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# MEETINGS

## Calendar of Meetings

**NOTE:** This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the *Notices* was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

<table>
<thead>
<tr>
<th>Meeting No.</th>
<th>Date</th>
<th>Place</th>
<th>Deadline for Abstracts*</th>
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<tbody>
<tr>
<td>658</td>
<td>August 26-30, 1968</td>
<td>Madison, Wisconsin</td>
<td>July 1, 1968</td>
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<td></td>
<td>(73rd Summer Meeting)</td>
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<tr>
<td>659</td>
<td>October 26, 1968</td>
<td>Baltimore, Maryland</td>
<td>Sept. 6, 1968</td>
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<tr>
<td>660</td>
<td>November 8-9, 1968</td>
<td>Clemson, South Carolina</td>
<td>Sept. 25, 1968</td>
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<td>661</td>
<td>November 16, 1968</td>
<td>Riverside, California</td>
<td>Sept. 25, 1968</td>
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<tr>
<td></td>
<td>(75th Annual Meeting)</td>
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<td></td>
<td>August 25-29, 1969</td>
<td>Eugene, Oregon</td>
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<tr>
<td></td>
<td>(74th Summer Meeting)</td>
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<td></td>
<td>January 22-26, 1970</td>
<td>Miami, Florida</td>
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<tr>
<td></td>
<td>(76th Annual Meeting)</td>
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<tr>
<td></td>
<td>August 1970</td>
<td>Laramie, Wyoming</td>
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<tr>
<td></td>
<td>(75th Summer Meeting)</td>
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<td></td>
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<tr>
<td></td>
<td>January 21-25, 1971</td>
<td>Atlantic City, New Jersey</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(77th Annual Meeting)</td>
<td></td>
<td></td>
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</tbody>
</table>

*The abstracts of papers to be presented in person at the meetings must be received in the Headquarters Offices of the Society in Providence, Rhode Island, on or before these deadlines. The deadlines also apply to news items. The next two deadlines for by-title abstracts will be June 24, 1968, and August 30, 1968.*

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The *Notices* of the American Mathematical Society is published by the Society in January, February, April, June, August, October, November and December. Price per annual volume is $12.00. Price per copy $3.00. Special price for copies sold at registration desks of meetings of the Society, $1.00 per copy. Subscriptions, orders for back numbers (back issues of the last two years only are available) and inquiries should be addressed to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904.

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The six hundred fifty-seventh meeting of the American Mathematical Society will be held on Saturday, June 15, 1968, at Reed College in Portland, Oregon, in conjunction with meetings of the Pacific Northwest Section of the Mathematical Association of America and the Society for Industrial and Applied Mathematics. The Association and SIAM will meet on Friday and Saturday, June 14 and 15.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, there will be two hour addresses at this meeting. Professor William G. Bade of the University of California, Berkeley, will address the Society at 11:00 a.m. on Saturday in Room 314, Eliot Hall. The title of his lecture is "Sources of equicontinuity in functional analysis." Professor W. A. J. Luxemburg of the California Institute of Technology will present a lecture entitled "On some recent developments in the theory of Riesz spaces" at 2:00 p.m. on Saturday in Room 314, Eliot Hall.

There will be sessions for contributed papers at 9:15 a.m. and 3:15 p.m. in Rooms 200, 314, and 416, Eliot Hall. Late papers may be added to the program. Information on late papers and program changes will be available at the registration desk.

REGISTRATION

The registration desk for the meeting will be located at the main entrance to Eliot Hall. It will be open from 3:00 to 6:00 p.m. on Thursday, June 13; from 8:30 a.m. to 5:00 p.m. on Friday, June 14; and from 9:00 a.m. to 12:00 noon on Saturday, June 15.

ACCOMMODATIONS

Dormitory space will be available at the rate of $2.75 per single room per night. Reservations for dormitory rooms should be sent to Professor Burrowes Hunt, Department of Mathematics, Reed College, Portland, Oregon 97202.

The first two of the following hotels and motels are on the east side of Portland (as is Reed College), while the last three are in downtown Portland.

Rose Manor Motel
- Single $8.00
- Double $10.50
- Twin $11.50
(These are commercial rates)

Sheraton Hotel
- Single $9.00 - $15.00
- Double 15.50 - 19.00
(Slightly lower rates are available to faculty members with a Sheraton ID card.)

Benson Hotel
- Single $12.00
- Twin $17.00
(These are faculty rates, requiring only personal identification.)

Heathman Hotel
- Single $7.50
- Double $11.00
- Twin $12.50
(These are faculty rates, requiring only personal identification.)

Hilton Hotel
- Single $9.00
- Double $16.00
(These are faculty rates, requiring only personal identification.)

MEALS

Luncheon will be available in the College Commons on the days of the meet-
ailings. If enough dormitory reservations are received, breakfast and dinner may also be served in the Commons. Information concerning local restaurants will be available at the registration desk.

TRAVEL

Portland is served by the major west coast airlines and railroad lines. Taxi service from the railroad station to Reed College costs approximately $3.00, and from the airport the cost is about $6.50. There is an airport limousine to all hotels.

Persons arriving by automobile from the north on Interstate 5 should follow signs toward Oregon City, then break off right at the sign to East Moreland and Reed College. Automobiles from the south on Interstate 5 should exit across the Ross Island Bridge and follow Powell to S. E. 39th, then right on 39th to Woodstock and right to Reed. Persons driving from the east on the Columbia River Freeway should exit at about 40th, take a sharp right on 46th, right to Gleason, right to 39th, then left to Woodstock, and right to Reed College.

PROGRAM OF THE SESSIONS

The time limit for each contributed paper is 10 minutes. The papers are scheduled at 15 minute intervals in order that listeners can circulate among sessions. To maintain the schedule, the time limit will be strictly enforced.

SATURDAY, 9:15 A.M.

Session on Topology, Room 314, Eliot Hall
9:15-9:25
(1) Some generalizations of the contraction mapping theorem. Preliminary report
Professor B. T. Sims, Eastern Washington State College (657-25)

9:30-9:40
(2) On a problem of Borsuk. Preliminary report
Professor N. R. Gray, Western Washington State College (657-14)

9:45-9:55
(3) On three problems of Franklin and Wallace
Professor L. E. Ward, Jr.*, and Mr. E. D. Tymchatyn, University of Oregon (657-6)

10:00-10:10
(4) Regularity for convergence structures
Mr. B. V. Hearsey, Washington State University (657-21)
(Introduced by Dr. D. C. Kent)

10:15-10:25
(5) A representation theorem for uniform convergence structures
Professor D. C. Kent, Washington State University (657-16)

SATURDAY, 9:15 A.M.

Session on Number Theory, Room 200, Eliot Hall
9:15-9:25
(6) The parity of the integer \( (2n - 2)! / n!(n - 1)! \)
Mr. D. M. Silberger, Western Washington State College (657-7)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.
9:30-9:40
(7) Prime divisors of the binomial coefficient
Mr. E. F. Ecklund, Jr., Western Washington State College (657-11)
(Introduced by Professor M. L. Faulkner)

9:45-9:55
(8) "Decimal" expansions to negative bases
Professor C. T. Long, Washington State University (657-5)

10:00-10:10
(9) The distribution on quadratic residues in fields of order $p^2$
Professor J. H. Jordan*, Washington State University, and Mr. N. R. Hardman, Harvard University (657-4)

10:15-10:25
(10) Pisot sequences and Pisot-Vijayaraghavan numbers. Preliminary report
Mr. M. J. DeLeon, Pennsylvania State University (657-15)

SATURDAY, 9:15 A.M.

Session on Analysis, Room 416, Eliot Hall

9:15-9:25
(11) A note on mini-gaps
Dr. B. G. Eke, University of California, San Diego (657-18)
(Introduced by Professor S. E. Warschawski)

9:30-9:40
(12) Bounds for the number of deficient values of entire functions whose zeros have angular densities
Professor Ki Choul Oum, State University College at Buffalo (657-19)

9:45-9:55
(13) Reduction of order for a system of differential equations at a turning point. Preliminary report
Dr. R. Y. Lee, University of Minnesota (657-22)

10:00-10:10
(14) Lower bounds for the essential spectrum of fourth order differential operators
Mr. Kurt Kreith, University of California, Davis (657-3)

10:15-10:25
(15) A theorem relating generalized Hankel and Whittaker transforms. Preliminary report
Professor H. M. Srivastava*, West Virginia University, and Mr. Om Dutta Vyas, Jodphur University, India (657-9)

10:30-10:40
(16) Uniform convergence of spherical harmonic expansions
Mr. D. L. Ragozin, Massachusetts Institute of Technology (657-29)

SATURDAY, 11:00 A.M.

Invited Address, Room 314, Eliot Hall

Sources of equicontinuity in functional analysis
Professor William G. Bade, University of California, Berkeley

SATURDAY, 2:00 P.M.

Invited Address, Room 314, Eliot Hall

On some recent developments in the theory of Riesz spaces
Professor W. A. J. Luxemburg, California Institute of Technology
SATURDAY, 3:15 P.M.

General Session, Room 314, Eliot Hall
3:15-3:25
(17) WITHDRAWN.

3:30-3:40
(18) Complementing pairs of subsets of the plane
Professor R. T. Hansen, Montana State University (657-20)

3:45-3:55
(19) Projections of f-vectors of 4-polytopes
Professor J. R. Reay, Western Washington State College (657-24)

4:00-4:10
(20) On finite decompositions of (2n - l)-spaces
Mr. Joseph Zaks, University of Washington (657-23)

4:15-4:25
(21) Concerning the ideal and order topologies on a lattice. Preliminary report
Mr. Charles Atherton, Washington State University (657-27)

SATURDAY, 3:15 P.M.

Session on Algebra, Room 200, Eliot Hall
3:15-3:25
(22) On finite groups with a cyclic Sylow subgroup
Professor Marcel Herzog, University of California, Santa Barbara (657-2)

3:30-3:40
(23) A class of modules over a left-noetherian ring
Professor G. R. Krause, Washington State University (657-28)
(Introduced by Professor J. H. Jordan)

3:45-3:55
(24) On commutative splitting rings
Mr. J. D. Fuelberth, University of Southern California (657-13)
(Introduced by Professor S. E. Dickson)

4:00-4:10
(25) Decomposition of atomic and orthogonally complete rings
Professor Alexander Abian, Iowa State University (657-31)

SATURDAY, 3:15 P.M.

Session on Analysis, Room 416, Eliot Hall
3:15-3:25
(26) Singular integrals in several variables over a local field
Professor Keith Phillips*, California Institute of Technology, and Professor Mitchell Taibelson, Washington University, St. Louis (657-32)

3:30-3:40
(27) On uniformly approximable Sidon sets. Preliminary report
Professor R. W. Chaney, University of California, Santa Barbara (657-1)

3:45-3:55
(28) On the essential set
Professor D. R. Chalice, Western Washington State College (657-26)

4:00-4:10
(29) Linear operators on spaces of continuous functions
Professor R. E. Atalla, Ohio University (657-30)
MEMORANDA TO MEMBERS

RESOLUTION ON THE DRAFT

The following resolution was passed at the Council meeting of April 19, 1968:
"The Council of the American Mathematical Society regrets the recent government decision about the drafting of graduating seniors and first year graduate students. The possible partial or total elimination of two successive graduate classes can cause a dangerous hiatus in the future production of scholars."

DIFFERENT CATEGORIES FOR BY-TITLE ABSTRACTS

By-title abstracts will be grouped according to the subject fields, starting with the August issue of *Notices*; abstracts classified in "other fields," or in more than one field, will appear in the miscellaneous category. This change is being made in response to requests by several members.

A new abstract form is available and will be sent to individuals upon request. The subject fields correspond to those already in use, and the old abstract form may be used. Authors who are not aware of this new procedure will be given an opportunity to provide the subject field when receipt of the abstract is acknowledged. Papers for which subject classifications are not provided by the author by the deadline for by-title abstracts will be placed in the miscellaneous group.

REQUEST FOR NAMES OF VISITING FOREIGN MATHEMATICIANS

The editors of these *Notices* would appreciate receiving the names and addresses of foreign mathematicians who will be visiting the United States in 1968-1969. Information should include the name and country of each mathematician, his U. S. address and the period of his visit.

The information will be published in the August issue of these *Notices*, in time for mathematics departments to extend speaking invitations for the coming academic year. The regular annual list of foreign mathematicians will appear in the November issue.
The American Mathematical Society will hold its seventy-third summer meeting in Madison, Wisconsin, from Tuesday, August 27, 1968, through Friday, August 30, 1968. All sessions will be held in lecture rooms and classrooms of the University of Wisconsin. Times are CENTRAL DAYLIGHT SAVING TIME throughout.

There will be two sets of Colloquium Lectures each consisting of four lectures. Professor Donald C. Spencer of Stanford University will speak on the subject "Overdetermined systems of partial differential equations." His lectures will be given on Tuesday, August 27, at 1:30 p.m., on Wednesday at 8:30 a.m., and on Thursday and Friday at 9:00 a.m. The other Colloquium Lecturer will be Professor John W. Milnor of Princeton University and the University of California, Los Angeles. His topic will be announced later. He will speak on Tuesday at 2:45 p.m., on Wednesday at 9:30 a.m., and on Thursday and Friday at 9:00 a.m. The initial lecture in each series will be delivered in the Union Theater, but the subsequent lectures will be given in Rooms B102 and B130 in Van Vleck Hall.

By invitation of the Committee to Select Hour Speakers for Annual and Summer Meetings, there will be three one-hour addresses. Professor W. C. Hsiang of Yale University will speak on Thursday, August 29, at 1:45 p.m. His talk will be entitled "Non-simply connected differential topology." Professor P. A. Griffith of the University of Houston and Princeton University will address the Society on Thursday, August 29, at 3:00 p.m. His subject will be "Some transcendental problems in algebraic geometry." Professor V. W. Guillemin of the Massachusetts Institute of Technology will speak on Friday, August 30, at 1:45 p.m. His topic will be "Recent developments in the theory of pseudogroups." All three talks will be given in the Union Theater.

There will be numerous sessions for the presentation of contributed ten-minute papers on Wednesday at 10:45 a.m., on Thursday at 10:15 a.m., and on Friday at 10:15 a.m. and 3:00 p.m. All of these sessions will be held in Van Vleck Hall. Abstracts of contributed papers should be sent to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904, so as to arrive prior to the deadline of July 1. Abstract blanks can be obtained on request from the same address. There will be no limit on the number of contributed papers. No provisions will be made for late papers.

This meeting will be held in conjunction with meetings of several other organizations. These include the Mathematical Association of America, the Society for Industrial and Applied Mathematics, the Institute of Mathematical Statistics, Pi Mu Epsilon, and Mu Alpha Theta. The Mathematical Association of America will meet from Monday through Wednesday. The Association will present Professor Hyman Bass of Columbia University as the Earle Raymond Hedrick Lecturer on the general topic of "Algebraic K-theory." His three lectures will be given on Monday, August 26, at 9:15 a.m. and at 1:30 p.m. and on Tuesday at 9:00 a.m. in the Union Theater. The individual titles will be "Stability theorems in topology and in algebra," "Grothendieck groups and Whitehead groups," and "Congruence subgroups and reciprocity laws." The Society for Industrial and Applied Mathematics will present Professor Peter D. Lax of the Courant Institute of Mathematical Sci-
ences, New York University, as the von Neumann Lecturer. He will speak on Wednesday, August 28, at 8:00 p.m. in Room 6210 of the Social Science Building on the subject "Nonlinear partial differential equations." The Institute of Mathematical Statistics will meet from Tuesday through Friday. The Institute will present Professor Herman A. Chernoff of Stanford University as the Wald Lecturer on the general subject "Continuous time stopping problems and optimal stochastic control." His three lectures will be given on Tuesday, August 27, at 4:00 p.m. and on Wednesday and Thursday at 3:00 p.m. in Room 1351 of the New Chemistry Building and will have the individual titles "Stopping problems and the heat equation," "Sequential analysis and the one-armed bandit problem," and "A stochastic control problem and the two-armed bandit problem." The Institute of Mathematical Statistics will also present a special invited address by Professor Gian-Carlo Rota of the Massachusetts Institute of Technology. He will speak on Tuesday, August 27, at 11:00 a.m. on "Recent advances in combinatorial theory." Pi Mu Epsilon and Mu Alpha Theta will meet concurrently with the Society.

COUNCIL AND BUSINESS MEETINGS

The Council of the Society will meet on Tuesday, August 27, at 5:00 p.m. in the Lake Shore Room of the Wisconsin Center. The Business Meeting of the Society will be held on Thursday, August 29, at 4:30 p.m. in the Union Theater.

REGISTRATION

The Registration Desk will be in the lobby of the Wisconsin Center. It will be open on Sunday from 2:00 p.m. to 8:00 p.m.; on Monday from 8:00 a.m. to 5:00 p.m.; on Tuesday, Wednesday, and Thursday from 9:00 a.m. to 5:00 p.m.; and on Friday from 9:00 a.m. to 1:00 p.m.

The registration fees will be as follows:

- Member: $3.00
- Member's family: 0.50
- Students: No charge
- Others: 7.50

EMPLOYMENT REGISTER

The Mathematical Sciences Employment Register will be held in Room 226 and the second floor Lake Lounge of the Wisconsin Center. It will be open Tuesday through Thursday, August 27 through August 29, from 9:00 a.m. to 5:00 p.m. on each of the three days. Attention is invited to the announcement of the Employment Register on Page 451 of the April Notices, in particular to the deadline for applications and to the necessity for prompt registration at the Employment Register desk by both applicants and employers.

EXHIBITS

Book exhibits and exhibits of educational media will be displayed in the Wisconsin Center on Tuesday, Wednesday, and Thursday.

BOOK SALE

Books published by the Society will be sold for cash prices somewhat below the usual prices when these same books are sold by mail on invoice.

DORMITORY HOUSING

Dormitory rooms will be available in both the University Residence Halls and in private dormitories. Reservations for dormitory rooms should be made in advance, using the form provided in these Notices. Although it is possible that rooms will still be available to persons who have not registered in advance, this is not guaranteed. Please note that the university dormitories are not air-conditioned, although some of the rooms in the private dormitories are. If for reasons of health, air-conditioning is necessary, please indicate on the housing form so special provisions can be made for an air-conditioned room.

The University Residence Halls (Sellery, Ogg, and Witte) are modern, attractive, quite new, and within walking distance of the various meeting rooms. Rates (including meals) are: $8.00/day/person for a double room; $9.75/day/person for a single room; children 2-11 $3.00/day; children 2 and under no charge.
1. Wisconsin Center
2. Lowell Hall
3. Carroll Hall
4. Wisconsin Memorial Union
5. Sellery Hall
6. Ogg Hall
7. Gordon Commons
8. Witte Hall
9. Lot 60
Cots and cribs cannot be supplied by the university, but participants may make arrangements for rental of this equipment by writing to: A to Z Rental, 2620 East Washington Avenue, Madison, Wisconsin, or AAA Rental Center, 4233 West Beltline Highway, Madison, Wisconsin. An infant may occupy the parents' room without additional charge if the parents provide cribs and bedding. Sleeping bags and pets are not permitted in the Residence Halls. Linen, blankets, towels, and soap are furnished, and daily maid service is provided. Please note that there is no air-conditioning. Dormitories may be occupied from Saturday, August 24, at 10:00 a.m. to Saturday, August 31, at 10:00 a.m. As meal service does not begin until dinner (evening) on Sunday and terminates with lunch on Friday, the charge for rooms only on Saturday, August 24, and on Friday, August 30, will be $5.75/day/person for a single room and $4.00/person in a double room. Charges will be adjusted for late arrivals and early departures on a pro rata basis. For those occupying the University Residence Halls for just a few days, one of those days being August 26, the day of the picnic, a small additional charge will be levied. (For example, a person arriving after lunch on Monday and leaving after lunch on Thursday will pay $24.85 for double occupancy. The extra $0.85 will go toward the picnic.) The University will not make refunds for meals that may be missed or accommodations not used during the period arranged for. There is limited parking in one of the university lots one to five blocks from the Southeast Residence Halls; permits will be issued for a charge of $1.00 for the meeting period to those residing in the University Residence Halls. If space in one of these lots is desired, please so indicate on the housing form on Page 668 of these Notice.

Carroll Hall, 620 North Carroll Street, is a complex of private dormitories all within easy walking distance of the Wisconsin Center. Rates (not including meals): $4.50/day/person in a double room; $6.50/day/person in a single room; children 11 and under are half price. Cots and cribs may be obtained for a small charge with advance notice. Many of these rooms are air-conditioned. Occupancy is the same as for the University Residence Halls.

Lowell Hall, 610 Langdon Street, is a private dormitory, also within easy walking distance of the Wisconsin Center. Rates (not including meals): $5.00/day/person in a double room; $7.00/day/person in a single room; children 2-11 are half price; children under 2 are free. Cribs can be rented with advance notice. All rooms are air-conditioned.

Upon arrival on campus all guests who have made advance dormitory reservations should go directly to the residence hall indicated on the confirmation slip. Guests who arrive without reservations should go directly to the Meetings Registration Desk in the Wisconsin Center if they arrive during the regular registration hours as indicated in the section of this announcement entitled 'Registration.' Those arriving when the Registration Desk is closed, should go to the main desk on the ground floor of Ogg Hall. Guests are urged, however, to arrive during the normal registration hours.

Payment should be made in the Southeast Halls Office for those staying in the University Residence Halls. Those staying in any of the halls in the Carroll Hall complex will make payment in the respective hall office.

FOOD SERVICE

For those staying in the University Residence Halls, meals will be served cafeteria style in the air-conditioned dining rooms of Gordon Commons (adjacent to the housing buildings) beginning with breakfast on Monday, August 26, and continuing through lunch on Friday, August 30. Hours for food service are as follows:

<table>
<thead>
<tr>
<th>Time</th>
<th>Breakfast</th>
<th>Lunch</th>
<th>Dinner</th>
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</thead>
<tbody>
<tr>
<td>7:30 a.m.</td>
<td>9:00 a.m.</td>
<td>11:30 a.m.</td>
<td>1:15 p.m.</td>
</tr>
<tr>
<td>8:30 a.m.</td>
<td>1:00 p.m.</td>
<td>5:00 p.m.</td>
<td>6:30 p.m.</td>
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A list of nearby restaurants will be available at the Registration Desk in the Wisconsin Center.
MOTELS AND HOTELS

There are a number of motels and hotels in the area, some of which are listed below with coded information which is explained at the end of the list. Those motels preceded by an asterisk (*) are within walking distance of the Wisconsin Center. Participants should make their own reservations with hotels and motels.

*TOWN-CAMPUS MOTEL (608) 257-4881
441 North Frances. 46 rooms
Single $9.00 - $12.00
Double 12.00 - 15.00
Twin 15.00 - 16.00
Extra person $3.00, Code: RT-FP-TV-AC
Located on campus (State and Frances)

*IVY INN MOTEL (608) 233-9717
2355 University Avenue. 60 rooms
Single $10.00
Double 12.00
Twin 14.00
Double-double $18.00
Extra person $4.00, rollaway.
Code: RT-CL-FP-TV-AC

RAMADA INN (608) 244-2481
3841 E. Washington Avenue. 198 rooms
Single $10.00
Double 13.00
Twin 15.00
Extra person $2.00,
Code: RT-CL-FP-TV-AC
4 miles from campus

STERLING MOTEL (608) 256-0691
915 W. Beltline. 40 rooms
Single $8.00
Double 10.00 - 14.00
Twin 12.00
Extra person $2.00,
Code: RT-CL-FP-TV-AC
2 miles from campus

HOWARD JOHNSON (608) 244-6265
4822 E. Washington Avenue. 80 rooms
Single $10.00 - $15.00
Double 13.00
Twin 17.00 - Extra person $3.00
Code: RT-CL-FP-SP-TV-AC
6 miles from campus

TRAVELodge (608) 256-8365
910 Ann Street. 100 rooms
Single $8.00 - $10.00
Double 10.00 - 12.00
Twin 12.00 - 14.00
Extra person $2.00,
Code: FP-SP-TV-AC
3 miles from campus

*EDGEWATER HOTEL (608) 256-9071
666 Wisconsin Avenue. 103 rooms
Single $11.00 - $22.00
Double 16.00 - 28.00
Twin 16.00 - 28.00
Extra person $1.00
Code: RT-CL-FP-TV-AC

NATIONAL MOTOR INN (608) 257-5341
350 W. Washington Avenue. 100 rooms
Single $10.00 - $12.50
Double 14.50 - 16.50
Twin 16.50 - 18.50
Extra person $3.00
Code: RT-CL-FP-SP-TV-AC
5 minutes drive from campus

*LORAINE HOTEL (608) 256-0231
123 W. Washington Avenue. 400 rooms
Single $6.50 - $10.50
Double 10.00 - 14.00
Twin 13.50 - 18.00
Code: RT-CL-TV-AC

MIDWAY MOTOR LODGE (608) 244-2424
3710 E. Washington Avenue. 100 rooms
Single $10.50 - $13.00
Double 14.00 - 16.00
Twin 16.00 - 20.00
Extra person $2.00,
Code: RT-CL-FP-SP-TV-AC
3 miles from campus

HOLIDAY INN (608) 244-4703
4 1/2 m NE on US 151. 208 rooms
Single $7.00 - $9.75
Double 9.00 - 11.75
Twin 13.75
Extra person $2.00,
Code: RT-CL-FP-SP-TV-AC
6 1/2 miles from campus

RT - Restaurant  SP - Swimming Pool
CL- Cocktail Lounge  TV - Television
FP - Free Parking  AC - Air-Conditioned

ENTERTAINMENT

There will be a bus excursion to Spring Green on Wednesday, August 28. The tour will be from 9:30 a.m. to 4:00 p.m. and will include a guided visit to the Frank Lloyd Wright School of Architecture and buildings designed by Wright of local sandstone, stucco, and natural wood combinations. Lunch will be served at Apple Hill Farm and will be followed by a visit to the "House on the Rock." The charge will be $6.00 for adults with a slight reduction for young people between the ages of 10 and 16. The trip is not recommended for children under 10. Tickets will be on

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sale in the registration area.

The traditional SIAM Beer Party will be held in the Wisconsin Memorial Union Cafeteria on Wednesday, August 28, at approximately 9:00 p.m. following the von Neumann Lecture. If weather permits, guests are welcome to use the Union Terrace. Tickets will be $2.00 and will be on sale at the registration area.

A picnic will be held on Wednesday, August 28, at 5:15 p.m. at the Athletic Field. In case of rain, the picnic will be held in the Gordon Commons Dining Rooms. Tickets will be $2.60 for adults and $1.30 for children 11 years of age and under and will be on sale at the registration area.

Conducted nature walks of the Wisconsin Arboretum will be available. Trail maps will be provided in the registration area.

Other diversions and facilities include: hiking and picnicking on Picnic Point, swimming in lakes or in natatorium, tennis courts, gymnasium facilities, canoeing and boating, excursion boat around the lake, Vilas Park Zoo with playground for children, Arboretum, golf courses, Historical Society Museum, dairy store at Babcock Hall, Madison Arts Center, and several art galleries.

TRAVEL

Madison is served by North Central, Northwest Orient, and Ozark Air Lines. In particular Northwest Orient Air Lines has direct flights to Madison from Chicago, Milwaukee, Minneapolis, New York, and Washington. The Milwaukee Road has several trains a day between Madison and Chicago; Badger Coach Lines has express bus service between Milwaukee and Madison; while Greyhound offers express bus service between Chicago and Madison, and also a special connection between O'Hare Field and Madison. For those coming from the east by car, an interesting possibility is presented by the sailings of the S. S. Milwaukee Clipper between Muskegon, Michigan, and Milwaukee, Wisconsin, and by the sailings of the Chesapeake and Ohio Ferry between Ludington, Michigan, and Milwaukee, Wisconsin. Both ferries carry cars, but advance reservations are desirable. The respective addresses of the two ferry companies are: Wisconsin and Milwaukee Steamship Company, 500 N. Harbor Drive, Milwaukee, Wisconsin; and Chesapeake and Ohio Autoferries, Jones Island Outer Harbor, Milwaukee, Wisconsin.

CAMPING

There is a large number of both private and public campgrounds within driving distance of Madison. There are no campgrounds within the city limits. For a complete list of public and private campgrounds, one should write to the Vacation and Travel Service, Wisconsin Conservation Department, Box 450, Madison, Wisconsin 53701. Those interested in summer cottages or resorts should write to the Madison Chamber of Commerce, Inc., 615 E. Washington Avenue, P. O. Box 71, Madison, Wisconsin, for their list entitled "Madison and Area Vacation Information." This list includes private accommodations within a 50-mile radius of Madison.

PARKING

Parking is extremely limited on the campus. However, most people staying in the university and private dormitories will be able to walk to the meetings. Alternatively, one can park in the city ramp at the corner of State and Lake (within one block of the Wisconsin Center and Wisconsin Memorial Union) and walk to the meetings. Another possibility is to park in the University Lot 60 or Lot 62 west of the campus and take the campus bus directly to Van Vleck Hall, the Wisconsin Memorial Union, or the Wisconsin Center. The buses run every 5 to 10 minutes during the day; the ride takes about 10 minutes, and costs ten cents.

WEATHER

For the last week of August temperatures in Madison typically range from a high of 75°-80° during the day to a low of 50°-55° during the night. Considerable variation from these figures is possible. Madison is not noted for its low humidity.

BOOKSTORE

Brown's Book Store (University Avenue branch) is open from 9:00 a.m. to 5:30 p.m., on Tuesday through Saturday and on Monday from 9:00 a.m. to 9:00
p.m. The State Street branch of Brown's is open from 9:00 a.m. to 5:00 p.m. on Monday through Saturday.

The University Book Store (University Avenue branch) is open from 9:00 a.m. to 5:00 p.m. on Monday through Friday and from 9:00 a.m. to 4:00 p.m. on Saturday. The State Street branch is open from 9:00 a.m. to 5:00 p.m. on Monday through Friday and is closed on Saturday.

LIBRARIES

The Mathematics-Physics Library in Van Vleck Hall will be open from 8:30 a.m. to 5:00 p.m. on Monday through Friday and from 10:00 a.m. to 1:00 p.m. on Saturday.

The Memorial Library (main library) will be open from 8:00 a.m. to 5:00 p.m. Monday through Friday and 8:00 a.m. to noon on Saturday.

MEDICAL SERVICES

The Student Health Service, located about one block from Van Vleck Hall, will offer emergency medical service only. There will be a charge of $5.00 per visit plus medication expenses.

ADDRESS FOR MAIL AND TELEGRAMS

Individuals may be addressed at Mathematical Meetings, Wisconsin Center, University of Wisconsin, Madison, Wisconsin 53706.

COMMITTEE

H. L. Alder, ex officio
P. T. Bateman, ex officio
R. H. Bing, Chairman
R. A. Brualdi
Sister Diane Drufenbrock
J. V. Finch
Simon Hellerstein
M. I. Knopp
Morris Marden
Mrs. Vera Nohel
J. Barkley Rosser
George Roussas
Mrs. Mary Ellen Rudin
Mrs. Jeanne Smith
M. B. Smith, Jr.
L. F. Wahlstrom
G. L. Walker, ex officio

Urbana, Illinois

Paul Bateman
Associate Secretary

NEWS ITEM

MANUSCRIPTS FOR MATHEMATICAL SURVEYS AND COLLOQUIUM PUBLICATIONS

Manuscripts are now being accepted for Mathematical Surveys and Colloquium Publications, two series of clothbound books published by the AMS. Mathematical Surveys is published to meet the need for careful expositions of fields of current interest in research; each book is designed to give a brief survey of a subject and an introduction to its recent developments and unsolved problems. Colloquium Publications deals with new contributions in fields of advanced mathematics; the colloquium series not only publishes colloquium lectures but will accept monographs not written in connection with lectures. The royalty rate for an author submitting a monograph or lecture to either series is 15 per cent of the list price of each book sold. The AMS royalty rate is substantially better than that offered by most publishers since it is based on list price, not the net sales price; and there is not a reduced rate that applies to foreign or other special categories of sales.

Manuscripts for Mathematical Surveys may be sent to: Bertram Yood (chairman), University of Oregon, Eugene, Oregon 97403; H. E. Brown, Jr., Brandeis University, Waltham, Massachusetts 02154; or Michio Suzuki, University of Illinois, Urbana, Illinois 61801. Manuscripts for Colloquium Publications may be sent to: George Mackey (chairman), Harvard University, Cambridge, Massachusetts 02138; Saunders Mac Lane, University of Chicago, Chicago, Illinois 60637; or N. E. Steenrod, Princeton University, Princeton, New Jersey 08540.
AMS Mathematical Offprint Service

The AMS has received excellent responses to its Mathematical Offprint Service. It was first introduced in the Notices in January 1968 on pages 44-48. In answer to requests from many mathematicians, we have incorporated several improvements into our system.

The most important improvement is an addition to the profile form. In the original form the subscriber made the specifications that he desired as to authors, languages, journals, and fields of interest. Now he may take the specifications that he has made in the four categories and combine them in any way that he chooses to produce a desired result. He may make exceptions to the rules he has already made or he may add to them. To achieve his desired result, the subscriber will formulate a Boolean expression which will state a condition for which the subscriber will indicate the result that he wishes--offprint, title listing, or nothing. A simple example for an exception would be the following: A subscriber has indicated that he does not wish to receive any items in Spanish, but if an article appears in Spanish written by Brown then he would like an offprint. He will make this specification by writing a Boolean equation. The computer will first comply with all of the subscriber's requests in the new free part of the form before it answers the general specifications. The diagram below explains the procedure used to produce, for the subscriber, an offprint or a title listing.

The subscriber may elect to have all the articles which meet his offprint specifications generated only as title listings for a period not to exceed six months; his statement will indicate which title listing would have been offprints. This option allows the subscriber to determine how closely his profile specifications reflect his actual interests and to see if he has been accurate or specific enough in filling out his profile form.

The subscriber's statements will be generated on the last Friday of every month and mailed to the subscriber on the following Wednesday. Each statement will include a listing of all offprints enclosed with the statement, and the title listings of all articles in which the subscriber has a peripheral interest or which match with his profile as a title listing. The sub-

![Diagram of the offprint service process]
scriber's account status will be printed at the bottom of the monthly statement. It will include: his balance before that month's selection, his balance after that month's selection, the number of offprints ordered for him that month, and the number of titles listed that month. The computer will calculate, for each subscriber, his average monthly subscription usage. When his balance is within two months average usage, he will be notified in the monthly statement.

One subscription, consisting of 100 offprints, will cost $30.00. Ten title listings equal one offprint. The subscriber may decide whether or not he wishes to have the titles of the offprints that have been ordered for him listed at the time they are selected; if he decides to receive such a listing, he will be charged an extra $0.03 per offprint. A subscriber must pay at least $30.00, but he may pay more if his interests are broad enough that the $30.00 will be depleted quickly. When he finds his balance is within two months average usage, he should make an additional payment of not less than $30.00 to assure that his subscription will not expire.

National Science Foundation procurement regulations permit grantees and contractors to charge information services related to their research projects directly against grants and contracts. NSF grantees should refer to NSF 63-27, "Grants for Scientific Research," June, 1963 (as amended December, 1963), p. 13.

The offprint service is designed to meet the needs of the individual mathematician. The AMS will welcome any suggestions that he might have to make this service more efficient and useful.

MOS will be in full operation by the beginning of July 1968. To order a subscription, complete the order form found on page 667 of these Notices and return it to the American Mathematical Society, Post Office Box 6248, Providence, Rhode Island 02904.

NEWS ITEM

CONFERENCE ON COMBINATORIAL THEORY

A Conference on Combinatorial Theory was held at Yale University on May 10-11, 1968, in honor of Professor Oystein Ore who is retiring this year. The conference was organized by Professors G. C. Rota and B. L. Rothschild of the Massachusetts Institute of Technology and Nathan Jacobson of Yale University, and included the following speakers:

Marshall Hall, Groups and combinatorial designs.
W. T. Tutte, Towards a theory of crossing numbers.
Frank Harary, Enumeration techniques in graph theory.
M. D. Plummer, Covering concepts in graphs.
A. J. Hoffman, Some integer polyhedra.
B. L. Rothschild, A matching theorem for graphs.
M. E. Watkins, Connectivity of vertex transitive graphs.
Gian-Carlo Rota, The many lives of lattice theory.
Melvin Dresher, Distribution of pure equilibrium points in n-person.
Claude Berge, Some problems of the composition of a permutation with the use of graph theory.
Garrett Birkhoff, Heterogeneous algebras.
J. W. Moon, The distance between points in random trees.
R. P. Dilworth, The partition lattice.
G. B. Dantzig, Complementary pivot theory, extensions and graph interpretation.
A. M. Gleason and J. G. Stemple also gave talks.
The Confinement of a Russian Mathematician

On February 15, 1968, Aleksandr S. Esenin-Vol'pin, a famous logician and mathematician, was taken from his home in Moscow and forcibly committed to a mental institution 45 miles from Moscow. Some 95 Soviet mathematicians and 24 American mathematicians have separately protested this action against a colleague, in written petitions. In order to present some of the facts leading up to Esenin-Vol'pin's commitment to the mental hospital at Stolbovaja, the following report has been compiled from articles in The New York Times of December 10 and 22, 1967, and January 11, 17, February 18, 22, April 24, and May 5, 14, 1968.

Four young members of Moscow's literary underground went on trial in January 1968 on charges of "agitation or propaganda carried out with the purpose of subverting or weakening the Soviet regime." They were tried under Article 70 of the Russian penal code dealing with state crimes. The trial followed a year of imprisonment during which time the three men and young girl were allowed no communication with anyone except their lawyers.

One of the young men was Aleksandr Ginzburg, a 30 year old poet, who had compiled a "white book" containing records and papers relevant to the case and trial of Andrei D. Sinjavski and Julii M. Daniel', writers who were tried and imprisoned in a labor camp in 1966 under the same article of the Russian penal code as were Ginzburg and his companions. This 400 page book was printed in Frankfurt, West Germany, in German, French, and Italian, and also in a small pocket edition in Russian for smuggling into the Soviet Union. There is no evidence, however, that Ginzburg took part in getting the book out of the Soviet Union to be printed. The charges against the other defendants arose from their reported role in producing a political literary review entitled Phoenix 1966 in which the trial of Sinjavskii and Daniel' was condemned. The two men contributed to the magazine and the girl supposedly helped in the typing of it. The prosecution also charged the four defendants with associating with agents of the N. T. S., Narodno-trudovo'f sojuz--People's Labor Alliance, a Russian anti-Soviet emigre organization in West Germany founded to carry out subversive activity against the Soviet government. The four were sentenced for from one to seven years.

According to an article in the January 11 issue of The Times, 31 writers and scientists signed an appeal urging the Moscow City Court to permit a "full public airing" of the trial in the press and urging that an impartial selection of witnesses for the defense be made. (At this time, the Soviet press had not mentioned the trial.) The appeal was sent to the chairman of the Moscow City Court on the eve of the trial. Among the 31 reported to have signed the appeal was Igor R. Šafarevič, a noted mathematician who won a Lenin Prize in 1959. Copies were sent to Premier Kosygin, Party Leader Brezhnev, and President Podgorny.

The appeal related to charges against Ginzburg, expressing concern over his year-long detention before the trial. (According to Article 97 of the Code of Criminal Procedure, detention before trial may not exceed nine months.) The appeal contended that the compilation of the material on the trial of Sinjavskii and Daniel' was not adequate ground for prosecution.

Shortly before the Ginzburg trial was concluded, Pavel M. Litvinov, grandson of the late Soviet Foreign Minister Maxim M. Litvinov, and the wife of Julii M. Daniel' stood outside the courthouse and denounced the trial and treatment of the four dissidents before foreign journalists.

On January 16 after the trial had ended, a long account appeared in Izvestia. This report dealt mainly with the charges that the four intellectuals were agents of the N. T. S.

The February 18 issue of The Times reported that internal security authorities
of the Soviet Union had warned relatives and friends of Ginzburg and his associates to "stop their protests or face the consequences." This issue also reported that Aleksandr S. Esenin-Vol'pin, a mathematician, author of poetry, and friend of Ginzburg, was taken from his home on February 15 and was not returned. (It was learned a week later that he had been confined to a mental institution at Stolbovaja. He had been incarcerated in the past as "mentally unstable.") Esenin-Vol'pin was reported to be the originator of one of the most current slogans of the protesters, "Respect Your Own Constitution." Pavel M. Litvinov was summoned by the K. G. B., the Soviet internal security police, to report to them the same day that Esenin-Vol'pin was taken, but did not go. The Times reported that he was still at liberty. Ginzburg's mother and his fiancée were called to the K. G. B. on February 10 and warned about spreading slanderous rumors about his trial. The same Times article stated that the action against the protesters was viewed as "a result of nervousness by the security agent over a number of petitions and letters of protest" circulating in Moscow.

On February 22, The Times stated that 200 Soviet political prisoners in two labor camps and one prison had gone on a hunger strike in support of their demands for better conditions. They demanded better living conditions, better food, abolition of forced labor, and the right to send and receive letters. Ninety-five Soviet mathematicians, reported elsewhere as 99, some of whom are members of the Academy of Sciences, protested to the Government against the confinement of Esenin-Vol'pin in a mental institution. Two of the signers of the Russian petition were P. S. Novikov and his wife Ljudmila Keldys, sister of the President of the Academy of Sciences; they were supposed to visit the University of Illinois, Urbana, for about two months, but their visits have been canceled. Their petition follows:

To the Minister of Public Health of the USSR
To the General Procurator of the USSR
Copy to the Chief Psychiatrist of the City of Moscow

"It has come to our attention that

the gifted Soviet mathematician Aleksandr Sergeevič Esenin-Vol'pin, a well-known expert in the field of mathematical logic, has been forcibly committed, without preliminary medical investigation and without the knowledge or consent of his relatives, to Psychiatric Hospital No. 5 at Stolbovaja, 45 miles from Moscow.

"The forcible commitment of a talented mathematician, in full possession of his powers, to a psychiatric hospital for the seriously ill, and the conditions to which he has been subjected by the very nature of such a hospital, are extremely harmful to his mental and physical health and to his dignity as a human being.

"In view of the humane purposes of our laws, and still more of our Public Health services, we regard this action as a gross violation of medical and legal standards.

"We urge you to intervene at once and to take such measures as will ensure that in the future our colleague can continue his work under normal conditions."

The following are the 95 Russian mathematicians who signed the petition:

Member of the Academy of Sciences of the USSR: Lenin Prize Laureate P. S. Novikov,

Corresponding Members of the Academy of Sciences of the USSR: Lenin and State Prize Laureate I. M. Gel'fand; State Prize Laureates Lazar’ Ljusternik, Dmitrii Men’šov; Lenin Prize Laureates S. P. Novikov, I. R. Šafarevič; Andrej Markov.


Doctors of Mathematical and Physical Sciences: M. S. Agranovič, Docent A. V. Arhangel’skiĭ.

Senior Research Associate: V. Ponomarev, Ja. G. Sinaĭ.

Candidates in Mathematical and


Candidate in Science: Senior Research Associate G. Tjurina.

Candidate in Pedagogical Sciences: Ju. A. Sihanovič.


Research Assistant: F. L. Varpa­hovskiǐ.

Senior Lecturer: G. A. Sestopal.

Lecturers: N. Vil'jams, Ju. A. Gastev.

Computer Engineer: I. G. Kristi.

Senior Engineer: V. K. Fink.

The address for the cosigners was given as: Department of Mathematics and Mechanics, Moscow State University (Lomonosov), Lenin Hills, Moscow 234, USSR.


"On February 18 and February 22 The New York Times printed stories about threats made to A. S. Esenin-Vol'pin, and about his confinement. These stories suggest that such actions were based upon his protests against the sentencing of Soviet intellectuals, and were designed to dis-courage further protests of this kind.

"The undersigned American mathematicians and logicians know of the excellent mathematical work of Esenin-Vol'pin, and of his helpful contributions to the mathematical community by way of translation and editorial work. We also know of his great integrity and are greatly concerned over the reports that he may have been subjected to pressure simply for expressing his views on public matters.

"We would greatly appreciate it, if you would advise us concerning the correctness of the published accounts about Esenin-Vol'pin and concerning his present status.

"Kindly address your reply to Professor Martin Davis, Courant Institute, New York University, 251 Mercer Street, New York, New York."


On April 24 The Times reported that "the Soviet Communist party expelled some of the country's leading scientists, including several working on military and space programs, for having participated in a protest against the detention of the mathematician Aleksandr Esenin-Vol'pin. Some of the scientists mentioned were: I. M. Gel'fand, famous for the use of mathematical methods in biology and winner of a Lenin Prize; Sergei Fomin, head of the cybernetics department at Moscow University; Jurij Manin, winner of a Lenin Prize; and Igor R. Šafarevič, who had signed the appeal concerned with the Ginzburg trial and also the petition concerning Esenin-Vol'pin.

As stated in The Times report, this was the first time that the Communist party had taken steps against "intellectual leaders of the scientific establishment whose military importance has usually given them privileged status. The authorities ignored their participation in protest..."
meetings and their demands for intellectual and artistic freedom."

The May 5 issue of The Times quoted part of a speech given by Party Leader Brezhnev a few weeks earlier in which he stated that foreign anti-Soviet organizations use, to further their propaganda, Soviet citizens who are "morally unstable, weak and politically immature people. Sometimes they catch in their net persons avid for self-publicity, those willing to make themselves known as loudly as possible...by whatever dubious political means they find at their disposal, and without scorning the praise of our ideological opponents. The Soviet public sharply condemns the shameful acts of these hypocrites. The renegades can not expect to go unpunished."

The same Times article stated that Mstislav Keldyš, President of the Academy of Sciences of the USSR, "warned protesting scientists that Soviet science could go ahead without them, and that therefore their scientific abilities would not protect them. His words were reportedly followed by the expulsion of several distinguished scientists...from the Communist party."

On May 8, 1968, a letter was written to Professor Martin Davis in answer to the March 19 letter of the 24 American mathematicians who questioned Esenin-Vol'pin's confinement. The letter was signed by nine of the most outstanding mathematicians in the Soviet Union: N. N. Bogoljubov, L. S. Pontrjagin, A. A. Markov, I. G. Petrovskii, P. S. Novikov, P. S. Aleksandrov, M. V. Keldyš, S. N. Mergeljan, A. N. Tihonov. Two of these were among the 95 protesters. The text of the letter follows:

"We acknowledge receipt of your letter concerning A. S. Esenin-Vol'pin and answer it herewith. The information you quote from the newspaper "The New York Times" about his present situation does not correspond to the facts.

"In reality, Esenin-Vol'pin has been under medical observation for mental illness during the course of several years and, as a result of his illness, he has been confined repeatedly to psychiatric hospitals. At the present time he is again hospitalized in the clinic of the Psychiatric Institute of the USSR Academy of Medical Sciences, where he is under the care of first-class specialists.

"We regret that this case has been brought to your attention. You will agree that if a person is ill and in need of medical assistance, it should be given to him."

The May 14 New York Times reported that Esenin-Vol'pin had been released from the mental hospital on the previous day.
NEWS ITEMS AND ANNOUNCEMENTS

FIFTH PRAGUE CONFERENCE ON INFORMATION THEORY,
STATISTICAL DECISION FUNCTIONS,
AND RANDOM PROCESSES

The Czechoslovak Academy of Sciences Institute of Information Theory and Automation is preparing the Fifth Prague Conference on Information Theory, Statistical Decision Functions, and Random Processes which will be held in Prague from September 9 to 15, 1968.

The purpose of the conference is the presentation of new results dealing with the above mentioned disciplines and discussions of important problems connected with them. From the invited scientists, the following have tentatively agreed to give papers:

A. A. Borovkov (Novosibirsk): Theorems on convergence to diffusion processes.


J. Gani (Sheffield): Recent developments in the theory of storage.


J. Havel (Prague): Special devices for stochastic problems solution.

V. Imedadze (Tbilisi): A set of specialised computing devices for the treatment of random processes.

M. Iosifescu (Bucharest): Sequential analysis of random systems with complete connections.


F. Kozin (Brooklyn): Approximation techniques for nonlinear stochastic systems.

K. Krinkeberg (Heidelberg): Recent results on mixing in topological measure spaces.

L. Le Lam (Berkeley): On the asymptotic behaviour of posterior distributions.

P. Mandl (Prague): The elimination of Killing measures by transformations of diffusion processes (presented in absentia).

P. A. Meyer (Strasbourg): Some recent advances in martingale theory.


A. Perez (Prague): Reduction procedures in statistical decision.

M. S. Pinsker (Moscow): New methods of information transmission through channels with feedback.

L. Telksnis (Vilnius): The determination of the change in the structure of random processes under uncertainty about statistical characteristics.

Those wishing to participate in the Conference are invited to present 15 minute communications in any of the above mentioned areas. Ample time will be allowed for discussions. The papers presented at the Conference will be published.

The summaries of lectures and communications should be submitted to the Organizing Committee, in English, not later than June 30, 1968. The address of the Committee is Prague 2, Vyšehradská 49. For more information write to Čedok-Intravel, Congress Department, Příkopy 18, Praha 1.

SYMPOSIUM ON COMPLEX ANALYTIC METHODS IN MATHEMATICAL PHYSICS

A symposium on complex analytic methods in mathematical physics, supported by the Air Force Office of Scientific Research, is being held at Indiana University, Bloomington, Indiana, on June 3 through June 6, 1968. The symposium stresses the use of complex analytic methods in problems of modern and classical physics, and the theory of partial differential equations. The program features one-
hour invited addresses by leading mathematicians and physicists working in this general area, and also a session of fifteen-minute contributed papers. The invited papers and selected contributed papers will be published as a volume.

The list of invited speakers and their tentative address titles is as follows:

V. Bargman, Princeton University, Group representations on Hilbert spaces of analytic functions.
L. Bers, Columbia University, Teichmüller spaces.
J. Bros, Institute for Advanced Study, Analyticity properties implied by two-particle structure in general quantum field theory.
R. J. Eden, Cambridge University, Consistency problems in S-matrix theory.
A. Erdélyi, University of Edinburgh, Uniform asymptotic expansions of integrals.
L. D. Faddeev, Leningrad University, Many-dimensional generalization of the inverse scattering problem.
P. Garabedian, New York University, Analytic methods for the numerical computation of fluid flows.
J. M. Jauch, University of Geneva, Scattering theory in general quantum mechanics.
M. Z. Krzywoblocki, Michigan State University, Integral operator methods in elasticity, fluid dynamics, and electromagnetics.
G. Mahoux, State University of New York at Stony Brook, Extensions of analyticity domains of functions with positivity properties.
F. Rohrlich, Syracuse University, The coherent state representation and quantum field theory.
A. Weinstein, American University, Some theoretical ramifications of the intermediate problems for eigenvalues.
J. A. Wheeler, Princeton University, Quantum geometrodynamics.
D. V. Widder, Harvard University, Homogeneous solutions of the heat equation.

NSF AWARDS FOR POSTDOCTORAL WORK IN THE SCIENCES

The National Science Foundation has awarded 120 Postdoctoral Fellowships to holders of doctoral degrees in the mathematical, physical, engineering, social, and life sciences. Each of the awards includes an annual stipend of $6,500, an allowance for dependents, and an allowance to help defray travel costs. Fellowship recipients in this program will study or carry on research at institutions in the United States and abroad.

The following are the recipients of the Postdoctoral Fellowships in mathematics: R. H. Berk, University of Michigan; L. J. Billera, City University of New York; P. R. Chernoff, Harvard University; L. J. Corwin, Harvard University; F. T. Farrell, Yale University; J. D. Lubin, Brown University; D. B. Meredith, Brandeis University; D. P. Niebur, University of Wisconsin at Madison; B. A. Taylor, University of Michigan; H. M. Taylor, Cornell University; and S. V. Ullom, University of Maryland.

DEPARTMENT OF STATISTICS UNIVERSITY OF MANITOBA

The University of Manitoba has created a separate Department of Statistics within the Faculty of Arts and Science. The Department presently offers courses leading to a B. S. Major in Statistics, a B. S. (Honours) in Statistics, and an M. S. degree. The planning for a Ph. D. program has been started. The emphasis is on both the theoretical and applied aspects of statistics. Enquiries concerning the program, financial assistance for study, and teaching or research positions may be directed to Dean A. L. Dulmage, Acting Head, Department of Statistics, University of Manitoba, Winnipeg 19, Manitoba, Canada.

TRAINING AIDS FOR EQUATION TYPING

Sandia Corporation has announced the reprinting of Sandia's Training Bulletin No. 1, Development of Training Aids for Equation Typing, SC-M-67-617. A free copy of the Bulletin may be had by sending requests, including full title and number of the Bulletin to Sandia Office of Industrial Cooperation, Sandia Corporation, P.O. Box 5800, Albuquerque, New Mexico 87115.
RECIPROCITY AGREEMENT WITH SVENSKA MATematikERSAMFUNDET

Svenska Matematikersamfundet has announced that the fee for membership has been raised and is now 15 Swedish Kronor for the fiscal year 1967-1968. Under reciprocity agreements, most foreign members pay half this amount or approximately $1.50. (The fee for lifetime membership is now 200 Swedish Kronor and is not subject to reductions under reciprocity agreements.) A member of the AMS wishing to become a member may send a check, payable to Svenska Matematikersamfundet, to Folke Eriksson, Svenska Matematikersamfundet, Nääckrosgatan 6, Mölndal, Sweden.

Your attention is also called to the following fact, taken from the inside cover of Mathematica Scandinavica: A sufficient condition to obtain the reduced subscription rate to Mathematica Scandinavica is to be a member of the AMS or any other society having a reciprocity agreement with Svenska Matematikersamfundet.

NSF AWARDS FOR ADVANCED SCIENCE GRADUATE STUDY

The National Science Foundation has announced awards of 1,925 Graduate Fellowships in the sciences, mathematics, and engineering for the academic year 1968-1969. The awards are made to accelerate the output of highly qualified scientists by encouraging outstanding students to obtain full-time advanced graduate training in the sciences. All fifty states, the District of Columbia, and Puerto Rico are represented. NSF Graduate Fellows may attend any appropriate nonprofit U. S. or foreign institution of higher learning.

Forty-two percent of the fellowships awarded are for tenure periods of two years. Continuation of the second year is dependent upon satisfactory academic progress of the Fellow during the first year and the availability of appropriated funds.

The new awards are in addition to continuation awards offered to 575 Graduate Fellows who received fellowships in March 1967.

Graduate Fellowships provide basic stipends (for 12 months) of $2,400 for the first year level of graduate study, $2,600 for study at the intermediate level, and $2,800 for the terminal year. They include additional allowances for dependents and for limited travel. In lieu of tuition and fees, United States institutions receive a standardized $2,500 cost-of-education allowance for each Fellow. At its discretion each fellowship institution may augment the stipend by not more than $1,000 per year.

The Foundation plans to reopen the Graduate Fellowship program for 1968-1969 in October 1968. Application forms will be available when the program is announced.

LATIN AMERICAN SCHOOL OF MATHEMATICS

The first Latin American School of Mathematics will be held at the Instituto de Matemática Pura e Aplicada, Rua Luiz de Camoes 68, Rio de Janeiro 58, GB, Brazil, from July 1 to 26, 1968. It is being organized and financed by the National Research Council of Brazil, with financial support of other institutions. The scientific program will consist of the following series of lectures at research and post-doctoral level:

Spectral theory of differential operators, Jean Dieudonné of the Université de Nice, France.

Desintegration of measures and applications to stochastic processes, Laurent Schwartz of the Université de Paris, France.

Singular integrals and elliptic operators, R. T. Seeley of Brandeis University.

Recent and classical results on the Cauchy problem, François Trèves of Purdue University.

There will also be one-hour lectures and sessions devoted to short research communications.

The organizing committee is J. Adem (Mexico), J. Barros-Neto (Brazil) and L. A. Santaló (Argentina). The chairman of the local committee is C. S. Höning (Brazil).
HISTORY OF MATHEMATICS AT THE UNIVERSITY OF TORONTO

With support from the Canada Council, a mathematical information bank is being developed at the University of Toronto under the direction of Kenneth O. May. Filed under key words are definitions, results, historical facts, citations to bibliographies, and titles of biographies, historical studies, expositions, and original sources. The bank will serve as a source for historical research, for answering inquiries on a cost basis, and for the preparation of reference publications.

The Ph.D. in mathematics may be earned at the University of Toronto with a thesis of an historical character. In addition to meeting the usual requirements for the doctor's degree, candidates will find facility in two or more foreign languages essential. The recently established Institute for the History and Philosophy of Science and Technology offers related graduate work. Inquiries about the program and fellowships should be addressed to Professor May, Department of Mathematics, Toronto 5, Canada.

INTERNATIONAL CONFERENCE ON COMPUTING METHODS IN OPTIMIZATION PROBLEMS

The second International Conference on Computing Methods in Optimization Problems will be held in San Remo, Italy, September 9-13, 1968, for the purpose of disseminating new computing techniques in the broad area of optimization problems. The conference will be sponsored by The Society for Industrial and Applied Mathematics with the cooperation of The University of California and The University of Southern California. There will be both invited and contributed papers. Special sessions will be devoted to specific application areas such as nuclear reactors and transportation systems.

Inquiries on travel arrangements should be addressed to L. W. Neustadt, Department of Electrical Engineering, University of Southern California, Los Angeles, California 90007.

NEW OFFICERS AND MEMBERS OF THE NATIONAL ACADEMY OF SCIENCES

The 105th annual meeting of the National Academy of Sciences was held April 22-24, 1968, in Washington, D. C. Emanuel R. Piore, Vice President and Chief Scientist of the International Business Machines Corporation, was elected to a four-year term as Treasurer of the Academy. John W. Tukey, Chairman of the Department of Statistics at Princeton University and Associate Executive Director of Research in Communication Sciences at Bell Telephone Laboratories, was elected as one of the new members of the Council.

Fifty new members were elected to the Academy in recognition of their distinguished and continuing achievements in original research. The following were elected for their contributions in mathematics: Garrett Birkhoff, Harvard University; A. P. Calderón, University of Chicago; William Prager, University of California, San Diego; I. M. Singer, Massachusetts Institute of Technology.

Carl Ludwig Siegel, Professor Emeritus at the University of Gottingen, was one of ten distinguished scientists who are not citizens of the United States to be elected as a foreign associate, one of the highest honors that can be bestowed by the Academy on a foreign scientist.

MAA INFORMATION BOOKLET

The Mathematical Association of America has released its new Information Booklet for the Spring 1968. It describes the services and materials available through the MAA for the improvement in mathematics and the teaching of mathematics in colleges and universities. The Booklet lists information on publications, films, employment, meetings of the Association, lecturers and consultants, and guidance of students. An address for further information on each category appears in the booklet. For a copy of the booklet write to Harry M. Gehman, Mathematical Association of America, State University of New York, Buffalo, New York 14214.
BACKLOG OF MANUSCRIPTS WILL BE ELIMINATED

On April 19, 1968, the Council approved the recommendation of the Committee to Monitor Problems in Communication that the backlog of manuscripts accepted for publication in AMS journals be eliminated. The recommendation contained the statement:

"The backlog does not reflect, as some would suggest, a flooding of the journals with low quality work. Rather, it is a simple consequence of the fact that the mathematical profession has grown more rapidly than its publishing mechanism. With the help of Federal funds, and in the conviction that there is a serious need for more mathematicians in our society, mathematicians have in recent years set to work to reproduce their own kind, with notable results; the annual production of Ph.D.'s in mathematics has grown from about 250 to about 850 in the past five years. These young people have all been imbued with the importance of publishing research, and they are proceeding to do so. In the view of the Committee, it is inconsistent deliberately to enlarge the profession while attempting to hold down the volume of research published. There are unquestionably more papers published now that are of no interest to any given reader than there were ten years ago, but it cannot be deduced that most of these papers are low quality. On the contrary, standards for acceptance have certainly risen during this period; the lower density of "interesting" papers is due instead to the larger corpus of knowledge and the proliferation of fields of specialization.

"With the help of page charges, larger sales to libraries and higher subscription prices to the latter, it is now possible to publish mathematics at no cost to the publisher. It is therefore recommended that the Society plan to publish as many pages in each of its journals as are necessary to keep the backlog essentially at zero, while keeping acceptance standards where they are now. This will of course have to be accompanied by a proportional rise in annual subscription prices.

"As a publishing house, the Society must worry about the backlog of papers piling up. But also, as a collection of publishing mathematicians, the Society has been concerned with the long lag between completion of a piece of research and its appearance in print, and indeed, one of the charges of this Committee was to investigate ways of speeding up publication."

Beginning in 1969, Proceedings and Transactions will be increased in size in order to eliminate the backlog. Once the backlog is depleted, the journals will publish manuscripts as soon as possible after acceptance by editors. The following is a comparison of the 1968 and 1969 sizes and prices of the two journals.

Proceedings is now a bi-monthly, one volume per year journal which will contain 1536 pages for 1968. In 1969 it will be published monthly, being divided into four volumes with 3960 pages budgeted for the year. The list price for 1969 will be $80.00 as compared with $12.00 in 1968:

Transactions is now a monthly journal divided into 5 volumes with 2800 pages for 1968. In 1969 it will consist of 12 volumes with 6720 pages budgeted for the year. The list price for 1968 is $60.00, and for 1969 will be $175.20.

The price to members for both journals is one-half the list price.

Mathematics of Computation and Bulletin are operated at zero backlog now and will continue to do so in the future. Mathematics of Computation will be increased in size to 1000 pages if enough manuscripts are received.

Erratum

ST. ANDREWS COLLOQUIUM, 1968

At the Edinburgh Colloquium, St. Andrews, Scotland, July 10-20, 1968, four courses will be given, each consisting of eight lectures. The four courses are Jordan Algebras, Normed Algebras, Fourier Analysis and Generalized Functions, and Weak Convergence Methods in Probability, and will be given by Professors Nathan Jacobson, F. F. Bonsall, N. G. de Bruijn, and P. Billingsley, respectively.
PERSONAL ITEMS

Professor STEFAN BERGMAN of Stanford University has been appointed to a visiting professorship at the College of France, Paris, for 2 months.

Professor WARD BOUWSMA of Pennsylvania State University has been appointed to an assistant professorship at Southern Illinois University.

Professor J. W. BREWER of Florida State University has been appointed to an assistant professorship at Virginia Polytechnic Institute.

Dr. W. J. ECKERT, former Director of the IBM Watson Laboratory at Columbia University, will receive the honorary Doctor of Science degree from Oberlin College on June 10.

Professor H. B. ENDERTON of the University of California, Berkeley, has been appointed Assistant Editor of Reviews, Journal of Symbolic Logic, and Lecturer at the University of California, Los Angeles.

Professor R. S. FREEMAN of the University of Maryland has been appointed to an associate professorship at the University of Oregon.

Dr. T. J. GRILLIOT of Duke University has been appointed to an assistant professorship at Pennsylvania State University.

Dr. R. F. JOLLY of the University of California, Riverside, has been appointed to an assistant professorship at Pennsylvania State University.

Dr. JEROME KARLE of the Naval Research Laboratory, Washington, D. C., has been named to the newly-created Chair of Science for the Structure of Matter at the Laboratory.

Professor D. E. KNUTH of the California Institute of Technology has been appointed Professor of Computer Science at Stanford University. He will be on leave during the 1968-1969 academic year as a staff mathematician at the Institute for Defense Analyses, Princeton, New Jersey.

Professor GUNTER LUMER of the University of Washington will be on leave as a visiting professor at the University of Strasbourg, France.

Dr. J. R. MCLAUGHLIN of Wayne State University has been appointed to an assistant professorship at Pennsylvania State University.

Professor ROBERT MOYER of Pennsylvania State University has been appointed to an associate professorship at the University of Kansas.

Dr. ARNOLD OBERSCHELP of the Technical University of Hannover, Germany, has been appointed to a professorship at the University of Kiel, Germany.

Mr. ALBERT SOGLIN of Soglin and Associates, Chicago, Illinois, has been appointed to an associate professorship at Chicago City College, Loop Campus.

Dr. G. J. TEE of the University of Lancaster, England, has been appointed Senior Lecturer at the University of Auckland, New Zealand.

Dr. C. L. VANDEN EYNDEN of Miami University, Oxford, Ohio, has been appointed to a visiting assistant professorship at Pennsylvania State University.

PROMOTIONS

To Professor. University of Oregon: R. F. TATE; Pennsylvania State University: PAUL AXT; University of Santa Clara: K. L. DE BOUVERE.

To Associate Professor. Pennsylvania State University: GEORGE ANDREWS, A. M. KRALL, MARIO PETRICH; University of Santa Clara: G. L. ALEXANDERSON; Sogan Jesuit College, Seoul, Korea: H. C. LEE.

To Assistant Professor. Pennsylvania State University: BERTHA MATHER.

INSTRUCTORSHIPS

Mary Washington College: C. A. KEMMLER; Purdue University, Calumet Campus: SIGRID WAGNER.

DEATHS

Professor B. A. AMIRA of the He-
brew University, Jerusalem, Israel, died on January 20, 1968, at the age of 72. He was a member of the Society for 21 years.

Dr. ELI GOURIN of New York, New York, died on February 4, 1968, at the age of 85. He was a member of the Society for 41 years.

Professor F. J. HAHN of Yale University died on January 14, 1968, at the age of 38. He was a member of the Society for 13 years.

Professor R. E. LANGER of the University of Wisconsin died on March 12, 1968, at the age of 73. He was a member of the Society for 47 years.

Dr. MILTON ROSENTHAL of Culver City, California, died on December 11, 1967, at the age of 50. He was a member of the Society for 8 years.

Dr. R. G. SANGER of Kansas State University died on March 13, 1968, at the age of 62. He was a member of the Society for 41 years.

Professor C. J. TREMBLAY of Bard College died on January 1, 1968, at the age of 53. He was a member of the Society for 21 years.

ERRATA

Professor L. L. LOWENSTEIN of Arizona State University died on August 23, 1967, at the age of 64. He was a member of the Society for 39 years.

Dr. C. E. STENARD of the Institute for Defense Analyses, Princeton, New Jersey, has accepted a position as a Member of the Technical Staff with the Bell Telephone Laboratories, Whippany, New Jersey.

NEWS ITEMS AND ANNOUNCEMENTS

INTERNATIONAL SUMMER COURSE ON ALGEBRAIC MODELS IN PSYCHOLOGY

The Netherlands Universities Foundation for International Co-operation (NUFFIC) announces its 1968 International Summer Course on Algebraic Models in Psychology to be held at 'Het Oude Hof', The Hague, the Netherlands, on August 5-17, 1968. The course is the third in a succession of summer courses in psychology organized by NUFFIC and sponsored by NATO. The Course, directed by Professor C. Flament, will deal with algebraic systems, mathematics and algebraic models as used in psycholinguistics, social psychology (graph theory), cognition and genetic epistemology. Applications are invited from young graduates in psychology and related disciplines who are interested in furthering their knowledge in this specific field. For an application write to Professor R. Doop, Course Registrar, Netherlands Universities Foundation for International Co-operation, 27 Molenstraat, The Hague.

188TH ANNUAL MEETING OF THE AMERICAN ACADEMY OF ARTS AND SCIENCES

The 188th annual meeting of the American Academy of Arts and Sciences was held May 8, 1968, in Boston, Massachusetts. A new Vice-President, Roger Revelle, Director of the Center for Population Studies at Harvard University, was elected to replace Garrett Birkhoff as Vice-President representing the Mathematical and Physical Sciences. Also elected were 103 new Fellows and 20 Foreign Honorary Members. Elected as Fellows for their endeavors in the field of mathematics were A. A. Albert, University of Chicago; H. P. Greenspan, Massachusetts Institute of Technology; J. G. Kemeny, Dartmouth College; Howard Raiffa, Harvard Business School; Stephen Smale, University of California at Berkeley.

Two other members of the AMS were elected: Valentine Bargmann of Princeton in physics, and P. C. Suppes of Stanford in philosophy and theology.
Editor, the Notices

There appeared in these Notices not long ago an expository article on the current state of mathematics in mainland China. It contained information of legitimate concern to large numbers of mathematicians, information which was not otherwise easily available. Among the items discussed were: The major centers of scientific activity; the work and influence of some of the principal mathematicians; the areas of specialization receiving greatest attention; the nature of state scientific policy, particularly as it affects conditions of research and international scientific communication.

There is no strong reason to commission a series of such reports, no more so than there is to reject competently reported information of this type when, in the course of events, it becomes available. After a recent trip to North Vietnam, as a guest of the Mathematical Society of Vietnam, A. Grothendieck gave a lecture, "La vie mathematique en Republique Democratique du Vietnam", at the invitation of the University of Paris. When a copy of it came to the attention of Serge Lang and myself, we agreed that it would be of serious interest to a large number of American mathematicians. With Grothendieck's consent, we therefore proposed its publication in one of the journals of the AMS. Those who have read the article, and with whom I have spoken, have endorsed this action, and the editors with whom we first corresponded concurred.

There was concern, nevertheless, that an undesirable political construction might be put on such an act of publication, and the proposal was therefore put to the Council of the AMS for final approval. That approval was denied at the April meeting of the Council in Chicago.

I feel that this matter involves issues of general importance to the Society, so I would like to present my assessment of the essential facts in the matter, and to take public issue with the vote of the Council.

The first point is that Grothendieck's article is not a political document, unless one wants to argue that an article about, say, mathematics in China, is a priori political. It is concerned with exactly what its title suggests. Indeed, one of its striking conclusions is that there does exist serious mathematical activity in North Vietnam. Moreover it is as perceptive and as independent in judgment as its authorship would lead one to expect. It covers facts of precisely the type discussed in the China article, on the basis of direct observation by a distinguished scientist.

It cannot be denied that political significance might, by some, be attached to the publication of Grothendieck's article, just as I now attach political significance to the Council's refusal to do so. That is a problem with which we all must, and do, live. Members of the Council do not escape it by pretending that negative action represents political innocence or neutrality.

The essential fact is that the article, which I contend to be of legitimate and timely scientific interest, was rejected, not by intrinsic or scientific criteria, but on the basis of speculation about its reception in the present political climate. Thus, a portion of the Council has transformed itself into a self appointed board of censors whose action constitutes precisely the improper intrusion of political criteria into areas of scientific communication which it is pretending to eschew.

Of course, certain members of the Council may contest my evaluation of the article as a non political document. Unfortunately, they have taken steps to guarantee that very few other mathematicians will have an opportunity to judge for themselves.

Hyman Bass
We are members of the Council of the AMS who attended the Council meeting, April 19. At this meeting there was much discussion about publishing in the Notices an article by A. Grothendieck, on his trip to North Vietnam and Hanoi, where he spent three weeks lecturing on mathematics last fall. Grothendieck wrote mainly about the state of affairs of North Vietnamese mathematics, and the effect of American bombing on the life of North Vietnamese mathematicians. By a very close vote, the Council decided not to publish Grothendieck's article in the Notices. (A majority of the members voting voted to publish, but the motion failed when, after an appeal to Council rules, fractional votes were counted.)

We would like clearly to disassociate ourselves personally from this act of the Council of the AMS, and we hope that the Council will reconsider the matter at a subsequent meeting.

P. T. Bateman  
M. Gerstenhaber  
J. J. Kohn  
S. Lang  
R. Palais  
M. Protter  
S. Smale  
B. Yood

The Secretary supplies background for the parenthetical remark.

By-laws, Article IV, Section 4: "The method for settling matters before the Council ... shall be by a majority vote of the members present. If the result of a vote is challenged, it shall be the duty of the presiding officer to determine the true vote by a roll call. ...Fractional votes shall be counted." (See Bulletin 73 (1967), 989.)

Council Minutes, April 19, 1968: "It was moved and seconded that the Editors be instructed to publish the article. Initially the Presiding Officer ruled that a majority vote of the members present favored the motion. The result was challenged and the true vote was determined, as specified in the By-laws, by a roll call in which fractional votes were counted. ---the motion lost."

The Grothendieck report mentioned in the foregoing letter appears as an attachment to the Council agenda for April 19, 1968, and will also be included in the minutes of the meeting. The Secretary of the Society has pointed out that these minutes are distributed to over 100 Society members. I urge that the recipients make the report available to their colleagues in order that as many mathematicians as possible may have an opportunity to inspect it.

Grothendieck's report has unfortunately been treated like a political hot potato rather than as an item of information of interest to mathematicians. Publication is not endorsement. The Society should put the report before every member, letting each, in his own wisdom, evaluate it for himself.

Murray Gerstenhaber
as the other is politically inspired. Arguments over whether the article is "political" or not end up in semantics: in particular, whether a document can be nonpolitical in content and yet be political in its effect. In this regard, we feel the overall impression the report makes is approximately that of Harrison Salisbury's well-known New York Times articles on life in Hanoi, except that it focuses on a narrower aspect of North Vietnamese life.

We voted against printing the report because as Pitcher suggests, we construe the Society's function narrowly and felt that the article was of minimal interest to mathematicians qua mathematicians. Indeed, it seems ludicrous to us to seriously compare it in mathematical interest to the report on Chinese mathematics. Such accounts of mathematical life in various small and underdeveloped countries have been appearing from time to time in the Monthly, and that seems to be a reasonable outlet for them, but we would vote in the same way against printing any of these reports in the Notices. If this means resigning ourselves to the inevitable crisis of "censorship," that's something with which editorial boards everywhere must, and do live.

Arthur Matuck
Gian-Carlo Rota
Editor, the Notices

I am writing about an experience which must be common to many editors of mathematical periodicals. After sending out an author's paper with the referee's suggestions, one receives a revised version from the author, with a letter expressing thanks to the referee, but no mention in the paper itself.

It seems to me that if the referee's work on the paper was such as to merit an acknowledgment, this should be a public one, even though he, of course, remains anonymous.

P. M. Cohn

NEWS ITEM

ASSOCIATION FOR MATHEMATICAL GEOLOGY

A thirteen member committee with international representation is working on the formation of the Association for Mathematical Geology. The Association will be affiliated with the International Union of Geological Sciences (IUGS) and will work closely with the International Association for Statistics in the Physical Sciences. Elections of officers for the new Association will be held in Prague this summer during the IUGS annual meeting. For more information write to Professor Richard Reyment, Paleontologiska Institutionen, Uppsala Universitet, Fack, Uppsala 1, Sweden.
654-34. BASIL GORDON, University of California, Los Angeles, California, and LORNE HOUTEN, Washington State University, Pullman, Washington 99163. Asymptotic results in plane partitions.

In Abstract 642-38, these (Notices) 14 (1967), 71-72, the notion of a plane partition, monotonic on rows, is defined. The authors have shown that the generating function, for \( b(n) \) the number of such partitions, is

\[
\sum_{n=0}^{\infty} b(n)x^n = \prod_{p=1}^{\infty} \left( 1 - x^p \right)^{-(\nu + 1)/2}.
\]

The authors obtain in this case that

\[
b(n) \sim 2^{-3/4} (3\pi \zeta(3))^{-1/2} (n/\zeta(3))^{-49/72} \exp \left[ \frac{3}{2} \zeta(3)(n/\zeta(3))^{2/3} + \pi^2 (n/\zeta(3))^{1/3}/24 - \pi^4/3456 \right] + \int_0^\infty (y \log y)/(e^{2\pi y} - y) \, dy \]

where \( \zeta(s) \) is the Riemann \( \zeta \)-function. (Received January 29, 1968.)

654-35. H. B. RIBEIRO, Pennsylvania State University, University Park, Pennsylvania 16802.
On atomisticity in dependence algebras.

Basic results immediately related to the notion of dependence for vector spaces had been established within the framework of Boolean algebras with an idempotent and monotone unary operator in a purely elementary way--arithmetically, non set theoretically, without the help of the finite character of dependence or the completeness of the Boolean algebra. However, the atomisticity of the Boolean algebra was used to obtain, within such framework, that each maximal free element generates. It is now shown that this ad hoc assumption may also be removed. (Received January 29, 1968.)

654-36. A. F. ROBERT, Stanford University, Stanford, California 94305. A numerical procedure for the determination of compressible fluid flows.

In J. Rational Mech. Anal. 4 (1955), 883-905, Bergman indicated a procedure for the determination of the flow pattern of compressible fluids. This method involves the evaluation of double integrals and solutions of a boundary value problem. The author discusses the computational program for solving the above problems using high speed computing machines. In this work he considers the case of subsonic flows near the sonic line. In many cases the flow can be continued into the super-sonic region. In this way one obtains flows of mixed type (transonic flows). (Received February 1, 1968.) (Author introduced by Professor Stefan Bergman.)

654-37. MACIEJ SKWARCZYNSKI, Stanford University, Stanford, California 94305. The invariant distance in the theory of pseudo-conformal transformations and the Lu-Qi-Keng conjecture.

For a bounded domain \( D \subset \mathbb{C}^n \) the correspondence \( t \to K(z,t^*) \) (where \( t \in D \) and \( K(z,t^*) \) is the Bergman kernel function of \( D \)) defines an analytic imbedding of the domain \( D \) into a Hilbert space \( L^2(H(D)) \). As was pointed out by Kobayashi [Geometry of bounded domains, Trans. Amer. Math. Soc. 92 (1959)] that the norm in \( L^2(H(D)) \) induces in a natural way the pseudo-conformally invariant distance
\( p(p,t) \) in D. The author shows that \( p(p,t) \) can be expressed by the kernel function, namely,

\[
p(p,t) = (2 - 2K(t,p^*)K(p,t^*)K(p,p^*)K(t,t^*))^{1/2}/Z.\]

This fact yields a new and simple proof of the theorem that the Bergman metric is positive definite. Also it yields a new formulation of the Lu-Qi-Keng conjecture on zeros of the kernel function. [See On Kaehler manifolds with constant curvature, Acta Math. Sinica (2) 16 (1966).] (Received February 1, 1968.)

654-38. H. M. LIEBERSTEIN, Wichita State University, Benton, Kansas 67017. Viscous flow properties in the operation of a nephron.

The salient feature of a nephron is its tiny size, six to ten microns radius, since the forth power law requires a tiny flow rate even from a large pressure gradient. Along closely folded hairpin loops, present in the nephrons of birds and mammals and called loops of Henle, a salt concentration gradient forms in the ambient medullary tissue. Urine collects in this tissue in ducts, equilibrates with it osmotically, and produces a final product hypertonic to blood. Other authors explain the mechanism of this loop in terms of an hypothesis of active extrusion of a small amount of sodium from one branch of the loop and operation of a countercurrent multiplication principle. By close attention to realistic physical principles, we construct a model that does not use this hypothesis but produces in numerical studies the observed concentration gradient and an amplification of this effect with length. A much weaker assumption than similar ones made in other models, that two salt concentrations take on stationary values, causes a linear initial value problem for a \((2 \times 2)\) first order ordinary differential equation system to replace a \((4 \times 4)\) first order partial differential equation system. Basic mechanisms used such as back diffusion were of no importance in previous square law models. Work supported by NASA Contract NSR 39080001. (Received February 20, 1968.)


It follows from a theorem of J. F. Adams (Topology 1 (1962), 67-72) and our theorem (below) that if a q-sphere bundle over the n-sphere is an H-space \((q, n)\) positive), then either both \(q\) and \(n\) belong to the set \([1,3,7]\), or \((q,n)\) is one of the pairs \((1,2), (3,5)\). (All of these possibilities are realized by either products of spheres or \(SO(3), SU(3)\), respectively.) Theorem. If the integral cohomology ring \(H^*(X)\), of the space \(X\), is a torsion-free, exterior algebra on two generators whose dimensions are either \((7,11)\) or \((7,15)\), then \(X\) is not an H-space. An interesting consequence of this theorem concerns Stiefel manifolds. The manifold \(W_{3,2}\) of \((3 \times 2)\) orthogonal quaternionic matrices, and the manifold \(Y_{2,2}\) of \((2 \times 2)\) orthogonal matrices over the Cayley numbers are 7-sphere bundles over the 11-sphere and the 15-sphere, respectively. Therefore, these Stiefel manifolds are not H-spaces. (Received March 18, 1968.)
The June Meeting in Portland, Oregon

June 15, 1968


Let G be a compact abelian group and let \( \Gamma \) be its character group. Suppose \( E \subset \Gamma \). \( E \) is a uniformly approximable (u.a.) Sidon set if it is a Sidon set and if there is a sequence of measures in the measure algebra \( M(G) \) whose Fourier-Stieltjes transforms converge uniformly to the characteristic function of \( E \). Then \( E \) is a u.a. Sidon set if and only if for every \( \epsilon > 0 \) and every bounded function \( f \) on \( E \) there exists \( \mu \in M(G) \) such that \( \hat{\mu} = f \) on \( E \) and \( |\hat{\mu}| < \epsilon \) off \( E \), or, equivalently, if and only if for every \( \epsilon > 0 \) there exists \( B > 0 \) such that if \( g \) is any function on \( E \) which vanishes at \( \infty \) then there exists \( f \) in \( L_1(G) \) so that \( \hat{f} = g \) on \( E \) and \( \|f\|_1 \leq B \|g\|_0 \) and \( \|\hat{f}\|_\infty < \epsilon \|g\|_\infty \) off \( E \). A third equivalent condition is that whenever \( f \) is a bounded function on \( \Gamma \) which vanishes off \( E \) and \( \{\lambda_n\} \) is a norm bounded sequence of measures in \( M(\Gamma) \) so that \( \lim \lambda_n = 0 \) pointwise on \( G \), then \( \lim \int_G f \lambda_n = 0 \). All known (to the author) Sidon sets are u.a. Sidon sets. (Received November 16, 1967.)

657-2. MARCEL HERZOG, University of California, Santa Barbara, California 93106. On finite groups with a cyclic Sylow subgroup.

A detailed information concerning the nonexceptional irreducible characters of a finite group \( G \) with certain cyclic Sylow subgroup is contained in: Theorem. Let \( G \) be a finite group containing a subgroup \( M \) of order \( m \) and satisfying the following conditions: (i) for every element \( h \) of \( M^\# \), \( C_G(h) \) is contained in \( M \); (ii) a Sylow \( p \)-subgroup of \( M \) is cyclic and nontrivial; (iii) \( N_G(M) \neq M,G \). Then the nonexceptional irreducible characters of \( G \), nonvanishing on \( M^\# \), are of one of the following two types: (I) \( P_i(i = 1, \ldots, y) \) where \( P_i(1) = r_i m + 1, P_i(h) = 1 \) for all \( h \) in \( M^\# \); (II) \( Q_j(j = 1, \ldots, q - y) \) where \( Q_j(1) = s_j m - 1, Q_j(h) = -1 \) for all \( h \) in \( M^\# \); where \( q = |N_G(M) : M| \) and \( r_i, s_j \) are nonnegative integers.

As a consequence, the following characterization of the simple groups \( PSL(2,p) \) and \( PSL(2,2b) \) is obtained: Corollary. Let \( G \) and \( M \) satisfy the assumptions of the theorem. In addition, let \( G' = G \) and \( [G : N_G(M)] < \frac{m^2 + 3m + 2}{2} \). Then \( G \) is isomorphic either to \( PSL(2,p), p > 3 \), or to \( PSL(2,2b) \), where \( m = 2^b + 1 > 3 \). (Received February 5, 1968.)

657-3. KURT KREITH, University of California, Davis, California 95616. Lower bounds for the essential spectrum of fourth order differential operators.

For a second order Sturm-Liouville operator \( k \) defined by \( ku = -(pu')' + qu \) on \( C^2(0,\infty) \), there is a well-known relation between the infimum of \( \sigma_e(k) \) (the essential spectrum of \( k \)) and the oscillation properties of \( ku = 0 \). By establishing a similar relation for the fourth order differential operator \( l \) defined by \( lu = (pu')'' + qu \) on \( C^2(0,\infty) \), one is able to establish estimates for inf \( \sigma_e(l) \) similar to those established by K. O. Friedrichs for inf \( \sigma_e(k) \). For example: If there exists a real \( a \) such that \( \lim_{x \to \infty} x^{-2} a_p(x) = p_0 > 0 \) and if \( Z(x) = q(x) + p_0 ((1 - a^2)^2/16)x^{a-2} \), then \( \lim \inf_{x \to \infty} Z(x) \leq \inf \sigma_e(l) \leq \lim \sup_{x \to \infty} Z(x) \). (Received February 22, 1968.)

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The distribution of quadratic residues in fields of order $p^2$.

Let $p$ be a prime of the form $4k + 3$. The lattice points in the square with vertices at $(\pm p/2, \pm p/2)$ form a model for the field of order $p^2$. The squares of this field are called quadratic residues of the field. If we define $(a/p) = 1$ if $a$ is a square and -1 otherwise, then the following are established by elementary methods. (i) $(a/p) = a^{(p^2-1)/2} \pmod{p}$. (ii) $(a/4) = (ia/p) = 1$. (iii) $(a \pm ai/p) = (\pm b \pm ai/p)$. (iv) $(a / p^2) = (a^2 + b^2 / p)$. (v) $\sum_{d \equiv 1 (p-1)/2} (c + di/p) = -1$, $b \neq 0, c \neq 0$. (vi) $(a + bi/p) = (a^2 + b^2 / p)$, (Legendre). (vii) $\sum_{d \equiv 1 (p-1)/2} (a^2 + b^2 / p) = -1$. (viii) If $\gamma$ is a primitive root of the field $p^2$, then $\gamma^p + 1$ is a primitive root of $p$. (Received March 8, 1968.)

Decimal expansions to negative bases.

Consider the following infinite sequences of integers $\{m_i\}_{i \geq 1}$, $\{M_i\}_{i \geq 1}$, $\{s_i\}_{i \geq 1}$, and $\{t_i\}_{i \geq 1}$, where, for each $i \geq 1$, $m_i \geq 2$, $M_i = \prod_{j=1}^{i} m_j$, $s_i = 0$ or 1 with infinitely many $i$ for which $s_i = 0$ and infinitely many $i$ for which $s_i = 1$, $t_i = (-1)^{s_i}$ and $u_i = 0$ for $s_i = 0$, $t_i = 0$ and $u_i = (-1)^{s_i}$ for $s_i = 1$, $v_i = \min \{-1, s_i - 1\} \sum_{k=1}^{\infty} (-1)^{s_k} y_k / M_k$, $0 \leq y_k < m_k$, and $V_i = \max \{-1, s_i - 1\} \sum_{k=1}^{\infty} (-1)^{s_k} y_k / M_k$, $0 \leq y_k < m_k$. The purpose of this paper is to show that every real number $a$ with $v_1 \leq a < V_1$ is representable in the form $a = \sum_{i=1}^{\infty} (-1)^{s_i} y_i / M_i$, $0 \leq y_i < m_i$, and that if $a = \sum_{i=1}^{\infty} (-1)^{s_i} y_i / M_i = \sum_{i=1}^{\infty} (-1)^{s_i} z_i / M_i$, are two such representations for $a$ where $y_i > z_i$ are the first unequal coefficients, then $y_r = z_r + 1$ and $y_i = 0, z_i = m_i - 1$ if $s_i \equiv s_r \pmod{2}$, $y_i = m_i - 1, z_i = 0$ if $s_i \neq s_r \pmod{2}$ for all $i > r$. (Received March 8, 1968.)

On three problems of Franklin and Wallace.

In their paper The least element map (Colloq. Math. 15 (1966), 217-221) Franklin and Wallace ask these questions. Let $X$ be a compact Hausdorff space, $R$ a reflexive relation on $X$ and $\Sigma$ the family of compact subsets of $X$ having an $R$-least element. (1) If $R$ is a closed quasi-order, is the set of maximal numbers of $\Sigma$ closed (in the Vietoris topology)? (2) If $R$ is a closed partial order, is the set of maximal $R$-chains closed? (3) If $R$ is a closed quasi-order, under what conditions will $R$ contain a closed partial order which is chain-equivalent to $R$? It is shown that (1) and (2) have negative answers. In (1) it can be shown that if $R$ is a partial order, then the answer is yes if and only if the set $N$ of $R$-minimal elements is closed and the map $x \rightarrow \{y : x \preceq y\}$ on $N$ is continuous. In (2) the answer is yes if $(X, R)$ is a topological lattice. A variety of information is available for (3), but none of these answers is sufficiently succinct to state here. (Received March 18, 1968.)

Analytic proofs have been given by P. Quarra and by J.H.M. Wedderburn that the number of formally distinct nth powers of an element \(x\) in a groupoid is \((2n - 2)/n!(n - 1)\). There is an elementary proof of this fact by Ivan Niven, and one of an equivalent fact by C. F. Gummer. We present yet another elementary proof thereof which relates this powers question to the number of cyclic permutations of \(n\) black objects and \(n - 1\) white objects by noting that each such permutation corresponds to exactly one formal nth power of \(x\), and that for each such permutation there is exactly one object from which to start counting clockwise so that the count of black objects always exceeds the count of white objects. We then prove the Theorem. The integer \((2n - 2)/n!(n - 1)\) is odd if and only if \(n\) is a power of 2. (Received March 18, 1968.)


Let \(E[t_1]\) and \(F[t_2]\) be (Hausdorff) locally convex topological vector spaces with \(Q(t_1)\) and \(Q(t_2)\) collections of seminorms generating \(t_1\) and \(t_2\) respectively. Let \(p \in Q(t_1)\). A continuous linear operator \(T: E[t_1] \to F[t_2]\) is said to be \(p\)-strictly singular if for each (closed) infinite-dimensional vector subspace \(M\) of \(E[t_1]\) and \(q \in Q(t_2), q(Tm) < p(m)\) for some \(m \in M\). A continuous linear operator \(T: E[t_1] \to F[t_2]\) is \(p\)-strictly singular if and only if for each (closed) infinite-dimensional vector subspace \(M \subset E[t_1]\) and \(q \in Q(t_2)\), there is an infinite-dimensional vector subspace \(N \subset M\) with \(q(Tn) < p(n)\) for all nonzero \(n \in N\). The collection \(pSS(E,F)\) of \(p\)-strictly singular operators \(T: E[t_1] \to F[t_2]\) is a linear subspace of \(C(E,F)\), the class of appropriate continuous linear operators. The following is also such a linear subspace: \(PSS(E,F) = \bigcup_{p \in Q(t_1)} pSS(E,F)\). In fact, \(PSS(E,E)\) is a two sided ideal in \(C(E,E)\). (Received April 1, 1968.)


\[
S_{q,k,m}^{(n)}(t; p) = \int_0^{\infty} (pt)^{\sigma-1/2} \exp (-1/2qpt) W_{k,m}(pt)f(t)dt, \text{ where, if } f(t) = O(t)e^{Lt}, \text{ then } R[(q + \rho)(p - 2\varepsilon)] > 0, \text{ and } R(\sigma + \delta + m + 1) > 0.
\]


The Sylvester-Schur Theorem implies that the binomial coefficient \( \binom{n}{k} \), with \( n \geq 2k \), has a prime divisor \( p > k \). The present paper considers "complementary" results: If \( n \geq 2k \), then the binomial coefficient \( \binom{n}{k} \) has a prime divisor \( p \leq f(n,k) \) where \( f(n,k) \) is a real valued function. The first result implies \( f(n,k) \) cannot be regarded as a function of \( k \) only; the second result solves a problem posed by P. Erdös. The theorems are: Theorem I. For each real number \( r \) and positive integer \( k \), there exists an infinite family, \( N \), of integers such that if \( n \in N \) and \( p \) divides \( \binom{n}{k} \), then \( p > r \).

Theorem II. If \( n \geq 2k \), then \( \binom{n}{k} \) has a prime divisor \( p \leq \max\{n/k, n/2 \} \), with the exception of \( \binom{7}{3} \).

(Received April 1, 1968.) (Author introduced by Professor M. L. Faulkner.)

657-12. W. L. BUCK and D. R. HORNER, Eastern Washington State College, Cheney, Washington 99004. The strict singularity of \( T \) and \( S \) does not imply the strict singularity of \( T + S \).

H. E. Lacey posed the question: Is the set \( SS(E,F) \) of all strictly singular operators from \( E \otimes_1 F \) a (closed) vector subspace of \( C(E,F) \), the set of continuous linear operators from \( E \otimes_1 F \)? The answer is known to be affirmative for \( E \otimes_1 F \) a normed space, \( SS(E,F) \) need not be closed in \( C(E,F) \). Furthermore, \( SS(E,F) \) need not be a linear space. Defined \( T, S : l^1 \rightarrow l^1 \otimes_1 (c_0, l^1) \times l^1 \) by \( Tx = (x,x) \) and \( Sx = (-x,0) \). The operators \( T \) and \( S \) are strictly singular while \( T + S \) is not. (Received April 4, 1968.)

657-13. J. D. FUELBERTH, University of Southern California, Los Angeles, California 90007. On commutative splitting rings.

The concepts of torsion and \( T \)-rings are as in S. E. Dickson's paper Decomposition of modules. II: Rings without chain conditions, Math. Z. (to appear). All rings are commutative with unit and all \( R \)-modules are unitary. Definition. A ring \( R \) is a splitting ring iff for all modules the torsion submodule is a summand. In his paper, Noetherian splitting rings are Artinian, J. London Math. Soc. 42 (1967), 732-736, Dickson conjectures that all splitting rings are torsion and shows that the conjecture holds for commutative Noetherian rings. The following theorems show that the conjecture is valid for a larger class of rings. Theorem. Let \( R \) be von Neumann regular. \( R \) is a splitting ring iff \( R \) is torsion. Theorem. Let \( R \) be a \( T \)-ring. Then the following are equivalent: (i) \( R \) is a splitting ring, (ii) \( R \) has a finite number of maximal ideals and is a splitting ring, (iii) \( R \) is torsion. (Received April 10, 1968.)
On a problem of Borsuk. Preliminary report.

A metric space \((X,d)\) is said to be SC (strongly convex) provided that, for every pair of points \(x,y \in X\), there is a unique point \(z \in X\) such that \(d(x,z) = d(z,y) = (1/2)d(x,y)\): A subset of a metric space is SC if it is SC in the relative metric. It is well known that a compact SC metric space is contractible and locally contractible, and thus, if finite-dimensional, is an absolute retract for the class of metrizable spaces. (Abbreviation: \(X \in \mathrm{AR}(\mathcal{M})\).) Also, a finite-dimensional compact metric space in which each point has at least one SC neighborhood is an absolute neighborhood retract for the class of metrizable spaces. (Abbreviation: \(X \in \mathrm{ANR}(\mathcal{M})\).) Borsuk asks (K. Borsuk, Theory of retracts, Ars Polona, 1967, p. 219) whether in either of these cases the condition of finite-dimensionality may be dropped. Introducing the concept loc SC (locally SC, meaning that every point of the space has a neighborhood basis consisting of SC neighborhoods), we get Theorem 1. If \(X\) is a locally compact metric space which is loc SC, then \(X \in \mathrm{AR}(\mathcal{M})\). Theorem 2. If \(X\) is a compact metric space which is both SC and loc SC, then \(X \in \mathrm{AR}(\mathcal{M})\). (Received April 10, 1968.)

Pisot sequences and Pisot-Vijayaraghavan numbers. Preliminary report.

Definition. A sequence of natural numbers \(a_0, a_1, \ldots, a_n, \ldots\) with \(2 \leq a_0 < a_1\) is called a Pisot sequence, denoted by \(PS(a_0, a_1)\), iff \(-1/2 < a_0 \cdot a_1)^2 \leq 1/2\). C. Pisot [Ann. Scuola Norm. Super., Pisa 7 (1938), 241-244] proved that if \(a_0 = 2\) or \(a_0 = 3\), then \(PS(a_0, a_1)\) satisfies a linear recurrence relation of order less than or equal to three. For arbitrary \(a_0 \geq 2\), we give various values of \(d\) (which depends on \(a_0\)) such that \(PS(a_0, a_1)\) satisfies a linear recurrence relation if \(a_1 \equiv d \pmod{a_0}\). Definition. A Pisot-Vijayaraghavan (PV-) number is an algebraic integer greater than one such that all of its conjugates, with the exception of itself, have moduli strictly less than one.

Definition. The polynomial \(P(z)\) is called a PV-polynomial iff it is the minimal polynomial of a PV-number. Vijayaraghavan [Proc. Cambridge Philos. Soc. 37 (1941), 350] showed that there exist PV-polynomials of all degrees. The roots of the polynomials he presented were not given explicitly. For every natural number \(n\), a PV-polynomial \(P_n(z)\) of degree \(n\) is given. Also the roots of \(P_n(z)\), two real if \(n\) is even but only one real if \(n\) is odd, are given explicitly. (Received April 12, 1968.)

A representation theorem for uniform convergence structures.

Theorem. The following statements about a convergence space \((S,q)\) are equivalent: (1) The space is uniformizable (in the sense of being compatible with the generalized uniformity defined by C. H. Cook and H. R. Fischer). (2) \(q\) belongs to the closure with respect to the order topology of the set of all completely regular topologies in the lattice \(C(S)\) of all convergence structures on the set \(S\). (3) \(q\) is a limit space with the property that \(\mathcal{F} \cap \mathcal{I} q\)-converges to \(x\) and \(y\) whenever \(\mathcal{F}\) is a filter which \(q\)-converges to \(x\), \(\mathcal{I}\) is a filter which \(q\)-converges to \(y\), and \(\mathcal{F}\) and \(\mathcal{I}\) fail to contain disjoint sets. (Received April 12, 1968.)
657-17. WITHDRAWN.

657-18. B. G. EKE, University of California, San Diego, La Jolla, California 92037.
A note on mini-gaps.

Suppose \( f(z) = \sum_{n=0}^{\infty} a_n z^n \) is a really mean univalent in \( |z| < 1 \), with \( a_n = 0 \), if \( n = n_k \) \((k \geq 0)\), and suppose \( n_{k+1} - n_k \leq C < +\infty \) if \( k \equiv k_0 \). A discussion of the behaviour of the sequence \( \{n_k\}_{k=0}^{\infty} \) will be given for the subclass of functions whose maximum moduli grow as swiftly as possible. (Received April 22, 1968.)

657-19. KI CHOUL OUM, State University College, Buffalo, New York 14214. Bounds for the number of deficient values of entire functions whose zeros have angular densities.

Let \( f(z) \) be an entire function of finite order \( \lambda \). Arakeljan has shown that, for every \( \lambda > 1/2 \), \( f(z) \) may have infinitely many deficient values in the sense of R. Nevanlinna. The author shows that this cannot happen if (i) the zeros of \( f(z) \) have an angular density (in the sense of Pfluger and Levin) and (ii) \( \lambda \) is not an integer. Under these two assumptions the number of deficient values cannot exceed \( 2\lambda + 1 \). If \( f(z) \) is of completely regular growth (in the sense of Levin), the result also holds for integral values of the order. The proofs rely heavily on a lemma of Edrei and Fuchs [Proc. London Math. Soc. 12 (1962), 321, Lemma 1]. (Received April 22, 1968.)


Let \( A \) and \( B \) be subsets of \( \mathbb{Z}^n \), where \( \mathbb{Z} \) denotes the set of integers and \( n \) is a natural number. Pair \((A,B)\) is called a complementing pair for \( A + B \) provided each element of the sum is uniquely represented and \( A \cap B \) contains the "zero" element of \( \mathbb{Z}^n \). Such a pair is denoted by \((A,B) \sim A + B\).

Complementing pairs for \( A + B \) equal to \( N = \{0,1,2,\ldots\} \) and \( N_m = \{0,1,2,\ldots, m-1\} \) have been characterized by N. G. de Bruijn [Publ. Math. Debrecen 1 (1950), 232-242; Nieuw Arch. Wisk. 4 (1956), 15-17] and C. T. Long [to appear in Pacific J. Math.]. Their addition theorems generalize to the so-called proper pairs in the plane which have the property that \((x,y)\) is in a component iff both \((x,0)\) and \((0,y)\) are in the same component. A complementing pair \((A,B)\) is proper iff the respective projections on each axis form a complementing pair for the axis interested with \( A + B \). All complementing pairs for \( N_n \times N_m \) are proper. Those complementing pairs of a quadrant for which the components are finite on distinct axes are also proper. Pair \((A,B) \sim C\) is called unique provided that for any pair \((A',B) \sim C\) we have \( A = A' \). Complementing pairs for subsets of \( N \times N \) are unique. Also, if \((A,B) \sim C \subset Z \times Z\) and \( C \) is finite, then \((A,B)\) is unique. (Received April 18, 1968.)


Let \( \mathfrak{C}(S) \) be the set of convergence structures on \( S \), ordered in the usual manner. Theorem 1. The closure with respect to the order topology on \( \mathfrak{C}(S) \) of the set of regular topologies on \( S \) is equal
to the closure of the completely regular topologies (which is known to be equal to the set of all Cook-Fischer uniformizable convergence structures). **Theorem 2.** The closures with respect to the order topology on $C(S)$ of the $T_2$, $T_3$, and $T_4$ topologies are equal, and equal to the set of all Cook-Fischer uniformizable convergence structures. Theorem 1 is used to define order regularity for a convergence structure. Four other possible definitions for regularity of a convergence structure are given, including the Cook-Fischer definition. These five definitions are compared for convergence structures and for pretopologies. In particular, it is shown that the Cook-Fischer concepts of regularity and uniformizability are independent.

On productivity and heredity the following is proved: **Theorem 3.** Strong, Cook-Fischer-Biesterfeldt, and order regularity are hereditary; $t$ and weak regularity are not hereditary. Cook-Fischer-Biesterfeldt and order regularity are productive; $t$, weak, and strong regularity are not productive. (Received April 18, 1968.)


Consider equation (1) $\epsilon dy/dx = A(x, \epsilon)y$, where $x$ is a complex variable, $\epsilon$ a small complex parameter, $y$ an $n$-dim. vector, $A(x, \epsilon)$ an $n$-by-$n$ matrix function holomorphic in $x, \epsilon$ in the domain $|x| \neq x_0, 0 < |\epsilon| \leq \epsilon_0, |\arg \epsilon| \leq \epsilon_0$. Let $A(x, \epsilon) = A_0(x) + \epsilon \hat{A}(x, \epsilon)$. **Assumptions.** (i) $A_0(x) = x^p \hat{A}_0(x)$, where $p$ is a positive integer and $\hat{A}_0(x)$ is holomorphic in $|x| \neq x_0$ with $\hat{A}_0(0) \neq 0$. (ii) $\hat{A}_0(x)$ is a 2-by-2 block diagonal matrix in which none of the eigenvalues of the upper left diagonal block of $\hat{A}_0(0)$ is equal to that of the lower right diagonal block. (iii) For a fixed real $\alpha$ and for every $\beta (0 < \beta < \pi/(p + 1))$, $\hat{A}(x, \epsilon)$ admits an asymptotic expansion, as $\epsilon \to 0$, valid uniformly in the region $S(x_0, \alpha, \beta) = \{x \in |x| \neq x_0, |\arg x - \alpha| \leq \beta\}$. **Assumption (i) asserts that all eigenvalues of $A_0(x)$ coalesce at $x = 0$, making $x = 0$ a turning point of (1).** **Theorem.** If assumptions (i), (ii), (iii) are satisfied, then there exists a function $P(x, \epsilon)$ such that the transformation $y = P(x, \epsilon) z$ takes (1) into (2) $\epsilon dz/dx = B(x, \epsilon) z$, where $B(x, \epsilon)$ is a 2-by-2 block diagonal matrix having the same partition as $\hat{A}_0(x)$ and $B(x, \epsilon) = A_0(x) + \epsilon \hat{B}(x, \epsilon)$. The matrices $\hat{B}(x, \epsilon)$ and $P(x, \epsilon)$ have asymptotic expansions as $\epsilon \to 0$, which are valid uniformly for $x$ in any proper subsectors of $S(x_1, \alpha, \beta), 0 < |x_1| \leq x_0$. An application of the theorem will be given. (Received April 25, 1968.)


It is known that if $G$ is an u.s.c. decomposition of $E^m$, with only finitely many nondegenerate elements, then the decomposition space $E^m/G$ of $G$ is embeddable in $E^{m+2}$ (L. V. Keldyš, Proc. Sympos. Prague, 1961, p. 230–234). The codimension 2 is the best possible one, at least for all odd dimensions $m$, in the following sense: **Theorem.** For each integer $n$, $n > 1$, there exists a monotone decomposition $G$ of $E^{2n-1}$, with only $2n + 3$ nondegenerate elements, such that $E^{2n-1}/G$ is not embeddable in $E^{2n}$; moreover, the nondegenerate elements of $G$ can be so chosen as to be p.w.l. homeomorphic to the $(n - 1)$ skeleton of the $(2n + 1)$-simplex (and hence of dimension $n - 1$). For $n = 2$, there are better results, for example D. Gillman's *A five circle decomposition of 3-space* (Abstract 636-65, these Notices 13 (1966), 594). (Received April 8, 1968.)

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Projections of f-vectors of 4-polytopes.

If \( P \) is a polytope in \( \mathbb{R}^n \), its \( f \)-vector is defined as \( f(P) = (f_0, f_1, \ldots, f_{d-1}) \) where \( f_i \) is the number of \( i \)-dimensional faces of \( P \). It is a well-known unsolved problem to determine the set of all \( (f_0, f_1, f_2, f_3) \) which represent a 4-polytope. Let \( \pi(0,2) = \{(f_0, f_2) | \exists \ 4\text{-polytope } P \text{ with } f_i \text{ } i\text{-faces}\} \).

Lemma. If \( (f_0, f_2) \notin \pi(0,2) \), then \( 10 \leq f_2 \leq f_0^2 - 3f_0 \) and \( f_0 \geq f_2 - \left[(1 + \sqrt{1 + 8f_2})/2\right] \). The converse is false, and the complete set of \( (f_0, f_2) \) which satisfy the equations but do not represent any 4-polytope are given for various cases. (Received April 29, 1968.)


The Contraction Mapping Theorem, which asserts that every contraction mapping on a complete metric space has a unique fixed point, has been generalized in a number of ways in recent years. J. Mathews showed in his Thesis (Iowa State University) that every contraction mapping on a developable space with a strong development has a unique fixed point. Two elementary generalizations of the Contraction Mapping Theorem, which require only minor alterations in the standard proof (cf. Kolmogorov and Fomin, Functional analysis, 1, p. 43), are the following results: Theorem 1. If \( f \) is a contraction on a bounded and complete semimetric space \( (S, d) \), then \( f \) has a unique fixed point. Theorem 2. If \( f \) is a contraction on an \( a \)-metric space \( (S, d) \), where \( a > 0 \), then \( f \) has a unique fixed point. Moreover, the nonnegative iterates of \( f \) determine for each point of \( S \) a finite semiorbit terminating at this fixed point. (Received April 29, 1968.)

On the essential set.

Define the essential set \( E \) of a function algebra \( A \) on a compact Hausdorff space \( X \) as the minimal closed set in \( X \) supporting every measure in \( A^\perp \). Using the fact that any representing measure has as closed support a set of antisymmetry, one finds that the result of Hoffman and Singer that every representing measure on \( X \) which is not a point mass lives on the essential set is immediate. From this the results of H. S. Bear follow immediately. If \( A \) is approximately normal on \( X \), then the essential set \( E = X - P \) where \( P = \{x \in X | \exists \ a \ closed \ nbhd \ V_x \ of \ x^\exists : A[V_x] = C(V_x^\perp)\} \). Using, this, it follows that if \( X = \bigcup_{i=1}^{\infty} F_i \) where the \( F_i \) are closed and \( A[F_i] = C(F_i) \), then \( A = C(X) \). This result has also been proved by Mullins and by Gamlin and Wilken in a different fashion. (Received April 29, 1968.)


If \( A \subseteq L \), let \( c(A) \) denote the closure of \( A \) in Frink's ideal topology. Theorem. Let \( B \) be a Boolean algebra and \( A \subseteq B \). (i) If \( A \) is closed under finite joins (meets), then \( x \in c(A) \iff x \in c(A \cap x^+) \) \((x \in c(A \cap x^+)) \). (ii) If \( A \) is closed under finite joins (meets), then \( x \in c(A \cup x^+) \) \((x \in c(A \cup x^+)) \) only if \( x \) is a finite meet (join) of elements from \( A \cap x^+ \) \((A \cap x^+) \) only. For a discussion of auto-topologies, see Rema (J. Indian Math. Soc. 30 (1966)). Theorem. Let \( B \) be a Boolean algebra, \( I \) an ideal of \( B \) and \( D = \{a^+ : a \in I\} \). The topology \( T_{ID} \) having a subbase of all sets of the form \( a^+ \cap b^+ \) \((a \in I, b \in D) \) is
auto-uniformizable and coincides with $T_D$. Corollary. The order topology on an atomic Boolean algebra is auto-uniformizable. Kent (Illinois J. Math. 10 (1966), 90-96) defined three conditions $(c_1, c_2, c_3)$ on a lattice. Theorem. A lattice $L$ satisfies $c_1$ and $c_3$ iff $L$ is bicomactly generated; the compact elements of $L$ form an ideal and the cocompact elements of $L$ form a dual ideal. Theorem. On any complete lattice satisfying $c_1$ and $c_3$ the ideal topology is finer than the order topology. (Received April 30, 1968.)


Let $R$ be a left-noetherian ring with identity and $M$ a unitary left $R$-module. A prime ideal $P$ of $R$ is called associated with $M$ if it is the left annihilator of all submodules $\neq 0$ of a nonzero submodule $N$ of $M$. If $P$ is furthermore left annihilator of all elements $\neq 0$ of $N$, $P$ is called strictly associated with $M$. $M$ is called admissible if all its associated prime ideals are strictly associated and it is shown that $M$ is admissible if and only if the injective hull $E(M)$ is the discrete direct sum of injective indecomposable modules of the form $E(R/P)$, where $P$ runs through the set of all primes associated with $M$. This result is a generalization of a result obtained by E. Matlis [Pacific J. Math. 8 (1958), 518]. Theorem. The following properties of $R$ are equivalent: (1) $R$ is a finite direct sum of local, left-artinian rings. (2) $R$ has the Artin-Rees-property for left ideals and all prime ideals are maximal left ideals. (3) All unitary left-$R$-modules $M$ are admissible and each prime ideal containing the left annihilator of $M$ is associated with $M$. (Received April 26, 1968.)


Let $M = S^q (q \geq 2)$ and decompose $L^2 (M)$ as $\bigoplus_1^\infty H_n$ where $H_n$ is the set of spherical harmonics of degree $n$. (Alternatively, $H_n = \varphi_n \cap \varphi^{k-1}_n$ where $\varphi_n = \{ p | M : p$ is a polynomial, deg $p \leq n \}$. Each $f$ in $L^2 (M)$ has a unique $L^2$-expansion $f = \sum_{j} f_j \in H_j$ with partial sums $L^2_n f = \sum_1^n f_j$. For $f \in C^k (M)$, let $\omega(t^{(k)}; \delta)$ be the sum of the moduli of continuity of some ample set of $k$th derivatives of $f$.

Previous work of the author yields Theorem 1. If $f \in C^k (M)$, then $\exists p_n \in \varphi_n$ such that $\| f - p_n \|_\infty = O(n^{-k} \omega(t^{(k)}; 1/n))$. The use of known estimates involving Jacobi polynomials gives Theorem 2. The map $L^k : C(M) \rightarrow C(M)$ has $\| L^k_n \| = O(n^{q-1/2})$. Following Lebesgue, these two results are combined to prove the main result of the paper which is the following analog of the Dini-Lipschitz theorem on uniform convergence of Fourier series. Theorem 3. If $k = [(q-1)/2]$ and $f \in C^k (M)$, then $L^k f \rightarrow f$ uniformly, provided either (i) $q$ is odd or (ii) $q$ is even and $\omega(t^{(k)}; \delta) = o(\delta^{1/2})$. (Received May 1, 1968.)

657-30. R. E. ATALLA, Ohio University, Athens, Ohio 45701. Linear operators on spaces of continuous functions.

Let $X,Y,Z$ be completely regular $T^2$-spaces, $C(X)$, $C(Y)$, $C(Z)$, the corresponding spaces of bounded complex-valued continuous functions. Let $A$ be a continuous linear operator from $C(X)$ to $C(Z)$, and $D$ a continuous linear operator from $C(X)$ to $C(Y)$ such that range $(D)$ is dense in $C(Y)$. 621
If $S \subseteq V$, where $V$ is a topological vector space, we define the k-hull of $S$, $[S]_k$, to be the closure of elements of the form $\sum_{i} t_i S_i$, $S_i \subseteq S$, $t_i$ real, $\sum_{i} |t_i| = k$, $1 \leq i \leq n$. **Theorem.** Let $A$ and $D$ be as above. Then the following are equivalent: (a) There is a linear operator $B$ from $C(Y)$ to $C(Z)$ such that $\|B\| = k$, $B1 = 1$, $Bf$ is real valued if $f$ is, and $A = BD$. (b) For each $f \in C(X)$, $Af(Z) \subseteq [Df(Y)]_k$, and this fails for $k' < k$. Letting $X = Y = Z$, $A = B$, $D$ be the identity, and $k = 1$ (1-hull = closed convex hull), we obtain a special case of R. R. Phelps [Proc. Amer. Math. Soc. 16 (1965), 381-382]. Using the representation of matrix operators on sequence spaces given in Sequential cores and a theorem of Phelps, with J. Bustoz [to appear in these Notices], we can prove Theorem. If $(a_{nm})$ is a real valued regular matrix, then $\limsup_{m \to \infty} \sum_{n=1}^{\infty} |a_{nm}|$ is the least number $k$ such that core $(Bx)$ $\subseteq$ [core $x)_k$ for each bounded sequence $x$ of complex numbers. (Received May 1, 1968.)

657-31. ALEXANDER ABIAN, Iowa State University, Ames, Iowa 50010. **Decomposition of atomic and orthogonally complete rings.**

Let $R$ be a ring in which for every $x \in R$ there exists a natural number $n(x) > 1$ such that $x^{n(x)} = x$. **Lemma.** The ring $R$ is partially ordered by $\preceq$ where $x \preceq y$ if and only if $xy = x^2$ for every $x, y \in R$. **Definition.** A nonzero element $a$ of $R$ is called an atom of $R$ provided $x \preceq a$ implies $x = a$ or $x = 0$, for every $x \in R$. Moreover, $R$ is called atomic if for every nonzero element $r$ of $R$ there exists an atom $a$ of $R$ such that $a \preceq r$. Furthermore, $R$ is called orthogonally complete if for every set $S$ of pairwise orthogonal elements of $R$ the sup $S$ exists. **Theorem.** The ring $R$ is isomorphic to a direct sum of fields if and only if $R$ is atomic and orthogonally complete. (Received May 1, 1968.)

657-32. KEITH PHILLIPS, California Institute of Technology, Pasadena, California 91109 and MITCHELL TAIBLESON, Washington University, St. Louis, Missouri. **Singular integrals in several variables over a local field.**

Singular integral transforms of the Calderon-Zygmund type are constructed for $K^d$, $K$ a non-discrete zero dimensional locally compact field. The transforms have the form $L \phi = \lim_{k \to \infty} \psi_k \ast f$, where the kernel $\psi_k$ vanishes in a neighborhood of 0, $\int_{|x|=1} \psi_k (x) dx = 0$, $\psi_k$ is homogeneous of degree 0, and $\psi_k$ satisfies certain smoothness conditions. In general, $\psi_k \notin L^1$. Convolution is with respect to additive Haar measure. Pointwise and $L^r$ ($1 < r < \infty$) convergence are proved. Relatively weak hypotheses on the kernels are found to work. The results substantially generalize the first author's paper Hilbert transforms for the $p$-adic and $p$-series fields, Pacific J. Math. 23 (1967), 329. (Received May 1, 1968.)
**ABSTRACTS PRESENTED BY TITLE**

**68T-411.** H. LAKSER, University of Manitoba, Winnipeg, Manitoba, Canada. *Free lattices generated by a partially ordered set. II.*

For the notation see Part I, Abstract 68T-356, *Notices* 15 (1968), 544. **Theorem 2.** A lattice \( L \) generated by a poset \( P \) is \( FL(P; M, N) \) if and only if (i) for all \( x \in L \), \( [x] \cap P \) is an \((M, N)\) ideal and dually; (ii) for all \( x, y \in L \), \( [x \vee y] \cap P = (\{a \in P \mid a \leq x_0 \} \cap P) \cup (\{a \in P \mid a \leq y_0 \} \cap P) \) (join of ideals in \( P \)) and dually; (iii) given \( x_0, x_1, y_0, y_1 \in P \) such that \( x_0 \wedge x_1 \leq y_0 \vee y_1 \), \( x_1 \nleq y_0 \vee y_1 \), then there is a \( p \in P \) such that \( x_0 \wedge x_1 \leq p \leq y_0 \vee y_1 \). The characterization of \( CF(P) \), R. A. Dean, *Trans. Amer. Math. Soc.* 83 (1956), 246, Theorem 6, is a corollary of this result. **Theorem 3.** Let \( L \) be a lattice and \( L^* \) a completion of \( L \). Any isotone \( f: P \rightarrow L \) has an isotone extension \( F: FL(P; M, N) \rightarrow L^* \); if all pseudo-principal \((M, N)\)-ideals and -dual ideals are finite set unions of principal ones, then \( Im(FL(P; M, N)) \subseteq L^* \). A special case is in Yu. Sorkin, *Mat. Sb. N. S.* 30 (1952), 677-694. Let \( L \) be the free product of lattices \( L_i \), \( i \in I \), amalgamated by a finite sublattice \( M \). **Theorem 4.** There exists an algorithm to determine whether \( A \leq B \) holds in \( L \) in terms of the lattice structure of the \( L_i \) and \( M \). (Received January 15, 1968.) (Author introduced by Professor G. A. Grätzer.)


Let \((X, \mathcal{U})\) be a symmetric generalized uniform space (Abstract 67T-612, *Notices* 14 (1967), 839). **Definition 1.** \((X, \mathcal{U})\) is totally bounded iff for every \( V \in \mathcal{U} \) there exists \( x_1, \ldots, x_n \) in \( X \) such that \( X = V(x_1) \cup \ldots \cup V(x_n) \). **Definition 2.** A filter \( \mathcal{F} \) in \( X \) is weakly Cauchy iff for every \( U \in \mathcal{G} \) there exists \( x \) in \( X \) such that \( U \cap [x] \in \mathcal{F} \). **Theorem 1.** If \((X, \mathcal{U})\) is a symmetric uniform space, then a filter \( \mathcal{F} \) in \( X \) is Cauchy iff it is weakly Cauchy. **Theorem 2.** If \((X, \mathcal{U})\) is a symmetric, connected topological space, then there exists a totally bounded, symmetric generalized uniformity \( \mathcal{U} \) on \( X \) such that \( \mathcal{F}(\mathcal{U}) = \mathcal{F} \) and every filter in \( X \) is weakly Cauchy. **Definition 3.** \((X, \mathcal{U})\) is complete iff every weakly Cauchy filter in \( X \) has a cluster point. **Theorem 3.** \((X, \mathcal{U})\) is compact iff it is complete and totally bounded. (Received February 2, 1968.)

**68T-413.** LUDVIK JANOS, University of Florida, Gainesville, Florida. *On equivalent families of pseudometrics.*

Let \( X \) be a compact Hausdorff space and \( \mathcal{D} = \{\rho(x, y; a) \mid a \in \mathcal{A}\} \) a family of pseudometrics on \( X \) inducing the given topology of \( X \). Let \( f: X \rightarrow X \) be a continuous map of \( X \) into itself and \( c > 0 \). For each \( a \in \mathcal{A} \), we define \( \rho^*(x, y; a, f, c) = \sup \rho(f(x)_a, f(y)_a) \) where the supremum is taken over the set of all nonnegative integers. For each \( f: X \rightarrow X \) and \( c > 0 \), the family \( \mathcal{D}(f, c) = \{\rho^*(x, y; a, f, c) \mid a \in \mathcal{A}\} \) is a family of pseudometrics on \( X \) and conditions have been investigated under which \( \mathcal{D}(f, c) \) induces the same topology as \( \mathcal{D} \). **Theorem 1.** If \( f: X \rightarrow X \) is continuous and \( c \in (0, 1) \), then \( \mathcal{D}(f, c) \) induces the same topology as \( \mathcal{D} \). **Theorem 2.** Let \( F: X \rightarrow X \) be continuous and such that the intersection \( \bigcap f^n(X) \) of all iterated images \( f^n(X) \) of \( X \) is a singleton. Then the preceding theorem is valid also for \( c = 1 \). (Received February 12, 1968.)
Maximal measures and the Dirichlet problem of the Choquet boundary.

Let $X$ be a compact $T_2$ space, let $H$ be a point-separating linear subspace of $C_R(X)$, with $E \in H$, and let $B$ denote the Choquet boundary of $X$ w.r.t. $H$. For $x \in X$, $f \in C_R(X)$, $g \in C_R(B)$ with $\sup |g(B)| < \infty$, let $f(x) = \inf \{ h(x) | h \in H, h \geq f \}$, let $\tilde{g}(x) = \inf \{ h(x) | h \in H, h|B \geq g \}$, let $f(x) = (\tilde{f} - f)(x)$, and let $g(x) = - (\tilde{g})(x)$. If $\mu$ is a regular Borel measure on $X$, $\mu$ is said to represent $x \in X$, written $\mu \rightarrow x$, if $\int f \, d\mu = h(x) \forall h \in H$. Lemma. Given $f \in C(X)$, there exists a maximal measure $\mu$ with $\mu \sim \epsilon_x$ and $\int f \, d\mu = \tilde{f}(x)$ if there exists a measure $\xi$ with $\xi \sim \epsilon_x$ and $\int f \, d\xi = \tilde{f}(x)$. Theorem. If $g$ is a bounded and continuous function on $B$, and $g \in \tilde{g} - \tilde{f} \over B = g|B$, then the following are equivalent: (1) $g$ can be extended to an $H$-harmonic function on $X$; (2) if $\mu_1$ and $\mu_2$ are maximal measures representing $x$, then $\int_B \tilde{g} \, d\mu_1 = \int_B \tilde{g} \, d\mu_2$; (3) $\tilde{g} = \tilde{g}$. Using this result, one can derive Alfsen's characterization of functions $g \in C_C(B)$ for some $h \in H$, where $H$ is a uniform closed linear subspace of $C_C(X)$. See Erik M. Alfsen, *On the Dirichlet problem of the Choquet boundary*, Math. Scand. (to appear). (Received February 21, 1968.)


Let $G = \mathbb{G} \cup \{ 0 \}$ be a group with zero, $e$ being the unit of $G$. By a $1$-$F$-system we mean a subgroup $F$ of $G$ and a system $I$ of left representatives modules $F$ verifying $Z \subseteq F$ where $Z = \mathbb{G}\backslash IF$. Theorem. A function $f$ of $G$ into itself, not identically 0 or $e$, is a solution of the functional equation $f(xf(y)) = f(x)f(y)$ if and only if there exists a $1$-$F$-system such that (1) $f(x) = 0$ if $x \in Z$; (2) $f(x) = i^{-1}x$ if $x \in IF$ ($i \in I$). A similar characterization is given for groups. Each solution generates a semigroup of solutions. If this semigroup is of finite order, it is a group. (Received February 21, 1968.) (Author introduced by Professor Ludvik Janos.)


In this paper, the following theorems are proved. Theorem 1. Suppose that $f(z) = z + a_2 z^2 + \ldots$ and $g(z) = z + b_2 z^2 + \ldots$ are regular in $|z| < 1$ and $g(z)$ is univalent and convex for $|z| < 1$. If the function $F$, defined by $F(z) = (2/z) \int_0^2 f(t)g(t)dt/t$, is univalent and starlike in $|z| < 1$, then $f(z)$ is univalent and starlike for $|z| < 2 - \sqrt{3}$. This result is sharp. Theorem 2. Let $f(z)$ and $F(z)$ be as in Theorem 1 with $g(z) = z + b_2 z^2 + \ldots$ regular and univalent in $|z| < 1$. If $F$ is univalent and starlike in $|z| < 1$, then $f(z)$ is univalent and starlike for $|z| < 1/5$. This result is sharp. Theorem 3. Suppose that $f(z) = z + a_2 z^2 + \ldots$ and $g(z) = 1 + b_1 z + \ldots$ are regular in $|z| < 1$ and $\Re \{g(z)\} > 0$ for $|z| < 1$. If the function $F$, defined by $F(z) = (2/z) \int_0^2 f(t)g(t)dt$, is univalent and starlike in $|z| < 1$, then $f(z)$ is univalent and starlike for $|z| < (5 - \sqrt{11})/4$. This result is sharp. (Received February 22, 1968.)
Let $E$ be a set of transformations (i.e., mappings) on the set $A$. $E$ is algebraic iff $E = E(\mathcal{A})$ (the set of endomorphisms of $\mathcal{A}$) for some $\mathcal{A}$, where $\mathcal{A} = \langle A; F \rangle$ is a universal algebra. For $a, b \in A$, $\langle a, b \rangle$ is an E-pair iff $as = at$ implies $bt = bs$ for all $s, t \in E$. For $a \in A$, define $\phi_a$ by $x \phi_a = a$ for all $x \in A$. Set $C_E = \{\phi_a \mid \text{for every } b \in A, a \neq b, \langle a, b \rangle \text{ is not an E-pair} \}$. Call $\phi_a$ an E-constant iff $E \subseteq E(\mathcal{A})$ implies $\phi_a \in E(\mathcal{A})$ for any $\mathcal{A}$. $E$ is locally closed (B. Jónsson) means that if $a \in E$ for every finite $X \subseteq A$ there is $\beta \in E$ such that $a|_X = \beta|_X$. Let $G$ be a group of permutations on $A$. Theorem. Let $K$ be a set of constant maps. $G \cup K$ is algebraic iff $G$ is locally closed $KG \subseteq K$, and $C_G \subseteq K$. Corollary 1. $G$ is algebraic iff $G$ is locally closed and $C_G = \emptyset$. Corollary 2. For any $G$, $C_G$ is the set of all $G$-constants. Remark. In proving the theorem one defines an operation $f_{ab}$ for every $G$-pair $\langle a, b \rangle$ with $\phi_a \in K$ by $f_{ab}(\phi) = bs$ for $\phi \in G$ and $f_{ab}(s) = s$ otherwise. The other operations used are from B. Jónsson, Algebraic structures with prescribed automorphism groups (to appear). (Received February 23, 1968.) (Author introduced by Professor G. A. Grätzer.)

Theorem 2. There exists a transformation semigroup $E$ and an $E$-constant $\phi_a$ such that $\phi_a \in C_E$. Theorem 3. Let a transformation group $G$ be transitive and locally closed. Then $G$ is algebraic iff there exists a $G$-pair. Let us call $G$ symmetric if there exists a $G$-pair $\langle a, b \rangle$ such that $\langle b, a \rangle$ is also a $G$-pair. For $X \subseteq A$, let $G_X = \{a|_X \mid a \in G, Xa \subseteq X\}$. Theorem 4. Let $G$ be transitive. Then the following conditions are equivalent: (i) $G$ is symmetric; (ii) if $|A| > 1$, then there exists $X \subseteq A, |X| > 1$, with $G_X$ abelian; (iii) same as (ii) with $G_X$ cyclic. If $A$ is finite, then $G$ is always symmetric, hence (ii) gives a result of P. Goralčík, Z. Hedrlín and J. Sichler (to appear). (Hint to Theorem 4: assume (i), and take a $\phi \in G$ with $\phi^2 = b$ where $\langle a, b \rangle$ is a symmetric $G$-pair, and set $X = \{a^n \mid n = 0, \pm 1, \pm 2, \ldots, \}$.) Theorem 5. There exist nonsymmetric transitive transformation groups. (Received February 26, 1968.)

On graphs with prescribed connectivities.

Let $\kappa(G)$, $\lambda(G)$, and $\mu(G)$ respectively denote the connectivity, line-connectivity, and minimum degree of a graph $G$. Chartrand and Harary [Graphs with prescribed connectivities, 1966 Sympos. on Graph Theory, Tihany, Acad. Sci. Hung., 1967, 61·63] showed that for all integers $b, c$, and $d$ with $0 < b \leq c \leq d$, there is a graph $G$ having $\kappa(G) = b$, $\lambda(G) = c$, and $\mu(G) = d$, and also constructed a smallest such graph in terms of the number of points. Let $G_1$ and $G_2$ be two graphs, where $G_1$ has $p_1$ points and $q_1$ lines, $i = 1, 2$. Then $G_1$ is smaller than $G_2$ if either (i) $p_1 < p_2$, or (ii) $p_1 = p_2$ and $q_1 < q_2$. A smallest graph $H$ with $\kappa(H) = b$, $\lambda(H) = c$, and $\mu(H) = d$ is constructed. (Received February 29, 1968.)
**Definition.** A sequence \( \{s_n\} \) (\( n = 1, 2, \ldots \)) is said to be slowly varying if \( \lim_{n \to \infty} \left( \frac{L_n}{L_m} \right) = 1 \) for every \( \infty > t > 0 \).

**Theorem 1.** Let \( (a_{n,k}) \) be a triangular matrix (i.e. \( a_{n,k} = 0, \ k > n \)). If
\[
\limsup_{n \to \infty} \left( n^\sigma \sum_{k=1}^{n} |a_{n,k}|^{\frac{1}{k^\tau}} \right) < \infty \quad \text{and} \quad \lim \inf_{n \to \infty} \left( |a_{n,n}| - n^{\sigma} \sum_{k=1}^{n} |a_{n,k}|^{\frac{1}{k^\tau}} \right) > 0 \quad \text{for some } \sigma > 0,
\]
and if \( \lim_{n \to \infty} \sum_{k=1}^{n} a_{n,k} = 1 \), then \( s_n \simeq L_n \Leftrightarrow \sum_{k=1}^{n} a_{n,k} s_k \simeq L_n (n \to \infty) \) for every slowly varying sequence \( \{L_n\} \). Theorem 1, which is a generalization of a result of R. P. Agnew [Eigenvalues of methods for evaluation of sequences, Proc. Amer. Math. Soc. 3 (1952), 550-556], is used in the proof of **Theorem 2.**

Let \( \alpha \) be a real number and \( |L_n| \) a slowly varying sequence. Then \( s_n \simeq L_n \Leftrightarrow \alpha s_n + \left( (1 - \alpha)/n \right) \sum_{k=1}^{n} s_k \simeq L_n (n \to \infty) \) if and only if \( \alpha > 0 \). Theorem 2 is an extension of Mercer's theorem. Theorems 1 and 2 were suggested by Professor R. Bojanic. (Received March 6, 1968.)


The core of a sequence of functions is defined in these *Notices* 15 (1967), 938. The bounded core theorem characterizes those regular matrices for which the core of a transform sequence at a point \( z_0 \) is contained in the core at \( z_0 \) of the original uniformly bounded sequence. This bounded core theorem is shown to be a special case of a theorem of R. R. Phelps [Proc. Amer. Math. Soc. 16 (1965), 381-382] as follows: The space of suitably defined equivalence classes of uniformly bounded sequences \( \{s_n\}, s_n \in C(D), D \) completely regular, is a Banach algebra with norm defined by
\[
\limsup_{n \to \infty} \|s_n\|,
\]
where \( \|s_n\| \) is the sup norm of \( C(D) \). This Banach algebra is isometric to \( C(X) \) for suitably defined compact \( X \) in such a way that limiting values of sequences are the same as values of corresponding functions in \( C(X) \). A matrix operator on sequences whose rows are a bounded subset of \( l^1 \) and whose columns are null sequences then defines an operator on \( C(X) \) with the same operator norm. If the matrix operator is \( (a_{ij}) \), the operator norm is \( \limsup (i \to \infty) \sum |a_{ij}| \).
Phelp's theorem is then applied. (Received February 26, 1968.)

68T-422. ETHAN COVEN, Wesleyan University, Middletown, Connecticut 06457. *Compactness of limit sets and semiorbit closures.*

Let \( T \) be a generative topological group and let \( K \) be a compact, symmetric neighborhood of the identity which generates \( T \). Let \( P \) be a replete semigroup in \( T \) and define an equivalence relation \( \sim \) on \( P \) as follows: \( a \sim b \) provided that there exists \( k_1, \ldots, k_n \in K \) such that \( (1) \ b = ak_1 \ldots k_n \) and \( (2) \ a k_1 \ldots k_i \in P \) for \( i = 1, 2, \ldots, n \). *Definition.* A subset \( Q \) of \( P \) is said to be \( K \)-connected provided that every element of \( Q \) belongs to the same equivalence class. *Theorem.* \( pP \) is \( K \)-connected for some \( p \in P \). *Theorem.* Let \( (X,T,\pi) \) be a transformation group where \( X \) is a locally compact Hausdorff space, and \( T \) is generative. Let \( P \) be a replete semigroup in \( T \) and let \( x \in X \). Then \( P_x \) is compact and nonempty if and only if \( \overline{xP} \) is compact. (Received February 26, 1968.)
Let Q be a compact set in the Euclidean space $\mathbb{R}^n$. Assume that the interior of the set Q is dense in the set Q. Let $C(Q)$ be the space of all continuous complex-valued functions on the set Q. Let $L(Q)$ be the space of all Lebesgue summable complex-valued functions on the set Q. Denote by $H(Q,D)$ the set of all continuous complex-valued functions $f$ on the product $Q \times D$ such that for every fixed $q \in Q$ the function $f(q, \cdot)$ is holomorphic on the set D being an open domain in the complex plane. This space of functions can be identified with the space of all holomorphic functions from the domain D into the Banach space $C(Q)$. Let $D_1$ be another domain in the complex plane such that $D \subseteq D_1$. The set $D_1$ is also assumed to be open. \textbf{Theorem.} Let $f$ be a function from the space $H(Q,D)$. If for every Lebesgue summable function $g \in L(Q)$ the function $\int g(q) f(q, \cdot) dq$ has a holomorphic extension on the domain $D_1$, then there exists a unique function $f_1$ in the space $H(Q,D_1)$ being the extension of the function $f$. The proof of the theorem makes use of the category argument. Some related results will soon follow. (Received February 26, 1968.)

68T-424. \textbf{ALBERT SADE, 364 Cours de la Republique, Pertuis Vaucluse, France. Quasigroupes parastrophiques: Groupe des automorphismes gauches.}

Pour qu'un quasigroupe soit isomorphe à son transposé il faut et il suffit qu'il soit isotope d'un quasigroupe possédant une autotopie gauche $(a,b,c)$ telle que l'équation $x^3 = cba$ ait une solution en $x$. Si un quasigroupe, $Q = E(\cdot)$, est isomorphe à son transposé, cet isomorphisme appartient au normalisateur du groupe d'automorphisme, $\mathcal{A}$, de $Q$ dans $G_E$, et les isomorphismes qui appliquent $Q$, soit sur lui-même, soit sur ses transposés forment un groupe dans lequel $\mathcal{A}$ est normal, d'indice 3 ou 1. L'ensemble de toutes les applications $J$ de $\mathcal{A}$ dans $\mathcal{A}$, $J = X \rightarrow (XT)^3$, $T$ = constante, $X$ décrivant $E$, est le produit par l'une d'elles du groupe $\{\Delta\}$ des translations de $\mathcal{A}$. Pour que cet ensemble soit un groupe - alors nécessairement isomorphe à $\mathcal{A}$ - il faut et il suffit qu'il existe une $J$ identique. Dans ces conditions tous les automorphismes de $Q$ sont involutifs et il existe un automorphisme gauche de $Q$ dont le cube est l'identique. (Received February 26, 1968.)


All modules are right modules for a ring $R$ with identity. \textbf{Definition 1.} A module $M$ is corationally complete if and only if the functor $\text{Hom}_R(M,\ldots)$ induces only onto maps when applied to epimorphisms $N \rightarrow N/V$ in the restricted situation that zero is the only $R$-homomorphism of $M$ to epimorphic images of $V$. \textbf{Definition 2.} A module $W$ is a corational extension by its factor module $W/T$ if and only if $T$ is a small submodule of $W$ and only the identity map among elements of $\text{Hom}_R(W,W)$ induces the identity map of $W/T$. \textbf{Definition 3.} A ring $R$ is right perfect if and only if every right $R$-module has a projective cover. \textbf{Theorem 1.} If $M$ is a corationally complete module, then there are no proper corational extensions by $M$. A partial converse is: \textbf{Theorem 2.} If $R$ is a right perfect ring and if there are no proper corational extensions by right $R$-modules, then every right $R$-module is corationally complete. Theorems 1 and 2 are shadows of the theorem of Findlay and Lambek (Canadian Bulletin of Mathematics, Vol. 1 (1958), 156): A module is rationally complete if and only if it has no proper rational extension. (Received February 26, 1968.)
Finsler (Comm. Math. Helv. 25 (1951), 75-90) and Doner-Tarski (Fund. Math. (to appear)) have set up hierarchies \( \mathcal{H}_C \) and \( \mathcal{H}_C^* \) resp. of binary operations on the ordinals. The operations of these hierarchies satisfy a generalization of the recursion relations \( a(b + 1) = ab + a \) and \( ab + 1 = ab \).

**Theorem 1.** If \( c \geq 4 \) and \( a \geq 2 \), then \( 0 \leq c < \omega; c^* = c + 1 \) if \( c = d + n, d \) is a limit ordinal, and \( 0 \leq n < \omega; c^* = c \) otherwise. Let \( \Gamma \) be the Schütte-Feferman ordinal, i.e. the least solution of the equation \( g(x) = x \) where \( g \) is defined inductively as follows: \( g_0(x) = \omega^x \), and \( g_s \) enumerates in order those numbers which are fixed points of \( g_v \) for all \( v < s \). Theorem 2. \( a, b, c < \Gamma \) implies \( \mathcal{H}_C(a, b) < \Gamma \). Furthermore, \( \omega < \delta < \Gamma \) implies \( d = \mathcal{H}_C(a, b) \) where \( a, b, c < \delta \). The author had previously shown (Comm. Math. Helv. 41 (1967), 273-286) that \( \delta \) has the same behavior with respect to Finsler's hierarchy. (Received February 20, 1968.)

**68T-427.** R. DE SAPIO, University of California, Los Angeles, California 90024. Differentiable structures on a product of spheres.

Let \( \mathcal{H}_n \) be the group of homotopy \( n \)-spheres, let \( + \) denote the connected sum operation, and let \( S^n \) denote the standard \( n \)-sphere. **Theorem 1.** If \( M \) is a differentiable manifold that is homeomorphic to \( S^k \times S^n \) (\( 2 \leq k \leq n, n + k \leq 6 \)), then there exist \( A_n \in \mathcal{H}_n, V^{n+k} \in \mathcal{H}_{n+k} \) such that \( M \) is diffeomorphic to \( (S^k \times S^n) + V^{n+k} \). (a) If \( B^n \in \mathcal{H}_n, U^{n+k} \in \mathcal{H}_{n+k} \) such that \( M \) is diffeomorphic to \( (S^k \times B^n) + U^{n+k} \), then \( S^k \times A^n \) and \( S^k \times B^n \) are diffeomorphic. (b) If \( k \geq n - 3 \), then \( M \) is diffeomorphic to \( (S^k \times S^n) + V^{n+k} \) and \( V^{n+k} \) is unique. Now let \( \tau_{n,k} : \mathcal{H}_n \otimes \pi_k(SO(n-1)) \rightarrow \mathcal{H}_{n+k} \) denote the pairing of Milnor-Munkres-Novikov. **Theorem 2.** If \( A_n \in \mathcal{H}_n, V^{n+k} \in \mathcal{H}_{n+k} \), then \( (S^k \times A^n) + V^{n+k} \) and \( S^k \times A^n \) are diffeomorphic if and only if there exists \( a \in \pi_k(SO(n-1)) \) such that \( V^{n+k} = \tau_{n,k}(A^n \otimes a) \). In particular, if \( k \geq n - 3 \), then \( \tau_{n,k} = 0 \). Now let \( \Phi_{n,k}^{k+1} \) denote the subgroup of \( \mathcal{H}_n \) consisting of those homotopy \( n \)-spheres that embed in \( (n + k + 1) \)-space with trivial normal bundle. **Theorem 3.** If \( A^n, B^n \in \mathcal{H}_n \) (\( k > 1, n > 4 \)), then \( S^k \times A^n \) and \( S^k \times B^n \) are diffeomorphic if and only if \( A^n = \pm B^n \mod \Phi_{n,k}^{k+1} \). Thus the differentiable structures on \( S^k \times S^n \) can be classified in terms of \( \mathcal{H}_n/\Phi_{n,k}^{k+1} \) and the pairing \( \tau_{n,k} \). (Received February 26, 1968.)


Define \( P_N(\Delta_x) = \prod_{k=1}^{N} (1 - \Delta_x/a_k^2) \) where \( x > 0 \) and \( \Delta_x \) is the differentiation operator \( (D^2 + (2\gamma/x)D) \). It is assumed that \( 0 < \gamma, 0 < a_1 \leq a_2 \leq a_3 \ldots \) and \( \sum_{k=1}^{\infty} (a_k^{-2} < \infty \). Let the functions \( \mu(x) \) and \( G(x,y) \) be the same as defined by D. T. Haimo in her work on Hankel convolutions. We then define a generalized function space \( H'(I) \) where \( I \) is the open interval \((0,\infty)\) and establish the following result. **Theorem.** Let \( F(x) = \langle \mu'(y)f(y), G(x,y) \rangle \) where \( f(y) \in H'(I) \). Then, \( (\mu(x)P_N(\Delta_x)^2F(x),\phi(x)) \rightarrow \langle \mu'\phi, \phi \rangle \) as \( N \to \infty \) for all \( \phi \in D(I) \). \( D(I) \) is the space of smooth functions defined on \( I \) having compact support. (Received February 21, 1968.) (Author introduced by Professor S. A. Naimpally.)
On entropy and mixing transformations.

Let \((X, \mathcal{F}, \mu)\) be the unit interval with Lebesgue measure. A class of ergodic measure preserving transformations on \(X\) were constructed by the author and D. S. Ornstein in (1): On induced transformations, Tech. Rep., Univ. of New Mexico, 1967. As in (1), let \(\tau(T)\) be the transformation corresponding to the tower \(T = \{l_{j,k} : 1 \leq j \leq n, 1 \leq k \leq m(j)\}. \) Theorem 1. The entropy of \(\tau(T)\) is given by \(h(\tau(T)) = (n \log n) / \sum_{j=1}^{n} m(j).\) Theorem 2. For each \(\tau = \tau(T),\) the sets \(A\) such that the induced transformation \(\tau_A\) is mixing are dense in \(\mathcal{F}\) with respect to the metric \(d(A,B) = \mu(A \Delta B).\)

(Received February 28, 1968.)

Cardinal expansions of bivariate functions.

Let the \(K\)-dimensional linear space \(\Phi_K\) of functions defined for \(x \in I\) be the null space of the linear differential operator \(D_{x,t}\) of degree \(K\) with constant coefficients, \(\Phi_K^\ast\) is the conjugate space of bounded linear functionals defined on \(\Phi_K.\) Let \(\{ l_i(\phi_k) = \delta_{ik} \}\) be any basis for \(\Phi_K\) and \(\{ \psi_i(x) \}\) is the corresponding cardinal basis for \(\Phi_K^\ast.\) Analogously define \(\Psi_K, D_y, \Psi_K^\ast, \{ m_j \}\) and \(\{ \psi_j(y) \}.\) Let \(\mathcal{D}_x, \mathcal{D}_y, L_1\) and \(M_1\) be the linear operators which are the bivariate extensions of \(D_{x}, D_{y}, L_1\) and \(M_1\) respectively, e.g. \(L_1[f] |_{y = y^*} = l_1(f(x,y^*))\) for each fixed \(y^* \in I.\) Theorem. The (hyperbolic) partial differential equation \(\mathcal{D}_x, \mathcal{D}_y, L_1[f] = \rho(x,y)\) subject to the compatible (i.e. \(L_1 M_1[f] = M_1 L_1[f]\)) auxiliary conditions \(L_1[f] |_{y = y^*} = A_1(y) \in C^K[I']\) and \(M_1[f] = B_1(x) \in C^K[I']\) has the unique solution \(f(x,y) \in C^K[I' \times I']\) given by \(f(x,y) = \sum_{i=1}^{K} L_1[f] \phi_i(x) + \sum_{j=1}^{K'} M_1[f] \psi_j(y) - \sum_{i=1}^{K} \sum_{j=1}^{K'} L_1 M_1[f] \phi_i(x) \psi_j(y) + \int \int K(x,s) K'(y,t) \rho(s,t) ds dt.\) Using this and a similar result for multivariate functions, it has been possible to derive numerous new, practical schemes for multivariate interpolation and quadrature. These results are an extension of those reported earlier in Abstract 68T-68, these \(Noucres\) 15 (1968), 211. (Received February 26, 1968.)

Linear axiom-systems.

There are axioms and axiom-systems \(\mathcal{A}\) which have the property that if two sets \(S_1\) and \(S_2\) are solutions (models) of \(\mathcal{A},\) then also the set \(S = S_1 \cup S_2\) is a solution of \(\mathcal{A}.\) P2, the second axiom of Peano, \((3f)(\forall x)(x \in S \Rightarrow f x \in S),\) is such an axiom. A set \(S\) is here defined as a class \(C\) together with a function \(f\) defined in \(C\) and associated to \(C:\) \(S = (C,f).\) A fundamental solution \(S\) of \(\mathcal{A}\) is a solution representable in the form of a sequence of pairs \((a_0, f_0), (a_1, f_1) = (a_2, \ldots)\) or abbreviated in the form of the sequence: \(a_0, a_1, \ldots,\) where \(a_{i+1} = f a_i.\) We suppose that if \(\mathcal{A}\) has a solution \(S,\) then each set isomorphic to \(S\) (with respect to \(f\)) is also a solution of \(\mathcal{A}.\) We write the union \(T = \bigcup a S_a,\) \(a \in M,\) where each \(S_a\) is isomorphic to one and the same set \(S,\) in the form \(\beta S,\) where \(\beta\) is the cardinal of \(M, \beta \leq \gamma (\gamma \text{ the cardinal of the continuum}).\) We call an axiom or axiom-system \(\mathcal{A}\) linear if together with the solutions \(S_1\) and \(S_2\) of \(\mathcal{A},\) also \(\beta_1 S_1 \cup \beta_2 S_2\) is a solution of \(\mathcal{A}.\) Then P2 is a linear axiom. (Received February 28, 1968.)
The $c^k$-classification of certain operators in $L_p$:

For $z$ complex, let $T_z = M + zJ$, where $M : f(x) \rightarrow xf(x)$ and $J : f(x) \rightarrow \int_0^x f(t) dt$ act in $L_p(0,1)$, $1 < p < \infty$. Using the terminology of our previous papers (Trans. Amer. Math. Soc. 115 (1965), 194-224; J. Math. Mech. 17 (1967), 181-188), we prove Theorem 1. $T_z$ is of class $C^n$ if $|\text{Re}z| \leq n$ and only if $|\text{Re}z| < n + 1$ (the $C^n$-operational calculus for $T_z$, $|\text{Re}z| \leq n$, is explicitly given). Theorem 2. The $W^k_1$-manifolds of $T_z$ are dense for $\text{Re}z < 0$ and trivial for $\text{Re}z \geq 1$ and $k < [\text{Re}z]$. Corollary 3. $T_z$ and $T_w$ are not similar if $|\text{Re}z| > |\text{Re}w|$ (they are similar if $\text{Re}z = \text{Re}w$; the stronger result "$T_z$ is similar to $T_w$ if and only if $\text{Re}z = \text{Re}w$" is obtained in another paper by a different method). Corollary 4. $T_z$ is not spectral for $|\text{Re}z| < 1$ (it is spectral for $\text{Re}z = 0$). (Received January 29, 1968.)

Recursion algebras:

The functions on $N \{0,1,2,\ldots\}$ to $N$ form an algebra with respect to the operations $\circ f + g$, $f \circ g$, and $Nf$ defined by $(Nf)(x) = f^x(0)$. By the results of Robinson, the set of primitive recursive functions is the subalgebra generated by $S$ and $E$, where $(Sx) = x + 1$ and $(Ex) = x - [x^{1/2}]^2$. The theory of degrees of recursive unsolvability can be formulated in terms of the inclusion relations among algebras of this type. The study of general recursive functions is the theory of a certain type of algebraic extension of these recursion algebras. This paper also initiates the study of the structure and representation of recursion algebras. (Received March 4, 1968.)

Mountain climbing:

Let function mean continuous function mapping the closed unit interval onto itself. For a function $f$ and a point $y$ in $[0,1]$, let $f^C(y)$ denote the set of all $x$ in $[0,1]$ for which there exists $\delta \geq 0$ such that: $x - \delta$ and $x + \delta$ are in $[0,1]$; $f([x - \delta, x + \delta])$ is a singleton; and for every $\epsilon > 0$, $y$ is in the interior of $f(x - \delta - \epsilon, x + \delta + \epsilon)$, either $x - \delta = 0$ or $f([x - \delta - \epsilon, x])$ is not a singleton, and neither $x + \delta = 1$ or $f([x,x + \delta + \epsilon])$ is not a singleton. Let $f$ and $g$ be functions. Technical necessary and sufficient conditions for the existence of functions $h$ and $j$ such that $fj = gh$ (i.e. $f(j(x)) = g(h(x))$ for each $x$ in $[0,1]$) offer the following results. Proposition. If $f$ and $g$ have fixed points 0 and 1, if $g$ locally constant at $x$ implies that $f^C(g(x))$ is a finite set, and if $g$ locally constant at $y$ implies that $g^C(f(y))$ is a finite set, then there exist functions $h$ and $j$ with fixed points 0 and 1 such that $fj = gh$. Proposition. If each of $f$ and $g$ is either a nowhere constant function or has the property that the inverse of each point in $[0,1]$ is a set with a finite number of components, then there exist functions $h$ and $j$ such that $fj = gh$. Proposition. For every real number $\epsilon > 0$, there exist functions $h$ and $j$ such that $\|fj - gh\| < \epsilon$. (Received March 1, 1968.)

Reducing the order of some nonlinear second order differential equations:

Use properties of the relative derivative in the monograph of M. Petrovic (Edan Differentialniye Algoritam i Negobe Primeneye, Srpska Kraljevska Akademija. Posebna Izdanja, Book CXI Belgrad,
and assume throughout this discussion that \( u, y, \) and \( Y \) are reversible functions on \( \mathbb{R}^1 \), with \( u \in \text{class } C^1 \), and \( y \in C^2 \), and \( Y \in C^2 \) such that \( y^{(1)}(x) = u(y) \), and for \( f \in C^0 \), \( Y^{(1)}(x) = u(Y) \cdot c \cdot \exp\left[-\int X f(t) \, dt\right] \), \( c \) const. If \( \beta \) is nonlinear, then \( y \) satisfies \( y^{(2)}(x) + c \cdot y^{(1)}(x) = \beta(y) \) iff \( c \cdot u(y) + u(y) \cdot \dot{u}(y) = \beta(y) \). Also, \( Y \) satisfies \( Y^{(2)}(x) + f(x) \cdot Y^{(1)}(x) = F(x, y) \) iff \( c^2 \cdot u(t) \cdot \dot{u}(t) = \exp\left[\int_X f(t) \, dt\right] F(x, t) \). See I. Bandic (Sur l'invariant d'une équation différentielle non-linéaire du second ordre, Glasnik Mat.-Fiz. Astron. Društvo Mat. Fiz. Hrvatske Ser. II. 16 (1961), 161-165).

In the second theorem, \( F \) is of class \( C^0 \) in \( \mathbb{R}^2 \).

68T-436. P. F. Duvall, Jr., University of Georgia, Athens, Georgia. Weakly flat spheres.

If \( \Sigma^k \) is a topologically embedded \( k \)-sphere in \( S^n \), \( \Sigma^k \) is said to be weakly flat if \( S^n - \Sigma^k \) is homeomorphic to \( S^{n-k-1} \times \mathbb{R}^{k+1} \). If \( X \) is a closed subset of a manifold \( M \), \( M - X \) is \( 1 \)-lc at \( X \) if for each open set \( U \) containing \( X \) there is an open set \( V \), \( X \subset V \subset U \), such that each loop in \( V - X \) is null homotopic in \( U - X \). **Theorem 1.** If \( \Sigma^k \subset S^n \) is an embedded \( k \)-sphere, \( 2 \leq k \leq n - 3 \), \( \Sigma^k \) is weakly flat if and only if \( S^n - \Sigma^k \) is \( 1 \)-lc at \( \Sigma^k \). **Theorem 2.** Let \( S \), \( Y \), \( Z \), and \( A \) be compact, \( 1 \)-connected ANR's, where \( X = Y \cup Z \), \( Y \cap Z = A \), and \( (Z, A) \) is homeomorphic to \( (CA, A) \) where \( CA \) is the cone over \( A \). If \( h: X \to S^n \) is an embedding, \( n \geq 5 \), \( \dim X \leq n - 2 \), and \( S^n - h(X) \) is \( 1 \)-lc at \( h(X) \), then \( S^n - h(Y) \) is \( 1 \)-lc at \( h(Y) \). Several facts about weak flatness and cellularity follow from Theorems 1 and 2, together with McMillan's cellularity criterion [Ann. of Math. 79 (1964), 327-337]. For example **Theorem 3.** If \( \Sigma^k \) is the boundary of a cellular \((k + 1)\)-cell in \( S^n \), \( 2 \leq k \leq n - 3 \), then \( \Sigma^k \) is weakly flat. (Received February 29, 1968.)

68T-437. Harry Pollard, Purdue University, Lafayette, Indiana 47906, and D. G. Saari, Yale University, New Haven, Connecticut 06520. Singularities of the \( n \)-body problem. I.

In the problem of \( n \) point masses governed by the Newtonian law of attraction, necessary and sufficient conditions for singularities to be due to collisions are found. The main result is the following **Theorem.** A singularity as \( t \to 0^+ \) is due to collisions if and only if \( U \sim \text{At}^{-2/3} \), \( t \to 0^+ \) for some positive constant \( A \). \( U \) is the self-potential of the system. (Received March 1, 1968.)


A homeomorphism \( f \) of a compact metric (d) space \( X \) is said to be expansive at \( x \) in \( X \) if there is a positive constant \( C(x) \) such that if \( y \neq x \) there is an integer \( n \) with \( d(x^n, y^n) \geq C(x) \). If a positive number \( C \) exists with the above property for each \( x \) in \( X \), \( f \) is called expansive. In answer to a question of T. S. Wu (Topological dynamics, Benjamin, to appear) examples of homeomorphisms, which are expansive at every point but not expansive, are constructed on spaces including the Cantor set and a continuum. **Theorem 1.** If \( f \) is expansive at each point of \( X \) and \( p, q \) are points of \( X \), only countably many points are positively asymptotic to \( p \) and negatively asymptotic to \( q \). **Theorem 2.** If \( f \) is expansive at each point of \( X \) and \( X \) is self-dense, there exist a positively proximal pair and a negatively proximal pair of points. If \( f \) is transitive, proximal can be replaced by asymptotic. (Received February 29, 1968.)
Theorem 1. The theory of finite elementary abelian groups with an additional unary predicate is hereditarily undecidable. Corollary. The theory of finite abelian p-groups with an additional unary predicate is hereditarily undecidable. Theorem 2. The theory of periodic abelian groups with an additional unary predicate is hereditarily undecidable. Theorem 3. The theory of finite cyclic groups with an additional predicate is hereditarily undecidable. (Received March 4, 1968.)

G. I. GAUDRY, Mathematics Institute, University of Warwick, Coventry, England. Changes of signs of restrictions of Fourier-Stieltjes transforms.

Let G be a compact Abelian group with ordered dual X, \( r_n \) the nth Rademacher function.

Theorem 1. Suppose that X is countable and that \( \phi \) is a function defined on \( X_+ \) and vanishing off \( S = \{x_1, x_2, \ldots \} \). Suppose that \( \phi \) has the following property: for each \( t \in \mathbb{S} \subset (0,1) \) where \( m(\mathbb{S}) > 0 \), the function which is 0 off S and \( r_n(t)\phi(x_n) \) when \( X = x_n \) is the restriction to \( X_+ \) of a Fourier-Stieltjes transform. Then \( \phi \in L^2(X_+) \). Theorem 2. If X is ordered but not necessarily countable and \( \phi \) has the property that \( \omega \phi \) is the restriction to \( X_+ \) of a Fourier-Stieltjes transform for every \( \pm 1 \)-valued function \( \omega \) on \( X_+ \), then \( \phi \in L^2(X_+) \). (Received February 29, 1968.)

R. C. GILBERT, California State College, Fullerton, California 92631. Symmetric operators with singular spectral functions.

Let A be a simple closed symmetric operator with deficiency indices (1,1) in the Hilbert space H. Let \( A_0 \) be a selfadjoint extension of A with a pure point spectrum having no finite limit points. Let \( A^+ \) be a minimal selfadjoint extension of A. Let \( R(\lambda) \) be the generalised resolvent of A corresponding to \( A^+ \). Then, \( (R(\lambda)f, h) = (R_0(\lambda)f, h)\frac{Q_1^2(\lambda) + 1}{Q_1^2(\lambda) + 1} - Q_1(\lambda)(f, g(\lambda))(g(\lambda), h)\frac{Q_1^2(\lambda) + 1}{Q_1^2(\lambda) + 1} + (f, g(\lambda))(g(\lambda), h)\frac{Q_1^2(\lambda) + 1}{Q_1^2(\lambda) + 1} \). Here \( \theta(\lambda), Q_1(\lambda), g(\lambda) \) are certain analytic functions in the upper halfplane. \( Q_1(\lambda) \) and \( g(\lambda) \) depend only on \( A_0 \). Substituting this expression for \( (R(\lambda)f, h) \) into the Stieltjes inversion formula, we obtain that \( A^+ \) is unitarily equivalent to the multiplication operator in \( L^2_\rho \), where \( \rho(\sigma) = (1/\pi)\lim_{\tau \to 0} \int_0^{\sigma} \Phi(\mu + i\tau) d\mu, \Phi(\lambda) = [\theta(\lambda)Q_1(\lambda) - 1] + [\theta(\lambda) + Q_1(\lambda)]^{-1} \). From this representation certain facts about the spectrum and spectral multiplicity of \( A^+ \) follow. In the case that A is a singular Sturm-Liouville operator one can assume that \( A_0 \) is a selfadjoint extension with a singular spectral function. (Received March 4, 1968.)

D. W. COHEN, University of New Hampshire, Durham, New Hampshire. Extension of locally compact local groups to global groups.

In a locally compact local group there are arbitrarily small neighborhoods of the identity which split into the direct product of a compact group and a local Lie group. This extension of a result for global groups by V. M. Gluskov [Amer. Math. Soc. Trans. (2) 15 (1960), 55-93] proves that every locally compact local group is locally isomorphic to a global group. (Received March 5, 1968.) (Author introduced by Professor A. R. Jacoby.)
Infinite exponential interpolation. Preliminary report.

Exponential approximation in the form \( f(x) \approx \sum_{m=1}^{n} a_m x^m \), by interpolation at \( x = 0, 1, 2, \ldots, n - 1 \), can be shown to result in \( |a_m| \to \infty \) as \( n \to \infty \) for the case \( f(x) = x \). A permanent biorthonormal system is provided by \( U_k(x) = \left[ \frac{\alpha^k}{(i-1)!} \right]^n \), where \( \alpha \) is taken with respect to \( n \), and \( L_k(f) = (ES)^{i-1} f(0) \), where \( S \) is the diagonal difference operation (Math. of Comp. 18 (1964), 113-118). Interpolation in this system yields \( f \approx \sum_{k=1}^{n} L_k(f) U_k(x) \) where now the coefficients remain finite as \( n \to \infty \), giving a formal series converging to \( f \) at \( x = 0, 1, 2, \ldots \). For \( f(x) = x \), this gives \( x \approx \sum_{k=1}^{n} (-1)^k (k-2)! U_k(x) \). A numerical check of the approximating ability of this expansion between interpolating points was made for selected values of \( x \in [0, 1] \). Double precision (16D) calculation suffices to calculate 40 terms giving as a typical result \( x = 1/2 \approx .4993 \). (Received March 4, 1968.)

Correction to "Completeness in valued spaces and algebras".

The assertion, in Quart. J. Math. Oxford Ser. 15 (1964), 346, to the effect that an incomplete valued module over a ring with projective ideals always admits an immediate extension is corrected as follows: The result holds as stated when the value set is subjected to an order theoretic hypothesis satisfied, for example, by linearly ordered sets (in conformity with Abstract 60T-21, these Notices 7 (1960), 999) and by partially ordered sets with the ascending chain condition. In the general case, projectivity of the ideals should be replaced by injectivity of the kernels of the projective system maps defining the valuation, and of their intersections. This is deduced from the possibility of mapping any element in a valued extension of such a module value homomorphically over it on an element immediate over it. Details as well as some further consequences will be furnished in a forthcoming Queen's Paper. (Received March 6, 1968.) (Author introduced by Mr. S. M. Fakhruddin.)

More about weak and strong cover compactness.

The statement that a space \( S \) is compact means that every infinite subset of \( S \) has a limit point. For definitions of weak cover compactness (w.c.c.) and strong cover compactness (s.c.c.), see Abstract 650-24, these Notices 14 (1967), 922. If a first countable (or locally compact) regular \( T_1 \) space is s.c.c. (w.c.c.), it is collectionwise normal with respect to any discrete collection of degenerate point sets. (A locally compact, s.c.c. (w.c.c.) regular \( T_1 \) space need not be normal; Abstract 68T-326, these Notices 15 (1968), 396.) If a first countable (or locally compact) regular \( T_1 \) space \( S \) is s.c.c. (w.c.c.), each open cover \( G \) of \( S \) has an open refinement \( H \) such that \( H' = [C(h)h] \in H \) is a s.c.c. (w.c.c.) closed refinement of \( G \). If a locally compact, regular \( T_1 \) space \( S \) is s.c.c. (w.c.c.), if \( B \) is any basis for \( S \), and if \( G \) is an open cover of \( S \), then some subcollection of \( B \) is a s.c.c. (w.c.c.) refinement of \( G \). (There exists a metric space \( S \) and a basis \( B \) for \( S \) such that no subcollection of \( B \) covering \( S \) is s.c.c.) If \( \prod_{\alpha \in A} S_{\alpha} \) is s.c.c. (w.c.c.), then each \( S_{\alpha} \) is s.c.c. (w.c.c.). However, the Cartesian product of two s.c.c. (w.c.c.) spaces need not be s.c.c. (w.c.c.). (Received March 6, 1968.)
Let $X_0, X_1, X_2, \ldots, X_n$ denote the random variables of a Markov chain with state space the nonnegative integers and transition probabilities given by $P(X_{n+1} = j | X_n = i) = P_{ij}$, $P(X_{n+1} = j | X_n = i) = q_j$ where $p_j = (1/2)(1 + \lambda/(i + \lambda))$ and $q_j = (1/2)(1 - \lambda/(i + \lambda))$, $p_0 = 1$, $X_0 = 0$ and $-1 < \lambda \leq 1$. Let $\Phi(\lambda, x) = \int_0^\lambda \phi(\lambda, t) \, dt$ where $\phi(\lambda, t) = 2t^2 e^{-t^2/2}/\Gamma(\lambda + 1/2)$. Let $P_{2j} = P(X_{2n} = 2j | X_0 = 0)$. Using the Karlin-McGregor integral representation for these random walks we have been able to obtain the following results. Theorem 1. \[ \sqrt{n} \sum_{j} P_{2j} \approx \phi(\lambda, t_j) \] where $t_j = 2j/\sqrt{n}$ and $\lim_{n \to \infty} t_j/\sqrt{n} = 0$. Theorem 2. $P(X_{2n}/\sqrt{2n} \approx t_j \mid n) = O(1 - \Phi(\lambda, t_j))$, $t_j \to \infty$, $j/\sqrt{n} \to 0$. Theorem 3. $P(\lim sup_{n \to \infty} (X_n/\sqrt{2n} \log \log n) \leq 1) = 1$. The case $\lambda = 1/2$ has been discussed by the author (cf. Ann. Math., Statist. (4) 37 (1966)). (Received March 6, 1968.)
generating functions for certain classes of generalized hypergeometric polynomials. Some of the formulas derived here extend the results presented by the author at the annual meeting of the Bharata Ganita Parishad at Lucknow (India) on April 16, 1967, while their specialized and limiting cases appear in the earlier works of N. Abdul-Halim and W. A. Al-Salam [Duke Math. J. 30 (1963), 51-62], F. Brafman [Pacific J. Math. 7 (1957), 1319-1323], T. W. Chaundy [Quart. J. Math. (Oxford), 14 (1943), 55-78], J. Meixner [Deutsch Math. 6 (1943), 341-349], L. Weisner [Pacific J. Math. 5 (1955), 1033-1039], and others. (Received March 6, 1968.)

68T-450. RAYMOND BALBES, University of Missouri, St. Louis, Missouri 63121.

On (J, M, m)-extensions of order sums of distributive lattices.

The order sum of a family \( \{L_a\}_{a \in S} \) of distributive lattices, indexed by a poset, was defined in Abstract 66T-488, these Notices 13 (1966), 738-730. In the first section of this paper a characterization of order sum is given which is analogous to the characterization of a free distributive lattice as one generated by an independent set. We then consider the collection \( Q \) of order sums obtained by taking different partial orderings on \( S \). A natural partial ordering is defined on \( Q \) and its maximal and minimal elements are characterized. Let \( J \) and \( M \) be collections of nonempty subsets of a distributive lattice \( L \), and \( m \) a cardinal. We define a \( (J, M, m) \)-extension \((\Psi, E)\) of \( L \), where \( E \) is a \( m \)-complete distributive lattice and \( \Psi: L \rightarrow E \) is a \( (J, M) \)-monomorphism. In the last section we define a \( m \)-order sum of a family of distributive lattices \( \{L_a\}_{a \in S} \). The main result here is that the \( m \)-order sum exists if the order sum \( L \) of \( \{L_a\}_{a \in S} \) has a \( (J, M, m) \)-extension, where \( J \) and \( M \) are certain collections of subsets of \( L \). These results are analogous to R. Sikorski's work in Boolean algebras. (Received March 7, 1968.)


A class of related Dirichlet and initial value problems.

Relationships are exhibited between solutions of the initial value problem \((P_1)\) \( u_t(x,t) = P(x,D)u(x,t) \), \( t > 0, u(x,0) = \phi(x) \), and the Dirichlet type problem \((P_2)\) \( v_{yy}(x,y) + P(x,D)v(x,y) = 0, \) \( y > 0, v(x,0) = \phi(x) \), where \( P(x,D) \) is a finite order linear partial differential operator. We obtain the following results: Theorem 1. Let \( u(x,t) \) be a solution of \((P_1)\) with \( |u(x,t)| \leq M \) or \( |u(x,t) - \phi(x)| \leq A t^a, A > 0, 0 < a < 1/2 \). Then there exists a solution of \((P_2)\) given by \( v(x,y) = \int_0^y e^{-y' \xi} \frac{1}{4t} u(x,1/4t) d \xi \). Theorem 2. Let \( \varphi(x,y) \) be a solution of \((P_2)\) such that \( s^{-1/2} v(x,s^{1/2}) \) and \( s^{-1/2} v(x,s^{1/2}) \) are invertible Laplace transforms in the classical sense. Then \( u(x,t) = \left( \frac{\sqrt{\pi} x}{2t} \right) s^{-1} \frac{1}{4t} s^{-1/2} v(x,s^{1/2}) \) satisfies the equation in \((P_1)\). If \((P_2)\) is well posed \( \lim_{t \to 0} u(x,t) = \phi(x) \). These results are applied to the heat equation and half space Dirichlet problems. See Bull. Amer. Math. Soc. 74 (1968), 375-378, for earlier results of the authors' that related initial-boundary value problems. (Received March 11, 1968.)


Let \( \Gamma \) be the Royden boundary of a Riemannian manifold \( R \) and \( \Delta \) its harmonic part. Denote
by \( \bar{G} \) a subregion of \( R \), by \( G \) its closure in \( R \cup \Gamma \), and by \( \partial G \) its relative boundary in \( R \). **Theorem.** Suppose that \( u \in HD(G) \). If \( \lim_{x \to a} u(x) \leq m \), for every \( a \in (\bar{G} \cap \Delta) \cup \partial G \), then \( u \leq m \). This seems to give the unique orthogonal decomposition of any Dirichlet finite function on \( R \) into an HD-function and one which vanishes on \( \Delta \), which is the counterpart of M. Nakai's result for Riemann surfaces (Nagoya Math. J. 17 (1960), 181-218). (Received March 8, 1968.)

68T-453. S. A. GREIBACH, Harvard University, Cambridge 38, Massachusetts. **Checking automata and one-way stack languages.** Preliminary report.

A checking automaton is a one-way nonerasing stack automaton which, once it has entered its stack, never writes on it again. **Theorem.** If \( L \subseteq a^* \) is an infinite cal (checking automaton language), then \( L \) contains an infinite regular set. **Corollary.** \( L = \{a^n \mid n \geq 1\} \) is a one-way nonerasing stack language that is not a cal. **Theorem.** The cal form a full AFL closed under substitution.

**Theorem.** Let \( \mathcal{L} \) be the one-way stack languages and let \( \mathcal{L}_1 \) and \( \mathcal{L}_2 \) be sub AFL of \( \mathcal{L} \). Then \( \mathcal{L} \) is closed under substitutions by members of \( \mathcal{L}_1 \) into members of \( \mathcal{L}_2 \) if and only if either \( \mathcal{L}_1 \) is a family of cal or \( \mathcal{L}_2 \) is a family of context-free languages. **Corollary.** The one-way stack languages are not closed under substitution. **Theorem.** The one-way stack languages properly contain both the one-way nondeterministic quasirealtime stack languages and the one-way nonerasing stack languages. (Received March 11, 1968.)

68T-454. W. O. J. MOSER, McGill University, Montreal, Canada, and MORTON ABRAMSON, York University, Toronto, Canada. **Enumeration of certain classes of restricted combinations.**

By elementary methods, the number of \( p \)-combinations \( 1 < x_1 < x_2 < ... < x_p \leq n \) satisfying the conditions \( 1 \leq n + x_1 + x_2 + ... + x_p \leq l \), \( k \leq x_1 \leq x_2 \leq ... \leq x_p \leq k' \), \( j = 2, ..., p \), is found to be \( \mathcal{O}_{k,k'}(n,p) = \sum_{i=0}^{\lfloor p \rfloor} (-1)^i \binom{\lfloor k - k' \rfloor}{i} \binom{n - (p - 1)(k - i)}{p - 1} \binom{n - (p - 1)(k - i) + 1}{p - 1} \). Recurrence relations are given. Special cases are noted, e.g., Fibonacci and Lucas numbers and their generalizations. Another class of \( p \)-combinations, satisfying the conditions \( x_1 = 1 + k_1 (\text{mod } m) \), \( x_j = x_{j-1} + 1 + k_j (\text{mod } m) \), \( j = 2, ..., p \), are enumerated and their number found to be \( \left\lfloor \frac{n + (m - 1)p - (k_1 + ... + k_p)}{m} \right\rfloor \). When the condition \( x_1 = 1 + k_1 (\text{mod } m) \) is deleted, the number is \( \left\lfloor \frac{n + (u + p)}{(u + p)} \right\rfloor \), where \( u = \left\lfloor \frac{n - p + (k_2 + ... + k_p)}{m} \right\rfloor \) and \( v = n - p - (k_2 + ... + k_p) - mu \). These are generalizations of Terquem's problem. (Received March 8, 1968.)

68T-455. N. A. TSERPES, Wayne State University, Detroit, Michigan 48202. **A characterization of idempotent probabilities on left groups.**

We consider a measure space \( (S,m) \) where \( S \) is a locally compact Hausdorff semigroup and \( m \) a probability on \( S \) satisfying \( \int_S f(x)m(dx) = \int_S \int_S f(xy)m(dx)m(dy) \) for every real valued function \( f \) on \( S \) with compact support. Let \( F \) be the support of \( m \). **Theorem 1.** If \( S \) is a left group (\( S \) has an idempotent element and \( Sx = S \) for all \( x \in S \)), then \( F \) is a closed left subgroup of \( S \) and \( m \) on \( F \) is \( r^* \)-invariant and right invariant. If in addition \( A^{-1}B \) is compact for every pair of compact \( A,B \subset S \), then \( F \) is also compact. **Corollary.** Let \( S \) be bicancelative and \( DA^{-1} \) be compact for all compact \( A,B \subset S \); then \( F \) is a compact subgroup of \( S \) and \( m \) is normed Haar measure on \( F \). (One shows that for some \( q \in F, qF \)
is compact, and since it is bicancelative it is a group, so that qPqx = qP or Fqx = F for all x ∈ F, and so F is a bicancelative left group and hence also a group.) The proof of Theorem 1 uses the result (J. G. Wendel) that the conclusion of Corollary 1 is valid for S = a compact group, and thus also provides a new proof of the well-known Corollary 2. Every idempotent probability on a locally compact group is the Haar measure on a compact subgroup and conversely. (This follows from Theorem 1 by observing that a left group with a two-sided identity is a group.) (Received March 7, 1968.)

68T-456. WITHDRAWN.


In Abstract 68T-169, these Notices 15 (1968), 349, Gandhi has announced: Theorem 2. The law [x, y]^2 = [x, y, x] in a group implies the law [x, y] = 1. Theorem 3. The law [x, y]^2 = [x_1, x_2, ..., x_n] in a finite group or in a soluble group implies the law [x, y] = 1. Theorem 2 is false since the dihedral group of order 8 satisfies the law [x, y]^2 = [x, y, x]. The conditions in Theorem 3 are superfluous as the law [x, y]^2 = [x_1, ..., x_n] trivially implies the law [x, y] = 1 (replacing z, x_1, ..., x_n by 1). (Received March 11, 1968.)


Let A be a commutative ring with unity and S be a set. Then each point b ∈ βS (S having its discrete topology) defines an S-ary operation ˘b : (spec A)^S → spec A. For τ ∈ (spec A)^S, the function ˘τ : βS → spec A defined by ˘τ(b) = ˘b(τ) is continuous and restricts to τ; it follows that b → ˘b is continuous. Let K : ˘S^* → K be the equational theory of compact, Hausdorff spaces (see F. E. J. Linton, Some aspects of equational categories, Proc. Conference on Categorical Algebra - La Jolla, Springer-Verlag, New York, 1966, pp. 84-94). Theorem 1. Spec A is a K-algebra. Specifically, ˘K(S) = (spec A)^S, ˘K(τ) = ˘τ ∘ f defines the necessary functor ˘K ∈ ˘S^K, and the K-operations ˘K(S → 1) on spec A are precisely the ˘b (b ∈ βS). Theorem 2. Let A = C(X), where X is a completely regular, Hausdorff space. The prime z-ideals are invariant under K-operations; the minimal primes under countable K-operations. (Received March 13, 1968.)

68T-459. K. F. BARTH and W. J. SCHNEIDER, Syracuse University, Syracuse, New York 13210. On a problem of Collingwood concerning meromorphic functions with no asymptotic values.

In Hayman's book Research problems in function theory, p. 34, Problem 5.19, the following problem (attributed to Collingwood) is posed: Must every function f(z), meromorphic in D = \{ |z| < 1 \}, with no asymptotic values take on every value (finite and infinite) in every neighborhood of every point of C = \{ |z| = 1 \}? (It is known (Collingwood and Cartwright) that globally f(z) must take on infinitely often every value (finite and infinite) and that locally f(z) can omit at most two values.) This question is answered in the negative by the following: Theorem. There exists a function f(z), meromorphic in D, whose zeros and poles cluster only to the point z = 1; and, in addition,
on a sequence of closed Jordan curves \( J \subset \text{Int} \ J_n \subset \subset \) with \( J_n \) tending to \( C \), it has the properties (i) \( \lim_{k \to \infty} \max_{z \in J_{2k}} \|f(z)\| = 0 \); (ii) \( \lim_{k \to \infty} \min_{z \in J_{2k+1}} \|f(z)\| = \infty \) (i.e., except at one point \( f(z) \) can omit the maximum number of values). The proof consists of writing \( f \) as the quotient of two functions \( g \) and \( h \) where \( g \) and \( h \) are constructed by a method quite similar to one used in a recent paper of the authors [On a problem of Bagemihl and Erdős concerning the zeros of an annular function, J. Reine Angew. Math. (to appear)]. (Received March 12, 1968.)

68T-460. A. H. SIDDIQI, Aligarh Muslim University, Aligarh, (UP), India. Fourier series with positive coefficients. I.

A continuous function \( f(x) \) will be said to belong to the class \( \Lambda^*(X(x)) \) if \( f(x + h) + f(x - h) - 2f(x) = O(|X(h)|) \), \( h \to 0 \) uniformly in \( x \), where \( X(x) \) is a function satisfying the condition \( \int_0^1 (|X(x)|/x)dx < \infty \). For \( X(x) = x \) the class \( \Lambda^*(X(x)) \) becomes \( \Lambda^* \) class. The following theorem has been proved which generalizes a theorem of Boas (J. Math. Anal. Appl. 17 (1967), 463-483); Theorem. Let \( \lambda(x) \) be a positive monotonic increasing function such that for \( \delta > 0 \), \( \lambda(x)/x \delta \lnot 0 \), \( \sum_{k \in \mathbb{N}} \lambda(k)/k^{1+\delta} = O(\lambda(n)/n\delta) \), and \( \sum_{k \in \mathbb{N}} \lambda(k)/k = O(\lambda(n)) \), \( n \to \infty \). If \( a_n \geq 0 \) and \( f = \sum a_n \cos nx \), then \( f \in \Lambda^*(X(x)) \) if and only if \( \sum_{k \in \mathbb{N}} \lambda(k)/k = O(\lambda(n)/n\delta) \), \( n \to \infty \), where \( \lambda(n) \) and \( n^2 |X(1/n)| \) satisfy the same condition, as \( \lambda(x) \) above for \( \delta = 1 \) and 2 respectively. (Received March 14, 1968.) (Author introduced by Dr. S. M. Mazhar.)

68T-461. D. R. CHILDS, General Dynamics, 97 East Howard Street, Quincy, Massachusetts 02169. Rigorous solution of the monoenergetic transport equation, I: Slab geometry.

A rigorous solution of the monoenergetic transport equation with constant source has been obtained. The macroscopic absorption, scattering and total cross sections, are assumed to be constant in the region of interest. Solutions of the form \( e^{axf(\mu)} \) and \( e^{-axf(\mu)} \) are obtained, thus transforming the integro-differential equation into a pure integral equation. The eigenvalue equation for the parameter \( a \) is exhibited explicitly for all orders of the scattering cross section in the form of a determinantal equation and the \( f(\mu) \) is shown to involve Legendre polynomials of the first and second kind. Previous work yields an eigenvalue equation for the isotropic scattering cross section. This paper extends the work for all orders of the scattering cross section and gives a simple scheme for determining the element in the determinantal equation. The determinant which becomes larger for each order of the scattering coefficient considered can be reduced and factored so that the eigenvalue equation involves the ratio of two series, the coefficients of the series being determined by a recursion relationship. (Received March 13, 1968.)


These results, the third chapter of a draft-monograph entitled Elementary Unique Factorization: Novae Disquisitiones Arithmeticae, extend the author's primitive-recursive-function-theory version of elementary unique prime factorization (see, e.g., Amer. Math. Monthly 74 (1967), 1100-1102) to intermediate algebra, viz, to (1) the elementary theory of ideals in the senses of Dedekind, Hilbert, and Artin and (2) to the theory of rings with special emphasis on regular local rings (e.g., the ring of \( p \)-adic integers) and free ideal rings (e.g., group algebras of free groups over a field),
so as to supplement the Euler-Kronecker infinite products for various zeta-functions, all analytic versions of unique prime factorization. Developed are new analogues of classical functions (e.g., a "mosaic analogue" of Möbius' function for Gaussian integers and its relation to the distribution of Gaussian primes). Further, use is made of the author's (1) ordinary modified Möbius' function and modified totient (these Notices 11 (1964), 680) for reestimations with sieve techniques and (2) modified Liouville and modified Möbius functions in reexaminations of Siegel's theorem on the class numbers of binary quadratic forms. (Received March 19, 1968.)

68T-463. C. L. BELNA, Michigan State University, East Lansing, Michigan 48823. A result on the n-segment property.

Let $f$ be a continuous complex-valued function defined on the open unit disk $D$, and let $C$ denote the unit circle. Let $S_1, \ldots, S_n$ be $n$ distinct rectilinear segments in $D$ terminating at $z = 1$, and denote the image of $S_k$ under the rotation $W(z) = e^{ia}z$ by $S_k(a)$. We say that $f$ has the $n$-segment property at $e^{ia} \in C$ relative to the $n$ segments $S_1, \ldots, S_n$ if \[ \bigcap_{k=1}^{n} C_{S_k(a)}(t, e^{ia}) = \emptyset, \] where $C_{S_k(a)}(t, e^{ia})$ denotes the cluster set of $f$ at $e^{ia}$ on $S_k(a)$. Theorem. Let $f$ be an arbitrary complex-valued function defined on $D$. Then the set of points $e^{ia} \in C$ at which $f$ possesses the $n$-segment property relative to a fixed set of $n$ segments is of measure zero. (Received March 18, 1968.) (Author introduced by Professor P. A. Lappan.)


"Sentence" means a sentence of the language $L(\omega_1, \omega)$ (i.e., a sentence formed by countable conjunction and disjunction and finite quantification). Let $\Delta$ be the set of sentences defined in Abstract 68T-24, these Notices 15 (1968), 196, and let $F$ be any sentence. The following is an analogue of Theorem 2 of loc. cit. Theorem 1. The class of direct factors of countable models of $F$ is identical with the class of countable models of a subset of $\Delta$. Similar results hold in case of other algebraic situations, e.g., those treated by H. I. Keisler [J. Symbolic Logic 30 (1965), 339-349]. Combinations of preservation statements, such as the following, are proved. Theorem 2. The following conditions on sentences $F_1, F_2, H$ are equivalent. (i) There is a model $A$ of $H$ such that $A$ is an homomorphic image of a (countable) model of $F_1$ and $A$ is a substructure of a (countable) model of $F_2$. (ii) If $F_1 \models G_1, F_2 \models G_2, G_1$ is positive and $G_2$ is universal, then $\{G_1, G_2, H\}$ is consistent. (Received March 18, 1968.) (Author introduced by Professor Robert Vaught.)

68T-465. L. R. ANDERSON, Case Western Reserve University, Cleveland, Ohio 44106. Integral manifolds of a class of 3rd order autonomous differential equations.

Consider the differential equation $y''' = ay'' + by' + g(y)$ where $a < 0$, $b > 0$, $g \in C^1$, $yg(y) \geq 0$, $g'(y) > 0$ for $y \neq 0$ and $|g(y)| \rightarrow \infty$ as $|y| \rightarrow \infty$, e.g. $g(y) = cy^3$, $c > 0$. There exist subsets $M_2$ and $M_1$ of $R^3$ homeomorphic to $R^2$ and $R$ respectively such that: (1) If $(y(0), y'(0), y''(0)) \in M_2$ ($\in M_1$), then $y(t)$ is continuous on $[0, + \infty)$ (on $(-\infty, 0]$) and $y^{(k)}(t) \rightarrow 0$ as $t \rightarrow + \infty$ (as $t \rightarrow - \infty$), $k = 0,1,2,3$. (2) If $(y(0), y'(0), y''(0)) \in R^3 - M_2$ ($\in R^3 - M_1$) either $y(t)$ is not continuous on $[0, + \infty)$ (on $(-\infty, 0]$) or $y(t)$ is unbounded as $t \rightarrow + \infty$ (as $t \rightarrow - \infty$). (3) $M_1 \cap M_2 = \{(0,0,0)\}$. (Received March 18, 1968.) (Author introduced by Professor A. C. Lazer.)
An extension of the Bernside problem is to determine those presentations of semigroups which, by virtue of the nature of their defining relations, permit a decision algorithm, and in particular, those that are finite. One such result is that of Green and Rees [On semigroups in which $x^r = x$, Proc. Cambridge Philos. Soc. 48 (1952), 35-40]. The present report deals with another special class of semigroups, extending the results announced earlier [Abstract 68T-237, these Notices 15 (1968), 370]. Let $S(n,r)$ be the semigroup generated by two elements $a$ and $b$ subject only to the relations (i) $aba = b^n$, (ii) $bab = a^r$. By symmetry, assume $n \neq r$. In a paper to appear in the Monthly, it was shown that $S(1,r)$ is finite with order $5r + 3$. \textbf{Theorem.} $S(2,r)$ is finite for $r = 2, 3, 4$, but not for $r = 5$. Specifically, $\text{ord}(S(2,2)) = 31$, $\text{ord}(S(2,3)) = 74$ and $\text{ord}(S(2,4)) \geq 2130$. All the semigroups $S(n,r)$, with $n \geq 3$, are infinite. If the commuting relation $ab = ba$ is adjoined, then the semigroup $S(n,r)$ becomes finite in every case except $n = 2, r = 5$. (Received March 20, 1968.)

\textbf{Flat regular quotient rings.}

Let $R$ be an associative ring with $1$ and singular right ideal zero. Let $S$ be a (von Neumann) regular ring, containing $R$ and such that $R_S$ is large in $S_R$. Let $M$ be a right $R$-module: (1) $M$ is essentially finitely generated (EFG) if $M$ contains a finitely generated large submodule, (2) $M$ is essentially finitely related (EFR) if there exists an exact sequence of right $R$-modules

$$0 \longrightarrow K \longrightarrow F \longrightarrow M \longrightarrow 0$$

with $F$ finitely generated free and $K$ EFG. \textbf{Theorem 1.} For arbitrary finitely generated right ideals $I$ and $J$ of $R$, the following are equivalent: (a) $S$ is flat as a left $R$-module, (b) $I$ is EFR, (c) $(I : a) = \{r \in R \mid ar \in I\}$ is EFG for every $a \in R$, (d) $(0 : a) = \{r \in R \mid ar = 0\}$ is EFG for every $a \in R$ and $I \cap J$ is EFG. \textbf{Theorem 2.} For any finitely generated free (right) $R$-module $F$ and finitely generated submodules $M$ and $N$ of $F$, the following are equivalent: (a) $S$ is flat as a left $R$-module, (b) $M$ is EFR, (c) $(M : x) = \{r \in R \mid xr \in M\}$ is EFG for each $x \in F$, (d) $(0 : x) = \{r \in R \mid xr = 0\}$ is EFG for each $x \in F$ and $M \cap N$ is EFG. (Received March 20, 1968.)

\textbf{Alexander's cell theorem without direct limits.}

J. W. Alexander's proof (that if $P$ is a $k$-cell in euclidean $n$-space $E^n$, then $\tilde{H}_q(E^n - P) \cong \tilde{H}_q(S^{n-1})$) is the usual proof given of this fact. It is inductive and explicitly uses a limiting argument. This is another proof. From the Mayer-Vietoris sequence if $E^{n+1} \supset E^n$, we have $\tilde{H}_q(E^{n+1} - P) \cong \tilde{H}_q(E^n - P)$. Whence $\tilde{H}_q(E^{n+r} - P) \cong \tilde{H}_q(E^n - P)$. A lemma of Klee ensures that for $r \geq k$, $P$ is equivalent to a $k$-simplex in $E^{n+r}$. So for $r \geq k$, $\tilde{H}_q(E^{n+r} - P) \cong \tilde{H}_q(S^{n+r-1}) \cong \tilde{H}_q(E^n - P)$. Since $\tilde{H}_q(S^{n+r-1})$ is zero except for $q = n + r - 1$, we have $\tilde{H}_q(E^n - P) \cong \tilde{H}_q(S^{n-1})$. (Received March 21, 1968.)
Some consistency questions in topology.

Let ZFC be set theory ZF along with the axiom of choice. **Theorem.** If ZF is consistent, so is ZFC & (no separable metric continuum is the union of $\aleph_1$ mutually disjoint closed sets).

**Theorem.** If ZF is consistent, so is ZFC & (no separable metric space of positive dimension is the union of $\aleph_1$ closed sets of dimension zero). Let $N^*$ be the Stone-Čech compactification of the integers without its isolated points; one calls a point of $N^*$ a $p$-point if every countable set of open sets about it has an open set in its intersection. **Theorem.** If ZF is consistent, so is $ZFC \cdot 2^{\aleph_0} = \aleph_2$ & (there are $p$-points in $N^*$). These propositions can be given the form of iterated Cohen extensions. (Received March 21, 1968.)

On a class of second-order differential systems with bounded coupling terms.

Consider the second order system (*) $x''_k + g_k(x_k) = h_k(t, x_1, \ldots, x_n, x'_1, \ldots, x'_n)$, $k = 1, \ldots, n$. Assume the functions $h_k$ are continuous and bounded and $h_k(t + 2\pi, x_1, \ldots, x_n, y_1, \ldots, y_n) = h_k(t, x_1, \ldots, x_n, y_1, \ldots, y_n)$, $k = 1, \ldots, n$. Assume the functions $g_k$ have continuous derivatives and there exist integers $m_k$, $k = 1, \ldots, n$, and $\delta > 0$ such that $(m_k + \delta)^2 \leq g_k'(x) \leq (m_k + 1 - \delta)^2$, $k = 1, \ldots, n$. The $g_k$ are not necessarily assumed to be odd. **Theorem.** There exists at least one $2\pi$-periodic solution of the system (*). If in the one-dimensional case $x'' + g(x) = h(t, x, x')$, $h$ depends only on $t$, the $2\pi$-periodic solution is unique. (see Abstract 68T-329, these Notices 15 (1968), 397). This partially generalizes a result of W. S. Loud (Abstract 636-119, these Notices 13 (1967), 611). (Received March 21, 1968.)
Let \( T_n \) be the \( n \)th degree Chebyshev polynomial normalized to the interval \([a,b]\) and let \( R_{n,m} \) be the set of rational functions with no poles in \([a,b]\) expressible as the ratio of a polynomial of degree \( n \) to one of degree \( m \). \( \textbf{Theorem.} \) Let \( P_{N}, P_{N}', P_{N''} \) denote polynomials of best approximation to \( f \in C[a,b] \) of degrees \( N, N', \) and \( N'' \), respectively, from \( R_{N,0}, R_{N-1,0}, \) and \( R_{N'-1,0} \) (provided \( N' > 0 \)), respectively. Suppose \( P_{N}' \neq f \) and that \( P_{N}' \), Chebyshev alternates with \( f \) \( N'+D'+2 \) times on \([a,b]\). Then \( D' = \max \{ d : P_{N}' \text{ is the best approximation from } R_{N',d} \} \) and the test can be iterated replacing \( N' \) by \( N \) and finding a new value of \( D' \).

\( \text{Remark.} \) By reversing the orders of the subscripts on \( R \), one obtains a similar theorem for reciprocals of polynomials. Several theorems in the paper of B. Boehm (Numer. Math. 6 (1966), 235-242) follow from this theorem as special cases. (Received March 21, 1968.)

Let \( X \) be a compact plane set, \( 0 \in \text{boundary } X \), \( R(X) \) the uniform closure on \( X \) of rational functions with poles off \( X \), \( \Lambda_{n} = \{ z : 1/2^{n+1} \leq |z| \leq 1/2^{n} \} \), and \( \gamma(U) \) the analytic capacity of any bounded plane set \( U \). A bounded point derivation for \( R(X) \) at \( 0 \) is a continuous linear functional \( D : R(X) \rightarrow \mathbb{C} \) such that \( D(fg) = f(0)Dg + g(0)Df \) for all \( f, g \in R(X) \). \( \textbf{Theorem.} \) There is a nonzero bounded point derivation for \( R(X) \) at \( 0 \) iff \( \sum_{n=0}^{\infty} 4^{n} \gamma(\Lambda_{n} \setminus X) < \infty \). The proof uses a result of Melnikov [Mat. Sb. 71 (113) (1966), 503-515] and a modified argument due to Curtis [Peak points for algebras of analytic functions (to appear)]. The result generalizes to higher order bounded point derivations. An example is constructed for which int \( X \neq \emptyset \), \( R(X) \neq A(X) \), such that there is no bounded point derivation for \( R(X) \) at any point on the boundary of \( X \). (Received March 21, 1968.)

Let \( D = \{ |z| < 1 \} \) and \( C = \{ |z| = 1 \} \) and let \( f(z), F(z) \) be functions defined in \( D \). We say that \( f(z) \) is subordinate to \( F(z) \) and write \( f(z) \prec F(z) \) if there exists \( \phi(z) \), analytic in \( D \) with \( |\phi(z)| \leq |z| \), such that \( f(z) = F(\phi(z)) \) everywhere in \( D \). J. V. Ryff [Duke Math. J. 33 (1966), 347-354] studied \( H^{p} \) functions in \( D \) related by subordination. Let \( G \) be a domain whose boundary \( \Gamma \) has positive logarithmic capacity. If \( f(z) \) is meromorphic in \( D \), takes all its values in \( G \), and at almost every point of \( C \) has a radial limit on \( \Gamma \), we call \( f(z) \) a function of class \( (L) \) (with respect to \( G \) and \( \Gamma \)). This class of functions was studied first by O. Lehto [Ann. Acad. Sci. Fenn. Ser A1 160 (1953), 1-14]. Results similar to Ryff's are obtained for functions of class \( (L) \). \( \textbf{Theorem 1.} \) If \( F(z) \) is of class \( (L) \) and \( f < F \), then \( f(z) \) is of class \( (L) \) if and only if \( \phi(z) \) is of Seidel's class \( (U) \), \( \phi(0) = 0 \), where \( f(z) = F(\phi(z)) \). In terms of Lehto's definitions of normal and exceptional values for functions of class \( (L) \), we prove \( \textbf{Theorem 2.} \) If \( f(z) \) and \( F(z) \) are of class \( (L) \) and \( f < F \), then every value in \( G \) exceptional for \( F(z) \) is exceptional for \( f(z) \). A value exceptional for \( f(z) \) need not be exceptional for \( F(z) \). (Received March 25, 1968.)
It is known that if $G$ is an u.s.c. decomposition of $E^n$, with only finitely many nondegenerate elements, then the decomposition space $E^n/G$ of $G$ is embeddable in $E^{n+2}$ (L. V. Keldysh, Proc. Sympos., Prague, 1961, pp. 230-234). The following is an extension to this theorem, and a partial affirmative answer to question #16 of S. Armentrout's Monotone decompositions of $E^3$ (Topology Seminar Wisconsin 1965): Theorem. If $G$ is an u.s.c. decomposition of $E^n$, $H = \bigcup_{\alpha \in A} G_\alpha$, the set of all the nondegenerate elements of $G$, $H^* = \bigcup_{\alpha \in A} G_\alpha^*$, then $E^n/G$ is embeddable in $E^{n+2}$ provided $H^*$ is closed, and for each $\alpha \in A$, there exists a neighborhood $V_\alpha$ of $G_\alpha$ such that $\alpha \neq \beta$ implies $V_\alpha \cap V_\beta = \emptyset$. (Received March 25, 1968.)

Let $X$ be a compact metric space with a closed partial order. For each $x \in X$, let $\Gamma(x) = \{y \in X | x \preceq y \text{ or } y \preceq x\}$. An antichain is a totally unordered subset of $X$. An order arc is a closed, connected, totally ordered subset of $X$. Let $L$ (resp. $M$) denote the set of minimal (resp. maximal) elements of $X$. Theorem 1. If $L$ and $M$ are closed and, for each $x \in X$, $\Gamma(x)$ is a nondegenerate order arc, then $X$ is homeomorphic to $M \times [0,1]$. Theorem 2. If $L$ and $M$ are closed and, for each $x \in X$, $\Gamma(x)$ is connected and nondegenerate, then the family of maximal order arcs of $X$ is a compact subspace of the space of compact subsets of $X$ (with the finite topology). Let $\mathcal{F} \times [0,1]$ have the partial order $(m,x) \preceq (n,y)$ iff $m = n$ and $x \preceq y$. There exists a map $f: \mathcal{F} \times [0,1] \to X$ such that for each $T \in \mathcal{F}$, $f(T \setminus [0,1])$ is an order preserving homeomorphism of $T \setminus [0,1]$ onto the maximal arc $T \subseteq X$ and, for each $y \in [0,1]$, $f(\mathcal{F} \times \{y\})$ is a compact maximal antichain in $X$. (Received March 25, 1968.)

A ring $R$ (with unit) is left QF-3 if it has a minimal faithful left module, i.e. a faithful left module which is (isomorphic to) a summand of every faithful left module. Theorem. The following are equivalent. (1) $R$ has zero left singular ideal and is left and right QF-3. (2) There exist idempotents $e, f \in R$, such that $Re$ and $fR$ are faithful projective-injective left and right ideals and $eRe$ is semisimple. (3) $R$ has zero left singular ideal, contains no infinite set of orthogonal idempotents, and has a faithful projective-injective left ideal and a faithful projective injective right ideal. (4) $R$ is a subring of a semisimple ring $Q$ and $R$ contains a left ideal $I$ and a right ideal $J$ such that $I$ and $J$ are, respectively, faithful left and faithful right ideals of $Q$. (5) $R$ has a two-sided semisimple generalized ring of quotients and both the left and the right socles of $R$ are essential. (Received March 25, 1968.)
If \( X \subseteq \omega \), let \([X]^{\omega} \) denote \( \{ Y \subseteq X | |Y| = \omega \} \). A partial function \( \Phi \) on \( 2^{\omega} \) (with standard topology) is Borel measurable if \( \Phi^{-1}(U) \) is Borel for every open set \( U \subseteq 2^{\omega} \). **Theorem.** Let \([\Phi_\epsilon]\) be any countable class of Borel measurable partial functions, and let \( A \in [\omega]^\omega \). Then there exists \( B \in [A]^\omega \) such that: (a) \( (T)(S)(T \subseteq [B]^\omega & S \subseteq [T]^\omega & \Phi_\epsilon(S) = T \Rightarrow T - S \) is finite). **Corollary.** If \( A \in [\omega]^\omega \), then there exists \( B \in [A]^\omega \) such that if \( S \subseteq T \subseteq B \) and \( T \) is arithmetic in \( S \), then \( S \) is arithmetic in \( T \). The proof of the theorem depends upon the recent result of F. Galvin and K. Prikry that every Borel set is Ramsey. This generalized a theorem of Ehrenfeucht that every open set is Ramsey which suffices to prove our theorem for continuous partial functions, and which we earlier used (Abstract 656-37, these Notices 15 (1968), 513) to obtain the special case of the Corollary for Turing degrees instead of arithmetic degrees. (Received April 1, 1968.)


The following result is proved: **Theorem 1.** Let \((S,\Sigma,m)\) be a probability space such that \( S \) is topological, and every \( f \in C(S) \), the space of real valued continuous functions on \( S \), is a random variable with finite expectation. Then every \( f \in C(S) \) is bounded almost surely; moreover, if \( S \) is realcompact or normal, admitting a complete uniform structure, then the support of \( m \), \( F = \{ x; \) every \( \Sigma \)-measurable open neighborhood of \( x \) has positive measure }, is a compact subset of \( S \). Hewitt (Fund. Math. 37 (1950)) proved the following: **Theorem 2.** Let \( S \) be completely regular and let \( I \) be a bounded linear functional on \( C(S) \) with \( I(1) = 1 \). Then: (a) There exists a Baire probability measure \( m^* \) on \( S \) such that \( I(f) = \int f(x)m^*(dx) \) for every \( f \in C(S) \). (b) Every \( f \in C(S) \) is bounded almost surely (\( m^* \)) and if in addition, \( S \) is realcompact, then the support of \( m^* \), \( F^* = \{ x; \) every Baire open neighborhood of \( x \) has positive \( m^* \)-measure } is compact. If part (a) of Theorem 2 is granted, then Theorem 1 implies clearly Theorem 2. Conversely, if \( m \) is outer regular relative to the open sets of \( S \), or if \( F^* \) is a \( G^0 \) (or equivalently a zero set), then also Theorem 2 implies Theorem 1. (Received March 26, 1968.)


Consider the triple, \((E,\Sigma,\mu)\), where \( E \) is a real linear space, \( \Sigma \) is a \( \sigma \)-algebra of subsets of \( E \), and \( \mu \) is a nonnegative, extended real valued, countably additive set function on \( \Sigma \), with \( \mu(\emptyset) = 0 \). **Definition.** A subset, \( S \), of \( E \) is said to be \( \mu \)-convex iff for \( x,y \in S \), \( N(x,y) = \{ ax + \beta y: \alpha, \beta \geq 0, \alpha + \beta = 1 \} \cap S^c \in \Sigma \) (where \( S^c \) is the set-theoretic complement of \( S \)) and \( \mu(N(x,y)) = 0 \). This definition gives the usual convexity if \( \Sigma \) is the power set of \( E \) and \( \mu \) is the measure which assigns to a set \( A \) in \( E \) the cardinality of \( A \) if \( A \) is finite and \( \infty \) if \( A \) is infinite, hence a generalization. **Theorem.** The intersection of a countable collection of \( \mu \)-convex sets in \( E \) is a \( \mu \)-convex set. (Received March 28, 1968.) (Author introduced by Professor George Berzsenyi.)

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A sufficiency condition for uniform continuity.

Concerning the last sentence in a paper by E. Elyash, G. Laush, and N. Levine [On the product of two uniformly continuous functions on the line, Amer. Math. Monthly 67 (1960), 265-267], the following more general sufficiency condition for uniform continuity was found. Theorem. Let \( X \) be a metric space with metric \( d_x \). If \( f: X \to [0, \infty) \) and \( f^2 \) is uniformly continuous on \( X \), then \( f \) is uniformly continuous on \( X \). The proof is based upon the Lemma. If \( a \geq 0 \) and \( b \geq 0 \), then \( \sqrt{a} \geq \sqrt{b} \leq \sqrt{a + b} \). (Received March 28, 1968.) (Author introduced by Professor George Berzsenyi.)

A note on the ideal transform and integral closure in Noetherian domains.

Let \( R \) be a commutative ring with identity having total quotient ring \( F \). If \( A \) is an ideal of \( R \), the transform of \( A \), \( T(A) \), is defined to be \( \bigcup_{n=1}^{\infty} B_n \) where \( B_n = R : A^n = \{ x \in F | xA^n \subset R \} \). We say that \( T(A) \) is a proper transform of \( R \) provided \( R \subset T(A) \). Theorem 1. Suppose that \( D \) is an integral domain with \( A \) a finitely generated ideal of \( D \). Let \( \{ P_a \} \) be the collection of all prime ideals \( P \) of \( D \) such that \( P \nsubseteq A \). Then \( T(A) = \bigcap_a D_{P_a} \). Corollary 1. If \( D \) is a Noetherian domain and \( A \) is an ideal of \( D \), then the transform of \( A \) is determined as the intersection of those \( D_{P_a} \)'s where \( P \) is prime and \( P \nsubseteq A \). The converse to Corollary 1 is false, Theorem 2. If \( D \) is a Noetherian domain which is not a local ring, then \( D \) is integrally closed if and only if each proper transform of \( D \) is integrally closed. The hypothesis that \( D \) is not local cannot be dropped. (Received March 27, 1968.)

The Leray spectral sequence for generalized cohomology.

The Leray generalization of the Mayer-Vietoris homology or cohomology exact sequence to a spectral sequence for the case of several subspaces is found to depend only on the first six Eilenberg-Steenrod axioms. Applying this to coverings of a space \( X \) by preimages of open sets in the range \( Y \) of a mapping \( f: X \to Y \), one obtains a new derivation of both the Leray-Serre homology or cohomology spectral sequence of a fibration and the original Leray sheaf-cohomology spectral sequence for an arbitrary mapping, all without using the dimension axiom. Convergence in the general case is insured by keeping the covering dimension of the space \( Y \) finite. (Received March 27, 1968.)

Some new classes of anticommutative algebras. Preliminary report.

The methods introduced by J. M. Osborn (Canad. J. Math. 17 (1965), 78-92) have been adapted to prove the following Theorem. Let \( A \) be an anticommutative algebra over a field of characteristic not two, and let \( A \) satisfy an identity of degree \( \leq 4 \) not implied by the anticommutative identity. Then \( A \) satisfies at least one of the following four identities: (1) \( ((yx)x)x = 0 \); (2) \( J(x,y,x,y) = 0 \); (3) \( a(xy)(xz) + \beta((xz)x)y - ((xy)x)z + \gamma((xy)y)x - ((xy)z)x + (\beta + \gamma)((zy)x)x = 0 \), where \( a, \beta, \gamma \in F \) are not all zero; (4) \( J(x,y,z)w - J(y,z,x)w + J(z,w,y) - J(w,x,y)z = 0 \), where \( J(x,y,z) = (xy)z + (yz)x + (zx)y \). There exist non-Lie, non-Malcev anticommutative algebras satisfying these identities. (Received March 28, 1968.)
A ring is semisimple if its Jacobson radical is zero. The following theorems give sufficient conditions on a group \( G \) in order that \( \mathbb{Q}[G] \) be semisimple.

**Theorem 1.** Let \( H \) be a group and \( K \) an Abelian group. Let \( \rho: H \rightarrow \text{Aut}(K) \) be a homomorphism. Via \( \rho \), we can form the semidirect product \( G = K \rtimes H \). Suppose the image of \( \rho \) is finite Abelian, and that \( \mathbb{Q}[H] \) is semisimple. Then \( \mathbb{Q}[G] \) is semisimple.

**Definition.** A group \( G \) is countably supersolvable if it has an invariant series with cyclic factors whose intersection is the identity.

**Theorem 2.** If \( G \) is countably supersolvable, then \( \mathbb{Q}[G] \) is semisimple.

**Theorem 3.** If \( G \) is countably nilpotent, then \( \mathbb{Q}[G] \) is semisimple.

**Theorem 4.** If \( G \) is any group and \( \mathbb{Z} \) is the group of integers, then \( \mathbb{Q}[G \times \mathbb{Z}] \) is semisimple. (Received March 29, 1968.)

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**Homomorphisms of minimal sets.**

Let \( T \) be a topological group, let \((M, T)\) be a universal minimal set for \( T \), and let \( G \) be the automorphism group of \((M, T)\). If \((X, T)\) is minimal, and \( \gamma: M \rightarrow X \) is a homomorphism, let \( G(X, \gamma) = \{ a \in G / \gamma a = \gamma \} \).

(i) If \( \gamma': M \rightarrow X \) is a homomorphism, then \( G(X, \gamma) \) and \( G(X', \gamma') \) are conjugate subgroups of \( G \). If \((Y, T)\) is minimal, and \( \pi: X \rightarrow Y \) is a homomorphism, \( \pi \) is called proper if \( G(X, \gamma) \neq G(Y, \pi \gamma) \) by (i) this is independent of \( \gamma \).

(ii) \( \pi \) is proper if and only if there exist \( x \) and \( x' \) not proximal such that \( \pi(x) = \pi(x') \) if and only if there exist \( x \neq x' \) in \( X \) such that \( (x, x') \) is an almost periodic point of \((X \times X, T)\), and \( \pi(x) = \pi(x') \), \( \pi \) is said to be of distal type if whenever \( x \neq x' \) and \( \pi(x) = \pi(x') \), \( x \) and \( x' \) are not proximal.

(iii) \( \pi \) is of distal type if and only if \( \pi^{-1}(y) \) is an almost periodic set (\( y \in Y \)).

(iv) \( G(X, \gamma) \) is a normal subgroup of \( G \) if and only if \((X, T)\) is a nonproper homomorphic image of a regular minimal set.

(v) \( G(X, \gamma) \subseteq G(X', \gamma') \) if and only if there is a minimal set \((Y, T)\) and homomorphisms \( \pi: Y \rightarrow X \), \( \pi': Y \rightarrow X' \), where \( \pi \) is nonproper.

(vi) (Notation as in Ellis, Group-like extensions of minimal sets, Trans. Amer. Math. Soc. 127 (1967), 125-135.) Let \( \mathcal{A} \subseteq \mathcal{B} \subseteq \mathcal{A}(u) \). Then \( \mathcal{B} \) is a group-like extension of \( \mathcal{A} \) if and only if the homomorphism of \( |\mathcal{B}| \) to \( |\mathcal{A}| \) induced by inclusion is of distal type. (Received March 29, 1968.)

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**Computable fields and arithmetically definable ordered fields.**

Computable (arithmetically definable, denoted AD) structures are algebraic structures whose relations can be viewed as recursive (arithmetical) number theoretic relations. Using the decidability of the elementary theory of the real numbers, we show Theorem 1. The fields of real algebraic, constructible and solvable numbers are computable fields. Also, concerning the fields of computable reals, denoted \( \mathcal{R}^C \), and arithmetical reals, denoted \( \mathcal{R}^D \), we prove the following. **Theorem 2.** If \( K \) is a computable (AD) subfield of the field of real numbers, then \( K \) is a proper subfield of \( \mathcal{R}^C (\mathcal{R}^D) \).

**Corollary 3.** \( \mathcal{R}^C (\mathcal{R}^D) \) is not a computable (AD) field. However, we have **Theorem 4.** \( \mathcal{R}^C \) is an AD-field. Finally, we prove **Theorem 5.** Let \( K \) be a (countable) subfield of real numbers which is not a proper subfield of \( \mathcal{R}^D \). No ordered field containing \( K \) (whose ordering is compatible with natural
ordering of $K$ is AD. Corollary 6. No ordered field containing $\mathcal{D}$ (whose ordering is compatible with the natural ordering of $\mathcal{D}$) is an AD-ordered field. (Received April 1, 1968.)

68T-488. J. W. HARDY and H. E. LACEY, University of Texas, Austin, Texas 78712. On the existence of Šilov boundaries.

In this paper, necessary and sufficient conditions are given to insure that a linear subspace of $C(X)$ ($X$ compact and Hausdorff) containing 1 has a Šilov boundary. Also, necessary and sufficient conditions are given in order that the Šilov boundary exist and be the closure of the Choquet boundary. These are given in terms of the smallest topology on $X$ that makes each element of $M$ continuous. In particular, if $M$ separates the points of the Choquet boundary of $M$, then the Šilov boundary exists and is the closure of the Choquet boundary. An example is given where the Šilov boundary exists, but is not the closure of the Choquet boundary. (Received April 1, 1968.)

68T-489. P. ROSENTHAL, University of California, Berkeley, California 94720. On complemented and quasi-complemented subspaces of quotients of $C(S)$ for Stonian $S$.

This work improves some of the results announced by the author in Abstract 68T-74, these Notices 15 (1968), 212, and in Proc. Nat. Acad. Sci. U.S.A. 59 (1968), 361-364. $m$ denotes the Banach of all bounded sequences of scalars (sometimes denoted by $l^\infty$). Let $S$ be an F-space ($S$ is a compact Hausdorff space such that disjoint open $F_\sigma$'s have disjoint closures), let $X$ be a Banach space which is a continuous linear image of $C(S)$, and let $Y$ be a closed linear subspace of $X$. Theorem 1. If $X/Y$ is not reflexive, there exists a bounded linear operator mapping $X/Y$ onto $m$. Theorem 2. If $Y^*$ is separable in the weak* topology, $Y$ is quasi-complemented. In particular, if $X^*$ is weak* separable, every subspace of $X^*$ is quasi-complemented. Corollary 3. Every subspace of $m$ is quasi-complemented. Theorem 4. If $S$ is $\sigma$-Stonian (the closure of every open $F_\sigma$ is open) and $X$ is not reflexive, then $m$ is isomorphic to a subspace of $X$. Corollary 5. If $X$ is an infinite-dimensional injective Banach space, $m$ is isomorphic to a subspace of $X$, and every subspace of $X$ with weak* separable dual is quasi-complemented in $X$. (Received April 1, 1968.)


Let $Y_1 = \{a,b,c,0\}, +$ be the semigroup where $a + a = b + b = c$ are the only sums unequal 0; let $Y_2 = \{a,b,c,0\}, +$ be the semigroup where $a + a = a + b = b + a = c$ are the only sums unequal 0. Then $Y_1$ and $Y_2$ are two nonisomorphic, subdirectly irreducible, commutative semigroups which generate the same primitive class of semigroups. This gives a solution for Problem 67 in G. Birkhoff, Lattice theory, 3rd edition (1967). (Received April 4, 1968.) (Author introduced by Professor K. W. Jänich.)


For $\Omega$ a field of characteristic $p$ and $H$ a subgroup of $G$, let $a(G,H)$ denote the Grothendieck group of $\Omega G$-modules, modulo $H$-split, $G$-exact sequences. This generalizes the notions of ordinary
Grothendieck groups and Green's ring of modular representations. Results concerning $\mathbb{Z}$-bases for $a(G,H)$ are therefore generalizations of the Krull-Schmidt and Jordan-Hölder theorems. Theorem. Let $H \Delta G$ be a cyclic $p$-group of order $h$, and $U_1, \ldots, U_s$ be the nonisomorphic principal indecomposable, $\Omega G$-modules. In each $U_j$ there exists a chain of $G$-submodules $V_{kj}$, $1 \leq k \leq h$, such that $\text{dim } [V_{kj}]: 1 \leq k \leq h$, $1 \leq j \leq s$, form a free $\mathbb{Z}$-basis for $a(G,H)$. As a ring, $a(G,H)$ has no nilpotent elements and can be described explicitly as an extension ring of $a(G,1)$. If $B$ is a $p$-complement of $G$, then the restriction functor induces a (ring) isomorphism $a(G,H) \cong a(H \cdot B, H \cdot B)$ if and only if each Brauer character of $B$ can be lifted to a generalized Brauer character of $G$. Among the corollaries, we note: For $H$ as above and $G$ a $p$-group, $a(G,H) \cong a(H)$ (and is thus independent of $G$). Other "excision" theorems can also be obtained. (Received April 3, 1968.)

68T-492. D. K. RAY-CHAUDHURI and R. M. WILSON, Ohio State University, Columbus, Ohio 43210. Solution of the Kirkman's school girl problem.

Let $b, v, r, k$ and $\lambda$ be positive integers such that $r = \lambda(v - 1)/(k - 1)$ and $b = \lambda v(v - 1)/k(k - 1)$. A $(v,k,\lambda)$-Balance Incomplete Block Design (BIBD) consists of a finite set $X$ of $v$ elements called treatments and $b$ subsets (called blocks) $X_1, X_2, \ldots, X_b$ such that (1) every treatment occurs in exactly $r$ blocks, (2) every block contains exactly $k$ treatments, and (3) every pair of treatments occurs together in exactly $\lambda$ blocks. A parallel class of blocks consists of a set of disjoint blocks such that every treatment occurs in one block of the class. A $(v,k,\lambda)$-BIBD is said to be resolvable if the $b$ blocks can be partitioned into $r$ parallel classes. A $(v,3,1)$-resolvable BIBD is said to be a Kirkman design of order $v$. In this paper it is proved that a necessary and sufficient condition for the existence of a Kirkman design of order $v$ is $v \equiv 3 \mod 6$. This problem is known as Kirkman's school girl problem and was introduced by Rev. Thomas J. Kirkman in an article, *On a problem in combinations*, Cambridge and Dublin Math. J. 2 (1847), 191-204, and attracted many mathematicians since 1847. (Received April 4, 1968.)


M. Skwarczynski in a recent paper entitled *The invariant distance in the theory of pseudo-conformal transformations and the Lu-Qi-Keng conjecture* defines a Lu-Qi-Keng domain as a domain $D \subset \mathbb{C}^n$ for which the Bergman function $K_D(z,t)$ does not vanish in $D \times D$. Further, he gave an example of a domain in $\mathbb{C}^1$ which is not a Lu-Qi-Keng domain. Using some known results from the theory of elliptic functions, the author is able to extend M. Skwarczynski's results to conclude every doubly-connected Lu-Qi-Keng domain in $\mathbb{C}^1$ is pseudo-conformally equivalent to a disc with the center deleted. (Received April 4, 1968.)


Let $R$ be any ring. Then an $R$-module $M$ is said to be a pseudo-injective $R$-module if for each submodule $N$ of $M$ every $R$-monomorphism of $N$ into $M$ can be extended to an $R$-endomorphism of $M$. The concept of pseudo-injective module was introduced by the present author and S. K. Jain in (On
pseudo-injective modules and self pseudo-injective rings, J. Math. Sci. 2 (1967), 23-31). In the above referred paper the problem that under what conditions a pseudo-injective module is quasi-injective was studied. The study of this problem is continued in this paper. In this paper all the rings are supposed to be with unity and the modules are taken to be unital right modules. Following are the main results proved. (1) Any pseudo-injective module over a commutative hereditary ring is quasi-injective. (2) Let \( K \) be any right Ore-domain, \( D \) the quotient ring of \( K \), \( n \) any positive integer and \( R \) any ring such that \( K_n \subseteq R \subseteq D_n \). Then any torsion free pseudo-injective \( R \)-module is injective. (Received April 8, 1968.) (Author introduced by Professor R. S. Varma)

68T-495. S. K. BASU, Southern University in New Orleans, New Orleans, Louisiana. A supplementary note on comparison of the total strength of some Hausdorff methods. II.

\( C^a \) and \( H^a \) being respectively the Cesàro and Hölder methods of order \( a > -1 \), the following result is proved as a supplement to a recent paper of the author (Math. Z. 103 (1968), 358-362).

**Theorem.** For \( 0 < \beta < a < 1 \), (i) \( C^\beta /H^\beta \) t.s. \( C^a /H^a \) when \( a + \beta \geq 1 \). (ii) \( C^\beta /H^\beta \) n.t.s. \( C^a /H^a \) when \( a + \beta < 1 \). (iii) \( C^a /H^a \) n.t.s. \( C^\beta /H^\beta \). (t.s. stands for 'totally stronger than' and n.t.s. for 'not totally stronger than'). (Received April 8, 1968.)

68T-496. S. M. MAZHAR, Aligarh Muslim University, Aligarh, India. On the local property of \( |R, \log n, 1|_k \) summability of Fourier series.

A sequence \( \{s_n\} \) is said to be summable \( |R, \log n, 1|_k \), \( k \equiv 1 \), if \( \sum_{k=1}^{n} |t_n - t_{n-1}|^k < \infty \), where \( t_n = (1/(\log(n + 1))) \sum_{k=1}^{n} s_k/k \). In this note the following theorem has been proved: **Theorem.** If \( \sum_{k=1}^{n} |A_n(x)|^k (\log n)^k < \infty \), \( k \equiv 1 \), then the summability \( |R, \log n, 1|_k \) of a Fourier series \( \sum A_n(t) \) depends only on the behaviour of its generating function \( f(t) \) in the immediate neighbourhood of the point \( t = x \).

For \( k = 1 \) it reduces to a theorem of Bhatt [Tohoku Math. J. 11 (1959), 13-19]. In particular, for \( k > 1 \), it follows that summability \( |R, \log n, 1|_k \) of a Fourier series is a local property. On the other hand it is well known that summability \( |R, \log n, 1|_1 \) is not so. (Received April 8, 1968.)

68T-497. WILLIAM VOXMAN, University of Iowa, Iowa City, Iowa. On the shrinkability of decompositions of 3-manifolds.

An upper semicontinuous decomposition \( G \) of a metric space \( M \) is said to be shrinkable if for each open covering \( U \) of \( H^G \), each \( \epsilon > 0 \), and each homeomorphism \( h \) from \( M \) onto \( M \), there exists a homeomorphism \( f \) from \( M \) onto \( M \) such that (1) \( f = h \) on \( M - U^* \) and (2) for each \( g \in G \), \( diam(f(g)) < \epsilon \) and there exists \( D \subseteq U \) such that \( h(D) \subseteq h(g) \cup f(g) \). \( G \) is said to be weakly shrinkable if for each open set \( W \) containing \( H^G \) and \( \epsilon > 0 \), there is a homeomorphism from \( M \) onto \( M \) such that \( h = id \) on \( M - W \) and for \( g \in G \), \( diam(h(g)) < \epsilon \). **Theorem 1.** If \( G \) is a cellular used of a 3-manifold \( M \), then \( M/G = M \) iff \( G \) is shrinkable. **Theorem 2.** If \( G \) is a weakly shrinkable 0-dimensional monotone used of an \( n \)-manifold \( M \), then \( G \) is cellular and \( M/G = G \). In fact, any shrinkable used of a manifold is cellular. A 0-dim. used \( G \) of a metric space \( M \) is locally shrinkable in case for each \( g \in G \), there exists an arbitrarily small open set \( U_g \) containing \( g \) such that (1) \( Bd H^G_g \cap U_g = \emptyset \) and (2) if \( K \) is compact, \( g \subseteq K \subseteq U_g \), and \( \epsilon > 0 \), then there exists a homeomorphism \( h \) from \( M \) onto \( M \) such that (a) if \( g' \in H^G_g \), \( g' \cap K \neq \emptyset \), then \( diam(h(g')) < \epsilon \) and (b) \( h(x) = x \) on \( M - U_g \). **Theorem 3.** Let \( G \) be a
0-dim., monotone used of an n-manifold M such that \( H_G^* \) is a \( G \). Then (1) if \( G \) is locally shrinkable, \( \gamma > 0 \), and \( G' \) is the used of \( M \) where \( H_G' = \{ g \in H_G : \text{diam } g \geq \gamma \} \), then \( M/G = M/G' = M \), and (2) if \( n = 3 \), then \( G \) is locally shrinkable iff \( G \) is weakly shrinkable. (Received April 10, 1968.)


Let \( N^n \) be an n-dimensional Poincaré-duality space. A splitting of \( N^n \) is a triad \( (X;Y,Z) \), \( X = Y \cup Z \), such that \( (Y,Y \cap Z) \) and \( (Z,Y \cap Z) \) are n-dimensional Poincaré-duality pairs, together with a homotopy equivalence \( \phi : N^n \cong X \). A k-dimensional Poincaré-duality-subspace of \( N^n \) is a splitting of \( N^n \) such that the inclusion \( Y \cap Z \subseteq Y \) is, up to homotopy, a fibration \( \xi \) with fibre \( \simeq S^{n-k-1} \).

If \( M^m \) is an m-dimensional Poincaré-duality space, \( f : M^m \to N^n \), then for a given splitting \( (X:Y,Z) \) of \( N^n \), we say that \( f \) splits if there is a splitting \( (A:B,C) \) of \( M^m \) and a map of triads \( (A:B,C) \to (X:Y,Z) \) homotopy consistent with \( f \). If \( (X:Y,Z) \) is a k-dimensional Poincaré-subspace of \( N^n \), we say that \( f : M^m \to N^n \) is homotopic to a transverse-regular map if \( f \) splits so that the map \( (B,B \cap C) \to (Y,Y \cap Z) \) is up to homotopy a map of \( S^{n-k-1} \)-fibrations (thus making \( (A:B,C) \) an \( (m-n+k) \)-dimensional sub-Poincaré-duality space of \( M^m \)).

Theorem. Let \( f : M^m \to N^n \) with \( (X:Y,Z) \) a k-dimensional Poincaré-subspace of \( N^n \) and \( \nu(M) \) the Spivak normal fibration for \( M^m \). If there exists a spherical fibration \( \eta \) over \( N^n \) with (1) \( f^* \eta - \nu(M) \) fibre-homotopically PL, (2) \( \xi + \eta|Y \) is fibre-homotopically PL, (3) \( 2m \equiv n + 2k + 2 \), then \( f \) is homotopic to a transverse regular map. (Received April 10, 1968.)


It is known that for any homotopy n-sphere \( \Sigma^n \), the product \( \Sigma^n \times S^p \) is diffeomorphic to \( S^n \times S^p \) provided \( p \equiv n \). But, for the exotic 16-sphere, \( \Sigma^{16} \times S^{12} \neq S^{16} \times S^{12} \). Calculations with the h-enclosability group \( \Omega_{n,p} \) lead to Theorem. Let \( n \neq 2 \) (mod 4) and \( i = 1,2 \) or 3. Then for any homotopy n-sphere \( \Sigma^n \), \( \Sigma^n \times S^{n-1} \approx S^n \times S^{n-1} \). Corollary. There are exactly eight exotic 17-spheres which imbed with nontrivial normal bundle in the stable range (we say these are strange) and eight which do not. Only two of those which are not strange bound \( \pi \)-manifolds. If we restrict to \( n = 1,3,5 \) (mod 8), then we can prove \( \Sigma^n \times S^{n-1} \approx S^n \times S^{n-1} \), \( 0 \equiv n \equiv 6 \). This would indicate that the strangest spheres occur for \( n = 0 \) (mod 4). (Received April 8, 1968.)

68T-500. ERIK ELLENTUCK, Rutgers, The State University, New Brunswick, New Jersey 08901. Almost combinatorial functions.

Let \( ZFU^0 = \text{set theory admitting urelemente and with AC replaced by AC}^0 = \text{AC for sets of finite sets} \); \( \Delta = \text{Dedekind cardinals, and } \omega = \text{integers.} \) Let \( \mathfrak{A} \) be an arbitrarily quantified positive first order sentence in functors for \( + \) and \( \cdot \). Let \( f_1, \ldots, f_k \) be function variables, and \( \mathfrak{A}_f \) the universal sentence obtained from \( \mathfrak{A} \) by replacing existential quantifiers by the \( f_i \) as Skolem functions.

Theorem. (i) \( (ZFU^0 \vdash \mathfrak{A} \text{ holds in } \Delta) \) if and only if (ii) \( (ZFU^0 \vdash \text{there exist almost combinatorial } f_i \)
such that $\mathcal{A}_f$ holds in $\Delta_l$. Using the methods of [Ann. of Math. 82 (1965), 225-248], (ii) can be reduced to prove-ability of a $\Sigma^1_1$ sentence about $\omega$. Several examples show that the theorem fails either (a) for Horn sentences, or (b) if we drop $AC^0$. We believe our result implies the 'naturalness' of almost combinatorial functions in axiomatic set theory. (Received April 12, 1968.)

68T-501. J. D. MASON, California State College, San Bernardino, California. A necessary and sufficient condition that the limit distribution of a sequence of normed, centered sums of independent random variables involving $r$ distributions be a convolution of $r$ stable distributions.

**Theorem.** Let $(X_n)$ be a sequence of independent random variables such that the distribution function of each $X_n$ is one of $r$ distribution functions $F_1, \ldots, F_r$. Assume that there exist normalizing coefficients $0 < B_n \to \infty$ and centering constants $\{A_n\}$ such that $B_n^{-1}(X_1 + \ldots + X_n) - A_n$ converges in law to a distribution function $G$. In order that $G$ be a convolution of $r$ stable distributions with characteristic exponents $0 < \lambda_1 < \ldots < \lambda_r \leq 2$, it is necessary and sufficient that (i) $F_i$ belongs to the domain of attraction of a stable law with characteristic exponent $\lambda_i$, $1 \leq i \leq r$ (possibly requiring a permutation of subscripts of $F_1, \ldots, F_r$); (ii) $n_r(n)/n \to 1$ as $n \to \infty$, where $n_r(n)$ denotes the number of random variables among $X_1, \ldots, X_n$ whose distribution function is $F_i$; and (iii) $B(i, n_r(n))/B(r, n_r(n)) - (\text{some}) p_i, 0 < p_i < \infty$, as $n \to \infty$, where $1 \leq i \leq r - 1$, and where $\{B(i, n), n = 1, 2, \ldots\}$ is the sequence of normalizing coefficients for $F_i$. In addition, $B_n$ may be taken as $B(r, n)$. (The necessity of (i) was proved by A. A. Zinger in Theor. Probability Appl. 10 (1965), 607-626 [SIAM transf]. The necessity of (ii) and (iii) is believed to be new. The sufficiency of (i), (ii), (iii) is trivial.) (Received April 12, 1968.)


Let $M^n$ be a closed, orientable topological manifold, $n \geq 2$, and let $Q^n$ be a simply connected P,L, manifold. If $M$ is $2n - q$ connected, $Q$ is $2n - q + 1$ connected and $2q > 3(n + 1)$, then any map $f: M \to Q$ is homotopic to a locally flat imbedding. If $2q > 3(n + 1) + 1$, $M$ is $2n - q + 1$ connected, and $Q$ is $2n - q + 2$ connected, then any two locally flat imbeddings $f, g: M \to Q$ which are homotopic are concordant, that is, there is a locally flat proper imbedding $H: M \times I \to Q \times I$ with $H(x, 0) = (fx, 0)$ and $H(x, 1) = (gx, 1)$. The proof of these facts makes use of a polyhedral imbedding theorem of A. V. Cernavski and engulfing theorems for topological manifolds due to M. H. A. Newman and J. A. Lees, and proceeds by induction on the number of coordinate neighborhoods of $M$. (Received April 12, 1968.)

68T-503. YAU-CHUEN WONG, University College of Swansea, Singleton Park, Swansea, Great Britain. Order-infrabarrelled Riesz spaces.

A locally convex Riesz space $(X, C, \mathcal{F})$ is said to be order-infrabarrelled if each barrel in $X$ which absorbs all order-bounded sets in $X$ is a $\mathcal{F}$-neighbourhood of $0$. There are bornological Riesz spaces which are not order-infrabarrelled. **Theorem 1.** An infrabarrelled Riesz space $(X, C, \mathcal{F})$ is order-infrabarrelled if and only if the topological dual $X'$ of $X$ is a band in the order-bound dual $X^b$ of $X$. **Theorem 2.** An order-infrabarrelled Riesz space $(X, C, \mathcal{F})$ is barrelled if and only if the dual cone $C'$, i.e. the set of all positive $\mathcal{F}$-continuous linear functionals on $X$, is a
\( \mathcal{D} \)-cone in \((X', C', \sigma(X', X))\) (for definition see H. H. Schaefer: Topological vector spaces, Macmillan, New York, 1966). Corollary. Let the order-bound dual \( X^b \) of a Riesz space \((X, C)\) be total over \( X \), and let \( \mathcal{F}_b \) be the order topology on \( X \) (for definition see Schaefer's book). Then \((X, C, \mathcal{F}_b)\) is barrelled if and only if \( C^* \) is a \( \mathcal{D} \)-cone in \((X^b, C^*, \sigma(X^b, X))\), where \( C^* \) is the set of all positive linear functionals on \( X \). Theorem 3. An infrabarrelled Riesz space \((X, C, \mathcal{F})\) is barrelled if and only if \( C^* \) is a \( \mathcal{D} \)-cone in \((X', C', \sigma(X', X))\) and \( X' \) is a band in \( X^b \). (Received April 12, 1968.)

68T-504. P. F. Duvall, University of Georgia, Athens, Georgia 30601. Topological embeddings in codimension one.

If \( X \) is a closed subset of a manifold \( Q \), we say that \( Q - X \) is \( 1 - lc \) at \( X \) if for each open set \( U \) containing \( X \) there is an open set \( V, X \subset U \subset V \), such that each loop in \( V - X \) is null homotopic in \( U - X \). Let \( M^n \) be a closed, \( 1 \)-connected topological manifold of dimension \( n \), \( n \neq 5 \), and suppose \( M \) is embedded in the interior of a \( PL \) \((n + 1)\)-manifold \( Q^{n+1} \). The main theorem of this paper is the following analog of the regular neighborhood theorem. Theorem. If \( Q - M \) is \( 1 - lc \) at \( M \), then every neighborhood of \( M \) in \( Q \) contains a compact \( PL \) submanifold \( W^{n+1} \) of \( Q \) such that (1) \( M \subset \text{int} \ W \), (2) \( W - M \) is \( PL \) homeomorphic to \( \partial W \times [0,1] \), (3) \( W \) is \( PL \) homeomorphic to the product of some closed \( PL \) manifold with \([0,1]\), and (4) \( M \) is a strong deformation retract of \( W \). Corollary. If \( M^n, n \neq 5 \), is a closed, \( 1 \)-connected \( PL \) submanifold in the interior of a \( PL \) \((n + 1)\)-manifold, then any regular neighborhood of \( M \) is \( PL \) homeomorphic to the product of a closed \( PL \) manifold with \([0,1]\). The proof of the theorem uses the Browder, Levine, and Livesay boundary theorem [Amer. J. Math. 87 (1965), 1017-1028]. (Received April 15, 1968.)

68T-505. WITHDRAWN.

68T-506. B. Treysbig, Tulane University, New Orleans, Louisiana 70118. An approach to the polygonal knot problem using projections and isotopies.

D. E. Penney has defined for an oriented polygonal knot \( K \) in regular position a "word" \( W(K) \). See these Notices 12 (1965), 793. We extend Penney's idea to define also a boundary collection \( C(K) \). For trefoil \( T \) with projection double points \( a, b, c \), \( W(T) = ab^{-1}ca^{-1}bc^{-1} \) and \( C(T) = \{ (c^{-1}a, ca^{-1}), (ab^{-1}, a^{-1}b), (bc^{-1}, b^{-1}c), (c^{-1}a, a^{-1}b, b^{-1}c), (ca^{-1}, ab^{-1}, bc^{-1}) \} \). \( C(T) \) describes the complementary domains of projection \( \pi(T) \) with ordered segments which indicate over and under crossings. Given \( K, K' \) there is natural definition of \( C(K) \cong C(K') \). Theorem A. \( C(K) \cong C(K') \Rightarrow K \cong K' \). Now define simple transformations \( S: K \rightarrow K' \) of type (a) \( I(I') \) add (delete) a small loop \( ... xx^{-1} ... \), (b) \( II(I') \) add (delete) a two crossing loop \( ... ab ... b^{-1}a^{-1} ... \), (c) \( III \) changing triangular domain \( ... xy ... y^{-1}z ... x^{-1}z^{-1} ... \) to \( ... rs ... zr^{-1} ... z^{-1}s^{-1} ... \). Theorem B. If \( S: K \rightarrow K' \)
is simple, then $K \cong K'$. We say $T : C(K) \to C(K')$ is of type * if there exist $L, L', S$ so that $C(L) \cong C(K), C(L') \cong C(K')$ and $S : K' - L'$ is of type $\ast$. Theorem C. If $h$ is an orientation preserving homeomorphism of $E^3$ onto $E^3$ taking $K$ onto $K'$, there exists $C(K) = T_0 C(K) T_1 \cdots T_n C(K_n) = C(K')$ or $C(-K')$ where each $T_p$ is simple. Conjecture. If $K, K'$ are as in Theorem C and $\pi K, \pi K'$ have $\leq N$ crossings, there is a Theorem C sequence where $\pi K_p$ has $\leq 2N$ crossings. (Received April 19, 1968.)

68T-507. R. J. DAVERMAN and W. T. EATON, University of Tennessee, Knoxville, Tennessee 37916. An equivalence for the embedding of cells in $E^3$. Let $B$ denote the set of points in $E^3$ whose norm is less than or equal to 1, and $\pi$ the map of $B$ onto $[-1,1]$ given by projection onto the first coordinate. Theorem. Given a topological 3-cell $K$ in $E^3$ and an open set $U$ containing $K$, there is a map $f$ of $E^3$ onto $E^3$ satisfying (1) $f(K)$ is an arc, (2) $f$ is a homeomorphism of $E^3 - K$ onto $E^3 - f(K)$, (3) $f|_{E^3 - U}$ is the identity, and (4) there are $E^3$-homeomorphisms $g$ of $B$ onto $K$ and $h$ of $[-1,1]$ onto $f(K)$ so that $h \pi = fg$. A similar result holds for topological 2-cells in $E^3$. Corollary. Let $C$ be a topological cell in $E^3$. Then there exists an arc $A$ in $E^3$ so that $E^3 - C$ is homeomorphic to $E^3 - A$. (Received April 19, 1968.)

68T-508. R. L. SEIFERT, JR., University of California, Berkeley, California, On prime binary relational structures. Let $\mathcal{B}$ be the class of structures $\langle A, R \rangle$ with one binary relation $R \subseteq A \times A$. If $\mathcal{H} \subseteq \mathcal{B}$, we say a structure $\mathfrak{A}$ is $\mathcal{H}$-prime, or $\mathfrak{A} \in \text{Pr}(\mathcal{H})$, iff: (i) $\mathfrak{A} \in \mathcal{H}$; (ii) $\mathfrak{A}$ has more than one element; and (iii) if $\mathfrak{A}, \mathfrak{B} \in \mathcal{H}$ and $\mathfrak{A} \not\cong \mathfrak{B}$, then either $\mathfrak{A} \not\cong \mathfrak{B}$ or $\mathfrak{A} \not\cong \mathfrak{B}$. [If $\mathfrak{A}, \mathfrak{B} \in \mathcal{H}$ means $\mathfrak{A} \cong \mathfrak{B}$ for some $\mathfrak{B}$.] Let $\mathcal{H} = \{ \langle A, R \rangle \in \mathcal{B} : \text{Dom}(R) \cup \text{Rng}(R) = A \}$, $\mathcal{D} = \{ \langle A, R \rangle \in \mathcal{B} : \text{Dom}(R) = A \}$, $\mathcal{F} = \{ \langle A, R \rangle \in \mathcal{B} : \text{Dom}(R) \cap \text{Rng}(R) = A \}$, $\mathcal{G} = \{ \langle A, R \rangle : x R x \text{ for all } x \in A \}$, and $\mathcal{L} = \{ \langle A, A \times A \rangle \in \mathcal{B} : A \text{ is infinite or has a prime number of elements} \}$. If $\mathcal{H} \subseteq \mathcal{B}$, let $\mathcal{H}_C$ be the class of connected structures in $\mathcal{H}$ (i.e. ones which are not disjoint unions of other structures), and let $\mathcal{H}^f$ be the class of finite structures in $\mathcal{H}$. Theorems. (1) $\text{Pr}(\mathcal{D}) = \text{Pr}(\mathcal{D}^f) = \text{Pr}(\mathcal{L}) = \text{Pr}(\mathcal{L}^f) = \text{Pr}(\mathcal{G}) = \text{Pr}(\mathcal{G}^f) = \text{Pr}(\mathcal{A}) = \text{Pr}(\mathcal{A}^f)$, (2) $\text{Pr}(\mathcal{F}) = \text{Pr}(\mathcal{F}^f) = \text{Pr}(\mathcal{C}) = \text{Pr}(\mathcal{C}^f)$, (3) $\text{Pr}(\mathcal{F}^f) = \text{Pr}(\mathcal{F}^f) = \text{Pr}(\mathcal{C}) = \text{Pr}(\mathcal{C}^f)$, (5) $\text{Pr}(\mathcal{C}^f) \subseteq \text{Pr}(\mathcal{C}^f)$, (6) $\text{Pr}(\mathcal{F}^f) \subseteq \text{Pr}(\mathcal{F}^f)$, (7) $\text{Pr}(\mathcal{C}) \cap \text{Pr}(\mathcal{C}) = \text{Pr}(\mathcal{C}) = \text{Pr}(\mathcal{C}) = \text{Pr}(\mathcal{C}^f)$. (8) The following structures are $\mathcal{H}^f$-prime: (i) the identity relations $\langle A, = \rangle$, where $A$ has a prime number of elements; (ii) reflexive closures $\langle A, f = \rangle$ of finite connected unary algebras $\langle A, f \rangle$ with more than one element; (iii) finite linearly ordered structures with more than one element. (Received April 19, 1968.)

68T-509. B. W. HUFF, University of California, Riverside, California 92502. The loose subordination of differential processes to Brownian motion. A differential process $\{X(T)/T \in [0, \infty]\}$ is said to be loosely subordinate to the standard Brownian motion $\{W(t)\}$ if it is independent of $\{W(t)\}$ and if there exists a loose subordinator $\{S(T)\}$, i.e. a differential process whose sample paths are a.s. nondecreasing, such that $W(S(T)) = X(T)$ a.s. Any differential process that has sample paths a.s. of bounded variation over $[0,1]$ is loosely subordinate to the standard Brownian motion. The Lévy parameters of the loose subordinator
have been obtained in terms of those of the loosely subordinate process when \( \mathcal{X}(T) \) is symmetric stable or is itself a subordinator. The common form of the Lévy spectral function of the loose subordinator in these cases is given by 

\[
M_{\mathcal{X}}(\lambda) = -\int_{-\infty}^{\infty} \left( \frac{2}{\sqrt{2\pi\lambda}} \right) \exp \{-t^2/(2\lambda)\} dt dM_x(x) \quad \text{for } \lambda > 0.
\]

(Received April 2, 1968.)

68T-510, H. J. KEISLER, University of Wisconsin and University of California, Los Angeles, California 90024. On the quantifier "There exist uncountably many".

Let \( L \) be a countable, first-order logic with identity, and form \( L(Q) \) by adding the quantifier \( (Qx) \) which means "There exist uncountably many \( x \) such that". Give \( L(Q) \) the usual first-order axioms and rules of inference, plus the Axioms: 1. \( (Qx)(x = y \lor x = z) \). 2. \( (\forall x)(\varphi \rightarrow \psi) \rightarrow ((Qx)\varphi \rightarrow (Qx)\psi) \). 3. \( (Qx)\varphi(x) \leftrightarrow (Qy)\varphi(y) \), where \( y \) does not occur in \( \varphi(x) \). 4. \( (Qy)(\exists x)\varphi \rightarrow (\exists x)(Qy)\varphi \lor (Qx)(\exists y)\varphi \). Completeness Theorem. A set \( T \) of sentences of \( L(Q) \) is consistent if and only if it has a model. The proof uses the completeness theorem for \( \omega \)-logic. A different completeness theorem for \( L(Q) \) is given by Ebbinghaus (these Notices 15 (1968), 547) using methods of Vaught and Fuhrken. Now form the "\( \omega \)-logic" \( L^\omega(Q) \) by adding constants \( 0,1,2,\ldots \) and a predicate \( N(x) \). Give \( L^\omega(Q) \), the infinite rule \( \varphi(0), \varphi(1), \ldots \rightarrow N(x) \rightarrow \varphi(x) \) and the new Axioms: 5. \( N(0), N(1), \ldots \)

\[ b. \quad \neg(Qx)N(x). \quad \text{\( \omega \)-completeness Theorem. A set \( T \) of sentences of \( L^\omega(Q) \) is consistent if and only if \( T \) has an \( \omega \)-model. Let \( \Gamma \) be a consistent theory in \( L^\omega(Q) \). By a class in \( T \) we mean a set \( \Gamma \subset T \) of sentences of \( L^\omega(Q) \) that is realized in all \( \omega \)-models of \( T \), is countable. (Received April 22, 1968.)


Regular matrices which transform almost convergent sequences into almost convergent sequences having the same \( F \)-limit were characterized by G. M. Petersen in Proc. Amer. Math. Soc. 11 (1960), 469-477. We extend his results to the class of almost regular matrices, defined by J. P. King in Proc. Amer. Math. Soc. 17 (1966), 1219-1225. Theorem. Let \( A = [a_{nk}] \) be an almost regular matrix and let \( A(x) = \{ \sum_{k} a_{nk} x_k \} \). Then \( F-\lim A(x) = F-\lim x \) for every almost convergent sequence \( x \) if and only if \( \lim_{q} \sum_{k=0}^{\infty} \left[ \frac{\left| \sum_{i=0}^{q} (a_{n+i,k} - a_{n+i,k+1}) \right|}{(q + 1)} \right] = 0 \) uniformly in \( n \). We base our proof on the following analogue of a theorem of Schur. Lemma. Let \( A(i) = [a_{nk}(i)] \), \( i = 0,1,\ldots \), be a sequence of infinite matrices such that \( \sum_{k} |a_{nk}(i)| \leq H < +\infty \) for all \( n \) and \( i \), and such that for each \( k \), \( \lim_{n} a_{nk}(i) = 0 \) uniformly in \( i \). Then, \( \lim_{n} \sum_{k} a_{nk}(i) x_k = 0 \) uniformly in \( i \) for each bounded sequence \( x \) if and only if \( \lim_{n} \sum_{k} |a_{nk}(i)| = 0 \) uniformly in \( i \). (Received April 2, 1968.)

68T-512, C. P. BRUTER, 33 Bld Dubreuil, 91-Orsay, France. Le Critère de Minimalité en combinatoire.

Nous désignons sous le nom de Critère de Minimalité une méta-conjecture qui apparaît comme une loi physique d'information minimale. Critère de Minimalité: Soient \( A \) et \( B \) deux classes d'objets définis sur des ensembles finis par la même axiomatique. On suppose \( A \subset B \), ce qui signifie que tous les objets de \( B \) ont les propriétés des objets de \( A \). Soit \( l(B) \) un ensemble d'indices qui caractérisent

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Designons par $B$ la famille d'objets minimaux de $\mathcal{A}$ par rapport aux caractères $i \in I(\mathcal{A})$, et qui ne sont pas objets de $\mathcal{A}$. Une classe $\mathcal{C}$ d'objets de $\mathcal{A}$ n'est composée que d'objets de $\mathcal{A}$ si et seulement si $\mathcal{C}$ ne contient pas d'objets de $B$. Le critère que nous venons d'énoncer manque de précision; il n'en est pas moins un guide très utile pour la recherche de caractérisation de classes d'objets. (Received April 19, 1968.)


In a real space $V_{n+1}$ of $(n + 1) \geq 2$ dimensions the possible solution of a first order partial differential equation $F(z; x; p) = 0$, where $F \neq 0$, is of at least class two in a region of the opulence $\Omega_{2n+1}$ and may be obtained by means of the group of contact transformations $T$ of the opulence. If such a $T$, for which $F(z; x; p)$ is a component, can be found, then the complete solution is given by its Lie directrix equation. If $z = x^{n+1}$ is considered to be an independent variable, then $F = 0$ may be written in a homogeneous form $G(x; p) = 0$, for which any solution is $u = u(x)$ with $p_i = \partial u / \partial x^i$, where $i = 1, \ldots, n$. In $V_{n+1}$ a contact transformation $T$ is considered in which the Lie directrix equation is of the form $u = u + H(x; x)$, where $H$ is of at least class two in a $(2n + 2) \geq 4$ dimensional region of points $(\xi; x)$, such that the matrix $(\partial^2 H / \partial x^i \partial x^j)$ is of rank $(n + 1)$. Then $T$ is given by the set of equations $\xi = \xi + H(x; p)$, $x^i = x^i(x; p)$, where $i = 1, \ldots, n$, for which $\partial \xi / \partial p \neq 0$, and all of the Poisson parentheses are zero. If $x^{n+1} = F$, then solving the systems $\xi = \xi(x; p)$, $x^{n+1} = 0$, for the $p_i$ in the form $p_i = p_i(x)$, the related Pfaffian equation $p_i dx^i = 0$ is integrable, of which an integral is the complete solution of $G(x; p) = 0$. (Received April 19, 1968.)

68T-514. PIERRE LEROUX, Université de Montréal, Montréal 3, Quebec, Canada. Une caractérisation de la catégorie des groupes. Il est bien connu que si $B$ est une variété de groupes satisfaisant le théorème de Shreier, alors $B$ est, ou bien la variété de tous les groupes, ou bien la variété de tous les groupes abéliens, ou bien la variété de tous les groupes abéliens d'exposant $p$ pour un entier $p$ premier [P. M. Neumann and M. F. Newman, On Shreier varieties of groups, Math. Z. 98 (1967), 196-199]. D'autre part, on sait que tout groupe est sous-groupe d'un groupe simple [W. R. Scott, Contributions to the theory of groups, University of Kansas, 1956]. La catégorie de tous les groupes est alors caractérisée comme étant une catégorie algébrique d'un type particulier, (en fait une variété de groupes) satisfaisant à l'axiome de Shreier et dans laquelle tout objet peut être plongé dans un objet simple, ces notions ayant été adéquatement généralisées. D'autres catégories de groupes sont aussi caractérisées. (Received April 19, 1968.)


Let $M$ be a topological manifold without boundary. A neighborhood of a subset $X$ of $M$ is called a p.l. neighborhood if it can be triangulated as an open p.l. manifold containing $X$ as a subcomplex. Theorem. Let $M^q$ be a closed simply connected, topological manifold, $q \geq 6$. Let $K^q$ be an $n$-complex,
Let $K$ be a topological space and $M$ be a manifold. If $f: K \to M$, then $M$ is triangulable as a p.l. manifold. The proof proceeds along the following lines. First, Newman's Engulfing Theorem for Topological Manifolds is used to show that $M$ is triangulable as a p.l. manifold. A splitting theorem of Browder and Stallings' proof of the Generalized Poincaré conjecture in dimensions $\geq 5$ are then used to show that $M$ is triangulable as a p.l. manifold. (Received April 18, 1968.)


Let $R$ be the ring of all continuous functions on $[0,1]$. Theorem. There exists a subring $S \subseteq R$ such that $\text{gl.d} (S) = 3$ and $S$ has a localization with zero divisors. Theorem. Let $f \in R$. Then $\text{hd} (fR) \leq 3$. Theorem. Let $f, g \in R$. Then the continuum hypothesis implies $\text{hd} (fR + gR) = 0$ or $3$. (Received April 18, 1968.)


If $M$ is a von Neumann algebra with commutant $M'$ and center $Z$, then $R(M, M')$ is type I. Let $e$ be a maximal abelian projection in $R$, $T(ze) = z$ the isomorphism between $Ze$ and $Z$, and $P(x) = T(exe)$ for $x \in M$. Identifying $Z$ and $C(K)$, and choosing $t \in K$, one defines a state on $M$ by $\varphi_t(x) = P(x)[t]$. Then $\varphi_t$ induces a C*-representation $\pi_t(M)$ on $H_t$. Using the decomposition of a type I algebra into a sum of type I algebras, one obtains Theorem 1. The direct sum representation $(\Sigma \pi_t)(M)$ on $\Sigma \oplus H_t$ is faithful. The definition of equivalent projections gives Theorem 2. If $e$ and $f$ are maximal abelian projections in $R$ with $ueu = f$, then $\varphi_{te} e$ and $\varphi_{tf}$ induce unitarily equivalent representations of $u^*Mu$ and $M$, respectively. Theorem 3. If $M$ is type $I_1$, then the weak closure of $\pi_t(M)$ is a type I factor, no matter what maximal abelian projection $e$ is chosen. Theorem 4. If $M$ is type $II_1$, then $e$ may be chosen so that $\pi_t(M)$ is a type $II_1$ factor. (Received November 20, 1967.)

68T-518. J. L. CHRISLOCK, University of California and Adlai E. Stevenson College, Santa Cruz, California 95060. A certain class of identities on semigroups.

If $f(x_1, \ldots, x_m) = g(x_1, \ldots, x_m)$ is an identity ($f$ and $g$ are words in the variables $x_1, \ldots, x_m$), then $V(f)$ and $V(g)$ will denote the set of variables used in $f$ and $g$, respectively. A product of variables $h(x_1, \ldots, x_m)$ will be called free if each variable in $V(h)$ occurs only once in $h(x_1, \ldots, x_m)$. Let $S$ be a semigroup. Theorem. The following are equivalent: (1) $S$ satisfies an identity $f(x_1, \ldots, x_m) = g(x_1, \ldots, x_m)$ with $V(f) \neq V(g)$. (2) $S$ is an ideal extension of a completely simple semigroup whose structure group satisfies $x^r = e$ by a semigroup that satisfies $y^r = 0$. (3) $S$ satisfies an identity $(z_1^w \cdots z_1^w)^r = x^r$. Theorem. The following are equivalent: (1) $S$ satisfies an identity $f(x_1, \ldots, x_m) = g(x_1, \ldots, x_m)$ with $V(f) - V(g) \neq \emptyset$ and free. (2) Some power of $S$ is a completely simple semigroup whose structure group satisfies $x^r = e$. (3) $S$ satisfies an identity $(z_1 \cdots z_1 w_1 \cdots w_1 z_1 \cdots z_1)^r = z_1 \cdots z_1 = z_1 \cdots z_1$ Theorem. The following are equivalent: (1) $S$ satisfies an identity $f(x_1, \ldots, x_m) =$
g(x_1,...,x_m) with V(f) - V(g) \neq \emptyset and f free. (2) Some power of S is a rectangular band. (3) S satisfies an identity w_1 ... w_t v_1 ... v_t z_1 ... z_t = w_1 ... w_t z_1 ... z_t. (Received April 19, 1968.)


A classical real Tauberian theorem of Hardy and Littlewood (1929) is proved here with remainder term (for a special case of this, see G. Freud, Acta. Math. Acad. Sci. Hungar. 2 (1951), 299-308). As an application, the following generalization of a theorem of E. Wirsing (Arch. Math. 7 (1956-1957), 263-272) is proved: Let T be an infinite set of primes p with \( \sum_{p \leq x, p \in T} p^{-1} \log p = (a + O((\log x)^{-\epsilon})) \log x \) and \( a = a(T) > 0, \epsilon = \epsilon(T) > 0; \) then \( \sum_{m \leq x, p | m = p \in T} m^{-1} = (c + O((\log \log x)^{-1})) \prod_{p \leq x, p \in T} (1 - p^{-1})^{-1} \) with \( c = c(a) > 0. \) (Received April 17, 1968.)


A link in a connected n-manifold M is an embedding \( f: Q \rightarrow M, Q \) being a finite disjoint union of spheres of various dimensions. Define \( G(f) = \pi_1(M - f(Q)) \) and let \( K(f) \) be the kernel of the inclusion homomorphism \( G(f) \rightarrow \pi_1(M). \) Then in the piecewise linear category, assuming \( \dim Q \leq n - 2: \)

Theorem. If \( f, g: Q \rightarrow M \) are isotopic links in M, then \( G(f)/K(f) \cong G(g)/K(g) \) for each \( q = 0, 1, 2, ..., \infty. \) Here \( K_0 \supset K_1 \supset K_2 \supset ... \supset K_\infty \) denotes the lower central series of the group K, defined by \( K_0 = K, \)

\( K_{q+1} = [K, K_q], K_\infty = \bigcap K_q. \) This extends results of K. T. Chen (dim \( Q = 1, M = \mathbb{R}^3, \) PL links) and J. Milnor (dim \( Q = 1, \dim M = 3, \) topological category). (Received April 17, 1968.)

68T-521. A. C. LAZER and L. R. ANDERSON, Case Western Reserve University, Cleveland, Ohio 44106. On the asymptotic behavior of a class of linear differential equations.

Consider the differential equation \( y^{(n)} + p(t)y = 0, \) where \( p \in C(-\infty, \infty), M \equiv p(t) \equiv m > 0, \) and where \( n \) is either odd or of the form \( 4k. \) Let \( S_1(S_2) \) denote the subspace of solutions which tend to zero along with their first \( n \) derivatives as \( t \rightarrow + \infty (t \rightarrow - \infty). \) If \( n = 4k, \) then \( S_1 \) and \( S_2 \) each have dimension \( 2k, \) and \( S_1 \cap S_2 \) consists of the null solution. If \( n = 4k + 1, S_1 \) has dimension \( 2k + 1, S_2 \) has dimension \( 2k, \) and \( S_1 \cap S_2 \) consists of the null solution. If \( n = 4k + 3, S_1 \) has dimension \( 2k + 1 \) and the dimension of \( S_2 \) is \( 2k + 2, \) with \( S_1 \cap S_2 = \{0\}. \) In each of the above cases, if \( y \notin S_1 (y \notin S_2), \) then \( y \) is unbounded as \( t \rightarrow + \infty (t \rightarrow - \infty). \) For \( n \) odd and \( 0 > m \geq p(t) \equiv m, \) the foregoing results are easily applied to characterizing \( S_1 \) and \( S_2 \) in this case. (Received April 22, 1968.)


Kaczmarz defined \( V^P \) to be the subspace of \( L^P = L^P(0,2\pi) \) consisting of those functions \( f(t) \) for which the sums \( \sum (f(b_k - t) - f(a_k - t)) \) remain bounded in \( L^P \) norm as \( (a_1, b_1), ..., (a_n, b_n) \) ranges over all finite collections of nonoverlapping subintervals of \( (0,2\pi). \) Theorem. \( V^P = V^1 \) for \( 1 \leq p \leq 2. \) (Received April 17, 1968.)
Odd $p$ groups as fixed point free automorphism groups.

The theorem stated here extends results announced earlier (Abstract 68T-14, these Notices 15 (1968), 192; also in Illinois J. Math. (to appear)). Theorem. Suppose $AG$ is a finite solvable group with normal subgroup $G$. Assume $A$ is an odd $p$ group. Suppose $p^c \not\equiv r^d + 1$ for any $p^c \equiv \exp A$ and any prime $r$ where $r^{2d+1}$ divides $|G|$ (and if $A$ is irregular we also require that $2p^c \not\equiv r^d + 1$ and $p^c \not\equiv r^d - 1$ where $r^d$ divides $|G|$). If $CG(A) = 1$, then the Fitting length of $G$ is bounded by the power of $p$ dividing $|A|$. The result was obtained by extending the methods of the earlier theorem. (Received April 22, 1968.)

Two consequences of topological engulfing.

Let $(W^n; V, V')$ be an $h$-cobordism between closed topological $(n - 1)$-manifolds $V$ and $V'$. Let $\approx$ denote homeomorphism. Theorem A. $W \times S^{2k-1} \approx V \times [0,1] \times S^{2k-1}$ except perhaps if $n = 3$, $k \equiv 5$ or $n = 4$, $k \equiv 3$. Theorem A follows from the case of the 1-sphere $S^1$ (i.e., $k = 1$) since $S^{2k-1}$ is the $k$-fold join of $S^1$s. The case of $S^1$ follows formally from Connell's engulfing result $W \approx V' \approx V \times [0,1]$ (Illinois J. Math. 11 (1967), 300-309). Hence Corollary. If $A^n$ is the closed region in $R^n$ between two locally flat $(n - 1)$-spheres, then $A^n \times S^{2k-1} = S^{n-1} \times [0,1] \times S^{2k-1}$.

Details will appear in Arch. Math. Some fresh engulfing proves Theorem B (cf. E. Luft, Invent. Math. 4 (1967)). If $M^n$, $n \equiv 5$, is a contractible open separable topological n-manifold, and $M^n$ is 1-LC at $\infty$, then $M^n \approx R^n$. The property 1-LC at $\infty$ is this proper homotopy invariant: for compact $K \subset M$, there exists compact $L \subset K$ so that every composed map $S^1 \approx M - L \Leftarrow M - K$ is contractible. A generalization of Theorem B tries to characterize open collars. (Received April 22, 1968.)

Maximal ideals in $OH^\infty(N)$.

Let $N$ be an $n$-dimensional Hilbert space. We denote by $OH^\infty(N)$ the Banach algebra of all norm bounded $N$-operator valued analytic functions in the unit disc, $B(N)$ the algebra of all bounded operators on $N$. Let $\Phi$ be a complex homomorphism of $H^\infty$. We define a map $\Phi_{\Phi, R}$ of $OH^\infty(N)$ into $B(N)$ by the relation $(\Phi_{\Phi, R}(A)\xi, \eta) = \Phi(R^{-1} A(x)R, \xi, \eta)$ where $\xi, \eta \in N$, $A \in OH^\infty(N)$ and $R$ an invertible element of $B(N)$. We write $A(\Phi)$ for $\Phi_{\Phi, I}(A)$. It is easy to see that $\bar{A}(\Phi) = A(\bar{\Phi})^*$ where $A(z) = A(\bar{z})^*$ and $\bar{\Phi}$ the complex homomorphism of $H^\infty$ defined by $\bar{\Phi}(t) = \bar{\Phi}(t)$. Theorem. $\Phi$ is a homomorphism of $OH^\infty(N)$ onto $B(N)$ if and only if $\Phi = \Phi_{\Phi, R}$ for some $\Phi$ and $R$ as above. Theorem. $M$ is a maximal left, right or two sided ideal of $OH^\infty(N)$ if and only if it is of the form $\{A|A(\Phi)\xi = 0\}$, $\{A|A(\Phi)^* = 0\}$ or $\{A|A(\Phi) = 0\}$, respectively, $\xi$ being any nonzero vector in $N$. A similar pair of theorems holds true for the algebra $OA(N)$, the subalgebra of $OH^\infty(N)$ of all functions having continuous extension to the unit circle. (Received April 22, 1968.)

68T-525. P. A. Fuhrmann, Tel-Aviv University, Tel-Aviv, Israel. Maximal ideals in $OH^\infty(N)$. (Received April 22, 1968.)
A note on nets and metrization,

A collection \( \mathcal{D} \) of subsets of a topological space \( X \) is a net for \( X \) if for each point \( x \) in \( X \) and open neighborhood \( U \) of \( x \), there exists a \( B \in \mathcal{D} \) such that \( x \in B \subseteq U \). A space with a \( \sigma \)-locally finite net is called a \( \sigma \)-space, and a \( T_3 \)-space with a countable net is called cosmic. For other background see Abstract 68T-111, these Notices 15 (1968), 224. For a \( T_3 \)-space \( X \) the following are equivalent:

1. \( X \) has a \( \sigma \)-closure preserving net;
2. \( X \) is a \( \sigma \)-space;
3. \( X \) has a \( \sigma \)-discrete net.

Improving A. Okuyama's theorem (Sci. Rep. Tokyo Kyoiku Daigaku Sect. A 9 (1967), 60-78), we can prove that for a \( T_1 \)-space \( X \) the following are equivalent:

1. \( X \) is metrizable;
2. \( X \) is \( T_3 \), an \( M \)-space and a \( \sigma \)-space;
3. \( X \) is collectionwise normal, a \( w \Delta \)-space and a \( \sigma \)-space;
4. \( X \) is collectionwise normal, a \( \sigma \)-space and has a point countable base. A space is a cosmic space iff it is \( T_1 \), collectionwise normal, separable, and a \( \sigma \)-space. (Received April 23, 1968.)

On monotonically complete subspaces of \( T_2 \)-spaces having \( \lambda \)-bases locally.

The authors have shown that the class \( C \) of \( T_2 \)-spaces having \( \lambda \)-bases locally is equivalent to the class of \( T_2 \) open continuous images of metrically topologically complete spaces and also to the class of \( T_1 \)-spaces which are locally monotonically complete and which have bases closurewise of countable order. (The phrase 'having property \( P \) locally' means that there exists a base for the space each of whose elements has property \( P \).) The purpose of this abstract is to state an analogue of the following theorem first proved by P. S. Alexandroff for the separable case [C. R. Acad. Sci. Paris 178 (1924), 185-187] and then by Vedenisov (who gives credit for an independent discovery by Hausdorff) for the general case [J. de Math. Ser. 9, 9 (1930), 377-381]. Theorem (Vedenisov-Hausdorff).

A subspace of a metrizable space is metrically topologically complete if and only if it has a countably monotonically complete base. Theorem. A subspace of a space in class \( C \) is in class \( C \) if and only if it has a monotonically complete base, or, equivalently, if and only if it has a countably monotonically complete base. (For terminology see Duke Math. J. 34 (1967), 255-272, 813-814; and Abstracts 66T-159, 66T-331, these Notices 13 (1966), 255, 510.) (Received April 23, 1968.)

Wiener-Hopf operators on locally compact groups.

Let \( G \) be a locally compact not necessarily Abelian group which has a Borel measurable linear order relation compatible with its group structure. Let \( M(G) \) be the Banach algebra of all complex bounded Radon measures on \( G \) with convolution as multiplication. For \( \eta \in \mathcal{G} \) let \( \chi^+ (\eta, \xi) \) and \( \chi^- (\eta, \xi) \) be the characteristic functions of the sets \( \{ \xi \in G : \xi \leq \eta \} \) and \( \{ \xi \in G : \xi \geq \eta \} \), respectively. For \( \xi \in M(G) \), define \( E^+ (\eta) \xi \) = \( \int_B \chi^+ (\eta, \xi) \xi(\xi \xi) \) for all Borel sets \( B \). Define \( E^- (\eta) \xi \) similarly. If \( E^+ (\eta) \mathcal{M}(G) \) = \( \{ \xi \in M(G) : E^+ (\eta) \xi = \xi \} \) and \( E^- (\eta) \mathcal{M}(G) \) = \( \{ \xi \in M(G) : E^- (\eta) \xi = \xi \} \), then \( E^+ (\eta) \mathcal{M}(G) \) and \( E^- (\eta) \mathcal{M}(G) \) are Banach spaces. For \( C \in \mathcal{G}(G) \) we consider the half-infinite Wiener-Hopf operators \( W_{\eta}^{+} \) and \( W_{\eta}^{-} \) acting on \( E^+ (\eta) \mathcal{M}(G) \) where \( W_{\eta}^{+} \mu = E^+ (\eta) \mu \) c, \( W_{\eta}^{-} \mu = E^- (\eta) \mu \) c, etc. Here \( e \) is the identity of \( G \). We show that if \( c = U_{\rho}^{-1} \ast V_{\lambda}^{-1} = U_{\rho}^{-1} \ast V_{\lambda}^{-1} \) where \( U_{\rho}, U_{\rho}^{-1}, U_{\lambda}, U_{\lambda}^{-1} \subseteq E^+ (\eta) \mathcal{M}(G) \) and \( V_{\rho}, V_{\rho}^{-1}, V_{\lambda}, V_{\lambda}^{-1} \subseteq E^- (\eta) \mathcal{M}(G) \), then the theory developed by I. I. Hirschman, Jr. (Trans. Amer. Math.
Soc. 121 (1966), 133-159), in the case where $G$ is Abelian, for finite section Wiener-Hopf operators, such as $W_C^* \rho(\eta) \mu = E^+(\eta)E^-(\eta)(\mu \ast c)$, $W_C^* \lambda(\eta) \mu = E^+(-\eta)E^- (c \ast \mu)$, etc., continues, with certain necessary modifications, to be valid. (Received April 23, 1968.)


Worrell and Wicke characterized the class of $T^2_\lambda$-spaces having $\lambda$-bases locally as a class of $T_2$ open continuous images of metrically topologically complete spaces and as the class of $T^1_\lambda$-spaces which are locally monotonically complete and which have bases closurewise of countable order. (For references and terminology see Abstracts 66T-159, 66T-331, 67T-249, these Notices 13 (1966), 255, 510; 14 (1967), 290; and Duke Math. J. 34 (1967), 255-272, 813-814. The term 'locally' here refers to the existence of a base whose elements have the property modified by the word 'locally'.) Theorem. If a $T^2_\lambda$-space $S$ is covered by a locally finite collection of closed subspaces having $\lambda$-bases locally, then $S$ has $\lambda$-bases locally. (Received April 23, 1968.)

68T-530. FRED GALVIN, University of California, Berkeley, California. Partition theorems for the real line.

I use the notation of Erdős and Rado, A partition calculus in set theory, Bull. Amer. Math. Soc. 62 (1956), 427-489. Theorem. If $\mathcal{P}(\omega)_1$ and $a < \omega_1$, then $\mathcal{P}(\omega_1, a)$. Corollary. If $\mathcal{P}(\omega_1)$, $\mathcal{P}(\omega)$, and $\mathcal{P}(\omega_1, \omega_1)$, then $\mathcal{P}(\omega_1, a)$. Corollary. If $|\mathcal{X}| > \aleph_0$, $\mathcal{P}(\omega_1)$, $\mathcal{P}(\omega_1, \omega_1)$, and $a < \omega_1$, then $\mathcal{P}(\omega_1, a)$. This generalizes Theorem 6 of Hajnal, Some results and problems on set theory, Acta. Math. Acad. Sci. Hungar. 11 (1960), 277-298. Theorem. The relation $\lambda \rightarrow (\lambda)^{\mathcal{P}}_{\aleph_0}$ holds with respect to Borel partitions, for all finite $r$ and $n$. More precisely, if $E$ is the real line, $[E]^r = K_1 \cup \ldots \cup K_n$ and if $\{(x_1, \ldots, x_r) : (x_1, \ldots, x_r) \in K_i\}$ is a Borel set in $E^r$ for each $i$, then $\lambda \in [K]_i$ for some $i$. (Received April 24, 1968.)

68T-531. P. M. GAUTHIER, Universite de Montreal, Montreal 3, Canada. Unbounded holomorphic functions bounded on a spiral.

Let $w = f(z)$ be an unbounded holomorphic function in the unit disc which is bounded on a spiral $a$, $\overline{a}$ denotes the tightness of $a$ in the sense of W. Seidel. A sequence of points $\{z_n\}$ of the unit disc is a sequence of $\rho$-points for $f(z)$ if $\{z_n\}$ are the centers of a sequence of non-Euclidean cercles de remplissage. Theorem. If $\overline{a} = 0$, then each sequence of points $\{z_n\}$, $|z_n| \rightarrow 1$, is a sequence of $\rho$-points. An analogous theorem holds in the plane. Bagemihl and Seidel have shown the existence of functions satisfying these hypotheses. Thus we have the existence of extremely wild holomorphic functions (in the disc or the plane). Our theorem generalizes a theorem of V. I. Gavrilov. (Received April 24, 1968.)


Since the spaces $L_p = L_p[0,1]$, $1 \leq p < \infty$, regarded simply as linear vector spaces, have the
property \( L_s \supset L_t \) if \( s < t \), it is natural to consider the spaces \( L_{p^+} = \bigcap \{L_t : 1 \leq t < p \} \) and \( L_{p^+} = \bigcup \{L_t : p < t \} \), which are distinct from \( L_p \). The spaces \( L_{p^\pm} \) are then topologized in a natural way as projective and inductive limits of the spaces \( L_t \), where \( L_t \) may be given either its strong or weak topology. \( L_p \) and \( L_q \) are mutually dual (\( 1 < p \neq \infty \), \( 1/p + 1/q = 1 \) and \( q = 1 \) if \( p = \infty \)) and the projective and inductive limit topologies on \( L_{p^\pm} \) can be identified as the appropriate strong and weak topologies. With their strong topologies, the spaces \( L_{p^\pm} \) are separable, barrelled and reflexive; \( L_p \) is complete but not normable; \( L_{p^+} \) is complete but not metrizable. With their weak topologies, the spaces \( L_{p^\pm} \) are separable, but neither complete nor metrizable. A set \( B \subset L_{p^+} \) is strongly bounded if and only if \( B \) is a strongly bounded subset of some \( L_t \), where \( p < t < \infty \). A similar result holds for \( L_p \). (Received April 25, 1968.)


Paracompactness as taken here does not require Axiom \( T_2 \). A space \( S \) is said to have Property Q if and only if for any collection \( H \) of open sets covering \( S \) there exists a collection \( K \) of open sets covering \( S \) such that if \( P \) is a point, there exist a finite subcollection \( F \) of \( H \) and an open set \( D \) containing \( P \) such that every element of \( K \) intersecting \( D \) is a subset of some member of \( F \). The fully normal spaces are those spaces \( S \) satisfying the strengthened form of Property Q in which it is required that \( \overline{F} = 1 \). Theorem. A topological space \( S \) is paracompact if and only if it has Property Q. Corollary. If \( \emptyset \) is a closed continuous mapping of a topological space \( S \) having Property Q and either (1) \( \emptyset \) is peripherally bicompact or (2) \( S \) is essentially \( T_1 \) and \( \emptyset \) is first countable or (3) \( \emptyset \) is open, then \( \emptyset(S) \) has Property Q. (Received April 24, 1968.)

68T-534. S. N. PATNAIK, University of Montreal, Montreal 3, Quebec, Canada. The Lefschetz number associated with set-valued maps. Let \( F : X \rightarrow Y \) be a upper semicontinuous set-valued map from a compact polyhedron \( X \) into another such space \( Y \). Let \( C(Y) \) denote the space consisting of a class of nonempty closed subsets of \( Y \) with a suitable topology. Let the continuous single-valued map \( f : X \rightarrow C(Y) \) be defined by \( f(x) = F(x) \) for all \( x \in X \). Then \( f \) induces the homology homomorphisms \( f_* : H_i(X, L) \rightarrow H_i(C(Y), L) \) (\( L \) a field of coefficients). With respect to suitable chosen basis, \( L_i \) has a rectangular matrix representation \( [M_{k'l'}] \). (If \( H_i(C(Y), L) \) is not finite dimensional, this could still be achieved by choosing proper minimal finite-dimensional subspaces.) Let \( p = \min(k, l) = \text{rank} [M_{k'l'}] \). Considering the square matrices \( p X_i p \) formed from the rectangular matrices \( [M_{k'l'}] \), we define the Lefschetz numbers of the set-valued map \( F : X \rightarrow X \) by \( L(F) = \sum \langle -1 \rangle^j \text{trace} p X_i p \). Theorem. If for a choice of the matrices \( p X_i p \) formed from the matrices \( [M_{k'l'}] \), \( L(F) \neq 0 \), there exists a fixed point under \( F \), i.e., a \( y \in F(y), y \in X \). Fixed-point theorems of Eilenberg-Montgomery and B. O'Neill follow from this. Corollary. If the space \( X \) is homologically acyclic and if the induced homomorphism \( f_* \) (as defined in above) is nontrivial, then there exists a fixed point under the set-valued map \( F : X \rightarrow X \). (Received April 24, 1968.)

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ANALYSIS I

BY SERGE LANG, Columbia University

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August 26-30, 1968

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