Notices

OF THE AMERICAN MATHEMATICAL SOCIETY

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Notices

OF THE

AMERICAN MATHEMATICAL SOCIETY

Edited by Everett Pitcher and Gordon L. Walker

CONTENTS

MEETINGS

Calendar of Meetings

NOTE: This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the *Notices* was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

Meeting No.	Date	Place	Deadline for Abstracts*
677	August 24-28, 1970 (75th Summer Meeting)	Laramie, Wyoming	June 30, 1970
678	October 31, 1970	Washington, D. C.	Sept. 10, 1970
679	November 20-21, 1970	Athens, Georgia	Oct. 6, 1970
680	November 21, 1970	Pasadena, California	Oct. 6, 1970
681	November 28, 1970	Urbana, Illinois	Oct. 6, 1970
682	January 21-25, 1971 (77th Annual Meeting)	Atlantic City, New Jersey	Nov. 5, 1970
683	March 26-27, 1971	Chicago, Illinois	
	April 7-10, 1971	New York, New York	
	August 30-September 3, 1971 (76th Summer Meeting)	University Park, Pennsylvania	
	January 17-21, 1972 (78th Annual Meeting)	Las Vegas, Nevada	

*The abstracts of papers to be presented in person at the meetings must be received in the Headquarters Offices of the Society in Providence, Rhode Island, on or before these deadlines. The deadlines also apply to news items. The next two deadlines for by-title abstracts will be June 23, 1970, and September 3, 1970.

OTHER EVENTS

September 1-10, 1970	International Congress of Mathematicians	Nice, France

The *Notices* of the American Mathematical Society is published by the American Mathematical Society, 321 South Main Street, P. O. Box 6248, Providence, Rhode Island 02904 in January, February, April, June, August, October, November and December. Price per annual volume is $10.00. Price per copy $3.00. Special price for copies sold at registration desks of meetings of the Society, $1.00 per copy. Subscriptions, orders for back numbers (back issues of the last two years only are available) and inquiries should be addressed to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904. Second class postage paid at Providence, Rhode Island, and additional mailing offices.

The Six Hundred Seventy-Sixth Meeting
Pacific Lutheran University
Tacoma, Washington
June 20, 1970

The six hundred seventy-sixth meeting of the American Mathematical Society will be held on Saturday, June 20, 1970, at Pacific Lutheran University in Tacoma, Washington. The Mathematical Association of America and the Society for Industrial and Applied Mathematics will hold Northwest Sectional Meetings in conjunction with this meeting of the Society. The Association and SIAM will have sessions on Friday and Saturday, June 19 and 20.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, there will be two invited hour addresses at this meeting. Professor Robert C. James of the Claremont Graduate School will address the Society at 11:00 a.m. on Saturday. The title of his lecture is "Geometry of normed linear spaces." At 2:00 p.m. there will be an address by Professor Charles J. Stone of the University of California at Los Angeles. He will speak on "Infinitely divisible processes and their potential theory." There will be sessions for contributed papers on Saturday morning and afternoon. Late papers can be accepted for presentation at this meeting. Information concerning late papers and program changes will be available at the registration desk.

All sessions of the meeting will be held in the Administration Building and in Xavier Hall.

REGISTRATION

Registration for the meeting will take place from 3:00 p.m. to 9:00 p.m. on Thursday and from 9:00 a.m. to 2:00 p.m. on Friday and Saturday. The registration desk will be located in Mortvedt Library on Thursday and Friday and in the foyer of Xavier Hall on Saturday.

ACCOMMODATIONS

Dormitory space will be available on Thursday, Friday, and Saturday nights. The rates at the dormitory are $3.50 per person per night on a double occupancy basis and $5.00 per person per night in a single room. Families can be accommodated in adjoining rooms. Towels and soap are not provided in the dormitories.

Dormitory, meal, and banquet reservations can be made by writing to Dr. John O. Herzog, Department of Mathematics, Pacific Lutheran University, Tacoma, Washington 98447. Applications for reservations should include the applicant's name, address, his expected time of arrival, the name and relationship of each member of his party, the type of accommodations desired (double, single, or family), and the number of banquet and dining facility reservations required. The deadline for making reservations is June 9, 1970.

The closest large motel with rooms available during the time of the meeting is the Rodeway Inn, located on Interstate 5, about 20 minutes driving time from campus. There are several hotels in downtown Tacoma, including the Winthrop Hotel at 9th and Broadway and the America West Tacoma Motor Hotel at 242 St. Helens. The rates of all of these hotels are approximately $11.00 to $12.00 for a single room and $14.00 to $16.00 for a double. Bus service is available from downtown Tacoma to the campus. Anyone who wishes to stay in a hotel or motel should make his reservations directly through a travel agent or write directly to the preferred hotel.

MEALS

Meals will be available on

Thursday and Friday at the Campus Dining Room. The prices are $1.10 for breakfast, $1.40 for lunch, and $2.25 for dinner. The campus Coffee Shop will be open daily. In addition, there are several restaurants within walking distance of the campus. The Mathematical Association of America will sponsor a banquet on Friday evening. Tickets for the banquet will cost $3.25. Persons who wish to attend the banquet or take their meals in the Campus Dining Room should make reservations with Dr. Herzog at the address given above.

TRAVEL

Air transportation to Tacoma arrives at the Seattle-Tacoma International airport. There is limousine service from the airport to downtown Tacoma. Pacific Lutheran University is located seven miles south of Tacoma, in the suburb of Parkland, a thirty-minute ride from downtown Tacoma by city bus. Members who drive to the meeting will find ample parking on campus across the street from the Mortvedt Library.

PROGRAM OF THE SESSIONS

The time limit for each contributed paper is 10 minutes. The papers are scheduled at 15 minute intervals in order that listeners can circulate among the sessions. To maintain the schedule, the time limit will be strictly enforced.

SATURDAY, 9:30 A.M.

Session on Analysis, Room A 208, Administration Building

9:30-9:40

(1) Topological algebras of formal power series. Preliminary report
Dr. Richard J. Loy, Carleton University (676-5)

9:45-9:55

(2) Interpolation of operators of weak type between rearrangement invariant function spaces
Dr. Mordecay Zippin, University of California, Berkeley (676-3)

10:00-10:10

(3) A homotopy argument for the solvability of nonlinear operator equations in nonseparable Banach spaces
Dr. Peter Hess, University of Chicago (676-9)

10:15-10:25

(4) On the linearity of generalized isometries
Mr. Andrew J. H. Vogt, University of Washington (676-13)

10:30-10:40

(5) On the subspaces of $L^p (p > 2)$ spanned by sequences of independent random variables
Professor Haskell P. Rosenthal, University of California, Berkeley (676-2)

SATURDAY, 9:30 A.M.

First General Session, Room A 215, Administration Building

9:30-9:40

(6) A note involving hypothesis H
Mr. E. F. Ecklund, Jr., Washington State University (676-15)

9:45-9:55

(7) A class of commutative semigroups in which idempotents are linearly ordered
Professor Motupalli Satyanarayana, Bowling Green State University (676-11)

10:00-10:10

(8) Decidability and nonfinite axiomatizability of the equational theory of the ordinal numbers with addition and multiplication. Preliminary report
Mr. Charles Martin, University of California, Berkeley (676-8)
(Introduced by Professor Leon A. Henkin)

10:15-10:25

(9) The problem of base decidability for equational theories of algebras. Preliminary report
Mr. George F. McNulty, University of California, Berkeley (676-7)

SATURDAY, 11:00 A.M.

Invited Address, Room X 201, Xavier Hall

Geometry of normed linear spaces
Professor Robert C. James, Claremont Graduate School

SATURDAY, 2:00 P.M.

Invited Address, Room X 201, Xavier Hall

Infinitely divisible processes and their potential theory
Professor Charles J. Stone, University of California, Los Angeles

SATURDAY, 3:30 P.M.

Session on Topology, Room A 208, Administration Building

3:30-3:40

(10) On factorizations of homeomorphisms
Professor Ludvik Janos, University of Florida (676-6)

3:45-3:55

(11) Some topological properties
Mr. V. Kannan, Madurai University, India (676-1)
(Presented by Professor M. Rajagopalan)

SATURDAY, 3:30 P.M.

Second General Session, Room A 212, Administration Building

3:30-3:40

(12) On (0,1)-matrix semigroups. Preliminary report
Professor Kim Ki-Hang Butler, George Washington University and St. Mary's College of Maryland (676-4)

3:45-3:55

(13) Tait colorings on generalized Petersen graphs
Professor Frank Castagna* and Professor Geert C. E. Prins, Wayne State University (676-10)

4:00-4:10

(14) On a theorem of Renyi and extensions
Professor Peter C. -C. Wang, Stanford University (676-14)

4:15-4:25

(15) On applications and theory of a new matrix-harmonic method
Professor Demetre John Mangeron*, and Professor M.N. Oguztoreli, University of Alberta, and Mr. V. F. Poterașu, Polytechnic Institute of Jassy, Iasi, Romania (676-12)

R. S. Pierce
Associate Secretary

Seattle, Washington

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.

PRELIMINARY ANNOUNCEMENT OF MEETING

The Seventy-Fifth Summer Meeting
University of Wyoming
Laramie, Wyoming
August 25-28, 1970

The seventy-fifth summer meeting of the American Mathematical Society will be held at the University of Wyoming, Laramie, Wyoming, from Tuesday, August 25, through Friday, August 28, 1970. All sessions of the meeting will be held on the campus of the University. The times listed below for events of the meeting are MOUNTAIN DAYLIGHT SAVING TIME throughout.

The Colloquium Lectures will be given by Professor R. H. Bing of the University of Wisconsin. The title of this series of lectures is "Topology of 3-manifolds." The Colloquium Lectures will be presented at 1:00 p.m. on Tuesday, August 25, in the Auditorium of the Arts and Sciences Building, and at 9:00 a.m. on Wednesday, Thursday, and Friday, August 26-28, in Room 302 of the Classroom Building.

By invitation of the Committee to Select Hour Speakers for Annual and Summer Meetings, there will be two or three invited hour addresses at the meeting. Professor Patrick P. Billingsley of the University of Chicago will speak on Thursday, August 27, at 1:00 p.m. in the Auditorium of the Arts and Sciences Building. The title of his lecture is "Some probability results connected with Diophantine approximati0n." Professor Srinivasa S. R. Varadhan of the Courant Institute of Mathematical Sciences, New York University, will present an address entitled "Diffusion processes: a martingale approach." This lecture will be given at 1:00 p.m. on Friday, August 28, in the Auditorium of the Arts and Sciences Building. There may be another invited hour address. If so, the name of the speaker and the title of his lecture will be listed in the program of the meeting, which will appear in the August issue of these *Notices*.

The first award of the Norbert Wiener Prize in Applied Mathematics will be made at 2:15 p.m. on Tuesday, August 25, in the Auditorium of the Arts and Sciences Building. There will be numerous sessions for contributed ten-minute papers. These are tentatively scheduled at 3:15 p.m. on Tuesday, 10:15 a.m. on Wednesday, and at 10:15 a.m. and 2:15 p.m. on Thursday and Friday. Abstracts of contributed papers should be sent to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904, so as to arrive prior to the June 30 deadline. It is requested that all abstracts be submitted on the new forms containing a subject classification scheme with 64 categories. These are available from most mathematics department offices, or they can be obtained by writing to the Society headquarters.

This meeting will be held in conjunction with meetings of the Mathematical Association of America, the Institute of Mathematical Statistics, and Pi Mu Epsilon. The Mathematical Association of America will meet from Monday through Wednesday. The Association will present Professor Harry Kesten of Cornell University as the Earle Raymond Hedrick Lecturer. The title of Professor Kesten's series of lectures will be "Escapades of a random walk." The Institute of Mathematical Statistics will meet from Monday through Friday. The Wald Lectures, sponsored by the IMS, will be given by Professor Murray Rosenblatt of the University of California, San Diego. The subject of these lectures is "Topics in estimation for stationary processes." Pi Mu Epsilon will meet concurrently with the Society and the Association.

COUNCIL AND BUSINESS MEETING

The Council of the Society will meet

on Tuesday, August 25, at 5:00 p.m. in the Rendezvous Room of the Washakie Center. The Business Meeting of the Society will be held on Thursday, August 27, at 4:00 p.m. in the Auditorium of the Arts and Sciences Building.

A proposed amendment to the By-laws, approved by the Council, is to be submitted to the membership at the Business Meeting. The effects are to make the Committee to Monitor Problems in Communications be a statutory committee, with members nominated by the nominating committee and elected by the membership; to make the chairman of the committee, elected by the committee, be a member of the Council; to make the current members of the appointed committee be the initial elected members with staggered terms; and to charge the committee with performance of such tasks in the field of communication of mathematics as are assigned to it by the Council.

REGISTRATION

The Registration Desk will be in the lobby of the Washakie Center. It will be open on Sunday from 2:00 p.m. to 8:00 p.m.; on Monday from 8:00 a.m. to 5:00 p.m.; on Tuesday, Wednesday, and Thursday from 9:00 a.m. to 5:00 p.m. and on Friday from 9:00 a.m. to 1:00 p.m. The telephone number will be 307-766-4340.

The registration fees will be as follows:

Members	$5.00
Students	$ 1.00
Nonmembers	$10.00

There will be no extra charge for members of the families of registered participants.

EMPLOYMENT REGISTER

The Joint Committee on Employment Opportunities have decided not to have an Employment Register at the Laramie, Wyoming, meeting.

EXHIBITS

Book exhibits and exhibits of educational media will be displayed in the Wisconsin Room of the Washakie Center on Tuesday, Wednesday, and Thursday.

BOOK SALES

There will be no book sale at this meeting.

DORMITORY HOUSING

Rooms are available in a new dormitory complex surrounding Washakie Center, registration headquarters. Dormitories have elevators and lounges, with kitchenettes on each floor. Coin-operated washers and dryers, as well as ironing boards and irons, are available.

Rooms may be occupied from 2:00 p.m. Saturday, August 22, until Saturday noon, August 29. Bed linens, blankets, towels and soap will be provided, but maid service is not available. The rates are: $4.00 per day, per person, double occupancy, and $6.00 per day, single. No additional charge is made for cribs in the rooms, but the university cannot provide them. Cribs can be rented from Smith's Outlet, 414 South Second Street (307-742-2510) at $3.00 per week. Guests are expected to pay for their rooms at the time they check in.

Upon arrival, all guests should check in at Washakie Center to register for dormitory rooms. Turn north from Grand Avenue (U. S. 30 and Business Interstate 80) on 15th Street, and then east at the first corner on Ivinson Avenue. The registration area is on the lower level of Washakie Center. Dormitory personnel will be on duty 24 hours a day to issue keys. Student bellhops, who will accept tips, will be available if desired.

Guests must register in advance to be assured of dormitory housing, using the form provided on page of these *Notices*. Dormitory reservations will be confirmed by the housing office of the University of Wyoming. It is probable that room will be available for those who do not register in advance, but this cannot be guaranteed.

FOOD SERVICES

The dining room in Washakie Center will be open for lunch on Sunday, August 23, and will serve meals through breakfast, Saturday, August 29, except during the Buffalo Roast Wednesday evening. Serving hours are:

Breakfast	7:00 a.m.-8:30 a.m.
Lunch	11:30 a.m.-1:00 p.m.
Dinner	5:30 p.m.-6:30 p.m.

The service is cafeteria style, on a cash basis, at the following prices:

	Breakfast	Lunch	Dinner
Adults	$1.25	$1.50	$2.00
Children (4-12)	0.65	0.75	1.00

Children (under 4) no charge

The Snack Bar in Washakie Center will be open as required to meet the demand.

Coffee and doughnuts will be available from a mobile unit just outside the Classroom Building while the meetings are in session.

MOTELS

There are a number of motels and a hotel in Laramie, some of which are listed below with following coded information: FP-Free Parking; SP-Swimming Pool; AC-Air Conditioned; TV-Television; CL-Cocktail Lounge; RT-Restaurant. Participants should make their own reservations.

BRANDING IRON MOTEL (307) 742-6808
1161 North Third-20 rooms

Single	$10.00
Double	$15.00

Some units for four and five people
Code: FP-TV
16 blocks from campus

BUCKAROOMOTOR LODGE(307) 742-2865
365 North Third-16 rooms

One double bed	$10.00
Two double beds	$14.00
Rollaway beds	$ 1.50

Code: FP-TV
8 blocks from campus

CIRCLE S MOTEL (307) 745-4811
2440 Grand-48 rooms

Double	$12.50-$13.50
Twin	$15.50
Family units	$18.50-$20.00
Rollaway beds	$ 2.00
Cribs	$ 1.00

Code: FP-TV-SP-AC
14 blocks from campus

DOWNTOWN MOTEL (307) 742-6671
165 North Third-30 rooms

Single	$12.00
Twin	$17.50
Two double beds	$19.50
Rollaway beds	$ 2.00
Free cribs	

Code: FP-TV-Some AC
7 blocks from campus

GAS LITE MOTEL (307) 742-6616
960 North Third-27 rooms

Single	$12.00
Double	$14.00
Twin	$17.00-$18.00
Rollaway beds	$ 3.00
Cribs	$ 2.00

Code: FP-TV-SP-AC
15 blocks from campus

HOLIDAY INN (307) 742-6611
180-U.S. 287-68 rooms

Single	$14.50
Double	$19.00
Rollaway beds	$ 2.00
Free cribs	

Code: FP-TV-SP-AC-CL-RT
3 miles from campus

HORSESHOE MOTEL (307) 742-2107
U.S. 30 North-20 rooms

One double bed	$10.00
Two double beds	$14.00
Extra person	$ 1.00

Code: FP-RT-TV-CL
3 miles from campus

LARAMIE TRAVELODGE (307) 745-4853
262 North Third-29 rooms

Single	$12.00
Double	$14.00
Twin	$16.00
Rollaway beds	$ 2.00
Free cribs	

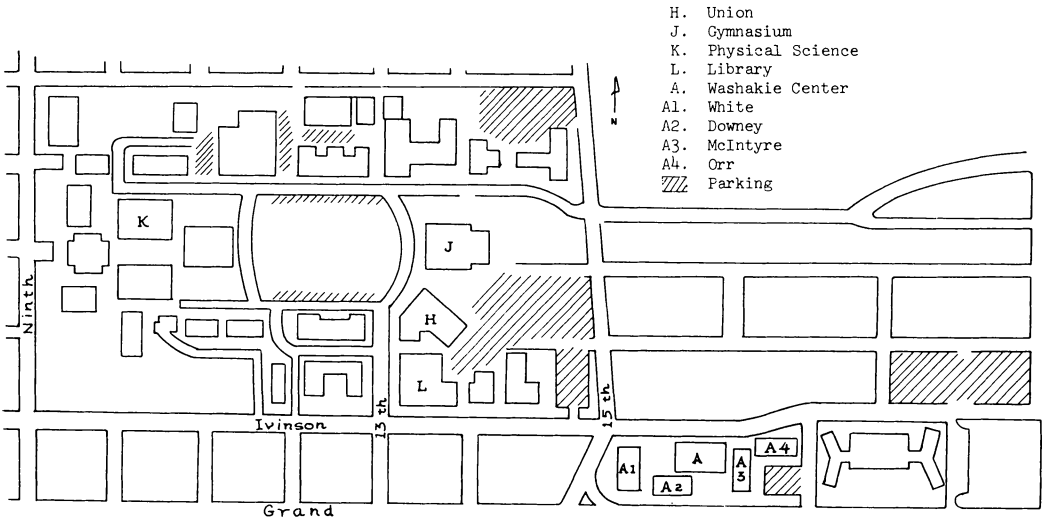
Code: FP-SP-TV-AC
8 blocks from campus

LAZY U MOTEL (307) 745-7322
1622 Grand-13 rooms

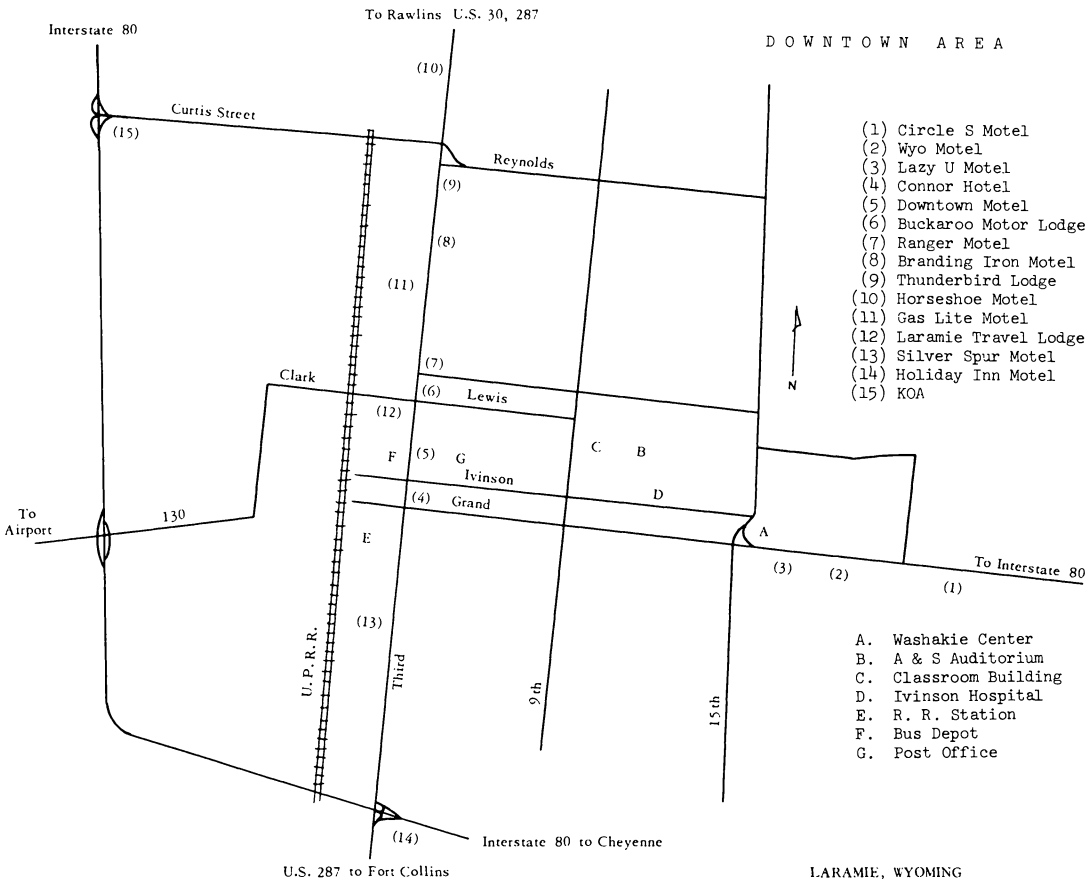
One double bed	$10.00
Two double beds	$14.00

Code: FP-TV
4 blocks from campus

CAMPUS AREA



DOWNTOWN AREA



LARAMIE, WYOMING

RANGER MOTEL (307) 742-6677
453 North Third-40 rooms

One double bed	$14.00
Twin	$16.00
Two double beds	$20.00
Cribs	$ 1.00

Code: FP-SP-TV-AC
10 blocks from campus

SILVER SPUR MOTEL (307) 742-3741
1104 South Third-24 rooms

One double bed	$12.00
Two double beds	$14.00-$18.00
Rollaway beds	$ 1.00

Code: FP-TV-Some AC
17 blocks from campus

THUNDERBIRD LODGE (307) 745-4871
1369 North Third-18 rooms

Double	$12.00
Twin	$14.00
Two double beds	$15.00-$16.00
Free cribs	

Code: FP-SP-TV-AC
19 blocks from campus

WYO MOTEL (307) 742-6633
1720 Grand-36 rooms

Single	$14.00
Double	$15.00
Rollaway beds	$ 3.00
Cribs	$ 1.00

Code: FP-SP-TV-AC
5 blocks from campus

PARKING

No permits for on-campus parking will be needed, and courtesy permits will be issued for downtown parking.

CAMPING

There is a Kampgrounds of America (privately-owned campground) at the Curtis Street exit of Interstate 80. There are four campgrounds 10 to 16 miles east of Laramie on Interstate 80 with a total of 87 campsites. There are four campgrounds located 30 to 40 miles west of Laramie on Wyoming 130 with a total of 66 campsites. Numerous other campgrounds are located further west in the Medicine Bow National Forest and south of Laramie in Northern Colorado.

For more detailed information on camping and facilities write:

Wyoming Travel Commission
2320 Capitol Avenue
Cheyenne, Wyoming 82001

BOOKSTORE

The University Bookstore, located in the Union, will probably be open Monday through Friday from 7:30 a.m. to 5:00 p.m. It sells souvenirs and personal articles as well as books. Two downtown bookstores are the West Book Store at 210 South Third and Books A Go Go at 408 University Avenue.

LIBRARIES

The University Library will be open Monday through Friday from 8:00 a.m. to 5:00 p.m. The Albany County Library at 405 Grand is open from noon to 9:00 p.m., Monday through Thursday and from noon to 6:00 p.m. on Friday.

MEDICAL SERVICES

Ivinson Memorial Hospital (phone 742-2141) at 1014 Ivinson Avenue (adjacent to the campus) is an 84 bed general hospital with an intensive care unit. A doctor is always on emergency call.

ENTERTAINMENT

There will be a Buffalo Roast on Wednesday at 5:30 p.m. on the mall north of the dormitories and west of the fieldhouse. In case of bad weather it will be held in Washakie Center. Prices are $2.75 for adults, and $1.25 for children from 4 to 12, with no charge for children under 4.

Tentative plans have been made for two trips--a Snowy Range trip leaving Washakie Center Tuesday at 8:45 a.m. and a historical bus tour, accompanied by Mr. Neal Miller, director of the State Archives and History Department, leaving Washakie Center at 8:45 a.m. Thursday. This trip will include a visit to the Sybille Game Refuge, and Fort Laramie National Monument. Bus fare is about $7.00. Washakie Center will fix box lunches for $1.25.

A guided walking tour of the University flower gardens will be led by Mr. Otto Dahl, University Horticulturist, leaving Washakie Center at 10:00 a.m. Wednesday.

Entertaining movies will be shown

in the Classroom Building at 7:00 p.m. on Monday, Tuesday, and Wednesday.

If there is sufficient interest, climbing expeditions in the Snowy Range or at Vedauwoo can be arranged.

Other diversions include: Geological Museum, the University Planetarium, the Laramie Plains Museum, the Hebard Western History Room in the University Library, campus golf and tennis courts, gymnasium facilities and equipment, enclosed city swimming pool, horse-back riding, fishing, and picnicking.

Parents can make arrangements to leave their children during the day at the Jack and Jill Nursery and Day School, 416 Hancock St., phone 307-745-7985. A list of baby sitters for evening hours will be available.

TRAVEL

During the summer, Laramie is on MOUNTAIN DAYLIGHT SAVING TIME.

Transportation to Laramie is available via regional airline, bus and train. Two major U. S. highways, U. S. 287 and Interstate 80, provide access by private auto. Frontier Airlines services Laramie with connections from Denver, Casper or Billings. Continental Trailways, Greyhound, and Colorado Motor Ways make scheduled stops in Laramie. Direct bus service from Denver to Laramie is available at the Denver Greyhound terminal. Union Pacific Railroad has passenger train service to Laramie from the east and west. Three or more people traveling together would find it cheaper to rent a car in Denver than to pay air fare. Rental cars available in Laramie are limited so that prior reservations would be advisable through Avis Rent-A-Car, 501 Garfield; Hertz Rent-A-Car, 401 Lewis; and National Car Rental System, 666 North Third, Laramie, Wyoming 82070.

National Car Rental System, 666 North Third, Laramie, Wyoming 82070.

WEATHER

The average maximum temperature during this week is 73° and the average minimum is 45°. Typically, mornings are sunny while afternoons may have some high cloudiness, possible light showers, and occasional gusty winds. Although highly improbable, snow is not an impossibility. Guests are advised to be prepared for cool weather.

ADDRESS FOR MAIL AND TELEGRAMS

Individuals may be addressed at Mathematics Meetings, Washakie Center, University of Wyoming, Laramie, Wyoming 82070. The telephone number of the Message Center will be 307-766-6422. This number will have two separate lines but will have a rotary on it so if one line is busy it will automatically place the call on the other line.

COMMITTEE

H. L. Alder, ex officio, R. M. Cooper, Rev. F.T. Daly, G.C. Gastl, J.H. George, Mrs. J.H. George, W.C. Guenther, J.R. Hanna, T.L. Jenkins, J.E. Kirk, Jr., R.W. McKelvey, R.S. Pierce, ex officio, A.D. Porter, J.H. Rowland, V.M. Sehgal, W.N. Smith, Chairman, P.O. Steen, J.W. Thomas, G.L. Walker, ex officio, Laurence Weinberg.

R. S. Pierce
Associate Secretary

Seattle, Washington

NOTE ON THE JOB MARKET

Reese T. Prosser

This is not a good year for mathematicians looking for a job. Applicants attending the recent winter meeting of the A.M.S. found that research positions available at major universities were scarce and the competition for them was ferocious. Teaching positions at lesser known colleges were in heavy demand, and institutions begging for applicants five years ago could pick and choose. Moreover, this year's graduates found themselves competing with those of previous years who had accepted temporary or undesirable offers and were now trying to improve their standing. Widespread rumors asserted that some departments counted over a hundred applicants for each available position.

The picture is no better in industry. Continued reductions in government support of research programs, together with the general disillusion of corporation management with the efficacy of pure mathematics, have shriveled the industrial market to the vanishing point.

What has become of the celebrated "critical shortage" of mathematicians? Has it already been superceded by a critical shortage of jobs? What can we expect in the future? Here is a brief review of the economics of the situation, intended only to provide a basis for further discussions. The figures presented here are based on the COSRIMS report entitled The Mathematical Sciences (NAS, 1968), together with revisions developed for a joint meeting of CGPE, CUE, and COSRIMS held May 27, 1969. The responsibility for their use here, however, is entirely my own, and cannot be attributed to any of these committees. A more carefully documented study will soon appear in an article by R. D. Anderson entitled "Are there too many Ph. D.'s?"

SUPPLY. On the basis of past experience and present estimates, we can expect that over the next five year period the mathematics departments in this country will produce nearly 10,000 new Ph. D.'s, including the 1200 graduating this year. Of these, 70% will want to go into teaching, 10% into industry, and 20% elsewhere.

DEMAND. On the same basis, we can expect that, over the next five year period, the universities and four-year colleges of the country will require around 3000 new conventional mathematics teachers. Of these, 1500 will serve as replacements, and 1500 as additions to the present mathematics departments.

In addition, these institutions will require an indeterminate number of new teachers to staff new programs in related fields. The number will depend essentially on how successfully these new programs can be designed, organized, and implemented.

If these figures are anywhere near the truth, then it is clear that we are facing a growing problem. The major universities for the most part have already completed their recent expansion programs and are now fully staffed. The four-year colleges are now in the process of upgrading their departments by adding new Ph. D.'s, and it seems likely that many of these positions will be filled within the next three years. By then we shall be producing nearly 2000 Ph. D.'s each year who will be competing for perhaps 500 conventional teaching positions.

What can we do? Here are some obvious alternatives:

LAISSEZ FAIRE. If we do nothing, then we can expect that by 1975 well over half of our Ph. D.'s will probably have to accept positions in some other field.

CURTAIL THE SUPPLY. Some universities are already considering reducing the size of their graduate programs, particularly in view of the fact that the government is already reducing its support. This may well be the best long-term solution for many institutions. It is no solution, however, for the projected 10,000 Ph. D.'s who are already " in the pipe" as graduate students.

INCREASE THE DEMAND. It may

be possible to create new positions, particularly in new programs, in interdisciplinary and related fields. Some optimistic estimates indicate that over the next five years we may be able to create as many as 6000 new positions in such fields as computer science, biological science, social science and management science. These estimates presuppose an aggressive sales campaign necessary to convince administrators, faculty and students of the market value of these new fields.

It may also be possible to create new positions in the two-year colleges and other disadvantaged institutions across the country. In view of the generally low salaries and heavy teaching loads at such institutions, however, this possibility probably requires massive federal support.

REDESIGN THE PRODUCT. Most of our present doctoral programs are now designed to produce specialized research mathematicians. Yet it is by no means clear that the country can support 2000 new research mathematicians each year by 1975. It is already apparent that the graduate with a broad background in modern mathematics, a strong interest and sound training in undergraduate teaching, and a nodding acquaintance with statistics, computers, or graphs, has a formidable advantage over the graduate with a single research specialty, no matter how good he is, in today's market. Any new position in an interdisciplinary or related field will require a flexibility of approach and a willingness to learn which we are now ruthlessly exterminating in our students.

All this suggests that it may be high time to review our doctoral programs to see how they can be made more responsive to the coming need to match supply and demand. It is no good baking cakes if pies are wanted.

Like all predictions of the future, this one is based on an extrapolation from the present under the canonical assumption that we'll have more of the same. Whatever happens, however, we can be pretty sure that it won't be more of the same, so that all of the above argument may well be nonsense. Nevertheless, it seems to me that we are now facing, not a shortage of jobs, but a mismatch of talents, which threatens to become serious if we don't do something substantive soon.

Editors' Note: The article by Professor Anderson referred to will appear in the June-July (vol. 77, no. 6) issue of the American Mathematical Monthly. The Society, of course, has also been very concerned with this problem, and a committee consisting of Professors R. D. Anderson, W. Duren (chairman), and G. Young is working on the problems raised by the preceding article. The membership is urged to communicate any ideas or opinions to the chairman of this committee.

LETTERS TO THE EDITOR

Editor, the *Notices*

I am writing concerning editorial policies of Math Reviews. For many years I have carefully read two sections of Math Reviews and I have been vaguely aware of a distinct difference in the coverage. The difference is in the number of papers without reviews or with unsigned extracts from the paper. I recently investigated this further with three recent issues of Math Reviews. Six topics, 3 "pure" and 3 "applied" were chosen and the reviews classified as tabulated below:

TOPIC	Signed Reviews	Unsigned Extracts	Title only
Category Theory	32	0	0
Algebraic Topology	58	1*	0
Approximations and Expansions	67	0	1**
	157 = 99%		
Applied Math (1st 3 sections)	43	8	1
Information Communication	65	37	3
Numerical Mathematics	139	33	6
	247 = 74%		

*paper on computing, **paper in Chinese

It appears that the editors of Math Reviews have adopted a policy of only partial reviewing for "applied" topics.

This extensive use of extracts raises the question of whether the review should summarize or evaluate a paper. The current editorial policy favors summaries. While I do not favor this policy, it at least allows us to look forward to the day when authors and referees have the responsibility for the summary.

I also object to the editor's policy of modifying or, worse, suppressing reviews. Specifically, I wrote the following review several years ago:

Editor, the *Notices*

"Contrary to the implications of the abstract, this paper presents some well-known results, some obvious observations and some uninteresting examples".

The paper's title was listed in Math Reviews without a review and without any consultation with me. One does not write such a review without considerable thought --after all, one is making a lifelong enemy! This is not the only case of suppression of which I am aware.

I believe that these policies do not reflect the wishes of the membership of the American Mathematical Society. I hope that the Council and membership of the Society will seriously consider giving new policy directives to the editors of the Math Reviews.

John R. Rice
Purdue University

Postscript: The executive editor has informed me that current editorial policy requires reviews to be "informative" and "sufficiently objective to preclude demands for equal time answers". This policy is then interpreted so as to uniformly exclude critical remarks. Since the mathematical world runs on critical evaluations of mathematical work, it seems unreasonable to me that Math Reviews should exclude critical remarks. This is apparently because such remarks might possibly be unfounded, yet I feel that the lack of perfect reviewers should not rule out this valuable type of review.

In spite of the above stated policy, the editors of Math Reviews engage in critical evaluations in an insidious way. The executive editor also informs me that papers not given signed reviews are judged by the editors not to be "reviewable". This then presumably means they are lacking in mathematical content and/or interest. It seems to me that to classify a paper in this group is the strongest criticism that one can make, clearly contradictory to the policy above stated. I know that many authors of papers that are so classified firmly believe their papers to have real mathematical content and interest. If one of the editors decides that a paper is unre-

viewable, he should be required to sign his name to this evaluation.

Editor, the *Notices*

The following two formal policy statements, adopted in 1967 and 1968 by earlier Boards of Editors for the Mathematical Reviews, reflect very closely the attitudes of the present Board towards coverage of "applied mathematics" and "critical reviews".

POLICY ON COVERAGE OF APPLIED MATHEMATICS (1967).

1. In order that an article on applied mathematics (say quantum mechanics) be reviewable, it should qualify for inclusion in one of the sections on pure mathematics (say group theory). If it does, its review should appear in the appropriate applied section.
2. In testing its qualifications for inclusion in a pure mathematics section, some consideration ought to be given to the probability that the article contains any new mathematics.
3. If an article contains new mathematics, or gives an outstanding exposition of known mathematical methods, or applies known mathematics in a striking new way, then it ought to be reviewed. Routine applications (no matter how arduous) of known mathematics (no matter how sophisticated) should not ordinarily be reviewed.

POLICY ON CRITICAL REVIEWS (1968).

A reviewer should be allowed to be critical (i.e., to use unpleasant adjectives) if the resulting review satisfies two conditions: (1) It is informative, and (2) it is expressed in terms sufficiently objective to preclude demands for "equal time" answers (which never have been allowed, and are still firmly against Board policy).

F. W. Gehring
O. Goldman
R. S. Pierce

Board of Editors
Mathematical Reviews

Editor, the *Notices*

I am writing in connection with the letter of Professor Rice and the statements of policy of the Board of Editors printed above.

The editors of MR attempt to follow the policy of the Board with regard to applied mathematics at all times. Probably the most important factor leading to the use of editorial (unsigned) reviews in the applied areas is the lack of mathematicians (as opposed to physicists) willing to evaluate the mathematical contributions of the articles in question, especially when this mathematical aspect may be either slight or well camouflaged, or both. Clearly in these cases the fact of an unsigned review does not necessarily reflect on the content of the work, mathematical or otherwise. (It should be remarked that most of these articles are considered more thoroughly in another reviewing (or abstracting) journal.)

There are numerous other reasons for the use of unsigned reviews (and these apply to all fields, although not equally). For example: (1) Use of author's summary or title is often advised by the reviewer to whom the article is assigned. (2) There may simply not be available an appropriate reviewer--appropriate with regard to field and/or language. (3) On occasions we simply cannot obtain a review by any other means (i.e., either from one of our reviewers or from another reviewing journal). Other factors include availability of an appropriate author's summary (e.g., in many Romanian journals and in the fields of operations research, information, automata); expository or preliminary nature of the work--although most such articles receive signed reviews; excessive age of a work--only recently obtained and reviewed primarily in the name of completeness.

From the above it should be clear that all articles mentioned in any form in MR are "reviewable", and that anything considered "unreviewable" is not included.

The remark in Professor Rice's postscript concerning critical reviews is the result of a misunderstanding, since many highly critical reviews satisfying the requirements of the Board's policy are published.

R. J. Crittenden, Executive Editor
Mathematical Reviews

INSTRUCTIONS FOR AUTHORS OF PAPERS FOR AMERICAN MATHEMATICAL SOCIETY JOURNALS

Each article submitted for publication in the Bulletin, Proceedings, Memoirs, Transactions, and Mathematics of Computation must be accompanied by a descriptive title, AMS subject classification numbers, and a list of key words and phrases. In addition, papers for the Proceedings, Transactions, Memoirs, and Mathematics of Computation must be accompanied by an abstract. The completeness and accuracy of these items will be taken into account in the refereeing and editorial process.

I. DESCRIPTIVE TITLE

Make your title as informative as possible. It must clearly identify both the general field of the paper and the particular branch of it under consideration. The title must not run to more than 10 or 12 words, and fewer words are better. Avoid jargonistic words that tell nothing and waste space, such as "concerning," "a remark about," "some contributions to the theory of." Also avoid proper names unless mathematical usage associates them with the work. An example of a nondescriptive title: "Concerning some applications of a theorem of John Doe." Note that titles may be used in information retrieval; therefore, every word in them counts. (See Section 2, paragraph 2 of "Manual for Authors.")

II. AMS SUBJECT CLASSIFICATION NUMBERS

These numbers classify the paper by field. They will be used both for information retrieval and for the Mathematical Offprint Service. In the latter, they direct your paper to subscribers; therefore, by assigning these numbers properly, you make sure that people interested in your work will see it. The numbers will be printed in a footnote on the first page of your article. A list of the numbers may be found in the Index to Volume 39 of Mathematical Reviews, June 1970. Be sure to use the AMS (MOS) subject classification numbers and not the MR subject classification numbers that will appear in the same volume. You should give numbers designating secondary as well as primary subject fields of your paper if they are applicable. Keep in mind that several primary (and secondary) numbers probably should be assigned. Since the classification scheme is likely to change with time, be sure to indicate the date of the scheme you are using.

III. KEY WORDS AND PHRASES

For the sake of nonspecialists (such as librarians), as well as to aid in future information retrieval, a list of key words and phrases must also be included. The list should be generous; however, it should not include words which would appear in almost any paper in the general field. Thus, "cohomology group" could be appropriate for a paper in Lie algebras, but not for a paper in algebraic topology.

