# Notices

# OF THE AMERICAN MATHEMATICAL SOCIETY

VOLUME 15, NUMBER 3 ISSUE NO. 105 APRIL, 1968

# *Notices*

# OF THE

# AMERICAN MATHEMATICAL SOCIETY

Edited by Everett Pitcher and Gordon L. Walker

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# MEETINGS

## Calendar of Meetings

**NOTE:** This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the *Notices* was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

| Meeting No. | Date | Place | Deadline for Abstracts* |
|---|---|---|---|
| 657 | June 15, 1968 | Portland, Oregon | May 1, 1968 |
| 658 | August 26-39, 1968 | Madison, Wisconsin | July 1, 1968 |
| 659 | October 26, 1968 | Baltimore, Maryland | |
| | January 23-27, 1969<br>(75th Annual Meeting) | New Orleans, Louisiana | |
| | August 25-29, 1969<br>(74th Summer Meeting) | Eugene, Oregon | |
| | January 22-26, 1970<br>(76th Annual Meeting) | Miami, Florida | |
| | August 1970<br>(75th Summer Meeting) | Laramie, Wyoming | |
| | January 21-25, 1971<br>(77th Annual Meeting) | Atlantic City, New Jersey | |

*The abstracts of papers to be presented in person at the meetings must be received in the Headquarters Offices of the Society in Providence, Rhode Island, on or before these deadlines. The deadlines also apply to news items. The next two deadline dates will be April 24, 1968 and June 24, 1968.

* * *

The *Notices* of the American Mathematical Society is published by the Society in January, February, April, June, August, October, November and December. Price per annual volume is $12.00. Price per copy $3.00. Special price for copies sold at registration desks of meetings of the Society, $1.00 per copy. Subscriptions, orders for back numbers (back issues of the last two years only are available) and inquiries should be addressed to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904.

Second-class postage paid at Providence, Rhode Island, and additional mailing offices. Authorization is granted under the authority of the act of August 24, 1912, as amended by the act of August 4, 1947 (Sec. 34,21, P. L. and R.). Accepted for mailing at the special rate of Postage provided for in section 34,40, paragraph (d).

Printed in the United States of America

# Six Hundred Fifty-Fifth Meeting

# Americana Hotel
# New York, New York
# April 10-13, 1968

## PROGRAM

The six hundred fifty-fifth meeting of the American Mathematical Society will be held at the Americana Hotel in New York, New York, from April 10 to April 13, 1968.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings the following four lectures will be presented:

Professor Henry P. McKean, Jr., of Rockefeller University will speak on "A Chapman-Enskog-Hilbert extension for the telegraph equation as a model of the derivation of fluid mechanics from the Boltzmann equation" at 11:00 a.m. on Friday, April 12.

Professor Frederick J. Almgren, Jr., of Princeton University will speak on "Measure theoretic geometry and elliptic variational problems" at 2:00 p.m. on Friday, April 12.

Professor Franklin P. Peterson of the Massachusetts Institute of Technology will speak on "Characteristic classes: old and new" at 11:00 a.m. on Saturday, April 13.

Professor George B. Seligman of Yale University will speak on "Algebraic Lie algebras" at 2:00 p.m. on Saturday, April 13.

A symposium on "Applications of categorical algebra" is scheduled for the afternoon of Wednesday, April 10, and both morning and afternoon of Thursday, April 11. The Organizing Committee consists of Professors Hyman Bass, Alex Heller (chairman), and John Moore.

There will be sessions for contributed ten-minute papers during both mornings and afternoons of Friday and Saturday, April 12 and 13.

### EMPLOYMENT REGISTER

The Mathematical Sciences Employment Register will be open from 2:00 p.m. to 5:00 p.m. on Thursday, April 11, and from 9:00 a.m. to 5:00 p.m. on Friday, April 12, in the Regency Ballroom. Information on the operation of this service may be found on page 43 of the January issue of these *Notices*.

### REGISTRATION

The registration desk will be in the Georgian Ballroom Foyer. It will be open from noon to 5:00 p.m. on Wednesday, April 5, and from 9:00 a.m. to 5:00 p.m. on Thursday through Saturday, April 6-8.

### ROOM RESERVATIONS

Persons intending to stay at the Americana should make their own reservations with the hotel. A reservation blank and a listing of room rates, is on page 416 of the February issue of the *Notices*.

### MAIL ADDRESS

Registrants at the meeting may receive mail addressed in care of the American Mathematical Society, Americana of New York, 801 7th Avenue (52nd between 6th and 7th), New York, New York 10019.

# SYMPOSIUM ON APPLICATIONS OF CATEGORICAL ALGEBRA

## WEDNESDAY, 2:00 P.M.

First Session, Georgian Ballroom A

2:00-2:30
Equality in hyperdoctrines and the comprehension schema as an adjoint functor
Professor F. W. Lawvere, The City University of New York

2:45-3:15
Categorical methods in the theory of computation
Professor Samuel Eilenberg, Columbia University

3:30-4:00
Construction of simplicial objects
Professor Michel André, University of Chicago and Battelle Institute, Geneva, Switzerland

4:15-4:45
On bicartesian squares
Professor Peter J. Hilton, Courant Institute, New York University, and Cornell University

## THURSDAY, 9:00 A.M.

Second Session, Georgian Ballroom A

9:00-9:30
Algebraic cohomology theories
Professor Murray Gerstenhaber, Institute for Advanced Study and the University of Pennsylvania

9:45-10:15
Cohomology of commutative rings
Professor Daniel Quillen, Massachusetts Institute of Technology

10:30-11:00
Generators and relations for certain special linear groups
Professor Richard G. Swan, University of Chicago

11:15-11:45
Groups of cohomological dimension 1
Professor John Stallings, University of California, Berkeley

## THURSDAY, 2:00 P.M.

Third Session, Georgian Ballroom A

2:00-2:30
Hopf algebras and multiplicative fibrations
Professor Larry Smith, Princeton University

2:45-3:15
Stable homotopy of finite complexes
Professor Peter J. Freyd, University of Pennsylvania

3:30-4:00
On a theorem of Wilder
Professor Jean-Louis Verdier, Columbia University and University of Strasbourg, France

4:15-4:45
To be announced.

# PROGRAM OF THE SESSIONS

The time limit for each contributed paper is 10 minutes. The papers are scheduled at 15 minute intervals in order that listeners can circulate among sessions. To maintain the schedule, the time limit will be strictly enforced.

## FRIDAY, 9:00 A.M.

Session on Functional Analysis I. Regency Foyer

9:00-9:10

(1) Weighted shifts. II: Cyclic vectors of the backward shift
Mr. R. A. Gellar, Columbia University and Fordham University (655-120)

9:15-9:25

(2) Rational approximation. II
Dr. William Saffern, New York, New York (655-110)

9:30-9:40

(3) Differential calculus of subconvex functionals
Professor E. B. Leach, Case Western Reserve University (655-60)

9:45-9:55

(4) Invariant subspaces containing all analytic directions
Professor M. J. Sherman, University of California, Los Angeles (655-57)

10:00-10:10

(5) Function algebras with one-point parts and zero point derivations. Preliminary report
Mr. Brian Cole, Yale University (655-77)

10:15-10:25

(6) Multiplier algebras of biorthogonal systems
Mr. R. J. McGivney*, Lafayette College, and Professor William Ruckle, Lehigh University (655-81)

10:30-10:40

(7) Universal ideals of operators. Preliminary report
Mr. Peter Falley, The City University of New York (655-107)

10:45-10:55

(8) Unitary equivalence and similarity of two-sided weighted shifts
Dr. R. L. Kelley, University of Miami (655-4)

## FRIDAY, 9:00 A.M.

Session on Analysis and Probability Theory. Vendome 11

9:00-9:10

(9) Extremal problems for functions with bounded boundary rotation. Preliminary report
Professor J. A. Pfaltzgraff, University of North Carolina (655-87)

9:15-9:25

(10) Some theorems concerning the directional derivative of a function of a complex variable
Professor M. O. Gonzalez, University of Alabama (655-35)

9:30-9:40

(11) On the Laplace transforms of functions of the integral, and the fractional, parts of X
Professor Ira Rosenbaum, University of Miami (655-108)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.

9:45-9:55

(12) Spherically symmetric measures in infinite dimensional spaces

Mr. R. H. Peterson and Dr. W. C. Taylor*, USA Ballistic Research Laboratories, Aberdeen Proving Ground, Maryland (655-28)

10:00-10:10

(13) A Strong Law of Large Numbers for weakly orthogonal sequences of Banach space valued random variables

Professor Anatole Beck and Mr. Peter Warren*, University of Wisconsin (655-6)

10:15-10:25

(14) The Hausdorff dimension of the sample path of a subordinator

Professor Joseph Horowitz, The University of Toledo (655-37)

10:30-10:40

(15) An axiomatic characterization of an algebra of conditional events

Professor Geza Schay, Jr., University of Massachusetts, Boston (655-52)

10:45-10:55

(16) On finite products of Poisson-type characteristic functions of several variables

Professor Roger Cuppens, The Catholic University of America (655-89)

(Introduced by Professor Eugene Lukacs)

## FRIDAY, 9:00 A.M.

Session on Foundations and Category Theory, Vendome 12

9:00-9:10

(17) On free monoids partially ordered by embedding

Professor L. H. Haines, University of California, Berkeley (655-88)

9:15-9:25

(18) On the existence of regular initial numbers

Professor Hidegoro Nakano, Wayne State University (655-24)

9:30-9:40

(19) Recursive functions modulo co-maximal sets. Preliminary report

Mr. Manuel Lerman, Cornell University (655-47)

9:45-9:55

(20) An example of an epic subcategory

Professor Johann Sonner, University of North Carolina, Chapel Hill (655-7)

10:00-10:10

(21) On a class of tricategories

Dr. J. V. Michalowicz, The Catholic University of America (655-9)

10:15-10:25

(22) The category of recursive functions

Professor Harry Gonshor, Rutgers, The State University (655-26)

10:30-10:40

(23) On a universal category of algebras and its embedding

Professor Z. Hedrlín, McGill University (655-66)

(Introduced by Professor J. Lambek)

10:45-10:55

(24) Functor categories and standard constructions (triples)

Dr. M. C. Bunge, McGill University (655-84)

## FRIDAY, 9:00 A.M.

Session on Algebra, Number Theory and Geometry, Chambord 14

9:00-9:10

(25) Lattice isomorphic solvable Lie algebras

Professor R. C. Glaeser, Temple University, and Professor B. Kolman*, Drexel Institute of Technology (655-68)

9:15-9:25

(26) The k-unitary convolution of certain arithmetical functions

Professor J. Chidambarashwamy, University of Toledo (655-19)

9:30-9:40

(27) Codes with simple automorphism groups. Preliminary report

Mr. E. P. Shaughnessy, Lafayette College (655-51)

9:45-9:55

(28) A note on character sums

Professor C. T. Whyburn, Louisiana State University, Baton Rouge (655-58)

10:00-10:10

(29) On a minimality property of complexes

Mr. Joseph Zaks, University of Washington (655-86)

10:15-10:25

(30) Regular hyperbolic subplanes

Professor R. J. Bumcrot, Ohio State University (655-76)

10:30-10:40

(31) On quadrics in projective spaces over finite fields

Dr. Karlhorst Meyer, Technische Hochschule Muenchen and University of Florida (655-44)

(Introduced by Professor A. R. Bednarek)

10:45-10:55

(32) Extension of order-functions in projective spaces. Preliminary report

Mr. D. P. K. Biallas, University of Florida (655-40)

(Introduced by Professor A. R. Bednarek)

### FRIDAY, 11:00 A.M.

Invited Address, Georgian Ballroom A

The Chapman-Enskog-Hilbert extension for the telegraph equation as a model for the derivation of fluid mechanics from the Bolzmann equation

Professor Henry P. McKean, Jr., Rockefeller University

### FRIDAY, 2:00 P.M.

Invited Address, Georgian Ballroom A

Measure theoretic geometry and elliptic variational problems

Professor Frederick J. Almgren, Jr., Princeton University

### FRIDAY, 3:15 P.M.

Session on Complex Analysis, Regency Foyer

3:15-3:25

(33) A characterization of the center of a circular domain which is invariant with respect to pseudo-conformal transformations

Professor Stefan Bergman, Stanford University (655-91)

3:30-3:40

(34) On the zeros of theta functions for superelliptic Riemann surfaces. Preliminary report

Professor R. D. M. Accola, Brown University (655-104)

3:45-3:55

(35) Welding Riemann surfaces and transmission problems with shifts

Mr. Donald Oarth, Princeton University (655-36)

4:00-4:10

(36) Bounds on holomorphic vector fields

Mr. E. R. Fernholz, University of Washington (655-67)

4:15-4:25

(37) Entire functions with widely spaced zeros and of bounded index

Mr. W. J. Pugh, University of Syracuse, and Professor S. M. Shah*, University of Kentucky (655-85)

4:30-4:40

(38) On a question of Seidel concerning holomorphic functions that are bounded on a spiral

Professor K. F. Barth and Professor W. J. Schneider*, Syracuse University (655-80)

4:45-4:55

(39) On the mean values of an entire function represented by a Dirichlet series

Professor P. K. Kamthan*, University of Waterloo, and Mr. P. K. Jain, Hans Raj College, Delhi, India (655-69)

5:00-5:10

(40) Flux in axiomatic potential theory

Professor Bertram Walsh, University of California, Los Angeles (655-82)

## FRIDAY, 3:15 P.M.

Session on Functional Analysis II. Vendome 11

3:15-3:25

(41) The analytic automorphisms of the Riemann sphere of a Banach algebra

Professor B. W. Glickfeld, University of Washington (655-61)

3:30-3:40

(42) A uniqueness principal for abstract Cauchy problems

Professor Matthew Hackman, University of Washington (655-115)

3:45-3:55

(43) The Hornich topology for meromorphic functions in the disk

Professor J. A. Cima* and Mr. J. A. Pfaltzgraff, University of North Carolina, Chapel Hill (655-95)

4:00-4:10

(44) On maps which preserve almost periodic functions

Dr. M. C. Tews, College of the Holy Cross (655-5)

4:15-4:25

(45) An existence theorem for fundamental solutions of an abstract differential operator

Mr. Paul Arminjon, University of Montreal (655-118)

4:30-4:40

(46) A generalization of Stone's theorem (and other theorems) in the theory of operators on a Hilbert module

Professor P. P. Saworotnow, The Catholic University of America (655-27)

4:45-4:55

(47) A Hilbert space inequality

Dr. Bertram Mond* and Dr. Oved Shisha, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio (655-14)

5:00-5:10

(48) Differential equations in linear topological spaces. III

Mr. H. O. Fattorini, University of California, Los Angeles (655-23)

## FRIDAY, 3:15 P.M.

Session on Analysis, Vendome 12

3:15-3:25

(49) Symmetrizable problems in regions with edges and corners

Professor Leonard Sarason, University of Washington (655-11)

3:30-3:40

(50) On the existence and uniqueness of generalized solutions of quasi-linear equations in two independent variables

Dr. R. B. Knight*, Catholic University of America, and Professor A. K. Aziz, University of Maryland (655-62)

3:45-3:55

(51) On nonlinear diffusion equation in $L_2(R^n)$. Preliminary report

Mr. B. M. Cherkas*, Georgetown University and Johns Hopkins University, and Dr. Choy Tak Taam, George Washington University (655-111)

4:00-4:10

(52) On the existence of weak solutions of a nonlinear wave equation

Professor J. M. Greenberg* and Professor R. C. MacCamy, Carnegie-Mellon University (655-113)

4:15-4:25

(53) Absolute continuity of harmonic measures with respect to arc length

Professor M. G. Arsove, University of Washington (655-99)

4:30-4:40

(54) Spin spherical harmonics. Preliminary report

Professor M. S. K. Sastry, Ohio University (655-49)

4:45-4:55

(55) Convergence-preservation criteria for a generalized Hausdorff mean

Professor C. W. Leininger, University of Dallas (655-55)

5:00-5:10

(56) Theorems on Cesàro summability of series

Professor S. Mukhoti, Memphis State University (655-74)

## FRIDAY, 3:15 P.M.

Session on Real Analysis, Chambord 14

3:15-3:25

(57) Bounds for the truncation error in sampling expansions

Professor J. L. Brown, Jr.,* and Professor H. S. Piper, Jr., Pennsylvania State University (655-16)

3:30-3:40

(58) Hausdorff dimension in sequence spaces

Professor Helmut Wegmann, Duke University (655-48)
(Introduced by Professor O. P. Stackelberg)

3:45-3:55

(59) Continuous nowhere differentiable functions

Professor J. R. McLaughlin, Pennsylvania State University (655-71)

4:00-4:10

(60) Fundamental sequences in ordered sets

Professor Michael Aissen, Fordham University and Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio (655-109)

4:15-4:25

(61) On the convergence of a sequence of Perron integrals

Mr. M. N. Manougian, University of Texas, Austin (655-121)
(Introduced by Professor H. J. Ettlinger)

4:30-4:40

(62) Solutions to a Perron type nonlinear integral equation

Mr. R. A. Northcutt, University of Texas, Austin (655-122)

## SATURDAY, 9:00 A.M.

Session on Topology, Regency Foyer

9:00-9:10

(63) Computer investigation of commuting functions. Preliminary report
Dr. W. M. Boyce, Bell Telephone Laboratories, Murray Hill, New Jersey (655-119)

9:15-9:25

(64) The idea of motion in a topological space and Cauchy's problem for generalized differential equation
Professor R. G. Lintz, McMaster University (655-2)

9:30-9:40

(65) Approximation spaces
Professor P. C. Hammer, Pennsylvania State University (655-3)

9:45-9:55

(66) The action of topological groups on Hausdorff continua
Professor W. J. Gray, University of Alabama (655-10)
(Introduced by Professor M. O. Gonzalez)

10:00-10:10

(67) Some properties of $\beta X$ - X for complete spaces
Professor S. M. Robinson, Cleveland State University (655-96)

10:15-10:25

(68) Uniformly continuous mappings defined by function systems
Dr. Kazumi Nakano, Loyola University (655-54)

10:30-10:40

(69) Sequential properties of ordered topological spaces
Professor P. R. Meyer, Hunter College of the City University of New York (655-93)

10:45-10:55

(70) A pair of noninvertible links
Professor W. C. Whitten, Jr., University of Southwestern Louisiana (655-25)

## SATURDAY, 9:00 A.M.

Session on Applied Mathematics, Vendome 11

9:00-9:10

(71) Petrov classification of orthogonal metric spaces
Mr. K. B. Marathe, University of Rochester (655-38)

9:15-9:25

(72) Geometry of linear four-dimensional spaces
Mr. Adolph Selzer, USN-USL, Fort Trumbull, New London, Connecticut (655-31)
(Introduced by Dr. S. H. Gould)

9:30-9:40

(73) On the validity of the geometrical theory of diffraction by convex cylinders
Professor C. O. Bloom and Professor B. J. Matkowsky*, Rensselaer Polytechnic Institute (655-15)
(Introduced by Professor G. H. Handelman)

9:45-9:55

(74) On a differential equation in elasticity
Professor R. C. MacCamy, Carnegie-Mellon University (655-101)

10:00-10:10

(75) On matrix approximations to the differential equations of electric networks with distributed elements
Professor Victor Lovass-Nagy, Clarkson College of Technology (655-50)

10:15-10:25
(76) Invariance of network functions with respect to orientation
Professor Paul Slepian, Rensselaer Polytechnic Institute (655-17)
10:30-10:40
(77) Stabilized numerical analytical prolongation with poles. Preliminary report
Professor Keith Miller, University of California, Berkeley (655-20)
10:45-10:55
(78) Singular points in products of semidynamical systems
Dr. P. N. Bajaj, Case Western Reserve University (655-72)

## SATURDAY, 9:00 A.M.

Session on Group Theory, Vendome 12
9:00-9:10
(79) Finite solvable groups containing maximal nilpotent subgroups
Dr. Hermann Simon, University of Miami (655-12)
9:15-9:25
(80) Commutative endomorphism rings
Professor J. M. Zelmanowitz, University of California, Santa Barbara (655-30)
9:30-9:40
(81) On solvable and supersolvable groups. Preliminary report
Professor J. R. Durbin, University of Texas, Austin (655-34)
9:45-9:55
(82) Representations of the general linear group over a finite field. Preliminary report
Professor A. J. Silberger, Bowdoin College (655-46)
10:00-10:10
(83) On automorphisms of finite simple groups
Dr. G. N. Pandya, University of Manitoba (655-59)
10:15-10:25
(84) Wielandt length of finite groups. Preliminary report
Professor A. R. Camina, University of Illinois (655-65)
(Introduced by Mr. J. J. Rotman)
10:30-10:40
(85) Direct product decompositions of class 2 groups of exponent p
Professor R. F. Spring, Ohio University (655-90)
10:45-10:55
(86) Generating groups of nilpotent varieties
Professor Frank Levin, Rutgers, The State University (655-114)

## SATURDAY, 9:00 A.M.

Session on Lattices, Semigroups and Matrices, Chambord 14
9:00-9:10
(87) On the construction of order ortho-homomorphisms. Preliminary report
Professor R. J. Greechie, Kansas State University (655-13)
9:15-9:25
(88) Section semicomplemented lattices. Preliminary report
Professor M. F. Janowitz, University of Massachusetts, Amherst (655-22)
9:30-9:40
(89) Direct decomposition of regular semigroups
Professor R. J. Warne, West Virginia University (655-106)
9:45-9:55
(90) A - P congruences on Baer semigroups
Mr. B. J. Thorne, Smith College (655-98)

10:00-10:10

(91) On automistic lattices with modular extensions

Professor Shuichiro Maeda, University of Massachusetts, Amherst, and Ehime University, Matsuyama, Japan (655-112)

(Introduced by Professor D. J. Foulis)

10:15-10:25

(92) Normal partial isometries closed under multiplication on unitary spaces

Professor Ivan Erdelyi, Kansas State University (655-33)

10:30-10:40

(93) Quasi-block-stochastic matrices and two theorems of A. Brauer

Mr. Werner Kuich, Michigan State University (655-73)

(Introduced by Professor C. P. Wells)

10:45-10:55

(94) An inequality involving a generalized inverse matrix

Dr. B. Mond and Dr. O. Shisha*, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio (655-105)

SATURDAY, 11:00 A.M.

Invited Address, Georgian Ballroom A.

Characteristic classes: old and new

Professor Franklin P. Peterson, Massachusetts Institute of Technology

SATURDAY, 2:00 P.M.

Invited Address, Georgian Ballroom A

Algebraic Lie algebras

Professor George B. Seligman, Yale University

SATURDAY, 3:15 P.M.

Session on Algebra, Regency Foyer

3:15-3:25

(95) Functional equations of modular forms

Professor A. P. Ogg, University of California, Berkeley (655-100)

3:30-3:40

(96) Class number and ramification in fields of algebraic functions

Mr. M. L. Madan, Ohio State University (655-1)

(Introduced by Professor R. P. Bambah)

3:45-3:55

(97) Global description of some deformations of rings

Dr. F. P. Callahan, General Electric Company, Philadelphia, Pennsylvania (655-32)

4:00-4:10

(98) On Krull overrings of an affine ring

Professor William Heinzer, Louisianan State University, Baton Rouge (655-42)

4:15-4:25

(99) Compact spaces of minimal prime ideals

Professor Joseph Kist, New Mexico State University (655-43)

4:30-4:40

(100) Orders in self-injective semiperfect rings

Professor A. C. Mewborn* and Mr. C. N. Winton, University of North Carolina, Chapel Hill (655-83)

4:45-4:55

(101) Normal bases of ambiguous ideals. Preliminary report

Professor S. Kuroda and Dr. S. Ullom*, University of Maryland (655-79)

5:00-5:10
(102) Lie algebras of genus 1 and 2
Professor J. W. Bond, Pennsylvania State University (655-102)

SATURDAY, 3:15 P.M.

Session on Algebraic Topology, Vendome 11

3:15-3:25
(103) The mod p cohomology of certain fibre spaces
Mr. David Beaucage, State University of New York at Stony Brook (655-18)

3:30-3:40
(104) From immersions to embeddings of smooth manifolds
Mr. F. X. Connolly, Columbia University (655-29)

3:45-3:55
(105) An extension of De Rham's theorem
Professor George Rublein, College of William and Mary (655-56)

4:00-4:10
(106) On the elementary ideals of link modules
Professor R. H. Crowell* and Mrs. D. P. Strauss, Dartmouth College (655-78)

4:15-4:25
(107) Extending homeomorphisms of $S^p \times S^q$
Professor Ralph Tindell, University of Georgia (655-94)

4:30-4:40
(108) Steenrod operations in the cohomology of algebras. Preliminary report
Mr. Andreas Zachariou, Oklahoma State University (655-116)
(Introduced by Professor John Jewett)

SATURDAY, 3:15 P.M.

Session on Differential Equations, Vendome 12

3:15-3:25
(109) Invariance for linear systems of ordinary differential equations
Professor Al Kelley, University of California, Santa Cruz (655-8)

3:30-3:40
(110) On the disconjugacy of second order, selfadjoint matrix differential equations
Professor G. J. Etgen, University of Houston (655-41)

3:45-3:55
(111) Stability of controlled motion
Professor V. Lakshmikantham, Professor S. Leela and Professor C. Tsokos*, University of Rhode Island (655-63)

4:00-4:10
(112) Finite difference forms containing derivatives of higher order
Dr. Manfred Reimer, University of Maryland and University of Tübingen, Germany (655-70)
(Introduced by Professor W. C. Rheinboldt)

4:15-4:25
(113) Reduction of integral boundary conditions to two-point boundary conditions
Professor W. R. Jones, Lafayette College (655-92)

4:30-4:40
(114) Two point boundary value problems for linear differential inequalities
Dr. M. L. Slater, Sandia Corporation, Albuquerque, New Mexico (655-97)

## SATURDAY, 3:15 P.M.

Session on Graph Theory, Chambord 14

3:15-3:25

(115) On category of graphs with a given subgraph
Professor Z. Hedrlín and Professor Eric Mendelsohn*, McGill University (655-64)

3:30-3:40

(116) Eigenvalues of the adjacency matrix of cubic lattice graphs
Dr. Renu Laskar, University of North Carolina, Chapel Hill (655-117)
(Introduced by Professor R. C. Bose)

3:45-3:55

(117) Bigraph topology and Skorokhod's M-convergence
Professor Y.-W. Kim, Wisconsin State University (655-75)

4:00-4:10

(118) On the total chromatic number of certain graphs
Mr. M. Rosenfeld, University of Washington (655-45)
(Introduced by Professor Branko Grunbaum)

4:15-4:25

(119) A characterization of a class of regular graphs
Professor Martin Aigner, Wayne State University (655-53)

Herbert Federer
Associate Secretary

Providence, Rhode Island

# Six Hundred Fifty-Sixth Meeting
# Sheraton-Chicago Hotel
# Chicago, Illinois
# April 16-20, 1968

## PROGRAM

The six hundred fifty-sixth meeting of the American Mathematical Society will be held at the Sheraton-Chicago Hotel, Chicago, Illinois, on April 16-20, 1968.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings there will be four one-hour addresses. Professor Stephen Smale of the University of California, Berkeley, will speak on Friday, April 19, at 11:00 a.m. His topic will be "Global stability questions in dynamical systems." Professor René Thom of the Institut des Hautes Études Scientifiques, Bures sur Yvette, will address the Society on Friday, April 19, at 2:00 p.m. His subject will be "Qualitative dynamics and morphogenesis." Professor Hans Weinberger of the University of Minnesota will speak on Saturday, April 20, at 11:00 a.m. His talk will be entitled "Some constructive bounds for eigenvalues." Professor Irving Reiner of the University of Illinois, Urbana, will address the Society on Saturday, April 20, at 2:00 p.m. His topic will be "Recent progress in the theory of integral representations." All four lectures will be given in the Tally-Ho Room on the ninth floor of the Sheraton-Chicago Hotel.

By invitation of the same committee there will be a special session of twenty-minute papers on Quasiconformal Mapping on Friday, April 19, at 3:15 p.m. This special session is being arranged by Professor Frederick W. Gehring of the University of Michigan. Sessions for the presentation of contributed ten-minute papers will be held at 3:15 p.m. on Friday, April 19, and at 8:30 a.m. and 3:15 p.m. on Saturday, April 20.

The Council of the Society will meet at 5:15 p.m. on Friday, April 19, in the Lake Ontario Room on the eighth floor of the Sheraton-Chicago Hotel.

### SYMPOSIUM ON NONLINEAR FUNCTIONAL ANALYSIS

With the support of the National Science Foundation there will be a symposium on Nonlinear Functional Analysis on April 16-19. The topic was chosen by the Committee to Select Hour Speakers for Western Sectional Meetings. The Organizing Committee of the Symposium, responsible for the planning of the program and the choice of speakers, consists of Felix E. Browder (chairman), James Eells, Jr., Tosio Kato, George J. Minty, Richard S. Palais, Jacob T. Schwartz, and Stephen Smale. On Tuesday, April 16, the sessions of the symposium will be held in the King Arthur Room on the third floor of the Sheraton-Chicago Hotel. On Wednesday, Thursday, and Friday the sessions of the symposium will be held in the Tally-Ho Room on the ninth floor of the hotel.

### REGISTRATION

On Tuesday, April 16, the registration desk will be located outside the King Arthur Room on the third floor of the Sheraton-Chicago Hotel. On Wednesday through Saturday, April 17-20, the registration desk will be located outside the Tally-Ho Room on the ninth floor. It will be open from 9:00 a.m. to 5:00 p.m. on all five days of the meeting.

## ROOM RESERVATIONS

Persons intending to stay at the Sheraton-Chicago Hotel should make their own reservations with the hotel. A reservation blank and a listing of room rates was given on page 416 of the February issue of the *Notices*. In conformity with Sheraton policy, the Sheraton-Chicago offers free valet parking for all registered guests. However, those coming by automobile should allow plenty of time at check-in and check-out for the disposition or retrieval of their cars.

## FOOD SERVICE

The Sheraton-Chicago Hotel contains a variety of restaurants, including the Camelot Room (a restaurant with Old English decor open for breakfast and lunch), the Brass Bull (a beef restaurant open for lunch and dinner), and the Kon-Tiki Ports (a Polynesian restaurant open for lunch and dinner). These restaurants operate on a somewhat reduced schedule on Saturday.

## MAIL ADDRESS

Registrants at the meeting may receive mail addressed in care of the American Mathematical Society, Sheraton-Chicago Hotel, 505 North Michigan Avenue, Chicago, Illinois 60611.

## TRAVEL INFORMATION

The Sheraton-Chicago Hotel is located on Michigan Avenue just north of the Chicago River. Those coming by way of O'Hare Airport should take the airport limousine marked "North Loop and Michigan Avenue"; this runs every half-hour until 9:00 p.m. and costs $2.00 one-way. Those coming by car should leave the Kennedy Expressway at the Ohio Street Exit and proceed east. Those coming by bus or train will find the Sheraton-Chicago to be within a short taxi ride of the rail and bus terminals.

## SYMPOSIUM ON NONLINEAR FUNCTIONAL ANALYSIS

### TUESDAY, 9:30 A.M.

Session A, King Arthur Room, Third Floor

9:30-10:30

On some nonlinear partial differential equations related to optimal control theory

Professor J. L. Lions, University of Paris, France

10:45-11:15

On some degenerate nonlinear parabolic equations

Professor Haim Brezis, University of Paris, France

11:20-11:50

On multiple solutions of nonlinear operator equations arising from the calculus of variations

Professor Melvin Berger, Courant Institute, New York University

### TUESDAY, 1:30 P.M.

Session B, King Arthur Room, Third Floor

1:30-2:30

Some aspects of monotonicity theory

Professor George J. Minty, Indiana University

2:45-3:15

Monotone convergence theorem

Professor Edgar Asplund, University of Washington

3:20-3:50

Perturbation of variational inequalities

Professor Umberto Mosco, University of Pisa, Italy

3:55-4:25

Monotone operators associated with saddle functions and minimax problems

Professor R. Tyrrell Rockafeller, University of Washington

WEDNESDAY, 9:00 A.M.

Session C, Tally-Ho Room, Ninth Floor

9:00-10:00

Remarks on nonlinear accretive operators in Banach spaces

Professor Tosio Kato, University of California, Berkeley

10:10-10:40

Fixed point theorems for nonexpansive mappings

Professor W. A. Kirk, University of Iowa

10:45-11:15

Nonlinear equations involving noncompact operators

Professor Walter Petryshyn, Rutgers, The State University

11:20-11:50

Remarks on evolution equations

Professor Walter A. Strauss, Brown University and Rockefeller University

WEDNESDAY, 1:30 P.M.

Session D, Tally-Ho Room, Ninth Floor

1:30-2:30

Nonlinear operators and nonlinear equations of evolution in Banach spaces

Professor Felix E. Browder, University of Chicago

2:45-3:15

On some nonexistence and nonuniqueness theorems for nonlinear parabolic equations

Professor Hiroshi Fujita, Courant Institute, New York University

3:20-3:50

Critical sets for noncompact quasi-autonomous nonlinear equations of equations

Professor G. Stephen Jones, University of Maryland

3:55-4:25

A general fixed point theorem

Professor Benjamin Halpern, University of California, Berkeley

THURSDAY, 9:00 A.M.

Session E, Tally-Ho Room, Ninth Floor

9:00-10:00

On regularity of solutions of some variational inequalities

Professor Guido Stampacchia, University of Pisa, Italy

10:10-10:40

The Bénard problem

Professor P. Rabinowitz, Stanford University

10:45-11:15

Some remarks on vector fields in Hilbert space

Professor Erich Rothe, University of Michigan

11:20-11:50

Existence theorems for Lagrange problems in Sobolev spaces

Professor Lamberto Cesari, University of Michigan

## THURSDAY, 1:30 P.M.

Session F, Tally-Ho Room, Ninth Floor

1:30-2:30

Manifolds of sections of fiber bundles and the calculus of variations
Professor Richard S. Palais, Brandeis University

2:45-3:15

Extensions of Lipschitzian mappings in normed spaces
Professor D. G. de Figuiredo*, University of Illinois, and
Professor Les Karlovitz, University of Maryland

3:20-3:50

An application of Leray-Schauder degree
Professor Jane Cronin, Rutgers, The State University

3:55-4:25

Degree theory for nonlinear Fredhold maps and its applications
Professor K. D. Elworthy, University of Manchester, England, and State University of New York at Stonybrook; and Professor A. J. Tromba*, Princeton University

## FRIDAY, 9:30 A.M.

Session G, Tally-Ho Room, Ninth Floor

9:30-10:30

Fredholm structures
Professor James Eells, Jr., Cornell University

## PROGRAM OF THE SESSIONS

The time limit for each contributed paper is ten minutes. The papers are scheduled at fifteen minute intervals in order that listeners can circulate among sessions. To maintain the schedule the time limit will be strictly enforced.

## FRIDAY, 11:00 A.M.

Invited Address, Tally-Ho Room

Global stability questions in dynamical systems
Professor Stephen Smale, University of California, Berkeley

## FRIDAY, 2:00 P.M.

Invited Address, Tally-Ho Room

Qualitative dynamics and morphogenesis
Professor Rene Thom, Institut des Hautes Etudes Scientifiques, Bures sur Yvette

## FRIDAY, 3:15 P.M.

Special Session on Quasiconformal Mappings, Tally-Ho Room

3:15-3:35

Kleinian groups and Eichler integrals
Professor Lars V. Ahlfors, Harvard University (656-122)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.

3:40-4:00

Quasiconformal vector fields

Professor Clifford J. Earle, Jr., Cornell University (656-121)

4:05-4:25

The nonuniqueness of quasiconformal maps

Professor Halsey L. Royden, Stanford University (656-123)

4:30-4:50

Quasiconformal mappings which keep boundary points fixed

Professor Edgar Reich*, University of Minnesota, and Professor Kurt Strebel, University of Zurich (656-120)

4:55-5:15

Existence theorem for a generalized Beltrami equation

Professof Olli E. Lehto, University of Helsinki and University of Minnesota (656-124)

FRIDAY, 3:15 P.M.

Session on Number Theory, Algebra, and Logic, East Room A

3:15-3:25

(1) On the representability of quaternary quadratic forms as sums of squares of two linear forms

Professor J. T. Hardy, University of Georgia (656-46)

3:30-3:40

(2) Splitting quadratic forms over Hasse domains

Professor L. J. Gerstein, University of California, Santa Barbara (656-16)

3:15-3:55

(3) A class equation under row equivalence for matrices over a principal ideal domain modulo m

Mr. B. R. McDonald, Michigan State University (656-23)
(Introduced by Professor B. M. Stewart)

4:00-4:10

(4) A combinatorial problem on finite Abelian groups

Mr. J. E. Olson, Mathematics Research Center, U. S. Army, University of Wisconsin (656-97)

4:15-4:25

(5) Representations of a group by transformations on its subgroups

Professor Hidegoro Nakano, Wayne State University (656-15)

4:30-4:40

(6) A characterization of the lattice orderings on a set which induce a given betweenness

Professor J. M. Cibulskis, Marquette University (656-7)

4:45-4:55

(7) Sets with no subsets of higher degree. Preliminary report

Mr. R. I. Soare, University of Illinois at Chicago Circle (656-37)

5:00-5:10

(8) Generalized completeness theorem and solvability of systems of Boolean polynomial equations

Professor Alexander Abian, Iowa State University (656-119)

FRIDAY, 3:15 P.M.

Symposium on Geometry and Topology, East Room B

3:15-3:25

(9) Unflat bundles and metrics of positive curvature. Preliminary report

Dr. A. D. Weinstein, Massachusetts Institute of Technology (656-83)

3:30-3:40

(10) Discs in negatively curved manifolds

Professor F. J. Flaherty, Oregon State University (656-82)

3:45-3:55

(11) Unknotting locally flat embeddings by isotopy. Preliminary report

Mr. T. B. Rushing, University of Georgia (656-78)

4:00-4:10

(12) Thom classes

Mr. J. C. Becker, University of Massachusetts (656-96)

4:15-4:25

(13) Automorphisms of meta-stably connected manifolds

Mr. J. P. E. Hodgson, University of Pennsylvania (656-33)

4:30-4:40

(14) On groups of isometries

Professor Ludvik Janos, University of Florida (656-95)

4:45-4:55

(15) Generators for topological entropy and expansiveness

Professor H. B. Keynes* and Professor J. B. Robertson, University of California, Santa Barbara (656-13)

5:00-5:10

(16) On uniformly $\mu$-complete mappings

Dr. H. H. Wicke* and Dr. J. M. Worrell, Jr., Sandia Corporation, Albuquerque, New Mexico (656-75)

## FRIDAY, 3:15 P.M.

Session on Differential Equations, East Room C

3:15-3:25

(17) A best approximate solution of the Riccati matrix equation

Mr. M. S. Henry* and Professor F. M. Stein, Colorado State University (656-61)

3:30-3:40

(18) A reduction algorithm for linear differential systems

Professor W. A. Harris, Jr., Professor Y. Sibuya, and Mr. L. Weinberg*, University of Minnesota (656-73)

3:45-3:55

(19) Comparison theorems for linear differential operators. Preliminary report

Mr. G. A. Bogar, University of Tennessee (656-86)

4:00-4:10

(20) Almost periodic solutions to second order differential equations

Professor A. M. Fink, Iowa State University (656-20)

4:15-4:25

(21) Oscillation and comparison theorems for a second-order linear differential equation

Mr. D. F. St. Mary, Iowa State University (656-25)

4:30-4:40

(22) Conditions for the existence of conjugate points for a fourth order linear differential equation

Professor J. S. Bradley, University of Tennessee (656-87)

4:45-4:55

(23) The distribution of zeros of fourth order differential equations. Preliminary report

Mr. Allan Peterson, University of Tennessee (656-88)

## FRIDAY, 3:15 P.M.

Session on Analysis, East Room D

3:15-3:25
(24) The invariance of the range of functions in certain rings of real functions
Professor L. E. Pursell, University of Missouri, Rolla (656-10)

3:30-3:40
(25) On integration-by-parts for a weighted integral and the Lebesgue-Stieltjes integral
Professor F. M. Wright, Iowa State University (656-112)

3:45-3:55
(26) On weighted integrals and the Lebesgue-Stieltjes integral
Professor F. M. Wright and Mr. J. E. Baker*, Iowa State University (656-114)

4:00-4:10
(27) Concerning the linearity of a certain transformation
Professor W. D. L. Appling, North Texas State University (656-14)

4:15-4:25
(28) Multiplicative perturbation of semigroups
Professor K. E. Gustafson, University of Minnesota (656-24)

4:30-4:40
(29) On the extensive derivative
Professor R. H. Bowman, Vanderbilt University (656-27)

4:45-4:55
(30) Representations of linear operators on quasicontinuous functions
Professor R. W. Reichhardt, Marquette University (656-109)

5:00-5:10
(31) An extension of the Gelfand-Mazur theorem
Sister Mary Raimonda Allard*, O. P., Rosary College, and Dr. P. P. Saworotnow, The Catholic University of America (656-6)

## SATURDAY, 8:30 A.M.

Session on Function Theory, Tally-Ho Room

8:30-8:40
(32) A note on entire functions of bounded index
Dr. A. K. Bose, Clemson University (656-76)

8:45-8:55
(33) The Paley-Wiener-Schwartz theorem for functions analytic in a half plane. Preliminary report
Mr. R. D. Carmichael, Duke University (656-26)

9:00-9:10
(34) A note on functions of bounded boundary rotation
Professor M. R. Gopal, Michigan Technological University (656-72)

9:15-9:25
(35) Mass functions of bounded variation and starlikeness
Professor C. D. Lustfield, Ohio University (656-79)

9:30-9:40
(36) An asymptotic analog of the F. and M. Riesz radial uniqueness theorems
Mr. K. F. Barth and Mr. W. J. Schneider*, Syracuse University (656-99)

9:45-9:55
(37) Triangular dilatation and quasiconformal mapping
Professor H. J. Renggli, University of New Mexico (656-41)

10:00-10:10
(38) Continuity of curve functionals and a technique involving quasiconformal mapping
Mr. Glenn Schober, Indiana University (656-102)

10:15-10:45

(39) Growth of Nevanlinna's characteristic function for various meromorphic functions

Mr. P. K. Kamthan, University of Waterloo (656-115)
(Introduced by Professor Taqdir Husain)

10:30-10:40

(40) Angular and oricyclic cluster sets for normal meromorphic functions. Preliminary report

Mr. R. S. Rodriguez, University of Tennessee (656-85)

10:45-10:55

(41) Proximate orders and exceptional values of a meromorphic function

Professor Hari Shankar, Ohio University (656-55)

## SATURDAY, 8:30 A.M.

Session on Algebra, East Room A

8:30-8:40

(42) New roots of unity generalizing commutative algebra and function theory

Mr. K. Demys, Montreal West, P. Q., Canada (656-93)

8:45-8:55

(43) Sums of complexes in torsion-free abelian groups

Mr. J. D. Tarwater*, North Texas State University, and Mr. R. C. Entringer, University of New Mexico (656-77)

9:00-9:10

(44) Homomorphism topologies and slender groups

Professor B. F. Hobbs, Olivet Nazarene College (656-54)

9:15-9:25

(45) Formally projective modules and ideal adic free modules

Professor Satoshi Suzuki, Queen's University (656-1)

9:30-9:40

(46) On ring structures determined by groups

Mr. K. E. Eldridge, Ohio University (656-104)

9:45-9:55

(47) Perfect rings over which no module has a proper corational extension. Preliminary report

Professor R. C. Courter, Wayne State University (656-91)

10:00-10:10

(48) Nil subrings of Goldie rings are nilpotent

Mr. Charles Lanski, University of Chicago (656-39)

10:15-10:25

(49) Invertibility of modules over Prüfer rings. Preliminary report

Mr. Howard Gorman, University of Chicago (656-19)

10:30-10:40

(50) Differentiably simple algebras

Professor R. E. Block, University of Illinois (656-111)

## SATURDAY, 8:30 A.M.

Session on Topology I, East Room B

8:30-8:40

(51) Concerning the separation of certain plane-like spaces by compact dendrons

Mr. J. W. Green, University of Texas (656-9)
(Introduced by Professor R. L. Moore)

8:45-8:55

(52) Diagonal cutting in Cartesian products

Professor Howard Cook, University of Houston (656-57)
(Introduced by Professor D. G. Bourgin)

9:00-9:10
(53) Continua which contain only degenerate continuous images of plane continua
Professor J. W. Rogers, Jr., Emory University (656-67)

9:15-9:25
(54) On the invariance of countable paracompactness under closed maps
Mr. Phillip Zenor, University of Houston (656-38)

9:30-9:40
(55) A fixed point theorem in uniform space
Professor S. P. Singh, Memorial University (656-105)

9:45-9:55
(56) Deficiency in infinite-dimensional spaces. II
Mr. D. W. Curtis, Iowa State University (656-32)

10:00-10:10
(57) Infinite-dimensional $\sigma$-compact sets
Professor R. D. Anderson, Louisiana State University (656-116)

10:15-10:25
(58) An infinite-dimensional Schoenflies theorem
Professor D. E. Sanderson, Iowa State University (656-107)

10:30-10:40
(59) Cells and cellularity in infinite-dimensional normed linear spaces
Mr. R. A. McCoy, Iowa State University (656-53)

## SATURDAY, 8:30 A.M.

Session on Functional Analysis I, East Room C

8:30-8:40
(60) Kernel functions for linear transformations
Professor J. T. Darwin, Jr., Auburn University (656-117)

8:45-8:55
(61) Vector sequence spaces and perfect summability matrices of operators in Banach spaces
Professor M. S. Ramanujan, University of Michigan (656-80)

9:00-9:10
(62) Solutions to $U_{12} = BU$ in a Banach space
Mr. J. W. Spellmann, Emory University (656-3)

9:15-9:25
(63) Approximations to nonlinear operator equations and Newton's method
Professor R. H. Moore, University of Wisconsin-Milwaukee (656-108)

9:30-9:40
(64) Factorization in Banach algebras and the general strict topology
Professor F. D. Sentilles and Professor D. C. Taylor*, University of Missouri (656-29)

9:45-9:55
(65) Absolute continuity and the Radon theorem
Professor S. G. Wayment, Weber State College (656-51)
(Introduced by Professor D. H. Tucker)

10:00-10:10
(66) An extended form of the mean-ergodic theorem
Professor L. C. Kurtz, Arizona State University, and Professor D. H. Tucker*, University of Utah (656-62)

10:15-10:25
(67) A maximal regular boundary for solutions of elliptic differential equations
Professor P. A. Loeb* and Professor B. Walsh, University of California, Los Angeles (656-81)

10:30-10:40
(68) Singular perturbation of eigenvalues. Preliminary report
Professor W. M. Greenlee, Northwestern University (656-21)

10:45-10:55

(69) Orthomodular lattices admitting no states
Professor R. J. Greechie, Kansas State University (656-120)

SATURDAY, 8:30 A.M.

Session on Measure Theory and Probability, East Room D

8:30-8:40

(70) On the existence and representation of integrals
Professor J. A. Reneke, Clemson University (656-68)

8:45-8:55

(71) Existence and characterization of the smallest and the greatest measures generating the same Lebesgue integral
Professor Witold Bogdanowicz, Catholic University of America (656-18)

9:00-9:10

(72) Concerning the extension of certain set functions
Mr. P. W. Lewis, University of Utah (656-49)

9:15-9:25

(73) On the range of unbounded vector valued measure
Mr. Czeslaw Olech, Brown University (656-92)
(Introduced by Professor F. M. Stewart)

9:30-9:40

(74) On compactness and vector measures
Dr. Jürgen Batt, Kent State University (656-30)

9:45-9:55

(75) Green's functions for generalized Schroedinger equations
Professor J. A. Beekman, Ball State University (656-42)

10:00-10:10

(76) On idempotent probabilities on semigroups
Professor A. Mukherjea, Eastern Michigan University (656-36)

10:15-10:25

(77) Functions of finite Markov chains with discrete or continuous parameter. Preliminary report
Mr. R. V. Erickson, Western Michigan University and University of Michigan (656-8)
(Introduced by Professor A. B. Clarke)

SATURDAY, 11:00 A.M.

Invited Address, Tally-Ho Room

Some constructive bounds for eigenvalues
Professor Hans F. Weinberger, University of Minnesota

SATURDAY, 2:00 P.M.

Invited Address, Tally-Ho Room

Recent progress in the theory of integral representations
Professor Irving Reiner, University of Illinois

SATURDAY, 3:15 P.M.

Session on Classical Analysis, Tally-Ho Room

3:15-3:25

(78) Concerning an integral and number sets dense in an interval
Mr. C. A. Coppin, University of Texas (656-11)
(Introduced by Professor H. S. Wall)

3:30-3:40
(79) Connectivity, semicontinuity, and the Darboux property
Professor J. B. Brown, Auburn University (656-17)
3:45-3:55
(80) Variation properties of sequences
Professor D. F. Dawson, North Texas State University (656-60)

4:00-4:10
(81) Characterizing Werkfelder of certain classes of summability methods. Preliminary report
Mr. J. R. Edwards, University of Utah (656-45)
4:15-4:25
(82) On Lambert summability
Professor Patrick Cassens*, University of Missouri, St. Louis, and Professor Francis Regan, Saint Louis University (656-59)
4:30-4:40
(83) Concerning the inclusion problem for a generalized Hausdorff mean
Professor C. W. Leininger, University of Dallas (656-63)
4:45-4:55
(84) Cardinal expansions of bivariate and multivariate functions
Mr. W. J. Gordon, General Motors Research Laboratories, Warren, Michigan (646-100)
5:00-5:10
(85) On an integral transform
Professor T. N. Srivastava, Loyola College (656-4)
5:15-5:25
(86) Certain fractional q-integrals and q-derivatives
Professor R. P. Agarwal, West Virginia University (656-94)
(Introduced by Professor A. M. Chak)

## SATURDAY, 3:15 P.M.

Session on Category Theory, East Room A
3:15-3:25
(87) A torsion theory for an Abelian category
Dr. J. L. Fisher, Radcliffe College (656-66)
3:30-3:40
(88) The Mayer-Vietoris sequence for higher projective and inductive limits
Mr. Oliver Pretzel, University of Illinois (656-90)
3:45-3:55
(89) The functor evaluation
Professor Pierre Berthiaume, Université de Montréal (656-70)
4:00-4:10
(90) Properties of Kan extensions
Professor Fritz Ulmer, Rutgers, The State University (656-52)
(Introducted by Professor Saunders Mac Lane)
4:15-4:25
(91) Pointwise limits in the category of adjoint pairs
Mr. P. H. Palmquist, University of Chicago (656-113)
4:30-4:40
(92) Exactness of direct and inverse limits
Dr. Ulrich Oberst, University of Chicago (656-118)
(Introduced by Professor Saunders Mac Lane)

## SATURDAY, 3:15 P.M.

Session on Topology II, East Room B

3:15-3:25
(93) Tameness of certain types of spheres
Professor W. T. Eaton, University of Tennessee (656-28)

3:30-3:40
(94) Tame subsets of spheres in $E^3$
Professor C. E. Burgess, University of Utah (656-58)

3:45-3:55
(95) Deforming simple closed curves into the complement of a surface in $E^3$
Professor R. J. Daverman, University of Tennessee (656-43)

4:00-4:10
(96) Spheres that are tame modulo tame sets
Mr. J. W. Cannon, University of Utah (656-56)
(Introduced by Professor C. E. Burgess)

4:15-4:25
(97) Decompositions of $E^3$ with a compact zero-dimensional set of nondegenerate elements
Professor J. P. Riley, Jr., Louisiana State University, New Orleans (656-40)

4:30-4:40
(98) Collapsible triangulations of the 3-cell with knotted spanning 1-simplexes
Professor W. B. R. Lickorish* and Professor J. M. Martin, University of Wisconsin (656-50)

4:45-4:55
(99) Two-spheres which avoid $I^3$ if $I^3$ contains a p-od
Professor E. H. Anderson, University of North Dakota (656-5)

5:00-5:10
(100) Free actions of $Z_4$ on S
Professor P. M. Rice, University of Georgia (656-69)

5:15-5:25
(101) Constructing three-manifolds from group homomorphism
Mr. William Jaco, University of Wisconsin (656-101)

## SATURDAY, 3:15 P.M.

Session on Functional Analysis II. East Room C

3:15-3:25
(102) One-dimensional branches of eigenvectors of nonlinear, compact operators. Preliminary report
(Mr. Harry Sedinger, Carnegie-Mellon University (656-103)

3:30-3:40
(103) A generalized Grynblum condition for Schauder bases
Professor J. A. Dyer, Iowa State University (656-44)

3:45-3:55
(104) A reducing map theorem
Mr. W. B. Johnson and Mr. R. A. Shive, Jr.*, Iowa State University (656-47)

4:00-4:10
(105) Generalizations of a Paley-Wiener theorem
Mr. W. B. Johnson, Iowa State University (656-48)
(Introduced by Professor J. A. Dyer)

4:15-4:25
(106) On the self dual locally compact Abelian groups with compact radical
Professor K. C. Ha, Illinois State University (656-110)

4:30-4:40
(107) Continued fractions over an inner product space. Preliminary report
Professor F. A. Roach, University of Georgia (656-64)

4:45-4:55
(108) Isometry between two-dimensional subspaces implies an inner product
Professor D. A. Senechalle, University of Georgia (656-89)

5:00-5:10
(109) Representation of linear functionals in Orlicz spaces. Preliminary report
Mr. C. E. Cleaver, University of Kentucky (656-74)

5:15-5:25
(110) Representation theory for locally convex $^*$-algebras
Mr. J. D. Powell, University of Kentucky and Centre College (656-84)

SATURDAY, 3:15 P.M.

Session on Applied Mathematics, Analysis, and Geometry, East Room D

3:15-3:25
(111) An explicit solution of a special class of linear programming problems
Professor A. Ben-Israel* and Professor Abraham Charnes, Northwestern University (656-12)

3:30-3:40
(112) Pascal's triangle and the exponential function
Professor R. L. Bhirud, Purdue University (656-106)

3:45-3:55
(113) Embedding homeomorphisms in differentiable flows
Professor Gordon Johnson, University of Georgia (656-2)

4:00-4:10
(114) A lower bound for the product of modules
Professor W. R. Derrick, University of Utah (656-71)

4:15-4:25
(115) On the spinornorm
Dr. Karlhorst Meyer, Technische Hochschule Muenchen and University of Florida (656-35)
(Introduced by Professor A. R. Bednarek)

4:30-4:40
(116) On incidence-groups with a weak-affine structure. Preliminary report
Dr. D. P. K. Biallas, University of Florida (656-31)
(Introduced by Professor A. R. Bednarek)

4:45-4:55
(117) Cohomology of discontinuous groups
Mr. Peter Curran, Fordham University (656-98)

Paul T. Bateman
Associate Secretary

Urbana, Illinois

# Symposium on Numerical Solution of Field Problems in Continuum Physics

## Durham, North Carolina

## April 5-6, 1968

An Applied Mathematics Symposium is being held in Durham, North Carolina, on Friday and Saturday, April 5-6, 1968, jointly sponsored by the AMS and SIAM. The AMS-SIAM Committee on Applied Mathematics, G. Birkhoff (chairman), Jim Douglas, Jr., C. C. Lin, W. H. Reid, A. H. Taub, and H. S. Wilf, has selected the topic "Numerical Solution of Field Problems in Continuum Physics." The symposium will concentrate attention on the relative power of numerical and analytical methods for solving problems having real physical or industrial importance. Support is expected under a grant from the U. S. Army Research Office (Durham). The Organizing Committee is composed of Professors Garrett Birkhoff (chairman), Jim Douglas, Jr., R. S. Varga, Calvin Wilcox, and Dr. Sidney Fernbach. Dr. Francis G. Dressel, Dr. A. S. Galbraith, and Dr. Gene B. Parrish are liaison representatives.

The four sessions, which comprise sixteen half-hour addresses, will be held in the Auditorium of the U. S. Army Research Office Building. This building is located behind the Engineering Building on the campus of Duke University. Registration will take place in the foyer of the Auditorium, and the registration desk will be open from 8:30 a.m. to 4:30 p.m. on both days. It is recommended that participants consider staying at the Downtowner Motor Inn. Those wishing reservations should write to U. S. Army Research Office (Durham), Attention: Dr. Gene B. Parrish, Box CM, Duke Station, Durham, North Carolina 27706.

Although there are no sessions for contributed papers, the Organizing Committee wishes to emphasize that not only are members of the Society invited to attend, but members of other mathematical organizations are cordially invited to attend the sessions. Participants may be reached by telephone at the following number: (919) 286-2285.

## PROGRAM OF THE SESSIONS

### FRIDAY, 10:00 A.M.

Session I, Fluid Mechanics: Weather Prediction

10:00

Design of difference schemes for evolutionary problems
Dr. K. W. Morton, U.K.A.E.A., Culham Laboratory, Berkshire, England

10:30

Numerical approximation of some equations arising in hydrodynamics
Professor J. L. Lions, University of Paris, France

11:00

Numerical simulation of the atmosphere: mathematical and computational aspects
Professor Akio Arekawa, University of California, Los Angeles

11:30

Problems of simulating three-dimensional flow in two dimensions
Professor D. K. Lilly, N.C.A.R., Boulder, Colorado

FRIDAY, 2:00 P.M.

Session II, Parabolic Problems

2:00

Some recent numerical methods for parabolic problems
Professor Jim Douglas, Jr., University of Chicago

2:30

Parabolic difference schemes
Professor Olof B. Widlund, Uppsala University and New York University

3:00

Numerical analysis of mathematical models of fluid flow in porous media
Dr. Harvey Price, Gulf Research and Development, Pittsburgh, Pennsylvania, and Professor R. S. Varga, Case Western Reserve University

3:30

Numerical methods for problems in reactor kinetics
Dr. A. H. Henry and Dr. Thomas A. Porsching, Bettis Atomic Power Laboratory, West Mifflin, Pennsylvania

SATURDAY, 9:30 A.M.

Session III, Elliptic Problems

9:30

Numerical solution of eigenfunction problems
Professor Garrett Birkhoff and Professor George Fix, Harvard University

10:00

Accurate numerical methods for nonlinear boundary value problems
Professor R. S. Varga, Case Western Reserve University

10:30

Factorization techniques for elliptic difference equations
Professor H. H. Rachford, Jr., Rice University, Professor Todd duPont and Dr. Herbert L. Stone, Esso Production Research, Houston, Texas

11:00

Optimal parameters for reactor criticality calculations
Dr. Eugene Wachspress, Knolls Atomic Power Laboratory, Schenectady, New York

SATURDAY, 2:00 P.M.

Session IV, Elliptic Problems; Finite Element Methods

2:00

On the convergence of difference quotients in elliptic problems
Professor Vidar Thomée, University of Maryland

2:30

Higher-order methods for elliptic problems
Professor James H. Bramble, University of Maryland

3:00

Finite element methods in scolic mechanics
Professor R. Clough and Professor C. A. Felippa, University of California, Berkeley

3:30

Finite-element stiffness methods by different variational methods in elasticity
Professor Theodore H. Pian, Massachusetts Institute of Technology

O. G. Harrold
Associate Secretary

Tallahassee, Florida

# PRELIMINARY ANNOUNCEMENTS OF MEETINGS

## Six Hundred Fifty-Seventh Meeting
## Reed College
## Portland, Oregon
## June 15, 1968

The six hundred fifty-seventh meeting of the American Mathematical Society will be held on Saturday, June 15, 1968, at Reed College in Portland, Oregon, in conjunction with meetings of the Pacific Northwest Section of the Mathematical Association of America and the Society for Industrial and Applied Mathematics. The Association and SIAM will meet on Friday and Saturday, June 14 and 15.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings there will be two hour addresses at this meeting. Professor William G. Bade of the University of California, Berkeley, will address the Society at 11:00 a.m. on Saturday. The title of his lecture is "Sources of equicontinuity in functional analysis." Professor W. A. J. Luxemburg of the California Institute of Technology will present a lecture entitled "On some recent developments in the theory of Riesz spaces" at 2:00 p.m. on Saturday. There will be sessions for contributed papers on Saturday morning and afternoon. All sessions of the meeting will be held in Eliot Hall and the Physics-Chemistry Lecture Hall.

The registration desk for the meeting will be located at the main entrance to Eliot Hall. It will be open from 3:00 p.m. to 6:00 p.m. on Thursday, June 13; from 8:30 a.m. to 5:00 p.m. on Friday, June 14; and from 9:00 a.m. to 12:00 noon on Saturday, June 15.

Dormitory space will be available at the rate of $2.75 per single room per night. Reservations for dormitory rooms should be sent to Professor Burrowes Hunt, Department of Mathematics, Reed College, Portland, Oregon 97202.

There are numerous hotels and motels in Portland. The Rose Manor Motel is about two miles from the Reed College Campus. The commercial rates at this motel range from $8.00 for a single room to $11.50 for a twin. The Sheraton Hotel near the Lloyd Center shopping area is on the east side of Portland, as is Reed College. The rates at this hotel run from $9.00 up for a single to a maximum of $19.00 for a double room. Persons who have Sheraton Corporation faculty identification cards can obtain a slightly lower rate. Hotels in downtown Portland include the Benson, and Heathman, and the Hilton. All of these hotels have reduced rates for faculty members with only personal identification required. The downtown area of Portland is approximately four miles from the Reed campus. Anyone who wishes to stay in a hotel or motel should make his own arrangements through a travel agent or directly with the chosen motel or hotel. Luncheon will be available in the College Commons on the days of the meetings. If enough dormitory reservations are received, breakfast and dinner may also be served in the Commons. Information concerning local restaurants will be available at the registration desk. There are tentative plans for a banquet on Friday evening. Details will be given in the program of the meeting which will appear in the June issue of these *Notices*.

