unity. Let $D(z) = D(z; a_0, a_1, \ldots, a_{p-1})$ be the determinant of the matrix $A = (w^j \exp(w^j a_n))$; $j = 0, 1, \ldots, p - 1$. Denote the zero of $D(z)$ nearest the origin by $c_0$. Then there exists an entire function $f \not= 0$ of exponential type $|c_0|$ such that $f(b_{n+k})(a_n) = 0$; $k = 0, 1, \ldots, p - 1; n = 0, 1, \ldots$. If $g$ is an entire function of exponential type $r < |c_0|$ and if $g(b_{n+k})(a_n) = 0$; $k = 0, 1, \ldots, p - 1; n = 0, 1, \ldots$, then $g = 0$. Until now
all upper bounds for the Whittaker W have been obtained by constructing functions $f \neq 0$ having the property that $f^{(n)}(e^{in\theta}) = 0; n = 0, 1, \ldots$ for some $\theta$. This theorem enables one to find the best possible value of $r$ associated with a given finite sequence of points $a_0, \ldots, a_{p-1}$ such that if $f$ is an entire function of exponential type $< \tau$ and $f^{(p\eta+k)}(a_k) = 0; k = 0, 1, \ldots, p - 1; n = 0, 1, \ldots$, then $f = 0$. Thus, any choice of the $a$'s on the unit circle gives an upper bound for $W$. So far, no choice of the $a$'s has been found which gives an upper bound lower than that already known, namely, .7378 to four decimal places. This tends to suggest that this number may perhaps be close to the true value of $W$. (Received October 29, 1965.)

630-98. T. A. VESSEY, University of Minnesota, Minneapolis, Minnesota. Some properties of oricyclic cluster sets.

An "oricyclic" cluster set $C_\Omega(f, e^{i\theta})$, for complex valued functions, $f(z)$, in the disk: $|z| < 1$, is defined at a boundary point, $e^{i\theta}$, by restricting sequences $\{z_n\}$ to lie in a collapsing sequence of disks, internally tangent to $|z| = 1$ at $e^{i\theta}$. Representation theorems for arbitrary and meromorphic functions are given in terms of the tangential-from-above, and tangential-from-below cluster sets relative to the tangent disks. A perimeter cluster set $C_p(f, e^{i\theta})$, defined in terms of the curvilinear cluster sets along the arcs forming the frontiers of the tangent disks is investigated. The difference $C_\Omega(f, e^{i\theta}) - C_p(f, e^{i\theta})$ is considered and a theorem of the Gross-Inversen type is proved. (Received October 29, 1965.)


Let $k$, $m$ and $n$ be positive integers and $g$ a non-negative integer such that $0 \leq g < k$. Let $A_1(x), A_2(x), \ldots, A_n(x)$ be $n \times n$ matrices of Lebesgue integrable functions on $a \leq x \leq b$ and let $a$ be any function with domain the set $\{1, 2, \ldots, m\}$ and range a subset of $\{1, 2, \ldots, k\}$. Then, consider the cyclicly related differential system: $Y_i = \sum_{j=1}^{m} A_j(x) Y_{g+i+a(j)}$, for all $a \leq x \leq b$, where $Y_1, Y_2, \ldots, Y_k$ are $n \times n$ matrices and the subscripts $g+i+a(j)$ are reduced modulo $k$. The general solution of this system is obtained and the exponential nature of the solutions is demonstrated. The results of W. M. Whyburn (Bull. Amer. Math. Soc. 36 (1930), 863-868) and P. Barnhard and E. J. Pellicciaro (J. Elisha Mitchell Sci. Soc. 77 (1961), 163-167) appear as special cases. The methods employed make use of the recent results of C. M. Ablow and J. L. Brenner on circulant matrices (Trans. Amer, Math. Soc. 107 (1963), 360-376). The case $m = \infty$ and other generalizations are also discussed. (Received October 29, 1965.)

630-100. BERTRAM MOND, ARL (ARM) Building, Wright-Patterson APB, Ohio. A Hilbert space generalization of Kantorovich's inequality.

Let $A$ be self-adjoint operator on a Hilbert space $H$ satisfying $mI \leq A \leq MI$, $0 < m < M$, where $I$ is the identity operator. Let $r$ and $s$ be real numbers, $r < s$, $rs \neq 0$. Theorem 1. $B = (M^s - m^s)A^r - (M^r - m^r)A^s - (M^s m^r - M^r m^s)I$ is a positive operator if $rs > 0$; while $-B$ is a positive operator if $rs < 0$. Theorem 2. Set $q = M/m$. Then $(A^s x, x)^{1/s}/(A^r x, x)^{1/r} \leq \{|x^{-1}(q^r - 1)^{1/r} - (s - r)^{-1}(q^s - q^r)(x, x)^{1/r} - (s - r)^{-1}(q^s - q^r)(x, x)^{1/s} - (1/r)}$ for all $x \in H(x \neq 0)$. [Cf. Cargo and Shisha, Bounds on ratios of...
means, J. Res. Nat. Bur. Standards 66B (1962), 169-170.] Letting \( s = 1 \) and \( r = -1 \), the above inequalities reduce to \( mM^{-1} + A \leq (m + M)I \), an inequality first given by Diaz and Metcalf [See Abstract 608-121, these Notices 11 (1964), 92] and \( (Ax,x)(A^{-1}x,x) \leq [(m + M)^2/4mM](x,x)^2 \), the well-known Kantorovich inequality. (Received October 29, 1965.)


Using additive notation for locally compact abelian groups, a topological torsion group is defined by \( (n!)x \to 0 \) for all \( x \in G \). A result of Braconnier may be paraphrased as follows: \( G \) is a topological torsion group if \( G \) is a local direct product of groups, each of which is a topological p-group (i.e., \( p^n x \to 0 \), all \( x, p \) a prime). A new characterization of connected groups is obtained, namely, \( G \) is connected if \( \hat{G} \) is topologically torsion-free (i.e., \( (n!)x \to 0 \) implies \( x = 0 \)). Mackey's classification of torsion-free divisible groups is used to show that \( G \) is torsion-free if \( \hat{G} \) has a dense subgroup which is divisible. An analysis of torsion-free divisible topological p-groups leads to a complete classification of homogeneous groups (i.e., the homeomorphic automorphisms act transitively on the nonzero elements). There are precisely four classes of homogeneous groups, and any particular homogeneous group can be specified by a locally compact field, and at most two cardinal numbers. (Received November 1, 1965.)


A Banach algebra \( \mathcal{M} \) is a convolution measure algebra if it may be represented as an algebra of measures under convolution multiplication on some topological semigroup and whenever \( \mu \in \mathcal{M} \) and \( \nu \) is absolutely continuous with respect to \( \mu \) then \( \nu \in \mathcal{M} \). If \( \mathcal{M} \) is a commutative convolution measure algebra then the maximal ideal space of \( \mathcal{M} \) has a natural representation as the set \( \hat{S} \) of all semi-characters on a topological semigroup \( S \). Information concerning the algebraic structures of \( S \) and \( \hat{S} \) should yield information about \( \mathcal{M} \). In particular, if \( \hat{S} \) is a group under pointwise multiplication then \( \hat{S} \) is the dual group \( \hat{G} \) of some locally compact abelian topological group \( G \) and \( \mathcal{M} \) may be imbedded in \( M(G) \). It seems reasonable to conjecture that, in this case, \( L_1(G) \subset \mathcal{M} \subset \vee(L_1(G)) \), where \( \vee(L_1(G)) \) is the intersection of all maximal ideals of \( M(G) \) containing \( L_1(G) \). The author has proven this to be true when \( G \) is discrete, compact, or the real line; however, the question remains open for general groups. (Received November 1, 1965.)


Lemma. Let \( M^n \) be a compact combinatorial manifold, \( P^k \) a finite polyhedron, \( n \geq 2k + 2 \), \( X \) a closed subset of \( P^k \), and \( \phi \) a fixed embedding of \( X \) into \( M^n \). Let \( \Phi \) be the set of embeddings \( f: P^k \to M^n \) such that \( f|x = \phi \) and \( f|P^k - X \) is a locally tame embedding of \( P^k - X \) into \( M^n \). Then, given \( \epsilon > 0 \), there is a \( \delta > 0 \) such that \( f, g \in \Phi \) and \( d(f,g) < \delta \) imply \( f \) and \( g \) are \( \epsilon \)-ambient-isotopic leaving \( M^n \) pointwise fixed. Let \( Q \) be an n-cell in \( S^n \), let \( E \) denote the set of points of \( \hat{Q} \) at which \( \hat{Q} \) fails to be locally flat in \( S^n \), and let \( D \) be an open-closed subset of \( E \). Theorem. There is no embed-
Corollary. If \( n \geq 6 \) (\( n \geq 8 \)) then \( D \) cannot be a Cantor set (Sierpinski curve) which is tame in both \( \hat{Q} \) and \( S^n \). (Received November 1, 1965.)

630-104. L. F. SHAMPINE, Sandia Corporation, Sandia Base, Albuquerque, New Mexico 87115. Quasilinearization and a new monotone iteration.

The method of quasilinearization provides a rapidly convergent monotone sequence for the solution of a variety of functional equations. Under suitable assumptions a new sequence can be developed from this sequence which converges from the other side of the solution. For the nonlinear boundary value problems for ordinary and partial differential equations which are treated as examples, quasilinearization and the new sequence provide quadratically convergent sequences of upper and lower bounds by a trivial change of existing routines. (Received November 1, 1965.)

630-105. B. E. RHOADES, Indiana University, Bloomington, Indiana 47405. On total inclusion for Norlund methods of summability, II.

Let \( \{p_n\} \) be a nonnegative sequence, \( p_0 > 0 \), \( p_n = p_0 + p_1 + \ldots + p_n \). The Norlund method of summability, denoted by \( (N,p_n) \), corresponds to a matrix \( A = \{a_{nk}\} \), with \( a_{nk} = p_{n-k}/P_n \) (\( k \leq n \)), and \( a_{nk} = 0 \) (\( k > n \)). Let \( x \) denote a sequence, \( A_n(x) = \sum_k a_{nk}x_k \). We say that, for two infinite matrices \( A \) and \( B \), \( A \) is totally stronger than \( B \), written \( A \text{ t.s.} B \), if \( B(x) \rightarrow L \) implies \( A(x) \rightarrow L, |L| \leq \infty. \)

Let \( C(a) \) denote the Cesaro method of order \( a \), \( (R,n,a) \) the discontinuous Riesz method of order \( a \). Then \( (C,a) = (N,p_n) \) with \( p_n = \Gamma(n+a)/\Gamma(a+1)\Gamma(n+1) \) and \( (R,n,a) = (N,q_n) \) with \( q_n = (n+1)^a - n^a \).

**Theorem 1.** \( (C,a) \text{ t.s.} (R,n,a) \) for \( 0 < a < 1. \)**

**Theorem 2.** \( (N,\cosh n^{1/2}) \text{ t.s.} (C,a) \) for \( 0 \leq a \leq \cosh 1. \)

These results answer questions raised by Debi [Bull. Calcutta Math. Soc. 47 (1955), 135-141] and answered in part by the author [ibid, 52 (1960), 123-125]. (Received November 1, 1965.)


A cosmic space is a \( T^3 \)-space which is the continuous image of a separable metric space. In [Pacific J. Math. 11 (1961), 105-125] Jack Ceder defined an \( M_3 \)-space to be a \( T^3 \)-space with a \( \sigma \)-closure preserving base and an \( M_3 \)-space to be a \( T^3 \)-space satisfying a slightly more general (or perhaps equivalent) condition. C. R. Borges has raised the questions: is every cosmic space an \( M_3 \)-space (answered negatively by the author in a paper to appear), and is a countable \( T^3 \)-space (which is necessarily a cosmic space) an \( M_3 \)-space? In this paper an example is given of a countable \( T^3 \)-space that is not an \( M_3 \)-space. Also it is proved that (1) any first countable cosmic space is a Lindelöf semi-metric space and (2) any separable first countable \( M_3 \)-space is a cosmic space. (A semi-metric space is a \( T_2 \)-space with a distance \( d \) such that (1) \( d(x,y) = d(y,x) \geq 0, \) (2) \( x \in \text{clos} M \) iff \( \inf \{d(x,y) : y \in M\} = 0 \). Note that a separable first countable \( M_3 \)-space is a Lindelöf semi-metric space but not conversely, and it is unknown whether a Lindelöf semi-metric space is a cosmic space. (Received November 1, 1965.)

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Let $S$ be a nondegenerate, locally arcwise connected, metric space which is not separated by any arc or point and which satisfies the Jordan curve theorem [R. L. Moore, Foundations of point set theory, Amer. Math. Soc. Colloq. Pub. XIII 1962]. Let $\{\mathcal{S}_n\}$ be the class of simple closed curves with associated complementary domains $I_n \subset I_{n+1}$ and $\bigcap_{n=1}^{\infty} I_n$ is at most one point. Define the equivalence relation $\sim$ on $\{\mathcal{S}_n\}$ by $\mathcal{S}_n \sim \mathcal{S}_\beta$ if and only if for every $n$ there exists an $m$ such that $I_{n,m} \subset I_n$ and vice versa. Let $[\mathcal{S}]$ denote the equivalence class of $\{\mathcal{S}_n\}$ containing $\mathcal{S}_n$ and define a base, $B$, for a topology on $[[\mathcal{S}]]$ by $E \in B$ if and only if there exists a complementary domain $I$ of a simple closed curve such that $E$ contains all $[\mathcal{S}]$ such that for some $n$, $I_n \subset I$. Theorem. $[[\mathcal{S}]]$ with the topology given by $B$ is homeomorphic to the 2-sphere. Using, the natural function available from $S$ into $[[\mathcal{S}]]$, most of the results of chapter 4 of the Foundations [cited above] can be shown to hold without axiom 5 (some need a natural modification). Also other results holding in the 2-sphere can be generalized to $S$. (Received November 1, 1965.)

Let $Y,Z,W$ be Banach spaces and $U$ the space of bilinear continuous operators $u: Y \times Z \to W$. Norms in $Y,Z,W,U$ are denoted by $|\cdot|$. Let $V$ be a Boolean ring of sets of a space $X$ and $\nu$ a non-negative finitely additive set function on $V$. Let $S(Y)$ be the set of all functions of the form: $s(x) = y_1 x_{A_1} + \ldots + y_k x_{A_k}$, where $y_i \in Y$ and $A_i \in V$ are disjoint sets. Denote by $M(Z)$ the set of all finite-additive functions $\mu: V \to Z$ such that $|\mu(A)| \leq c \nu(A)$ for all $A \in V$ and some $c$. The least such $c$ is denoted by $||\mu||$. Define $\int u(s,\mu) = u(y_1,\mu(A_1)) + \ldots + u(y_k,\mu(A))$ and $||s|| = |y_1| \nu(A) + \ldots + |y_k| \nu(A)$. A sequence $s_n$ is called basic if there exists a sequence $h \in S(Y)$ and a constant $M$ such that $s_n = h_1 + \ldots + h_n$ and $||s_n|| \leq M^2/n$ for all $n$. Let $L(Y)$ be the set of all functions $f$ such that there exists a basic sequence $s_n$ and a sequence of sets $A_n \subset Y$ such that $\nu(A_n) \to 0$ and $|f| \in X: |s_n(x) - f(x)| > \epsilon$ $\subset A_n$. Define $||f|| = \lim ||s_n||$ and $\int u(f,\mu) = \lim \int u(s_n,\mu)$. The above operators are well defined, that is their values do not depend on choice of the sequence $s_n$. Theorem. $(L(Y), ||\cdot||)$ is a semi-normed space and $\int u(f,\mu)$ is a trilinear operator from $U \times L(Y) \times M(Z)$ into $W$ such that $|\int u(f,\mu)| \leq |u| \|f\| \|\mu\|$ for all $u \in U$, $f \in L(Y)$, $\mu \in M(Z)$. Cf. W. M. Bogdanowicz, Proc. Nat. Acad. Sci. USA 53 (1965), 492-498 and 54 (1965), 351-354. (Received November 1, 1965.)

Let $x^t = f(t,x)$ be a system of $n$ almost periodic differential equations. Let $D$ be a diffeomorph of a standard $n$-disk. If $D \times [0,\infty)$ is an invariant set with respect to the flow $(1)$, then $D$ contains an almost periodic solution of $(1)$. To prove this result we use a result of Ezeilo, Contributions to differential equations 3 (1964), 337-349 and some results on the Liapunov stability of sets. As an application consider systems $(2) x^t = F(x)$ and $(3) x^t = F(x) + P(t,x)$ where $F$ is locally Lipschitz continuous in $x$ and $P$ is continuous in $(t,x)$ and almost periodic in $t$. Theorem 2. If $x = 0$
is a uniformly asymptotically stable critical point of (2) and \(|P(t,x)|\) is sufficiently small on a set \([|x| \leq r, |t| < \infty]\), then (2) has an almost periodic solution. (Received November 1, 1965.)


An involutorial algebra over a commutative ring \(\Phi\) consists of a pair \((\mathcal{A}, J)\), where \(\mathcal{A}\) is an associative algebra over \(\Phi\) (with identity), \(J\) is an involution of \(\mathcal{A}\), and \(\Phi\) is elementwise fixed by \(J\). An ideal of \((\mathcal{A}, J)\) is an ideal \(\mathcal{I}\) of \(\mathcal{A}\) for which \(J\mathcal{I} \subseteq \mathcal{I}\). The notions of subalgebra, involutorial rings, and homomorphism are defined in a similar fashion. \((\mathcal{A}, J)\) is simple if its only ideals are \((0, J)\) and \((\mathcal{A}, J)\). Let \(\gamma(\mathcal{A}, J) = \{A \in \mathcal{A} | A^J = -A\}\). Theorem. Let \((\mathcal{A}, J)\) and \((\mathcal{B}, K)\) be simple involutorial rings of characteristic not 2. Suppose that \((\mathcal{A}, J)\) contains a subring \((\mathcal{A}_0, J)\) for which the center of \(\mathcal{A}_0\) is the same as the center of \(\mathcal{A}\), and that \((\mathcal{A}_0, J)\) is a finite dimensional central simple involutorial algebra whose Lie algebra \(\gamma(\mathcal{A}_0, J)^1\) is of type \(A_l(p+1; \geq 3)\) and prime to the characteristic, \(B_{l \geq 2}, C_{l \geq 3}\) or \(D_{l > 4}\). Then (i) \(\gamma(\mathcal{A}, J)^1\) is a central simple Lie algebra, (ii) \(\gamma(\mathcal{A}, J)^1\) and \(\gamma(\mathcal{B}, K)^1\) are isomorphic iff \((\mathcal{A}, J)\) and \((\mathcal{B}, K)\) are isomorphic. (iii) Any isomorphism of \(\gamma(\mathcal{A}, J)^1\) onto \(\gamma(\mathcal{B}, K)^1\) is the restriction of an isomorphism of \((\mathcal{A}, J)\) onto \((\mathcal{B}, K)\). (Received November 1, 1965.)

630-111. A. K. BOSE, Box 1416, University of Alabama, University, Alabama. An extension of weighted average property.

Let \(\Omega\) be the class of all real-valued functions \(u\) defined in a given region \(R\) in \(E_n\), satisfying the weighted average property: (i) \(u(P)/u_0(P) = \int B(P, r) u \cdot w d\rho/\int B(P, r) u_0 w d\rho\) for each open sphere \(B(P, r)\) (with center at \(P\) and radius \(r\)) whose closure lies in \(R\), where \(w\) is a non-negative function defined in \(R\), \(u_0\) is a positive function belonging to class \(C^2(R)\) and \(d\rho\) stands for usual Lebesgue measure of \(B(P, r)\). Note that \(\Omega\) is non-empty (\(u_0\) belongs to \(\Omega\)). Theorem. If \(w\) belongs to class \(C^1(R)\), then each member of \(\Omega\) is a solution of the elliptic equation (2) \(w \Delta u + 2 \sum_{i=1}^n w_{x_i} u x_i + a \cdot u = 0\), where \(a\) is the function given by \(a u_0 = -[w u_0 + 2 \sum_{i=1}^n w_{x_i} u_{x_i}]\). Furthermore, if \(w \in C^2(R)\) is a solution of the elliptic equation (3) \(\Delta w + \beta w = a\), where \(\beta\) is some real constant, then each solution \(u\) of (2) which belongs to class \(C^2(R)\) is a member of \(\Omega\). Theorem. \(\Omega\) is a linear space over the reals and \(\Omega\) is infinite dimensional if and only if \(w\) belongs to class \(C^2(R)\) and is a solution of (3). Furthermore, if \(\Omega\) is finite dimensional, then \(1 \leq \dim \Omega \leq 2n - 1\). (Received November 1, 1965.)

630-112. C. A. HALL, Technical Services Division, Data Analysis Directorate, White Sands Missile Range, New Mexico 88002. Intrinsic functions on semi-simple algebras.

The concept of intrinsic function is defined in Elements of a theory of intrinsic functions on algebras by R. F. Rinehart [Duke Math. J. 27 (1960)]. In the same paper characterizations of intrinsic functions on the algebra of complex numbers over the real field, and the algebra of real quaternions are given. In a later paper, Intrinsic functions on matrices [Duke Math. J. 28 (1961050,)], Rinehart characterizes intrinsic functions on the algebra of \(n\) by \(n\) matrices with complex (real) elements as being essentially \(n\)-ary functions. C. G. Cullen in Intrinsic functions on matrices of real quaternions [Can. J. Math. 15 (1963)] characterizes intrinsic functions on the algebra of \(n\) by \(n\) matrices with real quaternion elements as being \(n\)-ary functions. These results are used along with those in Functions
on semi-simple algebras by Cullen and Hall [To appear in Amer. Math. Monthly] to characterize intrinsic functions on a general semi-simple algebra over the real or complex field as being essentially direct sums of \( n_1 \)-ary functions. This study was motivated by the attempt to extend the concept of an \( n \)-ary function to a general semi-simple algebra. (Received November 2, 1965.)


In this paper a new integral representation which was obtained by the author for the modified Bessel function is utilized in establishing the identities (1) \( K_{n+t/3}(\xi) = \frac{3^n n!}{2^n \Gamma(n + (4/3)/\sqrt{\xi})^{2n+1}} \times \sum_{r=0}^{n} C_{n,r}(\xi^{2r+1}/3^n \Gamma(n + \xi/3 + x))^{3r+1}/(4\xi + x) \) and (2) \( K_{n+t/3}(\xi) = \frac{3^n n!}{2^n \Gamma(n + \xi/3)/x^{3/2}} \times \sum_{r=0}^{n} C_{n,r}(\xi^{2r+1}/3^n \Gamma(n + \xi/3 + x))(3r+1)/x) \), where \( Ai(x) \) is Airy's function, \( \xi = (2/3)x^{3/2} \), and \( (\cdot)^n \) denotes the \( n \)th order derivative with respect to \( x \) of the expression enclosed by the parentheses. (Received November 1, 1965.)

630-114. STEPHEN SILVERMAN, New York University, University Heights, Bronx, New York 10453. Extensibility of outer measures.

Let \( \lambda \) be an outer measure on all subsets of a set \( X \). We ask what are the \( \sigma \)-algebras \( \mathcal{M} \) such that \( \lambda | \mathcal{M} \) (the restriction of \( \lambda \) to \( \mathcal{M} \)) is a measure. Such a \( \sigma \)-algebra \( \mathcal{M} \) will be called a \( \lambda \)-measure domain. It is well known that \( \mathcal{M}_{\lambda} \), (the Carathéodory \( \lambda \)-measurable sets) is a measure domain. We say \( \lambda \) is extensible if there exists a measure domain \( \mathcal{M} \), \( \mathcal{M} \nsubseteq \mathcal{M}_{\lambda} \), and \( \lambda \) is continuable if there exists a measure domain \( \mathcal{M} \nsubseteq \mathcal{M}_{\lambda} \). For finite outer measures extensible = continuable, and every measure domain is contained in a maximal measure domain. Also, necessary and sufficient conditions are given for continuability, and the existence of a unique maximal measure domain. The contents of unique maximal measure domains are studied. In the general case we have that any maximal measure domain contains \( \mathcal{M}_{\lambda} \). Methods are given for generating continuable and noncontinuable outer measures. (Received November 2, 1965.)

630-115. R. W. DEMING, Idaho State University, Pocatello, Idaho. The edge-path group and Čech homology groups of a generalized uniform space.

A space is a pair \((X,C)\) where \( X \) is a set and \( C \) is a filter of entourages in \( X \). (S. Lubkin, Theory of covering spaces, Trans. Amer. Math. Soc. 104 (1962), 205-238.) Let \((X,C)\) be a space with base point \( x \) and let \( u \in C \). A \textit{u edge-path} from \( x \) to \( y \in X \) is a sequence \( \{x_1, ..., x_n\} \) in \( X \) such that \( x_1 = x, x_n = y, \) and \( \{x_i\} \cap u \{x_i+1\} \neq \emptyset \) for \( i = 1, ..., n - 1 \). Our construction of the group \( E_u(X,C) \) (or, more briefly, \( E_u \)) from u edge-paths and an equivalence relation on u edge-paths is similar to the construction of \( \pi_1(X,x) \) from arcs and the homotopy relation in a topological setting. If \( C \) is directed by the relation: \( u > v \) iff \( u \subseteq v \), then \( \{E_u, 1_{uv} : u,v \in C, u \subseteq v\} \), where \( 1_{uv} : E_u \rightarrow E_v \) is induced by the identity function \( 1_X \) (nevertheless, \( 1_{uv} \) is not trivial), is an inverse system of groups. The \textit{edge-path group} \( E(X,C) \) of \((X,C)\) is the inverse limit of this system. If \((X,C)\) is a space and \( u \in C \), then \( \{u^{[x]} : x \in X\} \) is a cover of \( X \); hence there is an abstract simplicial complex associated with \( u \). The Čech homology sequence of \((X,C)\) is the inverse limit over \( C \) (directed as above) of the...
homology sequences of these complexes. Among the results that are obtained is: Theorem. Let (X,x,C) be connected. For each u ∈ C, the abelianization of E_u is isomorphic to the first homology group of the complex associated with u. (Received November 1, 1965.)


Let X be a simply connected space of constant curvature, $\mathcal{S}$ the space of closed, oriented, totally geodesic $l$-submanifolds of X and $\Phi$ a Haar measure on $\mathcal{S}$, invariant under the group of isometries of X. Suppose $0 \leq k \leq n = \dim X$, $0 \leq l \leq n$ and $k + l > n$. If T is a quasi-normal $k$-current in X, then $T \cap E$ is a quasi-normal $k + l - n$-current for almost all $E \in \mathcal{S}$. Theorem. T is normal if and only if $\int_{\mathcal{S}} M(\partial(T \cap E)) d\Phi < \infty$. (M is a norm induced on the space of currents by the metric on X; $\partial T$ is the boundary of T; T is normal if $M(T) + M(\partial T) < \infty$; quasi-normal currents are M-limits of normal currents.) For $X = R^n$, $k = n$, $l = 1$, this reduces to a statement concerning the existence of partial derivatives of a function of bounded variation which was proved independently by H. Federer and K. Krickeberg. (Received November 3, 1965.)


In this paper we shall study a difference method for the Dirichlet problem associated with the quasi-linear uniformly elliptic partial differential equations $a(x,y,u,u_x,u_y)u_{xx} + c(x,y,u,u_x,u_y)u_{yy} = g(x,y)$ and $a(x,y,u)u_{xx} + 2b(x,y,u)u_{xy} + c(x,y,u)u_{yy} + d(x,y,u)u_x + e(x,y,u)u_y - \gamma(x,y,u)u = g(x,y); u_{x...}$, denote the partial derivatives of u with respect to $x,..., (x,y)$ belongs to a bounded region $\Omega$, and a function $v(x,y)$ is given such that $u(x,y) = v(x,y)$ for $(x,y)$ on the boundary, $\partial \Omega$, of $\Omega$. We shall assume that the coefficients are sufficiently smooth functions of their arguments and that there exists constants $K_0, K'_0$ and $\mu$ such that $a(x,y,u,u_x,u_y)$ and $c(x,y,u,u_x,u_y)$ satisfy a relation of the form $K'_0 \geq a(x,y,u,u_x,u_y) \geq \mu > 0$, and $d(x,y,u), e(x,y,u)$ and $\gamma(x,y,u)$ satisfy a relation of the form $K'_0 \geq |d(x,y,u)|$: here the inequalities are to hold for all $(x,y) \in \Omega$ and for all u which satisfies the boundary data. It is then shown that certain associated difference equations have solutions by using the Banach Fixed Point Theorem. (Received November 3, 1965.)

630-118. MARVIN MARCUS and HENRYK MINC, University of California, Santa Barbara, California 93106. An inequality for the elementary symmetric functions of characteristic roots.

In a recent paper W. M. Frank obtained the following result: If $A_1,...,A_m$ are hermitian positive semidefinite n-square matrices and $\mu_1,...,\mu_m$ are arbitrary complex numbers then $|\det(\sum_{i=1}^m \mu_i A_i) | \leq |\det(\sum_{i=1}^m |\mu_i| A_i) |$. Clearly any complex n-square matrix A can be expressed (not uniquely) in the form $A = \sum_{i=1}^m \mu_i A_i$. In the present paper we generalize Frank's inequality to any elementary symmetric function of the characteristic roots. For any n-square matrix $X$ let $E_k(X)$ denote the kth elementary symmetric function of the characteristic roots of $X$. Theorem. Let $\lambda_1,...,\lambda_m$ be positive semidefinite hermitian n-square matrices and $\mu_1,...,\mu_m$ be any complex numbers. Set $A = \sum_{\nu=1}^m \mu_\nu A_\nu$ and $S = \sum_{\nu=1}^m |\mu_\nu| A_\nu$. Then $|E_k(A)| \leq E_k(S)$ for each $k = 1,...,n$. In case $\lambda_1,...,\lambda_m$ are positive definite, equality in the last inequality can occur if and only if the numbers $\mu_1,...,\mu_m$ are real nonnegative multiples of the same complex number. (Received November 3, 1965.)
Elastic-plastic torsion of a square bar.

Let \( Q \) be a square plane domain. Let \( \Psi \) be a function such that its value \( \Psi(q) \) at a point \( q \) in \( \overline{Q} \), the closure of \( Q \), is a constant multiple of the distance from \( q \) to \( \partial Q \), boundary of \( Q \). By elastic-plastic torsion we mean the following problem: to find the function \( \psi(x,y) \) such as to minimize the integral,

\[
I[u] = \int_Q \left[ (\partial u)^2 - 4\mu\theta u \right] \,dx \,dy,
\]
with \( \mu, \theta \) being positive constants. The admissible class consists of those functions which are continuous and are less than or equal to \( \Psi \) in \( \overline{Q} \) and which vanish on \( \partial Q \) and possess finite Dirichlet integrals over \( Q \). It has been found that (i) this problem has a unique smooth solution, (ii) the solution \( \psi \) depends continuously and monotonically upon the parameters \( \mu \) and \( \theta \), (iii) the elastic set \( E \) where \( \psi < \Psi \) is a simply connected region containing the diagonals of \( Q \), the plastic aset of \( Q - E \) consists of exactly 4 simply connected regions and they are separated by 4 analytic arcs contained in \( Q \). (Received November 8, 1965.)