Style. Use nouns, adjective-nouns, etc. in the natural groupings in which they occur in the paper, e.g. differential form, locally compact groups, deformations of algebras. Proper adjectives are especially useful in pinpointing the subject matter, e.g. Sobolev space, nondesarguesian planes. Reduce compounds to their components so that "convergence in mean or measure" becomes "convergence in mean, convergence in measure." The phrases and words should be taken from the body of the paper, the title, and the abstract. Avoid long phrases. In general, an item should have four words or less. Use as many items as you need to distinguish the field of the paper clearly and specifically.

Format for subject numbers and key words. These will be printed as a footnote to page one of the article. Include them, therefore, with the footnotes to your paper, but placed before the first footnote.

AMS 1970 subject classification. Primary 13G05; Secondary 13G05.

Key phrases: analytically unramified ring, semi-local ring, Dedekind domain, altitude formula, Jacobson radical,

Rees ring, analytically irreducible domain, unmixed domain.

1. This research was supported in part by the Ring-theory Foundation, under contract no. 25-35-7002.

2. . . . , etc.

IV. ABSTRACT

(Proceedings, Transactions, Memoirs, Mathematics of Computation only)

The abstract will be printed right after the title, in a different type size to separate it from the rest of the paper. The main purpose of the abstract is to enable readers to take in the nature and results of the article quickly, and to enable them to decide whether they wish to read further. Zentralblatt now publishes author's abstracts instead of reviews, so the abstract may also appear there. Of course, the abstract will also aid in retrieving information about your paper.

Length. For the Proceedings, an abstract should be at least one sentence, and at most 150 words; for the Transactions and Memoirs, at least one sentence, and at most 300 words; and for Mathematics of Computation, the abstract should be brief and reasonably self-contained. The length will depend primarily on the length of the paper itself so that the upper limits placed on abstracts for the Proceedings, Transactions, and Memoirs are meant for long papers only. To some extent, the difficulty of summarizing the material also determines the length.

Format. The abstract should be typed, double-spaced, on a separate page, and it should be placed at the front of the manuscript. Include the title.

Content. Try to state the object of the work, summarize the results, and give the principal conclusions as briefly as possible.

Style. Use full sentences. Avoid technicalities, since the abstract should be readable by anyone in the general field (analysis, algebra, etc.). Formulas are not desirable, but they may be included, and even numbered, if it seems best to do so. Do not cite bibliographic references, since the abstract should be able to stand alone. Similarly, do not refer by number to some theorem or formula in the body of the article. Incorporate the statements of theorems into complete sentences. "We prove that all hyperloops are quasi-regular" is preferable to "Theorem. All hyperloops are quasi-regular."

Relationship to introduction. Occasionally an abstract will make a further introduction unnecessary, and in that case the paper can proceed at once with the mathematics. Usually, however, the abstract will be too brief, and the introduction should enlarge upon it, provide more background, and describe earlier work.

EXAMPLES

D. O. BANKS and G. J. KUROWSKI, Computation of eigenvalues of singular Sturm-Liouville systems, Math. Comp. 22 (1968), 304-310.

Abstract. In recent papers, P. B. Bailey and M. Godart have used the Prüfer transformation to calculate the eigenvalues of nonsingular and some singular Sturm-Liouville boundary value problems. In the present paper, the authors establish the existence of a general class of a singular problem which may be solved in a straightforward manner using the Prüfer transformation. Some examples of the method are given. Finally, the class of problems to which the method is applicable is extended by introducing a modified transformation.

AMS 1970 subject classification. Primary 65L10, 65L15; Secondary 34B25.

Key words and phrases. Computation of eigenvalues, Sturm-Liouville systems, boundary value problems, Prüfer transformation, Newton-Raphson approximation method, Runge-Kutta fourth-order method, predictor-corrector method, Adams extrapolation method, Adams interpolation method.

J. L. SYNGE, Jets of radiation, Quart. Appl. Math. 26(1968), 153-165.

Abstract. A model source is constructed, the radiation from which (scalar, electromagnetic, or gravitational) is concentrated in a jet of small angle with an assigned target direction. This source is taken to be an infinite train of high-fre-

quency plane waves travelling in the target direction with the basic speed of propagation, the amplitude falling off exponentially with distance. The critical number, to be made large in this model, is the ratio a/λ, where a is a typical radius of the source and λ is the wave length. It is also shown that a jet of scalar radiation may be obtained from a source that possesses no frequency but consists of a single shock wave.

AMS 1970 subject classification. Primary 76Q05, 78A40, 83C35; Secondary 35L05.

Key words and phrases. Scalar radiation, electromagnetic radiation, gravitational radiation, plane wave, shock wave, space time, Ricci tensor, shock-jets.

A. SEIDENBERG, Derivations and integral closure, Pacific J. Math. 16(1966), 167-173.

Abstract. Let σ be an integral domain containing the rational numbers, Σ its quotient field, D a derivation of Σ, and σ' the ring of elements in Σ-quasi-integral over σ. It is shown that if $D\sigma \subset \sigma$, then $D\sigma' \subset \sigma'$.

AMS 1970 subject classification. Primary 13B10, 13B20, 13J05; Secondary 13E05.

Key words and phrases. Derivation, formal power series, integrally closed domain, quasi-integral dependence, Hasse-Schmidt differentiations, Noetherian domain.

PERSONAL ITEMS

Mr. JAYME MACHADO CARDOSO of the Federal University of Parana, Brazil, has been appointed to a professorship at the University of Campinas, Brazil.

Mr. DAVID M. CLARK of Georgia Institute of Technology has been appointed to an assistant professorship at the State University of New York, College at New Paltz.

Professor SPENCER E. DICKSON of Iowa State University of Science and Technology has been appointed to a visiting professorship at the University of Newcastle, New South Wales, Australia, from May 10 to August 31, 1970.

Mr. JOHN E. DONER of System Development Corporation has been appointed to an assistant professorship at Purdue University.

Professor RONALD G. DOUGLAS of the University of Michigan has been appointed to a professorship at the State University of New York at Stony Brook.

Dr. ALVIN N. FELDZAMEN, Director of Production at the Encyclopaedia Britannica Educational Corporation, has been elected to serve as Editorial Vice-President as well.

Dr. THEODORE E. FROST of the University of Wisconsin has been appointed to an assistant professorship at the University of Redlands.

Professor JAMES K. HIGHTOWER of Claremont Graduate College has been appointed to an associate professorship at California State College at Fullerton.

Dr. BILLY F. HOBBS of Olivet Nazarene College has been appointed to a professorship at Pasadena College.

Professor SYED A. HUQ, visiting at the University of Montreal, has been appointed a research member in the Institute for Advanced Studies of the Australian National University at Canberra.

Professor RONALD JACOBOWITZ of the University of Kansas has been appointed to a professorship at Arizona State University.

Dr. ROBERT R. KORFHAGE of Purdue University has been appointed to a professorship and has been named Director of the Computer Sciences-Operations Research Center at Southern Methodist University.

Mr. KING T. KWOK of Storrs, Connecticut, has been appointed to an assistant professorship at Southern Connecticut State College.

Professor JEAN PIERRE LAFON of the University of Toulouse, France, is a visiting lecturer at Mount Holyoke College for the 1969-1970 second semester.

Mrs. MARIJO O. LEVAN of Southeast Missouri State College has been appointed to an associate professorship at Eastern Kentucky University.

Mrs. HARRIET S. POLLATSEK of the University of Michigan has been appointed to an assistant professorship at Mount Holyoke College.

Professor JOHN A. RILEY of Lowell Technological Institute has been appointed to a professorship at the State University of New York, College at Plattsburgh.

Professor PHILLIP SCHULTZ of the University of Montana has been appointed a lecturer at the University of Western Australia, Nedlands, West Australia.

Dr. ALVIN D. WIGGINS of the University of California, Berkeley has been appointed to an assistant professorship at the University of California, Davis.

Professor JAMES S. W. WONG of Carnegie-Mellon University has been appointed to a professorship at the University of Iowa.

PROMOTIONS

To Professor. Kyoto University, Kyoto, Japan: SATOSHI SUZUKI; State University of New York at Albany: NURA D. TURNER; Texas A & M University: JOSEPH J. MALONE, JR.; University of Ottawa: ROOP N. KESARWANI.

To Associate Professor. Ohio University: KLAUS E. ELDRIDGE, HARI SHANKAR.

INSTRUCTORSHIP

Westminster College: HOWARD CHRISTENSEN.

DEATHS

Professor EZRA J. CAMP of Macalester College died on February 4, 1970, at the age of 63. He was a member of the Society for 35 years.

Mr. ROBERT H. CANTOR of Philadelphia, Pennsylvania, died on February 11, 1970, at the age of 33. He was a member of the Society for 6 years.

Mr. RANDALL M. CONKLING of New Mexico Highlands University died on July 6, 1969, at the age of 48. He was a member of the Society for 19 years.

Dr. TOMLINSON FORT of the University of South Carolina died on January 13, 1970, at the age of 84. He was a member of the Society for 40 years.

Dr. WALTER HOFFMAN of Wayne State University died in December 1969 at the age of 47. He was a member of the Society for 15 years.

Mr. C. D. JONES of Franklin, Michigan, died on December 25, 1969, at the age of 64. He was a member of the Society for 37 years.

Dr. JOSEPHINE J. MEHLBERG of the University of Chicago died on May 26, 1969, at the age of 54. She was a member of the Society for 11 years.

Dr. LEO MOSER of the University of Alberta died on February 9, 1970, at the age of 48. He was a member of the Society for 22 years.

Professor FEDERICO M. SIDSON of Iowa State University died on September 2, 1969, at the age of 40. He was a member of the Society for 18 years.

Mr. GEORGE T. VANCE of Clinton, Mississippi, died on August 23, 1969, at the age of 29. He was a member of the Society for 5 years.

Mr. CLARENCE R. WHITE of Fort Valley State College died on November 1, 1969, at the age of 62. He was a member of the Society for 19 years.

NEW AMS PUBLICATIONS

MEMOIRS OF THE AMERICAN MATHEMATICAL SOCIETY

Number 100

HOMOTOPY INVARIANTS IN DIFFERENTIAL GEOMETRY
By Tadashi Nagano

44 pages; List Price $1.60; Member Price $1.20

This Memoir is aimed at providing differential geometers with the fruits of algebraic topology. First, the de Rham theorem is proven for the relative cohomology groups of compact manifolds (with or without boundary) modulo compact submanifolds, based on the Morse theory. Then, the Thom class for an oriented (metric) vector bundle is given an explicit expression with a differential form. In the same spirit the Poincaré duality, the intersection number, the Euler class, and the Lefschetz number are discussed. Applications include localization theorems for the Euler class.

TRANSLATIONS OF MATHEMATICAL MONOGRAPHS

Volume 24

THEORY AND APPLICATIONS OF VOLTERRA OPERATORS IN HILBERT SPACE
By I. C. Gohberg and M. G. Kreĭn

440 pages; List Price $31.10; Member Price $23.32

This book can be considered as a second volume to Translations of Mathematical Monographs, Volume 18, "Introduction to the theory of linear nonselfadjoint operators." The authors point out that, while the present volume can be read independently, the content is essentially inseparable from that of Volume 18. During the period between the appearance of the two volumes, much progress was made in the theory of nonselfadjoint operators with continuous spectra. In fact, the abstract theory of triangular integration--the basis of the present volume--came into existence after 1959. The authors have set themselves the task of showing the results of these methods and the ideas in these new investigations, and have taken this opportunity to show unexpected and fruitful connections of the new theory with questions from classical analysis.

AMERICAN MATHEMATICAL SOCIETY TRANSLATIONS—SERIES II

Volume 88

TWELVE PAPERS ON REAL AND COMPLEX FUNCTION THEORY

330 pages; List Price $16.50; Member Price $12.38

Some properties of differentiable functions defined on an n-dimensional open set, S. M. Nikol'skiĭ; On uniform convergence of convex functions in a closed domain, I. Ja. Guberman; Inequalities with convex functions, E. K. Godunova; On systems of inequalities with convex functions in the left sides, I. I. Eremin; On coefficients of bounded univalent functions, M. I. Red'kov; On some extremal problems in the theory of univalent functions, Ja. S. Mirošničenko; An integral with respect to a semiadditive measure and its application to the theory of entire functions. I-IV, A. A. Gol'dberg; On the connection vetween the growth of the maximum modulus of an entire function and the moduli of the coefficients of its power series expansion, M.N. Seremeta; Growth of entire functions of several complex variables, L. I. Ronkin.

MEMORANDA TO MEMBERS

LETTERS TO THE EDITOR

During the past few months, the editors have received two good letters from unidentified writers. Neither has been published. The editors prefer that letters be signed and think that a letter gains force from being signed. They would consider publishing a letter over a pen name provided the writer identified himself in a covering letter. Letters from unidentified writers will ordinarily not be published.

LISTS OF PARTICIPANTS IN SYMPOSIA

A list of participants in the symposium on Computers in Algebra and Number Theory, held during the 673rd meeting of the Society in New York City in March 1970, and a list of participants in the symposium on Representation Theory of Finite Groups and Related Topics, held in conjunction with the 674th meeting of the Society in Madison, Wisconsin, in April 1970, may be obtained on request for a nominal charge of $1 to cover typing, copying, and mailing costs. These lists are being offered by the Society, as a continuation of the previously announced pilot experiment, to aid in communication among workers in areas of active interest in the mathematical sciences. Requests should be accompanied by a check or money order and should be addressed to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904.

ACTIVITIES OF OTHER ASSOCIATIONS

SOCIETY FOR INDUSTRIAL AND APPLIED MATHEMATICS 1970 FALL MEETING

The Society for Industrial and Applied Mathematics will hold the 1970 Fall Meeting at the Somerset Hotel, Boston, Massachusetts, on October 12-14, 1970. The program will include symposia on Scientific Computing, Nonlinear and Dispersive Waves and Singular Perturbations. Invited participants include D. J. Benney, Massachusetts Institute of Technology; F. P. Bretherton, Johns Hopkins University; J. D. Cole, University of California, Los Angeles; B. Coppi, Massachusetts Institute of Technology; W. Eckhaus, Techniche Hogeschool, Delft, Holland; C. A. Felippa, The Boeing Company; G. Fix, Harvard University; H. B. Keller, California Institute of Technology; M. D. Kruskal, Princeton University; R. E. O'Malley, Jr., New York University; S.V. Parter, University of Wisconsin; and W.C. Rheinboldt, University of Maryland. A panel discussion on Problems in Education in Applied Mathematics is planned, and the participants will be G. Birkhoff, Harvard University; T. E. Hull, University of Toronto; W. J. Jameson, Jr., Collins Radio Company; J. P. LaSalle, Brown University; and W. C. Rheinboldt, University of Maryland. The 1970 John von Neumann Lecture will be given at this meeting. Co-chairmen of the conference are G. Birkhoff and C. C. Lin. The Program Committee includes D. Anderson, G. Birkhoff, G. F. Carrier, R.C. DiPrima (chairman), G. Fix, C. C. Lin, and R. E. O'Malley, Jr.

SIAM standard abstract forms and further information on the meeting may be obtained by writing to the Society for Industrial and Applied Mathematics, 33 South 17th Street, Philadelphia, Pennsylvania 19103.

NEWS ITEMS AND ANNOUNCEMENTS

LECTURES IN APPLIED MATHEMATICS

The Society has obtained from John Wiley & Sons, Inc. their complete stock of three volumes of the Lectures in Applied Mathematics published in 1959. These books constitute three of the four volumes of the proceedings of the first summer seminar in applied mathematics organized by the AMS and held at Boulder, Colorado, in the summer of 1957.

Volume IA, Probability and Related Topics in Physical Sciences, by Mark Kac. xiii + 266 pp. List price $10.10; member price $7.58.

This book is an expanded version of 12 lectures delivered at the seminar. The volume retains the informality of style of the original lecture notes. It consists of four chapters written by Mark Kac, plus four appendices contributed by G.E. Uhlenbeck, A. R. Hibbs, and Balth van der Pol. The first chapter is titled "Nature of Probabilistic Reasoning" in which Kac considers a variety of problems arising in a number of fields to illustrate the power of probabilistic reasoning. The second chapter, "Some Tools and Techniques of Probability Theory," is concerned with a set of random walk problems. The author states that the third chapter, "Probability in Some Problems of Classical Statistical Mechanics," is a running commentary on some of the points raised in the first lecture of Uhlenbeck, and that all of the chapter has been directly influenced by discussions and correspondence with Uhlenbeck. The fourth chapter is concerned with some of the fundamental ideas introduced by Norbert Wiener and R. P. Feynman. The reviewer, M. Rosenblatt, comments in Mathematical Reviews that "The book is a stimulating addition to the literature on probability theory."

Volume IIA, Lectures in Fluid Mechanics, by Sidney Goldstein. xvi + 309 pp. List price $12.20; member price $9.15.

The author presented 12 lectures at the seminar, and this volume is the result of those lectures, plus several special lectures by J. M. Burgers. The book is divided roughly into three parts. The first part is devoted to a discussion of the equations governing the motion of fluids and of topics directly related to them; the second part is concerned with certain classes of solutions of the equations of fluid motion; and the third part offers an introduction to four topics in magnetohydrodynamics. The two special lectures of Professor Burgers give an excellent account of his work.

Volume IIIA, Partial Differential Equations, by Lipman Bers, Martin Schechter and Fritz John. ix + 334 pp. List price $12.50; member price $9.38.

This volume includes the original lecture notes, the edited notes, or the expanded notes of the lectures presented at the seminar . Lecturers included Lipman Bers, Martin Schechter, Fritz John, Lars Gårding, and the late A. N. Milgram to whom the volume is dedicated. The book is neither a textbook for beginners, nor a reference book for specialists. It is aimed, as the oral presentation was, at mathematicians not necessarily familiar with the field of partial differential equations who want to acquire an understanding of some of the problems and methods in this discipline.

NSF GRADUATE FELLOWSHIPS

The National Science Foundation has announced the awarding of 1,941 Graduate Fellowships for full-time study in the sciences, mathematics, and engineering in the academic year 1970-1971. More than 7,560 students competed for the NSF Graduate Fellowships which were awarded on the basis of merit. Panels of outstanding scientists, appointed by the National Research Council, reviewed each application; selections were made by the Foundation. Among those receiving awards this year were 330 in the mathematical sciences.

SLOAN RESEARCH FELLOWS

The Alfred P. Sloan Foundation has announced that 76 young physical scientists on the faculties of 43 colleges and universities have been awarded fellowships for basic research. Included in the list are 15 mathematicians: Jack H. Silver, H. Blaine Lawson, Jr., and Hung-hsi Wu of the University of California, Berkeley; Yiannis H. Moschovakis, University of California, Los Angeles; Christopher R. Lacher, Florida State University; Charles L. Fefferman, University of Chicago; Richard Hunt, Purdue University; Robert C. Hartshorne and John N. Mather of Harvard University; Steven A. Orszag, Massachusetts Institute of Technology; Ronald R. Coifman, Washington University; William K. Allard and Nicholas M. Katz of Princeton University; Jeff Cheeger and Hershel M. Farkas of the State University of New York at Stony Brook. The current group of Sloan Research Fellows was selected from 500 nominations by a committee of distinguished senior scientists.

CONFERENCE FOR ALGEBRAISTS

The Mathematical Institute, University of Aarhus, has announced an informal conference for algebraists to take place during June and July 1970. Arrangements are being kept informal in order that participants may do research, individually or jointly, have discussions, and present papers during the period they are in attendance. At the present time the list of participants, with approximate dates of attendance, include Robert M. Fossum, University of Illinois, June 15-July 15; Paulo Ribenboim, Queens University, two or three weeks; Klaus W. Roggenkamp, McGill University, June 15-July 3; Joseph T. Rotman, University of Illinois, June 24-August 5; Jan R. Strooker, University of Utrecht, June 15-July 15; Bo Stenström, Mathematical Institute, Göteborg, two weeks. Professor Roggenkamp will give a series of talks on K-theory and integral representations of finite groups during his period of attendance. Additional participants are expected to attend, including Professor Richard Brauer who will visit Aarhus for a few days in the beginning of July. A two-day symposium is planned for July 1-2 in which Chr. U. Jensen is expected to take part. Proceedings of the symposium, as well as the informal talks, will be printed during the conference.

Only limited funds are available for participant support. The Institute will assist in finding living quarters for those attending. For further information, please write to Svend Bundgaard, Matematisk Institut, Universitetsparken, Ny Munkegade, DK-8000, Aarhus C, Denmark.

AUDIO RECORDINGS OF MATHEMATICAL LECTURES

Several additional sets of taped lectures are now ready for distribution. No. 9 is "K_2 of global fields" by John T. Tate of Harvard University, and No. 10 is "K_2 of global fields" by Hyman Bass of Columbia. Both of these lectures were delivered at the six hundred sixty-eighth meeting of the Society in Cambridge, Massachusetts, on October 25, 1969. No. 11 is "Systems of ordinal functions and functionals" by Solomon Feferman of Stanford University. Professor Feferman's lecture was presented at the six hundred seventieth meeting of the Society in Claremont, California, on November 22, 1969. No. 12 is "Free boundary problems for parabolic equations" by Avner Friedman of Northwestern University. Professor Friedman's lecture was delivered at the six hundred seventy-first meeting of the Society in Ann Arbor, Michigan, on November 29, 1969. No. 15 is "Higher derivations and automorphisms of complete local rings" by Nickolas Heerema of Florida State University; his lecture was delivered at the six hundred sixty-ninth meeting of the Society in Baton Rouge, Louisiana, November 21-22, 1969.

The Audio Recordings of Mathematical Lectures may be purchased for $6 with the exception of the Colloquium Lectures (Nos. 5 and 6) which are $10 for each set of two tapes. Additional copies of the manual that accompanies the tapes may be purchased for $0.30 each. Standing orders for the entire series of lectures may be placed. Orders should be sent to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904.

INTERNATIONAL MATHEMATICAL UNION

The president of the National Academy of Sciences has appointed a delegation to the Sixth General Assembly of the International Mathematical Union in Menton, France, August 28-30, 1970, meeting immediately prior to the International Congress of Mathematicians, September 1-10, 1970 in Nice, France. The delegation consists of Charles B. Morrey, Jr. (chairman), Professor of Mathematics, University of California, Berkeley; A. Adrain Albert, Dean of the Division of Physical Sciences, University of Chicago; Nathan Jacobson, Professor of Mathematics, Yale University; William J. LeVeque, Professor of Mathematics, University of Michigan; and Henry O. Pollak, Director of the Mathematics Research Center, Bell Telephone Laboratories, Inc.

TOPICS IN WEAK CONVERGENCE

A regional conference, the title of which is Topics in Weak Convergence, will be held at the Department of Statistics, University of Iowa, Iowa City, Iowa, on August 31-September 4, 1970, under the sponsorship of the National Science Foundation. The principal lecturer will be Professor Patrick Billingsley, University of Chicago; he will give two lectures per day and later organize these lectures into an expository manuscript. The objective of the conference is to stimulate research in the theory of probability and statistics. Inquiries from professional statisticians and probabilists and very advanced graduate students are invited. It is anticipated that financial assistance for participants will be available for travel and/or subsistence. While priority on financial assistance will be given to persons from the central region of the country, the conference is open to others outside of this area. In the case of students who wish to attend, a letter should be sent by the major professor attesting to the student's preparedness for the lectures. Further information may be obtained by writing to Professor Robert V. Hogg, Department of Statistics, University of Iowa, Iowa City, Iowa 52240.

MATHEMATICAL OFFPRINT SERVICE

The Mathematical Offprint Service (MOS) has announced that the number of journals now included in the service has risen to 147. One year ago, June 1969, the number was only 90. Also a recent study showed that more than one-half of the articles reviewed in Mathematical Reviews are now available to subscribers. Copies of the new classification scheme and the new profile forms were mailed to subscribers in March. The new classification scheme will appear in the Index to Volume 39 of Mathematical Reviews, June 1970. Further information on MOS, as well as copies of the classification scheme and profile forms, may be obtained by writing to MOS, American Mathematical Society, P.O.Box 6248, Providence, Rhode Island 02904.

NEW YORK ACADEMY OF SCIENCES SCIENTIST-IN-RESIDENCE PROGRAM

The New York Academy of Sciences has announced a new Scientist-in-Residence Program to commence in the fall of 1970. Distinguished scientists in major research fields will each remain in residence at the Academy for a week, during which time they will meet with a small group of selected participating scholars for a series of colloquia, seminars, and informal discussions. Dr. Julian Schwinger, Nobel Laureate in Physics and Professor of Physics at Harvard University, will serve as the first scientist-in-residence during the week of October 19-23, 1970. The focus of discussion will be theories of particle physics. The second program, during the week of November 2-6, will feature Professor Paul Erdös of the Mathematical Institute of the Academy of Sciences in Hungary. Professor George Porter of London, Nobel Laureate in Chemistry, is scheduled to serve as scientist-in-residence in the spring of 1971. Professor Porter, whose field of special interest is photochemistry, is director of the Royal Institution of London and president of the Chemical Society of Great Britain.

Participating scholars in these programs are expected to be outstanding young investigators in the specified field of interest of the visiting scientist. A total of about 15 will be selected for each pro-

gram. Nominations of participating scholars may be made by academic department chairmen or directors of research institutions with which the candidate is associated. Academy members may also make recommendations. Nominations for the first two programs must be submitted by June 1, 1970. Further information and application blanks may be obtained from the Executive Director, The New York Academy of Sciences, 2 East 63rd Street, New York, New York 10021.

VOLUNTEER SPEAKERS SERVICE

The U.S. Information Agency provides embassies and consulates with information on visiting scholars, writers, and scientists who are willing to devote some time to participation in embassy-sponsored lecture programs and seminars. Volunteer Speakers are provided with an opportunity to extend their acquaintance, and sometimes their itineraries, and to help in the development of relations between their own professional communities and those of the host country. Per diem and the costs of regional travel undertaken in connection with USIS-sponsored activities are paid by the sponsoring embassies, but travel to and from the United States (or any other financial assistance) is not provided for. The role of the U.S. Information Service in Washington is limited to disseminating information on potential speakers, and no commitments are made for the actual arranging of programs. Mathematicians who will be overseas this year on sabbatical leave may be interested in this program. If so, further information may be obtained by writing to Dr. William F. DeMyer, Chief, Educational Support Branch, I.C.S., U.S. Information Agency, 1717H Street, N. W., Washington, D. C. 20547; telephone (202) 632-6738.

NSF SCIENCE DEVELOPMENT AWARDS

The National Science Foundation has announced the award of $5.4 million for science development to three universities: Brandeis University, Northwestern University, and the State University of New York at Stony Brook. These three-year science development grants are designed to assist a limited number of already good institutions to advance to a higher level of quality in their science activities.

CONFERENCE ON MATHEMATICAL ASPECTS OF GENERAL RELATIVITY

The Department of Mathematics of the University of Pittsburgh will hold a regional conference on general relativity theory July 13-17, 1970. Professor Roger Penrose will deliver a series of lectures entitled "Techniques of Differential Topology in Relativity." It is also hoped to schedule a limited number of contributed papers and discussions. Under a grant from the National Science Foundation, a limited number of travel and subsistence allowances will be available. Further information may be obtained by writing to the Organizing Committee, Conference on Relativity Theory, Department of Mathematics, University of Pittsburgh, Pittsburgh, Pennsylvania 15213.

1970 INTERNATIONAL SUMMER SCHOOL ON MANIFOLDS

The Netherlands Universities Foundation for International Cooperation (NUFFIC) has announced that the 1970 International Summer School on Manifolds will be held in the Mathematical Institute of the University of Amsterdam, August 17-30, 1970. The North Atlantic Treaty Organization has contributed a grant toward the support of the conference. The lecturers will include L. C. Glaser, University of Utah; W. C. Hsiang, Yale University; R. C. Kirby, University of California, Los Angeles, and University of Cambridge; L. Siebenmann, Princeton University; W. Browder, Princeton University; D. Sullivan, Massachusetts Institute of Technology; J. Cerf, University of Paris, Orsay; A. Haefliger, University of Geneva; M. Kervaire, New York University, Courant Institute; V. Poenaru, University of Paris; J. Mather, Harvard University; S. Smale, University of California, Berkeley; R. Thom, Institut des Hautes Etudes Scientifiques, Bures-sur-Yvette; E. C. Zeeman, University of Warwick, Coventry. Applications are invited from mathematicians (including graduate students) with an active interest in the theory of manifolds. Because accommodations are limited, admission may be res-

tricted. Participants will be housed in the International Centre of the Royal Tropical Institute at Amsterdam, and lectures will be held in the Mathematical Institute of the University of Amsterdam. There will be a charge of approximately $125 to cover the costs of housing, meals, service, and excursions. For further information, please write to the Course Registrar, Netherlands Universities Foundation for International Cooperation, 27 Molenstraat, The Hague, The Netherlands.

NSF POSTDOCTORAL FELLOWSHIPS

The National Science Foundation has announced the awarding of 169 Postdoctoral Fellowships. The mathematicians who received these fellowships were: Alexander J. Hahn, University of Notre Dame; Stanley O. Kochman, University of Chicago; John J. Hinrichsen, University of Michigan; William H. Jaco, University of Michigan; David A. Singer, University of Pennsylvania; Donald A. Drew, Rensselaer Polytechnic Institute; Jack A. Lees, University of Durham, England; Moss E. Sweedler, Cornell University; Gary C. Hamrick, University of Virginia; and James D. Kuelbs, University of Wisconsin. The Postdoctoral Fellowship holders were selected on merit from 1,294 applicants. The applications were evaluated by panels of outstanding research scientists appointed by the National Research Council; selections were made by NSF.

R. CREIGHTON BUCK TO SERVE ON NATIONAL ADVISORY COMMISSION

Professor R. Creighton Buck of the University of Wisconsin has been appointed by President Nixon to serve on the 15 member National Advisory Commission on Educational and Professional Development set up in 1967 to review and evaluate federal programs aimed at developing educational personnel. Professor Buck has been a member of the department of mathematics of the University of Wisconsin since 1950, and was chairman of that department from 1964 to 1966. He has been a member of the Society for 27 years.

REORGANIZATION OF NSF INSTITUTIONAL PROGRAMS

Dr. William D. McElroy, Director of the National Science Foundation, has announced the reorganization of the Foundation's Institutional Programs and staff. Dr. William V. Consolazio has been appointed Division Director for the new Division of Institutional Development, and will be responsible for the Science Development Program. The Science Development Program combines the former University Science Development and Departmental Science Development programs. Dr. Joshua M. Leise has been appointed Deputy Division Director.

The Science Development Program emphasizes basic science development that is responsive to the needs and plans of universities through continuance of support of development of individual departments, and assistance in the growth and improvement of university activities that couple research and education efforts in science to the solution of problems of social importance. This program will stress four areas: (1) a program for mathematics, the natural sciences, and engineering; (2) a program for the social sciences; (3) a program for interdisciplinary and multidisciplinary activities of science and engineering; and (4) a program for multidisciplinary areas of science that are directly related to solving society's problems.

Mr. Harold Horowitz has been appointed Acting Division Director for the newly created Division of Institutional Resources. This division is responsible for the Institutional Grants for Science Program, the Graduate Science Facilities Section, and the Architectural Services staff. Other staff changes in the Institutional Programs area are: Dr. J. Merton England has been appointed to the new position of Executive Assistant to the Deputy Assistant Director for Institutional Programs; Dr. Denzel D. Smith has been appointed to the new position of Special Assistant for Program Evaluation to the Assistant Director for Institutional Programs.

INTERNATIONAL CONFERENCE ON APPROXIMATION THEORY

The Department of Mathematics of the University of Maryland is organizing a five-day International Conference on Approximation Theory and Related Topics and Their Applications, October 26-30, 1970, on the campus in College Park. The conference will honor Professor Joseph L. Walsh on the occasion of his 75th birthday. The proceedings of the conference, as well as other manuscripts by authors who wish to dedicate them to Professor Walsh, will appear as a special issue of the Journal of Approximation Theory. The invitation committee consists of R. Creighton Buck, Philip J. Davis, Joseph L. Doob, David V. Widder, and Mishael Zedek (chairman). Further information may be obtained by writing to Professor Mishael Zedek, Department of Mathematics, University of Maryland, College Park, Maryland 20742.

REGIONAL CONFERENCE ON UNITARY REPRESENTATIONS OF GROUPS

A regional conference on unitary representations of groups, sponsored by a grant from the National Science Foundation, will be held at Dartmouth College, June 22-26, 1970. Professor Calvin C. Moore will deliver a series of expository lectures which will survey the area of group representations, give a picture of some of the problems in the subject and techniques used in their resolution, and devote special emphasis to applications in other areas of mathematics. The lectures will be complemented by informal seminars and discussion groups. Financial support will be provided for about 25 participants. Advanced graduate students and post-doctoral mathematicians are especially encouraged to apply. For further information, write to Professor Kenneth I. Gross, Department of Mathematics, Dartmouth College, Hanover, New Hampshire 03755.

SALEM PRIZE

The Salem Prize for 1970 was awarded to Dr. Yves Meyer, University of Paris at Orsay, for his work on algebraic numbers and harmonic analysis. The prize, established in 1968, is given every year to a young mathematician who is judged to have done an outstanding work on Fourier series and related topics. The recipient in 1968 was Dr. Nicholas Varopoulos; and in 1969, Dr. Richard Hunt received the prize. In all three years the jury consisted of Professors A. Zygmund, C. Pisot, and J.-P. Kahane.

SCRIPTA MATHEMATICA

Scripta Mathematica, which formerly accepted papers in the history, philosophy, and expository treatment of mathematics, has a new editorial board and will henceforth accept only papers devoted to new results in any branch of mathematics. Some expository articles of superior quality will also be accepted. The members of the new editorial board are A. Adrian Albert, University of Chicato (algebra); Edgar H. Brown, Jr., Brandeis (topology); S. S. Chern, University of California, Berkeley (geometry); Abe Gelbart, Managing Editor, Yeshiva University (analysis); Harold Grad, New York University (applied mathematics); Martin Schechter, Yeshiva University (analysis). Scripta Mathematica has, at present, no backlog of articles and will be able to publish accepted papers without delay. Those wishing to submit papers should send them to any of the editors or to the Scripta Mathematica office at Yeshiva University, 186th Street and Amsterdam Avenue, New York City 10033.

CONFERENCE ON APPROXIMATION THEORY AND FUNCTIONAL ANALYSIS IN NUMERICAL ANALYSIS

A conference on approximation theory and functional analysis in numerical analysis will be held at Boston University, July 20-24, 1970. Support for the conference will be provided by the National Science Foundation. The program will center around a series of lectures by Professor Richard S. Varga. Several other participants will present one or more lectures or talks in this area, and seminars and panels will be held. Travel and living expenses will be paid by the conference to approximately 25 participants. Applications for support should include an aca-

demic vita with a statement of research projects currently underway or planned. Advanced graduate students should include a list of relevant courses taken plus a letter from the research advisor or department chairman. All applications and inquiries should be addressed to Professor Robin E. Esch, Department of Mathematics, Boston University, 270 Bay State Road, Boston, Massachusetts 02215. (Telephone: 617-353-2560.)

STIPEND SUPPLEMENTS OF NSF FELLOWS AND TRAINEES

The National Science Foundation has revised its policy on supplementation of stipends of NSF graduate fellows and trainees. If it seems desirable, stipends may be augmented by the institution, with certain provisions, for NSF graduate fellowship and traineeship awards made in 1970. The basic stipend may be augmented by institutional funds by not more than $1000 for a tenure of a full calendar year, or $750 for a tenure of nine months, during the first year of residence regardless of level of graduate study. (Prior approval must be obtained from NSF for any exception to this limitation.) In the second or subsequent years of graduate residence at the same institution, stipends may be supplemented from institutional funds in such amounts as are in accordance with the supplementation policies of the individual institution. Funds that the institution has obtained from external (including Federal) sources may be considered as institutional funds for the purposes of these rules. NSF program grants may be used for supplementation, when appropriate, in an amount not to exceed $1000 a year for any federally assisted fellow or trainee. This restriction does not apply, however, to funds received by institutions from the NSF "Institutional Grants for Science" (base grants), from cost-of-education allowances, or from overhead costs allowed under program costs.

ATTENTION: MEMBERS
New Directions in Mathematical Communication

This article introduces a new section that will appear from time to time in these *Notices*. We hope that it will include letters from members expressing their opinions on the direction the AMS proposes to take in the development of its communication system. An open dialog with members will provide guidelines for AMS developmental activities and will keep the membership informed of the progress of the communication program.

Historically, the purpose of the AMS has been the "furtherance of scholarship and research in the mathematical sciences." A principal means of accomplishing this is to foster communication--to encourage the exchange of information among mathematical researchers and scholars. Information exchange is the Society's main business.

In its early days, the Society employed only one means to further scholarship and research--the mathematics meeting. A few meetings a year were sufficient for the needs of the existing community of mathematicians. In 1891 the Society established a second means of communication--the primary mathematics journal. In the eighty years following the first appearance of the BULLETIN of the then-New York Mathematical Society, the AMS has expanded its publication and meeting programs; at least partly as a result of its success at fostering the communication of mathematical ideas, the annual number of Ph.D. mathematicians produced by graduate institutions in the U. S. has increased tremendously. This in turn has created an even greater need for the wide dissemination of mathematical ideas. The variety of publications and services offered by the Society has grown in step with the need.

As the number of mathematicians has grown, the volume of mathematical information they produce has grown also. The information needs of research mathematicians have changed radically in the face of this information "flood."

The situation in mathematics does not correspond exactly to that in other disciplines. In physics, chemistry and other fields, the amount of publishable material being produced threatened to overwhelm the conventional media as well as the individual scientist. In mathematics, the conventional media can still handle the amount of information produced; however, it is becoming increasingly difficult for the individual mathematician to find in the media the small fraction of material that is relevant to his interests. This problem is not shared by all mathematicians. There is no problem for some of those at the very top of their profession, because they do not usually need to locate papers of interest in the literature--they receive preprints and reprints of important papers directly from their colleagues. However, some top mathematicians, such as Norman Steenrod and David Mumford, have been very enthusiastic about some of the recent innovations in mathematical information exchange. And these innovations are very important for the majority of mathematicians, who must spend hours finding the papers they need in the vast amount of mathematical literature being produced and published.

The Society has continued to introduce new means of communication to meet the changing needs of mathematicians. A permanent Committee to Monitor Problems in Communication (Comm.-Comm.) was established in 1966 to consider changing needs and problem areas, and to suggest, monitor and evaluate new services that help to solve information exchange problems. CONTENTS OF CONTEMPORARY MATHEMATICAL JOURNALS and the Mathematical Offprint Service (MOS) are two of the more recent innovations. The Committee has also been responsible for many other improvements in the AMS communication system.

In the area of subject classification,

the Committee has undertaken several projects. It directed the preliminary revision of the two-level MATHEMATICAL REVIEWS classification scheme, for use in MOS, in January 1969, and the creation of a new, three-level MOS scheme in late 1969 and early 1970. Comm.-Comm. requested mathematical publishers to print MOS classification numbers in their books as an aid to the Library of Congress in assigning library classifications. It also recommended the subject classification, in broad categories, of abstracts in these *Notices*. The Committee has been concerned with the revision of the Mathematics section of the Universal Decimal Classification system, used in some European and nearly all Eastern European scientific publications, and directed the translation into English of a recent Soviet proposal for such a revision.

Comm.-Comm. also suggested the establishment of the present requirement for documentation units, consisting of informative title, subject classification numbers, and key words and phrases, in all AMS journals; in all journals except the AMS BULLETIN, the documentation units include abstracts. In the future it is going to become increasingly important to have a means of clearly identifying the content of mathematical papers for information retrieval, and this is the reason Comm.-Comm. recommended the requirement.

In addition, the Committee initiated the establishment of the following AMS programs and projects: regular audio taping of invited lectures at AMS meetings; preparation of an author index for SOVIET MATHEMATICS--DOKLADY; preparation of two compilations of reviews in Number Theory and Group Theory, similar to the recent Reviews of Papers in Algebraic and Differential Topology edited by Norman Steenrod; regular notification of the availability of preprints for abstracts in these *Notices*; a symposia information center which collects and distributes information on symposia in the planning stages; preparation of back volumes of the AMS TRANSACTIONS in microfilm.

The Committee has recently turned its attention to the overall organization of information services. Comm.-Comm. feels that it is imperative for an overall, systematic study of the AMS information exchange program to be undertaken, to determine whether the existing projects are individually necessary; whether they are collectively sufficient to meet the needs of mathematicians now and in the near future; whether they are organized and managed in an efficient and economical way. The Committee has requested a grant from the National Science Foundation (which has supported Comm.-Comm.'s previous activities under NSF grants GN-731 and GN-784) for a comprehensive study of the AMS communication system.

Is the AMS offering too many different kinds of services? Are additional ones needed? Do present services address the needs of all groups of mathematicians, or are some needs being ignored? These and other questions will be investigated in the proposed program. The studies will be conducted in five major areas: primary publications, secondary publications, the data base, composition, and meetings.

We welcome questions, suggestions and criticisms from the membership regarding the proposed program. Because of space restrictions, communications should be limited to 200 words. Letters should be addressed to New Directions, the *Notices*, AMS, P.O. Box 6248, Providence, Rhode Island 02904.

