

## OF THE

## AMERICAN

## MATHEMATICAL

SOCIETY


## OF THE

## AMERICAN MATHEMATICAL SOCIETY

## Edited by Everett Pitcher and Gordon L. Walker

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## MEETINGS

## Calendar of Meetings

NOTE: This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the cNotices) was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

| Meeting No. | Date | Place | Deadline for Abstracts* |
| :---: | :---: | :---: | :---: |
| 668 | October 25, 1969 | Cambridge, Massachusetts | Sept. 9, 1969 |
| 669 | November 21-22, 1969 | Baton Rouge, Louisiana | Oct. 8, 1969 |
| 670 | November 22, 1969 | Claremont, California | Oct. 8, 1969 |
| 671 | November 29, 1969 | Ann Arbor, Michigan | Oct. 8, 1969 |
| 672 | January 22-26, 1970 | Miami, Florida | Nov. 6, 1969 |
| 673 | March 25-28, 1970 | New York, New York | Jan. 28, 1970 |
| 674 | April 14-18, 1970 | Madison, Wisconsin | Feb. 27, 1970 |
| 675 | April 25, 1970 | Davis, California | Feb. 27, 1970 |
|  | August 24-28, 1970 <br> (75th Summer Meeting) | Laramie, Wyoming |  |
|  | January 21-25, 1971 <br> (77th Annual Meeting) | Atlantic City, New Jersey |  |

*The abstracts of papers to be presented in person at the meetings must be received in the Headquarters Offices of the Society in Providence, Rhode Island, on or before these deadlines. The deadlines also apply to news items. The next deadline date for by-title abstracts will be October 30, 1969.

## OTHER EVENTS



The $\mathcal{C}$ Notices of the American Mathematical Society is published by the Society in January, February, April, June, August, October, November and December. Price per annual volume is $\$ 10.00$. Price per copy $\$ 3.00$. Special price for copies sold at registration desks of meetings of the Society, $\$ 1.00$ per copy. Subscriptions, orders for back numbers (back issues of the last two years only are available) and inquiries should be addressed to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904. Second-class postage paid at Providence, Rhode Island, and additional mailing offices.

# The Six Hundred Sixty-Eighth Meeting Massachusetts Institute of Technology Cambridge, Massachusetts October 25, 1969 

The six hundred sixty-eighth meeting of the American Mathematical Society will be held at the Massachusetts Institute of Technology on Saturday, October 25, 1969.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, Professor Hyman Bass of Columbia University will give an address entitled " $K_{2}$ of global fields" at 11:00 a.m., and Professor John T. Tate of Harvard University will give an address entitled " $K_{2}$ of global fields" at 2:00 p.m. Both lectures will be presented in the Compton Auditorium, Room 26-100.

There will be sessions for contributed papers at 9:30 a.m. and 3:15 p.m. in Rooms 2-190 and 2-390.

Registration will be on the first
floor of Building 2. It will open at 9:00 a.m.

Parking space will be available in the East Parking Garage on the M.I.T. grounds, at the corner of Main and Vassar Streets.
M.I.T. is an eight-minute walk from the Kendall Square station of the Cam-bridge-Dorchester subway; the convenient entrance is then at the northwest corner of the Hayden Memorial Library. Those coming by taxi or bus will find it convenient to use the main entrance, 77 Massachusetts Avenue. Taxi fare from Logan Airport is about $\$ 3.50$.

Lunch will be served in an M.I.T. cafeteria and the Student Center cafeteria. A list of nearby restaurants in Boston and Cambridge will be available.

PROGRAM OF THE SESSIONS

The time limit for each contributed paper is 10 minutes. The contributed papers are scheduled at 15 minute intervals. To maintain this schedule, the time limit will be strictly enforced.

> SATURDAY, 9:30 A.M.

Session on Analysis I, Room 2-190

## 9:30-9:40

(1) An estimate for general pseudo-differential operators

Professor Samuel Zaidman, Université de Montréal (668-3)
9:45-9:55
(2) A nonspectral generator of a bounded group

Professor Matthew Hackman, University of Washington (668-21)
10:00-10:10
(3) Functional-analysis identities for biadditive mappings on modules with nonassociative scalars

Professor Anthony J. Penico, University of Missouri, Rolla (668-19)
10:15-10:25
(4) Some characterizations of strictly convex Banach spaces. Preliminary report Professor Ellen Torrance, Mount Holyoke College (668-2)
15) WITHDRAWN.

Session on Algebra, Room 2-390
9:30-9:40
(6) An existence theory for pairwise balanced designs

Professor Richard M. Wilson, Ohio State University (668-10)
(Introduced by Professor D. K. Ray-Chaudhuri)
9:45-9:55
(7) Small regular local Noether lattices

Professor Kenneth P. Bogart, Dartmouth College (668-5)
10:00-10:10
(8) Structure of incidence algebras and their automorphism groups

Mr. Richard P. Stanley, Harvard University (668-8)
10:15-10:25
(9) Topologies on objects of categories

Professor Hayon Kim*, Loyola College, and Professor Basil A. Rattray, McGill University (668-9)
10:30-10:40
(10) An extension of classical Galois theory to inseparable fields Professor Nickolas Heerema, Florida State University (668-11)

> SATURDAY 3:15 P.M.

Session on Analysis II, Room 2-190
3:15-3:25
(11) On the Sheffer A-type of certain modified polynomial sets. Preliminary report Professor James Ward Brown, University of Michigan (668-12) 3:30-3:40
(12) Utilizing additive functionals to define stopping times to obtain preassigned distributions

Dr. Itrel E. Monroe, Dartmouth College (668-20)
3:45-3:55
(13) Borel vindicated

Professor Leon W. Cohen, University of Maryland (668-6) 4:00-4:10
(14) Semigroup theory and finite time stability for a class of partial differential equations

Professor Dahsoong Yu, University of Oklahoma (668-14) 4:15-4:25
(15) A constructive method for the solution of the stability problem

Professor James L. Howland*, University of Ottawa, and Professor John
A. Senez, Sir George Williams University (668-13) 4:30-4:40
(16) The stationary observer and the Klein-Gordon equation

Dr. Robert S. Strichartz, Cornell University (668-18)
SATURDAY, 3:15 P.M.
Session on Logic and Topology, Room 2-390
3:15-3:25
(17) Systems of ramified set theory using constructive ordinals

Mr. James R. Royse, San Francisco State College (668-17) 3:30-3:40
(18) Two hierarchies of reducibilities. Preliminary report

Mr. Alan L. Selman, Pennsylvania State University (668-15)
*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.
(19) Boolean algebras as functors

Mr. André Joyal, Université de Montréal (668-7)
(Introduced by Professor Gonzalo E. Reyes)

## 4:00-4:10

(20) Generalized reals

Professor F. Gonzalez Asenjo, University of Pittsburgh (668-4)
4:15-4:25
(21) Symmetric generalized topological structures

Dr. Charles J. Mozzochi, 18 Tuxis Road, Madison, Connecticut (668-1)
4:30-4:40
(22) Absolute reducibility of maps of at most 19 regions

Professor Ruth A. Bari, George Washington University (668-22)

Austin, Texas
Leonard Gillman
Associate Secretary

# NEWS ITEMS AND ANNOUNCEMENTS 

OPERATIONS RESEARCH<br>LECTURESHIPS

The Operations Research Society of America is continuing its Visiting Lectureship Program for the 1969-1970 academic year with sponsorship from the Na tional Science Foundation. The program has the following aims: to stimulate existing operations research courses and programs in universities and colleges which do not at present offer degrees in operations research; to aid and motivate the starting of new operations research courses and curricula; to motivate able students towards careers in operations research; to provide education and information on the nature and scope of operations research; and to provide information and advice to students, studentcounselors, and faculty members on the availability of advanced training in operations research.

Lecturers will not only present formal lectures, but will be available to discuss teaching problems and curricula matters with members of the staff and to advise students on future opportunities in study and employment. The lecturers are national leaders in operations research in universities, industry, and government.

Further information may be obtained by writing to Professor J. R. Borsting, Visiting Lectureship Series in Operations Research, Department of Operations Analysis, Naval Postgraduate School, Monterey, California 93940.

SPECIAL YEAR<br>IN FUNCTIONAL ANALYSIS

The following people will take part in the Special Year in Functional Analysis being held at Indiana University during its Sesquicentennial celebration, 1969-1970: September 15-21, S. K. Berberian; September 22-26, Peter Rosenthal and Charles Berger; September 29-October 4, Donald Sarason; October 6-10, Lewis Coburn; October 13-17, Henry Helson and Allen Shields; October 20-26, Tosio Kato (tentative); October 27-November 2, R. G. Douglas (tentative) and Harold Widom; Novem ber 3-9, Peter Lax and Shizuo Kakutani; November 10-16, Chandler Davis and James G. Glimm; November 17-23, Kenneth Hoffman (tentative); December l-6, William Donoghue, Jr. and Gunter Lumer (tentative); December 8-14, Arnold Lebow; December 15-21, Nachman Aronszajn and I. L. Glicksberg; January 5-11, William Arveson and Calvin Putnam; January l218, Carl Pearcy and Lawrence Wallen; February 2-8, Gerhard Kalisch and Oscar Lanford; February 9-15, Charles Rickart; February 16-22, R alph Phillips and Morris Scheiber; February 23-27, G. W. Mackey and Irving Segal (tentative). Most speakers will stay for a full week and will deliver lectures in the Sesquicentennial Seminar on Tuesday and Thursday afternoons. During the second semester, it is planned to have Professors Ciprian Foias, B. Sz.-Nagy, and M. A. Naimark in residence.

# PRELIMINARY ANNOUNCEMENTS OF MEETINGS <br> The Six Hundred Sixty-Ninth Meeting Louisiana State University Baton Rouge, Louisiana November 21-22, 1969 

The six hundred and sixty-ninth meeting of the American Mathematical Society will be held at Louisiana State University at Baton Rouge, Louisiana, November 21-22, 1969.

By invitation of the Committee to Select Hour Speakers, Professor Marvin Rosenblum of the University of Virginia, Professor Swarupchand M. Shah of the University of Kentucky, and Professor Nickolas Heerema of Florida State University will present hour talks. The title of Professor Shah's talk will be "Univalent functions with univalent derivatives"; the title of Professor Heerema's talk will be "Higher derivations and automorphisms of complete local rings"; the title of Professor Rosenblum's talk will be "Shifts and Hilbert space factorization problems."

Abstracts of contributed papers should be sent to the American Mathematical Society, Providence, Rhode Island, so as to arrive prior to the deadline date of October 8, 1969.

The registration desk will be located in the basement of the Mathematics Building, Lockett Hall, where all sessions will be held. Registration hours will be 9:00 a.m. to 5:00 p.m. Friday, November 21, and 9:00 a.m. to 12 noon Saturday, November 22.

Baton Rouge, which is approximately 75 miles northwest of New Orleans, is located on U.S. 61 and U.S. 190. It is served by Delta, Southern, and TransTexas Airlines and by Greyhound and Trailways Bus Companies. Passenger train service is extremely limited. From the New Orleans Airport to the Baton Rouge campus is a drive of less than an hour and a half. Several persons might wish to rent a car jointly at the airport and drive down.

Meals and snacks will be available at campus cafeterias and off-campus es-
tablishments. Coffee and doughnuts will be served each morning in the basement of Lockett Hall. A beer party at the Capitol House Hotel is planned for Friday evening. Tickets for this may be purchased at the time of registration.

Pleasant Hall is a university-owned hotel located on campus within a fiveminute walk of Lockett Hall. The accommodations are adequate but not luxurious. There are about 95 rooms with private baths and 45 rooms with hall baths available. Rates are as follows:

| With | v |  | \$ 7.00 | 1 person |
| :---: | :---: | :---: | :---: | :---: |
| " | " | " | 10.00 | 2 persons |
| " | " | ' | 12.00 | 3 persons |
| With | 1 |  | 4.00 | 1 person |
| " | " | ' | 7.00 | 2 persons |
| " | " |  | 9.00 | 3 persons |

Make reservations at Pleasant Hall Reservatıon Desk, Louisiana State University, Baton Rouge, Louisiana 70803.

Additional accommodations, convenient to the University, with approximate driving times to the university are given below:

JACK TAR CAPITOL HOUSE HOTEL
Lafayette at Convention
Baton Rouge, La. 70821
Single $\quad \$ 9.00$ to $\$ 15.00$
Double $\quad 12.00$ to 19.00 ( 2 persons, 1 bed)
Twin $\quad 14.00$ to 19.00 ( 2 persons, 2 beds)
PRINCE MURAT INN (5 minutes)
1480 Nicholson Drive
Baton Rouge, La. 70821
Single $\quad \$ 10.00$ up
Double $\quad 14.00$ up (two beds)
The following motels might not have accommodations available for Saturday night because of a home football game, but they do expect to have facilities available for Friday night.

BATON ROUGE TRAVELODGE MOTEL ( 10 minutes)
427 Lafayette Street
Baton Rouge, La. 70821

| Single | $\$ 11.50$ |
| :--- | :--- |
| Double | $13.50(2$ persons, 1 bed) |

HOLIDAY INN - SOUTH (15 minutes)
(Intersection of $\mathrm{I}-12$ and U.S. 61)
9940 Airline Highway
Baton Rouge, La. 70821

Single $\quad \$ 1.00$
Double $\quad 15.00$ ( 2 double beds) $\$ 2.00$ each additional person

Reservations should be made directly with Pleasant Hall, the hotel, or one of the motels. It is suggested that reservations be made as early as is practical.

O. G. Harrold<br>Associate Secretary

Tallahassee, Florida

# The Six Hundred Seventieth Meeting Claremont Graduate School Claremont, California November 22, 1969 

The six hundred seventieth meeting of the American Mathematical Society will be held at the Claremont Graduate School, Claremont, California, on November 22, 1969.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, there will be two hour addresses at this meeting. Professor Solomon Feferman of Stanford University will lecture at 1:00 a.m. on Saturday. The title of his talk is "Systems of ordinal functions and functionals." Professor R.D. Richtmyer of the University of Colorado will address the Society at 2:00 p.m. on Saturday. His lecture is entitled " $L^{2}$ spaces of distributions." There will be sessions for contributed papers on Saturday morning and afternoon. All sessions of the meeting will be held in Bauer Hall. The deadline for abstracts of contributed papers is October 8, 1969. However, late papers will be accepted for presentation at this meeting.

Registration for the meeting will begin at 8:30 a.m. on Saturday. The Registration desk will be located in Bauer Center at 9 th Street and Mills Avenue.

The list of motels near the Claremont Graduate School includes the following.

GRISWOLD MOTEL

$$
\begin{array}{cr}
555 \text { W. Foothill, Claremont } \\
\text { Single } & \$ 12.00 \text { up } \\
\text { Double } & 14.00 \mathrm{up}
\end{array}
$$

HOWARD JOHNSON'S MOTOR HOTEL 721 S. Indian Hill, Claremont

| Single | $\$ 10.50$ up |
| :--- | ---: |
| Double | 13.65 up |

UPLANDER MOTEL
Foothill and Euclid, Upland

| Single | $\$ 12.00 \mathrm{up}$ |
| :--- | ---: |
| Double | 16.00 up |

BONITA MOTEL 151 E. Bonita, Pomona

| Single | $\$ 8.50 \mathrm{up}$ |
| :--- | ---: |
| Double | 10.50 up |

TWILIGHT MOTEL
433 E. Foothill, Pomona
Single
\$ 6.30 up
Double
8.40 up

The first two motels listed above are a long walk from the Claremont Graduate School. Persons who stay in the other motels will need auxiliary transportation. All reservations should be sent directly to the preferred motel.

The Ontario Airport, which is about ten miles from Claremont, is serviced by direct flights from most west coast cities. There is also helicopter service from the Los Angeles International Airport to Pomona.

# The Six Hundred Seventy-First Meeting University of Michigan <br> Ann Arbor, Michigan November 29, 1969 


#### Abstract

The six hundred seventy-first meeting of the American Mathematical Society will be held at the University of Michigan, Ann Arbor, Michigan, on Saturday, November 29, 1969. All sessions of the meeting will be held in the Auditorium Unit of James B. Angell Hall. Angell Hall is a large building on the east side of State Street; the Auditorium Unit is on the ground floor on the far side from the street.


By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings there will be two one-hour addresses. Dr. Alan Baker of Cambridge University and the Universities of Michigan and Colorado will address the Society at 11:00 a.m. His subject will be "A survey of recent results in theory of diophantine equations." Professor Avner Friedman of Northwestern University will speak at l:45 p.m. on the topic, "Free boundary problems for parabolic equations."

By invitation of the same committee there will be two special sessions of selected twenty-minute papers, each of which will meet both morning and afternoon. Professor Donald J. Lewis of the University of Michigan is arranging one session on the subject of Number Theory; the list of speakers will include James B. Ax, Peter E. Blanksby, Harold G. Diamond, William Ellison, Takashi Ono, Carl R. Riehm, Wolfgang M. Schmidt, J. Roderick Smart, Harold M. Stark, and one or two others. The other special session is being arranged by Professor James B. Serrin of the University of Minnesota on the subject of Partial Differential Equations; the list of speakers will include Felix E. Browder, Todd Dupont, Jr., Eugene B. Fabes, Robert Finn, Keith Miller, Johannes C. C. Nitsche, Ralph S. Phillips, Joel A. Smoller, and one or two others.

There will be sessions for the presentation of contributed ten-minute papers both morning and afternoon. Those having time preferences for the presentation of their papers should so indicate on their abstracts. Abstracts should be submitted to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904, so as to arrive prior to the deadline of October 8, 1969. There will be a session for late papers if one is needed.

## REGISTRATION

The registration desk will be located in the lobby connecting Mason Hall, Haven Hall, and the Auditorium Unit of Angell Hall. This lobby faces the main University library and is known locally as "The Goldfish Bowl." The desk will be open from 9:00 a.m. to 4:00 p.m. on Friday, November 28, and from 8:30 a.m. to 3:30 p.m. on Saturday, November 29.

## ACCOMMOD ATIONS

The hotel headquarters for the meeting will be the Michigan Union, which is located on the west side of State Street. The Union will have sleeping accommodations for two hundred guests. The rates are $\$ 10.50$ for a single room, $\$ 12.50$ for a double-bedded room, and $\$ 13.00$ to $\$ 15.00$ for a twin-bedded room. All these rooms have private baths and television. In addition, a limited number of single rooms with semi-private baths at $\$ 6.50$ are available, and also dormitory-style accommodations at $\$ 4.50$ per bed. Please direct all requests for room reservations to Room Reservations, November Mathematics Meeting, Michigan Union, 530 South State Street, Ann Arbor, Michigan 48104 . It is important to mention the A.M.S. meeting in making reservations.

## F OOD SERVICE

The Michigan Union will provide breakfast at nominal cost for those staying there. In fact, for planning purposes the Union will assume that those staying in the Union will want breakfast unless the contrary is mentioned in the request for a room reservation. Lunch may be available in the Union, but dinner definitely will not be. However, there are many satisfactory restaurants within a half mile of the campus.

## TRAVEL AND LOCAL INFORMATION

Those coming by air should use Detroit Metropolitan Airport, which also serves Ann Arbor. Frequent bus and limousine connections are available to the Michigan Union. Ann Arbor can be reached from New York or Chicago by the New York Central Railroad; sleeping accommodations are available from New York. Ann Arbor is also served by Greyhound and Shortway Bus Lines.

Two major expressways intersect soutneast of the city: U.S. 23 is a northsouth route passing just east of Ann Arbor, while I-94 is an east-west Interstate Highway passing just south of Ann Arbor. Northbound drivers are advised to exit from U.S. 23 onto Washtenaw Avenue West. Those using I-94 should use the State

Road exit; State Road is a continuation of State Street.

Visitors may park in the Thompson Street Multistory Parking Structure one block east of the Union at a cost of at most one dollar per day. Theoretically it is necessary to pick up a special guest parking permit at the hotel desk of the Union or at the registration desk for the meeting before parking, but there is a reasonable chance that the magic words "mathematics meeting" will persuade the attendant at the structure to let cars enter directly.

Ann Arbor is on Eastern Standard Time throughout the year.

## SYMP OSIUM

On Friday, November 28, 1969, the day before the meeting itself, the University of Michigan will sponsor a symposium on Function Algebras and Rational Approximation, with the anticipated support of the National Science Foundation. There may be a limited amount of money available for travel expenses for specialists in the field who wish to attend this symposium. Inquiries may be addressed to Professor Allen L. Shields, Department of Mathematics, University of Michigan, Ann Arbor, Michigan 48104.

Paul T. Bateman
Associate Secretary
Urbana, Illinois.

# The Seventy-Sixth Annual Meeting Everglades Hotel Miami, Florida January 22-26, 1970 

The seventy-sixth annual meeting of the American Mathematical Society will be held at the Everglades Hotel in Miami, Florida, in conjunction with meetings of the Mathematical Association of America. Hour addresses will be given by Professor James B. Ax of the State University of New York at Stony Brook and Professor Morris W. Hirsch of the University of California, Berkeley. The Josiah Willard Gibbs Lecture will be given by Professor Walter H. Munk of the University of California, San Diego.

The general program is outlined as follows:

January 22-25: AMS contributed papers and invited addresses

January 24-26: MAA sessions
The Council of the American Mathematical Society has set no limit on the number of ten-minute papers that will be accepted for presentation in the regular sessions for contributed papers at the annual meeting. However, only the first 720 abstracts received will be assigned to day sessions because of space limitations. Abstracts received after the first 720 will be assigned to general evening sessions. The deadline for abstracts to be received in the Providence office is November 6, 1969. Because of the short period of time that is available to prepare the program for the annual meeting, the Providence office will not be able to accept changes in abstracts.

Authors are requested to notify the office of papers to be withdrawn.

## REGISTRATION

The registration desk for this meeting will be in the Everglades Hotel. The desk will be open from 2:00 p.m. to 8:00 p.m. on Wednesday, January 2l; from 8:00 a.m. to 5:00 p.m. on Thursday, January 22; from 9:00 a.m. to 5:00 p.m. on

Friday through Sunday, January 23-25; and from 9:00 a.m. to 3:00 p.m. on Monday, January 26.

The registration fees for the meetings are as follows:

| Member | $\$ 5.00$ |
| :--- | ---: |
| Non-member | 10.00 |
| Student | 1.00 |

The above fees include families.
EMPLOYMENT REGISTER

The Mathematical Sciences Employment Register will be maintained from 9:00 a.m. to 5:00 p.m. from Friday through Sunday, January 23-25, in The Roof of the Everglades Hotel.

## EXHIBITS

The book and educational media exhibits will be displayed in the Everglades Room of the Everglades Hotel. They will be open from Friday through Sunday, January 23-25, from 9:00 a.m. to 5:00 p.m.

## BOOK SALE

Books published by the Society will be sold for cash prices somewhat below the usual prices when these same books are sold by mail.

## SPECIAL MEMORANDUM TO MEMBERS

In an effort to minimize transportation problems, the regular sessions on Analysis and Applied Mathematics will be scheduled mainly at the Dupont Plaza. Sessions on Algebra will be concentrated mainly at the McAllister Hotel; and sessions on Foundations, Logic, and Topology will be mainly at the Everglades, Biscayne Terrace, and Columbus Hotels.

Members who return their reservation forms early may wish to take advantage of these plans. It is estimated that only a quarter of those attending will be able to find accommodations at the aforementioned hotels.

## ACCOMMODATIONS

Accommodations for the meeting will be handled by the City of Miami Convention Bureau. A form for requesting accommodations will be found on page 1000 of these $c$ (Votices). Persons desiring accommodations should complete this reservation form or a reasonable facsimile and send it to the Mathematical Meetings Housing Bureau, City of Miami Convention Bureau, 499 Biscayne Boulevard, Miami, Florida 33132. Reservations will be made in accordance with preferences indicated on the reservation form, insofar as this is possible, and all reservations will be confirmed. A deposit will be required by some hotels in Miami Beach. At time of confirmation, registrant will be informed of deposit requirement. No deposit is required by hotels in Miami proper. REQUESTS FOR RESERVATIONS SHOULD ARRIVE IN MIAMI NO LATER THAN DECEMBER 15, 1969. It is strongly recommended that members make reservations immediately on receipt of this issue, because of the large number of participants expected to attend this meeting which is scheduled at the height of the winter season. In conjunction with room accommodations, it might be well to make travel reservations at the same time.

| MIAMI |  |  |
| :---: | ---: | :---: |
| BISCAYNE | TERRACE |  |
| Singles | $\$ 12.00$ to $\$ 14.00$ |  |
| Twins | 16.00 to 18.00 |  |

COLUMBUS
Singles
$\$ 14.00$ to $\$ 18.00$
Twins
19.00 to 28.00

DUPONT PLAZA

Singles
Twins
$\$ 18.00$ to $\$ 21.00$
22.00 to 26.00

EVERGLADES
Singles
$\$ 10.00$ to $\$ 12.00$
Twins $\quad 16.00$ to $\quad 18.00$

| FOUR AMBASSADORS |  |
| :--- | ---: |
| Singles | $\$ 22.00$ to $\$ 28.00$ |
| Twins | 28.00 to 32.00 |

HOWARD JOHNSON

| Singles | $\$ 23.00$ |
| :--- | :--- |
| Twins | 26.00 to $\$ 32.00$ |


| LEAMINGTON |  |
| :--- | ---: |
| Singles | $\$ 9.00$ |
| Twins | 12.00 |


| McALLISTER |  |
| :--- | ---: |
| $\quad$ Singles | $\$ 9.00$ to $\$ 13.00$ |
| Twins | 15.00 to 22.00 |


| MIAMI COLONIAL |  |
| :---: | :---: |
| Singles | $\$ 12.00$ |
| Twins | 18.00 |
| MIRIMAR |  |
| Singles | $\$ 10.00$ |
| Twins | 14.00 |
| PARKLEIGH |  |
| Singles | $\$ 14.00$ |
| Twins | 20.00 |
| PATRICIA |  |
| Singles | $\$ 14.00$ to $\$ 18.00$ |
| Twins | 16.00 to 20.00 |
| PONCE de LEON |  |
| Singles | $\$ 15.00$ |
| Twins | 15.00 to $\$ 17.00$ |

ROYAL BISCAYNE
Singles $\quad \$ 45.00$

Twins 45.00
TOWERS
Singles
$\$ 16.00$
Twins $\quad 16.00$
MIAMI BEACH
ALGIERS
Singles $\quad \$ 34.00$
Twins $\quad 36.00$ to $\$ 41.00$
ATLANTIC TOWERS
Singles $\quad \$ 21.00$ to $\$ 22.00$
Twins 27.00 to 29.00
BARCELONA
Singles $\quad \$ 28.00$
Twins $\quad 33.00$


| CADILLAC |  |
| :---: | :---: |
| Singles | \$22.00 |
| Twins | 28.00 to \$32.00 |
| CARILLON |  |
| Singles | \$34.00 |
| Twins | 36.00 |
| CROWN |  |
| Singles | \$22.00 to \$24.00 |
| Twins | 24.00 to 28.00 |
| DEAUVILLE* |  |
| Singles | \$28.00 |
| Twins | 32.00 |
| DELANO |  |
| Singles | \$18.00 |
| Twins | 22.00 |
| Di LIDO |  |
| Singles | \$18.00 |
| Twins | 22.00 |
| HILTON PLAZA* |  |
| Singles | \$28.00 to \$36.00 |
| Twins | 28.00 to 36.00 |
| LUCERNE |  |
| Singles | \$18.00 |
| Twins | 22.00 |
| MOULIN ROUGE |  |
| Singles | \$30.00 |
| Twins | 30.00 |
| NAUTILUS |  |
| Singles | \$18.00 |
| Twins | 22.00 |
| SAXONY |  |
| Singles | \$24.00 to \$26.00 |
| Twins | 26.00 to 30.00 |
| SEA GULL |  |
| Singles | \$18.00 |
| Twins | 22.00 |
| SEA ISLE |  |
| Singles | \$18.00 |
| Twins | 22.00 |
| SEVILLE |  |
| Singles | \$38.00 |
| Twins | 40.00 |

*The DEAUVILLE and the HILTON PLAZA
require a check-out on January 25 th.
SHELBOURNE

Singles
$\$ 25.00$ to $\$ 27.00$
Twins $\quad 27.00$ to 31.00

| SHORE CLUB |  |
| :---: | ---: |
| Singles | $\$ 18.00$ |
| Twins | 22.00 |

SOVEREIGN
Singles
$\$ 26.00$
Twins
26.00

VERSAILLES
Singles
Twins
$\$ 22.00$ to $\$ 24.00$
24.00 to 28.00

There are a limited number of double rooms and suites available. The Convention Bureau will provide these rates on request.

## SHUTTLE BUS SERVICE

Shuttle bus service for those registrants staying at hotels in the Miami Beach area will be contracted for, providing there is an adequate demand for this service. Prepayment of $\$ 5.00$ per registrant must be received no later than December 22, 1969. The tickets will be mailed in advance of the meeting to those people requesting them by the Meeting Arrangements Department. Please make checks payable to the American Mathematical Society, and forward the check (be sure to include your return address) to the attention of Meeting Arrangements Department, Section SB, P.O.Box 6248, Providence, Rhode Island 02904. If there are not enough requests to warrant this service, you will be reimbursed.

## ENTERTAINMENT

Gray Line Sightseeing Tours, Inc. will set up a special sightseeing, information, and reservation desk in the Everglades Hotel for the mathematical meetings. Reservations will be made for regularly scheduled and special sightseeing tours by bus and boat in the greater Miami area.

An especially recommended tour is an all-day trip to Everglades National Park. The Everglades is a vast (almost 1-1/2 million acres) subtropical wilderness where land and water merge to form a natural wildlife haven of unusual beauty.

The park is about 70 miles southeast of Miami. Large flights of rare and exotic birds abound which are seldom seen anywhere else in the country. Gray Line Sightseeing Tours operates a regularly scheduled tour on Tuesday, Thursday, and Sunday. The Everglades tour includes a side excursion to a tourist attraction called the Orchid Jungle.

Bus trips to the Florida Keys and to Corkscrew Swamp Sanctuary under the auspices of the Tropical Audubon Society will be arranged if sufficient advance reservations are received. The deadline for advance reservations is December 24, 1969. More detailed information and reservations may be requested from Professor Robert Kelley, Department of Mathematics, University of Miami, Coral Gables, Florida 33124.

## TRAVEL

In the winter, Miami is on Eastern Standard Time.

There is regular airline service to Miami International Airport by the following airlines: Braniff, Delta, Eastern, National, Northeast, Northwest, Pan-American, TWA, United, and various foreign airlines. Miami is connected with the Bahamas and the Caribbean Islands by a number of airlines of which two of the domestic ones are Pan-American and Eastern airlines.

An airlines reconfirmation and information desk specifically assigned to the mathematical meetings will be set up by the Everglades Hotel. It will be staffed by trained airline personnel January 22-24, 1970. Florida has a reconfirmation rule for all arriving airline passengers holding return or continuing reservations. Participants should reconfirm on arrival in Miami. Since the meeting will be held at the height of the winter season, mem-
bers are urged to make their travel arrangements immediately.

## WEATHER

In January, the usual daily low temperature is about $55^{\circ}$ and the high around $75^{\circ}$. Immediately after the passage of a cold front, the early morning temperature may drop to as low as $40^{\circ}$ for a couple of days, and the high may be less than $65^{\circ}$. Rain is unlikely in January. The airport is almost never closed down by fog. The wind-shift after the passage of a cold front may bring smoke from fires in the Everglades for a few hours, but as far as is known this has never affected flight operations.

## CAMPING

There are two available campgrounds within fairly easy driving distance of Miami: (1) Mineral Springs Campground (private, AAA approved). Take State 27 (Krome Avenue) to 192 nd Street and follow signs. This is a "hammock" in the Everglades. Swimming in a pond fed by an artesian mineral spring. Fee: $\$ 8.00$ for 2; $\$ 0.50$ for each extra person. P. O. Box 761, Alhambra Circle, Coral Gables, Florida 33134. (2) Thompson Park (Dade County). On State 27, Krome Avenue Extension, between U.S. 41 and U.S. 27, nearer U.S. 27. Trailers and tents, 60 sites. Fee: $\$ 1.50$ for 6; $\$ 0.25$ for each extra person. P.O. RR 1, Box 659, Hialeah, Florida 33011.

There are two larger tent-trailer campgrounds in the Everglades National Park--Flamingo ( 241 sites) or Long Pine Key ( 108 sites). There are also a number of campgrounds in the Florida Keys from Key Largo on down to Key West. O. G. Harrold Associate Secretary
Tallahassee, Florida

# Some Mathematical Questions in Biology Boston, Massachusetts December 27, 1969 

The fourth annual symposium on Some Mathematical Questions in Biology will be held on December 27, 1969, in the State Suite of the Sheraton Plaza Hotel in Boston, Massachusetts. This symposium is cosponsored by the American Mathematical Society and the Society for Industrial and Applied Mathematics, and it is being held in cooperation with Section A (Mathematics) of the American Association for the Advancement of Science. The symposium will be supported by a grant from the Institute for Defense Analyses. Registration and hotel arrangements will be announced in SCIENCE.

This is the fourth in a series of annual symposia whose purpose is to stimulate direct contact between biologists
with some mathematical background and mathematicians. Most of the speakers are biologists who will address themselves to questions which are primarily of biological interest, but in which some mathematical analysis is involved. The morning session will be devoted to models of developing organisms and the afternoon to models of the brain.

The program will consist of six lectures, and it was arranged by the AMSSIAM Joint Committee on Mathematics in the Life Sciences. The members of this committee are Murray Gerstenhaber (chairman), Hans Bremermann, Robert MacArthur, Alston S. Householder, and R. C. Lewontin.
PROGRAM
9:00 a.m Chairman: Murray Gerstenhaber, University of Pennsylvania
Periodic wave propagation and pattern formation: a model
Morrel H. Cohen, Director, James Franck Institute, University of
Chicago
Periodic wave propagation and pattern formation: application to problems
in development
Brian C. Goodwin, Reader, University of Sussex, Brighton, Sussex,
England.
Metabolic stability, epigenesis, and self-replication in randomly construc-
ted macromolecular systems
Stuart Kauffman, Assistant Professor of Mathematical Biology, Uni-
versity of Chicago
2:00 p.m. Chairman: Jack D. Cowan, chairman, Committee on Mathematical Biology,
University of Chicago
Point processes and neural ensembles
George L. Gerstein, Professor of Physiology and Biophysics, Univer-
sity of Pennsylvania
Neuronal periodicity and frequency discrimination
Vernon Mountcastle, Professor of Physiology, Johns Hopkins University
Neurophysiological insight provided by mathematical boundary value prob-
lems, Wilfred Rall, Research Physicist, National Institutes of Health

Murray Gerstenhaber

# An Appeal for Preservation of Archival Materials 

By The Advisory Committee on History of the Mathematical Sciences

Our knowledge of mathematics and its history depends substantially on the preservation by our predecessors of manuscripts, notebooks, correspondence, apparatus, and other archival materials. This is obvious for ancient and medieval times, since what little we know depends on the few scraps that have survived. The invention of printing increased the diffusion and chance of survival for published materials, but private correspondence continued to be the most important mode of mathematical communication. Even after the founding of specialized journals, unpublished communications remained an essential part of the record.

Most highly creative mathematicians do not find the time to publish all their results, to say nothing of their ideas about mathematics and related matters. Gauss is an extreme case. Most of what he wrote has been published posthumously, and his famous journal did not come to light until 50 years after his death. If these ephemeral materials had not survived, many mathematical ideas would have been irretrievably lost and our picture of Gauss and of mathematics in the 19th century would be distorted. The situation is much less dramatic, of course, for most mathematicians, but it is not unimportant even for relatively "minor" figures. For example, only through the writings of others do we know who first made the four colour conjecture, because the discoverer did not write about it himself and his papers have not been preserved.

The twentieth century revolution in means of travel and communication has decreased the relative importance of printed material. The volume and slowness of publication has reduced the usefulness of journals as a means of communication. Private conversation and correspondence, notes, research reports, informal conference proceedings, preprints and various other ephemeral forms of communication
play a greater role. One symptom of this is the number of "well-known results" that are not published anywhere. Another is the many "rediscoveries" of published results. If in the future what we are doing now is to be known and understood in its scientific and social context, we must do a better job of preserving mathematical records.

The initial and primary responsibility must rest with the individual mathematician. He should not destroy correspondence to and from his mathematical colleagues. He should preserve a file of his own unpublished and semi-published material, including letters, drafts and other manuscripts, notebooks, diaries, bibliographies, preprints, reports, syllabi, notes for his own lectures, notes on the lectures of others, preliminary editions for local use, etc. He should keep photographs, sound recordings, apparatus, and momentos that throw light on the course of an investigation, the environment in which it took place, the organizations involved, etc.

Of course, not everything is worth preserving. But the mathematician should keep in mind that future generations will be interested in mathematicians as well as in their finished work, and that much of what is familiar to all of us will be unknown to the future unless some record of it is preserved. Not only historians of mathematics, but also a very wide scientific and lay public, are as interested in the origin and development of a mathematical theory as they are in specific mathematical results. Scholars generally are concerned with the whole range of mathematics as a component of civilization, and "personal" details of the lives of mathematicians frequently are of scientific and historical, as well as human interest.

We live in a period of rapid change in mathematics itself and in the many activities that make up the mathematical enterprise and its relations with other
aspects of culture. Many of these changes will be of the greatest interest to future generations. Organizations should take care to preserve their own records and to arrange for their deposit andpermanent accessibility. This applies especially to the many curricular reform groups, committees, and research groups whose records, memoranda, bulletins, and draft publications will be of the greatest interest to historians of the future and will enable them to understand events that today are familiar to all of us, but which may well appear quite mysterious some years from now.

It does little good for the mathematician to keep his papers unless he makes provision for their proper handling after his death. The best time to do this is in advance, by arranging for the deposit of papers in a library or other institution. In any case the mathematician should make provision in his will for materials that have not previously been provided for.

The families of mathematicians often play a key role in preserving documents. The best procedure for the layman is to consult mathematicians, historians of science, librarians, and archivists who can give advice based on knowledge of the materials and the means for preserving them.

An important responsibility rests also on departments of mathematics and mathematical organizations. They should establish their own archives which can serve as sources for the future historian. They should also see to it that the papers of deceased colleagues are not ignored until too late.

Where should materials be deposited? A natural place to consider first is an institution with which the mathematician has been associated. For example, government employees may approach the National Archives or the Library of Congress. Professors may find their own institutions appropriate, provided the library has an interest and adequate facilities. Often local museums, academies, or historical societies have archives. Many universities systematically
collect materials on the history of science. Some private organizations, such as the American Philosophical Society in Philadelphia, the Niels Bohr Library of the American Institute of Physics and the New York Library, collect documents. A good source of information on possible depositories is A Guide to Archives and Manuscripts in the United States, by P. M. Hamer (Yale University Press, 1961).

It happens sometimes that prompt steps to preserve archival materials are not taken because the task of arranging and sorting appears formidable. Actually this task should be done by experts in any case. The best procedure for the possessor of a collection is to consult the institution where the materials may be preserved and to encourage its staff to do the work. Arbitrary removal of materials, editing or rearrangement can be very damaging. Materials that are appropriately deposited will be properly organized and catalogued. Listing in The National Union Catalog of Manuscript Collections, published by the Library of Congress, will make them available to scholars. The donor can, of course, place limitations on use in terms of time and circumstance, but restrictions are awkward to administer and should be kept to a minimum.

To sum up, we ask all those who are in possession of archival materials in mathematics to arrange for their proper preservation so that our mathematical work will be firmly linked both to the past and to the future. Members of the Advisory Committee would be glad to advise and assist in placing collections and would welcome information about materials not yet listed in the National Union Catalogue.

Carl B. Boyer, Brooklyn College (CUNY); Churchill Eisenhart, National Bureau of Standards; Phillip S. Jones, University of Michigan; Kenneth O. May, Chairman, University of Toronto; Uta Merzbach, Smithsonian Institution; David Rosenblatt, National Bureau of Standards; Charles Weiner, American Institute of Physics.

# Can Mathematics be Saved? 

## By William G. Spohn, Jr.

How can this question be asked of mathematics, especially in view of its current flourishing state. The research journals are receiving more articles than they can handle. The number of Ph.D.'s and bachelors turned out is increasing rapidly. Mathematicians have found a place in industry. Modern mathematics is encompassing and linking varied branches of mathematics. Reforms in mathematics are taking place at all levels of the educational structure. Mathematics has never been held higher in public esteem. So--what could possibly be wrong?

In the midst of this prosperity it is not surprising that one does not see the danger signs. Articles in the American Mathematical Monthly over the past ten years show glimpses of these dangers, though I am sure few of the authors would think that the time had arrived to ask the title question of this article. Each of them sees some facet of the problem which involves the whole mathematical structure. The problem is concerned with the substance of mathematics, with teaching, with applications, with education, and with research.

Research exhibits various trends and emphases around the world. By sheer momentum there is still a mass of classical (nineteenth-century spirited) mathematics being published, mainly in the area of analysis. There is widespread research in applied areas stimulated by computation and the sciences, though much of this is in the form of isolated or specialized results not suitable for publication. However, the main force in mathematics today, particularly in the United States, is in the opposite direction, away from applications, towards the highly abstract. This we choose to call modern mathematics; it is dominated by modern algebra and topology.

On the surface modern mathematics seems impressive. Its abstractness and generality have given rise to a vast and growing structure. It has classified old subjects, indicating new paths of develop-
ment. It has linked apparently unrelated subjects, giving mathematics a new-found unity. It has put some difficult classical theorems in a new setting where they become more natural and meaningful. It has given mathematics a thinking and qualitative character, in contrast to the manipulative and quantitative character of much of classical mathematics. It has given rise to many new subjects and transformed or eliminated most of the old.

The common man seems unaware of the revolution in mathematics or of its implications. Modern mathematics began to take the spotlight in research in the 1920's. It rapidly dominated the mathematics departments in our graduate schools in the 1940's. It is sweeping elementary and collegiate levels in the $1960^{\prime} \mathrm{s}$. It is ironic that much of the change has been made possible by the favorable mathematical image engendered by space flights, computers, and scientific advances in general, whereas in fact the "new math" is replacing the very mathematics which made these things possible. We do grant that modern mathematics has a place in the mathematical structure, but we cannot tolerate the widespread intolerance of its advocates who would destroy all other mathematics.

Modern mathematics gains its significance secondhand; it is another step removed from "reality." It does not deal with most of the questions of the classical subjects; it has only an occasional point of contact. Its very qualitativeness precludes its effectiveness for most quantitative problems. Its viewpoint is limited; it has developed from isolated points of the old. The unity it supplies to mathematics is illusory, for the links to many subjects are weak and superficial. When it does apply to an old problem, there are usually other viewpoints which are important and essential to a full understanding of that problem.

Mathematics has become trapped by the delusion that it is a logical system
and that generalization is its goal. It has limited itself in scope and thinking. It has lost its intuition, the ability to see beauty in the particular, the challenge in solving problems. It belittles technique and has turned its back on applications and the mathematics that has become associated with applications. It thinks it is achieving an independent identity; while in reality, only empty isolation. It will miss the stimulation from nature, the classical links which gave it significance, and the financial support which made it possible.

Modern mathematicians have a strangle hold on mathematics, particularly in this country. Classical mathematicians find it increasingly difficult to get Ph.D.'s, to get their material published, to get positions or promotions in the "best" schools. As a result, classical mathematicians are becoming obsolete at a time when the demands from related applications would give them added status and new opportunities. The destruction has been so thorough that modern mathematics itself has felt the pinch. Witness the foreign mathematicians who are being im ported because they combine the modern view point with a classical background. How has this current state of self-destruction in the area of mathematics come about?

Throughout history one aspect of mathematics has disassociated itself from applications. Since 1800 this disassociation has been quite marked. Once the primary aim of mathematics ceases to be applications, it becomes difficult to assess the relative significance of its different branches or the "worth" of a given piece of research. This results in fads and fashions which may vary from time to time and from place to place. It is even more difficult to evaluate our mathematicians or to decide on what to base their status. Research has come to be our measure of mathematicians, even for those who are primarily teachers. The significance of the research, unfortunately, is often secondary to the number of pages or the number of articles. It is said Newton waited twenty years to publish his "Principia" for lack of the proof of a crucial theorem. Who waits today to carry through a research idea to completion? That would lead to only one paper or none at all if someone else published first. Submit results for publication as fast as
you get them. Then while you wait a year or so for them to be published, you can be well on your way towards getting some further development.

Many problems and inequities stem from making research a measure of men. Since there is a time lag in getting something published, friends or someone connected with the publication will by foreknowledge get a head start on the general researcher. Since there are too many journals for an individual to check, it may be an additional year or so until he becomes aware of a particular article through the "Mathematical Reviews". Because there are so many mathematicians all over the world working on problems, there is much duplication and it is often a matter of luck who gets to publish the result. The requirement that the results be new tends to drive research into new fields. The old fields have been worked over and as a rule only tougher problems remain. It is harder to determine what has been done in some old fields because of language and notation barriers, because of different naming or classification for the germ of an idea, or because the research appeared in the development of an entirely different subject matter. Thus the artificial requirement that research be new has a profound effect on the type of research that is being done.

Another flaw in the mathematical structure is having research men as teachers in our graduate schools. Teaching and research are all absorbing and require great dedication. It is the rare individual who can do both well, particularly at the same time. It is not clear what the objectives of our graduate schools are in the area of mathematics. It is said that graduate schools turn out scholars with a desire for learning who are broadly based and finely specialized to carry on original research. I submit that they have failed miserably. They have instead found a distorted criterion by which to sift out people for the "Ph.D. club." The candidate must have some facility for foreign languages, which may be antithetical to mathematical skill. He must have a superficial knowledge of a vast amount of mathematical material, which revolts his scholarly integrity. He moves rapidly to the frontier, without time for motivation,
insight, or tests of significance. It is unthinkable with such a background that he find his own dissertation topic. He doesn't have the perspective for that. He has to rely on his advisor for the problem and, perhaps, even for help with the solution. The evils of the system are obvious. The advisor may have no problem at the time; he may misjudge the difficulty of the problem or the capacity of the student. Personality conflict may be a significant factor. It is manifest that the graduate schools do not turn out research mathematicians, since over one-third of them publish nothing beyond the thesis.

There is no place for the teacher in our graduate schools and no place for the mathematical scholar in our society. The history of mathematics, the exposition of mathematics are no longer acceptable endeavors. What is the sense in developing mathematics, if once we make a new discovery, we catalogue it and forget it. Must we not organize it, polish it, relate it, teach it, apply it. These are all vital activities and all must be recognized as mathematics.

As a rule the Ph.D. mathematician makes a poor teacher for our colleges, because he has been "brain washed" in modern abstract mathematics. He either tolerates the course, getting as little involved as possible, or he will try to give the course a "meaningful" modern viewpoint. In the former case the student gets bored; in the latter he gets lost and discouraged. How many students give up mathematics and even college because of shattering experiences with early college mathematics courses? Basing the teacher's advancement on research rather than teaching is another self-destructive element of the system. Research, besides being all absorbing, will likely lead the teacher into areas of modern mathematics, where the thinking and viewpoint will be different from the classical subjects he will likely be teaching. This will necessitate constant adjustment and result in inferior activity in both areas.