630-120. NATHANIEL GROSSMAN, Institute for Advanced Study, Princeton, New Jersey 08540.
A polar Morse function for Riemannian manifolds of pinching greater than 1/4.

Let \( M^n \) be a smooth, simply-connected, complete Riemannian manifold of dimension \( n \geq 2 \) with sectional curvatures pinched \( > 1/4 \). Sphere Theorem. \( M^n \) is homeomorphic to a sphere. This well-known theorem is proved anew by directly constructing a smooth real-valued function \( f \) on \( M^n \) having only two singularities, both nondegenerate. The Sphere Theorem then follows from the so-called Theorem of Reeb. The construction of \( f \) involves a smooth fibration of \( M^n \) over \([0,1]\) into \((n - 1)\)-spheres, degenerating over 0 and 1 into antipodal points. This fibration is an extension of one introduced by D. Gromoll in his dissertation. Technical requirements are Toponogov's Comparison Theorem, Klingenberg's Lemma on the Injectivity Radius of the Exponential Map, and frequent use of the Inverse Function Theorem. (Received November 3, 1965.)

630-121. L. C. GROVE, Dartmouth College, Hanover, New Hampshire. Tensor products over H*-algebras.

If \( A, B, \) and \( C \) are (semi-simple)H*-algebras, with \( A \) a right \( C \)-module and \( B \) a left \( C \)-module, there is defined a tensor product \( A \otimes_C B \) that is again an H*-algebra. \( A \otimes_C B \) is shown to be isomorphic and isometric with a closed ideal in the ordinary tensor product \( A \otimes B \). In special cases the structure of \( A \otimes_C B \) is described more explicitly. For example, if \( A = L^2(G), B = L^2(H), \) and \( C = L^2(K) \), where \( G, H, \) and \( K \) are suitably related compact groups, then \( A \otimes_C B \) is isomorphic and isometric with \( L^2((G \times H)/M) \), where \( M \) is a homomorphic image of a "diagonal" subgroup of \( K \times K \). (Received November 3, 1965.)

630-122. BAYARD RANKIN, Case Institute of Technology, Cleveland, Ohio 44106. Laguerre functions with truncated domains.

We prove the following Theorem. For each non-negative integer \( n \), the real number \( x_k \) is a point where the Laguerre function, \( L_n(x) e^{-x}, 0 \leq x < \infty, \) assumes a local extremum, if and only if \( x_k \) is a solution of the equation:

\[
\int_0^\infty u L_n(x) e^{-u} du = (1 + 2n) \int_x^\infty L_n(2u) e^{-u} du.
\]
There are \( n + 1 \) such
solutions, \( x_0, x_1, \ldots, x_n \). The proof is built upon known identities that involve the Laguerre polynomials. We have found that this theorem plays a fundamental role when parametric time is replaced by quantum mechanical time in the description of the quantized, one-dimensional harmonic oscillator. [Quantized energy distributions for the harmonic oscillator, Phys. Rev. (1966) to appear.] The motivations for replacing parametric time with quantum mechanical time under fairly general conditions were presented before the Society at an earlier date [Abstract 614-62, these Notices, 11 (1964), 545] and appeared in [J. Math. Phys. 6 (1965), 1057-1071]. (Received November 4, 1965.)

630-123. ABOLGHASSEM GAFFARI, Mission Analysis Office, NASA Goddard Space Flight Center, Greenbelt, Maryland 20771. Association of calculus of variations with dynamic programming.

Let \( I[y] = \int_a^b F(x, y, y') dx \), \( y(a) = c \), \( y(b) = d \), (1) be a real functional with its associated Hamilton-Jacobi equation \( W_x + H(x, y, W_y) = 0 \), (2) subject to the boundary conditions \( W(a, c) = 0, \) \( W(b, d) = \min_y I[y] \), (4) and let \( F \) be continuously differentiable along with its partial derivatives up to the third order in a connected domain \( \Omega \) containing the two points \( A(a, c) \) and \( B(b, d) \). It is shown that the solution of (2), (3) is not uniquely determined unless (3) is modified. Equation (2) has been compared with the basic nonlinear partial differential of dynamic programming - \( W_a = \min_y \{ F(a, c, y) + y W_c \} \) (5) and some of their primary advantages in flight optimization problems are discussed and compared. Finally a derivation of (5) has been obtained. These results are the extension of the previously known results given by R. Bellman (RAND Report R-2, 1954), S. Dreyfus (J. Math. Anal. Appl. 1, (1960)), I. Gumowski (C. R. Acad. Sci. Paris 220 (1965)) and others. (Received November 4, 1965.)


Using order theoretic methods we obtain two selection theorems for multi-valued functions. The first result is a generalization of a result of Mioduszewski (Prace Mat. V (1961), 74-77) and the second theorem follows from an unpublished result of A. D. Wallace. Theorem 1. Let \( X \) be a tree, \( Y \) a compact, \( T_2 \), space and \( F : X \to Y \) a continuous, 0-dimensional function on \( X \) into \( Y \). Then for any \( x_0 \in X \), and \( y_0 \in F(x_0) \) there exists a continuous single-valued function \( f : X \to Y \) such that \( f(x) \in F(x) \) for all \( x \in X \) and \( f(x_0) = y_0 \). Theorem 2. Let \( X \) be a continuum with a continuous partial order such that each subcontinuum of \( X \) contains a zero. If \( F : Y \to X \) is a continuous continuum valued function, then there is a continuous single-valued function \( f : Y \to X \) such that \( f(x) \in F(x) \) for all \( x \in X \). A corollary to Theorem 2 is: If \( X \) has the f.p.p. for continuous single-valued functions, then \( X \) has f.p.p. for continuous continuum valued functions. (Received November 4, 1965.)


An extension theory in an additive category \( \mathcal{A} \) consists of a class \( \mathcal{E} \) of short exact sequences in \( \mathcal{A} \) satisfying certain conditions. An extension theory on \( \mathcal{A} \) produces extension functors \( \text{Ext}^n \) which enter into the usual exact sequences. Let \( \mathcal{E}^0 \) be the category of morphisms of \( \mathcal{A} \) and, for \( n \geq 1 \), let \( \mathcal{E}^n \) be the category of exact sequences of length \( n \) in \( \mathcal{A} \) which factor into sequences in \( \mathcal{E} \) and maps between such sequences. \( \mathcal{E} \) endows each \( \mathcal{E}^n \) with an extension theory. The resulting
extension functors are related by various exact sequences. For example if \( \mathcal{F} \) is abelian and \( \mathcal{S} \) consists of all exact sequences and \( f: A \rightarrow A^1, g: C \rightarrow C^1 \) are in \( \mathcal{S} \), we have an exact sequence

\[
\text{Ext}^n(f,g) \rightarrow \text{Ext}^n(A,C) \oplus \text{Ext}^n(A^1,C^1) \rightarrow \text{Ext}^{n+1}(f,g) \rightarrow.
\]

(Received November 4, 1965.)

630-126. D. L. OUTCALT, University of California, Santa Barbara, California 93106.

Simple \((n,n+1)\) - alternative rings.

The \( n \)-associator \((a_1, \ldots, a_n)\) is defined by \( (a_1, a_2) = a_1 a_2 \) and \( (a_1, \ldots, a_n) = \sum_{k=0}^{n-2} (-1)^k a_1 \cdots a_k a_{k+1} a_{k+2} \cdots a_n \). A ring is \( n \)-associative if the \( n \)-associator vanishes identically in the ring. A ring is \( n \)-alternative if \( (a_1, \ldots, a_n) = \text{sgn}(\pi)(a_{\pi(1)} \cdots a_{\pi(n)}) \) for \( \pi \) in \( S_n \) holds in the ring. A ring is \((n,n+1)\)-alternative if the ring is \( n \)-alternative and if the \((n+1)\)-associators \((a_1, a_1, a_2, \ldots, a_n)\) and \((a_1, \ldots, a_{n-1}, a_{n+1})\) vanish identically in the ring. A. H. Boers has shown that an \((n,n+1)\)-alternative ring of characteristic not 2 is \((n+1)\)-alternative and \((n+2)\)-associative. \( \text{Indag. Math., 27 (1965), 122-128} \). This paper establishes Theorem. A simple \((n,n+1)\)-alternative ring, \( n \geq 3 \), of characteristic not 2 is \( n \)-associative; there are no simple \((2,3)\)-alternative rings of characteristic not 2. (Received November 4, 1965.)


Let \( A \) be a noetherian local ring with maximal ideal \( m \). For any \( m \)-primary ideal \( q \) of \( A \) let

\[
e_0(q) = e^d(q)_{d-1} + \cdots + (-1)^{d-1} e_{d-1}(q)_{d+1} + (-1)^d e_d(q),
\]

where \( d = \dim(A) \), be the polynomial in \( n \) which, for sufficiently large \( n \), agrees with \( \text{length}(A/q^{n+1}) \). It is known that when \( A \) is a Cohen-Macaulay local ring \( e_1(q) = e_0(q) - \text{length}(A/q) \geq 0 \) (D. G. Northcott, J. London Math. Soc. 35 (1960), 209-214), and \( e_2(q) \geq 0 \) (M. Narita, Proc. Comb. Phil. Soc. 59 (1963), 269-275). A local ring \( A \) is called a complete intersection if it can be written as \( B/(Bx_1 + \cdots + Bx_s) \) where \( B \) is a regular local ring and \( x_1, \ldots, x_s \) are \( s = \dim(B) - \dim(A) \) elements from the maximal ideal \( m \) of \( B \). A complete intersection is a Cohen-Macaulay local ring. \( \text{Theorem.} \) If the sequence \( x_1, \ldots, x_s \) of leading forms in the form ring of \( B \) with respect to \( m \) is a regular sequence, then \( e_i(m) \geq 0 \) for every \( i \). \( \text{Corollary.} \) If \( A \) is the local ring of a point on a hypersurface and \( m \) is its maximal ideal, then \( e_i(m) \geq 0 \) for every \( i \). Similar results hold for modules over local rings. (Received November 4, 1965.)

630-128. PATRICK AHERN, University of California, Los Angeles, California and DONALD SARASON, University of California, Berkeley, California. Representing measures for function algebras.

Let \( A \) be a uniform algebra on a compact Hausdorff space and let \( \phi \) be a complex homomorphism of \( A \). Let \( S \) be the real linear span of the set of differences of representing measures for \( \phi \). It is assumed that \( (1) \) \( S \) has finite dimension, and \( (2) \) no nonzero element of \( S \) annihilates the logarithms of the moduli of all the invertible elements of \( A \). If \( A \) is a hypo-Dirichlet algebra (Wermer, Analytic disks in maximal ideal spaces, Amer. J. Math., Jan. 1964), then \( (1) \) and \( (2) \) are satisfied for every homomorphism of \( A \). From \( (2) \) it follows that \( \phi \) has a unique Arens-Singer measure \( m \). \( \text{Theorem 1.} \) If \( \rho \) is a representing measure for \( \phi \) then \( \rho \) is absolutely continuous with respect to \( m \) and \( d\rho/dm \) is bounded. Let \( H^2 \) be the \( L^2(dm) \) closure of \( A \), and let \( H^{20} = H^2 \cap L^{\infty}(dm) \). Then \( H^{20} \) is
a uniform algebra on the maximal ideal space of \( L^\infty(\mathbb{D}) \) and \( \phi \) induces a homomorphism \( \hat{\phi} \) on \( H^\infty \).

**Theorem 2.** Conditions (1) and (2) are satisfied by \( \hat{\phi} \). (Received November 4, 1965.)

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630-129. D. J. McGuinness, University of Maryland, College Park, Maryland. A nonhomogeneous differential equation with a second order turning point.

A class of linear second order ordinary differential equations of the form \( \frac{d^2u}{dt^2} = \lambda^2 \mathbb{P}(t) + Q(t, \lambda)u = \lambda^2 F(t, \lambda) \) is considered. It is assumed that \( \lambda \) is a large complex parameter, \( t \) belongs to some finite closed interval \( I \) and \( \mathbb{P}(t) \) is a real valued function having a single second order zero in the interior of \( I \). A formal solution is constructed in terms of solutions of

\[
\frac{d^2T_0}{dz^2} + [k - z^2]T_0 = -1 \quad \text{and} \quad \frac{d^2T}{dz^2} + [L - z^2]T = -z
\]

where \( k \) and \( L \) are functions of \( \lambda \) which are determined in the process. Properties of certain solutions of these simpler equations are investigated and they are used to show that, in suitable sectors of the \( \lambda \)-plane, the formal solution leads to an asymptotic expansion of some solution of the original equation. (Received November 4, 1965.)

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630-130. E. D. Conway, University of California, San Diego, California and J. A. Smoller, University of Michigan, Ann Arbor, Michigan. Global solution of the Cauchy problem for quasi-linear first order equations in several space variables.

Consider the problem (*) \( u_t + \sum_{j=1}^n (\partial/\partial x_j) \mathcal{F}_j(u) = 0, \ u(0,x) = u_0(x) \) in the region \( t \leq 0 \), where \( u = u(t,x), \ x = (x_1, \ldots, x_n). \) Let \( \mathbb{F} \) consist of all bounded functions having locally bounded (Tonelli-Cesari) variation in \( \mathbb{R}^n \), i.e., bounded functions whose gradients are measures in \( \mathbb{R}^n \) having locally finite total variation. Using finite differences we obtain: Theorem 1. If \( f_j \in C^1 \) and \( u_0 \in \mathbb{F} \) then there exists a weak solution of (*) having locally bounded \( T - C \) variation in \( t \geq 0 \) and is in \( \mathbb{F} \) for each fixed \( t > 0 \). Theorem 2. If \( u_0 \) is monotonic in one or more variables then for each fixed \( t \), \( u \) is monotonic in the same variables and the same sense. Theorem 3. If \( f_j \in C^2 \) and \( f''_j = a f''_j > 0, \ a_j > 0 \) then in the class of functions satisfying Oleinik's entropy condition in each space variable the weak solution of (*) is unique. These results extend results of O. A. Oleinik (Uspehi Mat. Nauk, 12 (1957), 3-72; Amer. Math. Soc. Transl., 26 (1963), 95-172). (Received November 1, 1965.)

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We give a formal asymptotic series which satisfies the reduced wave equation \( \Delta u + k^2 u = 0 \) up to arbitrary order in \( k^{-1} \) in the exterior of a smooth convex analytic body \( B \). The solution \( u \) vanishes to arbitrary order in \( k^{-1} \) on the boundary of \( B \), and \( u \) can be written in the form \( u = u_i + u_a \), where \( u_i \) is a prescribed incident field, and \( u_a \) satisfies the outgoing radiation condition. In the region of direct illumination, our solution yields the sum of the incident and reflected fields, as predicted by geometrical optics. In the region of deep shadow, we obtain the diffracted waves predicted by J. B. Keller's geometrical theory of diffraction. Near the shadow boundary our solution has a complicated behavior, analogous to the behavior of the exact solution for a circular cylinder or a sphere. Our results can be extended to apply to general differential equations and boundary conditions. (Received November 4, 1965.)

Shanks has shown that the normalized ballistic equations \( \ddot{s} = -s \), \( s(0) = 0 \), \( \dot{s}(0) = 1 \), remains invariant under the group of transformations \( s = -t, \ t = -\ddot{s}, \ n = 3 - \ddot{n} \). Here the general class of equations, \( \ddot{s} = F(s, s, t) \) is determined for which the same duality holds. This leads to a functional equation which among others is solved by means of generalizations of the notions of even and odd. (Received November 5, 1965.)


There are several essentially equivalent definitions of the height of one-dimensional subspaces of a finite-dimensional vector space over an algebraic number field. Using Grassmann-coordinates one can extend these definitions to subspaces of arbitrary dimension. Now let \( 0 < d < n, 0 < e < n \). Let \( A \) be a subspace of dimension \( d \) of euclidean \( E_n \). The author wants to find subspaces \( B \) of dimension \( e \) which are defined over a fixed number field, whose height is bounded by some constant \( h \), and which are "close" to \( A \). Most questions of classical diophantine approximations are special cases of this problem. If \( d = 1 \) or \( e = 1 \), the closeness of \( A \) and \( B \) can be measured by the "angle" between \( A \) and \( B \), and the problem can be solved. In general, there are several parameters measuring the closeness of \( A \) and \( B \), and a partial solution is obtained. (Received November 5, 1965.)


In this paper we continue the study of the replacement of continuity by \( \delta \)-continuity in the theory of AR's (see Abstract 629-17, these Notices, 12 (1965), 810). We obtain a new class of spaces, called strong-proximate absolute retracts (SPR). Some but not all of the important properties of AR's are extended to these spaces. Following are some definitions and sample theorems. Definition. If \( (M, \rho) \) is a metric space, \( X \subseteq M \) is called a strong proximate retract of \( M \) (abbreviated SPR) if for each \( \epsilon > 0 \), there exists a \( \delta > 0 \) such that for any \( \mu > 0 \) we can find a \( \mu \)-continuous mapping \( r: M \rightarrow X \) with the property that \( x \in M \) and \( \rho(x, X) \leq \delta \) implies \( \rho(x, r(x)) < \epsilon \). Definition. A metric space \( X \) is called a strong-proximate absolute retract (SPAR) provided it is compact and for every topological image \( Y \) of \( X \) which is a closed subset of a compact metric space \( M \), it is true that \( Y \) is a SPR of \( M \). Theorem. An AR is a SPR and a SPR is a WAPR. The converses of the above are false. Theorem. A space \( X \) is a SPR if and only if it is homeomorphic to a closed SPR of the Hilbert parallelogotope. Theorem. If \( X \) is a SPR then \( X \) has the p.f.p.p., and hence it has the f.p.p. Theorem. Let \( \{X_n\} \) be a sequence of SPR's each of diameter at most one. Then the product \( X_{n=1}^{\infty}X_n \) is a SPR. (Received November 5, 1965.)
Let $A$ be a finite dimensional associative algebra over a field $F$. A semigroup $S$ is a translate of $A$ if there are an extension $B$ of $A$ and an epimorphism $\phi: B \to F$ such that $A = \phi^{-1}(0)$ and $S = \phi^{-1}(1)$. Any such semigroup $S$ has a minimal ideal $K$ which is a union of isomorphic groups. After Stefan Schwarz the radical of $S$ is defined to be the union of all ideals $I$ of $S$ such that some power $I^n$ of $I$ lies in $K$. $A$ is said to be weakly semisimple if there is a translate $S$ of $A$ for which $R(S) = K$. Let $N$ denote the radical of $A$. **Lemma.** $R(S)$ is a translate of $N$. **Theorem.** $A$ is weakly semisimple iff $A = eAe \oplus N$ (vector space direct sum) for some (hence every) principal idempotent $e$ in $A$. This result yields a simple characterization of the algebras of class $Q$ investigated by R. M. Thrall (Canad. J. Math. 7 (1955), 382-390). (Received November 5, 1965.)

A strictly convex metric $D$ is a metric which has the additional property that for each pair of points $s, y$ there is a unique point $u$ such that $D(x, u) = D(u, y) = D(x, y)/2$. A topological space $S$ is SC-metrizable if it is possible to define a strictly convex metric on $S$ which preserves the topology of $S$. In the plane, the collection of SC-metrizable, compact continua is precisely the collection of all locally connected and point-like continua. Every dendrite is SC-metrizable. Every SC-metrizable continuum is unicoherent. In general, if a compact continuum $M$ can be expressed as the union of a discrete set and a countable collection of SC-metrizable continua $\{M_i\}$ such that for each integer $n$ the continuum $M_n$ intersects the union of those continua of the collection of index less than $n$ in a single point, which separates $M$, then $M$ is SC-metrizable. (Received November 8, 1965.)

Let $M_n(K)$ denote the algebra of $n$-square matrices over field $K$. We examine the following questions. I. Given exactly one of $A, B, C \in M_n(K)$ with $C \neq 0$, under what circumstances do the other two exist in $M_n(K)$ such that $C = AB - BA$, $CA = AC$? II. Given exactly one of $A, B, C \in M_n(K)$ with $C \neq 0$, under what circumstances do the other two exist in $M_n(K)$ such that $C = AB - BA$, $CA = AC$, $CB = BC$? III. Given nonsingular $C \in M_n(K)$, under what circumstances do $A, B$ exist in $M_n(K)$ with prescribed nonzero determinant such that $C = ABA^{-1}B^{-1}$? IV. If in III, $C$ commutes with both $A$ and $B$, what can be said about $C$? Full answers are obtained for II, and nearly full answers for I, III. Partial results are obtained for IV (with research still in progress). Our results extend results due to McCoy, Thompson, Sinha, and others. In II, when $C$ is the given matrix, it turns out that $B$ can usually be chosen to satisfy the same NASC as $C$, and sometimes this also holds for $A$, so that one can iterate to obtain elaborate chains of commutators. A similar situation holds in I. (Received November 8, 1965.)
Let \((X, S, \mu)\) be a nonatomic measure space with finite subspace property. If \(\Phi\) is a nonzero Young function and \(L_\Phi(X, S, \mu)\) is the associated Orlicz space the following theorems are obtained.

**Theorem 1.** If \(\mu(X) = \infty\), \(L_\Phi(X, S, \mu)\) is uniformly non-\(1 \Leftrightarrow 1\) if and only if there exists a \(\delta > 1\) such that for every \(n\)-set of nonnegative reals \((u_1, u_2, \ldots, u_n)\),

\[
\Phi(u_1 + u_2 + \ldots + u_n/n) \leq Z^{n-1}/n \sum_{i=1}^{n} \Phi(u_i)
\]

for some choice of signs. If \(0 < \mu(X) < \infty\) then \(L_\Phi(X, S, \mu)\) is uniformly non-\(1 \Leftrightarrow 1\) if and only if there exist two positive real numbers \(\delta, K > 1\), such that with \((u_1, 1 \leq i \leq n)\) as above, it is true that \(\sum_{i=1}^{n} \Phi(u_i) \leq L\) implies \(\Phi((u_1 + u_2 + \ldots + u_n/n)\delta) \leq Z^{n-1}/n \sum_{i=1}^{n} \Phi(u_i)\) for some choice of signs. **Theorem 2.** Every uniformly non-\(1 \Leftrightarrow 1\) Orlicz space \(L_\Phi(X, S, \mu)\) is reflexive. Theorem 2 solves the restriction of a conjecture of R. C. James to this class of Orlicz spaces in the affirmative. (Received November 8, 1965.)

**Height of uniformities.**

If \(\mathcal{U}\) and \(\mathcal{V}\) are two uniformities on a set, then \(\mathcal{U}\) is said to be less than or equal in height \((\leq h)\) to \(\mathcal{V}\) if, for \(U \in \mathcal{U}\), there exists \(V \in \mathcal{V}\) and a finite covet \(A_1, \ldots, A_n\) of \(X\) such that

\[
\bigcup_{i=1}^{n} (A_i \times A_i) \subset U \cup V^c.
\]

Then \(\leq h\) is a quasi-order and the resulting equivalence classes have order properties which are in some ways dual to those for proximity. **Theorem:** If \(\mathcal{U}\) and \(\mathcal{V}\) are in the same proximity class \((\mathcal{C}(P))\), and are related in height but not in the usual ordering, then \(\mathcal{U} \neq P \mathcal{U} \vee \mathcal{V}\). Other results and examples concerning the relationship between proximity and height are discussed. (Received November 8, 1965.)

Let \(T\) be a closed linear operator with domain in Banach space \(X\) and range in Banach space \(Y\). The following number \(\gamma(T) = \inf_{x \in D(T)} [\|Tx\|/d(x, N(T))]\), where \(d(x, N(T))\) is the distance from \(x\) to the kernel of \(T\), plays a fundamental role in Kato's study of bounded and \(T\)-bounded perturbations of \(T\). (See J. d'Analyse (1958), 273-322). We determine \(\gamma(T)\) exactly for \(T\) the maximal operator in \(\mathcal{L}_p(I)\) corresponding to any constant coefficient differential expression on an arbitrary unbounded interval \(I\). For \(T\) corresponding to very general differential expressions in \(\mathcal{L}_p(I)\), \(1 \leq p \leq \infty\), I bounded, an estimate for \(\gamma(T)\) is given. Using these estimates we obtain a number of classical existence theorems for operators of the form \(T + B\), where \(B\) is \(T\)-bounded; e.g., \(B\) is an integro-differential operator. In addition, the index of \(T + B\) is determined. (Received November 8, 1965.)

Let \(f\) be a real-valued function defined on \((a, b)\) such that \(n + 1\) derivatives of \(f\) exist on \((a, b)\) and the \(n + 1\)st has no zeros on \((a, b)\). Then if \(P\) is a polynomial of degree \(n\), \(f(x) = P(x)\) for at most
n + 1 values of x in (a,b). This fact is used to show that if h is continuous, real-valued and has three derivatives on [0,1], such derivatives positive on (0,1], h(0) = 0, and h(1) = 1, then 3 unique \( \xi \in (0,1) \) such that \( h'(\xi) = 1 \) and \( \xi > 1/2 \). Several natural generalizations are possible. (Received November 8, 1965.)


For definitions and notation, see Partially ordered algebraic systems, L. Fuchs, Pergamon Press, Oxford, 1963, and also the preceding abstract. (1) There exists a solvable \( O^* \)-group H such that \( FA_1 \) is a proper absolutely convex subgroup of H. Hence there exists an \( O \)-group other than \( FA_1 \) that does not admit a dense ordering. With the help of (1), one obtains the follow ing: (2) If G is an \( O \)-group and if G admits a linear ordering of type \( (\omega^* + 1 + \omega)_{\theta} \), where \( \theta = 1 \) or \( \theta = 2 \), then \( G = FA_1 \) or \( G = FA_2 \); on the other hand, for every ordinal \( \theta \geq 3 \), there exists a non-Abelian \( O^* \)-group \( G_{\theta} \) that admits an ordering of type \( (\omega^* + 1 + \omega)_{\theta} \). (Received November 8, 1965.)

630-143. R. B. SHER, University of Utah, Salt Lake City, Utah 84112. Toroidal decompositions of \( E^3 \).

Let \( \{T_i\}_1 \leq i \leq m \), be a linked chain consisting of \( m \geq 2 \) solid tori in the interior of a solid torus \( T_0 \) and let \( w(\{T_i\}, T_0) \geq 0 \) denote the winding number of this chain in \( T_0 \). (Definitions are not included here for "linked chain" and "winding number", but such definitions are suggested by the two examples cited below.) A linked chain \( \{T_i\} \) is defined to be simple in \( T_0 \) if there exist disjoint meridional disks \( D_1 \) and \( D_2 \) of \( T_0 \) such that for each \( i \) (1) either \( T_i \cap D_1 = \emptyset \) or \( T_i \cap D_2 = \emptyset \), and (2) if \( T_i \cap D_j \neq \emptyset \), \( T_i \cap D_j \) consists of a pair of disjoint meridional disks of \( T_j \). Let \( T_0 \) be a solid torus in \( E^3 \), \( \{T_i\} \) a linked chain of \( m \) solid tori in \( T_0 \) with \( w(\{T_i\}, T_0) = n \), \( \{T_{ij}\} \) a linked chain of \( m \) solid tori in \( T_i \) with \( w(\{T_{ij}\}, T_i) = n \), etc. Denote by \( G \) the upper semicontinuous decomposition of \( E^3 \) whose elements are components of \( \Lambda = T_0 \cdot T_1 \cdot T_2 \cdot \ldots \cdot T_m \), and points of \( E^3 - \Lambda \), and denote by \( E^3 / G \) the associated decomposition space. If the above conditions are satisfied, \( E^3 / G \) is said to be an \((m,n)\)-space. Bing has presented examples of a \((2,1)\)-space (Ann. of Math. 56 (1952), 354-362) and a \((2,2)\)-space (Fund. Math. 50 (1962), 431-453). Theorem 1. If \( m < 2n \), then \( E^3 / G \cong E^3 \). Theorem 2. If \( \{T_{ij}, \ldots, k\} \) is simple in \( T_{ij}, \ldots, k \), then \( E^3 / G \cong E^3 \). Examples are given to show that if \( m \geq 2n \), there exist \((m,n)\)-spaces \( S_1 \) and \( S_2 \) such that \( S_1 \approx E^3 \neq S_2 \). (Received November 8, 1965.)

630-144. D. R. BELDIN, Washington University, St. Louis, Missouri 63130. Some analysis on compact groups.

Let \( G \) be a compact, connected group and \( \Omega(G) \) the set of all equivalence classes of strongly continuous, irreducible, unitary representations of \( G \). For a finite, complex, central, regular Borel measure \( \mu \) on \( G \) and \( S \in \Omega(G) \) let \( \hat{S} = \int_G \text{trace} \, S \, d\mu(x) \). Let \( F_G^0 \) be the algebra under pointwise operations of all complex-valued, continuous, central functions \( f \) on \( G \) such that \( \|f\| = \sum_{S \in \Omega(G)} \dim S |\hat{S}| \) is finite. The algebra \( F_G^0 \) is a commutative Banach algebra under the norm \( \| \| \). Theorem 1. If \( \sigma \) is any complex-valued homomorphism of \( F_G^0 \), then there is an \( x \) in \( G \) such that \( \sigma(f) = f(x) \) for all \( f \).
A concept of positive definiteness is defined for complex-valued functions on $\mathbb{C}^1$. A theorem on $G$. The map $\mu \rightarrow \hat{\mu}$ sets up a one-one correspondence between the set of all finite, complex, central, regular Borel measures on $G$ and the set of positive definite functions on $\Omega$ satisfying the condition (i). There is a positive $M$ such that $|\phi(S)| \leq M \dim S$ for all $S$ in $\Omega$. (Received November 8, 1965.)

630-145. T. J. REED, University of Minnesota, Minneapolis, Minnesota 55414. On quasi-conformal mappings with given boundary values.

A continuous strictly increasing function $\mu$ mapping the real line onto itself is called $\rho$-quasisymmetric, $1 \leq \rho < \infty$ if $1/\rho \leq (\mu(x+t) - \mu(x))/(\mu(x) - \mu(x-t)) \leq \rho$ for all $t \neq 0$ and for all $x$. Let $\inf \{K \mid$ there exists a $K$-quasiconformal extension of $\mu$ to the upper half plane} and $\Psi(\rho) = \sup \{K(\mu) \mid \mu$ is $\rho$-quasisymmetric}. Theorem. $\Psi(\rho) = O(\rho)$, $\rho \rightarrow \infty$ and this is sharp with respect to order. This improves an estimate of Beurling and Ahlfors (Act. Math. 96 (1956), 125-142). (Received November 8, 1965.)

630-146. F. D. WILLIAMS, New Mexico State University, University Park, New Mexico. A necessary condition that an $H$-space be homotopy-Abelian.