ABSTRACTS OF CONTRIBUTED PAPERS

The March Meeting in New York
March 25-28, 1970

673-121. EDWARD B. SAFF and J. B. TWOMEY, University of South Florida, Tampa, Florida 33620. Critical points of polynomials.

Let P(a, n) be the set of all nth degree polynomials which have all of their zeros in γ: $|z| \leq 1$ and at least one zero at the point z = a. The following problem related to the Ilieff conjecture was suggested in a recent paper by Goodman, Rahman, and Ratti [Proc. Amer. Math. Soc. 21(1969), 273-274]: Describe a region D(a,n) such that (i) D(a,n) contains at least one critical point of every $p(z) \in P(a,n)$ and such that (ii) no proper subset of D(a,n) has property (i). For $0 \leq a \leq 1$, let $\Delta(a)$: $|z - a/2| \leq [(4 - a^2)/12]^{1/2}$ and let C(a) denote the circumference of $\Delta(a)$. It is shown that sets D(a,3) are given by $D(a,3) = \Delta(a) \setminus [C(a) \cap \{z: \operatorname{Im} z > 0\}]$ for $0 < a \leq 1$, $D(0,3) = \Delta(0)$. (Received March 16, 1970.)

The April Meeting in Madison, Wisconsin
April 14-18, 1970

674-99. AVIAD M. BROSHI, University of Massachusetts, Amherst, Massachusetts 01002. Galois correspondences between the irreducible characters and the conjugacy classes of finite groups.

Let G be a finite group, and let GAL be the Galois group of a splitting field for G. Then GAL acts as a permutation group of both the irreducible characters and of the conjugacy classes of G. Theorem. If the Sylow subgroups of G are abelian the two resulting permutation representations of GAL are permutation isomorphic. (Received March 2, 1970.)

674-100. URI FIXMAN, Queen's University, Kingston, Ontario, Canada and LIOR TZAFRIRI, University of Washington, Seattle, Washington 98105. The full algebra generated by a spectral operator.

Let T be a bounded spectral operator in a Banach space X. Let E(•), S, N, and $\sigma(T)$ denote respectively the resolution of the identity, the scalar part, the radical part and the spectrum of T; while A(T) is the full algebra generated by T (see N. Dunford, "Spectral operators," Pacific J. Math. 4(1954), 321-354). Theorem 1. If $\sigma(T)$ is totally disconnected, then A(T) is semisimple iff T = S. Theorem 2. If $E(\partial\sigma(T)) = 0$, then A(T) is semisimple. Theorem 3. Suppose: N is nilpotent. For $k \geq 1$, $N^k X$ is closed and the spectrum σ_k of the operator induced by T in this subspace has the property: If $(f_n)_1^\infty$ is a sequence of rational functions holomorphic on σ_k such that, uniformly on σ_k, $\lim_{n\to\infty} f_n = 0$ and $\lim_{n\to\infty} f'_n = g$ (g finite), then g = 0 on σ_k. Then: A(T) is semisimple. The structure of A(T) is determined in this case. There exist operators satisfying the conditions of Theorems 2,3 for which $N \neq 0$. (Received March 2, 1970.)

674-101. WITHDRAWN.

674-102. F. A. ROACH, University of Houston, Houston, Texas 77004. An algebraic operation for continued fractions over a vector space.

Let S be a real inner product space, u a point of S with unit norm, and for each z in S, let $1/z$ denote $[2((z,u))u - z]/\|z\|^2$ (with the usual conventions about a "point at infinity"). For x,y in S, let $x * y$ denote $1/\sqrt{-1/x} + 1/\sqrt{x} + 1/\sqrt{-y - 1/x}$. (This operation is analogous to a product xyx. However, it is shown that if S is finite dimensional, there is a distributive multiplication on S with $(cx)y = x(cy)$ for scalar c such that $x * y = x(yx)$ $[x * y = (xy)x]$ only in case $S = E^1$, E^2, E^4, or E^8.) Let x^2 denote $x * u$ and let xy denote cy if c is a scalar such that $cu = x$; otherwise, let xy denote $a * y$ where $a^2 = x$. (Each point not a scalar multiple of u has just two square roots, one of them the negative of the other.) For x, y, z in S, $x(y + z) = xy + xz$, $xu = ux = x$, and if $x \neq 0$, $x(1/x) = (1/x)x = u$ and $x[(1/x)y] = [x(1/x)]y$. Also $\|xy\| = \|x\| \, \|y\|$ and if $x \neq 0$ and $y \neq 0$, $1/(xy) = (1/x)(1/y)$. Through the use of this operation several results concerning continued fractions over S are obtained. For example, the results concerning "equivalence transformations" (as stated in Abstract 669-52, these *Notices* 17(1970), 82). (Received March 2, 1970.)

674-103. MURRAY SCHACHER, University of California, Los Angeles, California 90024. Cyclotomic splitting fields.

Suppose k is an algebraic number field and D a finite-dimensional central division algebra over k. It is well known that D has infinitely many maximal subfields which are cyclic extensions of k. From the point of view of group representations, however, the natural splitting fields are the cyclotomic ones. Accordingly it has been conjectured that D must have a cyclotomic splitting field which contains a maximal subfield. Our purpose is to show that the conjecture is false; we construct a counterexample of exponent p, one for every prime p. (Received March 2, 1970.)

674-104. MICHAEL G. CRANDALL and THOMAS M. LIGGETT, University of California, Los Angeles, California 90024. A nonlinear product formula in general Banach spaces. Preliminary report.

Let X be a Banach space and A be an accretive subset of $X \times X$. If $\lambda \geqq 0$, setting $J_\lambda(x + \lambda y) = x$ for $[x,y] \in A$ defines a function J_λ whose domain is denoted by D_λ. A simplified version of the main result is: Theorem. Assume that D_λ includes the closure of D_0 whenever λ is positive. If $x \in \overline{D}_0$ and $T \geqq 0$, then $\lim_{n\to\infty} J^n_{t/n}(x)$ exists uniformly in t for $0 \leqq t \leqq T$, and setting $S(t)x = \lim_{n\to\infty} J^n_{t/n}(x)$ defines a semigroup of contractions on $\overline{D}_0$. This result generalizes in various ways, e.g. $\lim_{k\to\infty} J^{n(k)}_{\epsilon(k)}(x)$ exists if $\lim_{k\to\infty} n(k)\epsilon(k)$ exists, A need not be exactly accretive, etc. The result is a significant extension of previous works, which would require, e.g., A to be linear or continuous, or X^* to be uniformly convex. The proof is quite elementary, utilizing a simple combinatorial estimate. Applications and examples will be given. (Received March 19, 1970.)

674-105. LARS INGE HEDBERG, University of Uppsala, Sysslomansgatan 8, Uppsala, Sweden and University of Michigan, Ann Arbor, Michigan 48104. Approximation in the mean by analytic functions.

Let $E \subset \mathbb{C}$ be compact, $L^p_a(E) \subset L^p(E)$ the subspace of functions analytic in the interior E^o of

E, and $R^p(E)$ the closure in $L^p(E)$ of rational functions with poles off E. V. P. Havin proved [Dokl. Akad. Nauk SSSR 178(1968), 1025-1028] that $R^p(E) = L^p_a(E)$, $1 < p < 2$, and gave a necessary and sufficient condition for $R^2(E) = L^2_a(E)$ in terms of logarithmic capacity, but remarked that the method of proof did not extend to $p > 2$. Definition. The analytic p-capacity, $p > 2$, of a compact set F is $\gamma_p(F) = \sup_f \lim_{z\to\infty} |zf(z)|$, f analytic off F, $f(\infty) = 0$, and $\int_{\mathbb{C}} |f|^p\, dx\, dy \leq 1$. Then there are constants such that $K_1 C_q(F)^{1/q} \leq \gamma_p(F) \leq K_2 M_q(F)^{1/q}$, where $q = p/(p-1)$, C_q is capacity with respect to r^{q-2}, and $M_q(F) = \inf \sum r_i^{2-q}$, r_i being radii of disks $K_{z_i}(r_i)$ which cover F. We denote the greatest non-increasing minorant of a function h by $h^\sim$. Theorem. $R^p(E) = L^p_a(E)$, $2 < p < \infty$, if $\lim_{\delta\to 0} \sup \gamma_p^q(K_z(\delta)\setminus E)\delta^{-2} = \infty$ for almost all $z \in \partial E\setminus\partial E^o$, and $\int_0 [\gamma_p^p(K_z(\delta)\setminus E)\delta^{1-p}]^\sim d\delta = \infty$ for almost all (M_q) $z \in \partial E^\sigma$. Theorem. If $R^p(E) = L^p_a(E)$, then $\gamma_p(K_z(\delta)\setminus E) = \gamma_p(K_z(\delta)\setminus E^o)$ for all z and δ, and $\int_0 [\gamma_p^q(K_z(\delta)\setminus E)\delta^{q-3}]^\sim d\delta = \infty$ for almost all (C_q) $z \in \partial E$. (Received April 3, 1970.) (Author introduced by Professor Peter L. Duren.)

674-106. JONATHAN K. LEE, Indiana University, Bloomington, Indiana 47401. On a class of functions in generalized harmonic analysis.

For a complex-valued measurable function f on $\mathbb{R}$, let $\|f\|^2 =_d \limsup_{h\to 0}(1/2h)\int_{-\infty}^{\infty}|f(x+h) - f(x-h)|^2 dx$. Then $Q = \{f: \|f\| < \infty\}$ is a pre-Banach space under the norm $\|\cdot\|$ when f, g such that $\|f - g\| = 0$ are identified. This identification will be tacitly understood. Theorem. Q is a Banach space. This is hard to show since $f \geq g \neq \|f\| \geq \|g\|$ (cf. Luxemburg and Zaanen, Indag. Math. 25 (1963), 143). To prove this theorem we show that the curve $x_t(\cdot) =_d (\tau_t f - f)(\cdot)$, where $(\tau_t f)(\lambda) = f(t+\lambda)$, is a helix in $L_2(\mathbb{R})$ (cf. Masani, Abstract 674-72, these *Notices* 17(1970), 541). Hence (loc. cit.) $\tau_b f - \tau_a f = T_\tau(a,b]\alpha$, where $T_\tau(a,b] = \{\tau_b - \tau_a - \int_a^b \tau_t dt\}/\sqrt{2}$ and $\alpha(\cdot) \in L_2(\mathbb{R})$ is uniquely determined by f. Lemma 1. $\|f - \alpha/\sqrt{2}\| = 0$. Let $\{f_n\}_{n=1}^\infty$ be a Cauchy sequence in Q, and pick f_{n_j} such that $\|f_{n_{j+1}} - f_{n_j}\| \leq 2^{-(j+1)}$, $j = 1,2,\ldots$. Let α_{n_j} be the function in $L_2(\mathbb{R})$ corresponding to f_{n_j}. Lemma 2. There exists an increasing sequence in $\mathbb{R}$ $\{M_j\}_{j=0}^\infty$ such that $M_0 = 0$ and $M_j \to \infty$ and such that if $\alpha = \text{l.i.m.}_{J\to\infty} \sum_{j=0}^J \alpha_{n_{j+1}} * \tilde{\chi}[M_j, M_{j+1}) \cup (-M_{j+1}, -M_j]$ in $L_2(\mathbb{R})$, then $\|f_n - \alpha\| \to 0$ as $n \to \infty$. The theorem is important because Q contains the important (nonlinear) space Q_0 of functions f of bounded quadratic variation q(f), arising in generalized harmonic analysis (cf. Wiener, Acta Math. 55(1930), 117-258), where $q(f) =_d \lim_{h\to 0}(1/2h)\int_{-\infty}^{\infty}|f(x+h) - f(x-h)|^2 dx$. (Received April 13, 1970.)

The April Meeting in Davis, California
April 25, 1970

675-27. DERRICK H. LEHMER, University of California, Berkeley, California 94720. On compounding means.

Two means M(u,v) and M'(u,v) are compounded to form a third mean $M * M'(u,v)$, as Gauss did with the arithmetic and geometric means, as follows. We define a_n and b_n recursively by $a_0 = u$, $b_0 = v$, $a_{n+1} = M(a_n, b_n)$, $b_{n+1} = M'(a_n, b_n)$. In case a_n and b_n approach a common limit as $n \to \infty$ then this limit is $M * M'(u,v)$. The operator $*$ is commutative nilpotent but not associative. To examine closure properties of $*$ we consider two classes of means: C_1 the class of all Minkowski means $\mu_p(u,v) = (u^p + v^p)^{1/p}$ and C_2 the class of all the rational functions $M_\Gamma(u,v) = (u^\Gamma + v^\Gamma)/(u^{\Gamma-1} + v^{\Gamma-1})$. These classes intersect in A G H, the arithmetic, geometric and harmonic means. For $\mu_p * \mu_q = \mu_s$ $(p \neq q)$ it is necessary and sufficient that $p + q = s = 0$. For example, $A * H = G$. For $M_p * M_q = M_s$

($p \neq q$) it is necessary and sufficient that $p + q = 2s = 0, 1, 2$. That is, $M_s = A$, G, or H. The paper concludes with a study of the function $M_1 * M_2$ as suggested by H. P. Robinson. When normalized and expanded in power series it generates coefficients with properties reminiscent of Bernoulli numbers. (Received March 2, 1970.)

675-28. NARENDRA K. GOVIL, Loyola College, Montreal, Quebec, Canada and BADRI N. SAHNEY, University of Calgary, Calgary, Alberta, Canada. On a sequence of Fourier coefficients.

The various summability methods for the sequences $\{nB_n\}$ have been considered, among other things, by R. Mohanty and M. Nanda [Proc. Amer. Math. Soc. 5(1954), 79-84], O.P.Varshney [Proc. Amer. Math. Soc. 10(1959), 790-795] and P. L. Sharma [Proc. Amer. Math. Soc. 15(1964), 337-340]. No necessary and sufficient condition for any of the summability methods is known. In order to obtain a necessary and sufficient condition for $(\Lambda) \cdot C_1$ method of summability of the sequence $\{nB_n\}$, the following theorem is proved here. Theorem. Let $\{\Lambda\} \equiv \{\lambda_{n,k}\}$ be a sequence of real, positive, increasing numbers with k and which defines a regular triangular matrix sequence. Also if $\Psi(t) \equiv \int_0^t |\psi(u)|du = o(t/Q(1/t))$ as $t \to +0$ where $Q(t)$ is positive and nondecreasing with t, then a necessary and sufficient condition that the sequence $\{nB_n\}$ be $(\Lambda) \cdot C_1$ summable to $\mathcal{L}/\pi$ is that $\int_1^n \bar{\Lambda}_n(t)/tQ(t)\, dt = O(1)$, where $\bar{\Lambda}_n \equiv \sum_{k=1}^n \Lambda_{n,k}$. (Received March 2, 1970.)

675-29. B. S. RAJPUT, University of North Carolina, Chapel Hill, North Carolina 27514. Unbounded determinable subsets of Banach spaces. Preliminary report.

Let D be a subset of a real Banach space B; then D is called determinable if $\forall \epsilon > 0$ $\exists$ continuous linear functionals $m_1,\ldots,m_n$ from B to the complex field C and a continuous map $f: C^n \to B$ (each m_j and f depending on ϵ) such that $\forall x \in D$ $\|x - f(m_1(x),\ldots,m_n(x))\| < \epsilon$. Let L(B) be the real Banach space (with the uniform norm) of all bounded linear operators on B, and let $D \subset L(B)$; then D is called determinable in principle if $\forall \epsilon > 0$ $\exists x_1,\ldots,x_n \in B$; $h_1,\ldots,h_n \in B^*$ (the conjugate space of B) and a continuous map $f: C^n \to L(B)$ (x_j's, h_j's and f depending on ϵ) such that $\forall T \in D$ $\|T - f(h_1(Tx_1),\ldots,h_n(Tx_n))\| < \epsilon$. The set D is called determinable in practice if $x_1 = x_2 = \ldots = x_n$. Characterizations are given of convex and circled subsets of B that are determinable. Equivalence of determinability and determinability in principle is shown for convex and circled subsets of L(B). Finally a characterization is given of convex and circled subsets of L(B) that are determinable in practice. (Received March 5, 1970.)

The June Meeting in Tacoma, Washington
June 20, 1970

676-1. V. KANNAN, Madurai University, Madurai-2, India. Some topological properties.

Definitions. Let P be a topological property. P is said to be nontrivial if there exists at least one space possessing P, which is not indiscrete. P is said to be productive, (hereditary, or divisible) if every product (subspace, or quotient) of spaces possessing P, also possesses P. Theorem 1. Let X be any zero-dimensional T_0 space. Let P be any nontrivial productive hereditary property. Then X possesses P. Theorem 2. Let P be any productive hereditary divisible property. Then P must be given by one of the following rules: (1) Every topological space possesses P. (2) A topological space X possesses P if and only if X is indiscrete. (3) A topological space X possesses P if and only if X has at most one point. (4) A topological space X possesses P if and only if X is an empty set. Theorem 3. Let X be any topological space. Then there exists a paracompact T_5 space Y such that X is a quotient of Y. (This Y can be chosen to be zero-dimensional also.) (Received March 12, 1970.)

676-2. HASKELL P. ROSENTHAL, University of California, Berkeley, California 94720. On the subspaces of L^p $(p > 2)$ spanned by sequences of independent random variables.

Let $2 < p < \infty$. The Banach space spanned by an infinite sequence of independent random variables in L^p, each of mean zero, is shown to be isomorphic (linearly homeomorphic) to ℓ^2, ℓ^p, $\ell^2 \oplus \ell^p$, or a new space X_p. The linear topological properties of X_p are investigated and some new linear topological invariants of Banach spaces are introduced; the inequalities developed may be of independent interest to probability theorists. Answers are also given to some of the questions raised in J. Lindenstrauss and H. P. Rosenthal, "The $\mathcal{L}_p$ spaces," Israel J. Math. 7(1969), 325-349. For example it is shown that X_p is an $\mathcal{L}_p$ space nonisomorphic to the previously known ones, and other new $\mathcal{L}_p$ spaces are constructed. It is also demonstrated that there exists a subspace A of ℓ^p isomorphic to ℓ^p such that ℓ^p/A is not isomorphic to any subspace of L^p (whence A is uncomplemented in ℓ^p). This paper has been accepted for publication by the Israel J. Math. (Received April 6, 1970.)

676-3. MORDECAY ZIPPIN, University of California, Berkeley, California 94720. Interpolation of operators of weak type between rearrangement invariant function spaces.

With each rearrangement invariant function space X (r.i. space, in short) on $(0,\infty)$ one can associate a pair $\{p_0(X), p_\infty(X)\}$ of numbers, $0 \le p_0(X) \le p_\infty(X) \le 1$, which plays the role of $1/p$ in the L_p spaces. A linear operator T from X to the measurable functions is said to be of weak type $\{X,X\}$ if there exists a constant c such that for every $f \in X$ $\sup_{t>0} (Tf)^*(t) \cdot \varphi_X(t) \le c \cdot \int_0^\infty f^*(s)d\varphi_X(s)$ where $\varphi_X(t) = \| \chi_{[0,t]} \|_X$ and $g^*(s)$ denotes the nonincreasing rearrangement of $|g(s)|$. Theorem. Let X_1, X_2 and X be r.i. spaces on $(0,\infty)$ and assume that $p_0(X_1) > p_\infty(X) \ge p_0(X) > p_\infty(X_2)$. Then every operator of weak type $\{X_i, X_i\}$ $i = 1,2$ is a bounded operator on X. This theorem strengthens a recent result of E. M. Semenov [Soviet Math. Dokl. 10(1969), 507-510]. (Received April 8, 1970.)

676-4. KIM KI-HANG BUTLER, St. Mary's College, St. Mary's City, Maryland 20686. On (0,1)-matrix semigroups. Preliminary report.

This paper presents some combinatorial results in connection with the Green's equivalence classes of semigroups of certain matrices. We classify all $n \times n$ matrices over a certain semiring (the Boolean algebra of two elements), which have row rank and column rank equal and less than four into $\mathcal{D}$-classes. We proved that the number of such $\mathcal{D}$-classes is independent of n (as long as $n \geq 3$). Furthermore, we analyze these $\mathcal{D}$-classes into $\mathcal{L}$, $\mathcal{R}$, and $\mathcal{H}$-classes and obtain a variety of results dealing with the number of elements in a given $\mathcal{D}$-classes, number of $\mathcal{L}$-classes, and number of $\mathcal{R}$-classes. Finally, we study the distribution and construction of idempotents in the various $\mathcal{D}$-classes establishing the complete relationship $\mathcal{D}$-classes and the idempotents they contain, after which we generalize some of our results. (Received April 14, 1970.)

676-5. RICHARD J. LOY, Carleton University, Ottawa 1, Ontario, Canada. Topological algebras of formal power series. Preliminary report.

Let K denote the real or complex field. Let A be an algebra with identity over K with a Fréchet space topology, $\underline{\underline{A}}$ an algebra of formal power series over A in a commutative indeterminate t, carrying a Fréchet space topology such that the coordinate projections $p_n: \underline{\underline{A}} \to A$ are all continuous. $\underline{\underline{A}}$ will be supposed to contain t, but not necessarily to have identity. Theorem 1. If $\underline{\underline{A}}$ is a (left) A-module, and is also a topological algebra, then $\underline{\underline{A}}$ is a topological (left) A-module. Theorem 2. Suppose that there is a sequence $\{\gamma_n\}$ of positive reals such that $\{\gamma_n^{-1} p_n\}$ is equicontinuous. Let B be a topological algebra over K with Fréchet space topology, $\varphi: B \to \underline{\underline{A}}$ a homomorphism with $t \in \varphi(B)$. Then φ is continuous. Corollary. Such an $\underline{\underline{A}}$ has unique Fréchet space topology as a topological algebra. This extends a previous result for the Banach algebra case ("Uniqueness of the complete norm topology and continuity of derivations on Banach algebras," Tôhoku Math. J. (1970)). (Received April 20, 1970.)

676-6. LUDVIK JANOS, University of Florida, Gainesville, Florida 32601. On factorizations of homeomorphisms.

In dealing with a homeomorphism $h: X \to X$ acting in a non necessarily metrizable space X it is sometimes useful to know whether h can be extended over a Cartesian product of metrizable spaces in such a way that its extension acts coordinatewise on the factors. Theorem 1. If X is compact Hausdorff and $h: X \to X$ a homeomorphism such that the family $\{h^n | n \geq 1\}$ is evenly continuous, then there exists a family $\{X_\alpha | \alpha \in \mathfrak{A}\}$ of compact metrizable spaces X_α, a corresponding family $\{h_\alpha | \alpha \in \mathfrak{A}\}$ of homeomorphisms $h_\alpha: X_\alpha \to X_\alpha$ and a topological embedding $i: X \to \pi_\alpha X_\alpha$ such that $i \bullet h = (\pi_\alpha h_\alpha) \circ i$. Combining this statement with certain results of J. de Groot we obtain the following: Theorem 2. Under the same assumptions as above there exists a topological embedding $i: X \to \pi_\alpha H_\alpha$, where H_α are copies of the separable Hilbert space, and unitary operators $u_\alpha: H_\alpha \to H_\alpha$ such that $i \bullet h = (\pi_\alpha u_\alpha) \circ i$. (Received April 20, 1970.)

676-7. GEORGE F. McNULTY, University of California, Berkeley, California 94720. The problem of base decidability for equational theories of algebras. Preliminary report.

For notation see Tarski, "Equational logic and equational theories of algebras" in "Contributions to math. logic" (Colloquium, Hannover, 1966), pp. 275-288, North-Holland, Amsterdam, 1968. Thus, the terms "algebra", "equational theory", "base of a theory", "finitely based" are assumed to be understood. A finitely based theory Θ is called base-decidable if the set of its finite bases is recursive, and base-undecidable otherwise; Θ is essentially base-undecidable if every finitely based theory $\Sigma \supset \Theta$, possibly with some new operation symbols, is base-undecidable. Examples of common theories which are base-decidable are not numerous. Among groupoids $\langle A, \cdot \rangle$ the theory of all commutative groupoids, that of all semigroups, and that of all commutative semigroups are base-decidable, as observed by J. Ng and A. Tarski. The author has shown that the same applies, for any $m,n = 0,1,\ldots$, to the theory of all commutative semigroups in which $x^{m+2} = x^{n+2}$ holds. It will be seen from a later abstract of the author that the theories of most interesting algebras are base-undecidable and, in fact, essentially base-undecidable. An example of a theory which is base-undecidable but not essentially so is provided by the theory of all groupoids satisfying both $(x \cdot x) \cdot x = x \cdot x$ and $(x \cdot x) \cdot (x \cdot x) = x \cdot x$. (Received April 22, 1970.)

676-8. CHARLES MARTIN, University of California, Berkeley, California 94720. Decidability and nonfinite axiomatizability of the equational theory of the ordinal numbers with addition and multiplication. Preliminary report.

For notation, see Tarski's article in "Contributions to math. logic," (Colloquium, Hannover, 1966), pp. 275-288, North-Holland, Amsterdam, 1968. Given any class X of ordinals closed under (ordinal) addition and any class Y closed under addition and multiplication, consider the algebras $\mathfrak{A}_X = \langle X,+\rangle$ and $\mathfrak{B}_Y = \langle Y,+,\cdot\rangle$. We can set e.g., X = OR or Y = OR, where OR is the class of all ordinals; we can also take any main number of addition for X, or any main number of multiplication for Y. For any given algebra $\mathfrak{C}$, let $T\mathfrak{C}$ be the first order theory of $\mathfrak{C}$ and $E\mathfrak{C}$ the equational theory of $\mathfrak{C}$. The following facts about $E\mathfrak{A}_{OR}$ are known: (i) ω^2 is the least ordinal γ for which $E\mathfrak{A}_{OR} = E\mathfrak{A}_\gamma$. (ii) $T\mathfrak{A}_{OR}$ and hence $E\mathfrak{A}_{OR}$ is decidable. (iii) $E\mathfrak{A}_{OR}$ is finitely based. (For a proof of decidability of $T\mathfrak{A}_{OR}$ see Büchi, Z. Math. Logik Grundlagen Math. 6(1960). A simple decision method for $E\mathfrak{A}_{OR}$ was designed by A. Selman and J. Zimbarg-Sobrinho. (i) is a corollary of this method. (iii) is a result of J. Karnofsky based upon the same method.) The corresponding, but not always analogous, results for $E\mathfrak{B}_{OR}$ obtained by the author are: (I) ω^{ω^ω} is the smallest ordinal γ for which $E\mathfrak{B}_{OR} = E\mathfrak{B}_\gamma$. (II) $E\mathfrak{B}_{OR}$ is decidable (although $T\mathfrak{B}_{OR}$ is not). (III) $E\mathfrak{B}_{OR}$ is not finitely based. (Received April 27, 1970.)

676-9. PETER HESS, University of Chicago, Chicago, Illinois 60637. A homotopy argument for the solvability of nonlinear operator equations in nonseparable Banach spaces.

Let T_0 be a continuous nonlinear mapping of a real reflexive Banach space X into its conjugate space X^*. A new existence theorem is given: The equation $T_0 u = 0$ admits a solution in a given open bounded subset G of X, provided T_0 is homotopic to a mapping T_1 having the property that for each finite-dimensional subspace F of X its approximant $T_{1,F}: F \to F^*$, has nonvanishing Brouwer degree on $G \cap F$ over $0 \in F^*$. Here $T_{1,F} = j_F^* T_1 j_F$, where j_F denotes the injection mapping of F into X, and

it is assumed that each mapping T_t, $0 \leq t \leq 1$, of the homotopy satisfies the condition (S) introduced by F. E. Browder in the study of nonlinear eigenvalue problems. By applying the above result, one can show that various recent existence theorems which have been obtained by homotopy arguments only for bounded operators in separable spaces, still hold for large classes of unbounded operators in nonseparable Banach spaces. (Received April 28, 1970.)

676-10. FRANK CASTAGNA and GEERT C. E. PRINS, Wayne State University, Detroit, Michigan 48202. Tait colorings on generalized Petersen graphs.

Mark Watkins [J. Combinatorial Theory 6(1969), 152-164] defined generalized Petersen graphs, conjectured that all but the original Petersen graph have a Tait cycle, and proved the conjecture in a number of cases. It is shown that the conjecture is true by constructing Tait cycles for the remaining cases. Three different schemata are used. The proofs depend strongly upon symmetry properties of the constructions. (Received April 29, 1970.)

676-11. MOTUPALLI SATYANARAYANA, Bowling Green State University, Bowling Green, Ohio 43402. A class of commutative semigroups in which idempotents are linearly ordered.

Let S be a commutative semigroup. S is called primary if every ideal in S is primary (the usual ring-theoretic definition). It is shown that idempotents form a chain in a primary semigroup. Clearly Archimedian semigroups are primary. For cancellative semigroups S is primary iff prime ideals are maximal. This result need not be true for noncancellative semigroups. But it is established that the only primary semigroups in which prime ideals are maximal are Archimedian. These observations led us to consider the interconnection between the semigroups in which prime ideals are maximal and the semigroups in which the idempotents form a chain and to determine when they are primary semigroups. A complete answer is found to this problem. (Received April 29, 1970.)

676-12. DEMETRE JOHN MANGERON and M. N. OGUZTORELI, University of Alberta, Edmonton 7, Alberta, Canada and V. F. POTERAŞU, Polytechnic Institute of Jassy, Iasi, Romania. On applications and theory of a new matrix-harmonic method.

The authors, starting from their previous research work devoted to the theory and applications of a new matrix-tensor method, subsequently used by various scientists (for instance, John J. Uicker, Jr., A. I. Jasiulionis, Milton Chase, P. A. Lebedev, C. Dragan, Chang Tsy-Siang, L. F. Litvin, M. F. Moroshkin,...) in their studies concerning the theory of mechanisms and the gearwheel transmission express the Fourier series $f(x) = a_0/2 + a_1 \cos x + b_1 \sin x + \ldots + a_n \cos nx + b_n \sin nx + \ldots$ in the form of the following matrix product $f(x) = \| A \| \{ T \}$, where $\| A \| = \| \ldots, a_n, \ldots, a_2, a_1, a_0/2, b_1, b_2, \ldots, b_n, \ldots \|$, $\{T\} = \{ \ldots, \cos nx, \ldots, \cos x, 1, \sin x, \ldots, \sin nx, \ldots \}$. This transcription allows us to obtain the matrix expressions concerning various operations with the Fourier series, the very suitable ones for the use of the DEC (digital electronic computers). A set of interesting applications concerning the mechanics of oscillations, the machine dynamics, and problems related with linear differential systems with variable periodic coefficients is also given. (Received April 29, 1970.)

676-13. ANDREW J. H. VOGT, University of Washington, Seattle, Washington 98105. On the linearity of generalized isometries.

Mazur and Ulam (see S. Banach, "Théorie des Opérations linéaires," Monografie Mat., PWN, Warsaw, 1932) proved that every isometry of one normed real vector space onto another is linear up to translation. If $\|fx - fy\| = \|x - y\|$ is replaced by $\|fx - fy\| = p(\|x - y\|)$ for an arbitrary function p, the same result holds true if the domain space is two-dimensional or more. If normed real vector spaces are replaced by real vector spaces equipped with nonsingular symmetric bilinear forms, isometries are again forced to be linear and in familiar cases the hypothesis of surjectivity may be dropped. (Received April 30, 1970.)

676-14. PETER C. -C. WANG, Statistics Department, Stanford University, Stanford, California 94305. On a theorem of Renyi and extensions.

Renyi (J. Appl. Probability 1(1964), 311-320) has considered an interesting model of traffic flow on a divided highway which extends to infinity in one direction without traffic lights or other inhomogeneities. It is assumed that each car travels at a constant speed which is a random variable and passing is always possible without delays. Among others, Renyi has obtained some asymptotic results regarding mainly the spatial distribution of cars along the highway when the temporal distribution of cars is assumed to be described by a Poisson process. It is the purpose of this paper to demonstrate that under Renyi's model, one can show that if the spatial distribution of cars is assumed to be Poisson process then the temporal distribution of cars (i.e., arrival times at some fixed position) is again Poisson process. This establishes the crucial structural property of Renyi's model on low-density traffic. (Received April 30, 1970.)

676-15. E. F. ECKLUND, JR., Washington State University, Pullman, Washington, 99163. A note involving hypothesis H.

The following conjectures are deduced as consequences of A. Schinzel's conjecture, hypothesis H. (A) There exist infinitely often eight consecutive odd numbers, each of which is the product of two distinct prime factors. (B) For every integer n, there exist infinitely many sets of n primes which are consecutive terms in an arithmetic progression. (Received April 30, 1970.)

676-16. WITHDRAWN.

ABSTRACTS PRESENTED TO THE SOCIETY

The next deadline for Abstracts will be June 30, 1970. The papers printed below were accepted by the American Mathematical Society for presentation by title. The abstracts are grouped according to subjects chosen by the author from categories listed on the abstract form. The miscellaneous group includes all abstracts for which the authors did not indicate a category.

One abstract presented by title may be accepted per person per issue of these *Notices*. Joint authors are treated as a separate category; thus, in addition to abstracts from two authors individually, one joint abstract by them may be accepted for a particular issue.

Algebra & Theory of Numbers

70T-A88. PHILIP KELENSON, Israel Institute of Technology, Haifa, Israel. Semi-primal algebras and clusters at most singular. Preliminary report.

We follow the terminology of Hu "On the fundamental subdirect factorization theorems of primal algebra theory," Math. Z. 112 (1969). A nontrivial algebra $\mathfrak{A}$ is primitive if for all $\mathfrak{B} \subseteq \mathfrak{A}$ either $\mathfrak{B} = \mathfrak{A}$ or $\mathfrak{B}$ is trivial. $\mathfrak{A}$ is normal if every nontrivial subalgebra of $\mathfrak{A}$ includes a primitive subalgebra of $\mathfrak{A}$. A normal algebra $\mathfrak{A}$ is regular if it has no one element subalgebra, singular if it has a smallest subalgebra of one element, and at most singular if it is either regular or singular. A pseudo-primal algebra is a locally semi-primal algebra, at most singular, with exactly one primitive subalgebra. Locally semi-primal algebras, at most singular, and their clusters are studied with particular emphasis on the pseudo-primal case. Characterizations and examples are provided which generalize results of Foster-Pixley, "Semi-categorical algebras. I," Math. Z. 83 (1964), 147-169; Foster, "Semi-primal algebras; characterization and normal-decomposition," Ibid. 99 (1967), 105-116; and Foster, "Normal semi-primals; a bounded constructive characterization," Ibid. (to appear). (Received January 12, 1970.) (Author introduced by Professor Stephan R. Cavior.)

70T-A89. LARRY R. NYHOFF, Calvin College, Grand Rapids, Michigan 49506. The influence on a finite group of the outer cofactors of its subgroups.

One can define outer cofactors of subgroups H in G as a kind of dual to the usual cofactors $H/\mathrm{cor}_G H$ and study their influence on G. The following are a few of the results obtained, extending some due to Deskins ("On maximal subgroups," Proc. Sympos. Pure Math., Amer. Math. Soc., 1959). Theorem. For θ a homomorphism-invariant subgroup-inherited property, if G has a θ-maximal subgroup having a θ-abnormal outer cofactor, or if the abnormal outer cofactors of each abnormal maximal subgroup of G are θ-groups, then $H/\mathrm{cor}_G H$ is a θ-group for all abnormal $H < G$. ("Self-normalizing" can replace "abnormal" throughout.) Results for ordinary cofactors (see Abstract 70T-A74, these *Notices* 17 (1970), 560) are then used. Theorem. The normal outer cofactors of a maximal subgroup S are isomorphic. Their order is called the normal index of S in G, $\eta(G:S)$. Theorem. G is p-solvable if-f (a) G has a p-solvable maximal subgroup with p-solvable normal outer cofactors, or (b) each (abnormal) maximal subgroup has p-solvable normal outer cofactors, or (c) $\eta(G:S)$ is p-power or prime to p for each (abnormal) maximal subgroup S of G. The intersection of the (abnormal) maximal subgroups not satisfying (c) is the largest normal p-solvable subgroup of G. Similar results are obtained for solvability. (Received February 2, 1970.)

70T-A90. GEORG J. RIEGER, Pötschener Str. 28, Gauting D-8035, Federal Republic of Germany and University of Munich, Federal Republic of Germany. On some binary Diophantine equations with infinitely many solutions. I.

Let c be a real number with $1 < c < 12/11$. Denote by $[A]$ the integer part of the real number A. Using van der Corput-type estimates of trigonometric sums, we prove that there exist infinitely many squares of the form $[x^c]$ with natural x; actually, we give an asymptotic formula for the number of natural numbers $x < t$ with $[x^c] = y^2$ for some natural y. Furthermore, for every natural number d, the ("Pell-like") equation $[x^c] - dy^2 = 1$ has infinitely many solutions. Also, every natural number $n > n(c)$ can be written (in many ways) as $n = [x^c] + y^2$ with natural x and y. (Received February 23, 1970.)

70T-A91. GEORGE A. GRÄTZER and H. LAKSER, University of Manitoba, Winnipeg 19, Manitoba, Canada. Some new relations on operators in general, and for pseudocomplemented distributive lattices in particular.

For a class K of algebras let $\underset{\sim}{H}(K)$, $\underset{\sim}{S}(K)$, $\underset{\sim}{P}(K)$, $\underset{\sim}{P}_P(K)$, and $\underset{\sim}{P}_S(K)$ denote the class of all algebras isomorphic to a homomorphic image, subalgebra, direct product, prime product, and subdirect product of algebras in K, respectively. Theorem 1. $\underset{\sim}{P}_P\underset{\sim}{P} \subseteq \underset{\sim}{P}_S\underset{\sim}{P}_P$. Corollary 1. $\underset{\sim}{S}\,\underset{\sim}{P}\,\underset{\sim}{P}_P(K)$ is the smallest implicational class containing K. (An implicational class, also called a quasivariety, is defined by a set of implications: $(x_1)(x_2)\ldots(x_n)\,(p_1 = q_1 \wedge p_2 = q_2 \wedge \ldots \wedge p_k = q_k) \rightarrow (p = q)$.) Corollary 2. If K consists of finitely many finite algebras, then $\underset{\sim}{S}\underset{\sim}{P}(K)$ is the smallest implicational class containing K. Corollary 3. There are implicational classes of pseudocomplemented distributive lattices which are not equational. For $\underset{\sim}{X}_1,\ldots,\underset{\sim}{X}_n \in \{\underset{\sim}{H},\underset{\sim}{S},\underset{\sim}{P}\}$ set $\underset{\sim}{X}_1\ldots\underset{\sim}{X}_n(K) = \underset{\sim}{X}_1(\ldots(\underset{\sim}{X}_n(K))\ldots)$, and $\underset{\sim}{X}_1\ldots\underset{\sim}{X}_n = \underset{\sim}{Y}_1\ldots\underset{\sim}{Y}_m$ in K if $\underset{\sim}{X}_1\ldots\underset{\sim}{X}_n(K_1) = \underset{\sim}{Y}_1\ldots\underset{\sim}{Y}_m(K_1)$ for all $K_1 \subseteq K$. Let $\mathcal{S}(K)$ denote the semigroup generated by $\underset{\sim}{H}$, $\underset{\sim}{S}$, $\underset{\sim}{P}$ over K. D. Pigozzi showed that $|\mathcal{S}(K)| \leqq 18$; S. Comer and J. S. Johnson showed that $|\mathcal{S}(B)| = 11$, where B is the class of Boolean algebras. Using the notation of Abstract 70T-A48, these *Notices* 17 (1970), 429, we prove Theorem 2. (i) $|\mathcal{S}(\mathcal{B}_i)| = 11$ for $i \leqq 2$, (ii) $|\mathcal{S}(\mathcal{B}_i)| = 12$ for $i = 3,4,\ldots$, (iii) $|\mathcal{S}(\mathcal{B}_\omega)| = 13$. (Received February 23, 1970.)

70T-A92. JOE W. FISHER, University of Texas, Austin, Texas 78712. Finite dimensional rings. Preliminary report.

Let R denote a ring with an identity and let Z(R) denote the left singular ideal of R. Theorem. If R has finite left dimension then R/Z(R) has both the ascending and the descending chain condition on left annihilators. In fact, each ascending and each descending chain of left annihilators in R/Z(R) has at most dim R proper inclusions. By making use of Herstein-Small's theorem which states that if a ring satisfies the ascending chain condition on both left and right annihilators then each nil subring of R is nilpotent, we obtain as a corollary the following theorem of Robert C. Shock [Abstract 672-638, these *Notices* 17 (1970), 267]. Corollary 1. If R has finite left dimension then each nil subring of R/Z(R) is nilpotent and has class of nilpotency $\leqq$ dim R + 1. In particular, if Z(R) is nilpotent and has class of nilpotency n, then each nil subring of R is nilpotent and has class of nilpotency $\leqq$ n(dim R + 1). Corollary 2 (Zelmanowitz). If R has finite left dimension and Z(R) = 0 then R has both the ascending and the descending chain condition on left annihilators. It should be noted that the proof of Shock's theorem which is obtained via this theorem and Herstein-Small's theorem is somewhat shorter than Shock's original proof. (Received February 26, 1970.)

70T-A93. M. V. SUBBARAO, University of Alberta, Edmonton, Alberta, Canada and M. VIDYASAGAR, Marquette University, Milwaukee, Wisconsin 53233. Two new combinatorial identities.