As a rule the Ph.D. mathematician is a "misfit" in industry. The problems are not worthy of his effort. He continues to work on "mathematics" and lends his prestige to the establishment. Even the advanced parts of applied mathematics are
as a rule only of occasional use. What is most needed is sound mathematical thinking and a mastery of elementary mathematical techniques. Often one must become acquainted with some other discipline to insure that a problem is properly formulated. The formulation and solution of the problem is a kind of research. It requires great attention to detail, seeing the problem from many points of view, getting it "out of the woods" onto one of the paths of organized mathematics. This is made to order for the classical mathematician, as is shown by many outstanding examples. Incidentally, the effective mathematician in industry is often part of a team, belying the doctrine that mathematical research is essentially an individual undertaking.

Another tragedy of the decline of classical mathematics is that scientists in other fields are forced to divert their energies to grapple with those parts of the mathematical heritage which are vital to their existence. In fact, most of the mathematics in science and industry today is being done by non-mathematicians. Not only is mathematics missing the grand opportunity, but the whole nation is suffering from the shoddy and wasteful mathematics that is being done, in most cases unavoidably, by non-mathematicians.

Reforms in the educational structure have received much thought in the last ten years. On the one hand, standard courses like college algebra and calculus have been reduced to trivia over a period of years, primarily from the pressures of mass education and, on the other hand, these courses at their best would not prepare the student for graduate courses in modern mathematics. If elementary and collegiate education goes abstract, where will business, engineering, and science students get their mathematics? Already these areas are setting up their own courses, with the resulting loss of beauty, generality, and power of mathematics. Unfortunately, there is no proper leadership for mathematical reform. The ranking organization, The American Mathematical Society, has concerned itself primarily with research. The Mathematical Association of America, with undergraduate mathematics for its alleged domain, is trying to cope with the problem;
but the scope of the problem is too broad for it to be effective, Leading mathematicians are so busy doing research or struggling to maintain status, that they do not see that mathematics as they know it is fading from the scene. They should be fighting for the kind of mathematics that had the broadness and depth to serve mathematics and science, teacher and researcher, amateur and professional.

We have quite a tradition of associating great mathematics with great mathematicians. We like to think of calculus as being developed by Newton (or Leibnitz); however, one finds much activity in this area for a century before. Some even carry the thread back two thousand years to Archimedes. Euclid had his forerunners, notably, Eudoxus. The fact that so many mathematical discoveries are made independently at about the same time, as was non-Euclidean geometry, for instance, indicates that the result is "in the air" waiting to be grasped. We make such a fuss about who is first. Who discovered determinants? Was it Vandermonde in 1771 when they first became well known, or Leibnitz in 1693 representing Westerncivilization, or Seki Kowa of Japan in 1683, or was it One Eye from the planet Far Away, who discovered them in $1,000,000$ B.C. With the many people all over the world working on the same problem, the "game" of who is first becomes quite ridiculous. Think of the tremendous waste in duplication of effort, in the struggle to get the problem in hand because of piecemeal, concise presentation, in the time spent checking the literature to determine that the result is new. It is time to end this charade and put the glory seekers "out of office." Let's return to mathematics that is worth doing for its own sake. We have no idea how much research potential there is in this country. The system has repelled the research man at every level.

What is needed therefore is a new attitude and a new approach to mathematical research. Once the necessity and urgency of this is realized, many solutions should be forthcoming. The most active areas of mathematics would perhaps be handled at research institutes. The results would not come out as a series of isolated, unreadable articles, but as an integrated readable account after five or ten years, say. There may be occasional reports in-
dicating progress or failures. Think of the effort to be saved by the enumeration of attacks which have failed. These institutes might serve as a clearinghouse for suggestions and partial results from outsiders. This would also provide a way for cutting down on the quantity of research published, which by sheer bulk is now overwhelming our libraries.

Once research is freed from the connotation of being new, it will become a significant force in our society. It will be recognized that the teacher is researching when he develops a novel way of presenting a topic to his class, that the student is researching when he is learning by discovery, that the layman is researching when he rediscovers a mathematical gem for his mathematics club. Why can't the layman share in the wonders of mathematics. Once the myth is destroyed of the superiority of certain mathematics and certain mathematicians, mathematics can be enjoyed by all. Local mathematics clubs could be under the guidance of some national organization. People would participate because mathematics is fun; newness, significance, usefulness would be secondary considerations in such clubs. There could be talks and solutions of take-home problems. The parent organization might supply a list of problems of various degrees of difficulty, including some at the research level. The much-neglected area of recreational mathematics should flower in this setting.

As soon as those primarily interested in research are happily housed in research institutes, graduate schools can be made a place to get an education. The courses would be broad--modern mathematics would be reduced to its proper status. Courses should not lead to the frontiers of knowledge, since these areas tend to be disorganized and specialized. Today great effort is wasted because graduate courses are too near the frontier. There are many false starts and changing viewpoints and definitions. This unstable quality is reflected in the great turnover of faculty in graduate-school mathematics departments. Courses stabilize because they are useful, well rounded, complete, because the material is exciting, gains in perspective, and relates to mathematics as a whole. Colloquia by guest lecturers, probably the research men, would outline
paths to various frontiers of knowledge. Note that it is only an outline; this allows the research student flexibility in finding his own paths, leading to broader and more solidly based research. We cannot hope that a graduate student turn out a piece of new research because the classical fields are too tough, the modern fields require too much preparation, and new fields involve too much luck. He can turn out original research, that is, his own research which could, perhaps, best be measured in a seminar on a broad topic with, perhaps, more than one faculty member. He may work only on one facet of a problem and may be required to take a second seminar if his contribution is not of sufficient significance. Elimination of the thesis and oral and language examinations should do much to remove the excessive element of gambling in the present structure. A graduate department may offer six or eight broad graduate courses spanning mathematical knowledge from philosophy to science. There may be little variation in the universities across the country and teachers would be used more efficiently than at present. The courses need not be jammed with material. They should convey the spirit of some branch of mathematics, including outstanding techniques, and important links. The student should have time to think and work things out on his own. He may take only one or two graduate courses at a time.

Examples of the teacher of the future are rare in our graduate schools today. He may appear in the following mold. He is a real scholar, broadly trained in classical mathematics. He is a devoted teacher, but does little research. He lacks prestige in the Department. His course is listed as Applied Mathematics or Partial Differential Equations or Mathematical Methods for Scientists. It covers Advanced Calculus, Introductory Analysis, Fourier Series, Laplace Transforms, Complex Variables, Differential Equations, Integral Equations, and the Calculus of Variations. On the one hand he touches applications; on the other, Real Variables, Modern Algebra, and Topology. The course is full of amazing techniques and ideas developed over the last two hundred years. Today this course is beneath mathematics students; fit fare only for scientists and engineers.

Once the graduate school is revamped, Ph.D.'s will come forth who are scholars and teachers, who can do research, both pure and applied. Once we get a flow of competent teachers whose graduate training provides proper background for their teaching, we can revamp undergraduate mathematics. Once we get broad and sound courses at that level which even prospective elementary and secondary teachers will find attractive, we can reform mathematics at the elementary and secondary levels as well.

The doctoral program which we have described would give excellent background for the sophisticated research of classical and modern mathematics, but it too would miss most of our research potential. Research oriented people should be encouraged to get as much educational background as possible; however, many will not make it through the graduate school framework or even to the end of college for that matter. Many are too specialized in their talents or too dedicated to their insights. Some learn by discovery, which is excellent research training, but slow education. In fact the research type tends to prolong his education indefinitely albeit at a slow pace. The research mind can be nurtured either by a special facet of our educational structure or as a part of an institute where he can work and learn as an individual or as part of a team. Some will discover new types of mathematics in the vigor of youth while unprejudiced by old mathematics. Surely in this class would go most of the great mathematicians of the past. Our research potential has been greatly underestimated because of our misunderstanding of the makeup of the research man and his educational needs. We don't have to teach research; the main thing is to recognize the potential and encourage it, not stifle and repel it.

With the advent of computers and the application of mathematics to many new areas, with the great public support and financial resources available, mathematics stands at the threshold of a bright future. However, it will take great leadership to grasp this moment of opportunity and stem the tide that is now sweeping mathematics to destruction. Let's return mathematics to the main stream so that it may be shared by all and assume its proper role as a dominant force in our society.

## DOCTORATES CONFERRED IN 1968-1969

The following are among those who received doctorates in the mathematical sciences and related subjects from universities in the United States and Canada during 1968-1969. The number appearing after each university is the number of doctorates listed for that institution: when applicable, this number includes the doctorates in both the department of mathematics and its related departments. Each entry includes the dissertation title. 130 universities are listed with a total of 1,156 individual names.

ALABAMA<br>UNIVERSITY OF ALABAMA (10)<br>Barnett, Edward Franklin<br>On a certain class of generalized analytic functions<br>Chen, Chao-Wen<br>On the matrix equation $A B=C B A, C$ commuting with both A and B .<br>Destito, Vito A.<br>Three-space problems in the calculus of variations with both end-points variable<br>Gregoria, Daniel George<br>On differential equations with discontinuous right members<br>Lovingood, Judson Allison<br>Dynamical polysystems as generalizations of control systems<br>Parker, Robert Russell<br>Generalized control systems in topological spaces<br>Smith, Jeannette McGinty<br>A study of functions invariant under certain groups and ternary semigroups of bilinear transformations<br>Steincamp, James William<br>On k-commutative matrices<br>Stover, James Anderson, Jr.<br>On structural properties of differential equations<br>Trotter, William Thomas<br>On universal subcontinua<br>AUBURN UNIVERSITY (3)<br>Maghsoodloo, Saeed<br>A Monte Carlo investigation of a frequency function related to rank correlation<br>Mazeres, Reginald Merle<br>On the relation between a semigroup and it's square<br>Moe, David Allen<br>Some properties of norm intervals

## ARIZONA

ARIZONA STATE UNIVERSITY (6)

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Aizley, Paul
    Structure theory for a class of convolution
        algebras
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Bresinsky, Henrik
Semigroup and analytic equivalence of algebroid branches in the plane
Catlin, Seth
Some theorems concerning regressive sets and regressive isols of order greater than or equal to one
Gersting, Judith Lee
Some results on T-regressive isols
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On defining $L F(2, q)$ as a quotient group of $(2,3, n)$
Richardson, Joan
Inductive invariants and dimension theory

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The densest lattice packing of tetrahedra
Stillman, Eugene Sims
Segmented and spline approximations

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Barksdale, James Bryan
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Blakemore, Carroll F.
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Brasher, Russell G.
The orientable double covering of a nonorientable manifold, and related spaces
Hansard, J. D., Jr.
Function Spaces: A study of the graph topology, the connected-open topology, the $\sigma$-topology, and the topology of uniform convergence
Simmons, David Rae
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Starling, Albert Gregory
Precosheaves and Cech homology
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Natural equivalences and lifting properties in categories

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$\lambda$-designs and related combinatorial configurations
Chow, Theresa Kee Yu (Miss)
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Davis, Daniel Lee
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Dodds, Peter Gerard
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Projections in a normed linear space and a generalization of the pseudo-inverse
Fisher, James Louis
Structure theorems for noncommutative complete local rings
Greene, Curtis
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Inaccessibility properties of cardinals
Lawson, Herbert Blaine, Jr.
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Lee, John Walter
Boundary value problems with cyclic totally positive Green's functions with applications to spline approximation and eigenvalue problems
Liggett, Thomas Milton
Weak convergence of conditioned sums of independent random vectors

MacKichan, Barry Bruce
A generalization to overdetermined systems of the notion of diagonal operators
Micchelli, Charles Anthony
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Problems in scattering theory
Reed, Michael Charles
On the definition and self-adjointness of Hamiltonians in different representations of the canonical commutation relations
Rounds, William Chesley
Trees, transducers, and transformations
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The explosion problem for branching Markov processes
Stanton, Charles Madison
The closed ideals in an algebra of analytic functions
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On scattering theory for the Schrödinger equation
Takahashi, Masakazu
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A numerical investigation of the simplex method
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Vicens, Pierre J.L.E.
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Department of Operations Research
Bent, Dale Harold
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Hopkins, David Stephen Prince
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Stidham, Shaler, Jr.
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Feldman, Louis Arnold
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Fields, Kenneth Lewis
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Tabatabaian-Kashani, Ali Mohammad
Some classes of quasi-invariant non-Gaussian random linear functionals on Hilbert space
Tartakoff, David Stephen
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Tewari, Udai Bhan
Harmonic analysis on compact semigroups
Tilson, Bret Ransom
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Tomter, Per
Anosov flows on infrahomogeneous spaces
Van Eps, John
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Wolf, Robert Alan
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Zalcstein, Yechezkel
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On minimal models of complex varieties

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Davies, Robert Braithwaite
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Permutations as products of large nondisjoint cycles
Cho, Tae Geun
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Gauld, David Barry
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Gittleman, Arthur Paul
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Hatcher, John Rudolph
Closed form solution of some singular integral equations
Iha, Franklin Takashi
Spectral theory of partial differential operators
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Joseph, Keith Stuart
Commutativity in non-Abelian groups
Kwon, Young Koan
Integral representations of harmonic functions on Riemannian spaces

Green's lines and HD-functions in Riemannian spaces
Maltz, Carl
Estimation of mixing distributions
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Turner, Edward Charles
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Rodriguez, Dennis Milton
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Schwabe, Robert Allan
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Shirley, Edward Dean
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UNIVERSITY OF CALIF ORNIA, Santa Barbara (6)
Kruse, Harriett (Botta)
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A generalization of the associated transfor-
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Pierce, Stephen Jay
Generalized isometries
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UNIVERSITY OF SOUTHERN CALIFORNIA (5)
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Implicit functions in locally convex spaces
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Stationary measures on branching processes

## COLORADO

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Properties and applications of absolutely continuous functions
Mallory, Curtiss Orville
An experiment using materials to supplement a mathematics content course for elementary education majors
Steffenson, Arnold Ray
Examples and counterexamples in point set topology
Teeters, Joseph L.
On generalizations of the Darboux property
Warnlee, Erlo Henry
The effect of practice in the fundamental arithmetic operations on teacher-education students
Young, Norman E.
Effectiveness of these approaches to the teaching of a methods course in mathematics for preservice elementary teachers as related to knowledge, understanding and attitude
Zamboni, Floyd Frank
A study of the effect of two different classroom procedures upon student achievement, anxiety, and attitudes of second year high school algebra

COLORADO STATE UNIVERSITY (5)
Halvorson, Kenneth Wade, Jr.
Sample editing in symmetric densities study of Mississippi River bends
Heiny, Robert Lowell
Stochastic models for surpluses and deficits
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Generalized inverse of a matrix
Nelson, Vern Alvin
Analyticity of solutions of elliptic equations
Williams, Michael Zane
The steady temperature interior to a semiinfinite rectangular parallelepiped with a discontinuous radiation condition

UNIVERSITY OF COLORADO (5)
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Jefferson, Thomas Hutton, Jr.
Some additional properties of $T$-fractions Johnson, James Stephen

Amalgamation of polyadic algebras and finitizability problems in algebraic logic
Mandell, Michael
Convergence of sequences of linear fractional transformations
Ramaley, William Charles
A new classification of ternary quadratic forms

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The ring of a continuous geometry
Fisher, Gerald
On the representation of formal languages using automata on networks
Mozzochi, Charles J.
Symmetric generalized uniform and proximity spaces
Small, Donald B.
On a functional equation arising in the metric theory of continued fractions

## Department of Statistics

Fridshal, Donald
Estimating ordered binomial parameters
Harrington, Leigh
The generalized discriminant function and nuisance parameters in classification procedures
Hatch, Lawrence Otis
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## WESLEYAN UNIVERSITY(4)

Brook, Robert Beaumont
On the least of traces and the greatest of ambits in topological dynamics
Hindman, Neil
On $P$-like spaces and their product with P-spaces
Huang, Ter-Jenq
On the theory of ambits and the proximal relation in topological dynamics
O'Connor, Patrick John
Generalized differentiation of functions of a real variable

## YALE UNIVERSITY(23)

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Faulkner, John Robert
Octonion planes defined by quadratic Jordan algebras
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On the automorphism group of a semi-simple Jordan algebra of characteristic zero
Hill, David Geoffrey B.
$\sigma$-finite invariant measures on infinite product spaces
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Petersen, Karl Endel
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Read, Thomas Thornton
Analytic structure in the spectrum of a Banach algebra
Shapiro, Leonard David
Distal and proximal extensions of minimal flows

Wilson, Robert Lee
Nonclassical simple Lie algebras
Department of Engineering and Applied Science
Chiang, John Hsi-teh
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Clover, Richmond Bennett, Jr.
Properties of magnetic insulators at radiofrequencies
Dabrowski, John Eugene
Surface hydration effects on the dehydration activity of gamma alumina
Date, Raghunath Vinayak
Kinetics of hydrochlorination of actyl and dodecyl alcohols in homogeneous and heterogenous systems
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Icsevgi, Aslan
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Kelvin, Norbert Victor Peter
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Klein, Robert Raymond
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Liu, Chih-Shen
Yield surfaces at elevated temperatures
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Ryan, Daniel Francis
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Taylor, James Hugh
Absolute stability criteria by Lyapunov's second method

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Mattamal, Martha Malliga
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Sornberger, George Clinton
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Zander, Vernon Emil
The generalization of the Fubini-Jenssen theorem to Orlicz spaces of Bochner measurable functions
Mureika, Roman Algirdas
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Gnugnoli, Giuliano
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Norris, Eugene Michael Some structure theorems for topological machines
Spears, William Thomas
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Embedding theorems for partial latin squares
Russell, Mary Jean
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Hanson, Thomas McH. Concerning vector semigroups

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Cooney, Sister Miriam P.
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Ferguson, Pamela A. A.
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Schultz, Reinhard Edward
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Thieleker, Ernest A.
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Locally flat imbeddings of topological manifolds

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Sanathanan, (Mrs.) Lalitha
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Petticrew, James W alter
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Rich, Michael
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Some topics in $n$ person game theory
Magid, Andy
Separable subalgebras of commutative algebras and other applications of the Boolean spectrum

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Miller, Charles Frederick III
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Morley, Larry John
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Pillai, Narayana
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Ridge, William Clayton
Composition operators

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Moran, Bryan Peter Solvable series of groups and their associated rings
Nowlan, Robert Anthony, Jr. $A_{n}$-actions on fibre spaces
Sullivan, Theodore Francis III
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Rickey, Vincent Frederick An axiomatic theory of syntax
Rush, Francis Thomas Subgroups of primary Abelian groups
Van Oss, Jeanne Alice Weakly-reflexive lattices

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Clancey, Kevin Francis
Spectral properties of semi-normal operators Cohoon, David Kent

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Danhof, Kenneth Joe
Definability and the Cantor method in model theory
Eifler, Larry Quin
Products of function algebras
Green, Euline Irwin
Banach limits and related topics
Hintz, Gerald Regis
Mixed problems for linear transport equations in slab geornetry
Jones, William Charles, Jr.
Finite groups with conditions on the subgroups lattice
Liu, Fon-che
Approximation--extension type properties of curves
Martin, Benjamin Joseph
On a new integral equation arising in the theory of radiative transfer
Miranda, Guillermo Massa
Application of singular integral equation methods to static problems of non-smooth elastic bodies
Moh, Tzuong-Tsieng
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Szeto, George
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Burchard, Hermann Georg
Interpolation and approximation by generalized convex functions
Desautels, Edouard Joseph
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Gibbs, Norman Edgar
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Symes, Lawrence Richard
A mathematical problem solving language and its interpreter

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Chang, Tseng Chi
On asymptotic expansions of the distribution of the characteristic roots of two matrices
Jouris, Gary M.
On some classical and complex multivariate normal distribution problems

McDonald, Gary Carl
On some distribution-free ranking and selection procedures
VanArman, Donald
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Engquist, Michael Lawrence
Existence of solutions for differential equations with multivalued right-hand side
Erisman, Albert Maurice
An algorithm for eigenvectors of nonHermitian matrices
Haddix, George Franklin
The lattice of intermediate fields of a purely inseparable extension
O'Brien, Joseph Nicholas
Stability and error analysis of linear multistep methods
Rognlie, Dale Murray
Generalized integral transforms
Shoultz, Werner William
Chains of minimal generating sets of inseparable fields
Sincovec, Richard Frank
Norm reduction algorithms for eigenvalues and eigenvectors of a matrix
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Loughlin, Timothy Arthur
Realization of classes of irreducible matrices
McGloin, Paul Arthur
On the duals of non-Hamiltonian maps
Millen, Jonathan Kays
On the capacity and homology of a discrete metric
Park, Robert Edward
On planning experiments for the approximation of response surfaces
Schoenstadt, Arthur Loring
Effect of boundary conditions in selected problems in acoustical and elastic wave propagation
Wernick, Robert Julius
Parametrized games and the eigenproblem
Wollkind, David Joseph
A nonlinear stability analysis of the freezing of a dilute binary alloy

THE ROCKEFELLER UNIVERSITY (2)
Kleinberg, Eugene M.
Strong partition properties for infinite cardinals
O'Neil, Patrick Eugene
Asymptotics and (0-1)-matrices
STATE UNIVERSITY OF NEW YORK AT BUFFALO (3)

Bartlow, Thomas
Power-associative quasigroups and loops
Luedeman, John
On the embeddings of topological rings
Department of Statistics
Kappenman, Russell Francis
Posterior regions for the ratios of means and for parallel profile differentials

STATE UNIVERSITY OF NEW YORK AT STONY BROOK (4)

Klee, Karl John
Summability of subsequences
Pigno, Louis
Some multiplier problems in Fourier analysis on groups
Roth, Herbert Neil
Limit theorems for continued fractions (probability)

## Department of Applied Analysis

## Chung, Sun Ho

Renormalization method and its application to elastic and electromagnetic waves in strong randomly fluctuating media

SYRACUSE UNIVERSITY (8)
Currano, Frank M., Jr.
Investigation of multiple zeros of a polynomial
DeGray, Ronald W.
Randomness and random number generators
Doud, Daniel P.
Dimension of finite partial ordered sets
Geramita, Anthony V.
Structure of projective modules over rings of global dimension less than or equal to two
Lim, Chin
Homological dimension
Lim, Clara
The strong Markov property
Orr, Richard
Remainder estimates for squarefree integers in arithmetic progressions
Prather, Ronald E.
A categorical unification of automata theory
UNIVERSITY OF ROCHESTER (6)
DiBello, Louis Vincent
Dedekind fields and some related classes of algebraic number fields
Hansell, Roger William
Borel measurable mappings for nonseparable metric spaces
Kibler, Dennis Francis
Isotopy and homeomorphism
Limaye, Balmohan V.
Ideals of analytic functions
Shannon, Richard T. Flat modules
Siegel, Martha Jochnowitz On birth and death processes

YESHIVA UNIVERSITY (4)
Gerber, Leon
Asymptotic relations between differential and difference equations

Ginsberg, Jonathan I.
Completeness and generator theorems in some Banach algebras
Kabak, Bertram Samuel
A Fatou type theorem for a special nonsymmetric tube domain
Williams, Vernon
Generalized interpolation spaces

## NORTH CAROLINA

DUKE UNIVERSITY (3)

Anderson, John Timothy
A generalization of the Bernoulli and Stirling numbers
Lucke, James Bennett
Commutativity in locally compact rings
Nelson, Roger Bain
Appointment streams with unpunctual customers

NORTH CAROLINA STATE UNIVERSITY (14)
Anthony, Joby Milo
Topologies for quotient fields of commutative integral domains
Byrd, Kenneth Alfred
A characterization of uniserial rings
May, Lawrence Edward
Perturbation problems in fully nonlinear systems
Richardson, Gary Douglas
On completions of uniform convergence spaces
Sox, Jason Loy, Jr.
Two classes of complete orthogonal sequences

## Department of Experimental Statistics

Ahsanullah, Mohammad
Consistency of least squares estimators for time series with lagged and regression variables with independent errors
Hwang, Frank Kwang-Ming
An inventory model with special sale
McMillan, Robert Glen
Tests for one or two outliers
Parish, Robert George
Minimum bias approximation of models by polynomials of low order
Samad, Abdus
Multilocation inventory model with special sale
Schofield, Thomas John
Estimating a mixture of noisy sinusoidal components
Suh, Moon Won
A study of the distribution and moments of bundle strength in sequential breakage of parallel filaments
Toro-Vizcarrondo, Carlos Enrique
Multicollinearity and the mean square error criterion in multiple regression: a test and some sequential estimator comparisons

## UNIVERSITY OF NORTH CAROLINA (8)

## Huff, Robert Edward

Measures on locally compact spaces
Stanford, David Phillip
Remarks on functions satisfying a weighted average property
Shock, Robert Charles
Rings with finiteness conditions
Department of Computer and Information

## Science

Oliver, Alfred
A measurement of the effectiveness of an interactive display system in teaching numerical analysis
Prokop, Jan Stuart
An investigation of the effects of computer graphics on executive decision making in an inventory control environment

## Department of Statistics

Kettenring, Jon R.
Canonical analysis of several sets of variables
Obenchain, Robert L.
Rank tests invariant only under linear transformations
Robillard, Pierre
Combinatorial problems in the theory of factorial designs and error correcting codes

## OHIO

CASE WESTERN RESERVE UNIVERSITY (24)
Carlson, David Helmuth
Extensions of dynamical systems via prolongations
Csordas, George Leslie
The Silov boundary and a class of functions in $\mathrm{H}^{\infty 0}$
Dailey, James W. Approximations by spline-type functions and related problems
Ebert, Ward Lyman
Endomorphisms, orders of tangency and applications to $\mathrm{H}^{\infty 0}$
Feiock, Ray Edward
Cluster set maximality theorems and joint continuity
Pierce, John George
Higher order convergence results for the Rayleigh-Ritz method applied to a special class of eigenvalue problems
Simanaitis, Dennis J.
Generalized limit sets in dynamical systems
Smith, Paul John
Structure of nonparametric tests of some multivariate hypothesis
Warner, James Ernie
Multivariate permutation tests with applications to signal detection

Systems Research Center
Jurdje vic, Velimir
Dynamical control systems

## Computing Center

## Walter, Kenneth

A name space environment for a program oriented computer system

Department of Operations Research
Corwin, Burton Decker
Some flow shop scheduling problems involving sequence dependent setup times
Kumin, Hillel
The design of Markovian congestion systems Mann, Stuart H.

A mathematical theory for the exploitation and control of biological population
McCuskey, William
Toward automatic design of data organization Nunamaker, Jay F., Jr.

On the design and optimization of information processing systems
Schoeman, Milton E. F.
Resource allocation for new product development
Shah, Arvind
Inventory control of substitute products
Silverman, Gary G.
Primal decomposition of mathematical programs by resource allocation
Singh, Vijendra $P$.
Queuing systems with balking heterogeneous servers

## Department of Statistics

Edmondson, Bennie Curtis
A statistical test for exponential smoothing
Fairlie, Andrew Miller, Jr.
Some topics on t-testing in multiple linear regression
Langman, Malcolm Keith
Some new trend tests
Suich, Ronald Charles
Truncated sequential t-test
OHIO STATE UNIVERSITY (13)
Block, Henry William
Convergence and error estimation for sums of dependent random variables
Girard, Dennis Michael
A general asymptotic formula for analytic functions
Haines, David C.
Quasi-orthogonality in P-rings
Heuvers, Konrad John
Clifford algebras, inner products, and spinors Hull, David L.

Limited theorems with weighted averages of random variables
Keck, David Nelson
On a transformation theory for measure spaces

Konvisser, Marc William
Metabelian p-groups which contain selfcentralizing elements
Liang, Joseph Jen-Yin
On interrelations of arithmetical invariants

## Parker, Donald Beshers

Wreath products and properties of groups Queen, Clifford Steven

Non-conservative function fields of genus one
Shook, Thurston Woolever
Directed metric spaces
Sternbach, Leonard Paul
Bases and quasi-reflexive spaces
Whitford, Leslie Eugene
On boundary behavior of power series

## OKLAHOMA

OKLAHOMA STATE UNIVERSITY (5)
Butler, George Malcolm
A new axiomatic approach for the Steenrod squaring operations
Finley, Richard Daniel
A theory of significance testing
Hunt, Hurshell Harvey
The distribution of certain functions of parameters: prior and posterior
Moore, Billy John
On a theory of significance testing
Yates, James Hartley
Nearest and farthest points of convex sets
UNIVERSITY OF OKLAHOMA (7)
Anderson, James Andrew
Tensor product of semigroups
Burris, Stanley Neal
Theory of pre-closures
Chance, Lelia Downing
Grammatical sets in half-ring morphologies
Frawley, William James
Locally disconjugate families of continuous functions
Johnson, Tony Maurice
Vector bundles and projection tensors
Department of Biostatistics and Epidemiology, School of Health
Harless, William Glenn
The use of computer-assisted instructions to test the total time hypothesis in verbal concept learning
Whorton, Elbert Benjamin, Jr.
The development and investigation of some extensions to the Ederer-Myers-Mantel procedure and test for clustering

## OREGON

OREGON STATE UNIVERSITY(13)
Attia, Farag Abdel-Salam
On the distribution junction of the integral between zero-crossings of a stationary Gaussian process

Bakhshi, Vidya Sagar
Boundary layer transition
Borah, (Chutia) Bolindra Nath
Thermo-elastic-elastic transition
Gamon, Kenneth Oakland
The propagation of waves and pulses in the presence of conical structures
Green, Beryl Mansfield
Characterizations of matrices for certain determinantal equalities
Johnson, Robert Carl
Equivariant Stiefel-Whitney classes
Lindstrom, Frederick Thomas
A mathematical model stimulating mass transport of chemicals in saturated porous media
Middleton, William Adams
Electromagnetic radiation and heat conduction fields near coaxial conical structures
Seethoff, Terrance Lee
Zero-entropy automorphisms of a compact Abelian group
Wong, James Tong
On the generalization of the distribution of the significant digits under computation

## Department of Statistics

Burnett, Thomas D.
Estimation of stochastically varying regression parameters
Fossum, Robert R.
Models for statistical dynamic prediction of the 500 -millibar surface
Phillips, Charles J.
Consistent empirical approximation of a priori distributions

## UNIVERSITY OF OREGON (13)

Bernhardt, Roert Lynn III
Purity and torsion in categories of modules
Brown, Ronald Paul
Irreducibility over complete rings
Clemons, Arthur James
Natural density algebras
Cunningham, Joel Luther
The quotient sheaf of a valuation ring and some results on primes
Dressler, Robert Eugene
On an extension of asymptotic and natural density
Fossum, Timothy Vaughn
Centralizer rings of induced linear representations
Garner, Lyan Evan
Category theoretic topics in local prime theory
Hamel, Ray Owen
Group algebras and exceptional characters
Johnsonbaugh, Richard Fred
I. Classical fundamental group and covering space theory in the setting of Cartan and Chevalley II. Spaces and algebras of vectorvalued differentiable functions

Mohler, Lee Knight
Orders and fixed points in hereditarily unincoherent continua
Pfaffenberger, William Elmer
Operator algebras and related topics
Richen, Forrest Allen
Modular representations in split BN pairs
Turnidge, Darrell Ray
Torsion theories and rings of quotients

## PENNSYLVANIA

CARNEGIE-MELLON UNIVERSITY (10)

Davis, Paul Lawrence
Existence, uniqueness, and stability of solutions of a nonlinear functional equation
Gordon, Robert Fred
Existence and nonexistence in the large for paired quasi-linear wave equations
Hsiao, George Chia-Chu
Singular perturbations of boundary value problems for a class of nonlinear differential equations with a small parameter
Imler, Linnea Darlene
Extensions of pseudometrics and linear space-valued functions
Kraynek, William Theodore
Interpolation of sub-linear operators on generalized Orlicz and Hardy spaces
Simons, William Harris
Disconjugacy criteria for self-adjoint systems of differential equations

## Department of Computer Science

Earley, Jay
An efficient context-free parsing algorithm Fikes, Richard

A heuristic program for solving problems stated as nondeterministic procedures
Wagner, Robert
Some techniques for alorithm optimization with application to matrix arithmetic expressions
Waldinger, Richard J.
Constructing programs automatically using theorem proving

## LEHIGH UNIVERSITY(11)

Balakrishnan, Subramanian
Equilibrium deformations of harmonic materials
Braidi, Siham Chukri, (Miss)
Submanifolds of spheres
Davitt, Richard Michael
The automorphism group and structure of certain classes of finite p-groups
Fleming, Daniel John
Continuous decompositions
Jessup, Peter Gordon
Dual spaces and the mixed topology
Kay, Edwin Joseph
Characterizations and applications of sequential norms

McGivney, Raymond John, Jr.
Multiplier algebras and sequence spaces Shaughnessy, Edward Paul

Associated $t$-designs and automorphism groups of certain linear codes
Swetits, John Joseph
Some results on doubly iterated matrix methods of summability
Taylor, John William
On certain topological spaces in which sequential convergence plays a large role
Williams, Scott Warner
The transfinite cardinal covering dimension
PENNSYLVANIA STATE UNIVERSITY (13)
Cable, Charles Allen
On the decomposition of a group ring
De Leon, Morris Jack
Pisot sequences and Pisot-Vijayaraghavan numbers
Gardner, Merritt Post
Minimal prime ideals of function spaces
Gendler, Stephen Ian
Hecke L-series and some lattice point problems
Ho, Grace Ping-Poo
The integral equation formulations for the low Reynolds number hydrodynamics
Leonard, Philip Andrew
Irreducible translates of quartics in GF [ $\mathrm{q}, \mathrm{x}]$
Marin, Francisco Lisandro
Convergence in E-completely regular spaces
McArthur, William George
The Hewitt realcompactification of a product of two spaces
McNeil, Phillip Eugene
The structure of certain semigroups with two idempotents
Sillars, Walter Anthony
Formal properties of certain classes of essentially context dependent languages
Smith, Wilbur Lee
On infinite product measures and semiregular measures

Department of Computer Science
Burger, Robert Thorton
Best approximation of functions with partitioned norms

Department of Statistics
Joshi, Sharadchandra Waman
Certain estimation and structural problems for a class of multivariate discrete distributions

## TEMPLE UNIVERSITY (1)

Tepper, David
On the radius of convexity and boundary distortion of schlicht functions

Moore School of Electrical Engineering, Computer Information and Sciences
Deily, Don
Computer generated movies
Lieblein, Edward
Automata theory
Litofsky, Barry
Automatic classification
McEowen, James Royce
Personal info storage and retrieval
Morton, Richard Phillip
Hierarchy of processes
Rehert, Allen Frank
Algorithm for reducing queuing in switching
Reigle, Earl William
Parallelism in computing machines
Simmons, Dick Bedford
Automatic documentation of programs
Tebbe, Horst
Switching theory
Weinberg, Paul Richard
Time shared chemical info system
Wolfberg, Michael S.
Graph theory system
UNIVERSITY OF PITTSBURGH (11)
Izen, Carole Sloane
Generalized limits in locally compact ocompact topological groups
Long, Celestine James (Rev.)
Intrinsic functions on semi-simple algebras
Luke, Stanley David
On certain generalized Taylor transforms
Neurendorf, Edward James
Some properties of transformations of double sequences
Pao, Chia-Ven
Stability theory of nonlinear operational differential equations in Hilbert spaces
Smerek, Edward James
The summability of Cauchy products of double-series
Stitzinger, Ernest Lester Group properties in Lie algebras
Wenger, Robert Benjamin The fine spectra of the summability operators $C^{p}$ and $T(a)$
Zee, Yun-Cheng Analysis of Jacobi sums of various orders

Systems Management Engineering and Operations Research
Delon, Gerald
Cost methodologies in the pathology laboratory
Weisman, Joel
Engineering design optimization under conditions of risk

## RHODE ISLAND

BROWN UNIVERSITY (22)

Grove, Edward Andrew
SO(n) actions on differentiable manifolds with vanishing first and second integral Pontrjagin classes
Kiley, William Thomas
Automorphism groups on compact Riemann surfaces
Markoe, Andrew Guy
New techniques in deformation of complex structure
Rosenberg, Alan Elliot
On the semisimplicity of the group algebra
Tannenwald, Ronald M.
Generic properties of dynamical systems on open manifolds

## Division of Applied Mathematics

Bar Ness, Yeheskel
On the discrete control problem
Billotti, Joseph E., S. J.
Dissipative nonlinear functional differential equations
Bosarge, W. Edwin, Jr.
Infinite dimensional iterative methods and applications
Cruz, Marianito Angeles
Existence and stability theorems for a class of neutral functional equations
Curtain, Ruth Frances
Stochastic differential equations in a Hilbert space
Gonzales, Enrique Alberto Velasco
Generic properties of polynomial vector fields at infinity
Grossman, Stanley
Existence and uniqueness of nonlinear Volterra integral and integrodifferential equations
Helliwell, William S.
The asymptotic behavior of the solutions to a nonlinear system of integro-differential equations occurring in reactor dynamics
Henry, Daniel Bauman
Linear functionals of differential equations: Adjoints and boundary value problems
Lewis, James Thomas Approximation with convex constraints
Nusayr, Abdul-Majid
On stress analysis of an inhomogeneous transversely isotropic elastic solid
Slemrod, Marshall
An invariance principle for dynamical systems in Hilbert space with applications to the asymptotic stability of equilibria
Strauss, Charles Michael
3DPDP--A three-dimensional piping design program
Sweet, Daniel
Periodic families of systems possessing a first integral

Wan, San
Optimality and strong stability of control systems
Yin, Wan-Lee
Kinematics of viscoelastic flows
York, Harold Lee
An approximate method for the solution of time-dependent boundary value problems in kinetic theory

## SOUTH CAROLINA

CLEMSON UNIVERSITY (4)
Bardwell, Donald Milton
Combinatorial designs and geometric constraints
Bowie, Patrick Carlton
Development of a Banach algebra for FourierBessel functions
Reiter, Harold Braun
On embedding of topological spaces in a product of real number lines
Sparks, Arthur Godwin
Intersections of maximal sets and the generalized convex kernel

## UNIVERSITY OF SOUTH CAROLINA (3)

Kim, Joong Ho
A note on complete local rings
Smith, James Reaves
Local domains with topologically T-nilpotent
$\quad$ radical
Stone, David Ross
Torsion-free and divisible modules over
matrix rings

## TENNESSEE

## GEORGE PEABODY COLLEGE FOR TEACHERS

 (3)Garnett, Emma Whitlock
A study of the relationship between the mathematical knowledge and the mathematical preparation of undergraduate elementary education majors
Johnson, Phillip E.
A history of Cantorian set theory
Norris, Fletcher Ragland
Pupil achievement as a function of an inservice training program on mathematics concepts for sixth grade teachers

## UNIVERSITY OF TENNESSEE (9)

Baily, John Lay
A class of decompositions of $E^{3}$ which are factors of $E^{4}$
Bogar, Gary Allan
Oscillation and distribution of zeros of solutions to nth order linear differential equations

Boyd, William Simonton, Jr.
Repairing embeddings of 3-cells with monotone maps of $E^{3}$
Cowan, Richard Elbert
Angular and one-sided cluster sets for meromorphic functions
Farley, Reuben William
Positive Clifford semigroups on the plane
Kork, John Oliver
Information and sufficiency in a sequential test
Peterson, Allan Clemens
Distribution of zeros of solutions of linear differential equations of order four
Smith, James Alan
Pseudo-ordered functions over a finite field satisfying a permutation condition
Stewart, Gilbert Wright III
Some topics in numerical analysis

## VANDERBILT UNIVERSITY (4)

Coleman, Norman Pendleton, Jr.
Projections in continuous function spaces Hill, Edward Teber

Ideals in the modular group ring of a $P$-group Metts, Isaac Spigner, Jr.

A correcting process for solving differential equations numerically by the predictorcorrector method and an investigation of its effects on propagated error
Wells, Carroll Glenn
Weak homotopy lifting properties

## TEXAS

RICE UNIVERSITY (4)
Belfi, Victor Allen
Non-tangential homotopy equivalances
Johnson, Raymond Lewis
A priori estimates and unique continuation theorems for second order parabolic equations
Morgan, John Willard
Stable tangential homotopy equivalences
Shreve, David Carr
Approximation of translation invariantoperators in $L^{p}$

## TEXAS A \& M UNIVERSITY (11)

Heatherly, Henry Edward
Embedding of near-rings
Kleiner, Alexander
Matrix methods, unbounded multipliers and summability of unbounded series
Landry, Gordon Joseph
A restricted Runge-Kutta method
Lich, Steve Chong Hong
On certain classes of near-rings

Institute of Statistics
Katiyar, Anand Singh
Contributions to variance estimation in sample surveys
Ruud, Paul G.
An algorithm for the optimization of response surface designs
Speed, Fred Michael
A new approach to the analysis of linear models

## Department of Industrial Engineering

## Carey, Gary Roland

An optimal dynamic policy for the design and maintenance of flexible pavements
Geldbach, Arthur Robert
Polynomial forecasting utilizing exponential smoothing on successive coefficient determinations
Mogg, Jack M. Dependent stage attribute acceptance sampling
Nolen, James H.
Development of a programming language for multistage optimization

## TEXAS CHRISTIAN UNIVERSITY (8)

Aslan, Farhad
Some generalizations of metric spaces
Berzsenyi, George
Products of monodiffric functions
Bolen, James Cordell
A reproducing kernel function and convergence properties for discrete analytic function
Drennan, David Lee
Value determining sets and basis of kernels for a functional Hilbert space
Harvey, James R.
Sequence spaces and the basis concept in Banach spaces
Howes, Norman Ray
Well ordered sequences
Marrache, Nazem M. Certain local properties of topological spaces
Mosher, James Roberts Generalized semirings of quotients

TEXAS TECHNOLOGICAL COLLEGE (3)
Amburgey, Jay K. A theory for rectangular matrices
Crossley, Eugene Semitopological properties and related topics
Lambert, Howard Topological characterization of an inner product space

## UNIVERSITY OF HOUSTON (3)

Hartfiel, Darald Joe
On the structure of certain matrix classes

Martens, Phillip Arthur
Homology and homotopy of infinite dimensional manifolds
Proctor, Clarke Wayne
Metrizability in Moore spaces

## UNIVERSITY OF TEXAS (14)

Baker, Johnnie W.
Some uncomplemented subspaces of $\mathrm{C}(\mathrm{X})$ of the type $\mathrm{C}(\mathrm{Y})$.
Chatfield, John A.
Existence of product integrals
Crim, Sterling C.
The two-dimensional $z$ transform with real domain and parameters
Evans, Noel D.
Functors in categories of Banach spaces
Garrett, Bert D.
Concerning pointwise discontinuous functions Huffman, Louie C.

Generalized functions of two discrete variables
Lamb, John F.
The structure of Hjelmslev space--a generalization of projective space
Proffitt, Michael H.
Concerning uncountable collections of mutually exclusive compact continua
Manougian, M. N.
On the Perron integral and solutions to partial differential equations
Sawey, Bennett C.
On a generalization of continuity
Turlington, Boyce L.
Numerical calculations of the Laplace transform by the use of Legendre polynomials
Vargas, Robert W.
On the $\beta$-topology on $\mathrm{C}(\mathrm{X})$
Wilks, Charles E.
The regularity of a product measure using regular conditional measure

Department of Computer Sciences
Darden, Stephen Charles
A contextual recognition system for formal languages

## UTAH

UNIVERSITY OF UTAH (6)
Billis, Mitchell James
Factor groups and complements in Cartesian products
Cannon, James Welden
Tame subsets of 2 -spheres in $E^{3}$
Edwards, John Roy
Characterizing Werkfelder of certainclasses of summability methods
Lether, Frank Gene
Cross-product cubature error estimates

Mansfield, Lois Edna
Optimal approximation and error bounds in spaces of multivariate functions
Newman, Stephen Edmond, Jr.
Harmonic analy-is on idempotent semigroups

## VIRGINIA

## UNIVERSITY OF VIR GINIA (10)

Bouldin, Richard Hindman Perturbed eigenvalues
England, William Tilford Spectral theory for systems of Toeplitz and singular integral operators
Kriete, Thomas Latimer III Commutants of a class of operators
Moore, Berrien III Outer factorization for vectorial Toeplitz operators
Morrel, Bernard Baldwin, Inversion of matrices of operators
Page, Lavon Barry
Bounded and compact vectorial Hankel operators
Wood, James Burnette
Torsion-free Abelian groups of rank 2
Department of Applied Mathematics and Computer Science
Hartberger, Ronald John
An analytic approach to calculus of variations and optimal control theory
Hord, Richard Anderson
A family of quadrature formulas and related numerical analysis
Wulf, William Allan
A notation for digital systems
VIRGINIA POLYTECHNIC INSTITUTE (13)
Argentiero, Peter D.
Euler sets in $E_{n}$
Ward, Frederick R.
On free and projective topological groups
Whitley, W. Thurman
Homotopy types of a certain subspace of the deleted product space

## Department of Statistics

Beus, Gary Boyd
Testing hypotheses involving categorical data
Bowen, Jacob Van, Jr.
Maximum likelihood estimation of parameters using grouped data samples
Cornell, John Andrew
A response surface approach to the mixture problem when the mixture components are categorized
Custer, Stephen Wampler
Stochastic model for pollution and dissolved oxygen in estuaries

Gibson, Allen Edward
Some aspects of time-dependent one-dimensional random walks
Hogan, John Wesley
Homomorphisms of $\omega^{n}$-right cancellations, $\omega^{\mathrm{n}}$-bisimple, and $\omega^{\mathrm{n}}$ I-bisimple semigroups
Lemon, Glen Hortin
Empirical Bayes analysis of some sequential experiments
Loadholt, Claude Boyd
Deterministic models of evolutionary processes involving genetic selection due to maternal-fetal genotypic interaction
Rencher, Alvin Clarence
The empirical Bayes approach to analysis of variance and linear regression
Thomas, Marlin Amos
A comparison of the classical and inverse methods of calibration in regression

## WASHINGTON

## UNIVERSITY OF WASHINGTON (16)

Cornelius, Eugene F. Jr.
The quasi-endomorphism algebra of a tor-sion-free Abelian group
Coury, John Edward
Sets of uniqueness and multiplicity on the infinite product of compact topological groups
Douglas, Robert James
Circuits in the d-cube
Kurshan, Robert Paul
Injective modules with finitely generated essential socle
Lightman, Denis Roger
The closed span of the translates of functions in $L_{p}(G)$
Renz, Peter Lewis
Smooth extensions and extractions in infinite dimensional Banach spaces
Schultz, Phillip
Periodic Abelian groups
Shorack, Roger
The slippage, selection, and detection problems
Topp, William Robert
On balanced-faithful rings
Vinsonhaler, Charles Irvin
On QF-3 rings
Werth, John St.Clair, Jr.
Subgroups of torsion complete Abelian pgroups
Williams, George Buchanan
S -objects and intrinsic classes in Abelian categories
Williams, Jerry Blake
On Yoneda rings
Zaks, Joseph
On minimal complexes and decompositions of $E^{n}$
Department of Biomathematics
Anderson, Gary Duane
A comparison of methods for estimating a probability density function

Gilbert, Richard Orrie
A Monte Carlo study of the robustness and power of analysis of variance and competing rank tests for Scheffe's mixed model

## WASHINGTON STATE UNIVERSITY (8)

Ali, Mir Kursheed
On integral stability
Atherton, Charles Russell, Jr.
Lattice ideals and intrinsic lattice topologies
Hearsey, Bryan Vandiver
On extending regularity and other topological properties to convergence spaces
Hewer, Gary Arthur
Topics in stability theory
Knoshaug, Clayton M.
Generalized sequences
Rabung, John R.
Some local distributions in number theory
Strand, Allen R.
Phase plane analysis for finite difference equations
Woo, Norman
On bases for the set of integers

## WISCONSIN

UNIVERSITY OF WISCONSIN (26)
Aschbacher, Michael
Collineation groups of symmetric block designs
Boyce, Stephen Scott
A formulation of Segal's model for quantum mechanics in terms of a proposition system
Carasso, Alfred
An analysis of numerical methods for parabolic problems over long times
Chambers, Graham, Jr.
p-normally embedded subgroups of finite soluble groups
Chee, Fak-Soong
Bounded holomorphic functions in several complex variables
Dorr, Fred
The asymptotic behavior and numerical solution of singular perturbation problems with turning points
Flytzanis, Elias
Eigenoperators of ergodic transformations Foster, James

Branching processes involving immigration
Goldstein, Martin Ivan
Critical age-dependent branching processes: single and multitype
Jensen, Richard
Cross sectionally connected spheres
Klatt, Gary
Pre-self-injective rings and projective modules over semiperfect rings
Leung, David Chilai
On nilpotent divisible groups

## Malkevitch, Joseph

Properties of planar graphs with uniform verrtex and face structure
Roosenraad, Cris
Inequalities with orthogonal polynomials
Row, William Harry
Compact subsets of 3 -manifolds definable by cubes-with-handles
Schindler, Susan
Some transplantation theorems for the generalized Mehler Transform and related asymptotic expansions
Smith, Kirby
On Jordan and associative rings
Smith, Peter
On the estimation of the Fourier coefficients of cusp forms for Hecke groups
Tall, Franklin David
Set-theoretic consistency results and topological theorems concerning the normal Moore space conjecture and related problems
Vasavada, Mahavirendra
Closed ideals and linear isometrics of certain function spaces
Yang, Chung-Chun
A generalization of the theorem of the Tumusa and Clunie and its application to the value distribution of meromorphic functions

Department of Computer Sciences
Douglas, Alan J.
Approximation of functions in relative error Shirey, Robert W.