Let $X$ be a homotopy-associative $H$-space. Let $(SX)_2$ denote the (James) two-fold reduced product of the suspension of $X$. Theorem. If $X$ is homotopy-Abelian, then there is a quasi-fibration $p: E \rightarrow (SX)_2$ with fiber $X$, such that: (a) $X$ acts on $E$ in such a way that the action restricted to the fiber coincides with the multiplication of $X$, and (b) the inclusion of $X$ in $E$ as fiber is null-homotopic. (The converse to this theorem is true without the assumption of homotopy-associativity on $X$, and is easily proven.) The existence of such a q.f. may be deduced from work of Stasheff (Proc. Cambridge Philos. Soc. 17 (1961), 734-745), but this derivation is rather round-about, depending on the geometry of the projective plane of $X$. Our proof, which does not depend on this previous work (but is certainly in the same spirit), is accomplished by a direct construction, using the multiplication and the associating and commuting homotopies for $X$. One virtue of such an explicit description for $p$ and $E$, in addition to clarifying the proof, is in facilitating the application of the following Corollary: If there exists a q.f., $p$, as above, then there is a map $j: X \rightarrow \Omega (SX)_2$ such that $\Omega p + j: \Omega E \times X \rightarrow \Omega (SX)_2$ is a homotopy equivalence. (Received November 8, 1965.)


Definition. Let $p$ be an interior point of an oriented arc $k$ in $S^3$. $p$ is a singularity of order 1 of $k$ if there exists a neighborhood of $p$ containing no wild points of $k$ except for $p$. Let $a$ be an ordinal ($a < c$). $p$ is a singularity of order $a$ of $k$ if there are arbitrarily small neighborhoods of $p$ containing singularities of $k$ of all orders less than $a$ and, with the exception of $p$, containing no other kind of wild point of $k$. Previously, algebraic invariants of singularities of order 1 were constructed (Abstract 619-43, this Noticei 12 (1965), 67). Algebraic invariants of singularities of every countable order are not constructed. (Received November 8, 1965.)
630-148. R. B. BENNETT, Knox College, Galesburg, Illinois 61401. **Selfreproductive sets.**

Define a metric set $M$ to be selfreproductive if and only if there is a positive number $\epsilon$ such that for each $\epsilon$-map $f$ (i.e., the diameters of the inverse sets of $f$ are not more than $\epsilon$) with domain $M$, $f(M)$ contains a subset homeomorphic to $M$. This is a topological property. It is easy to see that a 0-dimensional compact set is selfreproductive if and only if finite. **Theorem 1.** A 1-dimensional continuum is selfreproductive if and only if homeomorphic to a polyhedron with no points of order higher than 3. **Theorem 2.** A $d$-dimensional set is selfreproductive if and only if the following three conditions are satisfied. (1) Each component of $M$ is selfreproductive. (2) All but finitely many of the components of $M$ are arcs or points. (3) Each point of $M$ which is not an endpoint of the component of $M$ containing it is an interior point of that component. **Theorem 3.** There is no selfreproductive compact set which is 2-dimensional or 3-dimensional. The question of whether there are any $n$-dimensional selfreproductive sets for $n > 3$ is open, but there are good reasons to suspect that there are none. (Received November 8, 1965.)

630-149. E. W. CHAMBERLAIN, University of Vermont, Burlington, Vermont. **The univalence of functions asymptotic to non-constant logarithmic monomials.**

A logarithmic monomial is a function $M(z) = cz^m \log(z) \log_2(z) \log_3(z) \ldots (\log_\alpha(z))^m$ where $c$ is a complex number, $m_j$ real, and $\log_j(z) = \log(\log_{j-1}(z))$. For $-\pi < a < b \leq +\pi$, $F(a, b)$ is a system of neighborhoods of $\infty$ in the complex plane, each roughly resembling a sector $\{ z : a < \arg(z - z_0) < b \}$. We say $E(z) \to 0$ rapidly enough for $M$ over $F(a, b)$ if, for each $\epsilon > 0$, $|E|$ and $|M/M'|^\epsilon$ are $\leq \epsilon$ throughout some member of $F(a, b)$. **Theorem.** Let $E \to 0$ rapidly enough for $M$ over $F(a, b)$. Let $(a_1, b_1)$ $\subset (a, b)$ and $|m_0(b_1 - a_1)| \leq 2\pi$. Then $M(1 + E)$ is univalent in some member of $F(a_1, b_1)$. This result (to appear in Proc. Amer. Math. Soc.) justifies certain changes of independent variable in the asymptotic theory of ordinary differential equations. (Received November 15, 1965.)

630-150. G. F. CLEMENTS, University of Colorado, Boulder, Colorado 80304. **On a min-max problem of Leo Moser.**

With $A = (a_0, a_1, \ldots, a_n)$ a point in Euclidean $(n + 1)$-space, let $p(A, x) = \sum_{j=0}^n a_j x_j$, $q(A, x) = p^2(A, x) = \sum_{j=0}^n c_j(A) x_j$ and let $M(A) = \max_{j=0, 1, \ldots, 2n} c_j(A)$. Moser [Report of the institute in the theory of numbers, University of Colorado, 1959; p. 342] asks for the minimum of $M$ subject to the conditions $a_j \geq 0$, $j = 0, 1, \ldots, n$ and $\sum_{j=0}^n a_j = 1$. We conjecture that $\overline{X} = (\overline{a}_0, \overline{a}_1, \ldots, \overline{a}_n)$ with $\overline{a}_j = C_{-1/2, j}^{-1}(-1)^j$ is an absolute minimum point and prove that it is a relative minimum point. The proof involves an examination of the directional derivatives of the $c_j(A)$ at $\overline{X}$. For related results see Abstract 587-43 of Moser and Pounder, these Notices 8 (1961), 581. (Received November 9, 1965.)

630-151. E. C. JOHNSEN, University of California, Santa Barbara, California 93106. **Special Abelian group difference sets.**

If $G$ is a finite abelian group of $v$ elements and $D = \{d_j\}$ is a $k$-subset of $G$ such that in the set of differences $\{d_j d_k \}^{-1}$ each element different from the identity element in $G$ appears exactly $\lambda$ times,
where \(0 < \lambda < k < v - 1\), then we say that \((G, D)\) is a \(v, k, \lambda\) \textit{abelian group difference set} (AGDS).

A multiplier of \((G, D)\) is an automorphism \(\phi\) of \(G\) under which \(D^\phi = Da\) for some \(a \in G\). The main problem concerning AGDS's is their existence. For special classes of AGDS's we can analyze the equations satisfied by the group characters of a homomorphic image of \(G\) so as to handle a certain factorization of \(k - \lambda\) in the relevant cyclotomic number field. This factorization is a key to obtaining further information about these AGDS's. As a result, we obtain restrictions on the values of \(v, k,\) and \(\lambda\) and on the structure of \(G\) because of number theoretic and algebraic structural conditions. Two special classes for which this has been done are (1) the \((G, D)\)'s with the inverse multiplier \(\iota: g \to g^{-1}\), \(g \in G\), and (2) the \((G, D)\)'s which are skew-Hadamard, i.e., which satisfy \(g \in D\) if and only if \(g^{-1} \in D\), \(g \in G\). We discuss some constructions of these AGDS's. Further classes of AGDS's are under investigation. (Received November 3, 1965.)

630-152. R. S. FREEMAN, University of Maryland, College Park, Maryland 20742.

On the resolvent set and spectrum of homogeneous elliptic differential operators with constant coefficients.

Let \(A(\cdot)\) be a homogeneous elliptic polynomial of degree \(2m\) in \(n\) variables and let \(A\) be the associated homogeneous elliptic differential operator with constant coefficients. Let \(B_j, 0 \leq j \leq m - 1\), be homogeneous differential operators with constant coefficients and orders \(m_j < 2m\). Suppose \(\lambda \in \mathbb{C}\) and \(A(\xi) - \lambda \neq 0\) for \(\xi \in \mathbb{R}^n\). Write \(\xi = (\xi', \tau)\) with \(\xi' \in \mathbb{R}^{n-1}\) and \(\tau \in \mathbb{R}\) and for fixed \(\xi\) let \(\tau_1, \ldots, \tau_m\) be the complex roots of \(A(\xi, \tau) - \lambda = 0\) as a polynomial in \(\tau\). Let \(\tau'_1, \ldots, \tau'_m\) be the \(m\) roots with positive imaginary part and \(A_\lambda^+ (\xi, \tau) = \prod_{j=1}^m (\tau - \tau'_j)\). Let \(\Omega\) be the half space \(x_n > 0\) and \(\Gamma\) its boundary. Then \textbf{Theorem I}. The map \(u \to ((A - \lambda)u, B_0u, \ldots, B_{m-1}u)\) is a topological isomorphism of \(\mathbb{H}^{2m}(\Omega)\) onto \(\mathbb{H}^0(\Omega) \times \prod_{j=0}^{m-1} \mathbb{H}^{2m-m_j-1/2}(\Gamma)\) if and only if \(A(\xi) - \lambda \neq 0\) for \(\xi \in \mathbb{R}^n\) and the \([B_j]\) are linearly independent modulo \(A^+\) (so that the basic a priori estimates are valid). Then the map \(u \to ((A - \lambda)u, B_0u, \ldots, B_{m-1}u)\) is a topological isomorphism of \(\mathbb{H}^{2m}(\Omega)\) onto \(\mathbb{H}^0(\Omega) \times \prod_{j=0}^{m-1} \mathbb{H}^{2m-m_j-1/2}(\Gamma)\) if and only if \(A(\xi) - \lambda \neq 0\) for \(\xi \in \mathbb{R}^n\). (Received November 9, 1965.)

630-153. L. R. BRAGG, Case Institute of Technology, Cleveland, Ohio, 44106.

The radial heat equation and Laplace transforms.

Let \(\mu\) be real, \(\Delta_\mu = D_x^2 + [\mu - 1)/2]D_x\), and \(S_\mu(r, t) = (4\pi t)^{-1/2} e^{-r^2/4t}\). Let \(u(r, t)\) denote a solution of (*) \(u_t(r, t) = \mu u, u(r, t) = (4\pi t)^{-1/2} e^{-r^2/4t}\). Let \(u(r, t)\) be an entire function of growth \((1, \sigma)\) in \(r^2, \sigma > 0\). Then we have: \textbf{Theorem 1}. Let \(T_\mu(p, t) = \int_0^\infty \exp((1/4t - 1/p))^{-1} x^{H/2 - 1}\phi(x/2)dx\). Then the solution of (*) subject to \(u(r, 0) = \phi(r^2)\) is given by \(u(r, t) = \pi^{H/2} S_\mu(r, t)(r^2/16t^2)^{1/2} u - 1/2 1_{p - H/2} T_\mu(p, t))\), \(0 < t < 1/(4\sigma)\), with the variable in the inverse Laplace transform replaced by \(r^2/16t^2\). \textbf{Theorem 2}. Let \(\tilde{T}_\mu(p, t) = \int_0^\infty \exp(t + 1/p))^{-1} x^{H/2 - 1}\phi(x/2)dx\). Then if \(t \geq \sigma > 0\), a solution of (*) is given by \(u(r, t) = (r^2 - \mu/4\pi H/2) 1_{p - H/2} T_\mu(p, t))\) with the variable in the inverse Laplace transform replaced by \(r^2/4\). These integral representations replace ones involving Bessel functions and are meaningful even if \(\phi\) is not entire. They readily permit the determination of the behavior of solution functions of (*) for \(\phi\) having singularities or being a certain type of generalized function. (Received November 10, 1965.)

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Let $W$ be a locally compact Hausdorff space which is connected and locally connected but not compact. Let $\mathcal{S}$ be a class of real-valued continuous functions with open domains in $W$ such that for each open set $\Omega \subseteq W$ the set $\mathcal{S}_\Omega$ consisting of all functions in $\mathcal{S}$ with domains equal to $\Omega$ is a real vector space. Assume that a function is in $\mathcal{S}$ iff it is locally in $\mathcal{S}$ and that there is a base $\mathcal{D}$ for the topology of $W$ with the following property: If $\omega$ is a relatively compact set in $\mathcal{D}$, then there is a function $h_1 \in \mathcal{S}_\omega$ with $\inf_{x \in \omega} h_1(x) > 0$ and for any continuous $f$ on $\partial \omega$ there is a continuous $h$ on $\omega$ such that $h|\partial \omega = f$ and $h|\omega \in \mathcal{S}$. The Harnack principle is valid for $\mathcal{S}$ if for any region $\Omega \subseteq W$ and any increasing sequence $\{h_n\}$ in $\mathcal{S}_\Omega$ either $\lim_{n \to \infty} h_n$ is identically $\infty$ or $\lim_{n \to \infty} h_n \in \mathcal{S}$. The Harnack inequality is valid for $\mathcal{S}$ if for any region $\Omega$ and any compact subset $K \subset \Omega$ there is a constant $M \geq 1$ such that for every nonnegative $h \in \mathcal{S}_\Omega$ and every pair of points $x_1$ and $x_2$ in $K$ the relation $(1/M) \cdot h(x_1) \leq h(x_2) \leq M \cdot h(x_1)$ holds, and $M$ may be taken arbitrarily close to $1$ for "small enough" $K$. G. Mokobodski has established the equivalence of Harnack's principle and Harnack's inequality for the case in which the topology of $W$ has a countable base (not published). A proof is given showing that this restriction is unnecessary. (Received November 10, 1965.)

Eigenfunction expansions and scattering theory for the wave equation.

Let $\{U(t)\}$ denote the unitary group associated with the wave equation and zero Dirichlet boundary condition in the spatial domain $\Omega$ exterior to a $C^2$, compact surface $\Gamma$ in $\mathbb{R}^n (n \geq 2)$. It is shown that certain pairs of classical distorted plane waves form two complete sets of generalized eigenfunctions of $\{U(t)\}$. As a preliminary to this proof, it is shown that the distorted plane waves form two complete sets of generalized eigenfunctions of the self-adjoint operator in $L^2(\Omega)$ given by $-\Delta$ acting on functions which are zero at $\Gamma$. The eigenfunction expansions for $\{U(t)\}$ are shown to be closely related to the associated wave operators. This relationship establishes the existence of the scattering operator, which then is given explicitly in terms of the "transmission coefficient" occurring in the asymptotic expansion of the diffracted plane waves. This paper extends to arbitrary $n \geq 2$ results obtained through different methods by Lax and Phillips [Bull. Amer. Math. Soc. 70 (1964), 130-142] for the case of odd space dimension $n$. (Received November 10, 1965.)

Characteristic roots of M-matrices.

A square matrix $A$ is called an $M$-matrix if it has the form $kI - B$, where $B$ is a nonnegative matrix, $I$ is the identity matrix, and $k > p(B)$, the Perron maximal characteristic root of $B$. Estimates for $p(B)$ may be obtained by studying the characteristic roots of $M$-matrices. In [Abstract 622-17, these Notices 12 (1965), 330] this was done by generalizing a result of Ky Fan on $M$-matrices, [Quart. J. Math. (2) 11 (1960), 43-49]. Here a different generalization of Fan's result leads to further information about $p(B)$. (Received November 12, 1965.)
Simultaneous approximation of a function and its derivatives.

For \( r \) and \( n \) natural numbers \( n \geq r - 1 \), and \( f \in C^{r-1,1} \), let \( \pi_n^r \) be the class of polynomials

\[
P(x) = \sum_{\nu=0}^{n} \alpha_{\nu} x^{\nu}
\]

satisfying (*) \( \sum_{\nu=0}^{n} c_{\nu} \beta_{\nu} = \beta_j, \ j = 0, 1, \ldots, r - 1 \), with \( \text{Det} (c_{\nu})_{\nu=0,1,\ldots,r-1} \neq 0 \). Let

\[
w_k(x) = \begin{cases} 1 & k = 0, 1, \ldots, r, \\ 0 & \text{otherwise} \end{cases}
\]

be positive continuous on \([-1,1]\). Define (**)

\[
\rho_{r,p}(f) = \inf_{P \in \pi_n^r} \| f - P \|_{r,p}
\]

Theorem. The polynomial

\[
P(x) = \sum_{\nu=0}^{n} \alpha_{\nu} x^{\nu}
\]

for which the infimum in (**) is attained is unique, Several properties of the best polynomials in the (**) sense are discussed. A characterization is obtained in the \( L_p \)-norm when \( f(x) = 0 \) of \( x^{n+1} \) with suitable specifications of (*). (Received November 12, 1965.)

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Results in Post algebras.

For definitions and notations see e.g. Trackzyk (Coll. Math., Vol. XII, Fasc. 2, p. 155-166). In the present paper the representation theory of Epstein and Trackzyk is developed using another method. In addition, this representation theory is extended and some new theorems are proved, Sample results: (i) The correspondence between the category of Post algebras and Post homomorphisms, and the category of Post spaces and Post continuous maps (i.e. the dual maps of Post homomorphisms) is a one-one contravariant functor such that a Post homomorphism is epi(mono) iff its dual map is mono(epi). (ii) If \( P \) is a distributive lattice such that \( P \) is a Post algebra of order \( n \) with respect to the elements \( e_0, e_1, \ldots, e_{n-1} \) then the order and the elements \( e_i \) are uniquely determined. (iii) The afore mentioned method is also used to give a short proof of Trackzyk’s theorem that states that an \( \alpha \)-complete Post algebra is \( \alpha \)-representable iff its underlying Boolean algebra is \( \alpha \)-representable. (Received November 15, 1965.)

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Complete separation in the Stone topology.

Let \( L \) be the lattice of all ideals of a ring or the lattice of all \( 1 \)-ideals of an \( 1 \)-group and suppose \( S \) is a subset of \( L \) equipped with the Stone topology (c.f. R. L. Blair, Stone’s topology for a binary relation, Duke Math. J. 22 (1955), 271-280). F. W. Anderson (Approximations in systems of real-valued continuous functions, Trans. Amer. Math. Soc. 103 (1962), 249-271) gave a procedure for determining (in terms of elements of \( L \) when certain pairs of subsets of \( S \) were completely separated in \( S \). We extend Anderson’s procedure and results to an arbitrary complete lattice \( L \) and a subset \( S \) of \( L \) equipped with the Stone topology. (Received November 12, 1965.)

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On cancellative and abelian congruences in semigroups and groupoids.

We deal with algebraic systems with three operations \( \cdot, /, \backslash \). An element \( P \) of such a free system generated by a set \( E \) is but a polynomial in elements of \( E \) under the three operations, and we define the degree of \( P \) in \( x \in E \) by considering \( \cdot \) as a product and \( /, \backslash \) as quotient operations.

Theorem 1. In a quasigroup \( Q \) (with the usual three operations) the smallest cancellative abelian and
associative congruence is the set of all \((x,y): y = P(x)\) for some \(P\) of degree \(+1\) in \(x\) and \(0\) in any other element of \(Q\) involved. **Theorem 2.** In a semigroup \(S\) the smallest cancellative and abelian congruence for which a given \(H \subseteq S\) is contained in one class is the set of all \((x,y): y \in P(x)\) for some \(P\) involving at most \(H\) and elements of \(S\), with degree \(0\) in \(H\), \(+1\) in \(x\) and \(0\) in any other element of \(S\) involved (operations being in this case product of subsets, and the two quotients of \(A\) by \(B\): \(\{x; Ax \cap B \neq \emptyset\}\), \(\{x; xA \cap B \neq \emptyset\}\)). **Theorem 3.** In a semigroup \(S\), \(H\) is a class for some cancellative and abelian congruence if and only if \(P(H) \subseteq H\) for any \(P\) involving only \(H\) and elements of \(S\), with degree \(+1\) in \(H\) and \(0\) in elements of \(S\). **Theorem 1** has also an analogue in semigroups (take \(H = \emptyset\)), and this analogue and Theorems 2 and 3 are also valid in any groupoid for cancellative abelian and associative congruences. (Received November 12, 1965.)

630-161. **V. LAKSHMIKANTHAM** and **S. LEELAMMA**, University of Alberta, Calgary, Alberta, Canada. **Asymptotically self-invariant sets and conditional stability.** Preliminary report.

One has to consider, in many concrete problems like adoptive control systems, the stability of sets which are not self-invariant and this rules out Lyapunov stability because those definitions of stability imply the existence of a self-invariant set. LaSalle and Rath have introduced the notion of eventual stability to describe such situations. Although such sets are not self-invariant in the usual sense, they are so in the asymptotic sense. This leads to the new concept of asymptotically self-invariant sets. One of the authors has considered, recently, the conditional stability of self-invariant sets using several Lyapunov functions. Following similar technique, the conditional stability properties of asymptotically self-invariant sets are studied and this demands the use of Lyapunov functions with certain special properties. Consequently it is possible to deduce results parallel to Lyapunov theorems and this approach also eliminates the difficulty pointed out by LaSalle and Rath in their work. Examples are constructed to illustrate the results. (Received November 12, 1965.)

630-162. **V. LAKSHMIKANTHAM** and **S. LEELAMMA**, University of Alberta, Calgary, Alberta, Canada. **Parabolic differential equations and conditional stability.** Preliminary report.

The problem of stability of parabolic differential equations has been dealt with using a single Lyapunov function and differential inequalities [J.M.A.A. 9 (1964), 234-251]. The concepts of conditional stability and boundedness have been studied with respect to ordinary differential equations [J.M.A.A. 10 (1965), 368-377]. In the present paper, these concepts are extended to parabolic systems and with the help of several Lyapunov functions, sufficient conditions for the conditional stability of self-invariant sets are obtained. Further, defining asymptotically self-invariant sets corresponding to ordinary differential equations, the stability properties of such sets are also considered making use of Lyapunov functions having certain special character. (Received November 12, 1965.)

630-163. **J. C. C. NITSCHIE**, University of Minnesota, Minneapolis, Minnesota 55455. **Concerning the general maximum principle.**

For the study of certain quasi-linear elliptic differential equations--among them the minimal surface equation--a general maximum principle, recently proved in Math. Ann. 158 (1965), 203-214, is of importance: The difference \(z_2(x,y) - z_1(x,y)\) of two solutions satisfies the ordinary maximum
principle even if no assumptions are made about these solutions in the points of a compact sufficiently meager exceptional set $A$, a set of vanishing linear Hausdorff measure. Note that this set may extend to the boundary of the domain of definition and, in particular, may consist of boundary points only. Whether the vanishing of just the linear measure of $A$ also is a necessary condition for the general maximum principle to hold, was an open question. It is shown now by means of a concrete example--minimal surface equation in a square, $A$ an arbitrary set of nonvanishing linear measure on the boundary of the square--that the answer to this question is in the affirmative. (This settles in part Problem 3 on p. 256 in Bull. Amer. Math. Soc. 71 (1965), 195-270.) (Received November 12, 1965.)

630-164. H. W. LAMBERT, University of Utah, Salt Lake City, Utah 84112. Compact 0-dimensional decompositions of $E$

Some applications are made of techniques developed by S. Armentrout relative to certain decompositions of $E^3$. (See Abstract 614-46, these Notices) 11 (1964), 540). Assume $G$ is a point-like upper semi-continuous decomposition of $E^3$ with associated projection map $P$ and the closure of the image of the nondegenerate elements is compact and 0-dimensional. Let $Q$ be the points of $E^3/G$ which have neighborhoods homeomorphic to $E^3$ and let $C = E^3/G - Q$. Theorem 1. $C$ has no isolated point, that is if $C \neq \emptyset$ then $C$ is a Cantor set. Theorem 2. Let $U$ be an open set in $E^3$ such that $P(U)$ is open in $E^3/G$ and $U \cap P^{-1}(C) = \emptyset$. Let $G'$ be the decomposition of $E^3$ consisting of points of $U$ and elements of $G$ in $E^3 - U$. Then $E^3/G$ is homeomorphic to $E^3/G'$. For $i = 1,2$ assume $G_i$, $Q_i$ and $C_i$ are defined as $G$, $Q$ and $C$ are defined above, and let $P_i$ be the projection map associated with $G_i$. Theorem 3. Suppose that if a nondegenerate element of $G_1$ intersects a nondegenerate element of $G_2$ they are the same and that $G_1 + G_2 = G$, that is the nondegenerate elements of $G$ are those of $G_1$ and $G_2$. If the image under $P_1$ of an element $g_1$ of $G_1$ is in $C_1$ then $P(g_1)$ is in $C$. (Received November 12, 1965.)


Let $X$ be a compact metric space, and $C(X)$ the Banach space of continuous real (or complex) functions on $X$. Call a subspace of $C(X)$ a partition-subspace if it is spanned by a partition of unity \[\{\phi_1, \ldots, \phi_n\}\] on $X$ such that $\|\phi_i\| = 1$ for $i = 1, \ldots, n$. Theorem. There exists a sequence $E_1 \subset E_2 \subset \ldots$ of partition-subspaces of $C(X)$ whose union is dense in $C(X)$. Moreover each $E_n$ can be chosen $n$-dimensional, and any one $E_n$ can be specified in advance. This theorem has several applications: First, it implies that $C(X)$ is an $\mathcal{K}$-space in the sense of J. Lindenstrauss [Mem. Amer. Math. Soc. 48]. Second, it implies that $C(X)$ has a basis of norm 1 (cf. C. Bessaga [Bull. Acad. Polon. Sci. Cl. III 5 (1957), 11-14], where this is proved for certain special $X$), thereby strengthening F. S. Vaher's result [Dokl. Akad. Nauk. S.S.S.R. 101 (1955), 589-592] that $C(X)$ has a basis. Finally, it leads to a simplified proof of the simultaneous extension theorem obtained by the second author in [Studia Math. 25 (1964), 157-161]. (Received November 12, 1965.)
Inequalities on holomorphic functions omitting one value.

Let \( f \) be holomorphic in \( |z| < R \leq \infty \) and omit there the value 0. Set \( M(f, r) = \max_{|z| \leq r} |f(z)| \), \( m(f, r) = \min_{|z| \leq r} |f(z)| \). Using the techniques of harmonic measure we establish \( m(f, r)k(z) \leq |f(O)|k(z)M(f, 1)1 - k(z) \), \( k(z) = (2/\pi) \arcsin (1 - |z|)/(1 + |z|) \), where we give the result in the case in which \( R = 1 \). Applying this result we prove that if \( \{f_n(z)\} \) is a sequence of holomorphic bounded functions in \( |z| < 1 \), each omitting the value 0, and if \( |f_n(O)|M(f_n, 1) \leq K < \infty \), \( n = 1, 2, \ldots \), then the sequence \( \{f_n(z)\} \) is uniformly bounded by \( K \) on \( |z| \leq (\sqrt{2} - 1)^2 \). Also it can be shown that if \( f \) is an entire function omitting the value 0 with \( \lim_{r \to \infty} m(f, r)M(f, 7r) < \infty \) then \( f(z) \) is identically constant. (Received November 12, 1965.)

On pseudo-valued global fields.

A global field \( F \) is the quotient field of a Dedekind domain \( J \) on which almost all pseudo-valuations are bounded. If \( \langle \rangle \) is a fixed nontrivial non-Archimedean pseudo-valuation not belonging to the exceptional set, then \( \langle \rangle \) determines a finite set \( \{P_1, P_2, \ldots, P_f\} \) of proper prime ideals in \( J \) such that \( \langle \rangle(a) \) is small iff \( aJ_M \) is divisible by high powers of each \( P_i = P_iJ_M, 1 \leq i \leq \ell \), where \( J_M = \{\beta \in F|\beta J = R_1/R_2, (R_2, P_i) = J, 1 \leq i \leq \ell\} \). Two pseudo-valuations are topologically equivalent iff both determine the same set of primes. There exists an integral principal ideal \( C = \gamma J = p^a_1P^2_2 \ldots P^a_\ell \) and a set \( S = \{0 = a_0, a_1, \ldots, a_t\} \subset J \) of representatives of \( J/\gamma J \) such that the totality of formal Laurent series in \( \gamma \) with coefficients in \( S \) constitutes the completion \( \widetilde{F} \) of \( F \) with respect to \( \langle \rangle \). A series \( \sum_{j=0}^{\infty} \beta_j \gamma^{-j} \), \( \beta_j \in S, \beta_0 \neq 0 \), is \( \phi \)-convergent to an element \( \xi \) of \( F \) iff it is periodic; it is \( \phi \)-convergent to an element \( \xi \in J \) iff it terminates and \( l_0 \geq 0 \). The function \( \phi(\xi) = \phi(\sum_{j=0}^{\infty} \beta_j \gamma^{-j}) = \phi(\gamma)^{l_0} \) is a pseudo-valuation on \( \widetilde{F} \), topologically equivalent to \( \phi \) on \( F \). (Received November 12, 1965.)

Concerning a wild 3-cell described by Bing.

Andrews and Curtis [Ann. of Math. 75 (1962), 1-7] conjectured that if \( C \) is a \( k \)-cell in \( E^n \), then \((E^n/C) \times E^k = E^{n+k}\). If \( C \) is any cell in \( E^n \), \( E^n/C \) denotes the space obtained by collapsing \( C \) to a point. In connection with this conjecture, it would be desirable to have an example of a 3-cell \( C \) in \( E^3 \) such that \((E^3/C) \times E^1 \neq E^4 \). Meyer showed [Pacific J. Math. 13 (1963), 193-196] that if \( C \) is any 3-cell in \( E^3 \) such that \( Bdc \) is locally polyhedral modulo an arc on \( Bdc \), then \((E^3/C) \times E^1 = E^4 \). Thus if \( C \) is a 3-cell in \( E^3 \) such that \((E^3/C) \times E^1 \neq E^4 \), \( Bdc \) must be locally wild on a set not lying in any arc on \( Bdc \). A candidate for an example of the type sought is a cell \( B \) obtained from a construction due to Bing [Duke Math. J. 28 (1961), 1-15]. \( BdB \) is wild at each point of \( BdB \). However, the following result is established. Theorem. \((E^3/B) \times E^1 = E^4 \). (Received November 12, 1965,)

It is known that the deleted product of a triad has the homotopy type of a circle and that this is the only tree (finite, contractible, 1-dimensional polyhedron) with this property. In the first part of this paper, we obtain a formula for computing the number of isotopy classes of imbeddings of the triad in a tree and show that there is a definite relation between this number and the 1-dimensional Betti number of the deleted product of the tree. (It is known that the homology groups of the deleted products of trees are free abelian.) In the second part of the paper, we consider finite, contractible, 2-dimensional polyhedra. There are two such polyhedra which have the property that their deleted products have the homotopy type of a 2-sphere. However one of these can be imbedded in the other, and we present an algorithm for computing the number of isotopy classes of imbeddings of this one in a finite, contractible, 2-dimensional polyhedron. (Received November 12, 1965.)

630-170. E. Y. RODIN, Wyle Laboratories, Box 1008, Highway 20, Huntsville, Alabama. Solution of Burgers' equation with Cauchy data.