Let $\varphi(a,x) = \Pi_{n=1}^{\infty}(1 - ax^n)$; $\varphi_r(a,x) = \Pi_{n=1}^{r}(1 - ax^n)$ for r positive, and $\varphi_0(a,x) = 1$. We prove the following identities: I. $\varphi(a,x) = 1 - ax/(1 - x) + (a^2/(1 - x)) \Sigma_{r=1}^{\infty} x^{2r+1}\varphi_{r-1}(a,x)$; II. $1/\varphi(a,x) = 1 + ax/(1 - x) + (a^2/(1 - x)) \Sigma_{r=1}^{\infty} x^{2r}/\varphi_r(a,x)$. (Received February 27, 1970.)

70T-A94. C. R. HOWLETT, McMaster University, Hamilton, Ontario, Canada. Initial functors and algebraically small categories.

Let $T: \underline{Ens} \xrightarrow{op} \underline{\theta}$ be any algebraic theory over sets. The category of T-algebras in a category $\underline{A}$ is the full subcategory $(\underline{\theta}, \underline{A})$ of $\underline{Cat}(\underline{\theta}, \underline{A})$ determined by the product preserving functors. The underlying object functor $U_{\underline{A}}: (\underline{\theta}, \underline{A}) \to \underline{A}$ (evaluation at T1) preserves and constructs limits but examples of J. R. Isbell show that in general $U_{\underline{A}}$ need not have a left adjoint. Theorem. Let $H: \underline{A} \to \underline{B}$ be a functor which is faithful and initial (as in J. E. Roberts, J. Algebra 8 (1968), 181-193). Further suppose that $\underline{B}$ has products and that for each $b \in \text{Obj}\ \underline{B}$, $\{a \in \text{Obj}\ \underline{A}: Ha = b\}$ is a set. Then for each $a \in \text{Obj}\ \underline{A}$, if $U_{\underline{B}}$ admits a solution to its universal mapping problem at Ha, this solution lifts through H to a solution of the u.m.p. of $U_{\underline{A}}$ at a. Corollaries. (1) If $U_{\underline{B}}$ has a left adjoint $F_{\underline{B}}$, so does $U_{\underline{A}}$ and the latter may be chosen such that $(\underline{\theta},H)F_{\underline{A}} = F_{\underline{B}}H$. (2) If $\underline{B}$ is (finitely, boundedly) algebraically small (as in R. C. Davis, Dissertation, Tulane University, New Orleans, La., 1968) then $\underline{A}$ has the corresponding property. As applications, the categories of topological spaces, limit spaces, uniform spaces, and L-fuzzy sets (for complete lattices L) are algebraically small. (Received March 2, 1970.) (Author introduced by Professor Bernhard Banaschewski.)

70T-A95. GRAHAM F. LORD, Temple University, Philadelphia, Pennsylvania 19122. Two elementary partition identities. Preliminary report.

Let p(n) denote the number of partitions of n without restrictions and q(n) the number of partitions of n into distinct parts. Then $\Sigma(-1)^k p(n - k)$ = the number of partitions of n into distinct and even parts, where the sum is over all triangular numbers k such that $0 \leq k \leq n$. Hence for n even this sum is q(n/2); for n odd the sum is zero. Again summing over triangular numbers k such that $0 \leq k \leq n$, $\Sigma(-1)^k q(n - k) = 0$ unless $n = 2m(3m \pm 1)$ in which case the sum is equal to $(-1)^m$. Both identities are proven from an identity due to Jacobi and by considering the generating functions of p(n) and q(n). (Received March 4, 1970.)

70T-A96. BRUNO J. MÜLLER, McMaster University, Hamilton, Ontario, Canada. Duality for linear-topological modules.

For rings S and R with a Morita duality induced by a bimodule ${}_SU_R$ (K. Morita, Sci. Rep. Tokyo Kyoiku Daigaku Sect. A. 6 (1958), 83-142) we construct a duality between the categories of linear-topological Hausdorff modules, as follows: The dual X* is the module of continuous homomorphisms of the module X into U, carrying some linear topology equivalent to the finite topology. (Linear topologies on the same module are called equivalent if they have the same submodules closed.) Then the evaluation map $X \to X^{**}$ is a topological isomorphism up to equivalence, for all X. The classical

approach would be to select, from the equivalence class of the finite topology on each X^*, one representative topology in a consistent fashion. For any two such selections for R- and S-modules, each equivalence class contains at least one reflexive topology, and we conjecture that it never contains more. (This would mean that the particular selections are rather irrelevant.) We prove the conjecture for vector spaces (slightly more general for semisimple artinian S and R). Our results generalize and unify previous theories by S. Lefschetz, I. Kaplansky, H. Leptin and I. G. MacDonald (cf. Topology 1 (1962), 213-235). (Received March 5, 1970.)

70T-A97. ROBERT L. GRIESS, JR., University of Chicago, Chicago, Illinois 60637. A sufficient condition for a finite group of even order to have nontrivial Schur multiplicator.

There are various well-known techniques for giving upper bounds on the order or rank of the Schur multiplicator of a finite group. Establishing nontriviality, however, is often difficult because ad hoc methods are usually demanded. Proposition. Let G be a group of even order with no subgroup of index two. Suppose π is a faithful permutation representation of degree n, $n \geqq 4$, and that there is an involution i of G such that (i)π moves m letters, $m \equiv 4 \pmod 8$. Then there is a group H with a subgroup $B \leqq Z(H) \cap H'$, $|B| = 2$, $H/B \cong G$. This general criterion instantly gives nontriviality for M_{12}, $Sp_6(2)$, $G_2(4)$, the commutator subgroup of the Weyl group of E_8, the Hall-Janko group, the Higman-Sims group, and the Suzuki group of order $2^{13}3^7 5^2 7.11.13$. (Received March 5, 1970.)

70T-A98. RAYMOND E. SMITHSON, University of Wyoming, Laramie, Wyoming 82070. Fixed points of order preserving multifunctions.

Let $F: X \to X$ be a multifunction on a partially ordered set $(X, \leqq)$. Suppose for each pair $x_1 \leqq x_2$ and for each $y_1 \in F(x_1)$ there is a $y_2 \in F(y_2)$ such that $y_1 \leqq y_2$. Then sufficient conditions are given such that multifunctions F satisfying the above condition will have a fixed point. These results generalize the Tarski Theorem on complete lattices, and they also generalize some results of S. Abian and A. B. Brown (Canad. J. Math. 13 (1961), 78-82). By similar techniques two selection theorems are obtained. Further, some related results on quasi-ordered and partially ordered topological spaces are proved. In particular, a fixed point theorem for order preserving multifunctions on a class of partially ordered topological spaces is obtained. (Received March 6, 1970.)

70T-A99. WITHDRAWN.

70T-A100. KWANGIL KOH, North Carolina State University, Raleigh, North Carolina 27607. On one sided ideals of a prime type.

A right ideal I of a ring R is defined to be of a prime type if and only if $I \neq R$ and $AB \subseteq I$, A, B are right ideals, then either $A \subseteq I$ or $B \subseteq I$. By a chain of right ideals of a prime type we mean a finite strictly increasing sequence $P_0 \subset P_1 \subset \ldots \subset P_n$; the length of the chain is n. We define, the right dimension of a ring R, $\dim_r R$ to be the supremum of the length of all chains of right ideals of a prime type in R: it is an integer $\geqq 0$, or ∞. The left dimension of R, $\dim_\ell R$, is similarly defined. The main theorems in the present paper are as follows: (1) $\dim_r R = 0$ if and only if $\dim_\ell R = 0$ if and only if R modulo the prime radical is a strongly regular ring. (2) $\dim_r R = 0$ and R is right noetherian if

and only if R is right artinian and every right ideal of a prime type is a left ideal. (3) If R is a (right) primitive ring then $\dim_r R = n$ if and only if R is isomorphic to the $n \times n$ matrix ring over a division ring. (4) Every right ideal I of a prime type in a ring R is a left ideal if and only if, for any right ideal I of a prime type, the endomorphism ring of the quasi-injective hull of the right R-module R/I is a division ring and every nonzero submodule of R/I contains an isomorphic image of R/I.

(Received March 12, 1970.)

70T-A101. PAUL E. BLAND, Eastern Kentucky University, Richmond, Kentucky 40475.

A note on rationally complete modules.

All rings are associative rings with identity and all modules are unital. In [Canad. Math. Bull. 1 (1958), 77-85, 155-167] Findlay and Lambek introduced a ternary relation $\leqq$ among three right R-modules A, B and M. They wrote $A \leqq B(M)$ if A is a submodule of B and if for each $b \in B$ and each $m \in M$, $m \neq 0$, there exists an $r \in R$ such that $br \in A$ and $mr \neq 0$. Via this ternary relation they introduced a weakened form of injectivity--namely, rational completeness. The purpose of this note is to show that R is a test module for rational completeness. Definition. If M is a right R-module and K a right ideal of R, then K is said to be M-dense in R if $K \leqq R(M)$. Theorem. A right R-module M is rationally complete if and only if for each M-dense right ideal K of R and each $f \in \mathrm{Hom}_R(K,M)$ there exists an $m \in M$ such that $f(x) = mx$ for all $x \in K$. The proof is patterned after that of Baer's condition.

(Received March 19, 1970.)

70T-A102. HYMIE LONDON, McGill University, Montreal 110, Quebec, Canada. On the Diophantine equation $y^2 + 289 = x^3$. Preliminary report.

The only integer solution of the Diophantine equation $y^2 + 289 = x^3$, y positive, is $x = 17$, $y = 68$.

(Received March 13, 1970.)

70T-A103. MORRIS JACK DeLEON, Florida Atlantic University, Boca Raton, Florida 33432.

Characterizing the number theoretic functions which are polynomials.

A function defined on the positive integers is called a number theoretic function. Theorem. Let f(m) be a number theoretic function not identically equal to a constant and define $d_1(m) \equiv f(m+1) - f(m)$ and $d_{s+1}(m) \equiv d_s(m+1) - d_s(m)$ for $s = 1,2,\ldots$. There exists a polynomial $p(m) = p_0 + p_1 m + p_2 m^2 + \ldots + p_{n-1} m^{n-1} + p_n m^n$, where $p_n \neq 0$, such that $p(m) = f(m)$ for all positive integers m iff $d_n(m) \equiv c$ for some nonzero constant c. In fact, if p(m) exists, $p_n = c/n!$. Two of the lemmas used to prove this theorem are: Lemma 1. For every nonnegative integer n there exists a polynomial t(x) of degree n + 1 such that $t(x+1) - t(x) = x^n$. Lemma 2. If $d(x) \not\equiv 0$ is a polynomial of degree n - 1 then there exists a polynomial h(x) of degree n such that $h(x+1) - h(x) \equiv d(x)$.

(Received March 16, 1970.)

70T-A104. WILLIAM C. BROWN, Michigan State University, East Lansing, Michigan 48823.

A note on strong inertial coefficient rings. Preliminary report.

Let R be a commutative ring with identity and Jacobson radical p. Call R split if there exists a ring homomorphism $j: R/p \to R$ such that $\pi_0 j$ is the identity on R. Here π_0 denotes the projection of R onto R/p. The pair (R,j) is a strong inertial coefficient ring if any R-algebra A which is finitely

generated as an R-module and has A/N separable over R admits an (R/p)-algebra homomorphism $\zeta : A/N \to A$ such that $\pi\zeta$ is the identity on A. Here N and π denote the Jacobson radical of A and the projection of A onto A/N respectively. Using the main theorem of [W. C. Brown and E. Ingraham "A characterization of semilocal inertial coefficient rings", Proc. Amer. Math. Soc. (to appear)], one can completely determine all strong inertial coefficient rings. Theorem. The pair (R,j) is a strong inertial coefficient ring iff R is a finite direct sum of split Hensel rings. (Received March 9, 1970.) (Author introduced by Professor Edward C. Ingraham.)

70T-A105. EDWARD TZU-HSIA WANG, University of British Columbia, Vancouver 8, British Columbia, Canada. Diagonal sums of doubly stochastic matrices.

Let Ω_n denote the convex polyhedron of all doubly stochastic (d.s.) matrices. Theorem 1. Let $A,B \in \Omega_n$. Suppose more than $(n-1)(n-1)!$ corresponding diagonals of A and B have the same diagonal sums, then A = B. Theorem 2. Let $A \in \Omega_n$. Suppose more than $(n-1)(n-1)!$ diagonals of A have the same diagonal sums, then $A = J_n$, the d.s. matrix with all entries equal 1/n. Theorem 3. Let $A \in \Omega_n$, then $\Pi_\sigma \Sigma_{i=1}^n a_{i\sigma(i)} \leq 1$ with equality iff $A = J_n$, where the product is taken over all permutations σ of $\{1,2,\ldots,n\}$. Remark. Examples can be constructed to show that the number $(n-1)(n-1)!$ in Theorems 1 and 2 is best possible. (Received March 16, 1970.)

70T-A106. JOHN A. WRIGHT, University of Rochester, Rochester, New York 14627. There are 718 6-point topologies, quasi-orderings, and transgraphs.

The numbers of topologies, transitive directed graphs, and reflexive transitive relations on n points are the same. These relations may be denoted by certain (0,1)-matrices. The T_0 topologies correspond to the antisymmetric relations (partial orderings). Evans, Harary and Lynn, in Comm. ACM (1967) counted all these relations by computer for n distinct points, $n \leq 7$. We have counted the equivalence classes by computer for $n \leq 6$. Let h_n = number of classes; h_n^c = connected classes; h_n^0 = T_0 classes; h_n^{c0} = connected T_0 classes. Their values are respectively, for n = 5: 139, 94, 63, 44, and for n = 6: 718, 512, 336, 238. We use the following recursions: Let a partition of n be represented by $n = \Sigma r_i n_i$, $n_1 < \ldots < n_k$, $r_i \geq 1$. Then $h_n = \Sigma \Pi_{i=1}^k \mathrm{Comb}(h_{n(i)}^c + r_i - 1; r_i)$ where $\mathrm{Comb}(a; b) = a!/b!(a-b)!$ and summation is over all partitions of n. A similar relation holds between h_n^0 and $h_{n(i)}^{c0}$. A list of the connected 6-point quasi-orderings, in the form of diagrams, is available on request. (Received March 17, 1970.) (Author introduced by Professor Arthur H. Stone.)

70T-A107. BRIAN J. WINKEL, Indiana University, Bloomington, Indiana 47401. Orders in Noetherian rings. Preliminary report.

All rings have regular elements and all one sided conditions are left. Definition. A ring R is I-quorite for some ideal I if whenever a and b are elements of R with b regular and a in I there exist a' in I and b', regular in R, such that a'b = b'a. Definition. A ring R is I-Ore for some ideal I if whenever a and b are elements of R with b regular in R there exist a' and b' in R with b' regular in R such that a'b - b'a is in I. Noetherian rings which are orders are characterized as a corollary to the following: Theorem. Let R be a ring and N the sum of all nilpotent ideals of R. Then R is an order in a Noetherian ring if and only if (i) R is N-quorite, (ii) R is N-Ore, and (iii) R satisfies the as-

cending chain condition on closed left ideals. Using the method of proof in the theorem, the Ore condition is generalized as follows: Theorem. Let R be a ring and I a closed right ideal of R. Then R is an order in a ring Q(R) if and only if (i) R is I-quorite and (ii) R is I-Ore. (Received March 19, 1970.) (Author introduced by Professor Goro Azumaya.)

70T-A108. DAVID R. CECIL, Butler University, Indianapolis, Indiana 46208. Partial homotopisms.

Let G and $\bar{G}$ be two groups with identities e and $\bar{e}$ respectively. If $g: G \to \bar{G}$ is such that $g(xy) = g(x)[g(e)]^{-1} g(y)$ for all x, y in G then g is called a partial homotopism (p-homotopism). Any p-homotopism g is part of a homotopism triple (g, g_1, h), i.e. there exist g_1, $h = G \to \bar{G}$ such that $h(xy) = g(x)\, g_1(y)$ for all x, y in G. The p-homotopisms from G to G form a monoid under composition of functions having the constant functions for right zeros. Theorem 1. If g is a p-homotopism from G to $\bar{G}$ then kernel g (denoted K(g) and equal to $\{x \text{ in } G: g(x) = g(e)\}$) is a normal subgroup of G; and G/K(g) is p-homotopic to $\bar{G}$ if g is onto. Theorem 2. If $g(x_0) = \bar{e}$ then $\langle K(g) \cup \{x_0\} \rangle$ is equal to $\{x \text{ in } G: g(x) \text{ is in } \langle g(e) \rangle\}$. Corollary. If H is a subgroup of G containing K(g) and if $g(x_0) = \bar{e}$ for some x_0 in H then H contains all the preimages of the subgroup of $\bar{G}$ generated by g(e). (Received March 24, 1970.)

70T-A109. WITHDRAWN.

70T-A110. MOHAN SHARAD SHRIKHANDE, Indiana University, Bloomington, Indiana 47401. Hereditary and cohereditary modules. Preliminary report.

Let M be an R-module. We use the notion of M-projectivity and M-injectivity in the sense of G. Azumaya ("M-projective and M-injective modules," Trans. Amer. Math. Soc. (to appear)). Definition. M is hereditary (resp. semihereditary) if every (resp. f.g.) submodule of M is projective. Definition. M is cohereditary (resp. semicohereditary) if every (resp. co-f.g.) quotient of M is injective. (M is called co-f.g. if it has a f.g. essential socle; see Jans, J. London Math. Soc. (2) 1(1969), 588). Theorem 1. The following are equivalent for a projective module M: (i) M is hereditary; (ii) Every quotient of an M-injective module is M-injective. Theorem 2. Dual of Theorem 1. Similar characterizations of semihereditary (resp. semicohereditary) modules are obtained. Theorem 3. Let R be left artinian. Then the following are equivalent: (i) R is left hereditary; (ii) There exists a faithful hereditary module; (iii) There exists a faithful cohereditary module. (Received March 31, 1970.)

70T-A111. MICHAEL G. STONE, University of Calgary, Calgary 44, Alberta, Canada. A concrete characterization of automorphism and subalgebra structure in universal algebras. Preliminary report.

It is well known that given any abstract group G and any algebraic closure structure L there is a universal algebra $\mathfrak{A}$ with automorphism group Aut $\mathfrak{A}$ isomorphic to G and subalgebra structure Su($\mathfrak{A}$) isomorphic to L. If G is a group of permutations on some fixed set B, and L an algebraic closure structure on B, one can ask if there is an algebra $\mathfrak{B}$ over the set B for which the automorphism group Aut $\mathfrak{B}$ and the subalgebra structure Su($\mathfrak{B}$) are equal to G and L respectively. For G and

L to meet these criteria separately it is necessary and sufficient that G be locally closed, and only that L be an arbitrary algebraic closure structure. In order to jointly satisfy these conditions for a single algebra $\mathfrak{B}$, certain other obvious necessary conditions must be met. The exact nature of the interplay between Aut $\mathfrak{B}$ and Su $\mathfrak{B}$ in this context is given by the following Theorem. If $G \subseteq B^B$ is a group of permutations of B and $L \subseteq \mathfrak{P}(B)$ is an algebraic closure system, then there is an algebra with base set B such that G = Aut $\mathfrak{B}$ and L = Su $\mathfrak{B}$ iff the following conditions are satisfied: (1) G is locally closed (i.e. if a permutation $\varphi \in B^B$ agrees with some member of G on each finite subset of B then $\varphi \in G$). (2) $C \in L, \varphi \in G \Rightarrow \varphi(C) \in L$. (3) If $\varphi, \psi \in G$ agree on some finite $X \subseteq B$ then φ, ψ agree on the closure of X as well. (Received March 24, 1970.)

70T-A112. EUGENE W. JOHNSON and JOHN P. LEDIAEV, University of Iowa, Iowa City, Iowa 52240. A new characterization of Dedekind domains.

Let R be a Noetherian ring. An ideal A satisfies the cancellation law if AB = AC and $A \neq 0$ imply that B = C. It is well known that a Noetherian domain is a Dedekind domain if and only if each of its nonzero ideals satisfies the cancellation law. We show that it is sufficient to assume cancellability only for the maximal ideals. Theorem. A Noetherian ring R is a Dedekind domain if and only if every maximal ideal of R satisfies the cancellation law. (Received April 6, 1970.)

70T-A113. DAVID L. WINTER, Michigan State University, East Lansing, Michigan 48823. p-solvable linear groups of finite order. Preliminary report.

Theorem 1. Let p be an odd prime and let G be a finite p-solvable linear group which has a faithful representation of degree n over the complex number field. Let P be a p-Sylow subgroup of G. Then $|P: O_p(G)| \leq p^{\lambda}$ where $\lambda = \sum_{i=0}^{\infty} [n/\{p^i(p-1)\}]$. For certain cases it is possible to improve the result. Theorem 2. Let G satisfy the hypotheses of Theorem 1 with p non-Fermat and $n < p(p-1)$. Then $|P : O_p(G)| \leq p^{[n/p]}$. (Received April 9, 1970.)

70T-A114. STANLEY N. BURRIS, Department of Pure Mathematics, University of Waterloo, Waterloo, Ontario, Canada. Results on the equational theory of algebras.

Let τ denote a type of universal algebras and let μ be the associated multiplicity type. Theorem. (i) If $m_i = 0$, $i \geq 1$, then $A(\tau)$, the number of equationally complete systems, is 1; (ii) if $m_i = 0$ for $i \geq 2$ and $m_1 = 1$, then $A(\tau) = 2$; (iii) if $m_i = 0$ for $i \geq 2$ and $m_1 \geq 2$, then $A(\tau) = \mathrm{Max}\,(2^{\aleph_0}, 2^{m_1})$; (iv) if $m_i \neq 0$ for some $i \geq 2$, then $A(\tau) = \mathrm{Max}\,(2^{\aleph_0}, 2^{|O(\tau)|})$. This solves Problem 33 of Grätzer's "Universal algebra". Theorem. If $m_0 \geq \aleph_0$ or $m_1 \geq 2$ or $m_2 \geq 1$, then $\mathcal{L}(\tau)$ contains the dual of Π_∞. These results generalize Abstract 70T-A86, these *Notices* 17(1970), 564. (Received April 22, 1970.)

70T-A115. PATRICK J. FLEURY, McGill University, Montreal, 110, Quebec, Canada. Aspects of Harrison's homology theory.

Let R be a commutative algebra over the ring k, and let M be a left R-module. If k is a field of characteristic p, it is shown that there is a natural split epimorphism (monomorphism) e_n: $\mathrm{Hoch}_n(R,M) \to \mathrm{Harr}_n(R,M)$ (e^n: $\mathrm{Harr}^n(R,M) \to \mathrm{Hoch}^n(R,M)$) from Hochschild's (Harrison's) homo-

logy (cohomology) theory to Harrison's (Hochschild's) homology (cohomology) theory for $1 \leq n \leq p - 1$. It is further shown that by modifying Harrison's complex, there is a more complicated splitting of Hochschild's complex in all dimensions. Afterwards, it is found that $\mathrm{Harr}_n(R,M) = 0$ $(\mathrm{Harr}^n(R,M) = 0)$ if R is a polynomial algebra and $1 < n < 2p - 1$. If k is a ring containing the rationals, Harrison's theory coincides with the theory afforded by the symmetric algebra cotriple. (Received April 13, 1970.)

70T-A116. MATTHEW I. GOULD, University of Illinois at Chicago Circle, Chicago, Illinois 60680 and CRAIG R. PLATT, University of Manitoba, Winnipeg 19, Manitoba, Canada. Versatile monoids and versatile categories.

For terminology, see Abstract 70T-A40, these *Notices* 17(1970), 427. Theorem. A monoid M is versatile if and only if every element of M is either left cancellative or a left zero. Versatile categories are defined, and a similar characterization of versatile small categories is established. (Received April 13, 1970.)

70T-A117. C. R. HOWLETT and D. SCHUMACHER, McMaster University, Hamilton, Ontario, Canada. A class of finitely algebraically small categories.

The following result is established: if A is a cocomplete category with finite products and each of whose product endofunctors $a \times (\ )$, $a \in \mathrm{obj}(A)$, preserves injective limits then A is finitely algebraically small. That is to say, for any theory $T: S_\omega^{op} \to \Theta$ over the category S_ω of finite sets, the underlying object functor $A^T \to A$ of the category of models of T in A (i.e. product preserving functors $\Theta \to A$) has a left adjoint. For each $a \in \mathrm{obj}(A)$, the underlying object of the free algebra on a is constructed as the injective limit of a functor $D: (T,T1) \to A$. Here (T, T1) is the comma category, and D is defined as follows: for $\omega \in \Theta(Tn, T1)$, $D(\omega) = a^n$; for $\omega' \in \Theta(Tn', T1)$ and $f: n \to n'$ in S_ω with $\omega \circ Tf^* = \omega'$, put $D(f^*) = a^f: a^{n'} \to a^n$. An immediate corollary to this result is the fact that cartesian closed categories are finitely algebraically small. (Received April 13, 1970.) (Author introduced by Professor Eckehart Hotzel.)

70T-A118. WILLIAM G. BROWN, McGill University, Montreal 110, Quebec, Canada and JOEL L. BRENNER, University of Arizona, Tucson, Arizona 85721. On a hierarchy of generalized diagonal dominance properties for complex matrices.

This article concerns dominance conditions for an $n \times m$ matrix. In the simplest kind of dominance, the (absolute) value of the diagonal element exceeds the sum of the absolute values of the nondiagonal elements on the same row. This condition has been generalized in the literature in several ways, of which we consider ways in which the rows of the matrix cooperate. Our work amounts to a sorting out of certain dominance conditions that belong to a class $\mathcal{C}$ of dominance conditions. We prove a theorem characterizing all true statements of the form $C_1, C_2, \ldots, C_s \Rightarrow C_0$ where $C_i \in \mathcal{C}$ $(i = 0,1,\ldots,s)$. (Received April 14, 1970.)

70T-A119. JANICE R. ZEMANEK, University of Illinois, Urbana, Illinois 61801. Nilpotent elements in modular representation rings. Preliminary report.

Let G be a finite group, k any field of characteristic p, $p \neq 0$. The representation ring a(kG) is

defined as the free abelian group generated by symbols $[M]$, one for each isomorphism class of kG-modules, with relations $[M] = [M'] + [M'']$ whenever $M \cong M' \oplus M''$. Mulitplication is given by $[M][N] = [M \otimes_k N]$. Theorem. Let $p > 2$, and suppose that G has a noncyclic Sylow p-subgroup. Then a(kG) contains nonzero nilpotent elements. The proof depends on establishing the corresponding fact for a(RG), R = p-adic integers, and then using the obvious homomorphism $a(RG) \to a(\bar{R}G)$, where $\bar{R} = R/pR$. (Received April 15, 1970.)

70T-A120. DIETER HELD, Johannes Gutenberg-Universität, 6500 Mainz, Saarstrasse 21, Federal Republic of Germany and ULRICH F. SCHOENWAELDER, University of Missouri, St. Louis, Missouri 63121. A characterization of the simple group M_{24}.

Let E_0 be an elementary abelian group of order 16 and H_0 the centralizer of an involution of E_0 in the holomorph of E_0. It is known [D. Held, J. Algebra 13(1969), 253-296] that the groups M_{24}, $L_5(2)$, H_2 are the only finite simple groups G with an involution z whose centralizer $C_G(z)$ is isomorphic to H_0. In this situation $C_G(z)$ is a (splitting) extension of an extra-special group of order 2^7 by $L_2(7)$ and contains a Sylow 2-subgroup of G. The authors now prove that M_{24}, $L_5(2)$, H_2 are the only finite simple groups with a 2-central involution z whose centralizer $C_G(z)$ is an extension of an extra-special group of order 2^7 by $L_2(7)$. (Received April 8, 1970.)

70T-A121. BILL J. DULIN, Texas A & M University, College Station, Texas 77843. Embedding of ideals into the Kronecker function ring.

The purpose of this paper is to characterize an integrally closed domain D whose ideal structure can be embedded into its Kronecker function ring D*. We show that an ideal A of D is its own completion when A is finitely generated and $AD^* \cap D = A$. Our main result states the following are equivalent: (1) $AD^* \cap D = A$ for each ideal A of D, (2) $AD^* \cap A = A$ for each finitely generated ideal of D, (3) is Prüfer, (4) each torsion-free D-module is flat, and (5) D* is flat. This yields that D* is not flat when D is not Prüfer. (Received April 16, 1970.)

70T-A122. DAVID M. FOSTER, University of British Columbia, Vancouver 8, British Columbia, Canada. A general Cartan theory. Preliminary report.

Let U be a universal class of algebras over a field F, $f = f(x_1,\ldots,x_{n+1})$ be a linear homogeneous polynomial over F, $A \in U$, and $\dim A = m < \infty$. If $b_1,\ldots,b_n \in A$, we define a transformation $S(b_1,\ldots,b_n)$ of A by $xS(b_1,\ldots,b_n) = f(x,b_1,\ldots,b_n)$ for $x \in A$. A is f-nilpotent if for some k, $S(b_1,\ldots,c_1)\ldots S(b_k,\ldots,c_k) = 0$ for all $b_i,\ldots,c_i \in A$. A is f-nil if $S(b,\ldots,b)^m = 0$ for all $b \in A$. Set $B_b = \{x \in A: xS(b,\ldots,b)^m = 0\}$, and call B_b minimal Engel if the dimension of B_b is minimal. If R is a subalgebra of A, let $L_A(R)$ denote the Lie algebra generated by $\{S(b_1,\ldots,b_n): b_i \in R\}$. Call f an Engel function for U if A is f-nil iff A is f-nilpotent, for all $b \in A$, B_b is a subalgebra of A containing b, if R is a f-nilpotent subalgebra of A, $L_A(R)$ is nilpotent and the Fitting null component (FNC) of A relative to $L_A(R)$ is a subalgebra. Call H a Cartan subalgebra of A iff H is f-nilpotent and coincides with the FNC of A relative to $L_A(H)$. Following Barnes (Math. Z. 101 (1967)) we prove Theorem. Suppose f is an Engel function for U, $b \in A \in U$, F has at least n(dim A) elements, and B_b is minimal Engel. Then B_b is a Cartan subalgebra. Corollary. In the setting of the theorem, A contains Cartan subalgebras. (Received April 16, 1970.)

70T-A123. KARL K. NORTON, University of Michigan, Ann Arbor, Michigan 48104. A character-sum estimate and some applications. Preliminary report.

Let n, h be positive integers, and let χ be any nonprincipal Dirichlet character mod n. Write $S(n,h,\chi) = \sum_{m=1}^{n} |\sum_{x=1}^{h} \chi(x+m)|^2$. D. A. Burgess has proved that $S(n,h,\chi) \leq nh\{d(n)\log n\}^2$, where d(n) is the number of positive divisors of n (see my paper in J. Number Theory 1(1969), 398-418). Here we obtain the stronger result $S(n,h,\chi) < (7/6)nh$, which is best possible except for the constant. The proof is elementary but rather complicated, requiring a lemma due to Burgess and some ideas of Hooley. The constant 7/6 cannot be replaced by a constant less than 1, although it can be reduced a little. For an application, let k be a positive integer, C(n) the coprime residue-class group mod n, $C_k(n)$ the subgroup of kth powers, $\nu = [C(n): C_k(n)]$, $\alpha = \varphi(n)/\nu$, $h_0 < h_1 < \ldots < h_\alpha$ the $\alpha + 1$ smallest positive members of any given coset of $C_k(n)$ in C(n), and $\mathfrak{S}(n,\beta) = \sum\{(h_j - h_{j-1})^\beta : 1 \leq j \leq \alpha\}$. For any real $\beta \geq 1$, we have $\mathfrak{S}(n,\beta) \geq n(n/\alpha)^{\beta-1}$. Also, $\mathfrak{S}(n,\beta) = O_\beta(n(n/\alpha)^{2\beta-2})$ for $1 \leq \beta < 2$, and $\mathfrak{S}(n,2) = O(n(n/\alpha)^2 \log n)$. This improves some results in the paper mentioned above. (Received April 20, 1970.)

70T-A124. ELIZABETH WALL, Yale University, New Haven, Connecticut 06520. Groups admitting fixed-point-free automorphisms of order rs. Preliminary report.

Theorem I. Let G be a finite group admitting a fixed-point-free automorphism of order rs, where r and s are distinct primes. Assume that if G has even order, then $2^n + 1$ does not divide rs for any integer n and neither r nor s is equal to 2. Then G is solvable. (Received April 20, 1970.)

70T-A125. LOUIS J. RATLIFF, JR., University of California, Riverside, California 92502. Catenary rings and the altitude formula. Preliminary report.

Let (R,M) be a local domain. Theorem 1. R is catenary if and only if, for each prime ideal P in R, height P + depth P = altitude R. Theorem 2. R satisfies the altitude formula if and only if, for each locality (S,N) over R such that S dominates R, altitude S + trd (S/N)/(R/M) = altitude R + trd S/R. Half of each theorem is well known. The other half of Theorem 1 solves an open problem of long standing, and the other half of Theorem 2 shows the equivalence of the two definitions of altitude formula which have appeared in the literature. The terminology is the same as that in Abstracts 68T-299, these *Notices* 15(1968), 388, and 70T-A58, these *Notices* 17(1970), 432, and the proofs of these two theorems are based on results reported in these abstracts. (Received April 20, 1970.)

70T-A126. STANLEY N. BURRIS, University of Waterloo, Waterloo, Ontario, Canada and EVELYN M. NELSON, McMaster University, Hamilton 16, Ontario, Canada. The lattice of equational classes of commutative semigroups.

For notation and definitions see Abstract 69T-A92, these *Notices* 16(1969), 661. Let Π_N be the partitions of the set $\{1,2,\ldots,N\}$, and for $\pi \in \Pi_N$ define $\Sigma(\pi)$ to be the set of equations $\{((m,2N+1-m),(n,2N+1-n)): m,n \in A \text{ for some } A \in \pi\}$. Let e be $((a_1,\ldots,a_{4N+4}),(b_1,\ldots,b_{4N+4}))$ where $a_1 = \ldots = a_{2N+2} = b_{2N+3} = \ldots = b_{4N+4} = 1$ and $b_1 = \ldots = b_{2N+2} = a_{2N+3} = \ldots = a_{4N+4} = 0$. Using $[\ldots]$ to denote the deductive closure of a system of equations, let $\Gamma(\pi) = [\Sigma(\pi)] \cup [e]$. Theorem. The mapping $\pi \to \Gamma(\pi)$ is an embedding of the dual of Π_N into the lattice of equational

classes of commutative semigroups. Corollary. The lattice of equational classes of commutative semigroups does not satisfy any special lattice laws. (Received April 22, 1970.)

70T-A127. WALTER F. TAYLOR, University of Colorado, Boulder, Colorado 80302. The structure of compact algebras. Preliminary report.

Using similar methods, we continue the results of (Abstract 674-80, these *Notices* 17(1970), 544), to which we refer the reader for background information. Theorem. If $\mathfrak{A}$ is an equationally compact algebra, then there exist homomorphisms $p_i: \mathfrak{A} \to \mathfrak{A}_i$ $(i \in I)$, such that (1) each p_i is a retraction onto a subalgebra of $\mathfrak{A}$, (2) each $\mathfrak{A}_i$ is equationally compact, (3) each $\mathfrak{A}_i$ has power $\leq 2^n$, where n is the power of the first-order language of $\mathfrak{A}$, and (4) the map defined componentwise by the homomorphisms p_i is an isomorphism of $\mathfrak{A}$ onto a subalgebra of the product of the algebras $\mathfrak{A}_i$, onto which subalgebra the product retracts. If $\mathfrak{A}$ is a (totally disconnected) compact topological algebra, then we may replace (1) by (1') each $\mathfrak{A}_i$ is (totally disconnected) compact topological, with p_i continuous. Similar theorems are given for atomic-compact (topologically compact) relational structures. (Received April 22, 1970.)

70T-A128. PRABHA GAIHA, Northwestern University, Department of Industrial Engineering, Evanston, Illinois 60201. Spectral properties of complex matrices. Preliminary report.

Perron-Frobenius theory of nonnegative matrices has recently been extended by Vandergraft to matrices which leave invariant a solid pointed cone in R^n. It is pointed out here that his results can be carried over to complex matrices which leave invariant a solid pointed cone in C^n. (Received April 20, 1970.) (Author introduced by Professor Adi Ben-Israel.)

Analysis

70T-B102. SAMUEL ZAIDMAN, Université de Montréal, Montréal, Quebec, Canada. A result of weak almost-periodicity.

Let X be a Banach space, T_t, $t \geq 0 \to \mathfrak{C}(X,X)$ be a strongly continuous one parameter semigroup such that $\lim_{t\to\infty} T_t x = 0$, $\forall x \in X$. Suppose that there is a compact operator Q, $X \to X$ such that $QT_t = T_t Q$, $t \geq 0$, Q^{-1} exists on a dense domain in X and $(Q^{-1})^*$ exists on a dense domain in X^*. Let A be the infinitesimal generator of T_t, and $u(t)$, $-\infty < t < +\infty \to D(A)$ be a strong solution of the equation $u'(t) = Au(t) + f(t)$, where $f(t)$ is a weakly almost-periodic X-valued function. Then, if $\sup_{-\infty<t<\infty} \|u(t)\| = K < \infty$, $u(t)$ is weakly almost-periodic too. (Received November 14, 1969.)

70T-B103. ANDRE DE KORVIN and RICHARD J. EASTON, Indiana State University, Terre Haute, Indiana 47809. A generalized Lebesgue-type integral and a representation theorem.

See the previous *Notices* for the setting. In this more general setting a Lebesgue-type integration theory is developed along lines similar to those of R. J. Easton and D. H. Tucker [Abstract 642-153, these *Notices* 14 (1967), 108]. Some improvements are obtained, for example the space $L_k^1(H,X)$ is the completion of the space of all simple functions over the field generated by the closed subsets

of H. Theorem. Let Λ be a continuous linear transformation from $L_k^1(H,X)$ to a linear normed space Z, then there exists a finitely additive set function G, defined on F with values in $B(X,Z^{**})$, such that G satisfies the strong Lipschitz condition with respect to K, and such that $[\Lambda(f)]^{**} = \int_H dG \cdot f$, for all f in $L_k^1(H,X)$. Theorem. Let G be any finitely additive set function defined on F with values in $B(X,Z^{**})$, where G is strongly Lipschitz with respect to K. Then $\int_H dG \cdot f$ exists for all f in $L_k^1(H,X)$ and defines a continuous linear transformation from $L_k^1(H,X)$ to Z^{**}. (Received January 22, 1970.)

70T-B104. WITHDRAWN.

70T-B105. J. R. EDWARDS and STANLEY G. WAYMENT, Utah State University, Logan, Utah 84321. A V-integral representation for linear operators on spaces of continuous functions with values in topological vector spaces.

Suppose X and Y are topological vector spaces. In this paper we give an analytic representation of continuous linear operators from C into Y, where C denotes the space of continuous functions from the interval $[0,1]$ into X with the topology of uniform convergence. In order to obtain an integral representation theorem analogous to the ones given by R. K. Goodrich for the locally convex setting in Trans. Amer. Math. Soc. 131 (1968), 246-258, certain strong hypotheses on C must be assumed because of the need to be able to extend the operators to a subset of the double dual of C. However, by using the variational integral we are able to avoid this problem and give a representation theorem without additional hypothesis. (Received February 2, 1970.)

70T-B106. WILLIAM D. L. APPLING, North Texas State University, Denton, Texas 76203. Continuity and set function summability. Preliminary report.

The notions of integrability and summability are as in previous abstracts of the author. Suppos N is a positive integer and B is a function from E^N into the real numbers. Let W denote the statemen "$\{U,F,m\}$ is a finitely additive measure space and $\{H_k\}_{k=1}^N$ is a sequence of real-valued m-summable functions on F." Theorem 1. The following two statements are equivalent: (1) W implies that $\int_U B(H_1(I),\ldots,H_N(I))m(I)$ exists, and (2) B is continuous and bounded. Theorem 2. The following two statements are equivalent: (1) W implies that $B(H_1,\ldots,H_N)$ is m-summable, and (2) B is continuous and $\{|B(x_1,\ldots,x_N)|/[\sum_{k=1}^N |x_k|]|\sum_{k=1}^N |x_k| \geq 1\}$ is bounded. (Received January 19, 1970.)

70T-B107. RAYMOND L. JOHNSON, University of Maryland, College Park, Maryland 20742. Representation theorems and Fatou theorems for second order linear parabolic equations. Preliminary report.

For a second order linear uniformly parabolic equation on $R^n \times (0,T]$, under minimal hypothese on the coefficients, representation theorems are proven for a class of global constraints which includes positivity. It is then proven that solutions having such representations have nontangential limits almost everywhere. (Received February 23, 1970.)

70T-B108. LOUIS PIGNO, Kansas State University, Manhattan, Kansas 66502. A multiplier theorem.

Let G be a LCA group and φ a complex-valued function defined on the dual Γ. The Banach spaces $L^1(G) \cap L^\infty(G)$ and $L^1(G) \cap C(G)$ are defined in Rudin ["Fourier analysis on groups," Interscience, New York, 1962]. Definition. φ is said to be a multiplier of type $(L^1 \cap L^\infty, L^1 \cap C)$ if,

given $f \in L^1(G) \cap L^\infty(G)$, there corresponds a $g \in L^1(G) \cap C(G)$ such that $\varphi\hat{f} = \hat{g}$, where $\wedge$ denotes the Fourier transformation. Theorem. $\varphi \in (L^1 \cap L^\infty, L^1 \cap C)$ if and only if $\varphi = \hat{f}$ for some $f \in L^1(G)$ (to appear in Pacific J. Math.). (Received February 26, 1970.)

70T-B109 and B110. WITHDRAWN

70T-B111. DAVID BELL, Rice University, Houston, Texas 77001. Rational approximation of functions holomorphic on an annulus. Preliminary report.

A previous result of the author is: If α and β are two smooth simple closed curves in the complex plane (where β surrounds and may touch α) and V is the open set between α and β, and S is contained in the closure of W (where W is the open set bounded by α) then the set of rational functions whose poles are simple and are contained in S is L_1-dense in the L_1 holomorphic functions on V if and only if $\int_\alpha \log \operatorname{dist}(z, S \cup \beta) d|z| = -\infty$ or $\sum_{z \in S} \operatorname{dist}(z, \alpha) = \infty$. The sufficiency of this condition has been demonstrated in a previous paper. The necessity follows from a recent result of Taylor and Williams, and a recent paper of the author's relating such approximation to the existence of a nontrivial Lipschitzian holomorphic function on W which vanishes identically on S. (Received March 2, 1970.)

70T-B112. JAMES T. BURNHAM, University of Iowa, Iowa City, Iowa 52240. Closed ideals in subalgebras of Banach algebras. I. Preliminary report.

Definition. A dense left (right) ideal, B, in a Banach algebra $(A, \| \ \|)$ is an Abstract Segal Algebra (B is an ASA wrt A) iff (i) B is a Banach algebra wrt some norm $\| \ \|'$ and (ii) $\exists$ M, C > 0 so that $\|f\| \leq M\|f\|'$ and $\|fg\|' \leq C\|F\| \ \|g\|' \ \forall f,g \in B$. Theorem 1. If B is an ASA with an approximate identity, then every closed ideal I in B is = cl(I) $\cap$ B [cl(I) = closure of I in A] moreover, the map $I \to cl(I)$ is a bijection between the closed ideals of B and A. Theorem 1 is a generalization of the Fundamental Theorem of Segal Algebras due to Reiter, "Classical harmonic analysis" and "Locally compact groups", Oxford, 1968, p. 129. Theorem 2. If B is a proper ASA, then B can not have a bounded approximate identity. Suppose now that A is commutative. Theorem 3. The spaces of regular maximal ideals for B and A are homeomorphic and B is semisimple iff A is. Some applications of ASA's are given within the context of Beurling Algebras and "Normed ideals" [J. Cigler, Nederl. Akad. Wetensch. Proc. Ser. A. 72 (1969), 273-282]. (Received March 11, 1970.)

70T-B113. RAYMOND Y. T. WONG, University of California, Santa Barbara, California 93106. Homotopy negligible subsets of bundles.

A subset K of a space X is: (weakly) homotopy negligible if the inclusion $j: X - K \to X$ is a (weak) homotopy equivalence; locally homotopy negligible if each point of K has a fundamental neighborhood $\{U\}$ such that $j: U - K \to U$ is a homotopy equivalence and strongly homotopy negligible if $\{U\}$ is all neighborhoods of X. A Z-set is a set with property Z in the sense of Anderson. By a manifold we mean a metric space which has an open covering by sets homeomorphic to open subsets of C, where C is a closed convex subset of some metric locally convex TVS E. Theorem. Let $\xi: X \to B$ be a locally-trivial bundle space with fibre F such that F and B are manifolds. Then a closed subset K of X is strongly homotopy negligible in X if $K \cap \xi^{-1}(b)$ is locally homotopy negligible in each $\xi^{-1}(b)$. Furthermore, the same is true when "locally homotopy negligible" is replaced by "a Z-set". Special cases of ξ which are of interest: (1) $X = F \times B$ and (2) $X = E \times R$, where R is the real and $\xi: E \times R \to R$ is the projection. Sharper results can be obtained when $B = [0,1]^n$, $1 \leq n \leq \infty$. Theorem. Let C_1, C_2 be spaces; each is a closed convex subset of some E. Then a closed subset

K of $C_1 \times C_2$ is homotopy negligible in $C_1 \times C_2$ if $K \cap (x \times C_2)$ is weakly homotopy negligible in $x \times C_2$ for each $x \in C_1$. Other theorems follow. (Received March 2, 1970.)

70T-B114. STERLING K. BERBERIAN, University of Texas, Austin, Texas 78712. Conditions on an operator implying Re $\sigma(T) = \sigma(\text{Re } T)$.

It is shown that the equation of the title holds for (i) Toeplitz operators, (ii) operators whose spectrum is a spectral set, and (iii) operators that satisfy the growth condition (G_1) and have connected spectrum, and (iv) a new proof is given of C. R. Putnam's theorem that the equation holds for seminormal operators ["Commutation properties of Hilbert space operators and related topics," Springer-Verlag, New York, 1967, Theorem 3.4.1]. (Received March 16, 1970.)

70T-B115. WITHDRAWN.

70T-B116. CHAITAN P. GUPTA, Northern Illinois University, DeKalb, Illinois 60115. On the surjectivity of a nonlinear, noncompact mapping in a Banach space.

Let X be a real Banach space with dual X^* and denote the duality pairing between the element w of X^* and the element x of X by (w,x). Let μ be a continuous strictly-increasing function defined on the interval $[0, \infty)$ such that $\lim_{t\to\infty}\mu(t) = \infty$ and denote by J the duality mapping from X to 2^{X^*} corresponding to the gauge function μ. Theorem 1. Let $H: X \to X$ be a mapping such that (i) whenever $y_0 \in X$ and $R_0 > 0$ are such that $(w, y_0 - H(x)) \leqq \mu(\|x\|)\|x\|$ for each $x \in X$ with $\|x\| = R_0$ and $w \in J(x)$, there exists an $x_0 \in X$ such that $y_0 = x_0 + H(x_0)$ and (ii) $\lim_{\|x\|\to\infty;\, w\in J(x)} (w, x + H(x))/\mu(\|x\|) = +\infty$. Then $(I + H)(X) = X$. Theorem 1 generalizes the main theorems of S. Fucik (Comment. Math. Univ. Carolinae 10 (1969), 177-187) and has as corollaries the well-known fixed point theorems of Banach, Krasnoselskiĭ, Altman, Rothe, and Granas. (Received March 9, 1970.)

70T-B117. JEFFREY R. KROLL, University of Nebraska, Omaha, Nebraska 68101. Eigenvalues of the mean sigma and weighted mean integral operators related to a Gronwall inequality.

Schmaedeke and Sell have established a Gronwall inequality for the Stieltjes mean sigma and Dushnik integrals (Proc. Amer. Math. Soc. 74 (1968), 1217-1222). They indicate that the equation $f(x) = \int_0^x f(t)dg(t)$, $0 \leqq x \leqq T$, may have nontrivial positive solutions, where f is a bounded function, g is of bounded variation, and the integral is the Stieltjes mean sigma. We are able to prove the following Theorem. $\lambda > 0$ is an eigenvalue of the operator U, having a nonnegative eigenfunction, if and only if there is a $T' \in (0,T]$ such that $2\lambda = g(T') - g(T' -)$ and $g(t) - g(t -) < 2\lambda$ if $T' < t \leqq T$, where $Uf(x) = \int_0^x f(t)dg(t)$. The sufficiency part of the proof is done by constructing a sequence of step functions which converges uniformly to g and whose corresponding sequence of eigenfunctions converges to an eigenfunction of g. The proof uses Lemma 4.2a of R. E. Lane (Proc. Amer. Math. Soc. 5 (1954), 59-66). One may extend the results of Schmaedeke and Sell to obtain a Gronwall inequality for the weighted refinement integral (Wright and Baker, Proc. Amer. Math. Soc. 20 (1969), 42-52). This suggests a similar eigenvalue theorem for the weighted refinement integral with the condition that $\lambda = w_p \Delta g(T')$. This work comprised the author's master's thesis at University of Nebraska, at Omaha. (Received March 9, 1970.)

70T-B118. AMY C. MURRAY, University of California, Berkeley, California 94720. Asymptotic behavior of solutions of hyperbolic inequalities.

This paper treats the inequality (1) $|Lu| \leq k_1(t,x)|u| + k_2(t,x)\,\|\nabla u\|$, where L is a second order hyperbolic operator of the form $L = A - \partial^2/\partial t^2$ with $A \equiv \sum_{ij=1}^{n}(\partial/\partial x_i)a_{ij}(\partial/\partial x_j)$ a uniformly elliptic operator. The $a_{ij} = a_{ij}(t,x)$ are C^1 functions such that $a_{ij} = a_{ji}$ and $0 < m \leq \sum_{ij=1}^{n} a_{ij}\xi_i\xi_j \leq M^2$ whenever $\sum_{i=1}^{n}\xi_i^2 = 1$. Let $D(\epsilon,R)$ denote the unbounded conical region $\{(t,x): \epsilon \leq t,\ |x| \leq tM + R\}$. The main results provide upper bounds for the rate of decay of nonzero solutions of (1) in $D(\epsilon,R)$. These bounds depend on the asymptotic behavior of the functions k_1, k_2, and $\partial(a_{ij})/\partial t$ in $D(\epsilon,R)$ as $t \to \infty$. A typical Theorem. Suppose $k_1 = O(t^{-2})$, $k_2 = O(t^{-1})$, and $\partial(a_{ij})/\partial t = O(t^{-1})$ in $D(\epsilon,R)$. If u is a C^2 solution of (1) in $D(\epsilon,R)$ which decays so that for all $\alpha > 0$, $\lim_{T\to\infty} T^{\alpha}\int_{S(T)}(u^2 + \|\nabla u\|^2) = 0$, where $S(T) \equiv \{(T,x): |x| \leq TM + R\}$, then $u \equiv 0$ in $D(\epsilon,R)$. The proofs employ methods of Protter [Bull. Amer. Math. Soc. 68 (1962), 523-525] and Hörmander [Tolfte Skand. Matematikerkongressen, Lund, 1953, 105-115] (Received March 12, 1970.) (Author introduced by Professor Murray H. Protter.)

70T-B119. MANFRED BREUER, University of Kansas, Lawrence, Kansas 66044. A Riesz-Schauder decomposition theorem.

Let A be a semifinite properly infinite von Neumann algebra of operators of a complex Hilbert space H. Let $\mathfrak{M}$ be the norm closed ideal generated by the finite projections of A. Let $C \in \mathfrak{M}$. Let N_n resp. R_n be the null resp. range projection of $(1 - C)^n$. Then $\sup N_n = N_\infty$ is finite and $\inf R_n = R_\infty$ is cofinite. A consequence of this is that $\inf(N_\infty, R_\infty) = 0$ and $\sup(N_\infty, R_\infty) = 1$ (M. Breuer, Math. Ann. 178 (1968), 243-254). Moreover the algebraic direct sum $N_\infty(H) + R_\infty(H)$ is essentially a topological direct sum in the following sense: There is a nondecreasing sequence (E_n) of projections of A such that $\sup E_n = R_\infty$, $R_\infty - E_n$ is finite and $N_\infty(H) + E_n(H)$ is closed (topologically direct). (Received March 10, 1970.) (Author introduced by Professor Theodore W. Palmer.)

70T-B120. WITHDRAWN.

70T-B121. R. D. HOLMES and ANTHONY T. LAU, University of Alberta, Edmonton 7, Alberta, Canada. Nonexpansive actions of topological semigroups and fixed points.

Let K be a nonempty, bounded, closed convex subset of a Banach space B. If the action of a left reversible topological semigroup (i.e. any two nonempty closed right ideals have nonvoid intersection) S on K is separately continuous and nonexpansive, and one of the following holds: (1) K is weakly compact and has complete normal structure (e.g. when K is compact or B is uniformly convex). (2) There is a compact subset $M \subseteq K$ and an $a \in S$ which commutes with each element of S such that, for each $x \in X$, the closure of $\{a^n(x) | n = 1,2,\ldots\}$ contains a point of M; then S has a fixed point in K. This result generalizes certain theorems of L. P. Belluce and W. A. Kirk ["Fixed-point theorems for families of contraction mappings," Pacific J. Math. 18 (1966), 213-217; "Nonexpansive mappings and fixed-points in Banach spaces," Illinois J. Math. 11 (1967), 474-479] and T. Mitchell ["Fixed points of reversible semigroups of nonexpansive mappings," to appear]. The result is generalized to separated locally convex linear topological spaces and, as an application, it is shown that the space of strongly almost periodic functions of a left reversible topological semigroup has a left invariant mean. (Received March 16, 1970.)

70T-B122. JOSEPH DIESTEL, West Georgia College, Carrollton, Georgia 30117. Abstract-valued additive set functions of locally finite variations.

Let Y be a conditionally complete vector lattice with positive cone P. Let S be a ring of subsets of the set X. Denote by $BV^{loc}(S,Y)$ the collection of all $m: S \to Y$ which are finitely additive and such that given $A \in S$, the set $\mathcal{V}_A(m) = \{\sum_1^n |m(A_i)|: A_1, \ldots, A_n \in S, \text{disjoint}, \bigcup_1^n A_i = A, n \in N\}$ is a bounded set in Y. Denote by S(S,R) the collection of all S-simple realvalued functions defined on X; S(S,R) is a vector lattice with the usual definitions. Theorem. $BV^{loc}(S,Y)$ is isomorphic as a vector lattice to the space of order bounded linear transformations from S(S,R) to Y. (The ordering on $BV^{loc}(S,Y)$ is conditionally complete.) If $m \in BV^{loc}(S,Y)$ and $m(S) \subseteq P$ then m is called a positive measure if given a disjoint sequence $\{A_n\}$ of members of S such that $A = \bigcup_n A_n \in S$ then $m(A) = \sup_n \{\sum_1^n m(A_i)\}$. Theorem. Let $m \in BV^{loc}(S,Y)$, $m(S) \subseteq P$. Then there exists a measure m_c and a purely finitely additive function (in the sense of Hewitt and Yosida) m_p defined on S such that $m = m_c + m_p$. This decomposition is unique. (Received March 18, 1970.)

70T-B123. ARUNAVA MUKHERJEA and NICOLAS A. TSERPES, University of South Florida, Tampa, Florida 33620. Idempotent measures on locally compact semigroups.

A conjecture that the support of an r^*-invariant probability measure on a locally compact semigroup is a left group, is proven. (See these *Notices* 15(1968), 356 and 16(1969), 829.) Using this result we also prove that the support of a (regular) idempotent measure on a locally compact semigroup is completely simple, thus extending a well-known result of Pym and Heble and Rosenblatt on compact semigroups to the locally compact case. (Received March 18, 1970.)

70T-B124. WOLFGANG R. WASOW, University of Wisconsin, Madison, Wisconsin 53706. The central connection problem at turning points of linear differential equations.

A system of linear differential equations of the vectorial form $\epsilon\, dy/dx = A(x,\epsilon)y$ is considered, where ϵ is a positive parameter, and the matrix A(x) is holomorphic in $|x| \leq x_0$, $0 < \epsilon \leq \epsilon_0$, with an asymptotic expansion $A(x,\epsilon) \sim \sum_{r=0}^{\infty} A_r(x)\epsilon^r$, as $\epsilon \to 0$. The eigenvalues of $A_0(x)$ are assumed to become zero at x = 0, in such a way that $A_0(0) = \{\delta_{j,j+1}\}$ and $d/dx\, A_0(x)|_{x=0} \neq 0$. With the help of refinements of the representations for the inner and outer asymptotic expansions that were introduced by the author in Comm. Pure Appl. Math. 14(1961), 657-673 and 15(1962), 173-187, explicit connection formulas between these solutions are calculated in the form of asymptotic series that depend only on the coefficients of the series for the inner and outer solutions. As part of this derivation it is shown that only the diagonal entries of the connection matrix are asymptotically relevant. (Received March 19, 1970.)

70T-B125. TED W. GOODMAN, Virginia Polytechnic Institute, Blacksburg, Virginia 24061. Strong Lipschitz functions in topological vector spaces. Preliminary report.

In 1965, M. K. Ali introduced the concept of a local Lipschitz condition for mappings in l.c.s. ("Differential calculus for locally convex topological vector spaces," Department of Mathematics Preprints No. 28, University of Oklahoma, Norman, Oklahoma, 1965, p.6). In the present study we extend this definition to the notion of a Lipschitz condition, called a strong Lipschitz condition, for mappings in arbitrary t.v.s. Definition. Let X and Y be t.v.s. over a nondiscrete valuated field K.

A mapping f defined on a subset A of X into Y is called strongly Lipschitzian on A if for each 0-nbd U in Y there corresponds a 0-nbd V in X such that for all $x \in A$ and all $\lambda \in K$, $f((\lambda V + x) \cap A) - f(x) \subset \lambda U$. Results obtained include the following. Theorem. The composition of two strong Lipschitz mappings is again a strong Lipschitz mapping. Characterizations of strong Lipschitz mappings are given in various special settings. A particular characterization is: Theorem. Let X be a n.v.s. over F (R or C), let Y be a t.v.s. over F, and let $A \subset X$. A mapping f defined on A into Y is strongly Lipschitzian on A iff there exists $M > 0$ such that for all $x,y \in A$, $f(x) - f(y) \in M\|x - y\|U$. (Received March 24, 1970.)

70T-B126. FRANK DAVID LESLEY, University of California, San Diego, La Jolla, California 92037. Differentiability of minimal surfaces at the boundary. Preliminary report.

Let Γ be a Jordan curve in R^3 and $F(z) = (u(z), v(z), w(z))$: $\{|z| \leqq 1\} \to R^3$ a solution of Plateau's problem for Γ, where $z = x + iy$ are isothermal parameters. Then u, v, w are harmonic for $|z| < 1$; let λ, μ, ν be analytic functions whose real parts are respectively u,v,w. Using the Poisson integral and the defining properties of minimal surfaces we prove: Theorem 1. If $\Gamma \in C^{1,\alpha}$ for $0 < \alpha < 1$ then $\lambda, \mu, \nu \in C^{1,\alpha}$ for $|z| \leqq 1$. If $\Gamma \in C^{1,1}$ then λ', μ', ν' have modulus of continuity $Kt \log(1/t)$; K and the Hölder constants depend only on Γ. Theorem 2. Suppose $\Gamma \in C^{n,\omega(t)}$, $n \geqq 2$, that is, the nth derivative of the parametrization of Γ with respect to arclength has modulus of continuity $\omega(t)$ where $\int_0^\sigma (\omega(t)/t)dt < \infty$ for some $\sigma > 0$. Then $\lambda, \mu, \nu \in C^{n,\omega^*(t)}$ for $|z| \leqq 1$ where $\omega^*(\theta) = A(\int_0^\theta (\omega(\pi ct)/t)dt + \theta \int_\theta^\pi (\omega(\pi ct)/t^2)dt)$ and A and c depend only on Γ. In particular $\Gamma \in C^{n,\alpha}$ implies $\lambda, \mu, \nu \in C^{n,\alpha}$ for $0 < \alpha < 1$ and $\Gamma \in C^{n,1}$ implies $\lambda, \mu, \nu \in C^{n,\omega^*(t)}$ where $\omega^*(t) = Kt \log(1/t)$. Both theorems are true for subarcs of Γ. They generalize Kellogg's Theorem for conformal mapping and are an independently proved extension of the recent work of J. C. C. Nitsche (Invent. Math. 8(1969), 313-334). Theorem 2 for n = 1 was proved by S. E. Warschawski. (Received March 25, 1970.) (Author introduced by Professor Stefan E. Warschawski.)

70T-B127. F. GACS, McGill University, Montreal, Quebec, Canada. On polynomials whose zeros are in the unit disk.

The following is a conjecture due to A. W. Goodman, Q. I. Rahman, and J. S. Ratti [Proc. Amer. Math. Soc. 21(1969), 273-274]: If P, a polynomial of degree n, has all its zeros in $|z| \leqq 1$, and $P(a) = 0$, $0 \leqq a \leqq 1$, then P' must have a zero in $|z - a/2| \leqq 1 - a/2$. This is proved for $2 \leqq n \leqq 5$. It is further conjectured that the radius $1 - a/2$ can be replaced by $r_n(a)$, where $1/2 \leqq r_n(a) \leqq 1 - a/2$ and $r_n(a)$ is the zero of $(1/a)[(x + a/2)^n - (x - a/2)^n] - 1$. This is proved for n = 3. It is also proved that if, under similar conditions, $P(1) = 0$, $P'(1) \neq 0$, $n \geqq 4$ and P' does not have a zero in $|z - 1/2| < 1/2$, then $P(z) = c(z^n - 1)$. (Received March 26, 1970.) (Author introduced by Professor Robert Vermes.)

70T-B128. KEITH A. EKBLAW, University of Kentucky, Lexington, Kentucky 40506. Functions of bounded index as a subspace of a space of entire functions. Preliminary report.

Let E denote the class of entire functions, $B_n = \{f \in E \mid f \text{ is of bounded index } \leqq n\}$, and $N = \bigcup_{n=0}^\infty B_n$. For $f = \sum_{n=0}^\infty a_n z^n$ and $g = \sum_{n=0}^\infty b_n z^n$ in E define $d(f,g) = \mathrm{Sup}\{|a_0 - b_0|, \sqrt[n]{|a_n - b_n|}, n = 1,2\ldots\}$. It is shown that in the topology generated by this metric B_n is closed in E and $\bar{B} = (E \setminus \bar{B}) = E$. Also, B_n is nowhere dense in B and, hence, $B = \bigcup_{n=0}^\infty B_n$ is 1st category. For $B' = \{f \in B \mid f \text{ is transcendental entire}\}$ and $B'_n = B_n \cap B'$ it is shown that B'_n is nowhere dense in B' and, hence, $B' = \bigcup_{n=0}^\infty B'_n$ is 1st category. (Received March 18, 1970.)

70T-B129. A. R. REDDY, University of Toledo, Toledo, Ohio 43606. On an extension of a result of S. N. Bernstein. Preliminary report.

Let $f(x)$ be a real-valued, continuous function on $[-1,1]$; as usual, let $E_n(f) \equiv \inf_{p \in \pi_n} \|f - p\|$, for $n = 0,1,2,\ldots$, where the norm is the maximum norm on $[-1,1]$ and π_n denotes the set of all polynomials with real coefficients of degree at most n. Then the following result is due to S. N. Bernstein. Let $f(x) = \sum_{k=0}^{\infty} a_k x^k$ be an entire function. In addition let $\lim_{n\to\infty} n^{1/2}|a_n|^{1/n} = 0$. Then there exists a sequence $\{n_p\}_{p=1}^{\infty}$ such that $a_{n_{p+1}} \neq 0$ and $\lim_{p\to\infty}(2^n p E_{n_p}(f)/|a_{n_p} + 1|) = 1$. In this note we prove the following. Let $f(x) = \sum_{k=0}^{\infty} a_k x^k$ be a real-valued, entire function of perfectly regular growth (ρ, τ). Then for a sequence of integers $\{n_p\}_{p=1}^{\infty}$ such that $|a_{n_p} + 1| \neq 0$, $\lim_{p\to\infty}\{E_{n_p}(f)/|a_{n_p} + 1|\}^{1/n_p} = 1/2$. (Received April 3, 1970.)

70T-B130. THOMAS R. LUCAS, University of North Carolina, Charlotte, North Carolina 28205. M-splines. I. Hilbert space theory.

Let X be a real Hilbert space, Λ a family of continuous linear functionals on X, and M a continuous bilinear (not necessarily symmetric) functional on $X \times X$ such that $M(x,x) \geqq 0$ for all $x \in N(\Lambda) = \{n \in X: \lambda(n) = 0 \text{ all } \lambda \in \Lambda\}$. Any $s \in X$ is said to be an M-spline if $M(s,n) = 0$ all $n \in N(\Lambda)$, and if also $\lambda(s) = \lambda(x)$ for all $\lambda \in \Lambda$, s is said to be an $Sp(M, \Lambda)$-interpolate of x. The system $\{X, \Lambda, M, N(\Lambda), N_1, N_2\}$ is said to be N_1-posed if $N_1 = \{n \in N(\Lambda): M(n,n) = 0\}$; there exists a closed subspace N_2 and an $m > 0$ such that $M(n,n) \geqq m\|n\|^2$ for all $n \in N_2$; $N(\Lambda) = N_1 \oplus N_2$; and $M(x,n) = 0$ for all $x \in X$, $n \in N_1$. A theory of generalized splines is developed in this setting which, for N_1-posed systems, gives for any $f \in X$, the existence of a unique $Sp(M,\Lambda)$-interpolate $s \in f + N_2$. Any other Λ-interpolate of f, $\bar{s}$, is shown to be an $Sp(M, \Lambda)$-interpolate of f iff $\bar{s} - s \in N_1$. This development may be related to earlier approaches by the result: Let the system $\{X, \Lambda, M, N(\Lambda), N_1, N_2\}$ be N_1-posed. If M is symmetric, then for any $f \in X$, $M(s,s) = \text{Min}\{M(g,g): g \text{ is a } \Lambda\text{-interpolate of } f\}$, where s is a Λ-interpolate of f, iff s is an $Sp(M,\Lambda)$-interpolate of f. (Received March 27, 1970.) (Author introduced by Professor Joseph F. Schell.)

70T-B131. HILTON V. MACHADO, University of Chicago, Chicago, Illinois 60637 and Roosevelt University, Chicago, Illinois 60605. Metric spaces, normal structure and fixed points. Preliminary report.

Let X be a metric space, $\mathcal{A} \subset 2^X$ a family which (a) contains the closed balls of X, (b) is closed under intersections, (c) if A_i is a directed family in $\mathcal{A}$ then $\overline{\bigcup A_i} \in \mathcal{A}$. If $M \subset X$ one defines $\mathcal{A}$-span of M, $\mathcal{A}$-compactness and $\mathcal{A}$-normal structure ($\mathcal{A}$-n.s.) of M. An extension of a concept of Belluce-Kirk (Illinois J. Math. 11(1967)), the $\mathcal{A}$-complete normal structure ($\mathcal{A}$-c.n.s.) of a pair (X,K) is introduced. $N[X]$ indicates the set of nonexpansive self-mappings of X, $O(\mathfrak{F}, x)$ the orbit of $x \in X$ under the action of $\mathfrak{F} \subset N[X]$. If $\pi = f_1 f_2 \ldots f_n = f_2 f_3 \ldots f_1 = \ldots$, $\mathfrak{F} = \{f_1, f_2, \ldots, f_n\}$ is said to be cyclic commutative. Theorem. Let K be an $\mathcal{A}$-compact subset of X, $\mathfrak{F}$ a cyclic commutative finite family in $N[X]$. If $K \cap \mathcal{A}$-span $O(\pi, x) \neq \emptyset$ $\forall x \in X$ (π is the product), the sets $\mathcal{A}$-span $O(\mathfrak{F}, x)$, $x \in X$, have $\mathcal{A}$-n.s., and some orbit $O(\mathfrak{F}, x_0)$ is bounded then $\mathfrak{F}$ has a fixed point $x \in X$. Theorem. Let K be an $\mathcal{A}$-compact subset of X such that (X,K) has $\mathcal{A}$-c.n.s., $\mathfrak{F}$ a commutative semigroup in $N[X]$. If $K \cap \mathcal{A}$-span $O(f,x) \neq \emptyset$ $\forall x \in X$, $f \in \mathfrak{F}$, $f \neq$ identity, the sets $\mathcal{A}$-span $O(\mathfrak{F}, x)$, $x \in X$, have $\mathcal{A}$-n.s. and some orbit $O(\mathfrak{F}, x_0)$ is bounded then $\mathfrak{F}$ has a fixed point $x \in X$. One can also prove other theorems extending results of Belluce-Kirk (ibid) and of Kijima-Takahashi (Kōdai Math. Sem. Rep. 21(1969)). (Received April 8, 1970.)

70T-B132. GÜNTHER W. GOES, Illinois Institute of Technology, Chicago, Illinois 60616. Spaces of sequences with gaps. Preliminary report.

Let $\{n_j\}_{j=1}^{\infty}$ be an increasing sequence of positive integers and ω the linear space of all complex valued sequences $x = \{x_k\}_{k=0}^{\infty}$. Let $\Lambda = \{x \in \omega : x_k = 0 \text{ for } k \neq n_j, j = 1,2,\ldots\}$, let E be an FK-space, $E_N = \{x \in E: x \text{ has sectional convergence (AK)}\}$, $E_{1N} = \{x \in E: x \text{ has Cesàro-sectional convergence, i.e. } \{x_0, (1 - (n+1)^{-1})x_1, (1 - 2(n+1)^{-1})x_2, \ldots, (1 - n(n+1)^{-1})x_n, 0, 0, \ldots\} \to x \ (n \to \infty)\}$ $cs = \{x \in \omega : \sum_{k=0}^{\infty} x_k \text{ exists}\}$, $\sigma s = \{x \in \omega : \lim_{n\to\infty} \sum_{k=0}^{n} (1 - k(n+1)^{-1})x_k \text{ exists}\}$. Theorems. (i) If $\sigma s \cap \Lambda = cs \cap \Lambda$, then $E_{1N} \cap \Lambda = E_N \cap \Lambda$. (ii) Let $n_{j+1} \geqq n_j q$ where $q > 1$. If $\sum_{k=0}^{\infty} \hat{f}(k) e^{ikt} \sim f(t) \in L^1(T)$, then there exists a continuous function g on T with Fourier series of power series type i.e. such that $g(t) \sim \sum_{k=0}^{\infty} \hat{g}(k) e^{ikt}$ and such that $\hat{g}(n_j) = \hat{f}(n_j)$ for $j = 1.2,\ldots$. If $\mu \in M(T), \mu(t) \sim \sum_{k=-\infty}^{\infty} \hat{\mu}(k) e^{ikt}$ where $\hat{\mu}(k) = 0$ if $0 < k \neq n_j$, then there exists a continuous function h on T such that $h(t) \sim \sum_{k=-\infty}^{\infty} h(k) e^{ikt}$ where $\hat{h}(k) = 0$ if $0 < k \neq n_j$ and $\hat{h}(n_j) = \hat{\mu}(n_j)$ for $j = 1,2,\ldots$. (Received April 8, 1970.)

70T-B133. HEINRICH W. GUGGENHEIMER, Polytechnic Institute of Brooklyn, Brooklyn, New York 11201. Geometric theory of differential equations. I. Second order equations with generalized coefficients.

We develop Frenot equations of unimodular centro-affine geometry of plane curves with countably many cusps. Applications are to differential equations. In the following, Q is a function (not necessarily differentiable) on a real interval and P is a right continuous function, $P(t) = P(t + 0)$. (1) The weak solutions of $x'' + Q'x = F'$ are the first components of the absolutely continuous (Carathéodory) solutions of $x' + Qx - y - F = 0$, $y' = Qx'$. (2) For $x'' + P'x = 0$, P is the area function of the polar reciprocal of the graph of x for a pair of solutions of unit Wronskian. (3) $x'' + (\lambda - Q')x = 0$ with homogeneous boundary conditions is equivalent to a Fredholm problem with continuous symmetric kernel. (4) We prove equivalents of the Sturm comparison theorems, Liapounoff stability theorems, Weyl and Caligo boundedness theorems and a number of integral inequalities for Equation 2. (Received April 9, 1970.)

70T-B134. ALBERT E. HURD, University of California, Los Angeles, California 90024. Backward lower bounds for solutions of mixed parabolic problems.

Let H be a complex Hilbert space with norm $|\cdot|$ and inner product $(\cdot,\cdot)$. We consider H-valued generalized solutions u(t) of the equation $du/dt + A(t)u = f(t,u)$, where A(t) is an (in general unbounded) operator with variable domain $\mathcal{D}(t) \subset H$. It is assumed that A(t) can be written as $A(t) = A_0(t) + A_1(t)$, where $\mathrm{Re}(A_1(t)u(t), u(t)) = 0$ for all t in an interval (a,b), and that there exists a nonnegative differentiable function $\lambda(t)$ such that $\mathrm{Re}(A_0(t)u(t),u(t)) + \lambda(t)|u(t)|^2 > 0$ for all $t \in (a,b)$ such that $u(t) \neq 0$. We also suppose that the quantities $|A_1(t)u(t)|^2$, $|d/dt\ \mathrm{Re}(A_0 u,u) - \mathrm{Re}(A_0 u, du/dt)|$, and $|f(t,u)|^2$ are bounded above by a term of the form $\epsilon |A_0 u|^2 + K(t)\,[|\mathrm{Re}(A_0 u,u) + |u|^2]$ where K(t) is locally integrable on (a,b) and $0 \leqq \epsilon < 1/5$. Under these assumptions we prove Theorem. Suppose u(t) is a solution and let $p(t) = \mathrm{Re}(A_0(t)u(t),u(t)) + \omega(t)|u(t)|^2$ where $\omega(t) = \lambda(t) + \eta(t)$ for some positive, differentiable function $\eta(t)$ whose inverse and derivative are locally integrable. If $u(t) \neq 0$ on $(\tau,d) \subset (a,b)$ then $p(\tau) \geqq p(d) \exp(-\int_\tau^d M(t)dt)$ where $M(t) = 5(1 + \eta^{-1})\max\{K(t) + 2\epsilon\omega(t), (\omega(t) + 1)K(t) + \epsilon[\omega(t)]^2 + \omega'(t)\} + 2\omega(t)$. The theorem generalizes some of the results of A. Friedman, Arch. Rational Mech. Anal 17 (1964), 355-357. (Received April 9, 1970.)

70T-B135. ANDRE DE KORVIN, Indiana State University, Terre Haute, Indiana 47809. On functions of bounded variations. I. Preliminary report.

Let A be an abelian idempotent semigroup. Let S be a semigroup of semicharacters on A containing the identity. The notion of a function of bounded variation on S was defined by S. Newman [Bull. Amer. Math. Soc. 75(1969), 1396-1400]. F is of bounded variation on S if $\sup \Sigma_\sigma |L(X,\sigma)F| < +\infty$ where $L(X,\sigma)F = \Sigma_\tau m(\sigma,\tau)F(\Pi_{i=1}^n f_i^{\tau(i)})$, m is the Mobius function on n tuples whose components are 0 or 1. It is shown that an element of the dual space of absolutely continuous functions respectively to F is given by an integral, in fact $T(G) = v\int G \cdot dK$ where $v\int G \cdot dK = \lim_X \Sigma_\sigma L(X,\sigma)G\, K(B(X,\sigma))$ (see Newman's article for notations). K denotes here a set function with the property that $K(B(X,\sigma)) = \Sigma \lambda_i K(B(Y,\tau_i))$ if $\cup_i B(Y,\tau_i) = B(X,\sigma)$ with $\lambda_i = \mu_F(B(Y,\tau_i))/\mu_F(B(X,\sigma))$. μ_F is the measure corresponding to F. Moreover $\| T \| = \| K \|$ where $\| K \| = \sup |K(B(X,\sigma))|$ the sup is taken over all possible sets $B(X,\sigma)$. Conversely $v\int G \cdot dK$ defines a bounded linear functional on functions of bounded variation. This result generalizes a result by J. Edwards and S. Wayment (cf. Abstract 672-221, these *Notices* 17(1970), 146). (Received April 9, 1970.)

70T-B136. ANDRE DE KORVIN, Indiana State University, Terre Haute, Indiana 47809 and RICHARD ALO, Carnegie Mellan University, Pittsburgh, Pennsylvania 15213. On functions of bounded variations. II. Preliminary report.

For notations see S. Newman [Bull. Amer. Math. Soc. 75(1969), 1396-1400]. Define G to be Lipschitz respectively to F if there exists some $K > 0$ such that $|L(X,\sigma)G| < K|L(X,\sigma)F|$. Let K be a set function satisfying $K(B(X,\sigma)) = \Sigma \lambda_i K(B(Y,\tau_i))$ whenever $B(Y,\tau_i)$ forms a partition of $B(X,\sigma)$ with $\lambda_i = \mu_F(B(Y,\tau_i))/\mu_F(B(X,\sigma))$. K will be called bounded if $\sup |K(B(X,\sigma))| < \infty$ for all sets $B(X,\sigma)$. Then the set of F-Lipschitz functions is isomorphic, isometric as a normed space to the set of all functions K satisfying the previous conditions. This theorem in particular holds true for Lipschitz functions over [0,1]. The case of vector valued functions is studied. (Received April 9, 1970.)

70T-B137. JAMES WARD BROWN, 1014 Church Street, Apt. B-2, Ann Arbor, Michigan 48104. On Laguerre and related polynomials. Preliminary report.

Let $p_n^{(\alpha)}(x)$ be a simple polynomial set generated by a relation of the form $(1 - t^m)^{-\alpha}F(x,t) = \Sigma_{n=0}^\infty p_n^{(\alpha)}(x)t^n$ where m is a positive integer and F(x,t) is independent of the parameter α. The author has shown that $p_n^{(\alpha)}(x) = \Sigma_{k=0}^{[n/m]} \binom{\beta - \alpha + \gamma n - \gamma mk}{k} \{[\beta - \alpha + \gamma n]/[\beta - \alpha + \gamma(n - mk)]\}(-1)^k p_{n-mk}^{(\beta+\gamma(n-mk))}(x)$. In view of a generating relation due to T. W. Chaundy [Quart. J. Math. Oxford Ser. 14(1943), 55-78], the the hypergeometric polynomials $p_n^{(\alpha)}(x) = \binom{\alpha+n}{n} {}_{A+1}F_{B+1}[-n, (a); 1+\alpha, (b); x]$ with (a) and (b) independent of α satisfy the identity with m = 1. In particular, we have $L_n^{(\alpha)}(x) = \Sigma_{k=0}^n \binom{\beta-\alpha+\gamma k}{n-k} \{[\beta - \alpha + \gamma n]/[\beta - \alpha + \gamma k]\} (-1)^{n-k} L_k^{(\beta+\gamma k)}(x)$ for Laguerre polynomials occurring when A = B = 0. This is well known when $\gamma = 0$. The derivation of the general result is based on considerations of the generating relation $\{[1 - u^m(\beta;t)]^{1-\alpha}/[1 - (1+\beta m)u^m(\beta;t)]\} F(x,u(\beta;t)) = \Sigma_{n=0}^\infty p_n^{(\alpha+\beta n)}(x)t^n$ where $u(\beta;t)$ is the inverse of $v(\beta;t) = t(1 - t^m)^\beta$. (Received April 10, 1970.)

70T-B138. S. M. SHAH, University of Kentucky, Lexington, Kentucky 40506 and S. Y. TRIMBLE, Washington University, St. Louis, Missouri 63130. Univalent functions with univalent derivatives.

Let f(z) be regular in the unit disc D and normalized (f(0) = 0, f'(0) = 1) and let each $f^{(k)}$ (k = 0,1,...) be univalent in D. Then f is an entire function of exponential type. If this f has all zeros lying on a ray through the origin then Lindelöf's theorem shows that the genus of the canonical pro-

duct of f is zero. Hence $f(z) = ze^{\beta z}\prod_1^N(1 - z/z_k)\ldots(1)$ where $|z_1| > 1$ since f is univalent in D. Here N is finite or infinite, and if N = 0 the product $\prod_1^N$ is replaced by 1. Assume that βz_1 is real. Then a simple example shows that βz_1 must not be positive if f is univalent in D. Theorem. Let f(z) be any transcendental entire function of the form (1) where $\arg z_1 = \arg z_2 = \ldots$, $\beta z_1 \leq 0$, $|z_1| > 1$. If f is univalent in D, then (2) $|\beta| + \sum_{k=1}^N 1/(|z_k| - 1) \leq 1$. In fact, (2) holds if and only if f is starlike in D and all of its derivatives are close-to-convex there. Further, if $\{z_k^{(1)}\}_{k=0}^N$ are the zeros of f', then f and all its derivatives are univalent and convex in D if and only if $|\beta| + \sum_{k=0}^N 1/(|z_k^{(1)}| - 1) \leq 1$.
(Received April 13, 1970.)

70T-B139. DAVID L. LOVELADY, Georgia Institute of Technology, Atlanta, Georgia 30332.
A nonlinear variation-of-parameters formula.

Let X be a Banach space with norm N_1. Let H be the set of Lipschitz operators A from X to X such that A(0) = 0, with N_2 the strong Lipshitz norm. Let B be a locally N_2-Bochner integrable function from $[0,\infty)$ to H, and let W: $[0,\infty) \to H$ be that N_2-absolutely continuous function so that if p is in X, then h, given by $h(t) = W(t)[p]$, solves $h(0) = p$, $h'(t) = B(t)[h(t)]$ a.e. Let p be in X, and let f: $[0,\infty) \to X$ be locally Bochner integrable. Let $b > 0$, and let $s_0 = 0$, $0 = t_0 \leq s_1 \leq t_1 \leq s_2 \leq t_2 \leq \ldots \leq s_n \leq t_n = b$. Let $d_0 = p$, and if $1 \leq k \leq n$, $d_k = \int_{t_{k-1}}^{t_k} f(s)ds + W(s_k)W(s_{k-1})^{-1}[d_{k-1}]$. Let $\mathscr{J}_t[W,f,p] = d_n$. Let ${}_0\mathscr{J}^b[W,f,p]$ be the limit, in the sense of successive refinements of subdivisions t, of $\mathscr{J}_t[W,f,p]$. Let Z: $[0,\infty) \to X$ be an absolutely continuous function. Theorem. If $t \geq 0$, then ${}_0\mathscr{J}^t[W,f,p]$ exists. Furthermore, (i) and (ii) are equivalent. (i) $Z(0) = p$ and $Z'(t) = B(t)[Z(t)] + f(t)$ a.e. on $[0,\infty)$. (ii) $Z(t) = {}_0\mathscr{J}^t[W,f,p]$ whenever $t \geq 0$. Several stability-oriented inequalities follow as immediate consequences of the structure of ${}_0\mathscr{J}^t[W,f,p]$. (Received April 13, 1970.)