Implementation and analysis of efficient graph planarity testing algorithms
Su-Yueh-Wei, Stanley
A distributional theory of meaning
Timmreck, Eric M.
Advising by computers: Course advising, medical treatment, general advising
Williams, John H.
Bounded context parsable grammars
UNIVERSITY OF WISCONSIN-MILWAUKEE (3)

Deshpande, Madhukar Ganpatrao Right subdirectly irreducible rings
Dunn, Samuel Lee Rings with quasi-Frobenius quotient rings McMorris, Fred Raymond

The maximal quotient semigroup of a semigroup

## WYOMING

UNIVERSITY OF WYOMING (1)
Kreiling, Daryl
The hereditary property in general radical theory

## CANADA

## McGILL UNIVERSITY (4)

Kim, Hayon
Topological structures on categories
Mendelsohn, E.
Full embeddings and the category of graphs with applications to topology and algebra
Raphael, R.
Algebraic extensions of regular rings
Shuster, Jonathan
Properties of the inverse Gaussian distribution

McMASTER UNIVERSITY (3)
Gerhard, James Arthur
The lattice of equational classes of idempotent semigroups
Graves, William Henson
Representation theory of partially ordered vector spaces
Park, Young Lim Quotient-like extensions of rings of functions

## QUEEN'S UNIVERSITY (3)

Fakhruddin, Syed Mohamed Modules over a valuation ring
Gulliksen, Tor H. Homological invariants of local rings
Marshall, Murray A. The ramification filters of Abelian extensions of a local field

## UNIVERSITY OF ALBERTA (4)

Danyluk, Harry Theodore
The flow of an incompressible elastic-perfectly plastic solid
Kim, Yong-Woon Bitopological function spaces
Pandya, Guatam Nileshchandra On automorphisms of finite simple groups
Prasad, Jagdish
Some poised and non-poised problems of interpolation

UNIVERSITY OF BRITISH COLUMBIA (12)

Allegretto, Walter
Comparison and oscillation theorems for elliptic equations
Biggs, Richard G.
Some generalizations of nilpotence in ring theory
Chew, Kim-Peu
On certain rings of E-valued continuous functions
Headley, Velmer B.
Oscillation theorems for elliptic differential equations

Heinicke, Allan George
Some results in the theory of radicals of associative rings
Hillel, Joel S.
Existence of algebras of symmetry-classes of tensors with respect to translationinvariant pairs
Kerr, Charles Randall Inner equivalence of thick subalgebras
Lau, Anthony To-Ming
On topological semigroups with invariant means in the convex hull of multiplicative means
MacLean, Douglas W.
Differentiable engulfing and covering of manifolds
Nielsen, Ole A.
Maximal Abelian subalgebras of von Neumann algebras
Rennie, Robert R.
Finite mixtures of distributions with common central moments
Wong, James C. S.
Topological invariant means on locally compact groups

UNIVERSITÉ DE MONTRÉAL (4)
Arminjon, Paul
Quelques aspects de la théorie des equations Différentielles opérationnelles
Garancon, Maurice
Le rang de certaines variétés closes
Labelle, Gilbert
Sur certaines classes de fonctions analytiques
Roy, Roch
Processus stochastiques sur la sphère
UNIVERSITY OF OTTAWA (2)
Cohen, Rina S. (Mrs)
Cycle rank of transition graphs and the star height of regular events
Wong, Pak-Ken
Complemented Banach *-algebras

## UNIVERSITY OF TORONTO (11)

Baxter, John Robert
A class of ergodic automorphisms
Chan, Chiu Yeung
On some nonlinear initial-boundary value problems for the heat equation
Clements, John Carson
Perturbation methods for nonlinear differential operators
Lee, Yoong Sin
Invariant structures in inference
Malzan, Jerry
Real finite linear groups
Park, Ralph Allen
On Barner arcs and curves
Turgeon, Jean Maurice
On the rank numbers of an arc

Department of Computer Science
Ash, James Howard
Analysis of multistep methods fu. wrial second-order ordinary differential equations
Corneil, Derek Gordon
Graph isomorphism
Stewart, Neil F.
The comparison of numerical methods for ordinary differential equations

Department of Industrial Engineering
Shaw, Gordon Charlton
Contributions to transportation scheduling

## UNIVERSITY OF WATERLOO(6)

Department of Applied Analysis and Computer Science
Ehle, Byron Leonard
On Padé approximations to the exponential function and a-stable methods for the numerical solution of initial value problems
Taylor, Mark Adrian
Nets and binary systems
Williams, Hugh Cowie
A generalization of the Lucas functions

## Department of Combinatorics and Optimization Robertson, G. Neil <br> Graphs minimal under girth, valency and connectivity constraints

Department of Statistics
Kalbfleisch, John David
Likelihood methods
Lawless, Jerald F.
Quasi-residual balanced incomplete block designs

## UNIVERSITY OF WESTERN ONTARIO (4)

Dunham, Charles Burton
Chebyshev approximations by families of continuous functions
Mead, Ernest Roy
On the exact distribution of linear combinations of order statistics
Phillips, Robert John
Scales of logarithmic summability

## Department of Applied Mathematics <br> Zabransky, Friedrich <br> Numerical solutions for fluid flow in rectangular cavities

## UNIVERSITY OF WINDSOR (1)

Srivastava, Sadanand
Radiation conditions and uniqueness theorems for an $n$ dimensional wave equation in an infinite domain

## VISITING MATHEMATICIANS

The list of visiting mathematicians is being expanded this year to include both foreign mathematicians visiting in the United States and Canada, and Americans visiting abroad. Note that there are two separate lists.

FOREIGN MATHEMATICIANS VISITING IN THE UNITED STATES AND CANADA

| Name and Home Country | Host Institution | Field of Special Interest | Period of Visit |
| :---: | :---: | :---: | :---: |
| Abe, Eiichi (Japan) | Indiana University | Algebra | 9/68-6/70 |
| Abraham, Douglas B. (England) | Massachusetts Institute of Technology | Physics | 9/69-6/70 |
| Akagawa, Yasumasa (Japan) | Institute for Advanced Study | Algebraic Number Theory | 10/69-4/70 |
| Alemany, Ricardo (Argentina) | Rockefeller University | Biomathematics | 7/68-1/70 |
| Alexits, George D. (Hungary) | University of Alberta | Approximation Theory | 9/69-1/70 |
| Anh, Nguyen H. (Vietnam) | Institute for Advanced Study | Group Representation Theory | 10/69-4/70 |
| Armbrust, Manfred K. (Germany) | University of Houston | Algebra and Logic | 9/69-5/70 |
| Arscott, F. M. (England) | University of Calgary | Differential Equations | 9/69-4/70 |
| Aubert, Karl E. (Norway) | Tufts University | Harmonic Analysis | 9/69-9/70 |
| Baker, Alan (England) | University of Michigan | Number Theory | 8/69-10/69 |
| Balachandran, V.K. (India) | Institute for Advanced Study | Topology and Algebra | 10/69-4/70 |
| Balslev, Erik (Denmark) | SUNY at Buffalo | Applied Mathematics, Analysis, Differential Operators | 9/69-6/70 |
| Barner, M. (Germany) | University of Toronto | Geometry | 9/69-12/69 |
| Barth, Wolf (Germany) | Massachusetts Institute of Technology | Complex Analysis | 9/69-6/70 |
| Beck, Istvan (Norway) | University of Illinois | Commutative Algebra | 9/69-6/70 |
| Belousov, V.D. (Russia) | University of Waterloo | Universal Algebras | 9/69-6/70 |
| Bernard, Alain (France) | University of Kentucky | Banach Algebras, Complex Analysis | 6/69-8/69 |
| Bhattacharyya, Prodipeswar (India) | University of Minnesota | Complex Variables | 9/69-6/70 |
| Bieri, Hanspeter (Switzerland) | Lehigh University | Analysis | 9/69-6/70 |
| Birch, Bryan (England) | University of Michigan | Number Theory | $3 / 70-4 / 70$ |
| Bliedtner, Jürgen E. (Germany) | Yale University | Analysis | 9/69-6/70 |
| Bokobza, Julianne (France) | Purdue University | Partial Differential Equations | 9/68-6/70 |
| Bombieri, Enrico (Italy) | Columbia University | Mathematics | 9/69-7/70 |
| Borges, Rudolf (Germany) | New Mexico State University | Mathematical Statistics | 9/69-5/70 |
| Boughon, Pierre (France) | University of Kentucky | Algebra | 9/69-12/69 |
| Bruehlmann, Heinz (Switzerland) | University of Minnesota | Differential Geometry | 9/69-6/70 |
| Brunel, Antoine (France) | University of Minnesota | Ergodic Theory | 9/69-6/70 |
| Burghelea, Dan (Rumania) | Institute for Advanced Study | Differential Topology | 10/69-4/70 |
| Burns, Robert Geoffrey <br> (Australia) | McGill University | Group Theory | 7/69-7/70 |
| Cadogan, Charles C. (Jamaica) | University of Waterloo | Graph Theory | 9/69-9/70 |
| Calderon, Calixto (Argentina) | University of Minnesota | Real Variables | 9/69-6/70 |
| Cartwright, Mary (Great Britain) | Claremont Graduate School | Complex Analysis and Differential Equations | 9/69-3/70 |
| Christiansen, Peter (Denmark) | New York University, University Heights | Electromagnetic and Elastic Wave Propagation | 9/69-6/70 |
| Chuaqui, Rolando B. (Chile) | Institute for Advanced Study | Logic and Foundations | 10/69-4/70 |
| Collins, Michael J. (England) | University of Illinois, Chicago Circle | Group Theory | 9/69-6/70 |

Name of Home Country
Copas, John (England)
Davies, Edward B. (England)

Davis, Arthur W. (Australia)
Deleanu, Aristide (Rumania)
Doi, Koji (Japan)
Dolezal, Vaclav (Czechoslovakia)
Dou, Alberto, S. J. (Spain)

Dupont, J. L. (England)
Dvoretzky, Aryeh (Israel)
Ehrenfeucht, Andrzej (Poland)
Eichler, M. (Switzerland)
Elliott, Peter (England)
Ellison, William J. (England)
Endler, Otto (Germany)
Erle, Dieter H. (Germany)

Fairbairn, W. M. (England)
Fajtlowicz, Siemion (Poland)
Ferus, Dirk (Germany)

Fiedler, Miroslav
(Czechoslovakia)
Foias, Ciprian (Rumania)
Follmer, Hans D. (Germany)
Forte B. (Italy)
Frolik, Zdenek (Czechoslovakia)
Fukushima, Masatoshi (Japan)
Furstenberg, Harry (Israel) Gagliardo, Emilio (Italy)

Garcia-Diez, Jose (Spain)
Garling, Djh (England)
Godement, Roger (France)
Gopalakrishna, Avanche V. (India)
Grimeisen, Gerhard F. (Germany)
Grossman, A. (France)
Grunbaum, Francisco A.
(Argentina)
Grunenfelder, Luzius
(Switzerland)
Güntzer, U. (Germany)
Gupta, Hansraj (India)
Gupta, R. C. (India)
Hackenbroch, Wolfgang (Germany)
Hadeler, Karl P. (Germany)
Hájek, Jaroslav (Czechoslovakia)
Halford, W. D. (New Zealand)
Hannan, Edward J. (Australia)

Host Institution
SUNY at Buffalo
Massachusetts Institute of Technology
University of North
Carolina
Syracuse University
Institute for Advance Study
SUNY at Stony Brook
University of Notre Dame

Institute for Advanced Study
Columbia University
University of Southern California
University of Maryland
University of Colorado
University of Michigan
University of Rochester
Institute for Advanced Study
University of Waterloo SUNY at Buffalo

Massachusetts Institute of Technology

Auburn University
Indiana University
Massachusetts Institute of
Technology
University of Waterloo

SUNY at Buffalo
University of Illinois
Yale University
Oregon State University
Wichita State University
Lehigh University
Institute for Advanced Study
Wartburg College
University of Colorado
University of Toronto
Rockefeller University
University of Washington
University of Maryland
University of Alberta
Washington State University
University of Washington
University of Minnesota
Florida State University
University of Toronto
Yale University
$\frac{\text { Field of Special Interest }}{\text { Biometry and Time Series }}$
Functional Analysis
Statistics 8/69-8/70
Algebraic Topology $9 / 69-6 / 70$
10/69-4/70
9/69-9/70

2/69-2/70
10/69-4/70
7/69-9/69

9/68-6/70

1/70-6/70
9/69-6/70
8/69-7/70
9/69-2/70
10/69-4/70
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9/69-6/70
10/69-4/70
9/69-6/70
9/69-6/70
9/69-12/69
7/66-6/70
9/69-6/70
1/70-6/70
9/69-1/70
9/68-9/70
9/69-6/70
9/69-3/70
8/69-6/70
9/69-6/70
1/70-6/70

Name of Home Country
Hanner, Olof (Sweden)

Harada, Koichiro (Japan)

Hasse, Helmut (Germany) Hawkes, John (England)

Hedrlin, Z. (Czechoslovakia)
Heilmann, Ole Jan (Denmark)
Hengartner, Otto (Switzerland)
Hering, Franz (Germany)
Hervé, M. (France)
Holsztynski, Wlodek (Poland)
Horowitz, Shlomo (Israel)
Hotzel, Eckhart (Germany)
Itô, Seizô (Japan)
Ivkovic, Zoran (Yugoslavia)

Iwano, Masahiro (Japan)
Jacobs, Konrad (Germany)
James, Ioan M. (England)
Jauch, J. M. (Switzerland)
Jawa, M.S. (India)
Jech, Thomas (Czechoslovakia)
Jorgens, Konrad (Germany)
Kaiser, Klaus (Germany)
Kalton, Nigel J. (England)
Kapur, J. N. (India)
Kato, Mitsuyoshi (Japan)
Klingelhofer Klaus (Germany)
Kondo, Takeshi (Japan)
Koizumi, Shoji (Japan)
Kotzig, A. (Czekoslovakia)
Koutnik, Vaclav (Czekoslovakia)
Kreiss, Heinz-Otto (Sweden)
Kuiper, Nicolaas H. (Netherlands)
Kussmaul, Rainer (Germany)
Lal, Nand (India)
Lalitha, Ramanathan (India)
Lamont, Patrick John (Scotland)
Lazar, Aldo J. (Israel)
Lee, Ronnie (China)

Leslie, Joshua A. (Nigeria)
Leutwiler, Heinz (Switzerland)

| Host Institution | Field of Special Interest | Period of Visit |
| :---: | :---: | :---: |
| Western Washington State College | Convexity | 9/69-8/70 |
| Institute for Advanced Study | Group Theory | 10/69-4/70 |
| SUNY at Buffalo | Algebra, Number Theory | 1/70-6/70 |
| University of Michigan | Probability | 8/69-7/70 |
| Tulane University |  | 1/70-5/70 |
| Massachusetts Institute of Technology | Applied Mathematics | 8/69-1/70 |
| Indiana University | Statistics | 9/69-6/70 |
| University of Washington |  | 8/69-8/70 |
| University of Maryland | Several Complex Variables | 1/70-6/70 |
| University of Michigan | Topology | 8/69-7/70 |
| University of Minnesota | Probability | 9/69-6/70 |
| McMaster University | Semigroups | 1/69-8/70 |
| University of Wisconsin | Differential Equations | 9/69-8/70 |
| University of Missouri- | Probability Theory | 9/69-8/70 |
|  | Differential Equations | 8/69-8/70 |
| Ohio State University | Probability | 9/69-5/70 |
| University of Virginia | Topology | 4/70 |
| University of Colorado | Mathematical Physics | 9/69-10/69 |
| University of MissouriRolla | Continuum Mechanics | 1/69-8/70 |
| SUNY at Buffalo | Logic | 9/69-6/70 |
| University of Colorado | Mathematical Physics | 2/70-3/70 |
| University of Houston | Algebra | 9/69-5/70 |
| Lehigh University | Analysis | 9/69-6/70 |
| Carnegie-Mellon University | Applied Mathematics | 9/69-6/70 |
| Institute for Advanced Study | Topology | 10/69-4/70 |
| University of Minnesota | Partial Differential Equations | 9/69-6/70 |
| Institute for Advanced Study | Finite Groups | 10/69-4/70 |
| SUNY at Buffalo | Algebraic Geometry | 9/69-6/70 |
| University of Calgary | Graph Theory and Statistics | 8/69-8/70 |
| Kent State University | Topology | 9/69-6/70 |
| California Institute of Technology | Numerical Analysis | 1/70-6/70 |
| Institute for Advanced Study | Topology | 10/69-4/70 |
| University of Michigan | Numerical Analysis | 8/69-7/70 |
| Louisiana State University, Baton Rouge | Functional Analysis | 9/69-5/70 |
| McNeese State College | Analysis | 9/69-9/71 |
| Saint Mary's College | Number Theory | 9/69-6/70 |
| Louisiana State University, Baton Rouge | Analysis | 9/69-5/70 |
| Institute for Advanced Study | Differential Topology | 10/69-4/70 |
| Institute for Advanced Study | Algebraic Topology | 10/69-4/70 |
| University of Washington | Potential Theory and Subharmonic Functions | 9/69-6/70 |


| Name and Home Country | Host Institution | Field of Special Interest | Period of Visit |
| :---: | :---: | :---: | :---: |
| Lewis, John T. (England) | Institute for Advanced Study | Analysis | 10/69-4/70 |
| Lin, Sue-Chin (China) | Institute for Advanced Study | Perturbation Theory | 10/69-4/70 |
| Lin, Ta-Feng (China) | Rockefeller University | Probability Theory | 7/68-6/70 |
| Liron, Nadev (Israel) | New York University, University Heights | Numerical Analysis | 9/69-8/70 |
| Lorenz, Dan Hans (Israel) | University of Virginia | Partial Differential Equations and Generalized Functions | 9/69-6/70 |
| Lotz, Heinrich P. (Germany) | University of Illinois | Functional Analysis | 9/69-8/70 |
| Lusztig, Gheorghe (Rumania) | Institute for Advanced Study | Algebraic and Differential Topology | 10/69-4/70 |
| Maass, H. (Germany) | University of Maryland | Automorphic Forms | 9/69-6/70 |
| Macbeath, M. (United Kingdom) | University of Toronto | Geometry of Numbers | 9/69-11/69 |
| Malviya, Banshi Dhar (India) | McMaster University | Summability and Functional Analysis | 1/69-8/70 |
| Mangeron, Demetre (Rumania) | University of Alberta | Partial and Integro-Partial Differential Equations | 10/69-9/70 |
| Martin-Löf, Anders (Sweden) | Rockefeller University | Probability | 9/69-6/70 |
| Masuda, Kyuya (Japan) | Stanford University | Partial Differential Equations | 10/69-8/70 |
| Maumary, Serge (Switzerland) | Institute for Advanced Study | Algebraic and Differential Topology | 10/69-4/70 |
| McCartan, Sean D. (North Ireland) | Northern Illinois University | Theory of Semigroups | 9/69-6/70 |
| McCrudden, Michael (England) | University of Washington | Topological Groups | 9/69-6/70 |
| Mc Kay, John (Great Britain) | California Institute of Technology | Group Theory | 9/69-9/70 |
| Melchoir, H. (Germany) | University of Waterloo | Geometric Algebra | 9/69-12/69 |
| Michael, Ian McRae (Scotland) | McMaster University | Differential Equations | 9/69-8/70 |
| Milne-Thomson, L. M. (Australia) | University of Calgary | Applied Mathematics | 1/70-6/70 |
| Mishra, R.S. (India) | University of Waterloo | Relativity, Differential Geometry, Fluid Mechanics | 9/69-12/69 |
| Miyake, Toshitsune (Japan) | Institute for Advanced Study | Number Theory | 10/69-4/70 |
| Mogi, I. (Japan) | University of Saskatchewan, Regina | Differential Geometry | 9/69-5/70 |
| Munkholm, H. J. (Denmark) | University of Illinois, Chicago Circle | Algebraic Topology | 9/69-6/70 |
| Myhill, John (England) | University of Michigan | Logic | 9/69-5/70 |
| Naganuma, Hidehisa (Japan) | Institute for Advanced Study | Number Theory | 10/69-4/70 |
| Naimark, M. A. (Russia) | Indiana University | Functional Analysis | 2/70-6/70 |
| Natarajan, Subrahmanya (India) | Florida State University | Probability Theory | 6/69-6/70 |
| Necas, Jindrich (Czechoslovakia) | University of Illinois, Chicago Circle | Partial Differential Equations | 9/69-6/70 |
| Neukirch, Jurgen (Germany) | Massachusetts Institute of Technology | Algebraic Number Theory | 9/69-6/70 |
| Neumann, B. H. (Australia) | Vanderbilt University | Algebra | 9/69-1/70 |
| Neumann, Mrs. Hanna (Australia) | Vanderbilt University | Algebra | 9/69-1/70 |
| Neumann, Peter M. (England) | Vanderbilt University | Group Theory | 7/69-1/70 |
| Newman, M. H. A. (England) | University of Illinois | Topology | 9/69-1/70 |
| Novak, J. (Czechoslovakia) | University of California, Riverside | General Topology | 1/70-3/70 |
| Okuyama, Akihiro (Japan) | University of Pittsburgh | Topology | 8/69-7/70 |

Oosterhoff, Jacobus (The Netherlands)
Ostrowski, Alexander M. (Switzerland)

Petrovanu, Dan (Rumania)
Pilz, Günter (Austria)
Pittie, Harsh V. (India)

Porta, Horacio (Argentina)
Procesi Claudio (Italy)
Prolla, Joao Bosco (Brazil)

Radcliffe, John (England)
Radjaǔi, H. (Iran)
Rado, F. (Rumania)

Raghavan, T. E.S. (India)

Ramachandran, B. (India)
Rao, J.S. (India)
Rees, Elmer G. (England)

Reimann, Hans M. (Finland)
Remmert, R. (Germany)

Rhodes, Frank (England)
Richter, Michael (Germany)
Robertson, Alex P. (England)
Robinson, John (Australia)
Roggenkamp, Klaus W. (Germany)
Roquette, Peter (Germany)
Rosenfeld, Moshe (Israel)

Row, D. (Australia)
Roy, Amit (India)
Ruedy, Reto A. (Switzerland)
Sabbagh, Gabriel A. (France)
Saito, Tosiya (Japan)
Sasakura, Nobuo (Japan)

Scharlau, Winfried (Germany)

Schiffmann, Gerard (France)

Schinzel, Andre (Poland)
Schumacher, Dietmar (Germany)
Schwarz, Hubert F. (Germany)

Scott, Alastair J. (England)

Sedlacek, J. (Czeckoslovakia)
Segal, Graeme B. (England)

Seymour, Robert M. (England)
Host Institution
University of Oregon
SUNY at Buffalo
University of Michigan
University of Arizona
Institute for Advanced
Study
University of Illinois
Columbia University
University of Rochester

SUNY at Buffalo
University of Toronto University of Waterloo

University of Illinois, Chicago Circle

Catholic University
Indiana University
Institute for Advanced Study

University of Michigan
University of Maryland

Wesleyan University University of Texas

Swarthmore College
University of Connecticut
McGill University
University of Michigan
Louisiana State University Baton Rouge
University of Toronto
Columbia University
Columbia University
Yale University
University of Minnesota
Institute for Advanced Study
Institute for Advanced Study

Institute for Advanced Study
University of Michigan
McMaster University
California State College at Los Angeles

University of North Carolina

University of Calgary
Institute for Advanced Study
Institute for Advanced Study

Name and Home Country
Soedigdomarto, Moedomo (Indonesia)
Stepanek, V. (Czekoslovakia)
Stewart, William Brian (Scótland)
Steutel, Fred W. (Netherlands)
Stöhr, Karl Otto (Germany)
Storrer, Hans Heinrich (Switzerland)
Street, Ross Howard (Australia)
Sugiura, Mitsuo (Japan)

Swetharanyam, Sundaram (India)
Sz. -Nagy, Bela (Hungary)
Taylor, P.J. (England)
Teng, Tsing-houa (China)
Thöni, Werner (Switzerland)
Thedy, P. Armin (Denmark)
Trautmann, E. Gunther (Germany)
Traverso, Carlo (Italy)
Tweed, John (Scotland)

Tzafriri, Lior (Israel)
Ullom, Stephen V. (England)
Unterberger, André (France)
van Dijk, Gerrit (Netherlands)
Vesentini, Edoardo (Italy)
Vorel, Zdenek (Czekoslovakia)

Welsh, D. J. A. (England)
Wik, Ingemar (Sweden)
Wilkinson, James H. (England)
Wils, Willibrordus I. M.
(The Netherlands)
Wohlfahrt, K. (Germany)
Zajtz, A. (Poland)
Zama, Nobuo (Japan)
Zizler, Vaclav (Czechoslovakia)

Host Institution
University of Illinois

University of Calgary
University of Illinois
University of Texas
McGill University
Cornell University
Tulane University
New Mexico State University

McNeese State College Indiana University
University of Calgary
University of Notre Dame
Cornell University
Yale University
University of Notre Dame
Columbia University
North Carolina State University
University of Washington
Institute for Advanced Study
Purdue University
Institute for Advanced Study
University of Maryland
University of Southern California
University of Waterloo University of Washington

Stanford University
Massachusetts Institute of Technology

University of Illinois
University of Waterloo
University of Illinois
University of Washington

| Field of Special Interest | Period of Visit |
| :---: | :---: |
| Functional Analysis | 9/69-6/70 |
| Statistics | 9/69-9/70 |
| Group Theory | 9/69-6/70 |
| Probability \& Statistics | 9/69-5/70 |
| Algebra and Number Theory | 9/69-4/70 |
| Algebra | 7/69-9/70 |
| Category Theory | 9/69-5/70 |
| Lie Groups \& Lie Algebra | 9/69-8/70 |
| Computer Science | 9/69-9/70 |
| Functional Analysis | 2/70-6/70 |
| Computing Science | 7/69-7/70 |
|  | 3/70-5/70 |
| Topology | 4/69-3/70 |
| Algebra | 9/69-6/70 |
| Several Complex Variables | 9/69-2/70 |
| Mathematics | 9/69-7/70 |
| Elasticity and Transform Theory | 9/69-6/70 |
| Functional Analysis | 6/69-9/70 |
| Algebraic Number Theory | 9/69-6/70 |
| Partial Differential Equations | 9/68-6/70 |
| Harmonic Analysis on Lie Groups | 10/69-4/70 |
| Analytic Spaces | 9/69-1/70 |
| Ordinary Differential Equations Control Theory | ,9/69-6/70 |
| Graph Theory | 9/69-12/69 |
| Fourier Analysis | 9/69-6/70 |
| Numerical Analysis | 9/69-1/70 |
| C*-Algebras | 9/69-6/70 |
| Discontinuous Groups | 9/69-6/70 |
| Geometric Objects, Functional Equations, Linear Algebra | 9/69-6/70 |
| Automata Theory | 8/69-8/70 |
| Linear and Nonlinear Functional Analysis | 9/69-6/70 |

## AMERICANS VISITING ABROAD

| Ahern, Patrick (U.S.A.) | University of Nice, France | Analysis | 9/69-8/70 |
| :---: | :---: | :---: | :---: |
| Allendoerfer, Carl B. (U.S.A.) | University of Liverpool, England | Differential Geometry and Topology | 3/70-6/70 |
| Andrews, Fred C. (U.S.A.) | University of Tampere, Finland | Mathematical Statistics | 9/69-7/70 |
| Antman, Stuart (U.S.A.) | Oxford University, England | Applied Mathematics | 9/69-6/70 |
| Arnold, B. H. (U.S.A.) | National Taiwan Norman University | General Topology | 9/69-6/70 |
| Askey, Richard A. (U. S. A.) | Mathematische Centrum, The Netherlands | Analysis | 6/69-8/70 |
| Bebernes, Jerrold W. (U.S.A.) | Universitả Degli Studi Istituto Matematico 'Ulisse Dini', Italy | Ordinary Differential Equations | 9/69-6/70 |
| Boswell, Rupert Dean (U.S.A.) | University of Warwick, England | Homological Algebra | 6/69-6/70 |


| Name and Home Country | Host Institution | Field of Special Interest | Period of Visit |
| :---: | :---: | :---: | :---: |
| Brualdi, Richard A. (U.S.A.) | The University, Sheffield, England | Algebra \& Combinatorics | 6/69-7/70 |
| Burkholder, Donald L. (U.S.A.) | University of London, England | Statistics | 9/69-8/70 |
| Cavior, Stephan R. (U.S.A.) | Israel Institute of Technology | Number Theory | 9/69-1/70 |
| Cohen, Paul J. (U.S. A.) | Imperial College, London | Harmonic Analysis, Partial Differential Equations, Logic | 10/69-8/70 |
| Dicker, Daniel (U.S.A.) | Imperial College, London | Continuum Mechanics | 9/69-9/70 |
| Gould, S. H. (U.S.A.) | University of Moscow, Russia |  | 10/69-6/70 |
| Grizzle, James E. (U.S.A.) | World Health Organization, Geneva, Switzerland | Biostatistics | 9/69-8/70 |
| Halberg, Charles J. A., Jr. (U.S.A.) | University of Göteborg, Sweden | Functional Analysis | 7/69-6/70 |
| Harrison, Michael A. (U.S.A.) | Hebrew University in Jerusalem | Computer Science | 1/70-4/70 |
| Hawkins, Thomas W. (U.S. A.) | Eidg. Technische Hochschule, Zurich, Switzerland | History of Group Representation Theory | 9/69-7/70 |
| Herriot, John George (U.S. A.) | Mathematisches Institut der Technischen Hochschule, Munich, Germany |  | 9/69-6/70 |
| Hewitt, Edwin (U.S. A.) | Steklov Institute, Moscow, (Russia) | Harmonic Analysis on Groups, Measure Theory, Functional Analysis | 9/69-3/70 |
| Khabbaz, Samir A. (U.S.A.) | University of Liverpool, England | Topology | 9/69-6/70 |
| Knill, Ronald J. (U.S. A.) | University of Paris, France | Topology | 9/69-6/70 |
| Kolchin, Ellis (U.S.A.) | Tata Institute, Bombay, India | Differential Algebra | 2/70-3/70 |
|  | Kyoto University, Japan |  | 4/70-5/70 |
| Kundert, Esayas G. (U.S.A.) | University of Pisa, Italy | Algebraic Geometry | 7/69-12/69 |
| Lewin, Jacques (U.S.A.) | Australian National University; Queen Mary College, London | Group Theory | $\begin{aligned} & 9 / 69-12 / 69 \\ & 12 / 69-6 / 70 \end{aligned}$ |
| Lewin, Tekla (U. S. A.) | Australian National University, Queen Mary College, London | Group Theory | $\begin{aligned} & 9 / 69-12 / 69 \\ & 12 / 69-6 / 70 \end{aligned}$ |
| Lukacs, Eugene (U.S. A.) | Institute of Technology, Vienna, Austria | Probability Theory | 2/70-6/70 |
| Marcus, Marvin (U.S. A.) | University of Islamabad, Pakistan | Algebra | 3/70-6/70 |
| Martin, N. F. G. (U. S. A.) | University of Copenhagen, Denmark | Probability | 7/69-7/70 |
| McGregor, J. R. (Canada) | Imperial College, London | Statistical Inference and Applications | 7/69-6/70 |
| McShane, E.J. (U.S.A.) | University of Kyoto, Japan | Analysis | 7/69-1/70 |
| Meir, A. (Canada) | The Hebrew University, Jerusalem | Classical Analysis (Summability) | 9/69-8/70 |
| Minc, Henryk (U. S. A.) | Technion, Haifa, Israel | Matrix Theory | 10/69-4/70 |
| Nassar, Raja F. (U.S.A.) | University of Hamburg and University of Göttingen | Statistical Genetics | 6/69-9/70 |
| Nirenberg, Ricardo (U.S.A.) | University of Pisa, Italy | Analysis | 9/69-8/70 |
| Phelps, Robert R. (U.S.A.) | University of Paris, France | Convexity, Extreme Point Problems, Abstract BestApproximation | 9/69-6/70 |
| Rygg, Paul (U. S. A.) | University of Göteborg, Sweden | Algebra | 9/69-6/70 |
| Segal, Jack (U.S. A.) | University of Zagreb, Yugoslavia | Topology | 9/69-6/70 |
| Shorack, Galen (U.S.A.) | Math. Research Center, Amsterdam, Netherlands | Math Statistics (Distribution free statistics) | 9/69-6/70 |


| Name and Home Country | Host Institution | Field of Special Interest | Period of Visit |
| :---: | :---: | :---: | :---: |
| Smyth, Brian (U.S.A.) | University of Durham, England | Differential Geometry | 9/69-6/70 |
| Tait, R. J. (Canada) | University of Warwick, England | Applied Mathematics | 8/69-7/70 |
| Wainger, Stephen (U.S.A.) | University of Paris, France | Analysis | 8/69-8/70 |
| Wasow, Wolfgang (U.S.A.) | Federal Polytechnical Institute, Zurich, Switzerland | Differential Equations | 9/69-1/70 |
| Whyburn, Clifton (U.S.A.) | University of Mainz, Germany | Number Theory | 9/69-5/70 |
| Witz, Klaus G. (U.S.A.) | University of Geneva, Switzerland | Hilbert Spaces | 9/69-1/70 |
| Yood, Bertram (U.S.A.) | University of Edinburgh, Scotland | Mathematical Analysis | 9/69-7/70 |

# NEWS ITEMS AND ANNOUNCEMENTS 

NATIONAL BUREAU OF STANDARDS<br>POSTDOCTORAL RESEARCH ASSOCIATESHIPS


#### Abstract

Postdoctoral Research Associateships tor 1969-1970 have been awarded to 18 young men and women enabling them to further their studies at the National Bureau of Standards (U. S. Department of Commerce). The awarding of these research associateships is part of the postdoctoral programs sponsored each year by the National Research Council, the


National Academy of Sciences, the National Academy of Engineering, and the National Bureau of Standards. Among the recipients is Dr. Russell L. Merris of the University of California. Dr. Merris will continue work initiated in his doctoral thesis on the investigation of the coefficients of the matrix polynomial $\mathrm{K}(\mathrm{A}+\mathrm{B})$.

## ANNUAL SALARY SURVEY

This year's Annual Salary Survey is based on returns from 478 departments in mathematics and the mathernatical sciences, covering 7,307 academic positions held in 1968-1969 and 7,797 positions held in 1969-1970.

Institutions are divided into four groups according to the highest degree offered in mathematics.
(1) Ph.D. Granting institutions
(2) Institutions granting a master's degree in mathematics as its highest degree.
(3) Institutions granting a bachelor's degree in mathematics as its highest degree.
(4) Junior colleges and other institutions offering courses but not degrees in mathematics.

The Ph.D. granting universities have been subdivided into four groups. Groups I and II include the universities with leading mathematics departments according to the findings of a survey made by the American Council of Education in 1964* in which departments were rated according to the quality of their graduate faculty. Group $I$ is composed of institutions that were considered "Distinguished" and "Strong"; Group II are those considered "Good" and "Adequate Plus." Groups III and IV consist of all of the remaining Ph.D. granting institutions; Group III including those universities that have granted three or more doctorates during
the last three years; and Group IV consisting of institutions that have granted two or less doctorates during the last three years. The number of doctorates granted by universities was determined from the lists published in these cotices.

All Canadian universities maintaining Ph.D. granting programs are included in Groups III and IV.

Each institution submitted a minimum, median, and maximum salary figure for each of four academic ranks, both for staff members with master's degrees and for those with doctorates, creating 48 categories of salary figures. In some instances relatively few universities or colleges reported, and in as much as there were no significant figures available, salaries could not be listed.

In the following two pages the data in the parentheses gives the range of the middle $50 \%$ of salaries reported. The figures outside the parentheses represent the minimum and maximum salary listed by any reporting institution. Salaries are given in "hundreds of dollars."

All salaries refer to an academic year of 9 or 10 months. Grants and contracts are included but sabbatical payments and other part-time salaries are not.

This survey is the thirteenth in an annual series begun in 1957 by the Society's Committee on the Economic Status of Teachers.

[^0]PH．D．GRANTING INSTITUTIONS．Group I

| PH．D．GRANTING INSTITUTIONS．Group I |  |  |  |
| :---: | :---: | :---: | :---: |
|  | Total <br> of Staff M | mber <br> mbers |  |
| DOCTORAL DEGREE | 1968－1969 | 1969－1970 | Minimum |
| Instructor | 84 | 91 | 75（82－90） |
| Asst．Professor | 355 | 337 | 79（90－98） |
| Assoc．Professor | 217 | 235 | 104（114－130） |
| Professor | 402 | 442 | 132（150－170） |
|  | 1058 | 1105 |  |
| PH．D．GRANTING INSTITUTIONS．Group II |  |  |  |
| $\frac{\text { MASTER＇S DEGREE }}{\text { Instructor }}$ |  |  |  |
|  | 47 | 43 | 60（70－80） |
| DOCTORAL DEGREE |  |  |  |
| Instructor | 13 | 10 |  |
| Asst．Professor | 270 | 272 | 90（98－105） |
| Assoc．Professor | 164 | 181 | 107（115－130） |
| Professor | $\frac{207}{654}$ | $\frac{226}{689}$ | 99（148－164） |
| PH．D．GRANTING INSTITUTIONS．Group III |  |  |  |
| MASTER＇S DEGREE |  |  |  |
| Instructor | 224 | 220 | 54（66－81） |
| Asst．Professor | 77 | 73 | 72（83－107） |
| Assoc．Professor | 57 | 58 | 88（100－125） |
| Professor | 20 | 18 |  |
|  | 378 | 369 |  |
| DOCTORAL DEGREE |  |  |  |
| Instructor | 10 | 14 | －－－－ |
| Asst．Professor | 577 | 660 | 75（97－108） |
| Assoc．Professor | 351 | 398 | 92（110－133） |
| Professor | 415 | 441 | 117（145－162） |
|  | 1353 | 1513 |  |
| PH．D．GRANTING INSTITUTIONS．Group IV |  |  |  |
|  |  |  |  |
| Instructor | 93 | 75 | 62（67－82） |
| Asst．Professor | 59 | 69 | 72（85－95） |
| Assoc．Professor | 26 | 26 | 83（103－123） |
| Professor | $\frac{9}{187}$ | $\frac{8}{178}$ |  |

PH．D．GRANTING INSTITUTIONS．Group III MASTER＇S DEGREE

PH．D．GRANTING INSTITUTIONS．Group IV

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## PERSONAL ITEMS

Professor J. F. ADAMS of the University of Manchester, England, has been appointed a Lowndean Professor at the University of Cambridge, England.
lst Lieutenant J. T. ARNOLD of the United States Army has been appointed to an assistant professorship at Virginia Polytechnic Institute.

Dr. DONALD BALLOU of the University of Michigan has been appointed a research associate at Brown University.

Professor HERBERT S. BEAR of New Mexico State University has been appointed to a professorship at the University of Hawaii.

Dr. RICHARD BOURGIN of the University of Washington has been appointed to an assistant professorship at the State University of New York at Buffalo.

Mr. CHARLES H. BRASE of the University of Colorado has been appointed to an assistant professorship at the University of Hawaii.

Dr. GEORGE BRIDGMAN of the University of Minnesota has been appointed to an assistant professorship at Wartburg College.

Mr. JAMES W. CANNON of the University of Utah has been appointed a Visiting Post-Doctoral Fellow at the University of Wisconsin.

Dr. E. WILLIAM CHAPIN of Princeton University has been appointed to an assistant professorship at the University of Notre Dame.

Professor SHING-SHEN CHERN of the University of California, Berkeley, has been presented the honorary degree of Doctor of Science by the University of Chicago.

Dr. KIM-PEU CHEW of the University of British Columbia has been appointed to a visiting assistant professorship at the State University of New York at Buffalo.

Mr. DAVID COHOON of Purdue University has been appointed a Visiting PostDoctoral Fellow at the University of Wisconsin.

Professor ROBERT R. COLBY of the University of Kansas has been appointed
to an associate professorship at the University of Hawaii.

Professor BRIAN COLE of the University of California, Los Angeles, has been appointed to an assistant professorship at Brown University.

Dr. EDWARD A. CONNORS of the University of Notre Dame has been appointed to an assistant professorship at the University of Massachusetts.

Mr. JAMES J. COUNTRYMAN of the University of Notre Dame has been appointed to an assistant professorship at Purdue University, North Central Campus.

Professor MORTON L. CURTIS of Rice University has been appointed to a visiting professorship at the University of Hawaii for the second semester 1969-1970.

Mr. RONALD J. DETRY of the Illinois Institute of Technology has been appointed a Technical Staff Member at Sandia Laboratories, Albuquerque, New Mexico.

Professor ALBERTO DOU of the University of Madrid has been appointed to a visiting professorship at the University of Notre Dame, from February l, 1969 to January 3l, 1970.

Professor ALESSANDRO L. FIGATALAMANCA of the University of California, Berkeley, has been appointed a Visiting Lecturer at Yale University.

Professor W.J. FIREY of Oregon State University has been appointed to a visiting professorship at Michigan State University.

Dr. CHARLES E. FORD of the University of Toronto has been appointed to an assistant professorship at Washington University.

Professor DONALD A. S. FRASER of the University of Toronto has been appointed to a visiting professorship at the University of Hawaii.

Dr. LARRY J. GOLDSTEIN of Yale University has been appointed to an associate professorship at the University of Maryland.

ELLEN GOTTLIEB of New York University has been appointed to an assistant professorship at the University of Hawaii.

Mr. HUGH M. HILDEN of the Stevens

Institute of Technology has been appointed to an assistant professorship at the University of Hawaii.

Professor DENISE HUET of the University of Nancy has been appointed to a professorship at the University of Maryland.

Mr. Franklin T. IHA of the University of California, Los Angeles has been appointed to an assistant professorship at the University of Hawaii.

Professor JOHN R. ISBELL of Case Western Reserve University has been appointed to a professorship at the State University of New York at Buffalo.

Professor SEIZO ITO, of the University of Tokyo, Japan, has been appointed to a visiting professorship at the University of Wisconsin.

Dr. HERVE JACQUET of the Institute for Advanced Study has been appointed to an associate professorship at the University of Maryland.

Mr. DENNIS KIBLER of the University of Rochester has been appointed to an assistant professorship at the University of Hawaii.

Mr. DIMITRI KOUTROUFIOTIS of Courant Institute, New York University, has been appointed to an assistant professorship at the University of California, Santa Barbara.

Professor CHARLES KRUEGER of the University of Pittsburgh has been appointed to an assistant professorship at Montana State University.

Mr. KEE YUEN LAM of Brown University has been appointed to an assistant professorship at the University of British Columbia.

Professor ALAN LANDMAN of Columbia University has been appointed to an associate professorship at Brown University.

Dr. VORIS V. LATSHAW of Lehigh University has been retired as Associate Professor Emeritus.

Mr. EARL LAZERSON of Washington University has been appointed to an associate professorship at Southern Illinois University.

Mr. MATTHEW C. Y. LEE of the University of California, Berkeley, has been appointed to an assistant professorship at the University of Hawaii.

Professor JOHN J. LeTOURNEAU of

Fiske University has been appointed to an assistant professorship at Hampshire College.

Mr. ROY LEVOW of the University of Pennsylvania has been appointed to an asistant professorship at the University of Hawaii.

Dr. RONALD L. LIPSMAN of Yale University has been appointed to an associate professorship at the University of Maryland.

Professor ANDREW MARKOE of Brown University has been appointed to a visiting professorship at the University of Wisconsin.

Professor WILLIAM E. MARSH of Talladega College has been appointed to an assistant professorship at Hampshire College.

Professor KENNETH C. MILLETT of the Massachusetts Institute of Technology has been appointed to an assistant professorship at the University of California, Santa Barbara.

Professor HENRYK MINC of the University of California, Santa Barbara has been appointed to a visiting professorship at Technion, Haifa, Israel, first semester 1069-1970.

Dr. LEE K. MOHLER of the Mathematical Institute of the Polish Academy of Sciences has been appointed to a visiting assistant professorship at the State University of New York at Buffalo.

Mr. JOHN D. MOORE of the University of California, Berkeley has been appointed to an assistant professorship at the University of California, Santa Barbara.

Professor JURGEN K. MOSER of Courant Institute, New York University, has been presented the James Craig Watson Medal for contributions to the science of astronomy by the National Academy of Sciences.