R. S. Pierce
Associate Secretary

Las Cruces, New Mexico

# The Seventy-Third Summer Meeting
# University of Wisconsin
# Madison, Wisconsin
# August 27-August 30, 1968

The seventy-third summer meeting of the American Mathematical Society will be held at the University of Wisconsin, Madison, Wisconsin, from Tuesday, August 27, through Friday, August 30, 1968. All sessions will be held in lecture rooms and classrooms of the university.

There will be two sets of Colloquium Lectures each consisting of four lectures. Professor Donald C. Spencer of Stanford University will lecture on Tuesday, August 27, at 2:00 p.m. and on Wednesday, Thursday, and Friday at 9:00 a.m. Professor J. W. Milnor of Princeton and the University of California, Los Angeles, will lecture on Tuesday at 3:15 p.m. and on Wednesday, Thursday, and Friday at 9:00 a.m. The initial lecture in each series will be delivered at the Union Theater but the subsequent lectures will be delivered in Rooms B102 and B130 in Van Vleck Hall.

There will be several hour addresses. There will also be numerous sessions for the presentation of contributed ten-minute papers on Wednesday, Thursday, and Friday mornings beginning at 10:15 a.m. All of these sessions will be held in Van Vleck Hall. Abstracts of contributed papers should be sent to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904, so as to arrive prior to the deadline date of July 1. Abstract blanks can be obtained on request from the same address. There will be no limit on the number of contributed papers. No provision will be made for late papers.

This meeting will be held in conjunction with meetings of several other organizations, the Mathematical Association of America, the Institute of Mathematical Statistics, the Society for Industrial and Applied Mathematics, and Pi Mu Epsilon. The Mathematical Association of America will meet from Monday through Wednesday. The Earle Raymond Hedricks Lectures, sponsored by the Association, will be given by Professor Hyman Bass of Columbia University. The Society for Industrial and Applied Mathematics will present Professor Peter D. Lax of New York University as the von Neumann Lecturer at 8:00 p.m. on Wednesday. Pi Mu Epsilon will meet concurrently with the Association.

## COUNCIL AND BUSINESS MEETINGS

The Council of the Society will meet at 5:00 p.m. on Tuesday, August 27. The Business Meeting of the Society will be held on Thursday, August 29, at 4:30 p.m.

## REGISTRATION

The registration desk will be in the lobby of the Wisconsin Center. It will be open on Sunday from 2:00 p.m. to 8:00 p.m.; on Monday from 8:00 a.m. to 5:00 p.m.; on Tuesday, Wednesday, and Thursday from 9:00 a.m. to 5:00 p.m.; and on Friday from 9:00 a.m. to 1:00 p.m.

The registration fees will be as follows:

| | |
|---|---|
| Member | $2.00 |
| Member's family | $0.50 |

for the first such registration and no charge for additional registrations.

| | |
|---|---|
| Students | No charge |
| Others | $5.00 |

## EMPLOYMENT REGISTER

The Mathematical Sciences Employment Register will be held in Room 226 and the second floor Lake Lounge of the Wisconsin Center. It will be open Tuesday through Thursday, August 27 through August 29, from 9:00 a.m. to 5:00 p.m. on each of the three days. Attention is invited to the announcement of the Employment Register on Page 451 of this issue of these *Notices*, in particular to the deadline dates for application and to the necessity for prompt registration at the Employment Register desk by both applicants and employers.

### EXHIBITS

Book exhibits and exhibits of educational media will be displayed in the Wisconsin Center.

### BOOK SALE

Books published by the Society will be sold for cash prices somewhat below the usual prices when these same books are sold by mail on invoice.

Paul T. Bateman
Associate Secretary

Urbana, Illinois.

* * *

# NEW AMS PUBLICATIONS

## MEMOIRS OF THE AMERICAN MATHEMATICAL SOCIETY

Number 78

### MEASURABLE, CONTINUOUS AND SMOOTH VECTORS FOR SEMIGRAPHS AND GROUP REPRESENTATIONS

By Robert T. Moore

80 pages; List Price $1.60; Member Price $1.20.

A circle of classical "regularity" theorems for unitary representations of locally compact and Lie groups is generalized here to the setting of representations on locally convex topological vector spaces. Using the method of integrated representations ("smoothing by convolution"), the author gives several generalizations of the useful theorem: weakly continuous unitary representations are strongly continuous. The same basic method is used to extend Garding's theorem on the existence of $C^{\infty}$ vectors, and Nelson's similar theorem on analytic vectors, to suitable locally convex representations of Lie groups. Replacement of the hypothesis of weak continuity by that of weak measurability is studied. Finally, many of the methods and results are extended to treat representations of the additive semigroup $(0,\infty)$.

# AN INQUIRY INTO THE PROBLEM OF PAGE CHARGES

## Reese T. Prosser

The mathematics department at Dartmouth College made last year an informal inquiry into the problem of page charges. We did so because of our growing concern over the steady increase in the number of journals levying page charges and the accompanying increase in their rates per page. These increases have combined to create a budget problem for us which is no longer negligible.

We wrote to the chairman of each of some twenty-six departments, chosen to make up a representative cross section of the mathematical community, explaining our concern and asking them to comment. We deliberately included departments from both large and small state institutions and large and small private institutions, but we make no claim whatever that our sample was adequate in any statistical sense. Of these twenty-six departments, all but one responded, and most showed a lively interest in the problem.

The responses varied widely, and in some cases were quite discursive, but we believe that they can be summarized roughly as follows: (See Table 1 at the conclusion of this article.)

(1) A few (three) departments, devoted primarily to research, are so well-supported by contracts that they can cover nearly all of their page charges from contract money.
(2) A few (three) departments, devoted primarily to teaching, receive few if any page charges, and make no provision for them. In one case the charges are apparently passed on to the author.
(3) Most (nineteen) departments fall between these extremes. These departments are apparently finding their page charges an increasingly severe problem which they try to meet in various ways.

While individual differences are marked, the general pattern seems to be this: Page charges are

(a) paid by research contracts, if possible;
(b) otherwise, paid by AMS dues, if possible;
(c) otherwise, paid by department or institute funds, if possible;
(d) otherwise, rejected.

Our sample here falls into two roughly equal camps. Members of one camp (ten) feel that the journals ought to be supported in the public interest, and they meet page charges otherwise uncovered with institute funds, sometimes in considerable amounts. Those in the other camp (nine) feel that other commitments have higher priority on their institute funds, and reject page charges otherwise uncovered. Which camp a given department falls into seems to be determined by the personal views of the chairman, rather than by the size or character of its institution. No significant differences between state and private institutions appeared in our sample.

Most agreed that the present arrangement is unsatisfactory and that a better scheme is certainly needed. Here are a few representative comments:

"Page charges should be paid where possible to support the journals, but better means are needed."

"40% are paid by grants, 40% by AMS dues and 20% by department budget."

"We now have a budget deficit which we believe the university ought to cover."

"We ignore them unless they can be paid from a government grant."

"We are not a wealthy school, and I do not feel that this item should be first priority in the budget."

"I don't understand why journals prove to be such costly operations."

"All I can suggest for immediate relief is to publish more papers in foreign journals."

"I believe that the university administration must be tapped to support publication costs."

We have preserved here the anonymity we promised our respondents.

No one gave any indication that any author has been penalized because of his department's refusal to pay page charges.

In our own department we paid the following amounts during the calendar year 1967:

| | |
|---|---|
| $165.00 | from government grants |
| 90.00 | from AMS dues |
| 890.00 | from Dartmouth College funds |

This last figure is no longer negligible, and is liable to grow. We gather that our situation is typical. If so, then it may well be time to investigate alternate solutions.

| | | if possible | | otherwise | | |
|---|---|---|---|---|---|---|
| | Response | Pay from Contracts | Pay from AMS dues | Pay from Budget | Don't Pay | No Problem |
| Large State | 9 | 9 | 5 | 3 | 6 | |
| Small State | 1 | 1 | | 1 | | |
| Large Private | 9 | 9 | 7 | 3 | 3 | 3 |
| Small Private | 6 | 1 | 1 | 2 | 1 | 3 |

TABLE I

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Editorial Note: The problem of page charges was considered by an AMS committee consisting of Professor L. J. Paige, Chairman, Professor W. T. Martin, and Professor Alex Rosenberg. The final report of the committee, entitled "A Special Report on the Means of Financing Mathematical Journals," was prepared in February 1964 with financial assistance from the National Science Foundation under Grant GN-102. The report (P.B.177569) is available at a cost of $3.00 from the Clearinghouse for Federal Scientific and Technical Information, 5285 Port Royal Road, Springfield, Virginia 22151.

The committee concluded that "page charges, either as a means of direct financial aid or as an appropriate measure of subsidies, are the only realistic and practical method of supporting the major part of mathematical research publication."

The report contained statistical background information on several aspects of mathematical publication. The information was drawn from a survey of 23 leading mathematical journals, in which it was estimated that 90% of all mathematical research was published. This information has been updated recently, and a comparison of 1961 and 1967 data on the average length of the mathematical research paper, number and percentage of research papers over 25 pages in length, and the number and percentage written by joint authors appears below.

Information on the average length of articles in other disciplines was reported in an Office of Scientific Information Service Journal of 1963. At that time the average length of an article in chemistry, physics, and engineering was about 5 pages.

A statement of the Society's policy on page charges appeared in the January 1968 issue of these *Notices*, p. 52.

| | 1961* | 1967* |
|---|---|---|
| Number of pages | 15,056.0 | 17,787.0 |
| Number of articles | 1,299.0 | 1,779.0 |
| Average length of mathematical articles | 11.4 pp. | 9.9 pp. |
| Number of articles over 25 pages in length | 104.0 | 134.0 |
| Percentage of articles over 25 pages in length | 7.0% | 7.5% |
| Number of articles by joint authors | 211.0 | 339.0 |
| Percentage of articles by joint authors | 16.0% | 19.0% |

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*Figures for 20 journals.

# MEMORANDA TO MEMBERS

## MATHEMATICAL SCIENCES EMPLOYMENT REGISTER

The Mathematical Sciences Employment Register will again schedule interviews during the summer meeting at The University of Wisconsin. The Register will be located in the Wisconsin Center and will be open from Tuesday, August 27, through Thursday, August 29, from 9:00 a.m. to 5:00 p.m. on each of the three days.

In response to many requests, the Register, which is sponsored by the American Mathematical Society, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics, will have a literature display for employers wishing to have recruitment literature available to interested applicants. The charge for this service is $15.00 for a single poster or five hundred informational brochures.

Registration for the Employment Register is separate and apart from meeting registration. It is, therefore, most important that both applicants and employers sign in at the Employment Register desk as early as they can on Tuesday morning. A separate visual index will be maintained for Employment Register use only. Appointments will be scheduled only for applicants and employers who have actually signed in at the Register.

There is no charge for registration except when the late registration fee of $5.00 is applicable. Provision will be made for anonymity of applicants upon payment of $5.00 to defray the cost involved in handling such a listing.

Applicants and employers who wish to be listed with the Employment Register should write to the Mathematical Sciences Employment Register, Post Office Box 6248, Providence, Rhode Island 02904, for either applicant qualification forms or position description forms. These forms must be completed and returned to the Register not later than July 15, 1968, in order to be included in the August lists. Those forms which arrive too late to be included in the printed lists are taken to the meeting where they may be seen by applicants and/or employers who are interested in them. The printed lists will be mailed to subscribers during the first week in August. Lists can be ordered from the Register office in Providence. They will also be available at the meeting.

A subscription to the lists, which includes three issues (January, May, and August) of both the applicants list and the positions list, is available for $25.00 a year; the individual issues of both lists may be purchased in January, May, and August for $12.50. A subscription to the applicants list alone or single copies of that list are not available. Copies of the positions list only may be purchased for $3.00. Checks should be made payable to the American Mathematical Society and sent to the address given above. The 1968 List of Retired Mathematicians is free upon request and can be obtained from the Employment Register office.

## SUPPLEMENTARY PROGRAM

The Supplementary Program giving an alphabetizable listing by author of the Abstracts Presented by Title has been eliminated from these *Notices*. The institutional affiliation and address of the author and the name of the member introducing a nonmember will appear in the abstract itself.

One abstract presented by title per person may be accepted in any one issue. Joint authors are treated as a separate category; thus, in addition to abstracts from two authors individually, one joint abstract by them may be accepted for a particular issue.

# NEWS ITEMS AND ANNOUNCEMENTS

## INTERNATIONAL SYMPOSIUM ON PROBABILITY AND INFORMATION THEORY

An International Symposium on Probability and Information Theory will be held at McMaster University, Hamilton, Ontario, Canada, during April 4-5, 1968. The main addresses will be delivered by the following speakers: J. Aczel, M. Behara, T. Husain, R. Fortet, U. Krengel, K. Kricksberg, M. Rosenblatt-Roth, R. Ahlswede, M. Kac, J. H. B. Kemperman, F. Spitzer, J. Wolfowitz, R. L. Dobrushin, M. S. Pinsker, Yu. V. Prohorov, and I. A. Ovseievich.

Persons interested in contributing ten- to fifteen-minute papers should send abstracts to Professor I. Z. Chorneyko, Department of Mathematics, McMaster University.

The invited addresses and the abstracts of the contributed papers will appear in the proceedings.

The registration fee is $10.00, which includes a subscription to the proceedings. Student registration fee is $3.00. Hotel reservations and further information may be obtained from Miss. C. Finch, Department of Mathematics, McMaster University, Hamilton, Ontario, Canada.

## INTERNATIONAL SUMMER SCHOOL ON NEW TRENDS IN COMPUTER PROGRAMMING

An international summer school on New Trends in Computer Programming will be held on August 12-24, 1968, at the Technical University of Denmark (near Copenhagen). The school has been organized by J.-J. Duby, Scientific Center IBM France, and H. J. Helms, Technical University of Denmark. It will be held under the auspices of the Technical University of Denmark and sponsored by the NATO Science Committee.

The scientific program will include the following courses of lectures: Computer Graphics, A. van Dam; APL, K. Iverson; ALGOL 68, W. L. van der Poel; List Structures, J. Weizenbaum; Microprogramming, M. V. Wilkes; Computer System in Educational Environment, N. Wirth.

Further information and a pamphlet containing an application form are available from Professor Hans Jørgen Helms, Northern Europe University Computing Center, Technical University of Denmark, 2800 Lyngby, Denmark.

## LATIN AMERICAN SCHOOL OF MATHEMATICS

During the Sixth Brazilian Colloquium of Mathematics (July, 1967), a group of Latin American participants decided to organize a Latin American School of Mathematics. The provisional committee of the School is formed by Professors J. Adem (Mexico), J. Barros-Neto (Brazil) and L. Santaló (Argentina). The School is intended to organize instructional meetings, including courses and survey lectures at the research level, to be held in the various Latin American countries. The committee has approved that the first School should meet in Rio de Janeiro during July 1968 and be devoted to analysis. For further information on the scientific program of the first School, write to Instituto de Matemática Pura e Aplicada, Rua Luiz de Camoes 68, Rio de Janeiro 58, GB, Brazil.

## MAA COMMITTEE ON ASSISTANCE TO DEVELOPING COLLEGES

The MAA has recently established a committee to study ways of assisting developing colleges. One critical factor, particularly at Negro developing institutions, is faculty recruitment. Each of these colleges could benefit greatly from a semester or year spent there by a competent mathematician. The committee has agreed to serve temporarily as a clearinghouse. The Committee is compiling a list of vacancies sent to them by the colleges, and they will freely send this list to interested mathematicians. Please address correspondence to Professor George Springer, Chairman CADC, Department of Mathematics, Indiana University, Bloomington, Indiana 47401.

## CONFERENCE ON FUNCTIONAL ANALYSIS

An informal conference on functional analysis will be held May 3-4 at the University of Missouri, Columbia. The following have tentatively agreed to give hour talks: Errett Bishop, R. C. James, R. R. Phelps, and J. L. Taylor. Further information concerning this conference may be obtained by writing to R. M. Crownover, Department of Mathematics, University of Missouri, Columbia, Missouri 65201.

## THREE-DAY JOINT RESEARCH SESSION

The Faculté des Science de Strasbourg will hold a three-day joint research session on May 16-18, at the Université de Strasbourg. The program will include the following topics:

C*-algebras (exact title not yet known) --H. George
The problem of $N(< \infty)$ bodies in quantum mechanics--K. Hepp
The construction of interacting quantum fields--O. Landford
Classical mechanics of an infinite number of particles--O. Landford
Nilsson classes, their application to Feynman integrals--J. Leray
On the Gibbs phase rule--D. Ruelle.

Further information on the session may be obtained from Professor F. Norguet, Secrétariat de la R.C.P. No 25, Département de Mathématique, Rue René Descartes, 67 Strasbourg, France.

## FOREIGN SCHOLARS AVAILABLE FOR APPOINTMENTS IN U. S. UNIVERSITIES AND COLLEGES

The Committee on International Exchange of Persons, Conference Board of Associated Research Councils, has prepared a list of foreign scholars available under the provisions of the Fulbright-Hays Act for appointments in American universities and colleges during the academic year 1968-1969. This list, compiled annually, includes information about scholars nominated by the binational United States Educational Commissions or Foundations abroad for Fulbright-Hays travel grants covering costs of round-trip transportation from the home country to the United States, provided arrangements are completed for a lecturing or a research appointment with an appropriate stipend, at an American institution of higher learning.

A list and additional information on the scholars may be obtained from: Miss Grace E. L. Haskins, Program Officer, Committee on International Exchange of Persons, 2101 Constitution Avenue, N. W., Washington, D. C. 20418.

## NATIONAL SCIENCE FOUNDATIONS GUIDE TO PROGRAMS

A new brochure, Guide to Programs (NSF 68-6), has been released by the National Science Foundation giving details on its grant programs. A total of 53 support programs and 4 national research centers are described. An organizational chart of the Foundation is also included.

Single copies of the brochure are available at no cost by writing to Administrative Services, National Science Foundation, Washington, D. C. 20550. Bulk copies may be obtained at a cost of 50 cents each from the Superintendent of Documents, U. S. Government Printing Office, Washington, D. C. 20402.

## SUMMER CONFERENCE IN INTUITION AND PROOF THEORY

The State University of New York at Buffalo announces a Summer Conference in Intuitionism and Proof Theory, August 11-30, 1968. The first two weeks will include a 10-lecture course in proof theory given by Professor G. Takeuti of the University of Illinois, and a 10-lecture course in intuitionism given by Professor A. Troelstra of the University of Amsterdam. These courses will be introductory and of interest to graduate students in logic. More specialized talks will begin the first week and continue through the third. The list of speakers includes: O. Aberth, Rutgers, The State University; E. Bishop, University of California, San Diego; S. Feferman and H. Friedman, Stanford; P. Gilmore, IBM; A. Heyting, Amsterdam; W. Howard, University of Illinois at Chicago Circle; D. Isles, Tufts; S. C. Kleene, Wisconsin; M. Krasner, University of Paris; G. Kreisel, Stanford; H. Lauchli, University of Zurich; H. Levitz, New York University; P. Martin-Lof, University of Stockholm; C. Parsons, Columbia; D. Prawitz, Lunds; B. Scarpellini, University of Basel; W. W. Tait, University of Illinois at Chicago Circle; M. Takahashi, Tokyo University of Education; D. van Dalen, Rijkuniversteit; B. van Rootselaar, Landbouwhogeschool (Wageningen); and others. Graduate students wishing to attend should communicate with the Conference Director, Professor John Myhill, Department of Mathematics, State University of New York, 4246 Ridge Lea Road, Buffalo, New York 14226. A limited amount of graduate student support may be available. Those wishing to contribute papers also are asked to write Professor Mygill.

## CANADIAN JOURNAL OF MATHEMATICS

The Canadian Journal of Mathematics has been increased in price to $18.00 with a special price of $9.00 for individual members of AMS. The Journal also has been increased in size by 192 pages.

## CONFERENCE ON FUNCTIONAL ANALYSIS AND RELATED FIELDS

A Conference on Functional Analysis and Related Fields will be held at the University of Chicago at the Center for Continuing Education on May 20-24, 1968. This conference is being held in honor of Professor Marshall Stone on the occasion of his retirement from active service at the University of Chicago. The financial support of the conference will be provided by the Air Force Office of Scientific Research. The Organizing Committee consists of the following: F. E. Browder, chairman, A. P. Calderón, Saunders MacLane, R. G. Pohrer, Speakers will include the following: K. Chandrasekharan, S. S. Chern, J. L. Doob, P. R. Halmos, Harish-Chandra, Edwin Hewitt, R. V. Kadison, Tosio Kato, G. W. Mackey, Saunders MacLane, L. J. Nachbin, Edward Nelson, Louis Nirenberg, Dana Scott, I. E. Segal, I. M. Singer. Proceedings of the conference will be published.

# PERSONAL ITEMS

Dr. J. M. AARTS of the University of Amsterdam, Netherlands, has been appointed a Lecturer at the Delft Institute of Technology, Netherlands.

Professor E.-A. BEHRENS of the University of Frankfurt, West Germany, has been appointed to a professorship at McMaster University.

Dr. W. M. BOYCE of the National Aeronautical and Space Administration, Manned Spacecraft Center, Houston, Texas, has accepted a position as a member of the staff at the Mathematics and Statistics Research Center, Bell Telephone Laboratories, Incorporated, Murray Hill, New Jersey.

Professor R. M. COOPER of Arizona State University has been appointed to an assistant professorship at Chicago State College.

Mr. W. L. DICKEY, JR, of the Mellonics Systems Development, Division of Litton Industries, Sunnyvale, California, has accepted a position as a member of the staff with the Stanford Linear Accelerator Center, Stanford, California.

Professor N. V. FINDLER of the State University of New York at Buffalo has been appointed to a professorship in the Department of Computer Science and Mathematics at the University.

Professor MOSES GLASNER of the University of California, Los Angeles, has been appointed to an assistant professorship at the California Institute of Technology.

Mr. C. A. GREENHALL of the Jet Propulsion Laboratories at the California Institute of Technology has been appointed to an assistant professorship at the University of Southern California.

Professor R. W. HAKALA of Michigan Technological University has been appointed to a professorship at Oklahoma City University.

Professor L. W. JOHNSON of Oklahoma State University has been appointed to a professorship at the University of Alabama, Huntsville.

Mr. WILLIAM KARUSH of the System Development Corporation, Santa Monica, California, has been appointed to a professorship at San Fernando State College.

Professor KUNIHIKO KODAIRA of Stanford University has been appointed to a professorship at Tokyo University, Japan.

Dr. BENJAMIN LEPSON of the U. S. Naval Research Laboratories, Washington, D. C., has been appointed a consultant in the newly formed Mathematics and Information Sciences Division of the Laboratory. He retains his appointment as an adjunct professor at the Catholic University of America.

Professor LEOPOLDO NACHBIN of the University of Rochester has been appointed a visiting member at the Center for Theoretical Studies of the University of Miami.

Dr. R. R. PARKER of Brown Engineering, Huntsville, Alabama, has been appointed an associate director of research in its Advanced Systems and Technologies Group.

Dr. E. K. RITTER of the Lockheed Missiles and Space Company, Sunnyvale, California, has been appointed Chief of the Mathematics Department at the Naval Ordnance Laboratory, Silver Spring, Maryland.

Professor P. L. SADAGURSKY of George Washington University has been appointed to an assistant professorship at Queensborough Community College, City University of New York.

Dr. P. S. SCHNARE of Louisiana State University, New Orleans, has been appointed to an assistant professorship at the University of Florida.

Dr. D. R. SHIEVE of the University of Mississippi Medical Center has been appointed Director of Data Processing and Professor of Mathematics at Sam Houston State College.

Mr. SEYMOUR SINGER of Varian Associates, Palo Alto, California, has accepted a position as Senior Engineer with the Northrop Nortronics Corporation, Los Altos, California.

Dr. A. D. SOLOMON of New York University has been appointed Senior Lecturer in the Department of Applied Mathematics at Tel Aviv University, Israel.

Professor JOHANN SONNER of the University of South Carolina has been appointed to a professorship at the University of North Carolina, Chapel Hill.

Professor R. S. SPIRA of the University of Tennessee has been appointed to an associate professorship at Michigan State University.

Professor R. P. SRIVASTAV of Duke University has been appointed to an associate professorship at the State University of New York at Stony Brook.

Mr. J. T. WALLEN of the University of Nebraska has been appointed to an assistant professorship at Moorhead State College.

Professor E. T. WONG of Oberlin College has been appointed to research status for the 1968-1969 academic year. He will spend most of his time in Oberlin, with short trips to visit mathematicians at various universities here and abroad.

### PROMOTIONS

To Professor. Kent State University: T. N. BHARGAVA.

To Associate Professor. Florida State University: E. C. YOUNG; Pahlavi University, Iran: MEHDI BEHZAD.

To Assistant Professor. Nagaoka Technical College, Japan: YUTAKA IEI.

### DEATHS

Dr. B. I. BAIDAFF of Buenos Aires, Argentina, died on November 8, 1967, at the age of 79. He was a member of the Society for 24 years.

Mrs. M. P. HOLLCROFT of Wells College died on June 29, 1967. She was a member of the Society for 40 years.

Professor Emeritus T. R. HOLLCROFT of Wells College died on September 1, 1967, at the age of 78. He was a member of the Society for 50 years.

Mr. D. W. HULLINGHORST of Concord, New Hampshire, died on January 3, 1968, at the age of 55. He was a member of the Society for 19 years.

Professor S. H. KIMBALL of the University of Maine died on December 25, 1967, at the age of 66. He was a member of the Society for 41 years.

Professor Emeritus CHARLES LOEWNER of Stanford University died on January 8, 1968, at the age of 75. He was a member of the Society for 27 years.

Professor Emeritus C. N. MOORE of the University of Cincinnati died on December 12, 1967, at the age of 85. He was a member of the Society for 54 years.

Dr. G. A. SORMANI of the United Aircraft Research Laboratories and Trinity College, Hartford, Connecticut, died on December 2, 1967, at the age of 28.

Professor ROHN TRUELL of Brown University died on January 10, 1968, at the age of 54. He was a member of the Society for 20 years.

## Errata

The following are corrections of announcements in the January issue of these *Notices*.

Professor T. H. MACGREGOR of Lafayette College has been appointed to a professorship at the State University of New York at Albany.

Dr. HERMANN SIMON has been promoted to an associate professorship at the University of Miami.

Dr. W. W. S. CLAYTOR of Washington, D.C., died on July 14, 1967, at the age of 59. He was a member of the Society for 34 years.

# SUMMER INSTITUTES AND GRADUATE COURSES

The following is a list of graduate courses, seminars and institutes in mathematics being offered in the summer of 1968 for graduate students and college teachers of mathematics. The list was compiled from information received from graduate schools in the United States and Canada.

## Graduate Courses

### ALABAMA

AUBURN UNIVERSITY
Auburn, Alabama 36830
Application deadline: May 20
Information: L. P. Burton

June 10-August 20
Basic courses
Abstract Set Theory
Matrix Numerical Analysis
Ring Theory
Functional Analysis

### ARIZONA

ARIZONA STATE UNIVERSITY
Tempe, Arizona 85281
Application deadline: April 12
Information: Registrar and Director of Admissions

June 17-July 19
MA 404 Projective Geometry
MA 442 Vector Spaces and Matrix Theory
MA 461 Applied Complex Analysis
MA 462 Partial Differential Equations
MA 470 Foundations of Analysis
MA 591 Seminar (analysis)
MA 591 Seminar (topology)

July 22-August 23
MA 460 Applied Real Analysis
MA 471 Foundations of Analysis
MA 591 Seminar (analysis)
MA 591 Seminar (topology)

### ARKANSAS

UNIVERSITY OF ARKANSAS
Fayetteville, Arkansas 72701
Application deadline: June 3
Information: Dr. James E. Scroggs, Chairman, Department of Mathematics

June 3-August 23
Finite Dimensional Vector Spaces
Abstract Algebra I
Complex Variables
Point Set Topology
Rings and Ideals
Topics in Analysis

### CALIFORNIA

UNIVERSITY OF CALIFORNIA
Berkeley, California 94720
Application deadline: May 24
Information: Chairman, Department of Mathematics

June 24-September 6
202A General Topology
203 Measure and Integration
205 Theory of Functions of a Complex Variable
206A Linear Spaces
215A Algebraic Topology
225C Metamathematics
240A Differential Geometry
250A Groups and Rings
274 Topics in Algebra
276 Topics in Topology
278 Topics in Analysis

UNIVERSITY OF CALIFORNIA
Berkeley, California 94720
Application deadline: April 5
Information: Admission application: Graduate Division; General: Department of Statistics

June 20-September 6
Stat. 200B,M Introduction to Probability and Statistics at an Advanced Level.
Stat. 201 Mathematical Bases of Probability Theory.
Stat. 291G Monte Carlo Methods.
Stat. 298B Stochastic Models in Biology.

### COLORADO

UNIVERSITY OF COLORADO
Boulder, Colorado 80302
Application deadline: May 17
Information: Albert Lundell, Summer Chairman, Department of Mathematics

June 14-July 19
(First five week session)

July 22-August 23
(Second five week session)

June 14-August 23
(Ten week session)

A number of graduate courses in mathematics are offered.

## DISTRICT OF COLUMBIA

THE AMERICAN UNIVERSITY
Washington, D. C. 20016
Application deadlines: June 18 and 19
Information: Office of the Admission

June 20-July 24
Topology

## GEORGIA

EMORY UNIVERSITY
Atlanta, Georgia 30322
Application deadline: June 1
Information: Mary Frances Neff, Acting Chairman, Mathematics Department

June 17-July 25/July 26-August 17

| | |
|---|---|
| Math. 415 | Topological Semigroups |
| Math. 410 | Theory of Semigroups |
| Math. 405 | Normed Linear Spaces |

## ILLINOIS

UNIVERSITY OF ILLINOIS
Urbana, Illinois 61801
Application deadline: May 15
Information: H. J. Miles, Department of Mathematics

June 18-August 10

Courses for advanced undergraduate and beginning graduate students

Sets and Real Number System
Topics in Geometry
Selected Mathematical Topics for Secondary Teachers
Linear Transformations and Matrices
Introduction to Higher Algebra I, II
Introduction to Set Theory and Topology
Advanced Calculus
Differential Equations and Orthogonal Functions
Complex Variables and Applications
Introduction to Higher Analysis: Real Variables
Introduction to Higher Analysis: Complex Variables
Elementary Theory of Numbers
Theory of Probability
Advanced Statistics
Introduction to Numerical Analysis
Mathematical Methods in Engineering and Science

Purely graduate courses

Second Course in Abstract Algebra I, II
Advanced Topics in Abstract Algebra
General Topology
Theory of Functions of a Complex Variable
Real Analysis I, II
Partial Differential Equations
Banach Spaces
Mathematical Methods of Physics
Topics in Analysis

SOUTHERN ILLINOIS UNIVERSITY
Carbondale, Illinois 62901
Application deadline: March 30 for Admission to Graduate School
Information: A. M. Mark, Acting Chairman, Mathematics Department

June 18-August 30
Linear Algebra
Seminar in Algebra
Seminar in Analysis
Seminar in Topology

## KANSAS

KANSAS STATE TEACHERS COLLEGE
Emporia, Kansas 66801
Application deadline: May 31
Information: Marion P. Emerson, Head, Department of Mathematics

June 5-August 16

| | |
|---|---|
| MA 430 | Math. Programming |
| MA 450 | Probability and Statistics |
| MA 521 | Projective Geometry |
| MA 522 | Non-Euclidean Geometry |
| MA 525 | Abstract Algebra |
| MA 527 | Groups, Rings, and Fields |
| MA 528 | Vector Spaces |
| MA 531 | Differential Equations |
| MA 647 | Research Project in Mathematics |
| MA 535 | Advanced Calculus I |
| MA 536 | Advanced Calculus II |
| MA 541 | Mathematical Statistics |
| MA 544 | Introduction to Mathematical Logic |
| MA 555 | Complex Variables |
| MA 600 | Modern Mathematics in Secondary School |
| MA 615 | Topology |
| MA 650 | Thesis, MA or MS |
| MA 657 | Thesis, ED.S. |

## MISSISSIPPI

UNIVERSITY OF MISSISSIPPI
University, Mississippi 38677
Application deadline: May 15

June 6-July 12
Theory of Numbers I
Modern Algebra I
Advanced Calculus I
Mathematical Statistics I
Theory of Groups
Projective Geometry
Theory of Functions of Complex Variables I

July 15-August 15
Matrices: Basic Theorems on Matrix Manipulation
Modern Algebra II
Advanced Calculus II

Theory of Integrals
Theory of Numbers II
Mathematical Statistics II
Linear Algebra
Theory of Functions of Complex Variables II

## NEW YORK

NEW YORK UNIVERSITY
University Heights, New York
Application deadline: open
Informaton : J. B. Keller, Chairman, Department of Mathematics

June 17-July 26
T63.1101 Vector Analysis and Functions of Several Variables
T63.2201 Foundations of Mathematical Analysis
T63.2213 Linear Algebra and Matrices

July 29-September 6
T63.1103 Infinite Series with Applications to Ordinary Differential Equations
T63.2202 Functions of a Complex Variable
T63.2207 Partial Differential Equations (Methods)

STATE UNIVERSITY COLLEGE AT GENESEO
Geneseo, New York 14454
Application deadline: open
Information: William A. Small, Chairman, Department of Mathematics

June 24-August 2
573 Real Variables I
* 324 Advanced Calculus I
* 330 Modern Algebra
* 336 Higher Geometry
* 338 Introduction to General Topology
* 342 Mathematical Statistics I
* 331 Linear Algebra

SYRACUSE UNIVERSITY
Syracuse, New York 13210
Application deadline: open
Information: Summer Sessions

June 24-August 2
201 Intermediate Seminar
205a Functions of a Real Variable
206a Functions of a Complex Variable
231a Modern Algebra
261a Introduction to Point Set Topology
272a Fundamentals of Analysis
399 Dissertation

August 5-September 4
201 Intermediate Seminar
231b Modern Algebra
272b Fundamentals of Analysis
399 Dissertation

## OHIO

OHIO UNIVERSITY
Athens, Ohio 45701
Information: Robert K. Butner, Chairman, Graduate Committee

June 17-July 24
Advanced Calculus
Matrix Theory
Applied Complex Variables
Foundations of Mathematics
Point Set Topology
Topics in Geometry

July 25-August 31
Numerical Analysis
Advanced Calculus
Applied Complex Variables
Topology
Number Theory
Topics in Algebra

## PUERTO RICO

UNIVERSITY OF PUERTO RICO
Mayaguez, Puerto Rico 00708
Application deadline: April 15
Information: Eugene A. Francis, Chairman, Department of Mathematics

June 4-July 19
Complex Variables II
Mathematics of Modern Science
Elementary Partial Differential Equations

## TENNESSEE

EAST TENNESSEE STATE UNIVERSITY
Johnson City, Tennessee 37601
Application deadline: June 1
Information: Lester C. Hartsell, Chairman, Department of Mathematics

June 14-July 19
Introduction to Modern Algebra
Introduction to Modern Geometry
Introduction to Analysis
Topics from Ordinary Differential Equations
Foundations and Structure of Mathematics
Algebra for Elementary Teacher
Theory of Numbers
Modern Algebra
Advanced Differential Equations
Functions of Complex Variable
Functions of Real Variables
Special Problems in Mathematics

July 22-August 23
Introduction to Modern Algebra
Introduction to Modern Geometry
Introduction to Analysis
Topics from Partial Differential Equations
Foundations and Structure of Mathematics
Geometry for Elementary Teacher
Theory of Matrices

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* Graduate/undergraduate courses.

Modern Algebra
Partial Differential Equations
Functions of Complex Variable
Functions of Real Variables
Special Problems in Mathematics

UNIVERSITY OF TENNESSEE
Knoxville, Tennessee 37916
Application deadline: none
Information: Professor John H. Barrett, Head, Department of Mathematics

June 14-August 22
Real Variables (Continuation from Spring Quarter)
Credit Seminar in Topology
Credit Seminar in Algebra
Credit Seminar in Fourier Series
Credit Seminar in Non-linear Differential Equations
Credit Seminar in Computer Science

## UTAH

BRIGHAM YOUNG UNIVERSITY
Provo, Utah 84601
Application deadline: May 31
Information: J. C. Higgins or Hal G. Moore

June 17-August 23
Advanced Topics in Applied Mathematics
Special Topics in Analysis
Special Topics in Algebra
Matrix Analysis

## WASHINGTON

WASHINGTON STATE UNIVERSITY
Pullman, Washington 99163
Application deadline: June 1
Information: D. Bushaw, Mathematics Department

June 17-August 9
Advanced Topics in Applied Mathematics
Seminar in Topology

WASHINGTON STATE UNIVERSITY
Pullman, Washington 99163
Application deadline: May 17
Information: Dr. Ottis W. Rechard, Chairman, Information Science Program

June 17-August 9
Inf S 540 Information Storage and Retrieval

WESTERN WASHINGTON STATE COLLEGE
Bellingham, Washington 98225
Application deadline: June 10
Information: Joseph Hashisaki, Chairman, Department of Mathematics

June 17
Introduction to Abstract Algebra I, II
Advanced Calculus I, II
Mathematical Statistics
Advanced Abstract Algebra
Theory of Numbers
Analysis
Complex Variables II
History of Mathematics
Introduction to Topology
Modern Geometry
Computer Science with Application
Modern Higher Algebra
Topics in Differential Equations

## WISCONSIN

UNIVERSITY OF WISCONSIN
Madison, Wisconsin 53706
Information: Donald L. McQuillan Department of Mathematics

June 26-August 17
Linear Transformations in Hilbert Space
Abstract Algebra
Introduction to Algebraic Topology
Advanced Topics in Algebra
Advanced Topics in Point Set Topology

WISCONSIN STATE UNIVERSITY
Eau Claire, Wisconsin 54701
Application deadline: open
Information: R. Dale Dick, Dean, School of Graduate Studies

June 10-August 9
Geometry for Teachers
Introduction to Real Analysis
Modern Mathematics in the Junior High School

# Summer Institutes

## ALABAMA

AUBURN UNIVERSITY
Auburn, Alabama 36830

Institute for College Teachers of Mathematics
Dates: June 10-August 16
Sponsoring Agency: National Science Foundation
Subjects Covered: Basic Modern Algebra and Analysis.
Other Information: For minimally prepared teachers.
Information: Richard W. Ball, Department of Mathematics.

## CALIFORNIA

SAN JOSE STATE COLLEGE
San Jose, California 95114

Institute for Teachers of Freshman and Sophomore Mathematics.
Dates: June 24-August 2
Sponsoring Agency: National Science Foundation
Subjects Covered: Linear Algebra and Probability and Statistics emphasizing calculus applications.
Information: L. H. Lange, Department of Mathematics.

UNIVERSITY OF CALIFORNIA
Santa Barbara, California 93106

Institute for Predoctoral College Teachers of Mathematics
Dates: August 4-August 31
Sponsoring Agency: National Science Foundation
Subjects Covered: Linear Algebra and its applications, and a related program of supervised problem-solving.*
Information: Mrs. Ava Richards, Department of Mathematics.

## COLORADO

UNIVERSITY OF COLORADO
Boulder, Colorado 80302

Institute for Computer Science in Social and Behavior Science Research
Dates: June 17-July 19
Sponsoring Agency: National Science Foundation
Subjects Covered: Simulation of social and behavioral processes, theory and application.
Requirements for Admission: College teachers of Sociology, Psychology, Regional Economic Sciences (Urban Planning), and Computer Science.
Information: Daniel E. Bailly, Institute for Computing Science.

## GEORGIA

UNIVERSITY OF GEORGIA
Athens, Georgia 30601

Institute for College Teachers of Mathematics
Dates: June 17-August 23
Sponsoring Agency: National Science Foundation
Subjects Covered: Real Analysis, to include calculus of functions of several variables.
Information: B. J. Ball, Department of Mathematics.

## ILLINOIS

ILLINOIS INSTITUTE OF TECHNOLOGY
Chicago, Illinois 60616

Institute for College Teachers of Mathematics
Dates: June 24-August 16
Sponsoring Agency: National Science Foundation
Subjects Covered: Programs in Algebra, Analysis, Applied Mathematics, Geometry, Functional Analysis, Probability, Number Theory and Quadratic Forms.
Information: J. J. Mehlberg, Department of Mathematics.

UNIVERSITY OF ILLINOIS
Urbana, Illinois 61801

Insitute for College Teachers of Mathematics
Dates: June 18-September 8
Sponsoring Agency: National Science Foundation
Subjects Covered: The first summer (12 weeks) of a proposed four summer sequential program in Mathematics leading to the degree of Master of Arts in Mathematics.
Other Information: For college teachers of Mathematics who are responsible for the training of elementary and secondary teachers.
Information: Wilson M. Zaring, Department of Mathematics.

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*Those completing the course successfully are to be invited to join in a guided program of home study preparing for a more advanced course in Linear and Multilinear Algebra at Santa Barbara in 1969.

UNIVERSITY OF ILLINOIS
Urbana, Illinois 61801

Institute for Teachers of Electronics: Machine Design

Dates: June 17-August 10
Sponsoring Agency: National Science Foundation
Subjects Covered: Introduction to Machine Design, Advanced Applied Mathematics for Mechanical Systems, Advanced Circuits and Network Analysis, Advanced Electronics, and a Seminar on Technical Education.
Requirements for Admission: Teachers of electronics technology or machine design technology in technical institutes and junior colleges.
Information: Jerry S. Dobrovolny, 117 Transportation Building.

## INDIANA

INDIANA UNIVERSITY
Bloomington, Indiana 47401

Institute for College Teachers of Mathematics

Dates: September 29, 1968-June 6, 1969
Sponsoring Agency: National Science Foundation
Other Information: Individualized program for college teachers of Mathematics who have not completed their graduate training in Mathematics.
Information: Billy E. Rhoades, Department of Mathematics.

## KANSAS

KANSAS STATE TEACHERS COLLEGE
Emporia, Kansas 66801

Institute in Mathematics

Dates: June 5-August 16
Sponsoring Agency: National Science Foundation
Application: By invitation only.
Requirements for Admission: Major in Mathematics and 2 years teaching experience.
Other Information: A few replacements will be filled by new people in the Sequential Institute.
Information: M. P. Emerson, Head, Department of Mathematics.

## MARYLAND

OPERATIONS RESEARCH SOCIETY OF AMERICA
Transportation Science Section
428 East Preston Street
Baltimore, Maryland 21202

Theory of Vehicular Traffic Flow

Dates: June 7-June 11
Subjects Covered: Mathematical models useful for the description and control of traffic systems, and discussion of experiments and applications of these models.
Requirements for Admission: A degree in engineering or the physical sciences or equivalent.
Application deadline: May 22
Other Information: Enrollment is limited, and applicants are urged to apply early. This course is particularly suited for persons engaged in research or teaching.
Information: Denos C. Gazis, IBM Research Center, Box 218, Yorktown Heights, New York 10598.

## MICHIGAN

WAYNE STATE UNIVERSITY
Detroit, Michigan 48202

Institute for College Teachers of Mathematics

Dates: June 27-June 29
Sponsoring Agency: National Science Foundation
Subjects Covered: Lectures, short contributed research papers, and discussion periods on the current state of knowledge in the area of Semigroups.
Information: Karl W. Folley, Department of Mathematics.

## MINNESOTA

UNIVERSITY OF MINNESOTA
Minneapolis, Minnesota 55455

Institute for College Teachers of Mathematics

Dates: June 17-August 9
Sponsoring Agency: National Science Foundation
Subjects Covered: Probability Distributions, Theory and Methods of Inference.
Other Information: For college teachers of Mathematics who are or will be required to teach undergraduate courses in Probability Statistics but whose education has not included training in these areas.
Information: D. W. Lindgren, Department of Statistics.

## MISSOURI

UNIVERSITY OF MISSOURI
Rolla, Missouri 65401

Institute in Computer Science

Dates: June 10-August 2
Sponsoring Agency: National Science Foundation
Subjects Covered: Digital Computer Techniques, Numerical Analysis, Statistical Methods, Linear Programming, and Analog Computation.
Requirements for Admission: Teachers of Mathematics, Engineering or Physical Sciences.
Information: Ralph E. Lee, Computer Science Center.

## NEW JERSEY

PRINCETON UNIVERSITY
Princeton, New Jersey 08540

Institute for College Teachers of Electrical Engineering or Computer Science
Dates: June 19-July 2
Sponsoring Agency: National Science Foundation
Subjects Covered: Courses being developed for a Computer Science option in Electrical Engineering; Elements, Systems, and Computation; Introduction to Linear Algebra with Applications to Computer Science; Computation Structures; and Programming Method.
Information: William H. Surber, Department of Electrical Engineering.

RUTGERS, THE STATE UNIVERSITY
New Brunswick, New Jersey 08903

Institute for College Teachers of Mathematics
Dates: June 17-August 9
Sponsoring Agency: National Science Foundation
Subjects Covered: Predoctoral courses in Foundations of Analysis, Advanced Topics in Calculus, Abstract Algebra, Linear Algebra, Mathematical Theory of Statistics, Functions of a Real Variable and a Problem, and Curriculum Seminar.
Information: A. A. Austen, Director of the Summer Session.

## NEW MEXICO

NEW MEXICO STATE UNIVERSITY
Las Cruces, New Mexico 88001

Introduction to Modern Algebra and Analysis
Dates: June 10-August 2
Sponsoring Agency: National Science Foundation
Requirements for Admission: College teachers with little training in Abstract Mathematics.
Information: John B. Giever, Department of Mathematical Sciences.

## NEW YORK

SOUTHAMPTON COLLEGE, LONG ISLAND UNIVERSITY
Southampton, New York 11968

Institute for College Teachers of Logic, Mathematics, or Philosophy
Dates: June 23-August 3
Sponsoring Agency: National Science Foundation
Subjects Covered: Problems of ontology related to such logical concepts as individuals, classes, identity, properties and relations, existence, necessity and possibility, and modal logic.
Information: Jerome Shaffer, Department of Philosophy, University of Connecticut, Storrs, Connecticut 06828.

## NORTH CAROLINA

NORTH CAROLINA STATE UNIVERSITY
Raleigh, North Carolina 27607

Institute for Retired Military Officers
Dates: July 3, 1968-June 6, 1970
Sponsoring Agency: National Science Foundation
Subjects Covered: Refresher courses in Mathematics and Physics followed by adequate study and guided teaching to prepare for teaching Mathematics at the freshman and sophomore college level.
Other Information: A two-year program for retired military officers who have previously completed the study of Mathematics through the level of elementary differential equations.
Information: H. V. Park, Department of Mathematics, Box 5548, State College Station.

UNIVERSITY OF NORTH CAROLINA
Chapel Hill, North Carolina 27514

Institute for Junior College Teachers
Dates: June 10-July 19
Sponsoring Agency: National Science Foundation
Subjects Covered: Courses in Analysis and Linear Algebra, both designed to increase the competence of the participants to teach Calculus.
Information: E. A. Cameron, Department of Mathematics.

## OHIO

OBERLIN COLLEGE
Oberlin, Ohio 44074

Institute for Junior College Teachers of Mathematics
Dates: June 17-August 9
Sponsoring Agency: National Science Foundation
Subjects Covered: Designed, with CUPM recommendations for reorganization of the undergraduate curriculum in mind, for prospective teachers of courses in Linear Algebra or Probability and Statistics.
Information: Robert R. Stoll, Department of Mathematics.

## OKLAHOMA

UNIVERSITY OF OKLAHOMA
Norman, Oklahoma 73069

Institute for College Teachers of Mathematics
Dates: June 5-August 14
Sponsoring Agency: National Science Foundation
Subjects Covered: Programs in Abstract Algebra, Differential Equations, Calculus of Variations, Geometry, Differential Geometry, Mathematical Logic, Topology and Numerical Analysis.
Information: Richard V. Andree, Department of Mathematics and Astronomy.

## OREGON

UNIVERSITY OF OREGON
Eugene, Oregon 97403

Institute for College Teachers of Mathematics

Dates: June 17-August 9
Sponsoring Agency: National Science Foundation
Subjects Covered: Emphasis upon analysis and curriculum development.
Other Information: For junior, community, and (if appropriate) small college Mathematics teachers from the states of Alaska, Idaho, Montana, Oregon, and Washington.
Information: A. F. Moursund, Department of Mathematics.

## PENNSYLVANIA

PENNSYLVANIA STATE UNIVERSITY
University Park, Pennsylvania 16802

Institute in Systems Programming

Dates: June 17-August 16
Sponsoring Agency: National Science Foundation
Subjects Covered: Systems Programming, Computer Languages, Numerical Analysis and Computer Projects.
Requirements for Admission: College and Junior College Teachers of Mathematics or Computing.
Information: Alvin R. Grove, 214 Whitmore Laboratory, Pennsylvania State University.

## UTAH

BRIGHAM YOUNG UNIVERSITY
Provo, Utah 84601

Brigham Young University Summer School

Dates: June 17-August 23
Subjects Covered: Analysis, Algebra and Applied Mathematics.
Requirements for Admission: New students must make formal application before June 1, 1968.
Application deadline: May 31
Information: J. C. Higgins or Hal G. Moore

UTAH STATE UNIVERSITY
Logan, Utah 84321

Institute for College Teachers of Mathematics

Dates: June 17-August 9
Sponsoring Agency: National Science Foundation
Subjects Covered: Instruction in Analysis, Topology, and Modern Algebra and an introduction to Probability and Linear Algebra.
Requirements for Admission: For college and junior college teachers of Mathematics who do not have a master's degree in Mathematics.
Information: Neville C. Hunsaker, Department of Mathematics.

UTAH STATE UNIVERSITY
Logan, Utah 84321

Institute in Feedback Control Theory

Dates: July 1-August 23
Sponsoring Agency: National Science Foundation
Subjects Covered: Conventional and modern aspects of Feedback Control Theory including stochastic systems.
Requirements for Admission: For college, junior college and technical institute teachers of Engineering, Physics or Mathematics.
Information: Bruce O. Watkins, Department of Electrical Engineering.

## VIRGINIA

VIRGINIA POLYTECHNIC INSTITUTE
Blacksburg, Virginia 24061

Institute for College Teachers of Political Science

Dates: June 16-July 31
Sponsoring Agency: National Science Foundation
Subjects Covered: Mathematical applications in substantive areas of Political Science, utilizing matrix algebra, statistics of probability, scaling and factor analysis, and regression analysis.
Information: Joseph L. Bernd, Political Science Department.

## WISCONSIN

UNIVERSITY OF WISCONSIN
Madison, Wisconsin 53706

Conference on Qualitative Theory of Differential and Integral Equations

Dates: August 19-August 23
Sponsoring Agency: Office of Naval Research
Subjects Covered: Ordinary and Partial Differential Equations, Integral Equations, and Delay Equations (existence, stability properties, and existence and properties of periodic solutions).
Requirements for Admission: Advanced graduate standing, working in one of the above fields, or post-doctoral.
Application deadline: May 1
Information: John Nohel, Department of Mathematics.

# ABSTRACTS OF CONTRIBUTED PAPERS

## The April Meeting in New York
## April 10-13, 1968

655-1. M. L. MADAN, Ohio State University, Columbus, Ohio 43210. Class number and ramification in fields of algebraic functions.

Let F/K be a field of algebraic functions in one variable over a finite field K of constants. Let E/K be a finite separable extension of F/K of degree n. Let $\mathfrak{p}_1,\dots,\mathfrak{p}_\lambda$ be the finite set of primes of F/K which are ramified in E/K. The main results are the following two theorems. Theorem 1. Let $h_F$, $h_E$ denote the class numbers of F and E respectively; $\bar{e}_i$, $i = 1,\dots,\lambda$, be the greatest common divisors of the ramification indices of $\mathfrak{p}_i$ in E. Then $h_F \cdot \bar{e}_1 \dots \bar{e}_\lambda$ divides $n^2 \cdot h_E$. Theorem 2. For a rational prime q, let $R_q(C_0)$ be the q-rank of $C_0$, the group of divisor classes of degree zero of E. Let $\rho_q$ be the number of primes of F for which each of the ramification indices in E is divisible by q. Then the following holds $R_q(C_0) \geqq \rho_q - 1 - w_q(n)$, where $q^{w_q(n)}$ is the largest power of q which divides n. (Received August 18, 1967.)

655-2. R. G. LINTZ, McMaster University, Hamilton, Ontario, Canada. The idea of motion in a topological space and Cauchy's problem for generalized differential equation.

Recently the idea of derivatives in general topological spaces has been introduced (Abstract 648-76, these Notices 14 (1967), 651)and as a consequence of this it is possible to build a "dynamics" for a "particle" in a general topological space. More precisely, a g-path in a topological space is a continuous g-function $f: (I, \mathscr{V}_I) \longrightarrow (X, \mathscr{V})$ where $I = [0,1]$ and $\mathscr{V}_I$ and $\mathscr{V}$ are families of open coverings of I and X respectively. Now the velocity and the acceleration of an ideal point connected with the given g-path are given by the g-derivatives Df and $D^2 f$ of f. If $\Phi$ is a g-field the dynamics of the ideal point is determined by the equation $\mu D^2 f = \Phi \circ f$ where $\mu$ is a real constant attached to the ideal point in question. Now it is also possible to consider a Cauchy's problem, i.e. initial conditions problem, for the equation above. However, up to now only particular cases have been solved. (Received November 17, 1967.)

655-3. P. C. HAMMER, Pennsylvania State University, University Park, Pennsylvania. Approximation spaces.

Let E be a set of entities to be approximated. For each $p \in E$ let v(p) be a set of allowable approximants of p and let $V = \bigcup \{v(p) : p \in E\}$. Let T(p) be a reflexive order relation in v(p). An order relation is simply a transitive relation. Then the ternary relation $\mathscr{A} = \{(p, q_1, q_2) : p \in E, (q_1, q_2) \in T(p)\}$ is called an approximation space in (E,V). The statement $(q_1, q_2) \in T(p)$ is interpreted "$q_1$ is at least as good an approximant of p as $q_2$ is". A set $A \subseteq V$ is called a best approximant of $p \in E$ provided to each $q \in V(p)$ there is $q_0 \in A$ such that $(q_0, q) \in T(p)$. It is shown that each isotonic space is an example of an approximation space and hence, in particular, that all topological spaces are approximation spaces. Properties of approximation spaces are derived. Uniform approximation spaces are shown, in one context, to be essentially "metric" spaces. Examples illustrate applications. (Received November 17, 1967.)

655-4. R. L. KELLEY, University of Miami, Coral Gables, Florida 33124. Unitary equivalence and similarity of two-sided weighted shifts.

Let H be a separable infinite dimensional Hilbert space and let A and B be injective two-sided weighted shifts on H. Let $\{e_n\}$ and $\{f_n\}$ be orthonormal bases for H and let $\{\alpha_n\}$ and $\{\beta_n\}$ be sequences of nonzero complex numbers such that $Ae_n = \alpha_n e_{n+1}$ and $\beta f_n = \beta_n f_{n+1}$ for all n, $n = 0, \pm 1, \pm 2, \ldots$ . Theorem. A necessary and sufficient condition that S and T be unitarily equivalent is that there exist an integer k such that $|\alpha_n| = |\beta_{n+k}|$ for all n. A necessary and sufficient condition that S and T be similar is that there exist constants M and N, and an integer k, such that $0 < M \leqq \prod_{j=0}^{n-1} |\beta_{j+k}/\alpha_j| < N < \infty$ if $n < 0$ and $0 < M \leqq \prod_{j=1}^{-n} |\alpha_{-j}/\beta_{-j+k}| \leqq N < \infty$ if $n < 0$. Analogous results are known to hold for one-sided weighted shifts (see P. Halmos, A Hilbert space problem book, Van Nostrand, Princeton, N. J., 1967). (Received November 9, 1967.)

655-5. M. C. TEWS, College of the Holy Cross, Worcester, Massachusetts 01610. On maps which preserve almost periodic functions.

Let G and H be locally compact groups. A function $f: G \longrightarrow H$ is said to be a group-valued almost periodic function if (1) f is continuous and (2) for each neighborhood V of the identity of H there exists a finite subset X of G such that for each $y \in G$ there is an $x \in X$ such that $f(x + g) - f(y + g) \in V$ for each $g \in G$. The set of all such functions is denoted AP(G,H). When $H = \mathbb{C}$, the complex numbers, we denote $AP(G,\mathbb{C})$ by AP(G) and the definition is the usual one. A map $p: G \longrightarrow H$ is said to preserve almost periodic functions if $AP(H) \circ p \subset AP(G)$. Theorem 1. If G is LC, H is LCA and $p: G \longrightarrow H$ preserves almost periodic functions, then p is continuous. Theorem 2. Let G and H be LCA, $p: G \longrightarrow H$ and $cl(p(G))$ be compact. Then p preserves almost periodic functions iff $p \in AP(G,H)$. Theorem 3. Let G and H be LCA and let G be connected. Then $p: G \longrightarrow H$ preserves almost periodic functions iff $p = h + b$ where h is a continuous homomorphism from G to the connected component of the identity of H and $b \in AP(G, H)$. (Received November 9, 1967.)

655-6. ANATOLE BECK and P. WARREN, University of Wisconsin, Madison, Wisconsin 53706. A Strong Law of Large Numbers for weakly orthogonal sequences of Banach-space valued random variables.

Let $\mathfrak{X}$ be a Banach space with dual space $\mathfrak{X}^*$ and let $\{X_i\}$ be a sequence of $\mathfrak{X}$-valued random variables (an $\mathfrak{X}$-valued random variable is a strongly measurable function from a measure space $\Omega$ to the space $\mathfrak{X}$). We shall only consider $\mathfrak{X}$-valued random variables which have a variance $Var(X_i) = \int_\Omega \|X_i(\omega)\|^2 m(d\omega)$. A sequence of $\mathfrak{X}$-valued random variables is said to be weakly orthogonal if $\int_\Omega x^*(X_i(\omega))x^*(X_j(\omega))m(d\omega) = 0, \forall i \neq j, \forall x^*$ in $\mathfrak{X}^*$. In a Hilbert space, a weakly orthogonal sequence is also orthogonal in the usual sense. The following generalization of the Strong Law of Large Numbers for independent and identically distributed $\mathfrak{X}$-valued random variables is shown: If $\mathfrak{X}^*$ is a separable Banach space, then every sequence of $\mathfrak{X}$-valued random variables, with $Var(X_i) = \int_\Omega \|X_i(\omega)\|^2 m(d\omega)$

$< \infty$, which is weakly orthogonal and strictly stationary (in the sense of Doob) satisfies the Strong Law of Large Numbers. That is, $\|(1/N)\sum_{i=1}^{N} X_i(\omega)\| \longrightarrow 0$ almost surely. On the other side, an example is given of a Banach space $\mathfrak{X}$ having a separable dual and a weakly orthogonal sequence of uniformly bounded $\mathfrak{X}$-valued random variables having the same distribution but failing to be strictly stationary for which the Strong Law of Large Numbers does not hold. (Received November 5, 1967.)

655-7. JOHANN SONNER, University of North Cagolina, Chapel Hill, North Carolina 27514. An example of an epic subcategory.

Denote by X the subset of the reals consisting of the endpoints of the intervals occurring in the construction of Cantor's triadic set. Furthermore, denote by G the graph of the usual order on X, and consider G as a category. By changing the order on X one obtains an epic subcategory G' of G which is different from G. (Received November 13, 1967.)

655-8. AL KELLEY, University of California, Santa Cruz, California. Invariance for linear systems of ordinary differential equations.

In studying the existence and smoothness of invariant manifolds arising from nonlinear, perturbed systems of ordinary differential equations, one encounters the study of certain linear (in x), perturbation problems of the type $\dot{\theta} = a + \epsilon b(\theta, \epsilon)$, $\dot{x} = (A + \epsilon B(\theta, \epsilon))x$ where $\theta$ and x are vectors, A and B are matrices, b and B are multiply periodic in $\theta$, and $\epsilon$ is a perturbation parameter. Assuming A is a constant matrix consisting of square sub-matrices on the diagonal, $A = \mathrm{diag}(A_{11}, \ldots, A_{nn})$, with the maximum of the real parts of the eigenvalues of $A_{jj}$ less than the minimum of the real parts of the eigenvalues of $A_{kk}$ for $1 \leqq j < k \leqq n$; we construct a change of variables which reduces B to similar diagonal form. (Received November 29, 1967.)

655-9. J. V. MICHALOWICZ, The Catholic University of America, Washington, D. C. 20017. On a class of tricategories.

The objective of this paper is to develop the theory of the JTK-category, which concept is equivalent to the strongly pure tricategory with embeddings and quotients introduced by Heller, Embeddings and quotients in abstract categories, to be published, Pacific J. Math. The extent to which the categorical generalizations in the JTK-category of the noncategorical concepts of subobject, quotient object, one-to-one mapping, onto mapping, embedding and quotient map retain the properties of their progenitors is established. An investigation is made of categorical concepts, e.g. kernel, intersection, etc., in the JTK-category and new concepts are formulated, when necessary, to reflect the JTK-categorical notions of subobject and quotient object. Among the results obtained are a generalization of the Embedding Lemma of point-set topology and two versions of the Universality Theorem, proved by Semadeni, Projectivity, injectivity, and duality, Rozprawy Mat. 35 (1963), for bicategories in the JTK-category. Various examples of JTK-categories are also presented. (Received November 15, 1967.)

655-10. W. J. GRAY, University of Alabama, University, Alabama. The action of topological groups on Hausdorff continua.