70T-B140. SANDY GRABINER, Claremont Graduate School, Claremont, California 91711.
Multiplication on Banach spaces. Preliminary report.

Theorem 1. If E is a Banach space with weak*-separable dual, then it is possible to define on E a multiplication under which E becomes a Banach algebra and a commutative integral domain with identity. Theorem 2. For the algebra of bounded linear operators on E, there exists a maximal commutative subalgebra B and an element e of E for which the map $V \to Ve$ is a linear homeomorphism from B onto E. More can be proven in the case that E is itself separable. In this case, the multiplication of Theorem 1 can be chosen so that B becomes a Banach algebra of power series with a unique maximal ideal. Moreover, if T is the operation of multiplication by the indeterminate, then the algebra B of Theorem 2 is both the uniform closure of the polynomials in T and the algebra of operators which commute with T. Also, in the case that E is separable, the multiplication can be chosen so that T is unicellular. (Received April 14, 1970.)

70T-B141. M. ZUHAIR NASHED, Georgia Institute of Technology, Atlanta, Georgia 30332.
A characterization of higher order differentiability in locally convex spaces.

Let X be a linear space, H a topological linear subspace of X, Y a locally convex space, and β a system of bounded subsets of H containing all the bounded subsets of dimension n. Let f: $X \to Y$. We give necessary and sufficient conditions for f to be n times (H,β)-differentiable at x_0 in terms of a local representation of f in a neighborhood of x_0. For definitions and motivation, see Averbukh and Smolyanov (Russian Math. Surveys 22(1967), 201-258) and Nashed (Comment. Math. Univ. Carolinae 10(1969), 509-533 and 535-557). Let f be (H,β)-continuous at x_0. Then f is n times (H,β)-differentiable at x_0 if and only if there exist $P_0 \in Y$, $P_1 \in L(X,Y)$, P_k, $k = 2,\ldots,n$, where P_k is a symmetric

multilinear $(\beta,\ldots,\beta)$-hypocontinuous map, such that the function r_n defined in a deleted neighborhood N of θ by $r_n(h) = f(x_0 + h) - P_0 - P_1h - \ldots - P_nh^n$, satisfies the following conditions: (1) r_n is $(n - 1)$-times (H,β)-differentiable on N, and (2) $t^{j-n}r_n^{(j)}(x_0; th,\ldots) \to 0$ as $t \to 0$, uniformly with respect to $h \in B$ for each $B \in \beta$, for $j = 0,1,\ldots, n - 1$. This theorem gives in particular a characterization of higher order differentiability in the sense of Gâteaux (Fréchet, Hadamard) by taking β to be the collection of all finite (bounded, sequentially compact) subsets of H. (Received March 2, 1970.)

70T-B142. WILLIAM P. ZIEMER, Indiana University, Bloomington, Indiana 47401. Slices of maps and Lebesgue area.

Suppose X is a smooth k-manifold and let $f: X \to E^n$ be a continuous map whose Lebesgue area, L(f), is finite. Let H^α denote Hausdorff α-dimensional measure. If $U: E^n \to E^1$ is a Lipschitz function with constant 1, i.e. u is a contraction, and if $k = 2$ or $H^{k+1}[f(X)] = 0$, then it is possible to give a definition which yields a reasonable appraisal to the notion of the $k - 1$ area of f restricted to the boundary of $\{x: u[f(x)] < t\}$. This definition is in terms of the theory developed by H. Federer ["Currents and area," Trans. Amer. Math. Soc. 98(1961), 204-233]. Denote this $k - 1$ area of the "slice" of f by u at level t by $\lambda(f; u,t)$. Theorem. $L(f) = \sup\{\int_{-\infty}^{\infty}\lambda(f; u,t)dt: u \text{ a contraction}\}$. In the case X is a 2-cell, L. Cesari has a definition of the generalized length of the slice of f by u at level t, which is given in terms of prime ends ["Surface area," Ann. of Math. Studies, no. 35, Princeton Univ. Press, Princeton, N. J.]. While the generalized length of a slice rarely agrees with $\lambda(f; u,t)$, the above theorem remains valid when generalized length is employed. (Received March 30, 1970.)

70T-B143. LEONARD F. RICHARDSON, Yale University, New Haven, Connecticut 06520. Decomposition of the L^2-space of a general compact nilmanifold.

Let Γ be any cocompact discrete subgroup of a connected, simply connected, real, finite dimensional, nilpotent Lie group N. Let U represent N unitarily in $L^2(\Gamma\backslash N)$ by $(U(n_0)F)(\Gamma n) = F(\Gamma nn_0)$. Let π be an irreducible representation of N and f a linear functional on the Lie algebra of N related to π by the Kirillov correspondence. If $\bar{f}$ is a character on a subgroup $M \subset N$ determined by f, define $\bar{f}^n(p) = \bar{f}(npn^{-1})$, and call $\bar{f}^n$ an integral character iff $\bar{f}^n$ maps $\Gamma \cap n^{-1}Mn$ into 1. Theorem 1. If f is as above, then there is always a special subgroup $M \subset N$ such that f defines a character $\bar{f}$ on M, $\bar{f} \uparrow \pi$, induced in the sense of Mackey, and the multiplicity of π in U is the number of Γ-orbits among the integral characters in the N-orbit $\bar{f}^N$. Theorem 2. If $\bar{f}$ induces π and $\bar{f}$ is integral, then the map sending G(n) into $\tilde{G}(n) = \sum G(\gamma n)$, summed over $\Gamma \cap M\backslash\Gamma$, M special, determines a unitary injection of the induced representation space for $\bar{f} \uparrow \pi$ into $L^2(\Gamma\backslash N)$, and all the copies of π in $L^2(\Gamma\backslash N)$ are obtained by using this map for Γ-distinct integral characters $\bar{f}^n$. (Received April 17, 1970.)

70T-B144. RICHARD STAUM, 415 East 5th Street, Brooklyn, New York 11218 and Polytechnic Institute of Brooklyn, Brooklyn, New York 11201. The structure of non-Archimedean Banach algebras. Preliminary report.

Let X be a commutative non-Archimedean Banach algebra with identity over a nontrivially valued field F, with $\|X\| \subseteq \overline{|F|}$. Let $\mathfrak{M}$ be the family of maximal ideals of X; for each $M \in \mathfrak{M}$, let F_M denote the quotient normed algebra X/M. Let $h: X \to \pi(F_M: M \in \mathfrak{M})$ be given by $h(x)_M = x + M$ $(x \in X, M \in \mathfrak{M})$, and let $\hat{X}$ denote the normed algebra $\{h(x): x \in X\}$ with norm $\|h(x)\| = \sup\{\|x + M\|: M \in \mathfrak{M}\}$. Let $V(X) = \{x \in X: \|x\| \leq 1\}$; $P(X) = \{x \in X: \|x\| < 1\}$; $V(\hat{X}) = \{h(x) \in \hat{X}: \|h(x)\| \leq 1\}$; $P(\hat{X}) = \{h(x) \in \hat{X}: \|h(x)\| < 1\}$; $W(X) = \{x \in X: \|h(x)\| \leq 1\}$; and $J(X) = \{x \in X: \|h(x)\| < 1\}$. Let Rad(X) denote the radical of X. Relations among elementary properties of these structures are considered. Results

include the following. Theorem. The following are equivalent: $h:X \to \hat{X}$ is an isometry; $V(X) = W(X)$; $P(X) = J(X)$. Theorem. $h:X \to \hat{X}$ is a homeomorphism iff X is semisimple and $\hat{X}$ is complete. Theorem. The following are equivalent: X is semisimple; h carries W(X) isomorphically onto $V(\hat{X})$; h carries J(X) isomorphically onto $P(\hat{X})$. Theorem. $\hat{X}$ is complete iff the canonical mapping of X/Rad(X) onto $\hat{X}$ is a homeomorphism. Theorem. Every maximal ideal of $V(\hat{X})$ contains $P(\hat{X})$ iff every maximal ideal of W(X) containing Rad(X) contains J(X); if $\hat{X}$ is complete, these conditions hold. (Received February 26, 1970.)

70T-B145. PRATIBHA GAJENDRAGADKAR, Indiana University, Bloomington, Indiana 47401. Norm of a derivation on a von Neumann algebra. Preliminary report.

Let $\mathcal{H}$ be any Hilbert space. For $A, T \in B(\mathcal{H})$, $D_T(A) = AT - TA$. Stampfli has proved that $\|D_T\| = \sup\{\|AT - TA\|, A \in B(\mathcal{H}), \|A\| = 1\} = 2 \inf\{\|T - \lambda I\|, \lambda \in \mathbb{C}\}$ (to appear). In this paper, it is proved that, if $\mathfrak{A}$ is a von Neumann algebra on a separable Hilbert space $\mathcal{H}$, centre of $\mathfrak{A} = \mathcal{Z}$, then for every $T \in \mathfrak{A}$, $\|D_T|_{\mathfrak{A}}\| = 2 \inf\{\|T - Z\|, Z \in \mathcal{Z}\}$. For a selfadjoint T, this result is contained in a paper by Kadison, Lance and Ringrose (J. Functional Analysis 1(1967)), but the methods have little in common. (Received April 20, 1970.)

70T-B146. C. EDWARD MOORE, U. S. Naval Academy, Annapolis, Maryland 21402. An answer to Klee's question concerning separation of convex sets by semispaces. Preliminary report.

A semispace S at the zero vector θ in a real linear space E is a maximal convex subset of $E \sim \{\theta\}$. V. L. Klee [Math. Scand. 4(1956)] showed that all semispaces S at θ are represented by a collection F of linear functionals on E and an intrinsic order r on F, i.e. $S = S(F,r)$. Two disjoint convex subsets A and B of a locally convex Hausdorff space E are said to be concretely separated provided A - B is contained in a semispace S(F,r) where all members of F are continuous. Theorem. If A and B (as above) are concretely separated then either A - B is not dense in E or A - B has empty core. An example of disjoint closed convex subsets A and B in a pre-Hilbert space E such that A - B is dense in E and A - B has nonempty core has been obtained. The above theorem and this example provide a negative answer to a question of Klee [Boeing Research Report, May 1969] asking if disjoint closed convex subsets in a locally convex space are always concretely separated. (Received April 21, 1970.)

70T-B147. M. W. BARTELT, Rensselaer Polytechnic Institute, Troy, New York 12181. Multipliers and operator algebras on bounded analytic functions.

Let B denote the bounded analytic functions on the open unit disc D and let C be those functions in B which are uniformly continuous on D. Certain subalgebras of the bounded linear operators from B into B are introduced and the coefficient multipliers from B into B are characterized. Let (B,τ) denote B in the topology τ, where τ is σ, κ or β which are respectively uniform convergence on D, uniform convergence on compact subsets of D and the strict topology. For two of these topologies τ_1 and τ_2, define $\tau_1:\tau_2$ to be the set of all continuous linear operators from (B,τ_1) into (B,τ_2). Theorem. The continuity classes $\tau_1:\tau_2$ are algebras under composition, the only distinct ones are $\kappa:\sigma$, $\kappa:\kappa$, $\beta:\beta$ and $\sigma:\sigma$ and all the proper inclusions between them are given by $\kappa:\sigma \subset \beta:\sigma \subset \beta:\beta \subset \sigma:\sigma$ and $\kappa:\sigma \subset \kappa:\kappa \subset \beta:\beta$. Theorem. If $T(\sum a_n z^n) = \sum a_n c_n^z$ is a coefficient multiplier from B into B, then

$c_n = L(z^n)$ for some L in the dual of (C,σ). Conversely any such L determines such a multiplier and $\|T\| = \|L\|$. The multipliers from B into B (B into C) are shown to be the multipliers in $\varkappa : \varkappa(\beta : \sigma)$.
(Received April 23, 1970.)

70T-B148. KENNETH J. PRESKENIS, Newton College of the Sacred Heart, Newton, Massachusetts 02159. Approximations on disks. Preliminary report.

Let f be a complex valued function on $D = (|z| \leq 1)$, R_f = the uniform closure on D of rational functions in z and f which are finite. Theorem. If f is of class C^1 in a nbd of D and $|f_{\bar{z}}| > |f_z|$ everywhere (i.e., f is an orientation reversing immersion of D in the plane), then $R_f = C(D)$.
Let $F = (f_1,\ldots,f_n)$ be a map on B = unit polydisk in C^n with values in C^n, P_F = the uniform closure on B of polynomials in $z_1,\ldots,z_n$, $f_1,\ldots,f_n$. Theorem. If F is of class C^2 in a nbd of B, $F_{\bar{z}}$ is invertible and if for each a in B, there exist complex constants $\{c_j\}$, $\{d_{ij}\}$ $i,j = 1,\ldots,n$ such that $\sum c_j(z_j - a_j)(f_j(z) - f_j(a)) + \sum d_{ij}(z_i - a_i)(z_j - a_j)$ has positive real part for all $z \neq a$ then $P_F = C(B)$. Corollary. If $F = (f,g)$ where $f(z,w) = \bar{z} + cz\bar{z} + d\bar{z}^2 + q\bar{z}w$, $g(z,w) = \bar{w} + sw\bar{w} + t\bar{w}^2 + p\bar{w}z$ and the coefficients satisfy $|\bar{c} + d| + |d| + |q| < 1$ and $|\bar{s} + t| + |t| + |p| < 1$ then $P_F = C(B)$. Corollary. If $F(z) = \bar{z} + R(z)$ where $R = (R_1,\ldots,R_n)$ is of class C^2 and satisfies the Lipschitz condition $|R(z) - R(w)| \leq k|z - w|$ with $k < 1$ then $P_F = C(B)$. This corollary is a result of Hörmander and Wermer. The first theorem is proved using measures, the next is proved using methods from several complex variables. (Received April 23, 1970.) (Author introduced by Professor Andrew Browder.)

Applied Mathematics

70T-C21. SUDHANSHU KUMAR GHOSHAL, University of Toronto, Toronto-181, Ontario, Canada. Viscous and nonviscous flows past ellipsoid of revolution in the presence of a magnetic field. Preliminary report.

In Geomagnetics, the toroidal part of the magnetic field plays a dominant role. Now, the earth's shape is not exactly spherical, but ellipsoidal and the flow past an ellipsoid is much more complicated under these conditions. In the first part of the work, the inviscid and in the second part, viscous flow past ellipsoids of revolution have been considered. The flow is always assumed to be axially symmetric and the magnetic field toroidal, so that the velocity $\vec{q}$ and the magnetic field $\vec{H}$ are of the form $\vec{q} = \text{curl}[-\psi\hat{\varphi}/\bar{\omega}]$ and $\vec{H} = U\hat{\varphi}/\bar{\omega}$. In the nonviscous case Oseen's type approximation is applied to the governing equations and on the assumption R_m, the magnetic Reynolds number small, U and the stream function ψ are expanded in powers of R_m. The governing equations are solved considering the order of magnitude of the terms. In the viscous case, Stoke's approximation is applied to the equations. R_m and Hartman number M are assumed to be small, Reynold-number-effects are assumed to be negligible, ψ, U and U(inside) are expanded in powers of M and R_m. Substituting these in the equations and considering the order of smallness, the resulting equations are solved. (Received January 9, 1970.)

70T-C22. LAWRENCE E. LEVINE, Stevens Institute of Technology, Hoboken, New Jersey 07030. Generalized, self-similar, simple waves in a barotropic gas. Preliminary report.

Recently [Levine, Quarterly Appl. Math. 27 (1969), 399-404] some of the properties of two-dimensional, self-similar simple waves in a polytropic gas have been studied. By introducing the enthalpy $h(\rho) = \int dp/\rho$ and proceeding in a manner similar to that used in the article referred to above, it is possible to prove several analogous results for a barotropic gas. In particular, one has: Theorem 1. The flow in a self-similar, simple wave region in the $X = x/c_0t$, $Y = y/c_0t$ plane is irrotational. Theorem 2. The equation satisfied by the dimensionless velocity potential $\Phi(X,Y)$ is in general hyperbolic. More specifically, this equation cannot be elliptic anywhere in such a region, and it cannot be parabolic except possibly on a curve. Theorem 3. All physical quantities, i.e. velocity, sound speed, and enthalpy, are constant along one family of characteristics which consist of straight lines. These results are of interest in the light of a recent paper by A. F. Sidorov and O. B. Khairullina [Prikl. Mat. Meh. 33 (1969), 967-976]. (Received February 23, 1970.)

70T-C23. NATHANIEL COBURN and M. SOMMERFIELD, University of Michigan, Ann Arbor, Michigan 48104. The propagation of discontinuities along characteristic surface in nonequilibrium fluids.

We determine the ordinary differential equation which governs the growth and decay, along the bicharacteristic curves, of the discontinuities in the derivatives of the solutions to systems of quasi-linear partial differential equations. The technique will then be used to obtain the growth of discontinuities in the physical variables appearing in the equations of a compressible nonequilibrium fluid, We then formulate conditions which are sufficient to insure the growth of the discontinuities for a particular speed of propagation. (Received March 2, 1970.)

70T-C24. PADAM C. JAIN, I. A. BELOV and K. SANKARA RAO, Indian Institute of Technology, Powai, Bombay-76, India. Numerical study of flow around a circular cylinder at high Reynolds number.

The problem on the unsteady flow motion of a viscous incompressible fluid around a circular cylinder has been considered by Kawaguti and Jain [J. Phys. Soc. Japan 21 (1966)], Jain and Rao [Phys. Fluids 12 (1969)] and others for Reynolds number R up to 200. By introducing appropriate transformations, the authors have obtained numerical solutions of the problem at high $R = 2000$. It is found that, in addition to the mean steady-type drag force which is typical at $R < 200$, there is a fluctuating drag force acting on the cylinder at $R = 2000$. This phenomenon is explained by the shedding of vortices which causes the pressure to fluctuate in the downstream vicinity of the cylinder. The minimum total drag C_D is obtained as 1.17 during the first 100 time steps of calculations with an amplitude of C_D fluctuations of about 0.06. The calculated value of the drag due to friction is about ten percent of C_D. At $R = 200$, this numerical technique gives results which are in complete agreement with the previous results. (Received March 2, 1970.)

70T-C25. PADAM C. JAIN and K. SANKARA RAO, Indian Institute of Technology, Powai, Bombay-76, India. Effects of time-dependent fluctuations on the flow around a circular cylinder.

The effects of time dependent fluctuations in the magnitude of oncoming stream velocity on the flow field around a circular cylinder are examined numerically at Reynolds numbers R = 20, 30, and 40. The fluctuations in the external flow field have been found to reduce the drag and to retard the occurrence of the limiting steady states which have been computed for R = 20 and 30. Streamlines, equivorticity lines, drag, pressure distribution, vorticity distribution, and other flow characteristics are plotted and compared with the corresponding values for the uniform flow. (Received March 2, 1970.)

70T-C26. PETER M. GOORJIAN and W. J. McCROSKEY, U. S. Army Aeronautical Research Laboratory, Ames Research Center, Moffett Field, California 94035. Potential flow around a rotating blade.

The aerodynamic problem of potential flow around a rotating cylindrical blade is examined. First, it is shown that the problem does not have a unique solution as formulated by W. R. Sears', "Potential Flow Around a Rotating Cylindrical Blade," J. Aeronaut. Sci. 17 (1950), 183. However, the potential function he obtained for a circular cylinder is shown to be identical to the potential flow about a rotating ellipsoid in the limiting case of infinite length. Second, the Sears solution neglects a component of spanwise flow that a blade of finite length has. For example, in the case of the circular cylinder, the spanwise flow is zero on the surface of the cylinder, whereas the rotating ellipsoid has a nonzero spanwise velocity which is of the same order of magnitude as the crossflow velocities encountered in the flat plate case. This component may be important when the potential flow is used to calculate laminar boundary layer flow on rotating blades. (Received March 5, 1970.)

70T-C27. I. A. BELOV, PADAM C. JAIN and B. D. KHANWALKAR, Indian Institute of Technology, Powai, Bombay-76, India. Influence of periodical vorticity on the stagnation-point flow.

Problem on a two-dimensional stagnation-point flow of a viscous incompressible fluid with periodical external steady vortices is considered. The vortices are distributed in the direction along the flat plate with their axes normal to the plane of motion. The equation of the vortex transfer is linearized and solved by Galerkin's method. The numerical results show the occurence of the diffusion and successive amplification of the low-frequency external vortices in the shear layer of the flat plate. For a vortex nearest to the line of flow symmetry in the inviscid flow, rotating in the anticlockwise direction, the influence on the flow pattern in the shear layer is much more pronounced than in the case of the vortex rotating in the opposite direction. The results of the calculation are compared with those for the Hiemenz stagnation-point flow. (Received March 17, 1970.)

70T-C28. SEYMOUR SHERMAN, Indiana University, Bloomington, Indiana 47401. When is an Ising magnet a ferromagnet?

A subcollection of the correlation inequalities recently obtained by Ginibre [Phys. Rev. Lett. 23(1969), 828-830] for Ising ferromagnets is shown to be enough to guarantee that an Ising magnet in an external magnetic field and with only two-body interactions be a ferromagnet. Thus one now has a necessary and sufficient set of correlation inequalities to guarantee that an Ising magnet in a non-

negative magnetic field and with only two-body interactions be a ferromagnet. To appear in J. Math. Phys. Partially supported by NSF GP 7469. (Received March 24, 1970.)

70T-C29. ANDRE GLEYZAL, U. S. Naval Ordnance Laboratory, White Oak, Silver Spring, Maryland 20910. Absolute Newtonian coordinate calculus.

Let $ds^2 = z_{i\mathcal{A}\,j\mathcal{A}}\,dz^{i\mathcal{A}}dz^{j\mathcal{A}}$ and $d\sigma = u^{\mathcal{J}}_{\eta i\mathcal{A}}\,dz^{i\mathcal{A}}$, summed on repeated indices $i\mathcal{A}, j\mathcal{A}$, etc., be two fundamental invariant forms of a curved Newtonian space-time η. Let $z^{i\mathcal{A}} = z^{i\mathcal{A}}(z^{\alpha\mathcal{B}})$, $z^{4\mathcal{A}} = z^{4\mathcal{B}} = t$ = time express the class of all Newtonian transformations, where $\alpha\mathcal{A} = 1,\cdots,4$; $i\mathcal{A} = 1,\cdots,3$; and the symbols $\mathcal{A},\mathcal{B},\mathcal{C},\cdots,\mathcal{Z}$ denote arbitrary coordinate systems. Let $\mathcal{J}$ be motionless, let $u^{\mathcal{A}}_{\mathcal{B}} = u^{\mathcal{A}i\mathcal{A}}_{\mathcal{B}} = z^{i\mathcal{A}}_{4\mathcal{B}} = z^{i\mathcal{A}}_{,4\mathcal{B}} = \partial z^{i\mathcal{A}}/\partial z^{4\mathcal{B}}$, and let $u^{\mathcal{A}i\mathcal{C}}_{\mathcal{B}} = u^{\mathcal{A}i\mathcal{A}}_{\mathcal{B}}\,z^{i\mathcal{C}}_{i\mathcal{A}}$, where $z^{i\mathcal{C}}_{i\mathcal{A}} = z^{i\mathcal{C}}_{,i\mathcal{A}} = \partial z^{i\mathcal{C}}/\partial z^{i\mathcal{A}}$. Then $u^{\mathcal{A}}_{\mathcal{B}} + u^{\mathcal{B}}_{\mathcal{C}} = u^{\mathcal{A}}_{\mathcal{C}}$. This is the addition law of relative velocities, where the vector $u^{\mathcal{A}}_{\mathcal{B}}$ is the velocity of $\mathcal{B}$ relative to $\mathcal{A}$. Consider a tensor expression $\lambda^{\cdots}_{\cdots} = \lambda^{i\mathcal{A}\cdots i\mathcal{B};\, i\mathcal{S}\cdots i\mathcal{T};\, 4\mathcal{J}\cdots 4\mathcal{L}}_{i\mathcal{C}\cdots i\mathcal{D};\, i\mathcal{U}\cdots i\mathcal{X};\, 4\mathcal{K}\cdots 4\mathcal{M}}$ where $\lambda^{\cdots\cdots}_{\cdots i\eta\cdots} = \lambda^{\cdots\cdots i\mathfrak{M}}_{\cdots i\mathfrak{M}\cdots z_{i\eta}}$. For a vector $\lambda^{i\mathcal{A}}$ example: $\lambda^{i\mathcal{A}}_{4\mathcal{B}} = \lambda^{i\mathcal{A}}_{4\mathcal{A}} + \lambda^{i\mathcal{A}}_{j\mathcal{A}}\,u^{\mathcal{A}j\mathcal{A}}_{\mathcal{B}}$, $\lambda^{i\mathcal{A}}_{;j\mathcal{A}} = \lambda^{i\mathcal{A}}_{,j\mathcal{A}}$, $\lambda^{i\mathcal{A}}_{4\mathcal{A}} = \lambda^{i\mathcal{A}}_{,4\mathcal{A}} + \lambda^{j\mathcal{A}}u^{\mathcal{J}i\mathcal{A}}_{\mathcal{A}\,j\mathcal{A}} = \lambda^{i\mathcal{A}}_{,4\mathcal{A}} + \lambda^{j\mathcal{A}}(u^{\mathcal{J}i\mathcal{A}}_{\mathcal{A},j\mathcal{A}} + u^{\mathcal{J}k\mathcal{A}}_{\mathcal{A}}\,\Gamma^{i\mathcal{A}}_{k\mathcal{A}j\mathcal{A}})$, and $\lambda^{i\mathcal{A}}_{4\mathcal{C}} = \lambda^{i\mathcal{A}}_{4\mathcal{B}} + \lambda^{i\mathcal{A}}_{j\mathcal{A}}\,u^{\mathcal{B}j\mathcal{A}}_{\mathcal{C}}$. The "Christoffel time connection" is thus $\Gamma^{\mathcal{J}i\mathcal{A}}_{\mathcal{A}\cdot j\mathcal{A}} = u^{\mathcal{J}i\mathcal{A}}_{\mathcal{A}\,j\mathcal{A}}$ and is not a tensor due to the presence of three $\mathcal{A}$'s, although $u^{\mathcal{J}i\mathcal{B}}_{\mathcal{A}i\mathcal{C}}$ is a tensor. General tensor time derivatives of $\lambda^{\cdots}_{\cdots}$ are similarly defined with $\Gamma^{\mathcal{J}i\mathcal{A}}_{\mathcal{A}\cdot j\mathcal{A}}$ or $\Gamma^{\mathcal{J}\cdot j\mathcal{A}}_{\mathcal{A}i\mathcal{A}}$. The acceleration vector of $\mathcal{B}$ relative to $\mathcal{A}$ is $\eta^{\mathcal{A}i\mathcal{C}}_{\mathcal{B}} = u^{\mathcal{A}i\mathcal{C}}_{\mathcal{B}4\mathcal{B}}$, in coordinates of $\mathcal{C}$. Field equations and machine programs may thus be unified for all flow problems. (Received April 7, 1970.)

70T-C30. BORO DOERING, University of Düsseldorf, 4000 Düsseldorf, Federal Republic of Germany. On Chebyshev's and Halley's methods.

The assumptions of a theorem in a recent abstract of the author (Abstract 70T-C19, these *Notices* 17(1970), 574) can be weakened in case of Chebyshev's method (CM) $x_{n+1} := x_n - [I + 0.5f_n^{-1}F''(x_n)f_n^{-1}F(x_n)]f_n^{-1}F(x_n)$ $(f_n := F'(x_n))$ and Halley's method (HM) $x_{n+1} := x_n - [I - 0.5f_n^{-1}F''(x_n)f_n^{-1}F(x_n)]^{-1}f_n^{-1}F(x_n)$. Moreover, for these methods the error bounds can be sharpened and simplified. These turn out to be much better than the known bounds. (Results will soon be published in Numer. Math.) Let X,Y be arbitrary Banach spaces and $F: X \supset X_F \to Y$ a nonlinear operator defined on the (open) set X_F. Theorem. If $x_0 \in X_F$ can be chosen such that (1) $\exists f_0^{-1}$ with $\|f_0^{-1}\| \leq \beta_0$; (2) $\exists S := \{x \in X \mid \|x - x_0\| \leq 1.6\zeta_0\}$ with $\zeta_0 := \|f_0^{-1}F(x_0)\|$ where F is three times Fréchet differentiable with $\|F^{(j)}(x)\| \leq K_j\ \forall x \in S \wedge j = 2,3$; (3) $B_0K_j\zeta_0^{j-1} \leq 1/3$ for $j = 2,3$, then $\exists_1 x^*$ with $F(x^*) = \theta$ in S, CM and HM converge of order 3 to x^* and $\|x^* - x_{n+1}\| < (2/15)B_{n+1}\cdot(2K_3 + 7\beta_{n+1}K_2^2)\|x_{n+1} - x_n\|^3$ $\forall n \in \mathbb{N} \cup \{0\}$ where $\beta_{n+1} := \beta_n/(1 - \eta_n)$; $\eta_n := \alpha\beta_n\zeta_nK_2$; $\zeta_n := \|f_n^{-1}F(x_n)\|$; $\alpha := 7/3$(CM) and $12/5$(HM). At the cost of a worse bound (3) can still be weakened somewhat. It may, however, be remarked that in some papers of B. Jankó and his co-workers on CM and HM the assumption $B_0K_2\zeta_0 \leq 1/2$ is partly cancelled by other ones there, e.g. in Jankó and Balázs (Stud. Cerc. Mat. 18(1966), 817-828) it follows from assumption 5° that $B_0K_2\zeta_0 < (4 - \sqrt{7})/3 < 1/2$. (Received April 22, 1970.)

Geometry

70T-D10. KIN-ETSU ABE, Brown University, Providence, Rhode Island 02912. On the complex analogue of Hartman-Nirenberg's cylinder theorem. Preliminary report.

Let M^n be a complete Kähler hypersurface in $\mathbb{C}^{n+1}$ holomorphically and isometrically immersed by f. Let $\varphi : M^n \to \mathbb{C}P^n$ be the generalized Gauss map [for example, see K. Nomizu-B. Smyth, J. Math. Soc. Japan 20 (1968), 498-521]. Define the rank of φ to be the rank of the Jacobian φ_*. Theorem. The following conditions are equivalent: (i) $r \leq 2$ everywhere; (ii) $\varphi(M) \subset CP^1 \subset CP^n$; (iii) M is cylindrical: i.e., f(M) is a product of an (n - 1)-dimensional complex hyperplane C^{n-1} and a complex curve in the 2-dimensional complex plane orthogonal to $\mathbb{C}^{n-1}$ in $\mathbb{C}^{n+1}$. This result is the complex version of the Hartman-Nirenberg theorem [Amer. J. Math. 81 (1959), 901-920]. (Received November 7, 1969.)

70T-D11. JOSEPH A. ERBACHER, Brown University, Providence, Rhode Island 02912. Reducing the codimension of an isometric immersion. Preliminary report.

The following theorem extends some results of C. Allendoerfer (Amer. J. Math. 61 (1939), 633-644). Theorem. Let $\psi : M^n \to \tilde{M}^{n+p}(c)$ be an isometric immersion of a connected n-dimensional Riemannian manifold M^n into a (n + p)-dimensional space form $\tilde{M}^{n+p}(c)$ of sectional curvature c. Let $N_1(x)$ be the first normal space at x. If dimension $N_1 = \ell$ = a constant and if N_1 is invariant under parallel displacement with respect to the normal connection, then there exists a totally geodesic submanifold $N^{n+\ell}$ of $\tilde{M}^{n+p}$ such that $\psi(M^n) \subset N^{n+\ell}$. Remarks. (a) It is an easy consequence of Codazzi's equation that if dimension N_1 = a constant and if the type number of the immersion ≥ 2, then N_1 is parallel with respect to the normal connection. (b) A slightly more general theorem is true using the higher normal spaces as defined by Allendoerfer. (Received November 7, 1969.)

70T-D12. PATRICK BARRY EBERLEIN, University of California, Los Angeles, California 90024. Geodesic flow in certain manifolds without conjugate points. Preliminary report.

Let M be an n-dimensional C^∞ manifold, not necessarily compact, with complete metrics g, g* such that the following is true. (1) Sectional curvature for g* satisfies $K \leq c < 0$. (2) There are no conjugate points along any g-geodesic. (3) There exist positive constants a, b such that $a^2 g(x,x) \leq g^*(x,x) \leq b^2 g(x,x)$ for any vector x tangent to M. (4) If γ, σ are distinct maximal geodesics in the universal Riemannian covering of (M,g) then either $d(\gamma t, \sigma) \to \infty$ as $t \to \infty$ or $d(\gamma t, \sigma) \to \infty$ as $t \to -\infty$. Let SM(g) be the g-unit tangent bundle, $\{T_t\}$ the geodesic flow in SM(g), $\Omega(g)$ the nonwandering points of $\{T_t\}$ in SM(g). Theorem 1. One of the following must occur: (1) $\Omega(g)$ is empty. (2) $\Omega(g)$ is a single periodic orbit and the reverse orbit. (3) $\Omega(g)$ contains infinitely many distinct periodic orbits. Periodic points are dense in $\Omega(g)$. Either $\Omega(g)$ = SM(g) or $\Omega(g)$ is nowhere dense in SM(g). Theorem 2. If $\Omega(g)$ contains more than a single periodic orbit and the reverse orbit then $\{T_t\}$ has an orbit dense in $\Omega(g)$. For every g*-geodesic γ^* there is a unique g-geodesic γ of the "same type" and with the same endpoints, finite or infinite. One obtains results by comparing g-geodesics to g*-geodesics for which Theorems 1 and 2 are known. (Received March 6, 1970.)

70T-D13. AIDEN BRUEN, L. C. MORLEY and DAVID J. RODABAUGH, University of Missouri, Columbia, Missouri 65201. Veblen Wedderburn systems and spread sets. Preliminary report.

In this paper we show directly that the existence of a Veblen Wedderburn system Q is equivalent to the existence of certain collections σ of matrices known in the literature as t-spread sets. Various techniques for obtaining a new spread set from a given spread set σ, corresponding to a V-W system Q, are illustrated (for example if Q is finite the set of matrices obtained by transposing each matrix of σ is also a spread set). By examining the nuclei of V-W systems, viewed as spread sets, it is shown that the techniques above give rise to V-W systems which in general are not isotopic to Q. Some related results on isotopy are discussed. (Received March 11, 1970.)

70T-D14. WITHDRAWN.

70T-D15. ROBERT MALTZ, Université de Montréal, Montréal, Quebec, Canada. On the de Rham product decomposition.

We present a simple proof of the de Rham product theorem for semi-Riemannian manifolds, using an elegant method of constructing covering spaces due to Barrett O'Neill. As a matter of fact, we generalize the de Rham theorem to the case of metric connections with torsion. As a by-product, we give a simple proof of the Ambrose-Hicks theorem on parallel translation of curvature. (Received March 16, 1970.)

70T-D16. BERNHARD GANTER, Universität Bonn, Bonn, Federal Republic of Germany. The finite completion of finite partial Steiner systems of type (2,q + 1,n), where q is theorder of an affine plane.

A partial Steiner System (S,T) of type (h,m,n) is an n-set (i.e. a set with n elements) S and a set T of m-subsets of S, such that each h-subset of S is subset of at most one element of T. If each h-subset of S is a subset of exactly one element of T, (S,T) is called a Steiner System. A Steiner System of type $(2,q,q^2)$ is called an affine plane. The author shows by induction on n that any finite partial Steiner System of type (2,q + 1, n), where q is the order of an affine plane, can be embedded in a finite Steiner System. This is a generalisation of a note of C. Treash. (See Abstract 663-180, these *Notices* 16 (1969), 139.) (Received March 18, 1970.) (Author introduced by Dr. Rudolf Wille.)

70T-D17. JAYME M. CARDOSO, Instituto de Matematica, Universidade de Campinas, Caixa Postal 1170, Campinas -Sao Paulo, Brazil. Common tangents of two ellipses.

It is shown that the common tangents of two ellipses of same excentricity and parallel axis meet in the centers of similitude of its major circles (center of similitude in the sense of Veblen and Young, "Projective geometry", vol. 2, p. 162). (Received March 9, 1970.)

Logic and Foundations

70T-E29. PETER H. KRAUSS, State University of New York, New Paltz, New York 12561. Robinson's theorem for universal theories. Preliminary report.

Lemma. Let $\mathfrak{A}$ and $\mathfrak{B}$ be relational systems with index sets $I_{\mathfrak{A}}$ and $I_{\mathfrak{B}}$ respectively, where $I_{\mathfrak{A}} \subseteq I_{\mathfrak{B}}$. Then there exists a relational system $\mathfrak{C}$ such that $\mathfrak{B} \prec \mathfrak{C}$ and $\mathfrak{A}$ is embeddable into $\mathfrak{C} \upharpoonright I_{\mathfrak{A}}$, iff $\mathfrak{A}$ is a model of the universal theory of $\mathfrak{B} \upharpoonright I_{\mathfrak{A}}$. Theorem. Let $\mathfrak{A}$ and $\mathfrak{B}$ be relational systems with index sets $I_{\mathfrak{A}}$ and $I_{\mathfrak{B}}$ respectively. Let $I_{\mathfrak{A}} \cap I_{\mathfrak{B}} = J$, and suppose $\mathfrak{A} \upharpoonright J$ and $\mathfrak{B} \upharpoonright J$ are universally equivalent. Then there exists a relational system $\mathfrak{C}$ with index set $I_{\mathfrak{C}}$ such that $I_{\mathfrak{C}} = I_{\mathfrak{A}} \cup I_{\mathfrak{B}}$, $\mathfrak{A} \prec \mathfrak{C} \upharpoonright I_{\mathfrak{A}}$ and $\mathfrak{C} \upharpoonright I_{\mathfrak{B}}$ is a model of the universal theory of $\mathfrak{B}$. In particular, the union of the first order theory of $\mathfrak{A}$ and the universal theory of $\mathfrak{B}$ is consistent. Corollary. Let σ' and σ'' be universal sentences. If $K \not\models \sigma' \rightarrow \sigma''$, then for some sentence σ which is a Boolean combination of universal sentences and whose nonlogical constants occur both in σ' and σ'', $\models \sigma' \rightarrow \sigma$ and $\models \sigma \rightarrow \sigma''$. Remark. The lemma is a generalization of a theorem due to Keisler, and a syntactical proof of the corollary is due to Craig. (Received February 25, 1970.)

70T-E30. L. P. de ALCANTARA, University of Campinas, Campinas, Sao Paulo, Brazil. On a system of Houdebine.

In (C.R. Acad. Sci. Paris 260 (1965), 3805-3808), Houdebine describes a system of set theory apt to function as a basis for elementary category theory. Notwithstanding, it can be shown that this system is inconsistent. (In "Théorie des classes et théorie des catégories," thèse, Rennes, 1967, Houdebine studies a similar system, apparently consistent.) In this work we consider a new form of Houdebine's original system, denoted by H, using as underlying logic the predicate calculus of first order with equality (without the Hilbert symbol). We also modify H by the elimination of the schema of replacement, employing instead a strong version of the axiom of replacement; the new system so obtained is called H*. H* is weaker than H. H* is equivalent to a system of da Costa (cfr. N. C. A. da Costa, "On a set theory suggested by Dedecker and Ehresmann," Université de Lille, 1967), conveniently modified. H and H* can be used to found elementary category theory. Generalizing Houdebine's device, we introduce a stronger system C, in which there is a denumerable number of symbols of pertinence (Houdebine employs only two symbols of pertinence). (Received February 23, 1970.) (Author introduced by Professor N.C.A. da Costa.)

70T-E31. MANUEL LERMAN, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. Initial segments of the degrees of unsolvability.

Theorem. Every finite lattice is order-isomorphic to an initial segment of the degrees of unsolvability. (Received March 3, 1970.)

70T-E32. ERIK ELLENTUCK, Rutgers University, New Brunswick, New Jersey 08903. Weak second order geometry.

Weak second order plane Euclidean geometry E_2^1 (cf. A. Tarski, "What is elementary geometry? The axiomatic method. With special reference to geometry and physics," Proc. Internat. Sympos. (Berkeley, Calif., 1958), North-Holland, Amsterdam, 1959) is essentially nonfinitely axiomatizable and essentially hyperarithmetically undecidable. (Received March 10, 1970.)

70T-E33. WITHDRAWN.

70T-E34. BARUCH GERSHUNI, Frans van Mierisstraat, 37 huis, Amsterdam, Netherlands. On the representation of a singular in the theory of totalities.

The author has tried to determine the totality of all possible forms of representation of singulars in the "Elementary Totalities Theory" (E. T. T.). But it turned out that there is not such closed totality. The author has introduced (in mathematics) the concepts of "singular totality" with respect to the exterior and with respect to the interior of this totality. And it is not known which other forms of singular totalities may appear in the future. We define now the above concepts of singular. A totality is called a collectively connected singular with respect to its exterior iff it is provided with covers (or similar things) at its beginning and its end. These two covers are symbolized by a pair of parantheses" (" and ")". A totality is called a collectively connected singular with respect to its interior, iff there is a formula valid for all elements of the totality. This condition is symbolized by a stroke above the totality. So the two kinds of singular are written in the form (a,b,c,...) and $\overline{a, b, c, \ldots}$. The corresponding operators are ( ) and $\overline{}$. The united operator $\overline{(\ )}$ is called the set-operator and is written also in the form { }. The corresponding totality which is united both exteriorly and interiorly is the known totality "set" and is written in the form {a,b,c,...}. (Received March 18, 1970.)

70T-E35. GREGORY J. CHAITIN, Mario Bravo 249, Buenos Aires, Argentina. Computational complexity and Godel's incompleteness theorem. Preliminary report.

Given any simply consistent formal theory F of the state complexity L(S) of finite binary sequences S as computed by 3-tape-symbol Turing machines, there exists a natural number L(F) such that $L(S) > n$ is provable in F only if $n < L(F)$. On the other hand, almost all finite binary sequences S satisfy $L(S) > L(F)$. The proof resembles Berry's paradox, not the Epimenides nor Richard paradoxes. (Received April 6, 1970.)

70T-E36. CARL G. JOCKUSCH, Jr., University of Illinois, Urbana, Illinois 61801. Ramsey's theorem and recursion theory. Preliminary report.

Let N be the set of natural numbers, and let P be a variable ranging through partitions, into two classes, of all two-element subsets of N. For such partitions P, let H(P) be the collection of those infinite subsets of N, all of whose two-element subsets lie in the same class. A basic form of Ramsey's theorem states that H(P) is nonempty for every P. Theorem 1. If P is recursive, then

H(P) contains a Π^0_2 set. Theorem 2. There exists a recursive P such that H(P) contains no Σ^0_2 set. Theorem 3. If the sets A_i ($i \in N$) are uniformly of degree $\lesssim \underset{\sim}{0}'$, then there exists a recursive P such that H(P) contains a set of degree $\lesssim \underset{\sim}{0}'$ but does not contain any A_i. Theorem 4. If $\underset{\sim}{a}$ is a degree and $\underset{\sim}{0} \lesssim \underset{\sim}{a} \lesssim \underset{\sim}{0}'$, then there exists a recursive P such that H(P) contains a set of degree $\underset{\sim}{a}$ but no recursive set. Theorem 5. If P is recursive, then H(P) contains a set whose degree $\underset{\sim}{a}$ satisfies $\underset{\sim}{a}' \lesssim \underset{\sim}{0}''$. Most of these results are proved by direct construction, but the final one is derived from basis theorems of Soare and the author. (Received April 9, 1970.)

70T-E37. KENNETH KUNEN, University of Wisconsin, Madison, Wisconsin 53706. Two theorems on untrafilters. Preliminary report.

Let $\varkappa$ be any infinite cardinal. Theorem 1. There is a good ultrafilter on $\varkappa$. Theorem 2. If $\varkappa$ is regular and D is a descendingly $\varkappa^+$-incomplete ultrafilter on some set, then D is descendingly $\varkappa$-incomplete. For those $\varkappa$ such that $2^\varkappa = \varkappa^+$, these theorems are due, respectively, to H. J. Keisler (Ann. of Math. (2) 79 (1964), 338-359) and C. C. Chang (Trans. Amer. Math. Soc. 126 (1967), 108-118). (Received April 10, 1970.)

70T-E38. ALISTAIR H. LACHLAN, Simon Fraser University, Burnaby 2, British Columbia, Canada. The transcendental rank of a theory.

For each countable complete theory T, Morley ["Categoricity in power", Trans. Amer. Math. Soc. 114 (1965), 514-538] defined an ordinal $\alpha_T < (2^{\aleph_0})^+$ which is called the transcendental rank of T. It is shown that the possible values of α_T are the ordinals > 0 and $\leq \omega_1$. (Received April 16, 1970.)

70T-E39. THOMAS J. JECH, State University of New York at Buffalo, Amherst, New York 14226. Isomorphism types of trees. Preliminary report.

A tree means a normal tree of length ω_1 whose every level is countable. A tree is (a) Aronszajn (AT), (b) Kurepa (KT), (c) Suslin (ST) if (a) every branch (totally ordered subset) is countable, (b) there are at least $\aleph_2$ uncountable branches, (c) every set of incomparable elements is countable. A Suslin continuum (SC) is a counterexample to Suslin's conjecture. (I) Jensen's axiom ($\diamondsuit$) [Abstract 69T-E24, these *Notices* 16 (1969), 576] implies that the number of isomorphism types of ST (SC) is at least $\aleph_2$. (II) If V = L then there is a rigid KT, a KT with $\aleph_1$ automorphisms and a KT with $\aleph_2$ automorphisms; the number of isomorphism types of KT is $\aleph_2$. (III) The following is consistent relative to ZF: There is a ST (SC) with $\aleph_{17}$ automorphisms and $2^{\aleph_1} = \aleph_{324}$. (IV) In ZFC, there is an AT with $2^{\aleph_1}$ automorphisms; the existence of a rigid tree is open. (Received April 13, 1970.)

70T-E40. ALEXANDER ABIAN, Iowa State University, Ames, Iowa 50010. On a property of reduced powers.

Let $[Y,X,F]$ denote the reduced power Y^X relative to the filter F of the Boolean algebra 2^X. For every element y of Y let $[y]$ denote the equivalence class of $\{y\}^X$ in Y^X relative to F. Theorem. If Y has at least two elements and F is a nonvoid proper filter then $[Y,X,F] = \{[y] \mid y \in Y\}$ if and only if F is a $\overline{\overline{Y}}$-complete ultrafilter. (Received April 20, 1970.).

70T-E41. DAVID O. OAKLAND, Iowa State University, Ames, Iowa 50010. On properties of ultrapowers.

For notations see the preceding abstract. Theorem 1. If Y is simply ordered with at least two elements and F is a nonvoid proper filter then $[Y,X,F]$ is simply ordered if and only if F is an ultrafilter. Theorem 2. If Y is infinite and partially well ordered and F is an ultrafilter then $[Y,X,F]$ is partially well ordered if and only if F is σ-complete. Corollary. If Y is infinite and well ordered and F is a nonvoid proper filter then $[Y,X,F]$ is well ordered if and only if F is a σ-complete ultrafilter. (Received April 20, 1970.)

70T-E42. JAN MYCIELSKI, University of Colorado, Boulder, Colorado 80302 and University of California, Berkeley, California 94720. Undefinability of connected sets.

X denotes any finite set of points with integral coordinates on the plane. X is said to be connected if any two points of X can be joined by a polygonal line all edges of which have length 1 and all vertices of which are in X. Z denotes the set of all integers, $m = \{n : n < m\}$ for $m < \omega$. L_1 denotes the first order language with one binary function symbol and two binary relation symbols, L_2 denotes the first order language with two binary relation symbols. Theorem 1. There is no sentence $\sigma \in L_1$ such that $\langle Z, +, \leq, X\rangle$ satisfies σ if and only if X is connected. Theorem 2. There is no sentence $\sigma \in L_2$ such that for every positive $m < \omega$ and every $X \subseteq m \times m$ the structure $\langle m, \leq, X\rangle$ satisfies σ if and only if X is connected. (Received April 20, 1970.)

70T-E43. SAHARON SHELAH, Princeton University, Princeton, New Jersey 08540. On elementary classes containing categorical pseudo-elementary classes. Preliminary report.

Let T be a complete first-order theory. If $T_1 \supset T$, $PC(T_1, T)$ is the class of reducts of models of T_1 to $L(T)$. $PC(T_1, T)$ is categorical in a cardinal μ if all its models of cardinality μ are isomorphic. Definition. $P(T,\lambda,\mu)$ holds if there is $T_1 \supset T$, $|T_1| = \lambda$ such that $PC(T_1, T)$ is categorical in μ. Theorem 1. If $PC(T_1, T)$ is categorical in $\mu > |T_1|$, then every model in it of cardinality μ is saturated. From now on we assume T is countable. For definitions see Shelah, Israel J. Math. 7 (1969), 187, and Keisler, J. Symbolic Logic 32 (1967), 23-47, §4. Theorem 2. If $\mu > \lambda \geq 2^{\aleph_0}$ then $P(T, \lambda, \mu)$ holds iff T is superstable and has not the f.c.p. (In most cases we can replace $\mu > \lambda$ by $\mu \geq \lambda$.) Theorem 3. If $\mu > \lambda \geq \aleph_0$, $2^{\aleph_0} > \lambda$, then $P(T,\lambda, \mu)$ holds iff T is totally transcendental and has not the f.c.p. Theorem 4. $P(T, 2^{\aleph_0}, 2^{\aleph_0})$ holds iff T is superstable. (Received April 20, 1970.) (Author introduced by Professor Elias M. Stein).

70T-E44. DONALD ALVIN ALTON, Cornell University, Ithaca, New York 14850. Recursively enumerable sets which are uniform for finite extensions. Preliminary report.

For notation and background material, see Hartley Rogers, "Theory of recursive functions and effective computability," McGraw-Hill, New York, 1967. Let $W_0, W_1, \ldots$ be one of the usual enumerations of the recursively enumerable (r.e.) subsets of the set of natural numbers. Definitions. An r.e. set A is uniform for finite extensions if there is a recursive function f such that for all e, $W_{f(e)} \supset W_e \cap \bar{A}$ and $W_e \cap \bar{A}$ finite implies $W_{f(e)}$ finite; it is weakly uniform for finite extensions if there is a partial recursive function ψ such that for all e, if $W_e \cap \bar{A}$ is finite, then $\psi(e)$ converges,

$W_{\psi(e)}$ is finite, and $W_{\psi(e)} \supseteq W_e \cap \bar{A}$. Theorem 1. Every r.e. nonrecursive Turing degree contains a hypersimple set which is uniform for finite extensions. Theorem 2. If $\underline{a}$ is an r.e. Turing degree such that $\underline{a}' = \underline{0}''$, then $\underline{a}$ contains a simple set which is not weakly uniform for finite extensions. The proofs are priority arguments. (Received April 20, 1970.)

70T-E45. S. B. COOPER, University of Leeds, Leeds, England. Minimal upper bounds for ascending sequences of recursively enumerable degrees.

Clifford Spector showed that no infinite, ascending sequence of degrees has a least upper bound and G. E. Sacks showed that there are uncountably many minimal upper bounds for a given countable ascending sequence of degrees. Sacks asked whether there is an ascending uniformly recursively enumerable sequence of degrees with a r.e. minimal upper bound. It can be shown that $\underline{0}'$ cannot be such an upper bound, and, in fact, cannot be a minimal r.e. upper bound for such a sequence. Theorem 1. Every nonzero r.e. degree with jump $\underline{0}'$ is a minimal upperbound for an ascending sequence of r.e. degrees. Theorem 2. There is a r.e. degree which is a minimal r.e. upper bound for an ascending uniformly recursively enumerable sequence of degrees. (Received April 14, 1970.) (Author introduced by Dr. Frank R. Drake.)

70T-E46. BRUNO J. SCARPELLINI, Mathematics Institute, University of Basel, Basel, Switzerland. A model for bar recursion of higher type.

(A) Types. 0 is a type, if σ, τ are types then (σ/τ) is a type. The set N of natural numbers is the set S_0 of objects of type 0. If x_n, $x \in S_0$ then x_n converges against x ($x_n \Rightarrow_0 x$) iff $x_n = x$ apart from finitely many x_n's. Assume S_σ, S_τ and corresponding limit notions to be given. As $S_{(\sigma/\tau)}$ we take the set of continuous mappings from S_σ into S_τ ($f(x_n) \Rightarrow_\tau f(x)$ if $x_n \Rightarrow_\sigma x$). Define $\Rightarrow_{(\sigma/\tau)}$ as follows: $f_n \Rightarrow_{(\sigma/\tau)} f$ iff $f(x_n) \Rightarrow_\tau f(x)$ whenever $x_n \Rightarrow_\sigma x$ (Kuratowski, "Topologie. I," 1952). Theorem 1. $\bigcup_\sigma S_\sigma$ is a model for bar recursion of higher type. (B) Put $C_0 = S_0$. Let $C_\sigma \subseteq S_\sigma$, $C_\tau \subseteq S_\tau$ already be given. Assume that for all $x \in C_\sigma$, $y \in C_\tau$ their nonempty sets G(x), G(y) of Goedel numbers are known. Then $f \in C_{(\sigma/\tau)}$ iff $f \in S_{(\sigma/\tau)}$, $f(C_\sigma) \subseteq C_\tau$ and if there is a partial recursive φ such that: if $n \in G(x)$ then $\varphi(n) \in G(f(x))$. If e is a Goedel number of such a φ then $e \in G(f)$. Theorem 2. The bar recursive functionals are in $\bigcup_\sigma C_\sigma$. (C) A more constructive variant of $\bigcup_\sigma S_\sigma$ can be given, which can be reproduced in classical analysis. Shoenfield's result (Mathematical Logic, 1967, p. 226) can be extended to an equational calculus containing schematas for bar recursion of higher type. A detailed report appears elsewhere. (Received April 23, 1970.)

70T-E47. GEORGE McNULTY, University of California, Berkeley, California 94720. On algebras whose equational theories are essentially base-undecidable. Preliminary report.

For notation see [Abstract 676-7, these *Notices* 17 (1970), 638]. The main Theorem. (I) Let Θ be a finitely based equational theory containing some equation of the form $x = \tau$ where τ is a term such that its main operation symbol is of rank ≥ 2 or it contains at least two different operation symbols of rank 1 and no operation symbol of rank ≥ 2. Then Θ is essentially base-undecidable. (In case symbolism without parenthesis is used the main operation symbol of τ is its leftmost symbol.) Interesting particular cases of (I) are stated below: (II) The theory of each of the

following algebras is essentially base-undecidable (provided it is finitely based): (1) any semilattice $\langle A, \vee \rangle$, any lattice $\langle A, \vee, \wedge \rangle$, or any Boolean algebra $\langle A, \vee, \wedge, ^{-} \rangle$; (2) any group $\langle A, \cdot, ^{-1} \rangle$; (3) any ring $\langle A, +, -, \cdot \rangle$ or any ring with unit $\langle A, +, -, \cdot, 1 \rangle$; (4) any primal algebra; (5) any two-element groupoid $\langle A, \cdot \rangle$ in which $\cdot$ is not a constant operation; (6) any one-element algebra with at least (6') one operation of rank $\geqq 2$ or (6'') two operations of rank 1. (II(6')) can essentially be found in the Ph.D. thesis of Perkins (University of California, Berkeley, 1966). The conclusion of (II) extends to equational theories definitionally (rationally) equivalent to those of algebras listed in (II (1)), (II(2)). (Received April 23, 1970.)

Statistics and Probability

70T-F9. VIDYADHAR S. MANDREKAR and HABIB SALEHI, Department of Statistics and Probability, Michigan State University, East Lansing, Michigan 48823. Operator-valued wide-sense Markov processes and stochastic differential system.

The notion of wide sense Markov processes due to J. L. Doob for multivariate case is extended to the case of operator-valued processes using the work of Payen [Ann. Inst. H. Poincaré Sect. B3 (1967)]. The extensions to infinite-dimensional case of the results of F. Beutler on solutions of stochastic differential system (Ann. Math. Statist. 34 (1964)), and of V. Mandrekar, on the structure of wide-sense Markov processes (Nagoya Math. J. 33 (1968)) are obtained. Using the notion of the generalized inverse of an operator, the assumption of nonsingularity of covariances made earlier is removed. For purely nondeterministic Markov processes it is shown that the kernel of the Karhünen representation splits, giving exact analogues of earlier theorems of Doob (in stationary case) and Mandrekar (in nonstationary case). The operator-valued stochastic processes arise naturally in the study of random heat equation (see N. N. Vakhania, Theor. Probability Appl. 12 (1967)). (Received March 16, 1970.)

70T-F10. JOHN S. KALME, U. S. Naval Academy, Annapolis, Maryland 21402. Some analyticity properties of trajectories of stationary gaussian processes.

Let $L^p = \{f \mid \int_{-\infty}^{\infty} |f(x)|^p dx < \infty\}$, $\|f\|_p = (\int_{-\infty}^{\infty} |f(x)|^p dx)^{1/p}$, $p > 1$. Let $[V(t)f](x) = \pi^{-1} t \int_{-\infty}^{\infty} f(x-u)(t^2+u^2)^{-1} du$, $0 < t < \infty$, $[V(0)f](x) = f(x)$. Then $V(t)$ defines a semigroup of operators on L^p. The infinitesimal generator $Af = -\tilde{f}'$ where $\tilde{f}(x) = \pi^{-1} PV \int_{-\infty}^{\infty} f(u)(x-u)^{-1} du$. Theorem. Let $\{X_u\}$ be a real stationary gaussian process. Let $R(h) = E(X_{u+h} X_u)$ with $R(0) = 1$. Suppose $\exists$ a positive nondecreasing function $H(t)$ defined in $(0, \delta)$ such that $1 - R(t) = O(H(t)|\log|t||^{-1})$, $\int_0^{\delta} t^{-1-\sigma p} [H(t)]^{p/2} dt < \infty$, and $\exists\, k \ni \int_0^{\epsilon} t^{-1} [H(t)]^{1/2} dt < K[H(\epsilon)]^{1/2}$ for $0 < \epsilon < \delta$, where $0 < \sigma < 1$. Then $\exists$ a process $\{Y_u\}$ equivalent to $\{X_u\} \ni$ almost all trajectories of $\{Y_u\}$ are continuous and a.s. if $Y_u \in L^p$, $\int_0^{\infty} t^{-1-\sigma p} \|V(t)Y_x - Y_x\|_p^p dt < \infty$. Thus, a.s. $\{Y_u\} \cap L^p$ lies in the intermediate space between $D(A)$ and L^p. (Received March 17, 1970.)

70T-F11. MEIR SMORODINSKY, University of California, Berkeley, California 94720. A partition which is not "weak Bernoulli". Preliminary report.

In an attempt to further classify K-autohomomorphisms D. Ornstein suggested (orally) a stronger mixing property called weak Bernoulli (together with N. Friedman he proved that if a generator has this property then the transformation is isomorphic to a Bernoulli shift). I show that in every Bernoulli shift there is a partition which is not weak Bernoulli. I use the following Theorem. The shift on a regular stationary Gaussian process is isomorphic to a Bernoulli shift. (Received April 3, 1970.) (Author introduced by Jacob Feldman.)

70T-F12. WITHDRAWN.

70T-F13. DANIEL W. STROOCK, Courant Institute, New York University, New York, New York, 10012. On the range of degenerate diffusions, with applications to the strong maximum principle. Preliminary report.

Let $L = 1/2\, \sigma^* \nabla \cdot \sigma^* \nabla + b \cdot \nabla$, where $\sigma: [0,\infty) \times R^d \to R^d \times R^d$ and $b: [0,\infty) \times R^d \to R^d$ are bounded smooth functions and $\sigma^* \nabla \cdot \sigma^* \nabla$ denotes the operator $\sum_{j,k,\ell=1}^{d} \sigma^{k,j}(\partial/\partial x_j)\sigma^{k,\ell}(\partial/\partial x_\ell)$. It is shown that there is exactly one continuous Markov process $\{P_{s,x} : (s,x) \in [0,\infty) \times R^d\}$ with the property that $f(x(t)) - \int_s^t Lf(x(u))du$ is a $P_{s,x}$-martingale for all $f \in C_0^\infty(R^d)$. As a consequence, one can show that there is exactly one Markov semigroup $\{T_s : s \geq 0\}$ whose generator is an extension of $\partial/\partial t + L$ on $C_0^\infty([0,\infty) \times R^d)$. Also, it is shown that the support of $P_{s,x}$ is contained in the function-space closure of the set $\mathscr{S}(s,x)$ consisting of trajectories $\varphi(t)$ having the form $\varphi(t) = x + \int_s^t \sigma(\varphi(u))\psi(u)du + \int_s^t b(\varphi(u))du$, $t > s$, where $\psi : [s,\infty) \to R^d$ is piecewise constant. By making certain restriction on σ, one can show that $\mathrm{supp}(P_{s,x}) = \overline{\mathscr{S}(s,x)}$. As a consequence, it is possible to give an exact description of the region in which the strong minimum principle holds for functions which are excessive relative to $\{T_s : s \geq 0\}$. (Received April 14, 1970.)

70T-F14. STEPHEN S. WAGNER, Yerkes Regional Primate Research Center, Emory University, Atlanta, Georgia 30322. The maximum-likelihood estimate for contingency tables with zero diagonal.

Under a reasonable hypothesis about the distribution of threats among a group of K monkeys, or of irreflexive acts among any group of K individuals, the likelihood of obtaining a given table of observed frequencies n_{ij} is $L = (N!/\prod n_{ij}!)P_1^{n_1} \dots P_K^{n_K} Q_1^{m_1} \dots Q_K^{m_K}/(1 - P_1Q_1 - \dots - P_KQ_K)^N$ where $n_i = \sum_j n_{ij}$ and $m_i = \sum_k n_{ki}$ are the total number or threats made and received, respectively, by monkey i, N is the total number of threats altogether, and P_i and Q_i are the theoretical tendencies of monkey i to threaten and be threatened respectively and satisfy $\sum P_i = \sum Q_i = 1$. In the attempt to test the correctness of the hypothesis, a critical problem is that of finding the values of the P's and Q's maximizing L. We give all solutions and show that, for the usual table of observations, the solution is unique. (J. Amer. Statist. Assoc., to appear.) (Received April 20, 1970.)

Topology

70T-G77. WILLIAM F. LINDGREN, Southern Illinois University, Carbondale, Illinois 62901. Topological spaces with unique quasi-uniform structure.

A topological space $(X, \mathcal{T})$ is uqu (uqp) if and only if $\mathcal{T}$ has a unique compatible quasi-uniform structure (quasi-proximity). C. Barnhill and P. Fletcher have proved that a space with finite topology is uqu. The conjecture is made that every uqu topological space has finite topology. The set of real numbers with cofinite topology, however, is uqu with $\mathcal{T}$ infinite. Nonetheless, we have the following: Theorem. For an R_1 topological space $(X, \mathcal{T})$ the following are equivalent: (1) $\mathcal{T}$ is finite, (2) $(X, \mathcal{T})$ is uqu, (3) $(X, \mathcal{T})$ is uqp. Corollary. For an R_1, T_0 topological space $(X, \mathcal{T})$, the following are equivalent: (1) X is finite, (2) $(X, \mathcal{T})$ is uqu, (3) $(X, \mathcal{T})$ is uqp. (Received February 27, 1970.)

70T-G78. L. BRUCE TREYBIG, Tulane University, New Orleans, Louisiana 70118. Concerning a bound problem in knot theory.

In an earlier paper (Abstract 68T-506, these *Notices* 15 (1968), 652) the author shows that if two knot functions (see also these *Notices* 12 (1965), 793) f,g determine equivalent knots which are polygonal and in regular position, then f,g are the ends of a simple sequence α of knot functions. In an effort to bound the length of α in terms of f and g (1) a bound is found for the number of elements needed in a triangulation T of a tetrahedron Δ so that there is a simplicial isotopy $h: \Delta \times I \to \Delta$ which is fixed on Bd Δ, is affine on each element of T for all t, and takes a given polyhedral disk in Int(Δ) onto another; and (2) it is shown that two polygonal knots K,L in regular position can "essentially" be embedded as part of the 1-skeleton of a triangulation T of a tetrahedron, where (a) all 3 cells which are unions of elements of T can be shelled and (b) card.T is determined by K, L. A number of "counting" lemmas are proved. (Received February 27, 1970.)

70T-G79. JOHN R. MARTIN, Tulane University, New Orleans, Louisiana 70118. Determining the type of a knot from its word. Preliminary report.

D. E. Penney has defined for an oriented polygonal knot K in regular position a "word" W(K) (Abstract 627-25, these *Notices* 12 (1965), 793). It is shown that every word has a unique factorization into prime words and the natural concept of equivalent prime factorizations is defined. Two oriented knots K and L are called equivalent if there is an orientation preserving homeomorphism from $\mathbb{R}^3$ onto itself that carries K onto L so their orientations match. Results. (1) If K and L are oriented knots such that W(K) and W(L) have equivalent prime factorizations and certain double points of K and L have the same characteristic (in the sense of Reidemeister), then K is equivalent to L. (2) Using (1) together with the simple transformations I, I', II, II' and III defined by Treybig (Abstract 68T-506, these *Notices* 15 (1968),652), the author obtains necessary and sufficient conditions for two oriented knots to be equivalent. (3) If a conjecture of Treybig (Abstract 68T-506, these *Notices* 15 (1968), 652) is true, then there is a method by means of which it is possible to decide in a finite number of steps whether two given oriented knots are equivalent. (Received March 2, 1970.)

70T-G80. ARVIND K. MISRA, Indian Institute of Technology, Kanpur-16, India. Category of P-spaces and the functor P.

The full subcategory of P-spaces is epi-coreflective in the category $\mathcal{T}_1$ of T_1-spaces and maps. Let P be the epi-coreflective functor on $\mathcal{T}_1$. Theorem 1. The functor P preserves T_1, T_2, regularity, complete-regularity but even for a compact space X, PX need not be normal. Theorem 2. For any well-ordered space X, PX is orderable. (Received March 2, 1970.) (Author introduced by Professor Richard A. Alo.)

70T-G81. DANIEL R. McMILLAN, JR., Institute for Advanced Study, Princeton, New Jersey 08540. Decompositions of euclidean space yielding a manifold.

Throughout, let G denote an upper semicontinuous decomposition of E^k into compact, connected sets such that $Y^k = E^k/G$ is an open k-manifold. Notation. H_G = union of all nondegenerate elements of G; P = projection $E^k \to Y^k$. Theorem 1. If $P(H_G)$ is 0-dimensional (possibly noncompact) or if $H_G \subset E^{k-1}$, then each $g \in G$ is strongly acyclic (≡ "cohomologically trivial") over the integers. Theorem 2. If G is strongly acyclic and $k \neq 4$, then $Y^k \approx E^k$. Theorem 3. If G is strongly acyclic and some $g_0 \in G$ embeds in E^3 then g_0 has property UV^∞. In particular, if each $g \in G$ embeds in E^3 and $k \neq 4$, then G is cellular. Corollary. G is cellular if k = 3 and $P(H_G)$ is 0-dimensional; or if k = 4 and $H_G \subset E^3$. Remarks. There exists a piecewise-linear, strongly acyclic mapping of S^4 onto S^4 some of whose point-inverses fail to have property UV^∞. Theorem 2 follows easily from results of Alden Wright (for k = 3) and L. C. Siebenmann (for k > 4). The second part of the corollary requires the fact that a UV^∞ continuum in E^3 is cellular in E^4 ($G|E^3$ may not be cellular). We regard E^{k-1} as a hyperplane in E^k. (Received March 4, 1970.)

70T-G82. R. CHRISTOPHER LACHER, Florida State University, Tallahassee, Florida 32306. Suspending homotopy 3-spheres and embedding mapping cylinders in S^4.

It is known that if f is a mapping of the closed 3-manifold M onto itself such that Z_f (the mapping cylinder of f) embeds in S^4, then $Z_f \approx M \times I$. For maps between (possibly) distinct 3-manifolds, the situation is less clear. Theorem. The following are equivalent statements about a closed 3-manifold M: (a) there exists a mapping f of M onto S^3 such that Z_f embeds in S^4; (b) the suspension of M is S^4. Corollary. The following conjectures are equivalent: (1) If f is a mapping of the closed 3-manifold M onto S^3 such that Z_f embeds in S^4, then $M \approx S^3$. (2) Every triangulation of S^4 is combinatorial. (Received March 4, 1970.)

70T-G83. JEONG SHENG YANG, University of South Carolina, Columbia, South Carolina 29208. A class of topological groups with equal uniformities. Preliminary report.

A Hausdorff group G whose underlying space is a k-space is called a k-group. Let A(G) denote the group of all automorphisms of G endowed with the compact-open topology, and let I(G) denote the group of all inner automorphisms of G. Theorem 1. Every k-group with center Z such that G/Z is compact has equal left and right uniformities. Theorem 2. Let G be a k-group. Then a subgroup B of A(G) is relatively compact if and only if G has small B-invariant neighborhood of e and the B-orbits of points in G are relatively compact. Theorem 3. If G is a k-group such that G/Z is compact, then G has small I(G)-invariant neighborhood of e and the I(G)-orbit of x is relatively

compact for each x in G. Theorem 4. A k-group G has relatively compact I(G) if and only if G has small I(G)-invariant neighborhood of e and the I(G)-orbit of x is relatively compact for each x in G. (Received March 5, 1970.)

70T-G84. BEVERLY L. BRECHNER, University of Florida, Gainesville, Florida 32601.
Topological groups which are not full homeomorphism groups.

The following question is of interest: Given a topological group H, does there exist a metric space X such that the group of all homeomorphisms of X, under the compact-open topology, is both topologically and algebraically the same as H? This question is answered in the negative with the following theorem. Theorem 1. Let X be a metric space which admits a flow. Then G(X) contains a Hilbert cube and is therefore infinite dimensional. Corollary. Neither the topological group R of reals, nor any nontrivial compact, connected Lie group can be the group of all homeomorphisms of any metric space X. Theorem 2. Any Cantor group which is the countable direct product of finite cyclic groups is the full homeomorphism group of some metric continuum. (Received March 6, 1970.)

70T-G85. WILLIAM G. McARTHUR, Shippensburg State College, Shippensburg, Pennsylvania 17257.
Characterizations of zero-sets and realcompactness.

A family $\mathcal{B}$ of closed subsets of X is a G_δ-normal family if: (i) X and $\emptyset$ are members of $\mathcal{B}$, (ii) $\mathcal{B}$ is closed under finite intersections, (iii) disjoint members of $\mathcal{B}$ are contained in disjoint complements of sets in $\mathcal{B}$, and (iv) each member of $\mathcal{B}$ is the intersection of a countable collection of complements of sets in $\mathcal{B}$. Theorem 1. A subset of X is a zero-set if and only if it belongs to some G_δ-normal family of subsets of X. Theorem 2. A topological space is completely regular if and only if it has a base for the closed sets which is a G_δ-normal family. The family $\mathcal{B}$ is complete if every prime $\mathcal{B}$-filter with the countable intersection property converges to a point of X. Theorem 3. The $T_{3\frac{1}{2}}$ space X is realcompact if and only if X has a complete G_δ-normal family. (Received March 9, 1970.)

70T-G86. PERRIN WRIGHT, Florida State University, Tallahassee, Florida 32306.
The covering isotopy theorem for M^{n-1} in N^n, $n \geqq 5$.

Theorem. Let N^n be a closed manifold, $n \geqq 5$, and let M^{n-1} be a closed p.l. manifold. Let B be a locally compact finite dimensional metric space. If $\{h_b\}$, $b \in B$, is a continuous family of locally flat embeddings of M into N such that $h_b(M)$ separates N, then for each $b_0 \in B$ there is a neighborhood U of b_0 in B and a continuous family $\{H_b\}$, $b \in U$, of homeomorphisms of N onto N such that $h_b = H_b h_{b_0}$. Corollary. If $B = I^m$ we have a local covering m-isotopy theorem. If m = 1 we can patch local isotopies together and obtain a global covering isotopy theorem for isotopies of M in N. These results eliminate the need for any condition of local triviality on the family $\{h_b\}$, requiring only that each h_b be a locally flat embedding. We use previous results of Edwards-Kirby, Dyer-Hamstrom, and the author. (Received March 23, 1970.)

70T-G87. IVAN L. REILLY, University of Illinois, Urbana, Illinois 61801 and Eastern Illinois University, Charleston, Illinois 61920. On the metrization of quasi-metric spaces.

Bitopological methods are used to show that several sets of conditions on a quasi-metric space and its conjugate topology are sufficient for its metrizability. Among the results are: Theorem 1. Any quasi-metric space whose conjugate topology is compact is metrizable. Theorem 2. Any sequentially compact quasi-metric space whose conjugate topology is paracompact and sequentially compact is metrizable. Theorem 3. Any quasi-metric space which is coupled to its conjugate topology is metrizable. Theorem 4. Let p be a quasi-metric on the set X with conjugate q. If, for each $x \in X$, $q(x,y)$ is an upper semicontinuous function in y with respect to the quasi-metric topology (X,p), then (X,p) is metrizable. Examples are provided to show that the compactness condition of Theorem 1 cannot be significantly relaxed, and to show that a proof and theorem of Kim [Proc. Japan Acad. 44 (1968), 1009-1012] are false. (Received March 9, 1970.) (Author introduced by Professor Mary-Elizabeth Hamstrom.)

70T-G88. RICHARD A. ALO, Indian Institute of Technology, Kanpur, India, and HARVEY L. SHAPIRO, Northern Illinois University, DeKalb, Illinois 60115. Countably paracompact, normal, and collectionwise normal spaces. Preliminary report.

The concepts of strongly P-embedded and strongly C-embedded subsets are introduced. A subset is strongly P-embedded (respectively, strongly C-embedded) if every σ-locally finite (respectively, countable σ-locally finite) open cover of the subset has a refinement that extends to a locally finite cozero-set cover of the whole space. It is shown that these notions imply P-embedded (every continuous pseudometric extends) and C-embedded (every continuous real valued function extends). The following results are shown. Theorem 1. A space X is countably paracompact and collectionwise normal if and only if every closed subset is strongly P-embedded in X. Theorem 2. The space is countably paracompact and normal if and only if every closed subset is strongly C-embedded. Theorem 3. A space is countably paracompact if and only if every σ-locally finite (respectively, σ-discrete, σ-star finite) open cover of X has a locally finite refinement. Theorem 4. A normal space is countably paracompact if and only if every σ-discrete open cover of X has a point-finite (respectively, cushioned) open refinement. Additional characterizations of countably paracompact normal spaces are also given. (Received March 9, 1970.)

70T-G89. GEORGE M. BUTLER, Louisiana Polytechnic Institute, Ruston, Louisiana 71270. A new axiomatic approach to the Steenrod squaring operations.

Let X_n denote an Eilenberg-Maclane space of type (Z_2, n) and $\sigma^*: H^i(X_n, Z_2) \to H^{i-1}(X_{n-1}, Z_2)$ its cohomology suspension. It is well known that if one assumes σ^* is an isomorphism in small dimensions then the Steenrod squaring operations can be defined inductively. A new proof that σ^* is an isomorphism for $i < 2n$, $n \geqq 2$ is given. This proof uses Brown's generalization of the Eilenburg-Zilber theorem for fiber spaces in terms of the twisted tensor product. (Received March 13, 1970.)

70T-G90. HSU-TUNG KU and MEI-CHIN KU, Department of Mathematics and Statistics, University of Massachusetts, Amherst, Massachusetts 01002. Exotic free differentiable actions of S^1 and S^3 on homotopy spheres. Preliminary report.

Let (S^i, Σ^m), $i = 1$ or 3, be a free differentiable action of S^i on the homotopy m-sphere Σ^m, $m = 2n + 1$ or $4n + 3$, with orbit space $N = \Sigma^m/S^i$ and $f: N \to CP^n$ (resp. $f: N \to QP^n$) a homotopy equivalence which is transverse regular on the submanifold CP^{n-k} (resp. QP^{n-k}) with $n - k > 2$ (resp. $n - k > 1$). By a characteristic homotopy q-sphere Σ^q of Σ^m, we mean a homotopy q-sphere Σ^q which is S^i-invariant with $\Sigma^q/S^i = M$, where $M = f^{-1}(CP^{n-k})$, $q = 2n + 1 - 2k$; or $M = f^{-1}(QP^{n-k})$, $q = 4n + 3 - 4k$, and $f|M$ is a homotopy equivalence. Theorem 1. All characteristic spheres bound π-manifolds. Theorem 2. For any free differentiable S^3-action on homotopy $(4n + 3)$-sphere Σ^{4n+3}, the codimension 4 characteristic homotopy sphere is always standard. Theorem 3. There are infinitely many topologically distinct free differentiable actions of S^3 on homotopy $(4n + 7)$-spheres with codimension 4 characteristic spheres, so that none of them has a characteristic homotopy 8-sphere, for $n \geq 5$. Moreover, the qth rational Pontrjagin classes of the orbit spaces are all distinct for some $q \leq n$. Theorem 4. There are infinitely many topologically distinct free differentiable actions of S^3 on S^{4n+3} for every $n \geq 5$. Theorem 5. There are infinitely many topologically distinct free differentiable actions of S^1 on S^{2n+1} for every $n \geq 10$ (cf. Abstract 70T-G31, these *Notices* 17 (1970), 465). (Received March 10, 1970.)

70T-G91. M. S. GAGRAT and S. A. NAIMPALLY, Indian Institute of Technology, Kanpur-16, U. P., India. Proximity approach to extension problems. I.

For a separated Lodato space (X, δ) let Σ_X denote the family of all bunches in X with the absorption topology (M. W. Lodato, Pacific J. Math. 17 (1966), 131-135). Theorem 1. Every proximally continuous function f from (X, δ_1) to (Y, δ_2) has a continuous extension $f_\Sigma: \Sigma_X \to \Sigma_Y$. Theorem 2 (Generalization of Smirnov's theorem). Let (X, δ) be a separated Lodato space such that $A\ \delta\ B$ implies the existence of a bunch in X containing both A and B. Then (i) there exists a compact T_1-space $\underset{\sim}{X}$ (the space of all maximal bunches in X) containing a dense homeomorphic copy of X, (ii) $A\ \delta\ B$ iff $C\ell(A) \cap C\ell(B) \neq \emptyset$ in $\underset{\sim}{X}$, (iii) every proximally continuous function $f: (X, \delta) \to (Y, \delta')$ has a continuous extension $\bar{f}: \underset{\sim}{X} \to \Sigma_Y$. Theorem 3 (Generalization of Taimanov's theorem, Mat. Sb. 31 (1952), 459-463). Let X be a T_1-dense subspace of an R_0-space T and let X be assigned the Lodato subspace proximity induced by δ_0 on T namely, $A\ \delta_0\ B$ iff $\bar{A} \cap \bar{B} \neq \emptyset$ in T. Let (Y, δ) be a separated Efremovič proximity space and let $\underset{\sim}{Y}$ be its Smirnov compactification. Then a continuous function $f: X \to Y$ has a continuous extension $\bar{f}: T \to Y$ iff f is proximally continuous. The above result includes, as special cases, extension theorems of Blefko, Engelking, McDowell, Ponomarev and also yields several new ones. (Received March 15, 1970.)

70T-G92. M. S. GAGRAT, Indian Institute of Technology, Kanpur-16, U.P. India. On separation proximity spaces.

For a definition of separation or S-proximity spaces see S. B. Krishna Murti, J. Indian Math. Soc. 4 (1940), 116-119. In this paper the notion of a band which is analogous to that of a cluster or a bunch is introduced and is used to get generalizations of the well-known results of Smirnov and

Taimanov. Among others the following result is proved: Let X be a dense subspace of a T_1-space T and let Y be a locally compact Hausdorff space. Then a continuous function $f: X \to Y$ has a continuous extension $\bar{f}: T \to Y$ if and only if (i) for every pair of disjoint closed sets F_1, F_2 of Y, at least one of which is compact, $(C\ell_T f^{-1}(F_1) \cap f^{-1}(F_2)) \cup (f^{-1}(F_1) \cap C\ell_T f^{-1}(F_2)) = \emptyset$, and (ii) for each $t \in T$ there exists a compact subset K of Y such that $t \in C\ell_T f^{-1}(K)$. (Received March 16, 1970.) (Author introduced by Professor S. A. Naimpally.)

70T-G93. HERBERT C. LYON, University of Michigan, Flint, Michigan 48503. Incompressible surfaces in knot spaces.

All knots are tame and contained in S^3, and all surfaces are orientable. Theorem 1. There exist infinitely many distinct, prime, Neuwirth knots, each of which has the property that its complement contains closed, incompressible surfaces of arbitrarily high genus. Theorem 2. There exists a genus one knot which has incompressible spanning surfaces of arbitrarily high genus. Theorem 3. There exists a knot $K \subset S^3$ with unknotted incompressible spanning surfaces S_1 and S_2 which satisfy the following conditions: (a) $S_1 \cap S_2 = K$; (b) S_1 is not parallel to S_2; (c) S_1 is isotopic to S_2 in S^3. The proofs are constructive and geometric. (Received March 26, 1970.)

70T-G94. JAMES T. ROGERS, JR. and JEFFREY L. TOLLEFSON, Tulane University, New Orleans, Louisiana 70118. Maps between inverse limit spaces. Preliminary report.

