Notices

OF THE

AMERICAN

MATHEMATICAL

SOCIETY

VOLUME 17, NUMBER 5 ISSUE NO. 123 AUGUST, 1970

Notices

OF THE

AMERICAN MATHEMATICAL SOCIETY

Edited by Everett Pitcher and Gordon L. Walker

CONTENTS

MEETINGS

Calendar of Meetings

NOTE: This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the *Notices* was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

Meeting No.	Date	Place	Deadline for Abstracts*
678	October 31, 1970	Washington, D. C.	Sept. 10, 1970
679	November 20-21, 1970	Athens, Georgia	Oct. 6, 1970
680	November 21, 1970	Pasadena, California	Oct. 6, 1970
681	November 28, 1970	Urbana, Illinois	Oct. 6, 1970
682	January 21-25, 1971 (77th Annual Meeting)	Atlantic City, New Jersey	Nov. 5, 1970
	March 26-27, 1971	Chicago, Illinois	
	April 7-10, 1971	New York, New York	
	April 24, 1971	Monterey, California	
	August 30-September 3, 1971 (76th Summer Meeting)	University Park, Pennsylvania	
	January 17-21, 1972 (78th Annual Meeting)	Las Vegas, Nevada	

*The abstracts of papers to be presented in person at the meetings must be received in the Headquarters Offices of the Society in Providence, Rhode Island, on or before these deadlines. The deadlines also apply to news items. The next two deadlines for by-title abstracts will be September 3, 1970, and September 29, 1970.

OTHER EVENTS

September 1-10, 1970	International Congress of Mathematicians	Nice, France

The *Notices* of the American Mathematical Society is published by the American Mathematical Society, 321 South Main Street, P. O. Box 6248, Providence, Rhode Island 02904 in January, February, April, June, August, October, November and December. Price per annual volume is $10.00. Price per copy $3.00. Special price for copies sold at registration desks of meetings of the Society, $1.00 per copy. Subscriptions, orders for back numbers (back issues of the last two years only are available) and inquiries should be addressed to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904. Second class postage paid at Providence, Rhode Island, and additional mailing offices.

The Seventy-Fifth Summer Meeting
University of Wyoming
Laramie, Wyoming
August 25-28, 1970

The seventy-fifth summer meeting of the American Mathematical Society will be held at the University of Wyoming, Laramie, Wyoming, from Tuesday, August 25, through Friday, August 28, 1970. All sessions of the meeting will be held on the campus of the University. The times listed below for events of the meeting are MOUNTAIN DAYLIGHT SAVING TIME throughout.

The Colloquium Lectures will be given by Professor R. H. Bing of the University of Wisconsin. The title of this series of lectures is "Topology of 3-manifolds." The Colloquium Lectures will be presented at 1:00 p.m. on Tuesday, August 25, in the Auditorium of the Arts and Sciences Building, and at 9:00 a.m. on Wednesday, Thursday, and Friday, August 26-28, in Room 302 of the Classroom Building.

By invitation of the Committee to Select Hour Speakers for Annual and Summer Meetings, there will be two invited hour addresses at the meeting. Professor Patrick P. Billingsley of the University of Chicago will speak on Thursday, August 27, at 1:00 p.m. in the Auditorium of the Arts and Sciences Building. The title of his lecture is "Some probability results connected with Diophantine approximation." Professor Srinivasa S. R. Varadhan of the Courant Institute of Mathematical Sciences, New York University, will present an address entitled "Diffusion processes: a martingale approach." This lecture will be given at 1:00 p.m. on Friday, August 28, in the Auditorium of the Arts and Sciences Building.

The first award of the Norbert Wiener Prize in Applied Mathematics will be made at 2:15 p.m. on Tuesday, August 25, in the Auditorium of the Arts and Sciences Building. There will be numerous sessions for contributed ten-minute papers. These are scheduled at 3:15 p.m. on Tuesday, 10:15 a.m. on Wednesday, at 10:15 a.m. and 2:15 p.m. on Thursday, and at 10:15 a.m. on Friday. Late papers will not be accepted for presentation at this meeting.

This meeting will be held in conjunction with meetings of the Mathematical Association of America, the Institute of Mathematical Statistics, and Pi Mu Epsilon. The Mathematical Association of America will meet from Monday through Wednesday. The Association will present Professor Harry Kesten of Cornell University as the Earle Raymond Hedrick Lecturer. The title of Professor Kesten's series of lectures will be "Escapades of a random walk." The Institute of Mathematical Statistics will meet from Monday through Friday. The Wald Lectures, sponsored by the IMS, will be given by Professor Murray Rosenblatt of the University of California, San Diego. The subject of these lectures is "Topics in estimation for stationary processes." Pi Mu Epsilon will meet concurrently with the Society and the Association.

COUNCIL AND BUSINESS MEETING

The Council of the Society will meet

on Tuesday, August 25, at 5:00 p.m. in the Rendezvous Room of the Washakie Center. The Business Meeting of the Society will be held on Thursday, August 27, at 4:00 p.m. in the Auditorium of the Arts and Sciences Building.

A proposed amendment to the By-laws, approved by the Council, is to be submitted to the membership at the Business Meeting. The effects are to make the Committee to Monitor Problems in Communications be a statutory committee, with members nominated by the nominating committee and elected by the membership; to make the chairman of the committee, elected by the committee, be a member of the Council; to make the current members of the appointed committee be the initial elected members with staggered terms; and to charge the committee with performance of such tasks in the field of communication of mathematics as are assigned to it by the Council.

REGISTRATION

The Registration Desk will be in the lobby of the Washakie Center. It will be open on Sunday from 2:00 p.m. to 8:00 p.m.; on Monday from 8:00 a.m. to 5:00 p.m.; on Tuesday, Wednesday, and Thursday from 9:00 a.m. to 5:00 p.m. and on Friday from 9:00 a.m. to 1:00 p.m. The telephone number will be 307-766-4340.

The registration fees will be as follows:

Members	\$ 5.00
Students	\$ 1.00
Nonmembers	\$10.00

There will be no extra charge for members of the families of registered participants.

EMPLOYMENT REGISTER

The Joint Committee on Employment Opportunities has decided not to have an Employment Register at the Laramie, Wyoming, meeting.

EXHIBITS

Book exhibits and exhibits of educational media will be displayed in the Wyoming Room of the Washakie Center on Tuesday, Wednesday, and Thursday.

BOOK SALES

There will be no book sale at this meeting.

DORMITORY HOUSING

Rooms are available in a new dormitory complex surrounding Washakie Center, registration headquarters. Dormitories have elevators and lounges, with kitchenettes on each floor. Coin-operated washers and dryers, as well as ironing boards and irons, are available.

Rooms may be occupied from 2:00 p.m. Saturday, August 22, until Saturday noon, August 29. Bed linens, blankets, towels and soap will be provided, but maid service is not available. The rates are: \$4.00 per day / per person, double occupancy, and \$6.00 per day, single. No additional charge is made for cribs in the rooms, but the university cannot provide them. Cribs can be rented from Smith's Outlet, 414 South Second Street (307-742-2510) at \$3.00 per week. Guests are expected to pay for their rooms at the time they check in.

Upon arrival, all guests should check in at Washakie Center to register for dormitory rooms. Turn north from Grand Avenue (U. S. 30 and Business Interstate 80) on 15th Street, and then east at the first corner on Ivinson Avenue. The registration area is on the lower level of Washakie Center. Dormitory personnel will be on duty 24 hours a day to issue keys. Student bellhops, who will accept tips, will be available if desired.

Guests must register in advance to be assured of dormitory housing, using the form provided on page 704 of the June *Notices*. Dormitory reservations will be confirmed by the housing office of the University of Wyoming. It is probable that room will be available for those who do not register in advance, but this cannot be guaranteed.

FOOD SERVICES

The dining room in Washakie Center will be open for lunch on Sunday, August 23, and will serve meals through breakfast, Saturday, August 29, except during the Buffalo Roast Wednesday evening. Serving hours are:

Breakfast	7:00 a.m.-8:30 a.m.
Lunch	11:30 a.m.-1:00 p.m.
Dinner	5:30 p.m.-6:30 p.m.

The service is cafeteria style, on a cash basis, at the following prices:

	Breakfast	Lunch	Dinner
Adults	$1.25	$1.50	$2.00
Children (4-12)	0.65	0.75	1.00
Children (under 4) no charge			

The Snack Bar in Washakie Center will be open as required to meet the demand.

Coffee and doughnuts will be available from a mobile unit just outside the Classroom Building while the meetings are in session.

MOTELS

There are a number of motels and a hotel in Laramie, some of which are listed below with following coded information: FP-Free Parking; SP-Swimming Pool; AC-Air Conditioned; TV-Television; CL-Cocktail Lounge; RT-Restaurant. Participants should make their own reservations.

BRANDING IRON MOTEL (307) 742-6808
1161 North Third-20 rooms

Single	$10.00
Double	$15.00

Some units for four and five people
Code: FP-TV
16 blocks from campus

BUCKAROOMOTOR LODGE(307) 742-2865
365 North Third-16 rooms

One double bed	$10.00
Two double beds	$14.00
Rollaway beds	$ 1.50

Code: FP-TV
8 blocks from campus

CIRCLE S MOTEL (307) 745-4811
2440 Grand-48 rooms

Double	$12.50-$13.50
Twin	$15.50
Family units	$18.50-$20.00
Rollaway beds	$ 2.00
Cribs	$ 1.00

Code: FP-TV-SP-AC
14 blocks from campus

DOWNTOWN MOTEL (307) 742-6671
165 North Third-30 rooms

Single	$12.00
Twin	$17.50
Two double beds	$19.50
Rollaway beds	$ 2.00
Free cribs	

Code: FP-TV-Some AC
7 blocks from campus

GAS LITE MOTEL (307) 742-6616
960 North Third-27 rooms

Single	$12.00
Double	$14.00
Twin	$17.00-$18.00
Rollaway beds	$ 3.00
Cribs	$ 2.00

Code: FP-TV-SP-AC
15 blocks from campus

HOLIDAY INN (307) 742-6611
180-U.S. 287-68 rooms

Single	$14.50
Double	$19.00
Rollaway beds	$ 2.00
Free cribs	

Code: FP-TV-SP-AC-CL-RT
3 miles from campus

HORSESHOE MOTEL (307) 742-2107
U.S. 30 North-20 rooms

One double bed	$10.00
Two double beds	$14.00
Extra person	$ 1.00

Code: FP-RT-TV-CL
3 miles from campus

LARAMIE TRAVELODGE (307) 745-4853
262 North Third-29 rooms

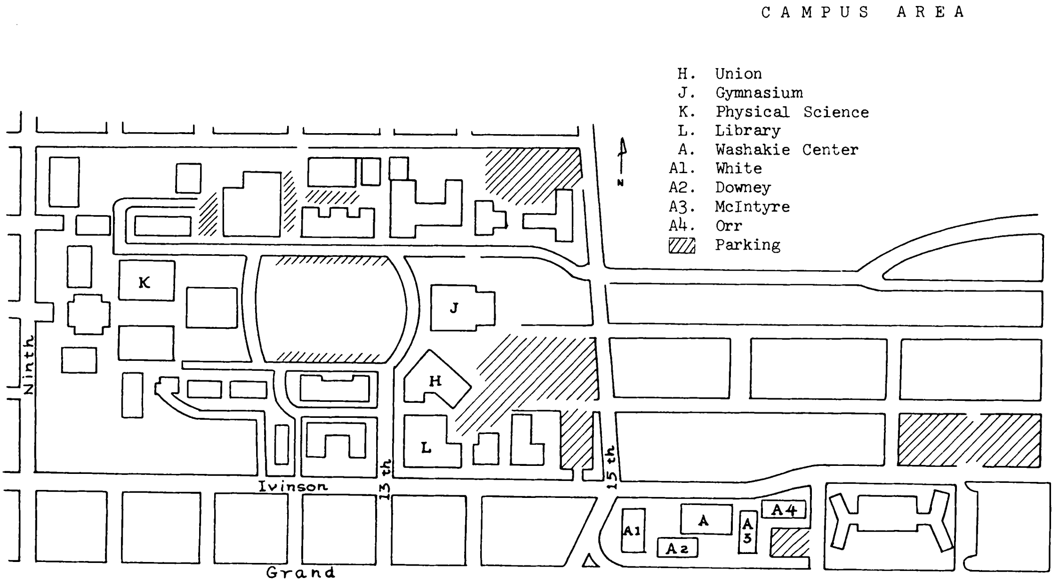
Single	$12.00
Double	$14.00
Twin	$16.00
Rollaway beds	$ 2.00
Free cribs	

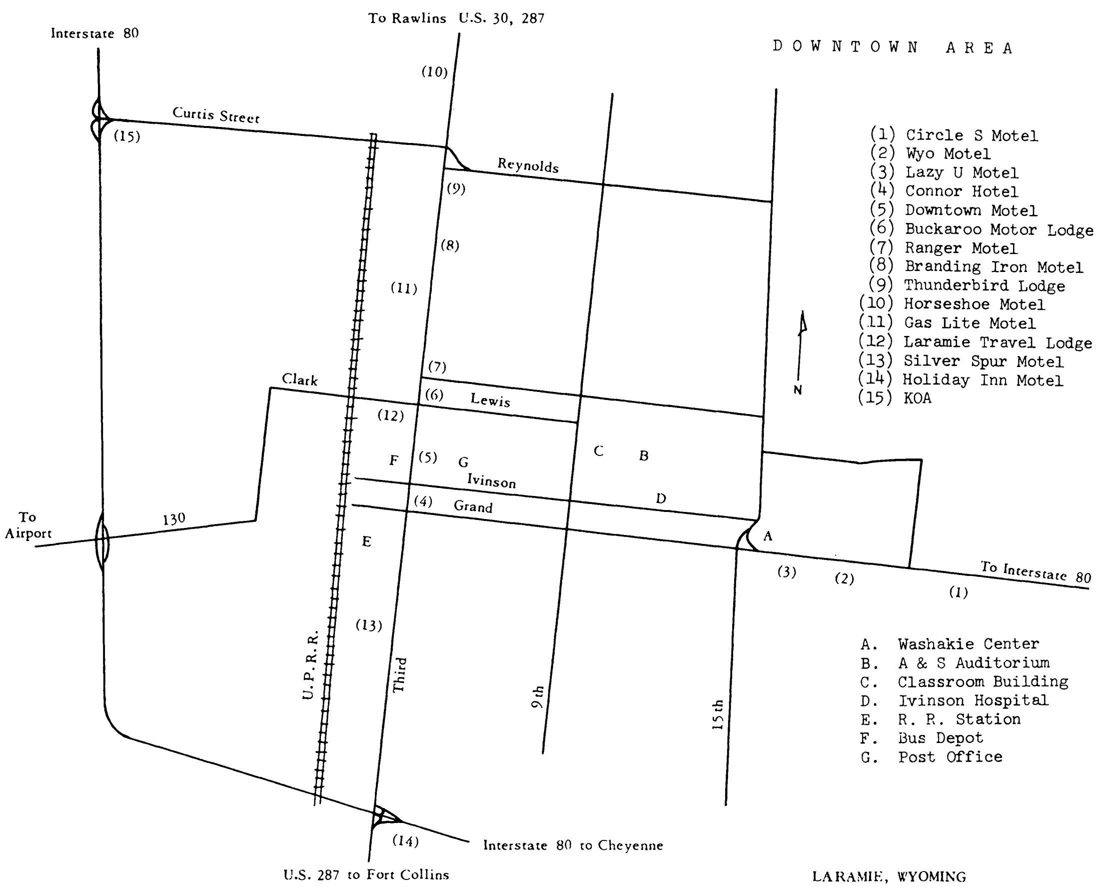
Code: FP-SP-TV-AC
8 blocks from campus

LAZY U MOTEL (307) 745-7322
1622 Grand-13 rooms

One double bed	$10.00
Two double beds	$14.00

Code: FP-TV
4 blocks from campus


CAMPUS AREA
H. Union
J. Gymnasium
K. Physical Science
L. Library
A. Washakie Center
A1. White
A2. Downey
A3. McIntyre
A4. Orr
Parking
Ninth
Ivinson
13 th
15 th
Grand
K
J
H
L
A1
A2
A
A3
A4
N


DOWNTOWN AREA
Interstate 80
To Rawlins U.S. 30, 287
Curtis Street
Reynolds
Clark
Lewis
Ivinson
Grand
To Airport
130
To Interstate 80
U.P.R.R.
Third
9th
15th
Interstate 80 to Cheyenne
U.S. 287 to Fort Collins
(1) Circle S Motel
(2) Wyo Motel
(3) Lazy U Motel
(4) Connor Hotel
(5) Downtown Motel
(6) Buckaroo Motor Lodge
(7) Ranger Motel
(8) Branding Iron Motel
(9) Thunderbird Lodge
(10) Horseshoe Motel
(11) Gas Lite Motel
(12) Laramie Travel Lodge
(13) Silver Spur Motel
(14) Holiday Inn Motel
(15) KOA
A. Washakie Center
B. A & S Auditorium
C. Classroom Building
D. Ivinson Hospital
E. R. R. Station
F. Bus Depot
G. Post Office
N
LARAMIE, WYOMING

RANGER MOTEL (307) 742-6677
453 North Third-40 rooms

One double bed	$14.00
Twin	$16.00
Two double beds	$20.00
Cribs	$ 1.00

Code: FP-SP-TV-AC
10 blocks from campus

SILVER SPUR MOTEL (307) 742-3741
1104 South Third-24 rooms

One double bed	$12.00
Two double beds	$14.00-$18.00
Rollaway beds	$ 1.00

Code: FP-TV-Some AC
17 blocks from campus

THUNDERBIRD LODGE (307) 745-4871
1369 North Third-18 rooms

Double	$12.00
Twin	$14.00
Two double beds	$15.00-$16.00
Free cribs	

Code: FP-SP-TV-AC
19 blocks from campus

WYO MOTEL (307) 742-6633
1720 Grand-36 rooms

Single	$14.00
Double	$15.00
Rollaway beds	$ 3.00
Cribs	$ 1.00

Code: FP-SP-TV-AC
5 blocks from campus

PARKING

No permits for on-campus parking will be needed, and courtesy permits will be issued for downtown parking.

CAMPING

There is a Kampgrounds of America (privately-owned campground) at the Curtis Street exit of Interstate 80. There are four campgrounds 10 to 16 miles east of Laramie on Interstate 80 with a total of 87 campsites. There are four campgrounds located 30 to 40 miles west of Laramie on Wyoming 130 with a total of 66 campsites. Numerous other campgrounds are located further west in the Medicine Bow National Forest and south of Laramie in Northern Colorado.

For more detailed information on camping and facilities write to: Wyoming Travel Commission, 2320 Capitol Avenue, Cheyenne, Wyoming 82001.

BOOKSTORE

The University Bookstore, located in the Union, will probably be open Monday through Friday from 7:30 a.m. to 5:00 p.m. It sells souvenirs and personal articles as well as books. Two downtown bookstores are the West Book Store at 210 South Third and Books A Go Go at 408 University Avenue.

LIBRARIES

The University Library will be open Monday through Friday from 8:00 a.m. to 5:00 p.m. The Albany County Library at 405 Grand is open from noon to 9:00 p.m., Monday through Thursday, and from noon to 6:00 p.m. on Friday.

MEDICAL SERVICES

Ivinson Memorial Hospital (phone 742-2141) at 1014 Ivinson Avenue (adjacent to the campus) is an 84 bed general hospital with an intensive care unit. A doctor is always on emergency call.

ENTERTAINMENT

There will be a Buffalo Roast on Wednesday at 5:30 p.m. on the mall north of the dormitories and west of the fieldhouse. In case of bad weather it will be held in Washakie Center. Prices are $2.75 for adults, and $1.25 for children from 4 to 12, with no charge for children under 4. A beer party sponsored by the Department of Mathematics will be held at the Connor Hotel at 8:00 p.m. Tickets may be purchased at the registration desk.

Tentative plans have been made for two trips--a Snowy Range trip leaving Washakie Center Tuesday at 8:45 a.m. and a historical bus tour, accompanied by Mr. Neal Miller, director of the State Archives and History Department, leaving Washakie Center at 8:45 a.m. Thursday. This trip will include a visit to the Sybille Game Refuge and Fort Laramie National Monument. Bus fare is about $7.00. Washakie Center will fix box lunches for $1.25.

A guided walking tour of the university flower gardens will be led by Mr. Otto Dahl, university horticulturist, leaving Washakie Center at 10:00 a.m. Wednesday.

Entertaining movies will be shown

in the Classroom Building at 7:00 p.m. on Monday, Tuesday, and Wednesday.

If there is sufficient interest, climbing expeditions in the Snowy Range or at Vedauwoo can be arranged.

Other diversions include the Geological Museum, the University Planetarium, the Laramie Plains Museum, the Hebard Western History Room in the University Library, campus golf and tennis courts, gymnasium facilities and equipment, enclosed city swimming pool, horse-back riding, fishing, and picnicking.

Parents can make arrangements to leave their children during the day at the Jack and Jill Nursery and Day School, 416 Hancock St., phone 307-745-7985. A list of baby sitters for evening hours will be available.

TRAVEL

During the summer, Laramie is on MOUNTAIN DAYLIGHT SAVING TIME.

Transportation to Laramie is available via regional airline, bus, and train. Two major U. S. highways, U. S. 287 and Interstate 80, provide access by private auto. Frontier Airlines services Laramie with connections from Denver, Casper, or Billings. Continental Trailways, Greyhound, and Colorado Motor Ways make scheduled stops in Laramie. Direct bus service from Denver to Laramie is available at the Denver Greyhound terminal. Union Pacific Railroad has passenger train service to Laramie from the east and west. Three or more people traveling together would find it cheaper to rent a car in Denver than to pay air fare. Rental cars available in Laramie are limited so that prior reservations would be advisable through Avis Rent-A-Car, 501 Garfield; Hertz Rent-A-Car, 401 Lewis; and National Car Rental System, 666 North Third, Laramie, Wyoming 82070.

WEATHER

The average maximum temperature during this week is 73° and the average minimum is 45°. Typically, mornings are sunny while afternoons may have some high cloudiness, possible light showers, and occasional gusty winds. Although highly improbable, snow is not an impossibility. Guests are advised to be prepared for cool weather.

ADDRESS FOR MAIL AND TELEGRAMS

Individuals may be addressed at Mathematics Meetings, Washakie Center, University of Wyoming, Laramie, Wyoming 82070. The telephone number of the Message Center will be 307-766-6422. This number will have two separate lines but will have a rotary on it so if one line is busy it will automatically place the call on the other line.

COMMITTEE

H. L. Alder (ex officio), Roger M. Cooper, Rev. Fred T. Daly, George C. Gastl, Mrs. Joanne George, John H. George, William C. Guenther, J. Ray Hanna, Terry L. Jenkins, Joe E. Kirk, Jr., Robert W. McKelvey, Richard S. Pierce (ex officio), A. Duane Porter, John H. Rowland, Virindra M. Sehgal, W. Norman Smith (chairman), P. O. Steen, James W. Thomas, Gordon L. Walker (ex officio), Laurence Weinberg.

R. S. Pierce
Associate Secretary

Seattle, Washington

TIMETABLE
(Mountain Daylight Saving Time)

Sunday August 23	American Mathematical Society	Mathematical Association of America	Institute of Mathematical Statistics
9:00 a.m. - 4:00 p.m.		Board of Governors Rendezvous Room Washakie Center	
2:00 p.m. - 8:00 p.m.	REGISTRATION - Lobby of Washakie Center		
		Auditorium, Arts and Sciences Building	
7:00 p.m.		Films	
		Harper and Row Calculus Films, by Harry M. Schey and Judah L. Schwartz	
7:00 p.m. - 7:10 p.m.		Taylor Series I Taylor Series II Newton's Method	
7:15 p.m.		Films of the MAA Individual Lectures Film Project (ILFP)	
7:15 p.m. - 8:15 p.m.		Nonstandard analysis with Abraham Robinson (b & w)	
8:25 p.m. - 8:50 p.m.		Unsolved problems in geometry with Victor Klee (color)	

Monday August 24	AMS	MAA	IMS
8:00 a.m. - 5:00 p.m.	REGISTRATION - Lobby of Washakie Center		
		Auditorium, Arts and Sciences Building	
9:00 a.m. - 9:15 a.m.		Welcome on Behalf of the University William D. Carlson, President University of Wyoming	
9:15 a.m. - 10:15 a.m.		The Earle Raymond Hedrick Lectures: Escapades of a random walk, Lecture I Harry Kesten	
10:25 a.m.		JOINT SESSION ON POPULATION ECOLOGY	
10:25 a.m. - 11:10 a.m.		Interaction between species N. G. Becker	
11:15 a.m. - 12:00 p.m.		Population growth models H. M. Taylor	
1:30 p.m. - 2:30 p.m.		The Earle Raymond Hedrick Lectures: Lecture II Harry Kesten	

Monday August 24	American Mathematical Society	Mathematical Association of America	Institute of Mathematical Statistics
2:40 p.m.		Panel discussion on problems in college teaching	
2:40 p.m. - 3:40 p.m.		Presentations by members of the panel: R. P. Boas Alex Rosenberg J. W. Brewer	
3:40 p.m. - 4:30 p.m.		General discussion by the panel and audience	
7:00 p.m.	MOVIES - Classroom Building		
		Auditorium, Arts and Sciences Building	
7:00 p.m.		Films	
		Films of the NCTM Series: Elementary mathematics for teachers and students (color)	
7:00 p.m. - 7:11 p.m.		Between rational numbers (Knights)	
7:11 p.m. - 7:19 p.m.		Extending multiplication to rational numbers (Clouds)	
7:20 p.m. - 7:29 p.m.		Solving pairs of equations (Pirates)	
7:30 p.m. - 7:56 p.m.		Operations extended to negative rationals	
8:10 p.m.		Films of the MAA ILFP project	
8:10 p.m. - 8:35 p.m.		Unsolved problems in geometry with Victor Klee (color)	
8:40 p.m.		ILFP films not previously shown	

Tuesday August 25	AMS	MAA	IMS
8:15 a.m. - 9:45 a.m.			Contributed papers: Rooms 210 and 304 Classroom Building
8:45 a.m.	BUS TOUR - Snowy Range		
9:00 a.m. - 5:00 p.m.	REGISTRATION - Lobby of Washakie Center		
9:00 a.m. - 5:00 p.m.	EXHIBITS - Wyoming Room - Washakie Center		
		Auditorium, Arts and Sciences Building	
9:00 a.m. - 9:50 a.m.		The Earle Raymond Hedrick Lectures: Lecture III Harry Kesten	
10:00 a.m. - 11:00 a.m.		Business Meeting Presentation of Lester R. Ford Awards; Presentation of a Special Honorary Title to a Member of the Association	Wald Lectures: Topics in estimation for stationary processes, Lecture I Murray Rosenblatt Classroom Building 304

Tuesday August 25	American Mathematical Society	Mathematical Association of America	Institute of Mathematical Statistics
11:10 a.m. - 12:00 p.m.		Public understanding of science and its implication for mathematics L. J. Paige	
11:10 a.m. - 12:30 p.m.			Invited papers: Room 304 Classroom Building
12:15 p.m.	PI MU EPSILON - Council Luncheon - Dining Room, Washakie Center		
1:00 p.m. - 2:00 p.m.	Colloquium Lectures: Topology of 3-manifolds, Lecture I R. H. Bing Auditorium, Arts and Sciences Building		
2:00 p.m. - 3:00 p.m.			Special invited address: Some exciting, new developments in geometrical models for use in data analysis J. B. Kruskal Room 304 Classroom Building
2:15 p.m.	Wiener Prize Session Auditorium, Arts and Sciences Building		
3:10 p.m. - 4:30 p.m.			Invited papers: Room 304 Classroom Building
3:15 p.m. - 5:15 p.m.	PI MU EPSILON - Contributed Papers - Room 201, Classroom Building		
3:15 p.m. - 5:10 p.m.	First General Session Classroom Building 103		
3:15 p.m. - 5:25 p.m.	Session on Complex Variables Classroom Building 105		
3:15 p.m. - 5:25 p.m.	Session on Topology and Differential Geometry Classroom Building 117		
3:15 p.m. - 5:25 p.m.	First Session on Functional Analysis Classroom Building 119		
4:35 p.m. - 5:50 p.m.			Contributed papers: Rooms 210 and 304 Classroom Building
5:00 p.m.	Council Meeting Rendezvous Room, Washakie Center		
6:30 p.m.	PI MU EPSILON - Banquet - Dining Room, Washakie Center		
7:00 p.m.	MOVIES - Classroom Building		
		Auditorium, Arts and Sciences Building	
7:00 p.m.		Films Films of the MAA Mathematics Today series	

Tuesday August 25	American Mathematical Society	Mathematical Association of America	Institute of Mathematical Statistics
7:00 p.m. - 8:01 p.m.		Let us teach guessing, a demonstration with George Polya (color)	
7:30 p.m.			Council Meeting Conference Room, Hoyt Hall
8:10 p.m. - 9:13 p.m.		John von Neumann (b & w)	

Wednesday August 26	AMS	MAA	IMS
8:00 a.m.	PI MU EPSILON - Dutch Treat Breakfast - Dining Room, Washakie Center		
8:15 a.m. - 9:45 a.m.			Contributed papers: Rooms 210 and 304 Classroom Building
9:00 a.m. - 5:00 p.m.	REGISTRATION - Lobby of Washakie Center		
9:00 a.m. - 5:00 p.m.	EXHIBITS - Wyoming Room, Washakie Center		
9:00 a.m. - 10:00 a.m.	Colloquium Lectures: Lecture II R. H. Bing Classroom Building 302		
10:00 a.m.	WALKING TOUR - University Flower Gardens - with Mr. Otto Dahl		
10:00 a.m. - 11:00 a.m.			Wald Lectures: Lecture II Murray Rosenblatt Classroom Building 304
10:15 a.m. - 12:10 p.m.	First Session on Algebra Classroom Building 103		
10:15 a.m. - 12:10 p.m.	Session on Ordinary Differential Equations Classroom Building 105		
10:15 a.m. - 11:40 a.m.	Session on Fixed Point Theorems Classroom Building 117		
10:15 a.m. - 12:10 p.m.	Second Session on Functional Analysis Classroom Building 119		
10:40 a.m. - 12:40 p.m.	PI MU EPSILON - Contributed Papers - Room 201, Classroom Building		
11:10 a.m. - 12:30 p.m.			Invited papers: Room 304 Classroom Building
		Auditorium, Arts and Sciences Building	
1:30 p.m.		Panel discussion on restructuring of college mathematics Moderator: H. J. Greenberg	
1:30 p.m. - 4:00 p.m.		Presentations and discussion by members of the panel: Thomas Kurtz J. P. Lasalle H. M. Schey	

Wednesday August 26	American Mathematical Society	Mathematical Association of America	Institute of Mathematical Statistics
2:00 p.m. - 4:30 p.m.			Invited papers: Room 304 Classroom Building
4:35 p.m. - 5:35 p.m.			Business Meeting Room 304 Classroom Building
5:30 p.m.	BUFFALO ROAST - Mall Area		
7:00 p.m.	MOVIES - Classroom Building		
8:00 p.m.	BEER PARTY - Connor Hotel		

Thursday August 27	AMS	MAA	IMS
8:15 a.m. - 9:45 a.m.			Contributed papers: Rooms 210 and 304 Classroom Building
8:45 a.m.	BUS TOUR - Historical Sites: Sybille Game Refuge and Fort Laramie National Monument		
9:00 a.m. - 5:00 p.m.	REGISTRATION - Lobby of Washakie Center		
9:00 a.m. - 5:00 p.m.	EXHIBITS - Wyoming Room - Washakie Center		
9:00 a.m. - 10:00 a.m.	Colloquium Lectures: Lecture III R. H. Bing Classroom Building 302		
9:30 a.m. - 12:30 p.m.	CBMS - Council Meeting - Rendezvous Room, Washakie Center		
10:00 a.m. - 11:00 a.m.			Wald Lectures: Lecture III Murray Rosenblatt Classroom Building 304
10:15 a.m. - 12:10 p.m.	Second General Session Classroom Building 103		
10:15 a.m. - 12:10 p.m.	First Session on Analysis Classroom Building 105		
10:15 a.m. - 12:10 p.m.	Session on Measure and Integration Classroom Building 117		
10:15 a.m. - 12:10 p.m.	Second Session on Analysis Classroom Building 119		
11:10 a.m. - 12:30 p.m.			Invited papers: Room 304 Classroom Building
1:00 p.m. - 2:00 p.m.	Invited address: Some probability results connected with Diophantine approximation, Patrick P. Billingsley Auditorium, Arts and Sciences Building		
2:00 p.m. - 4:30 p.m.			Invited papers: Room 304 Classroom Building

Thursday August 27	American Mathematical Society	Mathematical Association of America	Institute of Mathematical Statistics
2:15 p.m. - 3:40 p.m.	Second Session on Algebra Classroom Building 103		
2:15 p.m. - 3:40 p.m.	Third Session on Analysis Classroom Building 105		
2:15 p.m. - 3:25 p.m.	Session on Probability and Stochastic Processes Classroom Building 117		
2:15 p.m. - 3:40 p.m.	Fourth Session on Analysis Classroom Building 119		
4:00 p.m.	Business Meeting Presentation of L. P. Steele Prize for Expository Papers Auditorium, Arts and Sciences Building		
4:35 p.m. - 5:40 p.m.			Invited papers: Room 304 Classroom Building
7:30 p.m.			1971 Council Conference Room, Hoyt Hall

Friday August 28	AMS	MAA	IMS
8:15 a.m. - 9:45 a.m.			Contributed papers: Rooms 210 and 304 Classroom Building
9:00 a.m. - 1:00 p.m.	REGISTRATION - Lobby of Washakie Center		
9:00 a.m. - 10:00 a.m.	Colloquium Lectures: Lecture IV R.H. Bing Classroom Building 302		
10:00 a.m. - 11:00 a.m.			Special invited address: Large sample sequential testing of student's and other composite hypotheses G. E. Schwarz Classroom Building 304
10:15 a.m. - 12:10 p.m.	Third Session on Algebra Classroom Building 103		
10:15 a.m. - 12:10 p.m.	Session on General Topology Classroom Building 105		
10:15 a.m. - 12:10 p.m.	Session on Operator Theory Classroom Building 117		
11:10 a.m. - 12:30 p.m.			Invited papers: Room 304 Classroom Building
1:00 p.m. - 2:00 p.m.	Invited address: Diffusion processes: a martingale approach Srinivasa S. R. Varadhan Auditorium, Arts and Sciences Building		
2:00 p.m. - 4:30 p.m.			Invited papers: Room 304 Classroom Building
4:35 p.m. - 5:50 p.m.			Contributed papers: Room 210 Classroom Building

The time limit for each contributed paper is 10 minutes. The contributed papers are scheduled at 15 minute intervals. To maintain the schedule, the time limit will be strictly enforced.

TUESDAY, 1:00 P.M.

Colloquium Lecture I, Auditorium, Arts and Sciences Building

Topology of 3-manifolds
Professor R. H. Bing, University of Wisconsin

TUESDAY, 2:15 P.M.

Norbert Wiener Prize in Applied Mathematics, Auditorium, Arts and Sciences Building

TUESDAY, 3:15 P.M.

First General Session, Room 103, Classroom Building

3:15-3:25
(1) A theory of general machines and functor projectivity
Dr. David C. Rine, University of Iowa (677-68-1)
(Introduced by Professor Anthony J. Schaeffer)

3:30-3:40
(2) Addition in nonstandard models of arithmetic. Preliminary report
Professor Robert G. Phillips, University of South Carolina (677-02-1)

3:45-3:55
(3) Sectional representations over Boolean spaces
Professor Stephen D. Comer, Vanderbilt University (677-06-3)

4:00-4:10
(4) The vector lattice cover of an abelian lattice-ordered group
Dr. Jorge Martinez, University of Florida (677-06-1)

4:15-4:25
(5) Lattice characterization of spaces of functions measurable with respect to a sigma ring
Professor Witold M. Bogdanowicz, Catholic University of America (677-06-2)

4:30-4:40
(6) The descending chain condition in modular lattices
Professor Thomas G. Newman, Texas Technological University (677-06-4)

4:45-4:55
(7) Three theorems on geometric lattices
Professor Terrence J. Brown, University of Missouri-Kansas City (677-05-2)

5:00-5:10
(8) On Bollobás' number $k_r(n)$. Preliminary report
Professor John L. Leonard, University of Arizona (677-05-3)

TUESDAY, 3:15 P.M.

Session on Complex Variables, Room 105, Classroom Building

3:15-3:25
(9) Sums and products of normal functions and very normal functions
Professor David Winfield Bash, Jr., Purdue University, Fort Wayne Campus (677-30-3)

3:30-3:40

(10) Oscillatory behavior of u'' + hu - 0 for schlicht h. Preliminary report
Professor Joseph A. Cima and Professor John A. Pfaltzgraff*, University of North Carolina at Chapel Hill (677-30-5)

3:45-3:55

(11) An arclength problem for m-fold symmetric univalent functions
Mr. Sanford S. Miller, University of Kentucky (677-30-2)

4:00-4:10

(12) Value distribution of exponential sums
Professor Benjamin Lepson, U.S. Naval Research Laboratory, Washington D.C., and Catholic University of America (677-30-4)

4:15-4:25

(13) Entire functions of unbounded index and having simple zeros
Professor S. M. Shah, University of Kentucky (677-30-1)

4:40-4:40

(14) Distribution of roots and growth of a meromorphic function
Professor Hari Shankar*, Ohio University, and Professor S. K. Singh, University of Missouri-Kansas City (677-30-7)

4:45-4:55

(15) Divisors of poles with low order and 2-point support
Mr. John M. Kasdan, University of California, Los Angeles (677-30-8)
(Introduced by Professor Theodore S. Motzkin)

5:00-5:10

(16) A Stieltjes integral approach to the length of a curve on the Riemann sphere
Professor Fred M. Wright and Miss Nancy Heath*, Iowa State University (677-30-9)

5:15-5:25

(17) A doubly connected Riemann surface
Professor Joe E. Kirk, Jr., University of Wyoming (677-30-6)

TUESDAY, 3:15 P.M.

Session on Topology and Differential Geometry, Room 117, Classroom Building

3:15-3:25

(18) Whitehead torsion for free simplicial groups
Professor Robert P. Walker, University of North Carolina at Chapel Hill (677-55-1)

3:30-3:40

(19) Riemannian spaces of 2-recurrent curvature
Professor Alan H. Thompson, University of Pittsburgh (677-53-1)
(Introduced by Professor Charles G. Cullen)

3:45-3:55

(20) Bending and immersing strips and bands. Preliminary report
Professor Benjamin Halpern and Professor Charles S. Weaver*, Indiana University (677-53-2)

4:00-4:10

(21) Wild disks in E^n that can be squeezed only to tame arcs
Professor Robert J. Daverman, University of Tennessee (677-54-15)

4:15-4:25

(22) Higher genera for links
Professor Charles H. Goldberg, Trenton State College and Princeton University (677-54-7)

4:30-4:40

(23) On homeomorphism groups of free unions
Dr. Ludvik Janos, University of Florida (677-54-3)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.

4:45-4:55

(24) Finite products of Wallman spaces

Professor Frank C. Kost, State University of New York, College at Oneonta (677-54-1)

(Introduced by Professor Guy T. Hogan)

5:00-5:10

(25) Characterization of Kuratowski 14-sets

Professor Eric S. Langford, University of Maine (677-54-5)

(Introduced by Professor John C. Mairhuber)

5:15-5:25

(26) A note on Baire sets

Professor Virindra M. Sehgal and Mrs. Evelyn J. Morrison*, University of Wyoming (677-28-9)

TUESDAY, 3:15 P.M.

First Session on Functional Analysis, Room 119, Classroom Building

3:15-3:25

(27) On Banach spaces which are dual L-spaces

Professor H. Elton Lacey, University of Texas (677-46-15)

3:30-3:40

(28) The geometric Hahn Banach theorem for near topological vector spaces

Mr. Jimmy Ray Nanney, University of Mississippi (677-46-2)

3:45-3:55

(29) Convex filters and Cauchy filters

Professor Frank J. Wagner, University of Cincinnati (677-46-19)

4:00-4:10

(30) The metric half-space topology. Preliminary report

Professor Russell G. Bilyeu, North Texas State University (677-46-13)

4:15-4:25

(31) The completion of a B-convex normed Riesz space is reflexive

Professor Daniel P. Giesy, Western Michigan University (677-46-5)

4:30-4:40

(32) Existence theorems for Markuschevich bases in Banach spaces

Professor William B. Johnson, University of Houston (677-46-1)

4:45-4:55

(33) A norm characterization of continuous linear operators on locally convex spaces. Preliminary report

Mr. Charles F. Amelin, California State Polytechnic College, Kellogg-Voorhis Campus (677-47-11)

5:00-5:10

(34) Absolutely summing operators into $\mathcal{P}_p$ and $\mathcal{L}_p$-spaces. Preliminary report

Professor J. S. Morrell, University of Southern Mississippi (677-46-18)

5:15-5:25

(35) Characterizations of norms in finite dimensional spaces. Preliminary report

Dr. Ernest J. Eckert, California State College at Los Angeles (677-46-8)

WEDNESDAY, 9:00 A.M.

Colloquium Lecture II, Room 302, Classroom Building

Topology of 3-manifolds

Professor R. H. Bing, University of Wisconsin

WEDNESDAY, 10:15 A.M.

First Session on Algebra, Room 103, Classroom Building

10:15-10:25

(36) K_0 and K_1 of the category of short exact sequences. Preliminary report

Professor Irwin S. Pressman, Ohio State University (677-18-1)

10-30-10:40

(37) On the Krull-dimension of left noetherian left Matlis-rings
Professor Guenter Krause, University of Manitoba (677-16-4)

10:45-10:55

(38) A generalization of a theorem of Fröhlich
Dr. David W. Ballew, South Dakota School of Mines and Technology (677-16-1)

11:00-11:10

(39) Prime rings having one-sided ideal with polynomial identity coincide with special Johnson rings
Professor S. K. Jain, Ohio University (677-16-3)
(Introduced by Professor R. G. Helsel)

11:15-11:25

(40) On normal subgroups of modular group algebras. Preliminary report
Professor Klaus E. Eldridge, Ohio University (677-16-2)

11:30-11:40

(41) Isotopes of truncation algebras
Professor William E. Jenner, University of North Carolina at Chapel Hill (677-17-1)

11:45-11:55

(42) Semisimple classes and upper-type radical classes of narings
Professor Daryl Kreiling*, Western Illinois University, and Professor Terry L. Jenkins, University of Wyoming (677-17-2)

12:00-12:10

(43) Word problem for nonassociative rings with one defining relation. Preliminary report
Dr. David M. Clark, State University of New York at New Paltz (677-17-3)

WEDNESDAY, 10:15 A.M.

Session on Ordinary Differential Equations, Room 105, Classroom Building

10:15-10:25

(44) Iterative scheme for determining the oscillatory properties of $x'' + c(t)x = 0$
Professor Vadim Komkov, Texas Technological University (677-34-2)

10:30-10:40

(45) Disconjugacy criteria for nonselfadjoint differential equations of even order
Mr. Kurt Kreith, University of California, Davis (677-34-7)

10:45-10:55

(46) On the sign of the Green's function beyond the interval of disconjugacy. Preliminary report
Professor Allan C. Peterson, University of Nebraska (677-34-6)

11:00-11:10

(47) Hypergeometric solutions of second order linear ordinary differential equations with confluent regular singular points
Dr. Henry L. Crowson, IBM Corporation, Gaithersburg, Maryland (677-34-1)

11:15-11:25

(48) A generalized invariant imbedding equation: Nonlinear boundary conditions
Mr. Michael A. Goldberg, University of Nevada (677-34-4)

11:30-11:40

(49) Application of a limit theorem to solutions of a stochastic differential equation
Dr. John A. Morrison, Bell Telephone Laboratories, Murray Hill, New Jersey (677-34-3)

11:45-11:55

(50) Global theory of complex functional differential equations
Professor Robert J. Oberg, Knox College (677-34-8)

12:00-12:10

(51) Some effects of reordering the factorisation in a product semidynamical system. Preliminary report

Dr. Prem N. Bajaj, Wichita State University (677-34-9)

WEDNESDAY, 10:15 A.M.

Session on Fixed Point Theorems, Room 117, Classroom Building

10:15-10:25

(52) On an iterative test of Edelstein. Preliminary report

Dr. Sam B. Nadler, Jr., Loyola University, New Orleans (677-46-9)

10:30-10:40

(53) Fixed points of subcontractive mappings

Professor Jack D. Bryant* and Professor Lawrence F. Guseman, Jr., Texas A & M University (677-54-8)

10:45-10:55

(54) Sequential conditions for fixed and periodic points. Preliminary report

Professor Jack Bryant and Professor Lawrence F. Guseman, Jr.*, Texas A & M University (677-54-13)

11:00-11:10

(55) Subsequences of iterates with fixed points

Professor Virindra M. Sehgal, University of Wyoming (677-54-10)

11:15-11:25

(56) Fixed point theorems of the alternative for mappings with a contractive iterate

Professor Roger M. Cooper* and Professor Virindra M. Sehgal, University of Wyoming (677-54-4)

11:30-11:40

(57) Weak semicomplexes and the fixed point theory of tree-like continua

Professor Richard B. Thompson, University of Arizona (677-54-11)

WEDNESDAY, 10:15 A.M.

Second Session on Functional Analysis, Room 119, Classroom Building

10:15-10:25

(58) The Hausdorff moment problem is not equivalent to convergence preserving in the setting of linear normed spaces

Professor John R. Edwards* and Professor Stanley G. Wayment, Utah State University (677-40-1)

10:30-10:40

(59) A v-integral representation for the continuous linear operators on spaces of continuously differentiable vector-valued functions

Professor John R. Edwards and Professor Stanley G. Wayment*, Utah State University (677-47-7)

10:45-10:55

(60) Nonlinear evolution equations and product stable operators in Banach spaces

Professor Glenn F. Webb, Vanderbilt University (677-47-10)

11:00-11:10

(61) Exponential distribution semigroups are C_0 on dense Banach subspaces. Preliminary report

Professor Robert T. Moore, University of Washington (677-47-12)

11:15-11:25

(62) Two-sided operational calculus on an open interval. Preliminary report

Professor Gregers L. Krabbe, Purdue University (677-44-1)

11:30-11:40

(63) Convolution in certain spaces of generalized functions. Preliminary report

Dr. Charles W. Swartz, New Mexico State University (677-46-14)

11:45-11:55

(64) Translation-invariant linear forms on $\mathcal{D}$

Professor Gary H. Meisters, University of Colorado (677-46-11)

12:00-12:10

(65) Multiplication of singularity functions

Professor Chaman Lal Sabharwal, St. Louis University (677-46-3)

THURSDAY, 9:00 A.M.

Colloquium Lecture III, Room 302, Classroom Building

Topology of 3-manifolds

Professor R. H. Bing, University of Wisconsin

THURSDAY, 10:15 A.M.

Second General Session, Room 103, Classroom Building

10:15-10:25

(66) Oscillation of an error function associated with the k-free integers. Preliminary report

Mr. Graham F. Lord, Temple University (677-10-3)

10:30-10:40

(67) Rational approximation to $\sqrt[3]{2}$ and other algebraic numbers

Professor J. M. Gandhi, Western Illinois University (677-10-4)

10:45-10:55

(68) The distribution of kth power residues and nonresidues in the Euclidean domain $Z(\sqrt{-2})$

Dr. Gerald E. Bergum, Gonzaga University (677-10-2)

11:00-11:10

(69) Explicit solutions of pyramidial Diophantine equations

Professor Leon Bernstein, Illinois Institute of Technology (677-10-1)

11:15-11:25

(70) Subsquare complete Latin square. Preliminary report

Professor Raymond Killgrove and Mr. Frank Hiner*, California State College at Los Angeles (677-05-1)

11:30-11:40

(71) A geometric representation of some generalized hexagons

Professor Stanley E. Payne, Miami University (677-14-1)

11:45-11:55

(72) Penalty for violating dimensionality in generating n-metrics

Professor Sister M. Cordia Ehrmann, Villanova University (677-50-2)
(Introduced by Professor August A. Sardinas)

12:00-12:10

(73) On zero-extreme points and the generalized convex kernel

Professor Arthur G. Sparks, Georgia Southern College (677-50-1)

THURSDAY, 10:15 A.M.

First Session on Analysis, Room 105, Classroom Building

10:15-10:25

(74) Nonexistence of a continuous right inverse for surjective linear partial differential operators on special spaces of infinitely differentiable functions. II. Preliminary report

Dr. David K. Cohoon, University of Wisconsin (677-35-2)

10:30-10:40

(75) WITHDRAWN.

10:45-10:55

(76) On a theorem of Edelstein and generalizations

Dr. Vincent G. Sigillito, Johns Hopkins University (677-35-3)
(Introduced by Mr. David W. Fox)

11:00-11:10

(77) Vibration of plates bounded by elliptical and hyperbolic cylinders

Professor Willie R. Callahan, St. John's University (677-35-1)

11:15-11:25

(78) Rotating flow of non-Newtonian fluids

Professor P. Puri* and Professor Prem K. Kulshrestha, Louisiana State University in New Orleans (677-76-1)

11:30-11:40

(79) Thermoelastic stresses in the flexural deformation of a block

Professor K. L. Arora, Punjab University, India (677-73-1)
(Introduced by Professor R. P. Bambah)

11:45-11:55

(80) Differential of variational forms

Professor Mehdi S. Zarghamee, Arya-Mehr University of Technology, Tehran, Iran (677-49-1)
(Introduced by Dr. Morteza Anvari)

12:00-12:10

(81) Integral transforms for time varying systems

Professor Abdul Jabbar Jerri, Clarkson College of Technology (677-44-2)

THURSDAY, 10:15 A.M.

Session on Measure and Integration, Room 117, Classroom Building

10:15-10:25

(82) A Pettis-Dunford integral for topological group valued functions. Preliminary report

Mr. James J. Buckley, Georgia Institute of Technology (677-28-1)

10:30-10:40

(83) Fubini theorem for Orlicz spaces of Lebesgue-Bochner measurable functions

Professor Vernon E. Zander, West Georgia College (677-28-2)

10:45-10:55

(84) Vector measures with finite semivariation. Preliminary report

Mr. Paul W. Lewis, North Texas University (677-28-4)

11:00-11:10

(85) Operator-valued Feynman integrals of finite-dimensional functionals

Professor David L. Skoug* and Professor Gerald W. Johnson, University of Nebraska (677-28-5)

11:15-11:25

(86) Soap bubbles exist

Professor Henry C. Wente, Tufts University (677-28-6)

11:30-11:40

(87) The interior and exterior measures of the union and intersection of two arbitrary sets

Professor Max Shiffman, California State College at Hayward (677-28-7)

11:45-11:55

(88) A Cauchy-type condition concerning integrability

Dr. Michael Lee Steib, University of Houston (677-28-3)

12:00-12:10

(89) On the existence of (LR) $\int_a^b (udw + udv)$

Professor Fred M. Wright* and Mr. Dean Kennebeck, Iowa State University (677-28-8)

THURSDAY, 10:15 A.M.

Second Session on Analysis, Room 119, Classroom Building

10:15-10:25

(90) A connection between commutativity and separation of spectra of operators
Professor Mary R. Embry, University of North Carolina at Charlotte (677-47-3)

10:30-10:40

(91) Composition operators on B^p, the containing Banach space of H^p, $0 < p < 1$. Preliminary report
Mr. Steven J. Leon, Michigan State University (677-46-7)

10:45-10:55

(92) Continuity of homomorphisms and uniqueness of topology for F-algebras
Professor Ronn L. Carpenter, University of Houston (677-46-6)

11:00-11:10

(93) Concerning AW*-algebras. Preliminary report
Professor John Dyer, Louisiana State University (677-47-5)
(Introduced by Professor Pasquale Porcelli)

11:15-11:25

(94) CCR group extensions
Professor Irwin Schochetman* and Professor Harvey A. Smith, Oakland University, and Professor Robert C. Busby, Drexel University (677-43-1)

11:30-11:40

(95) Contractive and contractifiable semigroups
Professor Philip Meyers, Health Sciences Center, State University of New York at Stony Brook (677-46-16)

11:45-11:55

(96) Local groups of differentiable transformations
Dr. James R. Dorroh, Louisiana State University (677-58-1)

12:00-12:10

(97) Examples in the classification theory of Riemannian manifolds with respect to the equation $\Delta u = Pu$
Professor Moses Glasner, California Institute of Technology, Professor Richard Katz*, California State College at Los Angeles, and Professor Mitsuru Nakai, Nagoya University, Japan (677-58-2)

THURSDAY, 1:00 P.M.

Invited Address, Auditorium, Arts and Sciences Building

Some probability results connected with Diophantine approximation
Professor Patrick P. Billingsley, University of Chicago

THURSDAY, 2:15 P.M.

Second Session on Algebra, Room 103, Classroom Building

2:15-2:25

(98) Further results concerning Eulerian polynomials
Professor Leroy J. Derr, Louisiana State University in New Orleans (677-12-1)

2:30-2:40

(99) The number of solutions of certain equations in a finite field
Professor Robert G. Van Meter, Lawrence University (677-12-2)

2:45-2:55

(100) The structure of elementary functions. Preliminary report
Dr. Robert H. Risch, IBM Corporation, T. J. Watson Research Center, Yorktown Heights, New York (677-12-3)

3:00-3:10

(101) Valuation pairs and maximal partial homomorphisms
Mr. Patrick H. Kelly* and Professor Max D. Larsen, University of Nebraska (677-13-3)

3:15-325

(102) A generalization of the class group
Professor Elbert M. Pirtle, University of Missouri-Kansas City (677-13-1)

3:30-3:40

(103) Isomorphisms of polynomial rings
Professor Shreeram Abhyankar and Professor William J. Heinzer, Purdue University, and Mr. Paul Eakin*, University of Kentucky (677-13-2)

THURSDAY, 2:15 P.M.

Third Session on Analysis, Room 105, Classroom Building

2:15-2:25

(104) Doubly asymptotic Kuhn-Tucker conditions in mathematical programming. Preliminary report
Dr. Zlobec Sanjo, McGill University (677-90-1)
(Introduced by Professor Adi Ben-Israel)

2:30-2:40

(105) A geometrical approach to property (SAIN). Preliminary report
Professor Richard B. Holmes* and Dr. Joseph M. Lambert, Purdue University (677-41-4)

2:45-2:55

(106) On the regular behavior of orthogonal polynomials. Preliminary report
Professor Joseph L. Ullman, University of Michigan (677-41-1)

3:00-3:10

(107) Sequence spaces and interpolation problem for analytic functions
Professor Andrew K. Snyder, Lehigh University (677-40-2)

3:15-3:25

(108) Introduction to multiple asymptotic series with an application to elastic scattering
Dr. Kenneth D. Shere, U. S. Naval Ordnance Laboratory, White Oak, Silver Spring, Maryland (677-41-2)

3:30-3:40

(109) Approximations with spline functions and polyvibrating systems
Professor Demetre John Mangeron* and Professor M. N. Oguztoreli, University of Alberta (677-41-3)

THURSDAY, 2:15 P.M.

Session on Probability and Stochastic Processes, Room 117, Classroom Building

2:15-2:25

(110) Probability distributions on algebraic structures induced by groups of permutations
Professor Benon J. Trawinski, University of Alabama in Birmingham (677-60-2)

2:30-2:40

(111) Nonlinear prediction of generalized random processes
Professor George Yu-Hua Chi, University of Pittsburgh (677-60-3)

2:45-2:55

(112) Quasi-invariant, non-Gaussian random linear functionals on Hilbert space. Preliminary report
Professor Ali M. Tabatabaian-Kashani, San Francisco State College (677-60-1)

3:00-3:10

(113) Limit theorems for a class of stopping times. Preliminary report
Dr. Itrel E. Monroe, Dartmouth College (677-60-5)

3:15-3:25

(114) Ideal boundaries associated with elliptic differential operators of second order. Preliminary report
Professor Seizô Itô, University of Wisconsin (677-60-4)

THURSDAY, 2:15 P.M.

Fourth Session on Analysis, Room 119, Classroom Building

2:15-2:25

(115) Generalized Hausdorff-Young theorems. Preliminary report
Mr. Lynn R. Williams, University of Kentucky (677-42-1)

2:30-2:40

(116) Submodules of $C(X) \times \ldots \times C(X)$
Professor Ben G. Roth, University of Wyoming (677-46-12)

2:45-2:55

(117) On the geometric means of entire functions of several complex variables
Dr. Arun Kumar Agarwal, Grambling College (677-32-1)

3:00-3:10

(118) Generalized inverse and probability techniques leading to Valiron-Whittaker-type theorems and their applications in $\mathcal{C}^k$. Preliminary report
Professor J. Gopala Krishna*, University of Illinois, and Mr. I.H.N. Rao, Andhra University, India (677-32-2)

3:15-3:25

(119) The basic poweroids
Professor Anand M. Chak* and Professor Arun Kumar Agarwal, West Virginia University (677-33-1)

3:30-3:40

(120) Subgroups of rational functions of order six. Preliminary report
Dr. William M. Sanders, Madison College (677-33-2)

THURSDAY, 4:00 P.M.

Business Meeting, Auditorium, Arts and Sciences Building

FRIDAY, 9:00 A.M.

Colloquium Lecture IV, Room 302, Classroom Building

Topology of 3-manifolds
Professor R. H. Bing, University of Wisconsin

FRIDAY, 10:15 A.M.

Third Session on Algebra, Room 103, Classroom Building

10:15-10:25

(121) A class of subgroups of a finite p-group
Professor Deane E. Arganbright, Iowa State University (677-20-6)

10:30-10:40

(122) Self dual finite groups. Preliminary report
Professor Armond E. Spencer, University of Kentucky (677-20-4)

10:45-10:55

(123) Isotopy-isomorphy loops of prime order
Dr. Robert Wilson, Jr., University of Wisconsin (677-20-3)

11:00-11:10

(124) Compact totally $\mathcal{V}$ ordered semigroups
Dr. J. H. Carruth* and Dr. Charles E. Clark, University of Tennessee (677-20-1)

11:15-11:25

(125) Locally compact Clifford semigroups

Dr. James W. Stepp, University of Houston (677-20-2)

(Introduced by Professor Dennis P. Rodriguez)

11:30-11:40

(126) Character semigroups of locally compact inverse semigroups. Preliminary report

Professor Ronald O. Fulp, North Carolina State University (677-20-5)

11:45-11:55

(127) Density character of compact topological groups

Professor Gerald L. Itzkowitz, State University of New York at Buffalo (677-22-1)

12:00-12:10

(128) Extensions of topological groups

Professor Lawrence G. Brown, Stanford University (677-22-2)

FRIDAY, 10:15 A.M.

Session on General Topology, Room 105, Classroom Building

10:15-10:25

(129) WITHDRAWN.

10:30-10:40

(130) Bitopological spaces from quasi-proximities

Professor George C. Gastl, University of Wyoming (677-54-9)

10:45-10:55

(131) Perfect mappings and certain interior images of M-spaces. Preliminary report

Dr. Howard H. Wicke* and Dr. John M. Worrell, Jr., Sandia Laboratories, Albuquerque, New Mexico (677-54-12)

11:00-11:10

(132) Nonextendable classes and perfect maps

Professor Stanislaw G. Mrowka, State University of New York at Buffalo (677-54-14)

11:15-11:25

(133) Minimal convergence spaces

Professor Darrell C. Kent*, Washington State University, and Professor Gary D. Richardson, East Carolina University (677-54-18)

11:30-11:40

(134) On H-closedness and the Wallman H-closed extensions

Professor Chien Wenjen, California State College at Long Beach (677-54-6)

11:45- 11:55

(135) Paracompactness and elastic spaces

Professor Hisahiro Tamano, Texas Christian University, and Professor J. E. Vaughan*, University of North Carolina at Chapel Hill (677-54-17)

12:00-12:10

(136) Two closed cover sum theorems for the star-finite property

Professor Frank G. Slaughter, Jr., University of Pittsburgh (677-54-2)

FRIDAY, 10:15 A.M.

Session on Operator Theory, Room 117, Classroom Building

10:15-10:25

(137) Concerning the invariant subspace problem. Preliminary report

Professor John Dyer and Professor Pasquale Porcelli*, Louisiana State University (677-47-4)

10:30-10:40

(138) Spectral measures and separation of variables
Dr. David W. Fox, Johns Hopkins University (677-46-10)

10:45-10:55

(139) Spectral representation of selfadjoint extensions of a symmetric operator
Professor Richard C. Gilbert, California State College at Fullerton (677-47-2)

11:00-11:10

(140) Singular integrals and fractional powers of operators
Professor Michael J. Fisher, University of Montana (677-47-1)

11:15-11:25

(141) On two parameter singular perturbation of eigenvalues. Preliminary report
Professor Wilfred M. Greenlee, Northwestern University (677-47-8)

11:30-11:40

(142) Topological properties of paranormal operators on Hilbert space
Mr. Glenn R. Luecke, Iowa State University (677-47-9)

11:45-11:55

(143) A branching analysis of the Hartree equation
Professor Karl E. Gustafson* and Professor Duane P. Sather, University of Colorado (677-47-6)

12:00-12:10

(144) On the Fredholm alternative for asymptotically homogeneous unbounded mappings in Banach spaces
Dr. Peter Hess, University of Chicago (677-46-4)

FRIDAY, 1:00 P.M.