Solution of the boundary value problem defined by its Cauchy data \( v(0,t) = a(t) \), \( v_x(0,t) = b(t) \), for the Burgers' equation \( v_t + vv_x = \delta v_{xx} \) is presented. Sufficient conditions for uniqueness, in terms of the discriminant of the boundary values \( d = a^2 - 2b \), are given. Some applications of these results are discussed. (Received November 12, 1965.)

630-171. A. J. CHANDY, Southeastern Massachusetts Technological Institute, North Dartmouth, Massachusetts. The near-ring generated by the inner automorphisms of a group.

The endomorphisms of any group \( G \) generate a near-ring under the usual operations. The subnear-ring \( R \) generated by the inner automorphisms is a ring if and only if the group \( G \) is an L-group as defined by Levi [J. Ind. Math. Soc. 6 (1942), 87-97]. \( R \) is a commutative ring if and only if \( G \) is 2-nilpotent. (Received November 12, 1965.)


Let \( G \) be a LCA group and \( \Gamma \) its dual. Represent \( C^b(G) \) as operators on \( C_0(G) \). Let \( W_0(SO) \) be the weak (strong) operator topologies. Let \( f \in C^b(\Gamma) \). Then \( f \in M(G)^{A, \text{if}} \lambda^A \rightarrow \int fd\lambda \) is \( W_0 \) continuous on norm balls of \( M(\Gamma) \). Assuming \( G \) is \( \sigma \)-compact, one has \( f \in M(G)^{A, \text{if}} \) for \( f \in C^b(\Gamma) \) iff (*) \( \{ \lambda_n \} \subset M(\Gamma), \| \lambda_n \| \leq 1, \) and \( \lambda_n(x) \rightarrow 0 \) pointwise implies \( \int fd\lambda_n \rightarrow 0 \). If \( f \) satisfies (*), then on balls \( \lambda_x \rightarrow \int fd\lambda \) is sequentially continuous in the compact-open topology, and hence continuous in \( SO \) and therefore in \( W_0 \). This generalizes the reported theorem of Hewitt and Beurling, E. Hewitt, Surveys in applied mathematics, IV, John Wiley, New York, 1958. (Received November 15, 1965.)
We give an abstract treatment of "Fredholm elements" in Banach algebras. For a closed ideal I in a Banach algebra A, consider the usual quotient map \( \pi : A \to A/I \). We define Fredholm elements (with respect to I) to be elements \( a \) in A such that \( \pi(a) \) is invertible. This class is denoted by \( F(A,I) \). When \( A = B(E) \), the full algebra of bounded operators on a Banach space E, and I = K the ideal of compact operators, this definition gives the usual class of Fredholm operators. In general, if \( A \subset B(E) \) and \( I \subset A \cap K \), then \( F(A,I) \subset F[B(E), K] \cap A \). \textbf{Theorem.} If \( A \) is a C*algebra in \( B(H) \) (H, Hilbert space) then \( F(A, A \cap K) = F[B(H), K] \cap A \). The index of Fredholm operators as usually defined gives a homomorphism from the semigroup (actually a group) of connectivity components of \( F(A,I) \) into the integers. Denote this homomorphism by \( i \). \textbf{Theorem.} If \( A = B(E) \) and \( I = K \) then \( i \) is 1-1 if and only if the group of invertible elements of \( B(E) \) is connected. Finally, we study approximation by Fredholm operators in \( B(H) \) and determine the uniform closure of each component of Fredholm operators. (Received November 15, 1965.)

\[630-174. \text{W. E. RITTER, University of Washington, Seattle, Washington 98105. The least upper bound principle in hyperarithmetic analysis.}\]

Let \( G \) be an open set in \([0,1]\) expressed as the union (not necessarily disjoint) of a denumerable sequence \( I_n \) of open intervals with rational end-points. We say that \( G \) is hyperarithmetically open if \( I_n \) can be chosen to be hyperarithmetic (after a suitable encoding of the rationals in the nonnegative integers). (See e.g. D. Lacombe, C. R. Acad. Sci. Fr. 245 (1957), 1040-1043, for the recursive case.) \textbf{Theorem 1.} If \( G \) is nonvacuous and hyperarithmetically open, then it has hyperarithmetic extrema. However, the situation even for sets consisting entirely of hyperarithmetic reals is not so simple in general, according to \textbf{Theorem 2}. There are nonvacuous sets of hyperarithmetic reals which are definable in one-universal-function-quantifier form, which have complements in the set of all hyperarithmetic reals likewise definable, and yet which do not have hyperarithmetic extrema. (Received November 15, 1965.)

\[630-175. \text{J. M. DAY, Carnegie Institute of Technology, Pittsburgh, Pennsylvania and S. P. FRANKLIN, University of Florida, Gainesville, Florida. Spaces of continuous relations.}\]

Let \( CR(X) \) be the space of continuous real-valued relations on \( X \) provided with the compact-open topology. If \( X \) is Hausdorff, \( CR(X) \) is a topological semialgebra. Let \( KC(X) \) be the family of compact subsets of \( C(X) \) with the finite topology. If \( X \) is a Hausdorff space, \( KC(X) \) is a topological semialgebra; if \( X \) is also a k-space there is a continuous homomorphism \( \sigma : KC(X) \to CR(X) \) defined by \( \sigma(A) = \bigcup A \). Realcompact spaces \( X \) and \( Y \) are homeomorphic iff \( CR(X) \) and \( CR(Y) \) are isomorphic semi-algebras. \( CR(X) \) and \( CR(TX) \) are isomorphic for any Tychonoff space \( X \). \( CR() \) is a contravariant functor from the category of Tychonoff spaces and continuous functions into the category of real semi-algebras and homomorphisms with its expected properties. (Received November 15, 1965.)
630-176. RAY MINES, New Mexico State University, University Park, New Mexico.

Completeness and generalized primary groups.

Let $G$ be an abelian group divisible for all primes other than a fixed prime $p$ and $a$ be an
ordinal number. Let $G/P^aG \to E^a(P^aG)$ by $P^a$. Then $P^a(G)$ is independent of the group $E^a$. Let $U(G) = \text{Ext}(Z(P^aG), G)$ and $U^a(G) = U(G)/P^aU(G)$. For limit ordinals $a$, let $L_a(G) = \lim_{\beta < a} G/P^\beta G$. The following containments hold: $G/P^aG \subseteq U^a(G) \subseteq P^aG \subseteq L_a(G)$, and $G/P^aG \subseteq L_a(G) \subseteq L_aU^a(G)$. A necessary and sufficient condition that $P^a(G)$ be hereditary is that $P^a(G) = U^a(G)$, for all $G$. The group $G$ is called fully complete, if $L_a(G) = G/P^aG$ for all limit ordinals $a$. An example of a $p$-coprimary group which is not fully complete is given refuting a conjecture of Harrison. Call a group generally complete if $L_a(G)/(G/P^aG)$ is reduced for all limit ordinals $a$. A necessary and sufficient condition for $G$ to be generally complete is for $G$ to be cotorsion. (Received November 15, 1965.)

630-177. V. KRISHNAMURTHY, Birla Institute of Technology and Science, Pilani, India and JOAQUIN LOUSTAUNAU, New Mexico State University, University Park, New Mexico. On the State Diagram of a linear operator and its adjoint in locally convex spaces.

Let $T$ be a linear operator defined on a domain $D(T)$ dense in a locally convex (= l.c.) space and having values in another, $F$. The Strong (Mackey) State Diagram gives a schematic representation of the theorems about linear operators when we consider the dual spaces $E'$ and $F'$ in their strong (Mackey) topologies. A closed square in a state diagram indicates that the corresponding state of the pair $(T, T')$ can never occur. The remaining squares are called open squares. It is proved in this paper that there are only 23 open squares. When $T$ is continuous and $D(T) = E$ there are 23 open squares in the Strong State Diagram and 14 in the Mackey State Diagram. The open squares are supported by 43 examples thereby showing that each of the corresponding states does occur. Finally it is proved that if $T$ is closed, we can associate with it a continuous operator $S$ defined on all of a l.c. space in such a manner that, under reasonable hypotheses, $(S, S')$ has the same state as $(T, T')$. The State Diagrams of Taylor and Halberg (J. Reine Angew, Math, 198 (1957), 93-111), those of Goldberg (Pacific J. Math. 9 (1959), 69-79), and the theorem of Goldberg (Pacific J. Math. 12 (1962), 183-186) follow as particular cases of the results of this paper. (Received November 15, 1965.)


Consider the RODE (retarded ordinary differential equation) $y'(x) = f(x,y(x), y(a(x)))$, $a \leq x \leq b$, $y(a)$ given. Make the usual smoothness and continuation of solution assumptions.

Construct the solution in $[a,b]$ on the mesh $x_n = a + nh$ for $h > 0$, $n = O(1)$ $(b - a)/h$ by the algorithm $y_0 = y_0 = y(h), y_{n+1} = y_n + hf_n, z_{n+1} = y_n + hr_n$ where $q(n) = (a(x_n)) - a)/h$ (integer part) and $r(n) = ((a(x_n)) - a)/h - q(n)$. As the author has reported, then $y_n = y(x_n) + O(h)$ and $z_n = y(a(x_n)) + O(h)$ uniformly for $x_n \in [a,b]$. Further, $y_n = y(x_n) + e(x_n) + O(h^2)$ where $e(a) = 0$ and $e^x(x) = e(x)g_2(x) + e(a(x))g_3(x) - (1/2)y''(x)$ with $g_1(x)$ the partial of $f$ w.r.t. the $i$th variable evaluated at $x, y(x), y(a(x))$. Consider the above algorithm slightly modified by defining $z_{n+1} = y(q(n))$. Theorem. $y_n = y(x_n) + O(h)$ and $z_n = y(a(x_n)) + O(h)$, uniformly, thus implying convergence with the
same order. However, in general it is false that $y_n = y(x_n) + O(h^2)$. Conjecture. $y_n = y(x_n) + E(x_n)h + O(h^{1+s})$, where $E(a) = 0$ and $E'(x) = E(x)g_2(x) + E(a)g_3(x) - (1/2)y''(x) - (1/2)g_3(x)y'(a(x)), 0 < a < 1$ and $a'$ (derivative of $a$) vanishes nowhere in $[a,b]$ faster than polynomials of degree $1/s$. This conjecture is based upon other work of the author—a perturbation theorem for RODE and some asymptotic order estimates. (Received November 15, 1965.)


This equation $\frac{dx}{dt} + Ux = f(t,x,\lambda)$ is considered in a Banach space with an unbounded closed linear operator $U$ having a dense domain. The function $f$ satisfies a Lipschitz condition in $x$ with a variable coefficient $\theta(t)$; $f$ and $\theta$ are asymptotically periodic or almost periodic in $t$. Sufficient conditions for the existence, asymptotic-uniqueness, and stability of asymptotically periodic and almost periodic solutions are obtained. Other results include the continuity of these solutions with respect to the parameter $\lambda$, uniform approximation by curves in finite dimensional spaces and applications to nonlinear diffusion (heat) equations with an elliptic operator $U$. The method is to prove that the above equation is a periodic or almostperiodic equation with a perturbation, then apply a continuity theorem and other results which the author obtained in another paper Stability, periodicity and almost periodicity of the solutions of nonlinear differential equations in Banach spaces. (Research supported by the U. S. Army Research Office-Durham.) (Received November 15, 1965.)

630-180. C. E. BURGESS, University of Utah, Salt Lake City, Utah 84112. Cellular decompositions of $E^3$ which have only countably many nondegenerate elements.

Let $G$ be a cellular upper semicontinuous decomposition of $E^3$ such that the closure of the image, under the projection map, of the union of the nondegenerate elements of $G$ is a compact 0-dimensional set. Some further conditions on $G$ are given which imply that there exists a cellular upper semicontinuous decomposition $G'$ of $E^3$ such that $E^3/G \cong E^3/G'$ and $G'$ has only countably many nondegenerate elements. (These conditions are satisfied by Bing's decomposition of $E^3$ into points and tame arcs such that the decomposition space is different from $E^3$ [Ann. of Math. 65 (1957), 484-500].) Also, some conditions are given which imply that if $E^3/G \not\cong E^3$, then there exist cellular uppersemicontinuous decompositions $G_1$ and $G_2$ of $E^3$ such that (1) each nondegenerate element of $G_1$ is an element of $G$, (2) $G_2$ has only countably many nondegenerate elements, and (3) $E^3/G_1 \cong E^3/G_2 \not\cong E^3$. (Received November 15, 1965.)


Let $G$ and $F$ be saturated formations. $G$ is said to be strongly contained in $F$ if and only if for each solvable group $G$ with $G$-subgroup $E$ and $F$-subgroup $F$, some conjugate of $E$ is contained in $F$. For terminology see Gaschütz (Math. Z., 80 (1962-1963), 300-305). A characterization of strong containment is developed which has as its main application a wide generalization of the following Theorem. Let $N$ be the formation of nilpotent groups. If $N$ is strongly contained in $F$, then
either $\mathcal{N} = \mathcal{R}$ or $\mathcal{F} = \mathcal{L}$, the formation of all solvable groups. If $G$ is a solvable group, the $\mathcal{N}$-subgroups of $G$ are the Carter subgroups of $G$. Let $F(G)$ be the Fitting subgroup of $G$. Let $r$ be a prime, and $\mathcal{L}_r$ be the collection of all solvable $r'$-groups. **Theorem.** If $\mathcal{R}$ is the saturated formation, $\mathcal{R} = \{G[F(G) \leq L_r]\}$, then there are infinitely many formations which strongly contain $\mathcal{R}$. In fact, all such formations are characterized in this paper. If $G$ is solvable, an $\mathcal{R}$-subgroup of $G$ is the normalizer of a Sylow $r$-subgroup of $G$. (Received November 15, 1965.)

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630-182. J. E. dePILLIS, University of California, Riverside, California. **Noncommutative Markov processes.**

Let $\mathcal{A}$ be the algebra of linear operators on finite-dimensional Hilbert space $H$. Let $\eta_1$ be a state on $\mathcal{A}$, i.e. $\exists P_0 \in \mathcal{A}$ where $P_0$ is positive semi-definite (p.s.d.), and $\forall A \in \mathcal{A}$, $\eta_1(A) = \text{trace}(AP_0)$. The linear functional $\eta_n$ on $\mathcal{A}$ is defined by $\eta_n(A_1 \otimes A_2 \otimes \ldots \otimes A_n) = \eta_1(A_1 T(A_2 \ldots T(A_n)))$ for $A_i \in \mathcal{A}$, where $T$ is some linear map sending $\mathcal{A} \to \mathcal{A}$, constrained by the conditions that $T(A)$ is p.s.d. whenever $A$ is p.s.c. and $T(I) = I$, the identity operator on $H$. **Question.** What relations hold between $\eta_1$ and $T$ in order that the induced functional $\eta_n$ is a state? **Theorem 1.** If $rg(T)$, the range of $T$, is commutative, then $\eta_n$ is a state on $\mathcal{A}$. $\eta_1$ commutes with $rg(T)$. **Theorem 2** says that it is not unusual that $rg(T)$ is commutative. In fact suppose $\eta_n$ is a state for all $n$. Let $\eta_1(\cdot) = \text{trace}(-P_0)$, where $P_0$ has distinct eigenvalues (multiplicity 1). Then $rgT$ is commutative. **Theorem 3.** If $\eta_1(\cdot) = \text{trace}(-P_0)$, where $P_0$ is nonsingular, then $\eta_n$ is a state for all $n$, and $\eta_n$ commutes with $rg(T^*)$, where $T^*$ is the adjoint of $T$. (Received November 15, 1965.)

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630-183. P. C. FIFE, University of Minnesota, Minneapolis, Minnesota 55455. **Uniqueness for the Dirichlet problem in an unbounded domain, and related problems.**

Let $u(x)$ be a solution of the (not necessarily uniformly) elliptic equation $(a_{ij}(x)u_{ij})_j + b_j(x)u_i + c(x)u = 0$ with $C^1$ coefficients in an unbounded domain $D$, satisfying zero Dirichlet data. Let $M_p(r) = \int_{\Gamma_r} u_p(\Gamma_p > 1)$, where $\Gamma_r = D \cap \{x = r\}$. Let $\lambda(r)$ be the first eigenvalue of the Laplace-Beltrami operator on $\Gamma_r$. For a wide class of such operators and domains, one can exhibit a function $m_1(r)$ such that either $u = 0$ or $M_p(r) > Cm_1(r)$. In the special case of the Laplace equation in a sector, this result reduces to the known (Phragmen-Lindelöf) best possible result. There is also a function $m_0(r)$ depending (as did $m_1(r)$) only on the operator and $\lambda(r)$ such that if the Dirichlet data is not identically zero but has bounded support, and if $M_p(r) = o(m_1(r))$, then $M_p(r) = o(m_0(r))$. Similar growth and decay results hold near a singularity on the boundary. (Received November 15, 1965.)

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630-184. G. G. JOHNSON, University of Georgia, Athens, Georgia. **Concerning local symmetry in the moment problem.**

Suppose $\{c_n\}_{n=0}^{\infty}$ is a real number sequence such that the associated step function sequence $\{\phi_n\}_{n=1}^{\infty}$ is uniformly bounded on $[0,1]$, where for each positive integer $n, \phi_n(x) = 0$ if $x \leq 0, \phi_n(x) = c_0$ if $x \geq 1$ and $\phi_n(x) = \sum_{t=0}^{k}c(n,t)\sum_{l=0}^{n-t}(-1)^lC(n-t,l)(-1)^l\phi_{n+t}$ if $x \in [k/n, (k+1)/n) \cap (0,1)$. If $x \in (0,1)$ such that for each $\epsilon > 0$ there is a $d > 0$ and an integer $N > 0$ such that if $n > N$ then $y^2 - dy_n(x)$ -
\[ \phi_n(z) < \epsilon, \text{ then } \lim_{n \to \infty} \phi_n(z) \text{ exists.} \] (Received November 15, 1965.)


The author discusses some mathematical aspects of new models of the special theory of relativity. (Received November 15, 1965.)

630-186. L. L. LARMORE, University of Illinois at Chicago Circle Campus, Box 4348, Chicago, Illinois 60680. On the enumeration of oriented vector bundles over a finite complex.

Let \( X \) be an \( m \)-dimensional space and let \( \xi \) be a stable class of oriented vector bundles over \( X \).

It is possible to enumerate the \( n \)-dimensional realizations of \( \xi \) in terms of map-cohomology operations (a generalization of both cohomology operations and characteristic classes) determined by the relative Postnikov tower for \( BSO(n) \) over \( BSO \). If \( m \) and \( m - n \) are sufficiently small, the enumeration may be expressed as a quotient group of cohomology using only stable cohomology operations and Whitney classes. For example, if \( n = m = 5 \), the realizations may be put into a 1-1 correspondence with \( H^5(X;\mathbb{Z}_2)/L \), where \( L \) is the subgroup of all elements of the form \( x^5 + w_2x^3 + w_3x^2 + w_4x + Sq^5 \pi y + w_7 \pi y \) for any \( x \in H^1(X;\mathbb{Z}_2) \) and \( y \in H^3(X;\mathbb{Z}) \), where the \( w_i \) are Stiefel-Whitney classes of \( \xi \), and \( \pi \) is reduction mod 2. (Received November 15, 1965.)

630-187. STELIOS NEGREPONTIS, Indiana University, Bloomington, Indiana 47405.

Absolute Baire sets.

Theorem 1. Let \( X \) be a completely regular Hausdorff space. Then \( X \) is an absolute Baire set (i.e. \( X \) is an element of the \( \sigma \)-field generated by the family of zero-sets in the Stone-Čech compactification \( \beta X \)) if and only if there is a separable metric space \( \Gamma \) which is an absolute Borel space (in the class of separable metric spaces) and a proper (i.e. closed, continuous, onto and compact) mapping \( \phi: X \to \Gamma \). Theorem 2. Let \( X \) be a locally compact, \( \sigma \)-compact Hausdorff space. Then every Baire set of \( \beta X - X \) is \( C^* \)-embedded in \( \beta X - X \). (Received November 15, 1965.)


For terminology and some related results, see Abstract 622-46 (these Notices) 12 (1965), 339.

Let \( K \) be any consistent r.e. extension of R. M. Robinson's very weak subsystem \( R \) of arithmetic. Then the class of functions strongly representable in \( K \) is exactly the class of all partial recursive functions. The key to the proof is the theorem on exact separation of disjoint r.e. sets due to Putnam and Smullyan [Proc. Amer. Math. Soc. 11 (1960), 574-577] and to Shepherdson [Arch. Math. Logik Grundlagenforsch. 5 (1960), 119-127]. Classification of partial recursive functions by the alternation of quantifiers preceding the constructive arithmetic (or primitive recursive) formulas in the formulas strongly representing the functions leads, for each \( K \), to a classification with at most four classes. In particular, the \( \Sigma_1 \) class is a r.e. class of total recursive functions which contains all \( K \)-provable recursive functions. (Received November 15, 1965.)
A stochastic point process in $\mathbb{R}^n$ is a triple $(M, B, P)$ where $M$ is the class of all countable sets in $\mathbb{R}^n$ having no limit points, $B$ is the smallest sigma-algebra on $M$ which makes the functions $N_S(x)$, defined by $N_S(x) =$ the number of points of $x$ in $S$ where $x \in M$ and $S$ is a Borel set in $\mathbb{R}^n$, measurable, and $P$ is a probability measure on $B$. A variety of operations on point processes which yield new point processes can be defined, e.g., superposition, deleting points, random translations of points, and clustering of points. The sequence of processes produced by iteration of these operations on a specified point process will, under very general conditions and for a wide class of point process including the stationary ones, converge to a mixture of Poisson processes. These results are established via a generalization of a classical limit theorem for Bernoulli trials. (Received November 15, 1965.)
On mixed groups of torsion-free rank one.

Let $G$ be a mixed abelian group of torsion-free rank one and let $G_t$ denote its maximal torsion subgroup. For each $x$ in $G$, let $U_G(x)$ be the infinite matrix whose rows are the height sequences of $x$ for the various primes. There is then a simple and natural equivalence relation between such matrices (see J. Rotman, *Torsion-free and mixed abelian groups*, Illinois J. Math. 5 (1961), 131-143) such that $U_G(x) \sim U_G(y)$ whenever $x$ and $y$ have infinite order in $G$. Thus we assign to $G$ an equivalence class $U(G)$ of such matrices. **Theorem.** If $G$ and $K$ are countable mixed groups of torsion-free rank one, then $G \cong K$ if and only if $G_t \cong K_t$ and $U(G) = U(K)$. The theorem is false for uncountable groups, though it remains true under certain restrictions on $G_t$ (e.g., $G_t$ closed or a direct sum of countable groups). An existence theorem is established for mixed groups of torsion-free rank one with a given $U(G)$ and a given countable $G_t$. Certain applications are considered. (Received November 15, 1965.)

Undirected graphs realizable as graphs of semimodular lattices.

The Hesse diagram of a partially ordered set can be thought of as a directed graph in a natural way. We say an undirected graph $G_u$ can be realized as the graph of a semimodular lattice whenever there is some semimodular lattice whose Hesse diagram has $G_u$ as its associated undirected graph. The results of a recent paper by L. R. Alvarez [Canad. J. Math., to appear] can be generalized to yield a characterization of those undirected graphs that can be realized as the graph of a semimodular lattice of finite length. (Received November 15, 1965.)

On $t_m$-semireflexive locally convex spaces.

Let $E[t]$ be a locally convex space, $m$ a saturated covering of $E$ by bounded sets, $E'$ the topological dual of $E[t]$, and $t_m$ the topology on $E'$ of uniform convergence on sets of $m$. $E$ is said to be $t_m$-semireflexive if the topological dual, $Y$, of $E'[t_m]$ is $E$; that is, if every weakly closed set of $m$ is weakly compact (or weakly complete). **Theorem 1.** The $t_g$ ($E'$) closure of every set of $m$ is $t_c$ ($E'$) compact in $Y$. **Theorem 2.** If $E[t]$ is quasi-complete and $t$ is the Mackey topology, then $E[t]$ is $t_m$-semireflexive if and only if $t_k$ ($E'$, $Y$) induces the Mackey topology on $E$. **Theorem 3.** If $E'[t_m]$ is semireflexive, then $t_k$ ($E'$, $Y$) induces the Mackey topology on $E$, $t_{bs} (E', E) = t_k (E', E)$, and consequently $E$ is quasi-barrelled if and only if $t$ is the Mackey topology. All definitions and notations agree with those of G. Koth, *Topologische linear Raume*, I, Springer, Berlin, 1960. (Received November 15, 1965.)

On 1-factors in nonseparable graphs.

A 1-factor of a graph is a spanning subgraph in which each vertex has degree 1. Tutte (The factorization of linear graphs, J. London Math. Soc. 22 (1947), 107) has given a characterization
of graphs with 1-factors. **Theorem.** If a graph with a 1-factor is nonseparable (i.e., a block), then it has at least two 1-factors. The following result is closely related. A graph having a 1-factor has more than one if and only if there is a cycle of even length whose edges are alternately in and not in the given 1-factor. The multiplicity of 1-factors in n-connected graphs is also considered. (Received November 15, 1965.)


In an earlier report (Abstract 622-9, these Notices 12 (1965), 327) we proved global convergence of a class of iterative methods for solving polynomial equations provided the polynomial possessed a dominant zero. We now extend our results to the case of a pair of dominant zeros which are complex conjugate. We give an algorithm, easy to implement in practice, for generating an iteration function \( \phi(\lambda, t) \) for \( \lambda \) an arbitrary nonnegative integer. This iteration function is defined at \( \infty \). **Theorem.** Let the zeros \( \rho_{1} \) of the polynomial \( P \) be distinct, let \( \rho_{1} \) and \( \rho_{2} \) be complex conjugate, and let \( |\rho_{1}| > |\rho_{i}|, i = 3, 4, \ldots, n \). Let \( t_{0} \) be an arbitrary point in the extended complex plane such that \( t_{0} \neq \rho_{2}, \rho_{3}, \ldots, \rho_{n} \) and let \( t_{i+1} = \phi(\lambda, t_{i}) \). Then for \( \lambda \) sufficiently large but fixed, the sequence \( t_{i} \) is defined for all \( i \) and \( t_{i} \to \rho_{1} \). (Received November 17, 1965.)

630-197. JOSEF KRAL, Brown University, Providence, Rhode Island 02912. **Normal derivatives of potentials. Preliminary report.**

Let \( G \) be an open set in \( \mathbb{R}^{n} \) with a compact boundary \( B \) and let \( p(x) = |x|^{2-n}/n - 2 \) or \( p(x) = \ln |x| \) according as \( n > 2 \) or \( n = 2 \). Let \( C^{*}(B) \) be the Banach space of all finite signed Borel measures with support in \( B \); total variation is taken as a norm in \( C^{*}(B) \). With every \( \mu \in C^{*}(B) \) one associates the potential \( V_{\mu} = p * \mu \) and the distribution \( NV_{\mu} \) over \( \mathcal{D} \) defined by \( \langle \Phi, NV_{\mu} \rangle = \int_{G} \text{grad} \Phi(x) \cdot \text{grad} \, V_{\mu}(x) \, dx \). The support of \( NV_{\mu} \) is contained in \( B \). Let \( \theta \in \mathcal{S} = \{ \theta: \theta \in \mathbb{R}^{n}, |\theta| = 1 \} \), \( z \in \mathbb{R}^{n} \), \( t_{0} > 0 \). The point \( z + t_{0} \theta \) will be termed a hit of \( \mathcal{S}(\theta, z) = \{ z + t \theta: t > 0 \} \) on \( G \) provided both \( \mathbb{R}^{n} \setminus G \) and \( G \) meet every segment \( \{ z + t \theta: |t - t_{0}| < \delta \} \) \( (\delta > 0) \) in a set of positive linear measure. The number, to be denoted by \( n(\theta, z) \) \( (0 \leq n(\theta, z) \leq \infty) \), of all hits of \( \mathcal{S}(\theta, z) \) on \( G \) is a Baire function of the variable \( \theta \) on \( \mathcal{S} \) so that one may put \( v(z) = \int_{\mathcal{S}} n(\theta, z) \, d\mathcal{S}(\theta) \). The following result has application in connection with the Neumann problem. **Theorem.** In order that \( NV_{\mu} \in C^{*}(B) \) for every \( \mu \in C^{*}(B) \) it is necessary and sufficient that \( v(z) \) be bounded on \( B \). If this condition holds then \( \mu \to NV_{\mu} \) is a bounded operator on \( C^{*}(B) \). (Received November 17, 1965.)

630-198. J. M. ZELMANOWITZ, University of Wisconsin, Madison, Wisconsin. **Endomorphism rings of torsionless modules over semiprime rings.**

Let \( R \) be a semiprime (prime) ring and \( M \) a torsionless left \( R \)-module (i.e. a submodule of a complete product of copies of \( R \)). Set \( E = \text{Hom}_{R}(M, M) \), the elements of \( E \) being written as right operators. **Lemma.** \( E \) is a semiprime (prime) ring. If in addition the singular \( R \)-submodule of \( M \) is zero then so is the left singular ideal of \( E \). When this is the case, \( d_{R}(M) = d_{E}(E) \), where \( d_{R}(M) \) is the number of summands (finite or \( \infty \)) in a maximal direct sum of \( R \)-submodules of \( M \). **Theorem.**
If $M$ has zero singular submodule and $d(RM) < \infty$ then $E$ has a semisimple (classical) left quotient ring. Furthermore, this quotient ring is a division ring if and only if $M$ is a uniform $R$-module (i.e., any two nonzero $R$-submodules have nontrivial intersection). When $R$ itself has a semisimple left quotient ring $Q$, the quotient ring of $E$ can be obtained as the ring of $Q$-endomorphisms of the module of quotients $Q \otimes_R M$. One gets a previously announced result [Abstract 65T-287, these Notices 12 (1965), 481] as a corollary. Finally, an example is found to show that the theorem cannot be extended to include all finitely generated torsion-free $R$-modules. More specifically, there exists a uniform torsion-free module on two generators over a left Ore domain whose endomorphism ring satisfies neither right nor left quotient conditions. (Received November 17, 1965.)


A structure theorem for $n$-parameter semigroups.

Let $Y$ be a locally convex Hausdorff space and let $C(Y)$ denote the set of all nonempty, convex, compact sets of the space $Y$. The set $C(Y)$ forms a semilinear space under the operation of algebraic addition of sets and algebraic multiplication of a set by a scalar. Let the Euclidean $n$-dimensional space be denoted by $\mathbb{E}^n$ and let $Z_n$ be the positive cone in $\mathbb{E}^n$ that is, $Z_n = \{x \in \mathbb{E}^n : x = (x_1, \ldots, x_n), x_i \geq 0\}$. A one-to-one correspondence $x \rightarrow A(x)$ of the cone $Z_n$ into the space $C(Y)$ will be called an $n$-parameter semigroup provided that $A$ is additive. Theorem. Let $A(x)$ be an $n$-parameter semigroup. Then there exists an additive function $f(x)$ from the cone $Z_n$ into the space $Y$ and there exist sets $K_i \in C(Y)$, for $i = 1, \ldots, n$, such that $A(x) = f(x) + x_1 K_1 + \ldots + x_n K_n$ for all $x \in Z_n$. The above theorem is a generalization of a theorem by Rådström [See One-parameter semigroups of subsets of a linear space, Arkiv Mat., Band 4, Nr. 9 (1960), Sweden]. For related results in the case that the function $A(x)$ is continuous and defined on the base cone in a Banach space see Henney, Dagmar, A structure theorem of set-valued additive functions defined on the base cone in Banach spaces, Abstract 608-61, these Notices 11 (1964), 72. (Received November 17, 1965.)

