OF THE
AMERICAN MATHEMATICAL SOCIETY

Edited by Everett Pitcher and Gordon L. Walker

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### MEETINGS

#### Calendar of Meetings

NOTE: This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the *Notices* was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

<table>
<thead>
<tr>
<th>Meeting No.</th>
<th>Date</th>
<th>Place</th>
<th>Deadline for Abstracts*</th>
</tr>
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<tbody>
<tr>
<td>685</td>
<td>April 24, 1971</td>
<td>Monterey, California</td>
<td>Feb. 22, 1971</td>
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<tr>
<td>686</td>
<td>June 19, 1971</td>
<td>Corvallis, Oregon</td>
<td>May 5, 1971</td>
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<tr>
<td>687</td>
<td>August 30-September 3, 1971</td>
<td>University Park, Pennsylvania</td>
<td>July 7, 1971</td>
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<tr>
<td></td>
<td>(76th Summer Meeting)</td>
<td>January 17-21, 1972</td>
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<td></td>
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<td>Las Vegas, Nevada</td>
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</table>

*The abstracts of papers to be presented in person at the meetings must be received in the Headquarters Offices of the Society in Providence, Rhode Island, on or before these deadlines. The deadlines also apply to news items. The next two deadlines for by-title abstracts will be February 16, 1971 and April 28, 1971.*

The *Notices* of the American Mathematical Society is published by the American Mathematical Society, 321 South Main Street, P. O. Box 6248, Providence, Rhode Island 02904 in January, February, April, June, August, October, November and December. Price per annual volume is $10.00. Price per copy $3.00. Special price for copies sold at registration desks of meetings of the Society, $1.00 per copy. Subscriptions, orders for back numbers (back issues of the last two years only are available) and inquiries should be addressed to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904. Second class postage paid at Providence, Rhode Island, and additional mailing offices.

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The Seventy-Seventh Annual Meeting  
Chalfonte-Haddon Hall  
Atlantic City, New Jersey  
January 21-24, 1971

The seventy-seventh annual meeting of the American Mathematical Society will be held at the Chalfonte-Haddon Hall in Atlantic City, New Jersey. The meeting is being held in conjunction with the annual meeting of the Association for Symbolic Logic (January 21-22), the annual meeting of the Mathematical Association of America (January 23-25), and a meeting of the National Council of Teachers of Mathematics (January 23-24). The Conference Board of the Mathematical Sciences will present a panel discussion on Operations Research and Mathematics on Saturday, January 23, at 3:30 p.m. in the Vernon Room of Haddon Hall.

In response to a suggestion of the Council of the Society for richer fare at annual meetings, there will be more invited lectures than usual. As a consequence, there will be more evening lectures than usual.

By invitation of the Committee to Select Hour Speakers for the Summer and Annual Meetings, there will be four invited addresses. Professor Dennis P. Sullivan of the Massachusetts Institute of Technology will speak on Thursday, January 21, at 11:00 a.m.; the title of his lecture is "Symmetry in manifold theory." Professor Daniel G. Quillen of the Massachusetts Institute of Technology will speak on Thursday, January 21, at 1:30 p.m.; the title of his lecture is "Cohomology of groups and algebraic K-theory." Professor Leopoldo Nachbin of the University of Rochester and Instituto de Matematica Pura e Aplicada, Rio de Janeiro, Brazil, will speak on Friday evening, January 22, at 8:30 p.m.; the title of his lecture is "Recent developments in infinite dimensional holomorphy." Professor Harry Kesten of Cornell University will speak on Sunday, January 24, at 1:30 p.m.; the title of his lecture is "Some nonlinear stochastic growth models." All four lectures will be given in the Vernon Room of Haddon Hall.

The forty-fourth Josiah Willard Gibbs Lecture will be given by Professor Eberhard Hopf of Indiana University at 8:30 p.m., on Thursday, January 21, in the Pennsylvania Room of Haddon Hall; the title of his lecture will be "Ergodic theory and the geodesics on surfaces of negative curvature."

Professor Oscar Zariski of Harvard University will present the Retiring Presidential Address on Friday, January 22, at 11:00 a.m.; his title is "Some open questions in the theory of singularities." Dr. Yudell L. Luke of the Midwest Research Institute will speak on Sunday, January 24, at 5:00 p.m. in the Vernon Room. The title of his lecture will be "Information retrieval systems for mathematical journals."

The Society will award the Oswald Veblen Prize in Geometry on Saturday, January 23, at 1:15 p.m., in the Pennsylvania Room. There will be regular sessions for contributed ten-minute papers at 8:30 a.m. and 2:45 p.m. on Thursday, January 21, and Friday, January 22; at 3:45 p.m. on Saturday, January 23; and at 2:45 p.m. on Sunday, January 24.

COUNCIL AND BUSINESS MEETING

The Council of the Society will meet at 2:00 p.m., on Wednesday, January 20, in the West Room of Haddon Hall. The Business Meeting of the Society will be held on Saturday, January 23, at 2:30 p.m. in the Pennsylvania Room.

The agenda of the Business Meeting includes the following resolution:

That the Society shall poll its membership on the question: Are you in substantial agreement or disagreement with this statement:

I favor the prompt termination of American participation in
the War in Southeast Asia, and the withdrawal of our troops with the greatest speed possible, limited only by the availability of transport and the safety of said troops regardless of the consequences to the present governments in Saigon and Phnom Penh.

The above statement constitutes official notice under Article 10, Section 1 of the By-Laws, allowing final action on the resolution. The Secretary notes that if the resolution is passed, the opinion poll itself is a separate entity to be conducted by mail. Certain facts, estimates, and legal opinions collected by the Secretary for consideration by the membership appeared on page 1019 of the November 1970 issue of these Notices.

REGISTRATION

The registration desk for this meeting will be in the English Lounge of Haddon Hall, located on the Lounge and Dining Floor of the hotel. The desk will be open from 2:00 p.m. to 8:00 p.m. on Wednesday, January 20; from 8:00 a.m. to 5:00 p.m. on Thursday, January 21; from 9:00 a.m. to 5:00 p.m. on Friday through Sunday, January 22-24; and from 9:00 a.m. to 3:00 p.m. on Monday, January 25.

The registration fees for the meetings are as follows:
- Member $5.00
- Student $1.00
- Nonmember $10.00

There will be no extra charge for members of the families of registered participants.

EMPLOYMENT REGISTER

The Mathematical Sciences Employment Register will be maintained from 9:00 a.m. to 5:00 p.m. on Friday through Sunday, January 22-24, in the Carolina Room of the Chalfonte Hotel.

EXHIBITS

The book and educational media exhibits will be displayed in the Exhibit Hall from Friday through Sunday, January 22-24, from 9:00 a.m. to 5:00 p.m. The exhibit area is located in Haddon Hall adjacent to the Lobby.

BOOK SALE

Books published by the Society will be sold for cash prices somewhat below the usual prices when these same books are sold by mail.

MATHEMATICAL OFFPRINT SERVICE

A representative of the Mathematical Offprint Service will be available in the registration area to answer questions and to assist subscribers. Participants seeking information on the Mathematical Offprint Service are urged to stop at the MOS information desk during the regular registration hours.

ACCOMMODATIONS

Accommodations for the meeting will be handled by the Chalfonte-Haddon Hall Housing Bureau. A form for requesting accommodations will be found on the last page of the November Notices. Persons desiring accommodations should complete this reservation form or a reasonable facsimile and send it to the Mathematical Meetings Housing Bureau, Chalfonte-Haddon Hall, Atlantic City, New Jersey 08404. Reservations will be made in accordance with preferences indicated on the reservation form, insofar as this is possible, and all reservations will be confirmed. Deposit requirements vary from hotel to hotel, and participants will be informed of any such requirement at time of confirmation. REQUESTS FOR RESERVATIONS SHOULD ARRIVE IN ATLANTIC CITY NO LATER THAN JANUARY 8, 1971.

<table>
<thead>
<tr>
<th>HOTEL</th>
<th>SINGLES</th>
<th>TWIN (1 PERSON)</th>
<th>TWIN (2 PERSONS)</th>
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<tr>
<td>ABBEY MOTOR HOTEL</td>
<td>$12.00</td>
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<td>ARISTOCRAT HOTEL</td>
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<td>BARBIZON HOTEL</td>
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<td>BURGUNDY HOTEL</td>
<td>$12.00-$18.00</td>
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<td>CATALINA HOTEL</td>
<td>$14.00-$20.00</td>
<td>16.00</td>
<td>24.00</td>
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ACCOMMODATIONS
CAROLINA CREST HOTEL-MOTEL
Hotel: Singles $10.00-$14.00
Twin 14.00- 18.00
Motel: Singles $10.00-$14.00
Twin 16.00- 18.00

CHALFONT-HADDON HALL
Singles $12.00-$34.00
Twin 18.00- 36.00
Suites (Parlor, bath and 1 bedroom- twin beds) 48.00- 75.00
Suites (Parlor, bath and 2 bedrooms- twin beds) 72.00-109.00

COLONY MOTEL
Singles $12.00-$20.00
Twin 14.00- 24.00

COLTON MANOR HOTEL
Singles $12.00-$22.00
Twin 16.00- 26.00

CROWN MOTEL
Singles $12.00-$14.00
Twin 12.00- 14.00
Double 24.00- 32.00

LaFAYETTE MOTOR INN
Singles $12.00-$22.00
Twin 16.00- 22.00

RAMADA INN
Singles $12.00-$20.00
Twin 14.00- 30.00
Suites (Parlor, bath and 1 bedroom- twin beds) 55.00- 90.00
Suites (Parlor, bath and 2 bedrooms- twin beds) 125.00

SAHARA MOTEL
Singles $10.00-$12.00
Twin 14.00

SHERATON-DEAUVILLE
Singles $16.00-$32.00
Twin 18.00- 34.00
Suites (Parlor, bath and 1 bedroom- twin beds) 100.00
Suites (Parlor, bath and 2 bedrooms- twin beds) 135.00

SEASIDE MOTEL AND TOWER
Singles $14.00-$20.00
Twin 16.00- 22.00

TERRACE MOTEL
Singles $14.00-$20.00
Twin 16.00- 22.00

TRAYMORE
Singles $12.00-$20.00
Twin 16.00- 24.00

YMCA
Singles $4.50
Double 6.50

YWCA
Singles $5.00-$5.50
Double 4.50- 5.00

(YWCA associate membership fee of $5 must be paid)

NATIONAL SCIENCE FOUNDATION INFORMATION CENTER

NSF staff members will be available to provide counsel and information on all NSF programs of interest to mathematicians from 9:00 a.m. to 5:00 p.m. on January 22, 23, and 24, 1971, in the Sun Porch, Haddon Hall. This room is adjacent to the registration area on the Lounge and Dining Floor of the hotel.

ENTERTAINMENT

There are many things to see and do in the Atlantic City area. Among the attractions are walks along the Boardwalk; reconstructed colonial village of Smithville; the wine cellars, glass museum and hospitality center of the House of Renault; demonstrations of glassblowing; all day fishing trips; bus tours, etc. Brochures describing these and other attractions will be available at the hospitality desk in the registration area.

Atlantic City has a large number of excellent restaurants, and is particularly famous for its seafood. A guide to some of the better restaurants and nearby eating places will be available at the hospitality desk.

TRAVEL

In winter, Atlantic City is on Eastern Standard Time. There is regular airline service to the Philadelphia International Airport, about 60 miles northwest of At-
lantic City, by the following airlines: Allegheny, Air France, American, Delta, Eastern, Lufthansa, Mohawk, National, Northeast, Pan American, Trans World, and United.

From Philadelphia, participants may choose any one of the following modes of transportation. Nonstop limousine service is available from the Airport to Atlantic City. The Salem limousine Service desk is located near the baggage area at the Airport. The fare is $6. The current daily schedule lists departures at 10:30 a.m., 12:30 p.m., 2:30 p.m., 5:30 p.m., 7:00 p.m., and 10:30 p.m. Information on return schedules will be available in Atlantic City at the local information desk in the registration area. Driving time is approximately 65 minutes. Reservations for this service are accepted by all airline ticket agents. Excellent connections with Allegheny Commuter flights to Bader Field (Atlantic City’s in-town airport) can be arranged by any airline or travel agent. Allegheny Airlines Central Reservations offers confirmed reservations with checked baggage service through to Atlantic City. Flight time between Philadelphia International Airport and Bader Field is twenty minutes. The fare is $17 one way and $34 round trip.

Trailways Bus System, 13th and Arch Streets, Philadelphia, offers frequent departures to Atlantic City. Round trip from Philadelphia to Atlantic City is $3.90. The terminal is located at 13th and Arch which is near City Hall, Philadelphia, and can easily be reached by subway (City Hall stop). Bus schedules change frequently, so participants should check current departure times when making reservations. Specify "The Boardwalk Express" which takes 75 minutes. Buses are also available from New York to Atlantic City. Reservations are required and can be made by telephone. The trip takes approximately 2 hours and 15 minutes from the Port Authority Bus Terminal, 41st and 8th Avenue, New York City, via either the Lincoln Bus Company or the Public Service Coordinated Transport Company.

Rail service between Philadelphia and Atlantic City is available on a limited basis. The new PATCO Hi-Speed Line connects with the Pennsylvania Reading Seashore Line at Lindenwold. You may board the PATCO train at the Locust Street Station (between 15th and 16th) in Philadelphia. Travel time to Atlantic City is an hour and fifteen minutes. Schedules and fares should be checked directly with the railroad or with a travel agent.

Driving to Atlantic City from any point in the United States can be accomplished via a network of turnpikes. From CHICAGO, PITTSBURGH, AND WEST, take the Pennsylvania Turnpike to the Valley Forge exit. Continue on Schuylkill Expressway to the Walt Whitman bridge outside Philadelphia. After crossing the bridge, continue on the North-South Freeway, bear left onto the Atlantic City Expressway.

From points along the SOUTH ATLANTIC COAST, the direct route to Atlantic City is the Ocean Highway, over and through the Chesapeake Bay Bridge Tunnel. Follow US 13, right on US 113, and right on Delaware 18 to the Cape May-Lewes Ferry (1 hour and 30 minutes traveling time; price $3.25). In Cape May, follow the Garden State Parkway to the Atlantic City Expressway exit (East). An alternate route is to proceed to Wilmington, cross the Delaware Memorial Bridge, and take Route 40 directly into Atlantic City.

Those driving from PHILADELPHIA may cross the Walt Whitman bridge into New Jersey and connect either with the Atlantic City Expressway (tolls $1.25); Route 322, Black Horse Pike (no toll); or Route 30, the White Horse Pike (no toll).

Anyone driving from the NEW YORK CITY area may take the Holland Tunnel onto the New Jersey Turnpike and then take exit 11 (Garden State Parkway). Any of the three routes mentioned above, the Atlantic City Expressway, Black Horse Pike (Route 322), or White Horse Pike (Route 30) can be taken directly into Atlantic City.

WEATHER

During January one may expect an average daily high temperature of about 40° and a low temperature of around 29°. Rainfall and snow may occur one day out of every three, with rain more likely due to the proximity of the ocean.
MAIL AND MESSAGE CENTER

All mail and telegrams for persons attending the meetings should be addressed in care of Mathematical Meetings, Chalfonte-Haddon Hall, Atlantic City, New Jersey 08404. Mail and telegrams so addressed may be picked up at the Mail and Information Desk located at the registration area in the English Lounge of Haddon Hall.

A message center will be located in the same area to receive incoming calls for all members in attendance. The center will be open from January 21 through January 25 between 9:00 a.m. and 5:00 p.m. Messages will be recorded, and the name of any member for whom a message has been received will be posted until the message has been picked up at the message center. Members are advised to leave the following numbers with anyone who might want to reach them at the meeting: (609) 348-1292 and 348-1953.

LOCAL ARRANGEMENTS COMMITTEE


Leonard Gillman
Associate Secretary

Austin, Texas
<table>
<thead>
<tr>
<th>Time</th>
<th>American Mathematical Society</th>
<th>Association for Symbolic Logic</th>
</tr>
</thead>
<tbody>
<tr>
<td>2:00 p.m. - 8:00 p.m.</td>
<td>REGISTRATION - English Lounge - Haddon Hall</td>
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<tr>
<td>2:00 p.m.</td>
<td>Council Meeting</td>
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<td></td>
<td>West Room (H)</td>
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**THURSDAY, January 21**

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<tr>
<th>Time</th>
<th>AMS</th>
<th>ASL</th>
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<tbody>
<tr>
<td>8:00 a.m. - 5:00 p.m.</td>
<td>REGISTRATION - English Lounge - Haddon Hall</td>
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<tr>
<td>8:30 a.m. - 10:40 a.m.</td>
<td>Session on Analysis I</td>
<td></td>
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<td></td>
<td>Navajo Room (H)</td>
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<tr>
<td>8:30 a.m. - 10:40 a.m.</td>
<td>Session on Approximation Theory</td>
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<td>Pennsylvania Room, Section 3 (H)</td>
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<tr>
<td>8:30 a.m. - 10:40 a.m.</td>
<td>Session on Commutative Rings I</td>
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<td>Tower Rooms (H)</td>
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<tr>
<td>8:45 a.m. - 10:40 a.m.</td>
<td>Session on Differential Equations I</td>
<td></td>
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<td>Derbyshire Room (H)</td>
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<tr>
<td>8:30 a.m. - 10:40 a.m.</td>
<td>Session on General Topology I</td>
<td></td>
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<td>Rutland Room (H)</td>
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<tr>
<td>8:45 a.m. - 10:40 a.m.</td>
<td>Session on Functions of a Complex Variable I</td>
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<td>Roberts Room (C)</td>
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<tr>
<td>8:30 a.m. - 10:40 a.m.</td>
<td>Session on Nonassociative Algebras</td>
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<td>Mandarin Room (H)</td>
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<tr>
<td>8:30 a.m. - 10:40 a.m.</td>
<td>Session on Algebraic Topology</td>
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<td>Pavilion Room (H)</td>
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<tr>
<td>8:45 a.m. - 10:40 a.m.</td>
<td>Session on Number Theory I</td>
<td></td>
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<td>Music Room (C)</td>
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<tr>
<td>8:30 a.m. - 10:40 a.m.</td>
<td>Session on Group Theory I</td>
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<td>Garden Room (H)</td>
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<tr>
<td>9:00 a.m. - 11:50 a.m.</td>
<td>Invited Address:</td>
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<tr>
<td></td>
<td>Symmetry in manifold theory</td>
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<td></td>
<td>Dennis Sullivan</td>
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<td>Vernon Room (H)</td>
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<tr>
<td>11:00 a.m. - 12:00 p.m.</td>
<td>Invited Address:</td>
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<td>Cohomology of groups and algebraic K-theory</td>
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<td></td>
<td>Daniel Quillen</td>
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<td></td>
<td>Vernon Room (H)</td>
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<tr>
<td>1:30 p.m. - 2:30 p.m.</td>
<td>Invited Address:</td>
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<td>Cohomology of groups and algebraic K-theory</td>
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<tr>
<td></td>
<td>Daniel Quillen</td>
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Contributed Papers
Pennsylvania Room, Section 1 (H)
<table>
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<tr>
<th>Time</th>
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<th>Association for Symbolic Logic</th>
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<tr>
<td>2:00 p.m. - 2:45 p.m.</td>
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<tr>
<td>2:45 p.m. - 5:10 p.m.</td>
<td>Session on Modules</td>
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<td>Navajo Room (H)</td>
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<tr>
<td>2:45 p.m. - 5:10 p.m.</td>
<td>Session on Banach Spaces</td>
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<td>Pennsylvania Room, Section 3 (H)</td>
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<tr>
<td>2:45 p.m. - 5:10 p.m.</td>
<td>Session on Operator Theory I</td>
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<td>Tower Rooms (H)</td>
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<tr>
<td>2:45 p.m. - 4:55 p.m.</td>
<td>Session on Geometry</td>
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<td></td>
<td>Derbyshire Room (H)</td>
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<tr>
<td>2:45 p.m. - 5:10 p.m.</td>
<td>Session on General Topology II</td>
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<td></td>
<td>Music Room (C)</td>
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<tr>
<td>2:45 p.m. - 5:10 p.m.</td>
<td>Session on Measure and Integration</td>
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<td>Roberts Room (C)</td>
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<tr>
<td>2:45 p.m. - 5:10 p.m.</td>
<td>Session on Partial Differential Equations</td>
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<td>Mandarin Room (H)</td>
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<tr>
<td>2:45 p.m. - 5:10 p.m.</td>
<td>Session on Categories and General Systems</td>
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<td>Pavilion Room (H)</td>
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<tr>
<td>2:45 p.m. - 4:55 p.m.</td>
<td>Session on Lattices</td>
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<td>Blue Room (C)</td>
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<td>3:00 p.m. - 4:00 p.m.</td>
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<td>4:10 p.m. - 5:20 p.m.</td>
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<td>5:30 p.m. - 7:30 p.m.</td>
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<td>7:30 p.m. - 11:30 p.m.</td>
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<tr>
<td>8:30 p.m.</td>
<td>Gibbs Lecture:</td>
<td>Invited Address:</td>
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<tr>
<td></td>
<td>Ergodic theory and the geodesics on surfaces of negative curvature</td>
<td>Saturated ideals</td>
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<td></td>
<td>Eberhard Hopf</td>
<td>Kenneth Kunen</td>
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<td>Pennsylvania Room, Section 2 (H)</td>
<td>Vernon Room (H)</td>
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<td>Contributed Papers</td>
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<td>Pennsylvania Room, Section 1 (H)</td>
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<td>Social Hour</td>
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<td>Rutland Room (H)</td>
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<td>Council Meeting</td>
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<td>Rowsley Room (H)</td>
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<tr>
<td>Time</td>
<td>Session</td>
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<tr>
<td>9:00 a.m. - 5:00 p.m.</td>
<td>REGISTRATION - English Lounge - Haddon Hall</td>
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<tr>
<td>9:00 a.m. - 5:00 p.m.</td>
<td>EXHIBITS - Exhibit Hall - Haddon Hall</td>
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<tr>
<td>9:00 a.m. - 5:00 p.m.</td>
<td>EMPLOYMENT REGISTER - Carolina Room - Chalfonte Hotel</td>
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<tr>
<td>8:30 a.m. - 10:40 a.m.</td>
<td>Session on Harmonic Analysis</td>
<td>Navajo Room (H)</td>
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<tr>
<td>8:30 a.m. - 10:40 a.m.</td>
<td>Session on Function Spaces</td>
<td>Pennsylvania Room, Section 3 (H)</td>
</tr>
<tr>
<td>8:30 a.m. - 10:40 a.m.</td>
<td>Session on Commutative Rings II</td>
<td>Tower Rooms (H)</td>
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<tr>
<td>8:45 a.m. - 10:40 a.m.</td>
<td>Session on Differential Equations II</td>
<td>Derbyshire Room (H)</td>
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<tr>
<td>8:30 a.m. - 10:40 a.m.</td>
<td>Session on General Topology III</td>
<td>Rutland Room (H)</td>
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<tr>
<td>8:45 a.m. - 10:40 a.m.</td>
<td>Session on Functions of a Complex Variable II</td>
<td>Roberts Room (C)</td>
</tr>
<tr>
<td>8:30 a.m. - 10:40 a.m.</td>
<td>Session on Functions of Real Variables</td>
<td>Mandarin Room (H)</td>
</tr>
<tr>
<td>8:30 a.m. - 10:40 a.m.</td>
<td>Session on Mechanics</td>
<td>Pavilion Room (H)</td>
</tr>
<tr>
<td>8:45 a.m. - 10:40 a.m.</td>
<td>Session on Number Theory II</td>
<td>Music Room (C)</td>
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<tr>
<td>8:30 a.m. - 10:40 a.m.</td>
<td>Session on Finite Groups</td>
<td>Garden Room (H)</td>
</tr>
<tr>
<td>9:00 a.m. - 4:00 p.m.</td>
<td>Board of Governors</td>
<td>West Room (H)</td>
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<tr>
<td>9:00 a.m. - 10:00 a.m.</td>
<td>Invited Address: Non-standard models and their applications</td>
<td></td>
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<tr>
<td>10:10 a.m. - 11:45 a.m.</td>
<td>Contributed Papers</td>
<td>Pennsylvania Room, Section 1 (H)</td>
</tr>
<tr>
<td>11:00 a.m.</td>
<td>Retiring Presidential Address: Some open questions in the theory of singularities</td>
<td>Vernon Room (H)</td>
</tr>
<tr>
<td>1:30 p.m. - 2:30 p.m.</td>
<td>Survey Lecture: Applications of model theory to algebra</td>
<td>Simon Kochen</td>
</tr>
<tr>
<td>2:40 p.m. - 4:45 p.m.</td>
<td>Contributed Papers</td>
<td>Pennsylvania Room, Section 1 (H)</td>
</tr>
<tr>
<td>Time</td>
<td>American Mathematical Society</td>
<td>Mathematical Association of America</td>
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<tr>
<td>2:45 p.m. - 4:25 p.m.</td>
<td>Session on Associative Rings&lt;br&gt;Navajo Room (H)</td>
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<tr>
<td>2:45 p.m. - 5:10 p.m.</td>
<td>Session on Topological Vector Spaces&lt;br&gt;Pennsylvania Room, Section 3 (H)</td>
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<tr>
<td>2:45 p.m. - 5:10 p.m.</td>
<td>Session on Operator Theory II&lt;br&gt;Tower Rooms (H)</td>
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<tr>
<td>2:45 p.m. - 5:10 p.m.</td>
<td>Session on Differential Geometry&lt;br&gt;Derbyshire Room (H)</td>
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<td>2:45 p.m. - 5:10 p.m.</td>
<td>Session on Compact Spaces&lt;br&gt;Rutland Room (H)</td>
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<td>2:45 p.m. - 4:55 p.m.</td>
<td>Session on Probability and Statistics&lt;br&gt;Roberts Room (C)</td>
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<tr>
<td>2:45 p.m. - 5:10 p.m.</td>
<td>Session on Analysis II&lt;br&gt;Mandarin Room (H)</td>
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<tr>
<td>2:45 p.m. - 5:10 p.m.</td>
<td>Session on Algebraic Numbers and Fields&lt;br&gt;Pavilion Room (H)</td>
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<tr>
<td>2:45 p.m. - 5:10 p.m.</td>
<td>Session on Topological Groups&lt;br&gt;Music Room (C)</td>
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<tr>
<td>2:45 p.m. - 5:10 p.m.</td>
<td>Session on Combinatorics&lt;br&gt;Garden Room (H)</td>
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<tr>
<td>3:30 p.m. - 6:00 p.m.</td>
<td>ROCKY MOUNTAIN MATHEMATICS CONSORTIUM - Annual Meeting - Library (H)</td>
<td>Viking Theater (H)</td>
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<td>Film Program</td>
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<tr>
<td>7:30 p.m. - 7:40 p.m.</td>
<td></td>
<td>Sampler from the TOPOLOGY FILMS PROJECT</td>
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<tr>
<td>7:40 p.m.</td>
<td></td>
<td>FILMS OF THE MAA INDIVIDUAL LECTURES FILM PROJECT (ILFP)</td>
</tr>
<tr>
<td>7:40 p.m. - 8:05 p.m.</td>
<td></td>
<td>SHAPES OF THE FUTURE I -- SOME UNSOLVED PROBLEMS IN GEOMETRY&lt;br&gt;-- TWO DIMENSIONS with Victor Klee&lt;br&gt;(in color)</td>
</tr>
<tr>
<td>8:10 p.m. - 8:50 p.m.</td>
<td></td>
<td>SHAPES OF THE FUTURE II -- SOME UNSOLVED PROBLEMS IN GEOMETRY&lt;br&gt;-- THREE DIMENSIONS with Victor Klee&lt;br&gt;(in color)</td>
</tr>
<tr>
<td>9:00 p.m.</td>
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<td>IT'S HOW YOU COUNT THAT COUNTS&lt;br&gt;with Harold Shapiro (b&amp;w)</td>
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<tr>
<td>9:00 p.m. - 9:31 p.m.</td>
<td></td>
<td>PART I: NOTIONS AND NOTATION</td>
</tr>
<tr>
<td>9:32 p.m. - 10:10 p.m.</td>
<td></td>
<td>PART II: METHODOLOGY AND MACHINERY</td>
</tr>
<tr>
<td>10:11 p.m. - 10:47 p.m.</td>
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<td>PART III: APPLICATIONS</td>
</tr>
<tr>
<td>8:30 p.m.</td>
<td>Invited Address:&lt;br&gt;Recent developments in infinite dimensional holomorphy&lt;br&gt;Leopoldo Nachbin&lt;br&gt;Vernon Room (H)</td>
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<tr>
<td>Time</td>
<td>American Mathematical Society</td>
<td>Mathematical Association of America</td>
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<td>9:00 a.m. - 5:00 p.m.</td>
<td>REGISTRATION - English Lounge - Haddon Hall</td>
<td>National Council of Teachers of Mathematics</td>
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<tr>
<td>9:00 a.m. - 5:00 p.m.</td>
<td>EXHIBITS - Exhibit Hall - Haddon Hall</td>
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<td>9:00 a.m. - 5:00 p.m.</td>
<td>EMPLOYMENT REGISTER - Carolina Room - Chalfonte Hotel</td>
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<td>9:00 a.m. - 9:40 a.m.</td>
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<td>Pennsylvania Room, Section 2</td>
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<tr>
<td>9:40 a.m. - 10:20 a.m.</td>
<td></td>
<td>The Preparation of Mathematics Teachers in the United States During the Past Twenty Years C. R. Phelps</td>
</tr>
<tr>
<td>10:30 a.m. - 11:30 a.m.</td>
<td></td>
<td>The Program for the Preparation of Mathematics Teachers in Denmark Bent Christiansen</td>
</tr>
<tr>
<td>1:15 p.m.</td>
<td>Veblen Prize Session</td>
<td>General Discussion by the Panel and the Audience</td>
</tr>
<tr>
<td>2:30 p.m.</td>
<td>Business Meeting</td>
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</tr>
<tr>
<td>3:30 p.m. - 5:30 p.m.</td>
<td>C B M S PANEL DISCUSSION - Operations Research and Mathematics - Vernon Room (H)</td>
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<tr>
<td>3:45 p.m. - 5:10 p.m.</td>
<td>Session on Stochastic Processes I</td>
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<tr>
<td>3:45 p.m. - 5:40 p.m.</td>
<td>Session on Banach Algebras</td>
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<tr>
<td>3:45 p.m. - 4:55 p.m.</td>
<td>Session on Several Complex Variables</td>
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<tr>
<td>3:45 p.m. - 5:25 p.m.</td>
<td>Session on Differential Equations III</td>
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<tr>
<td>3:45 p.m. - 5:25 p.m.</td>
<td>Session on Metric Spaces</td>
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<td>3:45 p.m. - 5:25 p.m.</td>
<td>Session on Analytic Functions I</td>
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<tr>
<td>3:45 p.m. - 5:40 p.m.</td>
<td>Session on Numerical Analysis I</td>
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<tr>
<td>3:45 p.m. - 5:25 p.m.</td>
<td>Session on Games, Programming and Information</td>
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<tr>
<td>3:45 p.m. - 4:55 p.m.</td>
<td>Session on Convex Sets</td>
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<tr>
<td>3:45 p.m. - 5:40 p.m.</td>
<td>Session on Semigroups</td>
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</table>
### SATURDAY, January 23

<table>
<thead>
<tr>
<th>Time</th>
<th>American Mathematical Society</th>
<th>Mathematical Association of America</th>
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<tbody>
<tr>
<td>7:30 p.m.</td>
<td></td>
<td>ENCYCLOPAEDIA BRITANNICA SOUND FILMSTRIPS</td>
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<tr>
<td>7:30 p.m.</td>
<td></td>
<td>INTRODUCTION TO CALCULUS: LIMITS AND FUNCTIONS</td>
</tr>
<tr>
<td>7:30 p.m. – 7:42 p.m.</td>
<td></td>
<td>IDEAS, NUMBERS, AND LIMITS</td>
</tr>
<tr>
<td>7:44 p.m. – 7:56 p.m.</td>
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<td>MORE LIMITS</td>
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<tr>
<td>8:05 p.m.</td>
<td></td>
<td>INTRODUCTION TO CALCULUS: SEQUENCES AND CONVERGENCE</td>
</tr>
<tr>
<td>8:05 p.m. – 8:24 p.m.</td>
<td></td>
<td>IMAGINING SEQUENCES</td>
</tr>
<tr>
<td>8:26 p.m. – 8:46 p.m.</td>
<td></td>
<td>GETTING DOWN TO TERMS</td>
</tr>
<tr>
<td>9:00 p.m.</td>
<td></td>
<td>FILMS OF THE NCTM SERIES: ELEMENTARY MATHEMATICS FOR TEACHERS AND STUDENTS (in color)</td>
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<tr>
<td>9:00 p.m. – 9:11 p.m.</td>
<td></td>
<td>BETWEEN RATIONAL NUMBERS (KNIGHTS)</td>
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<tr>
<td>9:12 p.m. – 9:20 p.m.</td>
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<td>EXTENDING MULTIPLICATION TO RATIONAL NUMBERS (CLOUDS)</td>
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<tr>
<td>9:21 p.m. – 9:31 p.m.</td>
<td></td>
<td>THE BIGGEST RECTANGLE</td>
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<tr>
<td>9:32 p.m. – 9:40 p.m.</td>
<td></td>
<td>EXPLOITATION OF ERRORS (EDGAR’S GUESS)</td>
</tr>
<tr>
<td>9:41 p.m. – 9:50 p.m.</td>
<td></td>
<td>SOLVING PAIRS OF EQUATIONS (PIRATES)</td>
</tr>
<tr>
<td>9:51 p.m. – 9:59 p.m.</td>
<td></td>
<td>GRAPHING INEQUALITIES (MARVELOUS MARSHES)</td>
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<tr>
<td>10:00 p.m. – 10:11 p.m.</td>
<td></td>
<td>PROBABILITY (RAJAH)</td>
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### SUNDAY, January 24

<table>
<thead>
<tr>
<th>Time</th>
<th>AMS</th>
<th>MAA and NCTM</th>
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<tbody>
<tr>
<td>9:00 a.m. – 5:00 p.m.</td>
<td>REGISTRATION - English Lounge - Haddon Hall</td>
<td>Pennsylvania Room, Section 2</td>
</tr>
<tr>
<td>9:00 a.m. – 5:00 p.m.</td>
<td>EXHIBITS - Exhibit Hall - Haddon Hall</td>
<td>The Computer and the Calculus W. B. Stenberg</td>
</tr>
<tr>
<td>9:00 a.m. – 5:00 p.m.</td>
<td>EMPLOYMENT REGISTER - Carolina Room - Chalfonte Hotel</td>
<td>Business Meeting; the Association's Tenth Award for Distinguished Service to Mathematics; Award of the Chauvenet Prize</td>
</tr>
<tr>
<td>Time</td>
<td>Activity</td>
<td>Location</td>
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<tr>
<td>11:10 a.m. - 12:15 p.m.</td>
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<tr>
<td>1:30 p.m. - 2:30 p.m.</td>
<td>Invited Address: Some non-linear stochastic growth models</td>
<td>Vernon Room (H)</td>
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<tr>
<td>2:45 p.m. - 4:25 p.m.</td>
<td>Session on Stochastic Processes II</td>
<td>Navajo Room (H)</td>
</tr>
<tr>
<td>2:45 p.m. - 4:40 p.m.</td>
<td>Session on Functional Analysis</td>
<td>Pennsylvania Room, Section 3 (H)</td>
</tr>
<tr>
<td>2:45 p.m. - 4:40 p.m.</td>
<td>Session on Analysis III</td>
<td>Tower Rooms (H)</td>
</tr>
<tr>
<td>2:45 p.m. - 4:55 p.m.</td>
<td>Session on Linear Algebra</td>
<td>Derbyshire Room (H)</td>
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<tr>
<td>2:45 p.m. - 4:40 p.m.</td>
<td>Session on General Topology IV</td>
<td>Rutland Room (H)</td>
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<tr>
<td>2:45 p.m. - 4:25 p.m.</td>
<td>Session on Analytic Functions II</td>
<td>Roberts Room (C)</td>
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<tr>
<td>2:45 p.m. - 4:40 p.m.</td>
<td>Session on Numerical Analysis II</td>
<td>Mandarin Room (H)</td>
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<td>2:45 p.m. - 4:40 p.m.</td>
<td>Session on Logic and Foundations</td>
<td>Pavilion Room (H)</td>
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<tr>
<td>2:45 p.m. - 4:40 p.m.</td>
<td>Session on Near Rings</td>
<td>Music Room (C)</td>
</tr>
<tr>
<td>2:45 p.m. - 4:25 p.m.</td>
<td>Session on Manifolds</td>
<td>Garden Room (H)</td>
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<tr>
<td>5:00 p.m. - 6:00 p.m.</td>
<td>Special Address: Information retrieval systems for mathematical journals</td>
<td>Vernon Room (H)</td>
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<tr>
<td>7:30 p.m.</td>
<td>CONFERENCE BOARD - Council Meeting - West Room</td>
<td>Viking Theater (H)</td>
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<tr>
<td>7:30 p.m. - 9:30 p.m.</td>
<td>Film Program</td>
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<td>8:40 p.m. - 9:40 p.m.</td>
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<tr>
<td>9:00 a.m.</td>
<td>Registration - English Lounge - Haddon Hall</td>
<td>Pennsylvania Room, Section 2</td>
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<tr>
<td>9:00 a.m.</td>
<td>Model Theory</td>
<td>Abraham Robinson</td>
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<tr>
<td>10:00 a.m.</td>
<td>Set Theory</td>
<td>J.R. Shoenfield</td>
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<tr>
<td>11:00 a.m.</td>
<td>Recursion Theory</td>
<td>G.E. Sacks</td>
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<tr>
<td>2:40 p.m.</td>
<td>The Differentiation of Integrals</td>
<td>A.M. Bruckner</td>
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<td><strong>PRESENTORS OF TEN-MINUTE PAPERS</strong></td>
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<td>Following each name is the number corresponding to the speaker’s position on the program.</td>
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</table>
PROGRAM OF THE SESSION

The time limit for each contributed paper is ten minutes. The contributed papers are scheduled at fifteen minute intervals. To maintain this schedule, the time limit will be strictly enforced.

[Note. Ten percent of the abstracts were faulty (wrong forms, not classified, not typed, or not double-spaced). They were not rejected, however, but were worked into the program as fillers.]

THURSDAY, 8:30 A. M.

Session on Analysis. I, Navajo Room
8:30-8:40
(1) On sectionally bounded sequence spaces and \( \gamma \)-duality
Dr. Martin G. Buntinas, Loyola University of Chicago (682-46-45)

8:45-8:55
(2) The Bear metric in axiomatic potential theory. Preliminary report
Dr. Stuart P. Lloyd, Bell Telephone Laboratories, Inc., Murray Hill, New Jersey (682-31-1)

9:00-9:10
(3) On the mean-value property of harmonic functions. Preliminary report
Professor Myron Goldstein*, Arizona State University, and Professor Wellington H. Ow, Michigan State University (682-31-2)

9:15-9:25
(4) Vitali's theorem for harmonic functions on symmetric spaces of noncompact type
Mr. Manfred Stoll, Pennsylvania State University (682-31-3)

9:30-9:40
(5) Relationships between modules of the two and three dimensional Teichmuller ring domains. Preliminary report
Mr. Fredrick Sipinen, University of Utah (682-31-4)

9:45-9:55
(6) Bessel potentials on subcartesian spaces
Professor Nachman Aronszajn and Professor Pawel Szeptycki*, University of Kansas (682-31-5)

10:00-10:10
(7) The weak sequential continuity of the metric projection in \( L_p \) spaces
Professor Joseph M. Lambert, Pennsylvania State University, York Campus (682-41-4)

10:15-10:25
(8) The possibility of irregular behavior for orthogonal polynomials. Preliminary report
Professor Joseph L. Ullman, University of Michigan (682-41-6)

10:30-10:40
(9) Singular perturbations for nonlinear differential equations with a small parameter
Dr. George C. Hsiao, University of Delaware (682-41-9)

THURSDAY, 8:30 A. M.

Session on Approximation Theory, Pennsylvania Room, Section 3
8:30-8:40
(10) The general complex bounded case of the strict weighted approximation problem
Professor William H. Summers, University of Arkansas (682-46-7)

* For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.
8:45-8:55
(11) R-splines in Banach spaces. I. Interpolation of linear manifolds
Professor Richard B. Holmes, Purdue University (682-46-29)

9:00-9:15
(12) An algorithm for obtaining best approximate solutions to $Av = b$ in normed linear spaces
Professor Howard Anton* and Professor Charles S. Duris, Drexel University
(682-46-37)

9:15-9:25
(13) Approximation and selection
Professor Jorg Blatter, University of Texas (682-46-54)

9:30-9:40
(14) On the order of approximation of unbounded functions by positive linear operators
Professor Bruce Wood*, University of Arizona, and Professor S. M. Eisenberg,
University of Hartford (682-41-1)

9:45-9:55
(15) Sets of best approximation in certain classes of normed spaces
Professor Bruce L. Chalmers, University of California, Riverside, and Professor
Le Baron O. Ferguson*, Rensselaer Polytechnic Institute (682-41-2)

10:00-10:10
Professor Bruce L. Chalmers, University of California, Riverside (682-41-3)

10:15-10:25
(17) Recursive multivariate interpolation. Preliminary report
Professor Earl H. McKinney, Ball State University (682-41-5)

10:30-10:40
(18) n-widths, splines and singular differential equations. Preliminary report
Professor Joseph W. Jerome, Northwestern University (682-41-8)

THURSDAY, 8:30 A. M.

Session on Commutative Rings I, Tower Rooms

8:30-8:40
(19) Localization and colocalization in universal algebra
Professor Arthur L. Stone, Simon Fraser University (682-08-7)

8:45-8:55
(20) Equality of cardinal numbers of minimal generators under strong spans. Preliminary report
Dr. Japheth Hall, Jr., Stillman College (682-08-9)

9:00-9:10
(21) Number theoretic questions related to simple schemes
Dr. Vincent O. McBrien, College of the Holy Cross (682-13-3)

9:15-9:25
(22) The complete Baer extension of a semiprime ring. Preliminary report
Dr. David E. Peercy, University of West Virginia (682-13-4)

9:30-9:40
(23) u-rings for which each proper homomorphic image is a multiplication ring
Professor Craig A. Wood and Professor Dennis E. Bertholf*, Oklahoma State
University (682-13-5)

9:45-9:55
(24) On Krull dimension in power series rings
Dr. Jimmy T. Arnold, Virginia Polytechnic Institute and State University (682-13-6)

10:00-10:10
(25) Properties of the large quotient ring. Preliminary report
Dr. Monte B. Boisen, Jr., Virginia Polytechnic Institute and State University
(682-13-7)

10:15-10:25
(26) On valuation rings and containment relations between classes of ideals. Preliminary report
Professor Nick H. Vaughan, North Texas State University (682-13-10)

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10:30-10:40  
(27) Domains of Krull type and ideal transforms. Preliminary report  
Dr. John R. Hedstrom, University of North Carolina at Charlotte (682-13-12)  

THURSDAY, 8:45 A. M.  

Session on Differential Equations. I, Derbyshire Room  

8:45-8:55  
(28) A differential-difference equation leads to a generalized Fourier-Dirichlet representa-  
tion of the forcing function  
Professor Clifford H. Anderson, Ohio University (682-34-5)  

9:00-9:10  
(29) On the boundedness of solutions of a second order nonlinear differential equation.  
Preliminary report  
Mr. John W. Baker, University of Tennessee (682-34-7)  
(Introduced by Professor John S. Bradley)  

9:15-9:25  
(30) On the oscillation of a nonselfadjoint fourth order differential equation  
Professor Garret J. Etgen* and Professor John B. Scott, University of Houston  
(682-34-10)  

9:30-9:40  
(31) Geometric theory of differential equations. IV  
Professor Heinrich W. Guggenheimer, Polytechnic Institute of Brooklyn (682-34-12)  

9:45-9:55  
(32) On solutions of certain selfadjoint differential equations of fourth order  
Professor Marvin S. Keener, Oklahoma State University (682-34-15)  

10:00-10:10  
(33) Periodic solutions of certain Lienard equations with delay. Preliminary report  
Professor Robert B. Grafton, University of Missouri–Columbia (682-34-16)  

10:15-10:25  
(34) On the asymptotic behavior of the solutions of the third order nonlinear differential  
equation $u'''' - t^2 u''' = 0$. Preliminary report  
Mr. Paul Ohme, Florida State University and University of Rhode Island (682-34-18)  

10:30-10:40  
(35) A matrix analogue to a theorem of Leighton  
Professor Donald F. St. Mary, University of Massachusetts, and Professor Warren  
E. Shreve*, North Dakota State University (682-34-19)  

THURSDAY, 8:30 A. M.  

Session on General Topology. I, Rutland Room  

8:30-8:40  
Professor Joe A. Guthrie, University of Pittsburgh (682-54-20)  

8:45-8:55  
(37) Counterexamples to "extensions" of Lusin's separation principles. Preliminary report  
Mr. Eric J. Braude, Columbia University (682-54-21)  

9:00-9:10  
(38) The lattice of topologies: A survey  
Professor Roland E. Larson, Pennsylvania State University, Behrend Campus  
(682-54-27)  

9:15-9:25  
(39) Partitions of the Katetov extension. Preliminary report  
Professor Jack R. Porter* and Mr. Charles I. Votaw, University of Kansas  
(682-54-29)  

9:30-9:40  
(40) Preservation of topological properties under extensions of topologies. Preliminary report  
Dr. Donald F. Reynolds, West Virginia University (682-54-31)  

9:45-9:55  
(41) Restrictive semigroups of continuous selfmaps on spaces which are completely regular,  
Hausdorff and arcwise connected. Preliminary report  
Professor Robert D. Hofer, State University of New York, College at Plattsburgh  
(682-54-42)
10:00-10:10
(42) Lattices of lower semicontinuous functions and topological spaces determined by them
Professor Louis D. Nel, Carleton University (682-54-41)

10:15-10:25
(43) Sequence-covering and countably bi-quotient mappings
Mr. Frank Siwiec, St. John's University (682-54-34)

10:30-10:40
(44) Relations among certain mappings and conditions for their equivalence
Mr. Frank Siwiec and Professor Vincent P. Mancuso*, St. John's University (682-54-44)

THURSDAY, 8:45 A. M.

Session on Functions of a Complex Variable, I, Roberts Room
8:45-8:55
(45) Uniform finite generation of the rotation group
Professor Franklin Lowenthal, University of Oregon (682-30-3)

9:00-9:10
(46) A method of symmetrization and applications
Professor Dov Aharonov, Technion-Israel Institute of Technology, Haifa, Israel, and
Professor William E. Kirwan*, University of Maryland (682-30-4)

9:15-9:25
(47) Parseval's formula as a special case of the Leibniz rule for fractional derivatives
Professor Thomas J. Osler, Rensselaer Polytechnic Institute (682-30-5)

9:30-9:40
(48) Radial limits of functions with Hadamard gaps
Dr. Richard Hornblower, State University of New York at Albany (682-30-22)
(Introduced by Professor Joe W. Jenkins)

9:45-9:55
(49) Measure preserving homeomorphisms of the unit disc
Professor Hans M. Reimann, University of Minnesota (682-30-25)

10:00-10:10
(50) Weierstrass pairs for some modular groups
Mr. John M. Kasdan, University of California, Los Angeles (682-30-28)
(Introduced by Professor Martin A. Golubitsky)

10:15-10:25
(51) The strict inclusion $O_{PD} < O_{PE}$ in the classification of Riemannian manifolds. Preliminary report
Professor Young K. Kwon, University of Texas (682-30-29)

10:30-10:40
(52) Strong sequences of maximum indetermination. Preliminary report
Professor Kam-Fook Tse, Syracuse University (682-30-30)

THURSDAY, 8:30 A. M.

Session on Nonassociative Algebras, Mandarin Room
8:30-8:40
(53) Nilpotent groups of algebra automorphisms
Professor George F. Leger, Tufts University, and Professor Eugene M. Luks*,
Bucknell University (682-17-7)

8:45-8:55
(54) On near-generalized triple systems of even order with some applications
Professor Volodymyr Bohun-Chudyniv*, Morgan State College, and Mr. Boris
Bohn-Chudyniv, Seton Hall University (682-17-8)

9:00-9:10
(55) Computation of outer multiplicities on a computer
Professor Robert E. Beck*, Villanova University, and Professor Bernard Kolman,
Drexel University (682-17-6)
9:15-9:25
(56) On weak inverse property loops
Dr. Palaniappan Kannappan, University of Waterloo (682-17-2)

9:30-9:40
(57) Extensions of Chevalley algebras
Professor James F. Hurley, University of California, Riverside (682-17-3)

9:45-9:55
(58) A characterization of p-reductive Lie algebras
Dr. Robert Lee Wilson, Courant Institute, New York University (682-17-4)

10:00-10:10
(59) Generation closed classes of rings
Professor Richard L. Tangeman, Arkansas State University (682-17-5)

10:15-10:25
(60) Flexible algebras of degree two
Professor Joseph H. Mayne, Illinois Institute of Technology (682-17-1)

10:30-10:40
(61) On primitive and prime antiflexible rings
Mr. Hasan A. Celik, University of California, Santa Barbara (682-17-9)

THURSDAY, 8:30 A. M.

Session on Algebraic Topology, Pavilion Room
8:30-8:40
(62) The structure of automorphisms of planar triply-connected graphs
Dr. Herbert Fleischner, State University of New York at Binghamton (682-05-13)
(Introduced by Professor Hudson V. Kronk)

8:45-8:55
(63) Isotopy groups. Preliminary report
Professor Lawrence L. Larmore, California State College at Dominguez Hills
(682-55-5)

9:00-9:10
(64) On the A(p)-algebra structure of the Z(p)-cohomology of certain H-spaces. Preliminary report
Dr. W. A. Thedford, Virginia Commonwealth University (682-55-7)

9:15-9:25
(65) The Nielsen fixed point theory for Palais maps. Preliminary report
Dr. Uwe K. Scholz, University of Arizona (682-55-6)

9:30-9:40
(66) Realizing commutative diagrams of Abelian groups by corresponding homotopy commutative diagrams of Moore spaces. Preliminary report
Professor Paul C. Kainen, Case Western Reserve University (682-55-4)

9:45-9:55
(67) Orientability and Poincaré duality in generalized homology theories. Preliminary report
Professor James W. Vick, University of Texas (682-55-3)

10:00-10:10
(68) The third homotopy group of spun knots
Dr. S. J. Lomonaco, Jr., Texas Instruments, Inc., Dallas, Texas, and Southern Methodist University (682-55-2)

10:15-10:25
(69) A product theorem for H-group fibrations
Professor Frederick H. Croom, University of Kentucky (682-55-1)

10:30-10:40
(70) Spaces for which the fixed point property is characterized by homology groups
Professor Chung-wu Ho, Southern Illinois University at Edwardsville (682-55-8)

THURSDAY, 8:45 A. M.

Session on Number Theory, I, Music Room
8:45-8:55
(71) On the completeness of sequences of perturbed polynomial values
Dr. Stefan Andrus Burr, Bell Laboratories, Inc., Whippany, New Jersey (682-10-7)
9:00-9:10
(72) An approximation theorem for extended absolute values. Preliminary report
Dr. Ronald P. Brown, Simon Fraser University (682-10-9)

9:15-9:25
(73) Approximation properties of a complex continued fraction algorithm. Preliminary report
Professor Richard B. Lakein, State University of New York at Buffalo (682-10-8)

9:30-9:40
(74) An analogue of Minkowski's conjecture over the fields of Gauss and Eisenstein
Mr. James E. Sehnert, Ohio State University (682-10-10)
(Introduced by Professor A. C. Woods)

9:45-9:55
(75) The absolute value of the Riemann zeta function
Professor Robert S. Spira, Michigan State University (682-10-11)

10:00-10:10
(76) Minkowski convergents and the product of three linear homogeneous forms
Mr. Larry Rosenblum, Ohio State University (682-10-12)

10:15-10:25
(77) On expansions in nonintegral scales
Dr. J. Galambos, Temple University (682-10-14)
(Introduced by Professor Ivan N. Erdelyi)

10:30-10:40
(78) On a unifying proof technique in Diophantine approximations. Preliminary report
Professor Lawrence C. Eggan*, Illinois State University, and Professor Eugene A. Maier, University of Oregon (682-10-15)

THURSDAY, 8:30 A. M.

Session on Group Theory. I, Garden Room

8:30-8:40
(79) Automorphism groups. Preliminary report
Professor Hidegoro Nakano, Wayne State University (682-20-4)

8:45-8:55
(80) Alpha extensions of Abelian groups. Preliminary report
Dr. Donald Cook, Virginia Polytechnic Institute and State University (682-20-2)

9:00-9:10
(81) On the solvability question for groups of exponent 4
Mr. Tah Zen Yuan, Carroll College, and Professor Kenneth W. Weston*, University of Wisconsin–Parkside (682-20-11)

9:15-9:25
(82) Primary Abelian groups with semiduals
Professor Kai Faltins, University of Texas (682-20-14)

9:30-9:40
(83) On the splitting of mixed groups of torsion-free rank one
Professor Elias Toubassi, University of Arizona (682-20-15)

9:45-9:55
(84) Infinite groups which are the product of two abelian subgroups
Professor Bernhard Amberg, University of Texas (682-20-18)

10:00-10:10
(85) Abelian p–groups and normal torsion subgroups of their automorphism groups
Dr. Jutta Hausen, University of Houston (682-20-20)

10:15-10:25
(86) Automorphisms of group extensions
Professor Charles F. Wells, Case Western Reserve University (682-20-26)

10:30-10:40
(87) Defining relations for most integrally parameterized Chevalley groups
Professor W. P. Wardlaw, University of Georgia (682-20-28)

THURSDAY, 11:00 A. M.

Invited Address, Vernon Room
Symmetry in manifold theory
Professor Dennis P. Sullivan, Massachusetts Institute of Technology
THURSDAY, 1:30 P. M.

Invited Address, Vernon Room
Cohomology of groups and algebraic K-theory
Professor Daniel G. Quillen, Massachusetts Institute of Technology

THURSDAY, 2:45 P.M.

Session on Modules, Navajo Room
2:45-2:55
(88) Rings of quotients of endomorphism rings of projective modules
Professor Robert S. Cunningham and Professor Edgar A. Rutter, Jr., University of Kansas, and Professor Darrell R. Turnidge*, Kent State University (682-16-19)

3:00-3:10
(89) On perfect injectors and perfect projectors
Professor Robert S. Cunningham* and Professor Edgar A. Rutter, Jr., University of Kansas (682-16-1)

3:15-3:25
(90) Cofaithful modules and quasi-injectivity
Professor Robert R. Colby, University of Hawaii (682-16-3)

3:30-3:40
(91) The realization of modules and the Schur index
Professor George Szeto, Bradley University (682-16-2)

3:45-3:55
(92) On a special class of modules. Preliminary report
Dr. Augusto H. Ortiz, University of Puerto Rico (682-16-12)

4:00-4:10
(93) Rings classified by properties of modules. Preliminary report
Professor B. S. Chwe and Mr. Joe Neggers*, University of Alabama (682-16-20)

4:15-4:25
(94) Semiprime modules satisfying maximum conditions. Preliminary report
Professor Julius M. Zelmanowitz, Carnegie-Mellon University (682-16-22)

4:30-4:40
(95) On right PF X-rings
Professor Madhukar G. Deshpande, Marquette University (682-16-27)

4:45-4:55
(96) Finiteness of modules of integral differential forms
Professor In Young Chung, University of Cincinnati (682-16-23)

5:00-5:10
(97) The functor Qsp
Professor Hsiang-Dah Hou, Slippery Rock State College (682-16-24)

THURSDAY, 2:45 P. M.

Session on Banach Spaces, Pennsylvania Room, Section 3
2:45-2:55
(98) Weakly wandering vectors
Professor Ulrich Krengel, Ohio State University (682-46-5)
(Introduced by Professor Mustafa A. Akcoglu)

3:00-3:10
(99) Central sequences, generators, and direct integral decompositions of W-* algebras. Preliminary report
Professor Paul Willig, Stevens Institute of Technology (682-46-8)

3:15-3:25
(100) The Tschebyscheff inequality for Banach-space valued random variables. Preliminary report
Professor Sidney Birnbaum, California State Polytechnic College, Kellogg-Voorhis Campus (682-46-13)

3:30-3:40
(101) Integration generated by projective volumes over Hilbert spaces
Professor Witold M. Bogdanowicz and Professor John N. Welch*, Catholic University of America (682-46-18)
3:45-3:55
(102) The convex hull of the range of vector valued holomorphic mappings. Preliminary report
Professor Robert L. Hall and Professor D. L. Patil*, University of Wisconsin-
Milwaukee (682-46-21)

4:00-4:10
(103) Estimation of symmetric operators
Professor Yu-Lee Lee and Mr. Henderson C. H. Yeung*, Kansas State University
(682-46-27)

4:15-4:25
(104) Making $L_1$ uniformly convex in measure. Preliminary report
Dr. Merrill B. Goldberg, University of Colorado (682-46-35)

4:30-4:40
(105) Bifurcation of a nonlinear operator equation in a real Banach space. Preliminary report
Mr. David Westreich, Belfer Graduate School of Science, Yeshiva University and
Brooklyn College (682-46-38)

4:45-4:55
(106) Normalized iterations and nonlinear eigenvalue problems of variational type
Professor Charles V. Coffman, Carnegie-Mellon University (682-46-40)

5:00-5:10
Dr. Alan Radnitz, California State Polytechnic College, Kellogg-Voorhis Campus
(682-46-52)

THURSDAY, 2:45 P. M.

Session on Operator Theory, I, Tower Rooms
2:45-2:55
(108) A discrete Hille-Yosida-Phillips theorem
Professor Archie G. Gibson, University of New Mexico (682-47-3)

3:00-3:10
(109) Existence theorems for Hammerstein equations
Professor Herbert Amann, Indiana University and University of Freiburg, Federal
Republic of Germany (682-47-9)

3:15-3:25
(110) When is an operator the integral of a given spectral measure?
Professor Pesi R. Masani, Indiana University, and Professor Milton Rosenberg*,
University of Kansas (682-47-10)

3:30-3:40
(111) P. Levy's inversion formula for the Fourier-Stieltjes transform of a spectral measure
Professor Pesi R. Masani, Indiana University (682-47-11)

3:45-3:55
(112) Generalized invariant subspaces for linear operators
Professor Eberhard G. P. Gerlach, University of British Columbia (682-47-13)

4:00-4:10
(113) A spectral decompositon
Dr. William R. Parzynski, Montclair State College (682-47-15)

4:15-4:25
(114) Part of a weighted shift not similar to a weighted shift. Preliminary report
Dr. Ralph Gellar, North Carolina State University (682-47-20)
(Introduced by Professor Nicholas Rose)

4:30-4:40
(115) Commutators and the essential numerical range
Mr. Joel H. Anderson, Indiana University (682-47-14)

4:45-4:55
(116) On generalized inversion between Hilbert spaces
Professor Ivan N. Erdelyi, Temple University (682-47-16)

5:00-5:10
(117) On the fine structure of spectra
Professor C. J. A. Halberg, Jr., University of California, Riverside, and
Professor Åke Samuelsson*, Western Washington State College (682-47-17)
THURSDAY, 2:45 P.M.

Session on Geometry, Derbyshire Room
2:45-2:55
(118) On the translation groups of n-uniform translation affine Hjelmslev planes
Dr. David A. Drake, University of Florida (682-50-1)
3:00-3:10
(119) Generalized quadrangles as amalgamations of projective planes
Professor Stanley E. Payne, Miami University (682-14-1)
3:15-3:25
(120) Six element subgroups of real projectivities defined on the projective line. Preliminary report
Dr. William M. Sanders, Madison College (682-50-2)
3:30-3:40
(121) Three definitions of topological projective spaces
Dr. Kay S. Soerensen, University of Florida (682-50-6)
(Introduced by Professor James K. Brooks)
3:45-3:55
(122) Loop laws and collineations of planes
Professor Rafael Artzy, Temple University (682-50-4)
4:00-4:10
(123) Moebius planes of Hering type. I. Preliminary report
Dr. Nicholas K. Krier, University of North Carolina at Chapel Hill (682-50-8)
4:15-4:25
(124) Tangent properties and extremum problems of geometry. Preliminary report
Professor Necdet Ucoluk, Clarion State College (682-50-9)
(Introduced by Professor Sidney F. Mack)
4:30-4:40
(125) The segmentation of a set in euclidean space. Preliminary report
Dr. Stephen B. Cohen, Virginia Commonwealth University (682-50-5)
4:45-4:55
(126) A general geometric theory of surface area
Professor Leopoldo V. Toralballa, New York University, University Heights Campus (682-50-3)

THURSDAY, 2:45 P.M.

Session on General Topology, II, Music Room
2:45-2:55
Professor Francisco L. Marin, Universidad Simón Bolívar, Venezuela (682-54-2)
3:00-3:10
(128) Minimal Urysohn spaces
Professor Robert M. Stephenson, Jr., University of North Carolina at Chapel Hill (682-54-9)
3:15-3:25
(129) A generalization of semistratifiable and wΔ-spaces
Professor Richard E. Hodel, Duke University (682-54-11)
3:30-3:40
(130) Linearly stratifiable spaces
Professor Jerry E. Vaughan, University of North Carolina (682-54-19)
3:45-3:55
(131) Extension of the M-space concept
Professor Howard H. Wicke* Ohio University, and Dr. John M. Worrell, Jr., Sandia Laboratories, Albuquerque, New Mexico (682-54-51)
4:00-4:10
(132) A note on σ-spaces. Preliminary report
Professor Robert W. Heath, University of Pittsburgh (682-54-52)
4:15-4:25
(133) Some remarks on basically disconnected spaces. Preliminary report
Dr. Zong-hwe Tzeng, University of Florida and Wesleyan University (682-54-53)
(Introduced by Professor Anthony W. Hager)

4:30-4:40
(134) Concerning completable Moore spaces. Preliminary report
Mr. George Michael Reed, Auburn University (682-54-4)

4:45-4:55
(135) Completely pseudonormal and Moore spaces
Professor David Edwin Cook, University of Mississippi (682-54-15)

5:00-5:10
(136) Cyclicly descriptive Moore spaces
Mr. Jack D. Wilson, University of Mississippi (682-54-16)
(Introduced by Professor David Edwin Cook)

THURSDAY, 2:45 P.M.

Session on Measure and Integration, Roberts Room
2:45-2:55
(137) Partially-ordered distributions. Preliminary report
Mr. James F. Porter, Syracuse University (682-28-11)

3:00-3:10
(138) The regular decomposition of a measure. Preliminary report
Professor R. Bruce Mericle, Mankato State College (682-28-1)

3:15-3:25
(139) Invariant measures
Dr. James W. Roberts, University of South Carolina (682-28-4)
(Introduced by Professor James J. Buckley)

3:30-3:40
(140) Bimeasures and bilinear functionals on C(0,1)
Professor Thaddeus G. Dankel, Jr., Duke University (682-28-6)

3:45-3:55
(141) Integration-by-parts and substitution for integrals of interval functions
Dr. James D. Baker, Honeywell, Inc., Hopkins, Minnesota (682-28-2)

4:00-4:10
(142) A weak-Cauchy condition and isolated points of nonintegrability for continuous functions. Preliminary report
Dr. Michael Lee Steib, University of Houston (682-28-7)

4:15-4:25
(143) Necessary and sufficient conditions for the existence of Stieltjes integrals. Preliminary report
Mr. Gerald A. Kraus, Southern Illinois University (682-28-8)

4:30-4:40
(144) A direct approach to a product integral representation for a Gronwall inequality
Professor Fred M. Wright and Mr. Dean R. Kennebeck*, Iowa State University (682-28-9)

4:45-4:55
(145) The Gronwall inequality for weighted integrals
Professor Fred M. Wright* and Mr. Dean R. Kennebeck, Iowa State University (682-28-10)

5:00-5:10
(146) Complete ergodicity, weak mixing, and stacking methods
Mrs. Sarah L. Christiansen*, Drake University, and Dr. Julius R. Blum, University of New Mexico (682-28-5)

THURSDAY, 2:45 P.M.

Session on Partial Differential Equations, Mandarin Room
2:45-2:55
(147) Riemann's method generalized for second order hyperbolic systems of partial differential equations
Dr. Donald R. Snow, Brigham Young University (682-35-10)
3:00-3:10  
Professor Eutiquio C. Young, Florida State University (682-35-3)

3:15-3:25  
(149) A coerciveness inequality for a class of nonelliptic operators of constant deficit  
Professor John R. Schulenberger*, University of Utah, and Professor Calvin H. Wilcox, University of Geneva, Switzerland (682-35-6)

3:30-3:40  
(150) Removable sets for pointwise solutions of elliptic equations  
Professor James R. Diederich, University of California, Davis (682-35-4)

3:45-3:55  
(151) Remark on the Hadamard three circle theorem. Preliminary report  
Professor Rudolf Vyborny, University of Queensland, Australia, and Rensselaer Polytechnic Institute (682-35-11)

4:00-4:10  
(152) On the null-spaces of elliptic partial differential operators in $\mathbb{R}^m$  
Professor Homer F. Walker, Texas Tech University (682-35-13)  
(Introduced by Professor Vadim Komkov)

4:15-4:25  
(153) Periodic solutions of a nonlinear parabolic differential equation. Preliminary report  
Mr. David W. Bange, Colorado State University (682-35-12)  
(Introduced by Professor Robert E. Gaines)

4:30-4:40  
(154) On the stability of uniform solutions of the Navier-Stokes equations in n-dimensions  
Professor George H. Knightly, University of Massachusetts (682-35-8)

4:45-4:55  
(155) Solutions of the wave equation in an exterior characteristic quadrant  
Dr. Jeou-hwa Wang, Worcester Polytechnic Institute (682-35-7)

5:00-5:10  
(156) The bifurcation of periodic solutions in Banach spaces, II. Preliminary report  
Professor William S. Hall, University of Pittsburgh (682-35-5)

THURSDAY, 2:45 P.M.

Session on Categories and General Systems, Pavilion Room

2:45-2:55  
(157) Relativizing functors on rings and algebraic K-theory  
Dr. Michael R. Stein, Northwestern University (682-18-2)

3:00-3:10  
(158) Obstructions to liftings in commutative squares  
Professor Irwin S. Pressman, Ohio State University (682-18-3)

3:15-3:25  
(159) A commutative diagram for the generalized inflation-restriction exact sequence. Preliminary report  
Dr. Rudolph M. Najar, Wisconsin State University (682-18-4)

3:30-3:40  
(160) V-localizations and V-triples  
Professor Harvey E. Wolff, University of Texas (682-18-1)

3:45-3:55  
(161) An abstract extension. Preliminary report  
Professor James J. Buckley, University of South Carolina (682-08-5)

4:00-4:10  
(162) Decomposition theories. Preliminary report  
Professor Michael C. Gemignani, Smith College (682-08-1)

4:15-4:25  
(163) On the topological duality for primal algebra theory. Preliminary report  
Mr. Tah-Kai Hu, Western Washington State College (682-08-3)

4:30-4:40  
(164) Embedding properties  
Dr. Klaus Kaiser, University of Houston (682-08-6)
4:45-4:55
(165) Conservative algebras
   Dr. Jon Froemke, Oakland University (682-08-8)
5:00-5:10
(166) Infinite matroids
   Professor Samuel S. Wagstaff, Jr., University of Rochester (682-08-2)

THURSDAY, 2:45 P. M.

Session on Lattices, Blue Room
2:45-2:55
(167) The center of an orthogonic
   Mrs. Barbara Jeffcott, University of Massachusetts (682-06-1)
3:00-3:10
(168) Cancellability of chains. Preliminary report
   Dr. Simon C. Hsieh, University of South Carolina (682-06-2)
3:15-3:25
(169) Lattices of congruence classes
   Professor David Sachs, Wright State University (682-06-3)
3:30-3:40
(170) \(D_2\)-lattices
   Dr. Mary S. DeConge, Loyola University, New Orleans (682-06-4)
3:45-3:55
(171) Self-dual topological Boolean algebras
   Dr. Joel Kagan*, University of Hartford, and Dr. Robert W. Quackenbush, University of Manitoba (682-06-5)
4:00-4:10
(172) Cyclic atoms in orthomodular lattices
   Professor Donald E. Catlin, University of Massachusetts (682-06-6)
4:15-4:25
(173) Regular systems of left parameters in residuated lattice-ordered semigroups. Preliminary report
   Dr. Theodore J. Benac, U. S. Naval Academy, and Sister Claire Archambault*, Regis College (682-06-7)
4:30-4:40
(174) A characterisation of those abelian groups which can support a partial order with respect to which they are directed, interpolation groups
   Mr. Andrew Glass, University of Wisconsin (682-06-8)
4:45-4:55
(175) The orthomodular identity and metric completeness of the coordinatizing division ring
   Mr. Ronald P. Morash, University of Massachusetts (682-06-9)
5:00-5:10
(176) The Foulis-Randall sample space viewed as a generalized set. Preliminary report
   Mr. W. Robert Collins, University of Massachusetts (682-06-10)

THURSDAY, 2:45 P. M.

Session on Group Theory, II, Garden Room
2:45-2:55
(177) On the Engel margin
   Mr. Tommy K. Teague, Michigan State University (682-20-16)
3:00-3:10
(178) Some subgroups given by identities
   Professor Wolfgang P. Kappe, State University of New York at Binghamton (682-20-30)
3:15-3:25
(179) Factoring a free abelian group as a direct product with amalgamation. Preliminary report
   Professor Edward T. Ordman, University of Kentucky (682-20-12)
3:30-3:40
(180) Simple semigroups. Preliminary report
Professor Motupalli Satyanarayana, Bowling Green State University (682-20-32)

3:45-3:55
(181) A local theory of group extensions
Mr. Charles Edward Johnson, Ohio State University (682-20-33)

4:00-4:10
(182) Menger algebras and left ideals in associated semigroups. Preliminary report
Professor H. Ian Whitlock, Illinois Institute of Technology (682-20-27)

4:15-4:25
(183) Induced automorphisms of free metabelian groups
Dr. Orin N. Chein, Temple University (682-20-22)

4:30-4:40
(184) A characterization of idempotents and regular elements
Professor George Markowsky, St. Mary's College of Maryland (682-20-34)

4:45-4:55
(185) Fundamental, singly generated inverse semigroups
Professor Pierre Antoine Grillet, Kansas State University (682-20-3)

THURSDAY, 8:30 P. M.

Session on Harmonic Analysis, Navajo Room
8:30-8:40
(186) On the equation \( P(B) = \int P(Bx^{-1}) P(dx) \)
Mr. Arunava Mukherjea, University of South Florida (682-43-8)

8:45-8:55
(187) Operator and dual operator bases in linear topological spaces
Professor William B. Johnson, University of Houston (682-46-44)

9:00-9:10
(188) Banach spaces of compact multipliers and their dual spaces
Professor Gregory F. Bachelis*, Kansas State University, and Professor John E. Gilbert, University of Texas (682-43-2)

9:15-9:25
(189) Functions which are Fourier-Stieltjes transforms
Professor Stephen H. Friedberg, Illinois State University (682-43-4)

9:30-9:40
(190) Amerio-Doss theorems on Banach valued almost periodic functions
Mr. Wilbur P. Veith, Ohio State University (682-43-5)

9:45-9:55
(191) Some \( H^p \) spaces which are uncomplemented in \( L^p \). Preliminary report
Professor Samuel E. Ebenstein, Wayne State University (682-43-6)

10:00-10:10
(192) Compact induced representations
Professor Irwin Schochetman*, Oakland University, and Professor Robert C. Busby, Drexel University (682-43-7)

10:15-10:25
(193) Almost periodic function on a semidirect product of semigroups
Mr. Frank Dangello and Professor Robert J. Lindahl*, Pennsylvania State University (682-43-1)

10:30-10:40
(194) On the group algebra of an extension
Professor Arnold J. Insel, Illinois State University (682-43-3)
Session on Function Spaces, Pennsylvania Room, Section 3
8:30-8:40
(195) On commutators of Toeplitz operators. Preliminary report
Professor Matthew C. Y. Lee, University of Hawaii (682-46-6)
8:45-8:55
(196) Vector valued Köthe function spaces. Preliminary report
Mr. Alan L. Macdonald, Eastern Michigan University (682-46-12)
9:00-9:10
(197) The second duals of some function spaces
Professor Joel H. Shapiro and Professor G. D. Taylor, Michigan State University,
and Professor A. L. Shields*, University of Michigan (682-46-20)
9:15-9:25
(198) A characterization of zero sets for $A^\infty$
Professor James D. Nelson, Western Michigan University (682-46-31)
9:30-9:40
(199) On the existence of Schauder bases in spaces $C^k(M^n)$
Dr. Steven A. Schonefeld, Florida State University (682-46-30)
9:45-9:55
(200) Subspaces of certain function spaces. Preliminary report
Professor Joseph Diestel, University of Florida (682-46-33)
10:00-10:10
(201) On the monotonicity and subadditivity of norming perturbations on Banach function spaces
Dr. Joseph E. Quinn*, Loyola University, New Orleans, and Dr. Rozalind Reichard,
Morehouse College (682-46-46)
10:15-10:25
(202) A Banach space representation theorem. Preliminary report
Dr. W. R. Woodward*, Virginia Polytechnic Institute and State University, and Dr. R.
Rao Chivukula, University of Nebraska (682-46-47)
10:30-10:40
(203) Introversion in function spaces and conjugate convolution algebras
Dr. R. Rao Chivukula, University of Nebraska (682-46-24)

Session on Commutative Rings, II, Tower Rooms
8:30-8:40
(204) On the automorphism group of planar graphs
Dr. J. A. Zimmer, University of Waterloo (682-05-7)
8:45-8:55
(205) Covering relations among lattice varieties
Dr. Dang X. Hong, Vanderbilt University and California State College at San Bernardino (682-06-11)
9:00-9:10
(206) How changing $D[[x]]$ changes its quotient field
Dr. Philip B. Sheldon, Wisconsin State University (682-13-2)
9:15-9:25
(207) On the prolongations of a finite rank valuation
Dr. Michael J. Wright, Loyola University of Los Angeles (682-13-9)
9:30-9:40
(208) Algebraic Alexander-Spanier cohomology and separability
Professor Andrew T. Kitchen, St. John Fisher College and University of Rochester (682-13-11)
9:45-9:55
(209) Harrison's Witt ring of a ring of algebraic integers. Preliminary report
Professor Donald B. Coleman, University of Kentucky (682-13-13)
10:00-10:10
(210) Harrison's Witt ring of a commutative ring
Professor Donald B. Coleman and Professor Joel L. Cunningham*, University of Kentucky (682-13-14)

10:15-10:25
(211) On polynomial rings over a Hilbert ring. Preliminary report
Professor Robert Gilmer, Florida State University (682-13-1)

10:30-10:40
(212) Endomorphism of a formal power series ring over a polynomial ring. Preliminary report
Professor Joong Ho Kim, East Carolina University (682-13-8)

FRIDAY, 8:45 A. M.

Session on Differential Equations. II, Derbyshire Room
8:45-8:55
(213) A pairing of a class of evolution systems with a class of generators. Preliminary report
Professor James V. Herod, Georgia Institute of Technology (682-34-1)

9:00-9:10
(214) A characterization of a class of planar dynamical systems
Mr. Ronald A. Knight, Oklahoma State University (682-34-2)

9:15-9:25
(215) Oscillation of complex differential systems
Professor Donald F. St. Mary, University of Massachusetts (682-34-3)

9:30-9:40
(216) A generalization of the phase-shift-averaging method for oscillations in piecewise-linear systems
Professor Paul Davis*, Worcester Polytechnic Institute, and Professor Bernard A. Fleishman, Rensselaer Polytechnic Institute (682-34-4)

9:45-9:55
(217) Spectral theory for singular linear systems of Hamiltonian differential equations
Professor Churl S. Kim, Indiana University Southeast (682-34-8)

10:00-10:10
(218) Asymptotically invariant sets and perturbation results. Preliminary report
Professor S. Leela, State University of New York, College at Geneseo (682-34-11)

10:15-10:25
(219) Analytic simplification of a system of ordinary differential equations at an irregular type singularity
Professor Po-Fang Hsieh, Western Michigan University and Naval Research Laboratory, Washington, D.C. (682-34-14)

10:30-10:40
(220) Linear differential systems with infinitely many boundary points
Mr. Gary Brian Green*, Stanislaus State College, and Professor Allan M. Krall, Pennsylvania State University (682-34-17)

FRIDAY, 8:30 A. M.

Session on General Topology. III, Rutland Room
8:30-8:40
(221) Characterizations of metric spaces by the use of their midsets: intervals
Dr. Anthony D. Berard, Jr., Air Force Institute of Technology, Wright-Patterson Air Force Base, Dayton, Ohio (682-54-13)

8:45-8:55
(222) On the theory of ambits
Professor Ter-Jenq Huang, State University of New York, College at Cortland (682-54-48)

9:00-9:10
(223) On the product of F-spaces. Preliminary report
Professor Neil B. Hindman, California State College at Los Angeles (682-54-37)
9:15-9:25
(224) On fixed point theorems in metric space. Preliminary report
Professor Sankatha P. Singh*, Memorial University, and Mr. F. Zorzitto, Queen's University (682-54-38)

9:30-9:40
(225) Some Wallman compactifications determined by retracts
Professor H. L. Bentley, Bucknell University (682-54-39)
(Introduced by Professor H. W. Kim)

9:45-9:55
(226) Applications of infinite-dimensional manifolds to quotient spaces of metric ANR's
Professor David W. Henderson, Cornell University (682-54-49)

10:00-10:10
(227) On continuous change of fixed points
Professor Ludvik Janos, University of Florida (682-54-25)

10:15-10:25
(228) Characterization of 2-polyhedra using cone neighborhoods
Dr. Katherine L. Pedersen, Southern Illinois University (682-54-30)

10:30-10:40
(229) Decompositions of E\(^3\) that give E\(^3\). Preliminary report
Dr. Jacob H. Gerlach, Wisconsin State University, Whitewater (682-54-32)

FRIDAY, 8:45 A. M.

Session on Functions of a Complex Variable. II, Roberts Room
8:45-8:55
(230) On a problem of Erdős concerning the successive derivatives of an entire function
Professor Karl F. Barth* and Professor W. J. Schneider, Syracuse University (682-30-12)

9:00-9:10
(231) Extreme points in a class of polynomials having univalent sequential limits
Professor Ted J. Suffridge, University of Kentucky (682-30-14)

9:15-9:25
(232) On meromorphic functions with values distributed almost on a line
Dr. Chung Chun Yang, Naval Research Laboratory, Washington, D. C. (682-30-16)
(Introduced by Professor P. F. Hsieh)

9:30-9:40
(233) Meromorphic functions with prescribed asymptotic behavior. II
Professor J. L. Stebbins, University of Wisconsin-Milwaukee (682-30-17)
(Introduced by Professor Robert L. Hall)

9:45-9:55
(234) Exceptional values of entire and meromorphic functions. Preliminary report
Mr. Larry Earl Sherwood, University of Missouri-Kansas City (682-30-20)

10:00-10:10
(235) A method for determining the dilations of a family of quasiconformal homeomorphisms. Preliminary report
Mr. Stephen K. Parker, University of Utah (682-30-23)
(Introduced by Professor William R. Derrick)

10:15-10:25
(236) Locally univalent functions with locally univalent derivatives
Mr. Douglas M. Campbell, University of North Carolina at Chapel Hill (682-30-24)

10:30-10:40
(237) Pseudocontinuation of analytic functions by matching boundary values
Professor Robert L. Hall, University of Wisconsin-Milwaukee (682-30-26)

FRIDAY, 8:30 A. M.

Session on Functions of Real Variables, Mandarin Room
8:30-8:40
(238) Uniqueness of functions on a sphere from averages over sectors. Preliminary report
Professor George W. Batten, Jr., University of Houston (682-26-6)
8:45-8:55
(239) Gronwall-type inequalities for sum equations
Dr. Jet Wimp, Drexel University (682-40-2)

9:00-9:10
(240) Transfinite diameters and weighted averaging processes. Preliminary report
Dr. J. Clayton Shadeck, Philco-Ford Corporation, Houston, Texas (682-26-3)
(Introduced by Professor Dennis M. Rodriguez)

9:15-9:25
(241) A difference quotient norm for spaces of quasi-homogeneous Bessel potentials
Dr. Richard J. Bagby, New Mexico State University (682-26-1)

9:30-9:40
(242) Measurability and linear lattices of functions closed under convergence everywhere
Professor Witold M. Bogdanowicz, Catholic University of America (682-26-2)

9:45-9:55
(243) A nonlinear sequence to sequence transformation
Dr. Jon C. Helton, West Virginia University (682-40-1)

10:00-10:10
(244) An integral multiplication theorem
Professor Moses E. Cohen, Fresno State College (682-40-3)

10:15-10:25
(245) Necessary and sufficient conditions that a mapping of real-valued functions into the
reals be a limit
Professor William C. Bennewitz, Southern Illinois University at Edwardsville
(682-26-5)

10:30-10:40
(246) An integration by parts theorems for the \( \Psi \) integral. Preliminary report
Professor Ervin M. Eltze, Fort Hays Kansas State College (682-26-4)

FRIDAY, 8:30 A.M.

Session on Mechanics, Pavilion Room
8:30-8:40
(247) On MHD fluctuating flow in slip-flow regime with variable suction
Dr. V. M. Soundalgekar, Indian Institute of Technology, Bombay, India (682-76-3)
(Introduced by Professor S. A. Naimpally)

8:45-8:55
(248) Boundary layer methods in linear couple-stress elasticity
Mr. K. H. Chang*, Pennsylvania State University, and Professor Craig Comstock,
Naval Postgraduate School (682-73-3)

9:00-9:10
(249) On bound states in the continuum of N-body systems and the virial theorem
Mr. Sergio Albeverio, Princeton University (682-81-1)

9:15-9:25
(250) Finite flexure of an initially curved cuboid under nonuniform temperature distribution
Professor K. L. Arora, Panjab University, Chandigarh, India (682-73-1)
(Introduced by Professor Ram P. Bambah)

9:30-9:40
(251) Forced vibrations of anisotropic elastic sphere
Dr. Lokenath Debnath, East Carolina University (682-73-2)

9:45-9:55
(252) A vibration problem in polar coordinates. Preliminary report
Professor Willie R. Callahan, St. John's University (682-35-9)

10:00-10:10
(253) Similarity and Green's function to the Cauchy problem
Professor S. Manickam, Western Carolina University (682-76-1)

10:15-10:25
(254) Approximate method of solving the laminar boundary-layer equations
Dr. Gambhir M. Shrestha, Florida State University (682-76-2)
FRIDAY, 8:45 A.M.

Session on Number Theory, II, Music Room
8:45-8:55
(256) On the number of primes which divide a pair of relatively prime odd amicable numbers. Preliminary report
Dr. Peter Hagis, Jr., Temple University (682-10-13)

9:00-9:10
(257) On triangular numbers which are sums of consecutive squares
Professor Raphael Finkelstein*, Bowling Green State University, and Mr. Hymie London, McGill University (682-12-1)

9:15-9:25
(258) Pythagorean and triangular number triples
Professor Gregory Wulczyn, Bucknell University (682-10-4)

9:30-9:40
(259) A problem of Stöhr in additive number theory
Mr. Melvyn Bernard Nathanson, University of Rochester (682-10-3)

9:45-9:55
(260) Arithmetic formulas for some complementing sequences
Professor Charles A. Nicol, University of South Carolina (682-10-2)

10:00-10:15
(261) Generalized Dedekind-Rademacher sums. Preliminary report
Mr. James R. Krabill, University of North Carolina at Greensboro (682-10-6)

10:15-10:25
(262) Binary quadratic forms of determinant-pq
Professor Ezra Brown, Virginia Polytechnic Institute and State University (682-10-5)

10:30-10:40
(263) The factorization of $F_7$
Mr. Michael A. Morrison, University of California, Los Angeles, and Professor John D. Brillhart*, University of Arizona (682-10-1)

FRIDAY, 8:30 A.M.

Session on Finite Groups, Garden Room
8:30-8:40
(264) Properties of groups defined by normality conditions
Professor Luise Charlotte Kappe* and Professor Wolfgang P. Kappe, State University of New York at Binghamton (682-20-29)

8:45-8:55
(265) Abelian f.p.f. operator groups of type (p, p)
Professor James W. Richards, Kent State University (682-20-23)

9:00-9:10
(266) Finite groups with short nonnormal chains. Preliminary report
Professor Armond E. Spencer, University of Kentucky (682-20-25)

9:15-9:25
(267) Characterization of the simple components of the group algebras over the p-adic number field
Dr. Toshihiko Yamada, McGill University (682-20-13)

9:30-9:40
(268) On graphical regular representations of nonabelian groups. II
Professor Mark E. Watkins*, Syracuse University, and Mr. Lewis A. Nowitz, Programming Methods, Inc., New York, New York (682-20-21)

9:45-9:55
(269) The automorphism group of a p-group with the central quotient metacyclic
Dr. Albert D. Otto*, Illinois State University, and Dr. Richard Michael Davitt, University of Louisville (682-20-7)
10:00-10:10
(270) A class of 2-groups. Preliminary report
Professor Marc W. Konvisser, Illinois State University (682-20-8)

10:15-10:25
(271) The maximal subgroups of Conway's group $C_3$ and McLaughlin's group. Preliminary report
Dr. Larry Finkelstein, Wayne State University (682-20-9)

10:30-10:40
(272) On the Sylow $p$-subgroup of a finite group. Preliminary report
Mr. Paul B. Venzke, Minot State College (682-20-10)

FRIDAY, 11:00 A. M.

Retiring Presidential Address, Vernon Room
Some open questions in the theory of singularities
Professor Oscar Zariski, Harvard University

FRIDAY, 2:45 P. M.

Session on Associative Rings, Navajo Room
2:45-2:55
(273) Semiprime right Goldie rings which are direct sums of uniform right ideals
Professor Robert Gordon, University of Utah (682-16-13)

3:00-3:10
(274) Rings with zero singular left ideal and finite Goldie dimension. Preliminary report
Mr. Thomas J. Cheatham, University of Kentucky (682-16-25)

3:15-3:25
(275) Mappings on rings with involution. Preliminary report
Mr. Roy F. Riedlinger* and Dr. Willard E. Baxter, University of Delaware
(682-16-11)

3:30-3:40
(276) On generalized group rings. Preliminary report
Professor Gail L. Carns* and Professor Chong-Yun Chao, University of Pittsburgh
(682-16-7)

3:45-3:55
(277) A Hopf algebra generalization of the Wever-Specht formula
Dr. Gerald J. Giaccai, State University of New York, College at Fredonia (682-16-18)

4:00-4:10
(278) A generalization of the Zassenhaus-Taussky-Dade theorems to noncommutative orders. Preliminary report
Mr. Daniel A. Falk, Ohio State University (682-16-21)

4:15-4:25
(279) The Dedekind property for semirings
Professor Bill J. Dulin and Professor James R. Mosher*, Texas A & M University
(682-16-10)

(280) AND (281) WITHDRAWN

FRIDAY, 2:45 P. M.

Session on Topological Vector Spaces, Pennsylvania Room, Section 3
2:45-2:55
(282) Characterizations of uniform convexity and uniform smoothness. Preliminary report
Professor William Lee Bynum, College of William and Mary (682-46-2)

3:00-3:10
(283) Order characterizations of unconditional and absolute Schauder bases
Dr. Mark D. Levin* and Dr. C. W. McArthur, Florida State University (682-46-17)
3:15-3:25
(284) Norms arising from a multilinear functional. Preliminary report
Dr. Donald O. Koehler, Miami University (682-46-19)

3:30-3:40
(285) Strongly extreme points in normed spaces
Dr. Robert A. McGuigan, University of Massachusetts (682-46-25)

3:45-3:55
(286) Uniform convergence of positive operators
Professor Ralph L. James, Stanislaus State College (682-46-26)

4:00-4:10
(287) Preservation of closure in a locally convex space. Preliminary report
Mr. Isadore Brodsky, University of Maryland (682-46-36)
(Introduced by Professor John W. Brace)

4:15-4:25
(288) Varieties of linear topological spaces. Preliminary report
Professor Joseph Diestel and Professor Sidney A. Morris*, University of Florida
(682-46-49)

4:30-4:40
(289) A set-theoretic lemma with applications to locally convex topological vector spaces.
Preliminary report
Professor Stephen A. Saxon, University of Florida (682-46-50)

4:45-4:55
(290) Some results involving concrete semispaces and a question of Klee. Preliminary report
Professor C. Edward Moore, U.S. Naval Academy (682-46-16)

5:00-5:10
(291) Quasi-complementation in separable locally convex spaces
Professor James K. Weber, Jr., University of Hartford (682-46-11)

FRIDAY, 2:45 P. M.

Session on Operator Theory. II. Tower Rooms
2:45-2:55
(292) A fixed point theorem for certain functions on a complete Hausdorff uniform space
Professor Chi Song Wong, Southern Illinois University (682-47-1)

3:00-3:10
(293) Norm-sublinear transformations
Dr. Mary R. Embry, University of North Carolina at Charlotte (682-47-2)

3:15-3:25
(294) Strong concentration of the spectra of selfadjoint operators
Professor Chris Rorres, Drexel University (682-47-4)

3:30-3:40
(295) A remark concerning $B^\infty$-singular integral operators. Preliminary report
Professor Stuart M. Newberger, Oregon State University (682-47-6)

3:45-3:55
(296) On the extension of Lipschitz-Hölder maps on Orlicz spaces
Professor Charles E. Cleaver, Kent State University (682-47-7)

4:00-4:10
(297) Factoring operators which commute with a normal operator
Professor Lavon Barry Page, North Carolina State University (682-47-8)

4:15-4:25
(298) Purely imaginary powers of the indefinite integral
Professor Michael J. Fisher, University of Montana (682-47-12)

4:30-4:40
(299) Subnormality and quasinormality of Toeplitz operators. Preliminary report
Professor Takashi Ito and Professor Tin-Kin Wong*, Wayne State University
(682-47-19)

4:45-4:55
(300) The Hoffman-Wielandt theorem for Hilbert-Schmidt operators
Dr. James A. Cochran and Dr. Erold W. Hinds*, Bell Telephone Laboratories,
Inc., Whippany, New Jersey (682-47-18)
5:00-5:10
(301) On the mean ergodic theorem
Professor Jamil A. Siddiqi, University of Sherbrooke (682-47-21)

FRIDAY 2:45 P. M.

Session on Differential Geometry, Derbyshire Room
2:45-2:55
(302) Critical points of the length of a killing vector field
Professor Vilnis Ozols, University of Washington (682-53-1)
3:00-3:10
(303) Real trivial holomorphic principal fibre bundles
Professor Richard S. Millman, Ithaca College (682-53-2)
3:15-3:25
(304) Isoperimetric inequalities for manifolds with boundary
Professor Kit Hanes, Eastern Washington State College (682-53-3)
3:30-3:40
(305) Second order connections
Professor Robert H. Bowman, Arkansas State University (682-53-4)
3:45-3:55
(306) WITHDRAWN

4:00-4:10
(307) The de Rham cohomology of subcartesian structures. Preliminary report
Mr. Charles D. Marshall, University of Kansas (682-53-5)
4:15-4:25
(308) On the most general F-connections
Professor Krishan Lal Duggal, University of Windsor (682-53-7)
4:30-4:40
(309) An invariant approach to space-time symmetric
Professor George C. Debney, Jr., Virginia Polytechnic Institute and State University (682-53-8)
4:45-4:55
(310) Theory of special relativity in terms of absolute space and time. Preliminary report
Dr. Adolph Selzer, New London, Connecticut (682-83-1)

5:00-5:10
(311) On the structure of spaces admitting gravitation fields
Mr. Kishore B. Marathe, University of Rochester (682-83-2)

FRIDAY, 2:45 P. M.

Session on Compact Spaces, Rutland Room
2:45-2:55
(312) Compactness in R1-spaces
Dr. John M. Cibulskis, Northeastern Illinois State College (682-54-17)
3:00-3:10
(313) Wallman compactification as a functor
Professor Douglas Harris, Marquette University (682-54-14)
3:15-3:25
(314) Compact spaces homeomorphic to a ray of ordinals
Dr. John Warren Baker, Florida State University (682-54-28)
3:30-3:40
(315) Quasi-compactness and decompositions for arbitrary relations
Professor Stanley J. Wertheimer, Georgia Institute of Technology (682-54-12)
3:45-3:55
(316) The liberation of the Q-gaps. Preliminary report
Dr. Scott W. Williams, Pennsylvania State University (682-54-18)
4:00-4:10
(317) A note on paracompact p-spaces. Preliminary report
Professor Ronald A. Stoltenberg, Sam Houston State University (682-54-46)
4:15-4:25
(318) On \( \mathcal{J} \) -realcompactifications. Preliminary report
Professor Anthony J. D'Aristotle, State University of New York, College at Geneseo (682-54-3)

4:30-4:40
(319) Realcompactness and partitions of unity
Mr. G. De Marco, Universita di Padova, Italy, and Dr. Richard G. Wilson*
Carleton University (682-54-43)
(Introduced by Professor Louis D. Nel)

4:45-4:55
(320) Products of \( m \)-compact spaces
Mr. Victor Saks*, Windham College, and Professor Robert M. Stephenson, Jr.
University of North Carolina at Chapel Hill (682-54-8)

5:00-5:10
(321) Separation properties and C-compact spaces. Preliminary report
Mr. Thomas C. Lominac, Davidson College (682-54-33)

FRIDAY, 2:45 P.M.

Session on Probability and Statistics, Roberts Room
2:45-2:55
(322) Local limit theorems for nonidentically distributed lattice random variables
Professor J. David Mason, University of Georgia (682-60-2)

3:00-3:10
(323) Asymptotic properties of infinitely divisible distributions
Professor Roger A. Horn, Johns Hopkins University (682-60-5)

3:15-3:25
(324) On linear transformations of Yeh-Wiener measure
Professor Chull Park, Miami University (682-60-6)

3:30-3:40
(325) On the unimodality of distributions functions of class L
Dr. Stephen James Wolfe, University of Delaware (682-60-7)

3:45-3:55
Mr. Anthony G. Mucci, University of California, Irvine (682-60-16)

4:00-4:10
(327) Formulae for multiplying \( k_i \) by polykay products of weight 6. Preliminary report
Dr. P. N. Nagambal, Michigan Technological University (682-62-1)
(Introduced by Professor Hubert L. Hunzeker)

4:15-4:25
(328) On multiplication of \( k \)-statistics using ordered partitions. Preliminary report
Dr. Derrick S. Tracy*, and Mr. B. C. Gupta, University of Windsor (682-62-2)

4:30-4:40
(329) Certain algebraic properties of ordered partitions and their uses in statistics
Mr. B. C. Gupta* and Dr. Derrick S. Tracy, University of Windsor (682-62-3)

4:45-4:55
(330) On the distribution of the correlation coefficient for samples from the trivariate
normal distribution. Preliminary report
Dr. Frederick C. Durling, Medical University of South Carolina (682-62-4)
(Introduced by Dr. George E. Reves)

FRIDAY, 2:45 P.M.

Session on Analysis, II, Mandarin Room
2:45-2:55
(331) Spherical summability of conjugate multiple Fourier series and integrals at the
critical index
Professor Gary E. Lippman, Kenyon College (682-42-1)
3:00-3:10
(332) Factorization of Fourier transforms and Wiener-Hopf equations. Preliminary report
Professor Frank Stenger, University of Utah (682-42-2)

3:15-3:25
(333) Convolution, fixed point, and approximation in Stieltjes-Volterra integral equations
Professor Carl W. Bitzer, University of North Carolina at Greensboro (682-45-2)

3:30-3:40
(334) Algebraic structure for a set of nonlinear integral operations
Mr. David L. Lovelady, Georgia Institute of Technology (682-45-1)

3:45-3:55
(335) Optimality and a differential of variational forms
Dr. Mehdi S. Zarghamee, Arya-Mehr University of Technology, Tehran, Iran
(682-49-2)
(Introduced by Professor M. Anvari)

4:00-4:10
(336) Necessary conditions for control of functional differential equations of neutral type
Mr. George A. Kent, Brown University (682-49-1)

4:15-4:25
(337) A priori bounds in the Cauchy problem for coupled elliptic systems
Dr. Philip W. Schaefer, University of Tennessee (682-35-2)

4:30-4:40
(338) On the solution of a random integral equation by a method of stochastic approximation
Mr. W. J. Padgett* and Professor Chris P. Tsokos, Virginia Polytechnic Institute
and State University (682-60-20)

4:45-4:55
(339) On a stochastic integral equation of the Fredholm type
Mr. W. J. Padgett and Professor Chris P. Tsokos*, Virginia Polytechnic Institute
and State University (682-60-19)

5:00-5:10
(340) Existence results for some possibly noncoercive evolution problems with regular data
Mr. T. Mazumdar, University of Illinois (682-35-1)
(Introduced by Professor Robert W. Carroll)

FRIDAY, 2:45 P. M.

Session on Algebraic Numbers and Fields, Pavilion Room
2:45-2:55
(341) Functions over finite fields satisfying coordinate \( \psi \)-conditions
Dr. Robert M. McConnel, University of Tennessee (682-12-8)

3:00-3:10
(342) Systems of polynomial equations in finite fields
Professor Harald G. Niederreiter, Southern Illinois University (682-12-3)

3:15-3:25
(343) Nonuniquity of quadratic forms of class number 1. Preliminary report
Professor Larry J. Gerstein, Massachusetts Institute of Technology (682-12-5)

3:30-3:40
(344) Corresponding residue systems in normal extensions. Preliminary report
Professor William Ted Stout, Jr., University of Hawaii (682-12-6)

3:45-3:55
(345) Lifting Galois groups. Preliminary report
Mr. Richard D. Weiner, Washington University (682-12-9)
(Introduced by Professor Victor J. Katz)

4:00-4:10
(346) Representations of decomposable forms. Preliminary report
Professor Carter Waid, Texas Tech University (682-12-11)

4:15-4:25
(347) On the inverse of Carlitz's \( \eta \)-sum
Mr. K. Nageswara Rao, North Dakota State University (682-12-7)

4:30-4:40
(348) On elementary equivalence of \( \omega \)-pseudo-complete Hensel fields
Dr. Manfred Armbrust, University of Houston (682-12-10)
(349) Quadratic extensions of linearly compact fields
Dr. Ronald P. Brown, Simon Fraser University, and Professor Hoyt D. Warner*, Vanderbilt University (682-12-4)

(350) Reducible linear difference operators. Preliminary report
Dr. Charles H. Franke, Seton Hall University (682-12-2)

FRIDAY, 2:45 P. M.

Session on Topological Groups, Music Room
2:45-2:55
(351) Conjugacy classes of real Cartan subalgebras, revisited. Preliminary report
Professor Linda Preiss Rothschild, Tufts University (682-22-9)

3:00-3:10
(352) Invariant differential operators on a semisimple Lie algebra
Mr. Mohsen Pazirandeh, University of California, Los Angeles (682-22-8)

3:15-3:25
(353) Primitive actions and maximal subgroups of Lie groups
Professor Martin Golubitsky, University of California, Los Angeles (682-22-5)

3:30-3:40
(354) Reducing bundles in differentiable G-spaces
Mr. Louis A. Feldman, Stanislaus State College (682-22-4)

3:45-3:55
(355) Reflective subcategories and the generation of locally convex spaces. Preliminary report
Professor W. John Wilbur, Pacific Union College (682-22-7)

4:00-4:10
(356) On another radical in a topological ring. Preliminary report
Mr. Robert A. Massagli, University of Nebraska (682-22-6)

4:15-4:25
(357) On certain arcwise connected groups. Preliminary report
Professor Sigmund N. Hudson, Tulane University (682-22-2)

4:30-4:40
(358) A class of uniform spaces that admit an invariant integral
Dr. Gerald L. Itzkowitz, State University of New York at Buffalo (682-22-10)

4:45-4:55
(359) Uniform semigroups
Mr. Donald Marxen, University of Kentucky (682-22-3)
(Introduced by Professor John E. Mack)

5:00-5:10
(360) Ordered power associative groupoids (OPAGS)
Mr. Desmond A. Robbie, University of Florida (682-22-1)

FRIDAY, 2:45 P. M.

Session on Combinatorics, Garden Room
2:45-2:55
(361) A family of countable homogeneous graphs
Professor C. Ward Henson, Duke University (682-05-8)

3:00-3:10
(362) Nonregular polynomial graphs
Professor William G. Bridges, University of Wyoming (682-05-4)

3:15-3:25
(363) A Kuratowski-type theorem for the maximum genus of a graph
Professor Edward A. Nordhaus and Professor B. M. Stewart, Michigan State University, Professor Richard D. Ringeisen*, Colgate University, and Professor Arthur T. White, Western Michigan University (682-05-3)

3:30-3:40
(364) On merely-finitary pregeometries. Preliminary report
Professor Terrence Brown, University of Missouri-Kansas City (682-05-9)

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3:45-3:55  
(365) Disjoining permutations in Boolean algebras. Preliminary report  
Professor Douglas G. Kelly, University of North Carolina at Chapel Hill (682-05-5)

4:00-4:10  
(366) Generalized m-series in tree counting. Preliminary report  
Dr. Peter V. O’Neil, College of William and Mary (682-05-2)

4:15-4:25  
(367) A certain sequence of combinatorial identities  
Professor Jerome L. Paul, University of Cincinnati (682-05-10)

4:30-4:40  
(368) On coderived graphs  
Professor Lowell W. Beineke, Purdue University, Fort Wayne Campus (682-05-11)

4:45-4:55  
(369) The graphs of semirings. II  
Professor You-Feng Lin* and Professor Jogindar Singh Ratti, University of South Florida (682-05-12)

5:00-5:10  
(370) The number of 4 by 4 magic squares  
Professor David A. Smith, Duke University (682-05-1)

FRIDAY, 8:30 P.M.

Invited Address, Vernon Room  
Recent developments in infinite dimensional holomorphy  
Professor Leopoldo Nachbin, University of Rochester and Instituto de Matemática Pura e Aplicada, Rio de Janeiro, Brazil

SATURDAY, 1:15 P.M.

Veblen Prize Session, Pennsylvania Room, Section 2  
Titles: Unannounced

SATURDAY, 2:30 P.M.

Business Meeting, Pennsylvania Room, Section 2

SATURDAY 3:30 P.M.

CBMS Panel on Operations Research and Mathematics, Vernon Room

SATURDAY, 3:45 P.M.

Session on Stochastic Processes, I, Navajo Room

3:45-3:55  
(371) A limit theorem for age-dependent branching processes with arbitrary state space  
Professor Charles J. Mode, Drexel University (682-60-17)  
(Introduced by Professor Dale W. Lick)

4:00-4:10  
(372) Processes obtainable from Brownian motion by means of a random time change. Preliminary report  
Dr. Dennis M. Rodriguez, University of Houston (682-60-14)

4:15-4:25  
(373) An asymptotic 0-1 behavior of Gaussian processes  
Professor Clifford R. Qualls*, University of North Carolina at Chapel Hill and University of New Mexico, and Professor Hisao Watanabe, University of North Carolina at Chapel Hill and Kyushu University, Japan (682-60-21)

4:30-4:40  
(374) Embedding processes with independent increments in Brownian motion. Preliminary report  
Dr. Itrel E. Monroe, Dartmouth College (682-60-18)
4:45-4:55  
(375) On the extent of a random walk  
Professor David R. Beuerman, Queen's University (682-60-1)  
(Introduced by Dr. A. John Coleman)

5:00-5:10  
(376) Stochastic integrals in abstract Wiener space. Preliminary report  
Dr. Hui-Hsiung Kuo, Courant Institute, New York University (682-60-4)

SATURDAY, 3:45 P. M.

Session on Banach Algebras, Pennsylvania Room, Section 3  
3:45-3:55  
(377) Double centralizers on covariance algebras. Preliminary report  
Professor Robert C. Busby, Drexel University (682-46-14)

4:00-4:10  
(378) On S-property of C*-algebras. Preliminary report  
Professor Etang Chen, East Carolina University (682-46-22)

4:15-4:25  
(379) W*-algebras and nonabelian harmonic analysis  
Mr. Martin E. Walter, University of California, Los Angeles (682-46-23)

4:30-4:40  
(380) H-classes in a Banach algebra. Preliminary report  
Professor Jin Bai Kim, West Virginia University (682-46-42)

4:45-4:55  
(381) On Jordan *representations of Banach algebras. Preliminary report  
Mr. Satish Shirali, New Mexico State University (682-46-43)  
(Introduced by Professor Edward T. Kobayashi)

5:00-5:10  
(382) Continued fraction solution to the Riccati equation in a Banach algebra  
Dr. Wyman Fair, Drexel University (682-46-15)

5:15-5:25  
(383) Spectral theorem and lattice properties in Bp-algebras  
Mr. Milton Philip Olson, Lafayette, California (682-46-51)

5:30-5:40  
(384) Almost commutative Banach algebras  
Professor Bernard H. Aupetit, Université Laval (682-46-53)

SATURDAY, 3:45 P. M.