Invited Address, Auditorium, Arts and Sciences Building

Diffusion processes: a martingale approach
Professor Srinivasa S. R. Varadhan, Courant Institute of Mathematical Sciences, New York University

R. S. Pierce
Associate Secretary

Seattle, Washington

PRELIMINARY ANNOUNCEMENTS OF MEETINGS

The Six Hundred Seventy-Eighth Meeting
George Washington University
Washington, D. C.
October 31, 1970

The six hundred seventy-eighth meeting of the American Mathematical Society will be held at George Washington University, Washington, D. C., on Saturday, October 31, 1970.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, there will be two one-hour addresses. Professor James Ax of the State University of New York at Stony Brook will give an address entitled "On the algebraic relations among differentially-algebraic analytic functions."" Dr. Alan Baker of Cambridge University will announce his address at a later date.

Sessions for contributed papers will be held both morning and afternoon. Abstracts for contributed papers should be sent to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904, so as to arrive prior to the deadline of September 10, 1970.

Leonard Gillman
Associate Secretary

Rochester, New York

The Six Hundred Seventy-Ninth Meeting
University of Georgia
Athens, Georgia
November 20-21, 1970

The six hundred seventy-ninth meeting of the American Mathematical Society will be held at the University of Georgia at Athens, Georgia, on Friday and Saturday, November 20-21, 1970.

By invitation of the Committee to Select Hour Speakers for the Southeastern Meetings, there will be three one-hour addresses. Professor Erik Hemmingsen of Vanderbilt University will give an address entitled "Light open maps on n-manifolds." An address entitled "Representations of algebras by continuous sections" will be given by Professor Karl H. Hofmann of Tulane University, and Professor Ernest E. Shult of the University of Florida will give an address entitled "Recent results on doubly transitive groups."

There will be sessions for contributed papers both morning and afternoon. Abstracts for contributed papers should be sent to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904, so as to arrive prior to the deadline of October 6, 1970.

O. G. Harrold
Associate Secretary

Tallahassee, Florida

The Six Hundred Eighty-First Meeting
University of Illinois
Urbana, Illinois
November 28, 1970

The six hundred eighty-first meeting of the American Mathematical Society will be held at the University of Illinois, Urbana, Illinois, on Saturday, November 28, 1970. All sessions of the meeting will be held in classrooms of the university.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, there will be two one-hour addresses. Professor Richard S. Varga of Kent State University will address the Society at 11:00 a.m. His subject will be "How functional analysis and approximation theory mix today with numerical analysis." Professor O. Timothy O'Meara of the University of Notre Dame will speak at 1:45 p.m. His subject will be "Automorphisms of linear groups."

There will be sessions for the presentation of contributed ten-minute papers both morning and afternoon. Those having time preferences for the presentation of their papers should so indicate on their abstracts. Abstracts should be submitted to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904, so as to arrive prior to the deadline of October 6, 1970. There will be a session for late papers if one is needed.

There will be two special sessions of selected twenty-minute papers, each of which will probably meet both morning and afternoon. Professor Roger C. Lyndon of the University of Michigan is arranging one such session on the subject of Combinatorial Group Theory. The other special session is being arranged by Professor Walter V. Philipp of the University of Illinois on the subject of Probabilistic Methods in the Theory of Numbers. Most of the papers presented at these sessions will be by invitation. However, anyone contributing an abstract for the meeting who feels that his paper would be particularly appropriate for one of these special sessions should indicate this clearly on his abstract and submit it three weeks earlier than the above deadline, namely, by September 15, 1970, in order to allow time for the additional handling necessary.

Detailed information about travel and accommodations will appear in the October issue of the *Notices*. The Illini Union will have accommodations for almost a hundred guests.

The University of Illinois will sponsor a brief symposium on probability on Friday, November 27, 1970, the day before the meeting itself. Further details will be given in the October *Notices*.

Paul T. Bateman
Urbana, Illinois — Associate Secretary

LETTERS TO THE EDITOR

Editor, the *Notices*

One of the great continuing controversies among mathematicians concerns the requirements for the Ph. D. Recently I have conducted a survey to find out what the major schools in the United States are doing about these requirements. The response to this survey was excellent and reflects a very widespread interest in the questions asked. The purposes of this letter are to report the results of my survey and to ask for opinions about what should be required of a modern Ph. D. in mathematics.

So far in my survey I have received 75 written replies and have contacted 7 other schools by telephone. The questions were to indicate a school's testing procedure in its Ph. D. program (written and/or oral, qualifying and/or preliminary examinations) and to indicate the subjects required or optional in its program. There was a final question on whether real analysis was explicitly required for the Ph.D. while complex analysis was not. The results from the written responses are included below in the tabulation of the first two questions. Additional responses gathered by telephone are included along with the written responses in the tabulation of the final question. I wish to take this opportunity to thank every one who helped with my survey.

I was personally most interested in the final question in the survey. I feel that developing a student's ability for creative thinking should be the main object of his graduate training, but I do think there is a modest amount of mathematics which every modern Ph. D. in mathematics should know something about. Basically, these necessary topics are:

1. point set topology
2. linear algebra and matrices
3. additional algebra, such as rings, groups, fields, etc.
4. abstract (soft) analysis, such as Banach spaces, Hilbert spaces, rings of continuous functions, etc.
5. concrete (hard) analysis, such as finding a mapping function, evaluating an integral, solving a differential equation, etc.

It is my personal view that complex analysis is at least as basic and important as real analysis, and therefore I do not think that any department should suggest to its students that this is not so by explicitly requiring real analysis for the Ph. D. while not requiring complex analysis. Any reaction to this viewpoint would be appreciated in the form of letters to me and/or to the Editor of the *Notices*.

1. Check the following which are part of your testing procedure for the Ph. D.

<u>40</u> A written qualifying exam on subject matter
<u>26</u> An oral qualifying exam on subject matter
<u>10</u> An oral qualifying exam on research
<u>35</u> An oral preliminary examination
<u>29</u> A written preliminary examination

2. For each subject below, please indicate its appropriate status in your Ph. D. program

	Required	In a must select from group	Optional but pushed
Real analysis	58	9	5
Complex analysis	57	9	6
Topology	48	10	10
Algebra	58	10	5
Differential equations	0	2	28
Functional analysis	12	5	20
Numerical analysis	0	2	20
Set theory	4	2	19

3. Is real analysis explicitly required for the Ph. D. while complex ana- is not?

1 Yes 81 No

Fred M. Wright
Professor of Mathematics
Iowa State University
Ames, Iowa 50010

Editor, the *Notices*

The following is the report of donations for the late Professor H. Tamano's daughter, Akiko Tamano. (*Notices* AMS 17 (1970) No. 1, p. 74).

The foundation has received $5,826.40 in donations to which $31.02 interest has been added. Discounts for Canadian dollars and handling charges for foreign checks amounted to $15.80. $5,841.62 is currently on deposit in a Mellon Bank savings account. In addition, donations totaling 212,000 Japanese Yen have been received, out of which 10,720 Yen has been spent for business expenses in Japan (printing and postage). The remaining 201,280 Yen has been turned over to Akiko Tamano as the first of her high school scholarships.

Several more scholarships will be provided from the $5,841.62, plus interest, as Akiko's high school study progresses. The foundation will continue its operation until the date of Akiko's graduation from high school (perhaps in 1974) when the total balance of assets then will be turned over to her, and the foundation will dissolve.

We express our heartfelt thanks to all donors and to all people who helped us in various ways.

Sincerely,
Jun-iti Nagata
representing
The Foundation for Akiko Tamano

Editor, the *Notices*

The phrase "automata theory" is gaining currency. Would not "automaton theory" be preferable? After all we do not speak of "groups theory".

Arthur Sard

Editor, the *Notices*

When requests are made for reprints, I have been pleased on occasion to find enclosed with the request a pressure-sensitive label. This has been so helpful that I find myself sending a pressure-sensitive label to any correspondent from whom a reply or a reprint is required.

The label is of course addressed before it is dispatched.

J. L. Brenner

Editor, the *Notices*

Is it not possible that Professor Rice, who wrote protesting that his unfavorable review of a paper was omitted from the Mathematical Reviews, could have accomplished his objective and gained himself a friend rather than a "lifelong enemy" as he feared by slightly rewording his review? It would seem to me that it would be more informative both for the author and readers of the review if he had simply cited a suitable reference for the 'well-known results' and given a careful evaluation of how much more extensive the general knowledge of the paper's subject was supposed to be than the actual content of the paper. Most readers would have gotten the point, and the author's work would not have been entirely wasted.

It is not uncommon, at least in the literature with which I am reasonably familiar, to find that an author has labored away to obtain results which are finally published, only to find that Poincaré published much the same work in 1896, or that everybody at Stanford University knows the result, or some other embarrassing misfortune. Yet, the fact that the referee of the journal missed intercepting the article only attests the fact that the amount of literature in existence is so great, and the experience and preparation of authors so diverse, that it is possible to duplicate important bodies of research.

Yet, the author will go on living, and carrying out research; so that it would seem that reviewers such as Professor Rice have one final opportunity to orient the author in more productive channels. It is much, much more likely that a poor paper is written through ignorance than fraud, but in any event, one hopes that the careful wording of reviews could make the fact apparent in a constructive manner.

Harold V. McIntosh
National Polytechnic Institute
(Mexico)

PERSONAL ITEMS

Dr. OLIVER G. ABERTH of the City University of New York has been appointed to a professorship at Texas A & M University.

Professor RONALD ALTER of System Development Corporation has been appointed to an associate professorship at the University of Kentucky.

Professor ADI BEN-ISRAEL of Northwestern University has been appointed to a professorship at the Technion-Israel Institute of Technology, Haifa, Israel.

Dr. JOHN W. BERRY of Notre Dame University has been appointed to an assistant professorship at the University of Manitoba.

Professor G. R. BLAKLEY of the State University of New York at Buffalo has been appointed to a professorship and named head of the mathematics department at Texas A & M University.

Professor CHARLES KAM-TAI CHUI of the State University of New York at Buffalo has been appointed to an associate professorship at Texas A & M University.

Professor LINCOLN K. DURST of Claremont Men's College has been appointed Deputy Executive Director of the American Mathematical Society.

Dr. BURTON FEIN of UCLA has been appointed to an associate professorship at Oregon State University.

Dr. JAMES A. GERHARD of McMaster University has been appointed to an assistant professorship at the University of Manitoba.

Professor JAMES WALLACE GIVENS, JR. has resigned as Director of the Applied Mathematics Division at Argonne National Laboratory to resume his appointment there as Senior Scientist. He will continue his (concurrent) professorship at Northwestern University.

Professor DETLEF GROMOLL of the University of Bonn, Germany, has been appointed to a professorship at the State University of New York at Stony Brook.

Professor PAUL R. HALMOS has been given the title of distinguished professor by the Board of Trustees at Indiana University.

Dr. JOHN HARDY of the University of California, Livermore, has been appointed to an associate professorship at California State College, Bakersfield.

Dr. DAVID HERTZIG of Purdue University has been appointed to a professorship and named chairman of the department of mathematics at the University of Miami, Coral Gables, Florida.

Professor MICHAEL L. LEVITAN of Drexel University has been appointed to an assistant professorship at Villanova University.

Dr. H. B. MANN of the University of Wisconsin has been appointed to a professorship at the University of Arizona.

Professor EDWIN E. MOISE of Harvard University has been appointed to a visiting professorship at the Research Center of the National Polytechnic Institute in Mexico City for 1970-71.

Professor TAKAYUKI NÔNO of Hiroshima University has been appointed to a professorship at Fukuoka University of Education, Fukuoka, Japan.

Mr. PEDRO NOWOSAD of Porto Alegre, Brazil, has been appointed to an associate professorship at the University of Rochester.

Dr. CRAIG R. PLATT of Pennsylvania State University has been appointed to an assistant professorship at the University of Manitoba.

Professor J. BARKLEY ROSSER of the U. S. Army Mathematics Research Center at the University of Wisconsin has been awarded an honorary degree of Doctor of Science by the University of Florida.

Dr. WILFRIED SCHMID of the University of California at Berkeley has been appointed to a professorship at Columbia University.

Professor LEONARD C. SULSKI of the University of Sussex, England, has been appointed to an associate professorship at Holy Cross College.

Professor L. BRUCE TREYBIG of Tulane University has been appointed to a professorship at Texas A & M University.

Professor PETER C. WANG of the University of Iowa, visiting at Stanford University, has been appointed to a visiting associate professorship at Stanford University for the academic year of 1970-71.

Dr. KENNETH W. WESTON of Marquette University has been appointed to a visiting associate professorship at the University of Wisconsin-Parkside, Kenosha, Wisconsin.

Mr. JOSEPH J. WILLIAMS of the University of Toronto has been appointed a lecturer at the University of Manitoba.

Mr. ROBERT E. WILLIAMS of McGill University has been appointed a lecturer at the University of Manitoba.

Dr. J. D. ZUND of Virginia Polytechnic Institute has been appointed to an associate professorship at New Mexico State University.

PROMOTIONS

To Professor. University of Illinois at Chicago Circle: SHMUEL KANTOROVITZ; Lehigh University: EDWARD F. ASSMUS, JR.

To Assistant Professor. Lehigh University: GARY B. LAISON.

To Associate Professor. Lehigh University: ERIC P. SALATHE; University of Manitoba: NARAIN D. GUPTA, NORA E. LOSEY, PAPPUR N. SHIVAKUMAR, MASAMI WAKAE; Oklahoma State University: SHAIR AHMAD.

DEATHS

Dr. VERONICA HALL of the State University of New York, Downstate Medical Center died on March 9, 1970, at the age of 40. She was a member of the Society for 3 years.

Mr. GERALD M. PAVLIK of Warren, Michigan, died on January 5, 1970, at the age of 34. He was a member of the Society for 3 years.

ERRATA

Mr. PHILIP J. GREENBERG is an instructor at Kingsborough Community College (CUNY).

NEW AMS PUBLICATIONS

PROCEEDINGS OF THE STEKLOV INSTITUTE OF MATHEMATICS

Number 96

AUTOMATIC PROGRAMMING, NUMERICAL METHODS AND FUNCTIONAL ANALYSIS
Edited by V. N. Faddeeva

332 pages; List Price $22.40; Member Price $16.80

This volume of the Proceedings of the Steklov Institute of Mathematics consists mainly of studies carried out at the laboratory of approximate calculations of the Leningrad Branch of the Mathematical Institute of the Academy of Sciences of the USSR. Two studies by E. A. Volkov, whose subject lies within the scope of this volume, were carried out at the Theory of Functions section of the Institute. Six papers on automatic programming are devoted to the further development of the automatic programming system designed at the LBMI under the direction of Academician L. V. Kantorovič. The authors of these papers are L. V. Kantorovič, K. V. Šahbazjan, M. M. Lebedinskiĭ, T. N. Smirnova, and V. S. Sohranskaja. Nine of the papers dealing with numerical methods have diverse subjects, and the authors are D. K. Faddeev, V. N. Kublanovskaja, V. N. Faddeeva, M. N. Jakovlev, E. A. Volkov, L. N. Dovbyš, A. P. Kubanskaja, and L. T. Savinova. The final two papers, written by V. P. Il'in and N. K. Nikol'skiĭ, are devoted to functional analysis.

Number 103

BOUNDARY PROBLEMS FOR DIFFERENTIAL EQUATIONS. II
Edited by V. P. Mihaĭlov

224 pages; List Price $18.60; Member Price $14.00

This volume consists of a collection of papers given by participants in a mathematical physics seminar at the Steklov Institute of Mathematics in the Academy of Sciences of the USSR. The majority of the papers in this collection deal with the problem of the behavior of solutions, with increasing time, of various boundary-value problems for nonstationary partial differential equations. Also considered are certain boundary-value problems for equations of mixed type, L_p-estimates of the solution of the Cauchy problem for hyperbolic equations, and the solvability of the exterior oblique derivative problem for Helmholtz' equation. This publication should prove of value to researchers in the field of mathematical physics, both predoctoral and postdoctoral.

TRANSLATIONS—SERIES II

Volume 90

THIRTEEN PAPERS ON FUNCTIONAL ANALYSIS

258 pages; List Price $12.90; Member Price $9.64

Šilov, G. E., On some questions of analysis in Hilbert space. II; Ceĭtlin, Ja. M., Unconditional bases and semiorderedness; Nikol'skiĭ, S. M., Imbedding theorems for functions with partial derivatives considered in various metrics; Uspenskiĭ, S. V., Imbedding theorems for generalized Sobolev classes W_p^r; Barkar', M. A. and Gohberg, I. C., On factorization of operators relative to a discrete chain of projectors in Banach space; Barkar', M. A. and Gogberg, I. C., On the factorization of operators in a Banach space; Štraus, A. V., On one-parameter families of extensions of a symmetric operator; Agranovič, Z. S., An evolutionary equation and an expansion of open systems; Kreĭn, M. G., Analytic problems and results in the theory of linear operators in Hilbert space; Budjanu, M. S. and Gohberg, I.C., On multiplicative operators in Banach algebras. I. General propositions; Kuzel', O. V., The characteristic operator-function for an arbitrary bounded operator; Bahtin, I. A., On estimation of the spectrum of a class of positive linear operators; Bahtin, I. A., On continuous branches of semi-eigenvectors of nonlinear operators.

PROCEEDINGS OF SYMPOSIA IN PURE MATHEMATICS

GLOBAL ANALYSIS
Edited by S. S. Chern and Stephen Smale

Volume XIV
374 pages; List Price $13.40; Member Price $10.05

Volume XV
314 pages; List Price $11.80; Member Price $8.85

Volume XVI
256 pages; List Price $10.60; Member Price $7.95

The papers in these Proceedings grew out of lectures given at the fifteenth summer institute of the American Mathematical Society, the topic of which was Global Analysis. The institute was held at the University of California, Berkeley, in July of 1968. These volumes should provide an important start to the scientist who wishes to learn what is going on in that part of mathematics called global analysis. The first volume of the series, Volume XIV, contains papers by R. Abraham, Louis Auslander, Thomas F. Banchoff, Robert Bowen, Michael A. Buchner, R. W. Easton, John Franks, John Guckenheimer, Morris W. Hirsch, Nancy Kopell, O. E. Lanford III, Kenneth R. Meyer, Sheldon E. Newhouse, Zbigniew Nitecki, J. Palis, Julian Palmore, Charles C. Pugh, R. Clark Robinson, Michael I. Rosen, Richard Sacksteder, John Scheuneman, A. J. Schwartz, S. Shahshahani, Michael Shub, S. Smale, E. S. Thomas, Per Tomter, and R. F. Williams. Volume XV contains papers by Eugenio Calabi, J. Dowling, David G. Ebin, J. Eells, K. D. Elworthy, Robert B. Gardner, Alfred Gray, Phillip A. Griffiths, Ulrich Koschorke, H. Blaine Lawson, Jr., Jack Johnson Morava, Nicole Moulis, Hideki Omori, Robert Osserman, Richard S. Palais, Alan B. Poritz, Frank Quinn, A. J. Tromba, and Alan Weinstein. The last volume contains papers by R. Abraham, Michael F. Atiyah, Marcel Berger, Raoul Bott, Felix E. Browder, Clifford J. Earle, Halldor I. Eliasson, Robert B. Gardner, J. Glimm, Hubert Goldschmidt, Peter Greiner, V. W. Guillemin, Richard S. Hamilton, Tosio Kato, Takeshi Kotake, Masatake Kuranishi, Peter D. Lax, J. Marsden, L. Nirenberg, Bent E. Petersen, Ralph S. Phillips, I. E. Segal, Weishu Shih, D. C. Spencer, W. J. Sweeney, and K. Uhlenbeck.

ACTIVITIES OF OTHER ASSOCIATIONS

FOURTH ANNUAL SYMPOSIUM ON THE INTERFACE

The Southern California Chapter of the American Statistical Association will hold its Fourth Annual Symposium on the Interface at the University of California, Irvine, on September 17-18, 1970. The symposium is designed to bring participants together under circumstances conducive to both intensive and extensive discussion of the interplay of statistics and computer science. Workshop sessions will include statistics, hardware/software design and evaluation; computer languages for statisticians; medical statistics and computation; and computer science and statistics in secondary education. Keynote speakers will be Dr. F. J. Anscombe and Dr. R. D. Hamming. Further information may be obtained by writing to Dr. Mitchell O. Locks, C-E-I-R Professional Services Division, Control Data Corporation, 6060 W. Manchester, Los Angeles, California 90045, or Dr. Michael E. Tarter, Department of Mathematics and Department of Medicine, University of California, Irvine 92664.

MATHEMATICAL ASSOCIATION OF AMERICA

The fifty-first summer meeting of the Mathematical Association of America will be held at the University of Wyoming, Laramie, Wyoming, from Monday, August 24, to Wednesday, August 26, 1970. This meeting will be held in conjunction with meetings of the American Mathematical Society, the Institute of Mathematical Statistics, and Pi Mu Epsilon. There will be a joint session with the Institute of Mathematical Statistics on Population Ecology on Monday morning.

The nineteenth series of Earle Raymond Hedrick Lectures will be delivered by Professor Harry Kesten of Cornell University with the title "Escapades of a random walk."

At the business meeting of the Association on Tuesday, the Lester R. Ford Awards will be presented and a special honorary title conferred upon a member of the Association.

A complete program of the meeting is included in the timetable in this issue of these *Notices*.

PI MU EPSILON FRATERNITY

The Pi Mu Epsilon Fraternity will hold a banquet for members and guests in the dining room of the Washakie Center on Tuesday, August 25, at 6:30 p.m. Professor D. C. Kay of the University of Oklahoma will present a lecture entitled "How far is it from here to there?" A Dutch Treat Breakfast meeting for members and guests will be held on Wednesday, August 26, at 8:00 a.m., also in the dining room of the Washakie Center. Participants will go through the cafeteria line before convening for this meeting. The Pi Mu Epsilon Governing Council will meet on Tuesday at 12:15 p.m. in the dining room of the Washakie Center.

Sessions for contributed papers will be held on Tuesday at 3:15 p.m. and on Wednesday at 10:40 a.m. in Room 201 of the Classroom Building.

ASSOCIATION FOR COMPUTING MACHINERY

The Second ACM Symposium on Symbolic and Algebraic Manipulation will be held at the International Hotel, Los Angeles, California, on March 23-25, 1971. The Special Interest Group on Symbolic and Algebraic Manipulation (SIGSAM) is sponsoring the symposium, with the Special Interest Groups on Artificial Intelligence (SIGART) and Programming Languages (SIGPLAN) acting as co-sponsors. Other organizations are also expected to act as co-sponsors. The symposium will cover developments in the field in the past five years. A number of sessions of contributed papers are scheduled, and approximately ten invited tutorial papers will survey key subject areas. The symposium will be of interest to users of both symbolic and algebraic manipulation systems as well as to system developers.

Papers are invited on the following subjects: (1) systems for manipulating algebraic and symbolic structures; (2) algorithms for manipulating algebraic and symbolic structures; (3) software tools and techniques of use in manipulating symbolic and algebraic expressions; (4) languages for specifying symbolic and algebraic manipulation systems; (5) applications to mathematics, physics, celestial mechanics, chemistry, etc.; (6) computational group theory. Four copies of the paper and a 150-word abstract must be submitted by September 15, 1970, to the Program Committee Chairman, Joel Moses, Project MAC, Massachusetts Institute of Technology, 545 Technology Square, Cambridge, Massachusetts 02139. Authors will be notified of acceptance by November 15; accepted papers will be required in camera-ready form by December 15.

Further information on the symposium may be obtained by writing to the General Chairman, Robert Tobey, Applied Mathematics Division, Argonne National Laboratory, 9700 S. Cass Avenue, Argonne, Illinois 60439.

THE INSTITUTE OF MATHEMATICAL STATISTICS

The 126th meeting and 33rd annual meeting of the Institute of Mathematical Statistics will be held on the campus of the University of Wyoming, Laramie, Wyoming, on August 24-28, 1970. The Wald Lectures will be given by Professor Murray Rosenblatt of the University of California, San Diego. These lectures, entitled "Topics in estimation for stationary processes," will be given on Tuesday, Wednesday, and Thursday at 10:00 a.m. in Room 304 of the Classroom Building. A joint session with the Mathematical Association of America on Population Ecology will be held on Monday, August 25, in the auditorium of the Arts and Sciences Building. Professor N. G. Becker, lecturer in the Section of Ecology and Systematics, Division of Biological Sciences and the Center for Environmental Quality Management, Cornell University, will speak at 10:25 a.m. The title of his lecture will be "Interaction between species." At 11:15 a.m. a lecture entitled "Population growth models" will be presented by Professor H. M. Taylor, Environmental Systems Engineering and Operations Research, Cornell University. There will be two special invited lectures: J. B. Kruskal, Bell Telephone Laboratories, will present an address entitled "Some exciting new developments in geometrical models for use in data analysis" at 2:00 p.m., Tuesday. G.E. Schwarz, Hebrew University, will give an address entitled "Large sample sequential testing of students and other composite hypotheses" at 10:00 a.m., Friday. Both lectures will be in Room 304 of the Classroom Building. Numerous sessions for contributed papers have been scheduled at 8:15 a.m. and 4:35 p.m. on Tuesday, 8:15 a.m. on Wednesday and Thursday, and 8:15 a.m. and 4:35 p.m. on Friday. The complete program of the meeting is included in the timetable of this issue of these *Notices*.

MEMORANDA TO MEMBERS

MATHEMATICAL SCIENCES EMPLOYMENT REGISTER

The August issue of the Mathematical Sciences Employment Register is now available. The Register, sponsored by the American Mathematical Society, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics, is entering its eleventh year of service to mathematicians. In addition to the printed lists which are available in August, January, and May of each year, applicants and employers may list in any issue. There is no charge for listing. The next deadline for listing in the January issue is December 8, 1970.

A subscription to the lists, which includes both the Summary of Available Applicants and the Summary of Academic, Industrial, and Government Openings is $30. A single copy of the applicants lists, which includes the positions list, is $15, and a list of positions only is $5. All lists are mailed "Book Rate" unless a subscriber requests either "First Class" or "Air Mail" delivery. In this case, the postage fee for this special handling is charged to the subscriber. The Register also issues a List of Retired Mathematicians Available for Employment in January of each year. This list is free upon request.

The next open Register, at which interviews are scheduled for employers and applicants, will be held in conjunction with the Mathematics Meeting scheduled for Atlantic City, New Jersey, on January 21-25, 1971.

NEWS ITEMS AND ANNOUNCEMENTS

MATHEMATICAL OFFPRINT SERVICE

A novel use of the Mathematical Offprint Service has recently been brought to the attention of the MOS staff. University departments of mathematics can order a subscription to MOS in the name of the department. Each month MOS will mail to subscribing departments a list (no reprints) of current publications that cite works of the individual members of the faculty of mathematics. All that is required from each department is a list of its faculty members; the necessary details will be handled by MOS. This is an excellent way to make members of a department aware of further developments or application of the research that is being conducted within the department.

This is only one of a number of ways that MOS can be utilized. More details can be obtained by writing to the Mathematical Offprint Service, American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904.

SYMPOSIUM ON STATISTICAL DECISION THEORY AND RELATED TOPICS

The Department of Statistics of Purdue Universiy is organizing a Symposium on Statistical Decision Theory and Related Topics, November 23-25, 1970, as part of the special year in Mathematical Statistics and Probability. A program is planned for a number of invited talks by noted speakers and a limited number of contributed papers on new research in this field. The scheduling will allow ample time for formal and informal discussions. Abstracts for contributed papers should be submitted to the Symposium on Statistical Decision Theory, Department of Statistics, Purdue University, Lafayette, Indiana 47907.

SIXTH INTERNATIONAL CONGRESS ON CYBERNETICS

The International Association for Cybernetics has organized the Sixth International Congress on Cybernetics to be held in Namur, Belgium, on September 7-11, 1970. Subjects to be discussed at the Congress will be divided into the following groups: principles and methods of cybernetics, semantic machines, automation, cybernetics and human sciences, cybernetics and life. There will be general lectures by F. H. George, J. de Rosnay, L. Ectors, S. Beer, S. C. Dodd, C. Legendy, and A. David; a report by R. J. Van Egten on the proceedings of the round table conferences for the formal normalization of cybernetics; working sessions; and a plenary closing session devoted to synthesizing the work accomplished. The proceedings of the Congress will be published. Summaries of papers will be distributed to the participants at the opening of the conference, and copies may be obtained immediately by those not attending. Further information may be obtained by writing to the Secretariat of the International Association for Cybernetics, Palais des Expositions, Place A. Rijckmans, Namur, Belgium.

NATO SENIOR FELLOWSHIPS IN SCIENCE

The National Science Foundation and the Department of State have announced the awarding of 36 NATO Senior Foreign Fellowships in Science. These fellowships enable universities and nonprofit scientific research institutions to send senior staff members to research and educational institutions in other NATO nations, and the tenure is usually from one to three months. It is interesting to note that in this year of an International Congress of Mathematicians, no mathematician received a fellowship. Of the 68 applicants for these fellowships, none were mathematicians. Last year seven mathematicians received fellowships.

ASSISTANCE TO DEVELOPING COLLEGES SUMMER CONFERENCE

Under a grant from the National Science Foundation, Morgan State College, the MAA Committee on Assistance to Developing Colleges, and the University of Wyoming are sponsoring a summer conference. This conference, which will be held at the University of Wyoming on August 16-22, 1970, is concerned with mathematics at developing colleges which serve students many of whom have suffered from discrimination because of race, poverty, or geographical isolation. Its primary aim is to extend helpful relationships among mathematicians at such colleges and between them and the entire mathematical community. Fifty participants have been invited to attend. Reports of the proceedings will be prepared for distribution to participants and to others interested in the same questions.

AUDIO RECORDINGS OF MATHEMATICAL LECTURES

Several additional sets of taped lectures are now ready for distribution. No. 8 is "Vector fields and Gauss-Bonnet" by Paul F. Baum of Brown University. This is the recording of Professor Baum's invited address presented at the seventy-fourth summer meeting of the Society in Eugene, Oregon. No. 14 is "Shifts and Hilbert space factorization problems," an invited address by Marvin Rosenblum of the University of Virginia, presented at the six hundred sixty-ninth meeting in Baton Rouge, Louisiana. No. 17 is "Foliations and noncompact transformation groups" by Morris W. Hirsch of the University of California, Berkeley, and No. 18 is "Acyclicity in 3-manifolds" by Daniel R. McMillan of the Institute for Advanced Study and The University of Wisconsin. These invited addresses were presented at the seventy-sixth annual meeting in San Antonio, Texas. No. 19 is "Rigorous quantum field theory models" by James G. Glimm of the Courant institute, New York University; No. 21 is "Vector fields on manifolds" by Michael F. Atiyah of the Institute for Advanced Study; No. 22 is "Integration of complex vector fields" by Joseph J. Kohn of Princeton University. These three addresses were presented to the Society at the six hundred seventy-third meeting in New York City.

The Audio Recordings of Mathematical Lectures may be purchased for $6 with the exception of the Colloquium Lectures (Nos. 5 and 6) which are $10 for each set of two tapes. Additional copies of the manual that accompanies the tapes may be purchased for $0.30 each. Standing orders for the entire series of lectures may be placed. Orders should be sent to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904.

LEON BERNSTEIN

Dr. Leon Bernstein was one of two faculty members to receive an Excellence in Teaching Award from the Illinois Institute of Technology. The award was presented by the president of the university at the June commencement exercises.

CONFERENCE ON COMBINATORIAL GEOMETRY AND ITS APPLICATIONS

A conference on Combinatorial Geometry and its Applications (Geometria combinatoria e sue applicazioni) will take place on September 11-17, 1970, in Perugia, Italy. Further information may be obtained by writing to Professor Adriano Barlotti, Istituto di Matematica, Università degli Studi, 06100 Perugia, Italy.

ARTHUR B. COBLE MEMORIAL LECTURES

The Department of Mathematics of the University of Illinois at Champaign-Urbana is planning an annual series of public lectures by distinguished mathematicians, this series to be called the Arthur B. Coble Memorial Lectures. These lectures will be supported by the University of Illinois Foundation through a fund set up by Professor Coble's family. Arthur B. Coble (1878-1966) was professor of mathematics at the University of Illinois from 1918 to 1947; a member of the National Academy of Sciences from 1924 until his death; Colloquium Lecturer of the American Mathematical Society in 1928; and president of the Society in 1933 and 1934. His obituary appeared in the July 1970 issue of the Bulletin.

The first series of lectures, entitled "The Hilbert Problems," will be given by Professor Irving Kaplansky of the University of Chicago on September 23, 25, and

28, 1970. The title refers to the series of 23 research problems proposed by David Hilbert at the International Congress of Mathematicians held in Paris in 1900 (cf. Bull. Amer. Math. Soc., vol. 8(1901/02), pp. 437-479). Professor Kaplansky will discuss the progress made on these difficult problems during the last 70 years and the effect they have had on the development of mathematics.

MATHEMATICAL SURVEYS

The Mathematical Surveys series is published by the Society to meet the need for careful expositions of fields of current interest in research. Each book is designed to give a brief survey of the subject, an introduction to its recent developments, and a discussion of the unsolved problems. The Mathematical Surveys Editorial Committee, which consists at the present time of Edgar H. Brown, Jr., Michio Suzuki, and Bertram Yood, will welcome the opportunity to review manuscripts and to respond to inquiries from mathematicians who would like to have their work published in this series. Royalties are paid to authors at the rate of fifteen percent of list price on all sales, and books receive the widest possible distribution. Correspondence should be directed to the chairman of the committee, Professor Bertram Yood, Department of Mathematics, University of Oregon, Eugene, Oregon 97403.

3RD ANNUAL WORKSHOP IN MICROPROGRAMMING

The Association for Computing Machinery and the Institute of Electrical and Electronics Engineers are co-sponsoring the 3rd Annual Workshop in Microprogramming to be held October 12-13, 1970, in Buffalo, New York. There will be one session of submitted papers of general interest and two workshop sessions on such specific subjects as architectures, languages, and technology, followed by a panel and discussion on Microprogramming Prospects. Students are strongly encouraged to participate. Requests for invitations should indicate affiliation with SIGMICRO, IEEE, or ACM, and state areas of interest in microprogramming as well as understanding and/or involvement in the field. Requests may be submitted to Dr. Robert F. Rosin, Computer Science Department, State University of New York, 4226 Ridge Lea Road, Amherst, New York 14226.

COMMITTEE TO REVIEW SOCIETY ACTIVITIES

The Council of the Society has authorized the president to appoint a Committee to Review Society Activities; the members are Michael F. Atiyah, Morton L. Curtis, Samuel Eilenberg, Paul R. Halmos, William J. LeVeque, and Calvin C. Moore (chairman). This committee is charged "to investigate activities the Society now conducts which might be diminished or discontinued without loss to mathematics, or to determine if there are activities, not now being undertaken, which might be useful to the purposes of the American Mathematical Society." Comments from members of the Society concerning the questions raised in the above charge are welcomed by the committee and should be sent to Calvin C. Moore, Department of Mathematics, University of California, Berkeley, California 94720.

MATHEMATICAL SCIENCES PROPOSALS TO THE NATIONAL SCIENCE FOUNDATION

The Mathematical Sciences Section of the NSF has announced that for a variety of reasons, most of them obvious to all parties concerned, it would be desirable for the Mathematical Sciences Section to have committed the bulk of its funds for the Fiscal Year 1971 (July 1, 1970, to June 30, 1971) by February 1, 1971. Despite the very heavy work load involved, it is the Section's intention to do this. In order to insure full consideration, therefore, a proposal requesting support in the summer of 1971 should be in the hands of the cognizant Program Director by the autumn of 1970--certainly not later than November 1. Where this will not be possible, it would be advisable to discuss the problem with the Program Director concerned before this date.

CONFERENCE ON NUMERICAL RANGES

A research conference on Numerical Ranges of Linear Operators on Normed

Space will be held on July 5-10, 1971, at the University of Aberdeen, Scotland. The conference is being sponsored by the Edinburgh and London Mathematical Societies and the North British Functional Analysis Seminar. Professors F. L. Bauer, F. F. Bonsall, and G. Lumer are acting as advisers. The main part of the conference will consist of invited lectures; short talks and discussions will also be arranged. Further information may be obtained by writing to the organizing secretary, Dr. J. Duncan, Department of Mathematics, King's College, Aberdeen AB9 2UB, Scotland.

SYMPOSIUM ON NONLINEAR FUNCTIONAL ANALYSIS

The U. S. Army Mathematics Research Center has announced that a symposium on Nonlinear Functional Analysis will be held at the University of Wisconsin, Madison, Wisconsin, on April 12-14, 1971. This symposium will consist of three days of invited addresses on recent research on nonlinear functional analysis and applications, with particular emphasis on results obtained subsequent to the AMS symposium held in Chicago in April 1968. The following subjects will be discussed in the six sessions: Schauder-Leray and bifurcation theories; monotone, contractive, compact operators; convex functions, duality; spectral analysis; integral equations; equation of evolution; partial differential equations. For further information please write to Mrs. Gladys G. Moran, Symposium Secretary, Mathematics Research Center, The University of Wisconsin, Madison, Wisconsin 53706.

CONGRESS OF THE INTERNATIONAL FEDERATION FOR INFORMATION PROCESSING

The IFIP Congress 71 will be held in Ljubljana, Yugoslavia, August 23-28, 1971. Past congresses, which have been held in Paris, Munich, New York, and Edinburgh, have become the major international media for exchange of information among developers and users of information processing techniques and technology.

Travel grants will be available for some U. S. participants in a program administered by the National Academy of Sciences--National Research Council. Requests for applications can be addressed to the Division of Mathematical Sciences, National Research Council, 2101 Constitution Avenue, N.W., Washington, D. C. 20418.

Professor Herbert Freeman, chairman of the U. S. Committee for IFIP Congress 71, has announced a "Hot Line" committee to respond directly to inquiries. Richard V. Welch, UNIVAC, Division of Sperry Rand Corp., P. O. Box 8100, Philadelphia, Pennsylvania 19101, 215-MI 6-9000 (Exhibits); Eugene Grabbe, TRW, Inc., One Space Park, Redondo Beach, California 90278, 213-679-8711, Ext. 63337 (Technical Program); Arthur E. Hutt, Bowery Savings Bank, 110 East 42nd Street, New York City 10017, 212-697-1414 (Travel); Charles V. Freiman, IBM Corporation, Thomas J. Watson Research Center, Yorktown Heights, New York 10598, 914-945-1953 (Registration and Accommodations). Requests for information in specific areas may be obtained from these committee members, or queries may be addressed to U. S. Committee for IFIP Congress 71, Box 4197, Grand Central Post Office, New York, New York 10017.

CATALOG OF LECTURE NOTES

At the present time, NEW PUBLICATIONS, which is issued quarterly, carries a short catalog of lecture notes. Unfortunately, the list is not sufficiently complete, and, therefore, a systematic effort is being made to acquire information on more notes. The importance of expanding the list, making it as extensive as possible, cannot be overemphasized. Many such notes contain important summaries of recent research results, making them of great value to the mathematical community. Individuals and libraries have requested information on notes which have not been available to the AMS, and, as a result, the Committee to Monitor Problems in Communication has become interested in having a more complete list made available to mathematicians. European subscribers and correspondents are particularly urged to supply information.

The specific information needed is title, author, date, price, postage, number of pages, and address where notes may be obtained. If price and postage are to be prepaid, please so indicate. As the list is restricted to lecture notes (published and unpublished), please do not include preprints or theses. Send this information to NEW PUBLICATIONS, American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904.

SUBSCRIPTIONS TO THE MATHEMATICAL OFFPRINT SERVICE FROM FELLOWSHIP AWARDS

The National Science Foundation Division of Graduate Education in Science has been consulted on the possibility of supporting subscriptions to the Mathematical Offprint Service from fellowship awards. A number of NSF fellowship awards include in the financial terms a "special allowance" which is from $150 to $400, depending on the type of fellowship. This allowance is designated for special fellowship costs such as fees, research expenses, professional travel; and a subscription to MOS is an entirely appropriate use of these funds. NSF Graduate Fellowships and Traineeships and the NDEA Fellowships carry no such special allowance; however, a MOS subscription can be purchased out of the university's Cost-of-Education allowance. Grants are made each year to institutions under the NSF program of Institutional Grants for Science. The amount of these grants is based by formula on the total research awards to the institution for prior years. The Institutional Grants and the Cost-of-Education allowance are made to the institution which sets the policy for their expenditure.

PROFESSIONAL TRAINING IN MATHEMATICS

PROFESSIONAL TRAINING IN MATHEMATICS by F. A. Ficken and the late C. C. MacDuffee originally appeared in the *Notices*, volume 7, number 7, part 2, December 1960, pages 4-21. The revision, which is now ready for distribution, was written by Professor Ficken and includes paragraphs on computer science and a list of references which were not in the original article. Included also is the Selected List of Available Scholarships and Stipends in Mathematics which appeared in the December 1969 issue of the *Notices*.

This booklet is of inestimable value to both undergraduate and graduate students of mathematics. The authors discuss such subjects as the opportunities open to trained mathematicians, the need for graduate study, the qualifications and preparation for graduate study, choosing a graduate school, and being a graduate student. The subject matter is presented with great clarity from the viewpoint of two scholars of mathematics who have devoted their professional lives to both research and directing the studies of young mathematicians.

There is no charge for the book, except for a $0.25 fee for postage and handling. Requests should be sent to the American Mathematical Society, P.O.Box 6248, Providence, Rhode Island 02904, and should be accompanied by $0.25.

VISITING MATHEMATICIANS

The list of visiting mathematicians includes both foreign mathematicians visiting in the United States and Canada, and Americans visiting abroad. Note that there are two separate lists.

American and Canadian Mathematicians Visiting Abroad

Name and Home Country	Host Institution	Field of Special Interest	Period of Visit
Al-Salam, W. A. (Canada)	American University of Beirut	Classical Analysis	10/70 - 6/71
Auslander, Maurice (U.S.A.)	University of London, England	Algebra	2/71 - 6/71
Brown, Edward M. (U.S.A.)	University of Warwick, England	Topology	9/70 - 5/71
Chakravarti, I. M. (Resident, U.S.A.)	University of Geneva, Switzerland	Coding and Information Theory	9/70 - 8/71
Cheng, Hung (U.S.A.)	Desy (Laboratory) Germany	Applied Mathematics	8/70 - 7/71
Ciment, Melvyn (U.S.A.)	Tel Aviv University	Numerical Analysis	8/70 - 8/71
Conover, William Jay (U.S.A.)	University of Zürich	Nonparametric Statistics	8/70 - 8/71
Crowe, Donald (U.S.A.)	University College, England	Geometry	9/70 - 1/71
Day, Mahlon (U.S.A.)	University of Edinburgh	Functional Analysis	9/70 - 6/71
Doksum, Kjell A. (U.S.A.)	University of Oslo	Statistics: Statistical Testing Procedures	8/70 - 7/71
Dubinsky, Ed (U.S.A.)	Polish Academy of Sciences		9/70 - 6/71
Fusaro, B. A. (U.S.A.)	National Taiwan Normal University, Taipei	Partial differential equations	9/71 - 6/71
Friedman, Avner (U.S.A.)	Tel Aviv University	Partial Differential Equations	9/70 - 6/71
Gray, John (U.S.A.)	E.T.H., Switzerland	Category Theory	9/70 - 6/71
Hall, Marshall, Jr. (U.S.A.)	Trinity College, England	Group Theory and Combinatorics	1/71 - 6/71
Helgason, Sigurdur (U.S.A.)	Mittag Leffler Institute	Differential Geometry	9/70 - 6/71
Hellerstein, Simon (U.S.A.)	Imperial College, London University, England and Hebrew University of Jerusalem, Israel	Analysis	9/70 - 3/71 4/71 - 8/71
Hocking, J. G. (U.S.A.)	University of London, England	Topology	9/70 - 8/71
Ionescu Tulcea, Alexandra (U.S.A.)	Universities of Erlangen and Göttingen	Functional Analysis	4/70 - 6/71
Jogdeo, Kumar (U.S.A.)	Indian Statistical Institute	Statistics	9/70 - 1/71
Kahn, Daniel (U.S.A.)	Cambridge University, England	Topology	9/70 - 12/70
Kellogg, R. Bruce (U.S.A.)	Chalmers Institute of Technology, Sweden	Numerical Analysis	9/70 - 6/71

Name and Home Country	Host Institution	Field of Special Interest	Period of Visit
Kumpel, Paul Jr., (U.S.A.)	University of Hull, England	Topology	2/71 - 8/71
Levine, Arnold (U.S.A.)	World Health Organization, Switzerland	Bio-Statistics	6/70 - 8/71
Maskit, Bernard (U.S.A.)	University of Warwick, England	Analysis	9/70 - 8/71
McCoy, T. L. (U.S.A.)	University of Glasgow, Scotland	Analysis	9/70 - 8/71
McCulloh, Leon (U.S.A.)	Queen Mary College, London	Algebraic Number Theory	9/70 - 6/71
Moon, J. W. (Canada)	University of Cape Town, South Africa and University of Oxford, England	Graph Theory	7/70 - 1/71 1/71 - 8/71
Moran, D. A. (U.S.A.)	Cambridge University, England	Topology	9/70 - 8/71
Mumford, David (U.S.A.)	University of Warwick, England	Algebraic Geometry	9/70 - 6/70
Nagata, Jun-Iti (U.S.A.)	Tokyo University of Education	General Topology	1/71 - 8/71
Newhouse, Sheldon (U.S.A.)	Instituto de Matematica Pura e Aplicada, Brazil	Global Analysis	9/70 - 6/71
Pierce, Donald A. (U.S.A.)	University of Kent, England	Statistics	10/70 - 6/71
Robinson, R. Clark (U.S.A.)	Instituto de Matematica Pura e Aplicada, Brazil	Global Analysis	9/70 - 6/71
Rossi, Hugo (U.S.A.)	Université de Grenoble, France	Complex Analysis	9/70 - 6/71
Rotman, Joseph (U.S.A.)	Hebrew University of Jerusalem, Israel	Abelian Groups	9/70 - 1/71
Saari, Donald G. (U.S.A.)	Eidgenossische Technische Hochschule, Zürich	Celestial Mechanics	9/70 - 12/70
Steen, Lynn Arthur (U.S.A.)	Mittag-Leffler Institute	Analysis	9/70 - 5/71
Tewarson, R. P. (U.S.A.)	University of Oxford, England	Numerical Analysis	9/70 - 5/71
Thierrin, Gabriel (Canada)	Université de Paris	Automata and Formal Systems	9/70 - 8/71
Tsai, C. (U.S.A.)	University of Munich, F.R. of Germany	Algebra	9/70 - 8/71
Weichsel, Paul M. (U.S.A.)	Hebrew University of Jerusalem, Israel	Varieties of Groups	9/70 - 6/71
Williams, R. F. (U.S.A.)	Instituto de Matematica, Pura e Aplicada, Brazil	Global Analysis	4/70 - 6/71
Wong, P. K. (U.S.A.)	Tôhoku University, Japan	Differential Equations	9/70 - 8/71
Zelinsky, Daniel (U.S.A.)	Hebrew University of Jerusalem, Israel	Algebra	9/70 - 6/71

Foreign Mathematicians Visiting in the United States and Canada

Name and Home Country	Host Institution	Field of Special Interest	Period of Visit
Akagawa, Yasumasa (Japan)	Institute for Advanced Study	Algebraic Number Theory	10/70 - 4/71
Amann, Herbert (F.R. of Germany)	Indiana University	Numerical Analysis	9/70 - 6/71
Anh, Nguyen H. (South Viet-nam)	Institute for Advanced Study	Group Representation Theory	10/70 - 4/71
Aronsson, A. Gunnar (Sweden)	University of Michigan	Calculus of Variations and Theory of Optimal Control	9/70 - 5/71
Arsac, Jacques (France)	Université de Montréal	Programming Systems	9/70 - 8/71
Atkin, Oliver (England)	University of Arizona	Number Theory	9/70 - 6/71
Aubert, Karl E. (Norway)	Tufts University	Algebra	9/70 - 6/71
Aubin, Jean-Pierre (France)	University of Wisconsin	Functional Analysis and Partial Differential Equations	6/70 - 9/70
Baker, Alan (England)	Institute for Advanced Study	Number Theory	10/70 - 12/70
Balachandran, V. K. (India)	Institute for Advanced Study	Topology and Algebra	10/70 - 4/71
Bandelow, Christoph (F.R. of (Germany)	Florida State University	Probability Theory	9/70 - 6/71
Bandle, Catherine (Switzer-land)	Carnegie-Mellon Univer-sity	Analysis	9/70 - 8/71
Barsotti, Iacopo (Italy)	Yale University	Algebraic Groups and Algebraic Geometry	9/70 - 6/71
Beck, Istvan (Norway)	University of Illinois	Ring Theory	9/70 - 6/71
Behboodian, Javad (Iran)	University of North Carolina	Mixtures and Distributions	9/70 - 6/71
Bender, Helmut (F.R. of Germany)	University of Illinois at Chicago Circle	Group Theory	9/70 - 6/71
Bibel, Wolfgang (F. R. of Germany)	Wayne State University	Mathematical Logic, Com-puters	9/70 - 6/71
Björk, Jan-Erik (Sweden)	University of California, Los Angeles	Functional Analysis	9/71 - 6/71
Bohl, Erich (F.R. of Germany)	University of Calgary	Approximation Theory	7/70 - 4/71
Borosh, Itshack (Israel)	University of Illinois	Diophantine Approximation	9/70 - 6/71
Bosanquet, L. S. (England)	University of Western Ontario	Summability	5/70 - 10/70
Broman, Arne (Sweden)	Western Washington State College	Fourier Series	6/70 - 6/71
Buchwald, V. T. (Australia)	University of British Columbia	Solid Mechanics, Ocean Waves	1/71 - 6/71
Burghelea, Dan (Rumania)	Institute for Advanced Study	Algebraic and Differential Topology	10/70 - 4/71
Clancey, Kevin (Canada)	University of California, Los Angeles	Functional Analysis	9/70 - 6/71
Clark, Ronald S. (England)	University of Calgary	Differential Geometry	7/70 - 6/71
Cohn, P. M. (England)	Tulane University	Ring Theory	3/71 - 4/71

Name and Home Country	Host Institution	Field of Special Interest	Period of Visit
Corbett, J. V. (Australia)	University of Toronto	Mathematical Physics	8/70 - 5/71
Csiszár, Imre (Hungary)	Catholic University of America	Probability and Mathematical Statistics	9/70 - 12/70
Davie, A. M. (Scotland)	University of California, Los Angeles	Functional Analysis	9/70 - 6/71
Delange, Hubert (France)	University of Illinois	Analysis	9/70 - 1/71
Dolan, Patrick (England)	Skidmore College	Relativity	9/70 - 6/71
Dowker, C. H. (England)	University of Alberta	Topology	7/70 - 6/71
Dowker, Yael (England)	University of Alberta	Ergodic Theory	7/70 - 6/71
Dupont, Johan I. (Denmark)	Institute for Advanced Study	Topology	10/70 - 4/71
Ehrenfeucht, Andrzej (Poland)	University of Southern California	Logic, Automata Theory	9/70 - 6/71
Ejike, Uwadiegwu (Biafra)	University of Illinois	Applied Mathematics	9/70 - 6/71
Endo, Shizu (Japan)	McMaster University	Algebras Over Commutative Rings and Hopf Algebras	9/70 - 12/70
Epstein, David B. A. (United Kingdom)	University of Minnesota	Lie Groups	9/70 - 12/70
Erdös, Paul (Hungary)	Universities of Waterloo and Calgary	Combinatorics and Number Theory	9/70 -
Erle, Dieter (F.R. of Germany)	University of Massachusetts	Differential and Piecewise Linear Topology	9/70 - 6/71
Fischer, Gerd (F.R. of Germany)	University of Minnesota	Several Complex Variables	9/70 - 3/71
Fischer, Pal (Hungary)	University of Guelph	Applied Analysis	9/70 - 8/71
Fishel, B. (England)	Michigan State University	Analysis	9/70 - 8/71
Follmer, Hans (F.R. of Germany)	Dartmouth College	Probability	7/70 - 6/71
Fontanella, F. (Italy)	University of Alberta	Approximation Theory and Numerical Analysis	7/70 - 6/71
Fröhlich, Albrecht (England)	University of Arizona	Algebra, Number Theory	1/71 - 6/71
Frolik, Zdoenek (Czechoslovakia)	University of Pittsburgh	Measure Theory, Topology	5/70 - 12/70
Fukushima, Masotoshi (Japan)	University of Illinois	Probability	9/70 - 6/71
Gaier, Dieter (F.R. of Germany)	California Institute of Technology	Complex Analysis	10/70 - 6/71
Gamkrelidze, Revaz (USSR)	University of California, Los Angeles	Control Theory	1/71 - 3/71
Ghinelli, Dina (Italy)	University of North Carolina	Combinatorial Problems of Experimental Design, Information Theory	9/70 - 6/71
Gitler, Sam (Mexico)	Brandeis University	Topology	9/70 - 1/71
Goldie, Alfred (England)	Tulane University	Ring Theory	2/71 - 4/71
Grant, J. A. (United Kingdom)	University of Toronto	Numerical Analysis	8/70 - 5/71
Grunenfelder, Luzius (Switzerland)	Northwestern University	Algebra	9/70 - 6/71

Name and Home Country	Host Institution	Field of Special Interest	Period of Visit
Guichardet, A. (France)	Tulane University	Ring Theory	4/71
Haag, Rudolf (F.R. of Germany)	University of California, Los Angeles	Functional Analysis	3/71 - 6/71
Hajnal, Andras (Hungary)	University of Calgary	Set Theory	9/70 - 7/71
Harada, Koichiro (Japan)	University of Illinois	Group Theory	9/70 - 6/71
Haris, Stephen J. (Australia)	Institute for Advanced Study	Number Theory	10/70 - 4/71
Hasenjaeger, Gisbert (F.R. of Germany)	University of Illinois	Logic	9/70 - 8/71
Hasse, Helmut (F.R. of Germany)	San Diego State College	Algebra	9/70 - 6/71
Hawkes, T. O. (England)	University of British Columbia	Group Theory	8/70 - 7/71
Heilmann, Ole J. (Denmark)	Massachusetts Institute of Technology	Applied Mathematics	8/70 - 8/71
Henstock, Ralph (England)	University of British Columbia	Integration Theory	8/70 - 7/71
Heyer, H. (F.R. of Germany)	Tulane University	Representation Theory	2/71 - 3/71
Hoffmann-Jørgensen, Jørgen (Denmark)	Cornell University	Probability and Statistics	9/70 - 6/71
Hooley, Christopher (Wales)	Institute for Advanced Study	Number Theory	10/70 - 4/71
Hörmander, Lars (Sweden)	Institute for Advanced Study	Partial Differential Equations, Analysis	1/71 - 4/71
Huber, Peter J. (Switzerland)	Princeton University	Mathematical Statistics	9/70 - 8/71
Hudimoto, Hisao (Japan)	University of North Carolina	Statistical Inference	9/70 - 12/70
Huml, M. (Israel)	University of Toronto	Mathematical Physics	7/70 - 6/71
Ihara, Shin-ichiro (Japan)	SUNY, Stony Brook	Algebraic Geometry	9/70 - 6/71
Ihara, Yasutaka (Japan)	Stanford University	Algebra	9/70 - 3/71
Illusie, Luc (France)	Massachusetts Institute of Technology	Algebraic Geometry	9/70 - 2/71
Ivkovic, Zoran (Yugoslavia)	University of Missouri-Rolla	Stochastic Processes	9/69 - 6/71
Jategaonkar, Arun V. (India)	Rutgers University	Ring Theory	9/70 - 6/71
Johnson, Barry E. (England)	Yale University	Linear Operators and Operator Algebras	9/70 - 6/71
Johnson, David L. (England)	University of Illinois	Representation Theory	9/70 - 6/71
Jones, Antony J. (England)	Institute for Advanced Study	Number Theory	10/70 - 4/71
Kastler, Daniel (France)	University of California, Los Angeles	Functional Analysis	3/71 - 6/71
Katznelson, Yitzak (Israel)	Stanford University	Harmonic and Functional Analysis	9/70 - 8/71
Kawakubo, Katsuo (Japan)	Institute for Advanced Study	Topology	10/70 - 4/71

Name and Home Country	Host Institution	Field of Special Interest	Period of Visit
Klein, Abraham A. (Israel)	Yale University	Algebra	8/70 - 6/71
Koh, Kwangil (Korea)	Tulane University	Ring Theory	9/70 - 2/71
Kondo, Takeshi (Japan)	Institute for Advanced Study	Finite Groups	10/70 - 4/71
Krabs, W. (F.R. of Germany)	Michigan State University	Approximation Theory	1/71 - 8/71
Kulldorff, Gunnar (Sweden)	Purdue University	Statistics	11/70 - 6/71
Kurotschka, Viktor (F.R. of Germany)	University of Michigan	Mathematical Statistics	9/70 - 8/71
Lalitha, Ramanathan (India)	McNeese State College	Functional Analysis	9/69 - 9/71
Laursen, Kjeld B. (Denmark)	University of California, Los Angeles	Functional Analysis	9/70 - 6/71
Lelek, Andrew (Poland)	University of Houston	Topology	9/70 - 5/71
Lindgren, Georg (Sweden)	University of North Carolina	Stationary Normal Processes	9/70 - 6/71
Lindley, Dennis V. (England)	Oregon State University	Bayesian Inference	7/70
Linnik, Yu. V. (Russia)	University of California, Berkeley and University of California, Los Angeles	Statistics and Number Theory	1/71 - 3/71 3/71 - 6/71
van Lint, J. H. (The Netherlands)	California Institute of Technology	Combinatorial Analysis and Number Theory	9/70 - 3/71
Lusztig, Gheorghe (Rumania)	Institute for Advanced Study	Algebraic and Differential Topology	10/70 - 4/71
Malliavin, Paul (France)	Yeshiva University	Harmonic Analysis	9/70 - 12/70
Martineau, Robert P. (England)	University of Illinois at Chicago Circle	Group Theory	9/70 - 6/71
Maumary, Serge (Switzerland)	Institute for Advanced Study	Algebraic and Differential Topology	10/70 - 4/71
McKay, John (England)	California Institute of Technology	Group Theory	10/70 - 9/71
McLeod, J. Bryce (England)	University of Wisconsin	Partial Differential Equations	7/70 - 8/71
Medeiros, Luiz Adauto (Brazil)	Brown University	Partial Differential Equations	1/71 - 3/71
Michael, Ian (Scotland)	McMaster University	Partial Differential Operators	9/69 - 8/70
Michler, Gerhard (Germany)	Tulane University	Ring Theory	4/71 - 6/71
Miyake, Toshitsune (Japan)	Institute for Advanced Study	Number Theory	10/70 - 4/71
Molenaar, Wouter (The Netherlands)	Pennsylvania State University	Statistics	9/70 - 6/71
Mordell, L. J. (England)	University of Calgary	Number Theory	9/70 - 12/70
Munkholm, Hans J. (Denmark)	University of Illinois at Chicago Circle	Algebraic Topology	9/70 - 6/71

Name and Home Country	Host Institution	Field of Special Interest	Period of Visit
Narasimhan, Mudumbai S. (India)	University of California, Los Angeles	Functional Analysis	3/71 - 6/71
Nath, Prem (India)	McMaster University	Information Theory	5/70 - 12/70
Newman, Michael F. (Australia)	University of Illinois	Group Theory	9/70 - 12/70
Nickel, Karl (F.R. of Germany)	University of Wisconsin	Numerical Analysis	3/70 - 3/71
Okuyama, Akihiro (Japan)	University of Pittsburgh	General Topology	8/70 - 7/71
Olivier, Reinhard M. (F.R. of Germany)	Institute for Advanced Study	Topology, Differential Geometry	10/70 - 4/71
Oodaira, Hiroshi (Japan)	University of Minnesota	Probability	9/70 - 6/71
Ossa, Erich (F.R. of Germany)	Institute for Advanced Study	Equivariant Cobordism	10/70 - 4/71
Owen, Roger (England)	University of Michigan	Statistics	9/70 - 8/71
Pedersen, Gert K. (Denmark)	University of California, Los Angeles	Functional Analysis	3/71 - 6/71
Pełczynski, Aleksander (Poland)	University of California, Los Angeles	Functional Analysis	1/71 - 3/71
Peletier, V. (United Kingdom)	University of Minnesota	Applied Mathematics	9/70 - 6/71
Pillow, A. F. (Australia)	University of Toronto	Fluid Mechanics	1/71 - 6/71
Raghavan, T.E.S. (India)	University of Illinois at Chicago Circle	Statistics and Game Theory	9/70 - 6/71
Ramachandra, K. (India)	Institute for Advanced Study	Riemann-Zeta Functions	10/70 - 4/71
Rao, Bhamidi V. (India)	University of California, Berkeley	Statistics	9/70 - 8/71
Rathie, P. N. (India)	McGill University	Special Functions, Statistical Distributions and Information Theory	5/70 - 12/70
Révész, Paul (Hungary)	Indiana University and Catholic University of America	Probability and Mathematical Statistics	1/71 - 6/71
Richter, Michael (F.R. of Germany)	University of Texas at Austin	Logic and Foundations	9/69 - 8/71
Riemenschneider, O. (F.R. of Germany)	Institute for Advanced Study	Analytic Functions of Several Complex Variables	10/70 - 4/71
Ringrose, John R. (England)	Tulane University	Ring Theory	4/71 - 5/71
Ritter, Klaus G. (F.R. of Germany)	University of Wisconsin	Nonlinear Programming, Spline Functions and Approximation Theory	9/69 - 9/72
Robert, Alain (Switzerland)	Institute for Advanced Study	Automorphic Functions	10/70 - 4/71
Rosay, J. P. (France)	University of Kentucky	Functional Analysis	8/70 - 6/71
Rourke, Colin (England)	University of Wisconsin	Topology	9/70 - 6/71
Saito, Teishiro (Japan)	Tulane University	Ring Theory	9/70 - 5/71
Samuelson, Åke (Sweden)	Western Washington State College	Potential Theory	9/70 - 8/71

Name and Home Country	Host Institution	Field of Special Interest	Period of Visit
Sasakura, Nobuo (Japan)	Institute for Advanced Study	Moduli Theory of Algebraic Varieties	10/70 - 4/71
Schmidt, Bernd (F.R. of Germany)	California Institute of Technology	Geometric Operator Theory	9/70 - 5/71
Schumacher, Dietmar (F.R. of Germany)	McMaster University	Algebra, Category Theory	8/69 - 6/71
Serre, Jean-Pierre (France)	Institute for Advanced Study	Algebraic Geometry	10/70 - 12/70
Sewell, Michael J. (England)	University of Wisconsin	Nonlinear Elasticity	9/70 - 9/71
Seymour, R. M. (England)	University of Alberta	Algebraic Topology	9/70 - 8/71
Sharp, Rodney (Scotland)	Institute for Advanced Study	Commutative Algebra	10/70 - 4/71
Shih, Wei-shu (France)	University of Illinois	Algebraic Topology	9/70 - 6/71
Siefkes, H. (F.R. of Germany)	Purdue University	Automata Theory	9/70 - 6/71
Silberstein, J.P.O. (Australia)	University of Toronto	Analysis, Partial Differential Equations	9/70 - 5/71
Smythe, Neville F. (Australia)	Dartmouth College	Topology	9/70 - 6/71
Spence, Edward (Scotland)	University of Illinois	Number Theory	6/70 - 6/71
Srebro, U. (Israel)	University of Minnesota	Complex Analysis	3/71 - 6/71
Storer, Ray (Australia)	San Jose State College	Applied Mathematics	9/70 - 6/71
Storrer, Hans (Switzerland)	Tulane University	Ring Theory	9/70 - 5/71
Suita, Nobuyuki (Japan)	Washington University	Theory of Functions	9/70 - 6/71
Sukhatme, P. V. (India)	University of California, Berkeley	Populations and the Food Question	10/70 - 12/70
Suryanarayana, D. (India)	University of Alberta	Number Theory	9/70 - 8/71
Suwa, Tatsuo (Japan)	Institute for Advanced Study	Complex Analytic Geometry	10/70 - 4/71
Suzuki, Satoshi (Japan)	Florida State University	Algebra	6/70 - 9/70
Swetharanyam, Sundaram (India)	McNeese State College	Computer Science	9/68 - 9/70
Swinnerton-Dyer, H.P.F. (United Kingdom)	Harvard University	Number Theory	2/71 - 6/71
Szwarc, W. (Poland)	Carnegie-Mellon University		9/70 - 6/71
Takesaki, Masamichi (Japan)	Tulane University	Ring Theory	10/70 - 12/70
Tall, David O. (England)	Institute for Advanced Study	K-Theory	10/70 - 12/71
Tanaka, Hisao (Japan)	University of Illinois	Recursive Functions	9/70 - 6/71
Tijdeman, Robert (The Netherlands	Institute for Advanced Study	Number Theory	10/70 - 4/71
Trudinger, N. (Australia)	University of Minnesota	Partial Differential Equations	9/70 - 3/71
Uebe, Götz (F.R. of Germany)	University of Wisconsin	Integer Programming, Mathematical Economics	10/70 - 10/71
Verdier, Jean-Louis (France)	Brandeis University	Algebra	9/70 - 6/71
Viswanath, Kasturi (India)	University of Illinois	Functional Analysis	9/70 - 6/71
Vorel, Zdenek (Czechoslovakia)	University of Southern California	Applied Mathematics	9/70 - 6/71

Name and Home Country	Host Institution	Field of Special Interest	Period of Visit
Walker, Grant (England)	University of Virginia	Homotopy Theory	9/70 - 9/71
Watanabe, Hisao (Japan)	University of North Carolina	Stochastic Processes	9/70 - 6/71
Wirsing, Eduard (F.R. of Germany)	Institute for Advanced Study	Number Theory	10/70 - 4/71
Wojczynski, Wojbor (Poland)	Carnegie-Mellon University	Functional Analysis	9/70 - 8/71
Wong, James C.S. (Hong Kong)	McMaster University	Functional Analysis	9/69 - 6/70
Wong, Pak-Ken (China)	McMaster University	Banach Algebras and C*-Algebras	9/70 - 8/71
Wong, Yung-Chow (Hong Kong)	University of Hawaii	Differential Geometry	11/70 - 12/70
Yasuhara, Mitsuru (Japan)	Institute for Advanced Study	Mathematical Logic	10/70 - 4/71
Zama, Nobuo (Japan)	University of Illinois	Automata Theory	9/70 - 6/71
Zarantonello, Eduardo H. (Argentina)	University of Wisconsin	Hydrodynamics and Functional Analysis	1/71 - 6/71
Zieschang, Heiner (F.R. of Germany)	University of Michigan	Topology	9/70 - 5/71

BACKLOG FOR MATHEMATICAL RESEARCH JOURNALS

Information on the backlog of papers for research journals is published in the February and August issues of these *Notices* with the cooperation of the respective editorial boards. Since all columns in the table are not self-explanatory, we include further details on their meaning.