630-200. B. J. WALSH, University of California, Los Angeles, California 90024.

Atomicity of spectral measures on certain spaces.

A locally convex space $E[\mathcal{X}]$ is a quasi-Montel space (K. Kera, Proc. Jap. Acad. 40 (1964), 633-637) if its weakly compact subsets are $\mathcal{X}$-compact. Theorem. If $E$ is quasi-Montel and $\mu$ is a $\mathcal{X}$-equicontinuous spectral measure in $L(E)$, then $\mu$ (or a suitable extension of it) is purely atomic. (This extends a result of the author's Structure of spectral measures on locally convex spaces, appearing shortly in Trans. Amer. Math. Soc.) In particular, every spectral measure or $\sigma$-complete Boolean algebra of idempotents on $\ell^1$ or on any Köthe "gestufeter Raum" is purely atomic. The theorem is also used to characterize Köthe sequence spaces among complete locally convex vector lattices. (Received November 17, 1965.)

630-201. V. S. KRISHNAN, State University of New York at Buffalo, Buffalo, New York 14214.

Operations in certain categories.

The $C$-category over an $A$-category is the abstract correlate of a topological, or semi-uniform space or pre-ordered set, over an algebraic base. Using suitable characteristics of $A$-categories
and C-categories, certain operations are defined for certain 'sets' of objects and/or maps in the C-category giving rise to an object or map again (typical are 'products', limits of inverse directed sets, etc.). The problem is to characterize the resulting collection of objects and maps when one starts with a sub-category and uses certain combinations of these operators; certain combinations of operators give rise to the same final collection as certain others. The work is based on an earlier paper by the author on Closure operations in C-structures (Kon. Ned. Ak. Vet. Proc. A 56 (1953), 317-329). (Received November 17, 1965.)

Abstracts for Special Sessions


Let T be projective 3-space over a finite field GF(q). A spread of T is a set of $q^2 + 1$ lines of T which (as point-sets) partition the points of T. André by one construction and Bruck and Bose by another have shown that the theory of spreads is equivalent to the theory of translation planes of order $q^2$. And Bruck has proposed a method (not yet successful) for using spreads to construct projective planes of order $q^2(q^2 + 1)$. The present paper begins the classification (by geometric methods) of spreads according to isomorphism, two spreads of T being isomorphic if some collineation of T carries one into the other. A type of spread called subregular of index k is examined in detail; the index k can be any integer in the range $0 \leq 2k \leq q - 1$. This index is an isomorphic invariant but need not characterize a single isomorphism class. It seems likely that all spreads are subregular; this is proved for $q = 2, 3$. A packing of T is a set of $q^2 + q + 1$ spreads of T which (as line-sets) partition the lines of T. The paper gives a complete discussion of the packings for $q = 2$. (Received October 27, 1965.)

630-203. D. R. FULKERSON and O. A. GROSS, Rand Corporation, 1700 Main Street, Santa Monica, California. Incidence matrices and interval graphs.

According to present genetic theory, the fine structure of genes consists of linearly ordered elements. A mutant gene is obtained by alteration of some connected portion of this structure. By examining data obtained from suitable experiments, it can be determined whether or not the blemished portions of two mutant genes intersect or not, and thus intersection data for a large number of mutants can be represented as an undirected graph. If this graph is an "interval graph," then the observed data is consistent with a linear model of the gene. The problem of determining when a graph is an interval graph is a special case of the following problem concerning (0,1)-matrices: When can the rows of such a matrix be permuted so as to make the 1's in each column appear consecutively? Some results are obtained for this problem, culminating in a decomposition theorem which leads to a rapid algorithm for deciding the question, and for constructing the desired permutation when one exists. (Received December 3, 1965.)
The diameter of $P$ is the smallest number $k$ such that any two vertices of $P$ can be joined by a path formed from $k$ or fewer edges of $P$. Two functions $\Delta$ and $\Delta_b$ are studied, where $\Delta(d,n)$ is the maximum diameter of convex $d$-polyhedra with $n$ $(d-1)$-faces and $\Delta_b(d,n)$ is similarly defined for bounded polyhedra. The well-known Hirsch conjecture of linear programming asserts $\Delta(d,n) \leq n - d$, and the $d$-step conjecture asserts $\Delta(d,2d) \leq d$. The Hirsch conjecture was previously proved for $d \leq 3$, the bounded $d$-step conjecture (concerning $\Delta_b$) for $d \leq 4$. Here it is shown the bounded 5-step conjecture is true ($\Delta_b(5,10) = 5$) but the general 4-step conjecture is false ($\Delta(4,8) = 5$). It is also shown $\Delta_b(4,9) = 5$. Other new values of $\Delta$ and $\Delta_b$ are obtained from two reduction theorems, $\Delta(d+k, 2d+k) = \Delta(d,2d)$ and $\Delta_b(d+k, 2d+k) = \Delta_b(d,2d)$, which justify emphasis on the $d$-step conjecture. Some rough bounds for $\Delta$ and $\Delta_b$ are established; in particular, $\Delta(d,2d) \leq d + \lceil d/4 \rceil$, so the excess over the conjectured value tends to infinity with $d$. Also proved is the equivalence (though not on a dimension-for-dimension basis) of the bounded Hirsch conjecture, the bounded $d$-step conjecture, and the assertion that in a bounded $d$-polyhedron in which each vertex is of valence $d$, any two vertices can be joined by a path which does not revisit any $(d-1)$-face. (Received October 25, 1965.)

630-205. MARVIN MARCUS, University of California, Santa Barbara, California. Symmetry classes and combinatorial identities.

If $\dim V = n$, $H$ is a $p$ element ($|H| = p$) subgroup of $S_m$, and $X$ is a character of degree 1 on $H$, then a multilinear $f$ on $V$ with values in $P$ is symmetric w.r.t. $H$ and $X$ if $f(x_{\sigma(1)}^{\cdot}\cdot\cdot x_{\sigma(m)}) = X(\sigma)f(x_1^{\cdot}\cdot\cdot x_m)$ for all $\sigma \in H$ and all $x_i$, e.g., $f(x_1^{\cdot}\cdot\cdot x_m) = x_1^{\cdot}\cdot\cdot x_m$, the Grassmann product. The pair $P,f$ is a symmetry class of tensors over $V$ if the linear closure of range $f$ is $P$ and for any space $U$ and any multilinear $\phi$ on $V$ to $U$, symmetric w.r.t. $H$ and $X$, there exists a linear $h$ on $P$ to $U$ such that $\phi = hf$. If $A$ is linear on $V$ to $V$ define $K(A): P \rightarrow P$ by $f(Ax_1^{\cdot}\cdot\cdot Ax_m) = K(A)f(x_1^{\cdot}\cdot\cdot x_m)$, e.g., $K(A) = C_m(A)$, the $m$th compound of $A$ if $H = S_m$ and $X = \text{sgn}$. Theorem. $\det(K(A)) = (\det(A))^e$ where $e = (m/np)\sum_{\sigma \in H} X(\sigma)c(\sigma)$; $c(\sigma)$ is the number of disjoint cycles (counting cycles of length 1) in $\sigma$.

Corollaries. Sylvester-Franke theorem for $C_m(A)$, i.e., $e = C_{n-1,m-1}; e = C_{n+m-1,m-1}$ for induced transformations, i.e., $K(A) = P_n(A)$ (H. J. Ryser, Proc. Sympos. Appl. Math. 10 (1960), 141). Combinatorial corollaries. For sequences $a = (a_1^{\cdot}\cdot\cdot a_m)$ of $m$ integers, $1 \leq a_i \leq n$, define $a \sim \beta$ if $a_{\sigma(i)} = \beta_i$, $i = 1,\cdot\cdot\cdot,m$, for some $\sigma \in H$. If $\Delta$ is an S.D.R. for $\sim$ then $|\Delta| = (1/p)\sum_{\sigma \in H} n^c(\sigma)$. If $\Delta$ consists of those $a \in \Delta$ for which $\sum_{a_{\sigma(i)} = \alpha} X(\sigma) \neq 0$ then $|\Delta| = (1/p)\sum_{\sigma \in H} X(\sigma)n^c(\sigma)$. (Received October 18, 1965.)


Let $V$ be a sectionally continuous function on $(-\infty,\infty)$ with $|V| < M$, and let $T_t$ be the semi-groups on $C_0$ with strong infinitesimal generator $\Delta^2/2 + V$. Corresponding to each decomposition $V = V_1 + V_2$, $V_1 \leq 0$, $V_2 > 0$ there exists a measure space $(\Omega,\mathcal{F},\mu^2)$, where the elements $w \in \Omega$ are triples $w = (h,\xi,w(\xi))$, $0 \leq \eta < \xi \leq \infty$, $w(\xi)$ a continuous function of $t$, $0 \leq t < \xi$. If $\mu(t,\xi,dy)$ is the
transition function of $T_t$, i.e. $T_t f(x) = \int f(y) \mu(t,x,dy)$, then for $0 < t_1 < ... < t_n$ and Borel sets $B_1, B_2, ..., B_n$, let $\mu(t_1, x_1, dy_1) ... \mu(t_{n-1} - t_n, y_{n-2}, dy_{n-1}) \mu(t_n - t_{n-1}, y_{n-1}, B_n) = \mu^x_t [w : t < \tau, w(t_n) \in B_n, w(t_{n-1}) \in B_{n-1}, ..., w(t_1) \in B_1]$. If $V$ is not bounded, a similar space is obtained by truncating $V$ and passing to the limit. Application of these spaces is made to the semigroups $T_t$ and to some related questions. For example, it is immediate that if $\lim_{t \to \infty} T_t 1(x) = 0$ or $\infty$ for some $x$, then the same holds for all $x$. (Received November 15, 1965.)


Let $\{x_t\}$ be a Markov process with state space $[0, \infty)$ having a stationary transition probability function $p_t(x, B)$. The process is said to be "semistable of order $\alpha"$ if $p_{at}(x, B) = p_t(\alpha^{-a} x, a^{-\alpha} B)$ for each $a > 0$. (See Trans. Amer, Math. Soc. 104 (1962), 62-78, for an explanation of this definition.)

The problem is to determine in some sense the class of all such processes. Under some regularity conditions, $\Phi_t = \int_0^t \phi^{-1/a} x \, dt, x_0 > 0$, determines a continuous additive functional of the process; let $T(t) = \Phi^{-1}(t)$ and $y_t = x_{T(t)}$. Finally put $z_t = \log y_t$. Then $\{z_t\}$ is a process with independent increments on $(-\infty, \infty)$, whose behavior reflects that of $\{x_t\}$ up until the time of its first passage to 0. This correspondence yields, among other things, a simple new determination of the generator of a semistable diffusion. However, not every additive process can be obtained from a semistable one; the study of the inverse mapping is not complete. Some results on the two-sided case when $\{x_t\}$ has state space $(-\infty, \infty)$ have also been obtained. (Received December 2, 1965.)

630-208. S. C. PORT, 1700 Main Street, Santa Monica, California. A system of denumerably many transient Markov chains.

Let $P$ be the transition matrix of an irreducible, transient Markov chain with a denumerable set $\Omega$ for its state space. Assume the chain has an invariant measure $\mu$. At time 0 distribute particles in $\Omega$ according to a Poisson process with mean $\mu(x)$ at $x$, and then allow each of the particles to move, independently of the others, according to the transition law $P$. If for a finite nonempty set $B$, $M_n(B; r)$ is the number of particles which have hit $B$ exactly $r$ times by time $n$, $L_n(B)$ the number of distinct particles hitting $B$ by time $n$, and $S_n(B)$ the total number of particles which have been in $B$ by time $n$, then, with Problem 1, $M_n(B; r) \to \sum_{x \in B} \gamma_B(x) P(X = r - 1), L_n(B) \to C(B)$ and $S_n(B) \to \mu(B)$ where $\gamma_B(x)$ is the "capacitory measure" of $B$, $N(B)$ is the occupation time of $B$, and $C(B) = \gamma_B(B)$. Moreover, suitably normalized, these variables converge in law to the normal distribution. (E.g. $[S_n - n \mu(B)](n \sigma^2)^{-1/2} \to N(0, 1)$, where $\sigma^2 = \sum_{x \in \Omega} \gamma^2 x C_x(B)$.) Analogous results are shown to hold for $D_n(B)$, the number of particles which leave $B$ by time $n$, never to return. (Received October 29, 1965.)

630-209. Y. S. CHOW and H. E. ROBBINS, Purdue University, Lafayette, Indiana 47907. On optimal stopping for Markov processes.

Let $\Omega, \mathcal{F}, P$ be a probability space and $\{x_t\}^{\infty}_{n=1}$ be a sequence of random variables such that $E x_n^2 < \infty$. Let $\mathcal{F}_n$ be the Borel fields generated by $x_1, ..., x_n$. A stopping variable $t$ is a positive integer valued variable such that $[t = n] \in \mathcal{F}_n$ for each $n$. Let $C$ denote the class of all stopping...
variables $t$ for which $\text{E} x_t < \infty$, and $C_n$ the class of all $t$ in $C$ such that $P [t \geq n] = 1$. Define $\gamma_n = \text{ess sup}_{t \in C_n} E(x_t / \mathcal{F}_n)$. **Theorem.** If $\{x_n\}$ is Markovian, then $\gamma_n = E(\gamma_n | x_n)$. **Corollary.** If $\{x_n\}$ is independent, then $E(\gamma_n | \mathcal{F}_{n-1}) = E\gamma_n$. Both the theorem and the corollary are intuitively obvious. However our proof is rather lengthy. By applying a martingale convergence theorem we prove: **Lemma.** $\gamma_n = \lim_{b \to \infty} \lim_{a \to \infty} \lim_{n \to \infty} \gamma_n(a, b)$, where $\gamma_n(a, b) = \text{ess sup} P[\mathcal{F}_n] = 1 E(x(a, b) / \mathcal{F}_n)$ and $x(a, b) = a$ for $x < a$, $= a$ for $a \leq x \leq b$, and $= b$ for $x > b$. From the lemma and backward induction, we can prove the theorem. (Received December 2, 1965.)


The analysis of the standard random walk on $\mathbb{Z}_+ = \{0, 1, 2, \ldots\}$, is greatly facilitated by the study of the algebra $R(T_1)$ of trigonometric polynomials of the form $\sum_{n=0}^{\infty} a_n e^{i nt}$ (defined on the unit circle), and various completions thereof in different norms, which give rise to the $L_p$-spaces. See e.g. Spitzer, Principles of random walk, Van Nostrand, 1964. In the present paper, we present some preliminary results on a class of random walks (not necessarily spatially homogeneous) on $\mathbb{Z}_d = \{m_1, m_2, \ldots, m_d\}; m_i \in \mathbb{Z}_+$ and show how one may attach to each such walk an algebra of functions on an appropriate structure space. For each random walk in a certain subclass, this structure space turns out to be essentially a classical compact group and the algebra turns out to be a subalgebra of central representative functions on the group. Using this approach, results can be obtained for these random walks which generalize work of Karlin and McGregor [Ill. J. Math. 3 (1959), 66-81] and Kennedy [Proc. Roy. Irish Acad. 64 (1961), A, 89-100]. The state­ments of the results are too lengthy for this abstract. (Received December 3, 1965.)

630-211. B. JAMISON and STEVEN OREY, University of Minnesota, Minneapolis 14, Minnesota. Markov chains recurrent in the sense of Harris.

Let $\{X_n\}$, $n = 0, 1, \ldots$ be a Markov process with state space $(S, \mathcal{B})$ and stationary transition probabilities $P(x, \mathcal{B})$ which is recurrent in the sense of Harris [Third Berkeley Symposium on Mathematical Statistics and Probability, II, 113-124]. Harris showed the existence of a $\sigma$-finite stationary measure $Q$, unique up to constant multiple $S$ admits a unique modulo, $Q$-null sets partition into cyclically moving sets $C_0, \ldots, C_{d-1}$ and a $Q$-null set $F$. **Theorem.** The tail $\sigma$-field of $\{X_n\}$ is atomic, the atoms being of the form $[X_{nd} \in C_i$ for all but finitely many $n], i = 0, \ldots, d - 1$. For the denumerable case this result is due to Blackwell and Freedman [Ann. Math. Stat. (1964), 1291-1295]. From the theorem together with another result in Blackwell and Freedman follows the **Corollary.** If $x$ and $y$ belong to the same $C_i$, $P^x(x, \cdot) - P^y(y, \cdot) \to 0$ in variation as $n \to \infty$. An argument of Doob [Stochastic processes, Wiley, 1953] is modified to dispense with the usual assumption that $\mathcal{B}$ is separable. (Received December 2, 1965.)

630-212. R. T. SEELEY, Brandeis University, Waltham, Massachusetts. The powers $A^a$ of an elliptic operator $A$.

Let $A$ be an elliptic partial differential operator of order $m > 0$ on a compact manifold without boundary, and suppose there is some ray arg $\lambda = \theta$ in the complex plane such that for each cotangent
vector $\xi \neq 0$ we have for the top-order characteristic polynomial $\sigma_m(A)$ that $\arg(\sigma_m(A)(\xi)) \neq 0$. Suppose further that $A$ is invertible. Then there is an analytic group of operators $A^s$, with $A^s = (i/2\pi) \int_\Gamma \lambda^s(A - \lambda)^{-1}d\lambda$ when $\Re(s) < 0$; here $\Gamma$ is an unbounded path surrounding the spectrum of $A$. Further $A^1 = A$, $A^0 = I$, and $A^s$ is a CZO, or pseudo-differential operator, of order $ms$. The kernel $K_s(x,y)$ of $A^s$ extends analytically to an entire function if $x \neq y$, and to a meromorphic function if $x = y$. The poles of $K_s(x,x)$ are simple, and the residues are computed in terms of the complete characteristic polynomial of $A$. $K_s(x,x)$ is regular for those values of $s$ for which $A^s$ is a local operator, i.e., at $s = 0,1,2,...$ and its values there are similarly computed. Corresponding results are obtained if $A$ is a CZO acting on sections of a vector bundle. (Received December 1, 1965.)

630-213. JAMES SERRIN, University of Minnesota, Minneapolis, Minnesota. Removable singularities of solutions of elliptic equations.

There are basically two different types of removable singularity theorems available in the theory of partial differential equations, the first and most usual placing conditions on the behavior of the solution near the singular set. The second type applies more specifically to equations of the form, (*) $\text{div } A = B$, where $A$ and $B$ are, respectively, vector and scalar valued structure functions depending on the solution and its gradient. In this type suitable conditions are placed on the behavior of $A$ near the singular set, and it is then shown that the solution can be defined on this set so as to remove the singularity. The first result of this form, due to Bers (1951), states that isolated singularities of solutions of the minimal surface equation are always removable. Generalizations have since been obtained by Finn, Nitsche, Serrin, and de Giorgi and Stampacchia. Here we shall consider a general theorem which contains these previous results as special cases. Let $u$ be a continuous solution of (*) in a domain $D - Q$, where $D$ is open and $Q$ is a compact subset of $D$ having $(n - k)$-dimensional Hausdorff measure zero. Suppose that $A = A(u_x)$ and $B = B(u)$ are monotone functions of their arguments, and that $A(u_x)$ is in $L_k(D - Q)$, where $k^{-1} + k^{-1} = 1$. Then $u$ can be defined on $Q$ so that the resulting function is a continuous solution in all of $D$. Several applications and related results will also be discussed. (Received December 1, 1965.)

630-214. TOSIO KATO, University of California, Berkeley, California. Wave operators and similarity for some nonselfadjoint operators.

Let $H$, $H'$ be separable Hilbert spaces, $T$ a densely-defined linear operator in $H$ with its spectrum contained in the real axis. A densely-defined linear operator $A$ from $H$ to $H'$ is said to be smooth relative to $T$ if $D(D) \supset D(T)$ and $Z \rightarrow A(T + Z)^{-1}u$ is in the Hardy class $H^2$ in the upper and lower half-planes $\Omega_{\pm}$ for each $u \in H$. If $A$ is $T$-smooth, $B$ is $T^*$-smooth, and if $\|A(T - Z)^{-1}B\| \leq N < \infty$ for $Z \in \Omega_{\pm}$, then $T + kB^*A$ has a closed extension $T(k)$ similar to $T$ for $|k| < 1/N$. The existence of the wave operators $W_{\pm}(k) = \lim_{t \rightarrow \pm \infty} \exp \{-itT(k)\} \exp \{-itT\}$ is discussed. Applications. The Schrödinger operator $T(k) = -\Delta + k f(x)$ in $H = L^2(R^n)$ is similar to $T = -\Delta$ for small $|k|$ if $f \in L^p(R^n) \cap L^q(R^n)$ with $1 \leq p < n/2 < q \leq \infty$. For $n = 3$ it suffices that $\int \int |x - y|^{-2} |f(x)| |f(y)| dx dy < \infty$ or $f \in L^{3/2}(R^3)$. (Received December 1, 1965.)

The variational eigenvalue problem $Au = \lambda Bu$ is considered for $A$ and $B$ nonlinear differential operators obtained as Euler-Lagrange operators of multiple integral operators (or more generally of functionals on Sobolev spaces) and the existence of infinitely many normalized eigenfunctions established if $A$ is elliptic, $B$ positive, and both $A$ and $B$ odd. The proof uses the theory of the Lusternik-Schnirelman category on infinite dimensional manifolds. (Received December 1, 1965.)


One aspect of the existence problem of quantum field theory is the problem of the existence of solutions to equations of motion of quantized fields. The classical and quantized cases are compared. Existence theorems in the quantized case for $(\square + \mu^2)\phi = -4g\phi^3$ in two space-time dimensions and for a model of Schrödinger particles coupled to a quantized scalar field are discussed. (Received December 1, 1965.)


It is shown that the classical problem of coloring the vertices of a linear graph, subject to the requirement that no two adjacent vertices shall have the same color, is a special case of a very general class of combinatorial problems, all exhibiting the same kind of difficulties in varying degrees. The easiest of these problems can be solved with existing algebraic methods, and their solution points to new techniques which can be applied in attacking the coloring problems. Two results of a very general nature will be stated in the course of the talk. (Received December 9, 1965.)

630-218. Jun-Ichi Igusa, 911 Breezewick Road, Towson, Maryland 21204. Algebraic geometry and automorphic functions.

Algebraic geometry has been playing some role in the investigation of automorphic functions in the past, the present and probably also in the future. We shall illustrate this fact by explaining our recent contributions to this matter. (Received December 3, 1965.)


Let $G$ be a connected semisimple real Lie group having no compact normal subgroup of positive dimension. Let $\rho$ be a faithful representation of $G$ such that $\rho(K)$ consists of unitary matrices, $K$ being a maximal compact subgroup of $G$. Then the map $g \rightarrow \rho(g)^T \rho(g)$ defines a map of the symmetric space $X = G/K$ into the real linear space $S$ of hermitian matrices, which is injective. Accordingly one gets an embedding of $X$ into the real projective space $P$ associated with $S$. The topological closure $\bar{X}$ is the Satake $\rho$-compactification of $X$. Theorem. Let $\Gamma$ and $\Gamma'$ be closed subgroups of $G$ with $G/\Gamma$ and $G/\Gamma'$ having finite invariant measure. Let $\theta: \Gamma \rightarrow \Gamma'$ be an isomorphism and $\phi: \bar{X} \rightarrow \bar{X}$
a diffeomorphism such that \( \phi(\gamma x) = \theta(\gamma)\phi(x) \) for all \( \gamma \in \Gamma, x \in \mathbb{R} \). Then \( \theta \) is the restriction to \( \Gamma \) of an automorphism of \( G \). The proof rests on the Theorem. Let \( T \) be a Cartan subgroup of \( G \) containing a maximal \( \mathbb{R} \)-split subgroup \( T^* \) of \( G \). Any automorphism of \( T \) which stabilizes \( T^* \) and the set of roots having nonconstant restriction on \( T^* \) stabilizes the set of all roots. (Received December 2, 1965.)


We define a Tamagawa number for certain homogeneous spaces generalizing the definition of that number for algebraic groups and discuss its properties. (Received November 30, 1965.)

630-221. T. A. SPRINGER, University of California, Los Angeles, Los Angeles 24, California. Jordan decomposition in the Lie algebra of a linear group.

Let \( G \) be a connected linear algebraic group, defined over an algebraically closed field. Let \( \mathfrak{g} \) be the Lie algebra of \( G \). Define \( X \in \mathfrak{g} \) to be semisimple (resp. nilpotent) if \( X \) is contained in the Lie algebra of a subtorus (resp. a unipotent subgroup) of \( G \). Theorem. Any \( X \in \mathfrak{g} \) can be written as \( X = X_s + X_n \) with \( X_s \) semisimple, \( X_n \) nilpotent, \( [X_s, X_n] = 0 \); such a decomposition is unique. The proof uses the structure theory of linear groups. The theorem is used in a proof, by A. Borel and the author, of Grothendieck's rationality results for linear groups over arbitrary ground fields. (Received November 29, 1965.)

630-222. E. J. TAFT, 14 Vandeventer Avenue, Princeton, New Jersey 08540. Cohomology of algebraic groups and invariant splitting of algebras.

Let \( A \) be a finite-dimensional associative algebra over a field \( F \), with radical \( R \), center \( C \), and group \( G \) of automorphisms and anti-automorphisms. Assume \( A/R \) separable. Then the existence and uniqueness of \( G \)-invariant maximal separable subalgebras of \( A \) are related to the vanishing of \( H^1(G, M) \) for certain \( M \) formed from \( R \) using \( G \)-invariant subspaces and factor spaces, and letting the anti-automorphisms in \( G \) act via their negatives. For example, if \( R^2 = 0 \) and \( H^1(G, R/(R \cap C)) = 0 \), then existence is assured. If \( R^2 = 0 \) and \( H^1(G, R \cap C) = 0 \), then any two \( G \)-invariant maximal separable subalgebras are \( G \)-orthogonally conjugate, i.e., via a regular element which is a fixed point of the automorphisms in \( G \) and is inverted by the anti-automorphisms in \( G \). If \( R^2 \neq 0 \), the \( M \) which arise involve certain \( G \)-invariant Lie ideals in powers of \( R \). In general, the \( M \) are modules for the algebraic hull of \( G \), and we may assume \( G \) to be an algebraic group, with all modules and maps considered being rational. If characteristic \( F = 0 \), then techniques of Hochschild and Mostow may be used to show \( H^1(G, M) = 0 \) when \( G \) acts as a completely reducible group on \( R \). \( H^1(G, M) \) also vanishes if \( G \) is finite, of order not divisible by characteristic \( F \). Analogous results hold for Lie algebras over a field of characteristic 0. (Received November 15, 1965.)
ABSTRACTS PRESENTED BY TITLE

66T-1. R. L. IRWIN, University of Utah, Salt Lake City, Utah 84112. Absolute Hardy-Bohr factors.

Let $\hat{A}$ and $\hat{B}$ be absolutely regular (series-series) matrix transformations. Let $\hat{A}$ be normal, triangular and $\hat{A}_{nn} > 0$, $v \leq n$, $\hat{A}_{nn} \leq 0$ for $v < n$ (here $(\hat{A}_{nn})$ denotes the inverse of $(\hat{A}_{nn})$). Also assume $\hat{B}_{nn} \geq 0$, $v \leq n$, $\hat{B}_{nn}$ as $v \leq n$ and $\hat{B}_{nn} = O(\hat{A}_{nn})$. Theorem. When the above conditions are satisfied, a necessary and sufficient condition for $\sum_{v=0}^{\infty} \hat{a}_{vv}$ to be $|B|$-summable whenever $\sum_{v=0}^{\infty} \hat{a}_{vv}$ is $|A|$-summable is $E = \sum_{v=0}^{\infty} \hat{a}_{vv} c_{\mu}$ where $c_{\mu} = O(1)$. The theorem contains all Cesare methods $|C_{\alpha}|$ to $|C_{\beta}|$ with $0 \leq a \leq \beta \leq 1$. (Received June 7, 1965.)


Let $S = M^{O}(G,I,I,A)$ be a Brandt semigroup. The major result is the determination of all extensions (ideal) of $S$ by an arbitrary semigroup $T$ with zero. We determine such an extension by means of a partial homomorphism $\Lambda \rightarrow w_{A}$ of $T^{*} = (T \setminus 0)$ into $\mathcal{F}$, the full symmetric inverse semigroup on $I$ and mappings $\psi_{A}$ of the domain of $w_{A}$ into $G$. An extension of $S$ by a regular 0-bisimple semigroup $T$ is given by a partial homomorphism if and only if for some idempotent $E$ of $T^{*}$ there exists at most one idempotent of $S^{*}$ under $E$. If $T^{*}$ is a simple group and $S$ is finite such that $|T^{*}| > \max (|G|, |I|!)$, there are $2|I|$ extensions of $S$ by $T$. We also note that if $v$ is an extension of a semigroup $S$ by a semigroup $T$, then $V$ is an inverse semigroup if and only if $S$ and $T$ are inverse semigroups. (Received July 26, 1965.)

66T-3. HERBERT GROSS, Montana State University, Bozeman, Montana 59715. On Witt's theorem in the denumerable case.

Let $E$ be a $k$-vectorspace of denumerable dimension, $\Phi: E \times E \rightarrow k$ a nondegenerate trace-valued $\epsilon$-hermitian form. Let $V$ and $\bar{V}$ be isomorphic subspaces of $E(V \cong \bar{V})$. In order that there exists a metric automorphism of $E$ which maps $V$ onto $\bar{V}$ the following conditions are obviously necessary: (i) $V^{\perp} \cong \bar{V}^{\perp}$, (ii) $\dim (\text{rad } V)^{\perp} / \text{rad } V = \dim (\text{rad } \bar{V})^{\perp} / \text{rad } \bar{V}$, (iii) $\dim \text{rad } (V^{\perp})/(\text{rad } V)^{\perp} = \dim \text{rad } (\bar{V}^{\perp})/(\text{rad } \bar{V})^{\perp}$, (iv) $\dim V^{\perp} \perp V + \text{rad } (V^{\perp}) = \dim \bar{V}^{\perp} \perp \bar{V} + \text{rad } (\bar{V}^{\perp})$, (v) $\dim (\text{rad } (V^{\perp}))^{\perp} / V^{\perp} \perp + V^{\perp} = \dim (\text{rad } (\bar{V}^{\perp}))^{\perp} / \bar{V}^{\perp} \perp + \bar{V}^{\perp}$. Theorem. If $V$ and $\bar{V}$ satisfy these conditions then there exists such an automorphism of $E$ whenever the following (sufficient) conditions are satisfied: (1) If $\dim V / \text{rad } V = \aleph_{0}$ then an algebraic complement of $\text{rad } V$ in $V$ contains an infinite dimensional totally isotropic subspace, (2) same with $V^{\perp}$ in lieu of $V$. For large classes of fields (and corresponding forms) the last two conditions are always satisfied. (Examples: $k$ any Kneser field, or $k$ the uniquely determined skew field of rank 4 over $k_{0}$ whose center is $k_{0}$, $k_{0}$ an arbitrary local field with finite residue class field of characteristic $\neq 2$). The theorem gives in particular a proof for a conjecture by Kaplansky concerning alternate forms (Ann. Acad. Bras. Ci 22 (1950), 1-17). (Received August 18, 1965.)
Bilinear forms in the denumerable case and characteristic 2.

