Notices

OF THE AMERICAN MATHEMATICAL SOCIETY

VOLUME 15, NUMBER 1 ISSUE NO. 103 JANUARY, 1968

Notices

OF THE

AMERICAN MATHEMATICAL SOCIETY

Edited by Everett Pitcher and Gordon L. Walker

CONTENTS

MEETINGS

Calendar of Meetings

NOTE: This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the *Notices* was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

Meeting No.	Date	Place	Deadline for Abstracts*
654	March 21-23, 1968	Los Angeles, California	Jan. 24
655	April 10-13, 1968	New York, New York	Feb. 26
656	April 16-20, 1968	Chicago, Illinois	Feb. 26
657	June 15, 1968	Portland, Oregon	May 1
	August 26-30, 1968 (73rd Summer Meeting)	Madison, Wisconsin	
	January 23-27, 1969 (75th Annual Meeting)	New Orleans, Louisiana	
	August 25-29, 1969 (74th Summer Meeting)	Eugene, Oregon	
	January 22-26, 1970 (76th Annual Meeting)	Miami, Florida	
	August 1970 (75th Summer Meeting)	Laramie, Wyoming	

*The abstracts of papers to be presented in person at the meetings must be received in the Headquarters Offices of the Society in Providence, Rhode Island, on or before these deadlines. The deadlines also apply to news items. The next two deadline dates will be January 17, and February 19, 1968.

OTHER EVENTS

April 5-6, 1968	Numerical Solution of Field Problems in Continuum Physics	Durham, North Carolina

The *Notices* of the American Mathematical Society is published by the Society in January, February, April, June, August, October, November and December. Price per annual volume is $12.00. Price per copy $3.00. Special price for copies sold at registration desks of meetings of the Society, $1.00 per copy. Subscriptions, orders for back numbers (back issues of the last two years only are available) and inquiries should be addressed to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904.

Second-class postage paid at Providence, Rhode Island, and additional mailing offices. Authorization is granted under the authority of the act of August 24, 1912, as amended by the act of August 4, 1947 (Sec. 34,21, P. L. and R.). Accepted for mailing at the special rate of Postage provided for in section 34,40, paragraph (d).

The Seventy-Fourth Annual Meeting
San Francisco Hilton Hotel
San Francisco, California
January 23-26, 1968

PROGRAM

The seventy-fourth annual meeting of the American Mathematical Society will be held at the San Francisco Hilton Hotel, San Francisco, California, in conjunction with the annual meeting of the Mathematical Association of America. The Society will meet from Tuesday, January 23, through Friday, January 26. The Mathematical Association of America will meet from Thursday, January 25, through Saturday, January 27.

The forty-first Josiah Willard Gibbs Lecture, entitled "Symmetry principles in old and new physics," will be delivered by Professor Eugene P. Wigner of Princeton University in the Continental Ballroom of the San Francisco Hilton at 8:00 p.m. on Tuesday, January 23, 1968.

Dean A. A. Albert of the University of Chicago will give the Presidential Address in the Continental Ballroom at 1:30 p.m. on Tuesday, January 23. The title of his lecture is "On associative division algebras."

By invitation of the Committee to Select Hour Speakers for the Annual and Summer Meetings, there will be two invited addresses. Professor Wolfgang Wasow of the University of Wisconsin will speak at 1:30 p.m. on Thursday, January 25. The title of his address is "Connection problems for asymptotic series." Professor Louis Auslander of the City University of New York will present an address entitled "A survey of solvable Lie groups and applications" at 1:30 p.m. on Friday, January 26. Both of these invited addresses will be given in the Continental Ballroom.

The first George David Birkhoff Prize in Applied Mathematics will be awarded in the Continental Ballroom at 1:30 p.m. on Wednesday, January 24.

There will be two special sessions of twenty-minute papers. Papers will be given at these sessions by invitation and by selection from ten-minute papers submitted for the meeting. The topic for the first of these sessions is "Integration in function spaces," under the chairmanship of Professor Monroe Donsker, Courant Institute of Mathematical Sciences, New York University. This session will be held in the Continental Ballroom at 9:00 a.m. and 3:00 p.m. on Tuesday, January 23. The second special session of twenty-minute papers will be devoted to the topic of "Piecewise linear topology." The program chairman for this session is Professor Morris W. Hirsch of the University of California, Berkeley. This second special session will take place in the Continental Ballroom beginning at 9:00 a.m. on Wednesday, January 24.

There will be an informal meeting of persons interested in Category Theory at 3:00 p.m., Thursday, January 25, in the Walnut Suite. The purpose of this session is to discuss the most recent work on categorical algebra. The talks to be given will not be announced until the time of the meeting.

There will be regular sessions for contributed ten-minute papers during the mornings and afternoons of January 23 and 24, and during the afternoons of January 25 and 26. No sessions for late papers will be held.

There will be an open meeting of the Committee to Monitor Problems in Communication in the Mathematical Sciences on Wednesday evening, January 24, at 8:00 p.m. in the Continental Ballroom. The committee is concerned with new or better devices for communicating mathematics through research publications, reviewing journals, expository writing, meetings, films, etc. A number of topics will be introduced by the committee, with ample time between for full discussion from the floor. Anyone interested is invited to attend.

The business meeting of the Society will be held at 11:15 a.m. on Tuesday, January 23, in the Continental Ballroom.

The Council of the Society will meet at 2:00 p.m. on Monday, January 22, in the Walnut Suite on the fourth floor of the San Francisco Hilton.

EMPLOYMENT REGISTER

The Mathematical Sciences Employment Register will be maintained from 9:00 a.m. to 5:00 p.m. on Wednesday, Thursday, and Friday in the Imperial Ballroom of the San Francisco Hilton.

EXHIBITS

The book and educational media exhibits will be displayed in the North and West Lounges and the Garden Room on the ballroom floor of the San Francisco Hilton. The exhibits will be open from 9:00 a.m. to 5:00 p.m. on Tuesday, Wednesday, Thursday, and Friday.

REGISTRATION

The Registration Desk for this meeting will be in the East Lounge on the ballroom floor of the San Francisco Hilton. The Registration Desk will be open from 2:00 to 8:00 p.m. on Monday, January 22; from 8:00 a.m. to 5:00 p.m. on Tuesday, January 23; from 9:00 a.m. to 5:00 p.m. on Wednesday through Friday, January 24-26; and from 9:00 a.m. to 3:00 p.m. on Saturday, January 27. It will be helpful if persons attending the meetings will register as soon as possible after their arrival.

The registration fees for the meetings are as follows:

Member	$3.00
Member's family	0.50
(first additional person)	no fee
(others in member's family)	no fee
Student	no fee
Non-member	$7.50

ACCOMMODATIONS

Accommodations for the meeting will be handled by the San Francisco Convention and Visitors Bureau. A form for requesting accommodations may be found on page 966 of the November *Notices*. Persons desiring accommodations should complete this reservation form or a reasonable facsimile and send it to AMS-MAA Housing Bureau, 260 Fox Plaza, San Francisco, California 94102. All reservations will be confirmed. The San Francisco Convention and Visitors Bureau will make reservations in accordance with the preference indicated on the reservation form, insofar as this is possible. It will not be necessary to send a deposit with the registration form. If a person is assigned to a hotel that requires a deposit, then that hotel will request a first night's deposit to hold the room. Requests for reservations should arrive in San Francisco no later than January 12, 1968. The accompanying map shows the location of the various hotels which have reserved rooms for the meeting, including the headquarters hotel, the San Francisco Hilton. A list of room rates at these hotels follows:

BELLEVUE HOTEL

Doubles	$13.50 to $15.00
Twins	$15.00 to $18.00
Suites (parlor and 1 bedroom)	$25.00 to $35.00

CALIFORNIAN HOTEL

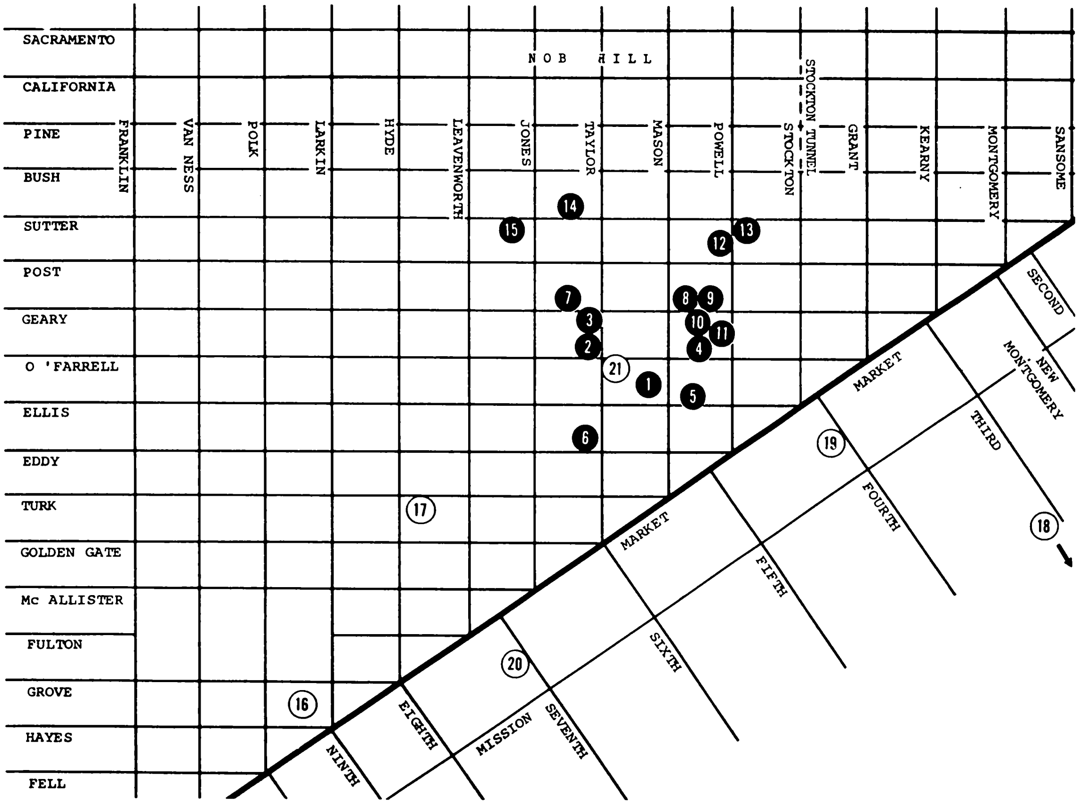
Singles	$12.00 to $13.00
Doubles	$14.50 to $15.50
Twins	$16.00 to $17.00
Suites (parlor and 1 bedroom)	$30.00

CANTERBURY HOTEL

Singles	$12.00 to $18.00
Doubles	$14.00 to $18.00
Twins	$16.00 to $21.00
Suites (parlor and 1 bedroom)	$35.00
Suites (parlor and 2 bedrooms)	$55.00

CHANCELLOR HOTEL

Doubles	$13.50
Twins	$15.00



1. San Francisco Hilton Hotel
2. Californian Hotel
3. Bellevue Hotel
4. Handlery Inn
5. Ramona Hotel
6. Olympic Hotel
7. El Cortez Hotel
8. Fielding Hotel
9. Franciscan Hotel
10. Stewart Hotel
11. Manx Hotel
12. Chancellor Hotel
13. Sir Francis Drake Hotel
14. Canterbury Hotel
15. Commodore Hotel
16. Civic Auditorium
17. YMCA Hotel (men and women)
18. Southern Pacific Depot)
    (Third & Townsend)
19. Santa Fe Bus Depot
20. Greyhound Bus Depot
21. Downtown Airlines Terminal

COMMODORE HOTEL

Singles	$12.00 to $18.00
Doubles	$15.00 to $20.00
Twins	$16.00 to $22.00

EL CORTEZ HOTEL

Singles	$11.00 to $14.00
Doubles	$12.00 to $15.00
Twins	$14.00 to $18.00
Suites	$28.00 to $36.00

(comb. parlor and 1 bedroom)
(sleeps 4, 2 baths)

FIELDING HOTEL

Singles	$12.00
Doubles	$15.00
Twins	$18.00
Suites	$40.00

(parlor and 1 bedroom)

FRANCISCAN HOTEL

Singles	$ 9.00 to $10.00
Doubles	$12.00
Twins	$14.00

HANDLERY MOTOR INN

Singles	$18.00 to $28.00
Doubles	$23.00 to $30.00
Twins	$23.00 to $30.00

SAN FRANCISCO HILTON HOTEL

Singles	$14.00 to $19.00
Doubles	$18.00 to $21.00
Twins	$18.00 to $21.00
Suites	$52.00 and up

MANX HOTEL

Singles	$14.00 to $15.00
Doubles	$16.00 to $18.00
Twins	$18.00 to $20.00

OLYMPIC HOTEL

Singles	$12.00 to $15.00
Doubles	$14.00 to $18.00
Twins	$15.00 to $18.00
Suites	$28.00 to $32.00

(parlor and 1 bedroom)

RAMONA HOTEL

Singles	$10.00
Doubles	$11.50
Twins	$13.50
Suites	$30.00

(parlor and 1 bedroom)

STEWART HOTEL

Singles	$12.00 to $18.00
Doubles	$16.00 to $20.00
Twins	$16.00 to $22.00
Suites	$40.00 to $75.00

(parlor and 1 bedroom)

SIR FRANCIS DRAKE HOTEL

Singles	$15.00 to $22.00
Doubles	$19.00 to $25.00
Twins	$22.00 to $26.00
Suites	$70.00

(parlor and 1 bedroom)

The San Francisco Hilton Hotel has three dining facilities: the Gazebo Restaurant (a coffee shop), which is open daily from 7:00 a.m. to 11:00 p.m. and offers a dinner buffet from 5:30 p.m. to 9:30 p.m.; the California Wine Garden and Bellagio Restaurant, which is open Monday through Saturday from 11:00 a.m. to 10:00 p.m. with a daily luncheon buffet and à la carte dinner menu, and on Sunday from 10:00 a.m. to 2:00 p.m. for Champagne Brunch and à la carte dinner to 10:00 p.m.; and the Chef's Table Restaurant, which is open Monday through Friday for luncheon from 11:30 a.m. to 2:00 p.m. and nightly for dinner from 5:00 p.m. to midnight.

ENTERTAINMENT

There will be a No-Host Get-Together from 5:00 p.m. to 7:00 p.m. on Thursday, January 25, in the Hilton Plaza. Mixed drinks will be available at a cost of one dollar per drink. This will be the only major social function of the meeting, and everyone is invited to it.

There are many things to see and much to do in the San Francisco Bay area. Brochures describing various tours around the city will be available at the Registration Desk. These will include walking and automobile trips, Gray Line bus rides, and Harbor Tours by boat. There will also be brochures in the registration area describing some of the major attractions of San Francisco, such as Chinatown, North Beach, Golden Gate Park, and Nob Hill.

San Francisco has numerous museums and art galleries. At night there is entertainment available to suit all tastes, from jazz and "highly original" nightclubs to legitimate theater and classical musical events.

Some of the finest restaurants in the nation are located in San Francisco. The Convention Bureau will provide a list of the outstanding dining places in the city. There will also be a guide to dining near the Hilton available at the Registration desk. This will list the places to eat (both plain and fancy) which are located within a few blocks of the San Francisco Hilton.

MAIL AND MESSAGE CENTER

All mail and telegrams for persons attending the meetings should be addressed in care of the American Mathematical Society. The San Francisco Hilton, Mason and O'Farrell Streets, San Francisco, California 94102. Mail and telegrams so addressed may be picked up at the Registration Desk.

Through the courtesy of the Pacific Telephone Company, a message center will be provided to receive incoming calls for all members in attendance. The center will be located in the East Lounge near the Registration Desk. It will operate from January 23 through January 27 between 9:00 a.m. and 5:00 p.m. Messages will be recorded, and the name of any member for whom a message has been received will be posted until the message is picked up at the message center. Members are advised to leave the following number with anyone who might want to reach them at the meetings: 415-776-1390.

TRAVEL AND LOCAL INFORMATION

The airlines serving San Francisco include American, Delta, National, Pacific, Pacific Southwest, Pan American, Trans-World, United, West Coast, Western, and various international carriers. There is bus transportation from the San Francisco International Airport to the downtown airport bus terminal which is next door to the Hilton.

Railroad service to San Francisco is offered by the Northern Pacific, Santa Fe, Southern Pacific, and Western Pacific Railroads. Taxi service is available from the various railroad depots to the Hilton Hotel.

The bus lines serving San Francisco include the Continental Trailways and the Greyhound Bus Lines. The bus terminals are located within a few blocks of the Hilton Hotel.

Those persons who come to San Francisco by car will find that several of those listed hotels offer free parking to their guests. The San Francisco Hilton has a parking garage within the hotel. Guests can register from their cars at the garage entrance. They will then be directed to a parking space on the same floor as their room.

During the month of January, San Francisco's average maximum temperature is 55 degrees and the minimum is 45 degrees. There is a likelihood of encountering some rain, so that rain coats, umbrellas, and rubbers or overshoes may prove useful. For clothing, medium weight wool suits or dresses are recommended.

COMMITTEE ON ARRANGEMENTS

D. W. Blakeslee (chairman), H. L. Alder, H. M. Bacon (acting chairman), W. G. Bade, N. H. Fisher, Mrs. Dorothy Friedman, R. S. Lehman, R. S. Pierce, P. E. Thomas, and G. L. Walker.

TIME TABLE

(Pacific Standard Time)

MONDAY, January 22	American Mathematical Society	Mathematical Association of America
2:00 p.m. - 8:00 p.m.	REGISTRATION - East Lounge	
2:00 p.m.	Council Meeting, Walnut Suite	

TUESDAY, January 23	AMS	MAA
8:00 a.m. - 5:00 p.m.	REGISTRATION - East Lounge	
9:00 a.m. - 5:00 p.m.	EXHIBITS - North Lounge, West Lounge, Garden Room	
9:00 a.m. - 10:20 a.m.	Special Session: Integration in function spaces I Monroe Donsker, Chairman Continental Ballroom	
9:00 a.m. - 10:55 a.m.	Session on Algebra I, Parlor 1	
9:00 a.m. - 10:55 a.m.	Session on Applied Mathematics I Parlor 3	
9:00 a.m. - 10:55 a.m.	Session on Geometry I, Parlor 7	
9:00 a.m. - 10:55 a.m.	Session on Topology I, Parlor 9	
9:00 a.m. - 10:40 a.m.	Session on Analysis I, Rosewood Suite	
9:00 a.m. - 10:55 a.m.	Session on Analysis II, Teakwood Suite	
11:15 a.m.	Business Meeting, Continental Ballroom	
1:30 p.m.	Presidential Address: On associative division algebras A. A. Albert Continental Ballroom	
3:00 p.m. - 4:20 p.m.	Special Session: Integration in function spaces II Monroe Donsker, Chairman Continental Ballroom	
3:00 p.m. - 5:40 p.m.	Session on Algebra II, Parlor 1	
3:00 p.m. - 5:55 p.m.	Session on Number Theory, Parlor 3	
3:00 p.m. - 5:10 p.m.	Session on Geometry II, Parlor 7	
3:00 p.m. - 5:40 p.m.	Session on Topology II, Parlor 9	
3:00 p.m. - 5:25 p.m.	Session on Analysis III, Rosewood Suite	
3:00 p.m. - 4:40 p.m.	Session on Analysis IV, Teakwood Suite	
8:00 p.m.	Josiah Willard Gibbs Lecture: Symmetry principles in old and new physics Eugene P. Wigner Continental Ballroom	

WEDNESDAY, January 24	AMS	MAA
9:00 a.m. - 5:00 p.m.	REGISTRATION - East Lounge	
9:00 a.m. - 5:00 p.m.	EXHIBITS - North Lounge, West Lounge, Garden Room	
9:00 a.m. - 5:00 p.m.	EMPLOYMENT REGISTER - Imperial Ballroom	
9:00 a.m. - 11:20 a.m.	Special Session: Piecewise linear topology Morris W. Hirsch, Chairman Continental Ballroom	
9:00 a.m. - 11:55 a.m.	Session on Algebra III, Parlor 1	
9:00 a.m. - 11:25 a.m.	Session on Applied Mathematics II Parlor 3	
9:00 a.m. - 11:55 a.m.	Session on Analysis V, Parlor 7	
9:00 a.m. - 12:10 p.m.	Session on Topology III, Parlor 9	
9:00 a.m. - 11:55 a.m.	Session on Analysis VI, Rosewood Suite	
9:00 a.m. - 11:25 a.m.	Session on Analysis VII, Teakwood Suite	
9:00 a.m. - 4:00 p.m.		Board of Governors, Toyon Suite

WEDNESDAY, January 24	American Mathematical Society	Mathematical Association of America
1:30 p.m.	George David Birkhoff Prize in Applied Mathematics Continental Ballroom	
2:30 p.m. - 5:40 p.m.	Session on Algebra IV, Parlor 1	
2:30 p.m. - 5:25 p.m.	Session on Analysis VIII, Parlor 3	
2:30 p.m. - 5:55 p.m.	Session on Graph Theory, Parlor 7	
2:30 p.m. - 4:55 p.m.	Session on Topology IV, Parlor 9	
2:30 p.m. - 5:40 p.m.	Session on Analysis IX, Rosewood Suite	
2:30 p.m. - 5:40 p.m.	Session on Analysis X, Teakwood Suite	
8:00 p.m.	Open Meeting: Committee to Monitor Problems in Communication in the Mathematical Sciences Continental Ballroom	

THURSDAY, January 25	AMS	MAA
9:00 a.m. - 5:00 p.m.	REGISTRATION - East Lounge	
9:00 a.m. - 5:00 p.m.	EXHIBITS - North Lounge, West Lounge, Garden Room	
9:00 a.m. - 5:00 p.m.	EMPLOYMENT REGISTER - Imperial Ballroom	
		Continental Ballroom
		What is Automata Theory? Samuel Eilenberg, Chairman
9:00 a.m. - 9:50 a.m.		Programs and Machines Dana S. Scott
10:00 a.m. - 10:50 a.m.		Grammars and Languages E. H. Spanier
11:00 a.m. - 11:50 a.m.		Theory of Computation Marvin Minsky
1:30 p.m.	Invited Address: Connection problems for asymptotic series Wolfgang Wasow Continental Ballroom	
3:00 p.m. - 4:55 p.m.	Session on Algebra V, Parlor 1	
3:00 p.m. - 5:10 p.m.	Session on Algebra VI, Parlor 3	
3:00 p.m. - 4:40 p.m.	Session on Probability and Statistics Parlor 7	
3:00 p.m. - 4:55 p.m.	Session on Topology V, Parlor 9	
3:00 p.m. - 4:55 p.m.	Session on Analysis XI Rosewood Suite	
3:00 p.m. - 4:55 p.m	Session on Analysis XII Teakwood Suite	
3:00 p.m.	Informal Session on Category Theory Walnut Suite	
5:00 p.m. - 7:00 p.m.	NO HOST GET-TOGETHER - Hilton Plaza	
		Continental Ballroom
		Film Program
7:30 p.m. - 8:03 p.m.		MR. SIMPLEX SAVES THE ASPIDISTRA, with Frank Kocher, Leon Henkin, and Julius H. Hlavaty (A CEM Level I film in color)

THURSDAY, January 25	American Mathematical Society	Mathematical Association of America
8:10 p.m. - 8:55 p.m.		Final Report of the CEM Level Project, by Professor C. B. Allendoerfer, including the presentation of some of the Level I films together with film strips not shown previously
9:10 p.m.		FILMS OF THE COLLEGE GEOMETRY PROJECT OF THE UNIVERSITY OF MINNESOTA (in color)
9:10 p.m. - 9:25 p.m.		SYMMETRIES OF THE CUBE by H. S. M. Coxeter and Wm. O. J. Moser
9:25 p.m. - 9:52 p.m.		ISOMETRIES by Wm. O. J. Moser and S. Schuster

FRIDAY, January 26	AMS	MAA
9:00 a.m. - 5:00 p.m.	REGISTRATION - East Lounge	
9:00 a.m. - 5:00 p.m.	EXHIBITS - North Lounge, West Lounge, Garden Room	
9:00 a.m. - 5:00 p.m.	EMPLOYMENT REGISTER - Imperial Ballroom	
		Continental Ballroom
9:00 a.m. - 9:50 a.m.		Chairman: R. L. Wilder What is Global Analysis? Stephen Smale
10:00 a.m. - 10:50 a.m.		Business Meeting; the Association's Seventh Award for Distinguished Service to Mathematics and the Award of the 1968 Chauvenet Prize
11:00 a.m. - 11:50 a.m.		Chairman: E. E. Moise Retiring Presidential Address: Evolution of the Concept of Rigor R. L. Wilder
1:30 p.m.	Invited Address: A survey of solvable Lie groups and applications Louis Auslander Continental Ballroom	
3:00 p.m. - 5:40 p.m.	Session on Algebra, Logic and Foundations, Parlor 1	
3:00 p.m. - 4:40 p.m.	Session on Algebra VII, Parlor 3	
3:00 p.m. - 4:55 p.m.	Session on Applied Mathematics III Parlor 7	
3:00 p.m. - 5:25 p.m.	Session on Topology VI, Parlor 9	
3:00 p.m. - 5:25 p.m.	Session on Analysis XIII, Rosewood Suite	
		Continental Ballroom Film Program
7:30 p.m. - 8:17 p.m.		MEASURES AND SET THEORY: A Lecture by Stanislaw Ulam (A CEM Individual Lectures film in black and white)
8:30 p.m. - 9:30 p.m.		WHO KILLED DETERMINANTS? A Lecture by Kenneth O. May (A CEM Individual Lectures film in black and white)

SATURDAY, January 27	American Mathematical Society	Mathematical Association of America
9:00 a.m. - 3:00 p.m.	REGISTRATION - East Lounge	
		<u>Continental Ballroom</u>
9:00 a.m. - 9:50 a.m.		Chairman: David Gale
		On Mathematical Reasoning in Science M. M. Schiffer
10:00 a.m.		Session on Applications of Mathematics in the Undergraduate Curriculum Chairman: A. H. Taub
10:00 a.m. - 10:25 a.m.		Is It Possible or Desirable to Introduce Applications into the Undergraduate Mathematics Curriculum? Ben Noble
10:25 a.m. - 10:35 a.m.		Discussion
10:35 a.m. - 11:00 a.m.		The Far-Flung Applications of Mathematics H. O. Pollak
11:00 a.m. - 11:10 a.m.		Discussion
11:10 a.m. - 11:35 a.m.		Applications of Mathematics and the Problems of the Small Department G. S. Young
11:35 a.m. - 11:45 a.m.		Discussion
11:45 a.m. - Noon		General Discussion by the Panel and Audience
1:45 p.m. - 2:35 p.m.		Chairman: Victor Klee
		Some Combinatorial Problems in the Theory of Convex Sets G. D. Chakerian
2:45 p.m.		Panel Discussion of an International Study of Achievement in Mathematics Moderator: J. G. Herriot
2:45 p.m. - 3:45 p.m.		Presentations by Members of the Panel: E. G. Begle, R. P. Dilworth, M. L. Hartung, B. W. Jones
3:45 p.m. - 4:30 p.m.		General Discussion by the Panel and the Audience

PROGRAM OF THE SESSIONS

The time limit for each contributed paper is ten minutes. The contributed papers are scheduled at 15 minute intervals. To maintain this schedule, the time limit will be strictly enforced.

TUESDAY, 9:00 A.M.

Special Session on Integration in Function Spaces I, Continental Ballroom

9:00-9:20
An integral calculus in function spaces
Professor R. H. Cameron, University of Minnesota

9:30-9:50
Classical analysis over infinite dimensional spaces
Professor Leonard Gross, Cornell University

10:00-10:20
Feynman path integrals
Professor Edward Nelson, Princeton University

TUESDAY, 9:00 A.M.

Session on Algebra I, Parlor 1

9:00-9:10
(1) A remark on class groups of extensions with certain types of Galois groups
Professor J. H. Smith, Cambridge, Massachusetts (653-205)

9:15-9:25
(2) A generalization of the Gauss bound for a certain class of biquadratic number fields
Dr. R. B. Lakein, University of Maryland (653-302)

9:30-9:40
(3) Extensions of lattices over orders
Dr. K. W. Roggenkamp, University of Illinois (653-86)

9:45-9:55
(4) On the divisibility of the group of divisor classes of degree zero
Professor W. H. Graves, University of North Carolina (653-362)

10:00-10:10
(5) Some results in the theory of primes
Mr. J. E. Schneider, University of Oregon (653-49)

10:15-10:25
(6) The converse to a well-known theorem on Noetherian rings
Mr. P. M. Eakin, Jr., Louisiana State University, Baton Rouge (653-1)
(Introduced by Professor J. E. Ohm)

10:30-10:40
(7) The unique primary decomposition theorem in commutative rings without identity
Professor Robert Gilmer, Florida State University (653-172)

10:45-10:55
(8) Analytic independence in Noetherian rings
Professor E. D. Davis, Purdue University (653-332)

TUESDAY, 9:00 A.M.

Session on Applied Mathematics I, Parlor 3

9:00-9:10
(9) Differentiation of functions of observational data
Dr. Mark Lotkin, General Electric Company, Cherry Hill, New Jersey (653-334)

9:15-9:25

(10) Numerical integration in higher dimensions

Professor R. B. Guenther, Oregon State University, and Professor E. L Roetman*, Stevens Institute of Technology (653-40)

9:30-9:40

(11) Randomized quadrature formulas

Dr. Seymour Haber, National Bureau of Standards, Washington, D. C. (653-67)

9:45-9:55

(12) On the convergence of a general class of iterative methods

Professor J. E. Dennis*, Jr., University of Utah, and Professor K. M. Brown, Cornell University (653-322)

10:00-10:10

(13) Investigation about the algebraic equations associated with Runge-Kutta -method

Professor D. Sarafyan* and Mr. E. Guillot, Louisiana State University in New Orleans (653-260)

10:15-10:25

(14) Behaviour of Padé fractions and the associated quantities

Professor R. L. Bhirud, Purdue University (653-154)

10:30-10:40

(15) Interpolation in n-dimensions

Professor R. B. Guenther*, Oregon State University, and Professor E. L. Roetman, Stevens Institute of Technology (653-41)

10:45-10:55

(16) On the location of the deviation points in Chebyshev approximation

Professor J. H. Rowland, University of Wyoming (653-184)

TUESDAY, 9:00 A.M.

Session on Geometry I, Parlor 7

9:00-9:10

(17) Metric characterization of bordered Riemannian manifolds; intrinsic regular border of a Riemannian manifold

Professor Nachman Aronszajan, University of Kansas (653-134)

9:15-9:25

(18) An application of the Cauchy-Kowalewski theorem in Riemannian geometry

Professor A. B. Poritz, University of Pennsylvania (653-299)

9:30-9:40

(19) A note on flat manifolds

Professor D. E. Blair*, Michigan State University, and Professor A. P. Stone, University of Illinois at Chicago Circle (653-261)

9:45-9:55

(20) Compact flat Riemannian four dimensional manifolds. Preliminary report

Professor L. S. Charlap* and Professor C. H. Sah, University of Pennsylvania (653-199)

10:00-10:10

(21) On holonomy groups of complex hypersurfaces. Preliminary report

Professor Katsumi Nomizu, Brown University, and Dr. Brian Smyth*, University of Notre Dame (653-34)

10:15-10:25

(22) Symmetries of surfaces of constant width

Professor J. P. Fillmore, University of California, San Diego (653-263)

10:30-10:40

(23) On holomorphic mappings of complex manifolds

Dr. Y. C. Lu, Bowling Green State University (653-158)

* For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.

10:45-10:55

(24) A characterization of the real miquelian Laguerre plane
Professor Hansjoachim Groh, University of Florida (653-132)
(Introduced by Professor A. D. Wallace)

TUESDAY, 9:00 A.M.

Session on Topology I, Parlor 9

9:00-9:10

(25) On "Lipschitz structures", structures which are relevant to the concept of contractivity
Professor Ludvik Janos, University of Florida (653-197)

9:15-9:25

(26) On a class of extremally disconnected spaces. Preliminary report
Mr. M. R. Kirch, Lehigh University (653-102)

9:30-9:40

(27) A locally convex topology on a preordered space
Professor M. D. Green, George Washington University (653-124)

9:45-9:55

(28) Infinite complementation in the lattice of topologies
Professor P. S. Schnare, University of Florida (653-108)

10:00-10:10

(29) Topologies compact modulo an ideal
Mr. R. L. Newcomb, University of California, Santa Barbara (653-347)

10:15-10:25

(30) Representation of relatively complemented distributive lattices
Professor Philip Nanzetta, University of Florida (653-130)

10:30-10:40

(31) Reflexive spaces and dual topologies
Professor F. J. Wagner, University of Cincinnati (653-177)

10:45-10:55

(32) Topologies generated by relations
Professor R. E. Smithson, University of Wyoming (653-284)

TUESDAY, 9:00 A.M.

Session on Analysis I, Rosewood Suite

9:00-9:10

(33) On convolution transforms
Dr. Zeen Ditzian, University of Alberta (653-304)

9:15-9:25

(34) On some integrals in operational calculus
Professor P. C. Consul, University of Calgary, (653-337)

9:30-9:40

(35) Generalizations of the Taylor transform generated by analytic functions
Professor T. K. Boehme, University of California, Santa Barbara, and Professor R. E. Powell*, University of Kentucky (653-56)

9:45-9:55

(36) The complex Hankel and I-transformations of generalized functions
Dr. E. L. Koh*, University of South Carolina, and Professor A. H. Zemanian, State University of New York at Stony Brook (653-85)

10:00-10:10

(37) On a subclass of harmonic functions defined by the Bergman-Whittaker operator
Professor P. L. Rosenthal and Mr. Maciej Skwarczynski*, Stanford University (653-266)

(38) Green's functions on the classical Cartan domains. I
Professor K. T. Hahn and Professor Josephine Mitchell*, Pennsylvania State University (653-220)

10:30-10:40

(39) A criterion for the proportionality of potentials with polar point support
Professor P. A. Loeb, University of California, Los Angeles (653-309)

TUESDAY, 9:00 A.M.

Session on Analysis II, Teakwood Suite

9:00-9:10

(40) On hypersurfaces of limit type
Professor Ubiratan D'Ambrosio, University of Rhode Island (653-330)

9:15-9:25

(41) Polynomial factors of light mappings on an arc
Professor S. W. Young, University of Utah (653-61)

9:30-9:40

(42) On fixed point theorem
Mr. K. L. Singh, Memorial University of Newfoundland (653-245)
(Introduced by Professor A. E. Fekete)

9:45-9:55

(43) On the differentiability structure of real functions
Professor A. M. Bruckner*, Mr. J. G. Ceder and Mr. M. L. Weiss, University of California, Santa Barbara (653-241)

10:00-10:10

(44) Generalized derivatives and monotonicity
Professor J. L. Leonard, University of Arizona (653-196)

10:15-10:25

(45) Means and minimization of errors
Professor Michael Aissen, Fordham University and Aerospace Research Laboratories (653-314)

10:30-10:40

(46) Behavior of the extended Hölder inequality with respect to the index set
Professor H. W. McLaughlin*, Rensselaer Polytechnic Institute, and Professor F. T. Metcalf, University of California, Riverside (653-190)

10:45-10:55

(47) Some complements of Hölders inequality
Professor D. C. Barnes, Washington State University (653-361)

TUESDAY, 11:15 A.M.

Business Meeting, Continental Ballroom

TUESDAY, 1:30 P.M.

Presidential Address, Continental Ballroom

On associative division algebras
Dean A. A. Albert, University of Chicago

TUESDAY, 3:00 P.M.

Special Session on Integration in Function Spaces II, Continental Ballroom

3:00-3:20

Integration in function spaces and its relation to some problems in statistical mechanics
Professor Mark Kac, Rockefeller University

3:30-3:50

Diffusion processes with a small parameter

Professor S. R. S. Varadhan, New York University

4:00-4:20

Nonlinear local functions of weak processes

Professor I. E. Segal, Massachusetts Institute of Technology

TUESDAY, 3:00 P.M.

Session on Algebra II, Parlor 1

3:00-3:10

(48) The number of graded partially ordered sets

Dr. D. A. Klarner, McMaster University (653-259)

3:15-3:25

(49) Generalizations of lattices with unique complements

Professor C. C. Chen, Queen's University, and Professor G. A. Grätzer*, University of Manitoba (653-234)

3:30-3:40

(50) States on orthomodular lattices

Professor M. K. Bennett, University of Massachusetts (653-294)

3:45-3:55

(51) On a square root function

Professor R. E. DeMarr, University of Washington (653-105)

4:00-4:10

(52) Extensions of pseudo lattice ordered groups. Preliminary report

Professor J. R. Teller, Georgetown University (653-141)

4:15-4:25

(53) Functions of bounded variation on a commutative idempotent semigroup

Professor J. E. Kist, New Mexico State University, and Professor P. H. Maserick*, Pennsylvania State University (653-29)

4:30-4:40

(54) Primal decomposition in noncommutative Noetherian systems. Preliminary report

Dr. T. J. Benac, United States Naval Academy, and Miss C. M. Murphy*, Catholic University of America (653-300)

4:45-4:55

(55) A generalization of Boolean rings

Professor H. G. Moore*, Brigham Young University, and Professor A. M. Yaqub, University of California, Santa Barbara (653-82)

5:00-5:10

(56) Some results on nonpotent, locally cyclic semigroups. Preliminary report

Professor R. G. Levin, Western Washington State College (653-155)

5:15-5:25

(57) Infinite inverse semigroups that are homomorphically finite

Professor B. A. Jensen, Portland State College (653-200)

5:30-5:40

(58) Strongly generalized periodic elements in a group

Professor H. A. Hollister, Bowling Green State University (653-26)

TUESDAY, 3:00 P.M.

Session on Number Theory, Parlor 3

3:00-3:10

(59) On the Euler and Bernoulli polynomials

Professor J. D. Brillhart, University of Arizona (653-156)

3:15-3:25

(60) Functions over finite fields preserving mth powers

Professor R. M. McConnell, University of Tennessee (653-145)

3:30-3:40

(61) Polynomials which after repeated division have remainders in arithmetic progression

Professor Gregory Wulczyn, Bucknell University (653-47)

3:45-3:55

(62) Uniform distribution in Galois fields

Professor L. Kuipers, Southern Illinois University (653-146)

4:00-4:10

(63) Irregularities in the distributions of finite sequences

Mr. E. R. Berlekamp and Mr. R. L. Graham*, Bell Telephone Laboratories, Incorporated, Murray Hill, New Jersey (653-162)

4:15-4:25

(64) Asymptotic formulae for multirowed partitions

Professor Basil Gordon, University of California, Los Angeles and Professor Lorne Houten*, Washington State University (653-186)

4:30-4:40

(65) Tauberian theorems for Dirichlet convolutions

Mr. S. L. Segal, University of Rochester (653-232)

4:45-4:55

(66) Primitive roots modulo a prime as consecutive terms of an arithmetic progression

Mr. Emanuel Vegh, U. S. Naval Research Laboratory, Washington, D. C. (653-176)

5:00-5:10

(67) The Pisano period, Fibonacci frequency and Leonardo logarithm of the positive integers

Mr. J. D. Fulton*, Oak Ridge National Laboratory, Oakridge, Tennessee, and Clemson University, and Mr. W. L. Morris, Oak Ridge National Laboratory, Oakridge, Tennessee (653-18)

5:15-5:25

(68) Primitive roots in certain intervals

Professor C. T. Whyburn, Louisiana State University, Baton Rouge (653-19)

5:30-5:40

(69) Distribution of quartic and quintic nonresidues

Professor J. H. Jordan, Washington State University, and Miss S. J. Kelley*, Western Washington State College (653-5)

5:45-5:55

(70) The distribution of kth power residues and nonresidues in the Gaussian integers

Professor J. H. Jordan, Washington State University (653-187)

TUESDAY, 3:00 P.M.

Session on Geometry II, Parlor 7

3:00-3:10

(71) On Hjelmslev planes and modular lattices with a homogeneous basis of chains

Dr. Benno Artmann, McMaster University (653-249)

(Introduced by Professor G. W. Bruns)

3:15-3:25

(72) Neighbor collineations of desarguesian Hjelmslev planes

Dr. D. A. Drake, University of Florida (653-160)

3:30-3:40

(73) The k-configuration. Preliminary report

Professor R. B. Killgrove, California State College at Los Angeles (653-28)

3:45-3:55

(74) On the nonexistence of a class of configurations which are nearly generalized n-gons

Professor S. E. Payne, Miami University (653-112)

4:00-4:10

(75) More about Radon's theorem

Professor J. R. Reay, Western Washington State College (653-73)

4:15-4:25

(76) An upper bound on the dimension of the convex kernel

Professor J. W. Kenelly*, Clemson University, and Mr. B. D. Evans, Oklahoma State University (653-182)

4:30-4:40

(77) Approximation of convex bodies by finite sums of line segments. Preliminary report

Professor N. F. Lindquist, Western Washington State College (653-207)

4:45-4:55

(78) Piecewise flatness and surface area

Professor L. V. Toralballa, New York University (653-95)

5:00-5:10

(79) A weighted volume-diameter inequality for n-cubes

Professor W. R. Derrick, University of Utah (653-231)

TUESDAY, 3:00 P.M.

Session on Topology II, Parlor 9

3:00-3:10

(80) Cellular arcs in 3-space have shrinking points

Professor C. D. Sikkema, Florida State University (653-100)

3:15-3:25

(81) Locally nice manifolds are tame in codimension three

Professor J. L. Bryant*, Florida State University, and Professor C. L. Seebeck III, Michigan State University (653-48)

3:30-3:40

(82) A uniform generalized Schoenflies theorem

Professor Perrin Wright, Florida State University (653-191)

3:45-3:55

(83) Homotopy properties of decomposition spaces

Professor Steve Armentrout, University of Iowa (653-344)

4:00-4:10

(84) Decompositions of S^3 and pseudo-isotopies

Professor T. M. Price, University of Iowa (653-201)

4:15-4:25

(85) A 2-sphere in E^3 is tame if it is 1-LC through each complementary domain

Mr. Warren White, University of Wisconsin (653-24)

4:30-4:40

(86) Piercing locally spherical spheres with tame arcs

Professor L. D. Loveland, Utah State University (653-12)

4:45-4:55

(87) Homeomorphic continuous curves in 2-space are isotopic in 3-space

Mr. W. K. Mason, University of Wisconsin (653-222)

5:00-5:10

(88) One-to-one continuous mappings into E^2

Mr. D. H. Pettey, University of Utah (653-293)

5:15-5:25

(89) Light open mappings on a torus with disk removed

Professor M. L. Marx, Vanderbilt University (653-193)

5:30-5:40

(90) Approximation of certain continuous functions of S^2 into E^3

Professor E. H. Anderson, University of North Dakota (653-223)

TUESDAY, 3:00 P.M.

Session on Analysis III, Rosewood Suite

3:00-3:10

(91) Existence and uniqueness of solutions of ordinary differential equations

Professor D. V. V. Wend, Montana State University (653-39)

3:15-3:25

(92) On the inverse problem for ordinary differential operators of even order

Professor J. B. Butler, Jr., Portland State College (653-20)

3:30-3:40

(93) A differential operator with no smooth functions in its domain

Professor R. J. Lindahl, Pennsylvania State University, and Professor R. D. Moyer*, University of Kansas (653-31)

3:45-3:55

(94) A separation condition for the zero solution of a system of differential equations

Professor A. M. Fink and Professor George Seifert*, Iowa State University (653-64)

4:00-4:10

(95) Spectral resolution of self-adjoint analytic differential operators. Preliminary report

Dr. A. L. Villone, International Business Machines, Los Angeles, California (653-257)

4:15-4:25

(96) On nonlinear elliptic boundary value problems of von Karman type

Professor M. S. Berger, Courant Institute of Mathematical Sciences (653-276)

4:30-4:40

(97) Comparison theorems for nonlinear vector differential equations

Professor W. T. Reid, University of Oklahoma (653-291)

4:45-4:55

(98) Stokes multipliers of subdominant solutions of the differential equation $y'' - (x^m + a_1 x^{m-1} + \ldots + a_{m-1}x + \lambda)y = 0$

Professor Herman Gollwitzer, University of Tennessee (653-37)

5:00-5:10

(99) On the matrix Sturm-Liouville equations

Dr. G. J. Etgen, National Aeronautics and Space Administration, Washington, D. C. (653-4)

5:15-5:25

(100) Quadratic eigenvalue problems

Professor Jerome Eisenfeld, Rensselaer Polytechnic Institute (653-328)

TUESDAY, 3:00 P.M.

Session on Analysis IV, Teakwood Suite

3:00-3:10

(101) Almost convergent positive linear operators

Professor J. P. King, Lehigh University (653-280)

3:15-3:25

(102) On sequences of contractions and their fixed points

Professor S. B. Nadler, Louisiana State University, Baton Rouge (653-204)

3:30-3:40

(103) Averaging iteration in a Banach space. Preliminary report

Professor C. L. Outlaw* and Mr. C. W. Groetsch, Louisiana State University in New Orleans (653-342)

3:45-3:55

(104) Matrix transformations between FK-spaces and sequences of Fourier coefficients

Professor Günther Goes, Illinois Institute of Technology (653-271)

4:00-4:10

(105) Matrix summability of convex sequences

Professor D. F. Dawson, North Texas State University (653-111)

4:15-4:25

(106) Replaceability of methods of summation

Professor H. I. Brown, State University of New York at Albany (653-44)

4:30-4:40

(107) On the absolute Nörlund summability of a Fourier series

Professor C. S. Rees*, University of Tennessee, and Professor S. M. Shah, University of Kentucky (653-8)

TUESDAY, 8:00 P.M.

Gibbs Lecture, Continental Ballroom

Representation theory in physics

Professor Eugene Wigner, Princeton University

WEDNESDAY, 9:00 A.M.

Special Session on Piecewise Linear Topology, Continental Ballroom

9:00-9:20

Mappings with contractible point inverses between p.l. manifolds

Professor Marshall Cohen*, Princeton University, and Professor Dennis Sullivan, University of California, Berkeley (653-363)

9:30-9:50

Constructing non-trivial h-cobordisms

Professor E. H. Connell, University of California, Berkeley (653-366)

10:00-10:20

K-theory and piecewise linear topology

Professor Dennis Sullivan, University of California, Berkeley (653-365)

10:30-10:50

Polyhedral monkey saddles

Professor T. F. Banchoff, Brown University (653-367)

11:00-11:20

An embedding without a normal microbundle

Dr. C. P. Rourke*, Institute for Advanced Study and University of Warwick, England, and Dr. B. J. Sanderson, University of California, Berkeley, and University of Warwick, England (653-364)

(Introduced by Professor M. W. Hirsch)

WEDNESDAY, 9:00 A.M.

Session on Algebra III, Parlor 1

9:00-9:10

(108) On generalized Baer groups

Mr. Phillip Griffith, University of Houston (653-258)

9:15-9:25

(109) Sums of automorphisms of a primary Abelian group

Mr. Frank Castagna, New Mexico State University (653-99)

9:30-9:40

(110) On certain classes of primary Abelian groups

Professor Paul Hill, University of Houston, and Professor Charles Megibben*, Vanderbilt University (653-93)

9:45-9:55

(111) Regular modules

Professor D. J. Fieldhouse, Queen's University (653-210)

10:00-10:10

(112) Algebraic compactness for modules. Preliminary report

Dr. R. B. Warfield, Jr., New Mexico State University (653-136)

10:15-10:25

(113) Projective ideals of finite type. Preliminary report

Professor W. W. Smith, University of North Carolina (653-57)

10:30-10:40

(114) On the free product of algebras

Professor T. W. Hungerford, University of Washington (653-52)

10:45-10:55

(115) On G-algebra extensions. Preliminary report

Professor K. C. Salter, University of Massachusetts (653-103)

11:00-11:10

(116) On the cohomology of completely primary rings

Dr. D. F. Sanderson, Western Washington State College (653-208)

11:15-11:25

(117) Semiprimary QF-3 rings

Professor R. R. Colby* and Professor E. A. Rutter, Jr., University of Kansas (653-292)

11:30-11:40

(118) The structure of QF-3 rings

Professor K. R. Fuller, University of Iowa (653-233)

11:45-11:55

(119) Some characterizations of quasi-Frobenius rings

Professor E. A. Rutter, Jr., University of Kansas (653-333)

WEDNESDAY, 9:00 A.M.

Session on Applied Mathematics II, Parlor 3

9:00-9:10

(120) Iteration procedures and Picard's criterion for equations of the first kind

Professor J. B. Diaz*, Rensselaer Polytechnic Institute, and Professor F. T. Metcalf, University of California, Riverside (653-142)

9:15-9:25

(121) Convergence of dynamic relaxation (second order Richardson's method)

Dr. R. B. Simpson, California Institute of Technology (653-89)

9:30-9:40

(122) Stability of mixed implicit difference schemes

Dr. Stanley Osher, Brookhaven National Laboratory, Upton, New York (653-239)

9:45-9:55

(123) Elliptic difference equations on a convex domain

Dr. W. H. Guilinger*, Bettis Atomic Power Laboratory, West Mifflin, Pennsylvania, and Dr. R. B. Kellogg, University of Maryland (653-32)

10:00-10:10

(124) Numerical solution of initial-boundary value problems for mildly nonlinear parabolic and hyperbolic equations

Professor Donald Greenspan, University of Wisconsin (653-23)

10:15-10:25

(125) The rate of convergence of parabolic difference schemes

Professor G. W. Hedstrom, University of Michigan (653-354)

10:30-10:40

(126) Boundary value problems for linear systems of ordinary differential equations involving many small parameters

Professor R. E. O'Malley, Jr., Mathematics Research Center, United States Army, and University of Wisconsin (653-217)

10:45-10:55

(127) An averaging scheme for some nonlinear resonance problems

Dr. J. A. Morrison, Bell Telephone Laboratories, Incorporated, Murray Hill, New Jersey (653-83)

11:00-11:10

(128) Elementary theory of differential systems. Preliminary report

Mr. R. B. McNeill, Pennsylvania State University (653-101)

11:15-11:25

(129) Isoperimetric bound for eigenvalue

Professor B. A. Troesch, University of Southern California (653-356)

WEDNESDAY, 9:00 A.M.

Session on Analysis V. Parlor 7

9:00-9:10

(130) Congruences defined by continuous real-valued functions

Professor Michael Friedberg, University of Houston (653-179)

9:15-9:25

(131) On imbedding medial topological groupoids

Professor Kermit Sigmon, University of Florida (653-139)

9:30-9:40

(132) Results concerning the Schutzenberger-Wallace theorem

Professor Tony Shershin, University of Southern Florida (653-358)
(Introduced by Dr. F. L. Cleaver)

9:45-9:55

(133) Compact semigroups with square roots

Professor J. M. Day, College of Notre Dame (653-27)

10:00-10:10

(134) Homomorphisms of topological semilattices

Professor J. D. Lawson, University of Tennessee (653-122)

10:15-10:25

(135) Fixed points and separately continuous actions of topological semigroups. Preliminary report

Professor Theodore Mitchell, Temple University (653-76)

10:30-10:40

(136) A class of convolution measure algebras. Preliminary report

Mr. S. E. Newman, University of Utah (653-286)
(Introduced by Professor J. L. Taylor)

10:45-10:55

(137) A maximal problem in harmonic analysis. III

Professor Edwin Hewitt, University of Washington, and Professor K. A. Ross*, University of Oregon (653-209)

11:00-11:10

(138) Measures with bounded convolution powers. Preliminary report

Mr. B. M. Schreiber, University of Washington (653-323)

11:15-11:25

(139) Almost periodic measures on a compact Abelian group

Professor J. W. Kitchen, Duke University (653-211)

11:30-11:40

(140) The existence of homomorphisms in compact connected Abelian groups

Professor G. L. Itzkowitz, State University of New York at Buffalo (653-214)

11:45-11:55

(141) Topological groups in the boundary of a locally compact connected Abelian group

Mr. James Stepp, University of Kentucky and Georgetown College (653-248)
(Introduced by Professor John Selden)

WEDNESDAY, 9:00 A.M.

Session on Topology III, Parlor 9

9:00-9:10

(142) Sum theorems for topological spaces

Professor R. E. Hodel, Duke University (653-11)

9:15-9:25

(143) Collectionwise normal subsets (continuation)

Professor C. E. Aull, Virginia Polytechnic Institute (653-272)

9:30-9:40

(144) A new extension of countable compactness

Professor W. M. Fleischman, State University of New York at Buffalo (653-53)
(Introduced by Professor Everett Pitcher)

9:45-9:55

(145) m-compactness, m-quasicompactness, and m-pseudocompactness

Professor J. H. Weston, University of Saskatchewan, Regina (653-175)
(Introduced by Professor C, L. Kaller)

10:00-10:10

(146) Pseudocompact spaces

Professor R. M. Stephenson, Jr., University of North Carolina at Chapel Hill (653-144)

10:15-10:25

(147) The Stone-Čech compactification for limit spaces

Professor Oswald Wyler, Carnegie-Mellon University (653-306)

10:30-10:40

(148) Extensions of totally bounded pseudometrics

Professor R. A. Alo and Professor H. L. Shapiro*, Carnegie-Mellon University (653-167)

10:45-10:55

(149) Some characterizations of the Freudenthal compactification of a semicompact space

Professor R. F. Dickman, Jr., University of Miami (653-7)

11:00-11:10

(150) H-closed spaces and reflective subcategories

Professor H. Herrlich and Professor G. E. Strecker*, University of Florida (653-74)

11:15-11:25

(151) On locally H-closed spaces

Professor J. R. Porter, University of Kansas (653-277)

11:30-11:40

(152) Homeomorphs of three subspaces of $BN \setminus N$

Professor W. W. Comfort*, Wesleyan University, and Professor S. Negrepontis, McGill University (653-51)

11:45-11:55

(153) Subspaces of F'-spaces

Professor Mark Mandelker, University of Kansas (653-169)

12:00-12:10

(154) The Stone-Čech compactification of an irreducibly connected space

Professor G. L. Pfeifer, University of Arizona (653-62)

WEDNESDAY, 9:00 A.M.

Session on Analysis VI, Rosewood Suite

9:00-9:10

(155) Polynomials in closed linear relations
Mr. M. J. Kascic, Dartmouth College (653-237)

9:15-9:25

(156) Partial isometries closed under multiplication on Hilbert spaces
Professor Ivan Erdelyi, Kansas State University (653-265)

9:30-9:40

(157) Wave operators and similarity for some operators in Banach spaces
Professor S.-C. Lin, University of Miami (653-212)

9:45-9:55

(158) The eigenfunctions of certain inner functions
Professor M. J. Sherman, University of California, Los Angeles (653-256)

10:00-10:10

(159) Variational properties of nonlinear spectra
Professor E. H. Rogers, Rensselaer Polytechnic Institute (653-326)

10:15-10:25

(160) The spectrum of an operator on an interpolation space
Professor J. D. Stafney, University of California, Riverside (653-129)

10:30-10:40

(161) On spectral permanence
Professor K. K. Oberai, Queen's University (653-305)

10:45-10:55

(162) Compact operators on Orlicz spaces
Dr. J. J. Uhl, Jr., Defense Intelligence Agency, ADPS Center, and Carnegie-Mellon University (653-164)

11:00-11:10

(163) On uniform convergence and positive operators
Professor L. C. Kurtz, Arizona State University (653-236)

11:15-11:25

(164) Approximation of C_0 semigroups
Professor T. I. Seidman, Carnegie-Mellon University (653-174)

11:30-11:40

(165) Semigroups of nonlinear transformations
Professor J. R. Dorroh, Louisiana State University, Baton Rouge (653-171)

11:45-11:55

(166) Some remarks on the evolution equation and semigroups. Preliminary report
Professor Pawel Szeptycki, University of Kansas (653-127)

WEDNESDAY, 9:00 A.M.

Session on Analysis VII, Teakwood Suite

9:00-9:10

(167) Weierstrass points and analytic submanifolds of Teichmueller space
Professor H. M. Farkas, Johns Hopkins University (653-3)

9:15-9:25

(168) Maximum term of a power series in one and several variables
Mr. J. G. Krishna, University of Kentucky (653-278)
(Introduced by Professor S. M. Shah)

9:30-9:40

(169) Convergence of complex Lagrange interpolation polynomials with nodes lying on a piecewise analytic Jordan curve with cusps
Mr. Patrick O'Hara, University of Miami (653-138)

9:45-9:55

(170) The boundary correspondence of quasiconformal mappings on quasicircles
Professor T. J. Reed, University of Colorado (653-268)

10:00-10:10

(171) On functions meromorphic in a disc
Professor J. E. McMillan, University of Wisconsin, Milwaukee (653-275)

10:15-10:25

(172) On a problem of Bagemihl and Erdös concerning the distribution of the zeros of an annular function
Professor K. F. Barth and Professor W. J. Schneider*, Syracuse University (653-180)

10:30-10:40

(173) On the zeros of a polynomial and its derivative. Preliminary report
Professor A. W. Goodman, Professor O. I. Rahman, and Professor J. S. Ratti*, University of South Florida (653-216)

10:45-10:55

(174) On univalent polynomials
Dr. D. A. Brannan, University of Maryland (653-269)
(Introduced by Professor J. A. Hummel)

11:00-11:10

(175) A coefficient inequality for certain classes of analytic functions
Mr. F. R. Keogh, University of Kentucky and Professor E. P. Merkes*, University of Cincinnati (653-121)

11:15-11:25

(176) Computer investigation of Landau's theorem
Dr. P. S. Chiang, Western Michigan University (653-355)

WEDNESDAY, 1:30 P.M.

George David Birkhoff Prize in Applied Mathematics, Continental Ballroom

WEDNESDAY, 2:30 P.M.

Session on Algebra IV, Parlor 1

2:30-2:40

(177) Finite groups whose powers have no countably infinite factor groups. Preliminary report
Mr. Mitchell Billis, University of Utah (653-288)
(Introduced by Professor W. R. Scott)

2:45-2:55

(178) On a class of solvable groups of even order
Professor Hermann Simon, University of Miami (653-91)

3:00-3:10

(179) An incomplete generalization of Frobenius's theorem. Preliminary report
Mr. J. W. Richards, Kent State University and Michigan State University (653-206)

3:15-3:25

(180) A note on solvable factorizable groups
Professor T. K. Seo, University of Kentucky (653-316)

3:30-3:40

(181) Solvable products of groups
Professor W. R. Scott*, and Professor Fletcher Gross, University of Utah (653-274)

3:45-3:55

(182) The near Frattini subgroups of finitely generated groups
Professor J. B. Riles, St. Louis University (653-351)

4:00-4:10

(183) Generalized Frattini subgroups of finite groups. II

Professor J. C. Beidleman, University of Kentucky (653-159)

4:15-4:25

(184) A note on generalized Frattini subgroups. Preliminary report

Mr. D. C. Dykes, University of Kentucky (653-152)
(Introduced by Dr. J. C. Beidleman)

4:30-4:40

(185) The unit group of a modular group algebra

Professor D. B. Coleman*, University of Kentucky and Professor D. S. Passman, Yale University (653-320)

4:45-4:55

(186) Automorphisms of integral group rings

Dr. S. K. Sehgal, University of Alberta (653-359)
(Introduced by Dr. R. Bercov)

5:00-5:10

(187) Polynomial functions and wreath products

Professor Joseph Buckley, University of Massachusetts (653-315)

5:15-5:25

(188) The endomorphisms of some one-relator groups

Professor Michael Anshel, University of Arizona (653-79)

5:30-5:40

(189) Groups defined by permutations of a single word. Preliminary report

Professor W. R. Emerson, New York University, Washington Square College (653-226)

WEDNESDAY, 2:30 P.M.

Session on Analysis VIII, Parlor 3

2:30-2:40

(190) Collectively compact sets which are totally bounded

Professor T. W. Palmer, University of Kansas (653-147)

2:45-2:55

(191) A linear space. Preliminary report

Professor Jack Nebb, University of Georgia (653-317)

3:00-3:10

(192) Seminorm-dual subspaces of the algebraic dual of a linear space

Professor D. R. Kerr, Jr., State University of New York at Albany (653-45)

3:15-3:25

(193) Banach spaces of Lipschitz functions with different metrics on the underlying space

Professor R. B. Fraser, Louisiana State University, Baton Rouge (653-303)

3:30-3:40

(194) Linear isometries on spaces of affine continuous functions

Mr. L. F. Guseman*, Dr. H. E. Lacey, NASA, Houston, Texas, and Dr. P. D. Morris, Pennsylvania State University (653-252)

3:45-3:55

(195) About compactness in Köthe spaces

Professor Sigrun Goes, DePaul University (653-281)

4:00-4:10

(196) Completion of norm linear spaces

Professor K.-W. Yang, Western Michigan University (653-185)

4:15-4:25

(197) A completeness theorem for locally convex spaces and some applications

Professor C. L. Devito, The University of Arizona (653-43)

4:30-4:40

(198) On the L_2 space of a Banach limit

Dr. S. P. Lloyd, Bell Telephone Laboratories, Incorporated, Murray Hill, New Jersey (653-153)

4:45-4:55

(199) Summability of vector sequences

Mr. J. B. Deeds, Louisiana State University, Baton Rouge (653-168)

5:00-5:10

(200) Complemented subspaces and lambda systems in Banach spaces

Professor W. J. Davis*, and Professor D. W. Dean, Ohio State University, and Professor Ivan Singer, Roumanian Academy of Sciences, Bucharest, and Florida State University (653-203)

5:15-5:25

(201) The analytic continuation of vector-valued holomorphic functions

Professor John Horváth, University of Maryland (653-329)

WEDNESDAY, 2:30 P.M.

Session on Graph Theory, Parlor 7

2:30-2:40

(202) On replications of incidence sequences

Mr. A. B. Owens, Naval Research Laboratory, Washington, D. C. (653-360)

2:45-2:55

(203) On the point-group and line-group of a graph

Professor Frank Harary*, University of Michigan, and Professor E. M. Palmer, Michigan State University (653-106)

3:00-3:10

(204) On reconstructing a graph

Professor R. L. Hemminger, Vanderbilt University (653-150)

3:15-3:25

(205) Expanding n-stars

Professor S. F. Kapoor, Western Michigan University (653-120)

3:30-3:40

(206) A sufficient condition for N-connectedness of graphs

Professor Gary Chartrand* and Professor S. F. Kapoor, Western Michigan University, and Professor H. V. Kronk, State University of New York at Binghamton (653-118)

3:45-3:55

(207) A tree counting problem

Mr. J. W. Moon, University of Alberta (653-282)

4:00-4:10

(208) On acyclic simplicial complexes

Professor Frank Harary, University of Michigan, and Professor E. M. Palmer*, Michigan State University (653-33)

4:15-4:25

(209) Minimal regular extensions of oriented graphs

Professor L. W. Beineke and Professor R. E. Pippert*, Purdue University (653-228)

4:30-4:40

(210) Higher-dimensional analogues of the four-color problem

Professor Branko Grünbaum, University of Washington (653-341)

4:45-4:55

(211) On uniquely colorable planar graphs

Professor Gary Chartrand, Western Michigan University, and Mr. Dennis Geller*, University of Michigan (653-70)

5:00-5:10

(212) On partitioning planar graphs

Professor S. T. Hedetniemi, University of Iowa (653-348)

(Introduced by Professor Frank Harary)

5:15-5:25

(213) On the point-dual acyclic numbers of a graph

Professor Gary Chartrand, Western Michigan University, Professor H. V. Kronk*, State University of New York at Binghamton, and Mr. C. E. Wall, Michigan State University (653-110)

5:30-5:40

(214) Connectivity and line-connectivity of graphs and their line-graphs

Professor Gary Chartrand, Western Michigan University, and Mr. M. J. Stewart*, Lansing Community College (653-117)

5:45-5:55

(215) On derived graphs and digraphs

Professor L. W. Beineke, Purdue University (653-195)

WEDNESDAY, 2:30 P.M.

Session on Topology IV, Parlor 9

2:30-2:40

(216) The Jiang subgroup for a map

Mr. W. J. Barnier, Dartmouth College (653-345)

2:45-2:55

(217) Primitive chains and $H_*(\Omega X)$

Professor D. P. Kraines, Haverford College (653-262)

3:00-3:10

(218) On higher Samelson products

Professor F. D. Williams, New Mexico State University (653-137)

3:15-3:25

(219) The isotopy type of certain finite polyhedra. Preliminary report

Mr. S. R. Clemens, University of North Carolina (653-287)

3:30-3:40

(220) Homotopy torsion in codimension two knots

Professor D. L. Sumners, Florida State University (653-50)

3:45-3:55

(221) On the homotopy theory of sphere bundles

Professor T. J. Kyrouz, University of Georgia (653-301)

4:00-4:10

(222) Standard spines of compact connected combinatorial n-manifolds

Professor B. G. Casler*, and T. J. Smith, Louisiana State University, Baton Rouge (653-198)

4:15-4:25

(223) A unique decomposition theorem for 3-manifolds with connected boundary

Mr. J. L. Gross, Dartmouth College (653-327)

4:30-4:40

(224) On suspending homotopy spheres

Professor P. W. Harley, University of Georgia (653-6)

4:45-4:55

(225) Some remarks on noncompact manifolds

Dr. Gudrun Kalmbach, University of Illinois (653-382)

(Introduced by Professor S. S. Cairns)

Session on Analysis IX, Rosewood Suite

2:30-2:40

(226) Application of the topological principle of Wazewski to control systems
Professor V. Lakshmikantham and Professor C. P. Tsokos*, University of Rhode Island (653-331)

2:45-2:55

(227) On a class of functional-integral equations. Preliminary report
Professor Constantin Corduneanu, University of Rhode Island and University of Iasi, Romania (653-297)
(Introduced by Professor V. Lakshmikantham)

3:00-3:10

(228) Variational problems involving functional differential equations
Dr. H. T. Banks, Brown University (653-343)

3:15-3:25

(229) A new differential inequality useful in control systems
Professor V. Lakshmikantham and Professor S. Leela*, University of Rhode Island (653-298)

3:30-3:40

(230) On the equivalence of certain stability properties
Mr. R. W. Gunderson*, Marshall Space Flight Center, Huntsville, Alabama, and Mr. J. H. George, University of Wyoming (653-59)
(Introduced by Dr. J. Horner)

3:45-3:55

(231) Estimates on the existence region for solutions of equations involving a small parameter
Professor H. I. Freedman, University of Alberta, (653-215)

3:00-4:10

(232) Periodic solutions of a second order nonlinear differential equation without damping
Professor D. F. Ullrich and Professor J. A. Marlin*, North Carolina State University (653-346)

4:15-4:25

(233) Existence of oscillatory solutions for a nonlinear odd order differential equation
Professor J. W. Heidel, University of Tennessee (653-240)

4:30-4:40

(234) On the generalized Liénard equation
Professor T. A. Burton and Professor C. G. Townsend*, Southern Illinois University (653-113)

4:45-4:55

(235) On the forced Liénard equation
Professor R. R. Stevens, University of Montana (653-264)

5:00-5:10

(236) Periodic solutions of nonlinear Sturm-Liouville problem
Professor J. H. Wolkowisky, University of Colorado (653-30)
(Introduced by Professor J. J. Stoker)

5:15-5:25

(237) Periodic solution of a third order equation
Miss Marcia Peterson*, and Professor William Swartz, Montana State University (653-17)

5:30-5:40

(238) Integral equations with nonnegative integrable resolvents
Professor R. K. Miller, Brown University (653-235)

WEDNESDAY, 2:30 P.M.

Session on Analysis X, Teakwood Suite

2:30-2:40

(239) Boole series representing functions and Boole functions of negative degree
Dr. G. O. Peters, General Electric Company, Philadelphia, Pennsylvania (643-313)

2:45-2:55

(240) Some new integral relations involving Bessel functions
Mr. H. E. Fettis, Wright-Patterson Air Force Base, Ohio (653-229)

3:00-3:10

(241) On zero type sets of Laguerre polynomials
Professor J. W. Brown, Oberlin College (653-42)

3:15-3:25

(242) The a-points of Faber polynomials for a special function
Professor H. S. Al-amiri, Bowling Green State University (653-230)

3:30-3:40

(243) Orthogonal polynomials whose zeros are dense in a half line. Preliminary report
Professor T. S. Chihara, Seattle University (653-353)

3:45-3:55

(244) Remarks on mixed Taylor-L_2 approximations
Professor Jay Leavitt*, and Professor Krzysztof Frankowski, University of Minnesota (653-311)

4:00-4:10

(245) Some uniqueness theorems for a series of Legendre polynomials
Mr. Paul Rosenthal, Stanford University (653-270)
(Introduced by Professor Charles Loewner)

4:15-4:25

(246) Hermite series singularities
Professor G. G. Walter, University of Wisconsin, Milwaukee (653-308)

4:30-4:40

(247) On I-series
Professor D. R. Lick, Western Michigan University (653-97)

4:45-4:55

(248) Quantitative polynomial approximation on certain planar sets
Professor D. J. Newman, Yeshiva University, and Professor Louis Raymon*, Temple University (653-81)

5:00-5:10

(249) Uniform approximation by polynomials with integral coefficients. I
Professor L. O. Ferguson, University of California, Riverside (653-16)

5:15-5:25

(250) A generalization of the Bernstein polynomial
Professor Bruce Wood, University of Arizona (653-161)

5:30-5:40

(251) Haar series
Professor J. R. McLaughlin, Pennsylvania State University (653-318)

WEDNESDAY, 8:00 P.M.

Open Meeting on Communication Problems in the Mathematical Sciences, Continental Ballroom

THURSDAY, 1:30 P.M.

Invited Address, Continental Ballroom

Connection problems for asymptotic series
Professor Wolfgang Wasow, University of Wisconsin

THURSDAY, 3:00 P.M.

Session on Category Theory, Walnut Suite

THURSDAY, 3:00 P.M.

Session on Algebra V, Parlor 1

3:00-3:10
(252) Left ideal axioms for nonassociative rings
Professor D. A. Lawver, University of Arizona (653-21)

3:15-3:25
(253) A characterization of intrinsic functions on quaternion matrices
Dr. R. E. Carlson and Professor C. G. Cullen*, University of Pittsburgh (653-72)

3:30-3:40
(254) A characterization of Borel and Cartan subalgebras of finite-dimensional Lie algebras
Professor O. H. Kegel, New Mexico State University (653-218)

3:45-3:55
(255) A group of a Lie algebra
Professor Nicholas Heerema, Florida State University (653-58)

4:00-4:10
(256) Scalar replacement in Lie algebras
Professor J. F. Hurley, University of California, Riverside (653-126)

4:15-4:25
(257) Derivation algebras of finite abelian group algebras
Professor J. W. Bond, Pennsylvania State University (653-13)

4:30-4:40
(258) A fusion theorem for semisimple groups. Preliminary report
Professor J. E. Humphreys, University of Oregon (653-335)

4:45-4:55
(259) Terminality of maximal unipotent subgroups of Chevalley and Steinberg groups
Dr. E. L. Spitznagel, Litton Scientific Support Laboratory, Fort Ord, California (653-202)

THURSDAY, 3:00 P.M.

Session on Algebra VI, Parlor 3

3:00-3:10
(260) Quotient structure of a semiring
Professor P. J. Allen, University of Alabama (653-246)

3:15-3:25
(261) Geometric interpretations of planar near-rings
Dr. Michael Anshel and Professor J. R. Clay*, University of Arizona (653-77)

3:30-3:40
(262) On near-rings of polynomials
Professor C. J. Maxson, State University College of New York, Fredonia (653-68)

3:45-3:55
(263) A near-ring embedding problem
Professor J. J. Malone, Jr., Texas A and M University (653-178)

4:00-4:10
(264) Coupled maps on groups and derived structures
Dr. Fritz Pokropp, University of Cincinnati (653-84)

4:15-4:25
(265) Lie isomorphisms of simple rings
Professor W. S. Martindale, 3rd, University of Massachusetts (653-80)

4:30-4:40

(266) A class of galois connections between a group and its automorphism group
Professor H. L. Egan, University of Maryland (653-36)

4:45-4:55

(267) Transvectors and endotensors. Preliminary report
Mr. Joseph Neggers, University of Alabama (653-267)

5:00-5:10

(268) On semiordered mathematical structures. Preliminary report
Professor L. G. Novoa, University of Alabama (653-219)

THURSDAY, 3:00 P.M.

Session on Probability and Statistics, Parlor 7

3:00-3:10

(269) A generalized Fourier-Stieltjes series representation of a singular bivariate probability measure. Preliminary report
Mr. J. D. Nelligan, General Electric Company, Syracuse, New York (653-244)

3:15-3:25

(270) On isomoment functional equations
Professor Janos Aczel, University of Waterloo (653-290)

3:30-3:40

(271) On the structure of equilibrium points of bimatrix games
Professor C. B. Millham, Washington State State University (653-324)

3:45-3:55

(272) The theory of infinitely divisible matrices and kernels
Professor R. A. Horn, University of Santa Clara (653-319)

4:00-4:10

(273) A converse of a random stable limit theorem
Professor S. R. Kimbleton, University of Pittsburgh (653-151)

4:15-4:25

(274) On spherical stochastic processes on a locally compact group
Professor Z. R. Pop-Stojanovic, University of Florida (653-349)
(Introduced by Professor A. R. Bednarek)

4:30-4:40

(275) General Griffiths inequalities on correlations in Ising ferromagnets
Mr. D. G. Kelly*, Jet Propulsion Laboratory, Pasadena, California, and Professor Seymour Sherman, Indiana University (653-15)

THURSDAY, 3:00 P.M.

Session on Topology V, Parlor 9

3:00-3:10

(276) On noncompact solvmanifolds. Preliminary report
Mr. A. W. Currier, University of Maryland (653-255)

3:15-3:25

(277) Imbeddings of compact Lie groups. Preliminary report
Professor L. N. Mann and Professor J. L. Sicks*, University of Massachusetts (653-238)

3:30-3:40

(278) Expansive homeomorphisms on manifolds and homogeneous spaces
Professor Erik Hemmingsen, Syracuse University, and Professor W. L. Reddy*, State University of New York at Albany (653-279)

3:45-3:55

(279) Local homogeneity
Professor G. S. Ungar, Louisiana State University, Baton Rouge (653-87)

4:00-4:10

(280) Lipschitz submanifolds

Professor F. W. Wilson, Jr., University of Colorado (653-104)

4:15-4:25

(281) A lower bound for the Δ-Nielsen number

Professor R. B. S. Brooks, Bowdoin College, and Professor R. F. Brown*, University of California, Los Angeles (658-98)

4:30-4:40

(282) Open mappings and closed zero-dimensional sets

Professor J. E. Keesling, University of Florida (653-131)

4:45-4:55

(283) Pseudo-circles and universal circularly chainable continua. Preliminary report

Mr. J. T. Rogers, Jr., University of California, Riverside (653-140)

THURSDAY, 3:00 P.M.

Session on Analysis XI, Rosewood Suite

3:00-3:10

(284) Entire functions of bounded index in several complex variables

Professor J. G. Krishna and Professor S. M. Shah*, University of Kentucky (653-253)

3:15-3:25

(285) On the boundary point principle for elliptic equations in the plane

Professor J. K. Oddson, University of California, Riverside (653-133)

3:30-3:40

(286) An ordering principle for generalized solutions of quasi-linear equations of the first order

Professor E. D. Conway, Tulane University, and Professor D. R. Smith*, University of California, San Diego (653-123)

3:45-3:55

(287) A Cauchy problem for a semiaxially symmetric wave equation

Professor E. C. Young, Florida State University (653-119)

4:00-4:10

(288) The Cauchy problem for an elliptic operator which changes classification on the boundary

Professor D. Sather, Cornell University, and Professor J. O. Sather*, University of Utah (653-336)

4:15-4:25

(289) Geometry of the complex characteristics in transonic flow

Professor E. V. Swenson, New York University, Courant Institute of Mathematical Sciences (653-125)

(Introduced by Professor P. R. Garabedian)

4:30-4:40

(290) On the singularities of biharmonic functions with discontinuous boundary values. Preliminary report

Professor N. M. Wigley, University of North Carolina (653-94)

4:45-4:55

(291) Existence-uniqueness theorems for nonlinear Dirichlet problems

Professor D. W. Lick, Brookhaven National Laboratory, Upton, New York (653-60)

THURSDAY, 3:00 P.M.

Session on Analysis XII, Teakwood Suite

3:00-3:10

(292) Some characterizations of approximate normality

Professor D. R. Chalice, Western Washington State College (653-63)

3:15-3:25

(293) Analytic structure in the spectrum of a Banach algebra. Preliminary report
Professor T. T. Read, Western Washington State College (653-213)

3:30-3:40

(294) A note on proposition observables
Professor Stanley Gudder, University of Wisconsin (653-35)

3:45-3:55

(295) Sequential convergence in the dual of a W*-algebra
Dr. C. A. Akemann, University of Pennsylvania (653-189)

4:00-4:10

(296) Commutators and certain II_1-factors
Professor Carl Pearcy, University of Michigan, and Professor D. M. Topping*, Indiana University (653-149)

4:15-4:25

(297) Fredholm theories in von Neumann algebras. II
Professor Manfred Breuer, University of Kansas (653-69)

4:30-4:40

(298) Faithful normal expectations on von Neumann algebras
Professor Andre de Korvin, Carnegie-Mellon University (653-242)

4:45-4:55

(299) Derivations and the action of functions on trace class. Preliminary report
Miss Frances Frost, University of Minnesota (652-243)

FRIDAY, 1:30 P.M.

Invited Address, Continental Ballroom

A survey of solvable Lie groups and applications
Professor Louis Auslander, City University of New York

FRIDAY, 3:00 P. M.

Session on Algebra Logic and Foundations, Parlor 1

3:00-3:10

(300) Some initial segments of the hyperdegrees
Professor S. K. Thomason, Simon Fraser University (653-71)

3:15-3:25

(301) Properties of L_p languages--languages rich enough to express that a set A is of less power than B
Professor Mitsuru Yasuhara, New York University (653-338)

3:30-3:40

(302) Galois theory and the amalgamation property in finite dimensional cylindric algebras. Preliminary report
Professor S. D. Comer, Vanderbilt University (653-88)

3:45-3:55

(303) Powers in generalized free products
Professor Seymour Lipschutz, Temple University (653-96)

4:00-4:10

(304) Some varieties of groupoids
Professor Marshall Saade, University of Georgia (653-90)

4:15-4:25

(305) Two remarks on unary algebras
Mr. W. A. Lampe, University of Manitoba (653-295)
(Introduced by Professor G. A. Grätzer)

4:30-4:40

(306) Extensions of congruences on infinitary partial algebras
Dr. G. H. Wenzel, Queen's University (653-340)

4:45-4:55

(307) A relation between two kinds of independence in Universal Algebra. Preliminary report

Professor Jon Froemke, Oakland University (653-251)

5:00-5:10

(308) Group-like extensions and similar algebras

Professor Brindell Horelik, State University of New York at Cortland (653-225)

5:15-5:25

(309) On a structure theory of a projective generator for an exact category with unions

Professor Kwangil Koh, North Carolina State University (653-75)

5:30-5:40

(310) Homogeneous morphisms in additive categories

Professor L.-C. Chern, University of Florida (653-296)

FRIDAY, 3:00 P.M.

Session on Algebra VII, Parlor 3

3:00-3:10

(311) Tensor products of F-vector spaces

Professor Morris Weisfeld, Duke University (653-325)

3:15-3:25

(312) On the nonsingular positive square matrix of an ordered ringoid. Preliminary report

Professor Maurice Chacron, University of Sherbrooke (653-166)

3:30-3:40

(313) Affinely variant sets in vector spaces of GF(2)

Professor Frederick Hoffman, Drexel Institute of Technology (653-321)

3:45-3:55

(314) Orthogonal similarity for symmetric matrices

Professor A. D. Porter, University of Wyoming (653-221)

4:00-4:10

(315) Inertia theory for simultaneously triangulable complex matrices

Mr. R. D. Hill, Idaho State University (653-247)
(Introduced by Dr. David Carlson)

4:15-4:25

(316) A separation theorem for nonsymmetric matrices

Dr. C. A. Hall*, and Mr. T. A. Porsching, Bettis Atomic Power Laboratory, Westinghouse Electric Corporation, West Mifflin, Pennsylvania (653-22)

4:30-4:40

(317) Spectral generalized inverses of singular square matrices

Professor T. N. E. Greville, Mathematics Research Center, U. S. Army, University of Wisconsin (653-114)

FRIDAY, 3:00 P.M.

Session on Applied Mathematics III, Parlor 7

3:00-3:10

(318) Green's function for a spherical cell

Professor H. W. Vayo, University of Toledo (653-312)

3:15-3:25

(319) Rotating time-like congruences in general relativity

Professor A. H. Thompson, University of Pittsburgh (653-357)
(Introduced by Dr. Charles Cullen)

3:30-3:40

(320) Steady Couette flow in a centrifuge

Professor P. K. Kulshrestha, Louisiana State University, New Orleans (653-170)

(Introduced by Professor D. Sarafyan)

3:45-3:55

(321) One-dimensional magnetohydrodynamic flow and the Monge-Ampère equation

Professor R. M. Gundersen, University of Wisconsin-Milwaukee (653-46)

4:00-4:10

(322) Frequency equations for the normal modes of vibration for an elliptical ring, including transverse shear and rotary inertia

Professor W. R. Callahan, St. John's University, (653-107)

4:15-4:25

(323) Uniform theory of diffraction of progressing waves

Professor R. M. Lewis and Professor D. S. Ahluwalia*, New York University, Courant Institute of Mathematical Sciences (653-165)

4:30-4:40

(324) Eigenfunction expansions and scattering theory for certain infinite domains

Dr. Charles Goldstein, Brookhaven National Laboratory, Upton, New York (653-273)

(Introduced by Mr. Stanely Osher)

4:45-4:55

(325) $O(h^{2n+2-m})$ bounds on some spline interpolation errors

Mr. B. K. Swartz, University of California, Los Alamos (653-116)

FRIDAY, 3:00 P.M.

Session on Topology VI, Parlor 9

3:00-3:10

(326) Some selection theorems for measurable functions

Professor C. J. Himmelberg and Professor F. S. Van Vleck*, University of Kansas (653-65)

3:15-3:25

(327) Bi-quotient maps and products of quotient maps

Professor Ernest Michael, University of Washington (653-78)

3:30-3:40

(328) Closed graphs and closed projections

Professor C. T. Scarborough, Mississippi State University (653-352)

3:45-3:55

(329) Retraction in m-paracompact spaces

Dr. V. J. Mancuso, St. John's University (653-181)

4:00-4:10

(330) On Ascoli theorems and the product of k-spaces

Professor R. C. Steinlage, University of Dayton (653-128)

4:15-4:25

(331) Quasi-hereditary properties, Baire category, and non-first-countable structure

Dr. H. H. Wicke*, and Dr. J. M. Worrell, Jr., Sandia Laboratory, Albuquerque, New Mexico (653-285)

4:30-4:40

(332) Characterizations of metric-dependent dimension functions

Dr. J. C. Smith, Jr., Virginia Polytechnic Institute (653-2)

4:45-4:55

(333) A metrization theorem

Professor P. A. O'Meara, Bowling Green State University (653-25)

5:00-5:10

(334) On quasi-metric spaces

Professor R. A. Stoltenberg, Washington State University (653-254)

5:15-5:25

(335) Metrizability of trees

Professor Carl Eberhart, University of Kentucky (653-157)

FRIDAY, 3:00 P.M.

Session on Analysis XIII, Rosewood Suite

3:00-3:10

(336) A theorem concerning product integrals

Professor L. L. Clarkson, Texas Southern University (653-289)

3:15-3:25

(337) Growth estimates of functions in L^1

Professor Nicolas Artémiadis, Southern Illinois University (653-192)

3:30-3:40

(338) Representation of a set as the union of two disjoint nonmeasurable sets

Professor Nand Kishore, University of Toledo (653-250)

3:45-3:55

(339) Representation of complete Lebesgue integrals by means of volumes

Professor W. M. Bogdanowicz and Mrs. M. M. Mattamal*, The Catholic University of America (653-173)

4:00-4:10

(340) Volumes generated by Daniell functionals on the space of all continuous functions on a normal topological space

Professor W. M. Bogdanowicz, The Catholic University of America, and Professor Hans Heyn*, University of Wyoming (653-38)

4:15-4:25

(341) Radon-Nikodym differentiation of one vector measure with respect to another

Professor W. M. Bogdanowicz, The Catholic University of America (653-143)

4:30-4:40

(342) Derivatives with respect to vector-valued measures

Professor Milton Rosenberg, University of Kansas (653-188)

4:45-4:55

(343) Regular additive set functions. Preliminary report

Mr. J. E. Huneycutt, Jr., University of North Carolina at Chapel Hill (653-307)

5:00-5:10

(344) A Jordan decomposition for weight functions

Professor J. K. Brooks*, The University of Florida, and Professor P. V. Reichelderfer, Ohio State University (653-92)

5:15-5:25

(345) On existence of solutions to a functional integro-differential equation in Banach spaces

Professor D. R. Anderson*, University of Wyoming, and Professor W. M. Bogdanowicz, The Catholic University of America (653-224)

R. S. Pierce
Associate Secretary

Las Cruces, New Mexico

PRELIMINARY ANNOUNCEMENTS OF MEETINGS

Six Hundred Fifty-Fourth Meeting
University of California
Los Angeles, California
March 21-23, 1968

The six hundred fifty-fourth meeting of the American Mathematical Society will be held at the University of California, Los Angeles, on March 21-23, 1968, in conjunction with a meeting of the Association for Symbolic Logic on March 22.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, Professor Rimhak Ree of the University of British Columbia will present a lecture entitled "Characters of finite Chevalley groups" on Saturday, March 23, at 2:00 p.m. All sessions and the invited address will be held in the Mathematical Sciences Building.

There will be sessions for contributed papers at 10:00 a.m. and 3:30 p.m. on Saturday, March 23. Abstracts of contributed papers should be sent to the American Mathematical Society, Box 6248, Providence, Rhode Island 02904. The deadline for receipt of abstracts is January 24, 1968.

SYMPOSIUM ON COMBINATORICS

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, there will be a Symposium on Combinatorics on Thursday and Friday, March 21-22. Following is a tentative schedule of the symposium: Session A, March 21, 9:30 a.m.; Combinatorial Analysis (Generating functions, asymptotic results); Session B, March 21, 2:30 p.m., Properties of Large Structures (Ramsey's theorem); Session C, March 22, 9:30 a.m., Homogeneity and Symmetry Properties; Session D, March 22, 2:30 p.m., Zero-one Matrices, Design and Incidence Geometries.

The Organizing Committee for this Symposium consists of Professors T. S. Motzkin (Chairman), Marshall Hall, and Gian-Carlo Rota. Financial support for the symposium is expected under a proposed grant from the National Science Foundation.

The program of the meeting in the February issue of these *Notices* will contain a complete listing of the symposium speakers and the titles of their lectures.

REGISTRATION

The registration desk for the meeting will be located in Room 5200 of the Mathematical Sciences Building. It will be open from 9:00 a.m. to noon on the days of the meeting.

ACCOMMODATIONS

The following hotels and motels are near the UCLA campus. Persons who wish reservations should write to the hotel or motel of their choice at the address given in Los Angeles, California 90024.

Cavalier Hotel and Apartments
10604 Santa Monica Boulevard

Single	\$ 9.00
Double	11.00

Claremont Hotel
1044 Tiverton Avenue

Single	\$ 6.00
Double	7.00
Twin	9.00

Seagull Motel
10811 Santa Monica Boulevard

Single	\$ 6.50
Double	7.50 - 10.00

Westwood Manor Hotel
10527 Wilshire Boulevard

Single	\$ 6.50
Double	8.50

Weyburn Hall
947 Tiverton Avenue
Single (double occupancy) $5.00 up.

Wilshire West Hotel
10990 Wilshire Boulevard

Single	$6.00
Double	8.00

MEALS

Meals can be obtained at the Student Union on Thursday and Friday. There are several restaurants in Westwood Village which can be reached by a short walk from the Mathematical Sciences Building. These restaurants will be open throughout the meeting.

TRANSPORTATION

The Los Angeles International Airport is located approximately 10 miles from the UCLA campus. Bus transportation from the airport is available on the "Airtransit" bus designated "Beverly Hills - Westwood." The fare from the airport is $1.15 per person.

R. S. Pierce
Associate Secretary
Las Cruces, New Mexico

Six Hundred Fifty-Fifth Meeting

Americana Hotel
New York, New York
April 10-13, 1968

The six hundred fifty-fifth meeting of the American Mathematical Society will be held at the Americana Hotel in New York, New York from April 10 to April 13, 1968.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings the following four lectures will be presented:

Professor Henry P. McKean, Jr. of Rockefeller University will speak on "A Chapman-Enskog-Hilbert extension for the telegraph equation as a model of the derivation of fluid mechanics from the Boltzmann equation" at 11:00 a.m. on Friday, April 12.

Professor Frederick J. Almgren, Jr. of Princeton University will speak on "Measure theoretic geometry and elliptic variational problems" at 2:00 p.m. on Friday, April 12.

Professor Franklin P. Peterson of the Massachusetts Institute of Technology will speak on "Characteristic classes: old and new" at 11:00 a.m. on Saturday, April 13.

Professor George B. Seligman of Yale University will speak on "Algebraic Lie algebras" at 2:00 p.m. on Saturday, April 13.

A symposium on "Applications of categorical algebra" is planned for the afternoon of Wednesday, April 10, and both morning and afternoon of Thursday, April 11. The Invitations Committee for this symposium consists of Professors Hyman Bass, Alex Heller (Chairman), and John Moore. Financial support of the symposium has been requested from the Office of Naval Research.

There will be sessions for contributed ten-minute papers during both mornings and afternoons of Friday and Saturday, April 12 and 13. The deadline for receipt of papers to be placed on the program is February 15, 1968. Abstracts of contributed papers should be sent to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904. Abstract blanks can be obtained by request from the same address. There will be no provision for late papers.

The Mathematical Sciences Employment Register will be open from 2:00 p.m. to 5:00 p.m. on Thursday, April 11, and from 9:00 a.m. to 5:00 p.m. on Friday, April 12. Information on the operation of this service may be found on page 43 of these *Notices*.

Travel instructions and hotel reservation blanks will appear in the February 1968 issue of these *Notices*.

Herbert Federer
Associate Secretary
Providence, Rhode Island.

Six Hundred Fifty-Sixth Meeting

Sheraton-Chicago Hotel
Chicago, Illinois
April 16-20, 1968

The six hundred fifty-sixth meeting of the American Mathematical Society will be held at the Sheraton-Chicago Hotel, Chicago, Illinois, on April 16-20, 1968. All sessions will be held in the Tally-Ho Room and the East Rooms on the ninth floor of the Sheraton-Chicago Hotel.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings there will be hour addresses on April 19-20, 1968, by Professor Irving Reiner of the University of Illinois, Urbana; Professor Stephen Smale of the University of California, Berkeley; Professor René Thom of the Institut des Hautes Etudes Scientifiques, Bures sur Yvette; and Professor Hans F. Weinberger of the University of Minnesota.

There will be sessions for contributed papers on April 19-20, 1968. Abstracts should be submitted to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904, so as to arrive prior to the deadline of February 26, 1968.

There will be a special session of twenty-minute papers on Quasiconformal Mapping under the chairmanship of Professor Frederick W. Gehring of the University of Michigan. Most of the papers to be presented at this session will be by invitation. However, anyone contributing an abstract for the meeting who feels that his paper would be particularly appropriate for this special session should indicate this clearly on his abstract and submit it two weeks earlier than the above deadline, namely by February 12, to allow time for additional handling.

As at recent Western Sectional Meetings, one of the sessions of contributed ten-minute papers will be devoted to Category Theory and will coincide with a meeting of an informal group called the Midwest Category Seminar. Similar informal arrangements in other fields can be made upon request to the Associate Secretary.

SYMPOSIUM ON NONLINEAR FUNCTIONAL ANALYSIS

With the anticipated support of the National Science Foundation, under a proposed grant, there will be a symposium on Nonlinear Functional Analysis on April 16-19. This topic was chosen by the Committee to Select Hour Speakers for Western Sectional Meetings. The Organizing Committee of the Symposium, responsible for the planning of the program and the choice of speakers, consists of Felix E. Browder (Chairman), James Eells, Jr., Tosio Kato, George J. Minty, Richard S. Palais, Jacob T. Schwartz, and Stephen Smale. The hour speakers at the symposium will be Professor Felix E. Browder of the University of Chicago; Professor James Eells, Jr. of Cornell University; Professor Tosio Kato of the University of California, Berkeley; Professor J. L. Lions of the University of Paris; Professor George J. Minty of Indiana University; Professor Richard S. Palais of Brandeis University; and Professor Guido Stampacchia of the University of Pisa. In addition the program of the symposium will also include approximately fifteen half-hour addresses.

Travel instructions and hotel reservation forms will appear in the February issue of these *Notices*.

Paul T. Bateman
Associate Secretary

Urbana, Illinois

Symposium on Numerical Solution of Field Problems in Continuum Physics

Durham, North Carolina April 5-6, 1968

An Applied Mathematics Symposium is planned for Durham, North Carolina, on Friday and Saturday, April 5-6, 1968. The title of the symposium will be "Numerical Solutions of Field Problems in Continuum Physics." The symposium will concentrate attention on the relative power of numerical and analytical methods for solving problems having real physical or industrial importance.

A tentative program has been set up as follows: Session I, Fluid Mechanics. Speakers, W. K. Morton, United Kingdom Atomic Energy Commission; J. L. Lions, University of Paris; J. Smagorinsky, United States Weather Bureau; Akio Arekawa, University of California, Los Angeles. Session II, Parabolic Problems and Reservoir Problems. Speakers, Jim Douglas, Jr., University of Chicago; Olof B. Widlund, Uppsala University and New York University; H. H. Rachford, Jr. and Todd duPont, Rice University; Harvey S. Price, Gulf Corporation. Session III, Elliptic Problems. Speakers, Garrett Birkhoff and George Fix, Harvard University; R. S. Varga, Case Western Reserve University; Eugene L. Wachpress, Knolls Atomic Power Laboratory; A. H. Henry and Thomas A. Porsching, Bettis Atomic Power Laboratory. Session IV, Initial Value Problems. Speakers, Vidar C. Thomée, University of Maryland; J. H. Bramble, University of Maryland; R. Clough and C. A. Felippa, University of California, Los Angeles; Theodore H. Pian, Massachusetts Institute of Technology.

There will be four sessions of two and one-half hours each, comprising sixteen half-hour addresses.

The Organizing Committee is composed of Professor Garrett Birkhoff (Chairman), Jim Douglas, Jr., R. S. Varga, Calvin Wilcox, and Dr. Sidney Fernbach. Dr. Francis G. Dressel, Dr. A. S. Galbraith, and Dr. Gene B. Parrish are liaison representatives. Support for the symposium is expected under a proposed grant from the U. S. Army Research Office (Durham).

Travel instructions and hotel reservation forms will appear in the February issue of these *Notices*.

O. G. Harrold
Associate Secretary

Tallahassee, Florida

ACTIVITIES OF OTHER ASSOCIATIONS

SIAM 1968 NATIONAL MEETING

A Symposium on Optimization will be held in conjunction with the 1968 National Meeting of the Society for Industrial and Applied Mathematics to be held in Toronto, June 11-14, 1968. The symposium will feature invited presentations on optimization, including papers in mathematical programming, control theory, and variational methods. One of the prime objectives of the symposium is to bring together researchers in these three different areas of optimization in the belief that these associations will be of mutual benefit.

The Conference Chairman is Dr. T. E. Hull, Department of Mathematics, University of Toronto; the Program Chairman is Dr. P. Wolfe, IBM Research, P. O. Box 218, Yorktown Heights, New York 10598; and the Local Arrangements Chairman is Mr. B. B. Goodfellow, IBM Company Limited, 1150 Eglinton Avenue East, Don Mills, Ontario, Canada.

FORTHCOMING MEETINGS OF THE CANADIAN MATHEMATICAL CONGRESS

A Summer Research Institute will be held from May 13 to August 9, 1968, at the following four branches: Université de Montréal and McGill University (Montreal); Queen's University (Kingston); University of Manitoba (Winnipeg) in conjunction with a combinatorial mathematics meeting; and University of British Columbia (Vancouver).

There will also be three summer schools. Three graduate courses, details of which will be announced in January 1968, will be offered at the University of British Columbia, Vancouver, from June 24 through August 16, 1968. Support for out-of-province students will be available. Further information may be obtained from Professor B. N. Moyls, Department of Mathematics, University of British Columbia, Vancouver 8, B.C.

Dalhousie University, Halifax, N. S., will offer a number of undergraduate courses intended primarily for high school teachers. The program is a cooperative effort of the Nova Scotia Department of Education and the Canadian Mathematical Congress. Tentatively, the following courses will be offered: Mathematics for Junior High School Teachers; Geometry, Algebra, and Calculus for Senior High School Teachers (three separate courses); and Mathematical Astronomy. It is also expected that two graduate-level courses will be offered, sponsored jointly by the Atlantic Provinces Inter-University Committee on the Sciences and the Canadian Mathematical Congress. All seven of the above-mentioned courses will be offered between July 2 and August 12, 1968. No tuition will be charged, although the registration fee is five dollars. Inquiries should be addressed to Professor A. J. Tingley, Secretary, Atlantic Provinces Committee, Department of Mathematics, Dalhousie University, Halifax, N. S.

The seventh session of the Séminaire de Mathématiques Supérieures, organized by the Department of Mathematics at the University of Montreal, will take place from June 24 to July 26, 1968, and will concern statistics and probability. The seminar will be supported jointly by the N.S.F. l'OTAN and the Société Mathématique du Canada. It is anticipated that support will be available for part or all of travel and living expenses of those who participate in the seminar. Further information may be obtained from M. Maurice L'Abbé, Department of Mathematics, University of Montreal, Montreal, Quebec.

THE MATHEMATICAL ASSOCIATION OF AMERICA
San Francisco, California
January 25-27, 1968

The fifty-first annual meeting of the Mathematical Association of America will be held in the San Francisco Hilton Hotel, San Francisco, California, from Thursday to Saturday, January 25 to 27, 1968, in conjunction with the annual meeting of the American Mathematical Society.

A complete program of the meeting is included in the time table in this issue of the *Notices*.

MEMORANDA TO MEMBERS

THE MATHEMATICAL SCIENCES EMPLOYMENT REGISTER

During the April meeting in New York City, the Mathematical Sciences Employment Register will be open for the scheduling of interviews between applicants and employers. The Employment Register, which is sponsored by the American Mathematical Society, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics, will be located in the Regency Ballroom of the Americana Hotel. The Register will be open from 2:00 p.m. to 5:00 p.m. on Thursday, April 11, and from 9:00 a.m. to 5:00 p.m. on Friday, April 12.

Registration for the Employment Register is separate and apart from meeting registration, and it is, therefore, most important that both applicants and employers sign in at the Employment Register desk as early as they can on Thursday afternoon. A separate visual index will be maintained for Employment Register use only. Appointments will be scheduled only for applicants and employers who have actually signed in at the Register.

There is no charge for registration except when the late registration fee of $5.00 is applicable. Provision will be made for anonymity of applicants upon payment of $5.00 to defray the cost involved in handling such a listing.

Applicants and employers who wish to be listed should write to the Mathematical Sciences Employment Register, Post Office Box 6248, Providence, Rhode Island 02904, for applicant qualification forms or position description forms. These forms must be completed and returned to the Employment Register not later than April 1, 1968, in order to be included in the May lists. The printed lists will be mailed to subscribers during the first week in May. The original forms, however, will be available at the meeting for those persons interested in reviewing them.

A subscription to the lists, which includes three issues (January, May, and August) of both the applicants list and the positions list, is available for $25.00 a year; the individual issues of both lists may be purchased in January, May, and August for $12.50. A subscription to the applicants list alone or single copies of that list are not available. Copies of the positions list only may be purchased for $3.00. Checks should be made payable to the American Mathematical Society and sent to the address given above.

NEW AMS MATHEMATICAL OFFPRINT SERVICE

The AMS has established an entirely new type of service for subscribers--Mathematical Offprint Service (MOS). This new concept in information distribution will offer the individual mathematician offprints of articles of specific interest to him from important AMS and non-AMS journals. Through MOS, the mathematician will be able to specify exactly which kinds of articles he wishes to receive and obtain them simultaneously with or shortly after publication. Thus, the new service will provide a single source of current information for the research mathematician.

In recent years, the amount of information published in the sciences has accelerated at a fantastic rate. The list of journals from which articles are reviewed in MATHEMATICAL REVIEWS includes nearly one thousand. This proliferation has created serious problems for the individual mathematician.

If he is to keep abreast of the latest material within his own specialized field, let alone his wider range of scientific interests, the mathematician must scan literally hundreds of journals. Much of his time will be wasted because the majority of articles in each of these journals will not be of interest to him. There is no current, classified source by which he can be directed to precisely those articles he seeks; MATHEMATICAL REVIEWS, which comes closest to providing this kind of source, may not publish reviews of articles for six to twenty-four months after they appear in primary journals.

MOS will eliminate the need for mathematicians to waste time locating important articles. By submitting a detailed interest profile, a subscriber will specify which articles he wishes to receive from about sixty journals chosen from the list of highest priority journals reviewed in MATH REVIEWS. Articles which satisfy the criteria established by the subscriber will be mailed to him on a continuing basis, along with titles of articles in which he has indicated a peripheral interest.

It must be emphasized that the new service does not threaten the existing structure of mathematical publication. The majority of subscribers to journals consists of institutions and libraries, and MOS subscriptions are not available to institutions. Rather, MOS is designed specifically for the individual and his particular interests, as the journal itself is not. The service will provide an additional dimension in mathematical publishing.

MOS will operate in the following way: When an individual agrees to subscribe, he will receive a form on which he will specify his particular interests. Parameters available for description include:

(a) Subject. Primary and secondary subject descriptors (P and S) will be assigned to each article using a revision, now being prepared for 1968, of the MATH REVIEWS subject classification scheme which appears in all MATH REVIEWS index issues. For the present, articles will be assigned subject classifications by a panel of classifiers, but eventually the classification scheme and scope notes will be sent to the participating journals in English, French, German, Italian, and Russian, and the classification will be done by authors and/or editors and referees.

The subscriber will also receive the MATH REVIEWS subject classification scheme, in whichever language he requests, with his interest profile form. He will indicate his fields of primary and secondary interest (p and s) on the profile. If he indicates p or s interest in a major classification (e.g., 10) he may exclude a subclassification (e.g., 10.3) within that classification.

(b) Author. The subscriber will indicate on his profile those authors whose articles in any subject he wishes to receive, and those authors whose articles he specifically does not wish to receive.

(c) Journal. A list of participating journals will be mailed to each subscriber along with the profile form and the classification material. The subscriber will indicate on his profile form those journals from which he does not wish to receive articles.