Session on Several Complex Variables, Tower Rooms  
3:45-3:55  
(385) A nonprincipal invariant subspace  
Mr. Chester Alan Jacewitz, Johns Hopkins University (682-32-1)

4:00-4:10  
(386) Subordination principle and distortion theorems on holomorphic mappings in the  
space C\^\infty  
Professor Kyong T. Hahn, Pennsylvania State University (682-32-2)

4:15-4:25  
(387) Whittaker constants for entire functions of several complex variables. Preliminary report  
Dr. John K. Shaw, Virginia Polytechnic Institute and State University (682-32-3)

4:30-4:40  
(388) Weighted spaces of entire functions. Preliminary report  
Mr. James J. Metzger, University of Georgia (682-32-4)

4:45-4:55  
(389) Theorems of Accola type on algebraic manifolds. Preliminary report  
Professor Su-shing Chen, University of Florida (682-32-5)  
(Introduced by Professor Sidney A. Morris)

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**SATURDAY, 3:45 P. M.**

**Session on Differential Equations, III, Derbyshire Room**

3:45-3:55

(390) Convergent integrals of solutions of a differential equation
Dr. Herman Gollwitzer, Drexel University (682-34-9)

4:00-4:10

(391) Absolutely stable differential equations which are not integrally stable
Professor James Kaplan, Northwestern University (682-34-13)

4:15-4:25

(392) Integral manifolds for perturbed nonlinear differential equations. Preliminary report
Dr. Noal C. Harbertson, North Carolina State University (682-34-21)

[Introduced by Professor Raimond A. Struble]

4:30-4:40

(393) Almost-periodic differential equations in Hilbert spaces
Professor Samuel Zaidman, Université de Montréal (682-34-6)

4:45-4:55

(394) Weighted differential operators in the limit-point case. Preliminary report
Professor Philip W. Walker, Virginia Polytechnic Institute and State University (682-34-22)

5:00-5:10

(395) A priori bounds for solutions of boundary value problems. Preliminary report
Professor Harry T. Sedinger, Point Park College (682-34-20)

[Introduced by Professor Helen B. Grimble]

5:15-5:25

(396) On singular points in semidynamical systems
Dr. Prem N. Bajaj, Wichita State University (682-34-23)

**SATURDAY, 3:45 P. M.**

**Session on Metric Spaces, Rutland Room**

3:45-3:55

(397) Spaces which admit only trivial little Lipschitz functions
Professor Robert B. Fraser, Louisiana State University (682-54-47)

4:00-4:10

(398) Remark on the dimension function d₃. Preliminary report
Dr. Joseph C. Nichols*, Radford College, and Dr. James C. Smith, Virginia Polytechnic Institute and State University (682-54-35)

4:15-4:25

(399) Dimension-theoretic properties of completions
Dr. Bruce R. Wenner, University of Missouri-Kansas City (682-54-40)

4:30-4:40

(400) The space of homeomorphisms of a compact 2-manifold onto itself is an absolute neighborhood retract. Preliminary report
Mr. R. Luke* and Dr. William K. Mason, Rutgers University (682-54-6)

4:45-4:55

(401) Aposyndetic properties of hyperspaces. Preliminary report
Mr. Jack T. Goodykoontz, Jr., University of Kentucky (682-54-50)

[Introduced by Professor John E. Mack]

5:00-5:10

(402) A note on closed subsets of symmetrizable spaces. Preliminary report
Mr. Dennis A. Bonnett*, Arizona State University, and Professor R. W. Heath, University of Pittsburgh (682-54-5)

5:15-5:25

(403) Cauchy sequences in semimetric spaces. Preliminary report
Dr. Dennis K. Burke, Miami University (682-54-22)
SATURDAY, 3:45 P. M.

Session on Analytic Functions. I, Roberts Room
3:45-3:55
(404) A representation theorem for functions holomorphic in the disk. Preliminary report
Mr. Philip J. Pratt, Grand Valley State College (682-30-1)
(Introduced by Professor Peter A. Lappan)

4:00-4:10
(405) On a theorem of Kaczmarzski concerning the equation \( f(z) = pf(a) \). Preliminary report
Professor Maxwell O. Reade*, University of Michigan, and Dr. E. J. Złotkiewicz,
M. Curie-Skłodowska University, Lublin, Poland (682-30-2)

4:15-4:25
(406) On univalent entire functions
Professor Boo Sang Lee, University of Kentucky (682-30-6)

4:30-4:40
(407) The Hardy class of Bazlevic functions and their derivatives
Mr. Sanford S. Miller, University of Kentucky (682-30-7)

4:45-4:55
(408) Holomorphic functions with areally mean \( p \)-valent derivatives
Professor Swarupchand M. Shah, University of Kentucky (682-30-8)

5:00-5:10
(409) Coefficients of functions of bounded boundary rotation. Preliminary report
Dr. James W. Noonan, U.S. Naval Research Laboratory (682-30-9)

5:15-5:25
(410) On polynomial approximation of \( H^p \) and \( \mathcal{A}^p \) functions
Professor Charles Kam-Tai Chui, Texas A & M University (682-30-10)

SATURDAY, 3:45 P. M.

Session on Numerical Analysis. I, Mandarin Room
3:45-3:55
(411) Minimum norm differentiation formulas
Dr. David Kenneth Kahaner, Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico (682-65-1)

4:00-4:10
(412) Finite-difference approximations to some odd-order partial differential equations. Preliminary report
Dr. A. J. Keeping, University of Missouri–Columbia (682-65-8)
(Introduced by Professor W. R. Utz)

4:15-4:25
(413) A numerical algorithm suggested by problems of transport in periodic media: the matrix case
Professor Richard C. Allen, Jr.*, Mr. P. Mundorff and Professor G. Milton Wing, University of New Mexico, and Professor James W. Burgmeier, University of Vermont (682-65-2)

4:30-4:40
(414) A difference equation based on the characteristics of quasi-linear hyperbolic systems
Professor Paul Gordon, Drexel University (682-65-15)

4:45-4:55
(415) Numerical integration with complex exponential kernels
Professor Meyer M. Kaplan* and Professor Roger S. Pinkham, Stevens Institute of Technology (682-65-9)

5:00-5:10
(416) Optimal quadrature formulas using generalized inverses
Professor Charles S. Duris, Drexel University (682-65-3)

5:15-5:25
(417) The method of Christopherson for solving free boundary problems for infinite journal bearings by means of finite differences
Professor Colin W. Cryer, University of Wisconsin (682-65-7)
5:30-5:40
(418) Applications of generalized Crank–Nicholson method
Mr. Thyrsa Frazier Sivager, Central State University (682-65-17)

SATURDAY, 3:45 P. M.

Session on Games, Programming, and Information, Pavilion Room
3:45-3:55
(419) Nondegeneracy and noncooperative games
Professor Joseph T. Howson, Jr., Boston College (682-90-2)
4:00-4:10
(420) Minimization of maximin machines
Professor Eugene S. Santos, Youngstown State University (682-94-1)
4:15-4:25
(421) Hybrid addition of matrices: A network theory concept
Professor Richard J. Duffin, Carnegie-Mellon University, and Professor George E. Trapp, Jr.*, West Virginia University (682-94-2)
4:30-4:40
(422) Optimality conditions in mathematical programming. Preliminary report
Professor Sanjo Zlobec, McGill University (682-90-1)
4:45-4:55
(423) Pseudo-recursiveness and pseudo-randomness within minimal program complexity hierarchies. Preliminary report
Mr. Robert P. Daley, Carnegie-Mellon University (682-68-1)
5:00-5:10
(424) Optimal drawing from a random urn
Dr. William M. Boyce, Bell Telephone Laboratories, Inc., Murray Hill, New Jersey (682-60-15)
5:15-5:25
(425) Smoothing data with tolerances by use of linear programming
Mr. Jonathan D. Young, Lawrence Radiation Laboratory, University of California, Berkeley (682-62-5)

SATURDAY, 3:45 P. M.

Session on Convex Sets, Music Room
3:45-3:55
(426) A generalized Kirchberger theorem
Mr. Steven R. Lay, University of California, Los Angeles (682-52-1)
4:00-4:10
(427) Maximizing the smallest triangle made by n points in a square
Mr. Michael Goldberg, Washington, D. C. (682-52-2)
4:15-4:25
(428) The Steiner point is not unique
Professor G. Thomas Sallee, University of California, Davis (682-52-3)
4:30-4:40
(429) On a conjecture of A. J. Hoffman. II
Dr. Joseph Y. Zaks, Michigan State University (682-52-4)
4:45-4:55
(430) Generalized convexity and globalization
Mrs. Jean B. Y. Chan Stanek, University of California, Los Angeles (682-52-5)
(Introduced by Professor Frederick A. Valentine)

SATURDAY, 3:45 P. M.

Session on Semigroups, Garden Room
3:45-3:55
(431) Finite invariant measure for semigroups of operators
Professor Usha Sachdeva, Ohio State University (682-47-5)
4:00-4:10 (432) Generalized homomorphism pairs. Preliminary report
Professor David R. Cecil, Texas A & I University (682-20-6)

4:15-4:25 (433) A generalization of the Rees theorem to a class of regular semigroups
Dr. Dennis P. Allen, Jr., Michigan Technological University (682-20-17)

4:30-4:40 (434) On the structure of the semigroup Ext (B,G) where B is a compact semilattice and
G is a compact abelian group. Preliminary report
Dr. James W. Stepp, University of Houston (682-20-19)

4:45-4:55 (435) Regular semigroups which are ideal extensions of groups
Professor Janet Elizabeth Ault, University of Florida (682-20-24)

5:00-5:10 (436) On the closure of the bicyclic semigroup in locally compact semigroups. Preliminary report
Dr. Michael F. Collins, Southwest Minnesota State College (682-20-31)

5:15-5:25 (437) On models of groupoid identities
Professor John Marshall Saade, University of Georgia (682-20-1)

5:30-5:40 (438) On Kim's conjecture
Professor Kim Ki-Hang Butler, Pembroke State University (682-20-5)

SUNDAY, 1:30 P. M.

Invited Address, Vernon Room
Some nonlinear stochastic growth models
Professor Harry Kesten, Cornell University

SUNDAY, 2:45 P. M.

Session on Stochastic Processes. II, Navajo Room
2:45-2:55 (439) Characterizations of a time distribution of a standard Brownian motion
Professor M. T. Wasan, Queen's University (682-60-9)

3:00-3:10 (440) Multiplicative operator functionals of a Markov process. Preliminary report
Professor Mark A. Pinsky, Northwestern University (682-60-3)

3:15-3:25 (441) Boundary conditions of a class of Markov chains
Mr. Chung-Tuo Shih, University of Michigan (682-60-8)

3:30-3:40 (442) Some ergodic theorems for Markov chains defined on an arbitrary state space
Mr. Richard W. Madsen, Iowa State University (682-60-10)
(Introduced by Professor J. C. Mathews)

3:45-3:55 (443) Last exit times and the Q-matrices of Markov chains
Mr. A. O. Pittenger, University of Michigan (682-60-13)

4:00-4:10 (444) Law of the iterated logarithm for a class of null recurrent and transient Markov chains
Mr. Haim Brezis and Professor Walter A. Rosenkrantz, Courant Institute, New
York University, and Mr. Burton Singer*, Columbia University (682-60-12)

4:15-4:25 (445) A degenerate elliptic-parabolic equation occurring in the theory of probability
Mr. Haim Brezis and Professor Walter A. Rosenkrantz*, Courant Institute, New
York University, and Mr. Burton Singer, Columbia University (682-60-11)
Session on Functional Analysis, Pennsylvania Room, Section 3

2:45-2:55
(446) Separating function algebras. I
Professor George L. Csordas* and Professor Harold B. Reiter, University of Hawaii (682-46-9)

3:00-3:10
(447) Separating function algebras. II
Professor George L. Csordas and Professor Harold B. Reiter*, University of Hawaii (682-46-10)

3:15-3:25
(448) Function algebras on the unit interval. Preliminary report
Mr. David M. Wells, University of Pittsburgh (682-46-34)
(Introduced by Professor David T. Brown)

3:30-3:40
(449) Systems of derivations and algebras of functions
Professor Ronn L. Carpenter, University of Houston (682-46-39)

3:45-3:55
(450) A lemma of deLeeuw's on extreme points
Professor Jerry A. Johnson, Oklahoma State University (682-46-1)

4:00-4:10
(451) A contribution to the theory of vector valued Orlicz spaces
Professor Michael S. Skaff, University of Detroit (682-46-4)

4:15-4:25
(452) Strictly increasing Riesz norms
Professor Lawrence C. Moore, Jr., Duke University (682-46-41)

4:30-4:40
(453) Extensions of basic sequences in Fréchet spaces
Professor William B. Robinson, Clarkson College (682-46-48)

SUNDAY, 2:45 P. M.

Session on Analysis, III, Tower Rooms

2:45-2:55
(454) An old conjecture and its implications for b-functions. Preliminary report
Professor Gloria Olive, Wisconsin State University (682-33-1)

3:00-3:10
(455) Some finite summation theorems and an asymptotic expansion for the generalized hypergeometric series. Preliminary report
Professor Chandra Mohan Joshi*, Texas A & M University, and Professor J. B. McDonald, Utah State University (682-33-2)

3:15-3:25
(456) On derivatives of biorthogonal polynomials
Professor Winchung A. Chai, Montclair State College (682-33-3)

3:30-3:40
(457) On the extreme eigenvalues of Toeplitz matrices associated with Laguerre polynomials
Professor John V. Baxley, Wake Forest University (682-33-4)

3:45-3:55
(458) Oscillations caused by retarded actions
Professor Gerasimos E. Ladas* and Professor V. Lakshmikantham, University of Rhode Island (682-39-1)

4:00-4:10
(459) On the asymptotic behavior of functional differential equations
Professor Thomas G. Hallam*, Florida State University and University of Rhode Island, Professor Gerasimos E. Ladas and Professor V. Lakshmikantham, University of Rhode Island (682-39-2)

4:15-4:25
(460) The solution of a difference system related to Sturm–Liouville polynomials
Dr. F. Lee Cook, University of Alabama in Huntsville (682-39-3)
4:30-4:40
(461) Barreled subspaces of Banach spaces and tensor products. Preliminary report
Dr. Forrest R. Miller, Kansas State University (682-46-3)

SUNDAY, 2:45 P. M.

Session on Linear Algebra, Derbyshire Room
2:45-2:55
(462) Commutators over finite fields
Dr. Larry S. Johnson*, Fort Lewis College, and Dr. A. Duane Porter and Dr.
Verne J. Varineau, University of Wyoming (682-15-7)

3:00-3:10
(463) The matrix equation $U_1 \cdots U_n A V \cdots V_m = B$ over a finite field. Preliminary report
Professor Ronald H. Dalla*, Eastern Washington State College, and Professor
A. Duane Porter, University of Wyoming (682-15-1)

3:15-3:25
(464) Uniqueness of symmetric matrix norms
Professor Joseph B. Deeds, University of South Carolina (682-15-2)

3:30-3:40
(465) Eigenvalues of complex tridiagonal matrices
Professor Peter M. Gibson, University of Alabama in Huntsville (682-15-3)

3:45-3:55
(466) Solutions of certain matrix equations. III
Professor John Jones, Jr., Air Force Institute of Technology (682-15-9)

4:00-4:10
(467) Cones which are topheavy with respect to a norm. Preliminary report
Dr. Emilie Haynsworth* and Dr. Miroslav Fiedler, Auburn University (682-15-8)

4:15-4:25
(468) On complete set of similarity invariants of certain pairs $(A, B)$ of commutative
nilpotent operators
Professor Hong Wha Kim, Bucknell University (682-15-4)

4:30-4:40
(469) Polynôme minimal d’un produit tensoriel d’applications linéaires
Professor Ghislain Roy, Université Laval (682-15-5)
( Introduced by Professor Mohammad Ishaq)

SUNDAY, 2:45 P. M.

Session on General Topology. IV, Rutland Room
2:45-2:55
(471) Disconnected spaces and strong normality conditions
Professor Richard A. Alo, Carnegie-Mellon University, Professor Harvey L.
Shapiro*, West Georgia College and Northern Illinois University, and Professor
Frank A. Smith, Kent State University (682-54-10)

3:00-3:10
(472) Expandability and collectionwise normality
Dr. J. Clarence Smith*, Virginia Polytechnic Institute and State University, and
Mr. L. L. Krajewski, University of Wisconsin-Milwaukee (682-54-36)

3:15-3:25
(473) $\alpha$ - weak normality and normality. Preliminary report
Dr. Eugene S. Ball, Tennessee Technological University (682-54-26)

3:30-3:40
(474) The normal linearly Lindelöf problem
Professor Norman R. Howes, University of Texas and Texas Instruments, Inc.,
Dallas, Texas, and Dr. Woodlea B. Sconyers*, Texas Christian University and
General Dynamics, Fort Worth, Texas (682-54-24)
3:45-3:55
(475) Transfinite Cauchy sequences characterize complete uniform spaces
   Professor Norman R. Howes, University of Dallas and Texas Instruments, Inc.,
   Dallas, Texas (682-54-23)

4:00-4:10
(476) Some quasi-uniform space examples
   Professor Troy L. Hicks*, University of Missouri-Rolla, and Professor John
   W. Carlson, Kansas State Teachers College (682-54-1)

4:15-4:25
(477) Some properties of quasi-uniform structures
   Professor Troy L. Hicks, University of Missouri-Rolla, and Professor John W.
   Carlson*, Kansas State Teachers College (682-54-7)

4:30-4:40
(478) Dense p-subspaces of proximity spaces
   Professor Don A. Mattson, Trinity College (682-54-54)

SUNDAY, 2:45 P.M.

Session on Analytic Functions. II, Roberts Room
2:45-2:55
(479) Boundary behavior of holomorphic functions
   Mr. David C. Haddad, West Virginia University (682-30-11)

3:00-3:10
(480) On the univalence of some analytic functions
   Dr. Gulan Mohamad Shah, University of Wisconsin-Waukesha County (682-30-13)

3:15-3:25
(481) Distribution of a-points for unbounded analytic functions. Preliminary report
   Dr. Daniel D. Bonar*, Denison University, and Dr. Francis W. Carroll, Ohio State
   University (682-30-15)

3:30-3:40
(482) Functions of bounded radius rotation. Preliminary report
   Professor Harry B. Coonce, Mankato State College (682-30-18)

3:45-3:55
(483) Commuting analytic functions without fixed points
   Professor Donald F. Behan, Union College (682-30-19)

4:00-4:10
(484) Odd functions of bounded boundary rotation
   Mr. Ronald J. Leach, Howard University (682-30-21)

4:15-4:25
(485) Starlike meromorphic functions. Preliminary report
   Professor James E. Miller, Texas A & M University (682-30-27)

SUNDAY, 2:45 P. M.

Session on Numerical Analysis. II, Mandarin Room
2:45-2:55
(486) Iterative solutions of Ax = \lambda x. Preliminary report
   Dr. William L. Morris, University of Houston (682-65-4)

3:00-3:10
(487) Error bounds for bicubic spline interpolation
   Dr. Ralph E. Carlson, Bettis Atomic Power Laboratory, West Mifflin, Pennsylvania,
   and Professor Charles A. Hall*, University of Pittsburgh (682-65-5)

3:15-3:25
(488) Multiple zeros of polynomials
   Professor Craig A. Wood, Oklahoma State University (682-65-6)

3:30-3:40
(489) Extension of minimal quadrature to functions of low-order continuity. Preliminary
   report
   Professor R. D. Riess*, and Professor Lee W. Johnson, Virginia Polytechnic
   Institute and State University (682-65-10)
3:45-3:55
(490) Probabilistic surface representation and computer graphics
Professor Gregory M. Nielson*, Arizona State University, Professor Robert E. Barnhill, University of Utah, and Mr. Robert M. Flegal, Xerox Research Laboratory, Palo Alto, California (682-65-11)

4:00-4:10
(491) On the use of perturbation in Galerkin's method
Dr. A. Zafarullah, Florida State University (682-65-12)
(Introduced by Professor De Witt L. Sumners)

4:15-4:25
(492) Strong minimum variance estimation of polynomial plus random noise
Professor William F. Trench, Drexel University (682-65-13)

4:30-4:40
(493) Hermite-Birkhoff interpolation problem in Haar subspaces
Professor Yasuhiko Ikebe, University of Texas (682-65-16)

SUNDAY, 2:45 P.M.

Session on Logic and Foundations, Pavilion Room
2:45-2:55
(494) A note on implicative models
Professor Edwin L. Marsden, Kansas State University (682-02-7)

3:00-3:10
(495) A construction in topological model theory. Preliminary report
Professor D. Randolph Johnson, Trinity College (682-02-8)

3:15-3:25
(496) First-order equality logic with weak existence assumptions
Professor Robert C. Wherritt, Wichita State University (682-02-9)

3:30-3:40
(497) Effective matchmaking (Recursion theoretic aspects of a theorem of Philip Hall)
Professor Joseph G. Rosenstein*, Rutgers University, and Professor Alfred B. Manaster, University of California, San Diego (682-02-2)

3:45-3:55
(498) The degree of the theory of addition of isols. Preliminary report
Professor Anil Nerode, Cornell University, and Professor Alfred B. Manaster*, University of California, San Diego (682-02-3)

4:00-4:10
(499) Projective geometry having solid as primitive. Preliminary report
Professor Theodore F. Sullivan, University of South Carolina (682-02-4)

4:15-4:25
(500) Completions of Boolean algebras with partial operators. Preliminary report
Mr. Yen-yi Wu, Pennsylvania State University (682-02-5)
(Introduced by Professor Hugo B. Ribeiro)

4:30-4:40
(501) A nonexistence theorem of differential equations
Professor Oliver G. Aberth, Texas A & M University (682-02-6)

SUNDAY, 2:45 P.M.

Session on Near Rings, Music Room
2:45-2:55
(502) The endomorphism near ring of $D_{2n'}$, n odd
Mr. Carter G. Lyons* and Professor Joseph J. Malone, Jr., Texas A & M University (682-16-4)

3:00-3:10
(503) D.g. near rings on $D_{2n}$, n odd
Professor Joseph J. Malone, Jr.*, and Mr. Carter G. Lyons, Texas A & M University (682-16-5)
3:15-3:25
(504) On groups and endomorphism rings
Dr. Carlton J. Maxson, Texas A & M University (682-16-6)

3:30-3:40
(505) Endomorphism near rings that are rings
Professor Bruce C. McQuarrie, Worcester Polytechnic Institute (682-16-8)

3:45-3:55
(506) Near rings with identities on certain groups
Professor Steve Ligh, University of Southwestern Louisiana (682-16-9)

4:00-4:10
(507) Radicals of endomorphism near-rings. Preliminary report
Professor Marjory Jane Johnson, University of South Carolina (682-16-15)

4:15-4:25
(508) Embedding of near-rings into near-rings with identity
Dr. Gerhard Betsch, University of Arizona and Mathematisches Institut der
Universitat, Federal Republic of Germany (682-16-16)
(Introduced by Professor James R. Clay)

4:30-4:40
(509) Near rings and block designs. Preliminary report
Dr. James R. Clay, University of Arizona (682-16-14)

SUNDAY, 2:45 P. M.

Session on Manifolds, Garden Room

2:45-2:55
(510) Proper maps of PL manifolds
Mrs. Christine B. Beaucage, State University of New York at Stony Brook (682-57-1)

3:00-3:10
(511) Obstructions to embedding. Preliminary report
Professor James W. Maxwell, Oklahoma State University (682-57-2)

3:15-3:25
(512) Triangulation of 3-manifolds: A PL approach. Preliminary report
Mr. Peter B. Shalen, Harvard University (682-57-3)

3:30-3:40
(513) Topological applications of transformations of group presentations. Preliminary
report
Professor Bernard W. Levinger and Professor Richard P. Osborne*, Colorado
State University (682-57-4)

3:45-3:55
(514) Hyperbolic \( \mathbb{R}^2 \) actions on 2-manifolds
Professor Chester R. Schneider, Oregon State University (682-58-1)

4:00-4:10
(515) Dissipative systems on manifolds. Preliminary report
Professor Siavash M. Shahshahani, Northwestern University (682-58-3)

4:15-4:25
(516) Necessary conditions for stability of diffeomorphisms. Preliminary report
Professor John M. Franks, Northwestern University (682-58-2)
(Introduced by Professor Robert F. Williams)

SUNDAY, 5:00 P. M.

Special Address, Vernon Room
Information retrieval systems for mathematical journals
Dr. Yudell L. Luke, Midwest Research Institute, Kansas City, Missouri

Leonard Gillman
Associate Secretary
The six hundred eighty-third meeting of the American Mathematical Society will be held at the University of Illinois at Chicago Circle, Chicago, Illinois, on Friday and Saturday, March 26–27, 1971. All sessions of the meeting will be held in the Lecture Center of the university. The university is located approximately one mile west and one-half mile south of the intersection of State and Madison Streets, the origin of coordinates in the Chicago street numbering system.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, there will be four one-hour addresses. Professor Marvin I. Knopp of the University of Wisconsin and the University of Illinois at Chicago Circle will speak on Friday, March 26, at 11:00 a.m.; his topic will be "Eichler cohomology and the Fourier coefficients of automorphic forms." Professor John H. Walter of the University of Illinois at Urbana-Champaign will address the Society on Friday, March 26, at 1:45 p.m.; his subject will be "The structure of the centralizers of involutions in finite simple groups." Professor Robert Ellis of the University of Minnesota will speak on Saturday, March 27, at 11:00 a.m.; his talk will be entitled "Recent results in the algebraic theory of minimal sets." Professor Allen L. Shields of the University of Michigan will address the Society on Saturday, March 27, at 1:45 p.m.; his topic will be "Spaces of analytic functions: some recent results."

By invitation of the same committee there will be five special sessions of selected twenty-minute papers. Professor Steven B. Bank of the University of Illinois at Urbana-Champaign is arranging one such session for Saturday, March 27, on the subject of Growth, Oscillation, and Asymptotic Properties of Solutions of Ordinary Differential Equations; the list of speakers will include Steven B. Bank, Harold E. Benzinger, William A. Harris, Jr., Po-Fang Hsieh, Zeev Nehari, Thomas L. Sherman, Gilbert Stengle, Walter C. Strodt, Robert K. Wright, Chung-Chun Yang, and possibly one or two others. Another special session is being arranged by Professor Jim Douglas, Jr., of the University of Chicago on the subject of Numerical Solution of Partial Differential Equations, to be held on Friday, March 26; the list of speakers will include James H. Bramble, Todd Dupont, Henry H. Rachford, Jr., W. Gilbert Strang, and a number of others. Professor Edward R. Fadell of the University of Wisconsin is organizing a session for Friday on Fixed Point and Coincidence Theory; the list of speakers will include Felix E. Browder, Andrzej Granas, John P. Huneke, Ronald J. Knill, and probably several others. Another session, to be held on Saturday, is being arranged by Professor Leon W. Green of the University of Minnesota on the subject of Flows; the list of speakers will be given in the February issue of these Notices. Finally, Professor Eben Matlis of Northwestern University is arranging a special session for Friday on the subject of Commutative Algebra; the list of speakers will include Shreeram Abhyankar, John A. Eagon, Robert M. Fossum, William J. Heinzer, Eben Matlis, Tsuqng-Tsieng Moh, Michael R. Stein, Roger P. Ware, and possibly one or two others.

There will be sessions for the presen-
tation of contributed ten-minute papers on both Friday and Saturday. Those having time preferences for the presentation of their papers should so indicate on their abstracts. Abstracts should be submitted to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904, so as to arrive prior to the deadline of January 27, 1971. There will be a session for late papers if one is needed, but, of course, late papers will not be listed in the printed program of the meeting.

REGISTRATION

The registration desk will be located in the Illinois Room Lobby on the third level of the Chicago Circle Center, the student center of the University of Illinois at Chicago Circle. The Center is located on the west side of Halsted Street, opposite the point at which Polk Street comes to a dead end. The registration desk will be open from 8:30 a.m. to 4:30 p.m. on Friday, March 26, and from 8:30 a.m. to 3:30 p.m. on Saturday, March 27.

ACCOMMODATIONS

The hotel headquarters for the meeting will be the Pick-Congress Hotel. However, those coming by car may prefer to stay at either the Holiday Inn (Kennedy Expressway) or the Ramada Inn (Downtown). Detailed information about these three hotels is given below. The rates quoted are subject to an 8% tax. A form which may be used for making reservations at the Pick-Congress Hotel can be found on the last page of these notes. Those making reservations at either the Holiday Inn or the Ramada Inn should simply specify that they will be attending the meeting of the American Mathematical Society as guests of the University of Illinois at Chicago Circle.

1. Pick-Congress Hotel, 520 South Michigan Avenue, Chicago, Illinois 60605. One and one quarter miles east of the university. Room rates are $18.00 to $27.00 for single occupancy and $26.00 to $35.00 for double occupancy. There is a delivery service garage.

2. Holiday Inn (Kennedy Expressway), 1 South Halstead Street, Chicago, Illinois 60606. One-half mile north of the university. Room rates are $15.00 for single occupancy and $19.00 for double occupancy. There is a free garage.

3. Ramada Inn (Downtown), 506 West Harrison Street, Chicago, Illinois 60607. One-half mile east of the university. Room rates are $15.00 for single occupancy and $17.00 for double occupancy. Free delivery-service parking is available.

FOOD SERVICE

The cafeteria on the ground floor of the Chicago Circle Center will be open for lunch on both days of the meeting. There are several Greek restaurants on Halsted Street just north of the campus. All three hotels listed have dining facilities. In addition, there are several restaurants on Wabash Avenue near the Pick-Congress Hotel. For those wishing to venture further afield, the list of dining possibilities is endless.

TRAVEL AND LOCAL INFORMATION

The University of Illinois at Chicago Circle is named after, served by, and located at the Chicago Circle cloverleaf formed by the three major expressways into the downtown area of Chicago from the north, south, and west. Those coming from the north on the John F. Kennedy Expressway or from the south on the Dan Ryan Expressway should exit at Taylor Street. Those coming from the west on the Dwight D. Eisenhower Expressway should use the Racine Avenue Exit. Parking is available for 50 cents in several coin parking lots on the campus.

There is direct limousine service between O'Hare Airport and the Pick-Congress Hotel. All three hotels and the Chicago Circle Campus are close to the major railroad stations.

The Holiday Inn and the Ramada Inn are within walking distance of the campus. There are two easy methods of getting to the campus by public transportation from the
Pick-Congress Hotel: (a) walk two blocks west and then take the Number 7 bus from its starting point on the west side of State Street between Harrison and Congress Streets, (b) walk three blocks west and two blocks north, enter the subway on Dearborn Street north of Van Buren Street, and take either the Congress or the Douglas train to the Halsted Street stop. Those preferring to go by taxi should ask to be taken to the intersection of Polk and Halsted Streets.

ENTERTAINMENT

The Chicago Symphony Orchestra will give performances of Verdi’s Requiem in Orchestra Hall on Thursday, March 25, at 8:15 p.m. and on Saturday, March 27, at 8:30 p.m. Tickets are priced at $5.50, $7.00, $8.00, $9.00, $9.50, and $10.00 and may be ordered from the Box Office, Chicago Symphony Orchestra, Orchestra Hall, 220 South Michigan Avenue, Chicago, Illinois, 60604. Orchestra Hall is three blocks north of the Pick-Congress Hotel.

The American Ballet Theatre will perform on Thursday, Friday, and Saturday evenings at 8:30 p.m. in the Auditorium Theatre, which is next door to the Pick-Congress Hotel. Tickets are priced at $2.50, $3.50, $5.00, $7.00, $8.00, $9.00, and $10.00 and may be ordered from the Auditorium Theatre, 70 East Congress Parkway, Chicago, Illinois 60605.

The Chicago Circle Campus contains the original site of the Jane Addams’ Hull House and Residents’ Dining Room. Both have been restored by the University of Illinois and are designated as National Historic Landmarks by the U.S. Department of the Interior. They are open to visitors from 10:00 a.m. to 4:00 p.m.

Paul T. Bateman
Associate Secretary

Urbana, Illinois
The six hundred eighty-fourth meeting of the American Mathematical Society will be held at the Waldorf-Astoria in New York City on April 7–10, 1971.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, Dr. Ted E. Petrie of the Institute for Defense Analyses will present an hour address. The title of his talk and information on three additional hour speakers will be announced in the February issue of these Notices.

There will be sessions for ten-minute contributed papers in the morning and afternoon of both Friday and Saturday. Provisions will be made for late papers.

SYMPOSIUM ON MATHEMATICAL ASPECTS OF STATISTICAL MECHANICS ON APRIL 7–8

With the expected support of the National Science Foundation, there will be a symposium on Mathematical Aspects of Statistical Mechanics on April 7–8. The AMS-SIAM Committee on Applied Mathematics, Hirsh G. Cohen, Jim Douglas, Jr., Joaquin B. Diaz, William H. Reid, Richard S. Varga, and Calvin H. Wilcox (chairman), chose the topic of the symposium and appointed the Organizing Committee which consists of Mark Kac (pro-tem), O. E. Lanford III, James C. T. Pool (chairman), Robert T. Powers, and Seymour Sherman.

REGISTRATION

The registration desk will be open from 9:00 a.m. to 5:00 p.m. on Wednesday through Friday, April 7–9, and 9:00 a.m. to 3:00 p.m. on Saturday, April 10.

ACCOMMODATIONS

Persons intending to stay at the Waldorf-Astoria should make their own reservations with the hotel. A reservation blank and a list of room rates will be included in the February issue of these Notices.

MAIL ADDRESS

Registrants at the meeting may receive mail addressed in care of the American Mathematical Society, The Waldorf-Astoria, 301 Park Avenue, New York, New York 10022.

Walter Gottschalk
Associate Secretary
Middletown, Connecticut
1971 Summer Institute on Partial Differential Equations

The American Mathematical Society will hold its Eighteenth Annual Summer Research Institute at the University of California, Berkeley, California, for three weeks starting on August 9, 1971. The topic for the institute will be "Partial Differential Equations." It is anticipated that the institute will be supported by a grant from the National Science Foundation. The Organizing Committee is composed of Professors Alberto P. Calderón, Lars V. Hörmander, Charles E. Morrey, Jr., Louis Nirenberg (chairman), James B. Serrin, Isadore M. Singer, and Donald C. Spencer.

The program of the institute will consist of several series of lectures: Lars Hörmander will speak on "New developments in the theory of Fourier integral operators and pseudodifferential equations"; Isadore M. Singer on "Differential equations and differential geometry"; François Treves on "Local solvability and regularity for linear partial differential equations"; James B. Serrin on "Some problems in nonlinear differential equations"; and P. D. Lax on "Scattering theory." In addition, a series of seminars under the direction of invited chairmen will be held. These will cover linear and nonlinear equations and geometric problems, as well as mathematical physics, including topics such as minimal surfaces, regularity for linear and nonlinear problems, hyperfunction methods in functional problems, bifurcation theory, singular integral operators, scattering theory, and asymptotic methods of geometrical optics.

Funds for participant support will be limited. It is hoped that a number of participants will find their own sources of support. The institute is open to all mathematicians specializing in partial differential equations and to advanced graduate students in this field. Those wishing to participate are invited to write to Dr. Gordon L. Walker, Executive Director, American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904. Recent Ph. D.'s and advanced graduate students who wish to be considered for support should write before March 1, 1971.

**NEWS ITEMS AND ANNOUNCEMENTS**

**NINTH ANNUAL SYMPOSIUM ON BIOMATHEMATICS AND COMPUTER SCIENCE IN THE LIFE SCIENCES**

The Division of Continuing Education of The University of Texas Graduate School of Biomedical Sciences at Houston has announced the Ninth Annual Symposium on Biomathematics and Computer Science in the Life Sciences to be held in Houston, Texas, March 22–24, 1971. Topics for sessions of the symposium are Theoretical Biology, Mathematical Models of Biological Systems, Mathematical and Applied Statistics in Biomedical Research, Experimental Design, Bioengineering and Simulation, Hospital and Biomedical Information Management, Biomedical Computer Applications including Time Sharing, Data Acquisition and Display, and Computer Assisted Instruction in the Biomedical Sciences. Further information may be obtained by writing to the Office of the Dean, The University of Texas Graduate School of Biomedical Sciences at Houston, Division of Continuing Education, P. O. Box 20367, Houston, Texas 77025.

**OSCAR ZARISKI, ADDRESS CHANGE**

The address of the ex-President of the Society, Professor Oscar Zariski, was printed in error in the 1970–71 Combined Membership List. Please note that his correct address is Department of Mathematics, Harvard University, 2 Divinity Avenue, Cambridge, Massachusetts 02138.
ACTIVITIES OF OTHER ASSOCIATIONS

CONFERENCE BOARD
OF THE MATHEMATICAL SCIENCES

As a feature of the annual mathematics meeting January 21–25, 1971, in Atlantic City, New Jersey, a panel discussion on Operations Research and Mathematics will be sponsored by the Conference Board of the Mathematical Sciences. This will be held from 3:30 p.m. to 5:30 p.m. on Saturday, January 23, in the Vernon Room of Haddon Hall. The panel has been organized under the direction of Professor Donald L. Iglehart of the Department of Operations Research of Stanford University, who will also serve as moderator of the panel discussion. Panelists will include Professor George B. Dantzig, Department of Computer Science and the Department of Operations Research, Stanford University; Professor Julian Keilson, Department of Statistics, University of Rochester; Professor Thomas L. Saaty, Department of Statistics and Operations Research, University of Pennsylvania; and Dr. Philip Wolfe, Mathematical Sciences Department, Thomas J. Watson Research Center, International Business Machines Corporation. The panel discussion will be concerned with both undergraduate and graduate education in operations research and with the undergraduate mathematical training appropriate for graduate work in operations research. It is also planned to touch on such aspects of operations research as career opportunities, current research trends, professional practice, and the prospects for future developments in the field.

MATHEMATICAL ASSOCIATION OF AMERICA

The fifty-fourth annual meeting of the Mathematical Association of America will be held at the Chalfonte-Haddon Hall, Atlantic City, New Jersey, in conjunction with the meetings of the American Mathematical Society, the National Council of Teachers of Mathematics, and the Association for Symbolic Logic.

A complete program of the meeting is included in the time table in this issue of these Notices.

ASSOCIATION FOR SYMBOLIC LOGIC

The annual meeting of the Association for Symbolic Logic will be held at the Chalfonte-Haddon Hall, Atlantic City, New Jersey, in conjunction with the meetings of the American Mathematical Society, the Mathematical Association of America, and the National Council of Teachers of Mathematics.

A complete program of the meeting is included in the time table in this issue of these Notices.
MEMORANDA TO MEMBERS

GROUP LIFE INSURANCE PLAN

Under a group insurance plan, insurance will be available to AMS members in early January. The plan will provide $10,000 group term life insurance coverage, regardless of health, for members who have been regularly at work at least 30 hours a week for 30 days immediately prior to enrollment and who are under 50 years of age. Coverage for members 50 through 69 years of age will be subject to insurance company acceptance on the basis of health. Coverage for dependents in lesser amounts will also be available. Additional amounts of coverage of $10,000, $20,000, or $30,000 may be requested but will be subject to acceptance by the insurance company. All coverage is reduced by 50% at 65 years of age and terminates after 70, when a permanent individual insurance policy can be obtained without regard for health.

Residents of Ohio, Texas, and Wisconsin will be eligible for individual policies with comparable coverage in accordance with the insurance laws of those states. Texas law requires that its residents submit a health statement to the insurance company for review.

Under the same group policy, insurance will also be available to members of MAA and SIAM. An AMS member is limited, therefore, to $10,000, regardless of health, and a maximum of $30,000 additional coverage even if he or she also belongs to MAA or SIAM, or both.

Continental Assurance Company of Chicago, Illinois, will underwrite the group insurance plan. If there is adequate participation and beneficiary payments do not exceed predetermined amounts based on actuarial tables, surplus funds (referred to as experience credits) will be used to reduce premiums or increase benefits, or both. Experience credits will be used solely for the benefit of insured members, and no part of any credit will be paid to or used for the benefit of the Society. These credits cannot, of course, be guaranteed.

The administrator of the plan has managed successfully a group insurance program for the American Society of Civil Engineers for the past 20 years. In addition, insurance programs have been administered for the American Statistical Association, the Association for Computing Machinery, the Institute of Electrical and Electronics Engineers, Inc., and many other scientific and technical associations. The Society has been assured that at no time will members be approached through personal solicitation.

Under this plan, insurance will be offered to members at rates substantially lower than the cost of similar coverage on an individual basis; its success will depend upon the interest and support of the membership. While the Trustees have approved of this plan for AMS members, no expense will be incurred by the Society in developing or continuing it. Full information is being mailed to the membership by the administrator of the plan.

RECIROCITY AGREEMENT WITH ALLAHABAD MATHEMATICAL SOCIETY

The Society has recently entered into a reciprocity agreement with the Allahabad Mathematical Society. Dues in the Allahabad Mathematical Society are $2.50 annually or $50 for a life membership. Members will receive the Indian Journal of Mathematics which is published three times a year; back volumes are available to members at a 25% discount. AMS members who wish to join the Allahabad Mathematical Society should apply to Dr. S. R. Sinha, Secretary, Allahabad Mathematical Society, Lakshmi Niwas, George Town, Allahabad-2, India; dues may be paid to Dr. D. P. Gupta, Treasurer, at the same address.
DOCTORATES CONFERRED IN 1969-1970
Supplementary List

The following is supplementary to the list of doctorates conferred in 1969–1970 which appeared in the October 1970 issue of these Notices. The numbers appearing in parentheses after each university indicate the following: the first number is the total number of degrees listed for that institution; the next six numbers are the numbers of degrees in the categories of 1. Pure Mathematics, 2. Applied Mathematics, 3. Computer Science, 4. Statistics, 5. Mathematics Education, 6. Other.

CALIFORNIA
CALIFORNIA INSTITUTE OF TECHNOLOGY (3;0,3,0,0,0,0)
Department of Applied Mathematics
Delaney, Michael E.
I. Singular perturbation problems involving singular points and turning points
II. On the averaged Lagrangian technique for nonlinear dispersive waves
Ellison, James A.
Existence, uniqueness, and stability of solutions of a class of nonlinear partial differential equations

COLORADO
UNIVERSITY OF NORTHERN COLORADO (5;5,0,0,0,0,0)
Brink, Allen L.
A problem in periodic groups and some related results
Earls, Robert W.
Nets in partially ordered spaces
Kieft, Raymond
A survey of integration theory
McKinley, William
Function space topologies
Smith, Kenneth
Function space topologies

FLORIDA
UNIVERSITY OF FLORIDA (6;5,1,0,0,0,0)
Brooks, Burrow P.
Corings in the category of rings
Chae, Younki
Topological multigroups
Khuri, Andrawas I.
Applications of Papkovitch functions to three-dimensional thermo elastic problems
Maxwell, Stephen J.
Certain well-factored categories
Smith, Linda W.
Natural compactifications of lattices
Williams, Louis F.
Computable functions

ILLINOIS
UNIVERSITY OF ILLINOIS (1;1,0,0,0,0,0)
Trombi, Peter C.
Fourier analysis on semi-simple Lie groups whose split rank equals 1

LOUISIANA
LOUISIANA STATE UNIVERSITY (3;3,0,0,0,0,0)
Lewis, Daniel Ralph
On the Radon-Nikodym theorem for vector measures
Spaht, Carlos G., II
Generalized quotient rings
Stegall, Charles Patrick
Sufficiently Euclidean Banach spaces and fully nuclear operators

MINNESOTA
UNIVERSITY OF MINNESOTA (16;14,2,0,0,0,0)
Adams, David Randolph
Exceptional sets for Bessel potentials of functions in \( L_p \).
Barger, Samuel Floyd
Generic perfection and the theory of grade

Bridgman, George Henry
Some results on the Fourier-Wiener transform

Bushard, Louis Bernard
Qualitative and quantitative description of locking-in

Fava, Norberto Angel
Weak type inequalities for iterated operators

Gruman, Lawrence Michael
The distribution of zeros of entire functions of several complex variables and their asymptotic growth

Hendricks, Walter James
Processes with stable components

Ibrahim, Samiha Shaker Tadros
Mathematical connection between solutions of the Navier-Stokes system and the Prandtl boundary layer system

Langworthy, Harold Frederick
Imprimitivity in Lie groups

Lorentz, Rudolph Alexander Henry
Gentleness versus trace class

Owen, Willis Lysle
Optimal stopping rules when the variance is infinite

Park, Won Joon
Multi-parameter Gaussian processes

Petty, David
An extension of an experimental memory model

NEW YORK UNIVERSITY (2;0,2,0,0,0,0)
Center for Applied Mathematics

Fetz, Bruce
The linearized Boltzmann operator as the generator of a semi-group

Weinberg, Laurence
Asymptotic distribution of eigenvalues for the boundary value problem

Woodruff, David Leslie
Vector fields and differential on analytic spaces

Zimmerman, Grenith Johnson
Topics in multiparameter processes

NEW YORK CITY UNIVERSITY OF NEW YORK, GRADUATE CENTER (5;5,0,0,0,0,0)

Evenchick, Elinor
Concerning free mappings of the plane

Goldsmith, Eleanor
Some extensions and applications of Weyl's identity

Halpern, Fred
Deductions in continuous logic

Kalmanson, Kenneth
Classes of combinatorial extrema in certain metric spaces

Weisman, David
Nuclear spaces of almost periodic functions

NEW YORK UNIVERSITY (1;1,0,0,0,0,0)

Feuer, Richard D.
Torsion-free subgroups of Fuchsian groups

OHIO

CASE WESTERN RESERVE UNIVERSITY (4;0,1,0,0,3,0)

Department of Operations Research

Balinsky, Warren Larry
Some manpower planning models based on educational attainment

Friedel, Donald Charles
Deterministic & stochastic R&D resource allocation models

Hartman, James Kern
A primal method for linear programs with coupling rows and columns

Lev, Benjamin
Noniteration algorithm for solving special types of transportation problems

CANADA

UNIVERSITY OF BRITISH COLUMBIA (5;4,1,0,0,0,0)

Kapoor, Jagmohan
Matrices which, under row permutations, give specified values of certain matrix functions
Noussair, Ezzat S.
Comparison and oscillation theorems for elliptic equations and systems
Qureshi, Hilal Ahmed
Constrained Hartree-Fock wave functions for atoms
Tam Ping-kwan
Inequivalence and equivalence of certain kinds of non-normal operators
Tan Kok-keong
Some fixed point theorems for non-expansive mappings in Hausdorff locally convex spaces

UNIVERSITY OF WATERLOO (2;0,0,1,0,1)

Department of Combinatorics and Optimization
Lawless, J. F.
Quasi-residual balance incomplete block designs
Robertson, Neil
Graphs minimal under girth, valency, and connectivity constraints

ERRATA

CARNEGIE-MELLON UNIVERSITY
(1;0,0,1,0,0,0)

Department of Computer Science
McCreight, Edward M.
Classes of computable functions defined by bounds on computation

Dr. McCreight was previously listed as receiving his degree from the University of Texas at Austin.

UNIVERSITY OF SASKATCHEWAN, SASKATOON (1;1,0,0,0,0,0)

To Ting-on
The Hahn-Banach type extension properties in ordered linear spaces and seminormed linear spaces

Dr. To was previously listed as receiving his degree from the University of Saskatchewan, Regina.

NEWS ITEMS AND ANNOUNCEMENTS

SUMMER SEMINAR IN NONLINEAR EIGENVALUE PROBLEMS

A Summer Seminar in Nonlinear Eigenvalue Problems will be held in Santa Fe, New Mexico, from June 14 to July 9, 1971. The seminar is being sponsored by the Rocky Mountain Mathematics Consortium, and it is expected that support will be provided by the National Science Foundation under the Advanced Science Seminar Program. A series of lectures will be presented by Felix E. Browder of the University of Chicago, Duane Sather of the University of Colorado, Paul Rabinowitz of the University of Wisconsin, Klaus Kirchgassner of the University of Bochum, and others to be announced at a later date. It is expected that funds will be available for transportation and support of participants on the pre-doctoral, postdoctoral, and senior levels. The seminar will stress both sophisticated mathematical tools and applied aspects of the theory of nonlinear eigenvalue problems. For further information and application forms, please write to Professor Paul Fife, Department of Mathematics, University of Arizona, Tucson, Arizona 85721.
VISITING MATHEMATICIANS
Supplementary List

The following is a supplement to the lists of visiting mathematicians printed in the August, October, and November issues of these (ûûûûûû). There are two lists: the first contains the names of American mathematicians visiting abroad during 1970–1971, and the second list contains the names of foreign mathematicians visiting in the United States and Canada.

American Mathematicians Visiting Abroad

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<thead>
<tr>
<th>Name and Home Country</th>
<th>Host Institution</th>
<th>Field of Special Interest</th>
<th>Period of Visit</th>
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<tbody>
<tr>
<td>Cortes, Heinz O. (U.S.A.)</td>
<td>University of Lund, Sweden</td>
<td></td>
<td>9/70 – 6/71</td>
</tr>
<tr>
<td>Hsiang, Wu-Yi (U.S.A.)</td>
<td>National Taiwan University</td>
<td></td>
<td>9/70 – 3/71</td>
</tr>
<tr>
<td>Hsiang, Wu-Yi (U.S.A.)</td>
<td>Bonn University</td>
<td></td>
<td>3/71 – 6/71</td>
</tr>
<tr>
<td>Lawson, H. Blaine (U.S.A.)</td>
<td>Institute for Pure and Applied Mathematics, Brazil</td>
<td></td>
<td>9/70 – 6/71</td>
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</table>

Foreign Mathematicians Visiting in the United States and Canada

<table>
<thead>
<tr>
<th>Name and Home Country</th>
<th>Host Institution</th>
<th>Field of Special Interest</th>
<th>Period of Visit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Almkvist, Gert (Sweden)</td>
<td>University of California, Berkeley</td>
<td></td>
<td>9/70 – 6/71</td>
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<tr>
<td>Dold, Albrecht (F.R. of Germany)</td>
<td>University of Washington</td>
<td></td>
<td>1/71 – 3/71</td>
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<tr>
<td>Doplicher, Sergio (Italy)</td>
<td>New York University, Courant Institute</td>
<td>Quantum Theory</td>
<td>2/71 – 6/71</td>
</tr>
<tr>
<td>Hörmander, Lars (Sweden)</td>
<td>New York University, Courant Institute</td>
<td>Partial Differential Equations</td>
<td>9/70 – 6/71</td>
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<tr>
<td>Kuo, Hui Hsuing (China)</td>
<td>New York University, Courant Institute</td>
<td>Functional Analysis</td>
<td>9/70 – 6/71</td>
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<tr>
<td>Leslie, Joshua (Nigeria)</td>
<td>University of California, Berkeley</td>
<td></td>
<td>9/70 – 6/71</td>
</tr>
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<td>Ohy, Yujiro (Japan)</td>
<td>New York University, Courant Institute</td>
<td>Partial Differential Equations</td>
<td>9/70 – 6/71</td>
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<tr>
<td>Osterwalder, Konrad (Switzerland)</td>
<td>New York University, Courant Institute</td>
<td>Field Theory</td>
<td>9/70 – 6/71</td>
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<tr>
<td>Parry, William (England)</td>
<td>University of California, Berkeley</td>
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<td>1/71 – 6/71</td>
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<td>Pefeyński, Aleksander (Poland)</td>
<td>University of California, Berkeley</td>
<td></td>
<td>3/71 – 6/71</td>
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<td>Sleeman, Brian D. (England)</td>
<td>New York University, Courant Institute</td>
<td>Fluid Dynamics</td>
<td>9/70 – 6/71</td>
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<tr>
<td>Sobczyk, Kazimierz (Poland)</td>
<td>New York University, Courant Institute</td>
<td>Mechanics and Probability</td>
<td>9/70 – 2/71</td>
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<tr>
<td>Stampacchia, Guido (Italy)</td>
<td>University of California, Berkeley</td>
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<td>1/71 – 3/71</td>
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<tr>
<td>Thompson, Mark (England)</td>
<td>New York University, Courant Institute</td>
<td>Perturbation Theory</td>
<td>9/70 – 6/71</td>
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</table>
The CBMS Regional Conference Series in Mathematics is a new series of monographs comprised of expository lectures presented at regional conferences sponsored by the Conference Board of the Mathematical Sciences with support from the National Science Foundation. The CBMS Regional Conferences feature a principal guest speaker who is required to develop his approximately ten lectures into a substantial expository paper. This series of monographs is the result. Standing orders for all forthcoming volumes in the series, as well as individual orders, may be placed with the American Mathematical Society, a 25% reduction being offered to all individuals and book dealers. Copies may also be ordered from the Conference Board of the Mathematical Sciences, 834 Joseph Henry Building, 2100 Pennsylvania Avenue, N.W., Washington, D.C. 20037.

Number 1, "Algebraic and analytic aspects of operator algebras" by IRVING KAPLANSKY (24 pages; list price $2.20; individual price $1.65), contains notes on the lectures presented at a conference held at the University of Hawaii on June 9–13, 1969. The theme is $C^*$-algebras, together with their numerous alphabetical progeny ($W^*$, $AW^*$, CCR, GCR). However, where appropriate, attention is paid to wider classes of Banach algebras, and the interplay between algebras and analysis receives recognition by occasional forays into pure ring theory. It fittingly begins with a simple characterization of von Neumann regular rings which promises to be quite useful.

Number 2, "Lecture notes on nilpotent groups" by GILBERT BAUMSLAG (76 pages; list price $3.10; individual price $2.33), is substantially the same as the notes prepared as an aid to the ten lectures given at the University of Texas on May 26–30, 1969. These lectures are concerned, in the main, with finitely generated nilpotent groups. The theory of these groups is rich and exciting. There seem to be three main parts to this theory. The first of these deals with the so-called commutator calculus, which was initiated by Philip Hall; the second aspect of the theory is, in a sense, governed by a single principle which may be likened to a well-known procedure in elementary number theory where one shows that a proposition about the integers holds modulo each prime $p$ and thence for the integers themselves; the third part of nilpotent group theory stems in the main from the connection between Lie groups and Lie algebras which was first discussed by A. I. Mal'cev.

Number 3, "Lectures in differentiable dynamics" by LAWRENCE MARKUS (52 pages; list price $2.70; individual price $2.03), is based on the notes of the lectures presented at Case Western Reserve University on June 2–6, 1969. The author discusses the history of differentiable dynamics; differentiable dynamics, definitions, and examples; basic problems of differentiable dynamics; comparison and contrast of topological and differentiable dynamics; elementary concepts of topological dynamics; contrast of topological and differentiable system theory; Morse-Smale, Anosov hyperbolic and generic dynamical systems; and the principal results of differentiable dynamics.

Number 4, "Twisted honeycombs" by H. S. M. COXETER (52 pages; list price $2.70; individual price $2.03), is based on lectures presented on August 18–27, 1969, at the University of Maine. The subject matter is described as follows: A honeycomb is defined as a symmetrical subdivision of a three-dimensional manifold into a number of polyhedral cells, all alike, each rotation that is a symmetry operation of a cell being also a symmetry operation of the whole configuration. The honeycomb is said to be twisted if it is not symmetrical by reflection; thus a twisted honeycomb, like a
screw, occurs in right-handed and left-handed varieties. The subject originated in 1933, when Weber and Seifert considered two one-celled honeycombs, each consisting of a regular dodecahedron with opposite faces identified. The work involves a combination of geometry and group theory, assisted by quaternions.

Number 5, "Recent advances in homotopy theory" by GEORGE W. WHITEHEAD (88 pages; list price $3.20; individual price $2.40), is based on lectures delivered at the New Mexico State University on December 27–31, 1969. Homotopy theory, which has experienced rapid development in the last twenty-five years, has had a profound effect on other branches of topology. The observation in its early history that many phenomena became much more regular in the "stable range" has led to a particularly rapid development of stable homotopy theory as a subject in its own right. This monograph is intended to describe some of the advances, particularly in the stable theory.

Number 6, "Lectures on the edge-of-the-wedge theorem" by WALTER RUDIN (36 pages; list price $2.40; individual price $1.80), contains the notes of lectures given at the University of Missouri, St. Louis, on June 1–5, 1970. The edge-of-the-wedge theorem deals with a question about analytic continuation of holomorphic functions of several complex variables. The problem arose in physics in connection with quantum field theory and dispersion relations, and the theorem was first proved by Bogolyubov in this connection. The present paper gives complete and self-contained proofs of several versions of this theorem and also contains some applications to function theory in a polydisc.

Number 7, "Holomorphic vector fields on compact Kähler manifolds" by YÔZO MATSUSHIMA (40 pages; list price $2.50; individual price $1.88), is the result of the lectures presented at Michigan State University, June 15–19, 1970. The volume contains seventeen chapters entitled Kähler geometry; Harmonic forms; The 1-form of type (0, 1) corresponding to a holomorphic vector field; Laplacian $\Delta'_i$; An integral formula; The case $C_1(M) \leq 0$; The case of $C_1 \geq 0$; Study of $a_i$; Theorems of Lichnerowicz; A remark on holomorphic vector fields on projective algebraic manifolds; The Albanese variety of a Kähler manifold and the Jacobi map; The case of Hodge manifold; G-sheaves; The action of $\text{Aut}_0(M)$ on complex line bundles over $M$; The Lie derivative of a complex line bundle; The kernel of the homomorphism $p_F$; Proof of Blanchard Theorem.
NEW AMS PUBLICATIONS

INDEX TO SOVIET MATHEMATICS — DOKLADY
List Price $8.00; Member Price $6.00

The Index to Soviet Mathematics — Doklady outlines the contents of the issues of Soviet Mathematics — Doklady published during the ten years from 1960 through 1969. It includes titles of all articles by a given author, Russian topic and page numbers, volume and page numbers of the AMS English translations, and Mathematical Reviews numbers. It is arranged alphabetically by author, contains approximately 130 pages, and is bound in a soft cover. The Index will be sold to Doklady subscribers as a separate item, independently of the annual subscriptions to the journal.

TRANSLATIONS OF MATHEMATICAL MONOGRAPHS

Volume 28
THE FUNCTIONAL METHOD AND ITS APPLICATIONS
By E. V. Voronovskaja
208 pages; List Price $17.80; Member Price $13.25

This book can be used as a textbook for graduate students of mathematics as well as for those preparing for examinations in the theory of functions, and in particular in the theory of best uniform approximation.

The material presented here may also be useful for engineers and students of radio and electrical engineering who are concerned with problems of the theory of filters, amplifiers, pulse technology, etc. To make it easier for engineers to read, the book contains a brief introductory section on the theory of moment sequences and the simplest theorems of functional analysis.

The first part of the book contains an exposition of the new theory which was announced in a series of papers by the author between 1934 and 1953; a detailed exposition has not been published previously. The second part illustrates the theory by solving a number of problems of Chebyshev type which could not be handled by classical methods.

MEMOIRS OF THE AMERICAN MATHEMATICAL SOCIETY

Number 104
OCTONION PLANES DEFINED BY QUADRATIC JORDAN ALGEBRAS
By John R. Faulkner
76 pages; List Price $1.50; Member Price $1.13

The purpose of this Memoir is to derive the theory of octonion (Cayley, Octave, Moufang) planes in a uniform fashion, valid for all characteristics and for both split and division octonion algebras, by using an exceptional quadratic Jordan algebra. In particular, it extends the previous theory to characteristic two. Among the subjects treated are norm semisimilarities of an exceptional quadratic Jordan algebra, an isomorphism of a spin group with a subgroup of norm preserving maps, the "fundamental theorem" for octonion planes, the harmonic point theorem, simplicity and isomorphisms of the "little projective group," automorphisms of order two of an octonion algebra in characteristic two, unitary groups of collineations commuting with a polarity, and the simplicity of the automorphism group of the exceptional quadratic Jordan algebra in characteristic two.

Number 105
COMPACTLY COVERED REFLECTIONS, EXTENSION OF UNIFORM DUALITIES AND GENERALIZED ALMOST PERIODICITY
By Michael H. Powell
244 pages; List Price $2.70; Member Price $2.03

This Memoir begins by proving that, in certain categories for whose objects there is an underlying uniform space
equipped with a directed covering, the compactly covered objects form a reflective subcategory. The theorem is general enough to prove the existence of Bohr compactifications and weakly almost periodic compactifications of groups and monoids as a special case, along with many others. The main object of the rest of the Memoir is to develop those tools which are necessary to give concrete constructions of reflections slightly more general than the special cases mentioned above and to elaborate on the relations between and properties of the various reflections treated.

AMS TRANSLATIONS—SERIES 2

Volume 91

EIGHTEEN PAPERS ON ANALYSIS AND QUANTUM MECHANICS

316 pages; List Price $15.80; Member Price $12.25

This volume in the AMS Translations series contains the following papers:

"Some properties of 2-increasing functions" by Ju. L. Šmul'jan;
"Indecomposable n-increasing functions" by Ju. L. Šmul'jan;
"Approximation of holomorphic functions by polynomials in a locally compact non-archimedeanly normed field" by M. Š. Stavskii; 
"On the approximation of functions in arbitrary norms" by K. K. Golovkin; 
"Imbedding theorems for fractional spaces" by K. K. Golovkin; 
"On saturation classes of linear operators in $L_p(-\infty, \infty)$" by R. G. Mamedov; 
"On the divergence of interpolatopn processes at a fixed point" by A. A. Privalov; 
"Properties of functions related to their rate of approximability by rational fractions" by A. A. Gončar; 
"On the variation of a function and its Fourier coefficients in the Haar and Schauder systems" by V. A. Matveev; 
"On norms of trigonometric polynomials and approximation of differentiable functions by linear averages of their Fourier series" by S.A. Teljakovskii; 
"On the existence of a solution of a class of systems of nonlinear integral equations" by L. I. Barklon; 
"Investigation of a nonlinear singular equation with Hilbert kernel" by H. Š. Muhtarov; 
"A method for the approximation of unbounded solutions of a system of quasilinear Volterra integral equations" by V. R. Vinokorov; 
"A direct method for solving integral equations" by B. G. Gabdulhaev; 
"On the construction of an S-matrix in accordance with the theory of perturbations" by B. M. Stepanov; 
"Selfadjointness of field operators and the moment problem" by V. P. Gačok; 
"On the spectrum of the energy operator for atoms with fixed nuclei on subspaces corresponding to irreducible representations of a group of permutations" by G. M. Žislín and A. G. Sigalov; and 
"On various mathematical problems in the theory of atomic spectra" by G. M. Žislín and A. G. Sigalov.

Volume 95

ELEVEN PAPERS IN ANALYSIS

256 pages; List Price $12.80; Member Price $9.60

This volume in the AMS Translations series contains the following papers:

"On the theory of fields of direction cones" by E. E. Viktorovskii; 
"A theory of integral funnels for dynamical systems without uniqueness" by M.I. Minkevič; 
"On the boundedness of solutions of a linear homogeneous equation of second order in Hilbert and Banach spaces" by K.S. Mamiľ; 
"On the spectrum of a linear differential operator of second order in a Hilbert space of vector functions with values in an abstract Hilbert space, I and II" by F. Z. Ziatdinov; 
"On unitary couplings of semi-unitary operators" by V. M. Adamjan and D. Z. Arov; 
"Parabolic convolution equations in a bounded region" by M. I. Višik and G. I. Eskin; 
"Summation methods for generalized Fourier series" by V. A. Marčenko; 
"On sets of uniqueness" by S. B. Steckin and P. L. Ul'janov; 
"The description of all solutions of the truncated power moment problem and some problems of operator theory" by M. G. Kreĭn; and 
"Approximate calculation of determinants" by D. F. Davidenko.

Volume 96

TEN PAPERS ON ALGEBRA AND FUNCTIONAL ANALYSIS

264 pages; List Price $13.20; Member Price $9.90

This volume in the AMS Translations series contains the following papers:
"Basic problems in the theory of projections of semi-lattices" by L. N. Ševrin; "Projection of lattices" by N. D. Filippov; "Partial idempotent operations associated with ordered sets" by V. V. Rozen; "Ordered algebraic systems" (two papers) by A. A. Vinogradov; "O-rings and L.A-rings" by B. M. Šaǐn; "On Čebyšev and almost Čebyšev subspaces" by A. L. Garkavi; "Almost Čebyšev systems of continuous functions" by A. L. Garkavi; "On multiplicative representations of J-nonexpansive operator-functions. I and II" (two papers) by Ju. P. Ginzburg.

Volume 97

ELEVEN PAPERS ON ALGEBRA, ANALYSIS AND TOPOLOGY

264 pages; List Price $13.70; Member Price $10.38

This volume in the AMS Translations series contains the following papers:

"Abelian varieties over a field of finite characteristic" by Ju. I. Manin; "On neutral polyverbal operations" by O. N. Macedonskaja; "On the removal of singularities of analytic functions" by E. P. Dolženko; "Entire functions bounded in a locally compact nonarchimedeanly normed field" by M. S. Stavskil; "On a generalization of the theorems of Wiener-Levy type of M. G. Krein" by I. C. Gohberg; "Fundamental aspects of the representation theory of Hermitian operators with deficiency index (m,m)" by M. G. Krein; "J-majorizing and modular operators in J-spaces" by Ju. L. Šmul'jan; "A classification of fixed points" by A. N. Šarkovskil; "The concept of motion in a generalized dynamical system" by B. M. Budak; "On dynamical systems without uniqueness as semigroups of non-single-valued mappings of a topological space" by I. U. Bronšteǐn; "Behavior of a mapping in the neighborhood of an attracting set" by A. N. Šarkovskil.
The history of science is full of amoebic splits; the philosopher of long ago became the natural philosopher of not so long ago, and he became the physicist of yesterday, who, in turn, became the quantum field theorist of today specializing in von Neumann algebras. Mathematics has been splitting too, and perhaps, before long, universities will have departments of algebra and offer Ph. D.'s in differential topology—disciplines whose scholars will see one another at meetings of the Faculty Senate only. Such splits may be regrettable, but they are facts of life, and, once they happen, we have to live with them.

Nevertheless, one of the greatest sources of the beauty and the utility of mathematics, despite the expansion and fragmentation that it has been undergoing, is its remarkable unity. A differential geometer must nowadays be an expert in algebra, an algebraist would be crippled without an understanding of notions whose motivation and techniques come from topology, and the modern analyst uses all the algebra and topology he can understand and wishes he could understand more.

Complex function theory is an old part of mathematics, a deep one, and one whose full implications and applications are still very far from completely understood. I wish I understood and could use more of it. My best students learned more of it than I ever did, and their work on functional analysis is, for that reason, universally considered to be penetrating and important. My own current work establishes a connection between Banach algebras (pure functional analysis) and transfinite diameter (pure complex function theory). The fashionable direction today is the theory of functions of several complex variables, a difficult and highly ramified descendant of the complex function theory of our predecessors, but one that couldn't exist and flourish without its classical version. And, incidentally, the ramifications of modern multivariable complex function theory connect it with topology and homological algebra and algebraic geometry as well; no part of this beautiful growth could live if another part were brutally excised.

I was moved to say these things by your letter in the August Notices; I wanted to stand up and be counted.

Sincerely yours,
Paul Halmos (signed)

Fred M. Wright
During the past two academic years, I enjoyed an appointment as visiting assistant professor at a university in Asia. On my way back to Canada this summer, I had the pleasure of paying a short social call at a second Asian university. In both cases I was very impressed by the agreeable working conditions and excellent library facilities.

The fact that I was surprised to discover such fine amenities leads me to suspect that many young North American and European mathematicians may be ill informed about the teaching and research opportunities on other continents. By finding out more about these opportunities and by seeking employment in Asia, Africa, and South America we could contribute valuable communication ties to the global mathematical community.

My own contacts developed through friends from graduate school. But a person interested in initiating correspondence about a position abroad could also draw on several library sources. The World of Learning (Europa Publications, 1970) and the International Handbook of Universities and Other Institutions of Higher Education (The International Association of Universities, fourth edition, 1968) both give a list of universities by country together with such basic information as the languages of instruction.

It also occurs to me that strategically located mathematicians from the Combined Membership List might not mind answering a few letters of serious enquiry. But we should certainly spare them the task of supplying details which are readily available through the Embassies.

J. B. Wilker

I want to add my support to the cogent arguments presented by Daniel H. Wagner and Oscar Zariski (Notices), October, 1970) concerning the dangers involved in the Society's speaking with one voice on political and social issues. Though I have strong personal views on many such issues, I am appalled at the thought of impressing my views on other Society members or having views of others attributed to me because I happen to be a Society member.

If such coercion should become a part of the Society's function, I would feel compelled to resign. Multiply that decision by not too many and the Society's ability to "facilitate free mathematical exchange" would most definitely be impaired.

Stephen S. Boyce

I should like to comment on the logic which is being used to support resolution B of the Society's recent referendum. First of all, the resolution itself is not at all "neutral" but rather a powerful endorsement of the status quo and those with an interest in maintaining it. Indeed the canard "Science is neutral" is precisely what the latter would like us to believe; and the implications of this are clear enough—we are technicians who should not meddle in the affairs of state but rather serve it unquestioningly. This logic and variations of it has, as we all know, been instrumental in all the major crimes against humanity in this century. Zariski, making comparisons with prewar Europe, claimed that no state of emergency exists. Disregarding the fact that his personal fortunes have undoubtedly improved, I can say, as one who experienced those same conditions, that this is simply false. All the phenomena which were present then are showing again now in a potentially much more dangerous form, disguised only by a certain level of economic affluence which may turn out to be evanescent. Zariski's answer to this is—let us all put our heads in the sand. However, from the fact that almost every learned society is having to face this issue, it is evident that there must be other alternatives.

M. C. Goodall
Editor, the Notices

It would be helpful to authors at institutions with financial problems if all journals printed a positive indication of their policy regarding page charges, e.g., "No page charges are assessed."

Colin C. Graham

Editor, the Notices

On page 890 of the Notices for October 1970, there is an announcement of an International Conference on the teaching of mathematics to be held by the Comprehensive School Mathematics Program in Carbondale, Illinois. The letter lists a number of mathematicians who "are being invited to participate." One of these names is mine. I was indeed invited last summer to participate. In August I wrote that I would not be able to do so. Under these circumstances it seems strange that my name should still be listed in this announcement in October.

There is another more significant point. The invitation from CSMP was accompanied by some materials used in their "Elements of Mathematics" program. On examination, these materials turned out to involve an extraordinary overemphasis on logic and formality. In my letter to CSMP in August, I called attention to this. My letter was acknowledged by CSMP without any comment on this substantive question and with no indication that this aspect of the EM program would be reexamined.

For these reasons I now make a public request that CSMP as a matter of first priority reexamine the formalistic direction of their EM program.

Saunders Mac Lane

Editor, the Notices

In view of the continuing efforts of some members to have the Society take a corporate position on a number of issues which many members do not consider to be of "direct professional concern" (Letters to the Editor, the Notices, October 1970), I make the following suggestion: that beginning with the next election the Council secure from each individual whose name will appear on the ballot an answer to the question "Does Resolution B have your unqualified support?" If his answer is in the negative, Council should further secure a brief statement of his position on Resolution B. This information should then be furnished to each member of the Society at the time he receives his ballot (or in advance thereof.)

Peter R. Weidner

EDITOR'S NOTE

In the October 1970 issue of these Notices, there appeared a letter from Mr. William B. Klein, mathematics editor for Barnes and Noble, Inc., suggesting a change in the present "limit" notation. Three mathematicians, Crosman Jay Clark, Kenneth O. May, and J. E. Kelley, have responded with suggestions carrying mathematical content as well as proposing notational improvement. They emphasize that the "x" in the usual notation \( \lim_{x \to a} f(x) = c \) is a dummy variable, is irrelevant, and may even interfere with the understanding of a concept. Each is proposing in some way to emphasize "lim" as a functional in the notation. For instance, corresponding to \( f \) and \( a \) there may exist \( \lim f(a) = c \). Alternatively, corresponding to \( f \) there is defined on an appropriate domain a function \( \lim f \), whose value \( c \) at \( a \) is then written \( \lim f(a) = c \).
Dean A. ADRIAN ALBERT of the University of Chicago has been elected vice president of the International Mathematical Union for a four-year term beginning January 1, 1971.

Mr. YALE ALTMAN of Blaisdell Publishing Company has been appointed mathematics and science editor of the Massachusetts Institute of Technology Press.

Dr. S. ANASTASIO of Fordham University has been appointed to an associate professorship at the State University of New York, College at New Paltz.

Professor MICHAEL ANSHEL of the University of Arizona has been appointed to an assistant professorship at City College (CUNY).

Dr. HARVEY J. ARNOLD of Oakland University has been named Acting Chairman of the Department of Mathematics.

Professor GEORGE BERGMAN will be on leave from the University of California, Berkeley, for the academic year 1970-1971. He will spend the year at Harvard University.

Professor ELIZABETH A. Berman of the University of Missouri at Kansas City has been appointed to an assistant professorship at Rockhurst College.

Dr. ISRAEL BERSTEIN of Cornell University has been appointed to a visiting professorship at SUNY at Buffalo.

Professor BARNARD H. BISSINGER of Lebanon Valley College has been named director of the Division of Mathematical Sciences at the Capitol Campus of Pennsylvania State University in Middletown.

Dr. STEPHEN L. BLOOM of Maplewood, New Jersey, has been appointed to an associate professorship at Stevens Institute of Technology.

Professor JAMES J. BOWE of the University of Kentucky has been appointed to an associate professorship at Madison College.

Mr. BEN-AMI BRAUN of Purdue University has been appointed to an assistant professorship at the University of South Florida.

Professor ALLEN U. BRENDER of Illinois Institute of Technology has been appointed to an assistant professorship at the University of Arizona.

Dr. LOUIS B. BUSHARD of the University of Minnesota has been appointed to an assistant professorship at North Dakota State University.

Professor KIM KI-HANG BUTLER of St. Mary's College of Maryland has been appointed to an associate professorship at Pembroke State University.

Professor HOWARD H. CAMPAIGNE of the Department of Defense has been appointed to a professorship at Slippery Rock State College.

Professor B. F. CARINESS of Duke University has been appointed to an assistant professorship at the University of Wisconsin.

Professor ROBERT CARMIGNANI of Rice University has been appointed to an assistant professorship at the University of Missouri at Columbia.

Dr. S. D. CHATTERJI of the University of Copenhagen has been appointed to a professorship at the École Polytechnique Fédérale de Lausanne.

Professor YUH-CHING CHEN of Wesleyan University has been appointed to an assistant professorship at Fordham University.

Professor FRANK A. CHIMENTI of Western Michigan University has been appointed to an assistant professorship at the State University of New York, College at Fredonia.

Professor WAI-MEE CHING of Louisiana State University in Baton Rouge has been appointed to an assistant professorship at Fordham University.

Professor KWANG-NAN CHOW of the California Institute of Technology has been appointed to an assistant professorship at San Fernando Valley State College.

Professor HARVEY COHN of the University of Arizona has been elected to
membership in the Institute of Advanced Study. He will be in residence there during the academic year 1970-1971, and then will return to the University of Arizona.

Dr. HUGH COOMES of SUNY at Albany has been appointed to an assistant professorship at Paterson State College.

Professor LAWRENCE A. COUVIL-LON of Louisiana State University has been appointed to an assistant professorship at Florida State University.

Professor PETER CSONTOS of the University of California, Berkeley, has been appointed to an assistant professorship at California State College at Hayward.

Professor EDWARD B. CURTIS of the Massachusetts Institute of Technology has been appointed to an associate professorship at the University of Washington.

Professor JAMES A. DEDDENS of the University of Michigan has been appointed to an assistant professorship at the University of Kansas.

Dr. JAMES DERR of the University of New Mexico has been appointed to an assistant professorship at West Virginia University.

Dr. MILOS DOSTAL of Montreal, Canada, has been appointed to an associate professorship at Stevens Institute of Technology.

Professor LESTER E. DUBINS of the University of California, Berkeley, will be on sabbatical leave for the academic year 1970-1971.

Professor THOMAS A. W. DWYER III of the University of Maryland has been appointed to an assistant professorship at Northern Illinois University.

Dr. PATRICK EBERLEIN of the University of California, Los Angeles, has been appointed a lecturer at the University of California, Berkeley.

Professor ROBERT D. EDWARDS of Princeton University has been appointed to an assistant professorship at the University of California, Los Angeles.

Professor DELBERT R. FULKER-SON of the Rand Corporation, Santa Monica, California, has been appointed to a visiting professorship at the University of Waterloo.

Professor STEPHEN J. GARLAND, on leave from Dartmouth College, has been appointed to a visiting assistant professorship at the University of California, Los Angeles.

Dr. FRANK GLASER of the University of California, Riverside, has been appointed a lecturer at California State Polytechnic College in Pomona.

Professor LEO A. GOODMAN of the University of Chicago has been appointed to the Charles L. Hutchinson Distinguished Service Professorship in Statistics and Sociology at the University of Chicago.

Mr. ELLIOT C. GOOTMAN of the Massachusetts Institute of Technology has been appointed to an assistant professorship at the University of Georgia.

Dr. WILLIAM J. GORDON of General Motors Research Laboratories is on leave for the academic year 1970-1971. He has been appointed to a visiting associate professorship at the University of Utah for the first semester. For the second semester, he has been appointed to an associate professorship at Syracuse University.

Dr. SEYMOUR HABER of the National Bureau of Standards has been appointed to a visiting professorship at the University of Maryland, Baltimore County.

Dr. DAVID C. HADDAD of Purdue University has been appointed to an assistant professorship at West Virginia University.

Professor ALFRED HALES of the University of California, Los Angeles, has been appointed a visiting lecturer at the University of Washington.

Professor MORRIS L. HAMILTON of St. Andrews Presbytery College has been appointed to an assistant professorship at Pembroke State University.

Professor FRANK HARARY of the University of Michigan has been appointed to a visiting professorship at the University of Waterloo.

Professor LAWRENCE H. HARPER of Rockefeller University has been appointed to an associate professorship at the University of California, Riverside.

Professor DONALD HARTIG of the University of California, Santa Barbara, has been appointed to an assistant professorship at Ohio University.

Dr. STUART HASTINGS of Case Western Reserve University has been appointed to an associate professorship at SUNY at Buffalo.

Dr. JON HELTON of the University of Texas has been appointed a postdoctoral fellow at West Virginia University.
Professor HERBERT K. HEYER of the University of Erlangen has been appointed to a professorship at the University of Tübingen.

Mr. PETER J. HILTON, on leave from Cornell University, has been appointed a visiting fellow at Battelle Memorial Institute, Battelle Seattle Research Center.

Professor NEIL B. HINDMAN of Wesleyan University has been appointed to an assistant professorship at California State College at Los Angeles.

Dr. CHUNG-WU HO of the Massachusetts Institute of Technology has been appointed to an assistant professorship at Southern Illinois University in Edwardsville.

Professor BING FUN IP of Wayne State University has been appointed to an assistant professorship at the University of Bridgeport.

Dr. RODNEY JOHNSON of Syracuse University has been appointed to an assistant professorship at Fordham University.

Dr. LOWELL JONES of Yale University has been appointed a lecturer at the University of California, Berkeley.

Professor SOJI KANEYUKI of Nagoya University has been appointed to a visiting professorship at Scuola Normale Superiore, Pisa, Italy.

Professor HARVEY B. KEYNES of the University of Minnesota has been appointed to a visiting associate professorship at the University of Maryland.

Professor AMY C. KING of the University of Kentucky has been appointed to an assistant professorship at Eastern Kentucky University.

Professor SHOSICHI KOBA YASHI of the University of California, Berkeley, will be on sabbatical leave during the fall quarter of 1970. He will spend the quarter at the Massachusetts Institute of Technology.

Dr. JERALD J. KOVACIC of Princeton University has been appointed to an assistant professorship at Fordham University.

Professor WILLIAM D. LAVERELL of Temple University has been appointed to an assistant professorship at Purdue University, Indianapolis Campus.

Professor H. BLAINE LAWSON of the University of California, Berkeley, has been named a Sloan Foundation Research Fellow at the University of California, Berkeley.

Professor FRANCIS R. S. LEE of Northern Michigan University has been appointed to an assistant professorship at the University of Massachusetts.

Professor PENG-YEE LEE of the University of Auckland has been appointed to an associate professorship at Nanyang University, Singapore.

Dr. EUGENE H. LEHMAN of Northern Michigan University has been appointed to a professorship at the University of Quebec.

Professor DERRICK H. LEHMER of the University of California, Berkeley, will be on sabbatical leave during the winter quarter of 1970. He will spend the quarter in research at the University of Arizona.

Professor LEO M. LEVINE of New York University has been appointed to an associate professorship at Queensborough Community College.

Mr. WILLIAM K. LINK, JR., of the University of North Carolina has been appointed information systems designer at Western Electric Company.

Professor GARY E. LIPPMAN of the University of California, Riverside, has been appointed to an assistant professorship at Kenyon College.

Professor WILLIAM D. McINTOSH of the University of Missouri at Columbia has been appointed to a professorship and to the chairmanship of the Department of Mathematics at Central Methodist College.

Professor RALPH McKENZIE will be on leave from the University of California, Berkeley, for the academic year 1970-1971. He will spend the year at the University of Colorado.

Dr. VED P. MADAN of the University of Alberta has been appointed to an instructorship at Red Deer College.

Professor MARK E. MAHOWALD of Northwestern University has been appointed to a visiting professorship at the University of Washington.

Professor HENRY B. MANN of the University of Wisconsin has been appointed to a professorship at the University of Arizona.

Dr. ALBERT W. MARSHALL of the Boeing Scientific Research Laboratories has been appointed to a visiting professor-
ship at the University of Washington.

Mr. ISAAC S. METTS of Vanderbilt University has been appointed a statistician at the Walter Reed Army Institute of Research.

Professor GEORGE J. MINTY of Indiana University has been appointed to a visiting professorship at the University of California, Berkeley, for the spring quarter of 1971.

Mr. FRED R. MONACO of Del Mar Engineering Laboratories has been appointed a cost analyst for Associated Products, Beverly Hills, California.

Professor M. SUSAN MONTGOMERY of DePaul University has been appointed to an assistant professorship at the University of Southern California.

Professor ANTHONY P. MORSE of the University of California, Berkeley, will be on sabbatical leave for the academic year 1970-1971.

Professor M. Z. NASHED of Georgia Institute of Technology has been appointed to a visiting professorship at the Mathematics Research Center at the University of Wisconsin.

Professor ANDREW P. OGG of the University of California, Berkeley, has been appointed to a research professorship in the Miller Institute for Basic Research in Science at the University of California, Berkeley, for the academic year 1970-1971.

Professor FRANK C. OGG, JR., of Johns Hopkins University has been appointed to an associate professorship at the University of Toledo.

Dr. BRUCE OLSEN of Columbia University has been appointed to a visiting assistant professorship at SUNY at Buffalo.

Dr. JAMES H. OLSEN of the U.S. Army has been appointed to an assistant professorship at North Dakota State University.

Professor PAUL OLUM of Cornell University has been appointed to a visiting professorship at the University of Washington.

Professor STANLEY J. OSHER of the University of California, Berkeley, has been appointed to an associate professorship at SUNY at Stony Brook.

Dr. ALEXANDER PAL of Pratt and Whitney Aircraft has been appointed to an associate professorship at Southern Illinois University in Edwardsville.

Dr. DAVID PEERCY of New Mexico State University has been appointed a postdoctoral fellow at West Virginia University.

Professor SAM PIERCE of the University of California, Los Angeles, has been appointed to an assistant professorship at California State College at Fullerton.

Professor SAMUEL PIERRE of the University of Paris has been appointed to a professorship at the University of Orsay.

Professor TOM PITCHER of the University of Southern California has been appointed to a professorship at the University of Hawaii.

Professor ROBERT PIZIAK of the University of Massachusetts has been appointed to an assistant professorship at the University of Florida.

Professor MURRAY H. PROTTER of the University of California, Berkeley, will be on sabbatical leave during the fall and winter quarters of 1970-1971.

Professor MALAYATTIL RABINDRANATH of the Massachusetts Institute of Technology has been appointed to an assistant professorship at the University of Michigan.

Professor JAMES F. RAMALEY of Bowling Green State University has been appointed to an assistant professorship at the University of Pittsburgh.

Professor PETER L. RENZ of Reed College has been appointed to an assistant professorship at Wellesley College.

Dr. DONALD REYNOLDS of Texas Christian University has been appointed a postdoctoral fellow at West Virginia University.

Mr. HENRY J. RICARDO is on leave from Manhattan College for the academic year 1970-1971. He has been appointed an NSF Faculty Fellow at Belfer Graduate School of Science, Yeshiva University.

Mr. BART FRANCIS RICE of Louisiana State University has been appointed an officer of research and development at the Defense Atomic Support Agency, Atmospheric Effects Division, Washington, D.C.

Professor JOHN R. RICE of Purdue University is on leave for the academic year 1970-1971. The first part of the year
he will be at the University of California, Santa Barbara.

Dr. DAVID RINE of the University of Iowa has been appointed a postdoctoral fellow at West Virginia University.

Dr. LON ROSE of New York University has been appointed a lecturer at the University of California, Berkeley.

Professor HASKELL P. ROSENTHAL of the University of California, Berkeley, will be on sabbatical leave during the spring quarter of 1971.

Professor HANNO RUND of the University of Waterloo has been appointed to a professorship and to the head of the Department of Mathematics at the University of Arizona.

Professor JACK P. SANDERS of the University of Virginia has been appointed an assistant professorship at the University of Missouri at Columbia.

Professor DONALD E. SARASON of the University of California, Berkeley, will be on leave during the academic year 1970-1971. He will carry on research at the University of Kentucky in the fall quarter, at Yeshiva University in the winter quarter, and at the University of California, Los Angeles, in the spring quarter.

Professor RICHARD L. SAYLOR of the University of California, Berkeley, has been appointed to an assistant professorship at the University of Miami.

Professor RICHARD M. SCHOR of Louisiana State University has been appointed a visiting lecturer at the University of Washington.

Professor MARY D. SCHRAT of Immaculate Heart College has been appointed to an associate professorship at California State College at Bakersfield.

Mr. PAUL A. SCHWEITZER of Harvard University has been appointed to a membership at the Institute for Advanced Study.

Professor JOHN L. SELFRIDGE of the University of Illinois has been appointed to a visiting professorship at the University of California, Berkeley.

Professor JOEL H. SHAPIRO of Queen's University has been appointed to an assistant professorship at Michigan State University.

Professor WARREN E. SHREVE of the University of Connecticut has been appointed to an assistant professorship at North Dakota State University.

Professor ERNEST E. SHULT of Southern Illinois University has been appointed to a professorship at the University of Florida.

Professor JACK H. SILVER of the University of California, Berkeley, has been named a Sloan Foundation Research Fellow for the period 1970-1972.

Dr. CARL SIMON of Northwestern University has been appointed a lecturer at the University of California, Berkeley.

Mr. BHAGAT SINGH of the University of Illinois has been appointed to an assistant professorship at the University of Wisconsin at Green Bay.

Professor KENNAN T. SMITH of the University of Wisconsin has been appointed to a professorship at Oregon State University.

Professor JACK SONN of City College (CUNY) has been appointed to an assistant professorship at Adelphi University.

Professor WALTER STRODT of Columbia University has been appointed to a professorship at St. Lawrence University.

Professor RICHARD R. SUMMERHILL, on leave from the University of Missouri at Columbia, is at the Institute for Advanced Study.

Professor MOSS E. SWEENER of Cornell University has been appointed to a visiting associate professorship at the University of California, Berkeley.

Dr. KANDIAH THANIGASALAM of Pennsylvania State University has been appointed to an assistant professorship at Fordham University.

Professor CARL WEINBAUM of the University of Hawaii has been appointed to an associate professorship at Hawaii Lao College.
Professor HANS F. WEINBERGER of the University of Minnesota has been appointed to a visiting professorship at the University of Arizona.

Dr. JAMES J. WOEPPLE of the University of Illinois has been appointed to a visiting assistant professorship at SUNY at Buffalo.

Professor JOSEPH A. WOLF of the University of California, Berkeley, will be on sabbatical leave during the spring quarter of 1971.

Professor ROGER L. WOODRIFF of Humboldt State College has been appointed to a professorship at Menlo College.

Professor HUNG-HSI WU of the University of California, Berkeley, has been named a Sloan Foundation Research Fellow for the period 1970-1972. Professor Wu will also be on leave from the University of California for the spring quarter of 1971 which he will spend at Princeton University.

Professor FRANK O. WYSE of Cleveland State University has been appointed to a professorship and to the chairmanship of the Department of Mathematics at Talladega College.

Dr. RAPHAEL ZAILER of the University of Chicago has been appointed a lecturer at the University of California, Berkeley.

Professor JOSEPH ZAKS of Wayne State University has been appointed to an assistant professorship at Michigan State University.

Dr. AVIGDOR ZAROMP of Temple University has been appointed to an assistant professorship at Oakland University.

Mrs. GRENIITH J. ZIMMERMAN of the University of Minnesota has been appointed to an assistant professorship at Loma Linda University.

Professor PHILIP W. ZIPSE of Lafayette College has been appointed to an assistant professorship at Montclair State College.

Professor A.C. ZITRONENBAUM of the University of California, Los Angeles, has been appointed to a professorship at Cornell University.

PROMOTIONS

To Chairman, Department of Mathematics, Vanderbilt University: BILLY F. BRYANT.

To Professor, University of California, Berkeley: WU-YI HSIAANG, DONALD E. SARASON; Harvard University: ARTHUR M. JAFFE; University of Marburg: WERNER SCHAA; University of Minnesota: WILLIAM F. POHL; Newark State College: JOAN L. LEVINE; University of Ottawa: ROOP N. KESARWANI; University of Paris: JEAN-PIERRE L. AUBIN; Saint John's University: GIDEON S. CARM; Salem College: A. T. CURLEE; University of Washington: JACK SEGAL; West Virginia University: I. DEE PETERS.

To Adjunct Professor, American University: GRACE S. QUINN.

To Associate Professor, University of Arizona: RICHARD B. THOMPSON; University of California, Berkeley: HASSELL P. ROSENTHAL, JACK H. SILVER; University of California, Irvine: F. B. CANNONITO; Carnegie-Mellon University: JOHAN G. F. BELNANTE; University of Houston: CLIFTON T. WHYBURN; University of Illinois at Chicago Circle: ROBERT J. SOARE; University of Maryland: UMBERTO NERI; University of Missouri at Kansas City: BRUCE R. WENNER; University of New South Wales: J. L. GRIPFITH; North Dakota State University: K. NAGAWARA RAO; University of Washington: LLOYD FISHER; University of Wisconsin: GRACE GOLDSMITH WAHBA.

To Assistant Professor, Clark University: ROBERT W. KILMOYER; Southwest Texas State College: JOHN J. EDGELL, JR.; University of Washington: DAVID F. PINCUS, ROBERT SMYTHE.

INSTRUCTORSHIPS

University of Arizona: JAMES W. THOMAS; University of Chicago: JAMES R. BUNCH; Fordham University: ELIZABETH RALSTON; Grande Prairie College: ANTON A. CSEUZ; Lewis College: MARY STEVENS RANADE; University of Minnesota: CARL D. MIRANDA; Oakland University: MARION ORTON; Wytheville Community College: DEANNA G. BOWMAN; Yale University: JAMES H. SCHMERL.

DEATHS

Sister MARGUERITE BONNER of Villanova, Pennsylvania, died on March 23, 1970, at the age of 29. She was a mem-
Professor MARY E. HALLER of Seattle, Washington, died on June 16, 1970, at the age of 68. She was a member of the Society for 39 years.

Mr. CHANDLER R. MURTON of Birmingham, Alabama, died on December 21, 1969, at the age of 64. He was a member of the Society for ten years.

Professor H. S. ZUCKERMAN of the University of Washington died on June 16, 1970, at the age of 58. He was a member of the Society for 34 years.
CBMS NEWSLETTER

The Conference Board of the Mathematical Sciences has announced that subscriptions to its NEWSLETTER are available, beginning with the January 1971 issue, at a special rate of $2 per year to individuals who belong to one or more of the member-societies. These include the American Mathematical Society, the Association for Computing Machinery, the Association for Symbolic Logic, the Institute of Mathematical Statistics, the Mathematical Association of America, the National Council of Teachers of Mathematics, the Operations Research Society of America, the Society for Industrial and Applied Mathematics, the Society of Actuaries, and The Institute of Management Sciences. To other individuals, libraries, and institutions, the subscription rate is $4 per year. Beginning in 1971, the NEWSLETTER will be published in four sixteen-page issues per year, in January, March, May, and October. Orders for subscriptions may be addressed to CBMS, 2100 Pennsylvania Avenue, N.W., Suite 834, Washington, D.C. 20037. Individual orders at the special $2 rate should be accompanied by prepayment and should list those member-organizations of CBMS to which the individual belongs.

ALFRED P. SLOAN FOUNDATION GRANT

The Alfred P. Sloan Foundation has just announced the award of a $50,000 grant to Princeton University. This grant was made to support partially five to seven visiting professors over the next five years in the Department of Statistics. The department, established on a small scale four years ago, seeks to maintain and expand its breadth of interests by inviting distinguished visitors for varying lengths of time. The chairman of the department is G. S. Watson.

SYMPOSIUM ON CLIFFORD ALGEBRA

A symposium on Clifford Algebra, Its Generalization and Applications will be held the last week of January 1971 at Ootacamund (Queen of Hill stations in South India, about 400 miles from Madras). Living expenses and round-trip travel from Madras will be provided for those who contribute papers. Among the participants will be Professor Takayuki Nono of Fukuoka University of Education, Professor Keijiro Yamazaki of the University of Tokyo, Professor Richard Bellman of the University of Southern California, and Dr. E. A. Lord of Durham University (England). For further information please write to Professor Alladi Ramakrishnan, Director, MATSCIENCE, Madras 20, India.

NEW PH.D. PROGRAM AT UNIVERSITY OF MONTANA

The University of Montana Department of Mathematics has announced a new Ph.D. program called "Mathematical Arts Option." The new program is designed to train college teachers, and the distinctive features are (1) a broader, less specialized curriculum than in the research-oriented program, with emphasis on the place of mathematics in the sciences and the historian's view of the modern mathematical era; (2) an expository thesis; and (3) a teaching "apprenticeship" program which emphasizes the responsibility of the college mathematics teacher to communicate knowledge. Details of this program may be obtained by writing to the Chairman, Mathematics Graduate Committee, University of Montana, Missoula, Montana 59801.

DIVISION OF STATISTICS
THE OHIO STATE UNIVERSITY

Formerly operated as a part of the Department of Mathematics at The Ohio State University, statistics is now formally recognized as the Division of Statistics in the College of Mathematics and Physical Sciences. The staff consists of P. O. Anderson, A. M. Barron, J. S. Rustagi, J. Singh, R. C. Srivastava, J. A. Sullivan, D. R. Whitney, T. A. Willke. The existing M.S. and Ph.D programs will continue, and an undergraduate degree program has been added. For further information, write to D. R. Whitney, 231 West 18th Avenue, Columbus, Ohio 43210.
The Mathematics Research Institute of Oberwolfach (Mathematisches Forschungsinstitut Oberwolfach) has recently announced the program that is planned for 1971.

**January 2-6**
Arbeitstagung  
Chairman: H. Salzmann, Tübingen

**January 10-16**
Arbeitstagung über Modelltheorie  
Chairman: G. H. Müller, Heidelberg

**January 17-23**
Kolloquium mit den Fachkollegen der Gymnasien des Fachbereichs Mathematik der Universität Konstanz  
Chairmen: W. Bos, Konstanz  
R. Fritsch, Konstanz

**January 24-30**
Wahrscheinlichkeitsrechnung und Statistik im Mathematikunterricht  
Chairmen: H. Athen, Elmshorn  
H. Dinges, Frankfurt

**January 31-February 6**
Kontinuumsmechanik  
Chairmen: W. Günther, Karlsruhe  
H. Lippmann, Braunschweig

**February 7-13**
Mathematische Modelle in der Biologie  
Chairmen: W. Bühler, Heidelberg  
J. M. Gani, Sheffield

**February 14-20**
Spezielle Funktionen  
Chairmen: C. Meyer, Köln  
F. W. Schäfke, Köln

**February 21-27**
Arbeitstagung über Funktionentheorie  
Chairmen: Ch. Pommerenke, Berlin  
H. Wittich, Karlsruhe

**February 28-March 6**
Partielle Differentialgleichungen  
Chairmen: W. Haack, Berlin  
E. Heinz, Göttingen  
G. Hellwig, Aachen

**March 7-13**
Medizinische Statistik  
Chairman: S. Koller, Mainz

**March 14-20**
Mathematische Statistik  
Chairman: W. Uhlmann, Würzburg

**March 21-27**
Wahrscheinlichkeitsrechnung  
Chairman: H. Dinges, Frankfurt

**March 28-April 3**
Mathematische Logik  
Chairmen: H. Hermes, Freiburg  
K. Schütte, München

**April 4-10**
Finite Geometries  
Chairmen: P. Dembowski, Tübingen  
D. R. Hughes, London

**April 11-17**
Arbeitstagung  
Chairmen: M. Kneser, Götingen  
P. Roquette, Heidelberg

**April 18-24**
Formale Sprachen und Programmiersprachen  
Chairmen: W. Händler, Erlangen  
G. Hotz, Saarbrücken  
H. Langmaack, Saarbrücken

**April 25-May 1**
Methoden und Verfahren der Mathematischen Physik  
Chairmen: B. Brosowski, Götingen  
E. Martensen, Darmstadt

**May 2-8**
Gruppen und Geometrien  
Chairmen: P. Dembowski, Tübingen  
D. Higman, Ann Arbor  
H. Salzmann, Tübingen

**May 16-22**
Allgemeine Gruppentheorie  
Chairmen: W. Gaschütz, Kiel  
K. Gruenberg, London

**May 23-29**
Differentialtopologie, speziell Blätterungen  
Chairmen: W. Klingenberg, Bonn  
J. Martinet, Strassburg  
G. Reeb, Strassburg
May 30-June 5
Algebraische Gruppen
Chairmen: T. A. Springer, Utrecht
J. Tits, Bonn

June 6-12
Diskrete Geometrie
Chairmen: H. S. M. Coxeter, Toronto
L. Fejes Tóth, Budapest
H. Zassenhaus, Columbus

June 13-19
Numerische Methoden der Approximationstheorie
Chairmen: L. Collatz, Hamburg
G. Meinardus, Erlangen

June 27-July 3
Ringe, Moduln und homologische Methoden
Chairmen: F. Kasch, München
A. Rosenberg, Ithaca

July 4-10
Aerodynamic and Structural Problems of Gliders
Chairman: R. Eppler, Stuttgart

July 11-17
Funktionenalgebren
Chairman: H. König, Saarbrücken

July 18-24
Grundlagen der Geometrie
Chairmen: F. Bachmann, Kiel
A. Barlotti, Perugia
H. Freudenthal, Utrecht
E. Sperner, Hamburg

July 25-31
Unternehmensforschung
Chairmen: R. Henn, Karlsruhe
H. P. Küni, Zürich
H. Schubert, Düsseldorf

August 1-7
Endliche Gruppen und Permutationsgruppen
Chairman: B. Huppert, Mainz

August 8-14
Universelle Algebra
Chairmen: W. Felscher, Halifax
G. Grätzer, Winnipeg

August 14-22
Lineare Operatoren und Approximation

Chairmen: P. L. Butzer, Aachen
J.-P. Kahane, Paris
B. Sz.-Nagy, Szeged

August 22-28
Algebraische Zahlentheorie
Chairmen: H. Hasse, Hamburg
P. Roquette, Heidelberg

August 29-September 4
Harmonische Analyse und Darstellungs-
theorie topologischer Gruppen
Chairmen: H. Leptin, Heidelberg
E. Thoma, München

September 5-11
Komplexe Analysis mehrerer Veränderlicher
Chairmen: H. Grauert, Göttingen
R. Remmert, Münster
K. Stein, München

September 12-21
Topologie
Chairmen: A. Dold, Heidelberg
D. Puppe, Heidelberg
H. Schubert, Düsseldorf

September 22-25
Fragen der Ausbildung in Mathematik an den Universitäten
Chairman: Not named

September 26-October 2
Geometrie
Chairmen: K. Leichtweiss, Stuttgart
K. H. Weise, Kiel

October 3-9
Funktionalanalyse
Chairmen: H. König, Saarbrücken
G. Köthe, Frankfurt
H. H. Schaefer, Tübingen
H. G. Tillmann, Mainz

October 10-16
Arbeitstagung
Chairmen: M. Kneser, Göttingen
P. Roquette, Heidelberg

October 17-23
Problemgeschichte der Mathematik
Chairmen: J. E. Hofmann, Ichenhausen
C. J. Scriba, Berlin
October 31-November 6
Fragen des Mathematikunterrichts an allgemeinbildenden Schulen
Chairman: Not named

November 7-13
Fortbildungslehrgang für Studienräte
Chairman: H. Salzmann, Tübingen

November 14-20
Optimierungsaufgaben
Chairmen: L. Collatz, Hamburg
W. Wetterling, Enschede

November 21-27
Fragen des Mathematikunterrichts an allgemeinbildenden Schulen
Chairman: Not named

November 28-December 4
Numerische Lösung Nichtlinearer partieller Differential-und Integrodifferentialgleichungen
Chairmen: R. Ansorge, Hamburg
W. Törmig, Jülich

Further information can be obtained by writing to the Director of the Institute, Professor Dr. Martin Barner, 78 Freiburg/Brsg., Albertstrasse 24, Federal Republic of Germany.

GEORGE WILLIAM HILL AND EMMY NOETHER RESEARCH INSTRUCTORSHIPS

The Department of Mathematics of the State University of New York at Buffalo has announced the establishment of the George William Hill and Emmy Noether Research Instructorships. (Professor Hill was the third president of the American Mathematical Society.) These instructorships will be awarded to young mathematicians who will have completed all requirements for the Ph.D. by September 1, 1971. Two awards will be granted yearly with appointments for two years. The 12-month stipend, beginning September 1971, is $15,300, including generous staff benefits. Teaching load will total two one-semester courses during the 12-month period. Each applicant should prepare both a summary of his or her post-high school educational background, as well as a sketch of past and projected research activity, and should request three mathematicians to send letters of recommendation. The applications and supporting letters should be sent to the Chairman, Department of Mathematics, SUNY at Buffalo, 4246 Ridge Lea Road, Amherst, New York 14226, so as to arrive by January 15, 1971. Appointments will be announced by February 15.
ITALIAN NATIONAL RESEARCH COUNCIL FELLOWSHIPS

The Italian National Research Council will award 20 fellowships to citizens of other countries who intend to do research in mathematics at Italian universities during the academic year 1971-72. Each fellowship will carry a monthly stipend of 180,000 Italian lire (approximately $288) for a maximum of 12 months. In addition, recipients will be given a reimbursement of their travel expenses. Applications, written in English, French, or Italian, should be addressed to Consiglio Nazionale delle Ricerche, Servizio Affari Scientifici e Tecnologici, Ufficio Attivita di Ricerca Sezione Borse di Studio, Piazza delle Scienze 7, Rome, Italy. Applications should include the following information: (a) date and place of birth, citizenship, and residence; (b) nature of the proposed research; (c) names of Italian mathematicians with whom the applicant would like to associate and collaborate; (d) knowledge of Italian (if any) or other foreign languages; (e) address. A brief curriculum vitae and two letters of reference should accompany the application. Applications should be submitted before March 10, 1971. Further information may be obtained by writing to Alessandro Figari Talamanca, Istituto Matematico, Universita di Genova, 16132 Genova, Italy.

RESEARCH CONFERENCE ON APPLICATIONS OF NUMERICAL ANALYSIS

As part of the Numerical Analysis Year sponsored by the Science Research Council, a research conference on "Applications of Numerical Analysis" will be held March 23-26, 1971, at the University of Dundee, Dundee, Scotland. Invited papers and a restricted number of submitted papers will be read.

Invited speakers from abroad who have agreed to present papers include: R. Ansorge, F. R. of Germany; C. Bardos, France; R. Bellman, U.S.A; J. Cea, France; A. Chorin, U.S.A.; L. Collatz, F. R. of Germany; G. Golub, U.S.A.; R. Gorenflo, F. R. of Germany; J. Greenstadt, U.S.A.; P. Henrici, Switzerland; R. Kalaba, U.S.A.; B. Noble, U.S.A.; M. Osborne, Australia; P. Raviart, France; H. Stetter, Austria; R. Temam, France; E. Wachspress, U.S.A.; and O. Widlund, U.S.A.

Submitted papers, which will be of 30 minutes duration, should be sent as soon as possible and at the latest by January 31, 1971, to Professor A. R. Mitchell, Department of Mathematics, University of Dundee, Dundee, Scotland. Further information and registration forms may be obtained from Professor Mitchell.

SYMPOSIUM ON RING THEORY

The University of Utah will sponsor a symposium on ring theory March 2-6, 1971, in Park City, Utah, with anticipated support from the National Science Foundation.


At most twenty forty-minute addresses will be given by speakers chosen from the invited participants. Ample time will be allotted for informal seminars and recreational activities.

Park City is a small ski resort town located in the heart of the Wasatch Mountains, about 30 miles east of Salt Lake City. The symposium will be housed in Park City's C'est Bon Hotel. March weather conditions usually promise mild daytime temperatures and excellent skiing conditions.

A limited amount of funds is available for travel and/or subsistence grants for younger mathematicians to attend the conference. Further information may be obtained by writing to R. Gordon, Department of Mathematics, University of Utah, Salt Lake City, Utah 84112.
INTERNATIONAL CONFERENCE ON MATHEMATICS OF CONTEMPORARY PHYSICS

An International Conference on Mathematics of Contemporary Physics will be held at Bedford College, London, from August 23 to September 11, 1971. The conference will be supported by the NATO Advanced Study Institute. This is to be an instructional conference designed to illustrate some of the applications of pure mathematics, especially functional analysis, in contemporary physics. The conference will not assume a knowledge of physics, as it is primarily intended for mathematicians who may wish to apply pure mathematics to physical problems. The following main courses have been provisionally arranged: J. Glimm, "Perturbation theory of selfadjoint operators"; R. Haag, "Axiomatics of quantum field theory"; K. Hepp, "Free and interacting quantized fields"; H. Hugenholtz, "Statistical mechanics." There will also be courses on "Group representations" and "C*-algebras." Parts of the main courses, and other courses, will be of an introductory nature. Accommodations, including full board, will be provided at Bedford College for 60 for the entire conference. Married couples are advised to arrange for hotel accommodations well in advance of the conference. A limited amount of funds will be available to participants who are unable to obtain financial support from other sources. Persons wishing to receive the second notice of this conference, and who desire further information should write to Dr. D. E. Cohen, Queen Mary College, Mile End Road, London E1, England.

INTERNATIONAL CONFERENCE ON STOCHASTIC POINT PROCESSES

An International Conference on Stochastic Point Processes; Statistical Analysis, Theory and Applications will be held at the IBM Research Center, Yorktown Heights, New York, on August 2-7, 1971, the week before the International Statistical Institute meetings in Washington, D.C. The aim of the conference is to bring together mathematicians and statisticians working in this field together with workers in applied fields, such as ecology, neurophysiology, traffic studies, reliability, geography, forestry, epidemiology, and geophysics. Three categories of papers will be presented: (1) survey papers on the mathematical theory, statistical analysis and models of univariate point processes, multivariate point processes, multidimensional point processes and line processes; (2) review papers on the types of problems involving point processes encountered in fields of application such as ecology, neurophysiology, physics, forestry, reliability, traffic, geography, etc.; (3) a limited number of contributed papers on new work in the field. It is hoped to have the survey papers available before the conference and also to print a compilation of open problems for discussion at the conference. Problems for inclusion should be submitted on not more than one typewritten 8 1/2 x 11 sheet no later than April 30, 1971. Further information may be obtained from and submissions may be sent to Dr. P. A. W. Lewis, Mathematical Sciences Department, IBM Research Center, P.O. Box 218, Yorktown Heights, New York 10598.