Professor STANISLAW MROWKA of Western Michigan University has been appointed to a professorship at the State University of New York at Buffalo.

Mr. DAVID NASH of the University of California, Berkeley, has been appointed to an assistant professorship at the University of Hawaii.

Professor STUART M. NEWBERGER of the University of California, Berkeley, has been appointed to an associate professorship at Oregon State University.

EDWARD PARBERRY of Pennsylvania

State University has been appointed to an assistant professorship at Wells College.

Dr. CARL A. PERSINGER of Michigan State University has accepted a position with Daniel H. Wagner Associates, Paoli, Pennsylvania.

Dr. MICHAEL H. POWELL of the University of California, Santa Barbara, has been appointed to an assistant professorship at the University of Maryland.

Professor PAUL RABINOWITZ of Stanford University has been appointed to an associate professorship at the University of Wisconsin.

Professor JAMES D. REID of Syracuse University has been appointed to an associate professorship at Wesleyan University.

Mr. GEORGE A. REILLY of Westinghouse Research has been appointed a Consultant at Westinghouse Tele-Computer Center.

Mr. HAROLD B. REITER of Clemson University has been appointed to an assistant professorship at the University of Hawaii.

Professor FRANK RHODES of the University of Southampton, England, has been appointed to a visiting professorship at Wesleyan University.

Dr. FRANCIS D. RYAN of the University of Notre Dame has been appointed to an assistant professorship at John Carroll University.

Professor GERALD E. SACKS of the Massachusetts Institute of Technology has been appointed to a visiting professorship at the University of Wisconsin.

Mr. D. JAMES SAMUELSON of the University of California, Santa Barbara, has been appointed to an assistant professorship at the University of Hawaii.

Professor DAVID A. SÁNCHEZ of the University of California, Los Angeles, has been appointed to a visiting assistant professorship at Brown University.

Professor HELMUT H. SCHAEFER of the University of Tubingen has been appointed to a professorship at the University of Maryland.

CEDRIC F. SCHUBERT of the University of California, Los Angeles, has been appointed to an associate professorship at Queen's University, Kingston, Ontario, Canada.

Dr. BRIAN SYMTH of the University
of Notre Dame has been appointed a Visiting Lecturer at the Durham University, England, during the academic year 19691970.

Professor LOUIS SOLOMON of New Mexico State University has been appointed to a professorship at the University of Wisconsin.

Professor H. M. SRIVASTAVA of West Virginia University has been appointed to an associate professorship at the University of Victoria, British Columbia.

Professor JAMES D. STASHEFF of the University of Notre Dame has been appointed a Sloan Fellow at the Institute for Advanced Study.

Mr. ARTHUR B. STEPHENS of the University of Maryland has been appointed to an assistant professorship at the University of Hawaii.

Dr. ROBERT G. TOBEY of the IBM Boston Programming Center has been appointed Assistant Director in the Applied Mathematics Division of Argonne National Laboratory.

Mr. SELDEN Y. TRIMBLE of the University of Kentucky has been appointed to an assistant professorship at Washington University.

Dr. AUGUST WALTMANN of Oklahoma State University has been appointed to an assistant professorship at Wartburg College.

Professor GEORGE WHAPLES of the University of Massachusetts has been appointed to a visiting professorship at the University of Notre Dame.

Professor W. WILEY WILLIAMS of Louisiana State University has been appointed to an assistant professorship at the University of Louisville.

Mr. MICHAEL J. WRIGHT of the University of Kansas has been appointed to an assistant professorship at Loyola University of Los Angeles.

Mr. JOEL ZEITLIN of the University of California, Los Angeles, has been appointed to an assistant professorship at Washingtọ University.

Professor JULIUS ZELMANOWITZ of the University of California, Santa Barbara, has been appointed to a visiting assistant professorship at the University of California, Los Angeles, winter and spring quarters 1969-1970.

## PROMOTIONS

To Associate Vice President for Academic Affairs. University of Notre Dame: F. L. BROWN.

To Assistant to the President for Planning and Analysis. University of Notre Dame: T. E. STEWART.

To Dean of the University of Wisconsin, College of Letters and Science. STEPHEN COLE KLEENE; Wells College: JOHN M. PERRY.

To George Herbert Mead Distinguished Service Professor. University of Chicago: IRVING KAPLANSKY.

To Louis Block Professor. University of Chicago: ALBERTO CALDERON.

To Landon T. Clay Professor. Harvard University: GEORGE W. MACKEY.

To Professor. Brown University' ROBERT D. M. ACCOLA; Stevens Institute of Technology: DAVID L. JAGERMAN; University of California, Santa Barbara: R. C. THOMPSON; University of Florida: A. R. BEDNAREK; University of Maryland: GERTRUDE EHRLICH, G. J. MALTESE; University of Notre Dame: GEORGE KOLETTIS, TADASHI NAGANO; University of Wisconsin: M. N. BLEICHER, D. W. CROWE, SIMON HELLERSTEIN, J. M. MARTIN, D. R. McMILLAN, DONALD MEQUILLAN, P. E. MILES, L. B. RALL; Washington University: J. -I. HANO, M. H. TAIBLESON; West Virginia University: H. W. GOULD.

To Associate Professor. Brown University: STANLEY SAWYER; Lehigh University:GERHARD RAYNA, A.K. SNYDER; University of Maryland: J. E. OSBORN, C. R. WARNER; Montana State University: R. D. EnGle, R. M. GILLETTE, H. G. RUTHERFORD, G. H.RYDER;St. Olaf College: L. A. STEEN; University of California, Santa Barbara: SEYMOUR BACHMUTH, MICHAEL CAMBERN, J.B. ROBERTSON; University of Florida: J. K. BROOKS, A. K. VARMA; University of Hawaii: L. J. WALLEN; University of Notre Dame: J. E. DERWENT, KARL KRON-

STEIN; University of Wisconsin: R. W. DICKEY; J. E. HALL, JAMES KUELBS, D. F. SHEA; Villanova University: L. C. ROBBINS, JR., L. R. ROY; Wesleyan University: A. W. HAGER, W. L. REDDY.

To Senior Lecturer. Hebrew University, Jerusalem: B. PELEG.

To Assistant Professor. Polytechnic Institute of Brooklyn: EMERIC DEUTSCH; State University of New York at Buffalo: N. D. GOODMAN, H. SUBRAMANIAN; University of California, Irvine: W. H. FELLNER, R. B. TARSY; University of Wisconsin: T. G. KURTZ; Villanova University: THOMAS BARTLOW, R. E. BECK, SR., M. C. EHRMANN.

To Associate Computer Scientist. Argonne National Laboratory: LOUIS JUST.

## INSTRUCTORSHIPS

Brown University: S. KARP, A. -M. SIMON; University of Notre Dame: J. R. SENFT, R. E. SOLAZZI, K. YAGI; University of Wisconsin: R. L. WILSON, JR.; Yale University: D. M. GOLDSCHMIDT, Z. H. NITECKI, R. TOLIMIERI.

## DEATHS

Professor THOMAS BUCK of the University of California, Berkeley, died on March 13, 1969 at the age of 87 . He was a member of the Society for 55 years.

Dr. EUGENE P. NORTHROP of the Ford Foundation, Ankara, Turkey, died on January 5, 1969 at the age of 60 . He was a

Professor GORDON T. WHYBURN of the University of Virginia, Charlottesville, died on September 8,1969 at the age of 65 . He was a member of the Society for 43 years and served as its President in 1953 and 1954.

## ERRATA

Professor L. M. Chawla of Government College, Lahore, West Pakistan, has been appointed Principal of the Central Training College, Lahore, West Parkistan.

# MEMORANDA TO MEMBERS 

## aUdIO RECORDINGS OF MATHEMATICAL LECTURES

The Society is issuing a series of taped lectures, the series to be entitled Audio Recordings of Mathematical Lectures. This series will include tape recordings, with accompanying manual, of Gibbs Lectures, Colloquium Lectures, and invited hour addresses. The first four lectures are now ready for distribution. No. 1 is "The role of vector and operator valued measures in functional analysis and probability" by Professor Pesi R. Masani of Indiana University. No. 2 is "On local solvability of linear partial differential equations" by Professor Francois Treves of Purdue University. No. 3 is "The theory of decision problems in group theory: a survey" by Professor William W. Boone of the University of Illinois. No. 4 is "Recent developments in fixed-point theory" by Professor Edward R. Fadell of the University of Wisconsin. These four lectures were presented at the six hundred sixty-fifth meeting of the Society held in

Cincinnati, Ohio, on April 18-19, 1969. Each lecture in the series is recorded on tape at a speed of $17 / 8^{\prime \prime} /$ second ( $4.75 \mathrm{~cm} / \mathrm{second}$ ) and can be played on standard tape recorders. Tapes at $33 / 4^{\prime \prime} / \mathrm{sec}-$ ond ( $9.5 \mathrm{~cm} / \mathrm{sec}$ ond) may be obtained if desired. Included with each tape is a manual which contains expressions and diagrams displayed during the lecture. The audio-tape contains references to the accompanying manual. The Audio Recordings of Mathematical Lectures may be purchased for $\$ 6$, and additional copies of the manual may be ordered for $\$ 0.30$ each. Standing orders for the entire series of lectures may be placed. There will be approximately 37 lectures recorded during the coming year. Orders should be sent to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904.

Partial support for this project is being provided by the United States Steel Foundation, Inc.

## TRAVEL GRANTS <br> INTERNATIONAL CONGRESS OF MATHEMATICIANS

Travel grants will be made to a num ber of qualified mathematicians for attendance at the International Congress of Mathematicians to be held in Nice, France, September 1-10, 1970. Selection of the grantees will be made by the Committee on Travel Grants of the NRC Division of Mathematical Sciences together with representatives of all mathematical societies cooperating with the Division and representatives of the various governmental agencies concerned. Younger mathematicians are en-
couraged to apply. Special efforts will be made to support their attendance, and they will have the opportunity to supplement their applications by submitting additional information. Applications can be obtained from the Division of Mathematical Sciences, National Research Council, 2101 Constitution Avenue N. W., Washington, D. C. 20418. Applications must be received on or before November 1, 1969.

The Society will not sponsor any group or charter flights to the Congress.

## SYMP OSIA INFORMATION CENTER

The Committee to Monitor Problems in Communication has recommended, and the Council has approved, the est ablishment
of a Symposia Information Center. Information on symposia in the early planning stages will be obtained from professional societies,
universities, and granting agencies. A file will be maintained by the Center, and information on prospective symposia will be available to any organization or individual planning a conference. If conflicts in subject matter, dates, or geographical area become apparent, the Center will notify the organizers. The announcement of definitely
scheduled symposia to potential participants will continue to becarried in these $($ Notices), this service being separate and apart from the Symposia Information Center.

Those planning symposia are requested to send all preliminary information to the headquarters of the Society as early in the planning as possible.

## COLLOQUIUM LECTURES

A limited number of the lecture notes of the two series of Colloquium Lectures presented at the summer meeting in Eugene, Oregon, in August 1969 are still available. These lectures were "On the periodicity theorem for the classical groups and some of its applications" by Raoul Bott of Harvard

University and "Harmonic analysis on semisimple Lie groups" by Harish-Chandra of the Institute for Advanced Study. Requests for the lecture notes should be accompanied by a check for $\$ 1$ each to cover the cost of handling.

## MATHEMATICAL SCIENCES EMPLOYMENT REGISTER

The Roof Room in the Everglades Hotel in Miami, Florida, will be the location of the Mathematical Sciences Employment Register during the annual meeting. The Employment Register will be open for three days, January 23 through January 25 , 1970, from 9:00 a.m. to 5:00 p.m.

The Employment Register is sponsored by the American Mathematical Society, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics for the purpose of establishing communication between mathematical scientists available for employment and employers with positions to fill. As part of the service, interviews between applicants and employers are arranged.

Registration for the Employment Register is separate and apart from meeting registration, and it is, therefore, most important that both applicants and employers sign in at the Employment Register desk as early as they can on Friday morning. No appointments will be scheduled for Friday, however. A separate visual index will be maintained for Employment Register use only. Appointments will be scheduled only for people who have actually signed in at the Register. Requests for appointments can be submitted on any or all of the days the Employment Register 'is open.

There is no charge for registration except when the late registration fee of $\$ 5.00$ is applicable. Provision will be made for anonymity of applicants upon payment of $\$ 5.00$ to defray the cost involved in handling such a listing. Applicants and employers who wish to be listed should write to the Mathematical Sciences Employment Register, Post Office Box 6248, Providence, Rhode Island 02904, for applicant qualification forms or position description forms. These forms must be completed and returned to the Employment Register not later than December 15, 1969, in order to be included in the January lists.

Those forms which arrive too late to be included in the printed lists are taken to the meeting where they may be seen by applicants and/or employers who are interested in them. The printed lists will be mailed to subscribers during the first week in January. Lists can be ordered from the Employment Register office in Providence. They will also be available at the meeting. A subscription to the lists, which includes three issues (January, May and August) of both the applicants list and the positions list is available for $\$ 30.00$ a year; the individual issues of both lists may be purchased in January, May, and August for $\$ 15.00$. A subscription to the applicants list alone or single copies of that list are not available. Copies of the positions list only
may be purchased for $\$ 5.00$. Checks should be made payable to the American Mathe-
matical Society and sent to the address given above.

## RETIRED MATHEMATICIANS

The List of Retired Mathematicians Available for Employment will once again be published in January 1970 and will be distributed to subscribers to the Employment Register lists when the January issue is mailed. Besides being available to subscribers, the list is available on request from the Employment Register office. Copies will also be available at the annual meeting in Miami, Florida, January 22-26, 1970. Retired mathematicians who are interested in being included in the list may
either request a form from the Employment Register office or send the following information: name, date of birth, highest degree earned and where it was obtained, most recent employment, present address, date available, references, preference for academic or industrial employment, and geographic location preferred. The deadline for receipt of either the completed form or the above information is Januaryl, 1970.

## MOS REPRINTS

The Mathematical Offprint Service (MOS) has announced that subscribers may now order copies of articles whose titles were selected for inclusion in their MOS monthly letters. This service is being instituted because of the large number of subscribers who have found that they want complete articles whose titles alone were selected on the basis of their stated secondary interests. Only copies of articles from those journals supplying offprints to MOS will be available. At the present time the supply of reprints will be limited to the number of extra copies (above regular
offprint orders) that are on hand. This means that subscribers must expect that some orders will not be filled. However, if the demand is sufficient to warrant it, the service will begin ordering additional copies.

Reprints will be priced at $\$ 1$ each and may be ordered only by MOS subscribers. MOS subscribers, however, may order for their colleagues who are not yet subscribers to the service. The Reprint Order Forms that have been mailed to all subscribers should be used, and prepayment should accompany orders.

## NEWS ITEMS AND ANNOUNCEMENTS

SYMP OSIUM ON<br>FUNCTIONAL ANALYSIS AND<br>RELATED TOPICS

The Naval Postgraduate School and the Office of Naval Research will sponsor a symposium on Functional Analysis and Related Topics to be held at Monterey, California, on October 16-17, 1969. The principal speakers will be Professors R. V. Chacon, A. M. Garsia, S. Kakutani, G. K. Kalisch, G.-C. Rota, and J. T. Schwartz. There will be a total of fourteen invited papers presented at the conference. Emphasis will be placed on the relation of functional analysis to other areas of mathematics, especially some which are of interest in applications. Further information may be obtained by writing to Professor Carroll O. Wilde, Department of Mathematics, Naval Postgraduate School, Monterey, California 93940.

## CHAUVENET MEMORIAL SYMP OSIUM

The United States Naval Academy has announced a symposium in honor of Professor William Chauvenet in conjunction with the dedication of the new Naval Academy Mathematics Building, Chauvenet Hall. The symposium will be held on October 17-18 at the Naval Academy, Annapolis, Maryland. The program will consist of five invited lectures by past winners of the Chauvenet Prize: Gordon T. Whyburn, "Recent developments in geometric topology," this lecture to be presented by E. E. Floyd of the University of Virginia as a memorial to Professor Whyburn; Saunders Mac Lane, "Hamiltonian mechanics"; Guido Weiss, "Harmonic Analysis"; Paul R. Halmos, "Finite dimensional Hilbert spaces"; Mark Kac, "A look at probability and analysis". The symposium will be cosponsored by the Office of Naval Research, and papers will be published in the American Mathematical Monthly. Further information may
be obtained by writing to Professor J. C. Abbott, Program Director, Mathematics Department, United States Naval Academy, Annapolis, Maryland 21402.

## ANTONI ZYGMUND

Professor Antoni Zygmund of the University of Chicago is among the 135 members recently elected to the American Academy of Arts and Sciences. Professor Zygmund, a native of Poland, joined the faculty of the University of Chicago in 1947. His special field of interest lies in mathematical analysis, primarily in the field of harmonic analysis, real and complex variables, and partial differential equations. He is a member of the National Academy of Sciences, the Polish Academy of Sciences, and the Argentine Academy of Sciences.

## CONFERENCE ON THE FOUNDATIONS OF MATTHEMATICS

The University of Waterloo will sponsor a conference on the Foundations of Mathematics on October 17-19, 1969. There will be addresses by six speakers on the mathematical and philosophical implications of problems in foundations. The speakers will be Professors Yehoshua Bar-Hillel, Solomon Feferman, Lászl6 Kalmár, Motokiti Kondo, Andrzej Mostowski, and John Myhill. Inquiries from those interested in attending this conference should be addressed to A. Kerr-Lawson, Department of Pure Mathematics, or to J. Van Evra, Department of Philosophy, University of Waterloo, Waterloo, Ontario, Canada.

## SPECIAL YEAR IN PARTIAL DIFFERENTIAL EQUATIONS

The Mathematics Division at Sussex University has made plans to hold a spe-
cial year in partial differential equations during the 1969-1970 session. There will be a program of lectures and seminars during the summer term (April, May, June, and July). The program will be supported by the Science Research Council (SRC). Distinguished experts in the field of partial differential equations, who will be in residence at Sussex, include Professors F. V. Atkinson, F. E. Browder, G. Fishera, and R. Finn. Those interested in spending any part of the year at Sussex may obtain further information from the Chairman of the Mathematics Division, The University of Sussex, Falmer, Brighton BN1 9QH. SRC support may be available for visitors from British universities.

## SYMPOSIUM ON <br> NONLINEAR PROGRAMMING

The Mathematics Research Center, U. S. Army, University of Wisconsin, will hold a symposium on Nonlinear Programming, May 4-6, 1970, in Madison, Wisconsin. The symposium will consist of approximately 15 invited lectures with emphasis on a closer rapport between theory and computation. The program committee consists of O. L. Mangasarian, K. Ritter, and J. B. Rosen (chairman).

## NSF RESIDENCIES IN COMPUTING AND COMPUTER ACTIVITIES

The National Science Foundation will award up to ten one-year faculty residencies at Systems Development Corporation, Santa Monica, California, for training in computing and computer applications. The residencies are intended to help institutions of higher education develop computing activities by providing advanced training to a faculty member of the institution's choice. Participants will spend from July 1, 1970, to June 30, 1971, in a program developed by SDC to provide three major types of activities: (l) training in the fundamentals of academic computer service; (2) seminars on the application of computers; and (3) research in the participant's own area of specialization with a research group at SDC.

Proposals for participation should be submitted by the institution of the proposed participant, with a dean or other equivalent academic officer acting as principal investigator. Since the objective of this program is to increase the number of institutions with strong academic computing activities, institutions already recognized as being outstanding in computing are not encouraged to apply. Preference will be given to those institutions granting 100 or more baccalaureates in science in any three-year period since 1963. The maximum award will be $\$ 50,000$ for each institution. Grant funds may be used for salary and travel expenses of the participant during the year of residency and for salary and expenses during the first year following the residency. Funds may not be used to pay indirect costs. The deadline for submitting proposals will be December 1, 1969, and awards will be announced by April 1, 1970. For additional information, write to the Education, Research and Training Section, Office of Computing Activities, National Science Foundation, Washington, D. C. 20550.

## FELLOWSHIP AND <br> RESEARCH OPP ORTUNITIES IN THE MATHEMATICAL SCIENCES

In its annual brochure on Fellowship and Research Opportunities in the Mathematical Sciences, the Division of Mathematical Sciences of the National Research Council calls attention to a number of fellowships and other kinds of support for research in the mathematical sciences at both the predoctoral and postdoctoral levels to be awarded during the year 1969-1970. Copies of this brochure are available from the Division of Mathematical Sciences, National Research Council, 2101 Constitution Avenue, N. W., Washington, D. C. 20418.

## SYMPOSIUM ON THEORY OF COMPUTING

The second annual symposium on Theory of Computing, sponsored by the ACM Special Interest Committee on Automata and Computability Theory, will be
held at Northampton, Massachusetts, on May 4-6, 1970. Some of the areas to be covered are automata and languages; computational complexity; parsing and translation; and theory of computer organization. Those wishing to present a paper in these or related fields should submit six copies of a detailed abstract (no word limit) to the chairman of the Program Committee, Professor Richard M. Karp, Department of Computer Science, University of California, Berkeley, by December 1, 1969. Authors will be notified of acceptance or rejection by January 15, 1970. A proceedings will be published, and accepted papers are due by March 1, 1970.

## RESEARCH FELLOWSHIP IN MECHANICS OF FLUIDS

A Boris A. Bakhmeteff Research Fellowship, offered by Humanities Fund, Inc. of New York City, will be available for the 1970-1971 academic year in an amount up to $\$ 3600$. It is intended to be a specific contribution for a definite research project of an original and creative nature in the general field of mechanics of fluids. The recipient shall be a graduate student who is a candidate for the master's or doctor's degree. He shall have no commitment that will interfere with his research work and study on a full-time basis. It is expected that the stipend will be applied principally toward fees, tuition, and subsistence. The study and research may be undertaken at an institution of the Fellow's choice. In the judgment of the committee, the adequacy of the facilities of the institution will have substantial weight in the selection of the Fellow.

Application forms should be filed by February 15, 1970, with Dean William Allan, School of Engineering, The City College of The City University of New York, New York 10031.

## SYMPOSIUM ON SINGULARITIES OF SMOOTH MAPS AND MANIFOLDS

A symposium on Singularities of Smooth Maps and Manifolds will be held in Liverpool, England, from October 1, 1969,
through August 31, 1970. The symposium will be supported by the Science Research Council and will be under the direction of Professor C. T. C. Wall. The themes to be developed will include classification of singularities of smooth maps between smooth manifolds; techniques of differential topology; relations between singularities of a smooth map and the topology of its domain and codomain; and manifolds with singularities (stratified sets). Many leading mathematicians from the field of singularity theory will participate in the symposium for periods varying from a few weeks to the full session. It is also hoped that a Summer School may be arranged for July or August 1970.

Further information may be obtained by writing to Dr. A. J. Ledger, Department of Pure Mathematics, The University, P.O. Box 147, Liverpool, L69 3BX, England.

## COLLOQUIUM ON

FUNCTIONAL ANALYSIS
In September 1970, following the Nice International Congress, a Colloquium on Functional Analysis is being planned by the Centre Belge de Recherches Mathématiques. Those interested in attending this conference should write to Professor H. G. Garnir, Department of Mathematics, University of Liège, 15, avenue des Liège, Belgium.

## DOCUMENTATION CENTER OF LAVAL UNIVERSITY

The Center of Documentation of Laval University, in cooperation with the Mathematics Department and the Faculty of Education, is compiling information on mathematics and the teaching of mathematics at the elementary, secondary, and college level and on the cultural and practical achievements, goals, and implications of mathematics and the teaching of mathematics. The Center plans to publish an alphabetical and methodological index of registered documents. For further information, please write to Professor Jacques Fortin, Department of Mathematics, Laval University, Quebec 10, Canada.

NSF CONFERENCES IN THE MATHEMATICAL SCIENCES

The National Science Foundation is seeking proposals for five-day regional conferences on subjects of current research interest in the mathematical sciences. The objective of the conferences is to stimulate and broaden mathematical research activity, particularly in regions of the country where such activity needs further development. The organization of the conferences, evaluation of proposals, and arrangements for publication of expository papers will be carried out by the Conference Board of the Mathematical Sciences (CBMS) under contract with the NSF.

At present ten conferences are projected, each to take place at a university during the summer of 1970 or during a recess of the academic year. Topics may include pure mathematics, applied mathematics, statistics, computer science, operations research and management science. Each conference is to have a lecturer and about 25 other participants, the latter to be drawn from the geographic region around the host institution. The lecturer would be expected to give two lectures a day during the five days of the conference, with the remainder of the time available for study, informal discussion, and exchange of ideas. All participants will receive allowances for travel and subsistence; and the principal lecturer will receive, in addition, a fee for delivering his lectures and for organizing these into a substantial expository paper. The CBMS will arrange for the editing and publication of these expository papers.

Inquiries regarding details of these regional conferences may be sent to the Conference Board of the Mathematical Sciences, 834 Joseph Heary Building, 2100 Pennsylvania Avenue, NW, Washington D.C. 20037. Proposals by prospective host institutions should be sent to the Mathematical Sciences Section, National Science Foundation, 1800 G Street, NW, Washington, D. C. 20550. Proposals will be evaluated by a panel of the CBMS, and awards of grants will be made by the NSF with the advice of the panel.

## SECOND JAP AN-UNITED STATES SEMINAR ON DIFFERENTIAL AND FUNCTIONAL EQUATIONS

After the success of the first joint Japan-United States seminar, which was held in June 1967 at the University of Minnesota, it is proposed to hold the second session of the seminar in Japan. The seminar is now being planned, and it is tentatively scheduled for one week at the end of August 1971 in Kyoto, Japan. M. Urebe and J. Nohel are co-chairmen of the organizing committee. It is hoped that the seminar will be sponsored jointly by the Japan Association for the Advancement of Science and the U. S. National Science Foundation. Approximately 20 one-hour lectures will be given by speakers to be invited by the organizing committee. In addition, there will be an opportunity for the presentation of a limited number of contributed papers. Further details will be published in these $($ Notices as soon as more information becomes available.

## SUMMER SCHOOL OF THEORETICAL PHYSICS

A summer school of Theoretical Physics will be held at the University of Grenoble, Haute Savoie, France, July 5 to August 29, 1970. The subject of the conference will be Mathematical Physics-Quantum Field Theory and Statistical Mechanics. Lectures will be given by J. Glimm, A. Jaffe, D. Ruelle, E. H. Lieb, R. B. Griffiths, K. Hepp, and H. J. Ginibre. The number of participants will be limited to 35 . A committee will select young researchers working in mathematical physics and experienced theoreticians wishing to study the mathematical problems encountered in theoretical physics. The cost will be approximately $\$ 215$, this fee to cover all expenses, apart from incidentals. The deadline for receipt of applications is March 15, 1970. For application forms and further information, write to Professor Cecile DeWitt, Department of Physics, University of North Carolina, Chapel Hill, North Carolina 27514 or Ecole d'été de Physique Theoreque, 74 Les Houches, France.

# ABSTRACTS OF CONTRIBUTED PAPERS 

## The October Meeting in Cambridge, Massachusetts October 25, 1969

668-1. CHARLES J. MOZZOCHI, 18 Tuxis Road, Madison, Connecticut 06443. Symmetric generalized topological structures.

This Abstract is a revised edition of an earlier abstract, "Symmetric generalized uniform and proximity spaces, " in which a number of typographical errors have been corrected, four open questions have been answered, and a chapter (about the contents of which the talk will be concerned) has been added in which the concept of a symmetric generalized topological group is defined and its relationship to the concept of a symmetric generalized uniform space is investigated. (Cf. Abstract 658-54, these CNotices 15 (1968), 733.) (Received June 16, 1969.)

668-2. ELLEN TORRANCE, Mount Holyoke College, South Hadley, Massachusetts 01075. Some characterizations of strictly convex Banach spaces. Preliminary report.

A complex Banach space ( $X,|\cdot|$ ) is strictly convex if each point of the unit sphere is an extreme point of the unit ball. A semi-inner-product on $(X,|\cdot|)$ is any map $[\cdot, \cdot]$ on $X \times X$ to $C$ such that (i) $[\lambda x+y, z]=\lambda[x, z]+[y, z]$, (ii) $[x, x]]>0$ if $x \neq 0$, (iii) $|[x, y]|^{2} \leqq[x, x][y, y]$, for all $x, y, z \in X, \lambda \in C$. Theorem. Let $[\cdot, \cdot]$ be any semi-inner-product on $X$. The following conditions are equivalent: (1) $(X,|\cdot|)$ is strictly convex. (2) If $|y+z| \leqq|y|$ and $[z, y]=0$, then $z=0$. (3) If $|y+z|=|y|$ and $[z, y]]=0$, then $z=0$. (4) $A \in L(X)$, if $|I+\lambda A| \leqq 1$ for some $\lambda \in C, \lambda \neq 0$ and if $[A x, x]=0$ for some $x \in X$, then $A x=0$. A similar theorem holds for real $B$ anach spaces. (Received June 18, 1969.)

668-3. SAMUEL ZAIDMAN, Universite de Montréal, Montreal, Quebec, Canada. An estimate for general pseudo-differential operators.

Consider complex-valued functions $a(x, \xi)$, defined for $x \in R^{n}, \xi \in R^{n}-\{0\} ; a(\infty, \xi)$ exists and $|a(\infty, \xi)| \leqq c,|a(\infty, \xi)-a(\infty, \eta)| \leqq c|\xi-\eta|(|\xi|+|\eta|)^{-1}$ if $a^{\prime}(x, \xi)=a(x, \xi)-a(\infty, \xi)$ its Fourier transform in $x, \tilde{a}^{\prime}(\lambda, \xi)$ has properties $\left|\tilde{a}^{\prime}(\lambda, \xi)\right| \leqq k(\lambda) ;\left|\tilde{a}^{\prime}(\lambda, \xi)-\tilde{a}^{\prime}(\lambda, \eta)\right|=k(\lambda)|\xi-\eta|(|\xi|+|\eta|)^{-1}$ where $\left(1+|\lambda|^{2}\right)^{p_{k}}(\lambda) \in L^{1}(R), \forall p=0,1, \ldots$. Suppose also that for any $e>0, b_{e}(x, \xi)=$ (Rea(x, $\xi)-c+\varepsilon)^{1 / 2}$ verifies above conditions, c being a real number $\leqq \operatorname{Re} a(x, \xi), x \in R^{n}$, $\xi \in R^{n}-\{0\}$. Then if $A(x, D)$ is the operator associated to $a(x, \xi)$ following Kohn-Nirenberg, we have for every e $>0$, there exists a constant $L_{\epsilon}$ such that $\operatorname{Re}(A(x, D) u, u)_{L^{2}}+L_{e}\|u\|_{H^{-1 / 2}}^{2} \geqq(c-e)\|u\|_{L^{2}}^{2}$, holds, $\forall U \in L^{3}$. (Received July 8, 1969.)

668-4. F. GONZALEZ ASENJO, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. Generalized reals.

Real numbers fill the line, thanks to a postulational dictum--the axiom of completeness of the line. But since dimension depends on structure and not on cardinality, such an axiom is justified only because of the simplicity it yields. In principle, there is no limit to the number of points that can be fitted on the line. To show this let us take the field of real numbers $R$ and apply to it the operation of ultraproduct, with R itself as an index set and with an ultrafilter that contains all cocountable subsets of $R$. We obtain a new field $R_{2}$ whose elements we may call reals of a second kind. We can prove that $R_{2}$ is of cardinality $2^{c}=c_{2}$ and obviously, then, is non-Archimedean and has infinites and infinitesimals. By iterating this operation one obtains fields $R_{k+1}$ of reals of $k i n d k+1$, forming the ultraproduct of $R_{k}$ with $R_{k}$ as an index set and with appropriate ultrafilters. $R_{k+1}$ is of cardinality $2^{c_{k}}=c_{k+1}$ and contains infinites and infinitesimals of $k$ kinds; centered on each real number $x$ of $R$ there is in $R_{k+1}$ a sequence of $k$ monads of reals of different kinds, a sequence which is nested in the sense that every element in $\mu_{n+1}(x)$ is closer to $x$ than every element in $\mu_{n}(x)$. This paper examines some properties of these generalized reals. (Received July 11, 1969.)

668-5. KENNETH P. BOGART, Dartmouth College, Hanover, New Hampshire 03755. Small regular local Noether lattices.

In "Structure theorems for regular local Noether lattices" [Michigan Math. J. 15 (1968), 167-176] the author introduced a collection $\left\{R L_{n}\right\}$ of structurally simple regular local Noether lattices, and described the relationship between the structure of the lattices $R L_{n}$ and the structure of regular local Noether lattices in general. In this paper we show that this relationship may be much stronger than expected by showing that if a regular local Noether lattice has precisely 3 minimal primes, then it is $\mathrm{RL}_{3}$. (Received July 22, 1969.)

668-6. LEON W. COHEN, University of Maryland, College Park, Maryland 20742. Borel vindicated.

With "interval" as an undefined term an axiomatic measure theory is developed for an abstract space which reflects the Borel process of 1898 . The resulting Borel classes $B a, a<\Omega$, are such that their union $\beta$ is an $\sigma$-algebra on which the measure is countably additive. No measurability condition is employed. In the context of a Hausdorff $T_{2}$ space this yields an integration theory as follows. A function $f$ is called almost continuous on an interval I if for $c>0$ there is a closed set $A \subset I$ such that $\mu(I-A)<c$ and $f$ is continuous on $A$ (Lusin's theorem). Egorov's theorem is proved in the following form: If $f_{n}$ is a convergent sequence of continuous functions on a compact set $A$, then for $\subset>0$ there is a closed set $B \subset A$ such that $\mu(A-B)<\epsilon$ and the convergence is uniform on $B$. A linear functional $I(f)$ is defined on a linear set of almost continuous functions with the property that if $f_{n}$ is a sequence of such functions converging almost everywhere on $I$ and $\left|I\left(f_{n}\right)\right| \leqq K$ for all $n$, then $I\left(\lim f_{n}\right)$ exists and $\lim I\left(f_{n}\right)=I\left(\lim f_{n}\right)$. The Lebesgue-Stieltjes theory in $\mathbb{R}^{n}$ is subsumed. All details are carried out within the Borel class of rank 3. (Received June 26, 1969.)

668-7. ANDRE JOYAL, Université de Montréal, Montréal, Canada. Boolean algebras as functors.

Let $\gamma$ be an infinite regular cardinal. A $\boldsymbol{\gamma}$-complete BA is a BA closed under sups of power $<\gamma$. A $\boldsymbol{\gamma}$-field is a BA isomorphic to a $\boldsymbol{\gamma}$-complete field of sets. A $\boldsymbol{\gamma}$-representable BA is a BA isomorphic to a quotient of a $\boldsymbol{\gamma}$-field by a $\boldsymbol{\gamma}$-ideal. Let $\mathrm{S}_{\infty}$ be the category of sets and let $\mathrm{S}_{\boldsymbol{\gamma}}$ be the full subcategory of $S_{\infty}$ generated by sets of power $<\gamma$. Then the category of functors (and natural transformations) from $S_{\gamma}$ to $\mathrm{S}_{\infty}$ which commute with projective limits is equivalent to the category of $\gamma$-fields with $\gamma$-isomorphisms. Restricting projective limit to diagrams of power $<\boldsymbol{\gamma}$, we obtain an equivalence with the category of $\gamma$-representable BA's (and $\gamma$-homomorphisms). A similar characterization of the category of $\boldsymbol{\gamma}$-complete $\mathrm{BA}^{\prime} \mathrm{s}$ is obtained. Denoting by $\underline{k}$ (with k integer $>1$ ) the full subcategory of $S_{\infty}$ whose objects are the cardinal $k^{n}(n \geq 0)$, then the category of BA's is equivalent to the category of functors from $k$ to $S_{\infty}$ commuting with finite products. For $k=2$, this is a result of W. Lawvere. A study of measure theory in this functorial context has been undertaken as well as the problem of representing functors from sets to sets. (Received August 14,1969 .)

668-8. RICHARD P. STANLEY, Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Drive, Pasadena, California 91103, and Harvard University, 2 Divinity Avenue, Cambridge, Massachusetts 02138. Structure of incidence algebras and their automorphism groups.

Let $P$ and $Q$ be locally finite partially ordered sets and $I(P), I(Q)$ their incidence algebras over a field $K$. Theorem 1. If $I(P)$ and $I(Q)$ are isomorphic as $K$-algebras, then $P$ and $Q$ are isomorphic. Theorem 2. If $P$ is finite, then the group $O(P)$ of outer $K$-automorphisms of $I(P)$ is isomorphic to the semidirect product of $\left(K^{*}\right)^{r-t}$ by $A(P)$, where $K^{*}$ is the multiplicative group of $K, r$ is the cycle rank of the Hasse diagram of $P$ considered as a graph, $t$ is the rank of the subgroup of cycles generated by cycles consisting of two unrefinable chains with the same endpoints, and $A(P)$ is the group of automorphisms of $P$. Corollary. If $P$ is finite with 0 or $l$, then $O(P) \cong A(P)$. (Received August 14, 1969.)

668-9. HAYON KIM, Loyola College, Montreal, Canada and BASIE A. RATTRAY, McGill University, Montreal, Canada. Topologies on objects of categories.

We extend the concept "topologies on a set" to "topologies on an object of a category.' Definition. A $\mathcal{C}$-topology on an object A of a category $\approx$ is a functor $\mathrm{X}: \mathcal{C}^{\mathrm{Opp}} \rightarrow \mathcal{J}$ such that (1) X preserves left roots, and (2) UX $=C(-, A)$ where $\mathcal{J}$ is the category of topological spaces and $U$ is the underlying set functor. Taking these as objects and natural transformations as maps we obtain a category $\theta(C)$. If $C$ is the category of sets our definitions reduce to the usual ones. If $C$ is the category of groups then $\theta(C)$ is isomorphic to the category of topological groups. Our main theorem is that if $\rho^{T}$ is the varietal category of Linton then $\theta\left(\rho^{T}\right)$ is equivalent to the category of models of the theory $T$ in the category of topological spaces. (Received August 18, 1969.)

668-10. RICHARD M. WILSON, Ohio State University, Columbus, Ohio 43210. An existence theory for pairwise balanced designs.

Given a set $K$ of positive integers and a positive integer $\lambda$, let $B[K ; \lambda]$ denote the set of $v>0$ for which there exists a pairwise balanced design on $v$ treatments with block sizes from $K$ and index of pairwise balance $\lambda$. A well-known conjecture on BIBD's is the existence conjecture (E. C.). There exists a constant $C=C(k, \lambda)$ such that $\{v \mid v \geqq C, \lambda(v-1) \geq 0(\bmod k-1), \lambda v(v-1) \geq 0$ $(\bmod k(k-1))\} \subseteq B[\{k\} ; \lambda]$. A set $K$ is said to be a closed set iff $K=B[K ; 1]$. The sets $B[K ; \lambda]$ are closed. Recursive construction methods yield the Main Theorem. Given a closed set $K$, there exists C such that for every $k_{0} \in K,\left\{v \mid v \geqq C, v \equiv k_{0}(\bmod \beta(K))\right\} \subseteq K$, where $\beta(K)=$ g.c.d. $\{k(k-1) \mid k \in K\}$. Corollary. Every closed set $K$ is finitely generated, i.e., there is a finite subset $K_{0} \subseteq K$ such that $K=B\left[K_{0} ; 1\right]$. Applying the Main Theorem results in Theorem. There exists $C=C(k, \lambda)$ such that $v \in B[\{k\} ; \lambda]$ for all $v \geqq C$ satisfying $v \equiv 1$ or $k(\bmod k(k-1) /(\lambda, k(k-1)))$. Theorem. The E.C. is valid for pairs $k$, $\lambda$ whenever either (i) $k /(\lambda, k)$ is one or a prime power, or (ii) $\lambda \geqq([k / 2]-1) \times$ ([k/2]-2). Theorem. If the E.C. is valid for $\lambda=1$, then it holds in general. Analogous results are given for the sets $B[K ; \lambda]$. (Received August 20, 1969.)

668-11. NICKOLAS HEEREMA, Florida State University, Tallahassee, Florida 32306. An extension of classical Galois Theory to inseparable fields.

Let $k$ be a field having characteristic $p, \neq 0$, let $R=k[x] / x^{n}+1 k[x]$, $x$ an indeterminate, and let $a$ be the group of all automorphisms of $R$ leaving the coset of $x$ fixed. Using results of M. E. Sweedler and R. L. Davis the following Galois correspondence is obtained, a correspondence which includes the purely inseparable theory of N. Jacobson and R. L. Davis and the classical finite separable theory. Let $\mathcal{K}$ denote the set of subfields $h$ of $k$ such that $[k: h]<\infty$. For $h \in \mathcal{K}$. let $A(h)=\{a \in a \mid a(a)=a \forall a \in h\}$ and for $\& \quad a \operatorname{subgroup}$ of $a$ let $I(\&)=\{a \in k \mid a(a)=a \forall a \in \mathcal{\&}\}$.
 simple extensions\}. In addition, the subgroups of $a$ having the form $I(h)$ for $h \in \mathcal{X}$ are intrinsically characterized as those groups \& such that $\&$ is a semidirect product of the inertial subgroup ${ }^{*}$, and a subgroup ${ }^{\&}{ }_{k}$ which is the extension to $a$ of a finite group of automorphisms on $k$. The invariant subgroup $\mathscr{A}_{0}$ is naturally isomorphic to a "Galois" group of higher derivations as characterized by R. L. Davis. Also, for $h \in \mathcal{K}_{I}, k / h$ is separable (purely inseparable) iff $\&=\&_{k}\left(=\&_{0}\right)$ where $\&=A(h)$. (Received August 25, 1969.)

668-12. JAMES WARD BROWN, University of Michigan, Dearborn, Michigan 48128. On the Sheffer A-type of certain modified polynomial sets. Preliminary report.

The author [Abstract 69T-B100, these CNotices) 16 (1969), 669-670] has introduced the polynominal set $\left\{p_{n}^{(a)}(x)\right\}$ defined by a generating relation of the form $(1-t)^{-a} F(x, t)=\sum_{n=0}^{\infty} p_{n}^{(a)}(x) t^{n}$ where $F(x, t)$ is independent of the parameter $a$. The object here is to study certain modifications of $\left\{p_{n}^{(a)}(x)\right\}$ in the context of generalized Appell representations introduced by R. P. Boas, Jr. and R. C. Buck [ "Polynomial expansions of analytic functions," rev. ed., Academic Press, New York 1964, p. 18] as well as the corresponding operators studied by C. A. Anderson [J. Math. Anal. Appl. 19 (1967), 475-491]. Theorem. If $\left\{p_{n}^{(a)}(x)\right\}$ has a generalized Appell representation (corresponding to the operator $J$ ), then so does $\left\{\mathrm{p}_{\mathrm{n}}^{(\mathrm{a}+\beta \mathrm{n})}(\mathrm{x})\right\}$ (corresponding to the operator $\mathrm{J}(1-\mathrm{J})^{\boldsymbol{\beta}}$ ). Applications
of this result are made in various special cases when $\left\{p_{n}^{(a)}(x)\right\}$ is, in addition, of some A-type as defined by I. M. Sheffer [Duke Math. J. 5 (1939), 590-622]. (Received September 4, 1969.)

668-13. JAMES L. HOWLAND, University of Ottawa, Ottawa 2, Ontario, Canada and JOHN A. SENEZ, Sir George Williams University, Montreal, Quebec, Canada. A constructive method for the solution of the stability problem.

The stability problem may be completely solved whenever the Liapunov matrix equation $S M+M^{T} S=-I$ may be solved for given matrices $M$ in upper Hessenberg form with nonzero codiagonal elements. In this case, the solution may be obtained as a linear combination of linearly independent matrices $S_{i}$, readily obtained from $M$, for each of which $S_{i} M+M^{T} S_{i}$ is diagonal. The $S_{i}$ are characterized by the property $S M=D+T$ where $D$ is diagonal and $T$ is skew-symmetric. Explicit formulae are available for the elements of the $S_{i}$, while the determination of the correct linear combination requires the solution of an $n \times n$ linear system. (Received September 5, 1969.)

668-14. DAHSOONG YU, University of Oklahoma, Norman, Oklahoma 73069. Semigroup theory and finite time stability for a class of partial differential equations.

In the application of semigroup theory to the stability analysis of linear partial differential equations, it is the Friedrich's extension operator rather than the partial differential operator itself that satisfies the conditions of the Hille-Yosida theorem in contrast with several recent applications. [C. Hsu, ANL-7322, 1967]. For a nonlinear continuous system whose nonlinearity is bounded, $\partial u(x, t) / \partial t=(-1)^{1+p} \sum_{|u|=0}^{2 p} a_{a}(x) \partial|a| u(x, t) / \partial x_{1}^{a_{1}} \ldots \partial x_{n}^{a_{n}}+g[t, u(x, t)]$, where $|a|=$ $\sum_{i=1}^{n} a_{i}, x=\left(x_{1}, \ldots, x_{n}\right)$ and the function $a_{a}(x)$ are real valued with $2 p$ continuous derivatives in a bounded domain $\Omega \subset R^{n}, g[t, u(x, t)]$ is a continuous, bounded, real-valued function of $t$ and $u(x, t)$ such that $g(t, 0)=0$, the nonlinear semigroup operator functions only within a finite interval of time, hence it is more meaningful to define stability in terms of finite time stability. [See L. Weiss and E. F. Infante, Proc. Nat. Acad. Sci. U.S.A. 54 (1965), 44-48]. Theorem 1. A sufficient condition for the finite time stability of the unperturbed motion is that the semigroup $\{T(\tau)\}$ be equibounded Theorem 2. A sufficient condition for the exponentially contractive finite time stability of the unperturbed motion is that there exist positive constants $M$ and $k$ such that $\left\|T(\tau) \varphi\left(t_{0} ; b\right)-T(\tau) \varphi\left(t_{0} ; a\right)\right\|$ $\leqq \mathrm{Me}^{-\mathrm{kt}}\left\|\omega\left(\mathrm{t}_{0} ; \mathrm{b}\right)-\varphi\left(\mathrm{t}_{0} ; \mathrm{a}\right)\right\|$. (Received August 13, 1969.)

668-15. ALAN L. SELMAN, Pennsylvania State University, University Park, Pennsylvania 16802. Two hierarchies of reducibilities. Preliminary report.

For each natural number $\mathrm{n} \geqq 1$, define two relations, $R_{n}$, and $\delta_{n}$, on subsets of $\omega$ by $A R_{n} B \leftrightarrow \forall X\left[X \in \Sigma_{n}^{A} \rightarrow X \in \Sigma_{n}^{B}\right]$ and $A \rho_{n} B \leftrightarrow \forall X\left[B \in \Sigma_{n}^{X} \rightarrow A \in \Sigma_{n}^{x}\right]$. Both $R_{n}$ and of are reflexive and transitive. A rec. $B \rightarrow A R_{n} B$ and $A R_{n} B \rightarrow A \in \sum_{n}^{B}$. Also, $A R_{n+1} B \leftrightarrow A^{(n)}$ rec. $B^{(n)}$. The following three theorems follow from this remark together with some new results that generalize well-known theorems about recursiveness to higher levels of the arithmetical hierarchy. Theorem. $R_{\mathrm{n}} \subsetneq R_{\mathrm{n}+1}$. Theorem. $R_{\mathrm{n}}$ is a maximal transitive subrelation on " $\Sigma_{\mathrm{n}}$ in ". Theorem. The $R_{\mathrm{n}}$ ordering is an upper semilattice. Consider now the sequence of relations $\mathscr{o}_{\mathrm{n}}$. Theorem. of $\varsubsetneqq \circ_{n+1}$.