The following theorem is a consequence of more general results by the authors. Let \( E \) be a \( k \)-vectorspace of denumerable (algebraic') dimension, \( \Phi: E \times E \to k \) a nondegenerate symmetric bilinear form, \( k \) a field of characteristic 2 of finite dimension over its subfield of squares. Let \( E_\star \) be the subspace of all \( x \in E \) with \( \Phi(x,x) = 0 \); for every subspace \( L \subset E \) let \( \| L \| \) be the "range" \( \{ \Phi(x,x) | x \in L \} \) (\( \| L \| \) is a \( k^2 \)-vectorspace contained in \( k \)). \( E \) admits an orthogonal decomposition of precisely one of the following types: (1) \( E = E_0 \oplus F \oplus G \) where \( F \) and \( G \) are finite dimensional, \( F \oplus G \) contains no isotropic vector and \( E_0 \) is spanned by a symplectic basis. (2) \( E = E_0 \oplus F \oplus F \oplus G \) where \( E_0 \) is an orthogonal sum of denumerably many copies of a finite dimensional space \( H \), \( F \) and \( G \) are finite dimensional and \( H \oplus F \oplus G \) contains no isotropic vector. \( E \) is uniquely determined, up to orthogonal isomorphism, by the invariants \( \| E \|, \| E_\star \| \) (\( = \| E_0 \| \)) and the finite dimensional space \( E_\star \). Choosing a finite \( k^2 \)-basis for \( k \) one can thus give complete lists of all nonisomorphic spaces \( E \). This generalizes a result by Kaplansky for the case \( [k:k^2] = 1 \). (Received August 18, 1965.)

Theorem 2. Let \( f(z) \) be a complex-valued function of a complex variable defined on a set \( S \) which is dense in the open unit disc \( D \). Then \( f(z) \) has an extension analytic in \( D \) iff it satisfies

(1): \( |f(x,y,z)| \) is not greater than \( M(R) \) whenever \( x,y,z \) are three distinct points in \( S \) of modulus not greater than \( R \), \( R \) is any number strictly between 0 and 1, \( M(R) \) is a finite real number depending only on \( R \). Here \( f(x,y,z) \) denotes the second order divided difference quotient of \( f(z) \) taken at the distinct points \( x,y,z \). Corollary 1. \( f(z) \) is analytic in \( D \) iff \( |f(x,y,z)| \) is bounded in every closed subset of \( D \) when \( x,y,z \), are distinct. Corollary 2 (Montel). If the functions of a class \( G \) are regular and locally uniformly bounded in a domain \( D \), then \( G \) has a subsequence converging to a function analytic in \( D \).

Corollary 3. If \( S \) is dense on every horizontal and vertical line in \( D \) and has the property that \( S \) contains \( p + \text{Re}(q - p) \) or \( p + i \text{Im}(q - p) \) whenever it contains \( p \) and \( q \) then condition (1) of Theorem 2 may be relaxed to (2): \( |f(z,z+a,z+ib)| \) is not greater than \( M(R) \) whenever \( z, z+a, z+ib \), are three distinct points in \( S \) of modulus not greater than \( R \), \( a \) and \( b \) are real, \( R \) and \( M(R) \) are as in Theorem 2. Conjecture 2. Corollary 3 holds whenever \( S \) is dense in \( D \). (Received September 13, 1965.)

Symmetry in non self-adjoint Sturm-Liouville systems.

Suppose \( a < b \) and \( C \) is the usual inner product space of continuous functions of \([a,b] \). Suppose also that \( p \) and \( q \) are in \( C \), \( p(x) > 0 \) for all \( x \) in \([a,b] \) and each of \( W \) and \( Q \) is a real \( 2 \times 2 \) matrix so that the only member \( f \) of \( C \) such that \( (pf')' - qf = 0 \) and \((*) \) \( W(f(a), p(a)f'(a)) + Q(f(b), p(b)f'(b)) = (0,0) \) is the zero function. Denote by \( S' \) the set of all \( f \) in \( C \) such that \( (pf')' - qf = 0 \) and by \( S \) the orthogonal complement of \( S' \). Denote by \( T \) the transformation on \( C \) such that if \( g \) is in \( C \), then \( Tg \) is the unique \( f \) in \( C \) such that \( (pf')' - qf = g \) and \((*) \) holds. Denote by \( P \) the orthogonal projection of \( C \) onto \( S' \). Theorem 1. If \( T \neq T^* \) then \( Tg = T^* g \) if and only if \( g \) is in \( S \). Theorem 2. If \( V \) is the restriction of \((I - P)T \) to \( S \), then \( V^* = V \). (Received August 31, 1965.)
66T-7. W. M. Causey, University of Kansas, Lawrence, Kansas. The univalence of an integral.

Let $S$ be the class of functions $f$ regular and univalent in $|z| < 1$ and normalized by $f(0) = 0$, $f'(0) = 1$. Suppose $f \in S$ is close-to-convex. Then $g(z) = \int_0^z f'(t)e^t dt$ is close-to-convex for $0 \leq \epsilon \leq 1$, and hence univalent in $|z| < 1$. Therefore $g_k(z) = \int_0^z k'(t)e^t dt$, where $k(z) = z/(1 - z)^2$ is the Koebe function, is univalent for $0 \leq \epsilon \leq 1$ and $|z| < 1$. The radius of convexity of the class of functions $g$, where $f$ is arbitrary in $S$, is $(2f - \sqrt{4f^2 - 2f + 1})/(2f - 1)$. (Received September 22, 1965.)


Subsets of a matroid (Whitney, Amer. J. Math., 57, 1935) are bigraded with respect to rank and dual rank. The algebraic properties of the associated two-variable rank generating function $\rho$ are set forth. $\rho(\xi, \eta) = a(\xi + 1, \eta + 1)$ where $a$ is the Tutte polynomial (Tutte, Canad. J. Math. 6, 1954). Partial evaluations $(-1)^p a(1 - n, 0)$ and $(-1)^q a(0, 1 - n)$ yield the Poincaré polynomials (Rota, Zeit. fur Wahrsch., 2, 1964) of the geometric lattices $G$ and $G^*$ of closed and dual-closed subsets of any matroid of rank $p$ and dual rank $q$. The Poincaré polynomials of an oriented matroid (Minty, Univ. of Mich., 1964) enumerate those coboundaries and cycles, with values in any group of order $n$, which are nowhere zero on the matroid. A matroid is oriented if and only if it is regular (Tutte, J. of Res. Nat. Bur. of Standards, 69B, 1965). (Received September 22, 1965.)

66T-9. WITHDRAWN


The language $L(\omega_1, \omega)$ differs from the first order finitary calculus in allowing countable conjunctions. Universal $L(\omega_1, \omega)$ formulas are built up from atomic formulas and their negations using only conjunction, disjunction, and universal quantification. Theorem. A sentence $\sigma$ of $L(\omega_1, \omega)$ is equivalent to a universal sentence of $L(\omega_1, \omega)$ if and only if all substructures of models of $\sigma$ are also models of $\sigma$. This is a positive answer to a question raised by D. Scott in Logic with denumerably long formulas and finitary quantifiers, Berkeley Symposium 1963, North Holland Publishing Company, Amsterdam, 1965. (Received September 23, 1965.)

Theorem. An r.e. set of natural numbers a is hh-simple iff it is coinfinite and for every r.e. superset β of a there exists an r.e. set γ such that β ∩ γ = a and β ∪ γ is the set of all natural numbers. (Received September 24, 1965.)


For each compact Hausdorff space X, let W be a collection of closed subsets which contains every single point set {x}, x ∈ X. Let C be a collection of triples (X, A: W) where A ∈ W, together with the collection of maps F : (X, A: W₁) → (Y, B: W₁), where F is subjected to the following conditions: (1) F : X → Y is an upper semicontinuous function such that F(u) ∈ W₂, for each u ∈ W₁, and (2) F(x) ∩ B ̸= ∅ implies B ⊂ F(x). Theorem 1. C is a category. Now, for each object (X, A: W) ∈ W, let Πₘ(X, A: W) be the set of upper semicontinuous functions f : (Πₘ, ∂Πₘ) → (X, A) such that f(x) ∈ W for each x ∈ Πₘ. This set is partitioned into homotopy equivalence classes. We denote the set of equivalence classes by M₁Πₘ(X, A: W) and give the notion of sum. Theorem 2. For m > 0, M₁Πₘ(X, A: W) is a group and M₁Πₘ is a covariant functor which is an extension of the ordinary homotopy functor Πₘ. Let T be the set of cellular subsets of Sⁿ. We give a topology to the set T (Strother, Duke Math. J. 22, 1955). Theorem 3. M₁Πₘ(Sⁿ, p: T) is isomorphic to Πₘ(Sⁿ, p) and T has the same homotopy group as Sⁿ. (Received September 24, 1965.)


Suppose \{c_t\}_{t=0}^{∞} is a real number sequence such that the associated step function sequence \{φₜ\}_{t=0}^{∞} is uniformly bounded on [0,1] where for each positive integer n, φₜ(x) = 0 if x ≤ 0, φₜ(x) = cₜ if x ≥ 1 and φₜ(x) = ∑ₖ₌₀⁻¹C(n,t)∑ᵢ₌₀⁻¹C(n₋₁, t₋₁)(⁻¹)ᵏ₋ₙ₋₁₋ᵢ₋₁ if x ∈ (0,1) ∩ [k/n, (k+1)/n). If 0 ≤ a < b ≤ 1 and \{Vₘφₜ\}_{t=1}^{∞} is a bounded sequence then limₘ→∞φₜ(x) exists for each x ∈ (a,b) and (Abstract 617-36, these Notices) 12 (1965), 53) \{φₜ\}_{t=1}^{∞} left slant converges at b and right slant converges at a to L_b and L_a respectively. If φ(x) = limₘ→∞φₜ(x) if x ∈ (a,b)φₜ(a) = L_b and φₜ(b) = L_a then φ is of bounded variation on [a,b]. (Received September 27, 1965.)

66T-14. ALAN HOWARD, Stanford University, Stanford, California. On the homotopy groups of an affine algebraic hypersurface.

Let V be a variety in complex projective space Pⁿ₊₁, with Vₘ⁺ = V ∩ \{z₀ = 0\}, and M = V - Vₘ⁺. Theorem 1. If V is nonsingular and nowhere tangent to the hyperplane \{z₀ = 0\}, then π₁(M) = 0 for 1 ≤ i ≤ n - 2; also π₋₁(M) = 0 or is finite cyclic. Theorem 2. Assume that there is a finite set S contained in Vₘ⁺ such that V - S is a manifold which is nowhere tangent to the hyperplane \{z₀ = 0\}. Then π₁(M) = 0 for 1 ≤ i ≤ n - 3; also π₋₂(M) = 0 or is finite cyclic. Theorem 1 complements a theorem of I. Fary on homology in Cohomologie des variétés algébriques, Ann. of Math. 65 (1957), 21-73. The transition from Theorem 1 to Theorem 2 is accomplished via: Theorem 3. Let V be an
r-dimensional subvariety of $\mathbb{P}^m$, and assume that there is a finite set $S$ contained in $V_0$ such that $V - S$ is a manifold which is nowhere tangent to the hyperplane $\{z_0 = 0\}$. Then for almost all hyperplanes $H$ in $\mathbb{P}^m$, the homomorphism $\pi_i(M \cap H) \to \pi_i(M)$ induced by inclusion is bijective for $i < r - 1$ and surjective for $i = r - 1$. The proof of theorem 1 is based on methods used by A. Andreotti and T. Frankel in The Lefschetz theorem on hyperplane sections, Ann. of Math. 69 (1959), 713-717 (as modified by J. Milnor); that of Theorem 3 is modelled on an unpublished proof by the same authors. (Received September 27, 1965.)


If $p$ is an arbitrary prime and $a$ is any positive integer, let $N(A,B,a)$ denote the number of incongruent solutions $U,V$ of the congruence $^*UA V = B (\mod p^a)$, where $A,B,U$ and $V$ are integral $m \times n$, $s \times t$, $s \times m$ and $n \times t$ matrices, respectively. $N(A,B,1)$ is known explicitly as a special case of a result of the author (Duke Math. J. 22 (1955), 497-510). In the present paper, an expression is obtained for $N(A,B,k+1)$ for all $k \geq 1$ as a sum involving the numbers of solutions of certain linear matrix congruences (mod $p$) in two unknowns associated with the solutions of $^*$. These latter numbers were determined by the author in (Annali di Mat. (IV) 44 (1957), 245-250). Using the general expression, the explicit value of $N(A,B,k+1)$ for all $k \geq 1$ is found when $A$ and $B$ are square and nonsingular (mod $p$). Finally, for $A$ and $B$ square and $|B| \neq 0$, a recursion formula for $N(A,B,k)$, for $k$ sufficiently large, is obtained. These results for $A$ and $B$ square are the bilinear analogs of some lemmas of C. L. Siegel on representations by quadratic forms (mod $p^a$) in (Ann. of Math. (2) 36 (1935), 527-506). (Received September 27, 1965.)

66T-16. JAN MYCIELSKI, Institute of Mathematics, Polish Academy of Sciences, Plac PKWN 6, m. 5, Wroclaw, Poland. A generalisation of the Bolzano-Weierstrass theorem.

A bounded closed set in a locally compact convex complete metric space is compact. (Received September 28, 1965.)

66T-17. C. A. PERSINGER, Air Force Institute of Technology, Air University, Wright-Patterson Air Force Base, Ohio. $m$-frames in tame n-books in $\mathbb{E}^3$.

An $n$-book $B^n$ in $\mathbb{E}^3$ is the union of $n$ disks in $\mathbb{E}^3$ meeting precisely along a common arc on the boundary of each. Theorem 1. Each mildly wild $m$-frame in $\mathbb{E}^3$ lies in a tame 3-book in $\mathbb{E}^3$. Theorem 2. Let $F_m$ be an $m$-frame in $\mathbb{E}^3$ which lies on a disk $D$ in $\mathbb{E}^3$ and suppose that $D$ is locally tame except at a finite number of points on $Bd D$. Then $F_m$ lies in a tame 3-book in $\mathbb{E}^3$. Theorem 3. Let $F_m = \bigcup_{i=1}^{m} A_i$ be an $m$-frame in $\mathbb{E}^3$ which is locally tame except at $p = \bigcap_{i=1}^{m} A_i$ and which has the property that $A_i$ is locally wild at $p$ for $i = 1,2,\ldots,m$. If $A_i$ and $A_j (i \neq j)$ are not locally linked in $\mathbb{E}^3$, then $F_m$ lies in a tame 3-book $B^3$ in $\mathbb{E}^3$. (Received October 1, 1965.)
Let \( n \) denote an integer greater than 1. \textbf{Theorem.} Let \( T_n \) be the \( n \)-torus and let \( X \) be a metric space admitting \( n \) bounded real-valued continuous functions one of which will distinguish between any two distinct points of \( X \) closer to each other than some positive constant. Then \( T_n \times X \times X \) admits an expansive homeomorphism. \textbf{Indication of proof.} Let \( \text{diag}(r_1, \ldots, r_n) \) induce an expansive homeomorphism on the \( n \)-torus, and let \( f_1, \ldots, f_n \) be the functions which distinguish between neighboring points of \( X \). If the ranges of these functions are small compared to the diameter of a fundamental region for the torus, then the homeomorphism of \( E_n \times X \) onto itself defined by \( F(y_1, \ldots, y_n, x) = (r_1 y_1 + f_1(x), \ldots, r_n y_n + f_n(x), x) \) induces an expansive homeomorphism on \( T_n \times X \). \textbf{Definition.} A metric space \( X \) is said to be uniformly immersed in a space \( Y \) if there exists a continuous map from \( X \) into \( Y \) whose restriction to any neighborhood of a certain positive radius is a homeomorphism. \textbf{Corollary.} If \( X \) can be uniformly immersed in a compact subset of \( E \), then \( T_n \times X \times X \) admits an expansive homeomorphism. (Received September 29, 1965.)


Let \( f \) be analytic in the open unit disk and \( 0 < R \leq 1 \). Put \( \|f\| = \sup \{|f(z)|: |z| < R\} \) and \( E_n(f) = \inf \{\|f - P\|: P \text{ is a polynomial of degree not exceeding } n\} \). Let \( g(z) = \sum_{k=0}^{\infty} a_k z^k \) where \( |a_k| \leq 1 \) and \( B(g) \) be the set of all functions representable in the form: \( f(z) = \sum_{k=0}^{\infty} a_k z^k \) and \( |h(z)| \leq 1 \) in \( |z| < 1 \). We also require \( a_n \geq 0 \) and \( \sum_{k=1}^{\infty} a_k r_k \cos kx \leq 0 \) for \( 0 \leq r < 1 \) and \( x \) real. Then \( f \in B(g) \) implies \( E_{n-1}(f) \leq a_n R^n \). The inequality cannot be improved for the class \( B(g) \) since \( f(z) = a_n z^n \in B(g) \) and \( E_{n-1}(f) = a_n R^n \). The proof is based on the Cauchy integral formula and a certain \( L_1 \) minimization problem. Applications of the theorem include a generalization of a theorem of Babenko (Izv. Akad. Nauk SSSR Ser. Mat. 22 (1958) 631-640): \( p \geq 0 \) and real, \( \|f^{(p)}(z)\| \leq 1 \) in \( |z| < 1 \) imply \( E_{n-1}(f) \leq (n + p + 1)^{(n+1)^{-1}} R^n \). Babenko proves this only for natural \( p \). As another application we cite: let \( A_q \) be the class of functions of the form \( f(z) = z^p \int_0^1 h(z) dq(t) \) where \( h \) is analytic and bounded in modulus by 1 in \( |z| < 1 \), \( p \) is natural, and \( q \) is increasing and bounded in \([0,1]\). Then \( f \in A_q \) implies \( E_{n-1}(f) \leq R^n \int_0^1 |z|^p dq(t) \) for \( n \geq p \). Again, this inequality cannot be improved for \( f \in A_q \). (Received October 4, 1965.)

\textbf{66T-20. HAROLD WIDOM, University of California, Berkeley, California, and H. S. WILF, University of Pennsylvania, Philadelphia, Pennsylvania. Small eigenvalues of large Hankel matrices.}

Denote by \( \lambda_N \) the smallest eigenvalue of the \((N+1) \times (N+1)\) Hankel matrix \( (c_{m+n})_{m,n=0,\ldots,N} \), where the \( c_n \) are the moments of a distribution function \( a(x) \) on the finite interval \([a,b]\) which satisfies \( \int_a^b [x - a] (b - x)^{-1/2} \log a'(x) dx \geq - \infty \). An asymptotic formula is found for \( \lambda_N \) as \( N \to \infty \). This formula takes the form \( \lambda_N \sim \rho_N^{1/2} \sigma - 2 N \) except for certain cases in which \( 1/2 \) becomes \( 1/4 \). The constant \( \sigma \) depends only on the interval \([a,b]\). For the special case of the Hilbert matrix \( (c_n = 1/n + 1) \) the result is \( \lambda_N \sim 2^{9/8} \pi^{3/2} (73 - 48 \sqrt{2})^{-1} N^{1/2} (3 + 2 \sqrt{2})^{-2} N^{-3/4} \). (Received October 4, 1965.)
Two weak bundle properties are defined. The second of these is as follows: A mapping \( p : T \to B \) is said to have the weak bundle property II with respect to a space \( K \) if and only if for each \( b \) in \( B \) and each mapping \( g_b : p^{-1}(b) \to K \), there exists an open set \( U_b \) containing \( b \) and a mapping \( \phi_{U_b} : p^{-1}(U_b) \to U_b \times K \) such that \( \phi_{U_b} \) extends \( g_b \) and \( p|_{p^{-1}(U_b)} = \pi \phi_{U_b} \) where \( \pi \) is the projection mapping of \( U_b \times K \) onto \( U_b \). The weak bundle property I is similar where \( \phi_{U_b} \) maps \( U_b \times K \) into \( p^{-1}(U_b) \).

**Theorem.** Suppose that \( p : T \to B \), each of \( T \) and \( B \) is a metric space, \( T \) is complete, covering \( \dim B \leq n + 1 \), and \( p \) is completely regular (Dyer-Hamstrom). Furthermore, \( K \) is a complete metric space such that the space \( G_b \) of all mappings of \( p^{-1}(b) \) into \( K \) is \( LC^n \) for each \( b \) in \( B \). Then \( p \) has the weak bundle property II w.r.t. \( K \). **Application.** Suppose that \( p \) is an open mapping from an \( n \)-manifold \( M^n \) to a metric space \( B \) such that \( p^{-1}(b) \) is homeomorphic to a Cantor set. Such is the case if the dyadic group acts freely on \( M^n \). Let \( K = [0, 1] \). Then \( p \) has the weak bundle property II. In particular, if \( g_b \) is a homeomorphism of \( p^{-1}(b) \) into \( K \), there is an extension \( \phi_{U_b} \) of \( g_b \) which takes \( p^{-1}(U_b) \) into \( U_b \times K \) so that \( p|_{p^{-1}(U_b)} = \pi \phi_{U_b} \) where \( U_b \) is open and \( b \in U_b \). (Received September 27, 1965.)


Let \( X \) be a locally compact Hausdorff space, \( Y \) a compact Hausdorff space, \( \mathbb{M}^+ \) the set of positive Borel measures on \( Y \) endowed with the \( w^* \)-topology, \( U_e \) the subset of \( \mathbb{M}^+ \) consisting of unit measures concentrated on points of \( Y \), and \( P \) a mapping from \( X \times \mathbb{M}^+ \) into the positive reals. If the mapping \( P \) is (i) continuous with respect to the product topology on \( X \times \mathbb{M}^+ \) and (ii) for each \( x \in X \), \( P(x, \mu) \) is additive and positive homogeneous in \( \mu \), then \( \sup \{ P(x, \mu)/P(y, \mu) : x, y \in K, \mu \in \mathbb{M}_e \} = \sup \{ P(x, \mu)/P(y, \mu) : x, y \in K, \mu \in U_e \} < +\infty \) for each compact subset \( K \) of \( X \). (Received October 7, 1965.)

66T-23. A. A. MULLIN, University of California, Box 808, Livermore, California 94551. On fixed-point results for some computable functionals.

Define the computable arithmetic functional \( \Psi_1(\psi^j, (\psi^*)^k, n) = \psi^j(n) + (\psi^*)^k(n) \), where \( n \) is a natural number and computable functions \( \psi^j \) and \( (\psi^*)^k \) are defined Proc. Nat. Acad. Sci. U.S.A. 50 (1963), 604-606. Let \( f \) be any function satisfying the condition that one of the (Cartesian) factors \( R \) of its range is a subset of one of the factors \( F \) of its domain. Then \( x \in F \) is called a quasi-fixed-point of \( f \) provided \( f^*(x) = x \), where \( f^* \) is obtained from \( f \) by keeping elements of the other factors, if any, of the domain and range of \( f \) fixed. **Lemma 1.** For every pair of natural numbers \( j \) and \( k \) ("exponents" of \( \psi \) and \( \psi^* \), respectively) there exists an \( n \) such that \( n \) is a quasi-fixed-point of \( \Psi_1 \). E.g., \( \Psi_1(\psi, \psi^*, 40) = 40 \). **Lemma 2.** If \( n > 1 \) is a quasi-fixed-point of \( \Psi_1 \) for some \( j \) and \( k \), then \( n \) isn't a quasi-fixed-point of \( \Psi_1 \) for any pair of "exponents" \( j' \) and \( k' \) with \( j' > j \) or \( k' > k \). **Lemma 3.** The asymptotic density of natural number quasi-fixed-points of \( \Psi_1 \) is zero; e.g., the square-frees \( > 1 \) are not quasi-fixed-points of \( \Psi_1 \). (Received October 11, 1965.)
Let $R$ be a commutative ring with identity and let $R[X]$ be the polynomial ring in one indeterminate over $R$. Each $R$-endomorphism of $R[X]$ is a substitution map determined by some element $f \in R[X]$. Denote by $\phi_f$ the $R$-endomorphism such that $\phi_f(g) = g(f)$ for all $g \in R[X]$. Let $f = \sum_{i=0}^{n} f_i X^i$.

**Theorem 1.** $\phi_f$ is onto if and only if $f_1$ is a unit of $R$ and $f_i$ for $i \geq 2$ is nilpotent. **Corollary.** $\phi_f$ is an $R$-automorphism of $R[X]$ if and only if $\phi_f$ is onto. Now let $S$ be an arbitrary commutative ring containing a nonzero regular element and let $T$ be the total quotient ring of $S$. Then each $R$-endomorphism of $R[X]$ is induced by a $T$-endomorphism of $T[X]$. For $f = \sum_{i=0}^{n} f_i X^i \in T[X]$, denote by $\psi_f$ the restriction of $\phi_f$ to $R[X]$.

**Theorem 3.** $\phi_f$ is one-to-one if and only if $\psi_f$ is one-to-one, $\psi_f$ is an $R$-endomorphism of $R[X]$ if and only if $f_1 R \subseteq R$ for all $i$, $\psi_f$ is an $R$-automorphism of $R[X]$ if and only if $f_1 R \subset R$, and $f_i$ for $i \geq 2$, is nilpotent. (Received October 13, 1965.)

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Let $\{p_n\}$ be a sequence of real positive constants such that $P_n \to \infty$, where $P_n = \sum_{m=0}^{n} p_m$.

**Theorem.** If $\{\lambda_n\}$ is a convex sequence such that $\sum p_n \lambda_n < \infty$, then $\{\lambda_n\}$ is a non-negative monotonic decreasing sequence tending to zero and $P_n \lambda_n = o(1)$, $n \to \infty$. This generalises a lemma of Chow (J. London Math. Soc. 16 (1941), 215-220). Two corollaries of this theorem have also been obtained which generalise various known results. (Received October 14, 1965.)

Consider Turing machines with one-way infinite tapes, $n$ numbered internal states, the tape symbols blank, 0, and 1, starting in state 1 and halting in state $n$, and in which the number $j$ of the next internal state after being in state $i$ satisfies $|i - j| \leq b$. $b$ can be chosen sufficiently large that any effectively computable infinite binary sequence is computable by such a machine. Such a Turing machine is said to compute a finite binary sequence $S$ if starting with its tape blank and scanning the end square of the tape, it finally halts with $S$ written at the end of the tape, with the rest of the tape blank, and scanning the first blank square of the tape. Define $L(S)$ for any finite binary sequence $S$ by: A Turing machine with $n$ internal states can be programmed to compute $S$ if and only if $n \geq L(S)$. Define $L(C_n)$ by $L(C_n) = \max L(S)$, where $S$ is any binary sequence of length $n$. Let $C_n$ be the set of all binary sequences of length $n$ satisfying $L(S) = L(C_n)$. Then (1) $L(C_n) \sim an$. (2) There exists a constant $c$ such that for all $m$ and $n$, those binary sequences $S$ of length $n$ satisfying $L(S) < L(C_n) - \lceil \log_2 n \rceil - m - c$ are less than $2^{n-m}$ in number. (3) For any $e > 0$ and $d > 1$, for all $n$ sufficiently large, if $S$ is a binary sequence of length $n$ such that the ratio of the number of 0's in $S$ to $n$ differs from $1/2$ by more than $e$, then $L(S) < L(C_{n \lceil \log_2 (1/2 + e, 1/2 - e) \rceil})$. Here $H(p,q) = -p \log_2 p - q \log_2 q$. We propose also that elements of $C_n$ be considered the most patternless or random binary sequences of length $n$. This leads to a definition and theory of randomness related to the R. von Mises A. Wald-A. Church theory, but in accord with some criticisms of K. R. Popper. (Received October 19, 1965.)
For each member of a set \( S \) of polygenic functions \( Z = Z(z) \), such that each one is of at least class \( C^3 \) on a region of the \( z \)-plane, and no one is either direct or reverse holomorphic, a generalized Schwarzian derivative \( [Z,z; a,\beta,\gamma,\delta] \) of the third order, is defined. The indices \((a,\beta,\gamma,\delta)\) vary on the set of two elements \((1,2)\), for which the first and second elements denote the mean and phase operators respectively. The two derivatives \([Z,z; 1,1,1,1] = [Z,z] \) and \([Z,z; 2,2,2,2] = \{Z,\bar{Z}\} \) represent the corresponding two Schwarzian mean and phase derivatives. Under a nondegenerate polygenic transformation \( T \) of at least class \( C^3 \), the Schwarzian derivative \( [Z,z] \) of \( T = T(\Gamma) \) relative to the arc length \( s \) of the original arc \( \Gamma \), is obtained in terms of these new symbols, and the intrinsic invariants of this arc \( \Gamma \). These symbols lead to characterizations of the conformal group. Finally, derivations are obtained in terms of these symbols, of the two Kasner conformal measures of first order and second order horn angles. See Kasner: The two conformal invariants of fifth order, Trans. Amer. Math. Soc. 44 (1938), 25-31. (Received October 20, 1965.)

Theorem 1. If \( f \) is a function on a uniform space \((X, \mathcal{U})\) with values in a uniform space \((Y, \mathcal{V})\) such that for each Cauchy net \( \{S_n, n \in D\} \) in \((X, \mathcal{U})\) \( \{f \circ S_n, n \in D\} \) is also a Cauchy net in \((Y, \mathcal{V})\), then \( f \) is continuous relative to the uniform topologies. Theorem 2. A function \( f \) from a uniform space to a uniform space preserves Cauchy nets iff \( f \) preserves Cauchy filter bases.

Theorem 3. If every continuous function \( f \) on the uniform space \((X, \mathcal{U})\) preserves Cauchy nets then \((X, \mathcal{U})\) is complete. (Received October 20, 1965.)


