NOTICES
OF THE
AMERICAN MATHEMATICAL SOCIETY

Edited by Everett Pitcher and Gordon L. Walker

CONTENTS

MEETINGS

Calendar of Meetings ............................................. 578
Program for the June Meeting in Corvallis, Oregon ............. 579
Abstracts for the Meeting: Pages 611-617

PRELIMINARY ANNOUNCEMENTS OF MEETINGS ................. 583

NEWS ITEMS AND ANNOUNCEMENTS ............................ 582, 589

MATHEMATICAL OFFPRINT SERVICE ............................ 590

LETTERS TO THE EDITOR ....................................... 592

SPECIAL MEETINGS INFORMATION CENTER ..................... 595

MEMORANDA TO MEMBERS ................................... 600

Policy on Recruitment

PERSONAL ITEMS .................................................. 601

NEW AMS PUBLICATIONS ....................................... 605

ABSTRACTS OF CONTRIBUTED PAPERS ......................... 608

ERRATA TO ABSTRACTS ....................................... 617

ABSTRACTS PRESENTED TO THE SOCIETY ....................... 618

INDEX TO ADVERTISERS ....................................... 687

RESERVATION FORM ............................................. 688
**MEETINGS**

**Calendar of Meetings**

**NOTE:** This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the *Notice* was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

<table>
<thead>
<tr>
<th>Meeting No.</th>
<th>Date</th>
<th>Place</th>
<th>Deadline for Abstracts*</th>
</tr>
</thead>
<tbody>
<tr>
<td>687</td>
<td>August 30-September 3, 1971 (76th Summer Meeting)</td>
<td>University Park, Pennsylvania</td>
<td>July 7, 1971</td>
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<tr>
<td>688</td>
<td>October 30, 1971</td>
<td>Cambridge, Massachusetts</td>
<td>Sept. 9, 1971</td>
</tr>
<tr>
<td>691</td>
<td>January 17-21, 1972 (78th Annual Meeting)</td>
<td>Las Vegas, Nevada</td>
<td>Nov. 4, 1971</td>
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<td>March 29-April 1, 1972</td>
<td>St. Louis, Missouri</td>
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<td></td>
<td>August 28-September 1, 1972 (77th Summer Meeting)</td>
<td>Hanover, New Hampshire</td>
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<td>January 25-29, 1973 (79th Annual Meeting)</td>
<td>Dallas, Texas</td>
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<td></td>
<td>August 20-24, 1973 (78th Summer Meeting)</td>
<td>Missoula, Montana</td>
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<td>January 13-19, 1974 (80th Annual Meeting)</td>
<td>San Francisco, California</td>
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<td></td>
<td>January 22-26, 1976 (82nd Annual Meeting)</td>
<td>San Antonio, Texas</td>
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*The abstracts of papers to be presented in person at the meetings must be received in the Headquarters offices of the Society in Providence, Rhode Island, on or before these deadlines. The deadlines also apply to news items. The next two deadlines for by-title abstracts will be September 2, 1971, and September 27, 1971.*

The *Notice* of the American Mathematical Society is published by the American Mathematical Society, 321 South Main Street, P. O. Box 6248, Providence, Rhode Island 02904 in January, February, April, June, August, October, November and December. Price per annual volume is $10.00. Price per copy $3.00. Special price for copies sold at registration desks of meetings of the Society, $1.00 per copy. Subscriptions, orders for back numbers (back issues of the last two years only are available) and inquiries should be addressed to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904. Second class postage paid at Providence, Rhode Island, and additional mailing offices.

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578
The six hundred eighty-sixth meeting of the American Mathematical Society will be held at Oregon State University in Corvallis, Oregon, on Saturday, June 19, 1971. The Mathematical Association of America will hold a Northwest Sectional meeting in conjunction with this meeting of the Society. The Association will have sessions on Friday and Saturday, June 18 and 19.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, there will be two invited addresses. Professor Ky Fan of the University of California, Santa Barbara, will address the Society at 11:00 a.m. on Saturday. He will speak on "Fixed point theorems in functional analysis." Professor Robert J. Blattner of the University of California, Los Angeles, will lecture at 2:00 p.m. on Saturday. The title of his lecture is "Linearly compact Lie modules." Both of these addresses will be given in room 151 of Weniger Hall. There will be sessions for contributed papers on Saturday morning and afternoon. The following program contains papers whose abstracts were received before May 5, 1971. Late papers will be accepted for presentation until a few days before the meeting. All sessions for contributed papers will be held in Kidder Hall. The registration desk for the meeting will be located in the first floor lobby of Kidder Hall, and will be open for the duration of the meeting.

Accommodations in residence halls will be available on Thursday, Friday, and Saturday nights. The rates are $3.50 per person per night on a double occupancy basis, and $5.00 per person per night in a single room. No maid service is provided. Reservations should be sent before June 11, 1971, to Mr. Edward M. Cracraft, Department of Mathematics, Oregon State University, Corvallis, Oregon 97331, and should be accompanied by a check (to cover the length of time desired) payable to Oregon State University. The room requests should include the expected time of arrival, names of all persons for whom space is required, and the type of accommodations desired.

There are several motels in Corvallis, including the following:

CITY CENTER MOTEL
315 S. W. Fourth St., 97330
Rates from $9.50 up

TOWNE HOUSE MOTOR INN
350 S. W. Fourth St., 97330
Rates from $9.00 up

COUNTRY KITCHEN MOTEL
800 N. W. Ninth St., 97330
Rates from $11.00 up

The City Center Motel and Towne House Motor Inn are located about a mile from the Oregon State University campus, and the Country Kitchen Motel is somewhat further from the campus. Reservations should be made directly with the desired motel.

Meals will be available in the cafeteria in the Memorial Union Building on campus. There are also many restaurants in Corvallis. The Mathematical Association of America will probably sponsor a salmon bake on Friday evening. Tickets for the salmon bake will be available at the registration desk.

The Corvallis Airport is served by Air West Airlines. Greyhound Bus Lines serves Corvallis. The Oregon State University campus is located directly west of downtown Corvallis. To get to Kidder Hall by car, drive west on Monroe Street and park on or near Monroe Street between 14th and 18th Streets. Kidder Hall is located on Campus Way between Benton Drive and Waldo Place.
PROGRAM OF THE SESSIONS

The time limit for each contributed paper is ten minutes. The contributed papers are scheduled at fifteen minute intervals. To maintain this schedule, the time limit will be strictly enforced.

SATURDAY, 9:30 A.M.

Session on Analysis, I, Room 274, Kidder Hall
9:30-9:40
(1) The v-derivative on measure algebra from functions on abelian idempotent semigroups
Miss Leiko Hatta, Utah State University (686-B2)
(Introduced by Professor Stanley G. Wayment)

9:45-9:55
(2) A derivative to match the v-integral
Professor Stanley G. Wayment, Utah State University (686-B1)

10:00-10:10
(3) On set correspondences into uniformly convex Banach spaces
Professor David Schmeidler, University of California, Berkeley (686-B7)

10:15-10:25
(4) The core topology and other affine topologies on finite dimensional spaces
Dr. Clifford A. Kottman, Oregon State University (686-B4)

10:30-10:40
(5) Generalized Abel expansion and a subclass of completely convex functions
Dr. S. Pethe*, University of Calgary, and Dr. Ambikeshwar Sharma, University of Wisconsin (686-B6)

SATURDAY, 9:30 A.M.

Session on Applied Mathematics and Probability, Room 276, Kidder Hall
9:30-9:40
(6) Generalized measures of deformation-rates in non-Newtonian hydrodynamics and their applications to torsional flow problems. Preliminary report
Mr. M. Park, Oregon State University (686-C2)
(Introduced by Professor Mysore N. L. Narasimhan)

9:45-9:55
(7) Stability of flows of thermo-viscoelastic fluids between rotating coaxial circular cylinders. Preliminary report
Mr. Nabil N. Ghandour, Oregon State University (686-C1)

10:00-10:10
(8) Quantization for minimum error. Preliminary report
Mr. Vishnu B. Jumani, Oregon State University (686-C4)
(Introduced by Professor William M. Stone)

10:15-10:25
(9) On the Shannon-Slepian estimates of probability of decoding error. Preliminary report
Mr. Warren G. Clendenin and Professor William M. Stone*, Oregon State University (686-C3)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.
The Levy distance between Wiener measure and the measure generated by a certain random walk
Dr. David F. Fraser, Brown University (686-F1)

SATURDAY, 11:00 A.M.

Invited Address, Room 151, Weniger Hall
Fixed point theorems in functional analysis
Professor Ky Fan, University of California, Santa Barbara

SATURDAY, 2:00 P.M.

Invited Address, Room 151, Weniger Hall
Linearly compact Lie modules
Professor Robert J. Blattner, University of California, Los Angeles

SATURDAY, 3:15 P.M.

Session on Analysis, II, Room 274, Kidder Hall
3:15-3:25
(11) Controllability of nonlinear systems using a growth condition
Mr. Jerald P. Dauer, University of Nebraska (686-B5)

3:30-3:40
(12) Certain double integrals involving hypergeometric functions, Preliminary report
Professor H. M. Srivastava, University of Victoria (686-B3)

3:45-3:55
(13) Values assumed by Hadamard gap series. Preliminary report
Dr. I-Lok Chang, American University (686-B8)

4:00-4:10
(14) Grunsky-Nehari inequalities for a subclass of bounded univalent functions
Dr. Duane W. DeTemple, Washington State University (686-B9)

4:15-4:25
(15) Coefficients of uniformly normal holomorphic functions defined in the unit disk. Preliminary report
Professor John H. Mathews, California State College at Fullerton (686-B10)

SATURDAY, 3:15 P.M.

Session on Topology and Algebra, Room 276, Kidder Hall
3:15-3:25
(16) Stronger forms of connectivity. Preliminary report
Mr. J. E. Leuschen and Professor Benjamin T. Sims*, Eastern Washington State College (686-G2)

3:30-3:40
(17) A topological proof of a theorem of Kneser
Mr. Bjorn O. Friberg, University of California, Los Angeles (686-G1)

3:45-3:55
(18) Manifold maps which commute with the Laplacian on p-forms
Mr. Bill Watson, University of Oregon (686-D1)
(Introduced by Professor John V. Leahy)
NEWS ITEMS AND ANNOUNCEMENTS

NSF POSTDOCTORAL FELLOWSHIPS

The National Science Foundation has announced the awarding of 185 Postdoctoral Fellowships. The following mathematicians, with their current affiliations, received awards: Pressley W. Millar, University of California, Berkeley; Ralph N. McKenzie, University of California, Berkeley; Peter B. Shalen, Harvard University; David A. Perin, University of Michigan; David A. Edwards, Columbia University; Lewis Leibovich, Cornell University; Peter J. Nyikos, Carnegie-Mellon University; Allen E. Hatcher, Stanford University; Gerald B. Folland, Princeton University.

NYU GREAT TEACHER AWARDS FOR 1971

Among the three professors named to receive New York University's Great Teacher Awards for 1971 was Professor Morris Kline, professor of mathematics at Washington Square College of Arts and Science. Professor Kline earned his B.S., M.S., and Ph.D. from New York University, and joined the faculty as an instructor in 1930. He served as director of the division of electromagnetic research at Courant Institute from 1946 to 1966, and as chairman of the department of mathematics at Washington Square College from 1959 to 1970.

INTERNATIONAL MATHEMATICAL UNION

The International Mathematical Union, in its present form, has completed twenty years of existence. Reports of the Union's activities during this period have been sent out to member-countries as required under the statutes and bylaws. Some of them have also been published, by arrangement, in the Bulletin of the Austrian Mathematical Society. These reports are now being carried in the new IMS Bulletin, the first issue of which was published in January 1971. All inquiries about the Bulletin should be addressed to the Secretary of the Union, Box 41, S-182 51 Djursholm, Sweden.

Kenneth A. Ross
Associate Secretary
The seventy-sixth summer meeting of the American Mathematical Society will be held at The Pennsylvania State University, University Park, Pennsylvania, from Tuesday, August 31, 1971, through Friday, September 3, 1971. All sessions of the meeting will be held on the campus of the university. The times listed for events of the meeting are EASTERN DAYLIGHT SAVING TIME throughout.

There will be two sets of Colloquium Lectures. Professor Lipman Bers of Columbia University will present four lectures entitled "Uniformization, moduli and Kleinian groups." These addresses will be given on Tuesday, August 31, at 1:00 p.m. and on Wednesday, Thursday, and Friday at 8:30 a.m. The other Colloquium Lecturer will be Professor Armand Borel of The Institute for Advanced Study. His topic will be "Algebraic groups and arithmetic groups." Professor Borel's four lectures will be given on Tuesday, August 31, at 2:15 p.m. and on Wednesday, Thursday, and Friday at 8:30 a.m. The first address of each series will be presented in the Schwab Auditorium; Professor Bers' remaining Colloquium Lectures will be presented in the Auditorium of the Conference Center and the remaining lectures by Professor Borel will be given in Room 102 of the Forum Building.

The AMS Committee on Employment and Educational Policy will present a panel discussion on Tuesday afternoon, August 31, at 3:30 p.m., in the Schwab Auditorium. The discussion will be concerned with current problems in the job market for Ph.D.'s, prospects for the future, and consequences for graduate programs in mathematics.

By invitation of the Committee to Select Hour Speakers for Annual and Summer Meetings, there will be at least five invited hour addresses at the meeting. Professor John M. Boardman of Johns Hopkins University will give a lecture entitled "Infinite loop spaces, trees, and the bar construction." Professor Felix E. Browder, University of Chicago, will speak on "Nonlinear functional analysis." A lecture entitled "Conjugacy problems in discrete dynamical systems" will be presented by Professor Joel W. Robbin of the University of Wisconsin, Madison. Professor Isadore M. Singer of the Massachusetts Institute of Technology will present an hour address entitled "Applications of elliptic operators." Professor Benjamin Weiss of the Hebrew University, Jerusalem, Israel, will give a lecture entitled "Recent progress on the isomorphism problem in ergodic theory." There may be two additional invited hour addresses. If so, the names of the speakers and the titles of their lectures will be listed in the program of the meeting, along with the dates and times of the lectures given above, in the August issue of these Notices. There will be numerous sessions for contributed ten-minute papers. These are tentatively scheduled at 11:00 a.m. on Wednesday, at 11:00 a.m. on Thursday, and at 11:00 a.m. and 4:00 p.m. on Friday. Abstracts of contributed papers should be sent to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904, so as to arrive prior to the July 7 deadline. Provision will be made for late papers. The abstracts should be submitted on abstract forms which are available in most departments of mathematics; forms can also be obtained by writing to the Society headquarters.

This meeting will be held in conjunction with meetings of the Mathematical Association of America, Mu Alpha Theta, and Pi Mu Epsilon. The Mathematical Association of America will meet
Monday through Wednesday. The Association will present Professor Abraham Robinson of Yale University as the Earle Raymond Hedrick Lecturer. The title of Professor Robinson's lectures will be "Nonstandard analysis and nonstandard arithmetic." Mu Alpha Theta and Pi Mu Epsilon will meet concurrently with the Society and the Association.

COUNCIL AND BUSINESS MEETING

The Council of the Society will meet on Tuesday, August 31, at 5:00 p.m. in Room 201 of the Conference Center. The Business Meeting of the Society will be held on Thursday, September 2, at 4:00 p.m. in the Schwab Auditorium.

REGISTRATION

The Registration Desk will be in the lobby of the Conference Center, the J. Orvis Keller Building. It will be open on Sunday from 2:00 p.m. to 8:00 p.m.; on Monday from 8:00 a.m. to 5:00 p.m.; on Tuesday, Wednesday, and Thursday from 8:30 a.m. to 4:30 p.m., and on Friday from 8:30 a.m. to 1:00 p.m. The telephone number will be 814-865-1422. The registration fee will be as follows:

- Members $5.00
- Students $1.00
- Nonmembers $10.00

There will be no extra charge for members of the families of registered participants.

EMPLOYMENT REGISTER

The Joint Committee on Employment Opportunities has decided not to have an Employment Register at the University Park, Pennsylvania, meeting.

EXHIBITS

Book exhibits and exhibits of educational media will be displayed in Rooms 114 and 115 on the main floor of the Conference Center on Monday, August 30, from 1:00 p.m. to 5:00 p.m., on Tuesday and Wednesday, August 31 and September 1, from 9:00 a.m. to 5:00 p.m., and on Thursday, September 2, from 9:00 a.m. to 12:00 p.m.

MATHEMATICAL OFFPRINT SERVICE

Information concerning the Mathematical Offprint Service (MOS) as well as assistance in completing MOS profile forms will be available at the MOS Information Desk.

BOOK SALES

Books published by the Society will be sold for cash prices somewhat below the usual prices when the same books are sold by mail.

RESIDENCE HALL HOUSING

Rooms are available in the West Residence Hall complex near the Conference Center registration headquarters. The residence halls have coin-operated washers and dryers as well as ironing boards. Irons are available.

Rooms may be occupied from 2:00 p.m., Saturday, August 28, until Saturday noon, September 4. Bed linens, blankets, towels, and soap will be provided.

The rates are:

- Adults Room and Breakfast $4.50 double room $5.50 single room
- Children (9 years of age and younger) $4.15 double room $5.15 single room

There is no charge for children in cribs who are sharing a room with their parents; however, the university cannot provide the cribs.

Upon arrival, all guests should check in at the Waring Hall Housing Desk to register for residence hall rooms. Guests are expected to pay for their rooms at the time they check in. Persons who preregister for residence hall accommodations will be sent a marked map showing the location of Waring Hall on the campus. Residence hall personnel will be on duty 24 hours a day to issue keys. Student bellhops will be available in the residence hall area and will accept tips.

Guests must register in advance to be assured of residence hall housing. Please use the form provided on the last page of these Notices. Residence hall reservation requests will be acknowledged by the Conference Center. It is probable that rooms may be available for those not
registering in advance, but this cannot be guaranteed.

**FOOD SERVICES**

The dining room in Waring Hall will be open for breakfast on Monday, August 30, for residents of the university dormitories. Lunch will be available from Monday through Friday on a cash basis. Lunch tickets may be purchased upon entering the dining room.

The prices are:

- **Adults**: $1.55
- **Children (9 years of age and younger)**: $1.20

The last meal will be breakfast on Saturday, September 4, 1971. Please note that the lodging charges include breakfast. Persons arriving prior to Monday, August 30, will be charged a reduced rate for their room because breakfast will not be available. Meals may be purchased in the nearby Student Union Building or the Nittany Lion Inn by those persons arriving before August 30. Dinner will not be available in Waring Hall.

The Student Union Building has a snack bar and cafeteria. The hours of operation are:
- **Lion's Den (snack bar)**: 7:00 a.m. - 2:00 p.m. breakfast and lunch
- **Cafeteria**:
  - 11:30 a.m. - 1:30 p.m. lunch
  - 4:45 p.m. - 6:45 p.m. dinner

A Cafeteria Snack-Coffee Bar will be open in the Conference Center during the meetings. The hours of operation for this snack bar and for the restaurants located in the Nittany Lion Inn will be posted in the registration area. Also available in the registration area will be a listing of local restaurants.

**MOTELS AND HOTELS**

There are a number of motels and hotels in the State College area, which are listed below with the following coded information: **FP** - Free Parking; **SP** - Swimming Pool; **AC** - Air Conditioned; **TV** - Television; **CL** - Cocktail Lounge; **RT** - Restaurant. Participants should make their own reservations. All prices are subject to change without notice.

**AUTOPORT MOTEL** (814)237-7666
- South Atherton Street - 60 rooms
- **Singles**: $10.00 - $12.00
- **Doubles**: 15.00 - 18.00
- **Suite**: 20.00 - 22.00
- Code: **FP-SP-AC-TV-RT**
- 2 miles from Conference Center

**COLLEGE COURT MOTEL** (814)238-0561
- North Atherton Street - 20 rooms
- **Singles**: $7.00 - $9.00
- **Doubles**: 10.00 - 12.00
- Twin 12.00 - 14.00 and up
- Code: **FP-AC-TV**
- 1 mile from Conference Center

**DUTCH PANTRY MOTEL** (814)238-8461
- South Atherton Street - 39 rooms
- **Singles**: $7.00 - $9.00
- **Doubles**: 10.00 - 12.00
- **Twin**: 12.00 - 14.00 and up
- Code: **FP-AC-TV**
- 1 1/2 miles from Conference Center

**HOLIDAY INN** (814)238-3001
- South Atherton Street - 311 rooms
- **Singles**: $13.00 - $14.00
- **Doubles**: 17.00
- **Twin Doubles**: 19.00
- **Studios**: 17.00 - 21.00
- **Suite**: 26.00
- Code: **FP-SP-AC-TV-CL-RT**
- 2 miles from Conference Center

**IMPERIAL "400" MOTEL** (814)237-7686
- South Atherton Street - 37 rooms
- **Singles**: $11.00 - $13.00
- **Twin Doubles**: 14.00 - 16.00
- 18.00 - 20.00
- **Studio**: 12.00 - 14.00 - 16.00
- Code: **FP-SP-AC-TV-RT**
- 1 mile from Conference Center

**KAR-MEL MOTEL** (814)355-5561
- Benner Pike, Route 26, Bellefonte, Pa.
- 45 rooms
- **Singles**: $8.00 - $10.00
- **Doubles**: 12.00 and up
- Code: **FP-AC-TV-RT**
- 8 miles from Conference Center

**NITTANY LION INN** (814)237-7671
- North Atherton Street - 150 rooms
- **Singles**: $13.00 and up
- **Doubles**: 15.00 and up
- **Twin**: 17.00 and up
- **Suite**: 22.00 and up
- Code: **FP-TV-CL-RT**
- Next to Conference Center
Key to Borough
1. Centre Hills Country Club
2. Penn Hi-Boy Motel
3. Holiday Inn Motel
4. Autoport Motel
5. Dutch Pantry Motel
6. Junior High School
7. Community Swimming Pool
8. Senior High School
9. Our Lady of Victory School
10. Holmes-Foster Park
11. Fairmount Park
12. Sheraton Motor Inn
13. Post Office
14. State College Hotel
15. Borough Hall
16. Imperial 400 Motel
17. V.F.W.
18. Nittany Lion Inn
19. Conference Center
20. Sunset View Park
21. Peters Motel
22. Nittany Manor Motel
23. Stevens Motel
24. College Court Motel
25. Pollock Rd.

Transportation Facilities:
14. Taxi
26. Interstate Bus Terminal
27. University Park Airport

Mid-State Airport Limousine
Pick up points:
(17) Imperial 400 Motel
(3) Holiday Inn
(4) Autoport Motel
(12) Sheraton Motel
(15) State College Hotel
(19) Nittany Lion Inn
NITTANY MANOR MOTEL (814)237-7638
NORTH AThERTON STREET - 37 ROOMS
SINGLES $9.00-$10.00
DOUBLES 12.00
TWIN DOUBLES 14.00 and up
CODE: FP-AC-TV-RT
3/4 MILE FROM CONFERENCE CENTER

PENN HI-BOY MOTEL (814)238-0571
SOUTH AThERTON STREET - 20 ROOMS
SINGLES $9.00
DOUBLES 12.00-14.00
TWIN DOUBLES 16.00
CODE: FP-AC-TV-RT
2 1/2 MILES FROM CONFERENCE CENTER

PETERS MOTEL (814)238-6783
NORTH AThERTON STREET - 20 ROOMS
SINGLES $9.00
DOUBLES 12.00
TWIN DOUBLES 14.00 and up
CODE: FP-AC-TV
1/2 MILE FROM CONFERENCE CENTER

SHERATON MOTOR INN (814)238-8454
SOUTH PUGH STREET - 98 ROOMS
SINGLES $12.00-$15.00
DOUBLES 17.00-21.00
SUITES 32.00 and up
CODE: FP-SP-AC-TV-CL-RT
3/4 MILE FROM CONFERENCE CENTER

STATE COLLEGE HOTEL (814)237-4350
WEST COLLEGE AVENUE - 40 ROOMS
SINGLES $7.00-$8.00
DOUBLES 10.00-12.00
TWIN 12.00-13.00
CODE: AC-TV-RT
1/2 MILE FROM CONFERENCE CENTER

STEVENS MOTEL (814)238-2438
NORTH AThERTON STREET - 16 ROOMS
SINGLES $8.00-$9.00
DOUBLES 11.00-13.00
FAMILY 14.00 and up
CODE: FP-AC-TV
3/4 MILE FROM CONFERENCE CENTER

PARKING
Permits for on-campus parking will be issued at check-in to the residence hall. Persons living in off-campus hotels and motels will be issued a parking permit at registration for the meetings.

CAMPING
There are three state parks with camping facilities within twenty-five miles of State College: Black Moshannon, Poe Valley, and Greenwood Furnace. Information on state parks may be obtained from Public Relations, Department of Forests and Waters, Commonwealth of Pennsylvania, Harrisburg, Pennsylvania 17120. There are several private campgrounds in the area: Bald Eagle Campsite, R.D. 3, Tyrone, Pennsylvania 16686; Bellefonte Kampgrounds of America, R.D. 2, Bellefonte, Pennsylvania 16823; Hartle Trailer Court, Drifting, Pennsylvania 16834; Snow Shoe Lions Club Park, Snow Shoe, Pennsylvania 16874.

BOOKSTORE
There is no bookstore on campus. However, there are several good bookstores in the downtown area which are open daily during normal working hours.

LIBRARIES
Pattee Library (main library) will be open from 7:00 a.m. to 10:00 p.m., Monday through Friday. It is convenient to the Conference Center. The Mathematics Reading Room houses the university's mathematics collection and is located in the McAllister Building. It will be open from 8:00 a.m. to 11:00 p.m. Monday through Friday.

MEDICAL SERVICES
The university's Ritenour Health Center will be open to treat emergencies twenty-four hours a day. This emergency service is available at no charge to registered participants. Centre Community Hospital, in Bellefonte (12 miles northeast), can handle any serious medical or surgical emergencies.

ENTERTAINMENT
One of the most interesting Amish communities in the United States is located twenty-five miles from State College. A lecture on the Amish entitled "Barndoor Britches and Shoo-fly Pie" will be given
by Dr. Maurice Mook on Tuesday, August 31, at 7:30 p.m. A tour of the Amish market in Belleville is scheduled for Wednesday, September 1, from 9:30 a.m. to 1:30 p.m. Tickets will be on sale in the registration area.

A beer party, sponsored by the Society for Industrial and Applied Mathematics, will be held on Wednesday, September 1, at 8:30 p.m. at the Skimont Lodge, five miles east of town. Transportation will be provided. Tickets will be sold in the registration area.

A chicken barbecue on Wednesday evening from 5:00 p.m. to 7:00 p.m. will be held in the University Skating Pavilion. Tickets, which are $3.25 per person for adults and $2.00 each for children under nine, will be on sale in the registration area.

A Chess Exhibition will be held. Grand Master Donald Byrne of the Penn State English Department will play twenty-five mathematicians simultaneously at 7:30 p.m. on Thursday, September 2, in the Hetzel Union Building. Persons interested in participating in this event should write to Scott Williams, Department of Mathematics, McAllister Building, University Park, Pennsylvania 16802. There may be a small fee.

Conducted tours of the university flower gardens will be available. If there is sufficient interest, bird-watching trips will also be arranged.

Other diversions and facilities include summer theatre, concerts; visits to the Mineral Industries Museum, university creamery, barns, and orchards; golf, tennis, bowling, ping-pong, gymnasium facilities, swimming pools (indoor and outdoor), picnicking, and movies.

Within a twenty-five mile radius there are state parks which have swimming beaches, marked hiking trails, camping, boating; fishing in Penn’s Creek, an internationally famed trout stream; Amish country, Indian Caverns, Penn’s Cave (an all-water cavern explored by boat), the Boal Mansion and Christopher Columbus Chapel, and Bear Meadows (a marshy area in the Seven Mountains southeast of State College) which supports botanical growth otherwise foreign to this area and climate.

Arrangements will be made to provide nursery care for children from two through five years old, a supervised playground program for the elementary level, and a few scheduled events for the junior and senior high group. A list of babysitters for evening hours will be available.

**TRAVEL**

Penn State is located at the geographic center of Pennsylvania in the borough of State College. Interstate 80 has three exits (Lamar, Jacksonville, and Milesburg) within twenty-five miles of State College. Greyhound and Continental Trailways bus lines serve State College from all major cities. Penn Central Railroad serves State College (at Lewistown depot, thirty miles southeast) from all major east and west points. Buses for State College meet some of the trains (the fare is $1.45); those trains with this connection are indicated on rail schedules. Pennsylvania Commuter Airline serves State College (at University Park Airport, five miles northeast) from Baltimore and Washington, D.C. (National Airport). Taxi fare from University Park Airport is $2.50 per person. Allegheny Airlines serves State College (at Midstate Airport, twenty-five miles northwest) from New York City, Philadelphia, and Pittsburgh. Limousines meet all Allegheny flights (the fare is $3.25). Cars can be rented at Midstate Airport and in several locations in State College. Persons completing the residence hall reservation form will receive additional travel information.

**WEATHER**

The average high temperature for this period is 80° and the average low is 58°. Rainfall in August amounts to 3.2". The humidity is usually not excessive.

**MAIL AND MESSAGE CENTER**

Individuals may be addressed at Mathematics Meeting, Conference Center, Keller Building, University Park, Pennsylvania 16802. The telephone number of the Message Center will be 814-865-1422.
NEWS ITEMS AND ANNOUNCEMENTS

SLOAN RESEARCH FELLOWSHIPS

The Alfred P. Sloan Foundation has announced the recipients of the Sloan Research Fellowships for the 1971-1972 academic year. Seventy-seven fellowships were awarded to young physical scientists on the faculties of forty-four universities and colleges. Among the Fellows are the following mathematicians and computer scientists: George M. Bergman, Manual Blum, Alan D. Weinstein, University of California, Berkeley; David W. Barnette, University of California, Davis; Gregory W. Brumfiel, Stanford University; Yum-Tong Siu, Yale University; Phillip A. Griffiths, University of Illinois, Urbana; Jonathan L. Rosner, University of Minnesota; Sylvain Cappell, Julius L. Shaneson, Princeton University; Nolan R. Wallach, Rutgers University; Donald A. Martin, Rockefeller University; Michael Fried, SUNY at Stony Brook; William A. Veech, Rice University; James W. Cannon, I. Martin Isaacs, University of Wisconsin.

NSF GRADUATE FELLOWSHIPS

The National Science Foundation has announced the awarding of 1,148 NSF Fellowships for the academic year 1971-1972. In addition, NSF confirmed the second year of support for 821 Graduate Fellows who received a two-year award in March 1970. Of the new Graduate Fellows, 204 are in mathematics.

ACE FELLOWS

The selection of thirty-nine American Council on Education Fellows of the Academic Administration Internship Program, supported by the Ford Foundation, has recently been announced. Among the Fellows for 1971-1972 are two mathematicians: Professor Laurence R. Alvarez of the University of the South and Professor D. Reginald Traylor of the University of Houston.

GEORGE WILLIAM HILL RESEARCH INSTRUCTORSHIPS

The State University of New York at Buffalo has announced the first awards of the George William Hill Instructorships. The two recipients are Jessi A. Ketonen of the University of Wisconsin and Dallas E. Webster, who has spent the past year at The Institute for Advanced Study.

NSF SENIOR POSTDOCTORAL FELLOWSHIPS

The February 1971 issue of these Notices announced the awarding of the NSF Senior Postdoctoral Fellowships to mathematicians, and the name of one mathematician was omitted from the list. Professor Glen E. Bredon of Rutgers University was also granted a Senior Postdoctoral Fellowship.
In January 1968, the first announcement of a new means of retrieval and information exchange, the Mathematical Offprint Service, was made. Over the preceding two decades, it had become evident that new means of communicating research results in mathematics were going to have to be devised. With the rapid increase in the number of research papers published yearly, the problems of retrieving information also increased. Computer technology was the obvious answer in solving some of these problems. MOS, by taking full advantage of computer techniques, and by acting as a guide to the literature and not as a substitute for the literature, has become one additional tool for the mathematician.

Six months after the first announcement, MOS was not only in active operation, but improvements and modifications were already being made in the original system. Many of the changes were suggested by subscribers, and some were developed by the Committee to Monitor Problems in Communication and the staff of MOS. Modifications continued to be made and new services added over the next two years. With the development of a new subject classification system in 1970, a mathematician was able to make very precise specifications on his profile form, and with the new scheme came additional options and services. New exclusion options were incorporated into the system; abstracts could be supplied; and a subscriber could obtain bibliographic references to specified authors (including himself).

The number of journals participating in MOS in June 1968 was 90, and by June 1971 the number has increased to approximately 200. The participating journals are among the most important mathematical journals now being published throughout the world. Journals published in the United States constitute thirty-one percent of the total; Canadian, one percent; and foreign, sixty-eight percent. It is estimated that the total cost of library subscriptions to these 200 journals would be approximately $6,000 a year.

The ways in which MOS can be used are many. The service is used by mathematicians preparing expository papers and in the compilation of cumulative reviews. A mathematician can broaden his relationships with the world community of mathematicians by receiving title listings of all papers in which the results of his own research have been cited. One of the more innovative ideas for the use of MOS was proposed by R.P. Boas in a letter to the editor of the Notices, February 1969, in which he stated: "A subscriber who teaches a specialized course could order for the period when he is preparing the course and teaching it, all papers classified in the field. I found that in this way I got many papers relevant to a course on approximation theory, and was able to use several as part of the course." Probably one of the most useful services that can be provided by MOS will be to young Ph.D.'s who, because of the present employment situation, may be isolated in small colleges and universities whose libraries are less extensive than those in the major research centers. To compensate for inadequate libraries, some colleges provide subscriptions to staff members, and these young mathematicians can be alerted to the important developments in mathematics.

The intention of the service is to remain flexible and responsive to the needs of the mathematical community, and among the recently instituted services is that of allowing a department of mathematics to subscribe in the name of the department. For these special subscriptions, MOS will mail to the departments a list of current publications that cite works of individual members of the faculty and all papers published by members of the department. All that the MOS staff needs to enter such a subscription is the names of the faculty members.

It has been recognized that some mathematicians have not subscribed to the service because of the necessity for completing a profile form. A new service
has been instituted that provides for the MOS staff to fill out the form after receiving the pertinent information from a subscriber by either letter or telephone. Under this system, the subscriber will receive only title listings for a trial period so that he can satisfy himself that the profile form was properly constructed to his needs. He can, of course, make changes at any time he wishes. And now it is possible for a subscriber to purchase a package profile. The package on ring theory is now complete. It is based on a profile form designed to identify current research results in ring theory published in the participating journals. Those who subscribe to the Ring Package will receive a monthly notice of title listings of papers directly concerned with ring theory. A sample copy of the monthly letter to Ring Package subscribers will be sent to anyone requesting it from MOS. Profiles are now being prepared on combinatorics and graph theory, Diophantine approximation, numerical solution of partial differential equations, numerical linear algebra, algebraic topology, and spaces of analytic functions. Anyone who wants to specify a package profile for some other field may submit a suggestion for the approval of the Committee to Monitor Problems in Communication.

In preparation now is the first issue of the Index of Mathematical Papers, which includes both subject and author indices of all articles processed by MOS. This book should prove of inestimable value to libraries as a means of retrieval of information on mathematical research. In fact, one of the by-products of the MOS system is the potential for full archival use.

The cost of this service is modest. A subscriber is asked to make an initial deposit of $30, and he is notified when the deposit is nearing depletion. Title references are $0.05 each; abstracts, $0.25; and offprint prices range from $0.45 to $0.85. If a subscriber completes his own profile form or makes any changes in one of the package profiles (the Ring Package, for instance), a fee of $4 is charged for the initial processing; no charge is made for subsequent changes in the profile form. There is a minimum monthly charge of $0.50. If the MOS staff prepares the profile for a subscriber, the initial charge is $10. Extra reprints may be ordered by subscribers either for their personal use or for the use of their colleagues. (These reprints are available only up to the number that MOS has in stock, and extra reprints are not always available for every paper.) The Index of Mathematical Papers is $30 per issue; there will be two issues per year.

Mathematicians who are working under government grants are advised that the National Science Foundation permits grantees to charge information services related to their research projects directly against their grants or contracts; this applies to a MOS subscription. (Refer to NSF 69-23, "Grants for Scientific Research," 1969, page 13.) Some other agencies allow and some encourage this same procedure under their grants. Subscribers to MOS should investigate the rules and regulations pertaining to their grants and contracts.

The MOS staff is always ready to answer questions about the service; in fact, both inquiries and suggestions are welcomed. Correspondence and orders for subscriptions should be sent to the Mathematical Offprint Service, P.O. Box 6248, Providence, Rhode Island 02904.
LETTERS TO THE EDITOR

Editor, the Notices

For many years the Society has held an annual meeting during April in New York. Most often, as this year, the days of the meeting overlap the first two days of Passover. Observant members of the Jewish faith celebrate this important holiday with their families and suspend their business and professional activities. For this reason, mathematicians who follow the Jewish tradition do not attend professional meetings on these days. I suggest that the Society, as a convenience to this group, schedule the April meeting in the future years so that it does not conflict with Passover. Many other normal business activities in New York are modified to accommodate the large Jewish population; I hope that the Society will do the same.

Simeon M. Berman
New York University

Editorial comment: There are several constraints that affect the schedule of the New York meeting. First, there are three April meetings, East, West, and Far West; by agreement, these are scheduled at different times. Second, the week prior to Easter coincides with the greatest number of academic spring vacations. Third, the week prior to Easter is a time of greatly reduced meeting costs; the Society saves perhaps $800 and the individual members about $6.00 per room per night in negotiated room rents. Since there are more days in the week than needed for the meeting, it has sometimes been possible to avoid Passover even when it also falls in that week. In the next seven years, 1972-1978, Passover falls in the week prior to Easter only twice, once on Thursday and once on Tuesday.

Everett Pitcher
Editor, the Notices

It is time to restore the Masters Degree in Mathematics to a status worthy of its name. It is highly unlikely that the U.S. government which largely created the Ph.D. demand (in its defense and related industries) and the Ph.D. supply (with NSF monies) now will be able to promote the “doctors of education” mathematics degree. This leaves us where we began with a need for something less than the research doctorate. Why not a bona fide Masters degree?

Carl Faith
Rutgers University

Editor, the Notices

As a result of surveys of the status of women on many campuses it becomes apparent that there is a pattern of discrimination against women in all fields. Discrimination of the following types is prevalent.

1) Women are predominantly at the bottom of the pyramid, irrespective of qualifications. There are very few women in higher faculty ranks, and women suffer a substantial salary inequity.

2) Many academic departments have no full-time female faculty at all. In many departments the percentage of female faculty is far below the percentage of females among qualified applicants.

3) In many departments women with Ph.D.’s hold positions below the rank of Assistant Professor, positions such as Lecturer, Instructor or Research Associate, and are kept at these low ranks without promotion or significant salary increase, while men are either started at a higher rank, or rapidly promoted to a higher rank.

4) Women tend to be hired on a marginal, temporary, or 1-year basis, far more often then are men. Women are also frequently hired on a part-time basis, when they have requested full-time employment. Often women teaching part-time have the same teaching load as men teaching full-time. Most men are hired on a regular or continuing basis but not most women.

5) There are departments which make it a policy not to appoint women who are married to members of the
I would like to recommend as a first step in improving the status of women in our profession, that in the forthcoming annual salary survey data be collected for both new Ph.D.'s and established mathematicians, comparing salary levels by sex.

I would like also to suggest that, as women are approximately 6% of awarded Ph.D.'s in mathematics over the last few years, an affirmative action plan for each mathematics department might well include giving top priority to raising the percent of women in the department to that level.

I feel it would also be appropriate for each department to scrutinize its records to see if there are women who should be given promotions and raises in pay to rectify any inequities.

There is also the possibility that more women of ability should be encouraged to go on for the Ph.D., so that the 6% figure could be raised, to be more in line with the much higher percentage of women among bachelor's and master's degrees in mathematics.

I would like to add, that certainly it would be desirable to pick faculty without regarding their sex at all, and that certainly this would be the fair way to proceed. But this method is appropriate only after inequities have been rectified.

Ruth Silverman
Washington University

Editor, the Notices

It is refreshing to read Professor B. Epstein's letter in the February Notices, which is common sense itself, but quite depressing to think that such obvious statements should have to be printed, in the face of the hopeless mental confusion which seems to prevail among so many members of the Society. As a foreigner, I consider it would be highly improper that I should discuss the merits of resolutions put before the Society and seeking to influence the actions of the U.S. government; and I would like to know if it has occurred to those who sponsor such resolutions that their adoption would put the foreign members of the Society in an awkward position, to say the least (and whatever they may think privately of the matter under discussion).

J.A. Dieudonné
Université de Nice

Two letters in the Notices, February 1971, from Robert D.M. Accola and Bernard Epstein merit comment. Epstein states: "mathematicians should act...as thinking members of their communities". Of course. But Accola points out that if that is all they do they have "little influence and invite reprisals by the government".

Mathematicians acting unitedly, however, could contribute materially to an immediate unilateral US cease fire and speedy withdrawal. This war is worse than what Epstein calls "a colossal strategic blunder as well as a moral atrocity". As for the "greater calamity" his colleagues believe we are "fending off", namely the independence and reunification of Indochina under the popular Communist dictatorship of Hanoi...is that really worth killing 300,000 civilians, kidnaping and relocating entire villages, burning 1/5 of the forests of South Vietnam, "1-2-many" My-Lai's, and the propping up by foreign armies of the corrupt military clique in Saigon the Americans have selected, installed, armed, and financed to rule their half of the country.

This is an even greater crime than the aggression against Mexico in 1848 when US took by force about 2 1/2 states, the 10 year near genocide of Filipino independentists in 1898-1908, and the obliteration of Pyong Yong in 1950-53 when the Korean reunification movement became strong.

AMS is democratically operated; if a majority wish to continue happily non-political, feeding at the government trough, all of us can do no more than be "thinking members of the community". The purpose of this letter is to convince a majority that the AMS itself should be a "thinking member of the community". This war against the small nations of Southeast Asia is an almost Hitlerian outrage. For 10 years AMS has done nothing to stop it.

Eugène H. Lehman
Université du Québec
à Trois-Rivières
Editor, the Notices

I think that all readers of the Notices should be aware that the mathematics meetings in Atlantic City were a turning point in the effort to have the mathematics professional organizations declare themselves on the war in Indochina. For the first time, the arguments against the war were put forward in strictly professional terms, and the response was substantial: over 37% of those present and voting at the MAA Board of Governors Meeting and 48 1/2% of the AMS Business Meeting gave their approval to resolutions which might be classified as "anti-war".

I think that six or twelve months of continued war, with its attendant costs to our profession (these costs are now accelerating) may well produce a majority for some sort of anti-war action at the University Park or Las Vegas meetings. In view of this, I feel it appropriate that the profession discuss the merits of such action, and especially the merits of the new arguments put forward at Atlantic City.

The Mathematics Society is in possession of a transcript of my remarks in support of the polling of the AMS membership, which was so narrowly defeated (276-294) at the Atlantic City meeting. I hope that these arguments will be made available to the membership in some way, so that we may discuss these issues among ourselves prior to the next national meetings. Publication of the remarks in the Notices would have that effect, or other means of circulation might be found. I hope that some way of informing the profession will be used, and in good time before August.

Anatole Beck
University of Maryland

Editorial comment: The Council has considered and rejected both the possibility of publishing the speech of Professor Beck at the Business Meeting in Atlantic City and the alternative of duplicating and distributing it in some other manner. On the other hand, the speech has been transcribed in the office of the secretary and edited by Professor Beck, who holds a typescript. An individual may obtain a copy for $1.00, to cover cost of duplication and mailing, payable with the request to Professor Anatole Beck, Department of Mathematics, University of Wisconsin, Madison, Wisconsin 53706.

Everett Pitcher

Editor, the Notices

I am inclined to believe knowledge is amoral and that the American Mathematical Society should avoid exhausting its energies debating moral issues and passing resolutions. But I believe that mathematicians as all other members of society have a moral obligation to express their views in the most effective way possible. For mathematicians I believe that would be through the Society. Therefore I suggest that this be done not by resolutions but rather in the form of opinion polls. A tearout page that could be machine evaluated could be part of the Notices. The returns would be a biased poll but it would give those who care a chance to register their concern and do it as mathematicians. It is important that people know that scientists are concerned.

Kermit G. Clemans
Southern Illinois University
SPECIAL MEETINGS INFORMATION CENTER

The purpose of this center is to maintain a file on prospective symposia, colloquia, institutes, seminars, special years, meetings of other associations, and to notify the organizers if conflicts in subject matter, dates or geographical area become apparent. A first announcement will be published in the Notices if it contains a call for papers, place, date, and subject, where applicable; a second announcement must contain reasonably complete details of the meeting in order for it to be published. Information on the pre-preliminary planning will be stored in the files, and will be available to anyone desiring information on prospective conferences. All communications on special meetings should be sent to the Special Meetings Information Center of the American Mathematical Society.

CONFERENCE ON THEORETICAL BIOLOGY AND BIOMATHEMATICS

A conference on Theoretical Biology and Biomathematics will be held at Proctor Academy, Andover, New Hampshire, on June 14-18, 1971. The program will include the following lectures: "Population genetics of multi-locus systems" by Richard Lewontin; "Logistic type models of competition" by Howard Levene; "Stability and evolution in exploitative systems" by Michael Rosenzweig; "Formal similarities between evolutionary theory and environmental planning" by Lawrence Slobodkin; "Deterministic and stochastic models of epidemics" by George Weiss; "The implications of some life history models for the real world" by Robert Ricklefs; "Niche parameters in bird communities" by Martin L. Cody; "Combinatorial and dynamic approaches to community structure" by Joel Cohen; "A statistical mechanics of interacting species" by Elliot Montroll; "Cooperative components, spatial localization, and oscillatory cellular dynamics" by Leon Glass; "Quantitative aspects of morphogenesis" by Jerome K. Percus; "Chemical organization in cells" by David Markowitz; "Models of development and regeneration of vascular systems" by Dan Cohen; "Ehemotaxis in E. Coli" by Evelyn Keller; "Theories of the control of development" by Morrel Cohen; "The making of neural connections" by Anthony Robertson; "Necessity and contingency in neurogenesis" by Marcus Jacobson; "Postnatal ontogenesis of synaptic organizations in the mammalian brain" by Dominick Purpura; "Detection of complex sounds at the level of single nerve cells" by Robert Capranica; "Stochastic cells" by Jack Cowan; "Characterization and plasticity of a neural group" by George Gerstein; "Theory of fuzzy systems and its applications" by L. A. Zadeh; "Linear systems analysis applied to the mammalian cerebral cortex" by Walter Freeman; "Beyond homeostasis and cybernetics: a speculation from studies of neural noise" by Edgar Gasteiger; "Current flow in dendrites: analysis of a simple cortical system" by David Hellerstein; "Transmission of visual information over several parallel channels" by F. A. Dodge, Jr.; "Encoding of nerve impulses in multiple channels" by Bruce W. Knight; "Empirical and theoretical analysis of the sources of human brain potentials" by Herbert G. Vaughan, Jr.; and lectures by Frank Harary, Michael Wilkins, and Thomas Schoener, titles to be announced. Additional information may be obtained from Dr. Alexander M. Cruickshank, Director, Gordon Research Conferences, Colby Junior College, New London, New Hampshire 03257.

CONFERENCE ON PROVING ASSERTIONS ABOUT PROGRAMS

The ACM Special Interest Group on Programming Languages (SIGPLAN) and Automata and Computability Theory (SIGACT) will jointly sponsor a conference on Proving Assertions about Programs on January 6-7, 1972, at New Mexico State
University, The program will include both contributed and invited papers. Appropriate topics include, but are not limited to, design of languages to facilitate proofs, relationship of formal language definition to proofs of assertions, equivalence to problems of logic, implications of undecidability results, proof methods based on induction. Contributed papers should be based on original research. The author must submit in triplicate a detailed abstract or a brief abstract and an initial draft (preferably abstract and draft) to Professor R. H. Stark, Computer Science Department, New Mexico State University, Las Cruces, New Mexico 88001, by July 30, 1971. Authors will be notified of the decisions of the program committee by September 30. Camera-ready complete manuscripts of accepted papers are required by October 31, 1971. Inquiries on registration should be sent to the Conference Chairman, Professor J. Mack Adams, Computer Science Department, New Mexico State University.

CONFERENCE IN ENTIRE AND MEROMORPHIC FUNCTIONS

A conference in Entire and Meromorphic Functions will be held at the University of British Columbia on June 28-30, 1971. There will be several principal speakers giving one or two hour addresses and shorter papers will also be presented, some by invitation. Those wishing to present their new results in entire and meromorphic functions are invited to apply in advance for consideration. Where practical, expenses of those presenting contributed papers will be borne by the conference. The cost of room and board in university accommodations, up to the extent of the available space, will be borne by the conference. Applications and requests for additional information can be made to Professor Afton H. Cayford, Department of Mathematics, University of British Columbia, Vancouver 168, Canada.

CONFERENCE IN OPERATOR THEORY

A conference in Operator Theory will be held at the University of New Hampshire on June 7-11, 1971. Professor Béla Sz.-Nagy, Bolyai Institute, Szeged, Hungary, will present a series of ten lectures on "Unitary dilations of Hilbert space operators and related topics." There will also be opportunities for informal discussion and for the presentation of short papers by conference participants. Support for this conference is expected from the National Science Foundation. If such support is forthcoming, travel, room, and board will be provided for a limited number of conference participants. For further information and application forms for support, write to Professor M. Evans Munroe, Department of Mathematics, Kingsbury Hall, University of New Hampshire, Durham, New Hampshire 03824.

REGIONAL CONFERENCE IN ANALYSIS ON LIE GROUPS AND HOMOGENEOUS SPACES

With expected support from the National Science Foundation, the Department of Mathematics, Dartmouth College, will hold a regional conference in Analysis on Lie Groups and Homogeneous Spaces on August 16-20, 1971. Professor Sigurdur Helgason will deliver a series of expository lectures, the aim of which is to stimulate, strengthen, and broaden interest in this area of mathematics. The lectures will be complemented by informal seminars and discussion groups. Financial support is anticipated for about twenty-five participants. Advanced graduate students and postdoctoral mathematicians are especially encouraged to apply. Further information may be obtained from Professor Kenneth I. Gross, Department of Mathematics, Dartmouth College, Hanover, New Hampshire 03755.

INTERNATIONAL CONFERENCE ON SYSTEM SCIENCES

The Information Sciences Program and the Department of Electrical Engineering of the University of Hawaii will hold the Fifth Hawaii International Conference on System Sciences and a special subconference on Computers in Biomedicine on January 11-13, 1972, in Honolulu, Hawaii. This is the fifth in a series of conferences devoted to advances in information and system sciences. It will broadly encompass the following areas: information sciences, computer sciences, communication theory, control theory, and system theory. Sessions will also be de-
voted to advances in interdisciplinary, ecological and environmental, social urban, transportation, and bio-engineering systems. Papers are invited in these and related areas, and three copies of a one-page, single-spaced abstract must be submitted by September 1, 1971. Authors will be notified of acceptance before October 15, 1971. Please send abstracts to HICSS-5, Information Sciences Program, 2565 The Mall, University of Hawaii, Honolulu, Hawaii 96822.

MATHEMATICAL SCIENCES REGIONAL CONFERENCE

A Mathematical Sciences Regional Conference will be held at the State University of New York at Buffalo on August 30-September 3, 1971. Support will be provided by the National Science Foundation. The principal lecturer will be Professor James Durbin, professor of statistics at the London School of Economics. His topic will be "Tests based on the sample distribution function." The mathematics used in tests based on the sample distribution function, which can be considered an interdisciplinary topic within the mathematical sciences, involves both the analytical and measure theoretic aspects of the theory of stochastic processes. Professor Durbin has been one of the most important recent contributors to research in this area.

Financial support for travel and subsistence for six days will be provided for twenty-five young faculty members and graduate students not currently in Buffalo. Interested persons are invited to write, enclosing a vita, to the Conference Director, Professor Emanuel Parzen, Department of Statistics, State University of New York at Buffalo, Room A1, 4320 Ridge Lea Road, Amherst, New York 14226.

CONFERENCE ON INJECTIVE AND PROJECTIVE MODULES

The American University will host a regional conference on Injective and Projective Modules on June 21-25, 1971. The principal speaker will be Professor Barbara Ososky of Rutgers University; in addition, there will be opportunities for informal talks and seminars. The National Science Foundation is providing support for the conference, including travel and subsistence for twenty-five participants. Additional information can be obtained by writing to Professor Mary Gray, Department of Mathematics, The American University, Washington, D.C. 20016.

CONFERENCE ON COMBINATORIAL GEOMETRY AND CONVEXITY

The National Science Foundation has approved a five-day regional conference at the University of Oklahoma, June 21-25, 1971, in the area of combinatorial geometry-convexity. The principal lecturer will be Professor Branko Grünbaum of the University of Washington, who plans to pursue several new topics in geometry ("arrangements," "spreads," and "continuous families") which have only recently been found to play a significant role in certain problems in convexity theory and in the study of topological projective planes. The conference will be planned so as to accommodate as many informal talks by the participants as possible. An expense and travel stipend will be awarded to all regional participants. Apply to Professor David C. Kay, Department of Mathematics, University of Oklahoma, Norman, Oklahoma 73069.

ADDENDA

The organizers of the Computer Science and Statistics: Fifth Annual Symposium on the Interface, announced in the April 1971 issue of these Notices, have requested that the following additions be made to the original announcement: a fifth workshop sequence entitled "extractive industries" has been added to the program; contributed papers and abstracts in any of the subject areas may be sent to the chairman to be considered for possible incorporation into the program. The chairman of the conference is Professor Mitchell O. Locks, College of Business Administration, Oklahoma State University, Stillwater, Oklahoma 74074.
INTERACTION PHENOMENA IN ENGINEERING SCIENCE

The ninth annual meeting of the Society of Engineering Science will hold a meeting at Rensselaer Polytechnic Institute on November 8–10, 1971. The topic of the meeting will be Interaction Phenomena in Engineering Science. Further information may be obtained by writing to Professor H. F. Tiersten, Mechanics Division, Rensselaer Polytechnic Institute, Troy, New York 12181.

SYMPOSIUM ON STATISTICAL AND PROBABILISTIC PROBLEMS IN METALLURGY

A symposium on Statistical and Probabilistic Problems in Metallurgy will be held at Battelle Northwest, Richland, Washington, on August 4–6, 1971. Support for this symposium will be provided by the Office of Naval Research. Further information can be obtained by writing to Dr. Wesley L. Nicholson, Battelle Northwest, P.O. Box 999, Richland, Washington 99352.

NAVY USERS CONFERENCE ON COMBINATORICS

A Navy Users Conference on Combinatorics will be held at the Naval Postgraduate School, Monterey, California, on July 26–30, 1971. Support for this conference will be provided by the Office of Naval Research. Further information may be obtained by writing to Professor Craig Comstock, Department of Mathematics, Naval Postgraduate School, Monterey, California 93940.

SYMPOSIUM ON NUMERICAL AND COMPUTER METHODS IN STRUCTURAL MECHANICS

A symposium on Numerical and Computer Methods in Structural Mechanics will be held at the University of Illinois, Urbana, on September 8–10, 1971. Support for this symposium will be provided by the Office of Naval Research. Further information may be obtained by writing to Professor Nathan Newmark, Department of Civil Engineering, University of Illinois, Urbana, Illinois 61801.

SYMPOSIUM ON DYNAMIC RESPONSE OF STRUCTURES

A symposium on Dynamic Response of Structures will be held at Stanford University, Stanford, California, on June 28–29, 1971. Support for this symposium will be provided by the Office of Naval Research. Further information may be obtained by writing to Professor George Herrmann, Department of Applied Mechanics, Stanford University, Stanford, California 94305.

THIRTY-EIGHTH SESSION OF THE INTERNATIONAL STATISTICAL INSTITUTE

The thirty-eighth session of the International Statistical Institute will be held at the Shoreham Hotel, Washington, D.C., on August 10–20, 1971. Support for this meeting will be provided by the Office of Naval Research. For further information, please write to H.E. Riley, American Statistical Association, 806 Fifteenth Street, N.W., Washington, D.C. 20005.

NINTH INTERNATIONAL SYMPOSIUM ON FUNCTIONAL EQUATIONS

The Istituto di Matematica Applicata of the University of Rome and the Italian Research Council C.N.R. are sponsoring a symposium on Functional Equations in Rome, Italy, and the Island of Elba on September 3–12, 1971. There will be thirty to fifty half-hour talks. In addition, ample time will be left for discussion and for the posing and/or solving of open problems in theory and applications of functional equations. Seminars on specialized topics will also be organized. This symposium is the ninth in a series of meetings held since 1962 in Europe and North America. Approximately seventy participants are being invited. Qualified mathematicians who are working in functional equations and their applications and who wish to participate should write to Professor Bruno Forte, Department of Applied Mathematics, Faculty of Mathematics, University of Waterloo, Waterloo, Ontario.
INTERNATIONAL SYMPOSIUM ON
COMBINATORIAL MATHEMATICS
AND ITS APPLICATIONS

An international symposium on Combinatorial Mathematics and its Applications will be held at Colorado State University on September 9–11, 1971. The objective of the symposium is to bring together leading workers from diverse areas of combinatorial theory, including such fields as foundations of combinatorial mathematics, enumerative techniques, various branches of graph theory, coding theory, design theory, finite geometries, search theory, number theory, combinatorial aspects of probability and statistical inference, automata theory, optimization techniques, and the applications of these to logistics, communication networks, and experimentation. In particular, there will be scientists from the last three fields who will explain some of the pressing combinatorial problems being faced in their fields. The symposium is being sponsored by the Air Force Office of Scientific Research, and it will be dedicated to Professor R.C. Bose. The Organizing Committee consists of Frank Harary, University of Michigan; C.R. Rao, Indian Statistical Institute; G.C. Rota, Massachusetts Institute of Technology; S.S. Shrikhande, University of Bombay; J.N. Srivastava, Colorado State University; and W.R. Trott, Air Force Office of Scientific Research. A partial list of invited speakers includes, apart from the members of the Organizing Committee, the following: A. Barlotti, University of Perugia; C. Berge, University of Paris; R.H. Bruck, University of Wisconsin; P. Erdős, Hungarian Academy of Sciences; W.T. Federer, Cornell University; J.M. Goethals Hoffman, IBM; G.O.H. Katona, Hungarian Academy of Sciences; H.B. Mann, University of Arizona; J. Ogawa, University of Calgary; D.K. Ray-Chaudhuri, Ohio State University; H.J. Ryser, California Institute of Technology; M.P. Schutzenberger, University of Paris; J.J. Seidel, Technical University Eindhoven, The Netherlands; E. Seiden, Michigan State University; and W.T. Tutte, University of Waterloo. The proceedings of the symposium will be published. All those interested are cordially invited to attend.

For further information, please write to Professor J.N. Srivastava, Department of Statistics, Colorado State University, Fort Collins, Colorado 80521.

CONFERENCE ON TEACHING
MATHEMATICS TO NONSPECIALISTS

The Institute of Mathematics and Its Applications is organizing a three-day conference in association with The Institute of Physics and The Institute of Biology on Teaching Mathematics to Nonspecialists at the Loughborough University of Technology on September 27–29, 1971. This conference is intended to deal primarily with the post-secondary level and aims to attract mathematicians, physicists, engineers, biologists, and social scientists. The opening address will be given by Professor H. Halberstan on "Why mathematics should be taught to nonspecialists for other than vocational reasons." Subsequent talks by engineers, physicists, biologists, social scientists, and mathematicians will cover "why, who, what, and how" of teaching mathematics for other needs. Professor M. J. Lighthill, FRS, will give the closing address on "More advanced applications of mathematics." One session will include contributed papers dealing with innovations in teaching mathematics to nonspecialists. The proceedings will be published. Residential accommodations will be available. Details of the conference and application forms can be obtained from The Secretary and Registrar, The Institute of Mathematics and Its Applications, Maitland House, Warrior Square, Southend-on-Sea, Essex SS1 2JY, England.

SYMPOSIUM ON PARTIAL
DIFFERENTIAL EQUATIONS AND
DISTRIBUTED PARAMETER CONTROL
SYSTEMS

A symposium on Partial Differential Equations and Distributed Parameter Control Systems will be held at the University of Warwick, Coventry, on July 13–16, 1971. The symposium is being organized by the Control Theory Centre at the University of Warwick in association with The Institute of Mathematics and Its Applications and
with the Institution of Electrical Engineers. The aim is to present the state of current research on partial differential equations with particular emphasis on distributed parameter control systems. Lectures will be given by A. Jeffrey, L. Markus, P.C. Parkes, D.L. Russell, B.M.E. de Silva, L. Meirovitch, C.F. Chen, P.K.C. Wang, C.N. Lashmore-Davies, D.H. Brereton, S.F. Bush, A. Friedman, J.L. Lions, A.G. Butkovskii, and P.L. Falb. Residential accommodations will be available. Details of the conference and application forms may be obtained by writing to The Secretary and Registrar, The Institute of Mathematics and Its Applications, Maitland House, Warrior Square, Southend-on-Sea, Essex SS1 2JY, England.

CONFERENCE ON RECENT DEVELOPMENTS IN FUNCTION ALGEBRAS

The University of Arkansas expects to host a regional conference on Recent Developments in Function Algebras, June 21–25, 1971. Professor Irving Glicksberg will be the lecturer, giving two lectures each day. Expenses for twenty-five participants are to be provided by the National Science Foundation. All inquiries concerning the conference should be directed to Professor Allan C. Cochran, Department of Mathematics, University of Arkansas, Fayetteville, Arkansas 72701.

SYMPOSIUM ON PARTIAL DIFFERENTIAL EQUATIONS

As part of the special year in Partial Differential Equations and Their Applications, sponsored by the Universities of Dundee, Edinburgh, Glasgow, Newcastle, and Strathclyde, an opening symposium is to be held at the University of Strathclyde on September 20–24, 1971. A series of survey lectures will be given to explain recent developments in partial differential equations to mathematicians who are not expected to be specialists in this field. The following have agreed to give one or more talks: Professor I. N. Sneddon, opening address; Professor N. W. Bazley, "Bifurcation theory"; Professor C. Cercignani, "Initial and boundary value problems for the Boltzmann equation"; Professors D. L. Colton and R. P. Gilbert, "Function theoretic methods"; Professor G. E. Latta, "Singular perturbations." Application forms, which must be returned not later than July 1, 1971, and further information may be obtained by writing to Dr. R. J. Cole, Department of Mathematics, University of Strathclyde, Glasgow C1, Scotland.

MEMORANDA TO MEMBERS

POLICY ON RECRUITMENT

The Council has directed that henceforth all forms for listing in the December issues of the Notices, devoted to assistantships and fellowships, and all advertising proof for advertising in the Notices shall contain the following statement: "All employers in the United States are required to abide by the requirements of Title VII of the Civil Rights Act of 1964, enunciating a national policy of equal employment opportunity in private employment, without discrimination because of race, color, religion, sex, or national origin. We assume that all institutions advertising positions abide by the spirit of this law, whether or not they are legally bound to do so." In addition, the rate card distributed to advertisers will carry notification of this fact.
PERSONAL ITEMS

RICHARD E. ALLAN of Texas Instruments, Inc., has been appointed president of the Computing and Educational Systems Company, Dallas, Texas.

HARRY P. ALLEN of Rutgers University has been appointed to an associate professorship at Ohio State University.

SAMUEL F. BARGER of the University of Minnesota has been appointed to an associate professorship at Youngstown State University.

KENNETH J. BARWISE of Yale University has been appointed to an associate professorship at Youngstown State University.

RAUL BRAVO of the University of California, Berkeley, has been appointed to a professorship at the University of Chile.

DEAN BROWN of Ohio State University has been appointed to an assistant professorship at Youngstown State University.

GERALD L. BROWN of the Sandia Corp. has been appointed a research staff member at the Institute for Defense Analyses.

JOHN BUONI of the University of Pittsburgh has been appointed to an assistant professorship at Youngstown State University.

ALLEN D. BURDOIN of Milton Academy has been appointed to a mastership at St. Paul's School.

JAMES R. CHOIKE of Wayne State University has been appointed to an assistant professorship at Oklahoma State University.

JACK W. CROSBY of Texas Southern University has been appointed a consultant at Bonner & Moore Associates, Houston, Texas.

BURHANDEENZ DAGHESTANI of North Texas State University has been appointed a programmer at the Continental National Bank Center, National Sharedata Corp., Ft. Worth, Texas.

ANTOINE DERIGHETTI of Harvard University has been appointed to a professorship at the University of Lausanne, Switzerland.

DAVID DOBBS of the University of California, Los Angeles, has been appointed to an assistant professorship at Rutgers University.

WYMAN G. FAIR of the Midwest Research Institute has been appointed to an assistant professorship at Drexel University.

BRUNO FORTE of the University of Pavia, Italy, has been appointed to a professorship at the University of Waterloo.

RONALD W. GOLLAND of the Illinois Institute of Technology has been appointed director of applications at the Computer Center of the University of Illinois.

SAMUEL GORENSTEIN of IBM, New York Scientific Center, has been appointed a research staff member at IBM, T. J. Watson Research Center.

THOMAS G. HALLAM of Florida State University has been appointed to a visiting associate professorship at the University of Rhode Island.

STANLEY C. HANSON of Kort Associates has been appointed manager of IR Engineering, Defense Systems Division, The Hallicrafters Company.

FREDERICK W. HARTMANN of Villanova University has been appointed a lecturer at the University of Western Australia.

MARTIN B. HERSKOVITZ of IBM has been appointed president of Hi Tor Systems, Inc., New City, New York.

JOHN R. HIGGINS of the University of Minnesota has been appointed a lecturer at the Cambridgeshire College of Arts and Technology, Cambridge, England.

SHISANJI HOKARI of the Tokyo Metropolitan University has been appointed to a professorship at the Josai University, Japan.

JEANNE S. HUTCHISON of the University of California, Los Angeles, has been appointed to an assistant professorship at the University of Alabama, Birmingham.

ARUN V. JATEGAONKAR of Cornell University has been appointed to a visiting professorship at Rutgers University.
CECIL L. KALLER of the University of Saskatchewan, Regina, has been appointed president of Notre Dame University, Nelson, British Columbia, Canada.

PATRICK KEAST of the University of St. Andrews has been appointed to an assistant professorship at the University of Toronto.

BRIAN KRITT has been appointed chairman of the Department of Mathematics at the University of Baltimore.

MORRIS LAI of the University of California, Berkeley, has been appointed a program assistant at the Far West Laboratory for Educational Research and Development.

DANIEL G. LAMET of the University of Oregon has been appointed to an assistant professorship at Boise State College.

PETER S. LANDWEBER of Yale University has been appointed to an associate professorship at Rutgers University.

ALAN C. LAZER of Case Western Reserve University has been appointed to an associate professorship at the University of Cincinnati.

YUDELL L. LUKE of the Midwest Research Institute has been appointed to a professorship at the University of Missouri, Kansas City.

PHILIP S. MARCUS of Shimer College has been appointed to an assistant professorship at Indiana University at South Bend.

ROBERT A. MELTER of the University of South Carolina has been appointed to an associate professorship at Southampton College, Long Island University.

TILLA K. MILNOR of Boston College has been appointed to a professorship and to the chairmanship of the Department of Mathematics at Douglass College, Rutgers University.

GUILLERMO MIRANDA of the Courant Institute of Mathematical Sciences has been appointed to an associate professorship at the Catholic University of Chile.

EDWIN E. MOISE of Harvard University has been appointed to a distinguished professorship at Queens College, CUNY.

W. DUANE MONTGOMERY of the Institute for Defense Analyses has been appointed to a visiting professorship at the University of Saskatchewan, Regina.

ROGER H. MORITZ of the Cornell Aeronautical Laboratory has been appointed to an assistant professorship at Alfred University.

MERVIN E. MULLER of the University of Wisconsin, Madison, has been appointed director of the newly formed Department of Computing Activities of the World Bank.

HYO CHUL MYUNG of Michigan State University has been appointed to an assistant professorship at the University of Northern Iowa.

MASAO NARITA of the International Christian University, Tokyo, Japan, has been appointed to a visiting professorship at Queen’s University for the year 1970-1971.

KOK LIP NG of the University of Singapore has been appointed managing director of Trans-Aire Electronics (S) PTE. Ltd. in Singapore.

L. BRUCE PALMER of the U. S. Naval Academy has been appointed a mathematician at the Acoustics Division of the Naval Research Laboratory.

TED PETRIE of the Institute for Defense Analyses has been appointed to a professorship at Rutgers University.

WALTER PRENOWITZ of Brooklyn College, CUNY, has been appointed to an adjunct professorship at Florida Atlantic University.

ANATOL RAPPORT of the University of Michigan has been appointed to a professorship at the University of Toronto.

LUIS RIBES of Queen’s University has been appointed to an assistant professorship at Carleton University.

WILLIAM M. SCHOFIELD of the Lockheed Georgia Company has been appointed president of The Bay Tree Company, Atlanta, Georgia.

ALBERT B. SCHWARZKOPF of the Naval Postgraduate School has been appointed to an assistant professorship at the University of Oklahoma.

YOSHIHIRO SHIKATA of Osaka University has been appointed to a professorship at Nagoya University, Japan.

RONALD SHONKWILER of the University of Colorado has been appointed to an assistant professorship at the Georgia Institute of Technology.

SHIEN-SIU SHU of Purdue University has been appointed to a professorship at and to president of the National Tsing Hua University, Sin Chu, Taiwan.
C. B. SMITH of the University of Florida has been appointed to a professorship at Trevecca Nazarene College.

MALCOLM SMITH of North American Rockwell has been appointed section manager of the Scott Research Laboratories, Inc.

ALAN SOLOMON of Tel-Aviv University has been appointed a senior lecturer at the University of Negev, Beersheba, Israel.

RICHARD K. STARK of the University of Florida has been appointed to an associate professorship at St. Mary's College of Maryland.

JAMES H. STODDARD of Kenyon College has been appointed to a professorship and to the chairmanship of the Department of Mathematics at Upsala College.

KENNETH E. SWICK of Occidental College has been appointed to an assistant professorship at Queens College, CUNY.

TOMMY K. TEAGUE of Michigan State University has been appointed to an assistant professorship at Gustavus Adolphus College.

TING ON TO of the University of Saskatchewan has been appointed a lecturer at the University of Canterbury, Christchurch, New Zealand.

FRANÇOIS TREVES of Purdue University has been appointed to a special professorship at Rutgers University.

V. R. R. UPPULURI of the Oak Ridge National Laboratory has been appointed to a visiting professorship at the University of Minnesota for the spring 1971 quarter.

ELDON JON VOUGHT of the California State Polytechnic College, Pomona, has been appointed to an associate professorship at Chico State College.

CHARLES R. WALL of the University of Tennessee has been appointed to an assistant professorship at East Texas State University.

BERTRAM WALSH of the University of California, Los Angeles, has been appointed to an associate professorship at Rutgers University.

PAUL A. WILLIS of the Mitre Corp. has been appointed president of Polytechnic Associates, Inc., Arlington, Virginia.

PROMOTIONS

To Professor. Boston College: GERALD G. BILODEAU; California State College, Long Beach: ANTHONY MARDELLIS; Colby College: LUCILLE ZUKOWSKI; Drew University: CHARLES W. LYTEL; University of Houston: DENNISON R. BROWN; Universidad Tecnica del Estado, Santiago, Chile: CESAR FERNANDEZ; M. A. College of Technology, Bhopal, India: KAILASH N. SRIVASTAVA; Rutgers University: ERIK ELLENTUCK, BENJAMIN MUCKENHOUPT; Wayne State University: MARTIN T. WECHSLER.

To Associate Professor. Boston College: RICHARD L. FABER; California State College, Long Beach: JOHN M. BACHAR, JR; Drew University: NORMA M. GILBERT; Rutgers University: WOLMER VASCONCELOS, NOLAN WALLACH; Seattle Pacific College: SAMUEL L. DUNN; Temple University: ORIN CHEIN, JANOS GALAMBOS, HALA PFLUGFELDER, JOHN SCHILLER; University of Washington: ROBERT B. WARFIELD; Wayne State University: GEERT PRINS, CHOON-JAI RHEE, BERTRAM M. SCHREIBER; University of Wisconsin-Milwaukee: DONALD W. SOLOMON.

To Assistant Professor. Youngstown State University: STAMAN RODFONG.

To Vice President and Editorial Director, Films and Publications, Encyclopaedia Britannica Educational Corporation: ALVIN N. FELDZAMEN.

To Research Mathematician. University of Plovdiv: PAVELO GEORGIEV TODOROV.

INSTRUCTORSHIPS

Columbia University: WILLIAM ABIKOFF; Davidson College: THOMAS C. LOMINAC; University of Minnesota: DONALD H. SINGLEY.

DEATHS

Professor LOUIS BRAND of the University of Houston died on January 27, 1971, at the age of 85. He was a member of the Society for 60 years.
Professor Emeritus BAILEY L. BROWN of Amherst College died on January 12, 1971, at the age of 67. He was a member of the Society for 44 years.

Dr. HERMAN E. ELLINGSON, retired research physicist with the Naval Ordnance Laboratory, died on April 25, 1970, at the age of 67. He was a member of the Society for 39 years.

Professor EBERHARD M. FELS of the University of Münich died on April 18, 1970, at the age of 46. He was a member of the Society for 9 years.

Professor Emeritus WALTER B. FORD of the University of Michigan died on February 24, 1971, at the age of 96. He was a member of the Society for 67 years.

Professor CHARLES A. HUTCHISON of the University of Colorado died on January 13, 1970, at the age of 72. He was a member of the Society for 41 years.

Professor VOJTĚCH JANÍK of Prague, Czechoslovakia, died on November 11, 1970, at the age of 72. He was a member of the Society for 46 years.

Dr. F. ELIZABETH LESTOURGEON of Bridgeton, New Jersey, died on February 6, 1971, at the age of 90. She was a member of the Society for 52 years.

Professor Emeritus WILLIAM E. MILNE of Oregon State University died on January 19, 1971, at the age of 81. He was a member of the Society for 54 years.

Professor RICHARD M. MONTAGUE of the University of California, Los Angeles, died on March 7, 1971, at the age of 40. He was a member of the Society for 9 years.

Professor Emeritus KAKUTARO MORINAGA of Hiroshima University died on November 20, 1970, at the age of 45. He was a member of the Society for 11 years.

Professor THEODORE S. MOTZKIN of the University of California, Los Angeles, died on December 15, 1970, at the age of 62. He was a member of the Society for 24 years.

Professor HENRY M. NODELMAN of the RCA Institute died on November 2, 1970, at the age of 58. He was a member of the Society for 28 years.

Professor HARRY S. POLLARD of Miami University died in January 1971 at the age of 70. He was a member of the Society for 44 years.

Professor ROBERT H. SHAW of Mary Washington College died on March 6, 1971, at the age of 54. He was a member of the Society for 26 years.

Mr. JOSEPH E. SOMMESE of Yale University died on November 3, 1970, at the age of 23. He was a member of the Society.

Professor CHARLES B. TOMPKINS of the University of California, Los Angeles, died on January 11, 1971, at the age of 58. He was a member of the Society for 36 years.

ERRATA

In the February 1971 issue of these Notices it was announced that ISADORE M. SINGER had been promoted to a professorship at the Massachusetts Institute of Technology. He has been a professor for many years. It should have been indicated that Professor Singer was named the newly established Norbert Wiener Professor of Mathematics at MIT.

In the February 1971 issue of these Notices it was announced that Mrs. Ann Yasuhara of New York University had been appointed a visiting member at The Institute for Advanced Study. The item should have read: MITSURU YASUHARA of New York University has been appointed a member at The Institute for Advanced Study for the academic year 1970-1971.
Suppose that $M$ is a 3-manifold and $G$ is a cellular decomposition such that the associated decomposition space is a 3-manifold $N$. In this Memoir, it is proved that $M$ and $N$ are homeomorphic. As a corollary, it follows that if a pointlike decomposition of $E^3$ yields a 3-manifold $N$, then $N$ is homeomorphic to $E^3$. The results of this Memoir may be stated in terms of compact mappings. The statement that a mapping $f$ from a 3-manifold $M$ onto a 3-manifold $N$ is cellular means that for each point $y$ of $N$, $f^{-1}[y]$ is a cellular subset of $M$. It follows that if $f$ is a compact cellular mapping from a 3-manifold $M$ onto a 3-manifold $N$, then $M$ and $N$ are homeomorphic. The techniques developed in this Memoir can be applied to a large number of problems concerning cellular decompositions of 3-manifolds that yield 3-manifolds.

Number 109
ASYMPTOTIC BEHAVIOR OF SOLUTIONS AND ADJUNCTION FIELDS FOR NONLINEAR FIRST ORDER DIFFERENTIAL EQUATIONS
By Walter Strodt and Robert K. Wright
288 pages; List Price $3.00; Member Price $2.25

This Memoir develops an abstract theory in which structures of asymptotic equivalence and asymptotic dominance are imposed upon differential field structures. For nonlinear first order differential equations, with coefficients in an asymptotically well-behaved differential field, solutions are sought which are adjoinable, that is, lie in an asymptotically well-behaved extension field, and thereby have asymptotic expansions. An application to the special concrete case where the equations have coefficients meromorphic in a sector leads to a broad existence theorem on adjoinable solutions. In particular, it follows that equations with rational coefficients always have adjoinable solutions.

Number 110
ABELIAN SUBALGEBRAS OF VON NEUMANN ALGEBRAS
By Donald Bures
132 pages; List Price $2.20; Member Price $1.65

This Memoir considers certain questions connected with the problem of classifying (up to equivalence by automorphisms of $A$) the abelian $W^*$-subalgebras of a $W^*$-algebra $A$. In part I, the question is: "When $A$ is a factor of type $II_1$, can the general case be reduced to the maximal abelian case?" With a small restriction, the answer yes is obtained for normal
subalgebras; in general, however, the reduction is to thick subalgebras (W*-subalgebras whose relative double commutant is maximal abelian). Part II is a generalized version of the von Neumann construction of factors, viewed as the construction of an extension of a given W*-algebra by a given group of automorphisms. This viewpoint is used in Part III to develop general methods for constructing and distinguishing thick subalgebras. For example, if A is the hyperfinite factor of type II₁ there exists a maximal abelian subalgebra which is the relative double commutant of a continuum of inequivalent thick subalgebras.

Number 111
MULTIPLIERS, POSITIVE FUNCTIONALS, POSITIVE-DEFINITE FUNCTIONS, AND FOURIER-STIELTJES TRANSFORMS
By Kelly McKennon
76 pages; List Price $1.90; Member Price $1.43

This paper has two principal aims. First, to extend the classical Cramer-Levy Convergence Theorem of probability theory to the more general settings of Fourier-Stieltjes transforms on a general locally compact Abelian group, positive-definite functions on a general locally compact group, and positive functionals on an adjoint (or *-) algebra. These generalizations are accomplished on the one hand by analysis of convergence in the conjugate space of an arbitrary C*-algebra, and on the other hand by investigation of the remarkable relationships which exist between positive functionals and multipliers.

The second aim of the paper is to record and develop some of the relationships between positive functionals and multipliers. One result in this direction is that a locally compact group is amenable if and only if the multipliers of its Fourier algebra may be identified with its Fourier-Stieltjes algebra.
These proceedings contain new results on wave propagation which have been obtained in recent years at the Leningrad Branch of the Steklov Institute of Mathematics of the Academy of Sciences of the USSR. In subject matter the book divides into two parts. The first part contains three papers on stochastic approaches to problems of the propagation of seismic waves. In respect of generality and the order of realization, there appear to be no analogous papers in the literature. The remaining papers are concerned with problems of the propagation and diffraction of waves in determinate media. This collection of papers can prove valuable to specialists interested in problems of wave propagation and to mathematicians concerned with the practical application of probability theory and mathematical statistics.

**CBMS REGIONAL CONFERENCE SERIES IN MATHEMATICS**

Number 8

**LECTURES IN HOMOLOGICAL ALGEBRA**
By Peter Hilton

82 pages; List Price $3.20; Member Price $2.40

This volume constitutes a record of the course in homological algebra given at the Virginia Polytechnic Institute in July 1970 under the auspices of the National Science Foundation's Regional Conference project. The nature of the audience required that the course begin with an introduction to the notion of modules over a unitary ring, but permitted rapid development of the theory from that starting point. The first three chapters may be regarded as containing material essential to any introductory course in homological algebra, while the later chapters reflect the choices actually made by the audience among many possible special topics accessible to those who had mastered the early material. Thus it may be claimed that the course achieved depth of penetration on a narrow front, while it is admitted that breadth of coverage of the entire domain of homological algebra was neither attempted nor achieved.

**MATHEMATICS INTO TYPE**
By Ellen Swanson

88 pages (approximately); List Price $4; Member Price $3

During the past two or three decades, when the amount of published mathematics was increasing at a rapid rate, several books were written to aid the mathematician in preparing his manuscript and to assist the compositor in typesetting it. Books and pamphlets were also written for the use of copy editors of nonscientific material, but not for copy editors of mathematics. This book is designed to fill this gap. The contents of "Mathematics Into Type" were incorporated originally in a manual that was used to standardize copy editing procedures and to serve as a guide in the training of editorial assistants at the American Mathematical Society. Sections were added over several years, and in 1970 it was completely rewritten as a guide to the preparation of mathematical copy for publication, particularly the publication of research mathematics. This book covers the publication of mathematics from manuscript to the printed book or journal, with emphasis on the preparation of the copy for the compositor and the proofreading and makeup of the publication. "Mathematics Into Type" will be useful to the author who is directly concerned in the editing of his book, and it should benefit any author who is preparing a manuscript for publication.
The 24th Mersenne prime M = 2^p - 1, and currently the largest known prime, is 2^{19937} - 1 = 4315424797...0968041471 (6002 digits). Primality was shown by the Lucas-Lehmer test in 39.44 minutes on an IBM 360/91 computer. As a corollary, the 24th even perfect number is (2^{19937} - 1) * 2^{19936} = 9311445590.....0271942656 (12003 digits). The three preceding Mersenne primes, found by Donald B. Gillies (Math. Comp. 18(1964), 93-97) were for p = 9689; 9941; and 11213. (Received March 18, 1971.)

Convergence of the coefficients in the kth power of a power series.

In this paper we investigate generalized convoluted numbers and sums by using the following recurring power-series \((1 + \sum_{v=1}^{m} a_v x^v)^{-k} = \sum_{n=0}^{\infty} u_n^{(k)} x^n\) (the coefficients \(a_v\) and \(u_n^{(k)}\) are integers, \(k = 1, 2, 3, \ldots\), and \(v_0^{(k)} = 1\)). The new results include formulas for the special cases of the generalized Tribonacci and Quatronacci numbers, as well as asymptotic formulas of \(u_n^{(k)}\) expressed in terms of \(u_1^{(1)}, a_v, n,\) and \(k\). As a natural result of the methods used in this paper we are also able to investigate a generalization of the binomial formula. (Received April 9, 1971.)

Uniform algebras constructed from the disk algebra. Preliminary report.

Let \(X\) be a perfect compact Hausdorff space, and let \(A\) be the disk algebra, taken on the unit circle \(T\). As noted in Abstract 667-178, these Notices 16(1969), 808, there exist maps \(\sigma\) of \(X\) onto \(T\), which when composed with \(A\) yield algebras on \(X\). Such algebras are in turn enlarged so as to become maximal with respect to possessing complex homomorphisms associated with the open unit disk and denoted \(\phi_z, z < 1\). Several properties of these enlarged algebras are examined, particularly as they depend upon the map \(\sigma\). In particular, it is shown that if the separation properties of \(\sigma\) are suitably restricted, then the homomorphisms \(\phi_z\) have unique representing measures on \(X\). (Received February 24, 1971.)

On jumping functions by connected sets. Preliminary report.

For real-valued functions of a real variable, this paper establishes a natural unifying relationship between Darboux functions, connected functions and continuous functions. This relationship concerns how a
connected set may "jump" a function. Employing it we establish analogues of known results about Darboux functions for connected functions and continuous functions. The results include characterizations of uniform limits and pointwise limits for these above families of functions. (Received March 11, 1971.)

684-F6. W. J. PADGETT, University of South Carolina, Columbia, South Carolina 29208 and CHRISP. TSOKOS, Department of Statistics, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. On the existence of a solution of a stochastic integral equation in turbulence theory.

In the theory of turbulence, the random position of a tagged point in a continuous fluid in turbulent motion, \( \mathbf{r}(t; \omega) \), is a vector-valued random function of time \( t \geq 0, \omega \in \Omega \), where \( \Omega \) is the supporting set of the underlying probability space \( (\Omega, B, P) \). If \( \mathbf{u}(t; \omega) \) is the Eulerian velocity field, then \( \mathbf{r}(t; \omega) \) satisfies the stochastic integral equation

\[
\mathbf{r}(t; \omega) = \int_0^t \mathbf{u}(T; \omega) \, dT, \quad t \geq 0.
\]

General conditions under which a random solution of this stochastic integral equation exists are given in the form of a theorem, and the theorem is proved using the concepts of admissibility with respect to an operator on a Banach space and fixed point methods of functional analysis. The results of the paper are generalizations of the work of J. L. Lumley (J. Mathematical Phys. 3(1962), 309-312.) (Received February 25, 1971.)

The April Meeting in Monterey
April 24, 1971

685-A15. JOHN C. HIGGINS, Brigham Young University, Provo, Utah 84601. A characterization of a class of commutative semigroups.

Let \( S \) be a semigroup. We call \( S \) an \( N \)-semigroup if \( S \) is commutative, cancellative, archimedean, and without idempotent. If \( S \) is an \( N \)-semigroup then \( S \) is said to be strongly power joined if \( S \) is both power joined and satisfies the property that every homomorphic image of \( S \) in the additive semigroup of positive rational numbers is finitely generated. Theorem. A semigroup \( S \) is a strongly power joined \( N \)-semigroup if and only if \( S \) is isomorphic to a subdirect product of a periodic abelian group and the semigroup of additive positive integers. (Received April 14, 1971.)


Let \( f(z) \) be an entire function of order \( \rho, 0 < \rho < \infty \), and represented by the gap-series,

\[
\sum_{n=0}^{\infty} a_n z^n = \lambda_0 < \lambda_1 < \ldots < \lambda_n < \lambda_{n+1} < \ldots.
\]

Write \( L = \liminf_{n \to \infty} \lambda_n^{\rho/n} / \lambda_{n+1}^{\rho/n} \), \( \theta = \liminf_{n \to \infty} a_n^{1/n} / \lambda_n^{\rho/n} \rho e \), \( \psi_n = (\log a_n / a_{n+1}) / (\lambda_{n+1} - \lambda_n) \), \( t = \liminf_{r \to \infty} (\log M(r)) / r^p \), where \( M(r) \) denotes the maximum modulus of \( f(z) \) on \( |z| = r \). The main results proved are as follows: if \( L > 0 \), then \( t = L \cdot \theta \); if \( \psi_n \) is eventually a nondecreasing function of \( n \), then \( \theta \geq t \). (Received April 7, 1971.)

609
A class of error functions \(e(x, y, z)\), called error measures, is defined, and a theory of approximation of elements \(f\) in \(C(a, b)\) by elements \(p\) from an \(n\)-dimensional subspace \(V\) with respect to the error
\[
\|e(x, f(x), p(x))\|_\infty
\]
is developed which has the usual Chebyshev error theory as a special case. Examples of error measures include the relative error \((y - z) / \max(|y|, |z|)\), Moursund's generalized weight functions (SIAM J. Numer. Anal. 5(1968), 127), and Dunham’s ordering function (J. Approximation Theory 2(1969)).

Results on existence, characterization, uniqueness and strong uniqueness of best approximations are given. The standard characterization of a Haar subspace (\(V\) is Haar iff best approximations are unique) and a generalization of the Ikebe characterization are established. A Remes-type algorithm is given and shown to converge which depends only on the solution of linear systems, thus providing an effective and practical method for the actual computation of the best approximation. (Received April 14, 1971.)

The Ekman Spiral (a phenomenon occurring in both oceanography and micrometeorology) is here treated as the following singular nonlinear free boundary problem. Find a \(\beta > 0\) and real analytic functions \(s\) and \(t\) on \([0, \beta)\) such that
\[
s'' = -t(\sqrt{r})^{-1} \quad \text{and} \quad t'' = s(\sqrt{r})^{-1}
\]
where \(r = s^2 + t^2 > 0\), \(\ell\) is a given real analytic function on \([0, \beta)\) such that \(\ell(0) = 0\), \(\ell'(0) = k = 0.4\) = universal Kármán constant, \(s(0)\) and \(t(0)\) are given, and \(r(\beta -) = 0\). Using an error distribution method of mixed type (L. Collatz, "The numerical treatment of differential equations," 3rd. ed., Die Grundlehren der Math. Wissenschaften, Band 60, Springer-Verlag, Berlin, 1960, p. 28).

The initial results were obtained, numerically, on an interval of the type \([.1, \beta)\). These results agreed quite well qualitatively, with actual observed physical phenomena. Analytical estimates have also been obtained to fit this method to an interval of the desired type \([0, \beta)\). (Received February 26, 1971.)

A Tietze type theorem on monotone increasing sets.

Let \(E^2\) be a two-dimensional Euclidean space with the conventional rectangular axes. For two points \(x\) and \(y\) in \(E^2\) not lying on a vertical or horizontal line, the line segment \(xy\) determines two convex right triangles having \(xy\) as the hypotenuse and each of the remaining sides parallel to one of the axes. Let \(T(x, y)\) denote the right triangle lying below \(xy\). **Definitions.** A set \(S\) in \(E^2\) is **monotone increasing** if for each pair of distinct points \(x \in S\), \(y \in S\) it is true that (1) if the line through \(x\) and \(y\) has real positive slope, then there exists a convex arc in \(S \cap T(x, y)\) joining \(x\) and \(y\); and (2) if the line through \(x\) and \(y\) does not have real positive slope, then \(xy \subset S\). The set \(S\) is **locally monotone increasing** if for each \(x \in S\), there exists a neighborhood \(N\) of \(x\) such that \(N \cap S\) is monotone increasing. **H. Tietze** proved that a closed connected set \(S\) in \(E^2\) is convex if and only if it is locally convex. A **Tietze type theorem** holds for monotone increasing sets.

**Theorem.** A closed connected set \(S\) in \(E^2\) is monotone increasing if and only if it is locally monotone increasing. (Received April 15, 1971.)
Another fixed point theorem for plane continua.

A continuum $M$ is said to be connected if every two points of $M$ can be joined by a hereditarily decomposable subcontinuum of $M$. It is known that every arcwise connected nonseparating subcontinuum of the plane has the fixed point property (C. L. Hagopian, "A fixed point theorem for plane continua," Abstract 71T-G3, these Notices 18(1971), 261). Here we prove that this theorem remains true if the word "arcwise" is replaced by "A". (Received April 5, 1971.)

The June Meeting in Corvallis
June 19, 1971

Algebra & Theory of Numbers


Let $R$ be an ordinary differential domain containing the rationals, integrally closed in its quotient field $K$. It is shown that if some derivative of an element $u$ of $K$ belongs to $R$ and if $u$ satisfies a differential polynomial $y^N - f(y) \in R[y]$ with $N > \deg f$, then $u \in R$. If $R$ is the coordinate ring of an irreducible Ritt manifold $M$, this last condition means that $u^{-1}$ does not specialize (differentially) to zero over $M$. It is not sufficient to assume that $u$ is defined on $M$. In the case where $u$ also satisfies a linear differential polynomial over $R$ with leading coefficient 1, the assumption about derivatives of $u$ is superfluous by a result of S. Morrison and this implies that projective models of linear differential extension fields are complete (cf. P. Blum, "Complete models of differential fields," Trans. Amer. Math. Soc. 137(1969), 309-325). There is an application to integration theory and some results due to the author and E. Kolchin on adherence values of rational differential functions on the differential affine line. (Received March 30, 1971.)

*686-A2. SETH L. WARNER, Reed College, Portland, Oregon 97202. Openly embedding local noetherian domains.

Let $m$ be a nonzero proper ideal of a noetherian integral domain $B$. Then $B$, equipped with its $m$-adic topology, is not an open subdomain of any strictly larger topological integral domain if and only if $m$ is not contained in any associated prime ideal of some (or equivalently, any) nonzero principal ideal contained in $m$. If $B$ is a complete local domain equipped with its natural topology and if $B$ is an open subdomain of a topological integral domain $A$, then either $A$ is a complete local domain, its topology is its natural topology, and $A$ is a finitely generated $B$-module, or else $\dim B = 1$, $A$ is a field, and its topology is defined by a discrete valuation. Consequently, an indiscrete locally compact integral domain that contains a compact open noetherian subdomain is either compact or a locally compact field. (Received April 27, 1971.)
It is shown that \((a, x, y) = (2, 3, 2)\) is the only solution of the title equation in which the integers \(a\), \(x\) and \(y\) all exceed one. The proof relies heavily on the papers of LeVeque (see W. J. LeVeque, "On the equation \(a^x - (a+1)^y = 1\)"; Amer. J. Math. 74(1952), 325-331) and Cassels (see J. W. S. Cassels, "On the equation \(a^x - b^y = 1\)"; Amer. J. Math. 75(1953), 159-162). The proof is elementary and is thought to be essentially different from previous proofs of the same result. (Received May 5, 1971.)

Analysis

A derivative to match the \(v\)-integral.

In [Abstract 672-221, these \(N\otice\) 17(1970), 146] the \(v\)-integral of a convex set function with respect to an absolutely continuous function is defined. The \(v\)-integral is used to give a characterization of the linear functionals on the space of absolutely continuous functions which are continuous in the BV norm. A major portion of the ordinary calculus and real variable theory is devoted to the study of determining to what extent the derivative and integral are inverse operators. This paper gives a definition of a \(v\)-derivative and sufficient conditions under which this derivative yields an inverse operator for the \(v\)-integral. (Received February 8, 1971.)

The \(v\)-derivative on measure algebra from functions on abelian idempotent semigroups.

By a parallel development, the results concerning the \(v\)-derivative in (S. G. Wayment, "A derivative to match the \(v\)-integral," Abstract 686-B1, these \(N\otice\) 18(1971)) are extended to the setting of (Abstract 70T-B135, these \(N\otice\) 17(1970), 661). After the work in "A derivative to match the \(v\)-integral" was completed, it was discovered that under certain conditions on \(K\) the \(v\)-derivative with respect to \(K\) is actually a \(v\)-integral with respect to the reciprocal measure \(1/K\). Also in this setting \(K\) and \(1/K\) are inverse operators of each other. (Received March 26, 1971.)

Certain double integrals involving hypergeometric functions. Preliminary report.

The problem of evaluation of double integrals of the type \((*) \Delta = \int_0^\pi \int_0^\pi \int_0^1 F_1(\alpha; \gamma; \zeta) \sin \psi d\psi d\theta\), where \(\zeta = \lambda_1^+ \lambda_2 \cos \psi + \lambda \cos \theta \sin \psi\) and the \(\lambda\)'s are constants, arises in a study of the collision terms of the Boltzmann equation in the kinetic theory of gases. Recently, S. M. Deshpande [Bull. Calcutta Math. Soc. 60(1968), 11-14] made use of a contour integral representation for Kummer's confluent hypergeometric function \(F_1(\alpha; \gamma; z)\), a certain Neumann expansion, the addition theorem for Legendre polynomials, and the orthogonality properties of the surface spherical harmonics involved to show that, if \(\text{Re}(\alpha) > 1\) and \(\text{Re}(\gamma) > 1\),
then (**): \[ \Delta = \left\{ \pi (\gamma - 1) (\alpha - 1) \right\} \sum_{i=0}^{\infty} \frac{1}{i!} \binom{\alpha - 1; \gamma - 1; \lambda_1 + i}{\lambda_1 + 1 / \lambda_2} \], where, for convenience, \( R^2 = \lambda^2 + \lambda_2^2 \). In the present paper the author proves formula (***) under the less restrictive condition \( \gamma - 1 \neq 0, -1, -2, \ldots \), by using only elementary identities involving series and integrals. The same elementary considerations are then applied to obtain a generalization of (***) with the \( \binom{1}{1; \lambda_1}(\zeta) \) in (*) replaced by the generalized hypergeometric function \( \binom{\nu}{\nu}(\sigma) \), where \( \nu, \sigma \) are nonnegative integers such that \( \nu \leq \sigma \), or \( \nu = \sigma + 1 \) and \( \max(|\lambda_1| + |\lambda_2| + |\lambda|, |\lambda_1 + R|) < 1 \). This paper is scheduled to appear in the first issue of Gyanabh 1(1971). (Received April 8, 1971.)

*686-B4. CLIFFORD A. KOTTMAN, Oregon State University, Corvallis, Oregon 97331. The core topology and other affine topologies on finite dimensional spaces.

A topology on a vector space \( V \) is an affine topology provided addition and scalar multiplication are continuous in each variable separately and lines are closed. The strongest affine topology is the core topology in which the open sets are precisely those which are radial (absorbing) at each of their points. Radial kernel(A) is the set of points at which \( A \) is radial and (radial kernel)\( ^{a} \) (A) is defined inductively for each ordinal number \( a \). The first uncountable ordinal is \( \omega \). **Theorem 1.** If \( A \subset V \) then (radial kernel)\( ^{\omega} \) (A) = core-interior(A) and no smaller ordinal suffices for all \( A \) if \( \dim V \geq 2 \). **Theorem 2.** The Euclidean topology on a finite dimensional vector space \( V \) is the weakest Hausdorff affine topology on \( V \) for which \( V \) is second category in itself. (Received April 23, 1971.)

*686-B5. JERALD P. DAUER, University of Nebraska, Lincoln, Nebraska 68508. Controllability of nonlinear systems using a growth condition.

Sufficient conditions for complete controllability of systems of the form \( \dot{x} = A(t)x + g(t, u) \) on \([t_0, \infty)\) are established in this paper. It is assumed that the inverse of the fundamental matrix solution of the homogeneous equation \( \dot{y} = A(t)y \) is uniformly bounded on \([t_0, \infty)\). The criterion used is growth condition similar to one used by LaSalle on linear control systems. Our results extend his concept of an asymptotically proper control system. (Received April 13, 1971.)


Following Schoenberg, we consider the expansion of a function \( f(x) \in C^\infty[0, 1] \), when \( f^{(2k)}(0), f^{(2k+1)}(1), \) \( k = 0, 1, \ldots, \) are prescribed. We call it a generalized Abel expansion. We obtain conditions under which a given function has an absolutely convergent generalized Abel-series expansion. These conditions are expressed in terms of a class of functions \( (c-c^* \text{ i.e. completely convex}) \) which turns out to be a subclass of completely convex functions. We show that a necessary and sufficient condition for a function to have an absolutely convergent generalized Abel expansion is that it is the difference of two minimal \( c-c^* \) functions. This is an analogue of a result of Widder (Trans. Amer. Math. Soc. 51(1942), 387-398). (Received April 19, 1971.)

613
Let $\Sigma$ be a nonempty $\sigma$-field of subsets of a set $S$. A (set) correspondence, say $\Gamma$, from $\Sigma$ to a Banach space $X$ maps, by definition, every element $E$ of $\Sigma$ to $\Gamma(E)$, a nonempty subset of $X$. The correspondence $\Gamma$ is additive if $\Gamma(E) + \Gamma(F) = \Gamma(E \cup F) + \Gamma(E \cap F)$ for all $E, F$ in $\Sigma$. In this paper, $E, F$, and $G$ denote elements of $\Sigma$ and $x, y, z$ denote elements of $X$. If $A$ and $B$ are nonempty subsets of $X$, then $A + B = \{x + y : x \in A \text{ and } y \in B\}$. The total variation (or simply variation) of $\Gamma$ is the extended real-valued set function, say $v$, on $\Sigma$ defined as follows: $v(E) = \sup\{\sum_{i=1}^{n} \|x_i\| : (x_i, E_i) \text{ is a finite sequence in } X \times \Sigma, \{E_i\} \text{ is a partition of } E \text{ and for all } i, x_i \in \Gamma(E_i)\}$. This is a direct generalization of the notion of total variation for Banach valued measures. Our main results are: Theorem 1. Let $\Gamma$ be an additive correspondence from $\Sigma$ to a uniformly convex Banach space $X$. If the total variation of $\Gamma$ is an atomless finite measure on $\Sigma$ then the closure of $\Gamma(E)$ is convex for all $E$ in $\Sigma$. (In particular, if $\Gamma(E)$ is closed, it is convex.) Theorem 2. Under the conditions of Theorem 1 the closure of the range of $\Gamma$ is convex. (Received April 30, 1971.)
Applied Mathematics


The stability problem of thermo-viscoelastic fluid flow between rotating coaxial cylinders is investigated using constitutive equations given by A. C. Eringen. In the course of the investigation, the solution set for the steady state Couette problem is first found. The velocity field is found to be identical with that of the classical viscous case and the case of a visco-inelastic fluid, but the temperature and pressure fields are different. By imposing some physically reasonable mechanical and geometrical restrictions on the flow, and by a suitable mathematical analysis, the equations of motion are reduced to a characteristic value problem. The resulting characteristic value problem has been solved in this paper and stability criteria are obtained in terms of critical Taylor numbers. In general, it is found that thermo-viscoelastic fluids are more stable than classical viscous fluids and visco-inelastic fluids under similar conditions of the problem. (Received April 21, 1971.)


Using the constitutive equations, introduced by Narasimhan and Sra (Internat. J. Non-Linear Mech. 4(1969), 361-372) and based on the concept of the generalized measures of deformation-rates involving not only velocity gradients but also acceleration gradients, the problem of torsional flows of non-Newtonian fluids between two infinite parallel planes has been investigated. Expressions for the velocity and pressure fields have been obtained to first- and second-order approximations. The normal stress differences, velocity profiles, apparent coefficient of viscosity and their behaviors depending on the rheological constants have been investigated and discussed in detail. The phenomena of reversed flows also have been treated. Numerical studies have been made for purposes of comparison with experiments. (Received April 30, 1971.)


Methods for determining bounds for the probability of error in decoding messages sent over a continuous Gaussian channel have been investigated by C. E. Channon (Bell System Tech. J. 38(1959)) and David Slepian (Bell System Tech. J. 42(1963)). For engineering purposes Slepian developed a triple recursion and a rather strong-arm numerical integration procedure for lower and upper bounds. Alternative methods are possible; several are discussed in the paper with some indication of the numerical difficulties. (Received May 5, 1971.)
P. Elias in his recent paper [Ann. Math. Statist. 41(1970), 1249–1259] discusses quantization process involving a vector-valued random variable \( x = (x_1, x_2, \ldots, x_n) \) with probability measure \( \mu \) defined on Lebesgue-measurable subsets of \( N \)-dimensional Euclidean space \( \mathbb{E}^N \). Quantization process, which gained importance in the pulse modulation problem as it became necessary to minimize the distortion in transmission of signal data, has its uses in the analysis of error in the Sheppard's Correction formula which involves analysis based on discrete space. This presentation will be concerned with certain formulations and estimations yielding this alternative estimate of error, in coordination with Sheppard's Correction formula. A precise statement of the results is not possible in the limited space of this abstract. (Received May 5, 1971.)

**Geometry**

*686-D1. BILL WATSON, University of Oregon, Eugene, Oregon 97403. Manifold maps which commute with the Laplacian on \( p \)-forms.*

All nonconstant \( C^\infty \) manifold maps \( f: M \to N \), denoted \( \Omega^p(M, N) \), which satisfy the equation \( f^* \Delta_N = \Delta_M f^* \) on smooth differential \( p \)-forms of \( N \) are shown to be Riemannian submersions. Hence, for nontrivial \( \Omega^p(M, N) \), \( M \) compact implies \( N \) compact and, moreover, \( \dim M \geq \dim N \). The sets \( \Omega^0(M, N) \) and \( \Omega^1(M, N) \) are completely characterized as well as the equal dimension case for all \( p \). For compact Einstein spaces, \( M \) and \( N \), a map \( f \in \Omega^1(M, N) \) is characterized in terms of the Laplacian operator \( \sim \) defined on vector-valued differential forms associated to the bundle \( \pi: M \to f^{-1}(T(N)) \) [Eells and Sampson, Amer. J. Math. 86(1964), 109–160]. In case \( p = 0 \): Theorem. For compact \( M \), a map \( f \in \Omega^0(M, N) \) if and only if \( f \) is a harmonic Riemannian submersion \( (\Delta f = 0) \). Many cohomology applications are made based on the Theorem. If \( \Omega^p(M, N) \) is nontrivial and \( M \) is compact, then \( b_p(N) \neq b_p(M) \). Examples are given where \( \Omega^p(M, N) \neq \Omega^q(M, N) \). (Received March 19, 1971.)

**Statistics and Probability**

*686-F1. DAVID F. FRASER, Brown University, Providence, Rhode Island 02912. The Levy distance between Wiener measure and the measure generated by a certain random walk.*

Let \( W \) be the (Wiener) measure on \( C[0,1] \to \mathbb{R}^m \) generated by standard \( m \)-dimensional Brownian motion. Let \( P^n \) be the measure on \( C[0,1] \to \mathbb{R}^m \) generated by \( \xi^n(t) \) is a random walk in \( \mathbb{R}^m \) with steps at time \( k/n \). The increments \( \Delta \xi^n_{k} = \xi^n(k/n) - \xi^n(k - 1/n) \) are assumed to have independent components in each of the coordinate directions, having exponential tails, i.e. \( P(|\Delta \xi^n_k| > x) \leq C \exp(-ax^\lambda) \). The Levy distance is then given by \( L(P^n, W) = O(n^{-1/4}(\log n)^{3/2} + 1/\lambda) \). Applications to the convergence of certain functionals, and to the Dirichlet problem are given. (Received May 5, 1971.)
Topology

*686-G1. BJORN O. FRIBERG, University of California, Los Angeles, California 90024. A topological proof of a theorem of Kneser.

Theorem. The space with C-O-topology of homeomorphisms of the plane deforms to the orthogonal group O(2). Kneser (Math. Z. 25(1926), 362-372) proved this result using complex function theory. This paper's proof uses techniques motivated by the local contractibility paper of Edwards and Kirby (Ann. of Math. (2) 93(1971), 63-88). Let \( h \) be an orientation preserving homeomorphism of \( \mathbb{R}^2 \) fixing the origin. \( h \) is canonically isotoped to the appropriate rotation \( g \) by lifting \( g^{-1} h \) to a bounded homeomorphism of \( \mathbb{R}^2 \) via the torus. A deformation of the homeomorphism group of the 2-sphere to \( O(3) \) is also obtained. Crossing the proof with \( \mathbb{R}^n \) gives a deformation of the homeomorphism group of \( \mathbb{R}^{n+2} \) fixing \( \mathbb{R}^n \), to \( O(2) \). (Received April 30, 1971.)


This paper is a comparative study of stronger forms of connectivity introduced by N. Levine in his paper, "Strong connectivity," Amer. Math. Monthly 72(1965) and by Steen and Seebach in their book, "Counterexamples in topology." Definition. If \( (X, \tau) \) is a topological space, then (a) \( A \subseteq X \) is strongly connected in the sense of Levine \( \text{s.c.}(L) \) iff whenever \( A \subseteq G_1 \cup G_2 \) with \( G_1, G_2 \in \tau \), then either \( A \subseteq G_1 \) or \( A \subseteq G_2 \); (b) \( A \subseteq X \) is strongly connected in the sense of Steen and Seebach \( \text{s.c.}(S^2) \) iff there do not exist any nonconstant continuous mappings \( f: A \rightarrow E^1 \). Clearly, \( \text{s.c.}(L) \) implies \( \text{s.c.}(S^2) \) implies connectivity. However, neither implication is reversible. E.g. the rational points in the upper half-plane with the Bing "sticky-foot" (irrational slope) topology is \( \text{s.c.}(S^2) \) but not \( \text{s.c.}(L) \). Moreover, \( E^1 \) is connected but not \( \text{s.c.}(S^2) \). Theorem 1. \( \text{s.c.}(S^2) \) and \( \text{s.c.}(L) \) are continuous invariants. Theorem 2. If \( X \) is connected but not \( \text{s.c.}(S^2) \), then \( X \) is uncountable. Theorem 3. (a) \( X \) is \( \text{s.c.}(L) \) iff each closed subspace of \( X \) is connected; (b) \( X \) is \( \text{s.c.}(S^2) \) if each open subspace of \( X \) is connected. Since the Bing example is not locally connected, the converse of Theorem 3(b) is clearly false. (Received May 3, 1971.)

ERRATA

Volume 18


The result (3) implies (1) of Theorem 2 was obtained previously by P. Zenor and published in Prace Mat. 13(1969), 23-32.


The theorem claimed is withdrawn on the grounds that it lacks a proof.

617
ABSTRACTS PRESENTED TO THE SOCIETY

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The papers printed below were accepted by the American Mathematical Society for presentation by title. The abstracts are grouped according to subjects chosen by the author from categories listed on the abstract form. The miscellaneous group includes all abstracts for which the authors did not indicate a category.

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**Algebra & Theory of Numbers**

*71T-A81. GEORGE A. GRÄTZER and HARRY LAKSER, University of Manitoba, Winnipeg 19, Manitoba, Canada. Modular lattices and the amalgamation property.*

An equational class $K$ is said to satisfy the amalgamation property if for $A, B_1, B_2 \in K$ and embeddings $f_i : A \rightarrow B_i$, $i = 1, 2$, there exist a $C \in K$ and embeddings $g_i : B_i \rightarrow C$, $i = 1, 2$, such that $f_1 g_1 = f_2 g_2$.

**Theorem 1.** Any nontrivial equational class $K$ of modular lattices that satisfies the amalgamation property is either the class of distributive lattices or the free lattice over $K$ on four generators is infinite. **Theorem 2.** If $K$ is an equational class of modular lattices and $K$ satisfies the amalgamation property, then each lattice in $K$ can be embedded in the subspace lattice of a projective geometry of dimension $\geq 3$. **Corollary.** Any equational class $K$ of modular lattices that satisfies the amalgamation property satisfies the Desargues identity. This corollary was first proved, in a different manner, by B. Jónsson (Abstract 71T-A42, these Notices 18 (1971), 400). (Received March 5, 1971.)

*71T-A82. ANDRÉ JOYAL, Université de Montréal, Montréal, Québec, Canada. Spectral spaces. II. Preliminary report.*

For notations and terminology see "Spectral spaces and distributive lattices," Abstract 71T-A18, these Notices 18 (1971), 393. Given a distributive lattice $D$ with 0 and 1, let $D^{opp}$ be the lattice obtained by interchanging $\lor$ and $\land$. If $X = \text{Spec}(D) \subseteq S_p$ corresponds to $D$ by our equivalence, then $\text{Spec}(D^{opp})$ is homeomorphic to $X$ with the topology $T_X^{opp}$ generated by all the complements of quasi-compact open subsets of $X$. **Theorem.** The least upper bound $C_X$ of the topologies $T_X$ and $T_X^{opp}$ is a compact totally disconnected topology over $X$ (i.e., a Stone space). **Theorem.** The functor $(X, T_X) \rightarrow (X, C_X)$ is a right adjoint to the inclusion functor of the category of Stone spaces into $S_p$. We define an order over $X$ by: $x \leq y$ iff $x \in [y]$. The graph of this (partial) order is closed with respect to the topology $C_X$ of $X$. Thus, we obtain a covariant functor $C : X \rightarrow (X, C_X)$ from $S$ to the category of ordered Stone (i.e., graph closed) spaces. **Theorem.** The functor $C$ is a full embedding. Furthermore, $Y$ is in the image of the functor $C$ iff $Y$ is a projective limit of finite (partially) ordered sets. **Remark.** Related results are known in algebraic geometry for the particular case of the underlying space of a quasi-compact scheme. (Received November 4, 1970.)
The lattice of equational subclasses of idempotent semigroups \( I \) and the \( p_1 \) sequence of its equational subclasses were completely described by J. A. Gerhard (J. Algebra 15(1970), 195-224 and Abstract 70T-A63, these Notices 17(1970), 434). The results are extended to the equational class \( \pi \) of idemconstant semigroups, i.e. the smallest equational class of semigroups containing all idempotent semigroups and all constant semigroups. \( \pi \) is defined by the identities \((xy)^2 = xy = x^2 y^2\), \(x(yz) = (xy)z\), and every idemconstant semigroup is a subdirect product of an idempotent semigroup and a constant semigroup. The free semigroups in \( \pi \) are described and the cardinal number of finitely generated free semigroups are determined. The lattice \( L(\pi) \) of equational subclasses of \( \pi \) is shown to be isomorphic to the direct product of \( L(I) \) and the two element chain. Those rings whose multiplicative semigroups are idemconstant are completely described by the Theorem. Let \( S \) be the equational class of all Boolean rings and \( C_0 \) be the equational class of zero rings. For a ring \( R \), the following statements are equivalent: (1) \( R \) satisfies \( xy = x^2 y^2 \). (2) \( R \) satisfies \( xy = (xy)^2 \). (3) \( R \) is in the equational class of rings generated by \( S \) and \( C_0 \). The representation of (3) is unique up to isomorphism. (Received February 15, 1971.) (Author introduced by Professor George A. Grätzer.)

A (not necessarily commutative) ring \( R \) with identity is residually finite if \( R/I \) is a finite ring for every nonzero ideal \( I \) of \( R \). Let \( R \subseteq R' \) be commutative rings with \( R \) noetherian such that the map \( \text{Spec} R' \to \text{Spec} R \) is surjective. Then if \( R' \) is residually finite, so is \( R \). Corollary. Let \( R - R' \) be a faithfully flat morphism of commutative rings; then if \( R' \) is residually finite, so is \( R \). K. L. Chew and S. Lawn claimed (Canad. J. Math. 22(1970)) that the completion of an infinite residually finite local ring is residually finite; this is however false (e.g. the local ring of a cusp over a finite field). Nevertheless: Theorem 1. Let \( R \) be a (commutative) noetherian ring and \( I \subseteq \text{rad} R \) and ideal such that (i) \( R \) is locally unibranch; (ii) \( \text{Spec}(R/I) \) is connected and for every \( M \in \text{max} R \) the formal fibres of \( R_M \) are geometrically normal. Then \( R \) is residually finite iff its completion is residually finite. Theorem 2. Same notation and hypotheses as in Theorem 1, suppressing the condition on the fibres; then \( R \) is residually finite iff its henselization is residually finite. Other results: (1) \( R - R' \) is a flat epimorphism of commutative rings; if \( R \) is residually finite, so is \( R' \). (2) \( R \) is a normal domain with quotient field \( K \), A is a finite simple separable \( K \)-algebra. If \( R \) is residually finite, so is any \( R \)-order inside A. (Received February 18, 1971.) (Author introduced by Professor Paulo Ribenboim.)

Let \( R \) be an associative ring which does not necessarily have a unity, let \( M \) be a right \( R \)-module, and let \( \text{End}_R(M) \) be the ring of \( R \)-endomorphisms of \( M \). Theorem 1. If \( M \) is Artinian, then each nil subring of \( \text{End}_R(M) \) is nilpotent. Theorem 2. If \( M \) is projective Artinian, then \( \text{End}_R(M) \) is semiprimary. In joint work,
Lance W. Small and the author have proved the following: **Theorem 3.** If \( M \) is injective Noetherian, then \( \text{End}_R(M) \) is semiprimary. It should be noted that 'projective' can be replaced by 'quasi-projective' in Theorem 2 and 'injective' can be replaced by 'quasi-injective' in Theorem 3. (Received March 3, 1971.)

*71T-A86. M. V. SUBBARAO and D. SURYANARAYANA, University of Alberta, Edmonton, Alberta, Canada. Some theorems in additive number theory.

Let \( K \) and \( r \) be fixed integers such that \( 1 < r < K \). In this paper we prove that every sufficiently large integer \( n \) can be expressed as the sum of a prime and an integer of the form \( a^Kb \), where \( a > 1 \) and \( b \) is an \( r \)-free integer. We also establish an asymptotic formula for the number of such representations of \( n \) with an error term \( O(n/\log^H n) \) where \( H \) is any positive number; and improve this \( O \)-term to \( O(n^{9/10} \log^{1/5} n) \) on the assumption of the extended Riemann hypothesis. (Received February 19, 1971.)

*71T-A87. THOMAS S. SHORES, University of Nebraska, Lincoln, Nebraska 68508. On the structure of Loewy modules. Preliminary report.

Let \( R \) be a commutative ring with unit, \( M \) an \( R \)-module and \( \alpha \) an ordinal. Let \( L_\alpha(M) \) be the \( \alpha \)-th term in the ascending Loewy series of \( M \) (so that \( L_1(M) = \text{socle of } M \) and \( L_\alpha(M, I) \) the \( \alpha \)-th term of the ascending Loewy series of \( M \) with respect to the maximal ideal \( I \) of \( R \). Then \( M \) is called a Loewy module if \( L_\alpha(M) = M \) for some \( \alpha \) and \( R \) is a Loewy ring if \( L_\alpha(R) = R \) for some \( \alpha \). Let \( H(M) \) be the injective hull of \( M \). **Theorem 1.** If \( M \) is a Loewy \( R \)-module with Noetherian socle, then \( M = \bigoplus \bigoplus L_\alpha(M, I) \), for various maximal \( I \) and some \( \alpha \).

**Theorem 2.** Let \( R \) be a Loewy ring with Jacobson radical \( J \). The following are equivalent: (i) \( R \) is Artinian. (ii) \( L_2(R) \) is Noetherian. (iii) \( R/J \) and \( L_2(J) \) are Noetherian. **Theorem 3.** \( R \) is locally Artinian iff \( L_2(H(R/I)) \) is Noetherian for all maximal ideals \( I \) of \( R \). **Theorem 4.** Let \( M \) be a Loewy module and \( \alpha \) an ordinal such that \( L_{\alpha+2}(M)/L_\alpha(M) \) is Noetherian. Then \( M = L_{\alpha+\omega}(M) \) and \( L_{\alpha+n+1}(M)/L_{\alpha+n}(M) \) is Noetherian for all positive integers \( n \). (Received February 19, 1971.)

*71T-A88. DAVID J. RODABAUGH, University of Missouri, Columbia, Missouri 65201. On antiflexible algebras.

In this paper we begin a classification of simple and semisimple totally antiflexible algebras (finite dimensional) over splitting fields of char. \( \neq 2, 3 \). For such an algebra \( A \), let \( P \) be the largest associative ideal in \( A^+ \) and let \( N^+ \) be the radical of \( P \). We determine all simple and semisimple totally antiflexible algebras in which \( N \cdot N = 0 \). Defining \( A \) to be of type \((m, n)\) if \( N^+ \) is nilpotent of class \( m \) with dim \( A = n \), we then characterize all simple nodal totally antiflexible algebras (over fields of char. \( \neq 2, 3 \)) of types \((n, n)\) and \((n-1, n)\) and give preliminary results for certain other types. (Received February 19, 1971.)
Let $V$ be the variety of algebras defined by the set $\Sigma$ of equations, and let $n$ be the number of operations of an algebra of $V$. Theorem. The following conditions are equivalent: (i) There exists a cardinal $m$ such that all subdirectly irreducible algebras in $V$ are of power $\leq m$. (ii) Every subdirectly irreducible algebra in $V$ is of power $\leq 2^m$. (iii) There is a set $K \subseteq V$ such that $|K| = 2^n$, $|x| = 2^n$ for $x \in K$, and every algebra in $V$ is a subalgebra of a product of algebras in $K$. (iv) Every algebra in $V$ is a subalgebra of some equationally compact algebra. (v) Every algebra in $V$ is a subalgebra of some equationally compact algebra in $V$. (vi) For every positive formula $\phi$ in the language of $\Sigma$ having four free variables such that $\forall x \forall y \forall z \phi(x,y,z) \rightarrow y = z$, there exists finite $n$ such that $\exists x_1 \exists x_2 \exists x_3 \exists x_4 \phi(x_1,x_2,y,z) \rightarrow y = z$.

For the notion of equational compactness, see J. Mycielski, Colloq. Math 13 (1964), 1-9, where the question is raised of which varieties satisfy (v). (Received February 22, 1971.)

Let $A$ be a nonassociative algebra with an identity element over a field $F$ and let $g: A \times A \times A \rightarrow F$ such that $(xy)z = g(x,y,z)x(yz)$ for all $x, y, z$ in $A$. Assume also that $(x,x,x) = 0$ holds for all $x$ in $A$. Theorem 1. If characteristic $F \neq 2, 3$ then $A$ is power-associative. Theorem 2. If $A$ is finite dimensional and semisimple and characteristic $F \neq 2, 3$ then $A$ is associative. (Received February 22, 1971.)

A ring $R$ is said to be a Kertesz ring if the intersection of any two modular right ideals in $R$ is modular. F. Szasz (Acta Math. Acad. Sci. Hungar. 20(1969), 211-216) has given the following two sufficient conditions, for a ring $R$ to be a Kertesz ring, viz., (1) $R$ has a left identity, (2) $Q_a \cap Q_b \neq \emptyset$ for any two elements $a, b$ in $R$ where $Q_a = \{a + x - ax | y \in R\}$. He asks 'Does there exist a Kertesz ring satisfying neither of the conditions (1) and (2) as stated above?' In this paper, we exhibit an example of such a ring. Also, we prove Theorem. $R$ is a ring without left identity. Suppose $R$ is a Kertesz ring. Then $Q_a \cap Q_b \neq \emptyset$ for any two elements $a, b$ in $R$, where $[Q_a]$ denotes the additive subgroup generated by $Q_a$ in $R$. An example is constructed to show the above stated condition is not sufficient. (Received February 15, 1971.) (Author introduced by Professor M. Rajagopalan.)

Let $B$ be a nondegenerate symmetric bilinear form on a vector space $V$ of finite dimension $n \geq 4$ over a finite field $F$ of characteristic $\neq 2$. Let index $V = n \geq 2$. Let $|F| = q$. Let $\Omega(V)$ be the commutator subgroup
of the group of linear transformations on $V$ preserving $B$. Let a one-dimensional subspace of $V$ be called a point. Suppose $S$ is a subset of $V$ so that $B(x, x) = 0$ for all $x$ in $S$, and suppose $S \neq 0$. Then say $S$ is singular. A group of root type 1 is a set of $\rho_{x, ku}$ of Tamagawa, Amer. J. Math. 80(1958), 191, where $u$ is singular as well as $x$ and $u$ and $x$ are fixed while $r$ runs over $F^*$; plus the identity. Each such group is isomorphic to the additive group of $F$, and the groups of root type 1 are in $1:1$ correspondence with the singular two-dimensional spaces of $V$.

Theorem. Let $G \leq \Omega(V)$ be transitive on singular points of $V$ and generated by groups of root type 1. Then

1. $t = 7$ and $G = S^2$, $G = S^2$;
2. $t = 4n + 2$, $v = 2n$, $n \geq 1$ or $t = 4n$, $v = 2n$, $n > 1$ and $G = SU(l/2, q^2)$; or
3. $G = \Omega(V)$.

(Received March 1, 1971.)

7IT-A93. PAUL PONOMAREV, Johns Hopkins University, Baltimore, Maryland 21218. Class numbers of definite quaternary forms with nonsquare discriminant. Preliminary report.

Let $V$ be a definite quadratic space over $Q$ of dimension four. Assume the discriminant $\Delta$ of the genus $Q$ of maximal integral lattices is not a square and the square-free kernel $D$ of $\Delta$ is odd. Suppose also $V_p$ is isotropic at every finite prime $p$ and the fundamental unit of $Q(D)$ has norm $= -1$. Hence $\Sigma = D(p_1 \cdot \cdot \cdot p_t)^2$, where $p_1, \cdot \cdot \cdot, p_t$ remain prime in $Q$. Set $d = p_1 \cdot \cdot \cdot p_t$, $M(\Delta) = (2^t \cdot 48D)^{-1} \pi_p (D^2 + 1) \cdot (\sum_{n=1}^D (D/n)^2)$, where $(D/n)$ is the Kronecker symbol. For any integer $m > 0$, set $\lambda(m) = \text{the number of primes dividing } m$, $h(-m) = \text{class number of } Q((-m)^{1/2})$. Let $\lambda_1 = 0$ if $3 \nmid \Delta$, $\delta = 1$ if $3 | \Delta$. Denote by $H$ the number of proper similarity classes in the "Iedalkomplex" containing $G$.

Theorem. If $D > 5$, then $H = M(\Delta) + \frac{c_1}{2} h(-D)/8 + 2 \delta h(-3D)/6 + \sum_{m \mid d, m \neq 1} c_m \lambda(m) - \alpha(m) h(-m) h(-mD)$, where:

(a) If $D = 1$ (mod 8), then $c_8 = 3/8$, $c_m = 1$ for $m \neq 3$, and $\alpha(m) = -2, 0, 2$ according as $m = 3$ (mod 8), $7$ (mod 8), $1$ (mod 4), respectively. (b) If $D = 5$ (mod 8), then $\alpha(3) = 1$ and:

(i) if $\Delta$ is even, then $c_1 = 3/2$, $c_m = 1$ for $m \neq 1$ and $\alpha(m) = 0, 2, 3$ for $m \neq 3$, according as $m = 2, 1$ (mod 4), resp.,

(ii) if $\Delta$ is odd, then $c_m = 1$ for all $m$ and $\alpha(m) = 0, 2$ for $m \neq 3$, according as $m = 3, 1$ (mod 4), resp. (Received March 1, 1971.)

*7IT-A94. STANLEY N. BURRIS, University of Waterloo, Waterloo, Ontario, Canada and University of California, Berkeley, California 94720. Equational classes of unary algebras.

Extending the results announced in Abstract 70T-A86, these Notice 17(1970), 564, the author has proved that the possible spectra for consistent equational theories of unary algebras are either of the form $\{m : m \geq 1\}$ or $\{1\} \cup \{m : m \geq n\}$, where $n$ is infinite. Furthermore, all of these possibilities can be realized. An equational theory $\Sigma$ of unary algebras with $m$ operation symbols is categorical in some $k$, $k \equiv m + N_0$, iff for every algebra $\mathfrak{A} = \langle A, \mathfrak{s} \rangle$ satisfying $\Sigma$, each $f \in \mathfrak{s}$ is either the identity function or a constant, and all constant functions are the same. (Received March 5, 1971.)

7IT-A95. HELEN SKALA, 1927 S. Racine, Chicago, Illinois 60608 and University of Massachusetts, Boston, Massachusetts 02116. Modular trellises.

Let $A$ be a set with a reflexive and antisymmetric relation $\equiv$. An element $c$ of $A$ is said to be the least upper bound (l.u.b.) of two elements $a$ and $b$ of $A$ and is denoted by $a \vee b$, if $a \equiv c$, $b \equiv c$ and whenever $a \equiv d$, $b < d$ for some element $d$ then $c \equiv d$. The greatest lower bound (g.l.b.) of $a$ and $b$, denoted by $a \wedge b$, is defined
dually. A is called a trellis if any two elements of A have a l.u.b. and a g.l.b. A is said to be modular if 

\[ x \leq y \text{ implies } (x \vee y) \wedge z = x \vee (y \wedge z) \text{ for any element } y. \]

An element of A is associative if, with any two other elements, it satisfies the associative laws for both \( \vee \) and \( \wedge \). Theorem. In a modular trellis the set of associative elements constitute an (associative) sublattice. Theorem. The subtrellis generated by three elements of a modular trellis, at least two of which are associative, is associative. (Received March 10, 1971.)

*71T-A96. PAUL-JEAN CAHEN, Queen's University, Kingston, Ontario, Canada. Module of the integral valued polynomials. Preliminary report.

One can ask: "Which polynomials over an algebraic number field map the integers into themselves?" (D. A. Lind, Amer. Math. Monthly 78(1971), 179-180). These "integral valued polynomials" obviously form a ring but one can first study their structure as a module over the ring of algebraic integers. Early in 1919, Polya established that, whenever the ring of integers is principal, this module is free and a base is supplied by polynomials of increasing degree (J. Reine Angew. Math. 149(1919), 96-116). This work proves that over any Dedekind domain, these polynomials form a projective module that can therefore be decomposed into a direct sum of fractional ideals; moreover one can make this decomposition with respect to polynomials of increasing degree. The fractional ideals occurring in such a decomposition generate a proper subgroup of the ideal class group.

(Received March 17, 1971.) (Author introduced by Professor Paulo Ribenboim.)

*71T-A97. MICHAEL DOOB, University of Manitoba, Winnipeg 19, Manitoba, Canada. A geometric interpretation of the least eigenvalue of a line graph.

A geometric interpretation of the least eigenvalue of a line graph was given in Abstract 71T-A139, these Notice 17(1970), 804. These results are generalized from the ring of complex numbers to an arbitrary integral domain by showing a correspondence between the eigenspace and a matroid of chains. Theorem. The eigenspace corresponding to the eigenvalue \(-2\) (the least possible of the ring is the complex numbers) is a matroid with a basis determined solely by the number of vertices and cyclic structure of each connected component. Theorem. If \(G\) is connected, the multiplicity of the eigenvalue \(=\) rank of the matroid and has the following value:

(i) \(|E| - |V| + 2\) if \(G\) is bipartite and \(|V| = 0 \mod p\),
(ii) \(|E| - |V| + 1\) if \(G\) is bipartite and \(|V| \neq 0 \mod p\), or
(iii) \(|E| - |V|\) if \(G\) is not bipartite. (Received March 11, 1971.)


Let \(H\) be a class of \(R\)-modules. Let \(F(H)\) be the set of ideals I of \(R\) such that there is an \(R\)-module \(A\) in \(H\) which has an element of order \(I\). \(F([A]) = F(A)\). Let \(U(H)\) be the smallest class containing \(H\) that is closed under direct sums, homomorphisms, subobjects and the union of chains. Theorem I. \(U(H) = H \iff H\) is the class of all \(R\)-modules \(A\) such that \(F(A) \subseteq F(H)\) and \(F(H)\) is a filter in the lattice of ideals of \(R\). Theorem II. Let \(A\) be an \(R\)-module such that (i) \(|A| > |R|\); (ii) \(C, B \subseteq A, |C| < |A|\) and \(\beta : C \rightarrow B\) implies that there exists \(\sigma \in \text{Aut}(A)\) such that \(\sigma | C = \beta\); (iii) \(I, J \in F(A)\) implies that there exist \(a, b \in A\) such that \(o(a) = I, o(b) = J\)
and \((a) \cap (b) = 0\); and (iv) \(F(U(A)) = F(A)\) then (i) \(B \in U(A)\) and \(A \subseteq B\) implies that \(A\) is a summand of \(B\);
(ii) \(A\) is injective in \(U(A)\); and (iii) if \(F(A)\) is a complete filter then \(A\) is quasi-injective in the class of all \(R\)-modules. (Received March 11, 1971.)


Let \(H\) be a Hadamard matrix of order \(4m\). Then there exist unimodular \(U\) and \(V\) such that \(UHV = \text{diag}(d_1, d_2, \ldots, d_{4m})\) is in Smith's normal form. Here \(d_1, \ldots, d_{4m}\) are positive integers such that \(d_i d_{i+1} = 4m\) \((1 \leq i \leq 2m)\) and \(d_1 \cdots d_1\) is the g. c. d. of the \(i\)-rowed minors of \(H\). Using the fact that \(HH^T = 4mI\) and that \(H\) and \(H^T\) have the same normal form, it is readily seen that \(d_1 d_{4m+1-i} = 4m\) \((1 \leq i \leq 4m)\). Moreover, it is clear that \(d_1 = 1, d_2 = 2\). The following theorem is immediately deduced. Theorem. Any two Hadamard matrices of order \(4m\), where \(m\) is squarefree, are integrally equivalent. This is a simpler proof than that originally given by W. D. Wallis and Jennifer Wallis, "Equivalence of Hadamard matrices," Israel J. Math. 7 (1969), 122-127. (Received March 12, 1971.)

*71T-A100. STANLEY S. PAGE, University of British Columbia, Vancouver, British Columbia, Canada. Flat covers.

The following theorem is proved. Let \(R\) be a ring with unit. If \(M\) is a left \(R\)-module there exist a flat left \(R\)-module, \(P\) unique up to isomorphism, and map \(f: P \rightarrow M \rightarrow 0\) such that for any flat \(P'\) and map \(g: P' \rightarrow M \rightarrow 0\) there exists a map \(h: P' \rightarrow P\) such that \(f \circ h = g\). This leads to the finding of the Jacobson radical of the \(\text{End}(P)\) for \(P\) projective and of course gives rise to homology. (Received March 12, 1971.)

*71T-A101. E. FRIED, GEORGE A. GRÄTZER and HARRY LAKSER, University of Manitoba, Winnipeg 19, Manitoba, Canada. Amalgamation and weak injectives in the equational class of modular lattices \(\mathcal{M}_n\).

For \(n \geq 5\), let \(M_n\) denote the modular lattice of length 2 with \(n-2\) atoms and let \(\mathcal{M}_n\) denote the equational class generated by \(M_n\). Theorem 1. Let \(B \in \mathcal{M}_n\), let \(A\) be a sublattice of \(B\), and let \(\Theta\) be a congruence relation on \(A\) such that \(A/\Theta\) contains no prime ideal. Then there is a congruence relation \(\bar{\Theta}\) on \(B\) whose restriction to \(A\) is \(\Theta\). The amalgamation class of \(\mathcal{M}_n\), Amal(\(\mathcal{M}_n\)), is the class of all algebras \(A\) in \(\mathcal{M}_n\) such that any two extensions of \(A\) in \(\mathcal{M}_n\) can be amalgamated over \(A\) in \(\mathcal{M}_n\) (Abstract 70T-A129, these Notices 17(1970), 801).

Theorem 2. If \(A \in \mathcal{M}_n\) contains no prime ideal then \(A \in \text{Amal}(\mathcal{M}_n)\). Theorem 3. If \(A \in \mathcal{M}_n\) is finite then \(A \in \text{Amal}(\mathcal{M}_n)\) iff \(A\) contains no prime ideal. A lattice \(C \in \mathcal{M}_n\) is a weak injective lattice if, for each lattice \(B \in \mathcal{M}_n\) and each sublattice \(A\) of \(B\), any homomorphism of \(A\) onto \(C\) can be extended to a homomorphism of \(B\) to \(C\). Theorem 4. A lattice in \(\mathcal{M}_n\) is weak injective in \(\mathcal{M}_n\) iff it is a Boolean extension of \(\mathcal{M}_n\) by a complete Boolean algebra. (Received March 15, 1971.)

624
Theorem 1. If every Sylow normalizer of a group $G$ has prime power index in $G$, then $G$ is solvable.

Theorem 2. Let $G$ be a solvable group with the property that every Sylow normalizer admits a cyclic supplement. Suppose, moreover, that $G$ does not map onto $\Sigma(4)$, the symmetric group on four letters. Then $G$ is supersolvable. Combining the above, we obtain: Theorem 3. If every Sylow normalizer of a group $G$ is supplemented by a cyclic group of prime power order, then either $G$ maps onto $\Sigma(4)$ or $G$ is supersolvable.

Theorem 4. Let $G$ be a group with the property that each Sylow normalizer admits an abelian Hall-supplement. Then $G$ is solvable. Theorem 5. Let $G$ be a group with the property that each Sylow normalizer of $G$ admits a cyclic Hall-supplement. Then $G$ is supersolvable. (Received March 15, 1971.)

*71T-A103. ALBERT A. MULLIN, USARV, HQ, USAICCV (LDSC), APO San Francisco, California 96384.


For definitions and terminology see these Notaia 17(1970), 802, 938. Consider the set $W$ of all one-place weakly relatively prime sequences. If $f \in W$, $g \in W$ then $f(g(\cdot)) \in W$. Lemma 1. Under the operation of composition of sequences, $W$ is an infinite, noncommutative monoid of sequences. Definition. Let $G$ be the infinite set of all one-place generalized relatively prime sequences. Let $f \in W \subset G$ and $g \in G$. Then $f(g(\cdot)) \in G$. Such a $G$ is said to be right weakly closed relative to $W$; clearly $G$, itself, is not closed under composition of sequences. Let $R$ be the (standard) set of all one-place relatively prime sequences and $T$ be the set of all one-place totally prime sequences. Lemma 2. Under the operation of composition of sequences, $R$ is an infinite, noncommutative semigroup, and $T$ is, also. Problems. Determine the semigroup structures of $T \subset R \subset W$. Two other nonabelian semigroups of sequences are constructed using the notion of a mosaic. (Received March 15, 1971.)

*71T-A104. V. S. RAMAMURTHI and K. M. RANGASWAMY, Madurai University, Madurai-2, Tamil Nadu, India. Generalised V-rings.

A ring $R$ with identity is called a right V-ring if every simple right $R$-module is injective. $R$ is called a generalised right V-ring (for short, a right GV-ring) if every simple right $R$-module is projective or injective. Let $Z(R)$ and $J(R)$ denote respectively the singular ideal and the Jacobson radical of $R$. Theorem 1. $R$ is a right GV-ring if and only if $Z(R) \cap J(R) = 0$ and every proper large right ideal of $R$ is an intersection of maximal right ideals. Theorem 2. Let $R$ be commutative. Then $R$ is a GV-ring if and only if $R$ is von Neumann regular. Theorem 3. Let $R$ be a ring with identity. Then $R$ is a semiprime right noetherian GV-ring if and only if every semisimple right $R$-module is injective. Theorem 4. Let $R$ be a right V-ring. Then, for every two-sided ideal $I$ of $R$, $R/I$ is left $R$-flat. Theorem 5. Let $R$ be a ring in which every large right ideal is two-sided. Then $R$ is a right GV-ring if and only if $R/I$ is left $R$-flat for every two-sided ideal $I$. (Received March 16, 1971.)

(Author introduced by Professor M. Rajagopalan.)
More on the generalized Macaulay theorem.

Let \( k_1 \leq k_2 \leq \ldots \leq k_n \) be given positive integers and let \( F \) denote the set of vectors \((i_1, \ldots, i_n)\) with integer components satisfying \( 0 \leq i_1 \leq k_1, \ldots, 0 \leq i_n \leq k_n \). If \( H \) is a subset of \( F \), let \((\mathcal{L})H\) denote the subset of those vectors with component sum \( \ell \), and let \( C((\mathcal{L})H) \) denote the smallest \( |(\mathcal{L})H| \) elements of \((\mathcal{L})F\). The generalized Macaulay theorem due to the author and B. Lindström [J. Combinatorial Theory 7 (1969), 230-238, MR 40 #50] shows that \( |(\mathcal{L})H| \geq |\Gamma(C((\mathcal{L})H))| \), where \( \Gamma((\mathcal{L})H) \) is the set of vectors in \( F \) obtainable by subtracting 1 from a single component of a vector in \((\mathcal{L})H\). A method is given for computing \( |\Gamma(C((\mathcal{L})H))| \) in this paper. It is analogous to the method for computing \( |\Gamma(C((\mathcal{L})H))| \) in the \( k_1 = \ldots = k_n = 1 \) case which has been given independently by Katona [in Theory of Graphs (Tihany, 1966) Academic Press, New York, 1968, pp. 187-207, MR 38 #1016] and Kruskal [in Mathematical Optimization Techniques, Univ. of California Press, Berkeley, Calif., 1963, pp. 251-278, MR 27 #4771]. (Received March 18, 1971.)

Chebyshev polynomials in several variables.

Let \( F(q) \) denote the finite field with \( q \) elements. Let \( a_1, \ldots, a_n, b \in F(q) \), and let \( z^{n+1} - g_1^{(k)}(a_1, \ldots, a_n, b)z^n + \ldots + (-1)^{n}g_n^{(k)}(a_1, \ldots, a_n, b)z + (-1)^{n+1}b \) be the polynomial over \( F(q) \) whose roots are the \( k \)th powers of the roots of \( z^{n+1} + a_1 z^n + \ldots + a_n z + b \). For fixed \( b \), the \( g_i^{(k)} \) are polynomials in \( a_1, \ldots, a_n \). Theorem 1. The vector \( g^{(k)} = (g_1^{(k)}, \ldots, g_n^{(k)}) \) is a PPV (permutation polynomial vector) iff \( (k, q^{s-1}) = 1 \) for \( s = 1, 2, \ldots, n + 1 \) (\( s \leq n+1 \) if \( b = 0 \)). Theorem 2. There are infinitely many primes \( p \) for which \( g^{(k)} \) is a PPV over \( F(p) \) iff for each prime \( p \) dividing \( k \), \( p - 1 \) does not divide \( s \) for \( s = 1, 2, \ldots, n + 1 \) (\( s \leq n+1 \) if \( b = 0 \)). Conjecture. If \( h \) is a PPV over \( F(p) \) for an infinite number of primes \( p \), then \( h \) is a composite of linear PPV's and PPV's of the form \( g^{(k)} \). (Fried proved this for \( n = 1 \) see "On a conjecture of Schur," Michigan Math. J. 17(1970), 41-55.) Theorem 3. For fixed \( b \), the \( g^{(k)} \) are closed under composition of polynomials iff \( b = 0 \) or \( b = \pm 1 \). Also an explicit formula for \( g_1^{(k)} \) is found. (Received March 18, 1971.)

Global dimension of triangular orders over DVR. Preliminary report.

Throughout, \((R, m)\) denotes a DVR with quotient field \( K \). An R-order \( \Lambda \) in \( M_n(K) \) is tiled if \( e_{ij} \Lambda e_{ij} \subseteq \Lambda \), where \( e_{ij} \) are usual matrix units in \( M_n(K) \) for \( 1 \leq i, j \leq n \). Thus \( \Lambda = (\Lambda_{ij}) \subseteq M_n(K) \) for some fractional ideals \( \Lambda_{ij} \) in \( R \). \( \Lambda = (\Lambda_{ij}) \) is triangular if \( \Lambda_{ij} = R \) for \( i \neq j \). \( \Omega^{(n)} = (\Omega^{(n)}_{ij}) \) denotes the triangular R-order in \( M_n(K) \) with \( \Omega^{(n)}_{ij} = m^{i-j} \) whenever \( i \neq j \). Theorem. Given a triangular R-order \( \Lambda \) in \( M_n(K) \), \( \mathrm{gldim} \, \Lambda < \infty \) iff \( \mathrm{gldim} \, \Lambda \approx n - 1 \) iff \( \Omega^{(n)} = \Lambda \). This was conjectured in R. B. Tarsy ["Global dimension of orders," Trans. Amer. Math. Soc. 151(1970), 335-340]. Another conjecture of Tarsy (loc. cit.) is disproved by constructing examples of successive triangular R-orders whose global dimensions differ exactly by 2 or by 3. (Received March 22, 1971.) (Author introduced by Professor A. V. Jategaonkar.)

626
Let $X$ be a topological space which is either a realcompact metric space or a realcompact space in which each of its closed subsets is a zero-set. Let $G$ be a topological abelian group. **Theorem 1.** The homomorphism, $\gamma$, maps $\mathcal{C}_n(X, G)$ isomorphically and homeomorphically onto $\mathcal{C}_n(C(X), G)$, and commutes with the boundary homomorphism $\partial$, i.e., $\delta \gamma = \gamma \partial$, where $\delta$ is from $\mathcal{C}_n(X, G)$ into $\mathcal{C}_{n-1}(X, G)$ and $\partial$ from $\mathcal{C}_{n-1}(X, G)$ onto $\mathcal{C}_{n-1}(C(X), G)$. **Theorem 2.** The homeomorphic isomorphism $\gamma : \mathcal{C}_n(X, G) \rightarrow \mathcal{C}_n(C(X), G)$ maps $\mathbb{Z}^n(X, G)$ onto $\mathbb{Z}^n(C(X), G)$ and $B_n(X, G)$ onto $B_n(C(X), G)$. Therefore, $\gamma$ induces a homeomorphic isomorphism onto, $\gamma'$: $H_n(X, G) \rightarrow H_n(C(X), G)$. **Theorem 3.** For any topological abelian group $G$, the $n$-dimensional Alexander-Kolmogorov-Lefschetz homology group $H_n(X, G)$ is isomorphic and homeomorphic with $n$-dimensional homology group $H_n(C(X), G)$. **Corollary.** If $G$ in Theorem 3 is compact or a field, then the homology groups $H_n(C(X), G)$ are isomorphic and homeomorphic with the corresponding Čech homology groups based on the finite open coverings. (Received March 22, 1971.)


Define a pre-order on a class of relational systems of the same type by (a) $A \equiv B \rightarrow A$ is embeddable in $B$, (b) $A < B \rightarrow A \equiv B$ and $7B \not\equiv A$, the case when $A$ and $B$ are incomparable is denoted by $A \not\sim B$. In this notation a relational system $A$, $|A| \equiv \omega$, is rigid iff, for all $a \in A$, $A \not\sim A - \{a\}$. If we let $X_1$ range over countable pseudocomplemented distributive lattices we show by explicit construction: (1) $\exists X$ such that $X$ is rigid. (2) $\exists \{X_i : i < \omega\}$ such that $X_1 > X_2 > X_3 > \ldots$. (3) $\exists \{X_i : i \in I\}$ such that $|I| = 2^\omega$ and $X_j (X_i)$ for $i \not= j$. This contrasts with the case for Boolean algebras where (1), (2), (3) are known to be false. (Received March 22, 1971.) (Author introduced by Dr. Brian Rotman.)

**71T-A110.** LEROY B. BEASLEY, University of British Columbia, Vancouver 8, British Columbia, Canada and LARRY J. CUMMINGS, University of Waterloo, Waterloo, Ontario, Canada. Permanent groups.

A permanent group is a subgroup of $GL_n(F)$ on which the permanent function is multiplicative. Let $A \cdot B$ denote the Hadamard product of matrices $A$ and $B$. The collection of those subgroups $G$ which contain the diagonal group $D$ and such that $A \cdot B^T \in D$ for every pair $A, B$ of matrices in $G$ is denoted by $A_n(F)$. **Theorem.** If $F$ has at least 3 elements then every group in $A_n(F)$ is a permanent group. **Theorem.** If a permanent group $G$ is generated by $D$ together with a set $S$ of elementary matrices and a set $Q$ of permutation matrices over any field $F$ then $G = H \cdot K$ where $H$ is the subgroup generated by $Q$ and $K$ is in $A_n(F)$. (Received March 22, 1971.)

**71T-A111.** JOHN FRIEDLANDER, Pennsylvania State University, University Park, Pennsylvania 16802. The distribution of power residues in algebraic number fields.

The work of various authors, notably Burgess (Proc. London Math. Soc. (3) 12(1962), 179-192) is considered in the setting of algebraic number fields. Amongst the results are the following. **Theorem 1.** Let
Let \( K \) be a fixed algebraic number field, and \( \mathfrak{p} \), a prime ideal. Let \( \epsilon > 0 \) be fixed. Let \( q_2 \) denote the least positive number which occurs as the absolute value of the norm of an integer of \( K \), which is a quadratic nonresidue mod \( \mathfrak{p} \). Then, for all primes of sufficiently large norm, \( q_2 < (N\mathfrak{p})^{\beta + \epsilon} \), where \( \beta = 1/4/\epsilon \). Similar results are developed for higher power residues. Theorem 2. Let \( K \) be the imaginary quadratic field of discriminant \( d \).

Let \( \mathfrak{p} \) be a prime ideal of \( K \) and let \( \chi \) be a nonprincipal character mod \( \mathfrak{p} \). Let \( 1/4 < b \leq 1/2, \epsilon > 0, \delta > 0 \). Let \( \mathcal{O} \) be the ring of integers of \( K \) and let \( \mathcal{O}_1 = \mathcal{O} \). Let \( \mathcal{J} = \{ \alpha \mathcal{O} \mid |N\alpha| \leq H \} \). Let \( |\mathcal{J}| \) denote the cardinality of \( \mathcal{J} \). Then, for all primes of sufficiently large norm, \( |\mathcal{J}| < \mathcal{O}^{1/2 - \rho_0 (N\mathfrak{p})^{-\delta}} \) where the implied constant depends only on \( \epsilon, \delta \) and \( b \), but not on \( d \), this result holding provided that: (I) If \( 1/4 < b < 2/7 \), then \( H > (N\mathfrak{p})^{b+\delta} |d|^{5/2+b/4b-1+\epsilon} \). (II) If \( 2/7 < b \leq 1/2 \), then \( H > (N\mathfrak{p})^{b+\delta} |d|^{9/2+b-1+\epsilon} \). A similar result is established for real quadratic fields. (Received March 23, 1971.) (Author introduced by Professor S. Chawla.)

**71T-A112. M. V. SUBBARAO,** University of Alberta, Edmonton 7, Alberta, Canada. **Graphical proofs of two identities.**

In a recent paper (Proc. Amer. Math. Soc. 26(1970), 23-27), the author (jointly with M. Vidyasagar) discovered the identities: \( \sum (\alpha x)^{n-1} \phi_{n-1}(\alpha, x) = \sum (-1)^{n-1} a^{2n-2} x^{(n+1)/2-1}/\phi_n(\alpha, x) = 1 \), \( \sum (-1)^n a^{3n-1} x^{(n+1)/2}(\alpha x - a^{-1} x^{-1}) \), where \( \phi_n(\alpha, x) = (1-\alpha x)(1-\alpha x^2) \cdots (1-\alpha x^n), n > 0: \phi_0(\alpha, x) = 1 \), and \( \sum \) denotes summation from \( n = 1 \) to \( n = \infty \). We here give graphical proofs of these analogous to Franklin's proof for the Eulerian expansion for the infinite product \( \Pi_{n=1}^{\infty} (1-x^n) \) (Hardy and Wright, Theory of numbers, 4th edition, 1960, pp. 286-287). (Received March 23, 1971.)

**71T-A113. KIM KI-HANG BUTLER,** Pembroke State University, Pembroke, North Carolina 28372. **Straddles and splits on semigroups.**

A straddle on a semigroup \( S \) is to be an ordered pair \((u, v) (u, v \in S)\) with the property that \( x = u x v \) for all \( x \in S \). A split of \( x \) in a semigroup \( S \) to be an ordered pair \((ab, cd) (a, b, c, d \in S)\) with the property that \( x = ab x cd \). We use the notation and terminology of Clifford and Preston ("The algebraic theory of semigroups. I, II," Math. Surveys, No. 7, Amer. Math. Soc., Providence, R. I., 1961) unless stated otherwise. In order to facilitate the subsequent discussion we introduce the following notations: (1) \( S T = \{(u, v) \in S \times S: x = u x v \) for all \( x \in S \) \}, (2) \( S p x = \{(ab, cd) \in S \times S: (ab, cd) \) is a split of \( x \} \). Theorem 1. If \( x \) and \( y \) belong to the same \( \beta \)-class \( D \) of a semigroup \( S \), then there exist \( a, b, c, d \in S \) such that \( (ab, cd) \) is a split of \( x \) and \( (ba, dc) \) is a split of \( y \).

Corollary 2. (i) If \( (ab, cd) \) is a split of \( x \), then \( ((ab)^2, (cd)^2) \) is a split of \( x \), (ii) \( S p x = S p x^2 \), (iii) \( S p x \neq \emptyset \) for all \( x \in S \). Theorem 3. \( S t S = Diag S = \{u, u\}: u \in S \} = S \) is a group such that \( x = x^{-1} \) for all \( x \in S \). (Received March 23, 1971.)

**71T-A114. MOTUPALLI SATYANARAYANA,** Bowling Green State University, Bowling Green, Ohio 43403. **Dual semigroups.**

If \( T \) is a semigroup with zero and if \( A \) is a subset of \( T \), then \( A^R = \{x \in T: Ax = 0\} \) and similarly \( A^L \) is defined. \( T \) is called a dual semigroup iff \( A^{RL} = A \) for every left ideal \( A \) and \( B^{LR} = B \) for every right ideal \( B \). Let \( S \) be a dual semigroup with 0. In this note we shall prove (1) \( S \) with identity is semisimple-like (every

628
right ideal is generated by an idempotent iff \( S \) is a group with zero. (2) If \( S \) has a nonzero idempotent then \( S \) has an identity iff \( S \) is right uniform (intersection of any two nonzero right ideals is nonzero). (3) If \( S \) is a right uniform 0-simple semigroup, it has no maximal proper right ideals or minimal proper right ideals. (4) If \( S \) is 0-simple, then \( S \) is a group with zero if \( S \) contains identity or \( S \) is right uniform and satisfies ascending chain condition on right ideals. (5) If \( S \) is right uniform and satisfies ascending condition on right ideals, then \( S \) has a nilpotent radical \( N \) such that \( S/N \) is left 0-simple and left regular semigroup. Finally the ideal lattice structure of dual semigroups is discussed. (Received March 26, 1971.)

*71T-A115. R. PADMANABHAN, University of Manitoba, Winnipeg 19, Manitoba, Canada. **Equational theory of idempotent algebras.**

A universal algebra \( \mathcal{U} = \langle A; F \rangle \) is idempotent if every \( f \in F \) is of positive arity (i.e. not nullary) and satisfies \( f(x, x, \ldots, x) = x \). **Theorem.** Every finitely based equational theory of idempotent algebras of type \( \langle m, n \rangle \) with \( m, n \geq 2 \) is two-based. (Received March 29, 1971.)

*71T-A116. JOEL H. SPENCER, The Rand Corporation, Santa Monica, California 90406. **Turan's theorem for k-graphs.**

Let \( 2 \leq k \leq b \leq p \) be positive integers. Set \( T(p, k, b) = \) the smallest \( q \) such that there exists a \( k \)-graph with \( p \) vertices, \( q \) edges, and no independent set of size \( b \). In other words, \( T(p, k, b) \) is the smallest number of \( k \)-subsets of a \( p \)-set \( V \) such that every \( b \)-subset of \( V \) contains at least one of these subsets. The determination of \( T(p, 2, b) \) is Turan's theorem. I prove \( T(p, k, b) \leq p^k b 1-k(k-1)k-1 k-k \) improving the previously known lower bounds when \( b \) is sufficiently greater than \( k \). (Received March 29, 1971.)

71T-A117. KHEE-MENG KOH, University of Manitoba, Winnipeg 19, Manitoba, Canada. **Idempotent algebras with \( P^3 = 3 \).** Preliminary report.

For notations and basic definitions, see Abstract 71T-A30, these *Notices* 18(1971), 397. **Theorem 1.** If the sequence \( \langle 0, 0, 3, m \rangle \) is representable, then \( m \geq 7 \). An example is provided to show that \( m = 7 \) is possible. **Theorem 2.** Let \( \mathcal{U} \) be an algebra representing the sequence \( \langle 0, 0, 3, 7 \rangle \). Then \( \mathcal{U} \) has a unique semilattice operation. **Corollary 3.** Let \( \mathcal{U} \) be an algebra representing the sequence \( \langle 0, 0, 3, 7 \rangle \). Then \( \mathcal{P} n(\mathcal{U}) \equiv 2^n - 1 \), for each \( n \geq 2 \). **Corollary 4.** The sequence \( \langle 0, 0, 3, 7 \rangle \) has the minimal extension property. (Received March 30, 1971.) (Author introduced by Professor George A. Grätzer.)

*71T-A118. JOHN CONWAY ADAMS, University of Colorado, Boulder, Colorado 80302. **G-rings and some of their properties.**

Let \( R \) be a commutative ring with identity. **Definition.** \( R \) is a G-ring if the total quotient ring of \( R \), \( Q(R) \), is finitely generated as a ring over \( R \). The following theorems list some of the results obtained in this paper. **Theorem 1.** The direct product of a collection of rings \( R_i \) is a G-ring iff each \( R_i \) is a G-ring and \( R_i \equiv Q(R_i) \) for all but finitely many indices \( i \). **Theorem 2.** \( R[X] \) is not a G-ring for any ring \( R \).
Theorem 3. Given a positive integer \( n \), there exists a G-ring \( R \) with at least \( n \) irreducible elements \( u_1, \ldots, u_n \) each satisfying \( R[1/u_i] = Q(R) \). The remaining theorems are proved for noetherian rings. Theorem 4. \( R \) is a G-ring iff \( R \) is semilocal and \( \text{rank}(P) = 1 \) for all regular prime ideals \( P \) in \( R \). Theorem 5. Given a positive integer \( n \), there exists a G-ring with Krull dimension \( n \). Theorem 6. If \( R \) is integrally closed then \( R \) is a G-ring iff \( R \) has only finitely many overrings. Theorem 7. \( R[[X]] \) is a G-ring iff the Krull dimension of \( R \) is zero. Theorem 8. If \( R' \) is the \( J(R) \)-adic completion of \( R \) then \( R \) a G-ring implies \( R' \) is a G-ring. (Received April 1, 1971.)


Let \( V \) (resp. \( A \)) be a variety of algebras (resp. an algebra). \( V \) (resp. \( A \)) is said to be equationally precomplete if \( V \) (resp. the variety \( v(A) \) generated by \( A \)) contains at most one equationally complete variety. \( V \) (resp. \( A \)) is said to be \( E \)-precomplete if \( V \) (resp. \( A \)) is equationally precomplete, \( E \) is equationally complete and \( E \in V \) (resp. \( E \in v(A) \)). We will now consider only associative rings. For a prime \( p \), let \( Z_p \) be the \( p \)-element group with trivial multiplication and \( F_p \) be the \( p \)-element field. Tarski proved that \( v(Z_p) \) and \( v(F_p) \) are precisely the equationally complete varieties of rings. We prove Theorem 1. If \( V \) is equationally precomplete, then it satisfies \( p^m x = 0 \) for some prime \( p \) and some natural number \( m \). Theorem 2. If \( A \) is \( Z_p \)-precomplete, then \( A = J(A) \); if \( A \) is \( F_p \)-precomplete, then \( J(A) = (0) \). (\( J(A) \) denotes the Jacobson radical of \( A \).) Theorem 3. \( V \) is \( Z_p \)-precomplete if and only if it satisfies \( p^m x = 0 \) and \( x^n = 0 \); \( V \) is \( F_p \)-precomplete if and only if it satisfies \( px = 0 \) and \( x^n = x \) (\( m \) and \( n \) are some natural numbers). Theorem 4. A \( Z_p \)-precomplete variety is generated by countably many finite (necessarily, nilpotent) rings; an \( F_p \)-precomplete variety is generated by finitely many finite fields. (Received April 5, 1971.)

*71T-A120. CHARLES R. WALL, East Texas State University, Commerce, Texas 75428. The fifth unitary perfect number.

A divisor \( d \) of an integer \( n \) is a unitary divisor if \( d \) and \( n/d \) are relatively prime. An integer is unitary perfect if it equals the sum of its proper unitary divisors. Subbarao and Warren [Canad. Math. Bull. 9(1966), 147-153] reported that 6, 60, 90 and 87360 are the first four unitary perfect numbers, and I have previously reported [Abstract 69T-A139, these Notices 16(1969), 825] that 146361946186458562560000 = \( 2^{18} \cdot 3^4 \cdot 7 \cdot 11 \cdot 13 \cdot 19 \cdot 37 \cdot 79 \cdot 109 \cdot 157 \cdot 313 \) is also unitary perfect. This last integer has been proved to be the fifth unitary perfect number, i.e., the next after 87360. The proof is based on the observation that an integer is unitary perfect if and only if it is the denominator of \( 2 = (1 + 1/x)(1 + 1/y) \cdots \), where \( x, y, \ldots \) are powers of distinct primes. (Received April 5, 1971.)

71T-A121. CHARLES R. WALL, DONALD B. JOHNSON and PHILIP L. CREWS, East Texas State University, Commerce, Texas 75428. Density bounds for the sum of divisors function.

Let \( A(x) \) be the density of the integers \( n \) for which \( \sigma(n)/n < x \), where \( \sigma(n) \) is the sum of the divisors of \( n \). The function is known to exist, and be continuous, for all real \( x \), and is clearly 1 if \( x = 1 \). Using a

630
refinement of the technique used by Behrend [Preuss. Akad. Wiss. Sitzungsber. 6(1933), 280-293] to bound the
density of the abundant numbers, we have obtained upper and lower bounds for $A(x)$ with $x$ in increments of
0.01 and $1 \leq x \leq 5$. Our calculations required 26 minutes on an IBM 360/40 computer. In particular, we
have shown that the density $A(2)$ of the abundant numbers is between 0.2441 and 0.2909 (Behrend gave 0.241
and 0.314). (Received April 5, 1971.)

71T-A122. ROBERT C. SHUCK, Southern Illinois University, Carbondale, Illinois 62901. Certain
Artinian rings are Noetherian. Preliminary report.

Necessary and sufficient conditions for certain Artinian structures to be Noetherian are given.

Theorem. An Artinian ring $R$ which need not have an identity element is Noetherian if and only if each cyclic
Noetherian submodule of $R$ is embedded in some Noetherian factor module of $R$. Corollary (Hopkins). An
Artinian ring with 1 is Noetherian. Theorem. For an Artinian module $M$ the following conditions are
equivalent: (1) $M$ is Noetherian. (2) There is a cyclic Noetherian submodule $A$ of $M$ such that $L(B) \leq L(A)$
for any cyclic Noetherian submodule $B$ of $M$ where, $L(A)$ (L(B)) denotes the length of the composition series
for $A$ (for $B$). (3) There is a bound for the set of lengths of the composition series for the cyclic Noetherian
submodules of $M$. Theorem. For a module $M$ over an Artinian ring $R$ which need not have an identity element
let each cyclic submodule of $M$ be embedded in some Noetherian factor module of $R$. T. A. E. (1) $M$ is
Artinian. (2) $M$ is Noetherian. (3) Each factor module is (Goldie) finite dimensional. (Received April 5,
1971.)

*71T-A123. JIRI SICHLER, University of Manitoba, Winnipeg 19, Manitoba, Canada. Testing categories.

There is a finite category $C$ such that the following two conditions are equivalent for any equational class
$\mathcal{S}$ of unary algebras: (i) $C$ is isomorphic to a full subcategory of $\mathcal{S}$; (ii) Every category $\mathcal{G}$ of algebras is
isomorphic to a full subcategory of $\mathcal{S}$. (Received March 30, 1971.)

algebras satisfying an identity of degree three.

Let $A$ be a finite dimensional algebra over a field $F$. Assume that there exists $g: A \times A \times A \to F$ such
that $(xy)z = g(x,y,z)x(yz)$ for all $x, y, z$ in $A$. The following is proved: Theorem. If $A$ is a nil power-
associative algebra then $A$ is a nilpotent algebra. (Received April 9, 1971.)

*71T-A125. TAH-KAI HU, Western Washington State College, Bellingham, Washington 98225. Locally
equational classes of universal algebras. Preliminary report.

Let $K$ be a class of algebras. $E_L(K)$ denotes the class of algebras $A$ having the property that, for each
finite subset $U$ of $A$, there exist a finite family $(B_i)_{i \in I}$ in $K$ and, for each $i \in I$, a finite subset $V_i$ of $B_i$
such that each identity which holds in $V_i$ for all $i$ also holds in $U$. $K$ is locally equational if $E_L(K) = K$. Let
$A$ be an algebra. If $\theta$ is a congruence on $A$ and $\lambda$ an endomorphism of $A$, set $\theta^\lambda = \{(a, b) \in A^2 :$
$(\lambda(a), \lambda(b)) \in \theta \}$. Let $C$ be a set of congruences on $A$. $C$ is fully invariant if, for each endomorphism of
A, \theta \in C implies \theta^A \in C. C is locally determined if a congruence \theta on A belongs to C whenever, for each finitely generated subalgebra B of A, there exists \eta \in C such that \eta \cap B^2 = \theta \cap B^2. S, H, P_F and D denote the operators of forming subalgebras, homomorphic images, direct products of finite families and directed unions. **Theorem 1.** \( E_L(K) = \text{DHSP}_F(K) \). K is locally equational if and only if K admits subalgebras, homomorphic images, direct products of finite families and directed unions. **Theorem 2.** Let A be an algebra of type \( \tau \) absolutely freely generated by an infinite set. Then there is an isomorphism between the lattice of locally equational classes of type \( \tau \) and the lattice of fully invariant and locally determined filters of the congruence lattice of A, under this isomorphism, equational classes correspond to principal filters generated by full invariant congruences. (Received April 9, 1971.)

71T-A126. BARRY M. MITCHELL, Dalhousie University, Halifax, Nova Scotia, Canada. The vanishing of the derived functors of the inverse limit functor.

Let R be any nonzero ring, and let I be a directed set. Let \( \lim(k) \) denote the kth right derived functor of the functor \( \lim \) from \( \mathcal{I} \)-projective systems of right R-modules to right R-modules. **Theorem.** If the smallest cardinal number of a cofinal subset of I is \( \aleph_1 \), then \( \lim(k) = 0 \) for \( k > n + 1 \), whereas \( \lim(n+1) \neq 0 \). The first part uses a theorem of B. Osofsky - I. Bernstein on the homological dimension of a direct limit, suitably generalized from rings to preadditive categories. This part was obtained previously by R. Goblot using another method. The second part is established by first reducing to the case where I is totally ordered, and then using another theorem of Osofsky on the homological dimension of directed modules (generalized to directed functors). (Received April 14, 1971.)

*71T-A127. CARTER WAID, Texas Tech University, Lubbock, Texas 79409. Torsion free abelian groups of finite rank. Preliminary report.

Using nonstandard analysis, a characterization of all rank-n subgroups of rational n-space is obtained. **Theorem.** With every rank-n subgroup G of \( \mathbb{Q}^n \) there is associated a positive integer d and a nonsingular lower triangular matrix A with nonstandard integer entries. An n-tuple \( x \in \mathbb{Q}^n \) is a member of G iff every coordinate of \( y = d^{-1} \cdot xA \) is a nonstandard integer. **Corollary.** With every rank-n subgroup G of \( \mathbb{Q}^n \) there is associated a positive integer d and a function A(k) defined on the positive integers whose values satisfy:

1. \( A(k) \) is a lower triangular \( n \times n \) matrix with integer entries reduced modulo \( kd \); (2) If \( m = \gcd(k, d) \), then \( A(k) \equiv A(\ell) \mod md \). If \( x \in \mathbb{Z}^n \) and k is a positive integer, then \( k^{-1}x \in G \) iff \( xA(k) = 0 \mod kd \). Conversely, every such pair \( (d, A(k)) \) determines a rank-n subgroup of \( \mathbb{Q}^n \). A complete set of invariants is also deduced from the theorem. An n-tuple \( \overline{b} = (b_1, \ldots, b_n) \) is a base in G (rank G = n) if \( \{b_1, \ldots, b_n\} \) is an independent subset of G. If A is an \( n \times n \) matrix with integer entries and \( A\overline{x} = \overline{b} \) has a solution \( \overline{x} \) that is a base in G, then \( \overline{b} \) is divisible by A. Using this idea of divisibility a matrix height sequence is determined by \( \overline{b} \). There is a natural equivalence on height sequences analogous to that occurring in the rank-1 case. The bases in G determine an equivalence class called the type of G. Two rank-n groups are isomorphic iff they are of the same type. (Received April 9, 1971.)
**71T-A128.** H. M. SRIVASTAVA, University of Victoria, Victoria, British Columbia, Canada and JOEL L. BRENNER, University of Arizona, Tucson, Arizona 85721. **Bounds for Jacobi and related polynomials derivable by matrix methods.** Preliminary report.

For a given matrix with dominant diagonal, a number of convenient upper and lower bounds for the corresponding determinant are available in the literature. Making use of these inequalities the authors first derive bounds for a general system of orthogonal polynomials for a specified range of values of the argument. The results thus obtained are then applied to the cases of the classical Jacobi and related polynomials. This article is a supplement to a recent paper of J. L. Brenner [Proc. Amer. Math. Soc., to appear; see also Abstract 685-A9, these Notices 18(1971), 537]. (Received April 16, 1971.)


Let $A$ be an abstract algebra, $E = (E, *, id_A) = \text{End}(A)$, and $(L(E), \leq)$ the lattice of all left ideals (including the empty ideal) of $E$. For $a \in E$ define $a', a : E \to E$ by $a(b) = a* b$, $a'(b) = b* a$ for all $b \in E$. Recall that for $E = (E, (a')_{a \in E})$ $E = \text{End}(E) = \{[a | a \in E], \}, id_A$. For $\varphi : B \to C$ let $R_\varphi$ denote its kernel $\varphi^{-1} \circ \varphi$, and for $S \subseteq E$ set $R_S : = \cap [R_a | a \in S], R_S : = \cap [R_a | a \in S]$, and consider $K : = [R_S | S \subseteq E] \subseteq \text{Cong}(A)$ and $\tilde{K} : = [R_S | S \subseteq E] \subseteq \text{Cong}(E)$. **Lemma.** $r_{\tilde{E}} : = (a')_{a \in E}$ induces similar unary algebraic structures on $L(E)$, $K$ and $\tilde{K}$, such that $L_{\tilde{E}} = (L(E), \leq, \to), K = (K, r_{\tilde{E}}, \leq)$ and $\tilde{K} = (K, r_{\tilde{E}}, \leq)$ are partially ordered algebras, and the operations commute with arbitrary infima for the indicated partial orderings. Thus, in considering the correspondences $\alpha : = \{(H, R_H | H \in L(E))\}, \tilde{\alpha} : = \{(H, R_H | H \in L(E))\}$ and $\beta : = \{(R_S, R_S) | S \subseteq E\}$, one gets the **Theorem.** $\alpha : L(E) \to K, \tilde{\alpha} : L(E) \to \tilde{K}$ and $\beta : K \to \tilde{K}$ are surjective homomorphisms, which preserve arbitrary infima, and $\beta * \alpha = \tilde{\alpha}$. This extends and strengthens Theorem I of W. A. Lampe's Abstract 658-114, these Notices 15(1968), 752. (Received April 19, 1971.)

**71T-A130.** KENNETH PACHOLKE, University of Colorado, Boulder, Colorado 80302. **Integral domains of almost finite character.** Preliminary report.

An integral domain $A$ has finite character if $A$ is the intersection of a family of valuation rings $R_v$ such that each nonzero element of $A$ is a nonunit in at most finitely many of the $R_v$. Some domains that have finite character are Dedekind, Krull, and generalized Krull domains, domains of finite real character, and domains of Krull type. However, there are domains $A$ which do not themselves have finite character, but do have the property that for each maximal ideal $M$ in $A$, the ring of quotients $A_M$ has finite character. This property is taken as the definition of almost finite character. Almost Dedekind and almost Krull domains provide known examples of domains of almost finite character, and in this paper almost generalized Krull domains (AGK-domains), domains of almost finite real character (AFRC-domains), and domains of almost Krull type (AKT-domains) are introduced. Any one-dimensional Prüfer domain is an AGK-domain, any Prüfer domain is an AKT-domain, the ring of all algebraic integers is an AGK-domain which is neither generalized Krull nor almost Krull, and the ring of entire functions is an AKT-domain which is neither a domain of Krull type nor an AGK-domain. Finally, the basic theory of domains of almost finite character is developed. (Received April 19, 1971.) (Author introduced by Professor Irwin Fischer.)
Let \( n \) and \( k \) be positive integers, \( C(n) \) the multiplicative group of residue classes (mod \( n \)) which are relatively prime to \( n \), \( C_k(n) \) the subgroup of \( k \)th powers, \( v \) the \( \alpha + 1 \) smallest positive members of any given coset of \( C_k(n) \) in \( C(n) \), and \( \Sigma \{ (h_j - h_{j-1})^2 : 1 \leq j \leq \alpha \} \). Then \( E(n, \beta) = O_k, \beta, \epsilon (n^{1+\epsilon} + \nu(3\beta+1)/2 + \epsilon) \) for \( \beta \geq 1, \epsilon > 0 \), where the implied constant depends at most on \( k \), \( \beta \), and \( \epsilon \). If \( n \) is cubefree or \( k \) is odd and squarefree, the exponent \((3\beta+1)/8\) can be replaced by \((\beta+1)/4\) (see Abstract 70T-Al31, these Notices 17(1970), 279). These results can be improved a little when \( n \) is a prime \( p \). Of particular interest is the case \( n = p, k = 2 \) (here the \( h_j \) are the quadratic residues or the quadratic nonresidues mod \( p \)); in this case, we have \( 2^\beta - 1 \leq E(p, \beta) = O(p + p(\beta+1)/4 (\log p)^{\beta-1}) \) for \( \beta \geq 1 \). The proofs involve some fairly simple manipulations of Burgess' deep inequalities for character sums, as well as an adaptation of a method due to Hooley. (Received April 19, 1971.)

*71T-A132. E. FRIED and JIRI SICHLER, University of Manitoba, Winnipeg 19, Manitoba, Canada.

Endomorphisms of commutative rings with identity.

Let \( \mathcal{R}_1 \) be the category of all rings with identity element 1 and all 1-preserving ring homomorphisms. **Theorem.** Every category of algebras can be fully embedded into \( \mathcal{R}_1 \). **Corollary 1.** Every monoid is isomorphic to the monoid \( \delta(R) \) of all 1-preserving endomorphisms of a commutative ring \( R \) with 1.

**Corollary 2.** For every monoid \( M \) there is a proper class \( C \) of pairwise nonisomorphic commutative rings with 1 such that, for each \( R \) in \( C \), \( \delta(R) \cong M \). **Corollary 3.** Let \( M \) be a monoid of transformations on the set \( X \). Then there is a commutative ring \( R \) with 1, \( R \) contains \( X \) and every \( m \in M \) extends uniquely to a 1-preserving endomorphism of \( R \). This extension is an isomorphism between \( M \) and \( \delta(R) \). (Received April 22, 1971.)


In an earlier paper (unpublished) with S. Fajtlowicz, we defined a variety of algebras (resp. an algebra) to be E-precomplete if the variety (resp. the variety generated by the algebra) contains a unique equationally complete variety \( E \). Kalicki and Scott have characterized all equationally complete varieties of (associative) semigroups as defined by \( SL = \{ x^2 = x; xy = yx \} \) or \( LF = \{ xy = x \} \) or \( RG = \{ xy = y \} \) or \( p - GR = \{ xy = yx; x^p y = y (p-prime) \} \) or \( CS = \{ xy = zt \} \). We prove **Theorem 1.** If \( V \) is an E-precomplete variety of semigroups, where \( E = SL, LF \) or \( RG \), then \( V = E \). **Theorem 2.** A variety of semigroups is \( p - GR \)-precomplete if and only if it satisfies \( \{ x^m = y^m \} \) \( x^p y = y \) for some natural number \( m \). **Corollary.** A \( p - GR \)-precomplete semigroup is a group. **Theorem 3.** A variety of semigroups is CS-precomplete if and only if it satisfies, for some natural number \( m \), the identities \( x^m = x^{m+1} \) and \( x^m = y^m \). **Corollary.** A CS-precomplete semigroup is a semigroup with zero. (Received April 22, 1971.)
GEORGE IVANOV, Australian National University, Canberra, Australia. Nonlocal rings whose ideals are all quasi-injective. Preliminary report.

For an integer \( m > 1 \), a sfield \( D \), and a null \( D \)-algebra \( V \) whose left and right \( D \)-dimensions are both one, let \( H(m, D, V) \) be the ring of all \( m \times m \) matrices whose only nonzero entries are arbitrary elements of \( D \) along the diagonal, and arbitrary elements of \( V \) at the places \((2,1), \ldots, (n,n-1), \ldots, (m,m-1), \) and \((1,m)\).

We show that the only indecomposable nonlocal rings whose left ideals are all quasi-injective are the simple Artinian rings and the rings \( H(m, D, V) \). (Received April 22, 1971.)

RAYMOND BALBES, 8001 Natural Bridge Road, St. Louis, Missouri 63121 and University of Missouri, St. Louis, Missouri 63121. On the partially ordered set of prime ideals of a distributive lattice.

A poset \( P \) is representable over a class \( C \) of distributive lattices if \( P \cong \mathcal{P}(L) \) for some \( L \in C \), where \( \mathcal{P}(L) \) is the poset of all prime ideals of \( L \) together with \( \emptyset \) and \( L \). A complete characterization is given for posets representable over the class \( \mathcal{M} \) of distributive lattices which are generated by their meet irreducible elements. Corollary. If \( P \) is a poset with \( 0 < 1 \) and \( [p] \) is finite for \( p \neq 0 \) then \( P \) is representable over \( \mathcal{M} \).

The problem of representing a poset \( P \) over the class of all distributive lattices is equivalent to the embeddability of \( P \) as the meet irreducible elements of a distributive algebraic lattice \( L \) in which the nonzero compact elements form a sublattice \( L \). Results concerning the degree to which \( \mathcal{P}(L) \) determine \( L \) are included: If \( \mathcal{P}(L) \cong 2^X \) then \( L \) is a free distributive lattice. (Received April 26, 1971.)

GEORGE A. GRATZER, HARRY LAKSER, University of Manitoba, Winnipeg 19, Manitoba, Canada and BJARNI JÓNSSON, Vanderbilt University, Nashville, Tennessee 37203. Nondistributive equational classes of modular lattices fail to have the amalgamation property.

Based on the results and methods of B. Jónsson, Abstract 71T-A42, these \( \text{Notices} \) 18(1971), 400, B. Jónsson, Math. Scand. 2(1954), 295-314, and G. Grätzer and H. Lakser, Abstract 71T-A81, these \( \text{Notices} \) 18(1971), we prove the following result: No equational class of modular lattices containing at least one nondistributive lattice has the amalgamation property. Let \( \mathcal{M} \) be the class of all modular lattices. Then no finite chain with more than one element belongs to Amal \( \mathcal{M} \). For the definition of Amal \( \mathcal{M} \) see G. Grätzer and H. Lakser, Abstract 70T-A129, these \( \text{Notices} \) 17(1970), 801. (Received April 26, 1971.)

J. ELI ROSENFIELD, University of Minnesota, Minneapolis, Minnesota 55455. The independence of natural and elementary equivalence of categories.

Two categories \( A, B \) are naturally equivalent iff there is a full faithful functor \( F: A \to B \) such that, for each object \( X \) in \( B \), there is a \( Y \) in \( A \) with \( F(Y) \cong X \) (see, e.g., Pareigis, "Categories and functors," p. 55). Two systems of the same type are elementarily equivalent iff the same first-order sentences are true in each. Example of two naturally, but not elementarily, equivalent categories: If \( A \) is nonskeletal, \( A \) has a (naturally equivalent) skeleton \( B \), i.e., \( B \) satisfies \( (XY)(YX)(X \equiv Y \equiv X = Y) \), a sentence false in \( A \). Proposition. Skeletal categories are naturally equivalent iff they are isomorphic. Examples of two elementarily, but not naturally...
equivalent categories: Let $A$ be the additive group of the real rationals, $B$ the additive group of the complex rationals. Alternatively, let $A$ be the ordered set of the rationals, $B$ that of the reals. In both cases, natural nonequivalence of $A$ and $B$ follows from the Proposition; for elementary nonequivalence, see Bell and Slomson, "Models and ultraproducts," pp. 175-181. (Received April 26, 1971.)

*71T-A138. IVO G. ROSENBERG, University of Saskatchewan, Saskatoon, Saskatchewan, Canada. The number of maximal closed classes in the set of functions over a finite domain. Preliminary report.

Let $E_k = \{0,1,\ldots, k-1\}$ and let $P_k$ be the set of all functions whose variables, finite in number, range over $E_k$ and whose values are in $E_k$. $A \subseteq P_k$ is a closed (iterative) class if $A$ is closed with respect to (1) composition of functions and (2) identification and permutation of variables. The number of all maximal (proper) closed iterative classes is

$$C(k,2) G^*(k-2) + a(k) + b(k) - k - 6 + d(k) + p_k + e(k)$$

where

1. $C(n,r)$ is the number of $r$ combinations of $n$ distinct objects,
2. $G^*(k)$ is the number of partial order relations on $E_k$,
3. $a(k) = \prod_{p \mid k} \frac{k!}{m!(p-1)m}$ where we sum over all prime divisors $p$ of $k$ and $m = k/p$,
4. $b(k) = \frac{(k-1)!}{(pC(m,2)(p-1)\cdots(p-1-m)!)}$ if $k = pm$ ($p$ prime, $m \geq 1$) and $g(k) = 0$ otherwise,
5. $d(k) = \sum_{l=0}^{k} (-1)^l \binom{k}{l} C^l(n,2)$ where we sum over all $h \geq 3$ and $m \geq 1$ such that $h^m \leq k$.

(Received April 26, 1971.)


The following generalizes Villamayor-Zelinsky's 'Galois theory for rings with finitely many idempotents' (see [Villamayor-Zelinsky, Nagoya Math. J. 27(1966), 721-731]). Let $S$ be a connected prescheme and $A$ a finite etale quasi-coherent $S$-algebra. **Lemma.** $A = \bigoplus_{i=1}^m A_i$, where each $A_i$ is a finite etale $S$-algebra and $m$ is bounded by $rk_A S$. For the following theorems let $G = \text{Aut}_{S^e} A$ and assume furthermore that $A^G = \bigoplus_{i=1}^m A_i$. **Theorem 1.** $G$ is finite and can be realized as a semidirect product of $\bigoplus_{i=1}^m \text{Aut}_{S^e} A_i$ and the permutation group on $m$-letters. Furthermore any two $A_i$'s are $S$-isomorphic. **Definition.** We define a groupoid $\mathcal{G} = \bigoplus_{i=1}^m \mathcal{G}_i$ and $\text{Mor}_{\mathcal{G}}(A_i, A_j)$ are $S$-isomorphisms from $A_i$ to $A_j$. **Theorem 2.** There is a bijective lattice-inverting correspondence: $[\text{subgroupoids of } \mathcal{G} \text{ whose objects are always } A_i\big|_{i=1,\ldots,m}] \cong [\text{Quasi-coherent finite etale } S\text{-subalgebras of } A]$. The correspondence is defined as a natural generalization of the one given by Villamayor-Zelinsky. (Received April 26, 1971.)

71T-A140. DAVID P. SUMNER, University of Massachusetts, Amherst, Massachusetts 01002. On indecomposable graphs. Preliminary report.

A graph is called indecomposable if it cannot be written as a nontrivial $X$-join (see Sabidussi, "Graph derivatives," Math. Z. 76(1961), 385-401). In this work we study those graphs which are indecomposable but fail to remain so upon the removal of an arbitrary edge. We show that every such graph is bipartite and has at least two endpoints. We also show that every point determining graph (i.e., one in which no two distinct vertices have the same set of adjacencies) with no isolated points contains two points such that the removal of either results in a point determining graph. We prove that a connected graph is complete if and only if it is point

636
determining but fails to remain so upon the removal of any edge. As a consequence of these results we prove that every connected, point determining graph is spanned by an indecomposable subgraph. (Received April 26, 1971.)

71T-A141. RAPHAEL M. ROBINSON, University of California, Berkeley, California 94720. Triple systems with prescribed subsystems.

It is shown that if S and T are triple systems of orders s and t, with s \equiv t \equiv 1 (mod 6) or s \equiv 3 (mod 6), then there is a triple system of order 2s + t containing disjoint copies of S and T. From this, it follows that if n \equiv 1, 3 (mod 6) and n \equiv 4s + 3, then a triple system of order s can be embedded in some triple system of order n. It is conjectured that this result holds for n \equiv 2s + 1. The construction also furnishes a new proof of the existence of triple systems of all possible orders. (Received April 28, 1971.)

Analysis

*71T-B103. ANDRE de KORVIN and LAURENCE E. KUNES, Indiana State University, Terre Haute, Indiana 47809. Some nonweak integrals defined by linear functionals. IV.

For notations and basic definitions the reader is referred to the previous abstracts (Abstracts 71T-B26, 71T-B28, these Notices 18(1971), 255, 406). Definition. Let f be a function from S into E, let m be an additive set function from \( \Sigma \) into \( L(E,F) \), assume that \( m(S) < \infty \) and that \( m_{y*} \) is countably additive. f is called \( m \)-integrable if (1) f is \( m_{y*} \) integrable and (2) for every A there exists \( y \in F \) such that y*(y) = \( \int_A f \ dm_{y*} \), where the integral is taken in the sense of Bochner. Theorem. Let \( f_n \) be a sequence of \( m \)-integrable functions which converge to f in measure. Assume f is \( m_{y*} \) integrable and that there exists a function g which is \( m_{y*} \) integrable and \( |f_n - g| < B \), and \( |f - g| < B \). Then f is \( m \)-integrable and \( \int_A f \ dm \) converges to \( \int_A f \ dm \) uniformly as \( A \in \Sigma \). Theorem. Let \( f_n \) be a sequence of \( m \)-integrable functions. Assume that m is countably additive, of finite variation, that \( f_n \) converges to f, a.e. and that there exists a \( \bar{m} \) integrable function g such that \( |f_n - g| < g \). Then f is \( m \)-integrable and \( \int_A f \ dm = \lim \int_A f_n \ dm \). (Received December 21, 1970.)

*71T-B104. PETRU MOCANU, Babes-Bolyai University, Cluj, Romania, MAXWELL O. READE, University of Michigan, Ann Arbor, Michigan 48104 and ELIGIUSZ ZLOTKIEWICZ, M. Curie-Skłodowska University, Lublin, Poland. On criteria for the univalence of analytic functions. Preliminary report.

In this note, the authors show that analytic functions satisfying certain criteria introduced by Mocanu [Mathematica (Cluj) 11(1969), 127-133] and Ogawa [J. Nara Gakugei Univ. 10(1961), 7-12], for the univalence of some analytic functions, are indeed Bazilewitsch functions [Mat. Sb. 37(1955), 471-476]. This complements certain results due to Reade [Publ. Math. Debrecen 11(1964), 39-43]. (Received January 20, 1971.)

*71T-B105. ANDRE de KORVIN, Indiana State University, Terre Haute, Indiana 47809 and RICHARD A. ALO, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213. A converse to Nikodym theorem.

For notations see Abstract 71T-B26, these Notices 18(1971), 255. Definition. An E valued function f will be called \( m \)-integrable (m is \( L(E,F) \) valued) if for each \( A \in \Sigma \) there exists some \( y \in F \) such that \( \langle y*,y \rangle = \int_A f \ dm_{y*} \). In this case \( y = \int_A f \ dm \). Theorem. Let U be a function from T into L(E,F). Assume
is integrable with respect to \( \gamma \) (\( \gamma \) a scalar measure) and that \( \langle Ux, y^* \rangle \) is \( \gamma \)-measurable. Then there exists some measure \( m : L(E, F) \rightarrow \) such that if \( f \in L_1(\|U\|, \gamma) \) then \( f \) is \( m \)-integrable and

\[
\int_A \int \langle Ux, y^* \rangle \, dm(y) \, dy = \int_A f(y) \, dm(y).
\]

Conditions can be obtained under which \( m \) is \( L(E, F) \) valued. (Received January 29, 1971.)

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The problem of photon capture by an elementary particle has occasionally been considered in the literature. Yukawa’s formula for the pion field of a nucleon suggests approaching field problems by using exponential functions. In this paper, an attempt is made to set up an exponential formula for the gravitational field of a free spinless particle which, for distances large compared to its diameter, approaches Newton’s formula. In the neighborhood of the particle, the potential has a potential barrier which prevents any particle from penetrating another one. The new solution can be substituted into Schwarzschild’s formula for the static spherically symmetrical gravitational field. The Riemann–Einsteinian curvature approaches zero for large distances. (Received February 3, 1971.)

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**71T-B107. BERNARD H. AUPETIT, Université Laval, Québec, Québec, Canada.** Continuity of the spectrum in Banach algebras with involution. Preliminary report.

Let \( A \) be a complex Banach algebra with an involution \( * \), \( H \) the set of hermitian elements (i.e. \( h = h^* \)), \( N \) the set of normal elements (i.e. \( xx^* = x^*x \)), \( \text{Sp} x \) the spectrum of \( x \) and \( \rho(x) \) its spectral radius. Theorem. If \( \rho \) is submultiplicative on \( H \), i.e. there exists \( \sigma > 0 \) such that \( \rho(xy) \leq \sigma \rho(x) \rho(y) \) for every \( x, y \) in \( H \) (and subadditive, i.e. there exists \( \beta > 0 \) such that: \( \rho(x+y) \leq \beta \rho(x) + \beta \rho(y) \) for every \( x, y \) in \( H \), if \( A \) has no identity), \( x - \text{Sp} x \) is continuous on \( N \). Corollary 1. If \( \rho \) is submultiplicative on \( H \) (and subadditive if \( A \) has no identity), the set of hermitian elements with real spectrum is closed in \( H \). Corollary 2. If there exists \( c > 0 \) such that \( \rho(h) \leq c \|h\| \) for every \( h \) in \( H \), the set of hermitian elements with real spectrum is closed in \( A \). This result of B. Yood (Bull. Amer. Math. Soc. 76(1970), 80-82) is also an easy consequence of a result of J. D. Newburgh (Duke Math. J. 18(1951), 165-176). Corollary 3. If \( A \) is symmetric, \( x - \text{Sp} x \) is continuous on \( N \). (Received February 11, 1971.)

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**71T-B108. RICHARD M. ARON, University of Kentucky, Lexington, Kentucky 40506.** The bornological topology for the space of holomorphic mappings.

For notation and terminology, see "Topology on spaces of holomorphic mappings", by Leopold Nachbin, Springer-Verlag, 1968. Let \( \theta \) be a holomorphy type between two complex Banach spaces \( E \) and \( F \), and let \( U \) be an open subset of \( E \). Let \( \mathcal{K}_\theta(U;F) \), \( \tau_\omega \theta \) be the space of holomorphic mappings of \( \theta \)-type from \( U \) into \( F \), endowed with the ported topology, \( \tau_\omega \theta \). A characterization of the bornological topology \( \tau_{b\theta} \) on \( \mathcal{K}_\theta(U;F) \) corresponding to \( \tau_\omega \theta \) is given. Using this characterization, we show that \( \mathcal{K}_\theta(U;F), \tau_{b\theta} \) is barreled and, assuming \( E \) to be separable, \( \mathcal{K}_\theta(U;F), \tau_{b\theta} \) is quasi-complete and induces the \( \tau_{\omega \theta} \) topology on each \( \tau_{\omega \theta} \)-bounded subset of \( \mathcal{K}_\theta(U;F) \). (Received February 18, 1971.)

\(\sigma\)-conservative and \(\sigma\)-regular matrices.

Let \(\sigma\) be a mapping of the set of positive integers into itself. For each bounded real sequence \(x = \{x_n\}\), set \(T_x = \{x_{\sigma(n)}\}\). Define \(V_\sigma\) to be the set of all bounded real sequences \(x\) for which \(\lim_{n \to \infty} (x + T_x + \ldots + T^n_x)/\binom{n}{p}/(p+1) = L_e\) for some real number \(L_e\), where \(e = \{1, 1, \ldots\}\). \(L_e\) is the \(\sigma\)-limit of \(x\). An infinite matrix \(A = (a_{nk})\) is called \(\sigma\)-conservative if \(Ax \in V_\sigma\) for all convergent \(x\), where \(Ax = \{\sum_k a_{nk}x_k\}\). The matrix \(A\) is \(\sigma\)-regular if it is \(\sigma\)-conservative with \(\sigma\)-limit of \(Ax = \lim x\) for all convergent sequences \(x\). Theorem 1. The matrix \(A\) is \(\sigma\)-conservative if and only if (1) \(\|A\| < \infty\), (2) \(a_{(k)} \in V_\sigma\) with \(\sigma\)-limit 0 for each \(k\), and (3) \(a \in V_\sigma\) with \(\sigma\)-limit +1. The proofs are straightforward generalizations of those given by J. P. King in "Almost summable sequences", Proc. Amer. Math. Soc. 17(1966), 1219-1225. These theorems extend some of King's notions to transformations considered by R. A. Raimi in "Invariant means and invariant matrix methods of summability", Duke Math. J. 30(1963), 85-94. (Received February 18, 1971.)

*71T-B110. WITHDRAWN.

71T-B111. SAMUEL ZAIDMAN, Université de Montréal, Montréal, Québec, Canada. The Runge approximation property for abstract differential equations.

Let \(A\) be a linear closed operator with dense domain in the Hilbert space \(H\); let us assume that for a real \(\sigma_0\) and arbitrary \(\tau \in (-\infty, +\infty)\) the resolvent operator \(\left((\sigma_0 + i\tau)^2 - A^*\right)^{-1}\) is estimated by a constant \(M\); also we suppose that the operator \(d^2/dt^2 - A^*\) has the support property (\(A^*\) is the adjoint of \(A\)). Then, the operator \(d^2/dt^2 - A\) has the Runge approximation property. Here the support property means that: \(u \in L^2_{\loc}(-\infty, +\infty; H)^{\sup \supp u \text{compact in } \mathbb{R}}\) \(\supp (d^2/dt^2 - A^*)u \subseteq [a, b]\) implies \(\supp u \subseteq [a, b]\). The Runge approximation property is defined by: for any pair of positive numbers \(T_1 < T_2\) the set \(W_\infty\) is dense in \(V_{T_2}\) in the norm \(L^2(-T_1, T_1; H)\). Here, \(W_\infty = \{u \in L^2_{\loc}(-\infty, +\infty; H), (d^2/dt^2 - A)u = 0\}\) in some weak sense; furthermore for any \(T > 0\), \(V_T = \{u \in L^2(-T, T; H), (d^2/dt^2 - A)u = 0\}\) in some weak sense. The same weak sense is taken in the definition of the support property. (Received November 18, 1970.)

*71T-B112. ALAN L. LAMBERT, University of Kentucky, Lexington, Kentucky 40506. Unitary equivalence and reducibility of invertibly weighted shifts.

Let \(H\) be a complex Hilbert space and let \(\{A_1, A_2, \ldots\}\) be a uniformly bounded sequence of invertible operators on \(H\). The operator \(S\) on \(L^2(H) = H \oplus H \oplus \ldots\) given by \(S(x_0, x_1, \ldots) = (0, A_1 x_0, A_2 x_1, \ldots)\) is called an invertibly weighted shift on \(L^2(H)\). After establishing a matricial description of the commutant of \(S\) it is shown that \(S\) is unitarily equivalent to an invertibly weighted shift with positive weights. Necessary and sufficient conditions are given for the reducibility of \(S\) and the following result is proved: Let \(\{B_1, B_2, \ldots\}\) be any sequence of bounded operators on an infinite dimensional Hilbert space \(K\). Then there is an operator \(T\) on \(K\) such that the lattice of reducing subspaces of \(T\) is isomorphic to the lattice of reducing subspaces of the \(W^*\)-algebra generated by \(\{B_1, B_2, \ldots\}\). Necessary and sufficient conditions are also given for the complete reducibility of \(S\) to scalar weighted shifts. (Received February 25, 1971.)

639
convex maps of the unit circle. III.

This paper is a continuation, with the same notation, of two earlier papers with the same title (Abstracts 70T-B54, 70T-B77). Let \( f(z) = z + \cdots \in \mathcal{K} \), with sections \( S_n(z) \); \( \Delta = \Delta f(z,w) \), \( g(z) = z + \cdots \in \mathcal{K} \); \( n \geq 0 \), \( S_n(0) = 0 \). It was shown in (*) that \( B_1 F(z) = (w/f(w)) \Delta f(z,w) g(z) \in \mathcal{K} \), and \( B_2 J(z) = (w/f(w)) \Delta f(z,w) = z + \sum_{n=1}^{\infty} \frac{b_n}{n!} z^n \in \mathcal{K} \). Applying (A_1) to (B_2) and the known bounds \( |b_n| \leq 1 \) gives (C) \( |f(z) - S_n(z)| \leq |z^n| |f(z)| \); in particular, \( \text{Re}(f(z)/S_n(z)) > 1/2 \), and \( |S_n(z)/(1+r^n)| \leq |f(z)| \). Applying (C) to (A_2) yields (D) \( |f^{(n)}(z)/n!| - |f(z) - S_{n-1}(z)|/z^n \leq |f(z)/z|[(1-r)^{-n-1}] \). Applying (E) \( |g^{(n)}(z)/n!| \leq |g'(z)| \) to the function \( J(z) \) gives

(E) \( |J(z)| \leq |g^{(n)}(z)/n!| \). It is also shown that (F) \( h(z) = \psi(z) F(z) \) is close-to-convex univalent with respect to \( F(z) \), and this leads to the inequalities \( z^2 f(z^2) - z f(z) \leq |z_1^2 f(z_1^2) - z_1 f(z_1)|; |z_1 f(z_1) - z_2 f(z_2)| \leq |f(z_2) - f(z_1)|/|z_1 z_2 + z_1 + z_2|; \) Re\( [\eta - \tilde{z}^{-2}]/z + \psi(z) (|z \Delta'/\Delta| + z g'(z)/g(z)] \geq 0 \). (Received February 26, 1971.)

*71T-B114. CARL L. DeVITO, University of Arizona, Tucson, Arizona 85721. On Grothendieck's characterization of the completion of a locally convex space.

Let \( E \) be a locally convex space with topology \( t \). A net \( \{x_\alpha \in A \} \) of points of \( E \) is said to be ultimately \( t \)-bounded if for each \( t \)-neighborhood \( U \) of zero there is a scalar \( \lambda \) and \( \alpha_0 \) in \( A \) such that: \( x_\alpha \in \lambda U \) for all \( \alpha \geq \alpha_0 \). Let \( t, t' \) be two locally convex topologies on \( E \) with \( t \) weaker than \( t' \). We shall say that \( E \) is \( (t, t') \)-complete if every net in \( E \) which is \( t \)-Cauchy and ultimately \( t' \)-bounded is \( t \)-convergent to a point of \( E \). Such spaces arise naturally in certain connections and many examples are known; see [C. L. DeVito, "On Alaoglu's theorem, bornological spaces and the Mackey-Ulam theorem," Math. Ann., to appear]. We can define, in a natural way, the \( (t, t') \)-completion of a locally convex space \( E \). If both \( t \) and \( t' \) are compatible with the dual system \( E' \), \( E \) we can give a simple characterization of this completion. In case \( t = t' \) our characterization reduces to the theorem of Grothendieck. This method of proving Grothendieck's theorem makes no use of the Grothendieck approximation lemma. (Received March 1, 1971.)


Goffman, Nishiura, and Neugebauer showed that if \( E \) is a Borel set and \( X \) a closed set in Euclidean \( n \) space \( E_n \), and if the ordinary metric density of \( E \) at every point of \( X \) is one, then there is a closed set \( F \) so that \( E \) has metric density one at each point of \( F \) (Duke Math. J. 28(1964), 497-506). Troyer and Ziemer established this result for outer Caratheodory measures on \( E_n \) (J. Math. Mech. 12(1963), 485-494). Using the technique of Goffman, Neugebauer, and Nishiura we are able to prove this theorem with suitable modifications, for an outer Caratheodory measure defined on a metric space. (Received March 1, 1971.)
Hartogs proved that every function which is holomorphic on the boundary of the unit ball in \( \mathbb{C}^n \), \( n > 1 \), can be extended to a function holomorphic in the ball itself. It is conjectured that a real k-dimensional \( \mathbb{C}^\infty \) compact submanifold of \( \mathbb{C}^n \), \( k > n \), is extendible over a manifold of real dimension \( (k+1) \). This is known for hypersurfaces (i.e. \( k=2n-1 \)) and submanifolds of real codimension 2. It is the purpose of this paper to prove this conjecture and to show that we actually get a stronger C-R extendibility. (Received March 1, 1971.)

GUNTER LUMER, University of Washington, Seattle, Washington 98105. Hardy spaces in star-shaped domains of \( \mathbb{C}^n \). Preliminary report.

Let \( G \) be a bounded domain in \( \mathbb{C}^n \), \( B \) its Bergman-Shilov boundary (Shilov boundary of \( G \) relative to the algebra \( A_G \) of functions in \( C(G) \), holomorphic on \( G \) ). Let \( G \) be star-shaped (in the sense: \( z_0 \in G \) s.t. \( z \in G \) if \( 0 \leq r < 1 \)). For \( f: G \to \mathbb{C} \), set \( f_r(z) = f(z_0 + r(z - z_0)) \). Let \( \text{hol} \ G \) (ph \( G \)), be the set of all holomorphic (pluriharmonic) functions on \( G \). We define, for \( p > 0 \), \( H^p(G) = \{ f \in \text{hol} \ G : |f|^p \) has a majorant \( \in \text{ph} \ G \} \). Let \( \|f\|^p = \inf \{u(z_0) : u \in \text{ph} \ G, u \equiv |f|^p \) in \( G \} \). Theorem. Let \( f \in \text{hol} \ G \), \( p > 0 \). Then \( |f|^p \) has a pluriharmonic majorant in \( G \) if and only if \( \sup \{ EM_{\mathbb{R}} \int_B |f_r|^p d\mu \leq \text{constant, for } 0 \leq r < 1 \), where \( M_{\mathbb{R}} \) is the set of representing measures (on \( B \) ) for \( z_0 \) relative to \( A_G \). Denote the above sup by \( I_r(f) \); it is increasing in \( r \); if finite, \( \lim_{r \to 1} I_r(f) = \|f\|^p \). For \( t \geq 0 \), let \( \omega(t) = \phi(\log t), \phi(.) \) a strongly convex function on \( [-\infty, +\infty), 0 \) at \(-\infty, > 0 \) for \( t > -\infty \). Define \( H^\omega(G) \) like \( H^p(G) \) replacing \( |f|^p \) by \( \omega(t) \). The theorem above still holds with \( \omega(|f|) \) in lieu of \( |f|^p \). Next, define \( H(G) \) as the union of all the \( H^\omega(G) \), \( \omega \) as above. Theorem. Suppose \( f \in H(G) \) and has radial limits \( M_{\mathbb{R}} \)-almost-everywhere on \( B \). Then \( f \in H(G) \) if and only if \( +\infty > \sup \{ EM_{\mathbb{R}} \int_B \omega(|f|) d\mu \), in which case the latter expression equals \( \psi(\|f\|), \| \psi \) defined like \( \| \| \) \). In particular if \( |f| \leq \text{constant} = c M_{\mathbb{R}}, a.e. \) on \( B \), then \( |f| \leq c \) in \( G \). Further results will be announced later. (Received March 11, 1971.)

THOMAS L. KRIETE III, University of Virginia, Charlottesville, Virginia 22903. Generalized Cesàro operators.

Let \( \psi \) be a complex-valued function continuous and univalent on the closed disk \( |z| \leq 1 \) and analytic in its interior with \( \psi(1) = 0 \). \( C \) is the operator defined formally on the Hardy space \( H^2 \) by \( (Cf)(z) = \psi(z)^{-1} \int_1^z \psi(\xi)H(f)d\xi \). If \( \psi(z) = 1 - z \), the usual identification on \( H^2 \) with \( \ell^2 \) takes \( C \) onto the discrete Cesàro operator \( C_0 \). With certain additional hypotheses on \( \psi \) it is shown that \( C \) is a bounded operator whose spectrum is the disk \( |z - 1| \leq 1 \). Other properties of \( C \), analogous to those of \( C_0 \), are found. In particular a condition on \( \psi \) is found which is necessary and sufficient for \( C \) to be subnormal; this condition is satisfied if \( \psi(z) = (1 + z)(1 + az)^{-1}, -1 < a < 1 \). Finally, suppose that \( \mu \) is a finite positive measure on the disk \( |z| \leq 1 \) such that the space \( H^2(\mu) \) has bounded point evaluations at every point \( z, |z| < 1 \). An inequality is found relating \( \mu \) to the norms of the point evaluations. This is applied to show that \( 1 - C \) is not similar to a weighted shift if \( C \) is subnormal. (Received March 2, 1971.)
Entire functions of strongly bounded index.

An entire function \( f(z) \) is said to be of strongly bounded index (s. b. i.) if there exist quantities \( x (0 < x < 1), r_0 > 0 \) and an integer \( s \geq 0 \), such that \( m \geq s + 1 \) and \( |z| \geq r_0 \) imply \( |f^{(m)}(z)|/m! \leq x \max_{0 \leq j \leq s} |f^{(j)}(z)|/j! \). The introduction of the class of s. b. i. functions is justified by the following new results. (I) If \( f \) is of s. b. i. then it is of bounded index in the accepted sense. (II) If \( f \) is of s. b. i. and if \( p(z) \neq 0 \) is a polynomial then \( f/p \) is of s. b. i. Moreover \( f/p \) is of s. b. i. provided \( f \) is entire. (III) Let \( \{a_n\}_{n=1}^\infty \) be a positive strictly increasing sequence such that \( a_{n+1} - a_n \geq b_n \) \( (n \geq 1) \) where \( \{b_n\}_{n=1}^\infty \) is positive nondecreasing and \( \sum (1/nb_n) < \infty \), then \( f(z) = \prod_{n=1}^\infty (1 + z/a_n) \) is of s. b. i. (IV) Given \( \rho, 0 < \rho \leq 1 \), there exists an entire function of order \( \rho \) such that it is of s. b. i. and of lower order zero. (V) Given \( \rho, 0 < \rho < 1 \), there exists an entire function \( f \) of order \( \rho \) such that it is (i) of bounded index, (ii) of irregular growth and (iii) \( f' \) is of unbounded index. (Received March 5, 1971.)

Martingales of Pettis integrable functions. Preliminary report.

Let \( (\Omega, \Sigma, \mu) \) be a finite measure space, \( X \) be a B-space with dual \( X^* \), and \( P^1(X) \) be the seminormed space of all strongly measurable Pettis integrable functions \( f: \Omega \to X \). A martingale \( (f_T, \mathcal{B}_T, T \in T) \) (\( T \) a directed set) in \( P^1(X) \) is convergent to \( f \in P^1(X) \) if and only if (i) the family \( \{x^* f_T : T \in T, x^* \in X^* \} \) is bounded and uniformly integrable in \( L^1 \) and (ii) for each \( \delta > 0 \) there exists a weakly compact convex set \( K = K(\epsilon) \) \( \subset X \) such that for any \( \delta > 0 \) there is an index \( T_0 \in T \), and \( E_0 \subset \mathcal{B}_{T_0} \) with \( \mu(\Omega - E_0) < \epsilon \) such that \( T \geq T_0 \) implies \( \int_{E_T} d\mu \subset K + \delta U \) for \( E \in \mathcal{B}_T, E \subset E_0 \). The condition (ii) may be considerably relaxed if \( X \) is reflexive or is a separable dual. The Orlicz space case is treated with a similar condition. A convergent martingale in \( P^1(X) \) converges strongly in measure to its limit, and if the index set \( T \) is the positive integers with natural order, a convergent martingale in \( P^1(X) \) converges strongly a.e. \( \mu \) to its limit. (Received March 5, 1971.)

On differentiation of integrals and the maximal operator.

Let \( |P| \) denote the Lebesgue measure of \( P \) in \( \mathbb{R}^n \). Let \( \mathcal{R}(0) \) be a collection of bounded measurable sets containing the origin 0. Assume \( \mathcal{R}(0) \) contains sequences contracting to 0, and that it is invariant by homotheties of center 0. Denote \( \mathcal{R}(x) = \{x + R : R \in \mathcal{R}(0)\} \), for \( x \in \mathbb{R}^n \), and \( \mathcal{R} = \bigcup \mathcal{R}(x) \). \( \mathcal{R} \) is a differentiation basis in \( \mathbb{R}^n \). The maximal operator \( M \) associated to \( \mathcal{R} \) is defined by \( Mf(x) = \sup_{|R| < \rho} \int_R |f| : R \in \mathcal{R}(x) \) for \( f \in L^p(\mathbb{R}^n) \), \( 1 \leq p < \infty \). That \( M \) is of weak type \( (p, p) \) means: There exists \( c > 0 \) such that for every \( \lambda > 0 \) and \( f \in L^p(\mathbb{R}^n) \), \( \{x : Mf(x) > \lambda\} \leq c \chi^{p} \|f\|^p \). Guzmán–Welland have proved that \( \mathcal{R} \) differentiates \( f \) for every \( f \in L^1(\mathbb{R}^n) \) and \( D(f, x) = f(x) \) a.e. if and only if \( M \) is of weak type \( (1, 1) \) [cf. 'On differentiation of integrals', to appear in Rev. Un. Mat. Argentina, 1971]. With a different method one can get the following Theorem.
$\mathcal{R}$ differentiates $f$ for every $f \in \mathcal{L}^p(\mathbb{R}^n)$ and $D(\mathcal{R}f, x) = f(x)$ a.e. if and only if $M$ is of weak type $(p, p)$. (Received March 8, 1971.) (Author introduced by Professor Miguel de Guzmán.)


Definition. Let $V_k(p)$ denote the class of functions $f(z)$ analytic in $|z| < 1$ such that: (i) for $|z|$ sufficiently near 1, $\int_0^{2\pi} \Re\{1 + zf''(z)/f'(z)\} \, dz = 2\pi r$ and (ii) $\lim_{r \to 1^-} \frac{1}{\pi} \int_0^{2\pi} \Re\{1 +zf''(z)/f'(z)\} \, dz \leq p k r^k$

If $p = 1$, $V_k(p)$ reduces to the class $V_k$ first defined by V. Paatero [Ann. Acad. Sci. Fenn. A, 33, 9(1931), 77pp].

We obtain an integral representation for $V_k(p)$ and use it to obtain the following results: Theorem. Let $f(z) = a_{-1}z^{p-1} + \ldots \in V_k(p)$ have real coefficients.

Then $p+1 k k \leq p 2 k^2 p-1$ and there is a function in $V_k(p)$ for which equality holds. Theorem. Let $f(z) = \ldots \in V_k(p)$.

Then $\lim_{n \to \infty} \frac{1}{n^{p((k+2)/2) - 2}} = \alpha / \Gamma(p((k+2)/2))$, where $\alpha = \lim_{r \to 1^-} (1 - r)^{p((k+2)/2)} M(r, f')$. (Received March 15, 1971.)

71T-B123. JOHN B. BUTLER, JR., Middle East Technical University, Ankara, Turkey. On the inverse problem for selfadjoint operators defined on a rigged Hilbert space. Preliminary report.

Let $\mathcal{L} = \mathcal{H} \otimes \mathcal{L}^2(\nu)$ be a rigged Hilbert space and let $\mathcal{F} = \int \mathcal{H}_0(1) \mathcal{M}_0(1), p = 0, 1,$ be two representation spaces for $\mathcal{L}$ with $\mathcal{F}_0$ the corresponding isomorphisms. Suppose that $H^0 = A^n$ where $A$ is positive definite on $\mathcal{H}$, $p = 0, 1,$ are operators on $\mathcal{F}$ whose representation on $\mathcal{F}$ is multiplication by $1$, and assume $B(\varphi, \psi) = \int (T^0 \varphi, T^0 \psi) \, d\mu_0(1) - (T^0 \varphi, T^0 \psi) \, d\mu_0(1))$ on $\mathcal{F} \otimes \mathcal{F}$ is continuous in each argument where $T^0 = F_1^0 \mathcal{M}_0^0$. By the abstract kernel theorem there exists $O^0_0$ on $\mathcal{H}$, $O^0_0 \mathcal{F}$ nuclear, such that $B(\varphi, \psi) = \langle O^0_0 \varphi, \psi \rangle$. In case $A$ is cyclic, $L_2(\nu) = \mathcal{F}$ the space on which $A$ acts as $(\text{id}/\text{d}x)$, $\mathcal{F}_0 \otimes L_2(\nu)$ functions with support in $[-a, a]$, define $\eta_0 P, \eta_0 P$ in $L(\mathcal{F}, \mathcal{F}')$ to be limits $-\infty$ of operators whose images in $L_2(\nu)$ restricted to $\mathcal{F}_0$ are integral operators with cut off kernels $\delta(x, y, a)\varphi(x, y, a)$ obtained by multiplying the kernel of $P \in L(\mathcal{F}, \mathcal{F}')$ with the kernels of the spectral measure $E([-a, a])$ of $A$ and of $A^{-1}E([-a, a])$ respectively.

Extend these definitions to general $A$. Theorem. If (i) $A_1^0 \mathcal{M}_0^0$ are nuclear, $\langle \eta_0 A_1^0 \mathcal{M}_0^0 \varphi, \psi \rangle = \langle A_1^0 \mathcal{M}_0^0 \varphi, \psi \rangle$, $i \neq n - 1, i = 0, 1, \ldots, 2n - 1$, then (ii) $K + O^0 + K \star O^0 = 0$ has a solution $K$ on $\mathcal{H}, K \mathcal{M}_0 \mathcal{F}$ nuclear, (iii) $\langle \eta_0 \varphi, \psi \rangle = \langle 1 \mathcal{M}_0 \mathcal{F}_0 \varphi, \psi \rangle$ defines $V$ such that $H^1 = H^0 + V$. (Received March 16, 1971.)

71T-B124. GEORGE M. MULLER, Stanford Research Institute, Menlo Park, California 94025. Linear iteration and summability. II.

A class of summability methods, together with a corresponding class of recursive schemes for solving the linear operator equation $y = Ly + f$ (see Abstract 68T-313, these Notices) 15(1968), 392), may be derived from the set of complex rational functions having 1 and $\infty$ as fixed points. Any such function can be written as $R(\zeta) = F(\zeta)/G(\zeta)$ where $F(\zeta) = \zeta^{k+m} + \sum_{m=1}^k \alpha_m \zeta^{k-m}$ and $G(\zeta) = \sum_{m=1}^k \beta_m \zeta^{-k}$, and the $\alpha$s and $\beta$s satisfy $\sum_{m=1}^k \alpha_m = -1$ and $\sum_{m=1}^k \beta_m = 1 + \sum_{m=1}^k \alpha_m$. Suppose $R(\zeta)$ to be such that $S$, the complement of the image of $|\zeta| \leq 1$ under $z = R(\zeta)$, is nonempty. Let $P_n(z), 0 \leq n \leq k - 1$, be arbitrary polynomials ($\neq 0$), and let $P_n(z) = \sum_{m=1}^k (-\alpha_m + \beta_m z)P_{n-m}(z)$, $n \geq k$. Then the transformation matrix $A = [a_{n,m}]$ defined by $P_n(z) = \sum_{m=0}^{N(n)} a_{n,m} z^m$, $643$
with \( a_{n,m} = 0 \) if \( m > N(n) \), sums the sequence of partial sums of the geometric series to \((1 - z)^{-1}\) for \( z \in \mathbb{S} \), uniformly on compact subsets of \( \mathbb{S} \). (If \( k = 1 \) and \( P_0(z) = 1 \), \( \Lambda \) is the Euler matrix of order \( \beta_1 \)). In any complex Banach space \( B \) with norm \( \| \cdot \| \), let \( L \) be a bounded linear operator whose spectrum lies in \( \mathbb{S} \), \( I \) the identity operator, \( f, v_0, v_1, \ldots, v_{k-1} \) arbitrary vectors. If \( v_n = -\sum_{m=0}^{N(n)} \alpha_m v_{n-m} + L(\sum_{m=0}^{N(n)} \beta_m v_{n-m}) + (\sum_{m=0}^{N(n)} \theta_m f) \), \( n \geq k \), then \( \| v_n - (I - L)^{-1} f \| \to 0 \). (Received March 22, 1971.)

*71T-B125. STYLIANUS K. PICHORIDES, University of Chicago, Chicago, Illinois 60637. On the best values of the constants in the theorems of M. Riesz, Zygmund and Kolmogorov.

Let \( f \) be a real \( 2\pi \)-periodic function and \( \tilde{f} \) its conjugate. Then: (i) the least value of the constant \( A_p \) in M. Riesz's theorem \( \| f \|_p \leq A_p \| f \|_p \), \( p > 1 \), \( f \in L^p \) is \( \tan(\pi/2p) \) if \( 1 < p \leq 2 \) (and hence \( \cot(\pi/2p) \) if \( p > 2 \)); (ii) the only possible values of the constant \( A \) in Zygmund's theorem \( \| f \|_1 \leq A(1/2\pi) (\int_{-\pi}^{\pi} |f| \log^+ |f|) + B \), \( f \in L^{\log^+} \) are those \( > 2/\pi \); (iii) for nonnegative functions, the least value of the constant \( B_p \) in Kolmogorov's theorem \( \| f \|_p \leq B_p \| f \|_1 \), \( p < 1 \) \( \in L^1 \) is \( (\cos(p\pi/2))^{-1/p} \); (iv) the constant \( \tan(\pi/2p) \) is also best possible for real nonperiodic functions on \( \mathbb{R}^1 \) (instead of the conjugate function it is considered now the Hilbert transform) and, in a more restrictive sense, for singular integral operators with odd kernel on \( \mathbb{R}^n \) (normalized so that the integral of the kernel over the unit sphere is \( 2/\pi \)). The proof of these results makes use of a refinement of the inequality on which pr. A. Calderón's proof of the theorem of M. Riesz is based (cf. A. Zygmund, "Trigonometric series", Cambridge Univ. Press, London-New York, 1968, Chapter VII, §2). (Received March 17, 1971.)

71T-B126. WILLIAM D. L. APPLING, North Texas State University, Denton, Texas 76203. Integrability and a class of continuous functions.

For each field \( F \) of subsets of a set \( U \), let \( R_{FB} \) \( R_{FAB} \) and the notion of integral be, for \( F \), what \( R_B \) \( R_{AB} \) and the notion of integral have been in previous abstracts, and let \( S(R_{FAB}) \) denote \( \{ g : g \in R_{FB} \}^{\mathbb{U}} \) if \( g \) is in \( R_{FAB} \). Theorem. If \( h \) is a continuous function from \( \mathbb{R}^1 \) into \( \mathbb{R} \), then the following two statements are equivalent: (1) If \( U \) is a field of subsets of a set \( U \) and \( g \) is in \( S(R_{FAB}) \), then \( \int_U h(g) \) exists and \( \int_U |h(g)| - h \int_U |g| \) = 0; (2) The following three conditions hold: (i) If \( 0 < c \), then there is a \( d > 0 \) such that if \( \{ x_k \}_{k=1}^n \) is a number sequence such that \( \sum_{k=1}^n |x_k| < d \), then \( \sum_{k=1}^n |h(x_k)| < c \); (ii) if \( 0 < c \) and \( 0 < M \), then there is a \( d > 0 \) such that if \( \{ (x_v, j) \}_{v=1}^m \) is a sequence of sequences of nonnegative numbers or nonpositive numbers such that \( \sum_{v=1}^m |x_v| < d \), \( v = 1, \ldots, m \), and \( \sum_{v=1}^m |x_v| \leq M \), then \( \sum_{v=1}^m |h(x_v, j)| < c \); and (iii) if \( 0 < c \) and \( 0 < M \), then there is a \( d > 0 \) such that if \( \{ (x_k, y_k) \}_{k=1}^n \) is either a sequence of nonnegative number pairs or nonpositive ones such that \( \sum_{k=1}^n |x_k| < M \), \( \sum_{k=1}^n |y_k| < d \) and \( |x_k| < d \) for \( k = 1, \ldots, n \), then \( \sum_{k=1}^n |h(x_k) - h(y_k)| < c \). (Received March 18, 1971.)

71T-B127. STANLEY J. POREDA and GEORGE S. SHAPIRO, Clark University, Worcester, Massachusetts 01610. Minimal norms for functions whose initial segments have norm one.

For \( \lambda = 0, 1, 2, \ldots \), let \( U_{\Lambda_\lambda} \) denote those functions \( R_\lambda(z) = \sum_{n=0}^\infty a_n z^n \) analytic in the open unit disc and for which \( \max_{|z| < 1} |\sum_{n=0}^\infty a_n z^n| = 1 \). It is not difficult to show that \( \bar{z} \) a largest \( M_\lambda \), \( 0 < M_\lambda \leq 1 \) such that 

644
max |z| = 1 |R(\lambda)| \leq M\lambda for all \lambda \in U. \quad \text{Theorem.} \quad \lim_{\lambda \to \infty} M\lambda = 0. \quad \text{This theorem follows as a corollary of the following: Example. If } F\lambda(z) = \lambda^\lambda \sum_{n=0}^{\infty} a_n z^n, \text{ where } a_n = (-1)^n [n \log n]^{-1}, \quad \text{then } \lim_{\lambda \to \infty} \max |z| = 1 |F\lambda(z)| < \infty \text{ and } \sum_{n=1}^{\infty} a_n \text{ diverges. (Received March 19, 1971.)}

*71T-B128. H. M. SRIVASTAVA and J. P. SINGHAL, University of Victoria, Victoria, British Columbia, Canada. Some generalizations of Mehler's formula.

In an earlier paper [H. M. Srivastava and J. P. Singhal, "Some extensions of the Mehler formula", to appear; see also Abstract 71T-B47, these Notices 18(1971), 412] the authors made use of certain operational techniques to prove an elegant unification of several interesting extensions of the well-known Mehler formula for Hermite polynomials given recently by L. Carlitz [Collect. Math. 21(1970), 117-130; see also Boll. Un. Mat. Ital. 3(1970), 43-46]. This paper is a sequel to the earlier work of the present authors. It shows, among other things, how their aforementioned results can be applied to derive a number of generalizations of Carlitz's formulas. Several interesting special cases are also discussed briefly. (Received March 22, 1971.)


In the present paper the author evaluates a contour integral involving the product of the H-function of C. Fox [Trans. Amer. Math. Soc. 98(1961), 395-429] and a generalized polynomial with arbitrary coefficients. By assigning suitable values to these coefficients, the main results of this paper can be reduced to contour integrals involving, for instance, the classical polynomials of Jacobi, Laguerre, and Hermite. Also since a large variety of functions that occur frequently in problems of analysis, both pure and applied, and mathematical physics are only particular cases of the H-function, involved in the main formulas, the contour integrals evaluated in this paper may prove to be of general interest. (Received March 22, 1971.)


Let A be a bounded operator on a separable Hilbert space. We say that A is a d.c. operator if the weakly closed algebra generated by A (and the identity) is equal to its double commutant. The following positive results about d.c. operators have been obtained: All algebraic operators (operators which satisfy a polynomial equation) are d.c. A normal operator is d.c. iff every invariant subspace is reducing. An isometry is d.c. provided its pure part does not vanish. (If its pure part does vanish, then it is unitary and the previous statement about normal operators applies.) All one-sided weighted shifts are d.c. A two-sided weighted shift is d.c. iff it is not invertible. The following negative results have also been obtained: Quasi-similarity does not preserve the property of being d.c.; i.e. if A is d.c. and B is quasi-similar to A, B need not be d.c. The direct sum of d.c. operators need not be d.c. (Received March 22, 1971.)

645
In this self-contained (modulo Hardy’s inequality) expository paper I establish in complete detail (by Lorentz space techniques) the fact (due to R. A. Hunt) that the $M$ operator is of type $(p, p)$ for $1 < p < \infty$ from the fact (due to Carleson and Hunt) that the $M^*$ operator is of restricted weak type $(p, p)$ for $1 < p < \infty$. No previous knowledge of Lorentz space theory or interpolation theory is assumed. Only a special case of Hunt’s Lorentz space interpolation theorem is established, and his original proof concerning the boundedness of the $M$ operator is modified in such a way as to eliminate the need for the nonelementary theorem of M. Riesz (that the Hilbert transform is of type $(p, p)$ for $1 < p < \infty$). Also a modification (communicated to me by Hunt) in the definition of the $M^*$ operator is employed together with other related modifications. (Received March 22, 1971.)


In this note we announce a result which is a generalization of a theorem by Kolmogorov and Arnol’d on the perturbation of invariant tori for Hamiltonian systems. Consider the real analytic Hamiltonian \( H(x, y, z) = h(y) + (z, \Omega z) \) where with conjugate variables \((x, y), (z, \omega)\) where \( x \in \mathbb{T}^n, y \in \mathbb{R}^n, z, \omega \in \mathbb{R}^m \) and \( \Omega \) is a constant matrix such that \( \text{Re} \gamma, \Omega \gamma > 0 \) for all complex vectors \( \gamma \neq 0 \). Theorem. Let \( \gamma \) be chosen such that \( h^{-1}(\gamma) = \omega \) where the components of \( \omega \) are rationally independent and satisfy the irrationality condition \( |\omega| \leq K^{(n+1)} \) for some \( K \) and for all integer valued vectors \( j \). Suppose \( h^{-1}(\gamma) \) exists and is real analytic. Then, the invariant torus \( y = \gamma, z = 0 \) may be continued analytically under any sufficiently small perturbation \( H(x, y, z) \) of the Hamiltonian \( H(x, y, z) \). The continued torus is represented by a real analytic embedding \( \xi : \gamma \rightarrow (x, y, z) \in \mathbb{T}^n \times \mathbb{R}^{n+2m} \) and \( \xi = \omega \). Moreover, the \( n + m \) dimensional invariant manifolds \( M_+: y = \gamma, z = 0 \) and \( M_-: y = \gamma, z = 0 \) may also be continued analytically. The existence of the continued torus is proved by using a Newton iteration procedure. No restriction on the multiplicity of the eigenvalues of \( \Omega \) is necessary. The results concerning the analyticity of the continuations of \( M_+ \) and \( M_- \) are an extension of work done by Kelly. In addition, the Hamiltonian character of the problem implies \( M_+ \) and \( M_- \) are Lagrangian manifolds. (Received March 25, 1971.)

**71T-B133.** STEVEN B. BANK, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801. A general theorem concerning the growth of solutions of first-order algebraic differential equations. This paper treats arbitrary equations, (*) \( \sum f_{kj}(z)y^k(y')^j = 0 \), where the \( f_{kj}(z) \) are either (1) entire functions, or (2) analytic functions in the unit disk. A growth estimate is obtained for meromorphic solutions (in the plane in case (1) and in the disk in case (2)), in terms of the growth of the coefficients \( f_{kj} \) and the distribution of poles of the solution. This estimate also shows how the growth of a solution is affected separately by the growth of the coefficients of terms of maximum total degree in \( y \) and \( y' \), and the growth of the other coefficients in the equation. As a special case, it is shown that a previous result of the author [Compositio Math. 22(1970), 369] can be improved as follows: Theorem. If in (*), the coefficients \( f_{kj}(z) \) of the terms of maximum total degree are polynomials, while the other coefficients are analytic functions of finite order in the
unit disk, then a meromorphic solution in the disk cannot be written as the quotient of two analytic functions \( h/g \), where \( h \) is of infinite order and \( g \) is of finite order in the disk. (Received March 25, 1971.)

*71T-B134. PREM K. KULSHRESTHA, Louisiana State University, New Orleans, Louisiana 70122. Structure of normalized typically real functions of order \( p \). Preliminary report.

Structural formulas are given in the form of integral representations for the classes \( T(p, z_0) \) and \( T'(p, z_0) \) of normalized typically real functions \( f(z) \) of finite integral order \( p \geq 1 \), which are regular in the unit circle \( K \), have real coefficients in their Maclaurin series expansions, and \( \text{Im} f(z) \) changes sign \( 2p \) times on \( |z| = r \) for each \( r \in (1-\delta,1) \), \( 0 < \delta < 1 \), and are such that for \( p = 1 \) they form the classes \( T(1, z_0) \) and \( T'(1, z_0) \) of typically real functions which satisfy the Montel normalization conditions \( f(0) = 0, f(z_0) = 1 \) and \( f'(0) = 0, f'(z_0) = 1 \) respectively in \( K \), where \( z_0 \neq 0 \) is a fixed point of \( K \). (Received March 25, 1971.)

*71T-B135. V. KANNAN and P. V. RAMAKRISHNAN, Madurai University, Madurai-2, India. Invariant means on Banach lattices.

Elegant characterisations are obtained for the amenability of a Banach lattice with respect to a collection of linear transformations; and Dixmier's criterion for left amenability of a semigroup is proved in a more general form, to hold in any monogenic Banach lattice. (Received March 29, 1971.) (Authors introduced by Professor M. Rajagopalan.)

*71T-B136. DANIEL RALPH LEWIS, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. Spaces on which each absolutely summing map is nuclear. Preliminary report.

For \( E \) and \( F \) Banach spaces, \( AS(E, F) \) is the space of absolutely summing operators from \( E \) into \( F \) with the absolutely summing norm, and \( N(E, F) \) is the space of nuclear operators with nuclear norm.

**Theorem.** Let \( E \) be a Banach space. The dual of \( E \) is isomorphic (respectively, isometric) to some space \( l^1(l) \) if and only if, for each Banach space \( F \), the injection of \( N(E, F) \) into \( AS(E, F) \) is an onto isomorphism (resp., isometry). (Received March 29, 1971.)


Consider the Cauchy problem for the equation \( u_{tt} - \Delta u = F(u) \) (\( x \in \mathbb{R}^n, t > 0 \)). For a certain compact set \( S \subset \mathbb{R}^n \), it is shown that there exists a finite \( T > 0 \) such that \( \sup_{x \in S} |u(x, t)| \) becomes infinite at \( t = T \). The assumptions are: (i) the data are subharmonic and have a positive lower bound \( \alpha \) on \( S \); (ii) \( F(s) \) is bounded below by a nonnegative, convex function \( f(s) \) which, for \( s \geq \alpha \), satisfies \( f(s) \geq ce^{s^\epsilon} \) \( (\epsilon > 0) \), \( f'(s) \leq K \), where \( c, K \) are positive constants depending on the data. This generalizes a theorem of J. Keller (see Comm. Pure Appl. Math. 10(1957), 523-530). Under certain circumstances, assumption (i) can be weakened considerably, and \( \Delta \) can be replaced by a uniformly elliptic second order variable coefficient operator in divergence form. Analogous theorems hold under similar hypotheses for mixed problems, where the spatial domain may be either a bounded open subset of \( \mathbb{R}^n \), or an exterior domain in \( \mathbb{R}^n \). (Received April 1, 1971.)
A complex valued function \( f \) continuous and not identically zero on \( \Gamma \) a closed Jordan curve is said to be approximable on \( \Gamma \) if for some \( n \in \mathbb{Z}^+ \) the polynomial \( p_n(f, \Gamma) \) of degree \( n \) of best uniform approximation to \( f \) on \( \Gamma \) is not identically zero. **Theorem.** Let \( f \) and \( \Gamma \) be as above, then \( f \) is approximable on \( \Gamma \) if \( \int_{\Gamma} d\arg f(z) \neq 0 \). The proof of this theorem makes use of the following: **Theorem.** If \( f \) is continuous, has constant modulus and is not identically zero on \( \Gamma \) a closed Jordan curve and if \( \int_{\Gamma} d\arg f(z) \neq 0 \) then for any \( \epsilon > 0 \) there exists a polynomial \( p(z) \) such that \( |\arg f(z) - \arg p(z)| < \epsilon \) for all \( z \in \Gamma \). (Received April 2, 1971.)

*71T-B139. R. P. SINGAL, Thapar College of Engineering, Patiala, India. A Watson type sum for the double hypergeometric series.*

For terminating double hypergeometric series \( F_{1,2}^{1,2}(1, 1) \) we have recently obtained Dixon and Watson type sums [Abstract 70T-B228, these *Notices* 17(1970), 959]. The object of this paper is to obtain a Watson type sum for the nonterminating double hypergeometric series \( F_{1,1}^{0,3}(1, 1) \) [P. Appell et J. Kampe de Feriet, "Fonctions hypergeometriques et hyperspheriques", Gauthier-Villars, Paris, 1926, p. 158] which has been expressed as difference of the product of gamma functions. From which follows immediately a Watson type sum for the terminating double series \( F_{1,1}^{0,3}(1, 1) \). (Received April 12, 1971.) (Author introduced by Dr. Brij M. Nayar.)


Uniqueness and norm convexity for smooth solutions to the Cauchy problem for classes of partial differential operators in Hilbert space are studied. Calderon proved that for equations of arbitrary order, if the real characteristic roots were simple, the complex roots at most double, and distinct roots remained distinct, one had uniqueness. It is shown here that for "solvable" equations of the form \( u_t(x, t) + (B(t, D) + iA(t, D))u(x, t) = 0 \), where \( D_x = -i\partial/\partial x, x \in \mathbb{R}^n \), and \( B \) and \( A \) are Hermitian pseudodifferential operators in the \( x \)-variables, such that \( B \) is semidefinite, one has a generalized convexity relation for \( \|u\|^2_{L^2(x)} \) and hence uniqueness. In the case that \( B \) and \( A \) are of order one this is an extension of Calderon's result to allow characteristic roots to wander from the real to the complex domains for such operators. Also, extensions of results of Agmon and Nirenberg are obtained to show uniqueness, norm convexity, and maximum allowable rate of norm decay (in \( t \)) for general first order elliptic and hyperbolic pseudodifferential equations, where \( B \) and \( A \) may depend on all three of \( x \), \( t \), and \( D_x \), but are restricted in other ways. Finally, the results are applied to certain higher order equations and systems. (Received April 7, 1971.)
Let $U^N$ denote the open unit polydisc in the space $C^N$ of $N$ complex variables. Let $T^N$ be the distinguished boundary of $U^N$ and $V^N = \{(z_1, \ldots, z_N) : |z_1| > 1, \ldots, |z_N| > 1\}$. A polynomial $P$ in $C^N$ is called a $T^{N*}$-polynomial if the only zeros of $P$ in $\overline{U}^N \cup \overline{V}^N$ lie on $T^N$. Every $T^{N*}$-polynomial is a $T^N$-polynomial, and the results of approximation by $T^N$-polynomials (in Abstract 684-B4, these $\textbf{Abstracts}$ 18(1971), 512) still hold if we replace $T^N$-polynomials by $T^{N*}$-polynomials. (Received April 8, 1971.)

**71T-B142.** STEFAN BERGMAN, Stanford University, Stanford, California 94305 and KYONG T. HAHN, Pennsylvania State University, University Park, Pennsylvania 16802. Some properties of pseudoconformal images of Reinhardt circular domains.

In S. Bergman, "The kernel function and conformal mapping," 2nd ed., Math. Surveys No. 5(1970), p. 183, and J. Analyse Math. 13(1964), 338, and 22(1969), 269, the invariants $J^{(\nu)}$, $\nu = 1, 2$, with respect to PCT's (pseudoconformal transformations) have been introduced and the problem when $J^{(1)}$ is constant has been considered. By relations $1/J^{(\nu+1)} = K^{-1} \det(3^{1/2}/3^{1/2})$, $m, n = 1, 2, \ldots$, infinitely many invariants are defined. In the case of two complex variables for a Reinhardt circular domain $R$, $1/J^{(\nu)}$ admit the developments $1/J^{(\nu)} = \sum_{n=0}^{\infty} (\sum_{m=0}^{\infty} E^{(\nu)}_{n-p, m} |z_1|^{2n-2p})$ at the center $O$ of $R$. The authors determine the values of $E^{(\nu)}_{n-p, m}$ in terms of $B_{mn}$, where $B_{mn} = \int_{R} |z_1|^{2m} |z_2|^{2n} d\omega$, $d\omega$ = volume element. For instance, $E^{(1)}_{10} = B_{00}^{-1} (B_{11} - 4B_{01}B_{00}) - B_{00}^{-1} B_{10} - B_{01}$. In this way one obtains the necessary and sufficient condition in order that $1/J^{(\nu)}_{R}$ is constant throughout $R$ in terms of infinitely many equations (polynomials in $B_{mn}$).

Let $t$ be a PCT of $R$ and let $B = t(R)$. In the case that $1/J^{(\nu)}_{R}$ is not constant, the authors determine the image $t_1 = t(O)$ of the center $O$ of $R$ in $B$. Using the theory of representative domains $R(B, t_1)$, the authors determine the pair $(v_{10}, v_{01})$ of functions which map $B$ onto $R$ so that $t_1$ goes into the center $O$ of $R$. (Received April 9, 1971.)

**71T-B143.** K. KUMARAN KUTTY, Queen's University, Kingston, Ontario, Canada. Dedekind $\sigma$ completion of a vector lattice.

If $E$ is an Archimedean vector lattice there exists a Dedekind $\sigma$ complete vector lattice $E'$ such that:

1. $E$ can be isomorphically embedded in $E'$ as a subvector lattice. (2) $E'$ is unique in the sense that if $L$ is a Dedekind $\sigma$ complete vector lattice in which $E$ can be isomorphically embedded as a subvector lattice, then $E'$ can be embedded in $L$ as a subvector lattice. (3) Every element of $E'$ can be obtained as sup$_i$ inf$_j a_{ij}$ and inf$_i$ sup$_j b_{ij}$ where $\{a_{ij}\}$ and $\{b_{ij}\}$ are double sequences in $E$ and $a_{ij} \leq b_{ij}$ for fixed $i$, $a_{ij} \leq b_{ij}$ for fixed $j$, $b_{ij} \leq b_{ij}$ for fixed $i$, $b_{ij} \leq b_{ij}$ for fixed $j$. (Received March 15, 1971.) (Author introduced by Professor Norman M. Rice.)

**71T-B144.** R. M. GOEL and VIKRAMADITYA SINGH, Punjab University, Patiala, Punjab, India. On radii of univalence of certain analytic functions. Preliminary report.

Let $S$ denote the class of regular, univalent functions $f(z)$ in $|z| < 1$ normalized by $f(0) = 0$, $f'(0) = 1$ and for $0 \leq \beta < 1$, $\alpha > 1/2$. Let $S(\alpha, \beta)$, $K(\alpha, \beta)$, $C(\alpha)$ and $D(\alpha)$ denote, respectively, the subclasses of $S$ which satisfy
in $|z| < 1$ the conditions (i) $|(zf'(z)/f(z) - \beta)/(1 - \beta) - \alpha| < \alpha$, $\text{Re}[zf'(z)/f(z)] > \beta$, (ii) $|(1 + zF''(z)/F'(z) - \beta)/(1 - \beta) - \alpha| < \alpha$, $\text{Re}[1 + zF''(z)/F'(z)] > \beta$, (iii) $|zF'(z)/G(z) - \alpha| < \alpha$, $\text{Re}[zG'(z)/G(z)] > 0$, and (iv) $|F'(z) - \alpha| < \alpha$. Further, for every $F(z) \in S$ and $C > -1$ let $f(z) = (1 + C)^{-1}[CF(z) + zF'(z)]$. Bernardi [Trans. Amer. Math. Soc. 135(1969), 429-446] has shown that for $C \geq 1$ an integer and $F(z) \in S(\infty, 0)$, $K(\infty, 0)$ or $C(\infty)$, $f(z)$ is, respectively, starlike with respect to the origin, convex, close-to-convex for $|z| < r_0 = (-2 + \sqrt{3 + C^2})/(C - 1)$. In this paper for any real number $C$ such that $c + \beta > 0$ we obtain precise upper and lower bounds for $\text{Re}[zf'(z)/f(z)]$, $F(z) \in S(\alpha, \beta)$. From the lower bound the corresponding generalizations of the results of Bernardi when $F(z) \in S(\alpha, \beta)$, $K(\alpha, \beta)$, $C(\alpha)$ or $D(\alpha)$ follow and the results of Bernardi are obtained by taking $\alpha = \infty$ and $\beta = 0$. (Received April 13, 1971.)


We study the cycle subspaces of a totally finite $L^p$ space, $1 < p < \infty$, $f$, and the two parameters which determine the cycles. Using the results of T. Ando, ("Contractive projections in $L^p$ spaces", Pacific J. Math. 17(1966), 391-405), this enables us to investigate commutativity of contractive projections in terms of the parameters. Necessary and sufficient conditions for the containment of one cycle subspace in another are given, and, for a number of cases, the commutativity problem is solved. For positive contractive projections the discussion is reduced to considering conditional expectations, where the commutativity question is shown equivalent to that of independence of certain subalgebras. In addition, those contractive projections with the averaging property are characterized, with results extending to $L^1$ as well. (Received April 15, 1971.)

*71T-B146. STANLEY E. WEINSTEIN, University of Utah, Salt Lake City, Utah 84112. Uniform approximation through partitioning. Preliminary report.

In this paper the problem of best uniform polynomial approximation to a continuous function on a compact set $X$ is approached through the partitioning of $X$ and the definition of strictly convex norms corresponding to the partition and each of the standard $L^p$ norms $1 < p < \infty$. For computational convenience a pseudonorm is defined corresponding to each partition. When the partition is chosen appropriately the corresponding best approximations (using both the norms and the pseudonorm) are arbitrarily close to a best uniform approximation. A characterization theorem for best pseudonorm approximation is presented, along with an alternation theorem for best pseudonorm approximation to a univariate function. (Received April 15, 1971.)

71T-B147. WITHDRAWN.
Consider the second order linear differential system \((*)\) \(x'' + \frac{\partial G(x)}{dx} = p(t) = p(t+2\pi)\) where \(x\) and \(p\) are \(n\)-vectors, \(p\) is continuous on the real line, and \(G\) is real valued function of class \(C^2\) on \(\mathbb{R}^n\). Assume there exist symmetric matrices \(A\) and \(B\) such that, for all \(x \in \mathbb{R}^n\), \(A \geq H(x) \geq B\), where \(H = \frac{\partial^2 G}{\partial x \partial x'}\), and \(A \geq M \geq B\), where \(M\) is a symmetric matrix, implies \(M\) has no eigenvalues of the form \(n^2\), \(n = 0, 1, 2, \ldots\). (This will be true for example if there exist integers \(m_k\), \(k = 1, \ldots, n\), such that \(m_k \leq \mu_k \leq (m_k + 1)^2\) where \(\mu_1 \leq \mu_2 \leq \ldots \leq \mu_n\) are the eigenvalues of \(A\) and \(\lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_n\) are the eigenvalues of \(B\).) Theorem. Under the above conditions there exists a unique \(2\pi\)-periodic solution of \((*)\).

The proof depends on comparison theorem for eigenvalues of the selfadjoint problem \(x'' + \lambda Q(t)x = 0\), \(x(0) = x(2\pi), x'(0) = x'(2\pi)\) where \(Q\) is symmetric positive definite, bounded, and measurable. This extends a result previously announced by the authors [Abstract 70T-B220, these Notices 17(1970), 957]. (Received April 19, 1971.)
the local solution. These estimates require the use of new nonlinear functionals. The two above theorems are also true for the periodic case; i.e. we specify \( u(x+1,t) = u(x,t) \) for all \( t \geq 0 \). The solution will lie in 

\[
L^\infty(0,T;V^{4(m+1)}((0,1)) \cap \cdots \cap C^m(0,T;V^0((0,1))) \quad \text{if} \quad u_0 \in V^{4(m+1)}.
\]

(Received April 21, 1971.)

71T-B151. ROBERT A. FONTENOT, Louisiana State University, Baton Rouge, Louisiana 70803. 

Approximate identities and double centralizers. Preliminary report.

Relationships between approximate identities for the function algebra \( C_0(S) \) and topological properties of a space \( S \) are studied, as well as properties of the double centralizer algebra \( M(A) \) of a \( B^* \) algebra \( A \) (always with the strict topology). Sample results: Theorem. The following are equivalent: (1) \( M(A) \) has a strict compact unit ball, (2) \( A \) is a subdirect sum of finite-dimensional algebras, (3) \( M(A) \) is semi-reflexive. Theorem. \( M(A) \) is nuclear iff \( A \) is finite-dimensional. Theorem. These are equivalent: (1) \( M(A) \) is DF, (2) \( M(A) \) is WDF, (3) \( J^*(A) \) is an essential two-sided \( A \)-module. Theorem. \( C_0(S) \) has a strict totally bounded approximate identity if and only if \( S \) is paracompact. In addition, well-behaved approximate identities, introduced by D. C. Taylor, and other types of approximate identities and relationships between them are being investigated. (Received April 22, 1971.)

*71T-B152. BRUCE DONALD CALVERT and KARL E. GUSTAFSON, University of Colorado, Boulder, Colorado 80302. Nonlinear product operators.

The following result is of interest for both (a) solution of noncompact Hammerstein integral equations, and (b) solution of nonlinear initial value problems. Theorem. Let \( X \) be a real reflexive Banach space, let \( A \) be a nonlinear, multi-valued, maximal monotone map between \( X \) and \( X^* \), let \( B \) be a nonlinear, multiple-valued, maximal monotone map between \( X^* \) and \( X \), and suppose that (1) either \( D(A) = X \) or \( R(B) = X \), and that (2) either \( D(B) = X^* \) or \( R(A) = X^* \). Then (a) if \( (I+BA)^{-1} \) is continuous, one has \( R(I+BA) = X \); and in particular, (b) if \( BA \) is accretive, it is \( m \)-accretive. (Received April 22, 1971.)


Using current terminology, one says that Weyl's theorem holds for a linear operator \( T \) if \( \sigma_e(T) = \sigma(T) - \pi_{00}(T) \), where \( \sigma_e \) denotes essential spectrum, \( \sigma \) denotes spectrum, and \( \pi_{00} \) denotes isolated eigenvalues of finite geometric multiplicity. We obtain 5 necessary and sufficient conditions that characterize those \( T \) for which Weyl's theorem holds, in terms of the behavior of the minimum modulus function, a certain decomposability of \( X \) in terms of \( T \), ascents and descents, geometric versus algebraic multiplicity, and spectral behavior within the boundary of the spectrum. These results hold for closed \( T \) in a Banach space. (Received April 22, 1971.)

71T-B154. DAVID ZEITLIN, 1650 Vincent Avenue North, Minneapolis, Minnesota 55411. On the solution of two polynomial identities.

This note solves Problem 5785*, p. 305, in the American Mathematical Monthly, March, 1971, namely: "Show that for each choice of the natural number \( m \) there are \( m \) positive numbers \( d_j \) \( (j=1,2,\ldots,m) \) with each
d_j > 1, such that \((x + 2)^{2m} + x^{2m} = 2\prod_{j=1}^{m}(x^2 + 2x + d_j)\) is an identity. The solution is \(d_j = \sec^2(\pi A_j)\), where \(A_j = (2j-1)/(4m)\), \(j = 1, 2, \ldots, m\). A new companion problem and identity is as follows: "Show that for each choice of the natural number \(m\) there are \(m\) positive numbers \(C_j (j=1, 2, \ldots, m)\) with each \(C_j > 1\) such that \((x + 2)^{2m+1} + x^{2m+1} = 2(x+1) \prod_{j=1}^{m}(x^2 + 2x + C_j)\) is an identity." The solution is now given by \(C_j = \sec^2((\pi)/(2m+1))\), \(j = 1, 2, \ldots, m\). The two solutions were obtained from the factorization of the left-hand sides of the proposed polynomial identities, using Chebyshev polynomials of the first and second kind, respectively. (Received April 22, 1971.)

*71T-B155. STANLEY H. BENTON, JR., Tulane University, New Orleans, Louisiana 70118. A general space-time boundary value problem for the Hamilton-Jacobi equation.

A simple but powerful lemma of Hopf is utilized in conjunction with the work on convex dual functions of Fenchel, as in the work of Conway, Hopf, Aizawa, and Kikuchi, to obtain a global solution of a general space-time boundary value problem for a Hamilton-Jacobi equation. Theorem. Let \(B\) be a closed set (boundary) in space-time with \(n\) space variables and \(b\) the least, or initial time. Let \(D\) be the complement of \(B\) in \((b, \infty) \times \mathbb{R}^n\). Let \(H\) be a strictly convex function on \(\mathbb{R}^n\) such that \(H(p)/|p| \to \infty\) as \(|p| \to \infty\). Let \(H^*\) be the convex dual of \(H\). Let boundary data be given in the form of \(\varphi \in C(B)\) with \(\varphi(t, x)\) bounded below for bounded time \(t\). Suppose that the following compatibility condition is satisfied: \((\varphi(t, x) - \varphi(s, y))/(t-s) \leq H^*((x-y)/(t-s))\) for \((s, y), (t, x) \in B, s < t\). Then \(u(t, x) = \text{min}[(t-s)H^*((x-y)/(t-s)) + \varphi(s, y) : (s, y) \in B, s < t]\) defines a locally Lipschitz function on \(D\) such that \(u(t, x) = \min((t-s)H^*((x-y)/(t-s)) + \varphi(s, y) : (s, y) \in B, s < t)\) for \((q, w) \to (t, x) \in B\) from \(D\) in such a manner that, for each \((q, w)\) there is an \((r, z) \in B, r < q\) with \((r, z) \to (t, x)\) and \((w - z)/(q - r)\) bounded, then \(u(q, w) \to \varphi(t, x)\). Improvements of known results on the Cauchy problem and plane boundary value problem follow as corollaries to this theorem. (Received April 23, 1971.)

*71T-B156. MARTIN E. MULDOON, York University, Downsview 463, Ontario, Canada. Some characterizations of the gamma function.

It is shown that if \(f(x) > 0\), if \(f(x) > 0\), if \((-1)^k x^{k+1} \log f(x) \geq 0 \) for \(k = 0, 1, 2, \ldots\) and \(a < x < \infty\), where \(a \geq 0\), and if \(f(x_k) = \Gamma(x_k)\) for each \(x_k\) in an increasing sequence of positive numbers for which \(x_k > a\) and \(\sum(1/x_k)\) diverges, then \(f(x) = \Gamma(x)\) for \(x > a\). This follows from a slight extension of a result due to W. Feller (Duke Math. J. 5(1939), 661-674) on interpolation by completely monotonic functions. (A function \(g(x)\) is said to be completely monotonic on an interval \(I\) if \((-1)^n g^{(n)}(x) \geq 0\) for \(x \in I\) and \(n = 0, 1, 2, \ldots\).) Thus the gamma function may be defined as the unique positive function \(f\) which satisfies \((-1)^k x^{k+1} \log f(x) \geq 0\) for \(k = 0, 1, 2, \ldots\) and \(a = 0\), and satisfies \(f(x) = \Gamma(x)\) for \(x > a\). This follows readily from this definition and the result above. Other characterizations may be based on the complete monotonicity on \((a, \infty)\) of \(\Gamma(x)/\Gamma(x)\) or of \(\Gamma(x)/\Gamma(x)\) where \(a = 1.46163\ldots\) is the unique positive zero of \(\Gamma(x)\). (Received April 26, 1971.)
A normed linear space being uniformly convex in every direction (u.c.e.d.) means that $\alpha_n \to 0$ as $n \to \infty$ whenever there exist sequences $\{x_n\}$ and $\{y_n\}$ and a nonzero member $z$ of $X$ for which $\|x_n\| = \|y_n\| = 1$ for every $n$, $x_n - y_n = \alpha_n z$ for every $n$, and $\|x_n + y_n\| - 2 \to 0$ as $n \to \infty$. It is shown that every separable Banach space can be renormed so as to be u.c.e.d. and that a Hilbert product of u.c.e.d. spaces is u.c.e.d. If a normed linear space $X$ is u.c.e.d., then each bounded subset of $X$ has at most one Čebyšev center and $X$ has normal structure. Thus every separable normed linear space can be renormed so that every bounded subset has at most one Čebyšev center and also (Zizler) so as to have normal structure. A reflexive Banach space has complete normal structure if it is u.c.e.d. (Received April 26, 1971.)

For $k \geq 2$ denote by $V_k$ the class of functions analytic in the unit disc and having boundary rotation at most $\frac{\pi}{k}$. Define $M(n, k) = \max\{|a_n|: f(z) = z + \sum_{j=2}^{\infty} a_j z^j \in V_k\}$, and let $F(z) = 1/k \left\{(1 + z)/(1 - z)\right\}^{k/2} - 1 = z + \sum_{j=2}^{\infty} A_j z^j$. The following results are proved: (i) If $n \leq k/2 + 1$, then $M(n, k) = A_n$. (ii) If $k$ is an even integer, then $M(n, k) = A_n$ for all $n$. (iii) For fixed $k$ and certain values of $\lambda$ (the values of $\lambda$ depend on $k$), $f \in V_k$ implies $\int_0^{2\pi} |f'(z)|^\lambda \, d\theta \leq \int_0^{2\pi} |F'(z)|^\lambda \, d\theta$. Also, $F$ and its rotations are the only extremal functions in (i), (ii), and (iii). Finally, if $f \in V_k$ with $k > 2$ and if $\beta = 2/(k - 2)$, then $g(z) = \left[\int_0^{\beta} t^{-1} f'(t)^\beta \, dt\right]^{1/\beta}$ belongs to $B(\beta)$, the class of Bazilevič functions of type $\beta$. (Received April 26, 1971.)

The second dual of any norm-closed subspace of compact operators on a Hilbert space is identified in a natural way with a subspace of bounded operators containing it. Necessary and sufficient conditions are found for a subspace of bounded operator to "be" the second dual of its compact part. Among subspaces having this property are the sets of selfadjoint operators, normal operators, and the set of all operators whose matrix with respect to a fixed o.n. basis is of the form $(w_{ij}^* b_j)$ where $(w_{ij})$ is some fixed double sequence. Theorem. If there is a sequence of compact operators $C_n$ converging to the identity operator in the weak operator topology, all $T_n$ commuting with some fixed operator $T$, then the commutant of $T$ is the second dual of its compact part. Among operators $T$ having this property are diagonal operators, but not weighted shifts. (Received April 27, 1971.)

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The Lebesgue integral of a measurable nonnegative function may be defined analytically or geometrically. Extending them to an arbitrary nonnegative function, not necessarily measurable, one may use interior, exterior, or average measure. It can be shown that both analytic and geometric definitions may be given for these three integrals, where 0 and +∞ are possible values, but they may be different. The interior and exterior integrals, whether analytic or geometric, can be shown to be upper and lower semicontinuous respectively, for uniformly bounded almost everywhere convergence in a bounded interval. Concerning the average integral, it can be shown to be lower semicontinuous for the analytic definition; of course, the average integral is \( \frac{1}{2} \) the sum of interior and exterior integrals. The geometric definitions are monotone increasing. It should be stated that the geometric definitions may also be formulated in an analytic manner. When taken over a bounded set X, and considered as set functions, the interior integral is superadditive for disjoint sets, while the exterior and average integrals are subadditive (all are additive over measurable sets X). Extensions can be made to unbounded sets, to multiple integrals, and also to linear combinations of interior and exterior integrals.

(Received April 28, 1971.)

**Applied Mathematics**


Relations are developed between the rate of convergence of whole step finite-difference approximations to solutions of parabolic partial differential equations together with initial and boundary data and the order of accuracy of the approximation and the smoothness of the data. (Received February 18, 1971.)

71T-C10. KENDALL E. ATKINSON, Computer Centre, Australian National University, Canberra, Australia. The numerical solution of an Abel integral equation by product integration.

Consider the Abel equation (1):

\[
\int_0^t \frac{f(s)}{\sqrt{t^2 - s^2}} \, ds = g(t).
\]

Let \( h > 0 \), \( t_j = jh \) for \( j \geq 0 \), and let \( f_n(s) \) be a piecewise linear function with nodes \( \{ t_j \} \). Determine \( f_n(s) \) by substituting it into (1) and collocating at the node points to obtain (2):

\[
\int_0^t \frac{f_n(s)}{\sqrt{t^2 - s^2}} \, ds = g(t_n) \quad \text{for} \quad n \geq 1.
\]

The result is a nonsingular triangular system for \( f_n(t_j), \quad j \geq 1 \); use \( f_n(0) = f(0) = (2/\pi) g(0) \). **Theorem.** Assume \( f \) is twice continuously differentiable and has a bounded third derivative on \([0, T]\). Then there is a constant \( B \) such that \( \max |f(t) - f_n(t)| \) on \([0, T]\) is \( \leq Bh^{3/2} \). Empirically, this result should probably be \( h^2 \). For the special case \( f(s) = s^2 \), it can be shown that \( \max |f(s) - f_n(s)| \) on \([0, T]\) is \( \leq Bh^2 \) log \( h \). (Received March 4, 1971.)
Product integration methods for a class of integral equations of the Abel type.

Consider the equation (1) \( g(t) = \int_{0}^{T} k(t, s)(t-s)^{-\alpha} f(s) ds \), \( 0 \leq t \leq T < \infty, \ 0 < \alpha < 1 \), where \( k(t, s) \) is continuous for \( 0 \leq s \leq t \leq T \) and \( k(t, t) \neq 0 \). Introduce the grid \( t_{i} = ih, i = 0, \ldots, I = T/h \). For each fixed \( t_{i} \), \( i \geq 1 \), let \( M_{h}(t_{i}, s) \) be a piecewise constant function defined by

\[
M_{h}(t_{i}, s) = k(t_{i}, t_{i}) f_{i} \text{ for } t_{i} \leq s < t_{i} + 1,
\]

where \( f_{i} \) denotes a numerical approximation to \( f(t_{i}) \). Replace in (1) \( k(t_{i}, s)f(s) \) by \( M_{h}(t_{i}, s) \) to obtain:

\[
g(t_{i}) = \frac{1}{h^{\alpha}} \sum_{j=0}^{I-1} M_{h}(t_{i}, t_{j}) (t_{i} - s) \alpha ds, i \geq 1.
\]

This gives a nonsingular triangular system for \( f_{j}, j = 0, \ldots, I-1 \). Theorem 1. If \( f(t) \) and \( k(t, s) \) are twice continuously differentiable for \( 0 \leq s \leq t \leq T \), then, for \( h \) sufficiently small, there exists a constant \( C \) such that

\[
| f(t_{j}) - f_{j} | \leq Ch, j = 0, \ldots, I-1.
\]

Similarly, let \( M_{h}(t_{i}, s) \) be a piecewise linear function, defined by

\[
M_{h}(t_{i}, s) = \frac{(t_{i+1} - s)k(t_{i}, t_{i}) f_{i} + (s - t_{i})k(t_{i}, t_{i+1}) f_{i+1}}{h}, \text{ for } t_{i} \leq s \leq t_{i+1}, 0 \leq j \leq I-1.
\]

Take

\[
\lim_{t \to 0} (g(t) t^{\alpha-1}) \frac{1}{h} = \frac{1}{h} f_{0} = \frac{1}{h} \int_{0}^{T} k(t, s) s^{-\alpha} ds.
\]

Substitution into (1) again leads to a nonsingular triangular system for \( f_{j}, j = 1, \ldots, I \). Theorem 2. If \( f(t) \) and \( k(t, s) \) are three times continuously differentiable for \( 0 \leq s \leq t \leq T \), then, for \( h \) sufficiently small, there exists a constant \( D \) such that

\[
| f(t_{j}) - f_{j} | \leq Dh^{2}, j = 1, \ldots, I.
\]

Numerical computations indicate that Theorem 2 holds for all \( \alpha \in (0,1) \).

(Received March 9, 1971.) (Author introduced by Professor Kendall E. Atkinson.)

71T-C12. HERBERT E. SALZER, 941 Washington Avenue, Brooklyn, New York 11225. Lagrangian interpolation at the Chebyshev points \( x_{n, \nu} = \cos(\nu \pi/n), \nu = O(1)n; \) some unnoted advantages. Preliminary report.

Besides many applications of the Chebyshev points \( x_{n, \nu} = \cos(\nu \pi/n), \nu = O(1)n \), in approximation, interpolation by Chebyshev series, numerical integration and numerical differentiation, there are advantages in their use in the barycentric form of the Lagrange interpolation formula and in checking by divided differences. When \( n = 2^{m} \), we obtain \( x_{2^{m}, \nu} \) with less than half the number of square roots that are required to find the other Chebyshev points \( x_{2^{m}, \nu} = \cos[(2\nu - 1) \pi/2^{m+1}], \nu = 1(1)2^{m} \). Also, the barycentric interpolation formula may be applied to the solution of a near-minimax problem so as to avoid extensive calculation of auxiliary polynomials, and in a numerical differentiation procedure that conveniently bypasses direct differentiation of the interpolation polynomial. (Received March 12, 1971.)


The study of gravity waves in general and Lee waves in particular in two dimensions has received a good amount of attention. Crapper (J. Fluid Mech. 16(1959),51, and Proc. Roy. Soc. A. 254, p. 73) first extended the investigation to three dimensions when the speed, direction and stability of the wind-stream are independent of the horizontal. In the present note an attempt has been made to generalize the theory when the motion is not restricted to two dimensions and the wind direction may vary with height. An equation giving the vertical variation of the amplitude of the waves is derived. Also the condition for the existence of Lee waves is established. (Received March 29, 1971.)
All differential games have value.

Using the concept of relaxed controls from control theory the approaches to (2 person, zero-sum) differential games of A. Friedman (J. Differential Equations 7(1970), 69–91) and W. Fleming (J. Math. Mech. 13(1964), 987–1008) are related. It is shown that if the 'Isaacs condition' (cf. Fleming, loc. cit.) is satisfied, then the game has a value in the sense of Friedman. Over the relaxed controls the Isaacs condition is always satisfied and so the game always has a value in this setting. Friedman's hypothesis that the two sets of control variables appear separated in the dynamical equations and payoff is not needed in the proof. The introduction of probabilistic ideas into differential games by relaxed controls thus gives a value, as the introduction of mixed strategies by von Neumann does for 2 person zero-sum games. (Received March 30, 1971.)


The concept of branch and bound and the combinatorial process are used to obtain an approximate solution of the AIP problems defined as, Maximize $Z = C \cdot X$ subject to (1) $A \cdot X \leq b$, (2) $X \geq 0$ an integer. The algorithm starts with selecting one set of variables with their limiting integer values that gives the maximum value of the objective function among the various sets. The value of the variable is truncated to the nearest integer values without violating the constraint. The limiting integer value of the variable is defined as the minimum value of the variable which satisfies all the constraints. The set of the variables consists of the prime variable and the covariables. The covariable utilizes the resources not consumed by the prime variable and the preceding covariables of the corresponding set. At each successive iteration, the variables of the initial set are traded-off to obtain another set of variables which improves the objective function. The value of the traded variables is updated at each iteration. The process ends when the variables of the initial set can not be traded-off further. The solution consists of all the variables obtained during the iterative process. The value of each variable from the corresponding set is added to obtain the final value of the particular solution variable. (Received March 31, 1971.) (Author introduced by Professor Robert E. Atalla.)

Locally testable events and semigroups. Preliminary report.

Let $\Sigma$ be a finite alphabet, and $E \subseteq \Sigma^+$. Let $k$ be a positive integer and for $w \in \Sigma^+$ of length $\geq k + 2$, let $L_k(w)$, $R_k(w)$ and $I_k(w)$ be, respectively, the prefix of $w$ of length $k$, the suffix of $w$ of length $k$ and the set of interior solid subwords of $w$ of length $k$. Definition. $E$ is locally testable iff $\exists k$ such that for all $w, w' \in \Sigma^+$ of length $\geq k + 2$, $L_k(w) = L_k(w')$, $R_k(w) = R_k(w')$ and $I_k(w) = I_k(w')$ imply that $w \in E \iff w' \in E$. Let $S(E)$ be the quotient semigroup of $\Sigma^+$ modulo the Myhill congruence. Theorem. Let $E \subseteq \Sigma^*$, $S = S(E)$ and let $I$ be the set of idempotents of $S$. The following are equivalent: (1) $E$ is locally testable. (2) There do not exist elements $s_1, s_2, \ldots, s_m, t_1, t_2, \ldots, t_n \in S$, $e_1, \ldots, e_m, f_1, \ldots, f_n \in I$ such that (a) $s_1 = t_1$, $e_1 = f_1$, $e_m = f_n$; (b) for all i,
\[ 1 \leq i \leq m, \quad s_1 s_2 \ldots s_i = s_1 s_2 \ldots s_i', \text{ and for all } j, 1 \leq j \leq n, \quad t_1 t_2 \ldots t_j = t_1 t_2 \ldots t_j'; \quad (c) \text{ for all } i, 1 \leq i \leq m - 1, \text{ there exists a } j, 1 \leq j \leq n - 1, \text{ and for all } j, 1 \leq j \leq n - 1, \text{ there exists an } i, 1 \leq i \leq m - 1, \text{ such that } e_i = f_j. \]

For all \( e \in E \), \( e \in S \) is a semilattice. **Corollary.** There is an algorithm for deciding whether a given regular event is locally testable. This solves an open problem of McNaughton and Papert. (Received April 19, 1971.)

**71T-C17. R. S. L. SRIVASTAVA and M. B. BANERJEE**, Indian Institute of Technology, Kanpur 16, India. **A mathematical criterion for a stable government.**

The paper presents a mathematical model of the problem of economic stability and provides a sufficient criterion which ensures the damping out of the effects arising due to arbitrary small changes in the variables characterising the state of a government at any time. The model assumes the prevalence of normal conditions, that is, times free from unforeseen calamities like severe earthquake, outbreak of epidemic on a large scale, invasion by another country, famine, etc. Some conclusions derived from the analysis of the system are: (a) In a socialist state, rise in population is conducive to stability if the people are engaged in productive employment. (b) In a socialist state, growth of population and increase in taxation are not compatible with stability if the people indulge in antisocial activities. (c) In general, if the people of a country are engaged in productive employment then decreasing government expenditure tends to make the system unstable. (Received April 26, 1971.) (Authors introduced by Professor S. A. Naimpally.)

**Geometry**

**71T-D8. JAYME M. CARDOSO**, Instituto de Matematica, Universidade de Campinas, Caixa Postal 1170, Campinas, Brazil. **Common tangents of two parabolas.** Preliminary report.

A principal circle of a parabola \( P \) is defined as the circle whose center is the focus of \( P \) and whose radius is the distance between the vertex and focus of \( P \). It is shown that the common tangents of two parabolas of parallel axis meet in the centers of similitude of its principal circles: (i) the interior center of similitude if the parabolas have real proper common points; (ii) the exterior center of similitude if the curves have no real proper common points. (See Abstract 70T-D17, these *Notices* 17(1970), 670.) (Received February 19, 1971.)

**71T-D9. JOHN DiCICCO**, Illinois Institute of Technology, Chicago, Illinois 60616 and ROBERT V. ANDERSON, Université du Québec à Montréal, Montréal 110, Québec, Canada. **Elementary properties of the index of an Appell transformation between two surfaces.**

An Appell transformation between two surfaces \( \Sigma \) and \( \tilde{\Sigma} \) is composed of a projective cartogram \( T \) and an index describing the time change. Conditions of the covariant derivatives of the index are developed which lead to a fundamental set of differential invariants. If \( \Sigma \) is a surface of constant Gaussian curvature \( K \) and if \( \Sigma \) is related to another surface \( \tilde{\Sigma} \) by a projective cartogram \( T \), then \( \tilde{\Sigma} \) is also of constant Gaussian curvature \( K \). Relationships are developed between the fundamental differential invariants of a projective cartogram \( T \) and
the Gaussian curvatures $K$ and $\bar{K}$ in the case where these curvatures are variable. In particular, the variation $d(\Delta_1(K))/ds$ along a Gaussian curve $\Gamma$ of a surface $\Sigma$ is studied. This study yields the result that under a projective cartogram $T$ between two surfaces $\Sigma$ and $\bar{\Sigma}$ of variable Gaussian curvature the orthogonal trajectories of its $\omega^1$ Gaussian curves $\Gamma$ are converted into the orthogonal trajectories of the $\omega^1$ Gaussian curves $\bar{\Gamma}$ of $\bar{\Sigma}$. (Received March 2, 1971.)

71T-D10. ISAAC CHAVEL, City College, City University of New York, New York, New York 10031.

Extremal lengths of Riemannian metrics in real projective spaces. Preliminary report.

Let $M$ be an-dimensional real projective space, $n \geq 2$, $g$ denote the Riemann metric tensor of constant curvature 1, $t$ a symmetric 2-tensor on $M$. Then for sufficiently small $\epsilon$, the tensor $g_{\epsilon} = g + \epsilon t$ determines a Riemannian metric on $M$. Let $\delta$ denote the formal adjoint of covariant differentiation with respect to $g$ and assume $\delta t = 0$. Furthermore let $L(\epsilon)$ be the length of the shortest geodesic in the nontrivial homotopy class of $M$, relative to $g$, $v(\epsilon)$ the volume of $M$ relative to $g$, and $\lambda(\epsilon) = (L(\epsilon))/v(\epsilon)$. Theorem. If $t$ is not a constant multiple of $g$, then there exists $\epsilon_0 > 0$ such that $\lambda(\epsilon) < \lambda(0)$ for all $\epsilon$ satisfying $0 < |\epsilon| < \epsilon_0$. The proof is based on the work of R. Michel on the corresponding "local" "wiedersehns" theorem for higher dimensions. (Received March 15, 1971.)

*71T-D11. ABRAHAM BERMAN, Centre de Recherches Mathématiques, Université de Montréal, Montréal 101, Québec, Canada. Linear inequalities in complex space.

Theorem. Let $A \in C^{m \times n}$, $C$ an Hermitian p.s.d. matrix of order $m$, $b \in C^m$, $S$ a polyhedral cone in $C^n$. Then the following are equivalent: (a) The system $Ax - Cy = b$, $x \in S$, $A^*y \in S^*$, $y^*Cy \leq 1$ is consistent. (b) $A^*z \in S^* \Rightarrow \Re(b, z) \geq -(z^*Cz)^{1/2}$. This theorem, applicable to nonlinear programming over cones, generalizes theorems of Ben Israel [J. Math. Anal. Appl. 27(1969), 367-389] and Kaul [Amer. Math. Monthly 77(1970), 956-960]. (Received March 31, 1971.)


The visibility function, $v$, assigns to each point $x$ of a fixed measurable set $E$ in a Euclidean space, the Lebesgue outer measure of $\{y : rx + (1-r)y \in E, \forall r \in [0, 1]\}$. If $E$ is a bounded open set the following results hold: Theorem. $v$ is lower semicontinuous. Theorem. If $E$ is planar and $E^c$ is locally connected, $v$ is continuous. Typical results when $E$ is compact are the following: Theorem. $v$ is upper semicontinuous. Theorem. $v$ is continuous on $E$ iff the visibility functions for the sets $\bigcup_{E \in E} \{y : |x-y| \leq 1/n\}$, $n < \omega$, when restricted to $E$ converge uniformly to $v$. Theorem. If $E$ is starshaped and $\dim(\text{convex kernel } (E)) \geq n - 1$, $v$ is continuous on $\text{Int}(E)$. Theorem. If $E$ is planar with simply connected components and locally connected boundary, then $v$ is continuous on $\text{Int}(E)$. (Received April 7, 1971.)
Asymmetric inequalities of the triangle.

In a previous note (Univ. Beograd. Publ. Elektrotehn. Fak. Ser. Mat. Fiz. 330(1970), 1-15), the author had derived an inequality relating the elements of two triangles. Its scope has now been extended to the master inequality $x^2 + y^2 + z^2 \geq (-1)^{n+1} (2yz \cos nA + 2zx \cos nB + 2xy \cos nC)$ where $x$, $y$, $z$ are real numbers, $n$ an integer and $A$, $B$, $C$ are angles of a triangle. There is equality iff $x/\sin nA = y/\sin nB = z/\sin nC$. For $n = 1$, we get Barrow's inequality; however, an equivalent form was given much earlier by Wohlenstenhome.

For $n = 2$, we get an inequality of Kooi and equivalently a previous one of the author (loc. cit., p. 13), i.e., $[(ax+by+cz)/A^2] \leq x/yab + y/zbc + zx/ca$. In particular, for $x = y = z$, $3/2 \geq (-1)^{n+1} \cos nA \geq -3$. For equality, the triangle is not unique for $n > 2$. This corrects some previous errors for the equality case (loc. cit., pp. 7-10). The master inequality is then specialized in many ways to obtain numerous well-known inequalities as well as a number of them which are believed to be new. (Received April 15, 1971.)

Logic and Foundations

*71T-E38. GIORGIO M. GERMANO and ANDEA MAGGIOLO-SCHETTINI, Laboratorio di Cibernetica del CNR, Via Toiano 2, 80072 Arco Felice, Italy. Eliminability of concluding formulas in Markov's normal algorithms. II.

For any function $f$ mapping natural numbers on natural numbers, the following statements are equivalent:

(a) $f$ is partial recursive; (b) there is a normal algorithm $F$ such that for every $n_1, \ldots, n_k$ it holds that: $F(n_1, \ldots, n_k) \sim f(n_1, \ldots, n_k)$ (see Detlovs, Trudy Mat. Inst. Steklov. 52(1958), 75-139); (c) there is a normal algorithm $F$ such that for every $n_1, \ldots, n_k$ it holds that: $F(n_1, \ldots, n_k) \sim f(n_1, \ldots, n_k)$; (d) there is a normal algorithm $F$ without concluding formulas such that for every $n_1, \ldots, n_k$ it holds that: $F(n_1, \ldots, n_k) \sim f(n_1, \ldots, n_k)$. (Received February 23, 1971.)

71T-E39. WILLIAM M. LAMBERT, JR., University of Detroit, Detroit, Michigan 48221. Equivalence of search computability and schematic definability.

The results announced in Abstract 678-E1 (these Notices 17(1970), 936), and at the meeting of the Association for Symbolic Logic (January 21; abstract forthcoming in J. Symbolic Logic) can be extended in that it is not necessary to take the equality relation as being distinguished. In passing from search computability to schematic definability, this follows from the fact that we employ (and even with equality present, generally could only get) a partial-sd relation among codes; in passing from schematic definability to search computability, we imitate the construction of "$\equiv_w$" in Moschovakis, "Abstract first order computability. I" (Trans. Amer. Math. Soc. 138(1969), 427-464). (Received February 24, 1971.)

660
A remark on the elementary theory of the hyperdegrees. Preliminary report.

Let PO be the elementary language for partial orderings. Theorem. There is a sentence of PO which is true in the partial ordering of hyperdegrees if a Ramsey cardinal exists, and false there if the universe is a generic extension of L. The sentence asserts the existence of a cone of minimal covers; see Jockusch and Soare, Proc. Amer. Math. Soc. 25(1970), 856-859, for an explanation of this terminology. The proof uses methods of the author's previous abstract, "Some results on hyperdegrees," Abstract 71T-E36, these Notices 18(1971), 565. The author suspects that the existence of $O^#$ does not follow from the existence of a cone of minimal hyperdegree covers. (Received February 26, 1971.)

Indice de vérité.

$f = u \Rightarrow v$ étant une formule implicationnelle, tautologique ou non, sur $\mu = \{x, y, \ldots\}$, de cardinal $c$, et $a$ le cardinal de l'ensemble des points $f = 1$, l'indice de vérité de $f$ est $a/2^c$. Aucune formule n'est identiquement nulle. L'IV d'une formule est au moins égal à celui de son conséquent. Si $u$ et $v$ n'ont aucun atome commun on a $IV_u = v = 1 - IV_v (1 - IV_v)$. Toute formule sur un seul atome, $x$, est tautologique ou équivalente à $x$. Le nombre des formules distinctes avec $m - 1$ symboles $\Rightarrow$, non isomorphes, d'une structure donnée sur $n$ atomes est $(1/n!)(\sum (-1)^i \binom{n}{i} (n-i)^m)$, $u = 0, \ldots, n$. Toute formule a un IV plus grand que $1/2$ ou est équivalente à un atome. Dans toute thèse il y a au moins un atome répété. La condition nécessaire et suffisante pour qu'une formule avec $k - 1$ signes $\Rightarrow$ ait tous ses atomes distincts est que son IV soit une fraction irréductible de dénominateur $2^k$. Une fonction est accessible si elle peut être définie par une formule implicationnelle. $a$ couvre $b$, $(a > b)$ si $a = 1$ toutes les fois que $b = 1$. Sur tout ensemble il existe une fonction accessible qui est nulle en un seul point, arbitrairement choisi, pourvu qu'il ait au moins une coordonnée nulle. (Received February 25, 1971.)

On the existence of Hanf numbers for higher order logics. Preliminary report.

Let $ZF_1$ be the theory which contains all the axioms of Zermelo-Fraenkel (with or without choice) except that replacement is assumed only for formulas which are $\Sigma_1$ in the power set operation. Theorem 1. One cannot prove the existence of the Hanf number for second order logic in $ZF_1$. Let $ZF_0$ be the subtheory of $ZF_1$ where full separation is replaced by separation for formulas which are $\Delta_0$ in the power set. Theorem 2. One cannot prove the existence of the Hanf number for $\omega$-logic in $ZF_0$. On the other hand, the usual computations of the Hanf number for $\omega$-logic can be carried out in $ZF_1$. Theorem 2 follows from the special case of Theorem 3 where $\alpha$ is Church-Kleene $\omega_1$. Theorem 3. For any countable admissible ordinal $\alpha$ there is a transitive model $M$ of $ZF_0$ with von Neumann ordinals exactly those less than $\alpha$. Similar results can be obtained for infinitary logics. (Received March 1, 1971.)

The following theorems are announced: Theorem 1. Let D be any countably complete ultrafilter over a cardinal \( \lambda \). Let \( f \) be the first function greater than all the constants less than \( \lambda \). Then, for any cardinal \( x \in \lambda \), the ultrafilter \( D \) is \((\lambda, x)\)-regular if and only if the function \( f \) has cofinality less than the constant function \( x \) in the ultrapower given by \( D \). As an application of Theorem 1 we get: Theorem 2. Let \( x \) be an uncountable regular cardinal so that every regular cardinal greater than \( x \) carries a uniform \( x \)-complete ultrafilter. Then \( x \) is strongly compact. (Received March 1, 1971.)

71T-E44. WILLIAM S. HATCHER, Université Laval, Québec 10, Canada and JOHN CORCORAN and JOHN HERRING, State University of New York, Buffalo, New York 14214. Variable-binding term operators.

We consider systems of first-order logic with identity to which we add variable-binding term operators (vbtos) as primitive notation. We define a semantics for the new systems in a natural way, and then prove that addition of the following axiom scheme to the usual deduction rules and axioms of first-order logic yields a sound and complete system relative to the modified semantics: 

\[
(y \, x \, y(x=y \Rightarrow F(x) \equiv G(y))) \Rightarrow (\forall x F(x)) = (\forall x G(x))
\]

where \( F(x) \) is a formula containing free occurrences of \( x \) but none of \( y \), and \( v \) is an arbitrary variable-binding term operator. The proof proceeds by first defining a semantics for an intermediate system and showing it to be sound and complete. The theorem is then proved by establishing that every interpretation in the new semantics gives rise to an equivalent model in the intermediate semantics and vice-versa. The technique illuminates the relationship between logic with and without vbtos. Finally, an analogous treatment of vbtos which bind a fixed finite number of variables free in a formula yields soundness and completeness with addition of a correspondingly generalized axiom scheme. (Received March 4, 1971.)

71T-E45. J. RICHARD BUCHI, Purdue University, Lafayette, Indiana 47907. The monadic second order theory of \( \omega_1 \). Preliminary report.

Let \( J \) be a filter in \( P(S) \). Let \( r \) be a finite-valued function from \( S. \) \( \sup(J, r) \) stands for the smallest set \( D \) of values of \( r \) such that \( \{ t ; rt \in D \} \in J \). For any limit-ordinal \( x \), let \( J_0 x \) denote the filter of terminal segments of \( x \); if \( x \) is not \( \omega_1 \)-accessible let \( J_1 x \) denote the filter of cofinal closed sets. We have shown Theorem 1 (without A. C.). If every \( x \in \alpha \) is \( \omega_1 \)-accessible then every formula \( F(i) \) in the monadic second order theory \( MT[\omega, \alpha, \nu] \) can be put into the form \( \left( \forall x \right) \left( A \land \left( \forall y \right) \left( r^0 = A \land \left( \forall y \right) \nu y = C[sup(J_0 x, r)] \right) \right) \land D[r^0] \). For limits \( y \) which are not \( \omega_1 \)-accessible the situation changes in two ways. First, the deterministic form must be abandoned, and second, the filter \( J_0 \) must be replaced by a stronger one. Theorem 2 (with A. C.). Every formula \( F(i) \) in \( MT[\omega, \nu] \) can be put into the form \( \left( \forall x \right) \left( A \land \left( \forall y \right) \nu y = C[sup(J_0 x, r)] \right) \land D[sup(J_1 \omega, r)] \). Corollary. \( MT[\omega, \nu] \) is decidable. The method will depend on whether \( \omega_1 \) is \( \omega_0 \)-accessible, and if not, on whether \( B_1 = P(\omega_1)/J_1 \) has atoms. Problem. Is there a model of set-theory in which \( \omega_1 \) is not \( \omega_0 \)-accessible but \( B_1 \) has an atom (\( B_1 \) is prime)? (Received March 8, 1971.)

(Author introduced by Professor John E. Doner.)

662
The common points of families of normal functions.

For every element i of a set A, let Fi be a normal function (i.e., an ordinal valued strictly increasing continuous function defined on the class of all ordinals). Using the illicit convenience of referring to \((F_i)_{i \in A}\) as a "family of normal functions" it is proved: Theorem 1. Every nonempty family of normal functions has arbitrary large common fixed points. Theorem 2. Every normal function has at least one regular cardinal in its range if and only if every nonempty family of normal functions has at least one common strongly inaccessible fixed point. (Received March 8, 1971.)

The ETT punctuation marks "spatium" and "comma".

While the empty space between the elements of a totality serves for the simultaneous spatial separation and connection of them, the comma serves for the material separation and connection of the same. We go out from the fact that the vertical stroke designates the separation and the horizontal dash the connection of the elements. So the dash at the end of a line serves for the connection of separated syllables. Now the comma the scheme of which is "7" has a vertical and a horizontal component. We interpret the comma so that the vertical component separates the elements of the totality, while the horizontal component connects them. Therefore the comma separates and connects simultaneously the elements. There are valid among others the following laws for the empty space and the comma: (I) Material separation and connection between the elements of a totality are possible only after a spatial separation-connection between the same has taken place. For matter needs space. (II) Spatial separation and material separation do not combine with each other to yield a resultant. (III) The same is valid for the spatial and material connection. (IV) Material separation and connection combine with each other and yield a resultant, as we have seen for the comma. (V) Material separation and connection are coexistent. (Received March 8, 1971.)

The closed unbounded filter over \(P_\lambda(\lambda)\).

Let \(x\) be a regular uncountable cardinal and let \(\lambda \geq x\), \(cf(\lambda) \geq x\). Let \(P_\lambda(\lambda) = \{P \subseteq \lambda : |P| < x\}\). A set \(C \subseteq P_\lambda(\lambda)\) is closed unbounded if (a) for every \(\gamma\)-directed system \(D \subseteq C\) with \(|D| < x\), \(\lim D \in C\), and (b) \(C\) is \(\gamma\)-cofinal in \(P_\lambda(\lambda)\). The closed unbounded filter \(F\) over \(P_\lambda(\lambda)\) is generated by the closed unbounded sets.

Theorem. (1) \(F\) is \(\lambda\)-complete, \((\chi, \lambda)\)-regular and closed under diagonal intersections. (2) If \(V = L\) and \(x\) is a successor cardinal then \(F\) is not \(2^\lambda\)-saturated. The proof of (2) uses a generalization of Jensen's axiom \((\ddagger)\). (Received March 9, 1971.)
For definitions see the paper of P. Erdős and A. Hajnal [Bull. Acad. Polon. Sci. Ser. Sci Math. Astronom. Phys. 14(1966), 19-23] (henceforth called EH). The following answers a question (due to J. Mycielski) raised in EH. Theorem 1. For any cardinal $K$, there is a locally finite Jonsson algebra of power $K$ just in case there is any Jonsson algebra of power $K$. Let $Fr(K, \lambda)$ mean that every algebra of power $K$ has a free subset of power $\lambda$. In EH it is shown that if there are no Jonsson algebras of power $K$, then $Fr(K, \omega)$; and clearly, if $Fr(K, K)$ then there can be no Jonsson algebras of power $K$. Theorem 2. $Fr(K, \lambda)$ iff whenever $F = [K]^<\omega \rightarrow [K]^<\omega$ there is $X \in [K]^{<\lambda}$ such that $Y \in [X]^{<\omega} \rightarrow f(Y) \cap X \subseteq Y$. Theorem 3. $Fr(K, \omega) \rightarrow [Fr(K, \omega)]^L$. Theorem 4. If $V = L$ then $Fr(K, \omega)$ iff $K > \omega$. (A combinational result equivalent to Theorem 4 was proved by J. Baumgartner before the author obtained this result by model-theoretic means.) Theorem 5. If $K$ is real-valued measurable, then $Fr(K, K)$. Theorem 6. $\lambda_1 < \lambda_2 \rightarrow [K][Fr(K, \lambda_1)] < [K][Fr(K, \lambda_2)]$. Theorem 7.

Consis(ZFC + Fr(K, \lambda)) \rightarrow Consis(ZFC + Fr(K, \lambda) + 2^{\aleph_0} \geq K). Theorem 8. Consis(ZFC + K has no Jonsson algebras) \rightarrow Consis(ZFC + K has no Jonsson algebras + 2^{\aleph_0} \geq K), answering a question raised in EH. Theorems 3, 4 and 5 strengthen results of J. Silver (Thesis) and Erdős-Hajnal (Acta Math. Acad. Sci. Hungar. 9(1958), 111-131). (Received March 12, 1971.) (Author introduced by Professor John C. Shepherdson.)

Let $M$ be a transitive model of ZF + DC (dependent choice) and $P(\omega) \in M$. (M may be a set or a proper class.) Theorem. $M$ satisfies the Axiom of Determinateness if and only if a cut elimination theorem holds in the determinate logic of $M$. (Received March 19, 1971.)

Let $\{A_i\}_{i \in I}$ be an indexed set of structures of a given type. Put a sheaf $P$ of structures on the discrete space $I$ as follows: for $u \subseteq I$, let $P(u) = \Pi_{i \in u} A_i$ and if $u' \subseteq u$, then we have the restriction homomorphism $P(u) \rightarrow P(u')$. Let Spec($I$) be the Stone space of the Boolean algebra of subsets of $I$. $P$ can be lifted to a sheaf $P^*$ on Spec($I$) by letting $P^*(X_u) = P(u)$ where $X_u$ is the basic open whose elements are all ultrafilters on $I$ containing $u$, and then extending $P^*$ to all opens by taking inverse limits in the usual manner. Then the stalk $P^*_F$ of $P^*$ at an ultrafilter $F$ is precisely the ultraproduct $\Pi_{i \in I} A_i / F$ w.r.t. $F$. That is, $P^*_F = \lim_{u \in F} P^*(X_u)$ over open $X \ni F = \lim_{u \in F} P^*(X_u)$ over basic opens $X_u \ni F$ (cofinality of such basic opens) = $\Pi_{i \in I} A_i / F$. Thus ultraproducts can be constructed naturally as direct limits and all the ultraproducts of an indexed set of structures can be obtained in one fell swoop as the stalks of a certain sheaf on the ultrafilter space of the index set. This natural construction of ultraproducts then leads to generalizations of the ultraproduct construction and the Końwodzinska Theorem (in a context general enough to handle Kripke structures.
and Boolean-valued models), which are developed in my thesis: "On topological model theory: sheaves and ultraproducts." (Received March 26, 1971.) (Author introduced by Dr. Edwin Weiss.)

Theorem. If ZFC + "There exists a supercompact cardinal" is consistent, then ZFC + "There exists a supercompact cardinal \( x \) and \( 2^\aleph_0 \neq \aleph_1^{\alpha+1} \) for arbitrarily large ordinals \( \alpha \) below \( x \)" is consistent. (Received March 29, 1971.) (Author introduced by Professor Azriel Levy.)

Mitotic r.e. sets. Preliminary report.

A recursively enumerable (r.e.) set is mitotic if it is the disjoint union of two r.e. sets of the same degree. Lachlan proved in "The priority method. I" [Z. Math. Logik Grundlagen Math. 13(1967), 1-10] the existence of a nonmitotic r.e. set. The following are theorems: (i) there exists a nonmitotic r.e. set of degree \( \aleph_0 \); (ii) there exists a nonmitotic r.e. set of degree \( \geq \) any given nonrecursive r.e. degree; (iii) there exists nonmitotic r.e. sets in incomparable degrees; (iv) there exists a nonrecursive r.e. degree in which every r.e. set is mitotic; (v) there exists a maximal set which is mitotic; and (vi) there exists a maximal set which is nonmitotic. All the results are proved using priority arguments. The fourth result is proved by an "infinite injury" priority argument. (Received March 29, 1971.)

A discrete chain of degrees of index sets.

Let \( \{ W_x \} \) be a standard enumeration of all r.e. sets. Definition. Let \( Z_{2m} = \{ x | \text{card } W_x = 2i \text{ for some } i \leq m \} \), \( Z_{2m+1} = \{ x | \text{card } W_x = 2i+1 \text{ for some } i \leq m \} \). The sets \( Z_m \) have the following properties in the partial ordering of index sets under 1-1 reducibility: Theorem. For each \( m \), (a) \( Z_{m+1} \) and \( Z_{m+1} \) are incomparable immediate successors of \( Z_m \) and \( Z_m \); (b) Any successor of both \( Z_m \) and \( Z_m \) is a successor of \( Z_{m+1} \) or \( Z_{m+1} \); (c) any predecessor of both \( Z_{m+1} \) and \( Z_{m+1} \) is a predecessor of \( Z_m \) or \( Z_m \). The proofs use the recursion theorem. The sets \( Z_m \) are recursively isomorphic to the sets \( Y_m \) previously used by the author to classify the index sets of finite classes of finite sets. (Received April 9, 1971.)

Some results in a language with a generalized quantifier. Preliminary report.

Let L be a first order language and let L(Q) be the language obtained from L by adding a new quantifier Q to L. The \( \alpha \)-interpretation of L(Q) (\( \alpha \)--an infinite cardinal number) is obtained by interpreting Q as "there exist at least \( \alpha \) elements". The \( \alpha \)-interpretation is denoted by \( \models_\alpha \). Let T denote an ordinary first order theory, and denote by T(\( \alpha \)) the set \{ \( \phi : \phi \) is a sentence in L(Q) such that for every model A of T, if \( |A| \geq \alpha \) then A \( \models_\alpha \phi \)\}. By generalizing Ehrenfeucht's notion (in Fund. Math. 49(1961), 129-141) to L(Q) one can prove:
(1) If $\mathbf{T}$ is the first order theory of $K$ unary relations then $\mathbf{T}(\alpha) = \mathbf{T}(\aleph_0)$ for every $\alpha$ (it was proved by Slomson that $\mathbf{T}(\aleph_0)$ is decidable). (2) If $\mathbf{T}$ is the first order theory of one equivalence relation then $\mathbf{T}(\alpha) = \mathbf{T}(\aleph_0)$ for every $\alpha$ ($\mathbf{T}(\aleph_0)$ is decidable). (3) If $\mathbf{T}$ is the first order theory of one unary function (formulated in a language that does not contain functions-symbols) then $\mathbf{T}(\alpha) = \mathbf{T}(\aleph_1)$ for every regular uncountable $\alpha$ ($\mathbf{T}(\aleph_1) \neq \mathbf{T}(\aleph_0)$).

(4) Denote by $\mathbf{T}_\alpha$ the set of all sentences in $L(\mathcal{Q})$ which are $\alpha$-valid in all well-ordered sets of cardinality $\geq \alpha$. Then (using theorems of Lipner (thesis), Slomson (a preprint from August, 1970) and the generalized notion of Ehrenfeucht) $\mathbf{T}_\alpha = \mathbf{T}_\beta$ for every $\alpha, \beta > \aleph_0$. (Received April 16, 1971.) (Author introduced by Professor Azriel Levy.)

**71T-E56. SAHARON SHELAH, University of California, Los Angeles, California 90024. Isomorphism of ultrapowers.**

Theorem 1. Let $\lambda, \mu$ be infinite cardinals such that $x < \mu$ implies $\lambda^x = \lambda$. Then there is an ultrafilter $D$ over $\lambda$ such that: (A) If $M, N$ are elementarily equivalent models of cardinality $< \mu$ then $M^\lambda/D, N^\lambda/D$ are isomorphic. (B) If $M$ is a model of cardinality $< \mu$, and $2^\mu \not\leq 2^\lambda$ then $M^\lambda/D$ is $\chi^\lambda$-saturated. (C) If $M_k, N_k$ are models of cardinality $\leq \chi < \mu$, for every $k < \lambda$, and $\prod_{k < \lambda} M_k/D, \prod_{k < \lambda} N_k/D$ are elementarily equivalent, then they are isomorphic. (This will appear in Israel J. Math.)

Definition. A model $M$ is $\lambda$-a.c. if: every set of cardinality $< \lambda$ of atomic formulas (with parameters from $M$) which is finitely satisfiable in $M$, is satisfiable in $M$. Definition. A filter $D$ over $\lambda$ is $\lambda$-g.o. if for every ordinal $\alpha$, $\langle \alpha, < \rangle^I/D$ is $\lambda$-a.c.

Theorem. Let $M, N$ be elementarily equivalent models (for simplicity, with finitely many relations), both of cardinality $\leq 2^{\lambda_1}$, $2^{\lambda_1} = 2^\kappa$. Let $D$ be an ultrafilter over $\lambda$, which contains a $\lambda$-g.o. filter. Then $M^\lambda/D, N^\lambda/D$ are isomorphic. (Received April 19, 1971.) (Author introduced by Professor Chen Chung Chang.)

**71T-E57. FRED GALVIN and SAHARON SHELAH, University of California, Los Angeles, California 90024. Negation of partition relations without $\text{CH}$.**

After Erdős, Hajnal and Rado, "Partition relations for cardinals," Acta Math. Acad. Sci. Hungar. 16(1965), 93, we define Definition 1. For a set $A$, natural number $n$, $[A]^n = \{B | B \subset A, \text{cardinality of } B \text{ is } n\}$.

Definition 2. $\lambda = [\mu]^{\kappa}_\delta$ means: for every set $A$, $|A| = \lambda$, $|A|^{\mu} = \bigcup_{i < \kappa} K_i$, where the $K_i$'s are disjoint then there are $j < \kappa$, $B \subset A$, $|B| = \mu$ such that $[B]^\delta \cap K_j = \emptyset$.

Definition 3. $\lambda \not\geq [\mu]^{\kappa}_\delta$ is the negation of $\lambda = [\mu]^{\kappa}_\delta$.

An old result of Sierpinski said that $2^{\aleph_0} \not\geq [\aleph_1]^{2}_{\aleph_0}$. Erdős, Hajnal and Rado prove with $\text{CH}$ that $\aleph_1 \not\geq [\aleph_1]^{2}_{\aleph_1}$.

Partially answering their questions are the following theorems. Theorem 1. $\aleph_1 \not\geq [\aleph_1]^{3}_{\aleph_1}$. Theorem 2. $\aleph_1 \not\geq [\aleph_1]^{2}_{\aleph_1}$. Theorem 3. $2^{\aleph_0} \not\geq [2^{\aleph_0}]^{2}_{\aleph_0}$. Remark. If $2^{\aleph_0}$ is real-valued-measurable then $2^{\aleph_0} > [2^{\aleph_0}]^{2}_{\aleph_1}$ (even more than this). Whether in Theorem 2 we can replace $4$ by a bigger cardinal is an open question. (Received April 19, 1971.)


Definition. $(S, \sim)$ is a typological measure iff $\sim$ is equivalence of atoms; $(S, \alpha)$, extensional equivalence iff partitions form sets of same atoms; $(S, \gamma)$, equipollence iff partitions form bi-univocal sets.
Extensionality.  \( \forall x (x \in A \supset x \in B) \supset A \sim B. \)  Equipollence.  \( \forall x, y (x \sim y \ (\exists F) \ (\text{Bijec } F : x \rightarrow y)) \supset x \sim y. \)  Definition.  \( \sigma \) is an atomized ZFS theory (extensionality written in \( \sigma \)) in first order logic with identity (=) either as syntactic identity, or synonymity transformable by logical rules into identity; \( \Gamma \) is a set theory iff a conservative extension of \( \sigma \) by adjunction of Equipollence.  (If \( T_2 \) is a first order theory derived from \( T_1 \) by adjoining a new function or predicate symbol, and defining axiom, \( T_2 \) is a conservative extension of \( T_1 \): "Mathematical logic," J. R. Schoenfield.)  Metalemma 1.  \( \Gamma \) is a conservative extension of \( \sigma \).  Definition.  \( \langle S, \sim \rangle \) is an extollence iff partitions form equicardinal equopic sets; \( P \) is a set theory iff a conservative extension of \( \Gamma \) with \( \sigma \sim, \gamma \sim \), \( \rho \)-theorems: under the logical rules and appropriate axiom(s), each is an =-equivalence.  Theorem. The \( \rho \)-powerset of any finite \( \rho \)-set forms a set lattice isomorphic to a brouwer algebra with boolean subalgebras.

(Rceived April 21, 1971.)

*71T-E59. RALPH MCKENZIE, University of Colorado, Boulder, Colorado 80302 and SAHARON SHELAH, University of California, Los Angeles, California 90024. In what cardinalities does an equational class have simple algebras?

Definition. An algebra \( \mathfrak{U} \) is simple if every homomorphism from \( \mathfrak{U} \) is either one-to-one or constant (= all elements have the same image). Let the similarity type of \( \mathfrak{U} \) be \( \tau(\mathfrak{U}) \), so \( |\tau(\mathfrak{U})| \) is the number of functions of \( \mathfrak{U} \).  Theorem 1. If \( \mathfrak{U} \) is a simple algebra of cardinality \( \geq (2^{\lambda})^+ \), \( \lambda = |\tau(\mathfrak{U})| + \aleph_0 \), then in every cardinality \( \geq \lambda \) there are simple algebras satisfying all the equations \( \mathfrak{U} \) satisfies.  Theorem 2. If \( \mathfrak{U} \) is a simple algebra of cardinality \( \geq \aleph_1 \), \( |\tau(\mathfrak{U})| \leq \aleph_0 \), then, in every cardinality \( \mu \), \( \aleph_0 \leq \mu \leq 2^{\aleph_0} \) there are simple algebras satisfying all the equations \( \mathfrak{U} \) satisfies.  Remarks. (1) There is an example showing Theorem 1 cannot be improved (for every \( \lambda \)). (2) We can replace "satisfying the equations" by "satisfying every positive universal sentence" and even more; but not by "every universal sentence." (Then we get the Hanf number of omitting type.) (3) In Theorem 2 we can replace \( \aleph_0 \) by e.g. \( \bigcup \omega \).  (Received April 28, 1971.)


We follow the terminology of P. Eklof and G. Sabbagh (Ann. Math. Logic. 2(1971), 251-295). In particular \( \Lambda \) denotes a fixed ring and module means (left) \( \Lambda \)-module.  Theorem. A submodule \( N \) of a module \( M \) is existentially closed in \( M \) if (and only if) \( N \) is a pure submodule of \( M \) and \( M \) and \( N \) satisfy the same universal sentences.  Corollary 1. A module \( M \) is existentially closed iff \( M \) is absolutely pure and Th(M) is model-consistent relative to the theory of modules.  Corollary 2. The theory of every absolutely pure module is model-complete iff \( \Lambda \) is coherent.  Corollary 3. The theory of every module is model-complete iff \( \Lambda \) is absolutely flat.  Corollary 4. Every inductive theory of abelian groups with the joint embedding property has a model-companion.  These results extend and explain much of the above-mentioned paper of Eklof and Sabbagh. A variant of Corollary 4 was communicated to the author by P. Eklof. An explicit description of complete model-complete theories of abelian groups may be given.  (Received April 23, 1971.)
Let \( f(x) \) be a recursive function and let \( D_f(X) \) be its canonical extension to the isols. Let \( A \) and \( Y \) each be isols and such that \( D_f(A) = Y \). Properties. If \( A \) is a regressive isol then \( Y \) will be regressive also. If \( Y \) is a regressive isol then \( A \) need not be regressive; yet in this event there will be a regressive isol \( B \) with \( B \preceq_A A \) and such that \( D_f(B) = Y \). (Here \( \preceq_A \) denotes the Nerode extension of the familiar relation \( \preceq \) to the isols.) (Received April 19, 1971.) (Author introduced by Professor Chen Tung Liu.)

We work in the context of A. Robinson's infinite forcing in model theory, where as usual \( K \) is a first-order set of sentences and \( \Sigma_K \) is the class of subsystems of models of \( K \). (All structures are in \( \Sigma_K \).) Definitions. (1) If \( S \) is either \( \forall \) or \( \exists \), \( S_n \) is the set of formulas in prenex form having \( < n \) blocks in the prefix, or starting with \( S \) and having \( n \) blocks in the prefix. (2) A structure \( \mathcal{U} \) is 0-existentially closed if all the constants of \( K \) are defined in \( \mathcal{U} \); for \( m \geq 1 \), \( \mathcal{U} \) is \( m \)-existentially closed (\( m \)-e. c.) if all the constants of \( K \) are defined in \( \mathcal{U} \) and for any extension \( \mathcal{V} \) of \( \mathcal{U} \), there exists an extension \( \mathcal{W} \) of \( \mathcal{V} \) which is \( (m-1) \)-e. c. and such that any \( \varphi \in \mathcal{V}_{2m} \) with constants from \( \mathcal{U} \) that holds in \( \mathcal{U} \) holds in \( \mathcal{W} \). Theorem 1. The structures which are \( m \)-e. c. for all \( m \) are cofinal in \( \Sigma_K \). Theorem 2. \( \mathcal{U} \) is generic iff \( \mathcal{U} \) is \( m \)-e. c. for all \( m \).

Theorem 3. Fix \( m \). Then \( \mathcal{U} \) is \( m \)-e. c. iff the constants of \( K \) are defined in \( \mathcal{U} \) and for any \( \varphi \in \mathcal{V}_{2m} \) with constants from \( \mathcal{U} \), \( \varphi \) holds in \( \mathcal{U} = \mathcal{W} \) forces \( \varphi \). The \( m \)-e. c. structures have several interesting properties; for example, the class of \( m \)-e. c. structures is inductive for each \( m \). And, using other properties, the behavior of generic structures with respect to elementary subsystems is very clearly and naturally explained. (Received April 26, 1971.) (Author introduced by Professor Dana Scott.)

F. W. Lawvere has developed a foundational basis for mathematics in the setting of category theory, i.e. the basic undefined object is "function" rather than "set". The category of sets is characterized by eight elementary axioms and one nonelementary axiom of completeness. Axiom 3 defines the natural numbers and an immediate consequence of this axiom is the existence of primitive recursive functions. The existence of total recursive functions requires the introduction of the operation of minimization. We define this operation in analogy with the standard definition of computability theory and show it to be equivalent to Lawvere's Axiom 5, the Axiom of Choice. The set of total recursive functions is not effectively enumerable. Thus the existence of a class of undecidable problems is equivalent to the introduction of the Axiom of Choice. (Received April 26, 1971.) (Author introduced by Professor Henry B. Cohen.)
Some formulas of a hologram-logic calculus. Preliminary report.

Let each expression A in a formal language be represented by a Gödel pattern: \( f_A(x,y) = 1 \) for \( a_{j-1} \leq x < a_j \), \( j = 1, \ldots, k \) where \( k \) is the number of primitive symbols in A and \( a_j \) is the index of the jth symbol. The hologram \( H_A(u,v) \) is the Fourier transform of \( f_A(x,y) \). Write: \( f_B(x,y) = f_B(x,y) \) for \( 0 \leq y < s \), \( s = 0 \) otherwise; \( f_B(x,y) = f_B(x,y) - f_B(x,y) \) for concatenation. \( H_{A,B}(u,v) = H_A(u,v) + \exp(ivJ)H_B(u,v) \). A \( \subset B \): Let \( k_1 \) and \( k_2 \) be the lengths of A and B, then for some \( s, 0 \leq s < k_2 - k_1 \), \( H_{B,J}(u,v) = H_{B,J}^0(u,v) \). Substitution of A for the unknown a in the expression A (if a appears as the last term in A): \( H_{A,a}(u,v) = H_A \exp(iv(J-1))/1 - \exp(ivJ) \). Hence substitution within itself yields a hologram with infinite intensities along the horizontal strips \( v = 2\pi/1 \). Formulas for subtraction, dissociation, synthesis etc. can be given. (Received April 6, 1971.)

**Topology**

**71T-G85.** JOHN P. HOLMES, Emory University, Atlanta, Georgia 30322. **Locally Banach semigroups with identity.** Preliminary report.

Suppose \( S \) is a topological semigroup with identity \( e \). If some neighborhood of \( e \) is homeomorphic to Euclidean \( n \)-dimensional space, P. S. Mostert and A. L. Shields ("Semigroups with identity on manifolds," Trans. Amer. Math. Soc. 91(1959), 380-389) have shown \( e \) is interior to the invertible elements of \( S \). This is to announce an example of a topological semigroup \( S \) with a neighborhood of its identity \( e \) homeomorphic to a Banach space having the following properties: (1) \( e \) is not interior to the invertible elements of \( S \). (2) No neighborhood of \( e \) consists entirely of elements having a square root. (Received February 18, 1971.)

**71T-G86.** W. WISTAR COMFORT, Wesleyan University, Middletown, Connecticut 06457 and STELIOS NEGREPONTIS, McGill University, Montréal 110, Québec, Canada. **Extending continuous functions on subspaces of product spaces.** Preliminary report.

Given completely regular, Hausdorff spaces \( X_i (i \in I) \) define \( X_J = \prod_{i \in J} X_i \) for \( J \subset I \), \( X = X_\emptyset \).

**Theorem.** Let \( Y \) be dense in \( X \). (a) If \( Y \) is \( \mathcal{N}_1 \)-pseudocompact (i.e., each of its locally finite families of open subsets is countable) then each \( f \) in \( C(Y) \) depends on countably many coordinates; (b) if in addition \( \pi_j(Y) = X_j \) for each countable \( J_j \), then \( Y \) is C-embedded in \( X \). **Remark.** The hypothesis of (a) holds if each \( X_i \) is separable or compact, or (via the lemmas below) if each \( X_F \) (\( F \) finite) is Lindelöf. Thus the theorem supplements results of Glicksberg, Corson, Corson-Isbell, Engelking, Gleason, Isbell, and Noble-Ulmer, which impose such conditions on the \( X_i \) and/or consider the case \( Y \supset \{ x \in X : |\{ i : x_i \neq p_i \} | = \mathcal{N}_0 \} \) for some \( p \) in \( X \). The chief tools are as follows. **Lemma 1** [N. Noble and M. Ulmer, Trans. Amer. Math. Soc., to appear]. If each \( X_F \) (\( F \) finite) is \( \mathcal{N}_1 \)-pseudocompact, so is \( X \). **Lemma 2.** If \( X \) is \( \mathcal{N}_1 \)-pseudocompact and \( \pi_F(Y) = X_F \) for each finite \( F \), then \( Y \) is \( \mathcal{N}_1 \)-pseudocompact. **Corollary** [N. Noble, to appear]. If each \( X_i \) is separable metric and \( Y \) is \( G_\delta \)-dense, then \( X = \nu Y \). (Received February 18, 1971.)

669
Removing coincidences of two maps.

It is known that if $f, g : X \to Y$ are maps of a topological space $X$ into a topological manifold $Y$, and that $f$ and $g$ can be deformed by homotopies to maps $f'$ and $g'$ which are coincidence-free, then $f$ may be deformed by a homotopy to a map $f''$ such that $f''$ and $g$ are coincidence-free. This result is generalized as follows: If $f, g : X \to Y$ are maps of a topological space $X$ into a topological manifold $Y$ and $f'$ and $g'$ are homotopic to $f$ and $g$ respectively, then for any homotopy $[g_t]$ from $g$ to $g'$ and any homotopy $[f_t]$ from $f$ to $f'$, there is another homotopy $[f_t']$ from $f$ to $f'$ such that, for any $t \in [0,1]$, $f'_t(x) = g(x)$ iff $f'_t(x) = g_t(x)$. There is also a homotopy $[f_t']$ from $f'$ such that the set of coincidences of $f_t'$ and $g_{1-t}$ is the same for all $t \in [0,1]$. Some applications of this result to fixed point theory and root theory are indicated. (Received April 5, 1971.)

Expansions of normal $T_1$ spaces.

Using methods developed in earlier work by the author (cf. Abstract 68T-198, these Notices 15(1968), 358, and Duke Math. J. 29(1962), 589), theorems are established concerning the large classes of normal $T_1$ spaces which cannot be expanded to $T_5$ spaces without changing the dispersion character. These results answer a question raised by E. Hewitt (Duke Math. J. 10(1943), 332). (Received February 19, 1971.)

On the existence of metacompact normal Moore spaces (perfectly normal spaces with $\sigma$-point-finite bases) which are not metrizable.

It is known that the classes of metacompact normal Moore spaces, perfectly normal spaces with $\sigma$-point-finite bases, and normal spaces with uniform bases are identical. Theorem. The existence of either of the following implies the existence of a nonmetrizable member of this class: (1) a locally compact, perfectly normal, metacompact Hausdorff space which is not paracompact, (2) a normal nonmetrizable Moore space which either is locally metacompact or locally satisfies the countable chain condition. (See also Abstract 663-689, these Notices 16(1969), 291.) It follows from known results that it is consistent with the axioms of set theory that there be a nonmetrizable member of this class. Two models of set theory are discussed, which behave oppositely with regard to Souslin spaces and certain normal nonmetrizable Moore spaces. (Received February 22, 1971.)

Quasi-topological groups. II.

Using methods and techniques given in (T. G. Raghavan: "Bitopological groups," 36th I. M. S. Conference, December, 1970, Abstract of papers No. 35) we establish: (1) If $(X, \tau_1, \tau_2)$ is a b.t.s. such that each point has a $\tau_1$-nhghd whose $\tau_2$-closure is $\tau_2$-countably compact and it is pairwise $E_1$ (= Each point is the countable intersection of $\tau_1$-closed $\tau_j$-nhghds, $i \neq j, i, j = 1, 2$) then $\tau_1 \subseteq \tau_2$. (2) If $(X, \tau_1, \tau_2)$ is pairwise Hausdorff b.t.s.
such that $\tau_1$ and $\tau_2$ are P-spaces and each point has a $\tau_1$- nghd whose $\tau_2$- closure is $\tau_2$- Lindelof, then $\tau_1 \subseteq \tau_2$. 

(3) If $(G, \tau, *)$ is a quasitopological group such that the identity is a $G_\delta$- point and has a nghd which is conjugate countably compact then it is a topological group. 

(4) If $(G, \tau, *)$ is a $T_1$ quasitopological group in which the identity is a P-point, and has a nghd which is conjugate Lindelof, then it is a topological group. 

(Received February 22, 1971.) (Author introduced by Professor M. Rajagopalan.)

*71T-G91. WOODLEA B. SCONYERS, Convair Aerospace, P. O. Box 748, Fort Worth, Texas 76101. Metacompact spaces and well-ordered open coverings. Preliminary report.

Compact spaces and paracompact spaces have been characterized by Mack using well-ordered open coverings in a natural way. The following theorem shows that this can be extended to certain generalizations of these spaces. **Theorem.** A space $X$ is metacompact (mesocompact; sequentially mesocompact) if and only if every well-ordered open covering of $X$ has a point-finite (compact-finite; convergent sequence-finite) open refinement. 

(Received February 24, 1971.) (Author introduced by Professor M. Rajagopalan.)


All spaces considered are Hausdorff. Let $E$ be a topological property defined in the class of Hausdorff spaces. Let $N_E$ associate to each space $X$, a collection of nets $N_E(X)$ in $X$ such that $X$ has $E$ if and only if every net in $N_E(X)$ is convergent in $X$. Then $N_E$ is called a determining type of nets for $E$. We characterise in this paper various properties of $E$ as properties of $N_E$. In particular, a property $E$ is closed-hereditary and productive if and only if there exists an 'imaginative' determining type of nets $N_E$ for $E$. The reflector $r_E X$ of any space $X$, in the sense of Herrlich and Van der Slot ('Properties which are closely related to compactness', Nederl. Akad. Wetensch. Proc. Ser. A 70(1967), 524-529) is characterised as the space $N_E(X)$ modulo a natural equivalence, where $N_E$ is the 'largest' imaginative determining type of nets for $E$. $E$-extensions for an $E$-regular space, and pre-order in the class of $E$-extensions are defined in a natural way. It is proved that all $E$-extensions of an $E$-regular space $X$ form a complete upper semilattice under the partial order generated; $r_E X$ characterised above is the 1-element; the $E$-extensions form a complete lattice when $X$ is 'locally $E'$ and 'strongly $E$-normal'. Many interesting results generalising those known for compactifications and realcompactifications are proved. (Received February 24, 1971.) (Author introduced by Professor M. Rajagopalan.)

*71T-G93. JOHN P. HEMPEL and WILLIAM H. JACO, Rice University, Houston, Texas 77001. Finitely presented normal subgroups of the fundamental group of a 3-manifold.

Let $M$ be a compact 3-manifold (with or without boundary, possibly nonorientable). Suppose there is an exact sequence $1 \to N \to \pi_1(M) \to Q \to 1$, where $N$ is a nontrivial finitely presented group and $Q$ is infinite. 

**Theorem 1.** Either $N$ is free or is isomorphic to the fundamental group of a closed 2-manifold. Let $\hat{M}$ be the 3-manifold obtained by capping off any 2-spheres in $\partial M$ with 3-cells. If $M$ is nonorientable assume that $\pi_2(\hat{M}) = 0$. We can write $M = M_1 \# H$ where $H$ is a homotopy 3-sphere and $M_1$ contains no "fake" 3-cells.
**Theorem 2.** In case $N$ is free assume that rank $N \geq 2$. Then either (i) $M_1$ is fiber bundle over $S^1$ with fiber a compact 2-manifold $F$ or (ii) $M_1$ is the union of two twisted $I$-bundles over a compact 2-manifold $F$ which intersect in the corresponding 0-sphere bundles. If $Q/[Q,Q]$ is infinite case (ii) does not occur. In either case $F$ is closed or bounded according as $N$ is the fundamental group of a closed 2-manifold or $N$ is free. A description for $\hat{M}$ is also given in the case $\hat{M}$ is nonorientable and $\pi_2(\hat{M}) \neq 0$. (Received February 25, 1971.)

*71T-G94. JOE A. GUTHRIE and H. EDWARD STONE, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. Subspaces of maximally connected spaces.*

The results of Donald F. Reynolds, Abstract 673-95, these Notices 17(1970), 419, are applied to obtain results about maximally connected spaces. **Theorem 1.** Every connected subspace of a maximally connected space is maximally connected. **Corollary 1A.** Let $\tau$ be a proper extension of a maximally connected topology for $X$. Then the $\tau$-components of $X$ are maximally connected. **Corollary 1B.** A necessary condition for a space $S$ to be a subspace of a maximally connected space is that the components of $S$ be maximally connected. Examples are given to show that no decomposition of a space into "maximally connected components" is possible in general. The maximally connected spaces are properly contained in the class of spaces in which every dense set is open; it is shown that every connected topology can be extended to a connected topology having this property. **Theorem 2.** In a maximally connected Hausdorff space (a) every compact set is finite, (b) no point has a countable local base. (Received February 25, 1971.)

*71T-G95. JASON GAIT and SHU-CHUNG KOO, Wesleyan University, Middletown, Connecticut 06457. Averages of functions and ergodic measures in F-spaces.*

A topological space $X$ is called an F-space if disjoint open $F_\sigma$-sets in $X$ have disjoint closures [Gillman-Jerison: "Rings of continuous functions"]. **Theorem 1.** Let $X$ be a compact F-space. Then for each infinite countable subset $\{x_i\}$ of $X$, there exists an $f \in C(X)$ such that the sequence of partial sums $n^{-1}\sum_{i=1}^{n} f(x_i)$ diverges as $n$ goes to infinity. **Theorem 2.** Let $M$ be an infinite compact minimal set in the discrete flow $(X,h)$ where $X$ is an F-space. Then $M$ supports at least two ergodic $h$-invariant Borel probability measures. In particular, Theorems 1 and 2 apply in the case of a discrete flow on $\mathbb{R}\setminus\mathbb{R}$, extending theorems of Rudin and Raimi for $\mathbb{N}\setminus\mathbb{N}$. (Received March 2, 1971.) (Authors introduced by Professor Walter H. Gottschalk.)


L. F. McAuley's definition of upper semicontinuous decompositions is compared with other segregation properties similar to the Hausdorff condition. The inheritance by the decomposition space of these and conditions such as first-countability are investigated. **Typical theorems.** A pseudo-open point-compact image of a first-countable space is Hausdorff if and only if it has unique sequential limits. Any pseudo-open point-compact image of a developable space is first-countable. (Received March 5, 1971.)
Theorem A. Suppose that $X$ is a regular Hausdorff space such that for each integer $n$, $X^n$ is Lindelöf (paracompact, metacompact; resp.). Then $X^\omega$ is Lindelöf (paracompact, metacompact; resp.) if and only if $X^\omega$ is countably metacompact. Theorem B. Suppose that $X$ is a regular Hausdorff space such that for each $n$, $X^n$ is Lindelöf (paracompact, resp.). Then the following conditions are equivalent: (i) $X^\omega$ is Lindelöf (paracompact, resp.), (ii) $X^\omega$ is normal, (iii) $X^\omega$ is countably paracompact, and (iv) $X^\omega$ is countably metacompact. (Received March 8, 1971.)

Let $G$ be a finitely generated group with $r$ generators and relators $\{R_i\}$. A topological space $X$ is constructed so that the group of homeomorphisms of $X$ is isomorphic to $G$. $X$ is a $T_\text{0}$ space with the property that any point has a minimal open set. If $G$ has finite order $n$, $X$ has $n(2r+1)$ points. A space $B_r$ is constructed which is the weak homotopy type of a wedge of $r$ circles. $X$ is the same homotopy type as the covering space of $B_r$ corresponding to the normal subgroup generated by $\{R_i\}$. (Received March 8, 1971.)

For notation see ["On totally bounded quasi-uniform spaces," Arch. Math. 21(1970), 396-401]. Let $(X, \mathcal{J})$ be a topological space and let $\mathcal{C}$ (\mathcal{G}) be the collection of all nested countable open $\mathcal{Q}$-covers (all nested countable point finite open covers). Let $U (U')$ be the collection of all u.s. c. functions (all u.s. c. functions which are bounded above). Then $\mathcal{U}_\mathcal{C} (\mathcal{U}_\mathcal{G}) = \mathcal{U} (\mathcal{U}')$ and every u.s. c. quasi-uniformity has a transitive base. The quasi-uniformity $\mathcal{U}_\mathcal{G}$ is always precompact and is totally bounded if and only if $\mathcal{U}_\mathcal{G} = \emptyset$. Moreover $\mathcal{U}_\mathcal{C}$ is [precompact] (totally bounded) if and only if every nested countable open $\mathcal{Q}$-cover [has a finite subcover] (is finite). If $(X, \mathcal{J})$ is a Tychonoff space then the following are equivalent: (i) the locally finite covering quasi-uniformity $\mathcal{L} \mathcal{J}$ is precompact. (ii) $\mathcal{J} \mathcal{J}$ is totally bounded. (iii) $\mathcal{L} \mathcal{J} = \emptyset$. If $(X, \mathcal{J})$ is regular then the following are equivalent: (i) The point finite covering quasi-uniformity $\mathcal{F} \mathcal{J}$ is totally bounded. (ii) $\mathcal{U}_\mathcal{J} = \emptyset$. (iii) $\mathcal{F} \mathcal{J} = \emptyset$. A space $(X, \mathcal{J})$ has a compatible transitive quasi-uniformity with the Lebesgue property if and only if every open cover of $X$ has a fundamental open refinement; hence every linearly ordered space has a compatible complete transitive quasi-uniformity. (Received March 9, 1971.)

For topological spaces $X$ and $Y$, let $Y^X$ denote the set of all functions on $X$ to $Y$ and $(X, Y)$ denote the set of continuous functions on $X$ to $Y$. $F \subset Y^X$ is said to have the property (G) if for each open set $U$ in $Y$
and each pointwise closed subset \( G \) of \( F \), \( \cap_{f \in G} f^{-1}(U) \) is open in \( X \). A property called regular has been defined by S. K. Kaul (Canad. Math. Bull. 12(1969), 461-466) for \( F \subset (X, Y) \). The following, among others, are obtained. **Theorem.** If \( Y \) is a uniform space, \( F \subset (X, Y) \) is regular at \( x \), and \( F(x) \) is a totally bounded subset of \( Y \), then \( F \) is equicontinuous at \( x \). Conversely, if \( F \) is equicontinuous at \( x \) and every open cover for \( \overline{F(x)} \) is uniform, then \( F \) is regular at \( x \). **Theorem.** If \( Y \) is Hausdorff or regular, and \( F \subset Y^X \) is compact relative to a jointly continuous topology, then \( F \) has the property (G). **Theorem.** If \( Y \) is regular, and \( F \subset (X, Y) \) is evenly continuous at \( x \) in \( X \), then \( F \) is regular at \( x \) if \( X \) is a P-space and \( \overline{F(x)} \) is Lindelof. A semi-uniform space has been defined by R. V. Fuller (Proc. Amer. Math. Soc. 26(1970), 365-368). A family \( F \) of functions from a topological space \( X \) to a semi-uniform space \( (Y, \mathcal{V}) \) is said to be semi-equicontinuous at \( x \) if for each \( \{V_1, V_2\} \) in \( \mathcal{V} \) there is a neighborhood \( U \) of \( x \) such that \( f(U) \subseteq V_1 \) or \( f(U) \subseteq V_2 \) for each \( f \in F \). **Theorem.** If \( F \subset (X, Y) \), where \( Y \) is a semi-uniform space, is compact relative to a jointly continuous topology, then \( F \) is semi-equicontinuous at each point. Ascoli theorems with weaker assumptions, and a condition for a regular semitopological group to be a topological group also proved. (Received March 8, 1971.)

71T-G101. PAUL R. SPARKS, University of Colorado, Boulder, Colorado 80302. **On products of spaces satisfying the countable chain condition.**

Let \( \text{CCC} \) be the countable chain condition, MA Martin's axiom, and SH Souslin's hypothesis. Let \( \text{MP} \) be the assertion that arbitrary products of \( \text{CCC} \) spaces are again \( \text{CCC} \). Let \( K \) be the assertion that for any uncountable family of open sets, there is an uncountable, pairwise intersecting subfamily. Let \( M \) be the assertion \( \text{CCC} \Rightarrow K \). The following theorem shows that \( M \) and \( \text{MP} \) are independent of ZFC. **Theorem 1.** MA + \( 2^{\aleph_0} > \aleph_1 \Rightarrow M \Rightarrow MP \Rightarrow SH \). Let \( \prod_{i \in I} X_i \) be a product of spaces. Let \( B \) be Bockstein's condition that every regular open set depends on only countably many coordinates. It is known that if each \( X_i \) satisfies \( K \), then \( \prod_{i \in I} X_i \) satisfies \( B \). **Theorem 1** shows that it is consistent to assume that if each \( X_i \) satisfies \( \text{CCC} \) then \( \prod_{i \in I} X_i \) satisfies \( B \). This result can be strengthened to the following theorem. **Theorem 2.** If each \( X_i \) satisfies \( \text{CCC} \), then \( \prod_{i \in I} X_i \) satisfies \( B \). That Theorem 2 cannot be essentially strengthened follows from the following theorem. **Theorem 3.** \( \aleph_1 \times \prod_{i \in I} \aleph_1 \) does not satisfy \( B \). (Received March 9, 1971.) (Author introduced by Professor J. Donald Monk.)


Let \( X \) be a finite \( H \)-complex with homotopy unit. We denote by \( p \)-tors \( A \) the subgroup \( \{a \in A \mid p^n a = 0, \text{some } k\} \) where \( A \) is an abelian group and \( p \) is an integer. **Theorem 1.** If \( X \) has no \( p \)-torsion and \( n < 2p \), then \( \pi_n \) has no \( p \)-torsion and \( p \)-tors \( \pi_{2p} \) is a vector space over \( Z_p \) with dimension equal to \( \dim \ker p^{1} \): \( H^{3}(X; Z_p) \oplus H^{3p+1}(X; Z_p) \rightarrow \mathbb{S}^{2} \) if \( p = 2 \). **Theorem 2.** If \( X \) is simply connected then \( \pi_4 \) is a vector space over \( Z_2 \) with dimension given by the formula in Theorem 1. **Theorem 3.** If \( X \) is simply connected and has no \( 2 \)-torsion, then \( \pi_5 \) is a vector space over \( Z_2 \) with the same dimension as \( \pi_4 \). Every element of \( 2 \)-tors \( \pi_6 \) has order \( \leq 4 \) and \( \dim \pi_6 \otimes Z_2 \) equals \( \dim H^3(X; Z_2) \). Proofs use the spectral sequence of Massey and Peterson, Mem. Amer. Math. Soc. No. 74 and the following lemma. If \( X \) has no \( p \)-torsion, there exists an unstable
module over the Steenrod algebra, $M$, such that $H^*(X;\mathbb{Z}_p) = U(M)$ and is primitively generated. The proof of Theorem 2 uses a result of Weingram, Abstract 70T-G129, these Notices 17(1970), 841. (Received March 3, 1971.)

*71T-G103. NADIM A. ASSAD, University of Iowa, Iowa City, Iowa 52240. A fixed point theorem for weakly uniformly strict contractions. Preliminary report.

Let $M$ be a complete, metrically convex, metric space, and let $K \subset M$. A mapping $\varphi : K \to M$ is said to be a weakly uniformly strict contraction [A. Meir and E. Keeler, J. Math. Anal. Appl. 28(1969), 326-329 ] if given $\epsilon > 0$ there exists $\delta > 0$ such that $\epsilon \leq d(x,y) < \epsilon + \delta$ implies $d(\varphi(x), \varphi(y)) < \epsilon$. Theorem. If $K$ is a nonempty closed subset of $M$ and if $\varphi : K \to M$ is a weakly uniformly strict contraction for which $\varphi(x) \notin K$ for all $x$ in the boundary of $K$, then there exists $x_0 \in K$ such that $x_0 = \varphi(x_0)$. (Received March 18, 1971.)

71T-G104. LUDVIK JANOS, University of Florida, Gainesville, Florida 32601. On the category of contractions.

Let $C$ denote the category whose objects are pairs $(X,f)$ consisting of a compact Hausdorff space $X$ and a map $f : X \to X$ such that the family $\{P_n/X\} = 1$ is evenly continuous and whose morphisms $\varphi : (X,f) \to (Y,g)$ are such continuous maps $\varphi : X \to Y$ for which $g \circ \varphi = \varphi \circ f$. If $\bigcap_{n=1}^\infty P_n(X)$ is a singleton, $(X,f)$ is said to be a contraction. Let $C^*$ be the full subcategory of all contractions and $C^{**}$ the full subcategory of those $(X,f)$ for which $f$ is a homeomorphism. There exist two naturally defined covariant functors $F : C \to C^*$ and $G : C \to C^{**}$, where $F(X,f)$ consists of the quotient space $X/A$ $(A = \bigcap_{n=1}^\infty P_n(X))$ and a map $f^*$ induced by $f$ on $X/A$, and $G(X,f) = (A,f|A)$. Theorem. The product functor $F \times G : C \to C^* \times C^{**}$ is a faithful functor (preserves nonisomorphism of objects) from $C$ onto $C^* \times C^{**}$. In this sense the concept of contraction can be viewed as complementary to that of homeomorphism. (Received March 18, 1971).


$C(X,E)$ denotes the group of all continuous functions from a $\theta$-dimensional Hausdorff space $X$ into a discrete abelian group $E$. Theorem 1. If $X$ is compact, then $C(X,E)$ is the direct sum of $m$ copies of $E$, where $m = \text{weight } X$. Corollary. The group $C(X,E)$ does not determine $X$. Theorem 2. If $X$ is compact and $X_0$ is a closed subset of $X$, then there exists an isomorphism $i$ of $C(X_0,E)$ into $C(X,E)$ such that (a) $i(f)$ is an extension of $f$ if $f \in C(X,E)$, (b) $C(X,E) = i(C(X_0,E)) \oplus C_{X_0}(X,E)$, where $C_{X_0}(X,E)$ is the group of all functions that vanish on $X_0$. Theorem 3. If $X_0$ is a compact subset of $X$ and $X_0$ either has a dense set of isolated points or is a generalized Cantor discontinuum, then $X_0$ is the smallest support of some homomorphism $\varphi : C(X,Z) \to Z$ (Z--the group of integers). The first two theorems and the first half of Theorem 3 are derived from a theorem of Nobeling [Invent. Math. 6(1968), 41-45] that the group of integer-valued bounded functions on an arbitrary set is free. (Received March 9, 1971.)
A bicom pactification of a pair $T_{3\frac{1}{2}}$ bitopological space. Preliminary report.

The product of a family $\{(X_\alpha, P_\alpha, Q_\alpha)\}_{\alpha \in A}$ is $(\prod_{\alpha \in A} X_\alpha, \pi P, \pi Q)$ where $\pi P$ and $\pi Q$ are the usual topologies for $\prod_{\alpha \in A} (X_\alpha, P_\alpha)$ and $\prod_{\alpha \in A} (X_\alpha, Q_\alpha)$ resp. A pair $T_{3\frac{1}{2}}$ space is bicom pact if it is pseudocompact (every pair continuous function from $(X, P, Q)$ into $(\mathbb{R}, R, L)$, the real line with right ray and left ray topologies, is bounded) and pair real-compact (pair homeomorphic to the intersection of a $\pi R$ closed subset with a $\pi L$ closed subset of a product of $(\mathbb{R}, R, L)$). This definition implies compactness in both topologies, but not conversely. Also under this definition, products of bicom pact spaces are bicom pact. According to a definition given by Fletcher, Hoyle and Patty [Duke Math. J. 36(1969), 325-332] $(\mathbb{R}, R, L)$ is pair compact. However not all pair continuous functions on $(\mathbb{R}, R, L)$ to itself are bounded (consider identity) and $(\mathbb{R}, R, L) \times (\mathbb{R}, R, L)$ is not pair compact. A bicom pactification for $(X, P, Q)$ is a bicom pact space $(X^*, P^*, Q^*)$ such that $(X, P, Q)$ is pair homeomorphic to a $P^*$-dense and $Q^*$-dense subset of $X^*$. It can be shown that a pair $T_{3\frac{1}{2}}$ space is pair homeomorphic to a subspace $A$ of a product of $([0,1], R, L)$, which product is bicom pact. Then $(\mathbb{A} \pi R \cap \mathbb{A} \pi L, \pi R, \pi L)$ can be shown to be bicom pact. (Received March 22, 1971.)

On resolvable spaces. Preliminary report.

We call a net one-to-one if, considered as a function, it is one-to-one and its directed set has no last element. For other definitions, see J. G. Cedar, "On maximally resolvable spaces," Fund. Math. 55(1964), 87-93. Theorem. Suppose that for each point $x$ of a space $X$ there is a one-to-one net $f_x$ converging to $x$. Then $X$ is $m$-resolvable, where $m$ is the least infinite cardinal for which there is a subspace $A$ of $X$ of cardinality $m$ such that $f_x$ is eventually in $A$ whenever $x$ belongs to $A$. Theorem. Let $X$ be any of the following: (1) a Hausdorff $k$-space without isolated points, (2) an infinite product, each factor having at least two points, (3) a linear topological space. Then each point of $X$ is a limit of a net which is both one-to-one and well-ordered, and hence $X$ is $m$-resolvable for some infinite $m$. Theorem. Suppose that a subset $A$ of $X$ is closed whenever each limit point of each convergent one-to-one net in $A$ belongs to $A$. Then $X$ is maximally resolvable. Example. There is a Hausdorff $\aleph_0$-resolvable space which is not maximally resolvable. (Received March 22, 1971.)

The diffeomorphism group of certain boundaries. Preliminary report.

The methods of (M. Kervaire and J. Milnor, "Groups of homotopy spheres," Ann. of Math. (2) 77(1963)) are applied to compute $\pi_0^e(Diff(\partial E))$ where $E$ is a smooth regular neighborhood of $K \subset E$, $dim E \equiv 2 \dim K + 3$, $dim K \equiv 2, dim E \equiv 0 \mod 4, E$ is parallelizable, $\pi_1^e(E) = 0$ and $\widetilde{KO}(E, \partial E) = 0$. Let $E'$ containing $E$ be such that $\pi_i^e(E') = \pi_i^e(E')$ for $i \leq \dim E - \dim K$ and $\pi_i^e(E') = 0$ for $i \equiv \dim E - \dim K$. Let $M$ be the double of $E$; for $n = \dim M$ let $j: \widetilde{KO}^{-1}(M) - \pi_n^e(E')$ be the analogue of the $J$ morphism and let $A$ be the cokernel of $j$. Let $\langle E \rangle$ be the space of homotopy equivalences of $E$. Let $\alpha$ denote semidirect product (the obvious one in
Theorem. \( \pi_0(\text{Diff}(E)) \cong (\pi_0(E) \cup K_0^{-1}(E)) \cup A \). Partial results for \( \pi_1(E) \not\equiv 0, K_0(E, \alpha E) \not\equiv 0 \) and \( n \equiv 0 \mod 4 \) are given; as applications of these we have: Example 1. Let \( E \) be a regular neighborhood of \( R^2 \subset R^4 \) where \( r \) is odd so \( K = R^2 \). Let \( C(\alpha E) \) be the group of diffeomorphisms \( \alpha E \to \alpha E \) concordant to the identity. Then \( \text{Diff}(\alpha E)/C(\alpha E) \cong \{Z^2 \} \times V \) where \( 1 \to \pi_4(E') \to V \to Z_2 \) is an exact sequence of groups.

Example 2. Let \( k \equiv 6 \mod 8 \) and \( r \equiv 5 \mod 8 \), \( k < r \). Set \( E = S^k \times D^{r+1} \) and \( K = S^k \times 0 \). Then \( \pi_0(\text{Diff}(S^k \times S^r)) \cong \pi_{k+r+1}(E') \) is a finite abelian group. (Received March 29, 1971.)

*71T-G109. V. KANNAN and T. THRIVIKRAMAN, Madurai University, Madurai-2, India.

A characterisation of the semilattice of compactifications.

A lattice-theoretic characterisation of the semilattice (respectively lattice) of all Hausdorff compactifications of a Tychonoff space (respectively locally compact space) is given. (Received March 29, 1971.)

(Authors introduced by Professor M. Rajagopalan.)

71T-G110. KOK-KEONG TAN and TECK-CHEONG LIM, Dalhousie University, Halifax, Nova Scotia, Canada. C-compact and C-closed spaces.

A \( T_2 \)-space \( X \) is (1) C-compact iff for any closed set \( A \) in \( X \), every open cover of \( A \) has a finite subfamily whose closures cover \( A \); (2) C-closed iff every continuous map from \( X \) into a \( T_2 \)-space is closed. Some characterizations on C-compact, C-closed and minimal \( T_2 \)-spaces are obtained. Examples are shown that (a) a C-closed space may not be C-compact and (b) the product of a compact \( T_2 \) space and a C-compact space may not be C-compact and thus, in particular, C-compactness and C-closedness are not productive. These answer two open questions of Giovanni Viglino's "C-compact spaces", Duke Math. J. (1969), 761-764. (Received March 30, 1971.)

*71T-G111. HAROLD R. BENNETT, Texas Tech University, Lubbock, Texas 79409 and DAVID J. LUTZER, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. A note on weak \( \theta \)-refinability. Preliminary report.

A space \( X \) is weakly \( \theta \)-refinable if each open cover \( \gamma \) of \( X \) has an open refinement \( \gamma' = \bigcup \{V(n) : n \in N \} \) such that if \( x \in X \), there is a \( \gamma'(n) \) with the property that \( \{V \in \gamma'(n) : x \in V \} \) is nonempty and finite. This property is strictly weaker than \( \theta \)-refinability [Worrell and Wicke, "Characterizations of developable topological spaces," Canad. J. Math. 17(1965), 820-830] in which each \( \gamma(n) \) is required to cover \( X \). One can prove: (a) \( X \) has a \( \theta \)-base \([\text{ibid.}]\) iff \( X \) is quasi-developable and hereditarily weakly \( \theta \)-refinable; (b) if \( X \) is weakly \( \theta \)-refinable and perfect (= closed sets are \( G_\delta \)'s), then \( X \) is subparacompact; (c) a linearly ordered topological space (or a subspace thereof) is paracompact iff it is weakly \( \theta \)-refinable; (d) if \( X \) is weakly \( \theta \)-refinable and locally quasi-developable, then \( X \) is quasi-developable. These results can be used to prove, for example, that the product of countably many copies of the Sorgenfrey line is subparacompact. (Received March 31, 1971.)
Semifields and positive linear functionals.

A theorem on extension of positive linear functionals is proved (defined in a linear space over the semifield $\mathbb{R}^I$ with values in $\mathbb{R}^I$) which implies Iseki-Kasahara (Proc. Japan Acad. 41(1965), 29-30) and Kleiber-Pervin (J. Austral. Math. Soc. 10(1969), 20-22) extended versions of Hahn-Banach theorem. Furthermore, the theorem is a consequence of Tychonov theorem for Hausdorff spaces. **Theorem.** Let $(E, \preceq)$ be a preordered linear space over the semifield $\mathbb{R}^I$, $V$ be a linear subspace of $E$ such that each element of $E$ is less than or equal to some element of $V$. If $f: V \to \mathbb{R}^I$ is a positive linear functional on $V$ (with the induced preorder) then there is a positive linear functional $g: E \to \mathbb{R}^I$ such that $g(x) = f(x)$ on $V$. (Received April 1, 1971.)

**Characterizations of $\sigma$-spaces.**

Let $(X, \mathcal{J})$ be a topological space, let $g$ be a function from $\mathbb{N} \times X$ into $\mathcal{J}$ such that for each $x$ in $X$, $x \in \bigcap_{n=1}^{\infty} g(n, x)$. Consider the following conditions on $g$: (A) if $x \in g(n, x)$ then $g(n, x) \subseteq g(n, y)$; (B) if $p \in g(n, x_n)$ for $n = 1, 2, \ldots$ then $x_n \prec p$; (C) if $p \in g(n, y_n)$ and $y_n \in g(n, x_n)$ for $n = 1, 2, \ldots$ then $x_n \prec p$; (D) if $p, x_n \in g(n, x_n)$ and $y_n \in g(n, x_n)$ for $n = 1, 2, \ldots$ then $x_n \prec p$; (E) if $p, x_n \in g(n, x_n)$ and $y_n \in g(n, p)$ then $g(n, x_n) \subseteq g(n, y_n)$ and $x_n \in g(n, p)$. **Theorem.** TFAE for a regular space $X$: (1) $X$ is a $\sigma$-space; (2) $X$ has a function $g$ satisfying (A) and (B); (3) $X$ has a function $g$ satisfying (C); (4) $X$ has a function $g$ satisfying (D); (5) $X$ has a function $g$ satisfying (B) and (E). (The characterization of $\sigma$-spaces in terms of (A) and (B) corrects a misprint in Abstract 682-54-52, these Notices 18(1971).) (Received April 8, 1971.)

**Actions by the bicyclic semigroup.**

For definitions, see Abstract 663-138, these Notices 16(1969), 125. Let $S$ be the bicyclic semigroup with discrete topology. An example is given of a compact space $X$ such that $S$ acts point-transitively ($Sx = X$ for some $x \in X$) on $X$. It is then shown that there is no compact space $X$ such that $S$ acts transitively ($Sx = X$ for all $x \in X$) on $X$. (Received April 8, 1971.)

**Fixed point theorems for noncompact approximative ANR's.**

A Lefschetz fixed point theory for certain upper semicontinuous multi-valued maps has been developed and the concept of MA-space has been defined (see Abstract 71T-G73, these Notices 18(1971)). A space $X$ is a (metric) approximative absolute neighborhood retract (A-ANR) if for each homeomorphism $h: X \to M$ with $M$ a metric space and $h(X)$ closed in $M$, there is a neighborhood $U$ of $h(X)$ and for each open cover $\alpha$ of $h(X)$ there is a continuous map $r_{\alpha}: U \to h(X)$ such that $r_{\alpha}|_{X}$ and $1_{X}$ are $\alpha$-near. A weak A-ANR is defined similarly, except that we only require a continuous map $r_{\epsilon}: U \to h(X)$ with $r_{\epsilon}|_{X}$ and $1_{X}$ $\epsilon$-near for each $\epsilon > 0$. A weak A-ANR satisfies condition $X$ if for each compact subset $K$ of $U$ it is possible to choose maps.
Every A-ANR is an A-space. (A-space is defined similarly to MA-space for single-valued maps; Fund. Math. 64(1969), 157–162.) Theorem 2. Every weak A-ANR satisfying condition \( \chi \) is an MA-space (rel. to the category of metric spaces). (Received April 16, 1971.)

*71T-G116. TSAU-YOUNG LIN, Purdue University, West Lafayette, Indiana 47907. Homological dimensions of Z-graded \( \pi_* \)-modules. Preliminary report.

In the Abstracts 70T-G37 and 70T-G104 (these Noticer 17(1970), 467, 686) we computed the homological dimensions of graded \( \pi_* \)-modules. In this note we extend the computation to Z-graded \( \pi_* \)-modules.

Theorem 1. Let \( A \) be a Z-graded \( \pi_* \)-module. Then the projective dimension of \( A \) in \( \pi_{\infty} \) is either
\[ P.d_{\pi_*} A \leq 2 \] or
\[ P.d_{\pi_*} A = \infty. \]

Theorem 2. Let \( A \) be a Z-graded \( \pi_* \)-module. Then the weak dimension
\[ W.d_{\pi_*} A \] of \( A \) in \( \pi_{\infty} \) is either 0 or \( \infty. \)

Theorem 3. Let \( A \) be a flat Z-graded \( \pi_* \)-module. Then \( A \) is realizable as a stable homotopy module \( \pi_*(L) \) for some spectrum \( L \).

Theorem 4. If \( A \) is a flat \( \pi_* \)-module, then
\[ P.d_{\pi_*} A \leq 2. \]
If we assume the T-nilpotency of \( \pi_* \) (a variant of Barratt's conjecture), then Theorem 1 can be improved, so that the projective dimensions of Z-graded modules are either 0, 1 or \( \infty. \) (Notations. \( \pi_{\infty} \) is the category of Z-graded modules. \( \pi_* \) is the ideal of elements of positive degrees in \( \pi_* \).) (Received April 19, 1971.)

*71T-G117. RICHARD J. TONDRA, Iowa State University, Ames, Iowa 50010. Monotone unions of compact manifolds minus a point.

Let \( M \) and \( N \) be connected \( n \)-manifolds. We will say that \( M \) is compactly equivalent to \( N \) if given any proper compact set \( K \subset M \) there is a proper embedding \( i: (K, K \cap \text{bd}M) \rightarrow (N, \text{bd}N) \), and given any proper compact set \( L \subset N \) there is a proper embedding \( j: (L, L \cap \text{bd}N) \rightarrow (M, \text{bd}M) \). Theorem 1. Let \( M \) be a compact connected \( n \)-manifold such that \( \text{bd}M \neq \emptyset \) and \( n \geq 6 \). If \( p \in \text{bd}M \), then \( M - p \) has the open monotone union property. Theorem 2. Let \( M \) be as above, \( p \in \text{bd}M \). There exists an open connected subset \( D \subset M - p \) such that if \( N \) is a noncompact connected \( n \)-manifold which is compactly equivalent to \( M - p \) and \( \text{bd}N = \text{bd}(M-p) \), then \( N \) is an open monotone union of \( D \). (Received April 21, 1971.)


We construct a tame knot in \( S^3 \) which has no unknotted incompressible spanning surface, thus establishing the existence of a new class of knots lying properly between the Neuwirth knots and all knots. The \( Zt \) module structure of the abelianized commutator subgroup of this knot is isomorphic to that of the square knot. This knot is also a counterexample to a finiteness conjecture stated by the author in his dissertation (University of Michigan, 1970). The basic construction yields a short proof of the known result (Gerhard Burde, Topology 5 (1966), 321–330) that there are infinitely many genus two Neuwirth knots. (Received April 21, 1971.)
Let X be a linearly ordered space with the usual open-interval topology, and let A be a closed subspace of X. We investigate the question: under what conditions is there a linear transformation E from C(A), the vector space of continuous real valued functions on A, to C(X) such that E(g) extends g for each g ∈ C(A). It is proved that E will exist if all but finitely many convex components of X have at least one endpoint in X; this hypothesis will be satisfied if A is compact or if X is conditionally complete (= has no interior gaps). If one considers C*(A), the space of bounded continuous real valued functions, instead of C(A), then the simultaneous extender E will always exist, and the image of E(g) will be contained in the convex hull (in the set of real numbers) of the set g(A). The case where X is a generalized ordered space is also studied. It is proved, for example, that simultaneous extenders E : C(A) → C(X) exist if X is the Sorgenfrey line, and that simultaneous extenders from C*(A) to C*(X) exist for any closed subspace of any generalized ordered space.

(Received April 22, 1971.)

The variety V(C) generated by a class C of topological groups is the smallest class of topological groups which both contains C and is closed under the operations of taking quotients, subgroups, arbitrary products and isomorphic images. It is shown that any topological group G in V(C) is isomorphic to a quotient of a subgroup of a product of members of C. For G Hausdorff, Hausdorff abelian or metric, finer results are established having several consequences, for example: (i) A locally compact Hausdorff group in a variety generated by compact groups is compact; (ii) the group of reals with its usual topology is in the variety generated by a class C of locally compact Hausdorff abelian groups if, and only if, it is a subgroup of a topological group in C; (iii) a metrizable group in a variety generated by separable metric groups is separable. (Received April 26, 1971.)

The concept of a sheaf of H-spaces \( \mathcal{H} \) over a Hausdorff space X is introduced. Using the Čech technique, a cohomology theory with coefficients in a sheaf of H-spaces is constructed in which the cohomology "groups", \( H^p(X, \mathcal{H}) \), are H-spaces. This theory is shown to satisfy Cartan's axioms for a sheaf cohomology theory in the corresponding formulation for sheaves of H-spaces. It is demonstrated that the theory includes the ordinary Čech sheaf cohomology theory. (These results are part of the author's dissertation.) (Received April 26, 1971.)

Let \( \sigma^m \) be the standard m-simplex. The author has proved the following two theorems: Theorem 1. If
$\mathbb{M}^k$ is a compact connected triangulated $k$-manifold, $k \geq 3$, then there exists a monotone open mapping of $\mathbb{M}^k$ onto $\sigma^m$. **Theorem 2.** If $m \geq 3$ and $\mathbb{M}^3$ is a compact triangulated 3-manifold, then there exists a light open mapping of $\mathbb{M}^3$ onto $\sigma^m$ such that each point-inverse set is homeomorphic to the Cantor set. Theorem 1 was announced in 1956 by R. D. Anderson. L. V. Keldys gave a proof (in Russian) in 1957. An immediate consequence of Theorem 2 is that there exists a light open mapping $f$ of any compact triangulated 3-manifold $\mathbb{M}^3$ onto $\sigma^3$ such that the branch set of $f$ is $\mathbb{M}^3$. (Received April 20, 1971.)

**Miscellaneous Fields**

71T-H1. OTOMAR HAJEK, Case Western Reserve University, Cleveland, Ohio 44106. **Pontrjagin's set of winning positions.**

For subsets $P, Q$ of $\mathbb{R}^n$ let $P \ast Q$ be the intersection of all $p - q$ as $q \in Q$ (L. S. Pontrjagin, "Linear differential games 2", Soviet Math. Dokl. 8(1967), 910-912). For a control system $\dot{x} = Ax + p$, $p(t) \in P$ in $\mathbb{R}^n$, let $\mathcal{R}_P(t)$ be the reachable set at time $t \geq 0$, i.e., the set of all $\int_0^t \exp(-As)p(s)ds$ for measurable $p : [0, t] \rightarrow P$. For a pursuit game $\dot{x} = Ax + p - q$, $p(t) \in P$, $q(t) \in Q$ in $\mathbb{R}^n$ let $W(t)$ be the alternating integral (loc. cit., modulo notational changes, with the origin as target) $\int_0^t \exp(-As)P ds \ast \exp(-As)Q ds$. Also let $W_0(t) = \mathcal{R}_P(t) \ast \mathcal{R}_Q(t)$. **Theorem.** Let $P, Q$ be compact, convex and contain 0. Then $W(t)$ coincides with (a) the largest, under inclusion, mapping $M : t \rightarrow M(t) \subset \mathbb{R}^n$ subject to $M(t) \subset W_0(t)$, $M(t) + \exp(-At)M(s) = M(t + s)$ for all $t, s \geq 0$; (b) the integral $\int_0^t \exp(-As)W_0(ds)$, defined as the limit of "Riemann sums" $\exp(-At_1)W_0(t_1) + \ldots$; and (c) the reachable set $\mathcal{R}_S(t)$ corresponding to constraint $S = P \ast Q$. (Received February 22, 1971.)

*71T-H2. JAMES M. McPHERSON, Australian National University, Post Office Box 4, Canberra, Australia. The Reidemeister-Schreier subgroup algorithm.

The theory of covering spaces is used to derive R. H. Fox's version of the Reidemeister-Schreier subgroup algorithm (vide R. H. Fox, "A quick trip through knot theory," in "Topology of 3-Manifolds," edited by M. K. Fort Jr., Prentice-Hall, Englewood Cliffs, N.J., 1963, §8). The proof uses only the most elementary theorems about group presentations and covering spaces. (Received March 8, 1971.)

*71T-H3. BENJAMIN VOLK, 13-15 Dickens Street, Far Rockaway, New York 11691. **Method.**

A sequence of steps was compiled as a method for problem-solving. This extends work of Professor Polya. The following is typical: Spend no more than one minute per step. Answer each question by a simple, one line, sentence. Review answers before cycling. What is the problem? What is your first impulse as to the solution? What items in the problem statement do you now have? What items in the problem statement do you now want? By what process will you go from what you have to what you want? What is given (input)? What is unknown (desired output)? What are the constraints (equations/inequalities)? What expansion with undetermined coefficients will solve problem? (Received March 17, 1971.)
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Subsequent chapters are concerned with differential equations and distributions on a manifold. The possibility of maximal solutions is a point of particular interest and here, for example, is a simple illustration of the influence of the topology of a manifold on its analysis.

In the last two chapters most of the previous work is used to develop the theory of Lie groups and Lie transformation groups. Some fundamental theorems are included whose detailed proofs are not easily available elsewhere.

A notable feature of this text is the provision of numerous interesting examples, many of which have been constructed or collected specially for this book. Each chapter contains problems for solution which are designed to reinforce the reader's comprehension of the text.

300 pages, 6 x 9, $19.95

Announcing

BANACH ALGEBRAS AND SEVERAL COMPLEX VARIABLES
by JOHN WERMER Brown University

The object of this book is to introduce the reader to an area of complex analysis which has flourished during the past fifteen years. It is the borderland between the theory of Banach algebras and classical function theory in one or more complex variables. The emphasis is on questions involving several complex variables, and particularly on problems concerning uniform approximation. The author's aim has been to make the exposition elementary and self-contained. Generality and completeness have cheerfully been sacrificed in many places in order to make it easier for the reader to understand the topic being discussed.

Publication Date 5/1/71
160 pp. $9.50 Hardcover

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RESERVATION FORM

THE PENNSYLVANIA STATE UNIVERSITY
University Park, Pennsylvania 16802
August 30 - September 3, 1971

In order to aid in planning, please return this form before July 30, 1971, to:

Mathematics Meeting
Conference Center
J. Orvis Keller Building
University Park, Pennsylvania 16802

NO DEPOSIT WILL BE ACCEPTED FOR RESIDENCE HALL RESERVATIONS.

Rates for room and breakfast:

<table>
<thead>
<tr>
<th>Category</th>
<th>Adults</th>
<th>Twin rooms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single rooms</td>
<td>$5.50</td>
<td>$4.50</td>
</tr>
<tr>
<td>Twin rooms</td>
<td>$4.50</td>
<td>$4.15</td>
</tr>
</tbody>
</table>

Children 9 years of age and younger:

<table>
<thead>
<tr>
<th>Category</th>
<th>Adults</th>
<th>Twin rooms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single rooms</td>
<td>$5.15</td>
<td>$4.15</td>
</tr>
<tr>
<td>Twin rooms</td>
<td>$4.15</td>
<td>$4.15</td>
</tr>
</tbody>
</table>

There are two single beds in each room.

Requests for residence hall housing will be acknowledged. For those wishing hotel or motel housing, a list of accommodations with rates appears in this issue of these Notices on pages 585, 587.

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NAME (please print, last name first)

MAILING ADDRESS (number and street)

CITY & STATE       ZIP CODE

SUMMER ADDRESS (if different from above)

I will arrive on ___________ at approximately ___________ a.m. p.m. via ___________
date hour car, bus, plane

I will depart on ___________ at approximately ___________ a.m. p.m.
date hour

Please reserve a single room for me. ( )

I wish to share a twin room with another person, name and address of preferred roommate listed below:

Name ___________________________ Address ___________________________

Number of rooms to be reserved for family ___________

These rooms will be occupied by the following: Husband ___________________________ Wife ___________________________

Children's names age sex Children's names age sex

______________________ ___ ___

______________________ ___ ___

______________________ ___ ___

I am interested in the following: I will be interested in eating lunch in the dining hall. ( ) yes ( ) no

# of tickets Event and date

I wish to share a twin room with another person, name and address of preferred roommate listed below:

<table>
<thead>
<tr>
<th># of tickets</th>
<th>Event and date</th>
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<tr>
<td></td>
<td>Amish Market Tour - September 1</td>
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<tr>
<td></td>
<td>Beer Party - September 1</td>
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<tr>
<td></td>
<td>Chicken Barbeque - September 1</td>
</tr>
</tbody>
</table>
ITERATIVE SOLUTION OF LARGE LINEAR SYSTEMS
by DAVID M. YOUNG, Center for Numerical Analysis, University of Texas at Austin, Austin, Texas
A Volume of Computer Science and Applied Mathematics
Series Editor: WERNER RHEINBOLDT
Provides a systematic development of a substantial portion of the theory of iterative methods for solving large linear systems with sparse matrices such as often arise in the numerical solution of elliptic partial differential equations by finite difference methods. The book also treats the successive over-relaxation method (SOR method) including several variants and related methods. Convergence properties of the various methods are studied in terms of the spectral radii of the associated matrices as well as in terms of certain matrix norms.
June 1971, about 560pp., in preparation

AN INTRODUCTION TO TRANSFORM THEORY
by DAVID V. WIDDER, Department of Mathematics, Harvard University, Cambridge, Massachusetts
A Volume in Pure and Applied Mathematics
Series Editors: PAUL A. SMITH and SAMUEL EILENBERG
Discusses some of the most fundamental aspects of the theory of integral transforms. It emphasizes the theoretical side of the subject, although practical applications of the Laplace transform are included in the introduction. Topics covered include Dirichlet's May 1971, 264pp., $15.00

FOURIER ANALYSIS AND APPROXIMATION
Volume I: One-Dimensional Theory
by PAUL L. BUTZER and ROLF J. NESSEL, both of Rheinisch Westfälische Technische Hochschule, Aachen, Germany
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Series Editors: PAUL A. SMITH and SAMUEL EILENBERG
Approximately half of this systematic treatment deals with the theories of Fourier series and Fourier integrals from a transform point of view. The underlying classical theory, therefore, is presented in a form that is directed towards the case of arbitrary locally compact abelian groups. The second half is concerned with the concepts making up the fundamental operation of Fourier analysis, namely convolution. It stresses the theory of convolution integrals and their use in "smoothing" functions, as well as the study of the corresponding degree of approximation. This approach not only embraces many of the topics of the classical theory, but also leads to significant new results, e.g., on summation processes of Fourier series, conjugate functions, fractional integration and limiting behavior of solutions of partial differential equations, and saturation theory.
1971, 490pp., $24.50

NUMERICAL SOLUTION OF PARTIAL DIFFERENTIAL EQUATIONS-II
SYNSPADE 1970
edited by BERT HUBBARD, Institute for Fluid Dynamics and Applied Mathematics, University of Maryland, College Park, Maryland
This book contains, in complete form, the papers presented at the Second Symposium on Numerical Solution of Partial Differential Equations, SYNSPADE, held May 11-15, 1970 at the University of Maryland. These papers range from talks on mathematical numerical analysis to descriptions of numerical methods applied to problems in fluid dynamics, meteorology, and mechanics.
1971, 658pp., $14.00

TRANVERSALE THEORY
An Account of Some Aspects of Combinatorial Mathematics by L. MIRSKY, University of Sheffield, Sheffield, England
A Volume of Mathematics in Science and Engineering
Series Editor: RICHARD BELLMAN
Provides the first systematic investigation of the range of questions which have their origin in Philip Hall's classical theorem on 'distinct representatives'.
1971, 264pp., $13.00

NONLINEAR FUNCTIONAL ANALYSIS AND APPLICATIONS
edited by LOUIS B. RALL, Associate Director, Mathematics Research Center, University of Wisconsin, Madison, Wisconsin
1971, 586pp., $11.00
something new has been added: cassettes

The Audio Recordings of Mathematical Lectures are recordings of lectures presented at meetings of the American Mathematical Society: Gibbs Lectures, Colloquium Lectures, and invited hour addresses. These audio recordings are now being prepared on cassettes as well as tapes. Each lecture is accompanied by a manual which contains displays referred to in the lecture, and the recordings contain editorial comments referring the listener to these displays.

The lectures are recorded on tape at a speed of 1 7/8"/second (4.75 cm/second) and can be played on standard tape recorders; tapes run at 3 3/4"/second are available if requested. Cassettes and tapes may be purchased for $6 each for hour addresses and $10 for Colloquium Lectures. Additional copies of the manual may be ordered for $0.30 each. At the present time, thirty-three lectures are available.

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