Theorem. $A \mathcal{d}_{n} B \rightarrow A \in \Sigma_{n}^{B}$, and $\mathcal{N}_{1}$ is a maximal transitive subrelation of $\Sigma_{1}$ in' . Theorem. A $d_{1} B$ if and only if there exist recursive functions $f$ and $g$ so that $x \in A \leftrightarrow \mathbb{Z} \forall z<f(y) g(x, y, z) \in B$. Theorem. The of ${ }_{\mathrm{n}}$-ordering is an upper semilattice, and the class of $\Sigma_{\mathrm{n}}$ sets is the 0 degree for the $\delta_{\mathrm{n}}$-degrees. (Received September 8, 1969.)

## 668-16. WITHDRAWN.

668-17. JAMES R. ROYSE, Department of Philosophy, San Francisco State College, San Francisco, California 94132. Systems of ramified set theory using constructive ordinals.

Using notations for constructive ordinals, we describe a hierarchy of first-order systems of ramified (type-free) set theory. $T_{a}$ is the system with all orders less than |a|, where a is a notation for |a|. We give a finite axiomatization of $T_{a}$ by using the predicates $\operatorname{Ord}(x)-x$ is a notation $<_{0} a$, $x<y-|x|<|y|, O(x, y)--y$ is of order $x$. The objects of order $O$ are to be the natural numbers, and the orders are cumulative. Axioms formalizing properties of the notations are included, a finite number of comprehension axioms, using Bernay's method, plus the principle of mathematical induction for sets of order < |a|. Let $T_{a+1}$ be $T_{2} a$, etc. Then: Theorem $1 . T_{a+4}-\operatorname{Cons}\left(T_{a+1}\right)$. Theorem 2. Mathematical induction on sets of order $|a|+3$ is not derivable from the other axioms of $\mathrm{T}_{\mathrm{a}+4}$. (Received September 8, 1969.)

668-18. ROBERT S. STRICHARTZ, Cornell University, Ithaca, New York 14850. The stationary observer and the Klein-Gordon equation.

Let $x \in E^{n}, t \in E^{1}, \square=-\partial^{2} / \partial t^{2}+\Delta_{x}$. We wish to determine solutions of $\square u=m^{2} u$ given $u(0, t)$ and certain special derivatives $P\left(D_{x}\right) u(0, t)$. Let $Y_{k 1}(x), Y_{k 2}(x), \ldots$ be an orthonormal basis for the spherical harmonics of degree $k$ in $E^{n}$. Theorem. Given tempered distributions $f_{k i}(t)$, arbitrary if $m=0$ and satisfying $\hat{f}_{k i}(s)=0$ for $-m<s<m$ if $m>0$, there exists a unique solution of $\square u=m^{2} u$ satisfying $Y_{k f}\left(D_{x}\right) u(0, t)=f_{k i}(t)$ (this makes sense because $-m^{2}$ is partially hypoelliptic in $x$ ). An explicit formula gives $u$ in terms of the $f_{k i}$ and the energy of $u$ is computed in terms of certain norms of the $\mathrm{f}_{\mathrm{k} \cdot}$. Some related equations are also examined. (Received September 8, 1969.)

668-19. ANTHONY J. PENICO, University of Missouri, Rolla, Missouri 65401. Functionalanalysis identities for biadditive mappings on modules with nonassociative scalars.

Let $A$ be a nonassociative algebra with 1 (over the field $F$ ) admitting an involution $a \rightarrow \bar{a}$, and obtainable, by the Cayley-Dickson construction (for terminology, cf. R. D. Schafer, "Nonnassociative algebras," Academic Press, New York, 1966), from one of its subalgebras. Let $M$ be a left A-module. Let ( , ) : $M \times M \rightarrow A$ be a mapping satisfying, for every $a$ in $A$ and $x, y$ in $M$ : (i) ( $x, y$ ) is additive in each argument; (ii) $(x, y)=\overline{(y, x)}$; (iii) $(x, a y)+(a y, x)=(x, y) \bar{a}+a(y, x)$; (iv) ( $x, x)$ is in the center of $A$. Then it can be shown that the identity (*) $(x+a y, x+a y)(y, y)=(x, x)(y, y)-(x, y)(y, x)+$ $[(x, y)+a(y, y)][(y, x)+a(y, y)]$ holds. Certain other familiar identities arising in classical functional analysis (e.g., polarization identity) can also be established. If $F$ is an ordered field, then with suitable restrictions on the properties of ( $\mathrm{x}, \mathrm{y}$ ), one can infer from (*) the Cauchy-Schwarz inequality
$(x, x)(y, y) \geqq(x, y)(y, x)$. The above-mentioned results appear to lay a foundation for a functionalanalysis theory for modules over the algebras $A$, and significantly extend results of a number of authors concerning linear spaces over the real quaternions and real Cayley numbers (octonions). (Received September 8, 1969.)

668-20. ITREL E. MONROE, Dartmouth College, Hanover, New Hampshire 03755. Utilizing additive functionals to define stopping times to obtain preassigned distributions.

Let $\left(X_{t}, F_{t}\right)$ be a symmetric stable process of index $a>1$ and $L_{t} X_{\text {be }}$ be local time for ( $X_{t}, F_{t}$ ) at $x$. Let $M$ be the set of measures $\mu$ on $R$ with the property that if $x \in U$ implies $\mu(U)=\infty$, then $\mu(\{x\})=\infty$, where $U$ is an open set. For $\mu \in M$ define the stopping time $\tau^{\mu}=\inf \left\{t: \int L_{t}^{x} \mu(d x)>1\right\}$ and the probability measure $\psi(\mu)$ on R by $\psi(\mu)(\mathrm{A})=\mathrm{P}\left\{\mathrm{X}_{\uparrow} \mu \in \mathrm{A}\right\}$. Theorem. A probability measure $\nu$ on $R$ can be realized as $\psi(\mu)$ for some $\mu \in M$ if and only if $\int|x|^{a-1} v(d x)<\infty$. Theorem. Let $Y$ be a random variable and $\left(X_{t}, F_{t}\right)$ be Brownian motion. If $E\{Y\}<\infty$ then there is a measure $\mu \in M$ such that $X{ }_{\tau} \mu$ has the same distribution as $Y$. If $E\{Y\}=0$ and $E\left\{Y^{2}\right\}<\infty$ then $\mu$ can be chosen such that $E\left\{Y^{2}\right\}=E\{\tau\}$. (Received September 9, 1969.)

668-21. MATTHEW HACKMAN, University of Washington, Seattle, Washington 98105. A nonspectral generator of a bounded group.

Theorem. Let $X \subset L^{\infty}(R)$ be the closed subspace of functions of exponential type 1. $X$ is reflexive; differentiation is bounded by 1 on $X$ and generates the group $\left\{T_{t}\right\}$ of translations; if $\delta \in X^{*}$ is evaluation at $0, \delta\left(T_{t} f\right)=f(t)$ is not the Fourier transform of a complex measure for every $f \in X$. (Received September 9, 1969.)

668-22. RUTH A. BARI, George Washington University, Washington, D. C. 20006. Absolute reducibility of maps of at most 19 regions.

Birkhoff and Lewis have proposed a strong form of the 4 -color conjecture in terms of chromatic polynomials. A configuration is said to be absolutely reducible if its presence in a map assures us that the Birkhoff-Lewis conjecture holds for the given map if it holds for all maps with fewer regions than the given map. Birkhoff and Lewis proved that, in a cubic map of simply connected regions, a proper 2-ring, a proper 3-ring, and a 4-sided region surrounded by a proper 4-ring are absolutely reducible configurations. A regular map which contains no regions with fewer than five sides is called a regular major map. Since every proper map which contains none of the above reducible configurations is a regular major map, a proof that all such maps satisfy the Birkhoff-Lewis conjecture would confirm the 4 -color conjecture. In this paper, certain properties of regular major maps are derived, and all such maps with at most 19 regions are determined within homeomorphism. The Birkhoff-Lewis conjecture is confirmed for each of these maps. It is thus shown that every proper map with at most 19 regions is absolutely reducible. (Received September 9, 1969.)

## ABSTRACTS PRESENTED TO THE SOCIETY

The next deadline for Abstracts will be October 30, 1969. The papers printed below were accepted by the American Mathematical Society for presentation by title. The abstracts are grouped according to subjects chosen by the author from categories listed on the abstract form. The miscellaneous group includes all abstracts for which the authors did not indicate a category.

One abstract presented by title may be accepted per person per issue of thesec $\mathcal{C}$ otices $).$ Joint authors are treated as a separate category; thus, in addition to abstracts from two authors individually, one joint abstract by them may be accepted for a particular issue.

## Algebra \& Theory of Numbers

69T-Al49. JIRI SICHLER, University of Manitoba, Winnipeg 19, Manitoba, Canada. Left universal categories.

A category $U$ is left universal if it has a small left adequate subcategory, and contains (up to equivalence) any category with small left adequate as a full subcategory. Any category with small left adequate is isomorphic to a full category of algebras. Therefore, (see Z. Hedrlifn, A. Pultr, "On full embeddings of categories of algebras," Illinois J. Math. 10 (1966), 392-406, and J. R. Isbell, "Small adequate subcategories," J. London Math. Soc. 43 (1968), 242-246) the category $2(1,1)$ of all algebras with two unary operations is left universal. The size of its left adequate is ( $1, K_{0}$ ) (see J. Sichler," Rich small categories," Abstract 69T-A106, these CNotices) 16 (1969), August issue). There is a left universal category with left adequate of size ( 1,5 ). No category with left adequate of smaller size is left universal. (Received May 5, 1969.) (Author introduced by Professor George A. Grätzer.)

69T-Al50. ALBERT A. MULLIN, USATACOM, Building 200A, Warren, Michigan 48089. On the number of representations as a sum of a prime number of prime numbers.

Definition l. Let $R^{*}(n)$ be the number of representations (up to commatativity) of a natural number $\underline{n}$ as a sum of a prime number of prime numbers. Lemma l. For each natural number $\underline{n}$, $\Pi(n / 2) \leqq R^{*}(n) \leqq n \cdot \Pi(n) \cdot\left(\sum_{m \mu}\lceil\mu(m) \mid / m) / \log ^{2} n\right.$, where $\Pi$ is the standard prime-counting function and $\mu$ is the standard Möbius function. Definition 2 . Let $V_{1}(n)$ be the greatest value (over all representations of $n$ as a sum of a prime number of prime numbers) of the least summands in such representations of $n$. Definition 3. Similarly, let $V_{2}(n)$ be defined by interchanging "greatest" and "least" in Definition 2. Lemma 2. For each natural number $n, V_{1}(n) \leqq[n / 2]_{p} \leqq V_{2}(n)$, where $[n] p$ is the greatest prime number not exceeding $\underline{n}$. Problems. Determine useful lower bounds to $V_{1}(n)$ and upper bounds to $V_{2}(n)$. (Received June 23, 1969.)

69T-Al51. RAYMOND BALBES, University of Missouri, St. Louis, Missouri 63121 and ALFRED HORN, University of California, Los Angeles, California 90024. Projective distributive lattices.

Let $L$ be a distributive lattice. Let $J$ (resp. $M$ ) be the set of join irreducible (resp. meet irreducible) elements of $L$. The main results are the following: If $L$ is countable, then $L$ is projective in the category of distributive lattices if and only if the following conditions hold:
(1) every element of $L$ is a finite sum of elements of $J$; (2) the product of any two elements of $J$
is in J ; (3) every element of $L$ is a finite product of elements of $M$. In the general case, necessary and sufficient conditions for projectivity are (1), (2), (3) and (4): if $F$ is a free distributive lattice and $f: F \rightarrow L$ is an epimorphism then there exists a function $g_{1}: J \rightarrow F$ such that $g_{1}$ preserves order and $\mathrm{fg}_{1}(\mathrm{x})=\mathrm{x}$ for all $\mathrm{x} \in \mathrm{J}$. (Received June 26, 1969.)

69T-Al52. IRVING GERST, State University of New York, Stony Brook, New York 11790. On the theory of nth power residues and a conjecture of Kronecker.

For rational integers a and $n>1$, let $P(a)$ denote the set of primes of which a is an nth power residue, and write $P(a)=P(b)$ when $P(a)$ and $P(b)$ differ at most in a set of primes having Dirichlet density zero. Theorem l. If $P(a)=P(b), a b \neq 0$, then there exists an integer $t$ with $0<t<n,(t, n)=1$ such that either (i) $a b^{t}=d^{n}$ for some integer $d$, or, if $8 \mid n$, (ii) $a b^{t}=2^{n / 2} d^{n}$ for some integer $d$. Conversely, if either (i) or (ii) holds, then $P(a)$ and $P(b)$ differ at most in a finite number of primes. The case $b=1$ had been given first by Trost (Nieuw Arch. Wiskunde $18(1934), 58-61$ ) and together with Theorem 1 implies Theorem 2. For $a b \neq 0, P(a)=P(b)$ iff for some integer $t$ with $0<t<n$, $(t, n)=1, P\left(a b^{t}\right)=P$, where $P$ is the set of all primes. These results are applied to determine when two monic, irreducible binomials which have essentially the same sets of prime divisors determine the same simple extensions over the rationals, thereby settling the validity of a conjecture of Kronecker in this case. As a result, counterexamples to the conjecture can easily be constructed, one simple example being $x^{8}-3.2^{4}, x^{8}-3^{7}$. (This paper will appear in Acta Arithmetica.) (Received June $30,1969$. )

69T-A153. JOHN H. COZZENS, Rutgers University, New Brunswick, New Jersey 08903. Homological properties of the ring of differential polynomials. II.

The ring of twisted polynomials $k[t, \rho]$, where $k$ is an algebraic closure of $Z / 2 Z$ and $\rho$ is the automorphism of $k$ defined by $z \mapsto z^{2}$, "localized" at the miltiplicative subset $\left\{t^{k} \mid k\right.$ an integer $\geqq 0\}$, provides a second example of a principal right and left ideal domain, not a field, that is a right V-ring (see "Homological properties of the ring of differential polynomials," Abstract 69T-A127, these CNotices) 16(1969), August issue). More generally, necessary and sufficient conditions in terms of $k$ and $\rho$ for $k[t, \rho]$ to be a right $V$-ring are obtained. (Received July 14, 1969.)

69T-A154. JIANG LUH, North Carolina State University, Kaleigh, North Carolina 27607. A generalization of the Chevalley-Jacobson Density Theorem.

Let $G$ and $H$ be vector spaces over division rings $\Delta$ and $\Delta^{\prime}$ respectively, where $\Delta \cong \Delta^{\prime}$. A group $N$ of semilinear transformations of $G$ into $H$ is said to be dense if, for every positive Integer $n$ and every $n$ linearly independent elements $g_{1}, g_{2}, \ldots, g_{n}$ in $G$ and every $n$ elements $h_{1}, h_{2}, \ldots, h_{n}$ in $H$, there exists $x \in N$ such that $g_{i} x=h_{i}, i=1,2, \ldots, n$. In this paper, we consider primitive T-rings in the sense of Nobusawa (see Luh, Osaka J. Math. 5 (1968), 165-174). Theorem. For a $\Gamma$-ring $M$ in the sense of Nobusawa, the following conditions are equivalent: (i) $M$ is $a$ primitive $\Gamma$-ring. (ii) There exist abelian groups $G$ and $H$ such that $M \leqq \operatorname{Hom}(H, G), \Gamma \leqq H o m(G, H)$ and $g \Gamma=H, h M=G$ for every $0 \neq g \in G$ and $0 \neq h \in H$. (iii) There exist vector spaces $G$ and $H$ over division rings $\Delta$ and $\Delta^{\prime}$ respectively, where $\Delta \cong \Delta^{\prime}$ such that $M$ is a dense group of semilinear
transformations of $H$ into $G$ and $\Gamma$ is a dense group of semilinear transformations of $G$ into $H$, and the compositions xay, $a x \beta$ for $x, y \in M, a, \beta \in \Gamma$ are compositions of mappings. Moreover, the socle of $M$ is the ideal of semilinear transformations of $H$ into $G$ of finite rank, and the socle of $\Gamma$, where $\Gamma$ is considered as a $\Gamma^{\prime}-$ ring with $\Gamma^{\prime \prime}=M$, is the ideal of semilinear transformations of $G$ into $H$ of finite rank. (Received July 7, 1969.)

69T-Al55. MICHAEL RICH, Temple University, Philadelphia, Pennsylvania 19122 and 5441 North Sawyer, Chicago, Illinois 60625. On a class of nodal algebras. II

Theorem. There do not exist any nodal algebras satisfying the identity $x(x y)+(y x) x=$ 2( xy ) x over any field F of characteristic zero. The proof is obtained by first showing that if x is a nilpotent element of a power-associative algebra satisfying the above identity over a field F of characteristic zero then the operator $L(x)-R(x)$ is a nilpotent operator. (Received July 14, 1969.)

69T-Al56. AHMAD SHAFAAT, Carleton University, Ottawa, Ontario, Canada. Subcartesian products of finitely many finite algebras.

Many varieties $\mathcal{K}$, like those of semilattices, distributive lattices, Stone algebras, abelian groups of exponent dividing $n$, normal idempotent semigroups, satisfy the following condition: $(\cdot)$ Every algebra in $\mathcal{K}$ is isomorphic to a subcartesian product of a family of certain finitely many finite algebras. All the known varieties satisfying (•) also have the property of having finitely many subquasivarieties. We partly explain this and show that if the lattice $L_{q}(\mathcal{K})$ of subquasivarieties of a quasivariety $\mathcal{X}$ of locally finite algebras has a finite maximal chain (in particular, if $\mathrm{L}_{\mathrm{q}}(\mathcal{K})$ is finite), then $\mathcal{X}$ satisfies ( $\cdot$ ). This result provides a general method of establishing ( $)$ for many varieties. Finiteness of $L_{q}(\mathcal{X})$ has another interesting consequence for locally finite $\mathcal{K}$ : Every subclass of $\mathcal{K}$ which contains subalgebras, isomorps and cartesian products of its algebras is a quasivariety. (Received July 14, 1969.)

69T-Al57. CHIN'THAYAMMA, University of Alberta, Edmonton, Alberta, Canada. Short definitions of lattices and semilattices. Preliminary report.

For each of the lattice operations join and meet, nine variants of the associative law and three variants of the absorption law can be obtained by permuting and grouping the variables in all possible ways in the two laws. By combining the associative and absorption laws or their variants another type of identities known as absorptio-associative laws (A. Petcu, Rev. Roumaine, Math. Pures Appl. $10(1965), 339-355$ ) can be formed. In this paper it is proved that 112 independent sets of four identities, 2 associative laws and 2 absorption laws or their variants, and 1128 independent sets of three identities, 2 absorptio-associative laws and 1 idempotent law, define a lattice and 48 sets of four identities of the first type do not define a lattice. Two independent sets of two axioms are also obtained for a semilattice which are much simpler than the set obtained by D. H. Potts (Canad. Math. Bull. 8 (1965), 519) and involve only three variables. None of the sets considered here includes the law of commutativity. (Received June 27, 1969.)

69T-A158. ADOLF MADER, University of Hawaii, Honolulu, Hawaii 96822. The fully invariant subgroups of reduced algebraically compact Abelian groups.

Let $A$ be a reduced algebraically compact group. Associated with each a $\in A$ and each prime $p$ is the Ulm sequence $U_{p}(a)=\left(H_{p}(a), H_{p}(p a), \ldots, H_{p}\left(p^{n} a\right), \ldots\right)$. Let $U(a)=\left(U_{2}(a), U_{3}(a), \ldots, U_{p}(a), \ldots\right)$. Partially order $\underline{U}=\{U(a) \mid a \in A\}$ by defining $U(a) \geqq U(b)$ iff $H_{p}\left(p^{n} a\right) \geqq H_{p}\left(p^{n} b\right)$ for all $n$ and all $p$. Theorem. $\underline{U}$ is a partially ordered set with meet. The set $\underline{U} *$ of dual ideals of $\underline{U}$ is a lattice under set inclusion, and if $\underline{N}$ is the lattice of fully invariant subgroups of $A$, then $a: \underline{U} * \rightarrow \underline{N}: U a=\{a \in A \mid U(a) \geqq h$ for some $h \in U\}$ is a lattice isomorphism. The most interesting case arises when $A=B *$, the $p$-adic completion of an unbounded direct sum $B$ of cyclic p-groups. Especially interesting are the pure fully invariant subgroups of $B^{*}$ containing $\bar{B}$, the maximal torsion subgroup of $B^{*}$. These groups (for $p \neq 2$ ) are characterized by the property that they are pure in $B^{*}$ containing $B$, and that every isomorphism between p-basic subgroups extends to an automorphism of the group. Two such groups are isomorphic iff they are equal. There are $2^{\mathcal{K}_{0}}$ different such groups generated by a single element. (Received June 25, 1969.)

69T-A159. REBECCA SLOVER, Virginia Polytechnic Institute, Blacksburg, Virginia 24061. A note on the radical of row-finite matrices.

Sexauer and Warnock have recently given a necessary and sufficient condition for an element to be a member of the Jacobson radical of the ring of all row-finite matrices over a ring $R$. The purpose of this paper is to give a slightly different characterization of this radical which will be more straightforward. Let $R$ be a ring, $J$ an infinite index set, $R_{J}$ the ring of all $\mathrm{J} \times \mathrm{J}$ row-finite matrices over $R$, and $\Gamma(R)$ the Jacobson radical of $R$. A matrix $A$ is diagonalized provided that if $\left\{a_{i_{1}} j_{1}, a_{i_{2}} j_{2}, \ldots\right\}$ is a sequence of entries of $A$ such that $\left\{j_{1}, j_{2}, \ldots\right\}$ contains infinitely many distinct elements, then there exists a positive integer $p$ such that $a_{i_{1} j_{l}} a_{i} j_{2} \ldots a_{i p} j_{p}=0$. Theorem. Let $A \in R_{J}$. Then $A \in I\left(R_{J}\right)$ if and only if $A \in[I(R)]_{J}$ and each element of the left ideal of $R_{J}$ generated by $A$ is diagonalized. (Received July 14, 1969.)

69T-A160. MICHAEL B. SLATER, University of Bristol, Bristol 8, England. On composing algebras without 1.

Say $A=\left(A,+,^{\cdot}, F, n\right)$ is a composing algebra iff ( $\left.A,+, \cdot, F\right)$ is a (nonassoc.) algebra over the (arbitrary) field $F$, and $n: A \rightarrow F$ satisfies $n(a x)=a^{2} n(x) ; n(x+y)-n(x)-n(y)=g(x, y)$ is F-bilinear; $n(x)=0$ and $g(x, A)=(0)$ imply $x=0 ; n(x \circ y)=n(x) n(y)$. Say $A$ is isotopic to $A_{0}=(A,+, \cdot, F, n)$ iff $x \bullet y=x U \cdot y V$ for some fixed invertible lin. trs. $U, V$ of $(A,+, F)$. If $A$ has a 1 , then $A$ is a composition algebra, and these, together with all their composing isotopes, are well understood (Kaplansky, Proc. Amer. Math. Soc. 4 (1953), 956). Theorem. The composing algebra $A$ is isotopic to a composition algebra iff $\mathbb{G}, c \in A$ so $R_{b}$ and $L_{c}$ are onto maps. Example. Let $H=(H,+, n, R)$ be real Hilbert sequence space with $n(x)=|x|^{2}$. There is a bicontinuous product • on $H$ so $(H,+, \cdot, n, R)$ is a composing algebra with left unity e (in particular $L_{e}$ is invertible), but not isotopic to any composition algebra. The construction is an easy modification of the one by Urbanik and Wright, Proc. Amer. Math. Soc. 11 (1960), 961. (Received July 15, 1969.)

69T-A161. JOHN D. DIXON, Carleton University, Ottawa, Ontario, Canada. The number of steps required in the Euclidean algorithm.

For any pair of integers $u, v$ with $1 \leqq u \leqq v$ the Euclidean algorithm defines a finite list of integers $r_{i}, q_{i}$ with the properties that all $q_{i}>0, r_{0} \geqslant r_{1}>\ldots>r_{n+1}=0, r_{0}=v, r_{1}=u$, and $r_{i-1}=q_{1} r_{i}+r_{i+1}($ for $i=1,2, \ldots, n)$. We call $n$ the length of the algorithm for $u$ and $v$ and denote it by $L(u, v)$. Then we can prove the following. Theorem. For all $\epsilon>0, c>0$ there exist $X$ and $K$ such that, for all $x \geqq x,\left|\left(12 \Pi^{-2} \log 2\right) \log v-L(u, v)\right|<(\log v)^{1 / 2+\epsilon}$ for all except at most $K x^{2}(\log x)^{-c}$ of the pairs $\mathrm{u}, \mathrm{v}$ with $1 \leqq \mathrm{u} \leqq \mathrm{v} \leqq \mathrm{x}$. (Received July 14, 1969.)

69T-162. PAUL F. CONRAD and JOHN DAUNS, Tulane University, New Orleans, Louisiana 70118. An embedding theorem for lattice ordered fields.

Consider a commutative lattice ordered field $G$ whose additive group has a finite lattice basis. (1) Then the special elements of $G$ form a multiplicative group if and only if for each special element $0<a \in G$, also $a^{-1}>0$. Assume (1). The archimedian classes of positive special elements of $G$ form a group $\Gamma$. Form the $\mathfrak{t}$-field $\mathrm{V}(\Gamma, R)$ of all functions $a: \Gamma \rightarrow R$ into the reals such that $\{\gamma \in \Gamma \mid a(\gamma) \neq 0\}$ is a finite union of sets satisfying the ascending chain condition. (2) If $\Gamma$ is torsion free, then there is an $\mathfrak{r}$-isomorphism $\pi$ : $G \rightarrow \mathrm{~V}(\mathrm{I}, \mathrm{R})$. These results will appear shortly in the Pacific Journal of Mathematics. (Received June 7, 1969.)

69T-Al63. GEORG J. RIEGER, State University of New York at Buffalo, Amherst, New York 14225. On linear algebra over the quaternions. I.

Denote by $K$ the division ring of the quaternions. The center of $K$ is the field $R$ of the real numbers. For natural $n$, consider $K^{n}$ as left vector space over $K$ with the symplectic inner product. For a linear map $A: K^{n} \rightarrow K^{n}$, the eigen value problem takes the form $x A=\lambda x$ with $\lambda \in K$ and $0 \neq x \in K^{n}$. For selfadjoint $A$, it follows $\lambda \in R$; the existence of eigen values $\lambda$ is proved by a compactness argument for Rayleigh quotients ( $\mathrm{xA}, \mathrm{x}$ )/( $\mathrm{x}, \mathrm{x}$ ), and the spectral theorem is given with several consequences, like the existence of $\sqrt{ } A$ and the polar decomposition of $A$. After introducing convergence, various iteration methods are discussed and special attention is given to speed of convergence; we study the methods of successive iteration and steepest descent for solving systems of linear equations and also the Jacobi method for computing eigen values of selfadjoint matrices. (Received July 22, 1969.)

69T-A164. BRIAN J. DAY, University of New South Wales, Kensington, New South Wales, 2033, Australia. Closed categories of functors.

The words category, functor, natural transformation, mean $q$ category, $q$-functor, $q$-natural transformation for a fixed (and suitably complete) symmetric monoidal closed category $v$. A pre$\underline{\text { monoidal category is a category } a \text { together with functors } P: a^{o p} \otimes a^{o p} \otimes a \rightarrow r, \mathrm{~J}: a \rightarrow r, \text { and }, ~}$ natural transformations satisfying certain axioms; a monoidal category is a special case. Theorem. To each premonoidal structure on a small category $a$ there corresponds an essentially unique biclosed monoidal structure on the functor category $[a, v]$ (that is, a monoidal structure with - $\otimes \mathrm{T}$ and $T \otimes$ - having right adjoints for each $T \in[a, v]$ for which $a(A-) \otimes l a(B-) \cong P(A B-)$. Examples of such closed functor categories are simplicial sets, complexes of abelian groups, modules over a commutative ring or over a hopf algebra, and bimodules over a small category. Other closely related
closed categories are those of sheaves over a topology and algebras over a (finitary) commutative theory. (Received July 18, 1969.)

69T-A165. STEPHEN H. BROWN, North Carolina State University, Raleigh, North Carolina 27607. On right quasi-duo rings.

Definition. A ring is called a right duo ring provided every right ideal is a left ideal. Definition. A maximal right ideal $M$ in a ring $R$ is called a quasi-modular maximal right ideal of $R$ if and only if $R^{2} \& M$. Definition. A ring $R$ will be called right quasi-duo provided every quasimodular maximal right ideal is a left ideal. Example. Let $R=\left(\begin{array}{l}D \\ j \\ D\end{array}\right)$. Where $D$ is a division ring. Then $R$ is right quasi-duo but not right duo. Proposition. $R$ is right quasi-duo if and only if for each pair of quasi-modular maximal right ideals, $M$ and $N, R / M \cong R / N$ implies $M=N$. Proposition. If $R$ is right quasi-duo and has a left identity then each maximal right ideal is a maximal left ideal. Theorem. Let $R$ be a right quasi-duo ring with 1 and let $R_{n}$ denote the $n \times n$ matrix ring over $R$. Then every simple right $R_{n}$-module is injective if and only if $R$ is a right duo regular ring. (Received July 28, 1969.)

69T-A166. R. PADMANABHAN, University of Manitoba, Winnipeg 19, Manitoba. Regular identities of lattices. II.

Theorem. The lattice of all equational classes of algebras defined by the regular equations of lattices (see Abstract 69T-A62, these $\mathcal{C}$ Notices) 16 (1969), 652) is isomorphic to the direct product of the lattice of all equational classes of lattices and the two-element chain. Hence, by a result of B. Jonsson ("Algebras whose congruence lattices are distributive," Math. Scand. 21 (1967), 110-121), this lattice is distributive. (Received July 28, 1969.)

69T-A.67. RICHARD M. WILSON, Ohio State University, Columbus, Ohio 43210. Difference families in elementary abelian groups.

Let $G$ be a finite additive abelian group of order $v$. $A(v, k, \lambda)$ - difference family in $G$ is a family ( $B_{i} \mid i \in I$ ) of subsets of $G$ (not necessarily distinct), each of cardinality $k$, and such that the list ( $a-b \mid a \neq b ; a, b \in B_{i} ; i \in I$ ) includes each nonzero element of $G$ precisely $\lambda$ times. The existence of such a difference family implies the existence of $a(v, k, \lambda)-$ - BIBD with $G$ as a regular group of automorphisms. The number of "base blocks" (the cardinality of $I$ ) is $\lambda(v-1) / k(k-1)$, which is necessarily an integer. Theorem. Let $k$ be given and let $q$ be a prime power, $q>\{(1 / 2) k(k-1)\}^{k(k-1)}$. Then there exists a $(q, k, \lambda)$ - difference family in the elementary abelian group of order $q$ if and only if $\lambda(q-1) \equiv 0(\bmod k(k-1))$. Corollary. Given $k$, there are infinitely many integers $v$ for which a ( $v, k, 1$ ) - BIBD exists. (Received June 19, 1969.) (Author introduced by Professor D. K. Ray Chaudhuti.)

69T-A168. KENNETH E. HUMMEL, The University of Texas, Austin, Texas 78712. Restricted semiprimary rings. Preliminary report.

Let $R$ be a ring with unity. $R$ is restricted semiprimary ( $R S P$ ) if and only if $R / A$ is semiprimary for every nonzero ideal $A$ of $R$. $R$ is restricted artinian ( $R M$ ) if and only if $R / A$ is left artinian for every nonzero ideal $A$ of $R$. Clearly: if $R$ is $R M$ then $R$ is $R S P$, but an example shows the converse is false. Theorem 1. $R$ is RSP if and only if (i) $R / M$ is artinian for every nonzero maximal ideal $M$ of $R$, and (ii) each nonzero ideal contains a product of maximal ideals. Theorem 2 . (a) A right perfect RSP ring is semiprimary, and (b) If R is RSP and not semiprimary then $R$ is prime. Additional properties of commutative RSP-domains are considered. (Received July 30, 1969.) (Author introduced by Mr. Efraim P. Armendariz.)

69T-Al69. R. J. WARNE, West Virginia University, Morgantown, West Virginia 26506. A class of regular bisimple semigroups. II.

Let $E$ be a band. Let $O(R)$ denote the collection of $R$-classes of $E$. Order $O(R)$ as follows: if $\left.R_{1}, R_{2} \in O_{( }^{\prime} R\right), R_{1}<R_{2}$ if and only if $e<f$ for all $e \in R_{1}$ and $f \in R_{2}$. If $O(R)$ is order isomorphic to $I^{0}$, the nonnegative integers, under the reverse of the usual order, $E$ is called a naturally ordered band. A bisimple semigroup $S$ is called $E$-bisimple if its set of idempotents form a naturally ordered band. Let N denote the natural numbers. Theorem. S is an E -bisimple semigroup if and only if $S \cong\left(\left(I^{0} \times\{0\}\right) \times(G \times K *)\right) \cup\left(\left(I^{0} \times N\right) \times(G \times K)\right)$, where $G$ is a group and $K$ and $K *$ are sets, under the multiplication $((n, k),(g, p))((r, s),(h, q))=\left((n, k)(r, s), g \theta^{r-t} h \theta^{k-t}, x\right)$, where $x=q$ if $r \geqq k$ and $x=p\left(h \theta^{k-r-1} \gamma\right)$ if $k>r, t=\min (r, k), \theta$ is an endomorphism of $G, \gamma$ is a homomorphism of $G$ into $G_{K}$, the full transformation group on $K$, and juxtaposition denotes multiplication in $G$ and $C$, the bicyclic semigroup. This theorem is the initial structure theorem (mod groups) beyond the Rees Theorem for regular (bisimple) semigroups which are not in general inverse semigroups. An isomorphism theorem is given. For some examples, see [R. J. Warne, Duke Math. J. 33 (1966), 190 ]. (Received July 29, 1969.)

69T-Al70. SAROJ JAIN, State University of New York, Buffalo, New York 14214. A generalisation of q-rings.

Jain, Mohamed and Singh ("Rings in which every right ideal is quasi-injective," Pacific J. Math. 30 (1969)) defined a right q-ring and characterised it as a right self injective ring in which every large right ideal is two-sided. In this paper, we study right continuous rings in which every large right ideal is two-sided. Such rings will be called right s-rings. Our object is to show that some of the results proved for right q-rings still hold for right s-rings. Some of the main results are: (i) a prime right $s$-ring is simple artinian, and (ii) a semiprime ring $R$ is a right $s-r i n g$ if and only if $R=S \oplus T$, where $S$ is a continuous strongly regular ring and $T$ is semisimple artinian. Examples are given to show that a right s-ring is not, in general, a right $q$-ring. (Received August 4, 1969.) (Author introduced by Professor R. S. Varma.)

69T-A172. KENT R. FULLER, University of Iowa, Iowa City, Iowa 52240 and D. A. HILL, University of Oregon, Eugene, Oregon 97403. On quasi-projective modules via relative projectivity.

In C. R. Acad. Sci. Paris Sér. A-B, 286(1969), Sér. A, 361-364]. E. de Robert has considered the concept of relative projectivity which extends that of quasi-projective modules. A module M is projective relative to a module $U$ in case $\operatorname{Hom}(M, \#)$ preserves the exactness of all sequences $\mathrm{O} \rightarrow \mathrm{S} \rightarrow \mathrm{U} \rightarrow \mathrm{T} \rightarrow \mathrm{O}$. Making use of Robert's results, the following answer to a question of C . Faith [Math. Reviews $36, \# 3817$ ] is given. Theorem. If every left $R$-module has a quasi-projective cover then $R$ is a left perfect ring. Other consequences of the methods of relative projectivity are: In order for every direct product of projective left R-modules to be projective it is sufficient that each direct product of copies of $R^{R}$ be quasi-projective. If $R^{M}$ has a projective cover then $M$ is quasi-projective if and only if $M$ is projective over a factor ring of $R$. Every direct sum of copies of a finitely generated quasi-projective module is quasi-projective. (Received August 7, 1969.)

69T-Al73. ANNE B. KOEHLER, MiamiUniversity, Oxford, Ohio 45056. Quasi-projective covers and direct sums.

A ring is associative with a unit element. The notion of projective cover has been defined by H. Bass ["Finitistic dimension and a homological generalization of semiprimary rings," Trans. Amer. Math. Soc. 95 (1960), 466-588], and the definition of quasi-projective cover has been given by L. E. T. Wu and J. P. Jans ["On quasi-projectives," Illinois J. Math. ll(1967), 439-447]. Theorem. If every module has a quasi-projective cover, then every module has a projective cover. Direct sums of quasi-projective and of quasi-injective modules are investigated. (Received August 7, 1969.)

69T-A174. DWIGHT W. READ, University of California, Los Angeles, California 90024. On m-extensions of Boolean algebras. Preliminary report.

Let $\left\{a_{t}\right\}_{t \in T}$ be an infinite indexed set of nondegenerate Boolean algebras. Let $\left\{\left\{i_{t}\right\}_{t \in T}, B\right\}$ be the Boolean product of $\left\{a_{t}\right\}_{t \in T}$. If $a$ is a Boolean algebra, let $K_{a}$ be the class of all m-extensions of $a, m$ a fixed, arbitrary, infinite cardinal number. Let $T$ ' be the set of all $t \in T$ such that $K_{a_{t}}$ contains more than one element. Theorem. $K_{\beta}$ does not contain a smallest element if $\overline{\mathrm{T}}^{\prime} \geqq \sigma_{\tau}$. Corollary. If $a_{t}=a_{t^{\prime}}$ for all $t, t^{\prime} \in T$, then $\mathcal{X}_{\beta}$ contains a smallest element if and only if an m -extension of $\beta$ is an m -completion. This answers the question posed by Sikorski on whether in
 p. 169] in the negative as Sikorski gives an example (in the case where $m=\sigma$ ) of a Boolean algebra $a$ such that ${ }_{a}$ contains more than one element ( p .172 ). A method of constructing, for an arbitrary infinite cardinal $m$, a Boolean algebra $a$ such that $k_{a}$ contains more than one element is also given. (Received August 8, 1969.)

69T-Al75. WITHDRAWN.

69T-A176. RAYMOND BALBES, University of Missouri, St. Louis, Missouri 63121. A representation theorem for distributive quasi-lattices.

In [Fund. Math. 60 (1967), 191-2001, J. Płonka introduces the notion of a distributive quasilattice (DQL). They are algebras which satisfy the usual axioms for a distributive lattice (considered as an algebra with two binary operations) except that the absorption laws are deleted. The purpose of this paper is to obtain a representation theorem for distributive quasi-lattices which is analogous to the representation of distributive lattices as rings of sets. Indeed, in the case when a $D Q L$ is a distributive lattice, our representation coincides with the classical one. (Received August 13, 1969.)

69T-A177. RICHARD D. WEINER, Washington University, St. Louis, Missouri 63130. Certain systems of field extensions.

Let $F$ be a field of characteristic $p \neq 0, K / F$ a Galois extension of degree relatively prime to $p$, and $G=G a l(K / F)$. For a fixed positive integer $n$, let $K^{n}$ denote the algebra of $n$-tuples of elements in $K$ with addition and maltiplication defined componentwise. G can be regarded as operating on $K^{n}$ componentwise. For each $X \in \operatorname{Hom}\left(G, G L\left(n, F_{p}\right)\right)$, where $F_{p}$ is the prime field of $F$, let $\mathrm{V}_{X}=\left\{\underline{\underline{V}} \in K^{\mathrm{n}}: \sigma(\underline{v})=X(\sigma)^{\mathrm{t}} \underline{\mathrm{v}}, \forall \sigma \in \mathrm{G}\right\}$ with t denoting transpose. Let P be the map defined on an algebraic closure $\Omega$ of $K$ by $P(x)=x^{p}$ - $x$. There exists (Jacobson, "Lectures in Abstract Algebra," Vol. III, pp. 132-140) a bijective correspondence between the abelian extensions $L / K, \ni L \subset \Omega$ and Gal(L/K) has order $p^{n}$ and exponent $p$, and the subgroups $Q / P(K)$ of $K / P(K)$ having order $p^{n}$ and exponent $p$. The principal result of this paper shows that such an extension $L$ of $K$ admits an action $\bar{G}$ of $G$ lifting $G$ and leaving $L$ invariant iff $\mathbb{G}$ an element $\underline{v} \in V_{X}$ for some $X \in \operatorname{Hom}\left(G, G L\left(n, F_{p}\right)\right) \ni$ the set $\left\{v_{1}+P(K), \ldots, v_{n}+P(K)\right\}$ generates the group $Q(L) / P(K)$ which corresponds to $L$. If this occurs and if $\underline{w} \in L^{n} \ni P(\underline{w})=\underline{v}, L=K\left(w_{1}, \ldots, w_{n}\right)$ and such an action $\bar{G}$ is defined by $\bar{\sigma}(\underline{w})=X(\sigma)^{t} \underline{w}$. A generalization of this result to the case where $\mathrm{Gal}(\mathrm{L} / \mathrm{K})$ is a finite abelian p-group will be forthcoming. (Received August 18, 1969.)

69T-Al78. E. W. JOHNSON and JOHN P. LEDIAEV, University of Iowa, Iowa City, Iowa 52240 and J. A. JOHNSON, California Technological Jet Propulsion Laboratory, Pasadena, California, 91109. Structure and embedding theorems for unique normal decomposition lattices.

Let $\mathcal{L}$ be a commutative multiplicative lattice in which each element is a join of join principal elements and in which $I$ is compact. $\mathcal{L}$ is a unique normal decomposition lattice (or UND lattice) if each element of $\mathcal{L}$ has a unique normal decomposition. A multiplicative lattice isomorphism $f: \mathcal{L} \rightarrow \mathscr{L}^{\prime}$ which maps primes into primes, primaries into primaries and 0 into 0 is called a dense embedding if for each $A^{\prime} \in \mathcal{L}^{\prime}$ which is not less than or equal to any isolated component of zero there exists a nonzero $A \in \mathcal{L}$ such that $f(A) \cong A^{\prime}$. Theorem $1 . \mathcal{L}$ is a UND lattice if and only if $\mathcal{L}$ is a finite direct sum of lattices having a nilpotent maximal, and one-dimensional lattices in which 0 is prime and in which each nonzero element is greater than or equal to a product of nonzero prime elements. Theorem 2. A lattice $\mathcal{L}$ in which each element has a normal decomposition is a UND lattice if and only if there exists a dense embedding of a lattice of ideals of a finite direct sum of Dedekind domains and special primary rings into $\mathcal{L}$ such that maximal ideals are mapped onto maximal elements of $\mathcal{L}$. Theorem 3. Let $\mathcal{L}$ be a UND lattice. Each primary element of $\mathcal{L}$ is a power of a prime if and only if $\mathcal{A}$ is represented as a lattice of ideals of a direct sum of Dedekind domains and special primary rings. (Received August 18, 1969.)

69T-Al79. C. J. EVERETT and NİCHOLAS C. METROPOLIS, Los Alamos Scientific Laboratory, Los Alamos, New Mexico 87544. Approximation of the $\nu$ th root of N .

Aclass of functions $g_{K}(w), K \geqq 2$, is defined, for which the recursive sequences $w_{n+1}=$ $g_{K}\left(w_{n}\right)$ converge to $N^{l / \nu}$, with relative error $e_{n+1} \cong C(K) e_{n}^{K}$. Newton's method results when $K=2$. The coefficients of the $g_{K}(w)$ are of the form $U_{K} / L_{r} U_{K-r}$, where $U_{r}, L_{s}$ are generalized "factorials," and form a triangle which is Pascal's when $\nu=2$. In this case, if $w_{1}=x_{1} / y_{1}$, where $x_{1}, y_{1}$ is the first positive solution of Pell's equation $x^{2}-N y^{2}=1$, then $w_{n+1}=x_{i n+1} / y_{n+1}$ is the $K^{n}$ pth or $2 K^{n}$ pth convergent to the continued fraction for $\sqrt{N}$, its period $p$ being even or odd. (Received August 22, 1969.)

69T-A180. CRAIG R. PLATT, University of Manitoba, Winnipeg 19, Manitoba, Canada. One-one and onto in algebraic categories.

An algebraic category is a category whose objects are universal algebras and whose morphisms are all the homomorphisms between them. Theorem. Let $K$ be a small category and I and $S$ subcategories of $K$. Then (a) and (b) are equivalent: (a) There exist an algebraic category $H$ and an isomorphism $F: K \rightarrow H$ of categories such that the image of $I$ is the set of $1-1$ homomorphisms of $H$, and the image of $S$ is the set of all onto homomorphisms of $H$; (b) the following hold: (1) $K$, $I$, and $S$ have the same objects, (2) I consists of monomorphisms of $K$, ( $2^{\prime}$ ) S consists of epimorphisms of $K$, (3) if $x y$ is in $I$, then $x$ is in $I$ (where $x y$ means $x$ followed by $y$ ), ( $3^{\prime}$ ) if $x y$ is in $S$, then $y$ is in $S$, (4) if $m$ is in $I, n$ is in $S, x$ and $y$ are in $K$, and $x m=n y$, then $x=n z$ for some $z$ in $K$. For the case of a one-object category, this was proved by M. Makkai (Acta Math. 15 (1964), 297-307). For small categories, the above solves Problem 10 of Grätzer ("Universal algebra," Van Nostrand, Princeton, N. J., 1968). (Received August 21, 1969.)

69T-A181. NARAIN D. GUPTA, University of Manitoba, Winnipeg 19, Manitoba, Canada. Metanilpotent varieties of groups.
A. L. Šmel'kin Dokl. Akad. Nauk SSSR 178 (1968), 307-310 = Soviet Math. Dokl. 9 (1968), 100103] has proved that if $\mathscr{F}$ is a subvariety of $\mathfrak{n}_{c} \mathscr{\ell}$ (nilpotent-of-class-c-by-abelian) such that $\mathfrak{B}$ does not contain the variety of all metabelian groups then every $\mathfrak{B}$-group is an exponent-by-nilpotent-by-
 August 25, 1969.)

69T-A182. CHARLES A. GREEN, University of Maine, Orono, Maine 04473. Upper semilattices in which prime and irreducible ideals coincide.

Let $S$ be an upper semilattice. Definitions are as in Abstract 68T-A55, these CNotices 15 (1968), 1040, except that the empty set is included among the ideals. Theorem 1 . The irreducible and prime ideals in $S$ coincide iff $S$ is a decomposition semilattice. Theorem 2. $S$ is a decomposition semilattice iff for every ideal A and every $x$ not in A there exists a prime ideal containing $A$ but not $x$; equivalently, iff every ideal is the intersection of the prime ideals containing it. (Received August 26, 1969.)