A weak solenoidal sequence is an inverse limit sequence where each factor space is a connected polyhedron and each bonding map is a covering map. Let (Y,g) be a weak solenoidal sequence with $Y_\infty = \lim(Y,g)$ and with projection maps $g_n: Y_\infty \to Y_n$. Theorem. Let (X,f) be an inverse limit system of continua with onto bonding maps. Then for any map $F: X_\infty \to Y_\infty$ and any $\epsilon > 0$ there is an induced map $\varphi: X_\infty \to Y_\infty$ that agrees with F on the set $V = (f_1 F)^{-1}\{v : v \text{ a vertex of } Y_1\}$. Moreover, there is an ϵ-homotopy $\varphi \cong F(\text{rel } V)$. A map $F: X_\infty \to Y_\infty$ is said to be fiber-preserving if there is an integer n such that for each $x \in X_n$ there is a $y \in Y_1$ with $F(f_n^{-1}(x)) \subset g_1^{-1}(y)$. Theorem. Let (X,f) be an inverse limit sequence of continua with onto bonding maps and each projection $f_n: X_\infty \to X_n$ an identification map. A map $F: X_\infty \to Y_\infty$ is an induced map if and only if F is fiber-preserving. Theorem. Let $\{n_i\}$ be a cofinal subset of the positive integers. A sequence of maps $\{\varphi_i: Y_{n_i} \to Y_i\}$ induces a local homeomorphism $\varphi: Y_\infty \to Y_\infty$ if and only if each φ_i is a covering map. (Received March 18, 1970.)

70T-G95. STANLEY P. FRANKLIN, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213 and J. K. KOHLI, Indian Institute of Technology, Kanpur 16, India. On open extensions of maps. Preliminary report.

Let f be a function, not necessarily continuous, from a topological space X into a topological space Y. A superspace X* of X and an open function f* from X* onto Y whose restriction to X is f is constructed. If the function f is continuous, then f* is also continuous. The space X* is obtained as a quotient of a disjoint topological sum of X and copies of Y. Definition. The space X* preserves property P if X and Y have property P implies X* has property P. The following are shown. (1) X* preserves (i) separation axioms up to complete regularity (ii) forms of normality and para-

compactness, (iii) any coreflexive property, (iv) (pathwise) connectedness. (2) X^* need not preserve metrizability, axioms of countability, weight, local weight, separability and forms of compactness. Also there are obtained sufficient conditions on the function f such that X^* preserves metrizability, weight, local weight and local compactness. Using these results improvements of recent theorems of Arhangelskiĭ, Čoban, Hodel and Proizvolov are obtained. Another method of extending a continuous function to an open continuous function is also given. (Received March 25, 1970.)

70T-G96. J. K. KOHLI, Indian Institute of Technology, Kanpur 16, India. On open extensions of maps, II. Preliminary report.

In this paper the work started in ([1] Abstract 70T-G95, these *Notices* 17 (1970)) is continued. Here f, X, Y, X^* and f^* carry the same meaning as in [1]. In [1] sufficient conditions on the function f are stated such that metrizability, weight, local weight and local compactness are preserved in X^*. Here it is shown that they are also necessary for the preservation of metrizability, local compactness and axioms of countability. However they are not necessary for the preservation of weight (local weight) $> \aleph_0$. Also many topological properties are considered, and either a necessary and sufficient condition or a sufficient condition on the function f such that the space X^* preserves a topological property is given. Using these results and that of [1] improvements (or analogies) of recent theorems of Čoban, Keesling and Nagami are obtained. Also there is given a new technique of extending a (continuous) function to an open (continuous) function by unifying domain and range. A modification of this technique is useful in obtaining partial improvements of recent theorems of Proizvolov and Čoban. (Received March 25, 1970.) (Author introduced by Professor Richard A. Alo.)

70T-G97. CHARLES L. HAGOPIAN, Sacramento State College, Sacramento, California 95819. An arc theorem for plane continua.

A continuum M is said to be semi-aposyndetic if for each pair of distinct points x and y of M, there exists a subcontinuum H of M such that the sets M - H and the interior of H (relative to M) each contain a point of $\{x,y\}$. In this paper the following is established. Theorem. If M is a semi-aposyndetic bounded plane continuum which does not separate the plane, then M is arcwise connected. (Received April 2, 1970.)

70T-G98. WILLIAM K. MASON, Rutgers University, New Brunswick, New Jersey 08903. The space of all self-homeomorphisms of a 2-cell which fix the cell's boundary is an absolute retract.

Let D be a 2-cell and H(D) the space (under the "sup norm" topology) of all homeomorphisms of D onto itself which are the identity on the boundary of D. Theorem. H(D) is an absolute retract. Corollary. If K and L are homeomorphic compact subsets of H(D), then the homeomorphism between them can be extended to a homeomorphism of H(D) onto itself. (Received April 2, 1970.)

70T-G99. ERIC M. BARTON, University of Illinois, Urbana, Illinois 61801. On some problems of Borsuk. Preliminary report.

A natural number n is said to be the index of r-proximity of two spaces X and Y with different r-types provided that there is a system of n + 1, but not less, spaces $X_0, X_1, \ldots, X_n$ with $X_0 = X$, $X_n = Y$ and such that X_i and X_{i+1} are r-neighbors for $i = 0, 1, \ldots, n - 1$. Otherwise the index of r-proximity of X and Y is infinite. [See Borsuk, "Theory of retracts", for definitions.] For each Peano space X a map h is defined, $h: (0,1] \to X \times I$ such that $\overline{h((0,1])} = h((0,1]) \dot{\cup} X \times 0$, (disjoint) and $h[t,1]$ is an embedding for $0 < t < 1$. If X and Y are two Peano spaces and h, h' are the corresponding maps, let $W = \overline{h((0,1])} \cup_f \overline{h'((0,1])}$ = the space $\overline{h((0,1])}$ attached to the space $\overline{h'((0,1])}$ by the map $f: h(1) \to h'(1)$. Lemma. The only retracts of $\overline{h((0,1])}$ are points, compact intervals, retracts of $X \times 0$ and homeomorphs of $\overline{h((0,1])}$. A similar lemma holds for W. Theorem. If X and Y are any two Peano spaces, then the index of r-proximity of X and Y is no greater than four. This theorem answers three questions posed by Borsuk, ["Theory of retracts," p. 208]. In particular, the first question asks if the index of r-proximity of the 2-sphere S^2 and the torus $S^1 \times S^1$ is finite. The theorem supplies an affirmative answer. (Received March 26, 1970.)

70T-G100. LUDVIK JANOS, University of Florida, Gainesville, Florida 32601. On maximal groups of isometries.

Let X be a metrizable topological space, G(X) the group of all homeomorphisms of X onto itself and M(X) the set of all metrics on X compatible with the topology of X. For each $\rho \in M(X)$ we denote by $G(\rho) \subseteq G(X)$ the subgroup of those homeomorphisms which preserve ρ. ($G(\rho)$ is the isometry group for ρ.) We say that the metric $\rho \in M(X)$ is optimal iff there is no metric $\rho' \in M(X)$ such that $G(\rho')$ would properly contain $G(\rho)$, which means that $G(\rho)$ is a maximal isometry group. If X admits at least one optimal metric we say X has the optimal metrization property. Theorem. If X is compact metrizable and if we topologize G(X) by compact open topology then X has the optimal metrization property if and only if there exists a maximal compact subgroup of G(X). (Received April 6, 1970.)

70T-G101. JEFFREY L. TOLLEFSON, Tulane University, New Orleans, Louisiana 70118. Imbedding free Z_k actions on 3-manifolds in SO(2) actions.

Let M be a closed, orientable, irreducible 3-manifold. Let Z_k be a cyclic group of prime order $k \geqq 2$. A Z_k action on M is proper if some homeomorphism generating the action is homotopic to the identity. Let $p: M \to M/Z_k$ be the projection to the orbit space. A free Z_k action on M is Z-classified if the covering space $p: M \to M/Z_k$ is induced by the standard k-sheeted covering of the circle by itself. In particular, this is always the case when $H_1(M/Z_k; Z)$ has no k-torsion. Theorem. A Z-classified free Z_k action on M is imbedded in an effective action of the circle group SO(2) on M if and only if it is proper. Corollary. If M admits a proper free Z_k action that is Z-classified, then the orbit space M/Z_k fibers over the circle and admits an effective SO(2) action without fixed points. Moreover, there is only one such Z_k action on M. (Received April 6, 1970.)

70T-G102. WILLIAM S. HATCHER, Université Laval, Ste. Foy, Quebec 10, Canada. Complete subcategories of the category of compact Hausdorff spaces.

Let $\underline{H}$ be the category of compact Hausdorff spaces with continuous functions as maps. A couple $\langle p,q \rangle$ of elements of an object $A \in Ob(\underline{H})$ is called an identity. An identity $\langle p,q \rangle$, p and $q \in A$, is satisfied by an object $B \in Ob(\underline{H})$ if $f(p) = f(q)$ for every continuous map $f: A \to B$. A full subcategory $\underline{C}$ of $\underline{H}$ is said to be defined by a class K of identities if $Ob(\underline{C})$ consists of the class of precisely those objects of $\underline{H}$ which satisfy all the identities of K. Theorem. A necessary and sufficient condition that a nonempty, replete, and full subcategory $\underline{C}$ of $\underline{H}$ be definable by a class K of identities is that $\underline{C}$ be closed under the operations of forming product spaces and taking closed subspaces (which implies that $\underline{C}$ is a complete subcategory of $\underline{H}$). This theorem bears an analogy to Birkhoff's theorem concerning equationally defined classes of abstract algebras. It is related to a certain generalization of Birkhoff's theorem to abstract categories which is due to the author but which is too complicated for statement here. We intend to publish the details of this result as well as the present theorem. (Note. There exist nontrivial, proper subcategories $\underline{C}$ of $\underline{H}$ which satisfy the conditions of the above theorem.) (Received April 1, 1970.)

70T-G103. VICTOR SAKS, Windham College, Putney, Vermont 05346. On $\mathfrak{m}$-bounded spaces. Preliminary report.

The following question has been raised by S. L. Gulden, W. M. Fleischman, and J. H. Weston (Proc. Amer. Math. Soc. 24 (1970), 197-203), and by R. M. Stephenson, Jr. (to appear): Do there exist infinite cardinal numbers $\mathfrak{m}$ (other than the trivial case $\mathfrak{m} = \aleph_0$) for which some topological space $X_{\mathfrak{m}}$ is not $\mathfrak{m}$-bounded (a space X is $\mathfrak{m}$-bounded provided each subset S of X, with $|S| \leq \mathfrak{m}$, lies in a compact subset of X) but for which cover of $X_{\mathfrak{m}}$ by $\mathfrak{m}$ open sets admits a finite subcover? In the present paper we show that for each infinite cardinal number $\mathfrak{m}$ such a space $X_{\mathfrak{m}}$ exists. Indeed, $X_{\mathfrak{m}}$ may be taken a topological group; alternatively, one may arrange $D \subset X_{\mathfrak{m}} \subset \beta D$, where D is discrete and $|D| = \mathfrak{m}$. (Received April 7, 1970.) (Author introduced by Professor W. Wistar Comfort.)

70T-G104. TSAU-YOUNG LIN, Purdue University, West Lafayette, Indiana 47907. Flat π_*-module and homological algebra of π_*. Preliminary report.

Notation as in Abstract 70T-G37, these *Notices* 17 (1970), 467. All the homology operations, or cohomology operations are assumed to be of degree ≥ 1. Theorem 1. P is a flat π_*-module if and only if P can be realized as a stable homotopy module $\pi_*(L)$ by a homology operational trivial spectrum L (i.e., all the higher order and primary homology operation on $H_*(L)$ are all zeros). Theorem 1'. F is a projective π_*-module if and only if F can be realized as a stable homotopy module $\pi_*(N)$ by a cohomology operational trivial spectrum N. (Note. A cohomology operational trivial spectrum is a wedge of spheres, therefore as a corollary F is free.) Theorem 2. The weak dimension of π_*-modules are either 0 or ∞, equivalently, F. W. $D(\pi_*) = 0$. Theorem 3. The projective dimension of π_*-modules are either 0, 1 or ∞, equivalently, F. P. $D(\pi_*) = 1$. Theorem 4. A π_*-module is flat if and only if its projective dimension ≤ 1. Theorem 5. Let $H_*(\pi_*; G)$ be the homology group of the argumented ring π_* with argumentation $\pi_* \to \pi_*/\pi_+ = Z$. Then for each n $H_n(\pi_*; G)$ and $H_{n+1}(\pi_*; G)$ can not vanish simultaneously for any Abelian group G, unless G is a direct sum of (additive group) rational numbers. (Received April 1, 1970.)

70T-G105. V. KANNAN, Madurai University, Madurai-2, India. Order of a quotient map. Preliminary report.

In this paper a method of associating a unique ordinal to each quotient map f from a topological space X onto a topological space Y is described and the following theorems are proved. We denote by $\sigma(f)$ the ordinal associated with f and call it the order of f. Theorem 1. A topological space X is a sequential space (k-space) with sequential order (k-order) α if and only if X is the image of a metric (locally compact) space under a quotient map of order α. Theorem 2. Let f be any quotient map. Then f is pseudo-open (i.e. hereditarily quotient) if and only if $\sigma(f) \leq 1$. Corollary (Arhangelskii). A topological space X is a Fréchet space (k'-space) if and only if it is a pseudo-open continuous image of a metric space (locally compact space). Theorem 3. Let $f: X \to Y$ and $g: Y \to Z$ be quotient maps. Then $\sigma(f) \leq \sigma(gf) \leq \sigma(g) \cdot \sigma(f)$. Corollary. Let X be a sequential space (k-space) with sequential order (k-order) α. Let Y be a pseudo-open continuous image of X. Then Y is either discrete or has sequential order (k-order) α. (Received April 8, 1970.) (Author introduced by Professor M. Rajogopalan.)

70T-G106. S. A. NAIMPALLY, Indian Institute of Technology, Kanpur-16, U.P., India. Wallman compactifications and Wallman realcompactifications.

(i) If $\mathcal{L}$ is a separating base on a T_1-space X, then $\delta = \delta(\mathcal{L})$ defined by "$A\ \delta\ B$ iff $A \subset L_1$, $B \subset L_2$, $L_1 \cap L_2 = \emptyset$, $L_1, L_2 \in \mathcal{L}$" is a compatible separated Lodato proximity on X. Further, if $\mathcal{L}$ is a normal base, then δ is an Efremovič proximity. (2) Let X be a Tychonoff space with a normal base $\mathcal{L}$ and let $\delta = \delta(\mathcal{L})$. Then the $\mathcal{L}$-Wallman compactification of X is homeomorphic to the δ-Smirnov compactification of X. (3) A necessary and sufficient condition for a Hausdorff compactification Y of X to be Wallman is that X has a normal base $\mathcal{L}$ such that (i) $C\ell_Y(L_1 \cap L_2) = C\ell_Y L_1 \cap C\ell_Y L_2$ for all L_1, L_2 in $\mathcal{L}$ and (ii) for $p,q \in Y$, $p \neq q$, there exist L_p, L_q in $\mathcal{L}$ such that $p \in C\ell_Y(L_p)$, $q \in C\ell_Y(L_q)$ and $L_p \cap L_q = \emptyset$. Suppose $\mathcal{L}$ is a countably productive normal base on X. (4) $\eta(\mathcal{L})$ is real compact if and only if $\bigcap_{n=1}^{\infty} C\ell_{X^{\mathcal{L}}}(L_n) = C\ell_{X^{\mathcal{L}}}(\bigcap_{n=1}^{\infty} L_n)$, $L_n \in \mathcal{L}$ where $X^{\mathcal{L}}$ is the $\mathcal{L}$-closure of X. (5) Let Y be an $\mathcal{L}^*$-realcompactification of X. Then Y is homeomorphic to $\eta(\mathcal{L})$ if and only if (i) $C\ell_Y(\bigcap_{n=1}^{\infty} L_n) = \bigcap_{n=1}^{\infty} C\ell_Y(L_n)$, $L_n \in \mathcal{L}$, (ii) $\{C\ell_Y(L) : L \in \mathcal{L}\}$ is a base for closed sets in Y and $p,q \in Y$, $p \neq q$ implies there exist L_p, L_q in $\mathcal{L}$ such that $p \in C\ell_Y(L_p)$, $q \in C\ell_Y(L_q)$ and $L_p \cap L_q = \emptyset$. (Received March 16, 1970.)

70T-G107. PHILLIP L. ZENOR, Auburn University, Auburn, Alabama 36830. On paracompact p-spaces.

Theorem 1. The completely regular T_1-space X is a paracompact p-space if and only if it is homeomorphic to the limit space of an inverse system of metric spaces with proper bonding maps. Theorem 2. If X is a paracompact p-space, then $\mathcal{C}X$, the space of all compact subsets of X with the Vietoris topology, is a paracompact p-space. (Received April 8, 1970.)

70T-G108. LESLIE C. GLASER, University of Utah, Salt Lake City, Utah 84112. On suspensions of homology spheres.

A gap has been found in one of the preliminary results leading to the work announced by the author in Abstract 70T-G53, these *Notices* 17 (1970), 472. Here, we reformulate this earlier work and note the significance of the following specific question: Does there exist, for some n, a contractible complex $K \subset E^n$ such that $\pi_1(E^n - K) \neq 0$ and $(E^n/K) \times E^1 = E^{n+1}$? Theorem 1. A negative answer to this question is equivalent to each of the results announced in the above. Corollary 2. If every triangulation of any n-manifold is locally flat on each 1-simplex, then it is locally flat on each open simplex. To obtain these results, we use a geometrical technique of Kirby and Siebenmann and show: Theorem 3. Suppose Z is a compact space containing a collared n-cell C such that $H_i(Z) \cong H_i(S^n)$, $\pi_1(Z) = 0$ and $Z \times E^k$ is an open $(n + k)$-manifold. If $n + k \geqq 5$ and $n > 2$, then $(Z - C) \times T^k = I^n \times T^k$, $\Sigma^k(Z - C) = \Sigma^k I^n$, and $\Sigma^k Z = \Sigma^k S^n$ (Σ denotes suspension). Theorem 4. If N^k and M^n are homotopy manifolds, N^k is a subcomplex of M^n, $n - k \neq 2$, and for n or k = 4 the corresponding complexes are topological manifolds, then N^k is locally flat in M^n. Moreover, this is the best result possible with respect to the polyhedral Schoenflies conjecture. (Received April 10, 1970.)

70T-G109. ROBERT F. BROWN, University of California, Los Angeles, California 90024. Divisibility in topological groups.

For a topological group G, denote by G_0 the component containing the identity element. Define $p_k : G \to G$ by $p_k(x) = x^k$. If G is a compact Lie group and $x \in G$, define the rank of xG_0 to be the dimension of a Cartan subgroup (in the sense of Siegel [Inst. Hautes Études Sci. Publ. Math. 34 (1968), 113-128]) generated by an element of xG_0. Theorem. Let G be a compact Lie group and $x,y \in G$ such that $p_k(xG_0) \subseteq yG_0$, then $p_k(xG_0) = yG_0$ if and only if xG_0 and yG_0 have the same rank. Theorem. Suppose either that a topological group G is compact or that G_0 is a compact manifold. If $x,y \in G$ such that $p_k(xG_0) \subseteq yG_0$ and the order of xG_0 in G/G_0 is relatively prime to k, then $p_k(xG_0) = yG_0$. (Received March 25, 1970.)

70T-G110. LOWELL E. JONES, University of Colorado, Boulder, Colorado 80302. A converse to the fixed point theorem of P. A. Smith.

Let K be a PL submanifold of D^r so that K intersects ∂D^r transversely in ∂K. Theorem. If $2 \cdot \dim(K) < r - 2$ and $\overline{H}_*(K, Z_n) = 0$, then $K \subset D^r$ is the fixed point set of a semifree PL homeomorphism on D^r having period n. (Received March 29, 1970.)

70T-G111. LOWELL E. JONES and JIGGERS PAULSON, University of Colorado, Boulder, Colorado 80302. Surgery on Poincaré spaces and applications.

Let τ_+^n (τ^n) be an n-universal BSF (BF) bundle. $L_*(0)$ denotes the Z_+-graded simply connected surgery obstruction group, and Ω_*^{SF} (Ω_*^F) is the Z_+-graded group of cobordism classes of oriented (unoriented) Poincaré spaces. By extending the techniques of surgery to Poincaré spaces the authors succeed in showing Theorem. (1) $(L_i(0) \oplus \lim_{n\to\infty}(\pi_{i+n}(T(\tau_+^n))) \otimes_Z Z(1/2) = \Omega_i^{SF} \otimes_Z Z(1/2)$ (for $i \geqq 13$). (2) The Z_2-vector spaces $\lim_{n\to\infty}(\pi_{i+n}(T(\tau^n)))$, Ω_i^F have dimensions which differ at most by 1 (for $i \geqq 13$). (Received March 29, 1970.)

70T-G112. CHARLES L. SEEBECK, III, Michigan State University, East Lansing, Michigan 48823. Tame arcs on wild disks. Preliminary report.

Theorem 1. Suppose M and N are topological m- and n-manifolds, $m < n$, $n \geqq 5$, M is embedded in N, and $G \subset M$ is a finite graph. Then the inclusion $i: G \to M$ can be approximated by embeddings $j: G \to M$ which equal i on the 0-skeleton and are locally tame in N. Theorem 2. Suppose that $M \subset N$ are as above with $m \geqq 5$, $m \leqq n - 2$ and that any embedding of the 2-disk into M can be approximated by an embedding into M which is locally tame in N. Then any embedding of a k-complex, $k < m$, into M can be approximated by an embedding that is locally tame in N and equivalent to the original embedding of the k-complex in M. (Received April 3, 1970.)

70T-G113. CLARENCE WILKERSON, Rice University, Houston, Texas 77001. $S^3 \times S^5$ bundles over S^7. Preliminary report.

Theorem. The total space of any $S^3 \times S^5$ bundle over S^7 is not an H-space. This question was prompted by results on SU(3) bundles over S^7 given in [M. L. Curtis and G. Mislin, "H-spaces which are bundles over S^7", to appear]. Let $S^3 \times S^5 \to E \to S^7$ be a fibration. $H^*(E,\mathbb{Z})$ is an exterior algebra on generators x_3, x_5, and x_7. If E is an H-space its projective plane P_2E is defined. As in [J. R. Hubbuck, "Generalized cohomology operations and H-spaces of low rank", to appear] $K^0(P_2E)$ contains a summand isomorphic as a filtered ring to a truncated polynomial algebra of height 3 over $\mathbb{Z}$, on generators y_4, y_6, and y_8. This summand is an algebra over the Adams operations $\{\psi^k\}$. Let $\langle \psi^k y_i, y_j^a \rangle$ be the coefficient of y_j^a in the expansion of $\psi^k y_i$. Using the relation $\psi^3\psi^2 - \psi^2\psi^3 = 0$, it is shown that $\langle \psi^2 y_4, y_6 \rangle \equiv 0 \bmod 4 \Rightarrow \langle \psi^2 y_6, y_6^2 \rangle \equiv 0 \bmod 2$. But by the axioms given for $\{\psi^k\}$ in [M. F. Atiyah, "On power operations in K-theory", Quart. J. Math. Oxford, Ser. 17 (1966), 165-193] $\langle \psi^2 x, x^2 \rangle \equiv 1 \bmod 2$. Hence E is not an H-space. The fact that $\langle \psi^2 y_4, y_6 \rangle = 0 \bmod 4$ is derived from $Sq^2 x_3 = 0$ in $H^*(E,\mathbb{Z})$ via a relation between the $\{\psi^k\}$ and the Pontrjagin powers given in [Atiyah, op. cit.]. (Received April 6, 1970.) (Author introduced by Professor David L. Rector.)

70T-G114. JAMES R. BOONE, Texas A & M University, College Station, Texas 77843. A space in which the convergence criterion determines the metric.

A topology is defined on $N \cup \{\omega\}$ which makes it a 0-dimensional σ-compact, separable, complete metric space which is not locally compact. This example is distinctly different from the usual one-point compactification of N. The ideals of the semiring N are used to describe the neighborhood base at ω and the Cauchy property is characterized by divisibility by factorials. The paper is intended to serve as a pedagogical exercise for students of topology. (Received April 13, 1970.)

70T-G115. YAICHI SHINOHARA, University of Toronto, Toronto, Ontario, Canada. Higher dimensional knots in tubes.

By an n-knot we mean a smooth oriented submanifold of the oriented (n + 2)-sphere S^{n+2} which is homeomorphic to S^n. Let K be an n-knot in S^{n+2}, V a tubular neighborhood of K and V' a tubular neighborhood of a trivial n-knot. Let f be a diffeomorphism of V' onto V which preserves the orientations induced by S^{n+2} in V' and V. If $n = 1$, we further assume that f transforms longitudes into longitudes. Let L' be an n-knot contained in V' and $L = f(L')$. Then L is an n-knot contained in V

and $L \sim \lambda K$ in V for some integer λ. Theorem 1. For $1 \leq q \leq n$, $\Delta_L^q(t) = \Delta_K^q(t^\lambda) \cdot \Delta_{L'}^q(t)$, where $\Delta_L^q(t)$, $\Delta_K^q(t)$ and $\Delta_{L'}^q(t)$ are the qth dimensional Alexander polynomials of L, K and L'. Let σ_1 and σ_2 denote the signatures of a (2m - 1)-knot defined by D. Erle [Topology 8 (1969), 99-114]. Theorem 2. If $n = 2m - 1$, then $\sigma_2(L) = \sigma_2(L')$ when λ is even and $\sigma_2(L) = \sigma_2(L') + (-1)^{(m+1)(\lambda-1)/2}\sigma_2(K)$ when λ is odd. Furthermore, if m is even, then $\sigma_1(L) = \sigma_1(L') + \lambda\sigma_1(K)$. For n = 1, Theorem 1 was proved by H. Seifert [Quart. J. Math. Oxford Ser. 1 (1950), 23-32] and Theorem 2 by the author [Abstract 660-16, these *Notices* 15 (1968), 1011-1012]. (Received April 15, 1970.)

70T-G116. VIRINDRA M. SEHGAL, University of Wyoming, Laramie, Wyoming 82070. A fixed point theorem for semigroups of mappings with contractive iterates.

Let (X,d) be a complete metric space and $M \subseteq X$. Let F be a commutative semigroup of self mappings (not necessarily continuous) on M. The semigroup is contractive in M, if for each $x \in M$, there is an integer $n(x) \geq 1$ and an $f_x \in F$ such that for each $y \in M$, $d(f_x^{n(x)}(y), f_x^{n(x)}(x)) \leq \psi(d(x,y))$, where ψ is some nonnegative real valued function. Theorem. If M is closed and bounded in X and F is contractive in M for some $\psi\,[0\ \infty) \to [0\ \infty)$ satisfying (1) ψ is right continuous, (2) nondecreasing and (3) $\psi(t) < t$ for all $t > 0$, then there is a unique $\xi \in M$ such that $f(\xi) = \xi$ for all $f \in F$, and there exists a sequence $f_n \in F$ such that $f_n(x) \to \xi$ for each $x \in M$. The above theorem generalizes a recent result of Browder. (Received April 17, 1970.)

70T-G117. RICHARD J. TONDRA, Iowa State University, Ames, Iowa 50010. A characterization of manifolds compactly equivalent to closed manifolds.

If M and Q are connected n-manifolds without boundary, then we will say that M is compactly equivalent to Q if any proper compact subset K of M can be embedded in Q and any proper compact subset L of Q can be embedded in M. Theorem. Let M and Q be connected n-manifolds, $n \neq 4,5$, such that M is open and Q is closed. Then M is compactly equivalent to Q if and only if there exists a bi-collared (n - 1)-sphere S contained in M such that M - S consists of two components U and V such that (i) U is homeomorphic to Q - p, p a point, and (ii) each compact subset of V can be embedded in Euclidean n-space. Corollary 1. Let M and Q be closed connected n-manifolds, $n \neq 4,5$. Then M is compactly equivalent to Q if and only if M is homeomorphic to Q. Corollary 2. Suppose that M, Q and n satisfy the hypotheses of the theorem. Then there exists an open connected subset D of Q such that M is an open monotone union of D. (Received April 17, 1970.)

70T-G118. RONALD C. O'NEILL, Michigan State University, East Lansing, Michigan 48823. On H-spaces and co H-spaces mod p. Preliminary report.

Let p be 0 or a prime. A map $f: X \to Y$ is a p-equivalence if $f_*: H_k(X; Z_p) \approx H_k(Y; Z_p)$, $k \geq 0$ (Z_0 = the rationals). X is p-dominated by Y if there are maps $j: X \to Y$ and $q: Y \to X$ so that qj is a p-equivalence. X is an H-space mod p if there is a map $m: X \times X \to X$ and a p-equivalence $h: X \to X$ such that $m(1,0) \cong m(0,1) \cong h$. Co H-space mod p is defined dually. The theorems below hold for 1-connected CW complexes of finite type. Theorem. The following are equivalent: (a) X is an H-space mod p. (b) X is p-dominated by an H-space. (c) X is p-equivalent to an H-space Y such that $\pi_k(Y) \approx \pi_k(X)$, $k \geq 0$. Theorem. The following are equivalent: (a) X is a co H-space

mod p. (b) X is p-dominated by a co H-space. (c) X is p-equivalent to a co H-space Y such that $H_k(Y) \approx H_k(X)$, $k \geqq 0$. The proofs involve Postnikov decompositions and homology decompositions. (Received April 20, 1970.)

70T-G119. TROY L. HICKS and JOHN W. CARLSON, University of Missouri, Rolla, Missouri 65401. On completeness in quasi-uniform spaces.

A quasi-uniform space $(X, \mathcal{U})$ is strongly complete (respectively complete) if every $\mathcal{U}$-Cauchy filter converges (respectively has an adherent point). Several constructions of completions of quasi-uniform structures are given. In particular, every quasi-uniform structure has a rather simple completion. It is shown that a Hausdorff quasi-uniform structure need not have a Hausdorff completion and a T_1 quasi-uniform structure need not have a T_1 strong completion. (Received April 22, 1970.)

Miscellaneous Fields

70T-H21. GEORGE F. CLEMENTS, University of Colorado, Boulder, Colorado 80302. Existence of distinct representative subsets of subsets of a finite set.

Let H be a set of $k_1 + k_2 + \ldots + k_n = K$ (billiard) balls, k_i of color i, $i = 1,2,\ldots, n$. $M = M(\ell, k; k_1, \ldots, k_n)$ denotes the largest integer for which $t \leqq M$ implies that for each system $\{A_1, \ldots, A_t\}$ of distinct ℓ-element subsets of H, there exists a system $\{B_1, \ldots, B_t\}$ of $(\ell-k)$-element subsets of H such that $A_i \subset B_i$, $i = 1,2,\ldots, t$; $1 \leqq k \leqq \ell \leqq K$. A method which combines the theorem of P. Hall [Ryser, "Combinatorial mathematics," The Carus Mathematical Monographs, No. 14, Math. Assoc. Amer., John Wiley and Sons, New York, 1963, p. 48] and the generalized Macaulay theorem due to the author and B. Lindström [J. Combinatorial Theory 7 (1969), 230-238] is given for producing M. In case $k_1 = k_2 = \ldots = k_n = b$, where b is a positive integer, formulas for certain of the M are given. The b = 1 case is due to G. Katona ["Theory of graphs," Proc. Colloq. (Tihany, 1966) Academic Press, New York, 1968]. (Received March 9, 1970.)

70T-H22. BERNT LINDSTRÖM, University of Stockholm, Box 6701, 113 85 Stockholm, Sweden. The optimal number of faces in cubical complexes. Preliminary report.

Consider the set M of all cubical complexes with a fixed number m of r-dimensional faces. It is required to find f(m; r,s), the maximum (minimum) number of s-dimensional faces of a complex in M, for $r < s$ $(r > s)$. The problem was suggested by J. B. Kruskal [J. Combinatorial Theory 6 (1969), 86-89]. A similar problem for simplicial complexes was solved by Kruskal in ["Mathematical optimization techniques," Univ. of California Press, Berkeley, 1963, pp. 251-278] and by G. Katona in ["Theory of graphs," Proc. Colloq. (Tihany, 1966) Academic Press, New York, 1968, pp. 187-207]. Kruskal introduced the concept of fractional pseudopower $m^{(s/r)}$ for this purpose. In analogy with this the author defines another pseudopower $m_{(s/r)}$ and proves that $f(m; r,s) = m_{(s/r)} + (m - m_{(r/r)})^{(s/r)}$. This agrees with a conjecture by Kruskal when $m = m_{(r/r)}$ if we put $0^{(s/r)} = 0$. The proof parallels the proof of the theorem by G. F. Clements and B. Lindstrom in [J. Combinatorial Theory 7 (1969), 230-238]. (Received March 12, 1970.)

70T-H23. OLE J. HEILMANN, DANIEL J. KLEITMAN, and ELLIOTT H. LIEB, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, and SEYMOUR SHERMAN, Indiana University, Bloomington, Indiana 47401. Some positive definite functions on sets.

Consideration of correlation inequalities for Ising ferromagnets with arbitrary spin has led to a class of positive definite functions on sets. A special case of these functions are linear combinations of the functions which enter into Muirhead's Theorem. Let $K =_{df} \{1,\ldots, k\}$ $(\forall_j \in K)$ $a^j = (a_1^j,\ldots, a_n^j) \geqq (0,\ldots,0)$ componentwise. $(\forall D \subset K)$ $a(D) =_{df} \sum_{j\in D} a^j$. Let $r = (r_1,\ldots, r_{2n})$ be a real 2n-tuple. Then $[r]$ means $\sum_{\pi\in G} \exp \sum_{i=1} r_i \sigma_{\pi(i)}$, where G is that subgroup of the symmetric group, S_{2n}, generated by the transpositions of 2i - 1 and 2i, i = 1,2,..., n. G has 2^n elements. If $\pi \in G$, then $\pi: i \to \pi(i)$. Let $r(D) = (r_1(D),\ldots, r_{2n}(D))$ where $r_{2i-1}(D) = a(K - D)_i$ and $r_{2i}(D) = a(D)_i$ and let σ be a real 2n-tuple. Theorem. $[r(D)]$ is a positive definite function of D in the group 2^K, where the group operation is symmetric difference. (Received March 13, 1970.)

70T-H24. E. GREGORY LEE, Reed College, Portland, Oregon 97202. A computer test for interval intersection graphs and consecutive 1's matrices.

A matrix A of 0's and 1's is said to have the consecutive 1's property iff. there exists a permutation matrix P with the property that PA has the 1's in each column appearing consecutively. D. R. Fulkerson and O. A. Gross in "Incidence matrices and interval graphs," Pacific J. Math. 15 (1965), 835-855, describe an algorithm for determining when a matrix has the consecutive 1's property. In addition they prove that a given undirected graph is the intersection graph of a family of intervals in the real line iff. a certain associated matrix, the dominant clique versus vertex matrix, has the consecutive 1's property. The program which has been written determines the dominant clique versus vertex matrix for a given graph and applies the consecutive 1's algorithm. The portion which actually utilizes the consecutive 1's algorithm may easily be used separately. Although the program is written in FORTRAN, version IV, for an IBM 1130 system, adaptation for other systems should be possible. A copy of the program with explanatory notes and necessary nonstandard subroutines will be sent on request. (Received March 19, 1970.) (Author introduced by Professor Peter L. Renz.)

70T-H25. JOSEPH T. BORREGO and HASKELL COHEN, University of Massachusetts, Amherst, Massachusetts 01002, and ESMOND DeVUN, Wichita State University, Wichita, Kansas 67208. Uniquely representable semigroups. II. Preliminary report.

A semigroup S is said to be uniquely representable in terms of two subsets X and Y if $X \cdot Y = Y \cdot X = S$, $x_1y_1 = x_2y_2$ is a nonzero element of S implies $x_1 = x_2$ and $y_1 = y_2$, and $y_1x_1 = y_2x_2$ is a nonzero element of S implies $y_1 = y_2$ and $x_1 = x_2$ for all $x_1, x_2 \in X$ and $y_1, y_2 \in Y$. In this paper we are concerned with semigroups S with no zero divisors and which are uniquely representable in terms of two subsets X and Y which are iseomorphic copies of the usual unit interval. It has been shown that the nonzero elements of the above semigroup form a cancellative semigroup. In this paper we will extend these results, and show that nonzero elements of the above semigroup can be embedded in a Lie group. As a corollary we have that every uniquely divisible semigroup on the two-cell whose set of idempotents consists of a zero and an identity can be embedded in a Lie group. (Received April 3, 1970.)

70T-H26. ALEXANDER HURWITZ, IBM Corporation, Los Angeles Scientific Center, Los Angeles, California 90067. Homogeneous 0-1 matrices. Preliminary report.

Let A be an $m \times n$ 0-1 matrix, $f_m(A)$ the number of columns consisting of all ones. If there is a $j \times n$ submatrix B of A, such that for every $j \times n$ submatrix C, $f_j(C) = f_j(B)$ then let $f_j(A) = f_j(B)$. A is j-homogeneous (a j-HM) if f_i is defined for $i = 1,2,\ldots, j$. A and B are equivalent if but for columns of all zeros, permuting the rows and then the columns of B gives A. Let G(s,t) be a directed graph whose vertices are the equivalence classes of 2-HM with $f_1 = s$ and $f_2 = t$, and the equivalence class $\mathbb{C}$ containing B a $1 \times s$ matrix, $f_1(B) = s$; an edge goes from one vertex to another if removal of a row from an element of the second gives an element of the first. Theorem. In G(s,t) there is exactly one infinite path $\mathbb{P}$ starting from $\mathbb{C}$ and the length of every path not included in $\mathbb{P}$ is $\leq \binom{s}{t}(s - t) + 1$. We exhibit mappings G(s,t) to G(s + x, t + x) and G(s,t) to G(2(s - t), s - t) which preserve edges, for $x > 0$, $s > t > 0$. Theorem. A vertex containing A, a 2-HM whose transpose B is a 1-HM, has no successors in G(s,t) if $f_1(A) \not\equiv f_2(A) \pmod{f_1(B)}$. If A and B have the same number of rows then define the juxtaposition AB to be the result from adjoining the columns of B at the right of A. Theorem. For $m \geq 2$ there exists a finite set S_m of 2-HM such that every 2-HM with m rows is equivalent to a juxtaposition of elements of S_m. (Received April 20, 1970.)

70T-H27. LON ROSEN, Courant Institute, New York University, New York, New York 10012. The $(\varphi^{2n})_2$ quantum field theory: higher order estimates. Preliminary report.

Let H(g) be the Hamiltonian with a spatial cutoff for the $(\varphi^{2n})_2$ field theory describing self-interacting bosons in two-dimensional space-time. Then for $0 < \alpha < 3$, $0 < \beta$ there are constants a,b, and γ such that $H_0^{\alpha} N^{\beta} \leq a(H(g) + b)^{\gamma}$ ...where H_0 is the free hamiltonian and N the number operator. It follows that H(g) is essentially selfadjoint on $C^{\infty}(H_0)$. The Heisenberg field $\varphi(x,t) = \exp(itH(g))\varphi(x,0)\exp(-itH(g))$ is a densely defined bilinear form which is C^2 in (x,t). $\varphi(x,t)$ satisfies the field equation $\varphi_{tt} - \varphi_{xx} + m^2\varphi + 2ng{:}\varphi^{2n-1}{:} = 0$. Moreover $\varphi(x,t)$ is independent of the cutoff function g provided that $g(x) = 1$ on a sufficiently large set. For suitable real functions f(x,t), $\varphi(f) = \int \varphi(x,t)f(x,t)dxdt$ is a selfadjoint operator. The C^*-algebra of local observables generated by the fields $\varphi(f)$ satisfies the Haag-Kastler Axioms with the possible exception of Lorentz covariance. (Received April 22, 1970.) (Author introduced by Professor James G. Glimm.)

ERRATA

Volume 16

DOUGLASS B. MORRIS. Choice and cofinal well-ordered subsets. Preliminary report, Abstract 69T-E87, Page 1088.

Reference, Line 9: "Ann. of Math. 'Logic'" should read "Annals of Mathematical Logic."

Volume 17

HECTOR O. FRATTINI. Bounded cosine functions in Hilbert space, Abstract 70T-B56, Page 442.

Line 7 from top of abstract says: $C(t) = Q^{-1} = Q^{-1}(\cos(tS))Q$,

should say: $C(t) = Q^{-1}(\cos(tS))Q$.

Line 1 from bottom of abstract says: joint,
should say: point.

THOMAS L. MARKHAM. Factorizations of completely positive matrices, Abstract 672-665, Page 274.

Line 5: Replace "cone" by "set".

Line 6: Replace "if and only if" by "only if".

DOUGLASS B. MORRIS. A model of ZF which cannot be extended to a model of ZFC without adding ordinals. Preliminary report, Abstract 70T-E27, Page 577.

Line 12: "$2^{\aleph_0}$" should be "$(2^{\aleph_0})^+$."

BRUNO SCARPELLINI. A note on Barinduction of higher type, Abstract 70T-E16, Page 455.

Line 3: "B is the theory obtained by adding to B all" should read:

"B is the theory obtained by adding to H_ω all".

JOHN K. TRUSS. Finite versions of the axiom of choice, Abstract 70T-E28, Page 577.

The theorem claimed is incorrect. The method of proof used does however give the sufficiency of $M(Z,n)$ for $[Z] \rightarrow [n]$ in these cases: - n = 22, 25, 26, 33, 34, 36, 44, 45. The general solution of Mostowski's problem appeared in Abstract 70T-E12, these *Notices* 16(1970), 454 (R.J. Gauntt).

WINSTON WALKER. Stationarity and stochastic differential equations. Preliminary report, Abstract 672-655, Pages 271-272.

This abstract should have two authors, Winston Walker and George Adomian.

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