Column 3. This is an estimate of the number of printed pages which have been accepted but are not necessary to maintain copy editing and printing schedules.

Column 5. The first (Q_1) and third (Q_3) quartiles are presented to give a measure of normal dispersion. They do not include misleading extremes, the result of unusual circumstances arising in part from the refereeing system. The observations are made from the latest issue of each journal received at the Headquarters Offices before the deadline for the appropriate issue of these *Notices*. Waiting times are measured in months from receipt of manuscript in final form to receipt of final publication at the Headquarters Offices. When a paper is revised, the waiting time between an editor's receipt of the final revision and its publication may be much shorter than is the case otherwise, so these figures are low to that extent.

	1	2	3		4	5		
JOURNAL	No. issues per year	Approx. No. pages per year	BACKLOG 6/30/69	12/31/68	Est. time for paper submitted currently to be published (in months)	Observed waiting time in latest published issue (in months) Q_1	Med.	Q_2
American J. of Math.	4	NR*	NR*	959		17	19	21
American Statistician	5	300	0	---	6	**	**	**
Annals of Math. Stat.	NR*	NR*	NR*	375	NR*	9	12	15
Annals of Math.	6	1200	NR*	1250	12	10	11	12
Canad. J. of Math.	6	1500	NR*	972	9-10	16	19	20
Comm. in Math. Physics	17	1500	1200	---	6	4	5	6
Comm. on Pure & Appl. Math	6	800	250	---	8-10	8	9	9
Duke Math. J.	4	800-840	1040	1252	18-21	24	25	26
Illinois J. of Math.	4	700	1700	1700	24-30	26	27	27
J. Amer. Stat. Assoc	4	2000	500	---	12	**	**	**
J. Assoc. for Comp. Mach.	4	700	40	---	12	7	9	11
J. of Diff. Geometry	4	530	600	---	12-14	20	22	25
J. Math. Anal. & Appl.	NR*	NR*	NR*	NR*	NR*	**	**	**
J. Math. Physics	12	2500	950	NR*	6	7	8	10
J. Math and Mech.	12	1200	1000	1000	NR*	12	12	13
J. Symbolic Logic	NR*	NR*	NR*	0	NR*	12	14	16
Linear Algebra and Appl.	4	500	6	---	9	11	15	20
Math. Biosciences	12	1350	0	---	6	**	**	**
Math. of Comp.	4	1000	0	0	8	10	11	13
Michigan Math. J.	4	400	200	180	12	12	15	17
Operations Research	6	1100	200	---	14	15	19	20
Pacific J. Math.	12	3300	900	1100	8	11	12	15
Proceedings of AMS	12	2500	200	50	9	8	10	12
Proc. Nat'l. Acad. Sci.	12	5000#	0	---	2	3	4	4
Quarterly of Appl. Math.	4	560	560	600	18	20	22	24
SIAM J. of Appl. Math.	NR*	NR*	NR*	0	NR*	9	11	13
SIAM J. on Control	NR*	NR*	NR*	0	NR*	8	9	14
SIAM J. on Numer. Anal.	NR*	NR*	NR*	0	NR*	11	12	15
SIAM Review	NR*	NR*	NR*	0	NR*	8	9	15
Transactions of AMS	12	5000	700	200	11	11	13	19

*NR means that no response was received to a request for information.
**Dates of manuscripts not indicated in this journal.
#For all services.
---This journal is new to this compilation. Figures re backlog as of 12/31/69 are not available.

ABSTRACTS OF CONTRIBUTED PAPERS

The June Meeting in Tacoma, Washington
June 20, 1970

676-17. ROGER B. EGGLETON, University of Melbourne, Victoria, Australia and RICHARD K. GUY, University of Calgary, Calgary, Alberta, Canada. The crossing number of the n-cube. Preliminary report.

The crossing number of the $\underline{n}$-cube is shown to be at most $5.4^n/32 - [(n^2 + 1)/2]2^{n-2}$. It is conjectured to be equal to this number (for $\underline{n} \geq 3$). Equality is established for $\underline{n} = 4$ and all nonisomorphic optimal drawings are obtained. Lower bounds are given for $\underline{n} > 4$. (Received May 4, 1970.)

676-18. DEMETRE JOHN MANGERON, M. N. OGUZTORELI, University of Alberta, Edmonton 7, Alberta, Canada and V. F. POTERAŞU, Polytechnic Institute of Jassy, Iasi, Romania. On the optimal problems in distributed parameter control systems.

The authors, starting from their previous results published in Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur. give a new demonstration of the transversality condition concerning the following optimal problem in distributed parameter control systems. Let $\dot{x} = f(x,u,w)$ be the considered system, where $x(t) = [x_1(t), x_2(t),\ldots,x_m(t)]$ is the vector of the state variables, $u(t) = [u_1(t),u_2(t),\ldots,u_r(t)]$ the control vector, $w = (w_1,w_2,\ldots,w_n) \in W$, w_k = constant, $k = 1,2,\ldots,n$, the parameter vector, $f = (f_1,f_2,\ldots,f_m)$ a given vector function, and $J = \int_{T_0=g_0(w)}^{T_1=g_1(w)} f_0(x,u,w)dt$ the performance criterion. Using the Pontryagin's maximum principle as well as some suitable transformations, one gets in a natural way the corresponding condition for optimal parameters. Various interesting examples, for instance, the optimization of a linear autonomous system depending on two parameters are subsequently discussed. (Received May 5, 1970.)

The Summer Meeting in Laramie, Wyoming
August 25-28, 1970

02 Logic and Foundations

677-02-1. ROBERT G. PHILLIPS, University of South Carolina, Columbia, South Carolina 29208. Addition in nonstandard models of arithmetic. Preliminary report.

Let $F = F(\alpha)$ denote a vector space of dimension α over the rationals Q. Let h be any mapping defined on F so that for all a and b in F, $h(a) + h(b) - h(a + b)$ is always an integer. Then for each such h we let F_hZ, Z the ring of integers, denote the group whose elements are $F \times Z$ and whose group operation is defined as $(a,x) + (b,y) = (a + b, x + y + h(a) + h(b) - h(a + b))$. In a previous paper we proved that for each nonstandard model $*Z$ of Z of cardinal α, there was an $h: F \to *Z$ such that F_hZ was the additive group of $*Z$, Z being embedded in F_hZ by the map $n \to (0,n)$. Also, we showed that if $h: F \to Z$ then F_hZ is never the additive group of a $*Z$. Here we prove that the additive group of every $*Z$ is a F_hZ for certain $h: F \to Q$. This is accomplished by embedding $*Z$ in $*Q$, $*Q$ a nonstandard model of Q of cardinality α, and then noting that the additive group of $*Q$ is the direct sum of F and Q. (Received June 19, 1970.)

05 Combinatorics

677-05-1. RAYMOND KILLGROVE and FRANK HINER, California State College, Los Angeles, California 90032. Subsquare complete Latin square. Preliminary report.

A Latin square is said to be subsquare complete iff every pair of distinct like marks is contained in a proper Latin subsquare. Such (abbr. S.C.) squares arise in the study of digraph complete representations of finite nonsingly generated projective planes (see Canad. J. Math. 16(1964), 70). Theorem 1. In any Cayley table for finite group, the smallest Latin subsquare containing any pair of distinct like marks is isomorphic to a cyclic group table. Theorem 2. An S. C. square all of whose proper subsquares are of order 2 is isomorphic to the Cayley table for some elementary Abelian group of order a power of 2. (See also, Keedwell, Math. Comp. 19(1965), 317, to show nongroup square with all proper subsquares of order 3.) Theorem 3. For every integer m greater than 2 there exists an S. C. square of (2 exp m) - 1 each of whose proper subsquares is of order 2 or order 3. Theorem 4. For every positive integer m, there exists an S. C. square of order (3 exp m) + 1, each of whose subsquares is of order 2 or 3. Furthermore, a process called semitensor product also generates S. C. squares, e.g. the only S. C. square of order 6, the Cayley table for the symmetric group on 3 letters, is a semitensor product of squares of orders 2 and 3. (Received June 22, 1970.)

677-05-2. TERRENCE J. BROWN, University of Missouri, Kansas City, Missouri 64110. Three theorems on geometric lattices.

Let J be a finitary closure relation with the exchange property on a set S, and let $S[J]$ be the family of J closed subsets of S (cf. Abstract 672-510 these *Notices* 17(1970), 230). Theorem 1. Suppose that for each $i \in I$ there exists $F_i \in S[J]$ of finite rank. If $\mathcal{B}$ is a collection of finite subsets of I with the property $B_1, B_2 \in \mathcal{B}$ imply that there exists $B_3 \in \mathcal{B}$ with $\Pi_{i \in B_1} J_{F_i}, \Pi_{i \in B_2} J_{F_i} \leq \Pi_{i \in B_3} J_{F_i}$, then $\sigma(\mathcal{B})$ defined by $\sigma(\mathcal{B})(G) = \bigcup_{B\in\mathcal{B}}(\Pi_{i\in B}J_{F_i}(G))$ for $G \in P(S)$ is a finitary closure relation with the exchange property on S. Let $\ell \geq 0$. Put $\Delta_\ell = \{\mathcal{D} \mid \mathcal{D} \subseteq S[J]$ and $\lambda(F) = \ell + 1$ for all $F\in\mathcal{D}\}$. For $\mathcal{D}\in\Delta_\ell$ put $\mathcal{D}^* = \{B \mid B \subseteq \mathcal{D}, |B| < \infty$, and $\Pi_{F\in B}J_F$ is a Q product$\}$. Theorem 2. If $\mathcal{D} \in \Delta_\ell$ then $\omega(\mathcal{D})$ defined by $\omega(\mathcal{D})(G) = \bigcup_{B\in\mathcal{D}^*}(\Pi_{F\in B}J_F(G))$ for $G \in P(S)$ is a finitary closure relation with the exchange property on S. Theorem 3. $(\{\omega(\mathcal{D}) \mid \mathcal{D} \in \Delta_\ell\}, \leq)$ is a geometric lattice (of possibly infinite dimension). (Received June 29, 1970.)

677-05-3. JOHN L. LEONARD, University of Arizona, Tucson, Arizona 85711. On Bollabás' number $k_r(n)$. Preliminary report.

The number $k_r(n)$ is defined as the smallest integer having the property that any graph (with neither loops nor multiple edges) having n vertices and $k_r(n)$ or more edges must contain at least two vertices joined by r independent paths. Known values are: $k_2(n) = n$, $k_3(2n) = 3n - 1$, $k_3(2n + 1) = 3n + 1$, and $k_4(n) = 2n - 1$. We establish the inequalities $k_5(n) \leq 3n - 5$, $k_5(2n) \geq 5n - 2$, $k_5(2n + 1) \geq 5n + 1$, and further show that given any integer r, for n sufficiently large, $k_5(n) > 5n/2 + r$. (Received June 29, 1970.)

06 Order, Lattices, Ordered Algebraic Structures

677-06-1. JORGE MARTINEZ, University of Florida, Gainesville, Florida 32601. The vector lattice cover of an abelian lattice-ordered group.

The essential content of the so-called "Hahn-embedding theorem for abelian ℓ-groups" is that every abelian ℓ-group can be embedded as an ℓ-subgroup of a vector lattice. The main object of this paper is to obtain a unique minimal embedding of a given abelian ℓ-group into a vector lattice. The construction of this minimal vector lattice cover is categorical: for each ℓ-group G one needs a vector lattice V(G) and an ℓ-homomorphism $\mu_G: G \to V(G)$ (which turns out to be 1 - 1); and whenever $\varphi: G \to W$ is an ℓ-homomorphism into the vector lattice W there is a unique lattice preserving linear transformation $\varphi^*: V(G) \to W$ such that $\mu_G\varphi^* = \varphi$. Actually, one must introduce the auxiliary notion of a partial vector lattice to insure that if W is a vector lattice, then V(W) = W. The existence and uniqueness of this cover is easy to prove. What is nontrivial and really the point of the whole construction is whether an ℓ-embedding $\varphi: G \to W$ always induces an ℓ-embedding φ^*. This is true if and only if V(G) is an essential extension of $G\mu_G$; that is, every nonzero ℓ-ideal of V(G) has a nontrivial meet with $G\mu_G$. It is shown that this is indeed the case for a large class of ℓ-groups which contains all the finite valued ℓ-groups. (Received May 14, 1970.)

677-06-2. WITOLD M. BOGDANOWICZ, Catholic University of America, Washington, D. C. 20017. Lattice characterization of spaces of functions measurable with respect to a sigma ring.

Let R be the space of reals. We do not include in it the infinities. Let X be any abstract space and F the space of all functions from X into R. A set $L \subset F$ is called a linear lattice if it is closed in F under addition and scalar multiplication and under the lattice operations defined by $(f \cup g)(x) = \sup\{f(x), g(x)\}$, $(f \cap g)(x) = \inf\{f(x), g(x)\}$ for all $x \in X$. The prefix S in front of the notion linear lattice will mean that it is closed under the Stone operation $f \to f \cap c$, where $c(x) = 1$ for all $x \in X$.

The prefix P will mean that it is closed under pointwise convergence everywhere. By the trace of a linear lattice L we mean the family of all sets $A \subset X$ the characteristic functions c_A of which belong to L. It will be denoted by tr L. By a sigma ring we mean as usual a ring of sets in X closed under countable unions. If V is a family of sets in X denote by V(X) the family of all functions $f \in F$ such that $f^{-1}(I) \in V$ for every interval I not containing the point zero. Theorem. There is one-to-one correspondence between SP-linear lattices in F and sigma rings of sets in X. Namely, the map $L \to \text{tr } L$ maps an SP-linear lattice into a sigma ring. The inverse map is given by the formula $V \to V(X)$. (See Bogdanowicz, "Theory of a class of locally convex vector lattices which include the Lebesgue spaces," Proc. Nat. Acad. Sci. U. S. A. (1970).) (Received June 12, 1970.)

677-06-3. STEPHEN D. COMER, Vanderbilt University, Nashville, Tennessee 37303. Sectional representations over Boolean spaces.

A sheaf of universal algebras (X,S) is called reduced if it is trivial or the following conditions hold: (i) X is a Boolean space; (ii) each stalk is nontrivial; (iii) the factor relations of $\Gamma(X,S)$ form a sublattice of $\Theta(\Gamma(X,S))$ which is isomorphic to the BA of all clopen subsets of X in a natural way. Theorem. Suppose K is an equational class of algebras such that every member A of K satisfies (I) the set $\Theta_0(A)$ of all factor relations of A form a sublattice of ΘA which is a BA and (II) the congruence relation generated by any proper BA ideal of $\Theta_0(A)$ is proper. Then, for every member A of K, there exist a unique (up to isomorphism) reduced sheaf (X(A), S(A)) of algebras in K such that $A \cong \Gamma(X(A), S(A))$. This theorem applies, in particular, to any variety of rings with unit, cylindric algebras, or lattices with 0,1. (Received June 25, 1970.)

677-06-4. THOMAS G. NEWMAN, Texas Technological University, Lubbock, Texas 79406. The descending chain condition in modular lattices.

It is shown that a modular lattice L satisfies the descending chain condition provided that the set of join-irreducibles in L satisfies the descending chain condition and each element of L is a join of finitely many join-irreducibles. This extends a similar result due to L. G. Kovács (J. Austral. Math. Soc. 10(1969), 1-4) which has the additional hypothesis of join-continuity on L. (Received June 29, 1970.)

10 Number Theory

677-10-1. LEON BERNSTEIN, Illinois Institute of Technology, Chicago, Illinois 60616. Explicit solutions of pyramidial Diophantine equations.

Let $\binom{x}{n,d} = x(x+d)(x+2d)\ldots(x+(n-1)d)$, (x,d,n rational integers, $n > 1$). In this paper Diophantine equations of the form $\binom{x}{n,d} + \binom{y}{n,d} = \binom{z}{n,d}$ are being investigated for the cases $n = 2,3,4$. For $n = 2$ and any $d \neq 0$ all solutions are stated explicitly by means of three parameters. For $n = 3$ two infinite solution classes are stated explicitly by means of two parameters, the first for any d, the second for a class of d. For the case $n = 4$ solubility criteria are stated. Methods of solution mainly lead to a Pellian equation $x^2 - Dy^2 = N$, N a rational integer. For $n > 2$ and $d = 0$ this is Fermat's conjecture. (Received May 21, 1970.)

677-10-2. GERALD E. BERGUM, Gonzaga University, Spokane, Washington 99202. The distribution of kth power residues and nonresidues in the Euclidean domain $Z(\sqrt{-2})$.

For a prime ρ in $(\sqrt{-2})$ and integer k such that $k|(N(\rho) - 1)$, an integer α in $Z(\sqrt{-2})$ is a kth power residue, or a kth power nonresidue depending on whether $\xi^2 \equiv \alpha (\mathrm{mod}\ \rho)$ is solvable or unsolvable in $Z(\sqrt{-2})$. A bound $B(\rho, k)$ is established so that there is a kth power nonresidue β such that $|\beta| < B(\rho, k)$ for ρ whose norm is sufficiently large. The function $B(\rho, 2) = N(\rho)^{a+\epsilon}$, for all $\epsilon > 0$ and $a = (8e)^{-1}$ is established. The function $B(\rho, k) = N(\rho)^{a+\epsilon}$ for all $\epsilon > 0$ and where $1/4a$ is the unique solution of $\Gamma(u) = 1/k$ where Γ is the Dickman-De Bruijn function is established. (Received June 1, 1970.)

677-10-3. GRAHAM F. LORD, Temple University, Philadelphia, Pennsylvania 19122. Oscillation of an error function associated with the k-free integers. Preliminary report.

Let $Q_k(x)$ denote the symmetrized summatory function of the k-free integers and let $E_k(x) = Q_k(x) - x/\zeta(k)$. A. M. Vaidya (J. Indian Math. Soc. 32(1968), 105-111) has shown that on the Riemann Hypothesis for each $\epsilon > 0$, $E_k(x) = \Omega_{+,-}(x^{1/2k-\epsilon})$. However, using a theorem of Grosswald on Dirichlet integrals ("On some generalisations of theorems of Landau and Polya," Israel J. Math. 3(1965), 211-220) the following result is proven: there exists a constant B such that for all nonnegative constants $C < B$ each of the two inequalities $E_k(x) \gtrless \pm Cx^{\theta/k}$ holds on an infinite set of real numbers tending to infinity, where $\theta = \sup\{\sigma | \zeta(\sigma + it) = 0\}$. (Received June 15, 1970.)

677-10-4. J. M. GANDHI, Western Illinois University, Macomb, Illinois 61455. Rational approximation to $\sqrt[3]{2}$ and other algebraic numbers.

Improving upon a theorem of Alan Baker [Quart. J. Math. Oxford Ser. (2) 15(1964), 375-383] we prove: Theorem. Suppose that m,n are integers such that $n \geq 1$ and $1 \leq m < n$. Let a,b be positive integers for which $(5/6)a \leq b < a$ and suppose that $a - b$ is divisible by n. Also let b be such that $n \not\equiv m \pmod b$ and that $\lambda = 4b(a-b)^{-2}\mu_n^{-1} > 1$ where for each positive integer n, μ_n is given by $\mu_n = \pi_{p/n;\ p\ \mathrm{prime}}[1/(p-1)]$ so that $1 \leq \mu_n \leq n$. Then $\alpha = (a/b)^{m/n}$ satisfies the equation $|\alpha - p/q| > c_1[\ln(2q/\lambda\mu_n)]/q^k$ where $c_1 = (1 - \sqrt{\pi}/5)c\mu_n^{k-2}\sqrt{\pi}/\sqrt{2(\ln\lambda)}$ for all p,q $(q > 0)$ where k and c are given by $\lambda^{k-1} = 2\mu_n(a+b)$, $c^{-1} = 2^{k+2}(a+b)$. From this theorem we obtain a better approximation for $\sqrt[3]{2}$, from which we also prove that all solutions in integers x,y of the equation $x^3 - 2y^3 = n$, $n \leq 10$, satisfy $|x| < M$, $|y| < M$, where $M = (2 \cdot 10^4|n|)^{16}$. Earlier Baker proved that for all n, $M = (3 \cdot 10^5|n|)^{23}$. (Received June 29, 1970.)

12 Algebraic Number Theory, Field Theory and Polynomials

677-12-1. LEROY J. DERR, Louisiana State University, New Orleans, Louisiana 70122. Further results concerning Eulerian polynomials.

The Eulerian polynomials $P_m(y)$, are intimately associated with the matrices G_m defined by, $g_{j,k} = 1/(k-j)! - m!/k!(m-j)!$, $j = 0,1,\ldots,m-1$, $k = 0,1,\ldots,m-1$, m a positive integer. Some results of this paper are (1) $yP_m(y)$ is the characteristic polynomial of G_m. (2) A generating function for the powers of G_m can be given in terms of Eulerian polynomials. (3) An interesting congruence relation arises, $P_m(y)P_{k-j}(y)/m!(k-j)! - P_k(y)P_{(m-j)}(y)/k!(m-j)!$ is divisible by $(1-y)^{k-j+1}$, $m \geq k \geq j \geq 0$. (Received 26, 1970.)

677-12-2. ROBERT G.VANMETER, Lawrence University, Canton, New York 13617. The number of solutions of certain equations in a finite field.

Let K be a finite field with q elements and let Z (Z^+) be the set of integers (positive integers). Let $f_1 =_{df} \Pi_{j=1}^n X_j^{k_j}$, where $k_j \in Z - \{0\}$ for all $j \in \{1,\ldots,n\}$ and $(k_1,\ldots,k_n) = 1$. Let $f_2 =_{df} \Sigma_{i=1}^t a_i \cdot \Pi_{j=1}^{n_i} X_{ij}^{k_{ij}}$, where $t \in Z^+$, $a_i \in K$, $n_i \in Z^+$, $k_{ij} \in Z - \{0\}$, $(k_{i1},\ldots,k_{in_i}) = 1$ for all $i \in \{1,\ldots,t\}$, $j \in \{1,\ldots,n_i\}$ and $n =_{df} \Sigma_{i=1}^t n_i$. Suppose $\underset{\sim}{x}$ $(=_{df}(x_1,\ldots,x_n))$ is a variable with domain K^n and $\underset{\sim}{y}$ $(=_{df}(y_1,\ldots,y_n))$ is a variable with domain K^m. L. Carlitz (Proc. Nat. Acad. Sci. U. S. A. 38(1952), 515-519) gave expressions for (1) $N[f_1(\underset{\sim}{x}) = g(\underset{\sim}{y})]$ $(=_{df} \#\{(c_1,\ldots,c_n,d_1,\ldots,d_m) \in K^{n+m}: f_1(c_1,\ldots,c_n) = g(d_1,\ldots,d_m)\})$ and (2) $N[f_2(\underset{\sim}{x}) = g(\underset{\sim}{y})]$ in terms of $N[g(\underset{\sim}{y}) = 0]$ and $\Sigma_{a \in K-\{0\}} N[g(\underset{\sim}{y}) = a]$ for any polynomial g in $Y_1,\ldots,Y_m$ over K. He also gave explicit expressions for (1) and (2) for various specific polynomials g. We generalize these results to the equation $f(\underset{\sim}{x}) = g(\underset{\sim}{y})$, where f is a $\varkappa$-polynomial in $X_1,\ldots,X_n$ over K ($=_{df}$ f is a polynomial in $X_1,\ldots,X_n$ over K and there exists $(A,B) \in Z^2$ such that $B \neq 0$ and $N[f(\underset{\sim}{x}) = a] = A + \varkappa(a)B$ for all $a \in K$, where $\varkappa(0) =_{df} q - 1$ and $\varkappa(a) =_{df} -1$ for all $a \in K - \{0\}$). The polynomials f_1 and f_2 are $\varkappa$-polynomials. (Received June 29, 1970.)

677-12-3. ROBERT H. RISCH, IBM Corporation, T. J. Watson Research Center, Yorktown Heights, New York 10598. The structure of elementary functions. Preliminary report.

Let $\langle \mathcal{B}' \rangle$ be a differential field of characteristic 0 with constant field K. $\mathcal{U}$ is a universal extension of $\mathcal{B}$ with constant field C. $\mathcal{B} = K(z,\theta_1,\ldots,\theta_n)$ where $z' = 1$ and each θ_i is either algebraic over $K(z,\theta_1,\ldots,\theta_{i-1})$ or else satisfies $\theta' = \theta f'$ or $f' = f\theta'$ for $f \in K(z,\theta_1,\ldots,\theta_{i-1})$, abbreviated $\theta = e^f$, $\theta = \log f$, respectively. Let $\{e^{\zeta_1},\ldots,e^{\zeta_r}, \log \eta_1,\ldots,\log \eta_s\}$ be the set of algebraically independent quantities among the θ_i. Then for $f \in \mathcal{B}$, e^f, $\log f$ is algebraic of $C \cdot \mathcal{B}$ iff $\exists$ integers $r_1,\ldots,r_r$, $s_1,\ldots,s_s$ such that f', respectively, $f'/f = \Sigma_1^r r_i \zeta_i' + \Sigma_1^s s_i(\eta_i'/\eta_i)$. Furthermore, if K is finitely generated, the r_i's and s_i's can be found in a finite number of steps. A typical consequence is that when one is restricted to real numbers, the functions of elementary calculus (exponential, log, tangent and inverse tangent) are irredundant, i.e., none can be expressed in terms of the others using only algebraic operations. The theorem enables one to determine the structure of the group of elementary functions with elementary inverse and to give a simple proof of Ritt's theorem that if the integral of an elementary function satisfies an elementary equation $F(z,w) = 0$, then the integral must be itself elementary. This last result is the only known way of proving the elliptic functions to be nonelementary. (Received June 29, 1970.)

13 Commutative Rings and Algebras

677-13-1. ELBERT M. PIRTLE, University of Missouri, Kansas City, Missouri 64110. A generalization of the class group.

Let R be an integral domain with quotient field L and let F be a family of valuations on L such that (1) $R = \cap\{R_v \mid v \in F\}$; (2) $R_v = R_{P(v)}$, $v \in F$; (3) Each $v \in F$ has rank one. When R is a Krull domain, it is well known that $C(R) \cong \mathcal{O}(R[X])$, where C(R) denotes the class group of R. We extend this theorem to all integral domains satisfying (1), (2), (3) above. This is done by constructing a semigroup S(R) which has C(R) as a homomorphic image. (Received May 22, 1970.)

677-13-2. SHREERAM ABHYANKAR and WILLIAM J. HEINZER, Purdue University, Lafayette, Indiana 47907 and PAUL EAKIN, University of Kentucky, Lexington, Kentucky 40506. Isomorphisms of polynomial rings.

Theorem. Let A be a 1-dimensional affine domain over an arbitrary ground field and let $x_1,\ldots,x_n$ be indeterminates over A. If $y_1,\ldots,y_n$ are indeterminates over a ring B and $\varphi: A[x_1,\ldots,x_n] \to B[y_1,\ldots,y_n]$ is an isomorphism, then A and B are isomorphic. Moreover, if A is not of the form $k[t]$, k a field, then $\varphi(A) = B$. (Received June 17, 1970.)

677-13-3. PATRICK H. KELLY and MAX D. LARSEN, University of Nebraska, Lincoln, Nebraska 68508. Valuation pairs and maximal partial homomorphisms.

Manis has developed a valuation theory on commutative rings with unity by generalizing three characterizations of valuation domains and showing their equivalence. His results did not extend the characterization of valuation domains as the domains of maximal partial homomorphisms. In this paper it is shown that Manis' theory also generalizes this aspect of valuation theory. In addition, a partial answer is given to a question concerning places raised by Harrison. Then it is shown that overrings of valuation rings are not necessarily valuation rings in any nice sense. However, a proper overring of a valuation ring V is always contained in a valuation ring which is a large quotient ring of V, and which is not the total quotient ring of V. It is then shown that if W is a large quotient overring of a valuation ring V, then there is a valuation ring containing W and differing from W at most by zero divisors. (Received June 30, 1970.)

14 Algebraic Geometry

677-14-1. STANLEY E. PAYNE, Miami University, Oxford, Ohio 45056. A geometric representation of some generalized hexagons.

Theorem. Let q be the point of $PG(3,s) = G$ with homogeneous coordinates $q = (c,0,0,0)$, $c \in F = GF(s)$, and let H be the plane $H = \{(x_0,x_1,x_2,0) \mid x_i \in F\}$. Then we construct a generalized hexagon P isomorphic to the one of Tits corresponding to a triality of type I_{id}. The points of P are of two types: points of type (i) are all the lines of G; points of type (ii) are all the flat pencils of lines which are centered in planes different from H and not containing q. The ranges of P are of two kinds. Let p be a point on any line L of H. Then all those points of P of type (i) consisting of lines in the pencil containing L and qp form the range of one line of P. The other kind of range consists of one point L_1 of type (i) not in H and not containing q, and certain of the points of type (ii) which contain L_1. Specifically, let $x = L \cap H$, z a point of L_1 different from x, and y a point of H not in the plane containing q,z, and x. For q,y,z normalized in a certain manner, the range in question contains in addition to L_1 those points L(t) of P of type (ii) containing (in G) L_1 and the line (of G) spanned by $y + tq$ and $z - tx$, $t \in F$. (Received February 2, 1970.)

16 Associative Rings and Algebras

677-16-1. DAVID W. BALLEW, South Dakota School of Mines and Technology, Rapid City, South Dakota 57701. A generalization of a theorem of Fröhlich.

Suppose A is a c.d.v.r. with q.f.K. Let R be a f.d. division K-algebra. Let Γ be a maximal A-order in R and let Λ be any A-order contained in Γ. For f.d. A-lattices M and N, let $[M:N]$ be the module index. Theorem. If M is a left Λ-module such that $M \otimes_A K \cong R^{(m)}$, then the following are equivalent. (i) $[\Gamma M:M] = [\Gamma:\Lambda]^m$. (ii) M is Λ-projective. (iii) M is Λ-free. This theorem is used to give a corresponding theorem for f.d. separable K-algebras. This is done by developing a Morita type correspondence between the orders in the separable K-algebra and the orders in the division algebra associated with it. (Received May 1, 1970.)

677-16-2. KLAUS E. ELDRIDGE, Ohio University, Athens, Ohio 45701. On normal subgroups of modular group algebras. Preliminary report.

For a fixed prime p let F be a field with p elements and let G be a locally finite p-group. Denote the group algebra of G over F by R and denote the group of units of R by *R. Theorem. Any subgroup H of G is normal in *R iff H is central in G. Outline of proof. Pick h in H and g in G, both different from 1. Using Theorem 2.3 of Losey [Michigan Math. J. 7 (1960), 237-240] observe that 1 - g - h is a unit in R. Thus for some h' in H, (1 - g - h)h = h'(1 - g - h). Now, using the fact that G is a basis of R over F, do a term by term comparison in this equation to obtain the desired result. The converse is obvious. (Received June 17, 1970.)

677-16-3. S. K. JAIN, Ohio University, Athens, Ohio 45701. Prime rings having one-sided ideal with polynomial identity coincide with special Johnson rings.

Throughout R is a prime ring and is regarded as an algebra over its centroid. Let R^{Δ} $({}^{\Delta}R)$ denote the right (left) singular ideal of R. R is called Johnson ring if it satisfies any of the two equivalent conditions: (a) $R^{\Delta} = 0 = {}^{\Delta}R$ and R possesses uniform right and left ideals. (b) The quotient ring of R (in the sense of Utumi) is $\mathrm{Hom}_D(V,V)$ where V is a vector space over a division ring D. In addition if D is finite dimensional over its center then R is called a special Johnson ring. Denote by C the center of Utumi's right quotient ring of R. The results shown are: (1) R is a special Johnson ring iff there exists a nonzero one-sided ideal with polynomial identity (PI). (2) R has generalised polynomial identity (GPI) nontrivial over C iff each nonzero right (left) ideal of R contains a nonzero right (left) ideal with PI. (3) If R has GPI over C then R cannot have nonzero nil one-sided ideals. (4) If R is integral domain then R has GPI over C iff R has PI. (5) R is a special Johnson ring with nonzero socle iff each non-nil right (left) contains an idempotent ($\neq 0$) and there exists a nonzero one-sided ideal with PI. (Received May 11, 1970.)

677-16-4. GUENTER KRAUSE, University of Manitoba, Winnipeg 19, Manitoba, Canada. On the Krull-dimension of left noetherian left Matlis-rings.

An associative ring R with identity is called a left Matlis-ring if for every injective indecomposable left R-module E there is a prime ideal P of R such that $E \cong E_R(R/P)$, the injective hull of the left R-module R/P. Let $\Gamma(R)$ denote the set of all pairs (L,M) of left ideals L and M od R with $M \subseteq L$

and define inductively $\Gamma_0(R) = \{(L,M) \in \Gamma(R);\ L/M \text{ artinian}\}$ and $\Gamma_\alpha(R) = \{(L,M) \in \Gamma(R);\ L \supseteq L_1 \supseteq \ldots \supseteq L_i \supseteq L_{i+1} \supseteq \ldots \supseteq M$ implies $(L_i, L_{i+1}) \in \bigcup_{\beta<\alpha} \Gamma_\beta(R)$ for almost all $i\}$ for ordinals $\alpha > 0$. If there is an ordinal α such that $\Gamma_\alpha(R) = \Gamma(R)$ then the smallest such ordinal is called the left Krull-dimension $\mathcal{L}$.K-dim(R) of R. Let spec(R) denote the set of all prime ideals of R and define inductively $\mathcal{P}_0(R)$ = set of all maximal ideals of R and $\mathcal{P}_\alpha(R) = \{P \in \text{spec}(R)$ such that $Q \in \bigcup_{\beta<\alpha} \mathcal{P}_\beta(R)$ for every prime ideal Q containing P properly$\}$ for ordinals $\alpha > 0$. If there is an ordinal α with spec(R) = $\mathcal{P}_\alpha(R)$ then the smallest such ordinal is called the classical Krull-dimension cl.K-dim(R) of R.

Theorem. cl.K-dim(R) = $\mathcal{L}$.K-dim(R) for every left noetherian left Matlis-ring R. (Received June 29, 1970.)

17 Nonassociative Rings and Algebras

677-17-1. WILLIAM E. JENNER, University of North Carolina, Chapel Hill, North Carolina 27514. Isotopes of truncation algebras.

Let A be an associative algebra over a field K, let H be a subalgebra and B a supplementary subspace. Then B is made into an algebra (nonassociative) by defining the product to be given by the projection on B of the product in A. Such an algebra is called a truncation algebra. In an earlier paper, Abstract 672-50, these *Notices* 17(1970), 98, examples were given of new simple nonassociative algebras obtained in this way. It is shown here by explicit constructions that these algebras are in general not isotopically simple in the sense of Albert. (Received April 7, 1970.)

677-17-2. DARYL KREILING, Western Illinois University, Macomb, Illinois 61455 and TERRY L. JENKINS, University of Wyoming, Laramie, Wyoming 82070. Semisimple classes and upper-type radical classes of narings.

Yu-Lee Lee (Proc. Amer. Math. Soc. 19(1968), 1165-1166) has shown that in the universal class of associative rings every class M determines an upper radical class. It is pointed out that Lee's proof is valid in a universal class of alternative rings but that a class M may not determine a unique upper radical class in a universal class of narings (not necessarily associative rings). This leads to the definition of an upper-type radical class determined by a class of narings. It is shown that every class M determines an (not necessarily unique) upper-type radical class in a universal class of narings. The example presented also shows that in a universal class of narings, the intersection of semisimple classes need not be a semisimple class. (Received April 9, 1970.)

677-17-3. DAVID M. CLARK, State University of New York, New Paltz, New York 12561. Word problem for nonassociative rings with one defining relation. Preliminary report.

An effective solution to the word problem for finitely presented nonassociative linear algebras is given by I. A. Zukov in Mat. Sb. Unfortunately, there does not appear to be any simple modification of Zukov's method which applies to finitely presented nonassociative rings (as modules over the integers). We give an effective solution to the word problem for nonassociative rings given by a finite number of generators and one defining relation, leaving the general problem still open. (Received June 24, 1970.)

18 Category Theory, Homological Algebra

677-18-1. IRWIN S. PRESSMAN, Ohio State University, Columbus, Ohio 43210. K_0 and K_1 of the category of short exact sequences. Preliminary report.

Let $\mathcal{E}_n = \mathcal{E}_n(A,B)$ denote the category whose objects are exact sequences $0 \to A \to X_1 \to \dots \to X_n \to B \to 0$ of (left) R-modules, and whose morphisms are those n-tuples $(f_1,\dots,f_n)$ $f_j: X_j \to X_j'$, which induce a commutative diagram between two sequences which are the identity on A and on B. The Baer sum operation can be defined so as to give a functor $\beta: \mathcal{E}_n \times \mathcal{E}_n \to \mathcal{E}_n$, which is a coherently associative and commutative product. The Grothendieck group $K_0(\mathcal{E}_n, \beta)$ can then be calculated: it is $\mathrm{Ext}^n(B,A)$. If $\varphi: A \to A''$ is a R-module homomorphism, and $\mathcal{E} = \mathcal{E}_1(A,B)$, $\mathcal{E}'' = \mathcal{E}_1(A'',B)$, then there is a product preserving, cofinal, E-surjective functor $\Phi: \mathcal{E} \to \mathcal{E}''$ which gives an exact sequence $K_1(\Phi) \to K_1(\mathcal{E}) \to K_1(\mathcal{E}'') \to K_0(\Phi) \to K_0(\mathcal{E}) \to K_0(\mathcal{E}'')$. We prove that the automorphism group of any short exact sequence $0 \to A \to X \to B \to 0$ is $\mathrm{Hom}(B,A)$, and that $K_1(\mathcal{E}) \cong \mathrm{Hom}(B,A)$. If φ is an epimorphism with kernel A', then $K_1(\Phi) \cong \mathrm{Hom}(B,A')$, and the exact sequence above agrees with the usual Hom-Ext exact sequence, except possibly at the fourth term. (Received June 29, 1970.)

20 Group Theory and Generalizations

677-20-1. J. H. CARRUTH and CHARLES E. CLARK, University of Tennessee, Knoxville, Tennessee 37916. Compact totally $\mathcal{H}$ ordered semigroups.

The relation $\leq(\mathcal{H})$ is defined as a semigroup S by $x \leq (\mathcal{H})\, y$ if $x \in yS^1 \cap S^1y$. If $\leq(\mathcal{H})$ is antisymmetric and each pair of elements of S compare relative to $\leq(\mathcal{H})$ then S is said to be totally $\mathcal{H}$ ordered. For $x,y \in S$ we let $[x,y] = \{z \in S \mid x \leq (\mathcal{H})\, z \leq (\mathcal{H})\, y\}$ Theorem. Let S be a compact totally $\mathcal{H}$ ordered semigroup with maximum element p and maximum idempotent e. Then $[0,e]$ is isomorphic to a generalized hormos $\mathrm{GHorm}\,(E(S), S_x, m_{xy})$ where S_x is either a usual interval, a nil interval, or a compact monothetic semigroup with zero having isolated identity adjoined. The entire semigroup S is isomorphic to the contact extension of $[0,e]$ by $[e,p]$ and $[e,p]$ is a compact monothetic semigroup with zero e. Moreover, each semigroup constructed in the above fashion is a compact totally $\mathcal{H}$ ordered semigroup. Corollary. Each compact totally $\mathcal{H}$ ordered semigroup is isomorphic to a closed subsemigroup of an I-semigroup. Hence, each such semigroup is abelian. (Received April 27, 1970.)

677-20-2. JAMES W. STEPP, University of Houston, Houston, Texas 77004. Locally compact Clifford semigroups.

Let S denote a locally compact Hausdorff semigroup which is a disjoint union of subgroups one of which is dense. It is the purpose of this paper to consider S when each subgroup of S is a topological group when given the relative topology and G (the dense subgroup) has the added property that it is abelian and G/G_0 is a union of compact groups. In particular, we show how to reduce such a semigroup to a semigroup which is a union of real vector groups. We also give the structure of S under the added assumption that E(S) is isomorphic to $E((R^x)^n)$, where $(R^x)^n$ denotes the n-fold product of the nonnegative multiplicative real numbers. (Received May 15, 1970.)

677-20-3. ROBERT L. WILSON, JR., University of Wisconsin, Madison, Wisconsin 53706. Isotopy-isomorphy loops of prime order.

Call a loop $(L, \cdot)$ an I-I loop if it is isomorphic to all of its loop isotopes. Bryant and Schneider (Canad. J. Math. 18(1966), 120-125) have shown that the set G of isomorphisms between principal loop isotopes of a loop $(L, \cdot)$ forms a group. Let A denote the group of automorphisms of $(L, \cdot)$. The paper proves two theorems: Theorem. If $(L, \cdot)$ is an I-I loop of order n, and n^2 does not divide $|G|/|A|$, then $N\mu(L, \cdot)$ (the middle nucleus) is not trivial. Theorem. A loop of prime order is an I-I loop if and only if it is a cyclic group. (Received June 5, 1970.)

677-20-4. ARMOND E. SPENCER, University of Kentucky, Lexington, Kentucky 40506. Self dual finite groups. Preliminary report.

A group G is called self dual if each subgroup of G is isomorphic to a factor group of G and each factor group of G is isomorphic to a subgroup of G. Theorem 1. A finite group G is self dual iff G is nilpotent and each Sylow subgroup of G is self dual. Theorem 2. If P is a finite p-group, then each subgroup of P is self dual iff P is abelian or $P = H \times K$ where H is the extra special group of order p^3 and exponent p and K is elementary abelian. (Received June 11, 1970.)

677-20-5. RONALD O. FULP, North Carolina State University, Raleigh, North Carolina 27607. Character semigroups of locally compact inverse semigroups. Preliminary report.

Let S denote a locally compact abelian continuous-inverse semigroup and for each idempotent e of S let H_e denote the maximal subgroup of S containing e. For idempotents e and f, $f \leq e$, let π_{fe} denote the mapping from the (Pontryagin) dual $H_f^\wedge$ of H_f into the dual $H_e^\wedge$ of H_e defined by $\pi_{fe}(\chi)(s) = \chi(sf)$. It is shown that if the set E of idempotents of S satisfies a certain separation hypothesis and λ is an idempotent character of S, then the maximal subgroup of the character semigroup $S^\wedge$ containing λ is isomorphic to inv lim $[\{H_e^\wedge\}_{\lambda(e)\neq 0}; \{\pi_{fe}\}]$. It is shown that the separation hypothesis on E is satisfied whenever E is compact, E is a chain, or E is totally disconnected. A corollary is obtained which gives necessary and sufficient conditions that there exists a compact semigroup topology on $\bigcup_{e\in E} S_e$ which relativizes to given topologies on E and on the various S_e whenever $\{S_e\}_{e\in E}$ is a family of compact abelian groups indexed by a compact totally disconnected similated E. (Received June 22, 1970.)

677-20-6. DEANE E. ARGANBRIGHT, Iowa State University, Ames, Iowa 50010. A class of subgroups of a finite p-group.

Let G be a finite p-group. Definition. A subgroup H of G is a maximal subgroup of exponent p if H is a subgroup of exponent p, and H is not a proper subgroup of a subgroup of exponent p. [Cf. Grün, Osaka Math. J. 5(1968), 117-146.] Theorem. Let G be a finite p-group, where $p > 2$. If H is a maximal subgroup of exponent p and the class of H does not exceed $(p - 1)/2$, then H is a normal subgroup of G. The theorem also holds for other subgroups of G (e.g. maximal subgroups of p^ith powers, maximal regular subgroups). Example. For each prime p there exists a finite p-group having p maximal subgroups of exponent p, all of which are conjugate and have class $p - 1$. (Received June 22, 1970.)

22 Topological Groups, Lie Groups

677-22-1. GERALD L. ITZKOWITZ, State University of New York at Buffalo, Amherst, New York 14226. Density character of compact topological groups.

Let G be a compact topological group, let $\omega(G)$ be the least cardinal of a basis of open sets for G. Theorem. If $\omega(G) = m$, then G contains a dense subset of cardinal n, when n is the least cardinal such that $2^n \geq m$. This generalizes a theorem of Kakutani for compact Abelian groups. (Received June 23, 1970.)

677-22-2. LAWRENCE G. BROWN, Stanford University, Stanford, California 94305. Extensions of topological groups.

Let G and A be polonais topological groups, A abelian. One considers two cohomology groups $H^2(G,A)$ and $H^2_V(G,A)$. $H^2(G,A)$ is defined by means of cochains which are Borel functions on $G \times G$, and $H^2_V(G,A)$ is obtained by identifying cochains which agree outside a first category set. It is shown that each of these cohomology groups classifies the topological group extensions of A by G. The proof makes use of a result of David Wigner: that in any extension there is a continuous section defined on a residual set in G. In the case where G is locally compact, it has been shown by Calvin Moore that the extensions are classified by $H^2(G,A)$ and also by the group obtained by identifying cochains which agree outside a set of Haar measure zero. The hypothesis that A be abelian can be eliminated. (Received June 29, 1970.)

28 Measure and Integration

677-28-1. JAMES J. BUCKLEY, Georgia Institute of Technology, Atlanta, Georgia 30332. A Pettis-Dunford integral for topological group valued functions. Preliminary report.

Let Y be a locally compact, Abelian topological group, Y' the character group of Y, μ a complex-valued measure, and $\int g d\mu$ the Lebesgue integral of $g: X \to C$. μ is 0-1 iff $\mu(A) \in \{0,1\}$, for every measurable A. If $y \in Y$, then $m(y) = Fy$ where $Fy(f) = f(y)$, for all $f \in Y'$. The domain of $\int^* \cdot d\mu$ (the extension), D, is all $h: X \to Y$ such that $f(h) \in L(\mu)$, for all $f \in Y'$. For $h \in D$, $F_h(f) = \int f(h) d\mu$, for all $f \in Y'$, and $\int^* h d\mu = F_h$. If $F_h \in \text{Range}(m)$, then $\int^* h d\mu = m^{-1}(F_h)$ also. Theorem 1. $\int^*(h_1+h_2)d\mu = \int^* h_1 d\mu \cdot \int^* h_2 d\mu$ on D iff μ is 0-1. Theorem 2. If all $F_h \in \text{Range}(m)$, then μ is 0-1. Theorem 3. $F_h \in \text{Range}(m)$ for all simple functions iff μ is 0-1. Theorem 4. If μ is 0-1 and either Y is compact or each $h \in D$ is the a.e. limit of simple functions, then every $F_h \in \text{Range}(m)$. Theorem 5. $\int^* \cdot d\mu = \int \cdot d\mu$ on $L(\mu)$ whenever Y = C under addition iff μ is 0-1. (Received April 13, 1970.)

677-28-2. VERNON E. ZANDER, West Georgia College, Carrollton, Georgia 30117. Fubini theorem for Orlicz spaces of Lebesgue-Bochner measurable functions.

Let (X,V,v) be the volume space formed as the product of the volume spaces (X_i, V_i, v_i) (i = 1,2). Let p,q be a pair of complementary (continuous) Young's functions, let Y, Z, Z_1, Z_2, W be Banach spaces, let w be a multilinear continuous operator on $Y \times Z_1 \times Z_2$ into W. Let $L_p(v,Y)$ be the Orlicz space of Lebesgue-Bochner measurable functions generated by p, and let $K_q(v,Z)$ be the associated space of finitely additive Z-valued set functions. Theorem. Let $f \in L_p(v,Y)$, $\mu_2 \in K_q(v_2, Z_2)$. Then

(a) the function $f(x_1, \cdot)$ is v_2-Orlicz summable v_1-a.e.; (b) the operator $r(f, \mu_2)$ defined by the expression $r(f,\mu_2)(x_1) = \int w_1(f(x_1,x_2), \mu_2(dx_2))v_1$-a.e. is bilinear and continuous from $L_p(v,Y) \times K_q(v_2,Z_2)$ into $L_p(v_1,Y_1)/N$, where $w_1(y,z_2) = w(y, \cdot, z_2)$, where Y_1 is the Banach space of bounded linear operators from Z_1 into W, and where N is the set of Y_1-valued v_1-measurable functions of zero seminorm; (c) the equality $\int w(f,d\mu_1,d\mu_2) = \int w_0(r(f,\mu_2),d\mu_1)$ holds for all $f \in L_p(v,Y)$, $\mu_i \in K_q(v_i,Z_i)$ $(i = 1,2)$, where $w_0(y_1,z_1) = y_1(z_1)$ for all $y_1 \in Y_1$, $z_1 \in Z_1$. This theorem enables one to establish a Jessen theorem for Orlicz spaces. (Received April 20, 1970.)

677-28-3. MICHAEL LEE STEIB, University of Houston, Houston, Texas 77004. A Cauchy-type condition concerning integrability.

It is shown that there is a counterexample to the conjecture that f is g integrable over $[a,b]$ with respect to the Riemann-Stieltjes refinement definition if for each $\epsilon > 0$ there is a subdivision D of $[a,b]$ such that if D' is a refinement of D then $\sum \{(fx) - f(y))(g(q) - g(p)): [p,q] \text{ in } D'\} < \epsilon$, where each of x and y is in $[p,q]$ for each $[p,q]$ in D'. (Received May 5, 1970.)

677-28-4. PAUL W. LEWIS, North Texas State University, Denton, Texas 76203. Vector measures with finite semivariation. Preliminary report.

Let E and F denote Banach spaces over the same scalar field, let $\underline{C}$ denote a ring of subsets of a universal space T, and let $\Omega(\underline{C})$ be the collection of all finitely additive, operator-valued set functions $m:\underline{C} \to B(E,F)$, where m has finite semivariation and B(E,F) is the space of bounded linear operators from E to F. In this paper, the regularity condition vsr (Proc. Amer. Math. Soc. 22(1969), 563-569) is related to the notion of regularity defined by Dinculeanu and Kluvanek in Proc. London Math. Soc. 17(1967), 505-512. It is shown that these two regularity conditions are equivalent for Baire measures but that neither implication necessarily holds if we extend the measures to the Borel sets. A hull-kernel type operator for these measures is defined, and this operator is used to give a "0 implies 0" characterization of the absolute continuity discussed by Tucker and Wayment in "Absolute continuity and the Radon-Nikodym theorem," J. Reine Angew. Math. (to appear). These topics are also related to vector-valued Riesz representation theorems. (Received June 22, 1970.)

677-28-5. DAVID L. SKOUG and GERALD W. JOHNSON, University of Nebraska, Lincoln, Nebraska 68508. Operator-valued Feynman integrals of finite-dimensional functionals.

Let $C[a,b]$ denote the space of continuous functions x on $[a,b]$. Let $\{\alpha_1,\ldots,\alpha_n\}$ be an orthonormal set of functions of bounded variation on $[a,b]$. Let $F(x) = f(\int_a^b \alpha_1(t)dx(t),\ldots,\int_a^b \alpha_n(t)dx(t))$. R. H. Cameron and D. A. Storvick in J. Math. Mech. 18(1968), 517-552, defined certain operator-valued function space integrals, and, in particular, an operator-valued Feynman integral. In their setting, existence theorems as well as explicit formulas for the function space integrals of functionals F as above are given. (Received June 25, 1970.)

677-28-6. HENRY C. WENTE, Tufts University, Medford, Massachusetts 02155. Soap bubbles exist.

Let γ be an oriented Jordan curve in E^3 homeomorphic to the unit circle $u^2 + v^2 = 1$. Let Δ denote the unit disk $u^2 + v^2 < 1$ and $\bar{\Delta}$ its closure. Let $\mathcal{J}(\gamma)$ be the set of vector valued functions $\bar{x}: \bar{\Delta}$

$\to E^3$ of class $C^0(\bar{\Delta}) \cap C^1(\Delta)$ whose boundary values are an admissible representation of γ, and such that $D(\bar{x}) = \iint_\Delta |\bar{x}_u|^2 + |\bar{x}_v|^2 \,du\,dv < \infty$. We suppose that $\mathcal{J}(\gamma)$ is not empty. This is true if γ is rectifiable, for example. Each $\bar{x} \in \mathcal{J}(\gamma)$ is a representation of a parametric surface of finite Lebesgue area, $A(\bar{x}) = \iint_\Delta |\bar{x}_u \times \bar{x}_v| \,du\,dv$ with $2A(\bar{x}) \leq D(\bar{x})$. Also, for each $\bar{x} \in \mathcal{J}(\gamma)$ the oriented volume functional $V(\bar{x}) = 1/3 \iint_\Delta \bar{x} \cdot (\bar{x}_u \times \bar{x}_v) du\,dv$ is well defined and finite. For K a given constant let $\mathcal{J}(\gamma, K)$ denote those $\bar{x} \in \mathcal{J}(\gamma)$ with $V(\bar{x}) = K$. Theorem. There exists an $\bar{x}_0 \in \mathcal{J}(\gamma, K)$ of minimum Lebesgue area and satisfying the following conditions: (1) $\Delta\bar{x} = 2H(\bar{x}_u \times \bar{x}_v)$ for some constant H. (2) $|\bar{x}_u| \equiv |\bar{x}_v|$, $(\bar{x}_u \cdot \bar{x}_v) \equiv 0$ (conformality). (3) $V(\bar{x}) = K$. (4) $\bar{x}: \partial\Delta \to E^3$ is a representation of γ. $\bar{x}_0$ is thus a conformal (except for possible isolated points) representation of a surface of constant mean curvature H, boundary γ, with $V(\bar{x}) = K$ and of minimum Lebesgue area. It is a consequence of (1) that $\bar{x}_0$ is analytic in Δ. (Received June 29, 1970.)

677-28-7. MAX SHIFFMAN, 318 Warren Avenue, San Leandro, California 94577 and California State College, Hayward, California 94542. The interior and exterior measures of the union and intersection of two arbitrary sets.

In an article soon to be published, the author has considered the interior and exterior measures of the union of two disjoint sets. The result can be expressed in terms of a certain collection of six independent nonnegative quantities. Thus was obtained a complete set of inequalities. In the present paper, two sets whether disjoint or not are considered. A complete set of conditions are obtained on the eight quantities $m_i(\ )$ and $m_e(\ )$ for the two sets, and their union and intersection. These form a complete set of inequalities. (Received June 29, 1970.)

677-28-8. FRED M. WRIGHT and DEAN KENNEBECK, Iowa State University, Ames, Iowa 50010. On the existence of (LR) $\int_a^b (udw + udv)$.

Let u, v, and w be real-valued functions on a closed interval $[a,b]$ of the real axis. For a partition $\Delta = \{a = x_0 < x_1 < x_2 < \dots < x_n = b\}$ of $[a,b]$, form the sum $S(\Delta)$ $= \Sigma_{i=1}^n \{u(x_{i-1}) \cdot [w(x_i) - w(x_{i-1})] + u(x_i) \cdot [v(x_i) - v(x_{i-1})]\}$. When these sums $S(\Delta)$ have a finite refinement limit, this limit will be denoted by (LR) $\int_a^b (udw + udv)$. This integral has been studied recently by B. Helton. Suppose that u is bounded on $[a,b]$. If there is a point p of $[a,b)$ such that $v(p^+)$ and $w(p^+)$ exist and are finite, such that $v(p^+) \neq v(p)$, and such that $u(p^+)$ does not exist, then (LR) $\int_a^b (udv + udw)$ does not exist. A result analogous to the preceding holds for left limits. If w is continuous and of bounded variation on $[a,b]$, if v is quasi-continuous on $[a,b]$, and if the Stieltjes integral $S\int_a^b udw$ does not exist, then (LR) $\int_a^b (udw + udv)$ does not exist. Suppose now that v and w are of bounded variation on $[a,b]$. Let s be a saltus function and φ be a continuous function on $[a,b]$ such that $v = s + \varphi$, and let t be a saltus function and ψ be a continuous function on $[a,b]$ such that $w = t + \psi$. Then, (LR) $\int_a^{b'} (udw + udv)$ exists iff the weighted refinement integrals $[F,(1,0)] \int_a^b udt$ and $[F,(0,1)] \int_a^b uds$ exist and the Stieltjes integrals $S\int_a^b ud\psi$ and $S\int_a^b ud\varphi$ exist. (Received June 29, 1970.)

677-28-9. VIRINDRA M. SEHGAL and EVELYN J. MORRISON, University of Wyoming, Laramie, Wyoming 82070. A note on Baire sets.

Let X be a locally compact Hausdorff space and S_0 be the class of Baire sets, that is the σ-ring generated by the class of compact G_δ sets of X. It is well known (see Halmos, "Measure theory,"

p. 221) that every compact Baire set is a G_δ set. In this note it is shown that every closed Baire set is a G_δ set. If S_1 is the σ-ring generated by the class of closed G_δ sets then, it is shown that every closed, σ-bounded element of S_1 is a G_δ set. (Received June 29, 1970.)

30 Functions of a Complex Variable

677-30-1. S. M. SHAH, University of Kentucky, Lexington, Kentucky 40506. Entire functions of unbounded index and having simple zeros.

