Notices

OF THE AMERICAN MATHEMATICAL SOCIETY

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Notices

OF THE

AMERICAN MATHEMATICAL SOCIETY

Edited by Everett Pitcher and Gordon L. Walker

CONTENTS

The Seventy-Sixth Summer Meeting
Pennsylvania State University
University Park, Pennsylvania
August 31-September 3, 1971

The seventy-sixth summer meeting of the American Mathematical Society will be held at The Pennsylvania State University, University Park, Pennsylvania, from Tuesday, August 31, 1971, through Friday, September 3, 1971. All sessions of the meeting will be held on the campus of the university. The times listed for events of the meeting are EASTERN DAYLIGHT SAVING TIME throughout.

There will be two sets of Colloquium Lectures. Professor Lipman Bers of Columbia University will present four lectures entitled "Uniformization, moduli and Kleinian groups." These addresses will be given on Tuesday, August 31, at 1:00 p.m. and on Wednesday, Thursday, and Friday at 8:45 a.m. The other Colloquium Lecturer will be Professor Armand Borel of The Institute for Advanced Study. His topic will be "Algebraic groups and arithmetic groups." Professor Borel's four lectures will be given on Tuesday, August 31, at 2:15 p.m., on Wednesday and Thursday at 10:00 a.m., and on Friday at 11:15 a.m. The first address of each series will be presented in the Schwab Auditorium; the remaining Colloquium Lectures will be presented in the Auditorium of the Conference Center.

The AMS Committee on Employment and Educational Policy will present a panel discussion on Tuesday afternoon, August 31, at 3:30 p.m. in the Schwab Auditorium. Professor William L. Duren, Jr., will serve as moderator of the panel. The remaining members of the panel will be Richard D. Anderson, John W. Jewett, and Gail S. Young. The panelists will discuss current problems of employment of Ph.D. mathematicians and seek the views of the mathematical public on these problems, prospects for the future, and consequences for graduate programs in mathematics.

By invitation of the Committee to Select Hour Speakers for Annual and Summer Meetings, there will be five invited hour addresses at the meeting. Professor John M. Boardman of Johns Hopkins University will give a lecture at 10:00 a.m. on Friday, September 3. His lecture will be entitled "Infinite loop spaces, trees, and the bar construction." Professor Felix E. Browder, University of Chicago, will speak at 1:30 p.m. on Thursday, September 2, on "Nonlinear functional analysis." A lecture entitled "Conjugacy problems in discrete dynamical systems" will be presented by Professor Joel W. Robbin of the University of Wisconsin, Madison, at 1:30 p.m. on Friday, September 3. Professor Isadore M. Singer of the Massachusetts Institute of Technology will present an hour address on Thursday, September 2, at 2:45 p.m. The title of his address will be "Applications of elliptic operators." On Friday, September 3, at 2:45 p.m. Professor Benjamin Weiss of the Hebrew University, Jerusalem, Israel, will give a lecture entitled "Recent progress on the isomorphism problem in ergodic theory." All hour addresses will be given in the Auditorium of the Conference Center. There will be numerous sessions for contributed ten-minute papers. These are scheduled at 11:15 a.m. on Wednesday and Thursday, and at 11:15 a.m. and 4:00 p.m. on Friday. The absolute deadline for receipt of late papers is August 20, 1971. Abstracts received in Providence after this date will be returned. All sessions for contributed papers will be held in the Conference Center.

This meeting will be held in conjunction with meetings of the Mathematical Association of America, Mu Alpha Theta, and Pi Mu Epsilon. The Mathematical Association of America will meet from

Monday through Wednesday. The Association will present Professor Abraham Robinson of Yale University as the Earle Raymond Hedrick Lecturer. The title of Professor Robinson's lectures will be "Nonstandard analysis and nonstandard arithmetic." Mu Alpha Theta and Pi Mu Epsilon will meet concurrently with the Society and the Association.

COUNCIL AND BUSINESS MEETING

The Council of the Society will meet on Tuesday, August 31, at 5:00 p.m. in Room 201 of the Conference Center. The Business Meeting of the Society will be held on Thursday, September 2, at 4:00 p.m. in the Schwab Auditorium. The Steele Prizes will be awarded prior to the Business Meeting.

REGISTRATION

The Registration Desk will be in the lobby of the Conference Center, the J. Orvis Keller Building. It will be open on Sunday from 2:00 p.m. to 8:00 p.m.; on Monday from 8:00 a.m. to 5:00 p.m.; on Tuesday, Wednesday, and Thursday from 8:30 a.m. to 4:30 p.m., and on Friday from 8:30 a.m. to 1:00 p.m. The telephone number will be 814-865-1422. The registration fees will be as follows:

Members	\$ 5.00
Students	\$ 1.00
Nonmembers	\$10.00

There will be no extra charge for members of the families of registered participants.

EMPLOYMENT REGISTER

The Joint Committee on Employment Opportunities has decided not to have an Employment Register at the University Park, Pennsylvania, meeting.

EXHIBITS

Displays of books and educational media will be exhibited in Rooms 114 and 115 on the main floor of the Conference Center on Monday, August 30, from 1:00 p.m. to 5:00 p.m.; on Tuesday and Wednesday, August 31 and September 1, from 9:00 a.m. to 5:00 p.m.; and on Thursday, September 2, from 9:00 a.m. to 12:00 p.m. All participants are encouraged to plan a visit to the exhibits sometime during the summer meeting.

MATHEMATICAL OFFPRINT SERVICE

Information concerning the Mathematical Offprint Service (MOS) as well as assistance in completing MOS profile forms will be available at the MOS Information Desk.

BOOK SALES

Books published by the Society will be sold for cash prices somewhat below the usual prices when the same books are sold by mail. The desk will be in operation during those hours listed under "Exhibits."

RESIDENCE HALL HOUSING

Rooms are available in the West Residence Hall complex near the Conference Center registration headquarters. The residence halls have coin-operated washers and dryers as well as ironing boards. Irons are available.

Rooms may be occupied from 2:00 p.m., Saturday, August 28, until Saturday noon, September 4. Bed linens, blankets, towels, and soap will be provided.

The rates are:

	Room and Breakfast
Adults	\$4.50 double room \$5.50 single room
Children (9 years of age and younger)	\$4.15 double room \$5.15 single room

There is no charge for children in cribs who are sharing a room with their parents; however, the university cannot provide the cribs.

Upon arrival, all guests should check in at the Waring Hall Housing Desk to register for residence hall rooms. Guests are expected to pay for their rooms at the time they check in. Persons who preregister for residence hall accommodations will be sent a marked map showing the location of Waring Hall on the campus. Residence hall personnel will be on duty 24 hours a day to issue keys. Student bellhops will be available in the residence hall area and will accept tips.

Guests must register in advance to be assured of residence hall housing. Please use the form provided on page 688 of the June *Notices*. Residence hall reservation requests will be acknowledged by the Conference Center. It is probable that rooms may be available for those not registering in advance, but this cannot be guaranteed.

FOOD SERVICES

The dining room in Waring Hall will be open for breakfast on Monday, August 30, for residents of the university dormitories. Lunch will be available from Monday through Friday on a cash basis. Lunch tickets may be purchased upon entering the dining room.

The prices are:

Adults	$1.55
Children (9 years of age and younger)	$1.20

The last meal will be breakfast on Saturday, September 4, 1971. Please note that the lodging charges include breakfast. Persons arriving prior to Monday, August 30, will be charged a reduced rate for their room because breakfast will not be available. Meals may be purchased in the nearby Student Union Building or the Nittany Lion Inn by those persons arriving before August 30. Dinner will not be available in Waring Hall.

The Student Union Building has a snack bar and cafeteria. The hours of operation are:

Lion's Den (snack bar)
7:00 a.m.-2:00 p.m. breakfast and lunch

Cafeteria
11:30 a.m.-1:30 p.m. lunch

Cafeteria
4:45 p.m.-6:45 p.m. dinner

A Cafeteria Snack-Coffee Bar will be open in the Conference Center during the meetings. The hours of operation for this snack bar and for the restaurants located in the Nittany Lion Inn will be posted in the registration area. Also available in the registration area will be a listing of local restaurants.

MOTELS AND HOTELS

There are a number of motels and hotels in the State College area, which are listed below with the following coded information: FP - Free Parking; SP - Swimming Pool; AC - Air Conditioned; TV - Television; CL - Cocktail Lounge; RT - Restaurant. Participants should make their own reservations. All prices are subject to change without notice.

AUTOPORT MOTEL (814)237-7666
South Atherton Street - 60 rooms

Singles	$10.00-$12.00
Doubles	15.00- 18.00
Suite	20.00- 22.00

Code: FP-SP-AC-TV-RT
2 miles from Conference Center

COLLEGE COURT MOTEL (814)238-0561
North Atherton Street - 20 rooms

Singles	$ 7.00-$ 9.00
Doubles	10.00- 12.00
Twin	12.00- 14.00 and up

Code: FP-AC-TV
1 mile from Conference Center

DUTCH PANTRY MOTEL (814)238-8461
South Atherton Street - 39 rooms

Singles	$ 9.00
Doubles	13.00- 16.00

Code: FP-SP-AC-TV-CL-RT
1 1/2 miles from Conference Center

HOLIDAY INN (814)238-3001
South Atherton Street - 311 rooms

Singles	$13.00-$14.00
Doubles	17.00
Twin Doubles	19.00
Studios	17.00- 21.00
Suite	26.00

Code: FP-SP-AC-TV-CL-RT
2 miles from Conference Center

IMPERIAL "400" MOTEL (814)237-7686
South Atherton Street - 37 rooms

Singles	$11.00-$13.00
Twin Doubles	14.00- 16.00
	18.00- 20.00
Studio	12.00- 14.00- 16.00

Code: FP-SP-AC-TV-RT
1/2 mile from Conference Center

KAR-MEL MOTEL (814)355-5561
Benner Pike, Route 26, Bellefonte, Pa. - 45 rooms

Singles	$ 8.00-$10.00
Doubles	12.00 and up

Code: FP-AC-TV-RT
8 miles from Conference Center

NITTANY LION INN (814)237-7671
North Atherton Street - 150 rooms

Singles	$13.00 and up
Doubles	15.00 and up
Twin	17.00 and up
Suite	22.00 and up

Code: FP-TV-CL-RT
Next to Conference Center

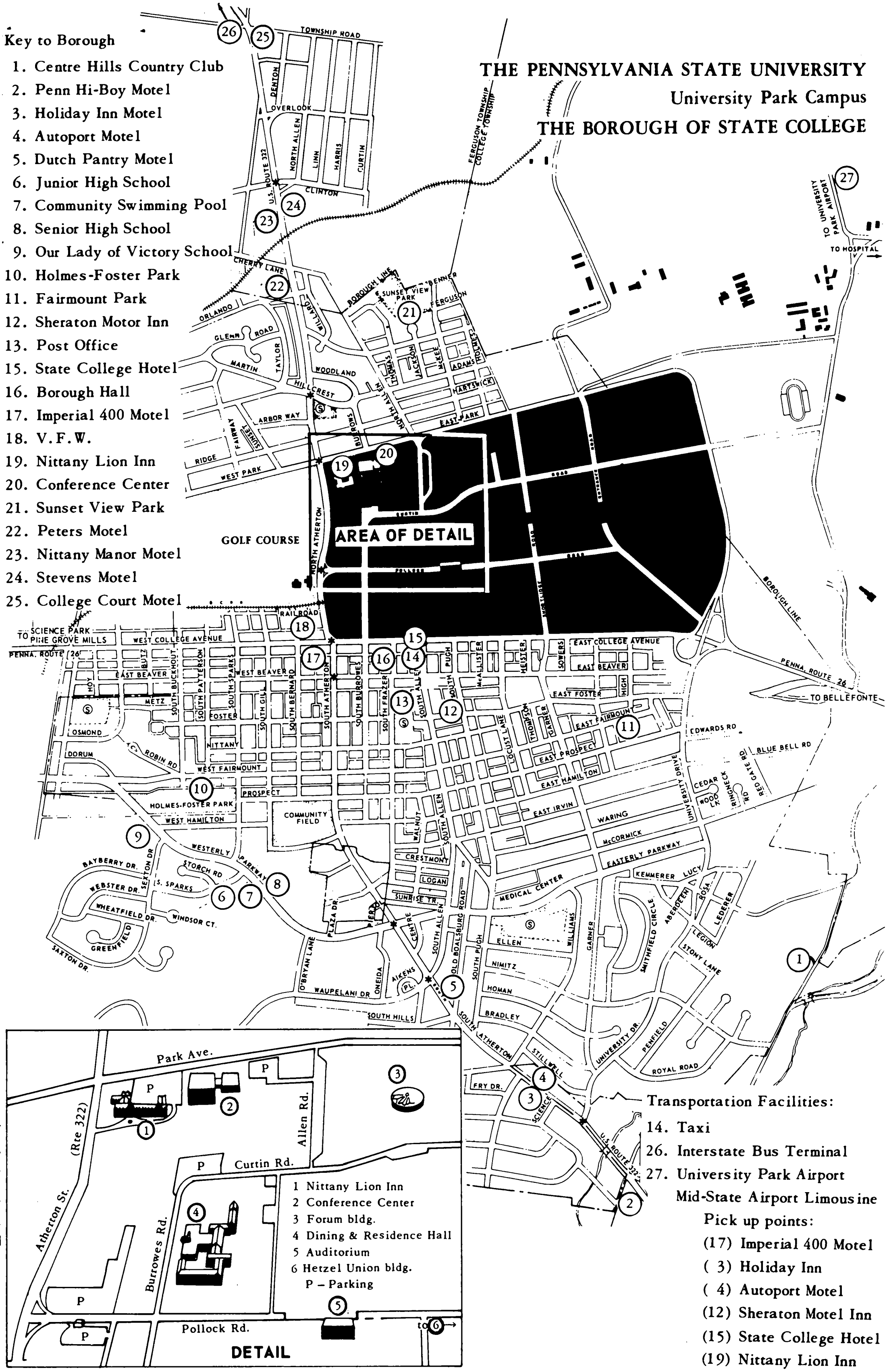
THE PENNSYLVANIA STATE UNIVERSITY

University Park Campus

THE BOROUGH OF STATE COLLEGE

Key to Borough

1. Centre Hills Country Club
2. Penn Hi-Boy Motel
3. Holiday Inn Motel
4. Autoport Motel
5. Dutch Pantry Motel
6. Junior High School
7. Community Swimming Pool
8. Senior High School
9. Our Lady of Victory School
10. Holmes-Foster Park
11. Fairmount Park
12. Sheraton Motor Inn
13. Post Office
15. State College Hotel
16. Borough Hall
17. Imperial 400 Motel
18. V.F.W.
19. Nittany Lion Inn
20. Conference Center
21. Sunset View Park
22. Peters Motel
23. Nittany Manor Motel
24. Stevens Motel
25. College Court Motel



Transportation Facilities:

14. Taxi
26. Interstate Bus Terminal
27. University Park Airport

Mid-State Airport Limousine

Pick up points:

(17) Imperial 400 Motel
(3) Holiday Inn
(4) Autoport Motel
(12) Sheraton Motel Inn
(15) State College Hotel
(19) Nittany Lion Inn

NITTANY MANOR MOTEL (814)237-7638
North Atherton Street - 37 rooms

Singles	$ 9.00-$10.00
Doubles	12.00
Twin Doubles	14.00 and up

Code: FP-AC-TV-RT
3/4 mile from Conference Center

PENN HI-BOY MOTEL (814)238-0571
South Atherton Street - 20 rooms

Singles	$ 9.00
Doubles	12.00- 14.00
Twin Doubles	16.00

Code: FP-AC-TV-RT
2 1/2 miles from Conference Center

PETERS MOTEL (814)238-6783
North Atherton Street - 20 rooms

Singles	$ 9.00
Doubles	12.00
Twin Doubles	14.00 and up

Code: FP-AC-TV
1/2 mile from Conference Center

SHERATON MOTOR INN (814)238-8454
South Pugh Street - 98 rooms

Singles	$12.00-$15.00
Doubles	17.00- 21.00
Suite	32.00 and up

Code: FP-SP-AC-TV-CL-RT
3/4 mile from Conference Center

STATE COLLEGE HOTEL (814)237-4350
West College Avenue - 40 rooms

Singles	$ 7.00-$ 8.00
Doubles	10.00- 12.00
Twin	12.00- 13.00

Code: AC-TV-RT
1/2 mile from Conference Center

STEVENS MOTEL (814)238-2438
North Atherton Street - 16 rooms

Singles	$ 8.00-$ 9.00
Doubles	11.00- 13.00
Family	14.00 and up

Code: FP-AC-TV
3/4 mile from Conference Center

PARKING

Permits for on-campus parking will be issued at check-in to the residence hall. Persons living in off-campus hotels and motels will be issued a parking permit at registration for the meetings.