SUMMER SEMINAR IN THEORY OF PROBABILITY AND MATHEMATICAL STATISTICS

The Mathematical Association of America will conduct a summer seminar in the theory of probability and mathematical statistics to be held at Williams College from June 21 to July 30, 1971. The staff for the seminar will be Professor Frank Spitzer, Cornell University, and Professor Geoffrey Watson, Princeton University. The program, funded by the National Science Foundation, is being conducted for teachers of probability or statistics in colleges and universities which offer an undergraduate major in mathematics but which do not have a Ph.D. program. Participants will be expected to hold the Ph.D. degree or have comparable qualifications. Further information can be obtained by writing to Professor Neil R. Grabois, Director of the 1971 Summer Seminar, Williams College, Williamstown, Massachusetts 01267.
ABSTRACTS OF CONTRIBUTED PAPERS

The November Meeting in Pasadena, California
November 21, 1970

680-A 11. RONALD ALTER and MARK B. VILLARINO, Department of Computer Science, University of Kentucky, Lexington, Kentucky 40506. A remark on primes in arithmetic progressions.

The authors prove that the excess of the number of primes of the form $10x \pm 3$ over the number of primes of the form $10x \pm 1$ is infinite. In 1853, Chebyshev conjectured the still unproven assertion that there exist more primes of the form $4y+3$ than of the form $4y+1$ in the sense that $\lim_{x \to \infty} \frac{x}{\log x} = \frac{4}{3}$. This paper concerns itself with a weaker form of Chebyshev's conjecture, proven by Landau, that if $\pi(x,k) = \sum_{p \leq x, p \equiv k} 1$ and $P(x,k,t_1,t_2) = \pi(x,k,t_1) - \pi(x,k,t_2)$ then $P(x,4,3,1) \to \infty$ as $x \to \infty$. In this paper, by generalizing Landau's technique, it is proven that $P(x) \sim P(x,10,\pm 3,\pm 1) \sim \sqrt{x}/\log x$. (Received October 23, 1970.)


A commutative semigroup $D$ is called a skewer if $D$ is a chain $\Lambda$ of commutative archimedean semigroups $D_\lambda$, $D = \bigcup_{\lambda \in \Lambda} D_\lambda$. Let $S = \bigcup_{\alpha \in \Gamma} S_\alpha$ be the greatest semilattice decomposition of $S$. If $\Lambda$ is a cofinal chain in $\Gamma$, then $\bigcup_{\lambda \in \Lambda} S_\lambda$ is called a cofinal subskewer of $S$. Theorem. A homomorphism of a cofinal subskewer of $S$ to a group $G$ can be uniquely extended to a homomorphism of $S$ to $G$. Theorem. If $S = \bigcup_{\alpha \in \Gamma} S_\alpha$ has a greatest group homomorphism $f: S \to G$, then there is a cofinal subskewer $W$ of $S$ such that $f|W$ is a greatest group homomorphism $W \to G$. By using these theorems, we can characterize the structure of commutative semigroups $S$ having greatest group homomorphism in terms of subskewers of the greatest separative homomorphic image $\bar{S}$ of $S$. (Received November 21, 1970.)


We prove that under certain conditions the flow of a Hamiltonian vector field on a possibly infinite-dimensional dynamical system exists for all time. (Received October 12, 1970.)


Sufficient conditions are obtained for the existence, uniqueness (in a neighborhood), and the applicability of a simple numerical procedure to compute a solution of the infinite nonlinear system of equations obtained by G. H. Hill in his study of the variational orbit in lunar theory. The conditions are slightly less stringent than A. Wintner's (Math. Z. 30(1929), 211-227) but the approach used here negates the possibility of proving that...
the solution depends analytically on Hill's constant \( m \). The conditions are obtained by modifying Wintner's equivalent system of equations and then applying a generalization of a theorem due to the author (Rend. Circ. Mat. Palermo (1962), 5–24) which provides sufficient conditions for such systems to be represented by a contraction in a suitable \( L_p \) space. (Received November 18, 1970.)


Let \( G \) be a compact Lie group acting differentiably on the paracompact \( C^\infty \)-manifold \( X \). For each subgroup \( K \) of \( G \) let \( X_K \) be the set of all points in \( X \) whose stability groups are conjugate to \( K \). Let \( \{ \mathcal{H}_\alpha \mid \alpha \in \Lambda \} \) be a collection of subgroups of \( G \). For each \( \alpha \in \Lambda \) let \( U_\alpha \) be a \( G \)-invariant component of \( \{ \mathcal{H}_\alpha \mid \alpha \in \Lambda \} \). Let \( \mathcal{U} \) be a \( G \)-invariant subset of \( \{ \mathcal{H}_\alpha \mid \alpha \in \Lambda \} \). If \( S \) is a \( G_y \)-slice at \( y \in \mathcal{U} \) with respect to \( X \), then \( S \) is a \( G_y \)-space. If \( L \) is a subgroup of \( G_y \), then \( S_L \) is the set of all points in \( S \) whose stability group with respect to \( G_y \) is conjugate to \( L \). Theorem 1. At every point \( y \in \mathcal{U} \) there is a Koszul \( G_y \)-slice \( S \) with respect to \( X \) such that for each \( \alpha \in \Lambda \) and each closed subgroup \( L \) of \( G \) with \( \mathcal{S}(\mathcal{U}) \neq \emptyset \), \( \mathcal{S}(\mathcal{U}) \subset \text{Bdry}(\mathcal{S}(\mathcal{U})) \).

Theorem 2. Every \( H \)-slice with respect to \( X \) at a point in \( \mathcal{U} \) contains a subslice with this property.

Theorem 3. If the orbit space of \( \mathcal{U} \) is connected then any two slices having this property have the same orbit types. (Received November 3, 1970.)

680-G6. JAMES T. ROGERS, Tulane University, New Orleans, Louisiana 70118. Embedding the hyperspaces of circle-like, plane continua.

**Theorem.** The hyperspace of subcontinua of a circle-like, plane continuum is embeddable in \( E^3 \).

(Received November 4, 1970.)

**The November Meeting in Urbana, Illinois**

**November 28, 1970**


Let \( A \) be a Jordan (noncommutative) algebra over a commutative ring \( R \), a bimodule \( M \) over \( A \) is then an \( R \)-module, together with two maps \( A: M \to M, M: A \to M \) both denoted multiplicatively and satisfying (rather complicated) conditions: \(((ab)a)b = (ab)(a)b = 0\), \(((ab)(c+d)+((cd)(a)+(bc)a)m + (ab)(mc)-(mb)(ca)-(cb)(am) = 0\), \(((ma)b)(c + ((ba)c)m + ((ca)m)b - (ma)(bc) + (ba)(cm) - (ca)(mb) = 0\). Now, we may consider the extensions of \( A \) by \( M \), i.e., the split (as \( R \)-modules) exact sequences \( 0 \to M \to E \to A \to 0 \), such that \( f \) is a Jordan algebra homomorphism and define the group of equivalence classes of extensions \( \text{Ext}(A, M) \). Take now for \( M \) the underlying module of \( A \). **Theorem.** There is a canonical isomorphism between the \( R \)-module of all extensions of \( A \) by \( M \) and the tangent space at the point \( A \), to the algebraic set \( S \) of all Jordan algebra structures on \( M \). Two such extensions are equivalent iff they are in the same orbit under the natural action of \( GL(M) \) on \( S \).
Corollary. The group $\text{Ext}(A, M)$ is the group of infinitesimal deformations of $A$, in particular if $\text{Ext}(A, M) = 0$ the algebra $A$ is rigid. Remark. The same situation holds in general for any "algebraically defined" type of algebras. (Received October 7, 1970.)


It is shown that it is possible to solve the word problem for the fundamental group $G$ of any of the above knots. The proof is obtained by replacing such a knot with an equivalent one having a knot-diagram with special properties. A presentation of $G$ and of another group $H$ is obtained from a knot-diagram for $K$, using the method in M. Dehn, Math. Ann., 69 (1910), 137-168. When the knot-diagram has the special properties, the presentation of $H$ (as a factor group $F/N$ of a free group $F$) falls into one of the categories for which R. C. Lyndon (Math. Ann., 166 (1966), 208-228) solved the word problem. An easily computed automorphism of $F$ yields a second presentation of $H$ in which one generator $x_0$ is not mentioned in the defining relations. Adding $x_0 = 1$ to this second presentation gives a presentation of $G$, and the result follows. (Received October 10, 1970.)


Let $P$ be a polynomial such that $k$ of the $n-1$ principal curvatures are different from zero at each point of $N(P) = \{ s \in \mathbb{R}^n : P(s) = 0 \}$; $N(P)$ is assumed to be nonempty, bounded, and $n-1$ dimensional. If $\text{Supp } \varphi \subset U_\delta = \{ s \in \mathbb{R}^n : |P(s)| < \delta \}$ with $\delta$ small and $\varphi \in C_c^\infty(\mathbb{R}^n)$, let $\varphi^\sigma$ be the integral of $\varphi$ over $N(P-q)$ if $q \in [-\delta, \delta]$ and $\varphi^\sigma(s) = \varphi^\rho(P(s))$ on $U_\delta$ and $= 0$ outside $U_\delta$. Then $\varphi^\sigma \in C_c^\infty(\mathbb{R}^n)$. We define the symmetrization $\varphi^\sigma$ of a distribution $\varphi$, with $\text{Supp } \varphi \subset U_\delta$, in a natural way. Setting $u = \mathcal{F}^{-1}[\varphi]$ and $u_0 = \mathcal{F}^{-1}[\varphi^\sigma]$, we prove that $u_0$ is the integral of the product of $u$ with some function $w(\cdot, \cdot)$ which depends only on $P$. This result is used to prove a Liouville type theorem for entire solutions of $P(-iD_x) u(x) = f(x)$, with $f \in C_c^\infty(\mathbb{R}^n)$. (Received October 19, 1970.)
A society, $S$, consists of a symmetric binary relation, $R$, with field $N$ which relates boys (evens) to girls (odds); $S$ is recursive if $R$ is - i.e., if one can effectively determine whether $b_i$ "knows" $g_j$. A solution to the marriage problem of $S$ is a 1-1 map $f$ from evens to odds such that $b_i$ knows his mate $g_{f(i)}$. See Halmos and Vaughan, "The marriage problem," Amer. J. Math. 72(1950), for Hall's criterion for the existence of a solution; our concern is with the recursiveness of solutions for recursive $S$. Theorem 1. There is a recursive society $S_d$ with a unique solution and that solution has degree $\delta \equiv \delta' \iff \delta \equiv \delta'$. Theorem 2. There exist solvable recursive societies with no solutions $\not\equiv \delta'$ yet (using a basis theorem of Jockusch and Soare) every such society has a solution whose degree has jump $\equiv \delta'$. Theorem 3. For any $n \not\equiv \delta_0$, there exist recursive societies with solutions in exactly $n$ pairwise incomparable degrees. Let $k \geq 2$. A $k$-society is one in which each person knows exactly $k$ others. Theorem 4. If a recursive $k$-society has a unique solution, that solution is recursive. Theorem 5. There exist recursive $k$-societies with no recursive solutions, yet every such society has a solution whose degree has jump $\delta'$. (Received October 28, 1970.)

Let $\Omega$ denote the recursive equivalence types, $\Lambda$ the isols, $N$ the natural numbers. Let $\equiv$, $+$, $\times$ denote the usual order relation, addition, multiplication, respectively in $\Omega$, $\Lambda$, or $N$. Let $\text{Th}(R)$ denote the usual first order theory of a relational structure $R$, $\text{Th}^2(R)$ its second order theory including set quantifiers. Erik Ellentuck has shown $\text{Th}^2(N, +, x)$ and $\text{Th}(\Lambda, +, x)$ and $\text{Th}(\Omega, +, x)$ have the same degree of unsolvability and that they all differ by recursive permutations. Using the coding techniques in our paper, "A universal embedding property of RETs," J. Symbolic Logic 35(1970), 51-59, $\text{Th}^2(N, +, x)$ is interpretable in (and hence one-one reducible to) each of $\text{Th}(\Lambda, \equiv)$, $\text{Th}(\Lambda, +)$, $\text{Th}(\Omega, +)$ and $\text{Th}(\Omega, \equiv)$. Thus each of these theories also differ by a recursive permutation from $\text{Th}^2(N, +, x)$. (Received October 28, 1970.)

Projective geometry having solid as primitive. Preliminary report.
geometry is considered. The name solld is added to Leśniewski's Mereology and a definition of point is given. It is then shown that there exists an interpretation of solid in ordered projective geometry such that the defined points correspond bijectively to the points of the geometry. (Received October 30, 1970.)


We refer definitions and notations to Henkin's paper in Pacific J. Math. 32(1970), 723-752. The fact that every Boolean algebra with operators \( \mathfrak{B} = (A, +, \cdot, - , 0, 1, f_1) \) has a completion \( \mathfrak{C} = (B, +, \cdot, - , 0, 1, f_1) \) if the Boolean part of \( \mathfrak{B} \) is a completion of the Boolean part of \( \mathfrak{A} \), and \( f_1 = f_1 \) for each \( i \in I \). Theorem. If \( \mathfrak{B} \) is a completion of a Boolean algebra with "partial operators" \( \mathfrak{B} = (A, +, \cdot, - , 0, 1, f_1) \) such that for each \( i \in I \), \( f_1 \) is completely \( p_1 \)-additive for some \( p_1 \), then a positive equation \( \tau = \rho \) holds in \( \mathfrak{B} \) iff it holds in \( \mathfrak{C} \). (Received November 2, 1970.)

682-02-6. OLIVER G. ABERTH, Texas A & M University, College Station, Texas 77843. A nonexistence theorem of differential equations.

In the \( x - y \) plane let \( R \) be the rectangle defined by the inequalities \( |x - a| \leq M_1 \), \( |y - b| \leq M_2 \), where \( M_1 \), \( M_2 \) are two positive numbers. If \( f(x, y) \) is a uniformly continuous function in \( R \) and satisfies \( |f(x, y)| \leq M_2/M_1 \), then in real analysis, by the theorem of Peano, there always exists a function \( y(x) \) satisfying the initial condition \( y(a) = b \) and the equation \( (1) \ y'(x) = f(x, y(x)), \ x \in \text{the interval } |x - a| \leq M_1 \). However, there is known no general constructive method for finding the function \( y(x) \). In the constructive analysis developed by the Russian mathematicians Markov, Zaslavskii, Ceitin, and others, this existence theorem fails. An example may be given of a constructive function \( f(x, y) \), uniformly continuous in a rectangle centered at the origin \( (0, 0) \), such that there is no constructive function \( y(x) \) satisfying \( (1) \) and the initial condition \( y(0) = 0 \), no matter how small an interval about \( 0 \) is prescribed for \( x \). (Received November 2, 1970.)

682-02-7. EDWIN L. MARSDEN, Kansas State University, Manhattan, Kansas 66502. A note on implicative models.

An implicative model (see L. Henkin, "An algebraic characterization of quantifiers", Fund. Math. 37(1950), 63-74) is of the form \( (X, /, 0) \) where \( X \) is a set, \( 0 \in X \), and \( / \) is a binary operation satisfying the axioms \( (1) \) to \( (4) \) listed below. Let \( \leq \) be defined by \( x \leq y \iff x/y = 0 \). The following hold for all \( x, y, z \in X \): \( (1) \ x/y \leq x \), \( (2) \ x/z \cdot y/z \leq (x/y)/z \), \( (3) \ 0 \leq x \), \( (4) \ leq \) is anti-symmetric. Brouwerian lattices and closure algebra are examples of implicative models. If in addition there is \( 1 \in X \) with \( x \leq 1 \) for all \( x \in X \), then \( X, /, 0, 1 \) is called
an implicative model with unit. Let \((X, \cdot, 0, 1)\) be an implicative model with unit, let \(x' = 1/x\), and let \(Y = \{x' \mid x \in X\}\). **Theorem.** \(Y\) is a Boolean algebra. Let \(\sim\) be the relation defined on \(X\) by \(w \sim x\) iff \((w/x)' = (x/w)' = 1\). **Theorem.** \(\sim\) is a congruence relation, and \(x \sim w\) iff \(x'' = w''\). It follows that \(X/\sim\) is isomorphic to \(Y\). These theorems generalize similar theorems for Brouwerian lattices. (See G. Birkhoff, "Lattice theory, III," Amer. Math. Soc. Colloq. Publ., Vol. 25, Amer. Math. Soc., Providence, R.I., 1967, Chapter V.11.)


Let \(\mathfrak{M} = (M, R, f)\) be a first order structure in which \(M\) has, in addition, a completely regular topology. A structure \(\mathfrak{M}' = (\mathfrak{M}, R, f)\) is constructed with the following properties: (1) the Stone-Čech compactification, \(\beta M\), and hence \(M\) itself, is a dense subset of \(\mathfrak{M}'\); (2) the value of a function or truth value of a relation on \(\mathfrak{M}'\) depends upon the values of the function or truth values of the relation at nearby points of \(M\); (3) \(\mathfrak{M}'\) is an elementary extension of \(\mathfrak{M}\). The construction is applied to relate the Stone-Čech compactification to compactifications of algebras and to the limit functions of Ascoli's theorem. (Received November 4, 1970.)

682-02-9. ROBERT C. WHERRITT, Wichita State University, Wichita, Kansas 67208. First-order equality logic with weak existence assumptions.

For any first-order language \(L\) with equality, individual constants, and function symbols, let \(STANEQ(L)\) be the standard first-order equality logic on \(L\), and let \(STREQ(L)\) be obtained from \(STANEQ(L)\) by retaining all rules (including the equality rules for all terms) except \(\gamma\) Elim, which is weakened to: \(\gamma x A \rightarrow A(y/x)\) for any variable \(y\). (\(A(t/x)\) denotes free substitution of the term \(t\) for \(x\) in the \(\psi A\).) In \(STREQ(L)\), \(\gamma \exists x(x = x)\) and \(\gamma \exists x(x = t) \land \gamma xA = A(t/x)\), but not \(\gamma x \exists y(y = f(x))\). A general semantic theory is formulated for equality logics by generalizing the standard notion of a realization to a semirealization, in which an inner domain \(D\), over which variables and quantifications range, is augmented by an outer domain \(D'\). Terms are interpreted recursively in \(D \cup D'\). **Theorem 1.** \(STREQ(L)\) is sound and semantically complete in the class of all semirealizations of \(L\). **Theorem 2.** \(STANEQ(L)\) is sound and semantically complete in the class of all full realizations of \(L\) (semirealizations so restricted as to amount to realizations in the standard sense). The results of H. Leblanc and R. Thomason for \(QC^3\) constitute a special case of Theorem 1, that for which the only terms are individual constants and variables. (Received November 5, 1970.)

05 Combinatorics

682-05-1. DAVID A. SMITH, Duke University, Durham, North Carolina 27706. The number of 4 by 4 magic squares.

Let \(H(n, r)\) denote the number of \(n\) by \(n\) matrices of nonnegative integers with all row and column sums equal to \(r\) (magic squares). An explicit formula for \(H(3, r)\) was published by MacMahon in 1916 ("Combinatory analysis", vol. II, p. 161). Anand, Dumir, and Gupta (Duke Math. J. 33 (1966), 757-769) conjectured that for each
n, $H(n, r)$ is a polynomial in $r$ of degree $(n-1)^2$ of a certain form. This conjecture has been proved correct for $r = 4$, namely, $H(4, r) = \binom{r+3}{3} + 20 \binom{r+4}{5} + 152 \binom{r+5}{7} + 352 \binom{r+6}{9}$. (This result has also been obtained independently, and by quite different methods, by Morris Abramson.) (Received October 19, 1970.)


Let $K_t$ be the complete graph with $t$ vertices. Form $n$ disjoint sets $S_1, \ldots, S_n$ of vertices and remove the branches connecting vertices of $S_i$ with vertices of $S_{i+1}$ for $1 \leq i \leq n-1$, and those connecting vertices of $S_1$ with $S_n$. A formula is derived for the number of spanning trees in the resulting graph. This generalizes the m-series of Bedrosian. Special cases include a recent formula due to Bercovici (IEEE Trans. Circuit Theory CT-13 (1969), 101-102) and a result related to formulas derived by Moon (see "Graph theory and theoretical physics," F. Harary, Editor, Academic Press, New York, 1967, pp. 261-272). (Received October 29, 1970.)


The maximum genus, $\gamma_M(G)$, of a connected graph $G$ is the largest integer $\gamma(N)$ for compact orientable 2-manifolds $N$ in which $G$ has a two-cell imbedding, where $\gamma(N)$ is the genus of the surface $N$. Let $S_k$ be a compact orientable 2-manifold of genus $k$. It is known that (Abstract 672-34, these Notices 17(1970), 787) a graph $G$ has a two-cell imbedding on $S_k$ if and only if $\gamma(G) \leq k \leq \gamma_M(G)$. We characterize all graphs $G$ for which $\gamma_M(G) = \gamma(G)$; i.e. those graphs whose two-cell imbeddings are limited to exactly one surface. The characterization is done in a manner reminiscent of the well-known Kuratowski conditions for planarity (Fund. Math. 15(1930), 271-283). Let $H$ be the graph and let $Q$ be the graph. Theorem. Let $G$ be a connected graph. Then the following statements are equivalent: (i) $\gamma_M(G) = \gamma(G)$. (ii) $G$ does not contain a subgraph homeomorphic to either $H$ or $Q$. (iii) $G$ is a cactus all of whose cycles are vertex disjoint. (iv) $\gamma_M(G) = \gamma(G) = 0$. (Received October 30, 1970.)


A directed graph $G$ on $n$ vertices is called an $f$-graph, for $f(x)$ a polynomial of degree at least two if the $n \times n$ incidence matrix $Z$ of $G$ satisfies $f(Z) = D + \lambda J$ where $D$ is diagonal, $\lambda \neq 0$ and $J$ is the matrix of ones. If $\lambda = 0$ call $G$ a degenerate $f$-graph. Theorem. A nonregular $f$-graph is a "sum" of $t$ regular $f$-graphs (possibly degenerate) where $t \leq \text{degree}(f)$. Here "sum" means $G$ can be decomposed into subgraphs $G_1, \ldots, G_t$ where all vertices of $G_i$ are joined to all vertices of $G_j$ for $i \neq j$. Necessary and sufficient conditions that $G$ should carry an $f$-graph for a fixed polynomial $f$ are given and all nonregular quadratic graphs, i.e. $f$-graphs with $f$ of degree two, are found. (Received November 2, 1970.)
Disjoining permutations in Boolean algebras. Preliminary report.

Let $\mathcal{B}$ be a Boolean algebra and $\mathcal{Y}$ an arbitrary subset of $\mathcal{B}$. A disjoining permutation of $\mathcal{Y}$ is a one-one map $p$ of $\mathcal{Y}$ onto $\mathcal{Y}$ such that $p(A) \cap A = \emptyset$ for $A$ in $\mathcal{Y}$. An order ideal in $\mathcal{B}$ is a subset $\mathcal{J}$ of $\mathcal{B}$ such that if $A \in \mathcal{J}$ and $B \subseteq A$, then $B \in \mathcal{J}$. We denote complementation in $\mathcal{B}$ by $A^\ast$. For $\mathcal{Y} \subseteq \mathcal{B}$, $\mathcal{Y}^\ast$ will denote the set of complements of members of $\mathcal{Y}$. If $\mathcal{Y}$ is an order ideal, then $\mathcal{Y}^\ast$ is an order filter. We prove the Theorem.

A result of F. J. Dyson (unpublished) provides that if $\mathcal{B}$ is finite and $\mathcal{Y}$ is itself an order ideal, then $\mathcal{Y}^\ast$ is an order filter. We further note that no weakening of the hypothesis that $\mathcal{B}$ be a Boolean algebra is possible. (Received November 2, 1970.)

682-05-6. WITHDRAWN.

682-05-7. J. A. ZIMMER, Department of Combinatorics and Optimization, University of Waterloo, Waterloo, Ontario, Canada. On the automorphism group of planar graphs.

A graph is understood to be undirected and without loops or multiple edges. A class, $\mathcal{C}$, of graphs which includes all planar graphs is defined. Two types, I and II, of graphical automorphisms are defined. Type I automorphisms are reminiscent of reflections and rotations of the complex plane. Type II automorphisms are reminiscent of inversions of the complex plane. The following theorem is proven: If $G$ is an element of $\mathcal{C}$ and $\alpha$ is an automorphism of finite order of $G$, then $\alpha$ can be expressed as the product of finitely many automorphisms of type I and at most one automorphism of type II. The present proof of this theorem is rather long and complicated. The notion of edge-contraction plays an important role both in the statement of and in the proof of the theorem. Also, much use is made of a certain graph whose vertex set is the quotient of the vertex set of $G$ by the relation $R$ which is defined on the vertex set of $G$ by setting $xRy$ if and only if $\alpha^kx = y$ for some integer $k$. References must be made only to Zorn's Lemma and to an edge-construction version of Kuratowski's characterization of planarity. (Received November 2, 1970.)


Let $\mathcal{X}_p$ be the class of all countable graphs which do not contain a complete subgraph with $p$ vertices ($p$ an integer $\geq 3$). $\mathcal{X}_p$ satisfies condition $D$ of (Fraïssé, Ann. Sci. École Norm. Sup. (3) 71(1954), 361 - 388). It follows that $\mathcal{X}_p$ has a member $G_p$ which is homogeneous (in the sense of Fraïssé) and which is universal for $\mathcal{X}_p$. ($G_p$ can also be given by a relatively simple direct construction.) Theorem 1. If $H$ is in $\mathcal{X}_p$ and has $\aleph_0$ vertices, then there is an induced subgraph $H'$ of $G_p$ such that $H \cong H'$ and every automorphism of $H'$ extends uniquely to an automorphism of $G_p$. Corollary. There is a maximal independent set of vertices in $G_p$ whose permutations all extend uniquely to automorphisms of $G_p$. Theorem 2. $G_p$ has an automorphism with a single orbit if and only if $p = 3$. Theorem 3. If $V_1, \ldots , V_n$ is a partition of the vertices of $G_p$ then there is $j = 1, \ldots , n$ so that the subgraph of $G_p$ induced by $V_j$ contains a copy of every finite member of $\mathcal{X}_p$. (Received November 4, 1970.)
Let $K$ be a merely-finitary pregeometry on $S$ and suppose that $J(A) = A$ for all $A \subseteq S$. If $a_1, \ldots, a_n \in S$ are independent and $K([a_1, a_{i+1}]) = F_i \subseteq S$, where $2 < |F_i| = \beta_i < \infty$, then $\prod_{1 \leq i < n} F_i^{J_{F_i}} = K$. There exists $\mathcal{B}$ such that $K(A) = \sum_{B \in \mathcal{B}} (\Pi_{i \in B} J_{F_i}(A))$ for all $A \subseteq S$, where $F_i \subseteq S$ for each $i \in \mathcal{B}$ and each $B \in \mathcal{B}$ is finite. If $K = \sum_{B \in \mathcal{B}} \Pi_{i \in B} J_{F_i}$, $F$ is $K$ closed, $\mathcal{B} = \{ D \subseteq S | K(D) = F \}$, then $K_F = \sum_{D \in \mathcal{B}} \sum_{B \in \mathcal{B}} (\Pi_{i \in B} J_{F_i})^x D$ if each $K$ independent set $A$ with $F \subseteq K(A)$ there exist finite $D \in \mathcal{B}$ and $B \in \mathcal{B}$ such that $E \subseteq \Pi_{i \in B} J_{F_i}(A)$. If $K$ is a projective geometry, then $K = \sum_{\mathcal{B} \in \mathcal{B}} F \subseteq R(x, y)$, where $\mathcal{R} = \{ R \mid 3 \text{ K independent } a_1, \ldots, a_n \in S, b \in K([a_1, a_{i+1}]) - \{ a_1, a_{i+1} \} \}$. If $F$ is $K$ closed, then $C$ is a circuit of $K_F$ if either $C$ is a circuit of $K$ and $F \subseteq K(C)$ or $C$ is a minimal $K$ independent set for which $F \subseteq K(C)$. (See Abstracts 672-510 and 677-05-2, these Notices 17(1970), 230, 758.) (Received November 3, 1970.)

Let $k$ be a nonnegative integer, and set $S_k(n) = 1^k + 2^k + \ldots + n^k$. In a note to appear in Amer. Math. Monthly, the author gives an elementary combinatorial proof of the identity $1 + \sum_{k=0}^{n-1} \binom{n}{k} T_k(n) = (n + 1)^r$. In the present paper, the method of proof is generalized and a sequence of combinatorial identities is obtained, with the above identity occurring as the first term in the sequence. (Received November 4, 1970.)

A graph is called coderived if both it and its complement are derived (or line) graphs. A characterization of coderived graphs in terms of ten forbidden induced subgraphs is obtained. This leads to a determination of all coderived graphs, of which only a finite number are neither complete nor null. (Received November 5, 1970.)

In supporting the previous conjecture: "The graph of a semiring with more than two elements is connected," the authors prove that the following classes of semirings with more than two elements have connected graphs: (1) the semirings with an idempotent, (2) the additively cancellative semirings, (3) the multiplicatively cancellative semirings, and (4) the semiring with an additive zero. Several open questions are given. (Received November 5, 1970.)

The representation of a planar triply connected graph on the 2-sphere is unique. Therefore every automorphism of such a graph belongs either to the set of orientation-preserving automorphisms $G^+(X)$ or to the set of orientation-reversing automorphisms $G^-(X)$. Theorem 1. If $\alpha \in G^+(X)$ has the order $n$, then $\alpha$ can be written as a product of pairwise disjoint pointcycles and linecycles each of which has length $n$. For $n > 2$ a fixline with respase to $\alpha$ does not exist. For $\beta \in B^-(X)$ a similar theorem can be stated. Let $F_{V}(\gamma)$, $F_{E}(\gamma)$, $F_{L}(\gamma)$ be the
number of fixpoints, fixlines, fixfaces respectively, with respect to \( \gamma \). Then the following theorem holds.

**Theorem 2.** (a) If \( \gamma \in G^+(X) \), then \( F(\gamma) = F_X(\gamma) + F_E(\gamma) + F_F(\gamma) = 2 \). (b) If \( \gamma \in G^-(X) \), then \( F(\gamma) = 0 \), if the order of \( \gamma \) is \( > 2 \). (c) If \( \gamma \in G^-(X) \) has the order 2, then either \( F(\gamma) = 0 \) or \( F(\gamma) \equiv 3 \). (Received November 5, 1970.)

### 06 Order, Lattices, Ordered Algebraic Structures

682-06-1. **BARBARA JEFFCOTT,** University of Massachusetts, Amherst, Massachusetts 01002. The center of an orthologic.

An orthologic is defined to be a system \((L, \preceq, 0, 1, \wedge)\) where (1) \((L, \preceq, 0, 1)\) is a bounded poset with \( 0 \neq 1 \), (2) \( \preceq \) is a symmetric relation on \( L \) satisfying the property that for all \( p \in L \), if \( p \preceq p \) then \( p = 0 \), (3) if \( p_1, p_2 \in L \) and \( p_1 \vee p_2 \) then \( p_1 \vee p_2 \) exists in \( L \), (4) if \( P_0, p_1 \) and \( p_2 \) are mutually \( \preceq \), then \( p_0 \wedge (p_1 \vee p_2) \), (5) if \( p \in L \) then there exists some \( q \in L \) (not necessarily unique) with \( p \preceq q \) and \( p \vee q = 1 \), and (6) \( p \preceq q \) if and only if for all \( r \in L \), if \( r \preceq q \) then \( r \preceq p \). If \( e, f \in L \), then one says that \( e \) commutes with \( f \), written \( e \circ f \), if there exist three mutually \( \preceq \) elements of \( L \), call them \( e_1, f_1 \) and \( d \), satisfying \( e = e_1 \vee d \) and \( f = f_1 \vee d \). The center of an orthologic is defined to be the set of all \( e \in L \) for which \( e \circ f \) for all \( f \in L \). This paper proves that the center of an orthologic is a Boolean Algebra. (Received August 24, 1970.)


Let \( C \) be a chain and let \( E \) and \( F \) be semilattices. It is proved that a chain \( C \) is a cancellable semilattice in the sense that \( C \times E \cong C \times F \) if and only if \( E \cong F \). (Received October 5, 1970.)

682-06-3. **DAVID SACHS,** Wright State University, Dayton, Ohio 45431. Lattices of congruence classes.

Let \( A \) be an abstract algebra with finitary operations, and let \( \Gamma \) be the lattice of congruence relations on \( A \). Associated with \( \Gamma \) is the lattice \( L \): the lattice of congruence classes. \( L \) is complete and atomic. An \( S \)-algebra is an algebra in which the congruence relations are determined by a single congruence class. A closed mapping of a complete, atomic lattice onto another is one which sends closed sets of atoms onto closed sets of atoms and in which the inverse image of a closed set is closed. **Theorem 1.** \( L \) has all possible closed mappings if \( A \) has permutative congruence relations. **Theorem 2.** If \( A \) is an \( S \)-algebra, and \( L \) has all possible closed mappings, then \( A \) has permutative congruence relations. **Theorem 4.** If \( A \) is an \( S \)-algebra with permutative congruence relations and \( L \) is geometric of length \( \leq 4 \), then \( L \) is the lattice of flats of an affine geometry where the points of the geometry correspond to the elements of \( A \). A homomorphism theorem generalizing the statement that normal subgroups and ideals are mapped onto their own kind under a homomorphism is also proved. Finally, a solution is given to Birkhoff's Problem 60 which deals with lattices of cosets of normal subgroups. (Received October 5, 1970.)

682-06-4. **MARY S. DECONGE,** Loyola University, New Orleans, Louisiana 70118. \( D_2 \)-lattices.

A definition of a \( D_2 \) lattice is given (a two norm defined on a lattice). It is proved that the \( D_2 \) lattice is a 2-metric space for a special function \( A \), defined by use of the norm on a \( D_2 \) lattice. It is also proved that there
exist a nondistributive as well as a distributive $D_2$ lattice. Several conditions for interiorness are proved for the special function $A$. (Received October 19, 1970.)

682-06-5. JOEL KAGAN, University of Hartford, West Hartford, Connecticut 06117 and ROBERT W. QUACKENBUSH, University of Manitoba, Winnipeg, Manitoba, Canada. Self-dual topological Boolean algebras.

Let $A$ be a topological Boolean algebra with $A_0$ and $A^*$ the set of open and clopen elements, resp., of $A$. For a pair of filters $\mathfrak{F}_1, \mathfrak{F}_2$ define $\mathfrak{F}_1 \equiv \mathfrak{F}_2$ iff the congruences generated by these filters are equal. Let $\varnothing, \mathfrak{F}_1^*, \mathfrak{F}_2^*$ represent filters, $\mathfrak{F}_{1\mathfrak{F}_2}$, maximal $\mathfrak{F}$-filters, and prime filters, respectively. Let $\text{Ker} \mathfrak{F}$ be the filter generated by the open elements in $\mathfrak{F}$. With these definitions the following conditions are equivalent: (1) $A$ is self-dual (i.e. $A_0 = A^*$). (2) For every pair $\mathfrak{F}_1, \mathfrak{F}_2$ of filters $\mathfrak{F}_1 \cap A^* \equiv \mathfrak{F}_2 \cap A^*$ then $\mathfrak{F}_1 \equiv \mathfrak{F}_2$. (3) Every $\mathfrak{F}_1^*$ is contained in a $\approx$-equivalent $\mathfrak{F}_{p^*}$. (4) The $\text{Ker} \mathfrak{F}_p$ is a $\mathfrak{F}_1^*$ for every prime filter in $A$. (5) If $\mathfrak{F} \subseteq \mathfrak{F}_1$ then there exists a Henle algebra $H$, a prime filter $P \subseteq H$, and an $h \in \text{Hom}(A, H)$ such that $\varnothing \subseteq h^{-1}(P)$ but $\varnothing \not\subseteq h^{-1}(P)$. (6) Every meet irreducible $\mathfrak{F}$-filter is a $\mathfrak{F}_1^*$. (7) If $\mathfrak{F} \subseteq \mathfrak{F}_1$ then there exists a $\mathfrak{F}_p^*$ such that $\mathfrak{F}_1 \subseteq \mathfrak{F}_p^*$ but $\mathfrak{F}_1 \not\subseteq \mathfrak{F}_p^*$. (8) If $\mathfrak{F} \not\subseteq \mathfrak{F}_1$ there exists a $\mathfrak{F}_p^*$ such that $\mathfrak{F}_1 \not\subseteq \mathfrak{F}_p^*$. (Received October 26, 1970.)

682-06-6 DONALD E. CATLIN, Department of Mathematics and Statistics, University of Massachusetts, Amherst, Massachusetts 01002. Cyclic atoms in orthomodular lattices.

Let $P(H)$ denote the projection lattice of a separable Hilbert space $H$. For each $x \in H$, let $P_x$ denote the projection onto the one dimensional subspace generated by $x$. If $B$ is a Boolean sublattice of $P(H)$, then it is a theorem that whenever $B$ is maximal in $P(H)$ there exists a vector $x_0 \in H$, called a cyclic vector for $B$, such that the join in $P(H)$ of all the $P_Q(x)$ as $Q$ ranges through $B$ is the identity operator $I$. We show that this theorem is an immediate corollary of a more general theorem about orthomodular lattices. In particular, if $B$ is a maximal Boolean sublattice of the complete atomic orthomodular lattice $L$ and $L$ satisfies a certain strong irreducibility condition, then denoting the Sasaki projection on $b$ by $\phi_b$, we have the existence of an atom $x_0$ such that $\bigvee_{b \in B} \phi_b(x_0) = 1$. (Received October 26, 1970.)

682-06-7. THEODORE J. BENAC, U. S. Naval Academy, Annapolis, Maryland 21402 and CLAIRE ARCHAMBAULT, Regis College, Weston, Massachusetts 02193. Regular systems of left parameters in residuated lattice-ordered semigroups. Preliminary report.

Let $L$ be a local, strongly left join principal Noetherian semigroup with $p \neq u$ the unique maximal element in $L$. Theorem. $L$ is regular of dimension one iff $p \neq 0$ is left join principal and not a zero divisor in $L$. Theorem. Let $(m_1, \ldots, m_n)$ be a system of left parameters of $L$ such that $m_1$ is not a zero divisor in $L/m_0 \cup \ldots \cup m_{n-1}$, $1 \leq 1 \leq n$ and $m_0 = 0$. Then, $L$ is regular of dimension $n$ iff $L/m_1 \cup \ldots \cup m_r$ is regular of dimension $n-r$, $1 \leq r \leq n$. (Received October 28, 1970.)

682-06-8. ANDREW GLASS, University of Wisconsin, Madison, Wisconsin 53706. A characterisation of those abelian groups which can support a partial order with respect to which they are directed, Interpolation groups.

A partially ordered set $S$ is said to have the interpolation property if for all $x, y, z, t \in S$ such that $x, y \leq z, t$,
there exists \( s \in S \) such that \( x, y \leq s \leq z, t \). A partially ordered group will be called weakly semi-isolated if whenever \( nx > 0 \) for some positive integer \( n \), then \( x > 0 \). Let \( G \) be an abelian group and \( T(G) \) be the subgroup comprising \( 0 \) and all torsion (periodic) elements of \( G \). Theorem. An abelian group \( G \) can support a partial order with respect to which it is a directed, interpolation group if and only if \( G \) is either torsion-free or its quotient by \( T(G) \) is noncyclic. As a corollary of the proof, an abelian directed, interpolation group can be reordered so as to be a weakly semi-isolated, directed, interpolation group. (Received October 29, 1970.)

682-06-9. RONALD P. MORASH, University of Massachusetts, Amherst, Massachusetts 01002. The orthomodular identity and metric completeness of the coordinatizing division ring.

Let \( F \) be any division subring of the real quaternions \( H \). Let \( l^2(F) \) denote the linear space of all square-summable sequences from \( F \) and let \( L \) denote the lattice of all "\( \perp \)-closed" subspaces of \( l^2(F) \), where "\( \perp \)" denotes the orthogonality relation derived from the \( H \)-valued form \( (a, b) = \sum_{i=1}^{\infty} a_i b_i \) where \( a, b \in l^2(F) \), \( a = (a_i; i=1, 2, \ldots) \), \( b = (b_i; i=1, 2, \ldots) \). Then, \( L \) is complete, orthocomplemented, irreducible, atomistic, and separable, but \( L \) is orthomodular and \( M \)-symmetric if and only if \( F \) is either the reals, the complex numbers, or the quaternions. (Received October 30, 1970.)

682-06-10. W. ROBERT COLLINS, Mathematics and Statistics Department, University of Massachusetts, Amherst, Massachusetts 01002. The Foulis-Randall sample space viewed as a generalized set. Preliminary report.

Let \((X, \perp, \mathcal{G})\) and \((Y, \#_\perp, \mathcal{B})\) be (generalized) sample spaces (as in D. J. Foulis and C. H. Randall, "Operational statistics. I," mimeographed paper, Univ. of Mass.). Define \( f: (X, \perp, \mathcal{G}) \to (Y, \#_\perp, \mathcal{B}) \) to be a Dacey-homomorphism iff \( f \) maps \( X \) to \( Y \) and (1) \( f(\mathcal{G}) \subseteq \mathcal{O}(Y, \#_\perp, \mathcal{B}) \), (2) \( \{ f^{-1}(y); y \in Y \} \) is a compatible orthopartition of \((X, \perp, \mathcal{G})\), and (3) \( f(X) \) is partitive in \( Y \). Theorem. There is a functor \( F \) from the category of Dacey-homomorphisms to the category of residuated maps on posets which (a) sends a sample space \((X, \perp, \mathcal{G})\) to its orthologic \( L(X, \perp, \mathcal{O}) \), and (b) sends a Dacey-homomorphism \( f \) to the map \( f^*: A \perp \mapsto f(A) \#_\perp \) between logics. Moreover, it can be shown that \( F \) generalizes that functor \( G \) from the category of functions on sets to itself (where \( G(X) = 2^X \) and \( G(f: X \to Y): A \mapsto f(A) \)), especially with reference to the action of \( G \) on epi and monomorphisms. (Received November 2, 1970.)

682-06-11. DANG X. HONG, California State College, San Bernadino, California 92407 and Vanderbilt University, Nashville, Tennessee 37203. Covering relations among lattice varieties.

A lattice \( M_{[n]} \) is a two-dimensional \( n \)-atomed modular lattice. Suppose we have \( k \) lattices \( M_{[n_i]} \), \( 1 \leq i \leq k \), and let us denote the atoms of \( M_{[n_i]} \) by \( x_{i,1}, x_{i,2}, \ldots, x_{i,n_i} \) and its greatest and smallest elements by \( u_i \) and \( v_i \) respectively. If we identify \( [x_{i,1}, u_i] \) with \( [v_{i+1}, x_{i+1}, 1] \), \( 1 \leq i < k \), then the lattice obtained is denoted by \( M_{[n_1, n_2, \ldots, n_k]} \). Given \( W \) a variety, or equational class, of lattices, the family \( C(W) \) of all varieties of lattices that cover \( W \) is said to strongly cover \( W \) if any variety properly containing \( W \) contains some member of \( C(W) \). Theorem. \( C(W) \) is finite, can be effectively found and strongly covers \( W \) if \( W \) is generated by \( \mathcal{O}_2, \mathcal{O}_3^\omega \cup \mathcal{O}_2, \) or \( \mathcal{O}_2 \cup \mathcal{O}_2^3 \), where \( \mathcal{O}_i^\omega \) is the family of all modular lattices of dimension less than or equal to \( i \) for \( i = 1, 2, \ldots, \) \( \mathcal{O}_i \) is any finite family of finite modular lattices of dimension less than
or equal to 3, and $\mathcal{A}$ is any finite family of finite lattices of the form $M_{[n_1, n_2, \ldots, n_k]}$. Consequently, each of these varieties is finitely based. This result includes, as special cases, the results of G. Grätzer (Duke Math. J. 33(1966), 613-622) and of B. Jónsson (Math. Scand. 22(1968), 187-196). (Received November 3, 1970.)

08 General Mathematical Systems


**Definition.** Let $X$ be a set. A collection $\mathcal{A}$ of ordered pairs $(A, \mathcal{S})$ is said to form a decomposition theory for $X$ if: (1) $(A, \mathcal{S}) \in \mathcal{A}$ implies $A \subseteq X$ and $\mathcal{S}$ is a partition of $X - A$. (2) $(A, [X - A]) \in \mathcal{A}$ for all $A \subseteq X$. (3) If $(A, \mathcal{S})$ and $(B, \mathcal{S})$ are in $\mathcal{A}$, then $(A \cup B, \mathcal{S} \cap \mathcal{S}) \in \mathcal{A}$, where $\mathcal{S} \cap \mathcal{S} = [U \cap V | U \subseteq A, V \subseteq B]$. (4) If $(A, \mathcal{S}) \in \mathcal{A}$ and $\mathcal{S}'$ is a partition of $X - A$ such that $\mathcal{S}$ refines $\mathcal{S}'$, then $(A, \mathcal{S}') \in \mathcal{A}$. Decomposition theories occur in a wide variety of contexts. Moreover, any family of partitions of subsets of a set $X$ generates a decomposition theory for $X$. A decomposition theory for $X$ can also be associated in natural ways with two topologies on $X$; it is not yet clear whether, or under what conditions, these topologies are identical. A "hierarchy" of "separation" properties can also be found among decomposition theories. (Received September 21, 1970.)


A matroid is representable if there is a dimension-preserving imbedding of it in a vector space. A matroid is constructed which is not the union of finitely many representable matroids. It is shown that a matroid is representable iff every finite subset of it is, and that if a matroid is representable in vector spaces over fields of characteristic $p$ for infinitely many primes $p$, then it is representable in a vector space over a field of characteristic 0. Representation in a field extension (the imbedding changes dimension into transcendence degree) is also considered. Every vector space is shown to be representable in a field extension and the analogs of the results obtained for vector space representation are investigated. (Received October 21, 1970.)


It is known that the equational class generated by a primal algebra is dually equivalent as a category to the category of Boolean spaces (Math. Z. 110(1969), 180-198). We prove that if an equational class is dually equivalent as a category to the category of Boolean spaces, then it is generated by a primal algebra; and we also give an "in-the-small" generalization of this result. (Received October 30, 1970.)

682-08-4. WITHDRAWN.
A class \( \mathfrak{g} \) of models has property \((E^*)\) iff there exists in \( \mathfrak{g} \) a model \( A \) such that every \( B \in \mathfrak{g} \) can be embedded in an ultrapower of \( A \). \( \mathfrak{g} \) has the embedding property \((E')\) iff every family \( (A_t \in \mathfrak{T}) \) (every finite family) can be embedded in a model of \( \mathfrak{g} \). Theorem. Assume that the type of \( \mathfrak{g} \) is countable and that \( \mathfrak{g} \) is closed under formation of ultraproducts and under the union of countable chains. Then \((E^*), (E)\) and \((E')\) are equivalent. Corollary. Under the same conditions as above, property \((E')\) implies that the class of all submodels of models in \( \mathfrak{g} \) is elementary. (Received November 4, 1970.)
Equality of cardinal numbers of minimal generators under strong spans. Preliminary report.

Let $P$ be a function $2^V - 2^V$ for some set $V$. If $S$ and $T$ are sets, let $|S|$ denote the cardinal number of $S$, and $S - T = \{ x \in S : x \notin T \}$. Let (A) be the property that if $X \subseteq V$, then $X$ has a maximal $P$-independent subset, that is, a maximal subset $Y$ such that $x \notin P(Y - \{x\})$ for all $x \in Y$; let (B) be the property that if $X$ and $Y$ are $P$-equivalent subsets of $V$, that is, $P(X) = P(Y)$, then $P(X)$ is a subset of $\bigcup \{P(Z) : Z \subseteq Y, |Z| \leq |X| \}$; let (C) be the property that if $X$ and $Y$ are $P$-independent $P$-equivalent subsets of $V$, then $|X| = |Y|$. If $X \subseteq V$, then the minimal generators of $P(X)$ are the subsets of $V$ which are $P$-independent and $P$-equivalent to $X$. $P$ is a strong span if $P$ is a closure structure (operation) having the exchange property and property (A). It is known that if $P$ is a closure structure having the exchange property, then any two $P$-independent $P$-equivalent subsets of $V$, one of which is finite, have the same cardinal number. Lemma 1. If $P$ has properties (A) and (C), then $P$ has property (B). Lemma 2. If $P$ is a closure structure having the exchange property and $U \subseteq V$, then a subset of $U$ is a maximal $P$-independent subset of $U$ if and only if it is $P$-independent and $P$-equivalent to $U$. Lemma 3. If $P$ is a closure structure having property (B) while $X$ and $Y$ are $P$-equivalent subsets of $V$ and $Y$ is $P$-independent, then $|Y| \leq |X|^2$. Theorem. If $P$ is a strong span, then $P$ has property (B) if and only if $P$ has property (C).

10 Number Theory

For some 65 years, since A. E. Western proved it composite, the seventh Fermat number $F_7 = 2^{27} + 1$ has withstood repeated factorization attempts. In 1965 G. D. Johnson proved that this 39 digit number is the product of two primes by searching unsuccessfully for factors to its cube root. On September 13, 1970 the factorization was finally achieved at UCLA on the IBM 360/91. The method and procedures that were used will be discussed. (Received September 28, 1970.)

Arithmetic formulas for some complementing sequences. If $C$ is a set of integers, two subsets $A$ and $B$ of $C$ are said to be complementing subsets of $C$ in case every $c \in C$ is uniquely represented in the sum $C = A + B = \{ x \mid x = a + b, a \in A, b \in B \}$. The structure of complementing subsets has been studied by de Bruijn, C. T. Long and others. Long characterized all pairs $A$ and $B$ when $C$ was the set $\{0, 1, \ldots, n - 1\}$ where $n$ is a positive integer. If $C(n)$ denotes the number of complementing subsets of $\{0, 1, \ldots, n - 1\}$ Long showed that $2C(n)$ was equal to $\sum C(d)$ where $d$ ranges over the divisors of $n$. We give an entirely different proof for the last relation and, using the technique developed for this proof, show that Long's formula can be extended to sets other than $\{0, 1, \ldots, n - 1\}$. (Received October 19, 1970.)
A problem of Stöhr in additive number theory.

A strictly increasing sequence of positive integers is a basis (asymptotic basis) of order \( h \) if every (every sufficiently large) positive integer can be represented as the sum of not more than \( h \) terms of the sequence. A basis (asymptotic basis) of order \( h \) is minimal if no proper subsequence of it is a basis (asymptotic basis) of order \( h \). Stöhr proved that minimal bases exist of all orders \( h \geq 1 \), and that every basis of order \( h \) contains a subsequence which is a minimal basis of order \( h \). The analogous questions for asymptotic bases were unsolved. In this paper it is proved, by the construction of examples, that minimal asymptotic bases exist of all orders \( h \geq 2 \), but that not every asymptotic basis of order \( h \) contains a minimal asymptotic basis of order \( h \). (Received October 27, 1970.)

Pythagorean and triangular number triples.

Let \( P(r, s, t) \) be a primitive Pythagorean integer triple with \( r = m^2 - n^2 \), \( s = 2mn \), \( t = m^2 + n^2 \). Let \( t(a, b, c) \) be an integer triangular number triple with \( t_a + t_b = t_c \). For every Pythagorean triple there exists two triangular number triples \( t(a, b, c) \) such that \( tkr+a + ksb = tkt+c \); \( k \) an integer. If \( t(a_1, b_1, c_1) \) and \( t(a_2, b_2, c_2) \) are two such triangular triples, then also \( a_1 + a_2 = r - 1, \ b_1 + b_2 = s - 1 \) and \( c_1 + c_2 = t - 1 \). (Received October 30, 1970.)

Binary quadratic forms of determinant-pq.

The author proves the following Theorem. If \( p \equiv q \equiv 1 \pmod{4} \) are primes such that \( (p | q) = 1 \), then \( x^2 - pqy^2 \) represents \(-1\) if \( (p | q)_4 = (q | p)_4 = 1 \); \( p \) if \( (q | p)_4 = 1 = - (p | q)_4 \); \( q \) if \( (p | q)_4 = 1 = - (q | p)_4 \). If \( (p | q)_4 = 1 = (q | p)_4 \), any of the three may be represented. The proof avoids the use of class field theory. (Received October 30, 1970.)

Generalized Dedekind-Rademacher sums. Preliminary report

H. Rademacher (Acta Arith. 9(1964), 97-105) introduced the sum \( s(h, k; x, y) = \sum_{m=0}^{k-1} (((m+y)/k) \cdot (((h(m+y)/k)+x)) \) with \( h, k \) integers; \( k > 0; x, y \) real; and \( ((x)) = 0 \) if \( x \) is an integer and otherwise \( ((x)) = x - [x] - \frac{1}{2} \). He obtained a reciprocity formula which generalized that for the Dedekind sum \( s(h, k) = s(h, k; 0, 0) \). In the present paper a new sum is defined \( s_u(h, k; x, y) = \sum_{m=0}^{k-1} \bigl( \frac{y}{k} \bigr) \cdot \bigl( \frac{(h(m+y)/k)+x}{k} \bigr) \). Here \( h, k, x, \) and \( y \) are as in the Dedekind-Rademacher sum and \( \bigl( \frac{y}{k} \bigr) = B_2(x - [x]) \). L. Carlitz (Math. Z. 85(1964), 83-90) considered the case \( t = 1 \). Extensions of many previously known results are obtained for more general cases. Among the results extended are many by Rademacher and Whiteman (Amer. J. Math. 63(1941), 377-405). The methods are principally arithmetic although analytic techniques are employed to obtain some of the results. (Received November 2, 1970.)
If $S$ is an arbitrary sequence of positive integers, define $P(S)$ to be the set of all integers which are representable as a sum of distinct terms of $S$. Call a sequence $S$ complete if $P(S)$ contains all sufficiently large integers, and subcomplete if $P(S)$ contains an infinite arithmetic progression. Then the following theorem is proved: Let the $n$th term of the integer sequence $S$ have the form $f(n) + t(n)$, where $f(n)$ is a polynomial and where for some $M > 0$ and $0 < \beta < 1/2$, $|t(n)| \leq Mn^\beta$ for every $n$. Then $S$ is subcomplete. It is further shown that $S$ is complete if, in addition, for every prime $p$ there are infinitely many terms of $S$ not divisible by $p$. These results are then extended to considerably more general sequences. (Received October 30, 1970.)
(c_1, c_2, \ldots, c_n) at the center of a fundamental parallelopiped. This has been proved only for n = 1, 2, 3, 4.

Let d be a squarefree positive integer, Q(\sqrt{-d}) the extension of the field Q of rational numbers by \sqrt{-d}, N(\sqrt{-d}) the ring of integers of Q(\sqrt{-d}). An n-dimensional lattice over N(\sqrt{-d}), called a d-lattice, is an N(\sqrt{-d})-module generated by n vectors A_1, A_2, \ldots, A_n in n-dimensional complex space \mathbb{C}^n, linearly independent over the complex field \mathbb{C}.

The determinant of a d-lattice is the determinant of the matrix whose columns are the A_i. An analogue of Minkowski's conjecture may be formulated for d-lattices. Mahler (J. London Math. Soc. 15(1940), 215-236) used the theory of reduction of Hermitian forms to prove such an analogue for 2-dimensional d-lattices for d = 1, 2, 3. The present author proves the Theorem. Let L be a 3-dimensional 1-lattice (resp. 3-lattice) of determinant 1. Given the point (c_1, c_2, c_3) of \mathbb{C}^3 there is a point (p_1, p_2, p_3) of L with the property that \[ \prod_{j=1}^{3} |p_j - c_j| \leq (\sqrt{8})^{-1} (\text{resp. } (3/3)^{-1}). \]

It is proved that the Riemann hypothesis holds if and only if, sufficiently high in the critical strip, \[ |\zeta(s)| \] increases as s moves left from the critical line. The method is the same as for the result in these Notices 17(1970), 949, and consists of showing, using the functional equation and the Hadamard factorization for \zeta(s), that on the Riemann hypothesis \[ \text{Re } \frac{\zeta'}{\zeta} < -\left(\log t\right)/2 + K + O(1/t) \] in the left half of the critical strip. (Received November 2, 1970.)

Let L be a lattice in R^3 and let P = (a, b, c) \in L. The notation x(P) = a, y(P) = b, z(P) = c is used. Let P, Q, R \in (L - 0). P is called a Minkowski convergent if \{(x, y, z): |x| < |x(P)|, |y| < |y(P)|, |z| < |z(P)| \} \cap L = 0.

(P, Q, R) is called a triple if \{(x, y, z): |x| < |x(P)|, |y| < |y(Q)|, |z| < |z(R)| \} \cap L = 0. A convergent Q is a neighbor of a convergent P if P and Q occur in some triple. The Hurwitz result on convergents in two dimensions is well known. The difficulty in applying convergents to three dimensional problems is discussed in Cassels ("An introduction to the geometry of numbers," Springer-Verlag, Berlin, 1959, p. 303). Conjecture. A Hurwitz-type result holds in three dimensions, i.e., if d(L) = 7 and P is a convergent, then P or one of its neighbors lies in \[ |XYZ| \leq 1. \]

Theorem 1. Let (P, Q, R) be a linearly independent triple and let d(L) = 1. At least one of P, Q, R lie in the region \[ |XYZ| \leq 1/3 \] and the result is best possible. Theorem 2. Let P be a convergent of L and let d(L) = 7. Let K: |XYZ| \leq 1. Applying an automorph of K, we may assume P = (A, A, A).

Then there is a point S = (x, y, z) \in L, S \neq 0, such that S \in K and \[ (x-y)^2 + (y-z)^2 + (z-x)^2 \leq 14/A. \]

It is still an open question whether or not a pair of relatively prime odd amicable numbers. Preliminary report.

On the number of primes which divide a pair of relatively prime odd amicable numbers. Kanold [Arch. Math. 4(1953), 399-401] proved that if m and n are a pair of relatively prime amicable numbers then
mn is divisible by at least 21 different primes. In the present paper this lower bound for the number of primes dividing mn is improved to 22 for the case mn odd. (Received November 3, 1970.)


Every real number \(0 \leq x \leq 1\) leads to the expansion \(x = \sum e_k g^{-k}\) where \(e_k = 0\) or \(1\) if \(1 < g \leq 2\) and the algorithm used to obtain the \(e_k\)'s is as follows. Let \(x = x_1\), \(e_k < gx_k \leq e_k + 1\) and \(x_{k+1} = gx_k - e_k\). It is well known that the digits \(e_k\), considered as random variables on the probability space \((X, R, P)\), are stochastically independent for \(g = 2\). This is no longer true if \(1 < g < 2\). To get information on the \(e_k\)'s, drop the terms in the expansion for which \(e_k = 0\), thus we get \(x = \sum g^{-n(k, x)}\). Since the \(e_k\)'s and the \(n(k, x)\)'s are related by \([\text{at least} k \text{ out of} e_1, e_2, \ldots, e_N \text{ are equal to} 1] = \{ n(k, x) < N \}\), it is sufficient to investigate the distribution of \(n(k, x)\).

Decompose \(n(k, x) = m_1(x) + m_2(x) + \ldots + m_k(x)\), i.e. \(m_j(x) = n(j, x) - n(j-1, x)\), with \(n(0, x) = 0\). The joint distribution of the \(m_k(x)\)'s is obtained and by that, a general formula is given for the distribution of \(n(k, x)\), and limit relations are deduced. The following neat result is obtained from it: the random variables \(m_1(x), m_2(x), \ldots\) are independent if, and only if, there exists an integer \(a \geq 1\) such that \(g^{a+1} - g^a = 1\). For the sufficiency part a direct and simple proof is given. In the general case, the method is to reduce the problem of distribution to a combinatorial problem of counting intervals of given types. This is made possible by a theorem of Parry (Acta Math. Acad. Sci. Hungar. 11(1960), 401-416) which gives a necessary and sufficient condition for a series \(\sum u_k g^{-k}\) to be the expansion of its sum by the algorithm specified earlier. (Received November 5, 1970.)


A proof technique first introduced by Ivan Niven ("Diophantine approximations", Interscience Publishers, New York, 1963) to prove results on the product of linear forms is shown to also yield similar proofs for the classical theorems of Hurwitz (real case) and Ford (complex case) on the approximation of irrational numbers by rational numbers. In each case, a geometric lemma is required. Extensions to other settings are anticipated. (Received November 5, 1970.)

12 Algebraic Number Theory, Field Theory and Polynomials

682-12-1. RAPHAEL FINKELSTEIN, Bowling Green State University, Bowling Green, Ohio 43402 and HYMIE LONDON, McGill University, Montreal 110, Quebec, Canada. On triangular numbers which are sums of consecutive squares.

In 1968 H. E. Thomas Jr. [Amer. Math. Monthly 75(1968), 1018] proposed the following problem: Find all the integer solutions \((n, r)\) of the equation \(\sum_{i=1}^{n} i = \sum_{j=1}^{r} j^2\). In this paper we shall prove the following Theorem. The only integer solutions of equation (*) are \((n, r) = (1, 1), (10, 5), (13, 6)\) and \((645, 85)\). (Received September 3, 1970.)

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682-12-2. CHARLES H. FRANKE, Seton Hall University, South Orange, New Jersey 07079. Reducible linear difference operators. Preliminary report.

Assume that \( f \) is a linear homogeneous difference polynomial of effective order \( n \) with coefficients in a difference field \( k \) and that \( M \) is a difference overfield of \( k \) obtained by adjoining a fundamental system for \( f \) to \( k \). \( f \) is reducible at \( q \) to order \( r \) if \( r < n \) and there is a \( g \in k[y] \), \( g(y) = \sum_{i=0}^{r} B(i) y^i \), \( g \neq 0 \), so that \( g(\beta) = 0 \) whenever \( f(\beta) = 0 \). The reduced order of \( f \) is the smallest \( r \) with \( f \) reducible at \( q \) to order \( r \) or \( n \) if no such \( r \) exists. \( f \) is reducible if the reduced order of \( f \) is less than \( n \). By direct computational methods it is shown that the reduced order of \( f \) is the maximal \( r \) so that \( r \) elements of a fundamental system for \( f \) are linearly independent over periodic elements. The concept of reducibility is shown to be independent of the ground field \( k \), and a test for reducibility at \( q \) to order \( r \) for fixed \( q \) and \( r \) is given. The Galois theory is applied to obtain a characterization of reducibility and a necessary condition for reducibility in the form of a set of matrices having a joint eigenvector. Sufficient results are obtained to describe reducible equations of order two or three. (Received October 13, 1970.)


A system of polynomials \( f_1(x_1, \ldots, x_n), \ldots, f_m(x_1, \ldots, x_n) \), \( 1 \leq m \leq n \), over a Galois field \( K = GF(q) \) is said to be orthogonal if the system of equations \( f_1(x_1, \ldots, x_n) = k_1, \ldots, f_m(x_1, \ldots, x_n) = k_m \) has exactly \( q^n - m \) solutions in \( K^n \) for each \( (k_1, \ldots, k_m) \in K^m \). This generalizes a notion of orthogonality by Kurbatov and Starkov and the notion of a permutation polynomial in several variables. Necessary and sufficient conditions for orthogonality in terms of character sums and permutation polynomials are given. If \( n = mr \) with an integer \( r \), then there is a one-to-one correspondence between orthogonal systems \( f_1(x_1, \ldots, x_n), \ldots, f_m(x_1, \ldots, x_n) \) in \( K \) and permutation polynomials in \( r \) variables over \( GF(q^m) \). Among other results, the total number of orthogonal systems of \( m \) polynomials in \( n \) variables is determined. The paper is related to the work of Carlitz [Trans. Amer. Math. Soc. 75 (1953), 405-427] and to results of the author on permutation polynomials in several variables [Proc. Japan Acad., to appear]. (Received October 23, 1970.)

682-12-4. RONALD P. BROWN, Simon Fraser University, Burnaby, British Columbia, Canada and HOYT D. WARNER, Vanderbilt University, Nashville, Tennessee 37203. Quadratic extensions of linearly compact fields.

A group valuation on an abelian group \( G \) (written additively) is a map \( W: G \to \mathbb{R} \cup \{0\} \) where \( \mathbb{R} \) is a linearly ordered set, satisfying \( W(a + b) \geq \min\{W(a), W(b)\} \), \( W(-a) = W(a) = \infty \) iff \( a = 0 \). Let \( F \) be a linearly compact (or, maximal) valued field, \( K \) a quadratic extension of \( F \). The field valuation \( v \) on \( F \) is shown to induce a group valuation on the norm factor group \( F^*/N_{K/F}(K^*) \) of \( K \) over \( F \), and the norm factor group is then calculated explicitly as a valued group—in fact as a naturally valued subdirect product of groups. Applications and generalizations of this structure theory are made to cyclic extensions of prime degree, to square (and pth power) factor groups, to generalized quaternion algebras, and to quadratic extensions of arbitrary fields. (Received October 27, 1970.)
682-12-5. LARRY J. GERSTEIN, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. Nonubiquity of quadratic forms of class number 1. Preliminary report.

Let $K$ be a totally real algebraic number field, and let $\mathfrak{o}$ be its ring of algebraic integers. Given a natural number $n$, the question is asked: Does there exist an $\mathfrak{o}$-lattice of rank $n$ whose class is equal to its genus and whose underlying quadratic $K$-space is definite? In the case $\mathfrak{o}$ is a principal ideal domain the following is shown: Theorem. There is a natural number $n_0$ such that for $n \geq n_0$ every $n$-ary definite $\mathfrak{o}$-lattice has class number at least 2. This result extends earlier work on quadratic forms over $\mathbb{Z}$ by Magnus and Watson.

The proof uses an extension of the notion of "minimum" to the setting of forms over number fields. (Received October 28, 1970.)

682-12-6. WILLIAM TED STOUT, JR., University of Hawaii, Honolulu, Hawaii 96822. Corresponding residue systems in normal extensions. Preliminary report.

Let $K$ and $K'$ be number fields with $L = K \cdot K'$ and $F = K \cap K'$. Suppose that $K/F$ and $K'/F$ are normal extensions of degree $n$. Let $\mathfrak{p}$ be a prime ideal in $\mathcal{O}_L$ and suppose that $\mathfrak{p}$ is totally ramified in $K/F$ and in $K'/F$. Let $\pi$ be a prime element for $\mathfrak{p}_K = \mathfrak{p} \cap K$, and let $f(x)$ be the minimum polynomial for $\pi$ over $F$. Suppose that $\mathfrak{p}_K \cdot \mathcal{O}_L = (\mathfrak{p}^e)$. Then $M(\mathfrak{p}^e; K, K') = \min\{m, e \cdot (t + 1)\}$, where $t = \min\{t(\mathfrak{p}_K; K/F), t(\mathfrak{p}_K; K'/F)\}$ and $m$ is the largest integer such that $(\mathfrak{p}_K)^{\lambda m/e} \cap \mathcal{O}_K \neq \{0\}$. (For terminology and notation, see L. R. McCulloh and W. T. Stout, Jr., "Corresponding residue systems in cyclic extensions of prime degree over algebraic number fields," J. Number Theory 1(1969), 312-325.) If we assume in addition to the above hypotheses that $n = p^s$, a prime power, that $\mathfrak{p}$ divides $p$ and is totally ramified in $L/F$, then $M(\mathfrak{p}^e; K, K') = p^{s-1}[\lambda p-1]t^p$, with $t = t(\mathfrak{p}; L/F)$. (Received October 29, 1970.)

682-12-7. K. NAGESWARA RAO, North Dakota State University, Fargo, North Dakota 58102. On the inverse of Carlitz's $\eta$-sum.

Carlitz [Duke Math. J. 14(1947), 1105-1120] introduced the Sum $\eta(A, H)$ over the polynomials with coefficients from $GF(p^n)$. In this paper the multiplicative property of this sum with respect to both the primary arguments $A$ and $H$ is used to find its Dirichlet's inverse $K(A, H)$ which satisfies the property that

$$\sum_{A} \eta(A/D, H/D)K(D, A)$$

is zero except when $A = H \equiv \text{the identity and is 1 in the latter case};$ here the double sum runs over all the primary divisors $D, \Delta$ of $A$, $H$ respectively. In particular it is shown that $K(A, H) = \sum_{A} \mu(A/D) \mu(D/D)$, where $\mu$ is the Möbius function and the summation is over all primary common divisors of $A$ and $H$. Several identities involving $K(A, H)$ are also obtained in terms of known arithmetical functions. (Received November 2, 1970.)

682-12-8. ROBERT M. McCONNEL, University of Tennessee, Knoxville, Tennessee 37916. Functions over finite fields satisfying coordinate $\psi$-conditions.

Let $F$ be a finite field of order $q = p^n$, $p$ a prime. Let $d_1, d_2$, and $d_3$ be any divisors of $q - 1$ greater than one. Set $q - 1 = m_1d_1 = m_2d_2 = m_3d_3$. Let $\psi_1(x) = x^{m_1}, \psi_2(x) = x^{m_2},$ and $\psi_3(x) = x^{m_3}$ for all $x \in F$.  

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Theorem. Let $\lambda_1 \cdot \lambda_2 \cdot \lambda_3$ be fixed elements of $F$ such that $d_1 = \lambda_2 \cdot \lambda_3$ and $d_3 = 1$. Let $f$ be any function from $f : F \times F \times F$ into $F$ such that $f(x,0,0) = f(0,y,0) = f(0,0,z) = 0$ for all $x, y, z \in F$. Then

$$
\psi_1 [f(x,y,z) - f(u,y,z) - f(x,y,v) + f(u,y,v)] = \lambda_1 \psi_1 [(x-u)(y-v)]
$$

and

$$
\psi_2 [f(x,x,z) - f(v,x,z) - f(x,y,w) + f(y,x,w)] = \lambda_2 \psi_2 [(x-v)(z-w)]
$$

and

$$
\psi_3 [f(x,y,z) - f(x,v,z) - f(x,y,v) + f(x,v,v)] = \lambda_3 \psi_3 [(y-v)(z-w)]
$$

for all $x, y, z, u, v, w \in F$ if and only if $f(x,y,z) = a_1 x^i + a_2 y^j + a_3 z^k$, where $0 \leq i, j, k \leq n$.

This theorem has been extended to functions of $m$ variables satisfying $m$ conditions. The proofs of these theorems use results and techniques found in [Acta Arith. 8(1963), 127-151; Duke Math. J. 36 (1969), 465-472]. (Received November 2, 1970.)

682-12-9. RICHARD D. WEINER, Washington University, St. Louis, Missouri 63130. Lifting Galois groups. Preliminary report.

Let $F$ be a field of characteristic $p \neq 0$, and let $\Omega$ be an algebraic closure of $F$. Let $K \subset \Omega$, where $K/F$ is a finite Galois extension with Galois group $G$ whose order $m$ is not divisible by $p$. There exists (Jacobson, "Lectures in abstract algebra," Vol. III, Van Nostrand, Princeton, N.J., 1964, Chapter III) a bijective correspondence between the finite abelian $p$-extensions $L/K$ where $L \subset \Omega$ and the subgroups of certain additive factor groups of Witt vector rings over $K$. In the principal result of this paper, we use this correspondence to develop a necessary and sufficient condition that a finite abelian $p$-extension $L$ of $K$ admit a lift $G$ of $G$ (i.e., a group $G = (a_1) \in G$ of automorphisms of $L/F \supset \sigma \mid K = \sigma, \forall \sigma \in G$) and we describe all such possible lifts. We also prove that $L$ admits a lift of $G$ iff $L/F$ is Galois. If $F$ possesses a non-Archimedean discrete rank one valuation, and if $F$ contains all the $m$th roots of 1, then we show that there are infinitely many cyclic extensions $L/K$ of degree $p$, where $L$ admits a lift of $G$. Let $A$ be a cyclic $p$-algebra over $K$ with a maximal cyclic subfield $L$ which admits a lift of $G$. We give a necessary and sufficient condition that $A$ admit a lift of $G$. Finally, we give a sufficient condition that a $p$-algebra be cyclic. (Received November 3, 1970.)

682-12-10. MANFRED ARMBRUST, University of Houston, Houston, Texas 77004. On elementary equivalence of $\omega$-pseudo-complete Hensel fields.

Ribenboim proved that two $\omega$-pseudo-complete Hensel fields with common valuation group and common residue class field $K$ of characteristic 0 are elementarily equivalent in the language of valued fields if $\text{Ext}^1_\mathbb{Z}(K', A) = 0$ for every torsion-free abelian group $A$. One can show that this condition can be replaced by the requirement that the two fields possess cross-sections. Since the existence of a cross-section for a Hensel field $(F, v)$ with residue class field $K$ of characteristic 0 follows from $\text{Ext}^1_\mathbb{Z}(K', v(F')) = 0$, and one can construct an even pseudo-complete Hensel field with a cross-section and residue class field of characteristic 0 such that $\text{Ext}^1_\mathbb{Z}(K', v(F')) = 0$, one obtains a strictly stronger theorem. (Received November 4, 1970.)

682-12-11. CARTER WAD, Texas Tech University, Lubbock, Texas 79409. Representations of decomposable forms. Preliminary report.

If $\varphi(X_1, \ldots, X_n)$ and $\psi(Y_1, \ldots, Y_m)$ are rational irreducible decomposable forms of degree $d$ and if
X = TY is a rational (integral) linear substitution that carries \( \phi \) into a rational (integral) multiple \( c \psi \) of \( \psi \). We say that \( T \) is a rational (integral) representation of scale \( c \) of \( \psi \) by \( \phi \). By an application of the unique factorization theorem for polynomial rings we show the following results. **Theorem.** The rational (integral) representations of \( \psi \) by \( \phi \), where the scale is allowed to vary, correspond to the members of a finite collection of linear subspaces (submodules) of the splitting field of \( \phi \) and \( \psi \). Moreover, if \( \phi \) and \( \psi \) do not represent zero, then \( m = n \) and the correspondence is one-to-one. **Theorem.** If \( \tau \) is a linear endomorphism of an algebraic number field \( K \) satisfying the identity \( \text{norm}(\tau x) = m \cdot \text{norm}(x) \), \( m \neq 0 \), then there is a unique automorphism \( \sigma \) of \( K \) and a unique nonzero \( t \) in \( K \) such that \( \tau x = t(\sigma x) \). This is a generalization of the basic lemma used in the theory of composition of binary quadratic forms. **Theorem.** If \( \phi(X_1, \ldots, X_n) \) does not represent zero and \( n \) and \( d \) are co-prime, then \( \phi \) has only a finite number of rational automorphs. (Received November 5, 1970.)

13 Commutative Rings and Algebras

682-13-1. ROBERT GILMER, Florida State University, Tallahassee, Florida 32306. On polynomial rings over a Hilbert ring. Preliminary report.

Let \( K \) be a field with algebraic closure \( \overline{K} \), and let \( \{X_\lambda\}_{\lambda \in \Lambda} \) be a set of indeterminates over \( \overline{K} \). These six conditions are equivalent. (1) \( K[\{X_\lambda\}] \) is a Hilbert ring; (2) \( K[\{X_\lambda\}]/M \) is algebraic over \( K \) for each maximal ideal \( M \) of \( K[\{X_\lambda\}] \); (3) each maximal ideal of \( K[\{X_\lambda\}] \) is of the form \((\{X_\lambda - t_\lambda\})\) for some \( t_\lambda \)'s in \( \overline{K} \); (4) \( |\Lambda| < |K|^{1/0} \); (5) if \( K[\{a_\lambda\}_{\lambda \in \Lambda}] \) is a field, then each \( a_\lambda \) is algebraic over \( K \); and (6) if \( \{f\} \cup \{f_a\} \) is a subset of \( K[\{X_\lambda\}] \) and \( f \) vanishes at each common zero of the \( f_a \)'s over \( \overline{K} \), then \( f \notin \{(f_a)\} \). More generally, if \( R \) is a commutative ring with identity and if \( \{X_\lambda\}_{\lambda \in \Lambda} \) is a set of indeterminates over \( R \), then these three conditions are equivalent. (1) \( R[\{X_\lambda\}] \) is a Hilbert ring; (2) \( R \) is a Hilbert ring and \( |\Lambda| < |R/M|^{1/0} \) for each maximal ideal \( M \) of \( R \); and (3) \( R \) is a Hilbert ring and \( (R/M)[\{X_\lambda\}] \) is a Hilbert ring for each maximal ideal \( M \) of \( R \). Both these results are well known when \( \Lambda \) is finite. (Received August 4, 1970.)

682-13-2. PHILIP B. SHELDON, Wisconsin State University, Whitewater, Wisconsin 53190. How changing \( D[[x]] \) changes its quotient field.

Let \( D[[x]] \) be the ring of formal power series over the commutative integral domain \( D \). It is shown that changing \( D[[x]] \) to \( D[[x/a]] \) changes (i.e. increases) the quotient field by an infinite transcendence degree over the original field whenever \( \cap_{i=1}^{\infty} a^i D = 0 \). From this it follows that if \( D_1 \) and \( D_2 \) are two distinct rings between the integers and the rational numbers, with \( D_1 \) contained in \( D_2 \), then the change in the ring of coefficients from \( D_1[[x]] \) to \( D_2[[x]] \) again yields a change in the quotient fields by an infinite transcendence degree. More generally, it is shown that \( D \) is completely integrally closed iff any increase in the ring of coefficients yields an increase in the quotient field of \( D[[x]] \). Moreover, \( D \) is a one-dimensional Prüfer domain iff any change in the ring of coefficients from one overring of \( D \) to another overring of \( D \) yields a change in the quotient field of the respective power series rings. Finally, it is shown that many of the domain properties of interest are really properties of their divisibility groups, and some examples are constructed by first constructing the required divisibility groups. (Received October 21, 1970.)
The geometric description of some important schemes has been illustrated by Mumford (cf. "Lectures on curves on an algebraic surface") using the division of points via the dimension of their stalks and via their images in Spec (Z). An important case is the arithmetic surface Spec (Z[X]) not only because it is a special case of the integral affine space \( A^n_Z \) but also because the prime ideals in \( Z[X] \) are well known to those interested in some simple number theoretic questions. While the generic point \([X^2+1]\) lies on the "straight line" containing all the points \([p,X+1]\) the generic point \([X^2+1]\) lies on the "curve" containing only some of the points \([p,X+k]\). However, the pattern followed by the values of \( k \) in \( X+k \) is interesting. The curve is "tangent" to the fibre of the subvariety \( V(2) \). The values of \( k \) occur in pairs whose sum is the prime \( p \). The first seven values of \( k \) are the first seven Fibonacci numbers which give rise to the possibility of a congruence relation to determine the values of \( k \). The determination of the values of \( k \) and similar constants occurring in other irreducible polynomials related to generic points defined by irreducible polynomials can be readily programmed. (Received October 23, 1970.)

682-13-4. DAVID E. PEERCY, University of West Virginia, Morgantown, West Virginia 26505. The complete Baer extension of a semiprime ring. Preliminary report.

It is well known that each Boolean algebra has a completion which is unique up to isomorphism. We extend this result to the class of semiprime rings (commutative with an identity). Specifically, let \( R \) be a semiprime ring. If \( a \) is an element of \( R \) and \( X_a(R) \) denotes the set of all minimal prime ideals \( x \) of \( R \) such that \( a \) is not in \( x \), then the family of all \( X_a(R) \), denoted by \( B(R) \), is a disjunctive semilattice of sets. The pair \((R^+, \phi)\) is called a complete Baer extension of \( R \) if (i) \( R^+ \) is a ring in which the annihilator of each subset is generated by an idempotent, and \( \phi \) is an isomorphism of \( R \) into \( R^+ \) which preserves equal annihilators, i.e., \( \phi^*(a) = \phi^*(b) \) whenever \( a^* = b^* \) for \( a, b \) in \( R \); (ii) the subring of \( R^+ \) generated by \( \phi(R) \) and the idempotents in \( R^+ \) is all of \( R^+ \); and (iii) the mapping which sends \( X_a(R) \) to \( X_{\phi(a)}(R^+) \) is a semilattice isomorphism of \( B(R) \) onto an order dense subsemilattice of \( B(R^+) \). Theorem. Each semiprime ring has a complete Baer extension which is unique up to isomorphism. For \( m \) an arbitrary cardinal number, we define in an analogous way the \( m \)-Baer extension of a semiprime ring and obtain the result that each semiprime ring has an \( m \)-Baer extension which is unique up to isomorphism. (Received October 28, 1970.)

682-13-5. CRAIG A. WOOD and DENNIS E. BERTHOLF, Oklahoma State University, Stillwater, Oklahoma 74074. u-rings for which each proper homomorphic image is a multiplication ring.

Let \( R \) be a commutative ring. \( R \) is called a multiplication ring if whenever \( A \) and \( B \) are ideals of \( R \) with \( A \) contained in \( B \), then there is an ideal \( C \) of \( R \) such that \( A = BC \). If every proper homomorphic image of \( R \) is a multiplication ring, we say that \( R \) satisfies (Hm). (See Abstract 663-358, these Notices, 16(1969), 189.) Commutative rings with identity satisfying (Hm) are characterized by Wood (J. Sci. Hiroshima Univ. Ser. A - I Math. 33(1969), 85-94). \( R \) is called a \( u \)-ring if the only ideal \( A \) of \( R \) such that \( \sqrt{A} = R \) is \( R \) itself. Theorem 1. If \( R \) is a primary \( u \)-ring, then \( R \) contains an identity. Theorem 2. Let \( R \) be a \( u \)-ring. If \( R \) is not a primary ring, then \( R \) satisfies (Hm) if and only if \( R \) is a multiplication ring. (Received October 28, 1970.)
Let $R$ denote a commutative ring with identity. We say that $R$ has dimension $n$ provided there exists a chain $P_0 \subset P_1 \subset \ldots \subset P_n$ of $n+1$ prime ideals of $R$, where $P_n \neq R$, but no such chain of $n+2$ prime ideals.

If $A$ is an ideal of $R$, then $A[[X]] = \{ f(X) = \sum_{i=0}^{\infty} a_i X^i \mid a_i \in A \text{ for each } i \}$ and $AR[[X]]$ denotes the ideal of $R[[X]]$ which is generated by $A$. Theorem. The following conditions are equivalent and imply that $R[[X]]$ has infinite dimension: (1) There exists an ideal $M$ of $R$ such that for any positive integer $k$ and any finitely generated ideal $B$ of $R$, where $B \subseteq M$, $a^k \notin B$ for some $a \in M$. (2) There exists an ideal $A$ of $R$ such that $A[[X]] \neq \sqrt{AR[[X]]}$. (3) There exists a prime ideal $P$ of $R$ such that $P[[X]] = \mathfrak{p}R[[X]]$. Example 1. If $V$ is a rank one nondiscrete valuation ring, then $\dim V[[X]] = \infty$. Example 2. If $D$ is an almost Dedekind domain which is not Dedekind, then $\dim D[[X]] = \infty$.

682-13-7. MONTE B. BOISEN, JR., Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. Properties of the large quotient ring. Preliminary report. Throughout this paper the term ring will denote a commutative ring with unity. Let $R$ be a ring with total quotient ring $K$. Define $R[A] = \{ x \in K \mid \text{there exists } y \in R \setminus A \text{ such that } xy \in R \}$ for any ideal $A$ of $R$. In case $A$ is a prime ideal, $R[A]$ is said to be a large quotient overring of $R$ [Griffin, Malcolm, "Prüfer rings with zero divisors," J. Reine Angew. Math. 239/240 (1970), 55-67]. Let $\mathfrak{C}$ be a set of ideals of $R$. The pair $(R, \mathfrak{C})$ is said to satisfy the containment property (property $c$) in case $A, B \in \mathfrak{C}$ if and only if $R[B] \subseteq R[A]$ whenever $A, B \in \mathfrak{C}$. Theorem 1. Let $R$ be a ring. Then there exists a set of ideals $\mathfrak{B}$ such that $\{ R[A] \mid A \in \mathfrak{B} \}$ is the set of all large quotient overrings of $R$ and $(R, \mathfrak{B})$ satisfies property $c$. Theorem 2. There exists a set of regular maximal ideals $\mathfrak{M}$ such that $R = \bigcap_{\mathfrak{M} \in \mathfrak{M}} R[\mathfrak{M}]$ and there are no containment relations among the elements of the set $\{ R[\mathfrak{M}] \mid \mathfrak{M} \in \mathfrak{M} \}$. There is an example of a ring $R$ for which $(R, \mathfrak{B})$ does not satisfy property $c$ where $\mathfrak{B}$ is the set of all regular maximal ideals of $R$. Hence the $\mathfrak{M}$ of Theorem 2 cannot always be the set of all regular maximal ideals of $R$. (Received October 30, 1970.)

682-13-8. JOONG HO KIM, East Carolina University, Greenville, North Carolina 27834. Endomorphism of a formal power series ring over a polynomial ring. Preliminary report. Let $A$ be a commutative ring with identity, $A[X]$ the polynomial ring in an indeterminate $X$ over $A$, and $A[X][[Y]]$ the formal power series ring in an indeterminate $Y$ over $A[X]$. Theorem. Let $\beta = \sum_{i=0}^{\infty} b_i Y^i \in A[X][[Y]]$ and $\rho$ an $A$-automorphism of $A[X]$. Then there exists an automorphism $\varphi$ of $A[X][[Y]]$ such that $\varphi(X) = \rho(X) + \sum_{i=0}^{n} a_i Y^i$ where $a_i \in A$ for each $i$, $\varphi(Y) = \beta$ and $\varphi(a) = a$ for all $a \in A$, if and only if the following conditions are satisfied: (i) The topological ring $A[X][[Y]]$ in the $(\mathfrak{B})$-adic topology is complete Hausdorff. (ii) $g_1$ is a unit of $A[X]$. This paper generalizes M. O'Malley and C. Wood's main result for $R$-endomorphism of $R[[X]]$ [J. Algebra 15(1970), 314-327]. (Received November 2, 1970.)
Some results on rank one valuations are extended to valuations of finite rank. Let \((K, v)\) be a valued field with Henselization \((K^h, v^h)\). Let \(K_a\) and \(K_s\) be the algebraic and separable algebraic closure of \(K\) respectively. Theorem 1. For any integer \(d > 1\), there exists a valued field \((K, v)\) of rank \(d\) such that \([K_a : K] = \omega\), while \(v\) has exactly two prolongations to \(K_a\). (This is impossible for \(d = 1\).) Theorem 2. If \((K, v)\) is a valued field of finite rank \(d \geq 1\) such that \(1 < [K^h : K] < \omega\), then \(K\) is a real field, \(K^h = K(L(\sqrt{-1}))\), and \(v\) has exactly two prolongations to \(K^h\) and to \(K_a\). Corollary. A valuation \(v\) of rank \(d \leq 1\) on a field \(K\) has exactly one, two (if \(K^h = K(L(\sqrt{-1}))\)), or an infinite number (if \([K^h : K] = \omega\)) of prolongations to \(K_a\). (Received November 2, 1970.)

Let \(D\) denote an integral domain with \(1 \neq 0\) and quotient field \(K\). The set of primary ideals of \(D\) will be denoted by \(\mathfrak{p}\) and the set of semiprimary ideals of \(D\) (i.e., ideals with prime radical) will be denoted by \(\mathfrak{p}\). If \(\pi\) is a general ring property, then an ideal \(A\) of \(D\) will be called a \(\pi\)-ideal provided there exists a \(\pi\)-domain \(J\) such that \(D \subseteq J \subseteq K\) and \(A = AJ \cap D\). The classes of Krull, Dedekind, almost Dedekind, Noetherian integrally closed ideals, and ideals which are an intersection of valuation ideals will be denoted by \(\mathfrak{K}, \mathfrak{D}, \mathfrak{A}, \mathfrak{N}\), and \(\mathfrak{Y}\), respectively. Necessary and sufficient conditions are given in order that \(\mathfrak{Y} \subseteq \mathfrak{J}\) and \(\mathfrak{K} \subseteq \mathfrak{J}\). In addition we prove Theorem 1. \(D\) is a rank one valuation ring if and only if \(\mathfrak{z} = \mathfrak{Y}\). Theorem 2. \(D\) is a rank one discrete valuation ring if and only if \(\mathfrak{z} = \mathfrak{K}\) and proper prime ideals of \(D\) are maximal. Theorem 3. \(D\) is a rank one discrete valuation ring if and only if \(\mathfrak{z} = \mathfrak{Y}\) and proper prime ideals of \(D\) are maximal. Theorem 4. \(D\) is a rank one discrete valuation ring if and only if \(\mathfrak{z} = \mathfrak{A}\) and proper prime ideals of \(D\) are maximal. (Received November 4, 1970.)

Watts ["Alexander–Spanier cohomology and rings of continuous functions," Proc. Nat. Acad. Sci. U.S.A. 54(1965), 1027-1028] has described a cochain complex \(C^*_K A\) for a commutative algebra \(A\) over a commutative ring \(K\), which is an algebraic analogue of the Alexander–Spanier cochain complex. In this paper it is shown that, if \(K\) is a field, then for any \(K\)-algebra \(A\) the separable algebraic closure of \(K\) in \(A\) can be identified with the zero dimensional homology of \(C^*_K A\). Furthermore, for a large class of local algebras over a field, the homology of \(C^*_K A\) is essentially trivial. These results generalize that of Greenleaf ["Watts cohomology of field extensions," Proc. Amer. Math. Soc. 21(1969), 208-210]. (Received October 29, 1970.)

Let \(D\) be an integral domain with quotient field \(K\). \(D\) is a \(v\)-domain if the set \(H(D)\) of finite \(v\)-ideals
under v-multiplication is cancellative. **Theorem 1.** Let $D$ be a v-domain, $D^v$ the Kronecker function ring of $D$. $D$ is a domain of Krull type if and only if (1) $D^v = D[X]_{\mathfrak{P}}$, $S = \{f(X) \in D[X] \mid (A_f)_v = D\}$ where $A_f$ is the ideal of $D$ generated by the coefficients of $f(X)$, and (2) each nonzero element of $D^v$ is in at most finitely many maximal ideals of $D^v$. **Definition.** An integrally closed domain $D$ with ascending chain condition on integral v-ideals is a pseudo Krull domain. **Theorem 2.** Let $D$ be pseudo Krull and assume $\mathcal{H}(D)$ is a group under v-multiplication. Then $D$ is a domain of Krull type. If in addition each maximal t-ideal is invertible then $D$ is a Krull domain. $D$ has property $\mathcal{T}$, (property $\mathcal{F}_T$) if each overring of $D$ is the ideal transform of an ideal (a finitely generated ideal) of $D$ [see Abstract 633-354, these *Notices* 16(1969), 187]. **Theorem 3.** Let $D$ be a domain of Krull type with defining family $\{D_{\mathfrak{p}_\alpha}\}$ of essential valuation rings. If $D$ has property $\mathcal{T}$ then $D$ is semi-quasi-local Prüfer and the maximal ideals of $D$ are among the prime ideals $\{\mathfrak{p}_\alpha\}$.

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For a commutative ring $R$ with 1 let $W(R)$ denote the ring given by generators $\langle x \rangle = \langle x \rangle_R$, $x \in R$, and relations (i) $\langle 0 \rangle = 0$, (ii) $\langle 1 \rangle = 1$, (iii) $\langle xy \rangle = \langle x \rangle \langle y \rangle$, (iv) $\langle x \rangle + \langle y \rangle = \langle x+y \rangle (1 + \langle xy \rangle)$. For a field $F$ of characteristic $\neq 2$, D. K. Harrison has characterized the Witt ring of anisotropic quadratic forms over $F$ by these generators and relations. T. A. Springer has proved that if $F$ is a field of characteristic $\neq 2$ and if $K/F$ is a field extension of finite odd degree, then $W(F) \to W(K)$ is injective. **Theorem.** Let $K$ be an algebraic number field with $(K:Q)$ odd, and let $R = \text{alg. int. } [K]$. The following statements are equivalent.

1. $W(\mathbb{Z}) \to W(R)$ is injective. 2. $\langle 2 \rangle_R \neq \langle 8 \rangle_R$. 3. $\mathbb{Z}/4\mathbb{Z}$ is a homomorphic image of $R$. The proof requires the following result, conjectured by Joel Cunningham and the author and proved by Harrison and Cunningham.

**Theorem.** An element in $W(\mathbb{Z})$ is zero if and only if it is in the kernels of all the following homomorphisms: $W(\mathbb{Z}) \to W(\mathbb{Q})$, $W(\mathbb{Z}) \to W(\mathbb{Z}/p\mathbb{Z})$, $p$ an odd prime, and $W(\mathbb{Z}) \to W(\mathbb{Z}/4\mathbb{Z})$. This is an analogue of the Hasse-Minkowski Theorem. (Received November 5, 1970.)

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*Harrison's Witt ring of a ring of algebraic integers.* Preliminary report.

Let $F$ be a field of characteristic not 2. A standard method of studying the quadratic forms over $F$ is to study the Witt ring of anisotropic quadratic forms over $F$. Recently Lorenz and Harrison have independently shown that $W(F)$ is given by generators and relations where the generators correspond to elements of $F$. It can be shown that $W(F)$ is given by generators $\langle a \rangle$ for $a \in F$ and relations (i) $\langle 0 \rangle = 0$, (ii) $\langle ab \rangle = \langle a \rangle \langle b \rangle$, and (iii) $\langle a \rangle + \langle b \rangle = \langle a+b \rangle (1 + \langle ab \rangle)$. Now let $R$ be any commutative ring and let $W(R)$ be the ring given by generators $\langle a \rangle$ for $a \in R$ and relations (i), (ii) and (iii). The main result is a characterization of the prime ideals of $W(R)$ in terms of the orderings of the integral factor rings of $R$. This result generalizes the results of Lorenz, Leicht and Harrison for $W(F)$. In particular if $P$ is any prime ideal of $W(R)$ then $W(R)/P$ is isomorphic to $\mathbb{Z}$ of $\mathbb{Z}/(p)$ for some prime $p$. (Received November 5, 1970.)
14 Algebraic Geometry

682-14-1. STANLEY E. PAYNE, Miami University, Oxford, Ohio 45056. Generalized quadrangles as amalgamations of projective planes.

The known generalized quadrangles with \( 1 + s \) points (lines) incident with each line (point) where \( s \) is even all contain an incident point-line pair \((x, L)\) for which both \( x \) and \( L \) satisfy a condition known as regularity. This permits such a quadrangle to be viewed as an "amalgamation" of "compatible" projective planes and suggests the search for new compatible pairs of planes. This point of view is also useful in calculating the collineation groups of the known quadrangles and is used in this paper to compute the collineation group of the quadrangle based on the permutation \( \alpha : x \rightarrow x(x+1)^{-1} \) for \( x \neq 1 \) in the field \( GF(q^5) \). (Received October 12, 1970.)

15 Linear and Multilinear Algebra, Matrix Theory (Finite and Infinite)

682-15-1. RONALD H. DALLA, Eastern Washington State College, Cheney, Washington 99004 and A. DUANE PORTER, University of Wyoming, Laramie, Wyoming 82070. The matrix equation \( U_1 \ldots U_n A V_1 \ldots V_m = B \) over a finite field. Preliminary report.

Let \( GF(q) \) denote the finite field with \( q = p^h \) elements. Let \( A(e, f) \) denote a matrix with \( e \) rows and \( f \) columns and \( A(e, f; p) \) a matrix of the same dimensions with rank \( 0 \). All of the matrices under consideration have elements from \( GF(q) \). Let \( A = A(e, f; p) \) and \( B = B(s, t; \omega) \). A. Duane Porter (Duke Math. J. 37(1970), 55-60) has determined the number of solutions \( U_i \), \( 1 \leq i \leq s \), \( V_j \), \( 1 \leq j \leq b+1 \), of the matrix equation \( U_1 \ldots U_n A V_1 \ldots V_m = B \) where \( \omega \leq \rho \) and \( U_1 \), \( 1 \leq i \leq s \), and \( V_1 \), \( 1 \leq j \leq b+1 \), are matrices of arbitrary size except that the product and equality must be defined. A generalization of this result is obtained in that the number of solutions \( U_1 \ldots U_n A V_1 \ldots V_m = B \) is obtained, where \( \omega \leq \min(p, k_1, s_2, \ldots, s_{n-1}, k_n, k_{b+1}, t_2, \ldots, t_{b+1}, k) \). In order to obtain this generalization, the number of solutions \( X_1 \ldots X_n = B \) is obtained, where \( \omega \leq \min(k_1, t_2, \ldots, t_{n-1}, k) \). (Received October 19, 1970.)


Let \( \chi \) be a finite dimensional normed space, and let \( \| \cdot \| \) be the lub matrix norm induced by \( \| \cdot \| \). Let \( A \) denote a typical \( n \) by \( n \) matrix with respect to the canonical basis and \( A^* \) its conjugate transpose.

Theorem. If \( \| A \| = \| A^* \| \) for every \( A \), then \( \| A \|_2 = \| A^* \|_2 \) and there is a positive constant \( k \) so that \( \| x \| = k \| x \|_2 \), i.e. \( \chi \) is an inner product space. More generally: The induced matrix norm \( \| \cdot \| \) of a finite dimensional normed space admits an isometric Hermitian involution \( (\| A \| = \| A^* \| \) for all \( A ) \) iff \( \chi \) is an inner product space. A standard method of constructing a matrix norm \( \nu \) which cannot be a lub norm is to define \( \nu(A) = \max(\| A(\onenorm) \|, \| A(\twonorm) \|) \), where \( \| \cdot \|_a, \| \cdot \|_b \) are distinct lub norms. In this situation, the following are equivalent: \( \| A \|_b = \| A^* \|_a \) for every \( A \), \( \nu(x \otimes y) = \nu(y \otimes x) \) for each rank one operator, \( \nu(A) = \nu(A^*) \) for every \( A \). (Received October 26, 1970.)

Let $\lambda$ be an eigenvalue of an $n$-square complex tridiagonal matrix $A = [a_{ij}]$. Theorem 1. If $a_{k,k+1}^2 a_{k+1,k} \leq 0$ for $k = 1, \ldots, n-1$, then \( \min\{\text{Re}(a_{11}^j) | i=1, \ldots, n\} \leq \text{Re}(\lambda) \leq \max\{\text{Re}(a_{11}^j) | i=1, \ldots, n\} \). Theorem 2. If $a_{k,k+1}^2 a_{k+1,k} \geq 0$ for $k = 1, \ldots, n-1$, then \( \min\{\text{Im}(a_{11}^j) | i=1, \ldots, n\} \leq \text{Im}(\lambda) \leq \max\{\text{Im}(a_{11}^j) | i=1, \ldots, n\} \). Arscott (Edinburgh Math. Notes 44, 5-7 [in Proc. Edinburgh Math. Soc. 12 (1961)]) proved Theorem 1 for $\lambda$ real, $A$ real, and $a_{k,k+1}^2 a_{k+1,k} < 0$. Jayne (Proc. Edinburgh Math. Soc. 16(1969), 251-253) generalized Arscott’s result by removing the requirement that $\lambda$ be real. Results analogous to Theorems 1 and 2 hold for the zeros of the permanent of the characteristic matrix of a tridiagonal matrix. (Received November 2, 1970.)

682-15-4. HONG WHA KIM, Bucknell University, Lewisburg, Pennsylvania 17837. On complete set of similarity invariants of certain pairs $(A, B)$ of commutative nilpotent operators.

Let $(A, B)$ be a pair of commutative nilpotent operators on a $n$ dimensional vector space over the complex field $K$. If the dimension of the kernel of $A$ is 2, then $B^2 = \rho(A, B)$, where $\rho(A, B)$ is a polynomial in $A$ and $B$ and the coefficients $A_{ij} \in K$ of $A^i B^j$ form a complete set of similarity invariants of $(A, B)$ $(i+j \neq 0, 0 \leq i \leq r-1, 0 \leq j \leq 1, r = \text{index of } A)$. (Received November 4, 1970.)

682-15-5. GHISLAIN ROY, Université Laval, Ste. Foy’s, Québec, Canada. Polynôme minimal d'un produit tensoriel d'applications linéaires.

Soient deux espaces vectoriels $U$ et $V$ sur le corps $C$. On considère deux endomorphismes linéaires $f$ et $g$, respectivement de $U$ et de $V$. On suppose que leurs polynômes minimaux sont respectivement $m_f(x) = (x-a)^m$ et $m_g(x) = (x-b)^n$. Alors le produit tensoriel de $f$ par $g$ a pour polynôme minimal $m_{fg}(x) = (x-ab)^{m+n-1}$, si $ab \neq 0$, $m_{fg}(x) = x^m$, si $a = 0$ et $b \neq 0$ ou si $a = 0$, $b = 0$ et $m \leq n$. D'où l'on généralise assez facilement au cas où $m_{f}(x)$ et $m_{g}(x)$ seraient des polynômes quelconques. (Received November 5, 1970.)


Let $GF(q)$ be the finite field of $q = p^f$ elements. In this paper, we obtain a formula for the number of nonsingular commutator matrices of order $n$ over $GF(q)$. This result is extended to matrices of arbitrary
trace, thus establishing the partition of the nonsingular n \times n matrices over GF(q) into sets of matrices
of given trace. The results are further generalized to sums of diagonal elements for diagonals other than
the principal one. Finally, we evaluate a sum which is a variation of a formula of Hodges (Duke Math J.
22(1955), 497-510). (Received November 5, 1970.)

682-15-8. EMILIE HAYNSWORTH and MIROSLAV FIEDLER, Auburn University, Auburn, Alabama 36830. Cone
s which are topoheavv with respect to a norm. Preliminary report.

Suppose the vectors in \mathbf{R}^n are partitioned into two components which are the projections on disjoint
subspaces \mathbf{R}^k and \mathbf{R}^{n-k}. Without loss of generality assume \mathbf{X} = (X_1, X_2)^T, X_1 \in \mathbf{R}^k, X_2 \in \mathbf{R}^{n-k}. Let g be
a norm on \mathbf{R}^{n-k} which maps \mathbf{R}^{n-k} into the positive orthant in an s-dimensional subspace, 1 \leq s \leq n-k. A
cone \mathbf{T} in \mathbf{R}^n is defined to be topoheavv with respect to g if, for vectors partitioned as above, \mathbf{T} = \{X | X_1 \geq Qg(X_2)\}
where Q is a k \times s nonnegative matrix. The case in which k = s = 1 is examined in particular and the set of
positive operators \mathbf{P}(\mathbf{T}) is characterized for certain topoheavv cones. For instance if \mathbf{T}_1 = \{X | x_1 \geq
\sum_{i>1} x_i \} and \mathbf{T}_2 = \{X | x_1 \geq \max_{i>1} x_i \}, then \mathbf{T}_1 and \mathbf{T}_2 are dual cones and \mathbf{P}(\mathbf{T}_1) =
\{A \in \mathbf{R}_+ | A + \theta_j A_j \in \mathbf{T}_1, j > 1\}, \mathbf{P}(\mathbf{T}_2) = \{A \in \mathbf{R}_+ | A + \sum_{j>1} \theta_j A_j \in \mathbf{T}_2\} where A_j is the jth column of A and \theta_j = \pm 1. (Received November 5, 1970.)

682-15-9. JOHN JONES, JR., Air Force Institute of Technology, Dayton, Ohio 45431. Solutions of
certain matrix equations. III.

Let A(z), B(z), C(z), D(z) be n by n matrices having elements which are holomorphic functions of a
single complex variable z for z \in \mathcal{R} a closed bounded region in the complex z-plane. Theorem 1. A necessary
condition that the matrix equation (*) A(z)X + XB(z) + C(z) = 0 have a solution X(z) having elements x_{ij}(z) which
are also holomorphic functions of a single complex variable z \in \mathcal{R} is that the pair of 2n by 2n matrices
\begin{align*}
(A(z) - C(z)) & \quad (A(z) 0) \\
0 & \quad (B(z) \\
0 & \quad (B(z))
\end{align*}
be similar. If the characteristic roots of A(z), B(z) are holomorphic functions of z for z \in \mathcal{R} then the above condition is sufficient for the existence of a solution X(z) of (*) having elements
x_{ij}(z) which are holomorphic functions of z for z \in \mathcal{R}. Theorem 2. Let f_\alpha(\lambda) be a polynomial of degree n \geq 1
in \lambda having complex coefficients such that B(z) = (\lambda - C(z) A(z))^{-1}, f_\alpha(R) = (U(z) M(z), V(z) N(z)). If U^{-1}(z) or M^{-1}(z)
exists for z \in \mathcal{R} then a common solution of the equations XU(z) + M(z) = 0, XM(z) + N(z) = 0 is a solution of the
quadratic matrix equation (***) XDX(z)X + A(z)X + XB(z) + C(z) = 0 for z \in \mathcal{R}. If M^{-1}(z) or N^{-1}(z) exists
for z \in \mathcal{R} then a common solution of the equations U(z) - M(z)X = 0, V(z) - N(z)X = 0 is also a solution of (***)
for z \in \mathcal{R}. (Received November 4, 1970.)

16 Associative Rings and Algebras

682-16-1. ROBERT S. CUNNINGHAM and EDGAR A. RUTTER, JR., University of Kansas, Lawrence, Kansas 66044. On
perfect injectors and perfect projectors.

Let \mathbf{P}_R be a finitely generated projective module with S = \text{End}_R(\mathbf{P}_R) and let \mathbf{P}_S^* = \text{Hom}_R(\mathbf{P}_R). Then the
functors \mathbf{F} = \mathbf{P}_R \otimes_{\mathbf{S}_R} \mathbf{M}_\mathbf{R}, and \mathbf{H} = \text{Hom}_S(\mathbf{P}_S), \mathbf{G} = \mathbf{P}_S^* \otimes_{\mathbf{S}_S} \mathbf{M}_\mathbf{S} \rightarrow \mathbf{R}_\mathbf{M} form an ad-
djoint triple (\mathbf{G}, \mathbf{F}, \mathbf{H}) of functors. F. W. Anderson ("Endomorphism rings of projective modules," Math.
Z. 111(1969), 322-332) obtained some necessary and sufficient conditions for the functor $F$ to preserve injective hulls and projective covers. We show that $G$ preserves projective covers (is a perfect projector) and that $H$ preserves injective hulls (is a perfect injector). We further show that $F$ always preserves projective covers and injective hulls for certain classes of $R$-modules characterized by their relation to the trace ideal $T$ of $P_R$ in $R$. As an application of these results we obtain a new and very natural proof of a result due to E. Mares and R. Ware: let $R^R M$ be a finitely generated projective module with $S = \text{End}_R(\_M)$. Then $S$ is a semiperfect (resp. left perfect) ring iff $R^R M$ is a semiperfect (resp. perfect) module. (Received September 25, 1970.)

682-16-2. GEORGE SZETO, Bradley University, Peoria, Illinois 61606. The realization of modules and the Schur index.

Let $R$ be a commutative ring with no idempotents except 0 and 1 and let $G$ be a group of order $n$ invertible in $R$. A splitting ring for $G$ is defined (G. J. Janusz, "Separable algebras over commutative rings," Trans. Amer. Math. Soc. 122(1966)). Then, the realization of an RG-module in a subring of $R$ is defined and its properties are obtained (G. Szeto, "The group character and split group algebras," Pacific J. Math. 34(1970)). In this paper, more facts about the realization of an RG-module are given. Also, the Schur index can be defined in the context of a commutative ring with no idempotents except 0 and 1, and some basic properties of the Schur index can be obtained. Definition 1. Let $R_0$ be a subring of $R$ and let $M$ be a finitely generated and projective RG-module; then we say that the character of $M$, $\chi_M$, is realizable in $R_0$ if and only if $M$ is realizable in $R_0$. Theorem 1. Let $R'$ denote $R[\{T^i\}] = R[T^i(g_1),\ldots,T^i(g_n)]$ where $G = \{g_1, g_2, \ldots, g_n\}$ and $T^i$ is the $i$th-group character for $G$. Then there is a strongly separable $R$-algebra $R''$ such that $\text{rank}(R''; R'[T^i])$ is realizable in $R'$. Definition 2. Let $R'$ be $R[\{T^i\}]$. Then a positive integer $M_{R}(T^i)$ is called the Schur index for $T^i$ if $M_{R}(T^i) = \{\text{min. rank} (R_0; R')$ such that $R_0$ is a strongly separable $R$-algebra and a splitting ring for $T^i\}$. Finally, some standard techniques for computing the Schur index are used in this paper. (Received September 28, 1970.)


An R-module $M$ is cofaithful if $R$ can be imbedded in a direct sum of copies of $M$. Theorem. (a) Every faithful module is cofaithful if and only if $R^1$ has finitely generated essential socle. (b) Every cofaithful module is a generator if and only if $R$ is injective. This gives a two-step approach to the Azumaya-Utumi theorem (Nagoya Math. J. 27 (1966), 697-708 and J. Algebra 6 (1967), 56-64). A module $M$ is $\pi$-quasi-injective if $M^1$ is quasi-injective for every index set $I$. Theorem. Suppose $R$ is right perfect. If every quasi-injective left $R$-module is $\pi$-quasi-injective, $R$ is left Artinian. This is a partial converse of a theorem of Fuller (Arch. Math. 20 (1969), 495-502). The proof uses the first theorem. (Received October 9, 1970.)

682-16-4. CARTER G. LYONS and JOSEPH J. MALONE, JR., Texas A & M University, College Station, Texas 77843. The endomorphism near ring of $D_{2n}$, $n$ odd.

Let $E(G)$ (A (G), I(G)) designate the (distributively generated) near ring generated by the endomorphisms (automorphisms, inner automorphisms) of a group $G$ and let $D_{2n}$ designate the dihedral group of order $2n$, $n$ odd.
It is shown that $E(D_{2n}) = A(D_{2n}) = I(D_{2n})$ and that $|E(D_{2n})| = 2^n$. Also, $D_{2n}$ embeds (as a group) in $E(D_{2n})$. The radical $J(E(D_{2n}))$ is characterized and it is shown that $E(D_{2n})/J(E(D_{2n})) \simeq I(q)$, where $q = 2p_1 \cdots p_m$ if $n = P_1^r \cdots P_m^r$. (Received October 13, 1970.)

682-16-5. JOSEPH J. MALONE, JR. and CARTER G. LYONS, Texas A & M University, College Station, Texas 77843. D. g. near rings on $D_{2n}$, $n$ odd.

$D_{2n}$ designates the dihedral group of order $2n$ written in additive notation. Theorem. The number of nonisomorphic distributively generated near rings whose group part is $D_{2n}$, $n$ odd, is $1 + 2r$, where $r$ is the number of distinct primes occurring in the factorization of $n$. In the proof it is shown that the set of elements of order 2 may be taken as the (d. g.) generating set, that $cd = 0$ if $|c| \neq 2$ and $d \in D_{2n}$, and that $fd = gd$ if $|f| = 2 = |g|$ and $d \in D_{2n}$. (Received October 13, 1970.)

682-16-6. CARLTON J. MAXSON, Texas A & M University, College Station, Texas 77843. On groups and endomorphism rings.

Recently, examples have been given which show that the near-ring, $E(G)$, generated by the endomorphisms of a nonabelian group $G$ may be a ring. One of the problems considered in this paper is that of determining necessary and sufficient conditions on a group $G$ in order that $G$ is abelian when $E(G)$ is a ring. For finitely generated groups a complete answer is given. Theorem. If $G$ is a finitely generated group and $E(G)$ is a ring then $G$ is abelian if, and only if, $G$ is a cyclic $E(G)$-module. Somewhat more general results are obtained by considering any semigroup $S$ of endomorphisms of $G$ and $S(G)$-modules. An internal characterization of those groups $G$ which are cyclic $E(G)$-modules is given. Theorem. $G$ is a cyclic $E(G)$-module if and only if, $\cup \{H \mid H$ is a proper fully invariant subgroup of $G\} \neq G$. (Received October 13, 1970.)


Let $R$ be a commutative ring with an identity, $G$ be a group, $t$ be a positive integer smaller than the order of $G$ and $RG_t$ be the free $R$-module generated by all $t$-complexes of $G$. On $RG_t$, a multiplication is defined by $XY = \sum_{i=1}^{t} X_i Y$ where $X = \{x_1, x_2, \ldots , x_t\}$ and $Y = \{y_1, y_2, \ldots , y_t\}$ are $t$-complexes of $G$. With the multiplication being extended to linearity and $r(\alpha C) = (rC)D = C(rD)$ being defined for all $r \in R$ and all $C$ and $D \in RG_t$. $RG_t$ is a ring called the generalized group ring of order $t$. $RG_1$ is the usual group ring. One of our purposes is to generalize the known results in group rings, e.g., Theorem 1. Let $\Delta_t = \{ \sum_{i=1}^{t} r_i x_i \in RG_t ; \sum_{i=1}^{t} r_i = 0 \}$ and $R$ be a field of char. $p$. Then the ideal $\Delta_t$ is nilpotent iff $G$ is a finite $p$-group. Theorem 2. Every element in $\Delta_t$ is a $R$-linear sum of left zero divisors iff $G$ is a torsion group. Let $G_t = RG_t \rightarrow RG_1$ be defined by $XG_t = \sum_{i=1}^{t} X_i$ and extended by linearity. Theorem 3. Let char. $R > 0$ and $R$ contain no nonzero nilpotent elements. If $RG_t$ has a nilpotent ideal containing $ker G_t$ properly, then $G$ contains a finite normal subgroup. Theorem 4. Let char. $R > 0$. If $G$ has a normal subgroup of order $t$ such that $t$ is divisible by some prime $p$ which also divides char. $R$, then there exists a nilpotent ideal in $RG_t$ containing $ker G_t$. (Received October 14, 1970.)
Endomorphism near rings that are rings.

It is well known that the set of endomorphisms of an abelian group forms a ring under pointwise addition and composition of endomorphisms. For a nonabelian group $G$ the set of endomorphisms generates a near ring $E(G)$. In this paper conditions are given under which $E(G)$ is a ring for certain types of groups. **Theorem 1.** Let $G$ be a group on which there exists a map $h: G \to G$ with $h + h = i$, the identity map on $G$. $E(G)$ is a ring if $(a + b) h - bh - ah = 0$ for any $a, b \in E(G)$. **Theorem 2.** Let $G$ be a torsion group which contains no element of order 2. Then $E(G)$ is a ring if and only if $(a + b) h = ah + bh$ for any $a, b \in E(G)$. **Theorem 3.** Let $G$ be a torsion group which contains no element of order 2. Then $h$ is right distributive over $E(G)$ if and only if $ha = ah$ for each $a \in E(G)$. In the above theorems, $E(G)$ can be replaced by $A(G)$, the near ring generated by the automorphisms of $G$ or by $I(G)$, the near ring generated by the inner automorphisms of $G$. (Received October 15, 1970.)

Near rings with identities on certain groups.

Near rings with identities on certain groups have been considered by Clay and Malone (Math. Scand. 19[1966], 146-150). In this note we extend their result to a larger class of groups. **Theorem.** Let $(G, +)$ be a finite non-abelian group with exactly one proper normal subgroup. Then $(G, +)$ cannot be the additive group of a near ring with an identity. **Theorem.** Let $(G, +)$ be an abelian group with exactly one proper normal subgroup. If $(G, +, \cdot)$ is a near ring with an identity, then $G$ is a commutative ring with an identity. It is known that the only ring that can be defined on a torsion divisible group is the zero ring. However, one can define a nontrivial near ring on a torsion divisible group. Near rings with identities on divisible groups will be discussed along with several open problems. (Received October 19, 1970.)

The Dedekind property for semirings.

In this paper the concept of Dedekind semidomain is defined, and it is shown that certain structures of this kind are Noetherian, have integral closure, and have the property that their prime $k$-ideals are maximal. Appropriate transportation theorems are given, the main result concerning Dedekind semidomains is given, and results concerning when a semiring is a ring are given. Examples are given throughout the paper to show that the hypotheses of certain theorems in the paper can not be greatly weakened. (Received October 26, 1970.)

Mappings on rings with involution. Preliminary report.

**Theorem.** Let $R$ be a two torsion free with $2R = R$ and with involution. Suppose $\varphi$ is an additive map from $R$ into $A$ where $A$ is a two torsion free ring generated by $\{\varphi(x) : x \in R\}$. Suppose further that $(A) \varphi(xx^*) = \varphi(x)\varphi(x^*)$ and $\varphi(k^2)$ is not in $Z_A$ for all $k$ in $K$. Then (1) if $A$ is prime and $\varphi$ is onto, then $\varphi$ is a homomorphism; (2) if $A$ is prime and there exists a nonzero two sided ideal in the subring generated by $\varphi(s)$ where
s in S, then ϕ is a homomorphism; (3) If A is semiprime and for some k in K, ϕ(k^2) is not a zero divisor, then ϕ is a Jordan homomorphism. (B) R = A is prime and ϕ(xx*) = xϕ(x)* + ϕ(x)x* or ϕ(xx*) = xϕ(x)x* + ϕ(x)x*. Then if W = {w in S : wx in S for all x in R} ≠ {0} and k^2 is not in Z for all k in K, ϕ is a derivation. The results of L. Small relative to Jordan maps on S are investigated for R, a prime ring with quotients, and similar results are obtained. (Received October 19, 1970.)


Given an arbitrary associative ring R; let Σ_R denote the class of R-modules M such that (O : M) is a prime ideal of R and such that the only submodule N of M with the property (O : N) = (O : M) is N = M. The intersection Σ(R) of all ideals I of R for which there is M ∈ Σ_R such that I = (O : M) is a radical of R. This radical contains the prime radical and is properly contained in the Jacobson radical. Σ is hereditary, in fact, every right ideal of a radical ring is a radical ring. Σ is a special radical. For the ring of n x n matrices over R, Σ(R_n) = (Σ(R))^n. (Received October 21, 1970.)

682-16-13. ROBERT GORDEN, University of Utah, Salt Lake City, Utah 84112. Semiprime right Goldie rings which are direct sums of uniform right ideals.

A complete characterization of the rings in the title of this abstract is given. This includes a characterization of such rings which (a) possess no proper idempotent ideals and (b) are simple. (These results are related to theorems of A. V. Jategaonkar and R. E. Johnson.) In another vein, it is shown that a necessary and sufficient condition for a right semihereditary ring R to be a right order in a semiprimary ring is for R to satisfy (1) the identity of R is a sum of orthogonal primitive idempotents and (2) aR is an essential right ideal in R whenever a is an element of R with 0 right annihilator. (In a recent joint paper, L. W. Small and the author characterized right semihereditary rings which are right orders in right artinian rings.) It follows that a right semihereditary ring satisfying (1) and (2) above is also left semihereditary. (Received October 28, 1970.)


In Abstract 653-77, these Notices 15(1968), 100, connections between planar near rings and geometric structures were given. G. Ferrero has extended these ideas to connect planar near rings with balanced incomplete block designs. These connections are further extended to construct balanced incomplete block designs with parameter v = p+1, b = ν(v-1)/(k-1), ν = k. (v-1)/(k-1), k = λ = t+1 where t is any proper divisor of p-1, for any prime p > 3. (Received October 29, 1970.)


Several different radicals have been defined for distributively generated near-rings with identity: nil-radical, ideal radical, quasi-radical, radical, and primitive radical. Let G be a finite group and let E(G)
denote the near-ring generated by the endomorphisms of $G$. Theorem 1. The above radicals are equivalent in $E(G)$. Theorem 2. The radical of $E(G)$ is a nonzero ideal if and only if $G$ is not equal to the sum of its minimal fully invariant subgroups. (Received October 30, 1970.)


It is known, that every near-ring can be embedded into a near-ring with identity (see e.g. Berman-Silverman, Pacific J. Math. 10(1960), 777-786). But while every ring $R$ can be embedded canonically into a ring $\overline{R}$ with identity such that $R$ is a two-sided ideal of $\overline{R}$, the corresponding result for near-rings is not true.

Examples of near-rings are given here, which cannot be right or left ideals in a near-ring with identity, because their additive groups can never be normal subgroups of the additive group of a near-ring with identity. We prove a necessary and sufficient condition for a near-ring $N$ to be a two-sided ideal in a near-ring $\overline{N}$ with identity. (Received October 30, 1970.)

682-16-17. WITHDRAWN.


Let $K$ be a commutative algebra over the rationals and let $H$ be a graded, connected Hopf algebra over $K$ with multiplication $\varphi$, comultiplication $\Delta$ and antipode $\gamma$. If $h$ is a homogeneous element of $H$ of degree $n$, let $\theta$ from $H$ to $H$ be the morphism of graded modules defined by $\theta(h) = nh$. Now a morphism of modules $\Gamma$ from $H$ to $H$ is defined by $\Gamma = \varphi(\gamma \otimes \theta) \Delta$. Theorem 1. Let $P(H)$ be the primitive elements of $H$, then $H$ is cocommutative if and only if $\text{image } \Gamma = P(H)$. Theorem 2 (Generalized Wever-Specht formula). If $H$ is cocommutative, then for all $h \in H$, $h \in P(H)$ if and only if $\Gamma h = \varphi h$. Theorem 3. Let $Q(H)$ denote the quotient module of indecomposable elements of $H$, then $H$ is commutative if and only if $\text{coimage } \Gamma = Q(H)$. Theorem 4. Let $\pi$ denote the natural morphism from $H$ to $Q(H)$. If $H$ is commutative then for all $h \in H$ there exists an $h' \in H$ such that $\pi h' = \pi h$ and $\Gamma h' = \varphi h'$. If $H$ is commutative, then by the Leray Theorem, $H$ is isomorphic to the symmetric algebra generated by $Q(H)$. The morphism $\Gamma$ gives a canonical such isomorphism that can easily be calculated. (Received November 2, 1970.)

682-16-19. ROBERT S. CUNNINGHAM and EDGAR A. RUTTER, JR., University of Kansas, Lawrence, Kansas 66044 and DARRELL R. TURNIDGE, Kent State University, Kent, Ohio 44240. Rings of quotients of endomorphism rings of projective modules.

Let $S$ be the endomorphism ring of a finitely generated projective right $R$-module. A hereditary torsion class $\mathcal{F}$ of $\mathcal{S}$ is a class of left $R$-modules closed under submodules, homomorphic images, extensions, and arbitrary direct sums. The functor $F = P \otimes_R (\ ): R^R \to S^S$ induces a one-to-one correspondence between the hereditary torsion classes in $\mathcal{S}$ and those in $R^S$ containing $\mathcal{F}_T = \text{Ker } F$. Theorem. If $Q_R$ and $Q_S$ are rings of left quotients of $R$ and $S$ with respect to corresponding hereditary torsion classes, then $P \otimes_R Q_R$ is a
finitely generated projective right \( Q_R \)-module and a left \( Q_S \)-generator and \( Q_R \cong \text{End}_{Q_S} (P \otimes_R Q_R) \) and \( Q_S \cong \text{End}_{Q_R} (P \otimes_R Q_R) \). Conditions under which the maximal rings of left quotients correspond in this manner and applications to rings with zero singular ideal are given. The results extend the work of Turnidge in "Rings of quotients of Morita equivalent rings," see Abstract 663-100, these Notices 16(1969), 114. (Received November 2, 1970.)

682-16-20. B. S. CHWE, and JOE NEGGERS, University of Alabama, University, Alabama 35486. Rings classified by properties of modules. Preliminary report.

All rings have 1 and all modules are right unitary. Techniques are based on material in "On the extension of linearly independent subsets of free modules to bases," by the authors, Proc. Amer. Math. Soc. 24(1970), 466-470. After the statement of the property, the class of rings with the property will appear in parentheses. (1) Every minimal generating set of a finite free module is a basis (local rings). (2) Every minimal generating set of a finite free module is a basis (local rings). (3) Every linearly independent subset of a finite free module can be extended to a basis by adjoining elements of a given basis (local rings such that every set of nonunits has a nonzero right annihilator). (4) Every linearly independent subset of a finite free module can be extended to a basis (same as 3 in the local ring case). (5) Every linearly independent subset can be extended to a basis (right Steinitz rings). (6) Every module has a minimal generating set (right Steinitz rings). (Received November 2, 1970.)

682-16-21. DANIEL A. FALK, Ohio State University, Columbus, Ohio 43210. A generalization of the Zassenhaus-Taussky-Dade theorems to noncommutative orders. Preliminary report.

Let \( k \) be a number field with integers \( R \). Let \( A \) be a semisimple algebra with center \( k \). Let \( O \) be an \( R \) order in \( A \) and let \( B \) be a two sided \( O \) ideal. Let \( \mathcal{B} = \{ x \mid x \in A \text{ and } BxB \subset B \} \). Definition. An ideal \( B \) is weakly invertible if \( B \mathcal{B} B = B \). Weak invertibility is equivalent to invertibility for commutative orders. Theorem. Given \( O \), there is an integer \( N(O) \), such that for any two sided \( O \) ideal \( B \), \( B^m \) is weakly invertible for all \( m \geq N(O) \). (Received November 3, 1970.)


A module \( V \) is semiprime if \( (vV^*)v = 0 \) implies that \( v = 0 \). Here \( V^* = \text{Hom}_R (V, B) \) and homomorphisms are written on the right. Suppose that \( V \) is a semiprime module and that \( R \) satisfies the maximum condition on annihilators of subsets of \( V \). Let \( \overline{V} \) denote the injective hull of \( R^* \), and set \( S = \text{Hom}_R (V, V) \). Then \( S \) is densely embedded in \( \overline{S} \) in the following sense: given a finite dimensional submodule \( U \) of \( V \) and \( \alpha \in \overline{S} \), there exists \( f \in S \) such that (i) \( f|_U \) is a monomorphism of \( U \) into itself, and (ii) \( f|_U \alpha \in S \). In particular, when \( V \) is itself finite dimensional, \( S \) is a semiprime left Goldie with semisimple left quotient ring \( \overline{S} \). For another application, we take \( V = R \) to learn that a semiprime ring satisfying the maximum condition on annihilator left ideals is densely embedded in its left injective hull \( R \); i.e., given \( \alpha \in \overline{R} \) and a finite dimensional left ideal \( I \) of \( R \), there exists an element \( r \in R \) with \( r\alpha \in R \), \( r \in I \) and \( \alpha r \cap I = 0 \). The same conclusions can be obtained using various other sets of hypotheses. (Received November 4, 1970.)
Let $\mathcal{R}$ be an integral domain, $K$ a field containing $\mathcal{R}$, $\mathcal{R}^*$ the $K$-module of all $\mathcal{R}$-derivations on $K$. Let $M(\mathcal{R})$ be the algebra of multilinear forms on $\mathcal{R}^*$, and $G(\mathcal{R})$ the Grassman algebra of $\mathcal{R}^*$. Let $\mathcal{M}(\mathcal{R})$ be the algebra of multilinear forms on $\mathcal{R}$, and $\mathcal{G}(\mathcal{R})$ the Grassman algebra of $\mathcal{R}$, which is essentially the algebra of alternating multilinear forms on $\mathcal{R}$. $M(\mathcal{R})$ and $G(\mathcal{R})$ are both regularly graded $K$-algebras such that the dual module $\mathcal{R}^*$ can be canonically imbedded into $M(\mathcal{R})$ and $G(\mathcal{R})$, onto $M(\mathcal{R})$ and $G(\mathcal{R})$ respectively. We define $d: K \rightarrow \mathcal{R}^*$ by $d(a)(D) = D(a)$ for all $a \in K$, $D \in \mathcal{R}$. Then $d$ is an $\mathcal{R}$-derivation. An element in $M(\mathcal{R})$ ($G(\mathcal{R})$) is called a (an alternating) differential form; homogeneous element homogeneous (alternating) differential form. Definition. A homogeneous element $x \in M_n(\mathcal{R})$ ($G_n(\mathcal{R})$) is called a homogeneous integral (alternating) differential form of degree $n$ if $x \in S(dS)^n$ for all valuation rings $S$ in $K$ containing $\mathcal{R}$. Theorem. If $\mathcal{R}$ is a Noetherian unique factorization domain and if $K$ is a finitely and separably generated extension field of the field of quotients of $\mathcal{R}$, the module of all homogeneous integral (alternating) differential forms of degree $n$ is a finitely generated $\mathcal{R}$-module. Remark. Homogeneous integral differential forms of degree zero are elements of $K$ which are integrally dependent over $\mathcal{R}$. And in this case this theorem still holds if $\mathcal{R}$ is a Noetherian integrally closed domain. (Received November 4, 1970.)
modules is torsionfree; the direct limit of torsionfree injective modules is injective; each absolutely pure torsionfree module is injective—a module is absolutely pure if it is a (Cohn) pure submodule of every module containing it; each module has a unique (up to isomorphism) torsionfree covering module—this gives a converse, in a special case, to a theorem of M. Teply [Pacific J. Math. 28(1969), 441-453]. (Received November 5, 1970.)

682-16-26. WITHDRAWN.

682-16-27. MADHUKAR G. DESHPANDE, Marquette University, Milwaukee, Wisconsin 53233. On right PF X-rings.

For a subdirectly irreducible module \( M_R \) over a ring \( R \) with 1, the intersection of all nonzero submodules is called the R-heart of \( M \). \( R \) is termed RSI if \( R_R \) is subdirectly irreducible and in this case, \( H, \hat{R}, \) and \( K \) will respectively denote the R-heart of \( R_R \), the injective hull of \( R_R \) and \( \text{Hom}_R(\hat{R}, \hat{R}) \). A ring \( R \) is called (right) PF if \( R_R \) is an injective cogenerator and \( R \) is called an X-ring if for every pair of primitive idempotents \( e \) and \( f \) such that \( e R \neq f R \); \( a \in e R \) and \( a^R \cap f R \neq 0 \) implies \( afR = 0 \).

**Theorem 1.** Let \( R \) be RSI. Then \( R \) is local iff (i) \( \hat{R} \) is subdirectly irreducible as a left \( K \)-module with heart \( H \) and (ii) each proper right ideal of \( R \) has nonzero left annihilator. **Corollary.** A (right) self-injective RSI ring is left subdirectly irreducible. **Theorem 2.** \( R \) is a PF X-ring iff it is isomorphic to finite direct sum of matrix rings over self-injective RSI rings. (Received November 5, 1970.)

682-16-28. WITHDRAWN.

### 17 Nonassociative Rings and Algebras


A new class of simple power-associative flexible algebras of degree two is constructed to obtain Theorem 1. There exist simple flexible algebras of degree two which are not commutative. The constructed algebras are not stable and thus are not noncommutative Jordan algebras. Now let \( A \) be a simple flexible power-associative algebra which is finite dimensional over an algebraically closed field of characteristic \( p \neq 2, 3, 5 \). Assume that \( A \) has degree two and is stable with respect to an idempotent \( u \neq 0, 1 \). **Theorem 2.** If \( A = A (1) + A (1/2) + A (0) \) is the Peirce decomposition of \( A \) with respect to \( u \), then the following are equivalent: (i) \( A \) is commutative; (ii) \( A (1/2) \) is commutative; (iii) \( ux = xu \) for all \( x \) in \( A (1/2) \). (Received October 8, 1970.)

682-17-2. PALANIAPPAN KANNAPPAN, University of Waterloo, Waterloo, Ontario, Canada. On weak inverse property loops.

One of the problems of varieties is to define some class of varieties by a single identity. This problem was solved for groups, Abelian groups, and loops with the inverse property. In this paper we give a characterization of the varieties of weak inverse property (WIP) loops as a subvariety of (i) left quasigroups with a single identity and (ii) right quasigroups with a single identity. Let \( G (\ast) \) be a loop with identity element \( 1 \). Then \( G \) is said to
be a loop with WIP if whenever three elements x, y, z of G satisfy the relation \(xy \cdot z = 1\), they also satisfy the relation \(x \cdot yz = 1\). Let \(G(*)\) be a groupoid. We say that \(G(*)\) is an iso-WIP loop provided there is a WIP loop \(G(o)\) which is a principal isotope of \(G(*)\) such that \(*\) and \(\cdot\) are connected by either of the relations \(x * y = x \cdot \rho(y)\) for all \(x, y \in G\), where \(\rho\) is the right inverse operator of \(G(*)\) or \(x * y = \lambda(x) \cdot y\), where \(\lambda\) is the left inverse operator of \(G(*)\). Theorem 1. A necessary and sufficient condition that a left quasigroup \(G(*)\) is an iso-WIP loop is that \([(t \cdot t) \cdot x] \cdot (z \cdot x) = (u \cdot u) \cdot z\) holds for all \(x, z, t, u \in G\). Theorem 2. A right quasigroup \(G(*)\) is an iso-WIP loop if and only if the following identity \((x \cdot z) \cdot [x \cdot (t \cdot t)] = z \cdot (u \cdot u)\) holds for all \(x, z, t, u \in G\). Theorem 3. A necessary and sufficient condition that a left quasigroup \(G(*)\) is an iso-WIP loop, in which the law \(w(x_1, \ldots, x_n) = e\) holds, is that \([((h \cdot t) \cdot w) \cdot x] \cdot (z \cdot x) = (u \cdot u) \cdot z\) is satisfied for all \(x, z, t, u \in G\). (Received October 16, 1970.)

682-17-3. JAMES F. HURLEY, University of California, Riverside, California 92502. Extensions of Chevalley algebras.

Let \(L\) be a complex finite dimensional simple Lie algebra with Cartan subalgebra \(H\) and Chevalley basis \(B\) relative to \(H\). Let \(R\) be a commutative ring with identity in which 2 and 3 are not zero divisors. Then the Chevalley algebra \(L_R\) of \(L\) over \(R\) is defined as \(R \otimes_L L_Z\), where \(L_Z\) is the free abelian group on \(B\) (see Abstract 653-126, these Notices 15 (1968), 114). If \(H_Z\) is the basis for \(H\) in \(B\), \(H_Z^1\) is the complex basis for \(H\) dual to the system of simple roots, \(H_Z\) is the free abelian group on \(H_Z^1\), and \(H_Z^1\) is the free abelian group on \(B_{H^1}\), then \(H_Z \subseteq H_Z^1\). Thus \(L_Z\) is a subset of \(L_Z^1\), the free abelian group on \((B \cup B_{H^1}) - B_{H^1}\). \(L_R^R = R \otimes_L L_Z^1\) is called an extended Chevalley algebra of \(L\) over \(R\). The ideal structure of \(L_R\) is determined using methods introduced in an earlier paper (Trans. Amer. Math. Soc. 137 (1969), 245-258), and the finitely many algebras intermediate between \(L_R\) and \(L_R^R\) are described explicitly. (Received October 19, 1970.)


Let \(L\) be a finite dimensional restricted Lie algebra over a field of characteristic \(p > 0\). \(L\) is said to be p-reductive if it has a completely reducible faithful finite-dimensional restricted representation. A subset \(S \subseteq L\) is said to be nil if for every \(x \in S\) there exists a positive integer \(n\) such that \(x^n = 0\). Theorem. Let \(L\) be as above. Then \(L\) is p-reductive if and only if \(L\) contains no nonzero nil ideals. (Received October 22, 1970.)

682-17-5. RICHARD L. TANGEMAN, Arkansas State University, State University, Arkansas 72467. Generation closed classes of rings.

W. G. Leavitt has considered in Proc. Amer. Math. Soc. 21(1969), 703-705, classes \(M\) of not necessarily associative rings satisfying property (a): If \(J \in M\) is an ideal of an ideal \(I\) of \(R\), and if \(J'\) is the ideal of \(R\) generated by \(J\), then \(J' \in M\). Theorem 1. If \(P\) is a radical class, then \(P\) satisfies property (a) iff \(I\) and ideal of \(R\) implies \(P(I)\) is an ideal of \(R\). Theorem 2. If \(M\) is homomorphically closed and satisfies property (a), then \(LM = M_2\), the second step in the lower radical construction, and \(LM\) satisfies property (a). A characterization is given of semisimple classes corresponding to such radicals. (Received October 26, 1970.)
Computation of outer multiplicities on a computer.

This paper describes a computer implementation of the Racah-Klymyk method for calculating the outer multiplicities of the tensor product of two irreducible representations of a complex simple Lie algebra. The techniques used are similar to those described in Abstracts 673-89 and 673-90, these Notices 17(1970), 417-418. (Received October 30, 1970.)

Nilpotent groups of algebra automorphisms.

We present conclusions about the structure of a nilpotent group of algebra automorphisms and about the influence of this nilpotence on the structure of the algebra. In particular, we indicate some corollaries to and generalizations of: Theorem. Let \( B \) denote an \( n \)-dimensional \((n > 1)\) nilpotent nonassociative algebra over a perfect field and suppose the automorphism group, \( G \), of \( B \) is nilpotent. Then \( G \) is the direct product of a finite cyclic group, \( C \), and a unipotent group; if \( B \) is a Lie algebra then \( C = \{1\} \). Examples are given which illustrate, among other things, that: (i) the automorphism group of a nonnilpotent Lie algebra over an algebraically closed field need not contain a torus of positive dimension; (ii) the property of nilpotence of the automorphism group of a Lie algebra is not independent of extensions of the base field, even in characteristic 0. (Received November 4, 1970.)

On near-generalized triple systems of even order with some applications.

The senior author has constructed a set of nonisotopic types of generalized triple systems (GTS) of even order (V. Bohun-Chudyniv, "On GTS of even order with some applications," Nachr. Österreich. Math. Ges., Beilage zu "Internat. Math. Nachrichten," No. 91, Jan. 1970). The aim of this paper is to construct near-generalized triple systems (NGTS) of even order and find some applications. Definition. A system of triplets is called a near-generalized triple system (NGTS) if not all pairs of elements appear in two different orders and some pairs are not in the system. Each triple system with additional relations represents right and left binary operations of a nonabelian loop. These operations (right and left) are isotopic and isomorphic, and are called g-operation loops. The authors prove that there exists only one type of NGTS of order 6, consisting of 20 systems. The NGTS of 12, 24 and, in general, of \( 6 \cdot 2^k \) order \((k > 0)\) comprise isotopic, isomorphic, and nonisotopic types of systems. The NGTS have applications to G-loop theory, as discussed by the senior author at the seminar "Solution of the G-loop problem," (Internat. Congress of Math., Nice, Sept. 8, 1970). (Received November 4, 1970.)
HASAN A. CELIK, University of California, Santa Barbara, California 93106. On primitive and prime antiflexible rings.

Let $R$ be an antiflexible ring of characteristic not 2, and let $M$, $N$, $Z$ denote the middle nucleus, nucleus and the center of $R$, respectively. **Theorem 1.** If $(x, x, x) = 0$ for all $x \in R$ and if $R$ is primitive, then either $R$ is associative or it is simple with an identity element. **Theorem 2.** If $R$ has the property $[R, R] \subseteq M$, then either $R$ is associative or $N = Z$. (Received November 5, 1970.)

18 Category Theory, Homological Algebra

682-18-1. HARVEY E. WOLFF, University of Texas, Austin, Texas 78712. $Y$-localizations and $Y$-triples.

Let $Y$ be a symmetric monoidal closed category. Let $A$ be a $Y$-category and $\Sigma \subseteq A_0$ a subcategory which is $Y$-localizable (see Abstract 70T-A169, these Notices 17(1970), 812). Let $T = (T, \eta, \mu)$ be a $Y$-triple on $A$. We give various conditions which imply the following: (*) there exists a $Y$-triple $\tilde{T}$ on $A[\Sigma^{-1}]$ and a subcategory $S \subseteq T$, which is $Y$-localizable, such that $A[\Sigma^{-1}] \tilde{T} \cong A[T[\Sigma^{-1}]]$. We prove, for example, that if $T_0(s) \subseteq S (S$ is the $Y$-saturation of $\Sigma$) for all $s \in \Sigma$ and the canonical $Y$-functor $\phi: A \to A[\Sigma^{-1}]$ has a $Y$-right adjoint then we obtain (*). We also prove similar results for the $Y$-Kleisli algebras. As an application we generalize to Grothendieck topologies the following result of Van Osdol (Thesis, University of Illinois, 1970). **Theorem.** If $B$ is finitarily tripleable over sets, then the category of sheaves with values in $B$ is tripleable over the category of sheaves with values in sets. (Received October 28, 1970.)


Let $F$ be a functor from rings to groups and suppose $q$ is a 2-sided ideal in the ring $A$. In many situations, one would like to define a "relative" group $F(A, q)$ so that the sequence $F(A, q) \to F(A) \to F(A/q)$ is exact. Moreover it is clear from the anticipated applications that $F(A, q)$ should depend functorially on the pair $(A, q)$ and that the left-hand arrow should not, in general, be injective. Such relative groups have been defined, ad hoc, for many such functors $F$. A uniform way to define such relative groups is described, which has the pleasant property of making certain theorems about these groups formal consequences of the corresponding theorems for the case $q = A$--the "absolute" case. It is then shown that this construction yields the relative groups as previously defined for a variety of functors in algebraic K-theory. Examples are given of theorems of algebraic K-theory whose relative versions thus follow formally from their absolute versions. (Received November 5, 1970.)

682-18-3. IRWIN S. PRESSMAN, Ohio State University, Columbus, Ohio 43210. Obstructions to liftings in commutative squares.

Suppose that $gf = kh$ are the morphisms of a commutative square in some fixed abelian category. A lifting is a morphism $\chi$ such that $h = \chi f$ and $g = k \chi$. A complete obstruction theory is developed which char-
acterizes those squares which admit such a morphism. This result has applications to algebraic topology. For example, given a commutative diagram of topological spaces, the nonexistence of a lifting might be established by simply applying one well-chosen functor to abelian groups, and verifying that the obstruction is nonzero in that instance. (Received November 5, 1970.)


Let \( G \) be a finite group with normal subgroup \( H \). Let \( M \) be a \( G \)-module. If \( H^i(H, M) = 0, \ 0 < i < n, \) fixed, then the inflation-restriction sequence \( 0 \rightarrow H^n(G/H, M^H) \rightarrow H^n(G, M) \rightarrow H^n(H, M)^G \rightarrow H^{n+1}(G/H, M) \rightarrow \) \( H^{n+1}(G, M) \) is exact. Berkson and McConnell (Trans. Amer. Math. Soc. 141 (1969), 403-413) showed that this exact sequence is part of a \( 2 \times 5 \) commutative diagram. It is shown that the Hattori generalization of the inflation-restriction exact sequence (J. Math. Soc. Japan 12 (1960), 65-80) is part of a \( 2 \times 5 \) commutative diagram which generalizes that of Berkson and McConnell. (Received November 5, 1970.)

20 Group Theory and Generalizations

682-20-1. JOHN MARSHALL SAADE, University of Georgia, Athens, Georgia 30601. On models of groupoid identities.

In this paper the following groupoid identities are investigated: \( xyz = uv, \) where \( u, v \in \{x,y,z\}, \) \( u \neq v, \) and \( xwyz = uv, \) where \( u, v \in \{x,w,y,z\}, \) \( u \neq v, \) and where \( xyz \) is \( x \cdot yz \) or \( xy \cdot z \) and \( xwyz \) is bracketed in any of the five ways. The purpose of this study is to characterize models of these identities as far as possible. For example, the subcollection of these identities which have as their only models the constant semigroups is determined. As another result, the models of another subcollection are characterized as unions of disjoint medial subgroupoids (where a medial (entropic) groupoid is one which satisfies the identity \( xw \cdot yz = xy \cdot wz \)). (Received August 11, 1970.)

682-20-2. DONALD COOK, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24060. Alpha extensions of Abelian groups. Preliminary report.

For each ordinal \( \alpha \) and each Abelian group \( A, \) there is a group \( p^\alpha A \) such that: (i) \( p^\alpha(p^\alpha A) = A. \) (ii) Any homomorphism from \( A \) to \( B \) extends to a homomorphism from \( p^\alpha A \) to \( p^\alpha B. \) (iii) The sequence \( 0 \rightarrow A \rightarrow B \rightarrow C \rightarrow 0 \) is totally pure if and only if the sequence \( 0 \rightarrow p^\alpha A \rightarrow p^\alpha B \rightarrow p^\alpha C \rightarrow 0 \) is totally pure. (iv) \( A \) is totally projective if and only if \( p^\alpha A \) is totally projective. (v) If \( A \) is an Abelian group and \( \alpha \) is an ordinal, then there is an Abelian group \( G \) such that \( p^\alpha G = A \) and \( G/A \) is totally projective. (vi) Any homomorphism from \( A \) to \( p^\alpha G \) extends to a homomorphism from \( p^\alpha A \) to \( G, \) (Received August 24, 1970.)

682-20-3. PIERRE ANTOINE GRILLET, Kansas State University, Manhattan, Kansas 66502. Fundamental, singly generated inverse semigroups.

Explicit descriptions are given. This includes a description of the free inverse semigroup on one generator. (Received September 8, 1970.)

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A transformation $M$ on a group $G$ is called an automoration of $G$ if there is $x \in G$ such that $MR_x = yx$ for $x, y \in G$. In this paper we discuss structures of automoration groups and their relation to representation theory in the paper of H. Nakano, "Representations of a group by transformations on its subgroups," Math. Ann. 181 (1969), 173-180. An automoration group $A$ on a group $G$ is said to be symmetric if $A$ contains both $Rx$ and $Lx$ for every $x \in G$ where $yRx = yx$ for $x, y \in G$. Finally we attempt to characterize symmetric automoration groups and prove the Characterization Theorem. A group $G$ is isomorphic to some symmetric automoration group if and only if $G$ has two invariant subgroups $U$ and $V$ such that $U = \{x : xv = vx \text{ for all } v \in V\}$ and $V = \{x : xu = ux \text{ for all } u \in U\}$, and a subgroup $A$ such that $G = UA = VA$ and $U \cap A = V \cap A = \{e\}$. (Received September 4, 1970.)

On Kim's conjecture.

In this paper we settle (with a counterexample) the conjecture raised by J. B. Kim, concerning the Miller and Clifford theorem. Kim's conjecture. If $a$ and $b$ are elements of a semigroup $S$, and if $R_b \cap L_a$ does not contain an idempotent, then $aHb = U(x, y) = (p_1f^#(x, y), p_2f^#(x, y))$ for all $x, y \in S_1$, where $p_1$ and $p_2$ are the projection functions, i.e. $f^#(x, y) = (p_1f^#(x, y), p_2f^#(x, y))$. Homomorphisms (F. Radó, Abstract 672-581, these Notices 17 (1970), 251) and p-Homotopisms (D. Cecil, Abstract 70T-A108, these Notices 17 (1970), 647) are examples, where $p_1f^#(x, y) = p_1f^#(x, z)$ for $z \in S_1$ and similarly for $p_2f^#$. If we define $(f^#, f) = (g^#, g)$ when $f = g$ and $f^# = g^#$, then the collection of all generalized homomorphism pairs (with $S_1 = S_2$) form a monoid $(M, *)$ under composition of functions. Let $\Delta_i$ be the diagonal of $S_i \times S_i$. The structure of the submonoid $M(\Delta) = \{f^#, f : f^#(\Delta_1) \subseteq \Delta_2\}$ is characterized, as are the structures of other subsemigroups, and properties of $M(\Delta) = \{f^#, f : \exists n \in \mathbb{Z} \text{ for which } f^n \text{ is a homomorphism}\}$ are exhibited. (Received October 14, 1970.)

Suppose $G$ is a finite $p$-group ($p > 2$) such that the central quotient is metacyclic and nontrivial. The automorphism group of a $p$-group with the central quotient metacyclic.

Theorem. The order of $G$ divides the order of the automorphism group of $G$. This result generalizes an earlier


All finite 2-groups G with the following property are classified: G has no normal abelian subgroup of rank 3, but the Frattini subgroup of G contains an abelian subgroup of rank 3. (Received October 19, 1970.)

682-20-9. LARRY FINKELSTEIN, Wayne State University, Detroit, Michigan 48202. The maximal subgroups of Conway's group $C_3$ and McLaughlin's group. Preliminary report.

Let $C_3$ be the smallest of the three new finite simple groups discovered by J. H. Conway and let $M$ be the finite simple group discovered by J. E. McLaughlin which is contained as a subgroup of index two in the automorphism group of a regular graph with 275 vertices. It has been verified by J. H. Conway that the automorphism group of $M$ admits $C_3$ as a transitive extension. The conjugacy classes of maximal subgroups and the isomorphism types in these classes are completely determined for both $C_3$ and $M$. In particular $C_3$ has fourteen conjugacy classes of maximal subgroups and $M$ has twelve conjugacy classes of maximal subgroups. (Received October 19, 1970.)


A subgroup $H$ of the finite group $G$ is called maximal abelian sensitive in $G$ if for each maximal abelian subgroup $A$ of $G$, $A \cap H$ is a maximal abelian subgroup of $H$. In this work it is shown that the finite group $G$ has a maximal abelian sensitive Sylow $p$-subgroup if and only if the Sylow $p$-subgroup of $G$ is a direct summand of $G$. (Received October 20, 1970.)


In analyzing the question of whether the freest group of exponent 4 is solvable, N. D. Gupta and K. Weston have shown [J. Algebra, in press]: Let $R$ designate the associative algebra generated by: $r_1, r_2, \ldots$ and satisfying only the following 3 conditions and their consequences: (1) $2r_1 = 0, i = 1, 2, \ldots$. (2) $r_1, \ldots, r_i = 0$ if some $j = j', i = i'$. (3) $r_1, \ldots, r_i, r_1^2 = 0$ where $a^b = a + b + ab$. If $\Delta(r_1, r_2) = r_1 r_2 + r_1 r_2$ and 

$$ \Delta(r_1, \ldots, r_{2k+1}) = (r_1, \ldots, r_{2k})(r_{2k+1}, \ldots, r_{2k+2}) + (r_{2k+1}, \ldots, r_{2k+2})(r_1, \ldots, r_{2k}) $$

then Theorem G is solvable if and only if for some $k$, $\Delta(r_1, \ldots, r_k) = 0$. Here we wish to announce that (3) is equivalent to

$$(3') 0 = x r_1 \ldots r_n + r_1 \ldots r_n x^{-1} r_1 \ldots r_n + r_1 \ldots r_n x^{-1} r_1 \ldots r_n x^{-1} r_1 \ldots r_n + \sum a A r_1 \ldots r_n A, \text{ where } x = r_1 \ldots r_n, x^{-1} = r_1 \ldots r_n, \text{ A projection of } x, A = \text{ closure of } A \text{ in } x; n \geq 2. $$ (Received October 9, 1970.)
682-20-12. EDWARD T. ORDMAN, University of Kentucky, Lexington, Kentucky 40506. Factoring a free abelian group as a direct product with amalgamation. Preliminary report.

While the group of integers \( \mathbb{Z} \) is not a direct product of groups, it can be factored as a direct product with an amalgamated subgroup: \( \mathbb{Z} = (p\mathbb{Z} \times q\mathbb{Z})_{pq\mathbb{Z}} \) for \( p \) and \( q \) relatively prime. Call a factorization \( G = (A \times B)_{C} \) coarse if there is a factorization \( G = (A' \times B')_{C'} \), with \( A' \leq A \), \( B' \leq B \), \( C' \) a proper subgroup of \( C \). Theorem. Every factorization \( F = (A \times B)_{C} \) of a free abelian group, with \( C \) nontrivial, is coarse. Further, if \( c \in C \), \( C' \) may be chosen so that \( c \in C' \). Of course, in many cases, such as the example above, \( C' \) cannot be trivial. (Received October 23, 1970.)

682-20-13. TOSHIHIKO YAMADA, McGill University, Montreal 110, Quebec, Canada. Characterization of the simple components of the group algebras over the p-adic number field.

It is an interesting problem to characterize the simple components of group algebras of finite groups over a field \( K \) of characteristic 0. In other words, the problem is to characterize the classes of central simple algebras \( A \) over \( K \) in the Brauer group \( Br(K) \), such that \( A \) is spanned by a finite multiplicative subgroup \( G \) of \( A \) over \( K \), i.e., \( A = \sum_{g \in G} \alpha_{g} \mathbb{Q}[\zeta_{p}] \). This question has been settled for the p-adic number field \( \mathbb{Q}_{p} \) (\( p \neq 2 \)). Theorem. Let \( p \) be an odd prime number. Then, a given (finite dimensional) division algebra \( \Delta \) over \( \mathbb{Q}_{p} \) is similar to a simple component of the group algebra \( \mathbb{Q}_{p}[\mathbb{G}] \) over \( \mathbb{Q}_{p} \) of some finite group \( G \) if and only if (i) the center \( k \) of \( \Delta \) is contained in some cyclotomic field \( \mathbb{Q}_{p}(\zeta_{n}) \), and (ii) the Hasse invariant of \( \Delta \) is of the form \( z/b \cdot (\text{mod } Z) \), \( z \in Z \), where \( Z \) is the ring of rational integers and \( b \) is the index of tame ramification of the extension \( k/\mathbb{Q}_{p} \), i.e., if \( \varrho \) is the prime ideal of the integer ring in \( k \) dividing \( p \), the ramification index of \( \varrho \) over \( p \) is \( bp^{\tau} \), \( (b,p) = 1 \), for a certain integer \( \chi \). (Received October 23, 1970.)

682-20-14. KAI FALTINGS, University of Texas, Austin, Texas 78712. Primary Abelian groups with semiduals.

For an Abelian group \( A \), let \( L_{0}A \) be the lattice of all \( X \leq A \) such that \( X \) or \( A/X \) is finitely generated. Theorem 1. The primary Abelian group \( A \) has a semidual if and only if \( A \) is torsion complete and every Ulm-invariant of \( A \) is finite. Theorem 2. Let \( A \) be a reduced Abelian p-group with \( p > 3 \) and rank \( \geq 2 \), and suppose that if \( A \) is bounded, then \( A \) has two independent elements of maximal order. Then \( A \) has a semidual if and only if, if \( \Gamma = Aut \mathbb{A} \) and \( \Pi = Aut \Gamma \), then \( \Pi \neq \Delta \mathbb{A} \), where \( \Delta \) is the centralizer of \( \Gamma/\mathbb{Z} \Gamma \) in \( \Pi \) and \( \mathbb{A} \) is the group of all inner automorphisms of \( \Gamma \). (Received October 23, 1970.)

682-20-15. ELIAS TOUBASSI, University of Arizona, Tucson, Arizona 85721. On the splitting of mixed groups of torsion-free rank one.

Let \( A \) be an abelian group of torsion-free rank one with \( T(A) \), its torsion subgroup, p-primary. Define the height slope of \( \tau \) to be \( \sup_{x \in A} (v(x) - \inf_{\ell \leq Z^{+}} h(p^{\ell}x)/\ell) \). Irwin, Khabbaz and Rayna, J. Algebra 14(1970), 423-442, defined \( \ell(A) \), the splitting length of \( A \), to be the least positive integer \( n \) (otherwise infinity) such that
A® . . . ®A (n factors) splits. **Theorem.** Let A be a mixed abelian group of torsion-free rank one, height slope α. T(A) p-primary and A/T(A) p-divisible. Then \( \lambda(A) = n \) if and only if one of the following holds:

(i) \( \alpha = (n-1)/(n-2) \) and \( h(p^k x) - \alpha = \infty \) for some \( x \in A \) of infinite order.

(ii) \( \alpha = n/(n-1) \) and \( h(p^k x) = \alpha \), except for some \( x \in A \) of infinite order.

**Corollary.** Same hypothesis as in the theorem. Then \( \lambda(A) = \infty = \alpha = 1. \)  
(Received October 30, 1970.)

682-20-16. TOMMY K. TEAGUE, Michigan State University, East Lansing, Michigan 48823. **On the Engel margin.**

For any word \( \lambda \) we denote by \( \lambda(G) \) the verbal subgroup of \( G \) and by \( \lambda^*(G) \) its associated marginal subgroup. (For the definitions see R. F. Turner-Smith, Proc. London Math. Soc. (3) 14 (1964), 321-341.)

Here we consider \( \lambda = [x, y, z] \). **Theorem.** \( \lambda^*(G) = \{ a \in G : [x, y, a] [a, y, x] = 1 \} \) for all \( x, y \in G \).

**Theorem.** For any \( a \in \lambda^*(G) \), \( [a, G, G] = 1 \). **Theorem.** \( Z_2(G) \subseteq \lambda^*(G) \subseteq Z_3(G) \) and \( \lambda^*(G)/Z_2(G) \) is an elementary Abelian 3-group of central automorphisms on \( G' \).

**Theorem.** If \( Z(G) \cap Z(G') \) has no elements of order 3, or if \( G' \) has no proper subgroup of finite index, then \( \lambda^*(G) = Z_2(G) \). **Theorem.** If \( [G : \lambda^*(G)] = m \) is finite, then \( \lambda(G) \) is finite with order which divides a power of \( m \). (Received October 29, 1970.)

682-20-17. DENNIS P. ALLEN, JR., Michigan Technological University, Houghton, Michigan 49931. **A generalization of the Rees theorem to a class of regular semigroups.**

A semigroup \( S \) such that each element of \( S \) is contained in a maximal principal right, a maximal principal left, and a maximal principal two-sided ideal of \( S \) is said to be max-principal. If \( S \) and \( T \) are semigroups, then then an epimorphism \( \psi : S \to T \) is said to be max-principal preserving if whenever \( I \) is a maximal principal right, a maximal principal left, or a maximal principal two-sided ideal of \( S \), then \( \psi(I) \) is a maximal principal right, a maximal principal left, or a maximal principal two-sided ideal of \( T \), respectively. Finally, a semigroup \( S \) having the property that whenever \( I_1 \) and \( I_2 \) are each maximal principal right or maximal principal left ideals of \( S \) which intersect nontrivially, then \( I_1 = I_2 \) is said to be principally separated. **Theorem.** Let \( S \) be a max-principal regular semigroup in which \( J = D \). Then one can construct a unique max-principal regular matrix semigroup \( W \) with \( J = D \), which is also principally separated, and a max-principal preserving epimorphism \( \psi : W \to S \) such that (i) \( \psi \) separates the D-classes of \( W \) and (ii) \( \psi \) is one-to-one on the intersection of any principal right with any principal left ideal of \( W \) (and hence on the H-classes of \( W \)). This extension of a previous result of the author was suggested by Professor A. H. Clifford. (Received October 30, 1970.)

682-20-18. BERNHARD AMBERG, University of Texas, Austin, Texas 78712. **Infinite groups which are the product of two abelian subgroups.**

A group \( G \) is said to be min-by-max if it is an extension of a group with minimum condition on subgroups by a group with maximum condition on subgroups. The following problem is discussed and partial solutions are given: Is every group \( G = AB \) which is the product of two abelian min-by-max subgroups \( A \) and \( B \) a min-by-max group? For instance the following may be proved. **Theorem.** If the group \( G = AB \) is the product of an abelian min-by-max subgroup \( A \) and an abelian subgroup \( B \) which satisfies the minimum condition or is cyclic, then \( G \) is an extension of a divisible abelian characteristic subgroup with minimum condition \( C \) by a torsionfree-by-finite
group with maximum condition; $C$ is a $p$-group if the maximal divisible subgroups of $A$ and $B$ are $p$-groups. Conditions are given under which similar results may be obtained when one of the two (abelian min-by-max) subgroups $A$ and $B$ satisfies the maximum condition or is accessible in $G$. Corresponding questions concerning the property 'max-by-min' are also discussed. (Received October 30, 1970.)

682-20-19. JAMES W. STEPP, University of Houston, Houston, Texas 77004. On the structure of the semigroup $\text{Ext}(B,G)$ where $B$ is a compact semilattice and $G$ is a compact abelian group. Preliminary report.

Let $B$ denote a compact semilattice with unit, let $G$ denote a compact abelian group, and let $\text{Ext}(B,G)$ denote the collection (of equivalence classes) of extensions of $G$ by $B$. In this paper we discuss the structure of the semigroup $\text{Ext}(B,G)$. Using the results we obtain about the structure of $\text{Ext}(B,G)$ we prove the following theorem. Let $p$ denote a prime and let $\mathbb{Z}_p$ denote the group of order $p$. Then the following are equivalent:

(a) $\text{Ext}(B,\mathbb{Z}_p)$ has a zero, (b) The unit of $B$ is an isolated point, (c) $\text{Ext}(B,\mathbb{Z}_p)$ admits a compact topology. (Received November 2, 1970.)

682-20-20. JUTTA HAUSEN, University of Houston, Houston, Texas 77004. Abelian $p$-groups and normal torsion subgroups of their automorphism groups.

For $p$ a prime, a group $H$ is called a $p'$-group, if $G$ is a torsion group without elements of order $p$. Let $G$ be an abelian $p$-group and $A(G)$ its automorphism group. Theorem. If $p > 5$, then every normal $p'$-subgroup of $A(G)$ is contained in the center of $A(G)$. (Received November 2, 1970.)


For background to the present paper see the previous abstract by the authors [Abstract 70T-A190, these Notices, 17(1970), 941]. Theorem. Every finite nonabelian group $G$ whose order is relatively prime to 6 has a graphical regular representation (i.e., there exists a graph $X$ with automorphism group $A(X) \cong G$ such that $A(X)$ is a regular permutation group on the vertex set of $X$). The proof invokes the W. Feit-J. G. Thompson Theorem [Pacific J. Math. 13(1963), 775-1029] to insure the solvability of $G$. Some nonabelian groups $G$ with $(|G|, 6) > 1$ do have graphical regular representations and some do not. (Received November 2, 1970.)


Let $\varphi$ be a free metabelian group of rank $q$, let $\varphi_n$ be the $n$th group of the lower central series of $\varphi$, and let $F$ be the free group of rank $q$. In a previous paper ["IA automorphisms of free and free metabelian groups," Comm. Pure Appl. Math. 21(1968), 605-629], the author proved that, for $q = 3$, there are automorphisms of $\varphi$ not induced by automorphisms of $F$ and that, for any integer $n \geq 2$, there are automorphisms of $\varphi/\varphi_n$ which are induced by automorphisms of $\varphi$ but not by automorphisms of $F$. In the present paper, the Bachmuth representation of the group of IA automorphisms of $\varphi$ [S. Bachmuth, "Automorphisms of free met-
abelian groups," Trans. Amer. Math. Soc. 118(1965), 93-104] is used to show that, for \( q > 3 \) and for any positive integer \( n \), every automorphism of \( \varphi/\varphi_n \) which is induced by an automorphism of \( \varphi \) is induced by an automorphism of \( F \). This reopens the question of whether every automorphism of \( \varphi \) is induced by an automorphism of \( F \) if \( q > 3 \). (Received November 2, 1970.)

682-20-23. JAMES W. RICHARDS, Kent State University, Kent, Ohio 44240. Abelian f.p.f. operator groups of type (p, p).

A group \( A \) of automorphisms on a finite group \( G \) is said to be fixed-point-free on \( G \) if \( g^a = g \) for all \( a \) in \( A \) implies that \( g = 1 \). The author shows that if \( A \) is an abelian f.p.f. group of automorphisms of type (p, p) on a finite solvable group \( G \) where the order of \( A \) is coprime to the order of \( G \), then the nilpotent length of \( G \) is bounded above 2, \( p \) is an arbitrary prime. This result has been known to be true for some time for the nonexceptional primes. Not long ago, H. Kurzweil indicated the result without proof. (Received November 2, 1970.)

682-20-24. JANET ELIZABETH AULT, University of Florida, Gainesville, Florida 32601. Regular semigroups which are ideal extensions of groups.

A semigroup \( V \) is an (ideal) extension of a semigroup \( S \) by a semigroup \( T \) if \( S \) is an ideal of \( V \) and the Rees quotient \( V/S \) is isomorphic to \( T \). A semigroup \( T \) with zero is called 0-categorical if \( abc = 0 \) implies that \( ab = 0 \) or \( bc = 0 \) for all \( a, b, c \in T \). In this paper all extensions of a group by a 0-categorical regular semigroup are described completely, insofar as semigroups are concerned. This is done using the minimal primitive congruence as defined by T. E. Hall (J. Austral. Math. Soc. 8(1968), 350-354). Finally, those semigroups which can be obtained as extensions of groups by 0-categorical regular semigroups are given an abstract characterization. Appropriate modifications are made in case the 0-categorical regular semigroup is in fact an inverse semigroup. (Received November 2, 1970.)


For a finite group \( G \), define \( h(G) = n \) if every upper chain in \( G \) of length \( n \) has a proper (\( \neq G \)) subnormal entry, and there is an upper chain of length \( (n-1) \) with no proper subnormal entry. Let \( \ell(G) \) denote the Fitting length of \( G \), \( k(G) \) denote the derived length of \( G \), and \( \pi(G) \) denote the number of distinct prime divisors of \( |G| \). Theorem. If \( G \) is a finite solvable nonnilpotent group then: (1) \( k(G) \leq h(G) \), (2) \( \ell(G) \leq h(G) - \pi(G) + 2 \). (Received November 2, 1970.)

682-20-26. CHARLES F. WELLS, Case Western Reserve University, Cleveland, Ohio 44106. Automorphisms of group extensions.

If \( 1 \to G \to E \to K \to 1 \) is an exact sequence of groups, with the second arrow inclusion, any automorphism \( \varphi \) of \( E \) which takes \( G \) onto itself induces automorphisms \( \tau \) on \( G \) and \( \sigma \) on \( K \). However, for a pair \((\varphi, \tau)\) of automorphisms of \( K \) and \( G \), there may not be an automorphism of \( E \) inducing the pair. The given extension induces a homomorphism from \( K \) to the outer automorphism group of \( G \); let \( Z \) be its kernel and \( Y \) its image. A
pair \((\sigma, \tau) \in \text{Aut} K \times \text{Aut} G\) is called compatible if \(\sigma\) fixes \(Z\) and the automorphism induced on \(Y\) by \(\sigma\) is the same as that induced by the inner automorphism of \(\text{Out} G\) determined by \(\tau\). Let \(C \subseteq \text{Aut} K \times \text{Aut} G\) be the group of compatible pairs. Let \(\text{Aut}(E;G)\) denote the group of automorphisms of \(E\) fixing \(G\). There is an exact sequence \(1 \rightarrow Z^1(K, ZG) \rightarrow \text{Aut}(E;G) \rightarrow C \rightarrow H^2(K, ZG)\). The last map is not in general surjective and is not even a group homomorphism, but the sequence is nevertheless 'exact' at \(C\) in the obvious sense. \((ZG\) is the center of \(G; Z^1\) and \(H^2\) have their usual meaning in extension theory.) (Received November 2, 1970.)


Let \(M\) be a set and \(S\) a semigroup of functions defined on \(M\) into \(M\). For each \(s\) in \(S\) let \(s^*: M \times M \rightarrow M \times M\) be the corresponding induced map such that for \(F, G\) in \(M\), \((F, G)s^* = (Fs, Gs)\). Let \(s^*: S \rightarrow S\) be the corresponding right multiplication. **Definition.** \(S\) is an associated semigroup iff there is a surjection \(\sigma: M \times M \rightarrow S\), called a (2-place) associator, such that for all \(s\) in \(S\), \(s^*: \sigma = \sigma s^*\). Every (2-place) Menger algebra (cf. Math. Ann. 157(1964), 167-178) has a corresponding associated semigroup and conversely every associator determines a corresponding Menger algebra \(M\) such that \(F(G, H) = F((G, H)\sigma)\), for \(F, G, H\) in \(M\). **Theorem.** Let \(S\) be an associated semigroup of functions containing the identity function. Then for any \(s, t\) in \(S\), the principal left ideals \((s)\) and \((t)\) are equal ifff the ranges of \(s\) and \(t\) are equal. (Received November 3, 1970.)

682-20-28. W. P. WARDLAW, University of Georgia, Athens, Georgia 30601. Defining relations for most integrally parameterized Chevalley groups.

For each faithful finite dimensional irreducible representation \(R\) of a finite dimensional simple Lie algebra \(L\) over the complex field, this paper treats the Chevalley group \(G\) over the rational field \(Q\) generated by \(\{x(\rho): \rho \in \Sigma, \rho \in Q\}\) (\(\Sigma\) is the set of nonzero roots of \(L\) and \(x(\rho) = \exp \rho R(X)\), where \(X\) is an element of a Chevalley basis of \(L\)) as defined by R. Steinberg in 'Lectures on Chevalley groups' (Yale University Notes, 1967) and its subgroup \(G_{Z_{Q}}\) generated by \(\{x(\rho): \rho \in \Sigma, \rho \in Q\}\). A finite set of defining relations of \(G_{Z_{Q}}\) in terms of the generators \(\{x(\rho): \rho \in \Sigma\}\) is explicitly stated for \(L\) of any type except \(G_2\). The paper utilizes a complete set of defining relations for \(\text{PSp}(4, Z)\) given by P. J. Gold ("On the mapping class and symplectic modular groups," Ph.D. Thesis, New York University, 1961) to extend the author's previous result for \(L\) of type \(A_n, D_n, E_n\) or \(F_n\) (see "Defining relations for certain integrally parameterized Chevalley groups," Abstract 642-145, these Notices) 14(1967), 106) to \(L\) of type \(B_n, C_n\) and \(F_4\). (Received November 3, 1970.)

682-20-29. LUISE CHARLOTTE KAPPE and WOLFGANG P. KAPPE, State University of New York, Binghamton, New York 13901. Properties of groups defined by normality conditions.

Let \(E\) be a subgroup closed property of finite groups, and \(N\) nilpotency. Define a derived property \(ME\) by: \(G \in ME\) if and only if the normal closure \(\langle x^G \rangle \subseteq E\) for all \(x \in G\). Familiar examples are the Dedekind groups and the variety \([x, y, y] = 1\). Let \(\alpha\) be an automorphism leaving a normal covering of \(G\) by \(E\)-subgroups invariant. The question whether the group of multipliers \(G^{-1}\alpha\) is an \(E\)-group is settled by showing \(E = ME\).

The following result is typical. **Theorem.** Let \(N \subseteq E = E(f) \subseteq N\) be a formation locally defined by the full and
integrated formation function $f$. Then $E = M_E$ if and only if $N \cap f(p)$ is exponentially closed for all $p$. (Received November 3, 1970.)

682-20-30. WOLFGANG P. KAPPE, State University of New York, Binghamton, New York 13901. Some subgroups given by identities.

Let $f = f(x; x_1, \ldots, x_k)$ be a word in $x, x_1, \ldots, x_k$ and $G$ a group. Denote by $f_G$ the set of all $a \in G$ such that $f(a; g_1, \ldots, g_k) = 1$ for all $g_1, \ldots, g_k \in G$. The terms of the ascending central series and the centralizers of iterated commutators are familiar examples where the set $f_G$ is actually a subgroup. Unlike the marginal subgroups however $f_G$ is not a subgroup in general as $f(x) = x^p$ and $f(x; x_1) = [x, x_1, x]$ show. It is proved here that $f_G$ is a subgroup for the following 3-variable words $f(x; x_1, x_2): [x, x_1, x_2] [x, x_2, x_1], [x_1, x_2, x] [x, x_1, x_2], [x_1, x_2, x] [x_2, x, x_1] [x, x_1, x_2]$. (Received November 3, 1970.)


Let $B$ denote either the bicyclic semigroup or its counterpart constructed from the additive semigroup of nonnegative reals. The elements of $B$ are ordered pairs of nonnegative integers or reals, resp., multiplication is given by $(a, b)(c, d) = (a + c - \min\{b, c\}, b + d - \min\{b, c\})$, and the topology is that induced from the real plane. A bicyclic closure is a locally compact semigroup containing (a copy of) $B$ as a proper dense subsemigroup. The well-known fact that there are no compact bicyclic closures is a special case of the Theorem. If $T$ is a bicyclic closure then $T - B$ is closed and noncompact. A semigroup $S$ is moderately $\Gamma$-compact if $\Gamma(x) = \{x, x^2, \ldots\}$ is compact whenever $x \in S$ and $\{x^i : i = 1, 2, \ldots\}$ clusters. Explicit descriptions are given for all moderately $\Gamma$-compact bicyclic closures $T$ which contain cluster points for either $\{(i, 0) : i = 1, 2, \ldots\}$ or $\{(0, i) : i = 1, 2, \ldots\}$ and which satisfy one of (a) the sequence $\{(i, i) : i = 1, 2, \ldots\}$ clusters in $T$, (b) $T$ is an inverse semigroup, or (c) $T = \bigcup_{x \in B} \Gamma(x)$. Each of the semigroups above which satisfy (a) or (b) contains the semigroup characterized by Eberhart and Selden [Trans. Amer. Math. Soc. 144(1969), 115-126] as the unique bicyclic closure which is an inverse semigroup with continuous inversion map. (Received November 4, 1970.)

682-20-32. MOTUPALLI SATYANARAYANA, Bowling Green State University, Bowling Green, Ohio 43402. Simple semigroups. Preliminary report.

A semigroup $S$ (with 0) is said to be simple (0-simple) if it has no ideals (nonzero ideals). We observe that a 0-simple semigroup $S$ has no zero-divisors if either one of the following conditions is satisfied. (i) $S$ has a right identity and $xy = 0 \Rightarrow x$ is nilpotent or (ii) $S$ satisfies ascending chain condition (a.c.c.) on right ideals and also satisfies left reversible property, $xS \cap yS \neq 0$ for every $x, y \in S$. In (i) the right identity condition cannot be dropped. Every 0-simple semigroup $S$ is shown to have nonzero idempotents if $S$ satisfies a.c.c. on right ideals and if $S$ has a proper minimal left ideal. This is the generalization of the result of the finite semigroups. In right simple semigroups every idempotent is a left identity. The existence of left identity or equivalently the existence of a nonzero idempotent is established for a right simple semigroup $S$ with a.c.c. on right ideals provided $S = f \cup fS$, $f \in S$. The structure of simple semigroups with finite numbers of idempotents and
with identity is determined and this leads to the description of rings with identity whose multiplicative semigroup is 0-simple. (Received November 4, 1970.)

682-20-33. CHARLES EDWARD JOHNSON, Ohio State University, Columbus, Ohio 43210. A local theory of group extensions.

Let $N$ and $F$ be finite groups, and $1 \rightarrow N \xrightarrow{i_1} E_1 \xrightarrow{j_1} F \rightarrow 1$, $1 \rightarrow N \xrightarrow{i_2} E_2 \xrightarrow{j_2} F \rightarrow 1$ two extensions of $N$ by $F$. Let $C$ be a class of subgroups of $F$. The two extensions are called $C$-equivalent if for each $D \in C$, the pre-images $D_1 = j_1^{-1}(D)$ and $D_2 = j_2^{-1}(D)$ are equivalent extensions of $N$ by $D$. Theorem. Let $E_1$ and $E_2$ be cyclic-equivalent. Then there are automorphisms and factor sets $\{x_i\}$ and $\{c_{i,j}\}$ for $E_1$, $\{y_i\}$ and $\{d_{i,j}\}$ for $E_2$ so that (i) $g^x_ig^y_j = g^z_{i,j}$ for all $g \in N$, (ii) $C_{i,j}d_{i,j}^{-1} = z_{i,j} \in C^2(F, Z(N))$, (iii) $E_1$ and $E_2$ are equivalent if and only if $z_{i,j} \in B^2(F, Z(N))$. (Received November 4, 1970.)

682-20-34. GEORGE MARKOWSKY, St. Mary's College of Maryland, St. Mary's City, Maryland 20686. A characterization of idempotents and regular elements.

Proceeding from an algebraic characterization of certain of the Green's equivalence relationships on a semigroup $G$ it is shown that an element $b$ of $G$ is regular iff there exist $x, y \in G$ such that $xRb$, $y,e_b$, and $xy = b$. If $b$ is regular an idempotent $m_b$ is associated with $b$ and it is shown that $b$ is an idempotent iff $yx = m_b$ (where $x$ and $y$ are as above). In general there are many ways to represent $b$ as a product $xy$ with $xRb$ and $y,e_b$, and conditions are given for uniqueness of representation in the semigroup of binary relations on a finite set. (Received November 5, 1970.)

22 Topological Groups, Lie Groups


Certain compact connected OPAGS with a zero for least element are considered. If the greatest element is an idempotent isolated in the set of idempotents and each nonzero element cancels, then the real thread is obtained. If the greatest element is a nonidempotent and each nonzero element cancels, and if as well there is mediality, $(ab)(cd) = (ac)(bd)$, then the real semigroup $[0, 1/2]$ is obtained. Two examples are included, one to show that some condition on a neighborhood of the nonzero idempotent in the first case above is needed to get a semigroup, even with the additional assumptions of commutativity and mediality. The other example shows that a compact connected ordered groupoid with zero, 0, for least element, an identity, 1, for greatest element, with no other idempotents and with the set of power associative elements clustering at 1, need not be either the real thread or the nil thread. (Received October 13, 1970.)

682-22-2. SIGMUND N. HUDSON, Tulane University, New Orleans, Louisiana 70118. On certain arcwise connected groups. Preliminary report.

The research reported here is a continuation of the author's "On connectivity properties of finite-dimen-
sional groups," Proc. Amer. Math. Soc. 23 (1969). Inasmuch as the topological and algebraic structure of an arcwise connected subgroup of a Lie group is known, arcwise connected groups are studied in the case that (a) they are finite-dimensional or (b) they are a subgroup of a locally compact group. In case (a), a local structure theorem is proved. A special case of the theorem is that there are small neighborhoods of the identity of the form $S \cdot D$, where $S$ is an open $n$-cell and $D$ is a totally disconnected, hereditarily paracompact, zero-dimensional, countable space (possibly noncompact and nonmetrizable). In case (b) the arc component of the identity of a locally compact group $G$ is determined. A special case of the theorem here is that if $G$ is compact, abelian, and has no circle subgroups, then the arc component of the identity of $G$ is the continuous injective, image of a product $H$ of copies of the real numbers (so that possibly $H$ may be a non-locally-compact group). (Received October 22, 1970.)

682-22-3. DONALD MARXEN, University of Kentucky, Lexington, Kentucky 40506. Uniform semigroups.

A uniform semigroup is a uniform space together with a uniformly continuous, associative binary operation. Theorem. If $(S, m, \tau)$ is a topological semigroup (not necessarily $T_2$) and $\gamma$ is a uniformity on $S$, then $\gamma$ is compatible with $\tau$ and $m$ if and only if $\tau$ is the uniform topology of $\gamma$ and the gage of $\gamma$ has a subbase of subinvariant pseudometrics. Theorem. If a topological semigroup, which is also an abstract group, has a compatible uniformity, then it is a topological group. A uniform semigroup $(S, \gamma)$ is called a metric semigroup if there is a subinvariant metric $d$ on $S$ that generates $\gamma$. Main Theorem. A Hausdorff uniform semigroup $(S, \gamma)$ is a dense subsemigroup of an inverse limit of metric semigroups. If $\gamma$ is complete or the uniform topology is Lindelöf, then $(S, \gamma)$ is an inverse limit of metric semigroups. (Received October 22, 1970.)


Let $G$ be a compact Lie group acting differentiably on the $C^\infty$-differentiable manifold $X$. For each subgroup $K$ of $G$ let $X_{(K)}$ be the set of all points in $X$ whose stability group is conjugate to $K$. It is well known that $X_{(K)}$ is a fibre bundle whose structural group is $N(K)/K$ where $N(K)$ is the normalizer of $K$ in $G$. Let $U$ be a $G$-invariant subspace of $X_{(H)}$ such that $U \subset \gamma X_{(K)}$. There is an obstruction theory which tells when the structural group of $U$ can be reduced to $[N(H) \cap N(K)]/H$. (Received October 29, 1970.)

682-22-5. MARTIN GOLUBITSKY, University of California, Los Angeles, California 90024. Primitive actions and maximal subgroups of Lie groups.

To date the efforts to classify the maximal (not necessarily connected) Lie subgroups of a connected Lie group has been concentrated on the case where the Lie algebra of the subgroup is itself a maximal subalgebra [see E. B. Dynkin, "The maximal subgroups of the classical groups," Amer. Math. Soc. Transl. (2) 6(1957), 246-378]. It is shown that this is a nontrivial assumption, e.g. the normalizer of a cartan subgroup of $sl(n, \mathbb{C})$. The main result is the classification of the reductive, maximal rank subalgebras of the classical (complex) Lie algebras which correspond to maximal Lie subgroups. It is also shown that if one assumes that the maximal Lie subgroup contains no proper normal subgroups of the whole group, then its Lie algebra is maximal if (a) it is
nonreductive, or (b) the containing group is not simple. The Lie algebras described above are shown to correspond exactly to the class of isotropy subalgebras of primitive, transitive, and effective actions of the Lie group in question. (Received October 30, 1970.)

682-22-6. ROBERT A. MASSAGLI, University of Nebraska, Lincoln, Nebraska 68508. On another radical in a topological ring. Preliminary report.

Wright [Amer. J. Math. 79(1957), 477-496] defines the notion of a radical in a topological Abelian group. Since every topological ring R may be regarded as an additive topological Abelian group, it will be interesting to reveal how Wright's radical denoted by $T(R)$, sits as a subset in R. Specifically, we demonstrate the following facts: Theorem 1. Let R be a topological ring. Then $T(R)$ is an ideal of R. Theorem 2. Let R be a connected locally compact topological ring. Then $T(R)$ is contained in the Jacobson radical of R. Theorem 3. Let R be a locally compact divisible ring with unity satisfying the second axiom of countability and containing no divisors of zero. If $T(R) \neq R$ is bounded, then $T(R)$ is contained in the Jacobson radical of R. (Received October 29, 1970.)


This is a study of reflective and coreflective subcategories of the category $\mathcal{C}$ of locally convex Hausdorff topological vector spaces and continuous linear transformations. A general category theorem characterizing reflective and coreflective subcategories is given and when applied to $\mathcal{C}$ it yields that a replete full subcategory is reflective if and only if it is closed under the formation of products and closed subspaces and is coreflective if and only if it is closed under the formation of direct sums and separated quotients. As a consequence any space E is contained in a smallest reflective subcategory $E_P$, a smallest coreflective subcategory $E_I$, and a smallest bireflective (both reflective and coreflective) subcategory $E_{PI}$ of $\mathcal{C}$ and we say E generates these subcategories. Simple conditions are given for a space to be in $E_P$ or $E_I$. If $E_{PI}$ contains all complete spaces, E is called universal. The universal metrizable spaces are characterized, from which it follows that the base field K is not universal. Thus $K_{PI}$ is a bireflective subcategory of $\mathcal{C}$ which does not include all complete spaces. An example is also given of a bireflective subcategory $E_{PI}$ larger than $K_{PI}$ which still does not include all complete spaces. These examples serve to indicate the limitations of inductive (generative) methods in $\mathcal{C}$. (Received November 2, 1970.)

682-22-8. MOHSEN PAZIRANDEH, University of California, Los Angeles, California 90024. Invariant differential operators on a semisimple Lie algebra.

The object is to give new proofs of some results of Harish-Chandara concerning radial components of invariant differential operators on a semisimple Lie algebra $\mathfrak{g}$ over $\mathbb{R}$ (Amer. J. Math. 79(1957) and 86(1964)). Our first result is a proof of Theorem 2 (loc. cit. 79(1957), 104) which uses Weyl's character formula. This theorem in combination with Weyl's unitarian trick is then used to determine the radial components of constant coefficient invariant differential operators. (Theorem 1, loc. cit. 79(1957), 100). Let $\mathfrak{g}$ be a semisimple Lie
algebra over \( \mathbb{R} \), \( \mathfrak{h} \) a Cartan subalgebra, \( \pi \) product of roots of a positive system. \( \mathfrak{h}' = \{ H : H \in \mathfrak{h}, \pi(H) \neq 0 \} \).

For any constant coefficient invariant differential operator \( D \) on \( \mathfrak{g} \), let \( \overline{D} \) be the constant coefficient differential operator on \( \mathfrak{h} \) which is the part coming from \( D \), and let \( \mathfrak{g}'(D) \) be the radial component of \( D \) on \( \mathfrak{g}' \). Then \( \mathfrak{g}'(D) = \pi^{-1} \overline{D} \cdot \pi \). These two theorems are then used to obtain a new proof of Theorem 1 (loc. cit. 86(1964), 547) which asserts that if \( D \) is an invariant differential operator on \( \mathfrak{g} \) with polynomial coefficients and \( \mathfrak{g}'(D) \) is its radical component on \( \mathfrak{g}' \), then \( \pi \cdot \mathfrak{g}'(D) \cdot \pi^{-1} \) is the restriction to \( \mathfrak{g}' \) of a polynomial coefficient differential operator on \( \mathfrak{g} \). (Received November 4, 1970.)


Let \( \mathfrak{g} \) be a complex semisimple Lie algebra of rank \( \ell \), and let \( \mathfrak{g} \) be the algebra of invariant polynomials on \( \mathfrak{g} \). By a well-known theorem of Chevalley, there exist \( \ell \) algebraically independent polynomials,

\[ u_1, u_2, \ldots, u_\ell, \]

which freely generate \( \mathfrak{g} \). The choice of such a set of generators gives a map \( u : \mathfrak{g} \to \mathbb{C}^\ell \), defined by \( u(x) = (u_1(x), u_2(x), \ldots, u_\ell(x)) \). Theorem. Let \( R \subset \mathfrak{g} \) be the subset of all regular semisimple elements. If \( \mathfrak{g}_0 \) is any real form of \( \mathfrak{g} \), then the image \( u(\mathfrak{g}_0 \cap R) \) consists of \( n \) connected components, where \( n \) is the number of distinct conjugacy classes of Cartan subalgebras of \( \mathfrak{g}_0 \). (Received November 4, 1970.)


We prove the following Theorem. Let \((X, \mathcal{U})\) be a locally compact uniform space with a weakly transitive group \( G \) of uniformly equicontinuous homeomorphisms acting on \( X \). Then: (i) \( G \) is nonexpansive with respect to some base \( B \) for \( \mathcal{U} \). (ii) \( X \) is uniformly locally compact. (iii) There exists a topology on \( G \) making \( G \) into a locally bounded topological group. (iv) \( X \) is isomorphic to \( W(G)/W(G)_p \) where \( W(G) \) is the locally compact Weil completion of \( G \), and \( W(G)_p = \{ g \in W(G) : g(p) = p \in X \} \). (v) There exists a unique \( G \)-invariant regular Borel measure on \( X \). These results contain a new proof of a theorem of I. E. Segal and explain why no examples other than those of topological groups and quotients of topological groups were found of the class of uniform spaces considered by Segal. (Received November 4, 1970.)

682-22-11. WITHDRAWN.

26 Functions of Real Variables

682-26-1. RICHARD J. BAGBY, New Mexico State University, Las Cruces, New Mexico 88001. A difference quotient norm for spaces of quasi-homogeneous Bessel potentials.

Let \( \alpha = (\alpha_1, \ldots, \alpha_n) \) with \( \alpha_1 \geq 1, \alpha_1 = 1 \) and let \( m/\alpha_1 \) be an even integer for each \( i \). For \( x \in \mathbb{R}^n \), let \( [x] = (\sum x_i m/\alpha_i)^{1/n} \). Define an operator \( J_\alpha^\gamma \) on tempered distributions by \( (J_\alpha^\gamma T)^w = (1 + [x] m)^{-\gamma/m} \), and let \( L_\alpha^p \) be the isometric image of \( L^p \) under \( J_\alpha^\gamma \). For \( 1 < p < \infty \) and \( 0 < \gamma < 1 \), an equivalent norm for \( L_\alpha^p \) is obtained in terms of a quasi-homogeneous difference quotient. This generalizes a result of Strichartz [J. Math. Mech. 16(1967)]. (Received September 8, 1970.)
Measurability and linear lattices of functions closed under convergence everywhere.

Let $X$ be any abstract space and $R$ the space of reals without the infinities. Let $F$ denote the product space $R^X$. In the space $F$ consider the usual operations of addition of functions, multiplication by a scalar, and supremum and infimum of two functions. Any subset $L$ of $F$ is called a linear lattice if it is closed in $F$ under these operations. If in addition the family $L$ of functions is closed in $F$ under convergence everywhere on the set $X$, then it is called a $P$-linear lattice. By the carrier $V$ of the family $L$ of functions we mean the collection of all sets of the form $f^{-1}(R_0)$, where $R_0 = R \setminus \{0\}$, and $f$ is a nonnegative function from $L$. For any $r_1, r_2 \in R$ define $(r_1 : r_2) = r_1 : r_2$ if $r_2 \neq 0$ and $(r_1 : r_2) = 0$ otherwise. Theorem. The family $L$ of functions in the space $F$ forms a $P$-linear lattice if and only if its carrier $V$ forms a sigma ring of sets in the space $X$ and the following is true: A function $f \in F$ belongs to the family $L$ if and only if there exists a nonnegative function $g \in L$ such that the support of the function $f$ is contained in the support of the function $g$ that is $f^{-1}(R_0) \subseteq g^{-1}(R_0)$ and the composed function $(f : g)$ is measurable with respect to the sigma ring $V$. (Received October 27, 1970.)

Transfinite diameters and weighted averaging processes. Preliminary report.

In 1923, M. Fekete introduced a set function defined for compact sets in the complex plane. He called this function the transfinite diameter. In the 1960's, E. Hille utilized the postulates for a general averaging process given by Kolmogorov (1930) and Nagumo (1930) to extend the notion of transfinite diameter to an arbitrary metric space. In this paper we give a set of postulates to define what we mean by a weighted averaging process and establish the analogue of the representation theorem proved by Kolmogorov and Nagumo in the unweighted case. We give three extensions of the transfinite diameter and one extension of a related notion, the Chebyshev constant, to an arbitrary metric space based on weighted averages. In each case, a passage to the limit is involved, and the corresponding convergence theorems are proved. Basic properties of the extensions of transfinite diameters are established, and inequalities between the extensions of transfinite diameters and the Chebyshev constant are also proved. Equalities between certain extensions are shown under mild restrictions on the weighted averages involved. (Received October 26, 1970.)

An integration by parts theorems for the $\Psi$ integral. Preliminary report.

A definition of $\Psi$ integrals is given in Abstract 672-200, these Notices 17(1969), 140. In this paper the following theorem is proved. Theorem. Let $f$ and $g$ be real-valued functions on the closed interval $[a,b]$, and suppose that the Stieltjes mean $\sigma$-integral, $\int_a^b f dg$, exists. Then the existence of one of the integrals $\int_a^b a f dg$ or $\Psi \int_a^b f dg$ implies the existence of the other integral. Also $\Psi \int_a^b f dg = f g|_a^b + \Psi \int_a^b f dg - 2M \int_a^b f dg$. (Received November 2, 1970.)
Necessary and sufficient conditions that a mapping of real-valued functions into the reals be a limit.

We first generalize the limit concept in McShane and Botts, "Real analysis," The University Series in Undergraduate Mathematics, Van Nostrand Co., Inc., Princeton, N. J., 1959. Consider, then, any nonempty collection $F$ of real-valued functions and any mapping $L: F \to R$ of $F$ into the reals. As our main result, we give necessary and sufficient conditions that there exist a direction $p$ such that for each function $f$ in the collection $F$, $f$ has a limit $\lim_{p} f$ with respect to $p$ and $Lf = \lim_{p} f$. As an application, we prove that if $F$ is a nonempty collection of continuous functions on some nonempty compact space, then every limit $L: F \to R$ is an evaluation mapping. (Received November 4, 1970.)

Uniqueness of functions on a sphere from averages over sectors. Preliminary report.

Let $f: S^{2} \to R$ be integrable and let $\alpha > 0$. For any point $p \in S^{2}$, let $(\theta, \psi)$ be colatitude and longitude relative to a north pole at $p$ and let $I(p, \varphi)$ denote the integral of $f$ over the sector $\{(\theta, \psi) : \varphi < \theta < \varphi + \alpha, 0 < \psi < \pi\}$. If $\alpha / 2\pi$ is not rational, then $f$ is uniquely determined by $I(p, \varphi)$. (Received November 5, 1970.)

28 Measure and Integration

The regular decomposition of a measure. Preliminary report.

A measure is a countably additive and nonnegative set function defined on a sigma-ring. A measure is said to be regular with respect to a given collection of sets, if it can be approximated by its values on that collection of sets, where that collection is closed under finite unions. The class of regular measures is closed under sums and arbitrary suprema. The regular part of a measure is the largest regular measure which is less than or equal to it. A measure is said to be weakly antiregular, if it is $S$-singular with respect to its regular part. See R. A. Johnson (Proc. Amer. Math. Soc. 18(1967), 628-632) for a discussion of $S$-singularity. It is shown that any measure can be uniquely expressed, up to $S$-singularity, as the sum of a regular measure and a weakly antiregular measure. One example of the above decomposition is the Lebesgue decomposition of a measure with respect to another measure. (Received November 2, 1970.)

Integration-by-parts and substitution for integrals of interval functions.

Suppose $h, f, g$ are real-valued functions on the closed interval $[a, b]$. For most types of Stieltjes integrals, if the formula (*) $\int_{a}^{b} h(fg) = \int_{a}^{b} (h(f)g + \int_{a}^{b} (hfg)df)$ holds, then integration-by-parts follows by letting $h = 1$. Also, there are conditions such that if integration-by-parts holds on each subinterval of $[a, b]$ then the formula (*) can be obtained. Since (*) is a form of substitution, there exists a relationship between this idea and integration-by-parts. In this paper, integration-by-parts and substitution are studied in terms of integrals.
of interval functions. A general substitution theorem is given and is used to obtain substitution formulas for the weighted integral and the \( \varphi \)-integral. Integration-by-parts is defined in terms of integrals of interval functions and some of the properties are discussed. It is then shown that integration-by-parts follows as a special application of substitution. (Received November 2, 1970.)

682-28-3. WITHDRAWN.

682-28-4. JAMES W. ROBERTS, University of South Carolina, Columbia, South Carolina 29208.

Invariant measures.

In the following we suppose that \( X \) is a compact Hausdorff space and that \( G \) is a group of homeomorphisms of \( X \) onto \( X \). We denote this pair by \((X,G)\). \((X,G)\) is said to satisfy condition I if for every finite sequence of open sets \( O_1, O_2, \ldots, O_m \) such that each \( O_1 \) has \( n_1 \) disjoint translates (with \( n_1 > 1 \)) there exists a subsequence \( O_{i_1}, O_{i_2}, \ldots, O_{i_k} \) such that \( \sum_{j=1}^{k} n_{i_j} \geq m \) and \( \bigcup_{j=1}^{k} O_{i_j} \neq X \). Theorem. Condition I is equivalent to the existence of a nontrivial finite Borel measure on \( X \) invariant under translation by members of \( G \). (Received November 3, 1970.)

682-28-5. SARAH L. CHRISTIANSEN, Drake University, Des Moines, Iowa 50311 and JULIUS R. BLUM, University of New Mexico, Albuquerque, New Mexico 87106. Complete ergodicity, weak mixing, and stacking methods.

Stacking methods using residuals, for constructing ergodic measure preserving transformations on the unit interval, are classified in 2 types (type 1, always cut in \( k \) pieces and add \( j \), and type 2 cut in \( kn \) and add \( jn \) at step \( n \)). Each type has 2 subtypes depending upon where the pieces are added. A transformation \( T \) is defined to be completely ergodic if and only if \( T^n \) is ergodic for all \( n \in \mathbb{N} \). The following results were obtained: A measure-preserving transformation of type 1 which is completely ergodic is weak mixing. A measure preserving transformation of type 2 which is completely ergodic and where \( \{j_n\} \) has a constant subsequence is weak mixing. In addition several theorems give sufficient conditions on the stacking method for the resulting transformation to be completely ergodic. (Received November 3, 1970.)

682-28-6. THADDEUS G. DANKEL, JR., Duke University, Durham, North Carolina 27706. Bimeasures and bilinear functionals on \( C(0,1) \).

A bimeasure on a measurable space \((X,\mathcal{F})\) is a function \( \mu : \mathcal{F} \times \mathcal{F} \to \mathbb{R} \) such that \( \mu(\cdot, A) \) and \( \mu(B, \cdot) \) are measures. A bounded bilinear functional \( B \) on \( C(0,1) \) defines a bimeasure on \([0,1], \) Borel sets) as follows: \( B(\varphi, \psi) = \int_0^1 \varphi(x) \mu_\varphi(dx) \) and \( \mu_\varphi(A) = \int_0^1 \varphi(y) \mu(A, dy) \). A bimeasure is extendable to a measure \( \mu^* \) if there is a measure \( \mu^* \) on \((X \times X, \mathcal{F} \times \mathcal{F})\) such that \( \mu(A, B) = \mu^*(A \times B) \). Theorem. A bounded bilinear functional \( B \) on \( C(0,1) \) is of the form \( B(\varphi, \psi) = \int_0^1 \int_0^1 \varphi(x) \psi(y) \mu^*(dx, dy) \) for a measure \( \mu^* \) on \([0,1] \times [0,1] \) if and only if the associated bimeasure is extendable to \( \mu^* \). By Morse and Transue (Rend. Circ. Mat. Palermo (2) 4, 270-300), we have: Corollary. There exist bimeasures on \([0,1]\) which are not extendable to measures. Theorem. The bimeasure \( \mu \) on \(([0,1], Borel sets)\) is extendable to a measure if and only if for every countable disjoint union
of rectangles $A_i \times B_i$ which is again a rectangle, $\sum_{i=1}^{\infty} \mu(A_i, B_i) = \sum_{i=1}^{\infty} \mu(A_i, B_i) \mu(A_j, B_j)$. (Received November 3, 1970.)


It is shown that there exists a pair $f, g$ of continuous functions over the interval $[u, v]$ such that $f$ is not $g$-integrable over $[u, v]$ with respect to the Riemann-Stieltjes integral and if $k$ is in $(u, v)$ then $f$ is $g$-integrable over $[k, v]$, but if $\epsilon > 0$ then there exists a positive number $\delta$ such that if $D$ is a subdivision of $[u, v]$ with norm less than $\delta$, then any two Riemann sums in terms of $D, f,$ and $g$ differ by less than $\epsilon$.

(Received November 3, 1970.)


Let $f$ be a bounded real valued function on the compact interval $[a, b]$. Let $g$ be a real valued function of bounded variation on $[a, b]$. For each $x \in [a, b]$ let $V(x)$ be the total variation of $g$ over $[a, x]$. $(S) \int_a^b dg$ is the ordinary Stieltjes integral of $f$ with respect to $g$. $(GS) \int_a^b df dV$ is the generalized Stieltjes integral discussed in Apostol ("Mathematical analysis," Addison-Wesley, Reading, Mass., 1957). Theorem 1. $(GS) \int_a^b df dV$ exists iff $(GS) \int_a^b df dV$ exists. Theorem 2. $(S) \int_a^b dg$ exists iff $(S) \int_a^b df dV$ exists. Theorem 3. $(GS) \int_a^b df dV$ exists iff each of the sets $\bigcup_{x \in L} [V(x)-, V(x)]$ and $\bigcup_{x \in R} [V(x), V(x+)]$ has Lebesgue measure zero. (Here $L$ and $R$ are the sets of left and right discontinuities of $f$, respectively.) Note. The definitions of the Stieltjes and generalized Stieltjes integrals as well as the above theorems are formally meaningful even when $f$ and $g$ are complex valued and $f$ is not bounded. If $f$ is unbounded, Theorems 1 and 2 as well as a slightly modified version of Theorem 3 remain true. If $f$ (bounded or not) and $g$ are complex valued and $(GS) \int_a^b df dV$ exists then $(GS) \int_a^b df dV$ exists.

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B. W. Helton (Proc. Amer. Math. Soc. 23(1969), 493-500) establishes a product integral representation for a Gronwall inequality for the refinement integral $(LR) \int_a^b (fH + fG)$. He uses ideas and results from an earlier paper of his (Pacific J. Math. 16(1966), 297-322). In the latter paper, Helton uses results from a paper by J. S. MacNerney (Illinois J. Math. 7(1963), 148-173). The purpose of this paper is to present a straightforward approach to Helton's result above. Also, the result is extended somewhat in this paper. It is observed that Helton's functions $H$ and $G$ may be replaced by nondecreasing real-valued functions $h$ and $g$, respectively, on $[a, b]$. There is a positive real number $c \leq 1$ such that $[g(p^+ - g(p)] \leq 1 - c$ for every $p$ in $[a, b]$ and such that $[g(p) - g(p^-)] \leq 1 - c$ for every $p$ in $(a, b]$. Let $f$ be a bounded real-valued function on $[a, b]$ such that the refinement integral $(LR) \int_a^b (fH + fG)$ exists and such that there is a positive real number $k$ for which $f(x) \leq k + (LR) \int_a^x (fH + fG)$ for all $x$ in $[a, b]$. It is shown directly that for each $x$ in $(a, b]$ the product refinement integral $\prod_{a}^{x} (k + dh)/(1 - dg)$ exists and does not exceed $\exp(3/c, \min([h(x) - h(a)] + [g(x) - g(a)]))$. Continuing thus, it is shown that $f(x) \leq k \prod_{a}^{x} (1 + dh)/(1 - dg)$ for all $x$ in $[a, b]$. (Received November 5, 1970.)
The Gronwall inequality for weighted integrals.

W. W. Schmaedeke and G. R. Sell (Proc. Amer. Math. Soc. 19(1968), 1217-1222) establish a Gronwall inequality for the Stieltjes mean sigma integral and the interior refinement integral. B. W. Helton (Proc. Amer. Math. Soc. 23(1969), 493-500) establishes a product integral representation for a Gronwall inequality for the (LR)-integral. J. V. Herod (Proc. Amer. Math. Soc. 23(1969), 34-37) establishes a Gronwall inequality for linear Stieltjes integrals. The Main Theorem of Schmaedeke and Sell is a special case of Theorem 4 of Helton. The linear function \( J(f) \) defined by Herod is more general than the linear function \( J(f) = (\text{LR}) \int_a^b (fH + fg) \) considered by Helton, but there are linear functions which satisfy the hypotheses of Helton’s Theorem 4 but do not satisfy the hypotheses of Herod’s theorem. This paper concerns a Gronwall inequality for the weighted refinement integral \( \int F(w_1, w_2, w_3) \int_a^b f(x)k(x)dg(x) \). The technique used is patterned after the proof given by Schmaedeke and Sell of their Main Theorem but is shorter and simpler. The ideas used are considerably less complicated than those used by Helton. When the assumptions of Helton’s Theorem 4 are weakened somewhat, this theorem contains the result in this paper when \( w_1 \) and \( w_3 \) are nonnegative real numbers such that \( w_1 + w_3 = 1 \) and \( w_2 = 0 \). (Received November 5, 1970.)


In the space \( C^n \) of n-times continuously differentiable functions on the unit circle \( \Gamma \), define \( f \) to be nonnegative if \((-1)^n \pi^{(n)}(x) + \int_0^\pi f \, dm \geq 0 \) for all \( x \), where \( m \) is Lebesgue measure. Then \( C^n \) becomes partially ordered with generating cone and its order dual \( M^n \) coincides with the strong dual of \( C^n \) in the derivative norm. The cone of \( M^n \) consists of those distributions which can be written \( \pi^n \sigma^{(n)} + \sigma \Gamma m \) for some positive measure \( \sigma \). \( M^n \) is a vector lattice generated by the linearly compact simplex corresponding to the probability measures, with extreme points corresponding to the point-measures. One thus obtains a characterization of the \( n \)th order distributions on \( \Gamma \) similar to the Riesz Representation Theorem for positive measures. The cones \( C^n \) (\( n = 1, 2, \ldots \)) form a decreasing sequence within the pointwise cone; correspondingly, the cones \( M^n \) (\( n = 1, 2, \ldots \)) are a succession of enlargements of the usual cone of positive measures, and give a unique form for the Structure Theorem for distributions of finite order. The method extends Choquet’s version of Herglotz’ Theorem on the open unit disc to include representations of harmonic functions with traces on \( \Gamma \) which are distributions. (Received November 5, 1970.)

30 Functions of a Complex Variable


Let \( \mathcal{H} \) denote the class of all holomorphic functions defined in the disk \(|z| < 1\) that have radial limit zero on a dense set of the circle \(|z| = 1\). Let \( \mathcal{H}_m \) be the corresponding class for meromorphic functions. **Theorem 1.** If \( h \) is holomorphic in the disk, then there exist functions \( f_0, f_1, g_0 \), and \( g_1 \) in
class $\mathcal{C}$ such that $h = f_0 + g_0$, $h = f_1 \cdot g_1$. Theorem 2. If $h$ is meromorphic in the disk, then there exist functions $f_0$ and $f_1$ in class $\mathcal{C}$ and functions $g_0$ and $g_1$ in class $\mathcal{C}$ such that $h = f_0 + g_0$ and $h = f_1 \cdot g_1$. (Received June 24, 1970.)

682-30-2. MAXWELL O. READE, University of Michigan, Ann Arbor, Michigan 48104 and E. J. ZLOTKIEWICZ, M. Curie Skłodowska University, Lublin, Poland. On a theorem of Kaczmarzski concerning the equation $f(z) = pf(a)$. Preliminary report.

Let $\mathcal{X}$ denote a fixed and compact class of functions analytic in the unit disc $\Delta$ satisfying $f(0) = 0$ and $f'(0) = 1$, and let $p$ and $a$ be fixed complex numbers, $p \neq 0$, $p \neq 1$, $0 < |a| < 1$. For $f \in \mathcal{X}$ the equation $f(z) = pf(a)$ may have a solution $z_f$ in the unit disc $\Delta$. Mocanu [Mathematica (Cluj) 6(29) (1964), 63-79] considered the problem of determining $m(p,a,\mathcal{X}) = \text{glb}\{ |z_f| | f \in \mathcal{X} \}$, for $\mathcal{X} \in \mathcal{F}$ the set of all normalized univalent functions, and Kaczmarzski [Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. 15 (1967), 245-251] considered the problem of determining $m(p,a,\mathcal{X})$ for $\mathcal{X} \equiv \mathcal{F}$, the set of all typically real univalent functions but with $p$ real. In this note we show that Kaczmarzski's result remains valid even for $\mathcal{X} \equiv \mathcal{F}$, the set of all typically real $f(z)$ for which $f'(0) = 0$, $f'(0) = 1$. Kaczmarzski used a variational method, while we use the notion of variability domains for the ratio $\frac{f(b)}{f(a)}$, $a \in \Delta, b \in \Delta$. In particular we use a result due to Pat [Ann. Univ. Marie Curie-Skłodowska Sect. A 18 (1964), 53-72]. (Received August 3, 1970.)

682-30-3. FRANKLIN LOWENTHAL, University of Oregon, Eugene, Oregon 97403. Uniform finite generation of the rotation group.

If a Lie group $H$ is generated by two one-parameter subgroups, one says $H$ is uniformly finitely generated by them if there exists a positive integer $n$ such that every element of $H$ can be expressed as a product of length at most $n$ of elements chosen alternately from the two one-parameter subgroups. Define the least such $n$ as the order of generation of $H$. It is shown that if $\psi$ denotes the angle between the axes corresponding to two different one-parameter rotation groups, then the order of generation of the rotation group is 3 if $\psi = \pi/2$ and if $\pi/(k + 1) \leq \psi < \pi/k$, the order of generation is $k + 2$ ($k \geq 2$). (Received September 10, 1970.)

682-30-4. DOV AHARONOV, Technion-Israel Institute of Technology, Haifa, Israel and WILLIAM E. KIRWAN, University of Maryland, College Park, Maryland 20742. A method of symmetrization and applications.

A method of symmetrization for plane domains is introduced that generalizes the methods of symmetrization considered by G. Szego and M. Marcus. It is shown that under this method of symmetrization the mapping radius of a fixed point is not decreased. It is also shown that this method of symmetrization preserves convex domains and Bieberbach-Eilenberg domains. Using these facts, some results are obtained concerning the covering properties of univalent convex functions in the unit disc and the Bieberbach-Eilenberg functions.
The fractional derivative of order $\alpha$ is a generalization of the familiar derivative $D^\alpha_z f(z) = \frac{d^\alpha f(z)}{dz^\alpha}$ to nonintegral values of $\alpha$. $D^\alpha_z f(z)$ is defined by generalizing Cauchy's integral formula

$$D^\alpha_z f(z) = \frac{\Gamma(\alpha + 1)/2\pi i}{z} \int_0^\infty \left( \frac{t-z}{t-z} \right)^{\alpha+1} dt,$$

where $\alpha$ is an arbitrary complex number. In the author's paper ["Leibniz rule for fractional derivatives generalized and an application to infinite series", SIAM J. Appl. Math. 18(1970), 658-674] the Leibniz rule for fractional derivatives $D^\alpha_z u(z)v(z) = \sum_{n=0}^\infty \frac{\alpha^n}{n!} D_z^n u(z) D_z^n v(z)$ is derived. Replacing the fractional derivatives appearing in this Leibniz rule by contour integral representations, and integrating over the appropriate contour, Parseval's formula emerges if $z$ is kept fixed. Thus Parseval's formula is a special case of the Leibniz rule for fractional derivatives in much the same way that a Fourier series is a special case of a Laurent series $f(t) = \sum_{n=-\infty}^{\infty} a_n (t-z)^n$ when $t-z = r \exp(i\theta)$ and $z$ and $r$ are kept fixed.

(Received October 12, 1970.)

682-30-6. BOO SANG LEE, University of Kentucky, Lexington, Kentucky 40506. On univalent entire functions.

In this paper the following two theorems have been proved. Theorem 1. Suppose $f(z)$ is a transcendental entire function such that $f(z) = c(\exp B z) \prod_{n=1}^N (1 - z/a_n)$, where $0 \leq N \leq \infty$ and $c, B, a_n$ are all complex numbers such that $c \neq 0$ and $|z_n| > 1$. Let $\gamma_n = |z_n| - (|z_n|^2 - 1)^{1/2}$. Then $f(z)$ is univalent in $D$ if $|B|/2 + \sum_{n=1}^N \gamma_n^2 + 2 \sum_{n=1}^N \gamma_n^2 \leq 1$. Theorem 2. Suppose $f(z)$ is a transcendental entire function such that $f(z) = c(\exp B z) \prod_{n=1}^N (1 - z/a_n)$, where $0 \leq N \leq \infty$ and $c, B, a_n$ are all real such that $C \neq 0$, $B \leq 0$ and $a_n > 1$. Then (i) $f'(z)$ has exactly $N$ zeros (each zero $> 1$) which are denoted by $b_1, b_2, \ldots, b_N$; (ii) $f(z)$ and all its derivatives are univalent in $D$ if $|B|/2 + \sum_{n=1}^N \gamma_n^2 + 2 \sum_{n=1}^N \gamma_n^2 \leq 1$, where $\gamma_n = b_n - (b_n^2 - 1)^{1/2}$. (Received October 19, 1970.)

682-30-7. SANFORD S. MILLER, University of Kentucky, Lexington, Kentucky 40506. The Hardy class of Bazilevic functions and their derivatives.

The Bazilevic function $f(z)$ defined in $D: |z| < 1$ by

$$f(z) = \sum_{n=0}^\infty a_n z^n,$$

where $f(z)$ is regular with $\Re P(\zeta) > 0$ in $D$ and $\alpha > 0$ is univalent. The class of such functions contains many of the special classes of univalent functions. The author determines the Hardy classes to which $f(z)$ and $f'(z)$ belong. In addition if $f(z) = \sum_{n=0}^\infty a_n z^n$ the limiting value of $|a_n|/n$ is obtained. (Received October 19, 1970.)

682-30-8. SWARUPCHAND M. SHAH, University of Kentucky, Lexington, Kentucky 40506. Holomorphic functions with areally mean $p$-valent derivatives.

Let $f(z) = \sum_{n=0}^\infty a_n z^n$ be holomorphic in the unit disc $D$. Theorem 1. If $f$ and each successive derivative $f^{(k)}$ are areally mean $p$-valent (a. m. p. v.) in $D$ and $p > \frac{1}{2}$, then $f$ is an entire function and $\lim sup_{r \to \infty} (\log M(r, f)/r) \leq B = A(p+2)^{2p}(p+1)$ where $P = \lfloor p \rfloor$ (integer part of $p$) and $A = A(p) = (p+2)^{2p-1} \exp(p^{1/2} + \frac{1}{2})$.

Theorem 2. Let $\lfloor n_j \rfloor_{j=1}^\infty$ be a strictly increasing sequence of positive integers. Let $f$ be defined in $D$ by the above power series and suppose that each $f^{(n)}$ is a. m. p. v. in $D$. Let $R$ be the radius of convergence of the power series.
(a) Then \( \lim \inf_{n \to \infty} (n_{j+1} - n_j)^{1/n_j} \geq R \). (b) If there is an integer \( M > p \) such that \( \lim \inf_{n \to \infty} (n_{j+1} - n_j)^{1/n_j} \geq R \), then \( \theta(n) \) and \( \phi(n) \) are slowly oscillating functions such that \( n_j \geq (1 + o(1)) \cdot \phi(j) \) and \( \lim \sup_{j \to \infty} \theta(j) = \alpha < 1 \), then \( f \) is an entire function of order not exceeding \( 1/(1 - \alpha) \). (Received October 19, 1970.)


Denote by \( V_k \) \( (k \geq 2) \) the set of all functions \( f(z) = z + \sum_{n=1}^{\infty} a_n z^n \) analytic in \( U = \{ z : |z| < 1 \} \) such that \( f'(z) \) is in \( V_k \) and such that \( f(U) \) is a domain with boundary rotation at most \( k \). Let \( S_n(k) = \max (|a_n| : f(z) = z + \sum_{j=0}^n a_j z^j \in V_k) \).

It is known (O. Lehto, Ann. Acad. Sci. Fenn. A 124 (1952)) that \( S_2(k) = k/2 \), \( S_3(k) = (k^2 + 2)/6 \), and (M. Schiffer and O. Tammi, Ann. Acad. Sci. Fenn. A1 396 (1967)) that \( S_4(k) = (k^3 + 8k)/24 \). In all cases the extremal function has been \( F(z) = (1/k) \left( \frac{1+z}{1-z} \right)^{1/2} - 1 \).

The following theorem is proved.

Theorem. Let \( f(z) = z + \sum_{n=1}^{\infty} a_n z^n \) be in \( V_k \) and let \( F(z) \) be as above. If \( n \geq (k+6)/4 \), then \( |a_n| \leq A(n, k) \), with equality for any such \( n \) if and only if \( f(z) \) is a rotation of \( F(z) \). Thus \( S_n(k) \approx A(n, k) \) for \( n \geq (k+6)/4 \). This theorem enables one to give a simple proof of Lehto's result that for fixed \( n \), \( S_n(k) \approx k^{n-1}/n! \) as \( k \to \infty \). The proof of the theorem is based on a representation of \( f(z) \) as a quotient of powers of starlike functions. (Received October 20, 1970.)

682-30-10. CHARLES KAM-TAI CHUI, Texas A & M University, College Station, Texas 77843. On polynomial approximation of \( H^p \) and \( \varphi^p \) functions.

Let \( C \) be the unit circle and \( D \) the open unit disc. Let \( H^p, 1 \leq p \leq \infty \), be the Hardy space with norm \( \| f \|_p \). and \( \varphi^p, 1 \leq p < \infty \), denote the class of functions \( f \) holomorphic in \( D \) such that \( \| f \|_p = (1/p) \int_D \| f(x+iy) \|^p \, dx \, dy \)^{1/p} < \infty \). A polynomial whose zeros lie on \( C \) is called a \( C \)-polynomial.

Theorem 1. If \( f \in H^p, 1 \leq p \leq \infty \), and is zero free in \( D \), there exist \( C \)-polynomials \( p_n \) which converge to \( f \) uniformly on each compact subset of \( D \) and which satisfy \( \| p_n - f \|_p \leq 2 \| f \|_p \) for all \( n \). Theorem 2. If the \( p_n \) are \( C \)-polynomials such that \( \| p_n - f \|_p \to 0 \) as \( n \to \infty \), then \( f \) is either a \( C \)-polynomial or the zero function. Theorem 3. If \( f \in \varphi^p, 1 \leq p < \infty \), and is zero free in \( D \), there exist \( C \)-polynomials \( p_n \) such that \( \| p_n - f \|_p \to 0 \) as \( n \to \infty \). (Received October 26, 1970.)

682-30-11. DAVID C. HADDAD, West Virginia University, Morgantown, West Virginia 26506. Boundary behavior of holomorphic functions.

Let \( D \) denote the open unit disc and let \( C \) denote the unit circle. G. R. MacLane ["Asymptotic values of holomorphic functions," Rice Univ. Studies 49 (1963), no. 1] defined the class \( \varphi \) of functions nonconstant and holomorphic in \( D \) that have asymptotic values at a dense subset of \( C \). The purpose of this paper is to give a necessary and sufficient condition for a function to belong to \( \varphi \). Let \( S \) be a nonvoid subset of \( D \). For each \( r, 0 < r < 1 \), let the components of \( S \cap \{ z : r < |z| < 1 \} \) be \( S_j(r), j \in J \). Let \( d_j(r) \) be the diameter of \( S_j(r) \), and let \( d(r) = \sup_j d_j(r) \). \( S \) is said to end at points of \( C \) if \( d(r) \) tends to 0 as \( r \) tends to 1. It is shown that if \( w = f(z) \) is a nonconstant function holomorphic in \( D \), then \( f \in \varphi \) if and only if there exists a line \( L \) in the \( w \)-plane such that the set \( \{ z : f(z) \in L \} \) ends at points of \( C \). (Received October 26, 1970.)
On a problem of Erdős concerning the successive derivatives of an entire function.

Theorem. Let \( \{S_k\} \) be any sequence of sets in the plane, each of which has no finite limit point. Then there exists a sequence \( \{n_k\} \) of integers and a transcendental entire function \( f(z) \) such that \( f^{(n_k)}(z) = 0 \) if \( z \in S_k \). This answers affirmatively a question of Erdős posed in Hayman's function theory problem book (p. 17, Problem 2.30). The construction depends on an infinite series argument and a judicious application and re-application of the following: Technical Lemma. Let (i) \( M > 0 \) and \( 0 < r < R \), (ii) \( A = \{a_j\} \) \( j \in \mathbb{C}, |z| < R \), (iii) \( B = \{b_{j,k}\} \) \( j = 0, 1, \ldots, n \), (iv) \( \{a_{j,k}, b_{j,k}\} \) \( j = 0, 1, \ldots, n \). Then given any \( \epsilon > 0 \), there exists a polynomial \( P(z) \) such that (i) \( P(z) = 0 \), (ii) \( |P(z)| < \epsilon \) if \( |z| < r \), (iii) \( P(z) = a_{j,k} \) for \( j = 0, 1, \ldots, n \). The lemma follows easily from material in Walsh's book on approximation and interpolation (pp. 310-312).


Let \( f(z) = z + \sum_{k=1}^{\infty} \alpha_k z^k \) and \( g(z) = z + \sum_{k=1}^{\infty} b_k z^k \) be analytic for \( |z| < 1 \) and \( \Re(f(z)/g(z)) > 0 \) or \( |f(z)/g(z) - (1 - \lambda) g(z)| < 1 \) for \( |z| < 1 \), where \( 0 < \lambda < 1 \). The author determines the values of \( R \) such that \( f(z) \) is univalent (and starlike) for \( |z| < R \) under the assumption that (i) \( \Re(g(z)/z) > 0 \) or (ii) \( \Re(g(z)/z) > \alpha \), \( 0 < \alpha < 1 \), for \( |z| < 1 \). The results obtained include some of the results obtained by MacGregor (Proc. Amer. Math. Soc. 14(1963), 514-520 and ibid, 521-524), Ratti (Math. Z. 107(1969), 241-248) and the author. A typical result follows. Theorem. Suppose that \( f(z) = z + \sum_{k=1}^{\infty} \alpha_k z^k \) and \( g(z) = z + \sum_{k=1}^{\infty} b_k z^k \) are analytic for \( |z| < 1 \) and \( \Re(g(z)/z) > 0 \) for \( |z| < 1 \). If \( \Re(f(z)/g(z)) \) \( > 0 \) for \( |z| < 1 \), then \( f(z) \) is univalent and starlike for \( |z| < \min((1 - \lambda)/(1 + \lambda), 1/n, R^{1/n}) \), where \( R = \left( \left(2n+\lambda - n\lambda\right)^2 + (1 - \lambda) \right)^{1/2} + \left(2n+\lambda - n\lambda\right) \right)/(1 + \lambda) \). (Received October 28, 1970.)

682-30-14. TED J. SUFFRIDGE, University of Kentucky, Lexington, Kentucky 40506. Extreme points in a class of polynomials having univalent sequential limits.

Let \( n \) be a fixed positive integer and let \( \alpha = n/(n+1) \). Let \( \mathscr{P}_n \) be the collection of polynomials of the form \( P(z) = z + \sum_{k=2}^{n} a_k z^k \) such that the equation \( \left[\sum_{k=2}^{n} (e^{i\theta} - e^{-i\theta}) a_k \sin(k\alpha) / \sin(k\alpha) \right]^j z^j = 0 \) has no roots in \( |z| < 1 \) for \( k = 1, 2, \ldots, n \). Then \( \mathscr{P}_n \) contains all appropriately normalized univalent polynomials of degree \( n \) or less. If \( \left\{ P_{n_k} \right\}_{k=1}^{\infty} \) is a sequence such that \( P_{n_k} \in \mathscr{P}_n \), \( n_k \to \infty \) as \( k \to \infty \) and \( P_{n_k} \) uniformly on compact subsets of the disk \( f \) is univalent. If \( f \not\in \mathscr{P}_n \) is extreme (i.e. \( f \) is not a proper convex combination of two distinct elements of \( \mathscr{P}_n \) then \( (n+1)/n \) \( P(z) = (1/n)z P'(z) \) is univalent. The collection of polynomials which are univalent in \( |z| < 1 \) and are of the form \( P(z) = z + a_2 z^2 + \cdots + 1/n z^n \) (so that all the zeros of \( P'(z) \) lie on the boundary) are dense in the class \( S \) of normalized univalent functions. These polynomials have the very striking geometric property that the tangent line to the curve \( P(e^{i\theta}) \), \( 0 \leq \theta \leq 2\pi \) turns at a constant rate (between cusps) as \( \theta \) varies. (Received October 28, 1970.)
For an analytic function $f$ on the unit disk $D$, let $L(f) = \limsup_{r \to 1} \{ \min(\log |f(re^{i\theta})|) \}^{1/2}$. We remark that if $L(f) > 0$ and $f$ is unbounded, then $f$ is strongly annular, and that $L(f) = 1$ for dominated term strongly annular (DTSA) functions (cf. Abstract 68T-149, these Notices 15(1968), 236 and Abstract 663-270, these Notices 16(1969), 163). For any complex number $a$, positive integer $p$, and $1 \leq \epsilon > 0$, let $A(e^{i\theta};\cos(p\theta) \geq \epsilon)$, $Z(f,a) = \{ z : f(z) = a \}$, $Z'(f,a)$ be the derived set of $Z(f,a)$ and finally, $S(e^{i\theta};a,p) = \{ f : Z'(f,a) \subset A(e^{i\theta};p) \}$. A result given in an article by Rubel (Duke Math. J. 30(1963), 437) aided in obtaining: Theorem. Let $f$ be an unbounded analytic function on $D$ such that $f \in S(e^{i\theta};a,p)$ for some $\epsilon$, $a$ and $p$. Then $L(f) \leq 1 - \epsilon$. Corollary 1. No DTSA function belongs to any $S(e^{i\theta};a,p)$. Let $g$ be the annular function constructed by Barth and Schneider for which $Z'(g,0) = \{ 1 \}$ (J. Reine Angew. Math. 234(1969), 179-183). Corollary 2. $L(g) \leq 0$. (Received October 29, 1970.)

Definition. A sequence of complex numbers $(a_n)_{n=1}^\infty$ is said to form a generalized A-set iff $\sum_{n} \left| \Im \left( \frac{1}{a_n}e^{-i\theta} \right) \right| = O(\log r)$, where $b = \left| \Im \left( \frac{1}{a_n}e^{-i\theta} \right) \right| > r^{-1}$. Then, by using the analogue theorems of Nevanlinna’s fundamental theorems for functions meromorphic in a half-plane, the following result is established: Theorem. Let $f$ be a meromorphic function on the whole finite plane with its number of poles satisfying the condition: $N(r,f) = O(r^\alpha)$ as $r \to \infty$, where $\alpha$ is a constant and is less than 2. Assume that there are three distinct values $a$, $b$, and $c$ (oo is permitted) such that the roots of the equations $f-a=0$, $f-b=0$, and $f-c=0$ form a generalized A-set, then the order of $f$ is no greater than one and in fact, $\tau(r,f) = O(r\log r)$ as $r \to \infty$. (Received October 22, 1970.)
Let $f$ be a function analytic in the unit disc with $f(0) = 0$, $f'(0) = 1$. Following Tammi [Ann. Acad. Sci. Fenn. Ser. A1 114 (1952)] the radius rotation for $f$ is defined to be $\tau = \lim_{r \to 1^-} \frac{2\pi}{\int_0^{2\pi} \left| \frac{zf'(z)}{f(z)} \right| \, d\theta}$, $z = re^{i\theta}$, $0 \leq r < 1$. Define $f \in W_k$ if $\tau \leq k\pi$. It is convenient to let $\alpha = (k+2)/4$ and $\beta = (k-2)/4$.

Theorem 1. If $f \in W_k$ then $f(z) = s_1(z)^\alpha/s_2(z)^\beta$ where $s_1$ and $s_2$ are starlike in $|z| < 1$. A variational formula for functions in $W_k$ is given. Theorem 2. If $f \in W_k$ then $r(1-r)^{2\beta}(1+r)^{-2\alpha} \leq |f(z)| \leq r(1+r)^{2\beta}(1-r)^{-2\alpha}$. Theorem 3. The radius of starlikeness of $W_k$ is $(k-(k^2-4)^{1/2})/2$. (Received November 2, 1970)

Let $D$ be the open unit disk in the complex plane. Let $A$ be the set of functions analytic in $D$ which map $D$ into itself, and are not the identity function on $D$. A linear fractional transformation which maps $D$ onto itself will be called a conformal automorphism of $D$ (abbreviated c. a.). Wolff [C. R. Acad. Sci. Paris 182 (1926), 200-201] showed that if $f \in A$ is not an elliptic c. a. of $D$, then the iterates $f^n$ of $f$ satisfy
\[ \lim_{n \to \infty} f^n(z) = w \] uniformly on compact sets for some $w$ with $|w| \leq 1$. In this case we denote $w$ by $T(f)$.

If $f$ is an elliptic c. a. of $D$, then $T(f)$ will denote the fixed point of $f$ in $D$. For $f \in A$, if $T(f) \in D$ it is the (unique) fixed point of $f$ in $D$, and if $f$ has a fixed point in $D$, then $T(f)$ is this fixed point. Theorem. If $f$ and $g$ are in $A$, $f$ is not a hyperbolic c. a. of $D$, and $f \circ g = g \circ f$, then $T(f) = T(g)$. This result is known when $T(f) \in D$. If $f$ and $g$ are assumed to have continuous extensions to the boundary of $D$, then a similar result is a corollary to a more general theorem of Shields [Proc. Amer. Math. Soc. 15 (1964), 703-706]. An example shows that the conclusion of the above theorem can fail when $f$ is a hyperbolic c. a. of $D$. (Received November 2, 1970.)

In this paper $T(r, f)$, $N(r, a)$, $n(r, a)$ have the usual meaning of Nevanlinna Theory. If $f(z)$ is an entire function of integral order $\rho \geq 1$, one calls $a$ an exceptional value $E_1$ (e. v. $E_1$) if $n(r, a) = o(\log M(r, f))$. The motivation to take a positive integer comes from the fact that for all entire functions of nonintegral order $n(r, a) \neq o(\log M(r, f))$, (a well-known result of Polya) and for zero order, there can exist uncountably many $a$'s for which $n(r, a) = o(\log M(r, f))$. Exceptional value $E_1$ for meromorphic functions is defined similarly replacing $\log M(r, f)$ by $T(r, f)$. In this paper results on e. v. $E_1$ analogous to e. v. B, e. v. V, e. v. N, e. v. L. e. v. E have been obtained. (For the definition of these exceptional values see S. M. Shah, Compositio Math. 9 (1951), 227-238.) A typical one is the following Theorem. A meromorphic function of integral order $\rho \geq 1$ can have at most two e. v. $E_1$. (Received October 26, 1970.)

Let \( V_k \) denote the class of all functions \( g(z) = z + \sum_{n=2}^{\infty} b_n z^n \) analytic in \( |z| < 1 \) for which
\[
\oint_0^{2\pi} \left| \operatorname{Re} \left[ 1 + zg''(z)/g'(z) \right] \right| \, dg \leq k \pi
\]
and let \( W_k \) be the class of odd functions in \( V_k \). **Theorem 1.** \( f(z) \in W_k \) if and only if there is a \( g(z) \in V_k \) with \( f'(z) = \sqrt{g(z)^2} \). Let \( A_{2n+1}(k) = \max |a_{2n+1}| \). **Theorem 2.** \( A_3(k) = k/6 \), \( A_5(k) = (k^2 + 4)/40 \) if \( k \leq 4 \) and \( A_5(k) = (7k+2)/10(10-k) \) if \( 2 < k \leq 4 \). **Theorem 3.** Let 
\[
f(z) = \sum_{n=0}^{\infty} a_{2n+1} z^{2n+1} \in W_k.
\]
Then (1) \( \alpha = \lim_{r \to 1} (1-r^2)(1-r^2)^{k+2}/4 \), \( M(r, \alpha) \) exists and \( \alpha \geq 2(k-2)/4 \), (2) \( \lim_{n \to \infty} \left( |a_{2n+1}|/(2n+1)^{k/4-3/2} \right) = \alpha/\Gamma((k+2)/4) \). (Received November 2, 1970.)


Using results of M. Weiss, Acta Math. 102(1959), 225-238, and K. G. Binmore, Bull. London Math. Soc. 1(1969), 211-217, the following result is proved: **Theorem.** If \( f(z) = \sum_{n=0}^{\infty} a_n z^n \) is unbounded in \( |z| < 1 \) and \( \lambda_{n+1} \cdot \lambda_n^{-1} \geq 3 \), then the cluster set of \( f \) is total at every point of \( |z| = 1 \). Using a standard result on cluster sets, it is then shown that under these hypotheses the function \( f \) can have radial limits only on a set of first category in \( |z| = 1 \). (Received November 4, 1970.)


Let \( D \) be a domain in \( \mathbb{R}^3 \), and \( f \in C^1(D) \), \( 0 \in D \), \( f(0) = 0 \). If \( F(P) = (f(P)P)/|P| \), \( P \in \mathbb{R}^3 \), then \( F \) maps the level surfaces of \( f \) into concentric spheres. Following the definition given by F. W. Gehring and J. Vaisala ("The coefficients of quasiconformality of domains in space," Acta Math. 114(1965), 4), we give sharp bounds on \( K_1^2(F) \) and \( K_0^2(F) \) and we achieve better bounds on the coefficients of quasiconformality of certain domains. In particular, \( K_1^2(\text{Cube}) \leq 6.854 \), \( K_0^2(\text{Cube}) \leq 3.959 \), and, if \( C \) is a right circular cylinder of height 2 and base radius 1, \( K_0^2(C) \leq 2.973 \), \( K_1^2(C) \leq 4.003 \). (Received November 4, 1970.)


S. M. Shah and S. Y. Trimble [Bull. Amer. Math. Soc. 75(1969), 153; Trans. Amer. Math. Soc. 144 (1969), 313; J. Math. Mech. 19(1969), 451] have discovered that the behavior of an analytic function \( f(z) \) is strongly influenced by the radii of univalence of its derivatives \( f^{(n)}(z) \) \( (n = 0, 1, 2, \ldots) \). In this paper many of Shah and Trimble's results are extended to large classes of locally univalent functions with locally univalent derivatives. The work depends on the concept of the \( \nu_{\mathcal{B}} \)-radius of a locally univalent function that is introduced and developed in this paper. Ch. Pommerenke's definition [Math. Ann. 155(1964), 108] of a linear invariant family of locally univalent functions, and the techniques of that theory are employed in this paper. It is proved that the universal linear invariant families \( \nu_{\mathcal{A}} \) introduced by Pommerenke are rotationally invariant. For fixed \( f(z) \) in \( \nu_{\mathcal{A}} \), it is shown that the function \( r \to \text{order}\left[ f(rz)/r \right] \) \( (0 < r \leq 1) \) is a continuous increasing function of \( r \). (Received November 5, 1970.)
Homeomorphisms of the closed unit disc $D$ onto itself which leave the 2-dimensional measure of any measurable set in $D$ invariant are being investigated. Such homeomorphisms will be shown to exist for any preassigned boundary correspondence $e^{i\phi} \rightarrow e^{i\psi(\phi)}$ with the property that $\psi(\phi): \mathbb{R} \rightarrow \mathbb{R}$ is bi-Lipschitzian (and $\psi(\phi) - \phi$ is periodic with period $2\pi$). (Received November 5, 1970.)

Let $I$ be an interval of the real axis which is contained in the intersection of the boundaries of disjoint Jordan domains $D$ and $E$. A function $g$, meromorphic in $E$, is the pseudocontinuation across $I$ of a function $f$, meromorphic in $D$, if $f$ and $g$ have the same nontangential boundary value $\phi(x)$ at almost every $x$ in $I$.

**Theorem.** In this situation, the boundary function $\phi$ has the property: for any circle $C$ on the Riemann sphere, either $m \{x \in I \cap \phi(x) \in C \} = 0$ or $m \{x \in I \cap \phi(x) \in C \} = m(S)$ denotes the Lebesgue measure of $S$.

There exists a homeomorphism of the closed upper half-plane, holomorphic in the open half-plane, which fails to have a pseudocontinuation across any interval of the real axis. The definition of pseudocontinuation was given by H. S. Shapiro who has also obtained holomorphic functions which have no pseudocontinuation ("Generalized analytic continuation," Symposia on Theoretical Physics and Mathematics, Vol. 8, (Symposium, Madras, 1967) Plenum Press, New York, 1968, pp. 151-163.) (Received November 5, 1970.)

**Theorem 1.** If $f \in \Sigma^*(p, w_0)$ there is a $p_0$, $.39 < p_0 < .6$, such that if $p < p_0$, then $\Re [\phi(z) / f'(z)] = 0$. **Theorem 2.** If $f \in \Sigma^*(p, w_0)$ and is univalent then $f(z) - w_0 = p w_0 (z - p)^{-1} (1 - p z)^{-1} \exp \left\{ \pi^{-1} \int_0^{2\pi} \log (1 - e^{-2t} \sin^2 \theta) \, dV(t) \right\}$ where $V(t)$ is a nondecreasing function on $(0, 2\pi)$.

**Theorem 3.** If $f \in \Sigma^*(p, w_0)$ and is univalent then $|f(z) - w_0| \leq 2^{-p} |w_0| (1 + r)^{2\pi} |1 - p z|^{-1} |r - p|^{-1}$ where $\alpha$ is the maximum jump of $V(t)$. (Received November 5, 1970.)

**Theorem.** If $\psi$ is an automorphism of $S$ onto itself there exists $\psi' \in \text{NSL}(2, \mathbb{R})(H)$ which gives rise to $\phi$. By consideration of the abelian differentials of the first kind of $S$ at elliptic fixed points of the group generated by $H$ and $\psi$, Weierstrass pairs are found for some modular groups. (Received November 5, 1970.)
Let $P$ be a nonnegative continuous function on an open Riemannian manifold $R$ with a positive definite metric tensor. We denote by $P_X(R)$ the class of $C^2$-functions $u$ on $R$ which satisfy the elliptic equation $\Delta u = Pu$ on $R$ and have the $X$-property. Here $\Delta$ is the Laplace-Beltrami operator associated with the Riemannian structure, and $X$ stands for $B$ (bounded), $D$ (Dirichlet-finite), $E$ (energy-finite), or their combinations. A pair $(R, P)$ belongs to the class $O_{PX}$ by definition, if the class $PX(R)$ consists of zero only ($P \neq 0$). The purpose of the present note is to announce the strict inclusion $O_{PD} < O_{PE}$ and thus to establish the complete string of inclusion relations: $O_G < O_{PB} < O_{PD} = O_{PBD} < O_{PE} = O_{PBE}$.

(Received November 5, 1970.)

31 Potential Theory

With the Brelot axioms holding on connected locally compact $R$, assume also Axiom IV: The set $H^+(R)$ of functions which are positive and harmonic on all of $R$ separates $R$. From the results of H. S. Bear and others, $R$ is metrized by $d(x, y) = \sup \{ |\log u(x) - \log u(y)| : u \in H^+(R) \}$, $x, y \in R$. Theorem. For each $x \in R$, and each $r > 0$ such that $B_{x, r}$ is compact, every point of $\{ y : d(x, y) = r \}$ is a regular boundary point of the open ball $B_{x, r}$. Corollary. Such balls can replace the base of regular regions of Axiom II. The problem of the existence of an axiomatic Laplacian and an axiomatic diffusion process is discussed. (Received September 4, 1970.)
A theorem of Epstein states the following: Let $D$ be a simply connected plane domain of finite area and $t$ a point of $D$ such that, for every function $u$ harmonic in $D$ and integrable over $D$, the area of $D$ equals $u(t)$. Then $D$ is a disk and $t$ its center. In this paper it is shown that the simple connectivity assumption in the above theorem may be replaced by a mild regularity condition on the boundary of $D$ and that the same conclusion still holds. It is also shown that if a plane domain with finite area has at least two boundary components which are continua then the mean-value property cannot hold for the class of all integrable harmonic functions with single-valued harmonic conjugates. (Received October 15, 1970.)

Let $G$ be a connected semisimple Lie group with finite center, $K$ a maximal compact subgroup, $G/K$ the corresponding Riemannian symmetric space of noncompact type. A function $f$ on $G/K$ is harmonic if $f$ is annihilated by every $G$-invariant differential operator on $G/K$ which annihilates constants. Vitali's theorem in one complex variable states that if \( \{f_n\} \) is a uniformly bounded sequence of holomorphic functions in a connected domain $D \subset \mathbb{C}$ having the property that $\lim f_n$ exists on an infinite subset $\sigma$ of $D$ having a finite limit point in $D$, then $\lim f_n$ exists everywhere in $D$ and is holomorphic. Similar subsets of $G$ and $G/K$ are defined, and a uniqueness theorem for analytic functions on $G$ and $G/K$ is given. Theorem. Let $\{f_n\}$ be a uniformly bounded sequence of harmonic functions on $G/K$ such that $\lim f_n$ exists on an infinite subset $\sigma$ of $G/K$ having the property that $f$ harmonic on $G/K$ and zero on $\sigma$ implies $f \equiv 0$ on $G/K$. Then $\lim f_n$ exists everywhere on $G/K$ and is harmonic. The proof uses the fact that any uniformly bounded family of harmonic functions on $G/K$ has compact closure. The results are then applied to harmonic functions on symmetric domains of noncompact type in euclidean space and also to holomorphic functions on bounded symmetric domains in $\mathbb{C}^n$. (Received October 19, 1970.)

Let $P > 0$. The $n$-dimensional Teichmüller domain, $T_n(P)$, is the ring domain in $\mathbb{R}^n$ whose boundary components consist of the segment $\{x = (x_1, x_2, \ldots, x_n) : -1 \leq x_1 \leq 0; x_i = 0, i \geq 2\}$ and the ray $\{x = (x_1, x_2, \ldots, x_n) : P \leq x_1 \leq 0; x_i = 0, 1 \leq i \leq 2\}$. A theorem by Gehring (Michigan Math. J. 9(1962), 149) established the relationship $\text{mod } T_3(P)/\text{mod } T_2(P) \geq 1$. In this paper the following additional inequalities are obtained: Let $h$ denote the 2-capacity of $T_2(P)$. Then $\max[1, \sqrt{n/2}] < \text{mod } T_3(P)/\text{mod } T_2(P) < \sqrt{2h + 16/n}$. Refinements to these inequalities and related asymptotic values are discussed. (Received October 19, 1970.)

A subcartesian space of class $C^\infty$ is a metrizable topological space $S$ with an atlas $\Phi$ satisfying the
following conditions: (1) For every \( \varphi \in \Phi \), \( \varphi \) is a homeomorphism with open domain \( V \subseteq S \) and image in some \( \mathbb{R}^{n_s} \). (2) For every \( \varphi, \psi \in \Phi \) and \( p \in V \cap V \), there exists open \( U \subseteq V \cap V \), \( p \in U \), such that \( \psi \circ \varphi^{-1} \) restricted to \( \varphi(U) \) can be extended to a \((C^\infty)\) diffeomorphism between open neighborhoods of \( \varphi(p) \) and \( \psi(p) \) in \( \mathbb{R}^n \) for some \( N \geq \max(n_s, n_s') \). We can assume that \( \Phi \) is a maximal atlas. A function \( u: S \to \mathcal{C} \) is of class \( P_{1/\nu}^{(v)}(S) \) (local Bessel potentials of reduced order \( \nu \)) if for every \( p \in S \) there is a chart \( \varphi \in \Phi \), \( p \in V \), such that \( u \circ \varphi \) can be extended to a Bessel potential of order \( \nu + n_s/2 \) on \( \mathbb{R}^{n_s} \). A subcartesian space \( S \) is of polyhedral type if for every \( p \in S \) there is a chart \( \varphi \in \Phi \) such that \( \varphi(V) \) is the intersection of an open cube with a geometric complex in \( \mathbb{R}^{n_s} \). It is shown that for a geometric complex \( K \subseteq \mathbb{R}^n \), \( u \in P^{(\nu)}(K) \) iff \( u \) is of class \( P^{\nu + k/2} \) on every maximal polyhedron of \( K \) of dimension \( k \) and if it satisfies together with its derivatives natural compatibility conditions at all points belonging to several maximal polyhedra of \( K \). It is shown, as a consequence, that if \( S_1 \subseteq S \) are \( C^\infty \) subcartesian spaces of polyhedral type such that the atlas on \( S_1 \) is the restriction of the atlas defining the structure on \( S \) then \( u \in P^{(\nu)}(S_1) \) iff \( u \) is the restrictions to \( S_1 \) of a function in \( P^{(\nu)}(S) \).

(Received November 4, 1970.)

### 31 Potential Theory

682-32-1. CHESTER ALAN JACEWITZ, Johns Hopkins University, Baltimore, Maryland 21218. A non-principal invariant subspace.

Let \( U^n \) be the open unit polydisc in complex \( n \)-space and \( H^2(U^n) \) the usual Hardy space (of index 2) of holomorphic functions on \( U^n \). A subspace of \( H^2(U^n) \) is invariant when it is closed under multiplication by the coordinate functions. Among problems left open in a paper of P. R. Ahern and D. N. Clark (J. Math. Mech. 19(1970)) and the book "Function theory in polydiscs" by Rudin is the following: Let \( M \) be a closed invariant subspace of \( H^2(U^n) \) such that for every \((z_1, \ldots, z_n)\) in \( U^n \) there is some \( f \) in \( M \) with \( f(z_1, \ldots, z_n) \neq 0 \). Is there an example of such a subspace \( M \) where for each \( g \) in \( M \) the smallest closed invariant subspace containing \( g \) is a proper subspace of \( M \)? The author has constructed an example of such a subspace. (Received October 30, 1970.)

682-32-2. KYONG T. HAHN, Pennsylvania State University, University Park, Pennsylvania 16802. Subordination principal and distortion theorems on holomorphic mappings in the space \( C^n \).

Generalizing the principle of subordination to the space \( C^n \) the author obtains various distortion theorems and inequalities for certain classes of holomorphic mappings. Let \( f \) and \( F \) be two holomorphic mappings of a domain \( D \subseteq C^n \) into \( C^n \). \( f \) is subordinate to \( F \) on \( D \) if \( F \) is a biholomorphic mapping of \( D \) onto \( B \) with \( f(D) \subseteq B \), and there exists a point \( t \in D \) such that \( f(t) = F(t) \). If \( f \) is subordinate to \( F \) in a bounded schlicht domain \( D \) with \( f(t) = F(t) \) at \( t \in D \), then at \( t \), \( (\partial f/\partial z) (\partial F/\partial z) * \equiv (\partial f/\partial z) (\partial F/\partial z) * \) and equality holds iff \( F^{-1} \cdot f \) is in the group of stability at \( t \); here \( (\partial F/\partial z) * \) denotes the conjugate transpose of the Jacobian matrix \( (\partial f/\partial z) \). Using this the author obtains: Theorem 1. If \( w = f(z) \) is a holomorphic mapping of a bounded homogeneous schlicht domain \( D \) into itself, then for \( z \in D \), \( |J_f(z)|^2 \equiv T_{D}(z, z)/T_{D}(f(z), f(z)) \) and equality holds iff \( f \) is an automorphism, where \( T_{D} = \det(T_{\alpha\beta}) = (I)_{\alpha\beta} \) is the Bergman metric tensor of \( D \). Theorem 2. Let \( D \) be a bounded homogeneous schlicht domain which is star-shaped with respect to \( 0 \in D \). If \( w = f(z) \) is a biholomorphic mapping of \( D \) onto \( f(D) \),
then \( z \in D \left| J_f(z) \leq d_w^n \right| J_z(0) \right|^{-1} \), where \( d_w \) denotes the largest number such that \( w + d_w z \in f(D) \) for \( z \in D \), and \( J_z \) the Jacobian determinant of automorphism which maps 0 into \( z \in D \). (Received November 2, 1970.)

682-32-3. JOHN K. SHAW, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. **Whittaker constants for entire functions of several complex variables.** Preliminary report.

Let \( f(z_1, z_2, \ldots, z_n) \) be an entire function. The exponential type of \( f \) is given by 
\[
\tau(f) = \limsup_{l \to \infty} \frac{1}{l} \left| f(a(0, \ldots, 0)) \right|,
\]
where \( a \) is a multi-index. The Whittaker constant \( W_n \) is defined to be the supremum of positive numbers \( c \) having the following property: if \( \tau(f) < c \) and if \( f(a) \) has a zero in the polydisc \( \left| z_k \right| < 1, 1 \leq k \leq n \) for each multi-index \( a \), then \( f = 0 \). Several characterizations of the Whittaker constant \( W_1 \) are known. In this paper, a determination of \( W_n \) is given for arbitrary \( n \). The determination depends on a system of generalized Goncharov polynomials. A precise relationship between \( W_n \) and \( W_1 \) is furnished and it is shown that \( W_1 > W_2 \approx W_3 \approx \ldots \). (Received November 2, 1970.)

682-32-4. JAMES J. METZGER, University of Georgia, Athens, Georgia 30601. **Weighted spaces of entire functions.** Preliminary report.

For \( \phi = \{ \varphi \} \) a class of real-valued functions on \( \mathbb{C}^n \), define \( E(\phi) \) to be the space of all entire functions \( f \) on \( \mathbb{C}^n \) which satisfy \( |f(z)| = O(\exp(\varphi(z))) \) for some \( \varphi \in \phi \). Let \( \Phi^* = \{ \varphi^*: \varphi \in \phi \} \), defined by \( \varphi^*(w) = \sup \{ \Re(z \cdot w) - \varphi(z): z \in \mathbb{C}^n \} \); also let \( D(\Phi^*) = \bigcup \{ D(\varphi^*): \varphi^* \in \Phi^* \} \) where \( D(\varphi^*) \) is the interior of the set in \( \mathbb{C}^n \) where \( \varphi^* < \infty \). Define \( E_p(\Phi^*) \) to be the space of all functions \( g \) analytic in \( D(\Phi^*) \) such that \( |g(w)| = O(\exp(\varphi^*(w))) \) for all \( \varphi^* \in \Phi^* \). With suitable assumptions on the class \( \phi \) -- the most important being that the functions in \( \phi \) are convex -- it is shown that under the natural inductive topology, \( E(\phi) \) is a Montel space whose dual can be identified with \( E_p(\Phi^*) \) via the Laplace transformation. In particular, \( E(\phi) \) is analytically uniform. Let \( n = 1 \), and let \( p > 1 \) and \( q > 1 \) be fixed. The above results hold for the class \( \bar{\phi} = \{ |x|^p + A |y|^q: A > 0 \} \) of functions of \( \mathbb{C} \). It is shown that in each of the spaces \( E(\phi) \) and \( E_p(\bar{\phi}) \) corresponding to this class, the local ideal generated by any one function is identical with the closed ideal generated by the function. This result is used to show that in each of these spaces every solution of a convolution equation can be approximated by finite sums of exponential-polynomial solutions of the equations. (Received November 4, 1970.)

682-32-5. SU-SHING CHEN, University of Florida, Gainesville, Florida 32601. **Theorems of Accola type on algebraic manifolds.** Preliminary report.

We extend the work of R. D. M. Accola, "Two theorems on Riemann surfaces with noncyclic automorphism groups," Proc. Amer. Math. Soc. 25(1970), 598-602. Several formulas are obtained on invariants (such as the genera and the Euler-Poincaré characteristics) of an algebraic manifold and its finite unramified covering manifolds. Another formula is on some invariants of quotients of a bounded homogeneous domain \( D \) by certain subgroups of a uniform discrete subgroup of the group of analytic automorphisms of \( D \). (Received November 5, 1970.)
33 Special Functions


Standard methods are used to show that the "old" conjecture: "All zeros of the $P_j^k(b)$ polynomials lie on the unit circle" implies that the formal b-functions corresponding to Exp, Cosh, Sinh, cos and sin are both generalized entire functions and entire functions when $b > 0$ [where $P_j^k(b)$ is defined in "Generalized powers", Amer. Math. Monthly 72(1965), 619-627; and the formal b-function $f_b(x) = f(x/b)$ is defined in Abstract 672-62, these Notices 17(1970), 101]. When $b > 0$ the conjecture yields: $\text{Exp}_b(x) = \text{Exp}[x\text{E}(b)]$, $\text{Cosh}_b(x) = \text{Exp}[x\text{C}(b)]$, $\text{Sinh}_b(x) = \text{Exp}[x\text{C}(b)]$ where $\text{E}(b) = \text{Exp}_b'(0)$, $\text{C}(b) = \text{Cosh}_b'(0) = \frac{1}{2} [\text{E}(b) - \text{E}(b^{-1})]$ and $\text{S}(b) = \text{Sinh}_b'(0) = \frac{1}{2} [\text{E}(b) + \text{E}(b^{-1})]$. Similar results hold for the b-functions $\text{exp}_b(ix)$, $\cos_b(x)$ and $\sin_b(x)$, where $\text{exp}_b(ix) = \cos_b(x) + i\sin_b(x)$. (Received September 17, 1970.)

682-33-2. CHANDRA MOHAN JOSHI, Texas A & M University, College Station, Texas 77843 and J. B. McDONALD, Utah State University, Logan, Utah 84321. Some finite summation theorems and an asymptotic expansion for the generalized hypergeometric series. Preliminary report.

For the generalized hypergeometric series

$$\sum_{n=0}^{\infty} \frac{((a_i)_n)}{((b_j)_n)x^n/n!} = F((ab); (b); x)$$

where $(a_i)_n$ stands for the set of $p$-parameters $a_1, a_2, \ldots, a_p$; $(b_j)_n$ stands for the set of $q$-parameters $b_1, b_2, \ldots, b_q$; and so on, and $(a_i)_n$ has the interpretation $\prod_{j=1}^{p} (a_{i_j})^{n_j}$, it has been shown that if $p + 1 < q$ or $p = q$, then

Theorem I. $p + 2 F_{q-1}^{1}(a, c+r; (b); (d); x) = 1/(c) \sum_{s=0}^{r} \binom{c-a}{s} \binom{c-a-1}{s} x^s F_{q-s}(c+r, s; (d); x)$

Theorem II. $p + 2 F_{q-1}^{1}(a, c; (b); (d); x) = 1/(c-r) \sum_{s=0}^{r-1} \binom{c-a}{s} \binom{c-a+1+s}{s} \binom{-c+r+s}{r} F_{q-s}(c+r, s; (d); x)$

These theorems lead to some interesting transformation formulas involving $2 F_1(x)$ and $3 F_2(1)$. One such transformation formula for $3 F_2(1)$ finds immediate application in proving a theorem on the asymptotic expansion of $F_{p}^1(x)$, that generalizes the well-known result (Slater, "Confluent hypergeometric functions," Cambridge Univ. Press, New York, 1960, p. 60). The asymptotic expansion thus obtained proves to be of considerable importance as it suggests the possibility of further extension of the Econometric problem investigated by the second author in the particular case, $p = 1$ [Ph.D. thesis (Econometrics), Purdue Univ., 1970] to be the general case of $p$-parameters. (Received October 30, 1970.)


Let $U_n(x) = x^n + \ldots; n = 0, 1, 2, \ldots$, be a set of polynomials together with its derivatives $U_n^{'}$ satisfying the biorthogonal conditions

$$\int_{a}^{b} p(x) U_n(x)x^{k} dx$$

is zero or not according as $i = 0, 1, \ldots, n - 1$ or $i = n$, and

$$\int_{a}^{b} q(x) U_n^{'}(x)x^{k} dx$$

is zero or not according as $i = 0, 1, 2, \ldots, n - 1$ or $i = n$, where $p(x)$ and $q(x)$ are admissible weight functions, i.e. $\int_{a}^{b} p(x) dx > 0$ and $\int_{a}^{b} q(x) dx > 0$. In this paper we show that $q(x)$ must satisfy the differential equation

$$\sum_{i=0}^{n} a_i x^{k-1} q^{(i)}(x) = C(x^{k-1}) x^{k-1} q(x) (C, 1, a_i's \text{ are constants})$$

with the boundary conditions $q(a) = 0$ and $q(b) = 0$. From this, the weight function $p(x)$ can be determined, up to a multiplicative constant from $q(x)$ by the relation $p(x) = (x^{k-1}/\sum_{i=0}^{n} a_i x^{k-1}) q(x)$. For $k = 1$, the above result reduces to the
orthogonal case which leads to the Jacobi polynomials. This result was first established by W. Hahn and later investigated by H. Krall (Bull. Amer. Math. Soc. 42(1936), 423–428). (Received November 3, 1970.)


Let \( f(x) \) be a real continuous function on \((0, \infty)\) and suppose \( \int_0^\infty |f(x)|^2 x^\alpha e^{-x} \, dx < \infty \). Here \( \alpha > -1 \) is a constant. Let \( T_n[f] \) be the generalized Toeplitz matrix of order \( n + 1 \) associated with \( f \) via the Laguerre polynomials \( L_n^{(\alpha)}(x) \) (see J. V. Baxley, Arch. Rational Mech. Anal. 30(1968), 308–320), and let \( \lambda_{j,n} = \lambda_{j,n}[f] \), \( j = 1, 2, \ldots, n + 1 \), be the eigenvalues of \( T_n[f] \) arranged in nondecreasing order. Let \( \omega \) be a positive integer and suppose (a) \( f(x) \sim m + \sigma x^\omega \) as \( x \to 0^+ \), where \( \sigma > 0 \), (b) \( f(x) > m \) for \( 0 < x < \infty \), (c) \( f(x) = o(x^N) \) as \( x \to \infty \) for some positive integer \( N \), (d) \( \lim \inf_{x \to \infty} f(x) > m \). Generalizing the methods of the above reference, we prove (1) \( \limsup_{n \to \infty} n^{-\omega} |\lambda_{1,n} - m| \approx \sigma \lambda_1 \), where \( \lambda_1 \approx \lambda_2 \approx \ldots \approx \lambda_n \approx \ldots \) are the eigenvalues, arranged in nondecreasing order, of a certain positive differential operator with compact inverse, (2) \( \lambda_{j,n} = m + O(n^{-\omega}) \), and \( O \) cannot be replaced by \( o \), (3) if, in addition, \( f(x) \sim m + \sigma x^\omega \) for \( 0 < x < \infty \), then equality holds in (1). (Received November 5, 1970.)

34 Ordinary Differential Equations

682-34-1. JAMES V. HEROD, Georgia Institute of Technology, Atlanta, Georgia 30332. A pairing of a class of evolution systems with a class of generators. Preliminary report.

With \( S \) a Banach space, the class \( OA \) consists of all functions \( V \) having the property that if \( a \leq b \) then \( V(a,b) \) is a function from \( S \) to \( S \) and (1) there is a continuous function \( \rho \) which is of bounded variation on each finite interval such that if \( a \leq b \) and \( \rho(a) - \rho(b) < 1 \) then \( 1 - V(a,b) \) has range all of \( S \) and, if \( P \) and \( Q \) are in \( S \), then \( 1 - V(Q,P)P \) = \( V(Q,P)Q \), (2) if \( x \leq y \leq z \) and \( P \) is in \( S \), then \( V(x,y)P + V(y,z)P = V(x,z)P \), (3) if \( a \leq b \) then there is a nondecreasing function \( \sigma \) such that if \( x \leq 0 \) and \( P \) is in \( S \) then there is a nondecreasing function \( \sigma \) such that if \( x > 0 \) and \( P \) is in \( S \) then there is a positive number \( \delta \) having the property that if \( Q \) is in \( S \) such that \( |Q - P| < \delta \) and \( a \leq x \leq y \leq b \) then \( V(Q) \leq V(Q)P \leq V(Q)P \), and (4) if \( a > b \) then there is a nondecreasing function \( \beta \) such that if \( x > 0 \) and \( P \) is in \( S \) then there is a positive number \( \delta \) having the property that if \( Q \) is in \( S \) such that \( |Q - P| < \delta \) and \( a \leq x \leq y \leq b \) then \( |\beta(x) - \beta(y)| \leq |V(Q,y) - V(Q,x)Q| \). A class \( OM \) is defined and a mapping \( \epsilon \) is found which "pairs" members \( V \) of \( OA \) and \( M \) of \( OM \) related by \( M(x,y)P = P + \int_x^y V[M(y,x)P] \) for all \( x \leq y \) and all \( P \) in \( S \). (Contrast J. S. Mac Nerney, "A nonlinear integral operation," Illinois J. Math. 8(1964), 621–638.) (Received June 4, 1970.)

682-34-2. RONALD A. KNIGHT, Oklahoma State University, Stillwater, Oklahoma 74074. A characterization of a class of planar dynamical systems.

For definitions and notations refer to those given by Ahmad (Pacific J. Math. 32(1970), 561–574). A flow \( (\mathbb{R}^2, \pi) \) is said to have characteristic 0 if and only if \( D(x) = K(x) \) for each \( x \) in \( \mathbb{R}^2 \). The following theorem characterizes planar flows of characteristic 0. Theorem. Let \( S \) be the set of critical points of a flow \( (\mathbb{R}^2, \pi) \). Then \( (\mathbb{R}^2, \pi) \) has characteristic 0 if and only if one of the following holds: (1) \( S = \emptyset \) and \( (\mathbb{R}^2, \pi) \) is parallelizable. (2) \( S \) consists of at most two Poincaré centers. For
each \( s \in S \), there exists an unbounded connected neighborhood \( N_s \) of \( s \) consisting of \( s \) and periodic orbits surrounding \( s \). Either \( s \) is a global Poincaré center or \( N_s \) is a single trajectory. The restriction of the flow to \( \mathbb{R}^2 - \bigcup_{s \in S} N_s \) is parallelizable. (3) \( S = \mathbb{R}^2 \). These conditions are sharp. (Received August 21, 1970.)

682-34-3. DONALD F. ST. MARY, Department of Mathematics and Statistics, University of Massachusetts, Amherst, Massachusetts 01002. Oscillation of complex differential systems.

This paper is concerned with first order linear matrix differential equations defined in the complex plane. It is shown that if such a system is oscillatory in a domain \( D \), that is, each component of a vector solution has a zero in \( D \), then there exist a solution and two points in \( D \) such that each component of the latter solution has a zero at one or the other of the points. It is also shown that some sufficient conditions for nonoscillation on the real line, recently developed by Z. Nehari, also hold in the complex plane. (Received October 15, 1970.)


The phase-shift-averaging method produces a canonical representation for periodic solutions of a class of piecewise-linear differential equations in terms of solutions of a related, simpler equation. The form of this representation may be regarded as a type of superposition result for the nonlinear equations in question, and the representation itself easily yields useful information about the behavior of the solution. Here the method is shown to be applicable to equations of the form \( Lx = (\text{sgn } a) g_m(x) + a \sin \omega t \), where \( L \) is an even-order, constant-coefficient ordinary differential operator, \( g_m(x) \) is an odd, piecewise-constant nonlinearity, and \( \text{sgn } a = a/|a| \) for some nonzero constant \( a \). One solution of this equation is shown to be represented by an average of \( m \) terms of the form \( (1/2) (y(t + t_k) + y(t - t_k)) \), \( k = 1, \ldots, m \), where \( y(t) \) is a solution of an equation similar to the given equation with the nonlinearity \( g_m(x) \) replaced by \( \text{sgn } x \). This result extends work of J. Chandra and Fleishman ("A canonical representation for solution of a class of nonlinear problems", Internat. J. Non-Linear Mech. 5(1970), 63-79) and simplifies considerably the proofs they gave. (Received October 15, 1970.)

682-34-5. CLIFFORD H. ANDERSON, Ohio University, Athens, Ohio 45701. A differential-difference equation leads to a generalized Fourier-Dirichlet representation of the forcing function.

Let \( m, n \) and \( p \) be fixed positive integers. Set \( h_0 = 0 \) and let \(-\infty < h_{-n} < \ldots < h_{-1} < h_0 < h_1 < \ldots < h_p < \infty \). Consider the equation \( D f \). The labeling of the subscripts has been chosen so that \( a_{i,j} \neq 0 \). We find a particular solution to Equation (1) which vanishes on a specified interval \([v + h_{-n}, v + h_p]\). The fact that the particular solution vanishes on the interval leads to a generalized Fourier-Dirichlet representation for a large class of functions. (Received October 16, 1970.)
Let us consider a Hilbert space $H$; then a linear bounded symmetric operator $A_0$ in $H$, such that the origin is not an eigenvalue for it. Then, consider another linear bounded operator $B$ in $H$, which commutes with $A$. Let $(E_n^0)_n$ be the spectral family of $A_0$ and $F_n = E_1/n^2 - E_1/(n+1)^2$, $G_n = E - 1/(n+1)^2 - E - 1/n^2$. Assume now that $\|\exp(tBF_n)\| \leq A_n$, $\|\exp(tBG_n)\| \leq A_n'$, $-\infty < t < +\infty$, and also that $\sum_n^{\infty} \|BF_n\| < \infty$, $\sum_n^{\infty} \|BG_n\| < \infty$ where we took the operator norm. Let us give now an arbitrary almost-periodic function (in Bochner's sense) $h(t)$ from $-\infty < t < +\infty$ to $H$ and let $u(t)$, $-\infty < t < +\infty$ be a strong solution of the differential equation $u'(t) = (A_0 + B)u(t) + h(t)$. Then, if $\sup_{-\infty < t < +\infty} \|u(t)\| = L < \infty$, then $u(t)$ is likewise almost-periodic Bochner. (Received October 8, 1970.)

For the second order differential equation $(\ast)$ $r(t)u'' + q(t)\phi(u, u') + p(t)f(u)g(u') = h(t, u, u')$ let (1) $q$ be nonnegative and continuous on $[0, \infty)$, (2) $\phi$ be continuous on $R^2$ and $h$ be continuous on $[0, \infty) \times R^2$, (3) $\phi(x, y)y > 0$ for all $y \neq 0$, (4) $f$ and $g$ be continuous on $R^1$, (5) $g(y) > 0$ for all $y \in R^1$, (6) $\lim_{|x| \to \infty} F(x) = \infty$, where $F(x) = \int_0^x f(s) ds \geq 0$, (7) $\lim_{|y| \to \infty} G(y) = \infty$, where $G(y) = \int_0^x (sds/g(s))$, (8) there exist an integrable function $e$ on $[0, \infty)$ such that $|h(t, u, u')| \leq e(t)$, (9) there exist a positive constant $M$ such that $y^2/g(y) \leq MG(y)$ for all $y \in R^1$. Theorem. Assume that (1)–(9) hold. If $p$, $r \in C([0, \infty)$) are such that (a) $0 < p_0 \leq p(t)$, $0 < r_0 \leq r(t)$, and (b) $\int_0^{\infty} (|r'(t)| + |p'(t)|)dt < \infty$, then, for each solution $u$ of $(\ast)$, both $u$ and $u'$ are bounded. By a modification of the assumptions on $p$ and $r$, other theorems, too numerous to state here, can be given. (Received October 22, 1970.)

Coddington and Levinson ["Theory of ordinary differential equations," McGraw–Hill, New York, 1955, Chapter 10, § 4, 5] obtained the Parseval equality and spectral expansion associated with an arbitrary $n$th order singular scalar differential equation directly, through the consideration of such a problem as a limiting case of corresponding selfadjoint two-point boundary value problems on compact subintervals of the reals. In this paper we employ the same general method to derive the Green's matrix, a spectral matrix, the Parseval equality, and spectral expansions associated with a singular linear Hamiltonian vector differential system. For the present treatment, the corresponding selfadjoint two-point boundary problems is a linear Hamiltonian vector differential system of the form $L_1[u, v'](t) = -v'(t) + C(t)u(t) - A^*(t)v(t) = \lambda K(t)u(t)$, $L_2[u, v'](t) = u'(t) - A(t)u(t) - B(t)v(t) = 0$, $t \in \delta$, $m_0[u, v] = 0$, where $\delta$ is a compact subinterval, while $u(t)$ and $v(t)$ are complex-valued $n$-dimensional vector functions, and $A(t)$, $B(t)$, $C(t)$, and $K(t)$ are complex-valued $n \times n$ matrix functions. (Received October 15, 1970.)

It is well known that if \( q \) is positive, continuous, monotone and unbounded on \([0, \infty)\), then \( \int_0^\infty y(t) dt \) exists, where \( y \) is any solution of the second order differential equation \( y'' + q(t)y = 0 \). We will show, by using new techniques, that \( \lim f(t) \int_0^t g(s)y(s)ds \) exists for an interesting class of weight functions \( f \), \( g \). This result gives qualitative information as to how strongly \( y \) approaches zero, or how evenly balanced the oscillations become near infinity. Extensions to nonlinear equations are immediate. (Received October 26, 1970.)

682-34-10. GARRET J. ETGEN and JOHN B. SCOTT, University of Houston, Houston, Texas 77004. On the oscillation of a nonselfadjoint fourth order differential equation.

This paper is concerned with the fourth order linear differential equation: (1) \( y^{(4)}(t) + p(x)y'' + r(x)y = 0 \), where it is assumed that each of \( p(x) \) and \( r(x) \) is a positive, differentiable function on \([a, \infty)\) and that \( p'(x) \neq 0 \) and \( r'(x) \geq 0 \) on this interval. The following results concerning the oscillatory behavior of solutions are obtained: (i) There exist two linearly independent, bounded, oscillatory solutions \( u(x) \) and \( v(x) \) of (1) which generate a strongly oscillatory subspace \( S \) of solutions. (ii) If \( y(x) \) and \( y^*(x) \) are each in \( S \), then their zeros separate on \((a, \infty)\). (iii) There exists a bounded, nonoscillatory solution \( w(x) \) of (1) such that \( w(x), w'(x), w''(x), w'''(x) > 0 \) on \((a, \infty)\), \( sgn w(x) = sgn w'(x) = sgn w''(x) = sgn w'''(x) \) and \( \lim_{x \to \infty} w(x) = 0 \). (iv) There exists a nonoscillatory solution \( z(x) \) of (1) such that \( z(x), z'(x), z''(x) \) are positive on \((a, \infty)\) and \( \lim_{x \to \infty} z(x) = \infty \). The paper concludes with a characterization of the points conjugate to \( a \) and with a discussion of the relationships between (1) and its adjoint: (2) \( (y'' + p(x)y'' - r(x)y = 0 \). (Received October 26, 1970.)


Some perturbation theorems are proved for nonlinear differential systems \( x' = f(t, x) \) with respect to which the asymptotically invariant set is uniformly stable in variation. Under different growth conditions on \( R(t, x) \), the behavior of the asymptotically invariant set with respect to the perturbed differential system \( x' = f(t, x) + R(t, x) \) is studied. (Received October 26, 1970.)


For (\( \ast \) \( x'' + \chi(t)x = 0, -\infty < t < + \infty, p(t) \geq 0 \), define \( \varphi_k(u, \lambda) \) as the location of the kth zero of a solution of (\( \ast \) \) with \( x(u) = 0 \). Similarly one defines \( \psi_k(u, \lambda) \) as the kth zero after \( u \) of \( x(t), x'(u) = 0 \). Let \( \lambda_k(u, a) \) be the kth eigenvalue of (\( \ast \) \), \( x(u) = x(u+a) = 0 \) and \( \mu_k(u, a) \) that of (\( \ast \), \( x'(u) = x'(u+a) = 0 \). (1) sign \( \left( \lambda_k(u, a) - \mu_k(u, a) \right) \) \( = \) sign \( \left( \psi_k(u, \lambda_k) - \varphi_k(u, \lambda_k) \right) = \) sign \( \varphi_k(u, \lambda_k, u) \). (2) Between any two stationary values of \( \lambda_k \) as function of \( u \) there is a stationary value \( u^* \) of \( \lambda_k(u, u^*) \) as a function of \( u \) and \( \lambda_k(u, u^*) = \mu_k(u^*, u) \). (3) \( \left( \varphi_k(u, u) - u \right) \int_u^x p(t) \, dt \geq 4(k+.5)^2 \cos^2 \frac{\pi}{4(k+2)} - .5(\varphi_k - u) (\varphi_k - u) \int_u^x p(t) \, dt \) if \( \varphi_k(u, u) \geq 0 \). Somewhat more complicated inequalities are obtained for \( \varphi_k(u, u) \leq 0 \), and for \( \left( \psi_k - u \right) \int_u^x p(t) \, dt \). (4) \( p(u) \geq \varphi_k(u, \lambda) \psi_k(u, \lambda) \). (Received October 26, 1970.)
Integral stability was defined in 1959 by I. Vrkoc to study the asymptotic behaviour of differential equations under integrable perturbations. Absolute stability was defined in 1964 by J. Auslander and P. Seibert, and arose in the study of generalized prolongations for dynamical systems on locally compact spaces. An example is given of a uniformly Lipschitz function \( f: \mathbb{R} \to \mathbb{R} \) such that 0 is absolutely stable for \( x' = f(x) \), but is not integrally stable for this equation. Based on this example, the relationship between absolute stability and integral stability is characterized for one dimensional autonomous systems and for two dimensional autonomous systems in which 0 is an isolated critical point. The analysis relies heavily upon the characterization of these two types of stability in terms of Lyapunov functions. (Received October 23, 1970.)

Analytic simplification of a system of ordinary differential equations at an irregular type singularity.

Let \( 1_n(\mu) = \text{diag}(\mu_1, \ldots, \mu_n) \) for given complex \( \mu_k (1 \leq k \leq n) \). Consider (I) \( x^{\sigma+1}y' = F(x, z)y, \ xz' = \mu(z) \), where \( y \) and \( z \) are vectors of \( m \)-dimensional and \( n \)-dimensional, respectively, \( \sigma \) is a positive integer, \( \text{Re} \mu_k \geq 0 (1 \leq k \leq n) \), and \( F(x, z) \) is an \( m \) by \( m \) matrix holomorphic in \( D(a, \theta_1, \theta_2; d) = \{ (x, z) \mid 0 < |x| \leq a, \theta_1 < \arg x < \theta_2, \|z\| \leq d \} \) and admits a convergent expansion in powers of \( z \) with coefficients having asymptotic expansions in powers of \( x \). Let \( \lim_{x \to 0} F(x, 0) \), as \( x \) tends to 0 in \( \theta_1 < \arg x < \theta_2, \lambda_i (1 \leq i \leq s) \) be the eigenvalues of \( F_0 \). If the sector \( \alpha < \arg x < \beta (\theta_1 \leq \alpha < \beta \leq \theta_2) \) is proper with respect to \( (\lambda_i - \lambda_j)(\sigma \times \sigma)^{-1} \) for \( 1, \ldots, s \), then, there exists an \( m \) by \( m \) nonsingular matrix \( P(x, z) \) such that (i) \( P(x, z) \) is holomorphic in \( D(a', \alpha, \beta, d') (\alpha' \leq a, d' \leq d) \) and has the property as that of \( F(x, z) \); (ii) the transformation \( (T) y = P(x, z)Y, z = Z \) reduces (I) to (II) \( x^{\sigma+1}y' = G(x, z)Y, xZ' = 1_n(\mu)Z \), where \( G(x, z) \) is block-diagonal agreeing with the Jordan canonical form of \( F_0 \). (Received October 23, 1970.)

On solutions of certain selfadjoint differential equations of fourth order.

Consider the selfadjoint linear differential equation \( [r(x)y'']' - p(x)y = 0 \), where \( r(x) \) and \( p(x) \) are positive and continuous on an interval \( [\epsilon, \infty) \). We study the behavior of oscillatory solutions of the above equation, in particular, the separation of zeros. Theorem. If \( u(x) \) and \( v(x) \) are essentially different oscillatory solutions (i.e., not constant multiples of each other) of the above equation and if \( u(a) = v(a) = 0 \) for \( a > \epsilon \), then there exists an oscillatory solution \( z(x) \) such that (1) \( z(a) = 0 \); (2) the zeros of \( u(x) \) and \( z(x) \) separate each other on \( (a, \infty) \); and (3) the zeros of \( v(x) \) and \( z(x) \) separate each other on \( (a, \infty) \). (Received October 30, 1970.)

Periodic solutions of certain Lienard equations with delay. Preliminary report.

An equation of the form \( d^2x/dt^2 + f(x)dx/dt + g(x) = 0 \) is called a Lienard equation; behavior of its solutions has been extensively studied. A delay can be introduced into the equation by replacing the "spring term", \( g(x) \),
by \( g(x(t- r)) \) \( (r > 0) \). Then the following conditions on \( f \) and \( g \) ensure the existence of a periodic solution with period greater than \( 2r \). (I) \( f(x) \) is continuous, \( f(0) = \alpha \), and for some \( a > 0 \), \( f(x) > 0 \) for all \( |x| \leq a \). (II) \( g(x) = bx + o(x) \) is continuous, and \( xg(x) > 0 \) for \( x \neq 0 \). (III) \( F(x) = \int_0^x f(s) \, ds \) is monotone increasing (decreasing) to \( \infty (\infty) \) as \( x \to \infty (\infty) \). (IV) \( F^{-1}(y)g(F^{-1}(y))/y \to 0 \) as \( y \to \pm \infty \). (V) The characteristic function, \( \lambda^2 - \alpha \lambda + b \exp(-r\lambda) \), has at least one zero with positive real part. This result can be proven by applying a theorem of the author which gives sufficient conditions for the existence of periodic solutions of nonlinear functional differential equations (J. Differential Equations 6(1969)). (Received October 30, 1970.)

682-34-17. GARY BRIAN GREEN, Stanislaus State College, Turlock, California 95380 and ALAN M. KRALL, Pennsylvania State University, University Park, Pennsylvania 16802. Linear differential systems with infinitely many boundary points.

Results previously known for boundary value problems involving only a finite number of boundary points are extended to those which involve an infinite number of (possibly dense) boundary points. The system \( Ly = y' + Py, \sum_{i=1}^{\infty} A_i y(t_i) = 0, 0 \leq t_i \leq 1, i = 1, \ldots, \infty, \) is discussed in the Hilbert space \( L^2(0,1) \). Conditions for inverting the operator \( L \) are found, and the Green's function is exhibited. It is shown to have the standard properties as well as some which are new, when considered as a function of its second variable. It is further shown to be the limit a.e. of Green's function for problems involving a finite number of boundary points, as those points increase in number. Finally it is shown that \( L^{-1} \) is compact. Using the Green's function, the domain of \( L \) is shown to be dense in \( L^2(0,1) \), and the adjoint \( L^* \) and its domain are found. The eigenvalues of \( L \) are shown to lie in a vertical strip with infinity as their only limit point. This implies that if \( L^{-1} \) fails to exist, a slight perturbation in \( P \) will result in an invertible \( L \), and the assumption made earlier concerning the existence of the Green's function is reasonable. (Received November 2, 1970.)

682-34-18. PAUL OHME, Florida State University, Tallahassee, Florida 32306 and University of Rhode Island, Kingston, Rhode Island 02881. On the asymptotic behavior of the solutions of the third order nonlinear differential equation \( u''' - t\sigma u^n = 0 \). Preliminary report.

The asymptotic behavior of the proper, nonoscillatory solutions of the nonlinear, third order, ordinary differential equation \( (*) u''' - t\sigma u^n = 0 \), when \( n > 1 \) and \( \sigma \) is an arbitrary real number, will be considered. Cases for \( \sigma \) and \( n \) are studied and the possible asymptotic behavior \( (t \to \infty) \) of the solutions of \( (*) \) are found and conditions for their existence are demonstrated. Motivation for this work is R. Bellman's study of the second order equation of the same form found in Chapter 7 of his book ["Stability theory of differential equations," McGraw-Hill, New York, 1953]. Although the results for the second and third order equations are similar, the proofs of the third order cases are not routine extensions of the second order cases due to the fact that some of the techniques used by Bellman are not readily extendable to the third order cases. (Received November 2, 1970.)

682-34-19. DONALD F. ST. MARY, University of Massachusetts, Amherst, Massachusetts 01002 and WARREN E. SHREVE, North Dakota State University, Fargo, North Dakota 58102. A matrix analogue to a theorem of Leighton.

Leighton (Proc. Amer. Math. Soc. 13(1962), 603-610) has shown for scalar equations that if \( r_1 > 0, r_2 > 0, \)
if $x$ is a nontrivial solution to $(r_1 x')' + p_1 x = 0$ such that $x(a) = x(b) = 0$, and if $\int_a^b (r_1 - r_2)(x')^2 + (p_2 - p_1)x^2 \geq 0$ then a nontrivial solution $y$ of $(r_2 y')' + p_2 y = 0$ either has a zero on $(a, b)$ or zeros at both $a$ and $b$. In the matrix case, if $\det R_1 \neq 0, R_2 > 0, R_2^* = R_2^*, P_2^* = P_2^*$, if $x(t)$ is a nontrivial vector solution of $(R_1 x')' + P_1 x = 0$ with $x(a) = x(b) = 0$, and if $\int_a^b x^* (R_1 - R_2)x' + x^* (P_2 - P_1)x \geq 0$ then every prepared matrix solution $Y$ of $(R_2 Y')' + P_2 Y = 0$ with $\det Y \neq 0$ has a focal point (zero of $\det Y$) on $(a, b)$ or focal points at both $a$ and $b$. (Received November 3, 1970.)


Let $0 < F(0, t) \leq F(z, t) \leq P(t)$, $F(z, t) \sim P(t)$ as $z \to \infty$. If $P(t) = \infty$, assume $K > 0$. Otherwise, assume that $K$ is greater than the $n$th eigenvalue of $y'' + kyP = 0, B_y = 0$, where $B$ is a homogeneous boundary condition for $C^1$ functions on $[0, 1]$. Then there exists a bound $C$, independent of $y$, such that if $y$ is a solution of $y'' + kyF(y^2, t) = 0, B_y = 0, k \geq K$, then $|y(t)| \leq C$ for every $t \in [0, 1]$. (Received November 4, 1970.)


For a system of perturbed nonlinear differential equations we prove an existence-uniqueness theorem for a two-sided integral manifold $X(\theta, t)$ defined for all $\theta$ and $t$ which emanates from a solution of the unperturbed system. We also prove an existence theorem for an $\alpha$-parameter family of right-sided integral manifolds $y(\alpha, \theta, t)$ which are defined for all $\theta$ and $t \geq \alpha$. The relationship between $X(\theta, t)$ and $y(\alpha, \theta, t)$ for $t \to \infty$ is then considered. In particular we show that solution curves lying on these right sided integral manifolds approach the two sided integral manifold exponentially as $t \to \infty$. Finally we give generalizations of these results. (Received November 4, 1970.)


Suppose that $L$ is a formally selfadjoint ordinary differential operator of the form $L(y) = y^{(n)} + \sum_{j=1}^n p_j y^{(n-j)}$ on $[a, \infty)$ with $n$ even and each $p_j$ complex valued and sufficiently differentiable. Then there is a positive valued continuous function $w$ on $[a, \infty)$ such that $L$ is in the $w$-limit-point case at $\infty$, i.e. if $\lambda$ is a complex number with $\text{Im} \lambda \neq 0$ then the dimension of the vector space of functions $y$ satisfying $L(y) = \lambda wy$ on $[a, \infty)$ and $\int_a^\infty |y|^2 w < \infty$ is $n/2$. (Received November 5, 1970.)

682-34-23. PREM N. BAJAJ, Wichita State University, Wichita, Kansas 67208. On singular points in semidynamical systems.

A singular point in a semidynamical system is defined as a point of nonunicity (Abstract 655-72; these Notices 15(1968), 486). This paper is, essentially, a continuation of the author's paper "Singular points in products of semidynamical systems," SIAM J. Appl. Math. 18(1970), 282-286, and discusses connectedness properties of the set of singular points in a product semidynamical system. (Received November 5, 1970.)
35 Partial Differential Equations


Let $H$ be a separable Hilbert space and $V(t) \subset H$ a family of Hilbert spaces dense in $H$ with continuous injections. Let $S(t)$ be the standard operator with domain $V(t)$ such that $((x,y)) = (S(t)x,S(t)y)$ and assume $S^{-n/2}(t)$ is weakly $C$ with $(S^{-n/2}(t)h,k) = (S^{-n/2}(t)h,k)$ $(n \geq 1)$. Define $W = L^2(V(t)) = \{ u \in L^2(H); Su \in L^2(H) \}$ and let $a(t,\cdot,\cdot)$ be a measurable family of sesquilinear forms on $V(t) \times V(t)$ with $|a(t,x,y)| \leq c|x||y|$. Let $\beta \leq n - 1$ with $\beta < n/2$ and assume $S^{-n/2}(t)S^\beta (t)$ extends to be a uniformly bounded operator family in $H$. Let $E \in L^2(H)$ with $u_0 \in D(S^{n/2}(0))$ and set $\Phi = S^{-n}F$ where $F = \{ v \in W; v' \in L^2(H); v(T) = 0 \}$. Assume $Re a(t,\phi,S^n\phi) \leq a(S^n\phi,\phi)$ then there exists $u \in W$ such that $-\int_0^T(a(t,v',\phi)dt + \int_0^T a(t,u,v)dt = \int_0^T (f,v)dt + (u_0,v(0))$ for all $v \in F$. This is a special case of a general theorem obtained jointly with R. Carroll (cf. Proc. Conf. Evol. Equation Functional Analysis, Univ. Kansas, 1970). There are various kinds of concrete examples where it applies with the noncoercive. The author also has extended the Lions projection theorem to apply to some noncoercive situations. Thus let $\Phi \subset W$ $(W$ Hilbert, $\Phi$ prehilbert) and let $E: W \times \Phi \rightarrow C$ be a sesquilinear form with $u \rightarrow E(u,\phi)$ continuous. Set $E(u,\phi) = (u,K\phi)$ and then $\|K\phi\|_W \leq \beta \|\phi\|_\Phi$ if and only if for all $L \in \Phi$ (antidual) there exists $u_L \in W$ such that $E(u_L,\phi) = L(\phi)$ for all $\phi \in \Phi$. (Received September 10, 1970.)

682-35-2. PHILIP W. SCHAEFER, University of Tennessee, Knoxville, Tennessee 37916. A priori bounds in the Cauchy problem for coupled elliptic systems.

By means of the logarithmic convexity of a suitable functional, an a priori inequality is developed for the sum of the squares of the solutions of the Cauchy problem for the coupled elliptic system $L_1 u = av + f$, $L_2 v = bu + g$, where $L_1$ and $L_2$ are uniformly elliptic differential operators, $a$, $b$, $f$, and $g$ are bounded integrable functions with $|b(x)| > b_0 > 0$, and only one of the functions, say $v$, satisfies a stabilizing condition. Upper bounds for the error in measurement of the Cauchy data on the initial surface are prescribed. From this a priori estimate uniqueness, stability, and pointwise bounds for the solutions $u$ and $v$ are simultaneously deduced. (Received October 15, 1970.)


Necessary and sufficient conditions are given for the uniqueness of solutions of the Dirichlet and Neumann problems for the singular hyperbolic equation $u_{tt} + \frac{k}{t}u_t - \sum_{i,j=1}^{m} (a_{ij} x_j x_j) + cu = 0$, $-\infty < k < \infty$, in cylindrical domains $Q = I \times D$ where $I$ is the interval $0 < t < T$ and $D$ is a bounded domain in $m$ dimensional space. (Received October 22, 1970.)


Let (1) $L_u = a_{ij} u_{ij} + b_u u_t + cu = 0$ (summation convention) on $\Omega - E$, $E$ dense with zero capacity. If no
information about \( u \) on \( E \) is allowed, the notion of \( u \) satisfying (1) needs to be specified. \( u \) is said to be a pointwise solution of (1) at \( x_0 \) if \( u \) has a total differential of order 2 in the \( L_1 \), Lebesgue class, sense at \( x_0 \) whose coefficients satisfy (1). For \( L \) an elliptic operator with Holder-\( \alpha \) coefficients, \( F \) sets are removable for the class of \( L_\infty \) solutions. This result contains a regularity theorem for a class of operators for which the adjoint is not necessarily defined. (Received October 29, 1970.)


Let \((D, \| \cdot \|)\) be a space of \( 2\pi \)-periodic functions of \( x \in \mathbb{R}_n \) and let \( X \) be its completion. Define \( M \) and \( N \) to be the respective null spaces of a partial differential operator \( L : D \to \mathbb{R} \) and its formal adjoint \( L^* \). Let \( f(x, u) \) be a continuous (nonlinear) map of \( X \) to its conjugate \( X^* \). Theorem. If the induced operator \( L^*: D/N \to X^* \) has a continuous inverse on the annihilator \( N^0 \) of \( N \), and if for each \( w \in X \), \( \exists y \in M \ni f(\cdot, y+w) \in N^0 \), then the equation \( Lu = f(\cdot, u) \) has a weak periodic solution in \( X \) provided \( f \) is "small". The proof depends on some results from Michael's selection theory and the Schauder fixed point theorem. As an example, it can be shown that the equation \( c^2u_{tt} + \Delta^2 u = b|u|^{r-1}\text{sgn }u + g(t, x) \) has periodic solutions of the same period as \( g \) under certain restrictions on the frequencies of \( g \), the dimension of \( \Delta \), and the value of \( r \in (2, \infty) \) for all integers \( p \geq 1 \). (Received October 29, 1970.)

682-35-6. JOHN R. SCHULENBERGER, University of Utah, Salt Lake City, Utah 84112 and CALVIN H. WILCOX, Institute of Theoretical Physics, University of Geneva, Geneva, Switzerland. A coerciveness inequality for a class of nonelliptic operators of constant deficit.

This paper presents a coerciveness inequality for a class of first-order matrix partial differential operators of the form \( A = -iE(x)^{-1}\sum_{j=1}^{n} A_j \frac{\partial}{\partial x_j} \). Here \( x \in \mathbb{R}^n \) and \( E(x), A_1, \ldots, A_n \) are \( m \times m \) Hermitian matrices with the following properties: the \( A_i \) \( (i = 1, \ldots, n) \) are constant, while \( E(x) \) is uniformly positive definite on \( \mathbb{R}^n \), and \( E(x) \) and \( \frac{\partial}{\partial x_j} E(x) \) \( (i = 1, \ldots, n) \) are continuous and bounded on \( \mathbb{R}^n \). The operator \( A \) is essentially selfadjoint on a Hilbert space \( \mathcal{H} \) of functions with inner product \( \langle u, v \rangle_E = \int_{\mathbb{R}^n} u(x)^*E(x)v(x)dx \). \( A \) is said to be coercive on \( N(A) \), the orthogonal complement of \( N \), if there exists a constant \( k > 0 \) such that \( \sum_{j=1}^{n} \left\| D_j u \right\|^2 < K\left( \left\| A u \right\|^2 + \left\| u \right\|^2 \right) \) for all \( u \in D(A) \cap N(A)^\perp \). In this paper it is shown that \( A \) is coercive on \( N(A) \) provided that \( \lim_{|x| \to \infty} E(x) = E_0 \) uniformly in \( x/|x| \), where \( E_0 \) is positive definite, and the symbol \( A(p, x) = E(x)^{-1}\sum_{j=1}^{n} A_j p_j \) satisfies rank \( A(p, x) = m - k \) for all \( p \in \mathbb{R}^n \setminus \{0\} \) and \( x \in \mathbb{R}^n \). Such operators \( A \) are said to have constant deficit \( k \). \( A \) is elliptic if and only if it has constant deficit \( k = 0 \). (Received October 23, 1970.)


In \((X, t)\)-space let \( X = (x', x) = (x_1, \ldots, x_n) \), \( C_+ = \{ x = - t \geq 0 \} \), and \( Q_E = \{ x > |t| \geq 0, -\infty < x_j < \infty, j = 1, 2, \ldots, n \} \). The notion of paraboloidal mean is introduced such that it satisfies the wave equation: \( \sum_{j=1}^{n} x_j x_{j\cdot} u_{x_j} + u_{xx} - u_{tt} = 0 \), and it is invariant under a subgroup of the group generated by Lorentz transformations and
translations. Representations of solutions in $Q_E$ is then obtained in terms of paraboloidal means of data of half of the boundary, $C_\infty$. The following theorem on uniqueness is proved: If a tempered solution with initial values and the initial values of its $t$-derivative growing no faster than a polynomial vanishes as $|x| \to \infty$ for all $x$, then it is uniquely determined by the values on $C_\infty$. This generalizes the result of Hersh and Chen for $n = 2$ (J. Math. Mech. 17(1967), 449-460). (Received November 2, 1970.)

682-35-8. GEORGE H. KNIGHTLY, University of Massachusetts, Amherst, Massachusetts 01002. On the stability of uniform solutions of the Navier-Stokes equations in $n$-dimensions.

Solutions $u(x, t)$, $p(x, t)$ of the Navier-Stokes equations: $\nu \cdot u = 0$, $u_t - \Delta u + u \cdot \nabla u + \nu p = 0$ are considered for $(x, t)$ in the half space $(n \geq 3) \mathbb{R}^n \times (0, \infty)$. It is shown that the uniform solutions ($u = u_\infty$, a constant vector, $p = p_\infty$, a constant scalar) are stable under initial disturbances $g$ of the class: $\nabla \cdot g = 0$, $|g(x)| \leq A \min \{1, |x|^{-s}\}$, $g \in C^1(\mathbb{R}^n)$, when $s \geq 1$ and $A$ is sufficiently small. For initial velocity $u_\infty + g$ a smooth, global solution $u$, $p$ is constructed and pointwise estimates are obtained for the rate of its decay to the uniform solution. When $1 \leq s < n - 1$, there holds $|u(x, t) - u_\infty| \leq \text{const} \ t^{-(1+s)/2}$ as $t \to \infty$, uniformly in $x$. (Received November 2, 1970.)


The partial differential equation derived by Reissner, Mindlin and Uflyand governing the motion of plate when transverse shear and rotatory inertia are considered are solved for plates bounded by circular contours and radial lines. Solutions are chosen which satisfy all of the eight different boundary conditions entering into the theory, and algorithms are given showing how to calculate the higher frequencies, and numerical data is given substantiating the new theory involved. (Received November 2, 1970.)


Let $u(x, y)$ be an $n$-vector, $A(x, y)$, $B(x, y)$ and $C(x, y)$ be $n \times n$ matrix functions, and $L[u] = u_{xy} + A x + B y + C u$. The adjoint operator is $L^*[v] = v_{xy} - (A^T v)_x - (B^T v)_y + C^T v$ where $^T$ indicates transpose. Let $V(x, y; s, t)$ satisfy the matrix characteristic initial value problem $L^*[V] = 0$, $V_x = B^T V$ on $y = t$, $V_y = A^T V$ on $x = s$, and $V(s, t; s, t) = I$, the identity matrix. Then the solution vector $u$ to the system of partial differential equations $L[u] = f(x, y)$ with $u$ and its partial derivatives given along a noncharacteristic curve $C$ can be expressed in terms of integrals of $V$, $f$, and the boundary data on $C$. In addition to solving second order hyperbolic systems this method can be used to solve systems of Volterra integral equations and has been used by the author to obtain a generalization of Gronwall's inequality for systems of partial differential equations. (Received November 2, 1970.)


Extensions of Hadamard three circle theorem to linear elliptic and parabolic equations are known (see Protter and Weinberger, "Maximum principles in differential equations", Prentice-Hall, Inc., Englewood Cliffs, 166
New Jersey, 1967, and references there). In this paper, another extension of the Hadamard three circle theorem will be discussed. (Received November 3, 1970.)


Consider the boundary value problem \( u_t = u_{xx} + f(x,t,u,u_x); u(0,t) = u(L,t) = 0 \). By the use of the Leray-Schauder fixed point theorem, the existence of a solution \( u(x,t) \) on \([0, L] \times R\), which is periodic in \( t \) of periodicity \( T \), is obtained. The conditions required are: (i) \( f(x,t,u,p) \) is periodic in \( t \) of period \( T \); (ii) \( f(x,t,u,p) \) is continuous and satisfies local Hölder conditions in all arguments; (iii) for \( |u| \leq M \), and all \( x,t,p, |f(x,t,u,p)| \leq \gamma_M(|p|) \) where \( \gamma_M(p) \) is positive, nondecreasing for \( p \leq 0 \) and \( \gamma_M(0) = 0(p^2) \) as \( p \to \infty \); and (iv) let \( f(x,u,p) = \min_{[0,T]} f(x,t,u,p) \) and \( T(x,u,p) = \max_{[0,T]} f(x,t,u,p) \). Assume that \( u'' + f(x,u,u') = 0 \) has a lower solution \( \alpha(x) \) on \([0,L]\) with \( \alpha(0), \alpha(L) < 0 \). Assume that \( u'' + \mathcal{F}(x,u,u') = 0 \) has an upper solution \( \beta(x) \) on \([0,L]\) with \( \beta(0), \beta(L) > 0 \). Moreover, assume \( \alpha(x) < \beta(x) \) on \([0,L]\). (Received November 4, 1970.)


The objects of interest are the null-spaces of linear first-order elliptic partial differential operators acting on functions in \( H^1(R^n; C^k) \), the first-order coefficients of which become constant coefficients of which vanish outside a compact set in \( R^n \). An example is given of an operator of this type which has a non-trivial null-space. It is shown that the dimension of the null-space of such an operator is finite for any number \( n \) of independent variables, and that this dimension is an upper-semicontinuous function of the operator in a certain sense. (Received November 5, 1970.)

39 Finite Differences and Functional Equations

682-39-1. GERASIMOS E. LADAS and V. LAKSHMIKANTHAM, University of Rhode Island, Kingston, Rhode Island 02881. Oscillations caused by retarded actions.

Oscillation theorems are proved for the retarded differential equations (1) \( y''(t) - p(t)y(t-\tau) = 0 \) and (2) \( y''(t) - p(t)y(t) - p(t)y(t-\tau) = 0 \). These oscillations are caused by the delay \( \tau \) and they disappear when the delay vanishes. Theorem 1. Assume that \( \tau > 0, p'(t) \leq 0 \) and \( \tau^2 p(t) \geq 2 \) for \( t \geq 0 \). Then all bounded solutions of (1) are oscillatory. Theorem 2. Assume that \( \tau > 0, p'(t) \leq 0 \) and \( \tau^2 p(t) \geq 2 \) for \( t \geq 0 \). The derivative of any solution of (2) is oscillatory. The above theorems are also generalized to equations where many delays occur which may be functions of time and to the case of differential inequalities. (Received October 14, 1970.)


Asymptotic relationships between the solutions of the nonlinear perturbed functional differential equation,
(1) \( x'(t) = A(t)x(t) + f(t, x_t) \), and the corresponding linear ordinary differential equation, (2) \( y'(t) = A(t)y(t) \), are studied. A generalized asymptotic equivalence is established between the solutions of (1) and (2). Conditions are found which imply that the asymptotic manifold of (1) generated by (2) is perturbable in the sense that solutions of (1) which are "initially close" to solutions of (1) that are in the asymptotic manifold must also be in the asymptotic manifold. These results extend known results for ordinary differential equations. (Received October 30, 1970.)


An earlier investigation by Ho, Jayne and Sledd [Duke Math. J. 33(1966), 131-140] yielded a necessary and sufficient condition specifying when each term of a three-term recursively generated polynomial sequence would be a Sturm-Liouville polynomial associated with a second-order ordinary differential equation. The condition requires that the coefficients in the generating recursion formula satisfy a system of two first-order difference equations. Recently the author and Jayne [J. Mathematical Phys. 10(1969), 1670-1680] have applied this condition to analysis of the natural (normal) frequencies of certain spring-mass chains, with chains of both finite and infinite length being considered. In this paper the closed-form solution of the aforementioned difference system is given, both in the singular and nonsingular cases. Singular solutions can occur when the difference system is solved over a finite index range, but not when solved over the infinite range. (Received November 2, 1970.)

40 Sequences, Series, Summability

682-40-1. JON C. HELTON, West Virginia University, Morgantown, West Virginia 26506. A nonlinear sequence to sequence transformation.

The purpose of this paper is to consider the nonlinear sequence to sequence transformation

\[
\left( \sum_{q=1}^{P} x_q \right) \left( \sum_{p=q}^{P} \frac{x_p}{q} \right)
\]

Necessary and sufficient conditions are given to transform sequences with absolutely convergent series into bounded sequences, convergent sequences, sequences with convergent series, sequences with absolutely convergent series and sequences of bounded variation, respectively. (Received October 14, 1970.)


We prove a number of inequalities for sequences which are analogous to inequalities occurring in the theory of integral equations. For special values of certain variables one of our inequalities reduces to a classical inequality (Gronwall's lemma) in the theory of difference equations. (Received October 19, 1970.)


In Abstract 675-7, these *Notices* 17(1970), 551, we obtained recurrence and differential relations for
a class of hypergeometric functions \( (*) \) \( _pF_q[\{a_k + k\};\{b_q + k\}, 2k + c, x] \) whose special cases include \( Q^n_k(x) \), the associated Legendre function of the second kind, \( J_{k+a}(x) \), the Bessel function, \( [Q^n_k(x)]^2 \), \( [J_{k+a}(x)]^2 \) and other functions. In this paper, a multiplication theorem is presented for this class of functions. The integral of the product of two generalized hypergeometric functions is expressed as an infinite series of the product of two \( (*) \) functions, with variables \( x \) and \( y \). Special cases of the integral are seen to be the known generating functions of \( Q^n_k(x) \), \( J_{k+a}(x) \) and others. Also, new expressions for the generating functions of \( [Q^n_k(x)]^2 \), \( [J_{k+a}(x)]^2 \) and others are presented in integral form. Finally, some integrals appear to be very similar to those that occur in electron collision problems [Math. Z. 108(1969), 191]. (Received November 3, 1970.)

### 41 Approximations and Expansions


H. Walk (Arch. Math. (Basel) 20(1969), 398-404) has employed positive linear operators to approximate functions which are continuous on a finite interval and have certain growth conditions on the real line. In this paper results are obtained on the order of approximation to unbounded functions by positive linear operators. The general results are applied to the Szasz-Mirakayan-Hille operators and to the variation-diminishing splines of Marsden and Schoenberg (Mathematica (Cluj) 8(31) (1966), 61-82). A typical theorem is the following: Let \( E_a \) denote the class of continuous functions of exponential type \( a \) on \([0,\infty)\). Let \( f \in E_a \), \( 0 < \epsilon < 1 \) and \( 0 \leq x < \infty \). Then, for \( n = 1,2,\ldots \), \( \| f(x) - P_n(f;x) \| \leq \omega((x/n)^{(1-\epsilon)/2};x) + C \exp(x/p(e^{ap} - 1))(x/n)^{\epsilon/p} + |f(x)|(x/n)^{\epsilon/p} \) where \( p > 1 \), \( 1/p + 1/p' = 1 \), \( C \) is a constant depending on \( f \), \( \omega \) is the modulus of continuity of \( f \) and \( P_n \) is the Szasz operator. (Received September 23, 1970.)


Let \( C^r I \) denote the space of \( r \)-times continuously differentiable functions on an interval \( I = [b,c] \). The question of uniqueness of best approximation of functions in \( C^r I \) by elements of a finite dimensional subspace, with respect to various norms, has been investigated in several papers. In this paper we consider spaces which include \( C^r I \) in the class of norms of the form \( \| f \| = \max \{ |f(a)|, |f^{(1)}(a)|, \ldots , |f^{(r-1)}(a)|, |f^{(r)}|_p \} \) where \( 1 \leq p \leq \infty \), \( \| \cdot \|_p \) is the \( L^p \) norm, and \( a \) is a fixed but arbitrary point of \( I \). An immediate consequence of the results in the case \( p = \infty \) is the following result, reminiscent of Haar's criterion for the case \( r = 0 \). Let \( C^r I \) be normed by the above norm with \( p = \infty \). (Here \( \| \cdot \|_p \) reduces simply to the uniform norm, of course.) Then approximation by elements of a subspace \( M \) of dimension \( n \) is unique if and only if for any \( p \) in \( M \) with \( p \neq 0 \) the number of \( j \)'s such that \( p^{(j)}(a) = 0 \) (\( 0 \leq j < r \)) plus the number of zeroes of \( p^{(r)} \) does not exceed \( n - 1 \). (Received October 13, 1970.)

A. L. Garkavi [Izv. Akad. Nauk. SSSR Ser. Mat. 23(1959)] obtained a necessary and sufficient condition for an n-dimensional subspace $V$ to be $p$-Tchebycheff with respect to the usual supremum norm $\|\cdot\|_\infty$ in $C^{r+1}$ (the space of $(r+1)$-times continuously differentiable functions on the compact interval $I$), $r \geq 0$. We extend the sufficiency part of Garkavi's result to the norm $\|f\| = \max\{\|f\|_\infty, \|f^{(1)}\|_\infty, \ldots, \|f^{(r)}\|_\infty\}$. We then show by use of results of D. R. Ferguson [J. Approximation Theory 2(1969)] that the polynomials of degree $\leq n-1$ satisfy these sufficiency conditions for any $r \geq 0$, thus obtaining a result due to D. G. Moursund [Math. Comp. 18(1964)] and Lee Johnson [Math. Comp. 22(1968)]; i.e., if $p_1$ and $p_2$ are two best polynomial approximations to a function $f$ in $C^r I$ and $\Sigma f^{(r+1)}$, then $p_1^{(r)} = p_2^{(r)}$, $r = 0, 1, \ldots$. In the case $r = 0$, Tchebycheff's classical result shows that the condition requiring the existence of the $(r+1)$st derivative of $f$ can be dropped. We give an example to show that this condition cannot be dropped if $r > 0$. (Received October 23, 1970.)

682-41-4. JOSEPH M. LAMBERT, Pennsylvania State University, York, Pennsylvania 17403. The weak sequential continuity of the metric projection in $L_p$ spaces.

Let $(Y, B, m)$ be a separable nonatomic measure space, $(Z, A, m)$ a purely atomic measure space. Let $M = \text{span}(x_i : i=1, \ldots, n)$ be a subspace of $L_p((x, \Sigma(B, A), m)$, $1 < p \leq 2 < \infty$, where $\Sigma(B, A)$ is the smallest $\sigma$-algebra containing $B$ and $A$. Let $P(M)$ denote the metric projection associated with $M$. Using the results of Holmes ("Continuity of best approximation operators," contributed to the Sympos. Infinite Dimensional Topology, Baton Rouge, 1967) one obtains $P(M)$ is weakly sequentially continuous if and only if the support of $x_i$ is purely atomic, $i=1, \ldots, n$. In particular there exist no finite dimensional subspaces in $L_p((Y, B, m)$, $1 < p \leq 2 < \infty$, which support a weakly sequentially continuous metric projection. In addition in the case directly above, the metric projection associated with any finite dimensional subspace has no points of weak sequential continuity. (Received October 23, 1970.)

682-41-5. EARL H. MCKINNEY, Ball State University, Muncie, Indiana 47306. Recursive multivariate interpolation. Preliminary report.

A generalized recursive interpolation technique for a set of linear functionals over a set of general univariate basis functions has been previously developed. This paper extends these results to restricted multivariate interpolation over a set of general multivariate basis functions. When the data array is a suitable configuration (e.g., an n-dimensional simplex), minimal degree multivariate interpolating polynomials are produced by this recursive interpolation scheme. By using product rules, recursive univariate interpolation applied to each variable produces multivariate interpolating polynomials (not of minimal degree) when the data is arranged in an n-dimensional rectangular array. By proper ordering of points in a data array, multivariate polynomial interpolation is accomplished over other arrays such as diamonds and truncated diamonds in 2-dimensions and their counterparts in n-dimensions. (Received October 26, 1970.)
The possibility of irregular behavior for orthogonal polynomials. Preliminary report.

Let \( \mu \) be a unit measure defined on the Borel subsets of \( I = [-1,1] \) and let \( c(\mu) \) denote the compact carrier. If \( c(\mu) \) is an infinite set, there are unique monic polynomials \( \{P_n(x)\} \) of degrees 0, 1, \ldots respectively satisfying \( \int P_m(x) P_n(x) \, d\mu = \delta_{m,n} N^2_n(\mu) \). The polynomial \( P_n(x) \) is the \( n \)th orthogonal polynomial associated with \( \mu \) and has \( n \) simple zeroes \( x_{i,n} \), \( i = 1, \ldots, n \), on \( I \). Let \( \nu_n \) be the measure assigning mass \( 1/n \) at \( x_{i,n} \), \( i = 1, \ldots, n \). Theorem. If \( \{\lambda_i\}, i = 0, 1, \ldots \), is a sequence of distinct points in \( I \), there are positive numbers \( \{a_i\}, i = 0, 1, \ldots \), such that if \( \mu \) is the measure assigning mass \( \lambda_i \) at \( x_{i,n} \), \( i = 0, 1, \ldots \), for any integer \( n > 0 \), the corresponding orthogonal polynomial \( P_n(x) \) has a simple zero in each of the intervals determined by \( \{x_{0}, x_{1}, \ldots, x_{n}\} \). By means of this theorem we can construct measures which help shed light on the asymptotic behavior of orthogonal polynomials. Application 1. There is a measure \( \mu \) with \( c(\mu) = I \) such that the measures \( \nu_n \) do not converge. Application 2. There is a measure \( \mu \) with \( c(\mu) = I \) such that the measures \( \nu_n \) do converge, say to \( \nu \), but such that \( c(\nu) \neq I \). Application 3. There is a measure \( \mu \) with \( c(\mu) = I \) such that the measures \( \nu_n \) converge to \( \nu(E) = \pi^{-1} \int_E dx / \sqrt{1-x^2} \), \( E \subset I \), but \( \lim (N_n(\mu))^{1/n} < 1/2 \). (Received October 29, 1970.)


For \( m \geq 1 \), let \( p \in C^m[a,b] \) satisfy \( p(x) > 0 \) for all \( x \in (a,b) \) with \( 1/p \in L^1(a,b) \). Consider the class \( \mathcal{C} \) of functions \( f \) satisfying \( D^{m-1} f \) absolutely continuous on \( [a,b] \) with \( \sqrt{p} D^m f \) square integrable on \( [a,b] \). The \( L^2(a,b) \) \( n \)-widths of the class \( \mathcal{C} = \{f \in \mathcal{C} : \|\sqrt{p} D^m f\|_{L^2} \leq 1\} \) are computed in terms of the eigenvalues of

\[
(1) \quad (-1)^m D^m p D^m u = \lambda u \quad \text{on} \quad [a,b] ; \quad D^j u(a) = D^j u(b) = 0, \quad m \leq j \leq 2m-1.
\]

The asymptotic distribution of the eigenvalues of the singular problem \( (1) \) is obtained and, hence, the asymptotic distribution of the \( n \)-widths of the class \( \mathcal{C} \). An application is given to show that certain spline approximations, determined by the singular operator of \( (1) \), are of optimal order. (Received November 5, 1970.)

682-41-9. GEORGE C. HSIAO, University of Delaware, Newark, Delaware 19711. Singular perturbations for nonlinear differential equations with a small parameter.

The validity of the method of inner and outer expansions (MIO) for treating a class of singular boundary value problems is discussed. As a model, one considers the boundary value problem \( P_\epsilon \) defined by \( y'' + x^{-1} y' = \epsilon y' \) on \( x > 1 \), \( y = 0 \) at \( x = 1 \) and \( y \rightarrow -1 \) as \( x \rightarrow \infty \). Here \( \epsilon > 0 \) is a small parameter. The problem \( P_\epsilon \) is singular in the sense that the degenerate problem \( P_0 \) has no solution. Based on a regular perturbation procedure by Finn and Smith (Arch. Rational Mech. Anal. 25(1967), 1-23), it is shown that the formal asymptotic expansions constructed by (MIO) are indeed in some sense the asymptotic expansions for the exact solution of \( P_\epsilon \) as \( \epsilon \rightarrow 0^+ \). (Received November 5, 1970.)
42 Fourier Analysis

42-1. GARY E. LIPPMAN, Kenyon College, Gambier, Ohio 43022. Spherical summability of conjugate multiple Fourier series and integrals at the critical index.

Theorem. Let \( f \) be a function in \( L_1(E^2) \) and \( K(r, \theta) = r^{-2} \sum (a_k \cos k \theta + b_k \sin k \theta) \) be a Calderón-Zygmund kernel satisfying \( \sum k^4 (a_k + b_k) < \infty \). Assuming that \( f \) satisfies a restricted Dini-type condition at \((x_0, y_0)\) the author proves that the difference between the spherical Bochner-Riesz means of order 1/2 of the conjugate Fourier integral of \( f \) and the principal-valued Hilbert transform of \( f \) converges to 0. The proof of this theorem depends upon facts about the Bessel functions of the first kind and on the one-dimensional Riemann-Lebesgue Theorem for Fourier integrals. The author has also proved a generalization of this localization theorem at the critical index of summability for Bochner-Riesz spherical means of Fourier integrals conjugate with respect to the spherical harmonic Calderón-Zygmund kernels in \( E_k \), \( k \geq 2 \), that localization fails in this case for spherical means taken below the critical index \((k - 1)/2 \) for such Fourier integrals, and that localization also fails at the critical index for Bochner-Riesz spherical sums of multiple Fourier series conjugate with respect to the spherical harmonic kernels. The methods for proving these results in \( E_k \) are similar to those Bochner used in his paper "Summation of multiple Fourier series by spherical means" (1936) and to those used in the two-dimensional theorem stated above. Used in the proofs are important estimates due to C. P. Chang. (Received November 2, 1970.)


Let \( \mathbb{R} \) denote the real line, and let \( \mathcal{R} \) denote the class of all functions \( F \in L^2(\mathbb{R}) \) which have the representation \( F(x) = \int_{\mathbb{R}} e^{iyt} f(t) dt \), where \( f \in L^2(\mathbb{R}) \). Let \( \mathcal{R}_+ \) denote the subset of functions \( F \in \mathcal{R} \) for which the corresponding function \( f(t) = 0 \) for \( t < 0 \), and let \( \mathcal{R}_- = \mathcal{R} - \mathcal{R}_+ \). Given \( p \in (2, \infty) \), it is shown that there exists a constant \( B_p \) depending only on \( p \), with the property that if \( \phi \in \mathcal{R} \cap L^\infty(\mathbb{R}) \) and \( \sup_{x \in \mathbb{R}} |\phi(x)| < B_p \), then there exist unique functions \( \psi \) and \( \omega \) with the following properties: (a) \( \psi \in \mathcal{R}_+ \cap L^p(\mathbb{R}) \), \( \omega \in \mathcal{R}_- \cap L^p(\mathbb{R}) \); (b) \( \log(1 + \psi) \in \mathcal{R}_+ \), \( \log(1 + \omega) \in \mathcal{R}_- \); (c) \( \exp \phi = (1 + \psi)(1 + \omega) \) a.e. on \( \mathbb{R} \). This result is applied to yield an explicit approximate solution \( f(h) (h \to 0) \) of a class of Wiener-Hopf integral equations, which converges (as \( h \to 0 \)) to the exact solution whenever a unique solution exists. (Received November 5, 1970.)

43 Abstract Harmonic Analysis

43-1. FRANK DANGELLO and ROBERT J. LINDAHL, Pennsylvania State University, University Park, Pennsylvania 16801. Almost periodic function on a semidirect product of semigroups.

In this paper we generalize certain results of K. deLeeuw and I. Glicksberg, "Almost periodic functions on semigroups", Acta. Math. 105(1961), 99-140. We first give an internal characterization of the tensor product of two function algebras. Using this we obtain our main results. Let \( S \) and \( T \) be semitopological semigroups with \( 1 \) and \( \sigma \) a semigroup homomorphism from \( T \) into the continuous endomorphisms of \( S \) such that \( \sigma_1 \) is the identity endomorphism and for each \( s_0 \in S \), \( (s,t) \to \sigma_t(s_0) \) is
continuous. Let \( S \circ_T \) denote the semidirect product of \( S \) with \( T \) induced by \( \sigma \). We show that there exists an \( M \)-introverted subalgebra, \( A^\sigma(S) \) of the Bochner almost periodic (B.a.p.) functions, \( A(S) \) on \( S \) having the property that \( A(S \circ_T) = A^\sigma(S) \otimes A(T) \). It follows that the B.a.p. compactification of \( S \circ_T \) is \( \overline{S \circ_T} \) where \( \overline{S} \) is the compactification of \( S \) induced by \( A^\sigma(S) \). \( \overline{T} \) is a B.a.p. compactification of \( T \), and \( \sigma \) is a certain homomorphism from \( \overline{T} \) into the continuous endomorphisms on \( \overline{S} \). We describe the largest weakly almost periodic compactification of \( S \circ_T \) subject to the condition that the compactification is a semidirect product. (Received August 20, 1970.)

682-43-2. GREGORY F. BACHELIS, Kansas State University, Manhattan, Kansas 66502 and JOHN E. GILBERT, University of Texas, Austin, Texas 78712. Banach spaces of compact multipliers and their dual spaces.

Let \( G \) be a compact group and let \( X \) and \( Y \) be one-sided Banach \( L^1(G) \)-modules, with \( Y \) reflexive.

(1) If \( X \) and \( Y \) satisfy some weak additional conditions, then the dual space of the compact multipliers from \( X \) to \( Y \) is the \( L^1(G) \)-tensor product of \( X^{**} \) and \( Y^* \), and hence the second dual space is all bounded multipliers from \( X^{**} \) to \( Y \). One can apply (1) in special cases to prove, for example: (2) If \( 1 < p < \infty, \), \( 1 < q < \infty, \) then \( L^q \) is the dual space of the compact multipliers from \( L^p \) to \( L^q \). (3) If \( G \) is abelian, \( 1 < p < \infty, \) then \( \mathcal{S}^{(p)} \), the Banach space of functions in \( L^p \) with unconditionally norm-convergent Fourier series, is reflexive. (Received October 7, 1970.)


G. P. Johnson (Trans. Amer. Math. Soc. 92 (1959), 411-429) characterized the maximal ideal space of \( L^1(G) \), where \( G \) and \( A \) are locally compact abelian groups. Let \( A, E \) and \( G \) be any separable locally compact groups such that \( A \) is a closed central subgroup of \( E \) and \( G = E/A \). Then \( E \) is called a central extension of \( G \) by \( A \). One can study the group algebra of \( E, L^1(E) \), applying the viewpoint of H. Leptin (Invent. Math. 3 (1967), 251-281). Then for each \( a \in \hat{A} \), the dual of \( A \), one can define a Banach algebra \( L^1(G, a) \) on the Banach space \( L^1(G) \) which depends on \( a \) and the structure of the extension, and a homomorphism \( h_a \) from \( L^1(E) \) onto \( L^1(G, a) \) with kernel \( K_a \). Theorem. For any closed maximal ideal \( M \) in \( L^1(E) \) there exists exactly one \( a \in \hat{A} \) such that \( M \supset K_a \). Consequently there is a one to one correspondence between the family of maximal closed ideals in \( L^1(E) \) and the totality of maximal closed ideals in \( L^1(G, a) \) for all \( a \in \hat{A} \). Corollary. If \( G \) is discrete every maximal closed ideal in \( L^1(E) \) is regular. A less immediate application leads to a 'Bochner' theorem involving the representation of any continuous positive definite function on \( E \) as an integral over a vector valued measure on \( \hat{A} \) with values in \( L^\infty(G) \). (Received October 8, 1970.)

682-43-4. STEPHEN H. FRIEDBERG, Illinois State University, Normal, Illinois 61761. Functions which are Fourier-Stieltjes transforms.

Let \( G \) be a locally compact abelian group, \( \hat{G} \) the dual group, \( M(G) \) the algebra of regular bounded Borel measures on \( G \), and \( M(G)^* \) the algebra of Fourier-Stieltjes transforms. Theorem. Let \( X \) be a closed subset
of G and f a continuous function on \( \hat{G} \). Then the following conditions are equivalent: (a) \( f \in M(X) \), where 
\[ M(X) = \{ m \in M(G) : \text{the support of } m \text{ is contained in } X \} \]; and (b) \( \{ \theta_n \} \subset M(\hat{G}) \), \( \theta_n(x) \leq C \), and \( \theta_n(x) \to 0 \) for every \( x \in X \) implies \( \int_{\hat{G}} fd\theta_n \to 0 \). The case where \( f \) is also assumed to be bounded and \( X = G \) was proved by Ramírez (Proc. Cambridge Philos. Soc. 64 (1968), 323-333) by using Grothendieck’s completion theorem. The author provides a short proof of the more general result by using a well-known result of Eberlein. (Received October 20, 1970.)

682-43-5. WILBUR P. VEITH, Ohio State University, Columbus, Ohio 43210. Amerio-Doss theorems on Banach valued almost periodic functions.

Let G be a topological group and B a separable, reflexive Banach space. Suppose \( f : G \to B \) is bounded and \( \Delta_t f(x) = f(x+t) - f(x) \) is almost periodic for each \( t \in G \). Then \( f \) is almost periodic. This result generalizes a theorem of Doss (MR23 A 3424). As a consequence of the above theorem, an affirmative answer is obtained to a conjecture of Amerio. That is, suppose \( f \) is an almost periodic function, \( f : R \to B \), where \( R \) is the real line and \( B \) is a separable, reflexive Banach space, and suppose \( F(t) = \int_0^t f(u) \, du \) is bounded. Then \( F \) is also almost periodic.

The theorem is proven by showing \( f \) has a continuous extension \( f' \) as a function from \( M \), the almost periodic compactification of \( G \), into \( (B, T^*) \) where \( T^* \) denotes the weak topology on \( B \). Using a result of M. K. Fort, Jr. (MR17, p. 1115), the subset of \( B \) with the norm topology, at each point of which \( f' \) is discontinuous, is a set of first category. Thus, the set of points of \( M \) where the oscillation of \( f' \) is less than an arbitrary \( \epsilon > 0 \) is shown to be a nonempty open set which is invariant under translation by elements of \( M \), hence is all of \( M \). (Received October 8, 1970.)

682-43-6. SAMUEL E. EBENSTEIN, Wayne State University, Detroit, Michigan 48237. Some \( H^p \) spaces which are uncomplemented in \( L^p \). Preliminary report.

Let \( T \) be the circle group and \( H^p(T) \) the classical Hardy space. Let \( T^\omega \) be the infinite dimensional torus and \( Z^\omega \) its dual group. The elements \( n \) of \( Z^\omega \) are of the form \( n = (n_1, n_2, n_3, \ldots) \) where the \( n_i \) are integers and only finitely many \( n_i \) are different from 0. Let \( A = \{ n \in Z^\omega : (n_1) \geq 0 \ \forall \ i \} \). Define \( H^p(T^\omega) = \{ F : F \in L^p(T^\omega) \text{ and } \hat{F} = \mathcal{F}X_A \} \). It is a classical result that \( H^p(T) \) is a complemented subspace of \( L^p(T) \) for \( 1 < p < \infty \). Theorem. \( H^p(T^\omega) \) is uncomplemented in \( L^p(T^\omega) \) for \( 1 < p < \infty \) unless \( p = 2 \). (Received October 26, 1970.)


Let \( G \) be a separable locally compact group, \( N \) a closed normal subgroup and \( G \times \hat{N} \to \hat{N} \) the canonical action. Let \( T \) be an element of \( \hat{N} \) such that the induced representation \( U^T \) of \( G \) is irreducible. We give necessary and sufficient conditions for \( U^T \) to be compact (i.e. CCR) in terms of the orbit of \( T \) in \( \hat{N} \), the canonical map of \( G/N \) into \( \hat{N} \) and the vanishing at infinity of a certain family of functions on \( G/N \). (Received October 30, 1970.)
On the equation
\[ P(B) = \int P(Bx^{-1})P(dx). \]

Let \( P \) be a nonnegative compact-finite regular measure on the Borel sets of a locally compact semigroup \( S \) satisfying the above equation, whenever the integral above makes sense. We allow \( P(S) \) to be \( \infty \). Then if \( P(AB^{-1}) < \infty \), \( P(A^{-1}B) < \infty \) and \( P(x^{-1}Ax^{-1}) < \infty \) for all compact sets \( A \) and \( B \) and points \( x \) in \( S \), the support of \( P \) is a closed completely simple semigroup. It follows then that a locally compact group does not admit an infinite regular compact-finite measure satisfying the above equation. (Received November 5, 1970.)

45 Integral Equations

682-45-1. DAVID L. LOVELADY, Georgia Institute of Technology, Atlanta, Georgia 30332. Algebraic structure for a set of nonlinear integral operations.

If \( V_1 \) and \( V_2 \) are in \( OA \) [see J. S. Mac Nerney, Illinois J. Math. 8(1964), 621-638], let \( V_1 \circ V_2 \) be the member of \( OA \) given by
\[ V_1 \circ V_2 (a, b) [P] = V_2 (a, b) [P] + \sum_{a}^{b} V_1 (l, V_2) [P]. \]
If \( W_1 \) and \( W_2 \) are in \( OM \), let \( W_1 \circ W_2 \) be that member of \( OM \) given by
\[ W_1 \circ W_2 (a, b) [P] = \Pi_{a}^{b} W_1 (l, W_2) [P]. \]
Theorem. If \( V_1 \) and \( V_2 \) are in \( OA \), \( \delta [V_1 \circ V_2] \) exists, and \( \delta [V_1 \circ V_2] = \delta [V_1] \circ \delta [V_2] \). Necessary and sufficient conditions are found for \( V_1 \circ V_2 = V_1 + V_2 \), and hence conditions are found for \( \delta [V_1 + V_2] = \delta [V_1] \circ \delta [V_2] \). It is found that recent coalescence results of J. V. Herod [Abstract 69T-B158, these Notices 16(1969), 834] can be used to determine the largest subgroup of the semigroup \( (OA, \circ) \). (Received October 15, 1970.)


\( N \) is the set of elements of a complete normed ring with unity; \( S \) is a set linearly ordered by \( \leq \); \( \mathfrak{g} \) is the class of functions from \( S \times S \) to \( N \) defined by the author in “Stieltjes-Volterra integral equations”, Illinois J. Math. 14 (1970), 438; \( I \) is the identity function on \( S \); and \( \psi \) is the reversible function from \( \mathfrak{g} \) onto \( \mathfrak{g} \) induced by the equation:
\[ M(t, x) = 1 + (L) \int_{x}^{t} F(t, l) \cdot M[l, x]. \]
If there is a number \( h > 1 \) such that \( h|x| \leq |x + x| \) for each \( x \) in \( N \), then the constant 1 function on \( S \) is the only fixed point of \( \psi \). Let a denote a number and let \( S \) denote \([a, \infty)\). \( \mathfrak{m} \) will denote the set to which \( f \) belongs only in case \( f \) is a function from \( S \) to \( N \) such that (i) \( f(a) = 1 \), (ii) \( f \) is of bounded variation on each interval \([a, b]\) of \( S \), and (iii) there is a number \( c > a \) such that \( f \) satisfies a uniform Lipschitz condition on \([a, c]\). For each \( f \) in \( \mathfrak{m} \) there is only one \( m \) in \( \mathfrak{m} \) such that \( m(t) = 1 + (L) \int_{a}^{t} df[a, t-1] \cdot m \) for each \( t \) in \( S \). Moreover, \( f(t) = 1 + (L) \int_{a}^{t} dm[a, t-1] \cdot f \) for each \( t \) in \( S \). If \( f \) is continuous then so is \( m \). Similar results are obtained for the nonhomogeneous case. If there is a number \( h > 1 \) such that \( h|x| \leq |x + x| \) for each \( x \) in \( N \), then \( f \) is \( m \) only in case \( f \) is the constant 1 function on \( S \). (Received October 29, 1970.)
46 Functional Analysis

682-46-1. JERRY A. JOHNSON, Oklahoma State University, Stillwater, Oklahoma 74074.
A lemma of de Leeuw's on extreme points.

In his doctoral thesis, T. M. Jenkins generalized a theorem of de Leeuw (see Studia Math. 21(1961/62), 55-66) that characterized the extreme points of the dual ball of $\text{lip}(S, d^a)$, $0 < a < 1$. This characterization depends upon a lemma, due to the latter author, which states: If $A$ is a subspace of $C(S)$ containing a function that peaks at $s \in S$ relative to $A$, then the functional $\epsilon_s : f \rightarrow f(s)$ is an extreme point of the dual ball of $A$. It is our purpose to present a very simple proof of de Leeuw's lemma that yields a stronger conclusion--namely that $\epsilon_s$ is a weak* exposed point of the dual ball of $A$. As a consequence, it follows that every extreme point of the dual ball of $\text{lip}(S, d^a)$ is a weak* exposed point. Both ideas can be extended to vector-valued functions. (Received September 14, 1970.)


Theorem. Let $X$ be a normed linear space and for each $t$ in $[0,1]$, let $\xi(t)$ and $\sigma(t)$ be the infimum and supremum, resp., of the set $\{\|x + ty\|^2 + \|x - ty\|^2 - 2; \|x\| = \|y\| = 1\}$. Then $X$ is uniformly convex if and only if for each $t$ in $(0,1]$, $\xi(t) > 0$, and $X$ is uniformly smooth if and only if $\sigma$ has a right-hand derivative $0$ at $0$. (Received September 24, 1970.)


Let $(E, \tau)$ be a vector space with a Banach topology, and let $F$ be a vector subspace of $E$. We say that $F$ is a barreled subspace of $(E, \tau)$ if $\overline{F} = E$ and $(F, \tau|F)$ is barreled. These arise at many interesting points in analysis and part of the following is a straightforward generalization while another requires some technical lemmas. Theorem 1. If $F$ is a barreled subspace of $(E, \tau)$ and $T: G \rightarrow E$ is continuous from the Banach space $G$ such that $T(G) \subset F$ then $T(G) = E$. If $(E, \tau)$ is any $T_2$ locally convex space then there exists a coarsest barreled topology for $E$ which is finer than $\tau$. Denote it by $\overline{\tau}$. Theorem 2. Let $F$ be a barreled subspace of a Banach space $(E, \tau)$ and let $\rho$ be a $T_2$ locally convex topology for $F$ which is coarser than $\tau$. Let $F_1$ be the completion of $(F, \rho)$. If $\overline{\rho} = \tau|F$ then the natural mapping $E \rightarrow F_1$ is an injection. Let $E$ and $F$ be Banach spaces. Let $\tau_\lambda$ and $\tau_\gamma$ be the topologies defined on $E \hat{\otimes} F$ by the least and greatest cross norms, respectively. It is known $\tau_\lambda \leq \tau_\gamma$ and that $\tau_\gamma$ is barreled. Corollary. There is no barreled topology between $\tau_\lambda$ and $\tau_\gamma$ if and only if the mapping $E \hat{\otimes} F \rightarrow E \hat{\otimes} F$ is an injection. (Received October 2, 1970.)


In this paper two of the conditions which define a generalized $N$-function $M(t, x)$ on $T \times E^n$ where $T$ is any space of $\sigma$-finite measure are relaxed (see, Pacific J. Math. 28(1969), 193-206; 413-430). First, $M(t, x)$ may vanish for nonzero vectors $x$ for each $t$ in $T$. Second, $M(t, x)$ may become infinite for vectors $x$ with finite

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magnitude. The resulting class of functions (called GJ-functions) is studied. Sufficient and necessary conditions are given for a GJ-function to satisfy a delta condition. Moreover, vector valued Orlicz classes and spaces are defined, and some topological and metric properties are proved. This paper generalizes some of the results of Šragin (Soviet Math. Dokl. 9 (1968), 2) for the class of Jung functions and extends the work of the author on GN-functions. (Received October 8, 1970.)

682-46-5. ULRICH KRENGEL, Ohio State University, Columbus, Ohio 43210. Weakly wandering vectors.

A vector f of a complex Hilbert space H is called weakly wandering for the isometry U of H if there exists a strictly increasing sequence \(0 = k_0 < k_1 < \ldots\) of integers such that the vectors \(U^k f\) are orthogonal to each other. Theorem. U has continuous spectrum if and only if the weakly wandering vectors span H. U has purely discrete spectrum if and only if there exist no nonzero weakly wandering vectors. In the first case the weakly wandering vectors are even dense in H. (Received October 16, 1970.)


Let \(dm\) be normalized Lebesgue measure on the unit circle and let \(H^2\) be the subspace of \(L^2 = L^2(dm)\) spanned by \(Z^n\) for \(n = 0, 1, 2, \ldots\), let \(P\) be the projection of \(L^2\) on \(H^2\). For \(f\) in \(L^\infty = L^\infty(dm)\), we define the Toeplitz operator \(T_f\) on \(H^2\) by \(T_f h = P(fh)\) for \(h\) in \(H^2\). Let \(A\) be the set of all functions \(g\) in \(L^\infty\) such that \(T_f T_g - T_g T_f\) is compact for every \(f\) in \(L^\infty\). Let \(Q\) be the set of all functions of the form \(\psi \phi\) where \(\psi\) is a function in \(H^\infty\) and \(\phi\) is inner. \(Q\) is norm-dense in \(L^\infty\) was shown by Douglas and Rudin. In this paper we show that (1) \(A\) is a norm closed subalgebra of \(L^\infty\) and \(A\) contains \(H^\infty + C\), where \(C\) stands for the space of continuous complex valued functions on the unit circle; (2) if \(g\) is in \(A\), then \(T_g\) is invertible if and only if \(g\) is invertible in \(A\) and the index of \(T_g\) is zero; (3) \(H^\infty + C\) contains \(A \cap Q\); and (4) the norm closed of \(A \cap Q\) is \(H^\infty + C\). (Received October 9, 1970.)

682-46-7. WILLIAM H. SUMMERS, University of Arkansas, Fayetteville, Arkansas 72701. The general complex bounded case of the strict weighted approximation problem.

The strict weighted approximation problem (see Nachbin [Ann. of Math. (2) 81 (1965), 289–302]) is reformulated in terms of antisymmetric sets in order to accommodate the general complex case, and several results are then obtained in the pivotal bounded case. Let \(X\) denote a completely regular \(T_1\)-space. If \(V\) is the set of nonnegative u.s.c. functions on \(X\) which vanish at infinity, then the weighted space \(CV(X)\) (notation as in [Trans. Amer. Math. Soc. 146 (1969), 121–131]) turns out to be the space \(C_b(X)\) of bounded continuous functions on \(X\) with the "strict" topology. Now assume \(A\) is a subalgebra of \(C(X)\) and that \(W\) is a linear subspace of \(CV(X)\) which is an \(A\)-module. Our main result states that \(W\) is always \(A\)-localizable in \(CV(X)\) in the bounded case of the weighted approximation problem. (Received October 19, 1970.)
Let $R$ be a $W^*$-algebra on a separable Hilbert space $H$. Then $R$ has a direct integral decomposition into factors $R(t)$. \textbf{Lemma 1.} If $(A_n(t))$ is a central sequence in $R$, then some subsequence (call it $(A_n'(t))$ again) is such that $(A_n'(t))$ is central in $R(t)$ for almost every $t$. \textbf{Lemma 2.} $R$ has a nontrivial central sequence iff $R(t)$ has a nontrivial central sequence for almost every $t$. \textbf{Theorem 1.} $R$ has property L iff $R(t)$ has property L for almost every $t$. \textbf{Theorem 2.} If $R$ is a noncommutative type I algebra, then $R$ has neither property L nor property A. \textbf{Theorem 3.} $R$ is singly generated iff $R(t)$ is singly generated for almost every $t$. \textbf{Corollary.} Every $W^*$-algebra on $H$ is singly generated iff every type $II_1$ factor is singly generated. (Received October 19, 1970.)

\textbf{Separating function algebras. I.}

Let $A$ be a sup-norm algebra of complex valued continuous functions on a compact Hausdorff space $X$. A closed subset $S$ of $X$ is an $L_A$-set or briefly an $L$-set if $L(S) = S$, where $L(S) = \bigcap f^{-1} f(S)$. An algebra $A$ on $X$ is a separating algebra if every closed subset of $X$ is an $L$-set. The authors discuss several properties of $L$-sets. For instance, it is shown that if $S$ is an $L$-set, then $S \cup T$ is an $L$-set for any finite subset $T$ of $X$. Furthermore, two equivalent conditions which characterize separating algebras are given. By way of application it is shown that Dirichlet and pervasive algebras are separating algebras. Finally the authors raise several important unanswered questions. (Received October 19, 1970.)

\textbf{Separating function algebras. II.}

For definitions and basic results see "Separating function algebras. I," above. \textbf{Theorem.} If $A$ is a maximal subalgebra of $C(X)$, then $A$ is a separating algebra. \textbf{Examples.} The bidisk algebra is an example of a nonseparating function algebra. An example of a nonseparating function algebra on the Riemann sphere, due to Rudin, Proc. Amer. Math. Soc. 7 (1956), 825-830, is given. Finally, an example is given of a nonseparating function algebra $A$ on a connected space $P$ having all of $P$ for its Silov boundary. This example fails to be essential, however. Thus, it seems unknown whether there exists an essential nonseparating algebra $A$ on $X$ having maximal ideal space $X$ and Silov boundary $X$. (Received October 19, 1970.)

\textbf{Quasi-complementation in separable locally convex spaces.}

The closed subspace $M$ of the topological vector space $E$ is said to be quasi-complemented if there is a closed subspace $N$ of $E$ such that $M \cap N = \{0\}$ and $M + N$ is dense in $E$. Murray (Trans. Amer. Math. Soc. 58 (1945), 77-95) proved that in a separable reflexive Banach space, every closed subspace is quasi-complemented. Mackey (Bull. Amer. Math. Soc. 52 (1946), 322-325) showed the condition of reflexivity could be dropped. These results are generalized by \textbf{Theorem 1.} Let $E$ be a separated locally convex space with dual $E'$ and sup-
pose both $E$ and $E'$ are weakly separable. If $M$ is a closed subspace of $E$ such that $M$ and $M'$ are separable in the topologies induced by $\sigma(E, E')$ and $\sigma(E', E)$ respectively, then $M$ is quasi-complemented in $E$. This implies

**Theorem 2.** Every closed subspace of a separable metrizable locally convex space is quasi-complemented. **Corollary.** If $E$ is any of the spaces $J_\mathcal{O}(K)$ for $K$ a compact subset of $\mathbb{R}^n$, $J_\mathcal{O}(\mathcal{O})$ for $\mathcal{O}$ an open subset of $\mathbb{R}^n$, $J(\mathcal{O})'$, $J_\mathcal{O}(\mathbb{R}^n)$, $J(\mathbb{R}^n)'$ or $H(\mathcal{O})'$, then every closed subspace of $E$ is quasi-complemented. (Received October 20, 1970.)


This paper generalizes Dieudonné's theory of Köthe function spaces ("Sur les espaces de Köthe," J. Analyse Math. 1(1951), 81-115) by considering spaces of vector valued functions. The function spaces considered are subspaces of the space $\Omega(E)$ which is the completion of the space of all measurable functions into a locally convex topological vector space $E$ which integrate locally against every continuous seminorm on $E$. The space $\Omega'(E')$ of functions into the topological dual of $E$ and satisfying a suitable condition is used to obtain a duality theory. Most of the results of Dieudonné have been generalized. A special class of spaces $\Lambda(E)$ where $\Lambda$ is a Köthe space is also studied. A $f \in \Omega(E)$ is in $\Lambda(E)$ if $p(f) \in \Lambda$ for each continuous seminorm $p$ on $E$. A conjecture of Cáceres ("Generalized Köthe function spaces," Proc. Cambridge Philos. Soc. 65(1969), 609) relating to the case where $E$ is a Banach space has been answered in the affirmative. The dual of $\Lambda(E)$ has been identified as an explicit subspace of $\Omega'(E')$ when $E$ is separable and the topological and Köthe duals of $\Lambda$ coincide. In particular, the dual of $\mathcal{L}^p(E)$ when $E$ is separable is known. (Received October 22, 1970.)


A Banach space, $X$, and its conjugate, $X^*$, are said to admit a conjugation when there is a pair of mappings, $x \rightarrow x^+$ and $x^* \rightarrow x^{*+}$ satisfying (1) $(\alpha x + \beta y)^+ = \alpha x^+ + \beta y^+$; $(\alpha x^* + \beta y^*)^+ = \alpha x^{*+} + \beta y^{*+}$; (2) $x^{++} = x$; $x^{*++} = x^*$; (3) $x^*(x^+) = x^*+(x)$. Let $X$, $X^*$ be such a pair with $X$ separable, let $(\Omega, \mathcal{F}, P)$ be a probability space and let $X : \Omega \rightarrow X$ be a strong random variable. The variance of $X$ is defined by $V(X) = E(XX^*)$, provided it exists. If $X$ is reflexive, then $V(X)$ is a positive hermitian mapping. It is shown that $P[|x^*(x)| > \varepsilon] \leq (1/\varepsilon)^2 \epsilon(x^*(V(X)x^*))$, a direct generalization of the Tschebytscheff inequality. Special results are obtained if $X$ is an $L^p$ space. (Received October 23, 1970.)


Let $A$ be a $C^*$-algebra, $G$ a metrizable, locally compact group, and $T$ a homomorphism of $G$ into the isometric $*$-automorphisms of $A$, the latter with the pointwise convergence topology. Define multiplication and involution on the space $L^1(A, G)$ of Bochner integrable $A$-valued functions by $(f \ast g)(x) = \int_G f(y)(T(y)g(y^{-1}x))d\mu(y)$; $f^*(x) = (f(x^{-1}))^* \Delta(x)^{-1}$ where $\mu$ is a left Haar measure with modular function $\Delta$. Let $\nu$ be a countably additive

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Borel measure on $G$ with values in the double centralizer algebra $M(A)$ of $A$, and define $\int_G f(x) d\mu(x) \in A$ and $\int_G (d\mu(x)) f(x) \in A$ for $f$ in $L^1(A,G)$, in the obvious way ($M(A)$ acts on $A$ by multiplication). Then the pair of maps $f \mapsto \int_G f(x) d\mu(x), f \mapsto \int_G (d\mu(x)) f(x)$ is a double centralizer on $L^1(A,G)$ and all double centralizers arise in this way. Such maps can be characterized by a "Riesz representation theorem". This result is a generalization of a result of Wendel. (Received October 23, 1970.)


A sequence of linear fractional transformations is used to obtain the formal continued fraction representation of the solution to the Riccati equation in a complex Banach algebra. Applications to the matrix Riccati equation and a related quadratic matrix equation are given. (Received October 19, 1970.)


See Abstract 70T-B146, these Notices 17(1970), 664, for definitions and references. A representation $S(F,\sigma)$ for a semispace $S$ at $\sigma$ is called a Klee representation. In a linear topological space a semispace $S$ with a Klee representation $S = S(F,\sigma)$, where each $f$ in $F$ is a continuous linear functional, is called a concrete semispace. Using theorems and techniques of V. L. Klee [Duke Math. J. 18(1951)], the following theorems have been proved: (1) A Banach space $E$ is reflexive iff any two disjoint bounded closed convex subsets of $E$ are concretely separated. (2) Every nonreflexive separable Banach space contains a pair of disjoint bounded closed convex subsets which cannot be concretely separated, but which can be separated by a closed hyperplane. (3) If $C$ is a closed convex cone in a locally convex Hausdorff space and $C \cap (-C) = \{ \sigma \}$, then $C \sim \{ \sigma \}$ is the intersection of all concrete semispaces at $\sigma$ which contain $C \sim \{ \sigma \}$. (Received October 26, 1970.)

682-46-17. MARK D. LEVIN and C. W. McARTHUR, Florida State University, Tallahassee, Florida 32303. Order characterizations of unconditional and absolute Schauder bases.

Let $E$ be a topological vector space with topological dual $E'$. By the wedge associated with a biorthogonal system $\{ x_i, f_i \}_{i=1}^\infty$ (where $x_i \in E$ and $f_i \in E'$ for $i = 1, 2, \ldots$, and $f_1(x) = \delta_{ij}$) is meant that subset of $E$ whose elements $x$ satisfy $f_i(x) \geq 0$ ($i = 1, 2, \ldots$). Theorem. Let $K$ be the wedge associated with a biorthogonal system $\{ x_i, f_i \}_{i=1}^\infty$ in a barrelled space $E$ such that $E$ is the closed linear span of $\{ x_i : i=1, 2, \ldots \}$. Then $\{ x_i \}_{i=1}^\infty$ is an unconditional basis for $E$ if and only if $K$ is a normal b-cone. Several other conditions equivalent to the biorthogonal system being an unconditional basis are also presented. The dual wedge to $K$ is that subset of $E'$ consisting of those continuous linear functionals $f$ such that $f(x) \geq 0$ for all $x \in K$. The dual wedge induces a partial ordering on $E'$. Theorem. Let $\{ x_i, f_i \}_{i=1}^\infty$ be a total biorthogonal system in a locally convex space $E$ whose dual $E'$ is weak* sequentially complete. Suppose that $E = K - K$, where $K$ is the wedge associated with $\{ x_i, f_i \}_{i=1}^\infty$. Then $\{ x_i \}_{i=1}^\infty$ is an absolute basis if and only if every equicontinuous subset of $E'$ has a least upper bound. (Received October 26, 1970.)
Let $V$ be a pre-ring of subsets of a space $X$, $H$ a Hilbert space and $P(H)$ the family of all orthogonal projection operators from $H$ to $H$. A mapping $p$ from $V$ into $P(H)$ is called a projective volume if (1) $A, B \in V$ and $A \cap B = 0$ implies $p(A)p(B) = 0$; (2) the function $p$ is strongly countably additive on $V$. This implies for each element $h \in H$ the function $v_h = |p(\cdot)|^2$ is a positive volume on $V$. Let $Y$ be a Banach space. Denote by $L_2(v_h,Y)$, $N(v_h)$, and $M(v_h)$ the space of Lebesgue–Bochner square summable functions, the family of null sets, and the sigma ring of measurable sets generated by the volume $v_h$. Let $Y$ be a Banach space. Denote by $L_2(v_h,Y)$, $N(v_h)$, and $M(v_h)$ the space of Lebesgue–Bochner square summable functions, the family of null sets, and the sigma ring of measurable sets generated by the volume $v_h$. Let $L(p,Y)$, $N(p)$, and $M(p)$ be the intersection of the spaces $L_2(v_h,Y)$, the families $N(v_h)$, and the sigma rings $M(v_h)$, respectively, over all $h \in H$. An integral of the form $\int f dp$ for $f \in L(p,Y)$, is developed. And it is proved that the space $L(p,Y)$ consists of all $M(p)$ measurable functions, $N(p)$ almost everywhere essentially bounded. For a development of the spaces $M(v)$ and $L_2(v,Y)$ generated by the positive volume see Bogdanowicz, Math. Ann. 164(1966), 251-269, and Bull. Acad. Polon. Sci. 13(1965), 793-800. (Received October 26, 1970.)


Let $X$ be a vector space over the reals; $n$ a positive integer; and $\varphi$ a multilinear mapping from $X^{2n}$ to the reals such that $\varphi$ is invariant under permutations of the variables, and $\varphi(x,\ldots,x) > 0$ if $x \neq 0$. Such a mapping is called a positive definite, symmetric, multilinear functional. [Marcus, Pierce, Pacific J. Math. 31(1969), 119]. Let $||x|| = \varphi(x,\ldots,x)^{1/2n}$. It is easy to show $|| |$ is not always a norm. Theorem 1. If $\varphi(x_1,\ldots,x_n, x_1,\ldots,x_n) = 0$ for all $x_1,\ldots,x_n$ then $|| |$ is a norm. The condition is not necessary. Suppose $X$ is finite dimensional and $T$ is a linear mapping from $X$ to $X$ such that $\varphi(T(x_1),\ldots,T(x_{2n})) = \varphi(x_1,\ldots,x_{2n})$ for all $x_1,\ldots,x_{2n}$. Then $T$ is called an isometry with respect to $\varphi$ [Pierce, Pacific J. Math. 33(1970), 183]. Let $P_2n(T)$ be the set of such $\varphi$. Theorem 2. For a given operator $T$, $P_2n(T)$ is nonempty if and only if $P_2n(T)$ contains some $\varphi$ that generates a norm and $T$ is an isometry with respect to that norm. Similar results can be formulated in the case the scalar field is the complex numbers. (Received October 29, 1970.)

682-46-20. JOEL H. SHAPIRO and GERALD D. TAYLOR, Michigan State University, East Lansing, Michigan 48823 and ALLEN L. SHEILDS, University of Michigan, Ann Arbor, Michigan 48104. The second duals of some function spaces.

Let $S$ be a locally compact Hausdorff space, let $B(S)$ denote the bounded, Borel measurable functions on $S$ with the supremum norm, let $C_0(S)$ denote the continuous functions vanishing at infinity, and let $M(S)$ denote the regular Borel measures on $S$. The $\alpha$-topology on $B(S)$ is the weak topology on $B(S)$ resulting from the pairing with $M(S)$: $(f, \mu) = \int fd\mu$. Theorem. If $E$ is a closed subspace of $B(S)$, if $F$ is a closed subspace of $E \cap C_0(S)$, if $F$ is $\alpha$-dense in $E$, and if the unit ball of $E$ is $\alpha$-compact, then $E$ may be naturally identified with the second conjugate space of $F$. This result includes a number of known results on spaces of analytic functions, of Lipschitz functions, and of operators on Banach spaces. (Received October 29, 1970.)
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The convex hull of the range of vector valued holomorphic mappings. Preliminary report.

It is proved that if \( f \) is a nonconstant holomorphic mapping of a domain \( G \) of the complex plane into a complex Banach space \( X \), then each point of \( f(G) \) is the center of a complex disc which is contained in the closed convex hull of \( f(G) \). The best constant for the radius of such a disc is obtained. It is also shown that there exists a holomorphic mapping of the unit disc \( D \) into the unit polydisc \( U^2 \) such that the closure of the convex hull of \( f(D) \) is the closure of \( U^2 \). (Received October 30, 1970.)


Given a C*-algebra \( \mathfrak{g} \) with identity on a Hilbert space \( \mathfrak{g} \). Let \( P \) and \( \Sigma \) be the positive cone and unit sphere of \( \mathfrak{g} \) respectively. If we denote by \( L = P \cap \Sigma \) and \( K \) the set of vector states of \( \mathfrak{g} \). Let \( L_0(\omega) \) be (resp. \( L_0(k) \)) the annihilator of \( \omega \in K \) (resp. \( k \subset K \)), and \( K_0(A) \) (resp. \( K_0(1) \)) the annihilator of \( A \in L \) (resp. \( 1 \in L \)).

\( L \) has S-property if for any \( A_1, A_2 \in L \), there exists \( A_3 \in L \) such that \( A_3 \supset A_1, A_3 \subset A_2 \) and \( K_0(A_3 \cap K_0(A_1)) \subset K_0(A_2) \). Theorem. Let \( \mathfrak{g} \) be a C*-algebra with identity acting on a Hilbert space \( \mathfrak{g} \), and \( L = P \cap \Sigma \) as defined above. Then \( \mathfrak{g} \) is a von Neumann algebra if and only if \( L \) has S-property. This theorem follows from the following lemmas. Lemma 1. \( L \) has S-property if and only if \( L_0(K) \) (for \( k \subset K \)) is a bounded, monotone-increasing directed sequence of \( L \). Lemma 2. For each \( A \in L \), \( K_0(A) \neq \emptyset \) if \( L \) has S-property. Lemma 3. Let \( \mathfrak{g} \) be a bounded, monotone-increasing directed sequence of \( L \). If \( K_0(A) \neq \emptyset \) for each \( A \in L \), then \( \mathfrak{g} \subset L_0(k) \) for some nonvoid subset \( k \) of \( K \). Lemma 4. Let \( \mathfrak{g} \) be a bounded, monotone-increasing directed sequence of \( L \). If \( \mathfrak{g} \subset L_0(k) \) for some nonvoid \( k \subset K \), then l.u.b. of \( \mathfrak{g} \) lies in \( \mathfrak{g} \). (Received October 30, 1970.)

MARTIN E. WALTER, University of California, Los Angeles, California 90024. W*-algebras and nonabelian harmonic analysis.

The Fourier-Stieltjes algebra \( B(G) \) of an arbitrary locally compact group \( G \) is the set of finite, complex-linear combinations of continuous, positive definite functions on \( G \), where addition and multiplication are defined pointwise and a Banach algebra norm (unique up to equivalence) can be specified. \( B(G) \) is thus a commutative, semisimple Banach algebra with unit. The main result is that \( B(G_1) \) and \( B(G_2) \) are isometrically isomorphic as Banach algebras iff \( G_1 \) and \( G_2 \) are topologically isomorphic as groups. The spectrum of \( B(G) \) is characterized as a *-semigroup of operators on Hilbert space, the subgroup of invertible elements (being precisely those elements which "preserve tensor products") is topologically isomorphic to \( G \). The Fourier-algebra \( A(G) \) is also shown to characterize \( G \), and the representation theory of the lattice of subgroups of \( G \) is studied. The main techniques of investigation come from the theory of C*- and W*-algebras. (Received October 30, 1970.)

R. RAO CHIVUKULA, University of Nebraska, Lincoln, Nebraska 68508. Introversin in function spaces and conjugate convolution algebras.

For a function \( f \) on a locally compact group \( G \) let \( \chi f \) be the left translate of \( f \) by \( x \in G \). A linear space \( X \) of functions is left invariant if \( \chi f \in X \) for all \( f \in X \) and \( x \in G \). A Banach space \( X \) of functions on \( G \) is left...
introverted if it is left invariant and if \( \lambda_t \in X \) for all \( \lambda \in X^* \) and \( f \in X \), where \( \lambda_t(x) = \lambda(x^t) \). For \( X \) left introverted and \( \lambda, \mu \in X^* \), let \( \langle \lambda \cdot \mu \rangle(f) = \lambda(\mu^f) \); then \( (X^*, \ast) \) is a Banach algebra. **Theorem 1.** If \( (a) l < p < \infty \) and \((1/p) + (1/q) = 1\), then \( L_p(G) \) is left introverted if and only if \( L_q(G) \) is closed under convolution. (b) \( L^1(G) \) is introverted if and only if \( G \) is compact. **Theorem 2.** Let \( C(G) \) be the space of all bounded continuous functions on \( G \); \( U(G) \subset C(G) \) uniformly continuous functions; \( AP(G) \subset C(G) \) almost periodic functions; \( C_0(G) \subset C(G) \) functions vanishing at infinity. Then \( (a) C(G) \) is introverted if and only if \( G \) is compact; \( (b) U(G), AP(G), \) and \( C_0(G) \) are introverted for any locally compact \( G \). Finally it is proved that in case the dual of any of the above spaces has a natural interpretation (e.g. \( C_0(G)^* = M(G) \), all bounded regular measures on \( G \)) then the operation \( \ast \) introduced above on \( X^* \) coincides with convolution (in the usual sense). (Received November 2, 1970.)

682-46-25. ROBERT A. McGUIGAN, Department of Mathematics and Statistics, University of Massachusetts, Amherst, Massachusetts 01002. **Strongly extreme points in normed spaces.**

Let \( S \) be a convex subset of a normed space. A point \( s \) in \( S \) is a strongly extreme point of \( S \) iff for every positive number \( r \) and every line segment \( v \) with length \( 2r \) and midpoint \( s \), the distance of one of the end points of \( v \) from \( S \) is greater than or equal to some positive number \( d_S(s, r) \). Results are obtained comparing strongly extreme points to other types of special boundary points of convex sets. An example is given of an exposed point which is not strongly extreme. A sufficient condition is given for every extreme point of the unit ball of a separable conjugate space to be strongly extreme. Using the notion of strongly extreme point, a convexity condition is defined which makes possible a unified treatment of both uniform nonsquareness and uniform convexity. (Received November 2, 1970.)

682-46-26. RALPH L. JAMES, Stanislaus State College, Turlock, California 95380. **Uniform convergence of positive operators.**

Let \( X \) and \( Y \) be ordered topological vector spaces and \( P \) a positive linear operator from \( X \) into \( Y \). **Definition.** A subset \( S \) of \( X \) is \( P \)-regular if for each neighborhood \( U \) of \( 0 \) in \( Y \) and each \( x \) in \( S \), there exist \( x^U, x_U \) in \( X \) such that \( x_U \leq x \leq x^U \), \( P(x^U - x_U) \in U \) and the sets \( \{ x^U : x \in S \}, \{ x_U : x \in S \} \) are totally bounded. **Theorem.** Let \( X \) be an ordered topological vector space which is barreled and \( Y \) a locally convex space ordered by a normal cone. Suppose \( \{ P_k \} \) is a sequence of continuous, positive, linear operators from \( X \) into \( Y \) such that \( P_k x \to Px \) for each \( x \) in \( X \). Then the convergence is uniform on each \( P \)-regular set. Each totally bounded subset of \( X \) is regular but the converse is false. However, every \( P \)-regular set is totally bounded in some weaker locally convex topology on \( X \). (Received November 2, 1970.)

682-46-27. YU-LEE LEE and HENDERSON C. H. YEUNG, Kansas State University, Manhattan, Kansas 66502. **Estimation of symmetric operators.**

**Theorem.** Let \( L^k_s(E, F) \) be the Banach space of multilinear symmetric operators with operator norm from Banach space \( E \) to Banach space \( F \) and \( f: U \to L^k_s(E, F) \) a function with \( U \) open in \( E \). If there exist constants \( a, b > 0 \) and \( c \geq 0 \) such that \( f(y) z^k / |z|^{k+c} \to 0 \) for \( a |y| < |z| < b |y| \) and \( z > 0 \), then \( f(y) / |y|^c \to 0 \) as \( y \to 0 \). This theorem can be used to prove the converse of Taylor's theorem and the fact that a multilinear symmetric operator is completely determined by its behavior on the diagonal. (Received November 2, 1970.)
Motivated by work in a Hilbert space setting of deBoor-Lynch, Atteia, Golomb, Jerome-Schumaker, and Laurent, we consider the following abstract optimization problem. Let \( X, Y \) be \( \mathbb{B} \)-spaces, \( R: X \to Y \) an \( \text{epimorphism} \), \( K \) a closed convex set in \( X \). Then \( (*) \): minimize \( \| R(x) \| : x \in K \) . Any \( x \) where this min is attained is called an \( R \)-spline interpolant of \( K \). In this paper a study is made of the case where \( K \) is a flat parallel to a given closed subspace \( M \). Topics considered include existence, uniqueness, and characterization of \( R \)-splines, approximation of an \( R \)-spline by \( R \)-spline interpolants of nearby flats, bases for the collection of \( R \)-spline interpolants of all flats parallel to \( m \) (\( \text{codim} M < \infty \)), and existence of relaxed \( R \)-splines (i.e., solutions of an extended problem \( (*) \) in an appropriate quotient space of \( X^{**} \)). Examples are given when \( X \) is an \( L^1 \)-space or a \( \text{W}^{p,m} \)-(Sobolev) space. Sample Theorem. Assume that \( \ker R \) is complemented or else is a continuous Chebyhev subspace. Then for any \( P \subset X \), \( R(P) \) is closed iff \( P + \ker R \) is closed. This result is useful for establishing the existence of \( R \)-splines. (Received November 2, 1970.)

If \( T \) is the one-dimensional torus, and \( k, n \) are natural numbers, then \( C^k(T^n) \) will denote the Banach space of all functions, \( f : T^n \to \mathbb{R} \), for which all the partial derivatives of order less than or equal to \( k \) exist and are continuous. It has been shown that \( C^k(T^n) \) has a Schauder basis. It follows from a result of Mitjagin that the space \( C^k(M^n) \) has a Schauder basis, where \( M^n \) is any \( n \)-dimensional compact \( C^k \) manifold. (Received November 2, 1970.)

Let \( U \) denote the open unit disk with boundary \( T \) and \( A^\infty \) the space of all functions \( f \) such that \( f^{[0]} \), the 0th derivative of \( f \), is bounded in \( U \), \( n = 1, 2, \ldots \). A closed subset \( E \) of \( T \) of measure zero is called a Carleson set if its complementary intervals \( \{I_n\} \) have lengths \( \epsilon_n \) satisfying \( \sum \epsilon_n \log \epsilon_n > -\infty \). Theorem 1. If \( \{z_k = r_k e^{i\theta_k}\} \) is a sequence of points in \( U \) such that \( \sum 1 - r_k < \infty \) and \( \{z_k\} \cap T = E \) is a Carleson set, then \( \{z_k\} \) is the zero set of a function in \( A^\infty \) if and only if \( E \cup \{e^{i\theta_k}\} \) is a Carleson set. Theorem 2. Let \( \{z_k = r_k e^{i\theta_k}\} \) be a sequence in \( U \) such that \( \sum 1 - r_k < \infty \) and \( \{z_k\} \cap T = E \) is a Carleson set. If for some \( \alpha \neq 1, \sum \text{dist} (e^{i\theta_k}, E)^\alpha < \infty \) then \( \{z_k\} \) is the zero set of a function in \( A^\infty \). (Received November 2, 1970.)

Let \( \mu \) be a finite nontrivial measure, and let \( 1 \leq r < s \leq \infty \). Theorem. \( L^s(\mu) \) is an \( \text{F}_{\sigma}^{-} \)-meager, uncountably codimensioned linear subspace of \( L^r(\mu) \); in fact, if \( K = \bigcup_{r < s} L^s(\mu) \), then the same holds for \( K \).
Somewhat more generally, we have Theorem. If \( \Phi \) is a Young’s function which satisfies the \( \Delta_2 \)-growth condition, and \( L^\Phi(\mu) \) is properly contained in another Orlicz space \( L^\varphi(\mu) \) then the conclusion of the above theorem holds for \( L^\Phi(\mu) \). Also, one has the Theorem. If \( \Phi \in \Delta_2 \) then any closed linear subspace \( L \) of \( L^\Phi(\mu) \) consisting of \( \mu \)-essentially bounded functions is finite-dimensional. Similar results hold in the situation of the \( L^p \) spaces as well as various vectorial generalizations of all of the above classes. (Received November 5, 1970.)


A function algebra \( A \) on a compact Hausdorff space \( X \) is a closed subalgebra of \( C(X) \) containing the constant functions and separating the points of \( X \). \( A \) is said to be local if each function in \( C(X) \) agreeing locally with an element of \( A \) must itself belong to \( A \). \( \text{Spec}(A) \) denotes the space of maximal ideals of \( A \), endowed with the Gelfand topology. \( A \) is said to be antisymmetric if \( A \) contains no nonconstant real-valued functions. Theorem. If \( A \) is a function algebra on the unit interval \( I \) and \( \text{Spec}(A) = I \), then \( A \) is local. Theorem. If \( A \) is a function algebra on the unit circle \( T \) and \( \text{Spec}(A) = T \), then \( A \) is either local or antisymmetric.
(Received October 26, 1970.)


The unit ball in the space \( L^1 \) is not rotund and no equivalent norm can yield uniform convexity. Here a family of spaces called \( L^\lambda \)-spaces are created which relate to \( L^1 \) and have a convexity property between rotundity and uniform convexity. This is done in essence by letting \( \lambda \) be a continuous, even, strictly convex (i.e. the graph of \( \lambda \) crosses any straight line at most twice) function with \( \lambda(0) = 0 \). An expression called a "\( \lambda \)-norm" is defined as \( \|f\|_\lambda = \lambda(f) \) and the \( L^\lambda \)-space consists of all functions with finite "\( \lambda \)-norm". The "\( \lambda \)-norm" is neither homogeneous nor subadditive, but does yield a neighborhood system for a topology. The space \( L^\lambda \) has a property called uniform convexity in measure which yields convergence in measure whenever the stronger property of uniform convexity yields norm convergence. By considering a function \( \lambda \) with \( \lambda(x) \leq x \), the space \( L^1 \) is contained in \( L^\lambda \) and so sequences in \( L^1 \) can be shown to converge in measure using the convexity properties of \( L^\lambda \). The method applies to prove the martingale theorem, the existence of Radon-Nikodym derivatives, and some ergodic theorems. (Received November 2, 1970.)

682-46-36, ISADORE BRODSKY, University of Maryland, College Park, Maryland 20742. Preservation of closure in a locally convex space. Preliminary report.

Let \( X \) and \( H \) be linear spaces with \( \langle X, H \rangle \) a dual system. Let \( G \) be a subspace of \( H \) with \( \langle X, G \rangle \) a dual system for the bilinear form from \( \langle X, H \rangle \). As an example of one of the basic theorems of this paper, we get a sufficient condition for when a proper \( \sigma(G, X) \)-closed subset \( A \) in \( G \) will be \( \sigma(H, X) \)-closed in \( H \). The sufficient condition is that \( A \) be a proper closed subset of \( G \) for a specified family of locally convex Hausdorff topologies defined on \( G \). This result is of interest because, if we ignore the linear

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structures on $H$ and $G$ and consider a given family of topologies on $H$, then $A$ closed in $G$ for all of the induced topologies in the given family does not necessarily imply that $A$ be closed in $H$ for any one of the topologies in the given family. The above machinery allows us to prove theorems as the following. Let $E$ be a locally convex space with topology $t$. Let $A$ be a proper closed subset of $E$. Suppose $A$ is closed in $E$ for every locally convex Hausdorff topology $t'$ on $E$ such that $t' < t$. Then $A$ is complete in $E$ for every locally convex Hausdorff topology $t'$ on $E$ such that $t' < t$. If $A$ also happens to be a bornivorous barrel in $E$, we further get that $E$ is semireflexive. (Received November 3, 1970.)


Let $V$ be a real linear space, $X$ a real, reflexive Banach space, and $A: V \to X$ a linear transformation with a closed range. This paper considers the problem of obtaining an algorithm which yields best approximate solutions to $Av = b$, i.e. solutions $\hat{v}$ for which $\|A\hat{v} - b\| \leq \|Av - b\|$ for all $v \in V$. It is shown that if appropriate convexity conditions are imposed on $X$, then the problem can be reduced to one of solving the dual maximization problem $\hat{f}(b) = \max_{f \in R(A)} \frac{\|f\|}{\|f\|}$ for the functional $\hat{f}$. An infinite algorithm is given which generates a sequence of functionals converging weakly to $\hat{f}$. This algorithm is modeled after an algorithm given by V. P. Sreedharan for determining solutions to overdetermined systems of linear equations which minimize the error in an abstract norm. (Received November 3, 1970.)


Let $x = Lx + T(\lambda, x)$ be a nonlinear operator equation in a real Banach space, where $I - \lambda_0 L$ is Fredholm with index 0, $\lambda_0$ is a simple eigenvalue of $L$ and $T \in C^1$ such that $\|T(\lambda, x)\| = o(\|x\|)$ and $\|T(\lambda, x)\| = o(\|x\|)$ uniformly for all $\lambda$ near $\lambda_0$. Theorem 1. $\lambda_0$ is a bifurcation point of the equation with respect to 0 and all solutions near $\lambda_0$ and 0 are of the form $\lambda = x(\phi)$ and $x = x(\phi)$, $0 < \phi < \sigma$, $\lambda(\phi)$, $x(\phi) \in C$. If $T \in C^n$ (is analytic) then $\lambda(\phi)$ and $x(\phi) \in C^{n-1}$ (are analytic) on $[0, \sigma)$. Theorem 2. If $T$ is analytic and $\lambda(\phi) \neq \lambda_0$ there are at most two solution branches, each branch analytic for $\lambda \neq \lambda_0$. If in addition $T$ is odd, for each $\lambda \in (\lambda_0 - \delta, \lambda_0 + \delta)$ (or $\lambda \in (\lambda_0 - \delta, \lambda_0)$) there exist two solutions near 0 and there are no solutions near 0 for $\lambda \in (\lambda_0 - \delta, \lambda_0 + \delta)$ (or $\lambda \in (\lambda_0 - \delta, \lambda_0)$). These results are proven by simple methods. Counterexamples to these conclusions are given when the hypotheses are weakened. (Received November 4, 1970.)


Let $D_0 \cdot D_1 \ldots$ be a system of derivations on a commutative F-algebra $A$. We show that if $A$ is semisimple and has an identity, then the derivations $D_0 \cdot D_1 \ldots$ are necessarily continuous. Now suppose that the F-algebra $A$ is uniform and that there is an element $f$ of $A$ and complex numbers $t_1 \ldots t_n$ such that polynomials in $f, (f-t_1)^{-1}, \ldots, (f-t_n)^{-1}$ are dense in $A$. We show that if $A$ has a derivation $D$ into the continuous functions on
its spectrum and if for any $x$ in the spectrum of $A$ there is a function $g$ in the range of $D$ such that $g(x) \neq 0$, then $A$ is topologically isomorphic to $\text{Hol}(U)$ for some open subset $U$ of the complex plane. (Received November 4, 1970.)


Normalized iterations and nonlinear eigenvalue problems of variational type.

Let $a(x)$ and $b(x)$ be real-valued even convex continuous functionals on the real Banach space $X$. Let $a(x)$ have a bounded strongly monotone hemicontinuous Gateaux derivative $A : X \to X^*$, and let $b(x)$ have a completely continuous Frechet derivative $B: X \to X^*$ such that $Bx = 0$ only if $x = 0$. Theorem. Let $c > a(0)$, then the problem $Ax = c$, $By$, $a(x) > 0$ has a unique solution $(x, 0)$ for every nonzero $y \in X$. The resulting mapping (normalized iteration) $x: Y \to X \setminus \{0\}$ has the following properties: (i) $x$ is odd and continuous. (ii) For any $\lambda > b(0)$, $x(\{x : a(x) \leq c, b(x) \leq \lambda\})$ is precompact in $X$. (iii) If $a(0) < a(x) \leq c$, then $b(x) = b(x)$ with equality only if $x$ is an eigenfunction of $(*) Ax = \mu Bx$, $a(x) = c$. This result yields a "spectral theorem" for $(*)$ which contains the full result as known for the case when $A$ and $B$ are linear. (Received November 4, 1970.)

682-46-41. LAWRENCE C. MOORE, JR., Duke University, Durham, North Carolina 27706. Strictly increasing Riesz norms.

A Riesz norm $p$ on a Riesz space $L$ is said to be strictly increasing if $u, v \in L$ and $0 \leq u \leq v$ implies that $p(u) < p(v)$. We extend a result of W. A. J. Luxemburg (Arch. Math. (Basel) 19 (1968)) by showing that every Riesz space with a strictly increasing Riesz norm has the countable sup property. On the other hand, every Riesz space with an $(A, ii)$ Riesz norm processes an equivalent strictly increasing Riesz norm. (Received November 4, 1970.)

682-46-42. JIN BAI KIM, West Virginia University, Morgantown, West Virginia 26505. H-classes in a Banach algebra. Preliminary report.

Let $B$ be a noncommutative complex Banach algebra. $B$ is regular if $e \in eBa$ for each $a$ in $B$. Under the multiplication $B$ forms a semigroup, and hence we can talk about $H$-classes of $B$. Assume $B$ is regular. Let $e$ be a nonzero and nonidentity idempotent of $B$. Let $Re$ be the $R$-class containing $e$ and let $H$ be an $H$-class contained in $Re$. Theorem. $H$ is open in the relative topology for $eBe$. (Received November 4, 1970.)

682-46-43. SATISH SHIRALI, New Mexico State University, Las Cruces, New Mexico 88001. On Jordan *representations of Banach algebras. Preliminary report.

The following is shown to be a necessary and sufficient condition for a Jordan *representation $T$ of a complex Banach algebra $A$, with identity and continuous involution, to be a direct sum of a *representation and a *antirepresentation: For any $x \in A$, the (necessarily real) spectra of $T(x^*)$ and $T(xx^*)$ have the same infimum and supremum. Methods of an earlier paper of the author [Duke Math. J. 34(1967), 741-746] are used to show that, if the condition is satisfied then $T$ can be regarded as defined on a suitable $B^*$-algebra; then the theorem of Størmer [Trans. Amer. Math. Soc. 120(1965), 438-447] is applied. The necessity is trivial. (Received November 4, 1970.)
A net \([S_d : d \in D]\) of continuous linear projections of finite rank on a Hausdorff linear topological space \(V\) is said to be a Schauder operator basis -- \(S.O.B.\) -- (resp. Schauder dual operator basis -- \(S.D.O.B.\)) provided it is pointwise bounded and converges pointwise to the identity operator on \(V\), and \(S_e S_d = S_d\) (resp. \(S_d S_e = S_e\)) whenever \(e \geq d\). \(S.O.B.'s\) and \(S.D.O.B.'s\) are natural generalizations of finite dimensional Schauder decompositions because a sequence of operators is both a \(S.O.B.\) and \(S.D.O.B.\) iff it is the sequence of partial sum operators associated with a finite dimensional Schauder decomposition. Also, a separable Fréchet space on which there is a continuous norm and which admits a \(S.O.B.\) also admits a finite dimensional Schauder decomposition. Many duality theory results concerning Schauder bases can be extended to \(S.O.B.'s\) or \(S.D.O.B.'s\).

In particular a locally convex space with a \(S.D.O.B.\) is semireflexive iff the \(S.D.O.B.\) is shrinking and boundedly complete. A number of results are proved which were previously unknown even in the Schauder basis case. For example, the strong dual of an evaluable space which admits a shrinking \(S.O.B.\) or shrinking \(S.D.O.B.\) is a complete barrelled space. (Received November 4, 1970.)

Unless defined herein, the notations are that of D. J. H. Garling [Proc. Cambridge Philos. Soc. 63(1967), 997-1019 and 963-981]. Let \(E\) be a \(K\)-space: \(E\) is the set of all sequences \(x\) whose sections \(P_n(x) = (x_1, x_2, \ldots, x_n, 0, \ldots)\) form a bounded subset of \(E\); \(E_N = \{x \in E : P_n(x) = x\}\). (1) \(E\) is a BS-space (i.e., \(E \subseteq E_N\)) if for each \(f \in E'\), the sequence \(y = (y_k)\) given by \(y_k = f(e_k)\) is an element of \(E' = E' \subset E_N = E' \subset E_N\).

(2) If \(E\) is barreled then, for each \(y \in E'\), there exists an \(f \in E'\) such that \(y_k = f(e_k)\), and \(E\) is a BS-space if and only if \(E = E'\). (3) If \(E\) is a sequentially complete BS-space then, under an appropriate topology on \(E' = E'_N\). These results generalize to the corresponding statements for Cesàro sections \(\sigma_n(x) = n^{-1}(P_1(x) + \ldots + P_n(x))\) and \(\sigma\)-duality. (Received November 4, 1970.)

An example is given of a function norm \(\rho\) such that \(\rho - \rho_L\) (\(\rho_L\) is the Lorentz function norm associated with \(\rho\)) is not monotone or subadditive. This answers a question posed by W. A. J. Luxemburg and A. C. Zaanan. Relationships between the monotonicity and subadditivity of the difference are also investigated.

Theorem. Let \(\rho\) be a saturated function norm and let \(\rho_L\) be the Lorentz norm associated with \(\rho\). Then \(\rho - \rho_L\) is monotone on \(L\) if and only if it is subadditive on \(L\). (Received November 4, 1970.)
A Banach space representation theorem. Preliminary report.

For any topological space \( S \) let \( C(S) \) be the Banach space of all bounded, real valued continuous functions on \( S \) with the sup norm; if \( S \) is also a topological semigroup, let \( UC(S) \) be the subspace of \( C(S) \) consisting of all uniformly continuous functions on \( S \). It is well known that each Banach space \( B \) is isometrically isomorphic to a subspace of \( C(S) \) where \( S \) can be taken to be a compact Hausdorff space. In this note the following generalization of this result is proved: Every Banach space \( B \) is isometrically isomorphic to a subspace of \( UC(S) \) where \( S \) can be taken to be a (metrizable) topological semigroup. If \( B \) is reflexive, \( S \) can be taken to be a compact topological semigroup (which need not be metrizable). (Received November 5, 1970.)

Extensions of basic sequences in Frechet spaces.

If \( E \) is a complete, barrelled space with a basis \( (x^n) \), a block basic sequence is a sequence \( (y^n) \) in \( E \) of the form \( y^n = \sum_{i=1}^{n} a_i x_i \) for each \( n \), where \( (p^n)_{n=0}^{\infty} \) is a sequence of integers such that \( p^0_0 < p^1 < \ldots \), and \( (a_i) \) is a sequence of scalars chosen so that \( y^n \neq 0 \) for each \( n \). The spaces \( E^n = \bigotimes_{n=1}^{p^n} E \) are called the \( n \)th block spaces associated with \( (p^n)_{n=0}^{\infty} \). We say that \( (y^n) \) has a block extension if there exists a Schauder basis \( (z^n) \) for \( E \) such that \( z^n = y^n \) for all \( n \), and \( z^i \in E^n \) if \( p^n_{n-1} < i \leq p^n_n \). M. Zippin has shown that in a Banach space with a basis, every block basic sequence has a block extension. It is shown that in a nuclear Frechet space with a basis such block extensions need not exist. The example is obtained in a nuclear echelon space of order 1. On the other hand, it is true that in the nuclear Frechet space \( \omega \), every block basic sequence has a block extension. (Received November 5, 1970.)


A class \( \mathcal{V} \) of Hausdorff linear topological spaces (l.t.s) is a variety if and only if it is closed under the operations of taking products, subspaces and (separated) quotients (e.g. class of all locally convex l.t.s. \([1.c.l.t.s.]\)). The variety \( \mathcal{V}(E) \) generated by the l.t.s. \( E \) is the smallest variety containing \( E \). Theorem. \( \mathcal{V} \) (reals) coincides with the class of all \( 1.c.l.t.s. \) having their weak topology. Theorem. Every variety contains \( \mathcal{V} \) (reals). Theorem. The variety \( \mathcal{V}(s) \), \( s \) the space of rapidly decreasing sequences, is the class of all nuclear spaces. Theorem. The class of all Schwartz spaces is a variety, which properly contains \( \mathcal{V}(s) \). Theorem. For any \( 1 \leq p < \infty \), \( \mathcal{V}(s) \neq \mathcal{V}(l^p) \neq \mathcal{V}(l^1) = \mathcal{V}([0,1]) \) \( \subset \mathcal{V}(l^1) \). Theorem. Normed linear spaces of different linear dimension generate different varieties. Theorem. The class of all varieties is not a set. Existence and identification of free objects are also considered as well as other relations between specific varieties. (Received November 5, 1970.)
The following Lemma appears to have a number of useful applications. **Lemma.** Let $D$ be an arbitrary set with $\text{card}(D) = 2^\aleph_0$. There exists a collection $\mathcal{B}$ of subsets $D$ such that $\text{card}(\mathcal{B}) = 2^\delta$ and such that if $B_1, \ldots, B_n, C_1, \ldots, C_m$ are distinct members of $\mathcal{B}$, then $\text{card}\left(\bigcup_{j=1}^n B_j \cup \bigcup_{k=1}^m C_k\right) = \delta$. Pelczynski (Studia Math. 30(1968), 240) proved that if the real Banach space $X$ contains a subspace isomorphic to $l_1(D)$, then the dual $X^*$ contains a subspace $Y$ isomorphic to $l_1(D_1)$ with $\text{card}(D_1) = 2^\delta$. The Lemma yields an independent proof, valid also for the complex case, which additionally shows that the isomorphism between $Y$ and $l_1(D_1)$, can be chosen so as to preserve the natural order relations. For both the real and complex case, the theorem holds with "isomorphic" replaced by "isometric". If $X$ is real and separable, the following are equivalent: (i) $X$ contains a subspace isomorphic to $l_1(D)$; (ii) $X^*$ contains a seminorming subspace isomorphic to $l_1(D_1)$; (iii) $X^*$ contains a norming subspace isomorphic to $l_1(D_1)$. The equivalence of (i) and (ii) is due to Pelczynski. The notion of a norming subspace has interesting generalizations to locally convex spaces. A perturbation theorem by Kato and uniform boundedness results of Brace and Nielsen are improved upon. (Received November 5, 1970.)

**Spectral theorem and lattice properties in $B_\rho$-algebras.**

$B_\rho$-algebras were introduced by C. E. Rickart. They are $C^*$-algebras $\mathfrak{A}$ such that if $T$ is a selfadjoint element of $\mathfrak{A}$, then there exists a projection $P$ in $\mathfrak{A}$ with the properties that $TP = T$ and for selfadjoint elements $S$, $TS = 0$ implies $PS = 0$. For such algebras, which include the AW*-algebras and the von Neumann algebras, the spectral theorem is obtained including the uniqueness and commutation statements. When the algebra is faithfully and properly embedded in the bounded operators on a Hilbert space, this spectral theorem becomes the usual one. Using the $\sigma$-completeness of the lattice of projections in a $B_\rho$-algebra, a partial order is introduced on the resolutions of the identity. It is found that with respect to this spectral order, the selfadjoint elements of a $B_\rho$-algebra form a $\sigma$-conditionally complete lattice. This order is characterized by $S \leq T$ if and only if $S^n - T^n$ is in the cone of elements of the form $RR^*$ for all positive integers $n$.

Conversely, it is shown that if the selfadjoint elements of a $C^*$-algebra are $\sigma$-conditionally complete in this order, then the algebra is $B_\rho$. (Received November 5, 1970.)

**The incomplete Cauchy problem in Banach spaces.** Preliminary report.

Let $E$ be a complex Banach space and $A$ a closed linear operator with dense domain. For integers $0 < m < n$, $\omega$ real, the $(m, \omega)$ Cauchy problem is to find a strong solution of $u^{(n)}(t) = Au(t)$ on $[0, \infty)$ having $m$ prescribed initial conditions. The case $(n, m, \omega)$ is said to be well posed if: (A) For any $u_1, u_2, \ldots, u_m$ in $D$, a dense subspace of $E$ and integers $0 \leq i_1 < i_2 < \ldots < i_m \leq n-1$ there is a solution such that $u^{(j)}(0) = u_j$, $j = 0, 1, \ldots, n-1$. (B) There is a constant $M$ such that all solutions satisfy $\|u(t)\| \leq e^{\int_0^t e^\omega s \|u^{(j)}(0)\|}$. It is shown that if case $(n, m, \omega)$ is well posed, $A^{-1}$ exists and if $\lambda$ has $m$ of its $n$th roots in $\{\lambda | \text{Re} \lambda < \omega\}$ then $-\lambda \in \rho(A)$. As a consequence of this the following cases $(n, m, \omega)$ cannot occur: $\omega \leq 0$, $m \geq (n+3)/2$ for $n$ odd; $\omega < 0$, $m \geq n/2 + 1$ for $n$ even; $\omega = 0$, $m = n$. (Received November 5, 1970.)
Let $A$ be a complex Banach algebra, $\text{Rad}(A)$ its Jacobson radical and $\rho(x)$ the spectral radius of $x$.

**Theorem 1.** Let $A$ with a unity, and $\| \|$ a seminorm on $A$ such that $\|x\| \leq \rho(x)$ for every $x$ in $A$, then $\|yx-yx\| = 0$ for every $x, y$ in $A$.

**Theorem 2.** The following properties are equivalent: (1) $A$ is almost commutative (i.e. $A/\text{Rad}(A)$ is commutative). (2) $\rho$ is subadditive (i.e. there exists $\alpha > 0$ such that $\rho(x_1 + \ldots + x_n) \leq \alpha(\rho(x_1) + \ldots + \rho(x_n))$ for every $x_1, \ldots, x_n$ in $A$). (3) $\rho$ is submultiplicative (i.e. there exists $\beta > 0$ such that $\rho(xy) \leq \beta \rho(x) \rho(y)$ for every $x, y$ in $A$). These results extend those of LePage, Hirschfeld and Żelazko and give a complete characterization of almost commutative algebras. They can be used to characterize commuting elements in a Banach algebra and normal elements in a Banach algebra with an involution, obtaining then an extension of a result of R. G. Douglas and P. Rosenthal in C*-algebras. (Received November 5, 1970.)

682-46-54. JORG BLATTER, University of Texas, Austin, Texas 78712. *Approximation and selection.*

The notions subalgebra, vector sublattice, G-subspace of the space of continuous real valued functions on a compact Hausdorff space are extended in a natural way to the case of Banach space valued functions. Closure theorems for these subspaces are obtained via a tensor product representation. Selection theorems provide the main tool for the development of the theory of best approximation by these subspaces. (Received November 5, 1970.)

### 47 Operator Theory

682-47-1. CHI SONG WONG, Southern Illinois University, Carbondale, Illinois 62901. *A fixed point theorem for certain functions on a complete Hausdorff uniform space.*

Let $f$ be a function of a complete Hausdorff uniform space $(X, U)$ into itself. Let $I$ be the identity function on $X$. Let $I f x$ be the function on $X$ defined by $(I f)^{-1}(x) = (x, f(x))$ for $x$ in $X$. Then the family of all sets $(I f)^{-1}(u) x (I f)^{-1}(u) U$, is a base for a uniformity $U_I$ on $X$. **Theorem.** If $f$ is a uniformly continuous function of $(X, U_I)$ into $(X, U)$ and if $(I f)^{-1}(u)$ is a nonempty closed subset of $X$ for each closed symmetric member $u$ of $U$, then $f$ has a unique fixed point. It generalizes Theorem 2 in the paper "On nonlinear contractions," Proc. Amer. Math. Soc. 20 (1969), 458-464, by D. W. Boyd and J. S. W. Wong. (Received October 15, 1970.)

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A norm-sublinear transformation is a function \( C \) from a complex (real) normed vector space \( X \) into a complex (real) normed vector space \( Y \) satisfying the following conditions: (i) \( \| C(ax) \| = |a| \| Cx \| \) for all scalars \( a \) and all \( x \) in \( X \) and (ii) \( \| C(\sum_{i=1}^{n} a_i x_i) \| \leq \| \sum_{i=1}^{n} a_i Cx_i \| \) for all scalars \( a_1, \ldots, a_n \) and all \( x_1, \ldots, x_n \) in \( X \). \( C \) is bounded if (iii) there exists a \( k \geq 0 \) such that \( \| Cx \| \leq k \| x \| \) for all \( x \) in \( X \). In this paper it will be shown that a norm-sublinear transformation is linear if and only if its range is a linear space, that a norm-sublinear transformation with range in an inner product space is necessarily linear, and that if \( Y \) is a reflexive space of dimension not equal to 2 and every norm-sublinear transformation with range in \( Y \) is linear, then \( Y \) is an inner product space. (Received July 23, 1970.)

The Hille-Yosida-Phillips theorem gives a nasc on the resolvent of a densely defined closed linear operator for it to be the infinitesimal generator of a strongly continuous semigroup \( T(\sigma) \) such that \( \| T(\sigma) \| \leq M \). The following theorem gives an analogous nasc on the resolvent \( R_z = (I - zT)^{-1} \) of an operator \( T \) for \( \| T^k \| \leq M, k = 0, 1, 2, \ldots \)

Theorem. If \( T \) is a bounded linear operator on a complex Banach space \( X \), then a nasc that \( \| T^k \| \leq M \) for all integers \( k = 0, 1, 2, \ldots \) is \( \| (R_z - I) R_z \| \leq M |z|^{k} (1 - |z|)^{k-1} \) for \( |z| < 1 \) and \( n = 0, 1, 2, \ldots \). Corollary. A nasc that \( \| T^k \| \leq 1 \) for all integers \( k = 0, 1, 2, \ldots \) is \( \| R_z - I \| \leq |z|^{k} (1 - |z|)^{k-1}, |z| < 1 \). (Received October 2, 1970.)

Let \( H \) be a selfadjoint operator with spectral measure \( E(\sigma) \) over the Borel sets \( S \) of the real line. The spectrum of \( H \) is said to be strongly concentrated on \( S \) if whenever \( H_n \) converges strongly to \( H \) in the generalized sense it is true that \( E_n(\sigma) \) converges strongly to the identity. Sufficient conditions on \( H \) are given for this to occur for a given arbitrary Borel set \( S \) and necessary and sufficient conditions when \( S \) is the spectrum of \( H \). In addition several more workable sufficient conditions are cited and a few examples illustrating the results are given. (Received October 12, 1970.)

This paper extends a few results of Dean-Sucheston (Z. Wahrscheinlichkeitstheorie und Verw. Gebiete 6(1966), 1-9), Neveu (Proc. Fifth Berkeley Symp. Math. Statist. and Probability (Berkeley, Calif., 1965/66) Vol. II, Part 2, pp. 461-472, Univ. California Press, Berkeley, Calif., 1967), and Granirer (Duke Math. J., to appear). Let \( \mathcal{J} = \{ T_{\sigma} : \sigma \in \Sigma \} \) be a representation of a semigroup \( \Sigma \) as a semigroup of positive linear contraction operators on \( L_1 \) of a probability space \( (X, \mathcal{G}, P) \). Theorem 1. If \( \Sigma \) is left amenable and countably generated, then there exists a positive fixed point for \( \mathcal{J} \) if the \( T_{\sigma} \) are all conservative and for each \( A \in \mathcal{G} \), all the left invariant means on the bounded function \( \int_{\mathcal{G}} T_{\sigma} \angle dp \) of \( \sigma \) coincide. Theorem 2. If \( \Sigma \) is left amenable, the following
four conditions are equivalent: (i) There exists a positive fixed point for \( \varphi \). (ii) \( p(A) > 0 \) implies that \( \inf_{\sigma \in \Sigma} \int_{\mathcal{T}} T \, dp = 0 \).

(iii) \( p(A) > 0 \) implies that \( \inf_{\sigma \in \Sigma} \sum_{n} \sigma_{n}^{a_{n}} h \in L_{\omega}^{+} \) for some sequence \( \sigma_{n}^{a_{n}} \) from \( \Sigma \) then \( h = 0 \).

Theorem 3. Let \( \varphi = \{ p(x, A) : \sigma \in \Sigma \} \) be an amenable semigroup of Markov transition probabilities, not necessarily null-preserving.

Then there exists a \( \varphi \)-invariant measure \( \mu \gg p \) iff \( A \in \sigma \) and \( \sum_{n} p(x, A) \in B(X) \) for some sequence \( \sigma_{n}^{a_{n}} \) from \( \Sigma \) imply that \( p(A) > 0 \), where \( B(X) \) is the space of all bounded functions on \( X \).

682-47-6. STUART M. NEWBERGER, Oregon State University, Corvallis, Oregon 97331. A remark concerning \( B^0 \)-singular integral operators. Preliminary report.

Let \( C \) be the \( B^0 \) singular integral operators as in the author's paper [Illinois J. Math. 9 (1965), 428-443].

It is now well known that \( Hu = \int e^{i x \cdot \xi} \sigma(H)(x, \xi) \hat{u}(\xi) \, d\xi \) holds for \( H \in C, u \in S \) (Schwartz space), and \( \sigma(H) \in BC(R^n \times S^{n-1}) \), the bounded continuous functions on \( R^n \times S^{n-1} \), equipped with the supremum norm.

It is easy to see that the formula continues to hold for \( H \in \sigma^{-1} \), the closure of \( C \) in the operator norm in the bounded operators on \( L^2(R^n) \). As a consequence \( \sigma^{-1} \cap K^{loc} = (0) \); for example \( \sigma^{-1} \) contains no nonzero compact operators.

It follows that \( \sigma \) is a one to one map of \( C^{-1} \) onto a dense subset of \( BC(R^n \times S^{n-1}) \). The author does not know whether \( \sigma(C^{-1}) = BC(R^n \times S^{n-1}) \).

(Received October 16, 1970.)

682-47-7. CHARLES E. CLEAVER, Kent State University, Kent, Ohio 44240. On the extension of Lipschitz-Hölder maps on Orlicz spaces.

Let \( X \) and \( Y \) be Banach spaces. The statement that "\( e(X, Y, \alpha) \) holds" means that for arbitrary \( D \subseteq X \), every Lipschitz map of order \( \alpha \) from \( D \) into \( Y \) has an extension of the same order on all of \( X \). Theorem 1. Let \( (X, A, \mu) \) be a \( \sigma \)-finite measure space, \( \varphi \) a Young's function satisfying a growth condition on \( X \), and \( \varphi_{0}(x) = |x|^2 \).

If \( 0 < \alpha \leq 1 \), and \( \varphi_{s} \) is the inverse of the function \( \varphi_{s}^{-1} = (\varphi^{-1})_{s} \), \( 0 \leq s \leq 1 \), then \( e(L^{\varphi_{s}}, H, \alpha) \) holds for \( 2-1/\alpha \leq s \leq 1 \) and \( H \) a Hilbert space. Theorem 2. Let \( (X, A, \mu) \) be a \( \sigma \)-finite measure space, \( \varphi_{1}, \varphi_{2} \) Young's functions satisfying a growth condition on \( X \) and suppose \( L^{\varphi_{2}} \) is reflexive. If \( 0 < \alpha < 1 \), \( 2-1/\alpha \leq t \leq 1 \), \( s = \alpha(2-t) \), \( \varphi_{s}^{-1} = (\varphi_{1}^{-1})_{s} \), \( \varphi_{t}^{-1} = (\varphi_{2}^{-1})_{t} \), then \( e(L^{\varphi_{1}}, L^{\varphi_{2}}, \alpha) \) holds.

This extends previously announced results for \( L^p \) spaces by Williams, Wells, and Hayden, Abstract 672-36, these Notices 17(1970), 95. (Received October 26, 1970.)

682-47-8. LAVON BARRY PAGE, North Carolina State University, Raleigh, North Carolina 27607. Factoring operators which commute with a normal operator.

Let \( T \) denote the unit circle. If \( f \in L^{1}(T) \), \( f \geq 0 \), and \( \log f \in L^{1}(T) \), then a classical result of G. Szegö asserts that \( f = |h|^2 \) where \( h \) is in the Hardy space \( H^{2}(T) \). Similar factorizations for operator-valued functions were obtained by Masani and Wiener, Helson and Lowdenslager, Lax, and Devinatz. We are interested in factorizations of the form \( F = H^{*}H \) where \( F \) and \( H \) are bounded operators which commute with some fixed normal operator \( N \). In the case that \( N \) is unitary, an analog of the Szegö theorem is available. For \( N \) nonunitary, we explore the conditions which can preclude such factorizations. (Received October 26, 1970.)
Let $X$ be a real Banach space, let $K$ be a linear operator from $X$ into its dual $X^*$ and let $F:X \to X$ be a nonlinear mapping. In this paper we present some new existence theorems for the Hammerstein equation $u + KF(u) = 0$ using monotonicity and compactness assumptions about $K$ and coerciveness, but no monotonicity assumptions upon $F$. (Received October 26, 1970.)

Let $\mathcal{H}$ be a complex, separable Hilbert space, and $E(.)$ a spectral (projection-valued) measure for $\mathcal{H}$ defined on a $\sigma$-algebra $\mathcal{B}$ over an arbitrary space $\Lambda$. Let $\gamma \in \mathcal{H}$, $S_{\gamma}$ be the subspace spanned by $E(B)\gamma$, $B \in \mathcal{B}$. Call a linear operator $T$ from $\mathcal{H}$ to $\mathcal{H}$ \textit{\$E$-subordinative}, iff $\gamma \in \mathcal{H}$, $S_{\gamma}$ $T(\gamma) \in S_{\gamma}$. \textbf{Theorem.} $T \in \int_{\Lambda} \phi(\lambda)E(d\lambda)$ for some $\sigma$-Borel measurable $\phi$ on $\Lambda$ to $\mathcal{C}$, iff $T$ is a closed, $E$-subordinative operator such that $E(B)\cdot T \subset T\cdot E(B)$, $\gamma B \in \mathcal{B}$. This theorem subsumes all known results of the kind. Letting $(\Lambda, \mathcal{B}) = (\hat{\mathbb{X}}, \hat{\mathcal{B}})$ where $\hat{\mathcal{B}}$ is the $\sigma$-algebra of Borel subsets of the character group $\hat{\mathbb{X}}$ of a LCA group $\mathbb{X}$, we easily get the result announced by Rosenberg, Abstract 664-113, these \textit{Notices} 16(1969), 641. Specializing further to $X = \mathbb{R}$, we get at once the result proved by Masani, Bull. Amer. Math. Soc. 71(1965), 546, which in turn readily yields the original SOM theorem, cf. Stone, J. Indian Math. Soc. 16(1951), 191, involving the well-known commutant condition $[H] \subset [T]$, where $H = \int_{\mathbb{R}} \lambda E(d\lambda)$. \textbf{Corollary.} (a) For continuous $T$, the requirement $E(B)\cdot T \subset T\cdot E(B)$, $\gamma B \in \mathcal{B}$, can be dropped. (b) For measures $E(.)$ of total multiplicity 1, the $E$-subordinate requirement can be dropped. (Received October 26, 1970.)

Let $E(.)$ be a spectral (projection-valued) measure for a complex Hilbert space $\mathcal{H}$ defined on the $\sigma$-algebra $\mathcal{B}$ of Borel subsets of the real number field $\mathbb{R}$. Let $U(.)$ be its Fourier-Stieltjes transform, i.e. $U(t) = \int_{\mathbb{R}} e^{it\lambda} E(d\lambda)$, $t \in \mathbb{R}$. (Obviously, $U(.)$ is a strongly continuous group of unitary operators on $\mathcal{H}$ onto $\mathcal{H}$.) \textbf{Theorem.} $\gamma a, b \in \mathbb{R}$ such that $a < b$, $E(a, b) + (1/2)[E(a] + [b)] = \lim_{T \to \infty} \int_{-T}^{T} \int_{\mathcal{B}(a, b)}(t)U(t)dt$ on $\mathcal{H}$, where the integral on the RHS is a strong Riemann integral and $\xi_{B}(t) = \int_{\mathcal{B}} e^{i\lambda t} d\lambda / (2\pi)$, $\gamma B \in \mathcal{B}$ with $\text{Leb}(B) < \infty$. This theorem extends P. Levy's classical inversion formula for the $FS$ transforms of probability measures. The proof depends on the corresponding result for a bounded, $\mathcal{H}$-valued, countably additive, orthogonally scattered measure $\rho(.)$ on $\mathcal{B}$, which in turn can be proved from a known Fubini-type theorem for $\rho \times \text{Leb}$, [Advances in Math. 2(1968), 86, 5.20]. \textbf{Application.} Let $E(.)$ be the spectral measure of the group of translations $\tau_{t}$, $t \in \mathbb{R}$, on $L_{2}(\mathbb{R})$. The Theorem shows at once (without appeal to the Plancherel theorem) that $\gamma f \in L_{2}(\mathbb{R})$ and $\gamma B \in \mathcal{B}$ with $\text{Leb}(B) < \infty$, $E(B)f = \xi_{B} \ast f$. (Received October 28, 1970.)
The imaginary powers, $i^{ic}$, of the indefinite integral, $I$, are defined in terms of singular integral operators of Muckenhoupt type. $i^{ic}$ is a bounded operator on $L^p(0, \infty)$ for $1 < p < \infty$. For $1 < p < \infty$, the $i^{ic}$ form a strongly continuous boundary value group for the analytic semigroup $i^D$, $\Re(\alpha) > 0$, acting on $L^p(0,1)$. As an operator on $L^p(0,1)$, $i^{ic}$ has purely continuous spectrum, and the spectrum of $i^{ic}$ lies in the annulus $\exp(-\pi|c|/2) \leq |z| \leq \exp(\pi|c|/2)$. Similar results hold for $J = \int_{\infty}^{\infty}$ and $J_1 = \int_{1}^{1}$. (Received October 26, 1970.)

The Banach space $\{X_2, \| \_ \|_2 \}$ is said to be an invariant Banach subspace for the bounded linear operator $A$ on the Banach space $\{X_1, \| \_ \|_1 \}$ if $X_2$ is continuously imbedded in $X_1$ and $AX_2 \subset X_2$. It is shown that every bounded linear operator $A$ on $X_1$ has a nontrivial invariant Hilbert subspace $X_2 = \mathcal{H}$ with compact imbedding into $X_1$, so that $A|\mathcal{H}$ is a simple unilateral shift. If $A$ is quasinilpotent then it has an invariant Hilbert subspace $\mathcal{H}$ compactly imbedded in $X_1$ on which $A|\mathcal{H}$ is compact; if moreover $A$ has a cyclic vector, then $\mathcal{H}$ will be dense in $X$, and $A^*$ will extend by continuity to a compact quasinilpotent operator on $\mathcal{H}^* = X_1^*$. If $X_1 = \mathcal{H}_1$ is a Hilbert space, then quadratic interpolation between $\mathcal{H}_1$ and $\mathcal{H}_2 = \mathcal{H}$ yields interpolation spaces $\mathcal{H}_{1,\alpha}$ and $\mathcal{H}_{2,\alpha}$ having the same properties with respect to $A$ as have the $\mathcal{H}$ and $\mathcal{H}^*$ above, but "arbitrarily close" to $\mathcal{H}_1 = \mathcal{H}_1^*$ (i.e. for all $\alpha \in [1, 2]$). (Received October 29, 1970.)

Let $Y$ be in $\mathfrak{B}(\mathcal{H})$, the algebra of bounded linear operators on a separable Hilbert space. If $Y$ is in the range of a selfadjoint derivation on $\mathfrak{B}(\mathcal{H})$, $Y$ is called a selfadjoint commutator. Let $W(Y)$ be the numerical range of $Y$. Then $W_e(Y) = \cap \{W(Y + F): F \text{ is of finite rank}\}$ is the essential numerical range of $Y$. **Theorem.** $Y$ is a selfadjoint commutator if and only if $0 \in W_e(Y)$. **Corollary 1.** (Radjavi, "Structure of $A^*A - AA^*$," J. Math. Mech. 16(1966), 19-26.) Let $B = B^* \in \mathfrak{B}(\mathcal{H})$. Then there exists $T \in \mathfrak{B}(\mathcal{H})$ such that $T^*T - TT^* = B$ if and only if $0 \in W_e(B)$. **Corollary 2.** Every commutator is similar to a selfadjoint commutator. **Corollary 3.** (Brown and Pearcy, "Structure of commutators of operators," Ann. of Math. (2) 82(1965), 112-127.) $Y \in \mathfrak{B}(\mathcal{H})$ is a commutator if and only if $Y$ is not of the form $\chi + K$ where $\chi \notin 0$ and $K$ is compact. **Corollary 4.** The set of selfadjoint commutators is uniformly closed. (Received November 2, 1970.)

Let $T$ be a closed linear operator defined on a dense subset of a Banach space $X$. Assume that the spectrum, $\sigma(T)$, of $T$ is a subset of the real line and that the resolvent, $R(z, T)$, of $T$ satisfies the growth condition: $(G_n): |\operatorname{Im} z|^{\frac{n}{2}} |R(z, T)| \leq K$, for $0 < |\operatorname{Im} z| \leq 1$, and $|\operatorname{Im} z|^{\frac{n}{2}} |R(z, T)| \leq K$, for $|\operatorname{Im} z| > 1$, for some positive integer $n$, some positive real number $K$ and all nonreal $z$. **Theorem 1.** If $X$ is reflexive and $T$ satisfies $(G_n)$
then the null space, \( N[(T-t)N] \), of \((T-t)N\) and the closure, \( Cl(R[(T-t)N]) \), of the range of \((T-t)N\) are quasi-complements in \( X \) for each real \( t \). **Corollary.** If \( X \) is reflexive and \( T \) satisfies \((G)\) then the residual spectrum of \( T \) is empty.

If we assume that \( N[(T-t)N] \) and \( Cl(R[(T-t)N]) \) are quasi-complements for each real \( t \), there is a set \([E_t : t \text{ real}]\) of closed densely defined idempotents which has many of the properties of a resolution of the identity for a selfadjoint operator in Hilbert space. Indeed, if \( T \) is a selfadjoint operator in Hilbert space \([E_t : t \text{ real}]\) is a resolution of the identity for \( T \). (Received November 2, 1970.)


A function analytic approach to the generalized inversion problem for arbitrary operators between Hilbert spaces is the topic of this work. Projective properties as well as extremal solutions for inconsistent linear equations are provided by the generalized inverse \( T^+ \) of the given operator \( T \). \( T^+ \) resulting to be domain-dense, it gives rise to \( (T^+)^* = T^+ \) and to \((T^*)^* = T^+ \)(* for the adjoint) which might improve the original operator \( T \), as shown by the following result: \( T^+ \) is such that \( D(T^+) = D(T) + \text{Ker} \ T \) and \( \text{Ker} \ T^+ = \text{Ker} \ T \), where \( D(A) \) and \( \text{Ker} \ A \) denote the domain and the kernel (null manifold) of the operator \( A \), respectively. As for the adjoint \( A^+ \) of the generalized inverse, both the kernel and the range \( R(·) \) turn out to be closed: \( \text{Ker} \ T^+ = \text{Ker} \ T \), \( R(T^+) = R(T) \cap (\text{Ker} \ T^+) \). Properties of \( T^+, T^*, \) and \( T^+ \) are developed to the extent of unbounded and nonclosed operators \( T \) between Hilbert spaces. (Received November 2, 1970.)

682-47-17. C. J. A. HALBERG, JR., University of California, Riverside, California 92502 and ÅKE SAMUELSSON, Western Washington State College, Bellingham, Washington 98225. **On the fine structure of spectra.**

A linear operator \( A \) with domain and range in a normed linear space \( X \), is classified I, II or III according as its range is all of \( X \); is not all of \( X \), but is dense in \( X \); or is not dense in \( X \). In addition \( A \) is classified 1, 2 or 3 according as \( A^{-1} \) exists and is continuous; exists, but is not continuous; or does not exist. The state of an operator is the combination of its Roman and Arabic numerical classifications and is denoted by the Roman numeral with the Arabic numeral as a subscript. For a specific operator \( A \) the complex plane is partitioned into subsets corresponding to the states of the operator \( µ1 - A \), where \( µ \) is a complex number. This partitions the spectrum of an operator \( A \) into a unique family of subsets which is called the fine structure of the spectrum of \( A \). The set of states corresponding to the elements in the fine structure is called the type of the fine structure. The principal result is that the only types of fine structure that can never occur for a bounded linear operator on a complex Banach space are \([I_3] \), \([III_1]\) and \([I_3, III_1]\). (Received November 2, 1970.)


In 1953 Hoffman and Wielandt (Duke Math. J. 20(1953), 37-40) proved that if \( A \) and \( B \) are normal matrices of finite order \( n \) with eigenvalues \( \lambda_\nu, \mu_\nu (1 \leq \nu \leq n) \) respectively, then there exists a suitable ordering of the eigenvalues\$ such that \( \sum_{\nu=1}^{n} |\lambda_\nu - \mu_\nu|^2 \leq \|A-B\|_e^2 \), where \( \|\cdot\|_e \) is the usual Euclidean matrix norm. In this paper
the Hoffman-Wielandt result is extended to the case of completely continuous normal operators of Hilbert-Schmidt type. The proof involves the use of an appropriate generalization of a result of Birkhoff (Univ. Nac. Tucuman Rev. Ser. A 5(1946), 147-150) concerning the representation of doubly stochastic matrices in terms of permutation matrices. (Received November 2, 1970.)

682-47-19. TAKASHI ITO and TIN-KIN WONG, Wayne State University, Detroit, Michigan 48202, Subnormality and quasinormality of Toeplitz operators. Preliminary report.

Every analytic Toeplitz operator is subnormal. Halmos has posed the following question: Is every subnormal Toeplitz operator either analytic or normal? (Bull. Amer. Math. Soc. 76 (1970), 887-933). In this paper, Halmos' question is investigated. The following theorems are proved. Theorem 1. If \( \varphi \) is a polynomial in an inner function and its complex conjugate, then the Toeplitz operator \( T_\varphi \) is subnormal if and only if it is either analytic or normal. Theorem 2. For a bounded analytic function \( \varphi \), the Toeplitz operator \( T_\varphi \) is quasinormal if and only if \( \varphi \) is a constant multiple of an inner function. Theorem 3. Let \( \varphi \) be a bounded almost analytic function. If the Toeplitz operator \( T_\varphi \) is quasi-normal, then \( \varphi \) is either a constant multiple of an inner function or \( \varphi \) is a nonanalytic trigonometric polynomial in which case \( T_\varphi \) is normal. Examples of hyponormal Toeplitz operators which are not subnormal are found. (Received November 3, 1970.)

682-47-20. RALPH GELLAR, North Carolina State University, Raleigh, North Carolina 27607. Part of a weighted shift not similar to a weighted shift. Preliminary report.

An operator \( T \) on a Hilbert space \( H \) is called a weighted shift iff there is an orthonormal basis \( \{ y_n \}_{n=0}^{\infty} \) of \( H \) with \( Ty_n = a_n y_{n+1} \) for nonzero complex \( a \)'s. Example. Suppose (1) \( H \neq \mathcal{L}(T-I)H \) and (2) \( \mathcal{L}(T-I)H = \mathcal{L}(T-I)^2H \). Then \( T|\mathcal{L}(T-I)H \) is not circularly symmetric and hence not similar to a weighted shift. (1) holds iff \( \sum |a_n|^2 < \infty \) and (2) iff \( \sum n^2 |a_n|^2 = \infty \) where \( a_n = \prod_{j=1}^{n} a_j \). (Received November 4, 1970.)

682-47-21. JAMIL A. SIDDIQI, University of Sherbrooke, Sherbrooke, Quebec, Canada. On the mean ergodic theorem.

A generalization of the mean ergodic theorem due to L. W. Cohen (Ann. of Math. (2) 41(1940), 505-509) asserts that if \( T \) is linear on a Banach space \( B \) to \( B \) such that \( \| T^n \| \leq A \) and \( \left( a_{kn} \right) \) is a regular matrix such that \( (*) \lim_{k \to \infty} \sum_{j=1}^{\infty} |a_{n,j+1} - a_{n,j}| = 0 \) uniformly in \( n \) and \( L_\infty x = \sum_{j=1}^{\infty} a_{n,j} x^j \) is a weakly compact set, then there is an \( x_0 \) in \( B \) such that \( Tx_0 = x_0 \). It is shown that this theorem holds under weaker hypotheses on the matrix. (Received November 5, 1970.)

49 Calculus of Variations and Optimal Control

682-49-1. GEORGE A. KENT, Center for Dynamical Systems, Division of Applied Mathematics, Brown University, Providence, Rhode Island 02912. Necessary conditions for control of functional differential equations of neutral type.

Optimal control problems involving systems governed by functional differential equations of the type
\[
(*) \frac{d}{dt} x(t) - \int_{\alpha_0}^{t} d_s \left[ \mu(t, s) x(s) \right] = f(x, u(t), t) \quad a.e. \text{ on } [t_0, s], \quad x(s) = \varphi(s), \quad s \in [\alpha_0, t_0],
\]
are considered,
where \( f(x(.), u(t), t) \) may depend on any or all of the values of \( x \) on the interval \([\alpha_0, t]\). Necessary conditions for optimality, including conditions for problems with a terminal manifold in function space or a fixed terminal function, are obtained utilizing an abstract maximum principle of Neustadt ["A general theory of extremals," J. Comput. System Sci. 3 (1969), 57-92]. Conditions for a fixed terminal function are obtained using methods employed in problems with bounded state variables. Examples of linear hyperbolic partial differential systems with boundary value controls which reduce to neutral systems included in the form (\( \epsilon \)) are discussed. (Received October 9, 1970.)

682-49-2. MEHDI S. ZARGHAMEE, Arya-Mehr University of Technology, Tehran, Iran. Optimality and a differential of variational forms.

Let us consider \( I(\mu, \mu^0) = \int_S \mathcal{L}d\mu \) and \( \overline{I}(\mu) = \min_{I(\mu, \mu^0)} I(\mu) \) where \( S \) is a Borel subset of \( R \), \( \mathcal{L} \) is a first order differential operator and \( \mathcal{L}(x) \geq 0 \) for all \( x \in S \), \( f \) belongs to continuously differentiable class of functions \( \mathcal{F} \), and \( \mu \) is a positive bounded regular Borel measure in \( R^n \). If \( M \) is the space of all positive bounded regular Borel measures on \( S \), we would want to know the necessary and sufficient condition for \( \mu_m \) to satisfy \( \mu_m(S) = c \) and \( \overline{I}(\mu_m) = \overline{I}(\mu) \) \( \forall \mu \in M \). To this end we first construct a differential which measures the variation of \( \overline{I} \) with respect to changes in \( \mu \). This derivative is constructed as the Radon-Nikodym derivative of a set function which is the Gâteaux differential of \( \overline{I} \) with a properly chosen increment. It is shown that this derivative is easily computable, in fact \( d\overline{I}/d\mu = \mathcal{L}0 \) where \( f_0 \) is the function in \( \mathcal{F} \) which minimizes \( I(f, \mu) \). It is then shown that \( \mu_m \), the solution to the above posed problem, is the one satisfying \( d\overline{I}/d\mu_m = \text{constant} \). This theorem has wide applications in the optimal design of structures. These results are generalized to more complicated variational forms. (Received November 4, 1970.)

50 Geometry


H. Lüneburg proved that a translation \( H \)-plane \( A \) with translation group \( T \) is isomorphic to an incidence structure \( J(T, \Pi) \) -- points being the elements of \( T \), lines being the cosets of subgroups which appear as elements of some collection \( \Pi \). If \( A \) is \( n \)-uniform, one sets \( 0 = t, 0 = t \) the set of points neighbor to \( 0 \) in the incidence structure induced by \( A \) in \( T, 1 \leq i < n \). By definition, \( 0 \) is an \( n \)-uniform affine \( H \)-plane, \( 1 \leq i \leq n \). There exists a prime \( p \) and integers \( x, j, l \) such that (i) the order of the affine plane \( 0 \) is \( p^x \); (ii) \( T = C_1 \oplus \cdots \oplus C_k \); (iii) \( C_1 \) is the direct sum of \( 2x \) cyclic summands \( \sigma \) \( i \leq 1 \) \( \leq 1 \); (iv) \( C_1 \) is the direct sum of \( 2x \) copies of \( (p^q) \) \( i \leq 1 \); (v) \( 0 = p^{q+1}(C_1 \oplus \cdots \oplus C_j) \oplus p^{q+1}(C_{e+1} \oplus \cdots \oplus C_k) \) where \( q, e \) satisfy \( n - i = kq + e, 0 \leq e < k \). Conversely, given \( p, x, j, k, l \), there always exists an \( n \)-uniform \((n = kj + l)\) translation \( H \)-plane (in fact, even a Pappian \( H \)-plane) satisfying (i) -- (v). (Received October 12, 1970.)
Let \( \mathcal{J} \) denote the group of projectivities defined on a projective line. Each linear rational function 
\[ f(x) = \frac{ax + b}{cx + d} \]
is the correspondent of one and only one projectivity 
\[ x_1' = ax_1 + bx_2, \quad x_2' = cx_1 + dx_2, \]
where \( a, b, c, d \) are real values meeting the restriction that \( ad - bc \) differs from zero. An algorithm developed by the author (cf. Abstract 677-33-2, these Notices 17(1970), 775), is used to identify infinitely many conjugate subgroups of \( \mathcal{J} \). Each subgroup thus obtained is isomorphic to the symmetric group \( S(3) \). (Received October 23, 1970.)

Let \( S \) be the locus in \( \mathbb{R}^3 \) of the functions, \( x = f(u, v), \ y = g(u, v), \ z = h(u, v) \) these functions being continuous on a set \( E \) consisting of the interior and the boundary of a simple closed polygon. We consider polyhedra inscribed on \( S \), each face of which has an angle between \( \phi \) and \( \pi - \phi \), \( 0 < \phi < \pi \). By the deviation of a face \( T \) we mean the L.U.B. of the acute dihedral angles between two admissible triangles which are inscribed in the portion of \( S \) which is subtended by \( T \). The deviation norm of a polyhedron inscribed on \( S \) is the greatest of the deviations of its faces. We show that if \( S \) is triangulable then for every sequence \( (\sigma_1, \sigma_2, \ldots) \) of admissible polyhedra inscribed on \( S \), such that the corresponding sequence of deviation norms converges to zero, the corresponding sequence of the polyhedral areas converges to the Lebesgue area of the surface. We also give the theory for nontriangulable surfaces. (Received October 26, 1970.)

Isostrophisms of loops (Artzy, Arch. Math. (Basel) 14 (1963), 95-101) are used to establish necessary and sufficient conditions for a projective plane to be coordinatizable by a quasifield whose multiplicative loop satisfies, respectively, the crossed-inversive, weak-inversive, right-inversive, or left-inversive law. For instance, the condition found for the crossed-inversive case reads: There are 4 points \( U, V, W, E \), no 3 collinear, such that there exists a collineation \( \alpha \) of the \( U-V-W \) net, with \( E\alpha = E, U\alpha = V, V\alpha = W, W\alpha = U \), and for each point \( X \neq V \) on \( EV \), \( X\alpha = XU \cap EW \). (Received October 28, 1970.)

The segmentation, \( S(A) \), of a set \( A \) in \( E^n \) is defined as the collection of all pairs \((p, q)\) of points of \( A \) for which the closed line segment \([p, q]\) joining these points is contained in \( A \). Theorem 1. \( A \) is convex iff \( S(A) \) is convex. Theorem 2. If \( A \subseteq E^1 \) is Lebesgue measurable, then so is \( S(A) \). However, this result does not hold for \( A \subseteq E^n, n \geq 2 \). Theorem 3. \( A \) is open (closed) iff \( S(A) \) is open (closed). (Received November 3, 1970.)
Let \( P \) be an \( n \)-dimensional \((2 \leq n < \infty)\) projective space, \( H \) its set of hyperplanes and \( \tau_n \) a topology on \( P \). Let \( \tau_n \) neither be the discrete nor the trivial topology. The following three definitions of topological projective spaces will be compared. (P1) \( \tau_n \) is the coordinate topology, i.e. there is a topology on \( H \) such that the mapping of \( n \) independent points onto the hyperplane spanned by them, and the mapping of \( n \) independent hyperplanes onto their point of intersection are continuous. (P2) Every central projection through a point \( p \in P \) onto a hyperplane not containing \( p \) is continuous (due to H. Lenz). (P3) \( \tau_n \) induces on each plane of \( P \) coordinate topology (due to H. Karzel). Theorem 1. A desarguesian \( n \)-dimensional projective space \((P, \tau_n)\) with \( (P1) \) is a projective space over a topological skew-field \( F \). \( \tau_n \) is the quotient topology of \( F^{n+1}/F^* \), where \( F^{n+1} \) has the product topology induced by the topology of \( F \). Theorem 2. There are projective spaces with (a) \( (P2) \) but neither \( (P1) \) nor \( (P3) \), (b) \( (P3) \) but neither \( (P1) \) nor \( (P2) \), (c) \( (P2) \) and \( (P3) \) but not \( (P1) \), (d) projective spaces with \( (P1) \) have always the properties \( (P2) \) and \( (P3) \). (Received November 4, 1970.)

682-50-7. WITHDRAWN.


The purpose of the paper is to prove that there are no finite Moebius planes of exactly Hering type I,3, I,4 or I,5. A stronger result is that if \( m \), the order of the plane, is greater than 5, then the associated groups cannot exist. Let \( M \) be a Moebius plane of order \( m \) and let \( G \) be the group generated by those dilatations assumed to exist in each case. In each case, except when \( M \) is of type I,3 and \( m \equiv 3 \text{ (mod 4)} \), a combination of elementary group theory and counting arguments suffices to produce nonexistence. In the exceptional case, the group \( G \) is doubly-transitive on the pairs \((x,x')\) on the fixed circle \( C \) for which \( M \) is \((x,x')\) transitive, and the stabilizer in \( G \) of one pair \((a,a')\) contains a normal subgroup regular on the remaining pairs. By invoking the deep theorem of Hering, Kantor and Seitz characterizing such groups a case argument eliminates each of the 5 possibilities. (Received November 5, 1970.)


A systematic approach to handle geometric extremum problems by means of tangent properties is considered. A method is developed to discover tangential properties of geometrically defined curves. Several new limit-related geometric problems are proposed and solved. Various well-known problems are unified and generalized. Thus, Fermat's problem "minimizing sum of distances of a variable point from the vertices of a triangle", Fagnano's problem "in a triangle, to inscribe a triangle with minimum perimeter", and their generalizations are treated as unified applications of the method. Some proposed and solved problems: If \( u, v, w \) denote distances of a variable point from the vertices of a triangle, to discuss the locations minimizing \( u + 2v + 5w, u^2 + v^2 + 3w^2, u \cdot v \cdot w, uv + w; \) Fagnano's problem with sides of inscribed triangle unequally
weighted or the perimeter sum replaced by the sum of the squares. As tangential property problems, some
"quasi-ellipses" are treated; namely, a focal distance is given weight, three foci are considered, foci re-
placed by circles, sum replaced by product, focal distance replaced by its cube. (Received November 5, 1970.)

52 Convex Sets and Geometric Inequalities

682-52-1. STEVEN R. LAY, University of California, Los Angeles, California 90024. A generalized
Kirchberger theorem.

Let $A$ be a nonempty compact convex subset of Euclidean $n$-space $E^n$, and let $F$ be a $k$-dimensional sub-
space of $E^n$. Then $C = A + F = \{a + f : a \in A$ and $f \in F\}$ is called the $k$-cylinder generated by $A$ and $F$. Theorem 1.
Let $P$ and $Q$ be compact subsets of $E^n$. Suppose for a fixed integer $k$ ($2 \leq k \leq n + 1$) that each subset of $k$ or
fewer points of $P$ can be strictly separated from $Q$ by a hyperplane. Then given any $(k - 1)$-cylinder $D = (\text{conv } Q) + F_1$ there exists a $(k - 2)$-cylinder $E = (\text{conv } Q) + F_2$ such that $E \subset D$ and $P \cap D$ is contained in one of the two
connected components of $D - E$. Theorem 2. Let $P$ and $Q$ be compact subsets of $E^n$. Suppose for a fixed integer
$k$ ($1 \leq k \leq n$) that each subset of $k$ or fewer points of $P$ can be strictly separated from $Q$ by a hyperplane. Then
given any $k$-cylinder $C = (\text{conv } Q) + F$ there exists a $(k - 1)$-cylinder $D = (\text{conv } Q) + F_1$ such that $D \subset C$ and $D \cap P$
$= \emptyset$. It is noted that Kirchberger's theorem can be readily deduced from Theorem 1 when $k = n + 1$. (Received
October 9, 1970.)

682-52-2. MICHAEL GOLDBERG, 5823 Potomac Avenue N.W., Washington, D. C. 20016. Maximiz-
ing the smallest triangle made by $n$ points in a square.

Place $n$ points in a square so that the smallest triangle, whose vertices are among these points, has the
largest possible area. Bounds for this area are established. Methods for obtaining likely solutions are de-
scribed. Conjectured solutions for $n$ less than 17 are submitted. (Received October 21, 1970.)

682-52-3. G. THOMAS SALLEE, University of California, Davis, California 95616. The Steiner point
is not unique.

The Steiner point $s(K)$ of any compact, convex set $K$ is known to possess the following properties:
(1) $s$ is continuous with respect to the Hausdorff metric; (2) $s(K+s(K'))=s(K+K')$ where $+$ denotes Minkowski
addition on the right; (3) $s(\tau K) = \tau s(K)$ for any rigid motion $\tau$. It is known that any function on the space of
all compact convex sets satisfying (1), (2) and (3) must be the Steiner point. We construct a function which
satisfies only properties (2) and (3), agrees with $s(K)$ if $K$ is rotund or a polytope, yet differs from $s(K)$ in
general. (Received October 29, 1970.)

682-52-4. JOSEPH Y. ZAKS, Michigan State University, East Lansing, Michigan 48823. On a conjecture
of A. J. Hoffman. II.

We have previously countered (see Abstract 673-93, these Notices) 17(1970), 418 few cases of a
conjecture due to A. J. Hoffman (who refers it to M, Rabin; see "On covering of polyhedra by polyhedra", Proc.
Amer. Math. Soc. 23(1969), 123-126). We have recently established the following Theorem. If $P$ is a d-polytope

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Let $E_n$ denote an $n$-dimensional Euclidean space and $p$ a fixed point in $E_n$. **Definitions.** A set $S \subset E_n$ is radially subconvex relative to $p$ if for each pair of points $x \in S$, $y \in S$, there exists a convex arc $C(x,y)$ in $S$ joining $x$ and $y$ lying in the convex hull of the set $\{x,y,p\}$. A set $S \subset E_n$ is weakly locally radially subconvex relative to $p$ if for each point $z \in S$, there exists a neighborhood $N$ of $z$ such that for each pair of points $x \in S \cap N$, $y \in S \cap N$, there exists a convex arc $C(x,y)$ in $S$ joining $x$ and $y$ lying in the convex hull of the set $\{x,y,p\}$. **Theorem.** Let $S \subset E_2$ be a closed connected set and $p$ a fixed point in $E_2$. The set $S$ is radially subconvex relative to $p$ if and only if $S$ is weakly locally radially subconvex relative to $p$. A stronger result holds if the point $p$ is assumed to be in $S$. **Theorem.** Let $S \subset E_n$ be a closed connected set and $p$ a fixed point in $S$. The set $S$ is starshaped relative to $p$ if and only if it is weakly locally radially subconvex relative to $p$. (Received November 4, 1970.)

**53 Differential Geometry**

682-53-1. VILNIS OZOLS, University of Washington, Seattle, Washington 98105. **Critical points of the length of a Killing vector field.**

Let $M$ be a complete compact Riemannian manifold, $X$ a Killing vector field on $M$, and $\phi_t$ its 1-parameter group of isometries of $M$. Denote by $\text{Crit}(|X|^2)$ (resp. $\text{Crit}(\phi_t)$) the critical point set of the function $|X|^2$ (resp. $\delta_t^2$, where $\delta_t(p)$ is the distance from $p$ to $\phi_t(p)$). It is shown that there is a number $a > 0$ such that $\text{Crit}(|X|^2) = \text{Crit}(\phi_t)$ for every $0 < |t| < a$. The proof makes use of a slight generalization of the period bounding lemma of ordinary differential equations. (Received October 22, 1970.)


Let $\text{Pic}(M,G)$ be the set of holomorphic equivalence classes of holomorphic principal fibre bundles which admit a $C^\infty$ section where $G$ is a complex Lie group and $M$ a complex manifold. Let $\mathfrak{g} = \text{Lie algebra of } G$. The author defines a set $\text{Exp} D(M,G)$ which is a quotient of the set of $\mathfrak{g}$-valued $(0,1)$ forms on $M$ and a map $\Xi : \text{Exp} D(M,G) \rightarrow \text{Pic}(M,G)$. **Theorem 1.** $\Xi$ is onto. **Theorem 2.** $\Xi$ is a (set) isomorphism if $G$ has the "exponential lift property" with respect to $M$. **Theorem 2** applies any time that the exponential map is a covering map and $\pi_1(M)$ is torsion. **Theorem 3.** If $(\mathfrak{g}, G) \neq \mathfrak{g}$ then $\text{Exp} D(M,G) = 0$ implies $D_{0,1}(M,C) = 0$. **Theorem 4.** If $G$ is nilpotent then $\text{Exp} D(M,G) = 0$ if and only if $D_{0,1}(M,C) = 0$. Applications to holomorphic connections will also be made. (Received October 22, 1970.)
Isoperimetric inequalities for manifolds with boundary.

Let $M$ be a compact orientable $n$-dimensional $C^\infty$ Riemannian manifold with boundary, $M$ embedded in $\mathbb{R}^{n+p}$. $X$ is the position vector from the origin $P$, $H$ is the mean curvature vector on $M$, while $\nu$ and $\sigma$ are $n$ and $n-1$ dimensional measures, resp. $\Delta$ is the Laplace-Beltrami operator on $C^\infty(\partial M)$, $\lambda$ is the first eigenvalue for $\Delta$, $\delta_p = \langle |X|, \Delta |X| \rangle / \langle |X|, |X| \rangle$, where $\langle \cdot, \cdot \rangle$ denotes the usual inner product on $C^\infty(\partial M)$, and $\delta = \{ \delta_p : P \in \partial M \}$.

Let $\rho$ be the radius of the $(n-1)$-sphere whose $\sigma$ measure is $\sigma(\partial M)$. Suppose $P \in \partial M$.

Results. The inequality $\sigma(\partial M) - c n v(M) + c \int_X H \, d\nu \geq 0$ holds provided $c \leq 2/\delta_p$. If equality holds for some fixed $c \leq 2/\delta$ and for each $P \in \partial M$ then $\partial M$ is an $n$-sphere with radius $1/c$. When $n=2$, $2/\lambda = 2/\delta = 1/\rho$. If $M$ is minimal then $\sigma(\partial M) - c n v(M) \geq 0$ provided $c \leq 2/\delta_p$. If $M$ is an $n$-ball and $c = 2/\delta_p$ then $c = 1/\rho$ and equality holds. If equality holds on $c \leq 2/\delta$ then $M$ is an $n$-ball and $c = 1/\rho = 2/\delta$. These results are obtained with the aid of an integral formula which also yields a general version of Minkowski's theorem: If $\sigma(\partial M) = 0$ then $\nu(M) = \int_M X \cdot H \, d\nu$. (Received October 26, 1970.)

Second order connections.

We investigate certain higher order structures on a $C^\infty$ manifold $M$, related to a second order connection on $M$. A second order connection on $M$ determines two types of geodesics on $M$ (called first and second order geodesics), and in the case of a second order connection induced from a linear (first order) connection we show that the first order geodesics are the usual geodesics (of the first order connection), but that there are second order geodesics which are not ordinary geodesics. Using second order geodesics we define families of exponential maps and associated normal coordinates of $M$. We define second order torsion, curvature and Riemann-Christoffel tensors of a second order induced linear connection, and the second order metric induced from a first order metric. In the case that the first order linear connection is Riemannian, we show that the induced second order connection is Riemannian with respect to the induced second order metric of the canonical first order metric of the first order connection. Finally, we define two invariants $K_I$ and $K_{II}$ which we call the 1st and 2nd second order curvatures of $M$. (Received October 30, 1970.)

Withdrawn.

The de Rham cohomology of subcartesian structures. Preliminary report.

A subcartesian structure $(S,\Phi)$ is a topological space $S$ with an atlas $\Phi$ whose charts map open subsets of $S$ into (not necessarily open) subsets of cartesian spaces $\mathbb{R}^n$ and such that for $\varphi, \psi \in \Phi$ the map $\varphi \circ \psi^{-1}$ has local $C^\infty$-extension to open sets. (See N. Aronszajn, Abstract 642-163, these Notices 14(1967), 111.) One can define covariant and contravariant tensor fields, and in particular, differential forms on a subcartesian structure. The differential forms form a graded anticommutative algebra $A$ over $\mathbb{R}$. Exterior differentiation $d$ in $A$ is well-defined modulo a homogeneous ideal $m$ of $A$. There is a Poincaré lemma modulo $m$. The
de Rham groups associated to \( d \) are proved to be isomorphic to the Čech cohomology groups of \( S \) with real coefficients. (Received November 4, 1970.)

682-53-7. KRISHAN LAL DUGGAL, University of Windsor, Windsor 11, Ontario, Canada. On the most general \( F \)-connections.

We define a differentiable structure, briefly \( G \)-structure, on a differentiable manifold by means of a vector valued linear function \( F \) such that \( \ddot{x} = a^2 x \) for arbitrary vector field \( x \), where \( \ddot{x} \defeq F(x) \) and \( a \) is any complex number. If \( G \)-structure is endowed with a hermite metric \( g \) such that \( g(\dddot{x}, \dddot{y}) + a^2 g(x, y) = 0 \), then we say that the manifold is equipped with a hermite structure subordinate to \( G \)-structure. A connection \( D \) in a manifold, endowed with \( G \)-structure, is said to be an \( F \)-connection if \( (D x F)(y) = 0 \). \( D \) is called half-symmetric if \( a^2 S(x, y) + S(x, y) + S(x, y) + S(x, y) = 0 \), where \( S \) is the torsion tensor of \( D \). In this paper we give various forms of the Nijenhuis tensor with respect to \( F \) and deduce some of its properties. We obtain the most general \( F \)-connection as a linear combination of an arbitrary connection and discuss the relation between their torsion tensors. Finally, we discuss the behaviour of the Nijenhuis tensor with respect to \( F \)-connection.
(Received November 4, 1970.)

682-53-8. GEORGE C. DEEBNEY, JR., Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. An invariant approach to space-time symmetries.

A study is made of Killing vector fields in nonflat vacuum Einstein spaces with a restriction primarily to those fields \( k = k^a e_a \) which have a nonnull Killing bivector \( B = K_{ab} e^a \wedge e^b \); i.e., \( K_{ab} e^a \wedge e^b \) and \( K_{ab} e^a \wedge e^b \) are not both zero. However, a theorem for null Killing bivector \((KBV)\) cases is mentioned: A vacuum Einstein space admitting a null KBV is necessarily algebraically special. Consequently, all algebraically general spaces admit a nonnull KBV if they admit a Killing vector at all. Study of integrability conditions for a nonnull KBV gives the following: Theorem. If a vacuum Einstein space admits a nonnull Killing vector field \( k \) with a nonnull KBV then \( k \) is not geodesic; i.e., trajectories of \( k \) are not translations. Further results involve a characterization of hypersurface orthogonal Killing vector fields in terms of the complex null tetrad formalism used. Invariants of the curvature tensor \( R \) and the KBV are utilized to partially classify results under this topic. (Received November 5, 1970.)

54 General Topology

682-54-1. TROY L. HICKS, University of Missouri, Rolla, Missouri 65401 and JOHN W. CARLSON, Kansas State Teachers College, Emporia, Kansas 66801. Some quasi-uniform space examples.

Let \( W \) denote the ordinals less than \( \Omega \), the first uncountable ordinal. \( t \) will denote the order topology for \( W \). It is well known that \((W,t)\) is normal, countably compact, not metrizable and not real compact. Dieudonné [C. R. Acad. Sci. Paris 209(1939), 145-147] showed that \((W,t)\) admits a unique compatible uniform structure \( \tau_t \) and \((W,\tau_t)\) is not complete. The main result of this paper is that \((W,t)\) admits a strongly complete [See Abstract 70T-G119, these Notices 17(1970), 691] quasi-uniform structure. Consequences of this result will also be noted. (Received September 28, 1970.)
All spaces under consideration are Hausdorff spaces. Let $\mathbf{X}$ be a topological space and $x$ a net in $\mathbf{X}$ ($\mathfrak{F}$ a filter in $\mathbf{X}$). A point of $x \in \mathbf{X}$ is a weak cluster point of $x$ (of $\mathfrak{F}$) iff the net $x$ is frequently in every closed neighborhood $\overline{V}$ of $x$ ($\overline{V} \cap F \neq \emptyset$, $F \in \mathfrak{F}$). Let $x, A$ be a point and a subset of $\mathbf{X}$ respectively. $x$ is a weak cluster point of $A$ if for every closed neighborhood $\overline{V}$ of $x$, $\overline{V} \cap A \neq \emptyset$. $A$ is strongly closed iff $A$ contains all of its weak cluster points. Theorem 1. $\mathbf{X}$ is H-closed iff every net $x$ (filter) in $\mathbf{X}$ has a weak cluster point. Theorem 2. Let $\mathbf{X}$ be a separated Hausdorff space, $\mathbf{X}$ is hereditary H-closed iff $\mathbf{X}$ is compact. Theorem 3. Let $\mathbf{X}$ be first countable. $\mathbf{X}$ is hereditary H-closed iff $\mathbf{X}$ is compact. Theorem 4. $\mathbf{X}$ is compact iff $\mathbf{X}$ is strongly closed in every superspace of $\mathbf{X}$. Theorem 5. $\mathbf{X}$ is minimal iff every net $x$ (filter) in $\mathbf{X}$ has at most one weak cluster point is convergent. Corollary 1. The cartesian product of a family $\mathbf{X}_\alpha$ of topological spaces is minimal iff each $\mathbf{X}_\alpha$ is minimal.

(Received September 28, 1970.)

Let $\mathfrak{P}$ be a countably productive normal base on a Tychonoff space $\mathbf{X}$, and let $\eta(\mathfrak{P})$ be the corresponding $\mathfrak{P}$-realcompactification of $\mathbf{X}$ [R. Alo and H. Shapiro, "$\mathfrak{P}$-realcompactifications," J. Austral. Math. Soc. 9 (1969), 489-495]. A real-valued function $f$ on $\mathbf{X}$ is countably $\mathfrak{P}$-uniformly continuous if corresponding to every positive epsilon there exists a finite or denumerable open cover of $\mathbf{X}$ by $\mathfrak{P}$-complements, on each of which the oscillation of $f$ is less than epsilon. Theorem. Every countably $\mathfrak{P}$-uniformly continuous function on $\mathbf{X}$ can be continuously extended to a real-valued function $f$ on $\eta(\mathfrak{P})$. By observing that a real-valued function on a topological space is continuous if and only if it is countably zero-set-uniformly continuous, we have the following well-known result as a Corollary. Every continuous, real-valued function on a Tychonoff space $\mathbf{X}$ can be continuously extended to a real-valued function on $\eta(\mathfrak{P})$, the Hewitt realcompactification of $\mathbf{X}$. We conclude by showing that not every realcompactification of a Tychonoff space is obtainable as a space $\eta(\mathfrak{P})$. This answers in the negative a question posed in the paper cited above. (Received October 5, 1970.)

Concerning completable Moore spaces. Preliminary report.

J. N. Younglove (Fund. Math. 48 (1959)) showed that each development for a complete Moore space satisfies Axiom C at each point of a dense subset. B. Fitzpatrick (Proc. Amer. Math. Soc. 16 (1965)) showed that each completable Moore space has a dense metrizable subset. Recently (Proc. Emory Top. Conf. (1970)) the author has given an example of a Moore space which has a dense metrizable subset but for which there exists no development satisfying Axiom C at each point of a dense subset. Thus the following theorem generalizes the above results. Theorem 1. Each completable Moore space has a development which satisfies Axiom C at each point of a dense subset. Corollary 2. Each completable Moore space in which there does not exist a discrete uncountable collection of mutually exclusive open sets is separable. James W. Ott, in a paper to appear, has shown
that, under the assumption of the continuum hypothesis, there exists a separable, noncompletable Moore space. Using the example mentioned and Theorem 1 the author is able to obtain the same result without the continuum hypothesis. (Received October 6, 1970.)


Symmetrizable spaces, as defined by Arhangel'skii [Russian Math. Surveys 21 (1966), 125], have recently generated a great deal of interest among those who are interested in generalized metric spaces. It is shown by an example that there exists a symmetrizable space having a closed subset which is not a $\mathcal{G}_\delta$. The existence of such an example provides an answer to a question of E. Michael as mentioned by S. Nedev [Soviet Math. Dokl. 8 (1967), 890]. This example is clearly also an example of a symmetrizable space that is not semistratifiable. (Received October 8, 1970.)

682-54-6. R. LUKE and WILLIAM K. MASON, Rutgers University, New Brunswick, New Jersey 08903. The space of homeomorphisms of a compact 2-manifold onto itself is an absolute neighborhood retract. Preliminary report.

Let $M$ denote a compact 2-manifold with boundary (the boundary may be empty). Let $H(M)$ denote the space, with the sup norm metric, of all homeomorphisms of $M$ onto itself which are fixed on the boundary of $M$. Theorem. $H(M)$ is an absolute neighborhood retract. (Received October 9, 1970.)

682-54-7. TROY L. HICKS, University of Missouri, Rolla, Missouri 65401 and JOHN W. CARLSON, Kansas State Teachers College, Emporia, Kansas 66801. Some properties of quasi-uniform structures.

Let $(X, \mathcal{U})$ be a quasi-uniform space. We say that a quasi-uniform space $(X, \mathcal{U})$ has property P if each $U \in \mathcal{U}$ is a neighborhood of the diagonal of $X \times X$ with respect to the product topology. We then show that if $(X, \mathcal{U})$ has property P then $\mathcal{U}$ is uniformizable. Also, if $(X, \mathcal{U})$ has property P, $Y \subset X$, and $(Y, \mathcal{U}_Y)$ is strongly complete, then $Y$ is closed. We provide an example to show that this is not the case in general. A necessary and sufficient condition for $\mathcal{U} \cap \mathcal{U}^{-1}$ to be a quasi-uniform structure is given. We define a class of separation axioms which parallel the usual separation axioms but are dependent upon the particular quasi-uniform structure rather than the generated topology. (Received October 12, 1970.)


The following question has been raised by S. L. Gulden, W. M. Fleischman, and J. H. Weston (Proc. Amer. Math. Soc. 24 (1970), 197–203): Do there exist infinite cardinal numbers $m$ (in addition to the case $m = \aleph_0$) for which some topological space $X_m$ is $m$-compact (each open cover of $X_m$ by $m$ open sets admits a finite subcover) but not $m$-bounded (a space $X$ is $m$-bounded provided each subset $S$ of $X_m$ with $\text{card } S \leq m$ lies in a compact subset of $X_m$). We show that for each infinite cardinal number $m$ such a space $X_m$ exists. Indeed, $X_m$ may be
taken to be a topological group; alternatively, one may arrange \( D \subseteq X_m \subseteq \beta D \), where \( D \) is discrete and \( \text{card } D = m \). We show also that if \( m \) is a regular cardinal (\( m \) is not expressible as the sum of less than \( m \) cardinals, each of which is less than \( m \)), then \( X_m \) can be constructed not strongly \( m \)-compact (\( X \) is strongly \( m \)-compact provided for every filter base \( \mathcal{F} \) on \( X \) with card \( \mathcal{F} \leq m \), there is a compact subset \( K \) of \( X \) such that \( \mathcal{F} \upharpoonright K \) is a filter base). (Received October 9, 1970.)


A topological space is said to be Urysohn if every pair of distinct points have disjoint closed neighborhoods, and a Urysohn space \((X, \mathcal{F})\) is called minimal Urysohn if there exists no Urysohn topology on \( X \) properly contained in \( \mathcal{F} \). In Pacific J. Math. 27 (1968), 611-617, C. T. Scarborough asks if the property minimal Urysohn is productive. **Theorem 1.** Let \( X \) be the minimal Urysohn space constructed by H. Herrlich in Math. Z. 88 (1965), 285-294. Then there exists a minimal Urysohn space \( S \) such that \( X \times S \) is not minimal Urysohn. **Theorem 2.** There exists a first countable minimal Urysohn space that is neither feebly compact nor of second category. **Theorem 2** answers a question raised by the author in Trans. Amer. Math. Soc. 138 (1969), 115-127. (Received October 15, 1970.)

682-54-10. RICHARD A. ALO, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213, HARVEY L. SHAPIRO, West Georgia College, Carrollton, Georgia 30117 and FRANK A. SMITH, Kent State University, Kent, Ohio 44240. Disconnected spaces and strong normality conditions.

A pseudo-ultrametric is a pseudometric that satisfies the additional condition \( d(x, y) \leq \max\{d(x, z), d(x, y)\} \). A subspace \( S \) of a topological space \( X \) is said to be almost ul-embedded in \( X \) (resp. ul-embedded in \( X \)) if every continuous pseudo-ultrametric on \( S \) can be extended to a continuous pseudometric (resp. continuous pseudo-ultrametric) on \( X \). The obvious generalizations to infinite cardinal numbers is also studied. The following results are shown: **Theorem 1.** A subspace \( S \) of a topological space \( X \) is ul-embedded in \( X \) if and only if every clopen partition of \( S \) can be extended to a clopen partition of \( X \). **Theorem 2.** A topological space \( X \) is ultra-collectionwise normal if and only if every closed subset of \( X \) is ul-embedded in \( X \). **Theorem 3.** A topological space \( X \) is collectionwise normal if and only if every closed subset is ul-embedded in \( X \). **Theorem 4.** A space \( X \) is ultranormal if and only if every closed subset of \( X \) is ul\(^{\mathbb{N}_0}\)-embedded in \( X \) if every closed subset is almost ul-embedded in \( X \). **Theorem 5.** If a pseudometric space is ultranormal then it is pseudo-ultrametrizable. **Theorem 6.** A normal \( T_1 \) space is ultranormal if and only if its Stone-Cech compactification is totally disconnected. (Received October 19, 1970.)

682-54-11. RICHARD E. HODEL, Duke University, Durham, North Carolina 27706. A generalization of semistratifiable and \( w^\Delta \)-space.

A topological space \( X \) is a \( \beta \)-space if there is a function \( g \) from \( \mathbb{N} \times X \) into the topology of \( X \) such that (1) for all \( x \) in \( X \) and \( n \) in \( \mathbb{N} \), \( x \in g(n, x) \); (2) if \( x \in g(n, x_n) \) for \( n = 1, 2, \ldots \) then the sequence \( (x_n) \) has a cluster point. Clearly every semistratifiable and every \( w^\Delta \)-space is a \( \beta \)-space. **Theorem.** The following are
equivalent for a regular space $X$: (a) $X$ is semistratifiable. (b) $X$ is a $\beta$-space with a $G^*_\delta$-diagonal. (c) $X$ is an $\alpha$-space and a $\beta$-space. (For definitions of an $\alpha$-space and the $G^*_\delta$-diagonal property, see Abstract 70T-G141, these Notices 17 (1970), 845.) (Received October 19, 1970.)

682-54-12. STANLEY J. WERTHEIMER, Georgia Institute of Technology, Atlanta, Georgia 30332. Quasi-compactness and decompositions for arbitrary relations.

Every semi-single-valued (ssv) surjective relation on a topological space $X$ to a topological space $Y$ defines a decomposition of $X$ into point inverses and a decomposition of $Y$ into point images. G. T. Whyburn has analyzed $T$ in terms of these decomposition spaces and their natural mappings, and defined quasi-compactness for ssv relations. In this paper Whyburn's analysis is extended to include all relations, quasi-compactness is defined for all relations so as to be consistent with the definition for ssv relations and Whyburn's analysis, and some results of Whyburn and E. Duda which are apparently known only for ssv relations or continuous functions are extended. For example: Theorem. If $T$ on $X$ to $Y$ is a surjection so that $T|S$ is quasi-compact for some cross-section $S$ for $T$, then $T$ is quasi-compact. Theorem. Let $T$ on $X$ to $Y$ be a doubly quasi-compact surjection with connected point images. If $X$ is locally $\Delta$-connected then $Y$ is locally $\sigma$-connected. Theorem. If $X$ is a Hausdorff $k$-space and $T$ on $X$ to $Y$ is a quasi-compact reflexive compact surjection of finite order with $T^{-1}$ semiclosed, then $T$ is compact. (Received October 19, 1970.)


If $(X, \rho)$ is a nontrivial metric space and $x$ and $y$ are distinct points of $X$, then \( \{ q \in X : \rho(x, q) = \rho(y, q) \} \) will be called the midset of $x$ and $y$ in $X$. If $X$ is a nontrivial metric space for which each midset consists of a unique point, then we will say that $X$ has the unique midpoint property (UMP). Theorem. If $X$ is a connected metric space with UMP, then $X$ is an interval and; (i) if $X$ has two distinct noncut points, $X$ is a closed interval; (ii) if $X$ has exactly one noncut point, $X$ is a half open interval; (iii) if $X$ has no noncut points, $X$ is an open interval. (Received October 5, 1970.)

682-54-14. DOUGLAS HARRIS, Marquette University, Milwaukee, Wisconsin 53233. Wallman compactification as a functor.

All spaces considered are $T_1$. A map $f:X \to Y$ is a wo-map if when $\nu$ is a finite open cover of $Y$ then there is a finite open cover $\omega$ of $X$ such that for each $W \in \omega$ there is $V \in \nu$ with $\text{cl} A \subset V$ whenever $A \subset X$ and $A \subset W$. Clearly identity maps and compositions of wo-maps are wo-maps, thus there is a category $\mathcal{W}$ of spaces and wo-maps, with subcategory $\mathcal{W}_c$ of compact spaces and wo-maps. Theorem. The category $\mathcal{W}_c$ is an epireflective subcategory of $\mathcal{W}$ with the Wallman compactification of a space as its reflection. Specifically for each space $X$ the inclusion $w_X$ of $X$ into its Wallman compactification is a wo-map, and if $f:X \to Y$ is a wo-map there is a unique wo-map $g:w_X \to w_Y$ with $w_X = w_Y f$. A proper subcategory $\mathcal{W}_c$ of $\mathcal{W}_c$ is also considered, being the class of maps $f:X \to Y$ having a closed extension $g:w_X \to w_Y$. The class $\mathcal{W}$ of maps having (not necessarily unique) extensions from $w_X$ to $w_Y$ is considered, and it is shown that the Wallman compactification is also a universal object here, in a suitably modified sense. (Received October 23, 1970.)
682-54-15. DAVID EDWIN COOK, University of Mississippi, University, Mississippi 38677. Completely pseudonormal and Moore spaces.

In Abstract 660-13, these Notice 15(1968), 1011, C. W. Proctor defines a space $S$ to be pseudonormal if and only if for each pair of mutually exclusive closed point sets $H$ and $K$ such that $H$ is countable, there exist mutually exclusive open sets $D$ and $E$ containing $H$ and $K$ respectively. **Theorem 1.** If $S$ is a pseudonormal Moore space such that (1) $H$ and $K$ are mutually separated point sets and (2) $|H|$ is countable, then there exist domains $D$ and $E$ containing $H$ and $K$ respectively such that either $D$ does not intersect $E$ or $D \cap E$ is $H \cap K$.

If condition (2) is replaced by "every infinite subset of $H$ has a limit point in $S$" the resulting proposition follows from Theorem 2 below and a theorem in Abstract 645-7, these Notice 14(1967), 382. However, if condition (2) is replaced by "$H+K$ is countable" the resulting proposition is not a theorem. See (R. L. Moore, "Foundations of point set theory", rev. ed., Amer. Math. Soc. Colloq. Publ. Vol. 13., Amer. Math. Soc., Providence, R. I., 1962) for definition of property D. **Theorem 2.** Every pseudonormal Moore space has property D.

(Received October 23, 1970.)

682-54-16. JACK D. WILSON, University of Mississippi, University, Mississippi 38677. Cyclicly descriptive Moore spaces.

A space is said to be cyclicly descriptive if there is a point $\Omega$, and a collection $G$ of simple closed curves such that (1) each element of $G$ contains $\Omega$, (2) no two elements of $G$ have three points in common, and (3) if $P$ and $Q$ are two points distinct from $\Omega$, then some element of $G$ contains them. Let $\Sigma$ be a space that is cyclicly descriptive and satisfies Axioms 0–4 of R. L. Moore's "Foundations of point set theory". Examples are given of both metrizable and nonmetrizable such spaces which are not topologically equivalent to a subset of a plane or a sphere. For each point $P$, distinct from $\Omega$, let $G_{P}$ denote the collection of all elements of $G$ which contain $P$. **Lemma.** Suppose for each nonnegative integer $i$, $J_{i}$ belongs to $G_{P}$, $X_{i}$ belongs to $J_{i}$, and $\alpha_{i}$ is that arc of $J_{i}$ having $\Omega$ and $P$ as endpoints and containing $X_{i}$. Then if $X_{0}$ is the sequential limit point of the sequence $X_{1}, X_{2}, \ldots$, $\alpha_{0}$ is the sequential limiting set of the sequence $\alpha_{1}, \alpha_{2}, \ldots$.

**Theorem.** If $\Sigma$ is locally peripherally compact, then $\Sigma$ is topologically equivalent to a sphere.

(Received October 23, 1970.)


If $(X, t)$ is a topological space and $A \subseteq X$, we define the semiclosure of $A$ by $\tilde{A} = \{ \tilde{a} : a \in A \}$. It is not hard to verify that semiclosure is a closure operator, and that one always has $\tilde{\tilde{A}} = \tilde{A}$ if, and only if $(X, t)$ is a $T_{1}$-space. **Theorem.** Let $A$ be a compact subset of the $R_{1}$-space $(X, t)$. Then, $\tilde{A}$ is closed. This extends the well-known result concerning compact subsets of $T_{1}$-spaces. **Theorem.** Let $(X, t)$ be a compact $R_{1}$-space, and let $A \subseteq X$. Then, $A$ is compact if, and only if $\tilde{\tilde{A}} = \tilde{A}$. (Received October 15, 1970.)
The liberation of the Q-gaps. Preliminary report.

In the paper "Completeness for all" [Proc. Washington State University Topology Conference, 1970] the author proposed the question, "what are the relationships between paracompactness (or metacompactness) and completeness?" The answer to this question for the class of linearly ordered topological spaces and their finite products is presented here. However, the answer to the same question for spaces with Wallman compactifications is considerably more difficult. The need thereby arose for a "liberated" definition of the Q-gap i.e., a definition free of the ordering structure so necessary to the topology of linearly ordered topological spaces. The author offers in this paper just such a definition some justification for its use, and a few results not accidental to the answering of the above question. (Received October 23, 1970.)

Linearly stratifiable spaces.

Let \( \alpha \) be an initial ordinal number and \( X \) a \( T_1 \)-space with topology \( \mathscr{J} \). \( X \) is called stratifiable over \( \alpha \) (or simply linearly stratifiable) if there is a map \( S: \alpha \times \mathscr{J} \rightarrow \mathscr{J} \) satisfying (where we write \( U_\beta \) instead of \( S(\beta, U) \)) for all \( U, W \in \mathscr{J} \) and all \( \beta < \gamma < \alpha \). (1) \( \bigcup_\beta U_\beta \subseteq U \), (2) \( \bigcup_\beta [U_\beta : \beta < \alpha] = U \), (3) \( U \subseteq W \) implies \( U_\beta \subseteq W_\beta \), (4) \( U_\gamma \subseteq U_\alpha \). In case \( \alpha = \omega \) (the first infinite ordinal) the above definition coincides with a popular definition of stratifiable space. Theorem. Linearly stratifiable spaces are hereditarily paracompact, preserved by closed continuous surjections, and preserved by locally finite closed unions. The finite product theorem holds for spaces stratifiable over the same \( \alpha \). Some closely related spaces are also considered. (Received October 26, 1970.)


A collection \( \mathscr{G} \) of subsets of a space \( X \) is a k-network for \( X \) if, whenever \( K \subseteq U \), with \( K \) compact and \( U \) open in \( X \), there exist finitely many elements of \( \mathscr{G} \) whose union contains \( K \) and is contained in \( U \). A collection \( \mathscr{G} \) of subsets of a space \( X \) is a cs-network for \( X \) if, whenever \( Z \subseteq U \), with \( Z \) a sequence converging to \( z \) and \( U \) open in \( X \), there exists an element of \( \mathscr{G} \) which contains \( z \) and all but finitely many other elements of \( Z \) and is contained in \( U \). E. Michael has called a regular space with a countable k-network an \( \mathcal{N}_0 \)-space. The author has previously characterized \( \mathcal{N}_0 \)-spaces as regular spaces with a countable cs-network. A regular space with a \( \sigma \)-locally finite cs-network is a cs-\( \sigma \)-space. Let \( C(X, Y) \) represent the mapping space from \( X \) to \( Y \) with the compact-open topology. Theorem. If \( X \) is an \( \mathcal{N}_0 \)-space and \( Y \) is a cs-\( \sigma \)-space, then \( C(X, Y) \) is a cs-\( \sigma \)-space. P. O'Meara has made the following Definition. A regular space with a \( \sigma \)-locally finite k-network is an \( \mathcal{N} \)-space. Theorem. Every cs-\( \sigma \)-space is an \( \mathcal{N} \)-space. Theorem. A paracompact space \( X \) is a cs-\( \sigma \)-space iff \( X \) is an \( \mathcal{N} \)-space. (Received October 26, 1970.)
Counterexamples to "extensions" of Lusin's separation principles. Preliminary report.

For a family \( \Omega \) of subsets of a set \( P \), let \( b\Omega \) be the smallest family of subsets which contains \( \Omega \) and is closed under countable unions and countable intersections. **Example 1.** There exists a Hausdorff topological space \( T \) containing a disjoint pair \( A, B \) of \( \mathcal{J}\)-Souslin sets such that \( A \cup B = T \) but \( A \not\subseteq b\mathcal{J} \). (\( \mathcal{J} \) denotes the family of open sets.) Thus the "de-topologised" version of Lusin's first separation principle appearing in "Compositions of set operations" (Canad. J. Math. 22(1970), 227-234) by D. W. Bressler and A. H. Cayford, is false. **Example 2.** There exists a compact Hausdorff space containing a pair \( A, B \) of analytic sets such that no disjoint pair \( A', B' \) of sets exist which are complements of analytic sets and have the properties \( A \sim B \subseteq A' \) and \( B \sim A \subseteq B' \). (Received October 26, 1970.)

Cauchy sequences in semimetric spaces. Preliminary report.

There are numerous examples of semimetric spaces \((X, d)\) in which there exist convergent sequences with no Cauchy subsequence. **Main Theorem.** Every semimetric space \((X, d)\) is semimetrizable with a compatible semimetric \( \rho \) where every convergent sequence in \( X \) has a subsequence which is Cauchy with respect to \( \rho \). **Corollary.** A \( T_1 \) space \( X \) is semimetrizable if and only if it is a pseudo open \( \Pi \)-image of a metric space. (Received October 28, 1970.)

Transfinite Cauchy sequences characterize complete uniform spaces.

The concepts of big coverings, heavy coverings, and arbitrarily large open sets are defined for uniform spaces. **Proposition.** A uniform space is paracompact if and only if each open covering that admits arbitrarily large open sets has a finite subcovering. **Proposition.** A uniform space is complete if and only if each heavy open covering has a finite subcovering. **Proposition.** A uniform space is preparacompact if and only if each open covering that admits arbitrarily large open sets is a subcovering of a big heavy open covering. As an application of the above we prove the following **Theorem.** A uniform space is complete if and only if each transfinite Cauchy sequence converges. **Theorem.** A Tychonoff space has a Lindeöf realcompactification if and only if it is preparacompact with respect to the countable normal coverings. Additional results about transfinite sequences have been announced previously. Of particular interest may be Abstract 654-33, these Notices 15(1968), 346 and p. 67, Proc. Washington State Topology Conference, 1970. (Received October 26, 1970.)

A recent solution of Dowker's problem by M. E. Rudin and the relationship between the normal linearly Lindelöf problem and Dowker's problem appear to make a few remarks on the normal linearly Lindelöf problem.
appropriate. It has previously been shown by the authors that if there exists a normal Hausdorff linearly Lindelöf space that is not Lindelöf then it is also a Dowker space. If $X$ is a space let $\chi(X)$ be the least infinite cardinal such that there is an open cover of cardinality $\chi(X)$ having no subcovering of smaller infinite cardinal. Theorem. A linearly Lindelöf space is Lindelöf iff its Lindelöf number $\chi(X)$ is a regular cardinal. Corollary. If there is a normal Hausdorff space that is linearly Lindelöf but not Lindelöf then its Lindelöf number must be a singular cardinal. Moreover since $\chi(X) \leq |X|$ such a space would necessarily have cardinality $2^\aleph_0$. The Dowker space exhibited by M. E. Rudin is of singular cardinality but it fails to be linearly Lindelöf. This problem is also related to an old problem of Alexandrov which asks for a space that is finally compact in the sense of complete accumulation points but is not Lindelöf. Mischenko exhibited such a space and M. E. Rudin showed that the space was not normal so the problem remains unsolved for the normal case. Theorem. Linearly Lindelöf $\Rightarrow$ finally compact in a sense of compl. acc. points. (Received October 26, 1970.)

682-54-25. LUDVIK JANOS, University of Florida, Gainesville, Florida 32601. On continuous change of fixed points.

We generalize to the nonmetrizable case some of our previous results. Let $X$ be a compact Hausdorff space and $X^X$ the set of all continuous selfmappings $f : X \to X$, endowed with the compact-open topology. Let $A \subset X^X$ be the set of all selfmappings $f$ having a unique fixed point $F(f) \in X$. Theorem. The mapping $F : A \to X$ is continuous. Corollary. Denoting by $B \subset X^X$ the set of all maps $f \in X^X$ such that $\bigcap_1^{\aleph_0} F(f)$ is a singleton and defining $F : B \to X$ by $F(f) \in F(f)$ we conclude that $F$ is continuous. (Received October 23, 1970.)

682-54-26. EUGENE S. BALL, Tennessee Technological University, Cookeville, Tennessee 38501. $\alpha$-weak normality and normality. Preliminary report.

Definition. A topological space $S$ is said to be $\alpha$-weakly normal, $\alpha$ an infinite cardinal, provided that if $\{H, \alpha \in A \} \subset A$ is a collection of closed sets well ordered by $A$ with no common part and $H$ is a closed set disjoint from $H, \alpha \in A$, then there are an open set $D$ and $C \subset A$ such that $H, C \subset D$ and $c1(D)$ is disjoint from $H$. If $\alpha = \aleph_0$ then $S$ is said to be weakly normal, defined in Abstract 669-8, these Notices 16(1969), 1045. Definition. A topological space $S$ is said to be $\alpha$-pseudonormal, $\alpha$ an infinite cardinal, provided that every two disjoint closed sets one of which has cardinality $< \alpha$ are contained in disjoint open sets. If $\alpha = \aleph_0$, then $S$ is pseudonormal defined by Proctor in Abstract 660-13, these Notices 15(1968), 1011. Theorem. If a topological space $S$ is regular and $\alpha$-weakly normal then $S$ is $\alpha$-pseudonormal. Corollary. If a topological space $S$ is regular and $\alpha$-weakly normal for each infinite cardinal $\alpha$, then $S$ is normal. Theorem. If $S$ is $\alpha$-weakly normal for each infinite cardinal $\alpha$ and metacompact, then $S$ has property B defined by Zenor in Proc. Amer. Math. Soc. 24(1970), 258-262. (Received October 29, 1970.)


This paper presents a survey of articles which have appeared since 1936 when Garrett Birkhoff first introduced the concept of the lattice of topologies. (Received October 29, 1970.)
Compact spaces homeomorphic to a ray of ordinals.

Denote by $\Gamma(\xi)$ the set of ordinals not exceeding $\xi$ with the interval topology. For an ordinal $\xi$, let $X(\xi)$ denote the derived set of order $\xi$. If $\lambda$ is the least ordinal $\xi$ such that $X(\xi)$ is finite and $n = \text{Card} \ X(\xi)$, then $(\lambda, n)$ is called the characteristic of $X$. A space is dispersed if it contains no perfect subsets. Definition. We say a topological space $X$ has property (D) if each $x$ in $X$ has a neighborhood base consisting of a decreasing, possibly transfinite, sequence $\{U_\alpha\}$ of closed and open sets with the additional property that for each limit ordinal $\gamma$ with $\alpha < \gamma$, $\bigcap_{\alpha < \beta} U_\beta$ contains at most one point. Theorem 1. Let $X$ be a compact, dispersed, Hausdorff space with characteristic $(\lambda, n)$. If $X$ has property (D), then $X$ is homeomorphic to $\Gamma(\omega^\lambda, n)$.

Definition. Let $\text{Ker}(X)$ denote the largest perfect subset of a topological space $X$. If $X$ has no nonempty, perfect subset, let $\text{Ker}(X) = \emptyset$. Theorem 2. Suppose $X$ is a first-countable, compact space and $\text{Ker}(X)$ is a $G_\delta$-set. Then $X$ is the union of two disjoint sets $P$ and $G$ where $P$ is perfect and $G$ is metrizable and countable.

Theorem 1 generalizes a well-known theorem by Mazurkiewicz and Sierpinski. Theorem 2 is a generalization of the Cantor-Bendixson Theorem restricted to compact spaces. (Received October 28, 1970.)

Partitions of the Katetov extension. Preliminary report.

Let $Y$ be an H-closed extension of a space $X$; there is a unique continuous surjection $f: X \to Y$ such that $f(x) = x$ for $x \in X$. The partition $\{f^{-1}(y) \mid y \in Y \setminus X\}$ of $X \setminus X$ is said to be induced by the H-closed extension $Y$. Theorem. Let $P$ be a partition of $X \setminus X$ (elements of $X \setminus X$ are open ultrafilters on $X$ with void adherence). $P$ is induced by some H-closed extension of $X$ iff for each $A \in P$, $\cap \{U \mid \forall U \in A\}$ has void adherence and $\forall y \in A$ whenever $y \in X \setminus X$ and $y \in \cap \{U \mid \forall U \in A\}$. A function $f: X \to Y$ is $\sigma$-continuous if for each $x \in X$ and neighborhood $N$ of $f(x)$, there is a neighborhood $M$ of $x$ such that $f(M) \subseteq f(N)$. Theorem. Two H-closed extensions $Y$ and $Z$ of a Hausdorff space $X$ induce the same partition of $X \setminus X$ iff they are $\sigma$-isomorphic (i.e., there is a bijection $f: Y \to Z$ such that $f(x) = x$ for $x \in X$ and both $f$ and $f^{-1}$ are $\sigma$-continuous). For each set $B$, let $\varphi(B)$ denote the set of all subsets of $B$.

Theorem. Let $X$ denote the set of isomorphism classes of H-closed extensions of an infinite discrete space $X$. Then $\text{card}(X) = \text{card}(\varphi(\varphi(X)))$. Theorem. $X$ is isomorphic to $\sigma\varphi X$ (the Fomin extension of $X$) iff $X$ has a finite proximate cover of open sets whose closures are almost H-closed.

Corollary. If $X$ is isomorphic to $\sigma\varphi X$, then $X$ is locally H-closed. (Received October 30, 1970.)

Characterization of 2-polyhedra using cone neighborhoods.

A space $M$ has the cone-neighborhood property if every point $p \in M$ is contained in an open set $U$ such that the closure of $U$ is a cone over the boundary of $U$ with vertex at $p$. A 1-dimensional compact metric space with the cone-neighborhood property is shown to be a linear graph. For a cone neighborhood $U$ of a point $p$ in $M$ where $M$ is a 2-dimensional compact metric space with the cone-neighborhood property, it is shown that the boundary of $U$ is locally a dendron, that each point in the boundary of $U$ has finite Menger Order relative the boundary of $U$, and that the boundary of $U$ contains only finitely many points with Menger Order greater than
or equal to 3 relative the boundary of U. Using a theorem of Kosinski (Fund. Math. 47(1959), 13, Theorem 3) it is shown that a 2-dimensional compact metric space is a 2-polyhedra if and only if it has the cone-neighborhood property. (Received October 23, 1970.)

682-54-31. DONALD F. REYNOLDS, West Virginia University, Morgantown, West Virginia 26506.

Preservation of topological properties under extensions of topologies. Preliminary report.

Let \((X, \tau)\) be a Hausdorff topological space. An extension of the topology \(\tau\) to a collection of subsets \(\sigma = \{A_\alpha \mid \alpha \in \Gamma\}\) is the topology generated by \(\tau \cup \sigma\) and is denoted by \(\tau(\sigma)\). An extension of \(\tau\) to a single set \(A \subseteq X\) is called a simple extension and is denoted by \(\tau(A)\). Theorem. Let \((X, \tau)\) be a regular (completely regular) space and let \(\sigma = \{A_\alpha \mid \alpha \in \Gamma\}\) be a collection of subsets of \(X\) such that \((X, \tau(A_\alpha))\) is a regular (completely regular) space for each \(\alpha \in \Gamma\). Then \((X, \tau(\sigma))\) is a regular (completely regular) space. Theorem. Let \((X, \tau)\) be a paracompact space and let \(\sigma = \{A_\alpha \mid \alpha \in \Gamma\}\) be a locally finite collection of subsets of \(X\) such that \((X, \tau(A_\alpha))\) is a paracompact space for each \(\alpha \in \Gamma\). Then \((X, \tau(\sigma))\) is a paracompact space. Theorem. Let \((X, \tau)\) be a metrizable space and let \(\sigma = \{A_\alpha \mid \alpha \in \Gamma\}\) be a locally finite collection of a \(\tau\)-closed subset of \(X\). Then \((X, \tau(\sigma))\) is a metrizable space. Theorem. Let \((X, \tau)\) be a normal space and let \(\sigma = \{A_\alpha \mid \alpha \in \Gamma\}\) be a \((\text{J/F})\)-point-finite \(\tau\)-closed cover of \(X\). Then \((X, \tau(\sigma))\) is a normal space. (Received October 30, 1970.)

682-54-32. JACOB H. GERLACH, Wisconsin State University, Whitewater, Wisconsin 53190.

Decompositions of \(E^3\) that give \(E^3\). Preliminary report.

Lemma. If \(G\) is a toroidal decomposition of \(E^3\), \(E^3/G\) is not homeomorphic to \(E^3\), and \(H\) is the set of nondegenerate elements in \(G\), then there is a toroidal decomposition \(G'\) of \(E^3\) such that \(H'\) (the set of nondegenerate elements in \(G'\)) is contained in \(H\), \(E^3/G\) is not homeomorphic to \(E^3\), and for each torus \(T\) used in defining \(G'\) there is an \(\epsilon_T^T\) such that \(T\) is not \(\epsilon_T^T\)-shrinkable with respect to \(G'\). Theorem. If \(G\) is a toroidal decomposition of \(E^3\) such that the set \(H\) of nondegenerate elements forms a continuous collection and is such that each element of \(H\) is locally connected then \(E^3/G\) is homeomorphic to \(E^3\). The main technique used in this theorem is an extension of the twisting used by Bing in his paper "A homeomorphism between the 3-sphere and the sum of two solid horned spheres." (Received October 30, 1970.)


These results are in response to a question raised by Viglino ("\(C\)-compact and seminormal spaces", Duke Math. J., to appear). A topological space \(X\) is \(C\)-compact if given a closed subset \(K\) of \(X\) and an open cover of \(K\), there is a finite subcollection dense in \(K\). A closed subset \(F\) of a space \(X\) is \(\text{regular closed}\) if for each point \(x\) not in \(F\) there exist disjoint nbds \(U\) and \(V\) of \(x\) and \(F\) respectively in \(X\). A space \(X\) is \((\text{weakly})\) \(\text{seminormal}\) if given any \(\text{regular}\) closed subset \(K\) of \(X\) and any open subset \(O\) containing \(K\), there is a regular open set \(R\) with \(K \subseteq R \subseteq O\). Theorem 1. There exists a \(C\)-compact \(T_2\) space which is not seminormal. Theorem 2. Every \(C\)-compact \(T_2\) space is weakly seminormal. (Received November 2, 1970.)
682-54-34. FRANK SWIETEC, St. John's University, Jamaica, New York 11432. Sequence-covering and countably bi-quotient mappings.

A Hausdorff space $X$ is strongly Fréchet if whenever $\{A_n : n \in \mathbb{N}\}$ is a decreasing sequence of sets in $X$ and $x$ is a point which is in the closure of each $A_n$, then for each $n$ there exists an $x_n \in A_n$ such that the sequence $x_n \rightarrow x$. Strong accessibility and strongly $k^*$-spaces are defined similarly. These spaces, as well as sequential and Fréchet spaces, are characterized by means of sequence-covering and countably bi-quotient mappings. A sequence-covering mapping has a definition analogous to that of a compact-covering mapping, while a countably bi-quotient mapping is a natural weakening of a bi-quotient mapping. (Received November 2, 1970.)


Let $(X, p)$ be a metric space and let $d_0(X, p)$ be the metric dimension of $X$ as defined by Katetov. K. Nagami and J. H. Roberts introduced the metric-dependent dimension function $d_3$ and proved that $d_3(X, p) \leq d_0(X, p)$ for all metric spaces. They posed the following question: Is $d_3(X, p)=d_0(X, p)$ for all metric spaces? J. C. Smith defined the dimension function $d^*_3$ and proved that $d_3(X, p) \leq d_3^*(X, p) \leq d_0(X, p)$. In this paper the authors give a partial solution to the question, is $d_3(X, p)=d_3^*(X, p)$ for all metric spaces, by introducing a function $d_3^{**}(X, p)$ and then proving that $d_3^{**}(X, p)=d_3(X, p)$. $d_3^{**}$ is defined as follows. An open cover $U$ of a set $X$ is called a $k$-star cover of $X$ if every element of $U$ intersects at most $k$ other elements of $U$. $d_3^{**}(X, p) \leq n$ if every $k$-star Lebesgue cover $U$ of $X$ has an open refinement $V$ such that order $V \leq n+1$. (Received November 2, 1970.)


A space $X$ is expandable if, for every locally finite collection $\{F_\alpha : \alpha \in A\}$ of subsets of $X$, there exists a locally finite open collection of subsets $\{G_\alpha : \alpha \in A\}$ such that $F_\alpha \subseteq G_\alpha$ for all $\alpha \in A$. This property of expandability has recently been studied by L. Krajewski. (See Abstract 672-632, these Notices 17 (1970), 265.) In this paper the authors investigate a number of different types of expandability properties referred to as boundedly expandable, almost expandable, almost discretely expandable, discretely expandable, H. C. expandable and discretely H. C. expandable. Two of the major results are the following: Theorem. (1) A space $X$ is almost expandable iff $X$ is almost discretely expandable and countably paracompact. (2) A space $X$ is almost expandable iff $X$ is almost discretely expandable and countably metacompact. Theorem. The following are equivalent in a regular discretely H. C. expandable space $S$. (1) $X$ is paracompact. (2) $X$ is subparacompact. (3) $X$ is metacompact. (4) $X$ is $\theta$-refinable. A number of generalizations concerning metrization theorems, sum theorems, product theorems and mapping theorems are obtained as applications of the above. (Received November 2, 1970.)

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Theorem 1. If $X$ is a weakly Lindelöf $F$-space then $X \times Y$ is an $F$-space for each $P$-space $Y$. Theorem 2. If $X$ is a CLWL $F$-space and there is a closed Lindelöf subspace $D$ of $X$ such that $X \setminus D$ is basically disconnected then $X \times Y$ is an $F$-space for each $P$-space $Y$. Remark. These theorems, neither of which is "best possible", are in response to a question of Gillman [Arch. Math. (Basel) 11 (1960), 53-55]. Theorems 1 and 2 remain valid if "$F$-space" is replaced by "$U$-space" where a (completely regular Hausdorff) space is a $U$-space provided disjoint cozero sets can be separated by an open-and-closed set. Still eluding proof is the following: Conjecture. $X \times Y$ is an $F$-space for each $P$-space $Y$ if and only if $X$ is a CLWL $F$-space. (Received November 2, 1970.)

Theorem 1. Let $X$ be a Hausdorff space and $T$ a continuous mapping of $X$ into itself. Let $F: X \times X \to [0, \infty)$ be a continuous mapping such that, for all $x, y$ in $X$, $F(Tx, Ty) \leq F(x, y)$ and when $x \neq y$, there is some $n$ depending on $x, y$ such that $F(T^n x, T^n y) < F(x, y)$. If there exists $x$ in $X$ such that $\{ T^k x \}$ has a convergent subsequence then $T$ has a unique fixed point. Theorem 2. Let $T: X \to X$ be a continuous mapping of a metric space $X$ into itself such that $d(x, y) \to 0$. If for some $x_0$ in $X$ the sequence of iterates $x = T^n x_0$ has a convergent subsequence $\{ x_n \}$ converging to $z$, then $z$ is a fixed point of $T$. A few known results we get as corollaries to our theorems. (Received November 2, 1970.)

The following theorem is proved: Theorem. Let $Y$ be a compactification of a locally compact space $X$ and let $K = Y - X$. Let $f: Y \to K$ be a continuous map such that for every closed subset $L$ of $X$ such that the complement of $L$ in $X$ has compact closure in $X$, we have $f[L]$ is a dense subset of $K$. Suppose further that $f$ reduces to the identity on $K$. Then $Y$ is a Wallman compactification of $X$. This theorem answers a question raised in the author's recent paper ["Some Wallman compactifications of locally compact spaces," submitted to Fund. Math.] and provides a tool for proving that some spaces which were not previously known to be Wallman are indeed so. (Received November 2, 1970.)

The results obtained deal with the introduction into a metric space of an equivalent metric which has certain dimension-theoretic properties, and they extend previous work of the author [Canad. J. Math. 21 (1969), 748-750, and Abstract 69 T-G64, these Notices 16 (1969), 691]. The following are examples. Theorem 1.
Let $\mathcal{F}$ be a locally finite collection of finite-dimensional closed subsets of a metric space $X$. Then there exists a completion $X^*$ of $X$ in which the closure of any member $F$ of $\mathcal{F}$ has the same dimension as does $F$.

**Theorem 2.** Let $\mathcal{F}$ be a closure-preserving collection of nonvoid finite-dimensional closed subsets of a metric space $X$. Then there exists a topologically equivalent metric for $X$ such that for any $F \in \mathcal{F}$, any $\epsilon > 0$, and any subset $A$ of $X$, $\dim [F \cap B(S_\epsilon (A))] < \dim F$. Other theorems deal with the extension of continuous functions to completions and metric compactifications which satisfy conditions analogous to that of Theorem 1. (Received November 2, 1970.)

682-54-41. LOUIS D. NEL, Carleton University, Ottawa 1, Ontario, Canada. **Lattices of lower semi-continuous functions and topological spaces determined by them.**

Let $\mathcal{L}(X)$ denote the lattice of all lower semicontinuous real-valued functions on the $T_0$-space $X$. No essential loss results from a restriction to $T_0$-spaces. A set of equivalence classes of closed prime ideals in $\mathcal{L}(X)$ can be formed and a topology be defined for this set by using only the lattice structure of $\mathcal{L}(X)$; thus a topological space $\Omega_\mathcal{L}$ is obtained, which is necessarily a pc-space (see L. D. Nel, Abstract 672-207, these Notices) 17 (1970), 142. $\Omega_\mathcal{L}$ is homeomorphic to $X$ iff $X$ is a pc-space. Hence for pc-spaces $X$, $Y$ the lattices $\mathcal{L}(X)$ and $\mathcal{L}(Y)$ are isomorphic iff $X$ and $Y$ are homeomorphic. The methods used yield similar results if $\mathcal{L}(X)$ is replaced by the sublattice of functions taking values in a real interval $(a, b]$, but they fail for an interval $[a, b]$. (Received November 2, 1970.)

682-54-42. ROBERT D. HOFER, State University College of New York, Plattsburgh, New York 12901. **Restrictive semigroups of continuous selfmaps on spaces which are completely regular, Hausdorff and arcwise connected.** Preliminary report.

Let $X$ be a topological space and $Y$ a nonempty subspace of $X$. Then the semigroup, under composition, of all continuous functions mapping $X$ into $X$ which also carry $Y$ into $Y$ is denoted by $S(X, Y)$ and is referred to as a restrictive semigroup of continuous selfmaps. **Theorem.** Let $X$ and $U$ be completely regular (normal), Hausdorff and arcwise connected spaces and let $Y$ and $V$ be compact (closed) subsets of $X$ and $U$ respectively such that both $Y$ and $V$ are either 0-dimensional or contain an arc. Then for each isomorphism $\varphi$ from $S(X, Y)$ onto $S(U, V)$ there exists a unique homeomorphism $h$ from $X$ onto $U$ carrying $Y$ onto $V$ such that $\varphi(f) = h \cdot f \cdot h^{-1}$ for each $f \in S(X, Y)$. This result has applications to near-rings of continuous selfmaps on topological groups. For example, let $G$ be an additive topological group and let $N_0(G)$ be the near-ring of all continuous selfmaps of $G$ which keep the zero fixed where addition is pointwise and multiplication is composition. **Corollary.** Let $G$ and $H$ be arcwise connected, Hausdorff groups and $\varphi$ an isomorphism from $N_0(G)$ onto $N_0(H)$. Then there exists a unique topological isomorphism $h$ from $N_0(G)$ onto $N_0(H)$ such that $\varphi(f) = h \cdot f \cdot h^{-1}$ for each $f \in N_0(G)$. (Received November 3, 1970.)
In this paper we study relationships between the ideals of $C(X)$ and partitions of unity on $X$. **Theorem.**

Let $X$ be a completely regular $T_1$-space. The following are equivalent: (1) $X$ is $\sigma$-compact. (2) Every maximal open cover of $X$ has a locally finite countable basic refinement. (3) Every maximal open cover has a locally finite basic refinement of nonmeasurable cardinal. (4) Every free maximal ideal of $C(X)$ contains a partition of unity of nonmeasurable cardinal. As corollaries we obtain new proofs of two well-known theorems of Katetov and Shiota. (Received November 3, 1970.)

**682-54-44.** FRANK SIWIEC and VINCENT J. MANCUSO, St. John's University, Jamaica, New York 11432. Relations among certain mappings and conditions for their equivalence.

The purpose of this paper is to organize the following mapping properties: almost-open, bi-quotient, compact-covering, compact-trace, countably bi-quotient, pseudo-open, $P_2$, and quotient. In addition, certain of these properties are used to give mapping characterizations of $k$-spaces, $k'$-spaces, and locally compact spaces. Several examples are given and some problems are posed. (Received November 3, 1970.)

**682-54-45. WITHDRAWN.**

**682-54-46.** RONALD A. STOLTENBERG, Sam Houston State University, Huntsville, Texas 77340. A note on paracompact $p$-spaces. Preliminary report.

It is known that a regular topological space $X$ is metrizable if and only if there is a sequence $\{\mathcal{V}_n\}$ of open covers of $X$ such that for each compact subset $C$ of $X$, $\{\operatorname{St}(C, \mathcal{V}_n)\}$ is a neighborhood base for $C$. A generalization of the above condition occurs in paracompact $p$-spaces. Let $X$ be a topological space; we say that $X$ has property $p^*$ if there exists a sequence $\{\mathcal{V}_n\}$ of open covers for $X$ such that for each compact subset $C$ of $X$, the set $P_C = \bigcap_{n=1}^{\infty} \operatorname{St}(C, \mathcal{V}_n)$ is compact and $\{\operatorname{St}(C, \mathcal{V}_n)\}$ is a neighborhood base for $P_C$. Now every paracompact $p$-space is a $p^*$-space and it is conjectured that at least for normal spaces the converse is true. The following preliminary result has been proved: **Theorem.** If $X$ is a normal subparacompact $p^*$-space, then $X$ is a paracompact $p$-space. (Received November 4, 1970.)

**682-54-47.** ROBERT B. FRASER, Louisiana State University, Baton Rouge, Louisiana 70803. Spaces which admit only trivial little Lipschitz functions.

Let $(X, d)$ be a compact metric space. **Definition.** A space $(X, d)$ is boundedly chainable if for $x, y \in X$, there exists $M$ such that for each $\varepsilon > 0$, $d(x, y) \leq M$, where $d_\varepsilon(x, y) = \inf \{ \sum_{i=1}^{n} d(x_1, x_{i+1}) | x_1 = x, x_{n+1} = y, d(x_i, x_{i+1}) \leq \varepsilon \}$. **Theorem 1.** A metric space $(X, d)$ admits only trivial little Lipschitz functions if it is boundedly chainable. **Theorem 2.** A locally compact complete metric space which is boundedly chainable is connected by rectifiable arcs. Examples are given to show that local compactness and completeness are (in some sense) necessary. (Received November 4, 1970.)
Let $T$ be a topological group. A $T$-space is defined to be a transformation group $(X, T)$ such that $X$ is compact Hausdorff. A $T$-space $(X, T)$ may be denoted simply by $X$. A $T$-ambit is defined to be an ambit whose acting group is $T$. As a general reference for ambits, we consult Gottschalk's paper: "Dynamical aspects of orbit-closures," Internat. Sympos. Topological Dynamics, Benjamin, New York, 1968. Theorem 1. Let $Aa_0$ be a mobile $T$- ambit, let $X$ be a $T$-space. Then $Aa_0$ is the ambience of $X$ if and only if the following conditions hold: (1) $Aa_0$ dominates each ambit in $X$, and (2) for any $a, b \in A$ such that $a \neq b$, there exists a morphism $\varphi$ of $A$ into $X$ such that $a \varphi \neq b \varphi$. Theorem 2. Let $XX_0$ be a $T$-ambit, let $M$ be a minimal subset of $X$ such that $XX_0$ dominates each ambit in $M$, and let $\mathfrak{N}$ be the class of all almost periodic $T$-ambits dominated by $XX_0$. Then: (1) $M$ is coalescent, and (2) the class $\{Mm \mid m \in M\}$ coincides with the class of all maximal members of $\mathfrak{N}$ up to isomorphisms. (Received November 4, 1970.)

Theorem 1. A space $X$ is a complete metric ANR if and only if $X$ can be embedded as a strong deformation retract of an open subset of some (not necessarily separable) Hilbert space. Using an extension of this theorem to noncomplete ANR's, a technique due to Borsuk (Bull. Acad. Polon. Sci. 18 (1970), 235-239), and many recent results from the theory of infinite-dimensional manifolds there is proved: Theorem 2. Let $X$ and $X'$ be metric ANR's and let $A \subseteq X$ and $B \subseteq X'$ be closed subsets. If there are homotopy equivalences $f: A \rightarrow B$ and $f': X \rightarrow X'$ such that $f$ is homotopic (in $X'$) to $f'|A$, then $X|A$ is homotopy equivalent to $X'|B$. ($X|A$ denotes the quotient space obtained by "shrinking $A$ to a point"). Also, Theorem 2 is extended to more general quotient spaces and various counterexamples and theorems relating to the converse of Theorem 2 are given. (Received November 4, 1970.)

Aposyndetic properties of hyperspaces. Preliminary report.

Let $X$ be a compact connected metric space and $2^X (K(X))$ denote the hyperspace of all nonempty closed (closed connected) subsets of $X$, with the finite topology. See Michael [Trans. Amer. Math. Soc. 71 (1951), 152-182]. Theorem. Each of $2^X$ and $K(X)$ is aposyndetic. Corollary. Each of $2^X$ and $K(X)$ is finite set aposyndetic. Theorem. $K(X)$ is mutually aposyndetic. Theorem. $K(X)$ is countable closed set aposyndetic. (Received November 5, 1970.)

Extension of the $M$-space concept.

In certain papers of the authors, conditions $\lambda_b', \lambda_c$, $\beta_c$, and $\beta_b$ have been introduced and studied. These conditions permit the characterization of open continuous images of certain kinds of $M$-spaces, e.g., paracompact Čech complete spaces, and have a number of other significant properties. Here a property is introduced which
can be used to define generalizations of M-spaces, of spaces satisfying the conditions just mentioned, and of q-spaces in the sense of Michael in such a way that analogues of results obtained by the authors and others may be obtained. For example, a regular T₀ space is an open continuous image of a regular T₀ generalized complete M-space if and only if it satisfies the appropriate generalization of condition λc. A generalized M-space in the above sense is always an M₀-space in the sense of Isiwata [Proc. Japan Acad. 45 (1969), 359-363]. The class of normal T₁ generalized M-spaces coincides with the class of normal T₁ M-spaces. Morita's example [Proc. Japan Acad. 43 (1967), 869-872] provides an example of a perfect mapping of a locally compact T₂ M-space having a completely regular T₀ range which is not a generalized M-space. (Received November 5, 1970.)


A collection η is a network for a space X provided that, for any point p and any open set R containing p, there is a member N of η such that p ∈ N ⊂ R. A T₃₂-space X is a σ-space if X has a σ-locally finite network. Theorem. A T₃₂-space X is a σ-space iff there corresponds to each point p of X a sequence {gₙ(p)} of neighborhoods of p such that (i) if p ∈ gₙ(xₙ) for n = 1, 2, ... then xₙ → p (ii) for all p, q and n, p ∈ gₙ(q) implies q ∈ gₙ(p). Characterizations of semistratifiable and σ-spaces in terms of a "distance function" are also given (e.g., X is semistratifiable iff for some function d from X × X into the nonnegative reals (i) d(yₙ, x) → 0 ⇒ yₙ → x = d(x, yₙ) → 0 for every point x and every point sequence {yₙ} in X; X is a σ-space iff there is a function d for X satisfying the above condition (i) plus (ii) [d(zₙ, yₙ) → 0 and d(yₙ, x) → 0] = d(zₙ, x) → 0). (Received November 5, 1970.)


Let X be a completely regular Hausdorff space. Let D(X) = {f: X → R ∪ {±∞} | f is continuous and f⁻¹(R) is dense in X}. The object D(X) has been used in the representation of an abstract φ-algebra. [See Henriksen-Johnson: "On the structure of a class of archimedean lattice-ordered algebras," Fund. Math. 50(1961), 73-74]. Definitions. A subset F of X is said to be D-embedded in X if ∀ f ∈ D(F), there exists g ∈ D(X) with g|F = f. F is said to have the D-restriction property in X if for each g ∈ D(X), g|F ∈ D(F). Theorem 1. Let X be a compact Hausdorff space such that D(X) is a φ-algebra. Then X is basically disconnected iff every closed subset is D-embedded in X. Theorem 2. The following are equivalent for compact basically disconnected X. (a) X satisfies the countable chain condition (hence, is extremely disconnected). (b) Each closed set with the D-restriction property is a clopen set. (c) X contains no nowhere dense closed set with the D-restriction property. (Received November 5, 1970.)
Let \((X, \delta)\) be a proximity space, where \(X\) is a dense (topological) subspace of a completely regular, Hausdorff space \(T\). In an earlier paper, "Extensions of proximity functions," Proc. Amer. Math. Soc. 26 (1970), 347-351, the author established a number of conditions equivalent to the following: every point \(x\) in \(T\) is a cluster point of a unique maximal round filter \(\mathcal{F}_x\) on \((X, \delta)\). In the present paper, we assume that each \(\mathcal{F}_x\) is also required to converge to \(x\). This property is shown to be equivalent to the condition that \(T\) admit a compatible proximity \(\delta_1\) for which \((X, \delta)\) is a \(p\)-subspace of \((T, \delta_1)\). Other equivalent conditions are also obtained.

S. Leader has shown that if \(T\) admits a compatible proximity \(\delta_1\) for which \((X, \delta)\) is a \(p\)-subspace of \((T, \delta_1)\), then each point \(x\) in \(T\) is a cluster point of a unique cluster \(\mathcal{C}_x\) from \((X, \delta)\). (See Theorem 3 of S. Leader, "Clusters in proximity spaces," Fund. Math. 47 (1959), 205-213.) An example is provided to show that the converse of this statement is false. (Received November 5, 1970.)

### 55 Algebraic Topology

682-55-1. FREDERICK H. CROOM, University of Kentucky, Lexington, Kentucky 40506. A product theorem for \(H\)-group fibrations.

Let \((E, p, B)\) and \((E', p', B)\) be \(H\)-group fiber structures with the weak covering homotopy property such that the basic fibers \(F\) and \(F'\) are \(H\)-subgroups of \(E\) and \(E'\) respectively. If there exist base point preserving fiber maps \(f: E \rightarrow E':g\) such that \(f\) is an \(H\)-homomorphism, then the product spaces \(E \times F\) and \(E' \times F\) have the same homotopy type. This result is obtained by means of a homotopy exact sequence induced by the map \(f\). If the restriction \(f|_F: F \rightarrow F'\) is a homotopy equivalence, it follows that \(f\) is a fiber homotopy equivalence. Under more restrictive hypotheses it is proved that \(f\) is a fiber homotopy equivalence provided that it induces a homotopy equivalence on the spaces of bases loops in \(E\) and \(E'\). (Received September 30, 1970.)

682-55-2. S. J. LOMONACO, JR., Texas Instruments, Inc., Dallas, Texas 75222 and Southern Methodist University, Dallas, Texas 75222. The third homotopy group of spun knots.

Let \(k(S^2)\) be a knotted 2-sphere in the 4-sphere \(S^4\) formed by spinning an arbitrary arc \(A\) about a standard 2-sphere \(S^2\). The second homotopy group of \(k(S^2)\) in \(S^4\) was computed in Topology 8 (1969), 95-98. The third homotopy group of \(k(S^2)\) in \(S^4\) is now determined and computed. Whitehead products and related invariants are also calculated. (Received October 2, 1970.)


Starting with an axiomatic cap product among generalized homology and cohomology theories, there is a corresponding product induced on the associated (Atiyah–Hirzebruch) spectral sequences. It is then possible to proceed from ordinary Poincaré duality at the \(E_2\) stage to the generalized duality theorem at the \(E_\infty\) stage. This leads to the characterization of general orientability of a manifold \(M\) as the requirement that the ordinary funda-
mental class $\sigma$ in $E_2$ be a permanent cycle in the spectral sequence. There are intermediate homology theories associated with the $E_n$ stage of the spectral sequence, $2 \leq n < \infty$, and orientability for these theories follows if $\sigma$ is an $n$-cycle. Examples may be cited that for arbitrarily large integers $n$, there are manifolds which are $n$-orientable, but not orientable in general. These results are highly analogous to those of Kan and Whitehead (Topology 3 (1965)). (Received October 19, 1970.)

682-55-4. PAUL C. KAINEN, Case Western Reserve University, Cleveland, Ohio 44106. Realizing commutative diagrams of Abelian groups by corresponding homotopy commutative diagrams of Moore spaces. Preliminary report.

If $\delta$ is a commutative diagram of Abelian groups, let $P(\delta)$ denote the problem of realizing $\delta$ by a corresponding homotopy commutative diagram $\delta'$ of $n$-dimensional Moore spaces so that $H_n(\delta') = \delta$. (We assume $n \geq 3$.) Theorem. Let $\delta$ be a commutative square. Then $P(\delta)$ is solvable. (This gives an affirmative answer to a question of Hilton.) Moreover a counterexample shows that we must have complete freedom to choose all four of the maps in any solution $\delta'$ of $\delta$. We believe that the method of proof of the theorem may be extended to prove the following: Conjecture. The Theorem is true for the diagram scheme on a $d$-dimensional cube. We may define an order relation on diagram schemes by saying $\Delta_1 \leq \Delta_2$ iff any commutative diagram on the scheme $\Delta_1$ can be extended to a commutative diagram on the scheme $\Delta_2$. We investigate the cofinality of the family $\{\Delta_{m_1}\}$ in the collection of all schemes $\Delta$, and then we apply this to the realization problem. (Received November 4, 1970.)

682-55-5. LAWRENCE L. LARMORE, California State College, Dominguez, California 90246. Isotopy groups. Preliminary report.

Let $M$ be an $n$-dimensional pointed differentiable manifold. Using obstruction theory, a group structure, which is Abelian, can be defined on $E_k(M)$, the set of base-point-preserving isotopy classes of inessential embeddings of the $k$-sphere into $M$, provided $3k \leq 2n-4$. Theorem 1. If $k \geq 2$, dim $M = 2k+1$, and $\pi_{k+1}(M)=0$, $E_k(M)$ is isomorphic to the direct sum of $a/2+b$ copies of $Z$ with $c$ copies of $Z_2$; where $a$ is the number of elements of $G=\pi_1(M)$ of order 2, $b+c$ is the number of elements of $G$ of order greater than 2, and $b$ is the number of those which preserve (respectively, reverse) the orientation of $M$ if $k$ is even (respectively, odd).

Theorem 2. If $k \geq 2$ and $3k \leq 2q-2$, $E_k(S^1 \times S^q)$ is isomorphic to the direct sum of countably infinitely many copies of the stable $(2k-q)$-stem. This group structure is thus isomorphic to that found by Hacon (Topology 7(1968), 1-10). In the computations we must evaluate the cohomology of the reduced deleted product of $S^k$ with coefficients in a sheaf which is not locally trivial, i.e., not a local system of groups. (Received November 4, 1970.)


Let $X$ be a metric ANR (absolute neighborhood retract). A map $f:X \to X$ is a Palais map if the set $cl(\cap f^k(X))$ is compact and each point has an invariant neighborhood $U$ such that $cl(f(U))$ is compact; a map $H:X \times I \to X$ is a Palais homotopy if the map $\hat{H}$ defined by $\hat{H}(x,t) = (H(x,t),t)$ is a Palais map. The familiar Nielsen theorem on compact metric ANRs is generalized to the following form: Theorem. To each Palais map
f: X → X, one can assign a nonnegative integer N(f) such that f has at least N(f) fixed points; if f and g are Palais homotopic then N(f) = N(g). **Theorem.** Let H be a Palais homotopy and define h_t(x) = H(x, t). Then N(H) = N(h_t) for all t ∈ I. (Received November 5, 1970.)


Let X be a countable CW-complex of finite type with an A_k-structure p_k: X • k • X → P(k-1) (Trans. Amer. Math. Soc. 108(1963), 275-312). H*(X;Z(p)) is primitively generated as a Hopf-algebra and has only a finite number of primitive generators in odd dimensions. An element x ∈ H*(X;Z(p)) is k-transgressive if it is transgressive with respect to p_k. **Theorem 1.** There is a group splitting, H*(P(k);Z(p)) = N + S with N a truncated polynomial algebra of height k + 1 with even dimensional generators, one for each odd dimensional k-transgressive primitive generator in H*(X;Z(p)), and S is an A(p)-algebra. (Cf. Theorem (1.1), Indian J. Math. (1963), 492-501.) Since S is an A(p)-algebra, N can be given an A(p)-algebra structure. **Theorem 2.** If X has an A_p-structure and satisfies the hypothesis of Theorem 1, then for each primitive p-transgressive x ∈ H*(X;Z(p)) with degree x > p^2 and p | deg x there is a primitive y ∈ H*(X;Z(p)) such that p^2(y) = x. (Cf. Theorem 1.1, Ann. of Math. (2) 77(1963), 306-312.) (Received November 5, 1970.)

682-55-8. CHUNG-WU HO, Southern Illinois University, Edwardsville, Illinois 62025. Spaces for which the fixed point property is characterized by homology groups.

By the Lefschetz fixed point theorem, a space X has the fixed point property (f.p.p.) if it satisfies the following condition: **Condition A.** X is compact, connected and the homology groups H_n(X) of integral coefficients are all torsion groups for n > 0. We now ask the **Question.** For which spaces, the f.p.p. is completely characterized by Condition A. Such spaces, if exist, will be called Lefschetz spaces. **Theorem 1.** All 1-dimensional simplicial complexes are Lefschetz spaces. **Theorem 2.** All topological manifolds of dimension 2 (with or without boundary) are Lefschetz spaces. **Corollary.** The only 2-manifolds which are a fixed point space are disk and projective plane. **Theorem 3.** Let X, Y be connected compact polyhedra and S(X) be the suspension of X. If X, Y are Lefschetz spaces, so are X × Y and S(X). Furthermore, X ∪ Y is also a Lefschetz space if X ∩ Y is a nonempty contractible space. (Received November 5, 1970.)

57 Manifolds and Cell Complexes

682-57-1. CHRISTINE B. BEAUCAGE, State University of New York, Stony Brook, New York 11790. Proper maps of PL manifolds.

A map f: M → N is proper if, for every compact set C ⊂ N, f^(-1)(C) is compact. If f is a piecewise linear (PL) map, then the branch set of f, B_f ⊂ M, can be defined as follows: x ∈ M - B_f if there exists a neighborhood U_x of x such that f(U_x) = V_x is a closed star neighborhood of f(x), and if there exists a PL homeomorphism h_x: U_x ∩ B_x × V_x, where B_x is a PL ball, such that f|_{U_x} = π_2 * h_x: U_x → V_x, where π_2 is the projection onto the second factor. **Theorem.** Let M^m, N^k be PL manifolds, and let f: M^m → N^k be a proper PL map with B_f = ∅. Suppose f^(-1)(y) = ∅. Then there exists a neighborhood N_y of y, and a PL homeomorphism g_y: f^(-1)(N_y) → f^(-1)(y) × N_y, such that f|_{f^(-1)(N_y)} = π_2 * g_y: f^(-1)(N_y) → N_y.
Thus, if $\beta(y) = \beta$ for every $y \in N^k$ and $N^k$ is connected, then $f$ is the projection map of a PL fibre bundle. The theorem is also proved in some cases when $M^n$ and $N^k$ are not manifolds. The proof uses a theorem of J. F. P. Hudson ["Extending piecewise-linear isotopies," Proc. London Math. Soc. (3) 16 (1966), 651-666, Theorem 3]. The author expects soon to eliminate the condition on $f^{-1}(y)$.

(Received August 28, 1970.)

682-57-2. JAMES W. MAXWELL, Department of Mathematics and Statistics, Oklahoma State University, Stillwater, Oklahoma 74074. Obstructions to embedding. Preliminary report.

A map $f: X \to Y$ is $k$-connected means $\pi_i(C_f \times X) = 0$ for $i \leq k$, where $C_f$ denotes the mapping cylinder of $f$. Suppose $M^n$ is a compact, $n$-dimensional PL manifold with empty boundary, $Q^q$ a $q$-dimensional manifold with empty boundary and $F$ is a $(2n-q)$-connected PL map of $M$ into $Q$. Hudson ("Piecewise linear topology", Benjamin Press, 1969) announced an obstruction theory for homotoping $F$ to an embedding. In this paper the more general case of a proper PL map between manifolds is considered for maps of $n$-dimensional manifolds in $(2n-1)$-dimensional manifolds. The following theorem is a result: Suppose $M^n$ is a PL manifold, orientable, $\partial M \neq 0$, and $f: (M, \partial M) \to (B^{2n-1}, \partial B^{2n-1})$ is a proper PL map such that $f|\partial M$ is an embedding ($B^{2n-1}$ denotes the $(2n-1)$-dimensional ball); then $F$ is homotopic, relative to $\partial M$, to an embedding provided $i^*: H_1(\partial M) \to H_1(M)$ is the zero map. (Received October 5, 1970.)


A rather short, conceptual proof is given of Moise's piecewise-linear approximation theorem (Ann. of Math. (2) 56 (1952), 97, Theorem 2). The result implies the triangulation theorem and Hauptvermutung for 3-manifolds. The present proof, which involves only piecewise-linear methods, proceeds by successively constructing the approximating homeomorphism on the surface, discs, and 3-cells of a generalized (n.n. compact) "Heegaard decomposition" of the domain manifold. The first step, which is the most delicate, involves the loop theorem, a 2-dimensional covering space argument, and an infinite multiple induction to keep track of cuts and pastes. The last two steps are simple applications of Dehn's lemma and the 3-dimensional PL Schoenflies theorem, respectively. A straightforward extension of the method would give a rel boundary approximation theorem which implies the triangulation theorem and Hauptvermutung for 3-manifolds with boundary. (Received October 21, 1970)


A Q-transformation on a free group $F$ is a product of mappings $Q: F^m \to F^m, Q(w_1, \ldots, w_m) = (w_1^{w_1}, \ldots, w_m^{w_m})$ where $w_i = w_j$, $i \neq j$, and $w_i = g^{-1}w_ig$, $g \in F$, $v$ a consequence of $(w_1, \ldots, w_{j-1}, w_{j+1}, \ldots, w_n)$. Two presentations of a group are called Q-equivalent if the relators are obtained from each other by Q-transformations.

Theorem 1. $\varphi$ and $\psi$ are Q-equivalent presentations of a group if and only if their corresponding 2-complexes $K_\varphi$ and $K_\psi$...
are spines of a common 3-complex. **Theorem 2.** If $\phi$ is a presentation of the trivial group and $K_{\phi}$ is the corresponding 2-complex, then $K_{\phi}S^2\vee\ldots\vee S^2$ and $S^2\vee\ldots\vee S^2$ are spines of a common 3-complex. Curtis and Andrews (Amer. Math. Monthly 73(1966), 21-28) have conjectured that any presentation of the trivial group is $Q$-equivalent to the trivial presentation. We show that a counterexample to this conjecture which is the spine of a 3-manifold must be a counterexample to the 3-dimensional Poincaré conjecture. Several presentations are given which might provide such a counterexample. (Received November 4, 1970.)

### 58 Global Analysis, Analysis on Manifolds

**682-58-1.** CHESTER R. SCHNEIDER, Oregon State University, Corvallis, Oregon 97331. **Hyperbolic** $R^2$ **actions** on 2-manifolds.

Let $X$ and $Y$ be commuting vectorfields ($[X,Y]=XY-YX=0$) on a compact 2-manifold $M$ and let $\phi$ denote the corresponding $R^2$ action on $M$. A definition for hyperbolic fixed points and $S^1$ orbits of such actions is given. With this definition $\phi$ can be "linearized" in a neighborhood of a hyperbolic fixed point or $S^1$ orbit. Call $\phi$ hyperbolic if it is hyperbolic at every fixed point and $S^1$ orbit; these $\phi$ are structurally stable. The main result is **Theorem 1.** There are no hyperbolic $R^2$ actions on a compact 2-manifold of negative Euler characteristic. $\phi$ is either effective or factors through an $R \times S^1$ action. The effective hyperbolic $R^2$ actions on $S^2$ and $T^2$ are determined. (Received November 3, 1970.)

**682-58-2.** JOHN M. FRANKS, Northwestern University, Evanston, Illinois 60201. **Necessary conditions** for stability of diffeomorphisms. Preliminary report. S. Smale has shown that if a diffeomorphism satisfies his Axiom A and the no-cycle property it is $\Omega$-stable, and conjectured the converse. In this paper we show that if $f$ is an $\Omega$-stable diffeomorphism then all periodic points are hyperbolic and there is a $\lambda \in (0,1)$ such that if $p$ is a periodic point of period $k$ then the $k$th roots of any eigenvalue of $df_p$ is either less than $\lambda$ or greater than $\lambda^{-1}$. A further result shows that certain stable and unstable manifolds must intersect transversely. For example any homoclinic point of an $\Omega$-stable diffeomorphism must be a transversal homoclinic point. (Received November 4, 1970.)

**682-58-3.** SIAVASH M. SHAHSHAHANI, Northwestern University, Evanston, Illinois 60201. **Dissipative systems on manifolds.** Preliminary report. Let $(M,\sigma)$ be a compact smooth Riemannian manifold, $TM$ its tangent bundle, and $\Gamma$ the set of second order ordinary differential equations on $M$ endowed with Whitney $C^\Gamma$-topology. Consider the smooth real-valued functions $K$, $V$, and $E$ on $TM$ where $K$ is the quadratic form associated to $\sigma$ (kinetic energy), $V$ is constant on the fibers of $TM$ (potential energy), and $E=K+V$. The symplectic form defined on $TM$ by $\sigma$ gives rise to a Hamiltonian system $X_E$. One can, and will, choose a Riemannian metric $\rho$ on $TM$ so that $\nabla K$ is vertical relative to $\rho$. A *dissipation force* on $TM$ is a vectorfield $D$ satisfying (i) $D$ is vertical, and (ii) $\rho(D, \nabla K) \leq 0$ with equality only at the zero-section. A dissipative system is a vectorfield of the form $X_E+D$.

**Theorem.** Suppose the Hamiltonian system $X_E$ has a finite number of zeros, all nondegenerate. There is an open and dense subset $\mathcal{D}$ of all dissipation forces on $TM$ so that if $D \in \mathcal{D}$, then $X_E+D$ satisfies (1) $X_E+D$ is $\Omega$-stable,
in fact \( \Omega \) consists of a finite number of hyperbolic zeros, (2) \( TM \) is the union of stable manifolds, and (3) at every zero, \( \dim(\text{stable manifold}) \geq \dim(\text{unstable manifold}) \). (Received November 5, 1970.)

### 60 Probability Theory and Stochastic Processes

**682-60-1.** DAVID R. BEUERMAN, Queen’s University, Kingston, Ontario, Canada. On the extent of a random walk.

Let \( X_1, X_2, X_3, \ldots \) be independent and identically distributed random variables which are attracted to a stable law \( G \) with \( \alpha \neq 1 \). Consider the random walk \( S_n \), where \( S_0 = 0, S_n = \sum_{i=1}^{n} X_i \) for \( n \geq 1 \). Define the extent of the random walk \( S_n \), \( e(S_n) = \max\{S_k - \min\{S_k\} \mid 1 \leq k \leq n\} \). In the case of drift (positive or negative), the results of Heyde (J. Appl. Probability 4 (1967), 144-150) are used to show that the limit distribution (l. d.) of \( e(S_n) \), suitably normed and centered, is \( G \). In the case of oscillation, weak convergence methods are used to obtain an invariance theorem which links the l. d. of \( e(S_n) \) to the distribution of \( \sup x(t) - \inf x(t) \) as applied to the corresponding stable process. Evaluation of the latter distribution is considered. In the Wiener process case, the result was obtained by Feller (Ann. Math. Statist. 22 (1951), 427-432). (Received October 2, 1970.)

**682-60-2.** J. DAVID MASON, University of Georgia, Athens, Georgia 30601. Local limit theorems for nonidentically distributed lattice random variables.

Let \( \{X_n\} \) be a sequence of independent integer-valued lattice random variables such that the distribution of \( \{X_n\} \) is one of the distinct nondegenerate distributions \( H_1, \ldots, H_r \) \( (r \geq 2) \). Assume there are sequences \( \{A_n\}, \{B_n\} \) \( (0 < B_n \to \infty) \) such that \( E_n = \sum_{i=1}^{n} X_i - A_n \to G \) (a nondegenerate distribution) in law. Let \( k_1 = \max \text{ span of } H_1 \), and let \( k \) be the greatest common divisor of \( k_1, \ldots, k_r \). A local limit theorem holds for \( \{X_n\} \) if one of the following is satisfied: (i) \( k = \max \text{ span of } H_i, 1 \leq i \leq p \); (ii) \( H_i \) is in the domain of attraction of a stable law, and \( G \) has a Gaussian component; (iii) \( G \) is the composition of \( p \) stable laws with distinct characteristic exponent, and \( G \) has a Gaussian component. (Received October 6, 1970.)


Let \( X = (X(t), \mathcal{G}, \mathbf{P}_t, \mathbf{P}_x) \) be a right continuous Markov process on a state space \( (E, \mathcal{G}) \). Let \( L \) be a fixed Banach space. A multiplicative operator functional (MOF) of \( (X, L) \) is a mapping \( (t, \omega) \to M(t, \omega) \) of \( \mathbb{R} \times \Omega \) to bounded operators on \( L \) for which \( t \to M(t, \omega) \) is a.s. right continuous, \( \omega \to M(t, \omega) \) is \( \mathcal{G}_t \) measurable, \( M(t+s, \omega) = M(t, \omega) M(s, \theta_t \omega) \), and \( M(0, \omega) \) is the identity operator. Theorem 1. Let \( E = \{1, 2, \ldots, N\} \) and let \( M \) be a MOF of \( (X, L) \) with \( \|M(t, \omega)\| \leq 1 \). Then there exist strongly continuous semigroups \( T_1(t), \ldots, T_N(t) \) on \( L \) and bounded operators \( \{\mathbf{P}_n\}_{n=0}^{\infty} \) on \( L \) such that for \( t \geq 0, M(t) = \mathbf{P}_n T_n(t) \mathbf{P}_{n-1}(t) \mathbf{P}_{n-2}(t) \cdots \mathbf{P}_1(t) \). \( 0 = \tau_0 < \tau_1 < \ldots \) are the jump times of \( X \) and \( N(t) \) is the number of jumps in \( [0, t] \).

Theorem 2. Let \( E = \{1, 2, \ldots, N\} \) and let \( M \) be a MOF of \( (X, L) \) such that \( t \to M(t, \omega) \) is a.s. continuous for
f ∈ L. Then the above representation simplifies to 

$$M(t) = (\prod_{i=1}^{N(t)} T_{X(t_{i-1})}^{N(t)}(t_{i-1})) T_{X(t_{N(t)})}^{N(t)}(t-t_{N(t)})$$

Finally we give applications to systems of parabolic equations and to storage theory. (Received October 19, 1970.)


Let $W(t, \omega)$ be the Wiener process on an abstract Wiener space $(\Omega, H, B)$ corresponding to the canonical normal distributions on $H$ (cf. L. Gross, J. Functional Analysis 1(1967), 135). We define stochastic integrals

$$I_{\xi} = \int_0^1 \xi(s, \omega) dW(s, \omega)$$

for nonanticipating transformations $\xi: \eta \rightarrow \sigma$ of $B \subset B^*$ and of $\eta: [0, \infty) \times \Omega \rightarrow H$. Ito's formula is generalized in the Theorem. Suppose $X(t, \omega) = x_0 + \int_0^t \xi(s, \omega) dW(s, \omega) + \int_0^t \sigma(s, \omega) dt$, where $\sigma: [0, \infty) \rightarrow H$ is nonanticipating. Let $f(t, x)$ be a continuous function on $R^+ \times B$, continuously twice differentiable in the $H$-direction for $x$ variable and once differentiable for $t$ variable. Then

$$f(t, X(t, \omega)) = f(0, x_0) + \int_0^t \left( \int_0^t \xi^*(s, \omega) f_x(s, X(s, \omega)) + \int_0^t \sigma(s, X(s, \omega)) \right) dW(s, \omega)$$

for all $t \geq 0$. Under certain assumptions on $A$ and $\sigma$ we prove the existence and uniqueness of solution of the stochastic integral equation

$$X(t, \omega) = x_0 + \int_0^t A(X(s, \omega)) dW(s, \omega) + \int_0^t \sigma(X(s, \omega)) ds$$

and show that the solution is a homogeneous strong Markov process. (Received October 21, 1970.)

682-60-5. ROGER A. HORN, The Johns Hopkins University, Baltimore, Maryland 21218. Asymptotic properties of infinitely divisible distributions.

If $F(x)$ is the cumulative distribution function of an infinitely divisible probability measure on the line, then the asymptotic behavior of $F(x)$ is sharply constrained. The following generalizations of recent results of Ruegg are obtained with elementary methods. Theorem. Assume that

$$1 - F(x) + F(-x) \leq (\exp(-xM(x)))$$

as $x \rightarrow \infty$, where $M(x)$ is a nonnegative measurable function. Then the distribution is normal (resp. degenerate) if and only if $M(x)$ can be chosen such that

$$M(x)/x \rightarrow \alpha$$

(resp. $M(x)/\ln x \rightarrow \alpha$) as $x \rightarrow \infty$ and such that $M(x)$ is strictly increasing and continuous for all large $x$. Similar methods yield the following one-sided result. Theorem. Assume

$$0 < C_1 \exp(-xM(x)) \leq F(-x) \leq C_2 \exp(-xM(x))$$

for all $x \geq 0$, where $m(x)$ and $M(x)$ are nonnegative measurable functions which are continuous and strictly increasing for all large $x$. If $\lim M(x)/\ln x = \infty$ then $\lim_+ m(x)/x > 0$. Thus, if $F(-x) \sim a \exp(-bx^{1+\epsilon})$ with $a, b > 0$ then if $\epsilon > 0$ it must be at least $1$. $0 < \epsilon < 1$ is not possible. Also, $F(-x) \sim a \exp(-b(x^{1/\alpha}))$ is not possible for any $\alpha > 1$. (Received October 22, 1970.)


Let $C_Y$ denote the Yeh-Wiener space, i.e., the space of all real-valued continuous functions $f(x, y)$ on

$$I^2 = [0, 1] \times [0, 1]$$

such that $f(0, y) = f(x, 0) = 0$, and a Gaussian measure defined on it so that the expected value

$$E[f(x, y)] = 0$$

and the covariance $E[f(x, t)f(y, t)] = \min(s, x) \cdot \min(t, y)$. Consider the Fredholm transformation

$$T[f(x, y)] = f(x, y) + \int_{I^2} K(x, y, s, t)f(s, t) dsdt$$

of $C_Y$ into $C_Y$. Under suitable assumptions on the kernel $K(x, y, s, t)$, the corresponding Radon-Nikodym derivative is found. The results include the Volterra transformation $T_1[f(x, y)] = f(x, y) + \int_0^x K(x, y, s, t)f(s, t) ds$ as a special case. (Received October 26, 1970.)
682-60-7. STEPHEN JAMES WOLFE, University of Delaware, Newark, Delaware 19711. On the unimodality of distribution functions of class L.

It is shown that a distribution function of class L is unimodal if its Lévy spectral function has support on the positive axis, and that this implies that every distribution function of class L is the convolution of at most two unimodal distribution functions of class L. Other results concerning the unimodality of distribution functions of class L and of other infinitely divisible distribution functions are also obtained. (Received October 29, 1970.)

682-60-8. CHUNG-TUO SHIH, University of Michigan, Ann Arbor, Michigan 48104. Boundary conditions of a class of Markov chains.

Let (Xt) be a minimal Markov chain with a countable number of exits. We characterize, using essentially hitting probabilities (in contrast to using the transition function as by K. L. Chung (1966) or using the resolvent as by E. B. Dynkin (1967)), all continuing chains satisfying the condition that almost surely only finitely many different exits are reached in finite time intervals. The characterization is such that for each distinguishable exit there exist two measures governing respectively the "reflection" at and the jumping away from the exit, which together with the hitting probabilities of (Xt) give the hitting probabilities of the continuing chain. Corresponding to each (permissible) set of "characteristics" there is a continuing chain in the above class. (Received November 2, 1970.)


A set of conditions are assigned to a strong Markov process which gives rise to a differential equation analogous to the Kolmogorov equation. However, in this case the duration variable is the net distance travelled and the state variable is a time, a situation precisely opposite to that of Brownian motion. By solving this differential equation under suitable boundary conditions, the density function of first passage time of standard Brownian motion process is derived. The first passage time process of the standard Brownian motion is also characterized by embedding Brownian motion in the infinitely divisible process. (Received November 3, 1970.)

682-60-10. RICHARD W. MADSEN, Statistical Laboratory, Iowa State University, Ames, Iowa 50010. Some ergodic theorems for Markov chains defined on an arbitrary state space.

Ergodic properties of nonhomogeneous Markov chains (NMC) with finite state space are well known. Recently Paz ["Ergodic theorems for infinite probabilistic tables," Ann. Math. Statist. 41(1970), 539-550] gave some ergodic properties for NMC's with countably infinite state spaces. Paz' work can be extended to an arbitrary state space. Let (Sn, E, µ) be a σ-finite measure space and let {Pn(x, y)} be a sequence of stochastic kernels with properties which guarantee existence of multistep stochastic kernels Pn,n+k(x, y). The ergodic coefficient a(P) is:

$$a(P) = 1 - \sup_{B \in E} \int_B |P(x, y) - P(z, y)|\mu(dy)$$

where the sup is taken over B \( \in E \) and x, z \( \in S \).

Two types of long run behavior of an NMC can be distinguished. If

$$\lim_{n \to \infty} [1 - a(P_{mn})] = 0$$

for all m, then the chain is called weakly ergodic. If there is a constant stochastic kernel Q (Q(x, y) = Q(z, y) for all z \( \in S \)) such that, for any m, lim \( n \to \infty \) \[ P_{mn} - Q \] = 0 (the norm of a kernel K is: \[ \|K\| = \sup_{x,y} |K(x, y)|\mu(dy) \]), then the chain is weakly ergodic.
called strongly ergodic. An equivalent condition to weak ergodicity is that there exist a subdivision of the chain into blocks of kernels \( \{ P_{i+1,j+1} \} \) such that \( \sum_{i=1}^{\infty} a_i P_{i+1,j+1} \) diverges. Other equivalent conditions for weak and strong ergodicity are also obtained. (Received November 3, 1970.)


We announce results on the differentiability properties of solutions to the evolution equation (1) \( v_t(x,t) = x v_{xx}(x,t) + a v_x(x,t), v(x,0) = g(x) \), \( 0 < x < +\infty \) and \( a > 0 \) which arises from the following change of variable: \( v(x,t) = u(x, t/2) \) where (2) \( u_t(x,t) = \frac{1}{2} u_{xx}(x,t) + (\gamma/x) u_x(x,t), u(x,0) = f(x), u_x(0,t) = 0, a = \gamma + 1/2 > 0. \)

When \( \gamma = (n-1)/2 \), the latter equation is the backward differential equation of the radial component of \( n \)-dimensional brownian motion. Set \( A g(x) = x g''(x) + a g'(x) \) and let \( B_0 \) denote the Banach space of \( k \)-times continuously differentiable functions \( g \) such that \( \lim_{x \to +\infty} g'(x) = 0, \lim_{x \to 0} g(x) = 0 \). Let \( g(1) = \sup_{x \in [0,1]} |g(x)| \).

Theorem. \( A \) is the infinitesimal generator of a strongly continuous contraction semigroup \( T(t): B_0 \to B_0 \) satisfying \( \| T(t)g \|_k \leq e^{-\alpha t} \| g \|_k \) for \( 0 < \alpha < \sigma_0 \), \( \sigma_0 \) depending only on \( a, \sigma, \) and \( k \). The results obtained for (1) yield corresponding results for (2). (Received November 3, 1970.)


Let \( X(m): m=0,1,2,\ldots \) be the random walk on the nonnegative integers with one-step transition probabilities \( p_i, i+1 = \frac{1}{2} (1 + \gamma / i + \epsilon), p_i, i-1 = \frac{1}{2} (1 - \gamma / i - \epsilon), p_i, i = 0, i \geq 1, i-1, p_0, 1 = 1, 1 - \gamma / i \geq \epsilon \geq 1 / (1 + \frac{1}{2}), \gamma \geq 0 \). The process is null recurrent for \( 0 \leq \gamma \leq \frac{1}{2} \) and transient for \( \gamma > \frac{1}{2} \).

We refine Trotter's operator theoretic methods (Pacific J. Math. 8(1958), 887-919) and apply the results of the previous announcement to establish Theorem 1. If \( \{ \sigma_n \}_{n=1}^{\infty} \) is a monotone increasing sequence such that \( \sigma_n \to +\infty, n \to +\infty \) and \( \sigma_n - (t \log n)^{2/3} / 4 \) then \( a \geq 0, P_{n+1,1}^{X([nt])} \sim P_n(X(t) \geq \sigma_n) \sim P_n(X(t) \geq \sigma_n) \), \( n \to +\infty \) where \( \{ Y(t), t \geq 0 \} \) is a diffusion process on \( [0, +\infty) \) corresponding to the backward differential equation \( U_t = \frac{1}{2} U_{xx} + (\gamma/x) U_x, U(x,0) = f(x), U_x(0,t) = 0 \).

Theorem 2. \( \lim_{n \to +\infty} \sup \frac{\log n}{2n \log \log n} \frac{1}{\sigma_n} \) with probability 1. (Received November 3, 1970.)


Let \( E \) be a countable state space, \( P(t) \) a semigroup on \( E, Q = P'(0), \) and \( X(t) \) an associated Markov chain. If \( A \) is any finite subset of \( E, I = E - A \) and \( F(t) \) the semigroup defined by \( X(t) \) killed at \( T_A \), then \( \{ R \}_{1}^{\infty} \) and \( G \) of \( P(t) \) and \( F(t) \) are related by \( R_{\lambda} = G_{\lambda} + \sum_{a,b} M_{ab}^{\lambda} w_{\lambda}^a \), where \( k_{\lambda}^{a} \) and \( w_{\lambda}^a \) are transforms of exit and entrance laws relative to \( F(t) \) and where \( M(\lambda) \) is an invertible matrix.

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on $A \times A$. Remarks. (1) If $P(t)$ is standard on $E$, the entrance law $(w^a(t)/\sum_i w^i(t))$ can be interpreted as a probability distribution given that the last exit from $A$ before $t$ occurs at the last exit from $a$ before $t$.

(2) There is a constant $w(a,A)$ such that $w(a,A) = P_a$ (time of last hit of a before $T_{A-a}$ equals $T_b$). Finally, the techniques of the decomposition can be reversed and applied to the question of when $Q$ determines $P(t)$ uniquely. Generalizations of recent results by Reuter and Williams are obtained thereby. (Received November 2, 1970.)


Suppose $\{X(t): t \in [0, +\infty)\}$ is a Brownian motion process. If $I \subset [0, +\infty)$ and $\{Y(\alpha): \alpha \in I\}$ is a collection of random variables it is said that $\{Y(\alpha): \alpha \in I\}$ can be obtained from the Brownian motion by means of a random time change provided there exists a collection $\{T_\alpha: \alpha \in I\}$ of nonnegative stopping times for the Brownian motion such that (a) for each $\omega$, $T_\alpha(\omega)$ is nondecreasing in $\alpha$ for $\alpha$ ranging in $I$ and (b) for each $\alpha \in I$, $X(T_\alpha) = Y(\alpha)$ a.s. Given a Brownian motion $\{X(t): t \in [0, +\infty)\}$ it is desirable to know what properties $\{Y(\alpha): \alpha \in I\}$ must have in order that it can be obtained from the Brownian motion by means of a random time change. In the discrete case the author shows that this can be done if $\{Y(n): n = 1, 2, \ldots\}$ is a sequence of random variables and if $\sum_{n=1}^\infty C_n$ is a sequence of positive constants such that for each $n$, $\sigma(1, 2, \ldots, n)$ has order in $1$ and $g(0) = 0$. Some results in the continuous case are also given. (Received November 4, 1970.)


We study stopping rules for optimal drawing without replacement from an urn containing $g$ good balls with value $+1$ and $b$ bad balls with value $-1$. When the probability distribution $\theta$ of the pairs $(g, b)$ is known and $E(g+b) < \infty$, we give a backward induction (dynamic programming) algorithm for the optimal strategy and value $V(\theta)$ of the urn which is well-suited to numerical computation. For the case where $g+b$ is known, various theorems and counterexamples are presented. (Received November 4, 1970.)


Let $q$ map the positive integers into $[0, 1]$ with $q(k) \downarrow 0$. Let a probability $(n!)^{-1}$ be attached to each permutation $\sigma = (\sigma(1), \ldots, \sigma(n))$ of the integers $1$ through $n$. For each $\sigma$, let $(X_1, \ldots, X_n)$ be the $n$-tuple of relative rankings determined by $X_k = j$ if $\sigma(k)$ has rank $j$ among $\sigma(1), \ldots, \sigma(k)$. We consider the Optimal Stopping Problem $v_n = \max_t E q(\sigma(t))$ where $t$ runs through stop rules over the space of relative rankings $(X_1, \ldots, X_n)$. If we define $R_k(\alpha) = \sum_{k=1}^{\infty} (k-1)! q(j) \alpha^j (1 - \alpha)^{k-j}$, then the differential equation $g'(\alpha) = -\frac{1}{(\alpha)^{-1}} \sum (R_k(\alpha) - g(\alpha))^+ + g(\alpha)$ has a unique solution on $[0, 1]$, and there exists a determined integer $n_0$ and a determined constant $\alpha_0 > 0$ such that if $n \geq n_0$, then $|v_n - g(0)| \leq 2\{\log n - n^{-1} + q(\alpha_0 \log n)\}$. The function $g$ also determines the optimal stop rule $t_n$ in the same sense for large $n$. Analogous results are obtained for a class of monotone but unbounded functions $q$. These results
extend the work of Chow, Moriguti, Robbins, and Samuels: "Optimum selection based on relative rank," Israel J. Math. 2(1964), 81-90. (Received November 5, 1970.)


In the theory of age-dependent branching processes with arbitrary state space integral equations of the form

\[ M(x, t) = f(x, t) + \int_0^t \int_{\mathcal{X}} F(x, y, ds)M(y, t-s)d\mu(dy) \]

arise, where \( X \) is some well-defined set, \( \mathcal{F} \) is a \( \sigma \)-algebra of subsets of \( X \), and \( \mu \) is a \( \sigma \)-finite measure on \( \mathcal{F} \). Suitable measurability conditions are placed on all functions in (1) so that the integral equation is well defined, and it is assumed that \( F(x, y, \cdot) \) on \([0, \infty)\) is nondecreasing and right continuous for all points \( (x, y) \in X \times X \). Put (2) \( f(x, y) = \lim_{t \to \infty} F(x, y, t) \) and (3) \( N(x, y) = \int_0^\infty 1F(x, y, dt) \), and suppose \( F \) and \( N \) belong to the space \( L^2(\mathcal{F}, \mathcal{F}, \mu) \) and the kernel \( F \) generates a strictly positive operator mapping \( L^2(\mathcal{F}, \mathcal{F}, \mu) \) into itself. Let \( \rho \) be the largest positive eigenvalue of \( F \), suppose \( \rho = 1 \), and let \( u, v \) be the right and left eigenvectors of \( F \) corresponding to \( \rho = 1 \) with the properties \( ||u|| = 1 \) and \( (u, v) = 1 \).

Then under conditions, which are too lengthy to give here, it can be shown that (4) \( \lim_{t \to \infty} M(x, t) = (u(x)/Nuv) \int_0^\infty f(y, s)v(y)u(dy) ds \). This result, among other things, makes precise a conjecture of T. E. Harris, (1963), "The theory of branching processes." Prentice-Hall, Englewood Cliffs, N.J., Chapter IX, § 7. The complete identification of the constant on the right in (4) seems to be new. (Received November 5, 1970.)


Let \( F \) be an infinitely divisible distribution with mean zero. Then there is a Wiener process \( (\Omega, \mathcal{F}, W_t) \) and a subordinator \( T(s) \) defined on \( \Omega \) such that \( W(T(s)) \) is a process with independent identically distributed increments and \( W(T(1)) \) has distribution \( F \). Moreover for each \( S, T(s) \) is a \( \mathcal{F}_t \) stopping time. (Received November 5, 1970.)


A study of a random or stochastic integral equation of the Fredholm type of the form \( x(t;\omega) = h(t;\omega) + \int_0^t k_0(t, \tau;\omega) \epsilon(\tau, x(\tau;\omega))d\tau, \ t \geq 0, \) is presented, where \( \omega \in \Omega \), the supporting set of the probability measure space \( (\Omega, \mathcal{A}, P) \). The existence and uniqueness of a random solution of the equation is considered by first investigating a stochastic integral equation of the mixed Volterra-Fredholm type of the form \( x(t;\omega) = h(t;\omega) + \int_0^t k(t, \tau;\omega)f(\tau, x(\tau;\omega))d\tau + \int_0^\infty k_0(t, \tau;\omega)\epsilon(\tau, x(\tau;\omega))d\tau, \ t \geq 0. \) A random solution, \( x(t;\omega) \), of an equation such as those above is defined to be a random function which satisfies the equation almost surely. Several theorems and useful special cases are presented which give conditions such that a random solution exists for each type of equation. This paper generalizes results of Anderson (Ph.D. Dissertation, University of Tennessee, 1966) and Tsokos (Math. Systems Theory 3(1969), 222-231) using the theory of admissibility of Banach (or Hilbert) spaces and methods of probabilistic functional analysis. (Received November 5, 1970.)

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A nonlinear stochastic integral equation of the Volterra type in the form \( x(t;\omega) = h(t;\omega) + \int_{0}^{t} k(t,\tau;\omega) x(\tau) d\tau \), \( t \geq 0 \), where \( \omega \in \Omega \), the underlying set in the probability measure space \((\Omega, A, P)\), was studied by Tsokos (Math. Systems Theory 3 (1969), 222-231) with respect to the existence of a unique random solution. A random solution of the stochastic integral equation is defined to be a second order stochastic process, \( x(t;\omega) \), which satisfies the equation with probability one. A stochastic approximation procedure of Burkholder (Ann. Math. Statist. 27 (1956), 1044-1059) is applied in order to approximate the random solution. Conditions on the functions \( h(t;\omega), k(t,\tau;\omega), \) and \( f(t, x) \) are presented for which the stochastic approximation procedure converges to a realization of the random solution with probability one. (Received November 5, 1970.)

62 Statistics


The \( k \)-statistics \( k_{p} \) of Fisher (Proc. London Math. Soc. 30 (1929)) are sample symmetric functions whose expected values are cumulants \( \kappa_{p} \) of the infinite population. Fisher introduced a combinatorial method to obtain sampling cumulants of \( k \)-statistics which was modified for finite populations by Wishart (Biometrika 39 (1952)). Dwyer and Tracy (Ann. Math. Statist. 35 (1964)) extended Fisher's combinatorial method to obtain products of polykays and gave the multiplication rules for \( k_{p} \). Nagambal and Tracy extended the above for \( \sum q_{i} = 5 \) (Ann. Math. Statist. 41 (1970)) and 6 (ASA meetings, Detroit, Dec. 1970)). The formulae for multiplying \( k_{p} \) by polykay products of weight \( \leq 4 \) were given by
Tracy (Ann. Math. Statist. 39 (1968)). Nagambal and Tracy (submitted for Annals) further developed the
formulae to cover the case when $\sum q_i = 5$. The aim of the present paper is to provide the formulae for multiply-
ing a polykay by polykay products of weight 6. Each formula gives the multiplication rule corresponding to the
particular polykay product. The formulae for specific polykay products can be obtained by specifying the \{ \},
which can be used to obtain finite moment formulae useful in obtaining the distributions of ratio statistics.
(Received October 26, 1970.)

682-62-2. DERRICK S. TRACY and B. C. GUPTA, University of Windsor, Windsor 11, Ontario,

Various methods for multiplication of k-statistics have been suggested by different authors in statistical
ordered partitions. In this paper, the method is further developed for multiple products and Kronecker products
of ordered partitions introduced. Certain rules which systematize the computation and drastically reduce the
sizes of matrices involved are devised. Products of k-statistics of weight 7 have been worked out with this
method and those of lower order verified. The above rules are found very effective in these computations. Using
ordered partitions, expressions for k-statistics in terms of s-functions and augmented monomial functions are
given for weight \( \leq m \). (Received October 29, 1970.)

Certain algebraic properties of ordered partitions and their uses in statistics.

A partition \( \alpha = (\alpha_1, \alpha_2, \ldots, \alpha_m) \) of any weight \( m \) is called an ordered partition if \( i \neq j \) or \( i \neq j \), for all
\((i, j), i, j = 1, 2, \ldots, m. \) Carney (Ann. Math. Statist. 39 (1968), 643-656) proved that the set \( L_m \) of ordered
partitions of weight \( m \), with the subpartition of partial ordering is a lattice. In this paper it is further proved
that this lattice is a complemented lattice. Also the set of ordered subpartitions of any ordered partition of \( L_m \)
forms a sublattice. The Kronecker product of ordered partitions of weights \( m_1, m_2, \ldots, m_n \) is defined and
it is proved that this product forms a sublattice of the lattice of ordered partitions of weight \( m_1 + m_2 + \ldots + m_n \).
This definition of Kronecker product of ordered partitions further facilitates the multiplication of k-statistics of
any degree. (Received October 29, 1970.)

682-62-4. FREDERICK C. DURLING, Medical University of South Carolina, Charleston, South
Carolina 29401. On the distribution of the correlation coefficient for samples from the trivariate normal
distribution. Preliminary report.

Let \( X_1, X_2, \ldots, X_N \) be a sample of size \( N \) from a p-variate normal population. If \( p = 2 \), the density of
\( \rho \), where \( \rho \) is the population correlation coefficient, has been derived by R. A. Fisher. For \( p = 3 \), the
joint density of the three sample correlation coefficients has been published for the special case when the
population correlation coefficients are zero. This joint density is a special case of the Wishart distribution.
However, the joint density has not been derived in general, even for a small sample. The joint density (for \( p = 3 \))
in general will be derived for a sample of four, utilizing the Wishart distribution of \( A = \sum_{k=1}^{N} (X_k - \bar{X})(X_k - \bar{X})' \),
where the \( X_k \)'s are independent, each with the distribution \( N(\mu, \Sigma) \). The joint density is expressed in terms of
elementary functions. (Received November 4, 1970.)

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We describe a method for smoothing the observed values of a function which has been measured for distinct values of its argument. It is assumed that the measurements are within some tolerance specified at each point of observation. Linear programming is employed to obtain optimal smoothing of the (local) degree desired subject to the restraints imposed by the tolerances. First and third degree smoothing are discussed, but the method may be extended to higher degree. (Received November 4, 1970.)

65 Numerical Analysis

682-65-1. DAVID KENNETH KAHANER, Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico 87544. Minimum norm differentiation formulas.

Numerical differentiation formulas of the form \( \sum_{i=1}^{N} w_i f(x_i) = f^{(m)}(a) \), \( a \leq x_i \leq b \), are considered. The roundoff error of such formulas is bounded by a value proportional to \( \sum_{i=1}^{N} |w_i| \). We consider formulas that have minimum norm \( \sum_{i=1}^{N} w_i^2 \), and converge to \( f^{(m)}(a) \) as \( b - a \to 0 \). The resulting roundoff error bounds can be several orders of magnitude less than corresponding bounds for high order differences. (Received May 18, 1970.)


A computational algorithm for obtaining the numerical solution to problems of the form \( u' (z) = A(z)u(z) + B(z)v(z) \), \(-v' (z) = C(z)u(z) + D(z)v(z)\), \( a \leq z \leq b \), \( u(a) = u_a \), \( v(b) = v_b \), is presented. Here \( u(z) \) and \( v(z) \) are \( n \) dimensional column vectors while the functions \( A(z) \), \( B(z) \), \( C(z) \) and \( D(z) \) are \( n \times n \) matrices. We assume that the coefficients \( A(z) \), \( B(z) \), etc. are piecewise continuous, periodic functions, all having the same period \( p \). The method is applicable to ordinary, linear, homogeneous differential equations of order \( 2n \) with periodic coefficients and quite general boundary conditions. The method of invariant imbedding is the basic mathematical tool used in the development of the algorithm and all results are rigorously established. A principle advantage of the algorithm is that integrations of the differential equations involved must be done over only one period; recursion formulas are used to obtain the remaining information over the entire interval \( a \leq z \leq b \). A numerical example is presented which serves to demonstrate the accuracy, efficiency, and stability of the algorithm. (Received October 12, 1970.)


This paper develops a procedure for deriving optimal quadrature formulas of the form \( \int_{a}^{b} f(x) \, dx = \sum_{i=0}^{N} w_i f(x_i) + E(f) \) given the nodes \( x_0, x_1, \ldots, x_N \), the interval \( [a,b] \), and the requirement that \( E(f) = 0 \) for \( f \in \varphi_n \). Here \( \varphi_n \) represents all polynomials of degree \( n \) or less, and \( n \leq N \). Two types of optimal formulas can be handled by this same general procedure; these are, (1) the smoothest or minimum variance formulas.
described by the condition that $\sum_{i=0}^{N} w_i^2$ be a minimum, (2) the Sard "best" formulas described by the condition that $J = \int_c K(t) dt$ be a minimum. Here $K(t)$ is the Peano kernel such that $E(f) = \int_c K(t)f^{(n+1)}(t) dt$. Using appropriately constructed generalized inverses, systems of differentiation formulas of the form $f(x_i) = \sum_{k=0}^{n} d_{ik} f(x_k) dx + R_i(f)$ for $i = 0, 1, \ldots, N$ are inverted to produce the desired optimal quadrature formulas. A FORTRAN program has been written which implements this procedure to compute both smoothest and Sard "best" formulas. (Received October 14, 1970.)

682-65-4. WILLIAM L. MORRIS, University of Houston, Houston, Texas 77004. Iterative solutions of $Ax = \lambda x$. Preliminary report.

It is known (Bauer and Householder, Numer. Math. 2(1960), 42-54) that if $A$ is a normalizable matrix then there is a scalar $\lambda(A)$ such that for each unit vector $y$ the disc $D(A; y) = \{ \lambda : |\lambda - y^H Ay| \leq \| \lambda \|_A \rho(A) \}$ contains an eigenvalue of $A$ where $\rho(A; y) = \| (A - y^H Ay)y \|$. This theorem is used to develop an algorithm to calculate the eigensystem of $A$ by considering the minimization of $\rho(A; y)$ over a sequence of two-dimensional subspaces. Applicable theorems are discussed. (Received October 19, 1970.)


The main result of this paper establishes the fourth order convergence of a sequence of bicubic spline interpolants to a function $f \in C^4[0,1]$, independent of the mesh ratios. In addition, explicit error bounds are given for bicubic spline interpolation and cubic spline blended interpolation. (Received October 26, 1970.)

682-65-6. CRAIG A. WOOD, Oklahoma State University, Stillwater, Oklahoma 74074. Multiple zeros of polynomials.

Let $P(X) = a_0 X^n + a_1 X^{n-1} + \ldots + a_n$ where each $a_j$ is a complex number and $a_0 \neq 0$. This paper is concerned with the numerical solution of such polynomials having multiple zeros. An algorithm based on the greatest common divisor of two polynomials is presented. Results of this algorithm are compared with the results of Newton's method and Muller's method on polynomials having multiple zeros. (Received October 28, 1970.)

682-65-7. COLIN W. CRYER, Computer Sciences Department, University of Wisconsin, Madison, Wisconsin 53706. The method of Christopherson for solving free boundary problems for infinite journal bearings by means of finite differences.

A method for solving free boundary problems for journal bearings by means of finite differences has been proposed by Christopherson. This method is analysed in detail for the case of an infinite journal bearing where the free boundary problem is as follows: Given $T > 0$ and $h(t)$ find $\tau \in (0, T)$ and $p(t)$ such that (i) $[h^3 p']' = h'$ for $t \in (0, \tau)$, (ii) $p(t) = 0$ for $t \in [\tau, T]$, (iii) $p(0) = 0$ and (iv) $p'(T-0) = 0$. First it is shown that the discrete approximation is accurate to $O(\frac{\Delta t^2}{2})$ where $\Delta t$ is the stepsize. Next it is shown that the discrete problem is equivalent to a quadratic programming problem. Then the iterative method for computing the discrete approximation is analysed. Finally, some numerical results are given. (Received October 29, 1970.)

The partial differential equations which are considered here have the form \[ \left( \frac{\partial}{\partial x} \right)^{2j-1} u = 0, \] where \( j \) is a positive integer. The stability of some finite-difference approximations to these equations is discussed. These approximations are then applied to certain half-strip problems. (Received October 29, 1970.)


A method for approximating integrals of the form \( \int_a^b e^{-\lambda x} f(x) \, dx \), more conveniently approximated in the form \( \int_0^n e^{-\lambda x} f(x) \, dx \), is presented. The sum \( \sum_0^n e^{-\lambda K} f(K) \) is used for the approximation and the error term, the focal point of interest, in the difference between the integral and its approximating sum is carefully examined. Half integer data points are also used and again the error structure is carefully examined. In addition integer and half integer sample points are used to investigate the integral when complex \( \lambda \) is used. This case for \( \lambda \) yields approximations for integrals of the form \( \int_a^b \cos (Kx) f(x) \, dx \) and \( \int_a^b \sin (Kx) f(x) \, dx \) which are of constant interest. The approximations to the integrals of this type are compared to the approximations for the same integrals obtained by the Filon method. (See, e.g., R. W. Hamming, "Numerical methods for scientists and engineers," McGraw-Hill, New York, 1962, p. 319.) Numerical results are presented to indicate the usefulness of the various formulas developed. (Received November 2, 1970.)


The concept of minimal norm quadratures for Hilbert spaces of analytic functions is extended to functions with a uniformly convergent Fourier-Chebyshev expansion on the interval \([-1,1]\) and for which the series of coefficients is absolutely convergent. If \( T_n(x) \) is the \( n \)th degree Chebyshev polynomial of the first kind and \( R_n \) denotes the \( n+1 \) point quadrature remainder functional, then for an arbitrary integer \( k \) and a fixed set of nodes, weights are chosen to minimize \( W(n,k) = \sum_{i=0}^k R_n(T_i) \). The solution of this problem leads to the answer of a question posed by Wilf (Numer. Math. 6(1964), 315-319). (Received November 2, 1970.)

682-65-11. GREGORY M. NIELSON, Arizona State University, Tempe, Arizona 85281, ROBERT E. BARNHILL, University of Utah, Salt Lake City, Utah 84112 and ROBERT M. FLEGAL, Xerox Research Laboratory, Palo Alto, California 94106. Probabilistic surface representation and computer graphics.

This research concerns the representation of surfaces, with probabilities incorporated, as well as the computer graphics implementation of the resulting surfaces. The numerical analysis draws on the work of A. Sard ("Linear approximation", Amer. Math. Soc., Providence, R. I., 1963), S. Golomb and H. F. Weinberger "On numerical approximation", edited by R. E. Langer, U. of Wisconsin Press, Madison, Wisc., 1959. (Received November 2, 1970.)
Let $E$ be a Banach space and $T$ a completely continuous operator on it. The equation $x = Tx$ may be approximated by $x_n = P_n Tx_n$ (Galerkin's method) or $x_n = P_n Tx_n + S_n x_n$ (perturbed Galerkin's method) where $P_n$ is a projection operator onto $F_n \subset E$ and $S_n$ is some perturbation operator on $E_n$. Valinikko ("Galerkin's perturbation method and the general theory of approximate methods for nonlinear equations," Z. Vycisl. Mat. i Mat. Fiz. 7(1969), 723-751 = U.S.S.R. Comput. Math. and Math. Phys. 7(1969)) has considered perturbed Galerkin's method and proved several convergence theorems. Here these results are applied to equations of the form $x = NTx$, $x_n = NT_n x_n$, where $\|T - T_n\| \to 0$ as $n \to \infty$, and $N$ is some nonlinear operator. These results are then used to give some methods for the numerical solution of nonlinear problems of the form $L(y) = F(s, y, y' - y(N - 1))$ with linear boundary conditions, $L$ being a linear self-adjoint differential operator of order $n$.

(Received November 2, 1970.)

Let $\mu$ and $\delta$ be the averaging and central difference operators and write the general linear functional on the space $H_{2k}^2$ of polynomials of degree $\leq 2k$ as $L(f) = (P(\delta^2) + \mu \delta Q(\delta^2)) f(0)$, where $P$ and $Q$ are polynomials of degree $k$ and $k - 1$ respectively. Suppose $v(r) = f(r) + \epsilon(r)$, where $f$ is in $H_{2k}^2$ and $\epsilon$ is a zero mean stationary random error with continuous spectral density $\phi(\lambda)$. For a fixed $n > k$, denote the operation defined by (1) $u = \sum_{-n}^{n} W_r v(r)$, where $W_{-n}, \ldots, W_{n}$ are chosen to minimize $\sigma^2(u)$ subject to the requirement that (2) $E(v) = L(f)$ for every $f$ in $H_{2k}^2$ by $MV(n, k; \phi; L)$ (minimum variance estimation of $L(f)$, with respect to the spectral density function $\phi$, of span $2n+1$ and degree $2k$). Let $C(\lambda) = \sum_{-n}^{n} W_r e^{i r \lambda}$ and $C_L(\lambda) = p(-4 \sin^2 \lambda/2) + i \sin \lambda Q(-4 \sin^2 \lambda/2)$. Then (2) implies that $C(\lambda) = C_L(\lambda) + O(\lambda^{2k+1})$, $\lambda = 0$; if in addition (3) $|C(\lambda)| < |C_L(\lambda)|$, $0 < |\lambda| \leq \pi$, then we say that (1) is a strong estimate of $L(f)$. (For an interpretation of (3) see [Abstract 69T-C24, these Notices] 16(1969), 681; where a strong estimate was called "stable.") Conditions are sought on $\phi$ and $L$ such that $MV(n, k; \phi; L)$ is strong for all $n \geq k+1 \geq 1$. Results obtained previously for $L(f) = f(0)$ [Abstract 672-661, these Notices] 17(1970), 273; to appear in J. Math. Anal. Appl.] are extended to a class of functionals which includes $L(f) = f(r)(0), 0 \leq r \leq 2k$. (Received November 3, 1970.)

WITHDRAWN.

This note describes a finite difference method, for the solution of first order quasi-linear hyperbolic systems that is based in a certain sense on the characteristics. Because knowledge of the diagonalizing matrices is required, its use involves more effort than might normally be expected of a finite difference approach. However, this also allows characteristic equations needed at a boundary to be translated into difference equations identical to those used at interior points. (Received November 4, 1970.)
Given an incidence matrix $E$ with $p$ rows and $q + 1$ columns such that $E_{ij} = 0$ or $1$, $i = 1, \ldots, p$, $j = 0, 1, \ldots, q$, $\sum E_{ij} = n$. Take an interval $[a, b)$ and a set of $p$ nodes $a < x_1 < \ldots < x_p < b$. Take an $n$-dimensional subspace $X$ of $C^q[a, b]$ and consider the problem of finding an element $\phi$ in $X$ such that $\phi^{(l)}(x_j) = d^l \phi / dx(x_j) = \gamma_j^{(l)}$ whenever $E_{ij} = 1$, where the $\gamma_j^{(l)}$ are arbitrarily given. In this work we present a generalization of the results given by Atkinson and Sharma (SIAM J. Numer. Anal. 6(1969), 230-235) and Matthews (SIAM Rev. 12(1970), 127-218). Assumptions. (a) $X$ satisfies the Haar condition on $[a, b]$; (b) $\dim X^{(q)} = \dim X - q$, $X^{(q)} = \{d^q \phi / dx^q; \phi \in X\}$. Such space $X$ necessarily contains $1, x, \ldots, x^{q-1}$. Theorem. The interpolation problem is uniquely solvable for any set of constants $\{\gamma_j^{(l)}\}$ if $E$ is conservative and satisfies the Polya condition. The matrix $E$ is conservative if each sequence starting at $(i_0, j_0)$ (say) of consecutive 1's in any row of $E$ satisfies the condition that at least one of the blocks $\{E_{ij}; i < i_0, j < j_0\}$ and $\{E_{ij}; i > i_0, j < j_0\}$ is free of 1's. The matrix $E$ satisfies the Polya condition if $m_0 + m_1 + \cdots + m_j = j + 1$ ($j = 0, \ldots, q$), where $m_j = \sum E_{ij}$ ($j = 0, \ldots, q$). (Received November 5, 1970.)

**Applications of generalized Crank-Nicholson method.**

It is well known that the finite-difference scheme $u_{i,j+1} - u_{i,j} = r(h^2 u_{i+1,j} + 2u_{i,j} + u_{i-1,j}) + (1 - r)(u_{i,j+1} - 2u_{i,j} + u_{i,j-1})$ approximating the equation $\partial u / \partial t + \partial^2 u / \partial x^2$ is unconditionally stable for $1/2 \leq r \leq 1$. (Generalized Crank-Nicholson Method.) We have written programs using BASIC for applications of this method with selected boundary value conditions. The results of these applications are reported in this paper. (Received November 5, 1970.)
682-73-1. K. L. ARORA, 582, Sector 11B, Chandigarh 11, India and Panjab University, Chandigarh, India. **Finite flexure of an initially curved cuboid under nonuniform temperature distribution.**

Using finite deformation theory, thermoelastic stresses produced as a result of flexure of an initially curved cuboid under nonuniform axially symmetric temperature distribution have been evaluated. The effect of temperature distribution on the stresses is shown by comparison of the corresponding force systems for a Mooney type material. Dislocation solution for a solid cylinder is obtained as a special case. (Received August 18, 1970.)

682-73-2. LOKENATH DEBNATH, East Carolina University, Greenville, North Carolina 27834. **Forced vibrations of anisotropic elastic sphere.**

A study is made of the dynamic response of a thick, homogeneous, and anisotropic elastic sphere under uniformly distributed internal and external pressure. The problem is solved exactly by using the Laplace transform treatment. Some characteristic features of the solution are explored. The frequency equations for the free vibration of isotropic and anisotropic spheres are obtained. It is shown that the solution of the corresponding isotropic problem considered by Tranter (Philos. Mag. 33 (1962), 1509) follows as a special case. (Received October 5, 1970.)


The microstructure theory of elasticity leads to a coupled system of elliptic singular perturbation equations of the boundary layer type. Extending the classical work of Višik and Lyusternik to such systems we derive a solution in terms of boundary layers and interior solutions. The validity of our solutions is proved using the estimates of Agmon, Douglass and Nirenberg for strongly elliptic systems (Comm. Pure Appl. Math. 17 (1964), 35-93). (Received November 2, 1970.)

76 Fluid Mechanics

682-76-1. S. MANICKAM, Western Carolina University, Cullowhee, North Carolina 28723. **Similarity and Green's function to the Cauchy problem.**

Similarity method of obtaining solutions to partial differential equations that remain invariant under a one-parameter group of transformations of the independent variables is adapted to finding Green's function to the Cauchy problem. The equations treated are Tricomi equation, Euler-Poisson-Darboux equation and a linearized version of Korteweg-deVries equation. The procedure, which is relatively uncomplicated with parabolic equations, involves suitable continuation of solutions across families of characteristics in hyperbolic problems. The method is then used to pick out singular parts of solutions corresponding to discontinuous initial values for problems in unsteady symmetric gas flow. (Received October 30, 1970.)
A two-parameter approximate method is obtained for solving the laminar boundary-layer equations for flow over a permeable surface with uniform suction. The method is based on physical ideas of dividing the boundary layer into two parts, the outer and inner. In the outer part of the boundary layer, where viscous effects are small, the development is given by the condition that the total head is constant along streamlines, apart from a second-order correction for viscosity. Near the wall, however, viscous forces must balance the pressure forces, and the profile adjust itself accordingly. Physically, suction affects the outer part of the boundary layer in that the streamlines are drawn towards the wall when the suction is applied. At the wall, the balance between viscous and pressure forces is influenced by the momentum of the fluid which is sucked away. The method is confirmed by comparison with the exact numerical solutions for some flows with and without pressure gradient. Also, it is found that the present method gives more accurate results for some cases than those obtained by the existing methods. (Received November 2, 1970.)

An analysis is presented for a two-dimensional flow of an incompressible, electrically conducting, rarefied gas past an infinite porous wall under the following conditions: (i) transverse magnetic field; (ii) the suction velocity normal to the plate oscillates in magnitude but not in direction about a nonzero mean; (iii) the free-stream velocity oscillates in time about a constant mean. The response of the skin-friction to the fluctuating stream and suction velocity is studied for variations in the suction parameter A, the rarefaction parameter $h_1$ and the frequency parameter $\omega$. It is observed that (i) the transient velocity increases with increasing A (suction parameter) or $h_1$ (rarefaction parameter) but decreases with increasing K (the magnetic field parameter). (ii) The amplitude of the skin-friction decreases with increasing $h_1$ or K, but increases with $\omega$ (the frequency). The phase of the skin-friction increases with increasing $\omega$ but decreases with increasing K. (Received November 4, 1970.)

This is a theoretical analysis of the unsteady Couette flow in magnetohydrodynamics. Two cases wherein (1) velocity of the moving plate varies as $(\text{time})^n$ and the plate is subjected to uniform suction and (2) velocity of the moving plate is uniform and the plate is subjected to a suction whose magnitude oscillates in time over a constant mean, are investigated. In the case of uniform suction, solutions are obtained for the momentum and magnetic field equations with the help of Laplace transform technique. The computed results have shown that suction retards the flow and decreases the induced field. The magnetic field decelerates the flow near the moving plate and accelerates near the stationary one. The skin friction on the moving plate increases with both suction and the magnetic field. In the case of periodic suction...
the amplitudes of skin friction and current density at the plates decrease, but the phase lags increase as the
frequency increases. They are little affected by frequency for higher values of magnetic parameter. The
amplitude of the skin friction on the moving plate decreases and that at the fixed plate increases while the
amplitudes of the current densities and the phase lags at both the plates decrease, with the increasing magnetic
field strength. (Received November 5, 1970.)

81 Quantum Mechanics

682-81-1. SERGIO ALBEVERIO, Department of Physics, Princeton University, Princeton, New Jersey
08540. On bound states in the continuum of N-body systems and the virial theorem.

We prove generalized versions of the quantum mechanical virial theorem and apply them to the investi­
gation of the spectrum of N-body Hamiltonians. We show in particular that for N particles interacting through
2-body potentials which may have singularities but "do not wiggle too much" no positive energy bound state can
exist. We also prove results on the absence of bound states with energy bigger than some value $E^0 < -\infty$ and
extend them to the case of N particles interacting through $\nu$-body forces ($\nu = 1, 2, \ldots, N$) and with an external
electromagnetic field. Also some remarks for the case of a Dirac electron in an external potential are given as
well as for some problems with boundary conditions. A by-product of this investigation is the unitarity of the
S-matrix and the strong asymptotic completeness for systems of N particles interacting by 2-body forces which
are not restricted to be purely repulsive. (Received November 3, 1970.)

83 Relativity

682-83-1. ADOLPH SELZER, 51 Linden Street, New London, Connecticut 06320. Theory of
special relativity in terms of absolute space and time. Preliminary report.

The clock paradox proved convincingly that the Minkowski-Einsteinian interpretation of four­
dimensional geometry is wrong. It will be shown in the presented paper that the Lorentz transforma­
tion can be explained as a relationship between two different states of motion of a test body observed
in the laboratory. This is an extension of a geometrical problem known in three-dimensional space
geometry, obtained by the introduction of the laboratory time as fourth coordinate. Since both the
space coordinates and the time are exclusively measured in the laboratory, they can be considered
as absolute, and not as relative as in the conventional theory of relativity. The wrong interpretation
of the Lorentz transformation in the latter is due to the fact that the ratio between the mass of the
earth, as the place of the laboratory, and the mass of the test body has not been taken into account.
(Received August 3, 1970.)

682-83-2. KISHORE B. MARATHE, University of Rochester, Rochester, New York 14627. On the structure
of spaces admitting gravitation fields.

Let $M$ be a pseudo-Riemannian manifold with fundamental tensor $g$ of signature $(+,-,-,+)$ and torsion-free
Levi-Civita connection $\Gamma^\mu_{\nu\lambda}$. Let $T$ be a symmetric covariant tensor of order two on $M$. We define gravitational
fields as the triple $(M, g, T)$, where $g$ satisfies Einstein's field equation with source (energy-momentum tensor) $T$. 241
Then there exists a linear transformation \( W \) of \( A^2(M) \)--the space of second order differential forms on \( M \)--such that \((M, g, T)\) is a gravitational field if and only if \( W \) commutes with the Hodge star operator on \( A^2(M) \).

(Received October 27, 1970.)

90 Economics, Operations Research, Programming, Games

682-90-1. SANJO ZLOBEC, McGill University, Montreal 110, Quebec, Canada. Optimality conditions in mathematical programming. Preliminary report.

Definition. Let \( \mathcal{L}: X \to Y \) be a continuous linear mapping between real Banach spaces \( X \) and \( Y \), \( B \in X^* \) a given vector, \( C \subseteq X^* \) and \( S \subseteq Y^* \) given nonempty closed convex cones. The system: \( \mathbf{b} - y \mathcal{L} \in C, y \in S \) is double-asymptotically consistent if there exist a sequence \( \{y^{(i)}, j \} \subseteq S, i, j = 1, 2, \ldots \), and a sequence \( \{c^{(i)} \} \subseteq C, i = 1, 2, \ldots \), such that \( \mathbf{b} = \lim_{i \to \infty} \lim_{j \to \infty} y^{(i)} + c^{(i)} \). Lemma. The system: \( \mathbf{b} - y \mathcal{L} \in C, y \in S \) is double-asymptotically consistent if and only if: \( \langle \mathcal{L}, z \rangle \in S^+ \), \( z \in C^+ \). This lemma is applied to the problem: maximize \( \{ \psi(x), x \in B, x \in D \} \). Theorem 1. If \( \mathbf{x} \) maximizes \( \psi \) over \( A = \{ x \in D : \mathcal{L}(x) \in B \} \) and if \( G \) is a closed convex cone in \( X \) such that \( K \cap G \subseteq P(A, \mathbf{x}) \), then \( \psi(\mathbf{x}) + y \cdot \psi(\mathbf{x}) \in G^-, y \in B^+[B, \mathcal{L}(\mathbf{x})] \) is double-asymptotically consistent. Theorem 2. If \( G \) is a closed convex cone in \( X \) such that \( x - x \in G \) for all \( x \in A \) or \( \Delta = \{ x : \mathcal{L}(x) \in B \} \) is pseudoconvex at \( \mathbf{x} \), \( \psi \) is pseudoconcave over \( A \) at \( \mathbf{x} \) or quasiconcave with \( \psi(\mathbf{x}) \neq 0 \), and if \( \psi(\mathbf{x}) \in \{ P(\mathbf{A}, \mathbf{x}) \cap H^+ \cap G \} \), then \( \mathbf{x} \) maximizes \( \psi \) over \( A \). \( M^+ \) is the polar of \( M, M^- = -M^+ \), \( P(M, \mathbf{x}) \) the pseudotangent cone to \( M \) at \( \mathbf{x} \), \( K = \{ x \in X : \langle \mathcal{L}(\mathbf{x}), k \rangle \in P[B, \mathcal{L}(\mathbf{x})] \} \). (Received October 21, 1970.)


A solution of a system of \( m \) equations (in at least \( m \) variables) is nondegenerate if it has at least \( m \) variables nonzero. A system of \( m \) equations is nondegenerate if no solution is degenerate and no two (distinct) solutions with exactly \( m \) nonzero variables have the same variables nonzero. A finite noncooperative game is nondegenerate if the system of equations in the corresponding complementarity problem is nondegenerate.

Theorem 1. A nondegenerate game is solvable if and only if it has a unique equilibrium. Theorem 2. For a polymatrix game, an equilibrium is stable if and only if it is nondegenerate. (Received November 3, 1970.)

94 Information and Communication, Circuits, Automata

682-94-1. EUGENE S. SANTOS, Youngstown State University, Youngstown, Ohio 44503. Minimization of maximin machines.

The minimization of maximin machines [E. S. Santos, "Maximin sequential-like machines and chains", Math. Systems Theory 3 (1969), 300-309] is examined with respect to various types of equivalence relations and minimal forms. Among the minimal forms considered are those which resemble the reduced form and minimal state form of stochastic machines. Full and effective solutions of the minimization problems are given with the aid of maximin algebra—a new type of algebra which resembles convex linear algebra. Algorithms for finding the minimal forms are provided. (Received October 19, 1970.)
Hybrid addition of matrices: A network theory concept.

Let A and B be the impedance matrices of passive resistive networks. It is well known that A and B are Hermitian semidefinite, abbreviated HSD. The impedance matrix of the series connection of 2 n-ports is given by $A + B$, the ordinary series sum. Anderson and Duffin (J. Math. Anal. Appl. 26 (1969), 576–594) have shown that the impedance matrix of the parallel connection is given by $A(A + B)^+ B$, where $C^+$ is the Moore-Penrose generalized inverse of C. A new matrix operation, hybrid addition is defined for HSD matrices using the hybrid connection of 2 n-port networks. The hybrid connection is part series and part parallel and contains each as special cases. Hybrid addition is shown to be commutative, associative and preserve the HSD partial order. If the hybrid sum of A and B is denoted $A * B$, then the series-hybrid inequality can be written:

$$(A + B) * (C + D) \geq A * C + B * D.$$  

ABSTRACTS PRESENTED TO THE SOCIETY

The next deadline for Abstracts will be January 20, 1971. The papers printed below were accepted by the American Mathematical Society for presentation by title. One abstract presented by title may be accepted per person per issue of these Notice. Joint authors are treated as a separate category; thus, in addition to abstracts from two authors individually, one joint abstract by them may be accepted for a particular issue.

Algebra & Theory of Numbers

71T-A1. WITHDRAWN.

71T-A2. VASANTI N. BHAT, University of Bombay, Bombay, India and S. F. KAPOOR, Western Michigan University, Kalamazoo, Michigan 49001. Minimally \((p - 6)\)-hamiltonian graphs.

A graph \(G\) on \(p\) points is minimally \(n\)-hamiltonian if it is \(n\)-hamiltonian (all induced subgraphs of \(G\) on \(p - n\) or more points are hamiltonian), but neither \(G\) is \((n + 1)\)-hamiltonian, nor \(G - x\) is \(n\)-hamiltonian for any line \(x\) of \(G\). Following the earlier results for \(n = 0, p - 5, p - 4\) and \(p - 3\), the case \(n = p - 6\) is studied here. Let \(M_p(n)\) denote the set of all minimally \(n\)-hamiltonian graphs on \(p\) points. Let \(p_1, 1 \leq i \leq 4\), denote the number of points having degree \(p - i\) in a graph \(G \in M_p(p - 6)\).

**Theorem.** Let \(G \in M_p(p - 6)\) and suppose \(p\) is even. Then \(p = p_4\) if and only if \(G\) is cubic and no subgraph of \(G\) is \(K_4\) or \(K_2,3\). Theorem. If \(G \in M_p(p - 6)\), then \(\delta(G) \geq p - 4\), and \(p_1 \leq 1, p_2 \leq 2, p_3 \leq 4\) and \(p_4 \neq 0\).

**Theorem.** (a) \(p_1 = 1 \iff p_2 = p_3 = 0\) and \(p_4\) is even; (b) \(p_2 = 2 \iff p_3 = 0\) and \(p_4\) is even; (c) \(p_2 = 1 \iff p_3 \leq 2\) and \(p_4\) is odd; and (d) \(p_1 = p_2 = 0 \iff p_3 \neq 4\) and \(p_4\) is even. Constructions are also described to show that each of the ten classes of minimally \((p - 6)\)-hamiltonian graphs is nonempty for every possible \(p\) for that class. (Received September 1, 1970.)

71T-A3. R. ARTHUR KNOEBEL, New Mexico State University, Las Cruces, New Mexico 88001 Criteria for primality of universal algebras containing a \(k\)-fold transitive group of permutations \((k = 1, 2)\). Preliminary report.

Let \(\mathcal{S} = (A; F)\) be a finitary finite algebra such that \(F\) contains a \(k\)-fold transitive group \(G\) of permutations. We give nsc for the primality of \(\mathcal{S}\) by stating the exceptional cases. \(\mathcal{S}\) is affine with respect to an Abelian group \(\mathcal{J} = (A; +)\) if every \(f \in F\) of \(n\) arguments can be written

\[
f(x_1, \ldots, x_n) = e_1(x_1) + \ldots + e_n(x_n) + e\]

for some endomorphisms \(e_i\) of \(\mathcal{J}\) and some \(e \in A\).

If \(\phi : A \to x_{i=0}^m H_i\), write \(\phi_i\) as the \(i\)th projection \(\phi_i : A \to H_i\); we agree that \(h = \{0, \ldots, h - 1\}\). \(\mathcal{S}\) is said to be special iff there are \(h \equiv 3, m \equiv 1, H_i (i = 0, \ldots, m)\) and a bijection \(\theta : A \to x_{i=0}^m H_i\) such that

\[
H_i = h (i = 1, \ldots, m)\]

and for all members \(f : A^n \to A\) of \(F\) and all \(k = 1, \ldots, m\), either \(\phi_k f : A^n \to h\) is not onto or \(\phi_k f^{-1} : (x_{j=0}^m H_j)^n \to h\) is a function only of one of the \(nm\) arguments \(x_{ji}\) which range over \(H_i = h (i = 1, \ldots, m; j = 0, \ldots, n - 1)\). (When \(\mathcal{S}\) is special, the kernel of \(\phi_0\) is a system of imprimitivity of \(P\).) **Theorem 1.** If \(k = 2\), then \(\mathcal{S}\) is not primial if (i) \(\mathcal{S}\) is affine w.r.t. some elementary Abelian group, or (ii) \(|A| = 2\) and \(\mathcal{S}\) has a nontrivial automorphism. **Theorem 2.** If \(k = 1\), then \(\mathcal{S}\) is not primial if (i) \(\mathcal{S}\) is affine w.r.t. some elementary Abelian group, (ii) \(\mathcal{S}\) has a nontrivial automorphism where each cycle has the same prime length, (iii) \(\mathcal{S}\) is not simple, or (iv) \(\mathcal{S}\) is special. (Received September 30, 1970.)
The well-known graph for Euler’s problem on the seven bridges of Königsberg (see, e.g., S. S. Cairns, "Introductory topology," The Ronald Press Co., New York, 1961) has the peculiar property that each of its vertices has an odd prime number of edges incident upon it and the total number of edges in the graph is an odd prime number. Definition 1. A vertex of a graph is said to be prime if it has a prime number of edges of the graph incident upon that vertex. Definition 2. A linear graph is said to be prim if it has odd prime vertices and only odd prime vertices. Definition 3. A linear graph is said to be highly prim if it is both prim and has an odd prime number of total edges.

Lemma 1. No prim graph has an odd number of vertices. Lemma 2. No prim graph with more than two vertices is unicursral. Lemma 3. There exist infinitely many (anisomorphic) highly prim graphs with $2n$ vertices for each positive integer $n$. Problem 1. Characterize the highly prim graphs. Problem 2. Which prim graphs with more than two vertices are Hamiltonian? Problem 3. Can the dual graph of a prim graph be prim? Problem 4. Can a self-dual linear graph be prim? (Received September 30, 1970.)

Let $R$ be a ring. If $N \subseteq M$ are right $R$-modules, define the closure of $N$ in $M$ to be $\{x \in M | (N : x) \text{ is large in } R\}$, denoted $\text{cl}_M(N)$. Let $Z(M) = \text{cl}_M(\{0\})$ and let $Z^*(M) = \text{cl}_M(Z(M))$, the singular and torsion submodules, respectively. $Z(R)$ and $Z^*(R)$ are two sided ideals of $R$, and $M/Z^*(M)$ is naturally an $R/Z^*(R)$-module. Let $E(M)$ be the injective hull of $M$. We can prove the following results.

Proposition 1. For any ring, $R$, and any $R$-module, $M$, we have the following $R$-isomorphisms:

$Z^*(E(M)) \cong E(Z^*(M)) \cong E(Z(M)) \cong \text{cl}_E(M)(Z^*(M))$. Proposition 2. If $R$ and $M$ are as in Proposition 1, then $E(M/Z^*(M)) \cong E(M)/E(Z^*(M)) \cong E_R/Z^*(R)(M/Z^*(M))$, where this last notation denotes the injective hull of $M/Z^*(M)$, considered as an $R/Z^*(R)$-module. Proposition 3. If $M = R$, then the objects in Proposition 2 are all rings whose multiplications extend the $R$-module multiplication. Moreover, these rings are regular, self-injective, Jacobson semisimple, and are ring isomorphic to their rings of $R$-endomorphisms. Remark. Proposition 3 implies that the injective hull of any ring contains—as an $R$-direct summand— a ring with the properties mentioned. Furthermore, this ring is zero if and only if $R = Z^*(R)$. (Received October 1, 1970.)

Let $G$ be a group such that every finitely generated subgroup of $G$ has an abelian subgroup of finite index. If $F$ is a field of characteristic zero and if $e$ is an idempotent in the group algebra $FG$ then the coefficient of the identity of $G$ in $e$ is rational. If $F$ is any field then left inverses in $FG$ are also right inverses. Examples of such groups are all abelian groups, all locally finite groups and all $FC$ groups. These results provide partial solutions to two of Kaplansky’s problems in ring theory. (See Amer. Math. Monthly 77(1970), 445-454.) (Received October 1, 1970.)

For a matrix with dominant diagonal, convenient upper and lower bounds for the determinant are available. Such bounds occur in the work of Price, Ostrowski, Hoffman, Haynsworth, and Brenner. In this article, the inequalities are used to bound the classical polynomials over a range of values of the argument. (Received October 2, 1970.)

71T-A8. ABRAHAM BERMAN, Centre de recherches mathématiques, Université de Montréal, Montréal 101, Québec, Canada, and ADI BEN-ISRAEL, Department of Applied Mathematics, Technion-Israel Institute of Technology, Haifa, Israel. A note on pencils of Hermitian or symmetric matrices.

Let A and B be two symmetric matrices of order n. A necessary and sufficient condition for the pencil \( \{A, B\} \) to contain a positive definite matrix is that the only positive semidefinite matrix of order n which is perpendicular to A and B is the zero matrix. A stronger result of Calabi, for \( n \geq 3 \), is derived. A similar condition is given for p symmetric matrices to have a positive definite linear combination. (Received October 2, 1970.)


The equivalence of (2) and (3) below does not require familiarity with our reference. Reference. Ben-Israel, Charnes, and Kortanek, "Duality and asymptotic solvability over cones," Bull. Amer. Math. Soc. 75 (1969), and erratum to same, ibid. 76 (1970). The ordered field \( \mathbb{R}(M) \) consists of the reals \( \mathbb{R} \) with a transcendental M adjoined which is larger than any \( r \in \mathbb{R} \). We are given a semi-infinite matrix (s.i.m.) interpreted as linear inequalities: \( u^T P_i \leq c_i \) for all \( i \in I \), where \( I \) is an arbitrary index set but, for some finite \( n \), each \( P_i \in \mathbb{R}^n \) and each \( c_i \in \mathbb{R} \) \( (u^T \) is the transpose of \( u \in \mathbb{R}^n \)). Theorem. The following are equivalent: (1) The s.i.m. is asymptotically consistent. (2) For every finite \( J \subseteq I \) the system \( u^T P_i \leq c_i \), \( i \in J \), is consistent. (3) The s.i.m. has a solution \( u \in \mathbb{R}(M)^n \). (4) The s.i.m. has a solution \( u \in \mathbb{R}(M)^n \) which has only polynomial part and degree \( \leq n \). (5) The s.i.m. has an asymptotic solution over the integers as net. Now let us minimize \( u^TP \) subject to the s.i.m. where \( P \in \mathbb{R}^n \). Corollary. The s.i.m. is IAC all solutions \( u \in \mathbb{R}(M)^n \) to it give positive infinite part in \( u^TP \). It is AC and AUBD iff for some solution \( u \in \mathbb{R}(M)^n \) the number \( u^TP \) has negative infinite part. (Received October 13, 1970.)

71T-A10. RAINER GÜTING, University of Zambia, Lusaka, Zambia. A system of axioms for Boolean algebras based on four simple relations.

Let a colony K be defined as a system consisting of an element 1 and at least one other element and let a binary operation \( "-" \) be defined on K such that K is closed under the operation \( "-" \) and that the following four simple relations hold for any elements \( a, b, c \in K \): (1) \( (a - b) - c = (a - c) - b \). (2) \( 1 - (1 - a) = a \). (3) \( a - a = 1 - 1 \). (4) \( a - (a - b) = a - (1 - b) \). Theorem. Every colony K can be transformed into a Boolean algebra by the definitions \( a' = 1 - a \), \( a \cap b = a - b' \) and \( a \cup b = (a' - b)' \) and vice versa every Boolean algebra is a colony with \( a - b \) defined by \( a - b = a \cap b' \). (Received October 19, 1970.)
Imbedding graphs in pseudosurfaces. Preliminary report.

Two problems posed by G. Ringel ("Combinatorial structures and their applications, " Guy et al, editors, Gordon and Breach, New York, 1970, 361-366) were: (i) determine if the genus of $K_{n,n,n,n}$ is $(n-1)^2$; (ii) consider imbedding graphs in the closed orientable 2-manifold of genus $\gamma$ (denoted by $S(\gamma)$) in which two distinct points have been identified. By an extension of the idea of (ii), a result consistent with the formula of (i) is obtained. Let $A$ denote a set consisting of $\sum_{i=1}^{t} n_i m_i \geq 0$ distinct points of $S(\gamma)$, $1 < m_1 < \ldots < m_t$. Partition $A$ into $n_i$ sets of $m_i$ points each, $1 = 1, \ldots, t$. For each set of the partition, identify all the points of that set. The result is called a pseudosurface, designated by $S(\gamma; n_1 (m_1), \ldots, n_t (m_t)) = S(\gamma; g)$, where $g = \sum_{i=1}^{t} n_i (m_i - 1)$.

Theorem. Let a graph, with $p$ vertices and $q$ edges, be 2-cell imbedded, with $r$ regions, in $S(\gamma; g)$; then $p - q + r = 2 - 2\gamma - g$. Thus $S = S(\gamma; g)$ has euler characteristic $\chi(S) = 2 - 2\gamma - g$. The pseudocharacteristic, $X'(G)$, of a graph $G$ is defined to be the largest integer $\chi'(S)$ for all pseudosurfaces $S$ in which $G$ can be imbedded. A triangular imbedding in a pseudosurface can be found for $K_{n,n,n,n}$ so that: Theorem. $\chi'(K_{n,n,n,n}) = 2n(2-n)$. Note that if $K_{n,n,n,n}$ is imbedded in $S(\gamma)$, $\gamma = (n-1)^2$, then the imbedding is triangular, and $S(\gamma)$ has characteristic $2n(2-n)$. (Received October 20, 1970.) (Author introduced by Professor Arthur Thomas White.)


Denoting by $H(n, r)$ the number of $n \times n$ matrices with nonnegative integer elements and common row and column sums $r$, and by $A(n, r)$ the number of such matrices with entries 0, 1 only, it is shown that these numbers are asymptotic to ((rn)!)/(r!)^(2n) exp(± (r-1)^2 / 2) respectively. The proof is elementary, both results being obtained simultaneously. The asymptotic form of $A(n, r)$ has been derived by P. E. O'Neil by a quite different method. (Received October 20, 1970.)


In Abstract 70T-A213, these Notices 17(1970), 947, we announced a stable K-theory of rings, where $\hat{K}^i(R) = \lim_{n \to \infty} K^{i-n}(R^n)$, and $\Gamma R = [t, \tau^{-1}] (t-1)$, the direct system being defined by cup product with $[t] \in K^{-1}(\Gamma R)$, the "fundamental class of the circle". In this paper we show that $\hat{K}$-theory is a multiplicative theory. The idea is to show that cup product with $[t]$ has all the formal properties of the suspension homomorphism in ordinary homotopy theory, and then to use an idea of G. Whitehead (Trans. Amer. Math. Soc. 102(1962), 256) for defining products in the limit. In addition, if $B$ is a ring which is finitely generated and projective as a right $A$-module, for the subring $A$, then there is a transfer homomorphism $\hat{K}^i(B) \to \hat{K}^i(A)$, $i \in Z$. If $B$ is in addition commutative, then we establish the multiplicative relation $\operatorname{trans}_f(x \cup y) = (\operatorname{trans} x) \cup f y$, where $x \in \hat{K}^i(B)$, $y \in \hat{K}^i(A)$, and $f: A \to B$ is the inclusion. This extends a construction of Bass and Tate. (Received October 28, 1970.)
Analysis


In the affine plane \( P \) over the field \( \mathbb{R} \) of real numbers, let there be given 4 different points \( p_j \) such that (i) they do not form a trapezoid, and (ii) no triple of them is collinear; let there also be given 4 different real numbers \( z_j \) (\( j = 0, 1, 2, 3 \)). A function \( f: P \rightarrow \mathbb{R} \) is called parabolic, if it can be written in a suitable affine coordinate system of \( P \) as a polynomial in one variable whose degree is at most 2. **Theorem.** There are at least one and at most 3 parabolic functions \( f: P \rightarrow \mathbb{R} \) with \( f(p_j) = z_j \) (\( j = 0, 1, 2, 3 \)). If we admit also nonreal parabolic functions, we get exactly 3 of them by our geometrical construction; their arithmetical mean turns out to be real and is considered the solution of the interpolation problem \( p_j - z_j \) (\( j = 0, 1, 2, 3 \)). (Received September 28, 1970.)


A method is developed for iteratively constructing solutions to multi-dimensional integral equations of the form

\[
\int_{0}^{\Omega} \ldots \int_{0}^{\Omega} k(x_1 - y_1, \ldots, x_n - y_n) f(y_1, \ldots, y_n) dy_1 \ldots dy_n = a f(x_1, \ldots, x_n) + g(x_1, \ldots, x_n),
\]

where \( a = 0 \) or \( a = 1 \), \( g \in L^2(\mathbb{R}^n) \), \( f \) is sought in \( L^2(\mathbb{R}^n) \), and \( k \) has the following properties: (1) \( k \in L^1(\mathbb{R}^n) \) or \( L^2(\mathbb{R}^n) \); (2) if the Fourier (or Fourier-Plancherel) transform of \( k \) is \( K \), then \( a - K(y) \), \( y \in \mathbb{R}^n \) lies in a closed, bounded, convex set not including the origin. The integral equation then possesses a unique solution which can be constructed iteratively. A bound on the error committed, in the \( L^2 \) norm, when the iterate is stopped at a particular step is given. Similar results are given for a problem related to the integral equation and involving boundary values of functions of several complex variables. (Received October 5, 1970.)

71T-B3. ANTHONY P. BLOZINSKI, Wright State University, Dayton, Ohio 45431. Some remarks on a convolution theorem for \( L(p,q) \) spaces. Preliminary report.

For notation and terminology see R. O'Neil, Duke Math. J. 30 (1963), 129-142 and L. Y. H. Yap, Duke Math. J. 36 (1969), 647-658. By a simple function we mean a function which can be written in the form \( f(x) = \sum_{j=1}^{N} c_j \chi_{E_j}(x) \), where \( c_1, \ldots, c_N \) are complex numbers, \( E_1, \ldots, E_N \) are sets of finite measure and \( \chi_{E_j} \) denotes the characteristic function of \( E_j \). Given three measure spaces \( (X_i, \mu_i), i = 1,2,3 \), a bilinear operator \( T \) which maps all pairs of simple functions on \( X_1 \) and \( X_2 \) into measurable functions on \( X_3 \) is called a convolution operator, if for simple functions \( f, g \), (1) \( \| T(f,g) \|_1 \leq \| f \|_1 \| g \|_1 \); (2) \( \| T(f,g) \|_\infty \leq \| f \|_1 \| g \|_\infty \); (3) \( \| T(f,g) \|_\infty \leq \| f \|_\infty \| g \|_1 \). The simple functions are dense in \( L^1 \) and \( L(p,q) \), \( 1 < p < \infty, 1 < q < \infty \). This information together with a splitting technique yields: **Convolution Theorem.** \( T \) can be uniquely extended so that: if \( f \in L(p_1,q_1), g \in L(p_2,q_2) \), where \( 1/p_1 + 1/p_2 > 1 \), then \( T(f,g) \in L(r,s) \) where \( 1/p_1 + 1/p_2 - 1 = 1/r \), and \( s \leq 1 \) is any number such that \( 1/q_1 + 1/q_2 \leq 1/s \). Moreover \( \| h \|_{(r,s)} \leq C(r) \| f \|_{(p_1,q_1)} \| g \|_{(p_2,q_2)} \). (Received October 5, 1970.)

On p. 237 in the paper: "On the convergence of Fourier series," Proc. Conference Orthogonal Expansions and Their Continuous Analogues (Edwardsville, Ill., 1967) Southern Illinois Univ. Press, Carbondale, Ill., 1968, Hunt states without proof: "It follows that \( \| M^* f \|_{p,\infty} \leq A_p \| f \|_{p,1}^* \) for all \( f \in L(p,1) \), \( A_p \equiv \text{Const} (p/p - 1) B_p \), \( 1 < p < \infty \)." Upon this statement rests all the results in his paper, and it is my opinion that whether or not it is true is a nontrivial open question. In this paper (maintaining the use of his Lorentz space interpolation techniques) I modify Hunt's proof in such a way as to eliminate the need for this statement. (Received October 5, 1970.)


Let \( \mathcal{H} \) be any Hilbert space and \( \mathcal{S} \) any closed, linear subspace of \( \mathcal{H} \). If \( A \) is any bounded operator on \( \mathcal{H} \) and \( P \) is orthogonal projection on \( \mathcal{S} \), the general Wiener-Hopf operator defined by A. Devinatz and M. Shinbrot (Trans. Amer. Math. Soc. 145 (1969), 467-494) is \( T_p(A) = PA \mathcal{S} \). Let \( \mathcal{J} \) be the collection of all bounded invertible operators on \( \mathcal{H} \) such that each operator takes \( \mathcal{S} \) onto itself. Let \( \mathcal{A} \) be the closed subalgebra of the bounded operators on \( \mathcal{H} \) generated by \( \mathcal{J} \) and \( A \). Then the following theorem can be proved. Theorem. If \( A \) is invertible and if \( T_p(A) \) is invertible then \( A^{-1} \) is in \( \mathcal{A} \). Let \( \mathcal{J} \) be a collection of operators on \( \mathcal{H} \) with the following properties: (i) there exist two positive numbers \( m \) and \( M \) such that for every \( S \in \mathcal{J} \) and for every \( f \in \mathcal{K} \), \( m \| f \| \leq \| SF \| \leq M \| f \| \); (ii) every element of \( \mathcal{J} \) commutes with \( A \) and \( A^* \); (iii) the set \( \{ Sg = S \in \mathcal{J}, g \in \mathcal{S} \} \) is dense in \( \mathcal{H} \). Then the preceding theorem yields immediately the spectral inclusion theorem of Devinatz and Shinbrot in a slightly improved form. Corollary. If there exists a family \( \mathcal{J} \) with the properties (i), (ii), and (iii), then the spectrum of \( T_p(A) \) contains the spectrum of \( A \) as an element of \( \mathcal{A} \). (Received May 28, 1970.)

71T-B6. ROBERT R. STEVENS, Loyola University, New Orleans, Louisiana 70118. Strictly oscillatory functions.

A function \( p \in C(a, \infty) \) is oscillatory if every solution of \( x + px = 0 \) has arbitrarily large zeros and is strictly oscillatory (s.o.) if \( p_+ - gp_- \) is oscillatory for all \( g \in C(a, \infty) \). Here \( 2p_+ = |p| + p \) and \( p_- = p_+ - p \). I.e., \( p \) is s.o. if it remains oscillatory under any (continuous) change in its negative part. Theorem. The functions \( \sin t, \sin t, (\alpha t/t) \sin t \) are, respectively, s.o. for every \( \alpha \neq 0, \alpha > 16/9, \alpha \leq a \). (This and similar results will appear in Arch. Math. (Basel).) (Received October 9, 1970.)

71T-B7. SAMUEL ZAIDMAN, Université de Montréal, Montréal, Québec, Canada. Stepanoff almost-periodic differential equations.

Let us consider a Hilbert space \( H \); then a continuous function \( f(t) \), \( - \infty < t < + \infty \rightarrow H \), which is almost-periodic with respect to the norm \( \left( \sup_{-\infty < a < \infty} \int_a^{a+1} \| f(u) \|_H^2 \, du \right)^{1/2} \) i.e. in the \( S^2 \)-norm. Let \( B \in \mathcal{B}(H,H) \) be a linear continuous symmetric operator in \( H \) and \( u(t) \), \( - \infty < t < + \infty \rightarrow H \), be a strong solution of the equation \( u'(t) = Bu(t) + f(t) \). Then, if \( u(t) \) is bounded on the real axis, it is almost-periodic in the sense of Bochner. (Received October 9, 1970.)
Let $S$ be a topological semigroup with identity, $W(S)$ the Banach space of all weakly almost almost periodic functions on $S$ (for definitions and notations, see K. DeLeeuw and I. Glicksberg, "Applications of almost periodic compactifications," Acta Math. 105 (1961), 63-97). Let $p(S)$ be the set of all $\varphi \in L^1(S)$ such that $\|\varphi\|_1 = 1$ and $\sigma: \varphi(\sigma) > 0$ is finite. Define $T_\varphi = \sum_\sigma \varphi(\sigma)R_\sigma$ for $\varphi \in p(S)$, where $R_\sigma f(\tau) = f(\tau\sigma)$, $f \in W(S)$ and $\tau \in S$. Theorem. $p(S)$ is a semigroup with convolution in $L^1(S)$ as multiplication $(\varphi \psi)(\sigma) = \sum_{\sigma_1, \sigma_2} \varphi(\sigma_1) \psi(\sigma_2) \psi(\sigma_1^{-1} \sigma_2)$, the map $T: p(S) \times W(S) \to W(S)$, where $T(\varphi, f) = T_\varphi(f)$ is a representation of $p(S)$ as a (convex) weakly almost periodic semigroup of operators in $W(S)$ and the weak operator closure $p(S)^\omega$ of $\{T_\varphi: \varphi \in p(S)\}$, is a compact topological semigroup in the same topology. Moreover, the following are equivalent: (1) $W(S)$ has a left (right) invariant mean, (2) The semigroup $p(S)$ has a right (left) zero, i.e., there exists $V \in UV = V (VU = V)$ for any $U$. (3) The continuous bounded functions $CB(p(S)^\omega)$ has a multiplicative left (right) invariant mean. Many other results in this direction are obtained. In particular, if $S$ is a locally compact group, we can also use $P(S) = \{\varphi \in L^1(S): \varphi \geq 0, \|\varphi\|_1 = 1\}$ and convolution in $L^1(S)$. Then the corresponding statements (2) and (3) above are always true since then $W(S)$ always has a two-sided invariant mean. (Received October 9, 1970.)


Contributions to non-archimedian integration theory. Preliminary report.

Let $K$ be a completely non-archimedian (n. a.) valued field of characteristic 0, $X$ a 0-dimensional locally compact Hausdorff space, $C^*_k(X)$ the algebra of all continuous $X \to K$ with compact support, $M(X, K)$ the $K$-vector space of all n. a. $K$-valued Radon measures on $X$, $M_1(X, K)$ all bounded $\mu \in M(X, K)$. For $\mu \in M(X, K)$ let $[\mu] = \{\lambda \in M(X, K) | L^1(\lambda) = L^1(\mu)\}$. The n. a. version of the Radon-Nikodym theorem is: Theorem. There exists $h \in E_{loc}^1(\mu)$ such that $[\nu] = [h\mu]$ iff there is an isometric linear $C^*_k$-morphism from $L^1(\mu)$ into $L^1(\mu)$. The support $S(\mu)$ of $\mu \in M(X, K)$ is the set of all $x \in X$ such that $\varphi_U \mu \neq 0$ for each open compact neighbourhood $U$ of $x (\varphi_U$ characteristic function of $U)$.

From now on let $K$ be spherically complete. Theorem. For each closed $C \subset X$ exists a $\mu \in M_1(X, K)$ such that $S(\mu) = C$. Let $S_k(X)$ be the family of all weakly closed $C^*_k$-invariant subspaces of $M(X, K)$, $S_k^c(X)$ the family of all $L \in S_k(X)$ such that there exists $L^c \in S_k(X)$ with $L \oplus L^c = M(X, K)$, $S_k^1(X)$ the family of all $L \in S_k(X)$ with $L \subset M_1(X, K)$. Theorem. (i) $S_k^c(X)$ is a Boolean ring with unit and $S_k^1(X)$ is an ideal in $S_k^0(X)$. (ii) Let $X, Y$ be 0-dimensional locally compact Hausdorff spaces. $X$ is homeomorphic to $Y$ iff there is an isomorphism $\sigma: S_k^c(X) \to S_k^c(Y)$ with $\sigma S_k^1(X) = S_k^1(Y)$. (Received October 1, 1970.) (Author introduced by Professor Gerhard J. Neubauer.)


Let $\Gamma$ be the unit circle. Let $A$ be the uniform algebra on $\Gamma$ consisting of those continuous functions which extend to be continuous on the closed unit disk and analytic on its interior. If $\varphi$ is a homeomorphism of $\Gamma$ onto $\Gamma$, let $A(\varphi) = \{f(\varphi) | f \text{ is in } A\}$. For any uniform algebra $B$ on $\Gamma$, put $\text{Re } B = \{\text{Re } f | f \text{ is in } B\}$. Theorem 1. Let $B$ be a uniform algebra on $\Gamma$ with $\text{Re } B = \text{ Re } A$. Then there exists a homeomorphism $\varphi$ of $\Gamma$ onto $\Gamma$ so that $B = A(\varphi)$. Theorem 2. If $\varphi$ is a homeomorphism of $\Gamma$ onto $\Gamma$ which extends to be conformal in a neighborhood
of \( \Gamma \), then \( \text{Re} A(\varphi) = \text{Re} A \). (Received October 12, 1970.)

71T-B11. SWARUPCHAND M. SHAH, University of Kentucky, Lexington, Kentucky 40506. Entire functions of irregular growth and of bounded index.

This paper is in continuation of an earlier paper [S. M. Shah, "On entire functions of bounded index whose derivatives are of unbounded index," Abstract 70T-B229, these Notices 17(1970), 960]. It is shown that there exist entire functions of bounded index whose lower order is less than the order \( \rho \). Theorem 1. Given \( \rho, 0 < \rho < (5/4 - 1)/2 \), there exists an entire function \( F(z) \) of bounded index and of order \( \rho \) and lower order less than \( \rho \).

Theorem 2. Given \( \rho, 0 < \rho \leq 1 \), there exists an entire function \( F(z) \) of bounded index and of order \( \rho \) and lower order zero. The proof of Theorem 1 (and also of Theorem 2) depends on the results of the paper mentioned above.

Let \( f(z) = \sum_{n=0}^{\infty} \frac{1}{1 - z/a_n}^\infty \) where \( a_1 = k_1 = 10^{10}, \gamma > 0 \) such that \( \frac{2(\rho - 1)}{\rho} < \frac{1}{\gamma} < 1 - \rho \), \( k_{n+1} = \lfloor k_n^\gamma + 1 \rfloor \), \( a_{n+1} = k_{n+1}/(n \geq 1) \), and \( F(z) = f(z) - 1 \). For Theorem 2 we take \( k_{n+1} = \lfloor k_n^{\log(n+10)} \rfloor, n \geq 1 \). (Received October 13, 1970.)


Let \( T \) be a set, \( \Sigma \) a \( \sigma \)-algebra of subsets of \( T \), \( X \) a Banach space, \( m: \Sigma \to X \) a (\( \sigma \)-additive) vector measure. \( R = \{ m(E) : E \in \Sigma \} \) its range and \( \overline{R} \) the closed convex hull of \( R \). A set \( E \in \Sigma \) is called an atom if \( m(E) \neq 0 \) and if \( F \subseteq E, F \in \Sigma \) imply \( m(F) = 0 \) or \( m(F) = m(E) \). If there are no atoms then the weak closure of \( R \) coincides with \( \overline{R} \) and all extremal points of \( \overline{R} \) belong to \( R \). (Received October 19, 1970.) (Author introduced by Professor J. Jerry Uhl, Jr.)

71T-B13. DOUGLAS W. WILLETT, University of Utah, Salt Lake City, Utah 84112. Generalized de la Vallée Poussin disconjugacy tests.

Assume \( Ly = y^{(n)} + p_1(t)y^{(n-1)} + \ldots + p_n(t)y = 0 \) is disconjugate on \([a, b]\). Then, \( \exists \) functions \( \gamma_k, k = 1, \ldots, n \), which can be explicitly determined in terms of the fundamental principal system of \( Ly \), such that \( Ly = r_1(t)y^{(n-1)} + \ldots + r_n(t)y \) is disconjugate on \([a, b]\) if \( \sum_{k=1}^{n} r_k(t) | \gamma_k(t) | dt \leq 1 \). For \( L = \Delta^n \), we obtain \( \gamma_1(t) = 2^{n-1}, \gamma_{k+1}(t) = (2^{n-1} - 1)(t - a)^k(b - t)^k(b - a)^{-k}/k! \) for \( k = 1, \ldots, n - 2 \), and \( \gamma_{n}(t) = (t-a)^{n-1}(b-t)^{n-1}(b-a)^{-n-1}/[(n/2)!/(n-1)/2!], \) where \( \lfloor x \rfloor \) is the greatest integer contained in \( x \).

(Received October 21, 1970.) (Author introduced by Professor John S. Alin.)

70T-B14. JOHN K. PERRYMAN, University of Texas, Arlington, Texas 76013. Integral transformations for functions which have no Laplace transformations. Preliminary report.

The author defines a set of integral transformations with \( H_n(st)e^{-s^2t^2 + 2st} \) as kernels, where \( H_n(t) \) is the Hermite polynomial of degree \( n \) in \( t \). Operational properties are developed for these transformations which make them useful tools in solving differential equations which contain functions for which there exist constants \( M, b \) and \( T \) such that these functions are bounded by \( Me^{bt^2} \) for each \( t > T \). Extensive tables have been developed for these transformations to facilitate their use in solving differential equations. (Received October 21, 1970.)

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71T-B15. HARI M. SRIVASTAVA, University of Victoria, Victoria, British Columbia, Canada. A formal extension of certain generating functions. II. Preliminary report.

This paper is a continuation of the earlier work of the present author [Glasnik Mat. Ser. III 5 (25) (1970), in press; see also Abstract 69T-B115, these Notices 16 (1969), 674] and it discusses the possibility of extending certain bilinear generating relations involving generalized hypergeometric functions to hold for Kampé de Feriet's double hypergeometric functions. Three such general formulas are proved by using the Laplace transform and its inverse in conjunction with the principle of multidimensional mathematical induction, and their interesting special forms are considered briefly. It is observed that the four bilinear generating relations, which the author proved a couple of years ago in an attempt to give extensions of the well-known Hille-Hardy formula for the Laguerre polynomials [see H. M. Srivastava, "An extension of the Hille-Hardy formula," Math. Comp. 23 (1969), 305-311; see also Abstract 68T-449, these Notices 15 (1968), 634-635], admit themselves of further generalizations leading, for instance, to the formal sums (*),

\[
\sum_{n=0}^{\infty} \left( \sum_{s=0}^{\infty} \left( \sum_{r=0}^{n} \left( \frac{s+n}{s!} \cdot \frac{r+n}{r!} \cdot \xi r s \cdot \gamma s \right) \right) \right) x^n,
\]

where \(p, \nu, \sigma\) and the \(\xi r s\) are arbitrary constants, real or complex. (Received October 23, 1970.)


The present paper incorporates a systematic study of linear, bilinear and bilateral generating functions, pure as well as mixed recurrence relations, and the differential equations associated with the class of polynomials \(G_n(\alpha)(x, r, p, k) | n = 0, 1, 2, \ldots\) defined by (*),

\[
G_n(\alpha)(x, r, p, k) = (x^{\alpha-k} / n!) \exp(px^r) \left( x^{k+1/dx} \right)^n \exp(-px^r),
\]

which evidently provides an elegant generalization of the various recent extensions of the classical Hermite and Laguerre polynomials. As an illustration of the results obtained in this paper, mention may be made of the following theorem which, when appropriately specialized, is capable of yielding a number of known bilinear and bilateral generating relations for the Hermite and Laguerre polynomials. Theorem. If \(F[x, t] = \sum_{n=0}^{\infty} c_n G_n(\alpha)(x, r, p, k) t^n\), where \(c_n \neq 0, n = 0, 1, 2, \ldots\), are arbitrary constants, then \((1-kt)^{-\alpha/k} \exp[px^r(1-(1-kt)^{-r/k})] F[x/(1-kt)^{1/k}, yt/(1-kt)^{1/k}] = \sum_{n=0}^{\infty} \gamma_n(y) G_n(\alpha)(x, r, p, k) t^n\), provided \(\gamma_n(y) = \sum_{m=0}^{n} c_m y^m, n = 0, 1, 2, \ldots\). (Received October 23, 1970.)

71T-B17. MORRIS MARDEN, University of Wisconsin, Milwaukee, Wisconsin 53201. Representation for the logarithmic derivative of a meromorphic function.

Generalizing the author's previous result for entire functions, a new representation is found for the logarithmic derivative of a meromorphic function \(f\) of finite order \(\rho\), in terms of the zeros and poles of \(f\). It differs from the one obtainable from the Hadamard representation in that it uses \(n = [\rho]\) critical points of \(f\) as parameters. It is applied to determining regions where at most \(n\) critical points of \(f\) may lie when the location of the zeros and poles is prescribed. (Received October 23, 1970.)
An entire function $f$ is the sum of its formal Bernoulli polynomial expansion if and only if $f^{(n)}(0) = o((2\pi)^n)$, $n \to \infty$. An entire function $f$ is the sum of its formal Lidstone series expansion if and only if $f^{(n)}(0) = o((\pi)^n)$. J. M. Whittaker [Proc. London Math. Soc. (2) 36(1933/34), 451-469] has shown that a Lidstone series which converges at one point which is not an integer converges uniformly on every compact set. For Bernoulli series, convergence at two points whose difference is not half an integer implies uniform convergence on every compact set. (Received October 22, 1970.)

If $G$ is a discrete group and $x \in G$ then $\bar{x}$ denotes the homeomorphism of $\beta G$ onto $\beta G$ induced by left multiplication by $x$. A subset $K$ of $\beta G$ is said to be invariant if it is closed, nonempty and $\bar{x}K \subset K$ for each $x \in G$. Let $ML(G)$ be the set of left invariant means on $G$. (They can be considered as measures on $\beta G$.)

**Theorem.** Let $G$ be a countably infinite amenable group and let $K$ be an invariant subset of $\beta G$. Then the non-empty $w^*$-compact convex set $M_K = \{ \varphi \in ML(G) : \text{supp} \varphi \subset K \}$ has no exposed points. Therefore, $M_K$ is infinite dimensional. It is known that $ML(G)$ has $2^C$ extreme points but the above theorem tells us that it has no exposed points. **Corollary.** Let $G$ be an infinite amenable group and let $K$ be an invariant subset of $\beta G$. Then the action of $G$ on $K$ is not equicontinuous. (Received October 22, 1970.)

For the second order differential equation \((*)\) $$(r(t)u')' + q(t)\varphi(u, u') + p(t)f(u) = h(t, u, u'),$$ let (1) $q$ be continuous and nonnegative on $(0, \infty)$, (2) $\varphi$ be continuous on $\mathbb{R}^2$ and $h$ be continuous on $(0, \infty) \times \mathbb{R}^2$, (3) $\varphi(x, y)y > 0$ for all $y \neq 0$, (4) there exists an integrable function $e$ on $(0, \infty)$ such that $|h(t, u, u')| \leq e(t)$, (5) $f$ be continuous on $\mathbb{R}$ and $\lim_{\xi \to 0} F(\xi) = \infty$, where $F(x) = \int_0^x f(s)ds \geq 0$. **Theorem 1.** Assume that (1)-(5) hold. If $p, r \in C(0, \infty)$ are such that (a) $pr$ is nonincreasing on $(0, \infty)$, (b) $0 < p_0 \leq p(t)$ and $0 < r_0 \leq r(t)$ for all $t \in (0, \infty)$, then, for each solution $u$ of $(*)$, both $u$ and $u'$ are bounded. **Theorem 2.** Assume that (1)-(5) hold. If $p, r \in C'(0, \infty)$ are such that (a) $0 < p_0 \leq p(t)$ and $0 < r_0 \leq r(t)$ for all $t \in (0, \infty)$, (b) $\int_0^\infty \left| (p(t)r(t))' \right| dt < \infty$, then, for each solution $u$ of $(*)$, both $u$ and $u'$ are bounded. By a modification of assumption (4), various theorems can be established for the boundedness of solutions of $(*)$ under different conditions on $p$ and $r$. (Received October 26, 1970.) (Author introduced by Professor John S. Bradley.)

polynomials," Plenum, New York, 1961) develop the theory of a general class of orthogonal polynomials related to a certain Toeplitz form. The author considers a similar class of orthogonal polynomials in several complex variables and then derives the identity (which is analogous to the Christoffel-Darboux identity) for the reproducing kernel, which in the case of one complex variable is given in the works cited above. (Received October 26, 1970.)

71T-B22. JONNIE BEE BEDNAR, University of Tulsa, Tulsa, Oklahoma 74104. A facial characterization of Choquet simplexes.

Let K be a compact convex set, EK the extreme points of K and A(K) the space of continuous affine functions on K with uniform norm. A face F of K is called a face of class B if (1) F is closed in the norm topology of A(K)\* and (2) F \cap EK is a Borel subset of EK. Here the Borel subsets of EK are the minimal \( \sigma \)-algebras generated by the relatively closed subsets of EK when K has the original topology. A face F of K is complemented if there is unique face F' disjoint from F such that each \( x \in K \setminus (F \cup F') \) has a unique expansion of the form \( x = r x_1 + (1-r)x_2 \) with \( x_1 \in F \) and \( x_2 \in F' \). Given a complemented face F one can easily define a bounded affine function f on K called the affine indicator function of F which satisfies \( F = \{ x : f(x) = 1 \} \) and \( F' = \{ x : f(x) = 0 \} \). If F is of class B, f is called a B-affine indicator function (B.A.I.F.). A compact convex set is a Choquet simplex if and only if (a) every face of class B is complemented, and (b) to each \( \epsilon > 0 \) and \( g \in A(K) \) there exist B.A.I.F.'s \( g_1, \ldots, g_n \) and scalars \( a_1, a_2, \ldots, a_n \) such that \( \| f - \sum_{j=1}^n a_j g_j \| < \epsilon \). (Received October 26, 1970.)


Let E and F be complex Hausdorff locally convex spaces, \( U \subseteq E \) be open nonvoid, \( \mathcal{H}(U, F) \) be the vector space of all holomorphic mappings \( f: U \rightarrow F \), on which we have the locally convex topologies \( T_{m,s} \) and \( T_{m,\omega} \) \( (m = 0, 1, \ldots, \infty), T_{\omega} \). Proposition 1. If \( F \neq 0 \), then \( T_{m,s} = T_{m+1,s} \) for some \( m \) finite if and only if every bounded subset of E is contained in the closed convex hull of some compact subset of E. Proposition 2. On \( \mathcal{H}(I, C) \), we have \( T_{0,s} = T_{\omega,s} \) for every set I; but we do have \( T_{\omega,s} = T_{\omega} \) if and only if I is finite or infinite denumerable. Proposition 3. \( T_{\omega,1} \) and \( T_{\omega} \) induce the same uniform structure, hence the same topology on every amply bounded subset of \( \mathcal{H}(U, F) \) provided the set of all continuous seminorms \( \alpha \) on E, such that the inductive limit topology on \( P(\mathcal{M}E,F) \) induces on \( P(\mathcal{M}E,F) \) its normed topology for every \( m \), is directed and it defines the topology on E. (In this last result, F is assumed to be normed, for simplicity.) (Received October 27, 1970.)

(Received Introduced by Professor Leopolda Nachbin.)


Let \( x = (x_n) \) be a complex sequence and for \( k = 0, 1, \ldots \) let \( (C, k) - \sum_{n=1}^{\infty} x_n \) denote the Cesàro sum of order k. The following theorem is proved: Given an integer \( k = 0, 1, \ldots \) and any complex null sequence x, there exist sequences \( y^{(s)} = (y^{(s)}_n) \) \( (s = 0, 1, \ldots, k+1) \) such that \( x = \sum_{s=0}^{k+1} y^{(s)}_n \) and \( (C, k) - \sum_{n=1}^{\infty} y^{(s)}_n \) \( \text{ exists.} \)

If \( k \) is an odd positive integer, then there exist sequences \( y^{(s)} = (y^{(s)}_n) \) \( (s = 0, 1, \ldots, k) \) such that \( x = \sum_{s=0}^{k} y^{(s)}_n \) and \( (C, k) - \sum_{n=1}^{\infty} e^{i \pi s / (k+1)} y^{(s)}_n \) \( \text{ exists.} \) For the proof properties of sequences of bounded variation of order k
and general properties of \((C, k)\)-dual Köthe spaces are used. (Received October 28, 1970.)

71T-B25. ROGER W. BARNARD, University of Maryland, College Park, Maryland 20742. Distortion theorems for subclasses of univalent functions containing a fixed disk. Preliminary report.

Let \(S_d^*\) be the class of functions \(f(z) = z + a_2z^2 + \ldots\) that are analytic and univalent in \(U = \{z : |z| < 1\}\) and map \(U\) onto a starlike domain that contains the disk \(\{w : |w| < d\}\) \((1/4 \leq d \leq 1)\). Let \(C_d\) denote the subclass of \(S_d^*\) that maps \(U\) onto a convex domain with \(1/2 \leq d \leq 1\). The problems of determining the max and min of \(|f(z)|\) on \(|z| = r\) are solved for the classes \(S_d^*\) and \(C_d\) using variational techniques due to T. J. Suffridge (cf. Michigan Math J. 16(1969), 33–42) and J. Krzyz (cf. Ann. Univ. Mariae Curie-Sklodowska Sect. A 14(1960), 7–18) resp. The result for \(S_d^*\) also gives the max for \(|f'(z)|\) and \(|zf'(z)/f(z)|\) (on \(|z| = r\)) and generalizes a result of T. J. Suffridge (cf. Michigan Math J. 16(1969), 33–42). For \(S_d^*\) (resp. \(C_d\)) the external function maps \(U\) onto a domain having as its boundary a single arc of \(\{w : |w| < d\}\) and two radial rays (resp. tangents) emitted from the endpoints of this arc. (Received October 28, 1970.)

(Author introduced by Professor William E. Kirwan.)

71T-B26. ANDRE DE KORVIN and LAURENCE E. KUNES, Indiana State University, Terre Haute, Indiana 47809. Some nonweak integrals defined by linear functionals. I.

Let \((S, \Sigma)\) be a measurable space. Let \(m\) be a finitely additive set function from \(\Sigma\) to \(L(E, F)\), where \(L(E, F)\) denotes all bounded linear operators from \(E\) into \(F\) (\(E\) and \(F\) are Banach spaces). \(\tilde{m}\) will denote the semivariation of \(m\). Let \(y^* \in F^*\) and let \(m_{y^*}\) denote the set function from \(\Sigma\) into \(E^*\) defined by \(\langle m_{y^*}(A), x \rangle = y^* m(A) x\). Lemma. Suppose all \(m_{y^*}\) are countably additive and that for every sequence of disjoint sets \(F_i \in \Sigma\) and every sequence \(y_i^* \in \sigma^*\) (\(\sigma^*\) is the unit sphere of \(F^*)\) \(\Sigma \tilde{m}_{y_i^*}(F_i) < \infty\), where \(\tilde{m}_{y_i^*}\) denotes the variation of \(m_{y_i^*}\). Then \(\tilde{m}(\lim E_n) = \lim \tilde{m}(E_n)\). Definition. \(m\) is called \(F^*\) regular if every \(m_{y^*}\) is regular. \(m\) is called \(E^*\) regular if for every \(\epsilon > 0\) and \(A \in \Sigma\) there exists \(A_1\) and \(A_2\) in \(\Sigma\) such that \(A_1 \subset A_2\) is compact and \(A_1 \subset A_2 \subset A_2^c \subset A_2\) and \(\tilde{m}(A_2^c - A_1) < \epsilon\). Proposition. If the conditions of the above lemma are satisfied and if \(m\) is \(E^*\) regular then \(m\) is regular. Corollary. Under the above conditions \(m\) is countably additive in the norm of \(L(E, F)\). (Received October 28, 1970.)

71T-B27. BRUCE L. R. SHAWYER, University of Western Ontario, London 72, Ontario, Canada and G. S. YANG, National Tsing Hua University, Sing-Tsu, Taiwan, Republic of China. On the relation between Abel-type and Borel-type methods of summability.

Let \(\lambda > -1, \alpha > 0, \beta\) a real number and \(N\) a nonnegative integer greater than or equal to \((\beta - 1)/\alpha\). Let \(\sigma(y) = (1 + y)^{\lambda - 1} \sum_{n=0}^{\infty} \alpha n^{\lambda} \langle \frac{y^n}{(1+y)^n} \rangle_n\) and \(S(y) = \sigma e^{-y} \sum_{n=0}^{\infty} s_n y^n \sigma^{n+\beta - 1}/(\alpha n + \beta)\). We say that \(s_n \rightarrow \lambda(A\lambda)\) if the series defining \(S(y)\) is convergent for all \(y > 0\) and tends to a finite limit \(\lambda\) as \(y \rightarrow \infty\); and that \(s_n \rightarrow \lambda(B, \alpha, \beta)\) if the series defining \(S(y)\) is convergent for all \(y \geq 0\) and tends to a finite limit \(\lambda\) as \(y \rightarrow -\infty\). Theorem. If \(s_n \rightarrow \lambda(A\lambda)\) and \(S(y)\) is slowly decreasing (i.e., \(\lim \inf (S(y) - S(x)) \equiv 0\) whenever \(y > x \rightarrow -\infty\) and \(y/x \rightarrow 1\)), then \(s_n \rightarrow \lambda(B, \alpha, \beta)\). The proof depends on a theorem of Pitt to show the boundedness of \(S(y)\) and then on a theorem of Wiener to obtain the final result. (Received October 28, 1970.)
Theorem 1 (Existence). Let \( \mathcal{A} \) be a Von Neumann algebra with a finite trace. Let \( A \) be a bounded Hermitian operator on \( L^2(\mathcal{A}) \) which takes nonnegative elements to nonnegative elements. If for some number \( p > 2 \) \( A \) is bounded from \( L^2(\mathcal{A}) \) to \( L^p(\mathcal{A}) \) then \( \| A \|_{L^2} \) is an eigenvalue of \( A \) of finite multiplicity. Theorem 2 (Uniqueness). Assume in addition to the hypotheses of Theorem 1 that \( A \) commutes with no nontrivial projection \( P_b \) on \( L^2(\mathcal{A}) \) of the form \( P_b \phi = b \phi b \) where \( b \) is a projection in \( \mathcal{A} \). Then \( \| A \|_{L^2} \) has multiplicity one.

Theorem 3. Let \( \mathcal{A} \) be as in Theorem 1. Let \( H_0 \) be a nonnegative selfadjoint operator on \( L^2(\mathcal{A}) \) whose kernel is spanned by the identity of \( \mathcal{A} \). Assume that for some \( t > 0 \) and some \( p > 2 \) \( \exp(-tH_0) \) takes nonnegative elements of \( L^2(\mathcal{A}) \) to nonnegative elements and is bounded from \( L^2(\mathcal{A}) \) to \( L^p(\mathcal{A}) \). Let \( \alpha \) be in \( \mathcal{A} \) and assume \( \alpha^* = \alpha \). Let \( B \alpha \phi = \alpha \phi + \phi \alpha \) for \( \phi \) in \( L^2(\mathcal{A}) \) and put \( H = H_0 + B \alpha \). Then \( \exp(-tH) \) satisfies the hypotheses of Theorem 2 and consequently \( \inf(\text{spectum } H) \) is an eigenvalue of \( H \) with multiplicity one. Application is made to the proof of existence and uniqueness of the physical vacuum for a Fermion interacting with an external scalar field via a Yukawa type interaction. The algebra \( \mathcal{A} \) is taken as a Clifford algebra over the one particle space. (Received October 21, 1970.)


Based on the assumption of linearization, an attempt is made to study an unsteady flow produced by a harmonic pressure distribution of frequency \( \omega (> 0) \) acting on the horizontal free surface of an inviscid, incompressible fluid which is rotating with uniform angular velocity \( \Omega \). Two cases (i) \( \omega > 2\Omega \) and (ii) \( \omega < 2\Omega \) are of particular interest. In accordance with Lamb's classification, it is shown that case (i) corresponds to "waves of the first class" which are qualitatively similar to the classical surface waves and case (ii), in contrast to the first class waves, corresponds to "waves of the second class" which are originated entirely due to rotation and have no counterpart in the nonrotating fluid. With the aid of the joint Laplace and generalized Hankel transformations together with asymptotic methods, the problem is analysed in considerable detail. Both the steady and the transient solutions are obtained explicitly. It is predicted that the solution asymptotically approaches the steady state as \( t \to \infty \). The significant effects of the Coriolis force and the characteristic features of the wave motions are investigated. (Received October 5, 1970.)

Let $M$ be a two-dimensional Riemannian manifold (a surface) immersed in $S^3$. The surface $M$ is said to be compassed if through every point in $M$ there passes an arc of a great circle which lies in $M$. In addition, $M$ is developable if a unit field normal to $M$ is parallel along each arc.

Theorem 1. The surface $M$ has Gauss curvature one if and only if $M$ is a developable compassed surface. Let $M$ (above) have Gauss curvature one. Lemma. Along great circle arcs through non-umbilic points in $M$, the relative mean curvature is a nonzero multiple of a secant function of arc-length. Theorem 2. If $M$ is complete and connected (if $M$ is a space form), then $M$ is a great sphere.

(Received September 8, 1970.)

71T-D2. VISHWA CHANDER DUMIR and DHARAM SINGH KHASSE, Panjab University, Chandigarath-14, India. On the minimal density of saturated systems of symmetric convex domains.

Let $K$ be a closed, bounded, symmetric convex domain with centre 0. A homothetic translate of $K$ with radius $r$ and centre $A$ means the set $rK + A$. A family $M$ of homothetic translates of $K$ is said to be a saturated system with minimal radius $r > 0$ if $r$ is the infimum of the radii of members of $M$, and any homothetic translate of $K$ of radius $r$ intersects at least one member of $M$. The lower density $\rho_M$ of $M$ is defined as $\rho_M = \lim_{t \to \infty} \inf (V(S(t))/V(tK))$ where $V(S(t))$ denotes the Lebesgue-measure of the point-set union of interiors of members of $M$ intersected with the set $tK$ and $V(tK)$ is the Lebesgue-measure of the set $tK$. The problem is to find the exact lower bound of the lower densities $\rho_M$ taken over all saturated systems $M$ of homothetic translates of $K$. Bambah and Woods proved that the lower bound is $\frac{1}{2} \mathcal{O}(K)$, where $\mathcal{O}(K)$ is the density of the thinnest covering of the plane by translates of $K$, when the lower bound is taken over all those saturated systems $M$ which form a packing and $K$ is strictly convex. In this paper it is proved that $\frac{1}{2} \mathcal{O}(K)$ is the exact lower bound when taken over all saturated systems $M$. The problem is reduced to that of finding the lower bound of the densities $\rho_M$ where $M$ is a saturated family of strictly convex domains with the same radius $r$ and with centres forming a discrete set. n (Received September 8, 1970.) (Authors introduced by Professor R. P. Bambah.)


In a recent paper [Topology 9(1970), 183-194] A. Haefliger has constructed a classifying space $BS^r$ for foliations which are transversely orientable of codimension $q$ on open manifolds. He obtains a map $\nu: BS^r \to BSO_q$ such that if $M$ is an open manifold with a foliation of codimension $q$ and $f: M \to BS^r$ classifies the foliation, then $\nu \circ f$ classifies the normal bundle. Theorem. Let $m$ be an odd integer. $\nu^*: H^*(BSO_q; \mathbb{Z}_m) \to H^*(BS^r; \mathbb{Z}_m)$ is injective. The proof employs the following. Lemma. There is a map $\overline{f}: BZ_m \to BS^2$ such that $(\nu \circ \overline{f})^*: H^*(BSO_2; \mathbb{Z}_m) \to H^*(BS^2; \mathbb{Z}_m)$ is injective. This lemma is proved by constructing explicit foliations on bundles over certain lens spaces. This lemma implies: Lemma. $\nu^*: H^*(BSO_2; \mathbb{Z}_m) \to H^*(BS^2; \mathbb{Z}_m)$ is injective. The second lemma and the Whitney sum formula imply the theorem. The result of Bott that $\nu^*: H^*(BSO_q; \mathbb{Q}) \to H^*(BS^r; \mathbb{Q})$ is the zero
Logic and Foundations

71T-E1. WITHDRAWN.


Let $M$ be an $L$-model ($L$-first-order), $|M| + |L| = \aleph_0 + \beta$, $\lambda > \text{cf}(\alpha)$ is regular, and $\mu < \aleph_\alpha = \mu^\lambda < \aleph_\alpha$.

Theorem 1. There is an elementary extension $N$ of $M$, $|N| = \aleph_0 + \beta$ such that: (A) If $p$ is a (consistent) type on $N$, $|p| = \lambda > \beta$, then some $q \subseteq p$, $|q| = \lambda$, is realized in $N$. (B) If $p$ is a type on $N$, and $q \subseteq p$, $|q| < \lambda = q$ is realized in $N$, then $p$ is realized in $N$. (C) If $\beta = \beta_0 + 1$, $\gamma = \gamma < \aleph_\alpha + \gamma'$, $p$ a type on $N$, $\lambda + |\beta| < |p| < \aleph_\beta + \beta'$ then some $q \subseteq p$, $|q| = \lambda$ is realized in $N$. (D) If $\beta = \beta_0 + 1$, $\gamma = \gamma < \aleph_\alpha + \gamma'$, $p$ a type on $N$, $\lambda + |\beta| < |p| < \aleph_\beta + \beta'$ and every $q \subseteq p$, $|p - q| > \lambda$ is realized in $N$, then $p$ is realized in $N$. Remark. There are minor improvements, and generalizations speaking on $|\{a \in |N| : a \text{ appears in } p\}|$ instead of $|p|$. If $Th(M)$ is unstable we have much freedom for $|\{p : p \text{ fails } A(B, \ldots)\}|$.

Definition. For $a \in |N|^a$ let $\nu_\lambda^a(a) = \{\phi(a) : N \models \phi[a]\}$, where $\nu = (\exists y_0, \ldots, y_i, \ldots, i < \lambda \wedge \Phi, \phi \subset L]$ and $\nu_\lambda(N) = \{\nu_\lambda^a(a) : a \in |N|^a\}$. Theorem 2. Let $\mu < \aleph_\alpha = \mu^{|L|} < \aleph_\alpha$. Then $M$ has an elementary extension $N$, $|N| = \aleph_0 + \beta$, $|\nu_\lambda(N)| \leq 2^\lambda + |L|^{|\beta|}$. (Received October 2, 1970.)

(Received October 15, 1970.)


It is shown that the problem of determining whether or not the Deduction Theorem holds for any arbitrarily given partial propositional calculus is recursively unsolvable. Furthermore, for every recursively enumerable degree of unsolvability $D$, there exists a class $C_D$ of partial propositional calculi such that the problem of determining whether or not the Deduction Theorem holds for an arbitrary member of $C_D$ is of degree $D$. The proof of first result parallels Yntema's (Notre Dame J. Formal Logic 5(1964), 37-50) proof of the recursive unsolvability of the completeness problem for partial implicational calculus with negation and also uses Singletary's (Notre Dame J. Formal Logic 9(1968), 193-211) construction of partial implicational calculi with recursively unsolvable decision problem. The second result follows from Boone's well-known result on degrees of unsolvability for semi-Thue systems. (Received October 5, 1970.)

71T-E4. ERIK ELLENTUCK, Rutgers University, New Brunswick, New Jersey 08903. Positive isolic sentences.

A positive sentence in a language whose relation symbols denote arbitrary relations $R \subseteq X^k *$ and whose function symbols denote $\sum^0_2$ functions $f : X^k \omega^* \rightarrow \omega^*$ is true in $\Lambda^*$ if and only if it has a Horn reduction true in $\omega^*$ with $\sum^0_2$ Skolem functions. If $R \subseteq X^{k+1}$ is an arbitrary relation then $(\forall x_0, \ldots, x_k \cdot) R(\bar{x}, y) \Lambda (x_0, \ldots, x_k, y)$ is true in $\Lambda^*$ if and only if $R$ is an almost recursive combinatorial function. (Received October 15, 1970.)

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Let $T$ be a countable first order theory which is stable and $P$ be a unary predicate symbol in the language of $T$. Theorem. If $A$ and $B$ are models of $T$ such that $P_A = P_B$ and $B$ is a proper elementary extension of $A$, then there exists a proper elementary extension $C$ of $B$ such that $P_A = P_C$. The proof uses Shelah's idea of restricting Morley's rank to a finite number of predicates [Israel J. Math. 7(1969), 187–202] and the fact that ranked types over the universe of a model all have degree 1. (Received October 21, 1970.)


Let $(\phi, \phi')$ be a measure of computational complexity (Blum, "A machine independent theory of recursive functions," J. Assoc. Comput. Mach. 14(1967), 322–336), where $\phi = [\phi_1]$ is an acceptable indexing and $\phi' = [\phi'_1]$ is the set of measuring functions. For any total $f: N \to N$, the set of $i$ such that $\phi_i$ is total and $\phi'_i(x) \equiv f(x)$ for all but a finite number of $x$ is the set of $f$-computable algorithms for $\phi'$, denoted $R_{f, \phi}'$. The complexity class (of $f$-computable functions for $\phi'$), $R_{f, \phi}'$, is $[\phi_i : i \in R_{f, \phi}']$. It has been shown that the family of complexity classes, for a fixed measure $\phi$, need not be closed under finite intersection. For any measure $\phi$, neither the family $[R_{f, \phi}' : f \in R_f]$ nor the family $[R_{f, \phi}' : f \in R_f]$ are closed under infinite intersection of nested sequences. These results contrast with McCreight and Meyer, "Classes of computable functions defined by bounds on computation," Assoc. Comput. Mach. Sympos. on Theory of Computing, May, 1969, pp. 79–88, where it is shown that the families are closed under infinite union of an increasing sequence of classes, although this family is not closed under finite union for any measure. (Received October 23, 1970.) (Author introduced by Professor Lawrence H. Landweber.)


Spector showed that there are minimal degrees of unsolvability, but very little is known about initial segments of the more comprehensive structure of partial degrees. John Case has shown that no minimal partial degree less that $0'$ can contain a total function. Theorem. There is a minimal partial degree less than $0'$. (Received October 22, 1970.) (Author introduced by Dr. Frank R. Drake.)

71T-E8. MENACHEM MAGIDOR, Hebrew University, Jerusalem, Israel. Super compacts and Skolem-Löwenheim theorem for higher order logic.

For definition of supercompact cardinal see Abstract 70T-E17, these Notices 17(1970), 455. Theorem 1. The existence of a supercompact cardinal is equivalent to the existence of ordinal $\mu$ such that for all $\beta \geq \mu$ $3\alpha < \beta$ such that $R(\alpha)$ can be elementary embedded in $R(\beta)$. The first such $\mu$ is the first supercompact cardinal. Theorem 2. The existence of a supercompact cardinal is equivalent to the existence of ordinal $\mu$ such that, for any structure $\langle M, R_1, \ldots, R_n \rangle$, $|M| \geq \mu$ and $\Sigma^1_1$ sentence $\phi$ such that $\langle M, R_1, \ldots, R_n \rangle = \phi$, there exists a substructure.
\( \langle M', R_1 | M', \ldots, R_n | M' \rangle \) such that \( | M' | < | M | \) and \( \langle M', R_1 | M', \ldots, R_n | M' \rangle = \varphi \). The first such \( \mu \) is the first supercompact cardinal. In Theorem 2 we could also have \( \varphi \) range over all finite order sentences. (Received October 29, 1970.) (Author introduced by Professor Azriel Levy.)

**Statistics and Probability**

71T-F1. RAVINDRA KUMAR JAIN, Department of Industrial and Systems Engineering, Ohio University, Athens, Ohio 45701. Moments of the matching problem.

The object of this paper is to show that certain identities hold among the moments of the well-known De Montmort matching problem. Let \( \mu_r(N) \) denote the \( r \)th moment about the origin (mean) for the population size \( N \). Some of the identities are: (a) \( \mu_r(N) = \mu_r(N) + \mu_{r+1}(N+1) \), if \( r < N \), (b) \( \mu_r(N+\lambda) = \mu_r(N) \), if \( \lambda \) is any positive integer and \( r < N \), (c) \( \mu_r(N) = \mu_r(N+1) - r \mu_{r+1}(N+1) + \cdots + \lambda \mu_r(-1) \), if \( r < N \), (d) \( \mu_{N+1}(N+1) - \mu_N(N+1) = 1 \), for all \( N \), (e) \( \mu_{N+1}(N+1) - \mu_N(N+1) = \frac{1}{2}(N^2 + N + 2) \), for all \( N \). An important and useful consequence of (a) and (b) is that given \( \mu_1(1) \), all the moments \( \mu_r(N) \) and \( \mu_r(N) \) are known when \( r < N \). There is considerable saving in computer time when the higher order moments are calculated with the techniques developed in the paper. With a given storage capacity of the computer, higher order moments which cannot be obtained ab-initio, can be obtained when programs are written using the above identities. (Received October 2, 1970.) (Author introduced by Professor Robert E. Atalla.)

**Geometry**

71T-G1. JACK D. BRYANT and LAWRENCE F. GUSEMAN, JR., Texas A & M University, College Station, Texas 77843. Fixed and periodic points of iterate contractive mappings.

We generalize two theorems of Edelstein (J. London Math. Soc. 37(1962), 74-79): Theorem. Let \( f \) be a selfmap of a metric space \( (X, d) \) and suppose that for some \( x_0 \) and \( u, f^m(x_0) \rightarrow u \). (A) If for some \( n, x \not= y \Rightarrow d(f^n(x), f^n(y)) < d(x, y) \) then \( u \) is the unique fixed point of \( f \). (B) If for some \( n \) and some \( \epsilon < 0, 0 \not= d(x, y) < \epsilon \Rightarrow d(f^n(x), f^n(y)) < d(x, y) \) then \( u \) is a periodic point of \( f \). The proof depends on a result announced here (Abstract 677-54-13, these Notices 17(1970), 795). We also comment on work of Bailey (J. London Math. Soc. 41(1966), 101-106). In particular, we raise the following question, motivated by his Theorem 1 (p. 102): Let a continuous selfmap \( f \) of a compact space \( X \) have a unique fixed point \( u \) and suppose that for each \( x \), some subsequence of \( \{ f^n(x) \} \) converges to \( u \). Is each pair of points proximal under \( f \)? (Received October 6, 1970.)

71T-G2. WŁODZIMIERZ HOLSZTYNSKI, University of Michigan, Ann Arbor, Michigan 48104. Approximation by homeomorphisms and solution of P. Blass problem on pseudo-isotopy.

For every map of \( f: S^1 \rightarrow S^1 = \{ z \in C : |z| = 1 \} \) of degree 1 existence of a pseudo-isotopy \( h: S^1 \times I \rightarrow R = \{ z \in C : |z| \geq 1 \} \) such that \( h(z, 0) = z \) and \( h(z, 1) = f(z) \) is established. On the other hand (i) maps of \( \Gamma^n \) into \( \Gamma^n \times 0 \subset E^{n+1} \) cannot be, in general, uniformly approximated by homeomorphic embeddings of \( \Gamma^n \) in \( E^{n+1} \) for
n > 1, and (ii) maps of $S^n$ into $S^n \subset E^n$ of degree 1 cannot be, in general, extended to a pseudo-isotopy of $S^n$ into $E^{n+1}$. (Received October 8, 1970.)


A set $X$ has the fixed point property if each map $f: X \to X$ leaves some point fixed (that is, there is a point $x \in X$ such that $f(x) = x$). In this paper the following is established: Theorem. If $M$ is an arcwise connected bounded plane continuum which does not separate the plane, then $M$ has the fixed point property. (Received October 9, 1970.)


R. W. Heath and E. Michael ("A property of the Sorgenfrey line," Compositio Math. (to appear)) recently proved that if $S$ is the Sorgenfrey line, then $S^0$, the product of countably many copies of $S$, is perfect (=closed sets are $G_\delta$). In this note, we prove that $S^0$ is subparacompact (=every open cover has a closed, $\sigma$-locally finite refinement) by establishing that (1) every finite product of $S$ with itself is perfectly subparacompact; (2) if $(X(n))$ is a sequence of spaces such that each finite product $\prod X(k) : 1 \leq k \leq n$ is perfectly subparacompact, then $\prod X(n) : n \neq 1$ is also perfectly subparacompact. (Received October 12, 1970.)


A theory of analytic sets for nonseparable metric spaces is developed by modifying the concept of $k$-analytic set introduced by A. H. Stone (Rozprawy Mat. 28(1962)). Calling a mapping co-$\sigma$-discrete if the image of every open cover has a (not necessarily open) $\sigma$-discrete refinement, we show that: A space $X$ of weight $\leq k$ is absolutely $k$-analytic (in the new sense) iff it is a continuous co-$\sigma$-discrete image of $B(k)$ (Baire space of wt. $k$). By a $k$-Borel subset of $X$ we mean a set in the smallest family of subsets of $X$ containing the open sets and closed under complementation and unions of $\sigma$-discrete families of at most $k$ sets. The absolute $k$-Borel sets of weight $\leq k$ are precisely the 1-1 continuous co-$\sigma$-discrete images of $B(k)$. This follows from our generalization, to $k$-analytic and $k$-Borel sets, of the famous theorem of Souslin on classical Borel and analytic sets; for separable spaces, $k$-Borel = Borel. Finally, we show that the image of a complete space under a continuous co-$\sigma$-discrete mapping is either $\sigma$-locally of weight $< k$ or contains a homeomorph of $B(k)$, thus generalizing the classical theorem that an absolutely analytic separable space is either countable or contains a homeomorph of the Cantor space. (Received October 12, 1970.)

71T-G6. ERNEST S. BERNEY, Idaho State University, Pocatello, Idaho 83201. A separable metric space which is chaotic.

A nontrivial topological space $X$ is said to be chaotic if for every two distinct points $p$ and $q$ of $X$, there exist open sets $U$ and $V$ such that $p \in U$ and $q \in V$ and no nonempty open subset of $U$ is homeomorphic to a
nonempty open subset of $V$. Theorem. There exists a subset $X$ of the unit interval of cardinality $C$ such that $X$ is chaotic. (Received October 16, 1970.)

71T-G7. V. KANNAN, Madurai University, Madurai - 2, India. $E$-Frechet spaces.

For a general class $E$ of topological spaces the concept of a topological space being $E$-Frechet is defined in this paper in such a way that the $E$-Frechet spaces are precisely the Frechet spaces when $E$ is the class of all metric spaces, and are precisely the $k'$-spaces when $E$ is the class of all compact spaces. Several characterisations are obtained and Franklin's characterisation of Frechet spaces and Arhangelskii's characterisation of $k'$-spaces are deduced as corollaries. The relationship between $E$-Frechet spaces, hereditarily $E$-determined spaces and pseudo-open maps are investigated. Finally a categorical characterisation of $E$-Frechet spaces is given using the concept of coreflexive subcategories. (Received October 23, 1970.) (Author introduced by Professor M. Rajagopalan.)

71T-G8. JOHN P. HEMPEL and WILLIAM H. JACO, Rice University, Houston, Texas 77001. 3-manifolds with fundamental group a direct product.

Theorem 1. Suppose $M$ is a compact 3-manifold and $\pi_1(M) \cong A \times B$. If $A$ is infinite and $B \neq 1$, then either $A$ or $B$ is infinite cyclic. Theorem 2. Suppose $M$ is a compact, irreducible 3-manifold and $\pi_1(M) \cong A \times B$ a nontrivial direct product. If $\pi_1(M)$ is infinite, then either (i) $\pi_2(M) \neq 0$ and $\pi_1(M) \cong \mathbb{Z} \times \mathbb{Z}_2$ or (ii) $\pi_2(M) = 0$ and $M = F \times S^1$ where $F$ is a compact 2-manifold ($F \neq S^2$, $P^2$, $B^2$). Epstein (Theorem 13, p. 117, "Topology of 3-manifolds," Prentice-Hall, Englewood Cliffs, N. J., 1962) proved Theorem 1 in the case $M$ is closed and orientable. Our proof uses different techniques and is independent of Epstein's result. (Received October 23, 1970.)

71T-G9. WILLIAM H. JACO, Rice University, Houston, Texas 77001. Finitely presented subgroups of 3-manifold groups.

Theorem 1. Let $M$ denote a 3-manifold. Suppose $G$ is a finitely presented subgroup of $\pi_1(M)$. Then there exists a compact 3-manifold $N$ with $\pi_1(N) \approx G$. A 3-manifold $M$ is said to be $P^2$-irreducible if every 2-sphere in $M$ bounds a 3-cell in $M$ and $M$ contains no 2-sided embeddings of $P^2$ (real projective 2-space).

Theorem 2. Let $M$ denote a $P^2$-irreducible 3-manifold and let $F$ denote a closed, injective surface in $M$ where $\chi(F) \neq 0$. Suppose $G$ is a subgroup of $\pi_1(M)$ such that $\pi_1(F) \subset G$ and the index of $\pi_1(F)$ in $G$ is finite. Then either (i) $\pi_1(F) = G$ or (ii) $\pi_1(F)$ has index two in $G$ and there is a twisted 1-bundle $N \subset M$ where $\pi_1(N) \approx G$ and $F = Bd N$. To prove Theorem 2 we use Lemma. Suppose $M$ is a compact, $P^2$-irreducible 3-manifold and $(\tilde{M}, p)$ is a covering of $M$. If $\tilde{M}$ is an $I$-bundle over the closed 2-manifold $F$ ($F \neq S^2$, $P^2$), then $M$ is an $I$-bundle over the closed 2-manifold $F^\dagger$ ($F^\dagger \neq S^2$, $P^2$). Sufficient conditions are given to determine if the fundamental group of a 3-manifold contains the fundamental group of a surface; i.e. the fundamental group is almost sufficiently large. (Received October 23, 1970.)
Two extension theorems for functions.

c is a closure operation on X if c is a single-valued map from \( p(X) \) to \( p(X) \) satisfying conditions: (i) \( c(\emptyset) = \emptyset \), (ii) \( A \subseteq cA \) for each \( A \in p(X) \), (iii) \( c(A \cup B) = cA \cup cB \) for A and B in \( p(X) \). The pair \((X, c)\) is called a closure space. A semi-quasi-uniformity for X is a filter \( \mathcal{U} \) on \( X \times X \) such that \( U \in \mathcal{U} \) implies \( \Delta_X \subseteq U \) and the pair \((X, \mathcal{U})\) is called a semi-quasi-uniform space. For a semi-quasi-uniformity \( \mathcal{U} \) of X, we define \( \mu : p(X) \rightarrow p(X) \) by setting \( \mu A = \{ x : x \in X, U[x] \cap A \neq \emptyset \ \text{for all} \ U \in \mathcal{U} \} \) for each \( A \in p(X) \). It can be shown that \( \mu \) is a closure operation on X. Continuity and uniform continuity for functions can be defined in a natural way. In the present paper, a theorem on uniformly continuous extension of functions and a theorem on continuous extensions of functions are proved. The former generalizes the well-known theorem on uniformly continuous extensions for functions from a uniform space to another one and the latter is concerned with functions from a dense subset of a general topological space to a certain semi-quasi-uniform space. (Received October 26, 1970.)

The exact couple of a mapping for twisted generalized cohomology.

We report that the Leray spectral sequence of (twisted) generalized cohomology arises from an exact couple. Consequences. Let \((X, A)\) be a space pair, \(f : X \rightarrow Y\) a map, and suppose that, for some integer \(d\), every open covering of \(Y\) has a refinement \(\{U_i\}\) such that the nerve of \(\{U_i - f(A)\}\) has dimension \(\leq d\). For any multiplicative generalized cohomology theory \(h\), \(h(X, A)\) is an \(h(Y)\)-module via \(h(Y) \otimes h(X, A) \rightarrow h(X) \otimes h(X, A) \rightarrow h(X, A)\), while the Leray sheaf of \(f\) is a sheaf of \(h(\text{pt.})\)-modules. Consider a family of members \(u_i (i \in I)\) of \(h(X, A)\):

Theorem 1. The \(u_i\) are a generating set if the corresponding cross sections of the Leray sheaf generate the sheaf.

Theorem 2. The \(u_i\) are linearly independent if the cross sections are. (Without the exact couple, the Leray spectral sequence can show only that the \(u_i\) freely generate \(h(X, A)\) if the cross sections freely generate the sheaf.) (Received October 26, 1970.)

Fixed points of symmetric product mappings for compact absolute neighborhood retracts.

Let \(X\) be a topological space and \(X^n\) the nth cartesian product with the usual topology. Let \(G\) be a group of permutations of the letters \([1, 2, \ldots, n]\). Then \(G\) can be considered as a group of homeomorphisms on \(X^n\) by defining, for \(g \in G\) and \((x_1, \ldots, x_n) \in X^n\), \(g(x_1, \ldots, x_n) = (x_{g(1)}, \ldots, x_{g(n)}).\) The orbit space under the action of \(G\) on \(X^n\) is denoted by \(X^n/G\) and called \(G\)-product of \(X\) (C. N. Maxwell, "Fixed points of symmetric product mappings," Proc. Amer. Math. Soc. 8 (1957), 808-815). If \(f : X \rightarrow X^n/G\) is a map then \(x \in X\) is called a fixed point of \(f\) if \(x\) is a coordinate of \(f(x)\). In this paper fixed point theorems of such maps are generalized to the case when \(X\) is a compact absolute neighborhood retract. The Lefschetz number \(\lambda(f)\) of the map \(f\) in case \(X\) is a compact absolute neighborhood retract is defined and it is shown that if \(\lambda(f) \neq 0\) then \(f\) has a fixed point. It is also proven that if \(X\) is a compact absolute retract then every map \(f : X \rightarrow X^n/G\) has a fixed point. (Received October 27, 1970.)
A set-theoretic proposition implying the metrizability of normal Moore spaces.

Definition. A collection $\mathcal{Y}$ of subsets of a topological space is normalized if for each $Y \subset \mathcal{Y}$ there exist disjoint open sets $U, U'$ such that $U \cap Y \subset U, U \cap (Y - \mathcal{Y}) \subset U'$. $\mathcal{Y}$ is separated if there exist mutually disjoint open sets $U, U'$, each $Y \in \mathcal{Y}$, $Y \subset U_Y$. Definition. Let $x$ and $\lambda, \lambda \prec x$, be infinite cardinals.

Let $G \subset X$. Let $\mathcal{Y} = \{Y \subset X\} \prec \lambda$ be a collection of disjoint subsets of $X$. $\langle G, \mathcal{Y} \rangle$ is doubly superior if $(\forall f \in X) (\exists G \in G \exists \gamma_0, \gamma_1 \in \lambda, \gamma_0 \neq \gamma_1) (\exists \alpha_0 \in Y_{\gamma_0}, \gamma_0) (\exists \alpha_1 \in Y_{\gamma_1}, \gamma_1) (\forall (\alpha_0) > f(\alpha_0), s(\alpha_0) > f(\alpha_0)).$

Definition. Let $p \subset \lambda \times 2$ be a function, domain $p \subset \lambda$, range $p \subset 2$. $p$ split $\langle G, \mathcal{Y} \rangle$ if $(\forall f \in X) (\exists G \in G \exists \gamma_0, \gamma_1 \in \lambda, \gamma_0 \neq \gamma_1) (\exists \alpha_0 \in Y_{\gamma_0}, \gamma_0) (\exists \alpha_1 \in Y_{\gamma_1}, \gamma_1) (\forall (\alpha_0) > f(\alpha_0), s(\alpha_0) > f(\alpha_0)).$ $\langle G, \mathcal{Y} \rangle$ splits if there is a $p$ which splits $\langle G, \mathcal{Y} \rangle$. Theorem. Every normalized collection in every first countable $T_1$ space is separated if and only if for every $\lambda$ and $\lambda$, every doubly superior $\langle G, \mathcal{Y} \rangle$ splits. Corollary. If for every $\lambda$ and $\lambda$, every doubly superior $\langle G, \mathcal{Y} \rangle$ splits, then every normal Moore space is metrizable. (Received October 27, 1970.)

71T-G14. WITHDRAWN.


For $n \geq 2$, a continuum $M$ is said to be $n$-mutually aposyndetic if given $n$ points in $M$, there exist $n$ disjoint subcontinua each containing one of the points in its interior. When $n = 2$, we obtain the notion of mutual aposyndesis [C. L. Hagopian, "Mutual aposyndesis", Proc. Amer. Math. Soc. 23(1969), 615-622]. Theorem 1. Let $M$ and $N$ be compact metric chainable continua. Then $M \times N$ is mutually aposyndetic if and only if both $M$ and $N$ are arcs. Also some relationships are shown between $n$-indecomposability and a strong form of non-$n$-mutual aposyndesis. (Received October 29, 1970.)


A locally separable Moore space is metrizable if and only if it is metacompact [D. R. Traylor, Canad. J. Math. 16(1964), 407-411]. An extended result is that a locally separable regular $T_0$ space $S$ is metrizable if and only if (1) $S$ has a base of countable order and (2) any collection of open sets covering $S$ is refined by a point-countable covering by open sets. The existence of nonnormal metacompact locally Lindelöfian Tychonoff $p$-spaces suggests, however, certain limitations in this type of approach for analysis of the Hausdorff perfect preimages of locally separable metric spaces (which become particularly conspicuous within the $M$-space framework). In contrast, the following approach does admit, with appropriate modifications, application for developing a theory for these preimages. Theorem. A regular $T_0$ locally separable space is metrizable if and only if it has a base of countable order and its scattered sets (ensembles clairsemés) are screenable in the global topology. Remark. Compare the author's abstracts [these Notices] 11(1964), 250 and 14(1967), 949. (Received October 29, 1970.)
$A_n$-classes in the cohomology of the total space of the primitive 2-stage Postnikov system $K(Z_2,p) \rightarrow E \rightarrow K(G,q)$.

$A_n$-classes was defined and studied by J. Stasheff in "Homotopy associativity of H-spaces. 1, II," Trans. Amer. Math. Soc. 108(1963), 275-312. Here we study the $A_n$-classes in $H^*(E;Z_2)$. For each primitive element $a$ in $H^*(G,q;Z_2)$ where $G$ is a finitely generated abelian group, we express $a$ uniquely by $\sum_I S^I_{ij} u_i$ where $\dim u_i = q$ or $q + 1$, and $S^I_{ij} u_i \neq u_j$ for any $i, j$. Let $N(a) = \text{max length of } I_{ij}$. Then by careful study of the Steenrod operations and the Moore spectral sequence of $\Omega E$, we can get the following Theorem. The Moore spectral sequence of $\Omega E$ collapses at $2^{N(k)}$ stage. Therefore if $x \in H^*(E;Z_2)$ is an $A_m$-class where $m \geq 2^{N(k)}$, then $x$ is a loop class. (Received September 28, 1970.) (Author introduced by Professor Clarence F. Stephens.)
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