If f is of bounded index, then f is at most of the exponential type of order one. But the converse is not true, since whenever f has zeros of arbitrarily large multiplicity then f is of unbounded index. On the other hand if f is of exponential type and has simple positive zeros a_n such that $a_{n+1}/a_n \geqq \gamma > 1$ then f is of bounded index. In this paper functions of unbounded index and having simple zeros are constructed. Theorem. Let $a \geqq 0$, $a_1 \geqq 1$ and $a_{k+1} \geqq \max\{3(k+1)a_k, a_k^{(k+1)/k}\}$ $(k \geqq 1)$, and let $f(z) = e^{az}\prod_{m=1}^{\infty}(1 - z/a_m)^m$. Then f(z) is an entire function and f(z) - c, Im $c \neq 0$, has all simple zeros, is of exponential type and is of unbounded index. (Received May 18, 1970.)

677-30-2. SANFORD S. MILLER, University of Kentucky, Lexington, Kentucky 40506. An arc-length problem for m-fold symmetric univalent functions.

Let S denote the class of normalized ($f(0) = 0$, $f'(0) = 1$) functions which are univalent in the unit disk. C, S* and K denote the subclasses of S which are convex, starlike and close-to-convex respectively. Let C_m denote the subclass of C which is m-fold symmetric ($f(e^{2\pi i/m}z) = e^{2\pi i/m}f(z)$, $m = 1,2,\ldots$) with $C_1 = C$. Similarly define S^*_m and K_m. Let $L_r(f) = \int_{|z|=r}|f'(z)|\,|dz|$ denote the arc-length of the image of $|z| = r$, $0 < r < 1$. The extremal problem $\max\{L_r(f) \mid f \in S\}$ remains unsolved. In recent papers the problems $\max\{L_r(f) \mid f \in C_m\}$, $\max\{L_r(f) \mid f \in S^*_m\}$, $\max\{L_r(f) \mid f \in K_m\}$ have been solved for the case m = 1 by F. R. Keogh, A. Marx and P. L. Duren respectively. The author solves these problems for any $m = 1,2,\ldots$. Some special cases are handled for which uniform bounds are obtained. (Received June 22, 1970.)

677-30-3. DAVID WINFIELD BASH, JR., Purdue University, Fort Wayne, Indiana 46805. Sums and products of normal functions and very normal functions.

valued Let $\rho(z,z')$ denote the hyperbolic distance between z and z' in $|z| < 1$. A complex, finite-valued function f defined in $|z| < 1$ is very normal if there exists a positive number P such that $|f(z) - f(z')| < P\rho(z,z')$ for each z, z' in $|z| < 1$. Very normal functions are normal functions and are closed under addition. Theorem. f is very normal if and only if there exists a positive number K such that $M(f(z)) \leqq K/(1 - |z|)$ for each z in $|z| < 1$ where M(f(z)) is the lim sup of the modulus of the usual difference quotient at z. In the case of holomorphic functions f, f is very normal if and only if f is uniformly normal [for definition, see Lappan, Comment. Math. Univ. St. Paul 12(1964), 46]. Theorem. Let f be normal in $|z| < 1$ with a well-defined, finite-valued, continuous logarithm in $|z| < 1$. Then log f is normal in $|z| < 1$. Theorem. Let f be very normal in $|z| < 1$. Then exp f is normal in $|z| < 1$. Example. The function f = exp(i Re $(1/(1 - |z|))$) is not normal but log f is normal in $|z| < 1$. Corollary. Let log f_1 and log f_2 be very normal in $|z| < 1$. Then the product of the normal functions f_1 and f_2 is normal. (Received June 22, 1970.)

677-30-4. BENJAMIN LEPSON, U.S. Naval Research Laboratory, Washington, D. C. 20390 and Catholic University of America, Washington, D. C. 20017. Value distribution of exponential sums.

By an exponential sum, we mean an entire function of the form $f = \sum_{j=1}^{n} P_j(z)\, e^{c_j z}$, where z is a complex variable, the c_j are complex constants, and the $P_j(z)$ are polynomials. The value distribution and, in particular, the distribution of zeros of these functions has been investigated by many authors. Dancs and Turán [Publ. Math. Debrecen 11(1964), 266-272] have obtained an upper bound for the number of zeros of f in an arbitrary square of the complex plane, which is independent both of the position of the square and of the coefficients of the $P_j(z)$. It follows easily that exponential sums are functions of bounded value distribution in the sense of Turán and Hayman [see W. K. Hayman,"Research problems in function theory," Athlone Press, Univ. of London, 1967, p. 17]. In this paper, these results, with the exception of the explicit estimate itself, are obtained in a simple manner as a consequence of previous results by the author on valency and uniqueness properties of families of analytic and meromorphic functions [these *Notices* 16(1969), 1068]. (Received June 26, 1970.)

677-30-5. JOSEPH A. CIMA and JOHN A. PFALTZGRAFF, University of North Carolina, Chapel Hill, North Carolina 27514. Oscillatory behavior of u'' + hu = 0 for schlicht h. Preliminary report.

Let h(z) be analytic in $E = \{z: |z| < 1\}$. Theorem 1. If h(z) belongs to the Hardy class H^p for $p \geq 1/2$ then (*) $u''(z) + h(z)u(z) = 0$ is nonoscillatory in E. Nehari proved this result for $p \geq 1$ (Amer. J. Math. 76(1954)). It is well known that if $h(z) = z + a_2 z^2 + \ldots$ is analytic and schlicht in E then h(z) belongs to H^p for all $p < 1/2$. The Koebe function $k(z) = z/(1-z)^2$ and the function $g(z) = ((1-z)^{-2} - 1)/2$ are schlicht in E and do not belong to $H^{1/2}$. It is shown that (*) is oscillatory for h(z) = k(z) and for $h(z) = g(\bar{z})$. Theorem 2. Let D be a domain on the Riemann sphere with boundary C, a Jordan curve passing through the point at infinity. Let h(z) be a complex valued analytic, univalent map of E onto D (h(0) = 0, h'(0) = 1). If C does not have a reentrant cusp at infinity then (*) is nonoscillatory in E. (Received June 26, 1079.)

677-30-6. JOE E. KIRK, JR., University of Wyoming, Laramie, Wyoming 82070. A doubly connected Riemann surface.

For each integer n the z-sphere S_n is joined to the sphere S_{n+1} along an interval of the positive real axis forming a first order branch cut of the surface. The interval joining S_{2n} to S_{2n+1} is left of the interval joining S_{2n} to S_{2n-1} and left of the interval joining S_{2n+1} to S_{2n+2}. The surface so formed is conformally equivalent to the twice punctured sphere. Infinite product representations are obtained for both the mapping function and its derivative. Use is made of the method of approximation by elliptic surfaces and results from Carathéodory kernel theory. (Received June 29, 1970.)

677-30-7. HARI SHANKAR, Ohio University, Athens, Ohio 45701 and S. K. SINGH, University of Missouri, Kansas City, Missouri 64110. Distribution of roots and growth of a meromorphic function.

Let f (z) be a meromorphic function and let $\bar{C}$ denote the extended complex plane. For notations and terminology, like n(r,a), $\bar{n}(r,a)$, N(r,a), T(r,f), order ρ, lower order λ, proximate order $\rho(r)$, exceptional values, etc. it is suggested to refer to W. K. Hayman, "Meromorphic functions," Oxford Mathematical Monographs, Clarendon Press, Oxford, 1964; B. Ja. Levine, "Distribution of zeros of entire functions," American Mathematical Society, Providence, R. I., 1964. For an entire function $f(z)$ denote $M(r,f) = \mathrm{Max}_{|z|=r}|f(z)|$, $M(r,f') = \mathrm{Max}_{|z|=r}|f'(z)|$ and let $M'(r,f)$ denote the derivative of $M(r,f)$ with respect to r. The results established are as follows. Theorem 1. Let $f(z)$ be a transcendental meromorphic function of finite nonzero order ρ and of proximate order $\rho(r)$ (with respect to $T(r,f)$). Then except possibly for $q - 1$ distinct values of $a \in \overline{C}$, where q is an integer greater than or equal to 3, $\limsup_{r\to\infty} \overline{n}(r,a)/r^{\rho(r)} \geq ((q-2)\rho)/q$. These exceptional values, if they exist, will be exceptional values in the sense of Valiron. Theorem 2. Let $f(z)$ be an entire function of order ρ having a as exceptional value in the sense of Nevanlinna and such that $\delta(a) = 1$. Then $\liminf_{r\to\infty} \log M(r,f)/T(r,f) \leq 4$ if $\rho = 1$; and $\leq 4\rho^{\rho}/(\rho - 1)^{(\rho-1)}$ if $\rho > 1$, and integer. Theorem 3. For entire functions of finite positive order ρ and mean type $M(r,f') \geq \rho\{\log M(r,f) + O(1)\}M(r)/r$, for a sequence of values of r tending to infinity. (Received June 29, 1970.)

677-30-8. JOHN M. KASDAN, University of California, Los Angeles, California 90024. Divisors of poles with low order and 2-point support.

Let S be a compact Riemann surface of genus $g \geq Z$. Let P be a point of S which is not a Weierstrass point. It is shown that $\exists$ only finitely many points Q such that $\ell(nP + mQ) > 1$, for m,n > 0, $m + n \leq g$. For $S = \overline{U/H}$, where U is the upper half plane and $H \subset SL(2,\mathbb{R})$ a criterion similar to Schoenberg's criterion for Weierstrass points if given. By use of this criterion examples of such "Weierstrass pairs" are given for some modular groups. (Received June 30, 1970.)

677-30-9. FRED M. WRIGHT and NANCY HEATH, Iowa State University, Ames, Iowa 50010. A Stieltjes integral approach to the length of a curve on the Riemann sphere.

Let S be the Riemann sphere in E^3 with center at the point $(0,0,1/2)$ and with radius $1/2$. Let T be the function with domain S and with range the extended complex plane such that $T(x_1,x_2,x_3) = (x_1 + ix_2)/(1 - x_3)$ for every (x_1,x_2,x_3) in $S - \{(0,0,1)\}$ and such that $T(0,0,1) = \infty$. The restriction of T to $S - \{(0,0,1)\}$ is the famous stereographic projection associated with S. If φ is a continuous complex-valued function with domain a closed interval $[a,b]$ of the real t-axis such that $V_a^b\varphi$ is finite and if $\tau(t) = T^{-1}(\varphi(t))$ for all t in $[a,b]$, a direct approach involving Stieltjes integral theory is employed to show that $V_a^b\tau$ equals the Stieltjes integral $\int_a^b \{1 + [\varphi(t)]^2\}^{-1} dV_a^t\varphi$. If K is a straight line in the complex plane together with the point ∞, a geometric approach and an analytic approach for determining a parametric representation for $T^{-1}(K)$ on the closed interval $[-\pi,\pi]$ of the real θ-axis are presented and compared. If φ is a complex-valued function on a closed interval $[a,b]$ of the real t-axis φ, not necessarily continuous, such that $V_a^b\varphi$ is finite, a physical interpretation for $V_a^b\varphi$ is given, and an interpretation and evaluation of $V_a^b T^{-1} \circ \varphi$ are discussed. (Received June 30, 1970.)

32 Several Complex Variables and Analytic Spaces

677-32-1. ARUN KUMAR AGARWAL, Grambling College, Grambling, Louisiana 71245. On the geometric means of entire functions of several complex variables.

Let $f(z_1,\ldots,z_n)$ be an entire function of the n (≥ 2) complex variables $z_1,\ldots,z_n$ holomorphic for

$|z_t| \leq r_t$, $t = 1,\ldots,n$. We have considered the case of only two complex variables for simplicity. Recently many authors have defined the arithmetic means of the function $|f(z_1,z_2)|$ and have investigated their properties. In the present paper, the geometric means of the function $|f(z_1,z_2)|$ have been defined and the asymptotic behavior of certain growth indicators for entire functions of several complex variables have been studied. The results are given in the form of theorems. (Received May 22, 1970.)

677-32-2. J. GOPALA KRISHNA, University of Illinois, Urbana, Illinois 61801 and I.H.N. RAO, Andhra University, Waltair, A. P., India. Generalized inverse and probability techniques leading to Valiron-Whittaker-type theorems and their applications in $\mathcal{C}^k$. Preliminary report.

The paper is ultimately concerned with the Valiron-Whittaker-type relations among the asymptotic behaviours of $\ell n\, \mathfrak{m}$, $\ell n\, \mu$ and $\sum \nu_j$ and their applications, where $\mathfrak{m} = \mathfrak{m}_f$, $\mu = \mu_f$ and $\nu = \nu_f$ are respectively the maximum modulus, maximum term and the cetnral index of an entire power series f over $\mathcal{C}^k$, the cartesian product of k-copies of the complex plane. The relations among the subfamilies of the standard families of functions dominated respectively by $\ell n\, \mathfrak{m}$ and $\ell n\, \mu$ present particular difficulties, and their proofs make use of some Wiman-type theorems, which in their turn, are arrived at through arguments with certain crucially located but rectifiable mistakes and admissible simplifications. We are thus led to critically examine Part I of the Ph. D. thesis: "Wiman-Valiron theory for entire functions of several complex variables" by Alan Schumitzky (Cornell University, 1965) and incidentally prove a Valiron-type relation (Theorem A) discussed in the same employing the notion of the generalized inverse of a matrix due to Moore. (Received June 29, 1970.)

33 Special Functions

677-33-1. ANAND M. CHAK and ARUN KUMAR AGARWAL, West Virginia University, Morgantown, West Virginia 26506. The basic poweroids.

I. M. Sheffer ("Some properties of polynomial sets of type zero," Duke Math. J. 5(1939), 590-622) and J. F. Steffènsen ("The poweroid, an extension of the mathematical notion of power," Acta Math. 73(1941), 333-366) studied the set of polynomials $\{P_n(x)\}$ satisfying the generalized Appell property $J\{P_n(x)\} = P_{n-1}(x)$, $n = 0,1,2,\ldots$, where $J = k_1D + k_2D^2 + \ldots$, $k_1 \neq 0$, and the expansion being convergent if the symbol of differentiation D is replaced by a sufficiently small number. In this paper we use the much more general linear distributive operator D_u of M. Ward ("A calculus of sequences," Amer. J. Math. 58(1936), 255-266) to generate our "basic poweroids". Two specific examples of these general poweroids have been given. The first follows from the famous q-difference calculus of Jackson and illustrates the case in which $u^n u^m = u^{n+m}$; here $u^n = u_{n+1} - u_n$ and $D_u x^n = u_n x^{n-1}$, $\{u_n\}$ being a fixed sequence of complex numbers with the only restrictions $u_0 = 0$, $u_1 = 1$, $u_n \neq 0$ for $n > 1$. The second example in which this law of exponents is not satisfied arises from the q-difference analogue of Schwarz derivative $\lim_{h\to 0}((F(X + h) - F(X - h))/2h)$. Incidentally, this is the third paper in which the authors (cf. these *Notices* 13(1966), 610 and 16(1969), 779) have utilized the operator D_u of Ward to generalize Appell polynomials. (Received June 17, 1970.)

677-33-2. WILLIAM M. SANDERS, Madison College, Harrisonburg, Virginia 22801. Subgroups of rational functions of order six. Preliminary report.

Let $R(x)$ denote the set of rational functions $F(x) = (ax + b)/(cx + d)$ with $ad - bc \neq 0$. Let * be an operation from $R(x) \times R(x)$ to $R(x)$ defined by substituting the first factor for each occurence of x in the second factor. It is well known that $(R(x),*)$ is a group. An algorithm identifying infinitely many subgroups of order six is obtained. The algorithm has the efficacy to establish obvious isomorphisms. A permutation on three symbols may be considered as a function on three integers and is capable of being represented by a rational function; thus the permutation $\binom{456}{546}$ may be associated with the rational function $r(x) = (84 - 16x)/(16 - 3x)$. (Received June 23, 1970.)

34 Ordinary Differential Equations

677-34-1. HENRY L. CROWSON, IBM Corporation, Gaithersburg, Maryland 20760. Hypergeometric solutions of second order linear ordinary differential equations with confluent regular singular points.

The second order linear ordinary differential equation $\sum_{j=0}^{2} p_j(z)u^{(j)}(z) = 0$, in which $p_j(z)$ are defined such that the equation has five regular singular points, was derived earlier by the author. It was shown that the application of Scheffé's criterion to $p_j(z)$ produced a set of confluent and nonconfluent equations, and that the nonconfluent equations had solutions expressible in terms of the hypergeometric function. In this paper, it is shown that the confluent equations also have hypergeometric solutions. The combination of these two pieces of information assists in a proof of the following: Theorem. A necessary condition for confluent and nonconfluent forms of $\sum_{j=0}^{2} p_j(z)u^{(j)}(z) = 0$ to have solutions in a neighborhood of any point, which are expressible in terms of the hypergeometric function, is that Scheffé's lemma be applicable to $p_j(z)$. (Received April 13, 1970.)

677-34-2. VADIM KOMKOV, Texas Technological University, Lubbock, Texas 79409. Iterative scheme for determining the oscillatory properties of $x'' + c(t)x = 0$.

It is known that by the use of Kummer-Liouville transformation the equation $(ax')' + cx = 0$ can be transformed to the form $y'' + \sigma y = 0$, in a way preserving the oscillatory behavior of solutions. By factoring out a suitable exponential function at each step, the author introduces an iterative procedure, consisting of a repeated application of the Kummer-Liouville transformation. The equation $z'' + \sigma_n z = 0$ obtained at the nth step of the procedure has the same oscillatory properties as the original equation. The iterative scheme is defined by the relation: $\sigma_{n+1} = [\sigma_n + \varphi'' + (\varphi')^2] \exp(4\varphi)$, when φ is an a priori chosen C^2 function. A class of oscillation criteria is obtained by a specific choice of $\varphi(t)$. Particular examples are given which generalize the Leighton-Wintner and the Kondrat'ev oscillation criteria. (Received May 25, 1970.)

677-34-3. JOHN A. MORRISON, Bell Telephone Laboratories, Murray Hill, New Jersey 07974. Application of a limit theorem to solutions of a stochastic differential equation.

A limit theorem of R. Z. Khas'minskii [Theor. Probability Appl. 11(1966), 390-406] is applied to the investigation of the solutions of the differential equations $u'_m(z) = v_m(z)$, $v'_m(z) = -\beta_0^2(1 + \epsilon N(z)) \cdot u_m(z)$, with $u_m(0) = \delta_{m1}$, $v_m(0) = \delta_{m2}$ $(m = 1,2)$, where $N(z)$ is a bounded, wide sense stationary stochastic process with zero mean, and β_0 and $\epsilon << 1$ are positive constants. The backward equation corresponding to an associated limit process, which is a diffusion process, is derived, leading to a formulation for the expectation of a function of u_1, v_1, u_2 and v_2, on an interval $0 \leq \epsilon^2 z \leq O(1)$. A transformation is introduced which leads to separation of variables in the backward equation. An application of the results is made to the problem of a plane electromagnetic wave normally incident on a randomly stratified dielectric plate, the wave numbers on both sides of the plate being generally different from β_0. An explicit expression is obtained, in the small ϵ limit, for the expected value of $\mathcal{T}\mathcal{T}^*$, where $\mathcal{T}$ is the amplitude transmission coefficient. Expressions are also obtained for the first and second order moments of u_1, v_1, u_2 and v_2, and the correlation functions, which are expressed in terms of the moments. (Received May 25, 1970.)

677-34-4. MICHAEL A. GOLBERG, University of Nevada, Las Vegas, Nevada 89109. A generalized invariant imbedding equation: Nonlinear boundary conditions.

This paper develops the theory of invariant imbedding for the boundary value problem, (1) $x'(t) = f(x(t),t)$, (2) $g(x(0)) + h(x(T)) = v$, where $x(t)$ takes its values in a Banach space X. Under appropriate differentiability assumptions on t, g and h we prove the following theorems. Theorem 1. Assume that (1), (2) have a unique solution for each $v \in X$ and $0 < T \leq s$; assume also that the linear variational equations corresponding to (1), (2) have unique solutions, and that the solutions are C^2 Frechét differentiable functions of (t,T,v). Then $x(t,T,v)$ and $R(T,v) = x(T,T,v)$ satisfy the Cauchy problems (3) $x_T(t,T,v) = -x_v(t,T,v)dg_{R(T,v)}f(T,R(T,v))$, $x(t,t,v) = R(t,v)$, (4) $R_T(T,v) + R_v(T,v)dg_{R(T,v)}f(R(T,v),T) = f(R(T,v),T)$, $R(0,v) = (g + h)^{-1}(v)$. (Subscripts denote partial Frechét differentials.) Theorem 2. Assume that the Cauchy problems (3), (4) have unique solutions then the function $x(t,T,v)$ satisfies (1),(2). Thus we show that solving a general class of boundary value problems is equivalent to solving Cauchy problems for hyperbolic partial differential equations. These results generalize those of Kalaba and Kagiwada, "Derivation and validation of an initial value method for certain nonlinear two point boundary value problems," J. Optimization Theory Appl. 2(1968), 378-387. (Received June 12, 1970.)

677-34-5. WITHDRAWN.

677-34-6. ALLAN C. PETERSON, University of Nebraska, Lincoln, Nebraska 68508. On the sign of the Green's function beyond the interval of disconjugacy. Preliminary report.

The sign of the Green's function for the $(m_1,\ldots,m_k)$-boundary value problem $y^{(n)} + p_1(x)y^{(n-1)} + \ldots + p_n(x)y = 0$, $y^{(i)}(x_j) = 0$, $1 \leq j \leq k$, $0 \leq i \leq m_j - 1$, $\Sigma_{j=1}^{k} m_j = n$, inside the interval of disconjugacy is well known. The author is concerned with the sign of the Green's function for various boundary value problems beyond the interval of disconjugacy. For $n = 4$, R. G. Aliev [Izv. Vysš. Učebn. Zaved. Mathematika 1964, no. 6(43), 3-9] proved that if $\alpha < \beta < \min[r_{31}(\alpha), r_{22}(\alpha)]$, then the Green's function for the (3,1)-boundary value problem is negative in $(\alpha,\beta) \times (\alpha,\beta)$. This theorem motivates the type of problem the author considers. (Received June 19, 1970.)

677-34-7. KURT KREITH, University of California, Davis, California 95616. Disconjugacy criteria for nonselfadjoint differential equations of even order.

Let $\eta_1(\alpha)$ and $\mu_1(\alpha)$ denote the first conjugate points of α with respect to (1) $\Sigma_{k=0}^{n}(-1)^k(p_k(x)u^{(k)})^{(k)}$

$+ \sum_{k=0}^{n-1}(-1)^k (q_k(x)u^{(k+1)})^{(k)} = 0$ and (2) $\sum_{k=0}^{n}(-1)^k (P_k(x)v^{(k)})^{(k)} = 0$, respectively. Theorem. If (i) $p_n(x) \geqq P_n(x) \geqq 0$ and $p_n(x) > P_n(x)$ on $\{x | q_{n-1}(x) \neq 0\}$ and (ii) the matrix (g_{ij}) is positive semi-definite, where $g_{ii} = p_{i-1} - P_{i-1}$ $(i = 1,\ldots,n-1)$; $g_{nn} = p_{n-1} - P_{n-1} - q_{n-1}^2/4(p_n - P_n)$; $g_{i(i+1)} = q_{i-1}$ $(i = 1,\ldots,n-1)$; $g_{ij} = 0$ otherwise, then $\eta_1(a) \geqq \mu_1(a)$. Using known disconjugacy criteria for the selfadjoint differential equation (2) it is possible to use this comparison theorem to establish disconjugacy criteria for the general linear nonselfadjoint differential equation of even order (1). (Received June 22, 1970.)

677-34-8. ROBERT J. OBERG, Knox College, Galesburg, Illinois 61401. Global theory of complex functional differential equations.

We study global existence theory for the linear functional differential equation (*) $f'(z) = a(z) \cdot f(g(z))$ where $a(z)$ and $g(z)$ are given functions analytic on a simply connected open set E that is mapped into itself by g. If E is a proper subset of $\underline{C}$ which contains a fixed point of g, there always exists a solution of (*) defined on all of E. Now suppose $E = \underline{C}$ and z_0 is an attractive fixed point of g (i.e. $g(z_0) = z_0$ and $|g'(z_0)| < 1$). By a previous result of the author (*) has a local solution $f(z)$ about z_0. We show that this solution continues over the "attractive domain" A (i.e. the component containing z_0 of the set of all z such that $g_n(z) \to z_0$, where $g_n = g \circ g \circ \ldots \circ g$ is the nth iterate of g). A is either the natural domain of definition of $f(z)$ or else $f(z)$ continues over the whole plane or the plane minus a point, possibly as a multiple-valued function. In the last case $w = f(z)$ can be uniformized by a variable u ranging over the whole plane: $z = P(u)$, $w = F(u)$. (*) then lifts to an equation of the same form $F'(u) = A(u)F(G(u))$ where $A(u)$ and $G(u)$ are entire and the solution $F(u)$ must be meromorphic. Several of our results are valid for more general equations, but all the essential phenomena appear to be already present in (*). (Received June 29, 1970.)

677-34-9. PREM N. BAJAJ, Wichita State University, Wichita, Kansas 67208. Some effects of reordering the factorisation in a product semidynamical system. Preliminary report.

Given a family (X_α, π_α), $\alpha \in A$, of semidynamical systems, product semidynamical system is defined in a natural way. In this paper, the nature of a start point or a singular point is shown to depend upon the manner of factorisation. (Received June 29, 1970.)

35 Partial Differential Equations

677-35-1. WILLIE R. CALLAHAN, St. John's University, Jamaica, New York 11432. Vibration of plates bounded by elliptical and hyperbolic cylinders.

The partial differential equations governing the motion of plates, when transverse shear and rotary inertia are considered, are solved and solutions are chosen which satisfy all of the eight different types of boundary conditions entering into the theory for various shapes of plates bounded by elliptical and hyperbolic cylinders. Algorithms are then presented showing how to obtain the numerical values for the frequencies. Using these algorithms certain numerical cases are actually worked out illustrating the theory presented. (Received April 29, 1970.)

677-35-2. DAVID K. COHOON, University of Wisconsin, Madison, Wisconsin 53706. Nonexistence of a continuous right inverse for surjective linear partial differential operators on special spaces of infinitely differentiable functions. II. Preliminary report.

The author uses the notation and definitions described in the abstract for Part I of the paper by the above title which appeared in these *Notices*. Let us suppose P(D) is a linear partial differential operator with constant coefficients having $n \geqq 2$ independent variables. Suppose N is a vector in $\mathbb{R}^n - \{0\}$ orthogonal to every characteristic. Then $1 < \delta \leqq m/(m-1)$ implies P(D) has a continuous right inverse in $\gamma^{(\delta)}(\mathbb{R}^n)$; this is a special case of Theorem 5.7.3 of Hörmander's "Linear partial differential operators." Suppose $Q(D_y)$ is the partial differential operator obtained from $P(D_x)$ by changing coordinates so that $y_n = N_1x_1 + \ldots + N_nx_n$. Suppose $X(\xi')$ is a zero of $Q(\xi', X(\xi'))$ which satisfies for some positive C and M and real B the condition that $\operatorname{Im} X(\xi') \leqq -C\|\xi'\|^M + B$ for all ξ' in $\mathcal{J}$, where $\xi' = (\xi_1, \ldots, \xi_{n-1})$, $\|\xi'\| = (\xi_1^2 + \ldots + \xi_{n-1}^2)^{1/2}$, and $\mathcal{J}$ is any unbounded set in $\mathbb{R}^{n-1}$. Then $M \leqq (m-1)/m$, and P(D) has no continuous right inverse in $\gamma^{(\delta)}(\Omega)$ for any nonempty open subset Ω of $\mathbb{R}^n$ and any $\delta > 1/M$. (Received June 10, 1970.)

677-35-3. VINCENT G. SIGILLITO, Applied Physics Laboratory, Johns Hopkins University, Silver Spring, Maryland 20910. On a theorem of Edelstein and generalizations.

Edelstein (Z. Angew. Math. Phys. 20(1969), 900-905) has derived a spatial decay estimate for a parabolic equation satisfying the condition of zero heat flux across a portion of the boundary. It is the purpose of this paper to obtain the same decay estimate for the more general boundary condition which allows heat-transfer to occur between the body and the surrounding medium. Edelstein's result is obtained as a special case. (Received June 25, 1970.)

677-35-4. WITHDRAWN.

40 Sequences, Series, Summability

677-40-1. JOHN R. EDWARDS and STANLEY G. WAYMENT, Utah State University, Logan, Utah 84321. The Hausdorff moment problem is not equivalent to convergence perserving in the setting of linear normed spaces.

In this paper we give a counterexample to the Theorem. A Hausdorff method is convergence preserving if and only if it is generated by a moment sequence as stated in "Vector valued summability methods on a linear normed space" by L. C. Kurtz and D. H. Tucker, Proc. Amer. Math. Soc. 16(1965), 419-428. New results are also obtained which extend those known on the equivalence of the generalized Hausdorff moment problem with a generalized Riesz Representation Theorem, and a class of normed spaces is given in which the above mentioned does hold. The key tool in establishing these is the v-integral. (Received June 5, 1970.)

677-40-2. ANDREW K. SNYDER, Lehigh University, Bethlehem, Pennsylvania 18015. Sequence spaces and interpolation problem for analytic functions.

For each sequence $w = \{z_n\}$ in the open unit disk with $\sum(1 - |z_n|) < \infty$ let $H^p(w)$ denote the set of sequences $\{f(z_n)\}$ where f ranges over the Hardy class H^p. The purpose of the present work is first to study the BK space properties of $H^p(w)$ and then to examine these properties in a general context. The motivation was supplied by certain interpolation problems for analytic functions, for instance the existence of interpolating sequences, and by known summability properties of $H^1(w)$. It is observed that $H^p(w)$ is a BK space congruent to H^p/S where $S = \{f \in H^p\colon f(z_n) = 0 \text{ for all } n\}$ and that $H^p(w)$, $p < \infty$, has the AD property and a 2-norm density property (i.e. there exists $\{x^n\} \subset E^\infty$ such that $x^n \to 1$ and $|x_k^n|$ is bounded). It is proved for $1 < p < \infty$ that $H^p(w)$ is conull if and only if $H^p(w)$ includes every sequence of bounded variation. Further, $\{\delta^n\}$ is a basis for $H^p(w)$ if and only if $\{z_n\}$ is an interpolating sequence, where $\delta_k^n = 0$ for $k \neq n$, $\delta_n^n = 1$. Finally, it is proved that $H^2(w)$ may include all bounded sequences without w being an interpolating sequence. Other applications to interpolation problems are considered. (Received June 30, 1970.)

41 Approximations and Expansions

677-41-1. JOSEPH L. ULLMAN, University of Michigan, Ann Arbor, Michigan 48104. On the regular behavior of orthogonal polynomials. Preliminary report.

Let $\{P_n(x)\}$ be the monic, orthogonal polynomials associated with a weight function p(x), non-negative and integrable on $[-1,1]$, and positive on a set of positive measure. Let $\nu_n(\alpha,\beta)$, $-1 \leq \alpha < \beta \leq 1$, be the number of zeros of $P_n(x)$ on $[\alpha,\beta]$. If $\nu_n(\alpha,\beta)/n \to (1/\pi)\int_\alpha^\beta dx/\sqrt{1 - x^2}$ we say that p(x) is a regular weight function. Let E be a measurable subset of $[-1,1]$. By the capacity of E, C(E), we mean the inner logarithmic capacity. We say that the capacity of E is stable if C(E') = C(E), where E' is any measurable subset of E of the same measure. Let S(p(x)), the support of p(x), be the set $\{x\colon x \in [-1,1],\ p(x) > 0\}$. Theorem. Let E be a measurable subset of $[-1,1]$. Every weight function p(x) satisfying S(p(x)) = E will be regular if and only if E has stable capacity 1/2. Remark. The interval $[-1,1]$ has stable capacity 1/2. Also for any $\epsilon > 0$ there is a measurable subset of $[-1,1]$ of measure $< \epsilon$ and stable capacity 1/2. Thus this theorem has as a special case the known result that if $p(x) > 0$ a.e. on $[-1,1]$ it is regular; and also shows that this condition is sufficient, but not necessary for p(x) to be regular. (Received May 21, 1970.)

677-41-2. KENNETH D. SHERE, U.S. Naval Ordnance Laboratory, Applied Mathematics Division, White Oak, Silver Spring, Maryland 20910. Introduction to multiple asymptotic series with an application to elastic scattering.

The concept of asymptotic power series is extended to multiple series of the form $\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} a_{m,n}x^{-n}e^{-\lambda mx}$. This theory is then applied to elastic scattering from the Yukawa potential. An asymptotic series for the solution at large distances is obtained which is valid for arbitrary wave number and angular momentum. (Received June 19, 1970.)

677-41-3. DEMETRE JOHN MANGERON and M. N. OGUZTORELI, University of Alberta, Edmonton 7, Alberta, Canada. Approximations with spline functions and polyvibrating systems.

G. Birkhoff and W. J. Gordon, in their papers published in a 1969 Academic Press volume "Approximations with special emphasis on spline functions", edited by I. J. Schoenberg, using some of the authors' previous results related with the theory and practice of the polyvibrating systems, continue their research work concerning the extension of the theory of splines to several dimensions. The authors, starting from the variational formulation of the polyvibrating problems exposed in their C. R. Acad. Sci. Paris (vols. 204, 253-255, 266, 270) set of papers, underline and study the intimate relationship between various types of splines of interpolation to the values $f_{i_1,i_2,\ldots,i_{2m}}$ on a mesh M in a hyperrectangle R: $[a_i \leq x_i \leq b_i;\ i = 1,2,\ldots,2m]$ and the solutions of some appropriate polyvibrating boundary value problems, both of them being characterized by the minimization in R of the square of the second order polyvibrating derivative of functions pertaining to the apposite functional space. (Received June 29, 1970.)

677-41-4. RICHARD B. HOLMES and JOSEPH M. LAMBERT, Division of Mathematical Sciences, Purdue University, Lafayette, Indiana 47907. A geometrical approach to property (SAIN). Preliminary report.

We use some ideas from the theories of best approximation and duality maps to study property (SAIN) in certain Banach spaces. This property was introduced by Deutsch and Morris [J. Approximation Theory 2(1969), 355-373], and is defined in terms of a triple (X,M,G) where X is a Banach space, M a dense subspace of X, and G a finite dimensional subspace of X'. Emphasis is placed on establishing nasc for (SAIN), thereby providing some answers to the question of Deutsch and Morris [ibid., Remark 2.6]. These conditions are in turn applied to a number of standard dense subspaces of $L^p(\mu)$ spaces. For example, the Deutsch-Morris necessary condition [ibid., Corollary 2.2] is sufficient for $(L^p[0,1],C,G)$ to have property (SAIN), but this is false for $(L^p[0,1],C^k, G)$, where $1 < p < \infty$, $1 \leq k \leq \infty$. The basic abstract result in this approach is Theorem. Suppose ${}^{\perp}G$ is an (E-F)-subspace of X. Then the following assertions are equivalent: (a) (X,M,G) has property (SAIN); (b) each nonzero $g \in G$ attains its norm solely on M (or not at all); (c) M contains the metric complement of ${}^{\perp}G$. (Received June 29, 1970.)

42 Fourier Analysis

677-42-1. LYNN R. WILLIAMS, University of Kentucky, Lexington, Kentucky 40506. Generalized Hausdorff-Young theorems. Preliminary report.

Let G be a compact connected abelian group with Haar measure dm and ordered dual Γ. A lacunary decomposition of Γ is a collection $\mathcal{D} = \{D_k\}_{k=-\infty}^{\infty}$ of subsets of Γ satisfying: (i) $D_j \cap D_k = \emptyset$ if $k \neq j$; and (ii) for each k there exists $a_k \in \Gamma$ such that $D_k = \{\gamma \in \Gamma: a_k < \gamma \leq 2a_k\}$. Let $\{J_k\}_{k=-\infty}^{\infty}$ be the lacunary decomposition of the reals determined by $\pm 2^k$. We prove the following two generalizations of the classical Hausdorff-Young theorem. Theorem 1. If $1 < p \leq 2$ there exists a constant A_p such that $\{\sum_{k=-\infty}^{\infty}(\sum_{\gamma \in D_k}|\hat{f}(\gamma)|^q)^{2/q}\}^{1/2} \leq A_p(\int_G |f(t)|^p dm(t))^{1/p}$, where $1/p + 1/q = 1$ and $f \in L^p(G,dm)$. Theorem 2. For $1 < p \leq 2$ and n a positive integer, there exists a constant A_p such that $\{\sum_{k=-\infty}^{\infty}(\int_{J_k \times R^{n-1}} \sum_{\gamma \in \Gamma}|\hat{f}(x,\gamma)|^q dx)^{2/q}\}^{1/2} \leq A_p(\int_{R^n}\int_G |f(x,t)|^p dm(t)dx)^{1/p}$, where $1/p + 1/q = 1$ and $f \in L^p(R^n \oplus G, dm\,dx)$. Results, analogous to the dual of the classical Hausdorff-Young theorem, also hold. (Received May 8, 1970.)

43 Abstract Harmonic Analysis

677-43-1. IRWIN SCHOCHETMAN and HARVEY A. SMITH, Oakland University, Rochester, Michigan 48063 and ROBERT C. BUSBY, Drexel University, Philadelphia, Pennsylvania 19104. CCR group extensions.

Let G be a separable locally compact group, N a closed normal subgroup which is type I and regularly embedded and $G \times \hat{N} \to \hat{N}$ the canonical action. It is a well-known result of G. W. Mackey that each irreducible representation of G is of the form U^T where T is an irreducible representation of the stability group H_L, for some L in $\hat{N}$, and T restricted to N is equivalent to a multiple of L. We give necessary and sufficient conditions for U^T to be CCR under the assumption that H_L is normal. If this is the case for each stability group, then we have necessary and sufficient conditions for the extension G to be CCR. (Received June 13, 1970.)

44 Integral Transforms, Operational Calculus

677-44-1. GREGERS L. KRABBE, Purdue University, Lafayette, Indiana 47907. Two-sided operational calculus on an open interval. Preliminary report.

Let (a,b) be an open subinterval of the reals such that $-\infty \leq a < 0 < b \leq \infty$. If $f_1(\ )$ and $f_2(\ )$ are functions on (a,b), let $f_1 \wedge f_2(\ )$ be the function on (a,b) defined by $f_1 \wedge f_2(x) = \int_0^x f_1(x-u)f_2(u)du$. Let W be the space of all the functions that are infinitely differentiable on (a,b) and whose every derivative vanishes at the origin. W serves as a space of test-functions to define a space $\mathcal{Q}$ of generalized functions analogous to $\mathcal{D}'(a,b)$: let $\mathcal{Q}$ be the space of all the linear operators A which map W into W such that $A(w_1) \wedge w_2(\ ) = A(w_1 \wedge w_2)(\ )$ for any $w_1(\ )$ and $w_2(\ )$ in W. The space $\mathcal{Q}$ is a commutative algebra of operators; its unit is the identity-operator I. Definition. If f( ) is a function, we denote by f the operator defined on W by $f(w)(\ ) = f \wedge w'(\ )$; note that the unit constant I( ) corresponds to the identity-operator I. The correspondence $f(\ ) \mapsto f$ is a linear injection of $\mathfrak{F}$ into $\mathcal{Q}$, where $\mathfrak{F}$ is the space of all the functions that are integrable on each compact subset of the open interval (a,b). The differentiation operator D belongs to $\mathcal{Q}$, it is invertible, and the function $f_1 \wedge f_2(\ )$ corresponds to the operator $f_1 D^{-1} f_2$. In case $(a,b) = (-\infty, \infty)$, the algebra $\mathcal{Q}$ contains all the distributions that are regular on the negative axis; the injection $f(\ ) \mapsto f$ is a useful extension of the two-sided Laplace transformation. (Received June 22, 1970.)

677-44-2. ABDUL JABBAR JERRI, Clarkson College of Technology, Potsdam, New York 13676. Integral transforms for time varying systems.

It is known that integral transforms, besides the well-known Fourier transform, were suggested for treating time varying systems. In this correspondence, we will consider integral transforms with kernel as a solution of nth order selfadjoint boundary value problems. We will present a modified definition of the time varying impluse response associated with such integral transforms but will coincide with the usual definition in the case of Fourier transform. This will enable us to give a physical interpretation for the sampling series of a signal represented by such transforms. The series will represent the output of a band limited filter with time varying impulse response and with input as the pulse train of the sampling points. The time varying impulse response will be described by the sampling function of the series and corresponds to the familiar low pass filter system function. Also, we will present a specific integral transform along with the necessary operational tools as an example. (Received June 30, 1970.)

46 Functional Analysis

677-46-1. WILLIAM B. JOHNSON, University of Houston, Houston, Texas 77004. Existence theorems for Markuschevich bases in Banach spaces.

Let X be a Banach space. A biorthogonal system $\{x_i,f_i\}_{i\in T}$ in (X,X^*) is a Markuschevich basis (M-basis) if $\{x_i\}$ is fundamental in X and $\{f_i\}$ is total over X. An M-basis $\{x_i,f_i\}_{i=1}^{\infty}$ is strongly series summable (s.s.s.)if there is a matrix $(\lambda_{i,n})$ of scalars such that for each $x \in X$, $x = \lim_{n\to\infty}\sum_{i=1}^{n}\lambda_{i,n}f_i(x)x_i$. X has the λ m.a.p. (where $\lambda \geqq 1$) if there is a net of operators of finite rank on X uniformly bounded by λ which converges in the strong operator topology to the identity operator on X. Theorem 1. Let X be complex and separable. If X^* has the λ m.a.p. for some λ, then X admits a s.s.s. M-basis. Theorem 2. Let X be complex. X admits a boundedly complete s.s.s. M-basis if and only if X has the λ m.a.p. for some λ and X is isomorphic to a separable conjugate space. X is a P space if X is complemented in every space which contains it as a subspace. Theorem 3. No infinite dimensional P space admits an M-basis. Theorem 4. If X admits a weakly compact fundamental set, then X admits an M-basis. (Received May 4, 1970.)

677-46-2. JIMMY RAY NANNEY, University of Mississippi, University, Mississippi 38677. The geometric Hahn Banach theorem for near topological vector spaces.

A near topological vector space is defined to be a pair (X,T) where X is a real vector space and T is a topology on X such that each of addition and scalar multiplication is separately continuous. Theorem. If X is a near TVS, U is an open set in X having just one boundary point b and the complement of U contains at least two points, then X is one dimensional. Theorem. If X is a near TVS and H is a hyperplane in X, then H is closed or dense in X. Theorem. If X is a near TVS, U is a convex open nonempty set in X and M is a subspace of X which does not intersect U, then there is a closed hyperplane H in X which contains M and does not intersect U. (Received May 18, 1970.)

677-46-3. CHAMAN LAL SABHARWAL, St. Louis University, St. Louis, Missouri 63103. Multiplication of singularity functions.

In this paper we generalize the theorems (SIAM J. Appl. Math. 18(1970), 503-510) to multiplication of the Dirac delta function with nonintegral powers of $1/x$ and $1/|x|$ also. Theorem 1. Let $t(x;k)$ be a real-valued function of x such that (i) $\int_{-\infty}^{\infty} t(x;k)\,(-\operatorname{sgn} x)^r\,dx = c_r$, $c_r = 1$, r even integer; (ii) $t(x;k)\,|x|^{-\alpha}$ is orthogonal to x^r on $(-\infty, 0)$, $(0, \infty)$ for $0 \leq r < \alpha \leq k$; r, α real, k integer. Then $nt(nx;k)x^{-\alpha}$ and $(ntnx;k)\,|x|^{-\alpha}$ converge weakly to scalar multiples of $\delta(x)$ and to $\delta(x)$ for $\alpha = 0$. Corollary 2. Let $t(x;k)$ satisfy the hypotheses of Theorem 1 and define $T_n(x;k) = nt(nx;k)(1 + \Sigma_{r=1}^{k} a_r |x|^{\alpha-p+r})$. Then weak limits of $T_n(x;k)x^{-\alpha}$ and $T_n(x;k)\,|x|^{-\alpha}$ are finite linear combinations of $\delta(x)$ and its derivatives for $\alpha > 0$; and $\delta(x)$ for $\alpha = 0$, $[\alpha] = p$. The above theorem is extended to N dimensional space where Ω_N is used for the surface area of unit ball. Theorem 3. Let $t(x;k)$ be even function of x and satisfy the hypotheses of Theorem 1. Define $t(\vec{x};k) = 2\,t(|\vec{x}|;k)/\Omega_N |\vec{x}|^{N-1}$ and $T_n(x;k) = nt(n\vec{x};k)$ $(1 + \Sigma_{r=1}^{k} a_r\,|\vec{x}|^{\alpha-p+r})$. Then $T_n(\vec{x};k)\,|\vec{x}|^{-\alpha}$ converges weakly to a linear combination of $D^P\delta(\vec{x})$ for $0 \leq |P| = (p_1 + \ldots + p_N) \leq p$. This reduces to $\delta(\vec{x})$ for $\alpha = 0$. Several new examples illustrating the above are given. (Received June 1, 1970.)

677-46-4. PETER HESS, University of Chicago, Chicago, Illinois 60637. On the Fredholm alternative for asymptotically homogeneous unbounded mappings in Banach spaces.

The range of a mapping of the form L + T + A from a real reflexive (nonseparable) Banach space X into its conjugate X* is investigated. Here L denotes a closed densely defined monotone linear operator with the property that L and L* are the closure of their restrictions to $D(L) \cap D(L^*)$, respectively; T is a bounded demicontinuous odd homogeneous operator satisfying condition (S): if $u_n \rightharpoonup u$ in X, $\lim\sup (Tu_n, u_n - u) \leq 0$, then $u_n \to u$; A is a bounded demicontinuous mapping with $\|Au\|/\|u\| \to 0$ as $\|u\| \to \infty$ and such that T + A satisfies condition (S). It is shown that if $(L + T)u = 0$ implies $u = 0$, then the mapping L + T + A is surjective. The proof is based on a homotopy argument; it seems to be the first time that an unbounded not everywhere defined operator having only weak continuity properties is discussed with such methods. This theorem permits one to extend some of the writers results on the solvability of the generalized Dirichlet problem for linear elliptic equations with degenerate coefficients (Ann. Acad. Sci. Fenn. A I (1969), 434) to the nonlinear case. (Received June 10, 1970.)

677-46-5. DANIEL P. GIESY, Western Michigan University, Kalamazoo, Michigan 49001. The completion of a B-convex normed Riesz space is reflexive.

A normed linear space $\mathfrak{X}$ is B-convex if there exist $k \geq 2$ and $\epsilon > 0$ such that for all $x_1, \ldots, x_k \in \mathfrak{X}$ with all $\|x_i\| \leq 1$, $\|\pm x_1 \pm \ldots \pm x_k\| \leq k(1 - \epsilon)$ for some choice of the + and - signs. A normed Riesz space is a real normed linear vector lattice in which norm and order are related by "$|x| \leq |y|$ implies $\|x\| \leq \|y\|$" ($|x| = \sup(x, -x)$). Call a space good enough if it is the Banach dual (conjugate) of a normed Riesz space, and hence is a norm-complete space with a natural Riesz space structure (Nakano, "Modulared semiordered linear spaces," Tokyo, 1950). Lemma. Every B-convex normed Riesz space is imbeddable in a good enough B-convex space. Lemma. If $\mathfrak{X}$ is a good enough B-convex space, so is $\mathfrak{X}^*$. Lemma. If $\mathfrak{X}$ is good enough and there exist $u_1 \geq u_2 \geq \ldots \geq 0$ in $\mathfrak{X}$ such that $\inf_n u_n = 0$ and $\{u_n\}$ is not Cauchy, then $\mathfrak{X}$ is not B-convex. The title theorem then follows from these lemmas, properties of good enough spaces (Nakano, loc. cit.) and Ogasawara's characterization of reflexive normed Riesz spaces (Luxemberg and Zaanen, "Notes on Banach function spaces. XIII", Nederl. Akad. Wetensch. Proc. Ser. A 67 = Indag. Math. 26(1964), 530-543, Theorem 40.1). (Received June 22, 1970.)

677-46-6. RONN L. CARPENTER, University of Houston, Houston, Texas 77004. Continuity of homomorphisms and uniqueness of topology for F-algebras.

Let A be an F-algebra and B be a commutative semisimple F-algebra such that the spectrum of B does not contain any isolated points. It is shown that any homomorphism of A onto B is necessarily continuous. Let A be a commutative semisimple F-algebra. It is shown that there is at most one topology with respect to which A is an F-algebra. (Received June 24, 1970.)

677-46-7. STEVEN J. LEON, Michigan State University, East Lansing, Michigan 48823. Composition operators on B^p, the containing Banach space of H^p, $0 < p < 1$. Preliminary report.

If φ is an analytic function mapping the unit disk into itself and f is analytic in the unit disk, the composition operator C_φ is defined by $C_\varphi(f) = f \circ \varphi$. The properties of C_φ as an operator on H^p, $1 \leqq p \leqq \infty$, have been studied by H. J. Schwartz (Abstract 672-339, these *Notices* 17(1970), 179). This article deals with composition operators on the spaces B^p, $0 < p < 1$, of functions f analytic in the unit disk satisfying $\|f\|_p = 2\pi^{-1}\int_0^1\int_0^{2\pi}(1-r)^{1/p-2}|f(re^{i\theta})|d\theta\, dr < \infty$. It is shown that C_φ is a bounded linear operator on B^p. Conditions are also given on φ in order that C_φ be a bounded operator from B^p into H^q, $0 < q \leqq \infty$. Compact and invertible operators are discussed and their spectra are determined. (Received June 22, 1970.)

677-46-8. ERNEST J. ECKERT, California State College, Los Angeles, California 90032. Characterizations of norms in finite dimensional spaces. Preliminary report.

A norm $\|\cdot\|$ in real n-dimensional vectorspace R^n is a function $F: R^n \to R$ satisfying, for all $x = (x_1, x_2, \ldots, x_n)$, $u = (u_1, u_2, \ldots, u_n) \in R^n$, $a \in R$, the conditions $F(0) = 0$, $F(x) > 0$ for $x \neq 0$, $F(ax) = |a|F(x)$ $F(x+u), \leqq F(x) + F(u)$. Theorem 1. Let ψ, φ be continuous one-to-one mappings of the set R_+ of nonnegative real numbers onto R_+. Then $F(x) = \psi(\sum_{k=1}^n \varphi(|x_k|))$ defines a norm on R^n if and only if $\varphi(x) = x^p$, $p \geqq 1$, and $\psi = \varphi^{-1}$. That is F is an l_p-norm. Theorem 1 generalizes a result by T. G. Newman, Amer. Math. Monthly 75(1968), 646. Theorem 2. Let $F: R^2 \to R$ be associative, $F(F(x_1,x_2),x_3) = F(x_1,F(x_2,x_3))$, and reducible, $u \geqq 0$, $v \geqq 0$, $F(t,u) = F(t,v)$ implies $u = \pm v$. Then F defines an l_p-norm, $p \geqq 1$. Theorem 2 is generalized to R^n. (Received June 25, 1970.)

677-46-9. SAM B. NADLER, JR., Loyola University, New Orleans, Louisiana 70125. On an iterative test of Edelstein. Preliminary report.

Conditions are given on the space in order that if the iterates at some point of a contractive $(d(f(x), f(y)) < d(x,y)$ for all $x \neq y)$ mapping do not converge, then it can be concluded that the mapping does not have a fixed point. Such conditions are (1) (X,d) is locally compact and connected or (2) X is a "line segment dense" subset of a finite dimensional Banach space. Several examples related to the above results and several unsolved problems are given. (Received June 25, 1970.)

677-46-10. DAVID W. FOX, Applied Physics Laboratory, Johns Hopkins University, Silver Spring, Maryland 20910. Spectral measures and separation of variables.

An expression is given for the spectral measure of a selfadjoint operator in a Hilbert space when separation of variables is possible. The construction uses the amalgamation theorem for normal operators in a natural way to obtain the required measure as a tensor convolution of the spectral measures of the part operators. (Received June 25, 1970.)

677-46-11. GARY H. MEISTERS, University of Colorado, Boulder, Colorado 80302. Translation-invariant linear forms on $\mathcal{D}$.

Let $\mathcal{D}$ denote the space of C^{∞} functions with compact support on the line $\mathbb{R}$. Theorem 1. Every translation-invariant linear form T on $\mathcal{D}$ is continuous, and hence has the form $T(\varphi) = c\int_{\mathbb{R}}\varphi(t)dt$. This is established by proving Theorem 2. If α/β is an irrational algebraic real number, then for every φ in $\mathcal{D}$ there exist u and v in $\mathcal{D}$ such that $\varphi' = u_{\alpha} - u + v_{\beta} - v$. If α/β has rational or certain transcendental values, then the corresponding statement is false. $u_{\alpha}(t) \equiv u(t+\alpha)$ for t in $\mathbb{R}$. More generally, Theorem 3. If α/β is an irrational algebraic real number, then the ideal generated in the convolution algebra $\mathcal{E}'$ by the two functions $\chi_{[-\alpha,0]}$ and $\chi_{[-\beta,0]}$ is equal to $\mathcal{E}'$. Here $\chi_{[-\alpha,0]}$ denotes the characteristic function of the interval $[-\alpha,0]$. (Received June 29, 1970.)

677-46-12. BEN G. ROTH, University of Wyoming, Laramie, Wyoming 82070. Submodules of $C(X) \times \ldots \times C(X)$.

Let C(X) be the ring of continuous real-valued functions on a compact Hausdorff space X with the sup norm topology. The closed submodules of the C(X)-module $C(X) \times \ldots \times C(X)$ are characterized and a necessary and sufficient condition that a finitely generated submodule of $C(X) \times \ldots \times C(X)$ be closed is established. (Received June 29, 1970.)

677-46-13. RUSSELL G. BILYEU, North Texas State University, Denton, Texas 76203. The metric half-space topology. Preliminary report.

Let N(x,y) denote the set of points nearer x than y in a Banach space B, and let T denote the topology generated by all such sets. Some results of Z. Opial in Bull. Amer. Math. Soc. 73(1967), 591-597, may be viewed as indicating a connection between T and the weak topology. Assume B is uniformly convex and has a weakly continuous duality mapping. Theorem. The weak interior of N(x,y) contains x. This is a topological version of a lemma of Opial. Theorem. On the unit ball, T contains the weak topology. The work of Opial shows that these results fail in L^p, $p \neq 2$. (Received June 29, 1970.)

677-46-14. CHARLES W. SWARTZ, Department of Mathematical Sciences, New Mexico State University, Las Cruces, New Mexico 88001. Convolution in certain spaces of generalized functions. Preliminary report.

A characterization of convolution operators in certain $K(M_p)$ spaces (see Gelfand and Shilov, "Generalized functions. II" for the definition) is given. The result contains as special cases the well-known characterization of the space $\mathcal{O}'_c$ of L. Schwartz and also a characterization of convolution operators in the space of distributions of exponential order. (Received June 30, 1970.)

677-46-15. H. ELTON LACEY, University of Texas, Austin, Texas 78712. On Banach spaces which are dual L-spaces.

It is shown that if A and B are separable Banach spaces whose duals are abstract L-spaces, then A^* is linearly isometric to B^* if and only if the cardinality of the extreme points of the unit spheres of A^* and B^* are the same. Under the assumption of the continuum hypothesis it is established that there are exactly two (up to isometric isomorphism) infinite dimensional dual L-spaces of cardinality c. (Received June 30, 1970.)

677-46-16. PHILIP MEYERS, Health Sciences Center, State University of New York, Stony Brook, New York 11790. Contractive and contractifiable semigroups.

A semigroup of operators $\{\mathcal{T}_t, t \geq 0\}$ on a metric space (X,d) is contractive, if for each T_t, $t > 0$, there is a metric d_t such that T_t is a contraction on (X, d_t). It is contractifiable if there is a single metric under which the semigroup is contractive. A semigroup is an $\mathcal{J}$ semigroup if for all $s \geq 0$, $\lim_{t \to s} \sup \{ d(T_t x, T_s x) \ x \in X\} = 0$. Theorem 1. An $\mathcal{J}$ semigroup is contractive if some T_t satisfies (i) $T_t \theta = \theta$ for some $\theta \in X$; (ii) $\forall x\ T_t^n x \to \theta$; (iii) $T_t^n U \to \{\theta\}$ for some nbhd U of θ. Theorem 2. A contractive $\mathcal{J}$ semigroup is contractifiable. For $\lambda \in (0,1)$ there is a metric d_λ such that each T_t is a contraction on (X, d_λ) with contraction constant λ^t. Letting X be the state space of a stationary system whose evolution is described by $\{\mathcal{T}_t\}$ we have Corollary. The metric d_λ generates a Lyapunov function $V(x) = d_\lambda(x, \theta)$. Also with this interpretation Theorem 3. The equilibrium state θ is uniformly asymptotically stable if θ has a nbhd $X_0 \subset X$ such that $\{\mathcal{T}_t\}$ restricted to X_0 is a contractifiable semigroup. (Received June 23, 1970.)

677-46-17. WITHDRAWN.

677-46-18. J. S. MORRELL, University of Southern Mississippi, Hattiesburg, Mississippi 39401. Absolutely summing operators into $\mathcal{B}_p$ and $\mathcal{L}_p$-spaces. Preliminary report.

Let X and Y be Banach spaces. Denote by B(X,Y) and $A_p(X,Y)$ the bounded linear operators and the p-absolutely summing operators ($p \geq 1$) from X into Y respectively. Definition. Given $p \geq 1$ and $\lambda \geq 1$, a Banach space X is called a $\mathcal{B}_{p,\lambda}$-space if for every positive integer n there is a subspace U of X with $d(U, \ell_p^n) \leq \lambda$. X is a $\mathcal{B}_p$-space if it is a $\mathcal{B}_{p,\lambda}$-space for some $\lambda \geq 1$. Theorem. Let X be a Banach space with the L-approximation property, $1 \leq p < \infty$, $\epsilon > 0$, and Y a $\mathcal{B}_{2p+\epsilon}$-space. Then $B(X,Y) = A_p(X,Y)$ if and only if X is finite dimensional. The notion of a $\mathcal{L}_p$-space ($p \geq 1$) was introduced by J. Lindenstrauss and A. Pelczynski [Studia Math. 29(1968)]. Theorem. Let Y be an infinite dimensional $\mathcal{L}_\infty$-space and $1 \leq p < \infty$. Then $B(X,Y) = A_p(X,Y)$ if and only if X is finite dimensional. (Received June 29, 1970.)

677-46-19. FRANK J. WAGNER, University of Cincinnati, Cincinnati, Ohio 45221. Convex filters and Cauchy filters.

For certain bounded sets B in a locally convex space E, the minimal Cauchy filters on B coincide with the maximal convex filters on B (for special definitions see Pacific J. Math. 15(1966), 1087-1092), and consequently $\bar{B}$ is complete in the second conjugate space E'' with the natural topology. (Received June 30, 1970.)

47 Operator Theory

677-47-1. MICHAEL J. FISHER, University of Montana, Missoula, Montana 59801. Singular integrals and fractional powers of operators.

Let H be a real separable Hilbert space and $L_p(H)$ be the p-power integrable functions with respect to the weak normal distribution on H. Let $y \to T_y$ denote the regular representation of the additive group of H acting on $L_p(H)$, $1 < p < \infty$, and let B be a Hilbert-Schmidt operator on H. Let A_h be the infinitesimal generator of the semigroup T_{tBh}, $t > 0$, and let $P_y(f)$ be the Poisson integral of f in $L_p(H)$; [see M. J. Fisher, Trans. Amer. Math. Soc. 137(1969), 387-405]. Set $J^\alpha(f) = \Gamma(\alpha)^{-1} \cdot \int_0^\infty P_t(f)t^{\alpha-1}e^{-t}dt$ for $\operatorname{Re} \alpha > 0$. J^α is the αth order Bessel potential on $L_p(H)$. $L_p^\alpha(H)$ is the range of J^α on L_p with norm $\|J^\alpha f\|_{\alpha,p} = \|f\|_p$. If $(-A)$ generates a bounded semigroup and if $\operatorname{Re} \alpha > 0$, let A^α denote the αth Komatsu-power of A; [see H. Komatsu, Pacific J. Math. 19(1966), 285-346]. $A_h^\alpha J^\alpha$ is a bounded operator on $L_p(H)$ if $\operatorname{Re} \alpha > 0$. If $n < \operatorname{Re} \alpha < n+1$, set $G^\alpha(f) = \int_0^\infty \int_H R_n(f,y,t)d\mu(y)t^{-\alpha-1}dt$ where μ is a Borel measure on H such that $\int_H \|y\|^{\operatorname{Re} \alpha} d|\mu|(y) < \infty$ and $R_n(f,y,t) = (n!)^{-1} \cdot \int_0^t (t-u)^n T_{uBy} A_y^{n+1} f\, du$. Then $G^\alpha J^\alpha(f) = M(\alpha)\int_H (-A_y)^\alpha J^\alpha(f)d\mu(y)$ and $\|G^\alpha J^\alpha\| \leq N(\alpha,p)\int_H \|y\|^{\operatorname{Re} \alpha} d|\mu|(y)$. If $\operatorname{Re} \alpha = n$, a nonnegative integer, a similar result holds when additional assumptions are made on μ. (Received April 22, 1970.)

677-47-2. RICHARD C. GILBERT, California State College, Fullerton, California 92631. Spectral representation of selfadjoint extensions of a symmetric operator.

Let A_1 be a simple closed symmetric operator with deficiency indices 1,1 and deficiency subspaces M(i), M(-i) in a Hilbert space H_1. Let A_0 be a fixed selfadjoint extension of A_1 in H_1 with resolvent $R_0(\lambda)$. Let $g_1 \in M_1(i)$, $\|g_1\| = 1$, and let $g_2 = g_1 - 2i R_0(-i)g_1$. Then $g_2 \in M_1(-i)$, and $\|g_2\| = 1$. If A is a minimal selfadjoint dilation of A_1 with spectral function E(t), A is unitarily equivalent to the multiplication operator in $L_P^2(-\infty,\infty)$, where P(t) is the matrix with elements $\rho_{jk}(t) = \int_0^t (1+s^2)d(E(s)g_j,g_k)$. The spectrum of A is determined by the behavior of $\rho(t) = \rho_{11}(t) + \rho_{22}(t)$ and the spectral multiplicity by the rank of the matrix with elements $d\rho_{jk}(t)/(d)\rho(t)$. Let $\Phi_{jk}(\lambda) = \lambda(g_j,g_k) + (\lambda^2+1)(R_1(\lambda)g_j,g_k)$, where $R_1(\lambda)$ is the generalized resolvent associated with A. Using Krein's formula for $R_1(\lambda)$, the $\Phi_{jk}(\lambda)$ can be determined in terms of two functions analytic in the upper half-plane, one associated with A_0 and one with A. From the relationship between the functions $\Phi_{jk}(\lambda)$ and the functions $d\rho_{jk}(t)/(d)\rho(t)$ and $\rho(t)$, theorems on the spectrum and spectral multiplicity of A can be obtained; e.g., if the spectrum of A_0 is singular, then the spectral multiplicity of A is 1. (Received June 1, 1970.)

677-47-3. MARY R. EMBRY, University of North Carolina, Charlotte, North Carolina 28213. A connection between commutativity and separation of spectra of operators.

Using a theorem of Marvin Rosenblum ("On the operator equation BX - XA = Q", Duke Math. J. 23(1956), 263-269) the following result is obtained: Theorem. If A and B are elements of a Banach algebra $\mathcal{B}$ and $\sigma(A) \cap \sigma(B) = \emptyset$, then A and B commute with an element of $\mathcal{B}$ if and only if A + B and AB commute with that same element of $\mathcal{B}$. Corollary 1. Under the hypotheses of Theorem 1, A and B commute if and only if A + B and AB commute. For the following corollaries, let A be a continuous linear operator on a Hilbert space. Corollary 2. If $\sigma(A) \cap \sigma(A^*) = \emptyset$, then A is normal if and only if

Re A commutes with AA*. Corollary 3. If A is unitary and $\sigma(A) \cap \sigma(A^*) = \emptyset$, then A and Re A commute with exactly the same operators. Corollary 4. If either Re A or Im A is invertible, then A is normal if and only if A commutes with (Re A) (Im A). (Received June 22, 1970.)

677-47-4. JOHN DYER and PASQUALE PORCELLI, Louisiana State University, Baton Rouge, Louisiana 70803. Concerning the invariant subspace problem. Preliminary report.

In what follows H denotes a complex, separable and infinite dimensional Hilbert space. A bounded linear operator, say A, on H is called completely normal if every invariant subspace for A is reducing for A. Theorem. The following two statements are equivalent: (1) every operator (bounded and linear) has a nontrivial invariant subspace, and (2) every completely normal operator on H is normal. Several corollaries are presented. (Received June 24, 1970.)

677-47-5. JOHN DYER, Louisiana State University, Baton Rouge, Louisiana 70803. Concerning AW*-algebras. Preliminary report.

A W*-algebra is a ring of operators in the sense of von Neumann. An AW*-algebra is a C*-algebra in which every maximal commutative *-subalgebra is generated by its projections, and which contains a least upper bound for any set of orthogonal projections. The existence of an AW*-algebra, B, which is not W* was established by Dixmier, "Sur certaines espaces considérés par M. H. Stone," Summa Brasil Math. 2(1951), 185-202. The example, B, is commutative, and it is stated as a standing conjecture that an AW*-algebra is W* if its center is W*: Kaplansky "Rings of operators", Benjamin, New York, 1968, p. 123. It is shown that this conjecture is false by demonstrating a type III AW*-algebra, a, with trivial center, which is not W*. For each integer $n \geqq 0$, a contains a subalgebra a_n isomorphic to the tensor product of B with the ring of $2^n \times 2^n$ matrices, $a_n \subset a_{n+1}$, and a_0 is a maximal commutative subalgebra of a. Every positive element $x \in a$ is the least upper bound of positive elements $x_n \in a_n$. It is shown that a is an AW*-algebra using the known properties of B. (Received June 24, 1970.)

677-47-6. KARL E. GUSTAFSON and DUANE P. SATHER, University of Colorado, Boulder, Colorado 80302. A branching analysis of the Hartree equation.

It is established that the Hartree equation for the Helium atom possesses a continuous branch of nontrivial solutions $w = w(\lambda)$ emanating from the first eigenvalue of the linearized equation. (Received June 24, 1970.)

677-47-7. JOHN R. EDWARDS and STANLEY G. WAYMENT, Utah State University, Logan, Utah 84321. A v-integral representation for the continuous linear operators on spaces of continuously differentiable vector-valued functions.

Suppose X and Y are linear normed spaces, and C_1 is the space of continuously differentiable functions from $[0,1]$ into X. The authors give a representation theorem for the linear operators from C_1 into Y in terms of the v-integral operating on the function as opposed to the derivative of the function. (Received June 5, 1970.)

677-47-8. WILFRED M. GREENLEE, Northwestern University, Evanston, Illinois 60201. On two parameter singular perturbation of eigenvalues. Preliminary report.

Let $V_0 \subset H$ be complex Hilbert spaces with V_0 dense in H and the corresponding injection compact. Let b(v) be a continuous and strictly positive definite quadratic form on V_0 with corresponding bilinear form b(v,w). Let a(v) be a nonnegative quadratic form which is closed and densely defined in V_0, with domain V_1. For $0 \leqq \tau \leqq 1$ denote by V_τ the τth quadratic interpolation space between V_1 and V_0. Let c(v) be a nonnegative quadratic form defined and continuous on V_γ. Denote by $\lambda_{n,0}$ the nth eigenvalue (in increasing order according to multiplicity) of $b(u_0,v) = \lambda_0(u_0,v)_H$ for all $v \in V_0$ and for $\epsilon, \mu > 0$ let λ_n be the nth eigenvalue of $\epsilon a(u,v) + \mu c(u,v) + b(u,v) = \lambda(u,v)_H$ for all $v \in V_1$. Theorem. If the eigenfunction $u_{n,0} \in V_\tau$ and is simple then: (i) for $\tau = 1$, $\lambda_n = \lambda_{n,0} + O(\epsilon) + O(\mu)$; (ii) for $\gamma \leqq \tau < 1$, $\lambda_n = \lambda_{n,0} + o(\epsilon^\tau) + O(\mu)$; (iii) for $0 \leqq \tau < \gamma$, $\lambda_n = \lambda_{n,0} + o(\epsilon^\tau) + o(\mu^{\tau/\gamma})$; all as $\epsilon \downarrow 0$, $\mu \downarrow 0$. Under additional hypotheses similar rate of convergence theorems hold for multiple eigenvalues and for eigenfunctions. Singular perturbation results for elliptic boundary value problems follow. (Received June 26, 1970.)

677-47-9. GLENN R. LUECKE, Iowa State University, Ames, Iowa 50010. Topological properties of paranormal operators on Hilbert space.

Let B(H) be the set of all bounded endomorphisms (operators) on the complex Hilbert space H. $T \in B(H)$ is paranormal if $\|(T - zI)^{-1}\| = 1/d(z, \sigma(T))$ for all $z \notin \sigma(T)$ where $d(z,\sigma(T))$ is the distance from z to $\sigma(T)$, the spectrum of T. If $\mathcal{P}$ is the set of all paranormal operators on H, then $\mathcal{P}$ contains the normal operators, $\mathcal{N}$, and the hyponormal operators; and $\mathcal{P}$ is contained in $\mathcal{L}$, the set of all $T \in B(H)$ such that the convex hull of $\sigma(T)$ equals the closure of the numerical range of T. Thus, $\mathcal{N} \subseteq \mathcal{P} \subseteq \mathcal{L} \subseteq B(H)$. Give B(H) the norm topology. The main results in this paper are: (1) $\mathcal{N}$, $\mathcal{P}$, and $\mathcal{L}$ are nowhere dense subsets of B(H) when dim $H \geqq 2$, (2) $\mathcal{N}$, $\mathcal{P}$, and $\mathcal{L}$ are arcwise connected and closed, and (3) $\mathcal{N}$ is a nowhere dense subset of $\mathcal{P}$ when dim $H = \infty$. (Received June 26, 1970.)

677-47-10. GLENN F. WEBB, Vanderbilt University, Nashville, Tennessee 37203. Nonlinear evolution equations and product stable operators in Banach spaces.

Let X be a Banach space, let N(X) be the set of nonlinear operators from all of X to X, and let L(X) be the Banach space of Lipschitz continuous operators from X to X. Let A be a function from $[0,\infty)$ to N(X) and let F be a function from $[0,\infty)$ to L(X) such that (1) F is absolutely continuous and almost everywhere strongly differentiable on $[0,\infty)$; (2) A is continuous as a function from $[0,\infty) \times X$ to X and A is bounded on bounded subsets of $[0,\infty) \times X$; (3) $\|(I - (F(t) - F(s))A(s))p - (I - (F(t) - F(s))A(s))q\| \geqq \|p - q\|$ for all $p, q \in X$ and $0 \leqq s \leqq t$; (4) range $(I - (F(t) - F(s))A(s)) = X$ for $0 \leqq s \leqq t$. Theorem. There is a function U from $[0,\infty) \times [0,\infty)$ to N(X) such that (1) $U(v,u)p = p$ for $p \in X$ and $0 \leqq v \leqq u$; (2) $\|U(v,u)p - U(v,u)q\| \leqq \|p - q\|$ for $p, q \in X$ and $0 \leqq u \leqq v$; (3) $U(v,w)U(w,u) = U(v,u)$ for $0 \leqq u \leqq w \leqq v$; (4) if $p \in X$ and $u \geqq 0$ then $d^+/dt\ U(t,u)p = F'^+(t)\,A(t)\,U(t,u)p$ for almost all $t \geqq u$. (Received June 29, 1970.)

677-47-11. CHARLES F. AMELIN, California State Polytechnic College, Pomona, California 91766. A norm characterization of continuous linear operators on locally convex spaces. Preliminary report.

Generalizing a construction of R. T. Moore (Bull. Amer. Math. Soc. 75(1969), 68-73) if $\Gamma_X(\Gamma_Y)$ is a saturated family of seminorms on X (Y) which generates the topology of X (Y) then for every connection $\varphi : \Gamma_Y \to \Gamma_X$ we may define the normed vector space $\mathfrak{I}_\varphi(X,Y) = \{ T: T$ is linear, $T:X \to Y$ and $\|T\|_\varphi = \sup \{p(Tx): p \in \Gamma_Y, \varphi p(x) \leq 1\} < \infty\}$. B. D. Craven observed that the set of continuous linear operators from X to Y is the union over all φ of $\mathfrak{I}_\varphi(X,Y)$. We obtain a similar result using normed subspaces generated by bounded sets: if $\mathcal{J} = \{g: g:\Gamma_Y \to [0,\infty)\}$, let $B_g = \{y \in Y: p(y) \leq g(p)\}$ and $B_{\varphi g} = \{x \in X: \varphi p(x) \leq g(p)\}$. Γ_Y can be chosen so that φ is surjective and then $\{B_g: g \in \mathcal{J}\}$ ($\{B_g: g \in \mathcal{J}\}$) is a cofinal family of bdd. sets in Y (X). Let $Y_g = \bigcup\{\lambda B_g: \lambda > 0\}$ ($X_{\varphi g} = \bigcup\{\lambda B_{\varphi g}: \lambda > 0\}$) be normed by the Minkowski functional $\|\cdot\|_g$ of B_g ($\|\cdot\|_{\varphi g}$ of $B_{\varphi g}$) and suppose that $T: X_{\varphi g} \to Y_g$, then we define $\|T\|_{\varphi g} = \sup\{\|Tx\|_g: \|x\|_{\varphi g} \leq 1\}$. Theorem. A linear $T:X \to Y$ is continuous iff there exists a connection $\varphi:\Gamma_Y \to \Gamma_X$ such that for all $g \in \mathcal{J}$, $T:X_{\varphi g} \to Y_g$ and $\sup\{\|T\|_{\varphi g}: g \in \mathcal{J}\} < \infty$. Then $T \in \mathfrak{I}_\varphi$ and $\|T\|_\varphi = \sup\{\|T\|_{\varphi g}: g \in \mathcal{J}\}$. This result permits us to discuss properties of adjoints $T^*: Y_{\beta^*} \to X_{\beta^*}$ between the strong duals in the spirit of a result of Moore (loc. cit., 85-90). (Received June 29, 1970.)

677-47-12. ROBERT T. MOORE, University of Washington, Seattle, Washington 98105. Exponential distribution semigroups are C_0 on dense Banach subspaces. Preliminary report.

Lions (Portugal Math. 19(1960), 141-164) defines an exponential distribution semigroup (EDSG) on a B-space X to be a homomorphism $G:D_0 \to B(X)$ of the convolution algebra D_0 of C^∞ functions compactly supported in $(0,\infty)$, such that for some $\omega \geq 0$, $\varphi \to G(e^{-\omega t}\varphi(t)) \in B(X)$ is a tempered distribution and other technical conditions hold. These often arise as integrated forms of one-parameter operator semigroups $\{T_t: t \in [0,\infty)\}$ (frequently not C_0 at 0) by $G(\varphi)u = \int_0^\infty \varphi(t)\, T_t u\, dt \quad \forall\, u \in X$. Theorem. Let G be an EDSG. Then there exists a dense subspace $Y \subset X$, invariant under $G(D_0)$, which is a B-space with respect to a stronger norm, and a unique C_0 semigroup $\{T_t\}$ on Y such that $G(D_0)$ restricted to Y is the integrated form of $\{T_t\}$. Either Y is first-category in X, or Y = X as B-spaces and G is exactly the integrated form of $\{T_t\}$. Generalizations to holomorphic semigroups and commutative n-parameter groups are discussed. An estimate due to Fujiwara (J. Math. Soc. Japan 18 (1966), 268-274) is used. (Received June 30, 1970.)

49 Calculus of Variations and Optimal Control

677-49-1. MEHDI S. ZARGHAMEE, Arya-Mehr University of Technology, Tehran, Iran. Differential of variational forms.

Let us consider $\bar{I}(\mu) = \min_{f \in \mathfrak{I}} I(f,\mu)$ where $I(f,\mu) = \int_S \mathcal{L} f\, d\mu$, $\mathfrak{I}$ is a closed subspace of continuously differentiable functions, μ is a positive bounded regular Borel measure in R^n, $\mathcal{L}$ is a first order nonnegative differential operator, and S is a compact subset of R^n. A quantity is defined which is the derivative of $\bar{I}$ with respect to μ at a point $p \in S$. It is proved that $dI/d\mu_0 = \mathcal{L} f_0(p)$ in which f_0 is the point in $\mathfrak{I}$ at which $I(f,\mu_0)$ achieves its minimum. The construction of this derivative is through a limiting process of a special Gâteaux differential of $\bar{I}$ at μ_0. For any positive bounded regular Borel measure $\lambda << \mu_0$, the Gâteaux differential with increment λ is then $\delta\bar{I}(\mu_0; \lambda) = \int_S (dI/d\mu_0)\, d\lambda$. Hence, one can obtain first order approximation to $\bar{I}(\mu_0 + k\lambda)$ for small values of k employing only f_0. This idea is used to approximate the natural frequency of an earth model with small lateral (nonradial) variations in the rigidity and the density from the natural frequency and the mode shape for the spherically symmetric case. (Received May 11, 1970.)

50 Geometry

677-50-1. ARTHUR G. SPARKS, Georgia Southern College, Statesboro, Georgia 30458. On zero-extreme points and the generalized convex kernel.

Definition 1. Let S be a compact set in E_m. A point p in S is called a 0-extreme point of S if and only if it is not interior to a segment in S. The set of 0-extreme points of S will be denoted by E(0,S). Let K(n,S) denote the nth order convex kernel of S. If $x \in S$, let K(n,x,S) denote the nth order convex kernel of x in S. Definition 2. A compact set in E_2 which is the closure of a Jordan domain is called a compact Jordan set. Theorem. Let S be a compact Jordan set. Then K(n,S) is obtained by intersecting the set of all K(n,x,S) such that $x \in E(0,S)$. (Received June 22, 1970.)

677-50-2. SISTER M. CORDIA EHRMANN, Villanova University, Villanova, Pennsylvania 19085. Penalty for violating dimensionality in generating n-metrics.

This paper takes the somewhat intuitive dimensionality laws of Euclidean 3-space and tests their applicability in the more abstract setting of the metric space, as well as in the less familiar 2-metric, 3-metric,..., n-metric space. Generalized axiomatic definitions for these spaces are presented. It is found that bonafide n-metrics can be generated from (n - k)-metrics by methods which seemingly violate the standard geometric dimensionality laws. (An n-metric so generated is termed a k-regenerated n-metric.) However, it is proved that k-regenerated n-metrics produced by the sum function or the maximum function lack interior points of all orders. (Received June 30, 1970.)

53 Differential Geometry

677-53-1. ALAN H. THOMPSON, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. Riemannian spaces of 2-recurrent curvature.

Let $\{x^a\}$ be a local holonomic coordinate system for a curved Riemannian space V_n, then V_n is said to be of 2-recurrent curvature if there exists a tensor field $\theta_{ef} \neq 0$ such that $\nabla_e \nabla_f R^a_{\cdot bcd} = \theta_{ef} R^a_{bcd}$, where R^a_{bcd} is the curvature tensor (cf. A. Lichnerowicz, Proc. Internat. Congress Math. (Cambridge, Mass., 1950) vol. 2, Amer. Math. Soc., Providence, R.I., 1952, pp. 216-223). W. Roter (Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. 12(1964), 207-211) showed that if the metric of such a V_n is positive definite, the V_n is necessarily a space of recurrent curvature. We consider the generalization of this result for the case of indefinite metric. The principal results are: If the scalar curvature of a 2-recurrent V_n is nonzero, and either the signature of the metric is $\pm$ (n - 2) or the Ricci tensor of the V_n is (positive) (or negative) definite then the space is of recurrent curvature. We present some results for the cases in which the scalar curvature is zero, and also consider the 2-recurrent space-times of general relativity. (Received March 24, 1970.)

677-53-2. BENJAMIN HALPERN and CHARLES S. WEAVER, Indiana University, Bloomington, Indiana 47401. Bending and immersing strips and bands. Preliminary report.

A C^2 immersion f(u,v) of the rectangle $0 \leq u \leq c$, $0 \leq v \leq h$ in E^3 is flat if $(\partial f/\partial u) \cdot (\partial f/\partial u) = (\partial f/\partial v) \cdot (\partial f/\partial v) = 1$ and $(\partial f/\partial u) \cdot (\partial f/\partial v) = 0$. It is a Moebius strip if f(u,v) = f(u + c, h - v); a cylinder if f(u,v) = f(u + c,v). We prove: Theorem 1. There exists a flat immersion of a Moebius strip in E^3

if and only if $c/h > \pi/2$. Theorem 2. The standard cylinder, $f(u,v) = ((c/2\pi)\cos(2\pi u/c), (c/2\pi)\sin(2\pi u/c), v)$, $0 \leq u \leq c$, $0 \leq v \leq h$, can be turned inside out via a regular isometric homotopy in E^3 if and only if $c/h > \pi$. We also consider the general problem of classifying regular isometric homotopy classes of flat cylinders. (Received June 30, 1970.)

54 General Topology

677-54-1. FRANK C. KOST, State University College of New York, Oneonta, New York 13820. Finite products of Wallman spaces.

O. Frink (Amer. J. Math. 86(1964), 602-607) asks if every Hausdorff compactification is Wallman-type. This question is unsettled. We show that if Z_1 and Z_2 are normal bases for the closed sets of X and Y then $\omega(Z_1) \times \omega(Z_2)$ is a Wallman-type compactification of $X \times Y$. As a result a class of Wallman-type compactifications of discrete X is generated. (Received April 27, 1970.)

677-54-2. FRANK G. SLAUGHTER, JR., University of Pittsburgh, Pittsburgh, Pennsylvania 15213. Two closed cover sum theorems for the star-finite property.

Recently Y. Yasui (Proc. Japan Acad. 43(1967), 263-268) obtained a locally finite closed cover sum theorem for the star-finite property (s.f.p.). In this paper a slight generalization of Yasui's theorem is obtained as a consequence of an apparently new theorem which gives a sufficient condition for the s.f.p. to be carried from the domain of a closed mapping to its range. We also prove the Theorem. Let X be a Hausdorff space and $\{F_i\}$ a countable closed collection with the interiors of the F_i's covering X. Let Bdry F_i be locally Lindelöf for each i. Then X has the s.f.p. iff F_i has the s.f.p. for each i. This theorem generalizes another theorem of Yasui. (Received May 27, 1970.)

677-54-3. LUDVIK JANOS, University of Florida, Gainesville, Florida 32601. On homeomorphism groups of free unions.

Let X be a topological space. We denote by G(X) the group of all its autohomeomorphisms. If $\{X_\alpha \mid \alpha \in \mathcal{A}\}$ is a family of topological spaces and $Y = \Sigma_\alpha X_\alpha$ its free union, it is of interest to know how G(Y) can be determined assuming that $G(X_\alpha)$ is known for each $\alpha \in \mathcal{A}$. Let H be an abstract group and $\mathcal{A}$ an abstract set (with the discrete topology). We define the group $(H * \mathcal{A})$ as the Cartesian product $H^{\mathcal{A}} \times G(\mathcal{A})$ with the group operation defined by: $(h_1,\pi_1) \cdot (h_2,\pi_2) = (h_1 \cdot h_2 \circ \pi_1, \pi_2 \circ \pi_1)$ for $h_1,h_2 \in H^{\mathcal{A}}$ and $\pi_1,\pi_2 \in G(\mathcal{A})$. Theorem 1. If the family $\{X_\alpha \mid \alpha \in \mathcal{A}\}$ consists of connected mutually non-homeomorphic spaces then G(Y) is isomorphic to $\Pi_\alpha G(X_\alpha)$. Theorem 2. If the above family consists of connected and mutually homeomorphic spaces ($X_\alpha \approx X$ for each $\alpha \in \mathcal{A}$) then G(Y) is isomorphic to the group $(G(X) * \mathcal{A})$. (Received June 4, 1970.)

677-54-4. ROGER M. COOPER and VIRINDRA M. SEHGAL, University of Wyoming, Laramie, Wyoming 82070. Fixed point theorems of the alternative for mappings with a contractive iterate.

Let (X,d) be a generalized complete metric space, and f a self-mapping of X. Diaz and Margolis (Bull. Amer. Math. Soc. 74(1968), 305-309), have proved a theorem of the alternative for contraction mappings on X. Here, we extend their result to mappings satisfying the following condition: (*) for each $x \in X$, there is a positive integer $n(x)$ such that $d(f^{n(x)}(x), f^{n(x)}(y)) \leq k\, d(x,y)$, for some $k < 1$ and all $y \in X$. Theorem 1. If f satisfies the condition (*), then for each $x_0 \in X$, the sequence of iterates $f^n(x_0)$ has the alternatives: (A) For each $f^k(x_0)$ with $m_0 = n(f^k(x_0))$, $d(f^k(x_0), f^{k+\mathcal{L}}(x_0)) = \infty$ for some $\mathcal{L} = 1,2,\ldots,m_0$. (B) The sequence $f^n(x_0)$ is d-convergent to some $\xi \in X$ and $f(\xi) = \xi$. Theorem 2. If f is a contraction, then for any $x_0 \in X$, the sequence of iterates $f^n(x_0)$ has the alternatives: (A') For each pair of positive integers m and n, $m > n$, $d(f^m(x_0), f^n(x_0)) = \infty$. (B') There exists a subsequence $f^{n_i}(x_0)$ which is d-convergent to a $\xi \in X$ and $f^{m-n}(\xi) = \xi$. (Received June 4, 1970.)

677-54-5. ERIC S. LANGFORD, University of Maine, Orono, Maine 04473. Characterization of Kuratowski 14-sets.

Suppose that S is a topological space. A famous problem of Kuratowski asserts that no more than 14 sets can be formed from a given subset X of S by using only the operations of closure and complementation, iterated in any order; moreover, there exist examples where 14 distinct sets are actually obtained. Such sets are called 14-sets. Let k, c, b, and i denote the operations of complementation, closure, boundary, and interior respectively. Theorem 1. X is a 14-set iff the following five conditions hold: (A) $Xbi \neq \emptyset$; (B) $X \cap Xckckck \neq \emptyset$; (C) $Xk \cap Xkckckck \neq \emptyset$; (D) $Xcib \neq \emptyset$; (E) $Xkcib \neq \emptyset$. Moreover, no four conditions imply the fifth. Theorem 2. If S is connected, then A, B, and C imply D and E. Theorem 3. The set $X \cap Xckckck$ is the largest relatively open subset of X which is nowhere dense in S. Theorem 4. Suppose that X is a subset of the real line with its usual topology. Then X is a 14-set iff the following: there exists a nonempty open interval I such that X and Xk are both dense in I, and both X and Xk contain nonempty, relatively open, nowhere dense subsets of R. (Received June 12, 1970.)

677-54-6. CHIEN WENJEN, California State College, Long Beach, California 90801. On H-closedness and the Wallman H-closed extensions.

A Hausdorff space is called H-closed if it is closed in every Hausdorff space containing it. (1) A Hausdorff space is H-closed if and only if each net of open sets in it has a cluster point. (2) The cartesian product of H-closed spaces is H-closed relative to the product topology. (3) A Wallman-type H-closed extension for a Hausdorff space is constructed. (4) The Stone-Weierstrass approximation theorem holds for Hausdorff spaces. (Received June 15, 1970.)

677-54-7. CHARLES H. GOLDBERG, Trenton State College, Trenton, New Jersey 08625 and Princeton University, Princeton, New Jersey 08540. Higher genera for links.

Although every spanning surface for a knot must be connected (we do not allow closed components), links of at least two components may have disconnected spanning surfaces. Given an oriented link $\mathcal{L}$, let $\nu(\mathcal{L})$ be the maximum number of components possible in any spanning surface for $\mathcal{L}$. Definition. For every positive integer $n \leq \nu(\mathcal{L})$, define $h^n(\mathcal{L})$ to be the minimum genus possible among the genera of those spanning surfaces for $\mathcal{L}$ which have at least n components. Clearly $h^1(\mathcal{L})$ is the usual genus of $\mathcal{L}$, and $0 \leq h^1(\mathcal{L}) \leq h^2(\mathcal{L}) \leq \ldots \leq h^{\nu(\mathcal{L})}(\mathcal{L})$. The product $\mathcal{L}_1 \cdot \mathcal{L}_2$ of two oriented links is defined as in Hashizume ("On the uniqueness of the decomposition of a link," Osaka Math. J. 10(1958), 283-300). Theorem. For any positive integer $n \leq \nu(\mathcal{L})$, the nth genus $h^n(\mathcal{L}_1 \cdot \mathcal{L}_2) = \min_{n_1+n_2=n+1}[h^{n_1}(\mathcal{L}_1) + h^{n_2}(\mathcal{L}_2)]$. This result extends both the formula for the genus of a product knot and Hashizume's Theorem 2. The proof is similar to the classical case. Corollary. If $\mathcal{L}$ is obtained by tying the small knot k in a component of $\mathcal{L}_1$, then $h^n(\mathcal{L}) = h^n(\mathcal{L}_1) + h^1(k)$ for every positive integer $n \leq \nu(\mathcal{L})$. (Received June 23, 1970.)

677-54-8. JACK D. BRYANT and LAWRENCE F. GUSEMAN, JR., Texas A & M University, College Station, Texas 77843. Fixed points of subcontractive mappings.

Let (X,d) be a compact metric space. Our main result gives fixed points of continuous mappings $f: X \to X$ which are subcontractive (if $x \neq f(x)$ then $d(f^2(x), f(x)) < d(f(x),x)$) and have a local contractive iterate at each fixed point. (Received June 24, 1970.)

677-54-9. GEORGE C. GASTL, University of Wyoming, Laramie, Wyoming 82070. Bitopological spaces from quasi-proximities.

Let ρ be a quasi-proximity on a set M. Define two functions c and k from P(M) into P(M) as follows: $c(A) = \{x: (\{x\},A) \in \rho\}$ and $k(A) = \{x: (A,\{x\}) \in \rho\}$. If ρ were symmetric, and hence a proximity, then c = k. The function c is enlarging and additive and leaves the empty set fixed. It may fail to be a topological closure since c(c(A)) may not be a subset of c(A). Theorem 1. If ρ is a quasi-proximity on M which also satisfies [P.1'] $A \subseteq M$ implies $(\emptyset, A) \notin \rho$ and [P.3'] $(A \cup B, C) \in \rho$ iff $(A,C) \in \rho$ or $(B,C) \in \rho$, then k is a topological closure on M. Theorem 2. If ρ is a quasi-proximity on M satisfying [P.3''] $(A,B) \in \rho$ iff $(\{x\},B) \in \rho$ for some $x \in A$, then $c(A) = A$ implies $k(M - A) = M - A$. Theorem 3. If ρ is a quasi-proximity on M which satisfies [P.3''] and is symmetric for points, then $k \subseteq c$. If M is finite, then k = c. (Received June 18, 1970.)

677-54-10. VIRINDRA M. SEHGAL, University of Wyoming, Laramie, Wyoming 82070. Subsequences of iterates with fixed points.

Let f be a continuous self-mapping of a metric space (X,d), $O(x_0)$ be the orbit of $x_0 \in X$ and $\rho(x_0)$ the diameter of the set $O(x_0)$. Assume throughout that the sequence $f^n x_0$ has a subsequence $f^{n_i} x_0 \to \xi$. Lemma. If there exists a $V: O(x_0) \to R^+$ satisfying (1) $V(fy) \leq V(y)$ for each $y \in O(x_0)$. (2) There exists a positive integer k such that if $\mathrm{Inf}_n V(f^n y) = V(y)$ for some $y \in \overline{O(x_0)}$, implies $f^k(y) = y$, then $f^k \xi = \xi$. As consequences of Lemma we have: Theorem 1. If f satisfies $x \neq y$, $d(fx,fy) < \max\{d(x,fx), d(y,fy), d(x,y)\}$, then $f\xi = \xi$ and $\lim_n f^n x_0 = \xi$. Theorem 2. If f satisfies for some $\epsilon > 0$, the condition: $x \neq y$, $\min\{d(x,fx), d(y,fy), d(x,y)\} < \epsilon$ implies $d(fx,fy) < d(x,y)$, then $f^k(\xi) = \xi$ for some positive k. Theorems 1 and 2 are generalizations of well-known results of Edelstein. Theorem 3. If f has a diminishing orbital diameter such that $\rho(x)$ is a continuous function of x on $\overline{O(x_0)}$, then $f\xi = \xi$. Theorem 3 generalizes a result of W. A. Kirk (J. London Math. Soc. 44(1969)). (Received June 26, 1970.)

677-54-11. RICHARD B. THOMPSON, University of Arizona, Tucson, Arizona 85721. Weak semicomplexes and the fixed point theory of tree-like continua.

R. H. Bing has called the question of whether or not all tree-like continua have the fixed point property, one of the most interesting unsolved problems in geometric topology. In this paper a geometric regularity condition on cofinal sets of tree chain covers is defined and the following result is proved. Theorem. The following are equivalent conditions on a tree-like continuum T: (a) T has a regular set of covers, (b) every cofinal collection of tree chains on T is regular, (c) T admits a weak semicomplex structure. It is noted that this provides a purely topological description of the largest class of tree-like continua which can be shown to have the fixed point property by existing algebraic methods. Theorem. If a tree-like continuum T has a regular family of covers, then T has the fixed point property. Finally, several examples are given which suggest that the converse of the last theorem may be valid. (Received June 23, 1970.)

677-54-12. HOWARD H. WICKE and JOHN M. WORRELL, JR., Sandia Laboratories, Albuquerque, New Mexico 87115. Perfect mappings and certain interior images of M-spaces. Preliminary report.

The authors have characterized those regular T_0-spaces which are open continuous images of regular T_0 complete M-spaces and those which are open continuous internally uniformly μ-complete images of regular T_0 M-spaces. (A complete M-space is a space which is a quasi-perfect pre-image of a complete metric space.) These characterizations are to appear in the Proceedings of the Pittsburgh International Topology Conference 1970. The authors show here that these two classes of spaces are closed under the actions of perfect mappings (actually under the actions of closed continuous peripherally bicompact mappings). These results may be contrasted with K. Morita's example of a Hausdorff locally bicompact M-space having a perfect image which is not an M-space [Proc. Japan Acad. 43(1967), 869-872]. (Received June 29, 1970.)

677-54-13. JACK D. BRYANT and LAWRENCE F. GUSEMAN, JR., Texas A & M University, College Station, Texas 77843. Sequential conditions for fixed and periodic points. Preliminary report.

Let f be a continuous self-map of a metric space (X,d), and let $\varphi_n^k(x) = d(f^{n+k}(x), f^n(x))$. Theorem. If $\lim_n \varphi_n^k(x) = \alpha_k(x)$ exists for some k, and if $\{f^n(x)\}$ has a cluster point u, then $d(f^{n+k}(u), f^n(u)) = \alpha_k(x)$ for each $n \geq 0$. Using this theorem we clarify results of Edelstein, Bailey, and Belluce and Kirk. (Received June 29, 1970.)

677-54-14. STANISLAW G. MROWKA, State University of New York at Buffalo, Amherst, New York 14226. Nonextendable classes and perfect maps.

For notation and terminology see S. Mrowka, "Further results on E-compact spaces. I," Acta Math. 120(1968), 161-185. Theorem. A class of functions $\{f_\xi : \xi \in \Xi\}$, $f_\xi : X \to E_\xi$, where X is a completely regular space, is $\{E_\xi : \xi \in \Xi\}$-nonextendable iff the corresponding parametric map is a perfect map of X onto a closed subspace of $X\{E_\xi : \xi \in \Xi\}$. Corollary 1. If E is completely regular and X is E-completely regular, then X is E-compact iff X admits a perfect map onto a closed subspace of some power of E; more exactly $\mathrm{def}_E X \leq m$ iff X admits a perfect map onto a closed subspace of E^m. Corollary 2. If E is completely regular, X is completely regular, $f: X \xrightarrow[\text{onto}]{} Y$ is perfect, and Y is E-compact, then X is E-compact; in fact, $\mathrm{def}_E X \leq \mathrm{def}_E Y$. Note. The above corollaries remain valid for generalized classes of compact spaces, but they, as well as the theorem, fail for Hausdorff spaces. (Received June 30, 1970.)

677-54-15. ROBERT J. DAVERMAN, University of Tennessee, Knoxville, Tennessee 37916. Wild disks in E^n that can be squeezed only to tame arcs.

Let Δ_2 denote the 2-cell $\{(x,y) \in E^2 | x^2 + y^2 \leq 1\}$, Δ_1 the 1-cell $\{(x,0) \in \Delta_2 | -1 \leq x \leq 1\}$, and π the projection map of Δ_2 onto Δ_1 sending (x,y) to (x,0). Suppose D is a disk topologically embedded in the interior of an n-manifold M. A map f of M onto itself is said to squeeze D to an arc A if and only if there exist homeomorphisms g_2 of Δ_2 onto D and g_1 of Δ_1 onto A = f(D) such that $fg_2 = g_1\pi$ and f is a homeomorphism of M - D onto M - A. It is known that for each disk D in the interior of an n-manifold M ($n \neq 4$) there exists a map f of M onto itself that squeezes D to an arc. Theorem. In case n = 3 or $n \geq 5$ there exists a wildly embedded disk D in Euclidean n-space E^n such that for each map f of E^n onto itself that squeezes D to an arc, f(D) is tamely embedded. (Received June 30, 1970.)

677-54-16. WITHDRAWN.

677-54-17. HISAHIRO TAMANO, Texas Christian University, Fort Worth, Texas 76129 and J. E. VAUGHAN, University of North Carolina, Chapel Hill, North Carolina 27514. Paracompactness and elastic spaces.

A relation R on a collection U of subsets of a set X is said to be a framing of U provided for every u,v $\in$ U if $u \cap v \neq \emptyset$, then (u,v) $\in$ R or (v,u) $\in$ R. If U and V are covers of a topological space X, we say U is an elastic refinement of V with elastic map f: U $\to$ V provided there exists a transitive framed relation R on U such that for every subcollection U' $\subset$ U which has an R-upper bound (i.e., there exists u $\in$ U such that (u',u) $\in$ R for every u' $\in$ U') we have $cl(\bigcup U') \subset \bigcup f(U')$. Theorem 1. A regular space is paracompact iff every open cover has an open elastic refinement. We call a collection P of pairs $p = (p_1,p_2)$ of subsets of a space X an elastic base provided (1) p_1 is open for every p $\in$ P, (2) for each x $\in$ X and each open set u containing x, there exists a p $\in$ P such that $x \in p_1 \subset p_2 \subset u$, (3) $\{p_1 : p = (p_1,p_2) \in P\}$ is an elastic refinement of $\{p_2 : p = (p_1,p_2) \in P\}$ with respect to the map $f(p_1) = p_2$. A T_1-space with an elastic base is called an elastic space. Theorem 2. Every stratifiable space is an elastic space (but not conversely). Every elastic space is paracompact. Every subspace of an elastic space is an elastic space. (Received June 29, 1970.)

677-54-18. DARRELL C. KENT, Washington State University, Pullman, Washington 99163 and GARY D. RICHARDSON, East Carolina University, Greenville, North Carolina 27834. Minimal convergence spaces.

A convergence space (S,q) is minimal P if the space has property P and each strictly coarser convergence structure on S fails to have this property; it is P-closed if it is a closed subspace of every extension space with property P. Given a filter $\mathfrak{F}$ on S and a positive integer n, let $\overline{\mathfrak{F}}^n$ be the nth closure of $\mathfrak{F}$ and let $a(\mathfrak{F})$ be the set of all adherence points of $\mathfrak{F}$. Conditions. (A) $x \in a(\mathfrak{F})$ iff $\mathfrak{F}$ q-converges to x; (B) $x \in a(\bigcap \overline{\mathfrak{F}}^n)$ iff $\mathfrak{F}$ q-converges to x; (C) $x \in a(\overline{\mathfrak{F}}^n)$, all n, iff $\mathfrak{F}$ q-converges to x; (D) Same as (A) but restrict $\mathfrak{F}$ to having a countable filter base. Regular, Urysohn, and first countable spaces are assumed to be Hausdorff. Theorem 1. (S,q) is minimal Hausdorff (resp., minimal regular, minimal Urysohn, minimal first countable) iff condition (A) (resp., (B), (C), (D)) is satisfied.

Theorem 2. (a) A Hausdorff space is Hausdorff-closed iff it is compact. (b) A first countable space is first-countable-closed iff it is countably compact. (c) A regular space is regular-closed iff, for each filter $\mathfrak{F}$, $\alpha(\bigcap \overline{\mathfrak{F}}^n) \neq \emptyset$. (d) A Urysohn space is Urysohn-closed iff, for each U-filter $\mathfrak{F}$, $\alpha(\mathfrak{F}) \neq \emptyset$; $\mathfrak{F}$ is a U-filter if x in $\alpha(\mathfrak{F})$ and $\mathcal{J} \to x$ imply that $\overline{\mathfrak{F}}^n \vee \overline{\mathcal{J}}^n \neq \emptyset$, all n. (Received June 30, 1970.)

55 Algebraic Topology

677-55-1. ROBERT P. WALKER, University of North Carolina, Chapel Hill, North Carolina 27514. Whitehead torsion for free simplicial groups.

Let Wh: (groups) → (abelian groups) denote the Whitehead group functor. Analogous to the theory developed by J. H. C. Whitehead for finite CW-complexes, a notion of torsion $\tau(f) \in Wh(\pi_1 Y)$ is defined for homotopy equivalences f: X → Y of finitely generated, free simplicial groups. It is proved that (i) if f is homotopic to f', then $\tau(f) = \tau(f')$; (ii) if g: W → X is another homotopy equivalence, then $\tau(fg) = \tau(f) + Wh(f)\tau(g)$; (iii) $\tau(f) = 0$ if and only if f is homotopic to a finite composition of certain elementary deformations; and (iv) two finitely generated, free simplicial groups G and G' have the same simple homotopy type (i.e., there is a homotopy equivalence between them with zero torsion) if and only if there exist contractible, finitely generated, free simplicial groups D and D' such that $G * D$ is isomorphic to $G' * D'$, where $*$ denotes free product. (Received June 29, 1970.)

58 Global Analysis, Analysis on Manifolds

677-58-1. JAMES R. DORROH, Louisiana State University, Baton Rouge, Louisiana 70803. Local groups of differentiable transformations.

Let S be a class $C^{(k)}$ Banach manifold, let r denote a positive integer with $r \leq k$, and let R denote the real number line. By a semi-$C^{(r)}$ local transformation group in S, we mean a continuous function T from an open subset M of R × S into S such that $(0,p) \in M$ and $T(0,p) = p$ for each $p \in S$, $T(u,T(t,p)) = T(t+u,p)$ for (t,p), $(u,T(t,p))$, $(t+u,p) \in M$, and T has continuous second place partial derivatives through the rth order. It is proven that every such function T is a class $C^{(r)}$ function (jointly in both variables) and is a local flow of a vector field on S. The vector fields on S which have class $C^{(r)}$ local flows are characterized. Even if S is an open subset of a Banach space, it is still necessary to consider nontrivial differentiable structures on S in order to characterize these vector fields. (Received June 22, 1970.)

677-58-2. MOSES GLASNER, California Institute of Technology, Pasadena, California 91109, RICHARD KATZ, California State College, Los Angeles, California 90032 and MITSURU NAKAI, Nagoya University, Nagoya, Japan. Examples in the classification theory of Riemannian manifolds with respect to the equation $\Delta u = Pu$.

Let R be a Riemannian manifold and P a nonnegative C^1 n-form on R. Under the assumption that there exists a subregion Ω of R such that Ω carries a nonzero harmonic function with finite Dirichlet integral vanishing on $\partial\Omega$ and $\int_\Omega P < \infty$ the following was recently shown (J. Analyse Math.

22(1969), 343-354): there exists a nonzero bounded solution of $\Delta u = Pu$ on R if and only if there exists a nonzero energy-finite solution on R. This paper establishes the necessity of this condition. (Received June 29, 1970.)

60 Probability Theory and Stochastic Processes

677-60-1. ALI M. TABATABAIAN-KASHANI, San Francisco State College, San Francisco, California 94132. Quasi-invariant, non-Gaussian random linear functionals on Hilbert space. Preliminary report.

A random linear functional (or weak distribution) X is a linear mapping from a linear space S to the space of measurable functions on some probability space. It induces a measure m on S^*, the algebraic dual of S. This measure is sometimes concentrated on a subspace of S^*. If M is a subset of S^*, the measure m is said to be quasi-invariant under M if it is equivalent to its translates by elements of M. The random linear functional (r.l.f.) is called quasi-invariant if m is, and is said to be Gaussian if X(s) has Gaussian distribution for every $s \in S$. The best known quasi-invariant random linear functionals are those which are Gaussian. In this paper certain classes of random linear functionals are constructed on Hilbert space which are realizable as measures on R^∞. They are quasi-invariant under ℓ^2-translations of R^∞, but are not equivalent to any Gaussian r.l.f. First, conditions are obtained for quasi-invariance of product measures on infinite product spaces. We then use quasi-invariant product measures to construct quasi-invariant random linear functionals on Hilbert space. Conditions are obtained for the r.l.f. not to be equivalent to any Gaussian one. Finally some examples of quasi-invariant, non-Gaussian random linear functionals are given. (Received June 22, 1970.)

677-60-2. BENON J. TRAWINSKI, University of Alabama, Birmingham, Alabama 35233. Probability distributions on algebraic structures induced by groups of permutations.

Let G_k be the group of permutation matrices of order k, and let v be an ordered k-tuple of distinct real numbers. Consider a random sample $M_1, M_2, \ldots, M_n$ from G_k, and for each n in the set of natural numbers N, denote by $S_{k,n}$ the range of $X_n = \Sigma_{\gamma=1}^{n} M_\gamma v$. It is shown that (1) for any m in N, there is a sequence $\{\psi(j)\}$, with $\psi(0) = m$, and a continuous mapping $h_{m,\psi(j)}$ such that $\{S_{k,\psi(j)}, h_{m,\psi(j)}\}$ is an inverse limit system with inverse limit space $S_{k,\infty}$, (2) subsets $I_{k,\infty}$ $[H_{k,\infty}]$ of $S_{k,\infty}$ have group structures isomorphic [homomorphic] to G_k, (3) there are probability distributions for which $\{X_{\psi(j)}, \psi(j) \in N\}$ is a stochastic process. Let π_n be the projection of $S_{k,\infty}$ on $S_{k,n}$, and let J_k $[K_k]$ stand for the union of isomorphic [homomorphic] structures in $S_{k,\infty}$ having projections in $S_{k,n}$. When the distribution is uniform, $P[\pi_n^{-1}(X_n) \in J_3] < 2(m+1)^{-1}$, and $P[\pi_n^{-1}(X_n) \in K_3] > (m-1)(m+1+m^{-1})^{-1}$ whenever $n \geq m$. (Received June 26, 1970.)

677-60-3. GEORGE YU-HUA CHI, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. Nonlinear prediction of generalized random processes.

Let (D) be the Schwartz space of infinitely differentiable scalar functions, φ, on the real line with compact supports, $\sigma(\varphi)$, and (Ω, Σ, P) be a fixed complete probability space. Let X: (D) $\rightarrow L^P(\Omega, \Sigma, P)$ with $1 < p < \infty$ be a generalized random process, $T = (-\infty, t)$, and $\mathcal{B}_T = \mathcal{B}(X(\varphi) : \sigma(\varphi) \subset T)$,

the σ-field generated by the random variables shown. Let $\{\tilde{\varphi}_i\}_{i=1}^{\infty} \subset (D)$ be such that $\sigma(\tilde{\varphi}_i) \not\subset T$, $i \geqq 1$. Then, relative to the pth moment of the error as the criterion of optimality, there exists a weak generalized random process (i.e., a closed linear transformation) $Y: \mathcal{D}(Y) \to L^p(\Omega, \Sigma, P)$, where $\mathcal{D}(Y)$ is dense in (D), such that $Y(\tilde{\varphi}_i)$ is the best nonlinear predictor for $X(\tilde{\varphi}_i)$, $i \geqq 1$, in the sense that $\|Y(\tilde{\varphi}_i) - X(\tilde{\varphi}_i)\|_p$ is a minimum for each $i \geqq 1$. (Received June 29, 1970.)

677-60-4. SEIZÔ ITÔ, University of Wisconsin, Madison, Wisconsin 53706. Ideal boundaries associated with elliptic differential operators of second order. Preliminary report.

Let A be the second order elliptic differential operator with variable coefficients on a manifold R and A* be the formally adjoint operator to A. Let S_M be the ideal boundary of R of Martin type associated with A and S_N be that of Neumann type associated with A*. Then both $R_M = R + S_M$ and $R_N = R + S_N$ are homeomorphically imbedded into a certain compact space $\hat{R} = R + \hat{S}$ in such a way that both homeomorphisms are identity mapping on R and accordingly that both S_M and S_N are imbedded in $\hat{S}$. The 'ideal boundary' $\hat{S}$ is divided into four parts--natural boundary, exit boundary, entrance boundary and regular boundary. Some partial results similar to a part of Feller's theory of classification of the boundary points associated with the one-dimensional diffusion equation are obtained. (Received June 29, 1970.)

677-60-5. ITREL E. MONROE, Dartmouth College, Hanover, New Hampshire 03755. Limit theorems for a class of stopping times. Preliminary report.

Let W(t) be the Wiener process, let X be a random variable with mean zero in the domain of attraction of a stable distribution of index $\alpha > 1$, and let T be a stopping time, defined in the manner proposed by Skorokhod, such that W(T) has the same distribution as X. Then if $\alpha < 2$, T is in the domain of attraction of the one sided stable distribution of index $\alpha/2$. If $\alpha = 2$, then there is a sequence $\{a_n\}$ such that $a_n^{-1}\Sigma_{i=1}^n T_i$ converges to 1 in probability where the T_i are independent copies of T. (Received June 30, 1970.)

68 Computer Science

677-68-1. DAVID C. RINE, University of Iowa, Iowa City, Iowa 52240. A theory of general machines and functor projectivity.

Let $\mathcal{M}$ be the category of monoids, $\mathcal{S}$ the category of sets, and $S: \mathcal{M} \to \mathcal{S}$ the forgetful functor. It is known that $M \in \mathcal{M}$ is free iff it is S-projective; input monoids M of machines must be free in order that the divisibility hypothesis of Krohn and Rhodes theory be satisfied. The author has replaced $\mathcal{M}$, $\mathcal{S}$, S by arbitrary $\mathcal{C}$, $\mathcal{B}$, F and has attempted to isolate an F-projective concept that is completely different from the free concept (yet having familiar projective properties) so that a new theory of divisibility with purely categorical tools can be considered. The class of all abstract machines $(\mathcal{M}, \mathcal{S})$ together with their morphisms is a pseudo-category that has not until now been explored. The author introduces a study of arbitrary pairs $(\mathcal{O}', \mathcal{O})$ with $\mathcal{O}' \subset \mathcal{O}$, a general machine theory, and is able to state a characterization theorem for these. Moreover, an interesting way of realizing a transition system for a general abstract machine is noted. (Received May 19, 1970.)

73 Mechanics of Solids

677-73-1. K. L. ARORA, 582, Sector 11B, Chandigarh 11, India and Pubjab Engineering College, Punjab University, Chandigarh, India. Thermoelastic stresses in the flexural deformation of a block.

Using finite deformation theory, thermoelastic stresses produced as a result of flexure and extension including axial dislocation of a block under nonuniform temperature distribution have been evaluated. The effect of temperature distribution on the stresses is shown by comparison of the corresponding force systems for a Mooney type material. (Received March 16, 1970.)

76 Fluid Mechanics

677-76-1. P. PURI and PREM K. KULSHRESTHA, Louisiana State University, New Orleans, Louisiana 70122. Rotating flow of non-Newtonian fluids.

The problem of an elastico-viscous fluid resting on a plate which moves with a time dependent velocity in its own plane (along negative x direction) and rotating with a constant velocity ω along with the fluid as a rigid body has been discussed. It is found that the elastic property of the fluid increases the drag and the lateral stress on the plate. The rotation reduces a fluid motion in the y direction. For a fixed time and distance from the plate, the velocity fluctuates with decreasing magnitudes with increasing ω, with the result that the boundary layer thickness reduces as ω increases. The stresses increase parabolically with respect to ω. The solution for an arbitrary velocity of the plate is also presented. In case of impulsive flows a secondary boundary layer whose thickness is of order $\sqrt{\nu t}$ (ν is the kinematic viscosity) is found to develop for very short times. (Received June 19, 1970.)

90 Economics, Operations Research, Programming, Games

677-90-1. ZLOBEC SANJO, McGill University, Montreal 110, Canada. Doubly asymptotic Kuhn-Tucker conditions in mathematical programming. Preliminary report.

For definitions see Guignard, SIAM J. Control 7(1969), 232-241. Consider the mathematical programming problem: maximize $\{\psi(x): \alpha(x) \in B, x \in C\}$, where $\psi: X \to R^1$ is a real function of X, $\alpha: X \to Y$ is a mapping between real Banach spaces X and Y, while $B \subset Y$ and $C \subset X$ are some non-empty sets. Let $A = \{x \in C: \alpha(x) \in B\}$, $P(A,\bar{x})$ be the pseudotangent cone to A at $\bar{x}$, P^+ the polar of P, $\nabla\psi(\bar{x})$ and $\nabla\alpha(\bar{x})$ the Frechét derivatives of ψ and α at $\bar{x}$ and $K = \{k \in X: \langle \nabla\alpha(\bar{x}), k\rangle \in P[B,\alpha(\bar{x})]\}$. Theorem. If G is a closed convex cone in X such that $K \cap G = P(A, \bar{x})$, then a necessary condition for $\bar{x}$ to maximize ψ over A is that there exist a sequence $\{u^{i,j}\}$, $u^{i,j} \in P^+[B, \alpha(\bar{x})]$ i,j = 1,2,... and a sequence $\{g^i\}$, $g^i \in G^+$ i = 1,2,... such that $\nabla\psi(\bar{x}) + \lim_{i\to\infty}\{\lim_{j\to\infty} u^{i,j}\nabla\alpha(\bar{x}) + g^i\} = 0$. This condition is also sufficient if G is a closed convex cone in X such that $x - \bar{x} \in G$ for all $x \in A$, if A or $\Delta = \{x \in X: \alpha(x) \in B\}$ is pseudoconvex at $\bar{x}$ and if either ψ is pseudoconcave over A at $\bar{x}$, or quasiconcave with $\nabla\psi(\bar{x}) \neq 0$. If both $K^- + G^-$ and $H = \{h \in X^*: h = u\nabla\alpha(\bar{x}), u \in P^-[B,\alpha(\bar{x})]\}$ are closed, then the above condition assumes the form $\nabla\psi(\bar{x}) + u\nabla\alpha(\bar{x}) \in G^-$, where $u \in P^+[B,\alpha(\bar{x})]$. (Received June 30, 1970.)

ABSTRACTS PRESENTED TO THE SOCIETY

The next deadline for Abstracts will be September 3, 1970. The papers printed below were accepted by the American Mathematical Society for presentation by title. The abstracts are grouped according to subjects chosen by the author from categories listed on the abstract form. The miscellaneous group includes all abstracts for which the authors did not indicate a category.

One abstract presented by title may be accepted per person per issue of these *Notices*. Joint authors are treated as a separate category; thus, in addition to abstracts from two authors individually, one joint abstract by them may be accepted for a particular issue.

Algebra & Theory of Numbers

70T-A129. GEORGE A. GRÄTZER and H. LAKSER, University of Manitoba, Winnipeg 19, Manitoba, Canada. The Amalgamation Property in equational classes of pseudocomplemented distributive lattices.

For notations and terminology see the Abstract 70T-A48, these *Notices* 17(1970), 429. A class K of algebras has the Amalgamation Property if, for algebras A, B, C $\in$ K, A a subalgebra of B and C, there exists an algebra D $\in$ K such that B and C are subalgebras of D and $B \cap C \supseteq A$. A class K of algebras has Property (P) if for algebras A_1, A, B_1, B, C_1, C $\in$ K, where C is the free K-product of A and B, A_1 is a subalgebra of A, B_1 is a subalgebra of B, C_1 the subalgebra of C generated by A_1 and B_1, then C_1 is the free K-product of A_1 and B_1. B. Jónsson proved that the Amalgamation Property implies (P). Theorem 1. For the classes $\mathcal{B}_n$ the following are equivalent: (i) the Amalgamation Property; (ii) Property (P); (iii) $n \leqq 2$ or $n = \omega$. The amalgamation class of a class of algebras K consists of all algebras A $\in$ K such that for all B, C $\in$ K, A a subalgebra of B and C, the amalgamation can be effected. A characterization of the amalgamation class of $\mathcal{B}_n$ is given which implies Theorem 2. A finite algebra A is in the amalgamation class of $\mathcal{B}_n$, $n < \omega$, iff A has no homomorphism onto $\overline{B}_i$ for any i, $2 \leqq i < n$. (Received February 27, 1970.)

70T-A130. THOMAS W. CUSICK, State University of New York at Buffalo, Amherst, New York 14226. Simultaneous Diophantine approximation of rational numbers.

For any real number x, let $\|x\|$ denote the distance from x to the nearest integer. Let n be any positive integer and let $\sigma = (s_1,\ldots,s_n)$ denote an arbitrary point in the set S^n of n-dimensional points all of whose coordinates are rational noninteger numbers. Define $\omega(n) = \inf_\sigma \sup_q \min_{1\leqq i\leqq n} \|qs_i\|$, where the infimum is taken over all σ in S^n and the supremum is taken over all integers q. If $z > 1$ is an integer divisible by k distinct primes, define $h(z) = k$. Then for each positive integer n define the function w(n) by $w(1) = 1/3$, $w(2) = 1/5$ and $w(n) = \max\{z: h(z) + (1/2)\varphi(z) \leqq n\}$ for $n \geqq 3$ (here φ is Euler's function). The main purpose of the paper is to propose the conjecture that $\omega(n) = 1/w(n)$ for every $n \geqq 1$, and to prove the conjecture for $n \leqq 7$. The problem of evaluating $\omega(n)$ originated in two papers of J. M. Wills (Monatsh. Math. 72 (1968), 254-263; 368-381); he proved $\omega(1) = 1/3$ and $(2n^2)^{-1} \leqq \omega(n) \leqq 1/w(n)$ for $n \geqq 2$. (Received April 24, 1970.)

70T-A131. JOHN B. WAGONER, University of California, Berkeley, California 94720. On K_2 of the Laurent polynomial ring. Preliminary report.

Let K_2 denote the algebraic K-theory functor defined by Milnor. Let R be an associative ring with unit and $R[t,t^{-1}]$ be the Laurent polynomial ring over R where $at^p \cdot bt^q = abt^{p+q}$. Let $R[t^\epsilon] \subset R[t,t^{-1}]$ (ϵ = +1 or -1) be the ring of polynomials in t^ϵ with coefficients in R. Theorem. There is a natural isomorphism $K_2(R[t,t^{-1}]) \cong K_2(R) \oplus K_1(R) \oplus (?)$. The unknown summand contains the image of $K_2(R[t^\epsilon])$ under the homomorphism $K_2(R[t^\epsilon]) \to K_2(R[t,t^{-1}])$. (Received April 24, 1970.)

70T-A132. JAMES W. PETTICREW, Indiana State University, Terre Haute, Indiana 47809. Multivalued homomorphisms between abelian groups.

Let $\mathcal{G}_i$ be abelian groups with operators Ω_i - $1 \in \Omega_i$ for i = 1,2. Let $(-1,-1) \in \Gamma \leqq \Omega_1 \times \Omega_2$. Denote by $\mathrm{Hom}_\Gamma(\mathcal{G}_1,\mathcal{G}_2)$ the set of all $\Phi \nmid R \leqq \mathcal{G}_1 \times \mathcal{G}_2$ such that for (x,y), (u,v) $\in$ R, $(\alpha,\beta) \in \Gamma$ we have (x + y, u + v), $(\alpha x, \beta y) \in R$. This is a generalization of MacLane's additive relation (Proc. Nat. Acad. Sci. U.S.A. 47(1961), 45-56). Next consider $\mathcal{R}_i = \mathrm{End}(\mathcal{G}_i)$ and let Γ' be the subring of $\mathcal{R}_1 \times \mathcal{R}_2$ generated by Γ. Theorem 1. $R \in \mathrm{Hom}_\Gamma(\mathcal{G}_1,\mathcal{G}_2)$ iff $R \in \mathrm{Hom}_{\Gamma'}(\mathcal{G}_1,\mathcal{G}_2)$. Henceforth assume $\Gamma = \Gamma'$. Define $\mathcal{D}_1(R)$ = Dom(R), $\mathcal{D}_2(R) = \mathcal{D}_1(R^{-1})$, $\mathcal{K}_1(R) = \{x \mid (x,0) \in R\}$, and $\mathcal{K}_2(R) = \mathcal{K}_1(R^{-1})$. It is known that $\mathcal{R}_i = \mathcal{D}_i(\Gamma)/\ {}_i(\Gamma)$ then $\mathcal{R}_1 \approx \mathcal{R}_2$. Theorem 2. $\mathcal{G}_i$ is an $\mathcal{R}_i$ module iff $\mathcal{K}_i(\Gamma) \leqq \mathcal{A}(\mathcal{G}_i)$, where $\mathcal{A}(\mathcal{G}_i)$ is the annihilator of $\mathcal{G}_i$ in $\mathrm{End}(\mathcal{G}_i)$. If this is the case for i = 1,2 and we identify $\mathcal{R}_2$ with $\mathcal{R}_1$, then we have MacLane's additive relations. For R, S $\in \mathrm{Hom}_\Gamma(\mathcal{G}_1,\mathcal{G}_2)$ define $R +_1 S = \{(x,y) \mid (x,y_1) \in R,\ (x,y_2) \in S,\ y = y_1 + y_2\}$, $R +_2 S = R^{-1} +_1 S^{-1}$, and $2R = R +_1 R$, $1/2R = R +_2 R$. Note that $+_1$ and $+_2$ are dual concepts. Theorem 3. $(\mathrm{Hom}_\Gamma(\mathcal{G}_1,\mathcal{G}_2), +_i)$ is an abelian semigroup. Further the following are equivalent: (1) $R \leqq S$; (2) $2R +_1 S = 2S +_1 R$, $0 +_1 S = 0 +_1 R +_1 S$, and $I +_1 R +_1 S = I +_1 R$ where $0 = \{(0,0)\}$ and $I = \mathcal{G}_1 \times \mathcal{G}_2$. (Received April 27, 1970.)

70T-A133. ALBERT A. MULLIN, USATACOM, Warren, Michigan 48090. Modified unitary divisors and totally prime sequences. Preliminary report.

This note supplements results of E. Cohen, M. V. Subbarao, and L. J. Warren on unitary divisors and results of R. Bellman, J. Lambek, and L. Moser on relatively prime sequences. Definitions. A divisor d of n is called modified unitary if the mosaics of d and n/d have no prime in common. Let s(n) be the sum of the modified unitary divisors of n. If s(n) = 2n call n a modified unitary perfect number (MUPN). Lemma 1. There exist MUPN; e.g., 6 and 60. Lemma 2. Every MUPN is a unitary perfect number; however, the unitary perfect numbers 90 and 87, 360 are not MUPN's. Lemma 3. There does not exist an odd MUPN. Hyperthesis. The existence of infinitely many MUPN's entails the infinitude of unitary perfect numbers. Conjecture. No MUPN > 60 exists (since it must have more than 25 digits, if it exists). Definition. A sequence of integers is totally prime if (1) $a_m \neq 0$ except for at most one m and (2) the mosaics of a_r and a_s have no prime in common, for every pair of distinct r and s. Lemma 4. There exist infinitely many totally prime sequences. Lemma 5. Every totally prime sequence is relatively prime, but not conversely. (Received April 29, 1970.)

70T-A134. STEPHEN W. SMOLIAR, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. A characterization of transversal pregeometries. Preliminary report.

Given a bipartite graph of a relation between two finite sets, S and T, one can define a pregeometry (sic matroid) on S whose independent sets are those subsets of S which dominate a matching--i.e. which possess a 1-1 map into T which can be embedded in the graph. Such pregeometries are called transversal pregeometries. By means of a special projection operation, called a free projection, one may determine whether or not an arbitrary pregeometry is transversal. The algorithm is constructive, in that if the pregeometry is transversal, it provides a bipartite graph which defines it, as well as providing a coordinatization of the pregeometry over a field of characteristic zero. (Received April 30, 1970.)

70T-A135. PRABHA GAIHA, Northwestern University, Department of Industrial Engineering, Evanston, Illinois 60201. A Perron-Frobenius theory for the generalized eigenvalue problem $Bx = \lambda Ax$ in complex matrices. Preliminary report.

A Perron-Frobenius theory for the generalized eigenvalue problem has recently been developed by Mangasarian. Here the generalized eigenvalue problem is reformulated in terms of a solid and pointed cone in the complex space C^n. It is shown that his results can be extended to this case. Also a notion of (A, S)-irreducibility for rectangular matrices is introduced and studied, where A is an $m \times n$ complex matrix of full column rank and S is a solid pointed cone in C^n. This notion is an extension of Vandergraft's definition of S-irreducibility given for square matrices. (Received April 20, 1970.) (Author introduced by Professor Adi Ben-Israel.)

70T-A136. GEORGE S. MONK, University of Washington, Seattle, Washington 98105. On the endomorphism ring of an abelian p-group and of a large subgroup.

In [Trans. Amer. Math. Soc. 141(1969), 99-105] P. Hill indicates how one would prove that if G is an abelian p-group with $p \neq 2$ and L is a large subgroup of G, then the endomorphism ring of G is generated by its units if this property holds for L. A counterexample to this assertion is found. The principal tool used in constructing the example is the following extension of a theorem of A.L.S. Corner [Quart. J. Math. Oxford Ser. (2) 20(1969), 277-296]: Theorem. Let B_1 be a large subgroup of a countable direct sum B_2 of cyclic p-groups and let C_i $(i = 1,2)$ be the torsion completion of B_i $(C_1 \subseteq C_2)$. Suppose R_0 is a countable unital subring of $E(B_1)$ with the property: If $\alpha \in R_0$ and $(p^nB_2)[p]\alpha = 0$ for some integer n, then $\alpha = p\beta + \sigma$, some $\beta \in R_0$ and $\sigma \in E_s(B_1)$. Let R_1 be the p-adic closure of $R_0 + E_s(B_1)$ in E(B) and $R_2 = \{\alpha \in E(B_2) | (B_1|\alpha) \in R_1\}$. Then there are subgroups G_i $(i = 1,2)$ pure in C_i, containing B_i such that $E(G_i) = E_s(G_i) + R_i$ and G_1 is large in G_2. (Received May 1, 1970.)

70T-A137. JOHN W. TAYLOR, University of Illinois, Urbana, Illinois 61801. An extension of Stone's partitioning theorem.

Let k, m be cardinals with $k \geq m \geq 1$ and $k \geq \aleph_0$. Suppose that M is a set of cardinal m and for each $\alpha \in M$ there is given a set X_α and a binary relation R_α on X_α. (The sets X_α need not be disjoint.) A subset S of $X = \bigcup\{X_\alpha : \alpha \in M\}$ is cofinal in X if for each $\alpha \in M$ and for each $x \in X_\alpha$ there exists $y \in S$ with $(x,y) \in R_\alpha$. Theorem. Assume that for each $\alpha \in M$ either R_α is transitive or $|X_\alpha| = k$ (or both). Then there is a partition of X into k cofinal subsets iff for each $\alpha \in M$ and for each $x \in X_\alpha$ there are at least k members y of X_α satisfying $(x,y) \in R_\alpha$. This extends a theorem of A.H. Stone (Mathematika 15(1968), 217-222) who proved the case m = 1 for a transitive relation. Our method of proof is quite different from that of Stone. (Received May 4, 1970.)

70T-A138. DRAGOMIR Ž. DJOKOVIĆ, Department of Pure Mathematics, University of Waterloo, Waterloo, Ontario, Canada. A determinantal inequality for projectors in a unitary space.

Let V be a finite dimensional unitary space and (1) $V = V_1 + \dots + V_k$ a direct decomposition. Let P_i be the orthogonal projector in V with range V_i. Theorem 1. If $A = P_1 + \dots + P_k$ then $0 < \det(A) \leq 1$ and $\det A = 1$ if and only if the decomposition (1) is orthogonal. Theorem 2. Let N_i, $1 \leq i \leq k$ be normal operators in V of rank r_i. Assume that $N = \sum_{i=1}^k N_i$ has rank $r = r_1 + \dots + r_k \leq n = \dim V$. If the nonzero eigenvalues of N (counting multiplicities) are the same as the nonzero eigenvalues of all N_i together, then $N_iN_j = 0$ for $i \neq j$. Theorem 2 generalizes a result of L. Brand (Proc. Amer. Math. Soc. 22(1969), 377). (Received April 27, 1970.)

70T-A139. MICHAEL DOOB, University of Manitoba, Winnipeg 19, Manitoba, Canada. A geometric interpretation of the least eigenvalue of a line graph. Preliminary report.

It has been shown by A. J. Hoffman that all eigenvalues of a line graph are greater than or equal to - 2. The following results will be given concerning the eigenvalue - 2: Theorem 1. If each edge in G is labelled by its component in x, then $A(L(G))x = -2x$ if and only if the sum of the labels around each vertex is zero. Theorem 2. The multiplicity of - 2 as an eigenvalue of A(L(G)) is equal to the number of independent even cycles. Theorem 3. L(G) has all its eigenvalues strictly greater than - 2 only when G is a tree or G has exactly one cycle and this cycle is odd. In this case the least eigenvalue asymptotically approaches - 2 as the diameter of the graph gets large. Theorem 4. The eigenspace corresponding to the eigenvalue - 2 has a basis with every entry equal to ± 1, ± 2, or 0. Every vector in the eigenspace is such that the sum of its coordinates is equal to zero. (Received May 4, 1970.)

70T-A140. PAUL E. HIMELWRIGHT and JAMES WILLIAMSON, Western Michigan University, Kalamazoo, Michigan 49001. A class of 1-factorable graphs. Preliminary report.

A 1-factor of a graph G is a spanning 1-regular subgraph of G. A graph is 1-factorable if it can be expressed as a collection of edge-disjoint 1-factors. The composition G[H] of a graph G with a graph H disjoint from G is the graph defined by: $V(G[H]) = V(G) \times V(H)$; $E(G[H]) = \{[(u_1,v_1)\cdot(u_2,v_2)] \mid u_1u_2 \in E(G)$ or $u_1 = u_2$ and $v_1v_2 \in E(H)\}$. The product $G \times H$ is defined by: $V(G \times H) = V(G) \times V(H)$; $E(G \times H) = \{[(u_1,v_1)(u_2,v_2)] \mid u_1 = u_2$ and $v_1v_2 \in E(H)$ or $v_1 = v_2$ and $u_1u_2 \in E(G)\}$. Theorem 1. If G is 1-factorable, then $G[\overline{K}_r]$ is 1-factorable. Theorem 2. If G and H are 1-factorable, then G[H] is 1-factorable. From these results, we have: Theorem 3. A complete n-partite graph G is 1-factorable if and only if it is regular of degree r(n - 1), where rn is even. Theorem 4. If G is 1-factorable and H is regular, then $G \times H$ is 1-factorable. (Received May 8, 1970.)

70T-A141. K. NAGESWARA RAO, North Dakota State University, Fargo, North Dakota 58102. On a congruence equation in $GF[p^n,x]$. Preliminary report.

R is a monic polynomial in $GF[p^n,x]$ and (X,R) denotes the greatest common divisor of X and R.

For a fixed N of GF$[p^n,x]$ the number of solutions of the congruence equation: $N \equiv X_1 + X_2 + \ldots + X_s \pmod{R}$ is obtained with the side condition that (X_i,R) $(i = 1,\ldots,s)$ is a square. The multiplicative properties of the related arithmetical functions are also obtained. (Received May 8, 1970.)

70T-A142. JONATHAN S. GOLAN, The Hebrew University, Jerusalem, Israel. Characterization of rings using quasiprojective modules. II. Preliminary report.

A left module over an associative ring R with 1 is quasiprojective iff, for every epimorphism $f: M \to N$, Hom (M,f): Hom$(M,M) \to$ Hom(M,N) is an epimorphism. A change-of-rings theorem for quasi-projectivity is proven and the results are applied to characterizing semiperfect, semihereditary, and hereditary rings. Theorem 1. The following are equivalent: R is semiperfect; (2) For all $n \geqq 1$, every cyclic R_n-module has a quasiprojective cover; (3) There exists an $n > 1$ such that every cyclic R_n-module has a quasiprojective cover. Theorem 2. The following are equivalent: (1) R is left semi-hereditary; (2) Every finitely-generated submodule of a projective left R-module is quasiprojective; (3) Every finitely-generated left ideal of R_n is quasiprojective for all $n \geqq 1$. Theorem 3. The following are equivalent: R is left hereditary; (2) Every submodule of a projective left R-module is quasipro-jective; (3) Every principal left ideal of E is quasiprojective, where E is the endomorphism ring of a free R-module. These results continue the author's previous results on characterization of rings (to appear, Israel J. Math.). (Received May 11, 1970.)

70T-A143. ROBERTO L. O. CIGNOLI, Instituto de Mathemática, Universidad Nacional del Sur, Bahía Blanca, Argentina. The representation of Moisil and Post algebras by continuous functions.

For definitions see Abstract 70T-A46, these *Notices* 16(1970), 428. L_n denotes the Moisil algebra of order n formed by the fractions $j/n - 1$, $j = 0,1,\ldots, n - 1$, with the natural lattice operations and - and s_i $(i = 1,2,\ldots, n - 1)$ defined as follows: $-(j/n - 1) = 1 - (j/n - 1)$ and $s_i(j/n - 1) = 0$ if $i + j < n$ and $= 1$ if $i + j \geqq n$. If X is a Boolean space (= totally disconnected compact Hausdorff space), $C_n(X)$ denotes the Moisil algebra of order n of all the continuous functions from X into L_n considered as a subspace of the real numbers, and K(X) the subalgebra of $C_n(X)$ of the functions with values 0 or 1. Theorem. For any Moisil algebra A of order n there exists a Boolean space X, unique up to homeo-morphisms, such that: (1) A is isomorphic to a subalgebra $\hat{A}$ of $C_n(X)$, and (2) $K(X) \subset \hat{A}$. A is a Post algebra if and only if $\hat{A} = C_n(X)$. As applications of this theorem, it is proved that the injective Moisil algebras of order n are just the complete Post algebras of order n, and that the free Post algebra of order n with c generators (c cardinal > 0) is isomorphic to $C_n(T)$, where T is the cartesian product of c copies of the discrete space with n points. (Received May 11, 1970.)

70T-A144. WILLIAM S. HATCHER, Université Laval, Québec 10, Canada. Quasiprimitive subcategories.

We say that the first isomorphism theorem is true in a category $\underline{C}$ if every map f in $\underline{C}$ has an image factorization me where e is the coequalizer of the pullback of f with itself. A Y-identity of $\underline{C}$ is a pair of maps $f: X \to Y$ and $g: X \to Y$ where X is some object in $\underline{C}$. An identity is a Y-identity for some object Y of $\underline{C}$. An object Z of $\underline{C}$ satisfies the Y-identity $\langle f,g \rangle$ if $hf = hg$ for every map $h: Y \to Z$. A full subcategory $\underline{A}$ of $\underline{C}$ is defined to be a class K of identities if Ob$(\underline{A}) = \{X \mid X$ satisfies every identity in K$\}$ Let $\underline{C}$ be a left complete category. Then a full subcategory $\underline{A}$ of $\underline{C}$ is quasiprimitive if it is closed under products and subobjects (this latter means that the domain of every mono whose codomain is in $\underline{A}$ is in $\underline{A}$). Theorem. Let $\underline{C}$ be a left complete category for which the first isomorphism theorem is true. A necessary and sufficient condition that a nonempty, full subcategory $\underline{A}$ of $\underline{C}$ be quasiprimitive is that it be definable by some class of identities K. Quasiprimitive classes of algebras are examples and identities can be taken to be pairs of points chosen from algebras in the basic category $\underline{C}$ of all algebras of the given similarity type. If $\underline{C}$ is the category of compact Hausdorff spaces, then we obtain as a special case the result of Abstract 70T-G102, these *Notices* 17(1970), 686. (Received May 11, 1970.)

70T-A145. GERALD E. SUCHAN, University of Houston, Houston, Texas 77004. Concerning some operators associated with a pair of nonnegative matrices. Preliminary report.

Let $A_{m\times n}$, $B_{m\times n}$, $X_{n\times 1}$, and $Y_{m\times 1}$ be matrices whose entries are nonnegative real numbers and suppose that no row of A and no column of B consists entirely of zeroes. Define the operators U, T and T' by $(U_x)_i = X_i^{-1}$ [or $(U_y)_i = Y_i^{-1}$], $T = UB^TUA$ and $T' = UAUB^T$. T is called irreducible if for no nonempty proper subsets S of $\{1,\ldots,n\}$ is it true that $X_i = 0$, $i \in S$, $X_i \neq 0$, $i \notin S$ implies $(T_X)_i = 0$, $i \in S$, $(T_X)_i \neq 0$, $i \notin S$. M. V. Menon ("Some spectral properties of an operator associated with a pair of nonnegative matrices", Trans. Amer. Math. Soc. 132(1968), 369-375) proved the following Theorem. If T is irreducible, there exists row-stochastic matrices A_1 and A_2, a positive number θ, and two diagonal matrices D and E with positive main diagonal entries such that $DAE = A_1$ and $\theta DBE = A_2^T$. Since an analogous theorem holds for T', it is natural to ask if it is possible that T' be irreducible if T is not. It is the intent of this paper to show that T' is irreducible if and only if T is irreducible. (Received May 1, 1970.) (Author introduced by Professor Richard D. Sinkhorn.)

70T-A146. V. S. RAMAMURTHI, Madurai University, Madurai 2, Tamil Nadu, India. Weakly regular rings.

In a ring R, every right ideal I satisfies the equation $I^2 = I$ if and only if, for every element a in R, (1) a belongs to aR, (2) the equation a = ax has a solution in the ideal RaR. Rings satisfying condition (2) are called right weakly regular rings. The class of right weakly regular rings strictly contains the class of (Von Neumann) regular rings and the class of biregular rings. The center of a right weakly regular ring is (Von Neumann) regular if and only if $I^2 = I$ for every right ideal I in R. A Goldie Ring is right weakly regular if and only if it is a direct sum of a finite number of simple rings with identity. Every right weakly regular ring can be imbedded as an ideal in a right weakly regular ring with identity. Right weak regularity is a hereditary, nonsemiprime radical property which admits an Amitsur Radical. (Received May 7, 1970.) (Author introduced by Professor M. Rajagopalan.)

70T-A147. N. VANAJA, Madurai University, Madurai 2, Tamil Nadu, India. A note on weakly regular rings.

A ring R with identity is called right weakly regular if for every $r \in R$, there exists an element x with a two sided ideal generated by r such that r = rx. Theorem 1. R is right weakly regular if and only if R/I is left R-flat for every two sided ideal I of R. Theorem 2. If R is a weakly regular ring then so is R_n, the ring of n by n matrices over R. Theorem 3. The group ring R = AG is weakly regular if and only if (i) A is weakly regular, (ii) every finitely generated subgroup of G is finite and (iii) the order of any finite subgroup of G is a unit in A. (Received May 7, 1970.) (Author introduced by Professor M. Rajagopalan.)

70T-A148. ROBERT P. KURSHAN, Bell Telephone Laboratories, Inc., Murray Hill, New Jersey 07974. <u>When the injective cogenerator has finite length.</u> Preliminary report.

Let U be a minimal injective cogenerator for the category $\mathfrak{M}$ of unital left modules over a ring R with identity; let $E = \mathrm{End}_R U$. If each $M \in \mathfrak{M}$ has nonzero socle and U has finite length, or if R is semi-primary and U/socle U is finitely embedded, then R and E are each left- and right-Artinian, and as both an R- and E-module U is a balanced minimal injective cogenerator of finite length. (Received May 18, 1970.)

70T-A149. WITHDRAWN.

70T-A150. ELBERT M. PIRTLE, University of Missouri, Kansas City, Missouri 64110. <u>Generalized factorial rings.</u>

Let R be a K domain. <u>Definition.</u> R is called a generalized factorial (GF) ring if C(R) = 0, where C(R) denotes the class group of R. Every factorial ring (UFD) is a generalized factorial ring but not conversely. GF rings are characterized and some of the properties of factorial rings are extended to GF rings. <u>Theorem</u>. If R is a GF ring, then $R[X_1,\ldots,X_n]$ is a GF ring. (Received May 22, 1970.)

70T-A151. EDWARD T. H. WANG, University of British Columbia, Vancouver 8, British Columbia, Canada. <u>Maximum and minimum diagonal sums of doubly stochastic matrices.</u>

Let Ω_n denote the convex polyhedron of all d.s. (doubly stochastic) matrices of order n. Let σ denote a diagonal of some matrix in Ω_n. <u>Theorem</u> 1. Let $S \in \Omega_n$ and σ be a diagonal with minimum diagonal sum, then $\max_i S_{i\sigma(i)} \leq 2/(n+1)$. If n = 2, an obviously better upper bound is 1/2. <u>Theorem</u> 2. Let $S \in \Omega_n$ and σ be a diagonal with maximum diagonal sum α. Suppose $\alpha \leq 2$, then $\max_i S_{i\sigma(i)} \leq [(n-1)(\alpha-1)+1]/n$ and $\min_i S_{i\sigma(i)} \geq (2-\alpha)/n$. <u>Theorem</u> 3. Let $S \in \Omega_n$, then $\max_\sigma \sum_{i=1}^n S_{i\sigma(i)} + \min_\sigma \sum_{i=1}^n S_{i\sigma(i)} \leq n$ with equality iff S is a permutation matrix or n = 2. <u>Remark.</u> In Theorem 2, if $\alpha > 2$ then the obvious upper and lower bounds are 1 and 0 respectively. If $\alpha \leq 2$, then both the upper and lower bounds are obtainable by taking $S = J_n$, the d.s. matrix with all entries equal 1/n. (Received May 22, 1970.)

70T-A152. JOHN DAUNS, Tulane University, New Orleans, Louisiana 70118. <u>Integral domains that are not embeddable in division rings.</u>

Totally ordered rings V are constructed such that $1 < \alpha \in V$ implies $1/\alpha \in V$, but such that V cannot be embedded in any division ring. V is a formal power series ring with exponents in a (non-commutative) totally ordered semigroup. For $\alpha \in V$, the support or supp α--the set of nonzero exponents of α--satisfies the ascending chain condition. If the biggest exponent of α is $< 0 \in \Gamma$, then $(1-\alpha)^{-1} = 1 + \alpha + \alpha^2 + \ldots$ is, firstly, well defined and, secondly, supp α satisfies the A.C.C. The known proof of the latter when Γ is a group does not generalize to semigroups. (Received May 25, 1970.)

70T-A153. ANTHONY V. GERAMITA, Queen's University, Kingston, Ontario, Canada. <u>Projective modules as a sum of projective ideals.</u>

All rings considered will be commutative noetherian domains and all modules unitary and

finitely generated. Definition. (a) We say that a projective R-module P has Serre dimension = t if (i) $P \cong P' \oplus R^s$ where rank P' = t, and (ii) $P \not\cong P'' \oplus R^{s+1}$ for any projective R-module P''.
(b) Serre dim R = sup { Serre dim P | P projective R-module } . Serre has shown that Serre dim R $\leq$ dim m-spec R. We show how to construct regular rings A_n ($n \geq 2$), with the following properties: (1) Serre dim A_n = dim m-spec A_n = gl. dim A_n = n. (2) A_n has a projective module P_n of rank = n and Serre dim P_n = n. (3) P_n completely decomposes into a sum of rank 1 projective modules. These examples show Serre's Theorem is the best possible general result. For $n \equiv 0 \pmod 2$ we further show that A_n has an indecomposable projective module of rank = n. The following question remains open. Does there exist a regular ring R with properties: (1) gl. dim R = Serre dim R > 1 and (2) R has no indecomposable projective modules? (Received May 25, 1970.)

70T-A154. AWAD A. ISKANDER, American University of Beirut, Beirut, Lebanon. On the lattice of integers. Preliminary report.

The set N of all positive integers is a distributive lattice under division. (1) A partially ordered set can be embedded into N iff all its principal left segments are finite. (2) A distributive lattice is isomorphic to a sublattice of N iff all its principal ideals are finite; in particular every finite distributive lattice is isomorphic to a sublattice of N; a free distributive lattice (Boolean algebra) is isomorphic to a sublattice of N iff it is finitely generated. (3) A distributive lattice is isomorphic to N iff it is the direct sum of denumerable copies of the chain $0 < 1 < 2 < \dots$.
(4) The two groups of all automorphisms of N considered as a lattice and as a multiplicative monoid are the same. (Received May 18, 1970.)

70T-A155. STEPHEN J. TILLMAN, Brown University, Providence, Rhode Island 02912. Binary integral quadratic forms over a certain class of function fields. Preliminary report.

Let p be an odd prime, and k an infinite algebraic extension of $\mathbb{Z}/p\mathbb{Z}$, such that k^* is 2-divisible. Assume x is transcendental over k. Let $Q(X,Y) = aX^2 + bXY + cY^2$ be a quadratic form such that $a,b,c \in k[x]$, $(a,b,c) = 1$. The main result is the following Theorem. Let h_Q be the class number of the form Q, and $\Delta = b^2 - 4ac$. Then if Δ is a nonsquare or a unit, and deg $\Delta \leq 2$, $h_Q = 1$ (direct calculation). If Δ is a square, $h_Q < \infty$ (lattice theory). If Δ is a nonsquare, and deg $\Delta \geq 3$, $h_Q = \infty$ (reduction theory if deg Δ is odd, ideal theory if deg Δ is even). (Received May 28, 1970.)

70T-A156. KHEE-MENG KOH, University of Manitoba, Winnipeg 19, Manitoba, Canada. Algebras representing $\langle 0,0,1,2 \rangle$. Preliminary report.

For definitions and notations, see Abstract 69T-A54, these *Notices* 16 (1969), 565.
Theorem 1. There exist two equational classes of algebras $\underset{\sim}{K}_1$ and $\underset{\sim}{K}_2$ such that an algebra $\mathfrak{A}$ represents the sequence $\langle 0,0,1,2 \rangle$ if and only if $\mathfrak{A}$ can be represented as an algebra $\langle A; \cdot, f \rangle$ where "$\cdot$" is a semilattice operation and f is a ternary operation belonging to either $\underset{\sim}{K}_1$ or $\underset{\sim}{K}_2$.
Theorem 2. Every algebra representing $\langle 0,0,1,2 \rangle$ contains one of eight algebras $\mathfrak{A}_i$, i = 1,2,..., 8, with $5 \leq |Ai| \leq 8$ for each i = 1,..., 8 as subalgebra. Theorem 3. Let $\mathfrak{A}$ be an algebra representing $\langle 0,0,1,2 \rangle$. Then there exists an algebra $\mathfrak{B}_i \in \underset{\sim}{K}_i$ for each i = 1,2 such that $p_n(\mathfrak{A}) \geq \min\{p_n(\mathfrak{B}_1), p_n(\mathfrak{B}_2)\}$ for n = 1,2,... . (Received May 28, 1970.) (Author introduced by Professor George A. Grätzer.)

70T-A157. RAYMOND BALBES, University of Missouri, St. Louis, Missouri 63121 and PHILIP DWINGER, University of Illinois, Chicago, Illinois 60680. Subdirect products of three element chains. Preliminary report.

The purpose of this investigation is to characterize the class C of distributive lattices which are subdirect products of three element chains. The class C contains all free distributive lattices on > 2 free generators and all chains of ≥ 3 elements, but does not contain any Boolean algebra. Theorem. Let L be a distributive lattice with 1 (0). L is in C if and only if $[x,1]$ ($[0,x]$) is not a Boolean sublattice for each relatively complemented $x < 1$ ($0 < x$). In particular, if L has a 1 (0) and no relatively complemented elements < 1 (>0) then L is in C. This is not generally true in case L has neither 0 nor 1. Structure Theorem. L is a member of C if and only if L does not have a non-degenerate Boolean algebra as a direct factor. (Received June 1, 1970.)

70T-A158. MELVIN F. JANOWITZ, University of Massachusetts, Amherst, Massachusetts 01002. On the natural ordering of a commutative semisimple ring.

Let R be a commutative semisimple ring with unity. In Abian, "Direct product decomposition of commutative semisimple rings," Proc. Amer. Math. Soc. 24 (1970), 502-507, it is shown that the relation $x \leq y$ if $xy = x^2$ is a partial order on R. The ring R is said to be idempotent ordered if this ordering coincides with the one given by $x \leq y$ if $x = ey$ for some idempotent e. Theorem 1. TAE: (i) R is a Rickart ring in the sense that the annihilator of each element is a principal ideal generated by an idempotent; (ii) R is idempotent ordered and $x \wedge 1$ exists for all $x \in R$; (iii) R is idempotent ordered and a meet semilattice; (iv) R is a meet semilattice such that $(x \wedge y)z = xz \wedge yz$ for all $x, y, z \in R$. Corollary. If R is a Rickart ring then it is a meet semilattice and every interval $[0,x]$ is a Boolean algebra; i.e., in the dual ordering, R is a semi-Boolean algebra. Theorem 2. The ring R is Rickart iff it admits a partial ordering "$\leq$" such that R is a meet semilattice. $(x \wedge y)z = xz \wedge yz$ for all $x, y, z \in R$, and every interval $[0,x]$ is complemented. There is at most one such order relation on R. (Received June 10, 1970.)

70T-A159. JAN MYCIELSKI, University of Colorado, Boulder, Colorado 80302 and University of California, Berkeley, California 94720. Independent sets in topological structures.

Let $\mathfrak{A} = \langle A, R_1, R_2, \ldots \rangle$ be a relational structure with at most countably many finitary relations such that A is a complete metric space without isolated points, for each n either $R_n = A^{r(n)}$ or R_n is of the first category in $A^{r(n)}$ (r(n) denotes the rank of R_n) and the set $\{R_1, R_2, \ldots\}$ is closed under "identification of variables" i.e. for every n with $r(n) \geq 2$ and $i < j \leq r(n)$ there exists m such that $r(m) = r(n) - 1$ and $R_m = \{(x_1, \ldots, x_{j-1}, x_{j+1}, \ldots, x_{r(n)}) : (x_1, \ldots, x_{j-1}, x_i, x_{j+1}, \ldots, x_{r(n)}) \in R_n\}$. Let C be the Cantor discontinuum and A^C the space of continuous functions $f: C \to A$ with the usual metric $\mathrm{dist}(f,g) = \max\{\mathrm{dist}(f(x), g(x)) : x \in C\}$. Theorem. The set $\{f \in A^C : f$ is 1-1 and the set $f(C)$ is independent in $\mathfrak{A}\}$ is residual in A^C (i.e. its complement is of the first category). This improves Theorem 1 of my paper "Independent sets in topological algebras", Fund. Math. 55 (1964), 139-147 and yields similar refinements of the other results of that paper. (Received May 25, 1970.)

70T-A160. KWANGIL KOH, North Carolina State University, Raleigh, North Carolina 27607. On quasi-simple modules and almost maximal right ideals.

Let R be a ring and M be a right R-module such that $MR \neq (0)$. M is quasi-simple if and only if the endomorphism ring of the quasi-injective hull of M is a division ring and every nonzero submodule of M contains an isomorphic image of M. A right ideal I is almost maximal if and only if R/I is quasi-simple. Theorem 1. The following statements are equivalent: (i) M is a quasi-simple R-module with zero singular submodule. (ii) M is isomorphic to a uniform right ideal U of R such that U intersection with the right singular ideal of R is zero and if A is a nonzero right ideal such that $A \subseteq U$ then $AU \neq (0)$. Theorem 2. Let R be a simple ring with 1 which has a uniform left ideal. If M is a quasi-simple right R-module with zero singular submodule and $K = \mathrm{Hom}_R(M,M)$ then $R \cong Kn$ if every essential K-submodule of M is an R-submodule of M. Theorem 3. If M is a projective quasi-simple R-module then M is finitely generated and the singular submodule of M is zero. Theorem 4. Let R be a ring with 1. Then R is a semiprime right Goldie ring if and only if there exists a finite number of almost maximal right ideals whose intersection is zero. (Received May 27, 1970.)

70T-A161. WITHDRAWN.

70T-A162. JOEL L. BRENNER, University of Arizona, Tucson, Arizona 85721. Gersgorin theorems for matrices over rings with valuation.

The collection of root-location theorems for matrices of complex numbers is now quite extensive. Since their proofs involve chiefly manipulation of absolute value inequalities, many of these theorems can be extended to noncommutative domains, in particular to quaternion matrices. Secondly, the ring of polynomials has a valuation with properties that differ slightly from those of the ordinary absolute value function. Using this valuation, a different type of regularity theorem is obtainable. With a suitable definition of proper value of a matrix of polynomials, these regularity theorems also lead to root-location theorems. Finally, bounds for determinants can be obtained. These bounds are given in terms of the valuation: for polynomials, they are bounds on the degree. (Received June 8, 1970.)

70T-A163. ABRAHAM BERMAN and PRABHA GAIHA, Department of Engineering Sciences, Northwestern University, Evanston, Illinois 60201. Irreducible monotonicity over cones.

Let K_1 and K_2 be closed convex pointed cones with nonempty interiors in R^n and R^m respectively. Define $A \in M(K_1, K_2)$ iff [$AX \in K_2$, $0 \neq X \in K_1$ is consistent and $AX \in K_2$, $0 \neq X \in K_1 \Rightarrow X \in \mathrm{int}\, K_1$]. If K_1 and K_2 are the nonnegative orthants, $M(K_1, K_2)$ reduces to the class M defined by Fiedler and Ptak. Properties of matrices in class M are generalized and developed for matrices in $M(K_1, K_2)$. It is shown that M(K, K) include irreducible M_k matrices studied by Haynsworth. (Received June 8, 1970.)

70T-A164. WITOLD M. BOGDANOWICZ, Catholic University of America, Washington, D. C. 20017. Characterization of linear lattices of functions closed under Stone's operation and dominated convergence.

Let R be the space of reals without the infinities and X any abstract space. Let $F = R^X$. A set $L \subset F$ is called a linear lattice if it is closed in F under linear combinations and the lattice operations defined by $(f \cup g)(x) = \sup\{f(x), g(x)\}$, $(f \cap g)(x) = \inf\{f(x), g(x)\}$ for all $x \in X$. By Stone's operation in F we mean the map $f \to f \cap c$ where c is the characteristic function of the set X. We shall say that L is closed under dominated convergence if f_n, $g \in L$ and $|f_n(x)| \leq g(x)$ for all $x \in X$ and $f_n(x) \to f(x)$ on X imply $f \in L$. If V is a sigma ring of sets in X denote by V(X) the space of V-measurable finite functions defined as usual. Theorem. Let L be a linear subset of F. The set L forms a linear lattice closed under the Stone operation and under dominated convergence if and only if there exists a sigma ring V of sets of X such that $L \subset V(X)$ and for every function $f \in V(X)$ such that $|f(x)| \leq g(x)$ for all $x \in X$ and some function $g \in L$ we have $f \in L$, i.e. L is a solid subset of the space V(X). (For applications see Bogdanowicz, "Theory of a class of locally convex vector lattices which include the Lebesgue spaces," Proc. Nat. Acad. Sci. U.S.A. 66 (1970).) Linear lattices of functions closed under Stone's operation dominated convergence will be called SD-linear lattices in the sequel notes. (Received June June 12, 1970.)

70T-A165. HERBERT S. GASKILL, Simon Fraser University, Burnay 2, British Columbia, Canada. A representation theorem for distributive semilattices. Preliminary report.

Definition. A semilattice $(L; \vee)$ is distributive, if and only if for all a,b,c in L such that $a \leq b \vee c$, there exists d, e in L such that $d \leq b$, $e \leq c$ and $a = d \vee e$; G. Grätzer, "Universal algebra", Van Nostrand, Princeton, N. J., 1968, p. 329. Theorem. $(L; \vee)$ is a distributive semilattice if and only if $(L; \vee)$ is a 1-1 direct limit of finite distributive lattices considered as semilattices. (Received June 15, 1970.)

70T-A166. MICHAEL G. STONE, University of Calgary, Calgary, Alberta, Canada. On endomorphism and subalgebra structure in universal algebras. Preliminary report.

M. Gould, Abstract 70T-A40, these *Notices* 17 (1970), 427, investigates those monoids M "compatible" with the two element lattice T. Theorem 1 here is a general result on the related problem of which monoids M are "compatible" with an arbitrary algebraic lattice L, in that $M \cong \mathrm{End}\,\mathfrak{A}$ and $L \cong \mathrm{Su}\,\mathfrak{A}$ simultaneously for some algebra $\mathfrak{A}$. For an arbitrary monoid M and $a, b \in M$ we say $a > b$ provided for each $c, d \in M$, $ca = da \Rightarrow cb = db$. The relation $\equiv$ defined on M by $a \equiv b$ iff $a > b$ and $b > a$ is an equivalence relation. Theorem 1. If M is compatible with L then there is a natural 1-1 map φ: $M/^{\equiv} \to L$, L, and under the natural partial orderings ($M/^{\equiv}$ inherits the $<$ relation), φ^{-1} is order preserving. Corollary. If M and L are compatible then $|M/^{\equiv}| \leq |L|$. A number of interesting results are accessible by means of this theorem. Among them: Theorem 2. For each positive integer n there is a monoid M which is compatible with the n-element chain N and which fails to be compatible with any lattice of length less than n. (Received June 15, 1970.)

70T-A167. HAROLD N. WARD, University of Virginia, Charlottesville, Virginia 22901. Representations of symplectic groups. Preliminary report.

Let h be a nondegenerate symplectic form on the space V of dimension 2n over GF(p^m), p an odd prime. Let K be a field containing a primitive pth root of unity e ($\neq$ 1). Put $f(x,y) = e^{\text{trace } h(x,y)}$, the trace from GF($p^m$) to GF(p). Form the twisted group algebra A of V over K, using f. Then A is a matrix algebra over K, the symplectic group Sp(V) gives automorphisms of A, and one gets a projective representation of Sp(V) of dimension p^{mn}. (An analog works for unitary groups.) The irreducible constituents have degrees $\frac{1}{2}(p^{mn} + 1)$ and $\frac{1}{2}(p^{mn} - 1)$ unless char K = 2; then the degrees are $\frac{1}{2}(p^{mn} - 1)$ (twice) and 1. The representations are thus ordinary. For finite K their fields can be determined, and invariant forms arise in certain cases. Thus one obtains homomorphisms of Sp(V) into certain linear groups; sometimes the image has small index (possibly 1). These well-known isomorphisms follow: PSL(2,5) with PSL(2,4); PSL(2,9) with A_6; PSL(2,7) with PSL(3,2); PSp(4,3) with PSU(4,2); S_6 with PSp(4,2). The embedding of PSL(2,7) in PSU(3,3) in the Hall-Wales group shows up; with a field automorphism of PSU(3,3) this leads to a symmetric design with v = 36, k = 15, λ = 6. (Received June 15, 1970.)

70T-A168. N. GANESAN, Annamalai University, Chidambaram, Tamil Nadu, India. Finite near-rings with zero divisors and regular elements. Preliminary report.

Certain ring-theoretic results of the author (Math. Ann. 161 (1965), 241-246) and of Koh (Math. Ann. 171 (1967), 79-80) are extended to the case of (left) near-rings. Theorem A. Any C-ring with a finite number n ($\geqq$ 2) of right zero divisors is finite and its order $\leqq n^2$. This upper bound n^2 is sharp. This theorem is not true for arbitrary near-rings. Definitions. In a near-ring any element that is not a left(right, two-sided) zero divisor is called left(right, two-sided) regular; a semigroup M is called a multiple group of the left (right) hand type if it has some left(right) identities and every element of M has a right(left) inverse w.r.t. every left(right) identity. Theorem B. N is an arbitrary finite near-ring with at least one left regular (right regular, right or anti-right distributive) element. Then such elements form a multiplicative multiple group of the left(right) hand type and the left(right) identities in M are the only left(right) identities for N. Theorem C. If N is a finite C-ring with at least one (two-sided) regular element that is right or anti-right distributive, then the regular elements of N form a multiplicative group whose identity is the identity for N as well. (Received June 22, 1970.)

70T-A169. HARVEY E. WOLFF, University of Illinois, Urbana, Illinois 61801. V-localizations and V-fractional categories.

Let V be a symmetric monoidal closed category. Let A be a V-category and $\Sigma \subset A_0$ (A_0 is the underlying category) a subcategory which contains the identities of A_0. A V-localization of A with respect to Σ consists of a V-category $A[\Sigma^{-1}]$ and a V-functor $\Phi: A \to A[\Sigma^{-1}]$ such that (i) $\Phi_0(s)$ is an isomorphism for every $s \in \Sigma$; (ii) $Ob(A) = Ob(A[\Sigma^{-1}])$ and Φ is the identity on objects; and (iii) if $F: A \to B$ is a V-functor such that $F_0(s)$ is an isomorphism for every $s \in \Sigma$, then there exists a unique V-functor $\overline{F}: A[\Sigma^{-1}] \to B$ such that $\overline{F} \cdot \Phi = F$. Theorem. If V is cocomplete and A is small then the V-localization of A with respect to any $\Sigma \subset A_0$ which contains the identities of A_0 exists.

(This result has also been proven independently by Benabou [unpublished].) A V-localization is a V-right fractional category of A if for every A,B $\in$ Ob($A[\Sigma^{-1}]$), $A[\Sigma^{-1}](A,B) = \text{inj lim}_{(\Sigma/A)^{op}} A(-, B)$ in V, where Σ/A is the category with objects $E \xrightarrow{s} A$, $s \in \Sigma$ and morphisms from $E_1 \xrightarrow{s_1} A$ to $E_2 \xrightarrow{s_2} A$ are morphisms $f: E_1 \to E_2$ in A_0 such that $s_2 f = s_1$. Sufficient conditions are given for the existence of V right fractional categories, which under suitable conditions are also necessary. Many of the results known in the Set-case are generalized to the V-case. (Received June 22, 1970.)

70T-A170. LOWELL W. BEINEKE and RAYMOND E. PIPPERT, Purdue University, Fort Wayne, Indiana 46805. Properties and characterizations of k-trees.

A k-graph is a simplicial complex of dimension at most k; a k-cell is a k-dimensional simplex. The graph theoretic concepts of circuit and connectedness have analogues in k-graphs as k-circuit and k-linkage. The class of k-graphs called k-trees is defined inductively: a (k - 1)-cell is a k-tree, and a k-tree with n + 1 vertices is obtained from a k-tree T with n vertices by adding a k-cell having precisely a (k - 1)-cell in common with T. In a chapter of "The many facets of graph theory" [Springer-Verlag, Berlin, 1969, pp. 263-270], the authors obtain some characterizations of 2-trees. The following theorem is a generalization of that work. Theorem. Let G be a k-graph, with p vertices, in which every r-cell, for $r < k$, is on a (k - 1)-cell. Then the following are equivalent. (1) G is a k-tree. (2) G has no k-circuits, p - k k-cells, and $kp - k^2 + 1$ (k - 1)-cells (N = 3,4,...,k + 2). G is k-linked and has $\binom{k}{r+1} + (p - k)\binom{k}{r}$ r-cells, where r = N - 2. (Received May 4, 1970.)

70T-A171. KENG TEH TAN, Queen's University, Kingston, Ontario, Canada. Ore conditions on group-rings. Preliminary report.

Let R be an Ore domain and G a group. Let (λ,μ) be a factor set for G over R and $S = R(G; \lambda,\mu)$ denote the group-ring with multiplication defined as $grhs = gh\lambda_{g,h} r^h s$ for $g, h \in G$; $r, s \in R$. Proposition 1. If G is finite, then S is an Ore ring. Proposition 2. If G is an infinite cyclic group, then S is an Ore domain. Theorem 3. If G is locally an extension of a polycyclic group by a finite group, then the ordinary group-ring RG is an Ore ring. Corollary 4. If G is an abelian group, then RG is an Ore ring. If furthermore, G is torsion-free, then RG is an Ore domain. (Received June 11, 1970.) (Author introduced by Dr. Anthony Vito Geramita).

70T-A172. JOEL KARNOFSKY, University of California, Berkeley, California 94720. Finite equational bases for semigroups. Preliminary report.

Let S be a semigroup, k, m, n be arbitrary positive integers and $S^n = \{s_1 s_2 \ldots s_n : s_i \in S\}$ the subsemigroup of n-fold products of S. Say S is FB if the equational theory of S has a finite basis. P. Perkins, in his doctoral dissertation ("Decision problems for equational theories of semigroups and general algebras," University of California, Berkeley, Calif., 1966) showed that if, for some n, S^n satisfies $x^m = x^{m+k}$ and $wxyz = wyxz$, then S is FB. He then checked all three element semigroups and showed them in FB. By similar methods it can be shown that any of the following conditions imply S is FB. (i) S^n satisfies $xyz = xy^{1+k}z$ and $wxyxz = wx^2yz$. (ii) S finite and satisfies $wxyx = wx^2y$. (iii) S finite and S satisfies $xy^3z = xy^{3+k}z$ and $wxyxz = wx^2yz$. (iv) Conditions (i), (ii), or (iii) with the asymmetric equations reversed. Using these conditions, all four element semigroups were checked and found in FB. Since all three element semigroups satisfy Perkins' condition or condition (i), if $|S^n| \leqq 3$, then S is FB. Perkins also gives an example of a six element semigroup not in FB, but the problem is open for five element ones. (Received June 22, 1970.) (Author introduced by Professor Leon H. Henkin).

70T-A173. JOHN JONES, JR., 5467 Mitchell Drive, Dayton, Ohio 45431 and Air Force Institute of Technology, Dayton, Ohio 45433. Solutions of certain matrix equations.

Let A, B, C denote n by n matrices with real elements and $f_\alpha(\lambda)$, $f_\beta(\lambda)$ polynomials of degree $n \geqq 1$ with real coefficients. The 2n by 2n matrices R, $f_\alpha(R)$, $f_\beta(R)$ are given by $R = \begin{pmatrix} -B & 0 \\ -C & A \end{pmatrix}$, $f_\alpha(R) = \begin{pmatrix} 0 & 0 \\ V & N \end{pmatrix}$, $f_\beta(R) = \begin{pmatrix} \hat{U} & 0 \\ \hat{V} & 0 \end{pmatrix}$, where the n by n matrices V, N, $\hat{U}$, $\hat{V}$ are real polynomials of the n by n matrices A, B, C of the matrix equation (*) $AX + XB + C = 0$. Theorem 1. If X is a solution of (*) then the pairs of matrices $f_\alpha(R)$, $\begin{pmatrix} N & V-NX \\ 0 & 0 \end{pmatrix}$, and $f_\beta(R)$, $\begin{pmatrix} 0 & X\hat{U}+\hat{V} \\ 0 & \hat{U} \end{pmatrix}$ are similar. Theorem 2. If $V \neq 0$ and $(N)^{-1}$ exists, then the solution X of $V - NX = 0$ is a solution of (*). If $\hat{V} \neq 0$ and $(U)^{-1}$ exists, then the solution X of $X\hat{U} + \hat{V} = 0$ is a solution of (*). Theorem 3. If the matrices $\tilde{R} = \begin{pmatrix} A & -C \\ 0 & -B \end{pmatrix}$, $\begin{pmatrix} A & 0 \\ 0 & -B \end{pmatrix}$ are similar, then there exists a solution X of (*) which satisfies the pair of equations $\overline{M} + X\overline{U} = 0$, $\tilde{M} - \tilde{N}X = 0$, where $f_\alpha(\tilde{R}) = \begin{pmatrix} \overline{N} & \overline{M} \\ 0 & \overline{U} \end{pmatrix}$, $f_\beta(\tilde{R}) = \begin{pmatrix} \tilde{N} & \tilde{M} \\ 0 & \tilde{U} \end{pmatrix}$, and hence $X = \tilde{N}^+\tilde{M} - \overline{M}\overline{U}^+ + \tilde{N}^+\tilde{N}\overline{M}\,\overline{U}^+$, where + denotes the generalized inverse of a matrix as defined by R. Penrose. (Received June 22, 1970.)

70T-A174. MARY RAYAR, Indiana University, Bloomington, Indiana 47401. Essential and small modules.

A left R-module M is called an essential R-module if M is isomorphic to a factor module A/B of an R-module A by an essential R-submodule B. Dually, M is called a small R-module if it can be imbedded as a small R-submodule of some R-module A: $M \Subset A$. Theorems. (1) M is essential iff $Z(M) = M$, where $Z(M)$ denotes the singular submodule of M. (2) M is small iff it is small in the injective hull $\hat{M}$: $M \Subset \hat{M}$. (3) Every commutative integral domain R which is not a field is as small as an R-module. (4) If R is a (noncommutative) integral domain which is a local ring then it is as small as an R-module whenever it is not self-injective. (5) Let M be cofinitely generated (J. P. Jans, J. London Math. Soc. (2) 1 (1969)). M is small iff $f(M) \Subset E_i$ for all $f: M \to E_i$, where E_i range over all injective hulls of nonisomorphic simple left R-modules. (6) Let R be left Artinian. Then (a) M is essential iff $r(J) \cdot M = 0$. (b) A finitely generated module M is small iff $\ell(J) \cdot M = 0$, where J is the radical of R and $r(J)$, $\ell(J)$ denote respectively the right, left annihilator of J in R. (Received June 23, 1970.) (Author introduced by Professor Goro Azumaya.)

70T-A175. LAWRENCE A. MACHTINGER, Illinois Institute of Technology, Chicago, Illinois 60616. Rings satisfying a not-necessarily transitive relation. Preliminary report.

Some of the results concerning linearly ordered rings can be extended to a wider class of rings which includes finite rings and rings with nonzero characteristic. Theorem. A ring $S \neq \{0\}$ is an integral domain iff there is a relation < on S such that for all a, b, c $\in$ S (1) if $a < b$ then $a + c < b + c$, (ii) if $a < b$ and $0 < c$, then $ac < bc$, and (iii) $a < b$ or $b < a$ iff $a \neq b$. Definition. If < is a relation on the ring S satisfying (i), (ii) and (iv) for all a, b $\in$ S one and only one of the statements $a < b$, $a = b$, $b < a$ holds, then (S, <) is a not-necessarily transitively ordered (nto) ring and < is a not-necessarily transitive order (nto) on S. Theorem. If S is an integral domain in which the product of squares is a

square and for all $x \in S$, there is a $y \in S$ such that $x = y^2$ or $x = -y^2$, then there is an nto on S iff (a) for all $x, y \in S$, $x^2 + y^2 = 0$ iff $x = y = 0$ (in which case the nto is unique). The nto is an order relation iff (b) for all $x, y, z \in S$, $x^2 + y^2 + z^2 = 0$ iff $x = y = z = 0$. Theorem. A finite ring $S \neq \{0\}$ has an nto (in which case it has only one) iff S is a field of order p^n where p is a prime of the form $4m + 3$ and n is odd. There are infinitely many nto's on the ring of integers which are not transitive.
(Received May 27, 1970.)

Analysis

70T-B149. JAMES T. BURNHAM, University of Iowa, Iowa City, Iowa 52240. Closed ideals in subalgebras of Banach algebras. II. Preliminary report.

For notation and terminology see J. T. Burnham, Abstract 70T-B112, these *Notices* 17(1970), 654. Let A be a commutative semisimple Banach algebra with an approximate identity. Suppose further that A is regular and the ideal of all elements of A with compactly supported Gelfand transforms is dense in A. Theorem. If A satisfies the condition D and B is an ASA, then B satisfies the condition D. As a corollary one obtains the Wiener-Ditkin-Shilov Tauberian theorem for B--see for instance Loomis, "Abstract harmonic analysis,"Van Nostrand, Princeton, N. J., 1953, p. 86. The above result takes care of every commutative Segal Algebra, thus generalizing a result of Yap, "Every Segal algebra satisfies Ditkin's condition," unpublished manuscript, Rice University, 1970.
(Received March 11, 1970.)

70T-B150. KURT MAHLER, Ohio State University, Columbus, Ohio 43210. On algebraic differential equations.

Let $f = \sum_{h=0}^{\infty} f_h z^h$ be a formal power series with complex coefficients which satisfies any algebraic differential equation $F(z;w,w',\dots,w^{(m)}) = 0$ (F a polynomial). Then there exist two positive constants C_1, C_2 such that $|f_h| \leq C_1(h!)^{C_2}$ for all h. This result is best possible. (Received April 24, 1970.)

70T-B151. GUILLERMO MIRANDA, Courant Institute, New York University, New York, New York 10012. On the integral equation solution of the Neumann-Poincaré problem in piecewise smooth domains and Carleman's estimate.

The Neumann-Poincaré problem for a region D in E_3 contains a parameter λ in the boundary conditions, and the values $\lambda = \pm 1$ correspond to the Neumann and Dirichlet problems. The problem is reduced to a pair of adjoint integral equations by means of single- and double-layer potentials, and when the boundary of D contains angular lines or corners, the kernel and its iterates become highly singular (compactness disappears). These equations can be regularized for $\lambda w < 1$ by means of a kernel decomposition, and Carleman showed (doctoral dissertation) that $w < 1$ in the case of a single angular line. Carleman overlooked the contribution of certain terms, but this estimate is shown to hold in spite of this fact. The extension to polyhedral corners and a class of conical points is discussed, as well as the behavior of the limiting values of a double-layer potential and its density near irregular points of the boundary surface. Also, the validity of Carl Neumann's series solution is established for such irregular domains without convexity restrictions. Carleman's estimate has been used by the author to solve elliptic and parabolic boundary-value problems (see Abstract 665-73, these *Notices* 16(1969), 646 and Abstract 672-312, these *Notices* 17(1970), 171. (Received April 30, 1970.)

70T-B152. DONALD A. EISENMAN, University of North Carolina, Chapel Hill, North Carolina 27514. Proper holomorphic self-mappings of the unit ball in C^n $(n \geq 2)$. Preliminary report.

The proper holomorphic self-mappings of the unit disk in C^1 and of the polydisks in C^n are well known, but the situation for the unit ball in C^n is as yet conjectural. The common conjecture is that all proper holomorphic self-mappings of the unit ball are automorphisms. The general problem is not solved here, but two special cases are treated. Theorem. All algebraic (defined by rational functions) proper holomorphic mappings from the unit ball to itself are automorphisms (for $n \geq 2$). Theorem. No proper holomorphic mapping of the unit ball to itself has covering number two (for $n \geq 2$). (Such a mapping presents the unit ball as an analytic cover of itself, and has a well-defined covering number.) (Received April 30, 1970.) (Author introduced by Professor Fred B. Wright.)

70T-B153. JAMES D. BAKER, Honeywell, Inc., Hopkins, Minnesota 55343 and ROBERT A. SHIVE, JR., Millsaps College, Jackson, Mississippi 39210. On the existence of Ψ-integrals.

If f, g are real-valued functions on $[a,b]$ and Ψ is a choice function on $[a,b]$ then $\Psi\int_a^b fdg$ is the integral defined in Abstract 672-200, these *Notices* 17(1970), 140. Theorem. Let g be a step function on $[a,b]$. There exists a choice function Ψ such that $\Psi\int_a^b fdg$ exists for each function f on $[a,b]$. Theorem. Suppose that g is a saltus function on $[a,b]$. There is a choice function Ψ such that $\Psi\int_a^b fdg$ exists for each bounded function f on $[a,b]$. Theorem. Let Ψ be a choice function on $[a,b]$. There is a bounded function f on $[a,b]$ such that $\Psi\int_a^b f(x)dx$ does not exist. Corollary. There is no choice function Ψ such that $\Psi\int_a^b fdg$ exists for each bounded function f on $[a,b]$ and each function g of bounded variation on $[a,b]$. Other existence theorems for this integral are given by Shive in the abstract mentioned above and by Baker in Abstract 672-475, these *Notices* 17(1970), 219. (Received May 1, 1970.)

70T-B154. MICHAEL G. HENLE, Yale University, New Haven, Connecticut 06520. A Lebesgue decomposition theorem for C* algebras. Preliminary report.

Notation. Let $\underline{B}$ be a C* algebra. For $p \in \underline{B}^*$, $p \geq 0$, let $\underline{H}_p$ denote the canonical Hilbert space associated with p, the completion of $\underline{B}$ in the norm $\|T\|_p = p(T^*T)^{1/2}$, $T \in \underline{B}$. Let $1_p \in \underline{H}_p$ be the element of $\underline{H}_p$ corresponding to the identity of $\underline{B}$, and π_p be the the canonical representation of $\underline{B}$ on $\underline{H}_p$. For f, $p \in \underline{B}^*$, f, $p \geq 0$, f is said to be almost dominated by p if for any sequence $\{T_n\} \subseteq \underline{B}$, $p(T_n^*T_n) \to 0$ and $f((T_n - T_m)^*(T_n - T_m)) \to 0$ together imply $f(T_n^*T_n) \to 0$. This is equivalent to the existence of a (possibly unbounded) positive operator A on $\underline{H}_p$, such that (1) A commutes with $\pi_p(\underline{B})$, (2) $1_p \in \mathcal{D}(A^{1/2})$, and (3) $f(T) = \langle \pi_p(T)A^{1/2}1_p, A^{1/2}1_p \rangle_{\underline{H}_p}$. f and p are said to be singular if the unique bounded operator A on $\underline{H}_{p+f}$ satisfying (1), (2) and (3), with p + f in place of p, is a projection. In this case $A\underline{H}_{p+f}$ may be identified with $\underline{H}_f$ and $(I - A)\underline{H}_{p+f}$ may be identified with $\underline{H}_p$, so that in this sense $\underline{H}_{p+f} = \underline{H}_f \oplus \underline{H}_p$. Theorem. Let p, $f \in \underline{B}^*$, p, $f \geq 0$. Then there exist f_1, $f_2 \in \underline{B}^*$, $f_1, f_2 \geq 0$ such that (a) $f = f_1 + f_2$, (b) f_1 is almost dominated by p, (c) f_2 and p are singular, and (d) f_1 and f_2 are singular. f_1 and f_2 are uniquely determined by these properties. The proof is a straightforward generalization of von Neumann's proof of the Lebesgue decomposition and Radon-Nikodym theorems. (Received May 1, 1970.)

70T-B155. HANS P. HEINIG, McMaster University, Hamilton, Ontario, Canada. Representation of functions as Laplace- and Laplace-Stieltjes transform.

The representation of functions f, holomorphic in the right half-plane is discussed by means of the complex inversion operator F_λ, $\lambda > 0$, defined by $F_\lambda(t) = (2\pi)^{-1/2} \int_{\mathbb{R}} k(\lambda, y) \exp[t(x+iy)] f(x+iy)dy$, $x > 0$, where $k(\lambda, y)$ is a Fourier summability kernel. Specifically, necessary and sufficient conditions for a holomorphic function f to be the Laplace-, respectively Laplace-Stieltjes transform of an L_p-function, $1 \leq p \leq \infty$, or a function of bounded variations are given. The results are extended further to certain weighted L_p-spaces and Lorentz-spaces $\Lambda(p,\varphi)$, $1 < p < \infty$ for some weight function φ. This generalizes results of H. Berens and P. L. Butzer "Uber die Darstellung holomorpher Funktionen durch Laplace- und Laplace-Stieltjes Integrale (Math. Z. 81(1963)). (Received April 27, 1970.).

70T-B156. RICHARD J. EASTON and ANDRE DE KORVIN, Indiana State University, Terre Haute, Indiana 47809. Linear operators on continuous functions with totally bounded range.

This is an improvement on some of the results of an earlier abstract (Abstract 70T-B78, these *Notices* 17(1970), 566-567). Theorem 1. Let H be a normal topological space. Then the set of all functions of the form $f \cdot x$, where $f \in C_B(H,R)$ and $x \in X$ is dense in $C_{TB}(H,X)$. Theorem 2. Let H be a normal topological space. If T is a continuous linear operator from $C_B(H,X)$ to Y, then there exists a unique, weakly regular, finitely additive, $B(X,Y^{**})$ valued, Gowurin set function K, defined in the field F, generated by the closed subsets of H, such that every f in $C_{TB}(H,X)$ is integrable with respect to K, and moreover $T^{**}(f) = \int_H dk \cdot f$. (Received May 11, 1970.)

70T-B157. GEORGE H. PIMBLEY, JR., Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico 87544. On secondary bifurcation of eigensolution branches with Hammerstein's operator. Preliminary report.

The eigenvalue problem $T(x) = \lambda x$, where $T(x) = \int_0^1 K(s,t)f(t,x(t))d\sigma(t)$, $K(s,t)$ is a symmetric oscillation kernel, $\sigma(t)$ is nondecreasing, $f(s,-x) = -f(s,x)$, $f(s,x) \in C$ in strip $-\infty < x < \infty$, $0 \leq s \leq 1$, $f_{xxxx} \leq M$, $f_x > 0$, $xf_{xx} > 0$ (superlinear), $0 \leq s \leq 1$, has continuous branches $x_p(s,\lambda)$ of eigenfunctions bifurcating from the trivial solution at the primary bifurcation points. Secondary bifurcation (s.b.) of a branch x_p occurs when $\lambda I - T'(x_p)$ becomes singular as the branch evolves, i.e., when λ becomes an eigenvalue $\mu_n(k, f_{x_p})$ of $T'(x_p) \cdot = \int_0^1 K(s,t)f_x(t,x_p) \cdot d\sigma(t)$. The functional $G_p(\varphi) = \max_{0 \leq s \leq 1}(\varphi(s)f_x(s,\varphi(s))/f(s,\varphi(s))) - \mu_p(K, f_x(s,\varphi))/\mu_{p+1}(K, f_x(s,\varphi))$ defines bifurcation zones in the positive cone of $C(0,1)$; when $|x_p|$ passes into a zone where $G_p > 0$, s.b. occurs. In the two-dimensional case ($\sigma(t)$ piecewise constant with two jumps), the zone $G_1 > 0$ lies along the main diagonal of the positive quadrant omitting a neighborhood of the origin. S.b. of the branch $x_1(\lambda)$ occurs when the kernel is continuously deformed to shift x_1 from one side of the bifurcation zone to the other. (Received May 11, 1970.)

70T-B158. MYRON S. HENRY, Montana State University, Bozeman, Montana 59715. Approximate solutions of differential equations: polynomial approximation on finite point sets. Preliminary report.

Suppose that F and G are elements of $C[I \times R^n, R^n]$, $I = [0,b]$, and that $Y(x)$ is the unique solution to $L[Y] \equiv Y' + F(x,Y) + G(x,Y) = H(x)$, $Y(0) = Y_0$, where $H(x) \in C[I,R^n]$. Let $\mathcal{P}_k = \{P_k(x)\}$, where $P_k(x) = Y_0 + \sum_{i=1}^k x^i \sum_{j=1}^n a_{ij}E_j$. Suppose that there exist scalar functions $u(x)$, $\varphi(Y)$, and $\mu(x,Y)$ such that $u(x)$ is bounded and has only ℓ distinct zeros on I; $\varphi \in C[R^n, R]$ and $\varphi(Y) = 0$ if and only if $\|Y\| = 0$; $\mu \in C[I \times R^n, R]$; and for all $r \geq 1$, $\|G(x,Y)\| \geq r^a|u(x)\varphi(Y/r)|$ and $\|F(x,Y)\| \leq r^\beta |\mu(x,Y/r)|$. Theorem 1. Let S be any subset of I that contains at least $k + 1 + \ell$ distinct points, including zero. If $a > \max(1,\beta)$, then there exists a $P_k^*(x) \in \mathcal{P}_k$ such that $\sup_S \|H(x) - L[P_k^*(x)]\| = \inf_{\mathcal{P}_k} \sup_S \|H(x) - L[P_k(x)]\|$. Suppose that $\{S_m\}$ is a sequence of subsets of I, that $S_m \subseteq S_{m+1}$, and that each S_m contains at least $k + 1 + \ell$ distinct points, including zero. Theorem 2. Let $S = \bigcup_{m=1}^\infty S_m$, and suppose that $cl(S) = I$. If $\rho_m = \inf_{\mathcal{P}_k} \sup_{S_m} \|H(x) - L[P_k(x)]\|$, and if $\rho = \inf_{\mathcal{P}_k} \sup_I \|H(x) - L[P_k(x)]\|$, then $\lim_{m\to\infty} \rho_m = \rho$. Theorem 3. Let $\{P_k^m(x)\}$ be a sequence of polynomials such that $\rho_m = \sup_{S_m} \|H(x) - L[P_k^m(x)]\|$. Then there exists a uniformly convergent subsequence $\{P_k^{m_\ell}\}$. Furthermore if $P_k^*(x)$ is the limit of the subsequence, then $\rho = \sup_I \|H(x) - L[P_k^*(x)]\|$. (Received May 12, 1970.)

70T-B159. LOUIS BRICKMAN, THOMAS H. MacGREGOR and DONALD R. WILKEN, State University of New York, Albany, New York 12203. Convex hulls of some classical families of univalent functions.

Let S denote the functions that are analytic and univalent in the unit disk Δ and satisfy $f(0) = 0$ and $f'(0) = 1$. Also, let K, St, S_R, and C be the subfamilies of S defined by $f(\Delta)$ is convex, $f(\Delta)$ is starlike with respect to 0, f is real on $(-1,1)$, f is close-to-convex, respectively. The closed convex hull of each of these families is determined as well as the extreme points for each. Moreover, integral formulas are obtained for each hull in terms of the probability measures over suitable sets. The extreme points for each family are particularly simple; for example, the Koebe functions $z \to z/(1 - xz)^2$, $|x| = 1$, are the extreme points of $\overline{co}$ St. These results are applied to discuss linear extremal problems over each of the four families. A typical result is: Let J be a "nontrivial" continuous linear functional on the functions analytic on Δ. The only functions in St that satisfy $\max\{\mathrm{Re}\ J(g): g \in St\} = \mathrm{Re}\ J(f)$ are Koebe functions and there are only finitely many of them. (Recieved May 14, 1970.)

70T-B160. THOMAS L. KRIETE, III, University of Virginia, Charlottesville, Virginia 22901 and DAVID TRUTT, Lehigh University, Bethlehem, Pennsylvania 18015. The Cesàro operator in ℓ^2 is subnormal. Preliminary report.

The Cesàro operator C_0 in ℓ^2 is defined by $C_0\{a_n\} = \{(a_0 + \ldots + a_n)/(n + 1)\}$. We explicitly determine a measure μ on the disk $|z| \leq 1$ such that $I - C_0$ is unitarily equivalent to the operator $T: F(z) \to zF(z)$ in $H^2(\mu)$, the closure in $L^2(\mu)$ of the polynomials. The spectrum of T is known to be the full disk $|z| \leq 1$. The spectrum of the normal extension operator $f(x) \to xf(x)$ in $L^2(\mu)$ consists of the circles C_n with centers at $1 - 1/(n + 1)$ and radii $1/(n + 1)$, $n = 0,1,2,\ldots$. The restriction of μ to each C_n is absolutely continuous with respect to arc length. (Received May 18, 1970.)

70T-B161. HARI M. SRIVASTAVA, University of Victoria, Victoria, British Columbia, Canada. A class of integral equations involving the H function as kernel. Preliminary report.

Integral equations of the type (*) $\int_0^x K(x/y)f(y)(dy/y) = g(x)$, $x \geq 0$, where g is prescribed and f is the unknown function to be determined, can be reduced fairly easily to the form (**) $\int_u^{\infty} K_1(t - u)f_1(t)dt = g_1(u)$, $u \geq 0$. In the present paper the author shows how the systematic use of the theory of the Mellin transform leads to a simple procedure by means of which this class of integral equations may be solved. The technique, which presupposes the existence of the Euler transform of the kernel as well as the Mellin transform of the Euler transform, is illustrated by obtaining a formal solution of the integral equation (***) $\int_y^{\infty} (x - y)^{-\alpha} H_{p,q}^{m,n}[x - y]f(x)dx = g(y)$, $y \geq 0$, where $H_{p,q}^{m,n}[\zeta]$ denotes the H function of C. Fox [Trans. Amer. Math. Soc. 98(1961), 395-429]. Since a large variety of functions that occur frequently in problems of analysis and mathematical physics are only specialized or limiting forms of the kernel used in (***), the inversion problem discussed in this paper may prove to be of general interest. (Received May 18, 1970.)

70T-B162. SHU-BUN NG, McMaster University, Hamilton, Ontario, Canada. On the continuity of positive and multiplicative linear functionals on Fréchet algebra.

Let A be a Fréchet algebra over complex field [W. Zélazko, "Metric generalization of Banach algebra," Rozprawy Mat. 47(1965)]. It is shown that: Theorem 1. Every complex multiplicative linear functional on A is continuous. This answers in the affirmative the well-known problem posed by E.A. Michael [Mem. Amer. Math. Soc., no. 11, 1952]. Theorem 2. Assume A has a continuous involution * and has an identity. Then every positive linear functional on A is continuous. (Received May 21, 1970.) (Author introduced by Professor Taqdir Husain.)

70T-B163. CORNELIS W. ONNEWEER, University of New Mexico, Albuquerque, New Mexico 87106. Adjustment on small sets of functions on certain groups.

Let G be a metrizable, compact, 0-dimensional abelian group. Then the charactergroup X of G is a discrete, countable, torsion group. Vilenkin (Amer. Math. Soc. Transl. (2) 28(1963), 1-35) developed part of the Fourier theory on such groups. Under the assumption that sup $p_n = p < \infty$ (see Vilenkin) we have the following Theorem. If f is a measurable function, finite a.e. on G, then for given $\epsilon > 0$ there exists a function g on G such that (i) $g(x) = f(x)$ except on a set of measure $\leq \epsilon$, (ii) the Fourier series of g converges uniformly. Our proof resembles the proof given by J. J. Price of a similar theory for Walsh-Fourier series (Illinois J. Math. 13(1969), 131-136). The theorem gives a partial answer to a question raised by Goffman and Waterman (Amer. Math. Monthly 77(1970), 119-134). (Received May 25, 1970.)

70T-B164. RAYMOND L. JOHNSON, University of Maryland, College Park, Maryland 20742. The equivalence of parabolic boundedness and parabolic limits for some second order parabolic equations. Preliminary report.

The theorem of Hattemer showing the equivalence for the heat equation of parabolic boundedness and having parabolic limits is extended to a class of second-order linear parabolic equations in divergence form with smooth coefficients. (Received May 25, 1970.)

70T-B165. EDWARD BECKENSTEIN and GEORGE BACHMAN, Polytechnic Institute of Brooklyn, Brooklyn, New York 11201 and LAWRENCE R. NARICI, St. John's University, Jamaica, New York 11432. Maximal ideals in topological algebras.

If M is a maximal ideal in a complex commutative Banach algebra with identity, then M has codimension one and consists entirely of singular elements. Gleason has shown the converse statement to hold as well. In this paper it is shown that the converse also holds in a certain class of complex commutative Hausdorff topological algebras with identity X: namely when X is a locally m-convex barreled complete Q-algebra. (X is called a Q-algebra if the set of invertible elements is open; X is called locally m-convex if there exists a neighborhood base at 0 of neighborhoods U such that UU is contained in U.) (Received June 11, 1970.)

70T-B166. R.S.L. SRIVASTAVA and S. K. BAJPAI, Indian Institute of Technology, Kanpur, India. α-spiral univalent function.

Let $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ be a regular and univalent α-spiral function in the unit disk $|z| < 1$. Set $F(z) = ze^{i\alpha} f'(z)/f(z)$ for some real α such that $|\alpha| \leqq \pi/2$. The following results have been obtained: (i) $(n - 1)^2 |a_n|^2 \leqq \sum_{k=1}^{n-1} k|a_k|^2 \cos^2 \alpha$. (ii) $\int_0^{2\pi} \log|f(re^{i\theta})/r| d\mu_f(\theta) = (\cos \alpha) \sum \alpha_\nu^2 \log(1 - r)^{-1}$ as $r \to 1$, where $\alpha_\nu = \lim_{r \to 1} (1 - r)/(1 + r)) F(re^{i\theta_\nu}) > 0, \sum \alpha_\nu \leqq 1$. (iii) If $\operatorname{Re}\{F(z)\} \leqq \alpha_1$, then $|a_n| \leqq (1/(n - 1)!) \Pi_{n=0}^{n-2} (m + \alpha'/\pi)$, where $\alpha' = \alpha_1 + 2(1 - \cos \alpha)$. As a corollary to the result in (i), it follows that $|a_n| \leqq n \cos \alpha$, which is a better estimate than that obtained by Bieberbach's conjecture for such a class of functions and if $\alpha = \pi/2$, the only α-spiral univalent function is the identity function. The results have been obtained by using Herglotz's integral representation for $\operatorname{Re}\{F(z)\}$. (Received June 2, 1970.) (Author introduced by Professor S. A. Naimpally.)

70T-B167. STANLEY J. POREDA, Clark University, Worcester, Massachusetts 01601. A connection between best approximation on an ellipse and best approximation on two circles.

Denote by $p_n(f, E, \mu)$ the polynomial of degree n, $n \in Z^+$, of best uniform approximation to the function $f(z)$, continuous on the compact set E in the plane, with respect to the weight function $\mu(z)$, positive and continuous on E. If $\mu(z) \equiv 1$ on E, set $p_n(f, E, \mu) = p_n(f, E)$. Also, for $a > 0$, let $E_a = \{z + a/z : |z| = 1\}$, $U_a = \{z : |z| = a\}$, $S_a = U_a \cup U_1$. Theorem. Let $F(\xi)$ be continuous on the ellipse E_a and let $f(z)$ be defined on the pair of concentric circles S_a by the relation: $f(z) = F(\xi)$ if $\xi = z + a/z$. Also let $\mu(z)$ be defined on S_a by: $\mu(z) \equiv |z|^n$, $n \in Z^+$. Then, $p_{2n}(u^n f(u), S_a, \mu)(z) = z^n p_n(F(\xi), E_a)(z + a/z)$. The special case where $a = 1$ gives a connection between best approximation on a circle and best approximation on a line segment. Several applications are given and some well-known examples of best approximation are derived. (Received June 4, 1970.)

70T-B168. WITHDRAWN.

70T-B169. HEINRICH W. GUGGENHEIMER, Polytechnic Institute of Brooklyn, Brooklyn, New York 11201. Geometric theory of differential equations. II. Analytic interpretation of a geometric theorem of Blaschke.

(1) Given a Hill equation of period T and collapsed second interval of stability: if f is continuous on $\tau \leq t \leq \tau + T$, $f(\tau) = f(\tau + T)$ and $\int_{\tau}^{\tau+T} f(t)x(t)dt = 0$ for all solutions x(t) of the Hill equation, then there exist at least three distinct values t_1, t_2, t_3 in $\tau \leq t < \varphi(t)$ for which $f(t_i)\varphi'(t_i)^{-3/4} = f(\varphi(t_i))\varphi'(\varphi(t_i))^{-3/4}$ where $\varphi(t)$ is Borůvka's dispersion function. (2) If the first interval of instability collapses and $f(\tau) = f(\tau + 2T)$, $\int_{\tau}^{\tau+2T} f(t)x(t)dt = 0$ then there exist $\tau \leq t_1 < t_2 < t_3 < \tau + T$ such that $f(t_i) = f(t_i + T)$. (3) Let $\varphi_2(t)$ be the second dispersion of a second order linear differential equation with integrable coefficients. Let f(t), g(t) be continuous functions, periodic of period $T \leq \varphi_2(t)$ and assume $g(t) > 0$, $\int_{\tau}^{\tau+T} f(t)x(t)dt = \int_{\tau}^{\tau+T} g(t)x(t)dt = 0$ for any absolutely continuous solution x(t). Then f/g has at least four relative extrema in $\tau \leq t < \tau + T$. (Received June 8, 1970.)

70T-B170. RICHARD STAUM, Polytechnic Institute of Brooklyn, Brooklyn, New York 11201. The algebra of continuous bounded functions into a nonarchimedean field. Preliminary report.

Let X denote the Banach algebra of continuous bounded functions on a topological space S into a nontrivially valued field F. S is called strongly T2 (ST2) if distinct points have distinct clopen neighborhoods; strongly regular (SR) if a disjoint point and closed set have disjoint clopen neighborhoods; mildly compact (MC) if every clopen cover has a finite subcover; and mildly countably compact (MCC) if every countable clopen cover has a finite subcover. Results include: Each x in X realizes its norm on some point in S iff F is discrete or S is MCC; F is the only quotient field of X iff F is locally compact or S is MCC. The canonical mapping of S into the maximal ideals of X (Gelfand topology) is continuous; it is one-to-one iff S is ST2; onto iff S is MC; a homeomorphism into iff S is ST2 and SR; a homeomorphism onto iff S is ST2 and compact. The canonical mappings relating the closed sets of S and the closed ideals of X are bijective and inverse to one another iff S is SR and MC. S is MC iff every closed subalgebra of X which separates quasicomponents is X itself or a fixed maximal ideal; if the only closed subalgebra of X which separates quasicomponents and contains the constants is X itself, then S is MCC. (Received June 8, 1970.)

70T-B171. AMY C. KING, University of Kentucky, Lexington, Kentucky 40506. A class of entire functions of bounded index. II.

This is a continuation of an earlier paper by the author [Abstract 672-95, these *Notices* 17 (1970), 110]. Let $P(z) = \Pi_{n=1}^{\infty}(1 - z/a_n)$, where P(z) is an entire function. Theorem 1. For $a_n = n^{\alpha}$, $2 < \alpha < 3$, P(z) is of bounded index. It is to be noted that for $\alpha = 2$, $P(z) = (\sin \pi\sqrt{z})/\pi\sqrt{z}$ and satisfies the linear differential equation $zP''(z) + (3/2)P'(z) + (\pi^2/4)P(z) = 0$. Thus by S. M. Shah, J. Math. Mech. 18 (1968), 131-136, P(z) is of bounded index when $a_n = n^2$. Theorem 2. Suppose $|a_1| \geq 4$ and $|a_{n+1}| \geq 4^n|a_n|$, then $|P^{(n)}(z)| \leq \max\{|P(z)|, |P'(z)|\}$. This gives more information than saying P(z) is of bounded index one. Theorem 3. If $|a_1| > 6.1$ and $|a_{n+1}| > (3.2)^n|a_n|$, n = 1,2,3,..., then for all z, $|P^{(n)}(z)| \leq \max\{|P(z)|, |P'(z)|\}$. Theorem 4. Let $F(z) = zP(z)$, $a_0 = 0$, $|a_1| > 32$, and $|a_{n+1}| > 2^n|a_n|$, n = 1,2,3,..., then for all z, $|F^{(n)}(z)| \leq \max\{|F(z)|, |F'(z)|\}$. Theorem 5. If $|a_1| > 18$ and $|a_{n+1}| > 3(3/2)^n|a_n|$, n = 1,2,3,..., then P(z) is of bounded index. (Received June 1, 1970.)

70T-B172. ANDRE DE KORVIN, Indiana State University, Terre Haute, Indiana 47809. On functions of bounded variations. III. Preliminary report.

For preliminary definitions and notations see Abstracts 70T-B135 and 70T-B136, these *Notices* 17 (1970), 661. Assume that μ_F and μ_G satisfy the following three conditions. (1) μ_G and μ_F can be extended as countably additive measures to $\bar{A}$, the σ-field generated by A. (2) $\mu_F(B(X, \sigma)) \neq 0$. (3) For every $\epsilon > 0$ there exists a set of the form $B(X, \sigma)$ such that $\mu_F(B(X, \sigma)) < \epsilon$. (Such is the case when S is indentifiable with an interval.) Definition. Let $\{C_n\}$ be a sequence of sets of the form $B(X, \sigma)$; $C_n \to p$ if (1) $\mu_F(C_n) \to 0$. (2) If B is a set of the form $B(X, \sigma)$ then $C_n \subset B$ for all by a finite number of n. (3) $p \in C_n$ for all n. Definition. Let $p \in A$ and assume there exists at least one sequence $\{C_n\}$ such that $C_n \to p$, let $(dG/dF)(p) = \lim_{C_n \to p} L(C_n)G/L(C_n)F$. Theorem. Assume that if $p \in A$ then p is the limit of at least one sequence $\{C_n\}$. Assume μ_F and μ_G satisfy the 3 conditions, then dG/dF exists on A - E where $\mu_F{}^*(E) = 0$. Corollary. The theorem holds if the first condition on μ_G is replaced by, G is Lipschitz respectively to F. (Received June 11, 1970.)

70T-B173. DAVID F. DAWSON, North Texas State University, Denton, Texas 76203. Matrix summability of divergent sequences.

It is shown that if a matrix $A = (a_{pq})$ has the property that $a_{pq} \to 0$ as $p + q \to \infty$, then A sums a divergent sequence of 0's and 1's to 0. This improves a result of Agnew [Bull. Amer. Math. Soc. 52 (1946), 128-132. MR 7, 292] to the effect that if A satisfies (1) $\sum_{q=1}^{\infty} |a_{pq}| < \infty$, $p \geq 1$, and (2) $\lim_p \max_q |a_{pq}| = 0$, then A sums a divergent sequence of 0's and 1's to 0. Equivalent to the first result mentioned above is the following: If $\lim_q a_{pq} = 0$, $p \geq 1$, and $\lim_p \max_q |a_{pq}| = 0$, then A sums a divergent sequence of 0's and 1's to 0. Thus Agnew's condition (1) is replaced with a condition which is both weaker and easier to check. It is also shown that if $\lim_p a_{pq}$ exists, $q \geq 1$, and $\lim_q \mathrm{lub}_p |a_{pq}| = 0$, then A sums a divergent sequence of 0's and 1's. (Received June 12, 1970.)

70T-B174. ANTOINE DERIGHETTI, Harvard University, Cambridge, Massachusetts 02138. Some results on functions on locally compact groups. Preliminary report.

Let G be an arbitrary locally compact group with left Haar measure dx. Let H be a closed subgroup of G and let q be a strictly positive continuous solution of the functional equation $q(x\xi) = q(x)\Delta_H(\xi)\Delta_G(\xi^{-1})$ for $x \in G$, $\xi \in H$; $d\dot{x}$ is the corresponding quasi-invariant measure on G/H. Theorem 1. If H has property P_1 and if there exists $0 \leq \lambda < 1$ such that, for every $\epsilon > 0$ and every finite subset F of G there is some $s \in L^1(G/H)$ with $s \geq 0$, $\|s\|_1 = 1$ and $\sup\{\int_{G/H} |q(yx)q(x)^{-1} s(y\dot{x}) - s(\dot{x})| d\dot{x} \mid y \in F\} < \lambda + \epsilon$, then G has the property P_1. Theorem 1 generalizes the case where H is normal and $\lambda = 0$ which is due to Reiter [C. R. Acad. Sci. Paris 267 (1968), 882-885]. For $f \in L^1(G)$ we denote by $\|f\|_\Sigma$ the norm of f as an element of the full C^*-algebra of G. Theorem 2. For every $\epsilon > 0$ and every compact subset K of G there is some $s \in L^1(G)$ with $s \geq 0$, $\|s\|_\Sigma = 1$ and $\sup\{\|{}_ys - s\|_\Sigma \mid y \in K\} < \epsilon$ (where ${}_ys(x) = s(yx)$). (Received June 12, 1970.)

70T-B175. DAVID K. COHOON, University of Wisconsin, Madison, Wisconsin 53705. Nonexistence of a continuous right inverse for surjective linear partial differential operators on special spaces of infinitely differentiable functions. I. Preliminary report.

Let $P(D)$ be a nontrivial linear partial differential operator with constant coefficients $n \geq 2$ independent variables. Let $\gamma^{(\delta)}(\Omega) = \{f \in C^{\infty}(\Omega)$: for every compact subset K of Ω and every $\epsilon > 0$ $\|f\|_{(\epsilon,K)} = \sup\{\epsilon^{-|\alpha|}|\alpha|^{-|\alpha|\delta}|D^{\alpha}f(x)|: \alpha \in \mathbb{N}^n, x \in K\}$ is finite$\}$. Suppose that there exist two linearly independent vectors in $\mathbb{R}^n$ which are orthogonal to every characteristic of $P(D)$. Then $P(D)$ has a continuous right inverse in $\gamma^{(\delta)}(\Omega)$ for some $\delta > 1$ if and only if $P(D)$ is a constant. This is significant because of the well-known result of B. Malgrange which states that if Ω is $P(D)$-convex and $\delta > 1$, then $P(D)$ is an epimorphism of $\gamma^{(\delta)}(\Omega)$. (Received June 8, 1970.)

70T-B176. WITHDRAWN.

70T-B177. G. D. LAKHANI, Indian Institute of Technology, Kanpur, India. An identity in $\mathcal{H}(B)$ spaces.

Let $\mathcal{C}(z)$ be a Hilbert space of formal power series $f(z) = \sum_{n=0}^{\infty} a_n z^n$ such that $\|f(z)\|^2 = \sum |a_n|^2 < \infty$, and $B(z)$ be a power series such that $B(z)f(z)$ belongs to $\mathcal{C}(z)$ and $\|B(z)f(z)\| \leq \|f(z)\|$ whenever $f(z)$ belongs to $\mathcal{C}(z)$. The set $\mathcal{H}(B)$ of elements $f(z)$ of $\mathcal{C}(z)$ for which $\|f(z)\|_B^2 = \sup_{g(z) \in \mathcal{C}(z)} [\|f(z) + B(z)g(z)\|^2 - \|g(z)\|^2] < \infty$, is a Hilbert space. The series $(f(z) - f(0))/z$ belongs to $\mathcal{H}(B)$ whenever $f(z)$ is in $\mathcal{H}(B)$ and the identity, $\|(f(z) - f(0))/z\|_B^2 \leq \|f(z)\|_B^2 - |f(0)|^2$, holds for all $f(z)$ of $\mathcal{H}(B)$. The equality is attained, if and only if, $B(z)$ does not belong to $\mathcal{H}(B)$. Theorem. Let $\mathcal{H}(B)$ be a given space which contains $B(z)$ in it. Then $(f(z) - f(0))/z$ belongs to $\mathcal{H}(B)$, and $\|(f(z) - f(0))/z\|_B^2 = \|f(z)\|_B^2 - |f(0)|^2 - (|\langle f(z), B(z)\rangle_B|^2)/(1 + \|B(z)\|_B^2)$ whenever $f(z)$ belongs to $\mathcal{H}(B)$. (Received June 15, 1970.) (Author introduced by Professor S. A. Naimpally.)

70T-B178. S. K. BAJPAI, Indian Institute of Technology, Kanpur, India. Coefficient estimates for univalent α-spiral functions.

Coefficient estimates for the function $f(z) = z + \sum_{n=k+1}^{\infty} a_n z^n$ belonging to the class $S_\rho(\alpha)$, $|\alpha| \leq \pi/2$, of α-spiral functions of order ρ, regular and univalent for $|z| < 1$ and characterized by the condition $\mathrm{Re}\{ze^{i\alpha}f'(z)/f(z)\} \geq \rho$, $(0 \leq \rho < 1)$ have been obtained. These include some earlier results due to Macgregor (Michigan Math. J. 10 (1963), 227-281), Libera (Canad. J. Math. 19 (1967), 449-456), Robertson (Ann. of Math. 37 (1936), 374-408), Golusin (Recueil Math. Moscow 36 (1929), 152-172 (Russian)) and Schild (Amer. J. Math. 87 (1965), 65-70). First a lemma is established by induction and then following the method of Clunie (J. London Math. Soc. 34 (1959), 215-216) and making use of the lemma, estimates for $\sum_{n=mk+1}^{(m+1)k}(n-1)^2|a_n|^2$ and $\sum_{n=mk+1}^{(m+1)k}(n-\rho)|a_n|^2$ have been derived. From these more refined estimates for $|a_n|$ have been obtained for functions of the class $S(\alpha)$, in general, and the class $S\rho$ (0) (starlike) in particular. (Received June 15, 1970.) (Author introduced by Professor S. A. Naimpally.)

70T-B179. R. S. L. SRIVASTAVA, Indian Institute of Technology, Kanpur, India. On the radii of convexity and starlikeness of univalent functions.

This paper investigates the radii of starlikeness and convexity of starlike functions of order β

and convex functions of order β. Converses of theorems of Bernardi (Proc. Amer. Math. Soc. 24 (1970), 312-318) have been established. These results also include theorems of Padmanabhan (J. London Math. Soc. (2) 1 (1969), 225-231). (Received June 15, 1970.) (Author introduced by Professor S. A. Naimpally.)

70T-B180. JAMES GUYKER, Lehigh University, Bethlehem, Pennsylvania 18015. Reducing subspaces of contractions with no isometric part. Preliminary report.

Let T be a contraction on a Hilbert space H and suppose that there is no nonzero vector f in H such that $\|T^n f\| = \|f\|$ for every n = 1,2,3,... . Theorem. If T is hyponormal and the rank of $1 - T^*T$ is finite, then T is normal. This result is best possible in the sense that the conclusion no longer holds if "finite rank" is replaced by "trace class". The proof is straightforward and elementary, and a similar technique yields the result that if the rank of $1 - T^*T$ is 1, then T is irreducible. In particular we have that the adjoint of the simple unilateral shift restricted to any invariant subspace is irreducible. (Received June 15, 1970.)

70T-B181. JERROLD W. BEBERNES, University of Colorado, Boulder, Colorado 80302, and JERRY D. SCHUUR, Michigan State University, East Lansing, Michigan 48823. Wazewski's method for contingent equations.

Let $W \subset R^1 \times R^n$ be open and let C be the set of nonempty, compact, convex subsets of R^n. We consider the contingent equation (1) $x' \in F(t,x)$ where $F: W \to C$ is upper semicontinuous. With $(t_P, x_P) = P \in W$, $\varphi(t,P)$ is a solution of (1) with $\varphi(t_P, P) = x_P$. Let $V \subset W$ be open. If $P \in V$ and if there exists t_1, t_2 such that $(t, \varphi(t, P)) \in V$ for $t_P \leq t < t_1$, $(t, \varphi(t, P)) \in W - \overline{V}$ for $t_2 < t < t_2 + \epsilon$, and $\{(t, \varphi(t, P)) : t_1 \leq t \leq t_2\} \overset{\text{def}}{=} C_\varphi(P) \subset \partial V$, then $\varphi(t, P)$ egresses strictly from V and $\bigcup\{C_\varphi(P) : \varphi$ is a solution of (1) through $P\} \overset{\text{def}}{=} C(P)$. Then $P \to C(P)$ defines a set-valued mapping from V into ∂V. We extend the Nagumo existence theorem to (1) and we use the concept of weak invariance to prove: Theorem 1. If all solutions of (1) through $P \in V$ egress strictly, then $C: P \to C(P)$ is upper semicontinuous and C(P) is compact and connected. We then define the concept of a set-valued retraction mapping and we prove a Wazewski-type theorem for the existence of solutions of (1) which remain in V on their maximal intervals of existence. (Received June 19, 1970.)

70T-B182. JOHN R. EDWARDS and STANLEY G. WAYMENT, Utah State University, Logan, Utah 84321. Extensions of the v-integral.

In these *Notices* 17 (1970) the v-integral is defined over finite intervals in E^1 and is used to give a representation for transformations continuous in the BV-norm. The functions f considered therein are real valued or have values in a linear normed space X, and the transformation T(f) has values in a linear normed space Y. In this paper the v-integral is extended in several directions: (1) the domain space of f to E^n; (2) the function space to (a) continuous over an interval, (b) C_c, (c) C_0, (d) C with uniform convergence on compact sets, (e) classical L^p spaces; (3) range spaces X for the functions and Y for the transformation to topological vector spaces (not necessarily convex); (4) when X and Y are convex spaces, then we represent transformation on C_1, the continuously differentiable functions with values in X. (Received June 9, 1970.)

70T-B183. WITHDRAWN.

70T-B184. LEO J. SCHNEIDER, Case Western Reserve University, Cleveland, Ohio 44106. On the oscillatory behavior of 2-2 disconjugate linear selfadjoint fourth order differential equations.

Let p, q, r be continuous and sufficiently differentiable on the half line $a \leq t < \infty$ with $r > 0$. Let $Ly = (ry'')'' - (qy')' + py$. $Ly = 0$ is said to be 2-2 disconjugate if no nontrivial solution has more than one double zero. Theorem. If $Ly = 0$ is 2-2 disconjugate, then either all solutions oscillate or no solutions oscillate. This extends a result by Leighton and Nehari (Trans. Amer. Math. Soc. 89 (1958), 367) for the differential equation $(ry'')'' + py = 0$, $r > 0$, $p > 0$ on $a \leq t < \infty$. (Received June 22, 1970.)

70T-B185. THOMAS A. W. DWYER, III, University of Maryland, College Park, Maryland 20742. Fischer spaces for the Hilbert-Schmidt holomorphy type. I.

E is a complex Hilbert space. The space of n-homogeneous Hilbert-Schmidt polynomials (n-H-S-polynomials) on E is the completion of the vector space generated by the polynomials $u'^n(x) = (x|u)^n$ given by each u in E, for the unique inner product such that $(u'^n|v'^n)_H = (v|u)^n$. The dual of the n-H-S-polynomials on E is the space of n-H-S-polynomials on E'. Given $r > 0$ the completion of the space of Hilbert-Schmidt polynomials $\Sigma_n p_n$, where p_n is an n-H-S-polynomial on E, for the inner product $(\Sigma_n p_n|\Sigma_n q_n)_r = \Sigma_n r^n n!(p_n|q_n)_H$ is a Hilbert space $F_r(E)$ of entire functions with reproducing kernel. The inverse limit inv lim $F_r(E)$ relative to r is a reflexive (F)-space, nuclear iff dim E is finite. The locally convex direct limit dir lim $F_r(E)$ is a reflexive bornological (DF)-space, (hence complete) conuclear iff dim E is finite. The Fourier-Borel transformation is a topological linear isomorphism from the dual of $F_r(E)$ (resp. inv lim $F_r(E)$) (resp. dir lim $F_r(E)$) onto $F_{r^{-1}}(E')$ (resp. dir lim $F_r(E')$) (resp. inv lim $F_r(E')$). If dim E is finite then $F_1(E)$ is the Fischer space of Bargmann, Newman, Shapiro and Treves. (Received June 22, 1970.)

70T-B186. GÜNTHER W. GOES, Illinois Institute of Technology, Chicago, Illinois 60616. A fundamental multiplier formula. Preliminary report.

Let A, B and C be subsets of ω, the space of complex valued sequences $x = \{x_k\}$, let ϕ be the space of all sequences with only finitely many nonzero terms and let $(A \to B) = \{x \in \omega : xa\{x_k a_k\} \in B$ for every $a \in A\}$ be the class of multipliers from A into B. For any $x \in \omega$ let $P_n(x) = \{x_1, x_2, \ldots, x_n, 0, 0, \ldots\}$. Then $(A \to B) = (A \cdot (B \to C) \to C)$ if one of the following sets of conditions is fulfilled: (i) $A \subset \omega$ arbitrary and $B = (D \to C)$ for some $D \subset \omega$. (ii) $A \subset \omega$ arbitrary, B is an FK-space with FAK i.e. such that, for every $\varphi \in B^*$ (= conjugate space of B) and every $x \in B$, $\lim_{n\to\infty} \varphi(P_n(x))$ exists. Furthermore $\phi \cap A \subset B$; and $x \in B$ if and only if $\{P_n(x)\}_{n=1}^{\infty}$ is bounded in B. $C = cs = \{x \in \omega : \Sigma_{k=1}^{\infty} x_k$ exists$\}$. (iii) A is an FK-space with AK, i.e. such that $P_n(x) \to x$ $(n \to \infty)$ for every $x \in A$, $\phi \cap A \subset B$, B is an FK-space with FAK, C = cs. (Received June 22, 1970.)

70T-B187. ROBERT M. HARDT, Brown University, Providence, Rhode Island 02912. Intersections of real algebraic chains. Preliminary report.

Let $n \geq t \geq m > 0$ be integers. A subset of R^n is semi-algebraic if it is defined by finitely many polynomial equalities and inequalities. An integral flat current [H. Federer, "Geometric measure theory", Springer-Verlag, 1969, p. 381] T is a t dimensional algebraic chain if there are t and t - 1 dimensional semi-algebraic sets C and D with spt $T \subset C$ and spt $\partial T \subset D$. If f is a polynomial map from R^n to R^m, then for almost all $y \in R^m$, the slice of T in $f^{-1}(y)$, $\langle T,f,y \rangle$, is a t - m current, defined by the relative differentiation of measures [Ibid., p. 435]. Let U be the set of $y \in R^m$ for which the dimension of spt $T \cap f^{-1}(y) \leq t - m$ and that of spt $\partial T \cap f^{-1}(y) \leq t - m - 1$. Theorem. The function sending y to $\langle T,f,y \rangle$ maps U into the t - m dimensional algebraic chains and is integrally flat continuous. An intersection product of algebraic chains is defined by slicing with the diagonal map, and a real algebraic intersection theory follows with various consequences of geometric and topological interest. (Received May 21, 1970.)

70T-B188. WILHELM FORST, Universität Konstanz, 775 Konstanz, Federal Republic of Germany. An analytic proof of Müntz's theorem. Preliminary report.

Müntz's theorem can be generalized in the following way: Theorem 1. Let X be the complex space $L^p(0,\infty)$ $(1 \leq p < \infty)$ or $C'[0,\infty] := \{f \in C[0,\infty] \mid f(\infty) = 0\}$, $\{\lambda_k\}_0^\infty$ a sequence of distinct positive numbers and $\{\nu_k\}_0^\infty$ a sequence of positive integers. If S denotes the set of all exponential polynomials of the form $\sum_{k=0}^n \sum_{l=0}^{\nu_k - 1} c_{kl} x^l e^{-\lambda_k x}$, the following properties are equivalent: (A) S is not dense in X; (B) $\sum_{k=0}^\infty \nu_k \lambda_k/(1 + \lambda_k^2) < \infty$; (C) There is a nontrivial function of the Paley-Wiener-class of the right half plane with zeros of order ν_k at λ_k. Theorem 2. Let X be the complex space $L^p(-a,a)$ $(1 \leq p < \infty)$ or $C[-a,a]$, $\{\lambda_k\}_0^\infty$ a sequence of distinct real numbers with $\lambda_0 = 0$, and $\{\nu_k\}_0^\infty$ a sequence of integers; $\nu_0 \geq 0$ and $\nu_k \geq 1$ for $k \geq 1$. Then the following properties are equivalent: (A) S is not dense in X; (B) $\sum_{k=1}^\infty \nu_k/|\lambda_k| < \infty$; (C) There is a nontrivial function of the Paley-Wiener-class to the degree a with zeros of order ν_k at the points λ_k. The proof is based on the representation theorems of Paley and Wiener and theorems of Carleman and Blaschke concerning the distribution of the zeros of analytic functions. (Received June 23, 1970.) (Author introduced by Professor Gerhard J. Neubauer.)

70T-B189. LAWRENCE A. SHULMAN, Georgetown University, Washington, D. C. 20007. Some cyclic vectors of the Cesàro operator.

The Cesàro operator C_0 is defined on the Hardy space H^2 of square summable power series as follows: If $f(z) = \sum a_n z^n$ belongs to H^2, then $C_0 f(z) = g(z)$ where $g(z) = \sum b_n z^n$ and $b_n = (1/(n+1))\sum_{k=0}^n a_k$, n = 0, 1, 2,... . If $|w| < 1$, the kernel function $K(w,z) = 1/(1 - \bar{w}z)$ is a cyclic vector for C_0. Moreover for any k = 0, 1, 2, ... the function $C_0^k K(w,z)$ is a cyclic vector for C_0. (Received June 1, 1970.)

70T-B190. VIDYADHAR S. MANDREKAR and HABIB SALEHI, Department of Statistics and Probability, Michigan State University, East Lansing, Michigan 48823. On Lebesgue decomposition theorem for operator-valued measures.

The concepts of absolute continuity and singularity for operator-valued measures are introduced and Radon, Nikodým and Lebesgue decomposition theorems for such measures are established. These theorems reduce directly to the classical results in the scalar case. (Received June 5, 1970.)

70T-B191. DAVID ZEITLIN, 1650 Vincent Avenue North, Minneapolis, Minnesota 55411. A new class of generating functions for hypergeometric functions.

Let Y_k and W_k be two sequences related by $W_n = \sum_{k=0}^{\infty} \binom{a+bn}{Mk+n} Y_k$, where a, b, a_i, b_i, and M are parameters independent of n. Case 1. $M = -m$, $m = 1,2,\ldots$. Define Y_k by (*) $k!(b_1)_k \ldots (b_q)_k Y_k = (-1)^{mk}(a_1)_k \ldots (a_p)_k x^k$. Then W_n is a generalized hypergeometric polynomial whose generating function, expressed as a Taylor series, is given by D. Zeitlin (Proc. Amer. Math. Soc. 25 (1970), 405-412). See Abstract 70T-B27, these *Notices* 17 (1970), 290. Case 2. $M = m$, $m = 1,2,\ldots$. Define Y_k by (*). Then W_n, $n = 0, \pm 1, \pm 2, \pm 3, \ldots$, is a generalized hypergeometric function whose generating function is expressed as a Laurent series (a key identity used for both Cases 1 and 2 is (6.1) on p. 196 in the paper by H. W. Gould (Duke Math. J. 28 (1961), 193-202)). Applications are given for the Whittaker function, Bessel coefficients, and the modified Bessel coefficients. Two independent extensions of both Cases 1 and 2 are also indicated. This paper will be published in Proc. Amer. Math. Soc. (Received June 25, 1970.)

70T-B192. KAM-FOOK TSE, Syracuse University, Syracuse, New York 13210. Interpolating sequences in a Stolz angle and interpolation by normal functions.

Let Ω, D and ρ be the Riemann sphere, the unit disk, and the non-Euclidean metric in D, respectively. If $S, S' \subset D$, $S \cap S' = \emptyset$, then $\rho(S, S') = \inf\{\rho(z,z'): z \in S, z' \in S'\}$. A sequence of points $\{z_n\}$ in D is called an interpolating sequence if for each $\{w_n\} \in l^{\infty}$ there exists $f \in H^{\infty}$ such that $f(z_n) = w_n$. Lemma 1. If S_j, $j = 1,2,\ldots,n$, are n disjoint interpolating sequences in D, then $S = \bigcup_{j=1}^{n} S_j$ is an interpolating sequence if and only if $\rho(S_i, S_j) > 0$ for all $i \neq j$. Our first result is Theorem 1. Let $\{z_n\}$ be a sequence of points contained in a Stolz angle in D. Then $\{z_n\}$ is an interpolating sequence if and only if $\rho(z_n, z_m) \geq \delta > 0$ for all $m \neq n$. To extend the interpolating problem to normal functions, we have to use careful construction to obtain two functions in H^{∞} such that their quotient is the needed function in Theorem 2. If $\{z_n\}$ is an interpolating sequence in D and $\{w_n\}$ is a sequence in Ω, then there exists a normal function H(z) (which is also of bounded type) such that $H(z_n) = w_n$ for all $n = 1,2,\ldots$. (Received June 17, 1970.)

70T-B193. H. RHEE, State University College of New York, Oneonta, New York 13820. Solutions of the wave equation in the interior of the characteristic cones. Preliminary report.

Definition. Let E(X,t) be the one-parameter family of confocal ellipsoids which have the origin 0 and a point $X = r\alpha$, $r = |X| \neq 0$, $X \in R^n$, as foci, and the line $\overline{OX}$ as the axis of rotational symmetry. A point $Y = \rho\gamma$, $\rho = |Y|$, will lie on E(X,t) of parameter t, if $\gamma = p\alpha + (1 - p^2)^{1/2}\beta$, $p = t/r + (r^2 - t^2)/2\rho r$, $t > r$, $(t - r)/2 \leq \rho \leq (t + r)/2$, β is a unit vector perpendicular to α. The ellipsoidal means, EM, of a continuous function f(X) over E(X,t) is defined as follows: $EM[f;X,t] = (\omega_{n-1} r)^{-1}(r^2 - t^2)^{-k} \int_b^c d\rho \int_\beta \rho^k f(\rho\gamma)(1 - p^2)^k d\omega_\beta$, where $k = (n - 3)/2$, $c = (r + t)/2$, $b = (t - r)/2$. Theorem. (I) Let f(X) be a continuous function in R^n with the property that $f(\rho\gamma)$ is of class C^2 with respect to γ. Then $EM[f;X,t]$ satisfies the wave equation in the cone $t > |X|$. (II) Let J(X) be a function of class C^{k+1}, such that (a) $w(Y) = \delta^{k+1}(\rho^{n-2}J(\rho\gamma))$ is of class C^2 with respect to γ, $\delta = \partial/\partial\rho$, (b) $\delta^i(\rho^{n-2}J(\rho)) = O(1)$ for small ρ, $1 \leqq i \leqq k$. Then $W(X,t) = (-2)^k(k!)^{-1}EM[w;X,t]$ satisfies the wave equation in $t > |X|$, and for n odd $\geqq 3$, we have $\mathrm{Lim}_{t\to r} W(X,t) = J(X)$. (Received June 4, 1970.)

Applied Mathematics

70T-C31. PADAM C. JAIN, I. A. BELOV, and K. SANKARA RAO, Indian Institute of Technology, Powai, Bombay-76, India. Effects of the directional flow periodicity on the flow around a circular cylinder.

Unsteady flow motion of a viscous incompressible fluid past a circular cylinder (cf. Jain and Rao, Phys. Fluids 12(1969), supplement 2) is considered when the cylinder is held stationary and fluctuating disturbances are imposed upon the direction of the velocity of the external stream. The problem is solved numerically at Reynolds number 40. It is found that the directional fluctuations exert a strong influence on the flow pattern in the shear layer and behind the cylinder. The value of the total drag is observed to increase for low frequency fluctuations with relatively increasing positive values of the amplitude ϵ, and to decrease for the same low frequency at relatively high negative value of ϵ. The flow pattern in the wake changes from the flow with alternate shedding of vortices to a steady one in which the separation streamline re-attaches to the line of symmetry giving rise to two closed recirculatory regions attached to the cylinder. Figures are drawn to show the flow patterns and the variations of the flow characteristics. Results are compared with the case of a flow without directional periodicity in the oncoming stream. (Received March 10, 1970.)

70T-C32. SUDHANSHU GHOSHAL, Jadavpur University, Calcutta-32, W. Bengal, India and ABHA GHOSHAL, University of Toronto, Toronto-181, Ontario, Canada. Numerical solution of coupled integro-differential equations of Fredholm type. Preliminary report.

A method based on the generalization of Runge-Kutta's method is applied to a system of coupled integro-differential equations of Fredholm's type. It is shown that, under certain conditions, the related integral, containing an unknown variable, in each integro-differential equation, can be approximated by a contraction operator, converging in the respective space concerned. Therefore they may be found out by the method of iteration, starting from arbitrary values, and hence, the given system may be solved with high degree of accuracy. (Received May 22, 1970.)

70T-C33. C. W. GROETSCH, 3650 Nicholson Drive, Apt. 1218, Baton Rouge, Louisiana 70803. Segmenting Mann Iterates.

Let $A = (a_{nk})$ be an infinite, lower triangular, regular stochastic matrix. We also assume that $a_{n+1,k} = (1 - a_{n+1,n+1})a_{nk}$ for $k \leqq n$ and $\Sigma_{k=1}^{\infty}(1 - a_{kk})a_{kk}$ diverges. $M(x,A,T) = \{v_n, x_n\}_{n=1}^{\infty}$ denotes the Mann process: $x_1 = x$, $v_n = \Sigma_{k=1}^{n} a_{nk}x_k$, $x_{n+1} = Tv_n$, where T is a nonexpansive mapping of a uniformly convex Banach space X into itself and $x \in X$. Theorem. If T has a fixed point then 0 is a cluster point of $\{v_n - Tv_n\}_{n=1}^{\infty}$. Corollary 1. If T maps a compact convex subset E of X into itself and has a unique fixed point p then M(x,A,T) converges to p for each $x \in E$. Corollary 2. If T is demicompact and maps a closed bounded convex subset E of X into itself then M(x,A,T) converges to a fixed point of T for each $x \in E$. Corollary 3. If T maps a closed convex subset E of X into itself and has a fixed point and if X has a weakly continuous duality mapping then $M(x,A,T)$ converges weakly to a fixed point of T for each $x \in E$. The corollaries generalize theorems of Outlaw (Bull. Amer. Math. Soc. 75 (1969), 430-432), Browder and Petryshyn (J. Math. Anal. Appl. 20 (1967), 197-228) and Opial (Bull. Amer. Math. Soc. 73 (1967), 591-597) respectively. (Received June 26, 1970.) (Author introduced by Professor Curtis L. Outlaw.)

Geometry

70T-D18. PATRICK BARRY EBERLEIN, University of California, Los Angeles, California 90024. Manifolds of hyperbolic type. Preliminary report.

Let H denote a complete, simply connected n-dimensional Riemannian manifold without conjugate points along any geodesic. H admits a Hadamard compactification if it can be imbedded as an open dense subset in a compact Hausdorff space $\overline{H}$ such that for any two distinct points of $\overline{H}$ there is a unique geodesic of H "joining" them which depends continuously on the endpoints. Let M denote a complete n-manifold without conjugate points whose universal cover admits a Hadamard compactification. Let SM denote the unit tangent bundle of M, $\{T_t\}$ the geodesic flow in SM and Ω the nonwandering points of $\{T_t\}$ in SM. Theorem 1. Ω can be classified as in Abstract 70T-D12, these *Notices* 17(1970), 669. Theorem 2. $\{T_t\}$ has an orbit dense in Ω unless Ω is a single periodic orbit and the reverse orbit. Theorem 3. If M is compact then $\{T_t\}$ is topologically mixing in SM, that is for any open sets U, V of SM there exists $A > 0$: $|t| \geq A$ implies $T_t(U) \cap V$ is nonempty. Corollary. Let (X,g) be a compact n-manifold without conjugate points such that if γ, σ are distinct maximal geodesics in the universal cover H then $d(\gamma t, \sigma)$ is unbounded. If X admits a metric g^* with $K < 0$ then the g-geodesic flow in SX(g) is topologically mixing. (Received May 4, 1970.)

70T-D19. GEORGE P. BARKER, University of Missouri, Kansas City, Missouri 64110. The lattice of faces of a convex cone.

If K is a closed convex pointed full cone in a real vector space V of dimension η, then a face of K is a set F such that $0 \leq x \leq y$ and $y \in F$ implies $x \in F$. ($x \leq y$ means $x - y \in K$.) The set $\mathfrak{F}(K)$ of all faces of K is a lattice if $F \wedge G = F \cap G$ and $f \vee G$ is the least face containing both. Theorem 1. $\mathfrak{F}(K)$ is a distributive lattice if and only if K has exactly n extreme rays. Theorem 2. The lattice $\mathfrak{F}(K)$ is complemented for any K. (Received April 27, 1970.)

70T-D20. JOHN DeCICCO, Illinois Institute of Technology, Chicago, Illinois 60616 and ROBERT V. ANDERSON, Université de Québec à Montréal, Montréal 110, Québec, Canada. Elements of density transformations of a Euclidean space E_n.

The set Γ of harmonic point density transformations of a Euclidean space E_n, $n \geq 2$, consists of those transformations T under which points correspond and every harmonic density is converted into a harmonic density. If, in a Euclidean space E_n, the point part of any such transformation T is a similitude, then the density transforms as $U = A_0\overline{U} + B(x)$, where $A_0 \neq 0$ is a constant and $B(x)$ is harmonic, and $\nabla^2(U) = m^2 A_0 \nabla^2(\overline{U})$. Let T be a harmonic point density transformation under which points correspond such that either the point part is not a harmonic correspondence, or the Laplacian of the density does not depend only on the partial derivatives of second order. Then the point part of T is an inversive map and the density transforms as $U = A_0\bar{U}/\rho^{n-2} + B(x)$, where $A_0 \neq 0$ is a constant, $\rho^2 = \delta_{\alpha\beta}(x^\alpha - x_0^\alpha)(x^\beta - x_0^\beta)$, and B(x) is harmonic. (Received May 25, 1970.)

70T-D21. RICHARD S. MILLMAN, Ithaca College, Ithaca, New York 14850. Geodesics in metrical connections.

Let M be a Riemannian manifold with metric g. A linear connection D is called metrical if $Xg(Y,Z) = g(D_XY,Z) + g(Y, D_XZ)$ for all vector fields X,Y,Z. It is well known that a metrical connection (hence its geodesics) is determined by its torsion. In this paper the author asks to what extent the geodesics of a connection are determined by the assumption that the connection is metrical. He defines a tensor of type (1,2), the Q-invariant of the connection D and then proves the Theorem. Two metrical connections have the same geodesics if and only if their Q-invariants are equal. As a Corollary. A metrical connection has the same geodesics as the Levi-Civitae (Riemannian) connection if and only if $g(T_YX,X) = 0$ for all X,Y (where T is the torsion transformation). The author concludes with an example to show that the Q-invariant is a genuinely weaker invariant than is the torsion. By geodesic, the author means a parametrized curve. (Received June 4, 1970.)

70T-D22. JAYME M. CARDOSO, University of Campinas, Caixa Postal 1170, Campinas, Brazil. Ellipse as homologous of the circle. Preliminary report.

Let C be a circle and E an ellipse. There are three degenerate conics in the pencil determined by C and E. Let r_i, i = 1,2,3,4,5,6, be the straight lines of these degenerate conics. At least two r_i are real and axis of the planar homologies in which E is the correspondent of C. In particular if E is a circle there are only two (one is the line at infinity) axes. (Received June 9, 1970.)

70T-D23. HUNG-HSI WU, University of California, Berkeley, California 94720. Complete noncompact hypersurfaces with nonnegative curvature. Preliminary report.

The theorem of van Heijenoort-Sacksteder states that if M is a hypersurface in R^{n+1} which is complete, noncompact and has nonnegative curvature, and if the curvature is positive at one point, then M is the boundary of an unbounded convex open subset of R^{n+1}. We further prove that under the same assumptions: (1) the spherical image of M in the unit sphere S^n has a geodesically convex closure, and hence lies in a closed hemisphere, (2) M is tangent to and lies above some hyperplane H such that the intersection with M of any hyperplane parallel to H and at a positive distance above H is a smooth (n -1)-sphere, and (3) M has infinite volume. If M has positive curvature everywhere, then (4) the spherical map is one-one and the spherical image is itself a geodesically convex open subset of the unit sphere, and (5) for the hyperplane H in (2), M is actually the graph of a function defined in some convex domain of H. These results completely generalize the classical theorem of Stoker on complete surfaces in R^3 with positive curvature. (Received June 22, 1970.)

70T-D24. ROBERT MALTZ, 1614 Crescent Place, Venice, California 90291. Isometric immersions in Euclidean space of manifolds with nonnegative Ricci curvature.

Theorem. Let $f: M^d \to R^{d+k}$ be an isometric immersion of a complete Riemannian manifold M with nonnegative Ricci curvature, into a Euclidean space R^{d+k} (assume everything of class $\mathcal{C}^{\infty}$). Let n denote the minimal value of the index of relative nullity. Then M is isometric to a Riemannian product $N^{d-n} \times R^n$, and f is an n-cylindrical immersion $f: N^{d-n} \times R^n \to R^{d+k}$. (Received June 22, 1970.)

Logic and Foundations

70T-E48. PETER H. KRAUSS, State University of New York, New Paltz, New York 12561. Universally complete universal theories. I. Preliminary report.

Consider relational systems $\mathfrak{A}$ with finitely many relations. $Th_{\forall}\mathfrak{A}$ denotes the (first-order) universal theory of $\mathfrak{A}$, and $\mathfrak{A} \equiv_{\forall} \mathfrak{B}$ means that $\mathfrak{A}$ and $\mathfrak{B}$ are universally equivalent. A set Σ of universal sentences is significantly consistent if Σ has an infinite model, and Σ is called universally complete if Σ is maximally significantly consistent (in the set of universal sentences). $\mathfrak{A}$ is called universally complete if $Th_{\forall}\mathfrak{A}$ is universally complete. For each $n < \omega$, let $\exists^{\leq n}$ denote the universal sentence stating that there are at most n objects. Theorem 1. Let Σ be a significantly consistent universal theory. Then the following are equivalent. (a) Σ is universally complete. (b) For every universal sentence σ, if $\sigma \notin \Sigma$ then there exists $n < \omega$ such that $\Sigma \models \sigma \leftrightarrow \exists^{\leq n}$. (c) For all $\mathfrak{A}, \mathfrak{B} \in \mathrm{Mod}\,\Sigma$, if $\bar{\bar{\mathfrak{A}}}, \bar{\bar{\mathfrak{B}}} \geq \omega$ then $\mathfrak{A} \equiv_{\forall} \mathfrak{B}$; and there exists $\mathfrak{A}$ such that $\bar{\bar{\mathfrak{A}}} \geq \omega$ and $\mathrm{Mod}\,\Sigma = \mathrm{Mod}\,Th_{\forall}\mathfrak{A}$. (d) Any two finite models of Σ of the same cardinality are isomorphic. Corollary 2. Let $\bar{\bar{\mathfrak{A}}} \geq \omega$. Then $\mathfrak{A}$ is universally complete iff any two finite subsystems of $\mathfrak{A}$ of the same cardinality are isomorphic. Remark. 1(b) relates our notion of universal completeness to the notion of universal completeness due to H. Ribeiro. R. Fraissé calls universally complete relational systems monotypic or monomorphic (see 2). (Received April 29, 1970.)

70T-E49. ROHIT J. PARIKH, Boston University, Boston, Massachusetts 02215. Length of proofs in first order arithmetic.

T is a consistent theory with symbols +, $\cdot$, S, 0 and containing Peano arithmetic. The length of a proof is the number of symbols. P(A) is the formula: "A is provable." $P^{k+1}(A) = P(P^k(A))$. Theorem 1. Let g be prim. rec. For all k, there are A, n such that $P^{k+1}(A)$ has a proof of length $\leq n$ and $P^k(A)$ is provable but has no proof of length $\leq g(n)$. Theorem 2. Consider formulae A: s = t where s, t are variable free terms formed from 0, +, $\cdot$, S, $k(x,i) = (x)_i$, $e(x,y) = x^y$. There are short formulae A such that there is no proof in ZF of length $\leq 10^{1000}$ (say) of A, $\neg$A or any $P^k(A)$ or $P^k(\neg A)$. Assume all axioms of T are true. Add the symbol F (feasible) and the axioms: F(0); $(\forall x)(F(x) \to F(S(x)))$; $\neg F(\theta)$ where θ is a fixed variable free term. T_1 is the extended theory. The value of θ is m. Assume T_1 formalised in the ϵ-calculus. Theorem 3. Let P be a proof in T_1 in tree form of a formula B not containing F. There are k formulae in P of the form $A(t) \to A(\epsilon_x A(x))$ where F occurs in A. There are ℓ formulae of the form $F(t) \to F(S(t))$ and $2^{k^2} \cdot \ell < m$. Then B is true. (Received April 29, 1970.)

70T-E50. STEPHEN G. SIMPSON, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. Omitting types in self-extending models.

The following improves a theorem of Morley, "Omitting classes of elements " in "The theory of models," North-Holland, Amsterdam, 1965. Theorem. Let T be a countable first order theory and S a set of formulas of L(T) in n free variables. Suppose T has models of all powers up to beth-omega-one omitting S. Then there are T* a conservative extension of T and a sequence S_i of sets of formulas of L(T*) in n_i free variables respectively such that (1) $S_0 = S$ and $n_0 = n$; (2) T* has a model omitting all the S_i; and (3) any model of T* omitting all the S_i has a "built-in" proper elementary extension omitting all the S_i. (Compare Vaught, "A Löwenheim-Skolem theorem for cardinals far apart," ibid.) The T* is inspired by recent work of Shelah on theories having k-like models for all uncountable k. The T* and the sequence S_i are obtained recursively from T and S. (Received April 29, 1970.) (Author introduced by Professor Gerald E. Sacks.)

70T-E51. OFELIA T. ALAS, University of São Paulo, Caixa Postal 8105, São Paulo, Brazil. Topological groups and the GCH.

Notations. For any two sets A and B, $A \leq B$ means that there is an injective map from A into B; $A < B$ means that $A \leq B$ and A and B are not equipotent sets. For any infinite sets M, Y and S, with $M < Y \leq 2^M \leq S$, let $G(S) = \{x \in \{0,1\}^S \mid \{s \in S \mid x_s = 1\} \leq M\}$ and $\tau(Y)$ be the set of all unions of sets $G(S) \cap \Pi_{s \in S} V_s$, where $V_s \subset \{0,1\}$, $\forall s \in S$ and $\{s \in S \mid V_s \neq \{0,1\}\} \leq Y$. The pair $(G(S), \tau(Y))$ is denoted by G(S,Y). Remark. If G(S) is a subgroup of $\{0,1\}^S$ and $\tau(Y)$ is a topology in G(S), then G(S,Y) is a topological group. (I) For any infinite sets M,Y and S, with $M < Y \leq 2^M \leq S$, if G(S,Y) is a topological group, then G(S,Y) has property K. (II) For any infinite sets M and Y, with $M < Y \leq 2^M$, if $G(2^M, Y)$ is a topological group, then $G(2^M, Y)$ has property K. Theorem. In the Zermelo-Fraenkel set theory, without the axiom of choice, (I), (II) and the Generalized Continuum Hypothesis are equivalent. Definition. A topological group has property K if it is a K-group (Abstract 69T-G33, these *Notices* 16 (1969), 434). (Received May 1, 1970.)

70T-E52. D. RICHARDSON, University of Iowa, Iowa City, Iowa 52240. Some theorems with short proofs. Preliminary report.

Let $\mathfrak{F}$ be any of the usual formalizations of first order arithmetic. $\vdash_K A$ means that the formula A has a proof in $\mathfrak{F}$ with $\leq$ K steps in it. It is shown that if $f(x_1,\ldots,x_n)$ is a Pressburger formula, there is a number K so that for any numerals $a_1,\ldots,a_n$ either $\vdash_K f(a_1,\ldots,a_n)$ or $\vdash_K \sim(F(a_1,\ldots,a_n))$. (Received May 8, 1970.) (Author introduced by Professor Robert H. Oehmke.)

70T-E53. STELIOS NEGREPONTIS, McGill University, Montréal, Québec, Canada. Adequate ultrafilters of special Boolean algebras.

Let S_α denote the special Boolean algebra of cardinality $\alpha = 2^\alpha$ (in the sense of Morley-Vaught). Theorem A. Let $\alpha = \alpha^\alpha > \omega$, $\omega \leq \delta < \alpha$. There is a countably incomplete, δ-uniform (in the natural sense), α-good (in the sense of Keisler) ultrafilter on the power S_α^δ. Theorem B. Let $\alpha = 2^\alpha > \omega$, $\omega \leq \delta < \alpha$. Then S_α has a specializing chain $\{\mathcal{C}_\beta, \beta < \alpha\}$, with $|\mathcal{C}_\beta| = 2^\beta$, and an ultrafilter p on S_α^δ which is adequate (i.e., p is countably incomplete, δ-uniform, and $p \cap \mathcal{C}_\beta^\delta$ is β^+-good for all β, $\delta \leq \beta < \alpha$). Theorem C. Let $\alpha = \alpha^\alpha > \omega$, and let $S_\alpha^{(\alpha)}$ denote the α-completion of S_α. There is a countably incomplete α-good ultrafilter on $S_\alpha^{(\alpha)}$. Let $\{\mathcal{C}_\beta, \beta < \alpha\}$ be a specializing chain for S_α, such that $|\mathcal{C}_\beta| = 2^\beta$, and let $S_\alpha^{[\alpha]}$ be the union of $\mathcal{C}_\beta^{(\beta^+)}$, $\beta < \alpha$, where $\mathcal{C}_\beta^{(\beta^+)}$ is the β^+-completion of $\mathcal{C}_\beta$. Theorem D. Let α be an uncountable strong limit cardinal. There is a chain of subfields of sets of $S_\alpha^{[\alpha]}$, such that $|\mathcal{B}_\beta| = 2^\beta$, $\mathcal{B}_\beta$ is β^+-complete field of sets, $\mathcal{C}_\beta^{(\beta^+)} \subset \mathcal{B}_\beta \subset \mathcal{C}_{2^\beta}^{((2^\beta)^+)}$ for $\beta < \alpha$, and there is an adequate ultrafilter p on $S_\alpha^{[\alpha]}$ (i.e. $p \cap \mathcal{B}_\beta$ is countably incomplete and β^+-good for $\beta < \alpha$). These results have model-theoretic applications, that will be described in a subsequent abstract. (Received May 22, 1970.)

70T-E54. DIANA BRIGNOLE and NEWTON C. A. da COSTA, Universidad Nacional del Sur, Bahía Blanca, Argentina. On supernormal Ehresmann-Dedecker universes.

This abstract is the sequel to a preceding one (Abstract 70T-E11, these *Notices* 17(1970), 453-454), in which we introduced the concept of supernormal Ehresmann-Dedecker universe and stated some of its properties. (For terminology, see the mentioned abstract.) Theorem. The existence of strongly inaccessible cardinal numbers $> \aleph_0$, implies the existence of supernormal complete Ehresmann-Dedecker universes which are not Sonner-Grothendieck universes. The underlying set theory is the Zermelo-Fraenkel system (Cohen, "Set theory and the continuum hypothesis," W. A. Benjamin, Inc., New York, 1966), with or without the axiom of regularity, but our results are true for other (usual) systems of set theory. (Received May 25, 1970.)

70T-E55. WILLIAM M. LAMBERT, JR., University of Detroit, Detroit, Michigan 48221. Comparing notions of abstract effectiveness. Preliminary report.

Modulo a natural translation algorithm, the class of primitive computable (pc) functions (Y. Moschovakis, "Abstract first order computability. I," Trans. Amer. Math. Soc. 138 (1969), 427-464) is strictly weaker than the class of schematically definable functions (W. Lambert, "A notion of effectiveness in arbitrary structures," J. Symbolic Logic 33 (1968), 577-602) when attention is confined to structures in the sense of the latter paper (Moschovakis' restriction that the number of generating functions be finite can be dropped here), Translation is needed because we deal with functions "stratified" to 3 sorts of arguments: natural numbers ω, the structure set B, and (with a compactness constraint) B^{ω}, and with values of just one of these sorts; Moschovakis works with functions on and to B^* generated from B and a separate object 0 by iterated pairing (1 = (0,0), 2 = (1,0), etc.). The translation generates a code number for the pair-structure of $z \in B^*$ which refers to a simultaneously-generated data string $\in B^{\omega}$. Translation can be omitted when this makes sense. "Purely" p.c. (combinatorial) number-theoretic functions are just the pr functions; all recursive functions are "purely" sd. (Received June 8, 1970.)

70T-E56. GIORGIO GERMANO, Laboratorio di Cibernetica del CNR, 80072 Arco Felice, Italy. A very short proof of a strong incompleteness theorem.

Let T be an arithmetical theory (a theory satisfied by a realization having the set of natural numbers as universe and representing every digit D_n on the number n) which is recursively enumerable and in which can be defined the elementary functions. It is very easy to show that there is a w.f.f. Θ and a term δ, both with at most one free individual variable, such that $\forall \Phi (\Phi \in T \text{ iff } \Theta(D_{g(\Phi)}) \in T)$, $\forall n(\delta(D_n) = D_{d(n)} \in T)$, g being a Gödel numbering of T and d its diagonal function (see my papers: 'Metamathematische Begriffe in Standardtheorien', Arch. Math. Logik Grundlagenforsch 13 (1970), and 'A short proof of the incompleteness theorem via truth theorems' submitted for publication to Theoria). For such a theory T, putting $n := g(\neg \Theta(\delta))$ and $r := d(n) = g(\neg \Theta(\delta(D_n)))$, we get therefore: $\Theta(D_r) \in T$ iff $\neg \Theta(\delta(D_n)) \in T$ iff $\neg \Theta(D_r) \in T$. Since T is consistent (as T is an arithmetic) it results therefore: $\Theta(D_r) \notin T$ and $\neg\Theta(D_r) \notin T$. (Received May 25, 1970.) (Author introduced by Professor Rudolf E. Kalman.)

70T-E57. SAHARON SHELAH, Princeton University, Princeton, New Jersey 08540.
Weak definability for infinitary languages. Preliminary report.

Let L be a language, P a predicate $\notin L$, L^* be L with P. Let ψ be a sentence in $L^*_{\varkappa^+,\omega}$. Definition. $\lambda \in K_1$ iff there is an L-model M, $\|M\| = \lambda$, and $|\{P : (M,P) \models \psi\}| > \lambda$. $\lambda \in K_1$ iff there is an L^*-model (M,P) of ψ, $\|M\| = \lambda$ and $|\{P^1 : (M,P) \cong (M,P^1)\}| > \lambda$. Theorem 1. If $\lambda^+ \in K$, $\lambda^{\aleph_0} > \lambda \geqq \varkappa$, then $\lambda \in K$. Theorem 2. If $\aleph_{\alpha+\beta} \in K$, $\beta < \omega_1$, $\lambda < \aleph_\alpha$ implies $\sum_{\chi < \mu(\varkappa)} \lambda^{\chi} \leqq \aleph_\alpha$, then $\lambda^{\aleph_0} > \lambda \geqq \varkappa$ implies $\lambda \in K_1$. ($\mu(\varkappa)$ is the Hanf number of $L_{\varkappa^+,\omega}$.) Theorem 3. If $\aleph_{\alpha+\beta} \in K$, $\beta < \omega_1$, $\aleph_\alpha = \sum_{n<\omega} \lambda_n$, $\mu = \sum_{m<\omega} \mu_m$, for every $m,n < \omega$, $\lambda_n^{\mu_m} = \lambda_n$, $\varkappa \leqq \mu < \mu^{\aleph_0}$, then $\mu \in K$. Theorem 4. If $\varkappa = \aleph_0$, $\aleph_{\alpha+\beta} \in K$, $\aleph_\alpha = \sum_{k<\mu} \lambda_k$, $\mu < \aleph_\alpha$, and there is γ such that: $\beta < \beth_\gamma$, and $\delta < \omega_1$, $k < \mu$ implies $\lambda_k^{\beth_{\gamma+\delta}} = \lambda_k$, then $\varkappa \leqq \lambda < \lambda^{\aleph_0}$ implies $\lambda \in K_1$ Corollary 5. (G.C.H.) If $\varkappa = \aleph_0$, and there is $\lambda \in K$, $\lambda \geqq \beth_{\omega_1}$, then every λ of cofinality ω belongs to K_1. Remark. (1) The proof implies a weak version of definability of P, in suitable cases. (2) There is an example insinuating that at least in Theorem 2, the restrictions are natural. (Received June 12, 1970.) (Author introduced by Professor Elias M. Stein.)

70T-E58. WITHDRAWN.

70T-E59. J. T. BALDWIN, Simon Fraser University, Burnaby 2, British Columbia, Canada.
$\aleph_1$ categorical theories whose models are the closure of strongly minimal sets. Preliminary report.

T is a countable complete $\aleph_1$ categorical theory. The notion of transcendence degree is due to Morley ("Categoricity in powers," Trans. Amer. Math. Soc. 114 (1965), 514), and those of closure and strongly minimal are explored in Marsh (Ph.D. Dissertation, Dartmouth, Hanover, N.H., 1966). A principal extension of T is an inessential extension by a finite number of constants satisfying a principal type. The following are equivalent: (i) There is a principal extension T' of T with a strongly minimal formula $\underset{\sim}{D}$ such that in every model $\mathcal{A}$ of T, $|a| = cl(D(\mathcal{A}))$. (ii) For some model $\mathcal{A}$ of T and every $n < \omega$, if p is first realized in $\mathcal{A}_n$ (an nth minimal prime extension of $\mathcal{A}$), then $p \in Tr^n(\mathcal{A})$. (iii) For every model $\mathcal{A}$ of T and every $n < \omega$, $p \in Tr^n(\mathcal{A})$ iff p is first realized in $\mathcal{A}_n$. That a_T is finite, can be proved either with a short argument from (i) or immediately from (iii). The methods are similar to those in Lachlan and Baldwin (Abstract 69T-E86, these *Notices* 16 (1969), 1087. (Received June 12, 1970.) (Author introduced by Professor Alistair H. Lachlan.)

70T-E60. R. G. JEROSLOW, University of Minnesota, Minneapolis, Minnesota 55455.
Homomorphisms of polyadic algebras of theories. Preliminary report.

Reference for notations: Jeroslow, "Uses of self-reference in arithmetic," Ph.D. thesis, Cornell, 1966, §3. All theories discussed are r.e., reflexive, extend first-order Peano, and are in the language of arithmetic; for parenthetically inserted "1-1," they are also 1-consistent. An n-schema ψ is a formula of the Predicate Calculus all of whose variables are among the first n variables. ψ occurs in a theory S if there is a theorem δ of S obtained from ψ by properly substituting arithmetic formulae for the predicates in ψ. ψ is provably consistent in S if S proves the (canonical) consistency of ψ as sole axiom. Theorem 1. There is a (1-1) homomorphism from $B_\infty(S)$ to $B_\infty(T)$ iff, for each n, there is a homomorphism from $B_n(S)$ to $B_n(T)$. Theorem 2. Let $n \geq 2$. There is a (1-1) homomorphism of $B_n(S)$ to $B_n(T)$ iff every probably consistent n-schema of S is a provably consistent n-schema of T. Theorem 3. Let $n \geq 2$. There is a (1-1) homomorphism of $B_n(S)$ to $B_n(T)$ iff every n-schema which occurs in S also occurs in T. Proofs utilize Orey's Arithmetical Compactness Theorem and Lemma 1. If S is relatively interpretable in T, S is interpretable in T. Lemma 2. S is interpretable in T iff there is a homomorphism of $B_\infty(S)$ to $B_\infty(T)$. (Received June 15, 1970.)

70T-E61. GADI MORAN, The Hebrew University, Jerusalem, Israel. Existence of nondetermined sets for some two person games over reals.

Players I and II choose alternately real numbers $x_0, x_1, \ldots$. I starts. S is a subset of the real line. $\Gamma_i(S)$ is the game subject to the following conditions: i = 1: $x_{2n+1} < x_{2n+3} < x_{2n+2} < x_{2n}$, I wins iff $\lim x_{2n} \in S$ (J. Mycielski, "On the axiom of determinateness. II," Fund. Math. 59 (1966), 4.2.(a));
i = 2: $|x_{n+1} - x_n| \leq 2^{-n}$, I wins iff $\lim x_n \in S$ (J. Mycielski, "Continuous games with perfect information" in "Advances in game theory," Princeton Univ. Press, Princeton, N. J., 1964, pp. 103-112);
i = 3: $0 < x_{2n} < \ell_n x_{2n-1}$, $0 < x_{2n+1} < k_n x_{2n}$, where ℓ_n and k_n are given sequences of positive numbers, I wins iff Σx_n converges and belongs to S_n (M. Reichbach and H. Hanani, "A generalization of games by Banach and Mazur"). Theorem 1. If neither S nor its complement includes a perfect subset then $\Gamma_i(S)$ is nondetermined, i = 1,2,3. Mycielski (J. Mycielski, "On the axiom of determinateness. II," Fund. Math. 59 (1966), 4.2.(d)) raised the question whether the games $\Gamma_i(S)$, i = 1,2, are determined for all S. By Theorem 1, the answer is negative. R. Solovay has proved in 1968 a sharper result for $\Gamma_1(S)$, namely: Theorem 2. $\Gamma_1(S)$ is a win for I iff S includes a perfect subset; it is a win for II iff S is at most denumerable. (Received June 18, 1970.) (Author introduced by Professor Azriel Levy.)

70T-E62. STEPHEN H. HECHLER, Case Western Reserve University, Cleveland, Ohio 44106. Independence results concerning a topological problem of Erdös. Preliminary report.

For any property $\mathcal{P}$ we define a function f from R (the set of reals) into its power set to be a $\mathcal{P}$-function iff $\forall x (x \notin f(x))$ and $\forall x (f(x)$ has property $\mathcal{P})$. For any set $S \subseteq R$ and any $\mathcal{P}$-function f we define S to be free for f iff $S \cap \bigcup \{f(s) : s \in S\} = \emptyset$. Erdös has asked if every nowhere-dense-function admits an uncountable free set, and Piranian has asked about not-everywhere-dense-measure-zero functions. Theorem. There exist models $\mathfrak{M}_1$, $\mathfrak{M}_2$, and $\mathfrak{M}_3$, of ZF + AC such that: (1) in $\mathfrak{M}_1$ every first-category-function admits a free set of cardinality $2^{\aleph_0}$, (2) in $\mathfrak{M}_2$ every measure-zero-function admits a free set of cardinality $2^{\aleph_0}$, (3) in $\mathfrak{M}_3$ there exists a nowhere-dense-function which admits no free set of cardinality greater than $\aleph_1$, but $\aleph_1 \neq 2^{\aleph_0}$. (Received June 22, 1970.)

70T-E63. DOUGLAS D. SMITH, Pennsylvania State University, University Park, Pennsylvania 16802. The nonrecursiveness of Tarski's S_3.

S_3, in A. Tarski's "Equational logic and equational theories of algebras" in "Contributions to Math. Logic" (Colloquium, Hannover, 1966), North-Holland, Amsterdam, 1968, pp. 275-288, is the set of finite sets of equations E such that Th(E), the equational theory of E, is one based. The proof that S_3 is not recursive is here obtained straightforwardly by the method employed, in other problems, by Peter Perkins, "Unsolvable problems for equational theories," Notre Dame J. Formal Logic 8 (1967), 175-185. Given a finite presentation of a semigroup β with unsolvable word problem, a set of equations $E(\beta)$ is defined, and for each pair U,V of β-words, a set of equations PUV $\supset E(\beta)$, in a language having two binary operation symbols and two constants, is defined so that Th(PUV) is one based if and only if U = V holds in β. (A personal communication just received from George F. McNulty indicates that this result follows from one he has obtained.) (Received June 22, 1970.)

70T-E64. DAVID O. OAKLAND, Iowa State University, Ames, Iowa 50010. On the axiom schema of strong infinity.

Based on the notion of a normal function (i.e., a strictly increasing and continuous ordinal-valued function) it is shown that the statement "Every normal function defined for all ordinals has a regular cardinal in its range" is equivalent to A. Lévy's axiom schema of strong infinity which states that "Every normal function defined for all ordinals has a strongly inaccessible cardinal in its range". (Received June 22, 1970.)

70T-E65. PAUL D. BACSICH, University of Bristol, Bristol, England. Compact injectives and Nonstandard Analysis.

Let $\underline{K}$ be the category of compact T_2 spaces, M the underlying set functor on $\underline{K}$. A compact object over a category $\underline{C}$ is a pair (D,T) where D is a $\underline{C}$-object and $T: \underline{C} \to \underline{K}$ is a functor with MT = (- , D). The compact objects over $\underline{C}$ form a category $\underline{KC}$. The following are theorems of ZF + Prime Ideal Theorem. Theorem 1. If $\underline{C}$ is an Isbell bicategory admitting sums and products, then (1) the product of compact injectives is injective, (2) the inverse limit of a $\underline{KC}$-system of compact injectives is injective, and (3) if $\underline{C}$ has a compact injective strict cogenerator then injectives are closed under powers. Theorem 2. If D is a compact object then the class of u such that D is u-injective is closed under direct limits. Theorem 2 replaces Nonstandard Analysis proofs of Extension Theorems (such as the Hahn-Banach Theorem) while Theorem 1(3) for Boolean algebras extends a result of Luxemburg (Fund. Math. 55 (1964), 239-247). (Received June 23, 1970.) (Author introduced by Professor John C. Shepherdson.)

70T-E66. WITHDRAWN.

70T-E67. ALEXANDER ABIAN and JAMES BAINBRIDGE, Iowa State University, Ames, Iowa 50010. On the consistency of the Generalized Continuum Hypothesis.

Let (K, ϵ) be a model for ZF. For every ordinal u, by virtue of transfinite induction, let the set A_u of (K, ϵ) be defined by : $A_0 = \{0, 1, 2, \ldots, \omega, \{1\}\}$, $A_u = \bigcup_{v<u} A_u$ if u is a limit ordinal, and, $A_{u+1} = A_u \cup P_u$, where P_u is the set of all the powersets of elements of A_u relativized to A_u. Consider the model (A, ϵ) where x is a set of (A, ϵ) iff $x \in A_u$ for some ordinal u. It is shown that (A, ϵ) is a model for the axioms of Extensionality, Powerset, Sumset, Infinity, Choice and Regularity. Moreover (A, ϵ) has one and only one infinite ordinal ω and therefore the Generalized Continuum

Hypothesis is valid in (A, ϵ). Furthermore, the domain A of model (A, ϵ) as well as every set of (A, ϵ) is a countable set of ZF. (Received June 23, 1970.)

70T-E68. HEINZ-DIETER EBBINGHAUS, Universität Freiburg, 78 Freiburg i. Br., Federal Republic of Germany. The Ehrenfeucht-Mostowski-theorem for L_Q.

Theorem. Let L be a denumerable first order language containing a unary predicate letter U, and $(\underline{A}, U^A)$ a structure of the same type as L having an elementary substructure $(\underline{B}, U^B)$ with $B \neq A$ and $U^B = U^A$. Furthermore let $(K, <)$ be a denumerable linear ordering. Then there is a structure $(\underline{C}, U^C)$ of the same type as L and a subset D of C such that: (1) $(\underline{C}, U^C)$ is elementarily equivalent to $(\underline{A}, U^A)$. (2) $K \subset C$. (3) $|C| = \aleph_1$. (4) $|D| = \aleph_1$. (5) $|U^C| = \aleph_0$. (6) Every automorphism f of $(K, <)$ can be extended to an automorphism f' of $(\underline{C}, U^C)$ with $f'(x) = x$ for $x \in D$. Using Fuhrken's reduction techniques this theorem yields as one corollary the Ehrenfeucht-Mostowski theorem for L_{Q_1} (Q_1 being interpreted by "there are at least $\aleph_1$") and for denumerable linear orderings $(K, <)$. Assuming GCH and using a recent compactness result of Fuhrken, the corollary can be generalized--for regular $\aleph_\alpha$--to $L_{Q_{\alpha+1}}$ (with at most $\aleph_\alpha$ nonlogical constants) and linear orderings $(K, <)$ of power $\leq \aleph_\alpha$. (Received June 16, 1970.) (Author introduced by Professor Walter Felscher.)

Statistics and Probability

70T-F15. STEVEN I. ROSENCRANS, Tulane University, New Orleans, Louisiana 70118. An extremal property of stochastic integrals.

Let $(t,b) \to e(t,b)$ be a nonanticipating Brownian functional that is essentially bounded with respect to both the time t and the Brownian paths b. Let y_t be the stochastic integral $y_t = \int_0^t e\, db$. Then for any convex function f satisfying $f(x) = O(e^{|x|^a})$, $|x| \to \infty$, for some $a < 2$, we have $Ef(y_t) \leq Ef(\|e\| b_t)$. In particular, taking $f(x) = |x|^k$, we have sharp bounds on all the moments of y_t. For $k = 4$ this result is $Ey_t^4 \leq 3\|e\|^4 t^2$, which improves results of Zakai and of Skorokhod. (But the results of these authors hold for a wider class of functionals.) (Received June 3, 1970.)

70T-F16. ADHIR KUMAR BASU, Queen's University, Kingston, Ontario, Canada. A note on Strassen's version of law of the iterated logarithm.

Recently V. Strassen (Z. Wahrscheinlichkeitstheorie und Verw. Gebiete (1964)) presented a beautiful and useful generalization of the classical law of the iterated logarithm. For his proof he appealed to a deep theorem of Skorokhod which permits one to realize a sequence of independent variables with finite variances in terms of random increments of Brownian motion. We extended Strassen's results to stationary ergodic martingales and to a nonidentically distributed case. The key idea is that Strassen's theorem holds for any random sequence for which there is a Skorokhod embedding on Brownian motion with stopping times T_n satisfying the strong law of large numbers. (Received April 24, 1970.) (Author introduced by Professor Jagdish N. Pandey.)

70T-F17. ROGER A. HORN, The Johns Hopkins University, Baltimore, Maryland 21218. On necessary and sufficient conditions for an infinitely divisible distribution to be normal or degenerate.

Recently, A. F. Ruegg [Abstract 672-543, these *Notices* 17 (1970), 240] has used deep results about entire characteristic functions to obtain elegant generalizations of the familiar fact that a nondegenerate infinitely divisible probability measure on the line cannot have compact support. This note generalizes Ruegg's results and contains simple asymptotic conditions which are both necessary and sufficient for an infinitely divisible distribution to be normal or degenerate. The methods used are quite elementary, viz., the Kolmogorov representation formula and an elementary inequality for Laplace transforms. Theorem. Let $F(x)$ denote the cumulative distribution function of a probability measure on the line, so that $1 - F(x) + F(-x) = O(\exp(-xM(x)))$ as $x \to \infty$, where $M(x)$ is a nonnegative measurable function. Assume that the measure is infinitely divisible. (a) The measure is normal (possibly degenerate) if and only if $M(x)$ can be chosen such that $M(x)/\ln x \to \infty$ as $x \to \infty$ and such that $M(x)$ is continuous and strictly monotone increasing for all sufficiently large x. (b) The measure is degenerate (i.e., a one-point measure) if and only if, in addition to the conditions in (a), $M(x)/x \to \infty$ as $x \to \infty$. The methods developed to prove this theorem also yield one-sided versions of it which generalize familiar properties of infinitely divisible nonnegative random variables. (Received June 15, 1970.)

70T-F18. ALAN F. RUEGG, University of Connecticut, Storrs, Connecticut 06268. A necessary condition for infinitely divisible distributions.

In a recent abstract (Abstract 672-543, these *Notices* 17 (1970), 240), the author has generalized the well-known result that an infinitely divisible (i.d.) distribution function (d.f.) cannot be finite unless it is degenerate; several conditions on the rate of decrease of the tail $T(x) = 1 - F(x) + F(-x)$ of an i.d. d.f. F are obtained. This note presents a partial generalization of these results by considering the one-sided asymptotic behavior of an i.d. d.f. F. Theorem. Let F be a d.f., not bounded to the left. Assume that (1) $\liminf_{x\to\infty} \ln \ln[1/F(-x)]/\ln x < 2$ and (2) $\liminf_{x\to\infty} [\{\ln \ln[1/F(-x)] - \ln x\}/\ln \ln x] > 1$. Then F cannot be i.d. (A similar result holds of course for $1 - F(x)$.) The proof is based on properties of characteristic functions which are analytic in the upper (lower) half-plane. It follows for example that F cannot be i.d. if $F(-x) \sim a \exp[-bx^c]$ as $x \to \infty$ with $a, b > 0$ and $1 < c < 2$. (Received June 19, 1970.)

70T-F19. BING FUN IP, Wayne State University, Detroit, Michigan 48202. Riesz decomposition of positive Banach space-valued supermartingales.

Meyer ("Probability and potentials," Blaisdell, Waltham, Mass., 1966) proved a Riesz decomposition theorem for real-valued supermartingales. In this note we prove the following Riesz decomposition theorem for positive Banach space-valued supermartingales: Let K be a regular cone in a separable Banach space $\mathfrak{X}$; and let $\{x_n(\omega), \mathcal{A}_n, n \geq 1\}$ be an $\mathfrak{X}$-valued supermartingale. If $\{x_n(\cdot\omega)\}$ majorizes an $\mathfrak{X}$-supermartingale $\{y_n(\omega), \mathcal{A}_n, n \geq 1\}$, then there exists an $\mathfrak{X}$-martingale $\{y_n(\omega), \mathcal{A}_n, n \geq 1\}$ and a potential $\{z_n(\omega), \mathcal{A}_n, n \geq 1\}$ such that $x_n(\omega) = y_n(\omega) + z_n(\omega)$ for every n, and the two processes are unique. (Received June 23, 1970.)

Topology

70T-G120. CHARLES J. MOZZOCHI, Yale University, New Haven, Connecticut 06520, S. A. NAIMPALLY and M. S. GAGRAT, Indian Institute of Technology, Kanpur 16, U.P., India. Symmetric generalized topological structures. II.

This monograph is a second revision of the monograph "Symmetric generalized uniform and proximity spaces" written by the first author (cf. Abstracts 658-54 and 668-1, these *Notices* 15(1968), 733 and 16(1969), 947). The main feature of this revision is a complete generalization of the Smirnov compactification for symmetric generalized proximity spaces. In addition a generalization of Taimanov's theorem concerning the extension of continuous mappings of topological spaces is produced which also yields generalizations of results of McDowell, Blefko, and Engelking. Analogous results for S-proximally continuous functions and further generalizations of the results of Taimanov and McDowell are also obtained. Finally, applications to the Wallman compactification and Wallman realcompactification are investigated. The results in this revision are due to the second and third author. (Received January 22, 1970.)

70T-G121. ARVIND K. MISRA, Indian Institute of Technology, Kanpur-16, India. Chain-net spaces.

A net whose domain is a chain is called a chain-net. A T_1- space is called a chain-net space provided in it a set A is open if and only if no chain-net outside A converges to a point in A. Besides some interesting categorical propositions the following structure theorem for such spaces is obtained. Theorem. Every chain-net space X is a quotient of a disjoint sum of a sequential space and a P-space on the same underlying set X. (Received March 2, 1970.) (Author introduced by Professor Richard A. Alo.)

70T-G122. R. CHRISTOPHER LACHER, Florida State University, Tallahassee, Florida 32306. Some cellularity criteria.

Theorem 1. Let X be a compact set in the interior of the topological n-manifold N, $n \geq 5$. Suppose that N is simply connected and that the Čech cohomology of X vanishes. If X satisfies the cellularity criterion of McMillan (i.e., N - X is 1 - LC at X) then X is cellular in N. Theorem 2. Let N be a compact, orientable topological n-manifold, $n \geq 6$, and let g be the minimum number of generators of $\pi_1 N$. If $X_1,\ldots,X_{g+1}$ are pairwise disjoint compact sets in N with trivial Čech cohomology, and if each X_i satisfies the cellularity criterion in N, then at least one X_i is cellular in N. In each proof, the deduction of property 1 - UV for X is a necessary step. (Received March 20, 1970.)

70T-G123. V. KANNAN, Madurai University, Madurai-2, India. On three questions of Arhangelskii and Franklin. Preliminary report.

In this paper, we give complete answers to three of the questions posed by A. B. Arhangelskii and S. P. Franklin in their paper "Ordinal invariants for topological spaces" (Michigan Math. J. 15 (1968)). Theorem 1. For each ordinal α, there exists a k-space K_α whose k-order is α. Theorem 2. There are no 'test spaces' for the k-order. Theorem 3. There exists a countable Hausdorff connected sequential space which is nowhere first countable. (Received April 8, 1970.) (Author introduced by Professor M. Rajagopalan.)

70T-G124. DANIEL R. McMILLAN, JR., Institute for Advanced Study, Princeton, New Jersey 08540. Cellular sets and a theorem of Armentrout.

Let M be a PL 3-manifold ($\partial M = \emptyset$) and $I = [0,1]$. Theorem. Let $X \subset M$ be compact. Suppose there is a map $f: X \to I$ (onto) such that (1) each $f^{-1}(t)$ is cellular in M ($t \in I$), and (2) each $X_t = f^{-1}([0,t])$ satisfies CC(the "cellularity criterion"). Then X is cellular in M. Proof. From known results, each X_t has property UV^{∞}. Hence, it may be assumed that M is orientable and is separated by each of its orientable, closed surfaces. Let B consist of all $t \in I$ such that X_t is cellular in M (equivalently, $t \in B$ iff a neighborhood of X_t embeds in E^3). Clearly, $0 \in B$, an open set. To show B closed, consider $b \in \bar{B} \cap (0,1]$. The hypotheses easily yield PL 3-cells C, D, in general position, such that $X_b \subset \mathrm{Int}\,(C \cup D)$. By Lemma 3 of Hempel and McMillan, Fund. Math. 64(1969), 102, $C \cup D$ embeds in E^3. Hence $b \in B$, a clopen set, and B = I. q.e.d. Corollary 1 (Armentrout). If a cellular u.s.c. decomposition of M yields E^3, then $M \approx E^3$. (Induct on i = 1,2,3 to show that the inverse image of a closed PL i-cell in E^3 is cellular in M; then take i = 3 and write M as a monotone union of open 3-cells.) Similarly: Corollary 2 (McMillan). A topological cell $X \subset M$ is cellular iff X satisfies CC. (Use Theorem 1 from Proc. Amer. Math. Soc. 19(1968), 154.) (Received April 10, 1970.)

70T-G125. ROBERT M. DIEFFENBACH, University of Iowa, Iowa City, Iowa 52240. Homotopic PL n-balls are ambient isotopic. Preliminary report.

Extending a result of Martin and Rolfsen [Proc. Amer. Math. Soc. 19(1968), 1290-1292] the following is proved. Theorem. Let B^n be a PL n-ball, Q^q a $(2n - q + 1)$-connected PL q-manifold, $q \geqq n + 2$. Suppose Q is either compact or open and that for i = 0,1, $H_i: B^n \to Q - \dot{Q}$ is a locally unknotted PL embedding. If there exists a homotopy $H: B^n \times I \to Q$ between H_0 and H_1 such that H_t is fixed on $\dot{B}^n$, then there exists a PL ambient isotopy $h_t: Q \to Q$ fixed on $H_0(\dot{B}^n) \cup \dot{Q}$ such that $h_1 H_0 = H_1$. Locally unknotted is taken here to mean that there exists a triangulation (L,K) of (Q,f(B)) with $\dot{K}$ full in K and (lk(v,L), lk(v,K)) an unknotted sphere pair for all vertices $v \in K - \dot{K}$. The proof uses the technique of Martin and Rolfsen, a modified version of Hudson's n-isotopy extension theorem and a theorem of Husch's concerning homotopy groups of function spaces. (Received April 24, 1970.) (Author introduced by Professor Thomas M. Price.)

70T-G126. W. JOHN WILBUR, Pacific Union College, Angwin, California 94508. Universal and permanence properties of locally convex spaces. I. Preliminary report.

Call a class $\mathcal{J}$ of spaces an I-class if it contains the base field and for each space E there is a space $I(E) \in \mathcal{J}$ and a map $f: I(E) \to E$ such that for any $I \in \mathcal{J}$ and $g: I \to E$ there is a unique $h: I \to I(E)$ with $g = f \circ h$. A P-class $\mathcal{P}$ is defined dually. A nontrivial class of spaces is an I-class if and only if it is closed under direct sums and quotients and is a P-class if and only if it is closed under products and closed subspaces. If E is a space and $\mathcal{J}$ an I-class the map $f: I(E) \to E$ is a bijection, while if $\mathcal{P}$ is a P-class the maps $g: E \to P(E)$ and $g': P(E)' \to E'$ are both bijections. Call the spaces I(E) and P(E) associates of E. Associated bornological, Mackey, weak, and nuclear topologies or spaces are well known. We study associated barreled semireflexive, and semi-Montel topologies. For any space E the associated semireflexive topology is the topology of uniform convergence on all convex, circled equicontinuous subsets of E'_β which are weakly compact for the barreled topology associated with E'_β. The associated barreled topology is realized as the topology of uniform convergence on all convex, circled subsets of E'_σ which are weakly compact for the semireflexive topology associated with E'_σ. If E is semireflexive the associated semi-Montel topology is that of uniform convergence on the equicontinuous precompact subsets of E'_β. (Received April 27, 1970.)

70T-G127. STEVEN FERRY, University of Michigan, Ann Arbor, Michigan 48104. A Sard's theorem for distance functions.

If A is a subset of the plane, define the ϵ-boundary of A to be the set of points whose distance from A is exactly ϵ. M. Brown has shown that for all but countably many ϵ the components of the ϵ-boundary of A are 1-manifolds. In the paper it is shown that the ϵ-boundary of A is itself a 1-manifold for all ϵ outside of a set of measure zero. However, an example is provided of a set A whose ϵ-boundary fails to be locally connected (and hence is not a 1-manifold) for uncountably many ϵ. An example is also provided of a set A in R^3 whose ϵ-boundary in R^3 has components which are not 2-manifolds for uncountably many ϵ, showing that Brown's result does not generalize. (Received April 27, 1970.) (Author introduced by Professor Morton Brown.)

70T-G128. MARLON C. RAYBURN, University of Manitoba, Winnipeg 19, Manitoba, Canada. Rimmed partitions of $\beta X - X$.

All spaces considered are completely regular T_2 and only T_2 compactifications are considered. The partitions of $\beta X - X$ which correspond to compactifications of X are here called E-partitions. A partition of space Y is "rimmed" if it decomposes Y into compact subsets, each of which is the intersection of open-closed sets. A "strongly" upper semicontinuous decomposition of Y is one for which for any neighborhood U of any partition set F, a saturated open-closed set S exists with $F \subseteq S \subseteq U$. A strongly upper semicontinuous partition into compact sets is called "strongly rimmed". Theorem. For any X, there is a space Y having a base of open-closed sets and a perfect map $f: X \to Y$ if and only if X admits a strongly rimmed partition. If X is Lindelöf, Y is 0-dimensional. Theorem. For any X, the following are equivalent: (a) The components of $\beta X - X$ form a (strongly) rimmed E-partition. (b) The quasi-components of $\beta X - X$ are components and there is a maximal compactification SX with SX - X totally disconnected (SX - X has a base of open-closed sets). (c) The quasi-components of $\beta X - X$ are components and there is a compactification αX with $\alpha X - X$ totally disconnected ($\alpha X - X$ has a base of open-closed sets). Corollary. If X is rimcompact, the maximal compactification SX exists and coincides with the Freudenthal compactification. (Received April 27, 1970.)

70T-G129. STEPHEN WEINGRAM, Division of Mathematical Sciences, Purdue University, Lafayette, Indiana 47907. On a conjecture about cross sections.

Theorem 1. If $f: E \to S^{2n+1}$ is a fibration with fibre a $(2n - 1)$-connected finite CW complex then f admits a cross section. In particular this proves that any fibration over S^3 with a finite 1-connected complex for fibre has a cross section. In the process of proving this theorem, we get a short proof of W. Browder's theorem that the first nonvanishing homotopy group of an H-space is in an odd dimension. We also prove Theorem 2. If X is a finite CW complex and $\alpha \in \pi_{2n}(X)$ is not zero under the Hurewicz map, then some higher order Whitehead product set $[\alpha, \alpha, \ldots, \alpha]$ must consist of nonzero elements. (Received April 27, 1970.)

70T-G130. WITHDRAWN.

70T-G131. FRANK G. SLAUGHTER, JR., University of Pittsburgh, Pittsburgh, Pennsylvania 15213. Some new results on inverse images of closed mappings.

The results of this paper supplement the author's previous ("A note on inverse images of closed mappings," Proc. Japan Acad. 44(1968), 628-632) study of conditions under which certain properties are enjoyed by the domain of a closed, continuous mapping when the range has the same property or a stronger one. Spaces are assumed to be at least T_1 and mappings are closed, continuous and onto. Theorem 1. Let $f: X \to Y$, with X a topological space and Y an M-space (M^*-space). Let $Y = Y_0 \cup Y_1$ with $f^{-1}(y)$ countably compact for y in Y_0 and Y_1 σ-discrete. Suppose that Bdry $f^{-1}(y)$ is countably compact for y in Y. Then X is an M-space (M^*-space) iff $f^{-1}(y)$ is an M-space (M^*-space) for y in Y. Corollary 1. Let $f: X \to Y$ with X normal and semistratifiable and Y metrizable. Then X is metrizable iff $f^{-1}(y)$ is metrizable for y in Y. Theorem 2. Let $f: X \to Y$ with X and Y metrizable and Y topologically complete. Then X is topologically complete iff $f^{-1}(y)$ is topologically complete for all y in Y. (Received May 13, 1970.)

70T-G132. HARRY LAKSER, University of Manitoba, Winnipeg 19, Manitoba, Canada. The homology of a lattice.

Let L be a lattice with 0,1. A subset $X \subseteq L - \{0,1\}$ is a crosscut of L if X intersects each maximal chain of L in exactly one element. A finite subset $\sigma \subseteq X$ spans if $\bigvee \sigma = 1$ and $\bigwedge \sigma = 0$. The simplicial complex K_X (G.-C. Rota, Z. Wahrscheinlichkeitstheorie und Verw. Gebiete 2(1964), 340-368) has as simplices all finite nonspanning subsets of X. The simplicial complex K_P (J. Folkman, J. Math. Mech. 15(1966), 631-636) has as simplices all finite chains in $L - \{0,1\}$. If K is a simplicial complex, then $|K|$ denotes the geometric realization of K with the C.W. topology. Theorem. If L is a lattice with 0,1, finite or infinite, and X is a crosscut of L, then the spaces $|K_X|$ and $|K_P|$ are homotopy equivalent. The proof uses only the most elementary properties of triangulable spaces. The theorem is a generalization of results of J. Folkman, loc. cit., and J. Mather, Proc. Amer. Math. Soc. 17(1966), 1120-1124. (Received May 11, 1970.) (Author introduced by Professor George A. Grätzer.)

70T-G133. W. THURMON WHITLEY, Virginia Polytechnic Institute, Blacksburg, Virginia 24061. Homotopy types of deleted products of unions of two simplexes. Preliminary report.

Let $X = A \cup B$, where A is an n-simplex, B is an m-simplex, and $A \cap B$ is a k-simplex, $k \geqq 0$. Let X^* denote the deleted product of X. Let S^p denote the p-sphere. Theorem. If $n = k + 1$, X^* has the homotopy type of S^{m-1}; if $m, n > k + 1$, X^* has the homotopy type of $S^{m-1} \cup S^{n-1}$; where $S^{m-1} \cap S^{n-1}$ has the homotopy type of S^k. From this result, the homology groups of X^* can easily be calculated. (Received May 18, 1970.)

70T-G134. T. THRIVIKRAMAN, Madurai University, Madurai-2, India. On localcompactifications of Tychonoff spaces. Preliminary report.

Definition. Let X be any space. A space Y is said to be a localcompactification of X if Y is a locally compact Hausdorff space and X is dense in Y. Theorem 1. The set L(X) of all localcompactifications of a space X under suitable order forms a complete upper semilattice and the semilattice K(X) of all Hausdorff compactifications of X is a subsemilattice of L(X). L(X) is complete if and only if X is itself locally compact. Theorem. Under a suitable equivalence, the localcompactifications of X are nothing but the open subsets of $\beta(X)$. (Received May 21, 1970.) (Author introduced by Professor M. Rajagopalan.)

70T-G135. BENNY EVANS and WILLIAM H. JACO, University of Michigan, Ann Arbor, Michigan 48104. Coverings of P^2-irreducible 3-manifolds.

A 3-manifold M is irreducible if every tame 2-sphere in M bounds a 3-cell in M. An irreducible 3-manifold M is P^2-irreducible if M does not contain any two-sided embeddings of real projective 2-space. Theorem 1. Suppose M is a P^2-irreducible 3-manifold. Then every covering space of M is P^2-irreducible. As an immediate corollary we have that each closed covering space of a prime, closed, orientable 3-manifold is prime. This partially answers a question of Tollefson (Michigan Math. J. 16(1969), 106). (Received May 25, 1970.)

70T-G136. ERIC J. BRAUDE, Columbia University, New York, New York 10027. Descriptive Baire and descriptive Z-Suslin sets. Preliminary report.

A subset X of a topological space T is called descriptive Z-Suslin in T if it is of the form $K[N^N]$, where K is a compact valued upper semicontinuous map of N^N into 2^T such that there exists a continuous map of N^N into C*(T) for which $K = Z \cdot F$. (Z denotes the map which takes functions onto their zero sets.) Theorem 1. An analytic (descriptive borel) subset X of T with representation $K[N^N]$ is descriptive Z-Suslin (descriptive Baire) with the same representation iff there is a separable metric space M and a continuous map q of T into M such that $q^{-1}\{q[K(i)]\} = K(i)$ for every i in N^N. Theorem 2. Every descriptive Z-Suslin in set in T is Z-Suslin in T. Theorem 3. The family of descriptive Z-Suslin subsets is closed under the operation (A). Theorem 4. The intersection of a descriptive Z-Suslin and a Z-Suslin subset of T is descriptive Z-Suslin in T. Theorem 5. The families of Z-Suslin and descriptive Z-Suslin subsets of T coincide iff T is descriptive Z-Suslin in itself. Theorem 6. If T is a completely regular Hausdorff space in which every open set is an F-Suslin set, then the families of analytic and descriptive Z-Suslin sets coincide. (Received May 28, 1970.)

70T-G137. BRUCE CONRAD, Temple University, Philadelphia, Pennsylvania 19122. Extending free circle actions on spheres to S^3 actions.

Let X be a PL (4k + 2) manifold with the homotopy type of CP^{2k+1}, corresponding by D. Sullivan's classification of such manifolds to the element $(N_1, \alpha_2, N_2, \ldots, \alpha_k, N_k)$ of $Z \oplus Z_2 \oplus Z \oplus \ldots \oplus Z_2 \oplus Z$. Theorem 1. The topological circle action on S^{4k+3} with orbit space X is the restriction of a free S^3 action iff $\alpha_i = 0$, $i = 2, \ldots, k$; and $N_1 \equiv 0 \bmod 2$; and $\sum(-1)^i N_i = 0$. Now suppose X admits a smooth structure and satisfies the hypotheses of Theorem 1. While it is not known that the corresponding S^3 action can be smoothed, certain smoothing obstructions vanish. Theorem 2. Suppose S^3 acts freely on S^{4k+3} with a triangulated orbit space Y. If the restriction to a circle action can be smoothed, then $\langle \hat{A}(Q), [Q] \rangle$ is an integer, where $Q \subseteq Y$ is the PL submanifold constructed as follows: Let $P \subseteq QP^k$ be an embedded spin manifold, suitably triangulated, and $h: Y \to QP^k$ be a homotopy equivalence transverse regular to P. Then $Q = h^{-1}(P)$. (Received June 1, 1970.)

70T-G138. R. CHRISTOPHER LACHER, Florida State University, Tallahassee, Florida 32306 and DANIEL R. McMILLAN, JR., Institute for Advanced Study, Princeton, New Jersey 08540. Partially acyclic mappings of manifolds.

One says that $X \subset M$ has q - uv(G) iff each neighborhood U of X in M contains a neighborhood V of X such that under inclusion $H_q(V; G) \to H_q(U; G)$ is zero. (H_* is reduced singular homology.) One says that $X \subset M$ has $uv^k(G)$ iff it has q - uv(G), $0 \leq q \leq k$. Henceforth, M and N denote (open or closed) topological manifolds of dimensions m and n, respectively, and $f: M \to N$ is a proper (compact), onto map. Let $A_q(f; G)$ consist of all $y \in N$ for which $f^{-1}(y) \subset M$ fails to have q - uv(G); $A(f; G) = \bigcup_{q=0}^{\infty} A_q(f; G)$; and $S(f) = \{x \in M: f^{-1}f(x) \neq x\}$. Read Z_* consistently as Z or Z_2, and assume $0 < k < n$. Theorem 1. If each $f^{-1}(y) \subset M$ has $uv^{k-1}(Z_*)$, then $A_q(f; Z_*) = \emptyset$ for $q > m - k$ and $A_{m-k}(f; Z_*)$ is locally finite in N. Theorem 2. If each $f^{-1}(y) \subset M$ has $uv^{k-1}(Z_2)$ and dim $A(f; Z_2) \leq k$, then m = n and deg(f) = 1. Theorem 3. If M and N are oriented, each $f^{-1}(y)$ has $uv^{k-1}(Z)$ and dim $A(f; Z) \leq k$, then m = n and deg(f) = ±1. Corollary 4. If $g: S^n \to N^n$ is onto, monotone, and dim $g(S(g)) \leq 1$, then g is a homotopy equivalence. Corollary 5. If $g: R^n \to N^n$ is a proper, onto monotone map, $n \geq 5$, and dim $g(S(g)) \leq 1$, then $N^n \approx R^n$. Remarks. Similar results are obtained for other coefficient modules. Cohomology is used in the proofs, and some cohomological conclusions are also possible. (Received June 1, 1970.)

70T-G139. JOHN L. BRYANT, Florida State University, Tallahassee, Florida 32306. Euclidean n-space modulo on (n - 1)-cell.

It is now known that a k-cell D in E^n is flat in E^{n+1} except possibly when $n \geq 4$ and $k = n - 1$. In [Fund. Math. 63 (1968), 43-51] we generalized the methods of Andrews and Curtis [Ann. of Math. (2) 75 (1962), 1-7] and Bing [ibid. 70 (1959), 399-412] and proved that $(E^n/D) \times E^1$ is homeomorphic to E^{n+1} provided this condition holds. Using the methods in the above papers together with radial engulfing and an engulfing theorem of Černavskiĭ [Soviet Math. Dokl. 9 (1968), 835-839], we establish this result in the remaining case. That is we prove that if D is an (n - 1)-cell in E^n ($n \geq 4$), then $(E^n/D) \times E^1 \approx E^{n+1}$. (We do not prove that such a D is flat in E^{n+1}.) (Received June 2, 1970.)

70T-G140. ROSS GEOGHEGAN, Institute for Advanced Study, Princeton, New Jersey 08540. Many function-spaces are infinite-dimensional manifolds. Preliminary report.

Let ℓ_2 be separable Hilbert spaces, ℓ_2^f the dense linear subspace of those sequences in ℓ_2 with only finitely many nonzero entries. X is an ℓ_2-manifold if metrizable and locally homeomorphic to ℓ_2. (X,Y) is an (ℓ_2, ℓ_2^f)-manifold pair if X is an ℓ_2-manifold and there is an atlas {U} for X and open subsets V_U of ℓ_2 such that the pairs $(U, U \cap Y)$ and $(V_U, V_U \cap \ell_2^f)$ are homeomorphic. Let

C((X,X'), (Y,Y')) be the space of pair-maps from (X,X') to (Y,Y') with compact-open topology, written C(X,Y) when X' = $\emptyset$. Let PL(X,Y) be the subspace of C(X,Y) consisting of all the p.l. maps (where appropriate). Theorem (1). Let A be compact, T locally compact, both separable; let each of A and T be either a polyhedron all of whose components have dimension > 0, or a metrizable topological n-manifold-with-boundary (n > 0); let A' be a closed proper subset of A and let $p \in T$; then C(A,T) and C((A,A'), (T,{p})) are ℓ_2-manifolds. Theorem (2). If T is a polyhedron as in Theorem (1) and I is the closed unit interval, then (C(I,T), PL(I,T)) is an (ℓ_2, ℓ_2^f)-manifold pair. Theorem (1) generalizes a theorem of Eells. Theorem (2) also holds for spaces of loops at any base point in T. The proofs involve many of the new theorems on indinite-dimensional topological manifolds. (Received June 2, 1970.)

70T-G141. RICHARD E. HODEL, Duke University, Durham, North Carolina 27706. Moore spaces and $\omega\Delta$-spaces.

The class of $\omega\Delta$-spaces was introduced by C. R. Borges [Canad. J. Math. 20 (1968), 795-804]. This paper is a study of conditions under which an $\omega\Delta$-space is a Moore space. A space X has a G_δ^*-diagonal if there is a sequence $\mathcal{G}_1, \mathcal{G}_2, \ldots$ of open covers of X such that, for any two distinct points x and y in X, there is an n in N such that $y \notin \overline{st(x, \mathcal{G}_n)}$. Theorem. Every $\omega\Delta$-space with a G_δ^*-diagonal is developable. A T_1-cover of a space X is a collection $\mathcal{V}$ of open subsets of X such that, for any two distinct points x and y in X, there is a V in $\mathcal{V}$ such that $x \in V$ and $y \notin V$. Theorem. Every regular, θ-refinable $\omega\Delta$-space with a point-countable T_1-cover is a Moore space. A space X is an α-space if there is a sequence $g_1, g_2, \ldots$ of functions from X into the topology of X such that (1) $\bigcap_{n=1}^{\infty} g_n(x) = \{x\}$; (2) if $y \in g_n(x)$ then $g_n(y) \subset g_n(x)$. Theorem. A regular T_1 $\omega\Delta$-space is a Moore space if and only if it is an α-space. (Received May 29, 1970.)

70T-G142. JOHN E. MACK, University of Kentucky, Lexington, Kentucky 40506 and MARLON RAYBURN and RUSSELL G. WOODS, University of Manitoba, Winnipeg, Manitoba, Canada. A one-point realcompactification of certain completely regular spaces.

The notation and terminology is that of the Gillman-Jerison text. Theorem. The following conditions on a completely regular Hausdorff space X are equivalent: (1) X is a proper open subset of its Hewitt realcompactification. (2) There exists a completely regular Hausdorff space *X uniquely determined by the following properties: (a) X is a dense subspace of *X, and *X - X has cardinality 1. (b) *X is realcompact. (c) If T is realcompact, if X is a dense subspace of T, and if T - X has cardinality 1, then there exists a continuous map from *X onto T whose restriction to X is the identity on X. Theorem. A zero-set Z_0 of X is closed in *X if and only if there exists a zero-set Z_1 of X such that $Z_1 \cap Z_0 = \emptyset$ and Z_1 is in every free z-ultrafilter on X with the countable intersection property. An investigation of the correspondence between the ideals M_ω and O_ω of C(*X) and certain ideals of C(X) has also been carried out ($\{\omega\}$ = *X - X). (Received June 4, 1970.)

70T-G143. IVAN L. REILLY, University of Illinois, Urbana, Illinois 61801. Pairwise Lindelöf bitopological spaces.

Definition 1. A cover of a bitopological space (X, T_1, T_2) is pairwise open if its elements are members of T_1 or T_2, and if it contains at least one nonempty member of each of T_1 and T_2. Definition 2. A bitopological space (X, T_1, T_2) is pairwise Lindelöf if each pairwise open cover of (X, T_1, T_2) has a countable subcover. Theorem 1. If (X, T_1, T_2) is pairwise Lindelöf then any proper subset of X which is T_1 closed is pairwise Lindelöf and T_2 Lindelöf. Theorem 2. Any pairwise regular pairwise Lindelöf bitopological space is pairwise normal. Theorem 3. Any second countable pairwise regular bitopological space is pairwise perfectly normal. (Received June 8, 1970.)

70T-G144. JOSEF BLASS, University of Michigan, Ann Arbor, Michigan 48108. On Wallman compactifications.

A base Z for the closed sets of a T_1 space X is said to be normal iff (a) Z is a ring of sets; (b) any two disjoint members A and B of Z are subsets respectively of disjoint complements $X \setminus C$ and $X \setminus D$ of members C and D of Z. O. Frink [Amer. J. Math. 86 (1964), 603] proved by canonically constructing from a normal base Z a compactification $\omega(Z)$ of X that the class of T_1 spaces with a normal base is identical with the class of Tichonoff spaces. His method raises the following problem: Given a compactification Y of X, can we find a normal base Z such that $\omega(Z) \cong Y$? In the following cases the answer is affirmative: (a) Y--zero dimensional. (b) Y--hereditarily normal space, $Y \setminus X$-zero-dimensional space. (c) $Y \setminus X$ zero-dimensional Lindelöf space. (d) X--pseudocompact space. (e) Y--metric space. In the case (e) we can find a normal base Z in Y such that if $X \subset \overline{X}$ then $Y \cong \omega(Z')$ where Z' is a normal base in X obtained by taking intersections of elements of Z and X. (Received June 15, 1970.)

70T-G145. JOHN E. COURY, University of Massachusetts, Amherst, Massachusetts 01002. Walsh sequences are nowhere dense.

For a nonnegative real number x less than one, call the sequence $\underline{x} = \{w_n(x)\}_{n=1}^{\infty}$ a Walsh sequence, where w_n is the nth Walsh function. Denote by $\mathcal{W}$ the set of all Walsh sequences as x ranges over $[0,1[$; then $\mathcal{W}$ may be viewed as a subset of $G = \{-1,1\}^{\omega}$, the countable product of two-element groups. It is proved that $\mathcal{W}$ is a perfect nowhere dense subset of G, of Haar measure zero. Utilizing $\mathcal{W}$, one can exhibit a perfect nowhere dense subset of $[0,1[$, of Lebesgue measure zero, which contains no dyadic rationals. This set, in contrast to Cantor sets, is not a symmetrical perfect set of constant ratio of dissection. (Received June 11, 1970.)

70T-G146. GEORGE K. GOSS, Wesleyan University, Middletown, Connecticut 06457. Weight and pseudo-weight of minimal Hausdorff spaces. Preliminary report.

Definition. The weight of a topological space X at a point p, $w(X,p)$, is the smallest cardinal m such that there exists a base of power m for the open neighborhoods of p. Definition. The pseudo-weight of a topological space X at a point p, $\Psi(X,p)$, is the least power of a family of open sets whose intersection is $\{p\}$. It is known that $w(x,p) = \Psi(X,p)$ for each point p in X when X is compact. I. I. Parovicenko has shown that there exists an absolutely closed space X with a point p such that $\Psi(X,p) < w(X,p)$. We show that for any space X having a point p such that $\Psi(X,p) < w(X,p)$ and $\Psi(X,p) \geq \aleph_0$ we can embed X into a minimal Hausdorff space Y such that $\Psi(X,p) = \Psi(Y,p)$ and $w(X,p) = w(Y,p)$. (Received June 16, 1970.)

70T-G147. PETER J. NYIKOS, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213. Not every 0-dimensional realcompact space is N-compact. Preliminary report.

An N-compact space is one which can be embedded as a closed subspace in a product of discrete countable spaces [Engelking and Mrowka, "On E-compact spaces," Bull. Acad. Polon. Sci. Ser. Sci. Math. Astr. Phys. 6 (1958), 429-436]. Theorem. P. Roy's space Δ [Trans. Amer. Math. Soc. 134 (1968), 117-132] is 0-dimensional and realcompact, but not N-compact. This theorem is proven by making use of the following (already known) result. Let X be 0-dimensional Hausdorff, and let α be an ordinal number. Let $\mathfrak{N}_\alpha$ be the uniformity on X whose base consists of all equivalence relations partitioning X into $< \aleph_\alpha$ clopen sets. Theorem. TFAE: (a) X is N-compact. (b) X is complete in $\mathfrak{N}_1$. (c) X is complete in $\mathfrak{N}_\alpha$ for any nonmeasurable cardinal $\aleph_\alpha > \aleph_0$. (d) Every ultrafilter $\mathfrak{U}$ on the Boolean algebra of clopen subsets of X having the c.i.p. is fixed. On Δ there are uncountably many clopen-set ultrafilters with the c.i.p. (Received June 15, 1970.)

70T-G148. WILLIAM ERB DIETRICH, JR., Northwestern University, Evanston, Illinois 60201. Dense decompositions of locally compact groups. Preliminary report.

A locally compact Abelian group which is not totally disconnected has a subgroup which decomposes every nonempty open set into two nonmeager (Category II), nonmeasurable sets. This is also true for nondiscrete LCA groups which are either: separable, compact, torsion-free and divisible, or compactly generated. In particular, such groups contain a nonmeasurable dense subgroup. (Received June 18, 1970.)

70T-G149. CHIEN WENJEN, California State College, Long Beach, California 90801. Mapping theorems for H-closed spaces.

A Hausdorff space is said to be H-closed if it has no proper Hausdorff extensions. In this article the Borsuk's homotopy extension theorem, the Hopf's extension theorem, and Hopf's classification theorem have been generalized to H-closed spaces. (Received June 15, 1970.)

70T-G150. ROBIN SOLOWAY, University of Wisconsin, Madison, Wisconsin 53706. Somewhere acyclic mappings are compact. Preliminary report.

A map $f: X \to Y$ is q-acyclic provided that for each $y \in Y$, $\check{H}_q(f^{-1}(y); Z_2) = 0$, f is acyclic if f is monotone and q-acyclic for all $q > 0$, and f is compact if the inverse image of each compact set is compact. Theorem 1. Let M^n and N^n be connected open manifolds with M triangulated. Let f be a monotone map of M onto N. If there is a nonempty open set 0 in N such that $f/f^{-1}(0)$ is acyclic, then f is compact. Using a result of Lacher and McMillan [Theorem 1 of Abstract 70T-G138, these *Notices* 17 (1970)] the following strengthening of Theorem 1 is obtained. Theorem 2. Let M and N be as in Theorem 1. Let f be a monotone map of M onto N. If there is a nonempty open set 0 in N such that $f/f^{-1}(0)$ is q-acyclic for $q = 1, \ldots, [(n - 1)/2]$, then f is compact. If an addition f is q-acyclic for $q \leq 4$ everywhere, then using a result of Hollingsworth and Sher the requirement that M is triangulated can be eliminated in Theorems 1 and 2. Remark. Jussi Väisälä has obtained similar results under the assumption that f was acyclic everywhere [Duke Math. J. 33 (1966), 679-681]. The method of proof of Theorem 1 is similar to that of A. C. Connor and S. L. Jones. (Received June 22, 1970.)

70T-G151. ANNE K. STEINER and EUGENE F. STEINER, Iowa State University, Ames, Iowa 50010. Binding spaces: A unified completion and extension theory. Preliminary report.

In order to provide a common setting for various completion procedures, we introduce a binding structure on a set X by asserting that certain finite families of subsets of X are bound together and by distinguishing a family of covers of X. These covers and bound families are then used to construct a topology on X and ideal elements called clusters. The set of all clusters is provided with a binding structure and serves as a completion. It is then proved that any structure preserving map into a complete, regular space may be extended to the completion. Stone-Čech and Wallman compactifications and Hewitt realcompactifications of topological spaces, the completions of uniform spaces, and the Smirnov compactification of proximity spaces are obtained as special cases, as are the associated universal mappings theorems. (Received June 22, 1970.)

70T-G152. SERGE MAUMARY, Institute for Advanced Study, Princeton, New Jersey 08540. The open surgery obstruction in odd dimensions. Preliminary report.

A locally finite complex X is said to be an open m-Poincaré complex if it has a cycle mod ∞ leading to an isomorphism $H_c^k(X; \pi_1 X) \approx H_{m-k}(X; \pi_1 X)$ and to an equivalence of inverse systems $H_c^k(X_n, \dot{X}_n; \pi_1 X_n) \to H_{m-k}(X_n; \pi_1 X_n)$ at each end (X_n subcomplex with $\overline{X - X_n}$ compact). Moreover, X is said to be tame if the ends are in finite number, isolated, finitely dominated, the inverse systems $\pi_1 X_n$ and $H_*(X_n; \pi_1 X_n)$ being stable at each end. Theorem. Let M be an open oriented tame differentiable manifold of odd dimension $m \geq 7$, and X be an open tame m-Poincaré complex with one end ϵ. Then the obstruction for a proper normal map $f: M \to X$ of degree 1 to be cobordant (by a tame proper normal cobordism) to a simple homotopy equivalence is the Grothendieck class of a projective quadratic $\pi_1 \epsilon$-module P such that $P \otimes_{\pi_1 \epsilon} Z\pi_1 X$ is a based form over $\pi_1 X$ with a preferred subkernel. This totalizes the Wall-Siebenmann-Farrell-Wagoner's obstructions, in the sense that the trivialisation of P over $\pi_1 X$ represents the obstruction for a proper homotopy equivalence to be simple, and that P is already based over $\pi_1 \epsilon$ when M,X are interior of compact objects with boundary. (Received June 22, 1970.)

70T-G153. CHARLES H. GOLDBERG, Trenton State College, Trenton, New Jersey 08625 and Princeton University, Princeton, New Jersey 08540. Genera of links with two components.

Given an oriented link $\mathscr{L}$, define $h^n(\mathscr{L})$ to be the minimum genus possible among the genera of those spanning surfaces for $\mathscr{L}$ which have at least n components (excluding closed components). Theorem. The pair of integers (m,n) can be $(h^1(\mathscr{L}), h^2(\mathscr{L}))$ for an oriented link $\mathscr{L}$ with two components if and only if (i) $0 \leq m \leq n$, and (ii) if $m = 0$, then n is even. The proof is largely geometric, involving the calculation of the genera of a fair profusion of links. A link of John Milnor ("Isotopy of links. Algebraic geometry and topology. A symposium in honor of S. Lefschetz," Princeton Univ. Press, Princeton, N.J., 1957, p. 305, Figure 6) is an example of a link with genera (1,2). (Received June 23, 1970.)

70T-G154. JOHN M. WORRELL, JR. and HOWARD H. WICKE, Sandia Laboratories, Albuquerque, New Mexico 87115. On open bicompact images of paracompact M-spaces. Preliminary report.

K. Morita gave an example [Proc. Japan Acad. 43 (1967), 869-872] of a perfect mapping of a locally bicompact M-space onto a space which is not an M-space. T. Ishii [Proc. Japan Acad. 43 (1967), 757-761] showed that a quasi-perfect mapping of an M-space onto a normal space has an M-space as its range. This abstract reports an example of an open bicompact mapping of a T_2 locally bicompact paracompact M-space onto a Hausdorff space Y which is not an M-space. The space Y admits no continuous Lindelöfian mapping onto any Hausdorff space having a base of countable order. [For details of such a Y see J. M. Worrell, Pacific J. Math. 30 (1969), 555-562.] Note also this Theorem. Any completely regular T_0-space which is an open bicompact image of a T_2 paracompact M-space is a metacompact p-space. This follows from the authors' theory of μ-spaces [Abstract 648-188, these *Notices* 14 (1967), 687]. In fact: if φ is an open countably compact mapping between completely regular T_0 spaces X and Y then if X is a μ-space so is Y [Abstract 649-21, these *Notices* 14 (1967), 820]. This, together with the proposition that a completely regular T_0 metacompact μ-space is a p-space, implies the result. (Received June 23, 1970.)

70T-G155. REUVEN PELEG, The Hebrew University, Jerusalem, Israel. Weak disjointness of transformation groups.

Two metric transformation groups (t.g.s) are weakly disjoint if their product is ergodic. This relation is denoted by X $\dot{\perp}$ Y. Theorem 1. If T is abelian X and Y minimal then X $\dot{\perp}$ Y iff $\Gamma(X) \perp \Gamma(Y)$. ($\Gamma(X)$ is the equicontinuous part of X.) The same theorem holds for the nonabelian case if we assume that the t.g.s support invariant ergodic measures. Theorem 2. If T is abelian, X $\perp$ Y and X' Y' are minimal t.g.s built from X and Y resp. by successive proximal and distal extensions then X' $\perp$ Y' iff their only common factor is the trivial t.g. Theorem 1 is used to generalize some disjointness relations of Furstenberg ("Disjointness in ergodic theory, minimal sets and a prohlem in Diophantine approximation," Math. Systems Theory 1 (1967), 1-49). (Received June 1, 1970.) (Author introduced by Professor Jonathan S. Golan.)

ERRATA

Volume 16

JOHN KASDAN. A Weierstrass point of a class of modular groups. Preliminary report, Abstract 667-162, Page 804.

Line 1: Replace "For prime p, the normalizer of $\Gamma_0^0(p)$ in $SL(2,\mathbb{C})$ is shown to be $\Gamma_0^0(p) \cup \Gamma_0^0(p)\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$."

by: "For prime $p \geq 5$, the normalizer of $\Gamma_0^0(p)$ in $SL(2,\mathbb{R})$ is shown to be $\Gamma_0^0(p) \cup \Gamma_0^0(p)\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$."

Volume 17

JOE W. FISHER. Finite dimensional rings. Preliminary report, Abstract 70T-A92, Page 642.

Line 1: Replace "with" by "without".
Line 10: Replace "Zelmanowitz" by "R. E. Johnson".

BARUCH GERSHUNI. On the representations of a singular in the theory of totalities, Abstract 70T-E34, Page 672.

Line 11: Replace "(a,b,c,...) and a,b,c,... " by "(a,b,c,...) and $\overline{a,b,c,\ldots}$".

JAMIL A. SIDDIQI. Infinite matrices summing every almost periodic sequence, Abstract 673-58, Page 408.

Line 4: After "exists for all t" add the phrase "$\mathrm{lub}_{n \geq 0} \sum_{k=0}^{\infty} |a_{n,k}| = M < \infty$".

JOHN A. WRIGHT. There are 718 6-point topologies, quasi-orderings, and transgraphs, Abstract 70T-A106, Page 646.

Line 7: h_6^0 should read 318, not 336.

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