CAMPING

There are three state parks with camping facilities within twenty-five miles of State College: Black Moshannon, Poe Valley, and Greenwood Furnace. Information on state parks may be obtained from Public Relations, Department of Forests and Waters, Commonwealth of Pennsylvania, Harrisburg, Pennsylvania 17120. There are several private campgrounds in the area: Bald Eagle Campsite, R.D. 3, Tyrone, Pennsylvania 16686; Bellefonte Kampgrounds of America, R.D. 2, Bellefonte, Pennsylvania 16823; Hartle Trailer Court, Drifting, Pennsylvania 16834; Snow Shoe Lions Club Park, Snow Shoe, Pennsylvania 16874.

BOOKSTORE

There is no bookstore on campus. However, there are several good bookstores in the downtown area which are open daily during normal working hours.

LIBRARIES

Pattee Library (main library) will be open from 7:00 a.m. to 10:00 p.m., Monday through Friday. It is convenient to the Conference Center. The Mathematics Reading Room houses the university's mathematics collection and is located in the McAllister Building. It will be open from 8:00 a.m. to 11:00 p.m. Monday through Friday.

MEDICAL SERVICES

The university's Ritenour Health Center will be open to treat emergencies twenty-four hours a day. This emergency service is available at no charge to registered participants. Centre Community Hospital, in Bellefonte (12 miles northeast), can handle any serious medical or surgical emergencies.

ENTERTAINMENT

One of the most interesting Amish Communities in the United States is located twenty-five miles from State College. A lecture on the Amish entitled "Barndoor Britches and Shoo-fly Pie" will be given by Dr. Maurice Mook on Tuesday, August 31, at 7:30 p.m. A tour of the Amish market in Belleville is scheduled for Wednesday, September 1, from 8:45 a.m. to 1:30 p.m. Tickets will be on sale in the registration area.

A beer party, sponsored by the Society for Industrial and Applied Mathematics, will be held on Wednesday, September 1, at 8:00 p.m. at the Skimont

Lodge, five miles east of town. Transportation will be provided. Tickets will be sold in the registration area.

A chicken barbecue on Wednesday evening from 5:00 p.m. to 7:00 p.m. will be held in the University Skating Pavilion. Tickets, which are $3.25 per person for adults and $2.00 each for children under nine, will be on sale in the registration area.

A Chess Exhibition will be held. Grand Master Donald Byrne of the Penn State English Department will play twenty-five mathematicians simultaneously at 7:30 p.m. on Thursday, September 2, in the Hetzel Union Building. Persons interested in participating in this event should write to Scott Williams, Department of Mathematics, McAllister Building, University Park, Pennsylvania 16802. There may be a small fee.

Conducted tours of the university flower gardens will be available. If there is sufficient interest, bird-watching trips will also be arranged.

There will be a social hour from 5:00 p.m. to 7:00 p.m. in the Assembly Room of the Nittany Lion Inn on Monday, Tuesday, and Thursday evenings. A cash bar will be set up and all participants are invited to attend.

Other diversions and facilities include summer theatre, concerts; visits to the Mineral Industries Museum, university creamery, barns, and orchards; golf, tennis, bowling, ping-pong, gymnasium facilities, swimming pools (indoor and outdoor), picnicking, and movies.

Within a twenty-five mile radius there are state parks which have swimming beaches, marked hiking trails, camping, boating; fishing in Penn's Creek, an internationally famed trout stream; Amish country, Indian Caverns, Penn's Cave (an all-water cavern explored by boat), the Boal Mansion and Christopher Columbus Chapel, and Bear Meadows (a marshy area in the Seven Mountains southeast of State College) which supports botanical growth otherwise foreign to this area and climate.

Arrangements will be made to provide nursery care for children from two through five years old, a supervised playground program for the elementary level, and a few scheduled events for the junior and senior high group. A list of babysitters for evening hours will be available.

TRAVEL

Penn State is located at the geographic center of Pennsylvania in the borough of State College. Interstate 80 has three exits (Lamar, Jacksonville, and Milesburg) within twenty-five miles of State College. Greyhound and Continental Trailways bus lines serve State College from all major cities. Penn Central Railroad serves State College (at Lewistown depot, thirty miles southeast) from all major east and west points. Buses for State College meet some of the trains (the fare is $1.45); those trains with this connection are indicated on rail schedules. Pennsylvania Commuter Airline serves State College (at University Park Airport, five miles northeast) from Baltimore and Washington, D.C. (National Airport). Taxi fare from University Park Airport is $2.50 per person. Allegheny Airlines serves State College (at Midstate Airport, twenty-five miles northwest) from New York City, Philadelphia, and Pittsburgh. Limousines meet all Allegheny flights (the fare is $3.25). Cars can be rented at Midstate Airport and in several locations in State College. Persons completing the residence hall reservation form will receive additional travel information.

WEATHER

The average high temperature for this period is 80° and the average low is 58°. Rainfall in August amounts to 3.2". The humidity is usually not excessive.

MAIL AND MESSAGE CENTER

Individuals may be addressed at Mathematics Meeting, Conference Center, Keller Building, University Park, Pennsylvania 16802. The telephone number of the Message Center will be 814-865-1422.

COMMITTEE

H.L. Alder (ex officio), Mrs. Patricia Axt, Raymond Ayoub, Walter H. Gottschalk (ex officio), Thomas Grilliot, Michael Keenan, Sidney Mack, Torrence Parsons, Mrs. Pilar Ribeiro, Donald C. Rung (chairman), Joel Schneider, Joseph Warren, Gordon L. Walker (ex officio), Scott Williams, Thomas Worgul.

TIMETABLE

CC-Conference Center

(Eastern Daylight Saving Time)

Sunday August 29	American Mathematical Society	Mathematical Association of America
9:00 a.m. - 4:00 p.m.		Board of Governors Rooms 402-403 Conference Center
2:00 p.m. - 8:00 p.m.	REGISTRATION - Lobby of Conference Center	
		Auditorium, Conference Center
7:00 p.m.		Films
		Films of the MAA Individual Lectures Film Project (ILFP)
7:00 p.m. - 7:25 p.m.		Shapes of the future I - Some unsolved problems in geometry - two dimensions with Victor Klee (in color)
7:35 p.m. - 8:15 p.m.		Shapes of the future II - Some unsolved problems in geometry - three dimensions with Victor Klee (in color)
8:25 p.m. - 8:54 p.m.		A Film of the Educational Broadcasting Corporation of New York: New world, new math (in color)

Monday August 30	AMS	MAA
8:00 a.m. - 5:00 p.m.	REGISTRATION - Lobby of Conference Center	
		Schwab Auditorium
9:00 a.m. - 10:00 a.m.		The Earle Raymond Hedrick Lectures: Nonstandard analysis and nonstandard arithmetic Lecture I Abraham Robinson
10:10 a.m. - 11:40 a.m.		Panel discussion: What undergraduate courses will be taught in 1984? A look into the future. Paul Axt (moderator) Garrett Birkhoff Murray Gerstenhaber J. B. Rosser
11:40 a.m. - 12:00 p.m.		General discussion by the panel and the audience
1:00 p.m. - 5:00 p.m.	EXHIBITS - Rooms 114-115 of Conference Center	
2:00 p.m. - 3:00 p.m.		The Earle Raymond Hedrick Lectures: Lecture II Abraham Robinson
3:10 p.m. - 4:40 p.m.		Panel discussion: Women in mathematics Christine W. Ayoub (moderator) Mary Gray Gloria C. Hewitt Mary E. Rudin
4:40 p.m. - 5:00 p.m.		General discussion by the panel and the audience
5:00 p.m. - 7:00 p.m.	SOCIAL HOUR - Assembly Room - Nittany Lion Inn	

CC-Conference Center

Monday August 30	American Mathematical Society	Mathematical Association of America
		Auditorium, Conference Center
7:00 p.m.		Films
		Films of the NCTM series: Elementary mathematics for teachers and students (in color)
7:00 p.m. - 7:11 p.m.		Between rational numbers (Knights)
7:12 p.m. - 7:22 p.m.		Reciprocals - multiplicative inverses (Sunglasses)
7:23 p.m. - 7:33 p.m.		The biggest rectangle
7:34 p.m. - 7:44 p.m.		Hidden treasure
7:45 p.m. - 7:56 p.m.		Exploitation of errors (Edgar's Guess)
7:57 p.m. - 8:07 p.m.		Solving pairs of equations (Pirates)
8:08 p.m. - 8:17 p.m.		Graphing inequalities (Marvelous Marshes)
8:18 p.m. - 8:30 p.m.		Probability (Rajah)
8:40 p.m. - 9:23 p.m.		A Film of the MAA Mathematics Today series (in color): Göttingen and New York, Reflections on a life in mathematics - Richard Courant

Tuesday August 31	AMS	MAA
8:30 a.m. - 4:30 p.m.	REGISTRATION - Lobby of Conference Center	
9:00 a.m. - 5:00 p.m.	EXHIBITS - Rooms 114-115 of Conference Center	
		Schwab Auditorium
9:00 a.m. - 9:50 a.m.		The Earle Raymond Hedrick Lectures: Lecture III Abraham Robinson
10:00 a.m. - 11:00 a.m.		Business Meeting Presentation of Lester R. Ford Awards
11:10 a.m. - 12:00 p.m.		History in the mathematics curriculum: its status, quality, and function. R. L. Wilder
12:15 p.m.	PI MU EPSILON - Council Luncheon - Waring Hall Dining Room	
1:00 p.m. - 2:00 p.m.	Colloquium Lectures: Uniformization, moduli and Kleinian groups, Lecture I Lipman Bers Schwab Auditorium	
2:15 p.m. - 3:15 p.m.	Colloquium Lectures: Algebraic groups and arithmetic groups, Lecture I Armand Borel Schwab Auditorium	
3:15 p.m. - 5:15 p.m.	PI MU EPSILON - Contributed Papers - Rooms 312-314, Conference Center	

CC-Conference Center

Tuesday August 31	American Mathematical Society	Mathematical Association of America
3:30 p.m. - 4:30 p.m.	Panel discussion: Employment and educational policy Richard D. Anderson William L. Duren, Jr. (moderator) John W. Jewett Gail S. Young Schwab Auditorium	
5:00 p.m.	Council Meeting Room 201, Conference Center	
5:00 p.m. - 7:00 p.m.	SOCIAL HOUR - Nittany Lion Inn - Assembly Room	
6:30 p.m.	PI MU EPSILON - Banquet - Dining Room B, Hetzel Union Building	
		Auditorium, Conference Center
7:00 p.m.		Films
		Allendoerfer geometry films (animated and in color)
7:00 p.m. - 7:10 p.m.		Geometric concepts
7:11 p.m. - 7:21 p.m.		Area and pi
7:22 p.m. - 7:32 p.m.		Geometric transformations
7:30 p.m.	LECTURE ON AMISH LIFE - "Barndoor britches and shoo-fly pie" Room 402, Conference Center	
7:40 p.m.		Films of the MAA Mathematics Today series
7:40 p.m. - 8:35 p.m.		Challenge in the classroom, the methods of R. L. Moore (in color)
8:45 p.m. - 9:28 p.m.		Predicting at random, a lecture by David Blackwell (in color)

Wednesday September 1	AMS	MAA
8:00 a.m.	PI MU EPSILON - Dutch Treat Breakfast - Waring Hall Dining Room	
8:30 a.m. - 4:30 p.m.	REGISTRATION - Lobby of Conference Center	
8:45 a.m. - 9:45 a.m.	Colloquium Lectures: Lecture II Lipman Bers Auditorium, Conference Center	
8:45 a.m. - 1:30 p.m.	TOUR - Amish Market, Belleville	
9:00 a.m. - 12:00 p.m.	MU ALPHA THETA - Council Meeting - Room 113 of Conference Center	
9:00 a.m. - 5:00 p.m.	EXHIBITS - Rooms 114-115 of Conference Center	
10:00 a.m. - 11:00 a.m.	Colloquium Lectures: Lecture II Armand Borel Auditorium, Conference Center	
10:40 a.m. - 12:40 p.m.	PI MU EPSILON - Contributed Papers - Rooms 312-314, Conference Center	
11:15 a.m. - 12:25 p.m.	Session on Algebra I Room 301 (CC)	