(d) Language. On the profile, the subscriber will indicate the languages in which he will accept articles.

Therefore, an interest profile will show p and s subjects; authors desired regardless of subject; authors excluded; journals excluded; and languages acceptable.

The subscriber will receive an offprint of:

(1) any article by a specified author, regardless of subject, if it is in an acceptable language and taken from a journal which is not specifically excluded;

(2) any article by an author neither specified nor excluded whose primary subject P coincides with the primary subject field p of the subscriber, if it is both in an acceptable language and taken from a journal not specifically excluded.

The subscriber will receive a title listing of:

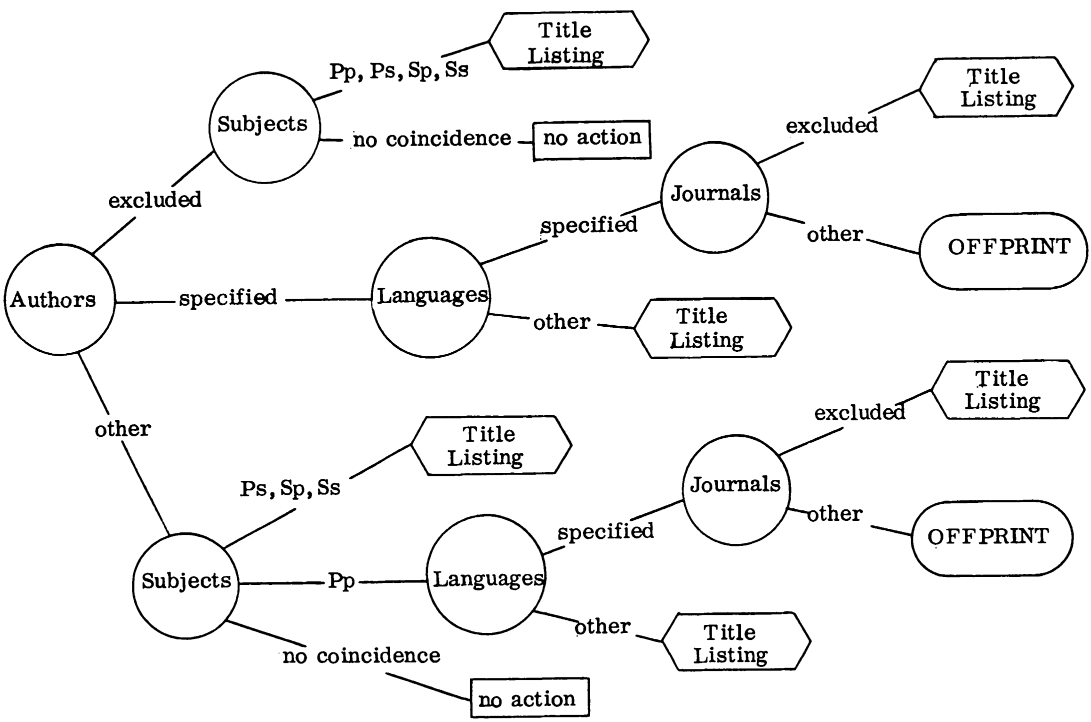
(1) an article by a specified author which is in an unacceptable language or from an excluded journal;

(2) any article by an excluded author, if the coincidence of subject fields is Pp, Ps, Sp, or Ss.

(3) an article by an author neither specified nor excluded if the coincidence of subject fields is Sp, Ss, or Ps; or if the coincidence of subject fields is Pp but the article is in an unacceptable language or taken from an excluded journal.

The procedure is clearer in the diagrammatic form shown below.

In the case of multiple authorship, not represented in the diagram, an article written jointly by a specified author and an excluded author, except when the excluded author is the subscriber himself, is treated exactly like an article written by a specified author alone; the subscriber receives an offprint if journal and language criteria are not violated. If the excluded author is the subscriber, he will receive a title listing. Articles written jointly by excluded authors and unspecified authors are treated as if they were written by excluded authors alone.



GINSBURG, SEYMOUR
SPANIER, E. H. BOUNDED REGULAR
SETS. PROC. AMER. MATH. SOC. 17 (1966) NO. 5. (SECONDARY
INTEREST).

TULLY, E. J. JR. A CLASS OF NATURALLY PARTLY
ORDERED COMMUTATIVE ARCHIMEDEAN SEMIGROUPS WITH MAXIMAL
CONDITION. PROC. AMER. MATH. SOC. 17 (1966) NO. 5. (SECONDARY
INTEREST).

CULVER, W. J. ON THE EXISTENCE AND UNIQUENESS OF
THE REAL LOGARITHM OF A MATRIX. PROC. AMER. MATH. SOC. 17
(1966) NO. 5. (SECONDARY INTEREST).

MORDELL, L. J. SOME QUADRATIC DIOPHANTINE EQUATIONS
OF GENUS 3. PROC. AMER. MATH. SOC. 17 (1966) NO. 5.
(SUBCLASSIFICATION OF INTEREST NOT WANTED).

BAUMSLAG, GILBERT. A REPRESENTATION OF THE WREATH
PRODUCT OF TWO TORSION FREE ABELIAN GROUPS IN A POWER SERIES
RING. PROC. AMER. MATH. SOC. 17 (1966) NO. 5. (SECONDARY
INTEREST).

PASSMAN, BETTY LOU
PASSMAN, D. S. GENERALIZED
FROBENIUS COMPLEMENTS. PROC. AMER. MATH. SOC. VOL. 17 (1966)
NO. 5. (SECONDARY INTEREST).

GAUSS, C. D. LINEAR APPROXIMATION. RUCKSCHRITTE
DER MATH. 7 (1966) NO. 2. (ORDER ENTERED TOO LATE).

ROSEN, MICHAEL. TWO THEOREMS ON GALOIS COHOMOLOGY.
PROC. AMER. MATH. SOC. 17 (1966) NO. 5. (ORDER ENTERED TOO
LATE).

SHULT, ERNEST. SOME ANALOGUES OF GLAUBERMANS
Z* THEOREM, PROC. AMER. MATH. SOC. 17 (1966) NO. 5.
(SECONDARY INTEREST).

MCCULLCH, L. R. CYCLIC EXTENSIONS WITHOUT RELATIVE
INTEGRAL BASES. PROC. AMER. MATH. SOC. 17 (1966) NO. 5.
(ORDER ENTERED TOO LATE).

GREENBERG, L. NOTE ON NORMAL SUBGROUPS OF THE
MODULAR GROUP. PROC. AMER. MATH. SOC. 17 (1966) NO. 5.
(ORDER ENTERED TOO LATE).

MONTAGUE, STEPHEN
THOMAS, GOMER. A CONDITION FOR
A FINITE GROUP TO BE NILPOTENT. PROC. AMER. MATH. SOC. 17
(1966) NO. 5. (SECONDARY INTEREST).

STARK, H. M. ON THE ASYMPTOTIC DENSITY OF THE K FREE
INTEGERS. PROC. AMER. MATH. SOC. 17 (1966) NO. 5. (ORDER
ENTERED TOO LATE).

AYOUB, RAYMOND. A NOTE ON THE RIEMANN ZETA FUNCTION.
PROC. AMER. MATH. SOC. 17 (1966) NO. 5. (ORDER ENTERED TOO
LATE).

THE FOLLOWING JOURNAL HAS JUST BEEN INCLUDED IN
THE OFFPRINT SYSTEM: J. LONDON. MATH. SOC., BEGINNING WITH
VOL. 75, NO. 3.
IF YOU DO NOT WISH TO RECEIVE OFFPRINTS FROM THIS
JOURNAL, PLEASE NOTIFY THE AMERICAN MATHEMATICAL SOCIETY.

The following is a fictitious example of how a subscription is processed each month.

Subscriber A. B. Newton indicated the following interests on his interest profile form, which was received and entered on August 5, 1967:

Primary interest--MR classification 10 (Theory of Numbers), excluding subclassification 10.10 (Diophantine Equations)

Secondary interest--MR classifications 15 (Linear Algebra) and 20 (Group Theory and Generalizations)

Author specified--C. D. Gauss

Author excluded--A. B. Newton

Journal excluded--none

Languages specified--English, German, Russian

The subscriber received three reprints accompanied by the following statement:

A M E R I C A N   M A T H E M A T I C A L   S O C I E T Y

P. O. BOX 6248   PROVIDENCE, R. I. 02904

06OCT67

PROFESSOR A B NEWTON
725 SMITH STREET
CHICAGO, ILL

THIS MONTHLY SHIPMENT OF OFFPRINTS INCLUDES THE FOLLOWING, SENT TO YOU AS BEING IN YOUR PRINCIPAL FIELD OF INTEREST.

SALIE, HANS. REICHWEITE VON MENGEN AUS DREI NATURLICHEN ZAHLEN. MATH. ANN. 165 (1966) NO. 1.

SIDLOVSKII, A.B. TRANSCENDENCE AND ALGEBRAIC INDEPENDENCE OF VALUES OF E FUNCTIONS SATISFYING LINEAR NONHOMOGENEOUS DIFFERENTIAL EQUATIONS OF THE SECOND ORDER. DOKL. AKAD. NAUK SSSR 169 (1966) NO. 1.

OLEINIKOV, V.A. ALGEBRAIC INDEPENDENCE OF VALUES OF E FUNCTIONS SATISFYING LINEAR NONHOMOGENEOUS DIFFERENTIAL EQUATIONS OF THE THIRD ORDER. DOKL. AKAD. NAUK SSSR 169 (1966) NO. 1.

YOU MAY ALSO BE INTERESTED IN THE FOLLOWING RECENT PAPERS WHICH ARE NOT BEING SENT TO YOU. YOU MAY CONSULT THESE ARTICLES IN THE JOURNALS IN WHICH THEY ARE PUBLISHED, OR REQUEST REPRINTS FROM THE AUTHOR.

OFFPRINTS ARE NOT AVAILABLE FROM THE AMERICAN MATHEMATICAL SOCIETY.

NEWTON, A. B. DISTRIBUTION OF PRIME NUMBERS. RUCKSCHRITTE DER MATH. 7 (1966) NO. 2. (AUTHOR NOT WANTED).

SIERPINSKI, W. SUR LES NOMBRES PSEUDOPARFAITS. MAT. VESNIK 17 (1965) NO. 2. (LANGUAGE NOT WANTED).

BAULIN, V. I. ON AN INDETERMINATE EQUATION OF THE THIRD DEGREE WITH LEAST POSITIVE DISCRIMINANT. TULSK. GOS. PED. INST. UCEN. ZAP. FIZ. MAT. NAUK VYP. 7 (1960) NO. 1. (SUBCLASSIFICATION OF INTEREST NOT WANTED).

SIBNER, R.J. HYPERBOLIC GENERATORS FOR FUCHSIAN GROUPS. PROC. AMER. MATH. SOC. 17 (1966) NO. 5. (SECONDARY CLASSIFICATION.)

One subscription will consist of a total of 100 offprints (ten title listings equal one offprint) and will cost $30. Shortly before each subscription is completed, the subscriber will be billed automatically for the next subscription.

The AMS welcomes subscription orders now. It is expected that MOS will be fully operative in early spring 1968, or as soon as a minimum number of journals agree to participate and the service has a sufficient number of subscribers. With each order, the subscriber should indicate whether he wishes to receive the subject classification schedule in English, French, German, Italian, or Russian. Shortly after his order is received, the subscriber may expect to receive an interest profile form, a list of participating journals, and the classification scheme.

To order a subscription to MOS, please complete the order form on page 280 and return it to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904.

NEWS ITEMS AND ANNOUNCEMENTS

EDUCATION COSTS AS DEDUCTIBLE BUSINESS EXPENSES

The following 1967 Internal Revenue Service Ruling (Rev. Rul. 67-421, I. R. B. 1967-48,11.), which came to our attention too late for inclusion in the section on tax status of grants in the December Special Issue of these *Notices* states that reasonable expenses for research and typing incurred in the preparation of a doctoral dissertation are tax deductible as business expenses:

"Advice has been requested concerning the deductibility, for Federal income tax purposes, of expenses incurred for research and typing under the circumstances described below.

"The taxpayer is employed as a full-time instructor at a state university. He is taking educational courses in order to maintain and improve his skills as a full-time instructor. The courses will also lead to a graduate doctoral degree. In the preparation of his dissertation to obtain the doctoral degree he incurs reasonable expenses for research and typing. It has been determined that the costs of the educational courses qualify as deductible ordinary and necessary business expenses under section 162(a) of the Internal Revenue Code of 1954 and section 1.162-5(a) of the Income Tax Regulations.

"Section 162(a) of the Code provides for the deduction of all ordinary and necessary expenses paid or incurred during the taxable year in carrying on any trade or business.

"The preparation of a doctoral dissertation is an integral part of an entire course of educational study pursued on the graduate level.

"Accordingly, since the costs of the related educational courses qualify as deductible business expenses, the taxpayer's reasonable expenses for research and typing incurred in the preparation of the dissertation to obtain his graduate doctoral degree are deductible as ordinary and necessary business expenses under section 162(a) of the Code."

THE CASE OF STEPHEN SMALE: CONCLUSION

The case of Stephen Smale is closed now, in that a grant renewal has been offered and accepted. One question remains --why was Smale's proposal challenged at all? A full account of the Smale controversy, insofar as the facts were available up to that time, was published in the October issue of these *Notices*. It concluded with the August 31 letter of William E. Wright, NSF Division Director for Mathematical and Physical Sciences, stating that Smale's grant would not be renewed unless he would resubmit a proposal for his own research alone; Smale's refusal to consider submitting another proposal; and a portion of the September 15 "Science" article by Daniel Greenberg which outlined the case the NSF was prepared to make against Smale's administration of his previous grant. The facts of the case which have become available since the publication of the October *Notices* appear below.

In a September 23 letter to Sanford Elberg, Dean of the Graduate Division at Berkeley, William E. Wright stated:

"In view of the numerous and widespread misinterpretations which have been placed on my letter to you of August 31..., I would like to reaffirm the position of the Foundation.

"The Foundation remains convinced that timely negotiations can result in a grant to the University of California with Professor Smale as principal investigator, which would support his research needs and those of his immediate collaborators in a manner completely consistent with our ability to sustain mathematical research generally."

Dean Elberg passed this letter on to Smale, who wrote him on September 27, stating "My own opinion is that it [Wright's letter] represents a significant concession on the part of the NSF. That letter can begin to ease the problems arising from the decisions of the NSF stated in Wright's August 31 letter." However, he went on to say that he was still concerned about clearing up reported NSF allegations about his administrative performance, which had been outlined in the article on the Smale case in the September 15, 1967, issue of "Science."

In one effort to do this, Smale wrote on September 29 to Philip Handler, Chairman of the National Science Board, who had issued a statement on September 12, quoted in its entirety in our last article, which said in part, "The Board...concurs with the Director [of NSF] that the management of this [Smale's] grant has been relatively loose and has not conformed to appropriate standards." In his letter Smale requested a bill of particulars on what was considered objectionable in his administration. He received no reply.

By this time, Smale had heard from an unofficial source that the NSF was now willing to award a grant to the University of California at Berkeley with Smale as principal investigator. On September 29 he wrote to Leland J. Haworth, Director of NSF, requesting verification of this information. He received no reply.

However, Smale's major concern at this point was still the criticism of his administration of his previous grant, as published in "Science." On October 15, 1967, he sent an article to "Science" requesting that it be published unedited in the News and Comment section. The article was a "reply to a journalistic account of NSF allegations." "Science" did not publish the article, principally because by this time the magazine had initiated its own careful investigation of the facts of the Smale case. In the November 3 issue, Daniel Greenberg published an article entitled "Smale: NSF's Records do not Support the Charges" ("Science," News and Comment Section, Vol. 158, pp. 618-619, November 3, 1967). The following are pertinent exerpts from the article:

"...on the basis of material that NSF has recently made available from its own files, two very disturbing facts are now clear concerning the Foundation's treatment of the professionally distinguished and politically left-wing mathematician from Berkeley:

"(i) NSF is unable, or at least unwilling, to provide any documentary evidence to support its allegations of impropriety or substandard performance on Smale's part in the administration of his government grant; but even more important, (ii) at the time NSF made these allegations, it was in possession of documentary evidence which either clearly contradicted the allegations, or showed them to be based on trivial and technical departures from ambiguous regulations.

"Both conclusions are drawn from voluminous files that NSF made available at the request of Science. The request for these materials was at first refused, but later was fulfilled when Science formally cited the recently enacted 'Freedom of Information Act' (P. L. 89-487), which requires federal agencies to make available upon request broad categories of government records that previously could be withheld from public inspection.

"... the allegations concerning Smale's administrative performance were based on the following:

"1) When Smale applied to the National Academy of Sciences-National Research Council (NAS-NRC) for a $400 travel grant to cover expenses of visiting Moscow--where he was to receive a prize and deliver a paper at the International Congress of Mathematicians--he failed to notify the Academy that he also had the use of $1,000 in travel funds that were included in the NSF grant to his Berkeley group.

"2) Smale failed to notify NSF that during the summer and fall of 1966 he would be away from Berkeley, where he was principal investigator on the NSF-supported project.

"3) In violation of regulations that require that American carriers be given preference for NSF-supported foreign travel, he returned to the United States on a French vessel.

"4) The time that he spent in Europe on NSF-supported summer salary was accounted for, but in a fashion that left considerable doubt as to whether he actually spent time as he said he did.

"Taking these points one by one, and referring to material that was in NSF's possession when the allegations were made, the following was found to be the case.

"1) Smale responded affirmatively and accurately to an NAS-NRC inquiry about other sources of funds for travel. The NAS-NRC form on which Smale applied for the $400 travel grant contained the question, 'Have you requested or been granted funds which *might* be used for travel to the 1966 Congress? If so, give details' (italics supplied). His reply was, 'On NSF contract application.' (At the time Smale filled out the form, October 1965, decision was still pending on the NSF grant that was to contain the $1,000 in travel funds.) The NAS-NRC award of $400 was accompanied by a letter, dated November 1965, that, in part, stated, 'There are a large number of meritorious applications which cannot be supported. It is hoped that you will promptly notify us *if you will not use this award either because you have other sources of travel funds or because you find it impossible to attend the Congress'* .(italics supplied).

"Smale, of course, planned to attend the Congress, so there was no need to notify NAS-NRC on that point. As for the $1,000 that came from NSF, this sum had been furnished him under a provision in his NSF-approved grant application which stated that travel funds are requested for the investigators to attend conferences.' Unlike many mathematicians who were going to the Moscow conference on a low-cost charter flight from the United States, Smale was going directly from Europe, where, with the use of NSF's $1,000 in travel funds, he conferred during parts of May, June, and July with other mathematicians at an institute in Paris, at the University of Geneva, and at a conference in Bonn. Thus there is little or no support for the contention that he improperly applied for and accepted the NAS-NRC travel grant to go to Moscow when he actually had another source of support for that journey. Perhaps the most that can be said is that he had two separate funds for legitimate travel and he neglected to compartmentalize the Moscow trip and the European travels.

"2) NSF puts great stock in its allegation that Smale failed to notify the Foundation that he would be absent from the Berkeley mathematics department during the summer and the fall. There is, however, a letter in NSF's possession, dated 11 May 1966, in which the administrative assistant of the Berkeley mathematics department wrote to NSF's program director for Analysis, Foundations, and Geometry as follows: 'As Professor Smale will be in Europe this summer and on leave in the fall quarter, he would like Professor S. Kobayashi to direct the project while he is away.' The materials furnished Science by NSF contain no reply to this letter, but the Berkeley mathematics department says it has a memorandum of a telephoned reply from NSF approving the request.

"3) The regulations that accompany NSF awards clearly specify that preference is to be given American carriers. No such regulations accompanied the \$400 that Smale received from the NAS-NRC. And it is his contention that the \$300 tourist class fare on the S.S. France, from Le Havre to New York, came out of the NAS-NRC grant. Since the overall NAS-NRC travel fund was in large part provided by a grant from NSF to NAS-NRC, it might be said that Smale was in violation of the spirit of the rules. But he had to get back from Moscow somehow, which justifies the use of the NAS-NRC Moscow travel grant for crossing the Atlantic. And, in the absence of any explicit restrictions on the NAS-NRC money as far as foreign carriers are concerned, it would appear that this part of the episode scarcely supports Handler's charge 'that management of the grant has been relatively loose and has not conformed to appropriate standards.'

"4) As for the time Smale spent in Europe on NSF-supported salary, he certified that he had an office at one or another institution or was in attendance at a conference from the last week in May through the end of July, thus meeting the requirement of 2 months of scholarly activity in return for 2 months' pay. NSF exudes skepticism, but when asked to provide something more substantial than strong hints of disbelief, it has nothing.

"After the NSF documents were examined by Science, Handler, NSF director Leland J. Haworth, and William E. Wright, director of the NSF division of mathematical and physical sciences, were informed in a letter that nothing could be found to substantiate the NSF allegations concerning Smale. They were asked whether such substantiation was to be found in certain categories of material that NSF said it was entitled to withhold under the Freedom of Information Act. Replies were forthcoming from Handler and from Clarence C. Ohlke, head of NSF's Office of Congressional and Public Affairs. Both these replies were to the effect that the substantiation was not recorded in documents, but rather had come in telephone conversations last year with various officials of the University of California. Science was advised to consult a certain one of these persons for details. On a nonattributable basis, this individual spoke freely and at length, pointing out, however, that the events in question took place a year ago and his memory was not fresh. 'NSF,' he explained, 'told us that Smale had not notified them that he would be away.' When this U.C. member was told that Smale had indeed advised NSF that he would be away, he seemed puzzled and said, 'Well, they objected to a lot of things he did, and I think you published all of them, but I can't recall details.'

"At this point, it must be said that there is something putrid about this whole business, and the aroma seems to come out of NSF headquarters."

Finally, Smale received a letter dated November 17 from Matthew P. Gaffney, NSF Program Director for Analysis, Foundations, and Geometry, which stated that the University of California had been awarded a grant with Smale as principal investigator. A copy of the actual notification of award from Leland Haworth, Director of NSF, was enclosed. The grant of \$87,500 is effective March 1, 1968, for a period of approximately two years, and it is awarded for the support of research in global analysis under the direction of Stephen Smale. The research group of which Smale is principal investigator includes three faculty associates and three others. S. Kobayashi, a member of Smale's previous research group, received a similar grant for support of a research group of the same size.

So Smale has his grant, and the controversy is over. But on November 9 the Washington Star* published an account of the conclusion of the Smale case which ended as follows:

"The affair was settled eventually in a way that partly satisfied everybody and wholly satisfied nobody. Smale got part of his grant, but not all, and does not control the whole research package as he had hoped. The foundation succeeded in splitting Smale's grant in two, but failed to force him to resubmit his application, as it originally demanded."

"Smale has not been badly damaged by his encounter with official Washington; indeed, he may be wiser and not necessarily sadder as a result. But the question is being asked in scientific circles whether the Smale case is a straw in the wind--whether potential dissenters are being put on notice that their political as well as professional credentials will be scrutinized before they qualify for federal support of academic research.

"If only conformists need apply, 1984 is here already. What can Stephen Smale do that a good computer can't do a lot faster? And besides, a computer wouldn't talk back."

NEWS ITEMS AND ANNOUNCEMENTS

PAGE CHARGES FOR AMS JOURNALS

Page charges for articles published in AMS journals will be increased from \$30 per page to \$40 per page, effective for all articles accepted for publication February 1, 1968, or thereafter. The increase has become necessary because of increases in printing costs which have taken place since the \$30 fee was instituted.

It must be emphasized that page charges in no way represent a financial obligation to authors. When a staff member of an institution publishes in a Society journal, the institution is billed for page charges either at the time the manuscript is received at the AMS headquarters, based on the estimated number of printed pages, or at the time of publication, based on the actual number of printed pages.

If the research was federally supported, a 1961 directive of the Federal Council of Science and Technology is applicable: "page charges for publication of scientific research results in scientific journals will be budgeted for and paid as a necessary part of research costs under Federal grants and contracts" within four criteria: 1) The research papers report work supported by the government; 2) The charges are levied impartially on all research papers published by the journal; 3) Payment of such charges is in no sense a condition for acceptance of manuscripts by the journal; 4) The journals involved are not operated for a profit.

AUSTRALIAN MATHEMATICAL SOCIETY--EIGHTH SUMMER RESEARCH INSTITUTE

The Eighth Summer Research Institute of the Australian Mathematical Society is being held at the Australian National University, Camberra, between January 9 and February 16, 1968.

LETTERS TO THE EDITOR

Editor, the *Notices*

One expects different practices in the publications of the Society than those of Time Magazine.

Who wrote the article entitled "The Case of Stephen Smale" appearing on page 778 of the October, 1967 issue of the *Notices*? Such controversial matters should be handled with the same degree of integrity as research papers. Anonymous authorship is generally unbecoming a professional society--and especially so for an article purporting to be an "unbiased account" of such matters.

A. N. Feldzamen

The Editors of the *Notices* jointly assume responsibility for the news items appearing in the journal and more generally for all unsigned material in the journal. Some of these items, for example an announcement of a symposium sponsored by a university, are either copied or cut down from material supplied by interested parties. In case one editor writes his own views, he signs the article. See The Changing Role of the Society, these *Notices* 14 (1967), 772-773 for an example. The fact that the article is signed by Gordon Walker means that he alone prepared it and that it appears with the consent of Everett Pitcher, who did not participate in writing it.

In the instance of the article The Case of Stephen Smale, the fact that it is unsigned means that the editors prepared it and are responsible for it. It should be clear on reading the article that the assistance of various people in divers ways was required. However the article is not their responsibility.

As in the case of joint authorship of scientific papers, the editors did not share equally in the writing and do not reveal how the work was divided. Each assumes full responsibility for the article.

The Editors

Editor, the *Notices*

What can be done to assist mathematics departments that show research promise? How can we encourage summer research institutes for graduate students? Is "regional development" a good thing?

There exists a committee whose purpose is to think about such problems; it is called the committee on Regional Development and Centers of Research (National Academy of Sciences--National Research Council). The members of the committee are J. W. Brace, A. M. Gleason, Edwin Hewitt, Seymour Sherman, W. J. Thron and the undersigned.

There surely are many members of the Society who have ideas about the answers to the sample questions above, and to other related questions, members who have suggestions that would be invaluable to the committee. The purpose of this letter is to ask for answers, suggestions, and help in general. Any help would be much appreciated; any member of the committee would be glad to receive it.

P. R. Halmos

NEWS ITEMS AND ANNOUNCEMENTS

SCHOOL OF MATHEMATICS INSTITUTE FOR ADVANCED STUDY MEMBERSHIPS

The School of Mathematics of the Institute for Advanced Study, Princeton, New Jersey 08540, will grant a limited number of memberships, in some cases with financial support, for research in mathematics at the Institute during the academic year 1968-1969. Candidates must have given evidence of ability in research comparable at least with that expected for the Ph.D. degree. Application blanks may be obtained from the Secretary of the School of Mathematics and should be returned (whether or not funds are expected from some other source) by January 15 or as soon thereafter as possible.

SUMMER MATHEMATICAL RESEARCH APPOINTMENTS

The National Science Foundation again supported a Research Participation for College Teachers Program in the summer of 1967 for teachers of mathematics and computer science. The program enabled fifteen professors from non-Ph.D. granting colleges and universities to devote ten weeks to uninterrupted mathematical study and research at The University of Oklahoma, Norman, Oklahoma. Each participant had an air-conditioned office, an air-conditioned study carrel in O. U.'s outstanding mathematical library, and ample opportunity for research. Each participant took part in at least one advanced seminar or course of his choice and gave one colloquium lecture. The rest of his time was spent in research with the guidance of a University of Oklahoma senior staff member when desired.

It is anticipated that support will be available in summer 1968 for ten post-doctoral ($1000 plus $125 for dependent, plus travel) and five pre-doctoral ($750 plus $125 for dependent plus travel) appointments. If ten weeks devoted to uninterrupted mathematical research in an outstanding library among highly motivated colleagues sounds tempting, you are encouraged to write to Dr. Richard V. Andree, The University of Oklahoma, Norman, Oklahoma 73069, for further details.

NSF SUMMER RESEARCH GRANTS

Some 161 participants in summer research programs for college teachers will be able to continue research projects at their home institutions as a result of $322,000 in National Science Foundation grants. The grants were made mostly to smaller colleges and universities, whose faculties generally have less chance to pursue research than their colleagues in larger schools.

To help these faculty members become involved in research, the Foundation supported 72 research projects this past summer in colleges and universities across the country. These projects enabled 469 college teachers to do basic research in such fields as mathematics, engineering, biology, chemistry, and other subjects while working with research scientists in laboratories of host universities.

Under the terms of the Foundation's programs, directors of the projects nominated almost 40% of the summer participants to receive "Academic Year Extensions" providing funds to continue research at their home institutions.

Extension grants for $2,000 are made available to each researcher's home institution and are usable over a two-year period. The research project is directly supported by $1,500 while the remaining $500 may be used by the institution either as back-up for the research or for other ways of improving education in the sciences.

POSTDOCTORAL FELLOWSHIPS AT THE UNIVERSITY OF NEW MEXICO

A recent grant from the National Science Foundation, the first departmental development grant to be awarded by the Foundation to a mathematics department, was made to the Department of Mathematics and Statistics of the University of New Mexico in recognition of past accomplishments and to encourage further development in the areas of applied analysis, probability and statistics. This grant enables the Department to offer several postdoctoral fellowships. Recent recipients of the doctorate and persons expecting to receive the doctorate before the beginning of the next academic year are eligible for these awards. The terms of the grant specify that applicants must work in analysis or related areas which include probability and statistics. Inquiries will be particularly welcomed from prospective candidates in these fields who wish to work on problems which originate or may be of interest in other areas of mathematics or in other disciplines.

In addition to a stimulating professional environment, the Department has available the use of the excellent facilities of the University Computer Center.

A minimum stipend of $10,000 for the academic year is attached to each award. No formal duties are imposed, but active participation in seminars and other research activities is expected.

EXPANSION OF GRADUATE PROGRAM IN MATHEMATICS AT TEMPLE UNIVERSITY

The Department of Mathematics at Temple University has announced an expansion of its graduate program in mathematics, starting in the academic year 1968-1969. The department will offer a series of courses leading to the M. A. and Ph.D. degrees in mathematics with specialization in probability theory and mathematical statistics. Fields of specialization include multivariate analysis and distribution theory, applied probability, information and coding theory, non-parametric methods, and design of experiment.

A limited number of assistantships and fellowships is available.

Further information may be obtained from Professor A. Schild, Chairman, Department of Mathematics, Temple University, Philadelphia, Pennsylvania 19122.

SYMPOSIUM ON SEMIGROUPS

The Department of Mathematics at Wayne State University will hold a symposium on semigroups on June 27-28, 1968, in Detroit, Michigan. The symposium will emphasize the three principal aspects of the theory of semigroups--harmonic analysis on semigroups, the algebraic structure of semigroups, and the structure of topological semigroups. Important subareas will also be considered; i.e., probability measures on semigroups and the theory of automata and machines as applied to the algebraic structures of semigroups. One-hour invited lectures will be given by L. W. Anderson, M. M. Day, E. Hewitt, R. J. Koch, W. D. Munn, J. L. Rhodes, M. Rosenblatt, and A. D. Wallace. Additional information on the symposium may be obtained by writing to the Symposium Secretary, Department of Mathematics, Wayne State University, Detroit, Michigan 48202.

AFOSR MATHEMATICS DIVISION RESEARCH PROGRAMS

The Mathematics Division of the Air Force Office of Scientific Research annually plans its research program during the period January 1 through June 30 of any given calendar year. Research proposals considered under this program may request support to begin no earlier than September 1 of the same year and no later than September 30 of the following year.

In addition to the Division's continuing research program in analysis, functional analysis, statistics and probability theory, special programs for the support of specifically-oriented research monographs, intensified research, conferences and symposia are emphasized in all areas of mathematics and statistics.

The research-monograph program is designed to bridge the gap between journal-level research mathematics and user-level understanding and applications. The intensified research program is designed to accelerate the development of mathematics, most often in conjunction with university leave. Both programs provide for a maximum of one-half time support for 9 months, plus full-time support for an adjoining 3 months on a non-recurring basis. Proposals submitted for consideration under either of these programs follow the outlines of a conventional research proposal, emphasizing the unique features of the anticipated results. It is helpful if research-monograph proposals include a sample chapter, in addition to a table of contents and an indication of publisher interest.

All proposals to be considered under the Division's current planning cycle should be submitted before June 30, 1968. Decisions on all proposals submitted under the research-monograph and intensified-research programs requesting support for some portion of the 1968-1969 academic year will be made no later than July 31, 1968. Decisions on all other proposals will be made as soon as possible after July 1, 1968. Proposals should be submitted to the Mathematics Division, Air Force Office of Scientific Research, 1400 Wilson Boulevard, Arlington, Virginia 22209.

TWENTIETH BRITISH MATHEMATICAL COLLOQUIUM

The Twentieth British Mathematical Colloquium will be held at the University of Leeds on April 2-6, 1968. There will be three morning sessions of three hours each. On April 3, A. Ledger, G. C. Shephard, and J. A. Tyrrell will speak on geometry, convex sets, and number theory. Lectures on analysis will be delivered by W. Parry, B. E. Johnson, and E. Thoma on April 4. The final three-hour session on April 5 will concern algebra, and J. L. Britton, H. Kupisch, and J. M. Howie will speak on various aspects of this subject.

In the afternoons there will be meetings of splinter groups on various topics. Members are invited to contribute short papers to these groups, and there will be opportunity for discussion.

Following these meetings, there will be a one-hour lecture on each of the three days by E. P. Specker, J.-P. Kahane, and P. Samuel, respectively. J.-P. Kahane will speak on perfect sets and trigonometric series, and P. Samuel will discuss Euclidean rings. E. P. Specker's topic has not yet been announced.

The membership fee for the colloquium is 10s. (5s. for research students) if the application is received by March 8; after that date, membership fees will be doubled. Each application must be accompanied by the appropriate fee.

Accommodations will be available in the halls of residence from Tuesday evening, April 2, until Saturday morning, April 6, at a charge of £9 5s. 0d. which will include all meals, gratuities, and transport between the hall and the university. For members who prefer to arrive on Wednesday morning and/or leave on Friday afternoon, this charge will be reduced.

Additional information may be obtained from E. W. Wallace, Secretary to the 1968 colloquium.

NSF SEEKS PROPOSALS ON USES OF COMPUTERS IN THE EDUCATIONAL PROCESS

The National Science Foundation invites grant proposals for unusual projects designed to develop or accelerate the applications of computers to all disciplines and to the educational process. In particular the Foundation seeks projects aimed at innovative developments in new computer-oriented curricula, computer-assisted instruction, specialized laboratories, conferences, and training for students and teachers in computer usage. Projects having the best chance of support are those showing imagination and originality and not those that simply extend current computer applications.

Institutions eligible to submit proposals include universities and colleges, associations of professional scientists, and nonprofit research organizations.

Applicants are encouraged to submit an informal written description of the proposed project to serve as a basis for a preliminary opinion on the possibility of support for the proposed work. A preliminary draft of this kind should discuss the need for the project, state the objectives, outline the contemplated work, and indicate the personnel to be involved, the estimated period of time required, and an approximate budget.

There are no specific budget limitations but, because of the limited funds available, projects must show unusual promise to be given high priority.

While proposals may be submitted at any time, several months should be allowed for evaluation and processing.

Communications should be addressed to the Education, Research and Training Section, Office of Computing Activities, National Science Foundation, Washington, D. C. 20550.

NATO POSTDOCTORAL FELLOWSHIPS IN SCIENCE

The National Science Foundation and the Department of State announced the award of 39 North Atlantic Treaty Organization (NATO) Postdoctoral Fellowships in Science.

Of the awards announced, 11 are in the life sciences, 25 in the physical sciences including mathematics and engineering, and 3 in the social sciences. Fellows will attend institutions in Belgium, Denmark, France, the Federal Republic of Germany, Italy, the Netherlands, Sweden, Switzerland, and the United Kingdom.

The United States citizens who are being offered awards were selected from among 462 applicants. All applicants were evaluated for NSF by panels of scientists appointed by the National Research Council. Final selections were made by the Foundation.

NATO Fellows will receive a stipend of $6,500 for 12 months, $4,875 for 9 months. In addition, dependency allowances and limited allowances for round-trip travel will be provided.

FORTHCOMING IUTAM SYMPOSIA

The International Union of Theoretical and Applied Mechanics is scheduling the following symposia for 1968-1969. Those who wish to participate actively in one of these symposia are advised to communicate with the chairman of its Scientific Committee.

THERMOANELASTICITY
East Kilbridge, Glasgow, Scotland
June 26-28, 1968
Chairman: Professor B. A. Boley
Columbia University
New York, New York 10027

HIGH-SPEED COMPUTING IN FLUID MECHANICS
Monterey, California
August 18-24, 1968
Chairman: Dr. F. N. Frenkiel
Navy Ship Research and Development Center
Washington, D. C. 20007

FLOW OF FLUID-SOLID MIXTURES
Cambridge, England
March 24-29, 1969
Chairman: Professor G. K. Batchelor
University of Cambridge
Cambridge, England

INSTABILITY OF CONTINUOUS SYSTEMS
Karlsruhe, Germany
During 1st week of September, 1969
Chairman: Professor H. Leipholz
Technische Hochschule
75 Karlsruhe, Germany

NINTH ANNUAL SYMPOSIUM ON SWITCHING AND AUTOMATA THEORY

The Ninth Annual Symposium on Switching and Automata Theory, sponsored by the Switching and Automata Theory Committee of the IEEE Computer Group and co-supported by the General Electric Company and Rensselaer Polytechnic Institute, will be held in Schenectady, New York, on October 15, 16, and 17, 1968. Papers describing original research results in the general areas of automata theory, switching theory, theory of computation, and theoretical aspects of logical design are being sought.

Authors are requested to send six copies of detailed abstracts (no word limit) to Mr. Sheldon B. Akers, Jr., Electronics Laboratory, General Electric Company, Syracuse, New York 13201, by May 17. Authors will be notified of acceptance or rejection by June 28. For inclusion in the Conference Record, a copy of each accepted paper typed on special forms will be due at the above address by August 17.

Local arrangements for the symposium are being handled by Dr. Philip M. Lewis, II, General Electric R D Center, Box 8, Schenectady, New York 12301.

McGILL UNIVERSITY POST DOCTORATE GRANTS

The Mathematics Department of McGill University announces the availability of a small number of Post Doctorate Grants for 1968-1969, open to young mathematicians with Ph.D.'s who show interest and promise in research. The grant is for $6,000, free of tax, for the academic year. A supplement of $1,200 is available for married persons with at least one child. No teaching duties are involved. There is an option for an additional remuneration of at least $2,000 for teaching one course. The appointment is renewable for an additional year.

Applicants should communicate with the Chairman of the Department of Mathematics, McGill University, Montreal 2, Canada, before March 1, 1968.

INSTITUTE OF ADVANCED MATHEMATICS AT THE UNIVERSITY OF PUERTO RICO

The Department of Mathematics of the University of Puerto Rico, Mayaguez Campus, proposes the institutionalization of research within the department by means of the establishment of the Institute of Advanced Mathematics. The Institute will contribute to the development of both basic and applied mathematical research. Its members will hold joint appointments as professors of the Department of Mathematics.

The Institute will sponsor well-known mathematicians who will organize courses and seminars of high caliber aimed at the formation of competent professional mathematicians. It is hoped that the Institute will be an important factor in raising the level of mathematical activity in the Central and South American countries.

NSF SENIOR POSTDOCTORAL FELLOWSHIP AWARDS

The National Science Foundation had announced the award of 55 Senior Postdoctoral Fellowships for 1968. The fellowships were awarded to permit these scientists to pursue further research and advanced training in their particular fields.

Senior Postdoctoral Fellows were selected from 384 applicants. Basic requirements included possession of a doctoral degree in science, mathematics, or engineering for at least the past five years and demonstrated ability in advanced research.

Seven grants were awarded in mathematics, all to members of the AMS. Recipients are Anatole Beck, University of Wisconsin; Edwin H. Connell, Rice University; William Craig, University of California at Berkeley; Wendell Fleming, Brown University; David Gale, University of California at Berkeley; Geoffrey S. S. Ludford, Cornell University; and Alex Rosenberg, Cornell University.

CONGRESS OF PHYSICISTS AND MATHEMATICIANS

Under the auspices of the Recherche Cooperative sur Programme N^{o} 25 du Centre National de la Recherche Scientifique, a congress of physicists and mathematicians is planned for May 16-18, 1968, in the Department of Mathematics of the University of Strasbourg. The program for the congress will be published at a later date.

SECOND SUMMER RENCONTRES IN MATHEMATICS AND PHYSICS

The Second Summer Rencontres in Mathematics and Physics of the Battelle Seattle Center will take place from July

29 to September 8, 1968. In order to provide a channel of communication between mathematicians and physicists, the Battelle Seattle Center will welcome about thirty individuals, approximately half from mathematics, half from physics in somewhat related areas. The center of interest for 1968 will be hyperbolic partial differential equations and wave mechanics.

Although it is expected that individual contacts and unscheduled discussions will play the major role in this exchange between mathematicians and physicists, a few frankly pedagogical courses will be organized as follows. 16 lectures will be given in mathematics, consisting of 12 lectures in strictly hyperbolic differential equations by Professor P. Lax, Courant Institute of Mathematical Sciences, and 4 lectures on nonstrict hyperbolic differential equations by Professor J. Leray, Collège de France. Of the 16 lectures given by physicists, 8 on wave mathematics will be delivered by Professor M. J. Lighthill, Imperial College. The 8 additional lectures have not yet been announced.

Professors F. J. Dyson, Institute for Advanced Study; L. Ehrenpreis, Courant Institute; and K. Thorne, California Institute of Technology will act as consultants to help organize informally the climate of mutual understanding.

NSF GRANTS TO IMPROVE
COLLEGE SCIENCE TEACHING

The National Science Foundation has awarded a total of $812,550 to ten colleges and universities to support as many Academic Year Institutes for college teachers of science and mathematics. Under these grants, more than 123 teachers throughout the U. S. will spend one academic year at colleges and universities where they will study in close contact with gifted teaching scientists and mathematicians. Fields of study to be pursued by the teachers include engineering, physics, chemistry, biology, mathematics, economics, and radiation science.

The Foundation has also awarded grants totaling $610,890 to support 36 Short Courses for College Teachers of science, mathematics, and engineering. The courses, aimed at familiarizing college and junior college teachers with the latest advances in their fields, are expected to draw 1,003 teachers from every state in the union to 32 campuses in 19 states. Participating teachers will study with outstanding scholars for periods of one to four weeks. The courses will be held in mathematics, engineering, and the physical, biological, and social sciences. A directory listing the institutions offering Short Courses for College Teachers may be obtained by sending a post card request to College Teacher Programs, National Science Foundation, Washington, D. C. 20550.

Under a third program, eight grants totaling approximately $82,000 provide for In-Service Seminars for College Teachers to be held during the summer of 1968 or during the academic year 1968-1969. More than 650 college, junior and community college, and technical institute teachers of science, mathematics, and engineering will benefit from the program. Of the eight grants, two were awarded in the field of chemistry, two in physics, three in biology, and the only one in the mathematical sciences was in the field of computing.

MEMO TO MEMBERS

PROCEEDINGS

The Editorial Committee regrets to announce that the PROCEEDINGS currently has a very large backlog.

VISITING FOREIGN MATHEMATICIANS

The foreign mathematicians listed below are in addition to those who were listed on pages 902-908 of the November, 1967 issue of these NOTICES.

Name and Home Country	Host Institution	Field of Special Interest	Period of Visit
Bressan, Aldo (Italy)	Carnegie-Mellon Institute	Continuous media	2/67-2/68
Burmester, M. (England)	University of Illinois, Chicago Circle	Projective Planes	9/67-6/68
Cabana, Enrique (Uruguay)	The Rockefeller University	Stochastic Intergrals	1/67-6/68
Cossey, Peter John (Australia)	City University of New York	Theory of Groups	9/66 6/68
Daguenet, Maryvonne (France)	University of California, Los Angeles	Mathematical categories and logic: Theory of Models	7/67-7/68
Doleans, Catherine (France)	University of Illinois	Probability theory and relations with potential theory	9/67-5/68
Eke, Barry George (United Kingdom)	University of California, La Jolla	Theory of functions and complex variable	9/67-6/68
Fowkes, Neville Donald (Australia)	Harvard University	Applied mathematics in oceanography	10/65-9/68
Lee, Keean (Korea)	University of Chicago	Homological algebra	7/67-6/68
Maczynski, M. J. (Poland)	University of Illinois, Chicago Circle	Boolean Algebra and Logic	9/67-6/68
Moro, Antonio (Italy)	Wayne State University	Differential Equations, Probability theory	9/67-6/68
Murthy, M. P. (India)	University of Illinois, Chicago Circle	Commutative Algebra	9/67-6/68
Neave, Henry R. (England)	University of Wisconsin	Statistics	6/67-6/68
Okubo, Kenjiro (Japan)	University of Minnesota	Theory of differential equations	9/66-6/68
Putter, Joseph (Israel)	University of Wisconsin	Statistics	9/67-7/68
Ringel, Gerhard (Federal Republic of Germany)	University of California, Santa Cruz	Graph theory	7/67-6/68
Swaminathan, S. (India)	University of Illinois, Chicago Circle	Abstract Analysis	9/67-6/68
Tsai, Ying-fan (Republic of China)	Oregon State University	Advanced mathematics and teaching methods	9/67-6/68

DOCTORATES CONFERRED DURING JANUARY-JUNE 1967

(Supplementary List)

ADELPHI UNIVERSITY (3)

Anshel (Orleck), Michael
Non-Hopfian groups with fully invariant kernels
Dunn, Joseph C.
Pontryagin's maximum principle, Bellman's equation, and the classification of extremals
Hirshon, Ronald
Some results on direct sums of Hopfian groups

UNIVERSITY OF UTAH (5)

*Dennis, John E., Jr.
Variations on Newton's method
Eaton, William Thomas
Side approximations in crumpled cubes
Hamilton, Wallace LeRoy
Categories of automata systems
*Lambert, Howard Wilson
Locally spherical decompositions of E^3
Lamoreaux, Jack Wayne
Decompositions of Metric spaces with a O-dimensional set of nondegenerate elements

*Doctorates conferred in 1966.

PERSONAL ITEMS

Professor H. J. ARNOLD of Bucknell University has been appointed to an associate professorship at Oakland University.

Mr. MARC ARONSON of the University of Miami has been appointed a Lecturer at Queensborough Community College.

Dr. LEONARD ASIMOW of the University of Washington has been appointed a Lecturer at the University of California, Berkeley, for the fall and winter of 1967-1968.

Mr. T. A. ATCHISON of Ling-Temco-Vought, Incorporated, has been appointed to an associate professorship at the Texas Technological College.

Dr. REINHOLD BAER of the University of Frankfurt, Germany, has been appointed a Distinguished Professor at New Mexico State University.

Dr. A. F. BARTHOLOMAY of Harvard University Medical School has been appointed to a professorship in the School of Medicine and Public Health at the University of North Carolina, Chapel Hill, and in the Department of Experimental Statistics at North Carolina State University at Raleigh.

Dr. PARANNARA BASAUAPPA of Dalhousie University has been appointed to an associate professorship at Algoma College.

Dr. H. S. BEAR, JR., of the University of California, Santa Barbara, has been appointed to a professorship at New Mexico State University.

Professor B. J. BEECHLER of Wheaton College has been appointed to an associate professorship at Pitzer College.

Professor R. F. BELL of Eastern Washington State College has been appointed to an associate professorship at West Virginia Institute of Technology.

Dr. G. M. BERGMAN of Harvard University has been appointed to an assistant professorship at the University of California, Berkeley. He will be on leave during the academic year 1967-1968 to pursue research on a Postdoctoral Fellowship of the Air Force Office of Scientific Research.

Professor LIPMAN BERS of Columbia University has been appointed a Research Professor in the Miller Institute for Basic Research, University of California, Berkeley, for the calendar year 1968.

Professor E. L. BETHEL of Clemson University has been appointed to an assistant professorship at Kent State University.

Professor R. E. BLOCK of the University of Illinois has been appointed to a visiting research associateship at Yale University for the fall semester.

Professor T. F. BRIDGLAND, JR., of the University of Alabama, Huntsville, has been appointed to a professorship at Florida State University.

Dr. J. L. BRITTON of the University of Kent at Canterbury, England, has been appointed to a visiting professorship at the University of Illinois from February 1 to April 1.

Professor R. B. BROWN of the University of California, Berkeley, will be on leave at the University of Utrecht, Netherlands, from January 1968 to December 1968.

Professor T. J. BROWN of Loyola University, Chicago, has been appointed to an assistant professorship at the University of Missouri, Kansas City.

Dr. R. C. BUSBY of the Drexel Institute of Technology has been appointed to an assistant professorship at Oakland University.

Professor W. M. CAUSEY of Mississippi State University has been appointed to an assistant professorship at the University of Cincinnati.

Professor R. W. CHANEY of Western Washington State College has been appointed to an assistant professorship at the University of California, Santa Barbara.

Professor CHARLES CHUI of the University of California, San Diego, has been appointed to an assistant professorship at the State University of New York at Buffalo.

Professor Y. H. CLIFTON of the University of California, Los Angeles, has been appointed Associate Editor with Mathematical Reviews, Ann Arbor, Michigan.

Professor M. L. CLINNICK of the California State Polytechnic College has accepted a position as Mathematician-Programmer with the Lawrence Radiation Laboratory, Berkeley, California.

Professor S. D. COMER of the University of Colorado has been appointed to an assistant professorship at Vanderbilt University.

Mr. H. B. COONCE of Delaware State College has been appointed to an assistant professorship at the United States Naval Academy, Annapolis, Maryland.

Professor H. O. CORDES of the University of California, Berkeley, will be on leave for the fall and winter quarters 1967-1968. He will carry on research at the University of Hamburg, Germany.

Dr. R. W. COTTLE of the Bell Telephone Laboratories, Holmdel, New Jersey, has been appointed to an assistant professorship in the Department of Operations Research at Stanford University.

Professor CASPAR CURJEL of the University of Washington will be on leave at the Swiss Federal Institute of Technology, Zurich, Switzerland.

Professor K. M. DAS of Michigan State University has been appointed to an associate professorship at Iowa State University.

Professor R. F. DE MAR of the University of California, Davis, has been appointed to an associate professorship at the University of Cincinnati.

Dr. KLAUS DIETZ of the University of Freiburg, West Germany, has been appointed a Lecturer at the University of Sheffield, England.

Professor JACOB FELDMAN of the University of California, Berkeley, will be on leave for the academic year 1967-1968. He will carry on research in Russia and Israel during the fall and winter and has been invited to lecture at the University of Paris during the spring.

Dr. A. N. FELDZAMEN of the Research Foundation of the State University of New York has been appointed Vice President and Treasurer of the Broadcasting Foundation of America, and a member of the Board of Trustees of this organization.

Professor P. C. FISCHER of Cornell University has been appointed to a visiting associate professorship at the University of British Columbia.

Professor MARSHALL FRASER of the University of Illinois has been appointed to an assistant professorship at Albion College.

Professor H. I. FREEDMAN of the University of Minnesota has been appointed to an assistant professorship at the University of Alberta.

Professor R. M. FREYRE of the Lowell Technological Institute has been appointed to a professorship at the State College at Boston.

Dr. JON FROEMKE of the University of California, Berkeley, has been appointed to an assistant professorship at Oakland University.

Professor THEODOR GANEA of the University of Washington will be on leave in Seattle and at the Institute Henri Poincare, France.

Professor RAMESH GANGOLLI of the University of Washington will be on leave in Seattle.

Professor Emeritus HILDA GEIRINGER of Wheaton College has been awarded an Honorary Doctorate from the University of Vienna, Austria.

Mr. R. M. GIBBS of the University of Alabama has been appointed to an assistant professorship at Armstrong State College.

Dr. B. W. GLICKFELD of Harvard University has been appointed to an assistant professorship at the University of Washington.

Dr. SIGRUN GOES of Northwestern University has been appointed to an assistant professorship at DePaul University.

Professor J. E. GOODMAN of New York University has been appointed to an assistant professorship at the City University of New York.

Professor R. J. GREECHIE of the University of Massachusetts, Boston, has been appointed to an assistant professorship at Kansas State University.

Professor F. P. GREENLEAF of the University of California, Berkeley, will be on leave during the fall and winter quarters 1967-1968. He will carry on research at the University of Pennsylvania for part of the leave and will be in residence in Berkeley for the remainder.

Professor P. A. GRILLET of the University of Florida has been appointed to an assistant professorship at Kansas State University.

Dr. F. D. GROSSHANS of the University of Chicago has been appointed to an assistant professorship at DePaul University.

Professor KLAUS HABETHA of the Technical University of Berlin, Germany, has been appointed to a visiting professorship at the Chalmers Institute of Technology and the University of Goteborg, Sweden, for the academic year 1967-1968.

Professor J. R. HANNA of the University of Colorado has been appointed to a professorship at the University of Wyoming.

Professor ROBERT HEATH of Arizona State University has been appointed a visiting Lecturer at the University of Washington.

Dr. L. B. HEILPRIN of the Council on Library Resources, Washington, D.C., has been appointed to a professorship at the University of Maryland.

Dr. MIGUEL HERRERA of the Institute for Advanced Study has been appointed to a visiting assistant professorship at the University of Washington for the fall and winter semesters.

Professor W. F. HILL of Tarleton State College has been appointed to a professorship at East Texas State University.

Professor A. L. HILT of Temple University has been appointed to a professorship at Albright College.

Professor P. J. HILTON of Cornell University has been appointed to a visiting professorship at the Courant Institute of Mathematical Sciences, New York University, for the academic year 1967-1968.

Mr. J. S. HINKEL of the University of South Carolina has been appointed to an assistant professorship at Armstrong State College.

Professor S. S. HOLLAND, JR., of Boston College has been appointed to an associate professorship at the University of Massachusetts.

Professor BRINDELL HORELICH of Lafayette College has been appointed to an associate professorship at the State University of New York College at Cortland.

Dr. J. M. HOWIE of the University of Glasgow, Scotland, has been appointed Senior Lecturer at the University of Stirling, Scotland.

Professor DENISE HUET of the Faculty of Science, France, has been appointed to a visiting professorship at the University of Maryland.

Professor F. N. HUGGINS of the University of Texas at Austin has been appointed to an assistant professorship at the University of Texas at Arlington.

Professor TERUO IKEBE of Kyoto University has been appointed to an assistant professorship at the University of Washington.

Mr. G. S. INNIS of the Los Alamos Scientific Laboratory, Los Alamos, New Mexico, has been appointed to an associate professorship at the Texas Technological College.

Dr. R. C. IRWIN of the Mitre Corporation, Colorado Springs, Colorado, has accepted a position as a Senior Research Mathematician with the Dikewood Corporation, Albuquerque, New Mexico.

Professor J. P. JANS of the University of Washington will be on leave in Seattle.

Professor H. H. JOHNSON of the University of Washington will be on leave in New York City.

Professor W. L. JOHNSON of the American University in Cairo has been appointed a Research Mathematician at the Stanford Research Institute.

Professor E. G. D. JONES of Hampton Institute has been appointed to a professorship at Norfolk State College.

Professor O. T. JONES of Stetson University has been appointed to a visiting assistant professorship at Louisiana State University.

Mr. S. F. KAPOOR of Michigan State University has been appointed to an assistant professorship at Western Michigan University.

Professor TOSIO KATO of the University of California, Berkeley, will be on sabbatical leave during the spring of 1968. He will carry on research in Paris, Germany and Switzerland.

Dr. O. H. KEGEL of the University of Cologne, Germany, has been appointed to a visiting professorship at New Mexico State University for the academic year 1967-1968.

Professor H. B. KELLER of the Courant Institute of Mathematical Sciences, New York University, has been appointed to a professorship in the Department of Applied Mathematics at the California Institute of Technology.

Professor RAYMOND KILLGROVE of San Diego State College has been appointed to an associate professorship at the California State College, Los Angeles.

Mr. HOWARD KLEIMAN of London, England, has been appointed to an assistant professorship at the Queensborough Community College.

Dr. DAVID KNUDSON of Cornell University has been appointed to an assistant professorship at the University of Washington.

Dr. E. T. KOBAYASHI of Haverford College has been appointed to an associate professorship at New Mexico State University.

Professor SAMUEL KOTZ of the University of Toronto has been appointed to a professorship at Temple University.

Professor L. S. KROLL of the University of California, Riverside, has been appointed to an assistant professorship at the University of California, Davis.

Professor A. V. LAGINESTRA of Rensselaer Polytechnic Institute has been appointed to an assistant professorship at New York University, University Heights.

Professor O. A. LAUDAL of the University of Oslo, Norway, has been appointed to a visiting associate professorship at the University of Illinois for the second semester.

Lt. E. S. LAVOIE of the New York City Police Department has been appointed to an assistant professorship at the John Jay College of Criminal Justice, City University of New York.

Professor RICHARD LEVIN of the University of California, Davis, has been appointed to an assistant professorship at Western Washington State College.

Professor D. W. LICK of the University of Tennessee will be on leave for one year at the Brookhaven National Laboratory, Upton, New York, as a visiting Assistant Research Mathematician in the Applied Mathematics Department.

Mr. T. L. LINCOLN of the National Institute of Health, Bethesda, Maryland, has accepted a position as a Staff Member with the Rand Corporation, Santa Monica, California.

Mr. N. E. LINDQUIST of Oregon State University has been appointed to an assistant professorship at Western Washington State College.

Professor MICHEL LOEVE of the University of California, Berkeley, has been appointed a Professor of Arts and Science for the fall and winter quarters 1967-1968.

Professor GUNTER LUMER of the University of Washington will be on leave at the University of Strausberg, France.

Professor R. E. LYNCH of the University of Texas has been appointed to an associate professorship in the Department of Computer Science and the Department of Mathematics at Purdue University.

Professor T. H. MACGREGOR of Lafayette College has been appointed to a professorship at the State University of New York at Buffalo.

Professor JOSIAH MACY, JR., of the Albert Einstein College of Medicine has been appointed Professor of Biomathematics and Director of the Division of Biophysical Sciences at the Medical Center of the University of Alabama.

Professor J. P. MALONEY of the University of Nebraska has been appointed to an assistant professorship at the University of Omaha.

Dr. O. L. MANGASARIAN of the Shell Development Company, Emeryville, California, has been appointed to an associate professorship at the University of Wisconsin in the Computer Sciences Department.

Professor HIDEYUKI MASTSUMURA of Kyoto University, Japan, has been appointed to a visiting associate professorship at Brandeis University.

Professor B. J. MCDONALD of Florida State University has accepted a position as a Mathematical Statistician with the Office of Naval Research, Mathematical Sciences Division, Washington, D. C.

Professor O. C. MCGEHEE of the University of California, Berkeley, has been awarded a Nato Fellowship at Orsay, France, for the academic year 1967-1968.

Mr. R. M. MEISEL of the Sperry Gyroscope Company, Great Neck, New York, has accepted a position as a Systems Engineer with the Grumman Aircraft Engineering Corporation, Bethpage, New York.

Professor R. A. MELTER of the University of Massachusetts has been appointed to an associate professorship at the University of South Carolina.

Mr. A. R. MEYER of Harvard University has been appointed to an assistant professorship at Carnegie-Mellon University in the Computer Science Department.

Professor PAUL MEYER of Washington State University has been appointed to a visiting associate professorship at the University of Washington.

Mr. J. V. MICHALOWICZ of the Research Analysis Corporation, McLean, Virginia, has been appointed to an assistant professorship at the Catholic University of America.

Professor S. S. MITRA of Clarkson College of Technology has been appointed to an associate professorship at Western Washington State College.

Professor J. D. MONK of the University of Colorado has been appointed a Visiting Professor and Research Mathematician at the University of California, Berkeley, for the academic year 1967-1968.

Dr. R. V. MOODY of the University of Saskatchewan has been appointed to an assistant professorship at New Mexico State University.

Professor C. C. MOORE of the University of California, Berkeley, will be on sabbatical leave for the fall and winter quarters 1967-1968. He will carry on research at Harvard University for part of the leave and will be in residence in Berkeley for the remainder.

Professor R. T. MOORE of the University of California, Berkeley, will be on leave during the fall and winter quarters 1967-1968. He will carry on research in Los Angeles, Pennsylvania and Berkeley.

Mr. JACK MORAVA of Oxford University, England, has been appointed a Member at the Institute for Advanced Study.

Professor ANNE MOREL of the University of Washington will be on leave for the spring quarter in Seattle.

Professor R. W. MORI of the United States Naval Academy has accepted a position as a mathematician with Hydrotronics, Falls Church, Virginia.

Mr. S. R. NEAL of the United States Naval Weapons Center, China Lake, California, has accepted a position as a Member of the Technical Staff with the Bell Telephone Laboratories, Holmdel, New Jersey.

Professor UMBERTO NERI of the University of Chicago has been appointed to an assistant professorship at the University of Maryland.

Professor J. A. NICKEL of Oklahoma City University has accepted a position as a Senior Research Mathematician with the Dikewood Corporation, Albuquerque, New Mexico.

Professor H. F. NIEMEYER of the Stevens Institute of Technology has been appointed to a professorship at the University of Marburg, West Germany.

Professor J. E. NYMANN of the University of Hawaii has been appointed to an associate professorship at the University of Texas at El Paso.

Professor RUFUS OLDENBURGER, Director of the Automatic Control Center, School of Mechanical Engineering at Purdue University, has been awarded the 1967 Donald P. Eckman Education Award from the Instrument Society of America.

Mr. P. V. O'NEIL of the University of Minnesota has been appointed to an assistant professorship at the College of William and Mary.

Professor SIDNEY PENNER of the City University of New York has been appointed to an assistant professorship at the Bronx Community College.

Professor R. S. PIERCE of the University of Washington will be on leave at New Mexico State University as a visiting professor.

Professor D. L. POWERS of the Universidad Santa Maria, Chile, has been appointed to an assistant professorship at Clarkson College of Technology.

Professor MURRAY PROTTER of the University of California, Berkeley, has been appointed a Research Professor in the Miller Institute for Basic Research, Berkeley, for the academic year 1967-1968.

Professor L. E. PURSELL of Grinnell College has been appointed to an associate professorship at the University of Missouri, Rolla.

Dr. C. H. RANDALL of the Knolls Atomic Power Laboratory, Schenectady, New York, has been appointed to an associate professorship at the University of Massachusetts.

Professor E. S. RAPAPORT of the Polytechnic Institute of Brooklyn has been appointed to a professorship at the State University of New York at Stony Brook.

Mr. W. N. REINHARDT of the University of California, Berkeley, has been appointed to an assistant professorship at the University of Colorado.

Professor J. L. RHODES of the University of California, Berkeley, has been awarded a Sloan Foundation Grant for the academic year 1967-1968. He will spend the year in Berkeley.

Professor G. J. RIEGER of the University of Munich, Germany, has been appointed to a visiting professorship at the State University of New York at Buffalo.

Dr. R. W. ROBINSON of Cornell University has been appointed to an assistant professorship at the University of California, Berkeley.

Professor SELBY ROBINSON of City College has been appointed to a professorship at Florida Atlantic University.

Professor S. M. ROBINSON of Union College has been appointed to a visiting professorship at Cleveland State University.

Professor G. M. ROSENSTEIN, JR., of Case-Western Reserve University has been appointed to an assistant professorship at Franklin and Marshall College.

Mr. H. L. ROSENZWEIG of the University of Virginia has been appointed to an assistant professorship at Haverford College.

Professor D. E. RYAN of Bowling Green State University has been appointed to an assistant professorship at San Diego State College.

Professor J. V. RYFF of the University of Washington will be on leave at the Institute for Advanced Study.

Professor H. J. RYSER of Syracuse University has been appointed to a professorship at the California Institute of Technology.

Dr. C. S. SADOSKY of the University of Buenos Aires, Argentina, has been appointed to a visiting assistant professorship at Johns Hopkins University.

Dr. RAFAEL SANCHES-DIAZ of the New Mexico Institute of Mining and Technology has been appointed to a professorship at Armstrong State College.

Professor T. V. SASTRY of the University of Rhode Island has been appointed to an associate professorship at Bradley University.

Professor ICHIRO SATAKE of the University of Chicago will be on leave for the academic year 1967-1968. He will be at the Institute for Advanced Study in the fall of 1967, and at the Institute Des Hautes Etudes, France, in the winter of 1968.

Professor KEN-ITO SATO of the Tokyo University of Education has been appointed to a visiting assistant professorship at the University of Minnesota.

Professor STANLEY SAWYER of the Courant Institute of Mathematical Sciences, New York University, has been appointed to an assistant professorship at Brown University.

Professor CAREL SCHEFFER of the University of Utrecht, Netherlands, has been appointed to a visiting assistant professorship at the University of British Columbia.

Professor ARNOLD SEIKEN of the University of Rhode Island has been appointed to an associate professorship at Union College.

Professor N. T. SHETH of Texas Southern University has been appointed to an associate professorship at the State University of New York College at Oswego.

Mr. M.-F. R. SHIU of Georgetown University has accepted a position as a Research Scientist with the National Biomedical Research Foundation, Silver Springs, Maryland.

Mr. H. B. SKERRY of Michigan State University has been appointed to an assistant professorship at Lehigh University.

Professor J. S. SKOCIK, JR., of West Virginia University has been appointed to an associate professorship at the California State College, California, Pennsylvania.

Professor STEPHEN SMALE of the University of California, Berkeley, has been appointed a Research Professor in the Miller Institute for Basic Research, Berkeley, for the academic year 1967-1968.

Dr. R. I. SOARE of Cornell University has been appointed to an assistant professorship at the University of Illinois at

Chicago Circle.

Professor Emeritus I. S. SOKOLNIKOFF of the University of California, Los Angeles, has received a Fulbright-Hays Award for 1967-1968. He will lecture at the Middle East Technical University, Ankara, Turkey.

Professor R. M. SOLOVAY of the University of California, Berkeley, has accepted a visiting appointment at Rockefeller University for the academic year 1967-1968.

Mr. R. P. SONI of the International Business Machine Corporation, Endicott, New York, has been appointed to a visiting associate professorship at the University of Tennessee.

Professor D. Z. SPICER of the University of California, Los Angeles, has been appointed to an assistant professorship at the University of Kentucky.

Professor T. P. SRINIVASAN of Panjab University, India, has been appointed a Visiting Professor and Research Mathematician at the University of California, Berkeley, for the academic year 1967-1968.

Dr. J. R. STALLINGS of Princeton University has been appointed to a professorship at the University of California, Berkeley.

Dr. O. N. STAVROUDIS of the National Bureau of Standards, Washington, D. C., has been appointed to a professorship in the Optical Sciences Center at the University of Arizona.

Mr. M. L. STEIB of the University of Texas has been appointed to an assistant professorship at the University of Houston.

Professor SATOSHI SUZUKI of Purdue University has been appointed to an associate professorship at Queen's University.

Professor M. E. SWEEDLER of the Massachusetts Institute of technology has been appointed to an assistant professorship at Cornell University.

Professor DOV TAMARI of the State University of New York at Buffalo has been appointed a Member at the Institute for Advanced Study for the academic year 1967-1968.

Sister R. J. TAUER of the College of St. Catherine has been appointed to a visiting assistant professorship at the University of Toronto.

Dr. J. D. THOMAS of New Mexico State University has been awarded an Atomic Energy Commission Faculty Fellowship at the Los Alamos Scientific Laboratory, Los Alamos, California, and will be on leave for the academic year 1967-1968.

Dr. J. R. THOMPSON of Vanderbilt University has been appointed to an assistant professorship at Indiana University.

Professor ALAN TROY of the University of Washington will be on leave at the University of Oregon.

Mr. A. E. VAGTS, JR., of the System Development Corporation, Dover-Foxcroft, Maine, has accepted a position as a Programmer with Informatics Incorporated, Rome, New York.

Mr. A. M. VAN DE WATER, JR., of the University of South Carolina has been appointed to an associate professorship at Winona State College.

Professor D. N. VERMA of New Mexico State University has been appointed a visiting member at the Institute for Advanced Study.

Mr. F. R. WADLEIGH of North American Aviation, Incorporated, Autonetics Division, Anaheim, California, has been appointed to an assistant professorship at the United States Naval Postgraduate School, Monterey, California.

Dr. J. B. WAGONER of Princeton University has been appointed to an assistant professorship at the University of California, Berkeley.

Dr. R. B. WARFIELD, JR., of Harvard University has been awarded a postdoctoral fellowship at New Mexico State University.

Professor JOHN WERMER of Brown University has been appointed a visiting Member at the Institute for Advanced Study for the academic year 1967-1968.

Professor M. B. WILLIAMS of the University of London, England, has been appointed to an assistant professorship at the Institute of Statistics at North Carolina State University at Raleigh.

Professor J. A. WOLF of the University of California, Berkeley, has been awarded a Sloan Fellowship. He will be on leave during the spring quarter 1968 and will carry on research at Cambridge and Berkeley.

Professor RAYMOND WONG of the

University of California, Los Angeles, has been appointed to a visiting assistant professorship at the University of Washington.

Professor R. L. WOODRIFF of Bozeman, Montana, has been appointed to an assistant professorship at Humboldt State College.

Professor D. E. WULBERT of Lund, Sweden, has been appointed to an assistant professorship at the University of Washington.

Mr. I. K. YALE of Morehouse College has been appointed to an assistant professorship at the University of Montana.

Professor OSCAR ZARISKI of the University of Rome, Italy, and Harvard University has been appointed to a visiting professorship at the University of California, Berkeley, for the winter quarter 1968.

Professor WILLIAM ZLOT of Yeshiva University has been appointed to an associate professorship at City College of the University of New York.

PROMOTIONS

To Professor. Armstrong State College: W. B. LAFFER, II; University of California, Berkeley: J. A. WOLF; University of Miami: E. F. LOW, J. D. MCKNIGHT, JR., HERMANN SIMON; New Mexico State University: J. E. KIST, LOUIS SOLOMON; Northeastern Illinois State College: L. M. WEINER; State University of New York at Buffalo: K. D. MAGILL, JR.; Tulane University: F. T. BIRTEL; University of Washington: RAMESH GANGOLLI, GUNTER LUMER.

To Associate Professor. University of Alberta: K. M. GARG; University of California, Berkeley: B. N. PARLETT, C. C. PUGH, D. E. SARASON; Hebrew University of Jerusalem: MICHAEL MASCHLER: University of Miami: HERMAN SIMON; State University of New York at Binghamton: BRUCE LERCHER; State University of New York at Buffalo: S. R. CAVIOR, AKIKO KINO, A. D. MACGILLIVRAY; Texas Southern University: L. L. CLARKSON; Wartburg College: W. L. WALTMANN; University of Washington: WILLIAM WOOLF.

To Assistant Professor. University of California, Berkeley: GENE LEWIS, O. C. MCGEHEE, RALPH MCKENZIE, WILFRIED SCHMID, J. H. SILVER; College of Santa Fe: B. A. DE VALCOURT; Vanderbilt University: V. W. NOONBURG; Wesleyan University: M. K. AGOSTON.

To Instructor. Arkansas State University: W. R. LIVINGSTON; Bristol Community College: C. A. PIASCIK; University of California, Berkeley: JEFF CHEEGER, DAVID EBIN, J. A. MORROW, PETER WALTERS; DePaul University: R. J. SHAKER; Fordham University: F. J. SERVEDIO; Harvard University: F. P. GADINER; University of Miami: G. L. MUSSER; University of Pittsburgh: H. E. SPAIN, JR.; Princeton University: BERNARD PINCHUK; New Mexico State University: FRANK CASTAGNA; State University of New York, Maritime College: E. J. RICH; University College of the University of Richmond: STEPHEN HARDIMAN; St. Mary's College: BARNABAS NAJAR; University of Washington: E. R. FERNHOLZ; Wellesley College: E. H. GOVER.

DEATHS

Professor J. W. BRADSHAW of Ann Arbor, Michigan, died on June 11, 1967, at the age of 89. He was a member of the Society for 53 years.

Dr. J. W. CELL of the North Carolina State University at Raleigh died on November 9, 1967, at the age of 60. He was a member of the Society for 36 years.

Mr. W. W. S. CLAYTON of Washington, D. C., died on July 14, 1967, at the age of 59. He was a member of the Society for 34 years.

Professor Emeritus P. H. LINEHAN of the City University of New York died on September 21, 1967, at the age of 88. He was a member of the Society for 54 years.

Professor L. L. LOWENSTEIN of Arizona State University died on August 23, 1967, at the age of 67.

Professor A. J. MACINTYRE of the University of California, Davis, died on August 4, 1967, at the age of 59. He was a member of the Society for 20 years.

Dr. MORRIS OSTROFSKY of the Westinghouse Defense and Space Center, Baltimore, Maryland, died on September 24, 1967, at the age of 58. He was a member of the Society for 20 years.

Mr. M. F. POLLACK of San Francisco, California, died on August 3, 1967, at the age of 78. He was a member of the Society for 10 years.

Dr. W. A. SHEWHART of the Bell Telephone Laboratories, Murray Hill, New Jersey, died on March 11, 1967, at the age of 76. He was a member of the Society for 45 years.

Professor Emeritus PAULINE SPERRY of the University of California, Berkeley, died on September 24, 1967, at the age of 82. She was a member of the Society for 51 years.

ERRATA

The following are corrections of announcements in the October issue of the *Notices*.

Dr. S. K. KNAPOWSKI of the University of Miami died on September 28, 1967, at the age of 36.

It was incorrectly stated in the October issue of the *Notices* that Professor H. D. LIPSICH of the University of Cincinnati died on August 4, 1956. Professor LIPSICH is presently a professor in the Department of Mathematics and has been a member of the Society for ten years.

SUPPLEMENTARY PROGRAM-Number 50

During the interval from September 27 to November 22, 1967, the papers listed below were accepted by the American Mathematical Society for presentation by title. After each title on this program there is an identifying number. The abstracts of the papers will be found following the same number in the section on Abstracts of Contributed Papers in this issue of these *Notices*.

One abstract presented by title may be accepted per person per issue of these *Notices*. Joint authors are treated as a separate category; thus in addition to abstracts from two authors individually one joint abstract by them may be accepted for a particular issue.

(1) Strongly branched coverings of closed Riemann surfaces
Professor R. D. M. Accola, Brown University (68T-155)

(2) Separation principles in difference hierarchies
Professor J. W. Addison, Jr., University of California, Berkeley (68T-154)

(3) Certain basic integral operators and hypergeometric transformations
Professor R. P. Agarwal, West Virginia University (68T-52)

(4) Cubic splines on the real line
Dr. J. H. Ahlberg, United Aircraft Research Laboratories, East Hartford, Connecticut, and Dr. E. N. Nilson, Pratt and Whitney Aircraft, East Hartford, Connecticut, and Professor J. L. Walsh, University of Maryland (68T-10)

(5) Split dilations of finite cyclic groups
Mr. Shair Ahmad, Case-Western Reserve University (68T-30)

(6) A note on compactifications and semi-normal spaces
Professor R. A. Alo and Professor H. L. Shapiro, Carnegie-Mellon University (68T-73)

(7) On a necessary condition for the validity of the Riemann hypothesis for functions that generalize the Riemann zeta functions
Dr. Ronald Alter, System Development Corporation, Santa Monica, California (68T-85)

(8) Near point theorem
Professor W. D. L. Appling, North Texas State University (68T-71)

(9) Fréchet differentiability of convex functions
Dr. Edgar Asplund, University of Washington (68T-146)

(10) Solution for nonlinear deflections in an elastic pressurized spherical shell under diametrical compression
Professor J. S. Bakshi, State University College at Buffalo (68T-41)

(11) A representation theory for prime and implicative semilattices
Professor Raymond Balbes, University of Missouri at St. Louis (68T-102)

(12) Calculus of set-valued functions. Preliminary report
Mr. Peter Bancroft, University of Colorado (68T-92)

(13) On the impossibility of extending the Riesz uniqueness theorem to holomorphic functions of slow growth
Dr. K. F. Barth and Professor W. J. Schneider, Syracuse University (68T-67)

(14) On rings with proper involution
Professor W. E. Baxter, University of Delaware (68T-129)

(15) Projections on affine spaces. Preliminary report
Mr. J. B. Bednar, Tracor, Incorporated, Austin, Texas (68T-82)

(16) An unusual matrix and its extensions
Professor Sami Beraha, Ball State University (68T-13)

(17) Class two p groups as fixed point free automorphism groups
Mr. T. R. Berger, University of Minnesota (68T-14)

(18) Convergence function algebras and c-embedded convergence spaces
Professor E. Binz, Queen's University (68T-107)

(19) Metric postulates for normed Boolean

algebra
Professor L. M. Blumenthal, University of Missouri, Columbia (68T-151)

(20) On Pettis-Dunford-Phillips property for vector-valued volumes
Professor W. M. Bogdanowicz, The Catholic University of America (68T-7)

(21) Representation of noncomplete Lebesgue integrals
Professor Witold Bogdanowicz and Mrs. Martha Mattamal, The Catholic University of America (68T-19)

(22) On annular functions. Preliminary report
Dr. D. D. Bonar, Ohio State University (68T-149)

(23) A metrization theorem
Mr. J. R. Boone, Texas Christian University (68T-100)
(Introduced by Professor Hisahiro Tamano)

(24) Locally convex topologies on function spaces
Professor J. W. Brace, University of Maryland; Professor G. D. Friend and Professor P. J. Richetta, Lehigh University (68T-62)

(25) Finite groups have automorphisms of large order. Preliminary report
Mr. J. K. Brown, Ohio State University (68T-138)
(Introduced by Professor W.P. Kappe)

(26) Representations and approximations by Darboux functions in the first class of Baire
Professor A. M. Bruckner, Professor J. G. Ceder and Mr. R. Keston, University of California, Santa Barbara (68T-153)

(27) Approximation of semigroups of operators in Fréchet spaces
Mr. A. B. Buche, Wayne State University (68T-46)

(28) Structure of a certain class of ordered groupoids. Preliminary report
Dr. Maurice Chacron, University of Sherbrooke (68T-159)

(29) Polynomial systems satisfying a special functional equation
Professor A. M. Chak, West Virginia University (68T-53)

(30) On pointwise convergence of Fourier series on the p-adic integers
Professor R. W. Chaney, University of California, Santa Barbara (68T-4)

(31) A theorem of nilpotent groups
Professor C. Y. Chao, University of Pittsburgh (68T-5)

(32) Randomly traceable graphs
Professor Gary Chartrand, Western Michigan University, and Professor H. V. Kronk, State University of New York, Binghamton (68T-78)

(33) Homotopy of algebras
Professor K.-T. Chen, University of Illinois (68T-150)

(34) On spline interpolation operators
Professor E. W. Cheney and Mr. F. Schurer, University of Texas (68T-139)

(35) Generalized temperatures and analytic functions
Professor F. M. Cholewinski, Clemson University, and Professor D. T. Haimo, Southern Illinois University (68T-136)

(36) Rings over which every right module is rationally complete
Professor R. C. Courter, Wayne State University (68T-50)

(37) Mobius inversion in lattice
Professor H. H. Crapo, University of Waterloo and Massachusetts Institute of Technology (68T-38)

(38) On chains of subalgebras. II. Preliminary report
Mr. Peter Csontos, University of Washington (68T-90)

(39) Glivenko-Cantelli type theorems and laws of large numbers for randomized sequences of random variables
Professor Miklos Csorgo, McGill University (68T-115)

(40) When isotopic submanifolds are ambient isotopic
Professor Jerome Dancis, University of Maryland (68T-51)

(41) Representation of l-groups and f-rings. Preliminary report
Professor John Dauns, Tulane University (68T-17)

(42) Any sewing of two crumpled cubes can be embedded in E^4
Professor R. J. Daverman and Professor W. T. Eaton, The University of Tennessee (68T-109)

(43) Uniqueness classes for periodic-type functionals
Professor R. F. DeMar, University

of Cincinnati (68T-59)

(44) Extensions of a theorem of Maxwell in the dynamical theory of gases
Professor J. L. Denny, University of Arizona (68T-114)

(45) Quasidynamical systems
Professor Vladimir Drobot, State University of New York, Buffalo (68T-93)

(46) The approximation property for some function algebras. Preliminary report
Mr. Larry Eifler, Purdue University (68T-36)

(47) Automorphisms of the semigroup of all polynomial functions over the reals
Professor A. G. Fadell and Professor K. D. Magill, Jr., State University of New York, Buffalo (68T-37)

(48) Pseudo- δ-mixing measure-preserving transformations on a probability space. Preliminary report
Mr. C. H. Farmer, Tracor, Incorporated, Austin, Texas (68T-77)

(49) Asymptotic estimates of Feynman integrals
Professor J. P. Fink, University of Pittsburgh (68T-157)

(50) What makes a positive measure the total variation measure of an analytic measure?
Professor Frank Forelli, Brown University (68T-20)

(51) Convex functions on convex polytopes
Professor David Gale, University of California, Berkeley, Professor Victor Klee and Professor R. T. Rockafellar, University of Washington (68T-75)

(52) Generalization of some partition function
Professor J. M. Gandhi, University of Manitoba (68T-161)

(53) On a function of Littlewood-Paley and Zygmund
Dr. George Gasper, Jr., University of Wisconsin (68T-113)

(54) A form of Plessner's theorem and identity theorem. Preliminary report
Mr. P. M. Gauthier, University of Montreal (68T-141)

(55) Fibre bundles and tensor products
Professor B. P. Gelbaum, University of California, Irvine (68T-57)

(56) Square roots of digraphs
Mr. D. P. Geller, University of Michigan (68T-79)

(57) Metric conditions that imply local invertibility. Preliminary report
Mr. Julian Gevirtz, New York University, Courant Institute of Mathematical Sciences (68T-32)

(58) A characterization of symbolic flows. Preliminary report
Mr. L. W. Goodwyn, University of Maryland (68T-9)

(59) Bivariate interpolation through curve networks
Dr. W. J. Gordon, General Motors Research Laboratories, Warren, Michigan (68T-68)

(60) The solution of mixed boundary value problems that involve time-dependent boundary regions for isotropic and homogeneous viscoelastic materials with one relaxation function
Dr. G. A. C. Graham, Simon Fraser University (68T-96)

(61) Functional analytic properties of extremely amenable semigroups
Professor E. E. Granirer, University of British Columbia (68T-54)

(62) Equationally compact semilattices
Professor G. A. Grätzer and Mr. H. Lakser, University of Manitoba (68T-26)

(63) Numerical solution of steady state Navier-Stokes problems for all Reynolds numbers
Professor Donald Greenspan, University of Wisconsin (68T-33)

(64) On 2-metabelian groups. Preliminary report
Dr. C. K. Gupta, University of Manitoba (68T-140)
(Introduced by Professor N. S. Mendelsohn)

(65) Commutation near-rings of a group. Preliminary report
Dr. N. D. Gupta, University of Manitoba (68T-126)

(66) On quasi-Frobenius local rings
Mr. R. N. Gupta, Ramjas College, Delhi, India, and Miss F. Saha, Delhi University, Delhi, India (68T-135)
(Introduced by Mr. K. N. Rao)

(67) A class of nonlinear forms on certain modules
Professor R. A. Guy, University of Montreal (68T-80)

(68) On a theorem of A. Brauer
Dr. C. A. Hall and Mr. T. A. Porsching, Bettis Atomic Power Laboratory, Westinghouse Electric Corpo-

ration, West Mifflin, Pennsylvania (68T-91)

(69) Tensor products of normed modules
Mr. R. E. Harte, University College of Swansea, Swansea, Wales (68T-28)

(70) Backward existence for retarded functional differential equations
Professor S. P. Hastings, Case-Western Reserve University and University of Dundee, Scotland (68T-15)

(71) Introduction to hyperboolean systems. III: Finite and transfinite fields; vector spaces and their duals
Professor John Hays, University of Maine (68T-42)

(72) Tensoring with a semigroup
Professor T. J. Head, University of Alaska (68T-1)

(73) On supports of regular Borel measures
Dr. D. J. Hebert and Dr. H. E. Lacey, University of Texas and NASA, Houston, Texas (68T-29)

(74) A generalization of Hugenholtz's theorem of types
Dr. R. H. Herman and Mr. G. G. Emch, University of Rochester (68T-97)

(75) Pointwise approximation by s.c. functions
Professor G. A. Heuer, Concordia College (68T-123)

(76) Can multiplication on $Sl(2)$ be extended?
Professor J. G. Horne, University of Georgia (68T-122)

(77) Strongly intrinsic extensions of left quotient semisimple rings
Mr. John Hutchinson, University of Kansas (68T-34)

(78) A Witt theorem for indefinite unimodular quadratic forms
Professor D. G. James, The Pennsylvania State University (68T-152)

(79) On self-maps which have a unique fixed point
Dr. Ludvik Janos, University of Florida (68T-40)

(80) Structure of left principal ideal rings
Mr. A. V. Jategaonkar, University of Rochester (68T-88)
(Introduced by Professor Newcomb Greenleaf)

(81) Nonconstructible patterns of degrees
Dr. R. B. Jensen, Stanford University (68T-6)
(Introduced by Professor Carol Karp)

(82) Continuous commuting functions
Professor G. G. Johnson, University of Georgia (68T-147)

(83) Covering semigroups. II
Mr. H. D. Kahn, Louisiana State University, New Orleans (68T-61)
(Introduced by Professor J. H. Abbott)

(84) Self-centralizing elements in finite p-groups. Preliminary report
Professor W. P. Kappe and Mr. M. W. Konvisser, Ohio State University (68T-141)

(85) Fractional integration and certain dual integral equations
Professor R. N. Kesarwani, University of Ottawa (68T-101)

(86) Ergodicity and mixing in topological transformation groups
Professor H. B. Keynes and Professor J. B. Robertson, University of California, Santa Barbara (68T-49)

(87) Folding a strip of stamps
Rev. J. E. Koehler, Seattle University (68T-160)

(88) On an additive decomposition of functions of several complex variables
Mr. Edgar Kraut, Mr. Stavros Busenberg and Mr. William Hall, Science Center of the North American Rockwell Corporation, Thousand Oaks, California (68T-43)

(89) Minimal exponential operators
Professor Erwin Kreyszig, University of Dusseldorf, Germany (68T-22)

(90) On a periodic solution of a Hamiltonian system
Mr. B. S. Lalli, University of Saskatchewan (68T-89)

(91) An engulfing theorem for topological manifolds
Mr. J. A. Lees, Rice University (68T-110)

(92) Some properties of dependence space
Mr. Pei Liu, Virginia Polytechnic Institute (68T-31)
(Introduced by Professor J. W. Layman)

(93) A note on inverse sequences of ANR's
Mr. Ivan Lončar and Professor Sibe Mardešić, University of Zagreb Yugoslavia (68T-12)

(94) Free groups of real two-by-two matrices
Professor R. C. Lyndon and Pro

fessor J. L. Ullman, University of Michigan (68T-104)

(95) On the closed graph theorem
Mr. A. G. R. McIntosh, Macquarie University, New South Wales, Australia (68T-11)

(96) Piecewise linear fibre bundles
Mr. J. N. McNamara, Columbia University (68T-145)

(97) Arcs in inverse limits on $[0,1]$ with only one bonding map
Professor W. S. Mahavier, Emory University (68T-70)

(98) Preservation theorems for infinitary logic
Dr. M. Makkai, Mathematical Institute of Hungarian Academy of Sciences, Budapest, Hungary (68T-24)
(Introduced by Professor Robert Vaught)

(99) A triple-theoretic construction of compact algebras
Professor E. G. Manes, Harvey Mudd College (68T-148)
(Introduced by Professor Courtney Coleman)

(100) Elementary divisors of associated transformations
Professor Marvin Marcus and Mr. Stephen Pierce, University of California, Santa Barbara (68T-95)

(101) A topological characterization of isometries
Mr. M. M. Marjanovic, University of Florida (68T-133)
(Introduced by Professor Ludvik Janos)

(102) A model for the inference mechanism of the mind and a weakness in present logics
Professor J. G. Maxwell, Kent State University (68T-108)

(103) Varieties associated with filtrations
Professor F. J. Mestecky, University of Maine (68T-118)

(104) An estimate on the number of periodic solution of an ordinary differential equation
Professor K. R. Meyer, University of Minnesota (68T-60)

(105) Bimeasurable functions. Preliminary report
Mr. J. G. Michaels, University of Rochester (68T-127)

(106) Banach sequence spaces
Professor V. J. Mizel, Carnegie-Mellon University, and Professor K. Sundaresan, University of Missouri (68T-143)

(107) A product formula for a tertiary characteristic class
Professor R. E. Mosher, California State College at Long Beach (68T-105)

(108) A generalization of a theorem of Alfsen and Fenstad
Mr. C. J. Mozzochi, University of Connecticut (68T-2)

(109) A divisor summation formula
Dr. C. Nasim, The University of Calgary (68T-45)
(Introduced by Professor B. N. Sahney)

(110) L^p estimates for certain nonlinear differential equations
Professor Umberto Neri, University of Maryland (68T-124)

(111) R-automorphisms of $R[[X]]$
Mr. M. J. O'Malley, Florida State University and NASA, Manned Spacecraft Center, Houston, Texas (68T-121)
(Introduced by Professor R. W. Gilmer, Jr.)

(112) On the four color problem
Professor Oystein Ore, Yale University, and Professor Joel Stemple, Queen's College, City University of New York (68T-25)

(113) Commuting recursive functions have a common fixed point. Preliminary report
Professor Jim Owings, University of Maryland (68T-128)

(114) An algebraic approach to axiom systems
Mr. Oswald Petrucco, Columbia, South Carolina (68T-21)
(Introduced by Professor Sidney Birnbaum)

(115) A comparison of two conditions against secondary bifurcation of eigenfunctions of Hammerstein operators
Dr. G. H. Pimbley, University of California and Los Alamos Scientific Laboratory, New Mexico (68T-56)

(116) Comparison of independence results in Mostowski's system (G) and in Zermelo-Fraenkel set theory
Mr. D. F. Pincus, Boston, Massachusetts (68T-142)

(117) Rings of continuous functions on open convex subsets of R^n

Professor L. E. Pursell, University of Missouri, Rolla (68T-48)

(118) Quasiconformal mappings which keep boundary points fixed
Professor Edgar Reich, University of Minnesota, and Professor Kurt Strebel, University of Zurich, Switzerland (68T-120)

(119) On multipliers of trigonometric series
Professor N. M. Riviere, University of Minnesota, and Mr. Y. Sagher, University of Illinois (68T-47)

(120) On quasi-complemented subspaces of Banach spaces. Preliminary report
Professor H. P. Rosenthal, University of California, Berkeley (68T-74)

(121) Extended operations and relations on the class of ordinal numbers. Preliminary report
Mr. A. L. Rubin and Mrs. Jean E. Rubin, Purdue University (68T-27)

(122) Approximation by rational functions of type (n, ν)
Mr. E. B. Saff, University of Maryland (68T-66)

(123) On the summability of a class of derived Fourier series
Dr. B. N. Sahney, The University of Calgary (68T-44)

(124) Henselization and rings of formal power series
Professor N. Sankaran, Queen's University (68T-55)

(125) Continuous functions which attain every level line at most finitely often
Professor S. A. Sawyer, Brown University (68T-76)

(126) An isomorphism theorem for a reduced Weyl group
Professor D. J. Schattschneider, University of Illinois, Chicago Circle (68T-119)

(127) The solution of equipment reliability and maintainability problems by queuing methods
Mr. Louis Schneider, North American Aviation, Incorporated, Anaheim, California (68T-112)

(128) A note on $\mathscr{M}$-paracompact spaces
Mr. W. B. Sconyers, Texas Christian University (68T-131)

(Introduced by Professor Hisahiro Tamano)

(129) A Hahn-Banach theorem for commutative semigroups with identity
Professor G. L. Seever, California Institute of Technology (68T-103)

(130) Entire functions satisfying a linear differential equation. II
Professor S. M. Shah, University of Kentucky (68T-81)

(131) On Abel-Hermite-Birkhoff interpolation
Mr. Ambikeshwar Sharma and Mr. J. Prasad, University of Alberta (68T-16)

(132) A note on classes of groups
Mr. T. S. Shores, University of Kansas (68T-39)

(133) Commuting functions and common fixed points
Mr. K. L. Singh, Memorial University of Newfoundland (68T-158)

(Introduced by Professor W. J. Blundon)

(134) Metrizability of M-spaces
Mr. Francis Siwiec, University of Pittsburgh (68T-111)

(135) Modularity implication in sets with projective laws
Miss Helen Skala, Illinois Institute of Technology (68T-65)

(136) A cellularity criterion in certain 3-manifolds
Mr. Stephen Slack, Wisconsin State University, Oshkosh (68T-86)

(137) Some radicals for alternative rings
Mr. M. B. Slater, University of Hawaii (68T-23)

(138) On the rank of incidence matrices in finite geometries. Preliminary report
Mr. K. J. C. Smith, University of North Carolina (68T-18)

(Introduced by Professor R. C. Bose)

(139) Denjoy-Khintchine integration in abstract spaces. I. Preliminary report
Professor D. W. Solomon, University of Wisconsin, Milwaukee (68T-106)

(140) Finite summation formulae associated with a class of generalized hypergeometric polynomials
Professor H. M. Srivastava, West Virginia University (68T-132)

(Introduced by Professor I. D. Peters)

(141) Characterization of contractible

open manifolds, and an equivalence between pointlike maps and cellular maps

Mr. J. L. Stern, University of Idaho (68T-86)

(Introduced by Dr. R. Osborne)

(142) Higher order conservation laws

Mr. A. P. Stone, University of Illinois, Chicago Circle (68T-63)

(143) The nature of a crucial inadequacy in the theory of relativity

Professor A. C. Sugar, Bradley University (68T-58)

(149) Some properties of a semidiscrete approximation to the beam equation

Professor R. A. Sweet, Cornell University (68T-35)

(Introduced by Professor R. J. Walker)

(145) On extended topologies

Professor C. D. Tabor, Louisiana State University, Shreveport (68T-134)

(146) Linearly cushioned refinements

Professor Hisahiro Tamano, Texas Christian University (68T-125)

(147) Torsionfree injective modules

Mr. M. L. Tepley, University of Nebraska (68T-3)

(148) Diagonal nuclear operators. Preliminary report

Mr. A. E. Tong, Tulane University (68T-94)

(149) Boundary value problems for linear differential systems

Professor D. H. Tucker, The University of Utah (68T-83)

(150) Odd perfect numbers: A search procedure, and a new lower bound of 10^{36}

Dr. Bryant Tuckerman, IBM, Thomas J. Watson Research Center, Yorktown Heights, New York (68T-116)

(151) Homogeneous identities on algebraic loops. Preliminary report

Mr. Carlos E. Vasco, St. Louis University (68T-156)

(Introduced by Professor J. F. Daly)

(152) Extensions of D. Jackson's theorem on best complex polynomial mean approximation

Professor J. L. Walsh and Mr. E. B. Saff, University of Maryland (68T-69)

(153) An estimation of the maximum modulus. Preliminary report

Mr. Joseph Warren, Wayne State University (68T-144)

(154) A topological problem in connectivity of graphs. II

Professor M. E. Watkins, University of Waterloo and University of North Carolina (68T-130)

(155) On sets measurable and nonmeasurable by a regular measure. Preliminary report

Mr. W. J. Wilbur, University of California, Davis (68T-99)

(156) The analytic continuation of the resolvent kernel in scattering theory

Professor C. H. Wilcox, University of Arizona (68T-117)

(157) A theorem on homotopically equivalent (2k+1)-manifolds

Professor Y.-F. Wong, DePaul University (68T-98)

(158) Structure of hereditarily infinite dimensional spaces

Professor J. M. Yohe, Mathematics Research Center, U. S. Army, University of Wisconsin (68T-8)

(159) Uniqueness of the Cauchy problem

Professor E. C. Zachmanoglou, Purdue University (68T-87)

(160) Logarithmic variation criteria for L_2-stability of solutions of a class of integral equations

Mr. Georges Zames and Mr. M. I. Freedman, NASA, Electronics Research Center, Cambridge, Massachusetts (68T-64).

ABSTRACTS OF CONTRIBUTED PAPERS

The November Meeting in Knoxville, Tennessee

November 17-18, 1967

650-58. JEAN POLLARD, Louisiana State University, Baton Rouge, Louisiana 70803. *On extending homeomorphisms on zero-dimensional spaces.*

A nondegenerate zero-dimensional separable metric space is said to be a 0-space. Consider the following types of 0-spaces: (A) perfect and compact; (B) perfect and locally compact, but not compact; (C) topologically complete (absolute G_δ) and nowhere locally compact; (D) perfect and countable. *Theorem.* If X and Y are 0-spaces, both of the same type A through D, and if H and K are closed proper subsets of X and Y respectively and if f is a homeomorphism of H onto K, then f has a homeomorphic extension F of X onto Y if and only if (1) f carries $\overline{X\setminus H}\cap H$ onto $\overline{Y\setminus K}\cap K$ and (2) $\overline{X\setminus H}$ and $\overline{Y\setminus K}$ are homeomorphic. Moreover, in case each of X and Y is of type A, C, or D, the second condition is unnecessary. This theorem is proved on the basis of a single sequence of lemmas and proofs applicable to all cases. For the special case where X and Y are compact and perfect 0-spaces, the theorem follows from Corollary 2 of B. Knaster and M. Reichbach in *Notion d'homogenéité et prolongements des homéomorphies,* Fund. Math. 40 (1953), 180-193. By specifying that H and K are null in the above theorem, we obtain, as a corollary, the known result characterizing 0-spaces of the above types. (Received October 4, 1967.)

The January Meeting in San Francisco
January 23-26, 1968

653-1. P. M. EAKIN, Jr., Louisiana State University, Baton Rouge, Louisiana. The converse to a well-known theorem on noetherian rings.

By ring we mean a commutative ring with identity. A ring S, with subring R, is said to be a finite integral overring of R if S is a finitely generated, unitary module over R. Also, a ring R is called an RMX ring if R/P is noetherian for every proper prime ideal P of R. Lemma. Let R be an RMX ring, S a noetherian, finite integral overring of R. Then if $P_1, P_2, \ldots, P_n$ are proper primes of R, $R/(P_1 P_2 \ldots P_n)^{ec}$ is noetherian. Lemma. Suppose D is an RMX domain with quotient field K and J is a finite integral overring of D. Then J is noetherian if and only if $D^* = J \cap K$ is noetherian. Moreover, if D^* is a finite D module, J is noetherian if and only if D is noetherian. Theorem. If S is a finite integral overring of R, then S is noetherian if and only if R is noetherian. Examples are given to show that the class of RMX rings is quite extensive and that none of the hypotheses of the theorem can be omitted. An example of a non-noetherian domain which is the intersection of two one dimensional, local subrings of its quotient field is given. (Received March 13, 1967.)

653-2. J. C. SMITH, Jr., Virginia Polytechnic Institute, Blacksburg, Virginia. Characterizations of metric-dependent dimension functions.

Let (X, ρ) be a metric space, and let $d_0(X, \rho)$ be the metric dimension of X. Let d_2 and d_3 denote the dimension functions for metric spaces introduced by Nagami and Roberts in their paper Study of metric-dependent dimension functions, Trans. Amer. Math. Soc. (to appear). V. I. Egorov and J. B. Wilkinson have characterized d_0 and d_3 respectively in terms of Lebesgue covers. Their results are as follows: (i) $d_0(X, \rho) \leqq n$ if and only if every Lebesgue cover of X has an open refinement of order $\leqq n + 1$. (ii) $d_3(X, \rho) < n$ if and only if every finite Lebesgue cover has an open refinement of order $\leqq n + 1$. In this paper the author introduces two new metric-dependent dimension functions d_6 and d_7, which are generalizations of the dimension function d_3. The main results of this paper are the following characterizations of d_2, d_6, and d_7 in terms of Lebesgue covers: (iii) $d_2(X, \rho) \leqq n$ if and only if every Lebesgue cover $\mathfrak{G} = \{G_1, G_2, \ldots G_{n+2}\}$ consisting of n + 2 members has an open refinement of order $\leqq n + 1$. (iv) $d_6(X, \rho) \leqq n$ if and only if every countable Lebesgue cover of X has an open refinement of order $\leqq n + 1$. (v) $d_7(X, \rho) \leqq n$ if and only if every locally finite Lebesgue cover has an open refinement of order $\leqq n + 1$. (Received May 15, 1967.)

653-3. H. M. FARKAS, The Johns Hopkins University, Baltimore, Maryland 21218. Weierstrass points and analytic submanifolds of Teichmueller space.

A theorem originally due to Rauch states that the locus of points in the Teichmueller space of genus g whose underlying surfaces possess a Weierstrass point whose Weierstrass sequence begins with $r < g$ is a complex analytic submanifold of the Teichmueller space of complex dimension $r + 2g - 3$ when $r + 1$ is a gap, and an $r + 2g - 4$ dimensional complex analytic variety when $r + 1$ is not a gap. It is shown here that the exceptional case of $r + 1$ not being a gap does not occur for $r = 3$ and $g \geqq 4$. (Received May 22, 1967.)

653-4. G. J. ETGEN, Headquarters, National Aeronautics and Space Administration, Washington, D. C. 20546. <u>On the matrix Sturm-Liouville equation.</u>

The second order matrix differential equation (1): $[P(x)Y']' + [\lambda F(x) + G(x)]Y = 0$ on X: $a \leqq x \leqq b$, L: $-\infty < \lambda < \infty$, where each of P, F and G is an $n \times n$ symmetric matrix of continuous real-valued functions on X with each of P and F positive definite, is called the matrix Sturm-Liouville equation. A solution $Y(x,\lambda)$ of (1) is conjoined and nontrivial provided $Y^*PY' - (PY')^*Y \equiv 0$ (* denotes transpose) on XL and $\det Y(x,\lambda)$ has at most a finite number of zeros on X for each λ on L. The values of λ for which there exists a nontrivial, conjoined solution $Y(x,\lambda)$ of (1) satisfying the two point boundary conditions (2): $AY(a,\lambda) - BP(a)Y'(a,\lambda) \equiv 0$ on L, (3): $\det [\Gamma Y(b,\lambda) - \Delta P(b)Y'(b,\lambda)] = 0$, where A, B, Γ and Δ are $n \times n$ constant matrices, A and B are symmetric, B is positive definite, $A^2 + B^2 = I$ (the $n \times n$ identity matrix), $AB = BA$, $\Gamma^*\Delta = \Delta^*\Gamma$, and $\Gamma^*\Gamma + \Delta^*\Delta = I$, are called the eigenvalues of the system (1), (2), (3). <u>Theorem.</u> There are an infinite number of eigenvalues $\lambda_0, \lambda_1, \lambda_2, \ldots$ of the system (1), (2), (3) forming an increasing sequence with $\lambda_m \longrightarrow \infty$ as $m \longrightarrow \infty$. Moreover, there is a nondecreasing sequence of nonnegative integers $h_0, h_1, h_2, \ldots$, with $h_m \longrightarrow \infty$ as $m \longrightarrow \infty$, such that $\det Y(x,\lambda_m)$ has exactly h_m zeros on $a < x < b$. (Received June 15, 1967.)

653-5. J. H. JORDAN, Washington State University, Pullman, Washington, and S. J. KELLEY, Western Washington State College, Bellingham, Washington. <u>Distribution of quartic and quintic nonresidues.</u>

Techniques previously established by Jordan (see J. H. Jordan, The <u>distribution of cubic and quintic nonresidues</u>, Pacific J. Math. 16 (1966), 77-85) are refined to improve the bounds on the size of the smallest positive integer in each class of quartic (quintic) nonresidues modulo p for sufficiently large primes $p \equiv 1 \pmod 4$ ($p \equiv 1 \pmod 5$)). We establish that each class of quartic (quintic) nonresidues contains a positive integer smaller than $\exp[(\alpha + \epsilon) \ln p]$ ($\exp[(\beta + \epsilon) \ln p]$). $0.2075 < \alpha < 0.210$ and $0.218 < \beta < 0.2195$. (Received July 3, 1967.)

653-6. P. W. HARLEY, University of Georgia, Athens, Georgia. <u>On suspending homotopy spheres.</u>

A <u>homotopy</u> n-<u>sphere</u> is a compact topological n-manifold having the homotopy type of S^n and a <u>fake</u> n-<u>cell</u> a compact, contractible, topological n-manifold whose boundary is homeomorphic to S^{n-1}. The object of this paper is to establish the following two propositions as regards such classes of manifolds. <u>Theorem</u> 1. <u>The suspension of a homotopy</u> 4-<u>sphere is homeomorphic to</u> S^5. <u>Theorem</u> 2. <u>The suspension of a fake</u> 4-<u>cell is homeomorphic to</u> I^5. (Received July 10, 1967.)

653-7. R. F. DICKMAN, Jr., University of Miami, P. O. Box 9085, Coral Gables, Florida 33124. Some characterizations of the Freudenthal compactification of a semicompact space.

Let X denote a semicompact Hausdorff space (i.e. every point of X has arbitrarily small neighborhoods with compact boundaries) and let γX denote the Freudenthal compactification of X as given by K. Morita in On bicompactifications of semibicompact spaces, Sci. Rep. Tokyo, Bunrika Daigaku Section A, 4 (1952), 222-229. Let $\mathscr{A}$ denote the set of all continuous maps of X into $I = [0, 1]$. For each $f \in \mathscr{A}$, let $B(f) = \{t \in I$: Fr $f^{-1}(t)$ contains a compact set that separates X into two disjoint open sets M and N where $f(M) \subset [0, t]$ and $f(N) \subset [t, 1]\}$. Finally let $F = \{f \in \mathscr{A}: B(f)$ is dense in $I\}$. Lemma 1. Every $f \in F$ has a unique continuous extension f' to γX. Lemma 2. For every pair of disjoint closed subsets A and B of γX there exists an $f \in F$ such that $f'(A) = 0$ and $f'(B) = 1$. Theorem 1. The Freudenthal compactification γX of X is the topologically unique compactification of X satisfying (a) every $f \in F$ has a continuous extension f' to γX; and (b) for every pair of distinct points x and y of $cl(\gamma X - X)$ there exists an $f \in F$ such that $f'(x) \neq f'(y)$. Let e denote the embedding of X into I^F defined by $e(x) = (f(x))$, $f \in F$, and let ψX denote the closure of e(X) in I^F. Theorem 2. The compactification ψX of X satisfies (a) and (b) of Theorem 1 and thus ψX is topologically equivalent to γX. Therefore the Freudenthal compactification of a semicompact space can be obtained by a Tychonnoff type embedding. (Received July 17, 1967.)

653-8. C. S. REES, University of Tennessee, Knoxville, Tennessee, and S. M. SHAH, University of Kentucky, Lexington, Kentucky. On the absolute Norlund summability of a Fourier series.

This paper generalizes a theorem of Fu Chen Hsiang (J. Austr. Math. Soc. 7(1967), 252-256). It is shown that the Hsiang test may fail to apply to some series for which the test given in the theorem below applies. Theorem. Let ϕ be an even, integrable function of period 2π with Fourier series $a_0/2 + \sum_1^\infty a_n \cos n x$. Suppose $\{p_n\}$ is a sequence of positive numbers such that $\{p_n - p_{n-1}\}$ is monotonic and bounded with $\sum_0^\infty 1/P_n < \infty$, where $P_n = \sum_0^n p_k$, and let $\{\theta_n\}$ be a sequence of real numbers with $\theta_n \uparrow \infty$ and $\sum_0^\infty \theta_n^2/P_n < \infty$. If f is a function which increases to infinity and satisfies $\sum_0^\infty n/P_n\ f(\theta_n) < \infty$ and $f(1/t)\ |\phi(t)| = O(1)$ as $t \to 0+$, then the series $a_0/2 + \sum_1^\infty a_n$ is summable $|N,P_n|$. Corollary. Let $\phi(t)$ satisfy the conditions of the theorem and suppose that $|\phi(t)| \downarrow 0$ as $t \downarrow 0$, $\phi(t) \neq 0$ for $0 < t < \delta$, and $\sum_k^\infty (|\phi(n^{-r})|/n) < \infty$, where $0 < r < 1/2$. Then the series $a_0/2 + \sum_1^\infty a_n$ is summable $|C,2|$. (Received July 20, 1967.)

653-9. 653-10. WITHDRAWN.

653-11. R. E. HODEL, Duke University, Durham, North Carolina. Sum theorems for topological spaces.

Let Q denote a class of topological spaces, this paper is a study of sum theorems for various Q. Specifically, suppose that X is a topological space and $\{F\alpha\}$ is a cover of X such that each $F\alpha$ is in Q. When can one assert that X is in Q? Perhaps the most interesting theorem of this type is the so called locally finite sum theorem, hereinafter denoted by (α). If X is a topological space and $\{F\alpha\}$ is a locally finite closed cover of X such that each $F\alpha$ is in Q, then X is in Q. It is well known that (α) holds when Q is the class of paracompact, stratifiable, or metrizable spaces. The author shows that (α) also holds for normal, collectionwise normal, and pointwise paracompact spaces. Several other sum theorems are proven, each of which holds for any class Q of topological spaces which satisfies (α) and is hereditary with respect to closed subsets. Example. Let X be a regular space and let $\mathscr{V}$ be a σ-locally finite open cover of X, each element of which is in Q and has compact boundary. Then X is in Q. (Received August 2, 1967.)

653-12. L. D. LOVELAND, Utah State University, Logan, Utah 84321. Piercing locally spherical spheres with tame arcs.

We define a 2-sphere S in S^3 to be locally spherical if for each point $p \in S$ and for each $\epsilon > 0$ there is a 2-sphere S' and a component K of S^3 - S' such that $p \in K$, diam $(S' \cup K) < \epsilon$, and $S' \cap S$ is a continuum. It is unknown whether a locally spherical 2-sphere is tamely embedded in S^3, although additional conditions have been imposed on $S' \cap S$ to insure that S is tame. For example, Burgess [Trans. Amer. Math. Soc. 114 (1965), 80-97] imposed the condition that $S \cap S'$ be a simple closed curve, and Loveland [Trans. Amer. Math. Soc. 123 (1966), 355-368] insisted that $S \cap S'$ satisfy a certain property $(*, S' \cap S, S)$. Theorem 1. If S is locally spherical at a point p of S, then S can be pierced by a tame arc at p. Theorem 2. If each component of S^3 - S is an open 3-cell and S is locally spherical, then S is tamely embedded in S^3. Theorem 3. If S is locally spanned in each component of S^3 - S, then S can be pierced by a tame arc at each of its points. See the above references for the definition of locally spanned in S^3 - S. (Received August 4, 1967.)

653-13. J. W. BOND, Pennsylvania State University, University Park, Pennsylvania. Derivation algebras of finite abelian group algebras.

Let G be a finite abelian group, F a field of char p, FG denote the group algebra of G over F, and $\mathscr{D}$(FG) the Lie algebra of derivations of FG into itself, where D is a derivation means a linear transformation satisfying D(ab) = D(a)b + aD(b) for all a,b in FG. Let G = PQ, P a p-group, Q a group with order relatively prime to p. Then $FQ = \sum_{i=1}^{k} F_i$, each F_i isomorphic to a field containing F. This decomposition leads directly to Theorem 1. $\mathscr{D}(FG) \cong \sum_{i=1}^{k} \mathscr{D}(F_iP)$ (as Lie algebras). Now let G^p be the group of p powers of elements of G. Then we have Theorem 2. $\mathscr{D}$(FG) has an ideal I such that $\mathscr{D}(FG)/I \cong \mathscr{D}(FG/G^p)$. Furthermore, if G is a finite abelian p-group I is a nilpotent ideal with class at most equal to the class of the augmentation ideal of FG^p. Jacobson has completely determined the structure of $\mathscr{D}(FG/G^p)$ in his earlier work on restricted p-algebras, so that Theorems 1 and 2 determine the structure of $\mathscr{D}$(FG). Theorem 1 follows directly from Jacobson's 1937 paper. Theorem 2 is proved by verifying ϕ: $\mathscr{D}(FG) \rightarrow \mathscr{D}(FG/G^p)$ is a Lie algebra homomorphism, where $\phi(D)(gG^p) = \sum \alpha(a_g^h)hG^p$ (sum over coset representatives of G/G^p), α the augmentation map of FG^p into F, a_g^h in FG^p determined by $D(g) = \sum a_g^h h$. (Received August 7, 1967.)

653 14. WITHDRAWN.

653-15. D. G. KELLY, Jet Propulsion Laboratory, 238-420, 4800 Oak Grove Drive, Pasadena, California 91103, and SEYMOUR SHERMAN, Indiana University, Swain Hall-East, Bloomington, Indiana 47401. General Griffiths inequalities on correlations in Ising ferromagnets.

The "generalized ferromagnetic" Hamiltonian $\mathscr{H}$ of a system $\{\sigma_1, \ldots, \sigma_n\}$ of Ising $(\sigma_i = \pm 1)$ is defined as follows: for each subset A of N = $\{1, \ldots, n\}$, a real number J_A is given, and $\mathscr{A}$ = $-\sum_{A \subset N} J_A \sigma^A$, where σ^A denotes $\Pi_{i \in A} \sigma_i$. The partition function is $Z = \sum_{\sigma_1 = \pm 1} \cdots \sum_{\sigma_n = \pm 1} \exp[-(kT)^{-1} \mathscr{A}]$, where k is Boltzmann's constant and T the absolute temperature. For any operator $O(\sigma_1, \ldots, \sigma_n)$, the thermal average is $\langle O \rangle = Z^{-1} \sum_{\sigma_1 = \pm 1} \cdots \sum_{\sigma_n = \pm 1} O(\sigma_1, \ldots, \sigma_n) \cdot \exp[-(kT)^{-1} \mathscr{A}]$. Theorem. If $J_A \geqq 0$ for all nonempty $A \subset N$, then (1) $\langle \sigma^R \rangle \geqq 0$ for all $R \subset N$ and (2) $kT \partial \langle \sigma^R \rangle / \partial J_S = \langle \sigma^R \sigma^S \rangle - \langle \sigma^R \rangle \langle \sigma^S \rangle \geqq 0$ for all R, $S \subset N$. These include the inequalities of Griffiths (J. Math. Phys. 8(1967), 478-489). The converse problem of finding sufficient conditions for "ferromagnetism", i.e. for $J_A \geqq 0$ (A nonempty), in terms of the thermal averages $\langle \sigma^R \rangle$, is examined; the problem is found to be expressible in terms of the real group algebra over the group $(2^N, \Delta)$ of subsets of N under symmetric difference. Nevertheless, the problem of finding more perspicuous sufficient conditions in terms of the $\langle \sigma^R \rangle$ remains open. (Received August 11, 1967.)

653-16. LE BARON O. FERGUSON, University of California, Riverside, California 92502. Uniform approximation by polynomials with integral coefficients. I.

Let A be a discrete subring of the complex number plane $\underset{\sim}{C}$ which is not contained in the ring of rational integers $\underset{\sim}{Z}$. The paper is concerned with characterizing those functions defined on a compact subset X of $\underset{\sim}{C}$ which can be uniformly approximated by polynomials with coefficients in A. We call this the complex case to distinguish it from the real case which we also consider here. The real case is that where X is a compact subset of $\underset{\sim}{R}$ and the coefficients of the approximating polynomials are taken from the ring of rational integers $\underset{\sim}{Z}$ or any discrete subring of $\underset{\sim}{R}$. The real case is completely solved in the sense that a necessary and sufficient condition in order that a function can be so approximated is found. The complex case is solved, if in addition to being compact. X either has transfinite diameter at least unity or a void interior and connected complement. (Received August 15, 1967.)

653-17. MARCIA PETERSON and WILLIAM SWARTZ, Montana State University, Bozeman, Montana 59715. Periodic solution of a third order equation.

Consider $x''' + \alpha x'' + \beta x' + g(x) = E(t)$ where E(t) is real, periodic of period 2π, and belongs to $L_2(0, 2\pi)$. Assume g'(x) is continuous and $0 < a \leqq g'(x) \leqq b$. For all β and all $\alpha \notin (0, 1]$ a constructive iterative proof is given of existence of a periodic solution of period 2π. For almost all α in (0, 1] existence of a periodic solution is proved providing α is properly related to the ratio a/b. (Received August 15, 1967.)

653-18. J. D. FULTON, Clemson University, Clemson, South Carolina 29631, and W. L. MORRIS, Oak Ridge National Laboratory, Oak Ridge, Tennessee. The Pisano period, Fibonacci frequency and Leonard logarithm of the positive integers.

Let M be the set of integers greater than one. For each $m \in M$, let $\pi(m)$ denote the period of the Fibonacci numbers modulo m. Then π (the Pisano period) maps M into itself. The existence of π was known to Lagrange. Fixed point theorem. $\pi(m) = m$ iff $m = (24)5^{\lambda - 1}$ for some positive integer λ. Iteration theorem. For each $m \in M$ $\exists$ a smallest positive integer ϕ such that $\pi^{\phi}(m) = \pi^{\phi+1}(m)$. Thus for each $m \in M$ there is a unique (smallest) ϕ and λ such that $\pi^{\phi}(m) = (24)5^{\lambda - 1}$. This defines two new functions on M, namely, $\phi(m)$ called the Fibonacci frequency of m and $\lambda(m)$ the Leonardo logarithm of m. (Received August 16, 1967.)

653-19. C. T. WHYBURN, Louisiana State University, Baton Rouge, Lousiana 70803. Primitive roots in certain intervals.

Using methods exposed in Abstract 644-38, these *Notices* 14(1967), 369, certain elementary estimates due to H. J. Kanold and A. Brauer for the least primitive root modulo certain primes are sharpened. In particular, if $1 > h > \pi/12^{1/2}$, $k = (6h^2/\pi^2 - 1/2)$, α is an integer, and if q is a prime $> \{k + h/(h^2 - 2k)^{1/2}\}\, 2^{\alpha-3}(k - h)^{-2}$; then for a prime $p = 2^{\alpha}q + 1$, there is a primitive root (mod p) in $(1, h(p - 1)^{1/2})$. (Received August 16, 1967.)

653-20. J. B. BUTLER, Jr., Portland State College, P. O. Box 751, Portland, Oregon 97207. On the inverse problem for ordinary differential operators of even order.

Let $L^0 = (d/dx)^n$, $L^1 = (d/dx)^n + q(x)$, $n = 2v$, be formal differential operators on $(-\infty, \infty)$ where q(x) is a continuous function, $q(x) = 0$, $-\infty < x \leqq 0$, $q(x) \in \mathscr{L}_2(-\infty, \infty)$. Assume that L^i determine self-adjoint operators H^i on $\mathscr{L}_2(-\infty, \infty)$, having continuous spectrums on $[0, \infty)$, with associated spectral measures $E^i(\Delta)$ of the form $E^i(\Delta)u = \int_{\Delta} s_j{}^i(x,l) \int_{-\infty}^{\infty} s_k{}^i(y,l)u(y)dyd\, \rho_{jh}{}^i(l)$, $\Delta \in [0, \infty)$, $u \in \mathscr{L}_2(-\infty, \infty)$, $i = 0, 1$. B will denote the set of (x,y) such that $0 < x < \infty$, $-x < y < x$ with boundary ∂B. For (x, y) in $B \cup \partial B$ define $F(x, y) = \int_0^{\infty}(\int_0^x s_k{}^0 dt)(\int_0^y s_k{}^0 dt)d\sigma_{jk}(l)$, $\sigma_{jk}(l) = \rho_{jk}{}^1(l) - \rho_{jk}{}^0(l)$, $0 < l < \infty$, j, k = 1,..., m, and $(\partial/\partial x)(\partial/\partial y)F = \Omega_c$. Let $C^p(B)$ denote the set of functions with continuous pth derivatives on B and $C^p(B \cup \partial B)$ the set of functions in $C^p(B)$ whose pth derivatives approach continuous limits along ∂B. Theorem. If F(x, y) is given such that (i) $F \in C^n(B)$, (ii) $F \in C^{n-1}(B \cup \partial B)$, (iii) $(\partial/\partial x)^{j+1}(\partial/\partial y)F|_{y=-x} = \alpha\delta(j, n-1)$, $j = 0, 1,\dots, n-1$, $n \geqq 4$, then the integral equation $k + \Omega_c + \int_{-x}^{x} k(x, y)\Omega_c(z,y)dz = 0$ has a unique solution k(x, y) in $B \cup \partial B$, $k \in C^n(B)$, $k \in C^{n-1}(B \cup \partial B)$ and $\rho_{jk}{}^1(l)$ is the spectral density matrix corresponding to H^1 determined by $q(x) = -(1/n)(d/dx) \cdot((\partial/\partial x)^{n-2}\, k|_{y=x})$, $x \geqq 0$. The proof follows the methods of I. M. Gel'fand, B. M. Levitan and L. A. Sahnovič. (Received August 18, 1967.)

653-21. D. A. LAWVER, University of Arizona, Tucson, Arizona 85721. Left ideal axioms for nonassociative rings.

In nonassociative ring theory the lattice of left ideals has not proven nearly as useful a tool as in the associative theory. A partial reason for this is that given an element x in a nonassociative ring R, Rx need not be a left ideal of R. We discuss a generalization of the Wedderburn structure theory for nonassociative rings under axioms which force Rx to be a left ideal for every x in the ring R. We also extend the concept of "module" to match an axiom under consideration and give structure theory for such "modules". (Received August 21, 1967.)

653-22. C. A. HALL and T. A. PORSCHING, Bettis Atomic Power Laboratory, Westinghouse Electric Corporation, P. O. Box 79, West Mifflin, Pennsylvania 15122. A separation theorem for nonsymmetric matrices.

A well-known separation theorem for symmetric matrices is the following: The eigenvalues of each principal (n - 1) by (n - 1) minor separate the eigenvalues of a given n by n symmetric matrix. This paper establishes the following related theorem for a class of matrices which are not necessarily symmetric: If A is a nonnegative, irreducible matrix with eignevalues $\lambda_1 > \lambda_2 > |\lambda_j|$, $(j = 3, 4, \ldots, n)$, then each principal minor has a real eigenvalue which separates λ_1 and λ_2. Examples are given which prevent further obvious generalizations of this theorem. (Received August 23, 1967.)

653-23. DONALD GREENSPAN, University of Wisconsin, Madison, Wisconsin. Numerical solution of initial-boundary value problems for mildly nonlinear parabolic and hyperbolic equations.

Given the points (x, y), (x + h, y), (x - h, y), (x, y + h), (x, y - h), (x, y - 2h), numbered, respectively, 0, 1, 2, 3, 4, 5, the heat operator is approximated at 0 by $(-2/h^2)u_0 + (1/h^2)u_1 - (1/2h)u_2 + (1/h^2)u_3 + (1/2h)u_4$, and the wave operator is approximated at 0 by $(-4/h^2)u_0 + (1/h^2)u_1 + (1/h^2)u_2 + (1/3h^2)u_3 + (3/h^2)u_4 - (4/3h^2)u_5$. Asymptotic estimates or periodicity results are used to convert initial-boundary problems to approximating boundary value problems on finite domains. Heat equation problems are then solved by the usual finite difference method available for elliptic problems. Wave equation problems are treated similarly after approximating the additional normal derivative condition on the first row. The problems treated can all have mildly nonlinear differential equations. The numerical method avoids completely the necessity of discussing the stability of the difference analogue. (Received August 23, 1967.)

653-24. WARREN WHITE, University of Wisconsin, Madison, Wisconsin 53706. A 2-sphere in E^3 is tame if it is 1-LC through each complementary domain.

Let S be a 2-sphere in E^3, V a component of E^3 - S. Theorem. S is tame from V if, for any $\epsilon > 0$, there is a $\delta > 0$ such that every loop in S of diameter less than δ can be shrunk to a point in an ϵ- subset of V. Theorem. S is tame from V if every Sierpiński curve in S can be deformed into V. (Received August 23, 1967.)

653-25. P. A. O'MEARA, Bowling Green State University, Bowling Green, Ohio 43402. A metrization theorem.

Definition 1. A family $\mathscr{P}$ of subsets of a topological space X is called a pseudobase for X if, whenever $C \subset U$ with C compact and U open in X, then $C \subset \bigcup_{i=1}^{n} P_i \subset U$ for some $\{P_1, P_2, \ldots, P_n\} \subset \mathscr{P}$. Definition 2. A Hausdorff space X is called an r-space if for each $x \in X$, there is a sequence of neighborhoods $\{U_n(x)\}_{n=1}^{\infty}$ such that if $x_n \in U_n(x)$, then $\mathrm{Cl}(\{x_1, x_2, x_3, \ldots\})$ is compact. Theorem. A topological space X is metrizable iff it is a T_3-space and an r-space having a δ-locally finite pseudobase. (Received August 23, 1967.)

653-26. H. A. HOLLISTER, Bowling Green State University, Bowling Green, Ohio 43402. Strongly generalized periodic elements in a group.

Let $S(x)$ denote the normal subsemigroup of the group G generated by x. x is said to be a generalized periodic (GP) element iff $e \in S(x)$. x is strongly GP iff every element of $S(x)$ is GP. The strongly GP elements of G form a normal subgroup H of G and G/H has no nontrivial strongly GP elements. If P is the cone of a maximal partial order for G, then an element $x \neq e$ is psuedo-positive with respect to P iff x is strongly GP. There is a one-to-one correspondence between maximal cones in G and those in G/H and several questions concerning maximal cones have the same answers in both groups. (Received August 17, 1967.)

653-27. J. M. DAY, 944 Round Hill Road, Redwood City, California 94061. Compact semigroups with square roots.

Let S be a compact semigroup with exactly two idempotents, a zero and an identity, denoted respectively by $0'$ and $1'$, with $S \neq 0' \cup 1'$. Let every $X \in S$ have at least one square root, let H denote the maximal group in S containing $1'$, and suppose that H is connected and contained in the center of S ($hx = xh$ for each $x \in S$ and $h \in H$). Let $[0,1]$ denote the usual closed real unit interval; a local one parameter semigroup in S is a homeomorphism $p: [0, 1] \longrightarrow S$ such that $p(x + y) = p(x)p(y)$ if $x, y, x + y \in [0, 1]$. Since H is a continuum abelian group with square roots, it is known that the set Q of points in H lying on one parameter groups in H is dense and if also H is Lie then $Q = H$. We prove analogously that the set P of points in S which lie on local one parameter semigroups in S is dense and if H is also Lie then $P = S$. As an application, we show that if S is also commutative, cancellative and finite-dimensional, then some neighborhood of H factors algebraically and topologically into the product of H and a local semigroup. (Received August 28, 1967.)

653-28. R. B. KILLGROVE, California State College, Los Angeles, California 90032. The k-configuration. Preliminary report.

Paige-Wexler's digraph complete set of Latin squares represents an affine plane thus: each square represents a parallel class and an element $a(i, j)$ of the square A represents the point $(j, a(i, j))$ on line, y-intercept i, slope m. If square A is not orthogonal to square B for slope n, then there exists $\alpha = a(i, j) = a(k, 1)$ of A and $\beta = b(i, j) = b(k, 1)$ of B. From this arises the k-configuration (resembling letter k) with lines $x = 0$, $x = j$, $x = 1$, $y = \alpha$, $y = \beta$, $y = f(x, m, i)$, $y = f(x, m, k)$, $y = f(x, n, i)$, $y = f(x, n, k)$, and points $(0, i)$, $(0, k)$, (j, α), (j, β), $(1, \alpha)$, $(1, \beta)$. In a projective plane let L have points p, q on it. Consider lines A, B, C, D such that $A = pab$, $B = pa'b'$, $C = qaa'$, $D = qbb'$. Pick c not on any of these 5 lines. Then one may determine c' such that triangle abc and triangle $a'b'c'$ are perspective from L. This configuration is confined iff it is the k-configuration or Theorem L. The Moulton plane has the usual lines $x = k$ and $y = mx + b$ for $m \leqq 0$, and some 'bent' lines. Let one such line be $y = x$ for $x \geqq 0$ and $y = Kx$ for $x \leqq 0$ where $0 < K < 1$. Then lines $x = 0$, $x = 1$, $x = 1/K$, $y = (K/K - 1)x + 1/1 - K$, $y = (K/K - 1)x + K/1 - K$, $y = x$ for $x \geqq 0$, $y = Kx - 1$ for $x \leqq 0$, $x = 0$, $x = -1$ and points $(0, 0)$, $(0, -1)$, $(1, 1)$, $(1, 0)$, $(1/K, 0)$, $(1/K, -1)$ form a k-configuration. (Received August 28, 1967.)

653-29. J. E. KIST, New Mexico State University, University Park, New Mexico, and P. H. MASERICK, Pennsylvania State University, University Park, Pennsylvania. Functions of bounded variation on a commutative idempotent semigroup.

Let $(A, +)$ be a commutative idempotent semigroup with identity 0 and consider the cone C of all completely monotonic functions on A equipped with the topology of simple convergence. The set $X = \{f \in C \mid f(0) = 1\}$ is a compact base for C which is shown to be a simplex (i.e., the linear span $C - C$ of C is a vector lattice). It is shown that A can be isomorphically embedded in the Borel field of a zero dimensional compact Hausdorff space S such that the nonnegative regular Borel measures on S are identified with the functions of C. The variation of a function on A is defined, and it is shown that this concept includes Birkhoff's definition of variation of a valuation on a distributive lattice. The elements of the vector lattice $C - C$ are characterized as those functions which are of finite variation. Birkhoff's characterization of the vector lattice spanned by the nonnegative and nonincreasing valuations on a distributive lattice follows as a special case. (Received August 28, 1967.)

653-30. J. H. WOLKOWISKY, University of Colorado, Boulder, Colorado 80302. Periodic solutions of nonlinear Sturm-Liouville problems.

The existence and multiplicity of real periodic solutions of (1) $[r(x)y']' + q(x)y + \lambda y[a(x) - h(x, y, y')] = 0$ is investigated. It is assumed that $r(x) > 0$, $a(x) > 0$, $r'(x)$, and $q(x)$ are continuous and periodic of period p over $(-\infty, \infty)$. Also that $r(x)$, $q(x)$ and $a(x)$ are even functions. In addition we require h to satisfy the following conditions for $-\infty < x, y, z < \infty$; (i) $h(x, y, z)$ is continuous and even in all variables, (ii) $h(x, y, z) \geqq 0$, (iii) $h(x, 0, 0) = 0$, (iv) $\lim_{c \to \infty} h(x, c\zeta, c\eta) = \infty$ for all $\zeta \neq 0$, $\eta \neq 0$, (v) $h(x, y, z)$ is periodic in x of period p. It is also assumed that the eigenvalues λ_k and $\bar{\lambda}_k$ $(k = 0, 1, 2, \ldots)$ of the "linearized" problem, $[r(x)y']' + qy + \lambda a(x)y = 0$, are nonnegative. Where the eigenfunction corresponding to λ_k is of minimal period p (type 1) and the eigenfunction corresponding to $\bar{\lambda}_k$ is of minimal period 2p (type 2). Theorem. For each $\lambda \in (l, \infty)$ there exists at least 2n type 1 and 2m type 2 real solutions of (1). Where if $l = \lambda_{2k}$ then $n = 2k + 1$ and $m = 2k$, if $l = \bar{\lambda}_{2k}$ then $n = 2k + 1$ and $m = 2k + 1$, if $l = \lambda_{2k+1}$ then $n = 2k + 1$ and $m = 2k + 2$, and if $l = \lambda_{2k+1}$ then $n = 2k + 2$ and $m = 2k + 2$. An analogous theorem is also proved when (ii) is replaced by $h(x, y, z) \leqq 0$. The proof of these theorems is based on a method used by the author previously (Comm. Pure Appl. Math. (3) 20 (1967), 549-561). (Received August 28, 1967.)

653-31. R. J. LINDAHL, Pennsylvania State University, University Park, Pennsylvania 16802, and R. D. MOYER, Univeristy of Kansas, Lawrence, Kansas 66044. A differential operator with no smooth functions in its domain.

Let D be the maximal operator in $\mathscr{L}_2(0, 1)$ for ordinary differentiation. A closed restriction of D is constructed with the property that its domain is dense in $\mathscr{L}_2(0, 1)$ but every smooth function in its domain is constant. (Received August 28, 1967.)

653-32. W. H. GUILINGER, Bettis Atomic Power Laboratory, P. O. Box 79, West Mifflin, Pennsylvania 15122, and R. B. KELLOGG, University of Maryland, College Park, Maryland. Elliptic difference equations on a convex domain.

An elliptic partial differential equation of the form $a(x, y)u_{xx} + b(x, y)u_{xy} + c(x, y)u_{yy} = f(x, y)$ is considered in a convex domain with Dirichlet boundary conditions. For a particular finite difference approximation of the problem, it is shown that a discrete L_2 norm of second divided differences of the discretization error and the maximum component norm of the discretization error are bounded by constants times the square root of the mesh spacing. The constants depend on properties of the domain and bounds on third derivatives of the solution. The approach used is to obtain a finite difference analog of the inequality, known in the theory of elliptic boundary value problems, which bounds the integral of the squares of the second derivatives of the solution u in terms of the integral of f^2. A finite difference analog of the Sobolev inequality is then used to obtain the pointwise error bound. (Received August 28, 1967.)

653-33. FRANK HARARY, University of Michigan, Ann Arbor, Michigan, and E. M. PALMER, Michigan State University, E. Lansing, Michigan. On acyclic simplicial complexes.

Pure simplicial complexes were introduced and enumerated in [F. Harary, Trans. Amer. Math. Soc. 78 (1955), 445-463]. A pure n-dimensional complex which is "connected and acyclic" is called an n-tree. Properties of 2-trees are found and a dissimilarity characteristic theorem is used to obtain formulas for the number of 2-trees with a given number of 2-cells. The enumeration technique, which is similar to that employed by R. Otter [Ann. of Math. 49 (1948), 583-599] for ordinary trees, can be specialized to count 2-trees with specified properties. We used this approach to count planar 2-trees, which correspond precisely to triangulations of the disk. These were enumerated by W. G. Brown [Proc. London Math. Soc. 14 (1964), 746-768] by entirely different methods. (Received August 28, 1967.)

653-34. KATSUMI NOMIZU, Brown University, Providence, Rhode Island, and BRIAN SMYTH, University of Notre Dame, Notre Dame, Indiana. On holonomy groups of complex hypersurfaces. Preliminary report.

For an n-dimensional complex hypersurface M in a Kählerian manifold of constant holomorphic curvature $\tilde{c}$, the following results are proved for the holonomy group H of M (with respect to the induced Kählerian structure): (1) If $\tilde{c} < 0$, H is always isomorphic to U(n). (2) If $\tilde{c} > 0$, H is isomorp either to U(n) or $SO(n) \times T^1$, the second case arising only when M is locally holomorphically isometri to the complex quadric Q^n in P^{n+1} (C). (3) If $\tilde{c} = 0$ and if the second fundamental form of M has maximal rank at least at one point, then H is isomorphic to U(n). The proof makes use of the formul and results obtained in B. Smyth [Differential geometry of complex hypersurfaces, Ann. of Math. 85 (1967), 246-266]. The result for c = 0 was obtained by a different method in Y. Kerbrat [Sous-variétés complexes de C^m, C. R. Acad. Sci. Paris 262 (1966), 1171-1174]. (Received August 29, 19(

653-35. STANLEY GUDDER, University of Wisconsin, Madison, Wisconsin 53706. A note on proposition observables.

We assume all observables are defined on a quite full logic satisfying conditions U and E. (See S. Gudder, Uniqueness and existence properties of bounded observables, Pacific J. Math. 19 (1966), 81-93 for notation and definitions.) An observable x is a proposition observable if $x^2 = x$. If x, y are bounded observables we define $x \circ y = (1/2)[(x+y)^2 - x^2 - y^2]$ and say that x and y are compatible if $x \circ (z \circ y) = (x \circ z) \circ y = (x \circ y) \circ z$ for all bounded observables z. Theorem 1. The following statements are equivalent. (i) $x_a \circ x_b$ is a proposition observable, (ii) $x_a \circ x_b = x_{a \wedge b}$, (iii) $a \leftrightarrow b$. Corollary. If x_a and x_b are compatible, then $x_a \leftrightarrow x_b$. Corollary. The following statements are equivalent. (i) $x_a + x_b$ is a proposition observable. (ii) $a \perp b$, (iii) $\sigma(x_a + x_b) \subset [0, 1]$, (iv) $x_a \circ x_b = 0$. A sequence of observables x_n converges weakly to an observable x if $\lim m(x_n) = m(x)$ for every state m. Theorem 2. The sequence of observables $(x_a \circ x_b)^n$ converges weakly to the proposition observable $x_{a \wedge b}$. (Received August 29, 1967.)

653-36. H. L. EGAN, University of Maryland, College Park, Maryland 20740. A class of galois connections between a group and its automorphism group.

Let G denote an arbitrary group, A(G) its group of automorphisms, and H any subset of G. Let $\Gamma(H,S) = \{\alpha \in A(G) | [S,\alpha] \leqq H\}$ (where $[g,\alpha] = g \cdot \alpha(g^{-1})$, for $g \in G$, and $[S,\alpha] = gp\{[x,\alpha] | x \in S\}$) for $S \leqq G$, and let $F(H,\emptyset) = \{g \in G | [g,\emptyset] \leqq H\}$, for $\emptyset$ any subset of A(G). For each fixed H the pair $\{\Gamma, F\}$ define a galois connection $2^G \rightleftarrows 2^{A(G)}$. If H is taken to be a characteristic subgroup of G then $\{\Gamma, F\}$ define a galois connection between the collection of characteristic subgroups of G and the collection of normal subgroups of A(G). If H is taken to be (1) $\{\Gamma, F\}$ yield a "galois theory" for groups: $\Gamma((1), S) = \{\alpha \in A(G) | \alpha \text{ fixes } S \text{ pointwise}\}$, $F((1), \emptyset) = \{g \in G | g \text{ is fixed by each } \alpha \in \emptyset\}$. In this case $\{\Gamma, F\}$ define a galois connection between the collections of all subgroups of G and of A(G). The concept of a galois (closed) subgroup is then defined in the usual way: $S \leqq G$ is galois if $F((1), \Gamma((1), S)) = S$. The galois theory for finite cyclic groups can be completely described. (Received August 31, 1967.)

653-37. HERMAN GOLLWITZER, The University of Tennessee, Knoxville, Tennessee 37919. Stokes multipliers of subdominant solutions of the differential equation $y'' - (x^m + a_1x^{m-1} + \dots + a_{m-1}x + \lambda)y = 0$.

The Stokes multipliers of subdominant solutions of the differential equation $y'' - (x^3 + \lambda)y = 0$, where λ is a parameter, have been studied recently by Y. Sibuya (Abstract 642-53, these *Notices* 1 (1967), 77). Consider the differential equation (DE) $y'' - (x^m + a_1x^{m-1} + \dots + a_{m-1}x + \lambda)y = 0$; $m \geqq 3$, a_j and λ complex. In this paper we shall study the Stokes multipliers as a function of λ when the coefficients a_j lie in some fixed compact set. Let $\omega = \exp(2\pi i/(m+2))$, $a = (a_1, \dots, a_{m-1})$ and $Ta = (\omega a_1, \dots, \omega^{m-1}a_{m-1})$. Let $f(x, a, \lambda)$ be a subdominant solution of (DE) in the sector $|\arg x| < \pi/(m+2) \cdot f(\omega^{-1}x, T^{-1}a, \omega^{-m}\lambda)$ and $f(\omega^{-2}x, T^{-2}a, \omega^{-2m}\lambda)$ are linearly independent solutions of (DE) and hence $f(x, a, \lambda) = c_1(a, \lambda)f(\omega^{-1}x, T^{-1}a, \omega^{-m}\lambda) + c_2(a, \lambda)f(\omega^{-2}x, T^{-2}a, \omega^{-2m}\lambda)$. We first prove that the Stokes multiplier $c_2(a, \lambda) = -\omega^{-1-2\nu(m,a)}$, $\nu(m, a)$ is a computable constant. Then we shall prove that $c_1(a, \lambda)$ is an entire function of (a, λ) such that $c_1(a, \lambda) =$

$\omega^{1/2-\nu(m,a)}$ $[(1 + o(1))\exp(L_m(a, \lambda) + L_m(T^{-2}a, \omega^{-m+2}\lambda)) + (1 + o(1))\exp(L_m(a, \omega^{m+2}\lambda) + L_m(T^{-2}a, \omega^4\lambda))]$ as λ approaches infinity in any given direction. $L_m(a, \lambda)$ is an analytic function for large $|\lambda|$ which is defined by a relatively simple continuation formula. (Received August 28, 1967.)

653-38. WITOLD BOGDANOWICZ, Catholic University of American, Washington, DC 20017, and HANS HEYN, University of Wyoming, Laramie, Wyoming. Volumes generated by Daniell functionals on the space of all continuous functions on a normal topological space.

Let X be a normal topological space and C the space of all real-valued continuous functions on X. Let J be a Daniell functional on C, i.e. a positive linear functional continuous under pointwise monotone convergence everywhere on X. By a volume generated by J we shall understand the volume v as defined in the paper by W. Bogdanowicz, An approach to the theory of integration generated by Daniell functionals and ... Math. Annalen 173 (1967), 34-52. See also W. Bogdanowicz, Proceedings of the Japan Academy 43 (1967), 186-191; 42 (1966), 1033-1037 and 1038-1043, and these *Notices* 13 (1966), 84. Let V consist of all sets being differences of open F_σ sets of the space X. One can prove that the family V of sets is a prering. A positive volume v on V is called compact if for every sequence of sets $A_n \in V$ with void intersection such that $\bar{A}_{n+1} \subset \operatorname{int} A_n$ for all n, $\bar{A}$ being the closure of A, there exist an index m such that $v(A_m) = 0$. Theorem. Let J be a Daniell functional on the space C and v the volume generated by it. Then v is a compact positive volume on the prering V. Conversely, if v is a compact positive volume on V then all functions $f \in C$ are v-summable (for definition see Bogdanowicz, Proc. Nat. Acad. Sci. USA 53 (1965), 492-498) and the restriction of the integral $\int f dv$ to C yields a Daniell functional J such that the volume generated by it coincides with the volume v. (Received August 31, 1967.)

653-39. D. V. V. WEND, Montana State University, Bozeman, Montana 59715. Existence and uniqueness of solutions of ordinary differential equations.

Strong monotonicity conditions are assumed to obtain existence and uniqueness theorems for the initial value problem (*) $y_k' = f_k(x, y_1, \dots, y_n), y_k(0) = a_k$, $k = 1, 2, \dots, n$. Theorem. Suppose for each k the function f_k in (*) is defined in the domain B: $0 \leqq x \leqq a$, $|y_j - a_j| \leqq b$, $j = 1, \dots, n$. If in addition each f_k is nonnegative and nondecreasing in each of $x, y_1, \dots, y_n$ in B, then there exists a solution in the extended sense of (*) to the right of $x = 0$. Theorem. Suppose each f_k in (*) is defined in the domain B above and is nondecreasing in each of $x, y_1, \dots, y_n$ in B. If in addition for each solution $(y_1(x), \dots, y_n(x))$ of (*) $f_k(x, y_1(x), \dots, y_n(x)) > 0$ for $x > 0$ for each k, then (*) has at most one solution to the right of $x = 0$. An application to an initial value problem for a general nth order nonlinear differential equation is made, and examples are given showing the necessity of some of the hypotheses in the theorem. (Received September 1, 1967.)

653-40. R. B. GUENTHER, Oregon State University, Corvallis, Oregon, and E. L. ROETMAN, Stevens Institute of Technology, Hoboken, New Jersey. Numerical integration in higher dimensions.

We develop cubature formulae for "diamond shaped" domains for arbitrary dimensions, which are up to twelfth order accurate. These formulae use a minimal number of grid points, e.g. the sixth order accurate formula given here uses $1 + 2n(2n^2 + 3n + 4)/3$ points. These formulae reduce to the standard Newton-Cotes formulae when the dimension is one. We illustrate how corresponding formulae can be developed for arbitrary, symmetric domains. (Received September 1, 1967.)

653-41. R. B. GUENTHER, Oregon State University, Corvallis, Oregon, and E. L. ROETMAN, Stevens Institute of Technology, Hoboken, New Jersey. Interpolation in n-dimensions.

We develop interpolation polynomials in n-dimensions using special sets of points, which are exact for polynomials of degree up to eight. In two dimensions we obtain interpolation polynomials of much higher order. In the case of one dimension, these formulae reduce to the usual Newton-Lagrange formulae using equally spaced grid points. We observe that more symmetric formulae are obtained by interpolating some points and least squares fitting others. (Received September 1, 1967.)

653-42. J. W. BROWN, Oberlin College, Oberlin, Ohio 44074. On zero type sets of Laguerre polynomials.

Sheffer (Duke Math. J. 5 (1939), 590-622) has defined a simple polynomial set $\{p_n(x)\}$ to be of type zero iff there is a differential operator $J \equiv \sum_{k=0}^{\infty} c_k D^{k+1}$, where the c_k are constants, such that $J[p_{n+1}(x)] = p_n(x)$ for all $n \geqq 0$. The operator J is said to be generated by the function $J(t) \equiv \sum_{k=0}^{\infty} c_k t^{k+1}$. Theorem. For any integer m the simple set $\{L_n^{(\alpha+mn)}(x)\}$ of Laguerre polynomials is of type zero corresponding to the operator generated by $-t(1 - t)^{-m-1}$. The proof is based on a generating function that is exhibited for $\{L_n^{(\alpha+mn)}(x)\}$. (Received September 5, 1967.)

653-43. C. L. DEVITO, The University of Arizona, Tucson, Arizona 85721. A completeness theorem for locally convex spaces and some applications.

Our completeness theorem is the following: Theorem 1. Let E be a locally convex space and suppose that E, when given the topology of uniform convergence on the convex, strongly bounded subsets of E', is both complete and separable. If the strong dual of E is complete, then E is Mackey quasi-complete. This result, together with some theorems of Dixmier [Duke Math. J. 15 (1948), 1057-1071] yields: Theorem 2. Let B be a separable Banach space and let Q be a norm closed, linear subspace of B'. If Q has positive characteristic, then $B[\sigma(B, Q)]$ is Mackey complete. Let L^1 be the Banach space of all real-valued, Lebesgue integrable functions on [0, 1]. Let C be the Banach space of all real-valued, continuous functions on [0, 1]. We may regard C as a norm closed, linear subspace of the dual, L^{∞}, of L^1. Theorem 3. The subspace C of L^{∞} has characteristic one. Because of Theorem 3 the method used to prove Theorem 1 yields: Theorem 4. Let F be a real-valued function of bounded variation on [0, 1], and suppose that F is continuous on the right. Then F is absolutely continuous iff $\lim \int_0^1 g_n dF = 0$ whenever $\{g_n\}$ is a sequence of elements of C which is weak* convergent to zero. (Received September 5, 1967.)

653-44. H. I. BROWN, State University of New York, Albany, New York 12203. Replaceability of methods of summation.

Let l represent the set of absolutely convergent series, $\sigma(x)$ the natural functional $\sum x_n$ on l, and l_0 the kernel of σ. If A is an l - l method of summation and l_A represents the summability field of A, then A is called absolutely regular if it preserves the functional σ on l. It is called replaceable if there exists an absolutely regular method B such that $l_B \supseteq l_A$. Refer to the paper by H. I. Brown and V. F. Cowling (Mich. Math. J. 12 (1965), 357-362) for terminology and notation. Theorem. The following statements are equivalent. (1) A is replaceable. (2) For each k = 1, 2,..., the sequence e^k lies outside of the l_A-closure of l_0. (3) l_0 is not l_A-dense in l. (4) σ is l_A-continuous on l. (5) There exists a member f in l'_A, the dual space of l_A, such that $f(e^k) = 1$ for k = 1, 2,.... (Received September 6, 1967.)

653-45. D. R. KERR, Jr., State University of New York, Albany, New York 12203. Seminorm-dual subspaces of the algebraic dual of a linear space.

Let X be a linear space, let $X^{\#}$ be the space of linear functionals on X, and for a seminorm p on X let $(X,p)'$ be the dual of X with the p-topology. Necessary and sufficient conditions on a subspace V of $X^{\#}$ are considered for there to exist a seminorm p on X such that $(X,p)' = V$. Theorem 1. There exists p so that $(X,p)' = V$ iff V has a w* bounded subset whose bipolar spans V. Define p* on $(X,p)'$ by $p^*(f) = \sup \{|f(x)|: p(x) \leq 1\}$, and call V bw* closed iff V contains the w* limit of each net in V which is p* bounded and which is w* convergent to an element of $(X,p)'$. Theorem 2. For each subspace V of $(X,p)'$, $(X,p_V)' = V$ iff V is bw* closed where p_V is the seminorm on X defined by $p_V(x) = \sup \{|f(x)|: f \in V \text{ and } p^*(f) \leq 1\}$. For any noncomplete (X,p), examples of proper total subspaces V of $(X,p)'$ such that $(X,p_V)' = V$ can be given. These are, of course, not norming. Theorem 3. If V is a p* closed subspace of $(X,p)'$, then there exists a seminorm q on X such that $(X,q)' = V$ iff V is bw* closed. If (X,p) is complete, several corollaries follow from the fact that V is bw* closed iff V is w* closed. (Received September 7, 1967.)

653-46. R. M. GUNDERSEN, University of Wisconsin, Milwaukee, Wisconsin 53201. One-dimensional magnetohydrodynamic flow and the Monge-Ampere equation.

Consider the one-dimensional, unsteady flow of an ideal, inviscid, perfectly conducting, compressible fluid, subjected to a transverse magnetic field. By the use of an analysis based on a Monge-Ampère equation, it is shown that general isentropic flows are governed by a second-order partial differential equation which reduces to the usual Euler-Poisson equation in the limit of vanishing magnetic field. Analogous results are obtained for the interaction of simple waves when the applied field is oblique in orientation. (Received September 7, 1967.)

653-47. GREGORY WULCZYN, Bucknell University, Lewisburg, Pennsylvania 17837. Polynomials which after repeated division have remainders in arithmetic progression.

The following classes of polynomials are studied: (1) Polynomials in r which after t divisions by r will have t remainers s, s, s, ... , s (monkey-cocoanut problem). (2) Polynomials in r, which after t divisions by r will have t remainders k, 2k, 3k, ... , tk. (3) Polynomials in r, which after t divisions by r will have remainders k, k + m, k + 2m, ... , k + (t - 1)m. There is a modified converse. (Received September 8, 1967.)

653-48. J. L. BRYANT, Florida State University, Tallahassee, Florida 32306, and C. L. SEEBECK, III, Michigan State University, East Lansing, Michigan. Locally nice manifolds are tame in codimension three.

An embedding f of an m-manifold M into the interior of a q-manifold Q, $q - m \geqq 3$, is said to be locally nice if Q - f(M) is 1-ULC. Theorem 1. Let M and Q be combinatorial manifolds of dimensions m and q, respectively, with $q \geqq 5$ and $q - m \geqq 3$. If f is a locally nice embedding of M into Int Q, then for each $\epsilon > 0$ there exists an ϵ-push h of (Q, f(M)) such that $hf:M \rightarrow Q$ is piecewise linear. Corollary 2. If P is a locally tame (q - 1)-complex in Q and $f|M - f^{-1}(P)$ is locally tame, then f is ϵ-tame. Corollary 3. If $f:M \rightarrow E^{q-1} \subset E^q$ then $f:M \rightarrow E^q$ is ϵ-tame. Corollary 4. If $M^m \subset E^q$ and $N^n \subset E^r$, then $M \times N$ is ϵ-tame in E^{q+r} provided $q - m \geqq 1$ and $r - n > 1$. Corollaries 2, 3, and 4 are just restatements of Theorem 1 for special locally nice embeddings. The proof uses the results of Homma and Hudson, together with some recent results of the authors. (Received September 11, 1967.)

653-49. J. E. SCHNEIDER, University of Oregon, Corvallis, Oregon. Some results in the theory of primes.

Primes for commutative rings with 1 were introduced by Harrison [Mem. Amer. Math. Soc., no. 68, 1966]. Call a commutative ring a C-ring iff every finite prime is an ideal. R is a C-ring iff Spec(R) = $X^{\#}(R)$ [the set of preprimes T of R such that, for each finite $E \subset R$, $T \cap E = \emptyset \Rightarrow$ there is a prime P of R with $T \subset P$ and $P \cap E = \emptyset$] iff R/P is a C-ring for each minimal prime ideal P of R. R is a C-domain iff R is absolutely integral or char(R) is not zero, the transcendence degree of R is one, and there is a unique valuation ring of the quotient field of R which does not contain R. In particular, the Krull dimension of a C-ring is zero or one. The set of finite primes of R is equal to Spec(R) iff R is a generalized Boolean ring. The C-domain result is used to show that if R is a domain of "adjusted" transcendence degree one, then the primes are exactly the nonzero preprimes with multiplicatively closed complements. Moreover, if the quotient field of R is a global field, then Y(R), the space of all primes of R, is homeomorphic to an open subset of Y(F). (Received September 11, 1967.)

653-50. D. L. SUMNERS, Florida State University, Tallahassee, Florida 32306. Homotopy torsion in codimension two knots.

The homology theory of the infinite cyclic covering space of a codimension two knot complement is reasonably well known, but little is known concerning the homotopy theory of the knot complement itself. Specifically, if $k: S^n \to S^{n+2}$ is a smooth embedding, and $T(kS^n)$ is an open tubular neighborhood of the embedding, let $S = S^{n+2} - T(kS^n)$. One of the module structures on the homotopy of S is the natural structure over the ring Λ = integral group ring of $\pi_1(S)$. We say that $\pi_i(S)$ $(i \geqq 2)$ has Λ-torsion if there exists $0 \neq c \in \pi_i(S)$, $0 \neq w \in \Lambda$ such that $0 = wc \in \pi_i(S)$. Theorem 1. For all $n \geqq 1$, given any integer p such that $1 \leqq p \leqq (n+1)/2$, there exists a slice knot (S^{n+2}, kS^n) such that (i) $\pi_i(S) = \pi_i(S^1)$, $1 \leqq i < p$, (ii) $\pi_i(S)$ has Λ-torsion $p \leqq i \leqq n+1-p$; $\pi_1(S) \neq Z$ $p = 1$. Let $k': B^{n+1} \to B^{n+3}$ denote proper smooth embedding of balls. Let B denote the knot complement. Theorem 2. For all $n \geqq 1$, given any integer p such that $1 \leqq p \leqq n$, there exists a knotted ball pair $(B^{n+3}, k'B^{n+1})$ such that (i) $\pi_i(B) = \pi_i(S^1)$ $1 \leqq i < p$, (ii) $\pi_i(B)$ has Λ-torsion $p \leqq i \leqq n$; $\pi_1(B) \neq Z$ $p = 1$. The proof is by surgery, and the homotopy torsion is related to the homotopy torsion in the infinite cyclic cover. One of the examples of the slice knots for n = 2 can be shown to be a counterexample to a theorem announced by Giffen. (Received September 11, 1967.)

653-51. W. W. COMFORT, Wesleyan University, Middletown, Connecticut 06457, and S. NEGREPONTIS, McGill University, Montreal, Quebec, Canada. Homemorphs of three subspaces of $\beta N \setminus N$.

Hypothesized spaces are completely regular Hausdorff; $\approx$ denotes homeomorphism; our proofs use the continuum hypothesis. An easy consequence of I. I. Parovičenko's theorem (Dokl. Akad. Nauk SSSR 150 (1963), 36-39 = Soviet Math. Dokl. 4 (1963), 592) is Theorem. Let X be 0-dimensional with $|C(X)| = c$ and let Z be a nonvoid zero-set in βX with $Z \cap X = \emptyset$. Then $Z \approx \beta N \setminus N$. Definition. For D discrete with $|D| = \aleph_1$, set $\Omega = \{p \in \beta D \setminus D\colon p \in cl_{\beta D} A$ for some countable $A \subset D\}$. Theorem. Let p be a P-point in $\beta N \setminus N$, Z^0 the interior of any nonopen zero-set in $\beta N \setminus N$. Then $\Omega \approx Z^0 \approx$

$\beta N \setminus N \setminus \{p\}$. Notation. For each X, P(X) denotes its space of P-points, X_π denotes X with the coarsest P-space topology containing the original. Λ is the lexicographically ordered space 2^{ω_1} of $\{0, 1\}$-"sequences" in its order topology. Theorem. If X is compact, then $X_\pi \approx \Lambda_\pi$ iff $|C(X)| = c$ and no point of X is a G_δ. Corollary. If X is locally compact, realcompact, noncompact with $|C(X)| = c$, then $(\beta X \setminus X)_\pi \approx \Lambda_\pi$. Theorem. If X is locally compact, σ-compact, noncompact with $|C(X)| = c$, then $P(\beta X \setminus X) \approx \Lambda_\pi$. Theorem. If X is noncompact, separable metric, then $(\beta X \setminus X)_\pi \approx \Lambda_\pi$. The case $\Lambda_\pi \approx (\beta N \setminus N)_\pi \approx P(\beta N \setminus N)$ was given by Parovičenko (loc. cit.). (Received September 11, 1967.)

653-52. T. W. HUNGERFORD, University of Washington, Seattle, Washington 98105. On the free product of algebras.

If A and B are differential graded augmented (DGA) algebras over a commutative ring K, then their free product $A * B$ is always defined (the definition is given by means of tensor products). $A * B$ is a DGA algebra such that $A \longrightarrow A * B \longleftarrow B$ is a universal diagram in the category of DGA-algebras; similarly for the free product of Hopf algebras. The relationships between various homologies H(A), H(B), $H(A \otimes B)$, $H(A * B)$ are investigated. Examples show that $H(A * B)$ does not determine $H(A \otimes B)$, and vice-versa. Theorem. If A and B are torsion-free DGA-algebras over Z, then $H(A * B)$ is completely determined by the homology spectra of A and B. Finally a Künneth Theorem of sorts is given, relating the additive structure of $H(A * B)$ (A, B as in Theorem) with H(A), H(B) and the derived functors of the n-fold tensor product (all $n \geqq 2$). (Received September 11, 1967.)

653-53. W. M. FLEISCHMAN, State University of New York, Buffalo, New York. A new extension of countable compactness.

A topological space X will be called starcompact provided that for each open covering $\mathcal{V}$ of X there is a finite set $A_{\mathcal{V}} \subseteq X$ such that $St(A_{\mathcal{V}}, \mathcal{V}) = X$. Every countably compact T_1 space is starcompact. Conversely, any regular starcompact space is countably compact. Relationships between starcompactness and both pseudocompactness and weak compactness have been explored. It is shown that a space which is both starcompact and point paracompact is, indeed, compact. These ideas lead to an extremely simple proof of the following generalization of a theorem of G. Aquaro (Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur 39 (1965), 19-21): Let $\underline{m}$ be an infinite cardinal, let X be a space which is $\underline{m}$-compact, and let $\mathcal{V}$ be a point-$\underline{m}$ open covering of X. Then $\mathcal{V}$ has a finite subcover. (Received September 11, 1967.)

653-54 and 653-55. WITHDRAWN.

653-56. T. K. BOEHME, University of California, Santa Barbara, California, and R. E. POWELL, University of Kentucky, Lexington, Kentucky 40506. Generalizations of the Taylor transform generated by analytic functions.

Let $r \in \{z: |z| < 1\}$. Let F_r be the collection of functions f such that f is analytic on $\{z: |z| \leqq 1\}$, $f(1) = 1$, and $|f(z)| < |r|^{-1}$ for $|z| \leqq 1$. Define $g(r, z) = (1 - r)f(z)/[1 - r f(z)]$. The $T(f; r) = (w_{n,k}(r))$ transform is defined by $\{g(r, z)\}^n = \sum_{k=0}^{\infty} w_{n,k}(r) z^k$. The Taylor transform, T(r), is obtained when $f(z) = z$. (i) Conditions are determined on r and f such that T(f; r) is regular. (ii) The region onto which a regular T(f; r) transform continues the geometric series analytically is found. (iii) Sufficient conditions are determined on r_1, r, and f such that a sequence which is $T(r_1)$ summable is also T(f; r) summable to the same value. (iv) Let F be a bounded function in C[0, 1). Define $P_n(F; r) = \sum_{k=0}^{\infty} w_{n,k}(r)F(t_{n,k})$ where $T(f; r) = (w_{n,k}(r))$, $t_{n,k} = [k - f^{(1)}(1)n]/k$ if $k \geqq f^{(1)}(1)n$, and $t_{n,k} = 0$ otherwise. Conditions on f and r are determined such that $P_n(F; r)$ converges uniformly to F. (Received September 13, 1967.)

653-57. W. W. SMITH, University of North Carolina, Chapel Hill, North Carolina 27514. Projective ideals of finite type. Preliminary report.

Let R be a commutative ring with unity and J be a finitely generated ideal of R. Denote the prime radical of R by rad R and the annihilator of J by $J^{\perp} = \{x | xJ = 0\}$. Theorem 1. If J is flat and $J^{\perp} \subset \text{rad } R$ then J is projective. Theorem 2. If $J^{\perp}$ is finitely generated and J is flat then J is projective. (Received November 6, 1967.)

653-58. NICKOLAS HEEREMA, Florida State University, Tallahassee, Florida 32306. A group of a Lie algebra.

Let L be a Lie algebra over a field of characteristic zero and let $\mathscr{A}$ be an enveloping algebra of L. Let $\mathscr{G}$ be a set of generators of L as an Abelian group. The subgroup of the multiplicative semigroup of the power series ring $\mathscr{A}[[x]]$ generated by $\{\exp ax\}_{a \in \mathscr{G}}$ is designated $\mathscr{S}_{\mathscr{G}}(L)$ and $\overline{\mathscr{S}(L)}$ denotes its completion in the topology of the lower central series. The following are proved. (1) The factor groups of the lower central series of $\mathscr{S}_{\mathscr{G}}(L)$ and of $\overline{\mathscr{S}(L)}$ are isomorphic to the additive groups of the corresponding derived algebras of L. Thus (2) $\mathscr{S}_{\mathscr{G}}(L)$ and $\overline{\mathscr{S}(L)}$ are nilpotent if and only if L is. (3) An element $\sum a_i x^i$ of $\overline{\mathscr{S}(L)}$ is in the nth derived subgroup of $\overline{\mathscr{S}(L)}$ if and only if $a_i = 0$ for $1 \leqq i \leqq n$. (4) $\overline{\mathscr{S}(L)}$ is independent of $\mathscr{G}$. So is $\mathscr{S}_{\mathscr{G}}(L)$ if L is nilpotent. They are both independent of $\mathscr{A}$ (depending on L only). (5) Conditions on $\overline{\mathscr{S}(L)}$ are given equivalent to simplicity (semisimplicity) of L. (6) Generalizations for sequences of exp and log are obtained as well as a variant of the Campbell-Hausdorff formula. (Received September 14, 1967.)

653-59. R. W. GUNDERSON, Marshall Space Flight Center, Huntsville, Alabama, and J. H. GEORGE, University of Wyoming, Laramie, Wyoming. On the equivalence of certain stability properties.

The vector differential equation $\dot{x} = f(x, t)$ is considered, where x and f belong to R^n, f is defined on $D_r = \{(x, t) : x < r, t \geqq 0\}$ and $f(0, t) = 0$ for $t \geqq 0$. Solutions of the differential equation on a t-interval J, with initial values $(x_0, t_0) \in D_r$, are required to satisfy the equation for all $t \in J$ and be differentiable on J. Solution properties are defined which correspond to explicit solution behavior requirements of several well known versions of stability in the sense of Liapunov. Examples are given to verify that the properties are different. It is then shown that the properties can be made equivalent by placing additional requirements on the right side. For example, it is shown that the equivalence of all the properties follows from the continuity of f, uniqueness of the trivial solution to the left of every $t_0 \geqq 0$ and uniqueness of the trivial solution to the right of $t_0 = 0$. (Received September 5, 1967.)

653-60. D. W. LICK, Brookhaven National Laboratory, Upton, New York 11973. Existence-uniqueness theorems for nonlinear Dirichlet problems.

Existence-uniqueness theorems are proved for Dirichlet problems for nonlinear equations of the form $\nabla^2 u - f(u) = 0$, $u \geqq 0$, with boundary condition $u = \phi$ where ϕ is continuous and positive. (Received September 14, 1967.)

653-61. S. W. YOUNG, University of Utah, Salt Lake City, Utah 84112. Polynomial factors of light mappings on an arc.

Theorem 1. If f is a continuous light function of $[a, b]$ onto $[c, d]$ and $\epsilon > 0$, there exists a factorization $f = Pg = P(g)$ such that P is a polynomial of $[a, b]$ onto $[c, d]$ and g is a continuous function of $[a, b]$ onto $[a, b]$ such that $|g(x) - x| < \epsilon$ for all $x \in [a, b]$. Theorem 2. If T is a continuous function of an arc onto an arc, then T is light if and only if T is topologically equivalent to a continuous function f of $[0, 1]$ onto $[0, 1]$ such that if $\epsilon > 0$, there exists a factorization $f = Pg$ where P is a polynomial of $[0, 1]$ onto $[0, 1]$ and g is a continuous function of $[0, 1]$ onto $[0, 1]$ such that $|g(x) - x| < \epsilon$ for all $x \in [0, 1]$. (Received September 15, 1967.)

653-62. G. L. PFEIFER, The University of Arizona, Tucson, Arizona 85721. The Stone-Cech compactification of an irreducibly connected space.

A connected topological space X is irreducibly (closed) connected about a subset A of X if no proper (closed) connected subspace of X contains A. Theorem 1. If the completely regular space X is irreducibly connected about A, then the Stone-Cech compactification βX is irreducibly connected about $A \cup (\beta X - X)$. Theorem 2. If the normal space X is irreducibly closed connected about A and either (i) locally connected or (ii) semilocally connected at each $x \in X - A$, then βX is irreducibly closed connected about A. (Received September 15, 1967.)

653-63. D. R. CHALICE, Western Washington State College, Bellingham, Washington 98225. Some characterizations of approximate normality.

Let A be a function algebra on a compact Hausdorff space X. The following generalizes a result of Ryff on the disk algebra. Lemma. Let μ be either a representing measure or an extreme point of the ball of $A^{\perp}$. If S is the closed support of μ, then A-hull (S) is connected. A is approximately normal on X if for any two disjoint closed sets E, F in X and $\epsilon > 0$, there is a function f in A such that $|f| < \epsilon$ on E and $|1 - f| < \epsilon$ on F. Theorem. The following are equivalent. (1) A is approximately normal on X. (2) Every representing measure for A concentrated on X has connected closed support. (3) For each pair of disjoint closed sets E, F in X, A-hull $(E \cup F)$ = A-hull (E) $\cup$ A-hull (F) and A-hull (E) $\cap$ A-hull (F) = $\emptyset$. (4) For each closed set F in X, the canonical map components (F) $\longrightarrow$ components (A-hull (F)) is 1-1, onto. (5) For each closed set F in X, the canonical map clopen sets (A-hull (F)) $\longrightarrow$ clopen sets (F) is 1-1 onto. Remark. It follows that every extreme annihilator must also have connected closed support and hence by the Krein-Milman theorem follows a "belonging" result of Wilkin. (Received September 18, 1967.)

653-64. A. M. FINK and G. SEIFERT, Iowa State University, Ames, Iowa 50010. A separation condition for the zero solution of a system of differential equations.

Consider a system of ordinary differential equations of the form (1) $x' = f(t, x)$ where x and f(t, x) are elements of real Euclidean n-space R^n, $R^1 = R$, and $t \in R$. Here $f(t, 0) = 0$ for $t \in R$, f is continuous and satisfies a local Lipschitz condition in x in $R \times U$, U an open subset of R^n such that $0 \in U$. Let V be a real valued function on $R \times U$ satisfying the same conditions as f there and in addition, $V(t, x) \longrightarrow 0$ as $x \longrightarrow 0$ uniformly for $t \in R$. Define $\dot{V}_{(1)}(t, x) = \limsup_{h \to 0+} [V(t + h, x + hf(t, x)) - V(t, x)]/h$. Theorem. Suppose there exists a real-valued function a defined and continuous on the nonnegative reals R^+ and such that $a(0) = 0$, $a(r) > 0$ for $r > 0$, and a function V as above such that $|\dot{V}_{(1)}(t, x)| \geqq a(|x|)$ for $(t, x) \in R \times U$. Then if K is any compact subset of U such that $0 \in K$, $x = 0$ is the only solution of (1) such that $x(t) \in K$ for all $t \in R$. (Received September 18, 1967.)

653-65. C. J. HIMMELBERG and F. S. VAN VLECK, University of Kansas, Lawrence, Kansas 66044. Some selection theorems for measurable functions.

Virtually all known (and some new) generalizations of the well-known lifting lemma of A. F. Filippov (J. Soc. Indust. Appl. Math. Ser. A Control 1 (1962), 76-84) can be deduced from a recent selection theorem of K. Kuratowski and C. Ryll-Nardzewski (Bull. Acad. Polon. Sci. Ser. Sci. Math. Astronom. Phys. 13 (1965), 397-403). We cite here some of the selection theorems we obtain, but omit the statement of the lifting theorem which can be easily obtained as a corollary in each case. Theorem 1. Let X be a set with σ-ring $\mathcal{S}$, Y a separable metrizable space, and F: $X \longrightarrow Y$ a point closed multifunction such that $F^{-1}(B) \in \mathcal{S}$ for each compact $B \subset Y$. Then there is a selector $f: X \longrightarrow Y$ for F such that $f^{-1}(B) \in \mathcal{S}$ for each compact $B \subset Y$. Theorem 2. Let X be a set with σ-algebra $\mathfrak{A}$. Y a Lusin space, F: $X \longrightarrow Y$ a point closed multifunction such that $F^{-1}(B) \in \mathfrak{A}$ for each Borel subset B of Y. Then there is a selector f: $X \longrightarrow Y$ for F such that $f^{-1}(B) \in \mathfrak{A}$ for each Borel subset B of Y. Theorem 3. Let X be a set with σ-algebra $\mathfrak{A}$, Y a separable metric space, and F: $X \longrightarrow Y$ a point complete multifunction such that $F^{-1}(B) \in \mathfrak{A}$ for each closed $B \subset Y$. Then there is a selector f: $X \longrightarrow Y$ for F such that $f^{-1}(B) \in \mathfrak{A}$ for each closed $B \subset Y$. (Received September 18, 1967.)

653-66. WITHDRAWN

653-67. SEYMOUR HABER, National Bureau of Standards, Washington, D. C. 20234. Randomized quadrature formulas.

A family of quadrature formulas is described, in which classical and Monte-Carlo ideas are combined. Asymptotic error formulas are derived, and it is seen that in some circumstances these formulas will allow more rapid convergence than either the simple Monte-Carlo method or the classical methods. A few specific formulas of this family are constructed. (Received September 18, 1967.)

653-68. C. J. MAXSON, State University College, Fredonia, New York 14063. On near-rings of polynomials.

Automorphisms, T, of the ring $R(= \langle F[x], +, \cdot \rangle)$ of polynomials in one indeterminant over a field F of characteristic zero are used to define multiplications $*_T$ on $F[x]$ in such a manner that $N_T = \langle F[x], +, *_T \rangle$ becomes a unitary (right) near-ring with two-sided zero. In general the polynomial near-rings N_T are not rings. It is shown that the N_T-subgroups of N_T are ideals of R and conversely. T is used to characterize those N_T-subgroups of N_T which are also (left) ideals. A new characterization of finite near-fields is given; a polynomial near-ring is used to show that this characterization does not extend to infinite near-fields. (Received September 18, 1967.)

653-69. MANFRED BREUER, University of Kansas, Lawrence, Kansas 66044. Fredholm theories in von Neumann algebras. II.

For the definitions used, see Abstract 648-61, these *Notices* 14 (1967), 647. In that abstract compact elements and Fredholm elements relative to a von Neumann algebra A of operators were defined. One can generalize the classical theorems of Atkinson, Cordes-Labrousse a.o. as follows. T is Fredholm relative to A iff it is regular modulo the compact elements of A. Let $\mathscr{F}(A)$ be the set of Fredholm elements of A and let ν: $\mathscr{F}(A) \to I(A)$ be the index map. Then $\nu(ST^*) = \nu S - \nu T$. Let A_1, A_2 be von Neumann algebras and let $B = A_1 \otimes A_2 \otimes Z(\mathbf{C}^2)$. There is a canonical imbedding $I(A_1) \otimes I(A_2) \subset I(B)$. Consider the map $(S_1, S_2) \to S_1 \# S_2$ of (A_1, A_2) into B (see Palais, Ann. Math. Studies 57, p. 207). If S_1, S_2 are Fredholm relative to A_1, A_2, then $S_1 \# S_2$ is Fredholm relative to B and $\nu(S_1 \# S_2) = \nu(S_1) \otimes \nu(S_2)$. Let A be properly infinite. Then ν induces an isomorphism of the group $\pi_0 \mathscr{F}(A)$ of connected components of $\mathscr{F}(A)$ onto I(A). (Received September 18, 1967.)

653-70. GARY CHARTRAND, Western Michigan University, Kalamazoo, Michigan 49001, and DENNIS GELLER, University of Michigan, Ann Arbor, Michigan 48104. On uniquely colorable planar graphs.

Any coloring of a graph G induces a partition of the set of points of G into color classes, each class consisting of all points having a given color. A graph G with chromatic number $\chi(G) = n$ is uniquely n-colorable if any two colorings of G with n colors induce the same partition into color classes. Some necessary conditions and other sufficient conditions are presented for planar graphs to be uniquely 2-, 3-, or 4-colorable. In addition, it is shown that there does not exist a uniquely 5-colorable planar graph. (Received September 18, 1967.)

653-71. S. K. THOMASON, Simon Fraser University, Burnaby 2, B. C., Canada. Some initial segments of the hyperdegrees.

The lattice of all finite subsets of a countable set is isomorphic to an initial segment of the hyperdegrees less than that of Kleene's 0. The proof uses a generalization of the forcing method of Gandy and Sacks A minimal hyperdegree, (to appear). (Received September 18, 1967.)

653-72. R. E. CARLSON and C. G. CULLEN, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. A characterization of intrinsic functions on quaternion matrices.

Let $\mathfrak{A}$ be an associative algebra over the field $\mathfrak{F}$ and let $\mathfrak{G}$ be the group of all automorphisms and antiautomorphisms of $\mathfrak{A}$ which leave $\mathfrak{F}$ elementwise invariant. A function $F: \mathfrak{A} \longrightarrow \mathfrak{A}$ is intrinsic on a domain $\mathfrak{D}$ if (i) $\Omega\, \mathfrak{D} = \mathfrak{D}\ \forall\ \Omega \in \mathfrak{G}$ and (ii) $F(\Omega\, Z) = \Omega\, F(Z)\ \forall\ \Omega \in \mathfrak{G}$ and $\forall\ Z \in \mathfrak{D}$. This paper provides a characterization of intrinsic functions for the case $\mathfrak{A} = Q_n = \{n \times n$ matrices of quaternions$\}$. Using an earlier result of the authors [Can. J. Math., 19 (1967)] it is shown that every intrinsic function F on Q_n is a poly-function $[\forall\ A \in \mathfrak{D},\ \exists\ p(x) \in \mathfrak{R}\,[x] \ni F(A) = p(A)]$ and an explicit stem function $\mathfrak{F}: V_n(\mathfrak{C}) \longrightarrow \mathfrak{C}$ is determined. (Received September 18, 1967.)

653-73. J. R. REAY, Western Washington State College, Bellingham, Washington 98225. More about Radon's theorem.

What conditions must be placed on a set $S \subset E^d$ to assure that it may be partitioned into r pairwise disjoint subsets whose convex hulls intersect in a set which is at least k-dimensional? The following partial answers are known (see Abstract 640-22, these *Notices* 13(1966), 841). (1) The cardinality of S must be at least $(d + 1)(r - 1) + (k + 1)$. This is always necessary and is sufficient if $k = 0$. If $k > 0$ then some type of independence is also necessary. (2) If $k \geqq 1$, a sufficient condition is that S be strongly independent (see reference for definition), and it has been conjectured that a weaker sufficient condition is the general position of S. The following newer results establish this conjecture for several special cases. (3) If $d = 2$ or if $r = 2$ then the general position of S (and the correct cardinality, of course) is a sufficient condition. Certain weaker conditions are sufficient in the case $d = 2$, namely: If $k = 1$, S is not contained in the union of two lines in E^2. If $k = 2$, there does not exist any line in E^2 which contains more than one third of the points of S. (The classic theorem of Radon is the special case $k = 0$, and $r = 2$.) (Received September 18, 1967.)

653-74. H. HERRLICH, Freie University, Berlin, Germany, and G. E. STRECKER, University of Florida, Gainesville, Florida 32601. H-closed spaces and reflective subcategories.

Every Hausdorff space X has an H-closed extension τX (called the Katětov extension of X) which is maximal in several respects. Thus for Hausdorff spaces, the H-closed extension τ is analogous to the Stone-Čech compactification β and to the Hewitt real compactification ν for completely regular spaces. The latter two extension processes are reflections for the category of all completely regular spaces and continuous functions. However, we have the following Theorem. The Katětov extension is not a reflection for the category of all Hausdorff spaces and continuous functions. Definition. A function $f: X \longrightarrow Y$ is called semi-open (resp. demi-open) provided that for each $A \subset X$,

$A^0 \neq \emptyset \Rightarrow (f(A))^0 \neq \emptyset$ (resp. $A^0 \neq \emptyset \Rightarrow (f(A))^{-0} \neq \emptyset$). Theorem. For the category of all Hausdorff spaces and continuous semi-open (resp. demi-open) functions, the Katětov extension is a reflection. Thus for this category τ is a functor such that the correspondence which assigns to each Hausdorff space X, the natural embedding r: $X \longrightarrow \tau X$ is a natural transformation from the identity functor to τ.
(Received September 18, 1967.)

653-75. KWANGIL KOH, North Carolina State University, Raleigh, North Carolina 27607. On a structure theory of a projective generator for an exact category with unions.

Let E be an exact category with zero and U be an object in E. Let S(U) be the family of subobjects of U. We say, $N \in S(U)$ is superfluous in U provided that $N \cup I = U$ for any $I \in S(U)$ implies that $I = U$. Theorem A. If f is a morphism from U to M for some $M \in E$ then f(I) is superfluous in M if I is superfluous in U. Theorem B. Suppose that if I is a proper subobject in U then there exists a maximal $J \in S(U)$ such that $J \geqq I$. Let Σ be the family of all maximal subobjects of U. If $\bigcap_{I_\alpha \in \Sigma} I_\alpha$ exists, then it is the largest superfluous subobject of U and U/J(U), where $J(U) = \bigcap_{I_\alpha \in \Sigma} I_\alpha$, is semisimple in the sense that zero is the only superfluous subobject of U/J(U). Theorem C. Let U be a projective generator of E in the sense that it is projective and if $M \in E$ then $M = \bigcup_{g \in [U,M]} g(U)$ where [U, M] = the set of morphisms from U to M. Furthermore, assume that if $M = A \cup B$, $A \cap B = 0$, then $A \cup B$ is a coproduct. If U is semisimple and satisfies the minimum condition on the subobjects then U is a coproduct of a finite number of minimal subobjects. If for each $M \in E$, S(M) is relatively complemented, in the sense that if $I \in S(M)$ there exists $J \in S(M)$ such that $I \cap J = 0$ and $I \cup J$ is essential, then every object of E is injective and projective. (Received September 19, 1967.)

653-76. THEODORE MITCHELL, Temple University, Philadelphia, Pennsylvania. Fixed points and separately continuous actions of topological semigroups. Preliminary report.

Let S be a topological semigroup with a Hausdorff topology and separately continuous product. Let LMC(S) (respectively, WLUC(S)) be the space of those bounded continuous real-valued functions f on S such that for each multiplicative mean $\mu \in C(S)^*$ (respectively, each element $\mu \in C(S)^*$ and each $s \in S$, if $s(n) \longrightarrow s$, then $\mu(l_{s(n)} f) \longrightarrow \mu(l_s f)$. Theorem 1. LMC(S) has a multiplicative left invariant mean iff whenever S acts on a compact Hausdorff space, where the map $S \times Y \longrightarrow Y$ is separately continuous, then Y contains a common fixed point of S. Theorem 2. WLUC(S) has a left invariant mean iff whenever S acts affinely on a convex compact subset Y of a locally convex space, where the map $S \times Y \longrightarrow Y$ is separately continuous, then Y contains a common fixed point of S. (For the jointly continuous cases, see Abstract 66T-444 and 66T-448, these *Notices* 13 (1966), 724 and 725.) (Received September 19, 1967.)

653-77. MICHAEL ANSHEL and J. R. CLAY, The University of Arizona, Tucson, Arizona 85721. Geometric interpretations of planar near-rings.

A near-ring $(N, +, \cdot)$ is planar if for every a, b, c $\in$ N such that at $\neq$ bt for some t $\in$ N, the equation $ax = bx + c$ has a unique solution for x in N. Many examples, both finite and infinite, have geometric interpretations. In particular, the points of some planar near-rings can be interpreted as the points of an affine plane. The following theorem is significant. Theorem. If $(N, +, \cdot)$ is a planar near-ring without zero divisors, then there are multiplicative subgroups $(B_a, \cdot)$ of the semi-group $(N^*, \cdot)$, where $N^* = N - \{0\}$, such that $N = \{0\} \cup (\bigcup_{a \in N^*} B_a)$, the B_a are pairwise disjoint, and the B_a are isomorphic as multiplicative groups. In some cases, each $B_a \cup \{0\}$ can be interpreted as a ray, a straight line, or a circle. (Received September 18, 1967.)

653-78. ERNEST MICHAEL, University of Washington, Seattle, Washington 98105. Bi-quotient maps and products of quotient maps.

Call a continuous surjection $f: X \to Y$ bi-quotient if, whenever $y \in Y$ and $\mathscr{U}$ is a covering of $f^{-1}(y)$ by open subsets of x, then finitely many f(U), with $U \in \mathscr{U}$, cover some neighborhood of y in Y. All continuous open surjections and all proper (= perfect) surjections are bi-quotient, and all bi-quotient maps are quotient maps. Theorem 1. If all $f_\alpha : X_\alpha \to Y_\alpha$ are bi-quotient, then so is their cartesian product $\prod_\alpha f_\alpha : \prod_\alpha X_\alpha \to \prod_\alpha Y_\alpha$. Theorem 2. Let $f: X \to Y$, with Y Hausdorff. (a) $f \times i_Z$ is a quotient map for any space Z if and only if f is bi-quotient. (b) $f \times g$ is a quotient map for any quotient map g if and only if Y is locally compact and f is bi-quotient. (Received October 30, 1967.)

653-79. MICHAEL ANSHEL, The University of Arizona, Tucson, Arizona 85721. The endomorphisms of some one-relator groups.

G. Baumslag and D. Solitar [Bull. Amer. Math. Soc. 68 (1962), 199-201] published the first examples of non-Hopfian, finitely generated one-relator groups. In this paper we characterize the endomorphisms of a class of groups studied in their paper. In the process we study some properties of conjugacy of these groups, compute the centralizers of certain special elements and apply these results to 'Generalized Hopfian Problems'. (Received September 18, 1967.)

653-80. W. S. MARTINDALE, 3rd, University of Massachusetts, Amherst, Massachusetts. Lie isomorphisms of simple rings.

A previous result of ours [Proc. Amer. Math. Soc. 14 (1963), 916, Theorem 5] is generalized as follows. Theorem. Let ϕ be a Lie isomorphism of a simple ring R onto a simple ring R', where the characteristic of R is different from 2 and 3 and R contains two nonzero orthogonal idempotents whose sum is the identity. Then ϕ is of the form $\sigma + \tau$, where σ is either an isomorphism or the negative of an anti-isomorphism of R onto R' and τ is an additive mapping of R into the center of R' which maps commutators into zero. (Received September 20, 1967.)

653-81. D. J. NEWMAN, Yeshiva University, New York, New York 10033, and LOUIS RAYMON, Temple University, Philadelphia, Pennsylvania 19122. Quantitative polynomial approximation on certain planar sets.

On the restriction of the point set of any algebraic curve in the plane to a square, the order of magnitude of the degree of approximation of properly normalized continuous functions (contractions) by nth degree polynomials (in any norm) is $1/n$. (See G. G. Lorentz, Approximation of functions, Holt, 1966, for a definition of the degree of approximation.) However, on the curve $y = e^x$, $0 \leqq x \leqq 1$, the order of magnitude of the error can be improved in the L^2 norm, to exactly $1/n^{3/2}$. A similar theorem is proved for the curve $y = x^{\alpha}$ for certain transcendental α. Furthermore, there is a class of curves on which the degree of approximation is improved to $1/n^2$ (even in the continuous norm)--best possible for any n^2-dimensional subspace of continuous functions. (Received September 20, 1967.)

653-82. H. G. MOORE, Brigham Young University, Provo, Utah 84601, and A. M. YAQUB, University of California, Santa Barbara, California 93106. A generalization of Boolean rings.

A ring R is called a B-ring if $x \in R$ implies x is nilpotent or x is idempotent. The following results are proved. Theorem 1. Let R be a B-ring, and let J be the Jacobson radical of R. Then R/J is a Boolean ring and the commutator ideal C of R is nil. Theorem 2. Suppose R is a B-ring and suppose R has at least one nonzero idempotent e. Suppose also that the idempotents of R commute with each other. Then R is a Boolean ring. Finally, an example is given of a B-ring R where R is not commutative. (Received September 20, 1967.)

653-83. J. A. MORRISON, Bell Telephone Laboratories, Incorporated, Murray Hill, New Jersey 07971. An averaging scheme for some nonlinear resonance problems.

A perturbed vector system of differential equations, containing a rapidly rotating phase, is considered. The zero order rate of change of a second variable is a function of the remaining, slowly changing, variables, and vanishes on a surface. Such a system arises, in particular, in the investigation of resonance phenomena in the essentially nonlinear forced vibrations of a one degree of freedom system which depends on slowly varying parameters. An averaging scheme is developed which involves expansion in the perturbation parameter, together with an expansion in the neighborhood of the resonance surface. This procedure provides a uniform approach for analyzing the motion of the system in the neighborhood of resonance, and it is shown how the averaging scheme presented here is related to other ones which have been used in resonance investigations. (Received September 20, 1967.)

653-84. FRITZ POKROPP, University of Cincinnati, Cincinnati, Ohio 45221. Coupled maps on groups and derived structures.

Let $G(\cdot)$ be a group, $S(G)$ its symmetric group. A map ϕ: $G \longrightarrow S(G)$; $a \longrightarrow a_{\phi}$ is called a coupled map of G iff $a_{\phi}b_{\phi} = (a \cdot a_{\phi}(b))_{\phi}$ for all $a, b \in G$. Define a new operation $\circ$ on G by $a \circ b = a \cdot a_{\phi}(b)$. What structure does the ϕ-derivative $G(\circ)$ have? The kernel $K(\phi) = \{a \in G\colon\ a_{\phi} = \text{identity}\}$ is a subgroup of G. The image $I(\phi) = \{a_{\phi}\colon\ a \in G\}$ is a subgroup of $S(G)$. If $K(\phi)$ is a normal subgroup then G(∘) is a quasigroup. G(∘) is a group iff I(ϕ) is a subgroup of the automorphismgroup of G(·). Some (particularly important) coupled maps are homomorphisms on G(·). All coupled maps on finite zyclic groups onto zyclic automorphismsubgroups are determined. An application for the construction of finite near-fields is mentioned. (Received September 24, 1967.)

653-85. E. L. KOH, University of South Carolina, Columbia, South Carolina, and A. H. ZEMANIAN, State University of New York at Stony Brook, Stony Brook, New York. The complex Hankel and I-transformations of generalized functions.

Let a be a positive real number and μ any real number. Let $S_\mu^k = (x^{-\mu-1/2}Dx^{2\mu+1}Dx^{-\mu-1/2})^k$, k = 0, 1, 2,... . Consider the space $J_{\mu,a}$ of smooth, complex-valued functions $\phi(x)$ on $(0,\infty)$ such that for every nonnegative integer k, $\tau_k^{\mu,a}(\phi) = \sup_{0<x<\infty} |e^{-ax}x^{-\mu-1/2}S_\mu^k(\phi)| < \infty$. $J_{\mu,a}$ is a first countable, complete, Hausdorff, locally convex, linear space topologized by the countable multinorm $\{\tau_k^{\mu,a}\}_k$. Its dual $J'_{\mu,a}$ consists of all continuous linear functionals on $J_{\mu,a}$. For $f \in J'_{\mu,a}$, the Hankel transform $H_\mu f$ is defined by $(H_\mu f)(y) = \langle f(x), (xy)^{1/2} J_\mu(xy)\rangle$ where y is restricted to the cut strip $\Omega_f = \{y: |\mathrm{Im}\, y| < a, y \neq 0$ or a negative number$\}$. This definition, while not as general as that given by Parseval's equation (Zemanian, J. Siam Appl. Math. 14 (1966), 561), is a direct extension of the classical transformation because the kernel appears explicitly. It is also more convenient for specific computation. Theorems on analyticity, boundedness, inversion and uniqueness, together with an operation-transform formula, are given. An apparently new transformation, viz., the μth order I-transformation is defined for $f \in J'_{\mu,a}$ by $(T_\mu f)(s) = \langle f(x), (xs)^{1/2} I_\mu(xs)\rangle$ where s is a suitably restricted complex parameter. The Hankel and I-transformations are related through a change of variable. An inversion theorem for the I-transformation is given. (Received September 21, 1967.)

653-86. K. W. ROGGENKAMP, University of Illinois, Urbana, Illinois 61801. Extensions of lattices over orders.

Let R be the ring of algebraic integers in an algebraic number field K, A a finite dimensional semisimple K-algebra, and G an R-order in A. A G-lattice is a unitary left G-module, finitely generated and torsion-free over R. We say that a G-endomorphism f of a G-lattice M is projective if $\mathrm{Ext}_G^1(M, X)\cdot f = 0$ for every G-lattice X. By L(M) we denote the set of projective G-endomorphisms of the G-lattice M. Let C be the maximal R-order in the center of A, and let $\{B_j\}$ be the maximal R-orders in A containing G. Define $J(B_i/G) = \{Z \in C: zB_i \subset G\}$ and let J(G) be the C-ideal generated by $J(B_i/G)$ for all B_i. Then $J(G)\cdot \mathrm{Ext}_G^1(M, N) = 0$ for all G-lattices M and N (cf. H. Jacobinski, Mich. Math. J. 13 (1966), 471-475). Put $F_L(B_j) = \{a \in A: B_j a \subset G\}$. Theorem. If for some B'_j, $F_L(B'_j)$ is a two-sided B_j-ideal, then $J(G) = \{x$: x in the center of G, $x\cdot \mathrm{Ext}_G^1(M,N) = 0\} = F_L(B'_j) \cap C = C \cap (\bigcap_M L(M))$, where the intersection is taken over all G-lattices M. (Received September 21, 1967.)

653-87. G. S. UNGAR, Louisiana State University, Baton Rouge, Louisiana 70803. Local homogeneity.

Two types of local homogeneity are defined and the following theorems are proved. (1) If X is a compact homogeneous uniform space then the induced topology is a reasonable topology (L. R. Ford, Homeomorphism groups and coset spaces, Trans. Amer. Math. Soc. 77 (1954), 490-497) for H(X) iff X is uniformly locally homogeneous. (2) The finite product of compact reasonable spaces is reasonable. (3) If X is a 2 locally homogeneous, locally arcwise connected compact metric space, then X is locally contractible. (4) If X is a 1 locally homogeneous Peano continuum, then X is a coset space of a topological group which is generated by a Peano continuum. (Received September 21, 1967.)

653-88. S. D. COMER, Vanderbilt University, Nashville, Tennessee 37203. Galois theory and the amalgamation property in finite dimensional cylindric algebras. Preliminary report.

For notation see S. D. Comer (Abstract 67T-251, these *Notices* 14 (1967), 291). Theorem 1. For $\alpha < \omega$ substitution operations can be defined in any simple CA_α of nonzero characteristic making it into an α-dimensional polyadic equality algebra (a PEA_α). Under this correspondence a CA_α set algebra becomes a PEA_α set algebra. As a corollary of Theorem 1 Krasner's generalized Galois theory can be extended from PEA_α's to finite dimensional CA_α's of nonzero characteristic. Let $\mathscr{K}_\alpha$ be the smallest class of CA_α's containing the full set algebras with base U where $0 < |U| < \alpha$ and which is closed under isomorphisms, direct products, and retracts. Theorem 2. For $1 < \alpha < \omega$ $\mathscr{K}_\alpha$ is the class of CA_α-injective algebras. A similar result holds for PEA_α's. Theorem 3. For $1 < \alpha < \omega$ the amalgamation property holds in the class of all CA_α's (PEA_α's) which satisfy the equation $d(\alpha \times \alpha) = 1$ (cf., above reference). (Received September 21, 1967.)

653-89. R. B. SIMPSON, California Institute of Technology, Pasadena, California 91109. Convergence of dynamic relaxation (second order Richardson's method).

Dynamic relaxation is the iteration process for solving the linear system $Au = f$ which, from two arbitrary starting values u_0 and u_1, proceeds according to (*) $(1 + c)u_{n+1} = 2u_n - (1 - c)u_{n-1} + \gamma(-Au_n + f)$. Here c and γ are parameters to be chosen. This method is proposed as an efficient stationary process for systems of finite difference equations for boundary value problems of higher than second order. For A positive definite and $c > 0$, (*) is convergent iff $0 < \gamma < 4(\lambda_N)^{-1}$ where λ_N, (λ_1) is the largest, (smallest) eigenvalue of A. For the optimal choice of c, γ(Frankel MTAC, pp. 65-75, 1950), we have $O(n\tau^n) = \|u_n - u\| \neq o(n\tau^n)$, in general, for $\tau = (\lambda_N)^{1/2} - (\lambda_1)^{1/2}/(\lambda_N)^{1/2} + (\lambda_1)^{1/2}$. If $\{\tilde{u}_n\}$ is the sequence calculated by (*) with the addition of a round off perturbation of norm less than ϵ, we have $\|\tilde{u}_n - u_n\|_M \leqq \epsilon(1 - \tau)^{-2}$. For each decomposition of $A = D_1 + D_2$, we obtain a successive iteration process of the form (**) $(1 + c)u_{n+1} = 2u_n - (1 - c)u_{n-1} + \gamma(- D_1u_{n+1} - D_2u_n + f)$ which includes various methods of using the new values as soon as they are computed. If the symmetric part of $D_1 - D_2^t$ is positive semi definite, (**) converges for any $\gamma > 0$ and $0 < c < 1$. Application to a model biharmonic problem is considered. (Received September 25, 1967.)

653-90. MARSHALL SAADE, University of Georgia, Athens, Georgia 30601. Some varieties of groupoids.

This paper extends a result of Evans (The spectrum of a variety, Z. Math. Logik und Grundlagen Math. (to appear)). Let n be an integer > 2 and let V be the variety of groupoids defined by the following two identities (using for simplicity of notation $u_1u_2 \ldots u_m$ to denote the product $(u_1(u_2(\ldots (u_{m-2}(u_{m-1}u_m)))\ldots)))$: (i) $x_1x_2 \ldots x_ny = x_1x_2 \ldots x_nz$ and (ii) $(x_1 \ldots x_{n-1}x)(x_n \ldots x_{2n-3}x) \ldots (x_{p-2}x_{p-1}x)(x_px)(xy) = x$, where $p = n(n-1)/2$. Theorem. Let $G \in V$ and let S be the set of idempotents of G. Then G is isomorphic to the groupoid on S^n, the set of all n-tuples of elements of S, with multiplication on S^n defined by $(a_1,\ldots,a_n) \cdot (b_1,\ldots,b_n) = (a_n,b_1,\ldots,b_{n-1})$. It now follows easily that V is precisely the class of all groupoids $(S^n, \cdot)$, where S is any nonempty set and "$\cdot$" is defined on S^n as in the theorem above. (Received September 22, 1967.)

653-91. HERMANN SIMON, University of Miami, Coral Gables, Florida 33124. On a class of solvable groups of even order.

Definition. Let $\underline{S}$ be a set of subgroups of a group G and T a subgroup of G; we say that $\underline{S}$ induces a partition on T iff the set of all $1 \neq S \cap T$ for all S in $\underline{S}$ is not empty and forms a partition of T. Theorem. The following properties of the finite group G and its maximal subgroup M with $Z(M) \neq 1$ and $G_2 < M$ are equivalent: (I) G is solvable and either (i) M is an HT-group whose HT-kernel K is normal in G and G/K is not cyclic of order p^2, or (ii) G is the holomorph of an elementary abelian p-group P with respect to a dihedralgroup M of automorphisms acting irreducibly on P, $p \nmid |M|$ and $|G| = 4u$, u odd. II. There exists a set $\underline{S}$ of subgroups of G such that $\underline{S}$ induces a nontrivial partition $\mathfrak{N}$ on M; $M^g \in \underline{S}$ for all $g \in G$; if $Y \in \underline{S}$ and $Y \cap M \in \mathfrak{N}$ is maximal in M, then $Y \cap M \neq Y$. If M is an HT-group whose HT-kernel K is not normal in G, then $Z(M) \nsubseteq M'$. (Received September 22, 1967.)

653-92. J. K. BROOKS, The University of Florida, Gainesville, Florida, and P. W. REICHELDERFER, The Ohio State University, Columbus, Ohio. A Jordan decomposition for weight functions.

Let T be a transformation from a measure space S onto a measure space S'. For a certain subset $\mathscr{D}$ of the measurable sets in S, the role of T may be described by a function W' defined on $S' \times \mathscr{D}$ having suitable properties (see Abstract 621-21, these *Notices* 12 (1965), 309); W' is called a weight function for T. In many applications it would be desirable to decompose a real valued weight function into nonnegative components. To accomplish this the total, positive, and negative variations (V', V'_+, and V'_-) of W' relative to $\mathscr{D}$ are defined. Conditions are given to insure that each of these functions is a nonnegative weight function for T and that they constitute a Jordan decomposition of W' in the sense that the relations $V' = V'_+ + V'_-$, $W' = V'_+ - V'_-$, hold almost everywhere uniformly with respect to $\mathscr{D}$. Applications to transformation theory are given. (Received September 22, 1967.)

653-93. PAUL HILL, University of Houston, Houston, Texas, and CHARLES MEGIBBEN, Vanderbilt University, Nashville, Tennessee 37203. On certain classes of primary abelian groups.

Let λ be a countable limit ordinal and let C_λ denote the class of reduced p-primary abelian groups G such that $G/p^\alpha G$ is a direct sum of countable groups for all $\alpha < \lambda$. The C_λ for $\lambda > \omega$ seem in some respects to be the suitably restricted classes to yield more powerful generalizations of the standard results for C_ω-- the class of all reduced abelian p-groups. Among the results obtained for C_λ we have p^λ-purification of infinite subgroups, an analog of the Kulikov criterion and a generalization of the notion of basic subgroups with the role of direct sums of cyclic groups being played by direct sums of countable groups. For example, a reduced abelian p-group G contains a subgroup B such that (i) B is a direct sum of countable groups of length $\leqq \lambda$, (ii) B is p^λ-pure in G and (iii) G/B is divisible if and only if G is a member of the class C_λ. Essential difficulties are encountered in attempting to extend considerations to the first uncountable ordinal. (Received September 22, 1967.)

653-94. N. M. WIGLEY, University of North Carolina, Chapel Hill, North Carolina 27514. On the singularities of biharmonic functions with discontinuous boundary values. Preliminary report.

Let u(x,y) be biharmonic ($\Delta\Delta u = 0$) in the domain $D: x^2 + y^2 < 1, y > 0$, and let u have the boundary values $u(x,0) = f_1(x)$, $u_y(x,0) = g_1(x)$ if $1 > x > 0$; and $u(x,0) = f_2(x)$, $u_y(x,0) = g_2(x)$ if $-1 < x < 0$. Let f_1 and g_1 be C^∞ on $[0,1]$ and f_2 and g_2 be C^∞ on $[-1,0]$. In addition let u(x,y) be bounded in D. Then u(x,y) is asymptotic (as $z \to 0$, $\operatorname{Im} z \geqq 0$) to $z/\bar{z}\, P_1 + \bar{z}/z\, P_2$ where P_1 and P_2 are formal power series in $\bar{z}$, z, $z \log z$ and $\bar{z} \log \bar{z}$. (Received September 22, 1967.)

653-95. L. V. TORALBALLA, New York University, Bronx, New York 10453. Piecewise flatness and surface area.

By the deviation on a triangle T inscribed on a nonparametric surface S is meant the LUB of the acute angles between the normal to T and the normal to any triangle T' inscribed on S whose xy projection is a subset of that of T. By the norm of a triangular polyhedron P inscribed on S is meant the greatest of the diameters of its faces. By the deviation norm of P is meant the greatest of the deviations on its foci. A sequence (P_n) of triangular polyhedra inscribed on S is regular if there exists α, $0 < \alpha < \pi/2$ and $m > 0$ such that (1) Each face of each polyhedron P_n has an angle between α and $\pi - \alpha$; (2) if θ is the angle between the z-axis and the normal to a face of P_n, then $\sec\theta < m$; (3) the corresponding sequence of norms as well as the corresponding sequence of deviation norms converges to zero. S is regular if S admits a regular sequence of inscribed polyhedra. The principal result is that if S is regular then the corresponding sequences of polyhedral areas converges to a unique limit. (Received September 22, 1967.)

653-96. SEYMOUR LIPSCHUTZ, Temple University, Philadelphia, Pennsylvania 19119. Powers in generalized free products.

Let G be the free product of free groups with an infinite cyclic group amalgamated. An algorithm is given to determine whether or not an arbitrary element of G is a power. This result generalizes previous results of Lipschutz (Powers in eighth-groups, Proc. Amer. Math. Soc. 16 (1965), 1105-1106) and Reinhart (Algorithms for Jordan curves on compact surfaces, Ann. of Math. (2) 75 (1962), 209-222). (Received September 22, 1967.)

653-97. D. R. LICK, Western Michigan University, Kalamazoo, Michigan 49001. On I-series.

The author defines an I-series to be a series of the type $\sum_{n=1}^{\infty} a_n I(z,\alpha,r_n,n)$, where $I(z,\alpha,r_n,n) = 1/(1 - zre^{-2\pi i n\alpha})$, α is a positive irrational, and $\{r_n\}$ is a Blaschke sequence of positive reals. Sets of convergence and sets of divergence are defined in the usual manner. The following two main theorems are proved. (1) Every set of type F_σ on the unit circle is a set of convergence of an I-series whose sum function is continuous in the closed unit disc. (2) Every denumerable set on the unit circle is a set of divergence of an I-series whose sum function is continuous in the closed unit disc. (Received September 26, 1967.)

653-98. R. B. S. BROOKS, Bowdoin College, Brunswick, Maine, and R. F. BROWN, University of California, Los Angeles, California. A lower bound for the Δ-Nielsen number.

Let $f,g: X \to Y$ be maps of compact ANR's, then there is a nonnegative integer $N(f,g,\Delta)$ (the Δ-Nielsen number) with the property that if f' is homotopic to f and g' is homotopic to g then there are at least $N(f,g,\Delta)$ points $x \in X$ such the $f'(x) = g'(x)$. Let $f_*, g_*: \pi_1(X) \to \pi_1(Y)$ be the induced homomorphisms and define an equivalence relation $\sim$ on $\pi_1(Y)$ by : $\alpha \sim \beta$ if $f_*(\gamma)\alpha = \beta g_*(\gamma)$ for some $\gamma \in \pi_1(Y)$. Let $R(f,g)$ be the number of equivalence classes then, as in fixed point theory, $N(f,g,\Delta) \leqq R(f,g)$. For maps $F,G: I \to \mathrm{Map}(X,Y)$ with $F(0) = F(1) = f$, $G(0) = G(1) = g$, define $a(t) = F(2t)(x_0)$, $0 \leqq t \leqq 1/2$, $a(t) = G(2t-1)(x_0)$, $1/2 \leqq t \leqq 1$. Let $T(f,g,\Delta) \subseteq \pi_1(Y)$ be all elements represented by such loops a. There is an index $\omega(f,g) \in G$ (a certain abelian group) generalizing the Lefschetz coincidence number. Let $J(f,g,\Delta)$ be the cardinality of $T(f,g,\Delta)/\sim$. Theorem. If $\omega(f,g) \neq 0$ then $J(f,g,\Delta) \leqq N(f,g,\Delta)$. An integer n divides $g \in G$ if $ng' = g$ for some $g' \in G$. Theorem. If $\omega(f,g) \neq 0$ and $\pi_1(Y)$ is abelian, then $J(f,g,\Delta)$ divides $N(f,g,\Delta)$, $R(f,g)$, and $\omega(f,g)$. Similar results are obtained in fixed point theory and in the study of the number of solutions to $f(x) = y_0$ for fixed $y_0 \in Y$. (Received September 25, 1967.)

653-99. FRANK CASTAGNA, New Mexico State University, Las Cruces, New Mexico 88001. Sums of automorphisms of a primary abelian group.

At the 1962 symposium on abelian groups held at New Mexico State University, L. Fuchs posed the following problem "For which abelian groups G does A(G) generate E(G)?" (Topics in abelian groups, Chicago, 1963, p. 16.) Here E(G) and A(G) are, respectively, the ring of endomorphisms and the group of automorphisms of G. In this paper it is shown that if $p > 2$ then for a large class of p-primary groups G it is true that A(G) generates E(G). Theorem. If G is a reduced p-primary,

($p > 2$), abelian group which is a direct sum of countable groups then every endomorphism of G is a sum of two automorphisms. As the Corollary of this result, it is also shown that if B is a direct sum of cyclic groups and $\bar{B}$ is the torsion completion of B then every element of $E(\bar{B})$ is a sum of two automorphisms. Finally, an example is given (for an arbitrary prime p) of a reduced p-primary abelian group G for which there are endomorphisms in E(G) that are not sums of automorphisms.
(Received September 25, 1967.)

653-100. C. D. SIKKEMA, Florida State University, Tallahassee, Florida 32306. Cellular arcs in 3-space have shrinking points.

Let X be a subset of R^n and let $x \subset X$. Then x is a shrinking point of X if given any neighborhood U of X - x there is a map g of R^n onto itself such that g is the identity outside of U and $X = g^{-1}(x)$ is the only nondegenerate inverse set for g. Theorem. An arc in R^3 is cellular if and only if it has at least one shrinking point. The proof uses some of the techniques of McMillan [A criterion for cellularity in a manifold, Ann. of Math. 79 (1964), 327-337]. Theorem. The set of shrinking points of an arc in R^3 is a single point or a subarc. (Received September 25, 1967.)

653-101. R. B. McNEILL, 703 Puddintown Road, State College, Pennsylvania 16801. Elementary theory of differential systems. Preliminary report.

Consider the nth order differential systems $(E_f)x' = A(t)x + f(t,x)$ and $(E)y' = - yA(t)$, where x and y are $n \times 1$ and $1 \times n$ vectors, respectively, f(t,x) is an $n \times 1$ vector, and A(t) is an $n \times n$ matrix whose elements, $a_{ij}(t)$, are real-valued functions of the real variable t, $t \geqq t_0$. It is assumed that the above systems are "well-behaved" enough to possess solutions. Using elementary means, Lagrange's Variation of Parameters Formula is derived independent of the usual linear considerations, and related results are given. These results are extended to certain systems of the form (E_{f+g}).
(Received September 25, 1967.)

653-102. M. R. KIRCH, Lehigh University, Bethlehem, Pennsylvania 18015. On a class of extremally disconnected spaces. Preliminary report.

Levine (Amer. Math. Monthly 70 (1963), 36-41) has defined a subset in a topological space to be semi-open if it is contained in the closure of its interior. Denote by L(T) the collection of semi-open sets in the space (X,T). (X,T) is said to be L-maximal if L(T) = T. Every L-maximal space is extremally disconnected (not necessarily Hausdorff). Theorem 1. If (X,T) is any topological space then there exists an L-maximal topology T' for X such that $T \subset T' \subset L(T)$. A topological space is said to be quasi-maximal if it has no isolated points and every strictly stronger topology has an isolated point. Quasi-maximal spaces are L-maximal. Theorem 2. Let (X,T) be a topological space with infinite dispersion character. There exists a quasi-maximal topology T' for X such that (a) T' is stronger than T, (b) the dispersion character of T' is the same as that of T, and (c) If S is any T'-open set then $cl_T S = cl_T \, int_T \, cl_T \, S$. (Received September 25, 1967.)

653-103. K. C. SALTER, University of Massachusetts, Amherst, Massachusetts 01002. On G-algebra extensions. Preliminary report.

Let G be a group. A G-algebra is an algebra over a commutative ring K on which G acts as automorphisms; a G-algebra homomorphism is an algebra homomorphism that commutes with the action of G. Let A and R be G-algebras with $R^2 = 0$. There is an obvious map, ϕ, from the group of equivalence classes of G-algebra extensions of R by A, $E_G(A,R)$, to the group of equivalence classes of algebra extensions, $E(A,R)$. The kernel of ϕ is $H^1(G,\mathrm{Der}(A,R))$ where $\mathrm{Der}(A,R)$ is the group of derivations from A to R. Let $E_G'(A,R)$ be the subgroup of $E_G(A,R)$ which consists of equivalence classes of extensions that split as KG-module extensions. The kernel of the map, ψ, from $E_G'(A,R)$ to $E(A,R)$ is zero if $\mathrm{Der}(A,R)$ is KG-injective or if $\mathrm{Hom}_K(A,R)$ is a completely reducible KG-module. (Received September 25, 1967.)

653-104. F. W. WILSON, JR., University of Colorado, Boulder, Colorado 80302. Lipschitz submanifolds.

It seems reasonable to call a topological manifold a Lipschitz manifold if the coordinate charts are regular Lipschitz homeomorphisms. There is then a problem of what to call a Lipschitz submanifold. By analogy with the case of differentiable manifolds we have a choice of either requiring that the coordinate charts be induced by Lipschitz implicit relations, or that the inclusion be a regular Lipschitz homeomorphism. Although these requirements are equivalent in the differentiable case, a simple example shows that they are not equivalent in the Lipschitz case. (Received September 25, 1967.)

653-105. R. E. DE MARR, University of Washington, Seattle, Washington 98105. On a square root function.

Let A be a partially ordered linear algebra with a positive multiplicative identity 1 and let K be the positive cone in A. If there exists an isotone function $f: K \longrightarrow K$ such that $f(1) = 1$ and $f(x)^2 = x$ for all $x \in K$, then A is order isomorphic to a partially ordered linear algebra of real-valued functions. (Received September 25, 1967.)

653-106. F. HARARY, University of Michigan, Ann Arbor, Michigan 48104, and E. M. PALMER, Michigan State University, East Lansing, Michigan. On the point-group and line-group of a graph.

The group $\Gamma(G)$ of a graph G is a permutation group which acts on the points of G. It induces another permutation group $\Gamma_1(G)$ which acts on the lines of G. Theorem 1. $\Gamma(G)$ and $\Gamma_1(G)$ are isomorphic (as abstract groups) if and only if G has at most one isolated point and K_2 (the complete graph on two points) is not a component of G. Theorem 2. If G is connected then $\Gamma(G)$ and $\Gamma_1(G)$ are identical (as permutation groups) if and only if (1) G is unicyclic and (2) if its cycle length is even, then (a) there are no reflections in $\Gamma(G)$ which fix more than two points and (b) the number of reflections in $\Gamma(G)$ which fix exactly two points is the same as the number of reflections in $\Gamma_1(G)$ which fix exactly two lines. (Received September 25, 1967.)

653-107. W. R. CALLAHAN, St. John's University, Jamaica, New York. Frequency equations for the normal modes of vibration for an elliptical ring, including transverse shear and rotary inertia.

The research work of Mindlin, Reissner, and Uflyand on plates is extended to the actual finding of the frequency equations for the normal modes of vibrations of an elliptical ring. Product solutions are assumed for the partial differential equations of motion and it is found that infinite series of product solutions satisfies the boundary conditions; the resulting frequency equation for each boundary condition appears as an infinite determinant, equated to zero, the elements of which are infinite series of terms involving Mathieu functions whose characteristic numbers are functions of the frequency. An algorithm is given showing how to calculate the roots of the infinite determinant that gives the normal modes of vibration. The present theory is more valid for higher modes of vibrations than the classical Lagrange theory of plates since it includes coupling between flexural and shear motions, and the numerical results, if carried out on high speed machines, should be valid for these high modes. (Received September 25, 1967.)

653-108. P. S. SCHNARE, University of Florida, Gainesville, Florida 32601. Infinite complementation in the lattice of topologies.

The author has shown [Multiple complementation in the lattice of topologies, Fund. Math. (to appear)] that every proper topology on an infinite set X has infinitely many principal complements in the lattice of all topologies, Σ, on X. The question naturally arises: what is the cardinality of this set of principal complements (resp., the set of all complements)? The complete answer is given in the following Theorem. Every proper topology on an infinite set X has at least $|X|$ complements (resp., principal complements) and at most $2^{2^{|X|}}$ complements (resp., $2^{|X|}$ principal complements). Moreover, these estimates are best possible. The difficult part of the proof, establishing that $|X|$ is a lower bound, although elementary, results from a series of reductions involving a fairly complicated case and subcase analysis. In proving that the bounds are best possible, the following is proved. Theorem. Every ultraspace on an infinite set X has $2^{2^{|X|}}$ complements and $2^{|X|}$ principal complements. (Received September 26, 1967.)

653-109. WITHDRAWN.

653-110. G. CHARTRAND, Western Michigan University, Kalamazoo, Michigan, H. V. KRONK, State University of New York at Binghamton, Binghamton, New York, and C. E. WALL, Michigan State University, East Lansing, Michigan. On the point-dual acyclic numbers of a graph.

The point-arboricity $\rho(G)$ of a graph G is defined as the minimum number of subsets in a partition of the point set of G so that each subset induces an acyclic subgraph. Dually, the tulgeity $\tau(G)$ is the maximum number of disjoint, point-induced, nonacyclic subgraphs contained in G. Several results concerning these numbers are presented, among which are formulas for the point-arboricity and tulgeity of the class of complete n-partite graphs. (Received September 26, 1967.)

653-111. D. F. DAWSON, North Texas State University, Denton, Texas. Matrix summability of convex sequences.

Matrices considered here have complex elements, and convex sequences are real sequences whose second differences are nonnegative. Theorem. A matrix $A = (a_{pq})$ sums every convergent convex sequence if and only if the following three conditions hold. (1) A has convergent columns, (2) the sequence of row sums of A is convergent, and (3) there exists a number k such that $|\sum_{j=1}^{n}\sum_{q=1}^{j} a_{pq}| < nk$, $p,n = 1,2,3,\ldots$. (Received September 27, 1967.)

653-112. S. E. PAYNE, Miami University, Oxford, Ohio 45056. On the nonexistence of a class of configurations which are nearly generalized n-gons.

Let P be a finite incidence plane with each point (line) incident with s + 1 lines (points). Let n be the smallest positive integer $k \geqq 3$ such that a closed irreducible chain of length 2k exists in P. If v is the number of points (and lines) in P, it is known that $v \geqq 1 + s + \ldots + s^{n-1}$ with equality holding if and only if P is a generalized n-gon (c.f. W. Feit and G. Higman, The nonexistence of certain generalized polygons, Journal of Algebra 1 (1964), 114-131). In this paper a study of the minimal polynomial of a certain incidence matrix for P yields conditions on n and s which imply the nonexistence of P when $v = 2 + s + \ldots + s^{n-1}$. Such P are then shown not to exist for n = 4,5,6, and that there are at most five values of s which might allow existence when n = 7. We conjecture that P never exists when $s > 3$ or $n > 3$. Simple cyclic examples exist for n = 3, s = 2,3. (Received September 27, 1967.)

653-113. T. A. BURTON and C. G. TOWNSEND, Southern Illinois University, Carbondale, Illinois 62901. On the generalized Lienard equation.

Consider the differential equation (1) $x'' + f(x)x' + g(x) = e(t)$ where $f(x) > 0$, $xg(x) > 0$ for $x \neq 0$, f(x) and g(x) are continuous for all x, $\int_0^t e(s)ds$ is continuous and periodic with smallest positive period T. Theorem 1. Suppose that $\int_0^x f(s)ds$ becomes unbounded as $x \to +\infty$ or as $x \to -\infty$. Then every solution of (1) together with its derivative is bounded as $t \to \infty$ and there exists a periodic solution if and only if $\int_0^{\pm\infty} [f(s) + |g(s)|]ds = \pm\infty$. Theorem 2. Every solution of (1) is oscillatory if and only if $\int_0^{\pm\infty} [f(s) + |g(s)|]ds = \pm\infty$. (Received September 27, 1967.)

653-114. T. N. E. GREVILLE, Mathematics Research Center, U. S. Army, University of Wisconsin, Madison, Wisconsin 53706. Spectral generalized inverses of singular square matrices.

A matrix A^S is called a spectral inverse of A if $AA^S A = A$, $A^S AA^S = A^S$, every generalized eigenvector of A of height p [see Scroggs and Odell, J. SIAM Appl. Math. 14 (1966), 796] for an eigenvalue $\lambda \neq 0$ is a generalized eigenvector of A^S of height p for λ^{-1}, and similarly for generalized "left" eigenvectors (row vectors x satisfying $x(A - \lambda I)^p = 0$), and a strong spectral inverse (s.s.i.) if also $A = P^{-1} JP$ and $A^S = P^{-1} J^+P$, where J is a Jordan form of A and J^+ its Moore-Penrose inverse. If the Wedderburn index k is 1, the Drazin pseudoinverse [Amer. Math. Monthly 65 (1958), 5] is the unique s.s.i. For $k \geqq 2$, a unique s.s.i. A^I with the property $(A^I)^I = A$ is determined by the following further condition. For any pair x,y of columns of P^{-1} that are generalized null vectors of A of heights p and q such that $x \notin \mathscr{R}(A)$ and $q < p$, or $q = p$ but $y \in \mathscr{R}(A)$, $x^*Ry = 0$, where $R = I + A^* A + (A^*)^2A^2 + \dots + (A^*)^kA^k$ and * denotes the conjugate transpose. (Received September 28, 1967.)

653-115. WITHDRAWN.

653-116. B. K. SWARTZ, University of California, Los Alamos Scientific Laboratory, P. O. Box 1663, Los Alamos, New Mexico 87544. $O(h^{2n+2-m})$ bounds on some spline interpolation errors.

It had been felt for perhaps three of four years that C^{2n}, 2n + 1 degree polynomial spline interpolation of a sufficiently smooth function yields $O(h^{2n+2-m})$ accuracy in approximating its mth derivative, $0 \leqq m \leqq 2n + 1$. This was recently shown to be so for $n \geqq 1$, for equally spaced joints, and under periodic boundary conditions [1, p. 151]. A bit more is now shown under two other boundary conditions: the error in spline interpolation is the error in local 2-point Hermite-Birkhoff interpolation [2] plus a higher order term. These higher order terms are zero when interpolating certain functions. Numbers are given for cubics with nonuniformly spaced joints; they are one to three orders of magnitude better than previously published bounds (including [3]). The role of the mesh ratio in these cubic cases is also discussed (and down-graded somewhat). [1] Ahlberg, Nilson, & Walsh, The theory of splines and their applications, 1967. [2] Schoenberg, On Hermite-Birkhoff interpolation, J. Math. Anal. Appl. 16 (1966), 538-543. [3] S. Nord, Approximation properties of the spline fit, BIT 7 (1967), 132-144. (Received September 28, 1967.)

653-117. G. CHARTRAND, Western Michigan University, Kalamazoo, Michigan, and M. J. STEWART, Lansing Community College, Lansing, Michigan 48914. Connectivity and line-connectivity of graphs and their line-graphs.

A graph G is n-connected if the removal of fewer than n points neither disconnects G nor reduces it to a single point, while G is m-line connected if it cannot be disconnected by the deletion of less than m lines. The line-graph L(G) has its points in one-to-one correspondence with the lines of G so that adjacency is preserved. Results are presented involving the two types of connectedness relationships which exist between graphs and their iterated line-graphs. In particular we show the following. Theorem. If G is n-connected then L(G) is n-connected and (2n - 2)-line connected while L(L(G)) is (2n - 2)-connected. (Received September 28, 1967.)

653-118. G. CHARTRAND and S. F. KAPOOR, Western Michigan University, Kalamazoo, Michigan 49001, and H. V. KRONK, State University of New York at Binghamton, Binghamton, New York. A sufficient condition for N-connectedness of graphs.

A graph is n-connected if it cannot be disconnected or reduced to a single point by the removal of fewer than n points. The following result is presented. Theorem. Let G be a graph with p points and let $1 \leqq n < p$. The following two conditions are sufficient for G to be n-connected: (1) For all k such that $n - 1 \leqq k < (p + n - 3)/2$, the number of points of degree not exceeding k does not exceed $k + 1 - n$, and (2) the number of points of degree not exceeding $(p + n - 3)/2$ does not exceed $p - n$. (Received September 28, 1967.)

653-119. E. C. YOUNG, Florida State University, Tallahassee, Florida 32306. A Cauchy problem for a semiaxially symmetric wave equation.

The partial differential equation $L(u) \equiv u_{tt} - u_{yy} - (k/y)u_y - \sum_{i=1}^{m-2} u_{x_i x_i} = 0$ arises when one seeks a solution of the wave equation $\sum_{i=1}^{m-2} u_{x_i x_i} + \sum_{i=1}^{n} u_{y_i y_i} = u_{tt}$ which depends only on the variables $x_1, \dots, x_{m-2}$, t, $y = (y_1^2 + \dots + y_n^2)^{1/2}$. Following an embedding method used by Young (J. Math. Anal. Appl. 16 (1966), 355-362), the Riesz kernel for the operator L can be determined. This leads to an explicit formula for the solution of the Cauchy problem $L(u) = f(x,y,t)$, $u(x,y,0) = g(x,y)$, $u_t(x,y,0) = h(x,y)$, $x = (x_1, \dots, x_{m-2})$. (Received September 28, 1967.)

653-120. S. F. KAPOOR, Western Michigan University, Kalamazoo, Michigan 49001. Expanding n-stars.

If G is a graph that can be expressed as $\bigcup_{i=1}^{\infty} S_i(n)$ where each $S_i(n)$ is homeomorphic to a star graph of degree n ($\geqq 2$) and $S_i(n) \subset S_{i+1}(n)$ for $i = 1,2,\dots$, then G is said to be an expanding n-star and is denoted by E(n). Let D(k,G) represent the number of vertices of G each of whose degree is greater than or equal to k and let ρ denote the maximum degree in G. Theorem 1. If $n \geqq 3$ then $(k - 2)(D(k,E(n)) - 1) \leqq n \leqq \rho$ for $3 \leqq k \leqq 2n$. Theorem 2. E(n) is planar for $n \geqq 2$. (Received September 28, 1967.)

653-121. F. R. KEOGH, University of Kentucky, Lexington, Kentucky, and E. P. MERKES, University of Cincinnati, Cincinnati, Ohio 45221. A coefficient inequality for certain classes of analytic functions.

Let $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ be analytic in $E = \{z : |z| < 1\}$ and let $J(f) = |a_3 - \mu a_2^2|$, where μ is a fixed complex number. (I) If $\mathrm{Re}\,\{zf'(z)/f(z)\} > \lambda$ for $z \in E$, where λ $(0 \leqq \lambda < 1)$ is fixed, then $J(f) \leqq (1 - \lambda)\max(1, |2(1 - \lambda)(2\mu - 1) - 1|)$. This is sharp. (II) If f is spiral-like in E, then $J(f) \leqq 2|\mu - 1| + |2\mu - 1|$ and this is sharp for real μ. (III) If f is close-to-convex in E, for real μ, $J(f) \leqq \max(1, 3|\mu - 1|, |4\mu - 3|)$. This is sharp for μ not in the interval (0,2/3). A sharp result of this type is obtained when there is a starlike function $g(z) = z + \dots$ in E such that $\mathrm{Re}\,\{zf'(z)/g(z)\} > 0$. The proofs are by elementary methods. (Received September 28, 1967.)

653-122. J. D. LAWSON, The University of Tennessee, Knoxville, Tennessee 37916. Homomorphisms of topological semilattices.

The unit interval I with multiplication defined by $xy = x \wedge y$ is an example of a topological semilattice (a commutative, idempotent topological semigroup). The set of continuous homomorphisms from a topological semilattice S into I is denoted by Hom(S,I). Theorem 1. If a locally compact topological semilattice S has a basis of neighborhoods that are subsemilattices of S, then Hom(S,I) separates points. Theorem 2. If S is a locally compact, finite dimensional topological semilattice in which $\{y : xy = x\}$ is connected for each $x \in S$, then Hom(S,I) separates points. Techniques similar to those found in the proof of Urysohn's Lemma are employed in the proof of Theorem 1. (Received September 28, 1967.)

653-123. E. D. CONWAY, Tulane University, New Orleans, Louisiana, and D. R. SMITH, University of California, San Diego, California. An ordering principle for generalized solutions of quasi-linear equations of the first order.

Let K be the family of weak solutions u(t,x) of (*) $u_t + \sum_{i=1}^{n}(\partial/\partial x_i)[f_i(u)] = 0$ which are smooth everywhere except on a countable number of sectionally smooth n-dimensional surfaces across which u has jump discontinuities. Theorem. Let $f_i'' > 0$ and let u and v be members of K which are nonincreasing in each of the variables $x_1,\dots,x_n$. Then if $u \geqq v$ at the base of a common cone of determinacy (classical) then $u \geqq v$ throughout the interior of the cone. The proof utilizes the same basic idea as does the uniqueness theorem of Haar. This partially generalizes a similar result, valid in the case where n = 1, due to A. Douglis (Comm. Pure Appl. Math. 12 (1959), 87-112). (Received September 28, 1967.)

653-124. M. D. GREEN, George Washington University, Washington, D. C. 20006. A locally convex topology on a preordered space.

It is shown that, with every preordered topological space $(X,\mathscr{T})$ there can be associated a locally convex topology $\mathscr{T}_c$ on X which has the property: if $\mathscr{T}_c$ is weaker than $\mathscr{T}$, then $\mathscr{T}_c$ is the l.u.b. of all locally convex topologies on X weaker than $\mathscr{T}$. A preordered Hausdorff space $(X,\mathscr{T})$ is said to be regularly preordered if, for every $x \in X$ and every decreasing open neighborhood U of x, there is a neighborhood V of x such that the closed decreasing hull of V is contained in U. Theorem. Let $(X,\mathscr{T})$ be a regularly ordered space with the additional property that the interior of every decreasing set is decreasing (condition *). Then $\mathscr{T}_c$ is the strongest regular locally convex topology on X weaker than $\mathscr{T}$. A function from a preordered space $(X,\mathscr{T})$ into a preordered space $(Y,\mathscr{F})$ is said to be c-continuous if it is continuous from $(X,\mathscr{T}_c)$ into $(Y,\mathscr{F}_c)$. Corresponding to the theorem of continuous extensions with respect to regular spaces, it is shown that a c-continuous function f from a dense subset A of a preordered space $(X,\mathscr{T})$ into a regularly ordered *-space has a unique c-continuous extension to X if and only if the c-limit of f restricted to A exists for every $x \in X$. (Received September 28, 1967.)

653-125. E. V. SWENSON, New York University, Courant Institute of Mathematical Sciences, 251 Mercer Street, New York, New York 10012. Geometry of the complex characteristics in transonic flow.

The numerical solution of an analytic quasi-linear partial differential equation in two variables and of mixed elliptic and hyperbolic type is considered. Analytic Cauchy data is prescribed on a noncharacteristic initial curve which cuts across both the hyperbolic and elliptic regions. The Cauchy problem becomes well-posed through complex extension of the independent variables. The solution may be obtained by the method of characteristics in the hyperbolic region by starting with real data on the given real curve and in the elliptic region by starting with complex data on symmetric paths in the complex extension of the initial curve. In some instances, such a collection of initial paths is insufficient for the complete solution of the problem because sections of the hyperbolic region lie outside the domain of influence of the real characteristics from the initial curve. An algorithm is presented for choosing a nonsymmetric path of data in the complex initial surface whose solution domain includes a real two-dimensional patch of the hitherto unsolved region. With this algorithm, the full solution to the two-dimensional inverse detached shock problem is obtained. (Received September 29, 1967.)

653-126. J. F. HURLEY, University of California, Riverside, California. Scalar replacement in Lie algebras.

Let L be a finite dimensional simple Lie algebra over the complex field, H a Cartan subalgebra. Using a Chevalley basis for L, we can obtain a new Lie algebra L_R over an arbitrary commutative ring R with identity. In the case where 2 and 3 are not zero divisors in R, we study the ideals I of L_R. Let E_R be the R-module generated by the Chevalley basis elements corresponding to nonzero roots and H_R the R-module generated by the Chevalley basis elements in H. Theorem 1. If I is not contained in H_R, then I contains JL_R for some ideal J in R. Theorem 2. If L is nonsymplectic, then $I = (I \cap E_R) \oplus (I \cap H_R)$. Theorem 3. Let m be the ratio of the squares of long to short root lengths. Let C be the Cartan matrix of L. Then a necessary and sufficient condition for every ideal in L_R to have the form JL_R for an appropriate ideal J in R is that both m and det C be invertible in R. The last result generalizes a 1964 theorem of W. Jehne for symplectic algebras (Sitzungsberichte Heidelberger Akad. Wiss. Math. Natur., Kl. 1962/64, 187-235). (Received September 29, 1967.)

653-127. PAWEL SZEPTYCKI, University of Kansas, Lawrence, Kansas 66044. Some remarks on the evolution equation and semigroups. Preliminary report.

Let A be a closed densely defined operator in a Banach space X. Consider the following equations: (a) $dx/dt = Ax$, $x(\cdot) \in C^1((0,T); X) \cap L^1((0,T); X)$, (a') $x(t') - x(t) = A\int_t^{t'} x(s)ds$, $x(\cdot) \in C((0,T); X) \cap L^1((0,T); X)$, and the initial conditions: (b) $x(t) \to x_0$ for $t \to 0$, (b') $(1/t)\int_0^t x(s)ds \to x_0$ for $t \to 0$. Theorem. (i) Problem (a), (b) has a unique solution (u.s.) for every $x_0 \in X$ iff A is the generator of a semigroup T(t) of class (C_0) such that $T(t)X \subset D(A)$, $t > 0$. (ii) (a'), (b) has a u.s. for every $x_0 \in X$ iff A is the generator of a semigroup of class (C_0). (iii) (a), (b') has a u.s. for every $x_0 \in X$ iff A is the generator of a semigroup T(t) of class (C_1) such that $T(t)X \subset D(A)$, $t > 0$. (iv) (a'), (b') has a u.s. for every $x_0 \in X$ iff A is the generator of a semigroup of class (C_1). Proofs of sufficiency of the conditions in (i) - (iv) can be found in or easily deduced from results of Hille, Philips, Functional analysis and semigroups. (Received September 29, 1967.)

653-128. R. C. STEINLAGE, University of Dayton, Dayton, Ohio 45409. On Ascoli theorems and the product of k-spaces.

In [Quart. J. Math. 5 (1956), 77-80] D. E. Cohen proved that the product of a locally compact Hausdorff space and a Hausdorff k-space is always a k-space. E. A. Michael in [Abstract 648-191, these *Notices* 14(1967), 688] observes that the assumption of local compactness cannot be weakened. We now show that the assumption of T_2 separation can be weakened. In fact, if X is a locally compact regular space and Y is a k-space then $X \times Y$ is a k-space. This result is then used to show that, even when X is not a Hausdorff space, the Ascoli Theorems [J. L. Kelley, General topology, van Nostrand, New York, 1955, Theorems 7.17 and 7.21] need not be altered as in [Kelley, Theorems 7.18 and p. 236] in order to cover k-spaces. This result was proved for Hausdorff k-spaces X by Bagley and Yang in [Proc. Amer. Math. Soc. 17 (1966), 703-705]. (Received September 29, 1967.)

653-129. J. D. STAFNEY, University of California, Riverside, California. The spectrum of an operator on an interpolation space.

Let B^0 and B^1 be Banach spaces continuously embedded in a topological vector space V so that the pair (B^0, B^1) is an interpolation pair as in (Studia Math. 24 (1964), 114). Let T be an operator on $B^0 \cap B^1$ and let $sp(T, \| \|)$ denote the spectrum of T when T is bounded with respect to the norm $\| \|$. We obtain the best possible upper bound (set) for $sp(T, \| \|_s)$ that depends only on $sp(T, \| \|_{B^k})$, $k = 0,1$ ($\| \|_s$ is the interpolated norm, $0 < s < 1$) To do this we associate with T an interpolation pair of commutative Banach algebras. We give a formula for obtaining the structure space H of the interpolated algebra from the structure spaces of the given algebras. It is H that is the upper bound for $sp(T, \| \|_s)$. (Received September 29, 1967.)

653-130. PHILIP NANZETTA, University of Florida, Gainesville, Florida 32601. Representation of relatively complemented distributive lattices.

A topological space X is called an L-space iff X is T_1, anti-Hausdorff (every pair of nonempty open subsets has nonempty intersection), nearly-Hausdorff (every closed proper subset is Hausdorff), and the compact-open subsets of X form a base for the topology of X. Let T be a relatively complemented distributive lattice, and let $\mathscr{M}(T)$ be the space of prime ideals of T with the hull-kernel topology. Theorem 1. If T has no least element, T is isomorphic to the lattice of nonempty compact-open subsets of the L-space $\mathscr{M}(T)$. Moreover, $\mathscr{M}(T)$ is compact iff T has a greatest element. Theorem 2. If X is an L-space, then X is homeomorphic to the space $\mathscr{M}(C)$ of prime ideals of the lattice C of nonempty compact-open subsets of X. These theorems, together with the Stone representation theorem for Boolean rings, indicate a certain duality between "compact" and "Hausdorff": $\mathscr{M}(T)$ is compact iff T has a greatest element and is Hausdorff iff T has a least element; if T' is the dual of T, then $\mathscr{M}(T)$ is compact iff $\mathscr{M}(T')$ is Hausdorff. (Received September 29, 1967.)

653-131. J. E. KEESLING, University of Florida, Gainesville, Florida 32601. Open mappings and closed zero-dimensional sets.

Throughout the following let X and Y be arbitrary metric spaces with $\dim Y = n < \infty$ and let f be an open continuous mapping from X onto Y. Then the following theorems hold. Theorem 1. Suppose that the following conditions hold: (1) $f^{-1}(y)$ is complete for all $y \in Y$ and (2) Y has a σ-locally finite basis of open sets $\{V_\alpha\}$ so that $f^{-1}(y)$ is perfect for all $y \in Fr(V_\alpha)$ for all α. Then there is a closed set K in X with $\dim K = 0$ and with the properties $f(K) = Y$ and $f|K$ is at most $n + 1$ to one. In case X has a complete metric, then $f|K$ can be required to be a compact mapping. Theorem 2. Let f be closed and $\dim X = m < n$. Then there is a closed set K in X with $\dim K = 0$ and $\dim f(K) \geqq n - m$. Theorem 3. If f has complete point inverses and $\dim X = m < n$ with X separable, then there is a closed set K in X with $\dim K = 0$ and $\dim f(K) \geqq n - m$. Theorem 4. If X is a separable Euclidean m-manifold with $m < n$, then there is a Cantor set K in X with $\dim f(K) \geqq n - m + 1$. (Received October 2, 1967.)

653-132. HANSJOACHIM GROH, University of Florida, 205 Walker Hall, Gainesville, Florida 32601. A characterization of the real miquelian Laguerre plane.

A Laguerre plane (W. Benz and H. Maurer, Jahresbericht DMV 1964) is called flat if the set S of spears is the cylinder $S_1 \times R$, the cycles are Jordan curves and the parallel classes are closed Jordan lines (i.e. subsets homeomorphic to the real line). Every flat Laguerre plane can be made in a unique way to a topological Laguerre plane. Using a Theorem of Melchior, we have: Theorem. For a flat Laguerre plane S,Z are equivalent: (1) S, Z is isomorphic to the real miquelian Laguerre plane (2) S, Z admits a group of automorphisms being simply transitive on the set of triples of not parallel spears and containing an element interchanging two parallel spears. (Received November 30, 1978.)

653-133. J. K. ODDSON, University of California, Riverside, California. On the boundary point principle for elliptic equations in the plane.

Let D be an open subset of the plane and $L = \sum_1^2 a_{ij} D_{ij}$ a second order differential operator with coefficients defined in D and satisfying there the normalization $\sum_1^2 a_{ii} = 1$ and the ellipticity condition $\sum_1^2 a_{ij}\xi_i \xi_j \geqq \alpha \sum_1^2 \xi_i^2$ for some constant α in the range $0 < \alpha \leqq 1/2$. Theorem. Let u(x,y) be a nonconstant twice differentiable function in D, continuous in $\bar{D}$, which satisfies $Lu \leqq 0$ in D. Suppose that u attains a local minimum of u_0 at a boundary point P_0 which subtends an open truncated sector S, $S \subset D$, of half angle θ_0. Then there exists a positive constant m and an explicitly determined positive constant μ such that $u(x,y) \geqq u_0 + mr^{\mu}$ on compact subsets of S, where r denotes distance measured from P_0. The exponent μ, as a function of α and θ_0, cannot be improved. (Received October 2, 1967.)

653-134. NACHMAN ARONSZAJN, University of Kansas, Lawrence, Kansas 66044. Metric characterization of bordered Riemannian manifolds; intrinsic regular border of a Riemannian manifold.

We restrict ourselves here to bordered C^{∞} Riemannian manifolds; open manifolds are considered as a special case with empty border. Normal coordinate patches are characterized in terms of the geodesic metric not only for inner points but also for border points. The following theorem results: For an abstract metric space $\mathscr{S}$ and any integer $n \geqq 0$ there exists a unique maximal open subset $\mathfrak{M} \subset \mathscr{S}$ locally isometric to a Riemannian bordered n-manifold. Applying to the abstract completion $\tilde{\mathfrak{M}}$ of a Riemannian n-manifold $\mathfrak{M}$ one arrives at the notion of an intrinsic regular border $\partial \mathfrak{M}$ of $\mathfrak{M}$ which contains the given a priori border of $\mathfrak{M}$. (Received October 2, 1967.)

653-135. WITHDRAWN.

653-136. R. B. WARFIELD, JR., New Mexico State University, Las Cruces, New Mexico 88001. Algebraic compactness for modules. Preliminary report.

A submodule A of a module B (over an associative ring with 1) is pure if for any finitely presented module F, the natural map $\mathrm{Hom}(F,B) \to \mathrm{Hom}(F,B/A)$ is surjective. A module C is pure-injective if for any module B and pure submodule A, any homomorphism of A into C extends to B. Theorem 1. The following are equivalent (1) C is pure-injective, (2) C is a retract of a compact topological module, (3) Every finitely solvable family of linear equations in C over R has a simultaneous solution in C. Such a module C is called algebraically compact. Theorem 2. Any module A can be imbedded as a pure submodule in an algebraically compact module C, and pure-injective envelopes exist. Theorem 3. If R is a valuation ring, then an R-module is algebraically compact iff it is of the form $A \oplus B$ where A is injective, $\bigcap rB = 0$ ($r \in R$, $r \neq 0$) and B satisfies the following completeness conditions: For any ideal J and descending family of ideals I_j, the maps $C \to \varprojlim C/I_jC$ and $C[J] \to \varprojlim C[J]/(C[J] \cap I_jC)$ are surjective (here C[J] is the annihilator of J), and for any ascending family of ideals I_j, the map $C \to \varprojlim C/C[I_j]$ is surjective. Theorem 4. A torsion-free algebraically compact module over a valuation ring is the pure-injective envelope of a direct sum of ideals. (Received October 2, 1967.)

65-137. F. D. WILLIAMS, New Mexico State University, Las Cruces, New Mexico 88001. On higher Samelson products.

Generalized higher order Whitehead products (GHOWP's) were introduced by G. J. Porter (Topology 3 (1965), 123-135) and he defined formally the generalized higher order Samelson products (GHOSP's) to be the adjoints of GHOWP's. Precisely, if $f_i : SA_i \to X$, $i = 1,\ldots,n$ are maps and $[f_1,\ldots,f_n]$ is their Whitehead product set, then the Samelson product set $\langle Tf_1,\ldots,Tf_n \rangle$ of the $Tf_i : A_i \to \Omega X$ is defined to be $T([f_1,\ldots,f_n])$, where $T: [SY; Z] \to [Y; \Omega Z]$ is the adjoint isomorphism. In the case $n = 2$ it is well known that the Samelson product defined in this way is equivalent to that defined in terms of commutators. In this paper we extend this equivalence to express GHOSP's of arbitrary order in terms of higher order commutativity. The higher order commutativity used is the C_n-commutativity introduced by the author (Abstract 636-21, these *Notices* 13 (1966), 581).
(Received October 2, 1967.)

653-138. PATRICK O'HARA, University of Miami, Coral Gables, Florida 33124. Convergence of complex Lagrange interpolation polynomials with nodes lying on a piecewise analytic Jordan curve with cusps.

Let C be a Jordan curve in the complex z-plane. Let $z = \phi(w)$, $\phi(w)$ analytic and univalent from $|w| > 1$, map $\{w; |w| > 1\}$ conformally onto Ext C so that $\infty \to \infty$. It is known that $\phi(w)$ has a continuous extension onto $\{w; |w| \geq 1\}$ which gives a topological mapping of $\{w; |w| = 1\}$ onto C. Define $S_n = \{\phi(\exp(2\pi ik/n + 1)); k = 0,1,\ldots,n\}$. Given f(z) continuous on C let $L_n(f;z)$ be the polynomial of degree n which interpolates to f(z) at the points, S_n. If $\phi'(w)$ can be extended so that it is absolutely continuous nonvanishing on C, J. H. Curtiss has shown that $\lim_{n\to\infty} L_n(f;z) = (1/2\pi i)\int_C (f(t)/(t - z))dz$ uniformly for z in any compact subset of Int C. His conditions imply that C has a continuously turning tangent and can have no corners or cusps. This paper extends Curtiss' result to a case in which C is the union of a finite number of analytic arcs which meet at "outward-pointing" cusps. The method of proof is substantially different from that used by Curtiss. (Received October 2, 1967.)

653-139. KERMIT SIGMON, University of Florida, Gainesville, Florida 32601. On imbedding medial topological groupoids.

A topological groupoid is called medial if $(xy)(uv) = (xu)(yv)$ for all x,y,u,v and an element e is called injective if the maps $x \to ex$ and $x \to xe$ are topological imbeddings. If f and g are continuous commuting endomorphisms of a commutative (topological) semigroup $(S,\circ)$ and if one defines $\cdot$ on S by $x \cdot y = f(x) \circ g(y)$ then $(S,\circ)$ is a medial topological groupoid. A topological groupoid so obtained will be called a transduct of the semigroup. Theorem. Let M be a compact or discrete medial topological groupoid containing an injective idempotent e such that $f^2(M) \subset f(M)^0$, where f is given by $f(x) = (ex)e$. Then M is topologically and algebraically isomorphic to a subgroupoid of a transduct of a locally compact commutative semigroup with unit. Other more specialized theorems are proven and an application to functional equations is given. In particular, a complete description of all medial, commutative, idempotent topological groupoids on an arc which contain an injective element is given. (Received October 2, 1967.)

653-140. J. T. ROGERS, JR., University of California, Riverside, California. Pseudo-circles and universal circularly chainable continua. Preliminary report.

Definition. A pseudo-circle is a nondegenerate, hereditarily indecomposable, circularly chainable, compact, metric continuum which is not snakelike. Theorem. There exist c pseudo-circles with the property that no one is the continuous image of another. Corollary. There exist c topologically distinct pseudo-circles. Theorem. There exists a pseudo-circle X which can be mapped continuously onto any circularly chainable continuum. Theorem. There exists a planar pseudo-circle Y which can be mapped continuously onto any planar circularly chainable continuum. Theorem. No pseudo-circle is a continuous image of a pseudo-arc. Theorem. A continuum is a continuous image of X(Y) iff it is q-chainable (monocyclically q-chainable). (Received October 2, 1967.)

653-141. J. R. TELLER, Georgetown University, Washington, D. C. Extensions of pseudo lattice ordered groups. Preliminary report.

Let A, Δ and G be pl-groups (Abstract 648-33, these *Notices* 14 (1967), 99). G is a pl-extension of A by Δ if there is a pl-homomorphism π of G onto Δ with kernel A such that π induces a pl-isomorphism of G/A with Δ. If G is a pl-extension then G is a partially ordered extension so for each $\alpha \in \Delta^+$, there is a subset $Q(\alpha) \subseteq A$ such that (i) $Q(\alpha) \neq \emptyset$, (ii) $Q(\theta) = A^+$ and (iii) if $\beta \in \Delta^+$, then $Q(\alpha) + Q(\beta) + f(\alpha,\beta) = Q(\alpha + \beta)$. Theorem. If A is a pl-group and Δ is lattice ordered then G is a pl-extension of A by Δ if and only if G is a Riesz extension (Abstract 619-35, these *Notices* 12 (1965), 65), and whenever $\alpha \wedge \beta = \theta$ and $b \in A$ there is an element $c \in A$ such that $Q(\alpha) \cap [Q(\beta) + b + f(\beta,\alpha - \beta)] = DI[c + K]$ where $K = H \cap \mathcal{M}^*(c,\alpha) \cap \mathcal{M}^*((c,\alpha) - (b, \alpha - \beta))$, and H is the 0-ideal of G generated by $\{(0,\theta) \leqq (x,\eta) | (x,\eta) \leqq (c,\alpha) \text{ and } (x,\eta) \leqq (c,\alpha) - (b,\alpha - \beta)\}$. Theorem. If G is a Riesz extension of a pl-group A by an l-group Δ and for each $\alpha \in \Delta^+$, $Q(\alpha) = DI(H(\alpha))$, $H(\alpha)$ a cardinal summand of A, then G is a pl-extension. (Received October 2, 1967.)

653-142. J. B. DIAZ, Rensselaer Polytechnical Institute, Troy, New York, and F. T. METCALF University of California, Riverside, California. Iteration procedures and Picard's criterion for equations of the first kind.

It is shown that certain iteration procedures for solving linear Fredholm integral equations of the first kind, $y(t) = \int_a^b K(t,s)x(s)ds$, are equivalent to Picard's [Rend. Circ. Mat. Palermo (1910), 79-97] criterion for the existence of a solution. Write the equation in the form $y = Ax$, where $A : H \longrightarrow H$ is a linear operator on the real infinite dimensional Hilbert space H. The following "chain of equivanences" is shown to hold for "suitable" A: Existence of a solution $x \Longleftrightarrow y \perp$ null space of A, plus a certain formula for the inverse of A at y is valid $\Longleftrightarrow y \perp$ null space of A, plus a certain iteration scheme converges

$\Leftrightarrow$ Picard's criterion. "Suitable" A means either (i) $A \not\equiv 0$, compact, self-adjoint, and positive semi-definite; or (ii) $A \not\equiv 0$ and both A^*A and AA^* are compact. (Received October 2, 1967.)

653-143. W. M. BOGDANOWICZ, The Catholic University of America, Washington, D. C. 20017. Radon-Nikodym differentiation of one vector measure with respect to another.

Let S(V) denote the family of all sets being finite disjoint unions of sets from the family V consisting of subsets of a space X. Let the family V be a refinement prering, i.e. for any $A,B \in V$ we have $A \cap B$ and $A \setminus B$ are in S(V). A function m from V into a B-space Y is called a vector volume if it is countably additive and has finite variation $|m|(A)$ on every set $A \in V$. Let R(V,Y) be the set of all vector volumes m such that there exists a function f summable with respect to $|m|$ on every set $A \in V$ and such that $m(A) = \int c_A f \, d|m|$, for definitions see Bogdanowicz, Proc. Nat. Acad. Sci. USA 53 (1965), 492-498. Let the pair (V,Y) have the Pettis-Dunford-Phillips property, i.e. every vector volume on V to Y' is in R(V,Y'). Let $v \in R(V,Z)$ and $m \in R(V,Y)$ and let v be absolutely continuous with respect to the volume m, i.e. for any $\epsilon > 0$ there exists $\delta > 0$ such that $\sum_j |v(A_j)| < \epsilon$ if $\{A_j\}$ is a finite family of disjoint sets from V and $\sum_j |m(A_j)| < \delta$. Let W be the space of all linear continuous operators from Y into Z. Theorem. There exists a function f on X to W such that f is $|m|$-summable on each set $A \in V$ and $v(A) = \int u(c_A f, dm)$, where $u(w,y) = w(y)$ for all $w \in W$ and $y \in Y$, and the integral is considered on the space of $|m|$-summable functions. (Received September 13, 1967.)

653-144. R. M. STEPHENSON, JR., University of North Carolina at Chapel Hill, Chapel Hill, North Carolina 27514. Pseudocompact spaces.

The terminology used here is the same as that in Products of nearly compact spaces, Trans. Amer. Math. Soc. 124 (1966), 131-147. We say that a topological space X has property A (B) provided that every countable regular filter base on X has an adherent point (if O(i), i = 1,2,..., is a collection of nonempty open subsets of X having disjoint closures, then $\bigcup O(i)$ is not closed). Theorem 1. Every feebly compact space has property B, and every space which has property B also has property A. No other implications hold among feeble compactness, A, and B, but a regular space X is feebly compact if and only if X has property A. On any space A implies pseudocompactness, but there exists a pseudocompact Stone space which does not have property A. Theorem 2. Every product of pseudocompact spaces, of which all but one (at most) are sequentially compact, is pseudocompact. Theorem 3. Let X and Y be pseudocompact spaces, and suppose that there is an infinite cardinal K such that every point of X has a fundamental system of neighborhoods containing K or fewer sets, and every open filter base on Y containing K or fewer sets has an adherent point. Then $X \times Y$ is pseudo-compact. (Received October 4, 1967.)

653-145. R. M. McCONNEL, University of Tennessee, Knoxville, Tennessee 37916. Functions over finite fields preserving mth powers.

Let F be a finite field of order $q = p^n$, p a prime. Let $d \neq 1$ be an arbitrary divisor of q - 1 and set q - 1 = md. Put $\psi_d(x) = x^m$. The following theorem and its proof is related to the work of L. Carlitz [Proc. Amer. Math. Soc. 11 (1960), 456-459; Acta Arith. 7 (1962), 167-172] and the author [Acta

Arith. 8 (1963), 127-151]. Theorem 1. Let λ be any element of F such that $\lambda^d = 1$. Let f be any function from F $\times$ F into F. Then $\psi_d[f(x,y) - f(u,y) - f(x,v) + f(u,v)] = \lambda\psi_d[(x-u)(y-v)]$ for all $x,y,u,v \in F$ if and only if $f(x,y) = ax^{p^i}y^{p^j} + g(x) + h(y)$ for some i and j such that $0 \leqq i < n$ and $0 \leqq j < n$, where g(x) and h(y) are arbitrary polynomials over F. Moreover $d|p^i - 1$, $d|p^j - 1$, and $\psi_d(a) = \lambda$. This theorem has been extended to functions of k variables. (Received October 4, 1967.)

653-146. L. KUIPERS, Southern Illinois University, Carbondale, Illinois 62901. Uniform distribution in Galois fields.

Let Φ = GF[q,x] denote the ring of polynomials in x over a field GF(q) of q elements, where $q = p^r$ and p is a prime number. Assume that GF(q) is determined by a zero μ of an irreducible polynomial of degree r in GF[p,x]. Hence $\alpha \in GF(q)$ means $\alpha = a_1\mu^{r-1} + \dots + a_r$ with $a_i \in GF(p)$. Define $t(\alpha) = a_1$. Let $M \in \Phi$ be a monic polynomial of degree $m \geqq 1$. Let A be an arbitrary element of Φ and let $A \equiv \alpha_1 x^{m-1} + \dots + \alpha_m$ (mod M). Now we define (following Y. H. Hodges, Uniform distribution of sequences in GF [q,x], Acta Arith. 12 (1966), 55-75): $e(A,M) = \exp(2\pi i \mathbf{a}_1/p)$ where $a_1 = t(\alpha_1) = t(\alpha_1(A))$. Let $\theta = \{A_k\}(k = 1,2,\dots)$ be a sequence of elements in Φ. We prove: The sequence θ is uniformly distributed modulo M if and only if $\lim_{n\to\infty}\sum_{k=1}^{n} e(A_kC,M)/n = 0$ for all $C \in \Phi$(reduced mod M) with $C \not\equiv 0$ (mod M). (Received October 4, 1967.)

653-147. T. W. PALMER, University of Kansas, Lawrence, Kansas 66044. Collectively compact sets which are totally bounded.

Let [X,Y] be the set of bounded linear operators between normed linear spaces X and Y. Then $\mathscr{K} \subseteq [X,Y]$ is collectively compact iff $\{Kx : K \in \mathscr{K}, x \in X, \|x\| \leqq 1\}$ has compact closure. Theorem. A collectively compact set is totally bounded iff $\mathscr{K}^* = \{K^* : K \in \mathscr{K}\} \subseteq [Y^*, X^*]$ is collectively compact. This settles affirmatively a conjecture of P. M. Anselone. The result was previously known only in two special cases: (1) X = Y = complex Hilbert space, (2) there is a common finite bound for the dimension of the range of all operators in $\mathscr{K}$. Its interest arises from the existence of a detailed theory (applicable to some classical approximations of integral operators) of strongly convergent sequences $T_n \to T$ with $\{T_n - T\} = \mathscr{K}$ collectively compact. Such a sequence converges in norm iff $\mathscr{K}$ is totally bounded. These considerations lead to several related characterizations of collectively compact sets and totally bounded sets of precompact operators. (Received October 4, 1967.)

653-148. WITHDRAWN.

653-149. CARL PEARCY, University of Michigan, Ann Arbor, Michigan, and DAVID TOPPING, Indiana University, Bloomington, Indiana 47401. Commutators and certain II_1-factors.

F. B. Wright (Ann. of Math. 60 (1954), 560-570) constructed a class of AW*-factors of type II_1, which Feldman later showed to be von Neumann factors. Theorem 1. Every self-adjoint operator in a Wright factor with trace zero is a commutator of two operators in the factor. Hence every operator in such a factor with zero trace is the sum of two commutators. For a von Neumann algebra $\mathscr{A}$ of type II_1, let $[\mathscr{A},\mathscr{A}]$ denote the linear span of the commutators. Theorem 2. With $\mathscr{A}$ a II_1-algebra, $[\mathscr{A},\mathscr{A}]$ is equal to (1) the set of finite sums of nilpotent operators of index two; (2) the set of all linear combinations $\sum_{i=1}^{n} a_i E_i$, where $\sum_{i=1}^{n} a_i = 0$ and for each i, E_i is a projection in $\mathscr{A}$ with $E_i \sim I - E_i$; (3) the linear span of all symmetries (= s.a. unitaries) S in $\mathscr{A}$ such that the projections $(1/2)(I \pm S)$ are equivalent in $\mathscr{A}$. Corollary 1. In a Wright factor $\mathscr{W}$, $[\mathscr{W},\mathscr{W}]$ is norm closed. Corollary 2. Bach Wright factor is the linear span of its projections E such that $E \sim I - E$. Corollary 3. If projections E with $E \sim I - E$ span a II_1-algebra $\mathscr{A}$ over its center, then $[\mathscr{A},\mathscr{A}]$ is the null space of the central trace. Conjecture. Every II_1-factor is spanned by its projections of dimension one-half. (Received October 4, 1967.)

653-150. R. L. HEMMINGER, Vanderbilt University, Nashville, Tennessee. On reconstructing a graph.

Let G be an unoriented finite graph without loops or multiple edges. If $a \in V(G)$ we will let G_a denote the graph obtained from G by deleting the vertex a and the edges adjacent to a. If $a \in E(G)$ we will let G^e denote the graph obtained from G by deleting the edge e. If a graph G is uniquely determined, up to isomorphism, by a given collection of subgraphs we will say that G can be reconstructed from the collection of subgraphs. It is well known that a tree can be reconstructed from its maximal subtrees. Harary has asked ["On the reconstruction of a graph from a collection of subgraphs," in Theory of graphs and its applications, M. Fielder (ed.), Prague, 1964, pp. 47-52] if a graph can be reconstructed from the collection $\{G^e | e \in E(G)\}$. We refer to this as the edge problem and to the reconstruction of G from the collection $\{G_a | a \in V(G)\}$ as the vertex problem. The line graph of G is the graph L(G) with $V(L(G)) = E(G)$ and with $(e_1, e_2) \in E(L(G))$ if and only if e_1 and e_2 are adjacent in G. We then prove the Theorem. The edge problem is equivalent to the vertex problem for line graphs, i.e. a solution to the edge problem would give a solution to the vertex problem for line graphs and conversely. (Received October 4, 1967.)

653-151. S. R. KIMBLETON, The University of Pittsburgh, Pittsburgh, Pennsylvania 15213. A converse of a Random stable limit theorem.

A random variable X is said to be of type α if $\alpha = \sup\{r > 0 | E[|X|^r] < +\infty\}$. The following theorem can be stated: Theorem. Let $\{X_j, j \geqq 1\}$ be independent, identically distributed random variables of type $0 < \alpha < 2$, $\alpha \neq 1$, with distribution function F. In addition, if $\alpha > 1$, assume that the random variables are centered at their expectations. Let $\{R(n), n \geqq 1\}$ be a sequence of positive integer valued random variables such that plim $R(n)/n = \pi > 0$. If there exists a sequence of constants $\{B_n\}$, $B_n > 0$ such that the limit in distribution G of $\sum_1^{R(n)} X_j/B_n$ exists and is nondegenerate, and the limit in distribution H of $\sum_1^{R(n)} X_j^s/B_n$ also exists and is nondegenerate where X_j^s is the symmetric random variable induced by X, then F is in the domain of attraction of a stable law of index α. This theorem may be viewed as a partial converse of a result of Wittenberg [Limiting distributions of random sums of random variables, Z. Wahrschein. 3 (1964), 7-18]. An application of this theorem to Markov chains is also discussed. (Received November 9, 1967.)

653-152. D. C. DYKES, University of Kentucky, Lexington, Kentucky 40506. A note on generalized Frattini subgroups. Preliminary report.

All groups considered are finite. A proper normal subgroup H of a group G is a generalized Frattini subgroup of G provided that $G = N_G(P)$ for each normal subgroup L of G and each Sylow p-subgroup, p a prime, P of L such that $G = HN_G(P)$, (J. Beidleman and T. Seo, Pacific J. Math., to appear). A weakly hypercentral subgroup of a nonnilpotent group is a generalized Frattini subgroup. (R. Baer, Amer. J. Math. 75 (1953), 633-644). In particular, the hypercenter of a nonnilpotent group is generalized Frattini. The product of two generalized Frattini subgroups is not necessarily a generalized Frattini subgroup, even if their orders are relatively prime. The following results are established. Theorem 1. If H is a generalized Frattini subgroup of G and K is a weakly hypercentral subgroup of G such that their orders are relatively prime and HK is proper in G, then HK is a generalized Frattini subgroup of G. Theorem 2. If H is a generalized Frattini subgroup of G and $K \lhd \lhd G$, then K is nilpotent iff $K/H \cap K$ is nilpotent. A characterization of H(G), the intersection of all maximal generalized Frattini subgroups of G, is forthcoming. (Received October 5, 1967.)

653-153. S. P. LLOYD, Bell Telephone Laboratories, Incorporated, Murray Hill, New Jersey 07971. On the L_2 space of a Banach limit.

Let $X = \{\chi_\theta, 0 \leqq \theta < 2\pi\}$ be the set of characters of the group Z of integers, extended to $C(\beta Z)$. R. G. Douglas [Proc. Amer. Math. Soc. 16 (1965), 30-36] has shown that if a Banach limit μ is not an extreme one then $\underline{X}$ fails to span $L_2(\beta Z, \mu)$, and asks whether $\underline{X}$ spans $L_2(\beta Z, \mu)$ when μ is extreme. The following example shows that the answer is no. Let $f \in C(\beta Z)$ be determined by $f(n) = e^{i\alpha n^2}$, $n \in Z$, with α/π irrational. Then for every Banach limit μ, $\int f \overline{\chi}_\theta d\mu = 0$ for all θ but $\int |f|^2 d\mu = 1$. (Received October 6, 1967.)

653-154. R. L. BHIRUD, 1125 E. 38th Street, Indianapolis, Indiana 46205. Behaviour of Pade' fractions and the associated quantities.

With each ordered pair [m,n] of nonnegative integers, there is associated a uniquely determined rational fraction $Y_{mn}(z)$, whose numerator and denominator are of degrees not exceeding m and n, respectively, and whose expansion in ascending powers of z agrees term by term with the given power series $\sum_{p=0}^{\infty} a_p z^p$ for more terms than that of any other such rational fraction. These rational fractions can be arranged in a table of double entry, by putting $Y_{mn}(z)$ in the mth row and nth column of the table (m,n = 0,1,2,...). These fractions and the table containing them are named after H. Pade . This paper contains some new results on the behaviour of Pade' fractions and some associated quantities. It is shown that the Pade' fractions $Y_{mn}(z)$ satisfy the following recurrence relations. $[Y_{mn} - Y_{m,n-1}] [Y_{m+1,n} - Y_{mn}] [Y_{m,n+1} - Y_{m-1,n}] = [Y_{m,n+1} - Y_{mn}] [Y_{mn} - Y_{m-1,n}] [Y_{m+1,n} - Y_{m,n-1}]$ and $[Y_{m-1,n} - Y_{m,n-1}] [Y_{m+1,n} - Y_{mn}] [Y_{m,n+1} - Y_{mn}] = [Y_{m,n+1} - Y_{m+1,n}] [Y_{mn} - Y_{m-1,n}] [Y_{mn} - Y_{m,n-1}]$. It is also shown that besides the Pade' fractions, there are five other associated qualities which satisfy the same pair of recurrence relations as the Pade' fractions. (Received October 6, 1967.)

653-155. R. G. LEVIN, Western Washington State College, Bellingham, Washington. Some results on nonpotent, locally cyclic semigroups. Preliminary report.

A semigroup S is called locally cyclic if for all pairs a, b $\in$ S, there exists c $\in$ S, and positive integers n and m such that $a = c^n$ and $b = c^m$. A semigroup is called an N semigroup if it is commutative, archimedean, cancellative and nonpotent (i.e., without an idempotent). A nonpotent locally cyclic semigroup is an N semigroup. The following theorem is used to determine a necessary and sufficient condition for two nonpotent, locally cyclic semigroups to be isomorphic. Theorem. A semigroup S is locally cyclic if and only if $S = \bigcup_{n=1}^{\infty} S_n$, where $S_n \subseteq S_{n+1}$ and S_n is a cyclic subsemigroup of S, for $n \geqq 1$. We also present a necessary condition for two nonpotent, locally cyclic semigroups to be isomorphic which uses the structure theorems on N semigroups derived by T. Tamura (see: Journal of Gakugei, Tokushima University, Volume VIII, 1957). (Received October 6, 1967.)

653-156. J. D. BRILLHART, University of Arizona, Tuscon, Arizona 85721. On the Euler and Bernoulli polynomials.

In a previous report (Abstract 623-2, these *Notices* 12 (1965), 350) it was shown that the Euler polynomial $E_5(x) = (x - 1/2)(x^2 - x - 1)^2$ is the only Euler polynomial with multiple roots, and that the Bernoulli polynomial $B_{2m+1}(x)$ has no multiple roots for any m. In a further investigation the interesting modular relationship: $(n + 1)E_n(x) \equiv 2B_{n+1}(x) \pmod 2$, $n \geqq 0$, was discovered. This relationship, which shows the Euler and Bernoulli polynomials are identical (mod 2), implies it is possible to carry over verbatim to the Bernoulli polynomials all the mod 2 arguments which were used for the Euler polynomials. In this way it follows directly that $B_{2m+1}(x)$ has no multiple roots and, if $B_{2m}(x)$ has a multiple factor, it can only be of the form $(x^2 - x - 1)^2$, where b is an odd, positive integer. Although no such factor is known, it is not difficult to show that $(x^2 - x - 1)^2$ is never a factor of $B_{2m}(x)$, as it is in the case of $E_5(x)$. While investigating this latter question, the following surprising factorization was discovered: $3B_{11}(x) = x(x - 1/2)(x - 1)(x^2 - x - 1)(3x^6 - 9x^5 + 2x^4 +$

$11x^3 + 3x^2 - 10x - 5$). This factorization is a counterexample to the conjecture that $B_{2m+1}(x)/x(x - 1/2) \cdot (x - 1)$ is always irreducible over the rational field. (Received October 6, 1967.)

653-157. CARL EBERHART, University of Kentucky, Lexington, Kentucky. Metrizability of trees.

A tree is a locally connected hereditarily unicoherent continuum. (See Ward, Mobs, trees, and fixed points, Proc. Amer. Math. Soc. 8 (1958), 798-804, and Whyburn, Analytic topology, Amer. Math. Soc. Colloq. Publ. Vol. 28, Amer. Math. Soc. Providence, R. I., p. 88.) Theorem 1. The weight of a tree is also its density. Corollary. Separable trees are metrizable. Definition. A point $x \in X$, a tree, is a branchpoint of X, if $X \setminus x$ has more than two components. B denotes the branchpoints of X. X is called a Souslin tree provided each arc in X is separable and every collection of pairwise disjoint open sets of X is countable. Theorem 2. A Souslin tree is separable iff it has a countable number of branchpoints. Theorem 3. Each closed set in a Souslin tree is G_δ. Definition. The core of a Souslin tree X, K, is the set of condensation points of the branchpoints of X. Corollary. A Souslin tree is separable iff the interior of K is void. (Received October 6, 1967.)

653-158. Y. C. LU, Bowling Green State University, Bowling Green, Ohio. On holomorphic mappings of complex manifolds.

Let M and N be two complex manifolds with dimensions m and n respectively. Let $f: M \longrightarrow N$ be a holomorphic mapping and u be the general elementary symmetric function of f. Denote Δu be the Laplacian of a function u on the manifold. Then $1/2\, \Delta u = E + R(\xi;\xi) - S(\xi;\xi)$ where E is a nonnegative quantity, $R(\xi;\xi)$ and $S(\xi;\xi)$ are sums of Ricci tensors on M and curvature tensors of N respectively. This gives the generalization of Chern's formula in his recent paper about holomorphic mapping between same dimensional complex manifolds. Besides, the following theorem can be proved. Let $f: D_m \longrightarrow N$ be a holomorphic mapping where D_m is the unit m-ball in C^m with the standard Kaehler metric and where N is a n-dimensional hermitian manifold with negative constant holomorphic sectional curvature ($= -2m(m + 1)$). Then f is distance-decreasing. (Received October 6, 1967.)

653-159. J. C. BEIDLEMAN, University of Kentucky, Lexington, Kentucky 40506. Generalized Frattini subgroups of finite groups. II.

A proper normal subgroup H of a finite group G is called a generalized Frattini subgroup of G if $G = N(P)$ for each normal subgroup L of G and each Sylow p-subgroup P, p a prime, of L such that $G = HN(P)$. Here N(P) is the normalizer of P in G. Several properties of generalized Frattini subgroups have been developed (J. C. Beidleman and T. K. Seo, Generalized Frattini subgroups of finite groups, Pacific J. Math. to appear). In the present paper we establish the following results. Theorem 1. A proper normal subgroup H of G is a generalized Frattini subgroup of G if and only if $F(G)/H = F(G/H)$, where F(G) is the Fitting subgroup of G. Theorem 2. Let G be a finite nonnilpotent group with Frattini subgroup $\phi(G)$. If every proper subgroup of $G/\phi(G)$ is nilpotent, then $\phi(G)$ is the unique maximal generalized Frattini subgroup of G. Theorem 3. Let G be a finite supersolvable group and let H be a self-normalizing nilpotent maximal subgroup of G. Then the core of H is a generalized Frattini subgroup of G. (Received October 6, 1967.)

653-160. D. A. DRAKE, University of Florida, Gainesville, Florida. Neighbor collineations of Desarguesian Hjelmslev planes.

In projective Hjelmslev planes (H-planes), points are said to be neighbor if they are joined by more than one line; a neighbor collineation is a collineation which maps each point onto a point neighbor to itself. Results are obtained for the full collineation groups of Desarguesian projective H-planes (DPH's) which generalize well-known results on the collineation groups of ordinary Desarguesian projective planes. Analogues are obtained for the neighbor collineation group C^N of a DPH, e.g. C^N is shown to be transitive on the sets of neighbor quadrilaterals. Two uniform projective H-planes are presented for which C^N is not transitive on neighbor quadrilaterals even though they possess Desarguesian affine H-planes. This suggests that C^N may be a sensitive tool for investigating projective H-planes. Let π be a uniform DPH, and let H be the ring coordinatizing π. Then $o(H) = t^2 = p^{2u}$ for some prime p. Then C^N is shown to be the semidirect product of a subgroup G by a subgroup G_δ. G is the direct product of 8u or 9u subgroups of order p (8u if and only if H is commutative). G_δ is cyclic, of order 1 or t - 1 depending upon another property of H. (Received October 6, 1967.)

653-161. BRUCE WOOD, The University of Arizona, Tucson, Arizona 85721. A generalization of the Bernstein polynomial.

Let f be analytic on $\Delta = \{z: |z| \leqq 1\}$ and, for $W \in \Delta$ and any complex number z, define the matrix $(a_{nk}(z))$ by $\{1 - z + zf(w)\}^n = \sum_{k=0}^{\infty} a_{nk}(f)w^k$, n = 1,2,..., and $a_{00}(f) = 1$, $a_{0k}(f) = 0$, k = 1,2,.... Define the linear operator H_n by $H_n(g,z) = \sum_{k=0}^{\infty} g(k/n)a_{nk}(z)$ for n = 1, z,... and any function g such that the series converges. The matrix $(a_{nk}(z))$ is a special case of a class of matrices introduced by Sonnenschein and studied by Clunie and Vermes {Acad. Roy. Belg. Bull. Cl. Sci. 45 (1959), 930-945}. The nth order Bernstein polynomial is obtained when f(z) = z for all z. Let $\Omega = \{f: f \in A(\Delta), f(1) = 1, f^{(1)}(1) = 1$, and $f^{(k)}(0) \geqq 0$, $k = 0, 1, 2, \ldots\}$. The following are established: (i) if $g \in C\{a,b\}$ and $f \in \Omega$ then $\lim_{n\to\infty} H_n(g,x) = g(x)$ uniformly on $\{a,b\}$; (ii) if $g(z) = \sum_{k=0}^{\infty} a_k z^k$ for $z \in \Delta$, $\sum_{k=0}^{\infty} |a_k| < \infty$, and $f \in \Omega$ then $\lim_{n\to\infty} H_n(g,z) = g(z)$ for each $z \in \Delta$ and $\lim_{n\to\infty} H_n(g,z) = g(z)$ uniformly on compacta of $\{z: |z| < 1\}$; and (iii) if g is analytic on the interior of the bounded component of the complement of an ellipse having foci 0 and 1 and $f \in \Omega$ then $\lim_{n\to\infty} H_n(g,z) = g(z)$ uniformly on any closed set contained in the interior of this bounded component. (Received October 2, 1967.)

653-162. E. R. BERLEKAMP and R. L. GRAHAM, Bell Telephone Laboratories, Incorporated, Murray Hill, New Jersey 07971. Irregularities in the distributions of finite sequences.

Let $X = (x_1, x_2, \ldots)$ be a sequence of points in the interval [0,1), I a subinterval, |I| its length and $X_n(I)$ the number of x_m in I with $m \leqq n$. Let $F_n(X)$ be the least upper bound of $|X_n(I) - n|I||$ for I varying in [0,1). $F_n(X)$ was first proved to be unbounded by van Ardenne-Ehrenfest, who showed $F_n(X) > c_1 \log\log n / \log\log\log n$. This was later improved by K. F. Roth who established $F_n(X) > c_2(\log n)^{1/2}$. We consider the following finite variant of this problem. For $n \geqq 1$, $0 \leqq k < n$, define $B_{n,k} = [k/n, k + 1/n)$. Fix an integer $d \geqq 0$ and suppose $(x_1, x_2, \ldots, x_{s+d})$ is a sequence with $x_i \in [0,1)$ and with s = s(d) chosen to be maximal such that for each $r \leqq s$ and each $k < r$, $B_{r,k}$ contains at least one point of the subsequence $(x_1, x_2, \ldots, x_{r+d})$. The fact that $s(d) < \infty$ follows from above. We establish the Theorem. (i) s(0) = 17; (ii) $s(d) < 4^{(d+2)^2}$. (Received October 3, 1967.)

653-163. WITHDRAWN.

653-164. J. J. UHL, JR., Apartment 806, 5055 Seminary Road, Alexandria, Virginia 22311.
Compact operators on Orlicz spaces.

Let Σ be a field of subsets of a set Ω, μ be a finitely additive nonnegative set function with finite subset property defined on Σ, and Σ_0 be the ring of sets of finite μ-measure. A partition $\pi = \{E_n\}$ is a finite disjoint collection of Σ_0-sets. Partitions are partially ordered by defining $\pi_1 \leqq \pi_2$ if $E \in \pi_1 \Rightarrow E$ is a union of members of π_2. $\mathfrak{X}$ and $\mathfrak{Y}$ are B-spaces; Φ is a continuous Young's function with complementary function Ψ. $L^{\Phi}(\mathfrak{X})$ is the Orlicz space of all totally μ-measurable functions f satisfying $\int_{\Omega}\Phi(\|f\|/k)d\mu < \infty$ for some k. $M^{\Phi}(\mathfrak{X})$ is the closed subspace of $L^{\Phi}(\mathfrak{X})$ determined by the μ-simple functions. Theorem. Let $\mathfrak{X}$ be reflexive, Ψ obey $\Psi(2x) \leqq N\Psi(x)$ for all x and some N. Then if $t: M^{\Phi}(\mathfrak{X}) \to \mathfrak{Y}$ is a bounded linear operator, t is compact if and only if (i) The operator $T(E): \mathfrak{X} \to \mathfrak{Y}$ defined by $T(E)[x] = t(x\chi_E)$ (χ_E is the indicator function of E) is compact for each $E \in \Sigma_0$, and (ii) if for each partition $\pi = \{E_n\}$ the operator t_π is defined for $f \in M^{\Phi}(\mathfrak{X})$ by $t_\pi(f) = t(\sum_\pi \int_{E_n} f d\mu/\mu(E_n)\chi_{E_n})$, then $\lim_\pi \|t - t_\pi\| = 0$ in the uniform operator topology. Moreover, if $\mathfrak{X}$ is finite dimensional, then each compact linear operator $t: M^{\Phi}(\mathfrak{X}) \to \mathfrak{Y}$ is the operator limit of linear operators with a finite dimensional range. (Received October 5, 1967.)

63-165. R. M. LEWIS and D. S. AHLUWALIA, Courant Institute of Mathematical Sciences, New York University, New York, New York. Uniform theory of diffraction of progressing waves.

A progressing wave is a solution of the wave equation $Lu = \Delta u - (1/c^2)u_{tt} = 0$ and is of the form $u(X,t) = \sum_{m=0}^{\infty} e_m[\phi(X,t)]z_m(X,t)$ where $e_0(\tau)$ is an arbitrary function and $e'_m(\tau) = e_{m-1}(\tau)$. The phase function $\phi(X,t)$ is constant along the bicharacteristics of the wave equation and the amplitude functions $z^m(X,t)$ satisfy first order ordinary differential equations which can be solved explicitly along the bicharacteristics. Our problem is to find the total field u(X,t) which satisfies the wave equation, the boundary condition $u = 0$ or $\partial u/\partial n = 0$ on the screen S and has finite limit at the edge of S. The screen consists of a portion of a plane, bounded by a smooth edge curve. $u_0(X,t)$, the incident progressing wave is prescribed by specifying initial conditions for the problem. A formal solution is constructed. Away from the shadow boundary the solution reduces to the sum of the incident, reflected and diffracted progressing waves. The diffracted wave corresponds to the diffracted rays which emanate from the edge. (Received October 2, 1967.)

653-166. MAURICE CHACRON, University of Sherbrooke, Sherbrooke, Canada. On the non-singular positive square matrix of an ordered ringoid. Preliminary report.

Let R be a ringoid with 0 and 1 in the sense of G. Birkhoff and let ($\leqq$) be a fixed order relation on R such that (i) for every positive element x($\geqq 0$) of R if $a \leqq b$ than $ax \leqq bx$ and $a + x \leqq b + x$. Assume that the binary multiplication and the binary addition in R are commutative. If $\mathfrak{M}_p$ denote the ringoid of the square matrix of order p on the ringoid R, it is shown that the order-product induced by ($\leqq$) in $\mathfrak{M}_p$ preserve incidently property (1). Again, assume that (ii) the order in R is with ascending chain condition and that for at least one natural number n we have $x^n \geqq x$ for all x in R, it is shown that the subringoid $\mathfrak{M}'_p$ of $\mathfrak{M}_p$ of the positive symmetric matrix satisfies properties (i) and (ii), which properties imply that the multiplicative semigroup of R and of $\mathfrak{M}_p$ are periodic semigroups. From this result, it follows that every nonsingular positive square matrix of an arbitrary order which commutes with its transpose is an inversible matrix in respect of the unity matrix. (Received October 4, 1967.)

653-167. R. A. ALO and H. L. SHAPIRO, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213. Extensions of totally bounded pseudometrics.

The concept of extending to the topological space X a continuous pseudometric defined on a subspace S of X has been studied by many authors. Recently H. L. Shapiro studied the problem by considering subspaces S for which every continuous pseudometric defined on S has a continuous pseudometric extension to X. Such subspaces S are said to be P-embedded in X. Shapiro showed that if S is P-embedded in X, then S is C-embedded in X, but not conversely. In this paper the authors introduce the concept of T-embedding, a particular case of P-embedding. A subset S is T-embedded in X in case every totally bounded continuous pseudometric on S has a totally bounded continuous pseudometric extension to X. The authors characterize T-embedding in terms of various types of open covers of the space X and show that in order for a subspace S to be T-embedded in X it is sufficient to extend every totally bounded continuous pseudometric on S to a continuous pseudometric on X. The authors prove that a subset is T-embedded if and only if it is C*-embedded (i.e., every bounded real-valued continuous function can be extended). Finally, using the concept of T-embedding, some new characterizations for a topological space to be normal are given. (Received October 5, 1967.)

653-168. J. B. DEEDS, Louisiana State University, Baton Rouge, Louisiana 70803. Summability of vector sequences.

In this note, the notion of almost convergence, as defined by G. G. Lorentz for scalar sequences, is generalized to sequences of vectors in Hilbert space. The almost convergence of such a sequence is equivalent to the norm convergence of the averages of the sequence, in a translation invariant fashion, provided its range is pre-compact. The particular cases of periodic and almost periodic sequences are discussed, and component-wise characterizations are developed, along with counter-examples. One result is the following. Let T be a continuous transformation mapping a Hilbert space into itself, such that T(0) = 0. Then T preserves almost convergent sequences iff T is additive and real homogeneous. (Received October 9, 1967.)

653-169. MARK MANDELKER, University of Kansas, Lawrence, Kansas 66044. Subspaces of F'-spaces.

A completely regular Hausdorff space X is an F'-space if for every point p in X, the ideal 0_p (of all real-valued continuous functions on X that vanish on a neighborhood of p) is prime. See [L. Gillman and M. Henriksen, Rings of continuous functions in which every finitely generated ideal is principal, Trans. Amer. Math. Soc. 82 (1956), 366-391] and [W. W. Comfort, N. Hindman, and S. Negrepontis, F'-spaces and their products with P-spaces, to appear]. A subspace Y is z-embedded in X if for every zero-set W of Y there is a zero-set Z of X such that $W = Z \cap Y$. In particular, C^*-embedded or Lindelöf subspaces are z-embedded. Theorem. Every z-embedded subspace of an F'-space is also an F'-space. Corollary. Every open subspace of a locally compact F'-space is also an F'-space. In particular, every open subspace of $\beta R - R$ is an F'-space. (Assuming the continuum hypothesis, it is shown in [N. J. Fine and L. Gillman, Extension of continuous functions in βN, Bull. Amer. Math. Soc. 66 (1960), 376-381] that every open subspace of $\beta R - R$ is in fact an F-space.) Theorem. If X is locally compact at infinity and σ-compact, then $\beta X - X$ is an F'-space. More generally, for any X, every locally compact G_δ in βX, disjoint from X, is an F'-space. (Received December 1, 1967.)

653-170. P. K. KULSHRESTHA, Louisiana State University, New Orleans, Louisiana 70122. Steady Couette flow in a centrifuge.

The problem of steady Couette flow of an incompressible viscous as well as an incompressible isotropic idealized elastico-viscous liquid in a centrifuge is considered. The rheological behavior of these liquids is assumed to be free from the Weissenberg effects at the boundaries of the centrifuge and the interaction between mechanical and thermal processes is disregarded. Exact solutions for the two types of liquids are obtained, the pressure and the extraneous force acting along the axis of rotation evaluated and the torques necessary to maintain the centrifugal motion calculated. Stability considerations of the flow impart relevant information on the problem of homogeneous mixing of miscible liquids of the two types. (Received October 9, 1067.)

653-171. J. R. DORROH, Louisiana State University, Baton Rouge, Louisiana 70803. Semigroups of nonlinear transformations.

We consider strongly continuous semigroups $\{G(t);\ t \geqq 0\}$ of nonexpansive transformations from S into S, where S is a subset of a Banach space X. S is required to be closed and convex and is subject to other restrictions as well, but S may be X, the closed unit ball in X, or a Saks space, and the setting is sufficiently general to cover the case of a semigroup giving the solution of a quasi-linear partial differential equation. Analogues are obtained for theorems about linear semigroups dealing with generators, resolvents, and exponential formulas. For instance, suppose that $G(\cdot)x$ is continuously differentiable for all x in some dense subset of S, and define $A_h x = (1/h)[G(h)x - x]$ for x in S and $h > 0$. Then A_h is the "infinitesimal generator" of a "uniformly continuous" semigroup $\{G^h(t)\}$ of nonexpansive transformations from S into S, and for each x in S, $G(t)x = \lim_{(h \to 0)} G^h(t)x$ uniformly for t in bounded intervals. (Received October 9, 1967.)

653-172. ROBERT GILMER, Florida State University, Tallahassee, Florida 32306. The unique primary decomposition theorem in commutative rings without identity.

A commutative ring R is a W-ring if each ideal of R has a unique shortest primary representation. Characterizations of W-rings with identity are known [Proc. Amer. Math. Soc. 14 (1963), 777-781]. Let R be a W-ring without identity. Theorem 1. If R is a belonging prime of (0), then R is the direct sum of a W-ring with identity and a nil ring; the converse also holds. Theorem 2. If R is not a belonging prime of (0), then R is the direct sum of a W-ring with identity and a W-domain; the converse is valid. Theorem 3. Let D be an integral domain without identity. Then D is a W-ring if and only if either (1) D is zero-dimensional, or (2) D is one-dimensional, the residue class ring of any maximal prime ideal of D is a field, and each nonzero element of D belongs to only finitely many maximal prime ideals of D. (Received October 9, 1967.)

653-173. W. M. BOGDANOWICZ and M. M. MATTAMAL, Catholic University of America, Washington, D. C. 20017. Representation of complete Lebesgue integrals by means of volumes.

A real valued functional $\int$ is called a Lebesgue integral if its domain $D(\int) = L$ consists of real-valued functions on a space X and the following conditions are satisfied: L is a linear lattice satisfying the Stone condition, i.e. L is a linear space and $f \cap g, f \cap 1 \in L$ for all $f,g \in L$; $\int$ is a finite valued positive linear functional such that if $f_n \in L$, $f_n(x) \geqq 0$ on X, $f(x) = \sum_{n=1}^{\infty} f_n(x) < \infty$ for $x \in X$ and $\sum_{n=1}^{\infty}\int f_n < \infty$, then $f \in L$ and $\int f = \sum_{n=1}^{\infty}\int f_n$. The Lebesgue integral is said to be complete if for every function g such that there exists $f \in L$ satisfying the condition $0 \leqq g(x) \leqq f(x)$ for all $x \in X$ and $\int f = 0$, we have $g \in L$. Theorem. If $\int$ is a (complete) Lebesgue integral, then $V = \{A \subset X : c_A \in L\}$ is a prering and the function v defined by $v(A) = \int c_A$ for $A \in V$ is a (complete) upper complete volume on V, where c_A is the characteristic function of the set A. If $\int$ is a complete Lebesgue integral, then $D(\int) = L(v,R)$ and $\int f = \int f dv$ for $f \in D(\int)$. For definitions see: Bogdanowicz, Proc. Nat. Acad. Sci. U.S.A. 53 (1965), 492-498 and Proc. Japan Acad. 43 (1967), 286-289. (Received October 9, 1967.)

653-174. T. I. SEIDMAN, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213. Approximation of C_0 semigroups.

Let $\{X_\alpha\}$ be a net of sequentially complete lctvs approximating the sclctvs X in the sense that there are continuous linear surjections $\pi_\alpha : X \to X_\alpha$ such that $\phi_\alpha \circ \pi_\alpha \to \phi$ strongly for every $\phi \in \Phi = \{$all continuous seminorms on X$\}$ where $\phi \longmapsto \phi_\alpha$ maps Φ into $\Phi_\alpha = \{$continuous seminorms on $X_\alpha\}$. Let $\mu : \Phi \to \Phi$ and call a linear operator T on X (or X_α) μ-continuous if $\phi(Tx) \leq |\mu\phi|(x)$, $x \in X$ (or $\phi_\alpha(Tx) \leqq [\mu\phi]_\alpha(x)$, $x \in X_\alpha$), for $\phi \in \Phi$. Let $S_\alpha = \{S_\alpha(t) : t \geqq 0\}$ be a C_0 semigroup of μ-continuous operators on X_α for each α with $R_\alpha(\lambda)$ as resolvent of the infinitesimal generator of S_α. Then $S_\alpha(t) \to S(t)$ (i.e., $\phi_\alpha(S_\alpha(t)\pi_\alpha x - \pi_\alpha S(t)x) \to 0$, $x \in X$, $\phi \in \Phi$) uniformly on compact t-intervals iff $R_\alpha(\lambda_0)$ converges strongly for some $\lambda_0 > 0$ (hence, for all λ_0 with Re $\lambda_0 > 0$), S being a μ-continuous C_0 semigroup on X. A similar result holds for discrete parameter semigroups on X_α converging to a C_0 semigroup on X. (Received October 9, 1967.)

653-175. J. H. WESTON, University of Saskatchewan, Regina Campus, Regina, Saskatchewan. m-compactness, m-quasicompactness, and m-pseudocompactness.

Let X be a topological space and m an infinite cardinal. X is m-compact if each open cover of X, of cardinality at most m, has a finite subcover. X is m-quasicompact if each open cover $\mathscr{U}$ of X by cozero sets, with $|\mathscr{U}| \leqq m$, has a finite subcover. X is called m-pseudocompact if each continuous function $F: X \rightarrow R^m$ has a compact range where R denotes the reals. Lemma. If X is m-pseudocompact then it is m-quasi-compact. If X is completely regular the converse is true. A space is called m-regular if for each open cover $\mathscr{U}$ with $|\mathscr{U}| \leqq m$ there is an open cover $\mathscr{V}$ with $|\mathscr{V}| \leqq m$ and such that for each $V \in \mathscr{V}$ there is a $U \in \mathscr{U}$ with $V^- \subseteq U$. Theorem. If X is a normal m-regular space then m-compactness, m-pseudocompactness, and m-quasicompactness are equivalent. Corollary. The theorem is also true if X is a completely regular, m-regular, $\aleph_0$-regular space. (Received October 9, 1967.)

653-176. EMANUEL VEGH, U. S. Naval Research Laboratory, Code 7820, Washington, D. C. 20390. Primitive roots modulo a prime as consecutive terms of an arithmetic progression.

Theorem 1. For each positive integer s there is a positive integer N (depending only on s) such that for each prime p greater than N there is an arithmetic progression that has at least s consecutive terms as primitive roots modulo p. Theorem 2. Let s be a positive integer and let $q_1 < q_2 < \dots < q_r$ be arbitrary primes. There is a positive integer N (depending on s and q_r) such that for each prime p satisfying the conditions (i) $p > N$ and (ii) $p - 1$ is of the form $q_1^{\alpha_1} q_2^{\alpha_2} \dots q_r^{\alpha_r}, \alpha_i \geqq 0$ - there is an arithmetic progression, with common difference an arbitrary primitive root modulo p, that has at least s consecutive terms as primitive roots modulo p. Corollary. Let each of s and m be a positive integer and let $q_1 < q_2 < \dots < q_r$ be arbitrary primes. There is an integer N (depending on s, q_r and m) such that for each prime p satisfying the conditions (i) and (ii) above and (iii) that the least primitive root modulo p is not greater than m- there is a sequence of s consecutive primitive roots modulo p. (Received October 9, 1967.)

653-177. F. J. WAGNER, University of Cincinnati, Cincinnati, Ohio 45221. Reflexive spaces and dual topologies.

For a topological space (X,τ), the dual topology τ' of τ is that which has as a base of open sets the closed sets of (X,τ). τ is reflexive if $\tau = \tau''$. Then (1) τ is reflexive if and only if every intersection of open sets is open; (2) τ' is reflexive; (3) a reflexive T_1 topology is discrete; (4) $\tau' = \tau$ if and only if τ' is weaker than τ if and only if every open set is closed. For x and $y \in X$, say $x \leqq y$ if x is in every neighborhood of y. Then $\leqq$ is a quasi-ordering of X and τ'' is the lower topology defined by $\leqq$. (Received October 9, 1967.)

653-178. J. J. MALONE, JR., Texas A and M University, College Station, Texas 77843. A near-ring embedding problem.

Near-rings of transformations are interesting because every near-ring may be embedded in a near–ring of transformations on some group. Let (G, +) be a group and T(G) its transformation near–ring, $T_0(G)$ the sub-near-ring of T(G) consisting of the transformations which commute multiplicatively with the 0 transformation. Theorem. (H, +) is embedded in (G, +) iff $T_0(H)$ is embedded in $T_0(G)$ by a kernel-preserving map. (For the definition of "kernel-preserving map" see J. C. Beidleman, On groups and their near-rings of functions, Amer. Math. Monthly 73 (1966), 981-983.) Corollary. H is embedded in G iff T(H) is embedded in T(G). Theorem. If (H, +) is a direct summand in (G, +), then $T_0(H)$ embeds in $T_0(G)$ as a direct summand which is a right ideal. (Received November 27, 1967.)

653-179. MICHAEL FRIEDBERG, University of Houston, Houston, Texas. Congruences defined by continuous real-valued functions.

Let S be a compact semigroup, C(S) the collection of continuous real-valued functions defined on S, and let $A \subset C(S)$. We define a relation R_A on S by saying that $(x,y) \in R_A$ if $f(axb) = f(ayb)$ for each $f \in A$ and $a, b \in S'$ (S' is S with an identity adjoined). R_A is a closed congruence and we denote the quotient space of S modulo R_A by S_A; S_A is a compact semigroup with the quotient topology. Theorem. If A is finite or denumerably infinite then S_A is metrizable with a subinvariant metric. Corollary. [Hofmann and Mostert]. A compact semigroup is (iseomorphic to) an inverse-limit of compact metric semigroups. Theorem. A compact, abelian, uniquely divisible semigroup is (iseomorphic to) an inverse limit of compact, metric, abelian, uniquely divisible semigroups. (Received October 9, 1967.)

653-180. K. F. BARTH and W. J. SCHNEIDER, Syracuse University, Syracuse, New York 13210. On a problem of Bagemihl and Erdös concerning the distribution of the zeros of an annular function.

Bagemihl and Erdös (A problem concerning the zeros of a certain kind of holomorphic function in the unit disk, J. Reine Angew. Math. 214/215 (1964), 340-344) have raised the question whether the zeros of an annular function (for definition see previous reference) must cluster to the whole unit circumference. This question is answered by the following Theorem. There exists an annular function f(z) whose zeros cluster only to a single point on the unit circumference. The function f(z) is constructed as a product of two functions g(z) and h(z) holomorphic in $D(= \{z : |z| < 1\})$ where (i) g(z) is constructed by means of Mergelyan's theorem and has the property that $\max_{z \in \gamma_n} |g(z) - n| < 1$ for all n where $\gamma_n = \{z : |z| = s_n, \epsilon_n \leqq \arg z \leqq 2\pi - \epsilon_n\}$ with $s_n \uparrow 1$ and $\epsilon_n \downarrow 0$, (ii) h(z) is constructed by a judicious repeated use of the following Lemma. Given any positive integer n, any positive number K, any complex number a ($|a| < 1$), any positive number $\epsilon > 0$ and any Jordan domain J ($\subset D$) which contains a and which has exactly one boundary point on $|z| = 1$, there exists a function w(z) holomorphic in D with $|w(z) - 1/K(z - a)^n| < \epsilon$ in D - J. The lemma follows from a construction similar to one used previously by the second author (Bull. Amer. Math. Soc. 72 (1966), 841-842). (Received October 10, 1967.)

653-181. V. J. MANCUSO, 161-34 28th Avenue, Flushing, New York 11432. Retraction in m-paracompact spaces.

Let C be a closed subset of a space X, and $f: C \to Y$ a continuous map from C into a space Y. Denote the adjunction space of X and Y via f by $X \cup_f Y$. Let W be a space and define $g: C \times W \to Y \times W$ by $g(c,w) = (f(c), w)$. Denote the adjunction space of $X \times W$ and $Y \times W$ via g by $(X \times W) \cup_g (Y \times W)$. Theorem. If W is locally compact, then $(X \times W) \cup_g (Y \times W)$ is homeomorphic to $(X \cup_f Y) \times W$. Corollary. If X and Y are m-paracompact normal, so is $X \cup_f Y$. In fact our methods show that the corollary holds for any class of normal spaces which can be characterized by the normality of their product with a compact (Hausdorff) space. Let Q be the class of m-paracompact normal spaces. Using well known techniques we can then establish Theorem. If $X \in Q$, X is an AR(Q), resp. ANR(Q), if and only if X is an ES(Q), resp. NES(Q). Theorem. X is an AR(Q) if and only if X is a contractible ANR(Q). (Received October 11, 1967.)

653-182. J. W. KENELLY, Clemson University, Clemson, South Carolina 29631, and B. D. EVANS, Oklahoma State University, Stillwater, Oklahoma. An upper bound on the dimension of the convex kernel.

Earlier papers have given conditions that establish the zero dimensionality of the convex kernel of a starshaped set. Hare and Kenelly (Nieuw Arch. Wisk. 14 (1966), 103-105), Foland and Marr (Pacific J. Math. 19 (1966), 429-432), and Larman (Proc. Camb. Phil. Soc. 63 (1967), 311-314). A forthcoming paper by F. A. Toranzos will give a theorem that generalizes these results. The following result extends the theorem of Toranzos. It is applicable to a larger collection of starshaped s sets, and it implies the previous point condition theorems. Statements are made in a finite dimensional linear space. Definition. The flats $L_1, \ldots, L_k$ assumed to have a common point are called intersectionally independent (i.i) provided the intersection of any $k - 1$ of them is not contained in the other. A single flat is taken to be an i.i. collection. Sets are i.i. if and only if their containing flats are i.i. Theorem. If a set S contains k intersectionally independent maximal convex subsets with the minimum dimension in the collection known to be l, then it is the case that the dimension of the convex kernel of S is at most $l - k + 1$. (Received October 11, 1967.)

653-183. WITHDRAWN.

653-184. J. H. ROWLAND, University of Wyoming, Laramie, Wyoming 82070. On the location of the deviation points in Chebyshev approximation.

Consider the problem of approximating continuous real-valued functions in the Chebyshev sense by polynomials of degree $\leqq n$ over the interval $[\alpha, \beta]$. Theorem. If $f^{(n+1)}$ and $g^{(n+1)}$ are positive and $f^{(n+1)}/g^{(n+1)}$ is strictly increasing on $[\alpha, \beta]$, then each interior deviation point of f is to the right of the corresponding point of g. Corollary. If $f^{(n+1)}$ is positive and strictly increasing, then each deviation point of f lies to the right of the corresponding oscillation point of the Chebyshev polynomial T_{n+1}. The same relationship is also true for the interpolation points. These results can be used to show that certain inequalities given by Shohat (Duke Math. J. 8 (1941), 376-385) are sharp. (Received October 11, 1967.)

653-185. K. -W. YANG, Western Michigan University, Kalamazoo, Michigan 49001. Completion of norm linear spaces.

Let F be the field of real numbers or the field of complex numbers. Let $N^{\cdot}$ ($B^{\cdot}$) denote the category whose objects are normed linear spaces (Banach spaces) over F and whose morphisms are contractions (continuous linear maps $f: X \to Y$ such that $|f| \leqq 1$). Let $\Lambda: N^{\cdot} \to B^{\cdot}$ be the "completion" functor. We say that a sequence in $N^{\cdot}$ (or $B^{\cdot}$) $\ldots \to X_{n-1} \xrightarrow{f_{n-1}} X_n \xrightarrow{f_n} X_{n+1} \to \ldots$ is normal exact if for each n, $f_n^{-1}(0) = f_{n-1}(X_{n-1})$ and $X_{n-1}/f_{n-1}^{-1}(0) \xrightarrow{f'_{n-1}} f_{n-1}(X_{n-1})$ (where f'_{n-1} is the map induced by f_{n-1})

is an isometry. Theorem. If $0 \to Y \xrightarrow{i} X \xrightarrow{p} Z \to 0$ is a normal exact sequence in N', then $0 \to \Lambda(Y) \xrightarrow{\Lambda(i)} \Lambda(X) \xrightarrow{\Lambda(p)} \Lambda(Z) \to 0$ is a normal exact sequence in B'. Corollary. Let $0 \to Y \to X \to Z \to 0$ be a normal exact sequence in N' Then X is complete iff Y and Z are. (Received October 11, 1967.)

653-186. BASIL GORDON, University of California, Los Angeles, California, and LORNE HOUTEN, Washington State University, Pullman, Washington. Asymptotic formulae for multirowed partitions.

In Abstract 642-38, these *Notices* 14 (1967), 71-72, a k-rowed partition of n, strictly monotonic on rows, is defined. The authors subsequently showed that the generating function $B_k(x) = \sum_{n=0}^{\infty} b_k(n)x^n$ for such partitions are $B_k(x) = \prod_{\nu=1}^{k-2}(1 - x^{\nu})^{[(k-\nu)/2]} P^{[k/2]} Q^{2\{k/2\}}$, where $P = \prod_{\nu=1}^{\infty}(1 - x^{\nu})^{-1}$, $Q = \prod_{\nu=1}^{\infty}(1 - x^{2\nu-1})^{-1}$ and $[\theta]$ and $\{\theta\}$ represent the integral and fractional parts of θ respectively. In this paper we determine a convergent series representation for $\rho_{a,b}(n)$; where $P^a Q^b = \sum_{n=0}^{\infty} \rho_{a,b}(n)x^n$ and a,b are integers, $a \geqq 1$. Combining this with the contribution from the polynomial $\prod_{\nu=1}^{k-2}(1 - x^{\nu})^{[(k-\nu)/2]}$ and taking the dominant term we obtain that $b_k(n)$ $\sim 2^{-1-(1/2)[k/2]-(1/2)\{k/2\}} \pi^{k^2/4-k/2+(1/2)\{k/2\}} (k/12n)^{(k^2-k+2)/8} \prod_{j=1}^{[(k-1)/2]}(k-2j)! \exp[\pi(kn/3)]^{1/2}$. (Received October 11, 1967.)

653-187. J. H. JORDAN, Washington State University, Pullman, Washington. The distribution of kth power residues and nonresidues in the Gaussian integers.

For the Gaussian prime γ and $k \mid N(\gamma) - 1$, the Gaussian integer α is a kth power residue or a kth power nonresidue depending on whether $\zeta^2 \equiv \alpha \pmod{\gamma}$ is solvable in Gaussian integers or not. A bound $B(\gamma,k)$ is established so that there is a kth power nonresidue β such that $|\beta| < B(\gamma,k)$ for γ whose norm is sufficiently large. The function $B(\gamma,2) = N(\gamma)^{a+\epsilon}$, for all $\epsilon > 0$ and $a = (8e)^{-1}$ is established. The function $B(\gamma,k) = N(\gamma)^{a+\epsilon}$ for all $\epsilon > 0$ and where 1/4a is the unique solution of $\rho(u) = 1/k$ with ρ the Dickman-DeBruijn function is established. (Received October 11, 1967.)9

653-188. MILTON ROSENBERG, University of Kansas, Lawrence, Kansas 66044. Derivatives with respect to vector valued measures.

Let X, Y be two arbitrary Hilbert spaces. Let $M(B) = \int_B M'(\omega)d\mu$, $N(B) = \int_B N'(\omega)d\mu$ be indefinite Bochner integrals (thus vector measures) over the measurable space $(\Omega,\mathscr{B})$, where $M'(\omega) \in X$, $N'(\omega) \in Y$. Let $\mathscr{S}(Y) = \{$submanifolds of Y$\}$, and $\mathscr{S}(X,Y) = \{$submanifolds of L(X,Y) (the bounded linear operators)$\}$. A function $S: \Omega \to \mathscr{S}(Y)$ is nicely measurable iff $\exists$ an at most countable family of strongly-measurable functions ξ_α, $\alpha \in \Lambda$, such that $A(\omega) = [\xi_\alpha(\omega)]: l^2(\Lambda) \to Y$, defined by $A(\omega)\cdot(a_\alpha)_{\alpha\in\Lambda} = \sum a_\alpha \xi_\alpha(\omega)$, $\sum |a_\alpha|^2 < \infty$, is bounded with closed range $S(\omega)$, $\forall\omega$. A function $\mathscr{L}: \Omega \to \mathscr{S}(X,Y)$ is nicely measurable relative to M' iff $\exists$ a family $(\Phi_\alpha)_{\alpha\in\Lambda}$ of strongly-measurable functions such that (i) $\mathscr{L}(\omega) = \{\sum_\alpha a_\alpha \Phi_\alpha(\omega) : \sum|a_\alpha|^2 < \infty\}$ (ii) $T(\omega) = \mathscr{L}(\omega)\cdot M'(\omega)$ is nicely measurable in terms of $\eta_\alpha = \Phi_\alpha \cdot M'_{\alpha\in\Lambda}$. R. N. Theorem. $N'(\omega) \in \mathscr{L}(\omega)M'(\omega)$ a.e. $(\mu) \Leftrightarrow \exists$ a strongly-measurable Φ such that (i) $\Phi(\omega) \in \mathscr{L}(\omega)$ a.e. (μ), (ii) $N'(\omega) = \Phi(\omega)M'(\omega)$ a.e. (μ). In this case, we say $N << M \bmod (\mathscr{L})$. We also obtain a Lebesgue decomposition theorem. These results are coordinate-free infinite-dimensional extensions of Rosenberg, Abstract 648-19, these *Notices* 14 (1967), 634. We make use of Desoer and Whalen, A note on pseudoinverses, J. SIAM 11 (1963), 442-447. (Received October 13, 1967.)

653-189. C. A. AKEMANN, University of Pennsylvania, Philadelphia, Pennsylvania. Sequential convergence in the dual of a W*-algebra.

Let A be a W*-algebra and A* its dual as a Banach space. Let A_0^* be the closed subspace of A* generated by the pure states of A. Theorem. Let $\{f_n\}$ be a sequence of positive functionals in A* and suppose that f_n converges to f in the weak* topology for some f in A_0^*. Then f_n converges to f uniformly. It is necessary here that we restrict to sequential convergence (as opposed to nets) and that A be a W*-algebra (as opposed to a C*-algebra). (Received October 13, 1967.)

653-190. H. W. McLAUGHLIN, Rensselaer Polytechnic Institute, Troy, New York 12181, and F. T. METCALF, University of California, Riverside, California. Behavior of the extended Holder inequality with respect to the index set.

Suppose that I and J are nonempty finite sets of positive integers; that a_{ij} $(i \in I,\ j \in J)$ are positive real numbers; and that λ_j $(j \in J)$ are positive real numbers. Then the "extended Hölder inequality" states that $R(I,J) \leqq 1$, where $R(I,J) = [\sum_{i\in I} \prod_{j\in J} a_{ij}^{\mu_j} / \prod_{j\in J}(\sum_{i\in I} a_{ij})^{\mu_j}]^{\sum_{k\in J}\lambda_k}$ and $\mu_j = \lambda_j / \sum_{k\in J} \lambda_k$ $(j \in J)$. Theorem. Suppose that I, J_1, and J_2 are nonempty finite index sets, with J_1 and J_2 disjoint; and that λ_j $(j \in J_1 \cup J_2)$ are positive real numbers. Then $R(I, J_1 \cup J_2) \leqq R(I,J_1) \cdot R(I,J_2)$, where equality holds if and only if there exists a real number τ such that $\prod_{j\in J_1} a_{ij}^{\mu_{1j}} = \tau \prod_{j\in J_2} a_{ij}^{\mu_{2j}}$ for all $i \in I$, with $\mu_{1j} = \lambda_j / \sum_{k\in J_1} \lambda_k$ and $\mu_{2j} = \lambda_j / \sum_{k \in J_2} \lambda_k$. (Received October 13, 1967.)

653-191. PERRIN WRIGHT, Florida State University, Tallahassee, Florida 32306. A uniform generalized Schoenflies theorem.

The generalized Schoenflies theorem proved by M. Brown [Bull. Amer. Math. Soc. 66 (1960), 74-76] states that a locally flat (n - 1)-sphere in E^n or S^n is flat. Equivalently, if $f: S^{n-1} \to E^n$ is a locally flat embedding, then f has an extension $F: B^n \to E^n$. Let $\theta(x,y)$ denote the angle in radians between two points x,y in E^n. Theorem 1. If $f: S^{n-1} \to E^n$, $n \geqq 5$, is a locally flat embedding such that for all $x \in S^{n-1}$, $\mathrm{dist}(x,f(x)) < \epsilon$ and $\theta(x,f(x)) < \epsilon$, then f has an extension $F: B^n \to E^n$ such that for all $x \in B^n$, $\mathrm{dist}(x,F(x)) < (36n\epsilon + 81\epsilon)(1+\epsilon) + 3\epsilon$. Theorem 2 is a similar result for embeddings of S^{n-1} into S^n. (Received October 13, 1967.)

653-192. NICOLAS ARTEMIADIS, Southern Illinois University, Carbondale, Illinois. Growth estimates of functions in L^1.

Let $f \in L^1(-\infty,\infty)$ and $\hat{f}(x) = \int_{-\infty}^{\infty} f(t)e^{itx}dt$, the Fourier transform of f. Then it is obviously true that if $\hat{f}$ belongs to $L^1(-\infty,\infty)$, f is essentially bounded; in fact we have $|f(t)| \leqq (1/2\ )\|\hat{f}\|$ a.e. In this paper we give a much weaker condition, than $\hat{f} \in L^1(-\infty,\infty)$, for f to be essentially bounded. By Λ_f we denote the Lebesgue set for f. Put $\phi(x) = \min\{R_e f(x), 0\}$. Theorem. Hypothesis. $f \in L^1(-\infty,\infty)$; $f(t) = 0$ for $t < 0$; $0 \in \Lambda_f$; $\phi \in L^1(-\infty,\infty)$. Conclusion. $|f(t)| \leqq 2 R_e f(0) + A(t)$; a.e., where A(t) is a nonnegative essentially bounded function with $A(t) \leqq (2/\pi)\|\phi\|$ a.e. There are applications of the above theorem, in the growth estimates of the coefficients of a Dirichlet or Taylor series. (Received October 13, 1967.)

653-193. M. L. MARX, Vanderbilt University, Nashville, Tennessee 37203. Light open mappings on a torus with disk removed.

Suppose δ is a continuous mapping of a Jordan curve J into Euclidean two-space. Consider J as the boundary of T, a 2-dimensional torus with a disk removed, and also as the boundary of D, a 2-cell. This paper is concerned with the relationship between the case where has a light open continuous extension to T and the case where has a similar extension to D. One such result is: If δ has a light open extension to T, then there exists a light open mapping $f: D \to S^2$ such that $f^{-1}(\infty)$ contains one or no points. (Received October 13, 1967.)

653-194. WITHDRAWN.

653-195. L. W. BEINEKE, Purdue University, Fort Wayne, Indiana 46805. On derived graphs and digraphs.

The derived graph of a graph G has the edges of G as its vertices, with adjacency determined by the adjacency of the corresponding edges in G. An analogous definition is made for digraphs. Characterizations of derived graphs and digraphs are given in terms of excluded subgraphs. Some results, corresponding to known results for graphs, are given for iterated derived digraphs. (Received October 13, 1967.)

653-196. J. L. LEONARD, University of Arizona, Tucson, Arizona 85721. Generalized derivatives and monotonicity.

Zahorski [Trans. Amer. Math. Soc. 69 (1950), 1-54] conjectured that Tolstoff's Theorem on the monotonicity of a real function could be generalized by weakening the hypothesis of approximate continuity to the property of being Darboux-Baire 1. He further stated that he did not know whether the theorem would hold if the approximate derivative were replaced with a weaker definition of the derivative. Bruckner [Mich. Math. J. 13 (1966), 15-26] verified the validity of the conjecture. Using Bruckner's results it is possible to show that the theorem is valid for far weaker derivatives, to wit: the preponderant derivative of Denjoy, the qualitative derivative of S. Marcus, as well as the ordinary unilateral derivative. (Received October 13, 1967.)

653-197. LUDVIK JANOS, University of Florida, Gainesville, Florida. On "Lipschitz structures", structures which are relevant to the concept of contractivity.

Let $\mathfrak{A} \subset P(X \times X)$ be a uniformity on X and $\mathscr{L}: \mathfrak{A} \to \mathfrak{A}$ a self-mapping on $\mathfrak{A}$ which we will call "Lipschitz operator", satisfying the following conditions: (i) $\forall U \in \mathfrak{A}[\mathscr{L}U \subset U]$, (ii) $\forall U \in \mathfrak{A}[\mathscr{L}U \circ \mathscr{L}U \subset U]$, (iii) $\forall V \in \mathfrak{A}\ \forall x, y \in X\ \exists k \exists U \in \mathfrak{A}[\mathscr{L}^k U \subset V \wedge (x,y) \in U]$. The triple $(X, \mathfrak{A}, \mathscr{L})$ satisfying these conditions we will call "Lipschitz structure" (L.S.) on X. Let $\phi: X \to X$ be a continuous self-mapping on X. Extending ϕ over $X \times X$ in usual way we can define contractivity of ϕ on $(X, \mathfrak{A}, \mathscr{L})$ requiring the existence of n such that $\forall U \in \dot{\mathfrak{A}}[\phi^n(U) \subset \mathscr{L} U]$. Defining a complete L. S. $(X, \mathfrak{A}, \mathscr{L})$ as a L. S. the underlying uniform structure $(X, \mathfrak{A})$ of which is complete, we can prove a generalized Banach fixed point theorem. Theorem. Let $(X, \mathfrak{A}, \mathscr{L})$ be a complete L. S. and $\phi: X \to X$ contractive on $(X, \mathfrak{A}, \mathscr{L})$. Then (a) ϕ has a unique fixed point $y \in X$ on X and (b) $\forall x \in X[\phi^n(x) \to y$ as $n \to \infty]$. (Received October 13, 1967.)

653-198. B. G. CASLER and T. J. SMITH, Louisiana State University, Baton Rouge, Louisiana 70803. Standard spines of compact connected combinatorial n-manifolds.

If M is a combinatorial n-manifold then it is possible to define a finite sub-polyhedron of M called an allowable polyhedron. If M is a compact connected combinatorial n-manifold with boundary, K is an allowable sub-polyhedron of M and M collapses to K, then K is a standard spine of M. The following theorems are proven. Theorem A. If M is a compact connected combinatorial n-manifold with boundary, $n > 2$, then there exists a standard spine of M. Theorem B. If K is a standard spine of a compact connected combinatorial n-manifold with boundary and f is a standard imbedding of K into the interior of a combinatorial n-manifold, then a regular neighborhood of f(K) is combinatorially equivalent to M. (Received October 13, 1967.)

653-199. L. S. CHARLAP and C. H. SAH, University of Pennsylvania, Philadelphia, Pennsylvania 19104. Compact flat Riemannian four dimensional manifolds. Preliminary report.

A complete classification (up to affine equivalence) is obtained of the compact flat riemannian 4-manifolds. There are 75. The methods used are those of Compact flat Riemannian manifolds, by L. S. Charlap, Ann. of Math. 81 (1965). Similar results (unpublished) have been obtained by E. Calabi by different methods. (Received October 13, 1967.)

653-200. B. A. JENSEN, Portland State College, Portland, Oregon 97023. Infinite inverse semigroups that are homomorphically finite.

An HF semigroup is an infinite semigroup such that each of its proper homomorphs is finite. Theorem. The Brandt semigroup $B(G;\Lambda)$ is HF iff either (i) G is an HF group and Λ is finite, or (ii) $|G| = 1$ and Λ is infinite. Theorem. If S is an HF inverse semigroup containing a primitive idempotent e, then $K = SeS$ is an HF Brandt ideal of S and either $S = K$ or S is formed by a finite chain of extensions of K by finite Brandt semigroups. Necessary and sufficient conditions are found for a chain of extensions of the latter type to yield an HF semigroup by applying the Lemma. Every congruence relation on an ideal of an inverse semigroup S can be extended to form a congruence on all of S.
(Received October 13, 1967.)

653-201. T. M. PRICE, University of Iowa, Iowa City, Iowa 52240. Decompositions of S^3 and pseudo-isotopies.

The main theorem in this paper is the following. Let G be a cellular upper semicontinuous decomposition of S^3 such that S^3/G is a 3-manifold. Then there is a pseudo-isotopy of S^3 onto itself that shrinks the elements of G to points. The main tool used is Armentrout's theorem that, under the above hypothesis, there exists a homeomorphism $h: S^3/G \to S^3$ such that $P \circ h: S^3/G \to S^3/G$ is close to the identity. A sequence of homeomorphisms $h_i: S^3/G \to S^3$ is chosen so that $\rho(P \circ h_i, \mathrm{id})$ gets small as i gets large. These h_i's are then used to squeeze the elements of G to points.
(Received October 13, 1967.)

653-202. E. L. SPITZNAGEL, Litton Scientific Support Laboratory, Fort Ord, California 93941. Terminality of maximal unipotent subgroups of Chevalley and Steinberg groups.

A nilpotent group G of class c is said to be "terminal" if whenever there exists a nilpotent $\widetilde{G}$ such that $\widetilde{G}/\Gamma_{c+1}(\widetilde{G}) \cong G$, then $\Gamma_{c+1}(\widetilde{G}) = 1$. Leonard Evens [Terminal p-groups, Illinois J. Math., (to appear)] has shown that for odd p different from the field characteristic, the nonabelian p-Sylow subgroups of finite classical groups are terminal. We prove the following complement. Theorem. Let U be a maximal unipotent subgroup of a Chevalley or Steinberg group of any of the following types: $A_n(n > 3)$, $B_n(n > 3)$, $C_n(n > 3)$, $D_n(n > 3)$, F_4, E_6, E_7, E_8, $A_n^1(n > 6)$, $D_n^1(n > 5)$, E_6^1. In the case B_n, C_n, D_n, F_4 and A_{2k}^1, let the field characteristic be odd. Then U is terminal. (Received October 13, 1966.)

653-203. W. J. DAVIS, D. W. DEAN, Ohio State University, 231 W. 18th Avenue, Columbus, Ohio 43210 and IVER SINGER, Roumanian Academy of Sciences, Str. M. Eminescu 47, Bucharest, Roumania. Complemented subspaces and lambda systems in Banach spaces.

For a B-space E, and a subspace X of E, define $K(X) = \inf \{\|P\| : P^2 = P, PE = X\}$. For a sequence (x_n) in E, let $G_n = K([x_1,\ldots,x_n])$ and $G_n' = K([x_1,\ldots,x_n] \mid [x_j])$, that is, G_n relative to the subspace $[x_j]$. Then (x_n) is called a (1) Λ-system if $[x_j] = E$ and $\lim G_n = \infty$, (2) Γ-system if $[x_j] = E$ and $\sup G_n = \infty$, (3) sub Λ-system if $\lim G_n = \infty$, (4) sub Λ-system if $\sup G_n = \infty$, (5) Λ-sequence if $\lim G_n' = \infty$, and a (6) Λ-sequence if $\sup G_n' = \infty$. Theorem 1. If E is a B-space containing subspaces X_n with $\dim X_n = n$ and $\sup K(X_n) = \infty$, then E has a noncomplemented subspace, F, which has a Schauder decomposition into finite dimensional subspaces. A converse to this theorem would be desirable, since it would make equivalent the existence of noncomplemented subspaces and the existence of n-dimensional subspaces with $K(X_n)$ unbounded. The authors have only partial converses. Theorem 2. If E is a separable B-space with a Λ-system, then E has a Λ-system. Theorem 3. The existence, in a separable B-space, of any of the systems ((1) - (6)) is equivalent to the existence in that space of all the systems. Further, E has a Λ-system if and only if there exist finite dimensional subspaces X_n of E with $K(X_n)$ unbounded. The original Λ-systems are the Lozynski-Kharshiladze systems in $C([0,1])$ (see, e.g., Natanson, Const. Fcn. Th.). In Uspekhi 18 (1963) no. 5 (113), Kadec showed that $L_p([0,1])(1 \leq p \neq 2)$ have Λ-systems. To form a larger list of spaces with Λ-systems, our Corollaries yield the following: The spaces l_p and $L_p([0,1])$ $(1 \leq p \neq 2)$, $C([0,1])$, c_0, and all separable universal B-spaces have Λ-systems. (Received November 10, 1967.)

653-204. S. B. NADLER, Louisiana State University, Baton Rouge, Louisiana. On sequences of contractions and their fixed points.

All spaces under consideration are complete metric spaces. A contraction mapping is a function $A: X \to X$ for which $\exists \alpha \in [0,1) \ni d(A(x), A(y)) \leq \alpha d(x,y) \forall x, y \in X$. Let $A_i : X \to X$ be a contraction mapping with fixed point a_i for each $i = 1,2,\ldots$ and let A_0 be a contraction mapping with fixed point a_0. Theorem 1. If the sequence $\{A_i\}_{i=1}^{\infty}$ converges uniformly to A_0, then the sequence $\{a_i\}_{i=1}^{\infty}$ converges to a_0. Theorem 2. If X is locally compact and the sequence $\{A_i\}_{i=1}^{\infty}$ converges pointwise to A_0, then the sequence $\{a_i\}_{i=1}^{\infty}$ converges to a_0. A construction is given, in any infinite dimensional separable Banach space, of a sequence of contraction mappings which converges pointwise to the zero mapping but whose fixed points do not converge. This construction, together with

Theorem 2, gives necessary and sufficient conditions that a separable Banach space be finite dimensional. Theorems 1 and 2 are used to obtain fixed point theorems for special mappings of product spaces. (Received October 13, 1967.)

653-205. J. H. SMITH, 100 Memorial Drive, Apartment 213-C, Cambridge, Massachusetts 02142. A remark on class groups of extensions with certain types of Galois groups.

If L/K is a normal extension of number fields, G(L/K) is either symmetric, dihedral or non-abelian simple, and $p \nmid [L:K]$ then some relations are deduced between the p-primary part of the class group of L and that of certain subfields. A typical result is: If G(L/K) is non-abelian simple then either p divides the class number of every subextension of codegree 2 or $(Cl_L)_p \cong (Cl_K)_p$. (Received October 13, 1967.)

653-206. J. W. RICHARDS, 548 East Summit, Apartment 104, Kent, Ohio 44240. An incomplete generalization of Frobenius's theorem. Preliminary report.

Let G be a finite group and H a subgroup with the property that $g \in G - H$ implies that $H \cap H^g = 1$. One can easily show that H must be a self-normalizing Hall subgroup if $H \neq 1$. Frobenius showed that H must have a normal complement in G. The author poses the following question: If instead of insisting that $H \cap H^g = 1$ one were to insist only that $H \cap H^g$ be a cylic p-group, then does H have a normal complement in G where we assume that H is a self normalizing Hall subgroup of G? The author is able to answer in the affirmative in the case that G is solvable or H is a p-group. (Received October 13, 1967.)

653-207. N. F. LINDQUIST, Western Washington State College, Bellingham, Washington. Approximation of convex bodies by finite sums of line segments. Preliminary report.

A convex body in E^n whose support function has a representation of the form $\int_\Omega |(u,v)|\Phi(d\omega(v))$, where Φ is a finite, nonnegative measure defined over the Borel sets on the unit spherical surface Ω, will be termed a sum of line segments. Theorem 1. If K is a sum of line segments, then K can be approximated by polytopes which are finite sums of line segments. Theorem 2. If K is the limit of a sequence of convex bodies which are sums of segments, then K is a sum of segments. Firey [Oregon State University, Department of Mathematics Technical report no. 19] has shown that if K is d-dimensional ($d \leqq n$), and a sum of segments, then K is a d-dimensional projection body. This gives a characterization of the class of convex bodies which can be approximated by polytopes which are finite sums of line segments. (Received October 13, 1967.)

653-208. D. F. SANDERSON, Western Washington State College, Bellingham, Washington. On the cohomology of completely primary rings.

As is well known, if A is a ring with ideal N, we may construct a graded ring F(A) = $\oplus_{i=0} N^i/Ni + 1$, where $N^0 = A$. When N is the radical of A and A/N is a division ring, we discuss ways of recreating A from F(A) by using resulting simplifications of Mac Lane's cohomological methods [Extensions and obstructions for rings, Ill. J. Math. 2 (1958), 316-345]. (Received October 13, 1967.)

653-209. EDWIN HEWITT, University of Washington, Seattle, Washington 98105, and K. A. ROSS, University of Oregon, Eugene, Oregon. A maximal problem in harmonic analysis. III.

Let G be a compact group. Let $\{U^{(\sigma)}: \sigma \in \Sigma\}$ be a complete system of inequivalent irreducible unitary representations of G with $U^{(\sigma)}$ acting on the finite-dimensional Hilbert space H_σ. For $f \in L_1(G)$, the Fourier transform $\hat{f}$ is an element of $E = \prod_{\sigma \in \Sigma} B(H_\sigma)$. The space E admits [possibly infinite] p-norms, $1 \leqq p \leqq \infty$; see R. A. Kunze [Trans. Amer. Math. Soc. 89 (1958), 519-540]. Using different but related p-norms, I. I. Hirschman, Jr. [Pacific J. Math. 9 (1959), 525-540] found all functions $f \in L_p(G)$, $1 < p < \infty$ and $p \neq 2$, such that $\|f\|_p = \|\hat{f}\|_{p'}$, where $1/p + 1/p' = 1$; such f are called maximal functions. Theorem. A function in $L_p(G)$, $1 < p < \infty$ and $p \neq 2$, is Kunze maximal iff it is a multiple of a translate of a subcharacter. A subcharacter is a function that equals a continuous [1-dimensional] character on an open subgroup G_0 and is zero off of G_0. Hirschman's maximal functions are just the Kunze maximal functions in the center of $L_1(G)$. (Received October 16, 1967.)

653-210. D. J. FIELDHOUSE, Queen's University. Kingston, Ontario, Canada. Regular modules.

In view of the fact that a ring A is (von Neumann) regular iff every left (or every rt.) ideal is pure (see Abstract 648-161, these *Notices* 14 (1967), 678 for the definition of purity), a left A-module M will be called regular iff all its submodules are pure. Theorem 1. A is a regular ring iff all its left (or all its rt.) A-modules are regular. Theorem 2. If $0 \to E \to F \to G \to 0$ is an exact sequence of left A-modules, then F is a regular module iff both E and G are regular modules and E is pure in F. Theorem 3. If $M = \sum M_i$ is a sum of submodules, then M is a regular module iff each M_i is regular. Theorem 4. The left module M is a regular projective module iff M is isomorphic to a direct sum of (principal) left ideals which are regular and are direct summands of A. Theorem 5. If A is commutative, the A-module E is regular iff its localization E_m is A_m-regular for all maximal ideals m of A. Theorem 6. If A is commutative and A/I is semiprime for every ideal I of A, then A is regular. A regular socle, analogous to the semisimple (= usual) socle, is defined, and its basic properties established. Additional characterizations of regular rings are given. (Received October 16, 1967.)

653-211. J. W. KITCHEN, Duke University, Durham, North Carolina 27705. Almost periodic measures on a compact Abelian group.

The almost periodic functions on a locally compact group are those having "small orbits" either under the action of the group (via translation) or its algebras (via convolution). The present paper is concerned with the orbits of measures on a compact abelian group. It is shown that the only such measures which deserve to be called almost periodic are those which are absolutely continuous with respect to Haar measure. (Received October 16, 1967.)

653-212. S.-C. LIN, University of Miami, Coral Gables, Florida 33124. Wave operators and similarity for some operators in Banach spaces.

Let X and Y be complex reflexive separable Banach spaces. Let $C_0(X,Y)$ be the class of all closed densely defined operators from X to Y. $C_0(X)$ stands for $C_0(X,X)$. Let T, $\widetilde{T}$ be in $C_0(X)$ with resolvent sets $\rho(T) \supset \Delta_+$, $\rho(\widetilde{T}) \supset \Delta_-$ ($\Delta_\pm = \{z \mid \mathrm{Im}\ z \gtrless 0\}$) and that $T \subset \widetilde{T}$. Let V be an operator which can be factored formally as the product BA of two operators $A \in C_0(X,Y)$ and $B \in C_0(Y,X)$. Conditions are given under which the perturbed operators $T(\kappa)$ and $\widetilde{T}(\kappa)$, formally given by $T + \kappa BA$ and $\widetilde{T} + \kappa BA$ respectively, are similar to T and $\widetilde{T}$ respectively. These similarities are established by explicitly constructing a nonsingular operator $W(\kappa)$ which actually implements the similarities. In his recent paper (Math. Ann. 162 (1966), 258-279) Tosio Kato considered the perturbation problem in which the X, Y above are separable Hilbert spaces and the resolvent set of the unperturbed operator T contains both the upper and the lower half planes $\Delta_{\pm}$. Our results are generalizations of his from Hilbert spaces to Banach spaces. Moreover, we consider the perturbation problem in which the resolvent set of the unperturbed operator T may contain only a half plane Δ_{+} or Δ_{-}. Applications are made to some interesting differential and integral equations. In particular, the theory of gentle perturbation of second kind, originated by K. O. Friedrichs (see Perturbation of spectra in Hilbert Spaces, Lectures in Applied Math. vol. 3, Amer. Math. Soc., Providence, R. I., 1965) and the results of Kato (in the paper cited above) follow. (Received October 13, 1967.)

653-213. T. T. READ, Western Washington State College, Bellingham, Washington 98225. Analytic structure in the spectrum of a Banach algebra. Preliminary report.

Let B be a commutative Banach algebra with identity, and let γ be an element of the spectrum, $\mathscr{S}(B)$, of B. Let $B\gamma$ denote the kernel of γ and $B\gamma^n$ the ideal generated by n-fold products of elements of $B\gamma$. If the linear space $B\gamma/Cl(B\gamma^2)$ is r dimensional, then we may associate with each $b \in B$ a multi-indexed series $\{\beta_{(i)}\}$, $(i) = (i_1,\dots,i_r)$, in a natural way. Theorem. If also $\lim\sup |\beta_{(i)}|^{1/|i|}$ is finite for each $b \in B$, then there is an analytic subvariety V containing the origin of a neighborhood of the origin in C^r such that $H(b) = \sum \beta_{(i)} z^{(i)}$ is a homomorphism of B into the ring O(V) of functions holomorphic on V. Thus H induces a one-to-one continuous map H^* of V into $\mathscr{S}(B)$ such that $H^*(0) = \gamma$ and $\hat{b} \circ H^* \in O(V)$ for each $b \in B$. The dimension of V at the origin is related to the structure of $B\gamma$. Moreover V is the largest subvariety which can be mapped into $\mathscr{S}(B)$ at γ in the sense that if $h^* : U \to \mathscr{S}(B)$ is an analytic disc at γ, then there is a neighborhood $W \subset U$ of the origin in C such that $h^*[W] \subset H^*[V]$. (Received October 16, 1967.)

653-214. G. L. ITZKOWITZ, 4246 Ridge Lea Road, Amherst, New York. The existence of homomorphisms in compact connected Abelian groups.

Let B denote the closure of the subgroup of elements of finite order in G where G is a compact connected Abelian topological group. Theorem 1. B is a topological direct summand of G, so that $G = B \times G/B$. This and a theorem of Braconnier implies Theorem 2. In order that the compact connected Abelian group G admit a nonzero continuous homomorphism into each nonzero compact connected Abelian group, it is necessary and sufficient that the torsion subgroup of G not be dense in G. (Received October 17, 1967.)

653-215. H. I. FREEDMAN, University of Alberta, Edmonton, Alberta, Canada. Estimates on the existence region for solutions of equations involving a small parameter.

Let $F(\xi,\epsilon)$ be a real n-dimensional vector function of the real n-dimensional vector ξ and real scalar ϵ. Let $F(0,0) = 0$, and let F have as many continuous partial derivatives as we need. The implicit function theorem is then modified to yield an estimate on the existence region for solutions of $F(\xi,\epsilon) = 0$ for ξ as a function of ϵ in the noncritical and several critical cases. These results are then applied to estimating the existence region of periodic solutions of ordinary differential equations involving a small parameter. (Received October 17, 1967.)

653-216. A. W. GOODMAN, O. I. RAHMAN and J. S. RATTI, University of South Florida, Tampa, Florida 33620. On the zeros of a polynomial and its derivative. Preliminary report.

Let $P(z)$ be a polynomial whose zeros $z_1,\dots,z_n$ lie in $|z| \leqq 1$. Ilieff conjectured (see W. K. Hayman, Research problems in function theory, p. 25) that $P'(z)$ always has a zero in $|z - z_1| \leqq 1$. In fact it seems that the region in which a root of the derivative must lie is smaller than the region conjectured by Ilieff. We conjecture that $P'(z)$ always has a zero in $|z - |z_1|/2| \leqq 1 - |z_1|/2$. We prove this stronger form of Ilieff's conjecture for some special cases. (Received October 17, 1967.)

653-217. R. E. O'MALLEY, JR., Mathematics Research Center-United States Army, University of Wisconsin, Madison, Wisconsin 53706. Boundary value problems for linear systems of ordinary differential equations involving many small parameters.

This paper considers the system of differential equations (1) $\Omega(\epsilon)y' = A(x,\epsilon)y$ where $\Omega(\epsilon) = \operatorname{diag}(\epsilon_1 I_{m_1} : \epsilon_2 I_{m_2} : \dots : \epsilon_{s-1} I_{m_{s-1}} : I_{m_s})$ and the small parameters ϵ_i are interrelated such that $\epsilon_j/\epsilon_{j+1} \to 0$ as $\epsilon_{s-1} \to 0$, $j = 1,\dots,s-2$. Under appropriate hypotheses on the matrix $A(x,\epsilon)$, a fundamental system of asymptotic solutions for (1) is obtained. Moreover, if (1) is considered on the interval $0 \leqq x \leqq 1$ subject to boundary conditions of the form (2) $R(\epsilon)y(0) + S(\epsilon)y(1) = c(\epsilon)$, a complete asymptotic solution for y, including "boundary layer terms", is explicitly obtained provided $R(0)$ and $S(0)$ are appropriately restricted. The solution of (1), (2), then, converges nonuniformly within $(0,1)$ as $\epsilon_{s-1} \to 0$ to the solution of the appropriate reduced boundary value problem. Lastly, results are related to previously known results concerning singular perturbations of boundary value problems for higher order scalar equations. (Received October 17, 1967.)

653-218. O. H. KEGEL, New Mexico State University, Las Cruces, New Mexico 88001. A characterization of Borel and Cartan subalgebras of finite-dimensional Lie algebras.

Let $\underline{\underline{f}}$ be a function defined on the class of all finite-dimensional Lie algebras over an algebraically closed field of characteristic 0 which associates with each such algebra L a class $\underline{\underline{f}}(L)$ of subalgebras which form an orbit under the group of automorphisms of L. Assume furthermore that $\underline{\underline{f}}$ has the following formal properties: (a) $\underline{\underline{f}}(L^\sigma) = (\underline{\underline{f}}(L))^\sigma$ for every epimorphism σ of L; (b) If $S \in \underline{\underline{f}}(L)$, then $S \in \underline{\underline{f}}(T)$ for every subalgebra T of L with $S \subseteq T$. The following three functions have these properties: $\underline{\underline{c}}(L)$ is the set of Cartan subalgebras of L, $\underline{\underline{b}}(L)$ is the set of the Borel subalgebras of L, $\underline{\underline{s}}(L)$ is the set of the Levi subalgebras of L. Call $\underline{\underline{f}}$ solvable, if for each L the set $\underline{\underline{f}}(L)$ consists of solvable subalgebras; $\underline{\underline{f}} \neq \underline{\underline{0}}$ if there is an L with $0 \notin \underline{\underline{f}}(L)$. Theorem. If the solvable function $\underline{\underline{f}} \neq 0$ satisfies the conditions (a) and (b) then either $\underline{\underline{f}} = \underline{\underline{c}}$ or $\underline{\underline{f}} = \underline{\underline{b}}$. (Received October 17, 1967.)

653-219. L. G. NOVOA, University of Alabama, University, Alabama 35486. On semiordered mathematical structures. Preliminary report.

A partial n-order on a set X is a reflexive, symmetric, and almost transitive relation defined on the family of oriented n-simplexes of X and which satisfies a certain "exchange axiom" (Pacific J. Math. 15, no.4, pp. 1338-1345). Almost transitive means that if the domain of the relation is restricted, by deleting the elements which are related to every other element, then the relation becomes transitive. A semiorder of dimension n is similarly defined except that the condition of almost transitivity is omitted. It is shown that the usual notions of order in different branches of mathematics, such as partial and full orders, cyclic, separation and betweenness orders can be derived from the above general notion of semiorder. A mathematical structure (algebraic, geometric, topological), in which a semiorder has been defined, is said to be semiordered if the characteristic auto-mappings, as usually defined, are order-isomorphisms. This notion includes f.i. the po-groups and cyclically ordered groups (Rieger), ordered fields, etc. Examples are presented of semiordered structures which are not orderable. In particular, a significant semiorder of dimension two for the projective plane is exhibited. (Received October 17, 1967.)

653-220. K. T. HAHN and JOSEPHINE MITCHELL, Pennsylvania State University, University Park, Pennsylvania 16802. Green's functions on the classical Cartan domains. I.

The classical Cartan domain R_I (ν = I,II) is given by $I - zz^* > 0$, where z is an m by n matrix of complex elements in case I and a symmetric matrix of order n in case II and z^* is the conjugate transpose of z. These domains possess the Bergman metric and the corresponding Laplace-Beltrami operator Δ_ν, both of which are invariant under biholomorphic mappings of R_ν. Fundamental solutions ψ of $\Delta_\nu \psi = 0$ are studied which are functions of various parameters. For the hypersphere H_1 such a solution ψ is obtained which is a function of the (invariant) Bergman distance and a Green's function is constructed from ψ. By means of this Green's function a boundary value problem is solved for the hypersphere H_R with $R < 1$. (Received October 17, 1967.)

653-221. A. D. PORTER, University of Wyoming, Laramie Wyoming. Orthogonal similarity for symmetric matrices.

Let $F = GF(q)$ be the finite field of $q = p^r$ elements, p odd. A number of the well known theorems concerning orthogonal similarity of real symmetric matrices do not hold in F. For example, the usual proof that every real symmetric matrix is orthogonally similar to a diagonal matrix is not valid in F. We discuss a number of theorems which give necessary and sufficient conditions for certain symmetric matrices to be orthogonally similar over F to a diagonal matrix. The theorems are obtained by placing various restrictions on the symmetric matrices under consideration. (Received October 17, 1967.)

653-222. W. K. MASON, University of Wisconsin, Madison, Wisconsin. Homeomorphic continuous curves in 2-space are isotopic in 3-space.

Theorem. Let E^2 be the standardly embedded plane in Euclidean three-space E^3. Let S and T be continuous curves in E^2. Let g be a homeomorphism of S onto T. Then there is a homeomorphism H of E^3 onto E^3 such that (a) H = g on S, and (b) H may be realized by an isotopy. This answers a question raised by Bing [see J. M. Kister, "Questions on isotopies in manifolds," in Topology of 3-manifolds and related topics, Prentice-Hall, 1962, p. 230]. (Received October 17, 1967).

653-223. E. H. ANDERSON, University of North Dakota, Grand Forks, North Dakota 58201. Approximation of certain continuous functions of S^2 into E^3.

Denote the usual metric for S^2 and E^3 by ρ. If f and g are continuous functions of S^2 into E^3, $\rho(f,g)$ is the least upper bound of the set of all numbers $\rho(f(x), g(x))$ for $x \in S^2$; d(f) is the greatest lower bound of all numbers $\rho(f,h)$ where h is a homeomorphism; K(f) is the closure of the set of all $x \in S^2$ with the property that there is $y \in S^2$, $y \neq x$, such that f(y) = f(x). Theorem. If f is a continuous function of S^2 into E^3 and (i) f(K(f)) is 0-dimensional, (ii) no component of K(f) separates S^2, then d(f) = 0. (Received October 17, 1967.)

653-224. D. R. ANDERSON, University of Wyoming, Laramie, Wyoming 82070, and W. M. BOGDANOWICZ, Catholic University of America, Washington, D. C. On existence of solutions to a functional integro-differential equation in Banach spaces.

Let (R,μ) be a finite measure space, T a compact interval and Y a Banach space. Let f be a mapping of $R \times T \times Y$ into Y such that for all fixed t,y the function $f(\cdot,t,y)$ is μ-Bochner measurable, for all fixed r,y the function $f(r,\cdot,y)$ belongs to the space C(T,Y) of all continuous functions on T to Y, and $|f(r,t,y) - f(r,t,z)| \leqq c|y - z|$ for all $r \in R$, $t \in T$, $y, z \in Y$ and some c, $|f(r,t,0)| \leqq g(r)$ for all $r \in R$ and $t \in T$, g being μ-summable. Let h be a mapping of $R \times T$ in T such that $|h(r,t) - t_0| \leqq |t - t_0|$ for $t \in T$ and $r \in R$, where $t_0 \in T$, and the map $r \longrightarrow h(r,\cdot)$ is μ-Bochner measurable from R into $C(T,Y_0)$, Y_0 the space of reals. Theorem. There exists a unique differentiable function $y \in C(T,Y)$ satisfying the functional integro-differential equation $y'(t) = \int_R f(r,t,y(h(r,t)))\mu(dr)$ for all $t \in T$ and the condition $y(t_0) = y_0$. If the space R consists of one point only and $\mu(R) = 1$ we get the following: Corollary. If f is a continuous mapping of $T \times Y$ into Y satisfying the Lipschitz condition in the second variable and h maps T into itself satisfying $|h(t) - t_0| \leqq |t - t_0|$ for $t \in T$, then there exists a unique differentiable function $y \in C(T,Y)$ such that $y'(t) = f(t,y(h(t)))$ for all $t \in T$ and $y(t_0) = y_0$, where y_0 is any point from the space Y. This result is related to the result: D. R. Anderson, SIAM Review 8 (1966), 359-362. (Received October 11, 1967.)

653-225. BRINDELL HORELICK, State University of New York, College at Cortland, Cortland, New York 13045. Group-like extensions and similar algebras.

For background and notation see R. Ellis, Trans. Amer. Math. Soc. 101 (1961), 384-395 and 127 (1967), 125-135. For each T-subalgebra $\mathscr{A} \subset \mathscr{A}(u)$ a T-subalgebra $\mathscr{A}^*$ is constructed with the property that $\mathscr{B} \subset \mathscr{A}^*$ if and only if $\mathscr{B} > \mathscr{A}$. New properties of the relation > are proved. Since $\mathscr{A}$ is distal if and only if $\mathscr{A} > R$, $\{\mathscr{A} | \mathscr{A} > \mathscr{F}\}$ may be regarded as a generalization of the set of distal algebras ("$\mathscr{F}$-distal" algebras). An analogous property "$\mathscr{F}$-proximal" is defined and studied. When $\mathscr{F}$ = R this reduces to "proximal". It turns out: $\mathscr{F}$-proximal is a closed equivalence relation on $\mathscr{A} \supset \mathscr{F}$ if and only if $G(\mathscr{F}^*) \subset G(\mathscr{A}) \subset G(\mathscr{F})$. Sets of similar algebras $[G(\mathscr{A}) = G(\mathscr{B})]$ with this property are studied, and a structure theorem relating these sets to each other by means of $>$ is proved. A generalization of Proposition 19 in the second reference above is proved which deletes the requirement that $\mathscr{D} = \mathscr{A}(G(\mathscr{D}))$. (Received October 17, 1967.)

653-226. W. R. EMERSON, New York University, Washington Square College, 251 Mercer Street, New York, New York 10012. Groups defined by permutations of a single word. Preliminary report.

Let $W = W(g_1,\dots,g_N)$ be a (reduced) word in the primitive symbols g_i, $1 \leqq i \leqq N$, not all necessarily occuring nontrivially. If P is any permutation in the symmetric group $\mathscr{S}_N$, P(W) denotes the word obtained from W by replacing every g_i by $g_{P(i)}$, $1 \leqq i \leqq N$. If $\mathscr{P}$ is any subset of $\mathscr{S}_N$ and W as above, we define $W^{\mathscr{P}}$ to be the group generated by the g_i, $1 \leqq i \leqq N$, subject to the relations $P(W) = 1$, all $P \in \mathscr{P}$. In the present report we discuss primarily the case when $\mathscr{P} = \mathscr{S}_N$, the full symmetric group. Detailed results are obtained for special cases, e.g. when N = 2 and the structure of $W^{\mathscr{S}_N}$ modulo its commutator subgroup. (Received October 17, 1967.)

653-227. WITHDRAWN.

653-228. L. W. BEINEKE and R. E. PIPPERT, Purdue University, Fort Wayne, Indiana 46805. Minimal regular extensions of oriented graphs.

Let G be an oriented graph and let r denote the maximum of the in- and out-degrees in G. A regular extension of G is an r-regular oriented graph of which G is an induced subgraph. The in-deficiency (resp., out-deficiency) of a vertex of G is the difference between r and its in-degree (resp., out-degree). Let s denote the sum of the in-deficiencies in G, and t the maximum of the combined deficiencies. Theorem. The minimum order of a regular extension of G is m + n, where n is the order of G and m is the least integer satisfying these conditions: (1) $m \geqq t$, (2) $mr \geqq s$, (3) $(m(m-1))/2 \geqq mr - s$. A similar result holds for directed graphs, while the result for ordinary graphs was first proved by Erdös and Kelly [Amer. Math. Monthly 70 (1963), 1074-1075]. (Received October 20, 1967.)

653-229. H. E. FETTIS, 432 Cushing Avenue, Kettering, Ohio. Some new integral relations involving Bessel functions.

By manipulation of well known differentiation formulae for Bessel functions, integrals of the "Schwartz" type $\int^x \exp(-at)t^p Z_p(zt)dt$, [where $Z_p(t)$ is any of the various Bessel functions] can be related to integrals of the form $\int^x Zq(xt)(a^2 + t^2)^{p-1/2}t^{q+1}dt$ with $q = p$ and $p - 1$. (Received October 19, 1967.)

653-230. H. S. AL-AMIRI, Bowling Green Staete University, Bowling Green, Ohio. The a-points of Faber polynomials for a special function.

Let $f(z) = z + a_0 + a_1/z + \ldots$, with $\overline{\lim}|a_n|^{1/n} < \infty$. The Faber polynomials $\{f_n(z)\}$, $n = 0,1,\ldots$, are the polynomial parts of the formal expansion of $(f(z))^n$ about $z = \infty$. Let $\triangle$ and $\triangle_a$, $a \neq 0$, be the derived sets of the zeros of $f_n(z)$ and $f_n(z)-a$, respectively. The object of this paper is the location of $\triangle$ and $\triangle_a$ for a special function $f(z) = z \exp 1/\lambda z = z + 1/\lambda + 1/2\lambda^2 z + \ldots$, where λ is an arbitrary positive number. Theorem. (a) $\triangle$ is the set $\Gamma: \{z \mid |z \exp 1/\lambda z| = e/\lambda$ and $|z| \geqq 1/\lambda\}$. (b) For $\lambda < e$, $\triangle_a$ is the set Γ in (a), while for $\lambda \geqq e$, $\triangle_a$ is the set $\Gamma_1 : \{z \mid |z \exp 1/\lambda z| = 1$ and $|z| \geqq 1/\lambda\}$. An asymptotic distribution for the derived sets along Γ and Γ_1 are discussed. The author utilizes and employs methods used by G. Szegö [Ube Reine Eigenschaft der Exponentialreihe, Sitzungsber, Ber. Math. Ges 23 (1924), 50-64]. (Received October 19, 1967.)

653-231. W. R. DERRICK, The University of Utah, Salt Lake City, Utah 84112. A weighted volume-diameter inequality for n-cubes.

Let A be a topological image of an n-cube in n-dimensional space and let w be a nonnegative upper semicontinuous real-valued function defined on n-space. The weighted distance λ_i between the images A_i and A_i' of a pair of opposite sides of the n-cube is given by $\inf \int_c w dH_n^1$, where the infimum is taken over all arcs c in A joining a point of A_i to a point of A_i', and H_n^k is the Hausdorff k-dimensional measure in n-dimensional space. Denote by $V(A) = \int_A w^n dH_n^n$ the weighted volume of A. Theorem. $V(A) \geqq \prod_{i=1}^n \lambda_i$. This theorem generalizes results presented by the author (Abstract 646-1, these *Notices* 14 (1967), 396). (Received October 19, 1967.)

653-232. S. L. SEGAL, University of Rochester, Rochester, New York 14627. Tauberian theorems for Dirichlet convolutions.

Ingham (J. Lond. Math. Soc. 20 (1945), 171-180) proved, among other results, the following Tauberian theorem, closely connected with prime number theory. Let F(x) be positive and non-decreasing (from some point on), and suppose (1) $\sum_{1 \leqq d \leqq x} F(x/d) = ax \log x + bx + o(x)$, a and b constants; then (2) $F(x) = ax + o(x)$. Variants and improvements of this result can be sought in several directions. Two types in particular are (i) Suppose $\sum_{d \leq x} F(x/d) = axg(x) + bx + E(x)$, where g(x) is positive-increasing, $g(x) = O(\log x)$, and $E(x) = o(x)$; if F(x) is positive nondecreasing, what can be concluded about F(x)? (ii) Suppose $\sum_{d \leq x} k(d)F(x/d) = ax \sum_{d \leq x}(k(d)/d) + bx + o(x)$ where k(d) is positive and nondecreasing and also subject to other restrictions, then if F(x) is positive and non-decreasing, $F(x) = ax + o(x)$. Theorems of both of these types are proved; for type (i) with $g(x) =$

log log x, $E(x) = ((cx)/\log x) + o(x/\log x)$; the conclusion (when $F(x) = \sum_{n \leq x} a_n$) of course being $F(x) = ((ax)/\log x) + o(x/\log x)$. (Received October 18, 1967.)

653-233. K. R. FULLER, University of Iowa, Iowa City, Iowa 52240. The structure of QF-3 rings.

Let R be a (two-sided) artinian ring with radical N. If e and f are primitive idempotents in R such that socle(Re) $\cong$ Rf/Nf and socle(fR) $\cong$ eR/eN then Re is said to be antistrophic to fR and Re is called a left antistrophic primitive for R. Thus Nakayama's original definition of QF rings can be restated: R is QF in case every primitive left ideal in R is antistrophic to a primitive right ideal. The indecomposable injective projective modules over a QF-3 ring R are (up to isomorphism) its antistrophic primitives. Moreover this notion allows a characterization of QF-3 rings strictly in terms of ideal structure. Theorem. An artinian ring is QF-3 iff each of its minimal left ideals is isomorphic to the socle of an antistrophic primitive. As a consequence Theorem. .Every artinian QF-2 ring is QF-3. (Received October 18, 1967.)

653-234. C. C. CHEN, Queen's University, Kingston, Ontario, Canada, and G. A. GRÄTZER, University of Manitoba, Winnipeg, Manitoba, Canada. Generalizations of lattices with unique complements.

The concepts of lattices with unique complements and almost uniquely complemented lattices (see e.g. Abstract 67T-458, these *Notices* 14 (1967), 694) can be generalized for an arbitrary cardinal number $\mathfrak{m}$: An $\mathfrak{m}$-complemented (resp. $\hat{\mathfrak{m}}$-complemented) lattice is a lattice L with 0 and 1 such that for each x in L, $x \neq 0,1$, the set of all complements of x in L has cardinality $= \mathfrak{m}$ (resp. $\leq \mathfrak{m}$). A homogeneous lattice is a lattice L with 0 and 1 such that the relation: ($x \sim y$ iff $x = y$ or x is a complement of y in L) is an equivalence relation. We can then modify the construction described in the abstract mentioned above to obtain the following results. (1) Every $\hat{\mathfrak{m}}$-complemented lattice with incomparable complements (i.e. if b,c are complements of a, then $b \geq c$ implies $b = c$) can be embedded into an $\mathfrak{m}$-complemented lattice with 0 and 1 preserved; (2) Every homogeneous $\hat{\mathfrak{m}}$-complemented lattice can be embedded into a homogeneous $\mathfrak{m}$-complemented lattice with 0 and 1 preserved; (3) Every lattice is a sublattice of an $\mathfrak{m}$-complemented (resp. a homogeneous $\mathfrak{m}$-complemented) lattice; (4) Existence and uniqueness (up to isomorphism) of free $\mathfrak{m}$-complemented lattices with $\mathfrak{n}$ generators. Note that results in the Abstract mentioned above are special cases of these ($\mathfrak{m} = 1$). (Received October 19, 1967.)

653-235. R. K. MILLER, Brown University, Providence, Rhode Island. Integral equations with nonnegative integrable resolvents.

Consider the resolvent equation $k(t) = a(t) - \int_0^t a(t-s)k(s)ds$. Conditions are given which insure that k(t) is nonnegative and of class $L'(0, \infty)$. This result can be applied in conjunction with invariance results of the author (Bull. Amer. Math. Soc. 72 (1966), 153-156) in order to study the nonlinear equation $x(t) = f(t) - \int_0^t a(t-s)g(x(s))ds$. In particular if f is periodic and g is a nonlinear spring, one can show that there exists a periodic function p(t) such that $x(t) - p(t) \to 0$ as $t \to \infty$. This generalizes some results of Levinson (J. Math. Anal. Appl. 1 (1960), 1-11) and of Friedman (J. Analyse Math. 15 (1965), 287-303). (Received October 19, 1967.)

653-236. L. C. KURTZ, Arizona State University, Tempe, Arizona 85281. On uniform convergence and positive operators.

Suppose A is a compact Hausdorff space and C(X), C(X*), and C(R) are the spaces of continuous functions from A to a linear normed space X, its conjugate X*, and the reals, respectively, with the uniform norm in each case. If $L_n \in B[C(X), C(X)]$, we define elements L_n^+ and T of $B[C(R), B[X, C(X)]]$ by $L_n(xf) = [L_n^+(f)](x)$ and $(Tf)(x) = xf$ ($f \in C(R)$, $x \in X$, $n = 1, 2, 3, \ldots$). Theorem. Suppose L_n and L_n^+ satisfy the conditions: (I) If $f, g \in C(R)$ and $|f(t)| \leqq g(t)$ for all $t \in A$, then $\|[L_n(xf)](y)\|_X \leqq \|[L_n(xg)](y)\|_X$ for all $y \in A$ and $x \in X$. (II) If $x^* \in X^*$ and $g \in C(X)$, then $L_n^+(x^*g) = T(x^*L_ng)$. Then if there exist $f_1, f_2, \ldots, f_m \in C(X)$ and $a_1, a_2, \ldots, a_m \in C(X^*)$ such that $p(t,y) = \sum_{i=1}^m a_i(y)[f_i(t)] \geqq 0$ with equality if and only if $t = y$, and if $L_n(f_i) \rightarrow f_i$ in C(X) ($i = 1, 2, \ldots, m$), then $L_ng \rightarrow g$ for all $g \in C(X)$. (Received October 20, 1967.)

653-237. M. J. KASCIC, Dartmouth College, Hanover, New Hampshire 03755. Polynomials in closed linear relations.

Let T be a linear relation, i.e. a linear subspace of $X \oplus X$ where X is a linear space. If X is a locally convex space, the algebraic and topological definitions of closedness of subspaces are equivalent and we may speak of a linear relation that is a closed subspace of $X \oplus X$, i.e. a closed linear relation. Such relations are generalizations of linear operators and it is possible to generalize the definition of a polynomial in an operator to a polynomial in a linear relation. Sufficient (and in a restricted case necessary and sufficient) conditions are derived which guarantee that a polynomial in a closed linear relation is closed. (Received October 20, 1967.)

653-238. L. N. MANN and J. L. SICKS, University of Massachusetts, Amherst, Massachusetts 01002. Embeddings of compact Lie groups. Preliminary report.

K. Hofmann and P. S. Mostert have raised the question of which compact connected Lie groups imbed in euclidean space with codimension 1. The following result provides such examples: Theorem. $G \times T^{n-1}$ differentiably imbeds with codimension 1 if and only if G imbeds with codimension n and trivial normal bundle (T^j denotes the j torus). Using representation theory it is easily established that: Theorem. SO(n), SU(n), Sp(n) imbed with codimensions n, 2n, 4n, respectively, and trivial normal bundle. Among the groups SU(n), Sp(n) and the exceptional Lie groups, we have shown that S^3 is the only one which imbeds with codimension 1. However, for SO(n) the result is much more interesting: Theorem. SO(n) imbeds with codimension 1 if and only if n = 1, 2, 4, 8. The last result is related to the vector field problem: We use secondary operations to handle $SO(2^\alpha)$ for $\alpha \geqq 4$; we use the parallelizability of S^3 and S^7 to show SO(4) and SO(8) imbed with codimension 1. (Received October 20, 1967.)

653-239. STANLEY OSHER, Brookhaven National Laboratory, 61 Brookhaven Avenue, Upton, Long Island, New York 11973. Stability of mixed implicit difference schemes.

Given a system of hyperbolic partial differential equations $u_t = Au_x$, $0 \leqq x, t < \infty$ with initial and boundary conditions, we approximate it by an implicit difference scheme with boundary conditions. This is the form $\sum_{k=-s}^{l} B_k v_{j+k}^{n+1} = \sum_{k=-s}^{l} C_k v_{j+k}^{n}$, $j = s, s+1, \ldots$ and $\sum_{k=0}^{r+s} a_{j+1,k+1} v_k^{n+1} = \sum_{k=0}^{r+s} \gamma_{j+1,k+1} v_k^{n}$, $j = 0, 1, \ldots, t$. We may replace these by the equivalent operator equations, $(T_0 + S_0)v^{n+1} = (T_1 + S_1)v^n$. T_0 and T_1 are Toeplitz Operators on l_2 and S_0 and S_1 are finite dimensional perturbations. Using operator theory, we obtain necessary and sufficient conditions for $(T_0 + S_0)^{-1}$ to exist. We then obtain sufficient conditions for $(T_0 + S_0)^{-1}(T_1 + S_1)$ to be power bounded. We thus have obtained sufficient conditions for stability of mixed implicit difference schemes using the Wiener-Hopf factorization of Toeplitz Matrices. These conditions are extensions of the Ryabenkii-Godunov criteria which were only necessary. (Received October 20, 1967.)

653-240. J. W. HEIDEL, The University of Tennessee, Knoxville, Tennessee 37916. Existence of oscillatory solutions for a nonlinear odd order differential equation.

The equation considered here is (1) $y^{(n)} + q(t)y^{\gamma} = 0$ where n is an odd integer $\geqq 3$, $q(t) \geqq 0$ and continuous on a half line $[a, \infty)$ and γ is the quotient of odd, positive integers. A solution $y(t)$ of (1) is called oscillatory if it does not have a last zero, i.e., $y(t_1) = 0$ implies there is a $t_2 > t_1$ such that $y(t_2) = 0$. A nontrivial solution of (1) is called singular if it is identically zero on some half line $[t_0, \infty)$. Theorem. Suppose that $\int^{\infty} s^{(n-1)\gamma} q(s)ds = \infty$ if $0 < \gamma < 1$ and that $\int^{\infty} s^{(n-2)+\gamma} q(s)ds = \infty$ if $1 < \gamma$. Then (1) has a nonsingular, oscillatory solution. (Received October 20, 1967.)

653-241. A. M. BRUCKNER, J. G. CEDER and M. L. WEISS, University of California, Santa Barbara, California 93106. On the differentiability structure of real functions.

Let f be a continuous real valued function defined on a nonempty perfect set P of real numbers. There exists a nonempty perfect set $Q \subset P$ such that the restriction of f to Q is monotonic and differentiable in the extended sense. Various related results are obtained. The proof depends on the following lemma which has other applications as well. Lemma. Let P be a nowhere dense perfect set of real numbers. Let $\mathscr{A}$ be the family of intervals complementary to P. Suppose $\mathscr{B}$ is any subfamily of $\mathscr{A}$ the endpoints of whose intervals are somewhere dense in P. Then, there exists a perfect subset Q of P such that whenever x and y are distinct points of Q, the longest interval of $\mathscr{A}$ between x and y belongs to $\mathscr{B}$. (Received October 20, 1967.)

653-242. ANDRE de KORVIN, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213. Faithful normal expectations on von Neumann algebras.

Let h and k be two Hilbert spaces. Let M be a von Neumann algebra acting on h. An ampliation of M in $h \otimes k$ is a map which to each T in M associates $T \otimes I_k$ in $L(h \otimes k)$. The result shown in this paper is that there exists an ampliation of M in $h \otimes k$ such that if N is any von Neumann subalgebra of M which is the range of a faithful, normal expectation Φ, then there exists an isometry V which commutes with $N \otimes I_k$, such that $\Phi \otimes I_k(T \otimes I_k) = V(T \otimes I_k)V^*$, VV^* is the identity. On putting

$V^*V = P$, P is in the commutant of $N \otimes I_k$, $\Phi \otimes I_k(T \otimes I_k)P = P(A \otimes I_k)P$. This result generalizes a result of Nakamura, Takesaki, Umegaki (Kodai Math. Sem. Rep. 12 (1960), 82-89). The proof depends on Stinespring's construction (Proc. Amer. Math. Soc. 6 (1955), 211-216) and on a result by Tomiyama (Tohoku Math. J. 10 (1958), 37-41). (Received October 20, 1967.)

653-243. FRANCES FROST, University of Minnesota, Minneapolis, Minnesota 55455. Derivations and the action of functions on trace class. Preliminary report.

Denote by C_1 the trace class of operators on a Hilbert space $\mathfrak{H}$. The action $T \longrightarrow f(T)$ of a function algebra M (dom $f \subseteq \mathfrak{C}$ for $f \in M$) on a closed subalgebra $\mathfrak{A}$ of C_1 is defined. When $f(T)$ is defined for T and $\lambda \in \sigma(T)$, $\lambda \neq 0$, then $f(T)E_\lambda(T) = \sum_{k=0}^{n-1}(1/k!)a_k(T - \lambda I)^k E_\lambda(T)$, where n is the index of λ and $E_\lambda(T)$ is the projection of $N_{(\lambda I - T)^n}$. An mth order system of derivations $D_0, D_1, \ldots, D_m$ from an algebra $\mathfrak{A}$ to an algebra $\mathfrak{B}$ is defined. Theorem. Let M be a subalgebra of $C(X)$, where $X = \bigcup_{n=1}^{\infty} X_n$, $X_n \subseteq \mathfrak{C}$, is compact and $X_n \subseteq X_{n+1}$. If there is an Nth order system of derivations from M to $C(X)$ with $D_0 f = f$, $f \in M$, and if dim $\mathfrak{H} = m \leqq N + 1$, then M acts on every algebra in $L(\mathfrak{H})$. Conversely, if M acts on $\mathfrak{A} \subseteq C_1$ and the coefficients a_k for $f(T)E_\lambda(T)$ above depend only on λ and the function f, the action determines a system of derivations whose order depends on $\mathfrak{A}$. A family of seminorms determined by the system of derivations and the sets X_n is defined for M. Let $\overline{M}$ be the completion of M in this topology. Given $x(t) = t$ ($t \in X$), $x \in M$ and $D_1 x(t) \neq 0$ for all $t \in X$. Then $C^N(X) \subseteq \overline{M}$ if the order of the system is N and $X \subseteq \mathfrak{R}$. If X is a region of the complex plane, then $\overline{M}$ contains the functions analytic on X. (Received October 20, 1967.)

653-244. J. D. NELLIGAN, General Electric Company, Building 3, Room 220, Electronics Park, Syracuse, New York 13201. A generalized Fourier-Stieltjes series representation of a singular bivariate probability measure. Preliminary report.

Let μ be a Lebesgue-Stieltjes measure on a subset D of R^2. Let μ_1 and μ_2 be the marginal measures of μ and $\{\phi_k\}$ and $\{\theta_k\}$, $k = 0, 1, 2, \ldots$ be complete orthonormal systems in $L_2(\mu_1)$ and $L_2(\mu_2)$ respectively. If $\phi_k \theta_n$ is μ-integrable for all pairs of nonnegative integers (k, n) then a generalized Fourier-Stieltjes series $\sum\sum a_{kn}\phi_k \theta_n$ can be associated with μ where $a_{kn} = \int \phi_k \theta_n \, d\mu$ and it is said to represent μ if $\mu([a,b]\times[c,d]) = \sum\sum a_{kn} \int_a^b \phi_k d\mu_1 \int_c^d \phi_n d\mu_2$ for all (finite) rectangles $[a,b]\times[c,d]$. In this note it is shown that the measure generated on $[-1,1]\times[-1,1]$ by uniformly distributing a unit mass on the $x = y$ diagonal of this square has such a representation with $\theta_k = \phi_k$ = the kth normalized Legendre polynomial. (Received October 20, 1967.)

653-245. K. L. SINGH, Memorial University of Newfoundland, St. John's, Newfoundland, Canada. On fixed point theorem.

The well-known conjecture "if f and g are continuous functions which map a closed interval of the real line into itself and which commute, then they have a common fixed point" has been disproved by Boyce and Huneke independently. In the present paper the following three theorems have been given. Theorem 1. Let I be the closed interval on the real line. Let f and g be continuous mappings from I into itself where f is surjective. Then there exists a point p in I for which $f(p) = g(p)$. Theorem 2. If f and g are continuous functions from a closed unit interval into itself, if they commute then they have a common fixed point provided $|f'(x)| < 1$. Theorem 3. Let f be a continuous function that maps a closed unit interval to itself, and let g be a contraction map from a closed unit interval to itself. Then if they commute, they have a common fixed point. (Received October 19, 1967.)

653-246. P. J. ALLEN, University of Alabama, University, Alabama. Quotient structure of a semiring.

A semiring is a nonempty set R on which there are defined two associative binary operations, called addition and multiplication, such that multiplication distributes over addition both from the left and from the right; addition is commutative; and R contains a zero. A nonempty subset I of a semiring R is called an ideal if $a + b \in I$, $ar \in I$ and $ra \in I$ for each $a, b \in I$ and for each $r \in R$. The notion of a Q-ideal was defined, and a construction process was presented by which one can build the quotient structure of a semiring modulo a Q-ideal. Maximal homomorphisms were defined and examples of such homomorphisms were given. Using these notions, the Fundamental Theorem of Homomorphisms for rings was generalized to include a large class of semirings. It was also shown that a proper Q-ideal I in the semiring R is primary if and only if every zero divisor in R/I is nilpotent. (Received October 19, 1967.)

653-247. R. D. HILL, Idaho State University, Pocatello, Idaho. Inertia theory for simultaneously triangulable complex matrices.

Let $A_1, A_2, \dots, A_s$ be simultaneously triangulable matrices of order n whose eigenvalues under a natural correspondence are $\lambda_k^{(1)}, \lambda_k^{(2)}, \dots, \lambda_k^{(s)}$ $(k = 1,2,\dots,n)$ and let $\mathscr{D} = (d_{ij})$ be Hermitian of order s. Then there exists a Hermitian $\mathscr{A}$ (of order n) such that $\sum_{i,j=1}^{s} d_{ij}\mathscr{A}_i\mathscr{A}\mathscr{A}_j^* > 0$ if and only if $\sum_{i,j=1}^{s} d_{ij}\lambda_k^{(i)}\bar{\lambda}_k^{(j)} \neq 0$ $(k = 1, 2, \dots, n)$. Furthermore, if $\pi(\mathscr{D}) \leqq 1$ and $\nu(\mathscr{D}) \leqq 1$, then $\mathrm{In}\ \mathscr{A} = \mathrm{In}\ \{A_1, A_2, \dots, A_s, \mathscr{D}\}$ where $\mathrm{In}\ \{A_1, A_2, \dots, A_s, \mathscr{D}\}$ is defined to be the ordered triple (π, ν, δ) where π, ν, and δ are the numbers of positive, negative and zero values respectively of $\sum_{i,j=1}^{s} d_{ij}\lambda_k^{(i)}\bar{\lambda}_k^{(j)}$. If $\pi(\mathscr{D}) > 1$ or $\nu(\mathscr{D}) > 1$, then there exist simultaneously triangulable matrices $\mathscr{A}_1, \mathscr{A}_2, \dots, \mathscr{A}_s$ of order n and a Hermitian matrix $\mathscr{A}_0$ of order n such that $\sum_{i,j=1}^{s} d_{ij}\mathscr{A}_i\mathscr{A}_0\mathscr{A}_j^* > 0$ and $\mathrm{In}\ \mathscr{A}_0 \neq \mathrm{In}\ \{\mathscr{A}_1, \mathscr{A}_2, \dots, \mathscr{A}_s, \mathscr{D}\}$. (Received October 19, 1967.)

653-248. JAMES STEPP, 200 Barberry Lane, Lexington, Kentucky. Topological groups in the boundary of a locally compact connected abelian group.

Let R^n denote the real n-dimensional vector group. Theorem 1. Let S be a locally compact semigroup with a dense group $G \cong R^n \times C$, where C is a compact connected abelian group. If H(e) is a maximal group which is topological, $H(e) \subseteq S\setminus G$, then $H(e) \cong R^p \times C_2$ where $p < n$ and C_2 is a compact connected abelian group. Further, $p = n - 1$ if and only if H(e) is open in $S\setminus G$. Theorem 2. Let S be as in Theorem 1. If $S\setminus G$ is a group, then there exists a one-parameter group $P \subseteq G$ so that $S \cong \times_{i=1}^{n-1} P \times \mathrm{Cl}\ (P \times C)$. Corollary. If S is as in Theorem 1 and $S\setminus G$ is a group, then $S\setminus G$ is compact if and only if $n = 1$. (Received October 19, 1967.)

653-249. BENNO ARTMANN, McMaster University, Hamilton, Ontario, Canada. On Hjelmslev planes and modular lattices with a homogeneous basis of chains.

A list of elements $a_1, \dots, a_n$ of a modular lattice L with greatest element U and least element N is said to be a homogeneous basis of L, if the a_i are independent, pairwise perspective and their union is U. We consider the case where n = 3 and the quotients $L(N, a_i)$ are chains. We define $A = \{p \in L | p$ is a complement of $a_1 \cup a_2$ or $a_1 \cup a_3$ or $a_2 \cup a_3\}$, $B = \{g \in L | g$ is a complement of a_1 or a_2 or $a_3\}$ and call A the set of points, B the set of lines. An incidence relation I in $A \times B$ is induced by the order relation of L. With further assumptions concerning the existence of certain relative complements, the system (A, B, I) turns out to be a Hjelmslev plane in the sense of W. Klingenberg, Math. Z., Vol. 60. A class of lattices is constructed such that every projective plane is a proper homomorphic image of the Hjelmslev planes derived from these lattices. This is a contribution to Problem 23, p. 166 of Skornjakov, Complemented modular lattices and regular rings. (Received October 18, 1967.)

653-250. NAND KISHORE, University of Toledo, Toledo, Ohio 43606. Representation of a set as the union of two disjoint nonmeasurable sets.

Let λ^* denote the Lebesgue outer measure on subsets of the real numbers. Then we have the following Theorem. If A is any subset of the real numbers with $\lambda^*(A) > 0$, then there exist two disjoint Lebesgue nonmeasurable sets P and Q such that $A = P \cup Q$. (Received October 20, 1967.)

653-251. JON FROEMKE, Oakland University, Rochester, Michigan 48063. A relation between two kinds of independence in universal algebra. Preliminary report.

By independence of algebras is meant independence in the sense of Foster (Math. Z. 62 (1955), 173). By independence of a set of elements of an algebra is meant independence in the sense of Marczewski (Colloq. Math. 14 (1966), 170). Theorem. Let $A, A_1, \dots, A_n$ be nontrivial, finite, similar algebras of finite, finitary species. Suppose $A_1, \dots, A_n$ are independent and $A \cong A_1 \times \dots \times A_n$. Then the elements $a_1, \dots, a_m$ in A are independent if and only if the elements $\theta_j(a_1), \dots, \theta_j(a_m)$ in A_j are independent for each $j = 1, \dots, m$, where θ_j denotes the natural projection of A onto A_j. The hypotheses of the theorem can be relaxed considerably to obtain one of the implications. Notation. Denote the maximal number of independent elements of the algebra A by i(A) and the order of A by $|A|$. Corollary. Under the hypotheses of the theorem, $i(A) = \min i(A_j) \leqq \min |A_j|$ where the minimums are taken over all $j = 1, \dots, m$. (Received October 23, 1967.)

653-252. L. F. GUSEMAN and H. E. LACEY, NASA - MSC - ED13, Houston, Texas 77058, and P. D. MORRIS, Pennsylvania State University, University Park, Pennsylvania. Linear isometries on spaces of affine continuous functions.

Throughout this paper K and L denote nonempty convex compact subsets of a locally convex Hausdorff topological linear space. The space A(K) (resp. A(L)) is the Banach space of all real-valued affine continuous functions on K (resp. L) under the supremum norm. The purpose of this paper is to study when a linear isometry from A(K) to A(L) implies K is affinely homeomorphic to L. Typical results are (1) if A(L) is order symmetric, then A(K) linearly isometric to A(L) implies K is affinely homeomorphic to L; and (2) if K is a simplex, then A(K) is order symmetric. (Received October 23, 1967.)

653-253. J. G. KRISHNA and S. M. SHAH, University of Kentucky, Lexington, Kentucky 40506. Entire functions of bounded index in several complex variables.

In this paper the concept of bounded index is extended to functions of several complex variables. It is known in the case of one variable that an entire solution of a differential equation with constant coefficients is of bounded index (see S. M. Shah, Abstract 67T-426, these *Notices* 14 (1967), 547). The direct extension of this theorem to systems of partial differential equations in several variables is false. However a generalization of the known result referred above is proved here. Theorem. Let C^n denote the space of n complex variables and let $D^{(j_1,\ldots,j_n)}$ denote the operation of taking $(j_1, \ldots, j_n)$th partial derivative in C^n. Consider a system of partial differential equations of the form $\sum a_{j_1,\ldots,j_n} D^{(j_1,\ldots,j_n)}(f) = 0$ for $m = 1, \ldots, k$, where the coefficients belong to C^1 and are not all zero for any particular m, and the summation extends over $(j_1, \ldots, j_n) \leq (p_1{}^{(m)}, \ldots, p_n{}^{(m)})$. Suppose that the system contains for m equal to some m_j and equation with $p_1^{(m_j)} = \ldots = p_{j-1}^{(m_j)} = p_{j+1}^{(m_j)} = \ldots = p_n^{(m_j)} = 0$ for each j among $1,\ldots,n$. Then any entire solution of the system is of bounded index. (Received October 23, 1967.)

653-254. R. A. STOLTENBERG, Washington State University, Bellingham, Washington. On quasi-metric spaces.

A quasi-metric space (X, d) is said to be strong if $\tau_d \subset \tau_{d'}$, where d' is the conjugate quasi-metric generated from d. If $(X, \mathscr{P}, \mathscr{Q})$ be a bitopological space, then $\mathscr{P}$ is said to be locally compact with respect to $\mathscr{Q}$ if for each $x \in X$ there is $\mathscr{P}$ neighborhood P of x such that the $\mathscr{Q}$ closure of P is compact relative to $\mathscr{Q}$. Theorem. If (X, d) is a quasi-metric space with $\tau_{d'}$ locally compact with respect to τ_d then $\tau_{d'} \supset \tau_d$. Corollary. Every compact quasi-metric space is strong. Theorem. Every regular strong quasi-metric space is a Moore space and every pointwise paracompact Moore space is a strong quasi-metric space. It follows that every compact T_2 quasi-metric space is metrizable. This is not generally true for locally compact T_2 quasi-metric spaces. There exists a locally compact T_2 strong quasi-metric space (X, d) which is not normal and hence not metrizable. Also $(X, \tau_{d'}, \tau_d)$ is a quasi-metrizable bitopological space with $(X, \tau_{d'})$ metrizable and (X, τ_d) regular but not metrizable. This example answers a question posed by Patty [Bitopological spaces, Duke Math. J. (to appear)]. (Received October 22, 1967.)

653-255. A. W. CURRIER, University of Maryland, College Park, Maryland 20742. On noncompact solvmanifolds. Preliminary report.

A solvmanifold is the quotient space G/S where G is a connected, simply connected solvable Lie group and S is a closed subgroup of G. The following results are obtained. (1) Every noncompact solvmanifold of rank 1 is a vector bundle over a circle. (2) Every noncompact solvmanifold of dimension less than 4 is a vector bundle over a compact solvmanifold. (Received October 23, 1967.)

653-256. M. J. SHERMAN, University of California, Los Angeles, California 90024. The eigenfunctions of certain inner functions.

Let $\mathscr{U}$ be an inner function in the sense of Lax. An eigenfunction of $\mathscr{U}$ is a scalar inner function q such that $\mathscr{U}(z) - q(z)I$ is not invertible for $|z| \leqq 1$ (except possibly for a set of linear measure 0 on $\{z : |z| = 1\}$). It is shown that if $\mathscr{U}$ has the form $(z - T^*)(1 - zT)^{-1}$ for T a bounded normal operator on the underlying Hilbert space, then q must be of the form $\alpha(z - \bar{\mu})(1 - \mu z)^{-1}$, where $\mu \in \sigma(T)$ and $|\alpha| = 1$. We also give estimates on α. (Received October 23, 1967.)

653-257. A. L. VILLONE, International Business Machines, 9045 Lincoln Boulevard, Los Angeles, California 90045. Spectral resolution of self-adjoint analytic differential operators. Preliminary report.

Let L be a formal differential operator whose coefficients are elements of the Hilbert space, H, of square summable analytic functions on the unit disk. Let A be a self-adjoint operator in H associated with L and $\{E_\lambda\}$ the corresponding resolution of the identity. Since H possesses a reproducing kernel, the projections $E_\Delta = E_b - E_a$ (where $\Delta = (a,b]$) are integral operators with analytic kernels. The projections E_Δ are then determined explicitly in terms of a spectral matrix, and a basis of solutions of the equation $(L - l)u = 0$. (Received October 23, 1967.)

653-258. PHILLIP GRIFFITH, University of Houston, Houston, Texas. On generalized Baer groups.

Let N be a nonvoid subset of the primes and let I_N be the subring of the rational numbers whose nonzero elements consist of those rationals having denominators prime to p for each $p \in N$. An abelian group G is called a B^N-group if Ext $(G,T) = 0$ for all torsion groups T whose p-primary components are zero for $p \notin N$. Theorem 1. G is a B^N-group if and only if $(tG)_p = 0$ for $p \in N$ ($(tG)_p$ denotes the p-primary component of the torsion subgroup of G) and G/tG is isomorphic to a subgroup of $\sum_\mu I_N$ where μ = rank (G/tG). If N consists of all primes except for a single prime p, we use the notation $B^{\hat{p}}$ to denote B^N. The group M is a direct sum of p-mixed groups if $M = \sum_p M_p$ where tM_p is p-primary. Theorem 2. Let G be a torsion free group. Then every extension M of a torsion group by G is a direct sum of p-mixed groups if and only if $G = \sum_p G_p$ where G_p is a $B^{\hat{p}}$-group for each prime p. (Received October 23, 1967.)

653-259. D. A. KLARNER, McMaster University, Hamilton, Ontario, Canada. The number of graded partially ordered sets.

A poset P is graded if there exists a mapping g sending P into the integers such that (i) $x < y$ implies $g(x) < g(y)$, and (ii) $1 + g(x) = g(y)$ if y covers x. The rank of a graded poset P is $\max\{g(x_1) - g(x_n) : (x_1,\ldots,x_n), x_i$ covers or is covered by $x_{i+1}\}$, where g is any rank function defined on P. We find an explicit formula for the number of graded posets of rank h that can be defined on a set X containing n elements. Also, we find the number of graded posets of length h having a greatest and least element that can be defined on X. The first result provides a lower bound for G*(m), the number of posets that can be defined on X; the second result provides an upper bound for the number of lattices satisfying the Jordan Dedekind chain condition that can be defined on X. (Received October 23, 1967.)

653-260. D. SARAFYAN and E. GUILLOT, Louisiana State University in New Orleans, New Orleans, Louisiana 70122. Investigation about the algebraic equations associated with Runge-Kutta method.

It is well known that with second, third, fourth and fifth order Runge-Kutta processes there are associated 2, 4, 8 and 16 nonlinear algebraic equations, respectively. These results suggest that 32 equations are associated with the sixth order process. Huta in 1956 has shown that actually there are 31 equations. And since the number of parameters in an s-stage process is given by $N = s(s + 1)/2$ he concluded that for the sixth order case $s = 8$ because then $N = 36 > 31$. Thus he established a sixth order formula requiring 8 substitutions. Butcher in 1964 has given sixth order formulas requiring only 7 substitutions. It will be shown that this reduction of stages is highly detrimental to resulting Runge-Kutta formulas. Indeed the obtained approximations are not only far inferior to those given by Huta's formula but are even inferior to certain fifth order formulas which require 6 substitutions. Furthermore adopting operational methods as employed by Huta and others and through the use of a 360-50 IBM computer the authors have established that for 7th, 8th,..., 14th order processes there are associated 59, 110, 201, 361, 639, 1114, 1917 and 3259 algebraic equations, respectively. (Received October 23, 1967.)

653-261. D. E. BLAIR, Michigan State University, East Lansing, Michigan, and A. P. STONE, University of Illinois, Chicago Circle, Chicago, Illinois. A note on flat manifolds.

Let M be a Riemannian manifold, E the module of vector fields on M, ∇ the covariant derivative operator with respect to the Riemannian connexion on M and $\mathbf{L}$ the Lie derivative operator. We first characterize flat Riemannian manifolds as follows. (1) M is flat if and only if there exists a vector-valued 1-form $h: E \longrightarrow E$ with constant distinct eigenvalues such that $\nabla_X h = 0$ and $\mathbf{L}_X h = 0$ for every $X \in E$. (2) M is flat if and only if there exists a cyclic vector-valued 1-form $h: E \longrightarrow E$ such that $\nabla_X h = 0$ and $\mathbf{L}_X h = 0$ for every $X \in E$. As an application we consider the holonomy group Φ_m at $m \in M$ of the Riemannian connexion. Then the holonomy group Φ_m of a flat Riemannian manifold has the following properties. (1) There exists a cyclic vector-valued 1-form h with generator $X_0(m)$ such that $gX_0(m)$ is also a generator for every $g \in \Phi_m$. (2) There exists a vector-valued 1-form h with constant distinct nonzero eigenvalues $\lambda_0,\ldots, \lambda_n$ with corresponding eigenvectors $\{X_i(m)\}$ such that $\{Y_i(m) = gX_i(m)\}$ are also eigenvectors of h for every $g \in \Phi_m$. (Received October 23, 1967.)

653-262. D. P. KRAINES, Haverford College, Haverford, Pennsylvania 19041. Primitive chains and $H_*(\Omega X)$.

Let $C_*(X;R)$ denote the normalized singular chain coalgebra of a simply connected space X over a P.I.D. R with the Alexander Whitney diagonal. Then $C^*(X,R)$ is an associative algebra under the usual cup product. Theorem. $C^*(X;R)$ is isomorphic to a free associative noncommutative algebra over R. Let PC_* and QC^* denote the subcomplex of primitive chains and the quotient complex of indecomposable cochains respectively. Using the cobar construction and elementary spectral sequence theory it is shown that $H_q(PC_*) \approx H_{q-1}(\Omega X;R)$ and $H^q(QC^*) \approx H^{q-1}(\Omega X;R)$ as R modules. (Received October 23, 1967.)

653-263. J. P. FILLMORE, University of California at San Diego, La Jolla, California. Symmetries of surfaces of constant width.

A closed convex hypersurface in Euclidean E^n is said to have constant width if the distance between the two supporting hyperplanes having a given normal vector is independent of that vector. A symmetry (resp. proper symmetry) of a closed hypersurface is a rotation of O(n) (resp. SO(n)) which, when combined with a translation, carries the surface onto itself. Theorem. There exists an analytic hypersurface of constant width in E^n which admits no symmetries other than the identity. Let G_2 be the two-component one-dimensional subgroup of SO(3) which consists of all rotations around a fixed axis and rotations of π around axis perpendicular to this fixed axis. Theorem. With the exception of subgroups conjugate to G_2, every closed subgroup of SO(3) is the group of proper symmetries of an analytic surface of constant width in E^3. A similar theorem classifies closed subgroups of O(3) as to whether or not they are the groups of symmetries of an analytic surface of constant width in E^3. (Received October 23, 1967.)

653-264. R. R. STEVENS, University of Montana, Missoula, Montana 59801. On the forced Liénard equation.

Lemma. If h(t) is measurable and $0 \leqq h(t) \leqq 1$ for $0 \leqq t \leqq 2\pi$ then $|\int_0^{2\pi} h(t)e^{it}dt| \leqq 2$. This inequality, which is of independent interest, implies the following generalization of a result due to Frederickson and Lazer (Abstract 644-16, these *Notices* 14 (1967), 364.) Theorem. If f(x) is continuous and g(t) is continuous and 2π-periodic then the differential equation $\ddot{x} + \dot{x}f(x) + x = g(t)$ has a solution x(t) such that $x(t) = o(t)$ and $\dot{x}(t) = o(t)$ as $t \to +\infty$ only if there exists a and b such that $2|\int_a^b f(x)dx| \geqq |\int_0^{2\pi} g(t)e^{it}dt|$. (Received October 22, 1967.)

653-265. IVAN ERDELYI, Kansas State University, Manhattan, Kansas 66502. Partial isometries closed under multiplication on Hilbert spaces.

This paper is concerned with the transmission of partial isometry through multiplication. The following main results are obtained: (1) Let U and V be partial isometries on a Hilbert space H, and W = UV. The following statements are equivalent: (i) W is a partial isometry; (ii) the initial space of U is invariant under the projection on the range of V, i.e. $VV^* \cdot R(U^*) \subset R(U^*)$; (iii) the range of V is invariant under the projection on the initial space of U, i.e. $U\ U \cdot R(V) \subset R(V)$. (2) Let $V_1, V_2, \ldots, V_n$ be n partial isometries on a Hilbert space H, and $W_i = V_1 V_2 \ldots V_i$, $i = 1,2,\ldots, n$. The following statements are equivalent: (i) $W_2, W_3, \ldots, W_n$ are partial isometries; (ii) the carrier of W_{i-1} is invariant under the projection on the range of V_i, i.e. $V_i V_i^* \cdot R(W_{i-1}^*) \subset R(W_{i-1}^*)$, for $i = 2,3,\ldots,n$; (iii) the range of V_{i+1} is invariant under the projection on the carrier of W_i, i.e. $W_i^* W_i \cdot R(V_{i+1}) \subset R(V_{i+1})$, for $i = 1,2,\ldots,n - 1$. (3) Let A and B be contractions, i.e. $\|A\| \leqq 1$, $\|B\| \leqq 1$, on a Hilbert space H, and M(X) a 2 by 2 operator matrix with X and $(I - XX^*)^{1/2}$ in the first row and zeros elsewhere. The product $M(A) \cdot M(B)$ on $H \oplus H$, is a partial isometry if and only if A is a partial isometry. (Received October 2, 1967.)

653-266. P. L. ROSENTHAL and MACIEF SKWARCZYNSKI, Stanford University, Stanford, California 94305. On a subclass of harmonic functions defined by the Bergman-Whittaker operator.

The Bergman-Whittaker operator $B_3(f) = (2\pi i)^{-1} \int_{|\zeta|=1} f(u,\zeta)(d\zeta/\zeta) = h(x,y,z) \in H$ transforms an analytic function $f(u,\zeta)$, $u = (1/2)(iy + z)\zeta + x + (1/2)(iy - z)\zeta^{-1}$ into a harmonic function $h(x,y,z)$. H becomes an algebra under the composition corresponding to pointwise multiplication of the associate functions $f(u,\zeta)$. In Duke Math. J. 30 (1963), 447-460, Bergman considers an entire function $g(\eta)$, where $\eta = u(B\zeta^P + D\zeta^S)$, r,P,S positive integers. Using the Nevanlinna theory, one obtains relations between the coefficients of the series development of $h = B_3(g)$ in spherical harmonics of the density of singularities of the reciprocal of $[h(x,y,z) - a]$. The authors show that this approach can be generalized to a larger class of harmonic functions, which can be defined by the substitution $\eta = u^r \sum_{\kappa=m}^{M} w_\kappa \zeta^\kappa$. They obtain relations in this general case between the coefficients in the development of h, and the density of singularities of the reciprocal of $[h(x,y,z) - a]$. (Here reciprocal is taken in the sense of Bergman's composition.) (Received October 11, 1967.)

653-267. JOSEPH NEGGERS, University of Alabama, University, Alabama. Transvectors and endotensors. Preliminary report.

Suppose A is an abelian group with a multiplication such that $x \cdot 0 = 0 \cdot x = 0$ for all $x \in A$. If B is a unitary left R-module and $f: A \times A \to R$ is such that $f(x,y)$ is an element in the center of R for all $x, y \in A$, then $\mathrm{Hom}_R^f(A,B) = \{\mu: A \to B \mid \mu(0) = 0, \mu(x + y) = \mu(x) + \mu(y) + f(x,y)\,\mu(xy) + f(y,x)\mu(yx)\}$ is an R-module. Thus if A has the trivial multiplication or $f(x,y) \equiv 0$, $\mathrm{Hom}_R^f(A,B) = \mathrm{Hom}_Z(A,B)$. If A is a Boolean ring, and $f(x,y) = -1$, $\mathrm{Hom}_R^f(A,B)$ is the collection of all additive set functions. We shall call elements $\mu \in \mathrm{Hom}_R^f(A,B)$, transvectors on A to B. We construct an R-module $\otimes_R^f A$ such that $\mathrm{Hom}_R^f(A,B) \cong \mathrm{Hom}_R(\otimes_R^f A,B)$, $\otimes_R^f A$ is the endotensor of A over R induced by f. The endotensor is constructed in a fashion analogous to an ordinary tensor product and the functors $\mathrm{Hom}_R^f(A,\cdot)$ and $\mathrm{Hom}_R(\otimes_R^f A,\cdot)$ are naturally equivalent. Using the concept of an endotensor we define categories of L-rings (cf. Abstract 650-630, these *Notices* 14 (1967), 924) and determine the morphisms, where a morphism $\phi: A_1 \to A_2$ is a mapping such that if $\mu \in \mathrm{Hom}_R^f(A_2,B)$, then $\mu\phi \in \mathrm{Hom}_R^f(A_1,B)$. Thus, e.g., in the category of Boolean rings, with the transvectors additive set functions ϕ is a morphism if and only if ϕ maps complemented ideals onto complemented ideals (hence atoms onto atoms) and if the inverse image of a complemented ideal is a complemented ideal. Corollary: There is an embedding of the category of sets and functions into a split exact category of Boolean algebras and certain homomorphisms. One can obtain many other results of a similar nature. (Received October 13, 1967.)

653-268. T. J. REED, University of Colorado, Boulder, Colorado 80302. The boundary correspondence of quasiconformal mappings on quasicircles.

A Jordan curve L is called a C-quasicircle ($C \geqq 1$) if the cross ratio inequality $|(z_1,z_2,z_3,z_4)| \leqq 1$ holds for any four points $z_1,\ldots,z_4$ in order on L. A homeomorphism f of a Jordan curve L onto a Jordan curve L' is called (A,α)- quasisymmetric ($A \geqq 1$, $0 < \alpha \leqq 1$) if $|(w_1,w_2,w_3,w_4)| \leqq A|(z_1,z_2,z_3,z_4)|^{\alpha}$ where $w_j = f(z_j)$, $j = 1,\ldots,4$ and $z_1,\ldots,z_4$ are any four points in order on L. Theorem 1. If f is an (A,α) - quasisymmetric homeomorphism of a C-quasicircle L onto a C'-quasicircle L' then there exists a K-quasiconformal homeomorphism of the plane onto itself, agreeing with f on L, and with K depending only on C, C', A, α. Theorem 2. If F is a K-quasiconformal mapping of the plane onto itself then its restriction to the mapping of a C-quasicircle onto a C'-quasicircle is (A,α)- quasisymmetric with A and α depending only on K, C, C'. These generalize results of A. Beurling and L. Ahlfors [Acta Math. 96 (1956), 125-142]. (Received October 2, 1967.)

653-269. D. A. BRANNAN, University of Maryland, College Park, Maryland 20742. On univalent polynomials.

Let P_n be the class of normalised univalent polynomials of degree n in $|Z| < 1$. Then $m(n) = M(1, P_n)|n^2$ is always bounded above as $n \to \infty$. However (1) there is a sequence $\{P_n\}$ such that m(n) does not tend to zero as $n \to \infty$, and (2) there is a sequence $\{P_n\}$ such that $(\log n)^2 m(n)$ does not tend to zero as $n \to \infty$ although $P_n(Z) \to Z$ locally uniformly in $|Z| < 1$. (Received October 23, 1967.)

653-270. PAUL ROSENTHAL, Stanford University, Stanford, California 94305. Some uniqueness theorems for a series of Legendre polynomials.

Assuming only the series of Legendre polynomials $\sum_{n=0}^{\infty} a_n P_n(x)$ converges pointwise to zero for all x, $1 \geqq x \geqq -1$ or $1 > x > -1$, where for all n, a_n is real and $P_n(x)$ is a Legendre polynomial, it is concluded for all n, $a_n = 0$. (Received October 20, 1967.)

653-271. GÜNTHER GOES, Illinois Institute of Technology, Chicago, Illinois 60616. Matrix transformations between FK-spaces and sequences of Fourier coefficients.

Let X and Y be FK-spaces, i.e. Fréchet spaces of complex sequences with the property that for $a = \{a_k\} \in X$ (resp. $a \in Y$) the mappings $a \to a_k$ are continuous for every k. Let T be a matrix transformation $X \to Y$ given by the infinite matrix (a_{kj}) such that for $a \in X$, $Ta = \{A_k\}$ where $A_k = \sum_{j=1}^{\infty} a_{kj} a_j \, \forall k$. Let T^* be the corresponding matrix transformation given by the transposed matrix (a_{jk}) of (a_{kj}) and let E be the greatest space of sequences to which T^* can be applied (i.e. $\sum_{k=1}^{\infty} a_{jk} b_k$ exists if and only if $\{b_k\} \in E$). If X has sectional convergence, i.e. if $a \in X$ implies $a^n \to a$ $(n \to \infty)$ where $a^n = \{a_1, a_2, \ldots, a_n, 0, 0, \ldots\}$ then the β-dual Köthespace of E, namely $E^{\beta^*} = \{\{d_k\} | \sum_1^{\infty} d_k b_k$ exists for every $\{b_k\} \in E\}$, is the smallest β-dual Köthespace into which X maps under T, i.e. $E^{\beta^*} \subset Y$ if Y is a β-dual Köthespace. Example: $A_k = k^{-1} \sum_{j=1}^{k} a_j$. Then $E^{\beta^*} = \{\{d_k\} | \sum_{k=1}^{\infty} |kd_k - (k+1)d_{k+1}| < \infty\}$. Generalizations of this theorem and applications to transformations of sequences of Fourier coefficients are considered. (Received October 24, 1967.)

653-272. C. E. AULL, Virginia Polytechnic Institute, Blacksburg, Virginia 24061. Collectionwise normal subsets (continuation).

To the collectionwise normality conditions discussed in a previous abstract (67T-624, these *Notices* 14 (1967), 843) the following condition is added: (e) Let $\{D_a\}$ be discrete with respect to (X,T) and D_a a subset of M. There exists a locally finite family of pairwise disjoint open sets $\{G_a\}$ such that $D_a \subset G_a$. Definition. A subset M of a topological space (X,T) is a generalized LF_σ if for every open set G there exists a set H, $M \subset H \subseteq G$, such that $H = \bigcup H_c$ where each H_c is an open F_σ subset with respect to (X,T) and $\{H_c\}$ is locally finite with respect to the relative topology for G. The following are proved. A generalized LF_σ set of a normal space satisfying b satisfies d. Generalized LF_σ subsets of collectionwise normal (paracompact) spaces satisfy d (are paracompact subspaces). The interior of a closed set satisfying c satisfies e. Locally finite unions of open sets satisfying e satisfy e. Countable unions of sets satisfying e satisfy e. (Received October 24, 1967.)

653-273. CHARLES GOLDSTEIN, Brookhaven National Laboratory, 61 Brookhaven Avenue, Upton, Long Island, New York. Eigenfunction expansions and scattering theory for certain infinite domains.

Let S be the two dimensional cone $r \geqq 0, \pi\alpha \geqq \theta > 0$ $(2 \geqq \alpha > 0)$ in polar coordinates. Let $\Omega \subseteq S$ denote a domain obtained from S by perturbing a finite portion of $\dot{S}$, the boundary of S. It is shown that A_0 is unitarily equivalent to A, where $A_0[A]$ denotes the self-adjoint operator given by $-\Delta$ acting on functions defined in $S[\Omega]$ and satisfying a zero boundary condition on $\dot{S}[\dot{\Omega}]$. A complete, orthogonal set of generalized eigenfunctions $w_n^0(x;\lambda)$ are explicitly given for A_0. These are obtained by separation of variables and yields a spectral representation for A_0. Two complete, orthogonal sets of generalized eigenfunctions $w_n^{\pm}(x;\lambda)$ are constructed for A, one set satisfying outgoing radiation conditions and the other incoming radiation conditions at infinity. It is then shown that the wave operators $w^{\pm}(\phi(A^{1/2}), \phi(A_0^{1/2}))f = \lim_{t\to\pm\infty} e^{it\phi(A^{1/2})}e^{it\phi(A_0^{1/2})}f$ exist, are unitary, and are independent of ϕ for a wide class of real valued functions $\phi(\lambda)$. (Received October 24, 1967.)

653-274. W. R. SCOTT and FLETCHER GROSS, University of Utah, Salt Lake City, Utah 84112. Solvable products of groups.

Theorem. Let $\mathscr{L}$ be the class of finite groups H such that every subgroup of a Sylow 2-subgroup of H is normal in H. If G = AB with $A \in \mathscr{L}$ and $B \in \mathscr{L}$, then G is solvable. The proof uses an unpublished result of Walter on the structure of groups with Abelian Sylow 2-subgroups, as well as the Feit-Thompson theorem and a result of Glauberman (George Glauberman, Central elements in core-free groups, J. Algebra 4 (1966), 403-420). (Received October 25, 1967.)

653-275. J. E. McMILLAN, University of Wisconsin-Milwaukee, Milwaukee, Wisconsin 53201. On functions meromorphic in a disc.

Let $w = f(z)$ be a nonconstant meromorphic function in $\{|z| < 1\}$, and let W denote the extended w-plane. Let $N(w,\delta)$ denote the set of all points of W at a chordal distance less than δ from w ($\delta > 0$), and define a closed set $B \subset W$ as follows: $w \in B$ if and only if $w \in W$ and for any $\delta > 0$ there exist $N(w_0, \delta_0) \subset N(w,\delta)$ and a component U of the preimage $f^{-1}(N(w_0,\delta_0))$ such that f(U) is not dense in $N(w_0, \delta_0)$. Theorem 1. Suppose V is a domain with $V \subset W - B$ and U is a component of $f^{-1}(V)$. Then either (i) for any $w \in V$ there exists $\delta > 0$ such that $U \cap f^{-1}(N(w,\delta))$ is relatively compact, or (ii) for any $w \in V$ either w is an asymptotic value of f along a path lying in U, or there exists $\delta > 0$ such that infinitely many relatively compact components of $f^{-1}(N(w,\delta))$ are contained in U. Theorem 2. Suppose that $w \in B$. Then for any $\delta > 0$ the set of points of $\{|z| = 1\}$ at which f has asymptotic values in $N(w,\delta)$ has positive linear measure, and the set of points of $N(w,\delta)$ that are asymptotic values of f at points of $\{|z| = 1\}$ has positive linear measure. (Received October 26, 1967.)

653-276. MELVYN S. BERGER, Courant Institute of Mathematacal Sciences, 251 Mercer Street, New York, New York 10012. On nonlinear elliptic boundary value problems of von Karman type.

Suppose L_1 and L_2 are bounded self-adjoint operators mapping a separable Hilbert space H into itself, with L_1 positive definite and L_2 compact such that $(L_2u,u) > 0$ for $u \neq 0$. Let N be a completely continuous strictly nonlinear gradient mapping of H into itself such that $(Nu,u) \geqq 0$ (with equality holding if and only if $u = 0$), $N(0) = 0$, and N is locally Lipschitzean. Theorem. The equation $(L_1 + N)u = \lambda L_2 u$ has (i) a one-parameter family of nontrivial solutions u(R) for all positive values R; (ii) a countably infinite number of distinct one parameter families of solutions (nontrivial) $\{u_n(R)\}$ for sufficiently small R, with associated numbers $\{\lambda_n(R)\}$ near each eigenvalue of $L_1 u = \lambda L_2 u$. For $H = \dot{W}_{2,2}(G)$ (the Sobolev space) with G a bounded domain in the plane, the above theorem can be specialized to yield results on the full von Karman equations for the buckling of thin elastic shells. (Received October 26, 1967.)

653-277. J. R. PORTER, University of Kansas, Lawrence, Kansas 66044. On locally H-closed spaces.

A Hausdorff space is locally H-closed iff every point has a neighborhood which is H-closed. Theorem. Let (X^*, τ^*) be a one-point H-closed extension of (X,τ) where $X^* - X = \{p\}$. Let $\alpha = \{U \cap X | p \in U \in \tau^*\}$. (X^*, τ^*) is the projective maximum one-point H-closed extension of (X,τ) iff $\alpha = \{V \in \tau | X - (\bar{V}) \text{ is H-closed}\}$. Theorem. Let (X,τ) be a locally H-closed space which is not H-closed. (X,τ) has a unique one-point H-closed extension iff every nowhere dense closed subset is contained in an H-closed subspace of (X,τ). A Hausdorff space is almost H-closed iff for every pair of disjoint open sets, the closure of one is H-closed. Theorem. The Katetov extension of a Hausdorff space (X,τ) is a one-point H-closed extension iff (X,τ) is almost H-closed and not H-closed. Corollary. An almost H-closed space is locally H-closed and pseudocompact. (Received October 26, 1967.)

653-278. J. G. KRISHNA, University of Kentucky, Lexington, Kentucky 40506. Maximum term of a power series in one and several variables.

Let $\Omega \subseteq \mathscr{C}^n$ be an open Reinhardt domain and let $K = K(\mu)$ stand for the set of all power series centered about the center of Ω, converging and with maximum term $\mu = \mu(|z_1|,\ldots, |z_n|)$ in Ω. It is shown in particular that K may be regarded as composed of equivalence classes which form a distributive lattice with a unique maximal and a unique minimal element. Valiron's theory of maximum term and rank of an entire series in one variable is extended to the case of a multiple power series, based on purely analytic considerations (which often systematise and simplify the discussions even when $n = 1$). It is shown that some results obtained or used by Bose and Sharma when n is 2 (Compositio Math. 15 (1963), 210-226) are not correct. The difficulties with the Valiron-type geometrical arguments particularly in several variables are indicated. (Received October 26, 1967.)

653-279. ERIK HEMMINGSEN, Syracuse University, Syracuse, New York 13210, and W. L. REDDY, State University of New York, Albany, New York 12203. Expansive homeomorphisms on manifolds and homogeneous spaces.

In every finite dimension greater than three there is a compact, connected manifold which is not an H-space (hence not a topological group) and which admits an expansive homeomorphism. In every finite dimension greater than one there is a compact, connected space, fibered over a manifold by a Cantor set, which is not an abelian group space and which admits an expansive homeomorphism. In every finite dimension greater than two, there are countably many different such spaces. The second class of examples is produced by finding positively expansive maps on the base manifolds. It is shown that such maps must be finite-to-one covering maps, and therefore neither the base space nor the total space can be simply connected. In previously published examples of expansive homeomorphisms on manifolds, the space has been a topological group space and the homeomorphism an automorphism. (Received October 26, 1967.)

653-280. J. P. KING, Lehigh University, Bethlehem, Pennsylvania. Almost convergent positive linear operators.

A sequence $x \in m$ is said to be almost convergent to s if $B(x) = s$ for each Banach limit B. A matrix $A = (a_{nk})$ is called almost regular if $x \in c$ implies $A(x) \in f$ and $A(x)$ is almost convergent to lim x, where f is the space of almost convergent sequences. Let e^i be defined by $e^i(x) = x^i$ for $i = 0,1,2$. The following result is an elementary analogue of a well-known result of Korovkin for convergent sequences of positive linear operators. Theorem 1. Let $\{L_n\}$ be a sequence of positive linear operators on $C[a,b]$. Then $\{L_n(f)(x)\}$ is almost convergent to $f(x)$, uniformly on $[a,b]$, for each $f \in C[a,b]$ if and only if $\{L_n(e^i)(x)\}$ is almost convergent to $e^i(x)$, uniformly on $[a,b]$, for $i = 0,1,2$. Theorem 1 may be used to establish relationships between almost convergent sequences of positive linear operators and almost regular matrices. A typical result in this direction is: Theorem 2. Let $L_n(f)(x) = \sum a_{nk}(x)f(x_{nk})$ be a sequence of positive linear operators on $C[a,b]$ with $a \leqq x_{nk} < x_{n,k+1} \leqq b$ for each n and k and $\lim_n x_{nk} = a$ for each k. If $\{L_n(f)(x)\}$ is almost convergent to $f(x)$ for each $f \in C[a,b]$ then the matrix $A = (a_{nk}(x))$ is almost regular for each $x \in (a,b]$. (Received October 25, 1967.)

653-281. SIGRUN GOES, DePaul University, 2332 North Kenmore Avenue, Chicago, Illinois 60614. About compactness in Köthe spaces.

Let E be a uniform space which can be written as the union of an increasing sequence of compact sets K_n, $\mathscr{U}$ a uniform structure base for E and μ a nonnegative Radon measure on E for which certain additional properties hold. Let Λ be a Köthe space over E and $\tau_{\mathfrak{H}}$ a Köthe topology on Λ given by seminorms p_H $(H \in \mathfrak{H})$. For each $f \in \Lambda$ with compact support and for each $U \in \mathscr{U}$, let $T_U(f)$ denote the obvious generalization of a Steklov function. Then the following 3 conditions are sufficient for a subset $\mathfrak{A}$ of Λ to be relatively compact: (1) $\mathfrak{A}$ is bounded; (2) $\forall H \in \mathfrak{H}\ \exists$ an index n_H such that $\lim_{U \in \mathscr{U}} \sup_{f \in \mathfrak{A}} p_H(T_U(\chi_{K_n} f) - \chi_{K_n} f) = 0$ if 0 if $n \geq n_H$; (3) $\lim_{n \to \infty} \sup_{f \in \mathfrak{A}} n_H(\chi_{K_n} f - f) = 0$ for every $H \in \mathfrak{H}$, Here χ_{K_n} denotes the characteristic function of K_n. Furthermore those Köthe spaces for which conditions (1) to (3) are also necessary for relative compactness are characterized. A similar result holds for translation invariant Köthe spaces Λ over a locally compact abelian group E with μ the Haar measure on E, if the functions $T_U(\chi_{K_n} f)$ are replaced by translates of $\chi_{K_n} f$. (Received October 25, 1967.)

653-282. J. W. MOON, University of Alberta, Edmonton, Alberta, Canada. A tree counting problem.

Let $p = (p_1,\ldots,p_k)$ and $e = (e_1,\ldots,e_k)$ denote partitions of n and m into k integers such that $0 \leqq e_i \leqq p_i - 1$ for each i (we assume that $1 \leqq k \leqq n$ and $0 \leqq m \leqq n - k$). Suppose n labelled points are split into k classes with p_i points in the ith class and that a tree T_n is formed on these n points such that e_i edges join points in the ith class to each other. Na and Rapoport [Ann. Mat. Statist. 38 (1967), 226-241] derived a formula for the number T(p,e) of labelled trees T_n that can be formed in this way by evaluating a certain determinant. Another derivation of their formula is given, based on the method of inclusion and exclusion. (Received October 25, 1967.)

653-283. GUDRUN KALMBACH, University of Illinois, Urbana, Illinois. Some remarks on noncompact manifolds.

Let M be a noncompact smooth manifold with countable base. Let f be a nondegenerate function on M with $f(q_i) \longrightarrow \infty$ for each sequence $q_i \in M$ without limit point on M. Let K be the union of all descending bowls associated with some critical point of f (see: Morse, Bowls of a nondegenerate function, a symposium in honor of M. Morse, 1964, Princeton). If f has finitely many critical points on M then K is a deformation retract of M. A weaker result holds if f has an infinite number of critical points on M. It is possible to choose f and the riemannian structure on M such that the following statements are true: K is a special cell-complex ($E \cap \bar{E}' = \emptyset$ or $= E$ for each pair of cells E, E' of the complex K). To each homology class and homotopy class of M there exists a suitable representative A such that the carrier of A is the union of a finite number of descending bowls. An alternative proof for Morse's theorem on the elimination of critical points can be given. (Received October 31, 1967.)

653-284. R. E. SMITHSON, University of Wyoming, Laramie, Wyoming 82070. Topologies generated by relations.

Let X be a set and R a relation on X. An antiset is a subset of X in which no two distinct elements are related. If A is an antiset let $D(A) = \{x \mid xRa \text{ for some } a \in A\}$ and $I(A) = \{x \mid aRx \text{ for some } a \in A\}$. If Ω is a collection of antisets, let $S = \{I(A) \mid A \in \Omega\} \cup \{D(A) \mid A \in \Omega\} \cup \{\emptyset\} \cup \{X\}$. Then S is a subbase for the closed sets of a topology which we denote by $\Upsilon(\Omega)$. Conditions are determined under which $(X, \Upsilon(\Omega))$ will be T_0, T_1, T_2, connected or compact. We also investigate relationship between an existing topology and $\Upsilon(\Omega)$. For example, if (X,Υ) is a tree, if R is the cutpoint order and if Ω is the finite antisets, then $\Upsilon = \Upsilon(\Omega)$. (Received October 30, 1967.)

653-285. H. H. WICKE and J. M. WORRELL, JR., Sandia Laboratory, P. O. Box 5800, Albuquerque, New Mexico 87115. Quasi-hereditary properties, Baire category, and non-first-countable structure.

One aspect of the so-called Baire Category Theorem suggests the following theorem pattern P: Let E denote a subspace of a Tychonoff space S. Then E has property Q if and only if E is the common part of the terms of a sequence $E_1, E_2, \ldots$ of subspaces of S having property Q. When Q is the property of being a metrically topologically complete space or a complete Moore space the resulting statement expresses classical theorems. Theorem. P becomes a theorem if Q is any of the following properties: being a μ-space, a complete μ-space, a p-space, a θ-refinable p-space, a metacompact p-space, or a paracompact p-space. The significance of the topological uniformization provided by the μ-space structure [cf. Abstract 651-5, these *Notices* 14 (1967), 935], as opposed to that possibly present in an open continuous image of a paracompact μ-space, is partly illuminated by the following example. There exists a bicompact T_2-space having dense paracompact subspaces E_1 and E_2 which are open continuous images of T_2 paracompact p-spaces and which have a common part with these properties: (1) $E_1 \cdot E_2$ has a point-countable base and, therefore, is an image of a metrizable space under an open continuous s-mapping. (2) The terms of any sequence of dense open subsets of $E_1 \cdot E_2$ have a common part dense in $E_1 \cdot E_2$. (3) $E_1 \cdot E_2$ is not normal. (Received October 30, 1967.)

653-286. S. E. NEWMAN, University of Utah, Salt Lake City, Utah 84112. A class of convolution measure algebras. Preliminary report.

Let M be a commutative, semisimple convolution measure algebra with structure semigroup S and maximal ideal space $\hat{S}$ (J. L. Taylor, Trans. Amer. Math. Soc. 119 (1965), 150-166). Theorem 1. The following statements are equivalent. (1) S is an idempotent semigroup. (2) $\hat{S}$ is an idempotent semigroup. (3) For every $f \in \hat{S}$, there exists a prime L-ideal N_f of M such that $M = N_f + N_f^{\perp}$ and such that $f(\mu) = \mu_2(S)$ where $\mu = \mu_1 + \mu_2$ ($\mu_1 \in N_f$, $\mu_2 \in N_f^{\perp}$). (4) For every $f \in \hat{S}$ and every nonnegative measure $\mu \in M$, $f(\mu) \geqq 0$. (5) $\mu + e$ is invertible for every nonnegative measure $\mu \in M$. A totally ordered semilattice is a commutative, linearly-ordered topological semigroup with maximum multiplication ($x \cdot y = \max(x,y)$) and is therefore an idempotent semigroup. The algebra of measures on a totally ordered semilattice is an algebra of the type described in Theorem 1 (E. Hewitt and H. S. Zuckerman, Pacific J. Math. 7 (1957), 913-941), as is the algebra of measures on a finite product of totally ordered semilattices. It is shown that there is an algebra of measures on an idempotent compact topological semigroup whose structure semigroup is not idempotent. (Received October 23, 1967.)

653-287. S. R. CLEMENS, University of North Carolina, Chapel Hill, North Carolina. The isotopy type of certain finite polyhedra. Preliminary report.

Suppose Γ is the category of piecewise linear imbeddings of finitely triangulable pairs. For each $X \in \Gamma$ and for each positive integer n, a certain class of imbeddings of I^n into X are singled out and called admissible. These admissible imbeddings are then used to define a functor W_n from Γ to Γ which is shown to be an isotopy functor. Hence W_n composed with the homology functor H yields an algebraic functor with the property that the groups $H \circ W_n(X)$ are isotopy invariants. However, the functor $H \circ W_n$ lacks being an isology theory since it does not satisfy the excision axiom. The functor $H \circ W_n$ together with dimension arguments is then applied to the problem of isotopy classification. A large class of contractible spaces is described and it is shown that no two spaces in the class are isotopically equivalent. Also a large class of spaces each with homotopy type of S^n are described, any two of which are of a different isotopy type. (Received October 19, 1967.)

653-288. MITCHELL BILLIS, The University of Utah, Salt Lake City, Utah 84112. Finite groups whose powers have no countably infinite factor groups. Preliminary report.

Let $\mathscr{P}$ be the class of finite groups G whose powers G^I have no countably infinite factor groups. Theorem. $G \in \mathscr{P}$ if and only if G is perfect. This theorem generalizes a result of B. H. Neumann and Sadayuki Yamamuro (Boolean powers of simple groups, J. Austral. Math. Soc. 5 (1965), 315-324), who showed that if G is a finite non-Abelian simple group, then $G \in \mathscr{P}$. (Received October 30, 1967.)

653-289. L. L. CLARKSON, Texas Southern University, Houston, Texas 77004. A theorem concerning product integrals.

In 1955, J. S. MacNerney [J. Elisha Mitchell Sci. Soc. 71] showed that if h is a simple graph of bounded variation on every interval, then if x,y is a number pair, ${}_x\prod^y(1 + dh)$ exists, with ${}_x\prod^x(1 + dh) = 1$, and that if a is a number and g the simple graph such that for each x, $g(x) = {}_a\prod^x(1 + dh)$, then g is of bounded variation on every interval. We say simple graph g is product expressible on [0,1], if $g(0) = 1$ and there exists a simple graph h with x-projection [0,1] such that $g(x) = {}_a\prod^x(1 + dh)$, for each x in [0,1]. Theorem. If g is a nondecreasing simple graph with x-projection [0,1] and $g(0) = 1$, then g is product expressible on [0,1]. We also show that for given g, the simple graph h with x-projection [0,1] such that $h(0) = 0$ and $g(x) = {}_0\prod^x(1 + dh)$ for each x in [0,1] is unique and that g is continuous at (x, g(x)), if and only if h is continuous at (x, h(x)), for x in [0,1]. (Received October 30, 1967.)

253-290. JONAS ACZEL, University of Waterloo, Waterloo, Ontario, Canada. On isomoment functional equations.

(I) $f(\sum^n x_k^m/n) = \sum^n f(x_k)^m/n$ ($n \geqq 2$ a fixed integer, $m \neq 0$ fixed, x_k variable, $k = 1,2,\ldots,n$) is an isomoment functional equation. Results. All real solutions of this equation are continuous a for $m \geqq 2$ integer (not for $m = 1$!) and $x_k \geqq 0$ or $x_k > 0$ ($k = 1,2,\ldots, n$); b for $m < 0$ integer, $f(x_k) \neq 0$ and $x_k > 0$; and c for m real, $|t|^m$ instead of t^m ($t = x_k$ or $t = f(x_k)$) in (I) and $x_k \geqq 0$ or $x_k > 0$ (in case of $m < 0$ real only $x_k > 0$, and $f(x_k) \neq 0$ has to be supposed again). $f(x) = 1$ and $f(x) = x$ are solutions in all cases, $f(x) = 0$ in the cases a and c for $m > 0$, and $f(x) = -1$ and $f(x) = -x$ for odd m in cases a and b. (The results in the cases b and c were found in collaboration with P. Fischer.) Similar theorems hold for the system $f(x + y) = f(x) + f(y)$, $f(x^m) = f(x)^m$ of functional equations. (Received October 30, 1967.)

653-291. W. T. REID, University of Oklahoma, Norman, Oklahoma 73069. Comparison theorems for nonlinear vector differential equations.

For a nonlinear real vector ordinary differential equation $E: y' = f(t,y)$ there is established a comparison theorem that provides a necessary and sufficient condition for a given solution $y^0(t)$, $a \leqq t \leqq b$ of E to possess a property which is a direct generalization of the Property (B) introduced by H.-W. Knobloch [J. Differential Equations 1 (1965), 1-26] for a scalar second order equation $x'' = g(t,x,x')$. This initial comparison theorem is further extended to a case, which for a vector equation E equivalent to an nth order scalar equation $E_n: x^{[n]} = g(t,x,\ldots,x^{[n-1]})$ presents as comparison criterion the condition that the linear equation of variation along a given solution of E_n has the Property (W) of Pólya [Trans. Amer. Math. Soc. 24 (1922), 312-324]. (Received October 30, 1967.)

653-292. R. R. COLBY and E. A. RUTTER, JR., University of Kansas, Lawrence, Kansas 66044. Semiprimary QF-3 rings.

Let R be a semiprimary ring. R is called a left QF-3 ring if R contains a faithful projective injective left ideal. Theorem. The following are equivalent: (i) R is left QF - 3; (ii) Essential extensions of torsionless R-modules are torsionless; (iii) The injective envelope of R is torsionless. Theorem. The following are equivalent: (i) R is left QF-3 and has zero left singular ideal; (ii) An R-module M is torsionless if and only if the singular submodule Z(M) of M is zero; (iii) If M is an R-module, then $\mathrm{Hom}_R(M,R) = 0$ if and only if $Z^2(M) = M$, where $Z^2(M) = \{m \in M | Em \subset Z(M)$ for some essential left ideal E of R$\}$. The first result was proved for Artinian rings by Wu, Mochizuki, and Jans (A characterization of QF-3 rings, Nagoya Math. J. 27 (1966), 7-13). (Received October 30, 1967.)

653-293. D. H. PETTEY, University of Utah, Salt Lake City, Utah 84112. One-to-one continuous mappings into E^2.

Theorem 1. If M^1 is a separable 1-manifold, f is a 1-1 continuous mapping of M^1 into $E^2(S^2)$, and $f(M^1)$ is closed in $E^2(S^2)$, then $f(M^1)$ separates $E^2(S^2)$. Theorem 2. If M^2 is a connected 2-manifold with boundary and h is a 1-1 continuous mapping of M^2 onto $E^2(S^2, I^2)$ then h is a homeomorphism. Glaser has proved a special case of Theorem 2 where Int M^2 is an open 2-cell and where there is an additional restriction on the mapping h [1-1 continuous mappings onto E^n, Amer. J. Math. 88 (1966), 237-243]. Counterexamples to the analogous theorem for dimensions greater than two have been given by Glaser (see paper cited above) and by K. Whyburn [A nontopological 1-1 mapping onto E^3, Bull. Amer. Math. Soc. 71 (1965), 533-537]. (Received October 30, 1967.)

653-294. M. K. BENNETT, University of Massachusetts, Amherst, Massachusetts. States on orthomodular lattices.

An orthomodular lattice (OML) is an orthocomplemented lattice L in which orthogonal pairs are modular. A state on a finite OML is a function $f: L \to 0,1$ such that $f(0) = 0$, $f(1) = 1$, and if $e \leqq g'$, $f(e \vee g) = f(e) + f(g)$. A set of states on L is called full iff $e \nleqq g \Rightarrow$ there exists f such that $f(e) > f(g)$. The 32-element nonmodular OML due to R. Greechie is shown not to have a full set of states. Moreover if there exists a finite orthomodular projective plane it admits exactly one state. If L_1 and L_2 are two finite OML's, the following are equivalent: (1) L_1 and L_2 each have a full set of states, (2) $L_1 \times L_2$ has a full set of states, and (3) $L_1 \circ L_2$ (horizontal sum) admits a full set of states. (Received October 30, 1967.)

653-295. W. A. LAMPE, University of Manitoba, Winnipeg 19, Canada. Two remarks on unary algebras.

An algebra $\mathfrak{A} = \langle A;F \rangle$ is unary if F is any set of unary and nullary operations on A. Theorem 1. A semigroup $\mathfrak{S}$ is isomorphic to the endomorphism semigroup of a simple unary algebra iff $\mathfrak{S}$ is (i) the group of order p, p = 1 or p a prime, or (ii) the two element semilattice, or (iii) isomorphic to the semigroup of all mappings on a two element set. This solves Problem 1 of G. Gratzer, Math. Ann. 170 (1967), 334-338. Remark. Each algebra constructed in the proof has one operation. --For $D \subseteq A^2$ set $D^* = \{\langle x,y \rangle \mid \langle y,x \rangle \in D\}$ and set $\mathfrak{S}(\mathfrak{A}^2)$ for the subalgebra lattice of $\mathfrak{A}^2 (= \mathfrak{A} \times \mathfrak{A})$. It is known that a lattice $\mathfrak{L}$ is isomorphic to the subalgebra lattice of a unary algebra iff $\mathfrak{L}$ is a complete sublattice of a complete atomic Boolean lattice. Theorem 2. Let $\mathfrak{L}$ be a lattice satisfying the above condition, let α be an automorphism of $\mathfrak{L}$ of order two, and $\mathfrak{m}$ any cardinal. Then (A) there exists a unary algebra $\mathfrak{A}$ such that $\mathfrak{m} \leqq |A|$ and $\mathfrak{L} \cong \mathfrak{S}(\mathfrak{A}^2)$, (b) there exists a unary algebra $\mathfrak{A}$ and an isomorphism ρ from $\mathfrak{L}$ onto $\mathfrak{S}(\mathfrak{A}^2)$ satisfying $(x\alpha) = (x\rho)^*$ for all $x \in L$, and (C) there exist $\mathfrak{A}$ and ρ satisfying both (A) and (B). Further, (C) is equivalent to the Axiom of Choice for two element sets, and (A) is independent of it. It is not known if (B) requires this axiom. (Received November 1, 1967.)

653-296. L. -C. CHERN, University of Florida, Gainesville, Florida 32601. Homogeneous morphisms in additive categories.

Let α be a category with an abelian group structure on each of its morphism sets and let β be a subcategory of α such that β is an additive category and such that $f \circ (\phi + \psi) = f \circ \phi + f \circ \psi$ for all $f \in [A \cdot B]_\beta \cdot \phi \cdot \psi \in [U \cdot A]_\alpha$. Consider the category R_β of all additive functors $(A \cdot \rho_A)$ from a ring R to β. Definition. A morphism $f: A \longrightarrow B$ in α is called a p-homogeneous morphism from $(A \cdot \rho_A)$ to $(B \cdot \rho_B)$ if (i) $f \circ \rho_A(r) = [\rho_B(r)]^{p} \circ f$, (ii) $\sum_{n=0}^{p} (-1)^{p-n} \sum_{0 \leqq j_0 < \cdots < j_n \leqq p} f \circ (\sum_{t=0}^{n} \phi_{jt}) = 0$ for all $\phi_i \in [U \cdot A]_\alpha$ and $r \in R$. When A is a free Z-module generated by $\{x_1, \ldots, x_m\}$, a function $f: A \longrightarrow Z$ is a p-homogeneous morphism if and only if $f(\sum_{i=1}^{m} \xi_i x_i) = \phi(\xi_1, \xi_2, \ldots, \xi_m)$ is a homogeneous polynomial of degree p. Theorem. The class of all additive functors from R to β together with all p-homogeneous morphisms between them for all positive integers p form a category $e(\alpha \cdot \beta \cdot R)$ which contains R_β as a subcategory. Suppose every $(A \cdot \rho_A)$ is the domain of at least one p-homogeneous morphism w satisfying the condition that if f is any p-homogeneous morphisms from $(A \cdot \rho_A)$ then there exists a unique 1-homogeneous morphism h such that $h \circ w = f$. Then we can construct a covariant epifunctor $\Gamma^p: R_\beta \longrightarrow R_\beta$ which preserves direct limits under certain conditions. By passing through direct limits and products we prove that under certain conditions a p-homogeneous morphism of presheaves induces a p-homogeneous morphism of the associated sheaves. (Received October 30, 1967.)

653-297. CONSTANTIN CORDUNEANU, University of Rhode Island, Kingston, Rhode Island 02881. On a class of functional integral equations. Preliminary report.

Let us consider the equation (E) $x(t) = h(t) + \int_0^t k(t,s) f(s; x_s) ds$, $t \in R_+$, where $x, h, f \in R^n$, $k(t,s) = \{k_{ij}(t,s)\}_{n \times n}$ and x_t is the restriction of the function $x(t)$ to $[0,t]$, $t > 0$, $x_0 \in R^n$. Such equations appear, for example, in the feed back control theory. If we have a dynamical system described by the input-output relation $x(t) = h(t) + \int_0^t k(t,s) u(s) ds$ and the feed back is given by $u(t) = f(t; x_t)$, then $x(t)$ will be a solution of (E). Some global results (existence theorems, behavior of solutions) are established by the use of Fourier transform and the methods of functional analysis. (Received October 30, 1967.)

653-298. V. LAKSHMIKANTHAM and S. LEELA, University of Rhode Island, Kingston, Rhode Island 02881. A new differential inequality useful in control systems.

Let $g(t, \xi, \eta)$ be continuous for $0 \leqq t < \infty$, $\xi, \eta \geqq 0$ and nondecreasing in η for each (t, ξ). Suppose that r(t) is the maximal solution of the scalar differential equation $\xi' = g(t, \xi, \xi)$, $\xi(t_0) = \xi_0 \geqq 0$, existing on $[t_0, \infty)$. Let $m(t) \geqq 0$ be continuous on $[0, \infty)$ and satisfy the differential inequality $D_- m(t) \leqq g(t, m(t), \eta(t))$ where $\eta(t) \geqq 0$ is continuous on $[0, \infty)$. Then, $m(t_0) \leqq r(t_0)$ implies $m(t) \leqq r(t)$, $t \geqq t_0$, for all $\eta(t)$ satisfying $\eta(t) \leqq r(t)$, $t \geqq t_0$. On the basis of this new differential inequality, the behavior of solutions of a control system of the form $x' = f(t, x, u)$, where u is the control function, is studied. This approach gives a convenient and unified way of specifying admissible controls corresponding to the desired behavior of solutions of the control system. (Received October 30, 1967.)

653-299. A. B. PORITZ, University of Pennsylvania, Philadelphia, Pennsylvania 19104. An application of the Cauchy-Kowalewski theorem in Riemannian geometry.

The concept of parallelism for curves in a Riemannian manifold M generalizes to higher dimensions in such a way that minimal immersions are self parallel and so correspond to geodisics. In brief, let f be an (not necessarily isometric) immersion of a Riemannian manifold N in M. Consider those tangent bundle morphisms $G: T(N) \longrightarrow T(M)$ that cover f and map fibers isometrically. For such G, a second fundamental form II_G is defined. G is said to be parallel if the trace of II_G vanishes. A parallel unit vector field X along a curve $\gamma: (a,b) \longrightarrow M$ is seen to be the image, under a unique parallel G, of the unit vector field d/dt on (a,b). The differential of an isometric immersion is seen to be parallel if and only if the immersion is minimal. Corresponding to the theorem on the existence of a unique parallel vector field extending to a curve data given at a point, there is a theorem that states the existence of a unique parallel G that locally extends to a neighborhood of a submanifold of codimension one, data given on the submanifold and partial data given everywhere. In the real analytic case the proof makes use of the Cauchy-Kowalewski Theorem on partial differential equations. (Received October 30, 1967.)

653-300. T. J. BENAC, United Stated Naval Academy, Annapolis, Maryland, and C. M. MURPHY, Catholic University of America, Washington, D. C. 20017. Primal decomposition in noncommutative Noetherian systems. Preliminary report.

$(R, \subseteq, \cdot)$ is a lattice-ordered semigroup such that for every a,b,c in R (i) the left residual $a:_l b$ and right residual $a:_r b$ exist, (ii) $ab \leqq b$, (iii) $a:_r(b \vee c) = (a:_r b) \wedge (a:_r c)$, (iv) the left (right) residuals of a satisfy the ascending chain condition. $a:_l b$ is a proper left residual of a if $b \nleq a$. $a:_r b$ is a proper right residual of a if $a < a:_r b$. An element a in R is p-primal if p is the maximum proper left residual of a. A decomposition $a = a_1 \wedge \ldots \wedge a_k$ is right-reduced if no a_i is superfluous nor replaceable by a proper right residual of itself; it is primal if the a_i are p_i-primal with $p_i \nleq p_j$, $i \neq j$. Theorem 1. Every element of R has a right-reduced primal decomposition. Theorem 2. Let $a = q_1 \wedge \ldots \wedge q_n = q'_1 \wedge \ldots \wedge q'_m$ be two right-reduced primal decompositions with $q_i p_i$-primal, $q'_j p'_j$-primal. Then n = m and $\{p_1, \ldots, p_n\} = \{p'_1, \ldots, p'_m\}$ = the set of maximal proper left residuals of a. Let $p_1, \ldots, p_n$, $n \geqq 2$, be the maximal proper left residuals of a and $a(p_i)$ = maximum $\{a:_r b \mid b \nleq p_i\}$. Theorem 3. $a(p_i)$ is the minimum p_i-primal element of R and $a = a(p_1) \wedge \ldots \wedge a(p_n)$. (Received October 30, 1967.)

653-301. T. J. KYROUZ, University of Georgia, Athens, Georgia. On the homotopy theory of sphere bundles.

In trying to extend the results of James and Whitehead [Homotopy theory of sphere bundles over spheres. I, Proc. Lond. Math. Soc., 1954] to sphere bundles over CW complexes, one is led to consider the following question. Suppose one has S^q fiberings E^1, E^2 over $B = B_0 \cup_f e^n$ with E_0^1, E_0^2 the restrictions to B_0, and a fiber homotopy equivalence $h_0: E_0^2 \longrightarrow E_0^1$; under what geometric conditions can h_0 be extended to a fiber homotopy equivalence $h: E^2 \longrightarrow E^1$? Theorem E. Let $q > \dim B$. Then h_0 extends to a fiber-homotopy equivalence if and only if h_0 extends to a homotopy equivalence. This and more general results are obtained from a study of the attaching maps θ and ϕ in a representation $E_0 \cup_\theta e^n \cup_\phi e^{n+q}$ of the total space E. The key idea is that θ and ϕ can be expressed in terms of the action $\Omega B \times S^q \longrightarrow S^q$ determined by the fibering. We also obtain the following: Theorem D. Let E^1, E^2 be S^q fiberings over B, with sections, and let $\alpha : E^2 \longrightarrow E^1$ be a section preserving homotopy equivalence which is fiberwise over B_0. Then α can be replaced by a fiber-homotopy equivalence $h : E^2 \longrightarrow E^1$ with $h|E_0^2 = \alpha|E_0^2$. (Received October 30, 1967.)

653-302. R. B. LAKEIN, University of Maryland, College Park, Maryland 20742.
A generalization of the Gauss bound for a certain class of biquadratic number fields.

Let $F = Q((-m)^{1/2})$, where Q denotes the rational number field and m = 1,2,3,7, or 11. (These are the Euclidean imaginary quadratic fields.) Let $K = F((\mu)^{1/2})$, $\mu \in F$, be a quadratic extension of F with absolute discriminant $D = D_K$. Then the classical Gauss bound for the minimal norm of the integral ideals in an ideal class of a quadratic field can be generalized to the biquadratic field K as follows. Theorem. Any ideal class of K contains an integral ideal $\mathfrak{A}$ whose norm $N\mathfrak{A}$ satisfies the inequality $N\mathfrak{A} \leqq (1/8)\cdot D^{1/2}$; and the stronger inequality $N\mathfrak{A} \leqq (1/12)\cdot D^{1/2}$ holds in case $F = Q((-7)^{1/2})$. This bound improves the usual Minkowski bound for K by a factor of about .55 in case $F = Q((-7)^{1/2})$, and by a factor of about .82 in the other four cases. Furthermore, it is shown that the bound given here is the best possible bound for the class of fields K. (Received October 30, 1967.)

653-303. R. B. FRASER, Louisiana State University, Baton Rouge, Louisiana 70803. Banach spaces of Lipschitz functions with different metrics on the underlying space.

Let X be a space with metrics d_1 and d_2. Lip (X, d_i) is the Banach space of bounded real- or complex-valued functions satisfying a local Lipschitz condition, with norm $\sup\{|f(x)| : x \in X\} \vee \sup\{|f(x) - f(y)|/d_i(x,y) : x \neq y;\ x,y \in X\}$. Theorem 1. Id : Lip $(X, d_1) \longrightarrow$ Lip (X, d_2) is a continuous imbedding iff Id : $(X, d_2) \longrightarrow (X, d_1)$ satisfies a local Lipschitz condition. Theorem 2. If d_2 is $o(d_1)$, then Id : Lip $(X, d_2) \longrightarrow$ Lip (X, d_1) is a compact operator. Similar theorems are proved for Lip $(X, \beta \circ d)$, where β is a modulus of continuity (possibly not subadditive). (Received October 30, 1967.)

653-304. . ZEEN DITZIAN, University of Alberta, Edmonton, Alberta, Canada. On convolution transforms.

The subject of this research is the class of convolution transforms (1) $f(x) = \int_{-\infty}^{\infty} G(x - t)\phi(t)dt$, where (2) $G(t) = (1/2\pi i)\int_{-\infty}^{\infty} e^{-st}[\prod_{k=1}^{\infty}(1 - s/a_k)]^{-1}ds$. The class is defined by the restrictions on a_k as follows: (a) $a_k \neq 0$ and $\min_n |n\pi - \arg a_k| \leqq \psi < \pi/2$. (b) For some natural l and $q > 1$, $|a_{k+l}| \geqq q|a_k|$ for $k \geqq k_0$. The inversion formula $P_m(D)f(x) = \phi(x)$, where $P_m(s) = \prod_{k=1}(1 - s/a_k)$ and $D \equiv d/dx$, is valid a.e. if the transform (1) converges and $|\int_0^t \phi(t)dt| \leqq ke^{M|t|}$ for $M \leqq \min|\mathrm{Re}\, a_k|$. To prove the above $|G_m(t)| = |(1/2\pi i)\int_{-i\infty}^{i\infty}[e^{st}/\prod_{k=m+1}^{\infty}(1 - s/a_k)]ds| \leqq M_1|a_{m+1}|\exp((-1/2)\cos\psi|a_{m+1}t|) + M_2|a_{m+2}|\exp((-1/2)\cos\psi|a_{m+2}t|)$ and similar estimates are established.
(Received October 30, 1967.)

653-305. K. K. OBERAI, Queen's University, Kingston, Ontario, Canada. On spectral permanence.

Let D be a directed set and let $\{E_\alpha : \alpha \in D\}$ be a family of complex, separated, barreled and quasi-complete locally convex spaces, all being subspaces of a vector space E. For $\alpha \leqq B$, $\alpha, \beta \in D$ we assume that $E_\alpha \subset E_B$ and that the topology induced on E_α by E_β is weaker than the topology of E_α. Let $E = \cup_{\alpha \in D} E_\alpha$ with the inductive limit topology. Let T be a continuous linear operator on E which leaves each E_α invariant. Let $T_\alpha (\equiv T/E_\alpha)$ be a spectral operator on E_α. Then T is a spectral operator on E. Conversely, let E be the strict inductive limit of E_α and let T have compact spectrum. Let T be a spectral operator on E with $P(\cdot)$ as the corresponding spectral measure. Then, if T and $P(\cdot)$ leave each E_α invariant, for all $\alpha \in D$, T_α is a spectral operator on E_α. (Received October 30, 1967.)

653-306. OSWALD WYLER, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15217. The Stone-Cech compactification for limit spaces.

A limit space is a set equipped with a convergence relation from filters to points, subject to certain axioms. A limit space X is separated if a filter on X converges to at most one point, compact if every ultrafilter converges, regular if X is separated and the closure of every convergent filter converges. A subspace of a compact regular space is called completely regular. Modifying a compactification of A. L. Cochran (Thesis, University of Oklahoma, 1966), we prove: Theorem. For every limit space X, there is a continuous mapping $j : X \longrightarrow \beta X$, where βX is regular and compact, with the following property. For every map $f : X \longrightarrow Y$, with Y regular and compact, there is a unique map $g : \beta X \longrightarrow Y$ such that $f = gj$. Corollary. Every product space of regular compact limit spaces is regular compact. Problem. Find an internal characterization of completely regular spaces. (Received October 30, 1967.)

653-307. J. E. HUNEYCUTT, JR., University of North Carolina, Chapel Hill, North Carolina 27514. Regular additive set functions. Preliminary report.

Let M be a lattice or a half-ring of sets and μ a modular (when M is a lattice) or additive (when M is a half-ring) function on M with values in a topological group G. The problem of extending μ to a countable additive function on the smallest σ-ring containing M is considered and sufficient conditions for this extension are discussed. Countable additivity on the ring generated by M is attained by placing certain regularity conditions on μ. These conditions reduce to ordinary regularity in the case μ is a nonnegative, real-valued function and the theorems of Von Neumann (when M is a half-ring), and of Pettis (when M is a lattice) are shown to be special cases of the results. When μ takes values in the complex numbers or a locally convex linear topological space, the theorems of Langlands or of Dinculeanu and Kluvanek are derived. Application is made to topological group valued functions on partially ordered sets combining works of Revuz and of Radu concerning generalized Stieltjes measures. (Received October 30, 1967.)

653-308. GILBERT WALTER, University of Wisconsin-Milwaukee, Milwaukee, Wisconsin 53201. Hermite series singularities.

The singular points of the function to which a Hermite series converges as well as the behavior of the function at infinity may be investigated by methods similar to those used by Nehari for Legendre series. One may deduce (1) that for series of Hermite polynomials $\sum(a_n H_n)/n!$ which converge to entire functions, the behavior at infinity depends on the location of singular points of $\sum a_n t^n$, (2) that the location of singular points of $\sum b_n H_n$ is reflected in the behavior of $\sum b_n t^n$ at infinity, (3) that for series of Hermite functions $\sum c_n h_{2n}(0) h_{2n}(x^{1/2})$ which converge to entire functions the behavior at infinity is reflected in the location of singular points of $\sum c_n t^n$. One may also obtain results about the singular points of the analytic representations $\sum d_n \widetilde{h}_n(z)$ of a distribution given by $\sum d_n h_n$ similar to results of the author for Legendre series. Here $\widetilde{h}_n(z)$ denotes a Hermite function of the record kind vanishing at $i\infty$ for $\operatorname{Im} z > 0$ and at $-i\infty$ for $\operatorname{Im} z < 0$. (Received October 30, 1967.

653-309. P. A. LOEB, University of California, Los Angeles, California 90024. A criterion for the proportionality of potentials with polar point support.

Let W be a locally compact Hausdorff space which is connected and locally connected but not compact, and let $\mathscr{H}$ be a harmonic class of functions on W in the sense of Brelot (Lectures on Potential Theory, Tata Institute of Fundamental Research, Bombay, 1960). Let Ω be a regular relatively compact region in W, and assume that the function 1 is $\mathscr{H}$-harmonic in Ω. Let x_0 be a polar point in Ω, and let P_1 and P_2 be two positive $\mathscr{H}$-harmonic functions in $\Omega - \{x_0\}$ such that for $i = 1$ and 2, $\limsup_{x \to x_0} P_i(x) = +\infty$ and $\lim_{x \to y} P_i(x) = 0$ at all $y \in \partial\Omega$. Then a well known, unsolved problem in the axiomatic potential theory of Brelot can be reduced to the question "Is P_1 proportional to P_2?". A necessary and sufficient condition for this proportionality is given. (Received October 31, 1967.)

653-310. WITHDRAWN.

653-311. JAY LEAVITT and KRZYSZTOF FRANKOWSKI, University of Minnesota, Minneapolis, Minnesota 55455. Remarks on mixed Taylor-L_2 approximations.

Given a function and values of its first n derivatives about the origin, we approximate this function in L_2 space in terms of 1, x, x,..., x^{n+m} over the interval $[-1,1]$ sh that the approximation P_{n+m} agrees with the first n terms of the Taylor Series. This problem gives rise to finite classes of orthogonal polynomials. Their properties are investigated and numerical examples are given. Functions defined by first order differential equations are also studied and a modification of Galerkin's scheme is proposed. These approximations combine the ease of obtaining the first few coefficients of the Taylor series with good convergence of L_2 expansions. The class of orthogonal polynomials are related to the hypergeometric function. (Received October 30, 1967.)

653-312. H. W. VAYO, University of Toledo, Toledo, Ohio 43606. Green's function for a spherical cell.

Electrical potential problems occurring in the electro-physiology of nerve and muscle differ from those usually considered in electrical theory for two reasons: first, the relation between current flow and polarization across a membrane of tissues is nonlinear, and second, because the interior of the tissue is not strictly equipotential. We suppose that u_e and u_i are the steady state potentials outside and inside a tissue covered by a closed polarized membrane S. Then u_e and u_i are both harmonic functions which satisfy the nonlinear boundary condition at S: $k_e \partial u_e/\partial n_e = -k_i \partial u_i/\partial n_i = F(u_e - u_i)$, where k_e and k_i are the conductivities of the external and internal media, $F(u_e - u_i)$ is some function of the membrane potential difference which determines the current flow across the membrane, and n_e and n_i are the outwardly and inwardly drawn normals on S. We introduce two harmonic functions H_e and H_i, defined in the exterior and interior respectively to find the Green's Functions G_e^e and G_i^e. (Received October 30, 1967.)

653-313. G. O. PETERS, General Electric Company, 6806 North 11th Street, Philadelphia, Pennsylvania. Boole series representing functions and Boole functions of negative degree.

Charles Jordan defined Boole polynomials of order one. Niels Norlund defined Bernoulli and Euler polynomials of higher order. The author (Abstract 62-1-24, Bull. Amer. Math. Soc. 62 (1956), 7) defined the Boole polynomials of higher and negative order, $\zeta_\nu^{(n)}(x)$ and $\zeta_\nu^{(-n)}(x)$. A polynomial f(x + u) can be expanded into a series of Boole polynomials (1) $f(x+u) = \sum_{i=0}^{\nu} \zeta_i^{(n)}(x)(1/i!)\,\Delta^i\nabla^n f(u)$ or with, - n. Some similar results are found for a function f(x + u), expandable into a Newton's series, ν becomes ∞. The Boole functions of negative degree, $-\nu$, $\zeta_{-\nu}^{(n)}(x)$ and $\zeta_{-\nu}^{(-n)}(x)$, are defined by the relations (2) $\zeta_{-\nu}^{(n)}(x) = \nabla^{-n} x^{(-\nu)}$ and $\zeta_{-\nu}^{(-n)}(x) = \nabla^{n} x^{(-\nu)}$ for $\nu > 0$, $x > -1$. Then (3) $\zeta_{-\nu}^{(-n)}(x+u) = \sum_{i=0}^{\infty} \binom{-\nu}{i} \zeta_i^{(-n)}(x) u^{(-\nu-i)}$, $x + u > \nu - 2$ and (4) $\zeta_{-\nu}^{(-n)}(u) = ((\zeta^{(-n)}/2) + u)^{(-\nu)}$, $u > \nu - 2$ and if $\theta(x)$ can be expanded as $\sum_{i=0}^{\infty} a_i x^{(i)}$, $\theta(\zeta^{(-n)}(x))$ is a solution of $\nabla^n f(x) = \theta(x)$. (Received October 31, 1967.)

653-314. MICHAEL AISSEN, ARL-ARM, Bldg. 450, Wright-Patterson AFB, Ohio 45433. Means and minimization of errors.

Let ϕ be a mean defined on $[a,b] \times [a,b]$, where $0 < a < b$. That is, $\phi(p,x)$ is between p and x when p and x are between a and b. We define the 'relative' error $E(p,x) = |p - x|/\phi(p,x)$. Let $\lambda(p) = \max_x E(p,x)$, and let $\lambda(\mu) = \min_p \lambda(p)$. If $\phi(t,u)$ and $\phi(u,t)$ are monotone (as functions of u) in each of $[a,t]$ and $[t,b]$ and if ϕ is continuous on the boundary of $[a,b] \times [a,b]$, $\lambda(p)$ will exist and μ will exist and be unique. In addition ϕ^* defined by $\phi^*(x,y) = \phi(y,x)$, will satisfy all the conditions imposed on ϕ. Let μ^* be defined from ϕ^* as μ was from ϕ. Result. If in addition to the restrictions on ϕ mentioned above, $\phi(kx,ky) = k\phi(x,y)$ for $k > 0$ then $\mu\mu^* = ab$. (Received October 31, 1967.)

653-315. JOSEPH BUCKLEY, University of Massachusetts, Amherst, Massachusetts 01003. Polynomial functions and wreath products.

Let B,A be groups with A abelian and written additively. Z[B] is the integral group ring of B and I the ideal generated by all b - 1, b in B. The functions from B to A form a B-module in the usual way. Let f denote both a function $B \longrightarrow A$ and its extension to a homomorphism $Z[B] \longrightarrow A$. Definition. $f: B \longrightarrow A$ is a polynomial function of degree $\leqq r$ if for each $b_1,\ldots,b_k$ in B there exist numerical polynomials $p_1,\ldots,p_t$ in k variables all of degree $\leqq r$ and $a_1,\ldots,a_t$ in A such that $f(b_1^{m_1}\ldots b_k^{m_k}) = \sum_1^t p_i(m_1,\ldots,m_k)a_i$ for all integers $m_1,\ldots,m_k$. Theorem 1. The following three conditions are equivalent. (i) f is a polynomial function of degree $\leqq r$. (ii) $I^{r+1} \subset \ker f$. (iii) I^{r+1} annihilates f. Theorem 2. If $f: B \longrightarrow A$ is a polynomial function of degree $\leqq r$ and $o(b) = n$, then $o(f(b))$ divides n^r. These results are applied to give a new proof of: Theorem (Baumslag). Let A,B be arbitrary groups. Then the wreath product AwrB is nilpotent if and only if B is a finite p-group and A is a nilpotent p-group of finite exponent. Finally the nilpotency of AwrB is discussed when A is abelian and shown to depend only on B and the exponent of A. (Received October 31, 1967.)

653 316. T. K. SEO, University of Kentucky, Lexington, Kentucky 40506. A note on solvable factorizable groups.

In this paper we are going to prove the following theorems. Theorem 1. If $G = AB$ is a finite group, A is cyclic, $[B:H] = p$, an odd prime p, and every subgroup of H is normal in B, then G is solvable. This theorem generalizes a theorem of Scott in Solvable factorizable groups, Ill. J. Math. 1 (1957), 389-394, and also Scott's Theorem 13.10.3, p. 415 in his book Group theory, Prentice Hall, New Jersey, 1964. As a generalization of the above theorem we get Theorem 2. If $G = AB$ is a finite group, A is abelian with cyclic Sylow 2-subgroup (possibly the identity subgroup). $[B:H] = p$, p is any prime, and every subgroup of H is normal in B, then G is solvable. (Received October 30, 1967.)

653-317. JACK NEBB, University of Georgia, Athens, Georgia 30601. A linear space. Preliminary report.

Denote $L = \{f | f$ is integrable on bounded intervals$\}$. Let $1 \leqq p < \infty$, $h \neq 0$, and denote $F_h^p = \{f | f \in L$ and $\int_{-\infty}^{\infty} |(1/h)\int_x^{x+h} f(t)dt|^p dx < \infty\}$. Define a function S on F_n^p by $S(f) = (\int_{-\infty}^{\infty} |(1/h)\int_x^{x+h} f(t)dt|^p dx)^{1/p}$. Define $D = \{(f,g) | f, g \in F_h^p$ and $(f - g)(x + h) = (f - g)(x)$ a.e.$\}$; $E = \{(f,g) | f,g \in F_h^p$ and $S(f - g) = 0\}$; $f^* = \{g | (g,f) \in E\}$; and $G_h^p = \{f^* | f \in F_h^p\}$. Define a function N on G_h^p by $N(f^*) = S(f)$. Theorem. (G_h^p, N) is a normed linear space. Also, D = E. Hence, the zero of G_h^p if $\{f | f \in F_h^p$ and $f(x + h) = f(x)$ a.e.$\}$. Denote $W(f^*) = f^* \cap L^p(-\infty, \infty)$. Either $W(f^*) = \emptyset$ or $W(f^*)$ is a singleton. Denote $K_h^p = \{f^* | W(f^*) \neq \emptyset\}$. Theorem. K_h^p is a proper subspace of G_h^p and K_h^p is not complete. (See Abstract 645-18, these *Notices* 14 (1967), 385.) (Received October 30, 1967.)

653-318. J. R. McLAUGHLIN, Pennsylvania State University, University Park, Pennsylvania 16802. Haar series.

Let $\{\chi_n^{(k)}(t)\}$ be Haar's orthonormal system [see Alexits, Conv. Prob. of orthog. series, p. 46]. We set $\chi_0(t) = \chi_0^{(0)}(t)$, $\chi_m(t) = \chi_m^{(k)}(t)$ for $m = 2^n + k$ $(0 \leqq k \leqq 2^n - 1,\ n = 0,1,\ldots)$. For each function $f(t) \in L(0,1)$ we set $a_m(f) = \int_0^1 f(t)\chi_m(t)dt$, $m = 0,1,\ldots$. Theorem 1. For every dyadic irrational number t_0 there exists an absolutely continuous function f such that $\sum |a_m(f)\chi_m(t_0)| = \infty$. Remark. Wang

Sei-Lei proved Theorem 1 for the special case $t_0 = 2/3$. Theorem 2. For every $t_0 \in [0,1]$, there exists a lacunary Haar series $\sum a_m \chi_m(t)$ which converges to zero for $t \neq t_0$ and diverges for $t = t_0$. Corollary. A nonempty set is a set of multiplicity for Haar series. Remark. G. Faber had previously shown that $\{1/2\}$ was a set of multiplicity for Haar series. Theorem 3. (i) If $0 < p \leqq 1$ and $\sum |a_m|^p m^{p/2} < \infty$, then $f(t) = \sum a_m \chi_m(t)$ is of bounded pth variation; (ii) If $\{\epsilon_m\}$ is a null sequence, then there exists a sequence $\{a_m\}$ such that $\sum |a_m m^{1/2} \epsilon_m| < \infty$ and $f(t) = \sum a_m \chi_m(t)$ is unbounded; (iii) For every $\epsilon > 0$ there exists a sequence $\{a_m\}$ such that $\sum |a_m| m^{1/2} (\log \log m)^{-1-\epsilon} < \infty$ and $f(t) = \sum a_m \chi_m(t)$ is differentiable almost nowhere. (Received October 30, 1967.)

653-319. R. A. HORN, University of Santa Clara, Santa Clara, California 95053. The theory of infinitely divisible matrices and kernels.

A matrix or integral kernel is said to be infinitely divisible if it has the property that all its (pointwise) positive fractional powers generate positive definite or semidefinite quadratic forms. Such matrices and kernels are characterized and it is shown that the location of their zeroes is restricted by conditions which in the matrical case are most easily stated in the language of graph theory. Methods for dealing with infinitely divisible matrices and kernels are developed and applications are made to probability theory, moment sequences, Green's functions, reproducing kernels, stochastic processes, schlicht analytic continuation, and conformal mapping. (Received October 30, 1967.)

653-320. D. B. COLEMAN, University of Kentucky, Lexington, Kentucky 40505, and D. S. PASSMAN, Yale University, New Haven, Connecticut. The unit group of a modular group algebra.

Let p be a prime, G a finite p-group, A the group algebra of G over a finite field of characteristic p, and U the set of elements $\sum a_i g_i$ in A such that $\sum a_i = 1$. Then U, the normalized group of units of A, is a p-group containing G. Theorem. If G is nonabelian, then U is irregular. (Received October 30, 1967.)

653-321. FREDERICK HOFFMAN, Drexel Institute of Technology, 32nd and Chestnut Streets, Philadelphia, Pennsylvania 19104. Affinely variant sets in vector spaces over GF(2) .

In a previous paper, Totally variant sets in finite groups and vector spaces (to appear), Lloyd R. Welch and the author answered the question of what finite groups and vector spaces contain totally variant subsets (i.e., subsets moved by every nonidentity automorphism, or in the vector space case, by every nonidentity nonsingular linear transformation). Here, the question of which finite dimensional spaces over GF(2) contain subsets moved by every nonidentity affine transformation is answered, although not completely. We prove that every vector space over GF(2) of dimension at least six contains an affinely totally variant subset. Combinatorial arguments are used to rule out the existence of such sets in dimensions four and below, and there is strong evidence against existence in dimension five. An affinely totally variant set is given in dimension six and a lemma gives an inductive means for construction of the sets in higher dimensions. (Received October 30, 1967.)

653-322. J. E. DENNIS, JR., University of Utah, Salt Lake City, Utah 84112, and K. M. BROWN, Cornell University, Ithaca, New York 14850. On the convergence of a general class of iterative methods.

Let F be defined on Ω, $\Omega \subset E^N$ and $F: \Omega \longrightarrow E^N$. Let Ω_0 be the closure of an open convex set in E^N with $\Omega_0 \subset \Omega$ and let Ω_0 be bounded. $\|\cdot\|$ will be l_2. Consider the class of iterative methods $x_{n+1} = x_n - G^{-1}(x_n)\cdot F(x_n)$, with $F = (f_1,\ldots,f_N)$ as above and $G(x_n)$ an $N \times N$ matrix which satisfies certain conditions (to be given below). Theorem. Let $\lambda(x)$ be defined as the positive square root of the smallest eigenvalue of $G(x)G(x)^*$, and let $\mu(x)$ be defined as the positive square root of the largest eigenvalue of $[J(x) - G(x)]\,[J(x) - G(x)]^*$, where $J(x)$ denotes the Jacobian of F at x. If (i) $F \in C^2(\Omega_0)$ and for every $x \in \Omega_0$, $\|F''(x)\| \leqq K$; (ii) for some $x_0 \in \Omega_0$, $\|F(x_0)\| \leqq \eta$; (iii) $\lambda(x) > 0$ for every $x \in \Omega_0$; (iv) $\lambda(x)/\mu(x) < 1$ for every $x \in \Omega_0$; then $G^{-1}(x) \equiv (G(x))^{-1}$ exists for every $x \in \Omega_0$, and $\|G^{-1}(x)\| \leqq 1/(\min_{x\in\Omega_0}\lambda(x)) \equiv B < \infty$. Furthermore, $\|I - J(x)G^{-1}(x)\| \leqq \max_{x\in\Omega_0}\mu(x)/\lambda(x) \equiv \delta < 1$. Corollary. If, in addition to (i) - (iv) of the previous theorem, it is true that (v) $h \equiv (B^2K\eta)/(1-\delta) < 2$ and $N(x_0,r) \subset \Omega_0$, where $r = (B\eta)/(1-\alpha)$, and $\alpha = \delta + (1-\delta)(h/2)$; then the iteration $x_{n+1} = x_n - G^{-1}(x_n)\cdot F(x_n)$ is defined for every n and converges to a point $\sigma \in \Omega_0$, with $F(\sigma) = 0$, at a rate given by $\|x_n - \sigma\| \leqq r\alpha^n$. (Received October 30, 1967.)

653-323. B. M. SCHREIBER, University of Washington, Seattle, Washington 98105. Measures with bounded convolution powers. Preliminary report.

Let G be a LCA group with character group Γ, and let M(G) denote the convolution measure algebra of G. Let $\mathscr{B} = \{\mu \in M(G): \sup_{n\geqq 1}\|\mu^n\| < \infty\}$, where μ^n is the nth convolution power of μ and $\|\cdot\|$ is the total variation norm. For $\mu \in \mathscr{B}$, it is clear that $\|\mu\|_\Gamma = \sup_{\gamma\in\Gamma}|\hat{\mu}(\gamma)| \leqq 1$, where $\hat{\mu}$ is the Fourier-Stieltjes transform of μ. The set $E_\mu = \{\gamma \in \Gamma: |\hat{\mu}(\gamma)| = 1\}$ is described, as is the function $\hat{\mu}|E_\mu$. These descriptions follow from results of Paul J. Cohen (Amer. J. Math. 82 (1960), 191-212, 213-226) and the following lemma. For an abelian group G, let $\mathscr{R}(G)$ denote the smallest Boolean algebra of subsets of G containing all cosets of subgroups of G. Lemma. If $h: G \longrightarrow H$ is a homomorphism and $A \in \mathscr{R}(G)$, then $h(A) \in \mathscr{R}(H)$. Further properties of $\mathscr{B}$ are investigated. For example, the following theorems are proved. $L^1(G)$ is considered as a subalgebra of M(G). Theorem 1. G has the property that every $f \in L^1(G)$ with $\|\hat{f}\|_\Gamma \leqq 1$ is in $\mathscr{B}$ iff G is compact. Theorem 2. If $f,g \in L^1(G)$, $E_f = E_g$ and $\hat{f} = \hat{g}$ on a neighborhood of E_f in Γ, then $f \in \mathscr{B}$ iff $g \in \mathscr{B}$. (Received October 30, 1967.)

653-324. C. B. MILLHAM, Washington State University, Pullman, Washington 99163. On the structure of equilibrium points of bimatrix games.

An $m \times n$ bimatrix game (A,B) has the $\alpha\beta$-interchangeability property if, given two equilibrium points (x,y) and $(\bar{x},\bar{y})$ with $xAy^T = \bar{x}A\bar{y}^T = \alpha$, $xBy^T = \bar{x}B\bar{y}^T = \beta$, it is then true that $(x,\bar{y})$ and $(\bar{x},y)$ are also equilibrium points and $xA\bar{y}^T = \bar{x}Ay^T = \alpha$, $xB\bar{y}^T = \bar{x}By^T = \beta$. Let $X_{\alpha\beta} = \{x|x \in R^m$ is a probability vector, (x,y) is an equilibrium point for some probability vector $y \in R^n$, $xAy^T = \alpha$, $xBy^T = \beta\}$, let $Y_{\alpha\beta} = \{y|y \in R^n$ is a probability vector, (x,y) is an equilibrium point for some probability vector $x \in R^m$, $xAy^T = \alpha$, $xBy^T = \beta\}$. A pure strategy $u_i \in R^m$ ($v_j \in R^n$) is essential for $X_{\alpha\beta}$ ($Y_{\alpha\beta}$) if $x_i > 0$ ($y_j > 0$) for some $x \in X_{\alpha\beta}$ ($y \in Y_{\alpha\beta}$). $A_{\alpha\beta}$, $B_{\alpha\beta}$ are the essential submatrices for $X_{\alpha\beta}$ and $Y_{\alpha\beta}$ if $A_{\alpha\beta} = (a_{ij})$, $B_{\alpha\beta} = (b_{ij})$ where u_i and v_j are essential for $X_{\alpha\beta}$ and $Y_{\alpha\beta}$ respectively. Let $\bar{m}(\bar{n})$ be the number of essential $u_i(v_j)$, let $\overline{X}_{\alpha\beta}(\overline{Y}_{\alpha\beta})$ be the linear space spanned by $X_{\alpha\beta}(Y_{\alpha\beta})$. Theorem. $\bar{n}$ - dim $\overline{Y}_{\alpha\beta} = \overline{m}$ - dim $\overline{X}_{\alpha\beta}$ + rank $A_{\alpha\beta}$ - rank $B_{\alpha\beta}$. Theorem. Let $e_{\alpha\beta}(f_{\alpha\beta})$ be the number of interior bounding faces of $X_{\alpha\beta}(Y_{\alpha\beta})$. Then $e_{\alpha\beta} \leqq n - \bar{n}(f_{\alpha\beta} \leqq m - \overline{m})$. The results generalize some well known theorems in 0-sum matrix games due originally to Bohnenblust, Karlin, and Shapley (Contr. to the theory of games, Vol. 1, pp. 51-72, 1950) and Gale and Sherman (Contr. to the theory of games, Vol. 1, pp. 31-71, 1950). (Received October 30, 1967.)

653-325. MORRIS WEISFELD, Duke University, Durham, North Carolina 27706. Tensor products of F-vector spaces.

Kuranishi (Nagoya Math. J. 15 (1959), 225-260) has defined the concept of an F-vector space. Each F-vector space determines a pair of numbers (m,p) called the characteristic. The characteristic constitutes a generalization to this class of vector spaces, which are generally infinite dimensional, of the notion of dimension for finite dimensional vector spaces. The latter correspond to F-vector spaces of characteristic (m,0) with m equaling the dimension. F-vector spaces are isomorphic if and only if they have the same characteristic. Except for a very special case, the construction of the space of formal curves, Kuranishi left open the question of forming tensor products of F-vector spaces. The paper demonstrates that the tensor product of two F-vector spaces having characteristics (m,p) and (m',p') respectively exists and is an F-vector space of characteristic (mm', p + p'). It is immediately seen that this generalizes the well-known result for finite dimensional vector spaces. (Received October 30, 1967.)

653-326. E. H. ROGERS, Rensselaer Polytechnic Institute, Troy, New York 12181. Variational properties of nonlinear spectra.

A linear map L_x of an inner product space W to itself depends on the real parameter x. A value k of x is in the "nonlinear spectrum" of L_x if L_k fails to have a continuous inverse on its range. A functional q is defined implicitly by $(L_{q(u)}u,u) = 0$. If this functional has a real branch p defined on all of W such that $(L'_{p(u)}u,u) > 0$ for all u, then simple continuity conditions yield a minimax theory for the nonlinear spectrum in the range of p. This leads to a separation theorem for eigenvalues of L_x constrained to a subspace of W. When I - L_x is compact, the spectrum in the range of p is shown to be a point spectrum whose members have finite multiplicity. This work generalizes a part of classical linear spectral theory from the point of view of the topological theory of critical points. (Received October 31, 1967.)

653-327. J. L. GROSS, Dartmouth College, Hanover, New Hampshire 03755. A unique decomposition theorem for 3-manifolds with connected boundary.

Let $\mathscr{L}$ be the class of connected, compact, oriented triangulated 3-manifolds with connected, nonvacuous boundary. If M and M' are 3-manifolds in $\mathscr{L}$, then one forms the disk sum $M \triangle M'$ by pasting a 2-cell on bd(M) to a 2-cell on bd(M'). This operation is well defined, associative, and commutative up to isomorphism. The 3-cell serves as an identity element. A 3-manifold P in $\mathscr{L}$ is $\triangle$-prime if P is not a 3-cell and whenever P is isomorphic to a disk sum $M_1 \triangle M_2$, either M_1 or M_2 is a 3-cell. The following theorem answers a question of Milnor [Amer. J. Math. 84 (1962), 6] in the affirmative. Theorem. Let M be a 3-manifold in $\mathscr{L}$ which is not a 3-cell. Then M is isomorphic to a sum $P_1 \triangle \ldots \triangle P_n$ of $\triangle$-prime 3-manifolds. The summands P_i are uniquely determined up to order and isomorphism. (Received October 31, 1967.)

653-328. JEROME EISENFELD, Rensselaer Polytechnic Institute, Troy, New York. Quadratic eigenvalue problems.

The simplest of the eigenvalue problems in which the parameter appears nonlinearly is the quadratic eigenvalue problem: $Ax = \lambda^2 Bx + \lambda Cx$. One seeks completeness theorems and numerical procedures for calculating the eigenvalues. Specializing to the types of problems which arise in the theory of hydrodynamic stability, we show that these have the equivalent linear form: $Kw = \kappa w$, where K has the desirable properties of being compact, symmetric and one to one. (Received October 31, 1967.)

653-329. JOHN HORVÁTH, University of Maryland, College Park, Maryland 20742. The analytic continuation of vector-valued holomorphic functions.

Let Λ and Λ_1 be two domains in the complex plane, $\Lambda \subset \Lambda_1$. Theorem 1. Let F be a quasi-complete, barelled, locally convex Hausdorff space and $E = F'$ its dual equipped with the strong topology. Let $f: \Lambda \to E$ be a bolomorphic function and assume that for every $y \in F$ the numerical holomorphic function $\lambda \mapsto < f(\lambda), y >$ can be extended to a holomorphic function in Λ_1. Then there exists a holomorphic function $f_1 : \Lambda_1 \to E$ such that $f_1(\lambda) = f(\lambda)$ for $\lambda \in \Lambda$ and $\langle f_1(\lambda), y\rangle = \langle f(\lambda), y\rangle$ for $\lambda \in \Lambda_1$ and $y \in F$. This generalizes a result due to Gelfand and Šilov (Generalized functions, vol. I, Chapter I, appendix 2, no. 3). The conditions on E are satisfied if E is reflexive. Theorem 2. (a) A function $\lambda \mapsto x(\lambda) = (\xi_n(\lambda))$ defined in Λ, with values in the Banach space c_0 of sequences converging to 0, is holomorphic iff (1) for every n the numerical function $\xi_n(\lambda)$ is holomorphic, (2) the sequence $(\xi_n(\lambda))$ converges to 0 uniformly on every compact subset of Λ. (b) Let $\lambda \mapsto x(\lambda) = (\xi_n(\lambda))$ be a holomorphic function $\Lambda \to c_0$ and assume that for every sequence $y = (\eta_n) \in l^1$ the holomorphic function $\lambda \mapsto \sum_n \eta_n \xi_n(\lambda)$ can be extended to a holomorphic function in Λ_1. Then there exists a holomorphic function $\lambda \mapsto \bar{x}(\lambda) = (\bar{\xi}_n(\lambda))$ defined in Λ_1 with values in c_0 such that $\bar{x}(\lambda) = x(\lambda)$ for $\lambda \in \Lambda$ and $\langle x(\lambda), y\rangle = \sum_n \eta_n \xi_n(\lambda)$ for $y \in l^1$ and $\lambda \in \Lambda_1$. (Received October 31, 1967.)

653-330. UBIRATAN D'AMBROSIO, University of Rhode Island, Kingston, Rhode Island 02881. On hypersurfaces of limit type.

Let $U = [0,1]$, N be an integer and α a real number. Consider mappings from U to R^n of the following type $(T,U;\ N,\alpha): w \in U \to \sum_{k=1}^{N} k^{-\alpha} \cdot p_k(w)$, where $p_k(w)$, all k, have components $p_k^i(w) = \prod_{j=1}^{r} x_{j,k}^i(u_j)$, all the x's are real valued continuous functions on $[0,1]$, and $w = (u_1, u_2, \ldots, u_r)$. If $J_{(N,\alpha)}(w)$ denotes the jacobian matrix of the transformation $(T,U;\ N,\alpha)$ at w, and $j_{(N,\alpha)}^{1\ 1_1 2 \ldots 1_r}(w)$ its determinants, we have the following Proposition. Let $x_{j,k}^i(u_j)$, $i = 1,2,\ldots,n$, $j = 1,2,\ldots,r$, all k, be functions of equally bounded total variation on $[0,1]$, and the curves $(x_{j,k}^1(u_j),\ldots,x_{j,k}^n(u_j))$, $j = 1,2,\ldots, r$, all k, have equally bounded lengths. Then, for any N, $J_{(N,\alpha)}(w)$ is componentwise summable in U, provided $\alpha > 1$, and in this case $|\int_U j_{(N,\alpha)}^{1\ 1_1 2 \ldots 1_r}(w)\, dw|$ are equally bounded. Now consider mappings $(T,U;\alpha)$ of the type $(T,U;\ \alpha) = \lim_{N\to\infty}(T,U;\ N,\alpha)$, and let $J_\alpha(w)$ be the jacobian of $(T,U;\ \alpha)$ at w.

Under the hypotheses of last Proposition, we have the following Theorem. If $\alpha > \delta + 1$, for a given $\delta > 1$, $J_{\alpha}(w) = \lim_{N\to\infty} J_{(N,\alpha)}(w)$ almost everywhere in U, and the hypersurface defined by $(T,U;\alpha)$ is rectifiable. (Received October 31, 1967.)

653-331. V. LAKSHMIKANTHAM and C. P. TSOKOS, University of Rhode Island, Kingston, Rhode Island 02881. Application of the topological principle of Wazewski to control systems.

Consider the control system of the form (*) $x' = F(t,x,u)$, where u is a control function and F is a mapping from $[0,\infty) \times R^n \times R^m \to R^n$. Let a continuous function $\psi(t) \geqq 0$ be given which is defined on $[0,\infty)$. Designate by $E = [u \in R^m : \|u\| = \psi(t),\ t \geqq 0]$. Using the topological principle of Wazewski, it is shown that for any control $u = u(t) \in E$ there exists a solution $x(t,u)$ of the control system (*) such that $\text{limit}_{t\to\infty} x(t,u) = 0$. (Received October 31, 1967.)

653-332. E. D. DAVIS, Purdue University, Lafayette, Indiana. Analytic independence in Noetherian rings.

A subset $\{x_1,\ldots,x_n\}$ of a commutative ring R is said to be analytically independent provided that $(x_1,\ldots,x_n)R$ is not the unit ideal, and for every f in the polynomial ring $R[X_1,\ldots,X_n]$ such that $f(x_1,\ldots,x_n) = 0$, the coefficients of f lie in the radical of $(x_1,\ldots,x_n)R$. In a previous paper [Pacific J. Math. 20 (1967), 197-205] we found a new proof of the theorem of "analytic independence of systems of parameters" by examining the homomorphism $R[X_1,\ldots,X_n] \to R[x_1/x,\ldots,x_n/x]$, where x is an appropriate regular element of R. The methods of that paper can be used to prove the converse result. Hence: Theorem. An ideal of a Noetherian ring is of the principal class if, and only if, it is generated by an analytically independent set; and furthermore, an ideal of the principal class is of height equal to the cardinality of any analytically independent set of generators. Of special interest is: Corollary. A set of dim(R) elements of a local ring R is a system of parameters of R if, and only if, the set is analytically independent. These results can be proved in a less elementary way by an application of Cohen's structure theory for complete local rings. (Received October 31, 1967.)

653-333. E. A. RUTTER, JR., University of Kansas, Lawrence, Kansas 66044. Some characterizations of quasi-Frobenius rings.

Let R be a ring and M an R-module. $M^* = \mathrm{Hom}_R(M,R)$ is a module of the "opposite hand" from M. There is a "natural" homomorphism $\sigma_M : M \to M^{**}$ defined by $\sigma_M(x)(f) = f(x)$ for all $x \in M$ and $f \in M^*$. The module M is torsionless (reflexive) if σ_M is a monomorphism (isomorphism). Theorem. Let R be a ring with minimum condition on left ideals. Then the following are equivalent: (i) R is quasi-Frobenius. (ii) Every finitely generated left R-module is torsionless. (iii) Every finitely generated left R-module is a submodule of a free R-module. (iv) Every cyclic left R-module is reflexive. (Received November 1, 1967.)

653-334. MARK LOTKIN, General Electric Company, Missile and Space Division, 115 Hedgerow Drive, Cherry Hill, New Jersey. Differentiation of functions of observational data.

In this paper there is discussed a method for the differentiation to any desired order of data obtained by measurements, and thus containing certain measurement errors. Minimum variance principles are employed for the determination of optimum data filtering span times, as a function of order of differentiation, degree of fitting polynomial, truncation and measurement errors. The relationships for optimum span times are illustrated by means of a specific case, applied to a set of prescribed test data. (Received November 2, 1967.)

653-335. J. E. HUMPHREYS, University of Oregon, Eugene, Oregon. A fusion theorem for semisimple groups. Preliminary report.

Let G be a semisimple algebraic linear group defined over an arbitrary field. Let $B = T \cdot U$ be a Borel group of G, with T a maximal torus and U a maximal connected unipotent subgroup of G. Theorem. If x and y are elements of U which are conjugate in G, then there exist elements $x = u_1, u_2, \ldots, u_m = y$ in U and closed T-invariant subgroups $U_1, \ldots, U_{m-1}$ of U satisfying: u_i and u_{i+1} lie in U_i and are conjugate in the normalizer $N(U_i)$, $1 \leqq i \leqq m - 1$. The proof involves the Bruhat decomposition of G, a lemma about the Weyl group, and results of Steinberg on regular unipotent elements. The Theorem reduces the study of fusion of elements of U to the study of T-invariant subgroups of U and their normalizers, and is the analogue of a recent result for finite groups [see Theorem 4.1 in J. L. Alperin, Sylow intersections and fusion, J. Algebra 6 (1967), 222-241]. (Received November 1, 1967.)

653-336. D. SATHER, Cornell University, Ithaca, New York, and J. O. SATHER, University of Utah, Salt Lake City, Utah 84112. The Cauchy problem for an elliptic operator which changes classification on the boundary.

The Cauchy problem for the operator $T_\alpha = (\partial^2/\partial y^2) + y^\alpha(\partial^2/\partial x^2)$, $\alpha > 0$, is considered in the elliptic region $y > 0$ with initial data prescribed on the line $y = 0$ where T_α changes type. This Cauchy problem is not well posed in the sense of Hadamard. For suitably restricted initial data $u(x,0) = f(x)$ and $(\partial/\partial y)u(x,0) = g(x)$, a solution of the equation $T_\alpha u = 0$ is obtained which, for each fixed $y > 0$, is in L^2 and takes on the initial data in the L^2 sense. Although the existence question associated with non well posed problems has been neglected in comparison with the corresponding uniqueness and continuous dependence questions, perhaps a more interesting fact is that the conditions sufficient to guarantee such a solution by Fourier transform methods are also necessary. A solution, with the above properties, exists in the strip $0 < y < (\gamma/\beta)^\beta$, $\beta = 2/(\alpha + 2)$, if and only if the function $h(x) = f(x) + \lambda_0 \phi * g(x)$ (ϕ is the Riesz potential of order β and λ_0 is a constant) is equal a.e. to the restriction to the real axis of a complex valued function $H = H(x + iy)$ such that H is analytic for $|y| < \gamma$, and $\sup_{|y|<\gamma-\delta} \int |H(x + iy)|^2 dx < \infty$ for every δ satisfying $0 < \delta < \gamma$. (Received November 1, 1967.)

653-337. P. C. CONSUL, University of Calgary, Calgary, Canada. On some integrals in operational calculus.

Mellin integral transforms pairs are virtually two sided Laplace transforms. A large number of such pairs can be obtained, by slight modifications, from them and from exponential fourier transforms. Long lists of such transform pairs have been given by Doetsch (1950), Doetsch and others (1947) and in the Bateman Manuscript Project by Erdelyi and others (1954). Consul (1966) has obtained some new inverse Mellin integral transforms. The object of this paper is to generalise one of those integrals and to improve the result. A number of other allied integrals have also been discussed. (Received November 1, 1967.)

653-338. MITSURU YASUHARA, New York University, University Heights, Bronx, New York 10453. Properties of Lp languages--languages rich enough to express that a set A is of less power than B.

An extension of a first order language is called an Lp language if there is a formula [A,B], having no individual variables and no predicate symbols except A and B (these are monadic), and sastisfying the following two conditions. (i) If [A,B] is true in an interpretation, and if A is a subset of B while the complements of A and B are of the same power, then A is of strictly less power than B. (ii) For every strictly ascending sequence of cardinalities $\langle \kappa_0, \kappa_1, \ldots, \kappa_\eta \rangle$, there is an interpretation of the variables in [*,*] so that [A,B] is true for all subsets A, B of a given domain D if and only if the cardinalities of A, B, D are κ_ζ, $\kappa_{\zeta+1}$, κ_η, respectively, where $\zeta + 1 < \eta$. Theorem 1. The set of valid formulae of any Lp language is not recursively enumerable. Theorem 2. No Lp language is compact. Theorem 3. For no Lp language, does the (modified) Lowenheim-Skolem-Tarski theorem hold. Examples of Lp languages in the literature are those considered by Härtig [Kolloq. über die Grundlagen der Math., math. Maxchinen u. ihre Anwendungen, Budapest, 1965], by Henkin [Infinitistic method, pp. 181-183, Warszawa, 1961] and by Thomason [J. Symbolic Logic 31 (1966), 700]. The above theorems solve some problems, extend some results, and refute some conjectures in the literature. (Received November 2, 1967.)

653-339. WITHDRAWN.

653-340. G. H. WENZEL, Queen's University, Kingston, Ontario, Canada. Extensions of congruences on infinitary partial algebras.

The paper solves problem 15 in G. Grätzer's book Universal algebra, © 67, D. Van Nostrand, which proposes to study the embeddability of partial infinitary algebras $\mathfrak{A} = \langle \{a_0, \ldots, a_\gamma, \ldots\}_{\gamma < \alpha}; F$ of type τ into factor algebras $\mathscr{L}$ of free algebras such that all congruences θ on $\mathfrak{A}$ can be extended to $\mathscr{L}$. The relevant results in the finitary case hinge on a constructively given congruence $\theta_{\bar{a}}$ on $\mathscr{P}^{(\alpha)}(\tau)$ (= free algebra of type τ on α generators). To settle the infinitary case one defines that congruence as kernel of a suitable epimorphism and obtains the following result. Theorem. (i) Every congruence θ on $\mathfrak{A}$ can be extended to a congruence θ' on $\mathscr{P}^{(\alpha)}(\tau)/\theta_{\bar{a}}$ such that $[A]\theta' = \mathscr{P}^{(\alpha)}(\tau)/\theta_{\bar{a}}$. (ii) θ is strong if and only if $[A]\theta' = A$ for some extension θ' on $\mathscr{P}^{(\alpha)}(\tau)/\theta_{\bar{a}}$. A transfinite argument shows that $\theta_{\bar{a}}$ defined in terms of a kernel of an epimorphism coincides with the constructive definition in the finitary case. Thus finitary results are implicit in the above theorem. (Received November 2, 1967.)

653-341. BRANKO GRÜNBAUM, University of Washington, Seattle, Washington 98105. Higher-dimensional analogues of the four color problem.

A d-dimensional simplicial complex is properly colorable by c colors if it is possible to assign its (d - 1)-simplices to c classes in such a way that any two (d - 1)-simplices contained in a d simplex belong to different classes. Theorem. For d ≧ 1, each d dimensional simplicial complex imbeddable in Euclidean (d + 1)-space is properly colorable by 6d colors. Corollary. For d ≧ 3, the 2-faces of each simple, d-dimensional convex polytope may be colored by 6d-12 colors in such a manner that 2-faces with a common edge have different colors. Using standard reductions each of the results leads to the "six colors theorem" for planar maps. (Received November 2, 1967.)

653-342. C. L. OUTLAW and C. W. GROETSCH, Louisiana State University, New Orleans, Louisiana 70122. Averaging iteration in a Banach space. Preliminary report.

Let T be a mapping of a Banach space X into itself. If A is an infinite regular matrix which is lower triangular and has each row sum equal to 1, let M(x,A,T) be the pair of sequences $x_1 = x$, $v_n = \sum_{k=1}^{n} a_{nk}x_k, x_{n+1} = Tv_n$. Theorem 1. If T is linear and $T_x^n \to p$ then $M(x,A,T) \to p$. Except for Theorem 6, assume that X is uniformly convex and T is nonexpansive; let F be the set of fixed points of T. Theorem 2. If X is the complex plane, $F = \{p\}$, $\sum_{n=1}^{\infty} a_{nn}(1 - a_{nn})$ diverges, and $a_{n+1,k} = (1 - a_{n+1,n+1})a_{nk}$, then $M(x,A,T) \to p$. If $0 < \lambda < 1$ and $f \in X$, let $S_\lambda = \lambda I + (1 - \lambda)T$ and $v_\lambda = \lambda I + (1 - \lambda)(T + f)$. Theorem 3. If T is linear and demicompact, then $S_{1/2}^n x \to p \in F$. Corollary. If X is n-dimensional and T is linear, then $S_{1/2}^n x \to p \in F$. Conjecture. The corollary holds with "n-dimensional" deleted. Theorem 4. If $f \in X$, a solution of $u = Tu + f$ exists if and only if $\{V_\lambda^n x\}$ is bounded for each x. Theorem 5. If T is bounded, linear, and asymptotically convergent, and $f \in$ range $(I - T)$, then $V_\lambda^n x \to u$ such that $u = Tu + f$. (Received November 2, 1967.)

653-343. H. T. BANKS, Brown University, Providence, Rhode Island 02912. Variational problems involving functional differential equations.

A variational problem with system equations $\dot{x}(t) = f(x(\cdot), u(t),t)$ on $[t_0,t_1]$, $x(t) = \phi(t)$ on $[\alpha_0,t_0]$, is considered. The n-vector function $f(x(\cdot), u(t),t)$ is a functional in x, and may depend on the values $x(\tau)$, $\alpha_0 \leqq \tau \leqq t$, where α_0 ($\alpha_0 < t_0$) is some fixed finite number. The function $f(x(\cdot), u,t)$ is assumed C^1 in x and Borel measurable in (u,t). The functional to be minimized over a class of measurable r-vector control functions u and a class of absolutely continuous initial functions ϕ is given by $J[\phi,x,u,t_1] = \int_{t_0}^{t_1} f^0(x(\cdot), u(t),t)dt$. Necessary conditions for a minimum are found. This generalizes a previous result by the author (see Abstract 642-193, these *Notices* 14 (1967), 120). (Received November 2, 1967.)

653-344. STEVE ARMENTROUT, University of Iowa, Iowa City, Iowa 52240. Homotopy properties of decomposition spaces.

It has been shown that if G is a cellular decomposition of a 3-manifold M such that the associated decomposition space is a 3-manifold N, then M and N are homeomorphic (Armentrout, Abstract 634-53, these *Notices* 13 (1966), 374). In this paper we apply results due to T. M. Price (Abstract 67T-197, these *Notices* 14 (1967), 274) and the author to establish theorems concerning cellular decompositions of higher dimensional manifolds. Theorem 1. If M is a compact manifold nd G is a cellular decomposition of M such that the associated decomposition space M/G is a manifold N, then M and N have the same homotopy type. Theorem 2. If n is a positive integer, $n \geqq 5$, and G is a cellular decomposition of S^n such that the associated decomposition space S^n/G is an n-manifold, then S^n/G and S^n are homeomorphic. A compact set M in a topological space has property UV^∞ if and only if for each open set U containing M, there exists an open set V such that $M \subset V$, $V \subset U$, and V is contractible in U. Theorem 3. Suppose n is a positive integer, $n \geqq 5$, and G is an upper semicontinuous decomposition of S^n into compact sets, each having property UV^∞. If S^n/G is a manifold of dimension greater than 3, then each element of G is cellular. (Received November 2, 1967.)

653-345. W. J. BARNIER, Dartmouth College, Hanover, New Hampshire 03755. The Jiang subgroup for a map.

Let $f: X \rightarrow X$ be any map (continuous function), where X is a finite connected complex. Jiang Bo-Ju has defined a subgroup, T(f), of the fundamental group $\pi_1(X)$. The following useful necessary and sufficient condition that an element of $\pi_1(X)$ also be an element of T(f) can be given for certain spaces X. The map $f: X \rightarrow X$ induces a homomorphism $f_*: \pi_1(X) \rightarrow \pi_1(X)$. Suppose X is aspherical. If $\alpha \in \pi_1(X)$, then $\alpha \in T(f)$ if and only if α commutes with all elements of $f_*\pi_1(X)$. The Nielsen number, N(f), is a lower bound for the number of fixed points of all maps homotopic to f. The Jiang subgroup, T(f), is interesting mainly because of its useful application to the approximation of N(f). Examples are given showing how to compute both T(f) and N(f) for certain maps $f: X \rightarrow X$, where X is a compact connected 2-manifold without boundary. (Received November 2, 1967.)

653-346. D. F. ULLRICH and J. A. MARLIN, North Carolina State University, Raleigh, North Carolina 27607. Periodic solutions of a second order nonlinear differential equation without damping.

In this paper sufficient conditions are given for the existence of T-periodic solutions of differential equations of the form (*) $\ddot{x} + f(t,x) = 0$ where f is periodic with period T in t, continuous in (t,x), and Lipschitzian in x on some set $(-\infty, \infty) \times [-R,R]$ for some $R > 0$. It is shown that if there exist numbers α, β which satisfy $-R < \alpha < \beta < R$ and are such that $f(t,x)f(t,y) < 0$ for $-\infty < t < \infty$, $-R \leqq x \leqq \alpha$ and $\beta \leqq y \leqq R$ then for T sufficiently small (*) always possesses at least one T periodic solution. The method of proof can also be generalized to systems of equations. (Received November 2, 1967.)

653-347. R. L. NEWCOMB, University of California, Santa Barbara, California 93106. Topologies compact modulo an ideal.

Let $\mathscr{I}$ be an ideal in the Boolean algebra $\{A \mid A \subseteq X\}$ and let $\mathscr{O}$ be a topology for X. $\mathscr{O}$ is called compact modulo $\mathscr{I}$ if any $\mathscr{O}$ open cover of X contains a finite subset $\mathscr{B}$ with $\bigcup \mathscr{B} = X[\mathrm{mod}\ \mathscr{I}]$ (equivalence in the Boolean algebra). The topological properties (1) maximal compact modulo an ideal, (2) minimal Hausdorff, and (3) Katĕtov are characterized. (Received November 2, 1967.)

653-348. S. T. HEDETNIEMI, University of Iowa, Iowa City, Iowa 52240. On partitioning planar graphs.

In [Amer. J. Math. 2 (1879), 193-200] Kempe presented an erroneous proof of the famous Four Color Conjecture. In this paper it is shown that the method which Kempe presented is very useful for obtaining results about partitioning the set of points or lines of planar graphs. Let P denote any property of a graph G. A P-partition of G of order n is a partition $\pi = \{V_1, V_2, \ldots, V_n\}$ of the set of points V(G) such that the subgraph induced by every subset V_i has property P. The P-chromatic number $\chi_P(G)$ of G is the smallest integer n such that G has a P-partition of order n. Using this terminology the Four Color Conjecture states: if P_0 denotes the property that a graph be totally disconnected, then for every planar graph G, $\chi_{P_0}(G) \leqq 4$. Theorem 1. If P_1 denotes the property that a graph be either disconnected or consist of a single point, then for every planar graph G, $\chi_{P_1}(G) \leqq 4$. A graph is outerplanar if it can be embedded in the plane in such a way that every point lies on the exterior region. Theorem 2. For every outerplanar graph G, $\chi_{P_1}(G) \leqq 3$. Additional results are given for planar graphs which provide upper bounds for $\chi_P(G)$, for other properties P of a graph G. (Received October 13, 1967.)

653-349. Z. R. POP-STOJANOVIC, University of Florida, Gainesville, Florida 32601. On spherical stochastic processes on a locally compact group.

Let G be a locally compact group and K its locally compact subgroup. Consider the spherical stochastic process $\{x_t\}$ on G, i.e., $B(k_1 g k_2) = B(g)$ for every k_1, k_2 in K, where $B(g) = (U_g x_0, x_0)$ and $g \to U_g$ is an irreducible unitary representation of G. Then, the following property holds: The function B of the process $\{x_t\}$ has the representation: $B(g) = \int_S B_M(g)\, dm(M)$, where S is the space of all maximal ideals of the ring of all functions $s(g) \in L^1(G)$ satisfying $x(k_1 g k_2) = x(g)$, for every

k_1, k_2 in K, m is a measure on S and B_M is the function mapping all symmetric maximal ideals different from ones in S into the set of the functions B. (Received October 13, 1967.

653-350. WITHDRAWN.

653-351. J. B. RILES, St. Louis University, 221 North Grand Boulevard, St. Louis, Missouri 63103. The near Frattini subgroups of finitely generated groups.

A new canonical subgroup, the near Frattini subgroup, is defined. Some properties of this subgroup are as follows. If G is a finitely generated group that is the direct product of certain subgroups G_i, then the near Frattini subgroup $\psi(G)$ of G is the direct product of the subgroups $\psi(G_i)$. If G is finitely generated and nilpotent, then $\psi(G)$ contains the derived subgroup of G and the torsion subgroup of G. Every finitely generated abelian group occurs as the near Frattini subgroup of some finitely generated group, which can be chosen nilpotent of class 2. The notions of near supplementation, of near complementation and of complete Z-reducibility (as ZG-module) are defined, and a theorem on near complementation and complete Z-reducibility of abelian normal subgroups is given. (Received November 3, 1967.)

653-352. C. T. SCARBOROUGH, Mississippi State University, State College, Mississippi. Closed graphs and closed projections.

(X,Y) has C.G.P. if every function on $A \subset X$ into Y with a closed graph in $X \times Y$ is continuous. (X,Y) has C.P.P. if $\pi_1 : X \times Y \longrightarrow X$ is closed. (X,Y) has C.P.P. w.r.t. open sets if π_1 maps the closures of open sets onto closed sets. Theorem 1. For each space Y, there exists a zero dimensional T_2 space $Y^* \ni$ (1) If Y is T_1 and (Y^*,Y) has C.G.P., then Y is compact. (2) If (Y^*,Y) has C.P.P., then Y is compact. Theorem 2. For each space Y, there exists a zero dimensional T_2 space T^* such that if (T^*,Y) has C.P.P. w.r.t. open sets, then Y is H(i). H(i) spaces are discussed in C. T. Scarborough and A. H. Stone, Nearly compact spaces, Trans. Amer. Math. Soc. 124 (1966), 131-147. Nadler has shown that C.P.P. implies C.G.P. We give an example to show that the converse is false. However, Theorem 3. Let X be first countable T_1 and Y be T_1. If (X,Y) has C.G.P., then (X,Y) has C.P.P. (Received November 3, 1967.)

653-353. T. S. CHIHARA, Seattle University, Seattle, Washington 98122. Orthogonal polynomials whose zeros are dense in a half line. Preliminary report.

Let an orthogonal polynomial sequence $\{P_n(x)\}$ be defined by (1) $P_n(x) = (x - c_n)P_{n-1}(x) - \lambda_n P_{n-2}(x)$, $P_{-1}(x) = 0$, $P_0(x) = 1$, c_n real and $\lambda_{n+1} > 0$ $(n = 1,2,\ldots)$. Let X denote the set of all zeros of all $P_n(x)$. Let σ and τ denote the smallest and largest limit points of the derived set of X. In 1898, O. Blumenthal proved: Theorem. If $\lim_{n\to\infty} c_n = c$ and $\lim_{n\to\infty}\lambda_{n+1} = \lambda > 0$, where c and λ are finite, then $\sigma = c - 2(\lambda)^{1/2}$ and $\tau = c + 2(\lambda)^{1/2}$ and X is dense in $[\sigma,\tau]$. Using the ideas of Blumenthal's proof, the following analogue for an infinite case is obtained. Theorem. If $\lim_{n\to\infty} c_n = +\infty$,

$\lim_{n\to\infty}\lambda_{n+1}/(c_n c_{n+1}) = 1/4$, and if $-\infty < \sigma < +\infty$, then X is dense in $[\sigma, \infty)$. Using the theory of chain sequences, criteria can be obtained for deciding when σ is finite. Example. In (1), let $c_n = an + b$ $(a > 0)$, $\lambda_n = dn^2 + fn + g > 0$. Then $\tau = +\infty$ and (i) if $a^2 > 4d$, then $\sigma = +\infty$, (ii) if $a^2 < 4d$, then $\sigma = -\infty$, (iii) if $a^2 = 4d$, then $\sigma = b - d^{1/2} - f/d^{1/2}$. General polynomial coefficients can also be discussed to a certain extent. (Received November 3, 1967.)

653-354. G. W. HEDSTROM, University of Michigan, Ann Arbor, Michigan 48104. The rate of convergence of parabolic difference schemes.

Let $\partial u/\partial t = P(x,D)u$, $u(x,0) = v(x)$, be a system which is parabolic in the sense of Petrowsky for $0 < t < T$ and $x \in R^d$. Let Λ_a be the Lipschitz space of functions on R^d for which the norm $\|v\|_a = \sup_{x,h\in R^d} |\sum_0^{[a]+1} \binom{[a]+1}{j} (-1)^j v(x - jh)|\, |h|^{-a}$ is finite. Let $u_\delta(x, t+\triangle) = \sum B_k(x,\delta) u_\delta(x - k\delta, t)$, $u_\delta(x,0) = v(x)$ ($\triangle\delta^{-m}$ = const., m the order of $P(x,D)$) be a difference scheme which approximates the system of differential equations with order of accuracy r. It is further assumed that the difference scheme is parabolic in the sense of Widlund. Then it follows that $\sup_x |u(x,t) - u_\delta(x,t)| \leqq C(t)\delta^\alpha \|v\|_a$ $(0 \leqq a \leqq r)$ if $v \in \Lambda_a$ and the coefficients of $P(x,D)$ are also in Λ_a. (Received November 3, 1967.)

653-355. P. S. CHIANG, Western Michigan University, Kalamazoo, Michigan 49001. Computer investigation of Landau's theorem.

Let $f(z) = a_0 + a_1 z + \ldots$ be regular for $|z| < 1$ and never takes the values 0 and 1, then $|a_1|$ has a bound depending only on a_0. J. A. Jenkins gave an explicit bound (Canad. J. Math. 8 (1956), 423-425) $|a_1| \leqq 2|a_0| \{|\log|a_0|| + 5.94\}$. The author investigates the shapes for the curves $|a_1| \leqq L(a_0)$ for given a_0 by the aid of a computer and shows that Jenkins' result is about right when a_0 is negative and that a much smaller estimate should be available when a_0 is positive or complex. (Received November 3, 1967.)

653-356. B. A. TROESCH, 523 North Elm Drive, Beverly Hills, California 90210. Isoperimetric bound for eigenvalue.

The minimum of the functional $\rho = \int_0^1 h(x)dx/\lambda_1$ is found for a class of nonnegative functions $h(x)$ defined for $0 \leqq x \leqq 1$, which are concave and satisfy $h_x(0) \leqq 0$. Here λ_1 denotes the lowest (nontrivial) eigenvalue of the boundary value problem $(hf_x)_x + \lambda f = 0$, $f_x(0) = h(1)f_x(1) = 0$, $\int_0^1 f(x)dx = 0$ (i.e., $\lambda \neq 0$), f continuous. For the eigenvalue λ_1 the isoperimetric inequality (*) $\lambda_1 \leqq (1/\rho_0)\int_0^1 h(x)dx$ is obtained. The result actually remains true for a wider class of functions $h(x)$: $h(x) \geqq 0$ for $0 \leqq x \leqq 1$, $h(x) \geqq h(a)$ for $0 \leqq x \leqq a$, $h(x) \leqq h(a)$ for $a \leqq x \leqq b$, no restriction on $h(x)$ for $b < x \leqq 1$. The constants a,b, $(1/\rho_0)$, and also c introduced below, are uniquely determined and obtained in a simple manner from the smallest root of $\tan x = x$, namely $a \doteq .2914$, $b \doteq .7317$, $c \doteq .05894$, $(1/\rho_0) \doteq 10.701$. Equality in (*) holds only if (**) $h(x) = h(a)$ for $0 \leqq x \leqq b$ and $h(x) = h(a)(1 - x)(x + c)/((1 - b)(b + c))$ for $b \leqq x \leqq 1$, i.e., $h(x)$ consists of a straight and of a parabolic arc. The inequality (*) sharpens a result for general $h(x)$ (Comm. Pure Appl. Math. 18, p. 319) where $(1/\rho_0) = 12$. For concave $h(x)$ the result also confirms a guiding principle for inequality constraints (cf. (**)) by Courant (Comm. Pure Appl. Math. 18, p. 339). (Received November 3, 1967.)

653-357. A. H. THOMPSON, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. Rotating time-like congruences in general relativity.

A particular anholonomic coordinate system adapted to a given rotating congruence of curves in the space-time of relativity, together with the concept of an anholonomic subspace (K. Yano and M. Ohgane, Ann. Math. 55 (1952), 318-327) is used to discuss the kinematics of a fluid in general relativity. This approach has the advantage of offering a direct geometrical interpretation of Cattaneo's "transverse covariant derivative" (C. Cattaneo, Nuovo Cimento 10 (1958)) as well as simplifying the computations usually encountered in discussing the above congruences. Sufficient conditions for the validity of the Herglotz-Noerther Theorem in a curved space-time are also obtained. (Received October 30, 1967.)

653-358. TONY SHERSHIN, University of South Florida, Tampa, Florida 33620. Results concerning the Schutzenberger-Wallace theorem.

The purpose of this paper is two-fold, namely, (1) to extend and topologize algebraic results due to J. R. Bastida concerning Green's relations and (2) to indicate that the Schutzernberger-Wallace Theorem is a consequence of these extensions. The results take the form of commutative diagrams. Bastida's results, among other things, associate in a natural fashion two groups with any three elements of a semigroup making use of a subset D of a particular H-slice. The generalizations in this paper indicate that under certain nonstringent conditions, one being that the semigroup is compact, we find that these groups are topological and that one of these groups is homeomorphic to D. (Received November 7, 1967.)

653-359. S. K. SEHGAL, University of Alberta, Edmonton, Alberta, Canada. Automorphisms of integral group rings.

The question of automorphisms of the integral group ring Z(G) of a finite group G is studied. It is a well known result of G. Higman that the only automorphisms of Z(G) if G is finite abelian are the ones induced by the group automorphisms. We prove Theorem 1. Let θ be an automorphism of Z(G) where G is finite nilpotent of class two. Then there exists an automorphism λ of G and a unit γ of Q(G), the group algebra of G with rational coefficients such that $\theta(g) = \pm \gamma g^{\lambda} \gamma^{-1}$ for all $g \in G$. We also prove Theorem 2. Let G and H be finitely generated abelian groups. Then $Z(G) \cong Z(H)$ implies $G \cong H$. The case when G is finite abelian is due to G. Higman. (Received November 1, 1967.)

653-360. A. B. OWENS, Code 7820, Naval Research Laboratory, Washington, D. C. 20390. On replications of incidence sequences.

To each sequence $\pi = (a_1,\dots,a_p)$ of positive integers with an even sum there corresponds a collection of linear graphs, $\mathscr{G}(\pi)$, with the property that each graph of $\mathscr{G}(\pi)$ has exactly p vertices $v_1,\dots,v_p$ with the incidence number of v_i equal to a_i $(i = 1,\dots,p)$. When k is a positive integer, define $k\pi = k\,(a_1,\dots,a_p)$ to be the sequence $(\underbrace{a_1,\dots,a_1}_{k},\dots,\underbrace{a_p,\dots,a_p}_{k})$. It is shown that there always exists a k such that $\mathscr{G}(k\pi)$ contains a graph which has no loops or multiple edges. ($\mathscr{G}(k\pi)$ is said to possess a simple graph realization of $k\pi$.) The smallest integer k such that $\mathcal{G}(k\pi)$ contains a simple graph is defined to be the replication number of π. The replication number of $\pi = (a_1,\ldots,a_p)$ is known when p = 1,2, or 3, and good approximations are known when p = 4,5, and 6. (Received October 13, 1967.)

653-361. D. C. BARNES, Washington State University, Pullman, Washington 99163. Some complements of Hölders inequality.

A simple method is given which in many cases allows one to calculate the best possible constant C such that the inequality (1) $\int_0^a fg dx \geq C[\int_0^a f^p dx]^{1/p}[\int_0^a g^q dx]^{1/q}$ holds for all functions f(x), g(x) which belong to certain kinds of function classes. For example: Theorem. Let f(x), g(x) be nonnegative concave functions on $[0,a]$. In case p, q ≥ 1 then the largest possible constant in (1) is $C = (1/6)a^{1-(1/p)-(1/q)}(1+p)^{1/p}(1+q)^{1/q}$. Equality holds in case $f(x) = \alpha x$, $g(x) = \beta(a-x)$ with $\alpha, \beta \geq 0$. If, however, $-1 < p, q \leq 1$ with p, q $\neq 0$ and the inequality (1) is reversed, then the smallest possible constant is $C = (1/3)a^{1-(1/p)-(1/q)}(1+p)^{1/p}(1+q)^{1/q}$. In this case equality occurs if $f(x) = \alpha x$, $g(x) = \beta x$, with $\alpha, \beta \geq 0$. The methods used to obtain results of this type depend only on some elementary integral inequalities and certain rearrangements of the functions f(x), g(x). See Hardy, Littlewood and Pólya, Inequalities, Cambridge Univ. Press, 1964. Theorems 250, 378. See also Beckenbach and Bellman, Inequalities, Springer-Verlag, 1965, pp.41-42. (Received November 10, 1967.)

653-362. W. H. GRAVES, University of North Carolina, Chapel Hill, North Carolina 27514. On the divisibility of the group of divisor classes of degree zero.

It is well known that the group of divisor classes of degree zero of an algebraic function field in one variable over an algebraically closed field is a divisible group. However, this divisibility is a consequence of the structure properties of abelian varieties. Because of the role played by this divisibility in class field theory, it seems appropriate to seek a proof of it in an algebraic setting. This is done in the case that the function field is of genus one (an elliptic function field). The proof relies upon the addition formulas for the group of divisor classes of degree zero for an elliptic function field. (Received December 4, 1967.)

653-363. MARSHALL COHEN, Princeton University, Princeton, New Jersey, and DENNIS SULLIVAN, University of California, Berkeley, California. Mappings with contractible point inverses between p.l. manifolds.

Theorem. Let M_1 and M_2 be closed piecewise linear n-manifolds, where $n \geq 5$ and $H^3(M_1, \theta_3) = 0$. Assume that there exists a piecewise linear mapping f from M_1 to M_2 such that $f^{-1}(x)$ is contractible for each x in M_2. Then M_1 and M_2 are piecewise linearly homeomorphic. (Here θ_3 is the group of h-cobordism classes of p.*l.* homotopy 3-spheres under the operation of connected sum.) (Received November 15, 1967.)

653-364. C. P. ROURKE, Institute for Advanced Study, Princeton, New Jersey, and B. J. SANDERSON, University of Warwick, England. An embedding without a normal microbundle.

A piecewise linear embedding of $S^{19} \times I$ in S^{29} with no topological normal microbundle (and hence in particular no piecewise linear one) is constructed. The restriction of the normal block bundle of this embedding to $S^{19} \times \{0\}$ gives a p.l. embedding of the 19-sphere in a 28-manifold with no (topological) normal microbundle. Since the 29-manifold is parallelizable this also gives a p.l. immersion of S^{19} in S^{28} without a normal microbundle. These results improve on results of Hirsch which showed existence of embeddings without normal disc bundles. Haefliger's work on links and Toda's calculations of some unstable 10-stems are used. (Received November 22, 1967.)

653-365. DENNIS SULLIVAN, University of California, Berkeley, California 94720. K-theory and piecewise linear topology.

It is possible to characterize the piecewise linear category (in the world of odd primes) by using K-theory. The periodicity in K-theory corresponds to a geometric periodicity in the classical surgery obstructions. There is a piecewise linear Riemann Roch Theorem and a piecewise linear Adams Conjecture (and proof). This connection arises from a homotopy equivalence $G/PL_p \to B0_p$, p p an odd prime (X_p means "X localized at p", where X is a CW complex). This homotopy equivalence is used to prove the Hauptvermutung for simply connected manifolds M^n satisfying $n \geq 6$ and $H_3 M$ has no 2-torsion. (Received November 29, 1967.)

653-366. E. H. CONNELL, University of California, Berkeley, California. Constructing nontrivial h-cobordisms.

Theorem. Suppose X^n is a regular neighborhood of a connected 2-complex K and $n \geq 6$. If W^n is any p.l. h-cobordism which has ∂X^n as one end, then W is homeomorphic to $M \times I$ iff the Whitehead torsion $\tau(W,M) = 0$. Thus W is topologically trivial iff it is p.l. trivial. Corollary. If Π is any finitely presented group with $Wh(\pi) \neq 0$, then $\exists$ an h-cobordism W with $\pi_1(W) = \pi$ which is not topologically trivial. The proof is based on the study of "geometric groups" (a joint work with J. Hollingsworth). The first topologically nontrivial h-cobordism was constructed by Farrell and Hsiang. (Received December 11, 1967.)

653-367. T. F. BANCHOFF, Brown University, Providence, Rhode Island 02912. Polyhedral monkey saddles.

Although critical point theory for height functions is basically the same for polyhedra in 3-space as for smooth surfaces, the degree theories are essentially different. For example, degree theory can be used to solve an exercise proposed by H. Hopf. There is no smooth embedding of a surface (other than the sphere) in 3-space and a height function on it with precisely three critical points (one of which would have to be a degenerate "monkey saddle"). On the other hand, this is not true for polyhedra: We exhibit for each orientable surface (other than the sphere) a polyhedral embedding in 3-space and a height function with precisely three critical points. However, we may still construct an analogue of the classical degree theory, and we show that for certain "transversal" embeddings, the polyhedral and the smooth results coincide. (Received November 2, 1967.)

Abstracts Presented by Title

68T-1. T. J. HEAD, University of Alaska, College, Alaska 99701. Tensoring with a semigroup.

Let $\mathscr{S}$ be the category of commutative semigroups and let F be the functor $F(A) = S \otimes A$ where S is a fixed commutative semigroup. Theorem. F is an additive covariant functor from $\mathscr{S}$ into $\mathscr{S}$ which preserves coproduct representations and direct limit representations. F preserves not only surjective homomorphisms but also epimorphisms. F can be characterized among additive covariant functors from $\mathscr{S}$ into $\mathscr{S}$ by the property that each homomorphism of F(N) into G(N) (N is the additive semigroup of positive integers) extends to a unique natural transformation of F into G. (Received August 17, 1967.)

68T-2. C. J. MOZZOCHI, University of Connecticut, Storrs, Connecticut. A generalization of a theorem of Alfsen and Fenstad.

Let X be a set with power set P(X). For every A,B in P(X) let $U_{A,B}$ equal $(X \times X) - ((A \times B) \cup (B \times A))$. Theorem. A uniform space $(X, \mathscr{U})$ is totally bounded iff for some proximity space $\mathscr{P}$ on X the family $\mathscr{S}$ of all sets $U_{A,B}$ where (A,B) is not in $\mathscr{P}$ is a subbase for $(X, \mathscr{U})$. Definition. A symmetric generalized uniform space $(X, \mathscr{U})$ is p-correct iff for some symmetric generalized proximity space $\mathscr{P}$ on X the family $\mathscr{S}$ of all sets $U_{A,B}$ where (A,B) is not in $\mathscr{P}$ is a subbase for $(X, \mathscr{U})$. Theorem. Let $\mathscr{P}_1$ be a symmetric generalized proximity space on X with proximity class $\pi(\mathscr{P}_1)$. There exists in $\pi(\mathscr{P}_1)$ one and only one p-correct symmetric generalized uniform space (denoted $\mathscr{U}(\mathscr{P}_1)^*$) on X such that for all U and V in $\mathscr{U}(\mathscr{P}_1)^*$ $U \cap V$ is in $\mathscr{U}(\mathscr{P}_1)^*$. Furthermore, there exists in $\pi(\mathscr{P}_1)$ a totally bounded symmetric generalized uniform space (denoted $\mathscr{U}(\mathscr{P}_1)$) on X such that it is the least element of $\pi(\mathscr{P}_1)$; and if $\mathscr{P}_1$ is the usual proximity for the reals, X, then $\mathscr{U}(\mathscr{P}_1)$ is properly contained in $\mathscr{U}(\mathscr{P}_1)^*$. Corollary (Alfsen and Fenstad). Let $\mathscr{P}$ be a proximity space on X with proximity class $\pi(\mathscr{P})$. There exists in $\pi(\mathscr{P})$ one and only one totally bounded uniform space on X. Remark. In this way a short, direct proof of the Alfsen-Fenstad result is obtained. (Received October 10, 1967.)

68T-3. M. L. TEPLY, University of Nebraska, Lincoln, Nebraska 68508. Torsionfree injective modules.

The notation and terminology used below follows that of J. S. Alin and S. E. Dickson in Goldie's Torsion Theory and its derived functor (Pacific J. Math. (to appear)). Theorem 1 gives nine conditions equivalent to the following property which a hereditary torsion theory may possess: (A) Any direct sum of torsionfree injective modules is injective. Theorem 2. If a hereditary torsion theory $(\mathscr{T}, \mathscr{F})$ satisfies (A), then the filter corresponding to $\mathscr{T}$ has a cofinal subset of finitely generated ideals. Theorem 3. If $(\mathscr{T}, \mathscr{F})$ is a hereditary torsion theory satisfying (A) and $R \in \mathscr{F}$, then every R-module has a unique torsionfree cover. The class $\mathscr{L} = \{B \mid B/\mathscr{C}(B) \text{ is injective}\}$ is studied.

Theorem 4. $\mathscr{L}$ is a torsion class if and only if the Goldie torsion theory satisfies (A). (Received September 18, 1967.)

68T-4. R. W. CHANEY, University of California, Santa Barbara, California 93106. On pointwise convergence of Fourier series on the p-adic integers.

The character group of the compact abelian group Δ_p of p-adic integers is isomorphic to the countable group $Z(p^\infty)$ and hence can be linearly ordered in several natural ways. (See Abstract harmonic analysis by Hewitt and Ross for descriptions of Δ_p and $Z(p^\infty)$.) We choose one such ordering for the character group and list the characters in order as $\gamma_1, \gamma_2, \ldots, \gamma_n, \ldots$. Given f in $L_1(\Delta_p)$ we define the Fourier series of f to be $\sum_{m=1}^{\infty} \hat{f}(\gamma_m)\gamma_m(x)$. We exhibit a continuous function on Δ_p whose Fourier series diverges on a dense, null set of the second category and we exhibit a function h in $L_1(\Delta_p)$ whose Fourier series diverges almost everywhere; since h agrees with trigonometric polynomials on certain neighborhoods of nonzero p-adic integers it follows that the principle of localization fails. Also we compute the Dirichlet kernel, estimate the Lebesgue constants, show that a certain subsequence of this kernel is an approximate unit, discuss "lacunary" Fourier series, and present a criterion for pointwise convergence. (Received September 25, 1967.)

68T-5. C. Y. CHAO, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. A theorem of nilpotent groups.

This is a "partial" generalization of a theorem of Burnside on p. 241 in Proc. Lond. Math. Soc. (2) 11 (1912). Let G be a nilpotent group. An invariant subgroup K of G is said to be a ψ-group of G if $G_2 \supseteq K \supseteq G_{r-1}$ where $G = G_1 \supset G_2 \supset \ldots \supset G_{r-1} \supset G_r \supset G_{r+1} = \{e\}$ is the lower central series of G. Theorem. A nonabelian group whose center is either a cyclic group of order p(a prime) or a cyclic group of infinite order cannot be a ψ-group of a nilpotent group. (Received September 27, 1967.)

68T-6. R. B. JENSEN, Stanford University, Stanford, California. Nonconstructible patterns of degrees.

Theorem 1. If ZF is consistent, then so is ZF + AC + "the lattice structure of the degrees of constructibility can be coded by a nonconstructible Δ_4^1 set of integers having maximal degree". Corollary. It is consistent relative to ZF to assume that the universe is the constructible closure of a nonconstructible Δ_4^1 set of integers. Theorem 2. If ZF is consistent, then so is ZF + "all constructible sets of integers are Δ_4^1" + the universe is the constructible closure of a Δ_4^1 set of integers". The Δ_4^1 set in question describes the lattice of degrees of "collapsing functions" f: $\aleph_0 \to \aleph_1^L$. Our proofs combine Sacks' perfect set construction of minimal degrees with the technique of iterated forcing. By another method, Solovay has independently shown the above corollary to hold with Δ_4^1 replaced by Δ_3^1. (Received September 28, 1967.)

68T-7. W. M. BOGDANOWICZ, The Catholic University of America, Washington, D. C. 20017. On Pettis-Dunford-Phillips property for vector-valued volumes.

Let S(V) denote the family of all sets being finite disjoint unions of sets from a family V consisting of subsets of a space X. Let the family V be a refinement prering, i.e. for any A, B $\in$ V we have A $\cap$ B and A$\backslash$B are in S(V). A function m from V into a B-space Y is called a vector-valued volume if it is countably additive on V and has finite variation $|m|(A)$ on every set A $\in$ V. Denote by V_σ the family consisting of countable unions of sets from V. Let R(V,Y) be the set of all vector volumes m on V to Y such that there exists a function f summable with respect to the variation $|m|$ on every set A $\in$ V and $m(A) = \int c_A f d|m|$. Cf. Proc. Nat. Acad. Sci. USA 53 (1965), 492-498. The pair (V,Y) has the Pettis-Dunford-Phillips property, PDP in short, if every vector-valued volume on V to Y' is in R(V,Y'). Theorem. Each of the following conditions assures that the pair (V,Y) has the PDP property: (1) X $\in V_\sigma$ and Y' is separable, (2) X $\in V_\sigma$ and Y is reflexive, (3) X is the Euclidean space R^n and the prering V consists of all sets of the form $A = J_1 \times \ldots \times J_n$, where J_k are half open intervals $\langle a_k, b_k \rangle$, $a_k \leq b_k$. (4) the refinement prering V consists of at most countable number of sets and X $\in V_\sigma$. (Received September 13, 1967.)

68T-8. J. M. YOHE, Mathematical Research Center, U. S. Army, University of Wisconsin, Madison, Wisconsin 53706. Structure of hereditarily infinite dimensional spaces.

A hereditarily infinite dimensional (HID) space is an infinite dimensional compact metric space with no positive dimensional compact subsets. The first example of such a space was constructed by D. W. Henderson in 1965. In this paper the structure of such spaces is studied and the following theorems are proved: Theorem 1. Each HID continuum contains uncountably many mutually exclusive hereditarily indecomposable HID subcontinua. Theorem 2. There are uncountably many topologically different HID continua. Theorem 3. Let X be an HID continuum, and let A be the set of points at which X has dimension 1. Then $X = A \cup \bigcup_{p \in X-A} M(p)$, where each M(p) is a maximal HID Cantor manifold containing p. If p,q are two points of X - A, then either M(p) = M(q) or dim $(M(p) \cap M(q)) = 0$. (Received October 2, 1967.)

68T-9. L. W. GOODWYN, 9709 Lorain Avenue, Silver Spring, Maryland 20901. A characterization of symbolic flows. Preliminary report.

Let X be a compact Hausdorff space, and let T be a self homeomorphism of X. Definition. An open cover of X separates points if for every x, y X, x $\neq$ y, there is an integer m such that T^m x A or T^my A for each A. Let k(X,T) be the smallest integer such that there is an open cover with k(X,T) elements which separates points. Let h(X,T) denote the topological entropy of T. (R. Adler, A. Konheim and M. McAndrew, Topological entropy, Trans. Amer. Math. Soc. 114 (1965), 309-319.) Proposition. h(X,T) = log k(X,T). Definition. (X,T) is said to have maximal entropy if h(X,T) = log k(X,T). Theorem. (X,T) is isomorphic to a symbolic flow iff (1) (X,T) is expansive, (2) X is totally disconnected, and (3) (X,T) has maximal entropy. Theorem. If X is totally disconnected, and if (X,T) is expansive, and if h(X,T) = 0, then (X,T) is coalescent. Theorem. If (X,T) is expansive and has maximal entropy, then (X,T) is topologically mixing, and there is a T-invariant. Borel measure on X such that T is measure theoretically equivalent to the Bernoulli shift on k(X,T) symbols with each symbol given equal weight. (Received October 2, 1967.)

68T-10. J. H. AHLBERG, United Aircraft Research Laboratories, East Hartford, Connecticut 06108, E. N. NILSON, Pratt and Whitney Aircraft, East Hartford, Connecticut 06108, and J. L. WALSH, University of Maryland, College Park, Maryland 20740. Cubic splines on the real line.

Let $\triangle: 0 = x_0 < x_1 < \dots$ partition $[0,\infty)$, $\|\triangle\| = \sup_i |x_i - x_{i-1}|$, $\delta_\triangle = \inf_i |x_i - x_{i-1}|$, and let $f_0', f_0, f_1, f_2, \dots$ be given. There is a one-parameter family of cubic splines $S_\triangle(\alpha;x)$ on $[0,\infty)$ satisfying $S_\triangle'(\alpha;0) = f_0'$, $S_\triangle(\alpha;x_i) = f_i$ $(i = 0,1,2,\dots)$, $S_\triangle''(\alpha;0) = \alpha$. They satisfy $S_\triangle(\alpha;x) \leqq f_0 + f_0'x + (1/2)Qx^2$, $S_\triangle'(\alpha;x) \leqq f_0' + Qx$ if $S_\triangle''(\alpha;x) \leqq Q$ for $x > 0$ and $S_\triangle(\alpha;x) \geqq f_0 + f_0'x + (1/2)\, Q_1x^2$, $S_\triangle'(\alpha;x) \geqq f_0' + Q_1x$ if $S_\triangle''(\alpha;x) \geqq Q_1$. Moreover, these inequalities are the best possible. If, in addition (1) $\|\triangle\|/\delta_\triangle < \infty$, and (2) $\sup_i |((f_{i+1} - f_i)/(x_{i+1} - x_i) - (f_i - f_{i-1})/(x_i - x_{i-1}))/(x_{i+1} - x_{i-1})| < \infty$, then there is a unique value of α for which $\sup_x |S_\triangle''(\alpha;x)| < \infty$. If $(-\infty,\infty)$ replaces $[0,\infty)$, then there is a two-parameter family of splines $S_\triangle(\alpha,\beta;x)$ with $S_\triangle(\alpha,\beta;x_i) = f_i$ $(i = 0, \pm 1, \pm 2, \dots)$ and analogous inequalities pertain. If (1) and (2) hold, then there is a unique pair α_0, β_0 such that $\sup_x |S_\triangle''(\alpha_0,\beta_0;\, x)| < \infty$. (Received October 2, 1967.)

68T-11. A. G. R. McINTOSH, Macquarie University, New South Wales, Australia 2122. On the closed graph theorem.

Definitions. Let $\mathscr{L}$ denote the family of pairs of locally convex spaces for which the closed graph theorem holds. That is, we say $(E,F) \in \mathscr{L}$ if every closed linear map of E into F is continuous. Let $\mathscr{L}_B = \{F \mid (E,F) \in \mathscr{L}$ for all Banach spaces $E\}$. Examples. An l.c.x. F is an element of $\mathscr{L}_B$ if one of the following conditions is satisfied: (i) F is (B_r)-complete; (ii) F is an (LF)-space; (iii) F is souslinien (see L. Schwartz, Comptes Rendu 263 Ser. A (1966), 602-605); (iv) $F \in \mathscr{D}$ where $\mathscr{D}$ is the class of spaces defined by Raikov (Sibersk. Mat. J. 7 (1966), 353-372). Definition. An l.c.s. E is called a Mackey space if its topology is the same as its Mackey topology. Theorem. $(E,F) \in \mathscr{L}$ if E is a sequentially complete Mackey space with complete strong dual, and F is a semi-reflexive l.c.s. which belongs to $\mathscr{L}_B$. Remark. The distribution spaces listed by Schwartz in the above-mentioned article are all semireflexive spaces which belong to $\mathscr{L}_B$. (Received October 2, 1967.)

68T-12. IVAN LONCAR and SIBE MARDESIC, University of Zagreb, Zagreb, p.p. 314 Yugoslavia. A note on inverse sequences of ANR's.

Let Π be a family of metrizable continua. Following S. Mardešić and J. Segal, Trans. Amer. Math. Soc. 109 (1963), 195-205, we say that a continuum X is Π-like provided for every $\epsilon > 0$ there exists an ϵ-mapping $f_\epsilon : X \to X_\epsilon$ onto some member X_ϵ of Π. Generalizing a result from the above quoted paper, we prove: Theorem 1. Let Π be a family of ANR's. Then every Π-like continuum X is the inverse limit of an inverse sequence $\{X_n ; f_{nm}\}$ with bonding maps f_{nm} onto and with X_n belonging to Π. The restriction to ANR's cannot be removed as it follows from the next result. Theorem 2. There exist two arc-like continua X and Y such that X is Y-like but there is no inverse sequence $\{Y_n ; f_{nm}\}$, $Y_n = Y$, with bonding maps onto, having X as its inverse limit. (Received October 2, 1967.)

68T-13. SAMI BERAHA, Ball State University, Muncie, Indiana. An unusual matrix and its extensions.

(1) This is a matrix of odd order whose n^2 elements have n^2 distinct numerical values. The n^2 elements of the inverse, however, have only 9 distinct values, one of which is common to $(n - 2)^2$ elements. (2) The arithmetic difference between the numerical values of the elements of the inverse bears a direct relation to that of the elements of the original matrix. (3) The greatest common divisor of the cofactors and the determinant (which is one for the order three) grows exponentially with the order of the matrix while the factors that remain after simplification are relatively small. The extension of the formalism from matrices of odd order to those of even order brings about a new type of matrix. (1*) These have (as in the case of ordinary matrices) as many elements as there are positions per row and per column, but certain positions are blank whereas others have two elements (called Resonant pair) in the same position. (2*) The inverse has this same property. (3*) Whereas the elements of the inverse are rational if the elements of the original are rational, this is neither the case of the determinant nor that of the cofactors. In fact, their huge common divisor is an irrational function of the elements of the original matrix. Matrices that have properties 1*, 2*, and 3* are called Room matrices in recognition of an analogous and inspiring precedent. (Received October 4, 1967.)

68T-14. T. R. BERGER, University of Minnesota, Minneapolis, Minnesota 55455. Class two p groups as fixed point free automorphism groups.

Theorem. Suppose that AG is a finite solvable group with normal subgroup G. Assume that A is a class $\leqq 2$ odd p group. If $p^c \neq r^d + 1$ for any $c \leqq \exp A$ and any prime r where r^{2d+1} divides $|G|$, and if $C_G(A) = 1$ then the Fitting length of G is bounded by the power of p dividing $|A|$. The theorem is proved by applying a fixed point theorem to a reduction of the Fitting series of G. The fixed point theorem is proved by reducing a minimal counterexample. If R is an extra special r subgroup of G fixed by B, a subgroup A, where B centralizes D(R), then all irreducible characters of BR which are nontrivial on Z(R) are computed. All nonlinear characters of a class two p group are computed. (Received October 6, 1967.)

68T-15. S. P. HASTINGS, The University of Dundee, Dundee, Scotland. Backward existence for retarded functional differential equations.

The following theorem is a result about the backward existence and uniqueness of solutions of functional differential equations of retarded type. It may also be interpreted as a theorem about forward existence and uniqueness for equations of advanced type. We use the notation of J. Hale in Contributions to Differential Equations 2 (1963), 291-319. Theorem. Consider the equation (1) $\dot{x}(t) = A(t)\,x(t - r) + g(t,x_t)$, where $x_t(\theta) = x(t + \theta)$, $-r \leqq \theta \leqq 0$, and A(t) is a continuous $n \times n$ nonsingular matrix function defined on some interval $[-C,0]$. Suppose that there is a real-valued nondecreasing function $M(\mu)$ defined for $-r \leqq \mu \leqq 0$ such that $M(-r) = 0$ and such that for each $\mu \in [-r,0]$ and for each ϕ,ψ continuous on $[-r,0]$ with $\phi(\theta) = \psi(\theta)$, $\mu \leqq \theta \leqq 0$ we have $|g(t,\phi) - g(t,\psi)| \leqq M(\mu)\|\phi - \psi\|$, $-C \leqq t \leqq 0$. Then the conditions (i) for some $\eta > 0$, $\dot{\phi}$ is continuous on $[-\eta,0]$, and (ii) $\dot{\phi}(0) = A(0)\phi(-r) + g(0,\phi)$ are necessary and sufficient for there to be a function $x(\cdot,\phi)$ which is continuous on $[-r-\epsilon, 0]$ for some $\epsilon > 0$ and which satisfies (i) on $[-\epsilon, 0]$ as well as the relation $x(\theta,\phi) = \phi(\theta)$, $-r \leqq \theta \leqq 0$. Also, $x(\bullet,\phi)$ is unique if it exists. More general results have also been obtained. (Received October 13, 1967.)

68T-16. A. SHARMA and J. PRASAD, University of Alberta, Edmonton, Alberta, Canada. On Abel-Hermite-Birkhoff interpolation.

Given k real numbers (*) $x_1 < x_2 < \ldots < x_k$, and an integer $n \geqq k$ the problem of determining a polynomial of degree $\leq n-1$ having a suitable number of assigned values and derivatives at these points is given by the incidence matrix E_n^k of order $k \times n$. If m_p denotes the number of 1's in the pth column and $M_p = \sum_0^p m_i$, then E_n^k is said to satisfy (P) Polya condition, if $M_p \geqq p+1$, $0 \leqq p \leqq n-2$, $M_{n-1} = n$. The incidence matrix E_n^k is said to be q - H if a nonzero entry in any row (except perhaps the first and last) is preceded by a nonzero entry. Schoenberg has proved that if $\dot{E}_n^k$ is q - H and satisfies (P), then the interpolation problem is poised, that is the nonhomogeneous interpolation problem has a unique solution for any choice of real nodes (*). In the present paper we first extend the result to Abel-Hermite-Birkhoff (AHB) matrices and then to weakly q - H matrices. The AHB matrices form a subset of weakly q - H matrices and it is shown that an interpolation problem corresponding to a weakly q - H matrix is poised if it satisfies condition (P). Some classes of matrices are shown to correspond to poised interpolation problems without being weakly q - H. The results do not seem to extend easily to trigonometric interpolation. (Received October 5, 1967.)

68T-17. JOHN DAUNS, Tulane University, New Orleans, Louisiana 70118. Representation of l-groups and f-rings. Preliminary report.

Let A be an f-algebra over the rationals Q, with $1 \in A$ and B its maximal l-ideals with $\bigcap B = \{0\}$. Define $\pi: E \equiv \bigcup\{A/M | M \in B\} \to B$, $\pi^{-1}(M) = A/M$; for $a \in A$, $\hat{a}: B \to E$ by $\hat{a}(M) = a + M \in A/M$. Set $\hat{A} = \{\hat{a} | a \in A\}$. Proposition. There are unique minimal topologies on E and B subject to (1) π and (2) all $\hat{a} \in \hat{A}$ are continuous. The l-group $\Gamma(B,E)$ of all continuous cross-sections $\sigma: B \to E$, $\pi \cdot \sigma = 1$, need not be a ring. Additively, A and $\Gamma(B,E)$ are topological groups which are not necessarily Hausdorff. Consider the following hypotheses: (α) $1 \leqq a \in A^* \equiv \{a \in A |\ |a| < r1$, some $0 < r \in Q\}$ implies $1/a \in A^*$; (α') $1 \leqq a \in A$ implies $1/a \in A$; (β) A is complete. Lemma. If (α) holds, these conditions are equivalent. (1) A^* is archimedean; (2) A is Hausdorff, (3) E is Hausdorff. Theorem. Consider $A \cong \hat{A} \subseteq \Gamma(B, E)$, then (1) ($\alpha$) implies A is dense in $\Gamma(B,E)$. (2) (β) implies $\hat{A} = \Gamma(B,E)$. (3) (α') implies all A/M, $M \in B$, are totally ordered division rings. A converse Theorem also holds. (Received October 4, 1967.)

68T-18. K. J. C. SMITH, University of North Carolina, Chapel Hill, North Carolina 27514. On the rank of incidence matrices in finite geometries. Preliminary report.

(For terminology, see R. D. Carmichael, Introduction to the theory of groups of finite order, Dover, New York, 1956.) Let $N_{t,d}$ be the incidence matrix of the points and d-spaces in PG(t,q), where q is a prime power, say $q = p^n$, and $t \geqq 2$, $1 \leqq d \leqq t$. Representing the points as the elements $1 = \alpha^0, \alpha, \ldots, \alpha^{v-1}$, where α is a primitive element of $GF(q^{t+1})$ and $v = (q^{t+1}-1)/(q-1)$, $N_{t,d}$ is given as $N_{t,d} = (n_{ij})$, where $n_{ij} = 1$, if the point α^j is incident with the d-space, $\sum_i$, $i = 0,1,\ldots, b-1$ and

$j = 0, 1, \ldots, v - 1$, where $b = (q^{d+1} - 1)/(q - 1)$; and 0, otherwise. The rank of $N_{t,d}$, over $GF(q)$, is denoted $r(t,d,q)$. Formulas for $r(t,t-1,q)$ and $r(t,d,p)$ are given. An upper bound is obtained for the general case of $r(t,d,q)$. Similar results are obtained for the incidence matrix of points and d-spaces in $EG(t,q)$, in which one point and all d-spaces incident with this point are omitted. (Received October 13, 1967.)

68T-19. WITOLD BOGDANOWICZ and MARTHA MATTAMAL, Catholic University of America, Washington, D. C. 20017. Representation of noncomplete Lebesgue integrals.

A real-valued functional $\int$ is called a Lebesgue integral if its domain $D(\int) = L$ consists of real-valued functions on a space X and the following conditions are satisfied: L is a linear lattice satisfying the Stone condition, i.e. L is a linear space and $f \cap g$, $f \cap 1 \in L$ for all $f, g \in L$; $\int$ is a finite valued positive linear functional such that if $f_n \in L$, $f_n(x) \geqq 0$ on X, $f(x) = \sum_{n=1}^{\infty} f_n(x) < \infty$ for $x \in X$ and $\sum_{n=1}^{\infty} \int f_n < \infty$, then $f \in L$ and $\int f = \sum_{n=1}^{\infty} \int f_n$. Let v be any upper complete volume on a prering V. Then the smallest extension of the volume v to a measure μ can be given by the formula: $M = \{A : A = \bigcup_{n=1}^{\infty} A_n, A_n \in V\}$, and $\mu(A) = \operatorname{Sup}\{v(B) : B \subset A, B \subset A, B \in V\}$ for $A \in M$. Theorem. Let $\int$ be the Lebesgue integral and v the upper complete volume: $v(A) = \int c_A$, where $c_A \in D(\int)$, and μ the smallest measure extending v. Then $D(\int) = L(\mu,R) = L(v,R) \cap M(\mu,R)$ and $\int f = \int f d\mu = \int f dv$ for all $f \in D(\int)$, where $M(\mu,R)$ is the space of measurable functions generated by the sigma-ring M as defined for instance in Halmos, Measure theory, and $L(v,R)$ is the space of v-summable functions. For definitions see Bogdanowicz, Proc. Nat. Acad. Sci. U.S.A. 53 (1965), 492-498 and Proc. Japan Acad. 43 (1967), 286-289. $L(\mu,R)$ denotes the classical space of finite-valued Lebesgue summable functions without identification of functions equal almost everywhere. (Received October 17, 1967.)

68T-20. FRANK FORELLI, 401 Wayland Avenue, Providence, Rhode Island 02906. What makes a positive measure the total variation measure of an analytic measure?

Let w be a unit vector in R^N (R is the real line and N is greater than 1) and (to make things interesting) assume that the half-line $R_+ w$ does not meet Z^N (R_+ is the set of positive real numbers and Z is the integer group). A complex Borel measure λ on T^N (T is the circle group) is called analytic (relative to w) if its Fourier coefficients $\hat{\lambda}(n)$ are 0 for each n in Z^N with $\langle n|w\rangle$ negative ($\langle x|y\rangle$ is the inner product of the vectors x and y). Now in addition let μ be a positive Borel measure on T^N, and for each t in R let M_t be the closed linear span in $L^2(\mu)$ of the characters $\exp(i\langle n| \ \rangle)$ with $\langle n|w\rangle$ greater than t. Then it is easy to see (and known) that when μ is the total variation measure of an analytic measure: (*) just the function 0 belongs to all of the subspaces M_t (t in R). Theorem. Assume (*). Then there is a Borel function g on T^N with $0 < |g| \leqq 1$ there and $g\mu$ analytic. When μ is absolutely continuous with respect to Haar measure on T^N, this is a Helson and Lowdenslager theorem (Prediction theory and Fourier series in several variables. II, Acta Math. 106 (1961), 175-213). (Received October 13, 1967.)

68T-21. OSWALD PETRUCCO, 14-C Woodland Terrace, Columbia, South Carolina. An algebraic approach to axiom systems.

Let $W = \{Pi, i \text{ in } J\}$ be a collection of axioms, indexed by the finite set J. We say that a category K is stable with respect to W if (a) the coproduct T of any family F of objects from K is an object of K and (b) T satisfies axiom P_i if and only if each member of F satisfies P_i. Now let Z_2^J denote the semigroup of all functions mapping J into the set Z_2 of integers modulo 2. We construct a subset $S[W]_K$ of Z_2^J in the following way: an element r of Z_2^J will belong to $S[W]_K$ whenever there exists an object 0 from K with the property that for any i in J we have r(i) = 1 if and only if 0 satisfies axiom P_i. We say that the pair (K,W) gives rise to a complete theory if for any category L which is stable with respect to W the members of $S[W]_L$ are found among those of $S[W]_K$. Theorem 1. $S[W]_K$ is an idempotent semigroup containing a uniquely determined subset $[W]_K$ of independent generators. Theorem 2. If the pair (K,W) gives rise to a complete theory then W is a consistent set of axioms if and only if $[W]_K$ contains the unit element of Z_2^J. Theorem 3. If W is a consistent set of axioms and if the pair (K,W) gives rise to a complete theory, then the axioms of W are independent if and only if $[W]_K$ has exactly n + 1 elements. (Received October 17, 1967.)

68T-22. ERWIN KREYSZIG, University of Dusseldorf, c/o Mathematisches Institut, University of Dusseldorf, Germany. Minimal exponential operators.

It is shown that the class of exponential operators, defined by $u = Af = \int_{-1}^{1} g(z, z^*, t) f(\phi) \cdot (1-t^2)^{-1/2} dt$, $\phi = z(1-t^2)/2$, $g(z,z^*,t) = \exp \sum_{\alpha=0}^{m} q_\alpha(z,z^*)t^\alpha$, f analytic, for generating solutions $u(z,z^*)$ of a partial differential equation $Lu = u_{zz^*} + b(z,z^*)u_{z^*} + c(z,z^*)u = 0$ contains a "minimal operator" A_0, having a smallest number of not identically vanishing terms in the exponent of g. This leads to a simplification of the theory of exponential operators (cf. S. Bergman, Integral operators in the theory of partial differential equations, Springer, Berlin, 1961) and its applications to the coefficient problem for series representations of those solutions $u(z,z^*)$. For given L, the operator A_0 is unique and can be obtained explicitly. (Received October 17, 1967.)

68T-23. M. B. SLATER, University of Hawaii, Honolulu, Hawaii 96822. Some radicals for alternative rings.

Let R be an alternative ring, B(R) its Baer (nilpotent) radical, L(R) the sum of all locally nilpotent ideals of R, and K(R) its Koethe (nil) radical. (i) If 2x = 0 implies x = 0 for x of the form $ab\cdot c - a\cdot bc$, then L(R) is itself locally nilpotent, and R - L(R) has no nonzero locally nilpotent right or left ideals. Hence (ii) under the same restriction $B(R) \subseteq L(R) \subseteq K(R)$. L(R) may justly be called the Levitzki radical of R. Now let V(R) be the largest ideal of R which is contained in its nucleus, and set R' = R - V(R). Then (iii) L(R') = K(R'). (iv) If 3x = 0 implies x = 0 for x of the form $ab\cdot c - a\cdot bc$, then B(R') = L(R') = K(R'). (Received October 17, 1967.)

68T-24. M. MAKKAI, Hungarian Academy of Sciences, Budapest, V. Reáltanoda u. 13-15. Hungary. Preservation theorems for infinitary logic.

By an extension of a method of L. Henkin and R. M. Smullyan analogues for $L(w_1, w)$ of results of H. J. Keisler (J. Symbolic Logic 30 (1965), 339-349) and some related results are proved. Example. Let $\langle v_{ij} : i,j < w \rangle$ be a doubly indexed set of variables, P a binary predicate symbol. Let Γ be the least set of formulas of $L(w_1, w)$ such that (i) $v_{ij} \approx v_{kl}$, $Pv_{ij}v_{jkl}$, $Pv_{ij}v_{kl} \wedge \neg Pv_{mj}v_{nl}$, $v_{ij} \approx v_{kl} \wedge \neg v_{mj} = v_{nl} \in \Gamma$ for all $i,j,k,l,m,n < w$; (ii) Γ is closed under conjunction, disjunction, and universal quantification; (iii) if $F \in \Gamma$, $i,j < w$ and v_{kj} does not occur freely in F for any $k \neq i$ then $\exists v_{ij} F \in \Gamma$. Let F, G be sentences of $L(w_1, w)$ containing only P as nonlogical constant. Theorem 1. If for any structures A, B, C, $A = B \times C$ and $A \models F$ imply $B \models G$ then there is a sentence H in Γ such that $F \models H \models G$. Theorem 2. For any sentence F in $L(w_1, w)$ there is a set Σ of sentences of $L(w_1, w)$ such that the class of countable substructures of models of F is identical to the class of countable models of Σ.

(Received October 9, 1967.)

68T-25. OYSTEIN ORE, Yale University, New Haven, Connecticut, and JOEL STEMPLE, Queens College, City University of New York, Flushing, New York 11367. On the four color problem.

It is shown that a planar map not colorable in four colors must have at least n = 40 countries. This improves on the result n = 36 due to C. E. Winn (1938). The rather elaborate calculations are based upon the Euler contributions of the faces in an irreducible graph and upon three new reducible configurations. (Received October 13, 1967.)

68T-26. G. GRATZER and H. LAKSER, University of Manitoba, Winnipeg 19, Canada. Equationally compact semilattices.

Let $\mathfrak{S} = \langle S; \vee \rangle$ be a semilattice, that is $\vee$ is a commutative, associative, and idempotent binary operation on S. An equation in X over $\mathfrak{S}$ is of the form $a \vee x_{i_0} \vee x_{i_1} \vee \ldots = b \vee x_{j_0} \vee x_{j_1} \vee \ldots$, where $x_{i_0}, x_{i_1}, \ldots, x_{j_0}, x_{j_1}, \ldots \in X$, $a, b \in S$. A set Σ of equations is locally solvable if every finite subset of Σ has a solution (in $\mathfrak{S}$). Following J. Mycielski (Colloq. Math. 13 (1964), 1-9, see also B. Weglorz, Fund. Math. 59 (1966), 289-298 and Colloq. Math. 16 (1967), 243-248) the semilattice $\mathfrak{S}$ is equationally compact if for any set X, a set of equations Σ in X over $\mathfrak{S}$ is locally solvable in $\mathfrak{S}$ if and only if it is solvable in $\mathfrak{S}$. Further, $\mathfrak{S}$ is n-equationally compact if the same condition holds, provided $|X| \leq n$. Theorem 1. A semilattice $\mathfrak{S}$ is equationally compact if and only if the following three conditions hold: (i) $\mathfrak{S}$ is join-complete; (ii) any chain C in $\mathfrak{S}$ has a meet; (iii) if $a \in S$ and C is a chain in $\mathfrak{S}$, then $a \vee \bigwedge(x | x \in C) = \bigwedge(a \vee x | x \in C)$. Theorem 2. Any 1-equationally compact semilattice is equationally compact. (Received October 19, 1967.)

68T-27. A. L. RUBIN and J. E. RUBIN, Purdue University, Lafayette, Indiana 47907. Extended operations and relations on the class of ordinal numbers. Preliminary report.

We have extended results of J. Doner and A. Tarski (An extended arithmetic of ordinal numbers, Fund. Math. (to appear)) and obtained some additional properties of the operations, O_γ. Definition 1. For each $\alpha, \beta \in On$, $\alpha \, O_0 \beta = \alpha + \beta$; if $0 < \gamma \in On$, $\alpha \, O_\gamma \beta = \bigcup_{\eta < \beta, \zeta < \gamma} (\alpha \, O_\gamma \eta) O_\zeta \alpha$.

Definition 2. For each $\alpha, \beta, \gamma \in$ On, α R_γ β iff $(\exists \delta)$ $(\delta > 0$ & α O_γ $\delta = \beta)$. Our results include the following: Theorem 1. If γ is a limit ordinal then $\langle \text{On}', R_\gamma \rangle$ (On' = On $\sim \{0\}$) is a complete lattice and each proper branch (i.e. $\{\beta : \alpha\ R_\gamma\ \beta\}$ for some $\alpha \geqq 2$) is isomorphic to $\langle \text{On}, \leqq \rangle$. Theorem 2. If $A = \{\alpha : \alpha\ O_\gamma\ \beta = \delta\}$ for some ordinal numbers $\beta > 0$, γ, and δ then $\bigcup A\ R_\gamma\ \delta$. Moreover, given α, β, and γ, we have obtained necessary and sufficient conditions on α' for the equality $\alpha\ O_\gamma\ \beta = \alpha'\ O_\gamma\ \beta$ to hold. (Received October 20, 1967.)

68T-28. R. E. HARTE, University College of Swansea, Singleton Park, Swansea, Wales, Great Britain. Tensor products of normed modules.

The 'mixed identities' of homological algebra are stated and proved for normed modules, together with associative and distributive laws. As an immediate application, the dual of a tensor product is represented as a space of operators, at once generalizing a theorem of Schatten and a theorem of Dixmier. Identification of products $L^p \otimes_{L^\infty} L^p$ as being of the form L^r leads to the result that the bounded L^∞-linear mappings from L^p to L^r consist of the L^q-multiplications, and in particular that L^∞ is 'maximal abelian' on each of the spaces L^p. The attempt to represent products of the form $\mathscr{Q}(E, G) \otimes_{\mathscr{Q}(E,E)} \mathscr{Q}(F,E)$ as subspaces of $\mathscr{Q}(F,G)$ leads to some of the theory of 'centralizers', and also to some 'noncommutative' analogues of the results about the integration spaces. (Received October 23, 1967.)

68T-29. D. J. HEBERT and H. E. LACEY, NASA-MSC-ED13, Houston, Texas 77058. On supports of regular Borel measures.

Let X be a compact Hausdorff space. The existence of a regular Borel measure whose support is X imposes definite structures on X, C(X), and C(X)*. In this paper a necessary and sufficient condition is given to insure that X is the support of a regular Borel measure. This is given in terms of the intersection number of a collection of open sets in X. Measures which vanish on a sigma ideal of a sigma field of subsets of X which contains a basis for the topology of X are also considered. In particular, necessary and sufficient conditions are given to insure the existence of a nonatomic regular Borel measure whose support is X. The final section of the paper is devoted to a study of normal measures, i.e., measures which vanish on meager Borel sets. Normal measures on X are shown to be related to normal measures on the projective resolution of X. (Received October 23, 1967.)

68T-30. SHAIR AHMAD, Case Western Reserve University, Cleveland, Ohio 44106. Split dilations of finite cyclic groups.

Let C be a cyclic group of order $|C| =$ de, d and e positive integers. Let ω be a fixed primitive eth root of unity of C, and let $U_{de}^{(r)} = \{x \in C \,|\, x^d = \omega^r\}$. Let K_{de} denote the set of mappings of the form $x \mapsto a_r x$ $(x \in U_{de}^{(r)}$, $r = 0, 1, \ldots, e - 1)$, where the a_r are elements of $U_{de}^{(0)}$. Let $\overline{K}_{de}$ denote the set of permutations of the above form where the a_r are arbitrary elements of C. K_{de} and $\overline{K}_{d_e}$ are finite groups of orders d^e and $e!d^e$ respectively (see Wells, Monatsh. Math. 71 (1967), 248-261). Theorem 1. An element ϕ of K_{de} has a cycle of length K if and only if there exists $\xi \in C$ such that $\phi(\xi) = \omega^m \xi$ and $k = d/(d,m)$. Theorem 2. Let A_{de} denote the alternating group on C. Then K_{de} is a subgroup of A_{de} if and only if d is odd. Theorem 3. If $|C| = d_1e_1 = d_2e_2$, then $\overline{K}_{d_1e_1} \cap \overline{K}_{d_2e_2} = \overline{K}_{de}$, where $d = [d_1, d_2]$ and $e = |C|/d$. Theorem 4. Let H be the set of elements of K_{de} of the form $x \mapsto \alpha x$ ($x \in C$), and let H_s be the subgroup of K_{de} whose elements fix $U_{de}^{(s)}$ elementwise. Then $K_{de/H}$ is isomorphic to H_s. (Received October 23, 1967.)

68T-31. PEI LIU, Virginia Polytechnic Institute, Blacksburg, Virginia 24061. Some properties of dependence space.

A dependence space $(X, \mathscr{E})$ is a set X together with a collection $\mathscr{E}$ of nonempty finite subsets of X satisfying the axioms: (AE_1) if E and F are distinct members of $\mathscr{E}$ and $x \in E \cap F$, then $E \cup F - \{x\}$ has a subset belonging to $\mathscr{E}$. (AE_2) if $e \in E$ but $e \notin F$, then e is in such a subset. (AE_1) is called the elimination axiom by Robertson and Weston (Proc. Edinburgh Math. Soc. 2 (1959), 139-141). In the following, A, B, and C will denote subset of X. A set A is $\mathscr{D}$-independent iff A has no subset which is a member of $\mathscr{E}$. An element x in X is $\mathscr{D}$-dependent on A, written $x \mathscr{D} A$, iff $x \in A$ or there is a subset A_1 of A such that $\{x\} \cup A_1 \in \mathscr{E}$. A set B is $\mathscr{D}$-dependent on set A iff for every $b \in B$, $b \mathscr{D} A$. Theorem 1. Every set A contains a maximal (w.r. to inclusion) $\mathscr{D}$-independent set. Theorem 2. If $\mathscr{A}_1$ is a maximal $\mathscr{D}$-independent subset of A, then $A \mathscr{D} A_1$. Theorem 3. If $A \mathscr{D} B$ and $B \mathscr{D} C$, then $A \mathscr{D} C$. Theorem 4. Let A and B be $\mathscr{D}$-independent sets, then $A \mathscr{D} B$ and $B \mathscr{D} A$ implies that A and B have the same cardinality. Without AE_2, the transitive property of $\mathscr{D}$, Theorem 3, does not hold. For example, let $X = \{a,b,c,d\}$ and define $\mathscr{E} = \{\{c\}, \{d\}, \{a,b\}, \{a,c,d\}\}$. This space X satisfies AE_1 but not AE_2, since $\{b\} \mathscr{D} \{a\}$ and $\{a\} \mathscr{D} \{c,d\}$, but $\{b\}$ does not depend on $\{c,d\}$. (Received October 9, 1967.)

68T-32. JULIAN GEVIRTZ, Courant Institute of Mathematical Sciences, 251 Mercer Street New York, New York 10012. Metric conditions that imply local invertibility. Preliminary report.

Let E^n be n-dimensional Euclidean space, $G \subset E^n$ be open and $f: G \to E^n$. Let $S^{n-1} = \{x \in E^n : \|x\| = 1\}$. $D^+f(x)$ and $D^-f(x)$ are respectively, the upper and lower limits of $\| f(x) - f(y)\| / \|x - y\|$ as $y \to x$. Jf(x) is the Jacobian determinant of f wherever defined. Theorem I. Let n = 3. Let (1) $D^+f(x) \leqq M < \infty$ for all $x \in G$, (2) $D^-f(x) \geqq m > 0$ for all $x \in G$, (3) $Jf(x) \geqq 0$ a.e. in G, [f is differentiable a.e. in G by (1)] and (4) $M/m < 2$. Then f is a local homeomorphism. In (4), 2 may be replaced by no larger number. Theorem I follows from Theorem II. If $g: S^2 \to S^2$ has degree $d \geqq 2$, then there is a rectifiable curve C on S^2 for which the length of f(C) is at least twice that of C. In E^2 Theorem I can be strengthened to: Theorem III. Let n = 2. Let (1) and (3) of Theorem I hold. Let (5) $D^-f(x) > 0$ for all $x \in G$ and (6) $D^+f(x)/D^-f(x) \leqq K < 2$ a.e. in G. Then f is a local homeomorphism. The proof is based on the theory of quasi-conformal mapping. (Received November 1, 1967.)

68T-33. DONALD GREENSPAN, University of Wisconsin, Madison, Wisconsin. Numerical solution of steady state Navier-Stokes problems for all Reynolds numbers.

A popular technique for the numerical solution of two dimensional steady state Navier-Stokes problems is modified so as to apply for all Reynolds numbers. The key to the modification is to take a new analogue of the vorticity equation as follows. In $-4\omega_0 + \omega_1 + \omega_2 + \omega_3 + \omega_4 + h^2R\{[(\chi_1 - \chi_3)/(2h)]\omega_y - [(\chi_2 - \chi_4)/(2h)]\omega_x\}$, substitute $\omega_y = (\omega_2 - \omega_0)/h$ if $\chi_1 - \chi_3 \geqq 0$, $\omega_y = (\omega_0 - \omega_4)/h$ if $\chi_1 - \chi_3 < 0$, $\omega_x = (\omega_0 - \omega_3)/h$ if $\chi_2 - \chi_4 \geqq 0$, $\omega_x = (\omega_1 - \omega_0)/j$ if $\chi_2 - \chi_4 < 0$. The resulting difference equation is then diagonally dominant for all R. (Received October 30, 1967.)

68T-34. JOHN HUTCHINSON, University of Kansas, Lawrence, Kansas 66044. Strongly intrinsic extensions of left quotient semisimple rings.

The problem of characterizing the left intrinsic extensions of various rings was stated by Faith (Lectures on injective modules and quotient rings, Rutgers University, (1964)). For definitions, see (Faith and Utuni, Intrinsic extensions of rings, Pacific J. Math. 14 (1964), 505-512). If R is a subdirect sum of nonzero rings $\{R_\alpha : \alpha \in A\}$ then the subdirect sum is essential if R (identifying R and its isomorphic image in $\prod_{\alpha\in A} R_\alpha$) is an essential left R-submodule of $\prod_{\alpha\in A} R_\alpha$. Theorem 1. Let S be a left quotient semisimple ring. Then R is a left strongly intrinsic extension of S iff (i) S(R) is an essential subdirect sum of rings S_1 and S_2 (R_1 and R_2) where $S_i \subseteq R_i$; (ii) if $R_2 \neq 0$ then $\overline{S}_2 = \overline{R}_2$ where $\overline{S}_2$ and $\overline{R}_2$ denote the left classical quotient rings of S_2 and R_2, and S_2 is left quotient semisimple; (iii) if $R_1 \neq 0$ then there exists left Ore domains $S'_1, \ldots, S'_n$, $R'_1, \ldots, R'_n$ such that $S_1(R_1)$ is an essential subdirect sum of the $S'_i(R'_i)$ and the R'_i are left intrinsic extensions of the S'_i. Theorem 2. If R and S are left Ore domains and Q is the left classical quotient ring of S and $S \subseteq R$, then R is a left intrinsic extension of S iff $R \otimes_S Q$ is a simple left $R \otimes_S Q$ module. (Received October 30, 1967.)

68T-35. R. A. SWEET, Cornell University, Ithaca, New York 14850. Approximation to the beam equation.

The equation $w(x)u_{tt} + (p(x)u_{xx})_{xx} = 0$, $0 < x < L$, $t > 0$, where w and p are positive on $[0,L]$, describes the free transverse vibrations of a beam with nonuniform cross-sectional area and moment of inertia. An approximation to the solution of this equation is obtained by the method of straight lines. The approximation has the form $HU'' + SU = F$, where H is a diagonal matrix of positive elements, F is a vector depending on the boundary conditions, and S is a pentadiagonal matrix whose ith row has the form $\{p_{i-1}, -2(p_{i-1} + p_i), p_{i-1} + 4p_i + p_{i+1}, -2(p_i + p_{i+1}), p_{i+1}\}$, where $p_i = p(x_i)$ and $\{x_i\}_{i=1}^n$ are the discretization points. Boundary conditions of the following form are considered: $au(x,t) + b(pu_{xx})_x(x,t) = cu_x(x,t) + du_{xx}(x,t) = 0$, for $x = 0$ and $x = L$. With appropriate choices for the constants these conditions include all the normal conditions encountered. It is shown that under these boundary conditions, S is similar to either an oscillation matrix [Gantmakher and Kreĭn, Oscillation matrices and kernels and small vibrations of mechanical systems, State Publishing House, Moscow, 1950] or to one which possesses all the properties of an oscillation matrix except nonsingularity. (Received October 30, 1967.)

68T-36. LARRY EIFLER, Purdue University, Lafayette, Indiana 47907. The approximation property for some function algebras. Preliminary report.

A Banach space B is said to have the approximation property if for each compact subset $\mathscr{K}$ of B and $\epsilon > 0$ there exists a continuous linear operator Φ of finite rank mapping B into B such that $\|x - \Phi(x)\| < \epsilon$ for each $x \in \mathscr{K}$. It was conjectured by A. Grothendieck (Mem. Amer. Math. Soc. No. 16 (1955)) that every Banach space has the approximation property and this conjecture remains open. Theorem. Let X be a compact subset of the plane whose complement consists of finitely many components and let R(X) denote the uniform closure in C(X) of the rational functions with no poles in X. Then R(X) has the approximation property. The proof follows from the original proof of Mergelyan's theorem which states that R(X) equals the space of continuous functions on X which are analytic on the interior of X and a theorem of J. Dugundji (Pacific J. Math. 1 (1951), 353-367) on the simultaneous extension of continuous functions. (Received October 30, 1967.)

68T-37. A. G. FADELL and K. D. MAGILL, JR., State University of New York, 4246 Ridge Lea Road, Buffalo, New York 14226. Automorphisms of the semigroup of all polynomial functions over the reals.

$\mathscr{P}$ denotes the semigroup, under composition, of all polynomial functions over the field of real numbers. Theorem. Every automorphism of $\mathscr{P}$ is inner. Corollary. The automorphism group of $\mathscr{P}$ is isomorphic to the group of all ordered pairs of real numbers (a,b) where $a \neq 0$ and multiplication is defined by $(a,b)(c,d) = (ac, ad + b)$. (Received October 25, 1967.)

68T-38. H. H. CRAPO, Massachusetts Institute of Technology, Cambridge, Massachusetts. Mobius inversion in lattice.

Consider functions f from a finite lattice L into a ring R and define an upper difference operator E by $Ef(x) = \sum_{y;x \leqq y} \mu(x,y)f(y)$, a lower difference operator D by $Df(x) = \sum_{y;y \leqq x} f(y)\mu(y,x)$, where μ is the Mobius function $\mu = \zeta^{-1}$ of the lattice L (G.-C. Rota, On the foundations of combinatorial theory, Zeits. Wahrsch. 2 (1964), 340-368). Theorem. For every supremum-homomorphism σ from a finite lattice P into a finite lattice L, and for any functions $f: P \longrightarrow R$, $g: L \longrightarrow R$, $\sum_{x \in P} Df(x)g(\sigma(x)) = \sum_{y \in L} f(\sigma^{\triangle}(y))Eg(y)$ where $\sigma^{\triangle}(y) = \sup \{x; \sigma(x) \leqq y\}$. Applying the above theorem to the lattice of intervals of a finite lattice, ordered by inclusion, we find $q_0 - q_1 + q_2 - \ldots = \delta_L(0,1) - \mu_L(0,1) + \sum \zeta(X \cap [x,y], Y)\mu(0,x)\zeta(x,y)\mu(y,1)$, where q_k is the number of k-element subsets A of X such that inf A = 0, sup A = 1. If X is a cross-cut of L and $Y = \Phi$, the above formula becomes the cross-cut theorem (Rota, op. cit.). If X = L and Y is the set $s^{\perp}$ of complements of a fixed lattice element s, then the above formula becomes the complementation theorem (H. H. Crapo, J. Comb. Theory 1 (1966), 126-131). (Received October 30, 1967.)

68T-39. T. S. SHORES, University of Kansas, Lawrence, Kansas 66044. A note on classes of groups.

Theorem. The only nontrivial group theoretic class that is closed under the operations of taking subgroups, quotients, cartesian products and finite normal products is the class of all groups. The theorem is false if any one of the conditions on the class is omitted. (Received October 30, 1967.)

68T-40. LUDVIK JANOS, University of Florida Gainesville, Florida 32601. On self-maps which have a unique fixed point.

Let S be a topological semigroup acting on a topological space X. Let $F \subset S$ be the set of all elements in S which have a unique fixed point in X, i.e. $s \in F$ iff there exists exactly one $x \in X$ such that $sx = x$. If we put $x = f(s)$ we have defined a map $f: F \longrightarrow X$, and it arises a question under which conditions f is continuous on F. Theorem. If X is compact and metrizable and $S = X^X$ endowed with the compact-open topology, then the mapping $f: F \longrightarrow X$ is continuous on F. Corollary. Let M denote the set of all metrics on X inducing the given topology. Let $\alpha \in (0,1)$, $\zeta \in M$, let $S(\alpha, \zeta)$ be the semigroup of all α-contractions on X with respect to $\zeta \in M$. Then because of $S(\alpha,\zeta) \subset F \subset X^X$ for each $\alpha \in (0,1)$ and $\zeta \in M$ it follows that fixed points vary continuously on the family $\bigcup \{S(\alpha,\zeta) | \alpha \in (0,1), \zeta \in M\}$ of all contractions on X. (Received November 2, 1967.)

68T-41. J. S. BAKSHI, State University College, Buffalo, New York 14222. Solution for nonlinear deflections in an elastic presurized spherical shell under diametrial compression.

The load P acting toward the center forms a surface with reversed curvature which increases with an increase in load. Displacements no longer remain small. To avoid the nonlinearity small increments to normal load P are applied. A simultaneous solution of equilibrium equations for element $ds = R_1 d\phi R d\theta$ in direction of P, stress strain relations and strain displacement relations $E\theta = (1/R_2)(u_\phi \cot\phi + u_R)$ $E_\phi = u_R/R_1 + (1/R_1)\partial u_\phi/\partial\phi + (1/2)((1/R_1)(\partial u_R/\partial\phi))^2$ we solve for u_R and u_ϕ. By adding u_R and u_ϕ to R_1 and R_2 initial radii of curvature one finds new radii of curvature. Next the load is increased slightly by ΔP so that linearity is preserved and new u_R and u_ϕ are found as before. These are added to the new R_1 and R_2 thus finding the shape of the deformed shell. (Received October 31, 1967.)

68T-42. JOHN HAYS, University of Maine, Orono, Maine 04473. Introduction to hyperboolean systems. III: Finite and transfinite fields; vector spaces and their duals.

Given hyperbooleans $\mathscr{B}$ ("bibooleans") constructed from boolean pairs (Abstract 67T-575, these *Notices* 14 (1967), 527), define $/B_1B_2 = B_3 \Leftrightarrow B_2 \neq \emptyset \wedge B_1 = \bigcap B_2B_3$. Theorem 7 ("division algorithm"). $\forall B_1, B_2 \subset \mathscr{B}: B_1 \subset B_2\ \exists!\ B_3, B_4: B_2 = \bigcup\bigcap B_3B_1B_4 \wedge \emptyset \subset B_4 \subsetneqq B_1$. A biboolean atom has form $\langle L,\emptyset\rangle$, for L a boolean atom. Theorem 8. For any chain subset $\mathscr{L}$ of $\mathscr{B}$, $\langle \mathscr{L}, \equiv, \bigcup\rangle$ is an abelian group iff $C_1 \equiv C_2 \pmod{C_3} \Leftrightarrow \exists!\ C_4: \backslash C_1C_2 = \bigcap C_4C_3$; for any atomic modulus $\langle \mathscr{L}, \equiv, \bigcup, \bigcap\rangle$ is a (finite) field. We construct quabooleans $\mathscr{Q} = \mathscr{B} \times \mathscr{B}$. $Q_1 = \langle B_{11}, B_{12}\rangle \subsetneqq Q_2 = \langle B_{21}, B_{22}\rangle \Leftrightarrow \bigcap B_{11}B_{22} = \bigcap B_{12}B_{21}$. $\bigcup Q_1Q_2 = \langle \bigcup\bigcap\bigcap B_{11}B_{22}B_{12}B_{21}, \bigcap B_{12}B_{22}\rangle . \backslash Q_1Q_2 =$

$\langle \bigcup \bigcap \bigcap B_{11}B_{22}B_{12}B_{21}, \bigcap B_{12}B_{22}\rangle$. $\bigcap Q_1Q_2 = \langle \bigcap B_{11}B_{21}, \bigcap B_{12}B_{22}\rangle$. $/Q_1Q_2 = \langle \bigcap B_{11}B_{22}, \bigcap B_{12}B_{21}\rangle \wedge Q_2 \neq \emptyset$. Theorem 9. $\langle \mathscr{Q}, \bigcup, \bigcap\rangle$ is a field. Form $\mathscr{Q}$-n-tuples $\mathscr{Q}_n$ with $\mathscr{Q}$-operations. Theorem 10. $\mathscr{Q}_n$ is a vector space over a $\mathscr{Q}$-field. Form appropriate "linear functionals" in $\mathscr{Q}_n$. Corollary. The set of linear functionals of $\mathscr{Q}_n$ is a dual space of $\mathscr{Q}_n$. (Received November 2, 1967.)

681-43. EDGAR KRAUT, STAVROS BUSENBERG and WILLIAM HALL, Science Center of the North American Rockwell Corporation, 1049 Camino Dos Rios, Thousand Oaks, California 91360. On an additive decomposition of functions of several complex variables.

The extension of the Wiener-Hopf technique to functions $f(z_1,\ldots,z_n)$ of more than one complex variable requires an additive decomposition of these functions which appeared to have been given by Bochner (Amer. J. Math. 59 (1937), 732-738). Bochner's theorem states that: if $f(z_1,\ldots,z_n)$, $z_j = x_j + iy_j$, is analytic in a tube T: $\gamma_i < x_i < \delta_i$, $i = 1,2,\ldots,n$, and if $\int_{-\infty}^{\infty}\cdots\int |f|^2 dy_1 \cdots dy_n$ converges in T, then $f = \sum_{i=1}^{2^n} f_i$, such that the f_i are analytic and bounded in octant shaped tubes T_i containing T. Moreover, the decomposition is unique up to additive constants. Here f_1 is analytic and bounded in $T_1: x_i > \gamma_i$, $i = 1,2,\ldots,n$, and f_2 in $T_2: x_1 < \delta_1$, $x_j > \gamma_j$, $j = 2,3,\ldots,n$. Let $g(z_2,z_3,\ldots,z_n) = \prod_{j=2}^{n}(z_j - \gamma_j + \epsilon)^{-1}$, $\epsilon > 0$, and construct the sum $\sum_{i=1}^{2^n} f_i' = \sum_{i=1}^{2^n} f_i$, where $f_1' = f_1 + g$, $f_2' = f_2 - g$, $f_j' = f_j$, $j = 3,4,\ldots,2^n$. The f_i' are bounded and analytic in the proper tubes T_i, and yet they do not differ from the f_i merely by a constant. This counter-example destroys the uniqueness claim of Bochner and renders the extension of the Wiener-Hopf technique more difficult than has been suggested (see: Radlow, Bull. Amer. Math. Soc. 70 (1964), 596-599). (Received November 3, 1967.)

68T-44. B. N. SAHNEY, The University of Calgary, Calgary, Alberta, Canada. On the summability of a class of derived Fourier series.

If $f(u)$ be even, $f(u) \in (-\pi,\pi)$ and defined by periodicity outside this range. Matrix summability of Fourier series has been discussed by Peterson (Proc. Amer. Math. Soc. 11 (1960), 994-998). Let $\psi_x(u) = f(x+u) - f(x-u)$, $g(u) = \psi_x(u)/(4 \sin u/2)$. Theorem. If (Λ) is a regular sequence to sequence triangular matrix such that $\sum_{k=0}^{n} |\Delta^2 \Lambda_{n,k}| = O(1)$ and if $(1/t)\int_0^t g(u)du$ is of bounded variation in $(0,\pi)$ and $g(u) \to 0$ as $u \to 0$, then $t_n = O(1)$. This generalizes a theorem due to Varshney (Boll. U. Mat. Italiana 16 (1961), 379-382) and includes a result due to Rath, as a particular case. (Received October 30, 1967.)

68T-45. C. NASIM, The University of Calgary, Calgary, Alberta, Canada. A divisor summation formula.

Let $\sigma_k(n)$ represent the sum of the kth powers of the divisors of n. Consider the class of functions $G_\lambda^2(0,\infty)$, defined by J. B. Miller [Proc. London Math. Soc. (3) 8 (1958), 224-241]. If a function $f(x)$ belongs to $G_{\lambda+t}^2$, then there exists $g(x)$, also belonging to $G_{\lambda+t}^2$, defined by $g(x) = \int_{\to 0}^{\to\infty} f(t)\, X_k(xt)dt$, $x > 0$, where $X_k(x)$ is a truncated Hankel kernel expressed in terms of the Bessel function $J_k(x)$, $Y_k(x)$ and $K_k(x)$. Further $f(x) = \int_{\to 0}^{\to\infty} g(t) X_k(xt)dt$, $x > 0$. Theorem. If k is a real number greater than 1, λ is an odd integer greater than $k + 1$ and τ is a real number greater than

$k + 2$, then $\lim_{N\to\infty}\{\sum_{n=1}^{N}\sigma_k(n)n^{(1/2)k}f(n)(1 - n/N)^{\tau} - \zeta(1 + k)\int_0^N x^{(1/2)k}f(x)(1 - x/N)^{\tau}dx$ $\zeta(1 - k)\int_0^N x^{(1/2)k}f(x)(\cos 2\pi x - \sum_{n=0}^{[(k-1)/4]}(-1)^n(2\pi x)^{2n}/(2n)!)(1 - x/N)^{\tau}dx\} =$ $\lim_{N\to\infty}\{\sum_{n=1}^{N}\sigma_k(n)n^{-(1/2)k}g(n)(1 - n/N)^{\tau} - \zeta(1 + k)\int_0^N x^{(1/2)k}g(x)(1 - x/N)^{\tau}dx - \zeta(1 - k)\int_0^N x^{-(1/2)k}$ $\cdot g(x)(\cos 2\pi x - \sum_{n=0}^{[(k-1)/4]}(-1)^n(2\pi x)^{2n}/(2n)!(1 - x/N^{\tau}dx\}$. Here $\zeta(x)$ is the Riemann zeta-function and $[x]$ denotes the greatest integer not greater than x. (Received November 2, 1967.)

68T-46. A. B. BUCHE, Center for Research in Probability, Mackenzie Hall, Wayne State University, Detroit, Michigan 48202. Approximation of semigroups of operators in Fréchet spaces.

In this paper approximation theorems for semigroups of operators in Fréchet spaces are obtained which generalize the Banach space results of Trotter (Pacific J. Math. 8 (1958), 887-919). Let $\mathfrak{X}$ be a Fréchet space with a countable family of seminorms $\{p_\gamma, \gamma \in \Gamma\}$; and let $\{\mathfrak{X}_n\}$ be a sequence of Fréchet spaces (with associated projections $P_n: \mathfrak{X} \to \mathfrak{X}_n$) approximating $\mathfrak{X}$. Let $\{T_n(t), t \geqq 0\}$ be a sequence of semigroups of class (C_0) with associated resolvent operators $\{R_n(\lambda)\}$; and let $R(\lambda)$ be a resolvent operator. A typical result can be stated as follows: If for each $x \in \mathfrak{X}$ and $\gamma \in \Gamma$, $p_\gamma(x - P_n x) \to 0$, $p_\gamma(T_n(t)P_n x) \leqq M_\gamma p_\gamma(P_n x)$, $p_\gamma(\lambda^m R_n^m(\lambda)P_n x) \leq M_\gamma p_\gamma(P_n x)$, and $R_n(\lambda) \to R(\lambda)$ densely, then strong $\lim_{n\to\infty} T_n(t) = T(t)$, where $\{T(t), t \geqq 0\}$ is a semigroup of class (C_0) in $\mathfrak{X}$ with resolvent operator $R(\lambda)$. As a concrete example, an approximation problem is considered in the Fréchet space of infinitely differentiable functions utilizing Chlodovsky's generalizations of Bernstein polynomials on an infinite interval (Compositio Math. 4 (1937), 380-393). (Received November 5, 1967.)

68T-47. N. M. RIVIERE, University of Minnesota, Minneapolis, Minnesota, and Y. SAGHER University of Illinois, Chicago, Illinois 60680. On multipliers of trigonometric series.

Let $T_u^p(x_0)$ be the space of all periodic (2π) functions in L^p, satisfying $\{(1/\rho)\int_{-\rho}^{\rho}|f(x + t) - P_m(t)|^p dt\}^{1/p} \leqq A\rho^u$ where P_m is a polynomial of degree m, and $m < u$. Let $t_u^p(x_0)$ be the space of all functions in $T_u^p(x_0)$ satisfying $\{(1/\rho)\int_{-\rho}^{\rho}|f(x + t) - P_m(t)|^p dt\}^{1/p} = o(\rho^u)$, where $m \leqq u$. These spaces were introduced by A. P. Calderón and A. Zygmund. Theorem 1. If $\Delta^j\{\lambda_n\} = O(1/nj)$ for $j = 0,1,\ldots, m + 2$, and if $f \sim \sum C_n e^{inx}$, $\Lambda f = \sum \lambda_n C_n e^{inx}$ then: Λ preserves $T_u^p(x_0)$, $t_u^p(x_0)$, for all $u \leqq m$. Theorem 2. If $\Delta^j\{\lambda_n\} = O(1/nj)$ for $j = 0,1,\ldots, m + 3$, and if $\sum C_n e^{inx}$ converges (C,α) almost everywhere in E, $\alpha \leqq m$, then so does $\sum C_n \lambda_n e^{inx}$. (Received November 5, 1967.)

68T-48. L. E. PURSELL, University of Missouri, Rolla, Missouri. Rings of continuous functions on open convex subsets of R^n.

Let X and Y be open convex subsets of R^n, C(X) and C(Y) be their respective rings of real continuous functions, and let $\phi: C(X) \to C(Y)$ be a ring isomorphism. We give a construction of the homeomorphism $h: X \to Y$ such that $\phi f = f \circ h^{-1}$ for all f in C(X) which does not involve ideals. Using the theorems: (i) A function in C(X) can be uniformly approximated on X by a function in the subring $C^\infty(X)$ of infinitely differentiable functions in C(X) (L. E. Pursell, Uniform approximation of real continuous functions on the real line by infinitely differentiable functions, Mathematics Magazine, (to appear)), (ii) If f is in $C^{\infty}(X)$ and $(a_1,\ldots,a_n)$ is in X, then there are $f_1,\ldots,f_n$ in $C^{\infty}(X)$ such that $f(r_1,\ldots,r_n) = f(a_1,\ldots,a_n) + \sum_{i=1}^{n}(r_i - a_i)f_i(r_1,\ldots,r_n)$ for all $(r_1,\ldots,r_n)$ in X. (H. Whitney, Differentiability of the remainder term in Taylor's formula, Duke Math. J. 10 (1943), 153-158.) and (iii) If a_X and a_Y are the constant functions in C(X) and C(Y) respectively such that $a_X(p) = a = a_Y(q)$ for all points p in X and q in Y, then $a_X = a_Y$ (L. Gillman and M. Jerison, Rings of continuous functions, Van Nostrand, 1960, p. 23); we show the desired homeomorphism $h = (x_1,\ldots,x_n)$ where $x_1,\ldots,x_n$ are the coordinate functions on X. (Received October 5, 1967.)

68T-49. H. B. KEYNES and J. B. ROBERTSON, University of California, Santa Barbara, California 93106. Ergodicity and mixing in topological transformation groups.

The notions of ergodicity and (weakly) mixing as introduced by H. Furstenberg, Math. Systems Theory 1 (1967), 1-51, are investigated with respect to eigenvalue properties of a related Banach algebra. Let X be a Baire Hausdorff space and $\mathscr{B}(X)$ the algebra of complex-valued bounded functions whose continuity points are a comeager set provided with the pseudo-norm $\|f\| = \sup\{|f(x)| : f \text{ continuous at } x\}$. Then f = g a.e. when f and g are in the same norm-induced equivalence class. Theorems analogous to measure-theoretic ergodic theory are obtained when a homeomorphism ϕ acts on X. Sample results are: Theorem 1. If ϕ^* is the induced isometry on $\mathscr{B}(X)$, then (X,ϕ^m) is ergodic iff $\phi^*(f) = \lambda f$ a.e. and $\lambda^m = 1$ implies f is a constant a.e. Theorem 2. If (X,ϕ) is weakly mixing and $\phi^*(f) = \lambda f$ a.e., then f is a constant a.e. A generalization of Furstenberg's construction in H. Furstenberg, Amer. J. Math. 85 (1963), 477-515, yields that if X is compact metric and (X,ϕ) is minimal, then (X,ϕ) is weakly mixing iff the only eigenvalues for ϕ^* are the constants. Generalizations of this last result with more complicated conditions are obtained. Applications to topological dynamics are given. A sample result is: Theorem 3. If X is a connected, simply-connected, compact metric space and (X,ϕ) is minimal, then (X,ϕ) is mixing. (Received November 6, 1967.)

68T-50. R. C. COURTER, Wayne State University, Detroit, Michigan 48202. Rings over which every right module in rationally complete.

Findlay and Lambek proved that a module is rationally complete (rational completeness is a weak form of injectivity by definition) if and only if it has no proper rational extension (Canad. Math. Bull. 1 (1958), 156). An R-module M is a rational extension of its submodule Y if and only if $\mathrm{Hom}_R(X/Y, M) = 0$ for all submodules X of M containing Y. The following is a consequence of Theorem A, Abstract 649-15, these *Notices* 14 (1967), 818. Theorem. In order for a ring R with identity to be such that every unital right R-module is rationally complete, it is necessary and sufficient that R be the direct sum of a finite set of R-ideals R_i where for each i R_i is a ring of n_i by n_i matrices over a ring S_i with identity element which satisfies (1) the nonunits of S_i form an ideal and (2) every nonzero homomorphic image of S_i has nonzero right socle. The proof depends largely on the fact that such rings have nil radicals. (Received November 6, 1967.)

68T-51. JEROME DANCIS, University of Maryland, College Park, Maryland. When isotopic submanifolds are ambient isotopic.

Theorem 1. Suppose $\{h_t/h_t: M \to Q, t \in [0,1]\}$ is an isotopy of a compact, topological m-manifold M into a topological q-manifold Q, $q - m \geqq 3$, such that $h_0(M)$ and $h_1(M)$ are locally-flat subsets of Int Q. Then there is an ambient isotopy $\{H_t/H_t: Q \to Q, t \in [0,1]\}$ and a compact set A such that $H_0 = 1$, $H_1 h_0 = h_1$ and $H_t|Q - A = 1$. Theorem 1 is a corollary of Theorems 2 and 3 which follow. Theorem 2. Let f be an embedding of a compact, topological m-manifold M into a q-manifold Q, $q - m \geqq 3$, and let $\epsilon > 0$ be given. Then there exists another embedding h of M into Q such that h(M) is locally-flat in Q and $d(f,h) < \epsilon$. Theorem 3. Let f, M and Q be as in Theorem 2. Given an $\epsilon > 0$, there is a $\delta > 0$ such that if h_0 and h_1 are two embeddings of into Int Q where $h_0(M)$ and $h_1()$ are locally-flat, $d(h_0,f) < \delta$ and $d(h_1,f) < \delta$, then there is an ambient isotopy $\{H_t/H_t: Q \to Q, t \in [0,1]\}$ such that $H_0 = 1$, $H_1 h_0 = h_1$, $d(H_t(x),x) < \epsilon$ for all $x \in Q$ and $H_t(x) = x$ whenever $d(x,f(M)) > \epsilon$ or $x \in \partial Q$. Bryant and Seebeck's corollaries to Homma's proof that Theorem 2 is valid when M and Q are combinatorial manifolds, $\partial M = \emptyset$ and h is a piecewise linear embedding, play an important role in the proofs of Theorems 2 and 3. (Received November 13, 1967.)

68T-52. R. P. AGARWAL, West Virginia University, Morgantown, West Virginia 26505. Certain basic integral operators and hypergeometric transformations.

The object of this paper is to exhibit the possibility of exploiting certain basic double integrals (as defined by W. Hahn, Math. Nachr. 2 (1949), 340-379) to find interesting integral formulae, series transformations and transformations between integrals for q-hypergeometric functions of one and two variables. In particular, the following two integral operators have been introduced $M^{s,t}_{x,y}[\Phi(-x,-y)] \equiv \int_0^\infty \int_0^\infty \Phi(-x,-y)x^{s-1}y^{t-1}d(q,x)d(q,y)$, $\Omega^{a;a}_{p;\eta,t,\lambda}[\Phi(x,y)] \equiv (1/(1-q)^2)\int_0^1\int_0^1 y^{p-1}(1-qy)_{\eta-1}x^{t-1}(1-qx)_{\lambda-1}(1-q^a xy)_{-a}\Phi(x,y)d(q,x)d(q,y)$. Suitable illustrations have been given to indicate the type of formulae one might obtain through such operators. (Received November 9, 1967.)

68T-53. A. M. CHAK, West Virginia University, Morgantown, West Virginia 26505. Polynomial systems satisfying a special functional equation.

Nielsen [Traité élémentaire des nombres de Bernoulli, Gauthier-Villars, Paris, 1923] and Ward [Ann. of Math. 31 (1930), 43-51] studied a set of polynomials $\{P_n(x)\}$ which satisfy the two functional equations (n = 0, 1, 2, 3,.... and a & b any complex numbers) $P_n'(x) = P_{n-1}(x)$; $P_n(ax+b) = \tau_n P(x)$. More recently Sharma and Chak [Riv. Mat. Universita di Parma 5 (1954), 325-337] and Al-Salam [q-Appel polynomials (in press)] studied the q-analogue of this set. It was a paper of Carlitz [Ann. Mat. Pura Appl., Ser. IV, 41 (1955), 359-373] which suggested the study of the properties of polynomial systems $\{H_n(x)\}$ satisfying a functional equation (k a real number; n = 0, 1, 2, 3,...) $D_q\{H_n(x)\} = H_{n-1}(q^k x)$ where $D_q f(x) = (f(qx) - f(x))/((q-1)x)$. In this paper we study these polynomial systems and also some subsets of this which have properties analogous to the regular sets of Nielsen and of Ward (for references see above). (Received November 9, 1967.)

68T-54. E. E. GRANIRER, University of British Columbia, Vancouver, B. C. Canada. Functional analytic properties of extremeley amenable semigroups.

S is an extremely left amenable (ELA) semigroup if there exists a multiplicative left invariant mean on the algebra of all bounded real functions on S, m(S). Theorem. The following are equivalent: (1) S is ELA. (2) For any normed space X and each anti-representation of S as linear operators T_s: $X \to X$ with $\|T_s\| \leqq 1$, dist(o,O(x)) = dist(o, Co O(x)) for all $x \in X$, where $O(x) = \{T_s x;\ s \in S\}$ and Co denotes convex hull. (3) For any ring A (not necessarily commutative) and any antirepresentation of S as ring homomorphisms T_s: $A \to A$, linear span $\{a - T_s a;\ a \in A,\ s \in S\} = \bigcup\{T_s^{-1}(0);\ s \in S\}$. (This set will necessarily be a two sided ideal.) (4) If A, B are algebras of bounded functions on X, Y resp with $1 \in A$, $1 \in B$ and if T_s: $A \to A$ is an antirepresentation of S as algebra homomorphisms then the convex set of all linear T: $A \to B$ with $T \geqq 0$, $T1 = 1$ and $TT_s = T$ for all $s \in S$ has extreme points, all of which are multiplicative (generalising a result of R. Phelps in Trans. Amer. Math. Soc. 108(1963), 265-274). Several other functional analytic properties of ELA semigroups including a version of the Alaoglu-Birkhoff ergodic theorem for the ELA case have been obtained. (Received September 28, 1967.)

68T-55. N. SANKARAN, Queen's University, Kingston, Ontario, Canada. Henselization and rings of formal power series.

A commutative ring R with identity is said to be Henselian at a maximal ideal M if R satisfies the Hensel's Lemma with respect to M. We prove Theorem 1. If R is a Henselian ring at the maximal ideal $M_1, M_2, \ldots$ then R[[X]] is also Henselian at the corresponding maximal ideal $M_1 R[[x]] + xR[[x]], \ldots$ where R[[x]] denotes the ring of formal power series in x on R. Theorem 2. If P is a prime ideal of R and $\bar{P}$ is the prime ideal of R[[x]] lying over P then the Henselization of R[[x]] at $\bar{P}$ is isomorphic with $\tilde{R}_{\bar{P}}[[x]]$ where $\tilde{R}_{\bar{P}}$ is the Henselization of R at P. (Received November 6, 1967.)

68T-56. G. H. PIMBLEY, University of California, Box 1663, Los Alamos, New Mexico 87544. A comparison of two conditions against secondary bifurcation of eigenfunctions of Hammerstein operators.

Let K(x,y) be an oscillation kernel, and consider the problem $\lambda w(x) = \int_0^1 K(x,y) f(y,w(y))dy$, where $f(x,-w) = -f(x,w)$, $f'_w(x,w) > 0$, $wf''_w(x,w) < 0$, $\lim_{w\to\infty}(f(x,w))/w = A(x) > 0$. The problem of secondary bifurcation of the branches of eigenfunctions was introduced in an earlier abstract [Abstract 639-5, these *Notices* 13 (1966), 830; see also J. Math. Mech. 12 (1963), 577]. The first condition states that there is no secondary bifurcation of the pth branch if f(x,w) is such that $wf'_w(x,w)/f(x,w) > \Lambda_p$, $0 \leqq w < \infty$, where $\Lambda_p = \sup_{\phi \in S_1^+} \mu_p(K,\phi)/\mu_{p-1}(K,\phi)$, $S_1^+ = [\phi \mid \|\phi\| = 1,\ \phi \in C(0,1),\ \phi \geqq 0]$, and the maximum is assumed for $\phi = \hat{\phi}$. Here $\mu_p(k,\phi)$ is the pth eigenvalue of the operator $\int_0^1 K(x,y)\phi(y)\cdot dy$. Now let $f(x,w) = \phi(x)g(x,w)$, where $\lim_{w\to 0}(g(x,w))/w = 1$, $\lim_{w\to\infty}(g(x,w))/w = A$, a constant. For any $\phi \in S_1^+$, the second condition against secondary bifurcation of the pth branch requires that $A > \mu_p(K,\phi)/\mu_{p-1}(K,\phi)$. If $\phi = \hat{\phi}$, the first condition is less restrictive than the second. If the second condition is satisfied for $f(x,w) = \phi(x)g(x,w)$, and $\mu_p(K,\hat{\phi})/\mu_{p-1}(K,\hat{\phi}) \leqq A$, then likewise the first condition is less restrictive. (Received November 6, 1967.)

68T-57. B. P. GELBAUM, University of California, Irvine, California 92664. Fibre bundles and tensor products.

Let E be a fibre bundle with fibre A, a commutative Banach algebra, base X, a compact Hausdorff space and group $\mathscr{A}$, the set of isometric $\mathbb{C}$-algebra automorphisms of A. The set $\Gamma(E)$ of continuous sections is a commutative Banach algebra in a natural (sup-like) norm and the maximal ideal space $\mathscr{M}_{\Gamma(E)}$ in the hull-kernel topology is a fibre bundle with fibre $\mathscr{M}_A$, in the hull-kernel topology, base X and group $\mathscr{A}$ of auteomorphisms of $\mathscr{M}_A$ in the hull-kernel topology. Since E is locally $A \times U$, U, a closed neighborhood in X, $\Gamma(E)$ is locally C(U;A), which for a suitable tensor product norm α is $A \otimes_\alpha C(U)$. Since C(U) is C(X)/N(U), where N(U) is the ideal of functions vanishing on U, we are led to a fibre-tensor bundle notion: A Banach algebra D obtained from (i) a pair A and B of commutative Banach algebras, (ii) a family $\{I\}$ of closed ideals such that int $h(I) \neq \emptyset$, $\bigcup_I$ int $h(I) = \mathscr{M}_B$, (iii) identifying elements in the disjoint union of the $A \otimes_\alpha B/$ by means of a cocycle of transition maps g_{IJ} in $\mathscr{A}$ (the last defined iff int $h(I) \cap$ int $h(J) \neq \emptyset$). Then $\mathscr{M}_D$ is a fibre bundle with fibre A, base $\mathscr{M}_B$ in the hull-kernel topology and group $\mathscr{A}$. (Received November 6, 1967.)

68T-58. A. C. SUGAR, Bradley University, Peroria, Illinois 61606. The nature of a crucial inadequacy in the theory of relativity.

If a point of light A moves to the left from a point source S and a point B simultaneously moves to the right, then the rate of separation of A and B is 2c, twice the velocity of light. We conceive of the notion of a set V of inviolate intuitions and construe the above phenomenon as an inviolate physical intuition which belongs to V. Clearly intellectural chaos would be a consequence of taking the point of view that, because some are, all intuition are unreliable. The kind of intuitions one accepts in intuitive logic is much more sophisticated than the above inviolate physical intuition. Yet with the exception of the intuitionists, mathematicians accept all of them, and we here construe them to be inviolate intuitions. Nor can one take refuge in mathematical logic since we use intuitive logic to construct mathematical logic. Since the Lorentz factor $(1 - v^2/c^2)^{-1/2}$ becomes complex for $v > c$, it is a theorem of relativity that no point can have a velocity with respect to another point greater than that of light. Furthermore, the Lorentz transformation measures the velocity of B relative to A as c, which contradicts the inviolate physical intuition. From this simple predictive failure we conclude that relativity must be considered to be a mathematical theory. It is probably as consistent as any accepted theory of mathematics. However, if relativity interpreted as a physical theory cannot predict in the simple environment of our inviolate physical intuition, how can one rely on it to predict in complex environments? (Received November 7, 1967.)

68T-59. R. F. DE MAR, University of Cincinnati, Cincinnati, Ohio 45221. Uniqueness classes for periodic-type functionals.

Let K be the class of entire functions of exponential type and let $K[\Omega]$ be the set of F in K whose Laplace transforms are analytic on the complement of the simply connected domain Ω. Let $\{L_n\}$ be a sequence of linear functionals defined on $K[\Omega]$ by (1) $L_n(F) = (2\pi i)^{-1} \int_\Gamma g_n(\zeta) f(\zeta) d\zeta$ where $\Gamma \leq \Omega$ encloses all singularities of f, the Laplace transform of F, and each g_n is analytic on Ω. Then $K[\Omega]$ is a uniqueness class for $\{L_n\}$ if $F \in K[\Omega]$ and $L_n(F) = 0$; $n = 0,1,\ldots$ implies $F = 0$. The main result is roughly that if $\{L_n\}$ is given by (1) with $g_{pn+k}(\zeta) = h_k(\zeta)[W(\zeta)]^{pn}$; $k = 0,1,\ldots, p-1$; $n = 0,1,\ldots$, and if $W(\Omega)$ has the property that $\alpha W(\Omega) = W(\Omega)$ where α is a primitive pth root of 1, then uniqueness depends on the univalence of W on Ω and the zeros of $\Delta(\zeta) = \det(h_k(Z(\alpha^j W(\zeta))))$ where Z is the inverse of W. If W is univalent on Ω and $L_n(F) = 0$, $n = 0, 1,\ldots$ for $F \in K[\Omega]$, then $F(z) = \sum_{j=1}^{N} u_j(z) \exp(\beta_j z)$ where the u_j are polynomials of degree less than the order of the zero of Δ at β_j and $\{\beta_j\}_{j=1}^{N}$ are all the zeros of Δ in Ω other than the origin. This generalizes the author's result for $g_n(\zeta) = [W(\zeta)]^n$ (Proc. Amer. Math. Soc. 16 (1965), 69-71). (Received November 6, 1967.)

68T-60. K. R. MEYER, University of Minnesota, Minneapolis, Minnesota. An estimate on the number of periodic solutions of an ordinary differential equation.

Let $\{\phi_t\}_{t\in R}$ be a one-parameter group of diffeomorphisms of a compact Riemannian manifold M. Let Ω be the set of nonwandering points. The set Ω has a hyperbolic structure (U structure) with respect to $\{\phi_t\}$ if there exists a continuous splitting of the tangent spaces $T_p = E_p^u \oplus E_p^s$, $p \in \Omega$ such that $d\phi_t : E_p^u \to E_q^u$ and $d\phi_t : E_p^s \to E_q^s$; $q = \phi_t(p)$ and $\|d\phi_t(u)\| \geq Ce^{\mu t}\|u\|$, $u \in E_p^u$; $\|\phi_t(v)\| \leq C^{-1}e^{-\mu t}\|v\|$, $v \in E_p^s$ where C and μ are constants $0 < C < 1$ and $\mu > 0$. Theorem. If $\{\phi_t\}$ has a hyperbolic structure on the set of nonwandering points then there exist constants K and β such that the number of periodic orbits of $\{\phi_t\}$ of period less that τ is less than $Ke^{\beta\tau}$. (Received November 5, 1967.)

68T-61. H. D. KAHN, Louisiana State University, New Orleans, Louisiana 70122. Covering semigroups. II.

This paper contains results on covering semigroups additional to those announced in Abstract 67T-377, these *Notices* 4 (1967), 531. Let $(\widetilde{S},\phi)$ be a connected, locally connected covering semigroup of the semigroup S with 1 the identity of S lifting to $\widetilde{1}$ the identity of $\widetilde{S}$. Then ϕ induces a bijective correspondence of the $\mathscr{H}$, $\mathscr{L}$, $\mathscr{R}$, $\mathscr{D}$, and $\mathscr{J}$ classes of $\widetilde{S}$ with the $\mathscr{H}$, $\mathscr{L}$, $\mathscr{R}$, $\mathscr{D}$, and $\mathscr{J}$ classes of S, respectively. Let H be an $\mathscr{H}$-class in S and $\widetilde{H} = \phi^{-1}(H)$ the corresponding $\mathscr{H}$-class in $\widetilde{S}$. If $\Gamma(\widetilde{H})$ and $\Gamma(H)$ are the left Schützenberger groups of $\widetilde{H}$ and H, then there exists a natural epimorphism $\theta : \Gamma(\widetilde{H}) \to \Gamma(H)$ with kernel isomorphic to P(S), the fundamental group of S. If K is the minimal ideal of S, then $\widetilde{K} = \phi^{-1}(K)$ is the minimal ideal of $\widetilde{S}$. If K is a retract of S, then $\widetilde{K}$ is a retract of $\widetilde{S}$. Moreover, $(\widetilde{K},\phi|\widetilde{K})$ is a simply connected covering space of K, and the fundamental groups of S and K are isomorphic. If K is also iseomorphic to a topological paragroup $[X,G,Y]_\sigma$ (Hofmann and Mostert, Elements of compact semigroups, Merrill, 1966), then P(S) is isomorphic to P(G), the fundamental group of any maximal subgroup of K. (Received November 6, 1967.)

68T-62. J. W. BRACE, University of Maryland, College Park, Maryland 20742, G. D. FRIEND and P. J. RICHETTA, Lehigh University, Bethlehem, Pennsylvania. Locally convex topologies on function spaces.

Every locally convex linear topology on a function space of scalar valued linear functions can be represented as convergence on a family of filters, each filter being composed of subsets from the domain. Application of this result to a dual pair of linear spaces gives the satisfying result that every locally convex topology on a member of the dual pair can be represented as convergence on a family of filters, each filter being composed of subsets from the other member of the dual pair.

Spaces of nonlinear and/or vector valued functions are treated by reduction to above case. The concept of convergence on a filter was originally presented by the first author in the Illinois J. Math. 9 (1965), 286-296. Convergence on a family of filters is developed in analogy with the theory of uniform convergence on a family of sets. Uniform convergence remains as a special case of convergence on filters. Representations of the Weak and the Mackey topology are obtained. (Received November 7, 1967.)

68T-63. A. P. STONE, University of Illinois, Chicago, Illinois 60680. Higher order conservation laws.

Let $\mathscr{S}$ be the localization of the A-module of differential forms on an analytic manifold where A is the ring of germs of analytic functions at a point of the manifold. The A-module is free and finitely generated of dimension n, and $\Lambda^{*}\mathscr{S}$ (the exterior algebra generated by $\mathscr{S}$) has the form $\Lambda^{*}\mathscr{S} = \Lambda^0\mathscr{S} \oplus \Lambda^1\mathscr{S} \oplus \dots \oplus \Lambda^n\mathscr{S}$, where $\Lambda^0\mathscr{S} = A$ and $\Lambda^1\mathscr{S} = \mathscr{S}$. If $\underline{h} \in \mathrm{Hom}_A(\mathscr{S},\mathscr{S})$ then $\underline{h}$ induces transformations $\Lambda^p\mathscr{S} \xrightarrow{h_p^{(q)}} \Lambda^p\mathscr{S}$, where $q = 0,1,2,\dots,p \leqq n$. If $\theta_i \in \mathscr{S}$ then these transformations are defined by setting $h_p^{(q)}(\theta_1 \wedge \dots \wedge \theta_p) = (1/(p-q)!q!)\sum_\pi |\pi| \cdot (\underline{h}\theta_{\pi(1)} \wedge \dots \wedge \underline{h}\theta_{\pi(q)}) \wedge \theta_{\pi(q+1)} \wedge \dots \wedge \theta_{\pi(p)}$ where π runs through all permutations of $(1,2,\dots,p)$ and $|\pi|$ denotes the signature of the permutation π. An element $\psi \in \Lambda^p\mathscr{S}$ is called a conservation law of order p for $h_p^{(0)}, h_p^{(1)},\dots,h_p^{(p)}$ if and only if ψ, $h_p^{(1)}\psi,\dots,h_p^{(p)}\psi$ are all locally exact p-forms. If the Nijenhuis tensor $[\underline{h},\underline{h}]$ of $\underline{h}$ vanishes identically and $\underline{h}$ has distinct eigenvalues then the existence of an eigenvector basis of exact forms $dv^1,\dots,dv^n$ for $\mathscr{S}$ is guaranteed and the following theorem is obtained. Theorem. If $[\underline{h},\underline{h}] = 0$ and $\underline{h}$ has distinct eigenvalues then $\psi \in \Lambda^p\mathscr{S}$ is a pth order conservation law if and only if $\psi = \sum_{i_1<\dots<i_p} f_{i_1\dots i_p}(v^{i_1},\dots,v^{i_p})dv^{i_1}\wedge\dots\wedge dv^{i_p}$. (Received November 5, 1967.)

68T-64. GEORGE ZAMES and M. I. FREEDMAN, NASA/Electronics Research Center, Code GOT, 575 Technology Square, Cambridge, Massachusetts 02139. Logarithmic variation criteria for L_2 stability of solutions of a class of integral equations.

The integral equation (*) $e(t) + k(t) \cdot (Ge)(t) = x(t)$ is considered, where $(Ge)(t)$ represents convolution of $e(t)$ by a kernel $g(t)$ defined and real-valued on $[0,\infty)$. The equation (*) will be termed L_2-stable if there exists $K > 0$ such that given any pair $(e(t), x(t))$ of real-valued functions on $[0,\infty)$ satisfying (*), with $x(t) \in L_2[0,\infty)$ and $e(t) \in L_2[0,T]$ for all $T > 0$, then $e(t) \in L_2[0,\infty)$ and, in fact, $\|e(t)\|_2 \leqq K \cdot \|x(t)\|_2$. Theorem. Let a and b be real constants with $a < \inf k(t)$ and $\sup k(t) < b$. Assume there is a $\mu > 0$ with $g(t)e^{-\mu t} \in L_1[0,\infty)$, and also a $\mu_1 > 0$ such that the Laplace transform $G(s)$ of $g(t)$ has a meromorphic continuation defined on $\mathrm{Re}\{s\} \geqq -\mu_1$ and satisfying the condition $\lim_{|s|\to\infty} G(s) = 0$ in $\mathrm{Re}\{s\} \geqq -\mu_1$. Let σ, with $0 < \sigma < \mu_1$, be such that, for any $k \in [a,b]$, $1 + kG(s)$ has no zeroes in $\mathrm{Re}\{s\} \geqq -\sigma$. Finally let there exist $T > 0$ such that for all $t \geqq 0$ $1/T\int_t^{t+T}(d/d\xi)\log(k(\xi)-a)/_{b-k(\xi)}\,d\xi < 4\sigma$. Under these hypotheses equation (*) is L_2-stable. (Received November 8, 1967.)

68T-65. HELEN SKALA, Illinois Institute of Technology, Chicago, Illinois 60608. Modularity implication in sets with projective laws.

Let $\cup$ and $\cap$ be two operations satisfying $X \cup ((X \cup Y) \cap Z) = X \cup ((X \cup Z) \cap Y)$ and $X \cap ((X \cap Y) \cup Z) = X \cap ((X \cap Z) \cup Y)$, "projective laws", for any three elements X, Y, Z of a set that includes two elements U and V such that $V \cup X = X = U \cap X$ and $V \cap X = V$, $U \cup X = U$ for each X. Then the operations are commutative, idempotent, absorptive (i.e., $X \cup (X \cap Y) = X = X \cap (X \cup Y)$), and alternative (cf. Menger, Ann. of Math. 37 (1936), 456-481 and C. R. Paris 206 (1938), 308-310). The operations also satisfy the modularity laws: $X \subseteq Z$ (i.e., $X \cap Z = X$ or, equivalently, $X \cup Z = Z$) implies $X \cup (Y \cap Z) = (X \cup Y) \cap Z$ for any Y. The associative law and the transitivity of the relation $\subseteq$ are equivalent under the above assumptions, but, as simple examples show, independent of them. (Received November 13, 1967.)

68T-66. E. B. SAFF, University of Maryland, College Park, Maryland. Approximation by rational functions of type (n,).

Let C be a Jordan curve of the z-plane; $D \equiv \operatorname{int} C$, and g(z) a function defined (finite) on C. Theorem 1. A necessary and sufficient condition for g(z) to be the boundary values on C of a function f(z) which is meromorphic in D with at most ν poles there and continuous on D + C is that there exist a sequence of rational functions $r_{n\nu}(z)$ of respective types (n,ν), i.e., $r_{n\nu}(z)$ is the quotient $(a_0z^n + \ldots + a_n)/(b_0z^\nu + \ldots + b_\nu)$, $\sum|b_k| \neq 0$. such that (*) $|g(z) - r_{n\nu}(z)| \leqq \epsilon_n(\longrightarrow 0)$ for z on C. Theorem 2. If inequality (*) holds and f(z) has precisely ν poles in D, then (1) For n large enough each $r_{n\nu}(z)$ has precisely ν finite poles which approach respectively the ν poles of f(z) in D. (2) If $S \subset \{D - [\nu \text{ poles of } f(z)]\}$ is closed, then $|f(z) - r_{n\nu}(z)| \leqq M(S)\epsilon_n$ for z on S, where M(S) is a constant dependent only on S and on the sequence $r_{n\nu}(z)$. Theorem 1 is proved by the known result for $\nu = 0$ and a theorem of S. Warschawski [Math. Z. 38 (1934), 669]. Theorem 2 follows from the theory of normal families. (Received November 13, 1967.)

68T-67. K. F. BARTH and W. J. SCHNEIDER, Syracuse University, Syracuse, New York 13210. On the impossibility of extending the Riesz uniqueness theorem to holomorphic functions of slow growth.

Theorem. Let (r) be a function on [0,1) satisfying $0 < \mu(r) \uparrow \infty$. Then there exists a non-constant function h(z), holomorphic in $|z| < 1$. Such that (i) $\max_\theta |h(re^{i\theta})| < \mu(r)$, (ii) for almost all θ, $\lim_{r \to 1} h(re^{i\theta}) = 0$. The proof consists of constructing, by means of Mergelyan's theorem, polynomials $P_n(z)$ with the properties (i) $|P_n(z)| < \alpha_n$ in $|z| < 1 - \beta_n$, (ii) Re $P_n(z)$ is never positive and eventually becomes large and negative on a set of unit radii of measure $2\pi - \gamma_n$ where α_n, β_n and γ_n are positive numbers, which are chosen to tend to zero sufficiently quickly. The proof now follows by setting $h(z) = e^{g(z)}$ where $g(z) = \sum_{n=1}^{\infty} P_n(Z^{m_n})$ and the integers m_n are chosen to go to infinity sufficiently fast. The authors first proof was a very computationally involved potential-theoretic argument. The above argument is a variation of a method used by G. R. MacLane (Michigan Math. J. 9 (1962), 21-24) to show the impossibility of extending the Fatou radial limit theorem to holomorphic functions of slow growth. (Received November 14, 1967.)

68T-68. W. J. GORDON, General Motors Research Laboratories, Warren, Michigan 48090. Bivariate interpolation through curve networks.

Let a network of functions $g_i(y)$ ($i = 0,1,\ldots,M$) and $f_j(x)$ ($j = 0,1,\ldots,N$) be defined over the grid $x = x_i$ ($i = 0,1,\ldots, M$), $y = y_j$ ($j = 0,1,\ldots,N$) on the x,y-plane, and let $g_i(y_j) = f_j(x_i)$. Also, on the perimeter of the rectangle $R = [x_0,x_M] \times [y_0,y_N]$, let compatible boundary conditions be specified on the first $k - 1$ normal derivatives, $\partial^\mu u/\partial n^\mu$ ($\mu = 1,2,\ldots,k - 1$), of a bivariate function $u(x,y)$. Now, if the set of functions $\{\phi_m(x)|m = 1,2,\ldots,M + 2k - 1\}$ is a basis for a linear space $V_{M+2k-1}(x)$ in which the univariate interpolation problem defined by the conditions $f(x_0)$, $f^{(1)}(x_0),\ldots,f^{(k-1)}(x_0)$, $f(x_1)$, $\ldots$, $f(x_M)$, $f^{(1)}(x_M),\ldots,f^{(k-1)}(x_M)$ is uniquely soluble, and if the set $\{\psi_n(y)|n = 1,2,\ldots, N + 2k - 1\}$ is a similarly defined basis for some linear space $V_{N+2k-1}(y)$, then there exists a unique function $Z(x,y)$ which is the sum of pairwise products of the given functions $f_j(x)$, $g_i(y)$ and the boundary conditions $\partial^\mu u/\partial n^\mu$ with the basis functions $\phi_m(x)$ and $\psi_n(y)$, and which satisfies $Z(x_i,y) = g_i(y)$ ($i = 0,1,\ldots,M$), $Z(x,y_j) = f_j(x)$ ($j = 0,1,\ldots,N$) and $\partial^\mu Z/\partial n^\mu = \partial^\mu u/\partial n^\mu$ on the boundary of R. The differentiability properties of the univariate functions used in the construction endow $Z(x,y)$ with analogous bivariate differentiability properties. Moreover, if the spaces $V_{M+2k-1}(x)$ and $V_{N+2k-1}(y)$ are spaces of $2k - 1$ degree polynomial splines with joints at the x_i and y_j respectively, then the function $Z(x,y)$ is the solution to the variational problem: Minimize $\iint_R\{\partial^{2k}/(\partial x^k\partial y^k)\,u(x,y)\}^2\,dxdy$ subject to the given auxiliary conditions. Although spline functions of odd-degree play a central role in the construction of this solution, the function $Z(x,y)$ is not, in general, a bivariate spline function. (Received November 13, 1967.)

68T-69. J. L. WALSH and E. B. SAFF, University of Maryland, College Park, Maryland 20740. Extensions of D. Jackson's theorem on best complex polynomial mean approximation.

Let $f(z)$ be of class L_p on an analytic Jordan curve C in the z-plane. Theorem. If $P_n(z)$ is a sequence of polynomials of respective degrees n of best qth power approximation to $f(z)$ on C, and $p_n(z)$ is any sequence of polynomials of respective degrees n, then for $0 < q < p \leqq \infty$ and norms on C, one has $\|f(z) - P_n(z)\|_p \leqq An^{1/q-1/p}\|f(z) - p_n(z)\|_p$, where A is a constant independent of n and z. In particular, a sufficient condition for the $P_n(z)$ to converge to $f(z)$ in the mean of order p on C is $n^{1/q-1/p}\|f(z) - p_n(z)\|_p \to 0$ as $n \to \infty$. This theorem includes Jackson's theorem [Bull. Amer. Math. Soc. 36 (1930), 851] as the special case $p = \infty$. Analogues exist for best approximation by rational functions, trigonometric polynomials, and bounded analytic functions. (Received November 13, 1967.)

68T-70. W. S. MAHAVIER, Emory University, Atlanta, Georgia 30322. Arcs in inverse limits on [0,1] with only one bonding map.

By a continuum we mean a nondegenerate, compact, connected metric space. It is well known that a continuum is chainable iff it is homeomorphic to the inverse limit of a sequence of maps from [0,1] onto [0,1]. We are interested in chainable continua which are homeomorphic to an inverse limit on [0,1] with only one bonding map. It is known that a pseudo arc is such a continuum and that there is a chainable continuum which is not homeomorphic to an inverse limit on [0,1] with only one bonding map. In this note we show that if f is a piece-wise monotonic map of [0,1] onto [0,1], then each sub-continuum of lim f contains an arc. Here lim f denotes the subspace of $[0,1]^{\infty}$ consisting of all number sequences $\{x_i\}$ such that for $i > 0$, $f(x_{i+1}) = x_i$. (Received November 13, 1967.)

68T-71. W. D. L. APPLING, North Texas State University, Denton, Texas. A near-point theorem.

U, F, R_A, A and $\|g\|$, for g in R_A, are as in Abstract 67T-501, these *Notices* 14 (1967), 707. Theorem. If f is in R_A and $\{g_i\}_{i=1}^{\infty}$ is a sequence of elements of the range of A and $\|f - g_n\| \to \|f - A(f)\|$ as $n \to \infty$, then $\|A(f) - g_n\| \to 0$ as $n \to \infty$. (Received November 13, 1967.)

68T-72. WITHDRAWN.

68T-73. R. A. ALO, H. L. SHAPIRO, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213. A note on compactifications and semi normal spaces.

Recently Orrin Frink [Compactifications and seminormal spaces, Amer. J. Math. 86 (1964), 602-607] gave a neat internal characterization of Tychonoff spaces. This characterization was given in terms of the notion of a normal base for the closed sets of a space X. In this note we give necessary and sufficient conditions for a Hausdorff compactification to be a Wallman-type compactification. These are given in terms of conditions imposed on the normal base $\mathcal{Z}$. Theorem 1. Let Y be a Hausdorff compactification of a T_1-space X, let g be the embedding of X into Y and let $\mathcal{Z}$ be a normal base on X that satisfies the following property: (P) For each y in Y and each neighborhood V of y there is a Z in $\mathcal{Z}$ such that $y \in \mathrm{cl}\, g(Z) \subset V$ and cl g(Z) is a neighborhood of y. Then there is a (closed) continuous map f of $\omega(\mathcal{Z})$ onto Y such that f agrees with g on X. Conversely, if f is a homeomorphism of $\omega(\mathcal{Z})$ onto Y that leaves X pointwise fixed, then condition (P) is satisfied. Theorem 2. Let Y be a Hausdorff compactfication of X. Then Y is homeomorphic to a Wallman-type compactification of X iff X has a normal base $\mathcal{Z}$ that satisfies: (a) $\mathrm{cl}_Y(A \cap B) = \mathrm{cl}_Y A \cap \mathrm{cl}_Y B$ for all A, B in $\mathcal{Z}$ and (b) For each y in Y and each neighborhood V of y there is a Z in $\mathcal{Z}$ such that $y \in \mathrm{cl}_Y Z \subset V$. (Received November 10, 1967.)

68T-74. H. P. ROSENTHAL, University of California at Berkeley, Berkeley, California 94720. On quasi-complemented subspaces of Banach spaces. Preliminary report.

Let $\underline{X}$ be a (real or complex) Banach space and A be a closed subspace of $\underline{X}$. A is said to be quasi-complemented if there exists a closed subspace B of $\underline{X}$ with A + B dense in $\underline{X}$ and $A \cap B = \{0\}$. Assume from now on that A* is separable in the weak* topology. (Note that this holds if A is separable.) Theorem 1. If $A^{\perp}$ contains an infinite dimensional reflexive subspace, then A is quasi-complemented. Theorem 2. Let S be an infinite compact Hausdorff space, and let $A \subset C(S)$. Then A is quasi-complemented if any of the following hold: (a) A is separable and S is extremely disconnected (the closure of every open set is open). (b) C(S)/A is isomorphic (linearly homeomorphic) to a conjugate space. (c) S is perfect and no infinite dimensional subspace of A* is isomorphic to Hilbert space. Theorem 2 is deduced from Theorem 1 by showing that in all the cases, $A^{\perp}$ contains an infinite dimensional reflexive subspace (provided in case (b), that A is not of finite codimension). A partial converse to

Theorem 1 and extensions of Theorem 2 hold in suitable spaces $\underline{X}$. Applications. c_0 is quasi-complemented in m and H^{∞} is quasi-complemented in L^{∞}. (Received November 10, 1967.)

68T-75. DAVID GALE, University of California, Berkeley, California, VICTOR KLEE and R. T. ROCKAFELLER, University of Washington, Seattle, Washington. Convex functions on convex polytopes.

The following results of the paper are useful in mathematical economics and other applications of optimization theory: A (real-valued) convex function on a convex polytope is upper semicontinuous. A convex function on the relative interior of a convex polytope admits a unique continuous extension to the entire polytope. More generally, the results of the paper show that the convex polytopes and closely related sets are exactly those domains (in finite-dimensional Euclidean spaces) whose convex functions have nice boundary behavior. Various aspects of this behavior (upper semicontinuity, upper Lipschitzianity, being bounded, attaining a maximum, and related conditions) are sorted into three groups, characterizing respectively the domains which are polytopes, those which are boundedly polyhedral, and those which can be expressed as the vector sum of a polytope and a closed convex cone. (Received February 27, 1967.)

68T-76. S. A. SAWYER, Brown University, Providence, Rhode Island 02912. Continuous functions which attain every level line at most finitely often.

Let $f(x) \in C[0,1]$ be such that $\text{card}\{x : f(x) = y\} < \infty$ for all y, or, more generally, is infinite only for a set of y of the first (Baire) category. Then every interval $(a,b) \subseteq [0,1]$ contains an interval $(c,d) \subseteq (a,b)$ on which f(x) is monotonic. In S. Sawyer, Some top. prop. of the function n(y), Proc. Amer. Math. Soc. 18 (1967), 35-40, we concluded that f(x) was of bounded variation on (c,d). It is an observation of A. P. Morse (2-25-67) that the method can be extended to prove that f(x) is actually monotone on a perhaps smaller interval $(c', d') \subseteq (c,d)$. (Received November 15, 1967.)

68T-77. C. H. FARMER, 1701 Guadalupe Street, Austin, Texas. Pseudo-δ-mixing measure-preserving transformations on a probability space. Preliminary report.

Mixing transformations are characterized in terms of pseudo-δ-mixing transformations. A weakened form of pseudo-δ-mixing transformations is introduced and is called almost-pseudo-mixing. If a transformation is pseudo-δ-mixing on a field, then the transformation is shown to be almost-pseudo-δ-mixing on the sigma-field generated by the field. A δ-ergodicity is introduced and analogous results for ergodic transformations are obtained. Finally, pseudo-δ-mixing is shown to be an invariant under isomorphism. (Received November 8, 1967.)

68T-78. GARY CHARTRAND, Western Michigan University, Kalamazoo, Michigan 49001, and H. V. KRONK, State University of New York, Binghamton, New York. Randomly traceable graphs.

A graph G is called randomly traceable if a hamiltonian path always results upon starting at any vertex of G and successively proceeding to any adjacent vertex not yet encountered. Theorem. A graph G with p vertices is randomly traceable if and only if it is one of the following: (i) the cycle

C_p with $p \geqq 3$ vertices, (ii) the complete graph K_p, (iii) the complete bipartite graph $K(p/2, p/2)$, where (iii) is possible only if p is even. (Received November 9, 1967.)

68T-79. D. P. GELLER, University of Michigan, Ann Arbor, Michigan 48106. Square roots of digraphs.

Mukhopadhyay (The square root of a graph, J. Combinatorial Theory 2 (1967), 290-295) has characterized graphs G for which there exist a graph H such that $G = H^2$. Such a graph H is a square root of G. A similar question may be answered for digraphs (directed graphs). Let S and T be two point sets, not necessarily disjoint or nonempty, and let $u \notin S \cup T$. The carrier-complete digraph $K(S,u,T)$ has point set $V = S \cup \{u\} \cup T$ and arc set $X = S \times \{u\} \cup \{u\} \times T \cup S \times T$. Theorem. A digraph D with p points $u_1, u_2, \ldots, u_p$ has a square root if and only if there exists a collection of p subdigraphs $K_i = K(S_i, u_i, T_i)$ associated with the points u_i such that (1) $D = \bigcup K_i$ and (2) $u_i \in T_j$ if and only if $u_j \in S_i$. (Received November 9, 1967.)

68T-80. R. A. GUY, Universite de Montreal, Montreal, Quebec, Canada. A class of nonlinear forms on certain modules.

We use notations and definitions given in Abstract 648-196, these *Notices* 14 (1967), 689. Let us assume the symbolic matrix associated with $u: E^p \to F^p$ is such that its "elements" $\mathscr{A}_{p_\alpha}^{p_\lambda}$ are mappings $(E_{p_1} \times \ldots \times E_{p_\alpha} \times \ldots \times E_{p_\lambda - p_\alpha}) \times E_{p_\alpha} \to F_{p_\lambda}$ arbitrary in the variables $x^{p_1}, \ldots, x^{p_\alpha}, \ldots, x^{p_\lambda - p_\alpha}$, but linear in the last x^{p_α}. We denote this class of maps by $\mathscr{A}_L(E^p, F^p)$. We denote by $G\mathscr{A}_L(E^p)$ the bijections of E^p of class $\mathscr{A}_L$. $G\mathscr{A}_L$ acts transitively in E^p and in its dual $(E^p)^*$, so they are $G\mathscr{A}_L$-homogeneous spaces. We call the $G\mathscr{A}_L$-space $(E^p)^*$ the space of $\mathscr{A}_L$-forms of E^p. It is possible, using elements of $\mathscr{A}_L(E^p, (E^p)^*)$, to establish a noncanonical isomorphism between the $G\mathscr{A}_L$-spaces E^p and $(E^p)^*$. (Received November 9, 1967.)

68T-81. S. M. SHAH, University of Kentucky, Lexington, Kentucky 40506. Entire functions satisfying a linear differential equation. II.

In this paper the following theorem is proved. Let $F(z)$ be a transcendental entire function and suppose that $F(z)$ satisfies a differential equation of the form $P_0(z)F^{(k)}(z) + P_1(z)F^{(k-1)}(z) + \ldots + P_k(z)F(z) = Q(z)$ where $P_j(z)$, $j = 0,1,\ldots,k$, and $Q(z)$ are polynomials and the degree of $P_0(z)$ ($\not\equiv 0$) is not less than that of any $P_j(z)$. Then $F(z)$ is of bounded index. (For terminology and earlier work see Abstract 67T-646, these *Notices* 14 (1967), 850.) In particular Bessel functions of nonnegative integer order and the confluent hypergeometric functions are of bounded index. (Received November 9, 1967.)

68T-82. J. B. BEDNAR, 6500 Tracor Lane, Austin, Texas 78721. Projections on affine spaces. Preliminary report.

In this paper, necessary and sufficient conditions that a weakly separating subspace [cf, D. E. Wulbert, Some complemented function spaces in C(X), to appear in the Pacific J. Math.] for a normed linear space to admit a projection of norm one are given. Let K be a compact convex subset of a

LCHTVS, A(K) be the Banach space of continuous affine functions on K, and suppose the subspace P of A(K) is weakly separating. Theorem 1. There is a projection of norm one on P if and only if P is linearly isometric to A(L) under the restriction map. Here, L is the closed convex hull of $\{X^* \in A(K)^* : X^*|P \in \text{ext}(S(P^*))\} \cap K$. Theorem 2. If B is a subspace of A(K) which has a weakly separating convex quotient, and there is a projection of norm one on B, then B is linearly isometric to A(F) for some compact convex set F. Theorem 3. If K is a metrizable simplex and the B-convex-quotient of K is lower semi-continuous then B is the range of a projection of norm one. Many of the results in the above quotation appear as corollaries. (Received November 9, 1967.)

68T-83. D. H. TUCKER, University of Utah, Salt Lake City, Utah 84112. Boundary value problems for linear differential systems.

Let C denote the space of $n \times n$ real matrix valued continuous functions on a finite interval $[a,b]$ with the topology of uniform convergence, U be a bounded linear transformation from C into the $n \times n$ real matricies and consider the boundary value problem (*) $Y' = AY + R$; $U(Y) = K$ where $A, R \in C$ and K is a real $n \times n$ matrix. We obtain necessary and sufficient conditions that (*) should have a solution for a fixed $R \in C$ and also for all $R \in C$ and in each case exhibit the general solution. It is not assumed that the homogeneous system is incompatible. An adjoint system is obtained and analogous theorems are established for it and finally a theorem of the Fredholm alternative type is established relating the two systems. The key structure is a certain right ideal in the $n \times n$ matricies which is uniquely determined by A and U. (Received November 15, 1967.)

68T-84. STEPHEN SLACK, Wisconsin State University, Oshkosh, Wisconsin 54901. A cellularity criterion in certain 3-manifolds.

A compact set X in the interior of a 3-manifold M^3 is cellular iff it is the intersection of a sequence of 3-cells each of which lies in the interior of its predecessor. A set X in M^3 is said to be a UV^∞ set iff for each open set U containing X there exists an open set V containing X, lying in U and contractible in U to a point. A set X in M^3 has the separation property iff there is a neighborhood N of X such that each polyhedral simple closed curve in N - X that bounds a polyhedral disk in N may be separated from X by a 2-sphere in M^3. Theorem. For a compact UV^∞ set in S^3 (the 3-sphere) to be cellular it is necessary and sufficient that it have the separation property. The theorem generalizes to certain 3-manifolds besides S^3, for example, those whose universal covering spaces are embeddable in S^3. The separation property alone is insufficient to imply the cellularity of a compact set. In the proofs use is made of McMillan's result that UV^∞ sets in S^3 are the intersection of a descending sequence of cubes-with-handles. (Received November 8, 1967.)

68T-85. RONALD ALTER, System Development Corporation, 2500 Colorado Avenue, Santa Monica, California 90406. On a necessary condition for the validity of the Riemann hypothesis for functions that generalize the Riemann zeta functions.

In 1927, Pólya stated a condition for the validity of the Riemann hypothesis for the ordinary Riemann zeta function. Grosswald generalized a formula of Hayman and used it to show that the Riemann zeta function actually satisfied this necessary condition for all sufficiently large integers n.

In this paper a class of functions, depending on several variable parameters, is studied. Each of these functions has a Riemann hypothesis that is associated with it. It is shown that Pólya's necessary condition for the validity of the ordinary Riemann hypothesis is also a necessary condition for the validity of the Riemann hypothesis that is associated with these functions that generalize the Riemann zeta function. It is also shown that these functions which generalize the Riemann zeta function actually satisfy this necessary condition. Among the functions studied are L-functions of real, primitive characters, Ramanujan's zeta function, Dedekind's zeta function over a field K of algebraic numbers, and the Epstein zeta function Z(s, Q), for certain values of the discriminant of the quadratic form Q(x,y). (Received November 6, 1967.)

68T-86. J. L. STERN, University of Idaho, Moscow, Idaho 83843. Characterization of contractible open manifolds, and an equivalence between pointlike maps and cellular maps.

The theorems listed below are proved using the engulfing methods of Newman and Connell, and standard connectedness arguments. Theorem 1. Every contractible open manifold $n > 4$, is the union of two open cells whose intersection is a contractible open manifold. Theorem 2. Let M be an open contractible manifold, $n > 4$. Let $M = O_1 \cup O_2$ where O_1 and O_2 are open cells. Assume for every compact set $F \subset M$ there exists an open cell O_3 and a closed cell B such that $F \subset O_3 \subset B \subset O_1$ and $O_3 \cup O_2$ is a 2-connected manifold that is 1-connected at infinity, then $M = E^n$. Theorem 3. Let M be an n-dimensional manifold without boundary (compact, or not), $n > 4$. Let f be a point-like map from M onto M, then f is a cellular map. (Received November 9, 1967.)

68T-87. E. C. ZACHMANOGLOU, Purdue University, Lafayette, Indiana 47907. Uniqueness of the Cauchy problem.

Let P(x,D) be a differential operator of order m with analytic coefficients defined in a neighborhood U of a point x^0 in R_n and having real coefficients in the principal part. Let Φ be a real-valued function in $C^k(U)$, k an integer $\geqq 2$, such that grad $\Phi(x^0) = N^0 \neq 0$ and suppose that the surface $S = \{x : x \in U, \Phi(x) = \Phi(x^0)\}$ is simply characteristic at x^0. Let $\mathscr{L} = \{x(t), \xi(t)\}$ be the bicharacteristic strip passing through (x^0, N^0) when $t = 0$. Suppose that $(d^i/dt^i)[\Phi(x(t)) - \Phi(x^0)]|_{t=0}$ equals 0 for $i = 0,\ldots, k-1$ and equals ρ for $i = k$, and $(d^i/dt^i)[\text{grad}\,\Phi(x(t)) - \xi(t)]|_{t=0} = 0$ for $i = 0,\ldots, [(k-1)/2]$. Then, if k is odd and $\rho \neq 0$ or if k is even and $\rho > 0$, uniqueness of the Cauchy problem holds: there is a nbhd $U' \subset U$ of x^0 such that every distribution u defined in U, satisfying $P(x,D)u = 0$, and vanishing when $\Phi(x) > \Phi(x^0)$, $x \in U$, must also vanish in U'. If k is even and $\rho < 0$, nonuniqueness holds: there is a nbhd $U' \subset U$ of x^0 and a function u in $C^{(m)}(U')$ satisfying $P(x,D)u = 0$, vanishing when $\Phi(x) > \Phi(x^0)$, $x \in U'$, and such that x^0 belongs to the support of u. Nonuniqueness also holds in the limiting case in which Φ is analytic and $\mathscr{L}$ is tangent to S in some neighborhood of x^0. These results contain and extend results of Malgrange, Hörmander and Treves. (Received November 6, 1967.)

68T-88. A. V. JATEGAONKAR, University of Rochester, Rochester, New York 14627. Structure of left principal ideal rings.

Let R be a primary principal left ideal ring (with unity), P the nil-radical of R, $\bar{R} = R/P$, $B_n = P^n/P^{n+1}$ considered canonically as a $\bar{R}$-bimodule and Q the Artinian simple left quotient ring of $\bar{R}$. This notation is mentioned throughout. R has large nil-radical if either $P^2 \neq 0$ or $P^2 = 0$, $P \neq 0$ and $rw = 0$ for some nonzero $w \in P$ implies $r + P$ is not regular in R. If $\phi : \bar{R} \longrightarrow \bar{R}$ is a unitary monomorphism, the $(1,\phi)$ $\bar{R}$-bimodule is the $\bar{R}$-bimodule on $(\bar{R},+) = M$ defined by the rule $r_1 * m * r_2 = r_1 m \phi(r_2)$ where $r_1, r_2 \in \bar{R}$, $m \in M$. The main results are the following: Theorem 1. $c \in R$ is left regular iff $c + P \in R$ is left regular. Theorem 2. R has an Artinian left quotient ring iff every left regular element of R is regular. Theorem 3. If R has large nil-radical then there exists a unitary monomorphism $\rho : Q \longrightarrow \bar{R}$. If $P^k \neq 0 = P^{k+1}$, then B_n is isomorphic with $(1, \rho^n | \bar{R})$ $\bar{R}$-bimodule for $1 \leq n \leq \max\{1, k-1\}$. B_k is a factor bimodule of the $(1, \rho^k | \bar{R})$ $\bar{R}$-module. Theorem 4. If R has large nil-radical and if $\bar{R} \cong M_n(D)$ where D is a pli-domain then $R \cong M_n(S)$ where S is a completely primary pli-ring. A counter-example is given to a conjecture of I. N. Herstein [Topics in ring theory, Univ. Chicago, Math. lecture notes, p. 75.] (Received November 8, 1967.)

68T-89. B. S. LALLI, University of Saskatchewan, Saskatchewan, Canada. On a periodic solution of a Hamiltonian system.

C. L. Siegel constructed a periodic solution for the system $\dot{x}_k = H_{y_k}$, $\dot{y}_k = -H_{x_k}$ ($\dot{} = d/dt$, $H_{x_i} = \partial H/\partial x_i$) ($k = 1, 2, \ldots, n$) under the assumption that the matrix of the linear terms has a pair of pure imaginary eigenvalues. If the said matrix has two pairs of pure imaginary eigenvalues we can still construct a periodic solution for the system on the similar lines provided we assume that the rest of the eigenvalues are of the form $\mu + iv$ ($\mu v \neq 0$) and form the set of 2n eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n; -\lambda_1, -\lambda_2, \ldots, -\lambda_n$, the first b eigenvalues are linearly independent over the rationals. (Received November 15, 1967.)

68T-90. PETER CSONTOS, University of Washington, Seattle, Washington 98105. On chains of subalgebras. II. Preliminary report.

Let $\underline{A} = (A, F)$ be an algebra with finitely many finitary operations. Call $\underline{A}$ a Jonsson algebra if for every proper subalgebra $\underline{B}$, $|B| < |A|$. Let $\mathfrak{J}$ be the class of Jonsson algebras, let $\mathfrak{D}$ be the class of algebras satisfying the descending chain condition. Galvin obtained a Jonsson algebra $\underline{A}$ with three unary operations and $|A| = \aleph_1$. By suitable modifications one can obtain the following: I. There exist algebras $\underline{A} = (A, f_1, f_2)$, where $|A| = \aleph_1$, and f_1 and f_2 are unary, with the following properties: (a) $\underline{A} \in \mathfrak{J} \cap \mathfrak{D}$, (b) $A \in \mathfrak{J}$ and $A \notin \mathfrak{D}$, (c) $A \notin \mathfrak{D}$ and $A \notin \mathfrak{J}$. II. Theorem I holds when (A, f_1, f_2) is replaced by $(A, +)$, where + is a binary operation. III. If $\underline{A} = (A, f_1, \ldots, f_1, \ldots, f_n)$, where f_i are unary operations and $|A| \geq \aleph_2$, then $\underline{A} \notin \mathfrak{D}$. (Received November 15, 1967.)

68T-91. C. A. HALL and T. A. PORSCHING, Bettis Atomic Power Laboratory, Westinghouse Electric Corporation, Box 79, West Mifflin, Pennsylvania 15122. On a theorem of A. Brauer.

A. Brauer (Recent advances in matrix theory, ed. H. Schneider, 1964) gives an iterative algorithm for computing to any degree of accuracy the Perron eigenvalue of a positive matrix. Brauer treats the more general class of nonnegative, irreducible matrices by taking powers of the given matrix until a positive matrix is obtained. Obviously, the amount of work involved may be prohibitive, if for example the given matrix is tridiagonal and of large dimension. Using the irreducibility of the given nonnegative matrix we are able to prove that Brauer's algorithm as given also converges for nonnegative, irreducible matrices, without resorting to taking matrix powers. (Received November 15, 1967.)

68T-92. PETER BANCROFT, University of Colorado, Boulder, Colorado 80302. Calculus of set-valued functions. Preliminary report.

Let $K^{(n)}$ denote the semigroup of nonempty compact convex subsets of E^n with the usual set addition. Topologize $K^{(n)}$ via the Hausdorff metric. Let $S^{(n)}$ denote the semigroup of support functionals to members of $K^{(n)}$, with the metric topology induced by $d(\underline{f}_1, \underline{f}_2) = \sup_{\|x\|=1} |\underline{f}_1(x) - \underline{f}_2(x)|$. The mapping, which assigns to each compact convex set in E^n its support functional, is an isometric isomorphism from $K^{(n)}$ onto $S^{(n)}$. If F is a continuous set-valued function from $[a,b]$ into $K^{(n)}$, then (following Aumann [J. Math. Anal. Appl. 12, August, 1965]) the integral of F is defined as $I(F) = \{\int_a^b h(\tau)d\tau\colon$ h is measurable, $h(t) \in F(t)\}$. I(F) is compact and convex. Let $f(\cdot, t)$ denote the support functional for F(t). The aforementioned isometric isomorphism may be used to show that the support functional for I(F) is $g(\cdot) = \int_a^b f(\cdot, \tau)d\tau$. This relationship is used to define a derivative for set-valued functions from $[a,b]$ into $K^{(n)}$. "Taylor Series" for set-valued functions and applications to contingent equations are given. (Received November 6, 1967.)

68T-93. VLADIMIR DROBOT, State University of NewYork, Buffalo, New York 14214. Quasidynamical systems.

Let X be a metric space and let F be a family of nonintersecting curves filling X. If the family F is geometrically continuous (regular), then the curves in F have many properties associated with the trajectones of a dynamical system. One can define the limit cycles, nonwandering points, minimal sets and the center of the space and obtain results analogous to the theorems in topological dynamics. The results thus obtained indicate the properties of a dynamical system which are independent of the group structure of such systems. (Received November 8, 1967.)

68T-94. A. E. TONG, Tulane University, New Orleans, Louisiana 70118. Diagonal nuclear operators. Preliminary report.

Let T be a bounded operator on normed spaces. $|||T|||$ will denote the sup norm of the operator. l_p^n denotes the normed space of all sequences $(a_1, a_2, \dots a_n)^{1/2}$ whose norm, denoted by $l_p(a_1, a_2, \dots, a_n)$, is defined as $(\sum_{1\leq i\leq n} |a_i|^p)^{1/2}$ where $p \geqq 1$. Theorem. Let $1 \leqq p, r \leqq \infty$. Let $T\colon l_p^n \to l_r^n$ be the linear operator with matrix representation (t_{ji}) and let D be the diagonal linear operator associated with T so that it has matrix representation $(\delta_{ji}t_{ji})$ where δ_{ji} is the Kronecker delta. Then $|||T||| \geqq |||D|||$ and equality holds if and only if $T = D$. Define nuclear operators on normed spaces as in A. Pietsch's Nucleare Lokalkonvexe Raume, Berlin, 1965, pp. 44-45. Denote the nuclear norm of an operator by $\nu(T)$. Theorem. Let $1 \leqq p, r \leqq \infty$ and let $T : 1_p^\infty \longrightarrow 1_r^\infty$ be a nuclear operator. Let D be the associated diagonal operator of T. Then $\nu(T) \geqq \nu(D)$. In fact $\nu(D)$ is $1_1(t_{11},\ldots t_{nn}\ldots)$ if $p \geqq r$ and $\nu(D)$ is $1_{pr/(pn-p+n)}(t_{11},\ldots t_{nn}\ldots)$ if $p < r$. Corollary. Let $1 \leqq p, r \leqq \infty$. The Banach space of all diagonal nuclear operators $D : 1_p^\infty \longrightarrow 1_r^\infty$ is isometric to 1_1^∞ if $p \geqq r$ and is isometric to $1^\infty_{pr/(pr-p+r)}$ if $p < r$. (Received November 6, 1967.)

68T-95. MARVIN MARCUS and STEPHEN PIERCE, University of California, Santa Barbara, California 93106. Elementary divisors of associated transformations.

Let V be an n-dimensional vector space over an algebraically closed field R of characteristic 0. If H is a subgroup of S_m, $m \leqq n$, and χ is a character of H of degree 1, then a multilinear function ϕ on V is symmetric w.r.t. H and χ if $\phi(v_{\sigma(1)},\ldots,v_{\sigma(m)}) = \chi(\sigma)\phi(v_1,\ldots,v_m)$, $\sigma \in H$, $v_i \in V$, $i = 1,\ldots,m$. Let P be a vector over R and μ a multilinear function symmetric w.r.t. H and χ with values in P. The pair (P,μ) is a symmetry class of tensors over V associated with H and χ if $\langle \text{rng}\ \mu \rangle = P$, and given any ϕ, symmetric w.r.t. H and χ with values in some vector space U over R, there exists a linear h: $P \longrightarrow U$ such that $\phi = h\mu$. If T: $V \longrightarrow V$ is linear, the associated transformation $K(T)$: $P \longrightarrow P$ is defined by $\mu(Tv_1,\ldots,Tv_m) = K(T)\mu(v_1,\ldots,v_m)$. Theorem 1. If $m < n$ and rank $T > m$, then T has linear elementary divisors iff K(T) does. If $\chi \equiv 1$, then the hypothesis that $m < n$ and rank $T > m$ may be dropped in Theorem 1. Theorem 2. Assume that $\chi \equiv 1$, T and S have nonnegative eigenvalues, and K(T) and K(S) have the same elementary divisors and these are linear. Then T and S have the same elementary divisors and these are linear. (Received November 5, 1967.)

68T-96. G. A. C. GRAHAM, Simon Fraser University, Burnaby 2, B. C., Canada. The solution of mixed boundary value problems that involve time-dependent boundary regions for isotropic and homogeneous viscoelastic materials with one relaxation function.

Let $\mathscr{R}$, with boundary $\mathscr{B}$, be a fixed region of space which is occupied by a homogeneous and isotropic linear viscoelastic material whose relaxation function in shear is a constant multiple of its relaxation function in isotropic compression. The material is undisturbed for negative time and (using the notation of Abstract 644-62, these *Notices* 14 (1967), 377) the boundary conditions $\underline{\sigma}_n(\underline{x},t) = \underline{A}(\underline{x},t)$, $\underline{x}$ on $\mathscr{B}_1(t)$; $\underline{\sigma}_s(\underline{x},t) = \underline{B}(\underline{x},t)$, $\underline{x}$ on $\mathscr{B}_2(t)$; $\underline{u}_n(\underline{x},t) = \underline{C}(\underline{x},t)$, $\underline{x}$ on $\mathscr{B} - \mathscr{B}_1(t)$; $|\underline{u}_s(\underline{x},t) = \underline{D}(\underline{x},t)$, $\underline{x}$ on $\mathscr{B} - \mathscr{B}_2(t)$, where $\mathscr{B}_1$ and $\mathscr{B}_2$ are time-dependent subregions of $\mathscr{B}$, are prescribed. It is shown that this problem may be reduced to the solution of a one-parameter family of elastic problems if $\mathscr{B}_i(t)$ $(i = 1,2)$ are either both monotone increasing for all time or both monotone decreasing for all time. (Received November 6, 1967.)

68T-97. R. H. HERMAN and G. G. EMCH, University of Rochester, Rochester, New York. A generalization of Hugenholtz's theorem of types.

The starting point for this paper is a C*-algebra $\mathfrak{A}$, of "observables" which is representative of physical systems occurring in Statistical Mechanics, and its Gelfand representation, under an "equilibrium state" ω. Letting $\mathscr{R}$ be the von Neumann algebra generated by the representation, we use the direct integral theory of von Neuman algebras to arrive at the conclusion that either $\mathscr{R}$ is of type III or the cyclic vector, Ω, for the Gelfand representation is a trace vector. In the latter case, $\mathscr{R}$ is isomorphic to the cartesian product of two von Neumann algebras, one of which is of type II_1 and the other being a direct integral of factor of type I_n with $n < \infty$. An additional condition on $\mathfrak{A}$ removes the type I part from the representation. (Received November 10, 1967.)

68T-98. Y.-F. WONG, DePaul University, Chicago, Illinois 60614. A theorem on homotopically equivalent $(2k + 1)$-manifolds.

Using Wall's results on surgery of covering manifolds, we are able to put a theorem of the author (Proc. Amer. Math. Soc. 16 (1965), 1022-1025) in a slightly generalized form. Theorem. Let M^{2k-1} and M_2^{2k-1} be two connected, compact differentiable homotopy equivalent manifolds with $k \geqq 3$ (not necessarily simple connected). (1) There is a connected differentiable manifold N^{2k} with boundary $\partial N = - M_1 \cup M_2$ such that the inclusion map i: $M_1 \subset N$ is $(k - 1)$-connected. (2) There is a continuous map g: $N \to M_2$ such that $g|M_2$ is the identity and $g|M_1$ is the homotopy equivalence. Then there is an obstruction μ (of Wall) ($\mu = 0$ if $\pi_1(M_1) = 0$) to realizing the $\pi_k(N, M_1)$ as an embedded handle body. And if $\mu = 0$, then M_2 is h-cobordant to $M_1 \# \sum$, M_1 connected-summed with a homotopy sphere. (Received November 16, 1967.)

68T-99. W. J. WILBUR, University of California, Davis, California 95616. On sets measurable and nonmeasurable by a regular measure. Preliminary report.

Let P be a locally compact Hausdorff topological space and let $(P, \mathscr{M}, \mu)$ be a measure space. By the term regular we shall mean regular in the sense of Hewitt and Stromberg (Real and abstract analysis, p. 177). Call a set $A \in \mathscr{M}$ an atom if and only if (i) $\mu(A) > 0$ and (ii) $B \in \mathscr{M}$ and $B \subset A$ implies $\mu(B) = 0$ or $\mu(B) = \mu(A)$. Then assuming that μ is regular and that $\mu(\{x\}) = 0$ for all $x \in P$ we have the following results. Theorem 1. The atoms of $\mathscr{M}$ are just the locally μ-null sets which are not μ-null. Theorem 2. If $\mu(P) > 0$, the axiom of choice implies there are subsets of P which are not $\mathscr{M}$-measurable. Theorem 3. Each $A \in \mathscr{M}$ with σ-finite measure and with $\mu(A) > 0$ has a perfect subset of positive measure. (Received November 16, 1967.)

68T-100. JAMES R. BOONE, Texas Christian University, Forth Worth, Texas 76129. A metrization theorem.

A family $\{F_\alpha | \alpha \in A\}$, of subsets of a topological space X, is said to be convergent sequence - finite (cs-finite) provided for each convergent sequence $\{p_i\}$ in X, $F_\alpha \cap \{p_i\} \neq \emptyset$ for at most finitely many $\alpha \in A$. A Hausdorff space is said to be sequentially mesocompact provided every open covering has a cs-finite open refinement. The following implications are valid: locally finite $\to$ cs-finite $\to$ point-finite and paracompact $\to$ sequentially mesocompact $\to$ metacompact. Lemma. In a first countable Hausdorff space, a family of subsets is cs-finite if and only if it is locally finite. Theorem 1. A topological space is metrizable if and only if it is regular and has a σ-cs-finite base. Theorem 2. A regular first countable space is paracompact if and only if every open covering has a cs-finite refinement. (Received November 16, 1967.)

68T-101. R. N. KESARWANI, University of Ottawa, Ottawa 2, Canada. Fractional integration and certain dual integral equations.

The space $L(0, \infty)$ of complex valued functions f integrable on $(0, \infty)$ forms a commutative and associative algebra over the field of complex numbers with the usual definitions of addition and scalar multiplication and the convolution, $(f * g)(x) = \int_0^\infty u^{-1} f(x/u)g(u)\,du$, as the definition of the product of two elements f and g in $L(0, \infty)$. It is known that the fractional integral operators $I_{xA}^{\eta,\alpha}$ and $K_{xA}^{\eta,\alpha}$ can be identified with the elements of this algebra [Buschman: Math. Japon. 9 (1964), 99-106]. This fact has been used to reduce the dual integral equations involving Meijer G-functions [Erdélyi, etc., Higher transcendental functions, vol. 1, p. 207] as kernels into two integral equations having a common kernel and the problem of dual integral equations then reduces to that of solving a single integral equation which has been discussed by the author in a series of earlier papers [Proc. Amer. Math. Soc. 13 (1962), 950-959; 14 (1963), 18-28, 271-277]. The importance of the discussed dual integral equations is due to the very general yet simple form of the kernels which include as particular cases many special functions used as kernels of dual integral equations in earlier studies. (Received November 16, 1967.)

68T-102. RAYMOND BALBES, University of Missouri, St. Louis, Missouri 63121. A representation theory for prime and implicative semilattices.

A large part of the theory of pseudo complemented lattices can be extended to pseudo complemented semilattices, as was pointed out by O. Frink. A similar observation can be made concerning implicative lattices and implicative semilattices. In the first part of this paper necessary and sufficient conditions are obtained for a semilattice L to be isomorphic with a family R of sets so that finite products in L correspond to intersections in R, and finite sums in L--when they exist--correspond to unions in R. Semilattices satisfying this condition will be called prime. A representation theory for prime semilattices is presented. It shows that a prime semilattice L is essentially a collection C of compact-open sets of a Stone space. As expected, L is a lattice if and only if C consists of all compact-open sets. Thus, a natural generalization of the Stone representation theorem for distributive lattices is obtained. As an example, the representation space of an implicative semilattice is determined. (Received November 6, 1967.)

68T-103. G. L. SEEVER, California Institute of Technology, Pasadena, California 91109. A Hahn-Banach theorem for commutative semigroups with identity.

Theorem. Let $(S, +)$ be a commutative semigroup with identity, 0, $p: S \to R \cup \{-\infty\}$ a subadditive function such that $p(0) = 0$, T a semisubgroup of S which contains 0, and let $\phi: T \to R \cup \{-\infty\}$ be a semigroup homomorphism such that $\phi(0) = 0$ and $\phi \leqq p|T$. Then there is a semigroup homomorphism $\Phi: S \to R \cup \{-\infty\}$ such that $\Phi|T \geqq \phi$ and $\phi\Phi \leqq p$. If X is a compact Hausdorff space, A a sup norm algebra on X, and $\sigma \in \sigma(A)$, then by taking for S the multiplicative semigroup of $C(X)$, for T the multiplicative semigroup of A, for p, $\log \|\cdot\|_\infty$, and for ϕ, $\log |\sigma(\cdot)|$, we obtain a Jensen measure for ϕ by setting $\mu(f) = \Phi(e^f) - i\Phi(e^{if})$, $f \in C(X)$, where Φ is as in the theorem. (Received November 13, 1967.)

68T-104. R. C. LYNDON and J. L. ULLMAN, University of Michigan, Ann Arbor, Michigan 48104. Free groups of real two-by-two matrices.

A method by Macbeath [Proc. Camb. Phil. Soc. 59 (1963), 555] is used to extend results of M. Newman [Mich. Math. J. 15 (1968), to appear]. Let A and B be real unimodular two-by-two matrices, acting as linear fractional transformations on the extended real axis $\underline{R}^*$. Then the group generated by A and B is the free product of the cyclic subgroups generated by A and B under the following circumstances. (1) A, B and AB have real fixed points, and the fixed points of each of these three lie in some interval of $\underline{R}^*$ containing no fixed point of either of the other two. (2) Each of A and B has two fixed points, those of either separating those of the other on $\underline{R}^*$, and $ABA^{-1}B^{-1}$ has real fixed points. (3) A or B elliptic of finite order, and subject to a slightly modified hypothesis. The special case of (1) for both A and B parabolic is subsumed under stronger results for the complex case which will be announced elsewhere. Results (1), (2), and (3) will appear in the Mich. Math. J. (Received November 10, 1967.)

68T-105. R. E. MOSHER, California State College, Long Beach, California 90801. A product formula for a tertiary characteristic class.

Let ξ be an SO(n)-bundle over B with integral Thom class U; suppose $n > 2$ and that the Euler class of ξ vanishes. Let $\alpha(\xi) \in H^{n+1}(B;Z_2)$ be the secondary characteristic class based on $(Sq^2 + w_2)(U) = 0$. Whenever $\alpha(\xi) \equiv 0$ one defines a tertiary class $\gamma(\xi) \in H^{n+2}(B;Z_2)$ modulo an indeterminacy Q. $\gamma(\xi)$ arises from a relation $(\Phi + \lambda w_3)(U) = 0$ for some $\lambda \in Z_2$, where Φ is the twisted secondary operation based on the relation $(Sq^2 + w_2)^2 = 0$, valid on integral classes. If ξ' is an SO(n')-bundle over B' with vanishing Euler class, then $\alpha(\xi \oplus \xi') \equiv 0$; hence $\gamma(\xi \oplus \xi')$ $\in H^{n+n'+2}(B \times B';Z_2)/Q$ is defined. Theorem. $\gamma(\xi \oplus \xi') = \alpha(\xi) \otimes \alpha(\xi')$ modulo indeterminacies. Corollary. Let $\xi + 1$ be the tangent bundle of S^{4q+1} and $\xi' + 1$ the tangent bundle of $S^{4q'+1}$ with $q,q' \geqq 1$. Then the $(4q + 4q' - 1)$-sphere bundle associated to $\xi \oplus \xi'$ over $S^{4q+1} \times S^{4q'+1}$ does not admit a section. (Received November 13, 1967.)

68T-106. D. W. SOLOMON, University of Wisconsin, Milwaukee, Wisconsin 53201. Denjoy-Khintchine Integration in abstract spaces. I. Preliminary report.

Let X be a second countable, locally compact metric space which has a base, $\mathscr{A}$, satisfying Romanovski's axioms I - X [Math. Sb. 9 (51) (1941)], and Y be either the real or complex numbers. This paper presents a descriptive and constructive definition of an integral of point functions, f, defined on a member of $\mathscr{A}$, and with range contained in Y, and studies some of the properties of this integral. The integral contains, as a special case, the integral of Čelidze (Tbl. Akad. Nauk., Radzm., vol. 25) and all known multiple-dimension restricted Denjoy integrals. The constructive process given can be compared with that presented for the Denjoy-Khintchine integral in Saks's Theory of the integral. As in the classical case, any integral can be attained in at most countably many applications of the construction process. In the case of real-valued functions of a real variable, the descriptive definition of this integral reduces to the classical descriptive definition of the Denjoy-Khintchine integral. (Received November 10, 1967.)

68T-107. E. BINZ, Queen's University, Kingston, Ontario, Canada. Convergence function algebras and c-embedded convergence spaces.

(For notations and terminology see E. Binz and H. H. Heller, Funktionenraume in der Kategorie der Limesraume, Ann. Acad. Sci. Fenn. A I 383 (1966).) Let A and A' be convergence algebras having unit elements e and e' respectively and Hom_c (A,A') the set of all continuous algebra homomorphisms (sending e into e'), with the continuous convergence structure. For a convergence space X we abbreviate $\mathscr{L}_c(X, \mathbb{R})$ by $\mathscr{L}_c(X)$ and $\mathrm{Hom}_c(\mathscr{L}_c(X), \mathbb{R})$ by $\mathrm{Hom}_c\mathscr{L}_c(X)$. Theorem 1. For any convergence space X the natural map $i_X: X \to \mathrm{Hom}_c\mathscr{L}_c(X)$ is surjective. A convergence space X is called c-embedded if i_X is a homeomorphism. The category of all c-embedded convergence spaces, which includes the category of all completely regular topological spaces is a proper subcategory of $\mathscr{L}$ and contains objects, which are not topological spaces. Theorem 2. Two c-embedded convergence spaces X and Y are homeomorphic iff $\mathscr{L}_c(Y)$ and $\mathscr{L}_c(X)$ are bicontinuous isomorphic. Theorem 3. The map $\mathscr{L}_c$: $\mathscr{L}_c(Y,X) \to \mathrm{Hom}_c(\mathscr{L}_c(X), \mathscr{L}_c(Y))$ defined by $\mathscr{L}_c(f) = f^*$ is a homeomorphism iff X is c-embedded. (Received November 6, 1967.)

68T-108. J. G. MAXWELL, Kent State University, Kent, Ohio 44240. A model for the inference mechanism of the mind and a weakness in present logics.

Information unified and stored in two areas A and B of the brain may be represented by characteristic functions (propositional functions) p(x) and q(x) on domains A and B respectively. Let 1 and 0 denote respectively true and false. If there is a relation ϕ (i.e., a possibly multiple valued correspondence) from A into B with the property that $\phi[p^{-1}(1)] \subseteq q^{-1}(1)$, then p is said to be accompanied (relative to ϕ) by q and we write $[p(\Rightarrow_\phi)q] = 1$. Theorem. If p and q are constant functions (i.e., involve no variables) then accompaniment coincides with material implication. If A = B and ϕ is the identity function, then accompaniment coincides in concept with both formal implication and Lewis' strict implication. Theorem. Let p, q, A, B and ϕ be as before the previous theorem. Let ϕ be onto and set $\eta = \phi^{-1}$. If there is $\zeta \in A \cap B$ for which $\zeta \in \phi(\zeta)$ and $p(\zeta) = q(\zeta)$, then $[\sim p(\Rightarrow_\phi)q] \wedge [q(\Rightarrow_\eta) \sim p] = 0$, i.e., $\sim p$ iff q is false. In view of this last theorem the "paradoxes" of the barber, of Russell, of Grelling, of Cantor and many others may now be seen to have a simple common explanation. Present formalized logics do not adequately represent accompaniment between propositional functions on differing domains. (Received November 6, 1967.)

68T-109. R. J. DAVERMAN and W. T. EATON, University of Tennessee, Knoxville, Tennessee 37916. Any sewing of two crumpled cubes can be embedded in E^4.

Theorem. If C and D are crumpled cubes, and h is a homeomorphism from Bd C to Bd D, then the sum of C and D sewn together by h can be embedded in E^4. The proof uses results of Gillman [Unknotting 2-manifolds in 3-hyperplanes of E^4, Duke Math. J. 33(1966), 229-246]. The embedding may be viewed as a portion of the final stage of a pseudo-isotopy of E^4 onto E^4. (Received November 17, 1967.)

68T-110. J. A. LEES, Rice University, Houston, Texas 77001. An engulfing theorem for topological manifolds.;

Let M^q be a $2n - q + 2$ connected topological manifold, K^n an n-complex, $q - n \geqq 3$, and let $f: K \longrightarrow M$ be an embedding, locally flat on every simplex of K. Then if fK is inessential in M, fK is contained in a q-disc, that is, a coordinate neighborhood in M. This result is analogous to a well known Engulfing Theorem in the piecewise linear category (See M. W. Hirsch and E. C. Zeeman, Engulfing, Bull. Amer. Math. Soc. 72 (1), 113-115). The proof makes use of piecewise linear techniques and M. H. A. Newman's Engulfing Theorem for Topological Manifolds, and proceeds by an induction on the number of simplexes of K. (Received November 29, 1967.)

68T-111. FRANCIS SIWIEC, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. Metrizability of M-spaces.

Generalizing Morita's M-space [see: A. Okuyama, On metrizability of M-spaces, Proc. Japan. Acad. 40 (1964), 176-179]. C. J. Borges in a forthcoming paper [On metrizability of topological spaces] defines a space X to be a wΔ-space if there exists a sequence $\{\mathscr{V}_n\}_{n=1}^{\infty}$ of open covers of X such that for each $x_0 \in X$, if $x_n \in st(x_0, \mathscr{V}_n)$ for $n = 1, 2, \ldots$, then the sequence $\{x_n\}$ has a cluster point. For other definitions see J. M. Worrell, Jr. and H. H. Wicke [Characterizations of developable spaces, Canad. J. Math. 17 (1965), 820-830]. The following improve some theorems of Borges. Lemma. A T_3, wΔ-space with a G_δ-diagonal (i.e., $\{(x,x) | x \in X\}$ is a G_δ in $X \times X$) has a base of countable order. Theorem. A space X is T_3, θ-refinable, wΔ and has a G_δ-diagonal iff X is a Moore space. Improving a theorem of Okuyama [loc. cit.], a T_2, θ-refinable, M-space with a G_δ-diagonal is metrizable. A T_2, semimetric, M-space is metrizable. A Moore, M-space is metrizable. Thus, a normal Moore space is an M-space iff a normal Moore space is metrizable. (Received November 17, 1967.)

68T-112. LOUIS SCHNEIDER, Automatics, P. O. Box 4171, 3370 Miraloma Avenue, D/348-2 B/221, Anaheim, California 92803. The solution of equipment reliability and maintainability problems by queuing methods.

This paper applies queuing methods to the solution of equipment reliability and maintainability problems. The queuing situation in which there are n machines and r repairmen is analogous to that in which a multi-component system has n components and r repair facilities. Such problems, referred to as servicing problems or machine-repairmen problems are discussed in this paper. One unique application of queuing theory, discussed in this paper, involves an investigation of the operational readiness of an ICBM missile complex. Tables and graphs are included which should prove helpful in the application of queuing theory to operating problems in fields other than reliability and maintainability. A set of graphs has been developed which depicts the relationship between the system utilization factor, and the probability that a system will neither be awaiting nor undergoing service for finite populations ranging from N = 5 to N = 50 where the number of service facilities or "channels" range from 1 to 4. (Received May 30, 1967.)

68T-113. GEORGE GASPER, JR., University of Wisconsin, Madison, Wisconsin. On a function of Littlewood-Paley and Zygmund.

Let $F(z)$ be a function holomorphic in the unit disc. It is well known that if $F \in H^p$, $p > 0$, then $\|g(F)\|_p \leqq A_p \|F\|_p$ and $\|s(F)\|_p \leqq A_{p,\delta} \|F\|_p$, where $g(F)$ is the Littlewood-Paley function and $s(F)$ is the Lusin function (see A. Zygmund, Trigonometric series, 1959, Chapter XIV). Recently the author [Proc. Nat. Acad. Sci. U. S. A. 57 (1967), 25-28] introduced a method by which these results could be extended to a class H^p of systems of conjugate harmonic functions in the unit sphere in E_n for values of p in a range reaching below 1; namely, $p > (n - 2)/(n - 1)$. The extension of these results to the class H^p in the half-space is presented in [On the Littlewood-Paley g-function and the Lusin s-function, Trans. Amer. Math. Soc. (to appear)]. E. M. Stein [Comptes Rendus Acad. Sci. Paris 264 (1967), Serie A, 107-108; Proposition 2] has announced an analogous result for a related function S^*_λ defined for the class H^p in the half-space (note that the function g^*_σ of Littlewood-Paley and Zygmund is, essentially, S^*_λ). In this paper it is shown how the above-mentioned method may be applied to obtain both Stein's Proposition 2 and the corresponding result for the class H^p in the unit sphere. (Received November 14, 1967.)

68T-114. J. L. DENNY, University of Arizona, Tucson, Arizona 85721. Extensions of a theorem of Maxwell in the dynamical theory of gases.

$(\mathfrak{X}, \mathscr{A}, \{P_t: t \in T\}, \{Q_t: t \in T\})$ is given where the Q_t and P_t are probabilities on $\mathscr{A}$ and $P_t(N) = 0 \Leftrightarrow P_{t'}(N) = 0$, $(t, t') \in T^2$. Theorem 1. Let $\Psi_n(f): \mathfrak{X}^n \to R^k$ and $\{P_t^n\}$ satisfy the hypotheses of Theorem 2 of the author [Proc. Acad. Sci. U. S. A. 57 (1967), 1184-1187]. If $\lim_n \sup \|Q_t^n - P_t^n\|_1 > 0$ then $Q_t(A) = \int_A c_0(t) \exp \sum_{i=1}^p c_i(t)\phi_i(x) Q_{t_0}(dx)$, $(t,A) \in T \times \mathscr{A}$, $p \leqq k$, $Q_t = P_t$. Theorem 2. Modify the assumptions of Theorem 1 as follows: (i) $\mathfrak{X} = R$, g replaces $\Psi_n(f)$, $g: R^n \to R$ is now only assumed to be continuous and each partial function of a real variable of g satisfies Lusin's condition (N); (ii) $P_t(N) = 0 \Leftrightarrow \lambda_1(N) = 0$ (Lebesgue measure). Then a.e. (λ_1) locally on open neighborhoods $dQ_t/dQ_{t_0} = c_0(t) \exp c_1(t)\phi_1(x)$ for continuous ϕ_1 and $P_t = Q_t$. Theorem 3. Modify the assumptions of Theorem 2 as follows: $\mathfrak{X}$ is a separable locally compact group, $g: \mathfrak{X}^n \to R^k$, $k \geqq 1$, is continuous and each partial function of g mapping $\mathfrak{X}^k$ into R^k satisfies a Lipschitz condition with respect to the Haar measure (H^k) and Lebesgue measure on R^k. Then a.e. (H) locally on open neighborhoods $dQ_t/dQ_{t_0} = c_0(t) \exp \sum_{i=1}^p c_i(t)\phi_i(x)$ for continuous ϕ_i, and $P_t = Q_t$. (Received November 13, 1967.)

68T-115. MIKLOS CSORGO, McGill University, Montreal 2, Canada. Glivenko-Cantelli type theorems and laws of large numbers for randomized sequences of random variables.

Let $X_1, X_2, \ldots$ be independent random variables and N_λ be a positive integer valued random variable whose distribution depends on the parameter λ. Let $F^*_\lambda(\cdot)$ be the empirical distribution function of M. Kac (see M. Kac, On deviations between theoretical and empirical distributions, Proc. Nat. Acad. Sci. U. S. A. 35 (1949), 252-257). Necessary and sufficient conditions are established for the behaviour of N_λ/λ and N_λ, as $\lambda \to \infty$, in terms of Glivenko-Cantelli type theorems. Also, laws of large numbers are proved for random sequences of random variables and in terms of these laws the behaviour of N_λ/λ and N_λ, as $\lambda \to \infty$, is again characterized. (Received November 8, 1967.)

68T-116. BRYANT TUCKERMAN, IBM Thomas J. Watson Research Center, Yorktown Heights, New York. Odd perfect numbers: A search procedure, and a new lower bound of 10^{36}.

It is an open question whether any odd perfect numbers exist. An infinite algorithm is described which, for each odd perfect number, would detect that number in a finite amount of computation. It generates a tree, by alternating the application of theoretical results with the computational factorization of particular large integers. It is then truncated to a finite algorithm which detects all odd perfect numbers below a given bound. The truncated algorithm was carried out with the aid of an IBM 7094, using a package of programs for arithmetic and number-theory computations with large ("multiword") integers. There resulted a finite tree showing that every odd perfect number must: (1) satisfy known restrictions at some one of the nodes of this tree; and hence (2) have either (a) some component $p^a > 10^{18}$, a even, or (b) no prime divisor < 7; and (3) be $> 10^{36}$. (The previous published bound is 10^{20}.) (Received November 14, 1967.)

68T-117. C. H. WILCOX, University of Arizona, Tucson, Arizona 85721. The analytic continuation of the resolvent kernel in scattering theory.

Let S be a compact surface in R^3 with exterior Ω. Let Δ denote the self-adjoint operator on $L_2(\Omega)$ corresponding to the Laplacian with Dirichlet boundary condition on S. The scattering problem for S asks for the solution u(x, k) of $\Delta u + k^2 u = \rho$ (ρ prescribed). Δ has a resolvent $R(k^2, \Delta)$ which is analytic in Im k > 0 and is an integral operator whose kernel is the Green's function G(x, y, k) for $\Delta + k^2$. Thus $u(\cdot, k) = R(k^2, \Delta)\rho = \int_\Omega G(\cdot, y, k)\rho(y)dy$ for Im k > 0. u(x, k) can be constructed by the integral equation method of W. D. Kupradse (Randwertaufgaben der Schwingungstheorie und Integralgleichungen, VEB, Berlin, 1956). In this paper the Fredholm determinant theory is used to solve the Kupradse equation and the following results are obtained. (1) An entire function D(k) is constructed such that $k \to D(k)\, u(\cdot, k)$ is an entire analytic function with values in the Fréchet space $C(\bar{\Omega})$. (2) The zeros of D(k) are shown to lie in Im $k \leqq 0$. (3) The poles of $u(\cdot, k)$ at the real zeros of D(k) are shown to be removable. P. D. Lax and R. S. Phillips (Scattering theory, Academic Press, New York, 1967) have obtained similar results by a different method based on semigroup theory. (Received November 13, 1967.)

68T-118. F. J. MESTECKY, University of Maine, Orono, Maine 04473. Varieties associated with filtrations.

Let (0,m) be a local domain with quotient field F. Let f be a filtration of m-primary ideals of 0; i.e., $f = (A_n)_{n \geq 0}$ where $A_0 = 0$, $A_{n+1} \subseteq A_n$, and $A_i \cdot A_j \subseteq A_{i+j}$. Following the methods of Rees, we define a graded ring R_f, consisting of all elements of the form $\sum_{i=-p}^{q} c_i t^i$, where t is an indeterminant over 0, and $c_i \in A_i$, with $A_i = 0$ for all $i \leqq 0$. For each relevant homogeneous prime P of R_f containing t^{-1}, a quasi-local ring Q(P) is defined. Let V_f be the collection of all such Q(P). We assume the following: 0 is a two dimensional normal spot over a pseudo-geometric Dedekind domain; A_n is a normal m-primary ideal; Q(P) is a local domain; R_f is a Krull domain; $\bigcup_n S(A_n)$ is finite, where $S(A_n)$ is the set of Rees valuations associated with A_n. Then V_f is a model, called the variety of f. (Received November 14, 1967.)

68T-119. D. J. SCHATTSCHNEIDER, University of Illinois, Chicago Circle, Chicago, Illinois 60680. An isomorphism theorem for a reduced Weyl group.

Let G be a connected semisimple algebraic group, T a maximal torus of G, $\mathfrak{r}$ the root system of G with respect to T, Γ a subgroup of Aut(G, T), S the subtorus of T left pointwise fixed by Γ, $\mathfrak{r}_0$ the subset of $\mathfrak{r}$ which annihilates S, W and W_0 the Weyl groups of $\mathfrak{r}$ and $\mathfrak{r}_0$ respectively, and $\overline{W} = N_G(S)/Z_G(S)$. Let $\overline{\mathfrak{r}}$ be the set of nonzero restrictions to S of elements of $\mathfrak{r}$. We study two (distinct) actions of Γ on W, and their corresponding subgroups of fixed points of W (denoted W^{Γ} and $W^{[\Gamma]}$). In particular, we show that when W contains all of the reflections with respect to elements of $\overline{\mathfrak{r}}$, there is a natural subgroup V of $W^{[\Gamma]}$, generated by involutions, such that $\overline{W} \cong V \circ W_0$, a semidirect product. (This study generalizes §2 of Steinberg's paper Variations on a theme of Chevalley, Pacific J. Math., Vol. 9, and uses a recent result of Hijikata.) (Received November 13, 1967.)

68T-120. EDGAR REICH, University of Minnesota, Minneapolis, Minnesota 55455, and KURT STREBEL, University of Zurich, Zurich, Switzerland. Quasiconformal mappings which keep boundary points fixed.

Let f(z) map $U = \{|z| < 1\}$ quasiconformally onto itself with complex dilatation $\mu(z) = f_{\bar{z}}/f_z$. Theorem 1. A necessary condition for $f(e^{i\theta}) = e^{i\theta}$, $0 \leqq \theta < 2\pi$, is that $|\iint_U \mu(z)[1 - |\mu(z)|^2]^{-1}g(z)dxdy| \leqq \iint_U |\mu(z)|^2[1 - |\mu(z)|^2]^{-1}|g(z)|dxdy$ for each function g(z) holomorphic in U with finite norm $\iint_U |g(z)|dxdy < \infty$. Corollary. Suppose w = F(z) is quasiconformal in U and $F^{-1}(w)$ has complex dilatation $p(w)\overline{\psi}(w)/\psi(w)$, where $p(w) \geqq 0$, and $\psi(w)$ is holomorphic and locally L^1. Then F(z) is extremal for its boundary values in the sense that F has no admissible local variation. Theorem 2. Suppose f(k; z) is the Teichmuller mapping with $\mu(z) = \overline{\phi}(z)/\phi(z)$, $\phi(z) \neq 0$, $z \in U$, normalized such that f(k; 1) = 1, f(k; i) = i, f(k; -1) = -1. If $(\phi(z))^{1/2}$ is rational and all its poles in the extended plane are on ∂U then $f(k; e^{i\theta}) = e^{i\theta}$, $0 \leqq \theta < 2\pi$, $0 \leqq k < 1$. An example of Theorem 2 is obtained when $f = \Psi^{-1} \circ A \circ \Phi$ where Φ and Ψ are holomorphic in U and $\Phi(U)$ is the "interior" of a parabola, and A is an affine stretching in the direction of the axis of the parabola. (Received November 14, 1967.)

68T-121. M. J. O'MALLEY, NASA, Manned Spacecraft Center, ED13, Houston, Texas 77058. R-automorphisms of R[[X]].

Let R be a commutative ring with identity, X an indeterminate over R. R. W. Gilmer (R-automorphisms of R[X], Proc. London Math. Soc. (to appear)) has determined all R-automorphisms of R[X]. We investigate the analogous problem in R[[X]], the formal power series ring. We define, for any i in ω_0, a function π_i: $R[[X]] \to R$ as follows: $\pi_i(f)$, for any $f \in R[[X]]$, is the coefficient of X^i in f. Let $\beta = \sum_{i=0}^{\infty} b_i X^i \in R[[X]]$, where $\bigcap_{n=1}^{\infty} (b_0)^n = (0)$, and suppose that R is complete in the (b_0)-adic topology. We define a mapping ϕ_β from R[[X]] into itself as follows: if $f = \sum_{i=0}^{\infty} f_i X^i \in R[[X]]$, $f^{(i)} = \sum_{j=0}^{i} f_j X^j$, and $p_k(f) = \lim_i \pi_k(f^{(i)}(\beta))$ for each i and k in ω_0, then $\phi_\beta(f)$ is defined to be $\sum_{i=0}^{\infty} p_i(f) X^i$. Theorem 1. ϕ_β is an R-endomorphism of R[[X]]. Theorem 2. Let $\beta = \sum_{i=0}^{\infty} b_i X^i \in R[[X]]$, where $\bigcap_{n=1}^{\infty} (b_0)^n = (0)$. There is an R-endomorphism ψ on R[[X]] such that $\psi(X) = \beta$ if and only if R is complete in the (b_0)-adic topology. When R is complete in the (b_0)-adic topology, $\psi = \phi_\beta$. Theorem 3. ϕ_β is an R-automorphism of R[[X]] if and only if b_1 is a unit of R. (Received November 13, 1967.)

68T-122. J. G. HORNE, University of Georgia, Athens, Georgia 30601. Can multiplication on $Sl(2)$ be extended?

Here, $Sl(2)$ denotes the universal covering group of the group $sl(2)$ of 2×2 real matrices of determinant 2. The title means: is there a locally compact topological semigroup S in which $Sl(2)$ sits as a proper dense subgroup? We do not now consider the question in this generality but rather ask the more special question: is there a semigroup on the set $\{(x,y,z): x \geqq 0\}$ such that the set $G = \{(x,y,z): x > 0\}$ forms a subgroup isomorphic to $Sl(2)$? We cannot yet say whether there is, but if there is we can pin down its nature very rigidly: there must be a two sided zero 0 in the boundary L of G such that $xy = 0$ if $x, y \in L$. Furthermore, for every $x \in L$, $x \neq 0$, $\dim Gx = \dim xG = 1$ and for every nonzero, $x, y \in L$, $(Gx)^{-}$ and $(yG)^{-}$ are half-rays from 0 such that yG spirals around 0 relative to $(Gx)^{-}$. (Received November 6, 1967.)

68T-123. G. A. HEUER, Concordia College, Moorhead, Minnesota 56560. Pointwise approximation by s.c. functions.

We use the definitions and notation of Abstract 67T-604, these *Notices* 14 (1967), 836. If $A \subseteq Y^X$, $\bar{A}$ will denote the p.c. closure of A. Theorem 1. $\bar{S} \subseteq K \Longleftrightarrow K = \bar{K} \Longleftrightarrow Y$ is Hausdorff or X is totally disconnected (every component is a singleton). Theorem 2. If the quasicomponents of X are its points, then $\bar{S} = K = Y^X$. Theorem 3. If Y is irreducible (every pair of nonempty open sets has nonempty intersection) then $\bar{S} = Y^X$. Theorem 4. If X has a quasicomponent with more than one point, and Y is reducible, then $\bar{S} \neq Y^X$. Corollary 5. When X is totally disconnected, the following are equivalent: (i) $\bar{S} = K$. (ii) $\bar{S} = Y^X$. (iii) Every quasicomponent of X is a singleton or Y is irreducible. Theorem 6. If ever component of X is a quasicomponent then $K \subseteq \bar{S}$. Theorem 7. If Y is reducible (hence, if Y is Hausdorff), and X has a component which is not a quasicomponent, then $K \nsubseteq \bar{S}$. Corollary 8. $\bar{S} = K$ iff at least one of the following obtains: (i) Every quasicomponent of X is a singleton. (ii) X is totally disconnected and Y is irreducible. (iii) Y is Hausdorff and every component of X is a quasicomponent. (Received November 14, 1967.)

68T-124. UMBERTO NERI, University of Maryland, College Park, Maryland 20742. L^p estimates for certain nonlinear differential equations.

Consider equations of the form $Pu = \sum_{|\alpha|=|\beta|=m} D^\alpha a_{\alpha\beta}(x) D^\beta u = f$, where $f \in L^p_{-m}(R^n)$, $1 < p < \infty$, $D = (2\pi i)^{-1}(\partial/\partial x)$, with coefficients $a_{\alpha\beta}$ bounded and uniformly Lipschitz continuous in R^n. We say that P is elliptic in R^n if $|P(x, \xi)| = |\sum a_{\alpha\beta} \xi^\alpha \xi^\beta| \geqq \epsilon > 0$ on $R^n \times \{|\xi| = 1\}$. Theorem. If P is elliptic in R^n, there exists a constant $c > 0$ such that $\|u\|_m \leqq c(\|Pu\|_{-m} + \|u\|_{m-1})$, for all $u \in L^p_m(R^n)$, where $\| \ \|_k$ is the norm in $L^p_k(R^n)$. This estimate follows readily from the representation $P = \Lambda^m K \Lambda^m$, where $\Lambda = F^{-1}|\xi|F$ is a Fourier multiplier and $K = H + S$ is a Euclidean singular integral operator with symbol $\sigma(K) = \sigma(H) = P(x, \xi)|\xi|^{-2m}$. (Received November 13, 1967.)

68T-125. HISAHIRO TAMANO, Texas Christian University, Fort Worth, Texas 76129. Linearly cushioned refinements.

Definition 1. A family $\mathfrak{V} = \{V_\lambda \mid \lambda \in \Lambda\}$ is said to be linearly cushioned in another family $\mathcal{U} = \{U_\alpha \mid \alpha \in A\}$ if there exist a well ordering of Λ and a mapping $\phi: \Lambda \to A$ such that $\overline{\bigcup\{V_\lambda \mid \lambda \in \Lambda^*\}} \subset \bigcup\{U_\alpha \mid \alpha \in \phi(\Lambda^*)\}$ for each bounded subset Λ^* of Λ. Theorem 1. A space is paracompact if and only if every open covering has a linearly cushioned open refinement. Defintion 2. A topology base $\mathcal{V} = \{V_\lambda \mid \lambda \in \Lambda\}$ is said to be a linearly cushioned refinement of a topology base $\mathcal{U} = \{U_\alpha \mid \alpha \in A\}$ if $\mathcal{V}$ is linearly cushioned in $\mathcal{U}$ and $\mathcal{U}^* = \{U_\alpha \mid \alpha \in \phi(\Lambda)\}$ forms a topology base. Definition 3. A space is said to be perfectly paracompact if there is a topology base having linearly cushioned refinement. Theorem 2. Every perfectly paracompact space is (hereditarily) paracompact. Theorem 3. The cartesian product of two perfectly paracompact spaces is paracompact. Every metrizable space is perfectly paracompact. Perfectly paracompact spaces have some additional properties similar to those of metrizable spaces. (Received November 6, 1967.)

68T-126. N. D. GUPTA, University of Manitoba, Winnipeg, 19, Canada. Commutation near-rings of a group. Preliminary report.

Define addition and multiplication of mappings of a group G into itself by: $g(\theta_1 + \theta_2) = g\theta_1 \cdot g\theta_2$ and $g(\theta_1\theta_2) = (g\theta_1)\theta_2$ for all $g \in G$. The zero and negative mappings are given by: $g0 = 1$ and $g(-\theta) = (g\theta)^{-1}$ for all $g \in G$. Let M(G) denote the left-distributive near-ring generated by all mappings of G into G and let N(G) be the subnear-ring of M(G) generated by all mappings $\rho(x)$ where $g\rho(x) = g^{-1}x^{-1}gx$ for all $g \in G$. Define $(\nu_1, \nu_2) = -\nu_1 - \nu_2 + \nu_1 + \nu_2$ and $(\nu_1, \nu_2, \nu_3) = ((\nu_1, \nu_2), \nu_3)$ for all ν_1, ν_2, ν_3 in N(G). Theorem 1. For $n \geqq 2$, $(\nu_1, \dots, \nu_n) = 0$ for all $\nu_1, \dots, \nu_n \in N(G)$ if and only if $(\rho(x_1), \dots, \rho(x_n)) = 0$ for all $x_1, \dots, x_n \in G$. As one of the consequence one gets Theorem 2. A group of all of whose $m + n + 1$ -generator subgroups are nilpotent of class at most $mn + m + n$ is locally nilpotent. (Received November 10, 1967.)

68T-127. J. G. MICHAELS, University of Rochester, Rochester, New York 14627. Bimeasurable functions. Preliminary report.

Let X and Y be absolute Borel metric spaces. A function f from X onto Y is bimeasurable if both f and f^{-1} preserve Borel sets. A theorem proved for separable metric spaces by R. Purves, Bimeasurable functions, Fund. Math. 58 (1966), 149-157, is generalized to nonseparable metric spaces as follows: Theorem. Let X and Y be absolute Borel metric spaces and let f be a bimeasurable function from X onto Y. Let $T = \{y \in Y: f^{-1}(y) \text{ is not } \sigma\text{-discrete}\}$. If T is Borel, then T is σ-discrete. (Received November 8, 1967.)

68T-128. JIM OWINGS, University of Maryland, College Park, Maryland 20742. Commuting recursive functions have a common fixed point. Preliminary report.

If $e \geqq 0$, let W(e) be the eth r.e. set; i.e., $n \in W(e) \leftrightarrow (Ey)T_1(e, n, y)$. If f is a recursive function, call f well-defined on the r.e. sets if for all e, e', $W(e) = W(e') \to W(f(e)) = W(f(e'))$. Theorem 1. If f, g are recursive and well defined on the r.e. sets and $W(f(g(e))) = W(g(gf(e)))$ for all

$e \geqq 0$ then for some e^*, $W(e^*) = W(f(e^*) = W(g(e^*))$. The proof makes use of the following Lemma. If f is well defined on the r.e. sets then, for all $e \geqq 0$, $W(f(e)) = \bigcup W(f(e'))$ where the union ranges over all e' such that W(e') is a finite subset of W(e). The proof of the lemma requires two applications of Myhill's fixed point theorem. Definition. If $e \geqq 0$, $e' \geqq 0$ let $e \sim e'$ mean e and e' enumerate the same r.e. set in the same order; i.e., $e \sim e'$ iff $(y)(Ey')(n)((Es)(s < y \ \& \ T_1(e, n, s)) \leftrightarrow (Es')(s' < y' \ \& \ T_1(e', n, s'))) \ \& \ (y')(Ey)(n)((Es)(s' < y' \ \& \ T_1(e', n, s')) \longleftrightarrow (Es)(s < y \ \& \ T_1(e, n, s)))$. Theorem 2. If f, g are recursive and for all e, e', $e \sim e' \longrightarrow (f(e) \sim f(e') \ \& \ g(e) \sim g(e') \ \& \ f(g(e)) \sim g(f(e')))$ then for some e^*, $e^* \sim f(e^*) \sim g(e^*)$. As a barrier to further refinement, there exist recursive functions f,g such that $f(g(e)) = g(f(e))$ for all $e \geqq 0$ but for no e^* is it true that $W(g(e^*)) = W(f(e^*))$. (Received November 13, 1967.)

68T-129. W. E. BAXTER, University of Delaware, Newark, Delaware 19711. On rings with proper involution.

A topological ring is said to have property (Y) if, and only if, $2A = A$; A has a proper continuous involution (with symmetric elements S) such that whenever the net $\{2x_\alpha\}$ tends to zero, so also does $\{x_\alpha\}$; A^3 is dense in A; and the left annihilator of a closed Jordan ideal U of S is zero if, and only if, U is S. One shows that for such rings and for annihilator rings with the first two properties above that every closed Jordan ideal of S is the intersection of S with a closed two-sided ideal. Also shown is the fact that $S \circ S$ is dense in S. A study is made of relations between the socle and Jordan ideals of S for topological rings. Finally, a new proof of Herstein's result for S in simple associative rings is given. (Received November 13, 1967.)

68T-130. M. E. WATKINS, University of Waterloo, Waterloo, Ontario, Canada. A topological problem in connectivity of graphs. II.

Let G denote a finite undirected graph without loops or multiple edges. V(G) denotes its vertex set. For each integer $n \geqq 2$, $\mathscr{Q}_n$ denotes the condition: $|V(G)| \geqq 2n$, and for any ordered n-tuples $(a_1, \ldots, a_n)$, $(b_1, \ldots, b_n)$ comprising 2n distinct vertices, there exist n pairwise-disjoint arcs $P_i[a_i, b_i] \subset G$ $(i = 1, \ldots, n)$. In Abstract 642-78, these *Notices* 14 (1967), 84, the author announced necessary conditions for G to satisfy $\mathscr{Q}_n$ and the particular case for $n = 2$ was investigated. Although a graph satisfying $\mathscr{Q}_3$ is necessarily 5-connected and nonplanar (see aforementioned abstract), the following results are now known: Theorem 1. The only graph satisfying $\mathscr{Q}_3$ which can be imbedded in the projective plane is the complete graph K_6. Theorem 2. For each $g \geqq 0$, there exists a 5-connected graph of genus g which does not satisfy $\mathscr{Q}_3$. (Received November 10, 1967.)

68T-131. W. B. SCONYERS, Texas Christian University, Forth Worth, Texas 76129. A note on $\mathscr{M}$-paracompact spaces.

Definition 1. A family $\{F_\lambda | \lambda \in \Lambda\}$ of subsets of a topological space is said to be hereditarily closure preserving if $\{F'_\lambda | \lambda \in \Lambda\}$ is closure preserving, where $F'_\lambda \subset F_\lambda$ for each $\lambda \in \Lambda$. If a space X has a hereditarily closure preserving closed covering, then X has the weak topology with respect to that covering. Theorem 1. A regular space X is paracompact if and only if every well-ordered open covering of X has a hereditarily closure preserving closed refinement. Theorem 2. In a topological space X the following conditions are equivalent: (a) X is $\mathscr{M}$-paracompact and normal. (b) Every open covering of X with power $\leqq \mathscr{M}$ has a linearly cushioned open refinement. (c) Every open covering of X with power $\leqq \mathscr{M}$ has a cushioned refinement. Definition 2. A family $\{F_\lambda | \lambda \in \Lambda\}$ is said to be linearly hereditarily closure preserving if Λ can be well-ordered in such a way that for each $\lambda \in \Lambda$, the family $\{F_\gamma | \gamma < \lambda\}$ is hereditarily closure preserving. Theorem 3. A normal space X is $\mathscr{M}$-paracompact if and only if for every well-ordered open covering $\{U_\alpha | \alpha \in A\}$ where card (A) $\leqq \mathscr{M}$, there is a linearly hereditarily closure preserving open covering $\{V_\alpha | \alpha \in A\}$ such that $\mathrm{Cl}(V_\alpha) \subset U_\alpha$ for each $\alpha \in A$. (Received November 6, 1967.)

68T-132. H. M. SRIVASTAVA, West Virginia University, Morgantown, West Virginia 26506. Finite symmation formulae associated with a class of generalized hypergeometric polynomials.

For the sequence of polynomials $\{\Phi_n^{(\lambda)}(x)\}$ introduced by means of the generating relation $E(x^\alpha t)\, G[f(x)(t/m)^m] = \sum_{n=0}^{\infty}(t^n/(\lambda+1)_n)\Phi_n^{(\lambda)}(x)$, where, as usual, E(Z) denotes the exponential function, and $G[z] = \sum_{k=0}^{\infty} g_k z^k$ $(g_k \neq 0)$, the author gives here a general formula that expresses $\Phi_n^{(\lambda)}(x)$ as a finite sum of $\Phi_n^{(\lambda)}(y)$, and discusses its numerous interesting special cases including scores of hitherto scattered results proved by various writers [cf., for instance, Proc. Amer. Math. Soc. 17 (1966), 552-556 and the bibliography quoted therein] in the theory of generalized hypergeometric polynomials. (Received November 14, 1967.)

68T-133. M. M. MARJANOVIC, University of Florida, Gainesville, Florida. A topological characterization of isometries.

Let M be a compact metrizable topological space and f: $M \rightarrow M$ a homeomorphism. Denote by D the set of all topologically equivalent metrics on M and let $f^n = f \circ f^{n-1}$, for n = 1, 2,..., f^0 identity and $f^{-n} = f^{-1} f^{-n+1}$, n = 1, 2,... . The topological concept of an evenly continuous family, due to T. L. Kelley and A. P. Morse, is used here to obtain a theorem which states when a homeomorphism is an isometry with respect to a suitably chosen metric. Theorem. Let f: $M \rightarrow M$ be a homeomorphism of a compact metrizable topological space. Then, there is a $\delta \in D$ such that f is an isometry with respect to δ if and only if the family $\{f^T\colon T \in I\}$ (I is the set of all integers) is evenly continuous. (Received November 20, 1967.)

68T-134. C. D. TABOR, 233 East Fifth Street, Homer, Louisiana. On extended topologies.

A simple extension of a topology is defined by Norman Levine as follows: Let $(X, \mathscr{T})$ be a topological space where $\mathscr{T}$ is some topology on X, and let A be a subset of X such that $A \notin \mathscr{T}$. Then the topology $\mathscr{T}(A) = \{U \cup (U' \cap A) | U, U' \in \mathscr{T}\}$ is called a simple extension of $\mathscr{T}$. A concept of (simultaneously) extending a topology by an arbitrary number of sets is introduced. Such an extension is denoted $\mathscr{T}[A_\alpha]$ where $\{A_\alpha | A_\alpha \subset X, A_\alpha \notin \mathscr{T}, \alpha \in \Lambda\}$ is the collection of subsets of X by which $\mathscr{T}$ is extended. It is shown that an extension of the form $\mathscr{T}[A_\alpha]$ is equivalent to a well-ordered succession of simple extensions. Theorem 1. Let $\mathscr{T}_1$ and $\mathscr{T}_2$ be topologies for a set X, $\mathscr{F}$ an arbitrary filter on X, and A_1 and A_2 the sets of convergence points of $\mathscr{F}$ with respect to $\mathscr{T}_1$ and $\mathscr{T}_2$ respectively. If $G_2 = \{T_\alpha | T_\alpha \notin \mathscr{T}_1, T_\alpha$ is a member of an open basis for $\mathscr{T}_2, \alpha \in \Lambda\}$, then

$A_1 \cap A_2$ is the set of convergence points of $\mathscr{F}$ with respect to $\mathscr{T}_3$ iff $\mathscr{T}_3 = \mathscr{T}_1[T_a]$. Theorem 2. $\mathscr{T}[A_a] = \mathscr{T}(\bigcup\{A_\alpha | a \in \Lambda\})$ iff $A_a \cap [\bigcup\{A_a | \alpha \in \Lambda\} - A_a] = \emptyset$ for every $a \in \Lambda$. Theorem 3. Let $(X, \mathscr{T})$ be a topological space and let A and B be subsets of X such that $A, B \in \mathscr{T}$. Then $\mathscr{T}(A) \subset \mathscr{T}(B)$ iff $[(\mathrm{Bdr}_{\mathscr{T}} A) \cap A] \subset [(\mathrm{Bdr}_{\mathscr{T}} B) \cap B]$ and there exists $U' \in \mathscr{T}$ such that $U' \cap B = A \cap B$ ($\mathrm{Bdr}_{\mathscr{T}} A$ denotes Bdr A with respect to $\mathscr{T}$). Some theorems concerned with the idea of extending one topology by the members of an open basis for another topology are also given. (Received November 20, 1967.)

68T-135. R. N. GUPTA, Ramjas College, Delhi-7, India, and F. SAHA, 9-C, Maurice Nagar, Delhi, India. On quasi-Frobenious local rings.

A semisimple Artinian ring which is not a division ring provides a counter example to the following result proved by Tol'skaja: A ring R is quasi-Frobenious iff every injective (right) R-module is free [MR 33 # 1329]. It is proved R is a quasi-Frobenious local ring iff every injective R-module is free. (Received November 20, 1967.)

68T-136. F. M. CHOLEWINSKI, Clemson University, Clemson, South Carolina, D. T. HAIMO, Southern Illinois University, Edwardsville, Illinois. Generalized temperatures and analytic functions.

The generalized heat equation is given by $\Delta_x u(x,t) = (\partial/\partial t)u(x,t)$, where $\Delta_x f(x) = f''(x) + (2\nu/x) f'(x)$, ν is a fixed positive number. Its fundamental solution is the function $G(x;t) = G(x,0;t)$ where $G(x,y;t) = \int_0^\infty e^{-tu} \mathscr{J}(xu) \mathscr{J}(yu)(u^{2\nu}/2^{\nu-1/2}\Gamma(\nu + 1/2))\,du$, with $\mathscr{J}(z) = 2^{\nu-1/2}\Gamma(\nu + 1/2) \times z^{1/2-\nu} J_{\nu-1/2}(z)$, $J_a(z)$ being the ordinary Bessel function of order a. Generalized temperatures are C^2 solutions of the generalized heat equation and those generalized temperatures u(x,t) which satisfy the condition $u(x,t) = \int_0^\infty G(x,y;t - t')u(y,t')(y^{2\nu}/2^{\nu-1/2}\Gamma(\nu + 1/2))\,dy$, with the integral converging absolutely for all t, t', $a < t' < t < b$, are said to have the Huygens property for $a < t < b$. The close analogy between the theory of generalized temperature functions and that of functions of a complex variable is developed. For example, a parallel to expansions in a Taylor series for analytic functions is that of expansions in terms of generalized heat polynomials $P_{n,\nu}(x,t)$ for functions with the Huygens property, where $P_{n,\nu}(x,t) = \sum_{k=0}^{n} 2^{2k}\binom{n}{k} \times \Gamma(n + \nu + 1/2)/\Gamma(n - k + \nu + 1/2))\, x^{2n-2k}t^k$. (Received November 20, 1967.)

68T-137. W. P. KAPPE and M. W. KONVISSER, Ohio State University, Columbus, Ohio 43210. Self-centralizing elements in finite p-groups. Preliminary report.

Call an element $X \in G$ self-centralizing if the centralizer of x in G consists of powers of x. For regular p-groups self-centralizing elements are elements of maximal order, but not every element of maximal order in a regular p-group having a self-centralizing element is self-centralizing. Theorem. Let G be a regular p-group having a self-centralizing element. Then G is generated by its self-centralizing elements and the minimum number of generators for G is bounded by p and the class of G. (Received November 20, 1967.)

68T-138. J. K. BROWN, Ohio State University, Columbus, Ohio 43210. Finite groups have automorphisms of large order. Preliminary report.

Let G be a finite group of order $|G|$, and m(G) the maximum of the orders of automorphisms of G. For an automorphism s of G, denote by G^{s-1} the subgroup generated by all $g^{-1}g^s$ for $g \in G$. Theorem. (1) $m(G) \leqq |G| - 1$ and equality implies that G is an elementary abelian p-group. (2) If $m(G) > (1/2)|G|$, then G is abelian of a very special type. (3) If $m(G) \geqq (1/4)|G|$, then G is solvable. (4) For an automorphism s of G we have $|s| \leqq |G^{s-1}|$ and equality implies that G^{s-1} is either cyclic or has the form $C_n \times C_2 \times C_2$, with n odd. (5) If k is an integer greater than 1, then the number of finite groups G having $m(G) = |G| - k$ is finite. Call an automorphism s of G simple if it involves a cycle of group elements whose length is equal to the order of s and let k(s) be the number of distinct primes dividing the order of s. Then (1) If $k(s) = 2$, then s is simple. (2) If $k(s) = 3$ or 4 and s is not simple, then G is not simple. (3) If G is simple and s is not simple, then $|G| > 9240$. (Received November 20, 1967.)

68T-139. E. W. CHENEY and F. SCHURER, University of Texas, Austin, Texas 78712. On spline interpolation operators.

Let C denote the space of continuous functions f on [0,1] such that $f(0) = f(1)$. Give C the supremum norm. If $0 = x_0 < x_1 < \ldots < x_n = 1$ then denote by S the subspace of C consisting of cubic spline functions with knots $x_0, \ldots, x_n$. To each f in C there corresponds a unique element Lf in S which interpolates to f at the prescribed knots. The operator L is a linear projection onto S. For a sequence $\{S_n, L_n\}$ of such spaces and projections it can happen (as in ordinary polynomial approximation) that $\|L_n\| \to \infty$, while other linear operators and nonlinear projections onto S_n converge pointwise to the identity. Theorem 1. $(1/42)\alpha\beta - 1 \leqq \|L\| \leqq (3/2)\alpha\delta + 1$, where $\alpha = \max(h_i^{-1})$, $\beta = \min(h_i + h_{i+1})$, $\delta = \max(h_i)$, and $h_i = x_i - x_{i-1}$. Theorem 2. For all $f \in C$, dist $(f, S) \leqq 18\omega(f; \delta)$. Theorem 3. There is a linear operator $A : C \to S$ such that $\|f - Af\| \leqq 18\omega(f; \delta)$ for all $f \in C$. (Received November 20, 1967.)

68T-140. C. K. GUPTA, University of Manitoba, Winnipeg 19, Manitoba, Canada. On 2-metabelian groups. Preliminary report.

A group is said to be n-metabelian if each of its n-generator subgroups is metabelian. Thus if $n \geqq 4$, n-metabelian is same as metabelian. But there is a 2-metabelian group which is not 3-metabelian and a 3-metabelian group which is not metabelian [B. H. Neumann, Proc. Glasgow Math. Assoc. 3 (1957), 13-17]. If a group is 3-metabelian, then it is centre-by-metabelian [I. D. Macdonald, Math. Z. 76 (1961), 270-282]. There is a centre-by-metabelian group which is not even 2-metabelian [C. K. Gupta, Thesis, Canberra (1967); or see M. F. Neuman, J. London Math. Soc. 41 (1966), 292]. In this paper, we exhibit a torsion-free group of 3×3 matrices over a commutative ring which is 2-metabelian but not centre-by-metabelian. We further deduce that for every prime $p \geqq 5$, there is a 2-metabelian group of exponent p which is not centre-by-metabelian. In particular, it follows that besides B. H. Neumann's example of the Sylow 2-subgroup of S_8, there are torsion-free groups and also groups of exponent p ($p \geqq 5$) which are 2-metabelian but not 3-metabelian. (Received November 20, 1967.)

68T-141. P. M. GAUTHIER, Case Postale 6128, Montreal 3, Canada. A form of Plessner's theorem and an identity theorem. Preliminary report.

Text of abstract. Let $w = f(z)$ be a function meromorphic in the unit disc D. For $S \subset D$ let $C(f, S)$ denote the cluster set of f on S. A boundary segment $\alpha \subset D$ is a Plessner segment if $C(f, \Delta)$ is total for each stoltz angle Δ containg α. Lemma. A boundary segment α is a Plessner segment for F(z) iff α satisfies the following condition: either α is a ρ-segment (Abstract 67T-448, these *Notices*), or $C(f, \alpha)$ is total. Theorem 1 (A form of Plessner's theorem). For almost every point z of the unit circle, either (a) z is a Fatou point for f(z); or (b) for every boundary segment α ending at z, α is a ρ-segment or $C(f, \alpha)$ is total. Theorem 2. Let f(z) tend to a value along a boundary curve α ending at a point z_0 of the unit circle. Then either z_0 is a Fatou point or there is a sequence of ρ-points whose limit is z_0. Theorem 3. Let f(z) tend to a value along a boundary curve α whose end is not a single point. Then either f(z) is identically constant, or for each point z_0 in the end of α, there is a sequence of ρ-points whose limit is z_0. (Received November 20, 1967.)

68T-142. D. F. PINCUS, 164 Longwood Avenue, Boston, Massachusetts 02115. Comparison of independence results in Mostowski's system (G) and in Zermelo-Fraenkel set theory.

Jech and Sochor (Bull. Polon. Sci., 1966) show that any statement of set theory with bounded scope has a model in Zermelo-Fraenkel (Z.F.) set theory whenever it has a model in Mostowski's system (G). The corresponding theorem fails for an arbitrary set theoretical statement. In particular, all axioms of multiple choice (see Zuckerman, Abstract 66T-422 of these *Notices*) are equivalent to the Axiom of Choice (A.C.) in Z.F., while A.C. is independent of many of them in (G). On the other hand, there is a fairly uniform method of taking a model of (G) and constructing a model of Z.F. with similar properties. For instance there is a model of (G) (Lauchli Fund. Math., 1964) such that (1) there is a canonical well-ordering of any well-orderable set, (2) there is a set which can not be linearly ordered. These properties are true in the corresponding Z.F. model. (Received November 20, 1967.)

68T-143. V. J. MIZEL, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213, and KONDAGUNTA SUNDARESAN, University of Missouri, Columbia, Missouri 65201. Banach sequence spaces.

Let (R^2, N_1) be a two dimensional normed linear space such that if U is the unit cell and P is the positive quadrant then $C\{(1,0), (0,1), (0,0)\} \subset P \cap U \subset C\{(1,0), (0,0), (0,1), (1,1)\}$ where C(X) denotes the convex hull of X. For $K \geqq 2$ define inductively the function N_K on R^{K+1} such that if $x = (x_1, x_2, \ldots, x_{k+1})$ then $N_K(x) = N_1(N_{k-1}(x_1, x_2, \ldots, x_K), |x_{K+1}|)$. Then N_K is a norm on R^{K+1} and the space of all sequences $x = \{x_n\}_{n\geqq 1}$ such that $N_0(x) = \sup_{k\geqq 1} N_K(x_1, x_2, \ldots, x_{k+1}) < \infty$ is a Banach space with N_0 as the norm. Denoting this Banach space as B_{N_0} obtain the following Theorem 1. If $N(1,a) = 1$ for some $a > 0$ then B_{N_0} is isomorphic to l_∞ while if $N_1(1,a) > 1$ for all $a > 0$ then every sequence in B_{N_0} is a null sequence. Theorem 2. B_{N_0} is separable if and only if the vectors $\{e^i\}_{i\geqq 1}$, where $e^i_j = 0$ if $j \neq i$ and $e^i_i = 1$, are a Schauder base of B_{N_0}. Theorem 3. B_{N_0} is reflexive if and only if B_{N_0} and B_{M_0} are separable where B_{M_0} is the sequence space determined by M_1 by the above iteration procedure, M_1 being the adjoint N_1. (Received November 16, 1967.)

68T-144. JOSEPH WARREN, Wayne State University, Detroit, Michigan 48202. An estimation of the maximum modulus. Preliminary report.

Let a boundary spiral, S, in D : $|z| < 1$ be defined by S : $z(t)$, $0 \leqq t < 1$, $z(t)$ continuous and simple, $z(0) = 0$, $\operatorname{Arg} z(t) > 0$ for $t > 0$, and limit $\operatorname{Arg} z(t) = +\infty$, as t tends to 1. Let $s = h(z) = -i \log z = (\operatorname{Arg} z + 2\pi) + i(-\log |z|)$ be defined in $D' = D - S$. For $\theta > 2\pi$, the line $\operatorname{Re} s = \theta$ meets $h(D')$ in at least one segment. Let ϕ_θ be the one which "separates $+i\infty$ from $+\infty$". Let k_θ be the preimage of ϕ_θ in D', $k(\theta)$ the length of k_θ. Define $K(\theta)$ to be the maximum of $|f(z)|$ on k_θ. Theorem. Let f(z) be holomorphic and unbounded in D, bounded on S. Then for any $\theta_0 > 2\pi$, $\liminf \log \log K(\theta)/\pi \int_{\theta_0}^{\theta} \{1/k(t)\}\, dt \geqq 1$, as θ tends to ∞. This theorem uses Ahlfors' distortion theorem [Acta Soc. Sci. Fenn. Ser. A, 1 (1930), 1-40] and the Phragmen-Lindelöf theorem in the proof. It is a generalization of a theorem of Schnitzer and Seidel [Math. Z. 88 (1965), 301-308]. (Received November 13, 1967.)

68T-145. J. N. McNAMARA, Columbia University, New York, New York 10027. Piecewise linear fibre bundles.

The category of quasi-topological spaces is constructed from the category of compact Hausdorff spaces (Spanier, Duke Math. J. 30 (1963), 1-14). If the same construction is applied to the category of finite simplicial complexes with piecewise linear maps, a category $\mathscr{A}$ results having the following properties: 1. The category of locally finite simplicial complexes with piecewise linear maps is a full subcategory of $\mathscr{A}$. 2. Each set of morphisms in $\mathscr{A}$ has a canonical structure as an object of $\mathscr{A}$. 3. If X is an object of $\mathscr{A}$, the set of automorphisms of X is a group object in $\mathscr{A}$. A theory of fibre bundles is examined in $\mathscr{A}$. Principal bundles with base a locally finite simplicial complex are classified by a universal bundle of the type constructed by Milnor. If G is the group of piecewise linear automorphisms of euclidean n-space, then the universal G-bundle also classifies the n-dim piecewise linear microbundles. (Received November 13, 1967.)

68T-146. EDGAR ASPLUND, University of Washington, Seattle, Washington 98105. Fréchet differentiability of convex functions.

Theorem. If E is a Banach space that can be provided with an equivalent norm such that its dual norm on E* is locally uniformly rotund, f is a convex function on E, and C is the largest open subset of E on which f is (finite valued and) continuous, then the set of points in E at which f is Fréchet differentiable is a dense G_δ subset of C. Remark. Previously, essentially the same conclusion was known to hold if E is a reflexive Banach space admitting an equivalent Fréchet differentiable norm (Lindenstrauss, On operators which attain their norm, Israel J. Math. 1 (1963)). This class of Banach spaces is contained in that of the theorem, as are also certain nonreflexive Banach spaces, e.g. c_0 of a countable set. (Received November 8, 1967.)

68T-147. G. G. JOHNSON, University of Georgia, Athens, Georgia 30601. Continuous commuting functions.

Theorem. If f is a monotone continuous function from [0,1] into [0,1], then there is a non-monotone continuous function g from [0,1] into [0,1] such that $f \circ g = g \circ f$. (Received November 13, 1967.)

68T-148. E. G. MANES, Harvey Mudd College, Claremont, California. A triple-theoretic construction of compact algebras.

Theorem. The category of compact Hausdorff spaces in tripleable over the category of sets (the triple is constructed explicitly). Each algebra over a triple in sets may be described (via the Yoneda lemma) by operations, recovering Birkhoff's approach to universal algebra (and generalizing it since operations may be of unbounded arity, e.g. compact spaces). Theorem. Given a category of algebras, the induced category of compact algebras (operations are continuous) is again tripleable over sets. Theorem. The category of compact Hausdorff topological transformation groups with a given topological phase group is tripleable over sets. (Received November 15, 1967.)

68T-149. D. D. BONAR, Ohio State University, Columbus, Ohio 43210. On annular functions. Preliminary report.

A function f on the unit disk D is annular if it is holomorphic and there exists a sequence of closed Jordan curves J_n in D converging outward to the boundary K of D such that the minimum of $|f|$ on J_n converges to ∞. If the J_n are circles concentric with D then f is strongly annular. Let $Z_a = \{ z : f(z) = a \}$, and Z'_a denote the limit points of Z_a. As a consequence of the minimum modulus theorem, for each a, Z_a is an infinite set. Also $Z'_a \subseteq K$. Theorem 8.6(i) of Collingwood and Lohwater, The theory of cluster sets, Cambridge Univ. Press, London, 1966, is used in obtaining Theorem 1. If f is annular then $Z'_a = K$ for all $a \notin S$ where S is at most a countable subset of the complex plane. If $a, b \in S$, $a \neq b$, then the complements of Z'_a and Z'_b are disjoint open sets in K. For each natural number n let R_n denote the set of points belonging to n equally spaced radii in D. A result of Rubel (Duke Math. J. 30 (1963), 437, Lemma 1) aided in obtaining Theorem 2. If f is strongly annular then Z_a cannot be a subset of R_n for any value of n. (Received November 16, 1967.)

68T-150. K.-T. CHEN, University of Illinois, Urbana, Illinois 61801. Homotopy of algebras.

Let k be a given commutative ring. All rings and modules are unitary. Two morphisms of k-algebras $\phi_i : A \rightarrow A'$, i = 0, 1, are said to be homotopic if there exist (i) a split exact sequence $0 \rightarrow k \xrightarrow{j} B \xrightarrow{\partial} N \rightarrow 0$ where j is the canonical morphism from k into a commutative k-algebra B and ∂ is a derivation from B into a B-module N, (ii) two augmentations s_0 and s_1 of the k-algebra B, and (iii) a morphism of k-algebras $\Phi: A \rightarrow A' \otimes B$ such that $\phi_i = (1 \otimes s_i)\Phi$. The homotopy relation gives rise to a homotopy category of k-algebras. There is a nontrivial covariant homotopy functor which assigns, to each cummutative k-algebra A, a cohomology ring obtained from the exterior algebra $\bigwedge_A(M)$, M being the A-module of universal differentials of A. (Received November 16, 1967.)

68T-151. L. M. BLUMENTHAL, University of Missouri, Columbus, Missouri 65201. Metric postulates for normed Boolean algebra.

Five points p, q, a, b, c of a metric space form (i) a θ-figure between p and q provided the betweenness relations B(p, a, q), B(p, c, q), B(a, b, c) subsist, but B(p, b, q) does not, and (ii) a tailed-Δ-figure with origin p provided B(p, q, a), B(p, q, c), and B(a, b, c) subsist, but B(p, q, b) does not, where B(x, y, z) denotes the relation: dist(x, y) + dist(y, z) = dist(x, z). The principal theorem proves that a metric space $\mathscr{M} = \{M; d\}$ is a normed Boolean algebra $\mathscr{B} = \{M; \cdot, +, ', \| \|\}$ if and only if $0 \in M$ such that (1) $\mathscr{M}$ does not contain a tailed-Δ-figure with origin 0, (2) $\mathscr{M}$ does not contain any θ-figure, (3) $a, b \in M$ imply the existence of $x_{a,b}, y_{a,b} \in M$ such that $B(a, x_{a,b}, b)$, $B(a, y_{a,b}, b)$ subsist, and if $x \in M$ with B(a, x, b) then $B(0, x_{a,b}, x)$ and $B(0, x, y_{a,b})$ subsist, (4) M contains at least one element 1 $(1 \neq 0)$ with B(0, x, 1) for every $x \in M$, and (5) $x \in M$ implies the existence of x' $(x' \in M)$ such that $B(x, 0, x')$ and $B(x, 1, x')$ subsist. Defining $a \prec b \equiv B(0, a, b)$ then $\mathscr{M}$ is a complemented distributive lattice $\mathscr{B}$, $(a \cdot b = x_{a,b}$, $a + b = y_{a,b}$, x' = complement of x) with $\|a\| = d(0, a)$ and $d(a, b) = \|a + b\| - \|a \cdot b\|$. (Received November 15, 1967.)

68T-152. D. G. JAMES, Pennsylvania State University, University Park, Pennsylvania 16802. A Witt theorem for indefinite unimodular quadratic forms.

Let L be a free Z-module of finite rank, and ϕ a unimodular symmetric bilinear map of $L \times L$ to the integers, Z. C. T. C. Wall, On the orthogonal groups of unimodular quadratic forms, Math. Ann. 147 (1962), pp. 328-338, gives necessary and sufficient invariants on two vectors $\alpha, \beta \in L$ for there to exist a mapping σ in the orthogonal group O(L, Z) of L over Z, such that $\sigma(\alpha) = \beta$(with a restriction on the rank r(L) and signature s(L) of L). The author has the following generalization. Theorem. Let $\sigma : J \longrightarrow K$ be a bijective linear transformation between the sublattices J and K of L, where $r(L) - |s(L)| \geqq 2(r(J) + 1)$. Then σ extends to an element of O(L, Z) if and only if, for each $\alpha \in J$, α and $\sigma(\alpha)$ have the same invariants (divisor, norm and type--see Wall for definitions). (Received November 16, 1967.)

68T-153. A. M. BRUCKNER, J. G. CEDER, and R. KESTON, University of California, Santa Barbara, California 93106. Representations and approximations by Darboux functions in the first class of Baire.

The following theorems are proven for a real valued function f defined on a real interval: Theorem A. If f is a Baire 1 function, then there exists a Darboux (i.e., maps intervals into intervals) Baire 1 function g such that f = g except on a first category set of measure zero. Theorem B. If f is a Baire 1 function, there exist Darboux Baire 1 functions g and h such that g = g + h. Theorem C. If f is a Baire 2 function, then there exists a sequence of Darboux Baire 1 functions converging pointwise to f. The corresponding facts for Baire α functions $(\alpha > 1)$ were recently obtained by A. B. Gurevič [DAN BSSR 10 (1966), 539-541] and L. Mišik [Rev. Math. Pures Appl. 12 (1967), 849-860]. (Received November 8, 1967.)

68T-154. J. W. ADDISON, JR., University of California, Berkeley, California 94720. Separation principles in difference hierarchies.

For any class R of sets and any n in ω let $\mathscr{D}_0(R) = \{\emptyset\}$, $\mathscr{D}_{n+1}(R) = \{A \sim B : A \in R, B \in \mathscr{D}_n(R)\}$, and $\check{\mathscr{D}}_n(R) = \{\bigcup R \sim A : A \in \mathscr{D}_n(R)\}$. Theorem. For any class R of sets closed under $\cap$ and any positive n in ω: if $\mathscr{D}_n(R)$ has the first separation property (relative to $\bigcup R$) -- i.e. for any disjoint A, B in $\mathscr{D}_n(R)$ there exists a C in $\mathscr{D}_n(R) \cap \check{\mathscr{D}}_n(R)$ such that $A \subseteq C$ and $C \cap B = \emptyset$ -- then so does $\check{\mathscr{D}}_{n+1}(R)$. Theorem. For any class S of sets closed under $\cap$ and any n in ω: if $\check{\mathscr{D}}_1(S)$ has the reduction property -- i.e. for any A, B in $\check{\mathscr{D}}_1(S)$ there exist A', B' in $\check{\mathscr{D}}_1(S)$ such that $A' \subseteq A$, $B' \subseteq B$, $A' \cup B' = A \cup B$, and $A' \cap B' = \emptyset$ -- and $\mathscr{D}_n(S)$ has the reduction property, then so does $\check{\mathscr{D}}_{n+1}(S)$. Theorem. For any class R of sets closed under $\cup$ and $\cap$ and containing $\emptyset$: $\mathscr{D}_n(R)$ has the reduction property for all n in ω if and only if R has the reduction property. Illustration. Let R be the set $\Sigma_1^0 \cap \mathscr{P}\omega$ of recursively enumerable subsets of ω. Then for any n $\check{\mathscr{D}}_n(R)$ has the first separation property and $\mathscr{D}_n(R)$ does not, so that the separation behavior of the "difference hierarchy" $\langle \mathscr{D}_n(\Sigma_1^0 \cap \mathscr{P}\omega) : n \in \omega \rangle$ generated by $\Sigma_{n+1}^0 \cap \mathscr{P}\omega$ parallels that of the arithmetical hiraracht $\langle \Sigma_{n+1}^0 \cap \mathscr{P}\omega : n \in \omega \rangle$. (Received November 22, 1967.)

68T-155. R. D. M. ACCOLA, Brown University, Providence, Rhode Island 02912. Strongly branched coverings of closed Riemann surfaces.

Let $\alpha : W_1 \to W_2$ be an n-sheeted covering of closed Riemann surfaces of genera g_1 and g_2. Call α strongly branched if $g_1 > n^2 g_2 + (n-1)^2$. Theorem 1. Suppose α is strongly branched. Then there are maps $\beta : W_1 \to W_3$ and $\gamma : W_3 \to W_2$ where $\alpha = \gamma \circ \beta$ and $\beta \neq \mathrm{id}$ so that if f_1 is meromorphic on W_1, of order $\leq n(g_2 + 1)$, then there is an f_3, meromorphic on W_3, and $f_1 = f_3 \circ \beta$. Definition. If G is a group of conformal self-maps of a surface W, G will be called strongly branched if the map $W \to W/G$ is strongly branched. Theorem 2. If G is strongly branched, then G contains a nonidentity subgroup which is normal in the full group of conformal self-maps of W. Corollary. If G is simple and strongly branched, then G is normal in the full group of W. (This generalizes the hyperelliptic situation.) Remarks. The proofs are elementary using mainly the Riemann-Hurwitz formula. The method gives explicit constructions of surfaces, of genera five or more, which admit only the identity as a conformal self-map. (Received November 22, 1967.)

68T-156. C. E. VASCO, Saint Louis University, St. Louis, Missouri 63103. Homogeneous identities on algebraic loops. Preliminary report.

Let $S_n = x_1 x_2 \ldots x_n$ be a string of n variables taken from a loop (L, *). Denote the operation in Lukasiewicz form: $xy\mu = x * y$. Then $S_n\mu^{n-1} = S_n\mu^{n-1}$ is a trivial homogeneous identity of degree n in standard form, denoted I_n. All nontrivial identities of this type can be obtained from I_n by applying to it pairs (α, β) of motions of the μ-operators, pairs $\{\pi, \rho\}$ of permutations of the variables, and identifications (k) of subsets of the variables. By means of this formalism, which provides a way of eliminating all trivial and isomorphic identities, the number of nonisomorphic identities of degree n in n distinct variables and the number of nonpermutational identities of degree n in any number of distinct variables are found. The cases n = 2, 3, 4, 5 are developed, Faragó's list of all third-degree identities is recovered, and all (150) fourth-degree identities (exclusive of permutations) are found and classified. The Moufang-Bol axioms, the Moufang identities, the Jordan identity, and Osborn's weak power-associativity are placed in a wider context, and many analogues of these identities are found. All third- and fourth-degree identities are studied in the 109 nonisomorphic loops of order 6 published by Bryant and Schneider, Canad. J. Math. 18 (1966), 120-125. (Received November 22, 1967.)

68T-157. J. P. FINK, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. Asymptotic estimates of Feynman integrals.

An asymptotic theorem is developed which gives the logarithmic asymptotic behavior of certain convergent integrals containing parameters. This theorem is then applied to the photon and electron self-energy graphs of quantum electrodynamics to obtain the following result: Theorem. If (a) the energy of contours of the Feynman integral corresponding to a photon or electron self-energy graph are rotated from the real axis to the imaginary axis, and (b) the momentum q of the photon or electron is replaced by tq, where t is a real scalar, then the asymptotic behavior of the photon or electron self-energy graph as $t \to \infty$ is given by $Ct^{\alpha}(\log t)^{\beta}$, where C is a constant, $\alpha = 1$ for electron self-energy graphs, and 2 for photon self-energy graphs, and β is the number of irreducible insertions in the graph. (Received November 22, 1967.)

68T-158. K. L. SINGH, Memorial University of Newfoundland, St. John's, Newfoundland, Canada. Commuting functions and common fixed points.

Theorem 1. If f and g are mappings of $[0,1]$ into itself such that $fg(x) = gf(x)$ for all $x \in [0,1]$ and $|f(x) - f(y)| \leqq |x - y|$ and $|g(x) - g(y)| \leqq \beta |x - y|$ for all $x, y \in [0,1]$ where β is any positive real number and $0 < \alpha < 1$, then there exists a common fixed point for both f and g. Theorem 2. If f and g are two continuous functions from $[0,1]$ into itself such that $fg(x) = gf(x)$ for all $x \in [0,1]$ and $|f(x) - f(y)| < \alpha |x - y|$ is a one to one and onto mapping of $[0,1]$ into itself, and $|y(x) - g(y)| \leqq \beta |x - y|$, where β is any positive real number and $\alpha > 1$, then f and g have a common fixed point. (Received November 22, 1967.)

68T-159. MAURICE CHACRON, University of Sherbrooke, Sherbrooke, Quebec, Canada. Structure of a certain class of ordered groupoids. Preliminary report.

Semigroups in which (1) $x^{n+1} = x$ has been studied particularly by J. A. Green and D. Rees, J. C. Brown, and more recently by L. Lesieur. He showed that the multiplicative semigroup of a ring with (1) is an abelian semigroup and (2) a disjoint union of periodic subgroups. In this note a suitable approaching of semigroups with (1) is proposed, namely, the class of the ordered groupoids R (in the sense of G. Birkhoff) such that every subgroupoid of R of one generator is at the same time finite (periodicity of R), associative (power associativity of R) and such that $x^{n+1} \geqq x$ for all x (n natural number depending on x). Some results related to (1) are given. For example, it is shown that the structure of R is nothing other than (3) a union (refined in a disjoint union in case of equality (1)) of suitable approaching of periodic groups, namely, periodic semigroups G_e with a unique idempotent e, maximal in G_e and weak right unity in G_e, that is $xe \geqq x$ for all x in G_e. Clearly, if G_e possess a one-side unity it must be a periodic group and this is the case of an ordered groupoid R such that

the order relation is the same as the relation of equality. If follows that decomposition (3) contains as a particular case not only (2) but also extends it to power associative groupoids with (1).
(Received November 17, 1967.)

68T-160. J. E. KOEHLER, S. J., Seattle University, Seattle, Washington 98122. Folding a strip of stamps.

This paper answers the problem of determining how many ways a strip of N stamps can be folded along the perforations so that the stamps are piled one on top of the other without destroying the continuity of the strip. Using combinatorial arguments, the problem is first solved by an inductive formula for N odd. Part of the computations involved are seen to yield an answer to the case when N is even. Three cases are considered: (1) both, (2) one of, (3) neither, the left end and top of the strip are labeled. If P(N) is the number of folds for a strip of N stamps of type (1), then (1/2)P(N) and (1/4)P(N) + (1/4)S(N) gives respectively the number of folds for a strip of N stamps of types (2) and (3), where S(N) is the number of symmetric folds of the strip. P(N) also equals the number of ways of joining N points on a circle by chords of alternating blue and red color without having any chords of the same color intersect. A computer lists the first few values of P(N), starting with P(3), as 6, 16, 50, 144, 462, 1392, 4536, 14060, 46310, 146376, 485914, 1557892, 5202690, 16861984 and of S(N), starting with S(3), as 2, 4, 6, 8, 18, 20, 56, 48, 178, 132, 574, 348, 1870, 1008. (Received November 8, 1967.)

68T-161. J. M. GANDHI, University of Manitoba, Winnipeg 19, Manitoba, Canada. Generalization of some partition function.

Let $A_k^{(s)}(n) = \sum_\rho (\omega_{\rho,k})^s e^{-2\pi n\rho i/k}$ where ρ ranges over those numbers which are less than k and prime to k. Here $\omega_{\rho,k}$ are certain 24kth roots of unity [for its definition see Lehmer, Trans. Amer. Math. Soc. 43 (1938), 271-295]. A large number of theorems have been proved for $A_k^s(n)$. Sone typical examples are: Theorem 1. If k be an odd integer then $A_k^s(n) = A_{2k}^{(s)}(4n + s(k^2 - 1)/8)$. Theorem 2. If k is odd then $A_2^{(s)}(n_1)A_k^{(s)}(n_2) = A_k^{(s)}(n_3)$ where $n_3 \equiv 4n_2 + kn_1 + s(k^2 - 1)/8 \pmod{2k}$. It is also proposed to study the function $A_k^s(n)$ defined by $A_k^{(s)}(n) = \sum_{(\rho,k^s)=1} \omega_{\rho,k} e^{-2\pi n\rho i/k^s}$ for all s, where ρ ranges over the nonnegative integers $< k^s$ such that ρ and k^s have no common sth power divisors other than 1. (Received December 3, 1967.)

Errata - Volume 14

JOAQUIN BUSTOZ. Gibbs sets and the generalized Gibbs phenomenon. Abstract 652-7, Page 938.

Line 5: Replace $\{s_n(z_0)\}$ by $\{s_n(z)\}$.

RANDOLPH CONSTANTINE, JR. A uniform boundedness theorem for nets. Abstract 651-6, Page 936.

The abstract was withdrawn prior to presentation at the meeting because of the discovery of an error in the purported proof.

D. W. DUBOIS. Topology of orders and modes. Abstract 651-4, Page 935.

Line 7: The last sentence should read: "If Q(X) is the one variable function field over the rationals then Arch is discontinuous on $\mathscr{M}(Q(X))$; but there exists a natural bijection of Arch $(\mathscr{M}(Q(X)))$ on the one point compactification of the reals."

N. F. LINDQUIST. Representations of central convex bodies. Abstract 648-54, Page 645.

Line 3: "... for all $u \in \Omega(2)\}$" should read "... for all $u \in \Omega(2)$ orthogonal to $\overline{v}\}$."

R. F. MILLAR. On the Rayleigh assumption in scattering by a periodic surface. Abstract 648-190, Page 687.

Line 7: Replace "(i) $\kappa b < 0.448$" by "(i) $\kappa b < \gamma$, where γ is some positive number no greater than 0.448".

Line 12: Replace "... within C" by "... within C and the exterior Green's function for C has no critical points outside C'".

Line 13: Replace "... outside C' when (i) obtains" by "... outside C' when $\kappa b < 0.448$ while the Green's function for C has no critical points outside C' when (i) obtains".

T. S. SHORES. On groups with category. Abstract 67T-613, Page 840.

Line 8: The statement of Theorem 3 should be: "If Σ is a subgroup, direct power and extension closed class of groups which contains all abelian groups, then $H \wr K \in C(\Sigma)$ iff $H \in \Sigma$ and $K \in C(\Sigma)$."

K. SUNDARESAN. The extended spherical image map. Abstract 648-18, Page 819.

Line 8: Replace "C^1-diffeomorphism if and only if" by "C^1-diffeomorphism if and only if the norm of its gradient is less than or equal to 1 at all x in the unit sphere."

Line 9: Replace "3" by "2".

Line 10: Replace "differentiable if and only if" by "differentiable with the norms of their derivatives at all x in the unit sphere less than or equal to 1 if and only if".

New and Forthcoming

CALCULUS AND ANALYTIC GEOMETRY, 2nd Edition

Abraham Schwartz, The City College of the City University of New York

A partial list of adoptions includes: MIT • Bemidji St. College University of Colorado • University of Pittsburgh • UCLA University of California, Santa Barbara • University of Washington • University of Nebraska • Stanford University • Rio Hondo Jr. College • Brigham Young University • Nassau Community College • Staten Island Community College • Suffolk Community College • Manhattan College, N. Y. • Cooper Union • CCNY • McGill University • Solutions Manual.

1967 1024 pp. $12.95

MATRICES AND LINEAR ALGEBRA

Hans Schneider and George P. Barker, both of the University of Wisconsin, Madison

Intended for students of mathematics as well as of other disciplines, this book combines concrete aspects of the subject with more abstract facets to provide a balance not found in other texts. While matrices are emphasized, a chapter is devoted to linear transformations, which are used again as another point of view in a chapter dealing with eigenvalues and similarity.

March 1968 352 pp. $7.00 (tent.)

POST-CALCULUS ANALYSIS

A. E. Labarre, Jr., Fresno State College

CONTENTS: Sets, Relations, and Functions; The Real Numbers; The Limit Concept; Three Important Theorems; Continuity; Differentiability; Integrability; Sequence of Functions, An Existence Theorem in Differential Equations; Fourier Series.

April 1968 256 pp. $8.25 (tent.)

paid announcement

MATHEMATICIANS: Job opportunities in war work are announced in the Notices, in the Employment Register, and elsewhere. We urge you to regard yourselves as responsible for the uses to which your talents are put. We believe this responsibility forbids putting mathematics in the service of this cruel war.

J. H. Abbott
Hugo d'Alarcao
W. Ambrose
Tim Anderson
D. G. Aronson
Gregory Bachelis
Hyman Bass
Paul T. Bateman
Anatole Beck
W. M. Boothby
David Blackwell
John A. Carpenter
Robert Carroll
R. V. Chacon
H. S. M. Coxeter
Chandler Davis
Martin Davis
J. L. Doob
Edward L. Dubinsky
David M. Eaves
John A. Ernest
Harley Flanders
Robert Finn
W. H. J. Fuchs
L. Terrell Gardner
I. Glicksberg
S. I. Goldberg
Leon Green
Henry Helson
Carl S. Herz
N. J. Hicks
Morris W. Hirsch
A. E. Hurd
Benton Jamison
Donald Kahn
N. D. Kazarinoff
M. I. Knopp
Jack Kiefer
John Lamperti
Serge Lang
R. K. Lashof
Lucien LeCam
Karel deLeeuw
Eugene H. Lehman, Jr.
Kenneth Leisenring
M. Loeve
Charles Loewner
Lee Lorch
Roger C. Lyndon
Kenneth O. May
Philip E. Miles
Edwin E. Moise
Edward Nelson
Peter E. Ney
J. Neyman
John A. Nohel
Paul Olum
Steven Orey
Donald Ornstein
Robert R. Phelps
Ralph S. Phillips
M. O. Reade
Michael Rosen
R. A. Rosenbaum
Alex Rosenberg
Joseph G. Rosenstein
Peter Rosenthal
Melvin Rothenberg
J. J. Rotman
G. Roy
Henry Scheffe
Frederic G. Schmitt
Seymour Schuster
Arthur J. Schwartz
Daniel Shea
Allen L. Shields
Marvin Shinbrot
Stephen Smale
J. R. Smart
J. A. Smoller
Frank Spitzer
George Springer
Charles M. Stein
Frank M. Stewart
Dorothy Stone
Dirk Struik
Gabor Szego
Donald H. Taranto
C. J. Titus
Howard G. Tucker
Michael Voichick
Hugh E. Warren
R. F. Williams
G. N. Wollan

paid announcement

Index to Advertisers

order form

AMS MATHEMATICAL OFFPRINT SERVICE

(Please complete this form and mail it to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904)

I wish to enter a subscription to the AMS Mathematical Offprint Service.

Name ______________________________

Mailing Address ______________________________

____
City (State and Zip of U.S.)

____
Country

I would like to receive the subject classification schedule in the following language:

English ______ French ______ German ______ Italian ______ Russian ______

The total cost of one subscription of 100 offprints (ten title listings equal one offprint) is $30.00.

A Modest Guide to San Francisco Bistros...

compliments of W. A. Benjamin, Inc.
plus texts and monographs
of a more purely mathematical
nature will be available at:

Booth 20

AMERICAN MATHEMATICAL SOCIETY
P.O. Box 6248
Providence, Rhode Island 02904

SECOND-CLASS POSTAGE
PAID AT
PROVIDENCE, RHODE ISLAND
AND
ADDITIONAL MAILING OFFICES

Return Requested

TIME VALUE