69T-A183. RONALD F. HIRSHON, Polytechnic Institute of Brooklyn, Brooklyn, New York 11201. Direct decompositions of a group into indecomposable direct factors.

A group which obeys the maximal condition for normal subgroups may not in general be decomposed in an essentially unique way as a direct product of directly indecomposable groups. Our main result indicates that if $G$ satisfies the maximal condition for normal subgroups then the possibility for an essentially unique decomposition for $G$ is dependent on the cancellation properties of homomorphic images of G. This result generalizes the well-known theorem that a group with a principal series may be decomposed in an essentially unique way. It is also shown that for certain groups $G$, the hopficity of homomorphic images of G plays a role in determining whether or not $G$ may be decomposed in an essentially unique way. (Received August 22, 1969.)

69T-A184. MARK L. TEPLY, University of Florida, Gainesville, Florida 32601. On noncommutative splitting rings.

The concepts of torsion and primary are as in S. E. Dickson's paper, "Decomposition of modules." II: "Rings without chain condition," Math. Z. 104 (1968), 349-357. All rings have identity, and all modules are left unitary $R$-modules. Definition. A ring $R$ is called splitting if each module contains its torsion submodule as a direct summand. In his paper, 'Noetherian splitting rings are Artinian," J. London Math. Soc. 42 (1967), 732-736, Dickson conjectures that all splitting rings are torsion and shows that the conjecture holds for commutative Noetherian rings. Theorem 1. Suppose each S-primary filter contains two-sided ideals $L_{s}, K_{s}$ satisfying: (i) $K_{s} \subseteq L_{s} \underset{\neq}{\subset}$. (ii) $L_{s}$ is finitely generated as a left ideal, and (iii) $K_{s}$ is finitely generated as a right ideal. Then $R$ is a splitting ring if and only if R is torsion. Corollary. Suppose each maximal left ideal is two-sided and is finitely generated as both a left and a right ideal. Then $R$ is splitting if and only if $R$ is torsion. Theorem 2 . Suppose $R / J(R)$ is torsion, where $J(R)$ denotes the Jacobson radical of $R$. Then $R$ is splitting if and only if R is torsion. (Received August 28, 1969.)

69T-A185. R. ALAN DAY, McMaster University, Hamilton, Ontario, Canada. Injectives in equational classes in Heyting algebras. Preliminary report.

A Heyting algebra is a bounded relatively-pseudo-complemented lattice considered as a (universal) algebra with operations: $\vee, \wedge, \rightarrow, 0,1$, where $x \leqq a \rightarrow b$ iff $a \wedge x \leqq b$. For $n<\omega$, let $A_{n}$ be the Heyting algebra obtained by adding a new unit to the Boolean algebra with $2^{n}$ elements. Let $\mathcal{F}$ be the equational class of all Heyting algebras. Lemma 1 . For $A, B \in \mathbb{A}, A \in S(B)$ then $B$ is an essential extension of $A$ iff $\forall b \in B, b<1$ implies $\mathbb{Z} a \in A \cdot \ni \cdot b \leq a<1$. Lemma 2. A non-Boolean $A$ is subdirectly irreducible iff $A$ is an essential extension of $A_{1}$. Theorem. For non-Boolean equational class $\{\in \mathcal{N}$, the following are equivalent: (1) $R$ contains a non-Boolean injective algebra, (2) $\left\{\right.$ has (enough) injectives, (3) $\left\{=\operatorname{HSP}\left(A_{n}\right)\right.$ for some $1 \leqq n<\alpha$. (Received August 28, 1969.)

69T-A186. K. B. REID and EZRA BROWN, Louisiana State University, Baton Rouge, Louisiana 70803. On the existence of doubly regular tournaments.

Let $e$ be an arc of a tournament $T$ and denote by $\tau(e)$ the number of 3 -cycles of $T$ which contain e. T is called homogeneous if there is an integer $k$ such that $\tau(e)=k$ for all arcs e of $T$. T is called doubly regular if every node dominates the same number of other nodes and every pair of distinct nodes dominates the same number of other nodes. Theorem 1 . $T$ is homogeneous if and only if $T$ is doubly regular. Theorem 2. Doubly regular tournaments exist for all prime power orders congruent to $3(\bmod 4)$. Theorem 3 . If there exists a doubly regular tournament of order $n$, then there exists a doubly regular tournament of order $2 \mathrm{n}+1$. (Received August 29, 1969.)

69T-A187. TREVOR EVANS, Emory University, Atlanta, Georgia 30322. Residually finite semigroups of endomorphisms.

In "Finitely presented loops, lattices, etc. are hopfian," (J. London Math. Soc. 44 (1969), 551-552) it was shown that Malcev's theorem that a finitely generated residually finite group is hopfian, generalizes to any variety of algebras. There is a similar generalization of Baumslag's theorem that the automorphism group of a finitely generated residually finite group is residually finite (J. London Math. Soc. 38 (1963), 117-118) to semigroups of endomorphisms of residually finite algebras in any variety. This generalization yields, by considering the semigroups of endomorphisms of free groupoids and free loops, new proofs of the well-known facts that free semigroups and free groups are residually finite. (Received August 29, 1969.)

69T-A188. PAUL E. SCHUPP, Courant Institute, New York University, New York, New York 10003. Small cancellation quotient groups of free products with amalgamation.

There has been considerable study of "small cancellation" quotient groups of free groups and free products. We have discovered that there is a small cancellation theory of quotient groups of free products with amalgamation. Many previous results from the free product situation, such as the generalized Greendlinger's Lemma and results on torsion elements, continue to hold in the free product with amalgamation case. The present theory has some immediate applications to embedding problems. Say that a group $G$ has property $Q$ if every countable group can be embedded in a quotient group of $G$. Theorem. Let $K$ be a free group. Let H be a finitely generated subgroup of K with $[\mathrm{K}: \mathrm{H}]=\infty$. Let $L$ be any group containing, as a proper subgroup, an isomorphic copy, $\mathrm{H}^{\prime}$, of H . Then the group $K * L$ amalgamating $H=H^{\prime}$, has property Q . Theorem. The group $G=$ $\left\langle a, b, c, d ; b^{-1} a b=a^{2}, c^{-1} b c=b^{2}, d^{-1} c d=c^{2}, a^{-1} d a=d^{2}\right\rangle$ used by $G$. Higman to prove the existence of a finitely generated infinite simple group, has property Q. (Received September 2, 1969.)

69T-A189. DEBORAH E. PRINCE, University of Kansas, Lawrence, Kansas 66044. X-hypercentral groups.

Let X be a class of groups closed under the taking of subgroups, quotient groups, and finite direct sums. For any group $G$, let $X_{1}(G)$ be the set of elements $x \in G$ such that $G$ induces an $X$ group of automorphisms on $x^{G}$. This set is a characteristic subgroup of $G$. By defining $X_{a+1}(G) / X_{a}(G)=$
$X_{1}\left(G / X_{a}(G)\right)$ and taking unions at limit ordinals, it is possible to define the $X$-hypercentral series of a group $G$. If $G=X_{\beta}(G)$ for some $\beta, G \in H X A$. Theorem. Let $X^{l}$ be the class of noetherian groups and $X^{2}$ the class of artinian groups. For $i=1,2$, the cartesian product of $H X^{i} A$ groups $\left\{G_{a}\right\}_{a \in \triangle}$ is an $H X^{i} A$ group if and only if all but finitely many of the $G_{a}$ are nilpotent of bounded class. An element $x \in G$ is called residually $\underline{X}$ if $x=e$ or if there exists $N \triangleleft G$ such that $e \neq x N \in X_{1}(G / N)$. A group $G \in R_{X}$ if each element is residually $X$. Theorem. If $G \in R_{X}$ and if $G$ satisfies the minimal condition for normal subgroups, then $G$ is an $X$ extension of a ZA group. In particular, $G \in H X A$. Theorem. If $G \in R_{X}$ and $N$ is a minimal normal subgroup of $G$, then $N \leqq X_{1}(G)$. Theorem. If the set $R=R(G)$ of residually $X$ elements of $G$ forms a proper subgroup of $G$, then: (i) $G=G$. (ii) G has no proper normal subgroups $N$ such that $G / N \in X$. (iii) $X_{1}(G)=X_{2}(G)=Z_{1}(G)=Z_{2}(G)$. Furthermore, if $G$ satisfies the minimal condition for normal subgroups, then $R=Z(G)$. (Received September 2, 1969.)

## Analysis

69T-B167. William D. L. Appling, North Texas State University, Denton, Texas 76203. Concerning part of the Bochner-Radon-Nikodym theorem.
$U, F, R_{B}, R_{A}, R_{A}^{+}$and the notion of integral are as in previous abstracts of the author. For each $f$ in $R_{A}$ let $Q_{f}$ denote $\left\{t \mid t\right.$ in $R_{B}, \int_{U} t f$ exists $\}$. For each $g$ in $R_{A}^{+}$, let $Z_{g}$ denote $\left\{t \mid t\right.$ in $Q_{g}$, $\left.0=\int_{U}\left\{\int_{V}|t(V)-t(I)| g(I)\right\}\right\}$. Theorem 1 . If $g$ is in $R_{A}^{+}$, $s$ is in $Q_{g}$, and for each $c>0$ there is some $t$ in $Z_{g}$ such that $\int_{U}|s(V)-t(V)| g(V)<c$, then $s$ is in $Z_{g}$. Theorem 2 . If $g$ is in $R_{A}^{+}$, then $Z_{g}$ includes $I_{g}=\left\{t \mid t\right.$ in $Q_{g}$, $t$ has no more than two numbers in its range $\}, Z_{g}$ contains every maximum function of every finite subset of $I_{g}$, and finally, by Theorem $\mathrm{l}, \mathrm{Z}_{\mathrm{g}}$ includes $\mathrm{Q}_{\mathrm{g}}$ and hence is $\mathrm{Q}_{\mathrm{g}}$. Theorem 3 . Suppose $T$ is a transformation from $R_{A}$ into $R_{A}$ such that $T(\underline{0})=\underline{0}$ and for some $K$ and all $f$ and $h$ in $R_{A}$ and $V$ in $F, \int_{V}|T(f)(I)-T(h)(I)| \leq K \int_{V}|f(I)-h(I)|$. If $f$ is in $R_{A}, t$ is in $Q_{f}$ and for each $I$ in $F$, $\mathrm{t}(\mathrm{I}) \mathrm{T}(\mathrm{f})=\mathrm{T}(\mathrm{t}(\mathrm{I}) \mathrm{f})$, then $\int_{\mathrm{V}} \mathrm{tT}(\mathrm{f})=\mathrm{T}(\mathrm{ft})(\mathrm{V})$ for all V in F . (Received April 9, 1969.)

69'T-Bl68. JAMES R. CHOIKE, Wayne State University, Detroit, Michigan 48202. An elementary proof of a theorem of Valiron.

Theorem. Let $f(z)$ be an unbounded holomorphic function in $|z|<l$ that is bounded on a boundary path $L_{l}$ whose end is $|z|=1$. Then there exists a boundary path $L$ along which $f(z) \rightarrow \infty$. In particular, if $L_{1}$ is a spiral, then $L$ is also a spiral. This theorem was proved by G. Valiron ["Sur les singularites de certaines fonctions holomorphes et de leurs inverses," J. Math. Pures Appl. 15 (1936), 423-435]. Outline of proof. Let $R$ be the simply-connected region $\{|z|<1\}-L_{1}$. We map $R$ in a one-to-one conformal manner onto the unit disk $|\zeta|<1$ in such a way that the initial point of $L_{1}$ corresponds to $\zeta=-1$ and the prime end $P$ of $R$, whose impression is $|z|=1$, corresponds to $\zeta=+1$. Denote this map by $\zeta=\zeta(z)$ and let $z=z(\zeta)$ be the inverse map. Let $F(\zeta)=f(z(\zeta))$. We now apply the following consequence of the Phragmen-Lindelöf theorem and the Gross-Iversen theorem. Theorem. Let $F(\zeta)$ be holomorphic in $|\zeta|<1$ and continuous for $|\zeta| \leqq 1$ with the exception of the point $\zeta=+1$. Suppose also that $\left|F\left(e^{i \theta}\right)\right| \leqq M<+\infty$ for $\theta \neq 0$, and that there exists $\zeta_{0}$ such that $\left|\zeta_{0}\right|<1$ and $\left|F\left(\zeta_{0}\right)\right|>M$. Then there exists a path $L$ ' in $|\zeta|<1$ terminating at $\zeta=+1$ with $F(\zeta) \rightarrow \infty$ along L'. (Received June 27, 1969.)

67T-B169. MITSURU NAKAI, University of California, Los Angeles, California 90024, and Nagoya University, Japan. On Royden algebras and quasi-isometries.

Let $M$ be an m-dimensional orientable separable manifold of class $C^{1}$ with Borel measurable metric tensor ( $g_{i j}$ ) such that in each parametric ball there exists a finite constant $k \geqq 1$ with $k^{-1}\left(\delta_{i j}\right) \leqq\left(g_{i j}\right) \leqq k\left(\delta_{i j}\right)$ a.e. The Royden algebra $R(M)$ associated with $M$ is the algebra of all bounded continuous functions $f$ with $\int_{M} d f \wedge * d f<\infty$. A homeomorphism $T$ of an $M$ onto another $M^{\prime}$ is a quasi-isometry if there exists a finite constant $k \geqq 1$ with $k^{-1} \rho(p, q) \leqq \rho^{\prime}\left(T_{p}, T_{q}\right) \leqq k \rho(p, q)$ for every $p$ and $q$ in $M$, where $\rho$ and $\rho^{\prime}$ are the induced metrics on $M$ and $M^{\prime}$. Theorem. $R(M)$ and $\mathrm{K}\left(\mathrm{M}^{\prime}\right)$ are algebraically isomorphic if and only if $\mathrm{M}^{\prime}$ and $\mathrm{M}^{\prime}$ are quasi-isometrically (resp. quasiconformally) homeomorphic for $m \geqq 3$ (resp. $m=2$ ). (Received June 30, 1969.)

69T-B170. JAMES H. WELLS, University of Kentucky, Lexington, Kentucky 40506. Some results concerning multipliers of $H^{\mathrm{p}}$.

Let $\mathrm{E}_{\mathrm{p}}$ and $\mathrm{E}_{\mathrm{q}}$ be subspaces of $\mathrm{L}^{\mathrm{p}}[-\pi, \pi]$ and $\mathrm{L}^{\mathrm{q}}\left[-\pi, \pi\right.$ Irespectively. Define $\mathrm{E}_{\mathrm{q}} \otimes \mathrm{E}_{\mathrm{p}}$ to be the space of all functions $h=\Sigma_{1}^{\infty} f_{i} g_{i}$ where $f_{i} \in L^{p}, g_{i} \in L^{q}$, * denotes convolution and ( $\dagger$ ) $\Sigma\left\|f_{i}\right\|_{p}\left\|g_{i}\right\|_{\mathrm{q}}<\infty$ with $\|\mathrm{h}\|$ the infimum of the numbers ( $\dagger$ ) over all such representations of h . Theorem 1. (i) The dual space of $H \otimes L^{p^{\prime}}(1 \geqq \mathrm{p}<\infty)$ is (linearly isometric to) the space ( $\mathrm{H}, \mathrm{H}^{\mathrm{p}}$ ) of multiplier transforms from $H$ into $H^{p}$. (ii) The dual space of $H \otimes H^{\infty}$ is $\left(H,\left(L / \vec{H}_{0}\right)\right)$. Theorem 2 . The following multiplier problems are equivalent: (i) $\left(\mathrm{H}, \mathrm{H}^{\mathrm{p}}\right)=\left(\mathrm{H}^{\mathrm{p}^{\prime}}, \mathrm{L}^{\infty} / \overline{\mathrm{H}}_{0}^{\infty}\right)=\left(\mathrm{H}^{\mathrm{P}^{\prime}}, \mathrm{C} / \overline{\mathrm{A}}_{0}\right)$ $(1<p<\infty)$, (ii) $(H, H)=\left(L^{\infty} / \widetilde{H}_{0}^{\infty}, L^{\infty} / \widetilde{H}_{0}^{\infty}\right)=\left(C / \bar{A}_{0}, C / \widetilde{A}_{0}\right)$, (iii) $(A, A)=M / \bar{H}_{0}=\left(M / \bar{H}_{0}, M / \bar{H}_{0}\right)$. Here $M$ denotes the finite measures, $C$ the continuous functions on the unit circle and $A$ those elements of $C$ with spectrum in the nonnegative integers. (Received June 30, 1969.)

69T-B171. NIEL SHILKRET, Polytechnic Institute of Brooklyn, 333 Jay Street, Brooklyn, New York 11201. Non-Archimedean measure and integration.

Measure and integration theory are discussed for set functions m:R $\rightarrow[0, \infty], R$ a ring, which have the "maxitivity" property $m\left(\cup E_{i}\right)=\sup m\left(E_{i}\right)$ instead of the usual additivity property. Several examples of maxitive functions are given in which $m(E)$ may reasonably be interperted intuitively as the "size" of $E$. The most important difference between maxitive and additive measures is that maxitive measures, unlike additive measures, may not be continuous from above at every set. The standard theory milst be modified to accomodate this deficiency. Thus, for example, Lebesgue's dominated convergence theorem is valid, provided the dominating function is a mean limit of simple functions. The integral is defined only for nonnegative functions, and is given by $\int f d m=\sup _{a>0} a m\{f \geqq a\}$. The theory of measure and integration of maxitive functions is applied to obtain a simple development of the theory integration for measures taking values in a non-Archimedean field or Banach space. (Received June 27, 1969.)

69T-B172. MARC A. RIEFFEL, University of California, Berkeley, California 94720. Unitary representations induced from compact subgroups.

Let $K$ be a compact subgroup of a locally compact group G. Mackey and Mautner, using definitions involving certain function spaces, showed in some now classical papers how to induce unitary
representations of $K$ up to $G$. In the present paper it is shown how these induced representations can be defined in terms of certain Hilbert module tensor products, or alternatively, certain spaces of Hilbert-Schmidt intertwining operators. Such definitions permit convenient derivations of the basic properties of these induced representations. The methods and results are quite analogous to those contained in "Induced Banach representations of Banach algebras and locally compact groups," J. Functional Analysis 1 (1967), 443-491, but they do not work if $K$ is not compact. An important role is played by the fact that the Hilbert space tensor product turns out to provide the left adjoint for the construction of spaces of Hilbert-Schmidt operators. (Received July 7, 1969.)

69T-B173. W. C. ROYSTER and T. J. SUFFRIDGE, University of Kentucky, Lexington, Kentucky 40506. Typically real polynomials.

Let $T R$ denote the class of normalized functions $f$ analytic and typically real in the unit disk $E$, that is, $\operatorname{Im}\{f(z)\} \cdot \operatorname{Im}\{z\} \geqq 0$. Let $P_{n}(z)=z+a_{2} z^{2}+\ldots+a_{n} z^{n}$ be a polynomial and let $R(u)$ be a polynomial such that $R(\cos \theta)=\sum_{k=1}^{n} a_{k}(\sin k \theta) / \sin \theta$. Then $P_{n} \in T R$ if and only if $R(\cos \theta) \geqq 0$ for all $\theta,-\pi \leqq \theta<\pi$. The following theorem is proved. Let $P_{n}(z)$ be a polynomial of degree $n$ and let $k$, $1<k \leq n$, be fixed. If among all polynomials of degree $n$ belonging to the class TR the $k$ th coefficient $a_{k}$ assumes its extreme value for $P_{n}(z)$, then $R(u)$ has the form $R(u)= \pm 2^{n-1} a_{n}(1 \pm u) \Pi_{j=1}^{(n-2) / 2}\left(u-\gamma_{j}\right)^{2}$ for $n$ even, where $-1 \leqq \gamma_{j} \cong 1,1 \leqq j \leqq(n-2) / 2$ and $R(u)=-2^{n-1} a_{n}\left(1-u^{2}\right) \Pi_{j=1}^{(n-3) / 2}\left(u-\gamma_{j}\right)^{2}$ or $R(u)=$ $2^{n-1} a_{n} \Pi_{j=1}^{(n-1) / 2}\left(u-\gamma_{j}\right)^{2}$ for $n$ odd, $1 \leq j \leq(n-1) / 2$. This theorem is used to obtain exact bounds on $a_{k}, k \leqq n, n \leqq 5$. The coefficient regions for the odd polynomials of degree three and five are obtained also. (Received July 11, 1969.)

69T-B174. ERNEST J. ECKERT, California State College, 5151 State College Drive, Los Angeles, California 90032. Equality of norms of homogeneous quadratic polynomials and their polars.

Let $U_{n}^{*}\left(x_{1}, \ldots, x_{n}\right)$ be a continuous symmetric $n$-linear operator from $E \times i \times \ldots \times E$ to $X$ with $E$ and $X$ real B:inach spaces, and $U_{n}(x)=U_{n}^{*}(x, \ldots, x)$. If $E=X=L^{2}(a, b)$, S. Banach (Über homogene Polynome in $L^{2}$, Studia Math. $7(1938), 36-44$ ) proved that $\left\|U_{n}^{*}\right\|=\left\|U_{n}\right\|$. In general $\left\|U_{n}^{*}\right\| \leqq n^{n} / n$. $\left\|U_{n}\right\|$ (A. Taylor, "Additions to the theory of polynomials in normed linear spaces," Tohuko Math. J. 44 (1938), 302-318; J. Kopec, J. Musielak, "On the estimation of the n-linear transformation," Studia Math. 15 (1955), 29-30). If $X=$ Reals with cone $K=\{x \mid x \geqq 0\}$, or $X=L^{P}(a, b)$ with $K=\{x \mid x(t) \geqq 0$ a.e. $\}$ or $X=C(T)=$ real bounded continuous functions on a topological space $T$ with $K=\{x \mid x(t) \geqq 0\}$, or $X=8$-space with a cone $K(M . G$ Kreĭn, S. G. Kreĭn, "Sur l'espace des fonctions continue definies sur un bicompact de Hausdorff es ses sousespaces semiordonnés," Mat. Sb. 13(55) (1943), 1-37) then we have the following Theorem. Let $U_{2}: E \times E \rightarrow X$ satisfy $U_{2}(x) \geqq 0$ for each $x \in E$. Then $\left\|U_{2}^{*}\right\|=$ $\left\|U_{2}\right\|$. (Preliminary Report) (Received June 30, 1969.)

69T-B175. PAUL ERDÖS, Hungarian Academy of Science, Budapest, Hungary and ARTHUR H. STONE, University of Rochester, Rochester, New York 14627. On the sum of two Borel sets.

It is shown that there exist Borel subsets $C, D$ of the real line whose linear sum $C+D$ $(=\{x+y \mid x \in C, y \in D\})$ is not Borel; one can even make $C$ a Cantor set and $D$ a $G_{6^{\circ}}$. The idea of
the construction is to imitate a known plane counterexample in $A \times B$, where $A$ and $B$ are suitable additive subgroups of $R$, and then transfer it to $A+B \subset R$. By using results of Mycielski (Fund. Math. 55(1964), 139-147), similar examples can be found in any connected topological group, with a complete metric, which is either locally compact or abelian. (Received July 17, 1969.)

69T-Bl76. THOMAS B. HOOVER, University of Michigan, Ann Arbor, Michigan 48104. Hyperinvariant subspaces for $n$-normal operators.

Let $\mathcal{X}$ be a (complex) Hilbert space, and let $A$ be a (bounded, linear) operator on $\mathcal{K}$. A nontrivial proper subspace $m$ of $K$ is said to be hyperinvariant for $A$ if $m$ is invariant under every operator on $K$ that commutes with $A$. An operator $B$ on the Hilbert space $X$ is said to be n-normal if there exists a Hilbert space $\mathcal{W}$ and $n^{2}$ mutually commuting normaloperators $A_{i j}(1 \leqq i, j \geqq n)$ on $\mathscr{N}$ such that $\mathcal{K}$ is the direct sum of $n$ copies of $\mathscr{\&}$ and $B$ is (unitarily equivalent to) the $n \times n \operatorname{matrix}$ ( $A_{i j}$ ) acting on K. Theorem. Every nonscalar n-normal operator has a hyperinvariant subspace. This theorem is proved using techniques for handling continuous matrix-valued functions on a Stonian space developed by Deckard and Pearcy in [Proc. Amer. Math. Soc. 14(1963), 322-328] and [Pacific J. Math. 14(1964), 857-869]. (Received July 2, 1969.)

69T-B177. VIDYADHAR S. MANDREKAR and HABIB SALEHI, Michigan State University, East Lansing, Michigan 48823. The square-integrability of operator-valued functions with respect to a trace-class valued measure.

Let $\mathcal{A}$ and $\mathcal{K}$ be two separable Hilbert spaces and $a$ be a $\delta$-ring of subsets of a space $\sqrt{ }$. . Let $M$ be a countably additive function defined on $a$ with values in the class of all nonnegative operators of traceclass on $\mathcal{*}$. The integral $\int \Phi \Phi_{1} \Psi^{*}$ for $\Phi, \Psi$ (not necessarily bounded) operator-values functions
 complete inner product space over the ring of all bounded operators on $\mathcal{K}$ into $\mathcal{K}$, and the bounded simple functions are dense in it. The stochastic integrals of the form $\int \boldsymbol{\Phi} d \boldsymbol{\xi}$, where $\Phi \in L_{2, M}$, and $\boldsymbol{\xi}$ is a countably additive orthogonally scattered measure on $a$ with values in the operators of HilbertSchmidt class are defined. As a consequence, the Kolmogorov Isomorphism Theorem for operatorvalued stationary stochastic processes is obtained. (Received July 25, 1969.)

69T-B178. B. ALAN TAYLOR, University of Michigan, Ann Arbor, Michigan 48104, and DAVID L. WILLIAMS, Syracuse University, Syracuse, New York 13210. The closed ideals of $A^{\infty}$.

Let $A^{m}$ denote the Banach space of all functions $f$ analytic for $|z|<l$ and such that $f(m)$ is continuous for $|z| \leqq 1$ with norm $\|f\|_{m}=\max \left\{\left|f^{(j)}(z)\right|:|z| \leq 1,0 \leq j \leq m\right\}$. Let $A^{\infty}=\cap\left\{A^{m}: 1 \leqq m \leqq \infty\right\}$ with the topology determined by the seminorms $\|\cdot\|_{m}, 1 \leq m<\infty$. For an ideal $I$ in $A$, let $I_{\text {loc }}$ denote the ideal of all functions $g \in A^{m}$ which vanish (counting multiplicities) on the common zeros in $|z| \leqslant 1$ of the functions in $I$. Also, let $S=S_{I}$ denote the greatest common divisor of the singular inner parts of the functions in I. Theorem I. An ideal I is closed in $A^{\infty}$ iff $I=S_{l o c}=\left\{S g: g \in I_{l o c}\right\}=\left\{h \in I_{l o c}: S\right.$ divides $\left.h\right\}$. Theorem 2. An ideal $I$ in $A, 1 \leq m<\infty$, whose functions have at most a finite number of zeros in common is closed iff $I=\left\{h \in I_{\text {loc }}\right.$ : S divides h\}. (See also B. A. Taylor and D. L. Williams, Abstract 663-203, these $\mathcal{C}$ (Notices) 16(1969), 144.) (Received August 4, 1969.)

69T-B179. C. T. BERENSTEIN, Courant Instutitute of Mathematical Sciences, New York University, New York, New York 10003, and MILAS A. DOSTAL, University of Montreal, Montreal, Quebec, Canada. Topological properties of analytically uniform spaces.

Analytically uniform spaces (AU spaces) which were introduced and studied extensively by L. Ehrenpreis (see, e.g., his forthcoming book "Fourier transforms in several complex variables", Interscience Publication), are duals of certain spaces $\underset{\sim}{W}$ of entire functions with the topology defined by some AU structure which is a family $\{k\}$ of functions $k(\zeta)\left(\zeta \in \mathbb{Z}^{n}\right)$ describing growth of elements in $\underset{\sim}{W}$. We show that such spaces $\underset{\sim}{W}$ are always nuclear. Nuclearity of a large class of AU spaces is also proved. Furthermore we study several topologies on the spaces of test functions for the Beurling distribution spaces $\mathcal{O}_{\omega}^{\prime}$ (cf. G. Björck, "Linear partial differential operators and generalized distributions," Ark. Mat. $6(1966)$, 351-407). Convenient AU structures on these spaces are exhibited. They represent a natural generalization of those studied in our article, Structures analytiques uniformes dans certains espaces de distributions, C. R. Acad. Sci. Paris Sér 268 (1969), 146-149. However the proofs in this more general case are entirely different. (Received August 4, 1969.)

69T-B180. E. WARD CHENEY and KENNETH HUGO PRICE, University of Texas, Austin, Texas 78712. Minimal interpolating projections.

A (bounded linear) projection $P$ of a normed space $X$ onto a subspace $Y$ is "minimal" if $\|P\| \cong\left\|P^{\prime}\right\|$ for every projection $P^{\prime}$ of $X$ onto $Y$. Let $T$ be a compact Hausdorff space, let $X=C(T)$, and let $\operatorname{dim}(Y)=n$. If there is a subset $T_{0}$ of $n$ points in $T$ such that $(x-P x) \mid T_{0}=0$ for all $x$, then $P$ is an "interpolating projection". Theorem. An interpolating projection is minimal if and only if for each bounded linear map $L: X \rightarrow Y$, the condition $L \mid Y=0$ implies that $0 \geqq \inf \{(L x)(t): t \in T$, $x \in X,\|x\|=1,(P x)(t)=\|P\|\}$. Corollary. Let $Y$ have the Haar property (i.e., 0 is the only member of Y having n or more roots). Let P be an interpolating projection of X onto Y of the form $\mathrm{Px}=$ $\Sigma_{i=1}^{n} x\left(t_{i}\right) y_{i}\left(t_{i} \in T, y_{i} \in Y\right)$. If $P$ is minimal, then either $\|P\|=1$ or there exist at least $n+1$ points at which the function $\sum_{i=1}^{n}\left|y_{i}\right|$ reaches its maximum on $T$. (Received August 4, 1969.)

69T-B181. CARL L. DeVITO, University of Arizona, Tucson, Arizona 85721. On a class of locally convex spaces. Preliminary report.

We shall use the terminology of J. Kelley, I. Namioka and coauthors ["Linear Topological Space," Van Nostrand, Princeton, N. J.7. All spaces mentioned will have their Mackey topologies. Let $E$ be a real, locally convex space and let $E^{\prime}$ be its dual. We shall say that $E$ is an ab-space if every net in $E^{\prime}$ which is weak ${ }^{*}$ Cauchy and eventually in some multiple of every strong neighborhood of zero, is weak ${ }^{*}$ convergent to a point of $E^{\prime}$. Every ab-space is quasi-barralled. Theorem l. Let $\left\{E_{\nu} \mid \boldsymbol{\nu}\right.$ in I$\}$ be a family of ab-spaces and let $E$ be their product. If $E$ is not an ab-space then the cardinal number of $I$ is strongly inaccessible. We prove several characterizations of ab-spaces. These lead to: Every bound space is an ab-space; the strong dual of an ab-space is complete. A sequence $\left\{x_{n}\right\} \subset E$ is said to be a local null sequence if there is a sequence $\left\{\lambda_{n}\right\}$, of real numbers such that: $\lim \lambda_{n}=\infty$ and $\left\{\lambda_{n} x_{n}\right\} \subset E$ converges to zero. The topology on $E^{\prime}$ of uniform convergence on the local null sequences of $E$ will be denoted by $t_{0}$. Theorem 2. E is a bound space iff every net in $E^{\prime}$ which is weak ${ }^{*}$ Cauchy and eventually in some multiple of every $t_{0}$-neighborhood of zero, is weak ${ }^{*}$ convergent to a point of $E^{\prime}$. Theorem 3. The strong dual of an ab-space is sequentially $\mathrm{t}_{0}$-complete. (Received August 6, 1969.)

69T-B182. BRUCE B, RENSHAW, University of California, Berkeley, California 94720. Ua the reduction theory for $W^{*}$ algebras. Preliminary report.

Let $Z=C(\Omega)$ be an abelian $W *$ algebra, and $A=\left(A_{*}\right)^{*}$ any $W *$ algebra. Let $Z \otimes A$ and $Z \bar{\otimes} A$ be the $C^{*}$ tensor product and $W^{*}$ tensor product respectively, and let $C(S C, A)$ and $C *(\Omega, A)$ be the Banach spaces of bounded functions $\varsigma_{6}$. A which are uniformly, resp. weakly continuous. Theorem. The known *isomorphism $Z \otimes A=C(\Omega, A)$ extends to a positive isometry $Z \bar{\otimes} A=C *(\Omega, A)$, $b \leftrightarrow\left\{b_{t}\right\}_{t \in \Omega}$ with the following properties: (1) If $b, c \in Z \bar{\otimes} A$ and either $b$ or $c \in Z \otimes A$, then (bc) $t=b_{t} c_{t}$ for all $t \in \Omega$. (2) If $b, c \in Z \bar{\otimes} A$ are arbitrary, then $b c$ is the unique element of $Z \bar{\otimes} A$ such that for all $\theta \in A_{*}, \quad \theta\left((b c)_{t}\right)=\theta\left(b_{t} \xi_{t}\right)$ a.e. (3) If $\left\{b_{a}\right\}$ is a bounded net in $Z \bar{\otimes} A$ and $b_{a} \rightarrow b$ weakly*, then $b$ is the unique element of $Z \bar{\otimes} A$ such that for all $\theta \in A_{*}, \theta\left(b_{a t}\right) \rightarrow$ $\theta\left(b_{t}\right)$ a.e. Here "a.e." is interpreted as "off a meager set". This result generalizes to the inseparable case the result of Sakai, Bull. Amer. Math. Soc. 70 (1964), 393-398. (Received August 12, 1969.) (Author introduced by Professor Jacob Feldman.)

69T-B183. FRIEDRICH WILLE, Universität der Freiburg, 78 Freiburg, Hebelstrasse 40, Germany. An existence theorem for monotone operators. Preliminary report.

Theorem. Let E be a reflexive Banach space and $\mathrm{E} *$ its adjoint. Let $\mathrm{F}: \mathrm{E} \rightarrow \mathrm{E} *$ be a hemicontinuous monotone operator satisfying the condition (a): $\lim _{\|x\| \rightarrow \infty}\|F(x)\|=\infty$. Then F maps E onto E*. Remark. The theorem generalizes a result of Minty, Browder, Kachurovskir a.o., who use coercitivity, i.e. $\lim _{\|x\| \rightarrow \infty}\langle F(x), x\rangle /\|x\|=\infty$, instead of (a) (cf. Kachurovskiil, Russian Math. Surveys. 23 (1968), 117-165). (Received August 12, 1969.)

69T-B184. ROBERT W. CARROLL, University of Illinois, Urbana, Illinois 61801, and JEFFREY M. COOPER, Northwestern University, Evanston, Illinois 60201. Remarks on variable domain problems in abstract evolution equations.

Let $H$ be a separable Hilbert space and $V(T) \subset H, 0 \leqq t<T$, a family of dense Hilbert spaces having continuous injections with $S(t)=R^{-1 / 2}(t)$ where $((R(t) x, y))_{t}=(x, y)_{H}$ for $x \in H$ and $y \in V(t)$. Assume first $S^{-2}(\cdot)$ is weakly $C^{1}$ in $H$. Set $W=L^{p}(V(t))=\left\{u \in L^{p}(H) ; u(t) \in V(t)\right.$ a.e.; Su $\left.\in L^{p}(H)\right\}$, $2 s p<\infty$, and lei $L: W \rightarrow W^{\prime}=L^{q}\left(V^{\prime}(t)\right)\left(V^{\prime}(t)=\right.$ antidual of $\left.V(t)\right)$ be defined by $L \mu=u^{\prime}$ with $\left.D_{i}^{\prime} L\right)=$ $\left\{u \in W ; u^{\prime} \in L^{q}(H) ; u(0)=0\right\},(l / p)+(l / q)=1$, while $L^{\prime}: W \rightarrow W^{\prime}$ is determined by $L^{\prime} u=-u^{\prime}$ with $D\left(L^{\prime}\right)=\left\{u \in W ; u^{\prime} \in L^{q}(H) ; u(T)=0\right\}\left({ }^{\prime}\right.$ in $\left.\mathcal{A}^{\prime}(H)\right)$. The main theorem asserts that $L^{\prime *}=L_{w}=L_{s}=\bar{L}$ and thus given a family $A(t): V(t) \rightarrow V^{\prime}(t)$ of monotone (nonlinear) operators such that $A u \in W^{\prime}$ for $u \in W$ it follows that solutions of the weak problem $L_{w} u+A u=f \in W^{\prime}, u(0)=0\left(o r u_{0} \in H\right.$ ) are unique (cf. (*) R. Carroll, Proc. Conf. Equations of Evolution and Nonlinear Semigroups, University of Kentucky, 1969, 11-24). Next a new class of weak linear uniqueness theorems is obtained which extends earlier work of the authors. The idea is to assume that a suitable family $G(t) \in \mathcal{L}(H)$ satisfies $R(G(t)) \subset V(t)$ with $G(\cdot)$ weakly $C^{1}\left(e . g . G(t)=S^{-a}(t)\right.$ in $\left.(*)\right)$; here $\langle A(t) x, y\rangle=a(t, x, y)$ with $A(t)$ linear and $p=2$. Then natural hypotheses on $S G, \dot{G}, S^{-1}, a\left(t, u, G G^{*} u\right)$ or $a(t, u, G u)$, etc., modeled on those of (*), § 2, yield weak uniqueness. (Received August 13, 1969.)

T-B185. SIDNE Y A. MORRIS, School of Physical Sciences, The Flinders University of South Australia, Bedford Park, South Australia 5042. A strong version of the Tychonoff fixed point theorem.
"Tychonoff's fixed point theorem is not immediately applicable in analysis because the domain is required to be compact, a situation rarely met in practice"--J. Dugundji (Topology). Accordingly the following is presented. Theorem. Let $f$ be a continuous mapping of a convex set $C$ of a Hausdorff locally convex linear topological space into a compact subset of $C$. Then there is at least one fixed point. This is established by a modification of Tychonoff's proof in his paper "Ein Fixpunktsatz" (Math. Ann. 3 (1935), 767-776). Under the additional assumption that $C$ is closed, the theorem is proved in Bonsall (Lectures on some fixed point theorems of functional analysis, Tata Institute) using a technique of Nagumo. It appears, however, that this extra condition can be dispensed with in that proof, thus giving an alternative proof of the result above. (Received August 12, 1969.) (Author introduced by Professor Brian Abrahamson.)

69T-Bl86. WITOLD M. BOGDANOWICZ, Catholic University of America, Washington, D. C. 20017 and VERNON E. ZANDER, West Georgia College, Carrollton, Georgia 30117. A vectorial integral with respect to an infinite product of vector-valued volumes.

Let $W, Y, Z_{t}(t \in T)$ be Banach spaces over a scalar field $F$. Let $\left(X_{t}, V_{t}, v_{t}\right)(t \in T)$ be a family of probability volume spaces and let ( $\mathrm{X}, \mathrm{V}, \mathrm{v}$ ) be the product probability volume space (Abstract 658-109, these $\mathcal{C}$ (Notices) 15 (1968), 750). Let $L(v, Y$ ) be a space of Bochner summable functions and let $K\left(v_{t}, Z_{t}\right)$ be spaces of vector-valued volumes (Bogdanowicz, Proc. Nat. Acad. Sci. U. S. A. 53(1965), 492-498). Let $z_{t}^{\prime} \in Z_{t}(t \in T)$ be a fixed $T$-tuple such that $\Sigma_{t}\left|\left\|z_{t}^{\prime}\right\|-1\right|$ is finite. Denote by $Z^{\prime}$ the family of all tuples $z=\left(z_{t}\right)$ such that $z_{t} \in Z_{t}$ for $t \in T$ and $\Sigma_{T}\left\|z_{t}-z_{T}^{\prime}\right\|$ is finite. Denote by $K\left(v, Z^{\prime}\right)$ the space of all functions $\mu$ such that (i) $\mu(A)=\mu_{t}\left(A_{\tau}\right)_{T}$ for all $A=X_{T} A_{t} \in V$, where $\mu_{t} \in K\left(v_{t}, Z_{t}\right)$ for $t \in T$, (ii) $\mu(X) \in Z^{\prime}$, and (iii) $\Sigma_{T} \mid\left\|u_{t}\right\|-1 \|$ is finite. Let $u: Y \times Z^{\prime} \rightarrow W$ be an infinitely-linear bounded continuous operator (Abstract 658-108, these CNotices) 15(1968), 750). Let $Z_{0}$ be the space of all bounded linear operators from $Y$ into $W$. Theorem. If for $\mu \in K\left(v, Z^{\prime}\right)$ we define a function $\mu^{\prime}$ by the formula $\mu^{\prime}(\mathrm{A})=u(\cdot, \mu(\mathrm{~A}))$ for all $\mathrm{A} \in \mathrm{V}$ then the function $\mu^{\prime}$ belongs to the space $K\left(v, Z_{0}\right)$ of vector-valued volumes. Thus an integral can be defined for $f \in L(v, Y), \mu \in K\left(v, Z^{\prime}\right)$ by $\int u\left(f,\left(d \mu_{t}\right)_{T}\right)=\int u_{0}\left(f, d \mu^{\prime}\right)$, where $u_{0}(y, z)=z(y)$ for all $y \in Y, z \in Z_{0}$. (Received August 14, 1969.)

69T-B187. STUART P. HASTINGS, Case Western Reserve University, Cjeveland, Ohio 44106. On solutions of the Falkner-Skan boundary value problem.

We consider the boundary value problem (1) $f^{\prime \prime \prime}+f^{\prime \prime}+\beta\left(1-f^{2}\right)=0,(2) f(0)=f^{\prime}(0)=0$, $(3) f^{\prime}(\infty)=1$, for negative $\beta$. Theorem. Suppose $f$ is a solution of $(1)-(2)$. If $\beta<0$ and $f^{\prime \prime}(0)=0$ or if $-1 \leqq \beta<0$ and $f^{\prime \prime}(0) \geqq 0$, then $f^{\prime}(\infty)=1$. For small $|\beta|$ there is a solution $f_{0}$ of (1)-(2)-(3) such that $\mathrm{f}_{0}^{\prime \prime}(0)<0,-1 \leqq \mathrm{f}^{\prime} \leqq 1$ on $\left.0, \infty\right)$, and $\left(1-f_{0}^{\prime}(t)\right) \mathrm{e}^{\epsilon \mathrm{t}} \rightarrow 0$ for some $\epsilon>0$. Remark. The proof is easy using the function $V=\mathrm{f}^{\prime \prime} / 2-B \cdot\left(\mathrm{f}^{\prime}{ }^{3} / 3-\mathrm{f}^{\prime}\right)$. An interesting question is whether there is only one solution with the properties of $f_{0}$. (Received August 14, 1969.)

69T-B188. HERBERT W. HETHCOTE, University of Iowa, Iowa City, Iowa 52240. Asymptotic bounds for zeros of special functions.

For $n \geqq 1$ let $a_{n}$ and $b_{n}$ be zeros (ordered by increasing values) of $u(x)$ and $v(x)$, respectively, which are nontrivial solutions of $u^{\prime \prime}+p(x) u=0$ and $v^{\prime \prime}+q(x) v=0$ with $p^{\prime}(x)$ and $q(x)$ continuous. It is shown that if $a_{n}$ is asymptotic to $b_{n}, p(x) \geqq q(x)$, and either $p(x)$ or $q(x)$ is nonincreasing, then $a_{n} \geqq b_{n}$ for $n \geqq 1$. Inequalities are obtained for the negative zeros $a_{n}$ of the Airy function $A i(z)$ and the zeros $j_{\nu, \mathrm{n}}$ of the Bessel function $\mathrm{J}_{\nu}(\mathrm{x})$. (Received August 18, 1969.)

69T-B189. KARL E. GUSTAFSON, University of Colorado, Boulder, Colorado 80302. T̈̈ceplitzHausdorff Theorem in Banach space.

Theorem. In a (real or complex) Banach space, the union of the numerical ranges of every linear operator (bounded or unbounded, perhaps not densely defined) is convex. Corollary. In a smooth Banach space, the numerical range is always convex. (Received August 25,1969 .)

69T-B190. SYED M. MAZHAR and R. S. KHAN, College of Engineering, Aligarh Muslim University, Aligarh, India. Some integrability theorems for Dirichlet series with positive coefficients.

Let $1=\lambda_{1}<\lambda_{2}<\lambda_{3}<\ldots<\lambda_{n} \rightarrow \infty$ and $f(t)=\bar{\Sigma}_{k=0}^{\infty} a_{k} e^{-\lambda k_{k}^{t}}, 0<t \leqq \infty$. In this note four theorems have been proved. These theorems generalise certain results due to Askey and Boas [MRC Technical Summary Report No. 850 (1968), Madison] for power series. A typical theorem proved by the authors is as follows. Theorem. If $\sum_{a_{k}}<\infty, a_{k} \geqq 0,1<p \leqq \infty$ and $r_{n}=\sum_{k=n}^{\infty} a_{k}$, then $\bar{\Sigma}_{n=1}^{\infty}\left(\lambda_{n+1}-\lambda_{n}\right) r_{n}^{p}<\infty$ if and only if $e^{-t / p}\left(1-e^{-t}\right)^{-2 / p}[f(0)-f(t)] \in L^{p}(0, \infty)$ where $\left\{\lambda_{n} / \lambda_{n+1}\right\}$ is an increasing sequence and $\Sigma\left(\left(\lambda_{n+1}-\lambda_{n}\right) / \lambda_{n}^{p}\right)<\infty$. (Received August 18, 1969.)

69T-B191. SAMUEL ZAIDMAN, Université de Montréal, Montréal, Québec, Canada. Uniqueness of bounded solutions for some abstract differential equations.

Let us consider: a Hilbert space $H$; a self adjoint operator $A$ in $H$, of domain $D(A) ; u(t)$, $-\infty<t<\infty \rightarrow D(A)$, continuously differentiable in $H$, and equation $u^{\prime}(t)=A u(t)$ is verified. Assume 0 is not an eigenvalue of $A$, and suppose (1) $\sup _{-\infty<t<\infty} \int_{t}^{t+1}\|u(s)\|^{2} d s<\infty$. Then $u(t)=\theta \forall t \in(-\infty, \infty)$. The result is true also for more general solutions $u(t)$; precisely for $u(t) \in L_{l o c}^{2}(-\infty, \infty ; H)$ such that $\int_{-\infty}^{\infty}\left(u(t), \varphi^{\prime}(t)+A \varphi(t)\right)_{H} d t=\theta, \quad \forall \varphi \in \theta\left(R^{1} ; D(A)\right)$. If (1) is verified, $u(t)$ is null almost everywhere on $-\infty<\mathrm{t}<\infty$. (Received August 21, 1969.)

69T-B192. JAMES W. NOONAN, University of Maryland, College Park, Maryland 20742. On the coefficients of functions of bounded boundary rotation. Preliminary report

Let $V_{k}$ denote the class of functions $f(z)=z+a_{2} z^{2}+\ldots$ that are analytic in the unit disc $U$, satisfy the condition $f^{\prime}(z) \neq 0$ in $U$, and map $U$ onto a domain with boundary rotation at most $k \pi$. V. Patero showed (Ann. Acad. Sci. Fenn. Ser. A, 33 (1931)) that $f \in V_{k}$ if and only if $\log f^{\prime}(z)=$ $\int_{0}^{2 \pi} \log \left(1-z e^{-i t}\right)^{-1} \mathrm{~d} \mu(t) \mid$ where $\mu(t)$ is real-valued and of bounded variation on $[0,2 \pi]$ with $\int_{0}^{2 \pi} d \mu(t)=2$ and $\left.\int_{0}^{2 \pi} \left\lvert\, \begin{array}{c}\mathrm{d} \mu(\mathrm{t}) \mid \\ \mathrm{k} / 2+1\end{array}\right.\right) \leqslant k$. Let $M\left(r, f^{\prime}\right)=\max |z|=r\left|f^{\prime}(z)\right|$. Theorem 1. Let $f(z) \in V_{k}$. Then the
limit $\omega=\lim _{r \rightarrow 1}(1-r) \quad M\left(r, f^{\prime}\right)$ exists finitely. If $\omega>0$, then there exists $\theta_{0} \in[0,2 \pi]$ such that $\omega=\lim _{r \rightarrow 1}(1-r)^{k / 2+1}\left|f^{\prime}\left(\mathrm{re}^{i \theta_{0}}\right)\right|$. Theorem 2. Let $f(z) \in V_{k}$ and $\omega$ be as above. Then $\lim \left|a_{n}\right| / n^{k / 2-1}=u / \Gamma^{(k / 2+1)}$. Corollary. Let $f(z)=z+a_{2} z^{2}+\ldots \in V_{k}$ and let $F(z)=$ $(1 / k)\left[((1+z) /(1-z))^{k / 2}-1\right]=z+A_{2^{2}} z^{2}+\ldots$. Then there exists a positive integer $n_{0}(f)$ such that $\left|a_{n}\right| \leqq A_{n}$ for $n \geqq n_{0}(f)$. Also, equality holds for infinitely many $n$ if and only if $f(z)$ is a rotation of $F(z)$ Parts of the proofs of Theorems 1 and 2 follow a method due to Hayman. (Received August 18, 1969.)

69T-B193. JAMES FRANCIS G. AUCHMUTY, University of Chicago, Chicago, Illinois 60637. The generation and topologization of the field of convolution quotients.

The set of continuous functions defined on the positive quadrant $R_{+}^{n}$ in $R^{n}$ is an integral domain with respect to addition and convolution and the quotient field of this set is the field of convolution quotients $Q$. It is proved that $g_{+}^{\prime}$, the set of distributions whose support is bounded below, its ideals and some of its subrings; including the set of all locally Lebesgue integrable functions whose support is either bounded below in $R^{n}$ or is contained in $R_{+}^{n}$ generate quotient fields isomorphic to $Q$. If $R$ is a topological integral domain with a unit element, $M(x)$ is the additive group of fractions $\{y / x: y \in R\}$ then $M(x)$ has a natural topology and the inductive limit of the family $\{M(x): x \in R\}$ with respect to the natural inclusion maps is isomorphic to the quotient field $Q R$ of $R$ and this is a strict inductive limit iff $R$ has no topological divisors of zero. Furthermore if $R$ is also a vector space, $Q R$ is a topological vector space by this construction and applying this to the rings that generate $Q$ one obtains a number of definitions of convergence similar to those given in Mikusinski's "Operational calculus", Pergamon Press, New York, 1958. (Received August 18, 1969.)

69T-B194. GODFREY L. ISAACS, Herbert H. Lehman College, City University of New York, Bronx, New York 10468. Some results in absolute summability.

With $A(u)$ of bounded variation over $[0, U]$, each $U>0$, and $A(0)=0$, we write $R_{k^{\prime}}(w)=$ $\left\{\Gamma\left(k^{\prime}+1\right)\right\}^{-1} \int_{w}^{\infty}(u-w)^{k^{\prime}} d A(u)\left(k^{\prime} \geqq 0\right)$ and $C(w, s)=\int_{0}^{w} e^{-u s} d A(u)$. It is shown that if $\underline{k}$ is positive and fractional and if $e^{-w s^{\prime}} R_{k}(w)$ is summable $|C, 0|$ for some $s^{\prime}$ whose real part is $<0$ (it being supposed that the integral defining $R_{k^{\prime}}(w)$ exists in the ( $C, k^{\prime}$ ) sense with, say, $\left.k^{\prime}=[k]+1\right)$ then $C(w, s)$ is summable $|C, k+\epsilon|(e a c h e>0)$ for each $\underline{s}$ whose real part is greater than that of $s$; if $\underline{k}$ is positive and integral the conclusion holds with $\epsilon=0$. Together with a 'converse' result, this may be used to show that if the abscissa of $|C, k|$ summability of $C(w, s)$ is negative then it has the value $\lim _{w \rightarrow \infty}{ }^{-1} \log \int_{\mathrm{w}}^{\infty}\left|\mathrm{dR}_{\mathrm{k}}(\mathrm{u})\right|$ except possibly for one (fractional) value of $\underline{k}$. (Received August 21,1969 .)

69T-B195. PAUL T. SCHAEFER, State University College, Geneseo, New York, 14454. Infinite matrices which map 1 into c. Preliminary report.

Let $A=\left(a_{n k}\right)$ be an infinite matrix, let $c$ denote the set of convergent sequences and let $c_{A}$ denote the set of all sequences $x$ such that $A(x)=\lim _{n} \Sigma_{k}{ }^{a}{ }_{n j} x_{k}$ exists. The matrix $A$ is an $\mathfrak{i}$ - matrix if $\mathfrak{l} \subset c_{A}$, where $\mathfrak{i}=\left\{x: \Sigma\left|x_{k}\right|<+\infty\right\}$. A regular $\mathfrak{i}$-c matrix is one such that $\Sigma x_{k}=A(x)$ for all $x \in \mathfrak{i}$. When $c_{A}$ is considered as an $F K$ space in the usual way, the $\mathfrak{l - c}$ matrix $A$ is called (1) perfect if $t$ is dense in $c_{A}$, (2) of type $M_{i}$ if $t \in \mathfrak{i}$ and $\Sigma_{n} t_{n} a_{n k}=0$ for all $k$ imply that $t_{n}=0$ for all $n$, (3) reversible if $A x=y$ has exactly one solution $x \in c_{A}$ for every $y \in c$, (4) replaceable if there is a regular $1-c$ matrix $B$ with $c_{B}=c_{A}$. The following results, analogous to the well-known $c-c$ and
$1-\mathfrak{l}$ cases, are obtained. Theorem 1. A reversible, row-finite, regular $\mathfrak{i - c}$ matrix is perfect if and only if it is of type $M$. Theorem 2. A necessary and sufficient condition that an $1-c$ matrix be consistent with every $1-c$ matrix $B$ such that $c_{B} \supset c_{A}$ and $A(x)=B(x)$ for all $x \in 1$ is that $A$ be perfect: Theorem 3. A matrix $A$ is replaceable if and only if $t_{0}=\left\{x \in 1: \sum x_{k}=0\right\}$ is not $c_{A}$-dense in 1 . Proofs of these results are standard, since continuous linear functionals on $c_{A}$ are representable by $\mathfrak{i - c}$ matrices when $A$ is an $\mathfrak{i - c}$ matrix. (Received July 14, 1969.)

69T-B196. WITHDRAWN

69T-B197. ROBERT H. MARTIN, JR., Georgia Institute of Technology, Atlanta, Georgia 30332. Uniform! y accretive operators and evolution equations.

Let $E$ be a Banach space and $E *$ be the dual of $E$. If $D \subset E$ and $A$ is a function from $D$ into $E$ then $A$ is said to be accretive if for each $x, y \in D$ and each $f \in E *$ such that $f(x-y)=\|x-y\|^{2}=\|f\|^{2}$, we have $f(A x-A y) \geqq 0$. Lemme 1 . A is accretive if and only if $\lim _{h \rightarrow+0}(\|x-y-h[A x-A y]\|-\|x-y\|) / h \leqq 0$ for each $x, y \in D$. Call A uniformly accretive if for each $\beta>0$ and each bounded subset $Q$ of $D$ such that $A$ is bounded on $Q$, the limit in Lemma 1 is uniform for $x, y \in Q$ with $\|x-y\| \geqq \beta$. Lemma 2. If $E *$ is uniformly convex then $A$ is accretive if and only if $A$ is uniformly accretive. Theorem l. If $A$ is a continuous (resp. demicontinuous and locally bounded) function from $E$ into $E$ for which each $z$ in $E$ has a neighborhood $V_{z}$ such that $A$ is uniformly accretive on $V_{z}$, then for each $z$ in $E$ there is a unique differentiable (resp. weakly differentiable) function $u_{z}$ from $[0, \infty)$ into E such that $u_{z}(0)=z$ and $u_{z}^{\prime}(t)+A u_{z}(t)=0$. Also, if $z$ and $w$ are in $E$ then $\left\|u_{z}(t)-u_{w}(t)\right\| \leqslant\left\|u_{z}(s)-u_{w}(s)\right\|$ whenever $0 \leqq s \leqq t$. Theorem 2. The results of Theorems 1 and 2 of Tosio Kato ("Nonlinear semi-groups and evolution equations," J. Math. Soc. Japan. 19 (1967), 508-520) are true if the assumption that E* is uniformly convex is replaced by the operators $A(t)$ are uniformly accretive and (a) of Lemma 2.5 in Kato's paper holds. (Received August 25, 1969.)

69T-B198. HARI M. SRIVASTAVA, University of Victoria, Victoria, British Columbia, Canada. A class of generating functions for generalized hypergeometric polynomials.

Let v be a function of t defined by $\mathrm{v}=\mathrm{t}(\mathrm{l}+\mathrm{v})^{\mathrm{b}+1}, \mathrm{v}(0)=0$. In the present note it is shown
 $1+v)^{a+1}\left(1-b_{V}\right)^{-1} \sum_{k=0}^{\infty} c_{k}\left[f(x)(-v)^{m}\right]^{k} / k^{\prime}$, where m is an arbitrary positive integer, the $c_{k}$ are arbitrary constants, and $f(x) \neq 0$ is a real function of $x$. Various specialized forms of the generating relation (*) occur throughout the literature. In particular, if the $c_{k}$ are given by (**) $c_{k}=$ $\Pi_{j=1}^{p}\left(a_{j}\right)_{k} / \Pi_{j=1}^{q}\left(\beta_{j}\right)_{k}, k=0,1,2, \ldots$, where $\beta_{j} \neq 0,-1,-2, \ldots ; j=1,2, \ldots, q$; then it readily yields a class of generating functions for generalized hypergeometric polynomials which for $\mathrm{m}=1$ lead to certain earlier results of the author [H. M. Srivastava, "Generating functions for Jacobi and Laguerre polynomials", Proc. Amer. Math. Soc. (to appear)]. (Received August 27, 1969.)

69T-B199. G. S. OLKHA, M. R. Engineering College, Jaipur, India, and P. N. RATHIE, Queen's University, Kingston, Ontario, Canada. On a generalized Bessel function and an integral transform.

The authors define a generalized Bessel function as, (*) $\tilde{\omega}_{\nu, \nu^{\prime} \cdot \lambda^{\prime}}^{\mu}(x)=x^{1 / 2} \int_{0}^{\infty} t^{\lambda-1} J_{\nu}^{\mu}(x t) J_{\nu^{\prime}}^{\prime}(1 / t) d t$,
under certain sets of conditions for the convergence of the integral. On specializing the parameters, this gives rise to several known Bessel functions: (i) $\tilde{\omega}_{\nu-1, \nu, 0}^{1,1}(x)=J_{2 \nu-1}\left(2 x^{1 / 2}\right)$ - -the well-known Bessel function, (ii) $\tilde{\omega}_{\nu, \nu+\mu / 2+1 / 2, ~}^{\mu / 2} / 2-1 / 2(x)=2^{(1-\mu)(1+\nu)} J_{2 \nu+1}^{\mu}\left(2 \mu / 2+1 / 2 x^{1 / 2}\right)-$ Wright's generalized Bessel function and (iii) $\tilde{\omega}_{\nu, v^{\prime}, 0}^{1,1}(x)=\tilde{\omega}_{\nu, \nu^{\prime}}(x)$-the well-known Bessel function introduced by G. N. Watson. Several recurrence relations and other properties of this generalized Bessel function (*) are derived. A generalized integral transform with the function (*) as the kernel is defined as follows. $\left.\quad \theta\left[\begin{array}{l}\mu, \mu^{\prime}, \sigma \\ \nu, \nu^{\prime}, \lambda\end{array}: f(x) ; y\right]=\int_{0}^{\infty}(x y)^{\sigma} \tilde{\omega}_{\nu, \nu^{\prime} \lambda^{\prime}}^{\mu, x^{\prime}}{ }^{2} y^{2}\right) f(x) d x$. The particular cases of this transform include the well-known integral transforms with $J_{\nu}(x y), J_{\nu}^{\mu}(x y)$ and $\left.\tilde{\omega}_{\nu}^{\mu}, \nu^{r(x}{ }^{2} y^{2}\right)$ respectively as the kernels. An inversion formula is established and other properties studied in detail. (Received August 28, 1969).

69'- B200. KEITH MILLER, University of California, Berkeley, California 94720. Finite difference approximants to exp(-tA) which are high order convergent in operator norm. Preliminary report.

Suppose $f$ is holomorphic on the sector $\{z: 0 \leqq \arg z \leqq \omega<\pi / 2\}$ with boundary $\Gamma$, and approximates $e^{-z}$ there as follows: (i) $\left|f(z)-e^{-z}\right| \leqq a|z|^{q+1}$ on $\Gamma$ for $|z| \leqq H$, (ii) $|f(z)|$ and $\left|e^{-z}\right|$ are $\leqq e^{-\beta|z|}$ on $\Gamma$ for $|z| \leqq H, 0<\beta \leqq 1$, (iii) $|f(z)|$ and $\left|e^{-z}\right|$ are $\leqq \gamma<1$ on $\Gamma$ for $|z|>H$, (iv) $|f(z)|$ and $\left|e^{-z}\right|$ are $\leqq \delta /|z|$ on $\Gamma$ for $|z|>H$, where $u, H, a, \beta, \gamma, \delta$ are positive constants, and $q \geqq 1$. Let $A$ be a "sectorial operator" on Hilbert space, with numerical range in $\{z: 0 \leqq \arg z \leqq \rho<\omega\}$ (see T. Kato, "Perturbation Theory," Springer-Verlag, New York, 1966, pp. 487-490. Theorem. There exists $K(\omega-\rho, q, H, a, \beta, a, \delta)$ such that $\left\|e^{-t A}-[f(t A / n)]^{n}\right\| \leqq K / n{ }^{q}, n=1,2, \ldots$, for all $t \geqq 0$. The proof involves merely estimates on the Cauchy integral formula for $e^{-t A}$. The result is even simpler for self adjoint $A$; Kato suggested extension to sectorial A. The major application is to parabolic PDE's, with - A an elliptic finite difference or finite element (see thesis, J. Blair, University of California, Berkeley, in progress) operator, or to "stiff" systems of ODE's (see work by Dahlquist, Widlund, and Gear). Condition (iii) fails for Crank-Nicolson and for the "diagonal Pade" approximants advocated by Varga. However, all "below diagonal Padé" approximants (see R. Varga, J. Math. and Phys. 40 (1961), 220-231) satisfy (i)-(iv) for sufficiently small $\omega$. (Received July 11, 1969.) (Author introduced by Professor Alessandro Figa-Talamanca.)

69T-B201. DONALD BRATTON, 2000 West Hemlock Street, Oxnard, California 93030. Spectral algebras.

Let A be a topological algebra (see Abstract 69T-B69, these CNotices) 16 (1969), 571) supplied with a continuous involution. The absolute value of $x \in A$ is the number $|x|=\sqrt{p(x * x)}$, where $p$ is the P-guage (Abstract 69T-B105, these Cotices) 16 (1969), 671). A is called a spectral algebra when it is Gelfand (Abstract 69T-B105, cited) and the set of $x \in A$ such that $|x| \leqq 1$ is bounded. Hypothesis. In each spectral algebra, $x \in U \Rightarrow x^{n} \in U$, where $U$ is the unit ball $|x| \leqq 1$. A is called regular when there exists a constant $K$ such that $|x y| \leqq K|x||y|$. If the topology of a spectral algebra $A$ is defined by a finite set of seminorms, then $A$ is regular. If $A$ is a regular spectral algebra then $U$ is a closed convex set stable for multiplication. Each commutative spectral algebra is regular and $U=P$. Let $A$ be a commutative powergrowth *-algebra and $B$ a spectral algebra. Assume that $A$ is Tonelle and
that B is Hausdorff and quasi-complete. For each continuous *-representation $\varphi: \mathrm{A} \rightarrow \mathrm{B}$ there exists a compact space $K$ and a decomposition $\varphi=\varphi_{2} \cdot \varphi_{1}$ where $\varphi_{1}$ is a continuous *-representation $A \rightarrow C(K)$ and $\Theta_{2}$ a continuous $*$-representstion $C(K) \rightarrow B$. Each commutative powergrowth spectral algebra which is Hausdorff and quasi-complete is topologically $*$-isomorphic to an algebra $C(K)$, where $K$ is a compact space. (Received September 2, 1969.)

## Applied Mathematics

69T-C32. HERMANE. GOLLWITZER, The University of Tennessee, Knoxville, Tennessee 37916. Bounds for a nonoscillatory solution of a nonlinear differential equation.

In the second order equation $\left(^{*}\right) y^{\prime \prime}+q(t)|y|^{a} \operatorname{sgn} y=0$, let $q$ be positive, continuous on $[a, \infty), a>0, a \neq 1$. Let $y(t)$ be a positive, nonoscillatory solution of (*). Theorem. If $a>1$, $\left\{_{t} \int^{\infty} \operatorname{sqds}\right\} y(t)^{a-1} \leqq C$, where $C$ is a constant. Corollary. If lim inf $t^{\lambda(a-1)} \bar{t} \int^{\infty} s q d s>0,0<\lambda<1$, then $y(t) \leqq C t^{\lambda}$. Basically, the theorem is proved by multiplying (*) by $t y(t)^{-a}$ and integrating by parts several times. This sharpens an estimate of Moore and Nehari. MR $22 \# 2755$, Theorem IX. This estimate is, in general, sharp since $t^{\lambda}$ is a solution of $y^{\prime \prime}-\lambda(\lambda-1) t^{-\lambda(a-1)-2}|y|^{a} \operatorname{sgn} y=0$. Analogous estimates, too lengthy to state here, are also true when $0<a<1$. These sharpen previous results of Belohorec, MR 35\#5703, Theorem 4. (Received July 22, 1969.)

69T-C33. PADAM C. JAIN and K. S. RAO, Indian Institute of Technology, Bombay - 76, India. Numerical studies of Kármán vortex street in the wake of a circular cylinder.

Jain and Rao [Supplement of Phys. Fluids, Vol. 12, 1969, to appear] have investigated numerically the existence of limiting steady state solutions at Reynolds numbers up to 60. In the present paper, the authors have obtained the formation of the Kármán vortex street by numerical computations of the unsteady problem at Reynolds number 200 by taking the boundary far from the cylinder. Figures have been drawn to show the development of the flow pattern and the Kármán vortex street, the vorticity distribution, the pressure distribution, and the drag, on the time. (Received June 25, 1969.)

69T-C34. WITHDRAWN.

## Geometry

69T-Di9. DONALD A. EISENMAN, University of California, Berkeley, California 94720. Intrinsic measures on complex manifolds and holomorphic mappings. III.

Theorem. Let $M$ be a hyperbolic (Kobayashi) manifold, $U$ open and relatively compact in $M, f: M \rightarrow M$ holomorphic, and suppose $U \subset f(U)$. Then $U=f(U)$ and $f$ is an automorphism of $M$. Let $D$ be a domain in $C^{n}, n \geqq 2$, and let $D_{0}$ be the union of $D$ with all compact components of $C^{n}-D$. Theorem. Let $U \subset B \subset D$, where $U$ is open and relatively compact in $B$ and $\bar{B}$ is homeomorphic to $\bar{B}^{n}$. Let $f$ be a holomorphic mapping from $D-\bar{U}$ to itself. Then either (i) $f_{*}\left(\pi_{2 n-1}(B-\bar{U})\right)=0$ in $\pi_{2 n-1}\left(D_{0}-\bar{U}\right)$ or (ii) fextends to automorphisms of $D$ and $D_{0}$. (Received June 13, 1969.) (Author introduced by Professor Shung-Shen Chem).

69T-D20. JOEL L. ROBERTS, Purdue University, Lafayette, Indiana 47901. Generic projection of algebraic varieties.

Let $k$ be an algebraically closed field, $V^{r} \subset \mathbb{P}^{n}$ a nonsingular closed subvariety ( $\mathbb{P}^{n}=$ projective
n-space over $k)$. Let $\sigma_{d}: \mathbb{P}^{n} \rightarrow \mathbb{P}^{N}\left(N=\binom{n+d}{d}\right.$ 1) be defined by global sections of the sheaf $O(d)$ on $\mathbb{P}^{n}$ Let $m$ be an integer, $r+1 \leqq m \leqq 2 r$. Theorem 1 . If $d \geqq 3 r /(m-r)$, then for $L$ in a dense open subset of the Grassmannian $G(N, N-m-1)$, the projection $\pi_{L}$ from $L$ has the properties:
(i) $\pi=\pi_{L} \mid V: V \rightarrow V^{\prime} \subset \mathbb{P}^{m}$ is finite and birational; (ii) if $V_{i}^{\prime}$ is the closure of $\left\{x \in V^{\prime} \mid x\right.$ is a closed point, and $\pi^{-1}(x)$ contains at least $i$ points of $\left.V\right\}$, then $V_{i}^{\prime}$ is purely of dimension $r-(i-1)(m-r)$, and $V_{2}^{\prime}=\operatorname{Sing}\left(V^{\prime}\right)$; (iii) for $x$ in a dense open subset of $V_{i}^{\prime}-V_{i+1}^{\prime},(*): V^{\prime}$ has exactly i branches at $x$, all of which are simple; (iv) conversely, if (*) holds, then $\hat{O}_{X, V} \cong \hat{O}_{O, W}$, where $W$ consists of i (linear) $r$-spaces through the origin 0 in affine $r$-space, which intersect exactly in an ( $r-(i-1)(m-r))$-space, and $x \in V_{i}^{\prime}-V_{i+1}^{\prime}$. Theorem 2. If $d \geqq 2$, then for $L$ in a dense open subset of $G(N, N-2 r)$, the projection $\pi_{L}$ has properties (i)--(iv) of Theorem $l$ (with $V^{\prime} \in \mathbb{P}^{2 r-1}$ ), and ( $v$ ) for the remaining finitely many closed points of Sing ( $V^{\prime}$ ), either: (a) $\operatorname{char}(k) \neq 2$, and $\hat{O}_{x, V^{\prime}} \cong k\left[\left[t_{1}, \ldots, t_{r-1}, t_{1} t_{r}, \ldots, t_{r}^{2}\right] \subset \subset\right.$ $k\left[\left[t_{1}, \ldots, t_{r}\right]\right]$, or $(b) \operatorname{char}(k)=2$, and $\hat{O}_{x, V^{\prime}} \cong k\left[\left[_{1}, \ldots, t_{r-1}, t_{1} t_{r}, \ldots, t_{r-1} t_{r}, t_{r}^{2}+t_{r}^{3}\right]\right]$. (Received August 25, 1969.)

69T-D21. R.E. HARRELL and LES A. KARLOVITZ, Institute for Fluid Dynamics and Applied Mathematics, University of Maryland, College Park, Maryland 20740. Nonreflexivity and the girth of spheres.

Let $X$ be a Banach space and B its unit ball. J. J. Schäffer ("Inner diameter, perimeter, and girth of spheres," Math. Ann. 173 (1967), 59-79) defined the girth of $B$ as the infimum of the lengths of centrally symmetric simple closed rectifiable curves lying in the boundary of B. Note that the girth of $B$ is always greater than or equal to 4. Remark. There exist spaces X (for example $\mathrm{C}[0,1]$ ) such that (a) the girth of B is 4 and such that (b) the girth is actually achieved by some curve. Theorem. If $X$ is a separable Banach space which satisfies (a) and (b), then $X$ is nonreflexive. (Received August 27, 1969.)

## Logic and Foundations

69T-E66. ALBERT J. V. SADE, 364 Cours de la République, (84) Pertuis, Vaucluse, France. Sur le premier systeme de Lukasiewicz.

Le $1^{\text {er }}$ système, (S), de Lukasiewicz, (L1) CCxyCCyzCCxz, (L2) CCNxxx, (L3) CxCNxy, avec M.P., admet en logique bivalente une solution est une seule, $C x y=x y+x+1, N x=x+1$. Toute solution de (L2), (L3), MP satisfait Ll, qui est inutile. En logique trivalente, (S) admet 8 solutions T, dont 4 sont isomorphes des 4 autres par $(0,2)=t \rightarrow 2 t+2$; ce sont, $N x=x^{2}+x+1, C x y=(00,01,02$; $10,11,12 ; 20,21,22 \rightarrow 111$, alb, 111, avec $(a, b)=1^{\circ}(0,0), 2^{0}(0,2), 3^{0}(2,0), 4^{0}(2,2)$. Chaque axiome de (S) est indépendant des autres, mais le systeme n'est pas strictement complet car la fonction CxCCyyNCzz, qui prend la valeur $x^{2}+x+1$ dans les 3 premiers cas, devient $2 x^{2}+2 x+1$ dans le $4^{\text {ième, }}$ la fonction CCxxx prend quatre valeurs differentes dans les 4 cas. Toutefois le caractère complet, au sens de Lukasiewicz, peut être demontré en quelques lignes en montrant, au moyen des polynômes Cxy, que toute formule tautologique par rapport à l'une des 4 solutions $T$ est aussi tautologique par rapport aux trois autres. Tl permet de demontrer toutes les thèses du système. (Received May 19, 1969.)

69T-E67. WITHDRAWN.

69T-E68. SAHARON SHELAH, The Hebrew University of Jerusalem, Jerusalem, Israel.
On unsaturated ultrapowers.
This notice answers several questions in Keisler, "Ultraproducts which are not saturated," J. Symbolic Logic 32 (1967), 23-43. We use his concepts. Stability is defined in Abstract 68T-E17, these $\mathcal{C}$ Notices 15 (1968), 930-931. For simplicity let $\mathrm{T}_{\mathrm{L}} \mathrm{T}_{1}$ be denumerable complete first-order theories. Theorem 1. $T$ is minimal iff it is stable or has not the f.c.p. Theorem 2 . If $T, T_{1}$ are stable and has the f.c.p., then they are equivalent by Keisler's order. Among the nonminimal theories they are minimal. Theorem 3. There exists a nonminimal nonmaximal theory $T$, iff there exists a nongood regular ultrafilter $D$ on $I$, such that $\pi_{i \in I} n_{i} / D \geqq \mathcal{N}_{0}$ implies $\pi_{i \in I} n_{i} / D>|I|$. There exists nonminimal nonmaximal nondenumerable complete theories. Theorem 4. If $M^{I} / D$ is ( $\left.2^{|I|}\right)^{+}$-saturated, and $M$ is $\lambda$-saturated, then $M^{I} / D$ is $\lambda$-saturated. Theorem 5 . If $T$, not necessarily denumerable, has a maximally $\lambda$-saturated model of power $>\lambda|T|$, and $\lambda>|T|$, then for every regular $\mu, \mu>|T|$, Thas maximally $\mu^{-s}$ saturated models of arbitrarily high powers. (There are generalizations for homogeneous and universal models.) (Received Febrauary 24, 1969.) (Author introduced by Professor Michael O. Rabin.)

69T-E69. T. G. McLAUGHLIN, University of Illinois, Urbana, Illinois 61801. Quasiretraceable sets and almost quasiretraceable sets.

Call a set a of natural numbers quasiretraceable it is regressive and is a finite union of retraceable sets; denote by $Q$ the class of such sets. Call a almost quasiretraceable $\Leftrightarrow$ it is regressive and is the union of finitely many recursive isomorphs of retraceable sets; denote by $A Q$ the class of such sets. Denote by $R$ the class of all regressive sets. Clearly, $Q \subseteq A Q \subseteq R$. By Abstract 66T-469, these $\mathcal{C}$ (otices) 13 (1966), 732-733, $A Q \neq R$. In view of Dekker's result that any co-r.e. regressive set is a recursive isomorph of a co-r.e. retraceable set, we complete the proof of the properness of the above inclusions as an immediate corollary to the Theorem. There exists a cor.e. regressive set not a member of $Q$. This theorem extends a previous result of the author (Proc. Amer. Math. Soc. 17 (1966), 427-429). (Received June 26, 1969.)

69T-E70. AUBERT DAIGNEAULT, Universite de Montreal, P. O. Box 6128, Montreal, Canada. Boolean ultrapowers.

Let $X$ be a nonempty set and $B$ a complete Boolean algebra with dual $B^{*}$. The Boolean power of $X$ over $B$ is the set $B(X, B)$ of all partitions $c: X \rightarrow B$ of $l \in B$ indexed by $X$. According to Ribenboim [Theorem 3 in "Boolean powers," forthcoming in Fund. Math.], $\mathcal{B}(\mathrm{X}, \mathrm{B})$ can be identified with the set of all continuous functions $\widetilde{c}$ valued in the discrete space $X$ and defined, each, on an open dense subset $Z_{c}$ of $B^{*}$ and which cannot be extended to larger domains in $B^{*}$. Let $D$ be an ultrafilter of $B$. The Boolean ultrapower $X_{D}=\beta(X, B) / D$ is obtained by identifying $c_{1}$ with $c_{2}$ iff the closure of $\left\{z \in Z_{c_{1}} \cap Z_{c_{2}} \mid \widetilde{c}_{1}(z)=\widetilde{c}_{2}(z)\right\}$ is in $D$. Similarly if $R \subset X^{n}$ it induces $R_{D} \subset X_{D}^{n}:\left\langle c_{1} / D, \ldots, c_{n} / D\right\rangle \in R_{D}$ iff the closure of $\left\{z \in Z_{c_{1}} \cap \ldots \cap Z_{c_{n}} \mid\left\langle c_{1}(z), \ldots, c_{n}(z)\right\rangle \in R\right\}$ is in $D$. Theorem. $\left\langle X_{D}, R_{D}\right\rangle$ is an elementary extension of $\langle X, R\rangle$. A similar result holds for $a$-bounded Boolean powers ( $a$ an infinite cardinal) in which one assumes only that each sum of less than a elements exists in $B$ and one takes only $a$-bounded partitions i.e. the partitions $c$ such that for less than a elements $x$ it is true that $c(x) \neq 0$. The proof is based on a study of the full functional $B$-values polyadic algebra with base X. (Received July 18, 1969.)

69T-E72. DOUGLASS B. MORRIS, Van Vleck Hall, University of Wisconsin, Madison, Wisconsin 53706. Existence of models of $Z \mathrm{~F}$ with involutions. Preliminary report.

Let M be any countable model of ZFC. A certain Cohen extension N of M has infinitely many sets of total indiscernibles and a property allowing the extension to a larger finite domain of any group of partial automorphisms satisfying certain conditions. Let $\mathrm{N}^{*}$ be any special elementary extension of N . The properties of N carry over to $\mathrm{N}^{*}$ in such a way that any countable group can be isomorphically embedded in the automorphism group of $\mathrm{N}^{*}$. In particular, $\mathrm{N}^{*}$ has automorphisms of order two. This answers a question of Cohen. (Keceived August 4, 1969.)

69T-E73. SOLOMON GARFUNKEL and HERBERT S. SHANK, Cornell University, Ithaca, New York 14850. On the undecidability of the theory of finite planar graphs.

We will say that an equivalence relation $E$ is of type ( $n_{1}, \ldots, n_{m}$ ) if its equivalence classes are of cardinality among $n_{1}, \ldots, n_{m}$ and there exist equivalence classes of $E$ of each cardinality. In addition we say that two equivalence relations on the same field N are disjoint iff the only pairs they have in common are the diagonal pairs $\{n, n\rangle, n \in N$. In particular we are interested in the theory $T^{\prime}$ whose models are of the form $\langle N, E, F\rangle$, where $N$ is finite and $E$ and $F$ are two disjoint equivalence relations of types ( $2 \mathrm{~m}-1$ ) and ( 1,2 ) on N respectively. Lemma. T ' is hereditarily undecidable. We prove the lemma by using the Rabin-Scott method of semantic embedding, taking as our base theory the theory of finite reflexive binary relations. Again applying this same method using $T^{\prime}$ as our base theory, we obtain -- Theorem. The elementary theory of finite planar graphs is hereditarily undecidable. (Received July 31, 1969.)

69T-E74. WALTER F. TAYLOR, University of Colorado, Boulder, Colorado 80302. Compact models.

Let $\varphi$ be any ( $\Lambda, v, \square)$-formila with two free variables. Let $\rho(n)$ be the first-order sentence which asserts that there are $n$ elements pairwise related by $\varphi$. Let $\varphi(\infty)$ be the first-order sentence ( $\Xi x) \varphi(x, x)$. Take $\psi$ to be any first-order sentence, and consider the infinitary rule of deduction: from all $\psi \supset \varphi(n)$, deduce $\psi \supset \varphi(\infty)$. Call a theory (WAC)-consistent if it does not entail a contradiction, using this new rule together with the usual rules. Theorem (Countable language). A theory $\Sigma$ has a weakly atomic-compact model iff $\Sigma$ is (WAC)-consistent. Assuming the continuum hypothesis, related results are as follows. Theorem (Countable language). If $थ$ is weakly atomic $\overline{\bar{\eta}}^{+}$compact, then $थ$ is weakly atomic-compact. Theorem. If the graph $G$ has chromatic number $\mathcal{K}_{2}$, then $G$ has elementary extensions of arbitrarily large chromatic number. The main tool is the partition calculus of Erdös and Rado (generalization of Ramsey's Theorem). For the compactness notions referred to above, consult [Fund. Math. 59 (1966), 289-298]. (Received August 12, 1969.)

69T-E75. WITHDRAWN.

69T-E76. JACOB M. PLOTKIN, Michigan State University, East Lansing, Michigan 48823. Nonalgebraic theories and models of ZF.

Let $T$ be a complete theory with equality, $F_{n}(T)$ the formulas of at most $n$ free variables, थ a model of $T$. For $a_{1}, \ldots, a_{k} \in|\mu|$ let $\Gamma\left(a_{1}, \ldots, a_{k}\right)=\left\{\varphi \in F_{k}(T)|\mu|=\varphi\left(a_{1}, \ldots, a_{k}\right)\right\}$. Definition. $a \in|थ|, B \subseteq|थ|$ a algebraic/B if there exist $b_{1}, \ldots, b_{n} \in B$ such that $\left\{y \in|थ| \mid \Gamma\left(b_{1}, \ldots, b_{n}, y\right)=\right.$ $\left.\Gamma\left(b_{1}, \ldots, b_{n}, a\right)\right\}$ is finite; $c l(B)=\{x \mid x$ algebraic $/ B\}$. Lemma. If $थ$ is $w$-nomogeneous then $c l(c l(B))=$ $c l(B)$. Lemma (Nerode). If ' $T$ is $\kappa_{0}$-categorical and $B \subseteq|थ|$ is finite then $c l(B)$ is finite. Definition. $T$ is nonalgebraic if for any infinite model 2 of $T$ and any $B \subseteq|2| c l(B)=B$. Theorem. Any nonalgebraic $\aleph_{0}$-categorical theory $T$ can be extended to a nonalgebraic $\kappa_{0}$-categorical theory $T$ ' having one new relation symbol denoting a dense linear ordering without end points. With the theorem and lemmas one can prove a support lemma in the manner of Pincus (Thesis, Harvard University) and by generic embedding (Abstract 68T-E13, these CNotices) $15(1968), 807$ ) obtain models of ZF where the universe is linearly ordered but various forms of choice fail. (Received August 18, 1969.)

69T-E77. LAWRENCE FEINER, State University of New York, Stony Brook, New York 11790. The strong homogeneity conjecture is false.

One can effectivize and relativize Hugill's construction [Proc. London Math. Soc.19(1969), 1-16] in order to imbed in the Turing degrees between $0^{(5)}$ and $0^{(6)}$, a $0^{(5)}$-recursive linear ordering which is not isomorphic to any $0^{(4)}$ recursive linear ordering. Since $\left\{d \mid 0 \leqq d \leqq 0^{(1)}\right\}$ is a $0^{(4)}$-recursive partial ordering, $\left\{\mathrm{d} \mid 0 \leqq \mathrm{~d} \leqq 0^{(1)}\right\}$ and $\left\{\mathrm{d} \mid 0^{(5)} \leqq \mathrm{d} \leqq 0^{(6)}\right\}$ are nonisomorphic. Thus, there is no jump preserving isomorphism from $\{d \mid d \geqq 0\}$ to $\left\{d \mid d \geqq 0^{(5)}\right\}$. (Received August 18, 1969.)

69T-E78. ALEXANDER ABIAN, Iowa State University, Ames, Iowa 50010. Synergistic models and the axiom of regularity.

Consider the Synergistic model (S, e) described in Abstract 69T-E64, these CNotices) 16 (1969), 844. It was shown that the axiom of choice ( $C$ ) is valid in ( $S, C$ ). Using induction on ordinal $u$ it is easily seen that there is no infinite descending e-chain in ( $S, C$ ). Thus, the consistency of the axiom of regularity ( $G$ ) with $Z F+C$ can be proved by means of Synergistic models without necessitating the construction of a model for the entire $\mathrm{ZF}+\mathrm{C}+\mathrm{G}$. (Received August 25, 1969.)

69T-E79. MIHÁLY MAKKAI, Mathematical Institute of the Hungarian Academy of Sciences, Budapest, V. Reáltanoda u.13-15, Hungary. Regular relations and Svenonius formilas.

Let $R$ be a regular binary relation between structures of a countable similarity type $\tau$, c.f. [P. Lindström, Theoria. 32 (1966), 172-185]. Let $C_{R}(K)={ }_{d f}\{B$ : ARB for some $A \in K\}$ and $K \omega_{1}=d f$ \{A:A $\in K$ and $A$ is countable\}. We construct a class $S_{R}$ of infinitary sentences of form $p \wedge \Sigma$ where $p$ is a prefix of length $\omega$ and $\Sigma$ is a (countable) set of finitary formulas of $\tau$ such that Theorem. (i) any element of $S_{R}$ is preserved under $R$, (ii) any finite approximation of any element of $S_{R}$ is logically equivalent to an element of $\Delta_{R}$ (for the definition of $\Delta_{R} c . f$. loc. cit.), (iii) for any countable $\tau^{\prime} \supset \tau$ and any set $T$ of finitary sentences of $\tau^{\prime}$, there is $\Phi \in S$ such that $C_{R}^{\omega_{1}}\left(\operatorname{Mod}(T) \Gamma_{\tau}\right)=\operatorname{Mod}{ }^{\omega_{1}}$ ( $\varepsilon$ ). This is a generalization of Theorem 2 in [M. Makkai, Abstract 69T-E17, these CNotices) 16 (1969), 425]. Corollary (P. Lindström loc. cit.). For sentences $\varphi, \psi$ of $\tau$, (for any $A$ and $B, A \neq \varphi$ and ARB imply $B \vDash \psi$ ] iff [there is $\vartheta \in \Delta_{R}$ such that $\varphi \vDash \vartheta \vDash \psi$ ]. (Received August 22, 1969.) (Author introduced by Professor Robert L. Vaught.)

69T-E80. A. M. SETTE, University of Campinas, Caixa Postal 1170, Campinas, São Paulo, Brazil. A note on Cw-algebras.

A $C_{\omega}$-algebra is an implicative lattice in Curry's sense ("Foundations of mathematical logic," McGraw-Hill, New York, 1963), $a=\langle A, \equiv, \wedge, V, \supset\rangle$, in which it is defined a unary operation ', such that $a \vee a^{\prime} \equiv 1$ and $a^{\prime \prime} \leqq$ for every $a \in A$. (The operations $\wedge, V$ and $\supset$ are monotone with respect to the equivalence relation $\equiv$.) This notion was introduced by N. C. A. da Costa as an algebraic version of the propositional calculus $C_{\omega}$ (C. R. Acad. Sci. Paris, Sér. A-B. 257 (1963), 3790-3792, and 263 (1966), 429-432). N. C. A. da Costa and the author have studied in ("Les algèbres $c_{\omega}, "$ to appear in C. R. Acad. Sci. Paris, Sér. A.-B.) several properties of these algebras, showing, in particular, that the operation' is (in general) nonmonotone with respect to $\equiv$. In this work we prove that it is (in general) impossible to define a nontrivial equivalence relation $R$ on $A$, such that $\left\langle A, R, \wedge, V, \supset,{ }^{\circ}\right\rangle$ is a $C \omega^{-a l g e b r a}$ and $\Lambda, V, \supset$ and ${ }^{\prime}$ are simultaneously monotone with respect to $R$. We present also a new algebraic version of the propositional calculus $C_{\boldsymbol{\omega}}$ and our algebraic methods may be applied to many other cases, for instance to the propositional calculi $C_{n}, 1 \leqslant n<\boldsymbol{L}(N$. C. A. da Costa, C. R. Acad. Sci. Paris, Sér. A-B. 257 (1966), 3790-3792). (Received August 13, 1969.) (Author introduced by Professor Newton C.A. da Costa.)

## Statistics and Probability

69T-Fl4. C. J. EVERETT, Los Alamos Scientific Laboratory, Los Alamos, New Mexico 87544 and S. M. ULAM, University of Colorado, Boulder, Colorado 80304. The entropy of interacting populations.

A study is made of interacting populations which closely parallels the Boltzmann kinetic theory and the Planck-Einstein-Tolman treatment of radiation interacting with matter. The analogues of the Boltzmann equation, H-theorem, and Maxwell-Planck steady state distributions are obtained. The analogy is perhaps surprising, since our formulation is in terms of a scalar "energy" rather than a vector "velocity". The "reversibility paradox" is thus avoided. (Received July 17, 1969.)

69T-F15. DAVID F. FRASER, Brown University, Providence, Rhode Island 02912. A probabilistic method for the rate of convergence to the Dirichlet problem.

The expectation $\mathrm{E}^{\mathrm{p}}(\Phi)$ approximates the solution $\mathrm{u}(\mathrm{z})=\mathrm{E}^{\mathrm{W}}(\Phi)$ of the Dirichlet problem for a plane domain $D$ with boundary conditions $\varphi$ on the boundary $\gamma$ of $D$, where $W$ is Wiener measure, $P$ is the measure generated by a random walk which approximates Brownian motion beginning at $z$, and $\Phi$ is the functional on paths which equals the value of $\varphi$ at the point where the path first meets $\boldsymbol{\gamma}$. This paper develops a specific rate of convergence. If $\boldsymbol{\gamma}$ is $C^{2}$, and $P^{n}$ is generated by random walks beginning at $z$, with independent increments in the coordinate directions at intervals $1 / n$, with mean zero, variance $1 / \sqrt{n}$, and absolute third moment bounded by $M$, then
 is the distance from $z$ to $\gamma$, and C is a constant depending only on $\gamma$. (Received July 1, 1969.) (Author introduced by Professor S. A. Sawyer.)

69T-Fl6. C. J. EVERET and P. R. STEIN, Los Alamos Scientific Laboratory, Los Alamos, New Mexico 87544. A Monte Carlo method for generating random sequences of prescribed expected density.

A "single transit" method of producing such sequences is given, employing a Poisson series of trials, which is simpler, faster, and more flexible than "sieve" devices. If $F(x)$ is a suitable function, a sequence $P(n), n \geqq 1$, may be defined such that, if $n$ is accepted for the random sequence $B$ with probability $P(n)$, then $P\{|B(N)-F(N)|<3 r F(N)\}>1-1 / r^{2}(1-r) F(N) \rightarrow 1$, where $B(N)$ is the number of $n \cong N$ accepted for $B$, and $3 r$ is any prescribed relative error. The "gap distribution" and "Goldbach property" are studied for various $P(n)$, including $P(n)=1 / 2$ (coin tossing) and $P(n) \sim 1 / \log n$ (prime distribution). Sample Theorem. For almost every sequence $\pi$ of "random primes", every sufficiently large even integer is a sum of two distinct integers of $\pi$. (Received July 18, 1969.)

69T-F17. NATHANIEL A. FRIEDMAN, State University of New York, Albany, New York 12203, and D. S. ORNSTEIN, Stanford University, Stanford, California 94305. On mixing transformations.

Theorem. Each ergodic measure preserving transformation on the unit interval induces mixing transformations on a dense class of measurable subsets. (Received July 22, 1969.)

69T-F18. WILLIS L. OWEN, University of Minnesota, Minneapolis, Minnesota 55414. Optimal stopping rules when the variance is infinite.

The following results were conjectured by Shepp in Ann. Math. Statist. 40 (1969), 993-1010. We have established this conjecture. Let $\left\{X_{n}\right\}$ be a sequence of independent identically distributed random variables which are in the domain of normal attraction of a stable law $V$ of index $a, 1<a<2$. Let $E\left(X_{i}\right)=0$ and $S_{n}=X_{1}+\ldots+X_{n}$. Let $Y_{t}, t \in[0, \infty)$, be a stable process of index $a$, with $Y_{0}=0$, and $E\left(Y_{t}\right)=0$. Theorem 1. There exists an increasing sequence of real numbers $\left\{b_{n}\right\}$, such that the stopping time $\mathrm{T}_{0}=\inf \left\{\mathrm{j}: \mathrm{S}_{\mathrm{j}} \geqq \mathrm{b}_{\mathrm{j}}\right\}$ is optimal in the sense that $\mathrm{E}\left(\mathrm{S}_{\mathrm{T}_{0}} / \mathrm{T}_{0}\right)=\sup \mathrm{E}\left(\mathrm{S}_{\mathrm{T}} / \mathrm{T}\right)$ where the supremum is taken over all stopping times for $S_{n}$. Theorem 2 . There exists a positive finite constant $y^{0}$, such that the stopping time $\nu=\inf \left\{t: Y_{t} \geqq y^{0}(1+t)^{1 / a}\right\}$ is optimal in the sense that $E\left[Y_{\nu} /(1+\nu)\right]=\sup E\left[Y_{T} /(l+T)\right]$ where the supremum is taken over all stopping times for the $Y_{t}$ process. Theorem 3. If the distribution of $Y_{1}$ is $V$, if $\left\{b_{n}\right\}$ is the sequence of Theorem 1 and $y^{0}$ is the constant of Theorem 2, then $\lim _{n \rightarrow \infty} b_{n} / n{ }^{1 / a}=y^{0}$. (Received July 3, 1969.)

69T-F19. EDNIN O. ELLJOTT, Bell Telephone Laboratories, Holmdel, New Jersey 07733. Extensions of measures and abstract stochastic processes.

An abstract stochastic process is considered with a probability measure $P$ defined on a sigma-field $F^{*}$ of cylinder sets in a function space $\Omega$ having a nondenumerable parameter set I (the cylinders are based on measurable sets in subspaces of denumerable dimension). If for each $t \in I, a_{t}=\left\{x \in \Omega: x(t) \in a_{t}\right\}$ where $a_{t}$ is a measurable subset of the th coordinate space, and if $a=U_{t \in I} a_{t}$ and $A \in F^{*}$, then the set $A a$ is called a nilset provided $P\left(A a_{t}\right)=0$ for each $t \in I$. With $\left\{\beta^{k}\right\}$ any sequence of nilsets and $B$ any member of $F^{*}$ it is shown that the outer measure $\bar{P}\left(B-U\left\{\beta^{k}\right\}\right)=P(B)$. The measure $P$ is then extended to $P^{\prime}$ where $P^{\prime}(A)=\operatorname{Inf}\left\{\bar{P}\left(A-U\left\{\beta^{k}\right\}\right):\left\{\beta^{k}\right\}\right.$
is a sequence of nilsets\} and $A$ is any subset of $\Omega$. This $P^{\prime}$ is an outer measure on $S \Sigma$ and it is shown that sets of the form $\left\{x \in \Omega: x(t) \in b_{t}, t \in I\right\}$ ( $b_{t}$ a measurable subset of the th coordinate) are measurable under $P^{\prime}$. Thus, events determined by conditions on nondenumerable collections of random variables are measurable. This differs from the usual extensions in stochastic theory since it is not based on a (P) nonmeasurable $\Lambda \subset \Omega$ of outer measure one (e.g., to obtain a separable process). (Received August 5, 1969.)

69T-F20. N. S. MENDELSOHN, University of Manitoba, Fort Garry Campas, Winnepeg 19, Manitoba, Canada. Intersection numbers for t-designs.

A t-design is a system consisting of $v$ points, b blocks, each block containing $k$ points and every subset of points appears in exactly $\lambda_{t}$ blocks. Every $t$-design is an $s$ design $s \leqq t$ and $\lambda_{s}=\binom{v-s}{t-s} /\binom{k-s}{v-s}$. If $B$ is a fixed block, intersection numbers $x_{0}, x_{1}, \ldots, x_{k}$ are defined where $x_{i}$ is the number of remaining blocks each of which intersects $B$ in exactly i points. These satisfy the equations $\sum_{i=u}^{k}\left(\begin{array}{l}i\end{array}\right) x_{i}=\left(\lambda_{u}-1\right)\binom{k}{u} u=0,1, \ldots, t$ and $b=\lambda_{0}$. These generalize to intersection numbers $x_{i}^{(u)}$ where $x_{i}^{(u)}$ is the number of sets of $u$ remaining blocks which intersect $B$ in $i$ points. The intersection equations are $\sum_{(i)}^{k}\binom{i}{i} x_{i}^{(s)}=\binom{\lambda_{u}-1}{s}\binom{k}{s}$. For symmetric $(v, k, \lambda)$ designs we can define similarly numbers $y_{i}^{(u)}$ as the number of sets of remaining u blocks, which intersect a fixed pair of blocks in exactly i points. The equations are $\left.\sum_{i=u}^{\lambda}\left(\begin{array}{l}i\end{array}\right) y_{x}^{(s)}={ }^{\boldsymbol{\lambda}}{ }_{u}{ }_{s}{ }^{-2}\right)\left(\begin{array}{l}\lambda_{2}\end{array}\right), u=0,1,2$. The equations are used to show the nonexistence of symmetric $t$-designs with $t \geqq 3$. (Received August 14, 1969.)

## Topology

69T-Gl25. DOUGLAS E. CAMERON, Virginia Polytechnic Institute, Blacksburg, Virginia 24061. Minimal $T_{2}$ Quasi-P spaces. Preliminary report.

Definitions. A quasi-P space is a $T_{1}$ space in which every $G_{d}$ is open; a $P$ space is a completely regular quasi-P space; a $T_{2}$ quasi-P space is $L(i)$ if every open filter base with the countable intersection property has at least one adherent point; a $T_{2}$ quasi- $P$ space is $L(i i)$ if every open filter base with the countable intersection property and unique adherent point converges. Theorem l. L(ii) implies $L(i)$ but not conversely; the converse does hold in $P$ spaces. Theorem 2. $A T_{2}$ quasi- $P$ space is minimal $T_{2}$ quasi- $P$ if and only if it is $L(i i)$. Theorem 3. Finite products are $L$ (ii) $[L(i)]$ if and only if each component space is $L(i i)$ [ $L(i)]$. Theorem 4. In a $T_{2}$ quasi- $P$ space ( $\mathrm{X}, \mathrm{T}$ ) the following are equivalent: (a) $\mathrm{L}(\mathrm{i})$; (b) $(\mathrm{X}, \mathrm{T})$ is closed in every $\mathrm{T}_{2}$ quasi- P space in which it may be embedded; (c) every open cover of the space has a countable proximate subcover. Theorem 5. Every minimal $\mathrm{T}_{2}$ quasi-P space is semiregular; a minimal $\mathrm{T}_{2}$ quasi- P space is regular if and only if it is maximal Lindelöf. (Not every minimal $T_{2}$ quasi- $P$ space is regular.) Theorem 7. A P-space is minimal $P$ if and only if it is maximal Lindelöf. (Received June 2, 1969.)

69T-Gl26. C. J. MOZZOCHI, 18 Tuxis Road, Niadison, Connecticut, 06443. On the completeness of a symmetric generalized uniform space III.

Let $(X, U)$ be a symmetric generalized uniform space. Let $\mathcal{J}$ be a filter in $(X, \mathcal{J}(U))$. Definition.
$\mathcal{Z}^{Z}$ is a $\Delta$-filter with respect to $U$ iff for every $F \in \mathcal{F}$ there exists $\mathrm{x} \in \mathrm{X}$ and $\mathrm{W} \in U$ such that $W[x] \subset F$. Let $\left(X_{a}, \mathscr{U}_{a}\right)$ and ( $X_{b}, \mathscr{l}_{b}$ ) be symmetric generalized uniform spaces. Definition. ( $\mathrm{X}, \mathcal{U}$ ) is $\Delta$-complete iff whenever $(X, U)$ is uniformly isomorphic to a dense subspace ( $X_{a}, u_{a}$ ) of ( $X_{b}, u_{b}$ ), then $X_{a}=X_{b}$. Theorem. Suppose ( $X_{a}, u_{a}$ ) is a dense subspace of $\left(X_{b}, u_{b}\right)$ and 3 is a $\Delta$-filter in ( $X_{b}, u_{b}$ ), then $B=\left\{F \cap X_{a} \mid F \in J^{Z}\right\}$ is a base for a $\Delta$-filter in $\left(X_{a}, U_{a}\right)$. Theorem. Suppose $(X, U)$ is separated. If $(X, U)$ is $\Delta$-complete, then every filter in $(X, \mathcal{U})$ is the neighborhood system of a point in $(X, \mathcal{J}(U))$. Theorem. Suppose $\left(x, u_{\text {}}\right)$ is uniformly isomorphic to a proper dense subspace ( $X_{a}, u_{a}$ ) of ( $X_{b}, u_{b}$ ). If ( $X_{b}, U_{b}$ ) is separated, then there exists a $\Delta$-filter in ( $X, U_{\text {) }}$ which is not the neighborhood system of a point in (X,J(U)) (cf. Abstracts 68T-412, 68T-G, these CNotices) 15(1968), 623,809). (Received May 7, 1969.)

69T-Gl27. CHARLES L. HAGOPIAN, California Institute of Technology, 107 Steele Laboratory Pasadena, California 9ll09. The existence of arcs in certain plane continua.

Let $M$ be a compact plane continuum such that for each pair of distinct points $p$ and $q$ in $M$, if $M$ is not aposyndetic at $p$ with respect to $q$, then $M$ is aposyndetic at $q$ with respect to $p$. In this paper the following results are established. Theorem. If $x$ and $y$ are distinct points of $M$ and $M$ is not aposyndetic at $x$ with respect to $y$, then $M$ contains an arc which has $x$ and $y$ as end points. Corollary. If there is a point $y$ in $M$ such that for each point $x$ in $M-\{y\}, M$ is not aposyndetic at $x$ with respect to $y$, then $M$ is arc-wise connected. (Received May 5, 1969.)

69T-G128. O. T. ALAS, Universidade de Sao Paulo, Caixa Postal 8105, São Paulo, Brazil. A characterization of collectionwise normal spaces.

Let ( $\mathrm{E}, \mathrm{T}$ ) be a normal Hausdorff space, w be an element which does not belong to E . Let $\mathrm{Y}=$ $E \bigcup\{w\}$ be the one point (=Alexandroff's) compactification of a discrete space of support $E$. Theorem. The space ( $\mathrm{E}, \mathrm{T}$ ) is countably paracompact and collectionwise normal if and only if the topological product space $\mathrm{E} \times \mathrm{Y}$ is normal. The proof is based in the following theorem proved by M . Katetov: A Hausdorff space is collectionwise normal and countably paracompact if and only if for each locally finite family $\left(F_{i}\right)_{i \in I}$ of closed subsets there is a locally finite family $\left(V_{i}\right)_{i \in I}$ of open subsets such that $\mathrm{F}_{\mathrm{i}} \subset \mathrm{V}_{\mathrm{i}}$, for any $\mathrm{i} \in \mathrm{I}$. (Colloq. Math. 6 (1958), 145-151--Russian.) (Received July 8, 1969.)

69T-Gl29. WITHDRAWN.

69T-Gl30. SIBE MARDIŠIĆ, University of Zagreb, Zagreb, p.p. 314, Yugoslavia. Mapping products of ordered compacta onto products of more factors.

An ordered compactum is a Hausdorff compact space $K$ provided with a total ordering $\cong$ and such that the topology of $K$ is the order topology induced by $\leqslant$ Theorem. Let $X_{1}, \ldots, X_{p+q}, p, q \geqq 1$, be infinite Hausdorff compact spaces such that there exist ordered compacta $K_{1}, \ldots, K_{p}$ and a mapping f of $K_{1} \times \ldots \times K_{p}$ onto $X_{1} \times \ldots \times X_{p+q}$. If for all $X_{j}, j \in\{1, \ldots, p+q\}$, the degree of separability $s\left(X_{j}\right) \leqq \mathcal{N}_{\tau}, \tau \geqq 0$, then the weight $w\left(X_{j}\right) \leqq \kappa_{\tau}$ for at least $q+1$ indices $j$. Corollary. If all $X_{j}, j \in\{1, \ldots, p+q\}$, are separable, then at least $q+1$ among the spaces $X_{j}$ are metrizable. W. W.

Babcock (Dissertation, Tulane University, New Orleans, 1964, pp. 1-29) obtained the weaker conclusion that at least one of the spaces $X_{j}$ is metrizable under the stronger assumptions that all $K_{1}, \ldots, K_{p}$ are separable and that $f$ is an open mapping. (Received July 14, 1969.)

69T-G131. RONNIE LEE, University of California, Los Angeles, California 90024 and The Institute for Advanced Study, Princeton, New Jersey 08540. Piecewise linear classification of some homotopy lens spaces.

Let $S^{2 n+1}$ be the unit ( $2 n+1$ )-sphere, each point of which is represented by ( $c_{0}, \ldots, c_{n}$ ) of complex numbers $c_{i}$ with $\Sigma\left|c_{i}\right|^{2}=1$. Let $\left(\mathbb{Z}_{p}, s^{2 n+1}\right)$ denote the $\mathbb{Z}_{p}$-action on $s^{2 n+1}$ given by the formula $\gamma \cdot\left(c_{0}, \ldots, c_{n}\right)=\left(\gamma c_{0}, \ldots, \gamma c_{n}\right)$ where $\gamma=\exp (2 \pi i / p)$. Its orbit space $S^{2 n+1} / \mathbb{Z}_{p_{n}}$ is the $(2 n+1)$-dimensional lens space $L^{n}(p)$. We claim the following: Theorem. Let $\operatorname{sht}\left(L^{n}(p)\right)$ be the set of simple homotopy triangulations of $L^{n}(p)$. If $2 n+1 \geqq 7$, and $n$ is odd, then there is an exact sequence of pointed sets $0 \rightarrow L_{2 n+2}\left(\mathbb{Z}_{p}\right)^{\sim} \rightarrow \operatorname{Aht}\left(L^{n}(p)\right) \rightarrow\left[L^{n}(p) ; G / P L\right] \rightarrow 0$ where $\left[L^{n}(p)\right.$; G/PL] is the group of $G / P L$-bundle over $L^{n}(p)$, and $L_{2 n+2}\left(\mathbb{Z}_{p}\right)^{\sim}$ is the reduced surgery obstruction group of Wall. (Received June 26, 1969.)

69T-Gl32. BRUCE R. WENNER, University of Missouri, Kansas City, Kansas City, Missouri 64110. Sums of finite-dimensional spaces.

Analogues are developed to the sum theorems in the dimension theory of metric spaces. It is shown that, within the class of metric spaces, any locally countable, $\sigma$-locally finite, or closurepreserving sum of finite-dimensional sets is countable-dimensional. Similar results are obtained under the more general hypothesis of countable-dimensional rather than finite-dimensional sets. (Received July 3, 1969.)

69T-G133. JEFFREY L. TOLLEFSON, Tulane University, New Orleans, Louisiana 70118. On characterizing 3 -manifolds that are products.

A free $Z_{k}$ action on a manifold $M$ is proper if a generator of the action is homotopic to the identity map. $M *$ denotes the orbit space $M / Z_{k}$. Theorem 1 . Let $M$ be a closed, connected, orientable, irreducible 3 -manifold. If $M$ admits a proper free $Z_{k}$ action (for some prime $k \geqq 2$ ) such that $H_{1}\left(M^{*} ; Z\right)$ has no element of order $k$, then (i) $M$ fibers over the circle, and (ii) $M$ admits an effective $S O(2)$ action without fixed points. A manifold $M$ is said to cover itself properly $k$ times if $M$ admits a proper free $Z_{k}$ action with $M^{*} \approx M$. Theorem 2. A closed, connected, orientable 3-manifold $M$ covers itself properly $k$ times, for every prime $k \geqq 2$, if and only if $M$ is the product of a 2 -manifold and $S^{1}$. A nonproduct example exists that meets all the conditions of Theorem 2 except that it doublecovers itself in only a nonproper manner. (Received July 17, 1969.)

69T-G134. MILTON ULMER, Wesleyan University, Middletown, Connecticut 06457. Functions on product spaces. Preliminary report.

For any infinite cardinal number $\mathcal{N}$, a space is said to be pseudo- $\mathcal{N}$-compact provided each locally finite family of open subsets is of smaller cardinality than $\mathcal{K}$. A function from a product space $\Pi_{a \in A} X_{a}$ into a space $Y$ is said to depend on fewer than $K$ coordinates provided there exists a
subset $B$ of $A$ such that $|B|<\mathcal{K}$ and $f(p)=f(q)$ for all $p, q \in \Pi_{a \in A} \quad X_{a}$ such that $p_{B}=q_{B}$. Now let $X=\Pi_{a \in A} X_{a}$, with $|A| \geqq K$ and each $\left|X_{a}\right| \geqq 2$. Paralleling and partially extending results by Mazur, Corson Isbell, Ross Stone, and Engelking (see Fundamenta Math. 59 (1966), 221-31) we show (provided $\mathcal{N}$ is not the supremım of a countable set of smaller cardinals) that any continuous function from $X$ into any space $Y$ such that $Y \times Y$ is normal and the diagonal of $Y \times Y$ is a $G_{\delta}$. set, depends on fewer than $\mathcal{K}$ coordinates provided $X$ is pseudo- $\mathcal{K}$-compact. Now suppose each $X_{a}$ is completely regular and Hausdorff. For each such $K$ we have: Each function in $C(X)$ depends on fewer than $K$ coordinates if and only if $X$ is pseudo- $\mathcal{K}$-compact. But if $K=\operatorname{Sup}_{n \in N} m_{n}$, each $m_{n}<K$, and if $X$ is pseudo- $N$-compact but not pseudo- $m_{n}$-compact ( $n \in N$ ), then there exists a function in $C(X)$ which does not depend on fewer than K coordinates. (Received July 25,1969 .) (Author introduced by W. W. Comfort.)

09T-G135. R. CHRISTOPHER LACHER, Florida State University, Tallahassee, Florida 32306. Cellularity criteria for maps.

A space $A$ is cell-like iff there exist a manifold $M$ and an embedding $\varphi: A \rightarrow M$ such that $\varphi(A)$ is cellular in M. A map is cell-like iff its point-inverses are cell-like spaces. Theorem 1. Let X and $Y$ be euclidean neighborhood retracts, $f: X \rightarrow Y$ a proper, onto map. Then the following are equivalent: (a) fis cell-like. (b) The mapping cylinder of $f$ is $L C{ }^{k}$ rel $Y$ for all $k$. (c) For any open set $U$ of $Y, f \mid f^{-1}(U): f^{-1}(U) \rightarrow U$ is a proper homotopy equivalence. Definition. A map $f: X \rightarrow Y$ is a $U V^{k}$-map iff, for any $y \in Y$ and neighborhood $U$ of $f^{-1}(y)$ in $X$, there is a neighborhood $V$ of $f^{-1}(y)$ in $U$ such that every map $S^{q} \rightarrow V$ extends to a map $B^{q+1} \rightarrow U^{-}, 0 \leqq q \leqq k$. Theorem 2. If $f: S^{n} \rightarrow S^{n}$ is an onto UV ${ }^{k}$-map, $2 k+2 \geqq n$, then $f$ is cell-like. Hence, if $n \neq 4$, $f$ is cellular. Theorem 3. If $f: M \rightarrow N$ is an onto $U V{ }^{k}$-map between closed orientable topological $n$-manifolds, $2 k+\overline{1 \geqq n \text {, then } f}$ is cell-like. Hence, if $n \neq 3,4, f$ is cellular. Note. Theorems 2 and 3 are best possible codimensionally. (Received July 14, 1969.)

69T-G136. HARVEY ROSEN, Florida State University, Tallahassee, Fiorida 32306, and MICHAEL D. TAYLOR, Florida Technological University, Orlando, Florida 32816. An equality for two-sided surfaces with a finite number of wild points. Preliminary report.

Let $S$ be a closed connected 2 -sided surface of genus $g(S)$ embedded in the interior of a 3 -manifold (with or without boundary) $M^{3}$. There is an open connected neighborhood $W$ of $S$ such that W - S consists of exactly two components, $U$ and $V$. We define $g(S, U)=n$ if there exists a point $x$ of $U$ and a sequence of closed surfaces $S_{1}, S_{2}, \ldots$ in $U$ such that (1) the genus of each $S_{k}$ is $n$, (2) each $S_{k}$ separates $x$ from $S$ in $U$, (3) $S=\lim S_{k}$, and (4) conditions (1) - (3) cannot be satisfied by any integer smaller than $n$. We suppose that $S$ is wild from $U$ just at the points $p_{1}, \ldots, p_{m}$. Without loss of generality, $S$ is locally tame from $V$ at these points. We define $g\left(p_{k}, U\right)$ to be the local enveloping genus at $p_{k}$ of any arc on $S$ which has $p_{k}$ as an endpoint. Theorem. If $m=1$ and $g(S)=0$, then $g(S, U)=0$; but for any other pair of nonnegative integer values for $m$ and $g(S)$, we have $g(S, U)=$ $g(S)+\Sigma g\left(p_{k}, U\right)$ if $S$ is orientable, and $g(S, U)=g(S)+2 \Sigma g\left(p_{k}, U\right)$ if $S$ is nonorientable. (Received July $22,1969$.

69T-G137. ROBERT F. BROWN and ALFRED W. HALES, Ūniversity of California, Los Angeles, California 90024. Primitive roots of unity in H-manifolds. II.

See Part I for definitions. Let ( $\mathrm{M}, \mathrm{m}, \mathrm{e}$ ) be an H -manifold and set $\pi=\pi_{1}(\mathrm{M}, \mathrm{e})$. Corollary 1 . If $\pi$ is infinite cyclic, then ( $M, m, e$ ) has $\varphi(k)$ totally primitive classes of kth roots of unity, where $\varphi$ is Euler's function. Corollary 2. If $\pi$ is infinite, then there exist primitive kth roots of unity in ( $M, m, e$ ) for each $k \geqq 2$. Corollary 3. If $\pi$ is finite and $p$ divides $|\pi|$ then there exist primitive $p^{k}$ th roots of unity in ( $M, m, e$ ) for each $k \geqq 1$. Example. For each $k=2,3, \ldots$, there is an $H$-manifold $\left(S^{3}, \mathrm{~m}^{(\mathrm{k})}, \mathrm{e}\right)$ with no primitive kth roots of unity. (Received July 3, 1969.)

69T-G138. RICHARD E. HODEL, Duke University, Durham, North Carolina 27706. A note on $\mathrm{F}_{\boldsymbol{\sigma}}$-screenable spaces.

According to McAuley (Proc. Amer. Math. Soc. 9 (1958), 796-799) a topological space is $F_{\sigma}$-screenable if every open cover has a $\sigma$-discrete closed refinement. In this paper two results about such spaces are proved. (1) Every metacompact space in which every closed set is a $\mathrm{G}_{\delta}$ is $\mathrm{F}_{\boldsymbol{\sigma}}$-screenable. (2) Every $\mathrm{F}_{\sigma}$-screenable space is countably metacompact. With regard to (1) it should be pointed out that Worrell (Portugal Math, 25 (1966), 175-179) has given an example of a metacompact space not $\mathrm{F}_{\boldsymbol{\sigma}}$-screenable. (Received July 28, 1969.)

69T-G139. ROBERT CONNELLY, University of Michigan, Ann Arbor, Michigan 4804. Unknotting close polyhedra in codimension three. Preliminary report.

A special case of the main result is: Theorem. Let $\hat{K} \subset K^{k}$ be compact polyhedra contained in the interior of a combinatorial manifold $M^{n}$, where $k \leqq n-3$. Let $\epsilon<0$ and $N$, a neighborhood of $\bar{K}-\hat{\mathrm{K}}$ in $\mathrm{M}^{\mathrm{n}}$, be given. Then there is a $\delta<0$ such that if $h: K \rightarrow M$ is a $P L \delta$-embedding such that
 of Lickorish's cone unknotting theorem, elementary engulfing techniques, and some new techniques involving shadows. This is a generalization of and independent of the results obtained by Miller (Abstract 69T-G42, these Cotices) $16(1969), 583$ ) as well as Černavskii (Topological imbeddings of polyhedra in Euclidean space, Dokl. Akad. Nauk SSSR 165 (1965), 1257-1260 = Soviet Math. Dokl. 6(1965), 1606-1609.) Using the recent stable homeomorphism theorem of Kirby, Siebenmann and Wall, (Abstract 69T-G27, these CNotices) 16 (1969), 112) and classical results of Connell and Stallings it is an immediate corollary that if a codimension three topological imbedding of a compact polyhedron in the interior of a combinatorial manifold is locally PL, then it is etame, where dimension of the manifold is $\geqq 5$. (Received August 4, 1969.)

69T-G140. F. THOMAS FARRELL and JOHN B. WAGONER, University of California, Berkeley, California 94720. A torsion invariant for proper h-cobordisms. Preliminary report.

Let $W^{n+1}$ be a connected $p .1$. or smooth proper $h$-cobordism between $M_{-}^{n}$ and $M_{+}^{n}$. Let $M_{-}$ have one stable end $\epsilon$ and let the relative Whitehead group of $f: \pi_{1} \epsilon \rightarrow \pi_{1} M_{\text {- }}$ be $\mathrm{Wh}(f)=K_{0}(f) /[ \pm g]$, where $[ \pm g]$ is the subgroup generated by triples $\left(Z\left[\pi_{1} \epsilon\right], \pm g, Z\left[\pi_{1} \epsilon\right]\right)$ for $g \in \pi_{1} M_{-}$. There is an exact sequence $\mathrm{Wh}\left(\pi_{1} \epsilon\right) \rightarrow \mathrm{Wh}\left(\pi_{1} M_{-}\right) \rightarrow \mathrm{Wh}(\mathrm{f}) \stackrel{\partial}{\rightarrow} \widetilde{\mathrm{K}}_{0}\left(\pi_{1} \epsilon\right) \rightarrow \widetilde{\mathrm{K}}_{0}\left(\pi_{1} M_{-}\right) . " \cong "$ means smooth or p.l. equivalence. Theorem $(n \geqq 5)$. (a) There is a torsion invariant $\tau=\tau\left(W ; M_{-}, M_{+}\right) \in W h(f)$
such that $W \cong M_{-} \times I$ iff $\tau=0$. (b) Any element of $W h(f)$ can be realized as the torsion of a proper $h$-cobordism on $M_{ـ}$. (c) $\partial T=0$ iff $W$ has an open neighborhood of infinity with a product structure. As in the Whitehead torsion theory for compact $h$-cobordisms one can prove sum, product, transfer, and duality formulae. The theory generalizes easily to the case where $M_{\text {_ }}$ has finitely many stable ends. L. C. Siebenmann has a theory of infinite simple homotopy types which yields some results for proper $h$-cobordisms on an arbitrary $M_{\ldots}$. Our methods produce an algebraic torsion invariant for his simple homotopy types of a locally finite, connected, finite dimensional CW-complex with finitely many stable ends. (Received August 1, 1969.)

69T-G141. BRUCE A. ANDERSON, Arizona State University, Tempe, Arizona 85281. A class of spaces with $T_{1}$-complements.

Call a topological space splitable if it has an infinite family of pairwise disjoint open sets and call a space a DN-space if for each point in the space, there is a net in the complement of the point that converges to the point and whose range is a discrete subspace. A technique of A. K. and E. F. Steiner is used to extend results of the author and D. G. Stewart in a paper to appear in the Proc. Amer. Math. Soc. Theorem l. If a $T_{l}$ space ( $X, T$ ) has a dense splitable DN-subspace, then $T$ has a $T_{1}$-complement that is compact on cofinite subsets of $X$. Corollaries of this result are that all Fréchet (hence first axiom) Hausdorff spaces, all locally compact Hausdorff spaces and all symmetrizable Hausdorff spaces have $T_{1}$-complements. Theorem 2. If ( $X, T$ ) is a $T_{1} D N$-space that can be expressed as the union of infinitely many pairwise disjoint open sets, then there are topologies $J$ and $K$ on $X$ such that any distinct pair of these three topologies are $T_{1}$-complements. Theorem 3 . Every $T_{1}$ space is an open and closed subspace of a $T_{1}$-space that has a $T_{1}$-complement. (Received August 4, 1969.)

69T'-G142. S. SWAMINATHAN and A. C. THOMPSON, Dalhousie University, Halifax, Nova Scotia. Browder-Göhde-Kirk theorem for a class of topological spaces.

Corresponding to a family $B$ of subsets of a topological space $X$, which is stable for closure and arbitrary intersections, a $\beta$-uniformity on $X$ is a collection $U$ of symmetric subsets of $X \times X$ containing $\Delta$ such that for each $x \in X$ and $U \in U$ the trace $U_{x}=\{y \in X:(x, y) \in U\}$ is a member of $B$. For each nonempty $A \subseteq X$, the functions $D(A)=\left\{U \in U: A \subseteq U_{x}\right.$ for every $\left.x \in A\right\}$ and $N(A)=$ $\left\{U \in U: A \subseteq U_{x}\right.$ for some $\left.x \in A\right\}$ serve to introduce the concepts of diameter and normal structure for $A$, e.g., A has normal structure (w.r.t. $\mu$ ) iff $D(B) \subset N(B)$ for every $B \in B$ which is contained in $A$ and is not a singleton. A mapping $f$ of $X$ into itself is called (i) nonexpansive iff $\left.D_{( }^{\prime} A\right) \subseteq D_{(f(A))}$ for every two-element subset $A$ of $X$, and (ii) normalising iff $D(B) \subset N(\overline{h u} \overline{1} f(B))$ for every nontrivial $B \in B$, where hull $A$ denotes the smallest closed member of $\beta$ which contains $A$. The following theorem places the Browder-Göhde-Kirk fixed point theorem for Banach spaces in the setting of topological spaces endowed with a $\beta$-uniformity. Theorem. Let ( $X, \beta, U$ ) be a compact space with a $\theta$-uniformity. Then every nonexpansive and normalising mapping of $X$ into itself has a fixed point. (Received June 30, 1969.)

69T-G143. ALDEN H. WRIGHT, University of Wisconsin, Madison, Wisconsin 53706, and University of Utah, Salt Lake City, Utah 84ll2. Mapping cylinder neighborhoods and a flatness condition for a 3 -sphere in a 4-sphere.

Let $S_{0}^{3}$ be a 3-sphere embedded in the 4-sphere $S^{4}$. Let $W$ be a complementary domain of $S_{0}^{3}$ in $S^{4}$. If $x \in S_{0}^{3}$, we will say that $x$ has a mapping cylinder neighborhood in $\bar{W}$ if there exist (1) an open neighborhood $V_{x}$ of $x$ in $S_{0}^{3}$, (2) an open 3-manifold $U_{x}$, (3) a proper (compact) map $\varphi_{x}$ of $U_{x}$ onto $\mathrm{V}_{\mathrm{x}}$, and (4) an embedding $\psi_{\mathrm{x}}$ of the mapping cylinder $\mathrm{Z}_{\varphi_{\mathrm{x}}}$ into $\overline{\mathrm{W}}$ which is the identity on $\mathrm{V}_{\mathrm{x}}$. Theorem. If each point of $\mathrm{S}_{0}^{3}$ has a mapping cylinder neighborhood in $\overline{\mathrm{W}}$, then $\overline{\mathrm{W}}$ is a 4-cell. R. C. Lacher ("Cell-like mappings, II," to appear) has also proved this theorem with the additional hypothesis that $W$ is ULC ${ }^{1}$ (uniformly locally l-connected). Lacher uses results of McMillan ("Strong homotopy equivalence of 3-manifolds," Bull. Amer. Math. Soc. 73 (1967), 718-722). To obtain the theorem we instead use a result of the author (Abstract 69T-G121, these CNotices) 16 (1969), 853-854) and the fact that $W$ is r-ulc for all integers $r \geqq 0$ (see Theorem II.5.35 of Wilder, "Topology of manifolds." Colloq. Publ. Vol. 32, Amer. Math. Soc., Providence, R. I., l1949; sec. ed., 1963). Otherwise the proof of the theorem is the same as Lacher's proof. The proof of the theorem is given in the author's Ph. D. thesis (University of Wisconsin, 1969). (Received August 6, 1969.)

69T-G144. JAN MYCIE LSKI, University of Colorado, Boulder, Colorado 80302. Some conditions for a mapping to be a homeomorphism.

Let $K$ be the closed $n$-cell, I the interior of $K$, $B$ the boundary of $K$ and $X$ any Hausdorff space. Theorem. If $f: K \rightarrow X$ is continuous, $f \mid I$ is an open local homeomorphism, $f \mid B$ is one-to-one and $f(I) \cap$ $f(B)=0$ then $f$ is one-to-one. Corollary. If $f: K \rightarrow R^{n}$ is continuous, $f \mid I$ is a local homeomorphism and $f \mid B$ is one-to-one then $f$ is one-to-one. E. Duda has shown that the Corollary is still valid if we replace " $f \mid 1$ is a local homeomorphism" by " $f \mid I$ is open and finite-to-one". A related result is the following. Theorem. If $f: R^{n} \rightarrow R^{n}$ is continuous, open and closed, finite-to-one and the set of points at which $f$ is not a local homeomorphism is bounded then $f$ is one-to-one. It is not known if the supposition "finite-to-one" in these two theorems is essential. (Received August 8, 1969.)

69T-G145. MARSHALL M. COHEN, Cornell University, Ithaca, New York 14850. P.l. n-balls in $R^{n}$ whose union is not a manifold.

Theorem. If $n \geqq 5$ there exist subpolyhedra $A$ and $B$ of $R^{n}$ such that $A, B$ and $A \cap B$ are p.l. $n$-balls while $A \cup B$ is not a topological manifold. Proof. L. C. Glaser has proved [1], [2]that, if $n \geqq 5$, there exists subpolyhedra $C$ and $D$ of $\partial^{n}$ such that $C, D$ and $C \cap D$ are p.l. $n$-balls while $C \cup D$ is a p.l. manifold with nonsimplyconnected boundary. Let $A=v C$ and $B=v D$ be the cones from the origin $v$. Then $A \cup B$ is a pseudomanifold with boundary $C \cup D \cup v(B d y(C \cup D))$. But the link of $v$ in $B d y(A \cup B)$--namely $B d y(C \cup D)$-- is not simply connected. Hence $B d y(A \cup B)$, and thus $A \cup B$ itself, is not a topological manifold, q.e.d. [1] Proc. Amer. Math. Soc. 16(1965), 1357-1364. [2]Trans. Amer. Math. Soc. 122(1966), 311-320. (Received August 8, 1969.)

69T-Gl46. P. E. CONNER and LARRY SMITH, University of Virginia, Charlottesville, Virginia 22903. On the homological dimension of $\Omega_{*}^{U}(X)$.

In a previous publication (to appear in the Inst. Hautes. Études Sci. Publ. Math.), we have studied the homological dimension of the bordism module $\Omega_{*}^{U}(X)$ over the complex bordism algebra $\Omega_{*}^{U}$. We also presented several methods of estimating this integral invariant. Here is one more such estimate. Theorem. Let $X$ be a finite complex and $p$ a prime integer. If there exist (l) $a_{1}, \ldots, a_{n}$, operations in the two sided ideal generated by the Bockstein $\beta$ in the Steenrod algebra $a(p)$, and (2) a class $x \in H^{*}\left(X ; Z_{p}\right)$ such that $a_{1} \ldots a_{n} x \neq 0 \in H^{*}\left(X ; Z_{p}\right)$ then hom dim $S_{\iota_{*}}^{U} \Omega_{*}^{U}(X) \geqq n$. (Received August 12, 1969.)

69T-G147. FRANCIS T. CHRISTOPH Jr., Temple University, Philadelphia, Pennsylvania 19122. Extensions of topological semigroups.

Particular cases of the following general problem are considered: Given a collection $S_{a}, a \in A$, of topological semigroups, place a semigroup structure on $U_{a \in A} U_{a}$ where $U_{a} \subseteq S_{a}$ for each $a \in A$ such that $U_{a \in A} U_{a}$ is a topological semigroup and certain factor semigroups of $U_{a \in A} U_{a}$ are topological semigroups and fulfill some specified conditions. This problem is specialized by considering topological ideal extensions (i.e., if $\mathrm{S}, \mathrm{T}$ are disjoint topological semigroups, T having a zero 0 , then a topological semigroup $H$ is a topological ideal extension of $S$ by $T$ if and only if $H$ contains $S$ as an ideal and $\mathrm{H} / \mathrm{S}$ is a topological semigroup which is topologically isomorphic to T ), adjunction spaces, semilattices of topological semigroups, and the catena construction of Hofmann and Mostert, "Elements of compact semigroups," Merrill, Columbus, Ohio, 1966. (Received August 14, 1969.)

69T-G148. ROBERT B. BROOK, School of Arts and Sciences, University of Hartford, 200 Bloomfield Avenue, West Hartford, Connecticut 06117. The trace relation in topological dynamics.

For a topological group $T$, the set of all its closed syndetic invariant subgroups is denoted by $\mathbb{X}(\mathrm{T})$. If $(X, T)$ is a transformation group, the trace relation on $(X, T)$ is defined to be the set of all ordered pairs $(x, y) \in X \times X$ such that $y \in \cap\{\overline{x H} \mid H \in \mathbb{N}(T)\}$ and is denoted by $\Lambda(X, T)$ or $\Lambda$. In case $\Lambda=\Delta$, the diagonal of $X$, we say that ( $X, T$ ) has singleton traces. Theorem 1 . For a minimal set ( $\mathrm{X}, \mathrm{T}$ ) the following statements are equivalent: (1) ( $\mathrm{X}, \mathrm{T}$ ) has singleton traces; (2) ( $\mathrm{X}, \mathrm{T}$ ) is embeddable in a product of perodic transformation groups; (3) ( $\mathrm{X}, \mathrm{T}$ ) is embeddable in a product of periodic coset transformation groups. Theorem 2. Let ( $\mathrm{X}, \mathrm{T}$ ) be a compact singleton trace transformation group, and let $x \in X$. Then ( $\overline{\mathrm{xT}}, \mathrm{T}$ ) is an equicontinuous minimal set. Theorem 3. Let ( $\mathrm{X}, \mathrm{T}$ ) be an equicontinuous minimal set where T is locally compact abelian, and let there exist $x_{0} \in X$ such that $\operatorname{card}\left(x_{0} T_{0}\right) \geqslant 2$, where $T_{0}$ is the connected component of the identity in $T$. Then ( $\mathrm{X}, \mathrm{T}$ ) has singleton traces. Theorem 4. An equicontinuous compact transformation group ( $\mathrm{X}, \mathrm{T}$ ) where $T$ is locally compact connected has singleton traces. An example shows the necessity of the hypotheses in Theorems 3 and 4. (Received August 18, 1969.)

69T-G149. MITSUYOSHI KATO, Institute for Advanced Study, Princeton, New Jersey 08540. Classification of compact manifolds homotopy equivalent to the sphere.

We consider a pair $(Q, a)$ consisting of a compact oriented $\&(=T O P, P L$ or DIFF) q-manifold
$Q$ (possibly $\pi_{1}(\partial Q) \neq 0$ ) homotopy equivalent to $S^{n}$ and a generator $a$ of $H_{n}(Q)$. ( $Q, a$ ) and ( $Q^{\prime}$, $a^{\prime}$ ) are $E$-equivalent, if there are orientation preserving \& embeddings $i: Q \rightarrow Q^{\prime}$ and $j: Q^{\prime} \rightarrow Q$ such that $i_{*} a=a^{\prime}$ and $j_{*} a^{\prime}=a$. Let \& $2_{n}^{q}$ be the set of E-equivalence classes of such pairs. Let $F \& C_{n}^{k}$ be the set of locally flat concordance classes of locally flat orientation preserving embeddings $\mathrm{f}: \mathrm{S}^{\mathrm{n}} \times \mathrm{D}^{\mathrm{k}} \rightarrow \mathrm{S}^{\mathrm{n}+\mathrm{k}}$. A map $v_{\&}^{\mathrm{n}, \mathrm{k}}: F \& \mathrm{C}_{\mathrm{n}}^{\mathrm{k}} \rightarrow \& \sum_{\mathrm{n}+1}^{\mathrm{n}+1+\mathrm{k}}$ is well-defined by essentially associating famanifold $D^{n+1+k} \bigcup_{f} D^{n+1} \times D^{k}$. We generalize Haefliger's Theorem (Ann. of Math. 83 (1966), 402-436). Theorem A. For $n+k \geqq 4$, if $\&=$ DIFF or $P L$, and for $n+k \geqq 5$, if $\&=T O P, v_{\&}^{n, k}$ is bijective. The abelian groups $F C_{n}^{k}=F D I F F C_{n}^{k}$ and FPLC ${ }_{n}^{k}$ have been studied by Haefliger, Levine ( $k \geqq 3$ ), Fox-Milnor, Kervair and Levine $(k=2)$. Theorem B. For $n+k \neq 3,4$, FTOPC $C_{n}^{k}$ is an abelian group and if $n \neq 3$, FPLC $_{n}^{k} \cong \mathrm{FTOPC}_{n}^{k}$. For $k \geqq 3$, FTOPC $_{3}^{\mathrm{k}} \cong \mathrm{FPLC}_{3}^{\mathrm{k}} \oplus \mathrm{Z}_{2}$. (Received August 25, 1969.)

69T-G150. WARREN WHITE, Pura e Aplicada, Rua Sao Clemente, 265, Rio de Janeiro, Brazil. On the envelope of a singular 2 -sphere.

Let $S$ be a 2 -sphere in $E^{3}, f: S \rightarrow$ int $S$ a map. The envelope, $E(f)$, of $f$ is the boundary of the unbounded component of $E^{3}-f(S)$. We may adjust $f$ slightly so that $E(f)$ is a PWL 2-manifold bounding a cube with handles. If $S$ is free and $T$ is a curvilinear triangulation for $S$, then there is a small map $f: S \rightarrow$ int $S$ with a small embedding $g$ of the $l$-skeleton of $T$ into $E(f)$. This paper studies methods of cutting or killing the handles of $E(f)$ in such a way that $g$ may be extended to a small embedding of $S$ into int $S$. Theorem. A free 2 -sphere with a nowhere-dense set of wild points is tame. Corollary. Every Sierpinski curve on a free 2 -sphere is tame. (Received September 2, 1969.)

## Miscellaneous Fields

69T-H52. MARLON C. RAYBURN, University of Manitoba, Winnipeg 19, Manitoba, Canada. On Borel fields.

Let $X$ have at least three points, $\Sigma$ be its lattice of topologies and $\Delta$ its lattice of $\sigma$-algebras. Set $\tau: \Sigma \rightarrow \Delta$ by $\tau(T)=B[T]$, the Borel field generated by topology $T$. Theorem. If $V_{\Gamma} \tau\left(T_{a}\right)$ is a topology (complete Boolean algebra), then $V_{\Gamma} \boldsymbol{T}\left(\mathrm{T}_{\mathrm{a}}\right)=\boldsymbol{(} V_{\Gamma} T_{a}$ ). Corollary l. If X is countable, then $\tau$ is a complete join homomorphism of $\Sigma$ onto $\Delta$. Corollary 2. Over any $X, \tau$ is a complete join homomorphism from the ideal of finite topologies onto the ideal of finite $\sigma$-algebras. Note $\tau$ is not a meet homomorphism on the ideal of finite topologies for any X. (Received August 20, 1969.)

69T-H53. SAMI BERAHA, Johns Hopkins University, Baltimore, Maryland 21218. A program for generating the chromials of regular maps by tracers.

The map is represented by an adjacency matrix whose square serves to: (1) determine the number of boundaries of each'region, (2) detect and eliminate 2 -rings and 3-rings, (3) canonize the matrices, (4) produce a key that adds to the flexibility of the program by allowing choices in the reduction of the maps, (5) order the matrices for storage and retrieval. In the absence of 2 -and 3 -rings to every $a_{i j}=1$ corresponds $a_{i j}^{(2)}=2$ (in the square of the matrix), and $a_{i i}^{(2)}$ equals the number of boundaries of the $i^{\prime}$ th region. In the presence of 2-and 3-rings flaws occur in the formalism that can easily be repaired but are kept in order to serve as tracers for detecting and eliminating such rings. The chromials are generated in Lewis form. (Received August 25, 1969.)

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David C. Kay, University of Oklahoma
The three geometries-spherical, Euclidean, and hyperbolic-are studied both axiomatically and from models, with an extensive development of non-Euclidean trigonometry and its consequences.
1969 / 384 pages / \$9.50

## TRIGONOMETRY: A Programmed Text

Mervin L. Keedy, Purdue University, and
Marvin L. Bittinger, Indiana University
This text incorporates a significant amount of reading material with frequent questions, demanding student response and participation in the learning process.
1969/272 pages / $\$ 5.95$ paper

TRIGONOMETRY: A Functional Approach<br>Bill Rice and Joe Dorsett, both of<br>St. Petersburg Junior College<br>All major trigonometric topics are developed through a distinctive classroom approach. Questions, problems, summaries, bibliographies, and appendixes are included.<br>1969 / 320 pages / \$7.95

## PLANE GEOMETRY, Third Edition

Frank A. Rickey, and J. Perry Cole, both of Louisiana State University
The authors present a concise, analytical development of plane trigonometry that provides the trigonometric background for modern courses in analytic geometry and the calculus of functions of real variables.
1969 / 272 pages / \$7.50

## APPLIED PROBABILITY

W. A. Thompson, Jr., University of Missouri

Designed for advanced students of engineering, mathematics, and statistics, this book focuses on probability in its applied context. Theory is developed from the beginning, but the relation of theory to application is emphasized.
September 1969 / 192 pages / \$10.50 (tent.)

## ELEMENTARY STATISTICAL METHODS, Third Edition

Helen M. Walker, Claremont Graduate School and University Center, and Josepn Lev,
New York State Education Department
New chapters added to this edition cover the chi-square and $F$ distributions, contingency tables, analysis of variance, partial and multiple correlation and multiple regression, and sampling from a finite universe with applications to survey sampling.
September 1969/384 pages / \$8.95 (tent.)


# SIMPLIFIED INDEPENDENCE PROOFS, BOOLEAN VALUED MODELS OF SET THEORY 

by J. BARKLEY ROSSER<br>Mathematics Research Center, United States Army, University of Wisconsin, Madison, Wisconsin

This monograph is an expository account of one of the most profound developments of mathematical logic in recent years. The author shows how to construct models for set theory in which the truth values of statements are elements of Boolean Algebra, and how by proper choices of Boolean Algebra one can arrive at independence proofs. The basic construction is presented in full detail with ample explanations. Cohen's original independence proofs were of such depth and conceptual novelty as to appeal only to pro-
fessional logicians. However, the proofs presented by Rosser are fairly simple and involve familiar concepts, so that they can be followed by interested mathematiciahs who have a modest familiarity with axiomatic set theory. Also, adequate explanations about Boolean Algebra are provided to enable the non-mathematical reader to comprehend the proofs fully. There is also a guide that explains how to skip most of the details and still understand the key ideas of the proofs.

## SEMIGROUPS

edited by KARL W. FOLLEY
Department of Mathematics, Wayne State University, Detroit, Michigan
The articles in this volume are based on lectures presented at the Wayne State University Symposium on semigroups. Emphasis is placed on analysis and probability of semigroups, the algebraic structure of semigroups, and the structure of topological semigroups. Technicians, research mathematicians, and computer scientists will find this work valuable. 1969, about 275 pp., $\$ 14.00$

## QUANTITATIVE AND QUALITATIVE GAMES, A GEOMETRIC APPROACH

## by AUSTIN BLAQUIERE

Francôis Gerard, Faculte des Sciences de Paris
GEORGE LEITMANN
Department of Electrical Engineering and Computer Science University of California, Berkeley, California
A volume of MATHEMATICS IN SCIENCE AND ENGINEERING
A Series of Monographs and Textbooks
This book is a record of joint research by the two authors in the field of two-person games and is based on lectures given by A. Blaquiere at the Institute Henri-Poincare. Two persongames are treated with perfect information. Both quantitative games (games of degree, zero-sum) and qualitative games (games of kind) are considered. Results, i.e. general properties, necessary conditions, and sufficient conditions are derived from a geometric point of view using properties of surfaces associated with optimal play.
1969, about 175 pp .


[^0]:    *The findings were published in a book by Allan Cartter entitled "An Assessment of Quality in Graduate Education." The information on mathematics was reprinted by the Society and can be found on pages 978-980 of the December 1966 issue of these $\mathcal{C N o t i c e s}$.