Let the subelementary functions be those which are obtained by addition, multiplication and composition from the real variable \( x \), the constants \( \pi \), \( \log 2 \), the rational numbers, and the functions \( e^x \) and \( \sin x \). It is shown that there is no effective method of deciding whether there is a real number \( x \) such that \( f(x) \) is less than zero as \( f \) ranges over the subelementary functions. Enlarging the class of functions by adding \( 1/x \) and \( |x| \), it is shown that the following statements are not decidable as \( g \) ranges over the functions in the enlarged class: (1) \( g(x) = 0 \); (2) \( \int_0^x g(x)dx = 0 \). (3) There is a function \( h \) in the enlarged class so that, for all \( x \), \( \int_0^x (x)dx = h(x) \). (Received October 20, 1965.)


If \( s^n \) is an \( n \)-simplex of a complex \( K \) which is not a face of any \((n + 1)\)-simplex of \( K \), and if \( s^{n-1} \) is an \((n - 1)\) face of \( s^n \) which is not a face of any other \( n \)-simplex of \( K \), the complex \( K - s^n - s^{n-1} \) is an elementary contraction of \( K \). If the complex \( L \) is the result of a sequence of elementary contractions of \( K \), \( L \) is a formal contraction of \( K \) (Whitehead, Proc. London Math. Soc., 1939). Two theorems are proved. (1) Any subdivided \( n \)-simplex contracts formally to a subcomplex of its \((n - 1)\) skeleton. (2) If \( s^{n-1} \) is an \((n - 1)\)-simplex of the subcomplex in (1), it has at most one \((n - 2)\) face which is not on some other \((n - 1)\)-simplex of the subcomplex. (Received October 21, 1965.)


Theorem. Let \( A \) be the infinitesimal generator of a contraction semigroup on a Banach space \( X \), and let \( B \) be a dissipative operator with \( D(A) \subseteq D(B) \). If there exist constants \( a \) and \( b \) with \( a < 1 \) such that for all \( u \in D(A) \), \( \|Bu\| \leq a \|Au\| + b \|u\| \), then \( A + B \) is the infinitesimal generator of a contraction semigroup. Corollary. Under the above conditions, except with \( a < \frac{1}{2} \), \( c(A + dB) \) is the infinitesimal generator of a contraction semigroup for all \( c \geq 0 \) and all \( 0 \leq d \leq \frac{a}{1 - a} \). Reference. See E. Nelson, Feynman integrals and the Schrödinger equation (J. Math. and Phys. 5 (1964), 332-343), where the above theorem is proved for \( a < 1/2 \). (Received October 22, 1965.)
Analogue of a theorem of Khintchine in a field of formal Laurent series.

Let $K$ be a finite field. Let $K\{t\}$ be the field of all formal Laurent series over $K$, i.e., $K\{t\} = \{ \sum_{i=-\infty}^{\infty} a_i t^i : a_i \in K, n \text{ some integer} \}$. We define a valuation $|\cdot|$ on $K\{t\}$ by $|0| = 0$, $|\sum_{i=-n}^{\infty} a_i t^i| = e^n$, and also define $\sum_{i=-n}^{\infty} a_i t^i = e_k$, $k$ the largest occurring negative exponent of $t$. Theorem. Given $n$ linear forms in $m$ variables over $K\{t\}$, say $L_j(X) = L_j(x_1, \ldots, x_m)$, $1 \leq j \leq n$, we have

$$(\max_{1 \leq j \leq n} |L_j(X)| - \alpha_j)^m \leq C$$

for some $\Lambda = (\alpha_1, \ldots, \alpha_n)$, $\alpha_i \in K\{t\}$, and for all nonzero $(x_1, \ldots, x_m)$, $x_1$ a polynomial in $t$, $C$ a constant. We may take $C = e^{-n-1}$ and this is best possible. A much weaker result (in particular, with a constant dependent on $m$) was announced by S. K. Aggarwal (Abstract 65T-357, these Notices 12 (1965), 620). (Received October 25, 1965.)

Lattice-embeddings in the degrees and hyperdegrees.

**Theorem 1.** There is a class $\{h_R : R \text{ recursive } \subseteq N\}$ of hyperdegrees such that (1) $h_R \leq h_S$ iff $R \subseteq S$, (2) $h_R \cup S = h_R \cup h_S$, (3) $h_R \cap S = h_R \cap h_S$, (4) $h_R \leq \text{hyp}(O)$ where $O$ is Kleene's set of constructive ordinal notations. **Corollary.** Any countable partial ordering $S$ is embeddable in the hyperdegrees $\leq \text{hyp}(O)$, in a way which preserves all finite joins (or all finite meets) which exist in $S$.

**Theorem 2.** There is a class $\{h_a : a \in 2^N\}$ of hyperdegrees such that 1 - 3 above hold for finite $R, S \subseteq 2^N$, $h_R$ being the join of the $h_a$ for $a \in R$. **Corollary.** The corollary above holds for partial orderings of cardinality at most $c$ in which each element has only finitely many predecessors. The theorems and corollaries hold also for degrees (with the same embedding), extending some results of Sacks [Degrees of unsolvability, Princeton, 1963]. All the results can be relativized. The proofs use an adaptation of the forcing method, cf. the author's previous abstracts [624-3 and 625-130, these Notices 12, (1965), 449-583], (Received October 27, 1965.)

Seminormed Riesz spaces and the Egoroff property.

Corresponding to each (monotone) seminorm $\rho (+ \infty \text{ admitted as value})$ on a Riesz space $L$, there is a seminorm $\rho_M$ which is "$\sigma$-Fatou" (i.e., $0 \leq u_{n}^\uparrow u \Rightarrow \rho_M(u_{n}^\uparrow u) \leq \rho_M(u)$) and which is maximal among those $\sigma$-Fatou seminorms dominated by $\rho$. Lorentz pointed out that, if $L$ is a Banach function space, $\sigma$-finiteness of the underlying measure space implies that $\rho_M$ may be calculated simply as $\rho_L$, where $\rho_L(u) = \inf \{ \lim \rho(u_n) : 0 \leq u_n^\uparrow u \}$. A weaker condition on $L$ is the Egoroff condition: $0 \leq u_{n,k}^\uparrow u \Rightarrow |3 \text{ sequence } u_m \text{ such that } u_m^\uparrow u \text{ and } (\forall n,m)(3k : u_m \leq u_{n,k})|$ (cf. Nakano's notion of "total continuity"). **Theorem.** If $L$ is Archimedean (a suitably modified theorem holds in general), $L$ is Egoroff if, and only if, $\rho_M = \rho_L$ for each seminorm $\rho$ on $L$. The necessity of the Egoroff condition is the important part; this shows, for example, that $C[0,1]$ possesses a Riesz norm $\rho$ with $\rho_{LL} \neq \rho_L$. In answer to some related questions of Luxemburg [Banach function spaces, Note XVI, Proc. Acad. Sci. Amsterdam, A68 (1965)] we construct the following: a non-Egoroff space which nevertheless possesses an integral Riesz norm; a space with an integral Riesz norm which is nowhere normal. (Received October 28, 1965.)
Let $S$ be a commutative semigroup and $I(S)$ the set of all identities of $S$. If $S$ is equationally nontrivial (e.n.t.) then there exists a least integer $m > 0$ and for that $m$ a least integer $k > 0$ such that $x^m = x^{m+k} \in I(S)$. **Theorem 1.** $I(S)$ always has a finite basis. **Theorem 2.** If $S$ is e.n.t. and has a set of $n$ generators then $I(S)$ has a finite basis involving no more than $2mn + 1$ distinct variables. **Definition.** $a \in S$ is prime if for all $b, c \in S$ $a \neq bc$. **Theorem 3.** If $S$ is e.n.t. and has exactly $p$ distinct primes then a rather crude bound on the number of distinct variables needed for a finite basis of $I(S)$ is $(pm + 2)(m + k)(m + 2k)/k$. **Theorem 4.** Given any $j > 0$ one can construct a finite commutative semigroup $S_j$ such that every basis of $I(S_j)$ involves at least $j$ distinct variables.

(Received October 28, 1965.)

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**66T-38.** EUGENE SPIEGEL, California Institute of Technology, Pasadena, California. **On the automorphisms of the unitary group over a field of characteristic $2$.**

Let $K$ be a field of char 2 which admits an involutary automorphism $--$. Let $V$ be an $n$ dimensional vector space over $K$ and $f$ a nonsingular hermitian form defined on $V \times V$. Denote by $U_n(K,f)$ the corresponding unitary group. Let $v$ be the dimension of a maximal totally isotropic subspace of $V$ and $K_0^v = \{k \in K | k^2 = 1\}$. **Theorem.** Let $\psi$ be an automorphism of $U_n(K_0^v)$. $\nu > 1$, $n > 5$ then $\psi(\lambda) = X(\lambda)\phi(\lambda)^{-1}$, $\lambda \in U_n(K_0^v)$ where $X$ is a "character" from $U_n(K_0^v)$ into $K_0^v$ and $\phi$ is a semi-linear transformation of $V$ onto itself, relative to an automorphism $\tau$ of $K$ which permutes with $--$, and $(\phi(x), \phi(y)) = \rho(x,y)\tau$ identically in $V$ for some $\rho \in K$. (Received October 28, 1965.)

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**66T-39.** ALBERT SADE, 86 Cours de la République, Pertuis, Vaucluse, France. **Quasigroupes demi-symétriques, II, Autotopies gauches.**

Si un quasigroupe $Q = E(\cdot)$ coincide avec son transposé par l'isotopie $I = (a,b,c)$ les paras- trophiques de $Q$ sont transportés sur leur transposé par des isotopides dérivées de $I$ par inversion et permutation circulaire, $(acb, bac, cba)$ est une autotopie de $Q$ et $(ac, ba, cb)$ applique $Q$ sur son transposé $G - D$. Si $a = b = 1$, le groupe de semi-automorphisme de $Q$ est le centralisateur de $c$ dans $E$. $I^{-1}$ applique l'adjoint de $Q$ sur son transposé et les isotopies qui préservent cette propriété sont $(Xr, YXr, r)$, $r$ arbitraire $\in \Theta_E(X,Y,Z)$, autotopie de $Q$. Chacune des 4 conditions $\Gamma_\alpha \Delta_\alpha = D^{-1}$, $E_\alpha = D^{-1}\Delta_\alpha D$, $(QD = Q)$; $E_\alpha \Gamma_\alpha = D^{-1}$; ou $(xy)x = (zy)y$, avec $\Theta \in \Theta_E$, $D = \text{Constante}$, $\Gamma, \Delta, E$ étant les translations à gauche, à droite et au milieu, est nécessaire et suffisante pour que $Q$ et son transposé soient identiques par une distorsion. Les deux conditions $E_\alpha \Gamma_\alpha = \Gamma_\alpha \Delta_\alpha$; ou $(xy)x = [xy]y(xy)$ sont seulement nécessaires. Le sous-ensemble des $(a,b,c)$ pour lesquelles $(acb, bac, cba)$ = Constante est contenu dans le produit par l'une d'elles du centralisateur de $I$ dans le groupe d'autotopie de $Q$. (Received October 28, 1965.)

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**66T-40.** LAWRENCE NARICI, Polytechnic Institute of Brooklyn, Brooklyn, New York. **On nonarchimedian Banach algebras.**

Let $X$ be a nonarchimedian Banach algebra with identity over a field $F$ with a nontrivial, nonarchimedian valuation, $| |$, (i.e., $\| x + y \| \leq \max (\| x \|, \| y \|)$) together with the customary axioms.

(Received October 28, 1965.)
In addition, it is assumed that for every maximal ideal $M$ of $M$, the set of all maximal ideals of $X$, $X/M$ is isomorphic to $F$. With this assumption, the usual Gelfand notation is adopted to designate the element $x(M) \in F$ corresponding to the coset $x + M$. The paper investigates structural properties of $X$. In particular the $\alpha$-modules, (where $\alpha$ is the valuation ring of $F$ associated with $| |$), $V = \{ x \in X: |x| \leq 1 \}$ and $W = \{ x \in X: |x(M)| \leq 1 \}$ for all $M \in M$, are investigated. It is shown, among other things, that if $M$ is compact in the Gelfand topology (which will be the case in particular if $F$ is complete with respect to $| |$, $| |$ is discrete and the residue class field is finite) then, if $V = W$, $X$ is congruent to $C_{F}(M)$: the algebra of continuous functions in $M$ with values in $F$ and with the usual supremum norm. The converse of this is also shown to be true. (Received November 3, 1965.)

66T-41. R. S. BUCY, University of Colorado, Boulder, Colorado. Linear positive machines.

Let $\mathcal{A}$ be a commutative Banach algebra with involution. A linear machine is a member of $\mathcal{A}^{*}$ the dual space of $\mathcal{A}$ considered as a Banach space. With every linear machine $f$ an ideal of $\mathcal{A}$, $\mathcal{I}_{f}$ is associated. The Krohn Rhodes definition of the state space is shown to lead to the state space of $f$ being identified with $\mathcal{A}/\mathcal{I}_{f}$. The following result holds; dimension $f$ = codimension $\mathcal{I}_{f} = \text{cardinality } S_{f}$ when $f$ is a positive linear functional and $S_{f}$ the support of its unique representing measure on the symmetric maximal ideal space. A notion of prime is introduced for positive machines and a decomposition theorem is obtained so that a positive machine is in the closed convex hull of its primes. When positivity is removed the above results are shown not to hold by counterexample. (Received November 3, 1965.)


Let $B$ be the system of set theory with a strong reflection principle proposed by P. Bernays. Kreisel has raised the question as to the consistency of $B$ with the continuum hypothesis (see Georg Kreisel, "Mathematical logic" in Lectures on modern mathematics, Wiley, 1965, especially pp. 112 for references). Within $B$ one can define an inner model within which $V = L$ and hence the general continuum hypothesis holds; further, within the model all classes are constructible in a natural sense, and thus there is a definable linear ordering of all classes which is essentially a well-ordering. These results are obtained from the methods developed in the author's doctoral thesis (M.I.T., 1965) where similar results for the impredicative extension of Godel-Bernay's set theory are proven. (Received November 2, 1965.)


Let $u$ satisfy $\ddot{u} + A(t)\dot{u} + B(t)u = 0$ with $A(t)$, $B(t)$ unbounded operators in a Hilbert space $H$ and let $u(s)$ and $\dot{u}(s)$ be presribed with $t \geq s$. The idea is to give conditions on $u(s)$ and $\dot{u}(s)$ (as well as on $A(t)$ and $B(t)$) which yield growth and convexity theorems for $Q = \|u\|^{2}$ or some $f(t, Q)$ as a function of some monotone function $w(t)$. Hypotheses are chosen to contain the case of correct problems and in particular some results in norm analogous to the pointwise results of Weinstein (Ann. Mat. 43 (1957), 325-340) are obtained. For example the condition (i) $\dot{Q}(s) \geq 0$ with $X(s) =$
\[ \|u(s)\|^2 + \text{Re}(u,Bu)(s) \leq 0 \text{ leads to } \dot{Q}(t) \geq 0 \text{ under realistic conditions. Thus when (2) } 2\text{Re}(u,Bu) = (u,Bu)^2 + N(u) \text{ holds and } A(t) \text{ is accretive one has } X(t) \sim X(s) - \int_0^t \text{Re} N(u) \text{ and then a further estimate (3) } 2\text{Re}(A\dot{u},u) \leq h(t)Q - G(u) \text{ with } G(u) + 2\int_0^t N(u) \geq 0 \text{ insures the conclusion. Convexity of } \log(gQ) \text{ of } gQ \text{ as a function of } w(t) \text{ follows also from these hypotheses and realistic inequalities between } g(t), w(t), h(t) \text{ and various of their derivatives.} \]

The equation \[ z + E(t)z = 0 \text{ with } E = B - \dot{A}/2 - A^2/4 \text{ is also examined in this context and corresponding results obtained.} \]

Realizations of the hypotheses are given. (Received November 4, 1965.)

66T-44. ALEXANDER ABIAN and DAVID DEEVER, The Ohio State University, 231 W. 18th Avenue, Columbus, Ohio 43210. On representation of partially ordered sets.

**Definition.** A subset D of a partially ordered set \((P, \leq)\) is called very-weakly-dense in \(P\) if for every two elements \(p\) and \(q\) of \(P\) with \(q \leq p\) there exists an element \(d_1\) of \(D\) such that \(d_1 \leq q\) and \(d_1 \neq p\). Moreover \(D\) is called weakly-dense in \(P\) if there exists an element \(d_2\) of \(D\) such that \(d_2 \leq q\) and \(d_2 \neq p\). **Theorem 1.** For every ordinal number \(\lambda\) a partially ordered set \((P, \leq)\) has a very-weakly-dense subset of power \(N_\lambda\) if and only if it has a weakly-dense subset of power \(N_\lambda\).

**Theorem 2.** Let \((P, \leq)\) be a partially ordered set with a very-weakly-dense subset of power less than or equal to \(N_\lambda\). Then \(P\) is isomorphic to a set of sequences of 0 and 1 of type \(\omega_\lambda\) partially ordered by the principle of strong first differences. **Theorem 3.** Let \(S\) be any set of sequences of 0 and 1 of type \(\omega_\lambda\), partially ordered by the principle of strong first differences. Then there exists a set \(S'\) of sequences of 0 and 1 of type \(\omega_\lambda\) with \(S \subset S'\) such that \(S = S'\) and \(S'\) has a weakly-dense subset of power less than or equal to \(N_\lambda\). (Received November 4, 1965.)

66T-45. HENRY TEICHER, Purdue University, Lafayette, Indiana and JACOB WOLFWITZ, Cornell University, Ithaca, New York 14850. Existence of optimal stopping rules for linear and quadratic rewards.

Let the independent, identically distributed random variables \(\{x_n' \mid n \geq 1\}\) be observed sequentially, i.e., for any positive integer \(n\), one may, having observed \(x_1, ..., x_n\), stop sampling and accept (case (i)) the linear reward \(c_n S_n = c_n \sum_{i=1}^n x_i\), (case (ii)) the quadratic reward \(c_n^2 S_n^2\). Let \(T = \{t\}\) denote the class of all stopping variables (rules), i.e., \(t\) is a positive integer-valued random variable such that \(P\{t < \infty\} = 1\), \(t = n\) means that sampling ceases after \(x_1, ..., x_n\) have been observed, and the event \(\{t = n\}\) is in the \(\sigma\)-algebra generated by \(x_1, ..., x_n\), for all \(n \geq 1\). For \(j = 1\) or 2, it is proved that, if \(E x_n = 0\), \(E x_n^2 < \infty\), and \(c_n > 0\), \(c_n^2 \leq c_n \cdot c_{n+2}\) \((n + 1)c_{n+1} \leq n c_n\) there exists a stopping rule \(t_j\) such that \(E c_t S_j^2 = \sup_{t \in T} E c_t S_j^2\). In the case \(j = 1\), \(c_n = n^{-1}\), this result was proved by Y. S. Chow and H. Robbins, Ill. J. Math. 9, No. 3, 444-454, when the distribution of the \(x_i\) is binomial, and extended to arbitrary distributions (with \(E x_1 = 0\), \(E x_1^2 < \infty\)) by A. Dvoretzky (to appear). (Received November 4, 1965.)

66T-46. PETER COLWELL, 1108 South Fourth Street, Apartment 21, Ames, Iowa 50010. On the boundary behavior of functions meromorphic in the unit disk.

Let \(G\) be a plane domain whose boundary \(\Gamma\) has a positive logarithmic capacity. Let \(f(z)\) be a function meromorphic in \(D: \{ |z| < 1 \}\) which is of class \((L)\) with respect to \(G\) and \(\Gamma\) (cf. D. A. Storvick, 139
Proc. Amer. Math. Soc. 8 (1957), 32-38). The following generalize a previous result (cf. Abstract 621-26, these Notices) 12 (1965), 311, Theorem 1). **Theorem 1.** Let \( f(z) \) be a function of class (L) with respect to \( G \) and \( r, \) and suppose that \( f(e^{i\theta}) = \lim_{r \to 1} f(re^{i\theta}) \) exists and belongs to \( r \) for every \( e^{i\theta} \) on \( C: \{ |z| = 1 \}. \) Then if \( a \) is any point of \( G \) and \( A = \{ z \in D : f(z) = a \}, \) the derived set of \( A \) is closed and nowhere dense on \( C. \) In the case where \( G \) is simply-connected and \( \Gamma \) is a Jordan curve, the following holds: **Theorem 2.** Let \( a \) be any point of \( G, \) and let \( E \) be any closed nowhere dense set on \( C. \) Then there exists a function \( f(z) \) analytic in \( D \) such that: (i) \( f(z) \) assumes its values in \( G; \) (ii) for every \( e^{i\theta} \) on \( C, \) \( \lim_{r \to 1} f(re^{i\theta}) = f(e^{i\theta}) \) exists and belongs to \( r; \) (iii) if \( A = \{ z \in D : f(z) = a \}, \) then the derived set of \( A \) is the set \( E. \) (Received November 5, 1965.)


This paper extends and strengthens convergence properties previously published and announced [J. L. Walsh, J. H. Ahlberg, E. N. Nilson, J. Math. Mech. 11 (1962), 225-234; G. Birkhoff, and C. deBoor, J. Math. Mech. 13 (1964), 827-836; A. Sharma, and A. Meir, Abstract 64T-496, these Notices Amer. Math. Soc. 11 (1964), 768] for periodic splines and for nonperiodic splines satisfying general end conditions. If \( \{ \Delta_k \} \) is a sequence of meshes on \( [a,b] \) \( (\Delta_k: a = x_k, 0 < x_{k,1} < \ldots < x_{k,N_k} = b) \) with \( \| \Delta_k \| \to 0 \) as \( k \to \infty, \) if \( f(x) \) is in \( C^q[a,b] \) \( (q = 0,1,2,3), \) and if \( S_k(x) \) is the spline of interpolation to \( f(x) \) on \( \Delta_k, \) then \( f^{(p)}(x) - S_k^{(p)}(x) = O(\| \Delta_k \|^q + \| a - p \|), \) \( p = 0,1,\ldots,q. \) If \( f^{(q)}(x) \) satisfies a Holder condition on \( [a,b] \) of order \( \alpha(p \leq a \leq 1), \) then \( f^{(p)}(x) - S_k^{(p)}(x) = O(\| \Delta_k \|^q + \| a - p \|) \) \( (\text{for } q = 0,3 \text{ we require } \| \Delta_k \|/\min(x_{k,j} - x_{k,j-1}) \leq \beta < \infty). \) For \( f(x) \) in \( C^4[a,b], \) with the two stated mesh conditions and the meshes becoming asymptotically uniform, we have \( \max_j \left[ \frac{S_k^{(p)}(x_{k,j} +) - S_k^{(p)}(x_{k,j} -)}{\| \Delta_k \|^{q+\| a - p \|}} \right] \to 0. \) It is further shown that the convergence rate of \( S_k^{(p)}(x) \) to \( f^{(p)}(x) \) can be no greater than \( O(\| \Delta_k \|^{q+\| a - p \|}) \) unless \( f(x) \) is a cubic. (Received November 8, 1965.)


The results of Abstract 600-11, these Notices 10 (1963), 266 are extended to torsion-free Abelian groups of arbitrary cardinality. For every ordinal \( \theta \geq 1, \) let \( (\omega^* + 1 + \omega^0_0) \) denote the order type of the antilexicographically ordered set of all \( \theta \)-termed sequences of integers that are almost all zero; let \( (\omega^* + 1 + \omega^0_0) = 1. \) For every ordinal \( \theta, \) let \( \overline{\theta} \) denote the cardinal of the same power as \( \theta. \) For every cardinal \( \phi \geq 1, \) let \( \mathbb{F}_\phi \) be the free Abelian group of rank \( \phi. \) (1) \( \mathbb{F}_1 \) obviously admits only one type of linear order, \( (\omega^* + 1 + \omega^0_0) \) \( (\text{2}) \) If \( \phi \geq 2, \) then \( \mathbb{F}_\phi \) admits precisely the following kinds of linear order: \( (\omega^* + 1 + \omega^0_0), \) where \( \overline{\theta} = \theta; \) \( (\omega^* + 1 + \omega^0_0) \cdot \delta_\theta, \) where \( 0 \leq \theta, \theta + 2 \leq \phi \) and \( \delta_\theta \) is a dense order type. (3) If \( G \) is not free, and if \( \phi \geq 1 \) is the smallest cardinal such that \( \mathbb{F}_\phi \) is not a pure sub-group of \( G, \) then \( G \) admits precisely the following kinds of linear order: \( (\omega^* + 1 + \omega^0_0) \cdot \delta_\theta, \) where \( 0 \leq \theta < \phi \) and \( \delta_\theta \) is dense. (Received November 8, 1965.)
Correction. The Theorem of Abstract 629-14 (these Notices 12 (1965), 809) should say that the range of f is A, not X. In the following, H is the particular infinite r.e. class of r.e. sets with no proper infinite r.e. subclass described in 629-14. Proposition 1. If a ∈ H, there exists a* ∉ H such that H* = (H - {a}) ∪ {a*} is r.e., but H ∩ H* is immune. The techniques used in proving the theorem together with techniques used by Pour-El and Putnam in a forthcoming paper yield Proposition 2.

There are r.e. classes S of theories of identity (of Abelian groups, of fields, rings, integral domains, etc.) such that if T ⊆ S, then there exists a theory T* of identity (of Abelian groups, of fields, etc.) such that T* ∉ S, S* = (S - {T}) ∪ {T*} is r.e., but S ∩ S* is immune. Finally, minor membership changes in H yield r.e. classes which have various numbers of infinite r.e. subclasses, but we have been unable to prove that for every natural number n there exists an r.e. class with exactly n distinct infinite r.e. subclasses. (Received November 10, 1965.)

Modification of intersections.

Let M, N, X be compact differential manifolds with M, N ⊆ X and ∂M, ∂N ⊆ ∂X and ∂M ∩ ∂N = ∅. Let dim M = r, dim N = s, dim X = m and n = r + s - m. In general, even if M, N and X are c-connected, c ≥ n + 1 and 2n + 3 ≥ r, s there does not exist an isotopy modulo boundaries λt : N → X such that λt(N) ∩ M = ∅. But in this case there does exist a generalized intersection number α(N, M : X) ∈ π1(Sr-n) such that Theorem. λ t exists if and only if α(N, M : X) = 0. The intersection number α(N, M : X) is defined by choosing an isotopy module boundaries φt : N → X of N ⊆ X such that φt is transverse regular along M and K = φt(N) ∩ M ⊆ U ⊆ M where U is diffeomorphic to Euclidean Rr. The connectivity allows one to extend the map KCφt(N) to a map C : C(K) → φt(N) of the cone of K. Then there is a unique framing (up to orientation) of C*φt(N) : X where v(A : B) is the normal bundle of A in B for manifolds A ⊆ B. Restriction defines a framing of v(K : U) = v(K : Rr) and so, by the Thom construction, an element α(N, M : X) of π1(Sr-n). The proof of the theorem is a step by step argument by spherical modifications. (Received November 12, 1965.)

Locally flat strings and half-strings.

A k-string (k-half-string) is a set homeomorphic to Rk(Rk-1 × [0,∞) = Hk). Theorem 1. Let X be a locally flat k-string (k-half-string) in the unbounded topological n-manifold M, k < n; then X has a neighborhood U in M such that (U, X) ≈ (Rn, Rk) (resp. (U, X) ≈ (Rn, Hk)). Theorem 2. Let X be a closed, locally flat k-half-string in Rn, k ≤ n, n > 3; then (Rn, X) ≈ (Rn, Hk). Theorem 3. Let M and N be topological manifolds, M ⊆ N, N without boundary, dim N > 3; assume that Int M is locally flat, and let E be the set of points of BdM at which M fails to be locally flat; then E has no isolated points, and hence E is either empty or uncountable. In particular, an (n - 2)-cell may not fail to be locally flat in Rn at precisely one point if that point is a boundary point, n > 3. (Received November 12, 1965.)
Consider the differential operator \( L = (d/dx)^n + q(x) \) on the interval \((-\infty, \infty)\). Assume that \( q(x) \) is continuous, \( q(x) = 0, x \leq 0 \), and suppose that \( L \) has an absolutely continuous spectrum on \( [0, \infty) \). Let \( D \) be the region: \( 0 < x < \infty, -x < \xi < x \) and let \( \Omega_c(\xi, \xi) \) denote the kernel of the operator \( (W_c - 1) \) where \( W_c \) is the weight operator (\( \Omega_c \) is known if the spectral density matrix corresponding to \( L \) is given). The following modification of the Gelfand-Levitan procedure for determining \( q(x) \) in terms of the weight operator is effective for certain of these operators: Theorem. Let (i) \( \Omega_c \in C^{2n-4}(D + \partial D), \Omega_c \in C^{2n-3}(D), \) (ii) \( (\partial/\partial \xi)^i \Omega_c = 0, i \neq n - 2, (\partial/\partial \xi)^i \Omega_c = 1, i = n - 2, i = 0, 1, 2, ..., 2n - 4, \xi = -x, \) and (iii) \( F(\partial/\partial \xi)k(x, \xi) = 0 \) for \( (x, \xi) \) in \( D \), \( F(p) = \prod_{n=1}^{2n-3} (p - dv) \). Then the equation \( F(\partial/\partial \xi)k(x, \xi) = -G(\partial/\partial \xi)k(x, \xi) \) has a unique solution \( k(x, \xi) \) satisfying the conditions (iv) \( (\partial/\partial \xi)^i k = 0, i \neq n - 2, (\partial/\partial \xi)^i k = -1, i = n - 2, i = 0, 1, 2, ..., 2n - 4, \xi = -x, \) where \( G(p) = \sum_{j=0}^{n-2} c_j p^{n-2-j} \) and \( c_j, j = 0, ..., n - 2, \) are the first \( (n - 2) \) coefficients in \( F(p) \). The function \( q(x) \) is given by the formula \( q(x) = -n(d/dx) F(\partial/\partial \xi)k(x, \xi) \). The proof uses the fact that \( k \) satisfies the Gelfand-Levitan integral equation. For a nontrivial example consider \( n = 4, dv = -1, \Omega_c = B(x + \xi) \) where \( B(x) = (1/2)(x^2 + x^3 + (1/2)x^4) - \exp(-x). \) (Received November 15, 1965.)

66T-53. R. A. GOLDSTEIN, Courant Institute of Mathematical Sciences, New York University, 251 Mercer Street, New York, New York 10012. Equality of minimal and maximal extensions of a partial differential operator in \( L_p(\mathbb{R}^n) \).

If \( P = P(D) \) is a linear partial differential operator with constant coefficients on \( C_0^\infty(\mathbb{R}^n) \cap L_p(\mathbb{R}^n) \) into \( L_p(\mathbb{R}^n) \), then the minimal and maximal extensions of \( P \) are equivalent for \( 1 \leq p < \infty. \) (Received November 15, 1965.)

66T-54. J. T. JOICHI, University of Minnesota, Minneapolis, Minnesota 55455. Normed linear spaces equivalent to inner product spaces.

Theorem 1. A normed linear space \( X \) (real or complex) is equivalent to an inner product space if there exists \( K > 0 \) such that for each finite dimensional subspace \( Y \) of \( X \), there exists a linear mapping \( T_Y \) of \( Y \) into \( H \) (Hilbert space) such that \( (1/K)|x| \leq |T_Yx| \leq K|x| \) for all \( x \) in \( X \). Utilizing a result which gives an upper bound to the measure of the "asphericity" of compact convex symmetric bodies in the plane in terms of the maximum of the ratios of the lengths of the projections of the body onto cross sections to the lengths of the cross sections, we are able to show: Theorem 2. If \( X \) is a real normed linear space endowed with two norms \( |\cdot|_1 \) and \( |\cdot|_2 \) where \( |\cdot|_2 \) is an inner product norm, and if there exists \( K \geq 1 \) such that each one-dimensional orthogonal projection in the space \( (X, |\cdot|_2) \) has norm at most \( K \) considered as an operator in the space \( (X, |\cdot|_1) \), then the two norms are equivalent. Further results are obtained by combining Theorems 1 and 2 and a result which tells us when a complete family of one-dimensional projections in \( X \) (one for each one-dimensional subspace of \( X \)) is the family of one-dimensional orthogonal projections with respect to some inner product norm in \( X \). (Received November 15, 1965.)
A nonassociative algebra is reductive Lie admissible if there exists a Lie subalgebra $H$ of the derivation algebra of $A^-$ such that $L = A^- \oplus H$ is a reductive Lie algebra relative to the multiplication given by $[xy] = x \cdot y + h(x,y)$, $[xh] = xh - (hx)$ and $[hh'] = hh' - h'h$ where $x, y \in A^-$, $h \in H$ and $h(x,y)$ is a suitable element in $H$ (and $K$ is the algebra with vector space $A$ and multiplication $x \circ y = xy - yx$). Thus a Lie admissible algebra $A$ with $h(x,y) = R(x \circ y)$ is a reductive Lie admissible algebra; alternative algebras are reductive Lie admissible but not Lie admissible. $L = A^- \oplus H$ is a reductive Lie algebra; that is, a Lie algebra of the form $1 \oplus \text{subspace direct sum}$ with subalgebra $\text{and complementary space}$ and (subspace direct sum) with $[mh] \subseteq m$. For a fixed decomposition define multiplication $xy$ on $m$. As in Abstract 65T-364, these $\text{Notices}$ 12 (1965), 623, Theorem. Let $g = m \oplus h$ be a reductive Lie algebra as above. Then there exists a reductive Lie admissible algebra $A$ with $A^- \cong m$ or $A^- / m$ (as algebras). Furthermore if $m$ is a simple algebra and $B$ a proper ideal of $A$, then $B^- \cong m$ or $B^- / m$ and $B$ is the only proper ideal of $A$. (Received November 15, 1965.)

For a space $X$, let $S(X)$ denote the semigroup of all continuous functions mapping $X$ into $X$ where the binary operation is that of composition. A class of spaces is $S$-admissible if for any two spaces $X$ and $Y$ of the class and any isomorphism $\phi$ from $S(X)$ onto $S(Y)$, there exists a homeomorphism $h$ from $X$ onto $Y$ such that $\phi(f) = h \circ f \circ h$ for each $f$ in $S(X)$. A space $X$ is referred to as an $S^*$-space if it is $T_1$ and for every closed subset $F$ of $X$ and each point $p$ in $X - F$ there exists a function $f$ in $S(X)$ and a point $y$ in $X$ such that $f(x) = y$ for $x$ in $F$ and $f(p) \neq y$. The class of $S^*$-spaces includes 0-dimensional, Hausdorff spaces and completely regular, Hausdorff spaces which contain two points joined by an arc. An example of an $S^*$-space which is not an $S$-space (Amer. Math. Monthly 71 (1964), 984) is given. Theorem. The class of $S^*$-spaces is $S$-admissible. (Received November 15, 1965.)

D. Raikov has shown that for a commutative Banach algebra $A$ with symmetric involution, the set $\mathcal{P}$ of positive linear functionals on $A$ having norm $\leq 1$ is isometrically isomorphic to the set of positive measures of norm $\leq 1$ defined on the maximal ideal space of $A$. Raikov's proof of this theorem depends on the Gelfand theory of commutative Banach algebras and the Riesz-Markov-Kakutani Theorem. Here we give a new and elementary proof of Raikov's result by first proving a Radon-Nikodym type theorem for positive functionals and then showing directly that the extreme points of the compact convex set of positive linear functionals in the unit ball of $A^\prime$ are exactly the set $M$ of positive multiplicative linear functionals. An application of the Krein-Milman Theorem makes possible the representation of every element of $\mathcal{P}$ as the centroid of a positive measure on $M$ and uniqueness of this representation is a consequence of the Stone-Weierstrass Theorem. Although $M$ can be identified with the symmetric portion of the maximal ideal space of $A$, our methods avoid
the use of maximal ideal theory and the representation theory usually associated with positive functionals. (Received November 15, 1965.)

66T-58. D. F. DAWSON, North Texas State University, Denton, Texas. Linear methods which sum sequences of bounded variation.

A complex sequence \( z = \{z_p\} \) is said to be of bounded variation provided \( \sum |z_p - z_{p+1}| < \infty \).

It is known [J. P. Brannen, Proc. Amer. Math. Soc. 15 (1964), 114-123; B. Kuttner, J. London Math. Soc., 37 (1962), 354-364] that a matrix \( A = (a_{pq}) \) sums every sequence of bounded variation if and only if the following three conditions hold: (1) \( \{a_{pq}\}_{p=1}^{\infty} \) converges, \( q = 1, 2, 3, \ldots \), (2) the sequence \( \{\sum_{q=1}^{\infty} a_{pq} p=1\}_{p=1}^{\infty} \) converges, and (3) there exists \( k \) such that \( |\sum_{q=1}^{j} a_{pq}| < k \) for all positive integers \( p \) and \( j \). This theorem is used to prove the following result. Theorem 1. If a matrix \( A \) sums every sequence of bounded variation, then \( A \) sums a convergent sequence not of bounded variation.

Theorem 1 is a special case of (but was instrumental in proving) the following result. Theorem 2. If \( M \) is a countable collection of matrices, each of which sums every sequence of bounded variation, then there exists a convergent sequence not of bounded variation which every matrix in \( M \) sums.

(Received November 17, 1965.)

66T-59. LOUISE HAY, Mount Holyoke College, South Hadley, Massachusetts 01075. Isomorphism types of index sets of partial recursive functions.

Let \( \theta f = \{n | a_n \simeq f\} \) in a Kleene enumeration of partial recursive functions. Isomorphism refers to equivalence under recursive permutation. Theorem 1. There are exactly 3 isomorphism types of sets \( \theta f \), determined by whether the domain of \( f \) is null, finite or infinite; in the first 2 cases, \( \theta f \) has Turing degree \( 0' \), in the last case degree \( 0'' \). Thus the set of possible "instructions" for computing \( f \) is essentially independent of the complexity of \( f \). The same result holds for r.e. sets; thus all infinite r.e. sets, independently of their Turing degree, have isomorphic index sets.

Definition. \( f \) has type \( T(f) = 0, 1 \) or 2 according as domain \( f \) is null, finite or infinite. Call \( f, g \) incomparable if neither extends the other. Theorem 2. Let \( f_0, f_1 \) be incomparable. Then \( (\theta f_0, \theta f_1) \) is doubly isomorphic to \( (\theta g_0, \theta g_1) \) if and only if \( T(f_0) = T(g_0), T(f_1) = T(g_1) \) and \( g_0 \upharpoonright g_1 \) are incomparable.

Corollary. There are exactly 3 isomorphism types of pairs of index sets of incomparable functions, corresponding to pairs of types \((1, 1), (1, 2) \) and \((2, 2) \). (Such pairs have relevance to the property of creativity of r.e. sets; see Abstract 64T-252, these Notices 11 (1964), 387.) By the theorem of Rogers on equivalence of "standard-type" enumerations, these results are independent of any particular enumeration. (Received November 18, 1965.)

66T-60. J. C. TAYLOR, McGill University, Montreal, Quebec, Canada. The Feller and Silov boundaries of a vector lattice.

Let \( E \) be a locally compact space and let \( H \) be a vector lattice of continuous real-valued functions on \( E \) with \( 1 \in H \). When \( H \) consists of bounded functions, it is shown that the Feller boundary is the space of connected components of the Silov boundary. The connection between lattice preserving real linear-functionals and the Silov boundary is extended, in the unbounded case, to the Feller total boundary by introducing linear functionals which admit infinite values. The adjunction of the
boundaries to $E$ is discussed, and necessary and sufficient condition given for $E$ to be dense in the resulting space. (Received November 18, 1965.)

66T-61. J. R. GOLDMAN, Harvard University, Cambridge, Massachusetts. **Infinitely divisible point processes in $\mathbb{R}^n$.**

Recent work on infinitely divisible point processes on the line is generalized to $\mathbb{R}^n$. Two special classes of infinitely divisible point processes, regular and singular processes, are singled out by dependency relations among disjoint sets of $\mathbb{R}^n$. Every stationary infinitely divisible point process is the superposition of a regular and a singular process and all regular processes can be realized as Poisson cluster processes. (Received November 18, 1965.)

66T-62. W. D. L. APPLING, North Texas State University, Denton, Texas. **Concerning some characterizations of absolute continuity.**

Suppose $F$ is a field of subsets of the set $U$, $R^+$ is the set of all real nonnegative valued functions on $F$, and $R^+_A$ is the set of all finitely additive elements of $R^+$. All integrals discussed are Hellinger type limits of the appropriate sums. **Theorem 1.** If each of $h$ and $m$ is in $R^+_A$, then the following four statements are equivalent: (1) If $P$ is a bounded element of $R^+$ such that $\int U P(I) m(I) = 0$ and $\int U P(I) h(I)$ exists, then $\int U P(I) h(I) = 0$. (2) If $Q$ is a bounded element of $R^+$ such that for some $K > 0$, $Q(I) [\max \{Q(I), K\} - Q(I)] = 0$ for all $I$ in $F$ and $\int U Q(I) m(I) = 0$ and $\int U Q(I) h(I)$ exists, then $\int U Q(I) h(I) = 0$. (3) If $T$ is in $R^+$ and $T(I)[T(I) - 1] = 0$ for all $I$ in $F$ and $\int U T(I) m(I) = 0$ and $\int U T(I) h(I)$ exists, then $\int U T(I) h(I) = 0$. (4) $h$ is absolutely continuous with respect to $m$. **Theorem 2.** In order that it be true that for each $q$ in $R^+_A$ with $q(U) > 0$, there is a bounded element $W$ of $R^+$ such that $\int U W(I) q(I)$ does not exist, it is necessary and sufficient that for each $h$ and $m$ in $R^+_A$ the following two statements are equivalent: (1) If $P$ is a bounded element of $R^+$ such that $\int U P(I) m(I)$ exists, then $\int U P(I) h(I)$ exists. (2) $h$ is absolutely continuous with respect to $m$. (Received November 17, 1965.)

66T-63. P. E. CONNER and E. E. FLOYD, University of Virginia, Charlottesville, Virginia 22901. **Bordism and complex K-theory.**

A $\mathbb{Z}_2$-graded generalized homology theory which is dual to complex K-theory may be directly described in terms of the weakly complex bordism functor $\{U_*(X, A), f_*, \partial\}$. Let $T: U_*(pt) \rightarrow Z$ be the ring homomorphism assigning to each closed weakly complex manifold its Todd genus. In this way $Z$ becomes a $U_*(pt) = U_*$-module. On the other hand $U_*(X, A)$ is a natural graded right $U_*$-module so we set $K_*(X, A) = U_*(X, A) \otimes U_* Z$. Since $f_*: U_*(X, A) \rightarrow U_*(Y, B)$ and $\partial U_*(X, A) \rightarrow U_*(A)$ are both $U_*$-module homomorphisms of degree 0 and -1 respectively there are induced homomorphisms $f! : K_*(X, A) \rightarrow K_*(Y, B)$ and $\partial! : K_*(X, A) \rightarrow K_*(A)$. The major question is the demonstration of exactness. This is not done directly, but instead we use a natural transformation between cohomology theories $\mu : U_*(X, A) \rightarrow K_*(X, A)$ defined on the cobordism functor to show that $K_*(X, A) \cong U_*(X, A) \otimes U^* K_*(pt)$, then the exactness of $\{K_*(X, A), f!, \partial\}$ will follow by an appeal to duality. (Received November 22, 1965.)

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Let $W$ be a noncompact connected, locally connected, locally compact Hausdorff space and $\mathcal{H}$ a class of harmonic functions with domains contained in $W$ in the usual setting of Brelot's axiomatic potential theory. Assume $f$ is $\mathcal{H}$-superharmonic. The results of Parreau, Smith and others on the relations between the nonexistence of positive and of various types of meanbounded harmonic functions defined on all of $W$, and Parreau's lattice decompositions of various vector spaces of harmonic functions, are established in the axiomatic theory and freed from their dependence on the Riemannian structure of the underlying $W$. \textbf{Sample theorem:} There exist bounded nonconstant $\mathcal{H}$-harmonic functions on $W$ iff there exist nonconstant $\mathcal{H}$-harmonic functions whose $p$th means $(p > 1)$ with respect to harmonic measure are bounded on all regular regions. Also investigated are the relations among the spaces of mean-bounded functions of two harmonic classes $\mathcal{H}$ and $\mathcal{H}'$ for which the positive $\mathcal{H}$-harmonic functions are $\mathcal{H}$-superharmonic on the complement of some compact $K \subseteq W$ (cf. Loeb, An axiomatic treatment of pairs of differential equations, Abstract 623-30, these Notices 12 (1965), 359. (Received November 22, 1965.)

\textbf{66T-65.} P. C. HAMMER, Pennsylvania State University, 426 McAllister Building, University Park, Pennsylvania 16802. \textit{Isotonic spaces in convexity. II.}

Let $\mu$ map each subset of a linear space $M$ into a set $L$ linearly ordered by $<$ such that $X \subseteq Y$ implies $\mu X \subseteq \mu Y$. Let $t \in L$ be given so that $\mu N \not< t$ ($N$ is the null set in $M$). Let $p \in \mu X$ provided $\mu (X \cap Y) \not< t$ for every semispase $Y$ at $p$. In this paper the isotonic functions $u_t$ under various specializations of $\mu$ are shown to embrace generalizations of Theorems of Dupin, Carathéodory, and Steinitz. In particular, $\mu X = |X|$, the cardinal number of $X$, and also defining $\mu X$ as the exterior measure of $X$ are appropriate. (Received November 22, 1965.)

\textbf{66T-66.} GEORGE GRATZER, Pennsylvania State University, McAllister Building, University Park, Pennsylvania. \textit{Congruence relations on partial algebras.}

Let $\mathcal{A} = (A; F)$ be a partial algebra, that is $A$ is a set and $F$ is a collection of partial operations on $A$. A binary relation $\Theta$ on $A$ is a congruence relation of $\mathcal{A}$, if $a_1 = b_1(\Theta), f(a_0, \ldots, a_{n-1})$ and $f(b_0, \ldots, b_{n-1})$ exist imply that $f(a_0, \ldots, a_{n-1}) = f(b_0, \ldots, b_{n-1})(\Theta)$. A congruence relation $\Theta$ on $\mathcal{A}$ is strong, if $a_1 = b_1(\Theta), f(a_0, \ldots, a_{n-1})$ exists imply that $f(b_0, \ldots, b_{n-1})$ exists. G. Gratzer and E. T. Schmidt (Acta Sci. Math. 24 (1963), 34-59) proved that if $\Theta$ is a congruence relation of $\mathcal{A}$, then there exists an algebra $\mathcal{B}$, which is an extension of $\mathcal{A}$, and there exists a congruence relation $\Phi$ on $\mathcal{B}$ such that $\Phi$ restricted to $A$ equals $\Theta$. Now we prove that $\Theta$ is a strong congruence relation if and only if $\mathcal{A}$ has an extension $\mathcal{B}$ which is an algebra, $\mathcal{B}$ has a congruence relation $\Phi$, such that every equivalence class of $\Theta$ in $A$ is also an equivalence class of $\Phi$ in $\mathcal{B}$. (Received November 22, 1965.)

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Orthogonal group matrices of hyperoctahedral groups.

Each irreducible orthogonal representation \( \{ \lambda; \mu \} \) of the hyperoctahedral group \( G_n \) of order \( 2^n \) is characterized by a two-part partition: \( [\lambda] \) of \( \ell = n - m \) and \( [\mu] \) of \( m \). The \( t \chi^T \) variables of this matrix representation \( g_k \rightarrow M^\lambda\mu(g_k) \) with group matrix \( M^\lambda\mu = \sum g_k^{-1} M^\lambda\mu(g_k) \) are associated with standard tableaux \( t^\lambda\mu \) whose rows and columns determine respectively the group sum \( \sum^\lambda\mu \) (of order \( \sum^\lambda\mu \)) and the alternating character group sum \( N^\lambda\mu \), from which a ring element \( E^\lambda\mu \) is built such that \( E^\lambda\mu / \ell^\lambda\mu \) is idempotent. From the element \( \sigma_d \) of \( G_n \) that changes the sign of digit \( d \), construct the \( n \)-factor product \( \sigma^\lambda\mu \) of idempotents \( (I + \sigma_d)/2 \) for \( d \) in \( [\lambda] \), and \( (1 - \sigma_d)/2 \) for \( d \) in \( [\mu] \). Let the transposition \( t_d \) of consecutive digits \( d, d+1 \) interchange tableaux \( t^\lambda\mu \) and \( t^\lambda\mu \) and let \( \gamma \) be the number of steps up and right from \( d \) to \( d+1 \) in \( t^\lambda\mu \). Set \( \gamma \) \( \gamma^{-1} = (\tau_d + \rho_v) (1 - \rho_v^2 \gamma^{1/2}) \), \( \gamma = 1 \). Then the \( u \)-entry of the group matrix \( M^\lambda\mu \) is \( \gamma \). (Received July 12, 1965.)

The automorphism group of nets.

It has recently been shown by L. Weinberg (SIAM J. to appear) that for every finite 3-connected planar graph \( X \), \( |G(X)| \leq 4a \), where \( G(X) \) is the automorphism group of \( X \), and \( a \) the number of edges of \( X \). This can be generalized as follows: Given a graph \( X \) a set \( M \) of finite circuits of \( X \) is called a set of meshes of \( X \) if (i) \( M \) is invariant, i.e., \( \phi M = M \) for every \( \phi \in G(X) \), (ii) given any edges \( e, e' \) of \( X \) there exist \( C_0, \ldots, C_n \in M \) such that \( e \in C_0 \), \( e' \in C_n \), and \( C_{i-1} \) and \( C_i \) have an edge in common, \( i = 1, \ldots, n \), and (iii) every edge of \( X \) belongs to at most two members of \( M \). A net is a pair \( (X, M) \) where \( X \) is a graph and \( M \) is a set of meshes of \( X \). Theorem. If \( (X, M) \) is a finite net, then \( |G(X)| \leq 2m \min |C| C \in M | \leq 4a \), where \( m = |M| \). From this, Weinberg's theorem is obtained by noting that for a finite 3-connected planar graph \( X \) the set of all circuits \( C \) such that \( X - C \) is connected is a set of meshes. (Received November 23, 1965.)

On the generalized inverse of products of matrices.

Let \( A^+ \) denote the generalized inverse of the matrix \( A \). The problem of finding conditions that \( (AB)^+ = B^+A^+ \) has received some attention. In particular, Penrose (A generalized inverse for matrices) has shown that \( (A^T A)^+ = A^+(A^T)^+ \) and Greville (Some applications of the pseudoinverse of a matrix, SIAM Rev. 2 (1960)) has shown that \( (AB)^+ = B^+A^+ \) if \( A \) is \( p \times n \), \( B \) is \( n \times q \) and both have rank \( p \). The present paper gives a class of matrices satisfying the aforementioned formula. \( B \) is an \( EPr_1 \) matrix, \( B \) an \( EPr_2 \) matrix and \( AB = BA \), then \( (AB)^+ = B^+A^+ \). As a corollary one obtains: if \( A \) is an \( EPr \) matrix, then \( (A^+)^+ = (A^+)^+ \). It is shown that the commutativity condition may not be dropped from the theorem and that the converse of the corollary is false. The proof uses a recent result of Pearl (A theorem on generalized inverses of matrices, Abstract 625-154, these Notices 12 (1965), 591). A consequence of this theorem is a short proof of the following: if \( A \) and \( B \) are matrices satisfying the conditions of the above theorem, then \( AB \) is an \( EPr \) matrix. (I. J. Katz, Wiegmann type theorems for \( EPr \) matrices). (Received November 23, 1965.)
Let there be given a differential operator \( L[u] = (au'')' - (bu')' + \beta u' + cu \) with \( a(x) > 0 (0 \leq x \leq 1) \), together with two "arbitrary" linear homogeneous boundary conditions at \( x = 0 \), and two boundary conditions at \( x = 1 \). Moreover, let \( M[u] = -apu'' + ap'u' + Pu \) with \( p(x) > 0 (0 < x < 1) \). Let the functions \( p(x) \) and \( P(x) \) satisfy a certain linear boundary value problem which consists of a linear differential inequality, a linear differential equation and corresponding boundary conditions. Then, the following statement is true: \( L[u](x) \geq 0 (0 \leq x \leq 1) \) implies \( u(x) \geq 0, \ M[u](x) \geq 0 (0 \leq x \leq 1) \) for all \( u \) satisfying the given boundary conditions if there exists a function \( z \) satisfying the given boundary conditions such that \( z(x) \geq 0, \ M[z](x) \geq 0, \ L[z](x) > 0 (0 \leq x \leq 1) \). The sufficient conditions mentioned above are closely connected with necessary conditions for the following implication: \( L[u](x) \geq 0 (0 \leq x \leq 1) \Rightarrow u(x) \geq 0 (0 \leq x \leq 1) \). This paper continues an earlier paper of the author [Math. Z. 90 (1965), 429-440]. (Received November 24, 1965.)

There is an Euler process for transforming convergent infinite series from alternating to positive terms (see K. Knopp, Theory and application of infinite series, Hafner, New York, (1947), 244 seq.). Theorem. This process has greater capabilities than indicated, when variations in the transformation process are used, and it can be extended to generate several additional types of significant results. Examples follow. Application 1. The known binomial expansion formula \( (1 - 1/X)^{1/2} = 1 - 1/2X - \sum_{n=1}^{\infty} \left( \frac{1}{r!} \frac{1}{(2r-1)!} X^{2r-1} \right) \) (where \( X > +1 \)) is transformed to \( (2X - 2)[1/(2X - 1) + \sum_{n=1}^{\infty} \left( \frac{1}{r!} \frac{1}{(2r-1)!} X^{2r} \right) \] (where \( X > +1 \)), which is a much more rapidly convergent series. Application 2. The known formula \( \sin X = \sum_{r=1}^{\infty} [(-1)^{r+1} X^{2r-1} / (2r - 1)!] \) (where \( X > 0 \)) is transformed to the highly accurate approximation \( \sin 1/X \sim 1/X! - [A_1 - 1/(A_2 X^2 + A_3) \cdot A_4 X^2] / (A_5 X^2 + 1)] \) (where \( X \geq 1; A_1 = C_3 / (C_2)^2; A_2 = 360 [C_2 C_4 / (C_3)^2 - 1]; A_3 = 5(C_2 C_5 / C_3 C_4 - 1), \) etc.; and \( C_n \) are the coefficients of the given series. This new formula gives result data which is accurate, at \( X = +1 \), to six decimal places and this accuracy improves as the variable increases in magnitude. This new approximation formula is good for computer programming usages. Also see the writer's Abstract 625-148, these \( \text{Notices} \) 12 (1965), 589 for another process yielding similar approximation formulae. (Received November 24, 1965.)

In the author's paper [1] (Commulative nonpotent archimedean semigroup with cancellation law, J. Gakugei, Tokushima Univ. 8 (1957), 5-11), commutative cancellative archimedean semigroup \( S \) without idempotent is determined by an abelian group \( G \), the set \( N \) of all nonnegative integers and a nonnegative integer valued function \( I(\lambda,\mu) \) defined on \( G \times G \) satisfying certain conditions. In this paper the author discusses how to construct \( S \) in the case where cancellation is not assumed. \( S \) is determined by (1) an abelian group \( G \), (2) the set \( N \) of all nonnegative integers, (3) \( I(\lambda,\mu) \) with the same conditions in [1], (4) a system of certain trees which satisfy the ascending chain condition but have no smallest element, (5) a commutative groupoid with identity. (Received November 26, 1965.)
A finite commutative $z$-semigroup $S$ is a finite semigroup which has a zero $0$ and has no idempotent except $0$. One of the authors constructed such semigroups by means of the extensions of a null semigroup of order 2 in his paper [M. Yamada; Proc. Japan Acad. 40, 94-98 (1964)]. The present paper gives a different way: Theorem 1. $T$ is a commutative $z$-semigroup of order $n + 1$ if and only if $T$ is a commutative extension of a commutative $z$-semigroup $S$ of order $n$ by a null semigroup $N$ of order 2. Let $\phi$ be a translation of $S$ such that $\phi^2$ is an inner translation of $S$. $T$ is determined by $S, \phi$ and an element $a \in S$ where $\phi^2 = \rho a^n x \rho = xa$ for $x \in S$. $T$ is denoted by $(S, \phi, a)$. The authors discuss how to determine $\phi$ and give a necessary and sufficient condition for two extensions to be isomorphic. In particular (1) If $S$ is a cyclic $z$-semigroup, then $(S, \phi, a) \cong (S, \phi', a')$ iff $\phi = \phi'$, $a = a'$. (2) If $S$ is a null semigroup, then $(S, \phi, a) \cong (S, \phi', a')$ iff there is an automorphism $\sigma$ of $S$ such that $\phi' = \sigma^{-1} \phi \sigma$, $a' = a \sigma$. (Received November 26, 1965.)

An $N$-semigroup as defined by Petrich (Czechoslovak Math. J. 14 (1964), 147-153) is a commutative cancellative archimedean semigroup without an idempotent. If $G$ is an abelian group and if $N_0$ is the nonnegative integers, then Tamura (J. J. Gakugei, Tokushima Univ. 8 (1957), 5 - 11) has proved that such a semigroup is the product $N_0 \times G$ with multiplication defined by $(m, a)(n, b) = (m + n + I(a,b), ab)$. See Clifford and Preston, The algebraic theory of semigroups (Amer. Math. Soc. Providence, R. I., 1961, page 136). Theorem. An $N$-semigroup is finitely generated if and only if $G$ is finite. Theorem. An $N$-semigroup satisfies the condition that for each $x, y \in S$ there exist integers $m$ and $n$ such that $x^m = y^n$ if and only if $G$ is periodic. (Received November 26, 1965.)

A locally cyclic semigroup $S$ is a semigroup which satisfies the following condition: For every $a,b \in S$ there are positive integers $m$ and $n$ and an element $c \in S$ such that $a = c^m$, $b = c^n$. $S$ is necessarily a commutative archimedean semigroup. Theorem 1. Every locally cyclic semigroup $S$ can be embedded into a homomorphic image $T$ of the semigroup $R$ of all positive rational numbers with addition ($T$ depends on $S$). In particular, a locally cyclic semigroup without idempotent can be embedded into $R$. Theorem 2. $S$ is a locally cyclic semigroup iff $S$ is the direct limit of cyclic semigroups $S = \bigcup_{i=1}^{\infty} S_i$, $S_i \subseteq S_{i+1} \subseteq \ldots$, where each $S_i$ is a cyclic semigroup. Theorem 3. A commutative semigroup $S$ can be embedded into $R$ iff $S$ is power-joined ($\forall a,b \in S \exists m,n > 0$ such that $a^m = b^n$) and $S$ is power-cancellative ($a^k = b^k$ for some $k$) $\Rightarrow a = b$. (Received November 26, 1965.)

Let $\{\eta_K\}$ be a strictly increasing sequence of positive integers satisfying (i) $\sum_{K=1}^{\infty} 1/\eta_K < \infty$,
and (ii) $\gamma_K^{\eta K+1}/(1 - \gamma_K) \leq 2^{-K}$, $K = 1, 2, \ldots, \infty$, where $\gamma_K$ is the positive solution of (iii) $\gamma_K^{\eta K} = 1 - 2^{-K}$.

Such sequences are easily constructed by induction and, without the requirement (i), have been considered earlier by D. G. Cantor [Proc. Amer. Math. Soc. 15 (1964), 335-336]. If, now, $\{b_K\}$ is any bounded sequence of complex numbers, then, in view of (i), the power series $f(z) = \sum_{K=1}^{\infty} b_K(z^{\eta K+1})/\eta_K + 1$ converges uniformly absolutely for $|z| = 1$, and hence represents a bounded analytic function on $U$, the unit disk of the complex plane. On the other hand, if $\{b_K\}$ does not converge to zero, then, in view of (ii), (iii) and Theorem 1 of Cantor (ibid.), $f'(z)$ does not have a radial limit at any point of the unit circle; a fortiori, $f'(z)$ is not of bounded type in $U$, thus providing yet another solution of a problem due to Bloch. (Received November 26, 1965.)

66T-77. J. L. WALSH, Harvard University, 2 Divinity Avenue, Cambridge, Massachusetts.

The convergence of sequences of rational functions of best approximation. III.

This paper continues recent work of the author. A typical new theorem is the following. Let the point set $E$ consist of a finite number of mutually exterior closed Jordan regions, each with rectifiable boundary. Let $F(z)$ be analytic and different from zero on $E$, meromorphic with precisely $\nu$ zeros in $E_r$, $1 < \tau \leq \infty$. Let the rational functions $R_{\nu n}(z)$ of respective types $(\nu, n)$ satisfy for the $q$th power norm on the boundary of $E$: $\limsup_{n \to \infty} \|F - R_{\nu n}\|_q^{1/n} \leq 1/\tau$, $q > 0$, where the $R_{\nu n}(z)$ have no limit point of poles on $E$. Then for the Tchebycheff norm on $E$ we have $\limsup_{n \to \infty} \|F - R_{\nu n}\|_\infty^{1/n} \leq 1/\tau$. More refined results exist if $1/F(z)$ is of class $H(k, a, p)$, $p \geq 1$, $0 \leq a < 1$ on $\Gamma_r$. (Received November 26, 1965.)


A space $S$ is here said to be regularly refinable if and only if for any collection $H$ of open sets covering $S$, there exists a collection $K$ of open sets covering $S$ such that if $x$ and $y$ are intersecting elements of $K$, some member of $H$ includes $x$ and $y$. Theorem 1. A Hausdorff space $S$ is paracompact if and only if $S$ is regularly refinable and every collection of open sets covering $S$ is refined by a point-countable collection of open sets covering $S$. Theorem 2. An Axiom 0 space of R. L. Moore's Foundations of point set theory is fully normal if and only if it is regularly refinable, countably metacompact, and satisfies the above point-countable refinement condition. Miščenko has announced the existence of a nonparacompact, regularly refinable Hausdorff space. (Received November 26, 1965.)
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