CC-Conference Center

Wednesday September 1	American Mathematical Society	Mathematical Association of America
11:15 a.m. - 12:25 p.m.	Session on Analysis Room 303 (CC)	
11:15 a.m. - 12:25 p.m.	Session on Complex Analysis I Room 305 (CC)	
11:15 a.m. - 12:25 p.m.	General Session I Room 306 (CC)	
11:15 a.m. - 12:25 p.m.	Session on Group Theory Room 311 (CC)	
11:15 a.m. - 12:10 p.m.	Session on Matrix Theory Room 401 (CC)	
11:15 a.m. - 12:25 p.m.	Session on Topology I Room 405 (CC)	
		Schwab Auditorium
1:30 p.m. - 2:40 p.m.		Panel discussion: Concerns of community colleges C. A. Lathan J. S. Mamelak (moderator) Ralph Mansfield R. D. Mazzagatti
2:40 p.m. - 3:00 p.m.		General discussion by the panel and the audience
3:10 p.m. - 4:10 p.m.		Panel discussion: Placement tests in mathematics - how valid are they? R. P. Boas (moderator) Marion G. Epstein Gene Murrow Alex Rosenberg
4:10 p.m. - 4:30 p.m.		General discussion by the panel and the audience
5:00 p.m. - 7:00 p.m.	CHICKEN BARBECUE - University Skating Pavilion	
8:00 p.m.	SIAM BEER PARTY - Skimont Lodge	

Thursday September 2	AMS	MAA
8:30 a.m. - 4:30 p.m.	REGISTRATION - Lobby of Conference Center	
8:45 a.m. - 9:45 a.m.	Colloquium Lectures: Lecture III Lipman Bers Auditorium, Conference Center	
9:00 a.m. - 12:00 p.m.	EXHIBITS - Rooms 114-115 of Conference Center	
10:00 a.m. - 11:00 a.m.	Colloquium Lectures: Lecture III Armand Borel Auditorium, Conference Center	
11:00 a.m. - 12:30 p.m.	MIDWEST CATEGORY SEMINAR — Room 312, Conference Center	
11:15 a.m. - 12:25 p.m.	Session on Algebra II Room 301 (CC)	
11:15 a.m. - 12:25 p.m.	Session on Combinatorics Room 303 (CC)	

CC-Conference Center

Thursday September 2	American Mathematical Society	Mathematical Association of America
11:15 a.m. - 12:25 p.m.	Session on Complex Analysis II Room 305 (CC)	
11:15 a.m. - 12:25 p.m.	Session on Functional Analysis I Room 311 (CC)	
11:15 a.m. - 12:25 p.m.	General Session II Room 306 (CC)	
11:15 a.m. - 12:10 p.m.	Session on Probability and Statistics Room 401 (CC)	
11:15 a.m. - 12:10 p.m.	Session on Topology II Room 405	
1:30 p.m. - 2:30 p.m.	Invited address: Nonlinear functional analysis Felix E. Browder Auditorium, Conference Center	
2:45 p.m. - 3:45 p.m.	Invited address: Applications of elliptic operators Isadore M. Singer Auditorium, Conference Center	
4:00 p.m.	Business Meeting Presentation of L. P. Steele Prizes Schwab Auditorium	
5:00 p.m. - 7:00 p.m.	SOCIAL HOUR - Nittany Lion Inn - Assembly Room	
7:30 p.m.	CHESS EXHIBITION - Hetzel Union Building - Donald Byrne, Grandmaster	

Friday September 3	AMS	MAA
8:30 a.m. - 1:00 p.m.	REGISTRATION - Lobby of Conference Center	
8:45 a.m. - 9:45 a.m.	Colloquium Lectures: Lecture IV Lipman Bers Auditorium, Conference Center	
10:00 a.m. -11:00 a.m.	Invited address: Infinite loop spaces, trees, and the bar construction John M. Boardman Auditorium, Conference Center	
11:15 a.m. - 12:15 p.m.	Colloquium Lectures: Lecture IV Armand Borel Auditorium, Conference Center	
11:15 a.m. - 12:25 p.m.	Session on Algebra III Room 301 (CC)	
11:15 a.m. - 12:25 p.m.	Session on Functional Analysis II Room 311 (CC)	
11:15 a.m. - 12:10 p.m.	Session on Logic Room 303 (CC)	
11:15 a.m. - 12:25 p.m.	Session on Ordinary Differential Equations Room 305 (CC)	

CC-Conference Center

Friday September 3	American Mathematical Society	Mathematical Association of America
11:15 a.m. - 12:10 p.m.	Session on Semigroups Room 401 (CC)	
11:15 a.m. - 12:10 p.m.	Session on Topology III Room 405 (CC)	
1:30 p.m. - 2:30 p.m.	Invited address: Conjugacy problems in discrete dynamical systems Joel W. Robbin Auditorium, Conference Center	
2:45 p.m. - 3:45 p.m.	Invited address: Recent progress on the isomorphism problem in ergodic theory Benjamin Weiss Auditorium, Conference Center	
4:00 p.m. - 4:55 p.m.	Session on Algebra IV Room 301 (CC)	
4:00 p.m. - 5:25 p.m.	Session on Applied Mathematics Room 401 (CC)	
4:00 p.m. - 4:55 p.m.	Session on Approximation Theory Room 303 (CC)	
4:00 p.m. - 4:40 p.m.	General Session III Room 306 (CC)	
4:00 p.m. - 5:25 p.m.	Session on Number Theory Room 311 (CC)	
4:00 p.m. - 5:25 p.m.	Session on Topology IV Room 405	

PROGRAM OF THE SESSIONS

The time limit for each contributed paper is ten minutes. The contributed papers are scheduled at fifteen minute intervals. To maintain this schedule, the time limit will be strictly enforced.

TUESDAY, 1:00 P.M.

Colloquium Lectures: Lecture I, Schwab Auditorium
Uniformization, moduli and Kleinian groups
Professor Lipman Bers, Columbia University

TUESDAY, 2:15 P.M.

Colloquium Lectures: Lecture I, Schwab Auditorium
Algebraic groups and arithmetic groups
Professor Armand Borel, Institute for Advanced Study

TUESDAY, 3:30 P.M.

Panel Discussion, Schwab Auditorium
Employment and Educational Policy
Professor Richard D. Anderson, Louisiana State University, Professor William L. Duren, Jr. (Moderator), University of Virginia, Professor John W. Jewett, Oklahoma State University, and Professor Gail S. Young, University of Rochester

WEDNESDAY, 8:45 A.M.

Colloquium Lectures: Lecture II, Auditorium, Conference Center
Uniformization, moduli and Kleinian groups
Professor Lipman Bers, Columbia University

WEDNESDAY, 10:00 A.M.

Colloquium Lectures: Lecture II, Auditorium, Conference Center
Algebraic groups and arithmetic groups
Professor Armand Borel, Institute for Advanced Study

WEDNESDAY, 11:15 A.M.

Session on Algebra I, Room 301

11:15-11:25
(1) Simple components of group algebras Q[G] which are central over totally real fields
Dr. Toshihiko Yamada, Queen's University (687-12-1)

11:30-11:40
(2) Generalized k-linear equations over a finite field
Dr. Robert G. Van Meter, State University of New York, College at Oneonta (687-12-2)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.

11:45-11:55

(3) On polynomials which commute with a given polynomial

Dr. William M. Boyce, Bell Telephone Laboratories, Inc., Murray Hill, New Jersey (687-12-3)

12:00-12:10

(4) Separation of nonassociates by valuations

Professor Max D. Larsen* and Dr. David E. Brown, University of Nebraska (687-13-1)

12:15-12:25

(5) Contracted ideals in power series rings

Professor Chin-Pi Lu, University of Colorado, Denver Center (687-13-3)

WEDNESDAY, 11:15 A.M.

Session on Analysis, Room 303

11:15-11:25

(6) Riemann function type solutions of boundary initial problems

Dr. Richard Kraft, National Bureau of Standards, Washington, D. C. (687-35-1)

11:30-11:40

(7) On the existence of not necessarily unique solutions of semilinear hyperbolic systems in two independent variables

Mr. Thomas J. Langan, Naval Ship Research and Development Center, Carderock, Maryland, and University of Maryland (687-35-2)
(Introduced by Dr. John W. Wrench, Jr.)

11:45-11:55

(8) On certain polynomials occurring in continued fractions

Dr. Reinhart Hitz, Old Dominion University (687-40-1)

12:00-12:10

(9) Riesz means and Hankel transform

Professor Kusum K. Soni, University of Tennessee (687-44-1)

12:15-12:25

(10) Some properties of continuous mean periodic functions. Preliminary report

Mr. Philip Laird, University of Calgary (687-46-5)
(Introduced by Professor James S. Muldowney)

WEDNESDAY, 11:15 A.M.

Session on Complex Analysis I, Room 305

11:15-11:25

(11) On the values omitted by univalent functions with two preassigned values. Preliminary report

Professor Maxwell O. Reade*, University of Michigan, and Professor Eliguisz Złotkiewicz, M. Curie-Skłodowska University, Lublin, Poland (687-30-1)

11:30-11:40

(12) On close-to-convex functions of higher order

Professor Adolph W. Goodman, University of South Florida (687-30-2)

11:45-11:55

(13) A note on the 2/3 conjecture for starlike functions

Dr. Carl P. McCarty*, LaSalle College, and Dr. David E. Tepper, Delaware State College (687-30-5)

12:00-12:10

(14) On the dual of a space of locally schlicht functions

Professor Joseph A. Cima, University of North Carolina at Chapel Hill (687-30-3)

12:15-12:25

(15) Branched structures and affine and projective bundles on Riemann surfaces

Professor Richard Mandelbaum, University of Massachusetts (687-30-4)

WEDNESDAY, 11:15 A.M.

General Session I, Room 306

11:15-11:25

(16) On the representations of primes by different binary quadratic forms

Professor Joseph B. Muskat, Bar-Ilan University, Ramat-Gan, Israel, and University of Pittsburgh (687-10-5)

11:30-11:40

(17) Primary ideals in rings of analytic functions

Dr. R. Douglas Williams, Washington, D. C. (687-13-2)

11:45-11:55

(18) Extension of axiomatic potential theory to the parabolic case. Preliminary report

Dr. Stuart P. Lloyd, Bell Telephone Laboratories, Inc., Murray Hill, New Jersey (687-31-1)

12:00-12:10

(19) Quantitative estimates for a nonlinear system of integrodifferential equations

Professor Thomas A. Bronikowski*, Marquette University, Professor James E. Hall and Professor John A. Nohel, University of Wisconsin (687-34-2)

12:15-12:25

(20) Duality in triangle inequalities

Mr. Murray S. Klamkin, Ford Motor Company, Dearborn, Michigan (687-52-1)

WEDNESDAY, 11:15 A.M.

Session on Group Theory, Room 311

11:15-11:25

(21) Automorphism groups of ultrafilters. Preliminary report

Mr. Richard A. Sanerib, Jr., University of Colorado (687-20-1)

11:30-11:40

(22) Amalgamated products of profinite groups

Professor Luis Ribes, Carleton University (687-20-4)

11:45-11:55

(23) A curious pairing of characters of $Sp_{2n}(2)$ and of its orthogonal subgroups. Preliminary report

Professor J. Sutherland Frame* and Dr. Arunas Rudvalis, Michigan State University (687-20-10)

12:00-12:10

(24) Solvable groups with abelian Carter subgroups. Preliminary report

Dr. Alphonse H. Baartmans, Southern Illinois University (687-20-12)

(Introduced by Dr. Worthen N. Hunsaker)

12:15-12:25

(25) A monomorphism theorem for groupoids. Preliminary report
Professor Richard H. Crowell* and Professor Neville F. Smythe, Dartmouth College (687-20-5)

WEDNESDAY, 11:15 A.M.

Session on Matrix Theory, Room 401

11:15-11:25

(26) The behavior of the determinant and permanent on sets of equimodular matrices
Mr. Gernot Michael Engel, University of Wisconsin (687-15-2)
(Introduced by Professor Hans Schneider)

11:30-11:40

(27) Localization of the zeros of the permanent of a characteristic matrix
Professor Peter M. Gibson, University of Alabama in Huntsville (687-15-3)

11:45-11:55

(28) Continuous dependence on A in the D_1AD_2 theorems
Dr. Richard D. Sinkhorn, University of Houston (687-15-5)

12:00-12:10

(29) WITHDRAWN

WEDNESDAY, 11:15 A.M.

Session on Topology I, Room 405

11:15-11:25

(30) A note on monotone normality
Professor Robert W. Heath and Professor David J. Lutzer*, University of Pittsburgh (687-54-1)

11:30-11:40

(31) The projection of a compact zeroset is a zeroset. Preliminary report
Dr. Eric J. Braude, Seton Hall University (687-54-2)

11:45-11:55

(32) Some theorems on topological expansions
Professor Mary E. Powderly, William Paterson College of New Jersey (687-54-7)

12:00-12:10

(33) Pseudo-expansive maps and transformation groups
Professor Lee H. Minor, Western Carolina University (687-54-14)
(Introduced by Professor John A. Bond, Jr.)

12:15-12:25

(34) No infinite dimensional compact group admits an expansive automorphism
Mr. Wayne M. Lawton, Wesleyan University (687-54-9)
(Introduced by Professor Walter H. Gottschalk)

THURSDAY, 8:45 A.M.

Colloquium Lectures: Lecture III, Auditorium, Conference Center
Uniformization, moduli and Kleinian groups
Professor Lipman Bers, Columbia University

THURSDAY, 10:00 A.M.

Colloquium Lectures: Lecture III, Auditorium, Conference Center
Algebraic groups and arithmetic groups
Professor Armand Borel, Institute for Advanced Study

THURSDAY, 11:15 A.M.

Session on Algebra II, Room 301

11:15-11:25
(35) Classgroups of p-groups
Professor Irving Reiner and Professor Stephen V. Ullom*, University of Illinois (687-16-1)

11:30-11:40
(36) The isomorphism problem for polynomial rings. Preliminary report
Mr. David Jacobson, Rutgers University (687-16-2)

11:45-11:55
(37) A generalization of the Clifford algebra to Hermitian world spaces and their related Euclidian world spaces
Dr. Konrad John Heuvers, Michigan Technological University (687-16-3)

12:00-12:10
(38) A general definition of injectivity
Dr. Sheila O. Collins, Tulane University (687-16-4)
(Introduced by Professor Laszlo Fuchs)

12:15-12:25
(39) The type of an ideal in the ring of power series
Professor Anthony A. Iarrobino, Jr., University of Texas (687-14-1)

THURSDAY, 11:15 A.M.

Session on Combinatorics, Room 303

11:15-11:25
(40) Two remarkable extensions of the Leibniz differentiation formula
Professor Henry W. Gould, West Virginia University (687-05-1)

11:30-11:40
(41) A characterization of s-systems with restrictions. Preliminary report
Professor Murray Hochberg, City University of New York, Brooklyn College (687-05-2)

11:45-11:55
(42) On existence of compound perfect squared squares
Professor Nicholas D. Kazarinoff and Mr. Roger Weitzenkamp*, University of Michigan (687-05-3)

12:00-12:10
(43) The maximum genus of the complete graph
Professor Gerhard Ringel, University of California, Santa Cruz (687-05-4)

12:15-12:25
(44) Self-orthogonal codes of half dimension over GF(2)
Dr. Vera S. Pless, Air Force Cambridge Research Laboratories, Bedford, Massachusetts (687-05-5)

Session on Complex Analysis II, Room 305

11:15-11:25

(45) Abel-Gončarov polynomial expansions
Professor James L. Frank and Professor John K. Shaw*, Virginia Polytechnic Institute and State University (687-30-6)

11:30-11:40

(46) Whittaker constants. II
Professor James D. Buckholtz, University of Kentucky, and Professor James L. Frank*, Virginia Polytechnic Institute and State University (687-30-7)

11:45-11:55

(47) On the maximum modulus and the mean value of an entire function of two complex variables
Dr. Arun Kumar Agarwal, Grambling College (687-32-1)

12:00-12:10

(48) The multiplicative Cousin problem with bounded data. Preliminary report
Professor Edgar Lee Stout, University of Washington (687-32-2)

12:15-12:25

(49) Coherence of Λ^{θ} and $\mathscr{I}_V$ when Λ infinite. Preliminary report
Mr. Hugh M. Collins, Tulane University (687-32-3)

THURSDAY, 11:15 A.M.

Session on Functional Analysis I, Room 311

11:15-11:25

(50) A representation theorem for precompact maps. Preliminary report
Dr. Daniel John Randtke, University of Georgia (687-46-1)

11:30-11:40

(51) Interpolation theorems for operators of weak type
Mr. Robert C. Sharpley, University of Texas (687-46-2)

11:45-11:55

(52) The 2n-dimensional disc algebra. Preliminary report
Mr. Garrett O. Van Meter II, University of Maryland (687-46-3)

12:00-12:10

(53) Some properties of a special class of perfect Banach sequence spaces. Preliminary report
Professor George W. Crofts, Virginia Polytechnic Institute and State University (687-46-6)

12:15-12:25

(54) Order summable families in vector lattices. Preliminary report
Dr. Charles G. Denlinger, Millersville State College (687-46-4)

THURSDAY, 11:15 A.M.

General Session II, Room 306

11:15-11:25

(55) A degenerate principal series of representations of $Sp(n+1,\mathbb{C})$
Professor Kenneth I. Gross, Dartmouth College (687-22-2)

11:30-11:40

(56) The structure of certain unitary representations of infinite symmetric groups
Professor Arthur Lieberman, University of South Florida (687-22-3)

11:45-11:55

(57) Derivation in Orlicz spaces

Professor Charles A. Hayes, Jr., University of California, Davis (687-28-1)

12:00-12:10

(58) On the definition and existence of an (LIR)-refinement integral

Professor Fred M. Wright* and Mr. Dean R. Kennebeck, Iowa State University (687-28-2)

12:15-12:25

(59) One-dimensional basic sets in the three-sphere

Professor Joel C. Gibbons, St. Procopius College (687-58-1)

THURSDAY, 11:15 A.M.

Session on Probability and Statistics, Room 401

11:15-11:25

(60) A probabilistic interpretation of complete monotonicity

Professor Clark H. Kimberling, University of Evansville (687-60-1)

11:30-11:40

(61) Overlapping recurrent patterns

Professor Peter C.-C. Wang and Professor Glenn A. Stoops*, Naval Postgraduate School (687-60-2)

11:45-11:55

(62) Covering the circle with random arcs

Dr. Lawrence A. Shepp, Bell Telephone Laboratories, Inc., Murray Hill, New Jersey (687-60-3)

12:00-12:10

(63) Procedures for nonparametric modal intervals. Preliminary report

Professor Bennet P. Lientz, University of Southern California (687-62-1)

THURSDAY, 11:15 A.M.

Session on Topology II, Room 405

11:15-11:25

(64) The Hausdorffication functor

Professor Eugene M. Norris, West Virginia University (687-54-5)

11:30-11:40

(65) A note on quasi-metric spaces. Preliminary report

Professor Robert W. Heath, University of Pittsburgh (687-54-11)

11:45-11:55

(66) A metrization theorem

Professor Charles C. Alexander, University of Mississippi (687-54-12)

12:00-12:10

(67) Linearly stratifiable spaces and a definition given by H. Tamano

Professor Jerry E. Vaughan, University of North Carolina at Chapel Hill (687-54-13)

THURSDAY, 1:30 P.M.

Invited Address, Auditorium, Conference Center

Nonlinear functional analysis

Professor Felix E. Browder, University of Chicago

THURSDAY, 2:45 P.M.

Invited Address, Auditorium, Conference Center
Applications of elliptic operators
Professor Isadore M. Singer, Massachusetts Institute of Technology

THURSDAY, 4:00 P.M.

Business Meeting of the Society, Schwab Auditorium
Presentation of the Steele Prizes

FRIDAY, 8:45 A.M.

Colloquium Lectures: Lecture IV, Auditorium, Conference Center
Uniformization, moduli and Kleinian groups
Professor Lipman Bers, Columbia University

FRIDAY, 10:00 A.M.

Invited Address, Auditorium, Conference Center
Infinite loop spaces, trees, and the bar construction
Professor John M. Boardman, Johns Hopkins University

FRIDAY, 11:15 A.M.

Colloquium Lectures: Lecture IV, Auditorium, Conference Center
Algebraic groups and arithmetic groups
Professor Armand Borel, Institute for Advanced Study

FRIDAY, 11:15 A.M.

Session on Algebra III, Room 301

11:15-11:25
(68) Selfadjointness with indefinite quadratic forms
Professor Lincoln E. Bragg, Florida Institute of Technology (687-15-1)

11:30-11:40
(69) The Gram-Schmidt process without iteration: applications. Preliminary report
Dr. Andreas Z. Zachariou, Johns Hopkins University (687-15-4)

11:45-11:55
(70) Cohomology theorems for Borel-like solvable Lie algebras
Professor George F. Leger, Tufts University, and Professor Eugene M. Luks*, Bucknell University (687-17-1)

12:00-12:10
(71) On loops satisfying Cayley relations
Dr. Volodymyr Bohun-Chudyniv, Morgan State College, and Mr. Boris Bohun-Chudyniv*, Seton Hall University (687-17-2)

12:15-12:25
(72) On algorithms for constructing distributive idempotent quasi-groups of order $2(3q + r)$ $(r = 1,2)$
Dr. Volodymyr Bohun-Chudyniv* and Dr. Walter R. Talbot, Morgan State College (687-17-3)

FRIDAY, 11:15 A.M.

Session on Functional Analysis II, Room 311

11:15-11:25

(73) A characterization of Hilbert modules
Dr. George R. Giellis, U. S. Naval Academy (687-46-7)

11:30-11:40

(74) Unitary implementation of groups of automorphisms of a von Neumann algebra
Professor Herbert Halpern, University of Cincinnati (687-46-8)
(Introduced by Professor Edward P. Merkes)

11:45-11:55

(75) On the spectra of mixed operators. Preliminary report
Professor Charles J. A. Halberg, Jr., University of California, Riverside (687-46-9)

12:00-12:10

(76) Maximal functional calculi
Professor Michael B. Dollinger, Louisiana State University, and Professor Kirti K. Oberai*, Queen's University (687-47-1)

12:15-12:25

(77) A noncommutative, noncompact group with tauberian group algebra
Mr. Peter R. Mueller-Roemer, East Carolina University (687-43-1)

FRIDAY, 11:15 A.M.

Session on Logic, Room 303

11:15-11:25

(78) U-extensions of countable structures
Mrs. Julia F. Knight, Pennsylvania State University (687-02-1)

11:30-11:40

(79) Canonical representations of formal systems. Preliminary report
Professor R. G. Jeroslow, University of Minnesota (687-02-3)

11:45-11:55

(80) An axiomatization of the set theory of Zadeh
Professor F. William Chapin, Jr., University of Notre Dame (687-02-4)

12:00-12:10

(81) A psychological view of the foundations of mathematics. Preliminary report
Dr. Robert F. Jolly, Davis, California (687-02-2)
(Introduced by Professor F. Burton Jones)

FRIDAY, 11:15 A.M.

Session on Ordinary Differential Equations, Room 305

11:15-11:25

(82) Closed orbit solutions to $x'' + g(x)(x')^2 + f(x) = 0$
Dr. G. Di Antonio, Pennsylvania State University, Capitol Campus (687-34-1)

11:30-11:40

(83) .The Poincaré-Lighthill perturbation technique and its generalizations
Professor Craig Comstock, Naval Postgraduate School (687-34-3)

11:45-11:55

(84) Criteria for the solvability of a nonlinear second order differential equation

Dr. Abolghassem Ghaffari, NASA Goddard Space Flight Center, Greenbelt, Maryland (687-34-4)

12:00-12:10

(85) Oscillation properties of third order differential equations

Professor Gary D. Jones, Murray State University (687-34-5)

12:15-12:25

(86) Disconjugacy of third order differential equations with nonnegative coefficients

Professor Garret J. Etgen* and Mr. C. D. Shih, University of Houston (687-34-6)

FRIDAY, 11:15 A.M.

Session on Semigroups, Room 401

11:15-11:25

(87) Homomorphisms between free contents

Dr. John Shafer, University of Massachusetts (687-20-6)

11:30-11:40

(88) Finite commutative subdirectly irreducible semigroups

Dr. Phillip E. McNeil, University of Cincinnati (687-20-7)
(Introduced by Professor Arthur E. Bragg)

11:45-11:55

(89) Irreducible $\mathfrak{N}$-semigroups. Preliminary report

Professor Takayuki Tamura, University of California, Davis (687-20-8)

12:00-12:10

(90) Left translations of the direct sum of semigroups

Professor R. B. Hora and Professor Naoki Kimura*, University of Arkansas (687-20-11)

FRIDAY, 11:15 A.M.

Session on Topology III, Room 405

11:15-11:25

(91) A Baire space extension

Professor Robert A. McCoy, Virginia Polytechnic Institute and State University (687-54-4)

11:30-11:40

(92) Sections and selections

Professor Louis F. McAuley, State University of New York at Binghamton, and Professor David F. Addis*, Texas Christian University (687-54-8)

11:45-11:55

(93) Local properties and a p-space analogue of a theorem of Smirnov. Preliminary report

Professor Howard H. Wicke*, Ohio University, and Dr. John M. Worrell, Jr., Sandia Laboratories, Albuquerque, New Mexico (687-54-10)

12:00-12:10

(94) Extending congruences on compact semigroups

Professor Albert R. Stralka, University of California, Riverside (687-22-1)

FRIDAY, 1:30 P.M.

Invited Address, Auditorium, Conference Center

Conjugacy problems in discrete dynamical systems
Professor Joel W. Robbin, University of Wisconsin

FRIDAY, 2:45 P.M.

Invited Address, Auditorium, Conference Center

Recent progress on the isomorphism problem in ergodic theory
Professor Benjamin Weiss, The Hebrew University, Jerusalem, Israel

FRIDAY, 4:00 P.M.

Session on Algebra IV, Room 301

4:00-4:10
(95) Group sequential topological groups. Preliminary report
Professor Stephen Baron, Clark University (687-18-1)

4:15-4:25
(96) Coalgebras, sheaves, and cohomology
Professor Donovan H. Van Osdol, University of New Hampshire (687-18-2)

4:30-4:40
(97) Dual connections and Galois connections. Preliminary report
Professor V. Sankriti Krishnan, Temple University (687-18-3)

4:45-4:55
(98) A finite non-Boolean algebra
Dr. Robert Willis Quackenbush, University of Manitoba (687-08-1)

FRIDAY, 4:00 P.M.

Session on Applied Mathematics, Room 401

4:00-4:10
(99) An exchange algorithm for determining the "strict" Chebyshev solution to overdetermined linear equations
Professor Charles S. Duris and Mr. Michael Temple*, Drexel University (687-65-1)

4:15-4:25
(100) Error estimates for Clenshaw-Curtis quadrature. Preliminary report
Dr. R. D. Riess, Virginia Polytechnic Institute and State University (687-65-2)
(Introduced by Professor George William Crofts)

4:30-4:40
(101) Determination of an optimal step-size for Runge-Kutta processes
Professor Diran Sarafyan, Louisiana State University in New Orleans (687-65-3)

4:45-4:55
(102) Flexure of an initally curved cuboid dielectric
Professor K. L. Arora, Punjab Engineering College, Chandigarh, India (687-73-1)
(Introduced by Professor T. P. Srinivasan)

5:00-5:10
(103) A single-label maximum-flow algorithm
Dr. Benjamin L. Schwartz, American University (687-90-1)

5:15-5:25

(104) Incidence matrix concepts for the analysis of the interconnection of networks

Dr. William N. Anderson, Jr., University of Maryland, Professor Richard J. Duffin, Carnegie-Mellon University, and Professor G. E. Trapp*, West Virginia University (687-94-1)

FRIDAY, 4:00 P.M.

Session on Approximation Theory, Room 303

4:00-4:10

(105) A unified approach to uniform real approximation by polynomials with linear restrictions

Professor Bruce L. Chalmers, University of California, Riverside (687-41-1)

4:15-4:25

(106) Strong unicity in approximation theory

Professor Martin W. Bartelt and Professor Harry W. McLaughlin*, Rensselaer Polytechnic Institute (687-41-3)

4:30-4:40

(107) Comparison of PLK and time scales uniformizations

Dr. A. Klimas and Dr. Guido Sandri*, Aeronautical Research Associates of Princeton, Inc., Princeton, New Jersey (687-41-2)

4:45-4:55

(108) Determination of extremal functions in H^p by a Fortran program

Professor Annette Sinclair, Purdue University (687-41-4)

FRIDAY, 4:00 P.M.

General Session III, Room 306

4:00-4:10

(109) On characterization of automorphism groups. Preliminary report

Professor Hidegoro Nakano, Wayne State University (687-20-2)

4:15-4:25

(110) A note on semigroups 0

Professor Kim Ki-Hang Butler, Pembroke State University (687-20-3)

4:30-4:40

(111) The four color theorem

Mr. J. M. Thomas, Philadelphia, Pennsylvania (687-50-1)

FRIDAY, 4:00 P.M.

Session on Number Theory, Room 311

4:00-4:10

(112) Complete residue systems in the quadratic domain $Z(e^{2\pi i/3})$

Dr. Gerald E. Bergum, South Dakota State University (687-10-1)

4:15-4:25

(113) A number theoretic sum involving divisors of polynomial values

Dr. William A. Webb, Washington State University (687-10-3)

4:30-4:40

(114) The reduced set of residues mod Πp_n

Mrs. Inda Lepson, University of Maryland (687-10-6)

(Introduced by Professor Benjamin Lepson)

4:45-4:55

(115) Bounds for consecutive kth power residues in the Eisenstein integers
Professor Richard B. Lakein, State University of New York at Buffalo (687-10-7)

5:00-5:10

(116) The evaluation of character series by contour integration
Professor Bruce C. Berndt, University of Illinois (687-10-4)

5:15-5:25

(117) On representations of generalized polygonal numbers as the sum of two such numbers in m ways
Professor Gregory Wulczyn, Bucknell University (687-10-2)

FRIDAY, 4:00 P.M.

Session on Topology IV, Room 405

4:00-4:10

(118) Disks in E^3/G
Dr. Edythe P. Woodruff, State University of New York at Binghamton (687-54-3)

4:15-4:25

(119) Euler characteristics of 2-manifolds and light open maps. Preliminary report
Dr. John D. Baildon, Pennsylvania State University, Worthington Scranton Campus (687-54-6)
(Introduced by Professor Louis F. McAuley)

4:30-4:40

(120) The group Eq(M), of self equivalances of certain manifolds. Preliminary report
Mr. David L. Smallen, University of Rochester (687-55-1)

4:45-4:55

(121) On Kneser's conjecture for bounded 3-manifolds. Preliminary report
Dr. Wolfgang H. Heil, Florida State University (687-55-3)

5:00-5:10

(122) Embedding the dunce hat in E^4
Professor John P. Neuzil, Kent State University (687-57-1)

5:15-5:25

(123) On Stasheff's fifth problem
Professor Casper R. Curjel, University of Washington, and Professor Roy R. Douglas*, University of British Columbia (687-55-2)

Walter H. Gottschalk
Associate Secretary

Middletown, Connecticut

PRELIMINARY ANNOUNCEMENTS OF MEETINGS

The Six Hundred Eighty-Eighth Meeting
Massachusetts Institute of Technology
Cambridge, Massachusetts
October 30, 1971

The six hundred eighty-eighth meeting of the American Mathematical Society will be held at the Massachusetts Institute of Technology, Cambridge, Massachusetts, on Saturday, October 30, 1971.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, there will be two one-hour addresses. Professor Gerald E. Sacks will present an hour lecture tentatively entitled "Differentially closed fields." The name of the second lecturer and the title of his address will be given in the October issue of these *Notices*.

There will be sessions for contributed papers both morning and afternoon. Abstracts for contributed papers should be sent to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904, so as to arrive prior to the deadline of September 9, 1971.

Walter H. Gottschalk
Associate Secretary
Middletown, Connecticut

The Six Hundred Eighty-Ninth Meeting
Auburn University
Auburn, Alabama
November 19–20, 1971

The six hundred eighty-ninth meeting of the American Mathematical Society will be held at Auburn University in Auburn, Alabama, on Friday and Saturday, November 19–20, 1971.

By invitation of the Committee to Select Hour Speakers for Southeastern Sectional Meetings, there will be three one-hour addresses, all of which will be presented in Room 307 of the Commons Building. Professor J. C. Cantrell of the University of Georgia will give an address entitled "Locally flat embeddings of manifolds." An address entitled "Developments in the theory of Schauder bases" will be given by Professor Charles W. McArthur of Florida State University, and Professor John Neuberger of Emory University will give an address entitled "Quasi-analyticity and semigroups."

There will be sessions for contributed papers both Friday afternoon and Saturday morning. Abstracts for contributed papers should be sent to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904, so as to arrive prior to the deadline of October 4, 1971.

The registration desk will be located in the entrance hall of the Commons Building, Physical Science Center, where all sessions will be held. Registration hours will be 10:00 a.m. to 5:00 p.m. on Friday, November 19, and 9:00 a.m. to 12:00 noon on Saturday, November 20.

Auburn is located on Interstate 85 approximately one-third of the way from Montgomery, Alabama, to Atlanta, Georgia.

Auburn is also accessible via U.S. 280, U.S. 29, and U.S. 80. The nearest commercial airline terminals are Atlanta, Georgia (two hours driving time); Columbus, Georgia, and Montgomery, Alabama (one hour driving time, each). It is likely that several pick-ups can be arranged at these airports for persons flying in and for whom car rental is impractical. Persons requiring such service should write to Professor L. P. Burton, Head, Department of Mathematics, by November 15.

Meals, except for snacks, must be taken at commerical establishments. Coffee and doughnuts will be served each morning in Room 244, Commons Building. A dutch-treat beer party is planned for Friday evening.

The two motels which are within walking distance of the meetings and which are holding blocks of rooms for reservations (deadline November 10) are:

University Motor Lodge (25-room block)

125 North College Street
P. O. Box 831
Phone: 887-6583

Single (one person per room)	$ 9.36
One Double Bed (two persons per room)	$12.48
Two Double Beds (two persons per room)	$14.56

(six blocks from Commons Building)

Heart of Auburn Motel (80-room block)

333 South College Street
P. O. Box 632
Phone: 887-3462

Single (one person per room)	$ 9.88
One Double Bed (two persons per room)	$12.48
Two Double Beds (two persons per room)	$15.60

(three blocks from Commons Building)

Additional accommodations are as follows:

Holiday Inn

Birmingham Highway 280
P. O. Box 551
Phone: 887-7065

Single (one person per room)	$ 9.45
One Double Bed (two persons per room)	$11.55
Two Double Beds (two persons per room)	$16.80

(five miles north of campus on Alabama 147, cocktail lounge)

Holiday Inn

Junction I-85 and U.S. 280
P. O. Box 391
Opelika, Alabama 36801
Phone: 745-6331

One Double Bed (one person per room)	$11.55
Two Double Beds (two persons per room)	$16.80

(five miles east of campus, cocktail lounge)

Stoker's Motel

1208 Opelika Highway
Auburn, Alabama 36830
Phone: 887-3481

Single (one person per room)	$ 8.40
One Double Bed (two persons per room)	$10.50
Two Double Beds (two persons per room)	$12.60

(four miles east of campus)

All reservations should be made directly with the motels as early as practicable.

O. G. Harrold
Associate Secretary
Tallahassee, Florida

The Six Hundred Ninetieth Meeting
University Of Wisconsin-Milwaukee
Milwaukee, Wisconsin
November 27, 1971

The six hundred ninetieth meeting of the American Mathematical Society will be held at the University of Wisconsin-Milwaukee, Milwaukee, Wisconsin, on Saturday, November 27, 1971. The sessions of the meeting will be held in the Science Complex, which houses the Department of Mathematics, and in the adjoining Physics and Engineering Building. These buildings are located near the corner of Cramer Street and Kenwood Boulevard in northeast Milwaukee.

By invitation of the Commitee to Select Hour Speakers for Western Sectional Meetings, there will be two one-hour addresses. Professor Raghavan Narasimhan of the University of Chicago will address the Society at 11:00 a. m. The title of his lecture will be announced in the October issue of these *Notices*. Professor Mary-Ellen Rudin of the University of Wisconsin-Madison will speak at 1:45 p. m. on the topic "Set theory and general topology."

There will be sessions for the presentation of contributed ten-minute papers both morning and afternoon. Those having time preferences for the presentation of their papers should so indicate on their abstracts. Abstracts should be submitted to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904, so as to arrive prior to the deadline of October 4, 1971. There will be a session for late papers if one is needed, but, since late papers will not be listed in the printed program of the meeting, it would be better if all the abstracts were to arrive before the deadline.

There will be three special sessions of selected twenty-minute papers. Professor Morris Marden of the University of Wisconsin-Milwaukee is arranging one such session on the subject of Function Theory. Another special session is being arranged by Professor Frank A. Raymond of the University of Michigan on the subject of Transformation Groups. The third special session is being arranged by Professor Thomas G. McLaughlin of the University of Illinois on the subject of Recursive Functions. Most of the papers presented at these sessions will be by invitation. Anyone contributing an abstract for the meeting, however, who feels that his paper would be particularly appropriate for one of these special sessions should indicate this emphatically on his abstract and submit it three weeks earlier than the above deadline, namely by September 13, 1971, in order to allow time for the additional handling necessary.

On Friday, November 26, 1971, the day before the meeting itself, the University of Wisconsin-Milwaukee will sponsor a brief symposium on the subject of Function Theory in the 1970's. The special session on Function Theory mentioned earlier will be an extension of this symposium.

Detailed information about travel and accommodations will appear in the October issue of these *Notices*. It is expected that convenient space in a university dormitory will be available. A beer party sponsored by one of the local breweries is in prospect.

Paul T. Bateman
Associate Secretary

Urbana, Illinois

THE PROBLEMS OF EMPLOYMENT IN MATHEMATICAL SCIENCES

Gail S. Young

At the meeting in Atlantic City in January 1971, the Council of the American Mathematical Society voted to expand the charge of the Committee to Advise on Analysis of Employment Data, as reported in the April 1971 issue of the *Notices*, page 486. The members of the committee are William L. Duren, Jr. (chairman), Richard D. Anderson, John W. Jewett, and Gail S. Young. The name of this committee has now been changed to the Committee on Employment and Educational Policy. The members of the committee were granted permission by the editors of the *Notices*, with the informal concurrence of the Council, to make editorial comments in the *Notices* on the data gathered by the committee from various sources, including the published data of the CBMS Survey Committee. The following article is the first such comment from a member of the Committee on Employment and Educational Policy; it is not, however, a committee document in any sense.

On January 28, 1968, the Committee on the Support of Research in the Mathematical Sciences finished its report [1]. Chaired by Lipman Bers, composed of some of the wisest and most illustrious members of the mathematical community, and assisted by several highly competent panels, COSRIMS prepared a document that is a model of what such a report should be. Besides other merits, it was unique among the series of similar studies of sciences prepared under the aegis of the Committee on Science and Public Policy in its concern for education.

We find on page 118 of the report the sentence, italicized, "Our main conclusion is that there is now a shortage of qualified college teachers of the mathematical sciences, and that this situation is likely to get worse before it gets better." Later, page 129, we find, "...we have estimated that over the period 1965-1970 some 8,000 new full-time staff members will be needed in the mathematical-science teaching in the four-year colleges and universities. We also have the estimate of net inflow of some 3,300 Ph.D.'s into such teaching during this period. Thus only about 41 percent of the new faculty will have doctorates; hence... the percentage of doctorate holders on mathematical-science faculties...will decline."

It is now, as I write, late June 1971. Virtually every graduate department I know about still has students with no positions; virtually daily I receive letters—I am a chairman—from students of obvious ability who must be writing their two hundredth or three hundredth letter of application. What went wrong? Are we suffering a temporary crisis, caused by the forces responsible for high national unemployment, or are our troubles deeper? My own belief is that governmental and economic causes have merely exacerbated our troubles, and perhaps brought them on a year or so earlier; if I am correct, the implications are grave.

In understanding our situation, some examination of the COSRIMS estimate is helpful. The prediction of great shortage of staff for 1970 is based on a projected increase of undergraduate enrollment in the mathematical sciences from 1965 to 1970 of a total of 796,000 enrollments. (See the table on page 125 [1].) I will break this into two parts, what the report calls "general growth" and what may be called "special growth". For general growth, a figure of 310,000 elections is given. That estimate comes from these considerations. We know as fact that from

1960 to 1965 the total undergraduate student body had increased by 48%. Data from the Lindquist Survey of 1960 and the CBMS Survey of 1965 [2] showed that in those five years undergraduate mathematics enrollment had increased by 44% in the sample of the Survey. It is approximately correct to say that in this period mathematics enrollment had gone up at the same rate as the general enrollment. The Office of Education has been quite accurate on short-range predictions, and for 1965-70, they forecast a 29% increase in undergraduate enrollments. If one applies the same rate of increase to enrollment in the mathematical sciences, one gets 310,000 enrollments.

Let me return for a moment and discuss that 44% growth in mathematics enrollment. It was one of the two data in the Survey Committee report that most surprised and disturbed me at the time; the other (p. 37 of [2]) was a clear indication that from 1962-63 to 1965-66 the percentage of mathematics faculty with doctorates had actually risen from 48% to 53%. Both of these were at complete variance with popular belief. However, what we had all noticed was that advanced calculus had become a multiple-section course; few paid much attention to the compensating dropping of sections of college algebra. If mathematics enrollments rose no faster than general enrollment, then we were certainly in for trouble. The Survey had found that, conveniently, there were almost exactly 100 enrollments in undergraduate mathematics for each full-time teacher. With that ratio, the 310,000 elections would provide for 3,100 jobs. One could take the projected number of new Ph.D.'s and arrive at an estimate of 3,300 new Ph.D.'s available for college teaching in 1965-70 (I am here using COSRIMS' estimate). There are graduate courses, and unqualified teachers to be replaced, so that the apparent excess did not mean unemployment. But one had to wonder, at least, would it be possible to absorb the very large number of Ph.D.'s predicted for the early 70's?

COSRIMS, of course, knew all this. They felt that mathematics enrollment would grow at more than 70 per cent from 1965 to 1970. This would come about from more people taking more mathematics. To give an indication of their thinking, I quote a few sentences from their carefully reasoned discussion. "...by 1970 half of the majors in [the biological sciences, psychology, and the social sciences] will be taking, during their undergraduate careers, one more mathematical science course than was typical for the majors in these fields in 1965." "...perhaps two-thirds of [the prospective elementary] teachers will by 1970 be taking one more mathematics course...than in 1965." Their estimates rested heavily on curricular recommendations from such groups as CUPM. It is obvious that these hopes have not been realized.

Indeed, the COSRIMS prediction would have led to a need for 8,000 new mathematical science faculty by 1970. This would have represented an amazing increase; there were in 1965-66 only 10,750 full-time four-year college teachers of mathematical sciences, and in 1966-67 a full-time-equivalent faculty of 3,100 in the junior colleges [2]. There have been no signs of such an increase.

By mid-Fall, we will know by how far we have failed to reach these estimates. The Survey Committee repeated its undergraduate survey in Fall 1970, and the data is now being processed. Elsewhere in this issue is the announcement of a discussion at the meeting of these problems, sponsored by the Society's Committee on Employment and Educational Policy and perhaps some preliminary data may be available then. Until we have the data and conceivably afterwards, we can only guess.

What we do know, with certainty, is the size of the 18-21 year old age group for the next 20 years. This will go from 14.4 million in 1970 to 17.0 million in 1980 and back to 14.5 million in 1990. College enrollments will necessarily show something similar. Allan Cartter [4,5] as early as 1965 predicted difficulties in employment in higher education from such demographic factors. In [5] he makes estimates for college enrollments to 1990 under generous assumptions. He estimates total full-time-equivalent enrollments of 6.3 million in 1970, 9.5 million in 1980, and 8.7 million in 1990 [6]. For the next decade, then, enrollments will increase by about 50%, equivalent to a 4% annual rate of increase. Let us play the game of assuming (1) mathematical science faculty will grow at a 4% rate to 1980; (2) all new teaching positions in our field in four-year colleges, two-year colleges, and technical institutes will be filled by Ph.D's; (3) there are 17,000 full-time positions at present. We then get an annual increase of 700–725 positions. I myself do not believe (1) or (2). And what happens from 1980 to 1990?

There are two studies, both reported in [3], that indicate clearly that last Fall the chairmen of mathematical science departments expected to have enough positions for Ph.D.'s for next year to provide one job per new Ph.D. As I write, it seems obvious that many of these have failed to materialize.

However, if the chairmen's expectations had been fulfilled, that is still alarming. If there is now no piled-up demand, prospects for future years become very poor. We are in trouble, and the ending of the Vietnam war, or of inflation, or of recession will not save us.

Nevertheless, our present troubles are much greater than the analysis of growth curves would indicate. It is worth saying something about the causes of these other difficulties, particularly to those not around in the late '50's and early '60's.

(a) One source of our troubles is the push for more Ph.D.'s. The Gilliland report of 1962 [7] was, perhaps, decisive in increasing Federal support of graduate instruction in the sciences. For mathematics, it set goals that were greeted with consternation [8] as being impossible, but which we have actually come very near to meeting. Re-reading the report now, I wonder why the statements of need seemed so convincing at the time. Perhaps it was because they fit into a post-Sputnik mood of a new era for science, signalized by the founding of NSF. There were also many people, I among them, who thought that, without great expansion of Ph.D. production, the extremely rapid expansion of college enrollment would result in a deterioration of the college faculties similar to that of the secondary school faculties in the '20's. In any case, throughout higher education the need for more scientists was accepted, and indeed welcomed.

The response of the Federal government was strong. In 1966-67, about a third of the full-time graduate students in mathematics were supported by Federal grants [9, p. 67]. The size of the direct contribution to the university represented by this support was not insignificant; there were cost-of-education allowances of $2,500 per fellowship or traineeship and overhead on research assistantships. Five NASA traineeships paid the salary of another assistant professor.

(b) All through the '60's, major foundations (Ford, NSF, others) put large sums of money into universities that showed promise of "excellence". To have a chance to get such grants, one had first to have departments that showed the prospects of excellence. Then one had to write a proposal that showed how one would take on new commitments, permanently, if the grant, for a small number of years, was awarded.

I believe now that the first serious signs of financial problems for higher education showed up about 1960. To the administrator, the possibility of moving his school up into a higher level where he could cash in on the rapidly increasing grant funds was a tempting solution, and an administrator who tried to resist these forces found his faculty being raided. There was no way to go but up. One of the most able administrators I have ever known told me then that his plan was to introduce as many Ph.D. programs as possible, and that, once started, money would come in to support them. With such beliefs, it is no wonder that administrators were willing to run deficits, take on expanded commitments, and bid high for faculty in salary and teaching loads.

State universities felt another pressure. The success of the Boston area and of the Palo Alto area in attracting nice, clean, high-salaried scientific industries had been noted, and every state wanted to duplicate that success. I remember a press release from one state university announcing the appointment of a distinguished pure mathematician in, let us say, homological algebra, which, the release said, "has many applications to guided missiles". Unfortunately, few scientific industries felt the need for consultants in homological algebra, and they remained on Route 128.

A word on the teaching-load question. Teaching loads began to drop with the introduction of research grants, which initially contained funding for released time for the academic year. With more grants, the pressure for lower loads generally became irresistible, particularly since one of the attractions a new institution could offer a young man was the same teaching load he could have on a grant. But this drop in load could be economically justified only if it meant that the additional research would be directly or indirectly financed an amount compensating for the loss of tuition or of state aid.

None of these things were bad in themselves. But they all came to an end at once, with no substitutes, at a time of great inflation. State legislatures made cuts in appropriations, or gave no increase, graduate support ended, foundations turned their attention elsewhere, boards of trustees demanded an end to deficit financing. The predictions of chairmen at the beginning of this year are not being fulfilled because of the desperate struggle of institutions to keep solvent.

(c) Certainly one of the reasons for the Gilliland report's estimates of need for mathematicians was a prediction of greater industrial and governmental employment of mathematicians. There has been a rise in absolute numbers, but in recent years only 15% or less of Ph.D. classes have gone into industry [9]. The collapse of the aerospace industry has cut into that, as well as causing many industrial mathematicians to look for academic jobs.

Whether a deliberate effort to train Ph.D.'s for industrial positions would have been proper or not, no large effort was made. Ph.D. programs in the mathematical sciences were and are focused on the academic market. Indeed, from 1958 to 1966, the percentage of Ph.D.'s in applied mathematics, computer science, and probability and statistics remained constant at 30% of the total.

How do we work our way out of our problems? The ones described in (a), (b), and (c) are out of our control, and one can only hope for improvements in university support that will abate their severity. The failure to meet the COSRIMS predictions is something we can do something about. My own skepticism about their predictions was based mostly on my observations that few departments, Dartmouth being the most notable exception, were then ready to take seriously enough their "service" functions. But the fact is that a social scientist or a biologist or an engineer will not take an additional course in mathematics unless a suitable course is offered. And in many universities those departments whose students are most apt to take additional mathematics have now added faculty that can teach such courses if we don't. The only hope for more academic positions for our young people is to have more students taught.

Replacement of non-Ph.D.'s by Ph.D.'s is not a humane solution, but is also not a lasting one. Two of the current Ph.D. classes could replace all the two-year college faculty, and a third could probably replace all non-tenured non-Ph.D.'s in the four-year institutions who are not themselves working for Ph.D.'s. In any case, this sort of "solution" merely shifts the suffering from young, able Ph.D.'s to older, less well-trained people. This is not to say that we should not upgrade faculties, but only that it cannot be done quickly, or have much effect on our job rate. I also see little evidence that the junior colleges are eager for our Ph.D.'s.

Industry? We do not really know what industry needs, and have no good way of finding out. But whatever it is, it doesn't seem to be people like the majority of our Ph.D.'s.

The future of the mathematical sciences depends on a steady supply of brilliant young scholars. We have now the most talented, best trained young mathematicians in history, but if we are not careful, the supply will be cut off, or will cut itself off. Whatever solutions we attempt to our problems, we must be sure not to let this happen.

Additionally, if the country really begins to face the problems of our society or of the world which mathematicians could help resolve, we may well find that, far from a surplus, we have nowhere near enough people.

I am most reluctant, therefore, to say that we should cut down on the number of Ph.D. programs in mathematics. I am reluctant to say, even, that no new ones should be created. Perhaps future study will make me see that this is necessary. What I do believe is that we must make fundamental changes in the nature of graduate work in mathematics which will prepare most of our students for something other than academic life. It is tempting to pretend to great wisdom and say what these changes should be. The answer or answers, however, will take the collective wisdom of us all, and will not come overnight.

References

[1] The Mathematical Sciences: A Report, National Academy of Sciences, Publication 1681, Washington, D.C., 1968.

[2] See Vol. I of the Survey Committee's report, Aspects of Undergraduate Training in the Mathematical Sciences, CBMS, Washington, D.C., 1967. This includes the relevant data from Lindquist.

[3] April 1971 *Notices*, page 486.

[4] Allan Cartter, Future Faculty Needs and Resources, in Improving College Teaching, American Council on Education, Washington, D. C., 1966.

[5] ____, Scientific Manpower for 1970-1985, Science, V. 172 (1971), pp. 132-140.

[6] ____, Cartter has a technical error in his predictions for mathematics, in that he uses an out-of-date figure for the percentage of Ph.D.'s in mathematics teaching. Except for that and the possibility of an increased share of undergraduate enrollment, as hoped for by COSRIMS, I find his data convincing.

[7] Graduate Training in Engineering, Mathematics, and the Physical Sciences, President's Science Advisory Committee, White House, 1963.

[8] Conference on Manpower Problems in the Training of Mathematicians, CBMS, Washington, D.C., 1963.

[9] Report of the Survey Committee, Vol. II, Aspects of Graduate Training in the Mathematical Sciences, CBMS, Washington, D.C., 1969. The figure for the other sciences and for engineering were nearly 50%.

University of Rochester
Rochester, New York

NEWS ITEMS AND ANNOUNCEMENTS

MATHEMATICIANS HONORED

Five mathematicians were awarded honorary degrees or medals in June 1971. All have served the Society as officers or committee members during their professional lives, and the Society wishes to congratulate them on the recognition that has just been bestowed upon them. A. Adrian Albert, dean of the division of physical sciences of the University of Chicago, received an honorary degree, doctor of humane letters, from the University of Illinois at Chicago Circle. Edward G. Begle, director of the School Mathematics Study Group and a member of the faculty at Stanford University, was awarded a Rosenberger Medal by the University of Chicago. Mina S. Rees, president of the Graduate Division of the City University of New York, received an honorary doctor of science degree from the University of Rochester, and a doctor of humane letters degree from New York University. J. Barkley Rosser, professor of mathematics and computer sciences and director of the Mathematics Research Center at the University of Wisconsin, was awarded an honorary doctor of science degree by Otterbein College. Donald C. Spencer, professor of mathematics, Princeton University, received an honorary doctor of science degree from Purdue University.

SOME SUPER-CLASSICS OF MATHEMATICS

Joseph A. Schatz

This collection of thirty-seven papers was chosen from the material which was collected by the author[*] in the course of compiling the Mathematics Citation Index. A citation index is a list of papers with each item on the list followed by a list of the papers which have cited (referred to) the given item. The Mathematics Citation Index was compiled from the references in approximately twenty-five thousand papers (roughly two-hundred thousand citations) taken from forty-eight serials published during the period 1950-1965. This is about five percent of the total mathematical literature, about ten percent of the mathematical literature published since Mathematical Reviews started, and about twenty percent of the periodical literature for the period covered. By definition, the super-classics are the papers which were cited fifty or more times in this corpus. The following thirty-seven super-classics were cited 2,549 times for an average of 68.9 citations per paper.

1) Lars Ahlfors and Arne Beurling, Conformal invariants and function-theoretic null-sets, Acta Math. 83 (1950), 101-129. MR 12, 171.
2) Armand Borel, Sur la cohomologie des espaces fibrés principaux et des espaces homogènes de groupes de Lie compacts, Ann. of Math. (2) 57 (1953), 115-207. MR 14, 490.
3) E. Čech, On bicompact spaces, Ann. of Math. (2) 38 (1937), 823-844.
4) I.S. Cohen, On the structure and ideal theory of complete local rings, Trans. Amer. Math. Soc. 59 (1946) 54-106. MR 7, 509.
5) Jean Dieudonné et Laurent Schwartz, La dualité dans les espaces $(\mathcal{F})$ et $(\mathcal{LF})$, Ann. Inst. Fourier (Grenoble) 1 (1949), 61-101 (1950). MR 12, 417.
6) Samuel Eilenberg and Saunders Mac Lane, Cohomology theory in abstract groups. I, Ann. of Math. (2) 48 (1947), 51-78. MR 8, 367.
7) Lars Gårding, Dirichlet's problem for linear elliptic partial differential equations, Math. Scand. 1 (1953), 55-72. MR 16, 366.
8) I.M. Gel'fand, Normierte Ringe, Mat. Sb. 9 (51) (1941), 3-24. MR 3, 51.
9) K. Gödel, Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme. I, Monatsh. Math. Phys. 38 (1931), 173-198.
10) Alexandre Grothendieck, Sur quelques points d'algèbre homologique, Tôhoku Math. J. (2) 9 (1957), 119-221. MR 21 #1328.
11) Marshall Hall, Projective planes, Trans. Amer. Math. Soc. 54 (1943), 229-277. MR 5, 72.
12) P. Hall, A contribution to the theory of groups of prime-power order, Proc. London Math. Soc. (2) 36 (1933), 29-95.
13) Edwin Hewitt, Rings of real-valued continuous functions. I, Trans. Amer. Math. Soc. 64 (1948), 45-99. MR 10, 126.
14) Lars Hörmander, On the theory of general partial differential operators, Acta Math. 94 (1955), 161-248. MR 17, 853.
15) Kenkichi Iwasawa, On some types of topological groups, Ann. of Math. (2) 50 (1949), 507-558. MR 10, 679.
16) N. Jacobson, The radical and semi-simplicity for arbitrary rings, Amer. J. Math. 67 (1945), 300-320. MR 7, 2.
17) Shizuo Kakutani, Concrete representation of abstract (M)-spaces. (A characterization of the space of continuous functions.), Ann. of Math. (2) 42 (1941), 994-1024. MR 3, 205.
18) M.G. Kreĭn and M.A. Rutman, Linear operators leaving invariant a cone in a Banach space, Uspehi Mat. Nauk

[*]This work was supported by the Atomic Energy Commission.

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19) J. Leray et J. Schauder, Topologie et équations fonctionnelles, Ann. Sci. École Norm. Sup. (3) 51 (1934), 45-78.

20) Edwin E. Moise, Affine structures in 3-manifolds. V. The triangulation theorem and Hauptvermutung, Ann. of Math. (2) 56 (1952), 96-114. MR 14, 72.

21) F.J. Murray and J. von Neumann, On rings of operators, Ann. of Math. (2) 37 (1936), 116-229.

22) Louis Nirenberg, Remarks on strongly elliptic partial differential equations, Comm. Pure Appl. Math. 8 (1955), 649-675. MR 17, 742.

23) D. Rees, On semi-groups, Proc. Cambridge Philos. Soc. 36 (1940), 387-400. MR 2, 127.

24) Jean-Pierre Serre, Homologie singulière des espaces fibrés. Applications, Ann. of Math. (2) 54 (1951), 425-505. MR 13, 574.

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32) A.D. Wallace, The structure of topological semigroups, Bull. Amer. Math. Soc. 61 (1955), 95-112. MR 16, 796.

33) H. Weyl, Über gewöhnliche Differentialgleichungen mit Singularitäten und die zugehörigen Entwicklungen willkürlicher Funktionen, Math. Ann. 68 (1910), 220-269.

34) George W. Whitehead, A generalization of the Hopf invariant, Ann. of Math. (2) 51 (1950), 192-237. MR 12, 847.

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36) J.H.C. Whitehead, Combinatorial homotopy. I, Bull. Amer. Math. Soc. 55 (1949), 213-245. MR 11, 48.

37) E. Witt, Treue Darstellung Liescher Ringe, J. Reine Angew. Math. 177 (1937), 152-160.

One of the most striking features of the list of super-classics is the heavy representation of algebraic topology. Since Mathematical Reviews had no heading for algebraic topology until volume 17 (1956), and the last paper I would classify as algebraic topology was reviewed in volume 16 (1955), none of these were classified as algebraic topology in Mathematical Reviews. The next largest representation of papers is in functional analysis, followed closely by algebra. The list of journals represented is even more strongly skewed than the list of subjects. The Annals of Mathematics accounts for eleven of the thirty-seven papers, and is represented by super-classics in functional analysis and algebra as well as those in algebraic topology. It is interesting to note that the Annals' popularity in citations is not a new fact. C.H. Brown [Scientific serials, ACRL Monograph Number 16, Association of College and Reference Libraries, Chicago, 1956] reports that in the period from 1929 to 1954 the Annals of Mathematics went from the tenth to the first most cited mathematical journal, and that the Transactions of the American Mathematical Society was the second most cited mathematical journal. Of the super-classics herein discussed, eleven appeared in the Annals, five in the Transactions, three in the Bulletin of the American Mathematical Society, two each in Acta Mathematica and Annales Scientifiques de l'École Normale Supérieure, and one each in fourteen other journals.

There were no papers in the super-classics with more than two authors, although six were papers by two authors. This is remarkably typical of the entire corpus.

Until the end of 1965, Mathematical Reviews had listed about 171,000 items. Kenneth O. May [Quantitative growth of the mathematical literature, Science 154 (1966), 1672-1673. MR 40 #7070] has estimated the number of items published before Mathematical Reviews started as 235,000. Thus, 24.7 percent of the journal-like citations in the Mathematics Citation Index were to the earlier 58.9 percent of the literature. For the super-classics, the corresponding figures are 664 citations to the ten papers published before Mathematical Reviews began (or 27 percent), and 1,885 citations to papers published afterwards. Of the citations of the super-classics 26.1 percent were citations of papers published before Mathematical Reviews started. These figures suggest that although mathematics has a substantial research front, it also has a healthy regard for at least some of the older periodical literature.

Citation counts furnish information on the nature of scientific literature which can be obtained in no other way. It would be useful to know, for instance, what percentage of the literature twenty or more years old will ever be used again. Citation counts give us a method of attacking questions like this. Unless care is exercised, however, the information obtained can be considerably distorted. In particular, failure to "clump" different descriptions of the same paper results in overestimates of the number of papers cited and underestimates for the average number of citations per paper. For example, there were 318 different descriptions of the thirty-seven super-classics; if these were thought to be 318 different papers, the conclusion would be that they were cited an average of 8.01 times. This is one of the major reasons why the small amount of information available on the use made of scientific literature should be viewed with considerable skepticism. The reasons why a given paper is described in many ways are the different ways of referring to a given journal, variations in an author's initials or name, difficulty of deciding which year a given volume of a journal was issued, missing information, and incorrect information. The question is not whether these things happen, but how frequently.

Twenty-seven of the super-classics were reviewed by Mathematical Reviews. Most of these reviews give the reader no indication of the "exalted" nature of these papers. It is interesting to speculate whether this is more a result of the unpredictability of the future, or of the intentional blandness of the reviews. If the latter is the case, perhaps this blandness has been overdone. This comment is offered with considerable trepidation, since I feel that one should exercise extreme caution in changing a product as excellent as Mathematical Reviews.

The author is aware that this report on the super-classics, while based on a large number of citations, is biased by the selection of journals used in compiling the information. They were the journals readily available to the author, and the results might have been quite different if a different selection had been made.

Sandia Corporation
Albuquerque, New Mexico

NEWS ITEMS AND ANNOUNCEMENTS

SALEM PRIZE

The Salem Prize for 1971 was awarded to Dr. Charles Fefferman of the University of Chicago for his work on harmonic analysis. The prize, established in 1968, is given every year to a young mathematician who is judged to have done outstanding work on Fourier series and related topics. The previous recipients were Dr. Nicholas Varapoulos in 1968, Dr. Richard Hunt in 1969, and Dr. Yves Meyer in 1970. The jury consisted of Professor A. Zygmund, C. Pisot, and J.-P. Kahane.

SUMMER STUDENT PROGRAM AT BROOKHAVEN NATIONAL LABORATORY

Seventy-eight students from forty-eight different colleges and universities throughout the United States are spending eleven weeks of their summer vacation at the Brookhaven National Laboratory. The program started Monday, June 14. The students were selected on the basis of their academic performance plus several letters of reference. Approximately 500 students applied for the openings this year, and of those selected, seven are majoring in mathematics. This program is funded by the Division of Nuclear Education and Training of the U. S. Atomic Energy Commission.

CONTENTS OF CONTEMPORARY MATHEMATICAL JOURNALS AND NEW PUBLICATIONS

Contents of Contemporary Mathematical Journals and New Publications will be combined into one journal beginning with the January 1972 issue of CCMJ. All of the information currently contained in New Publications, which lists new publications in mathematics and published and unpublished lecture notes, will appear in CCMJ. The new CCMJ will continue to be published biweekly, and subscriptions may be entered for twelve months, nine months, six months, or three months. An individual subscription for the full year beginning in January 1972 will be $12; for orders received after March 15, 1972, $9; for orders received after June 15, 1972, $6; and for orders received after September 15, 1972, $3. The equivalent list prices for institutional subscriptions are $24, $18, $12, and $6. Prepayment is required for nonmembers of the Society.

MEMORANDA TO MEMBERS

CONTRIBUTING MEMBERS

The Society acknowledges with gratitude the support rendered by the contributing members, who pay a minimum of $30 per year in dues. The extra dues paid by these members provide vital support to the work of the Society. In addition to those on the following list, four contributing members asked that their names remain anonymous.

Abbott, James H.
Albert, A. Adrian
Amir-Moez, Ali R.
Anderson, Richard D.
Andrews, Donald H.
Apostol, Tom M.
Arenstorf, Richard F.
Aroian, Leo A.
Ayer, Miriam C.
Babcock, William W.
Ballou, Donald H.
Barry, John Y.
Bauer, Frances B.
Beals, Richard W.
Beck, William A.
Beckenbach, Edwin F.
Beesley, E. Maurice
Bennett, Ralph B.
Bennewitz, William C.
Bernstein, Leon
Borisewich, John
Brickman, Louis
Bristol, Edgar H.
Brunswick, Natascha A.
Bryan, William S.
Bryant, Jack D.
Burington, Richard S.
Burke, James E.
Burt, Howard H.
Carson, Albert B.
Carter, Joan Cooley
Caywood, Thomas E.
Chaney, Robin W.
Clark, Harry E.
Clifford, Alfred H.
Cohen, Henry B.
Cohen, Teresa
Coleman, A. John
Collins, Heron S.
Colson, Henry D.
Cook, E. Allen, Jr.
Cooke, Roger Lee
Cornelius, Eugene F., Jr.
Coxeter, H. S. MacDonald
Cunkle, Charles H.
Danskin, John M.
Daus, Paul H.
DeMarr, Ralph E.
Devault, John L.
Donoghue, William F., Jr.
Durst, Lincoln K.
Eachus, J. J.
Earle, Clifford J., Jr.
Ellis, James W.
Embree, Earl O.
Epstein, Irving J.
Evans, George W., II
Farrell, Roger H.
Fass, Arnold L.
Feld, Joseph M.
Findley, George B.
Fuller, Leonard E.
Galant, David
Garrison, George N.
Gillman, Leonard
Gilmer, Robert
Gordon, Hugh
Gottschalk, Walter H.
Gould, Henry W.
Grace, Edward E.
Graves, Robert L.
Greif, Stanley J.
Guggenbuhl, Laura
Hacker, Sidney G.
Hamilton, Norman T.
Hardy, F. Lane
Harris, Charles D.
Hart, William L.
Hendrickson, Morris S.
Herwitz, Paul S.
Hilt, Arthur L.
Hochstadt, Harry
Hodges, John H.
Howell, James L.
Huff, Melvyn E.
Hufford, George A.
Hukle, George W.
Humphreys, M. Gweneth
Hutchinson, George A.
Ingraham, Mark H.
Isbell, John R.
Jackson, Stanley B.
James, R. D.
Jarnagin, Milton P., Jr.
Kalin, Theodore A.
Katzin, Martin
Keisler, James E.
Kelly, John B.
Kiernan, Bryce M.
Kirk, Joe E., Jr.
Kohls, Carl W.
Kossack, C. R.
Lanczos, Cornelius
Laning, J. H.
Laush, George
Lawrence, Sidney H.
Lemay, William H.
Levinson, Norman
Lewis, Hugh L.
Ling, Donald P.
Lipman, Joseph
Lubben, R. G.
MacNaughton, Lewis E.
Macy, Josiah, Jr.
Madi-Raj, Hagzl-Rao V.
Mansfield, Maynard J.
Mansfield, Ralph
Marchand, Margaret O.
Mattson, H. F., Jr.
Mayor, John R.
McBrien, Vincent O.
McGovern, Bernard J.
McIntosh, William D.

McLeod, Robert M.
McNaughton, Robert
Meder, Albert E., Jr.
Miller, Harlan C.
Miller, W. F.
Mitchell, Alfred K.
Mizel, Victor J.
Morrey, Charles B., Jr.
Morse, Anthony P.
Moursund, Andrew F.
Muller, David E.
Nelson, Eric J.
Newhouse, Sheldon E.
Norman, Edward
Norris, Donald O.
Norton, Karl K.
Offenbacker, Robert E.
Orloff, Leo N.
Outcalt, David L.
Owens, Owen G.
Paige, Eugene C., Jr.
Paige, Lowell J.
Palais, Richard S.
Pate, Robert S.
Pearson, Robert W.
Pell, William H.
Persinger, Carl A.
Peters, Stefan
Pflaum, C. W.
Plivka, Andrew D.
Poe, Robert L.
Potter, Meredith W.
Quade, Edward S.
Reardon, Philip C.
Rees, Carl J.
Rees, Mina S.
Rich, Ellis J.
Rinehart, Robert F.
Riney, John S.
Riordan, John
Roberts, J. H.
Rochon, Lloyd J.
Rose, Donald C.
Rosenblum, Marvin
Roth, Emile B.
Sampson, Charles H.
Sams, Burnett H.
Sawyer, Stanley A.
Saxon, Stephen A.
Schoenberg, Isaac J.
Schurrer, Augusta L.
Scott, Dana S.
Scott, Walter T.
Shampine, Lawrence F.
Shanks, Merrill E.
Sheffer, Isador M.
Shiffman, Max
Sinke, Carl J.
Sinn, Frederick W., Jr.
Sloan, Thomas D.
Smith, Duane B.
Smith, P. A.
Steenrod, Norman E.
Sternberg, David
Sudler, Culbreth, Jr.
Tellefsen, Carl R.
Thompson, Claude C.
Thompson, James M.
Thompson, Layton O.
Tucker, Albert W.
Turquette, Atwell R.
Vallarta, M. S.
Wallach, Sylvan
Walsh, Joseph L.
Weiss, Paul
Wendroff, Burton
Weston, James H.
Weyl, F. Joachim
Wheeler, Lyle Mary
Whitmore, William F.
Whitney, D. Ransom
Whitney, Hassler
Whittaker, James V.
Whyburn, W. M.
Widrewitz, Julius
Wilf, Herbert S.
Wilkins, J. Ernest W., Jr.
Wolman, Eric
Wonenburger, Maria J.
Zemel, Jacqueline L.
Zink, Robert E.

GRANTS FOR SCIENTIFIC RESEARCH

The Mathematical Sciences Section of the National Science Foundation once again announces that in order to insure full consideration, proposals requesting support beginning in the summer of 1972 should be in the hands of the cognizant program director not later than November 1, 1971. Since this is the second year for this procedure, the Section expects a closer adherence to this schedule than obtained last year. The Section also wishes to encourage greater attention to the NSF brochure "Grants for Scientific Research" (NSF 69-23) in the writing of proposals. Proposals recently have often been deficient in one or more of the following necessary items:

1. A full description of all other current research support or pending applications for such for all proposed investigators. In case there is no other support and no other application is pending or contemplated, the proposal must contain such a statement explicitly.

2. Curricula vitae of the proposed investigators, including lists of publications.

3. A bibliography of other pertinent publications (one's results are rarely based completely on one's own work).

4. Justification for any but the most usual items of support. In particular, requests for partial support of sabbatical leaves should be detailed.

5. An abstract of the proposed research, about one page in length, written in the third person, with a minimal number of symbols not on standard typewriters, and suitable for inclusion in the Science Information Exchange and for transmittal to several Foundation offices for information.

MATHEMATICAL SCIENCES EMPLOYMENT REGISTER

The August issue of the Mathematical Sciences Employment Register is now available. The Register is sponsored jointly by the American Mathematical Society, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics. Printed lists are available in August, January, and May of each year. Applicants and employers may list in any issue. There is no charge for listing either to employers or applicants, except employers who submit forms after the deadlines, but in time to be included in the printed publications, are charged a $5 fee for late listing. Applicants may be listed anonymously if they so request; there is a $5 charge for handling anonymous listings which must be paid when the form is submitted. The next deadline which is for listing in the January 1972 publications is December 8, 1971.

A subscription to the lists, which includes both the Summary of Available Applicants and the Summary of Academic, Industrial, and Government Openings, is $30. A single copy of the applicants lists, which includes the positions list, is $15; a list of positions only is $5. All lists are mailed "Book Rate" unless a subscriber requests either "First Class" or "Air Mail" delivery. The postage fee for this special handling is charged to the subscriber. The Register also issues in January of each year a List of Retired Mathematicians Available for Employment which is free upon request.

The next open Register, at which interviews are to be scheduled for employers and applicants, will be held in conjunction with the annual meeting scheduled for Las Vegas, Nevada, January 17-21, 1972. Interviews will be scheduled by computer in order to make it possible for applicants and employers to participate in as many interviews as possible. There is no registration fee for applicants; employers are charged $10 for registration. Applicants and employers must sign in at the Employment Register desk and obtain a code number if they wish to arrange interviews. It should be noted that registration for the Employment Register is separate and apart from meeting registration, as well as from the published lists, and it is mandatory that persons planning to participate in the open Register complete their general meeting registration first.

ANNUAL SALARY SURVEY

The fourteenth Annual Salary Survey will be published in the October issue of the *Notices*. This survey is conducted under the general supervision of the AMS Committee on Employment and Educational Policy which is composed of Richard D. Anderson, William L. Duren, Jr. (chairman), John W. Jewett, and Gail S. Young. The report, based on the results of this survey, will contain tables summarizing faculty salaries, graduate student support, and starting salaries of new Ph.D.'s. In view of the current and serious problems relating to employment in the mathematical sciences, the committee urges all chairmen to return these questionnaires as soon as possible in order that the information collected may be complete and accurate. This year, for the first time, a special questionnaire has been prepared for the chairmen of departments in two-year colleges. This new form was devised to make it possible to provide a better analysis of this important segment of the mathematical community.

Some recent developments underscore the urgency for assembling data as complete and accurate as possible. The National Science Foundation has announced plans to discontinue the biennial surveys of the National Register of Scientific and Technical Manpower. This means that the Annual Salary Survey represents the only current attempt to assemble and disseminate statistical data on salaries in the mathematical profession. In addition, the critical employment picture at present makes it even more important than usual to improve the statistical data on which a number of inferences are certain to be based—inferences with important consequences for the future development of mathematical education at the graduate level in particular and for the future growth of the entire profession.

CHANGE OF ADDRESS???????

Prompt notification of address changes will enable members to receive journals and other communications with the least possible delay. By attaching the mailing label to the notification of a change of address, clerks are supplied with the customer code by which they are able to process changes without undue delay. This also reduces the costs to the Society of implementing the change.

Clip or paste mailing label here, or copy the mailing label exactly as it appears, in its entirety.

Return this form or a copy to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904.

New Address

Name ______________________ Position ______________________

Institution or business ______________________

Address of institution or business ______________________

____

Mailing address (if different from above) ______________________

____

NEWS ITEMS AND ANNOUNCEMENTS

EXCHANGE PROFESSORSHIP

A French mathematician at a leading French university is interested in making an exchange of teaching positions with an American mathematician for the year 1972-1973. This mathematician is a professor of the Faculté des Sciences of his institution and is currently teaching a course in analysis (measure theory and distributions), and is also in charge of a postgraduate course in geometry (the geometrical methods of mechanics). His salary is 4,300 francs a month. Correspondence concerning this position should be placed in a plain sealed envelope and then inserted in an envelope addressed to FRENCH EXCHANGE, Dr. Gordon L. Walker, American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904. Such correspondence will be forwarded immediately.

NATO SENIOR FELLOWSHIPS IN SCIENCE

The National Science Foundation and the Department of State have announced the awarding of thirty-seven North Atlantic Treaty Organization (NATO) Senior Foreign Fellowships in Science. These fellowships enable universities and nonprofit scientific research institutions in the United States to send senior staff members to research and educational institutions in other NATO nations, or other countries participating in the program. The fellowships normally carry short-term tenures of from one to three months. Among the recipients of this year's awards are three mathematicians: Herman R. Gluck, University of Pennsylvania; Robert E. Barnhill, University of Utah; and Richard A. Johnson, University of Wisconsin.

LETTERS TO THE EDITOR

Editor, the *Notices*

There seems to be more and more talk about reducing the foreign language requirement for the Ph.D. degree in mathematics. To find out, what in fact, the current requirements are, I conducted a survey. The institutions approached were very cooperative, and many of them expressed a strong concern about the foreign language requirements. The purpose of this letter is to report the results of my survey of 120 institutions in the U.S. offering a graduate program in mathematical sciences, to which 101 responses were received (84%).

The current requirements are:

89 schools require two foreign languages (of which five indicated that one was enough if the proficiency was good enough),

9 only one language,

3 have no foreign language requirement at all.

(1 school has no graduate school requirement and states: "Department only requires that students be able to read whatever material in foreign languages is needed in their courses.")

The proficiency required:

52 schools require translation with dictionary,

31 require translation without dictionary,

14 have both possibilities.

In no response was a conversational knowledge indicated, reading ability only.

Ways of satisfying requirements:

54 schools definitely require an examination,

44 give a choice between credit for (reasonably good) college course work and examination,

2 indicated a possiblity of credit for high school course work.

Number of required semesters of courses in foreign language:

25 schools indicated less than 2 semesters in any language,

11 at least 4 semesters in 1 language,

8 at least 2 semesters in each of 2 languages,

4 at least 2 semesters in 1 language.

Examination administered by:

41 schools indicated definitely "other" (e.g. GSFLT),

32 have examination only by the department of mathematics,

17 have both of the above possibilities,

6 have examination by the graduate school.

In the response "other" for examination administered by:

15 schools indicated GSFLT (ETS),

11 Language Department,

4 both of the above.

The above results are concerning the Ph.D. degree. For the Master's, only 30 institutions have any foreign language requirement; and then it is one language only.

I would like to take this opportunity to express my gratitude to all who cooperated in this survey.

Rotraut C. Weiss
University of Santa Clara

Editor, the *Notices*

This communication differs from many others in that it is not concerned with ethical, moral or social problems, but only with mathematics. The problem in question has been open for some 15 years and has received the serious attention of at least six mathematicians who, on a basis of their contributions to the subject, may be regarded as experts. This fact is mentioned in order that you may know that the problem—though not comparable in importance with, exempli gratia, anti-Algerianism in France, apartheid in South Africa, neglect of oriental jewry in Israel, the vicious murder by Viet Cong of South Viet Nam civilians, murderous suppression of civil rights in Cuba, Haiti, and each of the two Chinas, deprivation of full civil rights of Gentiles in Israel, or the invasion of Cambodia by North Viet

Nam–is perhaps not trivial.

The problem is this–given a continuous associative multiplication on a 2-sphere $((x,y) \rightarrow xy)$ such that every element is a product, can there exist an element z satisfying $zx = z = xz$, whatever x?

Despite the non-inflamatory subject of this letter, I hope that it will be possible for you to publish it.

Alexander Doniphan Wallace
University of Florida

Editor, the *Notices*

Recently there has been some discussion in the *Notices* concerning any positions that the AMS might take concerning the Viet-Nam war. May I offer the following comments. Recently in my graduate algebra class, while doing fields and Galois theory, we discussed the Viet-Nam war. It is a problem not solvable by radicals.

Daniel Karpen
SUNY at Stony Brook

Editor, the *Notices*

Due to an unfortunate oversight, in my notice that the square of a nonseparable graph is Hamiltonian connected (71T-A79, *Notices*, Vol. 18, #3, April 1971), I failed to state that my results were dependent on Herbert Fleischner's three earlier papers "The Total Graph of a Block is Hamiltonian," "On Line-Critical Blocks," and "The Square of Every Non-Separable Graph is Hamiltonian" (to appear). I wish to apologize to Dr. Fleischner for this omission.

Arthur M. Hobbs
University of Waterloo

Editor, the *Notices*

As a member of the AMS, I deplore the attempts to politicize the Society. I vigorously object to the *Notices* being used as a vehicle for inflammatory letters urging political action. Recent letters have not only been hysterically anti-U.S., but have been historically inaccurate and unfair. I refer in particular to the letter by Eugene H. Lehman, which I feel should not have been published in the *Notices*.

There is indeed a great debate as to whether the AMS should take a stand on the war in Southeast Asia. Perhaps the membership should vote on this. I suggest instead that a vote be taken as to whether or not the Society should engage in politics in general. If the membership votes "yes" you may have my resignation, with regret.

I cast my vote now to keep politics out of the *Notices* and to keep the Society out of politics.

Henry E. Heatherly
University of Southwestern
Louisiana

SPECIAL MEETINGS INFORMATION CENTER

The purpose of this center is to maintain a file on prospective symposia, colloquia, institutes, seminars, special years, meetings of other associations, and to notify the organizers if conflicts in subject matter, dates or geographical area become apparent. A first announcement will be published in the *Notices* if it contains a call for papers, place, date, and subject, where applicable; a second announcement must contain reasonably complete details of the meeting in order for it to be published. Information on the pre-preliminary planning will be stored in the files, and will be available to anyone desiring information on prospective conferences. All communications on special meetings should be sent to the Special Meetings Information Center of the American Mathematical Society.

MATHEMATICAL ASSOCIATION OF AMERICA

The fifty-second summer meeting of the Mathematical Association of America will be held at The Pennsylvania State University, University Park, Pennsylvania, on August 30–September 1, 1971, in conjunction with meetings of the American Mathematical Society, Mu Alpha Theta, and Pi Mu Epsilon. The twentieth series of Earl Raymond Hedrick Lectures will be delivered by Professor Abraham Robinson of Yale University with the title "Nonstandard analysis and nonstandard arithmetic." The Lester R. Ford Awards will be presented at the business meeting of the Association on August 31, 1971, at 10:00 a.m. A complete program of the meeting is included in the timetable which appears on page 696 in this issue of these *Notices*.

CONFERENCE ON GROUP THEORY

The department of mathematics of York University is organizing a conference on Group Theory to be held in Toronto, Ontario, November 25–27, 1971. The program will consist of several invited addresses and a number of contributed papers. Topics will be on combinatorial group theory, and several experts in this field will be attending. Persons who wish to be invited to attend the conference should write to Dr. Trueman MacHenry, Group Theory Conference, Department of Mathematics, York University, Downsview, Ontario, Canada, indicating the amount of support required, if any.

INSTITUTE ON STATISTICAL ECOLOGY

With the financial support of the National Science Foundation, an advanced institute on Statistical Ecology in the United States will be held at The Pennsylvania State University on June 19–28, 1972, under the joint sponsorship of the International Statistical Institute and the International Association of Ecology. The purpose of this institute is basically to provide advanced and specialized education and research training on important topics in statistical ecology such as modeling and simulation of biological populations, chance mechanisms and statistical distributions in ecology, measurement and detection of spatial patterns, sampling biological populations, multivariate methods in ecology, quantitative populations, multivariate methods in ecology, quantitative population dynamics, and systems analysis in ecology. A major emphasis will also be given to individual study, individual problems and consultations, seminars by participants, and small group discussions and workshops. Invitations are being sent out to the prospective instructional staff and the guest lecturers. Under the NSF grant, support will be provided to speakers and participants. For further information, please write to Professor G.P. Patil, Director, Advanced Institute on Statistical Ecology, 330 McAllister Building, University Park, Pennsylvania 16802.

CONFERENCE ON THE THEORY OF ORDINARY AND PARTIAL DIFFERENTIAL EQUATIONS

A conference on the Theory of Ordinary and Partial Differential Equations will be held in the Department of Mathematics of The University of Dundee on March 28–31, 1972. A large number of distinguished speakers will deliver invited lectures, and any participant who wishes to give a short lecture on his own results will be given the opportunity to do so. Accommodations in Belmont Hall, one of the university's residence halls, will be available from the evening of March 27 to the morning of April 1. Further information may be obtained by writing to Professor I.M. Michael, Department of Mathematics, The University of Dundee, Dundee, Scotland.

A MODERN APPROACH TO THE SOLUTION OF THE PROBLEMS OF CONTINUUM MECHANICS

In the summer of 1972, a course lasting from August 21 through August 26 will be given at the University of Minnesota. The objective of this course is the exposition of the solutions of the problems of (1) describing the behavior of a plasma under the influence of an electromagnetic field, (2) describing the propagation of sound in a partially nonhomogeneous media such as the ocean, (3) solving parabolic equations describing diffusion processes and transition probabilities of states of systems which are subject to random disturbances, (4) determining the effect of an elastic disturbance on an inclusion or intrusion in an elastic or viscoelastic media, and (5) determining the scattering of electromagnetic waves by diamagnetic and metallic bodies. The techniques make use of recently developed tools of mathematical analysis and functional analysis. While specific applications have been listed, emphasis will be placed on solution procedures, and a broad area of applicability is anticipated. Those who are interested in contributing expository material to the course should write to Professor David K. Cohoon, School of Mathematics, 105 Vincent Hall, University of Minnesota, Minneapolis, Minnesota 55455. Those whose material is accepted for presentation in the course will be paid travel expenses plus an honorarium. For further information on attending this conference, please write to the Nolte Center for Continuing Education, Department of Conferences and Institutes, University of Minnesota, Minneapolis, Minnesota 55455.

THE INSTITUTE OF MANAGEMENT SCIENCES

The Institute of Management Sciences will hold a meeting on September 29–October 2, 1971, at the Detroit Hilton Hotel, Detroit, Michigan. The following areas will be discussed: health care, government administration, environmental management, transportation, population growth, housing, educational systems, and law enforcement. Registration information and a schedule of events may be obtained by writing to Harvey A. Shapiro, School of Economics and Management, Oakland University, Rochester, Michigan 48063.

CONFERENCE ON THE MATHEMATICS OF FINITE ELEMENTS AND APPLICATIONS

The Institute of Mathematics and its Applications is organizing a conference on the Mathematics of Finite Elements and Applications to take place at Brunel University, April 18–20, 1972. The principal lecturers will be A.R. Mitchell of the University of Dundee; J.T. Oden of the University of Alabama; O.C. Zienkiewicz of the University of Wales, Swansea; and M. Zlamal, Technical University, Brno. A number of half-hour submitted papers on the mathematical theory of finite elements together with methods of application and computational algorithms for implementation of these will be read. The proceedings of the conference will be published. Contributed papers should be submitted by November 1, 1971, to Dr. J.R. Whiteman, Department of Mathematics, Brunel University, Kingston Lane, Uxbridge, Middlesex, England. Accommodations will be available in a university residence hall, and application forms can be obtained from the Secretary and Registrar, The Institute of Mathematics and its Applications, Maitland House, Warrior Square, Southend-on-Sea, Essex SS1 2JY, England.

SYMPOSIUM ON PARTIAL DIFFERENTIAL EQUATIONS

As part of the Special Year in Partial Differential Equations and Their Applications, sponsored by the Universities of Dundee, Edinburgh, Glasgow, Newcastle, and Strathclyde, an opening symposium is to be held at the University of Strathclyde on September 20–24, 1971. A series of survey lectures will be given to explain recent developments in partial differential equations to mathematicians who are not expected to be specialists in this field. The following have agreed to give one or more talks: Professor I. N. Sneddon, opening address; Professor N. W. Bazley, "Bifurcation theory"; Professor C. Cercignani, "Initial and boundary value problems for the Boltzmann equation"; Professor D. L. Colton and R. P. Gilbert, "Function theoretic methods"; Professor G. E. Latta, "Singular perturbations." Application forms, which must be returned not later than July 1, 1971, and further information may be obtained by writing to Dr. R. J. Cole, Department of Mathematics, University of Strathclyde, Glasgow C1, Scotland.

CONFERENCE OF THE TEACHING OF NUMERICAL ANALYSIS

The Institute of Mathematics and its Applications is organizing a three-day conference on the teaching of numerical analysis, which will be held at the University of Sussex, Brighton, England, on January 3–5, 1972. Keynote speakers will be Professor L. Fox and Dr. K. Morton. The conference will cover the teaching of numerical analysis in schools and universities, and the usefulness of practical work with computers. Contributions from industrial mathematicians will also be included. Residential accommodations will be available to those attending the meeting. Further information may be obtained by writing to the Secretary and Registrar, The Institute of Mathematics and its Applications, Maitland House, Warrior Square, Southend-on-Sea, