NOTICES
OF THE
AMERICAN MATHEMATICAL SOCIETY

Edited by Everett Pitcher and Gordon L. Walker

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MEETINGS

Calendar of Meetings

NOTE: This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the *Notices* was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

<table>
<thead>
<tr>
<th>Meeting No.</th>
<th>Date</th>
<th>Place</th>
<th>Deadline for Abstracts*</th>
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<tr>
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<td>(74th Summer Meeting)</td>
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<td>668</td>
<td>October 25, 1969</td>
<td>Cambridge, Massachusetts</td>
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<td>November 22, 1969</td>
<td>Claremont, California</td>
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<td>January 22-26, 1970</td>
<td>Miami, Florida</td>
<td>Nov. 6, 1969</td>
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<td>(76th Annual Meeting)</td>
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<td>August 24-28, 1970</td>
<td>Laramie, Wyoming</td>
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<td>(75th Summer Meeting)</td>
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<td></td>
<td>January 21-25, 1971</td>
<td>Atlantic City, New Jersey</td>
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<td>(77th Annual Meeting)</td>
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*The abstracts of papers to be presented in person at the meetings must be received in the Headquarters Offices of the Society in Providence, Rhode Island, on or before these deadlines. The deadlines also apply to news items. The next two deadlines for by-title abstracts will be June 24 and September 2, 1969.*

OTHER EVENTS

December 1969
Symposium on Some Mathematical Questions in Biology
Boston, Massachusetts
PRELIMINARY ANNOUNCEMENT OF MEETINGS

The Seventy-Fourth Summer Meeting
University of Oregon
August 26-29, 1969

The seventy-fourth summer meeting of the American Mathematical Society will be held at the University of Oregon, Eugene, Oregon, from Tuesday, August 26, through Friday, August 29, 1969. All sessions of the meeting will be held on the campus of the university. The times listed below for the events of the meeting are PACIFIC DAYLIGHT SAVING TIME throughout.

There will be two sets of Colloquium Lectures. Professor Raoul Bott of Harvard University will present four lectures entitled "On the periodicity theorem of the classical groups and its applications." These addresses will be given on Tuesday, August 26, at 1:30 p.m. and on Wednesday, Thursday, and Friday at 8:30 a.m. The other Colloquium Lecturer will be Professor Harish-Chandra of the Institute of Advanced Study. His topic will be "Harmonic analysis on semisimple Lie groups." Professor Harish-Chandra's four lectures will be given on Tuesday, August 26, at 2:45 p.m. and on Wednesday, Thursday, and Friday at 9:40 a.m. The first two addresses of each series will be presented in the Ballroom of the Erb Memorial Union; the remaining Colloquium Lectures will be given in Room 150 of the Science Building.

By invitation of the Committee to Select Hour Speakers for Annual and Summer Meetings, there will be three one-hour addresses. Professor Robion C. Kirby of the University of California, Los Angeles, will speak on Thursday, August 28, at 1:30 p.m. The title of his lecture is "On the existence and uniqueness of triangulations of manifolds." Professor Murray Gerstenhaber of the University of Pennsylvania will present a lecture entitled "Algebraic deformation theory" at 2:45 p.m. on Thursday, August 28. The last invited address at the meeting will be given by Professor Paul F. Baum of Brown University at 1:30 p.m. on Friday, August 29. The title of his talk is "Vector fields and Gauss-Bonnet." All three lectures will be given in the Ballroom of the Erb Memorial Union.

There will be several sessions for contributed ten-minute papers. These sessions are scheduled at 10:50 a.m. on Wednesday, Thursday, and Friday, and at 2:45 p.m. on Friday. Abstracts of contributed papers should be sent to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904 so as to arrive prior to the July 1 deadline. Ab-stract blanks can be obtained from the same address. There will be no limit on the number of papers accepted for this meeting. No provisions will be made for late papers.

This meeting will be held in conjunction with meetings of the Mathematical Association of America, the Society for Industrial and Applied Mathematics, Pi Mu Epsilon, and Mu Alpha Theta. The Mathematical Association of America will meet from Monday through Wednesday. The Association will present Professor Errett Bishop of the University of California, San Diego, as the Earle Raymond Hedrick Lecturer. The topic of Professor Bishop's lectures is "The constructive point of view." The Hedrick Lectures will be given on Monday, August 25, at 9:15 a.m. and 1:30 p.m. and on Tuesday at 9:00 a.m. The Society for Industrial and Applied Mathematics will meet on Wednesday. The SIAM program will include the von Neumann Lecture, which will be given at 8:00 p.m. on Wednesday by Professor George Carrier of Harvard University. The title of his address will be "Singular perturbation theory in geophysics." Pi Mu Epsilon and Mu Alpha Theta will meet concurrently with the Society and the Association.
On Thursday, August 28, at 8:00 p.m., in the Ballroom, there will be a special panel on The Use of Hard-Won Information and Insight. This is a review of the National Academy of Sciences--National Academy of Engineering survey of Scientific and Technical Communication (SATCOM). The discussion will be on the recently published report of the Academy's Joint Committee on Scientific and Technical Communication and will be led by Dean F. J. Weyl and other panel members, who are yet to be selected.

COUNCIL AND BUSINESS MEETINGS

The Council of the Society will meet on Tuesday, August 26, at 5:00 p.m. in Room 101 of the Erb Memorial Union. The Business Meeting of the Society will be held on Thursday, August 28, at 4:00 p.m. in the Ballroom of the Erb Memorial Union.

At the Business Meeting, the Council will recommend changes in the By-Laws of the Society, consisting of the repeal of Article III, Section 3, and Article XI, Section 3, together with related editorial changes effective December 31, 1971. The Sections require a Committee on Printing and Publishing and define its function. The amendment would eliminate the committee and would not shorten the term of any Council member.

REGISTRATION

The Registration Desk will be in the Taylor Lounge of the Erb Memorial Union. It will be open on Sunday from 2:00 p.m. to 8:00 p.m.; on Monday from 8:00 p.m. to 5:00 p.m.; on Tuesday, Wednesday, and Thursday, from 9:00 a.m. to 5:00 p.m. and on Friday from 9:00 a.m. to 1:00 p.m. The telephone number will be 342-1411, Extension 2811.

The registration fees will be as follows:

- Member: $3.00
- Member's family: 0.50
- Students: No charge
- Others: 7.50

EMPLOYMENT REGISTER

At the January 1969 annual meeting, the Joint Committee on Employment Opportunities voted not to have an Employment Register at the Eugene, Oregon, meeting.

EXHIBITS

Book exhibits and exhibits of educational media will be displayed in Rooms 108-113 on the first floor of the Erb Memorial Union on Tuesday, Wednesday, and Thursday.

BOOK SALES

There will not be a book sale at this meeting.

DORMITORY HOUSING

Dormitory rooms will be available in both the University Residence Halls and in the College Inn, a private dormitory. Reservations for dormitory rooms should be made in advance, using the form provided on page 712 of these Natures. Although it is possible that rooms will be available for persons who have not registered in advance, this is not guaranteed.

The University Residence Halls (Carson, Earl, Hamilton, and Walton) are modern and very convenient to the various meeting rooms. Rates, including meals, are $11/day/person; children 2 through 11, half price; children 2 and under, no charge. Rooms contain two single beds and have sufficient floor space for a crib or a car bed. These cannot be supplied by the university, but participants may make arrangements for rental of this equipment by writing to Abbey Rents, 2750-11th Avenue West, Eugene, Oregon, or Valley Rental Service, 886 Sixth Avenue West, Eugene, Oregon. Rooms do not have private baths. Sleeping bags and pets are not permitted in the Residence Halls. Linen, blankets, towels, and soap are furnished; and daily maid service is provided. Due to limited space, single rooms may be available only in special cases. Rooms will be available from Saturday, August 23, at 2:00 p.m. to Saturday, August 30, at 9:00 a.m. Charges during periods when meals are not served will be adjusted on a prorata basis for late arrivals and early departures. The university will not make refunds for meals that may be missed or accommodations not used during the period arranged for. There is abundant parking space near the Residence Halls.

The College Inn, 1000 Patterson Street, is a modern private dormitory three blocks from the campus. Rates (not including meals) are: $8.50/day/person in
HOTEL-MOTEL AREA

11. City Center Lodge
12. Continental Motel
13. Eugene Hotel
14. Flagstone Motel
15. New Oregon Motel
16. Thunderbird Motel
17. Timbers Motel
18. Travel-Inn Motel

1. Erb Memorial Union
2. Carson Hall
2a. Earl Hall
2b. Walton Hall
2c. Hamilton Hall
3. Science Building
4. Science Library
5. Allen Building
6. Deady Building
7. Chapman Building
8. University co-op
9. Hospital
10. College Inn
a standard single; $9.50/day/person in a large single; $7.00/day/person in a double; $6.00/day/person in a triple or quadruple. Children 2 to 11 are half price. No charge for children under two. Cots and cribs are available for a small charge with advanced notice. All rooms have private baths. Linen, blankets, towels, and soap are furnished; and daily maid service is provided. Rooms in the College Inn will be available from noon Sunday, August 24, through noon Saturday, August 30. Limited off-street parking is available for residents of the College Inn.

Upon arrival on campus, all guests who have reservations or wish accommodations in the University Residence Halls should go to the main desk in Carson Hall for room assignments. Those with reservations at the College Inn should check in at the main desk of the Inn.

**FOOD SERVICE**

For those staying in the University Residence Halls, meals will be served cafeteria style beginning with lunch on Sunday, August 24, and continuing through lunch on Friday, August 29. Hours for food service are as follows:

- **Breakfast** 7:00 a.m. - 10:00 a.m.
- **Lunch** 11:00 a.m. - 1:15 p.m.
- **Dinner** 5:30 p.m. - 6:15 p.m.

On Wednesday, August 27, there will be a picnic on the Football Practice Field at 5:30 p.m. in lieu of the regular dinner. For residents of the College Inn, meals will be available cafeteria style beginning with breakfast on Monday, August 25, and continuing through breakfast on Saturday, August 30. Hours and prices for service are as follows:

- **Breakfast** ($1.00) 7:00 a.m. - 8:00 a.m.
- **Continental Breakfast** ($0.50) 8:00 a.m. - 9:00 a.m.
- **Lunch** ($1.50) 11:15 a.m. - 1:15 p.m.
- **Dinner** (2.50) 4:30 p.m. - 6:00 p.m.

The Erb Memorial Union will be open for coffee and snacks during the following hours:

- **Monday through Thursday** 7:00 a.m. - 10:00 p.m.
- **Friday** 7:00 a.m. - 11:00 p.m.
- **Saturday** 9:00 a.m. - 11:00 p.m.
- **Sunday** 9:00 a.m. - 10:00 p.m.

On Monday through Friday, the Erb Memorial Union cafeteria will serve meals at the following times:

- **Breakfast** 7:00 a.m. - 10:00 a.m.
- **Lunch** 11:00 a.m. - 1:15 p.m.
- **Dinner** 5:30 p.m. - 6:15 p.m.

On Saturday, August 23, and August 30, breakfast will be served from 9:00 a.m. to 11:00 a.m. and lunch from 11:00 a.m. to 1:15 p.m.

A list of nearby restaurants will be available in the registration area at the Erb Memorial Union.

**HOTELS AND MOTELS**

There are a number of motels and hotels in the area, some of which are listed below with coded information which is explained at the end of the list. Those motels preceded by an asterisk (*) are within walking distance of the University of Oregon. Participants should make their own reservations with hotels and motels.

- **CITY CENTER LODGE** (503) 344-5233
  - 476 East Broadway - 48 rooms
  - Single $7.50 - $9.00
  - Double 9.00 - 10.50
  - Twin 11.00 - 13.00
  - Extra person 1.00
  - Rollaway beds 1.50
  - Hide-a-beds 2.00
  - Code: RT-FP-SP-AC
  - 8 blocks from campus

- **CONTINENTAL MOTEL** (503) 343-3376
  - 390 East Broadway - 63 rooms
  - Single $9.00
  - Double 11.00
  - Twin 12.00 - 16.00
  - Extra person 2.00
  - Children 2.00
  - Code: RT-FP-SP-TV-AC
  - 5 blocks from campus

- **EUGENE HOTEL** (503) 344-1461
  - 222 East Broadway - 171 rooms
  - Single $8.00
  - Double 11.00
  - Twin 15.00
  - Code: RT-CL-FP-TV-AC
  - 10 blocks from campus
MANOR HOTEL (503) 345-2331
599 East Broadway - 26 rooms
Single $ 7.00
Double 8.00
Twin 9.00
Extra person 1.50
Code: FP-SP-TV-AC
6 blocks from campus

MOTEL FLAGSTONE (503) 343-7725
1601 Franklin Boulevard - 32 rooms
Single $ 7.00 - $ 8.00
Double 10.00 - 11.00
Twin 12.00
Extra person 1.00 - 2.00
Code: FP-SP-TV-AC
6 blocks from campus

NEW OREGON MOTEL (503) 345-8731
1655 Franklin Boulevard - 72 rooms
Single $11.00
Double 13.00
Twin 15.00
Extra person 2.00
Code: RT-FP-SP-TV-AC
3 blocks from campus

THUNDERBIRD MOTEL (503) 342-5201
205 Coburg Road - 130 rooms
Single $10.00
Double 13.00
Twin 16.00
Extra person 3.00
Code: RT-SP-CL-TV-FP-AC
2 miles from campus

THE TIMBERS (503) 343-3345
1015 Peal Street - 60 rooms
Single $ 9.00
Double 11.00
Twin 13.00
Extra person (Single) $2, (Twin) $1
Code: FP-TV-AC
8 blocks from campus

TRAVEL INN MOTEL
2121 Franklin Boulevard - 115 rooms
Single $ 8.50
Double 10.00
Twin 12.00 - $14.00 - $16.00
Suites to fit any need and priced according to number of people occupying same
Code: RT-CL-FP-SP-TV-AC
5 blocks from campus

*RT-Restaurant  SP-Swimming Pool
CL-Cocktail Lounge TV - Television
FP-Free Parking  AC-Air Conditioned

ENTERTAINMENT

There will be a reception on Tuesday, August 26, from 4:00 p.m. to 6:00 p.m. for adults only in the University Art Museum.

On Thursday, August 28, there will be a bus excursion to Crater Lake National Park. The tour, from 8:30 a.m. to 5:00 p.m., will follow different routes to and from the park. There will be a box lunch picnic on the rim of the lake. The charge for the tour will be $8.00, and tickets will be on sale in the registration area. Tickets will be slightly less for residents of the dormitories. This tour is not recommended for small children.

The traditional SIAM Beer Party will be held on Wednesday, August 27, at 9:00 p.m. following the von Neumann Lecture.

A picnic will be held on Wednesday at 5:30 p.m. on the Football Practice Field. In case of rain, it will be held at Hayward Field. Tickets are provided for residents of the University Residence Halls. For others, tickets will be on sale in the Registration area, $3.00 for adults and $1.50 for children 11 year of age and under.

Several tours will be available, including tours of the university collection of Oriental art, the Chase Gardens, and the Weyerhaeuser lumber and paper mill. Also open for visitors will be the Museum, the Miniature Wagon Museum, and the Museum of Natural History. Recreational facilities on campus include bowling, swimming, tennis, and canoeing. There are public golf courses nearby.

TRAVEL

During the summer, Western Oregon is on PACIFIC DAYLIGHT SAVING TIME.

Eugene is serviced by United Airlines and Air West, both with connecting flights from Portland and San Francisco. Greyhound and Trailway bus lines have frequent service to Portland with several express buses. Greyhound has good service to San Francisco. Southern Pacific main-
tains one passenger train a day from the north and one from the south. Those driving from the east have several interesting possibilities. For example, State Highway 242 over the McKenzie Pass from Bend to Eugene goes through spectacular lava beds, mountain, forest, and river scenery. For further information about other routes and about the many vacation opportunities in Oregon, write to the Travel Information Division, State Highway Building, Salem, Oregon 97310.

CAMPING

There are several excellent campgrounds and trailer parks within driving distance of Eugene. For a complete list of these facilities, write to the Convention Bureau, Eugene Area Chamber of Commerce, 230 East Broadway, Eugene, Oregon 97401.

Western Oregon is richly endowed by nature with outdoor recreational opportunities for the camper, fisherman, hiker and climber, as well as the sightseer. Advance information about these can be obtained from either of the above organization. Further information will be available in the registration area.

PARKING

At this time of year, there should be ample parking on campus within a few minutes' walk of the meeting rooms and the dormitories. Some of the larger lots have been indicated on the accompanying map. Parking permits will not be required.

WEATHER

Typical temperatures in Eugene for the last week of August range from highs of about 80°F to lows of about 50°F. Since it is usually rather cool in the evening, long sleeves or sweaters will be necessary. Rain is unusual in August, but it has occurred. At this time of year, Eugene can experience considerable smoke from agricultural burning.

BOOKSTORE

The University Co-operative Store, 895 E. 13th Avenue, is open from 8:15 a.m. to 5:00 p.m., Monday through Friday.

LIBRARIES

The Science Library, located in the basement of the Science Building, will be open from Monday through Thursday 8:00 a.m. to 10:00 p.m.; Friday, 8:00 a.m. to 9:00 p.m.; Saturday, 8:00 a.m. to 5:00 p.m.; Sunday, 2:00 p.m. to 10:00 p.m. The Main Library will be open from Monday through Friday, 7:30 a.m. to 9:30 p.m.; Saturday, 7:30 a.m. to 5:00 p.m.; Sunday, 2:00 p.m. to 9:30 p.m.

MEDICAL SERVICE

Emergency medical service will be available at Sacred Heart General Hospital, 12th and Alder Street, 24 hours a day.

ADDRESS FOR MAIL AND TELEGRAMS

Individuals may be addressed at Mathematical Meetings, Erb Memorial Union, University of Oregon, Eugene, Oregon 97403. The telephone number of the Message Center will be 503-342-1411, Extension 2812.

COMMITTEE


R. S. Pierce
Associate Secretary
Seattle, Washington
THOUGHTS ON GRADUATE EDUCATION AND THE DRAFT
by
Betty M. Vetter

[The following article is a slightly abridged version of an address delivered by Mrs. Vetter (Executive Director of the Scientific Manpower Commission) to the annual meeting of the Division of Mathematical Sciences of the National Research Council on March 10, 1969, in Washington, D.C. Copies of the full text may be obtained by writing to the Conference Board of the Mathematical Sciences, 834 Joseph Henry Building, 2100 Pennsylvania Avenue, N. W., Washington, D. C. 20037. Copies of the full text have been sent by the Conference Board to all chairmen of Ph.D. granting departments in the mathematical sciences in the United States and Canada and to the chairmen and members of the Senate and House Armed Services Committees. In addition, the Chairman of the Division of Mathematical Sciences of the NRC has sent a copy of this address to the President of the National Academy of Sciences, recommending that the attention of the President's Science Advisor be invited to the gravity of the situation.]

On June 30, 1967, President Johnson signed the Military Selective Service Act of 1967, thereby extending the power to induct men into military service for another four years. There were a few changes in the legislation from the previous act: (1) The title was changed from the Universal Military Training and Service Act (which it never was) to the Selective Service Act (which Congress apparently meant it to be); (2) Undergraduate student deferment was made mandatory upon request for all full-time students who completed the proper proportion of their degree objective each year; and (3) Legislation was provided for a prime age group from which to call Selective Service registrants (this legislation has never been implemented). It should be noted that provision for undergraduate deferment was accompanied by two penalties. Students who requested and received student deferment forfeited forever their right to automatic deferment if they became fathers (a right reserved to all other American men); and students who were deferred for college were to be placed immediately in the prime age group when deferment ceased, even to age 35. These penalties applied only to baccalaureate students, not to those deferred for any other cause such as farming, apprenticeship, or junior college.

The new draft law left to the President the power to implement the prime age provision or otherwise alter the order of call for induction, except that he could not choose to select by lottery without new legislation. Like all its predecessor legislation, the law left to the President the authority to provide regulations for the deferment of persons whose activities were found to be essential to the national health, safety, and interest, and of persons in training for such activities. Congress added that the National Security Council should advise the President as to which activities were critical so that deferment would be provided for men performing those activities or in training to perform them.

In February 1968, the National Security Council ruled that there was no need for occupational deferments based on any national shortages, and they suspended the Lists of Critical Occupations and Essential Activities which had been used for many years to provide guidance to local boards in determining which of their registrants possessed and were using skills that were in generally short supply and which were essential to the security and welfare of the country. The Security Council further stated that there was no necessity to provide deferment for any students in graduate school, since there were no critical occupations other than those in the medical fields already deferred by law. The Council noted their belief that such deferment was unfair to those persons who did not go to graduate school. As a result, no deferment was provided for students ready to begin their first or second
year of graduate study in the fall of 1968. Those students who had started a Ph.D. program prior to the enactment of the new legislation were to be allowed to finish their degree provided it could be completed within a total of five years past the baccalaureate, and provided their local board was willing to grant such deferment. Despite the fact that the Congress, the Department of Defense, Presidents Johnson and Nixon, the Marshall and Clark Panels, and most educators are on record as favoring an induction order that would focus on men at about their 19th year, it became apparent in February that the President was not going to change the order of call for involuntary induction. That order required drafting the oldest men first from among the combined age group of 19 through 25.

The societies of the Scientific Manpower Commission recently completed a survey* of the draft status of first and second year science graduate students in Ph.D. granting science departments in the fall of 1968. When the foreign nationals were removed from the total number of students reported, 46% of the remaining 17,000 U.S. students (and in particular, 46% of the 2,026 U.S. mathematical science students whose draft classifications were reported) were in draft-liable classification. Under present rules, few if any of these draft-liable students will be able to complete their graduate training to the Ph.D. level. In mathematics, 20% of the first year and 16% of the second year students were foreign nationals.

Only one draftee in 25 had a college degree in May 1968. (Most graduates entered the service as officers, but the officer openings have now long since been filled.) By July 1968, 10% of all new inductees had one or more college degrees; in August, 13.6% were graduates; in September, 19%; in October, 21%; and in November, 22%. The proportion fell to 15% in December, indicating both the higher calls and the large number of graduate students postponed to the end of the quarter or semester. Among the November graduate draftees, 15% had completed at least one year of graduate work. The Department of Defense expects the proportion of college graduates in the inductee population to rise rapidly at the end of the term, to 30% or more, as the postponed graduate students are inducted. [Note: February rise is not as great as anticipated—the great bulk in the graduate ratio (because of lower calls during the first semester) will hit in June and July.] The 17,500 college graduates inducted as draftees between July and January joined the 50,000 graduates who have entered the service as "volunteers" during that period. An additional 7,000 entered the Reserves in the first quarter of the fiscal year, and perhaps 4,000 during the second quarter. Thus, about 78,000 men with one or more college degrees entered military service in the first half of the fiscal year when draft calls were relatively low. It should be noted that this number is about 20% of the total bachelors and first professional degrees granted to men in the 1967-1968 school year. Even if military accessions of college graduates do not rise proportionally during the second half of the fiscal year, they would equal in number about 40% of a year's production of male baccalaureates and first professionals.

Since the military service has no use for the specialized training of most of the college trained inductees, most of our young scientists, engineers, and mathematicians will be out of their professions for at least the man-years of their combined military service. Many never will have practiced in their professions before entering the service, and some certainly will not come back to these professions, either for further training or for work. Among scientists and engineers, the problem of being out of the field for two or more years is more serious than in some of the other disciplines, since science and engineering change so rapidly that the men are, to some degree, both obsolete and rusty when they return to civilian life. This may not be as true for mathematics, but it is a factor for consideration.

For the graduate schools, some choices are possible. To some degree, the loss of U.S. male graduate students to the draft can be made up by the admission of

*A Survey of the Draft Status of First and Second Year Science Graduate Students (Fall 1968). Scientific Manpower Commission, 2101 Constitution Avenue, N.W., Washington, D.C., 20418 ($2.00 postpaid).

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more women, or of more foreign students as obviously has happened already. To some degree, a quality dilution also can occur which will keep graduate enrollment at a level which allows maintenance of the faculty. If enrollment is not kept up, staff losses are inevitable, and a good faculty cannot be reassembled on short notice when a sufficient number of returning veterans will require a full staff. The loss of a substantial number of Ph.D.'s in science and mathematics, as well as in other professions, will create particular problems in providing teachers. This will, of course, affect production of graduate trained professionals not only in the first half of the 1970's, but in the second half of the decade as well when teachers to train new teachers will be in short supply.

We might note the possibility that the technology gap, long acknowledged as a problem for developing countries, could become a problem for us. Russia presently is graduating 140,000 engineers each year compared to our 38,000, although her total population is not much larger than ours. Over the coming decade, she will produce a million more engineers than we, and Russia utilizes her engineers in engineering positions. They are not drafted to serve as truck drivers, stock clerks, riflemen, or typists. In our continuing struggle with communism over the next decade, we may find ourselves at a technological disadvantage because of our present utilization of our most vital natural resource--our trained brainpower.

Can these policies be changed before the cost becomes too high? The President is given by law the authority to provide regulations for the deferment of persons in essential activities and in training for such activities. He can do so at any time. The Congress can change the legislation for the draft to require that the President provide for such deferments. However, action either from the Executive or from the Legislative branch will require, I believe, three conditions that do not now exist. First, the educational community must believe and declare that its activities in the graduate training of professional manpower are essential to the national health, safety, and interest. Not a single group speaking for education has mentioned this in its policy statements about the draft. Such groups generally subscribe to the concept that "drawing straws" might leave them enough graduate students of random quality to carry out their mission. Second, attention from industry is required together with their support for the concept that provision must be made for the training and utilization of professional manpower in the national interest, both in and out of the services. Finally, a change must be made in public opinion by exposing the mythology that surrounds the draft.

Although it is certainly not true that "the poor go to Vietnam and the rich go to college," the phrase is catchy and widely believed. In actual fact, almost half of the young men in the country are found unqualified for service for physical, mental, or moral reasons; and the disqualification rate among the disadvantaged reaches as high as 80%. This myth also clouds another important fact. The preferential drafting of our best trained young men inadvertently denies help to some of our underprivileged, those with ability but with insufficient education and training to make a decent living. The Department of Defense operates the single biggest educational enterprise in the world, and through the period of its history has trained young men in many skills which later can be transferred or modified for use in the civilian world. The pressure of the draft has consistently induced many young men without opportunity or training to enlist in one of the services in order to participate in these training programs. Military service is not and should never be considered a punishment. For all it is an obligation, and for many it is an opportunity. But present draft rules throw the obligation principally on those young men who already have found their path to contributing to the nation with full training; and the opportunity is being lost to those who could most benefit from it. So much for "the poor go to Vietnam, the rich go to college."

Although Negroes in service represent a slightly lower proportion (8.9%) than their proportion within the draft age population (11.2%), many, if not most, persons believe the opposite to be true. What is true is that our very serious efforts to increase the number of Negroes entering graduate school are thwarted by the national decision to turn around and draft this qualified group out of school.

Although it is not true that today's students are a band of revolutionaries out to destroy the institutions of higher educa-
tion, the general public has an uneasy feeling that this is so, and that a little army discipline might just straighten them out. In truth, the small group of dedicated revolutionaries, which gets most of the student publicity, generally is composed of men not eligible for the draft.

Although it is not true that a man can serve his country only by donning the uniform of one of its military services, this theory now dominates the concept of deferment.

And one final myth: The draft "can somehow be made fair, if only we try hard enough." Let us look at this idea. Why is the only "unfair" deferment the one that was given for graduate study? Why are ministers and those in training for the ministry not only deferred but exempt? Do we need to protect all ministers, but no scientists or humanists? Why is there no outcry over the inequity created by disqualifying almost half of our young men because they are mentally, morally, or physically deficient, and sometimes by their own decision? Is fairness achieved by bringing Vietnamese and other foreign nationals here to get a graduate education but drafting American graduate students to fight in Vietnam? Does fairness result from providing permanent deferment to an untrained 19 year old who becomes a father, but denying it to a 23 year old who went to college before taking on the responsibilities of fatherhood?

How about a lottery? Casting lots will not change the inequity inherent in the draft; it will only change the individuals to whom the inequity is applied. Drafting everybody is the answer some people give--both boys and girls when they are 18 or 19, for two years of service to the country in the military or elsewhere--but what is gained by disrupting every life instead of just some? President Nixon has suggested the exact opposite--draft nobody and rely instead on a sufficient number of volunteers to meet our military requirements, by increasing incentives in the military service--but the voluntary army is a wishful dream for the United States, given today's world conditions and given our position in that world.

Will we not reach a better solution if we admit at the outset that there is no way to make the draft fair, when one man is asked to serve and another is not? Our search for equity has made us lose sight of the purpose of the draft which is to help us carry out our constitutional mandate to provide for the common defense. Since the need for our defense is one of many national needs, it should be considered as a part of the big picture, and resources (both human and other kinds) allocated in the most useful manner possible to best serve the national interest.

Remember, Congress renamed the draft legislation so that its title is Selective Service. Just as we select off at the bottom 45% or more that the military says it can't use because they are unqualified, we must select for deferment those whose service to the nation is more valuable temporarily or sometimes even permanently in some other capacity. We do this without question for fathers (except those who want to go to college) and for clergy, but selection for deferment for advanced education or for civilian work, even in direct support of the military, is seen by draft boards and other citizens as "draft dodging." If we will provide reasonable rules for selecting men for military service as well as for deferment or exemption, we will be substituting reason for the emotionalism that now pervades the draft issue. Our system must be flexible enough to serve our national needs in circumstances that vary between relative peace and total war, and to provide us with a continuing new supply of highly trained young people.

The solution that seems evident to me is that we must adopt a system of deferments based on national interest which will consider at any time whether the present activity of any individual is contributing more to the national interest than his present induction into the armed forces would contribute. I do not pretend that such a system would provide equality--only that it would provide the nation with armed forces of whatever size were presently needed, while maintaining a stable program for education at all levels and "fairness" in the form of opportunity for each young man to serve his nation in the place where he could contribute most at any given time. The wasteful use of human resources will cost a great deal more to repair than to prevent, and repair may not be possible at any price.
INSTRUCTIONS FOR AUTHORS OF PAPERS FOR AMERICAN MATHEMATICAL SOCIETY JOURNALS

After July 1, 1969, each article submitted for publication in the Bulletin, Proceedings, Memoirs, and Transactions of the American Mathematical Society and Mathematics of Computation must be accompanied by AMS subject classification numbers, and a list of key words and phrases as well as a descriptive title. In addition, papers for the Proceedings and Transactions must also be accompanied by an abstract. (Bulletin papers will not for the present have abstracts.) The completeness and accuracy of these items will be taken into account in the refereeing and editorial process. They are discussed below.

I. THE ABSTRACTS (Proceedings and Transactions only)

The abstract will be printed right after the title, in a different type face to separate it from the rest of the paper. The main purpose is to enable readers to take in the nature and results of the article quickly, and to enable them to decide if they need to read further. Another purpose is to aid in retrieving information: they may one day be published in a journal of abstracts.

1. Length. At least one sentence, and at most 150 words (for the Proceedings) or 300 words (Transactions). The length will depend primarily on the length of the paper itself so that these upper limits are meant for long papers only. But to some extent, the difficulty of summarizing the material also determines the length.

2. Format. Type it, double-spaced on a separate page, placed at the front of the manuscript. Include the title.

3. Content. Try to state the object of the work, summarize the results, give the principal conclusions. But keep it short.

4. Style. Use full English sentences. Avoid technicalities, since it should be readable by anyone in the general field (analysis, algebra, etc.). Formulas are not particularly desirable, but they may be included (and even numbered) if it seems best to do so. Do not cite bibliographic references, since the abstract should be able to stand alone. Similarly, do not refer by number to some theorem or formula in the body of the article. Incorporate the statements of theorems into complete sentences; thus, "We prove that all hyperloops are quasi-regular" is preferable to "Theorem. All hyperloops are quasi-regular."

5. Relationship to introduction. Occasionally an abstract will make a further introduction unnecessary, and in that case the paper can proceed at once with the mathematics. Usually however the abstract will be too brief, and an introduction enlarging upon it, providing more background, describing earlier work, etc., will be called for.

Currently, the new Journal of Number Theory and the Pacific Journal are printing abstracts. Those in the former journal are, for the most part, excellent. Abstracts in the Pacific Journal, though in general good, tend to be somewhat longer (compared to the length of the paper) than the type of abstracts we are asking for.

II. DESCRIPTIVE TITLE

Choose a title which is as informative as possible. It should identify clearly both the general field of the paper and the particular branch of it being considered. If you can include more, well and good, but it should not run to more than 10-12 words, and fewer words are better. Avoid jargon words which tell nothing and waste space, like "concerning", "a remark about", "some contributions to the theory of". Avoid proper names unless mathematical usage associates them with the work; "Concerning some applications of a theorem of J. Zilch" is a good example of a useless title. Note that titles may be used in information retrieval, so that every word in them ought to count.

III. AMS SUBJECT CLASSIFICATION NUMBERS

These numbers classify the paper by field. They will be used both for information retrieval and for the offprint ser-
vice. A list of the numbers may be found at the end of each volume of Mathematical Reviews. You should give numbers representing both the primary and secondary subjects of your article. Since this classifying scheme may be revised frequently, indicate the year of the Reviews from which you took the scheme. The numbers will be printed in a footnote on the first page of your article.

IV. KEY WORDS AND PHRASES

For the sake of nonspecialists (such as librarians), as well as to aid in future information retrieval, a list of key words and phrases should be also included. The list should be generous; however it should not include words which would appear in almost any paper in the general field. Thus "cohomology group" could be appropriate for a paper in Lie algebras, but not for one in algebraic topology.

Style. Use nouns, adjective-nouns, etc. in the natural groupings in which they occur in the paper (e.g., differential form, locally compact groups, deformations of algebras). Proper adjectives are especially useful in pinpointing the subject matter (e.g., Sobolev space, nondesarguesian planes). Reduce compounds to their components, so that "convergence in mean or measure" would reduce to "convergence in mean, convergence in measure". The phrases and words should be taken from the body of the paper, the title, and the abstract. Avoid lengthy phrases; in general an item should have four words or less. Use as many as needed to distinguish the field of the paper clearly and specifically.

Format for subject numbers and key words. These will both be printed as a footnote to page one of the article. Therefore, include them with the footnotes to your paper, but placed before the first footnote. Thus, after the bibliography, type:

Footnotes

AMS Subject Classifications: Primary 2307, 2354; Secondary 2204

Key Phrases: Analytically unramified ring, semi-local ring, Dedekind domain, altitude formula, Jacobson radical, Rees ring, analytically irreducible domain, unmixed domain

1. This research was supported in part by the Ring-theory Foundation, under contract no. 25-35-7002.

2. . . . etc.

EXAMPLES


Abstract: In recent papers, P. B. Bailey and M. Godart have used the Prüfer transformation to calculate the eigenvalues of nonsingular and some singular Sturm-Liouville boundary value problems. In the present paper, the existence of a general class of a singular problem which may be solved in a straightforward manner using the Prüfer transformation is established. Some examples of the method are given. Finally, by introducing a modified transformation the class of problems to which the method is applicable is extended.

AMS Subject Classifications: Primary 6540, 6562


* * * * *

**Abstract:** A model source is constructed the radiation from which (scalar, electromagnetic or gravitational) is concentrated in a jet of small angle with an assigned target direction. This source is taken to be an infinite train of high-frequency plane waves travelling in the target direction with the basic speed of propagation, the amplitude falling off exponentially with distance. The critical number, to be made large in this model, is the ratio \( a / \lambda \) where \( a \) is a typical radius of the source and \( \lambda \) the wave length. It is also shown that a jet of scalar radiation may be obtained from a source which possesses no frequency but consists of a single shock wave.

**AMS Subject Classifications:** Primary*7675,*7845,*8320,*7370; Secondary 7635, 7835, 8335, 8353

**Key Words and Phrases:** Scalar radiation, electromagnetic radiation, gravitational radiation, plane wave, shock wave, space time, Ricci tensor, shock-jets

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**Abstract:** Let \( \sigma \) be an integral domain containing the rational numbers, \( \Sigma \) its quotient field, \( D \) a derivation of \( \Sigma \) and \( \sigma' \) the ring of elements in \( \Sigma \) quasi-integral over \( \sigma \). It is shown that if \( D \sigma \subseteq \sigma \), then \( D \sigma' \subseteq \sigma' \).

**AMS Subject Classifications:** Primary 1360, 1393; Secondary 1315

**Key Words and Phrases:** Derivation, formal power series, integrally closed domain, quasi-integral dependence, Hasse-Schmidt differentiations, Noetherian domain

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LETTERS TO THE EDITOR

Editor, The Notices

I should like to express the opinion that use of the section, Letters to the Editor, in the Notices seems to be becoming increasingly abused in the form of advocacy of political views which bear only obliquely on the advancement of the mathematics profession. The letter from Professor Lang in the April issue is a striking case in point, and I suggest that he has previously received more than his share of space in this forum.

Daniel H. Wagner

Editors' Note:
The publication of the Letter to the Editor from Serge Lang on pages 483-484 of the April Notices was approved by the Council at its meeting of January 22, 1969. The approval covered the use of the pages of the Notices for a letter of length exceeding 1000 words and did not constitute an endorsement of the thoughts and sentiments expressed in the letter.

Editor, The Notices

Recently Professor Hans Rademacher passed away. He was world renowned for his work in Analytic Number Theory. Perhaps, it is not so well known that he was a mathematician of great breadth, a true mathematical scholar, and an excellent teacher. It is unfortunate that he did not publish more books, expository articles, and lecture notes to convey his excitement for mathematics and his unique ability to link problems and research, the pure and the applied, the classical and the modern. It would be a great service to the mathematical community if someone were to publish his works including lecture notes and problem collections.

Dr. William G. Spohn, Jr.

Editor, The Notices

I read all the review journals, Mathematical Reviews, Zentralblatt fur Mathematik, and Referativni Zhurnal Matematika, and study over hundreds of other research journals. I enjoy the study of various languages as well. But why is it that the Russian review journal persistently prints the Hebrew letter Aleph upside down? Anyone who will examine any current or past issue will see what I mean. Why does no one in the USSR complain? ... The situation has become so perverse that now the Zentralblatt seems to be copying the Russian upside-down transfinite numbers! See e.g. Zentralblatt, Vol. 154 (1968), page 252. See Referativnyi Zhurnal, 1968, reviews 5 A442, 443, 444 and 12 A65, for blatant examples.

We are all probably willing to admit that the aleph is confusing to most people, whether right-side up or left-side down. But after all the letter is used, and has been used since Cantor started it, I wish that anyone who can have any influence anywhere will urge those responsible to see that the aleph is restored to its rightful appearance. For once and all, it looks like this:

Henry W. Gould

Editors' comment: The Editors have abbreviated the letter presented. The head of our editorial department tells us that many years ago the matrix for "aleph" was constructed upside down by a Monotype company. The slug has an orientation based on grooves in a vertical face so that, once the error is made, it is perpetuated in subsequent uses because the slug will not fit into the matrix case in correct position; a hand operation is required to put the type into proper position. As a result of seeing the letter in two different positions, the untutored are confused; but this is no excuse for not getting it right.
PERSONAL ITEMS

Professor B. H. ARNOLD of Oregon State University will be on leave for the academic year 1969-1970 teaching at the National Taiwan Normal University, Taipei, Taiwan, under a Fulbright grant.

Professor D. D. AUFENKAMP, on leave from Oregon State University, has been appointed a staff associate at the National Science Foundation Office of Computing Activities, Washington, D. C.

Mr. PAZ AZARIA of the University of California, Berkeley, has been appointed a senior lecturer at Israel Institute of Technology, Haifa, Israel.

Dr. ROBERT BUMCROT of Ohio State University has been appointed to an associate professorship at Hofstra University.

Professor L. M. CHAWLA of Government College, Lahore, India, has been appointed Principal of the Central Training College, Lahore, India.

Professor T. R. CHOW of Oregon State University has been appointed to a visiting assistant professorship at the University of British Columbia for the academic year 1969-1970.

Professor JOEL DAVIS of Oregon State University has been appointed to a visiting assistant professorship in the Computer Science Department of the University of Illinois for the academic year 1969-1970.

Dr. R. S. DORAN of the University of Washington has been appointed to an assistant professorship at the University of Northern Iowa.

Mr. C. G. FAIN of the University of Oklahoma has been appointed a member of the technical staff at Communications and Systems, Inc., Falls Church, Virginia.

Dr. LAWRENCE FEINER of the Massachusetts Institute of Technology has been appointed to an assistant professorship at the State University of New York at Stony Brook.

Mr. J. D. FULTON of Oak Ridge National Laboratory has been appointed to an assistant professorship at Clemson University.

Mr. J. A. GOGUEN of the University of California, Berkeley, has been appointed to an assistant professorship at the University of Chicago.

Professor OTOMAR HAJEK of Caroline University, Czechoslovakia, has been appointed to an associate professorship at Case Western Reserve University.

Mr. H. M. HASTINGS of Princeton University has been appointed to an instructorship at Hofstra University.

Professor SEYMOUR HAYDEN of Clark University has been appointed to a professorship at the City University of New York, Herbert H. Lehman College.

Mr. HERMANN HEINEKEN of the University of Frankfurt, Germany, has been appointed to dozent at the University of Erlangen-Nuernberg, Germany.

Dr. M. S. HENRY of Colorado State University has been appointed to an assistant professorship at Montana State University.

Professor SIN HITOTUMATU of St. Paul's University, Tokyo, Japan, has been appointed to a professorship at the Research Institute for Mathematical Sciences, Kyoto University, Japan.

Mr. R. B. KIRK of the California Institute of Technology has been appointed to an assistant professorship at Southern Illinois University.

Professor M. I. KNOPP of the University of Wisconsin, Madison, has been appointed to a visiting professorship at the University of Basel, Switzerland, for the summer of 1969.

Dr. M. A. KNUS of Brandeis University has been appointed to a research assistantship at the University of Geneva, Switzerland.

Dr. G. E. LADAS of New York University has been appointed to an assistant professorship at Fairfield University.

Mr. N. T. LOSITO of St. John's University has been appointed to an assistant professorship at the State University of New York, Agricultural and Technical College at Farmingdale.

Mr. PEDRO NOWOSAD of Stanford University has been appointed to a visiting assistant professorship at the U. S. Army
Mr. R. D. POLLACK of Trinity College has been appointed to an assistant professorship at Queen's University.

Mrs. J. R. ROSENBLATT of the Applied Mathematics Division of the National Bureau of Standards has been named Chief of the Statistical Engineering Laboratory at the Bureau.

Dr. ROBERT SILBER of Clemson University has been appointed to an assistant professorship at North Carolina State University.

Dr. FRANK STENERG of The University of Michigan has been appointed to an associate professorship at the University of Utah.

Professor D. J. UHERKA of Arizona State University has been appointed to an associate professorship at the University of North Dakota.

Dr. DONALD WEHN of the State University of New York at Stony Brook has been appointed to an associate professorship at Hofstra University.

Mrs. DIANA YUN-DEE WEI of Marianopolis College, Montreal, has been appointed to an assistant professorship at Sir George Williams University.

PROMOTIONS

To Professor. Columbia University: C. K. CHU.

To Associate Professor. University of Oklahoma: D. C. KAY; Pennsylvania State University: F. P. CALLAHAN; Purdue University: RICHARD HOLMES; Texas A&M University: J. D. BRYANT.

To Assistant Professor. Florida Presbyterian College: V. W. MORRISON.

To Instructor. University of West Florida: J. J. LEESON.

To Director of Management Control Systems. U.S. Plywood-Champion Papers: L. E. DE NOYA.


To Operations Analyst. Los Angeles Technical Services Corporation: J. J. CAMPBELL.

To Numerical Analyst. Grumman Aircraft, Bethpage, New York: ANTHONY SPINGOLA.

DEATHS

Professor J. H. BARRETT of The University of Tennessee died on January 21, 1969, at the age of 47. He was a member of the Society for 21 years.

Mr. H. G. BRINKMAN of the University of Groningen, Germany, died in September, 1968, at the age of 61. He was a member of the Society for 7 years.

Professor EDWARD HALPERN of The University of Michigan died on January 31, 1969, at the age of 51. He was a member of the Society for 19 years.

Mr. BEN ISQUITH of Rockville, Maryland, died in April, 1968, at the age of 40. He was a member of the Society for 10 years.

Dr. H. C. LEVINSON of Kennebunk, Maine, died on September 20, 1968. He was a member of the Society for 41 years.

Professor Emeritus G. Y. RAINICH of The University of Michigan died on October 10, 1968. He was a member of the Society for 45 years.

Professor Emeritus H. S. THURSTON of the University of Alabama died on March 1, 1968, at the age of 73. He was a member of the Society for 44 years.

Mr. K. B. TUTTLE of Hughes Aircraft died on December 30, 1968, at the age of 62. He was a member of the Society for 8 years.
REFERENDUM
A REPORT TO THE MEMBERSHIP

The Council met in New York on April 4, 1969. They considered the following resolution:

B. Whereas the American Mathematical Society encourages all persons interested in mathematical research to be members and whereas these members hold a wide variety of political and social views and have been welcomed to membership without regard to these views, resolved that the Society shall not attempt to speak with one voice for the membership on political and social issues not of direct professional concern and shall adhere closely to the purpose stated in its Articles of Incorporation of "furtherance of the interests of mathematical scholarship and research."

It has been labeled "B" for reference. After extended discussion the Council voted unanimously to present resolution B to the membership for a referendum by mail ballot, with Council recommendation to the members to vote FOR the resolution B.

The Council considered five resolutions initiated at the Business Meeting of January 25, 1969. In so doing, they acted by virtue of the powers conferred in Article IV, Section 8, of the By-laws. The resolutions, numbered for reference, are the following:

1. Resolved, that since scientific discovery by its nature requires complete open channels of information, it follows that classified research is a contradiction in terms. Members should consider most seriously participation in any investigation under a contract restricting full exchange of information with learned men everywhere, and as a society we recommend that members seek to disengage themselves from such activity.

2. Resolved, that the American Mathematical Society urges each of its members to use his talents in ways that promote peace and to refrain from activities whose primary purpose is to promote warlike efforts.

3. Resolved, that a committee be appointed to study the causes and course of the current worldwide upheaval in relationships among faculty, students, and administration in higher education, with particular reference to the situation at San Francisco State College. This committee shall report to the members with recommendations for suitable action, in the Notices of the Society.

4. Resolved, that the Notices shall be open for letters and articles discussing issues which concern the members as scholars and citizens generally as well as mathematicians particularly.

5. Whereas the shortage of mathematicians in North American Universities is different and greater among black and brown Americans than among whites, and whereas this situation is not improving, be it resolved that the AMS appoint a committee composed of black and third world mathematicians to study this problem and other problems concerning black and third world mathematicians, and report their conclusions and recommendations to the Society.

The Council recommended, by a vote of 29 to 1, that each resolution 1 through 5 be presented to the membership for a referendum by mail, with Council recommendation to the members that they vote AGAINST each resolution 1 through 5 as a consequence of the position taken by the Council in resolution B.

The Council was of the opinion that the referendum should take place soon after the April Council meeting but did not include a schedule as part of their resolutions. The Executive Committee considered various schedules and favored holding the referendum immediately. As a result, the members will probably have received their ballots before they read this account.

The Secretary followed the relevant rules and practices that apply to an annual election of officers in preparing and distributing the ballots. Announcement of results can be expected no later than August 1969.

Everett Pitcher
Secretary
TRANSLATIONS OF
MATHEMATICAL MONOGRAPHS

Volume 18
INTRODUCTION TO THE THEORY OF LINEAR NONSELFADJOINT OPERATORS
By I. C. Gohberg and M. G. Kreǐn

396 pages; List Price $21.40; Member Price $16.05

The theory of nonselfadjoint operators in Hilbert space is a recent branch of functional analysis. In recent times, it has attracted the ever increasing attention of mathematicians and physicists, and sometimes of engineers also. The aim of this book is to present a number of achievements in this field, most of them related to the theory of completely continuous operators.

The authors discuss the well-known results of the general theory of bounded nonselfadjoint operators; the theory of symmetrically-normed ideals of the ring of bounded operators in a Hilbert space; the theory of perturbation of determinants and some of its applications; the various theorems on the completeness of the system of root (eigen- and associated) vectors of a completely continuous operator (operator bundle); the study of the spectral properties of a selfadjoint quadratic bundle; the simplest tests for the existence of a basis (of one kind or another) made up of the root vectors of a given linear operator.

The presentation in this book is carried out in the spirit of the abstract theory of operators. It is illustrated by various applications to the theory of integral equations. The reader who has some experience in the theory of boundary value problems for differential equations, or an acquaintance with the theory of linear vibrating systems with a finite or infinite number of degrees of freedom, will easily discover how many of the results discussed here find immediate application in each of these fields.

PROCEEDINGS OF THE STEKLOV INSTITUTE

Number 90
GEODESIC FLOWS ON CLOSED RIEMANN MANIFOLDS OF NEGATIVE CURVATURE
By D. V. Anosov

240 pages; List Price $15.20; Member Price $11.40

The methods and results of this monograph are chiefly based on the fact that a geodesic flow on a closed Riemannian manifold of negative curvature satisfies a so-called (U)-condition, roughly expressible as follows: near an arbitrary fixed trajectory of the dynamical system, the behavior of the neighboring trajectories is similar to that of trajectories close to a saddle. Numerous examples are given of (U)-flows (continuous time) and (U)-cascades (discrete time). Most important among the many results is the theorem that every (U)-system is structurally stable in the sense that for an arbitrary, sufficiently small perturbation, there exists a homeomorphism of the phase space which is close to the identity and takes the trajectories of the unperturbed system into those of the perturbed system.

Number 94
EXTREMAL PROBLEMS OF THE GEOMETRIC THEORY OF FUNCTIONS
Edited by Ju. E. Alenicyn

176 pages; List Price $11.80; Member Price $8.85

This volume is a collection of papers on various problems in the geometric theory of functions of a complex variable. For the most part, the papers are the work of students of Gennadiĭ Mihailović Goluzin and are related to problems with which he has been concerned. The authors are Ju. E. Alenicyn, S. A. Gel'fer, E. G. Goluzina, G. V. Kuz'mina, N. A. Lebedev, I. A. Aleksandrov, I. M. Milin, M. I. Revjakov, G. A. Skotnikova, and N. M. Gol'dina. The work of the last three authors is based on theses written at Leningrad State University.
TRANSLATIONS—SERIES II

Volume 77
FOURTEEN PAPERS ON SERIES AND APPROXIMATION
272 pages; List Price $13.60; Member Price $10.20

This volume contains the following papers: L. A. Balasov, Series with gaps; V. I. Berdyšev, Mean approximation of periodic functions by Fourier series; R. Bojanic and M. Tomic, On the absolute convergence of Fourier series with small gaps; I. I. Cyganok, A generalization of Jackson’s theorem; A. V. Efimov, On best approximation of classes of periodic functions by means of trigonometric polynomials; M. A. Jastrebova, On the approximation of functions satisfying a Lipschitz condition by the arithmetic means of their Walsh-Fourier series; P. I. Lizorkin, Estimates of trigonometric integrals and the Bernstein inequality for fractional derivatives; R. I. Osipov, On the representation of functions by orthogonal series; Ju. K. Suetin, Convergence and unicity constants for certain interpolation problems; S. A. Teljakovskiĭ, Two theorems on the approximation of functions by algebraic polynomials; M. F. Timan, The best approximation of a function and linear methods for the summation of Fourier series; G. C. Tumarkin, Approximation with respect to various metrics of functions defined on the unit circle by sequences of rational fractions with fixed poles, and Necessary and sufficient conditions for the possibility of approximating a function on a circumference by rational fractions, expressed in terms directly connected with the distribution of poles of the approximating fractions; I. M. Vinogradov, Estimation of trigonometric sums.

Volume 78
ELEVEN PAPERS ON TOPOLOGY
256 pages; List Price $12.80; Member Price $9.60

This volume contains the following papers: A. V. Arhangel’skiĭ, Closed mappings, bicompact sets and a problem of P. S. Aleksandroff; P. K. Belobrov, On the Čebyšev point of a system of sets; A. V. Černavskiĭ, Homeomorphisms of $\mathbb{R}^n$ are $k$-stable for $k \leq n - 3$, and Homeomorphisms of Euclidean space and topological imbeddings of polyhedra in Euclidean spaces. I; A. S. Grek, Regular polyhedra on a closed surface whose Euler characteristic is $\chi = 3$; Liao Shan-dao, Periodic transformations and fixed point theorem. I; F. I. Smidov, On the structure of point sets in three-dimensional space; A. B. Sosinskiĭ, Monotonically-open mappings of a sphere; N. V. Veličko, $H$-closed topological spaces; Wu Wen-jun, On the realization of complexes in Euclidean spaces, I, and On the realization of complexes in Euclidean spaces. II.

Volume 79
THIRTEEN PAPERS ON FUNCTIONAL ANALYSIS AND DIFFERENTIAL EQUATIONS
276 pages; List Price $13.80; Member Price $10.35

This volume contains the following papers: V. I. Arnol’d, On an a priori estimate in the theory of hydrodynamical stability; O. V. Besov, Continuation of functions beyond the boundary of a domain with preservation of differential-difference properties in $L^p$; V. I. Gurariĭ, Bases of spaces of continuous functions on compacta and some geometric problems; T. M. Karasëva, Rate of growth and boundedness of solutions of second-order differential equations with periodic coefficients; V. Ja. Lin, On equivalent norms in the space of square summable entire functions of exponential type; O. B. Lykova, Generalization of a theorem of Bogoljubov to the case of Hilbert space; Ju. A. Mitropol’skiĭ and A. M. Samoilenko, On the construction of solutions of linear differential equations with quasiperiodic coefficients by the method of accelerated convergence; V. V. Nemyčkiĭ and Ju. V. Maljšev, Weak structural stability of homogeneous systems; V. I. Paraska, On the asymptotics of eigenvalues and singular numbers of linear smoothing operators; I. I. Smulev, Periodic solutions of the first boundary problem for parabolic equations; Ju. K. Solncev, On the estimation of a mixed derivative in $L^p(G)$; I. V. Stankevič, On linear similarity of certain nonselfadjoint...
operators to selfadjoint operators and on the asymptotic behavior for $t \to \infty$ of the solution of a nonstationary Schrödinger equation; S. A. Teljakovskii, On the norms of linear polynomial operators.

Volume 80

THIRTEEN PAPERS ON FUNCTIONS OF REAL AND COMPLEX VARIABLES
288 pages; List Price $14.20; Member Price $10.65

This volume contains the following papers: V. S. Azarin, Generalization of a theorem of Hayman on subharmonic functions in an $m$-dimensional cone; N. N. Čaus, On the representation of continuous positive-definite kernels; Ju. F. Korobeǐnik, On completeness of a set of analytic functions; B. M. Levitan, Integration of almost-periodic functions with values in a Banach space; M. S. Mel'nikov, Estimate of the Cauchy integral along an analytic curve; S. M. Nikol'ski!, Inequalities for entire functions of exponential type and their application to the theory of differentiable functions of several variables, and Properties of certain classes of functions of several variables on differentiable manifolds; G. H. Sindalovskii, On total differentials; G. P. Tolstov, Differentiation and integration in abstract spaces; A. G. Vituškin, Estimate of the Cauchy integral; V. A. Zmorovič, On bounds of convexity for starlike functions of order $\alpha$ in the circle $|z| < 1$ and in the circular region $0 < |z| < 1$, and On a class of extremal problems associated with regular functions with positive real part in the circle $|z| < 1$, and On the bounds of starlikeness and univalence in certain classes of functions regular in the circle $|z| < 1$.

SIAM-AMS PROCEEDINGS

Volume 1
TRANSPORT THEORY
Edited by G. Birkhoff and R. Bellman
336 pages; List Price $11.00; Member Price $8.25

The industrial and military applications of atomic energy have stimulated much mathematical research in neutron transport theory. The possibility of controlled thermo-nuclear processes has similarly focused attention upon plasmas; and many classical aspects of kinetic theory and radiative transfer theory have been studied both because of their basic mathematical interest and because of their physical applications to areas such as upper-atmosphere meteorology. The mathematical difficulties in all of the areas cited are notoriously formidable. As a consequence, ingenious techniques have been developed to handle particular questions.

This volume is a collection of the methods and results of a number of experts in the field of transport theory. There are four main parts: Analytical Neutron Transport; Numerical Neutron Transport; Stochastic Aspects; Kinetic Theory and Plasma Transport. Each part contains four expository accounts of recent developments. These papers provide an excellent and comprehensive survey of the current state of transport theory and should serve as a stimulus for research in this field.

LECTURES IN MATHEMATICS IN THE LIFE SCIENCES

Volume 1
SOME MATHEMATICAL PROBLEMS IN BIOLOGY
Edited by Murray Gerstenhaber
122 pages; List Price $6.10; Member Price $4.58

A new series of publications is now being offered by the American Mathematical Society. The first volume, entitled Some Mathematical Problems in Biology, is available and contains articles by E.G. Leigh, R. C. Lewontin and Theodosios Pavlidis.

The text of the first volume is comprised of lectures given at a symposium held in conjunction with the December 1966 meeting of the American Association for the Advancement of Science. Sponsored by the American Mathematical Society and SIAM, the meeting dealt with mathematical theories from a biological point of view. The symposium was supported by a grant from the United States Steel Foundation, Incorporated.
The Australian Mathematical Society will begin publication of a Bulletin during the second half of 1969. One volume, consisting of three numbers, will be published in 1969, and two volumes will be published each year thereafter. The subscription price will be $7.50 (A) in 1969 and $15.00 (A) for subsequent years. For members of the Australian Mathematical Society (including members under reciprocity agreements), the price will be $4.00 (A) and $8.00 (A). Those wishing subscriptions should write to the Hon. Treasurer, Australian Mathematical Society, University of Queensland, St. Lucia, Queensland, Australia.

The Bulletin will publish research papers, detailed research announcements, and brief abstracts of Australian doctoral theses. The emphasis will be on rapid publication. Papers for publication should be sent to Professor B. H. Neumann, Department of Mathematics, Institute of Advanced Studies, Australian National University, POB 4, Canberra, ACT 2600, Australia.

BACK ISSUES OF THE TRANSACTIONS

The TRANSACTIONS, publication of which began in 1900, is devoted to the publication of research in pure and applied mathematics. As a result of a questionnaire distributed by the Society in 1968, it was decided to offer a microfilm edition of Volumes 1-77 (1900-1954). The projected time of release of this microfilm edition is the summer of 1969. The exact price has not yet been set, but it will be approximately one cent per page. Further information may be obtained by writing to University Microfilms, Ann Arbor, Michigan 48106.

These same volumes may also be obtained in either a clothbound or paperbound edition from the Johnson Reprint Corporation, 111 Fifth Avenue, New York, New York 10003. These paper reprints are obtainable for an average price of three cents per page and may be purchased as single volumes or in a full set.

ABSTRACTS FOR 1970 ANNUAL MEETING

As a result of Council action on April 4, 1969, there will be no limit on the number of abstracts that will be accepted for the annual meeting in January 1970. However, only the first 720 abstracts received will be assigned to day sessions because of space limitations. Abstracts received after the first 720 will be assigned to general evening sessions. The deadline for abstracts will be November 6, 1969. Because of the short period of time that is available to prepare the program for the annual meeting, the Providence office will not be able to accept changes in abstracts. Authors are requested to notify the office of papers to be withdrawn.

POLICY ON REPRINTS

In the February 1969 issue of the Notices, page 381, a new policy on reprints for authors was announced. This policy is being reconsidered and will not be put into effect at the present time.
In July 1969, the Mathematical Offprint Service (MOS) will have been in operation for one year. During this period, changes and modifications have been made in the system, many of them at the request of subscribers. As the service enters its second year, new services are being added that will make MOS more useful to subscribers. In addition, a number of additional services are planned for implementation on a pilot basis during the coming year.

The number of journals (90) now participating in MOS has limited the number of subscribers, and a decision has been reached to expand the service by removing the restriction on the number of participating journals. Coverage will be extended to most of the journals participating in Contents of Contemporary Mathematical Journals (CCMJ), these being the proceedings of Symposia in Pure Mathematics and the SIAM-AMS Proceedings of Symposia in Applied Mathematics are included in the service at the present time; and reprints from the proceedings of other meetings will soon be available.

At present time, a title is listed at the galley stage and sent to subscribers. This is often far in advance of the appearance of the journal, and some subscribers have requested that they receive title listings at the approximate time of the appearance of the journals. Some subscribers still want the earlier listing so they can obtain preprints from authors. It will soon be possible for them to select the date they receive title listings.

A minor change has been made in the pricing policy. Offprints, which have been priced at $0.30, will range from $0.25 to $0.50, the price depending upon the size of the offprint.

**NEWS ITEMS AND ANNOUNCEMENTS**

**ALSTON S. HOUSEHOLDER**

**DOCTORAL AWARD ESTABLISHED**

Dr. Alston S. Householder, director of the Mathematics Division of Oak Ridge National Laboratory, retired in May and was honored by the establishment of a $400 prize in his name. This prize is to go to the author of the best doctoral dissertation in numerical algebra. The award will be presented at the Fifth Gatlinburg Symposium in 1970; and the judging of the dissertations will be by an international committee composed of J. H. Wilkinson of the National Physical Laboratory in England, F. L. Bauer of the Mathematische Institut in Munich, and Olga Taussky-Todd of the California Institute of Technology.

Dr. Householder has been a member of the scientific staff of the Oak Ridge National Laboratory for more than 22 years. He will continue to serve as interim chairman of the mathematics department of the University of Tennessee, where he has also served as a professor for many years.

**INTERNATIONAL SUMMER SEMINAR ON THEORY OF ORGANIZATIONAL SYSTEMS**

An international summer seminar on Theory of Organizational Systems will be held in Dubrovnik, Yugoslavia, on August 4-16, 1969. Sessions will consist of one or two formal lectures followed by a discussion period. Two half-day sessions will be reserved for round table discussions, and there will be a closing session which will provide guidelines for further research in the area of organizational systems. All sessions will be held at the Arts Gallery, Fran Supila 19, Dubrovnik.

The number of participants will be limited. For registration forms and further information, write to L. Radanović, Center of Advanced Studies, P. O. Box 356, Belgrade, Yugoslavia.
The 12th Biennial International Seminar of the Canadian Mathematical Congress will be held at the University of British Columbia in Vancouver, Canada, from August 11 through August 27, 1969. This seminar will be followed by the annual meeting of the Canadian Mathematical Congress at the University of Victoria in Victoria, Canada, from August 28 through August 30.

The theme of the seminar will be Time Series and Stochastic Processes; Convexity and Combinatorics. Four series of research lectures at the postdoctoral level will be given: "Dual processes and potential theory" by R. K. Getoor; "Convexity and combinatorics" by V. L. Klee; "Current problems in time series" by E. Parzen; "Applications of probability theory to other areas of mathematics" by A. Rényi. Three series of instructional lectures at the predoctoral level will be given: "Frequency analysis of relations between stationary time series" by D. R. Brillinger; "Markov chains: potentials and boundaries" by D. A. Dawson; "Combinatorial graph theory: tree and extremal problems" by J. W. Moon.

Seminar participants, both graduate students and postdoctoral fellows, will be chosen primarily from Europe, the United States, and Canada. Limited participant support will be available. The lectures and living accommodations will be located in the University Totem Park residence of the University of British Columbia. Further information about the seminar and application forms may be obtained from Professor Ronald Pyke (program chairman), Department of Mathematics, University of Washington, Seattle, Washington 98105.

The Third Symposium on Inequalities will be held on September 1-9, 1969, at the University of California, Los Angeles. The symposium will be supported by the Mathematics Division, Air Force Office of Scientific Research and the Aerospace Research Laboratories, Wright-Patterson Air Force Base. Included in the program will be one-hour lectures, 15-minute talks, as well as informal workshops. The program will be devoted to topics in which inequalities play a fundamental role, as well as to classical inequalities, their generalizations, and related matters.
NEWS ITEMS AND ANNOUNCEMENTS

NSF DEPARTMENTAL SCIENCE DEVELOPMENT PROGRAM

The National Science Foundation has awarded $7,241,670 in 12 grants under the Departmental Science Development Program. This program is designed to improve the quality of research and education in individual areas of science and engineering at universities operating at the graduate level. The program is intended for institutions that have graduate programs in the areas of science, engineering, or mathematics at the master's or doctorate level; and each grant supports a specific area in which the institution already has sufficient strength to serve as a base for significant improvement to a higher level of capability. Grants are based on plans developed by institutions to implement long-term objectives for improvement of the scientific and engineering programs.

The State University of New York at Albany has received one of the grants for a three-year development plan that will strengthen the Department of Mathematics and its newly initiated Ph.D. program. The grant will provide funds for new faculty members, postdoctoral associates, graduate students, and secretarial help. The faculty will expand to 44 in three years. Funds will also be provided for purchasing supplies and library materials.

NATIONAL ACADEMY OF ENGINEERING

The National Academy of Engineering has announced the election of 44 engineers to the Academy. The National Academy of Engineering was established on December 5, 1964, as an organization of distinguished engineers, parallel to the National Academy of Sciences, autonomous in its administration and in the selection of members, and sharing with the Academy of Sciences the responsibility for advising the federal government. Among the new members is Dr. Brockway McMillan, Executive Director of the Military Research Division of Bell Telephone Laboratories, Inc., Whippany, New Jersey. Dr. McMillan was elected to membership because of his leadership in systems engineering in the field of communications and in large military projects in industry.

NYU'S GREAT TEACHERS AWARD FOR 1969

Professor Wilhelm Magnus of the Courant Institute of Mathematical Sciences was one of the three faculty members of NYU to be named a Great Teacher for 1969. Great Teachers have been selected annually since 1959 by a committee of elected University Senate members of NYU for "singular accomplishment in leading students to knowledge and understanding, and ... dedication and intellectual integrity representative of the highest ideals of the teaching profession." Professor Magnus received his doctorate in mathematics from Frankfurt University in 1931 and came to the United States in 1948 to join the staff of the California Institute of Technology as a research associate at the California Institute of Technology. He joined the staff of NYU in 1950.

RESEARCH ASSOCIATESHIPS IN MATHEMATICS

The Office of Naval Research has announced the award of 14 postdoctoral research associateships in mathematics for the academic year 1969-1970. These awards were made upon the recommendation of the ONR Advisory Committee on Mathematics. The following mathematicians were recipients of the awards: David K. Cohoon, University of Wisconsin; Jerome Dancis, University of Chicago; David E. Dobbs, University of California, Los Angeles; Robert D. Edwards, Princeton University; Frank L. Gilfeather, Indiana University; Gary R. Jensen, Washington-
ton University; Soji Kaneyuki, Washington University; James S. Milne, University of Michigan; Karl K. Norton, University of Michigan; Patrick E. O'Neil, Massachusetts Institute of Technology; John G. Pierce, New York University; Laurence C. Siebenmann, Princeton University; Rémi Vaillancourt, University of Chicago.

JAMES CRAIG WATSON MEDAL OF THE NATIONAL ACADEMY OF SCIENCES

At the 106th annual meeting of the National Academy of Sciences, one Australian and four U. S. scientists were honored for their outstanding contributions to research. Among them was Professor Jurgen K. Moser, Director of the Courant Institute of Mathematical Sciences. Professor Moser was awarded the James Craig Watson Medal for his mathematical contributions to dynamical astromony. His investigations relevant to the award are connected with his work in celestial mechanics. In 1968, Professor Moser was awarded the first George David Birkhoff Prize in Applied Mathematics by the American Mathematical Society and the Society for Industrial and Applied Mathematics.

NATIONAL SCIENCE FOUNDATION POSTDOCTORAL FELLOWSHIPS

The National Science Foundation has announced the names of the 130 recipients of Postdoctoral Fellowships. The recipients of these awards are all holders of doctoral degrees, or the equivalent, in the mathematical, physical, engineering, social, and life sciences; and they were selected from among 1,087 applicants. Each of the awards includes an annual stipend of $6,500, an allowance for dependents, and an allowance to help defray travel costs to the fellowship institution. Fellowship recipients in this program will study or carry on research at institutions in the United States and abroad.

Following are the mathematicians who received the Postdoctoral Fellowships: Allen Reiter, Lockheed Aircraft Corporation; Gintaras V. Reklaitis, Stanford University; John W. Bunce, Tulane University; Robert R. Kallman, Massachusetts Institute of Technology; Morris Goldfeld, Columbia University; Clifford S. Queen, Ohio State University; Eugene Switkes, Harvard University; Charles F. Miller, III, University of Illinois; Kenneth C. Land, University of Texas; and James W. Cannon, University of Utah.

NSF expects to reopen the program in October for awards to be made in March 1970.

ALFRED P. SLOAN FOUNDATION FELLOWSHIPS

Fellowships for basic research have been awarded by the Alfred P. Sloan Foundation to 76 young physical scientists. These fellows were selected from among 650 nominations. The committee which advises the Foundation on final selections, is composed of six senior scientists, including two mathematicians, Professor Lipman Bers and Professor R. H. Bing. The average age of the recipients is under 30; and the fellowships are designed to assist young scientists in carrying forward their research at an early stage of their careers when many of them could not yet find adequate research support from other sources. The two-year fellowships carry an average stipend of $8,750 a year; and funds may be used for purchase of equipment and supplies, support of technical and scientific assistance, professional travel, summer support, computer time, support of predoctoral and postdoctoral fellows, relief from teaching where this does not conflict with the needs of the fellow's department, and other purposes approved by the recipient's university.

Among the newly designated Sloan Research Fellows, the following are mathematicians: Srinivasa S. R. Varadhan, New York University; Charles C. Sims, Rutgers, The State University; Robert T. Powers and Gino C. Segre, University of Pennsylvania; Stephen Grossberg, Victor W. Guillemin, and Bernard Maskit, Massachusetts Institute of Technology; Charles H. Giffen, University of Virginia; William T. Eaton, University of Tennessee; Kenneth Kunen and Daniel G. Rider, University of
Wisconsin; Oscar E. Lanford, III, and Donald E. Sarason, University of California, Berkeley; Theodore W. Gamelin, University of California, Los Angeles; Leslie C. Glaser and Joseph L. Taylor, University of Utah.

CONFERENCE ON RIEMANN SURFACE THEORY

The Mathematics Department at the State University of New York at Stony Brook is sponsoring a ten-day conference on Riemann Surface Theory, June 23-July 2, 1969. The principle topics will be Moduli, Theta Constants and Kleinian Groups. Ten to 15 one-hour talks of general interest will be scheduled; and Professors Lars V. Ahlfors, Lipman Bers, and Harry E. Rauch have already accepted invitations to present talks. In addition, working seminars on specialized topics will be organized.

Approximately 40 participants are being invited. Qualified mathematicians who wish to participate in the conference are invited to write to the chairman of the Organizing Committee. Anyone who wishes to attend one or more of the sessions of the conference does not need to obtain a formal invitation. The Organizing Committee for the conference is composed of Professors Hershel M. Farkas, Irwin Kra, and James Simons (chairman).

CONFERENCE ON INTERNATIONAL MATHEMATICS PROGRAMS

The University of Arkansas will sponsor a conference on International Mathematics Programs, August 7-9, 1969. The conference will deal with secondary school curriculum materials and teacher training programs in various foreign countries and regions including India, Latin America, Africa, Spain, and Russia. Speakers will include Professors J. N. Kapur, Jose Tola, Albert Dou, Howard Fehr, Bruce Vogeli, Isaak Wirszup, and W. T. Martin. All interested persons are invited to attend. Further information may be obtained by writing to Professor W. R. Orton, Department of Mathematics, University of Arkansas, Fayetteville, Arkansas 72701.

REGIONAL CONFERENCE ON AUTOMATA THEORY AND COMPUTATIONAL COMPLEXITY

With financial support from the National Science Foundation, a summer research conference on Automata Theory and Computational Complexity will be held at the Valcour Educational Conference Center of the State University of New York, College at Plattsburgh, from June 23 to June 28, 1969. The conference will bring together 25 research level mathematicians interested in this area of applied mathematics. Dr. Juris Hartmanis, Chairman of the Department of Computer Science of Cornell University, will present a series of ten lectures and will join in discussions with the participants. These and other lectures resulting from other research conferences will be published by the Conference Board of the Mathematical Sciences.

The conference has several purposes: (1) to permit people with reasonable mathematical maturity to get a broad view of automata theory and understand some of the research problems; (2) to equip them to read some of the current literature in this field; and (3) to start them working in this area.

Complete financial support for all participants will be provided for by the grant. Those desiring to attend the conference may make application to Dr. William E. Hartnett, Department of Mathematics, State University of New York, College of Arts and Science, Plattsburgh, New York 12901.

WILLIAM PRAGER HONORED

Professor William Prager, Brown University, was honorary chairman of the Canadian Congress of Applied Mechanics. The congress met in May at the University of Waterloo, Waterloo, Ontario, at which time Professor Prager presented a formal address and received an honorary Doctor of Engineering degree from the university. He is one of the creators of the mathematical theory of plasticity, which has revolutionized the design procedures for structures, machine parts, and many other mechanical engineering processes.
TOPOLOGY OF MANIFOLDS
CONFERENCE

A Topology of Manifolds Conference, sponsored by the Air Force Office of Scientific Research, will be held at the University of Georgia, August 11-22, 1969. The purpose of this conference will be to bring together topologists working in the following areas: (1) Piecewise-linear and topological embedding and unknotting problems; (2) The Hauptvermutung, triangulation, and related problems; (3) Topology of 3-manifolds; (4) Group actions on manifolds. In addition to seminars on these topics, the program of the conference will include hour addresses by twenty invited speakers.

Inquiries from those interested in attending this conference should be addressed to Professors J.G. Cantrell and C. H. Edwards, Department of Mathematics, University of Georgia, Athens, Georgia.

LOGIC COLLOQUIUM

A conference in Mathematical Logic will be held in Manchester, England, August 3-23, 1969. This conference is being supported by NATO (as a NATO Advanced Study Institute) and the International Union of History and Philosophy of Science (Logic, Methodology, and Philosophy of Science Division). It is expected that the conference will be held jointly with a meeting of the Association of Symbolic Logic.

There will be four series of lectures of five to ten lectures each. Following is the tentative program: Recursion Theory, C. Karp (Maryland) and D. Lacombe (Paris); Problems of Decidability, M. Rabin (Jerusalem); Set Theory, J. R. Shoenfield (Duke); Model Theory, J. Silver (Berkeley). It is expected that the participants will have some knowledge of the rudiments of these subjects. The conference will also include a three-day seminar on Hierarchies and Generalizations of Recursion Theory, August 11-13, and a two-day seminar on Automata Theory and its Applications, August 21-22. There will be about twenty invited addresses, the nonseminar talks being mostly on Set Theory, as well as sessions for contributed papers and discussion of open problems.

Further information on the conference may be obtained by writing to 69 Logic Colloquium, Department of Mathematics, The University, Manchester M13 9PL, England. Those who wish to present a paper should submit, in duplicate, an abstract not exceeding 300 words by July 1, 1969.

LIST OF SYMPOSIA PARTICIPANTS

A list of the participants in the AMS-SIAM Symposium on Mathematical Aspects of Electrical Network Theory, held during the 664th Meeting of the AMS in New York in April, may be obtained on request for a nominal charge of $1.00 to cover typing, copying and mailing costs. The Society is offering the attendance list as a pilot experiment in aiding communication among workers in emerging areas of active interest in the mathematical sciences. If there proves to be sufficient interest in the list, the Society plans to offer attendance lists from all symposia, seminars, and institutes on a regular basis.

Requests should be addressed to the AMS, P.O. Box 6248, Providence, Rhode Island 02904.

NSF FELLOWSHIPS FOR ADVANCED SCIENCE GRADUATE STUDY

The National Science Foundation has awarded 1,929 Graduate Fellowships in the sciences, mathematics, and engineering for the academic year 1969-70. Of those receiving awards, 381 were in mathematics.

Forty-four percent of the fellowships awarded are for tenure periods of two years with continuation for the second year dependent upon satisfactory academic progress during the first year and the availability of funds. The Foundation expects to reopen the Graduate Fellowship program for 1970-71 in October 1969. Application forms will be available when the program is announced.
The Mathematics Research Institute of Oberwolfach (Mathematisches Forschungsinstitut Oberwolfach) has recently announced the program that is planned for 1969. Following is the program from mid-June through November.

June 15-21
Integralgeometrie und Wahrscheinlichkeitstheorie

June 22-28
Problemgeschichte der Mathematik
Chairmen: J. E. Hofman, Ichenhausen, and C. J. Scriba, Hamburg

June 29-July 5
Graphentheorie
Chairmen: G. Ringel, Berlin, and K. Wagner, Koln

July 5-7
Arbeitstagung des Heidelberger Seminars
Chairman: P. Roquette, Heidelberg

July 6-12
Spezielle Fragen aus den Grundlagen der Geometrie
Chairmen: A. Barlotti, Florenz, and E. Sperner, Hamburg

July 13-19
Differentialgeometrie im Groben
Chairmen: M. Barner, Freiburg, S. S. Chern, Berkeley, W. Klingenberg, Bonn

July 20-26
Arbeitstagung
Chairman: R. Baer, Frankfurt

July 27-August 2
Algebraische Zahlentheorie
Chairmen: H. Hasse, Hamburg, and P. Roquette, Heidelberg

August 3-9
Endliche Gruppen und Permutationsgruppen
Chairman: B. Huppert, Mainz

August 10-16
Systemtheoretische Probleme der Mechanik
Chairman: K. Magnus, Munchen

August 17-23
Himmelsmechanik
Chairman: E. Stiefel, Zürich

August 24-30
Harmonische Analyse und Darstellungstheorie topologischer Gruppen
Chairmen: H. Leptin, Heidelberg, and E. Thoma, Munster

August 31-September 6
Begriff der Zeit
Chairmen: G. H. Muller, Heidelberg, and G. J. Whitrow, London

September 7-20
Topologie

September 28-October 4
Funktionalanalyse
Chairmen: H. König, Saarbrücken, G. Kothe, Frankfurt, and H. G. Tillmann, Mainz

October 5-11
Arbeitsgemeinschaft
Chairmen: M. Kneser, Göttingen, and P. Roquette, Heidelberg

October 12-18
Formale Sprachen und Automatentheorie
Chairmen: J. Dörr, Saarbrücken, and G. Hotz, Saarbrücken

October 19-25
Mathematische Methoden in den Wirtschaftswissenschaften
Chairmen: R. Henn, Karlsruhe, H. P. Künzi, Zürich, and H. Schubert, Kiel

October 26-November 1
Didaktic der Mathematik
Chairman: Not yet known

November 3-8
Fachleitertagung für Mathematik
Chairmen: Not yet known

November 9-15
Fortbildungslehrgang für Studienrätte
Chairmen: J. André, Saarbrücken, and H. Salzmann, Tübingen

November 16-22
Iterationsverfahren in der Numerischen Mathematik
Chairman: L. Collatz, Hamburg

Further information can be obtained by writing to the Director of the Institute, Professor Dr. Martin Barner, 78 Freiburg/Brsg., Hebelstrasse 29.
COLLOQUIUM ON NUCLEAR SPACES AND IDEALS IN OPERATOR ALGEBRAS

On June 18-25, the Institute of Mathematics of the Polish Academy of Sciences will hold a colloquium on Nuclear Spaces and Ideals in Operator Algebras in Warsaw. The members of the Organizing Committee are C. Bessaga, A. Pełczyński (Chairman), S. Rolewicz, Z. Semadeni, W. Zelazko. The program will include both one-hour invited addresses and sessions for contributed papers. Further information may be obtained from the Secretary of the Organizing Committee, Professor Zbigniew Semadeni, Polish Academy of Sciences, Institute of Mathematics, Warsaw 1, Sniadeckich 8, Poland.

JUBILEE CONGRESS OF THE POLISH MATHEMATICAL SOCIETY

The Polish Mathematical Society is celebrating its fiftieth anniversary in 1969, and a Jubilee Congress will be held on September 3-9 in Cracow. Members of the Society are invited to participate. The scientific sessions will consist of a series of one-hour lectures to be presented by invited lecturers. For an application form and the guidebook of the Polish Travel Bureau "Orbis," which contains information on the travel of foreigners in Poland, please write to Professor Roman Sikorski, President, Polish Mathematical Society, Central Board, Warsaw, Sniadeckich 8, Poland.

NATIONAL ACADEMY OF SCIENCES

The National Academy of Sciences has announced the election of 50 new members in recognition of their distinguished and continuing achievements in original research. Among the new members are five mathematicians: Professor Stephen C. Kleene, University of Wisconsin; Professor Tjalling Charles Koopmans, Yale University; Professor Louis Nirenberg, New York University; Professor John T. Tate, Harvard University; and Dr. Warren Weaver, Special Advisor to the President, Sloan Foundation. Ten distinguished scientists from Finland, France, Japan, the Soviet Union, Sweden, Switzerland, and the United Kingdom were elected as foreign associates of the Academy. Among these new members was Professor N. N. Bogolyubov, head of the Department of Theoretical Physics of the Mathematics Institute of the Soviet Academy of Sciences. Professor Saunders Mac Lane of the University of Chicago has been elected to the Council of the Academy, his term to run for three years.

INTERNATIONAL SYMPOSIUM ON DESIGN AND APPLICATION OF LOGICAL SYSTEMS

The Société Royale Belge des Électriciens and the Industrial Electronics and Automatic Control Laboratories of the University of Brussels will sponsor a symposium on Design and Application of Logical Systems on September 15-20, 1969, in Brussels. The purpose of this symposium will be to create active cooperation between those working in the design of logical systems and those interested in their applications in research and industry. Further information may be obtained by writing to Dr. J. Florine, International Symposium, Laboratoire d'Électronique Industrielle, Université Libre de Bruxelles, 50, avenue F. D. Roosevelt, Bruxelles 5, Belgium.

NATIONAL RESEARCH COUNCIL DIVISION OF MATHEMATICAL SCIENCES

On July 1, Professor Lipman Bers, Columbia University, will succeed Professor R. H. Bing as chairman of the National Research Council's Division of Mathematical Sciences for a two-year term. Professor Bers is a member of the National Academy of Sciences and chairman of its Section of Mathematics and a fellow of the American Academy of Arts and Sciences. He has also served on the Executive Committee of the NRC Division of Mathematical Sciences and on the Advisory Committee on USSR and Eastern Europe (Office of the Foreign Secretary).
664-108. DIRAN SARAFYAN, Louisiana State University, New Orleans, Louisiana 70122.

**Predictor-corrector Runge-Kutta method.**

The Runge-Kutta method is used mostly for the generation of a few starting points and then is reverted for efficiency reasons to predictor-corrector type methods for the continuation of the process. A method based on special classes of fifth and sixth order Runge-Kutta formulas will be established which will possess the characteristics of predictor-corrector methods. This new method, from the standpoint of both efficiency and effectiveness, if not superior, is competitive with classical predictor-corrector type methods. Furthermore, it offers various other advantages relative to other known methods. (Received February 17, 1969.)

664-109. JAMES H. ABBOTT, Louisiana State University, New Orleans, Louisiana 70122.

**Variations on Lebesgue domination. Preliminary report.**

Certain variations of the Lebesgue Dominated Convergence Theorem appear in the literature, for example, Roydan, *Real analysis*, Second Ed., 1968, p. 89. Examples are easily constructed in which integrals converge, but the hypothesis of such a theorem is not satisfied. Therefore, useful equivalent conditions for the integrals to converge which are variations of the domination condition should be of interest. A few such conditions in arbitrary measure spaces are discussed under the hypothesis that the sequence of integrable functions converges to an integrable function. (Received February 17, 1969.)

664-110. RICHARD D. ANDERSON, Louisiana State University, Baton Rouge, Louisiana 70803.

**Dense sigma-compact subsets of infinite-dimensional spaces.**

Let $X$ be a separable metric space. A subset $K$ of $X$ has Property Z if for each nonempty homotopically trivial open set $U \subset X$, $U \setminus K$ is nonempty and homotopically trivial. A subset $M$ of $X$ has the (finite-dimensional) compact absorption property, the (f-d) cap, if (1) $M = \bigcup_{i>0} M_i$ where each $M_i$ is a (finite-dimensional) compactum having Property Z with $M_i \subset M_{i+1}$ and (2) for any $\epsilon > 0$, any $m > 0$ and any (finite-dimensional) compactum $K \subset X$ there exist an $n > 0$ and a homeomorphism $h$ of $K$ into $M_n$ such that $hK \cap M_m = id$ and $d(h, id) < \epsilon$. **Theorem.** Let $X$ and $X^1$ both be homeomorphic to the Hilbert cube or both be homeomorphic to Hilbert space. Let $M$ and $M^1$ be subsets of $X$ and $X^1$ respectively with $M$ and $M^1$ both having the cap or both having the f-d cap. Then there is a homeomorphism of $X$ onto $X^1$ carrying $M$ onto $M^1$. As applied to the Hilbert cube, this theorem topologically characterizes two types of dense $\sigma$-compact sets, for each of which the complement in the Hilbert cube is homeomorphic to Hilbert space. As applied to Hilbert space, this theorem topologically characterizes two types of $\sigma$-compact linear subspaces of Hilbert space. The author has been
informed that Bessaga and Pełczynski have recent similar results for the case of spaces homeomorphic to Hilbert space. (Received February 17, 1969.)

664-111. GEORGE T. RUBLEIN, College of William and Mary, Williamsburg, Virginia 23185. Obstructions to the integration of forms.

Let \( \theta \) be a closed \( p \)-form on the smooth \( n \)-dimensional manifold \( M \) with 0 periods. Let \( U \) be an open set in \( M \) and \( a \) a \((p - 1)\)-form on \( U \) such that \( da = \theta | U \). A relative version of de Rham's theorem (Abstract 655-56, these Notices) 15 (1968), 482) shows that \( a \) can be 'almost extended' to all of \( M \) if and only if \( \int_c \theta - \int_c a = 0 \) for all smooth singular chains \( c \) whose boundaries lie in \( U \).

Let \( J = \text{Image } H_p(M, U) \rightarrow H_p(U) \). \( \theta \) defines a homomorphism \( k: J \rightarrow Z \). Let \( A = \text{Image } H_{p-1}(V) \rightarrow H_{p-1}(U) \). We say \( H_{p-1}(U) \) is geometrically split by \( U - F \) if

\[ \text{there exists a homomorphism } h: H_{p-1}(U) \rightarrow Z \text{ extending } k \text{ and such that } h(A) = 0 \text{, and } h|\text{M} = 1-1. \]

Theorem. If \( 3 \leq p \leq n - 1 \), and \( U \) is connected, then there is a closed form \( \beta \) on \( U \) with integral periods such that \( \int_c \theta - \int_c a = -\int_c \beta \) (all smooth chains \( c \) as above) if and only if there is a connected \((n - p + 1)\)-dimensional closed submanifold \( F \) in \( U \) such that \( H_{p-1}(U) \) is geometrically split by \( U - F \). (Received February 17, 1969.)


A Hestenes ternary ring \( R \) is a slight modification of M. R. Hestenes' ternary algebra, Archive Rational Mech. Anal. 11 (1962). Let \( U \) and \( V \) be abelian groups; \( R \) is said to be a *-ternary ring in \( \text{Hom}(U, V) \) if \( R \) is a subgroup of \( \text{Hom}(U, V) \) and there is a map \( *: R \rightarrow \text{Hom}(V, U) \) such that \( (ab*c)* = c*ba* \) and \( (at b)* = a*t b* \) for all \( a, b, c \in R \).

Theorem 1 (Density Theorem). An irreducible *-ternary ring in \( \text{Hom}(U, V) \) is a dense ring of semilinear transformations of \( U \) into \( V \). The definition of a primitive ternary ring is a direct generalization of the ordinary binary notion. A right ideal \( A \) of \( R \) is a subgroup of \( R \) such that \( aRR \subseteq A \) for each \( a \in A \).

Theorem 2. \( R \) is primitive if and only if there is a maximal modular right ideal \( A \) of \( R \) such that \( (r \cap R \cap r \subseteq A) = (0) = (r \cap R \cap r \subseteq A) \). (Received February 17, 1969.) (Author introduced by Professor Malcolm F. Smiley.)

664-113. MILTON ROSENBERG, University of Kansas, Lawrence, Kansas 66044. Time invariant, subordinative operators in a Hilbert space. Preliminary report.

Let \( (U_x)_{x \in X} \) be a weakly continuous group of unitary operators on a separable Hilbert space \( H \) over a locally compact group \( X \) and \( E \) be its spectral measure on the Borel subsets \( B \) of its dual group \( \gamma (X) \) (of characters) such that \( U_x = \int_{\gamma (X)} [x, \omega] E(\omega) d\omega \) (Stone's Theorem). Let \( T \) be any linear operator on \( H \).

\[ \text{Theorem. } T = \int_\gamma \varphi(\omega) E(\omega) d\omega, \varphi(\omega) \text{ complex-valued and } \land \text{-measurable } \Rightarrow \]

\( (i) \) \( \varphi \in L^1(X, \omega) \), \( \varphi \) is \text{"time-invariant"}, \( \Rightarrow \)

\( (ii) \) \( T \) is \text{"subordinative"}, \( \exists \) \[ \mathcal{A} \text{ subspaces generated by } U_x f \] \( \text{"subordinate"}, \), \( \Rightarrow \)

\( (iii) \) \( T \) is closed and \( H = \text{closure } (B_T) \).

\[ \text{" } \Rightarrow \text{" is true without assuming separability, but the } \text{" } \Rightarrow \text{" is not } \text{[Nakano].} \]

This generalizes [Masani, Bull. Amer. Math. Soc. 71 (1965), 546-550], where \( X = (- \infty, \infty) \). We use for \" \Rightarrow \" : 1. If \( \mathcal{M} \) is a subspace of \( H \) which reduces \( U_x \) for each \( x \in X \), then \( \mathcal{M} \) reduces \( E(B) \) for each \( B \in \mathcal{B} \). 2. For each bounded \( \varphi \) and each \( f \in \mathcal{B}_T \), \( \int \varphi dEf \in \mathcal{B}_T \) and \( T(\int \varphi dEf) = \int \varphi dETf \). 3. Let \( \mathcal{L} \) be proj. onto \( \mathcal{A} \). Then \( \mathcal{L} \) is closed for each \( f \in \mathcal{B}_T \).

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4. The rest of the proof follows along the lines of [Riesz-Nagy, Functional analysis, 351-354].

(Received February 17, 1969.)


Let \([X; Y]\) denote the set of all linear continuous mappings from \(X\) into \(Y\) where \(X\) and \(Y\) are complex locally convex spaces. \(H(B)\), \(I\), and \(K\) are three testing function spaces as defined by Zemanian (SUNY Engrg. Coll. Tech. Rep. 111) where \(B\) is a complex \(B\)-space. \(W(B)\) is a subset of \([I; B]\) whose elements are \(B\)-valued functions. The mapping \(A : [I; B] \rightarrow [K; B]\) is passive on \(W(B)\) with respect to a prescribed semi-inner product \([\ , \]\) defined on \(B\) if for every \(V \in W(B)\), for \(u = Av\) and for every real number \(x\) we have \([u(t), v(t)]\) is L-integrable on \(- \infty < t < x\) and

\[
\text{Re} \int_{-\infty}^{x} [u(t), v(t)] \, dt \geq 0.
\]

Let \(B\) be generated from a real Banach space \(B_{R}\) through complexification. The space \(D_{L1}(B_{R})\) is the space of functions \(\phi\) from \(R\) into \(B_{R}\) equipped with the topology generated by the multinorm \(p(\phi)\). For \(y \in [D_{L1}(B_{R}); B_{R}]\) the mapping \(v \mapsto y * v\) from \([I; B]\) to \([K; B]\) is passive on \(D_{L1}(B_{R})\) and \(Y(s)\), the Laplace transform of \(y\), exists in \([B; B]\) as a positive-real mapping of \(B\) into \(B\) w.r.t. the prescribed s.i.p. The realizability theorem asserts that the converse also is true. Indeed it is a consequence of the realizability theorem that if the mapping \(A\) is passive on \(D_{L1}(B)\) with respect to one s.i.p. consistent with the norm of \(B\), then it is passive w.r.t. to any other such s.i.p.

(Received February 17, 1969.)


Some statistical tests of randomness and normality are made of the first 88062 binary digits (or equivalent in other bases) of \(J_{n}\) in various bases \(b; 2 \leq n \leq 15 \) with \(n\) square free and \(b = 2, 4, 8, 16\) and \(n = 2, 3, 5\) with \(b = 2, 3, 5, 6, 7,\) and \(10\). The statistical tests are the \(\chi^2\) test for cumulative frequency distribution of the digits, the lead test, and the gap test. The lead test is an examination of the distances over which the cumulative frequency of a digit exceeded its expected value. It is related to the arc sin law. The gap test (applied to the binary digits) consists of an examination of the distribution of runs of ones. The conclusion of the study is that no evidence of the lack of randomness or normality appears for the digits of the above mentioned \(J_{n}\) in the assigned bases \(b\). It seems to be the first statistical study of the digits of any naturally occurring number in bases other than decimal or octal. (Received February 17, 1969.)


Let \(\{X_{t}(\omega)\}\) be the Wiener process. Almost surely the local behavior at each \(t\) is one of seven varieties thus inducing a partition of \(( - \infty, + \infty)\) into seven disjoint \(B^{2}\) sets. For any continuous \(g(u, v)\) and any \(\delta > 0\), almost surely \(g(\min_{t - \delta \leq t \leq t + \delta} X_{t}(\omega), \max_{t - \delta \leq t \leq t + \delta} X_{t}(\omega))\) is almost everywhere locally constant. One can modify \(\{X_{t}(\omega)\}\) to get almost surely everywhere locally recurrent sample paths. (Received February 17, 1969.)
664-117. WITHDRAWN.

664-118. R. ARTHUR KNOEBEL, New Mexico State University, Las Cruces, New Mexico 88001. Finitely generated identities and subdirect products of universal algebras.

Theorem. Let $\mathfrak{B} = (B; o_1, o_2, \ldots, o_k)$ be a subdirect product of a finite independent collection of algebras $\mathfrak{A}_i (1 \leq i \leq m)$. If the identities of each algebra $\mathfrak{A}_i$ are finitely generated, then $\mathfrak{B}$ also has a finitely generated set of identities. Quantitatively, if each $\mathfrak{A}_i$ requires $n_i$ identities, then $\mathfrak{B}$ requires at most $n_1 + n_2 + \ldots + n_m + km + 1$ identities. (Received February 18, 1969.)

664-119. BRUNO H. BROSOWSKI, Max-Planck-Institut für Physik und Astrophysik, 8 München 23, Föhringerring 6, West Germany. On the necessity of the generalized Kolmogorov criterion.

Let $R$ be a normed space. We call $P_v(f)$ the set of best approximations for an element $f \in R$ by means of the elements of a subset $V \subset R$. If every $v \in V$ satisfies the inequality
\[
\min \{R e (l - v_0) : L \in E_{f - v_0} \} = 0,
\]
then $v_0$ is a best approximation for $f$ by means of the elements of $V$. Here $E_{f - v_0}$ is the set of the extremal points of the set $\{L \in R' : \|L\| \leq 1 \text{ and } L(f - v_0) = \|f - v_0\|\}$. This criterion which is called the generalized Kolmogorov criterion is, however, in general not necessary for a best approximation. The following theorem is true: Theorem. Let $P_v(f)$ be compact and convex for every $f \in R$. The following are equivalent: (A) For every element $f \in R$ there exists a best approximation which satisfies the generalized Kolmogorov-criterion. (B) The mapping $A_{(f,r)}(g) := f + r(g - P_V(g))/(r + E(f; V))$ has a fixed point for every $f \in R$ and every $r > 0$. (C) $V$ is a $\beta$-sum. (Received March 6, 1969.)


Let $E$ be the set of reflections of a Minkowskian plane $M$ over a commutative field $K$ of arbitrary characteristic. Then $E^3 \cap E^4 = \emptyset$ and $E^3 \cup E^4 = G = \text{motion-group of } M$. As usual, the group-space $E(G)((+),\cdot)$ of $M$ is a geometric structure with the point-set $E^4$, the plane-set $E^3$ and the incidence-relation $1 = \{(x, s) \in E^3 \times E^4 : x \cdot s \in E\}$. Let $(e_1, e_2, e_3, e_4)$ be a base of an associative algebra $(+, \cdot)$ over $K$, where the multiplication is defined by $e_1 e_1 = e_1, e_2 e_2 = e_4, e_2 e_3 = e_3 e_4 = e_3, e_4 e_2 = e_2 e_1 = e_2$ and $e_i e_j = 0$ otherwise. Then $A^{*}(\cdot) = \{\sum_{i=1}^{4} a_i e_i \in A | a_1 \cdot a_4 \neq 0\}$ is the group of units of $A$, and $K^{*}(\cdot) \subset A^{*}(\cdot)$. As a substructure of the projective space $A \setminus \{0\}/K^{*}$ the set $A^{*}/K^{*}$ has a geometric structure. Theorem 1. $A^{*}/K^{*}(\cdot, \cdot)$ and the substructure $E^4(\cdot, \cdot)$ of $E(G)(\cdot, \cdot)$ are isomorphic. Define an associative algebra $(A \times A)(+), \cdot)$ by $(X, Y) + (U, V) = (X + V, Y + V)$ and $(X, Y) \cdot (U, V) = (XU + Y \bar{V}, XV + YU)$, where $\bar{X} = \sum_{i=1}^{4} a_{2-i} e_i$ if $X = \sum_{i=1}^{4} a_i e_i$. Then $A^{*} \cup \{0, X | X \in A^{*}\} = B^{*}(\cdot)$ is a group, and $K^{*}(\cdot) \subset B^{*}(\cdot)$. Theorem 2. The geometric structure of $A^{*}/K^{*}$ can be extended to a geometric structure of $B^{*}/K^{*}$ such that $B^{*}/K^{*}(\cdot, \cdot)$ and $E(G)(\cdot, \cdot)$ are isomorphic. (Received March 14, 1969.) (Author introduced by Professor Alexander R. Bednarek.)

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Entire functions of arbitrarily fast upper and arbitrarily slow lower rates of growth.

It is well known that there exist transcendental entire functions of arbitrarily fast and of arbitrarily slow rates of growth (but which must, of course, grow faster than polynomials).

F. Gross, in an oral communication, raised the question as to whether or not there exist entire functions whose upper rates of growth are arbitrarily fast and (simultaneously) whose lower rates of growth are arbitrarily slow (with the above necessary restriction). The purpose of this note is to give an affirmative answer to this question. Specifically, we prove the following Theorem. Let $h(r)$ and $k(r)$ be positive functions of $r$ for $r > 0$ such that $\log k(r) \neq O(\log r)$ ($r \to \infty$). Then there exists an entire function $f(z)$ whose maximum modulus $M(r)$ exceeds $h(r)$ for one sequence of values of $r$ tending to infinity and is less than $k(r)$ for another such sequence. The construction involves entire functions of the form $f(z) = \sum_{n=1}^{\infty} (\pi/c_n)^{\lambda_n}$, where $\{\lambda_n\}$ is an increasing sequence of positive integers and $\{c_n\}$ is an increasing sequence of positive numbers tending to infinity.

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If $A$ is a function algebra on a compact Hausdorff space $X$, a closed subset $F$ of $X$ is an interpolating set if $A|F = C(F)$. Denote the maximal ideal space of $A$ by $M$. Then for $F$ closed in $X$, the $A$-convex hull of $F$ is the set of all $\varphi \in M$ such that $|\varphi(f)| \leq \|f\|_F$ for all $f \in A$. The essential set of $A$ is the smallest closed set $E$ such that for all $u \in A^+$, $u$ is supported on $E$. Let $P = \{x \in X : x$ is in the interior of some interpolating set}. The following theorem gives a necessary and sufficient condition for a point in $X$ to be in $E \cap P$. Theorem 1. $E \cap P = (X - P) \cap P$ and $M = (X - P) \cup (P - E)$, the intersection being disjoint. Corollary 1. If $A$ is approximately regular on $X$, then $E = X - P$. Corollary 2. If $X = M$, then $E = X - P$. Mullins [Proc. Amer. Math. Soc. 18 (1967), 271-273] proved this for metrizable $X$. Theorem 2. If $K$ is compact and $X - P \subseteq K \subseteq E$, then $A|K$ is closed in $C(K)$, $\hat{K} = \hat{E}$, and $A|K$ is an essential function algebra. (Received February 17, 1969.)

Order topology and the Egoroff property in Riesz spaces.

Let $L$ be a Riesz space. The sequence $\{f_n : n = 1,2,\ldots\}$ in $L$ is order convergent to the element $f \in L$ whenever there exists a sequence $u_n \downarrow 0$ in $L$ such that $|f - f_n| \leq u_n$ for all $n$. For any subset $S$ of $L$, the pseudo closure $S'$ of $S$ is the set of all $f \in L$ such that there exists a sequence $\{f_n\}$ in $S$ converging in order to $f$. A subset $S$ of $L$ is called ordered bounded if there exists an element $u \in L^+$ such that $|s| \leq u$ for all $s \in S$. A Riesz space $L$ is said to have the Egoroff property if, for every $f \in L$, the following condition holds: Given any double sequence of elements $\{u_{nk} : n, k = 1,2,\ldots\}$ in $L$ such that $0 \leq u_{nk} \uparrow |f|$ for $n = 1,2,\ldots$, there exists a sequence $0 \leq u_m \uparrow |f|$
such that, for every $m$, for every $n$, $u_m \neq u_{nk(m,n)}$ for some $k = k(m,n)$. Theorem. If $L$ is a Riesz space, then the following conditions are equivalent: (1) $L$ has the Egoroff property, (2) $S' = S''$ for every order bounded subset $S$ of $L$. (3) $S' = S''$ for every order bounded convex subset $S$ of $L$. (Received February 17, 1969.)

665-68. RUDY L. CURD, University of Kentucky, Lexington, Kentucky 40506. Note on interval clans with idempotent endpoints.

Let $C$ be the middle-third Cantor set on the real interval $[0,1]$. Let $J = [0,1]$ under the ordinary multiplication. Let $U$ be the $(1)$-semigroup whose set of idempotents is $C$ and such that if $P$ is a component in $[0,1] \setminus C$, then the closure of $P$ is isomorphic to $J$. Theorem 1. $U$ is a universal $(1)$-semigroup, i.e., if $S$ is an $(1)$-semigroup, then there is a continuous homomorphism $g$ from $U$ onto $S$. Using this result we have Theorem 2. The class of interval clans with idempotent endpoints has a universal element. (Received February 17, 1969.)

665-69. JAMES G. DOBBINS, University of Kentucky, Lexington, Kentucky 40506. A note on compactifiable semigroups.

Let $S$ be a locally compact semigroup. If there is a compact semigroup $T$ with $S \subseteq T$ then $S$ is compactifiable, and if the space $T$ is metrizable then $S$ is metric compactifiable. In this note, we give a necessary and sufficient condition that $S$ be metric compactifiable. Next we suppose $S$ is compactifiable and has minimal ideal $K$, and show that $K$ has a structure analogous to that for a minimal ideal in the compact case. Finally, we give some examples to show that if $S$ is not compactifiable and has a minimal ideal $K$, then the structure of $K$ can be considerably different from the compact situation. (Received February 17, 1969.)

665-70. PETER G. DODDS, California Institute of Technology, Pasadena, California 91109.

The Riesz space structure of an Abelian $W^*$-algebra.

Let $M$ be an Abelian $W^*$-algebra of operators in a Hilbert space $H$. By $M_0$ denote the set of linear, closed, densely defined transformations in $H$, which commute with every unitary operator of the commutant $M'$ of $M$. An elementary proof is given of the result of R. Pallu de la Barriere that every positive normal linear functional on $M$ is of the form $T \cdot (Tx,x)$ for some $x \in H$, $T \in M$. This result is then used to construct $M_0^+$; in particular, an elementary proof is given that every (unbounded) positive selfadjoint transformation in $H$ has a unique positive square root. When the algebraic operations are appropriately defined, $M_0^+$ becomes a partially ordered linear vector space. Denote by $\text{Re } M_0^+$, $\text{Re } M$ the set of selfadjoint transformations of $M_0^+$, $M$, respectively. Theorem. $\text{Re } M_0^+$ is a Dedekind complete, universally complete Riesz space which contains the Dedekind complete Riesz space $\text{Re } M$ as an order dense ideal. (Received February 17, 1969.)


Let $X$ be a uniformly convex Banach space in which orthogonality is right linear. If $A$ is a bounded linear operator it is possible to define the generalized adjoint, $A^+$, satisfying $[A(x),y] = \cdots$
\[ x, A^+(y) \], where \([\cdot, \cdot]\) is a semi-inner product. \( A^+ \) is not in general linear but it still possesses some interesting properties of the Hilbert space adjoint. The adjoint abelian operators of Stampfli (Adjoint abelian operators in Banach space, to appear in Canad. J. Math.) are characterized by the fact that they are the "selfadjoint" operators in the natural sense that \( A = A^+ \). Further, the invertible isometries are characterized by the fact that they preserve the semi-inner product and hence are those operators for which \( A^{-1} = A^+ \). Finally, it is easy to show that the spectrum of any operator on these spaces is included in the closure of its numerical range. (Received February 17, 1969.)

665-72. JAMES E. MILLER, NASA, Manned Spacecraft Center, Ed 13, Houston, Texas 77058.
Convex meromorphic mappings and related functions.

Let \( \Sigma(p), 0 < p < 1 \), denote the class of functions which are regular in \( E = \{ z : |z| < 1 \} \) except at \( z = p \) and satisfy (1) \( |z f''(z)/f'(z)| + 2p(z - p)^{-1} - 2pz(1 - p)^{-1} \neq 0 \) for all \( z \) in \( E \). A function \( f(z) \) in the class \( \Sigma(p) \) may have a logarithmic singularity; however, \( f(z) \) maps onto the exterior of a convex set in the sense that \( \lim_{r \to 1} |z f'(rei \alpha)/f'(rei \alpha)| < 0 \). Theorem 1. If \( f(z) \in \Sigma(p) \), then \( f(z) \) is convex in \( |z| < \rho_0(p) \), where \( \rho_0(p) = 1 + (1 + p)(1 + p - \sqrt{1 + p^2 + 4p})/2p \).

Theorem 2. If \( f(z) \in \Sigma(p) \) and \( f(z) \) is meromorphic in \( E \), then \( f(z) \) is convex in \( |z| < \rho_1(p) \) where \( \rho_1(p) \) is the smallest positive root of \( (p^3 + p) - (1 + 10p + p^2)^2 - (1 + 4p + p^2)^2 = 0 \). As \( p \to 1 \), both \( \rho_0(p) \) and \( \rho_1(p) \) tend to 3 - 2/\( \sqrt{2} \). A function \( f(z) \) belongs to the class \( C\Sigma(p) \) if \( f(z) \) is regular at each point of \( E \) except at \( p \) and if there is a \( g(z) \in \Sigma(p) \) such that \( \Re(f'(z)/g'(z)) > 0 \) for all \( z \) in \( E \). If we let \( p = 0 \), then \( C\Sigma(0) \) is the class of close-to-convex function with a pole at the origin. Theorem 3. The radius of convexity for the class \( C\Sigma(p) \) is the smallest positive root of \( (1 + 4p + p^2)^2 - (1 + 4p + p^2)^2 - (1 + 4p + p^2)^2 - (1 + 4p + p^2)^2 = 0 \). (Received February 17, 1969.)

665-73. GUILLAMO MIRANDA, Purdue University, West Lafayette, Indiana 47906.
Application of singular integral equation methods to static problems of nonsmooth elastic bodies.

This paper considers the first boundary-value problem of linear elastostatics (given displacements) for a class of nonsmooth regions. The partial differential equation for the displacement vector \( u \) can be written as \( (1 + k) \) grad div \( u - \) curl curl \( u = 0 \). By seeking the solution in the form of a double-layer elastic potential with unknown density, the problem is reduced to solving a system of singular integral equations of a type differing from those usually considered in the literature. These singular systems are reduced to an equivalent system of regular Fredholm equations. This reduction involves auxiliary singular equations, which are special cases of equations of a general kind to which successive approximations can be applied. The solution is then obtained for small values of the elastic parameter \( k \). (Received February 18, 1969.)

665-74. JOHN L. KELLEY, University of California, Berkeley, California 94720, and T. P. SRINIVASAN, University of Kansas, Lawrence, Kansas 66044. Extension of set functions on a lattice.

Theorem 1. If \( \mu \) is a real-valued nonnegative function on a lattice \( \sigma \) of sets containing \( \emptyset \), there is an extension of \( \mu \) to a measure on the \( \sigma \)-field generated by \( \sigma \) if and only if for any sequences.
\[ \{A_n\}_n \text{ and } \{B_n\}_n \text{ of members of } \mathcal{A} \text{ with } B_n \subset A_n \text{ and for each member } A \text{ of } \mathcal{A} \text{ with } \\
A \subset \bigcup_n (A_n \setminus B_n), \mu(A) < \sum_n (\mu(A_n) - \mu(B_n)). \]

Corollary 2. Each semiregular Borel content \( \mu \) in an arbitrary Hausdorff space \( X \) (not necessarily locally compact) has an extension to a regular Borel measure in \( X \). From a 'dual' extension theorem we derive Corollary 3. Let \( \mu \) be a finite-valued measure on a ring \( \mathcal{R} \) of sets and for each set \( A \), let \( \mu^*_\mathcal{R}(A) = \sup \{\lim_n \mu(E_n) : E_n \text{ is any decreasing sequence of members of } \mathcal{R} \text{ with } \bigcap_n E_n \subset A\}. \) Define a set \( E \) to be \( \mu^* \)-measureable if and only if for each set \( A \) with \( \mu^*_\mathcal{R}(A) < \infty, \mu^*_\mathcal{R}(A) = \mu^*_\mathcal{R}(A \cap E) + \mu^*_\mathcal{R}(A \setminus E). \) Then the family of all \( \mu^* \)-measurable sets is a \( \sigma \)-field containing \( \mathcal{R} \) and the restriction of \( \mu^*_\mathcal{R} \) to this \( \sigma \)-field is a measure extending \( \mu. \) (Received March 27, 1969.)

665-75. ROBERT EVERETT JACKSON, University of Texas, Austin, Texas. Concerning certain plane-like domains.

Let \( D \) denote a connected domain lying in a space satisfying R. L. Moore's Axioms 0, 1-5. \( D \) is said to be complete ['plane-like'] if and only if it is true that if \( A \) and \( B \) are boundary points of \( D \) and \( AB \) is an arc which lies, except for its ends, wholly in \( D \), then \( AB \) disconnects \( D \) [the closure of \( D \)]. Theorem. If \( \omega \) is a point of \( S - D \) and \( C(D) \) denotes the set of all points which may be separated from \( \omega \) by a simple closed curve lying in \( D \), then \( C(D) \) is a complete domain. Theorem. If \( A \) and \( B \) are boundary points of \( D \), \( AB \) is an arc which lies, except for its ends, wholly in \( D \), and \( D - AB \) is the sum of two mutually exclusive complete domains, then \( D \) is complete. A similar result is obtained for plane-like domains. Theorem. If the plane-like domain \( D \) is bounded by a nondegenerate compact continuum \( M \), then no proper subcontinuum of \( M \) disconnects \( M \). Example. There exists in \( E_2 \) a plane-like domain \( D \) such that \( \overline{D} \) is compact and separates \( S \). Theorem. If the common part of the two complete domains \( D_1 \) and \( D_2 \) is a connected subset of a simple domain lying in their sum, then \( D_1 + D_2 \) is complete. (Received March 31, 1969.)

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666-31. CLARENCE M. ABLOW, Stanford Research Institute, Menlo Park, California 94025. A characteristical finite difference method for the wave equation.

An explicit difference scheme of second order accuracy for obtaining approximate solutions to the wave equation in \( N \) space dimensions is presented. The scheme uses mesh cubes whose faces lie in characteristic surfaces. The consistency of the scheme is established for all \( N \); its stability for \( N = 2 \). (Received February 17, 1969.)


Let \( K \) be a compact set in \( C^n \). A function on \( K \) is of class \( C^k \) if it extends to a \( C^k \) function in a neighborhood of \( K \). Let \( \mathcal{G}_k \) be the sheaf of germs of functions on \( K \) which are of class \( C^k \) and holomorphic on int \( K \), \( 0 \leq k \leq \infty \). It is shown that for certain compact sets (e.g. polycylinders, analytic polyhedra, and in some cases strictly pseudo-convex sets) \( H^q(K, \mathcal{G}_k) = 0 \) for \( q > 0 \). If the set \( K \) is in addition convex, or a polyhedra, a similar theorem holds for the sheaf \( \mathcal{B} \) of germs of
functions on \( K \) which are defined, bounded and holomorphic on \( \text{int} \ K \) (and not necessarily defined on \( \text{bdry} \ K \)). In particular, Cousin problems on \( K \) involving holomorphic functions on \( \text{int} \ K \) with \( c^k \) boundary values (resp. bounded holomorphic functions) can be solved by functions with the same property. (Received February 17, 1969.)

666-33. RALPH S. TINDELL, University of Georgia, Athens, Georgia 30601. Unknotting improper imbeddings of PL manifolds.

Let \( M^m \) and \( N^n \) be PL manifolds and let \( f, g : M^m \to N^n \) be PL imbeddings with \( f^{-1}(\partial N) = g^{-1}(\partial N) = Q^{m-1} \), an \( (m-1) \)-dimensional submanifold of \( \partial M \). Suppose \( n - m \neq 3 \), \( M \) collapses to \( P \), and \( Q \) collapses to \( K \), with \( \dim P < n - m \) and \( \dim Q < n - m - 1 \). Then if \( f \) and \( g \) are properly homotopic, they are ambient isotopic. If \( f|Q = g|Q \) and \( f \) and \( g \) are homotopic rel \( Q \), they are ambient isotopic rel \( \partial N \). (Received February 17, 1969.)

666-34. JACOB BURBEA, Stanford University, Stanford, California 94305. Some properties of an invariant metric in domains with Bergman-Šilov boundary surface.

The author considers the kernel function \( \hat{K}_M \), obtained from a complete system of functions \( \{\varphi_\mu\} \) which are orthonormal when integrated over the Bergman-Šilov boundary surface of an analytic polyhedron \( \mathfrak{M} \). Using the kernel, we define an invariant metric \( d\hat{s}_M^2 = \int \frac{1}{m} \sum_{m,n=1}^{2} \hat{T}_{mn} \hat{d}z_m \hat{d}z_n^* \), \( \hat{T}_{mn} = (\hat{e}^2 \log \hat{K}/\partial z_m \partial \bar{z}_n), \hat{T}_1 = \hat{K}((\hat{T}_{11} \hat{Z}_1^2 - |\hat{T}_{12}|^2)^{1/2}, \hat{K} = \hat{K}_M(z,\bar{z}). \) We determine bounds for the volume of the indicatrix generated by the above metric. We show that under some additional assumptions an analogue of the Schwarz–Pick lemma holds in this case. (See also Bergman, Mat. Sb. 1 (43) (1936), 79-96.) (Received March 3, 1969.)

666-35. WITHDRAWN.


Consider a finite family of continuous self-mappings of a topological space \( X \), with a common fixed point. Suppose that for each member of the family, \( X \) has a metric for which that member is a contraction. It is shown that if the family is commutative, then \( X \) has a metric under which all members are (simultaneously) contractions. Additional hypotheses are given which ensure the same conclusion in the noncommutative case. (Received February 26, 1969.)

666-37. ROBERT W. ROBINSON, University of California, Berkeley, California 94720. A generalization of Sack's density theorem for recursively enumerable degrees.

Our notation is that of Sacks (Degrees of unsolvability, Princeton University Press, 1963). By generalizing the proof which Sacks gave for his density theorem (Ann. of Math. 80 (1964), 300-312),
we provide a proof of Theorem. Let a, b be r.e. degrees such that a \leq b, and let d, g be any degrees such that d \leq a and g < b. Then there is a r.e. degree c such that a \leq c \leq b, d \leq c, and c \leq g.

Corollary. If a, b are r.e. degrees such that a < b then there is an infinite set of mutually incomparable r.e. degrees between a and b. By making explicit some of the uniformities inherent in the proof of the theorem, it is easy to improve the theorem and its corollary so that in the latter an infinite r.e. sequence of mutually incomparable r.e. degrees between a and b is found. (Received March 31, 1969.)


Let E be a locally convex space of distributions on \( R^n \). E is admissible if it contains \( \mathcal{B} \) as a dense subspace and the injections \( \mathcal{B} \to E \to \mathcal{B}' \) are continuous. An admissible space E is of type (c) if it is a module (with respect to convolution) over \( \mathcal{B} \) and (i) for each \( u \in E \), the mapping \( \sigma - u \ast \sigma \) of \( \mathcal{B} \) into E is continuous, and (ii) for each \( \sigma \in \mathcal{B} \), the mapping \( u - u \ast \sigma \) of E into itself is continuous. If E is of type (c) then \( H_c(E, \mathcal{B}') \) is the space of continuous linear mappings of E into \( \mathcal{B}' \) which commute with convolution by elements of \( \mathcal{B} \). Then \( H_c(E, \mathcal{B}') \) is (isomorphic to) a subspace of \( \mathcal{B}' \). The following lemma is established: If E is a barrelled space of type (c) and \( w \in \mathcal{B}' \), then \( w \in H_c(E, \mathcal{B}') \) if and only if \( w \ast \sigma \in E' \) for each \( \sigma \in \mathcal{B} \). This lemma is fairly simple to prove and is useful in the study of \( H_c(E, \mathcal{B}') \). (Received April 4, 1969.)

666-39. LEONARD P. SASSO, Jr., University of California, Berkeley, California 94705. A minimal partial degree of unsolvability.

We define the class of functions g partial recursive in a given partial function f as the closure of f and the usual initial functions (successor, constants, and projections) under applications of composition, recursion, and the mu-operator (denoted \( g \equiv_T f \)). There is a primitive recursive indexing, a computation predicate, and a normal form for the class of functions partial recursive in a given partial function f. We define the partial degree of a partial function f (denoted \( d(f) \)) as the class of partial functions g such that \( g \equiv_T f \). We call \( d(f) \) total if \( d(f) \) contains a total function. Total degrees correspond to the usual degrees of unsolvability. A partial degree d is minimal if the only partial degree less than d is \( \mathbb{Q} \). We show that any minimal partial degree is nontotal and contains a function with range \([0,1)\). Finally, we show that there is a minimal partial degree. (Received April 14, 1969.)


Let \( f(n) \) be the maximal k for which the following is true: If \( G \) is an n-connected graph with a 1-factor, then \( G \) has k (different) 1-factors. Beineke and Plummer proved (J. Combinatorial Theory 2 (3), (1967), 285-289) that \( f(2) \equiv 2 \), and that for each \( n, f(n) \equiv n \). Theorem 1. For each \( n, f(n) \equiv n(n - 2)(n - 4) \ldots \), where the last factors are either \( 4 \ast 2 \), or \( 5 \ast 3 \). Equality holds if \( n \) is odd, or \( n \equiv 5 \). The proof of Theorem 1 uses the following Theorem 2. If \( G \) is 2-connected and has a 1-factor, then \( G \) contains a vertex V such that each edge of \( G \) incident to V belongs to some 1-factor of \( G \). (Received April 16, 1969.)

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Let \((z_i)\) be a sequence of points in the disk \(\{ |z| < 1 \}\). Then \((z_i)_{i=1}^{\infty}\) is called a Carleson sequence if \(\prod_{j \neq i}^{\infty} |(z_i - z_j)/(1 - \bar{z}_j z_i)| > \delta > 0, j = 1, 2, \ldots\). \((z_i)\) is called an exponential sequence if \((1 - |z_{j+1}|/(1 - |z_j|)) \leq r < 1, j = 1, 2, \ldots\). An exponential sequence is a Carleson sequence. Let \(\xi^1, \xi^2, \ldots\) be a sequence of continuous normalized linear functionals on the Hardy space \(H_2\), and let \(T_f = \{\xi^i\}_{i=1}^{\infty}\). H. S. Shapiro and A. L. Shields showed that if \(\xi^i\) is normalized pointwise evaluation at \(z_i\), then \(T(H_2) = \Xi^2 \circ (z_i)\) is a Carleson sequence (Amer. J. Math. 1961). J. T. Rosenbaum showed that if \(\xi^i\) is normalized pointwise evaluation of the nth derivative at \(z_i\), then \(T(H_2) = \Xi^2 \circ (z_i)\) is an exponential sequence (Mich. Math. J. 1967 and Pacific J. Math. 1968). The author shows that this latter result can be extended to the case where \((z_i)\) is a Carleson sequence and also that the \(\xi^i\) can be extended to certain linear combinations of the derivatives. (Received April 21, 1969.)
ABSTRACTS PRESENTED TO THE SOCIETY

During the interval from February 12 through April 25, 1969, the papers printed below were accepted by the American Mathematical Society for presentation by title. The abstracts are grouped according to subjects chosen by the author from categories listed on the abstract form. The miscellaneous group includes all abstracts for which the authors did not indicate a category.

One abstract presented by title may be accepted per person per issue of these Notice. Joint authors are treated as a separate category; thus, in addition to abstracts from two authors individually, one joint abstract by them may be accepted for a particular issue.

Algebra & Theory of Numbers


A ∈ C_m^n is EPr if N(A) = N(A*), e.g. M. H. Pearl, Proc. Cambridge Philos. Soc. 62 (1966), 673-677. Theorem 1. Let A ∈ C_m^n. Then A is EPr iff \lim_{\lambda \to 0} (\lambda I + A)^{-1} PR(A) = A^*.

Corollary. Let A ∈ C_m^n. Then \lim_{\lambda \to 0} (\lambda I + A^* A)^{-1} A^* = A^*. Theorem 2. Let A ∈ C_m^n, rank A = rank A^*, b ∈ C^n. Then b ∈ R(A) iff the limit \lim_{\lambda \to 0} (\lambda I + A)^{-1} b exists, in which case it = A^* b.

(A^* is the group inverse of A, e.g. I. Erdelyi, J. Math. Anal. Appl. 17 (1967), 119-132. Corollary. Let A ∈ C_m^n be EPr, b ∈ C^n. Then b ∈ R(A) iff the limit \lim_{\lambda \to 0} (\lambda I + A)^{-1} b exists, in which case it = A^* b. (Received February 5, 1969.)

69T-A60. IN Y. CHUNG, University of Cincinnati, Cincinnati, Ohio 45221. On Kähler's differential forms.

E. Kähler [Bericht über die Mathematiker-Tagung in Berlin, Januar 1953, pp. 58-163] defined his differential forms as follows: Let R be a commutative ring with 1, and A a unitary commutative R-algebra. (I, σ) is called an infinitesimal algebra over A if I is a commutative R-algebra containing A as a unitary subalgebra and σ = (σ_i)_{i ∈ N} = \{1, 2, ..., \}, where each σ_i : A → I is an R-algebra homomorphism such that (σ_i(a) - a)(σ_i(b) - b) = 0 for a, b ∈ A. Let (I, σ) and (J, τ) be infinitesimal algebras over A, then an R-algebra homomorphism f : I → J is called an infinitesimal algebra homomorphism if f|A is the identity mapping on A and f o σ_i = τ_i for all i ∈ N. Let (W, θ) be a universal infinitesimal algebra over A, i.e. a universal object in the category of infinitesimal algebras over A. If we define d_i = θ_i - i_A, where i_A is the identity mapping on A, then d_i is a derivation for each i ∈ N. An element of A(d_1 A) ...(d_k A) is called Kähler's differential form of degree k. Theorem. The A-module A(d_1 A) ...(d_k A) of Kähler's differential forms of degree k is isomorphic to the A-module T_k(U) of homogeneous elements of degree k of a tensor algebra T(U) of U, where (U, d) is a universal derivation module of A as R-algebra. (Received February 13, 1969.)

69T-A61. P. L. MANLEY, University of Windsor, Windsor, Ontario, Canada. Direct limits of free groups with finite rank.

The limit group of a direct system of free groups with finite rank is not, in general, a free group. The direct system of the additive groups of integers Z \mathbb{Z} \mathbb{Z} \mathbb{Z} \mathbb{Z} ... where p denotes multiplication by p has as its limit group the p-adic rational group which is not a free group. We determine the sufficient conditions for the limit group of a direct system of free groups with finite rank to be a free group.
group. \textbf{Theorem.} If $\{F_i, f_i\}$ is a direct system of free groups with finite rank, directed by a well-ordered set $I$, such that the image of $f_{ij} : F_i \rightarrow F_j$ is a subgroup closed under division in $F_j$ for $i < j$, $i, j \in I$, then the limit group $\varprojlim \{F_i, f_i\}$ is a free group. (Received February 14, 1969.)


Let $f$ and $g$ be two polynomials in an algebra. The equation $f = g$ is called regular (following J. Płonka, Fund. Math. 41 (1967), 183-189) if the set of variables occurring in $f$ is the same as that in $g$. It is called binary regular if, furthermore, the number of variables involved in it is at the most two. Płonka has shown (loc. cit.) that the set of all regular equations of lattices is finitely based. In this note a lattice-theoretic characterization of this set of equations is given. Namely, a set of eight regular equations--which describe (i) the semilattice character of the two operations $\lor$ and $\land$; and (ii) the substitution property of the natural partial order relation determined by one with the other--yields all the regular equations of lattices. From this result it is deduced that the set of all binary regular equations of lattices is also finitely based. Neither of these two sets of equations are one based. (Received February 17, 1969.) (Author introduced by Professor George A. Grätzer.)

69T-A63. ELBERT M. PIRTL, University of Missouri, Kansas City, Missouri 64110. Families of valuations and semigroups of fractional ideal classes.

Let $R$ be an integral domain with quotient field $L$ and let $F$ be a family of valuations on $L$ such that (i) each $v \in F$ has rank one, (ii) $R = \bigcap \{R_v | v \in F\}$, (iii) $R_v = R_p(v)$ for each $v \in F$. A partially ordered semigroup $\mathcal{O}(R)$ of fractional ideal classes is constructed using the family $F$. Necessary and sufficient conditions for $\mathcal{O}(R)$ and $\mathfrak{A}(R)$, the divisor group of $R$ constructed in Bourbaki's \textit{Commutative Algebra}, Chapitre 7, are determined. One of the main tools is the following \textbf{Proposition.} There is an order preserving homomorphism from $\mathcal{O}(R)$ onto $\mathfrak{A}(R)$. Almost-Krull domains with the property that $\mathcal{O}(R) \cong \mathfrak{A}(R)$ are studied. An example is given to show that $F$ need not be of finite character in order that $\mathcal{O}(R) \cong \mathfrak{A}(R)$. However, the domain $R$ in the example is not almost-Krull. The following question remains open: If $R$ is almost-Krull and $\mathcal{O}(R) \cong \mathfrak{A}(R)$, then is $R$ a Krull domain? This is true in the special case where $R$ is almost-Dedekind. (Received February 17, 1969.)


A group will be called a C- (respectively V, respectively CNV) group if it is the quotient group of a free group with respect to a characteristic (respectively verbal, respectively characteristic but not verbal) subgroup. \textbf{Lemma.} The direct sum of a CNV group and a C-group, both of which have a minimal set of generators of the same size, is a CNV-group. \textbf{Lemma.} Every direct summand of a C-group is a C-group. \textbf{Lemma.} The direct sum of two V-groups is a V-group. In dealing with finite nilpotent CNV-groups, these lemmas enable one to restrict consideration to finite CNV-groups which are P-groups. \textbf{Theorem.} If $G$ is a finite CNV p-group of nilpotence class 2 or 3, then $p = 2$ and $G$ has a minimal set of two generators and a presentation of the form $G = \langle x, y | x^{2^{2n+1}} = y^{2^{2n+1}} \rangle$. 

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\[(x,y)^{2^n} = (x,y,x) = (x,y,y) = 1\] where \(n \geq 0\), or else 
\[G = \langle x,y|x^m = y^m = (x,y)^q = (x,y,x)\rangle = \langle (x,y,y)\rangle = \langle (x,y,x,x)\rangle = \langle (x,y,x,y)\rangle = \langle (x,y,y,y)\rangle,\] where 
\[m = 2^u + v + 2, \quad q = 2^u + w, \quad p = 2^u, \quad a = 2^u + v + 1, \quad c = 0 \text{ or } 2^u + w + 1 \text{ with } c = 0 \text{ iff } w \neq v, \quad d = 0 \text{ or } 2^u - 1 \text{ and } u, v, \text{ and } w \text{ are nonnegative integers.}\] Here \((x,y)\) means \(x^{-1}y^{-1}xy\) and \((x,y,z)\) means \((x,y),z\), etc. In any of these cases, the quaternion group is a quotient group of \(G\). (Received February 12, 1969.)

Centralizers of permutation groups.

In the following, the centralizer of a permutation group on a set \(\Omega\) will always denote its centralizer in \(\text{Sym}^{\Omega}\). Two representations of a group \(G\) on \(\Omega\) and \(\Omega'\) are equivalent if \(\exists\) bijection 
\[b : \Omega \rightarrow \Omega'\text{ such that } \forall g \in G, \quad \forall \alpha \in \Omega, \quad \alpha^b = (\alpha^g)^b.\] A group \(G \not\subseteq \text{Sym}^{\Omega}\) is primary if all its transitive constituents are equivalent representations of \(G\). A primary constituent of \(G \not\subseteq \text{Sym}^{\Omega}\) is a maximal primary representation of \(G\) on a subset of \(\Omega\) (union of orbits). Theorem 1. \(G\) semiregular \(\Rightarrow G\) primary \(\Rightarrow G\) half-transitive, but neither converse implication is true. Theorem 2. The centralizer of a primary group \(G \not\subseteq \text{Sym}^{\Omega}\) is isomorphic to \(C \wr \text{Sym}(k)\) where \(C\) is the centralizer of (any) transitive constituent of \(G\) and \(G\) has \(k\) orbits. Theorem 3. The centralizer of a group \(G \not\subseteq \text{Sym}^{\Omega}\) is isomorphic to the direct product of the centralizers of its primary constituents. Several well-known theorems follow as corollaries; see references in my Abstract 663-739, these \(\text{Notices}\) 16 (1969), 302 (note: in the Theorem stated in that Abstract, \(H_i\) should have been defined as the centralizer of a transitive constituent on an orbit of length \(C_i\), instead of merely the transitive constituent). (Received February 12, 1969.)

A note on almost Krull domains.

Let \(D\) be an integral domain with quotient field \(K\). Following Pirtle in [\(J. \text{Sci. Hiroshima Univ., Ser. A - I, 32 (1968), 101-107}\)] we say that \(D\) is an almost Krull domain provided \(D_p\) is a Krull domain for each nonzero prime ideal \(P\) of \(D\). Theorem 1. In an almost Krull domain \(D\), the following are equivalent: (a) \(D\) is one dimensional, (b) \(D\) is Prüfer, and (c) \(D\) is almost Dedekind. With Griffin in [\(\text{Trans. Amer. Math. Soc. 130 (1968), 75-85}\)] we say that \(D\) is a domain of finite character provided there exists a family \(\{V_a\}\) of valuation rings each member of which contains \(D\) and with the additional properties that \(D = \cap_a V_a\) and each nonzero element \(x \in D\) is a nonunit in only finitely many \(V_a\)'s. Theorem 2. An almost Krull domain which is also a domain of finite character is a Krull domain. (Received February 13, 1969.)

Reciprocities between matrices with repeated elements.

Given an \(n \times n\) matrix \(A\) whose every row and column has \(p_i\) elements equal to \(a_i\), \(\Sigma_{i=1}^r p_i = n, 1 < r < n\). What are the conditions for \(B = A^{-1}\) to have in every row and column \(q_j\) elements equal to \(b_{ij}\), \(\Sigma_{i=1}^s q_i = n, 1 < s < n\). An earlier special solution \(r = 2 = s = 2\) for \((a_i, n) = 1\) appeared in the \(\text{Notices}\) and its significance is now enhanced by the proof that the generalization does not allow any
similar clear cut result: It can only be said that for n odd \( r \neq (n + 1)/2 \) \( \neq s \neq (n + 1)/2 \) and that for 

n even there is a choice between \( r \leq n/2 \) \( = s \leq n/2 \) and \( r \leq n/2 \) \( + 1 = s \neq n/2 + 1 \) but that no correspondence between specific values of \( r \) and \( s \) satisfying the strict inequality can be established except in dependence of the \( a_i \)’s. The privileged situation of the special case is due to a property defined as "equispacity" which specializes symmetry when \( r = 2 \) but as soon as \( r > 2 \), equispacity conflicts with any symmetry that reflects the regularity of the matrix. (Received February 25, 1969.)

69T-A68. CHARLES F. MILLER, III, and PAUL E. SCHUPP, University of Illinois, Urbana, Illinois 61801. Embeddings into hopfian groups.

Recall that a group \( K \) is hopfian if every epic endomorphism of \( K \) is an automorphism.

**Theorem.** Every countable group \( G \) is embeddable in a 2-generator, complete, hopfian group \( H \). (\( H \) depends on \( G \).) \( H \) can be chosen as a quotient of the group of 2 \( \times 2 \) unimodular matrices with integer entries \( \text{SL}(2,\mathbb{Z}) \). If \( G \) can be finitely presented, so can \( H \). Finally, if \( G \) is finitely generated, then the word and conjugacy problems for \( H \) have the same Turing degrees as those for \( G \).

**Corollary.** Let \( D \) be an r.e. degree of unsolvability. Then there is a finitely presented, hopfian group \( H \) (resp. \( K \)) with word problem (resp. conjugacy problem) of degree \( D \). Let \( G \) and \( H \) be as in the theorem. For \( n \neq 2,3 \), \( H \) has an element of order \( n \) if and only if \( G \) does. **Corollary 2.** \( \text{SL}(2,\mathbb{Z}) \) has continuously many nonisomorphic, complete, hopfian quotients. (Received February 25, 1969.)


A. A. Grau [**Ternary Boolean algebra**, Bull. Amer. Math. Soc. 53 (1947), 567-572] defines a ternary Boolean algebra as an algebraic system consisting of a set \( A \) and two operations on \( A \), one ternary \( (abc) \) and the other unary \( a' \), such that for all \( a,b,c,d,e \in A \) (i) \( (de(abc)) = ((dea)b(dec)) \), (ii) \( (baa) = a \), (iii) \( (abb') = a \), (iv) \( (aab) = a \), (v) \( (b'ba) = a \). In this paper it is proved that (i), (ii) and (iii) are sufficient to define a ternary Boolean algebra and that these three postulates are independent. A number of new sets of three independent postulates for a ternary Boolean algebra, e.g. (i) \( (baa) = a \), (ii) \( (b'ab) = a \), (iii) \( ((bde)(aed)c) = (b(edc)(aed)) \) and sets of two such independent postulates, e.g., (i) \( (abb') = a \), (ii) \( (de(abc)) = (b(dea)(edc)) \), and (i) \( (baa) = a \), (ii) \( ((bde)(aed)c) = (xz'(b(edc)(aed))x) \), are also obtained. (Received February 24, 1969.)

69T-A70. HOWARD E. GORMAN, Roosevelt University, Chicago, Illinois 60605. Invertibility and class number of orders.

Let \( L \) be a finite-dimensional, commutative, symmetric algebra with 1 over the quotient field \( K \) of a valuation ring \( R \). Let \( A \) be a finitely generated \( R \)-module contained in \( L \) which spans \( L \) over \( K \). Let \( P \) be an order in \( L \). Then, \( A \) is said to satisfy the Brandt Condition if \( N(A^\#)\Delta(A) \subset N(A) \), where \( A^\# \) is the dual of \( A \), \( \Delta(A) \) is the discriminant and \( N(A) \) is the norm of \( A \). Call \( A \) a Brandt module in this case, and \( L \) is called a Brandt algebra if modules are invertible if and only if they satisfy the Brandt Condition. Call modules \( A \) and \( B \) equivalent if there is \( x \in L \) such that \( A = xB \). The following theorems are proved: **Theorem 1.** The following are equivalent for \( P \): (i) \( P^\# \) is principal, (ii) \( P^\# \) satisfies the Brandt Condition with equality, (iii) all modules with order \( P \) are principal. **Theorem 2.**
L is a Brandt algebra if and only if any order in \( L \) has at most 2 equivalence classes of modules.

**Theorem 3.** If \( R \) is a discrete valuation ring with finite residue class field, then any order in \( L \) has only a finite number of equivalence classes of modules. (Received February 24, 1969.)


Call an integral domain \( R \) an M-domain if the homological dimension of its quotient field \( Q \) as an \( R \)-module is at most one. Then a result of Kaplansky [The homological dimension of a quotient field, Nagoya Math. J. 27 (1966), 139-142] is generalized as follows: **Theorem.** If \( R \) is an M-domain, then \( Q/R \) possesses a nonzero, countably generated, direct summand. Secondly a circle of ideas due to Matlis [Divisible modules, Proc. Amer. Math. Soc. 11 (1960), 385-391] is completed. **Theorem.** For a domain \( R \) the following are equivalent: (1) \( R \) is an M-domain, (2) the torsion submodule of a divisible \( R \)-module is always a direct summand, (3) every divisible \( R \)-module is the quotient of an injective. (Received February 20, 1969.) (Author introduced by Professor Irving Kaplansky.)

69T-A72. M. A. CREW, Midwestern University, Wichita Falls, Texas 76308. Semigroups which are groups.

A recent abstract of [A. R. Brown and D. R. Cecil, Abstract 69T-A25, these Notices 16 (1969), 405] gives a characterization of groups in terms of their closed sets. Let \( N \) be the intersection of all subsemigroups of a semigroup \( S \). Suppose (i) \( N \neq \emptyset \) and is finite and (2) if \( \emptyset \neq A \subseteq S \) and \( G \) is any cyclic subgroup of \( S \) then \( A \cap GA \neq \emptyset \). **Theorem.** \( S \) is a group. The converse is trivial. (Received February 24, 1969.) (Author introduced by Professor C. R. Williams.)

69T-A73. ROBERT J. PLEMMON and J. STEPHEN MONTAGUE, University of Tennessee, Knoxville, Tennessee 37916. The maximal subgroups of the semigroup of relations on a set.

**Theorem.** Let \( X \) be a countably infinite set. Then every countable group is a maximal subgroup of the semigroup \( B_X \) of relations on \( X \). In particular, each finite group is a maximal subgroup of some \( B_X \) where \( X \) is finite. It had been conjectured that the class of such maximal subgroups included only the symmetric groups. We obtain these results by first noting that for any countable group \( G \), there is a graph having \( G \) as its automorphism group. An incidence matrix for this graph is then embedded in the relation matrix for an idempotent relation \( a \) on a set \( X \). We then use a characterization of the Green's relations on \( B_X \) to show that \( a \) is the identity element of a maximal subgroup of \( B_X \), isomorphic to \( G \). Thus we obtain a new representation theorem for countable groups. (Received February 26, 1969.)

69T-A74. PHILIP KELENSON, University of California, Berkeley, California 94720. Generalized semicategorical algebras. Preliminary report. (1)

For similar algebras \( L, U S_w(A) \) denotes the set of all finite subsets of \( L, O(U) \) the set of all one-element subalgebras of \( U, \) and \( \text{Rep}(U, U) \) the class of all isomorphisms on \( U \) into subalgebras of direct powers of \( U \). \( U \to U \) means that there exists \( \Delta : S_w(B) \to S_w(A) \) such that for all \( x \in S_w(B) \) \( \Delta_x \neq \emptyset \) if
and every equation valid in $\mathcal{U}$ for all values of elements of $\Delta_x$ is also valid in $\mathcal{F}$ for all values of elements of $x$. $\mathcal{U}$ is semicategorical [wk] if $\mathcal{U} \not= \emptyset$ and every $\mathcal{F} \in \text{Rep}(\mathcal{U})$ and $\Delta : \mathcal{B} \rightarrow S_m(\mathcal{A})$ such that $\text{rng}(x) \subseteq \Delta_x [\Delta_x \cup \mathcal{O}(\mathcal{U})]$ for all $x \in \mathcal{B}$. $\mathcal{U}$ is finitarily semicategorical [wk] if $\mathcal{U} \not= \emptyset$ and finitarily generated $\Rightarrow$ (i) $\text{Rep}(\mathcal{U}) \not= \emptyset$ and (ii) $\exists \Delta : \mathcal{B} \rightarrow S_m(\mathcal{A})$ such that for any homomorphism $\sigma$ on $\mathcal{U}$ into $\mathcal{F}$ and all $x \in \mathcal{B}$, $\sigma x \in \Delta_x [\Delta_x \cup \mathcal{O}(\mathcal{U})]$. Theorem. If $\mathcal{U}$ is finitarily semicategorical then $\mathcal{U}$ is semicategorical and if $\mathcal{U}$ has only finitely many primitive operations of positive rank, then the above also holds "wk". This generalizes results of Astromoff and Foster-Pixley where finiteness restrictions were imposed on $\mathcal{U}$—cf. Foster-Pixley, Semicategorical algebras. I, Theorem 4A, and II Theorem 5.4, Math. Z. 83, 85 (1964). Classical examples of (infinite) finitarily semicategorical algebras may be found among subgroups of the group of all multiplicative roots of unity and "algebraic" fields.

(Received March 7, 1969.) (Author introduced by Professor Ralph McKenzie.)

69T-A75. JOEL L. BRENNER, University of Arizona, Tucson, Arizona 85721. $AB$ and $BA$ have the same characteristic equation.

The theorem of the title is not true. However, if $A$ and $B$ are square matrices it is true, and we give a simple and direct proof, which is even valid when the underlying field is noncommutative. The only lemma needed is that every matrix can be factored into the product of elementary (atomic) matrices. [The lemma is established by peeling off the coefficients of an arbitrary matrix one by one. An elementary matrix is by definition either $I + ae_{i1}$ (including $\text{diag}[1,1,...,1,0]$), or $I + be_{ij}$, i.e. a matrix obtained by adding any multiple of a matrix unit to the identity matrix.] With this lemma established, one first checks merely that $AE_k$ and $EkA$ have the same characteristic equation whenever $Ek$ is an elementary matrix. The proof is completed by applying induction, based on the number of factors $Ek$ needed (in the lemma) to produce $B = \Pi E_k$. (Received March 13, 1969.)

69T-A76. ALTON RAYMOND BROWN, North Texas State University, Denton, Texas 76203. Existence of a binary operation on an infinite set.

Let $\overline{H}$ be the set of all bijections on an infinite set $H$. A nonempty subset $\overline{C}$ of $\overline{H}$ is said to be pairwise distinct if for any $f$, $g \in \overline{C}$ either $f = g$ or, for every $x \in H$, $f(x) \not= g(x)$. A pairwise distinct set $\overline{C}$ is maximal if $\overline{C} \cup \{f\}$ is not pairwise distinct for any $f \in \overline{H} - \overline{C}$. Theorem. If $\overline{C}$ is a maximal pairwise distinct subset of $\overline{H}$ then card $\overline{C}$ = card $H$. The proof is accomplished by well-ordering $\overline{H}$ by a limit ordinal and appears to have no finite analogue. A binary operation is now easily obtained on $H$ by indexing $\overline{C}$ by $H$ and defining $a \circ b = f_a(b)$ for all $a, b \in H$. (Received February 17, 1969.) (Author introduced by Professor David R. Cecil.)


A subgroup $M$ of a finite group $G$ is a CC subgroup if $M$ centralizes each of its nonunit elements. The purpose of this paper is to prove the following Theorem. Let $G$ be a finite group with $M$ a CC Sylow three subgroup of $G$. Assume that the normalizer of $M$ in $G$ is a cyclic group of odd order greater than 1. Then either $M$ is abelian or $M$ is normal in $G$. The proof uses a knowledge of the infinite polyhedral group $\langle x, y, z | x^3 = y^3 = z^3 = xyz = 1 \rangle$, and character theory methods. (Received February 20, 1969.)
A homomorphism theorem for halfrings.

For background, see Abstract 665-556, these Notices 16 (1969), 249. Let \( \eta \) be a homomorphism of the halfring \( H \) onto the halfring \( K \), let \( \overline{\eta} \) be the induced homomorphism of the ring of differences \( \overline{H} \) onto \( \overline{K} \), let \( M \) be the kernel of \( \eta \) in \( H \), and let \( N \) be the kernel of \( \overline{\eta} \) in \( \overline{H} \). Let \( \nu \) be the natural homomorphism of \( \overline{H} \) onto \( \overline{H}/M \), and let \( \nu \) be the natural homomorphism of \( H \) onto \( H/N \). Theorem 1. \( H\nu \) is isomorphic to \( H/M \) and \( H\nu \) is isomorphic to \( K \). By the type \( \tau(M) \) of \( M \) is meant the family of all ideals \( I \) of \( H \) such that \( I \cap H = M \). Clearly \( M, N \in \tau(M) \). If \( \tau(M) = \{M\} \), \( M \) is said to be monotypic in \( H \). Theorem 2. In order that \( H\eta \) be isomorphic to \( H/M \) for every halfring homomorphism \( \eta \) with \( \text{Ker}(\eta) = M \), it is necessary and sufficient that \( M \) be monotypic in \( H \). (Received March 17, 1969.)

The semigroup of endomorphisms of a universal algebra \( \mathfrak{U} \) is denoted by \( E(\mathfrak{U}) \), its lattice of subalgebras by \( S(\mathfrak{U}) \), and its lattice of congruences by \( C(\mathfrak{U}) \). Let \( L \) be any lattice, and for each \( p \in L \) let \( M_p \) be a semigroup with identity. Theorem 1. There exist an algebra \( \mathfrak{U} \) and an isomorphism \( \phi \) of \( L \) onto a sublattice of \( S(\mathfrak{U}) \) such that (a) for every \( p \in L \), \( E(\phi(p)) \cong M_p \), and (b) if \( p \) covers \( q \) in \( L \) (i.e., \( p \supseteq q \) and there is no \( r \) in \( L \) such that \( p > r > q \)), then \( \phi(p) \) covers \( \phi(q) \) in \( S(\mathfrak{U}) \). Theorem 2. There exist an algebra \( \mathfrak{B} \) and a \( 1 \)-\( 1 \) join-preserving map \( \phi \) of \( L \) into \( C(\mathfrak{B}) \) such that \( E(\phi(p)) \cong M_p \) for all \( p \in L \). Remarks: Theorem 1(a) for a well-ordered class was proved by E. Mendelsohn and Z. Hedrlin (On the category of graphs with a given subgraph, to appear in Canadian Journal of Math.). Theorem 2 for a two element lattice was proved by Z. Hedrlin (On endomorphisms of graphs and their homomorphic images", to appear in Proof Techniques in Graph Theory, Academic Press). (Received March 14, 1969.)

Groups of weak isomorphisms.

If \( \mathfrak{U} = (A,F) \) is a universal algebra (all operations are finitary), \( A(\mathfrak{U}) \) will denote the group of automorphisms of \( \mathfrak{U} \), and \( W(\mathfrak{U}) \) will denote the group of weak automorphisms of \( \mathfrak{U} \) (for the definition of weak isomorphism see A. Goetz, On weak isomorphisms and weak homomorphisms of abstract algebras, Colloq. Math. 14 (1966), 163-167). Following the observation that \( A(\mathfrak{U}) \) is a normal subgroup of \( W(\mathfrak{U}) \), the following results were obtained: For any group \( G \) there is a unary algebra \( \mathfrak{U} \) such that \( W(\mathfrak{U}) \cong G \) and \( \mathfrak{U} \) has no nontrivial automorphisms. For any groups \( G,H \) there is an algebra \( \mathfrak{U} \) with \( A(\mathfrak{U}) = H \) and \( W(\mathfrak{U}) = G \times H \). For investigating \( W(\mathfrak{U}) \), the following concept has proved useful:

Definition. The algebra \( \mathfrak{U} = (A,F) \) is called \( A \)-saturated if the only finitary operations which can be added to \( F \) without changing \( A(\mathfrak{U}) \) are the operations derived from \( F \). We have the Theorem. If \( \mathfrak{U} = (A,F) \) is saturated then \( W(\mathfrak{U}) \) is the normalizer of \( A(\mathfrak{U}) \) in the group of permutations of \( A \). (Received March 17, 1969.)
69T-A81. SHEILA M. KAYE, McGill University, Montreal 110, Province of Quebec, Canada, On perfect group rings.

The group ring $AG$ of a group $G$ over a ring $A$ is perfect in the sense of Bass (Finitistic dimension and a homological generalization of semi-primary rings, Trans. Amer. Math. Soc. 95 (1960), 466-488) if and only if $G$ is finite and $A$ is perfect. (Received March 17, 1969.)

69T-A82. YOUNG LIM PARK, Laurentian University, Sudbury, Ontario, Canada, On the Jordan and Lie ideals of an associative ring.

Let $R$ be an associative ring. For subsets $A$ and $S$ in $R$, $(A)_S$ denotes the set of all $a \in A$ such that $a^s \in S$. Proposition 1. If $A$ is a Jordan or Lie ideal in $R$, then $(A)_R$ is the largest 2-sided ideal in $R$ contained in $A$, and if $A$ is a Lie ideal then $(A)_A$ is both a Lie ideal and a subring of $R$. Proposition 2. Let $A$ be a subring of $R$. If $A$ is a Jordan or Lie ideal in $R$, then either $A$ is commutative or $A$ contains a nonzero 2-sided ideal. Having these and the Lemma 3 of Herstein, Amer. J. Math. (2) 77 (1955), 279-285, one has Proposition 3. If $A$ is a Lie ideal in $R$, then either $ab = ba$ holds for any two elements $a$ and $b$ in $A$, or there exists a nonzero 2-sided ideal $I$ in $R$ such that $A \supseteq [I,R]$. (Received March 3, 1969.) (Author introduced by Dr. B. Banaschewski.)


This note generalizes an earlier result (concerning additive representations with a prime number of summands; Abstract 68T-A51, these Notices 15 (1968), 1039) so as to establish another variation of the Waring problem. Theorem. Let $N$ be the set of all strictly positive integers and $P$ be its subset of all prime numbers. For each $k \in N$, there exists $q \in P$ such that for each sufficiently large $n \in N$, $(*) n = \sum_i \sum p m_i^k$ for some $p \in P$ with $p \leq q$, where $m_i \in N$ and $i \in N$. No attempt is made here to replace $P$ with other proper subsets of $N$; e.g., the even integers. Let $G^*$ be defined by the relation: $G^*(k)$ is the least prime $q$ for which $(*)$ is solvable for every sufficiently large $n$. $G^*(k)$ is estimated and compared to its classical analogue $G(k)$ (R. Ayoub, An introduction to the analytic theory of numbers (1963)). Finally, $R(n,k)$ is defined to be the number of representations for which $(*)$ is solvable. This $R(n,k)$ is related to the earlier $R(n)$ (Abstract 69T-A16, these Notices 16 (1969), 313). (Received March 24, 1969.)


Let $K$ be a class of at most denumerable groups. A denumerable group $G$ is $(N_0,K)$-weakly universal if every $N \in K$ is isomorphically embeddable in $G$. Let $K_1$, $K_2$, $K_3$ respectively be the classes of at most denumerable groups which are 0-groups, locally nilpotent groups, metabelian groups. Lemma 1. There exists $2^{N_0}$ nonisomorphic finitely generated 0-groups. The proof of the lemma uses results of P. Hall [Finiteness conditions for soluble groups, J. London Math. Soc. (3) 4 (1954), 419-436] and B. H. Neumann [On ordered groups, Amer. J. Math. 71 (1949), 1-18]. Theorem 2. There is no group which is $(N_0,K_1)$-weakly universal. This theorem answers a question.

Theorem 3. There exists a group which is \((\mathbb{N}_0, K_2)\)-weakly universal.

Theorem 4. There exists a group which is \((\mathbb{N}_0, K_3)\)-weakly universal. (Received March 26, 1969.)

69T-A85. SURJEET SINGH and KAMLESH WASAN, Department of Mathematics, K. M. College, University of Delhi, Delhi-7, India. Self-injective rings and multiplication rings. II.

Any ring considered here is commutative, contains at least two elements but may not have unity. A ring \(R\) is said to have \((I)\)-property if each proper homomorphic image of \(R\) is self-injective. A ring \(R\) is said to have \((J)\)-property if for each nonzero proper prime ideal \(P\), \(R/P^2\) is a self-injective ring (see also, S. Singh and K. Wasan, Self-injective rings and multiplication rings, Abstract 69T-H16, these Notices 16 (1969), 436). The following results have been proved: (a) A noetherian ring with \((I)\)-property is either a simple trivial ring or has unity; (b) noetherian ring \(R\) satisfies \((J)\)-property if and only if one of the following holds: (i) \(R\) is a primary domain (an integral domain whose only prime ideals are \((0)\) and \(R\)), (ii) \(R\) is a nil-ring, (iii) \(R\) is a Dedekind domain, (iv) \(R\) is a PIR with d.c.c.; (c) for a ring \(R\) with \(1 \neq 0\), the following are equivalent: (i) \(R\) is an almost multiplication ring, (ii) for each proper prime ideal \(P\), the quotient ring \(R_P\) is a noetherian ring with \((J)\)-property, (iii) for each proper prime ideal \(P\), \(R_P\) is a noetherian ring with \((I)\)-property and \(R_P\) contains at most one minimal ideal. (Received March 3, 1969.) (Author introduced by R. S. Varma.)

69T-A86. GEORGE A. GRÄTZER and J. PÖNKA, University of Manitoba, Winnipeg 19, Manitoba, Canada. On the number of polynomials of a universal algebra. II.

For \(n > 1\), let \(p_n(\mathfrak{U})\) denote the number of \(n\)-ary polynomials of the universal algebra \(\mathfrak{U}\) depending on all \(n\) variables; let \(p_1(\mathfrak{U})\) be the number of nonconstant unary polynomials excluding \(p(x) = x\); let \(p_0(\mathfrak{U})\) be the number of constant unary polynomials. Theorem 1. Let \(p_0(\mathfrak{U}) = p_2(\mathfrak{U}) = p_3(\mathfrak{U}) = 0\). Then \(n\) divides \(p_n(\mathfrak{U})\) for all \(n > 3\). The special case \(p_1(\mathfrak{U}) = 0\) is equivalent to a result of G. H. Wenzel (Math. Z. 102 (1967), 205-215). Corollary. The sequence \((0, 0, 0, p_4, p_5, \ldots)\) with \(p_1 > 0\) is representable (i.e., there exists an algebra \(\mathfrak{U}\) with \(p_n = p_n(\mathfrak{U})\) for all \(n\)) iff \(n\) divides \(p_n\) for all \(n > 3\).

Theorem 2. Let \(\mathfrak{U}\) be an algebra with \(p_0(\mathfrak{U}) = p_1(\mathfrak{U}) = 0\). If for some integer \(k\) we have \(1 \neq p_n(\mathfrak{U}) \neq k\) for infinitely many \(n > 1\), then \(p_n(\mathfrak{U}) = 1\) for all \(n > 1\) (and \(\mathfrak{U}\) is equivalent to a semilattice). Corollary. The only bounded representable sequence starting with \(0, 0\) and with no further zeros is \((0, 0, 1, 1, \ldots, 1, \ldots)\). The proof of Theorem 2 uses an unpublished result of J. Dudek. (Received April 11, 1969.)


By extending some ideas of Schensted (Canad. J. Math. 13 (1961), 179-191) it is possible to obtain constructive proofs of the following Theorem (MacMahon). The generating function for plane partitions of at most \(r\) rows and \(c\) columns is \(\Pi_{0 \leq k < r} \Pi_{1 \leq i \leq c} (1 - x^{i+k})^{-1}\). Theorem. The generating function for plane partitions which are strict in rows \((a_{ij} > a_{i+1,j})\) and whose parts belong to a set \(S\) is \(\Pi_{1 \leq j \leq c} (1 - x^{i})^{-1} \cdot \Pi_{1 \leq j \leq c} (1 - x^{i+j})^{-1}\) where the second product is over all \(i, j \in S\) such that \(i < j\). (Received April 14, 1969.)
A ring $R$ is shown to be left artinian if and only if every finitely generated left $R$-module is cofinitely generated (i.e., has a finitely generated essential socle). A semiprimary ring $R$ is shown to be left artinian provided simple left $R$-modules are cofinitely presented (i.e., have a cofinitely generated injective resolution) and either $J(R)$ is central or $R$ is cofinitely generated. (Received April 11, 1969.)

The problem of giving a fruitful definition for prime ideals in Jordan rings has been considered by Tsai (The prime radical in a Jordan ring, Proc. Amer. Math. Soc. 19 (1968), 1171-1175, and The Lavitzki radical in Jordan rings, Abstract 663-451, these Notices 16 (1969), 217). Tsai has defined the prime radical and the Lavitzki radical for a Jordan ring and also has proved theorems characterizing these radicals as the intersection of a family of prime ideals. For the positive integer $s \geq 2$, define an $s$-naring as a naring in which $A^s$ is an ideal whenever $A$ is. An ideal $P$ in $N$ is a prime ideal provided that whenever $A_1, A_2, \ldots, A_s$ are ideals in $N$ such that $A_1A_2\ldots A_s \subseteq P$ then either $A_1 \subseteq P$, or $A_2 \subseteq P$, ..., or $A_s \subseteq P$. Using this definition the prime radical of an $s$-naring can be defined and theorems corresponding to those of Tsai can be proved. If, in addition, $N$ satisfies the Zavlakav property, then the Lavitzki radical of $N$ can be defined and characterized as the intersection of the prime ideals $P$ such that $N/P$ is Lavitzki semisimple. Since Lie, alternative, Jordan, and standard rings are 2, 2, 3, and 3-narings satisfying the Zavlakav property, the results apply to these classes of rings. (Received April 14, 1969.)

Let $R$ be a regular (von Neumann) commutative ring with an identity. Every simple module over $R$ is injective. R. P. Kurshan has shown that if every semisimple module over $R$ is injective, then $R$ is noetherian [Abstract 663-115, these Notices 15 (1969), 111], and thus there are only a finite number of nonisomorphic simple modules over $R$. Assume $R$ has an infinite number of nonisomorphic simple modules. Let $S = \sum_{i \in I} \Theta(R/m_i)$, where each $m_i$ is a maximal ideal, and let $E$ be the injective hull of $S$. If $S$ has only a finite number of nonisomorphic simple summands, it is easy to show that $S = E$. Theorem 1. Let $S$ have an infinite number of nonisomorphic simple summands. Then $S \neq E$. Theorem 2. Let $S = \sum_{k \in K} \Theta T_k$, where each $T_k$ is a homogeneous component of $S$. Then $E = \prod_{k \in K} T_k$ iff $R/((\cap_{j \in J} m_j)$ has nonzero socle for all $J \subseteq I$. (Received April 16, 1969.)
R is a ring with identity. \( R \) is quasiprojective iff, for every submodule \( N \) of \( M \), \( \text{hom}_R(M,M/N) = \text{hom}_R(M,M) \cap N \) (\( \nu_N \) the canonical projection). \( \text{[See Y. Miyashita, Quasiprojective modules, perfect modules, and a theorem for modular lattices, J. Faculty of Sciences, Hokkaido Univ., ser I, 19 (1966), 86-110; L. E. T. Wu and J. P. Jans, On quasiprojectives, Illinois J. Math. 11 (1967), 439-448.] \)

**Theorem.** Every left \( R \)-module is quasiprojective iff \( R \) is artinian semisimple. This dualizes a result of Faith and Utumi for quasi-injective modules \( \text{[Quasiinjective modules and their endomorphism rings, Arch. Math. 15 (1964), 166-174.]} \)

**Theorem.** Every left \( R \)-module is projective iff every left quasiprojective \( R \)-module is projective. (Received February 25, 1969.)

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69T-A91. JONATHAN S. GOLAN, The Hebrew University, Jerusalem, Israel. Rings over which all modules are quasiprojective.

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69T-A92. EVELYN NELSON, McMaster University, Hamilton, Ontario, Canada. \text{Equational classes of commutative semigroups.}

Let \( A \) be the lattice of all pairs \((r,s)\) of natural numbers with \( r \leq s \) and \( s \neq 0 \) ordered component-wise, i.e. \((r,s) \leq (t,u) \) iff \( r \leq t \) and \( s \leq u \). Let \( N' \) be the lattice of natural numbers \( \neq 0 \) ordered by division. Then \( A \times N \) is isomorphic to a sublattice of the lattice of equations classes of commutative semigroups; the isomorphism is given by \((s,r,n) \rightarrow \Omega_{s,r,n} \) where \( \Omega_{s,r,n} \) is the class of commutative semigroups satisfying \( x^s y^r = x^{s+n} y^r \) and \( x^r = x^{r+n} \). Moreover, for each equational class \( R \), there exist natural numbers \( r,s,t,n \) such that \( r \leq s \leq t \) and \( \Omega_{r,s,n} \leq R \leq \Omega_{r,t,n} \) and such that \((r,s,n)((r,t,n))\) is the largest (respectively smallest) element in \( A \times N \) with this property. For \( s \neq 1 \), the interval \([\Omega_{s,r,n}, \Omega_{s,t,n}]\) in the lattice of equational classes of commutative semigroups is isomorphic to a sublattice of \([\Omega_{s-1,r,n}, \Omega_{s-1,t,n}]\); the isomorphism is given by \( R \rightarrow R \cap \Omega_{s-1,t,n} \). If \( s = 1 \), this isomorphism maps onto \([\Omega_{0,r,n}, \Omega_{0,t,n}]\). If \( p | n \), then \([\Omega_{s,r,n}, \Omega_{s,t,n}]\) is isomorphic to a sublattice of \([\Omega_{s,r,n}/p, \Omega_{s,t,n}/p] \) via the map \( R \rightarrow R \cap \Omega_{s,t,n}/p \). (Received April 18, 1969.)

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69T-A93. FRED GALVIN and ALFRED HORN, University of California, Los Angeles, California 90024. Functions having the substitution property.

**Theorem.** Let \( S \) be a set of cardinal \( \beta > 2 \) and let \( f \) be a nonconstant \( \alpha \)-ary operation on \( S \), where \( \alpha \) is an ordinal. Then the following are equivalent: (1) \( f \) has the substitution property with respect to all equivalence relations on \( S \); (2) \( f \) has the substitution property relative to all equivalence relations of the form \( R_\alpha \), \( \alpha \in S \), where \( R_\alpha \) is the equivalence relation whose equivalence classes are \([a]\) and \( S - [a] \); (3) there exists a \( \beta^+ \)-complete prime filter \( F \) in the algebra of all subsets of \( \alpha \) such that \( f(s) = a \) if and only if \([i : s(i) = a] \in F \). In particular if \( \alpha \) is finite or if \( \beta \) is infinite and \( \alpha \) is less than the first measurable cardinal, (3) says that \( f \) is a projection. (Received April 21, 1969.)

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In \( \text{[Bull, Soc. Roy. Sci. Liège 36 (1968), 399-408]} \) J. Varlet characterizes a three-valued Lukasiewicz algebra as a bounded distributive lattice \( L \) fulfilling the conditions (1) \( L \) is pseudo-complemented, (2) \( L \) is dually pseudo-complemented and (3) the prime ideals of \( L \) occur in disjoint chains.

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of at most two elements; and asks whether (1) and (3) imply (2). The following example shows that it is not so. Let \( L \) be the set of all functions \( f \) on \( \omega + 1 \) to 3 such that either \( f(p) = 2 \) for all \( p \) in a final segment of \( \omega \) and \( f(\omega) = 1 \) or 2, or \( f(p) = 0 \) for all \( p \) in a final segment of \( \omega \) and \( f(\omega) = 0 \). C. Chang and A. Horn proved in (Proc. Sympos. Pure Math., vol. 2, Amer. Math. Soc., Providence, 1961, pp. 43-48) that \( L \) is a bounded distributive lattice satisfying (3). Defining \( (\forall f)(x) = 0 \) if \( f(x) = 0 \) and \( = 2 \) if \( f(x) \neq 0 \), \( \forall \) is a Boolean multiplicative closure on \( L \) (R. Cignoli, Proc. Japan Acad. 42 (1966), 1168-1174), hence \( L \) satisfies (1). Considering the function \( f(p) = 2 \) for all \( p \in \omega \) and \( f(\omega) = 1 \), it is shown that \( B(L) \) is not upper relatively complete, hence \( L \) is not a Lukasiewicz algebra (R. Cignoli and A. Monteiro, Proc. Japan Acad. 41 (1965), 676-680). (Received April 21, 1969.)

69T-A95. SAAD MOHAMED, University of Delhi, Delhi-7, India. Rings whose homomorphic images are q-rings.

Levy gave a complete characterisation of a noetherian commutative ring in which every proper homomorphic image is self-injective. (Pacific J. Math. 18 (1966)). The object of this paper is to generalise Levy's result to the noncommutative case, by studying right noetherian rings in which every proper homomorphic image is a right q-ring. Jain, Mohamed and Singh characterised a right q-ring as a right self-injective ring in which every large right ideal is two-sided. (Abstract 68T-A12, these Notices 15 (1968), 784). The following two theorems are proved: (A) Let \( R \) be a nonprime right noetherian ring. Then, every proper homomorphic image of \( R \) is a right q-ring if and only if (1) \( R = S \oplus T \), where \( S \) is semisimple artinian and \( T \) is a principal ideal duo ring with descending chain condition, or (2) \( R/J \) is regular and every nonzero ideal of \( R \) contains \( J \), or (3) \( R \) is a local ring whose maximal right ideal \( M \) satisfies \( M^2 = 0 \), and every proper homomorphic image of \( R \) contains at most one proper right (left) ideal. (B) Let \( R \) be a prime right noetherian ring with the property that every proper homomorphic image is a right q-ring. Then, every ideal of \( R \) is a product of prime ideals, and every nonzero prime ideal is a maximal right ideal. (Received April 23, 1969.) (Author introduced by Professor R. S. Varma.)

Analysis


In this paper we correct a slight error in the proof of a theorem in W. Rudin, An arithmetic property of Riemann sums, Proc. Amer. Math. Soc. (2) 15 (1964), 321-324. (Received February 17, 1969.)

69T-B77. S. E. HAYES, University of Texas, Austin, Texas. The relationships among certain Stieltjes integrals.

Let \( f \) and \( g \) be real functions whose domains include the interval \([a, b]\). If \( \int_a^b f \, dg \) exists, then \( \int_a^b |f| \, dg \) exists. (This theorem was stated in Abstract 68T-B57, these Notices 15 (1968), 922.) If \( \int_a^b |f| \, dg \) exists, then \( \int_a^b f \, dg \), \( \int_a^b f \, dg \), and \( \int_a^b |f| \, dg \) exist. If \( \int_a^b |f| \, dg \) exists, then \( \int_a^b |f| \, dg \) exists. If \( \int_a^b f \, dg \) and \( \int_a^b |f| \, dg \) both exist, then \( \int_a^b f \, |dg| \) exists. It can be shown by very simple examples that no other theorems of this type can be proved about the integrals considered here. (Received February 14, 1969.)
Determination of the modulus $\tau$ of a doubly-connected domain by using the Szegö kernel.

Let $\widehat{T}_D(z,\bar{z}) = (\partial^2 / \partial z \partial \bar{z}) \log \mathcal{K}_D / \partial z \partial \bar{z}$, where $\mathcal{K}_D = \mathcal{K}_D(z,\bar{z})$ is the Szegö kernel of a doubly-connected domain $D$. Define $\alpha_0 = \max z \in D(z,\bar{z})$, $T_{\alpha_0} = \{ z : \mathcal{K}_D(z,\bar{z}) = \alpha_0 \}$ and $N_D(T_{\alpha_0})$ is the non-Euclidean length of $T_{\alpha_0}$ with respect to $dg = \mathcal{K}_D(z,\bar{z}) dz \partial \bar{z}$. Then $\tau$ is determined uniquely from the transcendental equation $N_0(T_{\alpha_0}) = 2 \int_0^{\alpha_0} \frac{r^{n+1/2}}{(1-r^{2n+1})} dr$. Note that the right-hand side of this equation is a monotonic function of $\tau$, $0 < \tau < 1$. A similar method has been obtained by using the Bergman kernel function. However, the advantages of the present method lie in the fact that the Szegö kernel can easily be computed numerically and its error estimate is known, see Nehari [Proc. Nat. Acad. Sci. 37, 6 (1951), 369-372], while in the Bergman case this estimate is unknown. (Received January 15, 1969.)

Schwartz maps.

Let $N$ and $M$ be two von Neumann algebras with $N \subset M$. Let $G$ be a subgroup of the unitary group of $M$. By a Schwartz map relative to $(G,M)$ one means a linear map of $M$ into itself such that $P(X) = UP(X)U^{-1}$ for all $U$ in $G$ and $P(X)$ is in the weak closure of the convex hull generated by elements of the type $UXU^{-1}$ as $U$ ranges over $G$. Call two groups of unitaries equivalent if they generate the same von Neumann algebra. Let $S(G,L(h))$ be all Schwartz maps of $L(h)$ relative to $G$. Theorem. If $S(G,L(h))$ contains a normal map then $G$ is equivalent to a countable direct sum of finite groups. (Received February 11, 1969.)

On the Laplace transform. II.

The object of this paper is to prove a theorem and its converse on the Laplace transform. Special cases of this theorem were proved earlier by Buschman [A substitution theorem for the Laplace transformation and its generalization to transformations with symmetric kernel, Pacific J. Math. 7 (1967), 6] and by the author [On the Laplace transform. I, sent for publication]. The theorem is used in evaluating various multiple integrals involving the H function and in solving certain integral equations involving multiple integration. (Received February 3, 1969.)

Transportable forms. Preliminary report.

Let $\pi : V \to M$ be a differentiable (complex analytic) family of complex manifolds with quasi-hermitian metric. For $t_0 \in M$ let $w$ be a harmonic scalar form on $V_{t_0} = \pi^{-1}(t_0)$ of type $(p,q)$. We say that $w$ (and the cohomology class represented by $w$) is locally transportable at $t_0$ in the family $\pi : V \to M$ if there exists a neighborhood $U$ of $t_0$ and a differential form $w_t$ of type $(p,q)$ along the fibres of $\pi^{-1}(U)$ such that $\pi_t w_t = 0$ (or $\pi_t w = 0$ and $\nu_t w_t = 0$) for $t \in U$, and that $w_0 = w$. Set $\xi_t = Q_t - Q_t$, where $Q_t$ is the usual projection on $\pi_t$. Then $D_\xi \xi_t = [\xi_t, \xi_t] / 2$, and $\pi_t w_t = 0$ can be written as the equation $\pi_0 w_t = [\xi_t, w_t]$. As $\nu_t w_t = 0$ is equivalent with $\pi_t w_t = 0$, we get the equivalent equation $\pi_0 w_t = [\xi_t, w_t]$. Therefore, in order for $w$ to be transportable there has
to exist a simultaneous local solution \( w_t \) of the above two equations with \( w_0 = w \). From the principle of upper semicontinuity it follows that locally \( \dim H^p_q = \text{constant at } t = t_0 \) if and only if every harmonic form of type \((p,q)\) on \( V_0 \) is locally transportable. For this to be so there exist conditions in terms of the Dolbeault cohomology of \( V_0 \). The case of vector valued forms can be handled analogously using Serre duality. (Received February 17, 1969.)

69T-B82. LUDVIK JANOS, University of Florida, Gainesville, Florida 32601. On certain functional equation.

The equation \( g(f(x)) = cg(x) \) is investigated where \( f \) is a given function and \( c \) a given constant. Let \( R = (-\infty, \infty) \), \( G \) and \( G^+ \) denote the group of all and the subgroup of all increasing homeomorphisms of \( R \) onto itself. **Theorem 1.** If \( f \in G \) and there exists \( a \in (0,1) \) such that \( |f(y) - f(x)| \leq a |y - x| \) for all \( x, y \in R \), then for every \( c \in (0,1) \) the family of all continuous solutions \( g(x) \) forms a point-separating family of functions on \( R \). **Theorem 2.** If \( f \in G^+ \) and the remaining conditions are the same as in the theorem above, then for every \( c \in (0,1) \) there exists a solution \( g(x) \) belonging to \( G^+ \). (Received February 17, 1969.)

69T-B83. CHARLES A. COPPIN, University of Dallas, Irving, Texas 75060. A characterization of connected number sets.

The phrase "\( f \) is \( g \)-integrable on \( M \)" and \( I(f,g) \) are as in Abstract 656-11, these (1968), 505. Let \( g \) denote the identity function. **Theorem.** A number set \( S \) is connected if and only if there exists a real-valued function with domain \([0,1]\) such that \( I(f,g) = S \). (Received February 13, 1969.)

69T-B84. KENNETH O. LELAND, Illinois Institute of Technology, Chicago, Illinois 60616. A polynomial approach to topological analysis. II.

P. Porcelli and L. M. Weiner [Rev. Mat. Fis. Tucumán 11 (1957), 25-27] gave a derivation based on the fundamental theorem of algebra of the Cauchy inequality for polynomials which states that if \( P(z) = \sum_0^n a_k z^k \) is a polynomial such that \( |P(z)| \leq 1 \) if \(|z| = 1\), then \( |a_k| \leq 1 \) for \( k = 0, ..., n \). P. Porcelli and E. H. Connell [Bull. Amer. Math. Soc. 67 (1961), 177-181] later gave a greatly simplified argument. This paper shows how this result coupled with the Stone-Weierstrass Theorem (S.W.T.) can be made to give a development of complex variable theory for the case of a once continuously differentiable function \( f \). Let \( g \) be a continuous function on \( B = \{z : |z| = 1\} \). From the S.W.T. there exists a sequence of expressions of the form \( \sum_{-n}^{+n} a_k z^k \) which converges uniformly on \( B \) to \( g \). The Cauchy inequality forces convergence of the coefficients of this sequence yielding a formal power series \( \sum_{-\infty}^{+\infty} a_k(g) z^k \). If domain \( f \) is an annulus and such series are formed for \( f \) on concentric circles of the annulus, it readily follows that the coefficients of these series are independent of the choice of circle, thus giving a Laurent expansion for \( f \). If one sets \( L(g) = a_0(g) \), \( L \) can be interpreted as an integral. (Received February 18, 1969.)
Let $A$ be the (infinitesimal) generator of a semigroup of contractions on a Banach space $X$, such that $\text{Re} A < -a$, where $a > 0$. Let $B$ be an operator on $X$ with bounded inverse. Suppose $BA$ is dissipative, that $B(D(B)) \supset D(B(A))$, and that $B(D(A^\infty)) \subset D(A^\infty)$. Then we have the following results. **Theorem 1.** Let $C_1 = BA - AB$, $C_{k+1} = C_k B - BC_k$, $k \geq 1$. Suppose there is a positive integer $n$ such that $\|C_n x\| \leq \|B^{n+1} x\|$ and $\|BC_k x\| \leq \|B^{k+1} x\|$, $1 \leq k < n$, for all $x$ in $D(A^\infty)$. Then there is $\delta > 0$ such that $BA - \delta B$ generates a semigroup of contractions on $X$. **Theorem 2.** Let $C_1$ be as in Theorem 1, suppose $B^{-1}$ commutes with $C_1$, and that there is $\delta > 0$ such that $\|Ax\| \leq M\|C_1 x\| + \delta\|x\|$ where $M \leq \|B^{-1} + 1\|$. Then $BA$ generates a semigroup of contractions on $X$. **Theorem 3.** Let $C_1$ be as before, $E = C_1 B^{-1} - B^{-1} C_1$. Suppose $\|C_1 x\| \leq \alpha\|x\| + \delta\|Ax\|$ and $\|Ex\| \leq \gamma\|x\| + \delta\|Ax\|$, where $\|B^{-1}\|\beta + \delta < 1$. Further suppose that there is $\mu > 0$ such that $a \leq (\|B^{-1}\|\alpha + \gamma + \mu)/(1 - \|B^{-1}\|\beta - \delta)$. Then $BA$ generates a semigroup of contractions on $X$. (Received February 14, 1969.)
tion of $S$ when an appropriate topology is placed on $S$. Thus our algebraic isomorphism develops into a topological embedding. This result can be generalized to the following: The set $M$ of all nonzero homomorphisms of $G$ into a Banach algebra $B$ is said to be sufficient if for $f \in G$, $\pi(f) = 0$ for all $\pi \in M$ implies $f$ is zero. Under such condition $G$ is isomorphic to a subalgebra of the set of all continuous $B$-valued functions on $M$. (Received February 19, 1969.)

69R-B89. J. J. SEMBER, Simon Fraser University, Burnaby 2, British Columbia, Canada. Matrices that sum sequences of bounded variation as compact-like operators.

For both conservative and variation matrices, compact operators yield conull FK spaces [see A. K. Snyder, Math. Z. 90 (1965), 376-381, and J. J. Sember, Math. Z. 108 (1968), 1-6]. We consider here matrices $A$ that transform sequences of bounded variation into convergent sequences, a class including both the above. Definitions. Call $A$ standard in case $\sum_{k=1}^{\infty} \lim_{n \to 0} a_{nk}$ converges. Call $A$ almost compact in case there exists a subsequence $\{n_1^r\}$ of the natural numbers such that $\lim_{r \to \infty} \sum_{k=1}^{\infty} a_{nk}$ exists uniformly for $r = 1, 2, \ldots$. Then the following hold: (1) $C_A$ can be coregular when $A$ is compact, (2) $A$ need not be compact if $C_A$ is conull, (3) if $A$ is standard and almost compact, then $C_A$ is conull, (4) if $A$ is standard and compact, then $C_A$ is conull, (5) if $C_A$ is conull, then $A$ is weakly compact, (6) $C_A$ can be coregular when $A$ is weakly compact, (7) if $A$ has a continuous inverse on some infinite-dimensional subspace of the space of sequences of bounded variation, then $C_A$ is coregular. Question. What are the strictly singular matrices? (Received March 6, 1969.)


Motivated by Z. Semadeni's paper Categorical methods in convexity [Proc. Colloq. Convexity, Copenhagen 1965 (1967), 281-307] and by a function space defined by the author in [Proc. Amer. Math. Soc. 16 (1965), 967-971] we consider the category $\mathcal{W}$ in which the class of objects is the class of pairs $(X, H)$ where $X$ is a compact Hausdorff space and $H$ is a linear subspace of $C(X)$ (all continuous real-valued functions on $X$) that contains the constant functions and where if $(X_1, H_1), (X_2, H_2)$ are objects in $\mathcal{W}$, then a morphism $\alpha: (X_1, H_1) \to (X_2, H_2)$ is a continuous function $\alpha$ from $X_1$ into $X_2$ such that $h \circ \alpha \in H_1$ for all $h \in H_2$. The basic concepts of category theory are studied in $\mathcal{W}$ as well in certain subcategories of $\mathcal{W}$ and in particular it is shown that $\mathcal{W}$ is a complete and cocomplete category in the sense of Barry Mitchell [Theory of categories, New York 1965]. Also limits and colimits in $\mathcal{W}$ are related to limits and colimits in the category of compact convex sets and the category of Archimedean ordered vector spaces. Applications are given to the behavior of Choquet and Silov boundaries with respect to products and certain inverse limits in $\mathcal{W}$. (Received March 3, 1969.)

69T-B91. EDYTHE P. WOODRUFF, Rutgers University, New Brunswick, New Jersey 08903. Derivates of a function whose image is of Lebesgue measure zero.

Varbert (Amer. Math. Monthly 72 (1965), 831-841), has proved that if $f$ is defined and Lebesgue measurable on an interval and $E$ is the set of points where the derivative is zero, then the measure of $f(E)$ is zero. In this paper a converse theorem is proved. Theorem. If $f$ is an extended real-
valued function and $E$ is any set such that $m_f(E) = 0$, then $D_f \not\equiv 0 \not\equiv D^Ty$ and $D_f \not\equiv 0 \not\equiv D^Ty$ a.e. on $E$.

The proof involves observing that the inverse image of a point contains only countably many points and that, hence, one can define a countable number of inverse functions with ranges whose union is the domain of $f$. (Received March 4, 1969.)


In this paper we give a representation of a Gelfand-Pettis (weak) integral in terms of unconditionally convergent series. This solves the problem of obtaining a useful characterization of the weak integral (see Hille and Phillips, Functional analysis and semigroups, Colloq. Publ. Amer. Math. Soc. (1957), p. 77). For measurable functions this integral coincides with the Birkhoff integral and Dunford's integrals (the second integral and the absolutely continuous third integral); consequently, the structure of these integrals is also determined. Two applications of the representation are presented. The first is a simplified proof of the absolute continuity and the countable additivity of the indefinite weak integral. The second is to probability theory; a formal expression for the conditional expectation of a measurable weakly integrable function is given. Details will appear in the Proc. Nat. Acad. Sci. USA. (Received March 5, 1969.)


Let $S$ be a Banach space and let $A$ be a mapping from $S$ to $S$ such that (1) $A$ is monotone on $S$, i.e. if $\epsilon > 0$ and $p, q \in S$ then $\| (I - \epsilon A)p - (I - \epsilon A)q \| \geq \| p - q \|$, (2) $A$ is $m$-monotone on $S$, i.e. $A$ is monotone and if $\epsilon > 0$ then Range $(I - \epsilon A) = S$, and (3) $A$ is continuous on $S$. If $p \in S$ and $x \geq 0$ define the product integral of $A$ from 0 to $x$ w.r.t. $p$ as the point $z \in S$ such that if $c > 0$ there exists a chain $s$ from 0 to $x$ such that if $t = \{ t_i \}$ is a refinement of $s$ then $\| \sum_{i=0}^{n} (I - (t_i - t_{i-1})A)^{n} p - z \| < c$.

Theorem. Let $A$ be $m$-monotone and continuous on $S$. If $p \in S$ there is a continuous function $g_p$ from $[0, \infty)$ to $S$ such that $g_p(0) = p$ and if $x \geq 0$, $g'_p(x) = A(g_p(x))$ and $g_p(x)$ is the product integral of $A$ from 0 to $x$ w.r.t. $p$. (Received March 5, 1969.)

69T-B94. PETER MIKE GOORJIAN, University of California, Berkeley, California 94720. The uniqueness of the Cauchy problem for partial differential equations which may have multiple characteristics.

The uniqueness of solutions to the Cauchy problem is proven across convex surfaces for partial differential equations of order $m$ with constant leading coefficients, with complex characteristics of multiplicity at most $r$ and with real characteristics of multiplicity no greater than $(r + 1)/2$, $0 < r \leq m$. Only lower order derivatives of order no greater than $m - r/2$ with $L_\infty$ coefficients are allowed in the equation. Also uniqueness is proven across arbitrary surfaces for certain elliptic equations of order $m$ with Lipschitz continuous leading coefficients, with characteristics of multiplicity at most $r$ and with lower order derivatives with $L_\infty$ coefficients of order no greater than $m - r/2$. The principal part is derived from the product of $r$ elliptic operators with simple characteristics. If a lower order derivative of order higher than $m - r/2$ is allowed in these equations, then uniqueness fails by known
counterexamples. Uniqueness results are proven for some special equations with lower order terms of order greater than \( m \cdot r/2 \). Uniqueness of solutions of the Cauchy problem across convex surfaces is proven for parabolic equations in which the elliptic operator has constant coefficients and characteristics of multiplicity at most \( r \). (Received March 5, 1969.)


Definition. An annular function is a function holomorphic in \( D: |z| < 1 \) for which there exists a sequence of Jordan curves, \( J_n \), in \( D \) such that (a) \( J_n \) is contained in the interior of \( J_{n+1} \), (b) \( \min_{z \in J_n} |z| = 1 \) as \( n \to \infty \), (c) \( \min_{z \in J_n} |f(z)| \to \infty \) as \( n \to \infty \). **Theorem 1.** A holomorphic function is annular if and only if every level curve is a Jordan curve in \( D \). **Theorem 2.** A holomorphic function has a spiral boundary path on which \( f(z) \) tends to \( \infty \). For the definition of a spiral see Abstract 68T-144, these Notices 15 (1968), 235. **Theorem 3.** Let \( Z' \) be the derived set of the zeroes of an annular function and \( a \) an open component of \( C - Z' \) (\( C = \{z \mid |z| = 1\} \)). Then for every point \( \tau \) of \( a \), with one possible exceptions, there exists a sequence of arcs tending uniformly to an open arc of \( C \) containing \( \tau \) on which \( f(z) \) tends to zero. **Theorem 2** strengthens a theorem of Miller [Thesis, Case Western Reserve University (1968)]. **Theorem 3** extends a theorem of Lappan [J. Reine Angew. Math. 25 (1967)]. The proofs are by elementary methods. (Received March 12, 1969.)

69T-B96. SYED M. MAZHAR, Department of Applied Science, College of Engineering and Technology, A.M.U., Aligarh, India. On \( |L|_k \) summability of a Fourier series.

Let \( \Sigma a_n \) be a given infinite series with \( \{a_n\} \) as its nth partial sum. Let \( F(x) = (-1/\log(1-x)) \cdot \Sigma a_n x^n/n \), the series being convergent in \((0,1)\). If \( \int_0^1 (1-x)^{k-1} |dF(x)/dx|^k dx \) < \( \infty \), \( k \geq 1 \), for some \( \delta, 0 < \delta < 1 \), the series \( \Sigma a_n \) will be said to be summable \( |L|_k \). The summability \( |L|_k \) is the same as the summability \( |L|_1 \). It is shown that \( |A|_k = |L|_k, k \geq 1 \), while \( |L|_k \) and \( |L|_\infty \) are independent of each other. Concerning Fourier series a theorem has been proved for \( |L|_k \) summability which includes, as a special case, a theorem of Mohanty and Patnaik (J. London Math. Soc. 43 (1968), 452-456). (Received March 12, 1969.)

69T-B97. WILLIAM P. ZIEMER, Indiana University, Bloomington, Indiana 47401. Extremal length as a capacity.

If \( \chi \) is a family of continua in \( E^n \) and \( 1 < p < \infty \), the \emph{p-dimensional module} of \( \chi \) is defined as \( \inf \int_{E^n} \rho^p : f \land \chi \) where \( f \land \chi \) means that \( f \) is a nonnegative Borel function for which \( \int_{\partial E} f dH^1 \leq 1 \) for every \( \sigma \in \chi \). \( H^1 \) is Hausdorff linear measure. For \( A \subset E^n \), let \( \Gamma_p (E) \) be the \( p \)-dimensional module of all continua that intersect \( E \) and let \( \gamma_p (E) \) be the \( p \)-dimensional module of all continua that join \( E \) to \( \infty \). (For \( p \neq n \), this definition must be slightly modified.) \( \Gamma_p \) is an outer Caratheodory measure for which every Suslin set contains an \( F_\sigma \) set of equal measure and \( \gamma_p \) is a true capacity in the sense of Brelot. \( \Gamma_p \) and \( \gamma_p \) vanish simultaneously on Suslin sets. Let \( W^1_p \) be those distributions whose partial derivatives are in \( L^p \). Call \( u \in W^1_p \) precise if for every \( \epsilon > 0 \) there is an open set \( U \) such that \( \gamma_p (U) < \epsilon \) and \( u \) restricted to the complement of \( U \) is continuous. Every \( u \in W^1_p \) is equivalent to a volume.
precise function. In the terminology of Aronszajn and Smith, the precise function and the \( \gamma_p \) null sets form the perfect completion of smooth functions in \( W^1_p \). Finally, for every Suslin set \( A \subseteq E^n \), \( \gamma_p(A) = \inf \left\{ \int_A |u|^p \right\} \) where the infimum is taken over all precise functions \( u \) with compact support such that \( u(x) = 1 \) for \( \gamma_p \) almost all \( x \in A \). (Received March 12, 1969.)


Let \( L^\Phi(S) \) be an Orlicz space on \( (\Omega, \Sigma, \mu) \), \( \mu \)-finite. An element \( f_0 \in L^\Phi(S) \) is termed a weak unit if it is determined by (a linear combination of) \( \{X_{\lambda_n} : \lambda_n \|X_{\lambda_n} \| < \infty, \bigcup_{n=1}^{\infty} \Lambda_n = \Omega \} \). Such elements exist. A Reynolds operator \( R \) on \( L^\Phi(S) \) is a linear map satisfying the identity: (*) \( R(fg) = R(f)R(g) + R\left(\int (f - Rf)(g - Rg)\right) \), \( f, g \in L^\Phi(S) \), with \( f \) or \( g \) bounded. Theorem. Let \( \Phi(x) \leq C \Phi(x) + x \), \( 0 < C < \infty \), be a Young's function and suppose \( f_0 \) is a weak unit in \( L^\Phi(S) \) and in its adjoint \( L^{\Phi'}(S) \). Let \( R \) be a weakly compact contractive Reynolds operator on \( L^{\Phi'}(\sigma) \) with \( Rf_0 = f_0 \). Then there exist uniquely (i) a \( \sigma \)-field \( \mathcal{E} \subseteq S \) such that \( f_0 \) is \( \mathcal{E} \)-measurable, (ii) a conditional expectation \( E_{\mathcal{E}} : L^{\Phi'}(S) \to L^{\Phi'}(\mathcal{E}) \), and (iii) a strongly continuous semigroup of measure preserving operators \( \{V(t), t \geq 0\} \) on \( L^{\Phi'}(S) \), in terms of which \( R \) can be represented as: (\#) \( Rf = \int_0^\infty e^{-t}V(t)E^{\Phi'}(f)dt \), (Bochner integral), \( f \in L^{\Phi'}(S) \), where (a) \( R(L^{\Phi'}(\sigma)) \subseteq L^{\Phi'}(\mathcal{E}) \), (b) \( R \) and \( E_{\mathcal{E}} \) commute on bounded functions of \( L^{\Phi'}(\mathcal{E}) \) (= closure of the range of \( R \)).

Corollary. Let \( L^{\Phi}(S) \) be reflexive and \( f_0 \in L^{\Phi}(S) \cap L^{\Phi'}(S) \) be a weak unit. If \( R \) is a contractive Reynolds operator on \( L^{\Phi}(S) \) with \( Rf_0 = f_0 \), then \( R \) admits the representation (\#) of the theorem. This generalizes a result of G.-C. Rota's (Proc. Sympos. Applied Math., Amer. Math. Soc. 16 (1964), Theorem 2) who proved it with \( L^{\Phi} = L^2 \) and \( \mu(\Omega) < \infty \) so that \( f_0 = 1 \). (Received March 13, 1969.)


In a recent paper (Bull. Amer. Math. Soc. 74 (1968), 1083-1085) W. J. Davis proves the following Theorem. Let \( (M_i) \) be a sequence of closed subspaces of the Banach space \( E \) such that each \( (u_i) : E \to M_i \) is basic. Then there exists an integer \( N \) such that \( (M_1|_N \subseteq N) \) is a Schauder decomposition of \( (M_i|_N \subseteq N) \) i.e. the closed subspace generated by \( \bigcup_1 M_i \). Using a result of Ruckle (Schauder decompositions and bases, Dissertation, Florida State University, Tallahassee, Florida, 1963) and the well-known criterion for continuity of a linear operator \( T : E \to F \) where \( E, F \) are locally convex spaces in terms of pseudo-norms, Davis' results holds in a complete metrizable locally convex space. Hence the corollaries in Davis' paper hold in the more general setting as well. (Received March 4, 1969.)

69T-B100. JAMES WARD BROWN, University of Michigan, Dearborn Campus, Dearborn, Michigan 48120. A new class of polynomial sets.

Let \( \{p_n^{(a)}(x)\} \) be a polynomial set defined by a generating function of the form \( (1 - t)^{-a}F(x,t) = \sum_{n=0}^{\infty} p_n^{(a)}(x)t^n \) where \( F(x,t) \) is independent of the parameter \( a \). For special cases of \( F(x,t) \) a variety of well-known sets are generated. If, for example, \( F(x,t) = (1 - t)^{-1}\exp[-xt/(1 - t)] \), we have \( p_n^{(a)}(x) = l_n^{(a)}(x) \), the generalized Laguerre polynomials. The author has shown by a direct summation technique that \( \left[ \frac{1}{1 - u(\beta,t)} \right]^{1-a}/[1 - (1 + \beta)u(\beta,t)] \) \( F(x,u(\beta,t)) = \sum_{n=0}^{\infty} p_n^{(a\beta)}(x)t^n \) where \( u(\beta,t) \) is the inverse of \( v(\beta,t) = t(1 - t)\beta \); that is, \( v(\beta,u(\beta,t)) = u(\beta,v(\beta,t)) = t \). As \( \beta \) varies, particular cases of this generating
function occur in natural pairs in the sense that \( u(-1 - \beta; t) = -u(\beta; -t)/[1 - u(\beta; -t)] \). In the case of the generalized Laguerre polynomials, these results have already been exhibited by the author (Duke Math. J. 35 (1968), 821-823) when \( \beta \) is integral and subsequently by Carlitz (Duke Math. J. 35 (1968), 825-827) for arbitrary \( \beta \). (Received March 18, 1969.)


Theorem. Let \( \mathcal{R} \) be a von Neumann algebra on the separable Hilbert space \( \mathcal{H} \). Let \( s \rightarrow \varphi(s) \) be a one-parameter group of \(*\)-automorphisms of \( \mathcal{R} \) such that \( s \rightarrow \varphi(s)(T)x \) is continuous for each \( T \) in \( \mathcal{R} \) and \( x \) in \( \mathcal{H} \). Suppose that \( \varphi(s) \) is inner for each \( s \). Then there exists a strongly continuous one-parameter unitary group \( U(s) \) in \( \mathcal{R} \) such that \( \varphi(s)(T) = U(s)TU(-s) \), for all \( T \) in \( \mathcal{R} \). (Received March 25, 1969.)

69T-B102. PAUL M. KRAJKIEWICZ and WILLIAM W. BOSCH, University of Nebraska, Lincoln, Nebraska 68508. Polyanalytic functions with equal norm.

A function \( f(z) \) is said to be polyanalytic or \( n \)-analytic in a domain \( D \) if it has a representation of the form \( f(z) = \sum_{k=0}^{n-1} f_k(z)z^k \), where each \( f_k(z) \) is analytic in \( D \). If \( f(z) \) is \( n \)-analytic and \( g(z) \) is \( m \)-analytic in \( D \), then a necessary and sufficient condition for \( |f(z)| = |g(z)| \) in \( D \) is that there exist a polynomial \( p(z,\bar{z}) \neq 0 \) of degree \( \leq n - 1 \) in \( z \) and of degree \( \leq m - 1 \) in \( \bar{z} \) such that \( p(z,\bar{z}) \cdot f(z) = \overline{p(z,\bar{z})} \cdot g(z) \) for all \( z \) in \( D \). Necessary and sufficient conditions are also obtained in order that two polyanalytic functions have equal arguments modulo \( 2\pi \) at all points of \( D \) at which the functions do not vanish. (Received February 20, 1969.)


This paper describes a new algorithm for finding the unconstrained minimum of a function of \( n \) variables where explicit expressions are available for the first partial derivatives. Using the notation of Broyden (Math. of Comp. 21 (1967), 368-381) the matrix updating procedure is

\[
H_{i+1} = H_i - (p_iy_i^T H_i + H_i y_i^T p_i^T)(t_i + y_i^T H_i y_i / p_i^T y_i) p_i / y_i^T p_i.
\]

Theory indicates and preliminary experiments confirm that the algorithm is probably more stable than that of Fletcher and Powell, with the matrices \( H_i \) showing a reduced tendency to become singular. (Received February 3, 1969.) (Author introduced by Dr. John E. Dennis, Jr.)

69T-B104. ANDRE de KORVIN and R. J. EASTON, Indiana State University, Terre Haute, Indiana 47809. Measures on semigroups and some representations. I.

Let \( H \) be a locally compact, Hausdorff semigroup with identity. Let \( S \) be any linearly independent set of semi-characters of \( H \), closed under complex conjugation, which separates \( H \), and such that at no point of \( H \) do all members of \( S \) vanish. Let \( \mathcal{S} \) be the field generated by the open sets of \( H \), and \( M(\mathcal{S}) \) denote all bounded, regular, finitely additive, complex-valued set functions defined on \( \mathcal{S} \). It is shown that \( M(\mathcal{S}) \) is a semigroup under convolution product. Let \( B(\mathcal{S}) \) denote all bounded complex-valued functions of \( S \) which are extendable at continuous linear functionals to the
There exists an isometric isomorphism between $M(S)$ under convolution product and $B'(S)$ under pointwise multiplication, where the mapping is given by $u(f) = \int_S f(t)u(t)dt$ for $f \in S, u \in M(S)$. Corollary. If $H$ is an abelian, discrete semigroup with identity which is a union of groups, and $S$ is any linearly independent set of elements of $H$ which separate $H$. Then $M(S)$ is isometric isomorphic to $B'(S)$. In particular if $\int_H f(x)u(x)dx = 0$ for all $x \in S$, then $u = 0$. (Received March 31, 1969.)

A designates a topological algebra (see Abstract 69T-B69, Spectra in topological algebras, these Notices 16 (1969), 571). An element $a \in A$ is called powerbounded when the set \{a^n\} of the powers of $a$ is bounded in $A$. The spectrum of each powerbounded element is contained in the unit disc of the complex plane. $P$ designating the set of powerbounded elements, one has $xy \in P \Rightarrow yx \in P$. If there exists $k$ such that $x^k \in P$ then $x \in P$. If $A$ is commutative, then $P$ is convex and stable for multiplication. One has $r(x) \leq p(x)$ where $p$ designates the $P$-gauge. $A$ is called a Gelfand algebra when $r(x) = p(x)$ for all $x \in A$. $A$ is Gelfand iff $r(x) < 1 = x$ is nilpotent. Each subalgebra of a Gelfand algebra is Gelfand. In addition if $B$ is a subalgebra of $A$, $A$ being Gelfand, then $r_B(x) = r_A(x) = p(x)$ for each $x \in B$. If $A$ is the initial structure for a family of Gelfand algebras then $A$ is Gelfand. (Received March 31, 1969.)

An integral equation with Bessel function kernel.

It is proved that the integral equation $g(x) = (d/dx)\int_0^x J_0^{\sqrt{2(x-t)}} f(t)dt$ has a solution $f$ in $L^2(0, \infty)$ if and only if (i) $g(x)$ is in $L^2(0, \infty)$ and (ii) $g(x) = \int_0^\infty \sqrt{\frac{2}{\pi y}} g(y)dy$ vanishes in
0 \leq x < k. If the second condition is not satisfied, (i) $f$ neither belongs to $L^p(0, \infty)$ nor is the Hankel transform (of order zero) of a function in $L^p(0, \infty)$, $1 \leq p \leq 2$, (ii) although $f(x)e^{-cx}$ belongs to $L^2(0, \infty)$ for every $c > 0$, $x^{-n}f(x)$ need not be bounded as $x \to \infty$ for any $n > 0$. (Received April 2, 1969.)

69T-B108. JAMES D. FABREY, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. Exponential representations of the canonical commutation relations.

A class of representations of the canonical commutation relations is investigated. These representations are given by exploit formulas. An example is furnished by the representation acting on the renormalized Hilbert space constructed by Glimm for the $\sigma^4$ interaction for boson fields in three dimensions (Comm. Math. Phys. 10 (1968), 1-47), provided that the interaction terms which do not annihilate or create four particles are omitted. Exponential representations are apparently not tensor product representations, but they provide the possibility of computing useful criteria concerning various properties. In particular, a sufficient condition is obtained for two exponential Weyl systems not to be disjoint. They are all cyclic, locally Fock, and globally disjoint from the Fock-Cook representation. (Received by April 3, 1969.)


Let $\mathcal{B}(X)$ denote the Banach algebra of all continuous linear transformations from a Banach space $X$ into itself. Let $S(X)$ and $K(X)$ denote the ideals of strictly singular and compact operators of $\mathcal{B}(X)$, respectively. Let $\Pi$ denote the natural homomorphism of $\mathcal{B}(X)$ onto $\mathcal{B}(X)$ modulo $K(X)$. The ideal of inessential operators of $\mathcal{B}(X)$, denoted by $I(X)$, is equal to $\Pi^{-1}$ of the Jacobson radical of $\mathcal{B}(X)$ modulo $K(X)$. Subprojective spaces were introduced in [R. J. Whitley, Strictly singular operators and their conjugates, Trans. Amer. Math. Soc. 13 (1964), 252-261]. Theorem. If $X$ is a subprojective Banach space, then $S(X) = I(X)$. (Received March 26, 1969.)

69T-B110. JOHN A. BROWN, Montana State University, Bozeman, Montana 59715. Trigonometric series representation of certain singular functions.

Theorem. If $f(\theta) = \theta^{-n}$, $n$ a positive integer, then $f(\theta)$ has a trigonometric series representation, $\sum a_k \cos k\theta + b_k \sin k\theta$, which is $(C,n)$ summable to $f(\theta)$ for $0 < |\theta| < \pi$. If $n = 1$, the coefficients are given by the usual integral formulas for Fourier coefficients, except that principal values must be used. If $n \geq 1$, by expressing $f(\theta)$ in terms of the appropriate real or imaginary part of $(e^{i\theta} - 1)^{-n}$ the coefficients may be given in terms of residues, which reduces to the principal value integral representation for $n = 1$. Corollary. The Dirichlet problem for functions harmonic on $|z| < 1$ with boundary values $f(\theta) = \theta^{-n}$ ($\theta \neq 0$) has a solution. (Received March 25, 1969.) (Author introduced by Dr. Byron L. McAllister.)

69T-B111. PAUL R. CHERNOFF, University of California, Berkeley, California 94720. Representations of certain algebras of compact operators.

E, F are normed linear spaces. $K(E)$ is the norm closure of the bounded finite rank operators on E; $L(F)$ is the algebra of all bounded operators on F. Theorem 1. Suppose that $\pi : K(E) \to L(F)$ is
a nonzero continuous homomorphism. Then $E$ is topologically isomorphic to a subspace of $F$. Further, if $\pi$ is norm-decreasing it is isometric and $E$ is isometric to a subspace of $F$. \textbf{Theorem 2.}

Let $\pi$ be as in Theorem 1. Assume that $F$ is complete and that the ranges of the operators $\pi(A)$ span a dense subspace of $F$. Then there is a closed subspace $H$ of $F$ such that $F$ is isomorphic to the completion of $E \oplus H$ with respect to some cross-norm $\rho$, and $\pi$ corresponds to the natural representation of $K(E)$ on $E \oplus H$. ($H$ is the range of $\pi(P)$ for some rank one projection $P$.) The concrete specification of the norms $\rho$ which arise in this way seems difficult. However, if $H$ is finite dimensional, $\pi$ is equivalent to the direct sum of $\dim H$ copies of the identity representation. As an application of the purely algebraic ideas used in the proof, one has \textbf{Theorem 3.}

Let $\pi$ be as in Theorem 1. Assume that $F$ is complete and that the ranges of the operators $\pi(T)$ span a dense subspace of $F$. Then there is a closed subspace $H$ of $F$ such that $F$ is isomorphic to the completion of $E \oplus H$ with respect to some cross-norm $\rho$, and $\pi$ corresponds to the natural representation of $K(E)$ on $E \oplus H$. ($H$ is the range of $\pi(P)$ for some rank one projection $P$.)

On the failure of a decomposition. Preliminary report.

(1) The derivative of a continuous BVG* function (see, e.g., Saks, Theory of the integral) need not be Denjoy-Khintchine integrable. An example of a discontinuous BVG* function whose derivative is not Denjoy-Khintchine integrable is presented. Using a method of removal of discontinuities, a continuous BVG* function with (mod. 0) the same derivative function is produced. (2) The derivative of a continuous BVG* function may be Denjoy-Khintchine integrable without being Denjoy-Perron integrable. (Received April 7, 1969.)

On nonmeasurable continuous images. Preliminary report.

Let $S$ denote the unit circle, $I$ the unit interval. Let $f$ be a continuous real-valued function on $S \times I$ with $a = \min f < \max f = b$. Assume that for all but an at most countable set of $z \in \mathbb{I}$, all but an at most countable set of $(x_0, y_0) \in f^{-1}(z)$ possess neighborhoods $U(x_0, y_0)$ such that:

(1) $U(x_0, y_0) \cap f^{-1}(z)$ defines its first coordinates as a continuous function of its second coordinates and vice-versa; (2) the projections of $U(x_0, y_0) \cap f^{-1}(z)$ into both $S$ and $I$ contain nondegenerate intervals.

Then there is a set $E \subset S \times I$ of Hausdorff measure zero, with projections into $S$ and $I$ of measure zero, such that $f(E)$ is nonmeasurable. One therefore has, e.g., Corollary. There are sets $H$ and $Z$ of measure zero, contained in the real line, whose vector sum is nonmeasurable. (Received April 7, 1969.)

The asymptotic behavior of norms of powers of absolutely convergent Fourier series. Preliminary report.

Let $f(t)$ have an absolutely convergent Fourier series $f(t) = \sum a_k \exp(ikt)$ and let $\|f\| = \sum |a_k|$. If the conditions (i) $|f(t)| < 1$, $t \neq 0$, $f(0) = 1$; and (ii) $f(t)$ is analytic at $t = 0$ both hold, then exp(-ita) $f(t) = 1 + At^p + o(t^p)$, $t \to 0$ where $A \neq 0$ and $a = f'(0)$, defines $A$ and $p$, and $|f(t)| = 1 - \beta t^q + o(t^q)$, $t \to 0$, $\beta > 0$, defines $\beta$ and $q$. Our main result is the following Theorem. Let $f(t)$ be
absolutely continuous and \( f'(t) \) be of bounded variation. If (i) and (ii) hold, then (a) for \( p \neq q, \| F \| \sim (2/\pi)^{1/2} \delta(p)^{-1}(p(p-1))^{1/2} T(p/2q) \cdot |A|^{1/2} \rho^2|/2q_n(1-p/q)^{1/2} \) where \( \delta(p) = 0 \) if \( p \) is even, \( = 1 \) if \( p \) is odd; (b) for \( p = q, \lim_{n \to \infty} \| F \| = (2\pi)^{-1} \| \hat{F} \|_1 \) where \( \hat{F} \) is the Fourier transform of \( F(t) = \exp(\lambda t^2), Re \lambda = -\beta \). As a special case of this result we obtain: If \( f(z) \) is analytic on the closed unit disk, \( |f(z)| \leq 1, |z| = 1, z \neq 1 \) and \( f(1) = 1 \), then (a) or (b) holds. (Received April 7, 1969.)

69T-B115. HARI M. SRIVASTAVA, West Virginia University, Morgantown, West Virginia 26506.
A formal extension of certain generating functions.

In an attempt to give extensions of certain earlier results [H. M. Srivastava, Infinite series of certain products involving Appell's double hypergeometric functions, Glasnik Mat. Ser. III, 4 (24) (1969), 67-73; see also An infinite summation formula associated with Appell's function \( F_2 \), Proc. Cambridge Philos. Soc. 65 (1969), to appear] the author proves here a generating relation for the generalized Appell function in two arguments and shows how its specialized or limiting forms incorporate as particular cases scores of hitherto scattered results in the theory generalized hypergeometric polynomials [cf. H. M. Srivastava, Certain formulas associated with generalized Rice polynomials, Abstract 69T-B27, these Notices 16 (1969), 411]. The formal proof presented here involves the principle of multidimensional mathematical induction as well as the Laplace and inverse Laplace transform techniques illustrated, for instance, in the author's recent paper [An extension of the Hille-Hardy formula, Math. of Comp. 23 (1969), April issue; see also Abstract 68T-449, these Notices 15 (1968), 634-635]. (Received April 7, 1969.)

69T-B116. DAVID ZEITLIN, 1650 Vincent Avenue North, Minneapolis, Minnesota 55411. An operational formula for recurrent sequences of second order.

Let \( Z_n = (a^n - b^n)/(a - b) \) and \( V_n = a^n + b^n, n = 0,1, \ldots \) be solutions of \( W_{n+2} = AW_{n+1} - BW_n \), \( A^2 - 4B \neq 0 \), where \( a \neq b \) are roots of \( x^2 - Ax + B = 0 \). Let \( m \) and \( n_i, i = 1,2, \ldots, k \), be integers with \( k \geq 2, n \geq m \geq k \), \( \Pi_{i=1}^k a_i = 1 \) if \( r > n \), and \( E^nW_i = W_{i+n} \), where \( E \) is the shift operator. Theorem 1. Let \( R = (A^2 - 4B)^{m/2} (\Pi_{i=1}^m Z_{n_i}) (\Pi_{n=m+1}^n V_{n_i}) \) and \( p = n_1 - n_2 - n_3 - \ldots - n_k \). For \( m = 1,3,5, \ldots \), \( \Pi_{n=m+1}^n V_{n_i} \) and \( \Pi_{i=1}^m Z_{n_i} \) are omitted. Theorem 1 has as special case results given for the Fibonacci series by H. H. Ferns (Products of Fibonacci and Lucas numbers, Fibonacci Q. 7 (1969), 1-13). Using Theorem 1, we may now express a mixed product of Chebyshev polynomials (\( B = 1 \)) as a linear combination of Chebyshev polynomials of a given kind. Excluding a constant factor, \( R \) is a product of \( k \) factors chosen from \( Z_n \) and \( V_n \). \( E \) operates only on \( W_i \), namely, \( E^p(B^nW_i) = B^nW_{i+p} \). (Received March 26, 1969.)

69T-B117. ROBERT H. MARTIN, JR., Georgia Institute of Technology, Atlanta, Georgia 30332.
A bound for solutions of Volterra-Stieltjes integral equations.

Let \( S \) be the set of real numbers and \( R \) be a complete normed algebra over the real field with unity 1 so that \( |1| = 1 \). Let \( OA \) be the set of all order-additive functions \( V \) from \( S \times S \) to \( R \) (i.e. \( V(x,y) + V(y,z) = V(x,z) \) if \( x \neq y \neq z \) or \( x \neq y = z \) for which there is an order-additive function \( \alpha \) from \( S \times S \) to the nonnegative real numbers such that \( |V| \leq \alpha \) (see J. S. Mac Nerney, Illinois J. Math. 7

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If $V$ is in $OA$ and for each $(x,y)$ in $S \times S$, $\gamma[V](x,y) = x \sum y(1 + V) - 1$, then $\gamma[V]$ is in $OA$, $|\gamma[V]| \leq a$ and $|a \pi^b(1 + V)| \leq a \pi^b (1 + \gamma[V])$ for each $(a,b)$ in $S \times S$. If $x \pi^y(1 + V)$ has a multiplicative inverse in $R$ for each $(x,y)$ in $S \times S$, then $\gamma[V]$ is in $OA$, $|\gamma[V]| \leq a$ and $|a \pi^b(1 + V)| \leq a \pi^b (1 + \gamma[V])$ where $G[V](x,y) = - \sum y(1 + V)^{-1} V$. This theorem extends Theorem 3, page 58 of W. A. Cappel, Stability and asymptotic behavior of differential equations, Heath, Boston, 1965. (Received April 10, 1969.) (Author introduced by Professor James V. Herod.)


Let $K$ be a Choquet simplex and let $F$ be a face (i.e., a convex extremal subset) of $K$. Then $F$ is complemented if there exists a face $G$, disjoint from $F$, such that every element of $K \setminus (F \cup G)$ is a unique convex combination of an element of $F$ and an element of $G$. In this situation, the face $G$ is complementary to $F$. If $F$ is complemented, then its (unique) complementary face is equal to the union of all faces of $K$ disjoint from $F$ (see E. M. Alfsen, Math. Scand. 17 (1965), 169-176). Let the given topology on $K$ be denoted by $\tau$ and let $/\tau$ be the topology on $K$ determined by the metric $d(x,y) = \sup |f(x) - f(y)| : f$ is a real, affine, $\tau$-continuous function on $K$ with $|f(z)| \leq 1$ for all $z$ in $K$.

Theorem. For $F$ a face of $K$, the following are equivalent: (i) $F$ is complemented; (ii) $F$ is $/\tau$-closed; (iii) $F$ is the peaking set of a real, affine function on $K$. As a corollary, one has the known result (O. Hustad, Math. Scand. 11 (1962), 63-78; also see E. M. Alfsen, loc. cit.) that every $/\tau$-closed face is complemented. (Received April 10, 1969.)

69T-B119. NIEL SHILKRET, Polytechnic Institute of Brooklyn, 333 Jay Street, Brooklyn, New York 11201. Orthogonality in non-Archimedean Spaces I and II.

X and $Y$ are non-Archimedean $B$-spaces over the complete, nontrivially valued field $F$. Orthogonal is abbreviated $o$, or $\perp$. A linear transformation $A : X \to Y$ is $o$, if $x \perp y = Ax \perp Ay$.

Assume $X$ possesses a pair of $o$, vectors. Theorem. If $F$ has discrete valuation and prime element $p$, $A$ is $o$, iff (i) $|\sigma| < (|Ax|/|x|)(|Ay|/|y|) < |\sigma|^{-1}$ for all $x,y \in X \setminus \{0\}$; (ii) $|\|x\|| = |\|y\|| = |\|Ax\|| = |\|Ay\||$, $x \perp y$, and (iii) $|\|x\|| = |\|y\|| = |\|Ax\|| = |\|Ay\||$, $x \perp y$. If $F$ is nondiscrete, $A$ is $o$, iff $|\|Ax\|| = |\|y\||$, $x \neq 0$, is a nonzero constant. Corollaries include (1) an $o$, transformation is bounded; and (2) if $|\|x\|| = r|\|x\||$, $r > 0$, then the set $\mathcal{D}$ of $o$, transformations from $X$ onto $X$ is the group $\{F - \{0\}\} \mathcal{B}$, where $\mathcal{B}$ is the unitary group. Theorem. If $A \in L(X,Y)$ and $|\|A\|| = r$ if $A(\lambda) = |\lambda| > 0$ for some $\lambda \in \mathcal{P}(A)$, then $N(\lambda - \mu) \perp N(\lambda - \mu)$ for $\mu \in \mathcal{P}(A)$ such that $|\lambda - \mu| = |\lambda|$. $A$ is weakly $o$, if $A$ preserves the relation "$x \perp y$ or $x = 0$ or $y = 0$." A nonzero weakly $o$, transformation is $o$. If $X$ and $Y$ have finite $o$, bases, $L(X,Y)$ has an $o$, base, and the dual of an $o$, base $\{x_i\}$ in $X$ is an $o$, base $\{x_i\}$ such that $|\|x_i\|| = 1$. In $L(F^n,F^n)$, $\mathcal{B}/\{A : |\|Ax - x\|| < 1 \forall x\}$ is isomorphic to the general linear group $GL_n(F)$, where $F$ is the residue field of $F$. $A \in L(F^n,F^n)$ is $o$, iff $|\det A| = |\|A\||^n$. (Received April 11, 1969.)

69T-B120. NORBERTO L. KERZMAN, New York University, Courant Institute of Technology, New York, New York 10012. Taut manifolds and domains of holomorphy in $\mathbb{C}^n$.

H. Wu has introduced the concept of a taut complex analytic manifold and has proved that if an open set in $\mathbb{C}^n$, $n \geq 1$, is taut, then it must necessarily be a domain of holomorphy. (Acta Math. 119
The oscillation theorems which were established by Leighton and Nehari (Trans. Amer. Math. Soc. 89 (1958), 358-368) for the differential equation 
\[(ry'')'' + py = 0, \quad r \text{ and } p > 0 \text{ in } 0 \leq x < \infty\]
are extended here to a more general class of fourth order differential equations containing middle term i.e. 
\[Ly = (p_2y'')'' - (p_1y')' + p_0y = 0, \quad 0 \leq x < \infty.\]
Definition. We say that \(Ly = 0\) is of the LN type if there exist \(C^1[0,\infty)\) functions \(a_i\) and \(b_i\), \(i = 0,1,2\), such that for every nontrivial solution of \(Ly = 0\) the function 
\[F(y(x)) = y' \sum_{i=0}^{2} a_i y^{(i)} - y'' \sum_{i=0}^{2} b_i y^{(i)}\]
is strictly increasing for \(0 \leq x < \infty\). Theorem 1. The oscillation theory (pp. 358-368 of the above paper) is also true for 
\(Ly = 0\) where \(L\) is of the LN type. Theorem 2. \(Ly = (p_2y'')'' - (2p_1y')' + p_0y\) where \(p_2 > 0\) and \(p_0 + p_1 > p_2^2 / p_2', \quad 0 \leq x < \infty\), is of the LN type. Theorem 3. \(Ly = (p_2y'')'' - (p_1y')' + p_0y = 0\) where \(p_2 > 0, \quad p_0 > 0\) and \(p_2(p_1 + a') - a^2 \equiv 0\) where \(a\) is some \(C^1\) function is of the LN type. (Received April 15, 1969.)
serving transformation of $X$, $S = \{\varphi\}$, then $C^*(L^\infty, T_S)$ contains no nonzero compact operators. Hence $C^*(L^\infty, T_S)$ is not a type I $C^*$-algebra. For a compact abelian group $G$, $B$ its Borel sets, $S$ the set of translations of $G$ we have Theorem 2. $C^*(L^\infty, T_S)$ contains a nonzero compact operator iff $G$ is finite. Hence $C^*(L^\infty, T_S)$ is type I iff $G$ is finite. Theorem 2 establishes a correction and extension of Theorem 1 of Berger and Coburn, Bull. Amer. Math. Soc. 74 (1968), 1008-1012. (Received April 16, 1969.)

69T-B124. STERLING K. BERBERIAN, University of Texas, Austin, Texas 78712. Weyl’s theorem for some classes of not necessarily normal operators.

Theorem 1. If $T$ satisfies (a) and (b), then Weyl’s theorem holds for $T$. Theorem 2. If $T$ satisfies (a') and is reduced by each of its finite-dimensional eigenspaces, then Weyl’s theorem holds for $T$. Theorems 1 and 2 generalize results of V. Istratescu [Rev. Roumaine Math. Pures. Appl. 13 (1968), 1103-1105] and L. A. Coburn [Michigan Math. J. 13 (1966), 285-288]. Definitions. Weyl’s theorem holds for a bounded linear operator $T$ on a Hilbert space iff $\sigma(T) - \sigma_0(T) = \sigma_0(T)$, where $\sigma(T)$ is the spectrum of $T$, $\sigma_0(T)$ is the set of isolated points of $\sigma(T)$ that are eigenvalues of finite multiplicity, and $\sigma(T)$, the Weyl spectrum of $T$, is the complement of the set of complex numbers $z$ such that $T - z1$ is Fredholm of index 0. An operator $T$ satisfies (a) $\|T - z1\|^{-1}$ is normaloid for all $z$ not in $\sigma(T)$; (a') every isolated point of $\sigma(T)$ is an eigenvalue; (a) iff the restriction of $T$ to every reducing subspace satisfies (a); (a') iff the restriction of $T$ to every reducing subspace satisfies (a'); and (b) iff each eigenvalue of $T$ of finite multiplicity is a semi-bare point of $\sigma(T)$ (i.e., lies on the circumference of some closed disc that contains no other point of $\sigma(T)$). (Received April 17, 1969.)

69T-B125. JAMES KAPLAN, University of Maryland, College Park, Maryland 20742. On finite time stability. Preliminary report.

Let $\mathcal{J} = [t_0, t_0 + T)$, and let $\|\cdot\|$ be a norm on $\mathbb{R}^n$. Consider $(E)\dot{x} = f(t, x)$ where $f$ is locally Lipschitz from $[0, \infty) \times \mathbb{R}^n$ into $\mathbb{R}^n$. Weiss and Infante [Proc. Nat. Acad. Sci., U.S.A. 54 (1965), 44-48] made the following definitions: Definition 1. $(E)$ is stable with respect to $(a, \beta, \mathcal{J}, \|\cdot\|)$ if for every trajectory $x(t) = x(t; t_0, x_0)$ with $\|x_0\| < a$ we have $\|x(t)\| < \beta$ for all $t \in \mathcal{J}$. Definition 2. $(E)$ is contractively stable with respect to $(a, \gamma, \beta, \mathcal{J}, \|\cdot\|)$ if for $\|x_0\| < a$ we have $\|x(t)\| < \beta$ for all $t \in \mathcal{J}$ and if there exists $t_1 \in (t_0, t_0 + T)$ such that $\|x(t)\| < \gamma$ for all $t \in (t_1, t_0 + T)$. The author proves Theorem 1. $(E)$ satisfies Definition 1 if and only if there exists a locally Lipschitz function $V(t, x)$ satisfying (a) and (b) above, and (c) $V(t_0, x_0) < V(t_0 + T, x)$ for all $\|x_0\| < a$ and $\gamma \leq \|x\| < \beta$. Theorem 2. $(E)$ satisfies Definition 2 on $\mathcal{J}$ if and only if there exists a locally Lipschitz function $V(t, x)$ satisfying (a) and (b) above, and (c) $V(t_0, x_0) < V(t_0 + T, x)$ for all $\|x_0\| < a$ and $\gamma \leq \|x\| < \beta$. Theorem 3. A sufficient condition that solutions of $\dot{x} = Ax$ be unstable with respect to $(a, \beta, \mathcal{J}, \|\cdot\|)$ is that $\text{Re}(\lambda) > 1/T(\text{ln}\beta/a)$ for some eigenvalue $\lambda$. If $A$ is symmetric, this condition is necessary. Theorem 2 above is closely related to a result recently announced by Kayande [Abstract 664-78, these Notices] 16 (1969), 523]. (Received April 17, 1969.)


The inner product space $\{S_1, Q_1\}$ is continuously situated in the inner product space $\{S_2, Q_2\}$.
in case $S_1$ is a subset of $S_2$ and the identity function on $S_1$ is continuous from $[S_1, Q_1]$ into $[S_2, Q_2]$.

If $[S, Q]$ is a complete inner product space and the complete inner product space $[S_1, Q_1]$ is continuously situated in $[S, Q]$ and $S_1$ is a subset of $S_2$ but $S_2$ is not all of $S$, there exist complete inner product spaces $[S_1, Q_1]$ and $[S_3, Q_3]$ such that (1) $S_1$ is dense in $[S, Q]$, $[S_1, Q_1]$ is continuously situated in $[S_2, Q_2]$ and the orthogonal complement of $S_1$ in $[S_2, Q_2]$ is one dimensional, (2) $[S_2, Q_2]$ is continuously situated in $[S_3, Q_3]$ and the orthogonal complement of $S_2$ in $[S_3, Q_3]$ is one dimensional, and (3) $[S_3, Q_3]$ is continuously situated in $[S, Q]$ but $S_3$ is not all of $S$. This extends an earlier result by the author [these Notices] 16 (1969), 280; assertions (1) and (2) imply that $S_1$ is not dense in $[S_2, Q_2]$ and $S_2$ is not dense in $[S_3, Q_3]$; inductive application of the Theorem yields a doubly infinite sequence $[S_n, Q_n]$ [$n = 0, 1, 2, \ldots$] such that if $n$ is an integer then $[S_n, Q_n]$ is a complete inner product space, which is continuously situated in $[S_{n+1}, Q_{n+1}]$ and in $[S, Q]$, such that $S_n$ is dense in $[S, Q]$ but not in $[S_{n+1}, Q_{n+1}]$, its orthogonal complement in $[S_{n+1}, Q_{n+1}]$ being one dimensional. (Received April 17, 1969.)

69T-B127. SALVATORE D. BERNARDI, New York University, Bronx, New York. The radius of univalence or certain analytic functions.

Let $(S)$ denote the class of functions $F(z) = z + \ldots$ which are regular and univalent in $|z| < 1$ and which map $|z| < 1$ onto domains $D(F)$. Denote by $(C)$, $(S^*)$, and $(K)$ the subclasses of $(S)$ where $D(F)$ are, respectively, close-to-convex, starlike with respect to the origin, and convex. Theorem 1. Let $F(z) \in (C)$, $(S^*)$ or $(K)$, $f(z) = \frac{1}{1+(1+c)z}F(z)^c$, $c \equiv 1$. Then $f(z)$ is, respectively, close-to-convex, starlike with respect to the origin, convex, for $|z| < r_0 = \frac{1}{\sqrt{2} + \sqrt{3 + c^2}}/(1 - c)$. Theorem 2. Let $F(z)$ be the subclass of $(C)$ satisfying $\text{Re} \ F'(z) > 0$ for $|z| < 1$, $f(z)$ the same as in Theorem 1. Then $\text{Re} \ f'(z) > 0$ for $|z| < (-1 + \sqrt{2 + 2c + c^2})/(1 + c)$. All results are sharp and represent generalizations of corresponding results for the case $c = 1$ by A. E. Livingston (Proc. Amer. Math. Soc. 17 (1966), 352-357). (Received April 18, 1969.)


Suppose $X$ is a Banach space of functions with common domain over the complex scalar field, $A$ is a bounded linear operator on $X$ and one considers the functional equation $(A - \lambda I)x = y$. Let $c_n = \lambda^n y/\lambda^{n+1} y$ $(n = 0, 1, 2, \ldots)$ and $(d_n)_{n=0}^\infty$ be any sequence from $X$; consider the infinite process gotten by taking $\tilde{d}_{n+1} = \tilde{d}_{n+1}$, $\tilde{d}_k = (1/\lambda) + c_k - c_k/\lambda \tilde{d}_k + 1$ $(k = n, n - 1, \ldots, 0)$ and $\tilde{x}_{n+1} = y/\lambda + \tilde{d}_0$. The convergence of the $\tilde{x}_n$ to a solution $x$ of the above functional equation in the norm of $X$ is studied for families of test functions $\tilde{d}_n$ and a canonical representation of $\tilde{x}_n$ is derived. Typical Theorem. Suppose $y = y_1 + y_2$, $Ay_1 = \mu_1 y_1$, $\|c_n\| \equiv 1/\mu_2$ and $|\lambda| > \mu_1 \equiv \mu_2$ then the $\tilde{x}_n$ converge to a solution $x$ provided each $\tilde{d}_{n+1}$ is bounded in norm away from $1/\lambda - 1/\mu_2 + c_{n+1}$. Refer to Abstract 663-14, these Notices 15 (1968), 90-91, for references to previous results related to the above process. (Received April 21, 1969.)

69T-B129. STUART P. HASTINGS, Case Western Reserve University, Cleveland, Ohio 44106. On some boundary value problems related to the Falkner-Skan equation.

The following boundary value problem, due to Stewartson, arises in fluid mechanics:

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Here $\lambda$ is a real parameter and $a$ is a positive real number. When $a = 1$ it is clear that $h(t) = 1$ and (*) reduces to the well-known Falkner-Skan equation. The following result is probably new even in that special case; in any event the method of proof, using the Schauder-Tychonoff fixed point theorem, seems to be different from earlier approaches to the case of negative $\lambda$.

**Theorem.** For any $a \geq 1$ there is a $\lambda_0 < 0$ such that the above boundary value problem has at least one solution $(f,h)$ for any $\lambda$ in $(-\lambda_0, 0)$.

(Received April 21, 1969.)

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**69T-B130.** WILLIAM CARROLL CONNETT, University of Chicago, Chicago, Illinois 60637.

**Formal multiplication of trigonometric series and the notion of generalized conjugacy.**

The equiconvergence and localization theorems of Rajchman and Zygmund are extended to a large class of series $S = \sum \mu_n c_n e^{inx}$. The "formal conjugates" of $S = \sum c_n e^{inx}$, generated by the multipliers $\{\mu_n\}$. The case $\mu_n = i \text{sign} (n)$ is the usual conjugate; other interesting special cases are $\mu_n = |n|^{1/2}$ and $\mu_n = e^{i|n|^{1/2}}$. The equiconvergence theorems involve the formal multiplication of $S$ with a "good" series $T : \sum y_n e^{inx} \sim \lambda(x)$. There are two types of theorems here. For example: (A) if $c_n = o(|n|^k)$, $k > 0$, and the "good" series $T$ satisfies the conditions $\sum |y_n| |n|^{k+k'+2} < \infty$ (the least integer $\geq k$) and $\lambda'(x) = \lambda(2)(x) = \lambda(k+1)(x) = 0$ for $x \in E$, then (*) $(ST)_N - \lambda(x)S_N$ is uniformly summable $(C,k)$, $x \in E$, provided that $\gamma_n = O(|n|^{-\beta})$, $\beta > 0$ and $k' + 2$. (B) If $c_n = o(|n|^k)$ and $\sum |c_n| |n|^{-k-1}$ is convergent, then (*) is summable $(C,k)$ under weaker conditions than (A). There are parallel results for $-1 < k \leq 0$. Localization theorems are obtained by using the above and proving a generalization of Riemann's localization formula. (Received April 21, 1969.)

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**69T-B131.** ALEXANDER WEINSTEIN, Georgetown University, Washington, D. C. 20007.

**A counterexample to a statement of I. C. Gohberg and M. G. Krein.**

In the book, Introduction to the theory of non-selfadjoint operators (English translation, 1969, p. 25, Russian edition, 1965, p. 45) Gohberg-Krein consider the classical minimum-maximum theory for a symmetric nonnegative compact operator $A$ with eigenvalues $\lambda_1 \leq \lambda_2 \leq \ldots$ and eigenvectors $\varphi_1, \varphi_2, \ldots$. They remark that even in the case $\lambda_{j+1} < \lambda_j$ the minimum, $\lambda_{j+1}$, of the maximum Rayleigh quotient can be attained for orthogonality conditions defined by a subspace other than that generated by $\varphi_1, \varphi_2, \ldots, \varphi_j$. However, it was already shown [A. Weinstein, J. Math. Anal. Appl. 12 (1965), 50], that if $\lambda_{j+1}$ is the smallest eigenvalue of a $j+1 \times j+1$ positive definite matrix, then $\lambda_{j+1}$ can be attained only by the classical choice. It should be noted that Gohberg-Krein do not exclude finite-dimensional spaces, see p. 32 of Russian edition. For a further analysis of the statement of Gohberg-Krein see Abstract 69T-B13, these Notices 16 (1969), 317. (Received April 24, 1969.)

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**69T-B132.** W. JOHN WILBUR, Pacific Union College, Angwin, California 94508.

**Density theorems for locally compact groups.** Preliminary report.

Let $X$ be a locally compact group with identity $e$. Let $\mu$ be Haar measure on $X$ and $\mathcal{M}$ the $\sigma$-algebra of $\mu$-measurable sets. **Theorem 1.** If $B \in \mathcal{M}$ is a set of finite measure then there is a sequence $\{u_n\}_{n=1}^{\infty}$ of open neighborhoods of the identity $e$ such that if $\{A_n\}_{n=1}^{\infty}$ is any sequence of sets
from $\mathfrak{m}$ with $\mu(A_n) > 0$ and $A_n \subseteq u_n$ for each $n$, then for almost every $x \in B$, $\lim_n \frac{\mu((xA_n) \cap B)}{\mu(A_n)} = 1$. Now for any set $A \subseteq X$ let $A^n$ stand for the Cartesian product of $A$ with itself $n$ times. Then a relation $R \subseteq X^n$ will be called linear if for any $x \in X$ and $(a_j) \in R$, $(xa_j) \in R$. The relation $R$ will be called dense if $(e) \in R^\prime$ (the closure of $R$ in the product topology for $X^n$). Theorem 2. If $A \in \mathfrak{m}$, $R \subseteq X^n$ is a dense linear relation, and $\pi$ is the projection of $X^n$ onto its first coordinate, then $\mu(A - \pi(R \cap A^n)) = 0$. (Received April 24, 1969.)

**Applied Mathematics**


A discrete Newtonian mechanics is developed with velocity defined implicitly by $(v_i + v_{i-1})/2 = (x_i - x_{i-1})/\Delta t$, acceleration defined explicitly by $a_i = (v_i - v_{i-1})/\Delta t$, and with Newton's law assumed in the form $F(t_{i-1}) = ma_i$. The conservation laws are proved and computer studies are made of nonlinear pendulum and nonlinear string vibration problems. Existence and uniqueness theorems follow trivially from the arithmetic recursion formulas describing the motion of a discrete system. Computer examples demonstrate the strong stability of the formulation. (Received February 12, 1969.)

69T-C22. HARVEY T. BANKS and MARC Q. JACOBS, Center for Dynamical Systems, Division of Applied Mathematics, Brown University, Providence, Rhode Island 02912. A differential calculus for multifunctions.

Let $E$ be a Banach space, $F$ a reflexive Banach space. We consider multifunctions (point-to-set functions) $\Omega$ where $\Omega$ takes points of $E$ into closed, bounded, convex subsets of $F$. We use Rådström's embedding theorem [Proc. Amer. Math. Soc. 3 (1952), 165-169] to embed the collection $\mathcal{B}$ of all closed, bounded, convex subsets of $F$, metrized with the Hausdorff metric, into a normed linear space $\hat{\mathcal{B}}$. It is natural then to define $\Omega$ to be differentiable if and only if $\hat{\Omega}$ is differentiable, where $\hat{\Omega}$ is the image of $\Omega$ under the embedding. Thus the basic theory of differential calculus in normed linear spaces can be applied to the functions $\hat{\Omega}$. We then take advantage of special properties of the embedding and the algebraic structure of $\hat{\mathcal{B}}$ to induce an intuitively meaningful differential calculus for multifunctions $\Omega : E \to \mathcal{B}$. Applications to differential inequalities, contingent equations and control theory are discussed. A number of illustrative examples of differentiable multifunctions are given. (Received February 24, 1969.)

69T-C23. RICHARD H. FRANKE, University of Utah, Salt Lake City, Utah 84112. Best Chebyshev quadratures.

The remainder for the Chebyshev quadrature formula $Rx = \int_1^1 x(s)ds - u \sum_{k=1}^n x(a_{k-1})$ may be written as $Rx = \int_1^1 K(t)d^n x(t)/dt^n dt$, provided $Rx = 0$ for $i = 0, \ldots, n - 1$. Best quadratures are those which minimize $J = \int_1^1 [K(t)]^2 dt$. The equations for determining the nodes are developed for the general case. Specific numerical examples are computed for $n = 1$ and $n = 2$. The values obtained for $J$ are compared with the corresponding values for the classical formulas, and for another type of Chebyshev quadrature. (Received February 25, 1969.)
Let $L$ be a linear functional on the space $H_{2k}$ of polynomials of degree $\leq 2k$. If

$$(A) \sum_{j-n}^{n} W_j f(j) = Lf$$

for all $f$ in $H_{2k}$. Let $C(t) = \sum_{-n}^{n} W_j e^{ist}$ and

$C_L(t)$ be similarly defined for the unique solution $w_{-k},...,w_k$ of $(A)$ with $n = k$; then $C(t) = C_L(t) + O(t^{2k+1}) (t \to 0)$. If $|C(t)/C_L(t)| < 1 (0 < |t| \leq \pi)$, we call $(B)$ $u(r) = \sum_{-n}^{n} W_j v(j + r)$ a stable realization of $L$ in $H_{2k}$; this generalizes the definition of Schoenberg for $Lf = f(0)$ ($C_L(t) = 1$). If $v = f + \epsilon$, where $\epsilon$ is a stationary, zero mean error, then $Eu(r) = Lf_{(f(x) = f(x + r))}$; if $(B)$ is stable the variance of $u(r)$ is less than that of the "raw" estimate $u_L(r) = \sum_{-k}^{k} W_j v(j + r)$, regardless of the autocorrelation structure of $\epsilon$. A similar result holds for cyclic errors. Stability of minimum variance smoothing is investigated. The realization of $Lf = f^{(2r)}(0)$ ($0 \leq r \leq k < n$) for which $R_{y} = \sum_{-\infty}^{\infty} (\sum_{-\infty}^{\infty} (-1)^{y} G(r; l - a^2)) h_{a^2} = (x_1, ..., x_n)$, where $h(x)$ is harmonic in a star-like region (with respect to the origin) $D$, and $G(r; l - \sigma^2)$ = $-r R_{1}(r \sigma^2, 0; r, r)$, is a solution of $(**)$ $u + B(\psi)u = 0$, for $\chi \in D$. Furthermore, each regular solution of $(**)$ in $D$ has an integral representation of this form. This result improves the results of S. Bergman and I. Vekua concerning solutions of $(*)$ and also contains the special example given by Vekua for $(**)$ when $C(r^2) = \lambda^2$, a constant. (Received March 31, 1969.)
Geometry

69T-D15. MICHAEL J. KALLAHER, University of Manitoba, Winnipeg 19, Manitoba, Canada.

On finite affine planes of rank 3.

H. Lüneburg has proposed the problem of determining the finite affine planes admitting rank 3 groups of collineation. This is done in the following Theorem. If \( \mathcal{P} \) is a finite affine plane admitting a rank 3 group \( G \) of collineations then either (i) \( \mathcal{P} \) is a translation plane and \( G \) contains the group of translations, or (ii) \( \mathcal{P} \) is a dual translation plane and \( G \) contains the group of dual translations. The proof of this theorem will appear elsewhere. This corrects an earlier result of D. Higman [Math. Z. 104 (1968), 147-149] whose proof was incomplete. (Received March 12, 1969.)


If a complete Riemannian manifold \( M \) (for convenience, supposed simply connected) has \( k \) non-Euclidean factors, then any isometric immersion \( \psi \) of \( M \) in Euclidean space has codimension at least \( k \). Suppose \( \psi \) is an immersion of \( M \) having codimension exactly \( k \). Then \( \psi \) is cylindrical on \( M \)'s Euclidean factor. This result was proved under an additional hypothesis by the author (Reducibility of Euclidean immersions of low codimension, J. Differential Geometry, to appear). The present proof is similar, but makes use of a generalization of a formula of Chern and Kuiper concerning the geometry of an isometric immersion at a point. (Received March 21, 1969.)

Logic and Foundations

69T-E35. ALBERT J. V. SADE, 364 Cours de la République, Pertuis, Vaucluse, France.

Algèbre de Lukasiewicz trivalente.

Les 8 axiomes \( A_1 \ldots A_8 \) qui définissent une algèbre de Lukasiewicz (A. Monteiro, MR 33 #36, and Math. Japonicae (1) 12 (1967), 1-23; C. O. Sicoe, Proc. Japan Acad. 43 (1967), 733-736) forment un système d'équations fonctionnelles sur le corps \( k \) du 3e ordre qui a pour solution la L-algèbre de la logique trivalente définie par \( \sim x = N = 2x + 1 = M_7 \) (négation); \( \forall x = x^2 = \hat{0} = M_9 \) (possibilité); \( x \& y = 2xy + 2x^2y + 2xy^2 + x^2y^2 \) (copule); \( x \lor y = x + y + xy(1 + x + y + 2xy) \) (disjonction), de matrices \( \& = (00, 01, 02; 10, 11, 12; 20, 21, 22) \), \( \lor = (000; 012; 022); \) mais qui admet une seconde solution (et une seule), \( \sim x = 2x = (021) = R = M_6 = \sim (\sim N) \) de Günther; \( \forall x = 2x^2 + x + 1 = (112) = M_{22}; \) \( \forall y = 2x + 2y + 2x^2 + xy + 2y^2 + x^2y^2; \) \( x \& y = 2x^2 + 2y^2 + 2xy + y^2 + 2y^2 \), de matrices \( (010; 111; 012) \) et \( (002; 012; 222) \). \( A_1 \rightarrow A_5 \) suffisent à déterminer \& et \( \lor; A_8 \) est conséquence de \( A_1 \rightarrow A_7 \). Tout opérateur dyadique peut être défini au moyen des opérateurs monadiques \( \Sigma = [M_i], \ i = 0,1,\ldots, 26. \) On passe de la 1re à la 2me solution ci dessus par l'automorphisme de \( \Sigma, P_3 = (0,2)(1)(3)(4,5)(6,7)(8)(9,22)(14,24)(15,18)(10,21)(16,20)(12,26)(11,23)(13,25)(17,19), \) qui seul ne déplace pas les constantes. (Received February 14, 1969.)
**69T-E36. STEVEN K. THOMASON**, Simon Fraser University, Burnaby 2, British Columbia, Canada, and University of California, Berkeley, California 94720. On initial segments of the degrees of unsolvability.

**Definition.** A lattice \( L \) is well representable if there is a set \( S = \{ s_1, \ldots, s_m \} \) and dual isomorphism \( a \to \theta_a \) of \( L \) onto a sublattice of the partition lattice of \( S \) such that whenever \( u, v \in S \) and \( 1 \leq k, t \leq m \) and \( s_k \theta_{s_i} = u \theta_{s_i} v \) for all \( a \in L \), then there are \( t_1, \ldots, t_m \in S \) such that \( \theta_{s_i} = u, t_1 = v \), and \( s_k \theta_{s_j} = t_1 \theta_{s_j} t_i \) for all \( a \in L \) and \( 1 \leq i, j \leq m \). Theorem. Every well-representable lattice is isomorphic to an initial segment of the degrees. The lattice of all subspaces of a finite-dimensional vector space over a finite field is well representable. It seems possible that every finite lattice is well representable. (Many of the ideas of this paper are implicit in the work of Lerman, Abstracts 68T-E5 and 68T-E26, these Notices 15 (1968), 804, 933.) (Received January 15, 1969.)


It is sometimes necessary to decompose a given operation into a sequence of atomic (not further in the science in question decomposable) operations. Such a sequence is called a continuous sequence of operations. We consider here operations on classes and sets. In this field the concept of plural class plays an outstanding role. An instance is the conversion of an ordered pair. It consists of not less than eleven atomic operations.

\[
\begin{align*}
(a, b) &\rightarrow ((a), (a, b), (1), \ldots, (a), (a, b), (2), \ldots, a, a, b, (4), a, a, a, b, (5), a, b, b, a, (6), b, b, a, (7), b, b, a, (8), b, b, a, (9), b, b, a, (10), (b), (b, a), (11) \rightarrow (b, (b, a)) = (b, a). \end{align*}
\]

These operations are: (1) transformation of a singular class into a plural class; (2) simultaneous transformation of all the elements of a plural class into plural classes; (3) uniting; (4) inverting the uniting; (5) fusion of two equal elements into one element; (6) conversion of a plural class of two elements into one element; (7) doubling of an element; (8) uniting; (9) inverting the uniting; (10) transforming of elements in singular classes; (11) transforming the totality in a singular class. Another instance is uniting of two sets:

\[
\begin{align*}
\{m_1, m_2, \ldots, n_1, n_2, \ldots\} \cup \{m_1, m_2, \ldots, n_1, n_2, \ldots\} = \cup \{\{m_1, m_2, \ldots\}, \{n_1, n_2, \ldots\}\}. \end{align*}
\]

(Received February 7, 1969.)

**69T-E38. SAHARON SHELAH**, Hebrew University, Jerusalem, Israel. On generalization of categoricity.

**Theorem 1.** Let \( T \) be a complete first-order theory in the language with the generalized quantifier \( Q_{\leq} \cdot \) "there exists \( x \)'s of the power of the model". If \( |T| < 2^{\aleph_0} \), and \( T \) is categorical in \( \lambda \), \( \lambda > (2|T|)^+ \), and \( \lambda_0 < \lambda = \prod_{\kappa<\omega} \lambda_0 < \lambda = \aleph_0 \) then \( T \) is categorical in every power \( |T| = 2^{\aleph_0} \). (\( \delta \) is a limit ordinal).

**Definition 1.** (1) Let \( T_1 \) be a complete first-order theory with one place predicate \( Q \), then \( \langle \lambda, \mu \rangle \in C(T_1) \) if every two models, \( M, N \), of \( T_1 \) of power \( \lambda \), for which \( \lambda > |Q^M| = |Q^N| = \mu \), are isomorphic. (2) \( (\infty, \mu) \in C(T_1) \) if for some \( \lambda_0 \), \( \lambda \geq \lambda_0 \) implies \( \langle \lambda, \mu \rangle \in C(T_1) \). **Theorem 2.** Let \( \langle \lambda, \mu \rangle \in C(T_1) \), \( \lambda > \mu \geq |T_1|^+ \), \( \lambda, \mu \neq \inf \{ x : x < x \equiv |T_1| \} \), and \( \lambda^0 = \lambda, \mu^0 = \mu \) or

\[
\begin{align*}
\lambda &\leq 2_{\aleph_0} + \omega \geq |T| \geq |T| && \lambda \geq 2_{\aleph_0} + \omega \geq |T| \geq |T|. \end{align*}
\]

Then (1) \( \{ \mu : (\infty, \mu) \in C(T_1), \mu > |T_1| \} = \{ \mu : |T| > |T_1| \} \) where \( |T_1| < \mu \leq |T| \). (2) \( \langle \lambda_1, \mu_1 \rangle \in C(T_1), \lambda_2 \geq \lambda_1 \) implies \( \langle \lambda_2, \mu_1 \rangle \in C(T_1) \). **Theorem 3.** If a first order theory \( T_2 \) is categorical in \( |T_2| = 2^{\aleph_0} \), then \( T_2 \) has a model of power \( < |T_2| \). (This partially solves a question of Morley.) (Received February 11, 1969.)
69T-E40. DOV GABBAY, Hebrew University of Jerusalem, Israel. Decidability of some modal calculi I.

Let $K$ be the modal system with the axiom $\Box(\Diamond \varphi \rightarrow \Box \varphi) \rightarrow (\Box \varphi \rightarrow \Box \Diamond \varphi)$ and rule $\varphi \rightarrow \Box \varphi$. The following extensions of $K$ are decidable: (1) $m \vdash \Diamond \varphi \rightarrow \Box m \varphi$, (2) $m, n \vdash \Diamond m \varphi \rightarrow \Diamond n \varphi$, (3) $m \vdash \Diamond m \varphi$, (4) $m \vdash \Box \varphi \rightarrow \Box m \varphi$ and $\Diamond m \varphi \rightarrow \Box m \varphi$, (5) $\Diamond(\bigwedge_{i=1}^{k} (\Diamond \varphi_{i} \rightarrow \Box \varphi_{i}))$, $k \equiv 1$, (6) $m \vdash \varphi \rightarrow \Box m \varphi$, (7) $m \vdash \Box \varphi \rightarrow \Box m \varphi$. We use a theorem of M. O. Rabin concerning the decidability of monadic second order theory of $\mathbb{N}_{0}$ successor functions. (Received February 24, 1969.)

69T-E41. FRANK B. CANNONITI, University of California, Irvine, California 92664. Elementary permutations without provably recursive inverses.

Let $\mathcal{J}$ be a theory in which the Kleene $T$-predicate is expressed by a wf $T(a, b, c)$. Then A. Kino calls a recursive function $\varphi$ provably recursive in $\mathcal{J}$ if there is an index $e$ of $\varphi$ such that $\exists \mathcal{J} \forall x \forall y \exists z \mathcal{T}(e, x, y)$. (Cf. A. Kino, On provably recursive functions and ordinal recursive functions, J. Math. Soc. Japan 20 (1968), 456-476.) In a paper of the author with M. Finkelstein (to appear in J. Symbolic Logic) it is shown that each (singular) recursive $f$ is representable as $f = AB^{-1}C$ where $A, C$ are fixed elementary (in Kalmar's sense) and $B$ is an elementary permutation. Since not all recursive functions are provably recursive in $\aleph_{0}$ (formalized peano arithmetic), and since elementary functions are provably recursive and composition preserves provable recursiveness we have

Theorem. There are elementary permutations $\varphi$ that are provably recursive in $\aleph_{0}$ such that $\varphi^{-1}$ is not provably recursive in $\aleph_{0}$. (Received March 13, 1969.)

69T-E42. LAWRENCE FEINER, State University of New York, Stony Brook, New York 11790. Decidability and the Lindenbaum algebra isomorphism type.

By a theory, we mean a first order theory with a countable language. Two first order theories will be called isomorphic iff their Lindenbaum Algebras are of the same isomorphism type.

Theorem. Every theory with at most countably many completions and a $\Pi_{1}^{1}$ set of axioms is isomorphic to a decidable theory. Theorem. The theory of one binary relation has an axiomatizable extension which is not isomorphic to any decidable theory. (Received February 17, 1969.)

69T-E43. MARTIN M. ZUCKERMAN, City College of the City University of New York, New York, New York 10031. Choices from finite sets and choices of finite subsets.

Let $\sigma$ be Mostowski's set theory; $\sigma$ is of the Godel-Bernays type. It permits urelemente and does not include AC (the axiom of choice) among its axioms. Assume $\sigma$ is consistent. Let FS be the statement, "For every nonempty set $X$ of nonempty sets $x$, there is a function $f$ defined on $X$ such that for all $x \in X$, $f(x)$ is a nonempty subset of $x$." Let $I$ be the positive integers. For each $n \in I$, let $C(n)$ be the statement, "For every nonempty set $X$ of $n$-element sets $x$, there is a function $f$ defined on $X$ such that for each $x \in X$, $f(x) \in x$." For $Z = \{z_{1}, z_{2}, \ldots, z_{k}\} \subseteq I$, let $C(Z) = C(z_{1}) \wedge C(z_{2}) \wedge \ldots \wedge C(z_{k})$.

For every set $P$ of primes let $\text{Lin Comb } P = I \setminus \bigcup_{i=1}^{s} k \{p_{1}; s \equiv 1; k \equiv 0, p_{1} \notin P \text{ for } 1 \equiv i \equiv s \}$. Let $Y_{P} = I - \text{Lin Comb } P$. Theorem. For every set $P$ of primes there is a model for


σ ∪ {∀AC, FS, (∀n ∈ I) (C(n) iff n ∈ Y_p)}. (A Fraenkel-Mostowski model is constructed.) As an immediate corollary we have a direct proof of the necessity of Mostowski's Condition (M) for C(Z) → C(n) to be provable in σ. (Received March 5, 1969.)


Let z ∈ N and let x be a description of z relative to a universal (M. Davis, Proc. Amer. Math. Soc. 8 (1957), 1125) function [u], i.e. [u](x) = z. The shortest description of z relative to u is su(z). S = def {x ∶ z ∈ N, su(z) = x}. Hypothesis. There is a u such that of two equally long numbers (sequences) v and w, v is more randomized (lawless) than w when su(v) > su(w), and v is more lawish (regular, ordered) than w when su(v) < su(w). Although su(z) ∈ S and S ⊂ W, su(z) < S and S is recursive in the creative domain of [u]. Neither S nor S are themselves creative. S ∈ Σ_2 - Σ_1 in the Kleene hierarchy. ∀D: su[D part. rec. = card D < W. Let z(ν) denote a sequence of length ν. Define the randomization function z_rand(ν) by

\[ z_{\text{rand}}(\nu) = \begin{cases} \text{def} & \nu \in \mathbb{N} \\ z_{\text{rand}}(\nu) = su(z(\nu)) \end{cases} \]

Then z_rand(ν) < φ but z_rand(ν) < su(z), and S < S. No arbitrary long randomized sequence can be generated effectively. However, an oracle, powered with su(z), is capable of scoring in the von Mises’ game. These results extend earlier theorems in L. Lofgren "Recognition of order and evolutionary systems," pp. 165-175, in Computer and information sciences. II. Academic Press, New York, 1967. (Received March 17, 1969.)

69T-E45. EUGENE W. MADISON, University of Iowa, Iowa City, Iowa 52240. Real ordered fields with characterization of the natural numbers.

\[ \mathcal{R} = \{ \mathbb{R}, \mathbb{N}_0, +, *, \leq \} \] (abbreviated {R; \mathbb{N}_0}) denotes the ordered field of real numbers with the natural numbers \mathbb{N}_0 distinguished. Structures similar to \mathcal{R} are analyzed within LPC. Consider two well-known properties of \mathcal{R}: (a) Every proper elementary extension of \mathcal{R} enlarges \mathbb{N}_0. (b) Let \{R^*, \mathbb{N}^*\} be an elementary extension of \mathcal{R}, \mathbb{R}^* its ring of finite elements, and J its ideal of infinitesimals, then \mathbb{R}^*/J \cong \mathcal{R}. Theorem 1. Every subfield of \mathcal{R} (recursive reals) is elementary closed (relative to \mathbb{N}_0), hence satisfies (a). There exists a subfield of \mathcal{R} which is elementarily closed (relative to \mathbb{N}_0) and which is not arithmetically definably ordered. Theorem 2. Let \mathcal{R}_1 be a subfield of \mathcal{R}. Every subfield of real numbers which properly contains \mathcal{R}_1 obstructs elementary extension of \mathcal{R}_1. Corollary 3. Every computable ordered real field is obstructed by its proper real extensions.

Theorem 4. Let \mathcal{R}_0 \subseteq \mathcal{R}, \mathcal{R}^* = \{R^*, \mathbb{N}^*\} a nonstandard model of analysis. Every elementary extension \mathcal{E} = \{R^*, \mathbb{N}^*\} of \mathcal{R}_0 is isomorphic to a subfield of \mathcal{R}^*. Theorem 5. Let \mathcal{R}_0 be a proper subfield of \mathcal{R}. \mathcal{R}_0 satisfies (b) if and only if for any \mathcal{R}_1 (\mathcal{R}_0 \subseteq \mathcal{R}_1 \subseteq \mathcal{R}), \mathcal{R}_1 obstructs \mathcal{R}_0. Corollary 6. Every subfield of \mathcal{R} (satisfies (a) and (b). Corollary 7. Every computable ordered subfield of \mathcal{R} satisfies (a) and (b). (Received March 31, 1969.) (Author introduced by Professor W. A. Kirk.)


In his monograph, Construction order types, J. N. Crossley raised the question of whether the implication 2 + A = A ⇒ 1 + A = A is true for constructive order types. Using an earlier

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definition of constructive order type, A. G. Hamilton presented a counterexample in his paper, *An unsolved problem in the theory of constructive order types*, J. Symbolic Logic 33 (1968), 565-567. Hamilton left open the general question, however, since he pointed out that Crossley considers only orderings which can be embedded in a standard dense r.e. ordering by a partial recursive function, and that his counterexample fails to meet this requirement. We resolve the question by constructing a constructive order type \( A \) which meets Crossley's requirement and such that \( 2 + A = A \) but \( 1 + A \neq A \). The proof uses techniques we developed in *Recursion theory and Dedekind cuts*, Trans. Amer. Math. Soc., to appear in May, 1969. (Received March 28, 1969.)

69T-E47. CARL F. ECKBERG, Purdue University, Lafayette, Indiana 47907. A result in recursive analysis.

Let \( R \) be the real numbers between 0 and 1, and let \([0,1]\) be the set of recursive real numbers in \( R \). Definition. Let \((a_n)_n\) be an r.e. sequence of rational numbers in \([0,1]\) which converges classically to a limit \( \pi \in R - [0,1] \). We say \((a_n)_n\) satisfies property (A) if there exist recursive functions \( N_1, N_2 : N \to N \), defined for each index \( x \) of a recursive real number \( x \) in \([0,1]\), and such that for each such \( x, \bar{x} \in \langle N_1(x), N_2(x) \rangle \), and \( a_n \in \langle N_1(x), N_2(x) \rangle \) for \( n \equiv n_0 \). Definition. A real number \( \pi \in R \) is r.e. if it is the limit of an r.e. sequence of rational numbers. We say \( \pi \) is left (right) r.e. if each element of the sequence can be taken to be less than (greater than) \( \pi \). By a priority method we have Theorem 1. There exists an r.e. real number \( \pi \in [0,1] \) which (1) is neither left nor right r.e., (2) is the limit of an r.e. sequence of rationals satisfying property (A). Employing a construction due to O. Aberth we have Theorem 2. Let \( \pi \) be an r.e. real number in \([0,1]\) satisfying (1) and (2) above. Then there exists a bounded recursive operator \( F \), total on \([0,1]\), which (1) is neither left nor right continuous at \( \pi \), (2) is extendable to a continuous function on \( R - \{\pi\} \). (Received April 11, 1969.)

69T-E48. ALEXANDER ABIAN, Iowa State University, Ames, Iowa 50010. Synergistic models for Z-F.

Let Z-F denote the axiom system consisting of the axioms of Extensionality (E), Powerset (P), Sumset (S), Infinity (I) and the axiom scheme of Replacement (R) of Zermelo-Fraenkel (the underlying logic being the first order predicate calculus without equality). Definition. An axiom system is called Synergistic iff it consists of the axioms E, P, S, I and any finite (possibly zero) number of instances of the axiom scheme R. A model is called Synergistic iff it is a model for a Synergistic axiom system. By compactness theorem, a statement is consistent with (or independent of) Z-F iff it is consistent with (or independent of) every Synergistic axiom system. Thus, the proof of consistency of a statement (e.g., the axiom of choice, etc.) with Z-F can be given by constructing only Synergistic models in which the statement is valid, without necessitating the construction of a model for the entire Z-F. Also, the proof of independence of a statement from Z-F can be given by constructing only Synergistic models in which the negation of the statement is valid, without necessitating the construction of a model for the entire Z-F. Remark. It is obvious how to modify the stated definition in case Z-F is assumed to include axioms in addition to the ones mentioned above. (Received April 22, 1969.)

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Statistics and Probability

69T-F8. J. DAVID MASON, University of Georgia, Athens, Georgia 30601. A limit theorem concerning a sequence of normed, centered sums of independent random variable involving $r$ distributions.

Let $\{X_n\}$ be a sequence of independent random variables such that $F_{X_n} \in \{F_1, \ldots, F_r\}$ for all $n$, with $r \geq 2$. Assume $F_i$ is in the domain of attraction of a stable law with characteristic exponent $\lambda_i$, $1 \leq i \leq r$, with $0 < \lambda_1 < \ldots < \lambda_r \leq 2$. Assume $n_i(n)$ goes to infinity, where $n_i(n)$ is the number of times $F_i$ appears in $F_{X_1}, \ldots, F_{X_n}$. Theorem. A necessary and sufficient condition that there exist constants $\{A_n\}$ and $\{B_n\}$, with $0 < B_n < \infty$, such that $\sum_{i=1}^{n} (X_1 + \ldots + X_n) - A_n$ converges in law to a nondegenerate distribution $G$ is that for some set of indices $\{i_1, \ldots, i_1\}$, with $1 \leq i_1 < \ldots < i_1 \leq r$, $B(i,j,n)/B(i_i,n_i(n)) \to p_j$, where $p_j = 0$ for $j \notin \{i_1, \ldots, i_1\}$ and $0 < p_j < \infty$ for $j \in \{i_1, \ldots, i_1\}$, where $\{B(j,n)\}$ are normalizing coefficients for $F_j$. Furthermore, $G$ is a convolution of $r$ stable laws with characteristic exponents $\lambda_1, \ldots, \lambda_r$, and there is a constant $b > 0$ such that $B_n = bB(i_1,n_1(n))$ for all $n$.

(Received February 28, 1969.)

69T-F9. WOLFGANG J. KRIEGER, Ohio State University, Columbus, Ohio 43210. On the entropy and generators of ergodic measure-preserving transformations.

Rohlin proved that every aperiodic measure-preserving transformation with finite entropy has a generator (a-generator) with finite entropy. We prove that every ergodic measure-preserving transformation with finite entropy has a finite generator. We also obtain an estimate for the minimal number $\Delta(T)$ of elements that a generator of an ergodic measure-preserving transformation $T$ can contain. We show that $\Delta(T) \leq \max(2, e^{h(T)} + 1)$ where $h(T)$ is the entropy of $T$. (Received March 10, 1969.)


Consider the following random walk. Starting at the origin a particle proceeds to the right along straight lines in such a manner that at every noninteger $x$ the slope is $+1$, $-1$, or $+2$ with probabilities $p_0$, $p_2$, $p_1$ respectively. The probability of crossing the level $y = 0$ after $k$ steps is determined, also the probability of crossing the level $y = 0$ after $n$ steps starting from a height $y = k$ is determined as well as the probability of a first return to $y = 0$ at the nth step. The generating functions of these probabilities are then used to determine the number of level crossings of the random walk. (Received March 31, 1969.)

Topology

69T-G53. JOHN M. WORRELL, JR., Sandia Laboratories, Division 1721, P.O.Box 5800, Albuquerque, New Mexico 87115. Characterizations of certain subspaces.

For background, see Duke Math. J. 34 (1967), 255-271. Theorem 1. A topological space is a subspace of a regular space having a monotonically complete base of countable order if and only if it has a sequence $G_1, G_2, \ldots$ of bases for which it is true that each point of an open set $D$ lies in an open subset $D'$ of $D$ such that if $g_1, g_2, \ldots$ is a decreasingly monotonic sequence of sets, each $g_n$ intersecting $D'$ and belonging to $G_n$, then some $g_n$ is a subset of $D$. Theorem 2. A $T_0$-space is a subspace of a
complete Moore space if and only if it has a sequence $G_1, G_2, \ldots$ as above with the condition of monotonicity replaced with the finite intersection property. **Remarks.** (1) The first sequence condition has a base of countable order-like formulation for essentially $T_1$-spaces. (2) Any space satisfying the first sequence condition has a metrizable subspace dense in it. (3) Both subspace conditions are preserved under the actions of peripherally bicompact closed continuous mappings. (4) Neither condition is an invariant under the actions of uniformly monotonically complete, open continuous mappings between Tychonoff spaces. This work was supported by the United States Atomic Energy Commission. (Received February 12, 1969.)

69T-G54. ROBERT F. CRAGGS, University of Illinois, Urbana, Illinois 61801. **Extending homeomorphisms between approximating polyhedra.**

We prove the following: Theorem. Suppose $M$ is a 3-manifold with boundary, $K$ is a compact polyhedron, $K_a$ is a subpolyhedron of $K$, and $f$ is a homeomorphism of $K$ into $M$ such that $f^{-1}(\partial M) = K_a$. There is a $\delta > 0$ such that if $g$ is a homeomorphism of $K$ onto a tame set in $M$ with $g^{-1}(\partial M) = K_a$ and $d(f, g) < \delta$, and if $h$ is a $\delta$-homeomorphism of $g(K)$ onto a tame set in $M$ with $h^{-1}(\partial M) = g(K_a)$, then there are closed neighborhoods $N$ of $g(K)$ and $N'$ of $h(K)$ in $M$ and there is a homeomorphism $H$ of $N$ onto $N'$ such that $H(j_g(K)) = h$ and $H(\partial N \cap \partial M) = N' \cap \partial M$. If $M$ is a pwl manifold and $g$ and $h$ are pwl homeomorphisms, $N$ and $N'$ can be chosen to be polyhedra and $H$ to be pwl. The proof of the theorem relies on stronger results previously announced by the author for the special case where $K$ has no local cut points and $K_a$ no point components. (See Abstract 642-37, these Notices) 14 (1967), 71.) (Received February 13, 1969.)

69T-G55. GIOVANNI A. VIGLINO, Wesleyan University, Middletown, Connecticut 06457. **C-compact spaces.**

The following definition and Theorem 1-2 appear in C-compact spaces by the author (Duke Math. J., accepted for publication June 1968). It is similar (possibly equivalent) to the definition, "functionally compact" (f.c.) introduced in Abstract 69T-G29, these Notices) 16 (1969), 432. $(X, \tau)$ is C-compact (C-c) if given a closed set $Q$ and a $\tau$-open cover $0$ of $Q$, then there exists a finite number of elements $0_i$ of $0$ with $Q \subseteq C \cup_{i=1}^n 0_i$. **Theorem 1.** Every continuous function from a C-c space into a $T_2$ space is closed. **Theorem 2.** In a $T_2$ space compactness $\Rightarrow$ C-c $\Rightarrow$ minimal $T_2$ and neither implication is reversible. **Remark.** In "C-c" the author asks (1) Is the converse of Theorem (1) valid? (2) Is the product of C-c spaces C-c? (1) has apparently been resolved (69T-G29). The author resolved (2) by showing the product of a C-c space with $[0,1]$ need not be C-c. (nor f.c. It is clear from Theorem 3(b) below that C-c $\neq$ f.c.) This product result was submitted to Duke in July 1968. The paper was resubmitted in a longer work. The following definition and theorem are in the longer work. A filter base $F$ is (regular) adherent convergent (r.a.c.) if every (regular) open neighborhood of the adherent set of $F$ contains an element of $F$. **Theorem 3.** (a) $X$ is H-closed iff every open filter base is r.a.c. (b) $X$ is C-c iff every open filter base is a.c. (Received February 18, 1969.)

69T-G56. WITHDRAWN.
It is shown that the bounded Kneser conjecture (If M is a compact 3-manifold with nonvoid, connected boundary and \( \pi_1(M) \cong G_1 \ast G_2 \), a free-product, then there is a disk D regularly embedded in M and M - D has two components \( M_1 \) and \( M_2 \) with \( \pi_1(M_i) \cong G_i \), \( i = 1, 2 \) ) is false. (See J. Stallings, Abstract 559-165, these Notices 6 (1959), 531, for the closed Kneser conj.) Theorem 1. If M is a compact 3-manifold and \( \pi_1(M) \cong G_1 \ast G_2 \), then there are compact 3-manifolds \( M_1 \) and \( M_2 \) with \( \pi_1(M_i) \cong G_i \), \( i = 1, 2 \). Theorem 2. If M is a compact 3-manifold with nonvoid, connected boundary and \( \pi_1(M; \ast G \), then there is a disk D regularly embedded in M where M - D is connected and \( \pi_1(M - D) \cong G \). A group G is called freely reduced if G cannot be written as \( \mathbb{Z} \ast G' \). Theorem 3. If M is a compact 3-manifold with nonvoid, connected boundary and \( \pi_1(M) \cong G_1 \ast G_2 \) is freely reduced, then there is a disk D regularly embedded in M and M - D has two components \( M_1 \) and \( M_2 \) with \( \pi_1(M_i) \cong G_i \), \( i = 1, 2 \). Using Theorem 2 and Theorem 3, a proof of the closed Kneser conjecture is given. (Received February 14, 1969.)

A topological space \((X, \mathcal{T})\) is said to be minimal if every weaker topological space does not possess property P. The first two theorems give characterizations of minimal \( T_0 \) and minimal \( T_D \)-spaces, while the third gives a condition equivalent to bicom pactness in minimal \( T_0 \) and minimal \( T_D \)-spaces. (For a discussion of the \( T_D \) separation axiom, see C. E. Aull and W. J. Thron, Separation axioms between \( T_0 \) and \( T_1 \), Nederl. Akad. Wetensch. Proc. Ser. A65 = Indag. Math. 24 (1962), 26-37.)

Theorem. A \( T_0 \)-space \((X, \mathcal{T})\) is minimal \( T_0 \) iff the open sets in the topology are nested and the family \( \{[x]: x \in X\} \) is a base for \( \mathcal{T} \). Theorem. A \( T_D \)-space \((X, \mathcal{J})\) is minimal \( T_D \) iff the open sets in the topology are nested. Theorem. If \((X, \mathcal{J})\) is a minimal \( T_0 \) or minimal \( T_D \)-space, then the two following conditions are equivalent: (1) \((X, \mathcal{J})\) is bicom pact. (2) There exists exactly one singleton which is a closed set. (Received February 3, 1969.)

A net \( \mu = \{\mu_a: a \in D\} \) in a topological space \( X \) is said to be clusterable if and only if whenever \( p \) is a cluster point of \( \mu \), then \( p \) is a limit point of \( \mu \) (such nets are defined as 'maximal nets' by S. Clampa in his paper, On compactness in uniform spaces). A net \( \mu \) in a Tychonoff space \( X \) is said to be \( \mathcal{Z} \)-universal in case for each zero-set \( E \) in \( X \), either \( \mu \) is eventually in \( E \) or there exists a zero-set \( Z \) which is contained in \( X \setminus E \) such that \( \mu \) is eventually in \( Z \). A net \( \mu = \{\mu_a: a \in D\} \) is said to be closed under countable intersections in case for each countable family \( \{a_i: i \in \mathbb{N}\} \) of \( D \) there exists some \( a \in D \) such that \( a \equiv a_i \) for all \( i \in \mathbb{N} \). Lemma. Every \( \mathcal{Z} \)-universal net in a Tychonoff space \( X \) is clusterable. Theorem. A Tychonoff space \( X \) is realcompact if and only if every \( \mathcal{Z} \)-universal net in \( X \) that is closed under countable intersections converges if and only if every \( \mathcal{Z} \)-universal net in \( X \) that is closed under countable intersections has a clusterable convergent subnet. (Received February 24, 1969.)
Definition 1. A family of sets $\mathfrak{M}$ is a weak pseudo-base for a topological space $(X, \mathcal{T})$ if for $x$ and $T, x \in T \in \mathcal{T}$, there exists $M \in \mathfrak{M}$ such that $x \in M \subset T$. Definition 2. A topological space is coscreenable if every open cover has a $\sigma$-discrete closed refinement. Every Moore space and every perfectly $T_4$ space with a $\sigma$-point finite base has a $\sigma$-discrete closed weak pseudo-base. If a perfectly $T_4$ space has a $\sigma$-discrete closed weak pseudo-base and a point-countable base, the space is a Moore space. Hence a topological space is a normal metacompact Moore space if it is a perfectly $T_4$ space with a $\sigma$-point finite base. Every perfectly $T_4$ metacompact space is coscreenable. Coscreenable normal spaces are countably paracompact and coscreenable collectionwise normal spaces are paracompact. (Received February 19, 1969.)

An inequality on cycles in graphs.

Let $r$ be a 1-cycle over the integers modulo 2. Let $w$ index the $\gamma$-cycles of $r$, $j$ index the connected components of $w$ (in number), $a_j$ be the number of edges in the $j$th connected component of $w$, $a_0$ be the number of vertices in the $j$th connected component of $w$, $a_j$ be the number of edges of $r$ not in the $j$th connected component of $w$ but having both vertices in that component, and $b_j$ be the number of edges of $r$ not in the $j$th connected component of $w$ but having exactly one vertex in that component. Then $\Sigma \omega \Sigma_1 \Sigma 1 \gamma(\omega) 2a_j^{a_j} \geq \Sigma \omega \Sigma_1 \Sigma 1 \gamma(\omega)(2b_j^{b_j} + \beta_j^{b_j})a_j^{a_j}$. Generalizations include $\Sigma \omega \Sigma_1 \Sigma 1 \gamma(\omega) 2a_j^{a_j} \geq \Sigma \omega \Sigma_1 \Sigma 1 \gamma(\omega)(2b_j^{b_j} + \beta_j^{b_j})[a_0^{a_0} - a_j^{a_j}k]^{a_j}$ and $\Sigma \omega \Sigma_1 \Sigma 1 \gamma(\omega) 2a_j^{a_j} [C(a_0, k) - C(a_0 - a_j, k)]^{a_j}$, where $a_0$ is the number of vertices in $\Gamma$, $1 \leq k \leq a_0$, and $C(n, k)$ is the appropriate binomial coefficient. These inequalities arose in connection with a problem of ferromagnetism. (Received March 10, 1969.)

On the stable suspension homomorphism.

Let $s^q: \pi_1 X \rightarrow \pi_1+q S^q X$ denote the suspension homomorphism. If $q$ is large, we let $\pi_1^q X$ denote the stable group $\pi_{1+q} S^q X$ and $s = s^q$. Let $C$ be the class of finite groups. Theorem 1. Let $X$ be a $(k-1)$-connected space. (a) If $2k \equiv n + 3$, then $s^1(X, H_n S_{n+k} X)$ is finite, i.e. $s^1(S^q X)$ is a $C$-monomorphism. This theorem can be rephrased in terms of the Hurewicz homomorphism. Definition. A manifold $V^m$ is said to be spherical if the Hurewicz homomorphism $h: \pi_1(V, \partial V) \rightarrow H_1(V, \partial V)$ is not zero. Theorem 1 is proved using the following result: Theorem 2. Let $V^{n+k}$ be a $\pi$-manifold such that $H_i V = 0$ for $i > n$, $H_n V$ is torsion-free, and $\pi_1 \partial V = \pi_1 V$ is an isomorphism. Suppose $n \equiv 2$ and $k \equiv n + 3$. Set $t = [(n + k)/2]$. Then $V$ imbeds in a $(t-1)$-connected spherical $\pi$-manifold $W^{n+k+1}$ with $H_1 W = 0$ for $i > n$. If $\partial V$ is a homotopy sphere, then we may also assume that $\partial V \subset H Q$. (Received March 10, 1969.)
The main result is the following Theorem. Let \( X \) be a metric space, \( Y \) a complete metric space, and for each \( i = 1, 2, \ldots \), let \( X_i \) be a nonvoid closed finite-dimensional subset of \( X \) and \( f_i \) be a continuous mapping of \( X \) into \( Y \). Then there exists a complete metric space \( X^* \) such that (i) \( X \) is a dense subset of \( X^* \), (ii) \( \dim (\overline{cl} x_i(X_i)) = \dim X_i \) for each \( i = 1, 2, \ldots \), and (iii) each \( f_i \) can be continuously extended to \( X^* \). If \( X \) is a separable metric space, then the same theorem holds for \( X^* \) a compactification of \( X \), and a generalization is obtained of a well-known theorem of Hurewicz.

(Received March 5, 1969.)

Some theorems on completely regular mappings.

Let \( p: E \to B \) be a continuous surjection, where \( E \) and \( B \) are metric spaces. \( p \) is said to be completely regular if, given \( b \in B \) and \( \varepsilon > 0 \), there exists \( \delta(b, \varepsilon) > 0 \) such that if \( d(b, b') < \delta(b, \varepsilon) \), then there exists a homeomorphism \( h: p^{-1}(b) \to p^{-1}(b') \) with \( d(x, h(x)) < \varepsilon \) for all \( x \in p^{-1}(b) \). \( B \) is always assumed to be connected, so that all fibers are homeomorphic. \( \mathcal{K}(F) \) denotes the homeomorphism group of \( F \), with a natural topology. (If \( F \) is compact, \( \mathcal{K}(F) \) is given the compact-open topology, while if \( F \) is locally compact, \( \mathcal{K}(F) \) is identified with \( \mathcal{K}(\hat{F}, *) \subseteq \mathcal{K}(\hat{F}) \), where \( \hat{F} = F \cup \{ * \} \) is the one-point compactification of \( F \).) We now have the following results: Theorem 1. Let \( p: E \to B \) be completely regular with fiber \( F \), where \( F \) is locally compact and separable and \( \mathcal{K}(F) \) is \( LC^0 \). Then \( p \) is a Serre fibration. If in addition \( B \) is a finite-dimensional ANR, then \( p \) is locally trivial. Theorem 2. Let \( p: E \to B \) be completely regular with fiber \( F \), where \( F \) is a compact ANR and \( B \) is locally compact and finite dimensional. Then \( p \) is a Hurewicz fibration. (Received March 6, 1969.)

A metrization theorem for developable spaces.

An Hausdorff space, \( X \), is said to have property (\( \omega \)) if for each discrete collection of closed sets \( \{ F_a : a \in A \} \) in \( X \), there exists a cs-finite collection of open sets \( \{ G_a : a \in A \} \) such that \( F_a \subseteq G_a \) for each \( a \in A \) and \( G_a \cap F_{a'} = \emptyset \) if \( a \neq a' \). Every sequentially mesocompact space and every collection-wise normal space has property (\( \omega \)). Theorem. A regular space is metrizable if and only if it is developable and has property (\( \omega \)). This theorem is an extension of both Bing's theorem [Theorem 10, Canad. J. Math. 3 (1951), 175-186] and the theorem reported in Abstract 68T-G15, these \( \textit{Notiz} \) 15 (1968), 938. Characterizations of collectionwise normality are included, and some conditions under which spaces having property (\( \omega \)) are collectionwise normal and paracompact are presented. The notions of cs-finite collections and sequentially mesocompact spaces were defined previously [Abstract 68T-100, these \( \textit{Notiz} \) 15 (1968), 220]. (Received March 10, 1969.)

On the number of nonpiercing points of certain crumpled cubes.

If \( W \) is an open subset of \( E^3 \), the limiting genus of \( W \), denoted \( LG(W) \), is the least nonnegative integer \( n \) such that \( W \) is the monotone union of compact 3-manifolds \( H_1, \ldots, H_k, \ldots \) where the boundary
of each $H_k$ is the surface of genus $n$. If no such integer exists, $LG(W)$ is said to be infinite.

**Theorem 1.** Suppose $C$ is a crumpled cube such that $LG(\text{Int } C) = n$. Then there exists a finite set $Q$ of points in $\partial C$ such that for each open set $U$ containing $\partial C$, each point of $\partial C - Q$ has a neighborhood $V$ such that any loop in $V - \partial C$ is null-homotopic in $U - \partial C$. **Corollary.** If $C$ is a crumpled cube such that $LG(\text{Int } C) < \infty$ and $C$ has at most one nonpiercing point, then $\text{Int } C$ is homeomorphic to an open 3-cell. **Theorem 2.** If $C$ is a crumpled cube such that $LG(\text{Int } C) = n$ ($1 \leq n < \infty$), then $C$ has at most $n$ nonpiercing points. (Received March 10, 1969.)

69T-G68. WITHDRAWN.

69t-G69. MASAMI WAKAE, University of Manitoba, Winnipeg 19, Manitoba. On the $\Phi(k)$-index of a Euclidean space $\mathbb{R}^n$.

Let $\mathbb{R}^n$ be the $n$-dimensional Euclidean space. [1. M. Wakae: Some results on multiplicity of solutions in frame mappings, Math. Z. 98 (1967), 407-421] has established an upper bound for the $\Phi(k)$-index of $\mathbb{R}^n$ when $k = p^t$, a power of a prime number $p$. The exact value of the $\Phi(k)$-index of $\mathbb{R}^n$ has been obtained by finding an $(np^{t-1} (p-1) - 1)$-sphere which is invariant under the cyclic transformation group $S$ of order $k$, and using the exact sequence of the special cohomology groups of the above sphere. **Theorem.** The $\Phi(k)$-index of $\mathbb{R}^n$ is equal to $np^{t-1} (p-1)$. Using the above theorem, we may strengthen some results in the above mentioned paper. For example, [1, IV Theorem 2.8] can be replaced by "If $f$ maps $S^{n-1}$ into $\mathbb{R}$, then the ordered $n$-tuples $(w_1, w_2, ..., w_n)$ with $(f(w_1), f(w_2), ..., f(w_n)) \in F_n$ constitute a set $D$ considered imbedded in $SO(n)$. Let $D' = D/S$. Then $H_{N-j}(D') \neq 0$ for $(p-1)p^{t-1} - 1 \leq j \leq 4hp^{t-1} - 1$, where $N = \dim SO(n), n = k = p^t."$ (Received March 13, 1969.)

69T-G70. WITHDRAWN.
A theorem on representations of compact semigroups. Preliminary report.

Following J. H. Carruth and C. E. Clark [Abstract 663-535, these Notices 16 (1969), 242], a compact semigroup $S$ is an $L$-semigroup if the Schützenberger group of each $\mathcal{V}$-class of $S$ is a Lie group. A representation of a semigroup $S$ is understood to be a homomorphism of $S$ into an $L$-semigroup, and a representation $f$ is called $\mathcal{V}$-class preserving if $f(s) = f(t)$ implies $(s, t) \in \mathcal{V}$, the $\mathcal{V}$ relation on $S$. Further, a compact semigroup $S$ is said to admit a complete system of $\mathcal{V}$-class preserving representations if there are enough $\mathcal{V}$-class preserving representations of $S$ to separate points of $S$.

**Theorem.** Let $S = G \text{Horm}(X, S_X, m_{xy})$ [see Hofmann and Mostert, Elements of compact semigroups, Charles Merrill, 1966, p. 142] be a generalized hormos such that $\mathcal{V}$ is a congruence on $S$. If $S_X$ admits a complete system of $\mathcal{V}$-class preserving representations for each $x \in X$, then $S$ admits a complete system of $\mathcal{V}$-class preserving representations. **Corollary.** If $S = \text{Horm}(X, S_X, m_{xy})$ is a hormos, then $S$ admits a complete system of $\mathcal{V}$-class preserving representations. (Received March 17, 1969.)

**69T-G72. J. H. ROBERTS, Duke University, Durham, North Carolina. Dimension function $d_2$ covering dimension.**

In our paper A study of metric-dependent dimension functions (Trans. Amer. Math. Soc. 129 (1967), 414-435) K. Nagami and I introduced the metric dependent function $d_2(X, \rho)$, based on the Eilenberg-Otto characterization of dimension. We showed that $d_2(X, \rho) \leq \mu \dim(X, \rho)$, where $\mu \dim$ denotes the earlier introduced "metric dimension." Now Katetov (On the relations between the metric and topological dimensions, Czech. Math. J. 8 (1958), 163-166) has shown that for all metric spaces, $2\mu \dim(X, \rho) \equiv \dim X$ (covering dimension). The present paper proves the same result for the function $d_2$.

**Theorem.** For all metric spaces, $2d_2(X, \rho) \equiv \dim X$. (Received March 19, 1969.)

**69T-G73. JOSEPH C. NICHOLS, Duke University, Durham, North Carolina 27706. Realization of a particular metric-dependent dimension function.**

In their paper A study of metric-dependent dimension functions [Trans. Amer. Math. Soc. 129 (1967), 414-435], K. Nagami and J. H. Roberts introduced metric-dependent dimension functions $d_2$ and $d_3$ defined on the class of all metric spaces $(X, \rho)$. The following relations hold for all $(X, \rho)$:

$$d_2(X, \rho) \leq d_3(X, \rho) \leq \mu \dim(X, \rho) \leq \dim X,$$

where $\mu \dim$ is metric dimension, as defined by Katetov and $\dim X$ is covering dimension. Roberts and Slaughter [Fund, Math. 62 (1968)] have proved that if $\mu \dim(X, \rho) = r < n = \dim X$, then for every $k$ ($r \neq k \neq n$) there exists a topologically equivalent metric $\rho_k$ such that $\mu \dim(X, \rho_k) = k$. The present paper proves this same result for the function $d_3$. Whether the result is also true for the function $d_2$ remains an unsolved problem. (Received March 14, 1969.)

**69T-G74. L. WAYNE GOODWYN, University of Kentucky, Lexington, Kentucky 40506. Topological entropy bounds measure-theoretic entropy.**

Let $X$ be a compact Hausdorff space and let $T: X \to X$ be a continuous map. Let $\mu$ be a $T$-invariant probability measure on the Borel sets of $X$. It is proved that the measure-theoretic
entropy of $T$ with respect to $\mu$ is less than or equal to the topological entropy of $T$. This answers conjecture 1 of the paper: Topological entropy by R. L. Adler, A. G. Konheim and M. H. McAndrew, Trans. Amer. Math. Soc. 114 (1965), 309-319. (Received March 17, 1969.)

69T-G75. CHARLES L. HAGOPIAN, California Institute of Technology, Pasadena, California 91109. Concerning arcwise connectedness and the existence of simple closed curves in plane continua.

Let $M$ be a compact plane continuum. If $p$ is a point of $M$, define $K_p$ to be the set consisting of the point $p$ and all points $q$ in $M - \{p\}$ such that $M$ is not aposyndetic at $p$ with respect to $q$. In this paper the following is established. Theorem 1. If $M$ is semi-locally-connected at all except a finite number of its points and is such that for each point $x$ in $M$, $M$ is not semi-locally-connected at $x$ or $M$ is not aposyndetic at $x$, then $M$ is arcwise connected. Theorem 2. If for each point $x$ of $M$, the set $K_x$ is finite, and $M$ is either not semi-locally-connected at $x$ or not aposyndetic at $x$, then each point of $M$ is in a simple closed curve which is contained in $M$. Examples are also presented which rule out certain extensions of these results. (Received March 10, 1969.)

69T-G76. WITHDRAWN.

69T-G77. FRANK A. CHIMENTI, Western Michigan University, Kalamazoo, Michigan 49001. Concerning the limit inferior of sequences of sets. Preliminary report.

R. Engleking (Bull. Acad. Polon. Sci., Cl. III 4 (1956), 659-662) proved that the limit inferior over the class of sequences of nonempty closed subsets of the nonnegative rationals cannot be represented by a countable Boolean formula (i.e., a formula that can be written with the aid of countable Boolean operations and the closure operator). This result can be improved to: If $X$ is a topological space possessing at least one sequentially nonisolated point, then there exists no countable Boolean formula for the limit inferior over the class of sequences of nonempty closed subsets of $X$ (sets of at most two points are needed here and if the empty set is admitted, sets of at most one point will suffice). It is known that if every point of $X$ is isolated (i.e., $X$ is discrete), then there does exist a formula for the limit inferior. (Received March 24, 1969.)


Theorem. A connected and locally connected, separable regular space, in which every point is a strong cut point, is homeomorphic to the real line. This improves a result of A. J. Ward (The topological characterization of an open linear interval, Proc. London Math. Soc. 41 (1936). The proof rests on recent results of G. T. Whyburn (Cut points in general topological spaces, Proc. Nat. Acad. Sci. USA 61 (1968), 380-387). There exists a connected and locally connected, separable Hausdorff space with every point a strong cut point which is not homeomorphic to the real line. The counter-example is constructed from a countable, connected and locally connected Hausdorff space due to F. B. Jones. (Received March 21, 1969.)
Theorem. Suppose that \( M = M^{n-1} \) is a closed two-sided p.l. submanifold of the p.l. manifold \( V = \mathbb{V}^{0} \), where \( M \subset \text{Int} V \) and \( n \geq 5 \). Let \( N \) be the regular neighborhood of \( M \) in \( V \). Then there is a p.l. homeomorphism \( h: N \rightarrow M \times [-1,1] \). From the theorem and its proof we get the Corollary. If \( M \) and \( V \) are as above and if \( \epsilon > 0 \) then there is an ambient \( \epsilon \)-isotopy \( H: V \times I \rightarrow V \times U \) (in the topological category) such that \( H_0 = 1 \) and \( H_1(M) \) is a p.l. bicollied subpolyhedron of \( V \) which is p.l. homeomorphic to \( M \). Remarks. A similar theorem holds, with \( n \geq 5 \), for manifolds with boundary. The reader should note that we do not claim that \( h(M) = M \times 0 \) (i.e., that \( M \) is p.l. bi-collared). (Received March 21, 1969.)

Theorem. \( \pi_3(\text{TOP/PL}) \) is \( Z_2 \), not 0. 1st proof (of many). Let \( f: B^3 \times \mathbb{T}^n \rightarrow W^m \), \( m = n + 3 \geq 5 \), be the 'exotic' homotopy equivalence onto a compact PL manifold \( W^m \) where \( f|_{\partial} (\partial = \text{boundary}) \) is a PL homeo onto \( \partial W^m \), but \( f|_{\partial} \) does not extend to a PL homeo. By Wall's surgery \( f_{m+1} = f \times 1|S^1 \). By the s-cobordism theorem, in the covering \( \tilde{f} : B^3 \times \mathbb{R}^2 \times \mathbb{T}^{n-2} \rightarrow \tilde{W}^m \), \( \tilde{f}|_{\partial} \) does not extend to a PL homeo. By engulfing int \( \tilde{W}^5 \cong \mathbb{R}^5 \subset \mathbb{R}^5 \cup \infty = S^5 \). Make \( \tilde{f}_5 \) a PL homeo near \( \partial \) and choose a small collar \( C \) of \( \partial \) in \( B^3 \) so that \( \tilde{f}_5((C- \partial B^3) \times 0) \cup \infty \) forms a 3-disc \( D \subset S^5 \) locally flat on \( D - \infty \). Near \( \infty \), \( S^5 - D \) has the type of \( S^1 \). Thus \( \text{Černavskiǐ} \), Soviet Math. Dokl. 7 (1966), has shown that \( D \) is (topologically) locally flat, hence unknotted. This implies \( \tilde{f}_5(B^3 \times 0) \subset \tilde{W}^5 \) can be made a locally flat and homotopically unknotted disc \( D' \). If \( \pi_3(\text{TOP/PL}) \) were 0, then \( D' \times S^1 \) could be made PL locally flat in \( \tilde{W}^5 \times S^1 \cong \tilde{W}^6 \) (these \( \text{Notices} \) April 1969), whence \( \tilde{f}_6|_{\partial} \) would extend to a PL homeo, which it does not. So \( \pi_3(\text{TOP/PL}) = Z_2 \). If \( c = (W^5; f|_{\partial} B^3 \times \mathbb{T}^2, f|_{\partial} B^3 \times \mathbb{T}^2) \) be a product cobordism, then an easy proof extends \( \tilde{f}_5|_{\partial} \) to a homeo by an Alexander isotopy. Invertibility of \( c \) and an infinite meshing idea of \( \text{Černavskiǐ} \) make it valid. (Received March 3, 1969.)

Collaring an \((n-1)\)-manifold in an \( n \)-manifold.

Suppose that \( M \) is a closed \( \text{PL} \) \((n-1)\)-manifold, \( N \) is a \( \text{PL} \) \( n \)-manifold, \( M \) is topologically embedded in \( \text{Int} N \), and \( n \geq 5 \). Theorem. If \( M \) is 2-sided in \( N \) and can be pointwise approximated from one side by locally flat embeddings of \( M \), then \( M \) has a collar on that side. Theorem. If \( N - M \) is 1-LC at each point of \( M \) and \( M \) can be approximated pointwise by locally flat embeddings of \( M \), then \( M \) has a tubular neighborhood in \( N \). Furthermore, for any \( \epsilon > 0 \) there is a \( \delta > 0 \) such that for any locally flat embedding \( f: M \rightarrow \text{Int} N \) with \( \delta \) of \( \text{Id}|M \) there is an \( \epsilon \)-push \( H \) of \((N,M)\) such that \( Hf = \text{Id}|M \). (Received March 27, 1969.)
Theorem 1. If \( \Sigma \in \mathcal{R}_n \) is such that \( \Sigma \# M \) is diffeomorphic to \( M \) for some closed \( k \)-connected \( n \)-manifold \( M \), then \( \Sigma \) bounds a \( k \)-connected compact \( (n + 1) \)-manifold \( V \).

Theorem 2. If \( \Sigma \) is in the image of the Milnor-Munkres-Novikov pairing \( \pi_k(\text{SO}(n-k-1)) \otimes \mathcal{R}_{n-k} \rightarrow \mathcal{R}_n \) and \( n \geq 2k + 3 \), there exists a closed \( k \)-connected \( n \)-manifold \( M \) such that \( \Sigma \# M \) is diffeomorphic to \( M \). (Received March 26, 1969.)

69T-G83. JOHN L. BRYANT, Florida State University, Tallahassee, Florida 32306, and C. L. SEEBECK, III, Michigan State University, East Lansing, Michigan 48823. Locally nice embeddings in codimension three.

Suppose \( M \) and \( Q \) are PL \( m \)- and \( q \)-manifolds, respectively, \( q - m \geq 3 \), \( q \geq 5 \), and \( M \) has no boundary. Theorem. If \( M \) is embedded in \( \text{Int} Q \) as a closed set, \( Q - M \) is 1-LC at each point of \( M \), and \( M \) can be approximated by PL embeddings then \( M \) is \( \varepsilon \)-tame. (Received March 27, 1969.)

69T-G84. JACK GIROLO, Iowa State University, Ames, Iowa 50010. A fixed point theorem for connectivity functions.

Let \( X \) be a topological space and \( f : X \rightarrow X \). \( f \) is said to be a connectivity function if for any connected set \( C \subseteq X \), the set \( \{ x, f(x) : x \in C \} \) is connected in \( X \times X \). Theorem. Every connectivity function of the Hilbert cube into itself has a fixed point. This result follows by extending Theorem 4 (Stallings, Fund. Math. 47 (1959), 249-263) to a class of spaces which include the Hilbert cube and analogs of Theorems 4 and 5 (Hamilton, Proc. Amer. Math. Soc. 8 (1957), 750-756). (Received March 31, 1969.) (Author introduced by Professor James L. Cornette.)


Ishii, Tsuda, Kunugi [Proc. Japan. Acad. 44 (1968), 897-903] define a class \( C \) of topological spaces which contains locally compact \( M \)-spaces and first countable \( M \)-spaces. This author has shown that sequential \( M \)-spaces [see Franklin, Fund. Math. 57 (1965), 107-115] and, more generally, spaces which are both \( k \) and \( M \) belong to class \( C \). (Received April 4, 1969.)

69T-G86. OFELIA T. ALAS, Universidade de São Paulo, FFCL-Dep. de Matemática, Brasil, São Paulo, Caixa Postal 8105. Topological groups and uniform continuity. II.

Let \((G, \sigma)\) and \((G, \tau)\) be two nondiscrete Hausdorff groups, \( U \) be the right uniformity of \((G, \tau)\), \( e \) be the neutral element of \( G \) and \( \tau \supset \sigma \). Let \( J \) denote an infinite set such that (1) there is a family, \((M_j)_{j \in J} \), of \( \sigma \)-neighborhoods of \( e \), verifying \( \bigcap \{ M_j : j \in J \} \) is not a \( \tau \)-neighborhood of \( e \); (2) for any family, \((S_t)_{t \in T} \), of \( \sigma \)-neighborhoods of \( e \), with \( |T| < |J| \), the set \( \bigcap \{ S_t : t \in T \} \) is a \( \tau \)-neighborhood of \( e \). Theorem. If \( |J| = \aleph_0 \) and every \( \sigma \)-continuous real-valued function on \( G \) is uniformly continuous on \((G, U)\) into \( R \), then \((G, \sigma)\) is pseudo-compact. Example. If \((G, \sigma)\) is first countable, then \( |J| = \aleph_0 \).
Theorem. In general, if every \( \sigma \)-continuous real-valued function on \( G \) is uniformly continuous of \( (G, U) \) into \( R \), then, for any \( \sigma \)-neighborhood \( V \) of \( e \), the covering \( \{ V \_x \_x \in G \} \) has a subcovering of cardinality less than \( |J| \).

**Theorem.** If \( (G, \sigma) \) is a K-group, then the right and left uniformities of \( (G, \sigma) \) are equal. (See Abstract 69-G33, these *Notices* 16 (1969, 434.) (Received April 7, 1969.)

691-G87. WILLIAM S. MASSEY, Yale University, New Haven, Connecticut 06520. **Proof of a conjecture of Whitney.**

Let \( M \) be a closed connected nonorientable surface of Euler characteristic \( \chi \) which is embedded smoothly in \( S^4 \) with normal bundle \( v \). The Euler class \( e(v) \) is \( m \) times a generator of the cohomology group \( H^2(M) \) (with twisted integer coefficients) for some integer \( m \). In 1940 Whitney conjectured that \( m \) could only take on the following values: \( 2\chi - 4, 2\chi, 2\chi + 4, ..., 4 - 2\chi \) (see *On the topology of differentiable manifolds*, Lectures in Topology, Mich. Univ. Press, 1940, p. 113). This conjecture is proved in this paper by use of a theorem of Atiyah, Singer, and Hirzebruch (The index of elliptic operators, III, Ann. of Math. 87 (1968); cf. Proposition 6.15 on p. 583). In order to apply this theorem, one observes that \( H^1(S^4 - M) = \mathbb{Z}_2 \), hence there exists a unique 2-sheeted covering space of \( S^4 - M \). This covering space can be completed to a branched covering \( p : S' \to S^4 \) with \( M \) as the branch set. \( S' \) has an obvious involution \( T : S' \to S' \) with \( M \) as the fixed point set; the theorem is now applied to the pair \( (S', T) \). (Received April 9, 1969.)


A \( T_1 \) space \( (X, \mathcal{J}) \) is semistratifiable [Michael, Creede, Topology Conference, ASU, 1967] if there is a function, called a semistratification of \( X \), \( S : \{ \text{natural nos.} \} \times \mathcal{J} \to \{ \text{closed subsets of } X \} \) such that if \( U, V \in \mathcal{J} \) with \( U \subseteq V \), then \( S(n, U) \subseteq S(n, V) \) for each \( n \), and \( U = \bigcup \{ S(n, U) \_n \neq 1 \} \).

Creede proved that semimetrizable spaces are precisely the first countable semistratifiable spaces. R. W. Heath [Topology Seminar, Wisconsin, 1965, pp. 103-114] asked when a semimetrizable space is stratifiable in the sense of Borges. One can answer Heath's question as follows: **Theorem.** A semimetrizable \( T_1 \) space \( X \) is stratifiable if and only if there is a semistratification \( S \) of \( X \) such that whenever \( C \subseteq U \subseteq X \) with \( C \) compact and \( U \) open, then \( C \subseteq S(n, U) \) for some \( n \neq 1 \). Such a function \( S \) is called a \( k \)-semistratification of \( X \). For regular spaces, one can prove an analogous theorem for certain classes of nonfirst countable \( k \)-semistratifiable spaces. (Received March 26, 1969.)

69T-G89. R. C. LACHER, Florida State University, Tallahassee, Florida 32306. **Maps which preserve the structure of a space at infinity.**

For each integer \( k \neq 0 \) we define "\( e_k \)-map". An \( e_k \)-map is more general than, but similar to, a proper map of locally compact ANR's whose inverse sets near infinity are nonvoid \( k \)-connected ANR's. An \( e_0 \)-map extends to a map of end-point compactifications which is a homeomorphism on the set of end-points. Stability of \( \pi_1 \) and \( p_1 \) are preserved by \( e_k \)-maps, \( 1 \neq k \neq 1 \), as are the groups. Proper, cell-like maps on locally compact ANR's are \( e_\infty \)-maps, as are homeomorphisms. Two applications are: **Theorem 1.** If \( X \) is a connected, locally connected, compact metric space and if
C_1 and C_2 are compact, 0-dimensional sets of limit points which do not locally separate X such that X - C_1 \approx X - C_2, then (X, C_1) \approx (X, C_2). Theorem 2. If N is a topological n-manifold without boundary, n \neq 4, 5, and if f is a proper, cellular map of N onto the Hausdorff space Y, then the set \{y \in Y | Y is not locally euclidean at y\} contains no isolated points. (Received April 7, 1969.)

69T-G90. WITHDRAWN.

69T-G91. DAVID E. GALEWSKI, Michigan State University, East Lansing, Michigan 48823.
A weak codimension one approximation theorem for spheres.

Theorem. Let f : S^{n-1} \to S^n be a topological embedding of the (n - 1)-sphere in the n-sphere with n \neq 4 and N a neighborhood of f(S^{n-1}) in S^n. Then there exists a P. L. embedding g : S^{n-1} \to S^n such that g(S^{n-1}) \subset N and \{g\} \in \Pi n-1(N). (Received March 21, 1969.)

69T-G92. DOUG W. CURTIS, Louisiana State University, Baton Rouge, Louisiana 70803.
Property Z for function-graphs and finite-dimensional sets in \(l^\infty\) and \(s\).

The Hilbert cube \(l^\infty\) is the countable infinite product of compact intervals, the Fréchet space \(s\) the countable infinite product of lines. A closed subset \(K\) of a space \(X\) has Property Z in \(X\) if for every nonempty, homotopically trivial open set \(U\) in \(X\), \(U \setminus K\) is nonempty and homotopically trivial. (R. D. Anderson, On topological infinite deficiency, Michigan Math. J. 14 (1967), 365-383). Let \(G(f)\) be the graph of a map \(f : l^\infty \to l^\infty\), and let \(p \in l^\infty\). A subset \(K\) of \(l^\infty\) or \(s\) has finite deficiency \(k\) if its projections on at least \(k\) coordinate factors are single points. Theorem. \(G(f)\) has Property Z in \(l^\infty \times l^\infty\). Corollary. There exists a homeomorphism \(H\) of \(l^\infty \times l^\infty\) onto itself such that \(H(x, f(x)) = (x, p)\). Corollary. If a closed subset \(K \subset l^\infty\) has finite deficiency \(k\) and can be imbedded in \(k\)-dimensional Euclidean space, it has Property Z in \(l^\infty\). Theorem. A closed subset \(K \subset s\) with dimension \(k\) and finite deficiency \(2k + 1\) has Property Z in \(s\). (Received April 21, 1969.)

69T-G93. R. A. JENSEN and L. D. LOVELAND, University of Wisconsin, Madison, Wisconsin 53706. Spheres of vertical order 3 are tame.

We define a 2-sphere \(S\) in \(E^3\) to have vertical order \(n\) if each vertical line intersects \(S\) in no more than \(n\) points. Theorem. If \(S\) has vertical order 3, then \(S\) is tame. This theorem resolves a conjecture raised by John Cobb. It is the best theorem possible in the sense that there are wild spheres of vertical order 4. (Received April 25, 1969.)


Topological manifolds must now be studied in their own right. The classification theorem for piecewise linear (= PL) structures on a topological (= TOP) manifold of Kirby and Siebenmann, Bull. Amer. Math. Soc., July 1969, which contains a TOP-PL 'Cairns-Hirsch' theorem, yields readily the following: (1) Every TOP manifold triad \((W; V, V')\) (perhaps noncompact) of dimension \(\geq 6\) admits a (locally finite) TOP handle decomposition on \(V\). TOP handlebody theory works formally like the PL and DIFF theories in dim \(\geq 6\). But in dim 4 or 5 (one or both) there exists a closed manifold.
M with no TOP handle decomposition. M \times S^1 \times \ldots \times S^1 is never homotopy equivalent to a closed PL manifold. (II) A restricted relative TOP microbundle transversality theorem holds. It is like Williamson's PL theorems, Ann. of Math. 83, p. 9, but the source S must be a TOP open manifold, and the expected dimension of zero-section preimage must be \geq 5. (III) Surgery can be done on TOP manifolds of dim \geq 6 (\geq 5 if rel boundary). (IV) \mathcal{G}/TOP \cong \Omega^4(\mathcal{G}/TOP) \cong \Omega^4(\mathcal{G}/PL). In dim \geq 5 many TOP classifications can now be carried as far as for PL. (Received April 25, 1969.)

69T-G95. WILLIAM K. MASSEY, Rutgers University, New Brunswick, New Jersey 08903.

The space H(M) of homeomorphisms of a compact manifold onto itself is homeomorphic to H(M) minus any \sigma-compact set.

Let M denote a compact n-manifold with or without boundary. Let H(M) denote the space of homeomorphisms of M onto M with the sup norm metric. If the boundary of M is nonempty, let H'(M) denote the space of homeomorphisms of M onto M which are the identity on the boundary of M.

A set is \sigma-compact if it is the countable union of compact sets. Theorem 1. If C is a \sigma-compact subset of H(M), then H(M) - C is homeomorphic to H(M).

Theorem 2. If C is a \sigma-compact subset of H'(M), then H'(M) - C is homeomorphic to H'(M). (Received April 25, 1969.)

Miscellaneous Fields


Let E be a Hilbert space and let A: E \to E be compact, linear, symmetric, and positive. Let 0 < \lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_n \leq \ldots denote the reciprocals of the characteristic values of A, each \lambda_k occurring as often as the multiplicity of the corresponding characteristic value. Let g: E \to E have a continuous Fréchet derivative such that g'(x) is symmetric. Theorem. If there exists N and numbers \mu_N, \mu_{N+1} such that \lambda_N < \mu_N \leq \mu_{N+1} < \lambda_{N+1}, and \mu_N \leq g'(x) \leq \mu_{N+1} for all x \in E, then the mapping F = 1 - Ag is bijective and has a continuous inverse. (This partially extends a result of C. Dolph, Trans. Amer. Math. Soc. 66 (1949), 289-307.) Application. Let p \in C(R, R^n), G \in C^2(R^n, R) and H = (3^{2G}/2X_\lambda \lambda_j). If there exists an integer m and numbers \mu_m, \mu_{m+1} such that \mu_m < \mu_{m+1} < (m + 1)2, \mu_m \leq H(x) \leq \mu_{m+1}, x \in R^n, then the boundary value problem x'' + grad G(x) = p(t), x(0) = a, x(1) = b, a, b \in R^n arbitrary, has a unique solution. (Received November 21, 1968.)


Using techniques developed by Edmonds, Ringel, and Youngs, one can show that the genus of the cartesian product of K_{2m,2m} with itself is 1 + 8m^2(m - 1). Furthermore, if G_1 and G_2 are the complete bipartite graphs K_{r,s} and K_{2m,2m} respectively, where r \neq 2m and s \neq 2m, a formula is developed giving the genus of the cartesian product G_1 \times G_2 as a function of r, s, and m. As a further extension, let Q^{(n)}_1 be the graph K_{s,s} and recursively define Q^{(n)}_m = Q^{(n-1)}_m \times K_{s,s}, when n \geq 2. Then the genus of Q^{(n)}_n is 1 + 2^{n-3}s(n - 4), for all n, and s even; or when n > 1, and s = 1 or 3. The two
special cases \( n = 1, s = 2m \) and \( s = 1 \) give the genus of \( K_{2m,2m} \) and the \( n \)-cube \( Q_n \) respectively. (Received February 20, 1969.)

On a conjecture of Marshall Hall.

Let \( U(n;k) \) denote the class of all \( n \times n \) binary matrices with precisely \( k \) ones in each row and column. In respect of the van der Waerden conjecture that if \( A \in U(n;k) \) then per \( A \leq k^n (n! / n^n) \), Marshall Hall posed the question of whether one could prove that if \( k \geq 3 \) then
\[
\lim_{n \to \infty} \min_{A \in U(n;k)} \text{per } A = \infty. \tag{1}
\]
(This would be a consequence of the validity of the van der Waerden conjecture.) The following theorem, which answers the question in the affirmative, is proven inductively using combinatorial methods and the Fr"obenius-K"onig theorem to demonstrate the presence of required number of different permutations: Theorem. If \( A \in U(n;k) \) with \( k \geq 3 \), then per \( A \geq (k - 2)n + k \). (Received January 21, 1969.)

A generalization of Sperner's theorem on finite sets.

Following Sperner [Math. Z. 27 (1928), 544-548] call a set \( \sigma \) of subsets of a finite set \( M \) distinguished iff no set in \( \sigma \) is a subset of another set in \( \sigma \). Let \( M \) be a set of \( k_1 + k_2 + \ldots + k_n = \theta \) (billiard) balls with \( k_i \) balls of color \( i \), \( i = 1, 2, \ldots, n \), where \( k_1 \leq k_2 \leq \ldots \leq k_n \). Let \( \Sigma \) be a distinguished set of subsets of \( M \) with \( |\Sigma| \) maximal, and let \( \Sigma^j \) be the set of \( j \)-element subsets of \( M \), \( j = 0, 1, \ldots, \theta \).

Theorem. If \( \theta \) is even and \( 2k_n \neq \theta \), then \( \Sigma = \Sigma^{\theta/2} \). If \( \theta \) is odd and \( 2k_n \neq \theta \), then \( \Sigma = \Sigma^{(\theta+1)/2} \) or \( \Sigma^{(\theta-1)/2} \). If \( 2k_n > \theta \), then \( |\Sigma| = |\Sigma^{\theta/2}| \) and \( \Sigma \) can be realized in various ways. The special case of this theorem corresponding to \( k_1 = k_2 = \ldots = k_n = 1 \) is Sperner's theorem [op.cit.]. The proof involves the generalization of Macaulay's theorem due to the author and B. Lindstrom [Abstract 663-208, these Notices 16 (1969). 171]. (Received February 17, 1969.)

69T-H32. DON R. LICK, Western Michigan University, Kalamazoo, Michigan 49001.
Minimally \( n \)-line-connected graphs.

The line-connectivity \( \lambda(G) \) of a graph \( G \) is the minimum number of lines whose removal disconnects it. The graph \( G \) is said to be \( n \)-line-connected if \( \lambda(G) \geq n \). The graph \( G \) is said to be minimally \( n \)-line-connected if \( \lambda(G) = n \) and \( \lambda(G - x) = n - 1 \) for each line \( x \) of \( G \). Let \( \mu(G) \) denote the minimum degree of \( G \). The inequality \( \lambda(G) \geq \mu(G) \) is well known. The author has proved the following theorem. Theorem. There exists no minimally \( n \)-line-connected graph \( G \) with \( \mu(G) > n \). (Received March 10, 1969.)

69T-H33. GARY T. CHARTRAND and D. R. LICK, Western Michigan University, Kalamazoo, Michigan 49001, and AMES KAUGARS, Harvard University, Cambridge, Massachusetts 02138.
Critically \( n \)-connected graphs.

A graph \( G \) is said to be \( n \)-connected if the removal of fewer than \( n \) points neither disconnects \( G \) nor reduces it to the trivial graph consisting of a single point. The maximum value of \( n \) for which a graph \( G \) is \( n \)-connected is called its connectivity and is denoted by \( \chi(G) \). A graph \( G \) is said to be...
critically n-connected if \( \kappa(G) = n \) and \( \kappa(G - v) = n - 1 \) for each point \( v \) of \( G \). Let \( \mu(G) \) denote the minimum degree of \( G \). The inequality \( \kappa(G) \geq \mu(G) \) is well known. The authors have proved the following theorem. Theorem. There exists no critically n-connected graph with \( \mu(G) \geq (3n - 1)/2 \).

The authors have constructed examples to show that this inequality is best possible. (Received March 10, 1969.)

69T-H34. WALTER ALLEGRETTO, University of British Columbia, Vancouver 8, B.C., Canada. Comparison and oscillation theorems for elliptic operators. Preliminary report.

Let \( \lambda_0 \) denote the smallest real eigenvalue in a smooth bounded domain \( G \) of the uniformly elliptic operator \( L \), defined by \( Lu = - \sum_{i,j} a_{ij} D_{ij} u + \sum b_{ij} D_j u + cu \), with sufficiently smooth coefficients and vanishing boundary conditions. Let \( B = (b_{ij}/2, ..., b_{ij}/2) \), \( h = - \sum b_1 b_{ij}/2 \det (a_{ij}) \) where \( b_{ij} \) denotes the cofactor of \( b_{ij}/2 \) in the matrix \( (a_{ij}) \). Theorem 1. Let \( \mu \) denote the smallest eigenvalue of \( [(L + L^*)/2] + h \) in a smooth bounded domain \( D \) such that \( D \subset G \), then \( \lambda_0 < \mu \).

Here \( L^* \) denotes the formal adjoint of \( L \). Theorem 2. \( \lambda_0 \geq \mu_1 \) where \( \mu_1 \) denotes the smallest eigenvalue of \( (L + L^*)/2 \) for \( G \). A sufficient condition for \( \lambda_0 > \mu_1 \) is that \( \sum_{i,j} a_{ij} \) never vanish in \( G \). The monotonicity and continuity of \( \lambda_0 \) as a function of the domain are considered. Several strong oscillation theorems are an immediate consequence of the above results. (Received March 31, 1969.)

69T-H35. DONALD E. SARASON, University of California, Berkeley, California 94720. On prime ends and local connectivity.

Marie Torhorst ["Über den Rand der einfach zusammenhängenden ebenen Gebiete", Math. Z. 9 (1921), 44-65] has proved the following Theorem. If \( G \) is a simply connected domain in the plane and \( P \) is a prime end of \( G \), then \( \partial G \) fails to be locally connected at all except possibly one or two points of \( I(P) \) (the impression of \( P \)). These authors showed that the impression of a prime end breaks up into two "wings," and that there is a relation of "priority" among the points in either wing. The new proof establishes that any point of \( I(P) \) at which \( \partial G \) is locally connected is a point of lowest priority. (Received March 26, 1969.)


Definitions. For full AFL's \( \Sigma_1, \Sigma_2 \), let \( \Sigma_1 \supset \Sigma_2 \) be the family of all \( \tau(L), L \in \Sigma_1 \) in \( \Sigma_1 \) and \( \tau \) a substitution with \( \tau(a) \) in \( \Sigma_2 \) for \( a \) in \( \Sigma_1 \). Let \( \hat{\tau}(\Sigma) \) be the substitution closure of \( \Sigma \). Let \( \tau^{\Sigma_1}(a) = aL \) for \( a \) in \( \Sigma \). Let \( \rho \) be the regular sets, and \( C \) the context-free. Lemma. Let \( L_1 \in \Sigma_1 \), \( L_2 \in \Sigma_2 \), \( \Sigma_1 \cap \Sigma_2 = \emptyset \). Let \( \Sigma_1 \) and \( \Sigma_2 \) be full AFL's. (1) If \( \tau^{\Sigma_1}(L_1) \in \Sigma_1 \supset \Sigma_2 \), either \( L_1 \in \Sigma_1 \) or \( L_2 \in \Sigma_2 \). (2) If \( \tau^{\Sigma_1}(L_1) \in \rho \supset (\Sigma_1 \cup \Sigma_2) \), either \( L_1 \) is in \( \Sigma_1 \) or \( L_2 \) is in \( \Sigma_2 \). Theorem. If \( \Sigma \) is a nonsubstitution closed full AFL, so is \( \hat{\tau}(\Sigma) \supset \Sigma \), and \( \hat{\tau}(\Sigma) \) is not full principal. Theorem. If \( \Sigma_1 \) and \( \Sigma_2 \) are incomparable full AFL's, then \( \hat{\tau}(\Sigma_1 \cup \Sigma_2) \) is not substitution closed and \( \Sigma_1 \supset \Sigma_2 \) and \( \Sigma_2 \supset \Sigma_1 \) are incomparable and \( \hat{\tau}(\Sigma_1 \cup \Sigma_2) \) is not full principal. Corollary. If \( \Sigma \subset C \) is a nonsubstitution closed full AFL, \( \hat{\tau}(\Sigma) \not\subset C \). Corollary. If \( Q \) is a family of languages with \( \#(Q) = C \), then \( C \supset \hat{\tau}(L) \) for
some $L$ in $Q$. **Corollary.** For every $n$, there is a class of full context-free AFL's whose partial ordering under inclusion is isomorphic to the natural partial ordering of $n$-tuples of positive integers. (Received March 3, 1969.)

69T-H37. WALTER S. BRAINERD, Naval Postgraduate School, Monterey, California 93940. Parenthesis systems.

Let $A$ be a finite alphabet and $B = A \cup \{(, )\}$. $a \in B^*$ is balanced iff $a \in A$ or $a = (a_1, \ldots, a_n)$, $a_i$ balanced, $1 \leq i \leq n$. A semi-Thue system, $(B, \mathcal{P}, \Gamma)$, is a parenthesis system iff (1) for each production $\phi \rightarrow \psi$ in $\mathcal{P}$, $\phi$ and $\psi$ are balanced, and (2) each axiom in $\Gamma$ is balanced. These systems are generalizations of McNaughton's parenthesis grammars, which are context-free parenthesis systems.

**Theorem.** Given any parenthesis system, an equivalent context-free parenthesis grammar can be effectively constructed. Thus, each parenthesis system generates only a context-free set. This increases the class of grammars (but not the class of sets) which have all of the properties of finite state (type 3) languages. (E.g., the sets generated from a Boolean algebra and the natural problems concerning them are all solvable.) (Received April 18, 1969.)

69T-H38. ROBERT JAMES DOUGLAS, Statistics Department, University of North Carolina, Chapel Hill, North Carolina 27514. Tournaments that admit exactly one hamiltonian circuit.

A tournament is a directed graph where between each two vertices $v$ and $w$ either the ordered pair $(v, w)$ or $(w, v)$ is an edge, but not both. Those tournaments that admit exactly one Hamiltonian circuit are characterized, and it is shown that the number of nonisomorphic tournaments, with $n \geq 5$ vertices, that admit a unique Hamiltonian circuit is $1 + \sum_{k=1}^{n-3} \sum_{p=0}^{\min(n-1, n-k-3)} 2^{n-k-p-4} \cdot \left[ \binom{n-k-3}{p} + \binom{n-k-4}{p+1} \right]$. (Received April 24, 1969.)

**Errata - Volume 16**


Line 7: Replace "$\int_{t_0}^{t_{\infty}} |A_j(t)| dt < \infty$" by "$\int_{t_0}^{t_{\infty}} |A_j'(t)| dt < \infty$".

L. TAYLOR OLLMAN, Abstract 664-7 Intermediate theories with respect to Keisler's ultraproduct ordering, Page 501, has been withdrawn due to an error in one of the key proofs.
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University of Oregon
Eugene, Oregon 97403
August 25-29, 1969

Please return this form no later than July 31, 1969, to:

Mr. Kenneth R. Thomas
Division of Continuing Education
University of Oregon
Eugene, Oregon 97403

Rates for dormitory rooms:

University Residence Halls and Meal Service
(Carson, Earl, Hamilton, and Walton):
(Meals included)

Single room: $11/day/person
Double room: $11/day/person
(Children 2-11 half price; under 2, no charge)

The College Inn, 1000 Patterson Street:
(Meals not included)

Single room: $8.50/day/person
[Large] Single room: $9.50/day/person
Double room: $7.00/day/person
Triple or Quadruple room: $6.00/day/person
(Children 2-11 half price; under 2, no charge)

Reservations for dormitory housing will be confirmed by the individual dormitories. The payment for rooms at the dormitories (including The College Inn) must be made at check-in-time. For those wishing hotel or motel housing, a list of accommodations with rates appears in this issue of the Notice on page 64.

NAME (please print, last name first) ___________________________
MAILING ADDRESS (No. & Street) ___________________________
(City & State) ________________________ Zip Code _______
Summer address if different from above ___________________________
Arrival date and time ____________; Departure date and time ___________
Accompanied by: wife/husband (give name) children (give names and ages)

First and second preferences for accommodations are indicated below:

University Residence Halls
The College Inn

I wish to share a double room with another person; name and address of preferred roommate:
Name ___________________ Address ___________________

I will require dormitory reservations
Single (s) ___________________
Double (s) ___________________

PLEASE CHECK IN PROPER SPACES BELOW (Do not send payment.):
☐ I plan to attend the picnic on August 27.
☐ I plan to attend the SIAM Beer Party on August 27.
☐ I plan to attend the reception on August 26.
☐ I plan to take the tour on August 28.
The Bulletin of the
London Mathematical Society

Editor: J. E. Reeve

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The Bulletin will include a section devoted to shorter research papers of unusual interest or importance, and it is hoped that speedy publication can be maintained for this section. Contributions, which should not normally exceed five pages in print, should be sent to the Editorial Adviser whose mathematical interests are closest to those of the paper submitted.

The Journal of the London Mathematical Society (Series 2) contains research papers requiring not more than about 10 printed pages. Contributions to the Journal should be sent to Professor G. E. H. Reuter, Imperial College of Science and Technology, London, S.W.7. Papers requiring more than about 10 printed pages are published in the Proceedings of the London Mathematical Society (Series 3). Contributions intended for the Proceedings should be sent to Professor S. J. Taylor, Westfield College, London, N.W.3. Papers submitted to the Journal or Proceedings may be accompanied by abstracts (not normally more than 100 words in length for Journal papers; not normally more than 200 words in length for Proceedings papers) summarising their contents. Abstracts of recently accepted papers are published in the Bulletin.

The Society reserves the right to publish papers submitted for the Journal or Proceedings in either of these periodicals. It is possible that some papers submitted for the Bulletin will be considered more suitable for the Journal, but they will not be transferred without the author’s consent.

Nomination forms for Membership and Associate Membership of the Society can be obtained from Dr. D. E. Cohen, Queen Mary College, Mile End Road, London, E.1.
Three issues of the *Bulletin* will be published each year (March, July, November). Subscriptions of £6 are payable annually in advance on 1st January to the publisher, C. F. Hodgson and Son Ltd., Pakenham Street, London, W.C.1.

Members of the Society will receive the *Bulletin* and one other of the Society's publications free of charge. The Society has reciprocity agreements with the American Mathematical Society and with similar societies in several other countries. Information concerning these agreements and other activities of the Society, and nomination forms for Membership and Associate Membership, can be obtained from Dr. D. E. Cohen, Queen Mary College, Mile End Road, London, E.1.
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CRITICAL POINT THEORY IN GLOBAL ANALYSIS AND DIFFERENTIAL TOPOLOGY

by MARSTON MORSE, Institute for Advanced Studies, Princeton, New Jersey
and STEWART S. CAIRNS, Department of Mathematics, University of Illinois, Urbana, Illinois

This monograph is an introduction to a mathematical theory that has revolutionized the methods and aims of global analysis and differential topology and has opened up new possibilities in mathematical physics and engineering. It discusses the recent work of Bott on homotopy equivalence, Smale on the Poincare problem, and Milnor and Thom on cobordism, and provides a finite-dimensional introduction to Morse's critical point theory on differentiable manifold without any global triangulation of the manifold. Contents: ANALYSIS OF NON-DEGENERATE FUNCTIONS; ABSTRACT DIFFERENTIABLE MANIFOLDS; SINGULAR HOMOLOGY THEORY; and OTHER APPLICATIONS OF CRITICAL POINT THEORY.

LINEAR LIE GROUPS

by HANS FREUDENTHAL, Mathematisch Instituut der Rijksuniversiteit te Utrecht, Holland
and H. DE VRIES, Katholieke Universiteit te Nijmegen, Holland

This book treats the structure of Lie groups including the complex and real classification of semisimple groups, their maximal solvable and non-semisimple maximal proper subgroups, and their finite-dimensional linear representations. Also discussed are the fundamental groups, wrappings and automorphisms, symmetric spaces, Weyl's Character and Dimension Formula, Invariant Bilinear and Sesquilinear Forms—Unitary Representations, Minimally Compact Dressing, and Tit's geometries. By using a progressive approach, the authors lead the reader from simple to more complex situations, and present a self-contained work. August 1969, about 590 pp.

MATHEMATICAL METHODS OF RELIABILITY THEORY

by B. V. GNEDENKO, YU. K. BELAYEV, and A. D. SOLOVYEV
all at the Division of Technology and Mathematics
Department of Probability Theory
Moscow State University, Moscow, USSR
1969, 506 pp., $24.50

PICTURE PROCESSING BY COMPUTER

by AZRIEL ROSENFELD
Computer Science Center, University of Maryland
College Park, Maryland
1969, about 150 pp.

STOCHASTIC INTEGRALS

by H. P. MCKEAN, JR.
Rockefeller University, New York, New York
1969, 140 pp., $9.00

COMPUTING METHODS IN OPTIMIZATION PROBLEMS II

by LOTFI ZADEH
University of California, Berkeley, California
LUCIEN W. NEUSTADT
Univ. of Southern California, Los Angeles, Calif.
and A. V. BALAKRISHNAN
University of California, Los Angeles, California
1969, in preparation