Let (X,T) be a topological transformation group, where X is a Hausdorff continuum. X is T-irreducible if no proper subcontinuum of X is T-invariant. Kelley, Wallace, and the author have shown that if X is T-irreducible and T is either compact, abelian, or T is pointwise regularly almost periodic and X is locally connected, then X contains no cut point. The author has shown that if T is cyclic and X is T-irreducible, then X is not the union of two proper universal subcontinua. We present these results: Theorem 1. If T is abelian and X is T-irreducible, X is not the union of two proper universal subcontinua. Corollary 1 (Wallace). If T is abelian and X is T-irreducible, X contains no cut point. Corollary 2. If T is abelian and X is T-irreducible and hereditarily unicoherent, X is indecomposable. Theorem 2. These are valid: (1) If T is almost periodic and X is T-irreducible, X contains no cut point. (2) If T is connected and either almost periodic or pointwise regularly almost periodic, and if X is locally connected, T leaves every cut point of X fixed. Some unsolved problems are discussed. In particular, assume cod(X) $\leqq$ n, in the sense of Cohen, and X is T-irreducible with $H^n(X) \neq 0$. Can X be separated by any subcontinuum of cohomological dimension $\leqq n - 2$ if T is abelian? (Received November 5, 1967.)

655-11. LEONARD SARASON, University of Washington, Seattle, Washington 98105. Symmetrizable problems in regions with edges and corners.

Boundary value problems for symmetrizable first order linear operators of the type treated by Friedrichs and Lax [Comm. Pure Appl. Math. 18 (1965), 355-388] are studied. The operator $K = A^i(x)\,\partial_i + C$ is symmetrizable if there is a nonsingular symbol $r(x, \xi/|\xi|)$ which is smooth in $R^n \times S^{n-1}$ and such that $rA^i\xi_i$ is symmetric. The basic estimate $\|u\| \leqq c\|Ku\|$, satisfied by smooth functions u satisfying appropriate boundary conditions and lying in a subspace of finite codimension, is extended to regions of the above type under the additional hypothesis that $r(x,n) = 1$ at all boundary points x; here n is the unit normal to the boundary. Some extra positivity is required of K. The method of proof is this: one extends u to vanish outside the region and imagines a smooth portion of the boundary to be extended smoothly beyond its edges and corners. Now the methods of Friedrichs and Lax are applied in the larger region, taking into account the extra hypothesis and that Ku involves some delta functions. (Received December 4, 1967.)

655-12. HERMANN SIMON, University of Miami, Coral Gables, Florida 33124. Finite solvable groups containing maximal nilpotent subgroups.

Note. All groups considered here are finite. Definition. A partition JI of G is a nonempty set of subgroups of G such that each $1 \neq g \in G$ is contained in one and only one $X \in$ JI. If S is a subgroup of G, then S is called JI-admissible iff $S \cap X = 1$ or $= X$ for all $X \in$ JI. Theorem 1. The following properties of the group G and its maximal and nilpotent subgroup M are equivalent: (I) G is solvable. (II) The derivatives of $M/\bigcap_{g\in G} M^g$ are strongly closed in $M/\bigcap_{g\in G} M^g$. As an application of Theorem 1 one obtains the following Theorem 2. The group G is solvable if (a) G contains a maximal 2-Sylow group M, and (b) M admits a nontrivial partition JI such that the conjugates of M intersect M in JI-admissible subgroups. (Received November 17, 1967.)

655-13. R. J. GREECHIE, Kansas State University, Manhattan, Kansas 66502. On the construction of order ortho-homomorphisms. Preliminary report.

N. Zierler and M. Schlessinger have proved that every orthocomplemented poset P may be represented by the set H(P) of all order ortho-homomorphisms from P to the two element Boolean lattice (the orthocomplementation, ordering, zero and unit are set theoretic complementation, inclusion, $\emptyset$ and H(P), respectively). We provide an algorithm for generating the order ortho-homomorphisms in case the poset is a member of a certain class $\mathscr{L}$. The algorithm is of most interest when P has finite cardinality; $\mathscr{L}$ contains all such posets. The construction is based on the following definition: a $\Gamma$-sequence is a function $\gamma : N \longrightarrow P \times 2^P$ such that if $\gamma(i) = (b_i, B_i)$, then (1) $B_1 = P$, (2) $b_i = 0$ if and only if $B_i = \emptyset$, (3) $B_i \neq \emptyset$ implies $b_i \in B_i$, (4) $b_i = 0$ implies $b_{i+1} = 0$, and (5) $b_i \neq 0$ implies $B_{i+1} = B_i - \{x \in P : b_i \leqq x \text{ or } x \leqq b_i'\}$. (Received November 6, 1967.)

655-14. BERTRAM MOND and OVED SHISHA, Aerospace Research Laboratories, Office of Aerospace Research, Bldg. 450, Wright-Patterson AFB, Ohio 45433. A Hilbert space inequality.

Recently, the authors proved the following inequality. Let A be a selfadjoint operator on a Hilbert space H satisfying $0 < m \leqq A \leqq M$. Then, for every x of unit norm in H, $(Ax,x) - (A^{-1}x,x)^{-1} \leqq (M^{1/2} - m^{1/2})^2$. This result is now generalized as follows: Suppose that T is an operator on H satisfying $\|T\| \leqq M$, $\|T^{-1}\| \leqq 1/m$. Then, $|(Tx,y)| - |(T^{-1}x,y)|^{-1} \leqq (M + m)[(x,x)(y,y)]^{1/2} - 2(Mm)^{1/2}$ for all x and y in H. (Received December 19, 1967.)

655-15. C. O. BLOOM and B. J. MATKOWSKY, Rensselaer Polytechnic Institute, Troy, New York. On the validity of the geometrical theory of diffraction by convex cylinders.

We consider the scattering of a circular cylindrical wave normally incident on a smooth convex cylinder of cross section C. C is formed by joining a pair of smooth convex arcs to the exterior of a circle $C_0$; one on the illuminated side of $C_0$ and one on the dark side, so that C is circular in neighborhoods of the points of diffraction. By a rigorous argument we establish the asymptotic behavior for high frequency of the field in a certain portion of the shadow S, determined by the geometry of C in S. The leading term of our asymptotic expansion is the field predicted by the geometrical theory of diffraction. Previous authors have derived asymptotic expansions in S, in a limited number of special cases where separation of variables is possible. Others, who have considered more general shapes, have only been able to obtain bounds on the field in S. In contrast, our result is believed to be the first rigorous asymptotic solution in the shadow of a nonseparable boundary, whose shape is frequency independent. (Received November 9, 1967.)

655-16. J. L. BROWN, JR., and H. S. PIPER, JR., Pennsylvania State University, State College, Pennsylvania 16801. Bounds for the truncation error in sampling expansions.

Let $f(t) = (1/2\pi)\int_{-\pi r}^{\pi r} F(\omega)e^{i\omega t}d\omega$ for $-\infty < t < \infty$, where $F(\omega) \in L_2[-\pi r, \pi r]$ and $0 < r < 1$. Then, it is shown by elementary real-variable techniques that for $|t| \leqq 1/2$, the error $e(t) \equiv f(t) - \sum_{-N}^{N} f(n)(\sin \pi(t-n))/(\pi(t-n))$ satisfies $|e(t)| \leqq (M|\sin \pi t|)/(N \cos \pi r/2)$, where M is a constant depending on the $L_2$-norm of f(t). Previous proofs of the result by J. B. Thomas et al have been accomplished by bounding the growth of the entire function f(t) and using contour integration in the complex t-plane. The present direct method can also be extended to estimating the error when the partial sum is extended over the asymmetrical index range from $-N_1$ to $N_2$ with $N_1, N_2 > 0$. (Received December 4, 1967.)

655-17. PAUL SLEPIAN, Rensselaer Polytechnic Institute, Troy, New York 12181. *Invariance of network functions with respect to orientation.*

To obtain the branch voltages and branch currents in an electrical network by the processes of network analysis, an orientation must be assumed for each branch of the network. On the other hand, when an electrical network is introduced into a circuit, no orientation of all branches is needed to develop the branch voltages and branch currents. Thus, one would expect theorems stating that the processes of network analysis are indeed invariant under change in the orientation of the branches of the network, but unfortunately, no such theorems appear in the standard references. In this paper it is proved that in the case of resistive networks the branch voltages and branch currents are independent of any change in orientation of the branches. The topological formulas of Kirchhoff's Third and Fourth Laws are used in the argument. It is expected that these results can be extended to apply to networks with more complicated branch elements. (Received December 26, 1967.)

655-18. DAVID BEAUCAGE, State University of New York, Stony Brook, New York. *The mod p cohomology of certain fibre spaces.*

The results of W. S. Massey and F. P. Peterson [*The cohomology structure of certain fibre spaces.* I, Topology 4 (1965), 47-65] can be extended to the case of an odd prime characteristic. The appropriate definition is: a sequence $x_1, x_2, \ldots$ in an algebra S is a *prime sequence* if, for each i, the kernel of the endomorphism $S/(x_1,\ldots,x_{i-1}) \longrightarrow S/(x_1,\ldots,x_{i-1})$ induced by multiplication by $x_i$ is 0 if the degree of $x_i$ is even, $(\bar{x}_i)$ if odd. Their result [*The mod 2 cohomology structure of certain fibre spaces*, Mem. Amer. Math. Soc. 74 (1967)] on the unstable Adams spectral sequence can also be extended. (Received December 26, 1967.)

655-19. J. CHIDAMBARASHWAMY, University of Toledo, Toledo, Ohio 43606. *The k-unitary convolution of certain arithmetical functions.*

A divisor d of n is called a k-unitary divisor of n, if $(d,n/d)_k = 1$, where $(a,b)_k$ is the largest kth power divisor common to both a and b. For any arithmetical functions g(n) and h(n) let f(n) be their k-unitary convolution, i.e. $f(n) = \sum_{d\delta=n;(d,\delta)_k=1} g(d)h(\delta)$. It is proved that (i) if $g(n) = O(n^\epsilon)$ $0 \leqq \epsilon < 1/2$, $h(n) = n^r$, then for $r \geqq 1$ the average order of f(n) is $(\sum_{m=1}^{\infty}(g(m)\phi_k(m))/m^{r+2})n^r$ where $\phi_k(n)$ is the number of numbers in a residue system mod n which have 1 as their kth power G.C.D. with n, and (ii) if $g(n) = O(n^\epsilon)$ $0 \leqq \epsilon < 3/4$, $h(n) = n^r\mu^2(n)$, then for $r \geqq 1$, the average order of f(n) is $(1/\zeta(2))(\sum_{m=1}^{\infty}(g(m)\phi(m))/mJ(m))n^r$, J(n) being the Jordan function of order 2 if $k = 1$, and is $(1/\zeta(2))(\sum_{m=1}^{\infty}g(m)/m^{r+1})n^r$ if $k > 1$. The results when $k = 1$, $r = 1$, $\epsilon = 0$ were obtained by Eckforc Cohen. (Math. Z. 74 (1960), 66-80, Theorems 4.1 and 5.1). We also obtain some of the properties of the generalised Euler's function $\phi_k(n)$ defined above. (Received December 4, 1967.)

655-20. KEITH MILLER, University of California, Berkeley, California 94720. *Stabilized numerical analytical prolongation with poles.* Preliminary report.

We consider a problem of considerable interest for nuclear scattering theory: the approximate determination of a meromorphic function f on the unit disc D, and the number, location, and weight of its poles, from approximate values h(z) for f(z) given on an interior data set $\alpha$. We suppose that (i) $\alpha$ has nonzero capacity, (ii) f has only N poles $\xi_1, \dots, \xi_N$, all simple and in a compact $D_1 \subset D$, (iii) the poles are well separated, $\min|\xi_i - \xi_j| \geqq d > 0$, (iv) the weights $W_i$ are well bounded from zero, $|W_i| \geqq k > 0$, and (v) $|F - Bh| < \epsilon$ on $\alpha$, where $B(z) = (z - \xi_1) \dots (z - \xi_N)$ and $f = F/B$. Finally, we require that F satisfy a known global bound, say $|F| < 1$ on D, in order to restore stable dependence on the data. *Theorem.* Suppose (i) $g = G/C$ where $C(z) = (z - z_i) \dots (z - z_M)$, G is holomorphic in D, (ii) $z_1, \dots, z_M \in 2D$, (iii) $M \leqq N$, (iv) $|G - Ch| < \epsilon$ on $\alpha$, and (v) $|G| < 1$ on D. If $\epsilon < \epsilon_0$ then N = M and $|\xi_i - z_i| \leqq K\epsilon^{\lambda}$, $0 < \lambda < 1$, where $\epsilon_0$, $\lambda$, K depend only on $\alpha$, $D_1$, N, d, k. Similar results hold when data is given only on a finite subset of $\alpha$, thereby establishing convergence for a least squares numerical method (SNAPP) which finds rational approximations $g = G/C$ for f. Moreover, SNAPP includes precise computer generated a posteriori bounds for the error in pole locations and weights. (Received November 27, 1967.)

655-21. WITHDRAWN.

655-22. M. F. JANOWITZ, University of Massachusetts, Amherst, Massachusetts 01002. *Section semicomplemented lattices.* Preliminary report.

A lattice L with 0 is called *section semicomplemented* (SSC) if a < b implies the existence of an x such that $0 < x \leqq b$ and $x \wedge a = 0$. The relation $a \nabla b$ in a lattice with 0 denotes that $(a \vee x) \wedge b = x \wedge b$ for all x. The element a is *subperspective* to b if $a \leqq b \vee x$ and $a \wedge x = 0$ for some x; it is *subprojective* to b if there exist $x_0, \dots, x_n$ such that $x_0 = a$, $x_n = b$, and $x_{i-1}$ is subperspective to $x_i$ for all i. *Theorem* 1. If L has 0 and is dual SSC, TAE: (i) $e \nabla f$; (ii) $e \vee x = 1$ implies $f \leqq x$; (iii) x =

$(x \vee e) \wedge (x \vee f)$ for all x. If p,q are points in L, then p is perspective to q iff $p \nabla q$ fails. Theorem 2. A lattice L with 0 is SSC iff its completion by cuts is SSC. Theorem 3. Let L be complete, SSC and dual SSC. Then: (i) the central cover of a is the join of all elements subprojective to a; (ii) the lattice of congruences of L is a Stone lattice; (iii) the center of L is a complete sublattice. Theorem 4. Let L be complete, atomistic and dual SSC. Then two points p,q are projective iff their central covers are equal. Hence L is the direct sum of irreducible sublattices $L(0,z_i)$ where two points p and q belong to the same component iff they are projective. (Received January 10, 1968.)

655-23. H. O. FATTORINI, University of California, Los Angeles, California 90024. Differential equations in linear topological spaces. III.

Notations, definitions and assumptions are those from Abstract 67T-41, these *Notices* 14 (1967), 140. Theorem 1. A generates a c.f. $S(\cdot)$ of type $\leqq w$ iff $R(\lambda^2; A)$ exists for $\lambda > w$, is $C^\infty$, and the sets $\{(n!)^{-1}(\lambda - w)^{n+1}(d/d\lambda)^n \lambda R(\lambda^2; A)u;\ \lambda > w,\ n = 1,2,\ldots\}$ are bounded in E for all $u \in E$. Theorem 2. Assumption 2 is satisfied when E is a $L^2$-space, $1 < p < \infty$. Corollary. The C. P. for $u'' = Au$ is u.w.p. iff $A = B^2 + CI$, B the generator of a strongly continuous group. For E a Hilbert space, $b \geqq w$, $u(\cdot)$ a generalized solution (g.s.) of $u'' = Au$, define $E_b(u(\cdot); t) = (1/2)(|u'(t)|^2 + |A_b^{1/2}u(t)|^2 + b^2|u(t)|^2)$ (the b-energy of $u(\cdot)$). Theorem 3. Let $u(0) \in D(A_b^{1/2})$. Then the b-energy of $u(\cdot)$ is finite for all t. Theorem 4. Assume $S(\cdot)$ is uniformly bounded and that the 0-energy of every g.s. with $u(0) \in D(A_b^{1/2})$ is uniformly bounded. Then A is equivalent to a nonpositive selfadjoint operator. Theorem 5. Same hypotheses as Theorem 2. Then A generates a c.f. of type $\leqq w$ iff $R(\lambda; A)$ exists for $\lambda > w^2$ and $\{e^{-w(\lambda)|t|}S_\lambda(t);\ \lambda > w'^2,\ -\infty < t < \infty\}$ is bounded, $w' > w$, $\lim w(\lambda) = w$, $S_\lambda(t) = \sum_{n=0}^{\infty}(t^{2n}/(2n)!)(\lambda^2 AR(\lambda; A)^2)^n$; moreover $S(t)u = \lim_{\lambda \to \infty} S_\lambda(t)u$. The same result is valid for general E if Assumption 2 is assumed to hold. (Received January 15, 1968.)

655-24. HIDEGORO NAKANO, Wayne State University, Detroit, Michigan 48202. On the existence of regular initial numbers.

For any space S we obtain a well-ordered space $\Delta_S$ of all ordinal numbers in S. For any well-ordered space $\Lambda$ and for any system of well-ordered space $S_\lambda$ $(\lambda \in \Lambda)$ we obtain a well-ordered space $\sum_{\lambda \in \Lambda} S_\lambda$ as the sum of them. A well-ordered space S is said to be accessible if we can obtain S by these two processes from the natural number space. For two well-ordered spaces S and R we write $S \leqq R$ if there is an isomorphism from S onto a segment of R. A well-ordered space S is said to be inaccessible if $A \leqq S$ for any accessible space A. We can prove that there is a regular initial number in a well-ordered space S if and only if S is inaccessible. All of the accessible spaces form a Cantor set but not a space, just like all of the Dedekind's natural numbers. However we can define the least inaccessible space by an axiom system, as we can define the natural number space by the Peano's axiom system. The detail is found in the book, H. Nakano, Set theory (to appear). (Received January 15, 1968.)

655-25. W. C. WHITTEN, JR., 117 Brentwood Boulevard, Lafayette, Louisiana 70501.
A pair of noninvertible links.

An oriented, ordered link L of $\mu$ components tamely imbedded in the oriented 3-sphere S will be called invertible if and only if there is an orientation-preserving autohomeomorphism of S which takes each component of L onto itself with reversal of orientation. While the existence of noninvertible knots quarantees noninvertible links, it is of interest to have examples of such links with components all of which are invertible. In this paper a pair of prime, noninvertible links is given, each link consisting of 2 components. All components are of knot type $5_1$ so that each component admits inversion. One of the links is interchangeable, while the other is not. The proof of noninvertibility in each case reduces to showing that two particular elements belonging to a certain factor group, isomorphic to $Z_2 * Z_5$, of the knot group of $5_1$ are not conjugate. (Received January 15, 1968.)

655-26. HARRY GONSHOR, Rutgers, The State University, New Brunswick, New Jersey.
The category of recursive functions.

It is first shown that under suitable conditions the isomorphism theorems of universal algebras are valid for recursive maps. Then it is shown that many of the concepts of the theory of recursive equivalence types can be expressed categorically. For example, the sum and product of types in the sense of Dekker correspond to the category sum and product respectively. Also the Dekker min function is a special case of the category intersection. Finally, it is shown that inclusion maps correspond to extremal monics. (Received January 17, 1968.)

655-27. P. P. SAWOROTNOW, The Catholic University of America, Washington, D. C. 20017.
A generalization of Stone's theorem (and other theorems) in the theory of operators on a Hilbert module.

This is a continuation of the study of generalized Hilbert spaces [Abstract 644-59, these *Notices* 14 (1967), 376]. It is shown that the Closed Graph Theorem and the Spectral Theorem for unbounded selfadjoint operators are valid also for the case of a Hilbert module. Also there is a generalization of Stone's Theorem in the theory of generalized Hilbert spaces: Each representation $s \longrightarrow T_s$ of an Abelian group G by $\mathfrak{A}$-linear unitary operators Ts on a Hilbert $\mathfrak{A}$-module H is of the form $T_s = \int(\overline{s,a})dP_a$, where $E \longrightarrow P_E$ is a measure on G whose range consists of $\mathfrak{A}$-linear projections ($\mathfrak{A}$ denotes a proper $H^*$-algebra; an additive operator P on H is $\mathfrak{A}$-linear if $P(fa) = (Pf)a$ for all $f \in H$, $a \in \mathfrak{A}$). (A statement of Stone's theorem can be found on p. 147 of L. H. Loomis, Abstract harmonic analysis, 1953.) (Received January 19, 1968.)

655-28. R. H. PETERSON and W. C. TAYLOR, USA Ballistic Research Laboratories, Aberdeen Proving Ground, Maryland 21005. Spherically symmetric measures in infinite dimensional spaces.

In an infinite dimensional space whose finite dimensional subspaces are euclidean, let a probability measure be called spherically symmetric about the origin if its projections in every finite dimensional subspace has spherical symmetry in the common origin. Theorem. In such an infinite dimensional space a measure is spherically symmetric in the origin if and only if it is contained in the convex hull of the gaussian measures symmetric in the origin. The result does not hold for finite dimensional spaces. (Received November 30, 1967.)

655-29. F. X. CONNOLLY, Columbia University, New York, New York 10027. From immersions to embeddings of smooth manifolds.

When is an immersion $f : V^n \longrightarrow M^m$ of closed smooth manifolds homotopic to an embedding? We study the self intersection manifold of the immersion f, and show that, in favorable circumstances it can be altered by surgery, simply by changing f by a regular homotopy. Then, for any generalized homology theory $\widetilde{H}$, for which V and M are orientable we define a class $\widetilde{\gamma}(f)$ in $\widetilde{H}_r(V)$, $r = 2n - m$. We then show: Theorem A. If V is $[(r + 1)/2]$-connected, M $r + 1$-connected, $\nu(f)$ has a cross section and $m \geqq 3(n + 1)/2$, then f is homotopic to an embedding provided $\widetilde{\gamma}(f) = 0$ in ${}^k\widetilde{\Omega}_r(V)$. If $\widetilde{\gamma}(f) \neq 0$, f is not homotopic to an embedding with cross section. Theorem B. $\widetilde{\gamma}(f)$ is computable simply in terms of the fundamental classes of V and M and the induced homology map of f. Relative and unstable versions of Theorem A are also proved together with similar embedding theorems. ${}^k\Omega$ above is a fixed homology theory. (Received November 20, 1967.)

655-30. J. M. ZELMANOWITZ, University of California, Santa Barbara, California 93106. Commutative endomorphism rings.

While considering the problem of classifying torsion-free abelian groups with commutative endomorphism rings, the following result was obtained. Theorem 1. The only torsionless (i.e., embeddable in some strong direct sum of copies of infinite cyclic groups) abelian group with commutative endomorphism ring is the infinite cyclic group. The following ring-theoretic generalizations hold. Theorem 2. Let R be a ring with left quotient ring $Q = (D_1)_{n_1} \oplus \dots \oplus (D_i)_{n_t}$ where $(D_i)_{n_i}$ is the ring of $n_i \times n_i$ matrices over a division ring $D_i$. Let M be a torsionless left R-module, and set $E(M) = \mathrm{Hom}_R(M,M)$. Then the following conditions are equivalent: (1) E(M) is commutative. (2) Each $D_i$ is a field, and for every pair of submodules A and B of M with $A \cap B = 0$ one has $\mathrm{Hom}_R(A,B) = 0$. (3) Each $D_i$ is a field and M contains an essential submodule $N_1 \oplus \dots \oplus N_s$ where $s \leqq t$, each $N_i$ is uniform, and $\mathrm{Hom}_R(N_i,N_j) = 0$ whenever $i \neq j$. Theorem 3. When $t = 1$, the following conditions are equivalent: (1) E(M) is commutative. (2) $D_1$ is a field and E(M) has no zero divisors. (3) $D_1$ is a field and M is uniform. Corollary. Over a Dedekind domain R, the only torsionless modules with commutative endomorphism ring are the ideals of R. (Received January 22, 1968.)

655-31. ADOLPH SELZER, 36 Hawthorne Drive North, New London, Connecticut 06320. Geometry of linear four-dimensional spaces.

In the special theory of relativity, the line element of a moving body has the form $ds^2 = c^2dt^2 - (dx^2 + dy^2 + dz^2)$, where the length ds is assumed to be invariant but not independent of the moving particle. In order to account for the line element of light, the concept of the null line was introduced. This is a strange assumption which can be avoided if one considers ds as an independent parameter. In the case of light, however, this leads to infinite increments of the coordinates, a fact which does not agree with the measurement of the speed of light. The difficulty does not appear if one multiplies the above formula by the factor, $(1 - v^2/c^2)$, where v is the speed of the particle, and c the speed of light. Rearranging terms, the new line element formula is $c^2 dT^2 = (dX^2 + dY^2 + dZ^2) + dS^2$, where T is the absolute world time (no coordinate) and S the eigentime (no invariant but depending on the speed). This change agrees with all physical observations confirmed by the theory of relativity and, at the same time, avoids all difficulties, e.g., the clock paradox. Reference: Revised mechanics, by Adolph Selzer, 2nd ed., 1967. (Received January 15, 1968.)

655-32. F. P. CALLAHAN, 632 Deaver Drive, Blue Bell, Pennsylvania. Global description of some deformations of rings.

For background refer to Abstract 649-25, these *Notices* 14 (1967), 821, Deformation of a ring having two generators. An outline of the proofs of the following two theorems will be given: Theorem 1. Let k be a field, $k_t$ and R (commutative) power series rings in transcendentals t and x,y, resp., with coefficients in k. Let S be the free (noncommutative) ring in X and Y with coefficients in $k_t$ and let $I_W$ be the two-sided ideal in S generated by $XY - YX - tW$, W being in S but not involving t. Then any deformation of R is isomorphic to $S/I_W$ for a suitably chosen W. Theorem 2. Let k be a field, char $k = p \neq 0$, $k_t$ as in Theorem 1. Let $(u_i)^p$ be in k, and $k(u_i)$ be the extension of k for this $u_i$, $i = 1,\dots,N$. Let $K = k(u_1) \otimes \dots \otimes k(u_N)$. Let S be the (commutative) power series ring in transcendentals $U_1,\dots,U_N$ with coefficients in $k_t$, and let $P_1,\dots,P_N$ be polynomials in $U_1,\dots,U_N$ with coefficients in k. Let $I_P$ be the ideal in S generated by $(U_i)^p - P_i(U_1,\dots,U_N)$, $i = 1,\dots,N$. (Here P stands for the collection of polynomials $P_1,\dots,P_N$.) Then any commutative deformation of K is isomorphic to $S/I_P$ for a suitably chosen P. (Received November 29, 1967.)

655-33. IVAN ERDELYI, Kansas State University, Manhattan, Kansas 66502. Normal partial isometries closed under multiplication on unitary spaces.

The extension of isometry to the noninvertible case entails the loss of the closure under multiplication. The present paper is concerned with classes of normal partial isometries closed under multiplication on finite-dimensional unitary spaces. The following main results are obtained: (1) The product $P = AB$ of two normal partial isometries A and B, both of rank r, is a normal partial isometry of rank r if and only if they have equal ranges, $R(A) = R(B)$. (2) Let $A_1, A_2,\dots,A_m$ be m normal partial isometries of rank r. Any partial product $P_{ij} = A_i A_{i+1} \dots A_j$ $(1 \leqq i < j \leqq m)$ is a normal partial isometry of rank r if and only if each factor has the same range: $R(A_1) = R(A_2) = \dots = R(A_m)$. (Received February 1, 1968.)

655-34. J. R. DURBIN, University of Texas, Austin, Texas 78712. On solvable and supersolvable groups. Preliminary report.

In an earlier paper (Abstract 648-59, these *Notices* 14 (1967), 646), we characterized finite nilpotent groups as those finite groups in which each element is residually central. In the present paper we give analogous results for solvability and supersolvability. Theorem. A finite group G is solvable (respectively supersolvable) iff for each nonidentity element $x \in G$ there exists a pair H,K of subgroups of G such that $K \lhd H \lhd G$ (respectively $K \lhd H \lhd G$ and $K \lhd G$), $x \in H \setminus K$, and H/K is solvable (respectively cyclic). These results yield very broad, and apparently new, classes of generalized solvable and supersolvable groups. (Received February 5, 1968.)

655-35. M. O. GONZALEZ, University of Alabama, University, Alabama 35486. *Some theorems concerning the directional derivative of a function of a complex variable.*

Let $f(z) = u(x,y) + iv(x,y)$ be a complex function having differentiable components u and v in some open set A of the complex plane, and let $f'_0(z) = f_z + f_{\bar{z}} e^{-2i\theta}$ be the directional derivative of f at z in the direction of the regular arc $C : z = z(t)$, $\alpha \leqq t \leqq \beta$, contained in A and passing through z. The following theorems are proved: (1) If a function f(z) has a vanishing directional derivative along a regular arc C, then the function is a constant along C; (2) Let g(z) be a continuous function of z along C, and let $f(z) = {}^{(C)}\int_{z_0}^{z} g(\zeta)d\zeta$. Then f(z) has a directional derivative at each point of C, and $f'_\theta(z) = g(z)$; (3) If g(z) is continuous along C, and if ${}^{(C)}\int_{z_0}^{z} g(\zeta)d\zeta = 0$ for every $z \in C$, then $g(z) = 0$ along C; (4) If $\phi(z)$ is such that $\phi'_\theta(z) = g(z)$, $z \in C$, then ${}^{(C)}\int_{z_0}^{z_1} g(\zeta)d\zeta = \phi(z_1) - \phi(z_0)$, the function $\phi$ depending in general on C. (Received February 2, 1968.)

655-36. DONALD ORTH, Princeton University, Princeton, New Jersey. *Welding Riemann surfaces and transmission problems with shifts.*

Let X be an H-manifold, i.e. a not necessarily orientable Riemann surface, and S a closed subse of X which is locally a star of $C^{1+a}$ curves ([R] H. Röhrl, Über das Riemann-Privalovsche Randwertproblem, Math. Ann. 151 (1963), 365-423). Form the bordered H-manifold $\bar{\bar{X}}$ ([R]) with projection $pr : \bar{\bar{X}} \rightarrow X$. Let $\{\sigma_\lambda^1 : \lambda \in \Lambda\}$ be the 1-simplices of a simplicial structure on $\bar{\bar{X}}_\beta = pr^{-1}(S)$ for which $pr(\sigma_\lambda^1)$ is a $C^{1+a}$ curve for every $\lambda \in \Lambda$. Let $\phi : \Lambda \rightarrow \Lambda$ be a bijective map for which $\phi(\lambda) \neq \lambda$ and $\phi \circ \phi(\lambda) = \lambda$, and $\alpha_\lambda : \sigma_\lambda^1 \rightarrow \sigma_{\phi(\lambda)}^1$ a $C^{1+a}$ homeomorphism such that $\alpha_{\phi(\lambda)} \circ \alpha_\lambda =$ identity, again for all $\lambda \in \Lambda$. Form the quotient space $X^\alpha$ by identifying two points $\bar{\bar{x}}, \bar{y} \in \bar{\bar{X}}_\beta$ whenever there are finitely many maps $\alpha_{\lambda_1}, \ldots, \alpha_{\lambda_n}$ for which $\alpha_{\lambda_n} \circ \ldots \circ \alpha_{\lambda_1}(\bar{\bar{x}}) = y$. Let $\chi : \bar{\bar{X}} \rightarrow X^\alpha$ be the quotient map. Assume that $\chi^{-1}(x)$ is a finite set for every $x \in X^\alpha$. $X^\alpha$ is then a $C^\infty$ manifold. A condition on the maps $\{\alpha_\lambda : \lambda \in \Lambda\}$ is given and shown to be both necessary and sufficient in order that $X^\alpha$ can be given a unique H-structure for which $\chi \circ pr^{-1} : X \backslash S \rightarrow X^\alpha \backslash \chi(\bar{\bar{X}}_\beta)$ is an H-homeomorphism and $\chi(\bar{\bar{X}}_\beta)$ is locally a star of $C^{1+a'}$ curves on $X^\alpha$ for some $a' \leqq a$. This result and those in [R] are then used to describe the solution spaces of a large class of transmission problems with shifts generalizing those in the sense of Haseman and Carleman. The welding procedure and transmission problems with shifts on holomorphic families of Riemann surfaces are also discussed. (Received February 5, 1968.)

655-37. JOSEPH HOROWITZ, University of Toledo, Toledo, Ohio. *The Hausdorff dimension of the sample path of a subordinator.*

Let T(s), $s \geqq 0$, be a subordinator, i.e. a random process with stationary independent increment and a.s. increasing sample paths defined on a probability space $(\Omega, F, P)$. Suppose $T(0) = 0$ a.s. Let $g(\lambda) = \alpha\lambda + \int_0^\infty (1 - e^{-\lambda y}) n(dy)$ be the subordinator exponent, where $\alpha \geqq 0$ is the drift, n is the Lévy measure of T, and $g(\lambda)$ satisfies $E(e^{-\lambda T(s)}) = e^{-sg(\lambda)}$. Define $H(y) = n(y, \infty)$ for $y > 0$. Blumenthal and Getoor (J. Math. Mech. 10 (1961), 493-516) have shown that if $\alpha = 0$, then $\sigma \leqq \dim Q \leqq \beta$ a.s., where $\sigma = \sup\{0 \leqq \gamma \leqq 1 : \lambda^{-\gamma} g(\lambda) \rightarrow \infty$ as $\lambda \rightarrow \infty\}$ and $\beta = \inf\{0 \leqq \gamma \leqq 1 : \lambda^{-\gamma} g(\lambda) \rightarrow 0$ as $\lambda \rightarrow \infty\}$, $Q = Q(\omega)$ is the range of $T(s, \omega)$, and dim Q denotes the Hausdorff dimension of Q. Using the theory of semilinear Markov processes, developed in the author's doctoral dissertation (University of Michigan, 1967), the following result is established: if $\alpha = 0$, then $\dim Q \leqq \sigma$ a.s. Conclusion: $\dim Q = \sigma$ a.s. (Received February 5, 1968.)

655-38. K. B. MARATHE, 316 Meigs Street, Rochester, New York 14607. Petrov classification of orthogonal metric spaces.

The Petrov classification has been used, following the matrix notation, to examine the general metric of the orthogonal type, referred to as Dingle's metric here. The algebraic conditions that have to be satisfied in transition from local anisotropy to local isotropy through the intermediate stages of the Penrose diagram have been obtained. The algebraic conditions are really differential equations the solution of which present a series of new problems. The equivalence of neighbourhoods as considered here can be of considerable significance in the exploration of new gravitational fields. The conditions (22) and (23) do not seem to have been studied before. They have been considered with reference to some special metrics of interest. (Received November 9, 1967.)

655-39. WITHDRAWN.

655-40. D. P. K. BIALLAS, University of Florida, Gainesville, Florida 32601. Extension of order-functions in projective spaces. Preliminary report.

Let P be a desarguesian projective space and K the coordinate-field of P. Every order in P defined for quadrupels of colinear points and inducing a subgroup of index 2 in the multiplicative group of K can be extended to an order on a certain class of quadrupels of subspaces of P. This "generalized" order has essentially the same properties as the original one. Proofs are using the concept of a generalized cross-ratio and determinants in skew-fields. References. E. Sperner, Beziehungen zwischen geometrischer und algebraischer anordnung, S.-B. Heidelberger Akad. Wiss. Math.-Nat. Kl. 1949, no. 10, (1949), 413-448. D. Biallas, Verallgemeinerte Doppelverhältnisse und Endomorphismen von Vektorräumen, Abh. Math. Sem. Univ. Hamburg 29 (1965). (Received February 8, 1968.)

655-41. G. J. ETGEN, University of Houston, Houston, Texas 77004. On the disconjugacy of second order, selfadjoint matrix differential equations.

Consider the second order, selfadjoint matrix differential equation (1) $(P(x)Y')' + Q(x)Y = 0$, on $X : c \leqq x \leqq d$, where each of $P(x)$ and $Q(x)$ is an $n \times n$ symmetric matrix of continuous, real-valued functions on X and $P(x)$ is positive definite. A solution pair $\{Y(x), P(x)Y'(x)\} \equiv \{Y(x), Z(x)\}$ of (1) is conjoined and nontrivial provided $Y^*Z - Z^*Y \equiv 0$ (* denotes transpose) on X and det $Y(x)$ has at most a finite number of zeros on this interval. Equation (1) is said to be disconjugate on X if and only if no nontrivial, conjoined solution pair $\{Y(x), Z(x)\}$ has the property that det $Y(x)$ vanishes more than n times on X, multiple zeros of det $Y(x)$ being counted according to their multiplicity. In this paper, sufficient conditions that (1) be disconjugate on X are obtained and, in addition, the methods employed are extended to provide estimates of the oscillation of solutions of (1). These results are also interpreted in terms of the conjugate points of c relative to certain two point boundary problems associated with (1) where the disconjugacy of (1) implies that the interval $c < x \leqq d$ contains no points conjugate to c. (Received February 8, 1968.)

655-42. WILLIAM HEINZER, Louisiana State University, Baton Rouge, Louisiana 70803. On Krull overrings of an affine ring.

By an overring of an integral domain A we mean a ring containing A and contained in the quotient field of A. Theorem. Let A be a normal affine ring of absolute dimension two defined over a field or pseudogeometric Dedekind domain. If A has torsion class group and D is a Krull overring of A, then D is Noetherian. Corollary. If A is a polynomial ring in two variables over a field or in one variable over the ring of integers, then each Krull overring of A is Noetherian. (Received February 9, 1968.)

655-43. JOSEPH KIST, New Mexico State University, Las Cruces, New Mexico 88001. Compact spaces of minimal prime ideals.

Let R be a commutative ring with identity, and let X be the space of minimal prime ideals of R. It is shown that if X is compact, and if R has no nonzero nilpotents, then there is a sheaf of integral domains with base space the Boolean space X such that the ring $\Gamma$ of continuous sections over X is generated by the image of R in $\Gamma$ and the characteristic functions of open-closed subsets of X. The ring $\Gamma$ has the property that the annihilator of each finite set is a direct summand, and is, in a precise way, the smallest ring with this property containing a copy of R. If R has the property that the annihilator of each finite subset is a direct summand, then, as is known, it has no nonzero nilpotents, and X is compact. It follows from the above representation that R and $\Gamma$ are isomorphic. (Received February 9, 1968.)

655-44. KARLHORST MEYER, University of Florida, Gainesville, Florida. On quadrics in projective spaces over finite fields.

Let K (a finite field of any characteristic $\neq 0$) have o elements and $V^{n+1}$ be the vector space over K of dimension n + 1. Then the projective, n-dimensional space $V'^n \cong [V^{n+1} - (0,\ldots,0)]/K^*$ contains $N^n = (o^{n+1} - 1)/(o - 1)$ different points $\xi'$. Let $q(\xi) = \sum_{i,k=1}^{n+1} a_{ik}x_i x_k$ be any quadric form in $V^{n+1}$; then Q is defined as the set of singular vectors $\xi$ of $V^{n+1}$ with respect to the quadratic form q ($q(\xi) = 0$). We call $Q' = \{\xi' : q(\xi) = 0\}$ a quadric. (a) Let Q' be a quadric containing no straight line but at least 2 points. Then it follows: for $n \geqq 2$, Q' has exactly $Q^n = N^{n-1} + 1 - N^{n-2}$, $Q^1 = 2$ different points. (b) There is a method for calculating the number P of points of any quadric if one knows the dimension s of the vertex of the quadric and the maximal dimension r of a linear subspace belonging to Q'. Let L'(Q') be the projective space spanned by Q'. Then it follows that $Q' = L'(Q')$, if $L'(Q') \neq V'^n$; $P = Q^{n-r} \cdot (N^r - N^s) + N^s$, if $L'(Q') = V'^n$. (Received February 8, 1968.)

655-45. M. ROSENFELD, University of Washington, Seattle, Washington 98105. On the total chromatic number of certain graphs.

A total coloring of a graph is a coloring of its edges and vertices in such a way that no two adjacent elements have the same color. The total chromatic number $\tau(G)$ of a graph G is the minimal number of colors in a total coloring of G. It was conjectured by M. Behzad that for any graph G, $\tau(G) \leqq v(G) + 2$, where v(G) is the maximal valence of the vertices in G. This conjecture is verified for some families of graphs. Theorem 1. If G is a bipartite graph, then $\tau(G) \leqq v(G) + 2$. Theorem 2. If G is a complete 3-partite graph, then $\tau(G) \leqq v(G) + 2$. Theorem 3. G is a complete balanced k-partite graph, then $\tau(G) \leqq v(G) + 2$. (Received February 9, 1968.)

655-46. A. J. SILBERGER, Bowdoin College, Brunswick, Maine 04011. Representations of the general linear group over a finite field. Preliminary report.

J. A. Green (Trans. Amer. Math. Soc. 80 (1955), 402-447) has shown how to compute the characters of the finite general linear groups. Let $G_n = GL(n, GF(p^m))$. This note will indicate how, knowing the traces, one may compute the matrix coefficients of the irreducible representations of $G_n$. Assume a knowledge of the matrix coefficients for the representations of $G_k$, $k < n$. $G_n$ contains subgroups isomorphic to $G_{k_1} \times \ldots \times G_{k_r}$ for any $k_1 + \ldots + k_r = n$ (diagonal blocks). Let $G_{k_1} \times \ldots \times G_{k_r}{}^*$ denote those matrices with anything to the right of the "blocks". Call "degenerate" any representation of $G_n$ whose restriction to $G_{k_1} \times \ldots \times G_{k_r}{}^*$ contains the lift of a representation of $G_{k_1} \times \ldots \times G_{k_r}$ for some $k_1 + \ldots + k_r = n$. Our problem is easy to solve for degenerate representations. Theorem. Let U be a nondegenerate representation of $G_{n-1}$, let $\alpha$ be a representation of $G_1$, and let $U^{**}$ be any nondegenerate representation of $G_n$. Form $U \otimes \alpha$ and consider the induced representation of $G_{n-1} \times G_1{}^*$. It decomposes into $U_1 + U_2$, where, if $U_1$ denotes the lift of $U \otimes \alpha$ to $G_{n-1} \times G_1{}^*$, then $U_2$ is the restriction to $G_{n-1} \times G_1{}^*$ of $U^{**}$. Knowing the trace of $U^{**}$, one easily computes its matrix coefficients from those of $U_2$. (Received February 9, 1968.)

655-47. MANUEL LERMAN, Cornell University, Ithaca, New York 14850. Recursive functions modulo co-maximal sets. Preliminary report.

Let $M_1, M_2, \ldots$ be maximal r.e. sets, $\overline{M}_1, \overline{M}_2, \ldots$ their complements. Let $\mathscr{R}/\overline{M}_i$, $i = 1,2,\ldots$, denote the model obtained from the recursive functions of one variable under the equivalence relation $f \backsim g$ iff $f(x) = g(x)$ for all but at most finitely many $x \in \overline{M}_i$. In Abstract 556-31 (these *Notices* 6 (1959), 173), Feferman, Scott and Tenenbaum announced that no such model is elementarily equivalent to the elementary theory of the integers. Theorem 1. If there exist recursive functions f and g such that $f(\overline{M}_1) \cap \overline{M}_2$ and $\overline{M}_1 \cap g(\overline{M}_2)$ are both infinite, then $fg|_{\overline{M}_2} = I|_{\overline{M}_2}$ and $gf|_{\overline{M}_1} = I|_{\overline{M}_1}$ where I is the identity function. Furthermore, such functions f and g exist iff $\overline{M}_1$ and $\overline{M}_2$ have the same many-one degree. Theorem 2. $\mathscr{R}/\overline{M}_1 \equiv \mathscr{R}/\overline{M}_2$ iff $\overline{M}_1$ and $\overline{M}_2$ have the same many-one degree. Theorem 3. There is a Turing degree $\underline{a}$ of a maximal set, and a class of $\aleph_0$ maximal sets $\overline{M}_1, \overline{M}_2, \ldots$ of degree $\underline{a}$, but of pairwise incomparable many-one degrees. Furthermore, for any n, there exist maximal sets $M_1, \ldots, M_n$ of the same WTT-degree (weak truth-table degree), but of pairwise incomparable many-one degrees. (Received February 8, 1968.)

655-48. HELMUT WEGMANN, Duke University, Durham, North Carolina 27706. Hausdorff dimension in sequence spaces.

Let (X,d) be a metric space and Y the space of sequences in X. Generalizing methods of Cigler and Volkmann (Abh. Math. Sem. Hamburg 26 (1963)), the Hausdorff dimension for subsets of Y is defined and estimates for the dimension of various subsets are given, such as Cartesian product sets $\prod_{i=1}^{\infty} A_i, A_i \subset X$, the set of convergent sequences, and the set of sequences which have all limit points in a given subset of X. (Received February 8, 1968.)

655-49. M. S. K. SASTRY, Ohio University, Athens, Ohio 45701. Spin spherical harmonics. Preliminary report.

Let $H_2$, $\mathscr{E}_3$ and $\nabla$ be as defined in The spin model of Euclidean 3-space, W. F. Eberlein, Amer. Math. Monthly 69 (1962), 587-598. We identify $H_2$ with two rowed complex column matrices. Definition. Let $\psi: \mathscr{E}_3 \longrightarrow H_2$ with $\psi(x) = \binom{U(x)}{V(x)}$, where $U, V: \mathscr{E}_3 \longrightarrow \mathbf{C}$ and $x = (x_1, x_2, x_3) \in \mathscr{E}_3$. Then $\psi$ is a spin spherical harmonic (SSH) of degree $l$ ($l$ a nonnegative integer) if (i) U and V are homogeneous polynomials of degree $l$ in $x_1, x_2, x_3$ with complex coefficients, and (ii) $(\nabla\psi)(x) = 0$ for all $x \in \mathscr{E}_3$. Theorem. The SSH's of degree $l$ form a complex vector space of dimension $(2l + 2)$. We denote this vector space by $W_l$. Let SU(2) denote the special unitary group of $H_2$ and let $\mathscr{F} = \{\psi : \psi: \mathscr{E}_3 \longrightarrow H_2\}$. For $u \in SU(2)$ we define $\hat{u} \in \mathrm{End}(\mathscr{F})$ by $(\hat{u}\psi)(x) = u\psi(u^{-1}xu)$ $(x \in \mathscr{E}_3)$. Theorem. Let $\sigma: SU(2) \longrightarrow \mathrm{End}(W_l)$ with $\sigma u = \hat{u}$ $(u \in SU(2))$. Then $\sigma$ is an irreducible representation of SU(2). Corollary. By using SSH's we obtain all the even-dimensional irreducible representations of SU(2) and hence all the irreducible double-valued representations of the rotation group of $\mathscr{E}_3$. (Received February 7, 1968.)

655-50. VICTOR LOVASS-NAGY, Clarkson College of Technology, Potsdam, New York 13676. On matrix approximations to the differential equations of electric networks with distributed elements.

Approximating the derivatives with respect to the spatial variables by difference quotients and leaving the time variable continuous, the governing partial differential equations of linear electric networks with distributed elements can be approximated by ordinary linear matrix differential equations. The solution of these equations can be greatly facilitated by partitioning the coefficient matrices into submatrices. Solution formulae of matrix differential equations describing transient behaviors of one-phase and multiphase nonhomogeneous transmission lines and networks with non-lumped elements are derived in terms of eigenvalues of submatrices. (Received February 16, 1968.)

655-51. E. P. SHAUGHNESSY, Lafayette College, Easton, Pennsylvania 18042. Codes with simple automorphism groups. Preliminary report.

For $l$ an odd prime, we consider the $(l + 1, (l + 1)/2)$ extended quadratic-residue code over GF(q) (as described by E. F. Assmus, Jr., and H. F. Mattson, Jr., in New 5-designs, which will appear in J. Comb. Theory). In the cases where $l = 11$, $q = 3$ and $l = 23$, $q = 2$, the automorphism groups of the codes are the Mathieu groups $M_{12}$ and $M_{23}$ respectively (Assmus and Mattson, Perfect codes and the Mathieu groups, Arch. Math. 17 (1966)). In this paper it is shown that when $(l - 1)/2$ is an odd prime, then the automorphism group of the code is simple. (Received February 12, 1968.)

655-52. GEZA SCHAY, JR., University of Massachusetts, Boston, Massachusetts 02116. An axiomatic characterization of an algebra of conditional events.

The author has defined an algebraic structure on pairs of sets that formalizes the intuitive notions of conditional events A|B as occur in conditional probabilities. (For a brief partial description see Abstract 649-16, these Notices 14 (1967), 818.) This algebra has five basic operations; of these, two pairs define two distributive lattice structures. These are strangely interlaced, for instance the fifth operation--complementation--changes each of the two partial orders into the other one. Analogously to Stone's representation theorem for Boolean algebras, a theorem is presented which states that any algebra satisfying a certain set of axioms is isomorphic to an algebra of the above type on pairs of sets. (Received February 12, 1968.)

655-53. MARTIN AIGNER, Wayne State University, Detroit, Michigan 48202. *A characterization of a class of regular graphs.*

Let us define a cubic lattice graph as a graph G, whose vertices are identified with the $n^3$ ordered triplets on n symbols, such that two vertices are adjacent if and only if the corresponding triplets have two coordinates in common. Let $d(x,y)$ denote the distance between two vertices x and y, and $\Delta(x,y)$ the number of vertices adjacent to both x and y, then a cubic lattice graph G is readily seen to have the following properties: (P1) The number of vertices is $n^3$. (P2) G is connected and regular of degree $3(n - 1)$. (P3) If $d(x,y) = 1$, then $\Delta(x,y) = n - 2$. (P4) If $d(x,y) = 2$, then $\Delta(x,y) = 2$. (P5) If $d(x,y) = 2$, then there exist exactly $n - 1$ vertices z, such that $d(x,z) = 1$ and $d(y,z) = 3$. R. Laskar succeeded in proving that for $n > 7$ the conditions (P1) - (P5) characterize cubic lattice graphs. In the present paper, we supplement her work, using a different method (which applies to all n), by showing that except for $n = 4$, in which case exactly one exceptional graph exists, any graph G possessing the properties (P1) - (P5) must be a cubic lattice graph. (Received February 12, 1968.)

655-54. KAZUMI NAKANO, Loyola University, New Orleans, Louisiana 70118. *Uniformly continuous mappings defined by function systems.*

Let $(\underset{\sim}{R}, \mathfrak{A})$ and $(\underset{\sim}{S}, \mathfrak{L})$ be spaces with unities $\mathfrak{A}$ and $\mathfrak{L}$ respectively. A unity is a collection of sets of bounded real-valued functions on a given space. An order between two unities is defined. (Abstract 632-37, these *Notices* 13 (1966), 336). *Definition.* A mapping $M:\underset{\sim}{S} \rightarrow \underset{\sim}{R}$ is called uniformly continuous if $\mathfrak{A}M < \mathfrak{L}$ where $\mathfrak{A}M = \bigcup\{\bigcup_{\phi \in \Phi} \phi M; \Phi \in \mathfrak{A}\}$. The following theorems are immediately derived from the definition. *Theorem* 1. If M is a mapping from $\underset{\sim}{S}$ to $(\underset{\sim}{R}, \mathfrak{A})$, then $\mathfrak{A}M$ is the weakest function system on $\underset{\sim}{S}$ such that M is uniformly continuous. There exists a function system $\bigvee_{\lambda \in \Lambda} \mathfrak{A}_\lambda$ such that (1) $\mathfrak{A}_\lambda < \bigvee_{\lambda \in \Lambda} \mathfrak{A}_\lambda$ for all $\lambda \in \Lambda$ and (2) $\mathfrak{A}_\lambda < \mathfrak{L}$ for all $\lambda \in \Lambda$ implies that $\bigvee_{\lambda \in \Lambda} \mathfrak{A}_\lambda < \mathfrak{L}$. *Lemma.* $\bigvee_{\lambda \in \Lambda} \mathfrak{A}_\lambda = \{\Phi; \Phi < \mathfrak{A}_\lambda \text{ for some } \lambda \in \Lambda\}$ and $\bigvee_{\lambda \in \Lambda} \mathfrak{A}_\lambda \sim \bigcup_{\lambda \in \Lambda} \mathfrak{A}_\lambda$. *Theorem* 2. If M is a mapping from $(\underset{\sim}{S}, \mathfrak{L})$ to $\underset{\sim}{R}$, then $\bigcup\{\mathfrak{A}; \mathfrak{A}M < \mathfrak{L}\}$ is the strongest function system on $\underset{\sim}{R}$ such that M is uniformly continuous. (Received February 13, 1968.)

655-55. C. W. LEININGER, University of Dallas, Dallas, Texas 75061. *Convergence-preservation criteria for a generalized Hausdorff mean.*

Let $H^{(s)}(d)$ denote the generalized Hausdorff mean defined in Abstract 67T-99 (these *Notices* 14 (1967), 158), and assume that $s_n \leqq n$, $n = 1,2,3,\ldots$ . Let $R^*$ denote the space of sequences d such that $d_n = \int_{[0,1]} I^n dg$, $n = 0,1,2,\ldots$, and on $[0,1]$ g is Riemann-integrable but not of bounded variation. *Theorem* 1. If $d \in R^*$ and $H^{(s)}(d)$ is conservative, then it is multiplicative. *Theorem* 2. If $d \in R^*$, then $H^{(s)}(d)$ is regular over the space $c_0$ if and only if $\|H^{(s)}\|$ exists. *Theorem* 3. If $d \in R^*$, then $H^{(s)}(d)$ is multiplicative if and only if there exist numbers K and L such that (i) $\sum_{p=0}^{n} |H_{np}^{(s)}| < K$, $n = 0,1,2,\ldots$; (ii) $\lim_n \sum_{p=0}^{n} H_{np}^{(s)} = L$. *Theorem* 4. If $d \in R^*$ and there is a number M such that

$\sum_{p=0}^{n} \prod_{k=p+1}^{n} s_k/k < M$, then $H^{(s)}(d)$ is regular over $c_0$, and furthermore if g(1-) exists, then $H^{(s)}(d)$ is multiplicative. (Received February 13, 1968.)

655-56. GEORGE RUBLEIN, College of William and Mary, Williamsburg, Virginia 23185. An extension of DeRham's theorem.

Let M be a paracompact differentiable manifold. Let $\theta$ be a closed p-form on M. Let U be an open set in M, and $\omega$ a (p - 1)-form on U with $d\omega = \theta|U$. If c is a differentiable singular p-chain denote by $|c|$ its support, and $\partial c$ its boundary. Then $\int_c \theta - \int_{\partial c} \omega = 0$ for all differentiable p-chains c with $|\partial c| \subset U$ if and only if: given V open, $\bar{V} \subset U$, there exists a (p - 1)-form $\omega_1$ on M with $d\omega_1 = \theta_1$ and $\omega_1|\bar{V} = \omega|\bar{V}$. Thus $\int_c \theta - \int_{\partial c} \omega$ serves as an obstruction to "extending" $\omega$. These pairs $(\theta, \omega)$ with 0 obstruction are treated as trivial in Allendoerfer-Eells [Comment. Math. Helv. 32 (1958), 165-179]. (Received February 14, 1968.)

655-57. M. J. SHERMAN, University of California, Los Angeles, California 90024. Invariant subspaces containing all analytic directions.

Let $H^2_{\mathscr{A}}$ denote the Hilbert space of functions $F(e^{i\theta})$ defined on the unit circle with values in the separable Hilbert space $\mathscr{A}$, and which are weakly in the Hardy class $H^2$. A closed subspace $\mathscr{M} \subset H^2_{\mathscr{A}}$ is invariant if it is invariant under the right shift operator. $\mathscr{M}$ is said to contain all directions if for every $F \in H^2_{\mathscr{A}}$ there is a scalar function f such that $f F \in \mathscr{M}$. It is shown that if either (i) $q_e e \in \mathscr{M}$ for all $e \in \mathscr{A}$, where $q_e$ is a finite Blaschke product, or (ii) $\mathscr{M}$ arises from a bounded operator on $\mathscr{A}$ using constructions due to Rota or Potopov and $\mathscr{M}$ contains all directions, then $\mathscr{M} \supset q H^2_{\mathscr{A}}$ for some fixed finite Blaschke product q. The general question as to whether invariant subspaces containing all directions must contain $q H^2_{\mathscr{A}}$ for some scalar inner function q remains open. (Received February 15, 1968.)

655-58. G. T. Whyburn, Louisiana State University, Baton Rouge, Louisiana 70803. A note on character sums.

In Abstract 644-38, these *Notices* 14 (1967), 369, an estimate for the density of power residues (mod p) in $[1,q] = Q$ with $q < p$ was obtained by elementary methods. Unknown to the author, A. Schinzel and M. Bhaskaran had previously used a portion of these methods to obtain an estimate for $|\sum_{a<k\leq b} \chi(k)|$ where $0 \leq a < b$, $h^2 - a^2 \leq m$ and $\chi$ is a nonprincipal real character (mod m). In this note, using the methods of Abstract 644-38, a similar bound to that of Schinzel and Bhaskaran is obtained without the restriction that $\chi$ be real. (Received February 15, 1968.)

655-59. G. N. PANDYA, University of Manitoba, Winnipeg 19, Manitoba, Canada. On automorphisms of finite simple groups.

Let G denote a finite Chevalley group or a group of "Twisted type" of R. Steinberg. If Inn (G) and Au (G) denote respectively the inner automorphism and the automorphism group of G, we wish to find a complement C for Inn (G) in Au (G). A sufficient condition is developed for the existence of this C and it has been verified for the following cases. Let K denote the basefield for the corresponding Lie algebra and let $|K| = p^n$, p a prime. Then the condition holds for: (1) The groups of the type $B_l$, $C_l$, $E_7$ and $D_l$ ($l$ even) provided $p^n \equiv 3 \pmod 4$. (2) The groups of the type $E_6$ provided $p^n \equiv 4$ or $7 \pmod 9$. (3) The groups of the type $A_l$ provided $(p^n - 1) = d\,u$ with g.c.d $(d,u) = 1$, where $d =$ g.c.d. $(p^n - 1, l + 1) =$ odd number. (4) The Steinberg groups $E_6^1$ provided $p^n \not\equiv 2 \pmod 3$ or $(p^n)^2 \not\equiv 8 \pmod 9$. (5) The Steinberg groups $A_i^1$ provided g.c.d. $(l + 1, p^n + 1) = d =$ an odd number and $(p^n + 1) = d\,u$ with g.c.d. $(d,u) = 1$. (Received February 15, 1968.)

655-60. E. B. LEACH, Case Western Reserve University, Cleveland, Ohio 44106. Differential calculus of subconvex functionals.

A differential calculus is developed for subconvex (= convex) functionals on a normed vector space X. Among other properties is the property that if such a functional f has a second degree polynomial approximation near any point, then the differential of f has a Fréchet differential at that point. This is used in the proof of a generalization of the Sundaresan-Bonic-Reis theorem that if X and X* have $C^2$ norms, then X is equivalent to an inner product space. The hypothesis of smoothness is reduced simply to the hypothesis that the norms in X and X* have second degree polynomial approximations about one corresponding pair of points in X and X*. The calculus is also applied in proving Whitfield's results on nonexistence (in certain cases) of Fréchet differentiable functions with bounded nonempty support. (Received February 15, 1968.)

655-61. B. W. GLICKFELD, University of Washington, Seattle, Washington 98105. The analytic automorphisms of the Riemann sphere of a Banach algebra.

Let R denote a complex commutative Banach algebra with unit, with Riemann sphere R*. (The definitions used here may be found in Abstracts 609-14 and 64T-94, these *Notices* 11 (1964), 209, 215.) Let $R_p^*$ denote the connected component of R* which contains R; $R_p^*$ is an analytic R-manifold. (It is possible that R* is not connected, e.g. when R is the continuous functions on the three-sphere.) Theorem. Each analytic automorphism of $R_p^*$ is given by a fractional linear transformation. The proof proceeds via Theorem 2 of Abstract 64T-94, and an application of the classical monodromy theorem. (Received February 15, 1968.)

655-62. R. B. KNIGHT, Catholic University of America, Washington, D. C. 20017, and A. K. AZIZ, University of Maryland, College Park, Maryland 20742. On the existence and uniqueness of generalized solutions of quasi-linear equations in two independent variables.

The paper concerns the existence and uniqueness of generalized solutions of the Dirichlet problem connected with a quasi-linear elliptic differential equation of the form $a(x,y,u,u_x,u_y)u_{xx} + 2b(x,y,u,u_x,u_y)u_{xy} + c(x,y,u,u_x,u_y)u_{yy} + d(x,y,u,u_x,u_y) = 0$, $u|_\Gamma = \phi(x,y)$ where $\Gamma$ denotes the boundary of a simply-connected domain G. Solutions are sought in the Sobolev space $W_2^p(G)$, $p \geqq 2$. The coefficients, along with other regularity conditions, are assumed to satisfy a growth condition of the form $|a(x,y,u,p,q)| \leqq M[1 + |u| + |p| + |q|]$. By an appropriate transformation, the problem is reduced to an equivalent problem involving certain functional equations in the complex plane. The main tools used are the theory of generalized functions, the Banach contraction mapping principle, and

Schauder's fixed point theorem. The existence of a solution is proved for $p \geqq 2$, and sufficiently close to 2. With the additional assumption that the coefficients a,b,c,d satisfy a Lipschitz condition, the solution is shown to be unique. (Received November 27, 1967.)

655-63. V. LAKSHMIKANTHAM, S. LEELA and C. TSOKOS, University of Rhode Island, Kingston, Rhode Island. Stability of controlled notion.

Consider the control system (*) $x' = f(t,x,u)$, where u is a control vector. Let $\Omega$ be the admissible control set given by $\Omega = [u : U(t,u) \leqq r(t), t \geqq 0]$, where $U(t,u) \geqq 0$ is continuous on $[0,\infty) \times R^m$ and $r(t)$ is the maximal solution of the scalar differential equation $y' = g(t,y,y)$. Necessary and sufficient conditions are given for the stability of motion described by the control system (*). (Received February 14, 1968.)

655-64. Z. HEDRLIN and ERIC MENDELSOHN, McGill University, Montreal 2, Canada. On category of graphs with a given subgraph.

It is proved that for any graph G without loops, the category of graphs can be fully embedded into its full subcategory formed by those graphs containing G as a full subgraph. Applications. For any pair of groups H, $H_1$, there exists a topological space T with subspace $T_1$ such that the group A(T) of all autohomeomorphisms of T is isomorphic with H, $A(T_1)$ is isomorphic with $H_1$. For every pair of monoids M, $M_1$, there exists a semigroup S with a subsemigroup $S_1$ such that the monoid E(S) of all endomorphisms of S is isomorphic with M, $E(S_1)$ is isomorphic with $M_1$. More general theorems of this type are proved. (Received February 15, 1968.)

655-65. A. R. CAMINA, University of Illinois, Urbana, Illinois 61801. Wielandt length of finite groups. Preliminary report.

For a finite group G a characteristic subgroup W(G) can be defined as the intersection of the normalisers of all subnormal subgroups of G. As G is finite, $W(G) \neq 1$ (H. Wielandt, Math. Z. 69 (1958), 463-465). Thus a series $\{W_n(G)\}$ can be defined by $W_1(G) = W(G)$ and $W_n(G)/W_{n-1}(G) = W(G/W_{n-1}(G))$. This series terminates at G and if this is $W_n(G)$, n is called the Wielandt length of G. Some preliminary results are discussed concerning groups with bounded Wielandt length. One such result is that if G is a soluble group of Wielandt length $\leqq$ n, then G has Fitting length at most n + 1. (Received February 14, 1968.)

655-66. Z. HEDRLÍN, McGill University, Montreal 2, Canada. On a universal category of algebras and its embedding.

A concrete category U is defined such that every category of algebras or any concrete category with small components can be fully embedded in U by a functor preserving underlying sets. Assuming there is no measurable cardinal, it is proved that U can be fully embedded into the category of graphs and, consequently, into the category of semigroups. If every object in a category C is a range of only a set of morphisms in C, then C can be fully embedded in U (proved by A. Pultr). Thus, every such category can be fully embedded into the category of semigroups. (Received February 15, 1968.)

655-67. E. R. FERNHOLZ, University of Washington, Seattle, Washington 98105. Bounds on holomorphic vector fields.

Let X be a reduced analytic space and H(X) be the algebra of global holomorphic functions on X. Let $t_1, \dots, t_n$ be holomorphic vector fields on X. The $t_i$ are operators on H(X). Let $t^k$ represent any k-fold iterate of the $t_i$. Let K be a compact subset of X. Then there exist a compact set $K' \subset X$ and constants $c > 0$ and $m > 0$ such that for all $f \in H(X)$ and for all integers $k > 0$, $\|t^k f\|_K \leqq c^k m(k+1)!\, \|f\|_{K'}$. This shows that the operators $t^k$ are continuous in the Fréchet topology on H(X). (Received February 15, 1968.)

655-68. R. C. GLAESER, Temple University, Philadelphia, Pennsylvania, and B. KOLMAN, Drexel Institute of Technology, Philadelphia, Pennsylvania 19104. Lattice isomorphic solvable Lie algebras.

Theorem. Let L and M be lattice isomorphic Lie algebras with derived algebras L' and M' nilpotent. Then the orders of solvability of L and M differ by at most one. (Received February 15, 1968.)

655-69. P. K. KAMTHAN', University of Waterloo, Waterloo, Ontario, Canada, and P. K. JAIN, Hans Raj College, Delhi 7, India. On the mean values of an entire function represented by a Dirichlet series.

Taking f(s) to be an entire function represented by a Dirichlet series, we define mean values $A_\delta(\sigma)$ and $m_{k,\delta}(\sigma)$ of f(s): $A_\delta(\sigma) = \lim_{T\to\infty} \int_{-T}^{T} |f(\sigma + it)|^\delta dt$; $m_{\delta,k}(\sigma) = (\int_0^\sigma (A_\delta(x))^{1/\delta} dx)/e^{k\sigma}$. The main result established in this paper is that $\log A_\delta(\sigma)$ is a convex function of $\sigma$ for all $\delta$, $0 < \delta < \infty$. Taking derivatives into consideration, the next result of ours is $A(\sigma, f^{(s)}) \leqq M A_\delta(\sigma + \eta)/\eta^{s\delta}$, for $\eta > 0$; M is a constant. With the help of these results, we establish some results expressing order $\rho$ and lower order $\lambda$ in terms $A_\delta(\sigma)$, $A_\delta(\sigma, f^{(s)})$; $M_{\delta,k}(\sigma)$; $M_{\delta,k}(\sigma, f^{(s)})$ and also some results on asymptotic relations between $A_\delta(\sigma)$ and $M_{\delta,k}(\sigma)$. We also establish some results estimating the comparative growth of these functions with respect to a function involving a proximate order $\rho(\sigma)$. One of the results is $\lim_{\sigma\to\infty} {\sup \atop \inf} \{(A_\delta(\sigma))^{1/\delta} / M_{\delta,k}(\sigma)\}^{1/\sigma} = {e\rho \atop e\lambda}$; $0 < \delta < \infty$, $0 < k < \infty$. (Received February 26, 1968.)

655-70. M. J. REIMER, University of Maryland, College Park, Maryland 20742. Finite difference forms containing derivatives of higher order.

The use of finite difference forms for the numerical treatment of initial value problems of ordinary differential equations depends on two concepts, namely stability and degree of approximation. The latter can be improved significantly if derivatives of higher order are used. The stability problem is solved for such generalized finite difference forms. (Received February 16, 1968.)

655-71. J. R. McLAUGHLIN, Pennsylvania State University, University Park, Pennsylvania 16802. Continuous nowhere differentiable functions.

The simplest examples of continuous nowhere differentiable functions are functions represented by the series of sawtooth functions. The present author has noticed that many of the series of sawtooth functions previously considered are simply formally integrated Rademacher series. Also there is a close parallel between lacunary trigonometric series $\sum_{m=1}^{\infty} a_m \int_0^t \sin(b_m x)dx$ and integrated Rademacher type series $\sum_{m=1}^{\infty} a_m \int_0^t r_1(b_m x)dx$, where $\{a_m\}$ is a sequence of real numbers, $\{b_m\}$ is an increasing sequence of positive integers such that $b_m/b_{m-1}$ is even, and $r_1(t)$ denotes the first Rademacher function. The author has studied integrated Rademacher type series and obtained results whose proofs are, in general, simpler that either the corresponding ones for trigonometric series or those previously given for series of sawtooth functions. For example, by utilizing integrated Rademacher type series we are able to: (A) easily generalize all the well-known examples of continuous nowhere differentiable functions which are represented by series of sawtooth functions; (B) give a simple example of an absolutely continuous function possessing a proper minimum on a dense set; (C) give a simple proof that Lip 1 is the weakest condition of its kind that implies differentiability almost everywhere. (Received February 19, 1968.)

655-72. P. N. BAJAJ, Case Western Reserve University, Cleveland, Ohio 44106. Singular points in products of semidynamical systems.

Let $(X_0,\Pi_0)$ be a semidynamical system. Let $x_0 \in X_0$. If there exists $t > 0 \ni 0 < \tau \leqq t \Rightarrow$ Card $\{z : z\tau = x_0\} \geqq 2$, we call $x_0$ singular. Let $(X_\alpha,\Pi_\alpha)$, $\alpha \in \mathscr{A}$, be semidynamical systems, and $(X,\Pi)$ their product defined in the obvious way. Let $x \in X$, $x = \{x_\alpha\}$, be a singular point. x is called improper singular relative to the factorisation $\Pi X_\alpha$ if there does not exist any $\alpha \in \mathscr{A} \ni x_\alpha$ (in $X_\alpha$) is singular; otherwise call it proper. (See Bajaj, Ph.D. Thesis, Case Western Reserve University, p. 91-108, for a similar notion regarding start points.) Theorem. Let $x \in X$, $x = \{x_\alpha\}$. Let $\theta'_\alpha =$ Sup $\{\theta_\alpha \geqq 0 : \{z_\alpha : z_\alpha\tau = x_\alpha\}$ is a singleton $\forall\tau \ni 0 \leqq \tau \leqq \theta_\alpha\}$. If x is an improper singular point, then Inf $\{\theta'_\alpha : \alpha \in \mathscr{A}\} = 0$. Moreover, if Inf $\{\theta'_\alpha : \alpha \in \mathscr{A}\} = 0$, then x is either a start point or a singular point. Theorem. If there exists an improper singular point, then the set of singular points is dense in X. (Received February 19, 1968.)

655-73. WERNER KUICH, Michigan State University, East Lansing, Michigan 48823. Quasi-block-stochastic matrices and two theorems of A. Brauer.

Theorems similar to two well-known theorems of A. Brauer, concerned with characteristic vectors and characteristic roots of arbitrary matrices, are valid also for quasi-block-stochastic matrices. In particular if the $(l \times l)$-matrix A is quasi-block-stochastic, i.e. if there exist a $(l \times k)$-matrix F of a certain form and a $(k \times k)$-matrix $S_A$ such that $AF = FS_A$, then (i) $yF = 0$ for some characteristic vectors y with regard to the columns of A. (ii) A-FH has at least $l$-k roots in common with A, H being an arbitrary $(k \times l)$-matrix. (Received November 27, 1967.)

655-74. S. MUKHOTI, Memphis State University, Memphis, Tennessee 38111. Theorems on Cesàro summability of series.

We consider the Cesàro summability, for integral orders of the series $\sum_{\nu=0}^{\infty} a_\nu d_\nu$, and establish equivalence theorems. Theorem 1 (the case $k = 0$). Suppose that $d_n > 0$ for $n \geqq 0$, and (i) $d_{n+1} = o(1)$ as $n \longrightarrow \infty$, (ii) $d_{n+1}\sum_{\nu=0}^{n} |\Delta(1/d_{\nu+1})| = O(1)$, where $\Delta d_\nu = d_\nu - d_{\nu-1}$. Then necessary and sufficient conditions for (I)$\sum_{\nu=0}^{\infty} a_\nu d_\nu$ to be convergent to S are that (II) $-\sum_{\nu=0}^{\infty} S_\nu \Delta d_{\nu+1}$ should be convergent to S and $S_n d_{n+1} = o(1)$, where $S_n = \sum_{\nu=0}^{n} a_\nu$. Theorem 2 (the case $k \geqq 1$, k integer). Suppose that $d_n > 0$ for $n \geqq 0$, and (i) $(1/B_k^n)\sum_{m=0}^{n} B_k^m |\Delta^k\{\Delta(1/d_{m+k+1})\sum_{\nu=m+k}^{n} B_{k-1}^{n-\nu} d_{\nu+1}\}| = O(1)$ ($\Delta$ operating on m), (ii) $d_{n+1}^k/n^k = o(1)$ as $n \longrightarrow \infty$, where $d_{n+1}^k = \sum_{\nu=0}^{n} B_{k-1}^{n-\nu} d_{\nu+1}$, and where $B_k^n = \binom{n+k}{k}$, $B_k^m = \binom{m+k}{k}$, $B_{k-1}^{n-\nu} = \binom{n-\nu+k-1}{k-1}$. Then necessary and sufficient conditions for (I) $\sum_{\nu=0}^{\infty} a_\nu d_\nu$ to be summable (C,k) to S are that (II) $-\sum_{\nu=0} S_\nu \Delta d_{\nu+1}$ should be summable (C,k) to S and $S_n d_{n+1} = o(1)$ (C,k). (Received November 10, 1967.)

655-75. Y. -W. KIM, Wisconsin State University, Eau Claire, Wisconsin 54701. Bigraph topology and Skorokhod's M-convergence.

The graph topology was initiated by S. A. Naimpally [Trans. Amer. Math. Soc. 123 (1966), 267-272]. Skorokhod discussed several different types of topologies on D, the space of all discontinuous functions of the first kind defined on [0,1] (Theor. Prob. Appl. 1 (1956), 261-290). One observes that Skorokhod's M-convergence topology is a special case of the bigraph topology which is a generalization of graph topology in bitopological spaces, and that bigraph topology has interesting relationships with pairwise compact open topology (Abstract 68T-225, these *Notices* 15 (1968), 366-367). Definition. Let $(x, \tau_1, \tau_2)$ and $(Y, \sigma_1, \sigma_2)$ be bitopological spaces and let $\Gamma_{\tau_l \times \sigma}$ be the topology generated by the basis $\Gamma_u$, $u \in \tau_l \times \sigma$, $l = 1,2$. Then $\widetilde{\Gamma}_{12}^{\tau} = \{Y^X, \Gamma_{\tau_1 \times \sigma}, \Gamma_{\tau_2 \times \sigma}\}$ is said to be the bigraph topological space generated by $\tau_1$ and $\tau_2$. Theorem. If $(X, \tau_1, \tau_2)$ has at least two points, then $\widetilde{\Gamma}_\sigma^{\tau 12}$ is pairwise Hausdorff iff $(X, \tau_1, \tau_2)$ is pairwise Hausdorff and $(Y, \sigma)$ is $T_1$. Theorem. If $(X, \tau_1, \tau_2)$ is pairwise compact and pairwise Hausdorff, then the bigraph topology coincides with pairwise compact open topology for pairwise continuous function. (Received January 15, 1968.)

655-76. R. J. BUMCROT, Ohio State University, Columbus, Ohio 43210. Regular hyperbolic subplanes.

A hyperbolic plane H is an incidence structure in which: every two points are on a unique line; given a nonincident point and line, there are two lines through the point that do not meet the line; every set of three noncollinear points generates H; there exist noncollinear points. H is regular of order $(\pi, \lambda)$, where $\pi$ and $\lambda$ are integers, if each line is on exactly $\pi$ points and each point is on exactly $\lambda$ lines. Theorem. Let H be a regular hyperbolic plane of order $(\pi, \lambda)$ and let H' be a regular proper subplane of H, of order $(\pi', \lambda')$. Then every line of H meets H' if and only if $\lambda = \lambda'(\pi' - 1) - \pi' + 2$, and not every line of H meets H' if and only if $\pi\pi' \geqq \lambda'(1 + \lambda'(\pi' - 1))$. A construction of finite hyperbolic planes due to R. Sandler (Amer. Math. Monthly 70 (1963), 853-854) is studied. In showing that this construction never yields a regular plane, we obtain the following Theorem. Let P be a finite projective plane of order n and let k be an integer such that $1 \leqq k \leqq n$. Then it is impossible to remove exactly k points from each line of P. (Received February 23, 1968.)

655-77. BRIAN COLE, Yale University, New Haven, Connecticut 06520. Function algebras with one-point parts and zero point derivations. Preliminary report.

Let A be a function algebra on a compact space X. A construction is given which associates to A a function algebra $\tilde{A}$ on $\tilde{X}$ and an onto map $\pi$: $\tilde{X} \longrightarrow X$ with the following properties: (1) $f \circ \pi \in \tilde{A}$ for $f \in A$; (2) every function in $\tilde{A}$ has a square root in $\tilde{A}$; (3) the mapping $\mu \longrightarrow \hat{\mu}$ defined by $\int f d\hat{\mu} = \int f \circ \pi d\mu$ for all $f \in C(X)$ takes $M(\tilde{X})$ onto $M(X)$ in such a way that Ball $\tilde{A}^{\perp}$ maps onto Ball $A^{\perp}$; (4) $\partial_{\tilde{A}}$, the Shilov boundary of $\tilde{A}$, is $\pi^{-1}(\partial_A)$. The second of these ensures that every part in the maximal ideal space $\mathfrak{M}_{\tilde{A}}$ of $\tilde{A}$ consists of a single point and that any (possibly unbounded) derivation at a point of $\mathfrak{M}_{\tilde{A}}$ vanishes identically on $\tilde{A}$. By properties (3) and (4), $\tilde{A} \neq C(\tilde{X})$ or $\partial_{\tilde{A}}$ is proper in $\tilde{X}$ whenever the corresponding condition holds for A. In particular, this gives a negative answer to the conjecture of Gleason (Seminars on Analytic Functions, Inst. for Adv. Study, Princeton, 1957) that any function algebra with one-point parts must be C(X). (Received February 21, 1968.)

655-78. R. H. CROWELL and D. P. STRAUSS, Dartmouth College, Hanover, New Hampshire. On the elementary ideals of link modules.

Let $\{t_1, \ldots, t_m\}$, $m \geqq 2$, be a basis for the free abelian multiplicative group H, and let I(H) be the augmentation ideal of the integral group ring Z(H). Consider an exact sequence $0 \longrightarrow B \longrightarrow A \longrightarrow I(H) \longrightarrow 0$ of Z(H)-morphisms, which is a link module sequence relative to $\{t_1, \ldots, t_m\}$ as described in [R. H. Crowell, Torsion in link modules, J. Math. Mech. 14 (1965), 292]. If $L \subset S^3$ is a link with $m \geqq 2$ components and if $G = \pi_1(S^3 - L)$, then such a sequence arises in which $H = G/G'$, the module B is the group $G'/G''$ written additively with the operation of H defined by conjugation, and A is the module having as relation matrix the Alexander matrix of G. Let $\Delta_1$ be the Alexander polynomial of A, and let $E_0(B)$ be the 0th-elementary ideal of B. J. W. Milnor has conjectured that $E_0(B) = \Delta_1 I(H)^k$, where $k = (m - 2)(m - 3)/2$. In this paper we prove that Milnor's conjecture is correct. (Received February 23, 1968.)

655-79. S. KURODA and S. ULLOM, University of Maryland, College Park, Maryland 20742. Normal bases of ambiguous ideals. Preliminary report.

Hilbert's theory of root numbers is generalized to root numbers for normal bases of ambiguous ideals. Let Q(n) be the cyclotomic field of nth roots of unity over the rationals, $l$ an odd prime, prime $p \equiv 1 \bmod l$. F denotes the subfield of Q(p) with $[F : Q] = l$; P is the prime ideal of F dividing p. The ideal $P^j$ for $j \equiv 0, 1, 1 + (l - 1)/2 \bmod l$ has a normal basis. This gives information about the action of the Galois group of $Q(l)/Q$ on the ideal class group of $Q(l)$. We have also, e.g. the following Theorem. If $l$ is regular, the ideal $P^2$ has a normal basis for every prime $p \equiv 1 \bmod l$ if and only if $Q(l)$ has class number one. It follows that there exist ambiguous ideals of certain fields Q(p) which are projective but not free as ZG-modules, $G = G(Q(p)/Q)$. (Received February 26, 1968.)

655-80. F. BARTH and W. J. SCHNEIDER, Syracuse University, Syracuse, New York 13210. On a question of Seidel concerning holomorphic functions that are bounded on a spiral.

Let $D = \{|z| < 1\}$ and $C = \{|z| = 1\}$. Theorem. Let S be the spiral $z = r(\theta)e^{i\theta}$ where $0 \leqq \theta < \infty$, $r(\theta) \geqq 1/2$ and $r(\theta) \uparrow 1$. Then there exists a function w, holomorphic in D, such that w is bounded on S and w has $\infty$ as its only asymptotic value. This answers affirmatively a question of Seidel. Outline of proof. Let $\{b_n\}$ $(n \geqq 0)$ denote the set of points at which S intersects the positive real axis. Let $c_n = b_{2n}$, $d_n = b_{2n+1}$, $a_n = c_n + (1/2)(d_n - c_n)$, $\sigma_n = \{|z - a_n| = (1/8)(d_n - c_n)\} \cap \{\operatorname{Im} z \leqq 0\}$, $\alpha_n = \{z: c_n \leqq |z| \leqq c_n + (3/8)(d_n - c_n), \arg z = 0\}$, and $\beta_n = \{z: c_n + (5/8)(d_n - c_n) \leqq |z| \leqq d_n, \arg z = 0\}$. Let $\gamma_n$ be the subarc of S between $c_n$ and $d_n$ (note $\bigcup \gamma_n \neq S$). By Mergelyan's theorem f, holomorphic and nonzero in D, is constructed so that f is bounded on S, $f \approx 1$ on $\gamma_{2n} \cup \alpha_{2n} \cup \beta_{2n}$ and $f \approx -1$ on $\gamma_{2n+1} \cup \alpha_{2n+1} \cup \beta_{2n+1}$. By use of functions of the form $1/(z - a)^n$ and in $D - S$ the pole sweeping technique in Bull. Amer. Math. Soc. 72 (1966), 841-842, a holomorphic function g is constructed so that $\operatorname{Re} g > 0$ on $\alpha_n \cup \beta_n$, $g \approx 0$ on S, and $|f \cdot (1 + g)| \geqq n$ on $\sigma_n$. It follows that $w = f(1 + g)$ is the desired function. (Received February 23, 1968.)

655-81. R. J. MCGIVENY, Lafayette College, Easton, Pennsylvania 18042, and WILLIAM RUCKLE, Lehigh University, Bethlehem, Pennsylvania. Multiplier algebras of biorthogonal systems.

Let $\{e_i, E_i\}$ be a total biorthogonal system in a linear topological space X. The multiplier algebra of X with respect to $\{e_i, E_i\}$, written M(X), is the set of all scalar sequences $(t^{(i)})$ such that for each $x \in X$ there is $y \in X$ with $E_i(y) = t^{(i)}E_i(X)$. The form of M(X) is determined when $\{e_i, E_i\}$ is a norming complete biorthogonal system in a Banach space or a basis in a complete barreled space. It is shown that a sequence space is the multiplier algebra for a basis in a Banach space if and only if it is a $\gamma$-perfect BK-algebra. (Received February 21, 1968.)

655-82. BERTRAM WALSH, University of California, Los Angeles, California 90024. Flux in axiomatic potential theory.

On a locally compact Hausdorff space W, let $\mathscr{U}$ be a complete presheaf of functions satisfying the well-known axioms of Brelot and in addition the axiom: $1 \in \mathscr{U}$. A complete study is made of the problem of defining an analogue of the classical flux integral in the axiomatic setting. For elliptic (i.e. those with W compact) and parabolic sheaves $\mathscr{U}$, the flux functional is essentially unique (a fact related to Sario's construction of normal operators, and containing the solution of a problem posed by Nakai). For hyperbolic $\mathscr{U}$, however, the linear space of flux functionals is in order-preserving isomorphic correspondence with the space of global sections of the Hervé adjoint sheaf $\mathscr{U}^*$; choice of a flux functional corresponds to selecting a normalization of $\mathscr{U}^*$ for which $1 \in \mathscr{U}^*$. Extremal functionals are "normal derivatives at points of an ideal boundary." Every flux functional on a hyperbolic space can be associated with at least one normal operator in the sense of Sario. (Received February 21, 1968.)

655-83. A. C. MEWBORN and C. N. WINTON, University of North Carolina, Chapel Hill, North Carolina. Orders in self-injective semiperfect rings.

Let R be a ring containing a regular element. R is a right order in a quasi-Fröbenius ring if and only if (1) $R_R$ is finite-dimensional, (2) if $x \in I(R_R)$, the injective hull of $R_R$, there exists a regular element $b \in R$ such that $xb \in R$, and (3) R satisfies the ascending chain condition for annihilators of subsets of $I(R_R)$. This sharpens a theorem of Jans [J. Algebra 7 (1967), 35-43]. Also, if R satisfies (1), (2), and (3') R satisfies the ascending chain condition on annihilator right ideals, then R is a right order in a self-injective semiprimary ring. (Received February 23, 1968.)

655-84. M. C. BUNGE, McGill University, Montreal, Canada. Functor categories and standard constructions (triples).

For an autonomous category $\underset{\sim}{P}$ with a representable underlying set functor, fix a set $\underline{I}$ and denote by $\mathscr{L}_{\underset{\sim}{P},\underline{I}}$ the category whose objects are the $\underset{\sim}{P}$-categories with a set of objects of the same cardinality as $\underline{I}$ and whose morphisms are functors between them whose object functions are isomorphisms. Under the further assumption that $\underset{\sim}{P}$ has all finite limits as well as $\underline{I}$-fold products and sums, it is shown that $\mathscr{L}_{\underset{\sim}{P},\underline{I}}$ is equivalent to the category of all $\underset{\sim}{P}$-adjoint triples in the product category $\underset{\sim}{P}^{\underline{I}}$ and triple morphisms. Moreover, for any $\underset{\sim}{C} \in |\mathscr{L}_{\underset{\sim}{P},\underline{I}}|$ and its corresponding $\underset{\sim}{P}$-adjoint triple $\underset{\sim}{T}_{\underset{\sim}{C}}$ in $\underset{\sim}{P}^{\underline{I}}$, the category of $\underset{\sim}{T}_{\underset{\sim}{C}}$-algebras is isomorphic to the category $\underset{\sim}{P}^{\underset{\sim}{C}}$ of all $\underset{\sim}{P}$-valued $\underset{\sim}{P}$-functors on $\underset{\sim}{C}$ and natural transformations. Call a category coregular if every epimorphism in it is a coequalizer. Assuming that $\underset{\sim}{P}$ is also coregular, the above results are applied to prove the following characterization theorem: a $\underset{\sim}{P}$-category $\mathscr{X}$ is equivalent to a category of the form $\underset{\sim}{P}^{\underset{\sim}{C}}$ for some $\underset{\sim}{C} \in |\mathscr{L}_{\underset{\sim}{P},\underline{I}}|$ if and only if $\mathscr{X}$ is coregular with coequalizers and there exists a faithful $\underset{\sim}{P}$-functor $\underline{U}: \mathscr{X} \longrightarrow \underset{\sim}{P}^{\underline{I}}$, having an adjoint and a coadjoint. Examples are considered. (Received February 2, 1968.)

655-85. W. J. PUGH, University of Syracuse, Syracuse, New York, and S. M. SHAH, University of Kentucky, Lexington, Kentucky. Entire functions with widely spaced zeros and of bounded index.

The authors investigate entire functions of bounded index. Their first result indicates that there exist entire functions of bounded index and arbitrarily slow growth. More precisely they prove (I): The entire function f(z) satisfies the condition $|f^{(k)}(z)| \leqq \max \{|f(z)|, |f^{(1)}(z)|\}$ for all z and all $k = 1, 2, \ldots$ provided (i) f(z) is of order zero and has only simple negative zeros $\{a_n\}$, and (ii) these zeros satisfy the inequalities $|a_1| > 16$, $|a_{n+1}| > 2^n |a_n|$. (II) Given a sequence $\{\Psi_n\}$ such that $\Psi_n$ increases to $\infty$, however rapidly, with n, there always exists a transcendental entire function f(z) satisfying the condition $\Psi_k |f^{(k)}(z)| \leqq \max \{|f(z)|, |f^{(1)}(z)|\}$ for all z and all $k = 2, 3, \ldots$ . Their result on functions of unbounded index shows that their exist entire functions of unbounded index having an asymptotically prescribed growth. (III) If g(z) is an arbitrarily given transcendental entire function, there always exists an entire function f(z), of unbounded index, such that $\log M(r,g) \sim \log M(r,f)$ $(r \longrightarrow +\infty)$. (The authors would like to thank Professor Edrei for the help he has given them in this result.) (Received February 21, 1968.)

655-86. JOSEPH ZAKS, University of Washington, Seattle, Washington 98105. On a minimality property of complexes.

Let $C_q^p$ denote the p-skeleton of the (q - 1)-simplex, and let $K * L$ be the join of the complexes K and L. B. Grünbaum (Graphs and complexes, lecture notes, 1967) proved that if n, p, $n_1, \ldots, n_p$ are positive integers, then $C_{2n_1+1}^{n_1-1} * \ldots * C_{2n_p+1}^{n_p-1}$ is an n-complex, called nice, which is not embeddable in $E^{2n}$. K is called (p.w.l., geometrically) minimal in $E^d$ if each proper subcomplex of K is (p.w.l., geometrically) embeddable in $E^d$. Theorem 1. Each nice n-complex is p.w.l. minimal in $E^{2n}$, and is geometrically minimal in $E^{2n}$ provided p = 1 or all $n_i = 1$, except possibly one of them. Theorem 2. If K is geometrically minimal in $E^d$, then $K * C_3^0$ is geometrically minimal in $E^{d+2}$; and if K and L are p.w.l. minimal in $E^d$ and $E^h$, then $K * L$ is p.w.l. minimal in $E^{d+h+2}$. (Received February 23, 1968.)

655-87. J. A. PFALTZGRAFF, University of North Carolina, Chapel Hill, North Carolina 27514. Extremal problems for functions with bounded boundary rotation. Preliminary report.

Let $\mathscr{S}(g,M)$ denote the class of functions S(z) regular in the unit disk and represented by a Stieltjes integral $S(z) = \int_0^{2\pi} g(z,t)\, dm(t)$, where $m(t) \in$ B.V.$[0,2\pi]$, $m(0,2\pi) = 1$, $V_0^{2\pi}(m(t)) = M$, and g(z,t) is a given function analytic in z with derivatives continuous in t. Let J be a real L.C.S. functional on $\mathscr{S}$, i.e. $J(S + \epsilon T) = J(S) + \epsilon J_1(S;T) + o(\epsilon)$, where $J_1$ is real, linear and continuous in T. Theorem. Let J be a real L.C.S. functional on $\mathscr{S}$ such that $J_1(S;g_t(z,t))$ has 2k zeros in $0 \leqq t < 2\pi$. Then any solution $S_0 \in \mathscr{S}$ of the problem $\max(J(S): S \in \mathscr{S})$ must be of the form $S_0(z) = 2^{-1}(M + 1) \cdot \sum_{\nu=1}^{\alpha} p_\nu g(z,t_\nu) - 2^{-1}(M - 1) \sum_{\nu=1}^{\beta} n_\nu g(z, \tau_\nu)$ where $t_\nu, \tau_\nu \in [0,2\pi)$, $\alpha, \beta \leqq k$, $p_\nu, n_\nu > 0$ and $\sum p_\nu = \sum n_\nu = 1$. The t's must be points of max and the $\tau$'s points of min for $L(t) \equiv J_1(S;g(z,t))$. The proof is by a method of Goluzin (Amer. Math. Soc. Transl. (2)18(1961)). We show that the step function method of approximation yields the same results, and we apply both methods to various extremal problems for regular and meromorphic functions of bounded boundary rotation. Some sharp bounds for curvature are obtained. The results of this paper were obtained before the author was aware of the work of B. Pinchuk, Bull. Amer. Math. Soc. 73 (1967). (Received February 22, 1968.)

655-88. L. H. HAINES, University of California, Berkeley, California. On free monoids partially ordered by embedding.

Let $\Sigma^*$ be the free monoid with null word $\epsilon$ generated by a finite alphabet $\Sigma$. Let $\leqq$ partially order $\Sigma^*$ by embedding (i.e. $x \leqq y$ iff $x = x_1x_2 \ldots x_n$ and $y = y_1x_1y_2x_2 \ldots y_nx_ny_{n+1}$ for some integer n, where $x_i$ and $y_j$ are in $\Sigma^*$ for $1 \leqq i < j \leqq n + 1$). Theorem 1. Each set of pairwise incomparable elements of $\Sigma^*$ is finite. For any $A \subset \Sigma^*$ define $\widetilde{A} = \{x \text{ in } \Sigma^* : y \leqq x \text{ for some } y \text{ in } A\}$ and $\underset{\sim}{A} = \{x \text{ in } \Sigma^* : x \leqq y \text{ for some } y \text{ in } A\}$. Theorem 2. Let $A \subset \Sigma^*$. Then there exist finite subsets F and G of $\Sigma^* \ni : \widetilde{A} = \widetilde{F}$ and $\underset{\sim}{A} = \Sigma^* - \widetilde{G}$. Theorem 3. $\widetilde{A}$ and $\underset{\sim}{A}$ are regular sets for any $A \subset \Sigma^*$. For any $R \subset \Sigma^* \times \Sigma^*$ define $\hat{R} = \{(x,y) : (x,z) \text{ in } R \text{ for some } z \geqq y\}$ and (trace of R) tr(R) = all words $x_1x_2 \ldots x_n$ where $x_i$ in $\Sigma$ and $(x_iy_i, x_{i+1}y_{i+1})$ is in R for some $y_1, y_2, \ldots, y_n$ in $\Sigma^*$. Theorem 4. L is a context-sensitive (CS) language iff there exists a length preserving homomorphism h, a context-free language K and a regular relation $R \ni : L = h(\mathrm{tr}(\hat{R}) \cap K)$. This representation theorem reduces certain important open problems about CS languages to languages of the form $\mathrm{tr}(\hat{R})$ where R is a regular relation (e.g. every CS language is deterministic iff $\mathrm{tr}(\hat{R})$ is deterministic for every regular relation R). Research sponsored by ghe National Science Foundation under Grant GP-6945. (Received February 22, 1968.)

655-89. ROGER CUPPENS, The Catholic University of America, Washington, D. C. 20017. On finite products of Poisson-type characteristic functions of several variables.

Let f be the product of p Poisson-type characteristics functions of the two variables $(t_1,t_2)$ defined by $f(t_1,t_2) = \exp\{ik_1t_1 + ik_2t_2 + \sum_{j=1}^{p} [\lambda_j(\exp(i\alpha_jt_1)-1) + \mu_j(\exp(i\beta_jt_2)-1) + \nu_j(\exp(i\alpha_jt_1 + i\beta_jt_2)-1)]\}$ where $k_1,k_2,\alpha_j,\beta_j$ are real, $\lambda_j,\mu_j,\nu_j$ are nonnegative. It is well known that in the univariate case the product of two Poisson-type characteristic functions has no indecomposable factor. But if $p = 2$, $\alpha_1 = 1$, $\beta_1 = 2$, $\alpha_2 = 2$, $\beta_2 = 1$ and if all the $\lambda_j,\mu_j,\nu_j$ are positive, then f has an indecomposable factor. On the other hand, the following theorems are valid: Theorem 1. If $p = 2$ (we can suppose that $0 < \alpha_1 < \alpha_2$, $\beta_2 > 0$) and if the $\alpha_j$ and $\beta_j$ satisfy one of the conditions (I) $\beta_1\beta_2 \leqq 0$, (II) $0 < \beta_1 \leqq \beta_2$, (III) $0 < \beta_2 < \beta_1$, $\alpha_2\beta_2 + \alpha_1\beta_1 - \alpha_2\beta_1 > 0$, (IV) $\beta_1$ and $\beta_2$ are incommensurable, then f has no indecomposable factor. Theorem 2. If $p > 2$, if $\alpha_1,\ldots,\alpha_p$ are rationally independent, and if $\beta_1,\ldots,\beta_p$ have the same property, then f has no indecomposable factor. Conditions assuring in the cases $p = 3$ and $p = 4$ that f has no indecomposable factor and the generalization of these theorems to the case of n (>2) variables are also given. (Received February 21, 1968.)

655-90. R. F. SPRING, Ohio University, Athens, Ohio 45701. Direct product decompositions of class 2 groups of exponent p.

Let G be a class 2 group of exponent p (p = an odd prime) with d generators. Let $p^t$ and $p^k$ be the orders of G and the commutator subgroup of G respectively. Then $t = k + d$ and the maximum value for K is C(d,2). Theorem 1. If $k > C(d - 1,2)$ then G is irreducible. If $k > C(d - 1,2) - (d - 3)$, then G is not reducible to a direct product in which every factor is a class 2 group. Theorem 2. If $t > C(d - n, 2) + (2d - n)$, then every direct product factorization of G contains fewer then n factors. If $t > C(d - 2n, 2) + 3(d - n)$, then every direct product factorization of G contains fewer than n class 2 factors. Theorem 3. If d is odd, then G is reducible to a direct product in which at least one factor is abelian if $k = 1$. (Received February 22, 1968.)

655-91. STEFAN BERGMAN, Stanford University, Stanford, California 94305. A characterization of the center of a circular domain which is invariant with respect to pseudo-conformal transformations.

Extending his previous considerations (see J. Analyse Math. 13 (1964), 317-353; and also G. Springer, Duke Math. J. 18 (1951), 411-424), the author determines certain properties of the center of a circular domain which remain invariant under pseudo-conformal transformations. These results yield the necessary and sufficient conditions to determine whether or not a given domain $B \in C^2$ is a pseudo-conformal image of a circular domain C. If this is the case, the pair of functions mapping B onto C is the pair which maps B onto the representative domain. (Received February 22, 1968.)

655-92. W. R. JONES, Lafayette College, Easton, Pennsylvania 18042. Reduction of integral boundary conditions to two-point boundary conditions.

Let $y' - A(x)y = \lambda B(x)y$, $s[y] \equiv My(a) + Ny(b) + \int_a^b F(x)y\,dx = 0$ be the vector-matrix form under the customary transformation of the scalar differential equation $\sum_{i=0}^n p_i(x)y^{(i)} = \lambda q(x)y$ with boundary conditions $\sum_{j=1}^n [m_{ij}y^{(j-1)}(a) + n_{ij}y^{(j-1)}(b) + \int_a^b f_{ij}(x)y^{(j-1)}dx] = 0$ $(i = 1, \ldots, n)$ with complex-valued coefficient functions on the real interval $[a,b]$, complex parameter $\lambda$, and complex constants $m_{ij}$ and $n_{ij}$. Let $n \times n$ matrix $F(x)$ be considered as comprised of n column vectors, i.e. $F(x) = [F_1\ F_2 \ldots F_n]$. Lemma. If the vectors $F_i(x)$ satisfy the relation $F_1 - F_2' + F_3'' - \ldots (-1)^{n-1}F_n^{(n-1)} = 0$ on $[a,b]$, then the conditions $s[y] = 0$ can be replaced by two-point boundary conditions, $M_1y(a) + N_1y(b) = 0$. Theorem. If the transformed problem $y' - A(x)y = \lambda B(x)y$, $s[y] = 0$ is $\mathscr{T}$-summetrizable (for definition see Jones, J. Differential Equations (2) 3 (1967)), then the conditions $s[y] = 0$ can be replaced by two-point boundary conditions. (Received February 21, 1968.)

655-93. P. R. MEYER, Hunter College of the City University of New York, Bronx, New York 10468. Sequential properties of ordered topological spaces.

The concepts of Fréchet space and sequential space are known to provide successive proper generalizations of first countable spaces (e.g. see Franklin, Fund. Math. 57 (1965), 107-115). It is also known that neither the Fréchet nor the sequential property is productive. In the present paper we show that, for products of ordered spaces (i.e. products in which each coordinate space has the order topology arising from some total order), the situation is quite different: all three properties are equivalent and are preserved under the formation of countably infinite products. The results are formulated more generally, for an arbitrary infinite cardinal number m, in the terminology of Abstract 648-116, these *Notices* 14 (1967), 664. Theorem. Let X be the product (with the product topology) of a family $\{X_i : i \in I\}$ of nontrivial topological spaces. (i) In order that X be m-sequential, it is necessary that each $X_i$ be m-sequential and card $I \leqq m$. If, in addition, each $X_i$ is ordered, then these conditions are also sufficient. (ii) If each $X_i$ is ordered, then X is m-sequential iff X is m-Fréchet iff each point of X has a neighborhood base of cardinality $\leqq m$. (Received February 21, 1968.)

655-94. RALPH TINDELL, University of Georgia, Athens, Georgia 30601. Extending homeomorphisms of $S^p \times S^q$.

We point out the following proposition: denote by C(M) the group of concordance classes of PL homeomorphisms of M onto itself, and define the subgroup $G_{p,q}$ of $C(S^p \times S^q)$ to be the concordance classes of homeomorphisms which extend to $S^{p+q+1}$. If $p > q \geqq 1$, then $G_{p,q}$ is cyclic of order 2, generated by an orientation reversing homeomorphism, i.e. any orientation preserving homeomorphism which extends is concordant to the identity. If $p \geqq 2$, then $G_{p,p}$ is isomorphic to $Z_2 + Z_2$, the additional generator being the map $(x,y) \longrightarrow (y,x)$. (Received February 19, 1968.)

655-95. J. A. CIMA and J. A. PFALTZGRAFF, University of North Carolina, Chapel Hill, North Carolina. *The Hornich topology for meromorphic functions in the disk.*

Let $H_k$ be the set of all functions $f(z) = \sum_{n \geq k} a_n z^n$ holomorphic in $|z| < 1$ for k a positive integer. Let $\|f\| = \sup_n |a_n|^{1/n}$ be the Hornich norm; and for $\phi_1$, $\phi_2$ in $[0, 2\pi]$ and $r < 1$, define the map T from $H_k$ into $H_{k-1}$ by $Tf = \sum_{n \geq k} a_n z^{n-1}(\sin n\phi / \sin \phi)$ where $\phi = 1/2(\phi_1 - \phi_2)$. *Theorem.* For fixed $\phi$ and $k > 1$, the map T is one to one and onto if and only if $\phi$ is not a rational multiple of $\pi$. *Theorem.* Every continuous linear functional F on $H_1$ is of the form $F(f) = \sum_{n=1}^{\infty} c_n a_n$, $f(z) = \sum_1^{\infty} a_n z^n$ where $\limsup |c_n|^{1/n} < 1$. (Received February 20, 1968.)

655-96. S. M. ROBINSON, Cleveland State University, Cleveland, Ohio. *Some properties of $\beta X - X$ for complete spaces.*

The following two theorems generalize well-known results of Fine and Gillman: *Theorem* 1. If X admits a complete uniform structure, every zero set of $\beta X - X$ is the closure of its interior. *Theorem* 2. If X admits a complete uniform structure and if H is a subset of $\beta X - X$, $X \cup H$ is pseudocompact implies H is dense in $\beta X - X$. We employ these two theorems to prove (assuming the continuum hypothesis) the following generalization of a well-known theorem of Rudin: *Theorem* 3. If X is locally compact and complete $\beta X - X$ contains a dense set of P-points. A *remote* point of $\beta X$ is defined by Fine and Gillman to be a point not in the closure of any discrete subset of X. Using the continuum hypothesis, they construct remote points in $\beta R$. Plank [*On a class of subalgebras of C(X) with applications to* $\beta X$, Fund. Math. (to appear)] proves the existence of remote points in $\beta X$, where X is a separable metric space. We prove: *Theorem* 4. If X is locally compact, metrizable, and without isolated points, $\beta X - X$ contains a dense set of remote points. (As do the Fine-Gillman and Plank results, ours requires the continuum hypothesis.) (Received February 19, 1968.)

655-97. M. L. SLATER, Sandia Corporation, P. O. Box 5800, Albuquerque, New Mexico 87115. *Two point boundary value problems for linear differential inequalities.*

The following result is typical. Let I be the interval $[0,1]$, and for $x \in C_2(I)$ define $L_2(x) \equiv a_0(t)x'' + a_1(t)x' + a_2(t)x$, where the $a_i$ are real and $\in C_{2-i}(I)$, and $|a_0| > 0$ on I. Let $c \in L(I)$. *Theorem.* The following two statements are equivalent: (1) for all $x \in C_2(I)$, if $x \geq 0$ and $-L_2(x) \geq 0$ on I, then $\int_0^1 cx\,dt \leq 0$; (2) there exists a function $u = u(t)$, $t \in I$, such that (i) u' exists and is absolutely continuous on I, (ii) $u \geq 0$ on I, $u(0) = u(1) = 0$, $a_0(0)u'(0) \geq 0$, and $a_0(1)u'(1) \leq 0$, and (iii) $L_2^+(u) \geq c$ a.e. on I, where $L_2^+$ is the adjoint of $L_2$. The principal tool is the Namioka-Bauer theorem on the extension of positive linear forms. (See H. H. Schaefer, *Topological vector spaces*, Macmillan, New York, 1966, p. 227.) The theorem of the abstract is the differential analog of the Minkowski-Farkas lemma for matrix inequalities. (Received February 26, 1968.)

655-98. B. J. THORNE, Smith College, Northampton, Massachusetts 01060. *A-P congruences on Baer semigroups.*

See M. F. Janowitz, *A semigroup approach to lattices* [Canad. J. Math. 18 (1966), 1212-1223], for notation used but not defined here. An anihilator preserving (A-P) congruence on a semigroup S is a congruence $\rho$ on S such that $L(x\rho) = L(x)\rho = \{y\rho \in S/\rho : y \in L(x)\}$ and $R(x\rho) = R(x)\rho$. If S is a Baer semigroup and $\rho$ an A-P congruence on S, then $\rho$ induces a lattice congruence $\Theta$ on $\mathscr{L}(S)$ such that $\mathscr{L}(S/\rho) \cong \mathscr{L}(S)/\Theta$. This congruence satisfies ($\alpha$) $Se \equiv Sf(\Theta) \Rightarrow Se\phi_x \equiv Sf\phi_x(\Theta)$ and $Se\phi_x^+ \equiv Sf\phi_x^+(\Theta)$ for all $x \in S$. A congruence $\theta$ on a lattice L is called compatable with a Baer semigroup S coordinatizing L if ($\alpha$) holds. Any congruence compatable with S is induced by an A-P congruence on S. A compatable congruence $\Theta$ is uniquely determined by its kernel which in turn is uniquely determined by the kernel of any A-P congruence inducing $\Theta$. Further, for any lattice L the lattice of congruences compatable with a fixed Baer semigroup forms a subcomplete sublattice of the lattice of all congruences on L. (Received February 26, 1968.)

655-99. M. G. ARSOVE, University of Washington, Seattle, Washington 98105. *Absolute continuity of harmonic measures with respect to arc length.*

For a plane region $\Omega$ bounded by finitely many disjoint analytic Jordan curves, it is well known that Green's function $G_z$ possesses a normal derivative at each point of the boundary and that the harmonic measure $m_z$ has linear density $-(1/2\pi)\partial G_z/\partial\nu$ ($\nu$= exterior normal). A more general situation will be considered here, namely that in which $\partial\Omega$ is only required to consist of finitely many disjoint rectifiable closed curves. It is then shown that $m_z$ remains absolutely continuous with respect to arc length and that the linear density is equal to $-(1/2\pi)\partial G_z/\partial\nu$ almost everywhere on $\partial\Omega$. Moreover, this property persists when the Euclidean boundary is replaced by the prime end boundary. The above results extend earlier work of M. Tsuji [*On the Green's function,* Japanese J. Math. 18 (1942), 379-383] and S. Verblunsky [*On a fundamental formula of potential theory,* J. London Math. Soc. 26 (1951), 25-30]. (Received February 26, 1968.)

655-100. A. P. OGG, University of California, Berkeley, California 94720. *Functional equations of modular forms.*

Let f be a modular form of dimension - k for the group $\Gamma_0(N)$, where N is square-free, and assume f is an eigenfunction for the Hecke operators T(p) for $p|N$, say $f|T(p) = a_p \cdot f$. Let $f_1(\tau) = N^{-k|2}\tau^{-k}f(-1/N\tau)$. Then the following are equivalent: (1) f satisfies the functional equation $f_1 = \pm f$; (2) $a_p^2 = p^{k-2}$ for all $p|N$; (3) $f_1|T(p) = a_p \cdot f_1$ for all $p|N$. (Received February 26, 1968.)

655-101. R. C. MACCAMY, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213. *On a differential equation in elasticity.*

The partial differential equation $u_{tt} = [\sigma(u_x) + \lambda(u_x)u_{xt}]_x$ is studied. $\sigma$ is a monotone increasing function, $\lambda$ a positive function. It is shown that there exists a unique solution of the equation such that $u(0,t) = u(1,t) = 0$, $u(x,0) = f(x)$, $u_t(x,0) = g(x)$. This solution is stable with respect to the initial data and tends to zero as t tends to infinity. This generalizes results of Greenberg, MacCamy and Mizel (to appear in J. Math. Mech.) in which $\lambda$ was assumed constant. The equation is a model for so-called linearly viscous materials and contains, as a special case, the one-dimensional Navier-Stokes equation for compressible flow. (Received February 26, 1968.)

655-102. J. W. BOND, Pennsylvania State University, University Park, Pennsylvania. *Lie algebras of genus 1 and 2.*

The genus of a finite-dimensional algebra is the difference between its dimension and the number of elements in any of its minimal generating sets. Proposition. The genus of any subalgebra or quotient algebra is less than or equal to the genus of the algebra. The following can be proven by induction on the dimension of L: Theorem. If L is a finite-dimensional genus 1 or 2 Lie algebra over any field, then L is either solvable or contains a simple subalgebra. Next we show Proposition 1. If S is a simple subalgebra of a genus 1 Lie algebra, then S = L and dim L = 3. Proposition 2. If S is a simple subalgebra of a genus 2 Lie algebra L, then $S = L^2$, dim S = 3, dim L = 4. Next, Theorem. A nonminimal solvable genus 1, genus 2 Lie algebra L has a 1,2 dimensional ideal J, respectively, with genus 0 quotient. This theorem leads to the classification of nonminimal genus 1 and genus 2 Lie algebras through the study of the different possible L-modules structures for J. (Received February 26, 1968.)

655-103. WITHDRAWN.

655-104. R. D. M. ACCOLA, Brown University, Providence, Rhode Island 02912. On the zeros of theta functions for superelliptic Riemann surfaces. Preliminary report.

Definition. A closed Riemann surface will be called superelliptic if it can be represented as a two-sheeted covering of a torus. Theorem. Let W be a nonhyperelliptic Riemann surface of odd genus g, $g \geqq 5$. Let $\theta[\mu]$ be the usual first order theta-function associated with the Jacobian of W with characteristic $[\mu]$. The following condition is necessary and sufficient that W be superelliptic. There are four theta characteristic $[\mu_i]$, i = 1,2,3,4, $[\mu_4] = [\mu_1\mu_2\mu_3]$ and $\theta[\mu_i]$ has order (g - 1)/2 at zero. The proof of the sufficiency for $g \geqq 9$ follows from work of H. Martens on linear series with low Clifford index. The cases g = 5,7 are reduced to those of $g \geqq 9$ by considering certain smooth coverings of W. The necessity follows from methods used in the hyperelliptic case. Remarks. (1) In general, there are other zeros for superelliptic theta functions analogous to the hyperelliptic case. (2) For sufficiently high even genus, analogous results hold. (3) For g = 5, the codimension of the superelliptic locus in Teichmueller space is four. (4) For g = 5, the methods yield a characterization of surfaces admitting certain automorphism groups isomorphic to $Z_2 \times Z_2$. (5) For $g \geqq 4$, a surface cannot be hyperelliptic and superelliptic at the same time. (Received February 26, 1968.)

655-105. B. MOND and O. SHISHA, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio 45433. An inequality involving a generalized inverse matrix.

Recently, the authors proved the following inequality. Let A be a positive definite hermitian matrix. Then for every column vector X of unit norm, $(AX,X) - (A^{-1}X,X)^{-1} \leqq (\Lambda^{1/2} - \lambda^{1/2})^2$, where $\Lambda$ is the largest, $\lambda$ the smallest eigenvalue of A. This result remains true if "semidefinite" is substituted for "definite", "smallest positive" for "smallest", and the generalized (Penrose) inverse of A for $A^{-1}$ (we assume $A \neq 0$). (Received February 26, 1968.)

655-106. R. J. WARNE, West Virginia University, Morgantown, West Virginia. Direct decomposition of regular semigroups.

If X is a semigroup, let $E_X$ denote the set of idempotents of X. Theorem. S is a regular semigroup such that $E_S \cong K \times B$, where K is a semilattice and B is a rectangular band, if and only if

$S \cong T \times B$, where T is an inverse semigroup with $E_T \cong K$. S is bisimple if and only if T is bisimple. Certain applications of the theorem to the bisimple inverse semigroups studied in R. J. Warne, *Bisimple inverse semigroups mod groups* [Duke Math. J. 34 (1967), 787-811] are indicated. (Received February 26, 1968.)

655-107. PETER FALLEY, The City University of New York, New York, New York 10040. *Universal ideals of operators*. Preliminary report.

In the space $\mathscr{B}(E,F)$ of bounded operators from a Banach space E into a Banach space F certain well-known ideals are defined: The ideal of operators of finite-rank, the ideal of completely continuous operators, and the ideal of weakly compact operators. One remarks that these are defined for any two Banach spaces, and always share a number of properties. These observations lead to the concept of a *universal ideal* $\mathscr{U}$, a procedure which to every pair (E,F) of Banach spaces associates a definite ideal $\mathscr{U}(E,F)$ in $\mathscr{B}(E,F)$ called a realization of $\mathscr{U}$. The author employs the notions of Abstract 648-160, these *Notices* 14 (1967), 99, to characterize the universal ideals of completely continuous operators, and to show that every two-sided ideal of completely continuous operators on a Hilbert space is a realization of a universal ideal. (Received February 26, 1968.)

655-108. IRA ROSENBAUM, University of Miami, Miami, Florida 33157. *On the Laplace transforms of functions of the integral, and the fractional, parts of* X.

In the author's *Topics in analysis.* I: *Papers on the integral, and the fractional, parts of a real number* (Miami, Fla., 1967) are given, among other things, the Laplace transforms of a variety of exponential, hyperbolic, and trigonometric functions of the integral, and/ or fractional, parts of x, as well as of products of such functions. Also given in the cited work are the Laplace transforms of $(\mathrm{Fr}\ x)^n$, $(\mathrm{In}\ x)^k$, and $(\mathrm{Fr}\ x)^n(\mathrm{In}\ x)^k$ for positive integral n and k. Subsequent work has yielded the Laplace transforms of products of $(\mathrm{In}\ x)^k$, and $(\mathrm{Fr}\ x)^n$, with various exponential, hyperbolic, and trigonometric functions of the integral, and/or fractional, parts of x. In addition to presenting these results, the present paper gives a number of special transforms of simple and noteworthy forms, e.g. those of $(\mathrm{In}\ x)^k \cos((\pi/2)\mathrm{In}\ x)$ and $(\mathrm{Fr}\ x)^n \cos(\pi\,\mathrm{In}\ x)$. Operations giving new Laplace transforms from old are also considered. (Received February 26, 1968.)

655-109. MICHAEL AISSEN, Fordham University and Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio 45433. *Fundamental sequences in ordered sets.*

A sequence in a linearly ordered set X is a constant if and only if there is exactly one element of X which is a lower bound and an upper bound to the sequence. Notions of weak lower bounds and weak upper bounds of a sequence are introduced. A sequence is called "fundamental" if it has at most one simultaneous weak upper and weak lower bound, and "convergent" if it has exactly one such bound. X is called "complete" if every "fundamental" sequence "converges". *Theorem.* Every linearly ordered set X can be embedded in a "complete" linearly ordered set X*. The technique mimics the Cantor completion process for metric spaces, but includes some situations which are nonmetrizable. The set X* is order isomorphic with X** and with the usual completion of X by cuts. (Received February 26, 1968.)

655-110. WILLIAM SAFFERN, 382 Wadsworth Avenue, New York, New York 10040. *Rational Approximation*. II.

See these *Notices* 14 (1967), 372, for terminology. *Definition*. If X is a compact subset of the plane, the inner boundary of X is the set of all boundary points of X which do not belong to the boundary of some component of the complement of X. *Example*. There exists a compact subset of the plane X whose interior is a simply connected region dense in X, and whose boundary is a rectifiable, nonanalytic, locally simple curve satisfying: (a) the inner boundary of X has positive analytic capacity and zero analytic C-capacity (Vituskin, Soviet Math. Dokl. 7 (1966), 1622); (b) A(X) is not a Dirichlet algebra; (c) A(X) = B(X). *Theorem*. If X is a compact subset of the plane whose interior is a simply connected region dense in X, and whose boundary is locally simple curve which is analytic, then A(X) = B(X). (Received February 26, 1968.)

655-111. B. M. CHERKAS, Johns Hopkins University, Silver Spring, Maryland 20910, and CHOY TAK TAAM, George Washington University, Washington, D.C. *On nonlinear diffusion equation in* $L_2(R^m)$. Preliminary report.

The equation (*) $\partial u(t,x)/\partial t = Au(t,x) + \sum_{i=0}^{l} d_i(t,x)u^i(t,x)$ is considered, where $A = \sum_{i,j=1}^{m} a^{ij}(x)(\partial^2/\partial x_i \partial x_j) + \sum_{i=1}^{m} b^i(x)(\partial/\partial x_i) + c(x)$ is a strictly elliptic operator with bounded $C^\infty(R^m)$ coefficients which satisfy other boundedness conditions. In [*Functional analysis*, Springer-Verlag, New York, 1965, pp. 413-418], K. Yosida shows that the smallest closed extension $\widetilde{A}$ in complex $L_2(R^m)$ of A is the infinitesimal generator of a holomorphic semigroup $\{\exp(\widetilde{A}t) : t \geqq 0\}$ of class $\{C_0\}$ in $L_2(R^m)$ which satisfies $\|\exp(\widetilde{A}t)\|_2 \leqq e^{(\lambda_0 + \eta_0)t}$, where $\lambda_0$ and $\eta_0$ are ellipticity constants. The basic approach is to treat the differential equation (**) $du(t,\cdot)/dt = \widetilde{A}u(t,\cdot) + \sum_{i=0}^{l} d_i(t,\cdot)u^i(t,\cdot)$ in $L_2(R^m)$. Let $c_0 = \text{lub}\{c(x) : x \in R^m\}$. It is shown that $\|\exp(\widetilde{A}t)\|_2 \leqq e^{(\lambda_0 + \eta_0 + c_0)t}$. When $c_0 < -(\lambda_0 + \eta_0)$, this a priori estimate is used to establish the existence, uniqueness, and stability of bounded, periodic, almost periodic, and compact solutions for (**). Under suitable conditions on the coefficients $d_i(t,x)$, this solution is shown to satisfy (*) in t and x. (Research of the first author was partially supported by U.S. ARO-Durham, Contract No. DA-31-124-ARO-D-271, and research of the second author was supported by U.S. ARO-Durham, Contract No. DAHC-04-67-C-0056 and NSF Grant GP-6707.) (Received February 26, 1968.)

655-112. SHUICHIRO MAEDA, University of Massachusetts, Amherst, Massachusetts 01002. *On atomistic lattices with modular extensions*.

Let $\Lambda$ be a complemented modular lattice and let S be a subset of $\Lambda$ satisfying the following conditions: (1) $0 \notin S$, (2) $a,b \in S$ implies $a \cup b \in S$, and (3) $a \in S$ and $0 < a_1 \leqq a$ imply $a_1 \in S$. Then, $L = \Lambda - S$ forms a lattice by the same order as in $\Lambda$. We call L a Wilcox lattice and we call $\Lambda$ the modular extension of L. If S has a greatest element i, then, i is called an imaginary unit for L. *Theorem* 1. Any nonmodular Wilcox lattice $L = \Lambda - S$ is irreducible. *Theorem* 2. If a Wilcox lattice $L = \Lambda - S$ is atomistic (i.e. any nonzero element is the join of atoms) and if the length of L is not 2, then its modular extension $\Lambda$ is uniquely determined up to isomorphism and then $\Lambda$ is atomistic. *Theorem* 3. If a Wilcox lattice $L = \Lambda - S$ where S is not empty is compactly atomistic (i.e. upper continuous and atomistic), then (1) its modular extension $\Lambda$ is irreducible and compactly atomistic,

(2) there exists the imaginary unit for L, and (3) L is not modular and satisfies Euclid's parallel axiom of a general form. (Received February 26, 1968.)

655-113. J. M. GREENBERG and R. C. MACCAMY, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213. On the existence of weak solutions of a nonlinear wave equation.

We discuss the existence of weak solutions of the initial boundary value problem: (E) $u_{tt} = E(u_x)u_{xx}$, $(x,t) \in R_T \overset{\text{def}}{=} \{(x,t) | 0 < x < 1, 0 < t < T\}$; (IC) $u(x,0) = u_0(x)$ and $u_t(x,0) = v_0(x)$; and (BC) $u(0,t) = 0$ and $u(1,t) = 0$, $t \in (0,T)$, where (i) $\xi \longrightarrow E(\xi)$ is a 'smooth' function on $(-\infty, \infty)$ satisfying $\inf_{\xi \in (-\infty,\infty)} E(\xi) \geqq E_0 > 0$; (ii) $u_0 \in C^4[0,1]$ and satisfies $u_0(0) = u_0(1) = u_0''(0) = u_0''(1) = 0$; and (iii) $v_0 \in C^2[0,1]$ and satisfies $v_0(0) = v_0(1) = 0$. For each fixed $T > 0$ we establish the existence of a function $u \in L_2(R_T)$ with strong $L_2(R_T)$ derivative $u_x$ and weak $L_2(R_T)$ derivative $u_t$ which satisfies the problems (E), (IC), and (BC) in the following weak sense: $\iint_{R_T} [u\, \phi_{tt} + \sigma(u_x)\phi_x] dx\, dt = \int_0^1 [-v_0(x)\phi_t(x,0) + u_0(x)\phi_t(x_0)]dx$. Here $\phi \in C^\infty(R_T)$ and has compact support in $\{x,t | 0 < x < 1, 0 \leqq t < T\}$, and $\sigma(\mu) \overset{\text{def}}{=} \int_0^\mu E(\xi)d\xi$. The existence theorem is established by a 'viscosity' method. Using results previously established by the authors for the equation $u_{tt} = E(u_x)u_x + \lambda u_{xxt}$, $\lambda > 0$, together with (IC) and (BC), the authors demonstrate that solutions of the viscous problem $u^{(\lambda)}$ converge in the sense indicated previously to the desired weak solution. (Received February 26, 1968.)

655-114. FRANK LEVIN, Rutgers University, New Brunswick, New Jersey 08903. Generating groups of nilpotent varieties.

Let $N_c$ denote the variety of all nilpotent groups of class $\leqq c$ and $d(c)$ denote the smallest value of k such that $N_c$ is generated by its k-generator groups. Then $d(c) = c - 1$, for $c \geqq 3$. This is proved by explicitly constructing a law for the $(c - 2)$-generator groups of $N_c$ which is not a law in $N_c$. (Received February 26, 1968.)

655-115. MATTHEW HACKMAN, University of Washington, Seattle, Washington 98105. A uniqueness principal for abstract Cauchy problems.

Let X be a Banach space, and let $A(t)$, $0 \leqq t < a$, be closed densely defined operators in X. Say that $A(\cdot)$ is analytic with respect to $f \in X^*$ if, on $0 \leqq t_1 \leqq \ldots \leqq t_n < a$, measurable functions $A(t_1)^* \ldots A(t_n)^* f$ with essential bounds $b_n$ exist such that the numbers $|b_n|^{1/n}/n$ are uniformly bounded. Theorem. If $A(\cdot)$ is analytic with respect to every element of a separating set $S \subset X^*$, the only solution $u(\cdot)$ of $\forall g \in X^*$, $0 \leqq t < a$, $g(u(t)) = \int_0^t g(A(s)u(s))ds$ is the trivial one. (Received February 26, 1968.)

655-116. ANDREAS ZACHARIOU, Oklahoma State University, Stillwater, Oklahoma 74074. Steenrod operations in the cohomology of algebras. Preliminary report.

The results below generalise those announced in Abstract 68T-202 (these Notices 15 (1968), 359). Let K be a commutative ring (with 1) and A an augmented (graded) algebra over K which is connected. Consider K as an A-module and let C be an A-module-free resolution of K, with boundary $\partial$ and contracting homotopy S. Let $C^* = \mathrm{Hom}_A(C,K)$ be the dual of C with boundary $\delta$, the dual of $\partial$. Let $H^{**}(A)=\mathrm{Ext}_A^{**}(K,K)$ be the cohomology of A, obtained by using $(C^*, \delta)$. Let $\tau$ be the "twist" map for $C \otimes C$ or $C^* \otimes C^*$. Theorem 1. There are maps $\Delta_i : C \longrightarrow C \otimes C$, $i = 0,1,2, \ldots$ (diagonals) -- constructed inductively by using s (and the associated contracting homotopy for $C \otimes C$) -- such that: $\Delta_0$ is a chain map; for $i > 0$, $\Delta_i$ is a chain homotopy for $\Delta_{i-1}, \tau\Delta_{i-1}$. The maps $\Delta_i$ induce maps $\upsilon_i : C^* \otimes C^* \longrightarrow C^*$ (cup-i products), such that : $\upsilon_0$ is a chain map; for $i > 0$, $\upsilon_i$ is a chain homotopy for $\upsilon_{i-1}, \tau\upsilon_{i-1}$. Finally the maps $\upsilon_i$ induce maps $Sq^i : H^{**}(A) \longrightarrow H^{**}(A)$ (Steenrod squares), which satisfy most of the properties of the $Sq^i$'s in the topological case. Of special interest is the case when $K = Z_2$ and C is the bar construction on a Hopf algebra A with commutative diagonal (See A. Zachariou, On cup-i-products in the cobar construction $F(A^*)$, Master of Science Thesis, Manchester, England, 1966). (Received February 26, 1968.)

655-117. RENU LASKAR, University of North Carolina, Chapel Hill, North Carolina 27514. Eigenvalues of the adjacency matrix of cubic lattice graphs.

A cubic lattice graph is defined to be a graph G, whose vertices are the ordered triplets on n symbols, such that two vertices are adjacent if and only if they have two coordinates in common. If $n_2(x)$ denotes the number of vertices y, which are at distance 2 from x, and A(G) denotes the adjacency matrix of G, then G has the following properties: $(P_1)$ The number of vertices is $n^3$. $(P_2)$ G is connected and regular. $(P_3)$ $n_2(x) = 3(n-1)^2$. $(P_4)$ The distinct eigenvalues of A(G) are $-3$, $3(n-1)$, $n-3$, $2n-3$. It is shown that if $n > 7$, then any graph G (with no loops and multiple edges) having the properties $(P_1)$ - $(P_4)$ must be a cubic lattice graph. An alternative characterization of cubic lattice graphs has been given by the author (J. Comb. Theory (4) 3 (1967), 386-401). (Received February 26, 1968.)

655-118. PAUL ARMINJON, University of Montreal, Montreal, Canada. An existence theorem for fundamental solutions of an abstract differential operator.

Let Y be a locally convex complete space, and A a closed operator with domain D(A) dense in Y. Suppose that the resolvent $R(\lambda,A)$ exists in the region $\Sigma : |\mathrm{Im}\ \lambda| \leqq (1/\epsilon) \log |\mathrm{Re}\ \lambda|$ with $|\lambda| \geqq N_0$, where $N_0$ and $\epsilon$ are positive constants, as well as on a Jordan arc $\Gamma$ joining $(-N_0,0)$ to $(N_0,0)$. If the operators $\{|\lambda|^{-s} \exp(\Delta|\mathrm{Im}\ \lambda|)R(\lambda)\}_{\lambda \in \Sigma\cup\Gamma}$ are equicontinuous in $\lambda$, the abstract differential operator $L = (1/i)d/dt - A$ has fundamental solutions $E \in C^k[|t| > (s+k+1)\epsilon + \Delta; \mathscr{L}(Y,Y)]$. (Received February 26, 1968.)

655-119. W. M. BOYCE, Bell Telephone Laboratories Incorporated, Murray Hill, New Jersey 07971. Computer investigation of commuting functions. Preliminary report.

This report summarizes my further development of the techniques which led to my counterexample to Dyer's commuting functions problem. (Huneke discovered the same counterexample by different techniques -- see Abstracts 67T-231 and 67T-218, these *Notices* 14 (1967), 284, 280.) The concept of an S-commuting, S-disjoint pair of continuous functions on the unit interval is presented along with algorithms for generating such pairs, starting with the basic Baxter permutations (see Math. Algorithms 2 (1967), 19-26). Data are presented to support the conjecture that in the limit $1/2\pi$ of the Baxter permutations have no fixed point; the corresponding ratio for unrestricted permutations is known to be $1/e$. (Received February 26, 1968.)

655-120. R. A. GELLAR, Columbia University and Fordham University, New York, New York. *Weighted shifts.* II: *Cyclic vectors of the backward shift.*

Let $y_0, y_1, \ldots$ be a Schauder basis of a reflexive Banach space. Assume $T^*y_i = a_i y_{i-1}$, $i \geqq 0$, (a's complex), and assume $T^*y_0 = 0$ define a bounded linear operator. Let $f = \sum_{n=0}^{\infty} b_n y_n$. *Theorem.* Assume $\overline{\lim}_n |b_n \prod_{i=1}^{n} a_i|^{1/n} < \underline{\lim}_n |\prod_{i=1}^{n} a_i|^{1/n} = I_T$ or equivalently that the analytic function defined by $f(z) = \sum_{n=0}^{\infty} (b_n \prod_{i=1}^{n} a_i) z^n$ has radius of convergence greater than $(I_T)^{-1}$. Then either f(z) is a rational function or f is cyclic (that is, $\{T^{*n}f\}$ $n \geqq 0$ spans B). R. G. Douglas, H. S. Shapiro, and A. L. Shields in *On cyclic vectors of the backward shift*, Bull. Amer. Math. Soc. 73 (1967) announced this result for $y_0, y_1, \ldots$ an orthonormal basis in Hilbert space and all $a_i = 1$. (Received February 26, 1968.)

655-121. M. N. MANOUGIAN, University of Texas, Austin, Texas. *On the convergence of a sequence of Perron integrals.*

The following theorem concerns the convergence of a sequence of Perron integrals where the integral is defined in terms of major and minor functions. *Theorem.* Let $\{f_n(x)\}$ be a sequence of Perron integrable functions on $[0,1]$ where n is a counting number, and $f_n(x) \geqq g(x)$ for each n a.e. (almost everywhere) on $[0,1]$ where g(x) is Perron integrable on $[0,1]$. Let $\lim_n f_n(x) = f(x)$ a.e. on $[0,1]$. Then f(x) is Perron integrable on $[0,1]$ and $\lim_n (P)\int_0^1 f_n(x)dx = (P)\int_0^1 f(x)dx$ on $[0,1]$ if and only if the sequence of integrals $\{(P)\int_0^1 [f_n(x) - g(x)]dx\}$ is EAC (equiabsolutely continuous) on $[0,1]$. The proof is based on Vitali's theorem on the convergence of a sequence of Lebesgue integrals [Rendiconti del Circolo Matematico di Palermo 23 (1907), 137-155]. (Received February 26, 1968.)

655-122. R. A. NORTHCUTT, University of Texas, Austin, Texas. *Solutions to a Perron type nonlinear integral equation.*

*Theorem.* *Hypothesis* 1. $f(t,y(t))$ is a Perron integrable function in t on $I = \{x \mid 0 \leqq x \leqq 1\}$ for y(t) continuous in t on I. *Hypothesis* 2. $f(t,y(t))$ is continuous in y for t almost everywhere on I. *Hypothesis* 3. There is a g(t) which is Perron integrable on I such that $f(t,y(t)) \geqq g(t)$ for t almost everywhere on I. *Hypothesis* 4. $\{(P)\int_0^x [f(t,y(t)) - g(t)]\,dt\}$ is an equiabsolutely continuous set on I for all y(t). *Conclusion.* There is a continuous and locally absolutely continuous function $y^*(x)$ on I such that $y^*(x) = (P)\int_0^x f(t,y^*(t))dt$. The proof follows from Ascoli's Theorem on a uniformly bounded set of equicontinuous functions on I and from the theorem *On convergence of a sequence of Perron integrals* by M. N. Manougian (see Abstract 655-121). y(x) can be interpreted as a (vector) function in $S^k$, the Euclidean space of k dimensions, where k is a counting number greater than 1. The set of solutions $\{y^*\}$ is a uniformly bounded equicontinuous set of functions on I and every limit function $\bar{y}(x)$ of a sequence of solutions $\{y_n^*\}$ is a solution of the given integral equation. (Received February 26, 1968.)

# The April Meeting in Chicago
## April 16-20, 1968

656-1. SATOSHI SUZUKI, Queen's University, Kingston, Ontario, Canada. Formally projective modules and ideal adic free modules.

Terminologies are taken from Grothendieck's EGA IV and Suzuki's papers in J. Math. Kyoto Univ. (2) 2 (1963) and (3) 5(1966). We always consider modules with ideal adic topologies. We compare formally projective modules and ideal adic free modules more deeply than we did on one of cited papers above and prove the flatness of complete formally projective modules in our case. Meanwhile we prove the separatedness of modules which are obtained by finite coefficients extensions of complet formally projective modules. (Received November 13, 1967.)

656-2. G. G. JOHNSON, University of Georgia, Athens, Georgia 30601. Embedding homeomorphisms in differentiable flows.

Theorem. If f is a homeomorphism of $[0,1]$ onto $[0,1]$ which has a positive continuous derivative on $[0,1]$ and $f(x) > x$ if $x \in (0,1)$ then the following two statements are equivalent. (1) There is a homeomorphism h of $[0,1]$ onto $[0,1]$ which has a positive derivative on $[0,1]$ and a linear fractional transformation $l$ of $[0,1]$ onto $[0,1]$ where $l(x) > x$ if $x \in (0,1)$ such that $f(x) = h^{-1} \circ l \circ h(x)$ if $x \in [0,1]$. (2) There is a flow $F_t$ on $[0,1]$ such that $f = F_1$, $F_t$ has a positive continuous derivative on $[0,1]$ for each number t, $\theta(x) = \prod_{k=0}^{\infty} [f'(F_k(x))/f'(1)]$ is positive and continuous on $(0,1]$ and $\alpha(x) = \prod_{k=-1}^{-\infty} [f'(0)/f'(F_k(x))]$ is positive and continuous on $[0,1)$ and $f'(0) = 1/f'(1)$. (Received February 23, 1968.)

656-3. J. W. SPELLMANN, Emory University, Atlanta, Georgia 30322. Solutions to $U_{12} = BU$ in a Banach space.

Let S be a real Banach space. Let E(S) be the set to which B belongs if and only if B is a closed linear transformation from a subspace of S to S. If $B \in E(S)$, D(B) denotes the subset of S to which p belongs if and only if (i) p is in the domain of $B^k$ for all positive integers k and (ii) there is a number a in (1,2) and a positive integer M so that $\|B^k p\| \leqq M^a \ldots (M + k - 1)^a M^k \|p\|$ for $k = 1,2,\ldots$ . If $B \in E(S)$, $P_{D(B)}$ denotes the set of all functions g for which there is a nonnegative integer n and a sequence $p_0, p_1, \ldots, p_n$ each term of which is in D(B) so that $g(x) = p_0 + xp_1 + \ldots + x^n p_n$ if $x \geqq 0$. Let $\{T(x) | x \geqq 0\}$ be a strongly continuous, linear, nonexpansive semigroup in S. Let C denote the infinitesimal generator of T. Theorem. If A is a bounded linear transformation from S to S which commutes with C then D(AC) is a dense subset of S. Theorem 2. Suppose B is in E(S), $d > 0$, and each of g and h is a member of $P_{D(B)}$ so that $g(0) = h(0)$. Then there is a function U from $[0,d] \times [0,d]$ to S so that $U_{12} = BU$, $U(x,0) = g(x)$ if $x \in [0,d]$, and $U(0,y) = h(y)$ if $y \in [0,d]$. Theorem 2 may be extended to higher order equations, e.g. $U_{112} = BU$, $U_{123} = BU$, $U_{123} = B^2U$, etc. Analogous results w with nonconstant coefficients have also been obtained. (Received November 29, 1967.)

656-4. T. N. SRIVASTAVA, Loyola College, Montrèal, Canada. On an integral transform.

A generalization of the classical Laplace transform $\Phi(p) = p\int_0^\infty \exp(-(1/2)pt)f(t)dt$ has been given by Varma [On the generalization of a Laplace integral, Proc. Nat. Acad. Sci. India, Part A 20 (1951), 209-216] in the form $\Phi(p) = p\int_0^\infty \exp(-(1/2)pt)\cdot(pt)^{m-1/2}\omega_{k,m}(pt)f(t)dt$. Sharma [Math. Z. 89 (1965), 94-97] introduced a new integral transform in the form $\chi(p) = \int_0^\infty \exp[-(1/4)npt]$ $G^{4,n}_{m+n+2,m+n+2}((1/4)p^2t^2 \,|\, {a_1 \dots a_n;\ \alpha_1 \dots \alpha_m \atop b_1 \dots b_4;\ \beta_1 \dots \beta_{m+n-2}})\cdot f(t)dt$ where $n \geqq 0$, $m + n \geqq 2$, $\int_0^\infty x^{2b_i}f(x)dx$ exists for $i = 1,2,3,4$, $0 < m < 3$, and $|\arg p| < \min[\pi/2, ((3-m)/2)\pi]$. The object of this paper is to obtain three inversion formulas for the generalised Laplace transform and a close relationship which exists between the Laplace transform and the Sharma transform. (Received November 29, 1967.)

656-5. E. H. ANDERSON, University of North Dakota, Grand Forks, North Dakota 58201. Two-spheres which avoid $I^3$ if $I^3$ contains a p-od.

A p-od k is a generalized triod. Denote the end-points of k by $e_i$, $i = 1,\dots, p$. Let $I^3$ be the set of all points in $E^3$ whose distance from the origin is less than or equal to 1. The usual metric for $E^3$ will be denoted by $\rho$. Theorem. Suppose k is a p-od contained in $I^3$ with end-points only on $Bd(I^3)$, f is a homeomorphism of $S^2$ into $E^3$ such that $f(S^2)\cdot k = \emptyset$, $0 < \epsilon < \min\{\rho(f(S^2), k), (1/2)\rho(e_i, e_j)\}$ and T is a component of $f(S^2) - (I^3 + \epsilon)$. Then, there is a homeomorphism g of $S^2$ into $E^3$ such that $T \subset g(S^2) \subset (f(S^2) - (I^3 + \epsilon)) + ((I^3 + \epsilon) - I^3)$, $f = g$ on $g^{-1}(g(S^2) - (I^3 + \epsilon))$ and $\rho(g(S^2), k) > \epsilon$. (Received December 7, 1967.)

656-6. Sister MARY RAIMONDA ALLARD, Rosary College, River Forest, Illinois 60305, and P. P. SAWOROTNOW, Catholic University of America, Washington, D. C. An extension of the Gelfand-Mazur theorem.

Let A be a possibly noncommutative algebra over the complex field C such that the underlying linear space is locally convex and such that right multiplication $R_a : x \longrightarrow xa$ is continuous everywhere in A. If A is a division algebra and if for each x in A the resolvent function $x(\lambda) = (x - \lambda e)^{-1}$ is continuous wherever it is defined then A is isomorphic to the field of complex numbers. We need not assume that A is complete nor that inversion $(x \longrightarrow x^{-1})$ is continuous in A. The set E of all bounded complex-valued functions defined on the complex field with the product space topology furnishes an example of an algebra which satisfies the requirements of the theorem and in which resolvent functions are continuous but inversion is not continuous. (Received December 1, 1967.)

656-7. J. M. CIBULSKIS, Marquette University, Milwaukee, Wisconsin 53233. A characterization of the lattice orderings on a set which induce a given betweenness.

Let $(X, \leqq)$ be a lattice with 0 and I. Following E. Pitcher and M. F. Smiley, we define betweenness on X by (abc) if and only if $ab + bc = b = (a + b)(b + c)$. M. F. Smiley and W. R. Transue have introduced, for a fixed point $p \in X$, a relation $\leqq_p$ defined by: $a \leqq_p b$ if and only if (pab). It follows easily from a result of Padmanabhan that $\leqq_p$ is a partial ordering of X for each $p \in X$ if and only if $(X, \leqq)$ is modular. Our results include Theorem 1. $(X, \leqq_p)$ is a lattice if and only if p is a central element of $(X, \leqq)$. In such a case, $\wedge$ and $\vee$ in $(X, \leqq_p)$ are given by the following formulas: (1) $a \vee b = abp + (a + b)p'$; (2) $a \wedge b = (a + b)p + abp'$. Theorem 2. Let p belong to the center of

$(X, \leqq)$ so that $(X, \leqq_p)$ is a lattice. Then betweenness on $(X, \leqq_p)$ coincides with betweenness on $(X, \leqq)$. Theorem 3. Let $(X, \leqq')$ be an ordering of X as a lattice with zero which induces the same betweenness as $(X, \leqq)$. Then there exists a unique central element p in $(X, \leqq)$ so that $\leqq_p$ coincides with $\leqq'$. (Received November 15, 1967.)

656-8. R. V. ERICKSON, Western Michigan University and University of Michigan, Kalamazoo, Michigan 49007. Functions of finite Markov chains with discrete or continuous parameter. Preliminary report.

Necessary and sufficient conditions for an arbitrary stochastic process to be a function of a finite Markov chain have been sought by various authors. Among these Dharmadhikari (Ann. Math. Statist. 34 (1963), 1022-1032 and 1033-1041) gives sufficient conditions for the discrete parameter case and Heller (Ann. Math. Statist. 36 (1965), 1286-1291) extends these to necessary conditions as well. Leysieffer makes some progress in the continuous time case (Ann. Math. Statist. 38 (1967), 206-212). In attempting to extend Leysieffer's work, we have found a unified approach, applying equally well to discrete or continuous time, which gives necessary and sufficient conditions in both cases. This method, using nothing more than finite dimensional vector spaces, matrices, and polyhedral cones, yields three additional dividends: (1) it shows that the two cases differ in an unexpected way, (2) it introduces a new class of processes which include all those treated by the above authors, and which seem more likely to extend to the countable state space case, and finally (3) it shows how to construct all possible Markov chains of which a certain given process is a function. We also obtain necessary and sufficient conditions for a function of a finite Markov chain to be Markovian. (Received November 27, 1967.)

656-9. J. W. GREEN, University of Texas, Austin, Texas 78705. Concerning the separation of certain plane-like spaces by compact dendrons.

Suppose S is a space satisfying R. L. Moore's axioms 0, 1-4. Among the results obtained are the following: If a compact dendron M separates S but does not have infinitely many complementary domains, then M has two complementary domains U with connected boundaries such that there is an arc in M containing the boundary of the closure of U. If M is a compact dendron containing three, but not infinitely many, end points of S and each arc in M whose end points are end points of S has three complementary domains, then (1) there are three complementary domains U of M such that some arc in M contains the boundary of the closure of U and (2) every complementary domain of M has a connected boundary. (Received November 29, 1967.)

656-10. L. E. PURSELL, University of Missouri, Rolla, Missouri 65401. The invariance of the range of functions in certain rings of real functions.

We prove: Lemma. If F(X) is a ring of real functions on a nonempty set X which contains all real constants on X, G(Y) is a ring of real functions on a nonempty set Y, and $\phi : F(X) \rightarrow G(Y)$ is a ring homomorphism, then $\phi r_X = r \cdot (\phi 1_X)$ for all real numbers r, where $r_X$ is the constant on X with value r and $1_X$ is the constant on X with value 1. (Our proof is based on the proof outlined by L. Gillman and M. Jerison, Rings of continuous functions, Van Nostrand, 1960, p. 23, for the special case: F(X) is the ring C(X) of all real continuous functions on X.) Theorem. If also f in F(X) implies

$1_X/f$ in $F(X)$ iff $f(x) \neq 0$ for all x in X, $G(Y)$ contains $1_Y$, and $\phi 1_X = 1_Y$, then the range $f(X)$ contains $(\phi f)(Y)$. Hence $f \geqq 0_X$ implies $\phi f \geqq 0_Y$. Corollary. If also $\phi$ is a ring isomorphism, then $f(X) = (\phi f)(Y)$. (Received November 15, 1967.)

656-11. C. A. COPPIN. University of Texas, Austin, Texas 78756. Concerning an integral and number sets dense in an interval.

Suppose $[a,b]$ is a closed number interval and $\Delta = \{M | \bar{M} = [a,b],\ a \in M,\ b \in M\}$. D is an M-partition of $[a,b]$ means that D is a partition of $[a,b]$ and, if u is an end point of a member of D, then $u \in M$. D' is an M-refinement of D means D' is a refinement of D and D' is an M-partition of $[a,b]$. f is a g-integrable on M means that each of f and g is a real-valued function with domain including $[a,b]$ and there is a number W (denoted by $\int_M f dg$) such that, if $\epsilon > 0$, there is an M-partition D of $[a,b]$ such that, if D' is an M-refinement of D, then $|W - \sum f(x)[g(q) - g(p)]| < \epsilon$ where the sum is taken over all $[p,q]$ in D' and x is any member of $[p,q] \cap M$. If f is g integrable on some member of $\Delta$, $I(f,g) = \{W | W = \int_M f dg,\ M \in \Delta\}$. Theorem. If each of f and g is a bounded real-valued function with domain $[a,b]$, f and g have no common discontinuities from the left nor common discontinuities from the right, and $I(f,g)$ exists, then $I(f,g)$ is connected. Theorem. A member M of $\Delta$ is countable if and only if there is a real-valued function f and a real-valued function g each with domain $[a,b]$ such that f is g-integrable on M but no other member of $\Delta$. (Received November 27, 1967.)

656-12. A. BEN-ISRAEL and ABRAHAM CHARNES, Northwestern University, Evanston, Illinois 60201. An explicit solution of a special class of linear programming problems.

Consider the problem: (LP) Maximize $(c,x)$ subject to $a \leqq Ax \leqq b$ where $A = (a_{ij})$, $c = (c_j)$, $a = (a_i)$, $b = (b_i)$ $(i = 1,\dots,m;\ j = 1,\dots,n)$. Theorem. Let (LP) be feasible and possess bounded optimal solutions. Let T be any $n \times m$ matrix, with columns $t_i$ $(i = 1,\dots,m)$, satisfying $ATA = A$. Let A have full row rank. Then the optimal solutions of (LP) constitute the manifold $\sum_{i \in I_-} t_i a_i + \sum_{i \in I_+} t_i b_i + \sum_{i \in I_0} t_i(\theta_i b_i + (1 - \theta_i)a_i) + N(A)$ where $I_{-,+,0} = \{i : (c,t_i) <, >, = 0\}$, $0 \leqq \theta_i \leqq 1$ is arbitrary $(i \in I_0)$, and $N(A)$ is the null space of A. Extensions to general linear programming problems are discussed. (Received January 4, 1968.)

656-13. HARVEY KEYNES and J. B. ROBERTSON, University of California, Santa Barbara, California 93106. Generators for topological entropy and expansiveness.

Let X be a nonempty compact Hausdorff space and $\mathcal{U}$ an open cover of X. Let $N(\mathcal{U})$ denote the minimal cardinality of a subcover and $H(\mathcal{U}) = \log N(\mathcal{U})$. If $\mathcal{V}$ is another cover, then $\mathcal{U} \vee \mathcal{V} = \{U \cap V | U \in \mathcal{U},\ V \in \mathcal{V}\}$. Let $\phi$ be a homeomorphism of X and set $h(\phi,\mathcal{U}) = \lim(1/n)N(\mathcal{U} \vee \phi^{-1}\mathcal{U} \vee \dots \vee \phi^{-n+1}\mathcal{U})$. Then the topological entropy $h(\phi)$ is $\sup\{h(\phi,\mathcal{U}) | \mathcal{U}$ open cover of X$\}$. We study here the topological analog of a generator and examine the structure of such flows. Definition. A finite open cover $\mathcal{U}$ is a generator for $(X,\phi)$ if for every bisequence $(A_i)$ of elements of $\mathcal{U}$, $\bigcap_{-\infty}^{\infty} \phi^{-i}(\bar{A}_i)$ is at most one point. Theorem 1. If $(X,\phi)$ has a generator $\mathcal{U}$, then (a) $h(\phi) = h(\phi,\mathcal{U})$; (b) there exists a d-ary symbolic flow $(Y,\sigma)$ and a closed invariant subset Z of Y for which the transformation group $(X,\phi)$ is a homomorphic image of $(Z,\sigma)$. Corollary 1. If X is 0-dimensional, then $(X,\phi)$ is imbedded in $(Z,\sigma)$. Theorem 2. $(X,\phi)$ has a generator iff $(X,\phi)$ is expansive. Generalizations are given when a discrete group T acts on X or simply when a continuous map $\psi$ acts on X. In the latter case, the notion of one-sided generator is defined, and a short proof is given of the fact that homeomorphisms on infinite compact metric spaces never have one-sided generators. (Received January 4, 1968.)

656-14. W. D. L. APPLING, North Texas State University, Denton, Texas 76203. *Concerning the linearity of a certain transformation.*

U, F, $R_A$, $R_A^+$ and the notion of integral are as in Abstract 65T-320, these *Notices* 12 (1965), 609. Suppose $M \subseteq R_A$ and M has the following two properties: (1) If f is in M and k is in $R_A$ and $\int_V |k(I)| \leq \int_V |f(I)|$ for all V in F, then k is in M, and (2) if h is in $R_A^+$ and for each V in F, $g(V) = \sup\{z \mid z = w(V)$, w in $M \cap R_A^+$, $h - w$ in $R_A^+\}$, then g is in M. For each f in $R_A$, let c(f) denote the element s of M such that if q is in M and q is not s, then $\int_U |f(I) - s(I)| < \int_U |f(I) - q(I)|$. *Theorem* 1. M is a linear space iff for each f in $R_A^+$, $c[f - c(f)](U) = 0$. *Theorem* 2. If M is a linear space, then c is linear. (Received January 15, 1968.)

656-15. HIDEGORO NAKANO, Wayne State University, Detroit, Michigan 48202. *Representations of a group by transformations on its subgroups.*

For a subgroup S of a group G a system of transformations $T_u$ ($u \in G$) on S is called a representation of G on S, if $T_u T_v = T_{uv}$ for $u, v \in G$ and $xT_y = xy$ for $x, y \in S$. A subgroup H of G is called an *adjoint* of a subgroup S of G, if $S \cap H = \{e\}$ and $HS = G$. For a subgroup S of G there is a representation of G on S if and only if S has an adjoint, and there is a true representation of G on S if and only if S has an adjoint H such that $\bigcap_{x \in S} xHx^{-1} = \{e\}$. (Received January 17, 1968.)

656-16. L. J. GERSTEIN, University of California, Santa Barbara, California 93106. *Splitting quadratic forms over Hasse domains.*

For F a global field, a *Hasse domain* $\mathfrak{O}$ in F is a subring of F obtained as the intersection of almost all the valuation rings in F. This is a natural generalization of the ring of integers of an algebraic number field. *Theorem.* Given any Hasse domain $\mathfrak{O}$ in any global field F, there is a natural number $n_0$ with the property that if L is an $\mathfrak{O}$-lattice on any regular indefinite quadratic F-space, and rank $L \geq n_0$, then L has an orthogonal splitting $L = L_1 \perp L_2$. For definitions, see O. T. O'Meara, *Introduction to quadratic forms*, Academic Press, New York, 1963. The number $n_0$ is computable, and in fact $n_0 = 7$ always works when F is a function field. That no $n_0$ can be found that works simultaneously for *all* Hasse domains is seen in *Theorem.* Given any $m \in \underset{\sim}{N}$, there exists a Hasse domain $\mathfrak{O}$ in some algebraic number field and an indecomposable $\mathfrak{O}$-lattice L with rank $L > m$ and indefinite (indeed, *isotropic*) underlying quadratic space. The methods evolve from the local integral theory of quadratic forms and globalize via relationships between the genus, spinor genus, and class. (Received January 19, 1968.)

656-17. J. B. BROWN, Auburn University, Auburn, Alabama. *Connectivity, semicontinuity, and the Darboux property.*

The word "graph" means graph of a real function, and if f is a graph, the X-projection of f is the set of all abscissas of points of f. If f and g are graphs, then the statement that *g cuts f* means that g has X-projection an interval, and there are points P and Q of f, P higher than Q, such that (1) the abscissas of P and Q are in the X-projection of g, (2) every point of Cl(g) is lower than P and higher than Q, and (3) f and Cl(g) do not intersect. A graph has the Darboux property if and only if no subset of a horizontal line cuts it. Theorem. There is a disconnected graph with X-projection 0,1 which no continuous graph cuts, but if f is a graph with X-projection 0,1 and no lower semicontinuous graph cuts f, then f is connected. (Received January 22, 1968.)

656-18. WITOLD BOGDANOWICZ, The Catholic University of America, Washington, D. C. 20017. Existence and characterization of the smallest and the greatest measures generating the same Lebesgue integral.

Let $\int$ be a Lebesgue integral over a space X, i.e. $\int$ is a Daniell functional on a linear lattice L of real-valued functions on the space X such that: $f \cap 1 \in L$ if $f \in L$, and if $f_n \in L$, $\int f_n < M$ for all n and $f_n$ increasingly converges to a finite function f, then $f \in L$. Let $V = \{A \subset X : c_A \in L\}$ and $v(A) = \int c_A$ for $A \in V$. Let $M_0$ denote the family of sets being the countable unions of sets from V and let $M_\infty = \{A \subset X : A \cup B \in V \text{ if } B \in V\}$. Put $\mu_i(A) = v(A)$ if $A \in V$ and $\mu_i(A) = \infty$ if $A \in M_i \setminus V$, $i = 0$ or $\infty$. If $\mu$ is a measure over the space X, denote by $\int_\mu$ the Lebesgue integral $\int f d\mu$ restricted to the family $L_\mu$ of all finite-valued $\mu$-measurable functions with finite integral $\int |f| d\mu$. Let D denote the family of all measures $\mu$ generating the integral $\int$, that is such that $\int = \int_\mu$. Theorem 1. The set functions $\mu_0$ and $\mu_\infty$ belong to the family D of measures. Theorem 2. If $\mu$ is a measure from the family D then $\mu$ is an extension of the measure $\mu_0$ and $\mu_\infty$ is an extension of the measure $\mu$, i.e. the measure $\mu_0$ is the smallest measure generating the integral $\int$ and $\mu_\infty$ is the greatest measure generating the integral $\int$. Theorem 3. There exists one and only one measure $\mu$ generating the integral $\int$ if and only if the whole space X belongs to the family $M_0$, i.e. the space X is totally sigma-finite with respect to a measure $\mu$: $\int = \int_\mu$. (Received January 24, 1968.)

656-19. HOWARD GORMAN, University of Chicago, Chicago, Illinois. Invertibility of modules over Prüfer rings. Preliminary report.

Let L be a finite-dimensional algebra with 1 over the quotient field K of a domain R. We adopt the usual definitions for orders and for the invertibility of R-modules contained in L. We obtain the following results. Theorem. If R is a Prüfer ring, L is commutative and $(L : K) = n$, then $A^{n-1}$ is invertible for any module A. This generalizes a result of Dade, Taussky and Zassenhaus (Math. Ann. 148 (1962), 31). Theorem. If L is not quadratic over K, it contains a noninvertible module. If R is a Prüfer ring, then all modules in L are invertible if and only if L is 2-dimensional, $K \oplus$ trivial algebra or a certain generalization of the 2 by 2 triangular matrices. (Received January 25, 1968.)

656-20. A. M. FINK, Iowa State University, Ames, Iowa 50010. Almost periodic solutions to second order differential equations.

Let K be a compact set in $E^2$ and $f(t,y,z)$ be almost periodic in t, uniformly for $(y,z) \in K$. We consider the differential equation $y'' = f(t,y,y')$. Say L is in the hull of f if there is a sequence $t_n$ such that $L(t,y,z) = \lim_n f(t + t_n,y,z)$ uniformly on $E^1 \times K$. Theorem. If each function L in the hull of f is strictly increasing in y for each fixed t and z, then the existence of a solution $\phi$ of $y'' = f$ such that $(\phi(t), \phi'(t)) \in K$ for all t implies the existence of an almost periodic solution. (Received January 30, 1968.)

656-21. W. M. GREENLEE, Northwestern University, Evanston, Illinois 60201. *Singular perturbation of eigenvalues.* Preliminary report.

Let $V_1 \subset V_0 \subset H$ be complex Hilbert spaces with the inclusions algebraic and topological, $V_1$ dense in $V_0$ and in H, and the injection of $V_0$ into H compact. Let $a(u,v)$ (resp. $b(u,v)$) be a bounded bilinear form on $V_1$ (resp. $V_0$). Assume for some $\beta > 0$, $b(v,v) \geqq \beta|v|_0^2$, $v \in V_0$; that $a(v,v) \geqq 0$, $v \in V_1$; and that for $0 < \epsilon \leqq \epsilon_0$ there are $\alpha$, $\delta > 0$ such that $\epsilon a(v,v) + b(v,v) \geqq \epsilon\alpha|v|_1^2 + \delta|v|_0^2$, $v \in V_1$. Let $\lambda_n(\epsilon)$ (resp. $\lambda_n$) be the nth eigenvalue (counted in increasing order by multiplicity) of $\epsilon a(u_\epsilon,v) + b(u_\epsilon, v) = \lambda(\epsilon)(u_\epsilon, v)_H$ for all $v \in V_1$ (resp. $b(u,v) = \lambda(u,v)_H$ for all $v \in V_0$). Let B be the operator in H with domain $D(B) = \{u \in V_0 | v \longrightarrow b(u,v)$ is bounded on $V_0$ in the topology of $H\}$ and $(Bu,v)_H = b(u,v)$ for all $v \in V_0$. Denote the quadratic interpolation spaces between $V_1$ and $V_0$ by $V_\tau$, $0 \leqq \tau \leqq 1$. *Theorem. If* $\tau \in [0,1)$ *and, with the graph norm on* $D(B)$, $D(B) \subset V_\tau$ *topologically, then* $\lambda_n(\epsilon) = \lambda_n + o(\epsilon^\tau)$, $\epsilon \downarrow 0$. Together with results on eigenfunctions, this supplements work of Kato (Math. Ann., 1953, and Springer-Verlag, 1966) and Huet (Ann. Inst. Fourier (Grenoble), 1960). Applications to elliptic differential eigenvalue problems supplement results of Moser (Comm. Pure and Appl. Math., 1955) and Višik and Lyusternik (Uspehi Mat. Nauk, 1957). (Received January 29, 1968.)

656-22. WITHDRAWN.

656-23. B. R. MCDONALD, Michigan State University, East Lansing, Michigan 48823. *A class equation under row equivalence for matrices over a principal ideal domain modulo* m.

In 1955 L. E. Fuller developed a canonical set for matrices under row equivalence over a principal ideal domain modulo m. Assuming the residue class field is finite (for example the rational integers), this paper partitions the canonical matrices modulo $p^n$ into classes. Enumeration formulas are given for the number of canonical matrices in each class and for the number of matrices row equivalent to each canonical matrix. These formulas are summarized in a class equation under row equivalence for the matrices modulo $p^n$. The counting formulas are multiplicative and by the Chinese Remainder Theorem may be extended to the case modulo m. (Received January 31, 1968.)

656-24. K. E. GUSTAFSON, University of Minnesota, Minneapolis, Minnesota 55455. *Multiplicative perturbation of semigroups.*

The question of when one can multiply the infinitesimal generator A of a contraction semigroup by another operator B (bounded or unbounded) such that the product remains a generator is investigated. In particular, criteria for an accretive operator product are obtained. (Received February 2, 1968.)

656-25. D. F. ST.MARY, Iowa State University, Ames, Iowa 50010. *Oscillation and comparison theorems for a second-order linear differential equation.*

Let $p(t)$, $q(t)$ and $r(t)$ be real valued continuous functions on the interval $[a,\infty)$. *Theorem* 1. Let $r(t) > 0$ on $[a,\infty)$, $\int_a^\infty [1/r(s)]ds = \infty$ and $\int_t^\infty p(s)ds \geqq 0$ for large t, then all solutions of the differential equation $(r(t)y')' + p(t)y = 0$ oscillate if and only if there is a sequence of intervals $[a_n,b_n]$, with $a_n \longrightarrow \infty$ as $n \longrightarrow \infty$, such that the least positive eigenvalue $\lambda_n$ of the differential system $(r(t)y')' + \lambda_n p(t)y = 0$, $y(a_n) = y'(b_n) = 0$ satisfies $\lambda_n \leqq 1$, $n = 1,2,\ldots$ . *Theorem* 2. Let $\lambda$ and $\mu$ denote, resp., the least positive eigenvalues of the differential systems $(r(t)y')' + \lambda p(t)y = 0$, $y(a) = y'(b) = 0$; $(e(t)z')' + \mu q(t)z = 0$, $z(a) = z'(b) = 0$. If $\int_t^b q(s)ds \geqq |\int_t^b p(s)ds|$ for $a < t < b$, then $\mu \leqq \lambda$, and $\mu < \lambda$ unless $p(t)$ and $q(t)$ coincide. These theorems generalize results of Z. Nehari (Trans. Amer. Math. Soc. 85 (1957), 428-445). (Received February 2, 1968.)

656-26. R. D. CARMICHAEL, Duke University, Durham, North Carolina 27706. *The Paley-Wiener-Schwartz theorem for functions analytic in a half plane.* Preliminary report.

We prove the following theorem of Paley-Wiener-Schwartz type. *Theorem* 1. Let $f(z)$ be analytic in the half plane $\mathrm{Im}(z) > 0$ and continuous on the real line. Let $|f(z)| \leqq C(1 + |z|)^N e^{2\pi A|\mathrm{Im}\, z|}$ for some constant C, nonnegative integer N, and real number A. Then there is an element $U \in \mathscr{S}'$ such that $\mathrm{supp}\,(U) \subseteq (-\infty, A)$ and $f(z) = \langle U, e^{-2\pi izt} \rangle$, $\mathrm{Im}(z) > 0$. In proving the above theorem we make use of the *Theorem* 2. Let $f(z)$ be analytic in $\mathrm{Im}(z) > 0$ and continuous on the real line. Let $|f(z)| \leqq C(1 + |z|^2)^{-1} e^{2\pi A|\mathrm{Im}\, z|}$. Then there is a function $g(x)$ with support in $(-\infty, A)$ such that $g(x) \in L^2(-\infty, \infty)$, and $f(z)$ is the Fourier transform of $g(x)$ in $\mathrm{Im}(z) \geqq 0$. We note that analogous theorems hold in the half plane $\mathrm{Im}(z) < 0$. We also extend a result of Bremermann and Durand (J. Math. Phys. 2 (1961), 240-258). This extension is the *Theorem* 3. Let $U \in \mathscr{S}'$ and $\mathrm{supp}(U) \subseteq (-\infty, A)$, $A \neq \infty$. Then the function $f(z) = \langle U, e^{-2\pi izt} \rangle$ satisfies $|f(z)| \leqq C(1 + |z|)^N e^{2\pi(A+\epsilon)|\mathrm{Im}\, z|}$, $\mathrm{Im}(z) > 0$, for some constant C, nonnegative integer N, and $\epsilon > 0$. Again a similar theorem holds in $\mathrm{Im}(z) < 0$. Combining Theorem 1 and its related theorem for the lower half plane, we have the hard direction of the Paley-Wiener-Schwartz Theorem which is *Theorem* 4. Let $f(z)$ be an entire analytic function, and let $|f(z)| \leqq C(1 + |z|)^N e^{2\pi A|\mathrm{Im}\, z|}$ for some constant C, nonnegative integer N, and real number A. Then there is an element $U \in \mathscr{S}'$ such that $\mathrm{supp}\,(U) \subseteq (-A, A)$ and $f(z) = \langle U, e^{-2\pi izt} \rangle$. (Received November 6, 1967.)

656-27. R. H. BOWMAN, Vanderbilt University, Nashville, Tennessee 37202. *On the extensive derivative.*

The problems of extending extensive differentiation to higher order extensors is investigated. Three special cases are treated to motivate the definition of extensive differentiation for extensors of arbitrary order and range, and the main theorem. *Theorem.* If a class $C'$ extensor of type $(Q_1,Q_2,Q_3,Q_4,0)$ and range $M$ is defined along a parameterized arc $\gamma$ of class $C^{M+1}$, then it has a unique extensive derivative along $\gamma$. (Received February 5, 1968.)

656-28. W. T. EATON, University of Tennessee, Knoxville, Tennessee 37916. *Tameness of certain types of spheres.*

*Theorem.* A topological 2-sphere $S$ in $E^3$ is tame if for each horizontal plane $P$, $P \cap S$ is empty, is a point, or is a simple closed curve. *Definition.* A sphere $S$ is said to be pierced by a disk $D$ at arc $A \subset S$ if $\mathrm{Int}\, A \subset \mathrm{Int}\, D$, $\mathrm{Bd}\, A \subset \mathrm{Bd}\, D$ and the two components of $D - A$ lie in opposite complementary domains of $S$. *Theorem.* A topological 2-sphere $S$ in $E^3$ is tame if and only if $S$ can be pierced by a tame disk at each of its subarcs. (Received February 5, 1968.)

656-29. F. D. SENTILLES and D. C. TAYLOR, University of Missouri, Columbia, Missouri 65201. *Factorization in Banach algebras and the general strict topology.*

Let $X$ be a Banach space and let $B$ be a commutative semisimple Banach algebra of operators in $X$. The strict topology on $X$ induced by $B$ is that locally convex topology on $X$ generated by the seminorms $x \longrightarrow \|T(x)\|$ for each $T \in B$. The strict topology in this setting includes the several special cases studied by other authors, notably R. C. Buck and J. Wang. Briefly, if $B$ has an approximate unit, results completely analogous to those obtained by Buck are obtained in the general setting. More specifically, necessary and sufficient conditions are given in order that $X$ with the strict topology be a Banach or Fréchet space, complete, or barrelled. It is shown that the dual space is always complete and the equicontinuous subsets of the dual are characterized in terms of the approximate unit. Furthermore, necessary and sufficient conditions are given in order that the norm and strictly bounded sets in $X$ are the same, thus making the dual a Banach space which is a closed subspace of the dual of the Banach space $B \cdot X = \{T(x) : T \in B,\ x \in X\}$. This development is a consequence of the existence of an approximate unit in $B$ which is equivalent to the factorization of the finite subsets of $B$ by a common factor. (Received February 5, 1968.)

656-30. JÜRGEN BATT, Kent State University, Kent, Ohio 44240. *On compactness and vector measures.*

*Theorem 1.* Let $E,F$ be Banach spaces, $U_j \in L(E,F)$, $j = 0,1,2,\ldots$, and $P \subset F$ the set of all finite sums $\sum_j U_j x_j$ with $x_j$ in the unit ball $S \subset E$. Then $P$ is [weakly] conditionally compact if and only if every infinite series $\sum_j U_j x_j$, $x_j \in S$, converges weakly to an element of $F$ and each $U_j$ is [weakly] compact. We construct a sequence of $U_j \in L(c_c, l_\infty)$ for which the set $Q \subset F$ of all finite or infinite sums $\sum_j U_j x$, $x \in S$, is conditionally compact and $P$ is bounded but $P$ is not weakly conditionally compact. *Theorem 2.* Let $U$ be an additive regular set function of bounded semi-variation on the Borel-field $\mathscr{B}$ of a compact metric space T with values in L(E,F) such that every U(B), $B \in \mathscr{B}$ is weakly compact. If no subspace of F is isomorphic to $c_0$ and $E^{**}$ is separable, then the set $P \subset F$ of all finite sums $\sum_j U(B_j)x_j, B_j \in \mathscr{B}$ disjoint, $x_j \in S$, is weakly conditionally compact. We construct the example of such a U on $\mathscr{B}$ of $T = [0,1]$ with $E = F = l_1$ for which the set $Q \subset F$ of all $U(B)x, B \in \mathscr{B}, x \in S$, is conditionally compact (and P is bounded) but P is not weakly conditionally compact. This answers two questions of C. Foiaş and I. Singer (Rev. Math. Pures Appl. 5 (1960), 729-752) and extends previous results (Math. Ann. 174 (1967), 291-304). (Received February 6, 1968.)

656-31. D. P. K. BIALLAS, University of Florida, Gainesville, Florida 32601. On incidence-groups with a weak-affine structure. Preliminary report.

A set $G(\cdot, \Upsilon)$ provided with a group-structure "$\cdot$" and a certain geometrical structure "$\Upsilon$" is called an incidence-group if each mapping $G \longrightarrow G$ defined by $x \longrightarrow ax$ $(a \in G)$ is an isomorphism with respect to $\Upsilon$ (cf. [2]). If and only if G is commutative and $\Upsilon$ is the structure of weak affine space (cf. [1]) possessing an affine base, G may be coordinatized by a Veblen-Wedderburn system. This result may be considered as a generalization of a theorem due to J. Andre concerning affine planes with a group of translations. References. [1] E. Sperner, On non-desarguesian geometries, Sem. Inst. Nazionale Alta Math. 1962-1963, Anal. Alg. Geom. e Topol., vol. 2, Inst. Naz. Alta Mat., pp. 574-594. Edez. Cremonese, Rome, 1965. [2] H. Karzel, Bericht über projektive Inzidenzgruppen, Jber. Deutsch. Math.-Verein. 67 (1964), Abt. 1, 58-92. (Received February 8, 1968.)

656-32. D. W. CURTIS, Iowa State University, Indianola, Iowa 50125. Deficiency in infinite-dimensional spaces. II.

Definition. A closed subset A of a metric space X is homotopically deficient if for every $n \geq -1$, every map $f: E^{n+1} \longrightarrow X$ with $f(S^n) \cap A = \emptyset$, and every $\epsilon > 0$, there exists a map $g: E^{n+1} \longrightarrow X$ with $f = g/S^n$, $d(f,g) < \epsilon$, and $g(E^{n+1}) \cap A = \emptyset$. Theorem. (i) In metric linear spaces, finitely deficient sets are homotopically deficient. (ii) In complete metric spaces, closed countable unions of homotopically deficient sets are homotopically deficient. R. D. Anderson's characterization [Michigan Math. J. 14 (1967), 365-383] of topological infinite deficiency as Property Z, in the separable case, leads to Corollary 1. In separable infinite-dimensional Fréchet spaces, topological infinite deficiency and finite deficiency are equivalent properties and are preserved under closed countable unions. Corollary 2. Separable infinite-dimensional Fréchet space is homogeneous w.r.t. closed $\sigma$-compact sets. Proposition. Let M be a separable metric space such that the complement of every locally compact ($\sigma$-compact) subset is nonempty homotopically trivial. Then $M \not\approx F \times C$, where F is finite-dimensional and C is locally compact ($\sigma$-compact). Corollary 3. Separable infinite-dimensional Fréchet space is not homeomorphic to any product of a finite-dimensional space and a $\sigma$-compact space. (Received February 9, 1968.)

656-33. J. P. E. HODGSON, University of Pennsylvania, Philadelphia, Pennsylvania 19104. *Automorphisms of meta-stably connected manifolds.*

Let $M^m$ be a closed, connected, piecewise linear manifold of dimension m; we say M is meta-stably connected if it is connected and if $\pi_i(M) = 0$, $1 \leqq i \leqq [m/3] + 1$. The manifold $P^m$ obtained by removing a disc from M has the homotopy type of a CW-complex K of dimension $k \leqq [2m/3]$. Define the quasi-homeotopy group of M, $\mathscr{A}(M)$, to be the group of concordance classes of PL-automorphisms of M, keeping a given disc $D^m \subset M$ fixed, which are homotopic to the identity. *Theorem.* Let $M^m$ be a closed PL and meta-stably connected manifold of dimension m, then $\mathscr{A}(M) = \text{Image}\,(\mathscr{T}^{m+1}(sk) \xrightarrow{s} \mathscr{T}^{m+2}(sk))$ where $\mathscr{T}^{m+1}(sk)$ is the group of $(m + 1)$-thickenings of the suspension of K, and s is the map induced by taking the product with the unit interval. Similar results may be obtained in the smooth case but there are complications due to the fact that $\Gamma_n$ is not zero. (Received February 8, 1968.)

656-34. WITHDRAWN.

656-35. KARLHORST MEYER, University of Florida, Gainesville, Florida. *On the spinornorm.*

Let V be a n-dimensional vector space over a field K of any characteristic. In the case of char K = 2, we assume that K contains $o(K) \geqq 2n$ different elements. Let q be a quadratic form so that $V^{\perp} = 0$. A reflection $S_{\alpha_i}$ is an orthogonal transformation with a $(n - 1)$-dimensional fixspace which is orthogonal to $\alpha_i$. To every chain of vectors $(\alpha_1, \ldots, \alpha_j) = P$, one can associate $S_{\alpha_1} \ldots \cdot S_{\alpha_j} \in O_n(K,q)$. In the set of chains an equivalence relation is obtained by so-called elementary transformations. They are obtained from the three-reflection-theorem (Bachmann) and trivial extensions and reductions. *Theorem* 1. Every relation $(\alpha_1, \ldots, \alpha_{2r})$ (i.e. $S_{\alpha_1} \cdot \ldots \cdot S_{\alpha_{2r}} = 1 \in O_n$) is equivalent to the empty chain. *Theorem* 2. Different decompositions of an orthogonal transformation have equivalent chains of vectors. In this way it is possible to define the spinornorm independent of the characteristic without using the theory of Clifford-algebras: *Definition.* The Norm N(P) of any chain P is defined as $N(P) \equiv \prod_{i=1}^{j} q(\alpha_i) \bmod K^{*2}$. Let P be any chain which is associated with $U = S_{\alpha_1} \cdot \ldots \cdot S_{\alpha_j} \in O_n(K,q)$. Then the spinornorm $\theta(U)$ is defined as $\theta(U) = N(P)$. (Received February 8, 1968.)

### 656-36. A. MUKHERJEA, Eastern Michigan University, Ypsilanti, Michigan 48197. On idempotent probabilities on semigroups.

Let S be a locally compact Hausdorff (or just metric) semigroup. Then a regular (relative to open sets from outside and compact sets from inside) Borel (generated by open sets) probability $\phi$ is idempotent if $\int f(s)\phi(ds) = \iint f(st)\phi(ds)\phi(dt)$ for every continuous f with compact support (for every bounded continuous f when S is metric). We say S has condition (i) weak (L) if $Kx^{-1} = \{y \text{ in } S : yx \text{ is in } K\}$ is compact whenever K is so, (ii) (L) if for any two compact sets A and B there exists a compact $K = K(A,B)$ such that $x \notin K$ implies $xA \cap B = \emptyset$. There are corresponding conditions (R) and weak (R) referring to multiplication on the right by x and y respectively. Heble and Rosenblatt characterised idempotents $\phi$ on compact Hausdorff semigroups as the product of a normed Haar measure and two regular probabilities. Pym obtained the same result when S is locally compact Hausdorff with (L) and left cancellation. It is shown in this paper that the same result can be obtained if S is locally compact Hausdorff (or just metric) with (L) and weak (R) (or with (R) and weak (L) in any case). Since compactness in S implies the existence of (L) and (R), but not conversely (as is shown here), this result extends Heble-Rosenblatt's result. It may be remarked that the above result is obtained by showing that such a semigroup admits of an idempotent if and only if it has a compact subsemigroup. (Received February 7, 1968.)

### 656-37. R. I. SOARE, University of Illinois, Chicago, Illinois 60680. Sets with no subsets of higher degree. Preliminary report.

Theorem. There is an infinite set of natural numbers containing no subset of higher (Turing) degree. (In fact, every infinite set contains an infinite subset with this property.) This problem was first posed by W. Miller, and was brought to our attention by C. G. Jockusch, Jr. Our proof makes use of a combinatorial result formulated by C. G. Jockusch, Jr., and recently proved by F. Galvin. (Received February 8, 1968.)

### 656-38. PHILLIP ZENOR, University of Houston, Houston, Texas 77004. On the invariance of countable paracompactness under closed maps.

A topological space S is said to be weakly normal provided that if $\{H_i\}$ is a monotone decreasing sequence of closed sets with no common part and H is a closed set that does not intersect $H_1$, then there is an integer N and an open set 0 containing $H_N$ such that $\overline{0} \cap H = \emptyset$. Let $\mathscr{S}$ denote the class to which X belongs if and only if X is a $T_2$-space such that each closed subset of X is a $G_\delta$-set. Theorem. In $\mathscr{S}$ the following statements are equivalent: (a) If $X \in \mathscr{S}$ is then X is normal. (b) If $X \in \mathscr{S}$ and f is a closed mapping taking X onto Y, then Y is countably paracompact. (c) If $X \in \mathscr{S}$ then X is weakly normal. (Received February 8, 1968.)

### 656-39. CHARLES LANSKI, University of Chicago, Chicago, Illinois 60637. Nil subrings of Goldie rings are nilpotent.

Define a Goldie ring to be a ring having the ascending chain condition on left annihilators and having no infinite direct sum of left ideals. Using the results of Herstein and Small in their paper

Nil rings satisfying certain chain conditions, Canad. J. Math. 16 (1964), 771-776, one can show that given a Goldie ring R and a nil subring N which is not nilpotent, there exists a subring S of R with the following properties: (1) S is generated as a ring by elements $x_1, x_2, x_3, \ldots, x_n, \ldots$, (2) $x_1 x_2 \ldots x_k \neq 0$ for any k, (3) $x_1 x_2 \ldots x_k x_n x_{n+1} \ldots x_{n+k_0} = 0$ for any $n \leqq k$ where $k_0$ is fixed. However the existence of such a subring gives rise to a contradiction. Hence one obtains the main result that every nil subring of a Goldie ring is nilpotent. (Received February 8, 1968.)

656-40. J. P. RILEY, JR., Louisiana State University, New Orleans, Louisiana. Decompositions of $E^3$ with a compact zero-dimensional set of nondegenerate elements.

A compact 0-dimensional set C in $E^3$ is tame if there is a homeomorphism of $E^3$ onto $E^3$ taking C into a line. This paper first proves a theorem about building spheres that avoid certain sets and then applies this theorem to extending several theorems of Bing (Tame Cantor sets in $E^3$, Pacific J. Math. 11 (1961), 435-446) to compact 0-dimensional sets. Let G denote a monotone decomposition of $E^3$ whose set of nondegenerate elements H is compact and 0-dimensional, and where $\phi$ is the decomposition map. Assuming $E^3/G$ is $E^3$, this paper is mainly concerned with giving some sufficient conditions for $\phi(H)$ to be tame. Theorem. If H is a set of straight line intervals pointing in one direction, then $\phi(H)$ is tame. Theorem. If (1) $H = \bigcup_{p=1}^{\infty} H_p$ where each $\phi(H_p)$ is closed and (2) each $H_p$ has the property that if $h \in H_p$ and $\epsilon \supset 0$, there is a 2-sphere $S_h \subset E^3 - (\bigcup H_p)$ such that $h \subset \operatorname{Int} S_h \subset N(h, \epsilon)$; then $\phi(H)$ is tame. Corollary. If H is a collection of straight line intervals pointing in only a countable number of directions and whose lengths are bounded away from zero, then $\phi(H)$ is tame. Theorem. If H is as in previous corollary and C is a Cantor set whose points lie in distinct elements of H, then C is tame. (Received February 9, 1968.)

656-41. H. J. RENGGLI, University of New Mexico, Albuquerque, New Mexico 87106. Triangular dilatation and quasiconformal mapping.

Let S be a well-chained subset of the complex plane E, and let $f: S \longrightarrow E$. Let p,q,r be three points in S and let d(p,q) denote the corresponding distance. Definition. f is said to have bounded triangular dilatation (bd. tr. dil.) iff (i) f is not a constant and (ii) there is a number C, $C \geqq 1$, such that for all such triples the condition $d(p,q) \leqq d(p,r)$ implies $d(f(p), f(q)) \leqq C \cdot d(f(p), f(r))$. It is easy to show that such a function f is one-to-one and continuous on S. Furthermore it has a unique continuous extension $f^*$ to the closure $S^*$ of S such that in case S is not bounded $f^*(\infty) = \infty$ and $f^*$ is a homeomorphism of $S^*$ onto $f(S^*)$. Theorem 1. Let S be dense in a subdomain D of E. If $f: S \longrightarrow E$ has bd. tr. dil., then either the extension g of f to D or its conjugate $\bar{g}$ is quasiconformal (q.c.) in D. Conversely a q.c. mapping of a subdomain D of E into E has bd. tr. dil. on every compact subset of D. Theorem 2. An increasing homeomorphism f of the real line R onto itself is quasisymmetric iff f has bd. tr. dil. If $f: R \longrightarrow E$ has bd. tr. dil., then f(R) is a quasiline. Theorem 3. A q.c. mapping f of a half-plane H into E can be extended to a q.c. mapping of E onto itself iff f has bd. tr. dil. (To appear in Comment. Math. Helv.) (Received February 9, 1968.)

656-42. J. A. BEEKMAN, Ball State University, Muncie, Indiana 47306. Green's functions for generalized Schroedinger equations.

Let $\{X(w), s \leqq w \leqq t\}$ be a Gaussian Markov stochastic process with continuous sample functions, and such that $X(s) = x$, $X(t) = y$, with probability one. For appropriate $F[X]$'s, conditional expectations $E\{F[X]|X(s) = x, X(t) = y\}$ are shown to be equal to "sequential" integrals, defined in terms of finite-dimensional Riemann integrals. As examples of sequential integrals, the mean and covariance functions for the conditioned Wiener process are computed. A formula for $E\{F[X]|X(s) = x, X(t) = y\}$ is given in terms of a Fourier transform of $E\{G[X]|X(s) = 0\}$ for an appropriate $G[X]$. $E\{F[X]|X(s) = x, X(t) = y\}$ is shown equal to an expected value of a functional $H[X]$ over the Wiener process, conditioned by $X(0) = 0$, $X(1) = 0$. Let $V(x,t) = x^2 - f(t)x$, the forced harmonic oscillator. An appropriate conditional expectation of this potential is shown to satisfy a pair of Schroedinger equations plus Dirac delta function conditions. (Received February 12, 1968.)

656-43. R. J. DAVERMAN, University of Tennessee, Knoxville, Tennessee 37916. Deforming simple closed curves into the complement of a surface in $E^3$.

Theorem. If $S$ is a surface in $E^3$ satisfying (a) each arc of $S$ is tame and (b) each simple closed curve on $S$ can be deformed into either component of $E^3 - S$, then $S$ is tame. Definition. A 2-sphere $S$ in $E^3$ has Property D at a point $p$ of $S$ if for each $\epsilon > 0$ there exists an $\epsilon$-disk $E$ on $S$ and an annulus $A \subseteq E$ such that $p \in \operatorname{Int} E$, $\operatorname{Bd} E \subset A$ and each simple closed curve in $A$ can be deformed into $\operatorname{Int} S$. Theorem. If the sphere $S$ in $E^3$ has Property D at each point and if $\operatorname{Int} S$ is an open 3-cell, then there exists a point $q$ in $S$ such that $S$ is locally tame from $\operatorname{Int} S$ at each point of $S - q$. (Received February 12, 1968.)

656-44. J. A. DYER, Iowa State University, Ames, Iowa 50010. A generalized Grynblum condition for Schauder bases.

The following theorem is a generalization of a theorem of J. R. Retherford and C. W. McArthur in Some remarks on bases in linear topological spaces, Math. Ann. 164 (1966), 280-285. Theorem. Suppose $V$ is a linear Hausdorff space and $B = \{b_i\}_{i=1}^{\infty}$ is a sequence of nonzero elements in $V$. A sufficient condition that $B$ be a Schauder basis for its closed linear span is that for every neighborhood $U$ of the origin there exists a neighborhood $W_U$ of the origin such that if $p$ and $q$ are integers with $p < q$ and $\sum_{i=1}^{q} t_i b_i \in W_U$, then $\sum_{i=1}^{p} t_i b_i \in U$. If $\overline{spB}$ is barrelled or has the t-property this condition is also necessary. (Received February 12, 1968.)

656-45. J. R. EDWARDS, University of Utah, Salt Lake City, Utah 84112. Characterizing Werkfelder of certain classes of summability methods. Preliminary report.

In this paper, summability method from a group $G$ to a topological group $G'$ means a summability method in the sense of D. L. Prullage [Math. Z. 96 (1967)]. The problem of characterizing Werkfelder of summability methods is considered in two settings. In the first, $G$ is an ordered group and $G'$ is a topological group. In this setting a characterization is given for the Werkfelder of a class of methods which, in the special case where G and G' are the reals, includes the Cesaro methods and also weighted and Nörlund means which have all positive entries. In the second setting, G is a first countable topological group and G' is a commutative, first countable topological group. In this setting a characterization is given for the Werkfelder for a class of methods which, in the Kurtz-Tucker setting [Proc. Amer. Math. Soc. 14 (1963)], includes convergence preserving methods. (Received February 26, 1968.)

656-46. J. T. HARDY, University of Georgia, Athens, Georgia 30601. On the representability of quaternary quadratic forms as sums of squares of two linear forms.

Necessary and sufficient conditions are obtained in order that an integral quaternary quadratic form be expressible as a sum of squares of two linear forms with integral coefficients. The number of such representations is infinite, if not zero. (Received February 12, 1968.)

656-47. W. B. JOHNSON and R. A. SHIVE, JR., Iowa State University, Ames, Iowa 50010. A reducing map theorem.

Let V be a locally convex linear topological space and let L be a linear transformation of V into itself. L is called a reducing map iff there exists a local base B of convex circled neighborhoods of $\theta$ such that for each $U \in B$ there is a t, $0 \leqq t < 1$, such that $L(U) \subset tU$. Theorem. Suppose V is a separated, locally convex, sequentially complete, linear topological space. If L is a reducing map of V into V, then I - L is a linear homeomorphism of V onto V. And $(I - L)^{-1}(x) = \sum_{n=0}^{\infty} L^n(x)$. For an application of this theorem see Generalizations of a Paley-Wiener theorem by W. B. Johnson, these *Notices*, below. (Received February 12, 1968.)

656-48. W. B. JOHNSON, Iowa State University, Ames, Iowa 50010. Generalizations of a Paley-Wiener theorem.

Let V be a complete (resp. sequentially complete), locally convex, separated linear topological space. Let $\{x_k : k \in K\}$ be a linearly independent set with dense linear span (resp. sequentially dense linear span). Let $\{y_k : k \in K\}$ be a subset of V, and let T be a local base of open, convex, circled sets. A reducing map theorem (these *Notices*, R. A. Shive, Jr., and W. B. Johnson) is used to prove a Theorem. If for all U in T, there is t, $0 \leqq t < 1$, such that $\sum_{k \in F} a_k x_k \in U$ implies $\sum a_k (x_k - y_k) \in tU$, for all finite subsets of F of K and scalars $a_k$, then there is a linear homeomorphism, M, of V onto V such that $M(x_k) = y_k$, for all k in K. A similar theorem is obtained if V is assumed to be a complete linear metric space. Corollary. If $\{x_k : k \in K\}$ is a dual generalized basis (resp. Schauder basis), then $\{y_k : k \in K\}$ is a dual generalized basis (resp. Schauder basis) similar to $\{x_k : k \in K\}$. This generalizes a Paley-Wiener theorem of M. G. Arsove and R. E. Edwards [Generalized bases in topological linear spaces, Studia Math. 19 (1960), 103]. (Received February 12, 1968.)

656-49. P. W. LEWIS, University of Utah, Salt Lake City, Utah 84112. *Concerning the extension of certain set functions.*

This paper is concerned with extending some results of N. Dinculeanu [*Vector measures*, Pergamon Press, Berlin, 1967, Section 5]. Extension problems for both finitely and countably additive, vector valued set functions having finite variation are discussed. The total variation of the extension is completely characterized in terms of the variation of the original set function in the finitely additive case. In the countably additive case, following a counterexample and restatements of several of Dinculeanu's results, attention is turned to a certain uniqueness problem pertaining to Theorem 3, Section 5 of this same book. Dinculeanu asserts that this uniqueness is an open question. The main result of this note shows that one of the conditions of Theorem 3 is both necessary and sufficient to guarantee uniqueness. (Received February 12, 1968.)

656-50. W. B. R. LICKORISH and J. M. MARTIN, University of Wisconsin, Madison, Wisconsin 53706. *Collapsible triangulations of the* 3-*cell with knotted spanning* 1-*simplexes.*

Suppose that T is a triangulation of a 3-cell and $\alpha$ is a spanning 1-simplex of T. Bing has shown that if $\alpha$ has the knot type of an n-bridge knot, $n \geqq 3$, then T is not collapsible. The following theorem shows that, in a sense, Bing's result is the best possible. *Theorem.* Suppose that K is a 2-bridge knot (e.g. the trefoil, the four knot). Then there is a triangulation T of the 3-cell and a spanning 1-simplex $\alpha$ of T such that (1) the knot type of $\alpha$ is the knot type of K, and, (2) T collapses. This result can be extended to show that Bing's results on triangulations of the 3-cell whose $r$th derived subdivisions do not collapse are also the best possible. (Received February 12, 1968.)

656-51. S. G. WAYMENT, Weber State College, Ogden, Utah 84403. *Absolute continuity and the Radon theorem.*

Let C be the set of all continuous functions f from a compact Hausdorff space H into a Banach space X. Let T be a continuous linear transformation from C into a Banach space Y. Recent works by D. H. Tucker and D. J. Uherka have shown that a Reitz representation theorem holds in this setting where integration is with respect to a finitely additive operator valued measure K defined on a ring $\xi$ of sets in H. R. J. Easton has shown that relative to such a measure one can define a Lebesgue-type integral. Such an integral defines from a measure K a new measure G. This paper gives a definition of absolute continuity and shows that such a G is absolutely continuous with respect to K. The concept of an integrable function is replaced by the notion of an integrable sequence of simple functions on a set E in $\xi$. Such a sequence is said to be a slur and if B(E) is a slur for each E in $\xi$, then B is said to be a blur. If G is absolutely continuous with respect to K, then there exists a blur B so that the integral over E with respect to K gives G(E). An example is given to show that the blur cannot in general be replaced by a function. A Helley-Brey theorem and other related theorems are proved. (Received February 12, 1968.)

656-52. FRITZ ULMER, Rutgers, The State University, New Brunswick, New Jersey. *Properties of Kan extensions.*

Continuous properties of Kan functor extensions are studied. In certain cases Kan extensions can be characterized by continuous properties. (Received February 12, 1968.)

656-53. R. A. McCOY, Iowa State University, Collins, Iowa 50055. *Cells and cellularity in infinite-dimensional normed linear spaces.*

Let E be a normed linear space. An *open cell in* E is an open subset of E which is homeomorphic to E. *Theorem 1 (Monotone Union Theorem).* The union of an increasing sequence of open cells in E is an open cell in E. A closed subset C of E is a *closed cell in* E if there exists a homeomorphism from $(B_1, \mathrm{Bd}\, B_1)$ onto $(C, \mathrm{Bd}\, C)$, where $B_1$ is the closed unit ball in E. A *collar* of C is $h(B_1 - \mathrm{Int}\, B_{1/2})$, where h is some homeomorphism of $(B_1, \mathrm{Bd}\, B_1)$ onto $(C, \mathrm{Bd}\, C)$ and $B_{1/2}$ is the closed ball of radius 1/2 in E. A subset A of E is *cellular in* E if there exists a decreasing sequence, $\{C_i\}_{i=1}^{\infty}$, of closed cells in E such that $\bigcap_{i=1}^{\infty} C_i = A$ and $C_i - C_{i+1}$ contains a collar of $C_i$ for each i. In addition, if for each open set U containing A, $C_n \subset U$ for some n, then A will be called *strongly cellular in* E. A closed subset A of E is *point-like in* E if $E - A$ is homeomorphic to $E - \{0\}$. *Theorem* 2. If A is cellular in E, then A is point-like in E. The converse is false for E infinite-dimensional. *Theorem* 3. A is strongly cellular in E if and only if E/A (decomposition space) is homeomorphic to E. (Received February 13, 1968.)

656-54. B. F. HOBBS, Olivet Nazarene College, Kankakee, Illinois 60901. *Homomorphism topologies and slender groups.*

Let H and G be groups with G discrete, and let A be any nonempty subset of Hom(H,G). Then the weak topology on H induced by A, denoted t(A), is called a *homomorphism topology* on H; the topology t(Hom(H,G)) is usually denoted t(H,G). Homomorphism topologies are group topologies, and they may be used to provide a new characterization of slender groups (see R. J. Nunke, *Slender groups*, Bull. Amer. Math. Soc. 67 (1961), 274-275). Let I be an indexing set of cardinality m, where m is any infinite cardinal number less than the first cardinal number of measure not zero. Let $P = Z^I$ be the direct product group of m copies of the discrete integers Z, let $t_p$ be the product topology on P, and for each j in I define $e^j = (e_i^j)_{i \in I}$ in P by setting $e_j^j = 1$ and $e_i^j = 0$ for all $i \neq j$ in I. Call a discrete torsion-free abelian group G an m-*slender* group if every h in Hom(P,G) maps all but finitely many of the $e^j$ onto 0 in G. *Theorem.* A discrete torsion-free abelian group G is m-slender if and only if $t_p = t(P,G)$. (Received February 12, 1968.)

656-55. HARI SHANKAR, Ohio University, Athens, Ohio 45701. *Proximate orders and exceptional values of a meromorphic function.*

For notations and terminology cf. W. K. Hayman, *Meromorphic functions*, Oxford Univ. Press, New York, 1964. Let f(z) be a meromorphic and nonconstant function in the open complex plane, and let it be of finite, nonzero order $\rho$. Let $a$ be any complex constant, and let $\rho(r)$ be the Proximate order of f(z). Denote $\underline{n}$, $\overline{n}$, $\underline{N}$, and $\overline{N}$, respectively, the lim inf and lim sup as $r \to \infty$, of the ratios

$n(r,a)/r^{\rho(r)}$, $N(r,a)/r^{\rho(r)}$. It is known that $\bar{n} \leqq e\rho$. Theorem. For any integer $q \geqq 3$ and any $a$, $\bar{\bar{n}} \geqq (\rho \cdot (q - 2))/q \geqq \rho/3$, except possibly at most for $(q - 1)$ values of $a$. These exceptional values, if exist, are exceptional values of $f(z)$ in the sense of Valiron with Nevanlinna's defect $\delta(a) > 2/q$. (Received February 12, 1968.)

656-56. J. W. CANNON, University of Utah, Salt Lake City, Utah 84112. Spheres that are tame modulo tame sets.

Doyle and Hocking [Proc. Amer. Math. Soc. 11 (1960), 832-836] and Bing [Michigan Math. J. 11 (1964), 33-45] have shown that a sphere is tame if it is tame modulo a tame finite graph or tame Sierspinski curve. Theorem 1. A sphere is tame if it is tame modulo a tame, nondegenerate, locally connected continuum. A closed subset of a sphere which shares this "taming" property with tame, nondegenerate, locally connected continua is called a taming set. A recent result of C. E. Burgess is used to establish Theorem 2. A closed, 1-dimensional, nonseparating, tame subset of a sphere is a taming set if and only if it has no point as a component. The problem of characterizing taming sets is reduced to the problem of removing the word "nonseparating" from the above theorem. (Received February 15, 1968.)

656-57. HOWARD COOK, University of Houston, Houston, Texas 77004. Diagonal cutting in Cartesian products.

Theorem. The nondegenerate, hereditarily decomposable, compact, metric continuum M is chainable if, and only if, for each two points a and b of M, the diagonal in $M \times M$ weakly cuts (a,b) from (b,a). Theorem. If a and b are two points of the diadic solenoid $\Sigma_2$, the diagonal in $\Sigma_2 \times \Sigma_2$ weakly cuts (a,b) from (b,a). Theorem. There exists an hereditarily decomposable circle-like continuum M containing two points a and b such that the diagonal in $M \times M$ weakly cuts (a,b) from (b,a). (Received February 14, 1968.)

656-58. C. E. BURGESS, University of Utah, Salt Lake City, Utah 84112. Tame subsets of spheres in $E^3$.

Let K be a 1-dimensional closed subset of a 2-sphere S in $E^3$ such that $S - K$ is connected and the diameters of the components of K have a positive lower bound. Theorem. The set K is tame (i.e. K is a subset of a tame 2-sphere) if and only if it can be described with trees of topological 3-cells. Corollary. If K is tame and S is locally tame modulo K, then S is tame. This corollary has recently been extended by J. W. Cannon to the case where no component of K is a point, and it can be extended to the case where K separates S and the diameters of the components of $S - K$ have a positive lower bound. A special case of the corollary has been announced by Hosay [these *Notices* 9 (1962), 117]. (Received February 15, 1968.)

656-59. PATRICK CASSENS, University of Missouri, St. Louis, Missouri 63121, and FRANCIS REGAN, St. Louis University, St. Louis, Missouri 63103. On Lambert summability.

Let $f(t) = t/(1 - t)$. For q a positive integer let the operator $H^q = (z\, d/dz)^q$ denote q operations $(z\, d/dz)$ on some analytic function, where z and $(d/dz)$ are not commutative. Let $F(z) = \sum_{n=1}^{\infty} a_n f(b_n z^n)$ and $L(z) = \sum_{n=1}^{\infty} c_n f(z^n)$ converge uniformly for all $|z| \leqq r < 1$. Conditions are found on $\{a_n\}$ and $\{b_n\}$ so that F(z) may be expanded into a series of the form L(z). It was found that when $c_n = o(n^p)$ for some $p > 0$ and $|b_n| \leqq 1$, F(z) and its corresponding L(z) represent the same function within the unit circle. Also, if the sequences $\{a_n\}$ and $\{b_n\}$ of F(z) are such that $|b_n| \leqq 1$ and $a_n b_n = \sum_{d|n} S(n/d) \sum_{m|d} c_m$, where for some positive integer k there exist k nonnegative integers $p_0, \dots, p_{k-1}$ such that (1) $C_{p_0} - \sum c_{km}/km = s$, (2) for $t = 1, \dots, k - 1$, the $p_t$th Cesàro sum of the series $\sum_{j=1}^{m} c_{kj-t}$ is $o(n^{p_t+1})$, then if $z^*$ is any primitive kth root of 1 and q is any positive integer, $\lim_{r \to 1-} (1 - r)^{q+1} H^q F(rz^*) = s \cdot q!$. ($S(n|d)$ is as defined in Doyle, Ann. of Math. 40 (1939), 353-9.) Other generalized Lambert summability techniques are discussed. (Received February 14, 1968.)

656-60. D. F. DAWSON, North Texas State University, Denton, Texas 76203. Variation properties of sequences.

Let $\theta_0(1) = 1$, $\theta_0(p) = 0$, $p = 2,3,4,\dots$, and if k is a positive integer, $\theta_k(n) = \sum_{p=1}^{n} \theta_{k-1}(p)$, $n = 1,2,3,\dots$ . Let S denote the set of all real sequences and B denote the set of all bounded sequences in S. If k is a positive integer, let $M_k = \{x \in B : \Delta^k x_p \geqq 0,\ p = 1,2,3,\dots\}$ $D_k = \{x \in S$: there exist $y, z \in M_k$ such that $x = y - z\}$, and $BV_k = \{x \in B : \sum_{p=1}^{\infty} \theta_k(p) |\Delta^k x_p| < \infty\}$. Lemma. If $x \in BV_{k+1}$, then (1) $x \in BV_p$, $p = 1,2,\dots, k$, (2) $\theta_{j+1}(p)\Delta^j x_p \longrightarrow 0$ as $p \longrightarrow \infty$, $j = 1,2,\dots, k$, and (3) $\sum_{p=1}^{\infty} \theta_j(p) \Delta^j x_1 = x_1 - \lim_n x_n$, $j = 1,2,\dots, k + 1$. Theorem. If $x \in S$, then $x \in BV_k$ if and only if $x \in D_k$. For $k = 1$, this theorem is the well-known result that a sequence is of bounded variation (or absolutely convergent) if and only if it is the difference of two convergent nonincreasing sequences. For $k = 2$, the theorem includes some known facts concerning bounded convex sequences. (Received February 15, 1968.)

656-61. M. S. HENRY and F. M. STEIN, Colorado State University, Fort Collins, Colorado 80521. A best approximate solution of the Riccati matrix equation.

If H(x) is an $r \times r$ matrix, let $\|H(x)\| = \sum_{i,j=1}^{r} |h_{ij}(x)|$ define an absolute value function. Let Y(x) be the unique solution to the $2n \times 2n$ associated Riccati matrix equation $L[Y(x)] \equiv Y(x) - \int_a^x K(t)Y(t)dt = Y_a$, where K(x) has A(x), B(x), C(x), and $-D(x)$ as continuous $n \times n$ submatrices over the interval $[a, a + \alpha]$. If W(x) is the unique solution of the Riccati matrix equation $W'(x) + W(x)A(x) + D(x)W(x) + W(x)B(x)W(x) = C(x)$, with initial condition $W(a) = W_a$ in a specific generalized rectangle, and if $P_k(x)$ is a minimizing matrix polynomial for (*) $\max \|L[Y(x)] - L[P_k(x)]\|$, over the interval $[a, a + \alpha_1]$, $\alpha_1 \leqq \alpha$, where the coefficients in $P_k(x)$ are such that $P_k(a) = Y_a$, then the $n \times n$ matrix $Q_k(x) = P_{21}^k(x)[P_{11}^k(x)]^{-1}$, obtained from $P_k(x)$, is such that (i) $Q_k(a) = W_a$ and (ii) the expression $\|W(x) - Q_k(x)\|$ approaches zero as k becomes infinite. Furthermore $Q_k(x)$ is the best approximation to W(x) in the sense that (*) is minimized. (Received February 15, 1968.)

656-62. L. C. KURTZ, Arizona State University, Tempe, Arizona 85281, and D. H. TUCKER, University of Utah, Salt Lake City, Utah 84112. *An extended form of the mean-ergodic theorem.*

Suppose X is a reflexive Banach space and V is a continuous linear map on X into X such that $\|V^n\| \leqq M$ for $n = 0,1,2,\dots$, suppose N is the null space of I - V, R is the closure of the range of I - V, and P is the projection on X associated with N,R. Denote by C(X) the space of continuous functions from $[0,1]$ into X, and by $T_1$ the continuous linear transformation from C(X) into X given by $T_1(f) = \int_0^1 f(t)dt$. In this setting, the mean-ergodic theorem states that for $p_n(t) \cdot x = \sum_{k=0}^n \binom{n}{k} t^k (1 - t)^{n-k} V^k \cdot x$ it is true that $T_1(p_n(t) \cdot x) \rightarrow T_1(P \cdot x)$ for each $x \in X$. The main result of this paper states a much stronger type of convergence, namely, that for any bounded linear operator T from C(X) into a Banach space Y such that the generating function for T is continuous at $t = 0$ and $t = 1$, it is true that $T(p_n(t) \cdot x) \rightarrow T(P \cdot x)$ for each $x \in X$. (Received February 14, 1968.)

656-63. C. W. LEININGER, University of Dallas, Texas 75061. *Concerning the inclusion problem for a generalized Hausdorff mean.*

Let $H^{(s)}(d)$ denote the generalized Hausdorff mean defined in Abstract 67T-99 (these *Notices* 14 (1967), 158), and assume that $s_n \leqq n$, $n = 1,2,3,\dots$ . Let BV denote the space of sequences d such that $d_n = \int_{[0,1]} I^n dg$, $n = 0,1,2,\dots$, and on $[0,1]$ g is of bounded variation. The Hausdorff mean generated by d is denoted by H(d). *Theorem* 1. If $d \in BV$ and $\lim_n \prod_{k=1}^n s_k/k > 0$, then $H^{(s)}(d)$ and H(d) are equivalent. *Theorem* 2. If $d \in BV$, $\lim_n \prod_{k=1}^n s_k/k > 0$, x is a sequence and $H^{(s)}(d)$ is not H(d), then the two methods are consistent for x if and only if $x_0 \lim_n H_{n0} = 0$. *Theorem* 3. If $d \in BV$ and $H^{(s)}(d)$ and $[H(d)]^{-1}$ are conservative, then $H^{(s)}(d)$ and H(d) are equivalent. *Theorem* 4. If $d \in BV$, $\lim d = 0$ and H(d) is regular, then there is a mean $H^{(s)}(d)$ which includes H(d). (Received February 13, 1968.)

656-64. F. A. ROACH, University of Georgia, Athens, Georgia. *Continued fractions over an inner product space.* Preliminary report.

Let S denote a real inner product space. The following notation will be used: (1) $C(x,y) = 2((x,y))y - x$ for every x,y in S, (2) $I(x) = x/\|x\|^2$ for every x in S distinct from 0, and (3) $u/x = I[C(x,u)]$ for every x, u in S with $\|u\| = 1$ and $x \neq 0$. (Notice that for certain choices of S and u, u/x is the ordinary reciprocal.) If, for $n = 0,1,2,\dots$, $b_n$ is a point of S, the expression (*) $b_0 + \underline{u}/\overline{b_1} + \underline{u}/\overline{b_2} + \dots$ will be referred to as a continued fraction over S and said to converge if and only if the sequence $b_0$, $b_0 + u/b_1$, $b_0 + u/(b_1 + u/b_2)$,... converges. Theorems analogous to certain convergence theorems for continued fractions with complex elements are obtained. For example, if S is complete, $b_0 = 0$, and $\|b_n\| \geqq 2$, $n = 1,2,3,\dots$, then (*) converges and has norm not greater than 1. (Received February 14, 1968.)

656-65. WITHDRAWN.

656-66. J. L. FISHER, Radcliffe College, Cambridge, Massachusetts 02138. A torsion theory for an Abelian category.

Let $\mathscr{D}$ be a Grothendieck Ab(5) category. Subclasses $\mathscr{T}$ and $\mathscr{F}$ of the objects of $\mathscr{D}$ are defined, and the couple $(\mathscr{T},\mathscr{F})$ is shown to be a torsion theory for $\mathscr{D}$ in the sense of Dickson (Trans. Amer. Math. Soc. 121 (1966), 223-235). $\mathscr{F}$ contains an object D such that $D \xrightarrow{n} D$ has nonzero cokernel for every integer $n > 1$ if and only if there is an exact imbedding $\mathscr{A}b \longrightarrow \mathscr{D}$. If such an imbedding exists, then for every small Abelian category $\mathscr{C}$ a functor $G: \mathscr{C}^0 \longrightarrow \mathscr{A}b$ is left exact if and only if $\otimes G: [\mathscr{C},\mathscr{D}] \longrightarrow \mathscr{D}$ is exact. (Received February 16, 1968.)

656-67. J. W. ROGERS, JR., Emory University, Atlanta, Georgia 30322. Continua which contain only degenerate continuous images of plane continua.

Lemma. If f is a continuous function such that if $\theta \geqq 0$, $f(\theta) > f(\theta + 2\pi) > 0$, and K is the set of all points in the plane with polar coordinates $(f(\theta),\theta)$ where $\theta \geqq 0$, and $\overline{K}$ (the closure of K) is the continuous image of some chainable continuum (compact, connected metric space); then $\overline{K} - K$ is either degenerate or an arc. It is known that continua obtained as limits of inverse sequences satisfying Anderson and Choquet's Lemma 2 (Proc. Amer. Math. Soc. 10 (1959), 345-353) are tree-like, but contain only degenerate chainable continua. Using the above lemma it is shown that one such continuum contains only degenerate continuous images of chainable continua. Moreover, there is a plane continuum that contains only degenerate continuous images of plane continua which do not separate the plane. Similarly, it is shown that H. Cook's one-dimensional continua $M_1$ and $M_2$ (Fund. Math. 60 (1967), 241-249) contain only degenerate continuous images of plane continua. The respective properties of these continua are inherited by product spaces with factors all homeomorphic to some one of them. Thus $M_1 \times \ldots \times M_1$ (n factors) is an n-dimensional continuum which contains only degenerate continuous images of plane continua. (Received February 16, 1968.)

656-68. J. A. RENEKE, Clemson University, Clemson, South Carolina 29631. On the existence and representation of integrals.

Suppose that $\Omega$ is a set, R is a nonempty collection of subsets of $\Omega$, and D is the collection of finite nonempty subsets of R to which M belongs only in case $M^*$, the union of all the members of M, is in R and the members of M are relatively prime in R, i.e. if A and B are in M then there is no nonempty member of R which is contained in both A and B. We will assume that each nonempty A in R contains a point x such that if M is in D and A is in M then no other member of M contains x.

Let $B(\Omega, R)$ denote the closure in the space of functions from $\Omega$ to the number-plane which have bounded final sets of the linear space spanned by the characteristic functions of members of $R$ with respect to be supremum norm $|\cdot|$. We will assume then $B(\Omega, R)$ is an algebra. An *integral* on $B(\Omega, R) \times R$ is a function $K$ from $B(\Omega, R) \times R$ to the number-plane such that (1) for each $(f,A)$ in $B(\Omega, R) \times R$, $K[\ ,A]$ is linear on $B(\Omega, R)$ and $K[f,\ ]$ is *additive* on $R$, i.e. $K(f, M^*) = \sum_{H \text{ in } M} K(f, H)$ for each $M$ in $D$, and (2) there is an additive function $\lambda$ from $R$ to the nonnegative numbers such that $|K(f, A)| \leqq |1_A f|\lambda(A)$, for each $(f,A)$ in $B(\Omega, R) \times R$. This paper is concerned with the existence and representation of integrals on $B(\Omega, R) \times R$. (Received February 15, 1968.)

656-69. P. M. RICE, University of Georgia, Athens, Georgia 30601. *Free actions of $Z_4$ on $S^3$* .

G. R. Livesay proved in *Fixed-point-free involutions of the 3-sphere*, Ann. of Math. 72 (1960), 603-611, that every free action of $Z_2$ on $S^3$ is equivalent to the orthogonal action. This paper proves the same theorem for free actions of $Z_4$ on $S^3$. The proof is geometric in character and relies heavily on Livesay's result. (Received February 16, 1968.)

656-70. PIERRE BERTHIAUME, Université de Montréal, Montréal, Quebec, Canada. *The functor evaluation.*

Let $D: \mathscr{B} \to \mathscr{C}$ be a functor, $\mathscr{B}_d$ the category of all diagrams in $\mathscr{B}$, $D_d: \mathscr{B}_d \to \mathscr{C}_d$ the functor induced by D. Then D is said to be functorially codense (f.c.) iff there exists a functor D' such that $\mathscr{C} \xrightarrow{D'} \mathscr{B}_d \xrightarrow{D_d} \mathscr{C}_d \xrightarrow{\text{colim}} \mathscr{C}$ is naturally equivalent to the identity endofunctor of $\mathscr{C}$, where it suffices to assume that $\mathscr{C}$ is $(D_d \cdot D')(\mathscr{C})$-cocomplete (just write everything in words!). (1) The left adjoint $e^b: \mathscr{B} \times \mathscr{A}^* \to (\mathscr{A},\mathscr{B})$ of evaluation $e: \mathscr{A} \times (\mathscr{A},\mathscr{B}) \to \mathscr{B}$, $\mathscr{A}$ small, $\mathscr{B}$ with "enough" coproducts, is f.c. This implies: (2) Any functor $G: \mathscr{A} \to \mathscr{B}$, $\mathscr{A}$ small, which is left adequate and generates $\mathscr{B}$, is f.c., and thus in particular, the Yoneda functor is f.c. (c.f. also Lambek, *Completion of categories*, Lecture Notes in Mathematics, No. 24, Springer-Verlag, Berlin, 1966). Conversely, if $\mathscr{B}$ is also small, $e^b$ is left adequate and generates $(\mathscr{A}, \mathscr{B})$. (3) *Theorem*. Let $D: \mathscr{B} \times \mathscr{I} \to \mathscr{C}$ be f.c. and such that for all I in $\mathscr{I}$, D( ,I) has a right adjoint $D^{\#}_I$. If $\mathscr{D}$ is $(G_d \cdot D')(\mathscr{C})$-cocomplete and $H: \mathscr{D} \to \mathscr{C}$ and $G: \mathscr{B} \times \mathscr{I} \to \mathscr{D}$ are such that for all I in $\mathscr{I}$, G( ,I) is left adjoint to $D^{\#}_I \cdot H$, then H has a left adjoint (which is constructed). (1) $\Rightarrow$ Theorems 5.2 and 6.2 of M. André (*Cagegories of functors and adjoint functors*, Amer. J. Math. 88 (1966), (using (3)) $\Rightarrow$ The Kan Extension Theorem $\Rightarrow$ (1). (C.f. also P. Berthiaume, *On adjoints of functors between functor categories*, Abstract 67T-505, these *Notices* 14 (1967), 708. André's notions of globalisation and localisation are also generalized and the Yoneda lemma is proved from the existence of $e^b$. (Received February 6, 1968.)

656-71. W. R. DERRICK, University of Utah, Salt Lake City, Utah 84112. *A lower bound for the product of modules*.

Let A be a topological image of an n-cube in n-dimensional Euclidean space and $w_1, \ldots, w_n$ be n nonnegative $L_n$-integrable Borel-measurable real valued functions defined on A. *Theorem* 1. $\int_{\text{Int } A} (\prod_{i=1}^n w_i) dm \geqq \prod_{i=1}^n (\inf_{\Gamma_i} \int_c w_i ds)$, where $\Gamma_i$ is the family of arcs joining the images $A_i$ and $A_i'$ of the ith pair of opposite sides of the n-cube, and m is Lebesgue n-measure. *Theorem* 2. $\prod_{i=1}^n M(\Gamma_i) \geqq 1$, where $M(\Gamma_i)$ is the module of the family $\Gamma_i$. Theorem 1 answers and extends a conjecture posed in the plane by R. J. Duffin (J. Math. Anal. Appl. 5 (1962), 200-215), and Theorem 2 extends to n-space the well-known plane identity $M(\Gamma_1)M(\Gamma_2) = 1$. (Received February 16, 1968.)

656-72. M. R. GOPAL, Michigan Technological University, Houghton, Michigan 49931. A note on functions of bounded boundary rotation.

Let $V_k$ denote the class of functions $f(z) = z + a_2z^2 + \ldots$, which are analytic and satisfy $f'(z) \neq 0$ for $|z| < 1$ and map the unit disk onto a domain with boundary rotation $2k\pi$. (The boundary rotation of a domain with continuously differentiable boundary curve is defined as the total variation of the direction angle of the boundary tangent under a complete circuit.) O. Lehto (Ann. Acad. Sci. Fenn. A I 124 (1952)) showed that for the class $V_k$, the function $f_0(z) = (e^{it}/2k)[((1 + ze^{-it})/(1 - ze^{-it}))^k - 1]$, t real, maximizes $|a_2|$ and $|a_3|$ for each k. Using variational methods due to Goluzin (Amer. Math. Soc. Transl. (2) 18 (1961), 1-15), it is shown that the function $f_0(z)$ maximizes $|a_2|$ and $|a_3|$ for each k, and $|a_4|$ for $k \geqq 3$. (Received February 19, 1968.)

656-73. W. A. HARRIS, JR., Y. SIBUYA, and L. WEINBERG, University of Minnesota, Minneapolis, Minnesota 55455. A reduction algorithm for linear differential systems.

Consider the linear differential system (1) $A(z)dy/dz = B(z)y + f(z)$, where z is a complex variable, y and f are n-dimensional vectors, A and B are $n \times n$ matrices, and the elements of A,B, and f are holomorphic at $z = 0$. We are concerned with the existence of solutions of the differential system (1) and in the number of solutions which have prescribed regularity properties, in particular, solutions that are holomorphic or solutions that have at most a pole at $z = 0$. If $A(z) \equiv 0$, then (1) is a linear algebraic system and, if $\det A(z) \not\equiv 0$, then (1) is a differential system to which classical singular point theory is applicable. This paper considers the intermediate case, $A(z) \not\equiv 0$, $\det A(z) \equiv 0$. We develop an algebraic algorithm by which the existence of solutions can be decided--through algebraic necessary conditions-- and (i) constructed through solutions of algebraic systems of (ii) the system (1) can be algebraically reduced to a similar problem of lower order with corresponding $\det A \not\equiv 0$. Our results overlap recent results of R. J. Hanson (Funkcial. Ekvac. 10 (1967), 123-131). (Received February 19, 1968.)

656-74. C. W. CLEAVER, University of Kentucky, Lexington, Kentucky 40506. Representation of linear functionals in Orlicz spaces. Preliminary report.

Let X be a set and $\mu$ a measure defined on a $\sigma$-ring of subsets of X. A generalized Lax-Milgram theorem can be used to find a representation for linear functionals on certain Orlicz spaces, even on spaces where the Radon-Nikodym theorem does not apply. Theorem. Let $\Phi$ and $\Psi$ be complementary N-functions with the properties that $\Psi(v)$ is continuous and for each $0 < \epsilon < 1/4$ there exist constants K and $R_\epsilon$, $1 < R_\epsilon < K$, satisfying the following conditions alternatively: (a) If X is of infinite measure, then for $0 < u$, $\Phi(2u)/\Phi(u) \leqq K$ and $R_\epsilon < \phi(u)/\phi((1 - \epsilon)u)$. (b) If X is of finite measure, then $\lim\sup_{u\to\infty} \Phi(2u)/\Phi(u) \leqq K$ and $\lim\sup_{u\to\infty} \phi(u)/\phi((1 - \epsilon)u) > R_\epsilon$. Then for each linear functional F on $L_\phi$ there exists a unique $v \in L_\Psi$ such that for each $u \in L_\phi$, $F(u) = \int_X u(x)v(x)d\mu$. The theorem of McShane [Proc. Amer. Math. Soc. 1 (1950), 402-408] concerning $L_p$-spaces follows as a corollary to this theorem. (Received February 23, 1968.)

656-75. H. H. WICKE and J. M. WORRELL, JR., Sandia Corporation, P. O. Box 5800, Albuquerque, New Mexico 87115. On uniformly $\mu$-complete mappings.

The notion of a uniformly $\mu$-complete mapping mentioned in Abstracts 649-21 and 67T-695, these *Notices* 14 (1967), 820, 949, is an analogue of that of a uniformly monotonically complete mapping (Abstract 66T-333, these *Notices* 13 (1966), 511). For results related to the concept of $\mu$-spaces cf. Abstracts 648-188 and 67T-666, these *Notices* (1967), 687, 857. Theorem. A continuous mapping of a Tychonoff $\mu$-space S is uniformly $\mu$-complete if and only if for some $T_2$ bicompactification bS of S there exists a complete $\mu$-space E which is a subset of bS including S such that the inverse image of any point of the range of S is a closed subset of E. If there exists a $T_2$ bicompactification bS of S with this property, then all $T_2$ bicompactifications of S have the property. Theorem. Any Tychonoff $\mu$-space S is an open continuous image of a $T_2$ paracompact p-space which lies densely in a paracompact Čech complete space $\Sigma$ such that the inverse image of any point of S is closed in $\Sigma$. (Received February 19, 1968.)

656-76. A. K. BOSE, Clemson University, Clemson, South Carolina 29631. A note on entire functions of bounded index.

Let $f(z) \not\equiv 0$ be an entire function of bounded index with index $\leqq$ n and $c_f(z) = \operatorname{Max}\{|f(z)|, |f'(z)|, |f^{(2)}(z)|/2!, \ldots, |f^{(n)}(z)|/n!\}$. Let $\Omega_N$ denote the class of all entire functions of bounded index with index $\leqq$ N. The function $c_f(z)$, where f is a member of $\Omega_N$, satisfies the following inequality: Theorem 1. If D be a bounded set, then there exists a positive number $\mu$, depending only on D and N, such that the inequality $c_f(z) \leqq \mu c_f(w)$ is true for any two points z and w of D and any f belonging to $\Omega_N$. As an application of this inequality we get the following theorem: Theorem 2. Let $\{f_n\}_1^\infty$ be a sequence such that for each n, $f_n \in \Omega_N$: (i) $\{f_n\}_1^\infty$ is normal if and only if there is a subsequence $\{c f_{n_k}\}_1^\infty$ of the sequence $\{c f_n\}_1^\infty$ which is bounded at a single point. (ii) If $\{f_n\}_1^\infty$ converges uniformly on every compact subset of the plane to an entire function g, then g is of bounded index with index $\leqq$ N. (Received February 23, 1968.)

656-77. J. D. TARWATER, North Texas State University, Denton, Texas 76203, and R. C. ENTRINGER, University of New Mexico, Albuquerque, New Mexico 87106. Sums of complexes in torsion-free abelian groups.

Let G be a torsion-free abelian group and let d(A) be the maximum number of independent elements in the finite subset A of G. Theorem. If A and B are nonempty subsets of G with $d(B) \leqq d(A)$, then there are at least d(A) elements of A + B which have unique expressions a + b, where a is in A and b is in B. (Received February 22, 1968.)

656-78. T. B. RUSHING, University of Georgia, Athens, Georgia. Unknotting locally flat embeddings by isotopy.

Definition. Let $f: M^k \to E^n(S^n)$ be an embedding of a k-dimensional manifold M and let $D^m$, $m \leqq k$, be an m-cell such that (1) either $D^m \subset \mathring{M}$ and is locally flat in $\mathring{M}$ or $D^m \subset \dot{M}$ and is locally flat in $\dot{M}$, (2) $f|D^m$ is locally flat, and (3) $D^m$ has a neighborhood U in M such that f is locally flat at each point of $U - \mathring{D}^m$. Then $D^m$ is called an m-*cell of singularity*. *Theorem* 1. Suppose $X^k \in \{E^k, E^k_+, B^k, S^k\}$ and let $f: X^k \longrightarrow E^n$, $n - k \geq 3$, be a closed embedding such that either (1) f is locally flat modulo cells of singularity, or (2) f is locally nice (i.e. $E^n - f(X)$ is 1 - LC) and $n \geq 5$. Then there is an isotopy $e_t : E^n \twoheadrightarrow E^n$ such that $e_0$ = identity and $e_1 f$ is the inclusion $i : X \subset E^n$. *Theorem* 2. Suppose $(Y^n, X^k) \in \{(I^n, I^k), (E^n_+, E^k_+)\}$ and let $f: X^k \longrightarrow Y^n$, $n - k \geq 3$, be a proper, closed embedding such that (1) $f|\mathring{X} : \mathring{X} \longrightarrow \mathring{Y}$ are either (i) locally flat modulo cells of singularity or (ii) locally nice, and (2) $f|\dot{X} : \dot{X} \longrightarrow \dot{Y}$ are either (i) locally flat modulo cells of singularity and $n \geq 5$ or (ii) locally nice and $n \geq 6$. Then there is an isotopy $e_t : Y \twoheadrightarrow Y$ such that $e_0$ = identity and $e_1 f$ is the inclusion $i : X \subset Y$. Furthermore, if $f|\dot{X}$ is the identity, then $e_t$ is the identity on $\dot{Y}$. The proofs use recent results of Bryant and Seebeck, and Hudson. Assuming the locally flat hypotheses, we can use the techniques of Chernavskiĭ rather than the work of Bryant and Seebeck to prove both theorems in the stable range and also to prove Theorem 2 up to codimension 3 if f is the identity on $\dot{X}$. (Received February 23, 1968.)

656-79. C. D. LUSTFIELD, Ohio University, Athens, Ohio 45701. *Mass functions of bounded variation and starlikeness.*

Let B[N] denote the class of functions represented by $f(z) = z \cdot \exp\{-\int_0^{2\pi} \log(1 - e^{-it}z)d\mu(t)\}$, $|z| < 1$; where $\mu(t) \in BV[0,2\pi]$ satisfying $\mu(t) = P(t) - N(t)$, $\int_0^{2\pi} dP(t) = N + 2$, $\int_0^{2\pi} dN(t) = N$, $N \geqq 0$, and the normalizations $\mu(t) = (1/2)[(t + 0) + (t - 0)]$, $\int_0^{2\pi} \mu(t) = 2\pi$. An extension of a theorem of F. R. Keogh [Proc. London Math. Soc. (3) 20 (1959), 481-491] concerning the existence of the radial limit of arg f(z) is obtained. Necessary and sufficient conditions are determined for a function to belong to B[N] for some $N \geqq 0$. The radii of univalence and starlikeness for $f(z) \in B[N]$ are determined and shown to be equal, and a lower bound for the radius of convexity is found. Distortion theorems for $|f(z)|$ and $|f'(z)|$ are given. The bounds obtained are all sharp with the exception of the upper bound for $|f'(z)|$. (Received February 23, 1968.)

656-80. M. S. RAMANUJAN, University of Michigan, Ann Arbor, Michigan 48104. *Vector sequence spaces and perfect summability matrices of operators in Banach spaces.*

In this paper we formulate the concepts of perfectness and type m for matrices of operators between Banach spaces and characterize matrices of either of the above types; these discussions are based more on the duality theory of vector sequence spaces than on the actual conditions for the matrix to be a summability matrix of a particular kind and as such are applicable to matrices which are regular for $l_p$ ($p \geqq 1$). Typical of the definitions is the following: suppose E and F are Banach spaces and $\lambda(E)$, $\lambda(F)$ are sequence spaces over E and F, with $\lambda$ being a solid sequence space, with the topology $T_k(\lambda, \lambda^*)$ of the duality $(\lambda, \lambda^*)$. The matrix $A \equiv (A_{nk})$ of linear continuous operators on E into F satisfying $A[\lambda(E)] \subset \lambda(F)$ is said to be perfect if $\lambda(E)$ is dense in $\lambda_A$, the summability field of A, topologized in a natural way; the matrix A with $A[\lambda(E)] \subset \lambda(F)$ is said to be of type m if $\sum_n A^*_{nk} y'_n = \theta$ for each k and for $(y') \in \lambda^*(F') \Rightarrow (y') = \theta$ (the primes are used for dual spaces and * for the adjoint operators). A typical result is the following: if A is reversible then it is perfect if and only if it is of type m. (Received February 23, 1968.)

656-81. P. A. LOEB and B. WALSH, University of California, Los Angeles, California 90024. A maximal regular boundary for solutions of elliptic differential equations.

Let W be a locally compact Hausdorff space and $\mathscr{H}$ a complete presheaf of real-valued "harmonic" functions on W such that W and $\mathscr{H}$ satisfy Axioms 1,2,3 of Brelot's potential theory, along with Axiom 4: $1 \in \overline{\mathscr{H}}$. A criterion for regularity of points on ideal boundaries of W is given in terms of systems of barriers. Let $\mathscr{B}\mathscr{H}_W$ denote the bounded harmonic functions on W; a construction is given which assigns to every Banach sublattice $\mathfrak{H}$ of $\mathscr{B}\mathscr{H}_W$ an ideal boundary $\Delta_{\mathfrak{H}}$ which compactifies W. Each $\Delta_{\mathfrak{H}}$ contains a "harmonic boundary" $\Gamma_{\mathfrak{H}}$ consisting of regular points possessing systems of barriers, and $\mathfrak{H}$ is isometrically isomorphic to $\mathscr{C}(\Gamma_{\mathfrak{H}})$. The choices $\mathfrak{H} = \mathscr{B}\mathscr{H}_W$ and $\mathfrak{H} = \mathscr{B}\mathscr{H}_{W\setminus A}$ (A an outer-regular compact set) yield homeomorphic spaces $W^*_{\mathfrak{H}} = W \cup \Gamma_{\mathfrak{H}}$ and establish an isometric isomorphism of $\mathscr{B}\mathscr{H}_W$ and $\mathscr{B}\mathscr{H}_{W\setminus A}$. If W is an open Riemann surface and $\mathfrak{H}$ is the uniform closure of the bounded harmonic functions with finite Dirichlet integral, then $\Gamma_{\mathfrak{H}}$ is the harmonic part of the Royden boundary. Finally, it is shown that when $\mathscr{H}$ and $\mathscr{K}$ are harmonic presheaves such that the positive functions in $\mathscr{H}$ are superharmonic with respect to $\mathscr{K}$, then $\Gamma_{\mathscr{K}}$ is a compact open subset of $\Gamma_{\mathscr{H}}$. Thus a known isometric isomorphism of $\mathscr{B}\mathscr{H}_W \to \mathscr{B}\mathscr{H}_W$ is reestablished. (Received February 19, 1968.)

656-82. F. J. FLAHERTY, Oregon State University, Corvallis, Oregon 97331. Discs in negatively curved manifolds.

A Riemannian manifold M is said to be negatively curved if the sectional curvatures of M are not all zero and lie in the interval $[-k,0]$ for some positive k. Theorem. Let $M^{n+1}$ be a simply connected, negatively curved manifold and hence diffeomorphic $R^{n+1}$. Let $N^n$ be an orientable hypersurface in M, where n is at least two. Suppose that the eigenvalues of the second fundamental form that points into the bounded component lie in the interval $[a,b]$ $(\sqrt{k}, 2\sqrt{k})$ and $2/b$ is greater than $1/\sqrt{k}$ arc coth $a/\sqrt{k}$, then the closure of the bounded component U has the homology of a point. Moreover, if n is at least five, then U is diffeomorphic to the standard disc. (Received February 22, 1968.)

656-83. A. D. WEINSTEIN, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. Unflat bundles and metrics of positive curvature. Preliminary report.

Let G be a Lie group, H a proper closed subgroup of G, P a smooth principal G-bundle. A G-invariant metric on P induces a connection $\phi$ on P and a metric on the associated G/H-bundle $B = P/H$. If B has strictly positive curvature in this metric, then $\phi$ must be what we call an H-unflat connection. Roughly speaking, an unflat connection is one whose curvature form is very nondegenerate. For example, an $\{$identity$\}$-unflat connection on an $S^1$-bundle corresponds to a symplectic structure on the base manifold. Though this example shows that the problem of determining all bundles which admit unflat connections is extremely difficult, we can give some consequences of unflatness and can say more in specific situations. Theorem. If P admits an H-unflat connection, then: (1) $\dim G/H \leq M(\dim P/G - 1)$, where $M(k)$ is the Radon-Hurwitz number of linearly independent vector fields on $S^k$; (2) if G and P/G are compact, then some characteristic number of P is nonzero, so P admits no flat connection. We also consider in detail the case $G = SO(4)$, $H = SO(3)$, $P/G = S^4$ and show that all but one of the bundles for which B is an exotic 7-sphere admit no H-unflat connection. The omitted case is unsolved. (Received February 22, 1968.)

656-84. J. D. POWELL, University of Kentucky and Centre College, Danville, Kentucky 40422. Representation theory for locally convex$^*$-algebras.

Let A be a locally convex$^*$algebra. (G. R. Allen, On a class of locally convex algebras, Proc. London Math. Soc. 17 (1967), 91-114.) Let $A_0$ be the class of elements for which the set of powers $(\lambda x)^n$ is bounded for some $\lambda \neq 0$. Definition. Positive functionals F which satisfy the following conditions will be called admissible: (1) $\sup\{F(x^*a^*ax)/F(x^*x) : x \in A\} < \infty$ for all $a \in A_0$, and (2) for each $x \in A$ there exists $x_0 \in A_0$ such that $x' = x_0'$, where $x' = x + \alpha_F$ and $\alpha_F = \{x \in A : F(y^*x) = 0$ for all $y \in A\}$. The main theorem is as follows: Let F be an admissible positive hermitian functional on the commutative l.c.$^*$-algebra A. Then there exists a representation $a \longrightarrow T_a$ of A on a Hilbert space, $\mathscr{H}$, such that $T_a^* = T_{a^*}$ for each $a \in A_0$. Corollary. If $A_0$ is also an algebra, then the restriction of the representation to $A_0$ is a $^*$-representation of $A_0$ on $\mathscr{H}$. The definition of a representable function is extended and sufficient conditions are given for a hermitian functional on a sequentially complete commutative$^*$-algebra to be admissible. (Received February 22, 1968.)

656-85. R. S. RODRIGUEZ, University of Tennessee, Knoxville, Tennessee 37916. Angular and oricyclic cluster sets for normal meromorphic functions. Preliminary report.

Let f be normal and meromorphic in $D = \{z \mid |z| < 1\}$. Let $h(z_1,z_2)$ denote the non-Euclidean distance between $z_1,z_2 \in D$ and $D_h(a,r) = \{z \mid h(a,z) < r\}$. If $E \subseteq D$ and 1 belongs to the boundary of E, let $C(f,1,E)$ denote the cluster set of f at 1 along E. A result of W. Seidel (Trans. Amer. Math. Soc. (1) 34 (1932), 1-21) is extended as follows: Theorem 1. If $\{a_n\}_1^\infty$ and $\{b_n\}_1^\infty$ are sequences in D that converge to 1 and satisfy $h(a_n,b_n) < \delta$ for each n, then for each $\epsilon > 0$ the set $C(f,1,\bigcup a_n)\backslash C(f,1,\bigcup b_n)$ is included in the interior of $\bigcap_{m=1}^\infty \{\bigcup_{n=m}^\infty f[D_n(a_n,\epsilon)]\}$. A simple arc at 1 is angular if it is included in a Stolz angle at 1. Define the principal angular cluster set of f at 1, $\pi_A(f,1) = \bigcap\{C(f,1,\lambda) \mid \lambda$ is an angular arc at $1\}$. Define the angular cluster set of f at 1, $C_A(f,1)$, and the angular range set of f at 1, $R_A(f,1)$, in the usual way, and let $E^o$ denote the interior of E. Theorem 1 is used to prove the following: Theorem 2. $C_A(f,1) = [R_A(f,1)]^o \cup \pi_A(f,1)$. Similar results are proved for left and right oricyclic cluster sets. (Received February 23, 1968.)

656-86. G. A. BOGAR, University of Tennessee, Knoxville, Tennessee 37916. Comparison theorems for linear differential operators. Preliminary report.

In this paper we will be concerned with the differential operator L studied by Hinton (J. Differential Equations 2 (1966), 420-437). By establishing that Theorem 1 of Cičken (Izv. Vysš. Učebn. Zaved. Matematika 1962, no. 2 (27), 170-179) holds for this operator, several comparison theorems are established. This operator, being a generalization of the classical operator and an operator studied by Nehri (Trans. Amer. Math. Soc. 129 (1967), 500-517), the following theorem is a generalization of Theorem 4.1 of the above paper and Corollary 1 of Levin (Dokl. Akad. Nauk SSSR 148 (1963), 512-515 = Soviet Math. Dokl. 4 (1963), 121-124). Theorem. Let the operators $L_i$ be defined by $L_i[y] = (D_{n-1}(y))' + q_1D_{n-1}(y) + \ldots + q_{n,i}y$, $i = 1,2,3$, with $q_{n,1}(t) \leq q_{n,2}(t) \leq q_{n,3}(t)$. Then any interval of nonoscillation for $L_1$ and $L_3$ is an interval of nonoscillation for $L_2$. (Received February 23, 1968.)

656-87. J. S. BRADLEY, University of Tennessee, Knoxville, Tennessee 37916. *Conditions for the existence of conjugate points for a fourth order linear differential equation.*

For the fourth order differential equation (*) $y^{iv} - py = 0$ with p continuous, the following theorem is proved: *If p is concave down on* $[a,b]$ *and* $(b - a)^4(p(b) + p(a)) > 6^5 \cdot 40/271$, *then there is a nontrivial solution of* (*) *having two double zeros on* $[a,b]$. A theorem of A. Ju. Levin is used to obtain a similar necessary condition that a solution of (*) have two double zeros on an interval $[a,b]$. If, in addition, $p(x) > 0$ on $[a,b]$, then these conditions provide upper and lower bounds for the first conjugate point of a. For example, if $p(x) = x$, then it can be shown that the first conjugate point of zero is between 3.2 and 4.1. (Received February 23, 1968.)

656-88. ALLAN PETERSON, University of Tennessee, Knoxville, Tennessee 37919. *The distribution of zeros of fourth order differential equations.* Preliminary report.

Consider the fourth order linear differential equation $L(y) \equiv (D_3y)' + q_3D_2y + q_4y = 0$, see J. Barrett, these *Notices* 9 (1962), 469. An extremal solution of $L(y) = 0$ for $\eta_1(t)$ (first conjugate point of t) is a nontrivial solution of $L(y) = 0$ which has four zeros on $[t, \eta_1(t)]$. The functions $r_{ij}(t)$ for $i + j = 4$, $r_{ijk}(t)$ for $i + j + k = 4$, and $r_{1111}(t)$ have been defined by R. G. Aliev [Izv. Vysš. Učebn. Zaved. Matematika 1964, no. 5 (42), 3-7] as extensions of the nonoscillation numbers of Azbelev and Caljuk [Mat. Sb. 51 (93) 1960, no. 4, 476-486; Amer. Math. Soc. Transl. (2) 42 (1964), 233-245]. Analogous definitions are given for the functions $Z_{ij}(t)$, $Z_{ijk}(t)$, and $Z_{1111}(t)$, with the additional requirement that the first zero be at $x = t$. Let $Z_1(t) \equiv \max[Z_{31}(t), Z_{13}(t), Z_{22}(t)]$. The distribution of the zeros of extremal solutions of $L(y) = 0$ for $\eta_1(t)$ is investigated under the conditions $\eta_1(t) < Z_1(t)$ and $\eta_1(t) = Z_1(t)$. Aliev has given some ordering theorems concerning $r_{ij}(t)$, $r_{ijk}(t)$, and $r_{1111}(t)$ and a more complete picture is presented here. (Received February 23, 1968.)

656-89. D. A. SENECHALLE, University of Georgia, Athens, Georgia. *Isometry between two-dimensional subspaces implies an inner product.*

If L is a normed linear space whose dimension is at least three, then L is an inner product space if and only if for every pair $(E_1,E_2)$ of two-dimensional subspaces of L there is a linear isometry from $E_1$ onto $E_2$. (Received February 20, 1968.)

656-90. OLIVER PRETZEL, University of Illinois, Urbana, Illinois 61801. *The Mayer-Vietoris sequence for higher projective and inductive limits.*

This sequence does not necessarily hold for pushout diagrams of partially ordered sets. Conditions are discussed under which the projective or inductive sequences, or both, exist. (Received February 22, 1968.)

656-91. R. C. COURTER, Wayne State University, Detroit, Michigan 48202. *Perfect rings over which no module has a proper corational extension.* Preliminary report.

A ring R with identity is left perfect if and only if for every left R-module M there is a projective left module P such that an exact sequence $0 \longrightarrow K \longrightarrow P \longrightarrow M \longrightarrow 0$ exists, where K is a small submodule of $P$. A module $V$ is a corational extension of its factor module $V/Y$ if and only if zero is the only homomorphism of $V$ into an epimorphic image of $Y$. A module $V$ is a rational extension of its submodule $X$ if and only if zero is the only homomorphism from a submodule of $V/X$ into $V$. Theorem. The following statements are equivalent on a ring $R$ with identity: (1) No right or left $R$-module has a proper rational extension. (2) $R$ is right and left perfect and no right or left module has a proper corational extension. (3) $R$ is a direct sum of a finite set of ideals $T_i$ where, for each $i$, $T_i$ is isomorphic with an $n_i$ by $n_i$ matrix ring over a ring $S_i$ which satisfies (i) the nonunits of $S_i$ form an $(S_i)$-ideal and (ii) for each $(S_i)$-ideal $X$, the right and left socles of $S_i/X$ are nonzero. (Received February 26, 1968.)

656-92. CZESLAW OLECH, Brown University, Providence, Rhode Island 02912. On the range of unbounded vector valued measure.

Let $\Sigma$ be a $\sigma$-field of subsets of a space $S$ and $\mu: \Sigma \longrightarrow R^n$ a nonatomic measure. A. A. Liapunov proved that the range of such measure is compact and convex. This result is extended to unbounded measures, that is, $\mu: \Sigma \longrightarrow R^n \cup \{\infty\}$. Denote by $P$ the range of $\mu$. Then $P$ is convex, the closure $\bar{P}$ of $P$ does not contain a line and each compact extreme face of $\bar{P}$ is contained in $P$. (Received February 22, 1968.)

656-93. K. DEMYS, 101 Brock Avenue S., Montreal West, Province of Quebec, Canada. New roots of unity generalizing commutative algebra and function theory.

The countercomplex plane's analogue of the Wessel-Argand diagram is the four branches of two rectangular hyperbolas with the two asymptotes at ± 45° to the real axis. We call a countercomplex number one of the form $a + tb$ or $ta + b$; $|t| = 1$, $|a| > |b|$, $a,b$ real, $t^{\pm 1}$ representing a $\pm 180°$ rotation about the asymptote $\pm c(1 + t)$; and $(- t)^{\pm 1}$, about $\pm c(1 - t)$; $t^2 = 1 = t^0$; $t \neq \pm 1$. By expanding, $(e^{\theta t})^n = \cosh n\theta + t \sinh n\theta$ and $(te^{\theta t})^n = t^n(\cosh n\theta + t \sinh n\theta)$, where $n$ may be real, complex or countercomplex, and $(\cosh \theta, \sinh \theta) = (a,b)/(a^2 - b^2)^{1/2}$. Also, where $i = \sqrt{-1}$, $e^{\theta ti} = \cos \theta + ti \sin \theta$; $\operatorname{Log} ti = (\pi/2)ti = \log t + \log i$; $t^{-n} = t^n$ and $(ti)^n = t^n i^n$ only if $n$ is an integer. Thus $t$ and $i$ are not power-distributive, here indicating that $ti$ is a new vector perpendicular to the real, imaginary and counterimaginary axes. The quasi-4-space $(1,t,i,ti)$ has physical applications as does the theory of the countercomplex variable $(x + ty)$, which implies that of real and complex variables, since the roots of $t$ contain $1$ and $i$, e.g. $(+ t)^{\pm 1/2} = (1 + t \mp i(1 - t))/2$ and $(t \cosh \theta + \sinh \theta)^{1/2} = \pm t^{\pm 1/2}(\cosh \theta + t \sinh \theta)$. For real, complex or countercomplex $n$, $1^{1/n} = t^{2k/n} = \cos^2(\pi k/n) + t \sin^2(\pi k/n) + i(t - 1)\sin(\pi k/n) \cos (\pi k/n)$; $k = 1,2,3,\ldots$, furnishing new roots of unity; $\log(t^{\pm 1}) = \pm \pi i (t - 1)/2$; $\operatorname{Sin}(t\theta) = t \sin \theta$; $\cos(t\theta) = \cos \theta$. Also $1^{1/n} = (ti)^{\pm 4k/n}$ yields another set of roots. (Received February 20, 1968.)

656-94. R. P. AGARWAL, West Virginia University, Morgantown, West Virginia 26506. Certain fractional q-integrals and q-derivatives.

In a recent paper W.A.Al-Salam (Proc. Edinburgh Math. Soc. 15 (1966)) has defined a fractional q-integral operator by the basic integral $K_q^{\eta,\alpha}f(x) = (q^{-\eta}x^{\eta}/\Gamma_q(\alpha))\int_x^{\infty}(y - x)_{\alpha-1}y^{-\eta-\alpha}f(yq^{1-\alpha})d(y;q)$ where $\alpha \neq 0,-1,-2,\ldots$ . The object of the present paper is to define a fractional q-operator similar to the above corresponding to the q-analogue of $\int_a^x f(t)dt$. The operator defined is $I_q^{\eta,\alpha}f(x) = (x^{-\eta-\alpha}/\Gamma_q(\alpha))\int_0^x(x - tq)_{\alpha-1}t^{\eta}f(t)d(t;q)$. A study of this operator has been made and its relationship with certain q-analogues of the Laplace transform given by W. Hahn (Math. Nachr. 2 and 3 (1949-1950)) are established in this paper. It has been pointed out that such a study is useful in the development of certain types of identities in Combinatory Analysis. (Received February 26, 1968.)

656-95. LUDVIK JANOS, University of Florida, Gainesville, Florida. On groups of isometries.

Let X be a compact metrizable space. D denotes the set of all metrics on X inducing the topology of X, $S = X^X$ the semigroup of all continuous mappings of X into itself endowed with compact-open topology, A* the closure of $A \subset S$ in S, and $H \subset S$ the group of all homeomorphisms of X onto itself. A subgroup $G \subset H$ is called an isometry group iff there is $\rho \in D$ such that $\rho(g(x), g(y)) = \rho(x,y)$ for all $x, y \in X$ and all $g \in G$. Theorem 1. A subgroup $G \subset H$ is an isometry group iff G* is compact in S. In Abstract 68T-133, these *Notices* 15 (1968), 231, M. M. Marjanovic announced the result that $f \in H$ is an isometry iff the subgroup generated by f is an evenly continuous family. Due to the result of A. D. Wallace (Swelling lemma) this leads to the: Theorem 2. A homeomorphism $f \in H$ is an isometry iff $\Gamma(f) = \{f^n | n \text{ positive integer}\}^*$ is compact in S. (Received February 21, 1968.)

656-96. J. C. BECKER, University of Massachusetts, Amherst, Massachusetts 01002. Thom classes.

Let $\mathscr{P}$ denote the category of finite CW-pairs, and $\mathscr{P}(B(0))$ the category of triples (X,A,f) where $(X,A) \in \mathscr{P}$ and $f: X \longrightarrow B(0)$. For each spectrum F a cohomology theory H( ;F) on $\mathscr{P}(B(0))$ is constructed, which is an extension of the usual cohomology theory h( ;F) on $\mathscr{P}$. Let $\mathscr{Y} = (Y,X,r)$ be on (n - 1)-sphere bundle and form its fibrewise suspension $\Sigma(Y) = (\Sigma(Y), X, \Sigma(r))$. For $f: X \longrightarrow B(0)$ and F a ring spectrum, a Thom class for $(\mathscr{Y},f)$ is an element $u \in H^n(\Sigma(Y), X^+, f\Sigma(r); F)$ whose restriction to each fibre $(S^n,+)$ is the n-fold suspension of a unit of $H^0(*;F)$. Theorem. If f is a classifying map for $\mathscr{Y}$, then $(\mathscr{Y},f)$ has a Thom class. Let D be an F-module. We have the following Thom Isomorphism Theorem. Theorem. For $g: X \longrightarrow B(0)$, $\Phi_u : H^k(X,g;D) \longrightarrow H^{n+k}(\Sigma(Y), X^+, \mu(f \times g)\Sigma(r); D)$, by $\Phi_u(v) = u \cup \Sigma(r)^*(v)$, is an isomorphism. Here $\mu: B(0) \times B(0) \longrightarrow B(0)$ is a multiplication map for B(0). Let f be a classifying map for $\mathscr{Y}$. From the Thom class for $(\mathscr{Y},f)$, an Euler class $\chi_F \in H^n(X,f; F)$ may be defined in the usual way. Let S denote the sphere spectrum. Theorem. Suppose Dim (X) < 2n - 1. Then $\mathscr{Y}$ admits a cross-section if and only if $\chi_S = 0$. (Received February 26, 1968.)

656-97. J. E. OLSON, Mathematics Research Center, U. S. Army, University of Wisconsin, Madison, Wisconsin. A combinatorial problem on finite Abelian groups.

Let G be a finite Abelian group. Define $s = s(G)$ to be the smallest positive integer such that, for any sequence $g_1, g_2, \dots, g_s$ (repetition allowed) of group elements, there exist indices $1 \leqq i_1 < \dots < i_t \leqq s$ for which $g_{i_1} g_{i_2} \dots g_{i_t} = 1$. The problem of finding s(G) was proposed by H. Davenport (Midwestern Conference on Group Theory and Number Theory, Ohio State University, April 1966) in the following connection. If G is the class group of an algebraic number field F, then s(G) is the maximal number of prime ideals (counting multiplicity) in the decomposition of an irreducible integer in F. In this paper, s(G) is determined for all finite Abelian p-groups G and upper estimates for s(G) are obtained in the general case. (Received February 21, 1968.)

656-98. PETER CURRAN, Fordham University, Bronx, New York 10458. Cohomology of discontinuous groups.

In view of recent results of Bers, the cohomology of discontinuous groups is of interest in Riemann surface theory. The following theorem is obtained. Let $\Gamma$ be a group of Möbius transformations with generators $a_1, b_1, \dots, a_g, b_g, c_1, \dots, c_n, d_1, \dots, d_m$ and defining relations $k_1 \dots k_g c_1 \dots c_n d_1 \dots d_m = d_r^{l_r} = 1$, $r = 1, \dots, m$, where $k_i = a_i b_i a_i^{-1} b_i^{-1}$. $\Gamma$ acts on $V_{2q-2}$, the space of complex polynomials in one variable of degree $\leqq 2q - 2$, as follows (q is an integer $\geqq 2$): For $P \in V_{2q-2}$ and $\gamma \in \Gamma$, $(P\gamma)(z) = P(\gamma(z))/\gamma'(z)^{q-1}$ ($\gamma'$ is the derivative of $\gamma$). If $2g - 2 + n + \sum_r (1 - 1/l_r) > 0$, then $\dim H^1(\Gamma, V_{2q-2}) = (2g - 2 + n)(2q - 1) + 2 \sum_r [q - q/l_r]$. (Brackets denote the greatest integer function.) The dimension of $H^1$ is also found for the cases in which $2g - 2 + n + \sum_r (1 - 1/l_r) \leqq 0$. (Received February 21, 1968.)

656-99. K. F. BARTH and W. J. SCHNEIDER, Syracuse University, Syracuse, New York 13210. An asymptotic analog of the F. and M. Riesz radial uniqueness theorem.

It is known (Privalov, Randiegenschaften analytischer Funktionen, VEB Deutscher Verlag, Berlin, 1956, p. 214) that given any set M of measure zero on $|z| = 1$ (in particular for some sets of second category) there exists a nonconstant function f(z), bounded and analytic in $|z| < 1$, with the property that $\lim_{r \to 1} f(re^{i\theta}) = 0$ if $e^{i\theta} \in M$. Theorem. Let $\mu(r)$ be any arbitrary monotone decreasing function on $[0,1)$ such that $\lim_{r \to 1} \mu(r) = 0$. Let S be any set of second category on $|z| = 1$. If f(z) is any function bounded and analytic in $|z| < 1$ with the property $|f(re^{i\theta})| = o(\mu(r))$ for each $e^{i\theta} \in S$, then $f(z) \equiv 0$. The proof involves multiplying f(z) by a nonconstant analytic function g(z) with the properties: (i) $\mathrm{Max}_\theta |g(re^{i\theta})| < 1/\mu(r)$. (ii) $\lim_{r \to 1} g(re^{i\theta}) = 0$ if $e^{i\theta} \in P$ and P has measure $2\pi$ (such functions exist: see authors' Abstract 68T-67, these *Notices* 15 (1968), 210). Now $h(z) = f(z) \cdot g(z)$ has radial limit zero on $P \cup S$ which by the Privalov radial uniqueness theorem implies $h(z) \equiv 0$. One can also obtain theorems analogous to the one above for positive harmonic functions and also for meromorphic functions of bounded characteristic. (Received February 23, 1968.)

656-100. W. J. GORDON, General Motors Research Laboratories, Warren, Michigan. Cardinal expansions of bivariate and multivariate functions.

Let $\Phi_K(x)$ be a K dimensional linear space of functions defined for $x \in I$, and let $\Psi_{K'}(y)$ be a K' dimensional linear space of functions defined for $y \in I'$. $\Psi_K^*$ and $\Psi_{K'}^*$ are, respectively, the dual (conjugate) spaces of linear functionals defined on $\Phi_K(x)$ and $\Psi_{K'}(y)$. Let $\{l_i\}$ be a basis for $\Phi_K^*$ and $\{m_j\}$ a basis for $\Psi_{K'}^*$; and let $\{\phi_i(x)\}$ and $\{\psi_j(y)\}$ be the unique (cardinal) bases for $\Phi_K(x)$ and $\Psi_{K'}(y)$ which are biorthonormal with respect to the $\{l_i\}$ and $\{m_j\}$ respectively, i.e. $l_i[\phi_k(x)] = \delta_{ik}$ for $i,k = 1,2,\ldots,K$ and $m_j[\psi_l(y)] = \delta_{jl}$ for $j,l = 1,2,\ldots,K'$. For bivariate functions $f(x,y)$, let the linear operator $L_i$ be the extension of $l_i$ and let $M_j$ be the extension of $m_j$, i.e. for each fixed $y = y^* \in I'$, $L_i[f]\,|_{y=y^*} = l_i(f(x,y^*))$. The images $L_i[f]$ and $M_j[f]$ are, respectively, univariate functions of y and of x. Theorem. Let $f(x,y)$ be any function for which $L_i[f]$ $(i = 1,2,\ldots,K)$ and $M_j[f]$ $(j = 1,2,\ldots,K')$ are defined and for which the $L_i$ and the $M_j$ commute: $L_iM_j[f] = M_jL_i[f]$ for all $i = 1,2,\ldots,K$ and $j = 1,2,\ldots,K'$. Then the function $Z(x,y) = \sum_{i=1}^{K} L_i[f]\phi_i(x) + \sum_{j=1}^{K'} M_j[f]\,\psi_j(y) - \sum_{i=1}^{K}\sum_{j=1}^{K'} L_iM_j[f]\phi_i(x)\psi_j(y)$ satisfies the interpolation conditions $L_i[Z] = L_i[f]$ and $M_j[Z] = M_j[f]$ for $i = 1,2,\ldots,K$ and for $j = 1,2,\ldots,K'$. (Received February 23, 1968.)

656-101. WILLIAM JACO, University of Wisconsin, Madison, Wisconsin 53706. Constructing three-manifolds from group homomorphism.

Let S be a closed, orientable surface of genus $n > 0$ and let T be a wedge at $t_0$ of n simple closed curves. If $u_1$ and $u_2$ are homomorphisms of $\pi_1(S, s_0)$ onto $\pi_1(T,t_0)$, then the homomorphism $u_1 \times u_2$ of $\pi_1(S,s_0)$ into $\pi_1(T,t_0) \times \pi_1(T,t_0)$ is called a splitting homomorphism. Equivalence classes of splitting homomorphisms and equivalence classes of Heegaard splittings of a closed, orientable 3-manifold are defined. Each class of Heegaard splittings induces a unique class of splitting homomorphisms. Theorem 1. Let $u_1 \times u_2$ be a splitting homomorphism. Then there is a unique class of Heegaard splittings so that the induced splitting homomorphism is equivalent to $u_1 \times u_2$. Theorem 2. Conjecture B is true iff Poincaré's conjecture is true iff Conjecture D is true. (Conjectures B and D appear in "How not to prove the Poincaré Conjecture", J. Stallings, Top. Sem. Univ. Wis. (1965), Princeton Univ. Press, Princeton, N. J.) (Received February 21, 1968.)

656-102. GLENN SCHOBER, Indiana University, Bloomington, Indiana 47401. Continuity of curve functionals and a technique involving quasiconformal mapping.

Let C be a Jordan curve on the Riemann sphere with complementary domains G and $\widetilde{G}$. Define a curve functional $\lambda$ by $\lambda^{-1} = \sup_{h \in H} |D_G(h) - D_{\widetilde{G}}(h)|/[D_G(h) + D_{\widetilde{G}}(h)]$ where D is the Dirichlet integral over the indicated set and H is the family of all nonconstant functions h which are harmonic in $G \cup \widetilde{G}$, are continuous on C, and have $D_{G \cup \widetilde{G}}(h) < \infty$. For sufficiently smooth curves it is known that $\lambda$ agrees with the least positive nontrivial Fredholm eigenvalue of the classical Neumann-Poincaré integral operator of 2-dimensional potential theory. In general, $1 \leqq \lambda \leqq \infty$, and it is known that $\lambda > 1$ iff C is a quasicircle. Moreover $\lambda = \infty$ iff C is a circle. With a slight restriction on the limit curve (local rectifiability is sufficient), the functional $\lambda$ is shown to be upper semicontinuous under Fréchet convergence of curves. Under more restrictive convergence, the functional $\lambda$ is shown to be continuous through a technique using quasiconformal mapping. The technique involves mapping a fixed curve successively onto a sequence of curves converging to it by means of plane quasiconformal mappings. Conditions are developed and imposed which allow these mappings to be chosen with maximal dilatations which converge to one. (Received February 23, 1968.)

656-103. HARRY SEDINGER, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213. *One-dimensional branches of eigenvectors of nonlinear, compact operators.* Preliminary report.

Let H be a real Hilbert space and $f: H \longrightarrow H$ nonlinear, compact, and of class $C^2$. It is known that if $f(x) = \lambda x$, $\lambda \neq 0$ and not an eigenvalue of $f'(x)$, then there exists a unique branch of eigenvectors passing through x. A similar result is given. *Theorem.* Let $f(x) = \lambda x$, $x \neq 0$ and $\lambda$ not an eigenvalue of $P(x)f'(x)$ where $P(x)y = y - \|x\|^{-2}(x,y)x$. Then there exists a unique branch of eigenvectors passing through x. For $f'(x)$ selfadjoint and $\lambda$ an eigenvalue of $f'(x)$, necessary and sufficient conditions are given for the applicability of this theorem. The proof of this theorem leads to a second result. Let $f(0) = 0$ and $f'(0)$ be selfadjoint. It is known that if $\lambda$ is an eigenvalue of $f'(0)$ with multiplicity one, then $\lambda$ is a bifurcation point of f. *Theorem.* The branch of eigenvectors of f corresponding to $\lambda$ is one-dimensional. This theorem is applicable to nonlinear Sturm-Liouville Problems. (Received February 26, 1968.)

656-104. K. E. ELDRIDGE, Ohio University, Athens, Ohio 45701. *On ring structures determined by groups.*

By specifying properties of the quasiregular group of an Artinian ring, we determine some of its structure. *Proposition* 1. If the quasiregular group is simple, the ring is finite of order $2^m$ or an odd prime. *Proposition* 2. If the ring is semisimple and its quasiregular group is solvable, then the torsion free part is commutative while the noncommutative components of the torsion part are isomorphic to the ring of $2 \times 2$ matrices over GF(2). *Proposition* 3. If the quasiregular group is finitely generated and all of its elements have finite order, then the ring is finite. The proofs follow the pattern established in Abstracts 625-57 and 636-35, these *Notices* 12 (1965), 561, and 13 (1966), 585, respectively, using known results for division rings and the torsion-torsion free decomposition of Artinian rings. (Received February 26, 1968.)

656-105. S. P. SINGH, Memorial University, St.John's, Newfoundland, Canada. *A fixed point theorem in uniform space.*

The following theorem has been given in uniform space: *Theorem.* Let M be a metric space and let f be a nonexpansive mapping of M into itself which has diminishing orbital diameters. Suppose for some $x \in M$ a subsequence of the sequence $\{f^n(x)\}$ of iterates of x has limit z. Then $\{f^n(x)\}$ has limit z and z is a fixed point of f. (Received February 26, 1968.)

656-106. R. L. BHIRUD, Purdue University, Indianapolis, Indiana 46204. *Pascal's triangle and the exponential function.*

The famous Pascal's triangle of binomial coefficients seems to bear an intimate relationship with the exponential function $e^z$. For example, if we denote the binomial coefficient by $\binom{m}{n} = m!/n!(m-n)!$ for $m \geqq n \geqq 0$, then $\mathrm{Lim}_{m\to\infty} \sum_{k=0}^{m} \binom{m}{k}(z/m)^k = e^z$. The purpose of this paper is to reveal a new relationship between the Pascal's triangle and the exponential function. (Received February 26, 1968.)

656-107. D. E. SANDERSON, Iowa State University, Ames, Iowa 50010. *An infinite-dimensional Schoenflies theorem.*

Let E be an infinite-dimensional normed linear space, $B_t$ the ball in E of radius t and $S_t$ its boundary. A subset C of a space X is a collared E-cell (bicollared E-sphere) in X if there is an embedding of $B_2$ ($B_3$ - Int $B_1$) in X preserving boundary points and mapping $B_1$($S_2$) onto C. Adaptations of Morton Brown's proof of the finite-dimensional Schoenflies theorem combined with well-known results of Victor Klee produce the following results: *Theorem* 1. *Collared E-cells in E are tame.* That is, E is homogeneous with respect to such cells. *Theorem* 2. *If X is the union of two E-cells, collared in X, then X is homeomorphic to E.* *Theorem* 3. *A bicollared E-sphere S in E splits E.* That is, E - S has two components whose closures are E-cells, which by Theorem 1 must be tame. *Corollary.* If X is the union of two open sets, U and V, each homeomorphic to E, and if $U \cap V$ contains a bicollared E-sphere which separates U - V from V - U, then X is homeomorphic to E. (Received February 26, 1968.)

656-108. R. H. MOORE, University of Wisconsin, Milwaukee, Wisconsin 53201. *Approximations to nonlinear operator equations and Newton's method.*

In the Banach space X let K and $\widetilde{K}$ be continuously Fréchet differentiable, compact operators. Consider (1) $Px \equiv (I - K)x - z = 0$ and (2) $\widetilde{P}x \equiv (I - \widetilde{K})x - z = 0$, where $z \in X$ is given. Conditions are given relating $\widetilde{K}$ and K under which the presence of a suitable approximate solution $\widetilde{x}$ of (2) guarantees the existence of a neighboring (exact) solution of (1). The conditions include the case in which $\widetilde{K}$ is one of a sequence of operators $K_m$ satisfying $\|K_m x - Kx\| \to 0$ and $\|K'_m(x)h - K'(x)h\| \to 0$ for each x,h instead of operator convergence. These results are obtained using Newton's method and a generalization of the Kantorovič theorem. For, this $I - K'(x)$ and $I - K'_m(x)$ and their inverses are related using the assumed collective compactness of the operators $K_m$. The results apply in particular to numerical quadrature approximations for nonlinear integral equations in $\mathscr{L}[a,b]$. A simple illustration of this is given. (Received February 26, 1968.)

656-109. R. W. REICHHARDT, Marquette University, Milwaukee, Wisconsin 53233. *Representations of linear operators on quasicontinuous functions.*

It has been shown by Lane [Trans. Amer. Math. Soc. 89 (1958), 378-394] that every bounded and stationary linear operator on the space of all quasicontinuous functions can be represented as the sum of two mean Stieltjes integrals. The generalization of this representation theorem which omits the requirement that the operator be stationary is obtained by a method which uses Lane's result for the stationary case. Special forms of the representation are obtained under restrictions of the domain and image space of the operator. (Received February 26, 1968.)

656-110. KWANG CHUL HA, Illinois State University, Normal, Illinois 61761. *On the self dual locally compact Abelian groups with compact radical.*

*Definition.* Let G be a locally compact Hausdorff Abelian group with character group $G^{\wedge}$. G is called self dual if there is a topological isomorphism $\alpha : G \longrightarrow G^{\wedge}$ from G onto $G^{\wedge}$. *Lemma.* Let G be an LCA group, T(G) its radical. If $ny \in T(G)$ for some positive integer n, then $y \in T(G)$. Hence G/T(G) is torsion-free. *Theorem.* Let G be a locally compact Abelian group with compact radical. Then G is self dual if and only if G is the direct product $R^n \times D \times D^* \times H$, where $R^n$ is the real n-dimensional Euclidean space ($n \geqq 0$) and D is a discrete torsion-free group and D*, its dual, is compact connected, and H is a finite discrete group. (Received February 26, 1968.)

656-111. R. E. BLOCK, University of Illinois, Urbana, Illinois 61801. *Differentiably simple algebras.*

Let A be a finite-dimensional algebra (associative or not) over a field F. If D is a set of derivations of A, A is called D-simple if $A^2 \neq 0$ and no proper ideal of A is invariant under D; A is called differentiably simple (d.s.) if D-simple for some D (and hence for the set of all derivations). Jacobson noted that if F has characteristic $p > 0$, S is a simple algebra over F and $G \neq 1$ is an elementary abelian p-group, then the group ring SG ($\cong S \otimes_F B_n(F)$, where $B_n(F)$ is the truncated polynomial ring) is d.s. but not simple (and is associative or Lie etc. according as S is). *Theorem 1. If A is d.s., then either A is simple or A = some SG with S,G as above.* This generalizes Harper's result (Trans. Amer. Math. Soc. 100 (1961), 63-72) for the case A commutative associative and F algebraically closed. In the Lie case it solves a long-standing conjecture (Zassenhaus, Abh. Math. Sem. Univ. Hamburg (1940)) and gives the structure of the semisimple algebras by means of the result of Seligman (Proc. Amer. Math. Soc. 7 (1956)) reducing this to the determination of the d.s. algebras. *Theorem 2. If A is D-simple where D is closed under addition, commutation and multiplication by elements of the centroid of A, then A is $\{d\}$-simple for some d in D.* (Received February 26, 1968.)

656-112. F. M. WRIGHT, Iowa State University, Ames, Iowa, 50010. *On integration-by-parts for a weighted integral and the Lebesgue-Stieltjes integral.*

Let g be a real-valued function on the real axis of bounded variation on every closed interval, and let f be a real-valued function on the real axis bounded on every closed interval. For p and integer $\geqq 2$, for $(w_1, w_2, \ldots, w_p)$ an ordered p-tuple of real numbers with $w_1 + w_2 + \ldots + w_p = 1$, and for [a,b] a closed interval of the real axis, a weighted refinement integral $[F,(w_1,w_2,\ldots,w_p)]\int_a^b f(x)\,dg(x)$ is defined. It is shown that if this weighted integral exists, then the Lebesgue-Stieltjes integral $LS\int_{[a,b]} f(x)\,dg(x)$ exists, and a formula is presented relating these two integrals. For $(v_1,v_2,v_3)$ an ordered triple of real numbers with $v_1 + v_2 + v_3 = 1$, an integration-by-parts theorem is presented relating the weighted integrals $[F,(v_1,v_2,v_3)]\int_a^b f(x)\,dg(x)$ and $[F,(v_1,v_2,v_3)]\int_a^b g(x)\,df(x)$. This therem is used to show that if f is of bounded variation on every closed interval, then $LS\int_{[a,b]} g(x)\,df(x) = LS\int_{[a,b]} f(x)\,dg(x) - 2 \cdot [F,(1/2,1/2)]\int_a^b f(x)\,dg(x) + [f(b^+) + f(b)] \cdot g(b) - f(b) \cdot g(b^+) - [f(a) + f(a^-)] \cdot g(a) + f(a) \cdot g(a^-)$. The right side of this formula is defined if $[F,(1/2,1/2)]\int_a^b f(x)\,dg(x)$, $f(b^+)$, and $f(a^-)$ exist, and it might then be used to define $LS\int_{[a,b]} g(x)\,df(x)$. If f is of bounded variation on every interval, then $LS\int_{[a,b]} g(x)\,df(x) = -[F,(0,1,0)]\int_a^b f(x)\,dg(x) + f(b^+) \cdot g(b) - f(a^-) \cdot g(a)$. (Received February 26, 1968.)

656-113. P. H. PALMQUIST, University of Chicago, Chicago, Illinois 60637. *Pointwise limits in the category of adjoint pairs.*

The author defines a category $\mathscr{J}(\underline{A},\underline{B})$ whose objects are pairs of adjoint functors between $\underline{A}$ and $\underline{B}$. The morphisms are pairs of adjoint natural transformations. In this category, pointwise limits are defined, and conditions for their existence are given. Also, applications are given when $\underline{A} = \underline{B}$ is autonomous. (Received February 26, 1968.)

656-114. F. M. WRIGHT and J. E. BAKER, Iowa State University, Ames, Iowa 50010. *On weighted integrals and the Lebesgue-Stieltjes integral.*

Let f and g be real-valued functions on the real axis. Let $(v_1,v_2,v_3)$ be an ordered triple of real numbers such that $v_1 + v_2 + v_3 = 1$. For $[a,b]$ a closed interval of the real axis, a weighted refinement integral $[F,(v_1,v_2,v_3)]\int_a^b f(x)\,dg(x)$ is defined and examined for existence. It is shown that if $v_2 = 0$ and $[F,(v_1,v_2,v_3)]\int_a^b f(x)\,dg(x)$ exists, then $[F,(v_3,v_2,v_1)]\int_a^b g(x)\,df(x)$ exists and equals $f(b)g(b) - f(a)g(a) - [F,(v_1,v_2,v_3)]\int_a^b f(x)\,dg(x)$. If $v_2 \neq 0$, if g is of bounded variation on every closed interval, if f is bounded on $[a,b]$, and if $[F,(v_1,v_2,v_3)]\int_a^b f(x)\,dg(x)$ exists, it is shown that $[F,(v_3,v_2,v_1)]\int_a^b g(x)\,df(x)$ exists, and a formula is obtained relating these two integrals. This result is used to obtain Schaerf's theorem on integration-by-parts for the Lebesgue-Stieltjes integral $LS\int_{[a,b]} f(x)\,dg(x)$ when f and g are of bounded variation on every closed interval. If f and g are of bounded variation on every closed interval, Schaerf's theorem is used to show that $LS\int_{[a,b]} g(x)\,df(x) = -[F,(0,1,0)]\int_a^b f(x)\,dg(x) + f(b^+) \cdot g(b) - f(a^-) \cdot g(a)$. The right side of this formula is defined if the weighted integral $[F,(0,1,0)]\int_a^b f(x)\,dg(x)$ exists and both limits $f(b^+)$, $f(a^-)$ exist and are finite, and this formula could then be used to define $LS\int_{[a,b]} g(x)\,df(x)$. (Received February 26, 1968.)

656-115. P. K. KAMTHAN, University of Waterloo, Waterloo, Ontario, Canada. *Growth of Nevanlinna's characteristic function for various meromorphic functions.*

Let f(z) be a meromorphic function in the entire complex plane. In this paper, we have compared the growth of $T(r,f)$ with respect to $N(r,a)$, $n(r,a)$ and $r^pN_p(r,a)$ under various sufficient conditions, where $N_p(r,a) = \int_0^r n(x,a)\,dx/x^{p+1}$, p is a nonnegative integer. Examples have been given to show that if the sufficient conditions are violated, the corresponding result does not necessarily hold. Comparisons of the growth of $n(r,a)$ with respect to certain functions like $r^{\rho}L(r)$ or $r^{\rho(r)}$ is also derived from the growth of $T(r,f)$. For functions of integral order, our main result is $(1 + o(1))T(kr,f)/r^GN_G(r) \leqq H(G)k^{G+1}\int_1^{\infty} y^{\delta}dy/(y + k)^2 + Ik^G/(A - \epsilon)$, r is arbitrarily large. (Received February 19, 1968.)

656-116. R. D. ANDERSON, Louisiana State University, Baton Rouge, Louisiana 70803. Infinite-dimensional $\sigma$-compact sets.

Let $I^{\infty} = \prod_{j>0} I_j$ where for each $j > 0$, $I_j$ is the closed interval $[-1, 1]$. Let s be the countable infinite product of lines and let f be an imbedding of s in $I^{\infty}$ obtained by shrinking each coordinate axis in s onto the open interval $(-1, 1)$. Clearly, $B(I^{\infty}) = I^{\infty}\backslash f(s)$ is $\sigma$-compact (i.e. is the countable union of compact sets). We consider the following sets to be subsets of s with the inherited topology: (1) $\sigma = \bigcup_{i>0} \sigma_i$ where for each $i > 0$, $\sigma_i = \prod_{j>0} I_j(i)$ with $I_j(i) = [-i, i]$, and (2) for each p, $0 < p < \infty$, $M_p = \{(x_j)_{j>0} \mid \sum_{j>0}|x_j|^p < \infty\}$. Clearly $M_p$ is the space consisting of the points of the well-known space $l_p$ but with a topology inherited from s. A theorem somewhat more general than the following is proved. Theorem. Let X denote any particular $M_p$ space or $\sigma$. Then there exists a homeomorphism h of $I^{\infty}$ onto itself such that $hf(X) = B(I^{\infty})$. Using earlier results of the author [Topological properties of the Hilbert cube and the countable infinite product of open intervals, Trans. Amer. Math. Soc. 126 (1967), 200-216; On topological infinite deficiency, Michigan Math. J. 14 (1967), 365-383], it follows that for such space X, f(X) and f(s) have dual properties in $I^{\infty}$ as follows: (1) for any $\sigma$-compact set $K_1 \subset f(s)$, $f(s)\backslash K_1$ is homeomorphic to f(s) and for any $\sigma$-compact set $K_2 \subset I^{\infty}\backslash f(X)$, $f(X) \cup K_2$ is homeomorphic to f(X); (2) for any compact set $K_3 \subset I^{\infty}\backslash f(s)$, $f(s) \cup K_3$ is homeomorphic to f(s) and for any compact set $K_4$ in f(X), $f(X)\backslash K_4$ is homeomorphic to f(X). (Received February 26, 1968.)

656-117. J. T. DARWIN, JR., Auburn University, Auburn, Alabama 36830. Kernel functions for linear transformations.

For $1 < p < \infty$, let L denote the Banach space of (equivalence classes) real-valued measurable functions $f \ni \int_0^1 |f|^p < \infty$, with the usual norm. Let $H_p$ denote the Banach space of all functions $F \ni F(x) = \int_0^x f$ for some $f \in L_p$, with norm $N_p(F)$ given by the $L_p$ norm of f. Theorem. If T is a continuous linear transformation from $L_p$ to $L_\alpha$, $1 < p, \alpha < \infty$, then there is only one function K from $[0,1] \times [0,1] \ni$ (1) $K(x,0) = 0$, (2) $K(0,t) = 0$, (3) $K(x, \cdot) \in H_q$, where $p + q = pq$, (4) $\exists\ m > 0 \ni N_q(K(x, \cdot)) \leqq m$, (5) $K(\cdot, t) \in H_\alpha$, and (6) $Tf = (d/dx)\int_0^1 f(t)K(x,dt) \ni (x,t) \in [0,1] \times [0,1]$. The integrals are all Lebesgue or Lebesgue-Stieltjes integrals. Theorem. If K is such a function as given in (1)--(5), then K generates a continuous linear transformation from $L_p$ to $L_\alpha$, as given in (6). (Received February 26, 1968.)

656-118. ULRICH OBERST, University of Chicago, Chicago, Illinois 60637. Exactness of direct and inverse limits.

The following result is proven: Let X be a small category. Then the inverse limit $\varprojlim_X : Ab^X \longrightarrow Ab$ is exact if and only if for every abelian category $\mathfrak{A}$ with exact direct products, the inverse limit $\varprojlim_X : \mathfrak{A}^X \longrightarrow \mathfrak{A}$ is exact. If these equivalent conditions are satisfied, then the direct limit functor $\varinjlim_{X^0} : \mathfrak{A}^{X^0} \longrightarrow \mathfrak{A}$ is exact for every abelian category $\mathfrak{A}$ satisfying the axiom (AB 5). If moreover X is an ordered set, then X is filtered from below. The proof is easy, and uses the formal tensor product and hom-functor. (Received February 26, 1968.)

656-119. ALEXANDER ABIAN, Iowa State University, Ames, Iowa 50010. *Generalized completeness theorem and solvability of systems of Boolean polynomial equations.*

Denoting by $\{0,1\}$ the two-element Boolean algebra, it is proved (without use of the Axiom of Choice) that the Generalized Completeness Theorem of Goedel, i.e. the statement *Every consistent generalized first-order theory has a model* is equivalent to the statement *Every consistent system of Boolean polynomial equations over $\{0,1\}$ has a solution in $\{0,1\}$*, where a system of Boolean polynomial equations over $\{0,1\}$ is called *consistent* if every finite subsystem has a solution in $\{0,1\}$. (Received February 26, 1968.)

656-120. R. J. GREECHIE, Kansas State University, Manhattan, Kansas 66502. *Orthomodular lattices admitting no states.*

Let $L = \bigcup \{B_\alpha : \alpha \in I\}$ be such that (1) $(B_\alpha, \leqq_\alpha, '^\alpha)$ is a Boolean lattice for all $\alpha \in I$, (2) if $\alpha, \beta \in I$, and $x \in B_\alpha \cap B_\beta$, then $x'^\alpha = x'^\beta$, (3) if $\alpha, \beta \in I$, $\alpha \neq \beta$, then $B_\alpha \cap B_\beta = \{0,1\}$ or $\{0,1,a,a'\}$ where $a$ is an atom of both $B_\alpha$ and $B_\beta$, $a' = a'^\alpha = a'^\beta$, and (4) $B_\alpha \neq 2^1$, $B \neq 2^2$ for all $\alpha \in I$. If $n \in \mathbb{Z}$, $n \geqq 3$, we call $\{B_{\alpha_0}, \ldots, B_{\alpha_{n-1}}\}$ (where $\alpha_i \in I$) *an atomistic loop of order* $n$ in case, for $n-1 \geqq i > j \geqq 0$, $B_{\alpha_i} \cap B_{\alpha_j}$ has cardinality 4 if and only if $i - j = n - 1$ or $i - j = 1$. *Theorem.* $L$, with the induced order and orthocomplementation, is an orthomodular lattice (resp. poset) if and only if the order of every atomistic loop in $L$ is at least 5 (resp. 4). *Definition.* By a *state* on an orthomodular lattice $L$ we mean a mapping $\alpha : L \longrightarrow [0,1]$ such that $0\alpha = 0$, and for $x,y \in L$, $x \perp y \Rightarrow (x \cup y)\alpha = x\alpha + y\alpha$. Via the above theorem and a novelle diagramatic representation of orthomodular lattices we exhibit infinitely many orthomodular lattices which admit no states. (Received February 26, 1968.)

656-121. EDGAR REICH, University of Minnesota, Minneapolis, Minnesota 55455, and KURT STREBEL, University of Zurich. *Quasiconformal mappings which keep boundary points fixed.*

The authors obtain some partial results in the direction of characterizing those quasiconformal mappings of the disk onto itself which keep all boundary points fixed, extending the work reported on in these *Notices)* 15 (1968), 226, Abstract 68T-120. (Received February 22, 1968.)

656-122. C. J. EARLE, JR., Cornell University, Ithaca, New York 14850. *Quasiconformal vector fields.*

A vector field $V$ on a Riemannian manifold $X$ produces an infinitesimal change of structure. If that change is bounded on $X$, $V$ is said to be quasiconformal. Let $t \longrightarrow \phi(t)$ be a smooth map from $(-\delta, \delta)$ to the group of diffeomorphisms of $X$. Suppose that $\phi(0)$ is the identity and that there is a number $K$ such that $\phi(t)$ is $(1 + K|t|)$-quasiconformal for $t \in (-\delta, \delta)$. Then the vector field $\phi'(t)$ is quasiconformal. There is a partial converse. (Received March 4, 1968.)

656-123. L. V. AHLFORS, Harvard University, Cambridge, Massachusetts 02138. *Kleinian groups and Eichler integrals.*

This is a study of the importance of Eichler integrals for finitely generated Kleinian groups. (Received March 4, 1968.)

656-124. H. L. ROYDEN, Stanford University, Stanford, California. *The nonuniqueness of extremal quasiconformal maps.*

If $W_1$ and $W_2$ are two finite Riemann surfaces, and [f] a homotopy class of homeomorphisms of $W_1$ onto $W_2$, then the Ahlfors-Bers-Teichmuller theory asserts the existence of a quasiconformal mapping in [f] whose dilation is minimal and that this mapping is unique. Strebel has given an example of a mapping of the circumference of the unit disc onto itself for which there is more than one extension to a mapping of the disc onto itself with minimal dilation. The purpose of this talk is to give examples of two Riemann surfaces $W_1$ and $W_2$ (of infinite connectivity) and a homotopy class [f] of homeomorphisms of $W_1$ onto $W_2$ which contains more than one extremal quasiconformal mapping. Two examples are given. In both of them $W_1$ and $W_2$ are plane domains. The first is characterized by great simplicity of construction, and in the second $W_1$ and $W_2$ are each formed by removing a discrete sequence of points from the plane. (Received March 4, 1968.)

656-125. O. E. LEHTO, University of Minnesota, Minneapolis, Minnesota 55455. *Existence theorem for a generalized Beltrami equation.*

Let E be a compact set of two-dimensional measure zero in the extended complex plane $\Omega$, and $\mu$ a measurable function in $\Omega$ such that $\sup|\mu(z)| < 1$ in every compact subset of $\Omega - E$. A function w is called a $\mu$ homeomorphism if w is homeomorphic in $\Omega$ and locally $\mu$-quasiconformal in $\Omega - E$. Let $\psi\mu(z,r)$ be the mean value of $|1 - e^{-2i\theta}\mu(z + re^{i\theta})|^2/(1 - |\mu(z + re^{i\theta})|^2)$ over $0 \leqq \theta < 2\pi$. Assume that, for every $z \in E$, the integral of $1/r(1 + \psi\mu(z,r))$ over any interval $(r_1,r_2)$ is positive and tends to $\infty$ as either $r_1 \to 0$ or $r_2 \to \infty$. Then $\mu$-homeomorphisms exist. In the special case where E is a Jordan curve, this existence theorem can be applied to a sewing problem for conformal mappings. (Received March 4, 1968.)

# Abstracts Presented by Title

68T-345. R. D. M. ACCOLA, Brown University, Providence, Rhode Island 02912. *On certain groups of automorphisms of closed Riemann surfaces.*

*Theorem.* Let $G_0$ be a finite group of order $n_0$. Let $G_i$, i = 1,2,...,s, be a set of subgroups (of order $n_i$ resp.) so that $G_0 = \bigcup_{i=1}^{s} G_i$ and if $i \neq j$, $i,j > 0$, then $G_i \cap G_j = \langle id \rangle$. Suppose W, a closed Riemann surface of genus g, admits a group of conformal self-maps isomorphic to $G_0$. For each i = 0,1,...,s let $g_i$ be the genus of $W/G_i$. Then $(s - 1)g + n_0 g_0 = \sum_{i=1}^{s} n_i g_i$. *Proof.* The Riemann-Hurwitz formula gives for each i : $2g - 2 = n_i(2g_i - 2) + r_i$. The conditions on $G_0$ imply that $r_0 = \sum_{i=1}^{s} r_i$ and $n_0 = \sum_{i=1}^{s} n_i - s + 1$. The formula now follows. *Examples* of groups $G_0$ are elementary abelian groups, dihedral groups, and other semi-direct products including Frõbenius groups. *Applications.* In the following let $W_k$ stand for a surface of genus k. (1) (H. L. Farkas) A $W_3$ which covers a $W_2$ must be hyperelliptic. (2) A $W_3$ admitting a group isomorphic to $Z_2 \times Z_2 \times Z_2$ must be hyperelliptic. (3) If $G_0 = Z_2 \times Z_2$ on a nonhyperelliptic $W_3$, then $g_0 = 0$ and $g_1 = g_2 = g_3 = 1$. (4) Let $W_{4g+1}$ $(g \geq 1)$ admit two automorphisms $S_1$, $S_2$ each of order 2 so that $W_{4g+1}/\langle S_i \rangle$ has genus g for each i. Then $S_1$ and $S_2$ commute. (Received November 2, 1967.)

68T-346. C. J. MOZZOCHI, University of Connecticut, Storrs, Connecticut. *A generalization of a theorem of Wolk.*

Let X be a set with power set P(X). Let $(Y,\mathcal{P})$ be a symmetric generalized proximity space with proximity class $\Pi(\mathcal{P})$. Let $(Y,\mathcal{U})$ be a symmetric generalized uniform space. Let $\{f_n, n \in D\}$ be a net of members of $Y^X$. *Definition (Leader).* $f_n$ converges to f with respect to $\mathcal{P}$ (notation: $(f_n,f;\mathcal{P})$) iff for every A in P(X) and B in P(Y) (f[A], B) not in $\mathcal{P}$ implies there exists $m_0$ such that if $n \geq m_0$ then $(f_n[A], B)$ not in $\mathcal{P}$. *Definition.* $f_n$ converges to f with respect to $\mathcal{U}$ (notation: $(f_n,f;\mathcal{U})$) iff for every U in $\mathcal{U}$ there exists $m_0$ such that for every x in X if $n \geq m_0$ then $f_n(x)$ is in U[f(x)]. *Theorem.* Let $\mathcal{U}(\mathcal{P})^*$ be p-correct in $\Pi(\mathcal{P})$, and let $\mathcal{U}(\mathcal{P})_n$ be p-correct of degree n in $\Pi(\mathcal{P})$. Then $(f_n,f;\mathcal{P})$ implies $(f_n, f; \mathcal{U}(\mathcal{P})_n)$ and $(f_n,f; \mathcal{U}(\mathcal{P})^*)$. *Corollary (Wolk).* Let $(Y,\mathcal{P})$ be a proximity space with proximity class $\Pi(\mathcal{P})$. Let $(Y,\mathcal{U}_0)$ be a uniform space where $\mathcal{U}_0$ is the Alfsen-Fenstad uniformity in $\Pi(\mathcal{P})$. Then $(f_n,f;\mathcal{P})$ implies that $f_n$ converges to f uniformly. *Remark.* There exist spaces $\mathcal{P}$ for which $\mathcal{U}(\mathcal{P})_1$ is properly contained in $\mathcal{U}(\mathcal{P})^*$. (Received October 5, 1967.)

68T-347. R. P. GUPTA, University of North Carolina, Chapel Hill, North Carolina 27514. *Bounds on the chromatic and achromatic numbers of complimentary graphs.*

The graphs considered are finite, undirected and have no loops and no multiple lines. Two graphs G and $\bar{G}$ defined on the same set of points are called complimentary if any two points are adjacent in one of G or $\bar{G}$ but not in both. Let $a_1, a_2, \ldots, a_k$ represent k distinct colors. Given a graph G, any function f which assigns to each point v of G a unique color $f(v) \in \{a_1,a_2,\ldots,a_k\}$ is called a k-coloring of G. The chromatic number $\chi(G)$ of G is the smallest number k for which there exists a k-coloring f of G such that for any two distinct points v and u, f(v) = f(u) implies v and u are not adjacent. The achromatic number $\psi(G)$ of G is the largest number k for which there exists a k-coloring of G such that for any two distinct colors $\alpha_i$ and $\alpha_j$, there exist points v and u with $f(v) = \alpha_i$ and $f(u) = \alpha_j$ which are adjacent. We have proved the following theorem which also solves a problem proposed by Hedetniemi [*Homomorphisms of graphs and automata*, (1966), 24-25]. *Theorem*. *If* G *and* $\bar{G}$ *are complimentary graphs defined on a set of p points, then* (1) $\chi(G) + \psi(\bar{G}) \leq p + 1$, (2) $\psi(G) + \psi(\bar{G}) \leq [4p/3]^{+}$. *For any* $p \geq 1$, *the bounds* (1) *and* (2) *are attainable.* As a corollary to (1), we obtain the upper bound $\chi(G) + \chi(\bar{G}) \leq p + 1$, due to Gaddum and Nordhaus [Amer. Math. Monthly 63 (1956), 175-177]. (Received November 27, 1967.) Introduced by Professor R. C. Bose.

68T-348. ROBERT MALTZ, University of California, Irving, California. *Isometric immersions into spaces of constant curvature.*

*Theorem*. Let $M^d$ be a complete d-dimensional Riemannian manifold isometrically immersed in a complete d + k-dimensional Riemannian manifold $\bar{M}^{d+k}(K)$ of constant sectional curvature K. Assume further that for each point of M the index of relative nullity $\nu$ is positive. Then there exists a totally geodesic complete submanifold L of $M^d$ which has constant sectional curvature K and is immersed totally geodesically in $\bar{M}(K)$. (Received November 30, 1967.)

68T-349. R. J. DAVERMAN and W. T. EATON, University of Tennessee, Knoxville, Tennessee 37916. *The Cartesian product of a line with the union of two crumpled cubes.*

Let X denote a topological space obtained by sewing together two (3-dimensional) crumpled cubes with a homeomorphism between their boundaries. *Theorem*. $X \times E^1$ *is homeomorphic to* $S^3 \times E^1$. X is the decomposition space associated with an upper semicontinuous decomposition G of $S^3$ whose nondegenerate elements are the fibering arcs of a 3-dimensional annulus. Let G' denote the decomposition of $S^3 \times E^1$ given by $\{g \times w \mid g \in G \text{ and } w \in E^1\}$. If $\epsilon$ is a positive number, techniques due to J. L. Bryant [*Euclidean space modulo a cell*. II, Abstract 650-1, these *Notices* 14 (1967), 916] establish the existence of an isotopy $h_t$ of $S^3 \times E^1$ onto itself such that (1) for each nondegenerate element g' of G', $h_1(g')$ has diameter less than $\epsilon$, (2) $h_t(g')$ $(0 \leq t \leq 1)$ is contained in the $\epsilon$-neighborhood of g' and (3) $h_t$ satisfies all the conditions of Bing's criterion in [R. H. Bing, *The Cartesian product of a certain nonmanifold and a line is* $E^4$, Ann. of Math. 70 (1959), 399-412]. These conditions are sufficient to guarantee the existence of a pseudo-isotopy $f_t$ of $S^3 \times E^1$ onto itself such that the nondegenerate elements of $f_1$ are precisely the nondegenerate elements of G'. (Received December 7, 1967.)

68T-350. P. L. ANTONELLI, University of Tennessee, Knoxville, Tennessee 37916. *On the stable diffeomorphism question for homotopy spheres.*

The stable diffeomorphism question for homotopy spheres asks: for which $\Sigma^n$ and $S^p$ is it true that $\Sigma^n \times S^p$ is diffeomorphically distinct from $S^n \times S^p$? A solution of this problem is given for the stable range $n < 2p + 1$, via construction of an h-cobordism group $\Omega_{n,p}$ whose order is just the number of distinct $\Sigma^n \times S^p$. Of central importance in the construction is the notion of h-enclosability of homotopy spheres. As usual, the order of $\Omega_{n,p}$ is given in terms of a homotopy question about which something is known. Calculations are made for $n < 20$. *Theorem*. *In the stable range,*

$\Sigma^n \times S^p = S^n \times S^p$ *iff* $\Sigma^n$ *has trivial normal bundle in* $R^{n+p+1}$ (we suppose $\Sigma^n \subseteq R^{n+p+1}$). *Corollary. If* $\Sigma^n$ *bounds a* $\Pi$-*manifold, then* $\Sigma^n \times S^p = S^n \times S^p$ *in the stable range. Theorem.* $\Omega_{n,n-4} = Z_2$, *for* $n = 16, 24, 32$. We conjecture that this is the case for an infinite number of $n = 0 \pmod 8$. *Theorem.* $\Omega_{n,1} = bP^{n+1}$, *the group of homotopy spheres which bound* $\Pi$-*manifolds.* This last is essentially a theorem of M. Hirsch and J. Milnor. (Received December 26, 1967.)

68T-351. W. D. L. APPLING, North Texas State University, Denton, Texas 76203. *Continuity and continuous products.*

We prove an analogue of a special case of a theorem of J. S. Mac Nerney (*Integral equations and semigroups*, Illinois J. Math. 7 (1963), 148-173). Suppose F is a field of subsets of a set U and $R_A^+$ is the set of all nonnegative-valued finitely additive functions defined on F. All "integrals" considered are refinement-wise limits of the appropriate sums or products. *Theorem.* If g is in $R_A^+$, then the following four statements are equivalent: (1) $\prod_U [1 + g(I)] = \exp[g(U)]$; (2) $\int_U g(I)[1 - (1/[1 + g(I)])] = 0$; (3) $\int_U g(I)^2 = 0$; and (4) if $0 < c$, then there is a subdivision D of U such that if I is in a refinement of D, then $g(I) < c$. (Received December 6, 1967.)

68T-352. ALBERT SADE, 364 Cours de la République, Pertuis 84, France. *Autotopies des systèmes associatifs.*

Le groupe $A_P$ des autotopies principales d'un système associatif, S, a pout éléments $(\Delta_m, \Gamma_m^{-1}, 1)$ où m décrit le groupe $\mathscr{B}$ des éléments nonsinguliers de S. Le groupe des autotopies fondamentales, $A_F$, a pout éléments $(\Gamma_u, \Delta_r, \Delta_r\Gamma_u)$ où u et r décrivent $\mathscr{B}$. Il est isomorphe à $\mathscr{B}^2$ et égal au produit direct des sous-groupes $(1, \Delta_r, \Delta_r)$ et $(\Gamma_u, 1, \Gamma_u)$. Pour que B, groupe d'automorphisme, soit normal dans A, groupe d'autotopie de S, il faut et il suffit que tout élément de B soit permutable avec tout élément de $A_F$ et alors A est le produit direct de B et de $A_F$. Aux isomorphismes près, $A_P$ est un invariant isotopique; il est normal dans A et pour que $A_P$ soit dans le centre de A il faut et il suffit que B appartienne au centralisateur de $A_P$ et alors $\mathscr{B}$ est abélien. $A_F$ est normal dans A, le groupe quotient est B. L'intersection de $A_P$ et de $A_F$ a pout éléments $(\Delta_m, \Gamma_m^{-1}, 1)$ où m parcourt l'ensemble des éléments centraux non-singuliers de S. On a $A_P \subset A_F \rightleftarrows \mathscr{B} \subset \mathscr{Z}_S \rightleftarrows A_P \lhd A_F$. Pour que A contienne le produit direct $A_P \times A_F$ il faut et il suffit que le seul élément central non-singulier de S soit e, unité de Q. Pour que A soit égal à ce produit direct il faut et il suffit que S soit complet (système associatif n'ayant que des automorphismes internes et dont le seul élément central nonsingulier est neutre). (Received December 11, 1967.)

68T-353. D. J. FIELDHOUSE, P. O. Box 101, Kingston, Ontario, Canada. *Flat covers.*

A submodule D of the left A-module E will be called *impure* in E iff $D \neq E$ and D contains no pure submodules of E other than 0. Any exact sequence $0 \longrightarrow K \longrightarrow F \longrightarrow E \longrightarrow 0$ will be called a *refinement* of E, which is *proper* iff $K \neq 0$. The refinement is *flat* (resp. *impure*) iff F is flat (resp. K is impure in F), and *impure flat* iff it is both impure and flat. A refinement $u : F \longrightarrow E$ is *minimal* iff for any factorization $u = vw$, w epi $\Rightarrow$ w iso. A minimal flat refinement is called a *flat cover. Theorem* 1. A flat refinement is a flat cover iff it is impure flat. *Theorem* 2. Every module has a flat cover, and every flat refinement can be factored through a flat cover. *Theorem* 3. If A is left perfect, then any refinement is a flat cover iff it is a Bass projective cover (and therefore unique up to isomorphism). Examples are given to show that the flat cover is not always unique up to isomorphism. *Theorem* 4 (Localization). Let A be commutative and $0 \rightarrow K \rightarrow F \rightarrow E \rightarrow 0$ be a refinement of E. If the localized refinement is an $A_m$ flat cover of $E_m$ for all maximal ideals m of A, then the original refinement is a flat cover of E. (Received December 20, 1967.)

68T-354. Y. GIVE'ON and M. A. ARBIB, Stanford University, Stanford, California 94305. *Algebra automata.* II: *The categorical framework for dyamic analysis*.

For a theory T a T-automaton (with output) is $M = \langle Q, \mathscr{A}, Y, \lambda\rangle$ : Q & Y sets, $\mathscr{A}$ a T-algebra and $\lambda : Q \longrightarrow Y$ a map. A *response function* on T is any map $f : T_0 \longrightarrow Y$ where $T_0 = T([1], [0])$. The *response function* $f_M$ on M is $f_M = \lambda\mathscr{A}_0$ where $\mathscr{A}_0 : T_0 \longrightarrow Q$ is the restriction of $\mathscr{A}$ to $Q^{[0]} \times T_0$ and $T_0$ is identified with $Q^{[0]} \times T_0$. Thus, we replace the category of monoids by the category of theories Th, and right actions of monoids on sets by T-algebras. This analogy carries through and we derive the construction of a minimal realization and of a normal form of any given response function $f : T_0 \longrightarrow Y$ as it is done with mappings $f : X^* \longrightarrow Y$ in ordinary automata theory. Also the basic division lemmata which underlie Krohn and Rhodes' decomposition theory for ordinary automata are true for response functions on free theories. In fact, like the free monoids, the free theories are characterized as the U-*projective* objects in Th where U is the forgetful functor on Th which assigns to each theory T its *carrier*, the graded set $\underset{\sim}{T} = (T_0, T_1, \ldots, T_n = T([1], [n]), \ldots)$. (An object P of a category C with a functor $U : C \longrightarrow B$ is said to be U-*projective* iff for each morphism $g : P \longrightarrow R$ and any morphism $e : Q \longrightarrow R$ in C such that U(e) *is epic in* B, there exists a morphism $f : P \longrightarrow Q$ in C with $ef = g$.) (Received November 27, 1967.)

68T-355. LUDVIK JANOS, University of Florida, Gainesville, Florida. *A converse of the generalized Banach's contraction theorem.*

Let X be a completely regular space and $f : X \longrightarrow X$ a self map of X. Let $c \in (0,1)$. We will say f is a topological c- contraction iff there exists on X a generating family of pseudometrics $\{\rho_a | a \in \mathfrak{A}\}$ such that $\forall a \in \mathfrak{A}\ \forall x, y \in X[\rho_a(f(x), f(y)) \leqq c\rho_a(x,y)]$. *Theorem.* Let X be compact Hausdorff and $f : X \longrightarrow X$ continuous. Then f is a topological c-contraction for some $c \in (0,1)$ if and only if $\bigcap_n f^n(X) = \{a\}$ for some $a \in X$, i.e. iff the intersection of all iterated images of X under f is a singleton. This is a generalization of the author's previous result [Proc. Amer. Math. Soc. (2) 18 (1967), 287-289] where metrizability of the space X is assumed. (Received January 15, 1968.)

68T-356. H. LAKSER, University of Manitoba, Winnipeg 19, Canada. *Free lattices generated by a partially ordered set.* I.

Let M,N be families of finite nonempty subsets of a poset P such that sup X exists for all $X \in M$ and inf X exists for all $X \in N$. An (M,N)-*ideal* of P is a subset $I \subseteq P$ such that (i) $X \in M$, $X \subseteq I$ imply sup $X \in I$; (ii) $x \leqq y$, $y \in I$ imply $x \in I$. The (M,N)-ideals form a lattice. A *pseudo-principal* (M,N)-*ideal* is an (M,N) ideal obtained by taking a finite sequence of joins and meets of principal (M,N)-ideals, (M,N)-*dual ideals* and *pseudo-principal* (M,N)-*dual ideals* are defined dually. FL(P; M,N) is the free lattice generated by P such that the sup of elements of M and inf of elements of N is preserved. *Theorem* 1. There is an algorithm to determine for any pair of lattice polynomials A, B over P whether $A \leqq B$ holds in P in terms of whether or not the set intersection of certain pseudo-principal (M,N)-ideals and -dual ideals is empty. Trivial special cases are the construction of CF(P), R. A. Dean, Trans. Amer. Math. Soc. 83 (1956), 238-249, and the construction of free products, G. Grätzer, C. R. Platt, and the author, *Free products of lattices*, Abstract 68T-252, these *Notices* 15 (1968). (Received January 15, 1968.) Introduced by Professor G. Gratzer.

68T-357. ALEXANDER ABIAN, Iowa State University, Ames, Iowa 50010. *On solvability of Boolean polynomial equations in one unknown.*

*Theorem*. A Boolean ring R is complete if and only if every (finite or infinite) system $\mathscr{P}$ of Boolean polynomial equations in one unknown with coefficients in R has a solution in R provided every subsystem of $\mathscr{P}$ consisting of two equations has a solution in R. (No use of the axiom of choice is made in the proof). (Received January 17, 1968.)

68T-358. D. W. SOLOMON, University of Wisconsin, Milwaukee, Wisconsin 53201. *On nonmeasurable sets.*

Let $(X, \mathscr{A}, \mu)$ be a complete measure space, $\mu^*$ the outer measure associated with $\mu$. A set $A \subseteq X$ is called a *nonmeasurable kernel* if it has positive outer measure and contains no measurable set of positive measure. Under certain conditions on $\mathscr{A}$, *Theorem*. *If* $\mu^*(A) > 0$, *then* $A = K_1 \cup K_2$, *where* $K_1$ and $K_2$ *are nonmeasurable kernels and* $\mu^*(K_1) = \mu^*(A)$. *If* A *is measurable, then* $K_1$ *and* $K_2$ *may be chosen so that* $\mu^*(K_1) = \mu(K_2) = \mu(A)$. Under certain conditions, e.g. X is a Romanovski space [see, e.g. Abstract 68T-106, these *Notices* 15 (1968), 222], a complete characterization of nonmeasurable sets and nonmeasurable kernels can be obtained using points of outer density. Nonmeasurable kernels may be used to characterize the structure of nonmeasurable sets and to completely determine the nonmeasurable behavior of such sets. The behavior of a set $K \subseteq X$ on the members of certain classes of subsets of X provides a criterion for determining whether or not K is a nonmeasurable kernel. Other properties of nonmeasurable kernels, too numerous to list here, are available. (Received January 19, 1968.)

68T-359. G. K. WHITE, University of British Columbia, Vancouver 8, British Columbia, Canada. *Distributively related left groups.*

A semigroup with left identity and right inverse is known as a left group, and can be exhibited as the direct product $G \times E$ of a group G and a right zero semigroup E (see Clifford and Preston, *The algebraic theory of semigroups*. Vol. I, Math. Surveys, no. 7, Amer. Math. Soc., Providence, R. I., 1961, p. 37 ff). Let $+_0 = +$ denote ordinary addition, and define $+_1$ on the positive reals by $\ln(a +_1 b) = \ln a +_0 \ln b$. Then there are in a sense only 3 types of continuous near-rings $R(+_0, +_1)$: a field $(a +_1 b = ab)$, a trivial ring $(a +_1 b = 0)$ and a near-ring which with 0 deleted is also a left group $(a +_1 b = |a|b)$. From the latter the author constructs an infinite sequence of operations $\{+_i : i \in Z\}$, over an extension $\mathscr{S}(+_i : i \in Z)$ of the reals, with $\mathscr{S}(+_i)$ a semigroup and $\mathscr{S}(+_i, +_{i+1}) \cong \mathscr{S}(+_0, +_1)$. There is a logarithm L which is bijective on $\mathscr{S}$, and $L(a +_{i+1} b) = L(a) +_i L(b)$. (Received December 1, 1967.)

68T-360. K. L. SINGH, Memorial University of Newfoundland, St. John's, Newfoundland, Canada. Contraction mappings and fixed point theorems.

Theorem I. Let $T^n$ (n is a positive integer) be a function from a nonempty set X into itself, and let K be another function also defined on X into itself such that K possesses a right inverse $K^{-1}$ (that is a function $K^{-1}$ such that $KK^{-1} = I$, where I is the identity mapping of X). Then the function $T^n$ has a fixed point if and only if the composite function $K^{-1}T^nK$ has a fixed point. When $n = 1$, we get a well-known result due to Chu and Diaz (Atti della Accad. Sci. Torino C l. Sci. Fis. Mat. Natur. 99 (1964-1965), 351-363). Theorem II. Let X be a complete $\epsilon$-chainable metric space. Let T be a self mapping of X into itself such that there exists a mapping K of X into itself which has the right inverse $K^{-1}$ and which makes the mapping $K^{-1}TK$ $(\epsilon, \lambda)$-uniformly locally contractive (i.e. there exists a real number $\lambda$ with $0 \leqq \lambda < 1$ such that $0 < d(x,y) < \epsilon \Rightarrow d(K^{-1}TK(x), K^{-1}TK(y)) < \lambda d(x,y)$, $x, y \in X$ $x \neq y$). Then T has a unique fixed point. Theorem III. Let X be a metric space. Let T be a self mapping of X into itself such that there exists a mapping K of X into itself which has the right inverse $K^{-1}$ and which makes the mapping $K^{-1}TK$ contractive, further assume that there exists a subset $M \subset X$ and a point $x_0 \in M$ such that $d(x, x_0) - d(K^{-1}TK(x), K^{-1}TK(x_0)) < 2d(x_0, K^{-1}TK(x_0))$ for every $x \in M$, and $K^{-1}TK$ maps M into a compact subset of X; then there exists a unique fixed point for T. (Received January 22, 1968.) Introduced by Professor A. E. Fekete.

68T-361. W. S. MARTINDALE, III, University of Massachusetts, Amherst, Massachusetts 01002. When are multiplicative mappings additive?

A result of Rickart [Bull. Amer. Math. Soc. 54 (1948), 761, Theorem II] is generalized as follows. Theorem. Let R be a ring containing a family of idempotents $\{e_i\}$ satisfying (1) $xR = 0$ implies $x = 0$, (2) for each i, $e_ixe_iR(1 - e_i) = 0$ implies $e_ixe_i = 0$, and (3) $e_iRx = 0$, for each i, implies $x = 0$. Then any one-one multiplicative mapping of R onto an arbitrary ring S is necessarily additive. (Received January 22, 1968.)

68T-362. W. A. LAMPE, University of Manitoba, Winnipeg, Manitoba. On the interdependence of structures associated with a universal algebra. Part I. Preliminary report.

Let $\mathfrak{A} = \langle A;F \rangle$ be a universal algebra, $\mathfrak{G}(\mathfrak{A})$ the automorphism group of $\mathfrak{A}$, $\mathfrak{S}(\mathfrak{A})$ the subalgebra lattice of $\mathfrak{A}$, and $\mathfrak{C}(\mathfrak{A})$ the congruence lattice of $\mathfrak{A}$. An algebraic lattice $\mathfrak{L}$ is said to have property (*) iff there is an $a \in L$, $a \neq 0$, such that $a \leqq \bigvee(x_i | i \in I)$ implies that there exists an $i \in I$ such that $a \leqq x_i$. Theorem. Given a group $\mathfrak{G}$ and algebraic lattices $\mathfrak{L}_1$ and $\mathfrak{L}_2$ such that $|L_1| > 1$ and $\mathfrak{L}_2$ has property (*), there is an algebra $\mathfrak{A}$ such that $\mathfrak{G}(\mathfrak{A}) \cong \mathfrak{G}$, $\mathfrak{S}(\mathfrak{A}) \cong \mathfrak{L}_1$, and $\mathfrak{C}(\mathfrak{A}) \cong \mathfrak{L}_2$. (Received January 22, 1968.) Introduced by Professor G. Grätzer.

68T-363. T. J. SUFFRIDGE, University of Kentucky, Lexington, Kentucky 40506. On univalent polynomials.

If $P_n(z) = \sum_{k=1}^{n} a_kz^k$ is univalent for $|z| < 1$ where we assume $a_1 = 1$, $|a_n| = 1/n$ and $a_k$ is real, $k = 2,3,\ldots,n$; then $|a_k| \leqq [(n - k - 1)/n]\,[\sin (k\pi/(n + 1))/(\sin \pi/n + 1)] = B_k$. The inequality is sharp as shown by the polynomial $\sum_{k=1}^{n} B_kz^k$ which is univalent for $|z| < 1$. (Received January 24, 1968.)

68T-364. H.-D. EBBINGHAUS, Abt. f. math. Logik, 78 Freiburg, Albertstrasse 30, Germany. On the logic with the added quantifier Q (there are uncountably many).

In Fund. Math. 54 (1964) Vaught has shown the axiomatizability of the logic $L_Q$ based on a (countable) first order language with the added quantifier Q. A slight modification of Vaught's argument yields a complete Gentzen-type calculus. We permit the language to contain function symbols. For wff's $\alpha$ and binary K we define wff's $\tau(\alpha)$ and $\delta_K(\alpha)$ inductively with $\tau(\alpha) \equiv \delta_K(\alpha) \equiv \alpha$ for atomic $\alpha$; $\tau$ and $\delta_K$ homomorphic relative to $\neg, \wedge, \bigwedge$; $\tau(Qx\alpha) \equiv \bigvee x \neg x = x$ and $\delta_K(Qx\alpha) \equiv \bigwedge y \bigvee x(Kyx \wedge \delta_K(\alpha))$ (y new). For unary P and S and binary f we set $\pi_{PS} \equiv \bigwedge x_0(Px_0 \rightarrow Sx_0) \wedge \bigvee x_0 \neg Sx_0$ and $\alpha_{KPf} \equiv \mathrm{Ord}(K) \wedge \bigwedge x_0x_1(Px_1 \wedge Kx_0x_1 \rightarrow Px_0) \wedge \bigwedge x_0x_1 Pfx_0x_1 \wedge \bigwedge x_0x_1x_2(Kx_1x_0 \wedge Kx_2x_0 \wedge fx_1x_0 = fx_2x_0 \rightarrow x_1 = x_2)$ (where Ord(K) is a first order sentence characterizing K as an ordering). $\Pi$ may stand for $\bigwedge$-prefixes, $\Pi^S$ for the relativization of $\Pi$ to S and $\Delta$ for finite sequents of wff's. Let R be the rule $\Delta_1 \neg\alpha_{KPf} \vdash \tau(\alpha)$, $\Delta_2 \alpha_{KPf} \pi_{PS}, \Pi_1^S(\alpha_1 \leftrightarrow \alpha_1^S) \ldots \Pi_n^S(\alpha_n \leftrightarrow \alpha_n^S) \vdash \delta_K(\alpha) \Rightarrow \Delta_1\Delta_2 \vdash \alpha$ with K,P,S, f not in $\Delta_1\Delta_2\alpha$ and Q, S not in $\alpha_1, \ldots, \alpha_n$ $(n \geqq 0)$. Theorem. A complete system of rules of inference for $L_Q$ is obtained by adding R to a complete system of rules of inference for the first order predicate calculus. (Received December 27, 1967.) Introduced by Professor Walter Felscher.

68T-365 B. GARDNER, Memorial University of Newfoundland, St. John's, Newfoundland, Canada. On contraction mappings.

Definition. Denote by F the family of functions $\alpha(x,y)$ satisfying (1) $\alpha(x,y) = \alpha(d(x,y))$, (2) $0 \leqq \alpha(d) < 1$ for every $d > 0$, (3) $\alpha(d)$ is a monotonically decreasing function of d. Theorem 1. If $d(Tx, Ty) \leqq \alpha(x,y)d(x,y)$ for every $x, y \in X$, where X is a complete $\epsilon$-chainable metric space, then there exists a unique fixed point for T. Theorem 2. For any $\alpha \in (0,1)$ there exists a distance function $d^*$ in the metric space X such that $d^*(Tx,Ty) \leqq \alpha(x,y)d^*(x,y)$, where $\alpha(x,y) = \alpha(d^*(x,y))$. Corollary. When $\alpha$ is constant we get the result of Ludvik Janos, A converse of Banach's contraction theorem, Proc. Amer. Math. Soc. 18 (1967), 287-289. (Received January 22, 1968.) Introduced by Dr. S. P. Singh.

68T-366. WILLIAM VOXMAN, University of Iowa, Iowa City, Iowa. Decompositions of 3-manifolds and pseudo-isotopies.

Suppose M is a 3-manifold with boundary and G is a cellular decomposition of M such that M/G is a 3-manifold with boundary. Then there is a pseudo-isotopy of M onto itself which shrinks the elements of G to points. This theorem is an extension of a result of Price (Abstract 653-201, these *Notices* 15 (1968), 103), and various techniques used in Price's result are essential to the proof of the above theorem. (Received January 24, 1968.)

68T-367. G. E. REYES, University of Montreal, Montreal, Canada. Generalized definability for countable models.

Our main result is the following strengthening of the Chang-Makkai's theorem on generalized definability for countable theories (see Chang, Bull. Amer. Math. Soc. 70 (1964), 808-813, and Makkai, Acta Math. Acad. Sci. Hungar. 15 (1964), 227-235): Theorem. Let T be a countable theory in $\mathscr{L}(\underline{P})$. Then the following are equivalent: (i) For every infinite countable $\mathscr{L}$-structure

$\mathfrak{A}$, $|\{P : (\mathfrak{A}, P) \in \text{Mod } T\}| < 2^{\omega}$. (ii) For every infinite countable model $(\mathfrak{A}, P)$ of T, $|\{P^1 : (\mathfrak{A}, P^1) = (\mathfrak{A}, P)\}| < 2^{\omega}$. (iii) There are formulas $\theta_i(x_1 v_1, \ldots, v_k)$, $1 \leqq i \leqq n$, of $\mathcal{L}$ such that $T \vdash \bigvee_{1 \leqq i \leqq n} \exists v_1, \ldots, v_k \; \forall x(\underline{P}x \leftrightarrow \theta_i(x_1 v_1, \ldots, v_k))$. The theorem settles in the positive a conjecture of M. Makkai, who had previoüsly proved (unpublished) a special case. A generalization of the theorem for uncountable theories is also obtained. The proof uses the method of diagrams and special models combined with a topological method developed by the author in his dissertation (Typical and generic relations in a Baire space for models, Ph.D. Thesis, University of California, Berkeley, Calif., 1967). (Received January 24, 1968.)

68T-368. FRED GALVIN, University of California, Berkeley, California 94720. A generalization of Ramsey's theorem.

Theorem. If $\mathcal{S}$ is a collection of finite subsets of $\omega$, and if every infinite subset of $\omega$ contains a member of $\mathcal{S}$, then there is an infinite subset X of $\omega$ such that every infinite subset of X has an initial segment which belongs to $\mathcal{S}$. From this one can easily derive Nash-Williams' generalization of Ramsey's theorem (C. St. J.A. Nash-Williams, Proc. Cambridge Philos. Soc. 61 (1965), 33, Theorem 1). (Received January 26, 1968.)

68T-369. D. D. BOOTH, University of Wisconsin, Madison, Wisconsin 53706. Increasing sets of degrees.

Let ZF be Zermelo Fraenkel set theory and ZFC be set theory with the axiom of choice. One can consider the degree of nonconstructibility of a set of integers by defining two such sets to be equivalent when they are interconstructible. Theorem. If ZF is consistent, then ZFC & $\forall a \subseteq \omega$ [there exists an increasing set of degrees ordered like the reals in L(a)] is consistent. Such a model can be obtained by the adjunction of Cohen reals. If there is a measurable cardinal then there is a well-ordered increasing sequence of degrees ordered like the countable ordinals, but no sequence that is longer. (Received January 26, 1968.)

68T-370. K. L. SINGH and B. GARDNER, Memorial University of Newfoundland, St. John's, Newfoundland, Canada. Contractive mapping in uniform spaces.

Theorem 1. If f is a contractive mapping of a Hausdorff uniform space X into itself and there exists a subset $M \subset X$ and a point $x_0 \in M$ such that (1) $d(x, x_0) - d(f(x); f(x_0)) < 2d(x_0, f(x_0))$ for every $x \in M$, and f maps M into a compact subset of X, then there exists a unique fixed point. Theorem 2. Let X be a Hausdorff uniform space and let f be a contractive mapping of X into itself which has diminishing orbital diameters. Suppose for some $x \in X$ a subsequence of the sequence $\{f^n(x)\}$ of iterates of x has limit $\xi$. Then $\{f^n(x)\}$ has a limit $\xi$ and $\xi$ is a fixed point of f. Theorem 3. Let X be a complete Hausdorff uniform space. Let f be a contractive mapping such that there exists a subset $M \subset X$ and a point $x_0 \in M$ satisfying the following (1) $d(x, x_0) - d(f(x); f(x_0)) < 2d(x_0, f(x_0))$ for every $x \in M$. (2) $d(f(x), f(y)) \leqq \alpha(x,y)d(x,y)$ for every $x, y \in M$ where $\alpha(x,y) = \alpha(d(x,y))$. Then f has a unique fixed point. (Received January 26, 1968.) Introduced by Professor A. E. Fekete.

68T-371. WAI-MEE CHING, University of Toronto, Toronto 5, Canada. New nonhyperfinite nonisomorphic factors.

A von Neumann algebra is called hyperfinite if it is weakly generated by an increasing sequence of finite-dimensional $*$-subalgebras. Schwartz (Two finite, non-hyperfinite non-isomorphic factors, Comm. Pure Appl. Math. 16 (1963), 19-26; Non-isomorphism of a pair of factors of type III, Comm. Pure Appl. Math. 16 (1963), 111-120) proved that there exists a pair of nonisomorphic nonhyperfinite factors of type $II_1$, and of type III respectively. We establish the existence of three nonisomorphic nonhyperfinite factors of type $II_1$, and of type III respectively by introducing the following algebraic property of a von Neumann algebra: Definition. A von Neumann algebra R is said to have property C, if for each sequence $U_k$ $(k = 1,2,\ldots)$ of unitary operators in R with the property that strong lim $U_k^*TU_k = T$ for each $T \in R$, there exists a sequence $V_k$ $(k = 1,2,\ldots)$ of mutually commuting operators in R such that strong lim $(U_k - V_k) = 0$. (Received January 29, 1968.) Introduced by Sister R. J. Tauer.

68T-372. J. R. SHILLETO, University of California, Berkeley, California. The arithmetic of standard models of ZF.

Define an $\omega$-model M of second-order arithmetic to be $\beta_n$ iff for every function f in M the true $\Sigma_n^1$ properties of f hold in M (for n = 1 this gives the notion of $\beta$-model introduced by Mostowski). Let $\delta_n^f$ be the least ordinal not the order-type of a $\Delta_n^{1,f}$ relation. Proposition 1. The following are equivalent for M an $\omega$-model: (a) M is $\beta_2$; (b) M is closed under relative $\Delta_2^1$-ness; (c) for each f in M: $\exists g \forall h \phi(f,g,h) \Rightarrow \exists g$ in $M \forall h \phi(f,g,h)$ for arithmetical $\phi$. Proposition 2. For a standard model M of ZF and f a number-theoretic function in M, every $\Sigma_2^{1,f}$ and $\Pi_2^{1,f}$ true sentence holds in M iff $\delta_2^f \subseteq M$ (cf. Shoenfield's absoluteness theorem). For a standard model M of ZF let $M_\beta$ denote the corresponding $\beta$-model of analysis. Let $\Omega^M$ $(On^M)$ be the least ordinal in M uncountable in M (the set of ordinals in M). Corollary 3. (a) M is $\beta_n \Rightarrow \Omega^M \geqq \delta_n$; (b) $On^M \geqq \Omega \Rightarrow M_\beta$ is $\beta_2$; (c) $M_\beta$ is $\beta_2 \Rightarrow (L \cap M)_\beta$ is $\beta_2$. Proposition 4. By results of Cohen it is consistent to assume there are standard models M but the $M_\beta$ is never $\beta_2$ (assuming Consis (ZF + there is a standard model)). Proposition 5. Assuming V = L, any $\omega$-model closed under relative $\Delta_n^1$-ness is $\beta_n$ (whence the $\Delta_n^1$ functions do not satisfy full comprehension). (Received January 29, 1968.)

68T-373. WITHDRAWN.

68T-374. R. A. HUNT, Princeton University, Princeton, New Jersey 08540, and M. H. TAIBLESON, Washington University, St. Louis, Missouri 63130. On the almost everywhere convergence of Fourier series on the ring of integers in a local field. Preliminary report.

Using the methods of Carleson (Acta Math. 116 (1966), 135-157) as modified by the first author (to appear in Orthogonal expansions and their continuous analogues, Southern Illinois Univ. Press, Carbondale, Ill., 1968), and the analysis of Fourier series on the ring of integers $\mathfrak{O}$ in a local field as outlined by the second author (Bull. Amer. Math. Soc. 73 (1967), 623-629), it is proved that the Fourier series of a function in $L^p(\mathfrak{O})$, $p > 1$, converges a.e. As in the real case, maximal function norm inequalities are proved. These inequalities actually imply a.e. convergence for functions in classes larger than $L^p$. These results have been obtained in the case of the 2-series field ($\mathfrak{O}$ is the Walsh-Paley group) by Billard ($p = 2$) and Sjölin ($p > 1$). (Received January 31, 1968.)

68T-375. NICK METAS, Queen's College, Flushing, New York 11367. Some further reductions in the definition of a projective Banach space. Preliminary report.

This paper extends the results announced in Abstract 64T-290, these *Notices* 11 (1964), 453. Theorem. Let P be a fixed Banach space and suppose P has the property that for every injective Banach space X (see Definition 1, Abstract 64T-290), every closed subspace $X_0$ of X, and every bounded linear transformation T from P to $X/X_0$, there exists a bounded linear transformation $\tilde{T}$ from P to X such that $Q\tilde{T} = T$ where Q is the quotient map from X onto $X/X_0$. Then P is projective (see Definition 1, Abstract 64T-290). Remark. We can change our hypothesis that X is injective to the hypothesis that X is an $l_\infty(S)$ space or to the hypothesis that X is congruent to a C(S) space, S compact Hausdorff, and the conclusion that P is projective will still hold. (Received February 1, 1968.)

68T-376. R. K. GUY, University of Calgary, Calgary, Alta, Canada, and P. A. KELLY, University of Waterloo, Waterloo, Ontario, Canada. The no-three-in-line problem.

An old puzzle (Dudeney, "Rouse Ball") can be generalized to ask for sets of 2n points chosen from an n by n array of $n^2$ points of the unit lattice with no three of them in a straight line. Some evidence and a probabilistic argument are given which support the following Conjectures. 1. There is no solution which has the symmetry of a rectangle, without also having the full symmetry of the square (?). 2. If $n > 10$, there is no solution having the full symmetry of the square (?). 3. There is only a finite number of solutions to the no-three-in-line problem (?). 4. For large n, one may expect to be able to choose approximately $(2\pi^2/3)^{1/3}n$ points with no three in line, but no large number (?). (Received February 1, 1968.)

68T-377. K. E. ATKINSON, Indiana University, Bloomington, Indiana 47401, and A. SHARMA, University of Alberta, Edmonton, Canada. A partial characterization of poised Hermite-Birkhoff interpolation problems.

Following the notation of [I. J. Shoenberg, On Hermite-Birkhoff interpolation, J. Math. Anal. Appl. 16 (1966), 538-543], we prove two results which partially characterize poised incidence matrices. An incidence matrix $E_n$ is called irreducible if it satisfies $M_p > p + 1$, $p = 0,1,\ldots,n - 2$,

$M_{n-1} = n$. The horizontal sum of incidence matrices is defined, as well as the notion of a conservative matrix, an easily computable concept. *Theorem* 1. If $E_n$ satisfies the Polya condition, then it has a unique and maximal decomposition as the horizontal sum of irreducible incidence matrices; moreover, $E_n$ is poised if and only if all of its irreducible components are poised. *Theorem* 2. If $E_n$ is conservative and satisfies the Polya condition, then it is poised. *Conjecture*. If $E_n$ is poised and irreducible, then it is conservative. (Received February 5, 1968.)

68T-378. R. S. DORAN, University of Washington, Seattle, Washington 98105. *Construction of uniform* CT-*bundles*. Preliminary report.

An S-*bundle* (resp. T-*bundle*) is a triple $(\mathscr{B},\pi,Q)$ where $\mathscr{B}$ and Q are nonempty sets (resp. topological spaces) and $\pi$: $\mathscr{B} \longrightarrow Q$ is a surjective (resp. continuous surjective) map. A *local cross-section* of a T-bundle over an open U in Q is a continuous map $\psi: U \longrightarrow \mathscr{B}$ satisfying $\pi \circ \psi = 1_U$. A CT-*bundle* is a T-bundle such that each point of $\mathscr{B}$ lies in the image of some local cross-section. A $\mathscr{B}$-*uniformity* on an S-bundle is a uniform structure on $\mathscr{B}$ restricted to $\mathscr{B} \oplus \mathscr{B} = \{(a,b) : \pi(a) = \pi(b)\}$ in $\mathscr{B} \times \mathscr{B}$. Let $(\mathscr{B},\pi,Q)$ be a T-bundle and $\mathscr{L}$ a $\mathscr{B}$-uniformity; if $b \in \mathscr{B}$, $F \in \mathscr{L}$, and $\psi$ is any local cross-section with domain $D_\psi$ satisfying $b \in \mathrm{Image}(\psi)$, define $F[\psi] = \{a \in \mathscr{B}: \pi(a) \in D_\psi$ and $(\psi(\pi(a)),a) \in F\}$. *Definition.* Let $(\mathscr{B},\pi,Q)$ be a CT-bundle, $\mathscr{L}$ a $\mathscr{B}$-uniformity, and let $\Gamma^*$ denote the set of all local cross-sections. The quadruple $(\mathscr{B},\pi,Q,\mathscr{L})$ is a *uniform* CT-*bundle* if for each $b \in \mathscr{B}$ a neighborhood base at b is given by $\mathscr{N}_b = \{F[\psi]: F \in \mathscr{L}, \psi \in \Gamma^*$ with $b \in \mathrm{Image}\ (\psi)\}$. *Theorem*. Let $(\mathscr{B},\pi,Q)$ be an S-bundle, $\mathscr{L}$ a $\mathscr{B}$-uniformity, and $\Gamma \subset \{\psi: Q \longrightarrow \mathscr{B} \mid \pi \circ \psi = 1_Q\}$ any set of maps with $\mathscr{B} = \bigcup_{\psi \in \Gamma} \psi(Q)$. Then there exist unique smallest topologies on $\mathscr{B}$ and Q making $(\mathscr{B},\pi,Q,\mathscr{L})$ into a uniform CT-bundle with $\Gamma$ contained in all continuous cross-sections. *Remark*. Numerous other results have been obtained, and applications to the representation theory of topological algebras (in particular C*-algebras) are planned in the future. (Received February 5, 1968.)

68T-379. W. T. WHITLEY, Virginia Polytechnic Institute, Blacksburg, Virginia 24061. *On f-ideals in rings of continuous functions.*

Let C(X) denote the ring of continuous real-valued functions on a topological space X. A z-*ideal* in C(X) is an ideal I such that if the zero set of a function f in C(X) equals the zero set of a member of I, then f is in I [L. Gillman and M. Jerison, *Rings of continuous functions*, Van Nostrand, 1960]. An ideal I is an f-*ideal* if for each a,b in I, there exists c in I such that cx = a and cy = b have solutions in C(X) [C. E. Aull, *Ideals and filters*, Compositio Math. 18 (1967), 79-86]. In this note the following theorem is proved. *Theorem*. Every z-ideal in C(X) is an f-ideal. However, the converse is false. (Received February 2, 1968.)

68T-380. B. L. BRECHNER, Louisiana State University, New Orleans, Louisiana 70122. *Homeomorphism groups of dendrons.*

It is the purpose of this paper to answer the following question: Is the space of homeomorphisms of a contractible, locally contractible space necessarily locally contractible? We answer this question in the negative by constructing examples of certain dendrons, whose groups of homeomorphisms are zero-dimensional and nowhere discrete. Thus they are not locally contractible. (Received February 2, 1968.)

68T-381. M. B. SLATER, University of Hawaii, Honolulu, Hawaii 96822. The socle of an alternative ring.

Let R be a semiprime alternative ring; no restriction on characteristic. If A is a minimal right ideal of R, then A = eR for e a nuclear idempotent. Conversely, if e is a nuclear idempotent, eR is minimal iff eRe is a C. D. algebra or an associative division algebra. Let $S_r(R)$ ($S_e(R)$) be the sum of the minimal right (left) ideals of R. Then $S_r(R) = S_e(R) = S(R)$, say, is a two-sided ideal, the socle of R. S(R) is a weak direct sum of C. D. algebras and simple associative rings having minimal right ideals. On the class $\mathcal{R}$ of (isomorphism classes of) semiprime alternative rings, let S be the mapping sending $R \longrightarrow S(R)$; let $D: R \longrightarrow D(R)$, the associator ideal of R, and $U: R \longrightarrow U(R)$, the maximum nuclear ideal of R. Then a complete set of defining relations for the semigroup of mappings generated by $\{S,D,U\}$ is UU = U; SS = S; SD = DS = DSD; SU = US; DU = UD = 0. If R is subdirectly irreducible with heart M, and $M^2 \neq (0)$, then R is associative or a C. D. algebra. (Received February 5, 1968.)

68T-382. STEVEN BANK, University of Illinois, Urbana, Illinois. On the structure of a fundamental set of solutions near an irregular singularity.

In this paper we treat nth order homogeneous ordinary linear differential equations whose coefficients are complex analytic functions defined in an unbounded sectorial region R, and which have asymptotic expansions there, as $z \longrightarrow \infty$, in terms of real powers of Z and/or trivial functions (i.e. functions which as $Z \longrightarrow \infty$ in R are of smaller rate of growth than all powers of Z). In [J. Math. Anal. Appl. 16 (1966), 138-151], we showed that if p is the critical degree, then, in subsectorial regions, the equation possesses p linearly independent solutions $\phi_1,\ldots,\phi_p$, each asymptotically equivalent ($\sim$) as $z \longrightarrow \infty$ to a function of the form $cz^{\alpha}(\log z)^{\beta}$. In this paper the behavior of the remaining n - p solutions in a fundamental set is investigated. Associated with the differential equation is an algebraic polynomial G(y) of degree n - p. Of importance are those functions $cz^r$ (for real r) which are critical of G(y) (i.e. for which there exists $h \sim cz^r$ such that G(h) is not $\sim G(cz^r)$). (See [Ann. of Math. 74 (1966), 83-112].) If each such $cz^r$ is not critical of $\partial G/\partial y$, then we show that in subsectorial regions the differential equation possesses n - p solutions $g_1,\ldots,g_{n-p}$, such that for each j, either $g_j$ or $1/g_j$ is trivial, and such that $\{\phi_1,\ldots,\phi_p\ g_1,\ldots,g_{n-p}\}$ is a fundamental set. (Received February 7, 1968.)

68T-383. WITHDRAWN.

68T-384. C. J. EVERETT, Los Alamos Scientific Laboratory, 1334 43 Street, Los Alamos, New Mexico. On Veblen-Wedderburn systems.

Let g generate the $(\cdot)$ group of $F = GF(p^n)$, $p \geqq 2$, $n \geqq 2$, and suppose $1 < d \mid p^n - 1$. To each $j = 1, \ldots, d$, assign an $a = a(j)$ on $\{1, \ldots, n\}$ with $a(d) = n$, $a(j) < n$ for at least one j, and so that $d \mid p^a - 1$ for each $a = a(j)$. This is always possible for p odd, and for $p = 2$ iff n is composite. On the elements of F, define $(+)$ as usual; and $(\circ)$ by $0 \circ y = 0$, and for $e \equiv j \bmod d$, $g^e \circ y = g^e \cdot y^A$ where $A = p^a$ and $a = a(j)$. The result is a Veblen-Wedderburn (V-W) system without the law $(x + y) \circ z = x \circ z + y \circ z$. Examples (e.g. for $p = 3$, $n = 2$) show that more such pathological systems exist, not based on the above automorphism scheme. Construction is in terms of matrices, via the equivalence of a V-W system with a set of $p^n$ $n \times n$ matrices $A_k$ over GF(p), including 0 and I, with the difference of any two nonsingular. (Received February 9, 1968.)

68T-385. D. W. CURTIS, Iowa State University, Indianola, Iowa 50125. Deficiency in infinite-dimensional spaces. I.

Definition. A closed subset of a topological linear space has finite deficiency if there exist space homeomorphisms taking it into closed subspaces of every finite codimension. The following theorem and corollary extend earlier results given in Abstract 67T-587, these *Notices* 14 (1967), 831. Theorem. Closed $\sigma$-compact (compact) subsets of infinite-dimensional Banach (Fréchet) spaces have finite deficiency. Corollary. Homeomorphisms between closed, finite-dimensional, separable, locally compact (compact) subsets of infinite-dimensional Banach (Fréchet) spaces can be extended to stable space homeomorphisms. (Received February 9, 1968.)

68T-386. C. F. WELLS, Case Western Reserve University, Cleveland, Ohio 44106. Split translations of groups. Preliminary report.

Let G be a group with normal subgroup H of finite index, and let $G/H \cong \Gamma$. An H-split translation of G is a mapping $\phi : G \rightarrow G$ such that $x^{-1}\phi(x) = y^{-1}\phi(y)$ whenever x and y are in the same coset of H. The H-split translations of G form a semigroup ST(G;H) on composition. A natural generalization of work of B. H. Neumann (J. London Math. Soc. 35 (1960), 184-192) gives a definition of wreath product of semigroups of transformations so that $ST(G;H) \cong H \text{ wr } T(\Gamma)$, where $T(\Gamma)$ is the semigroup of all transformations of $\Gamma$ into itself. The collection of H-split translations which are permutations of G form a group $PT(G;H) \cong H \text{ wr } \mathrm{Sym}(\Gamma)$. (This generalizes my Abstract 67T-510, these *Notices* 14 (1967), 710.) PT(G;H) is also isomorphic to Ore's "symmetry of degree $|\Gamma|$ over H". Theorem. If G is Abelian and $G \cong H \times \Gamma$, then $ST(G;H) \cap \mathrm{Aut}(G)$ is isomorphic to the split extension of $\mathrm{Hom}(\Gamma,H)$ by $\mathrm{Aut}(\Gamma)$ determined by defining $\psi^{\sigma} = \psi \circ \sigma$ for $\sigma \in \mathrm{Aut}(\Gamma)$, $\psi \in \mathrm{Hom}(\Gamma,H)$. (Received December 1, 1967.)

68T-387. J. A. GERHARD, McMaster University, Hamilton, Ontario, Canada. The lattice of equational classes of idempotent semigroups.

The lattice of equational classes of idempotent semigroups is described as follows. The sublattice generated by the atoms is known to be the eight element Boolean lattice. Let the unit of the

Boolean lattice be $(1,0)$ and let two of its coatoms be $(0,3)$ and $(0,4)$. The remaining elements may be given by pairs $(n,i)$, $n = 1,2,3,\ldots$, $i = 0,1,2,3,4$, and an element 1, to represent the unit of the lattice. The following is a list of the nontrivial order relations among the elements. For every $n \geqq 1$, $(n-1,3) < (n,0)$, $(n-1,4) < (n,0)$, $(n-1,3) < (n,1)$, $(n-1,4) < (n,2)$, $(n,1) < (n,3)$, $(n,0) < (n,3)$, $(n,2) < (n,4)$, $(n,0) < (n,4)$. Each element of the lattice is determined by a single equation. A characterization of all the equations which determine an element of the lattice is also given. (Received February 8, 1968.)

68T-388. P. C. FISCHER, University of British Columbia, Vancouver 8, British Columbia, Canada. A multidimensional generalization of the firing squad problem.

Consider a finite set of identical finite-state sequential machines placed at various integral lattice points in n-space, i.e. associate with each automaton an n-tuple with integral components. Two lattice points are adjacent if they differ in only one component and by exactly 1. Inputs to a given machine are 2n-tuples, each component of which is either a special null symbol if the appropriate adjacent point has no machine placed on it, or the state of the adjacent machine if one is so located. A special start signal is given to an arbitrarily chosen machine at time 0. If at some later time all of the machines simultaneously and for the first time enter a special "firing" state, the set will be said to be synchronizable. The problem for the plane was suggested by Z. A. Melzak. Theorem. There exists a finite automaton such that any finite connected set in n-space of copies of this machine is synchronizable. The time for synchronization is no greater than 4k, where k is the number of machines in the set. (Received February 8, 1968.)

68T-389. WILLIAM JACO and D. R. MCMILLAN, JR., University of Wisconsin, Madison, Wisconsin 53706. Retracting 3-manifolds onto finite graphs.

Let $M^3 \subset S^3$ be a polyhedral cube-with-holes of genus n, i.e. Bd $M^3$ is connected and of genus n. Call $M^3$ retractable if there is a retraction of $M^3$ onto a wedge of n simple closed curves. If such a wedge can be chosen in Bd $M^3$, call $M^3$ boundary-retractable. Let $G = \pi_1(M^3)$ and let $G = G_1 \supset G_2 \supset G_3 \supset \ldots$ be the lower central series for G (i.e. $G_{n+1} = [G_n, G]$). Theorem 1. $M^3$ is retractable iff $G/G_\omega$ is a free group where $G_\omega = \bigcap_{i=1}^{\infty} G_i$. Theorem 2. $M^3$ is boundary-retractable iff there is a mapping f of $M^3$ onto a cube-with-handles H of genus n such that $f|\text{Bd } M^3$ is a homeomorphism onto Bd H. Theorem 3. For each $n \geqq 3$, there is a cube-with-holes $M^3$ of genus n such that $G_\omega = 1$, yet G is not a free group. Hence, $M^3$ is not retractable. Lambert has given non-boundary-retractable examples for each $n \geqq 2$. Theorem 4. If $n = 2$, $M^3$ is retractable iff each mapping of $M^3$ into the torus $T = S^1 \times S^1$ is homotopic to a mapping into a proper subset of T. (Received February 12, 1968.)

68T-390. EDWARD CLINE, University of Minnesota, Minneapolis, Minnesota. An application of transfer for the prime three.

We call a group G an SR group if (1) G contains a subgroup $P_1$ of order three such that a $S_3$-subgroup $P_2$ of $N_G(P_1)$ is elementary of order nine, and (2) $N_G(P_2)/P_2$ is semiregular on the conjugates (under G) of $P_1$ which are contained in $P_2$. By applying the Hall-Wielandt transfer theorem, we prove Theorem 1. If G is an SR group, then $O^3(G) < G$. Theorem 1 is, in general, false for primes $p \neq 3$. The concept of Zassenhaus group can be generalized by considering groups G which contain a Fröbenius subgroup MQ with kernel M such that M and Q are TI sets in G. In this case, MQ acts on its cosets with one fixed point, a orbits of length $|M|$ and b orbits of length $|MQ|$. Call such groups *(a,b) groups*. We apply Theorem 1 to the problem of the existence of simple (2,b) groups. *Theorem* 2. If G is a simple (2,b) group on the cosets of MQ, then $|Q|$ is prime to six. (Received February 12, 1968.)

68T-391. J. A. DYER and W. B. JOHNSON, Iowa State University, Ames, Iowa 50010. *Similar generalized bases and isomorphisms.*

The notation and terminology used below follows that of O. T. Jones and J. R. Retherford, *On similar bases in barrelled spaces*, Proc. Amer. Math. Soc. 18 (1967), 677-680. The theorem given here generalizes one given by M. G. Arsove and R. E. Edwards, *Generalized bases in linear topological spaces*, Studia Math. 19 (1960), 95-113, and gives a partial answer to a question raised by Jones and Retherford. *Theorem.* Suppose that E and F are linear Hausdorff spaces and that $(x_i,f_i)_{i\in I}$ and $(y_i,g_i)_{i\in I}$ are generalized bases for E and F respectively. If $(x_i)$ and $(y_i)$ are similar, then there exists an algebraic isomorphism T from E onto F such that $Tx_i = y_i$ for each i in I, and T has a closed graph. *Corollary.* If E and F satisfy the hypotheses of the theorem and both the closed graph and open mapping theorems hold for linear maps from E onto F, then $(x_i)$ and $(y_i)$ are similar if and only if there is a linear homeomorphism T from E onto F such that $Tx_i = y_i$ for each i in I. This corollary will hold, for example, if each of E and F is a barrelled, fully complete space or if each of E and F is the separated inductive limit of a sequence of fully complete Baire spaces (c.f. A. P. and W. J. Robertson, *Topological vector spaces*, Cambridge Tracts in Math. and Math. Phys., no. 53, Cambridge Univ. Press, New York, 1964, Chapter VI. (Received February 12, 1968.)

68T-392. N. C. A. da COSTA and A. I. ARRUDA, University of Campinas, Campinas, Sao Paulo, Brazil. *Further considerations on the postulate of separation.*

This paper is a sequel to the one announced in Abstract 68T-336, these *Notices* 15 (1968), 399-400, where we have constructed a propositional calculus, here called $\mathscr{T}_1$; the objective of the present work is to study new propositional calculi, $\mathscr{T}_2$, $\mathscr{T}_3$ and $\mathscr{T}_4$ (and the corresponding restricted predicate calculi, with and without equality), serving to the same purposes of $\mathscr{T}_1$ and stronger than this last calculus. $\mathscr{T}_2$, $\mathscr{T}_3$ and $\mathscr{T}_4$ are obtained from $\mathscr{T}_1$ by strengthening the notion of negation. In $\mathscr{T}_2$ we have: $\rightarrow (A \supset B) \supset ((A \supset \neg B) \supset \neg A)$, $\neg A \,\&\, \neg B \rightarrow \neg(A \vee B)$ and $\rightarrow A \,\&\, \neg A \supset B$. If F is a classical tautology, then $\rightarrow F$ is provable in $\mathscr{T}_3$; this is not true for $\mathscr{T}_4$, but in this calculus is valid a generalized version of the replacement theorem. In $\mathscr{T}_1$, $\mathscr{T}_2$, $\mathscr{T}_3$ and $\mathscr{T}_4$ are not valid the rule of modus ponens and, for instance, the following sequents: $A \,\&\, \neg A \rightarrow B$, $A \,\&\, \neg A \rightarrow \neg B$ and $A \supset B, A \supset \neg B \rightarrow \neg A$. The systems of set theory based on those calculi, using the customary postulates of the Zermelo-Fraenkel theory (the postulate of separation formulated without any restriction), are not trivial, but they are inconsistent in a generalized sense. (Received February 16, 1968.)

68T-393. J. W. CANNON, University of Utah, Salt Lake City, Utah 84112. *Singular side approximations of 2-spheres.*

Singular side approximations are suggested by the results of Bing's paper [*Pushing a 2-sphere into its complement,* Michigan Math. J. 11 (1964), 33-45]. *Definition.* Let S be a 2-sphere in $E^3$, and let $f: S \to S \cup \mathrm{Int}\, S$ be an $\epsilon$-map such that (1) $f(S) \cap S$ and $f^{-1}(f(S) \cap S)$ are 0-dimensional and (2) $f|[S - f^{-1}(F(S) \cap S)]$ is a homeomorphism. Then f(S) is called a *singular $\epsilon$-approximation to S from Int S.* *Theorem* 1. If F is an $F_\sigma$ set on S, then the following are equivalent: (A) For each $\epsilon > 0$ there is a singular $\epsilon$-approximation f(S) to S from Int S so that $f(S) \cap F = \emptyset$. (B) (*, F', Int S) is satisfied for each closed subset F' of F. (C) There exist closed subsets $\{F_n\}$ of F such that $F = \bigcup_{n=1}^{\infty} F_n$ and, for each n, (*, $F_n$, Int S) is satisfied. That (*, F', S) is satisfied means roughly that S can be side approximated from Int S with nonsingular spheres so as to miss F'. (See Loveland's paper [Pacific J. Math. 19 (1966), 489-517] for a precise definition of (*, F', Int S).) Theorem 1 is used to establish *Theorem* 2. A 2-sphere S in $E^3$ is tame if for each $p \in S$, each $\epsilon > 0$, and each component V of $E^3 - S$, there is a tame loop which links p on S and can be shrunk to a point in an $\epsilon$-subset of V. This extends a result announced by White [Abstract 653-24, these *Notices* 15 (1968), 84]. (Received February 15, 1968.)

68T-394. J. M. BOYTE, Virginia Polytechnic Institute, Blacksburg, Virginia. *Open convergence and closure sets.*

*Definition.* See J. M. Boyte, *Open convergence,* Abstract 650-34, these *Notices* 141 (1967), 926. The notation $N_A$ is an open set containing the set A. The following theorems have been proven, and we assume in each (X,T) satisfies the first axiom of countability with the exception of next to the last. *Theorem*. (X,T) is regular if and only if each $\{x_i\} \xrightarrow{\text{openly}} a$ implies $N_a \cap (\bigcup_{i=1}^{\infty}\{x_i\}) \neq \emptyset$ for each $N_a$. *Theorem*. (X,T) is regular if and only if each $\{A\} \xrightarrow{\text{openly}} a$, A countable, $a \notin A$ implies $\{A\} \to a$. *Theorem.* If $\bar{A}$ is countable for each countable set A, then (X,T) is regular if and only if for each countable closed set F and $a \notin F$ we have $N_a$ and $N_F$ such that $N_F \cap N_a = \emptyset$. *Definition.* If there exists a closed set A such that a sequence $\{x_n\} \to A$ and if $B \subset A$ such that $\{x_n\} \to B$ implies $\bar{B} = A$, then A is called the closure set of the sequence $\{x_n\}$. *Theorem*. If (X,T) is a $T_3$ space, then (X,T) is sequentially compact if and only if each sequence has a closure set. *Theorem.* If (X,T) is a $T_3$ sequentially compact space, then (X,T) is separable if and only if there exists a sequence $\{x_n\}$ having a closure set A such that $(X - \bigcup_{n=1}^{\infty}\{x_n\}) \subset A$. *Theorem*. In a $T_3$ sequentially compact space each sequence has a unique closure set. (Received February 16, 1968.) Introduced by Dr. E. P. Lane.

68T-395. K. V. T. CHESS, University of Kansas, Lawrence, Kansas 66044. *Direct sums and normal systems.*

The group properties N, $\widetilde{N}$, Z, ZA, $\overline{SI}$ are defined in A. G. Kurosh, *The theory of groups*, Vol. II, Cheasea, New York, 1960. *Definitions*. A group G is a $\bar{Z}$ group iff every invariant system of G can be refined to a central system. G is a $\bar{\bar{Z}}$ ($\bar{\bar{SI}}$) group iff every subgroup of G is $\bar{Z}$ ($\overline{SI}$). If K, H are subgroups of G with $K \lhd H$, the factor group H/K is termed a factor of G. Let A, B be groups with $A \neq E \neq B$, and let S be a set with cardinal number $\geqq 2$. *Theorem* 1. $A \oplus B$ is an N group iff (a) A and B are N groups and (b) if F(A), F(B) are isomorphic non-E factors of A, B, resp., then the center of $F(B)$ is not $E$. *Corollary* 1. $A^{(S)}$ is an $N$ group iff $A$ is $ZA$. *Corollary* 2. The standard restricted wreath product $A$ wr $B$ is an $N$ group iff there is a prime $p$ such that $A$ and $B$ are $p$-groups, $A$ is $ZA$, and $B$ is finite. Hence, $A$ wr $B$ is an $N$ group iff $A$ wr $B$ is $ZA$. *Theorem* 2. The direct sum of $\overline{\overline{Z}}$ $(\overline{\overline{SI}})$ groups is $\overline{\overline{Z}}$ $(\overline{\overline{SI}})$. *Theorem* 3. $A \oplus B$ is an $\widetilde{N}$ group iff (a) $A$ and $B$ are $\widetilde{N}$ groups and (b) if $F(A)$, $F(B)$ are isomorphic factors of $A,B$, resp., then every minimal normal non-$E$ subgroup of $F(B)$ is central in $F(B)$. *Corollary.* $A^{(S)}$ is $\widetilde{N}$ iff $A$ is $\widetilde{N}$ and $\overline{\overline{Z}}$. (Received February 15, 1968.)

68T-396. DAVID ZEITLIN, 1650 Vincent Avenue North, Minneapolis, Minnesota 55411. *On a class of generating functions.*

Theorem 1 for Laurent series is the companion result for Taylor's series given by D. Zeitlin, *On generating functions and a formula of Chaudhuri*, Amer. Math. Monthly 74 (1967), 1056-1062. *Theorem* 1. With $E(u) = \sum_{n=0}^{\infty} u^n/n!$ and $G(u) = \sum_{k=0}^{\infty} g_k u^k$, $M(x) \not\equiv 0$ and $Q(x) \not\equiv 0$ as real functions, let $E(M(x)t^q)G(t^{-p}Q(x)) = \sum_{n=-\infty}^{\infty} R_n(x)t^n$, where $p,q = 1,2,\ldots$ . Then $R_n(x) = (Q(y)/Q(x))^{n/p} \cdot \sum_{k=0}^{\infty}(P(x) - P(y))^k Q^{-qk/p}(y)R_{n-kq}(y)/k!$, with $P(x) \equiv M(x)Q^{q/p}(x)$; and $R_n(x)$, for $n + pk \geqq 0$ and $n + pk \equiv 0 \pmod{q}$, $= \sum_{k=0}^{\infty}(g_k Q^k(x)(M(x))^{(n+pk)/q})/((n + pk)/q)!$. With $' \equiv d/dx$, we have $pQ(x)R_n'(x) + nQ'(x)R_n(x) = (pQ(x)M'(x) + qQ'(x)M(x))R_{n-q}(x)$. Other results, too complicated for enumeration, have also been obtained. (Received February 15, 1968.)

68T-397. J. L. BAILEY, University of Tennessee, Knoxville, Tennessee 37916. *The product of a certain class of decomposition spaces and $E^1$ is $E^4$.*

R. H. Bing (Ann. of Math. 70 (1959), 300-412) showed that the cartesian product of the "dogbone" decomposition space and $E^1$ is $E^4$. This result is generalized using techniques similar to Bing's. *Theorem.* Let $G$ be an upper semicontinuous decomposition of $E^3$, all of whose nondegenerate elements are arcs. Further, require that the set of nondegenerate elements of $G$ be homeomorphic to the cartesian product of the Cantor set with a unit interval. Let $E^3/G$ denote the decomposition space corresponding to $G$; $E^3/G \times E^1$ is homeomorphic to $E^4$. (Received February 13, 1968.)

68T-398. C. P. BRUTER, 33 Boulevard Dubreuil, 91 Orsay, France. *Construction des matroides.*

Le présent texte fait suite à celui paru dans les *Notices* de Février. Soient $M$ un matroide défini sur un ensember fini $E$ et $\underset{\sim}{S}(M)$ la famille de ses stigmes. Soient $E_1$ un ensemble qui contient $E$, $e' \in E_1 - E$, et $e$ un élément fixé de $E$. Appelons opération d'adjonction simple de $e'$ à $M$, par l'intermédiaire de $e$, l'opération qui consiste à construire la famille $\underset{\sim}{S}(M')$ des sous-ensembles suivants de $E' = E \cup e$: (i) $S' = \{e,e'\} \in \underset{\sim}{S}(M')$, (ii) si $S \in \underset{\sim}{S}(M)$, $S \in \underset{\sim}{S}(M')$, (iii) si $S \in \underset{\sim}{S}(M)$, $e \in S$, $S'' = S \cup e' - e \in \underset{\sim}{S}(M')$. On vérifie alors trivialement le *Thèorème* 3. Par adjonction simple d'un èlèment $e'$ au matroide $M$ defini sur l'ensemble fini $E$, on obtient un matroide $M'$ défini sur l'ensemble $E' = E \cup e$. Des Théorèmes 2 et 3, on déduit simplement le théorème de H. Crapo, *Bonds of single element extensions*, J. Res. Nat. Bur. Standards, Sect. B 69 (1965), 55-65. (Received February 14, 1968.)

68T-399. S. J. TRAMEL, Louisiana State University, Baton Rouge, Louisiana 70810. On Artin equivalence and Krull rings. II. Preliminary report.

(For definitions and notation, see Abstract 67T-606, these *Notices* 14 (1967), 837.) Lemma: If A, B are nonzero ideals in a Krull ring D, then $A \sim B$ if and only if $AD_P = BD_P$ for each minimal prime ideal P of D. Corollary. If D is a Krull ring and A, B are ideals in D such that $A \sim B$, then $A : (x) \sim B : (x)$ for each $x \in D$. Theorem 4. If D is a Krull ring, then the following are equivalent: (1) D is Noetherian; (2) if A is an ideal in D such that $A \sim P$ for some minimal prime ideal P of D, then A is finitely generated. (Received February 15, 1968.)

68T-400. T. G. H. SKARD, University of Oslo, Oslo 3, Norway. Axiomatic algebra of regular expressions.

Let P, Q and R be regular expressions over the alphabet $\{a_1, \dots, a_r\}$, and let p and q be different regular expressions of the form $\emptyset$ or $a_i$. Axioms. $P \cup Q = Q \cup P$, $P \cup (Q \cap R) = (P \cup Q) \cap (P \cup R)$, $P \cap P' = \emptyset$, $(P' \cup Q')' = P \cap Q$, $P(QR) = (PQ)R$, $P(Q \cup R) = PQ \cup PR$, $(Q \cup R)P = QP \cup RP$, $P\emptyset = \emptyset$, $P\emptyset^* = P$, $P^* = \emptyset^* \cup PP^*$, $P^* = (\emptyset^* \cup P)^*$, $\emptyset' = (\bigcup_{i=1}^{r} a_i)^*$, $pP \cap pQ = p(P \cap Q)$, $pP \cap qQ = \emptyset$, and $pP \cap \emptyset^* = \emptyset$. Rules of inference. $P = (Qq)P \cup R \Rightarrow P = (Qq)^*R$, and rules stating the properties of equality. This is proved to be a consistent, complete and decidable axiom system for the algebra of regular expressions with the operators $\cup$, $\cap$, $'$, $*$ and $\cdot$. The proof of the completeness of the axiom system, which is the main task of the paper, is based upon the following facts: Certain sets of regular expression equations have unique solutions. A regular expression is uniquely characterized by its derivatives (and the $\delta$-function). A regular expression has only a finite number of dissimilar derivatives. (Similarity is defined by the equations $\emptyset P = \emptyset$, $\emptyset^* P = P$, $\emptyset \cup P = P$, $P \cup P = P$, $P \cup Q = Q \cup P$, $P \cup (Q \cup R) = (P \cup Q) \cup R$.) (Received February 15, 1968.) Introduced by Professor Stål Aanderaa.

68T-401. N. A. TSERPES, Wayne State University, Detroit, Michigan, and A. MUKHERJEA, Eastern Michigan University, Ypsilanti, Michigan. On semi-invariant and idempotent measures on semigroups. Preliminary report.

Let S be a Hausdorff space and $\phi$ a Borel measure on S. When S is a topological semigroup and $\phi$ is regular (probability), we call $(S, \phi)$ a MS (PMS). $\phi$ is (i) idempotent in a PMS if $\int f(s)\phi(ds) = \iint f(st)\phi(ds)\phi(dt)$ for every bounded continuous function f, (ii) semi-invariant in a MS if $\phi(Ca) \geqq \phi(C)$ for every compact C and a in S, (iii) r*-invariant if $\phi(t_a^{-1}(B)) = \phi(B)$ for every Borel B and $t_a$, right translation by a. A PMS is P*MS if every continuous function is integrable w.r.t. $\phi$. F is the support of $\phi$. Theorem 1. If S is realcompact or paracompact in a P*MS, then F is compact. Corollary. If $\phi$ in Theorem 1 is (i) r*-invariant, then F is a compact left group, (ii) idempotent, then F is a compact kernel. Theorem 2. Let S be normal with both cancellations in a P*MS with $\phi$ semi-invariant. Then F is a compact group. Theorem 3. Let $(S\phi)$ be an MS with S a metric space such that $t_a$ is closed for every a in S. Then if $\phi$ is r*-invariant, F is a left group. This generalizes Proposition 4.2 in Argabright's A note on invariant integrals on locally compact semigroups, Proc. Amer. Math. Soc. 17 (1966), 377-382. We remark that Argabright's conjecture that the support of a r*-invariant measure in a locally compact semigroup is a left group is not true for infinite measures. (Received February 15, 1968.)

68T-402. B. F. RYAN, Cornell University, Ithaca, New York 14850. *An embedding in the isols.*

An almost recursive combinatorial function is called an almost recursive combinatorial polynomial if all but a finite number of its Stirling coefficients are zero. Let S be the set of all almost recursive combinatorial polynomials. If f, g $\in$ S, define f $\sim$g iff for some integer k, $f(x_1 + k,\dots,x_n + k) = g(x_1 + k,\dots,x_n + k)$. *Theorem.* There exists a map $\phi: S \longrightarrow \Lambda$ such that for $f,g,g_1,\dots, g_n \in S$: (1) $f \sim g$ iff $\phi(f) = \phi(g)$. (2) $\phi(f(g_1,\dots,g_n)) = f_\Lambda(\phi(g_1),\dots,\phi(g_n))$. (3) If $k_1,\dots,k_{i-1}, k_{i+1},\dots,k_n \in E$, $x \in \Lambda$ and $x \leqq \phi(f(k_1,\dots,k_{i-1}, x_i,k_{i+1},\dots,k_n))$, then there exists $h \in S$ with $\phi(h) = x$. The proof uses Ramsey's theorem (Proc. London Math. Soc. (2) 30 (1929), 264-286) to construct a set $\alpha$ such that for any subset $\beta$ of $\alpha$ and any $n \in E$, if $x + y = \binom{\langle\beta\rangle}{n}$, then for some $k \in E$ either $\binom{\langle\beta\rangle-k}{n} \leqq x$ or $\binom{\langle\beta\rangle-k}{n} \leqq y$. By a category argument, we can find a sequence $X_1,X_2,\dots$ of subsets of $\alpha$ such that $f \not\sim g$ implies $f_\Lambda(\langle X_1\rangle,\dots, \langle X_n\rangle) \neq g_\Lambda(\langle X_1\rangle,\dots,\langle X_n\rangle)$. Then define $\phi(f) = f_\Lambda(\langle X_1\rangle,\dots,\langle X_n\rangle)$. (Received February 19, 1968.) Introduced by Mr. Thomas A. Ryan, Jr.

68T-403. J. B. BROWN, Auburn University, Auburn, Alabama. *Nowhere dense Darboux graphs.*

The word "graph" shall refer to the graph of a real function, and if f is a point set in the plane, the X-projection of f is the set of all abscissas of points of f. The statement that a graph f is a Darboux graph means that if C is a connected subset of its X-projection, then f(C), the image of C under f, is connected. *Theorem.* If f is a Darboux graph with X-projection and interval, then f is nowhere dense in the plane if and only if f is either continuous or else discontinuous only over a set of the first category. On the other hand, there is a transformation T from the plane E into E such that (1) if C is a connected set in the plane, then T(C) is connected, (2) T is totally discontinuous, and (3) T, considered as a subset of E × E, is nowhere dense. If f is a Darboux graph with X-projection and interval, then the additional stipulation that f be nowhere dense in the plane is not sufficient to insure that f be connected or that it even have a nondegenerate component. (Received February 19, 1968.)

68T-404. C. J. EVERETT and N. METROPOLIS, Los Alamos Scientific Laboratory, P.O.Box 1663, Los Alamos, New Mexico 87544. *Symmetrizability of incidence matrices.* Preliminary report.

An n × n matrix A of 0's and 1's is *symmetrizable* if PA is symmetric for some permutation matrix P, and satisfies *condition* (N) in case $Q(AA^T)Q^T = A^TA$ for some permutation matrix Q. It is trivial that (N) is necessary for symmetrizability, and the problem is posed to determine the minimal order $n_0$ (certainly exceeding 4) for which it is not sufficient. The question is of particular interest since all matrices A of type $(v,k,\lambda)$ are *normal:* $AA^T = A^TA$ [H. J. Ryser, *Combinatorial mathematics,* Carus Math. Monographs, no. 14, Math. Assoc. Amer., 1963 (distributed by Wiley, New York)]. All those tried appear to be symmetrizable. In particular, every projective geometry based on a Veblen-Wedderburn system with commutative multiplication is so. Algorithms developed for *construction* of all normal matrices A and of their *symmetrization* may succeed in determining $n_0$ if "small". What additional conditions insure symmetrizability is a contingent open question. (Received February 19, 1968.)

68T-405. R. M. VANCKO, University of Manitoba, Winnipeg 19, Canada. *On the system of locally independent subsets of a universal algebra.*

A set I in a universal algebra $\mathfrak{A}$ is *locally independent* if every map $\phi : I \to [I]$ can be extended to an endomorphism of $[I]$, where $[I]$ is the subalgebra generated by I. *Theorem.* A system $\mathscr{S}$ over a finite set A is the family of locally independent sets of some algebra defined on A if and only if $\mathscr{S}$ is hereditary and satisfies one of the conditions: (1) $\mathscr{S}$ contains either no singletons or more than one singleton; (2) the set A can be partitioned into three or more nonvoid sets $A_i$ so that $\{x,y\} \in \mathscr{S} \Rightarrow \{x,y\} \subseteq A_i$ for some i; (3) $\mathscr{S}$ contains exactly one singleton $\{a\}$ and there exists a mapping $\alpha : A \to A$ satisfying (i) $\alpha(b) = c$ for some $b \neq c$, $a \notin \{b,c\}$, (ii) $\{x,y\} \in \mathscr{S}$, $\alpha^m(x) = \alpha^n(y) \Rightarrow \alpha^m(w) = \alpha^n(z)$ for all w, z in $\{u : u = \alpha^k(x) \text{ or } u = \alpha^k(y) \text{ for some } k\}$, (iii) $\{x_1,\dots,x_n\} \in \mathscr{S} \Rightarrow \{\alpha(x_1), x_2,\dots,x_n\} \in \mathscr{S}$. Moreover, each of (1), (2), (3) is sufficient if A is infinite and $\mathscr{S}$ has finite character. The sufficiency of (1) was shown by J. Schmidt (see G. Grätzer, *Universal algebra*, Van Nostrand, Princeton, N. J., 1968, Section 35, and Problem 53). (Received February 19, 1968.)

68T-406. GEORGE KOZLOWSKI, University of Michigan, Ann Arbor, Michigan 48105. *A generalization of a theorem of Smale.*

By means of star-refinements of open covers the following generalization of a theorem of Smale (Proc. Amer. Math. Soc. 8 (1957), 604-610) can be established: *Theorem.* Let $f : X \to Y$ be a closed map of a paracompact (Hausdorff) $LC^n$ space onto a metric space Y with the property that $f^{-1}y$ is $LC^{n-1}$ and $(n - 1)$-connected for each $y \in Y$. Then Y is $LC^n$, and the induced homomorphism $\pi_k(X,x) \to \pi_k(Y,fx)$ is monic for $k < n$ and epic for $k \leqq n$ for each $x \in X$. (Received February 19, 1968.)

68T-407. PAUL GAUTHIER, Université de Montréal, Montréal, P. Q., Canada. *Some identity theorems for mevomorphic functions.*

For $0 < R \leqq +\infty$ we denote by D the set $(|z| < R)$, and we denote by W the Riemann sphere. We consider W and D as metric spaces, where W has the spherical metric, D has the Euclidean (parabolic) metric when D is parabolic, and D has the non-Euclidean (hyperbolic) metric when D is hyperbolic. *Definition.* A function $w = f(z)$, meromorphic in D, is normal if and only if it is uniformly continuous when considered as a mapping from the metric space D to the metric space W. In case D is hyperbolic, P. Lappan has shown that this definition is equivalent to the Lehto-Virtanen-Noshiro definition of normalcy. If D is parabolic, then Lehto and Virtanen have shown that there are no nonconstant functions normal in D in their sense. However, there are many nonconstant functions normal in a parabolic domain in our sense. Indeed it can be shown, using Lappah's technique, that the class of such functions coincides with Yosida's class A which includes, for example, the elliptic functions. A. L. Shaginian has shown that if $\alpha$ is a boundary curve in D, then there is a positive function $\lambda(r)$, $0 \leqq r < R$, such that for all $f(z)$ bounded and holomorphic in D, if $|f(z)| \leqq \lambda(|z|)$, $z \in \alpha$, then $f(z) \equiv 0$. This theorem is extended to normal meromorphic functions and analogous results are obtained for general meromorphic functions. (Received February 19, 1968.)

68T-408. R. F. DICKMAN, University of Miami, Coral Gables, Florida 33124. Real compactifications of arbitrary topological spaces.

Let X denote an arbitrary topological space and let C(X) denote the ring of all continuous real valued functions on X. We say that X is realcompact if every real maximal ideal in C(X) is fixed. A realcompactification of X is a realcompact space in which X is dense. We let $\rho_X$ denote the mapping of X into $R^{C(X)}$ defined by $\rho_X(x) = (F(x))$, $F \in C(X)$. Theorem. There exists a topologically unique realcompactification $\tau X$ of X satisfying: (a) X is C-embedded in $\tau X$, i. e. for every $F \in C(X)$ there exists a continuous extension $F^1$ of F to $\tau X$; (b) for every pair of distinct points a and b in $\tau X - X$ there exists an $F \in C(X)$ such that $F^1(a) \neq F^1(b)$; (c) X is open in $\tau X$; (d) $\rho_{\tau X}$ is a compact mapping; and (e) a closed subset A of X is closed in $\tau X$ if and only if $\rho_X|A$ is a compact mapping. The construction employs the technique of unifying the domain and range of a mapping due to G. T. Whyburn [A unified space for mappings, Trans. Amer. Math. Soc. 74 (1953), 344-350]. Corollary 1. If X is completely regular and realcompact $\tau X \approx X$. Corollary 2. For any space X, $\tau X \approx \tau^2 X$. (Received February 19, 1968.)

68T-409. J. M. WORRELL, JR., and H. H. WICKE, Sandia Corporation, P. E. Box 5800, Albuquerque, New Mexico 87115. On uniformly monotonically complete mappings.

The concept of a uniformly monotonically complete mapping is defined in Abstract 66T-333, these *Notices* 13 (1966). Applications of the concept can be found in an article of the authors in Duke Math. J. 34 (1967), 255-272. Recall that the class of $T_2$-spaces having λ-bases locally is the class of open continuous $T_2$ images of metrically topologically complete spaces [Duke Math. J. loc. cit.]. Theorem. Suppose $\phi$ is a continuous mapping of a subspace R of a $T_2$-space S having λ-bases locally onto a $T_1$-space. Then $\phi$ is uniformly monotonically complete if and only if there exists a subspace E of S which has λ-bases locally such that $\phi^{-1}(P)$ is closed in E for all P in $\phi(S)$. Theorem. Any $T_1$-space S having a base of countable order is an open continuous image of a metrizable space which lies densely in some metrically topologically complete space Σ such that the inverse image of any point of S is closed in Σ. (Received February 19, 1968.)

68T-410. J. D. HARRIS, University of Kansas, Lawrence, Kansas 66044. Completion of a space with respect to a class of spaces. Preliminary report.

For each class E of topological spaces, the class of E-complete spaces is defined. Using a naturally arising family of open filters, the E-completion $\beta_E X$ of a space X with respect to an arbitrary class E of spaces is constructed. The space $\beta_E X$ is E-complete, every continuous function from X into an E complete space can be extended to $\beta_E X$, and it is the smallest principal E-extension of X. When all spaces are Hausdorff, E-complete spaces are shown to be identical with the E-compact spaces described by Herrlich [Math. Z. 96 (1967), 228-255] and the E-completion identical with the E-compactification. The present paper provides an alternative construction for E-compactifications and a generalization of E-compactness. It also leads to a new construction of the Stone-Čech compactification. Further properties of E-complete spaces and of E-completions are established. An interesting class of spaces, which includes the regular spaces and the Hausdorff spaces, is introduced in which E-completions have especially pleasant properties. (Received February 19, 1968.)

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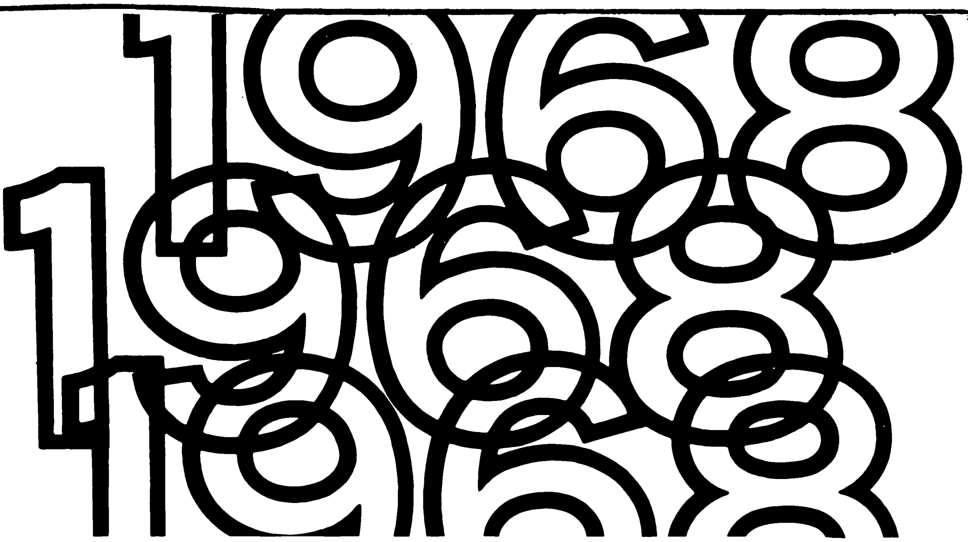

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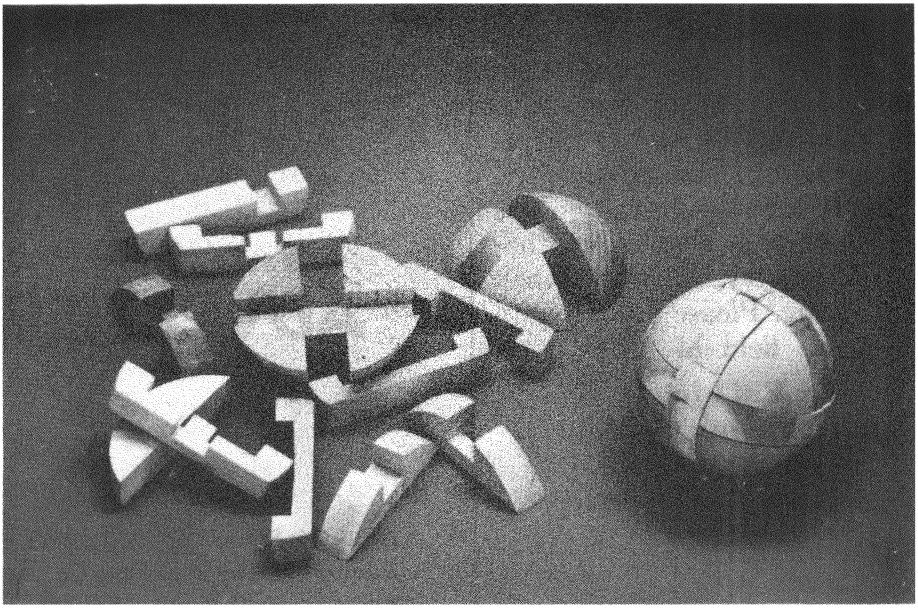

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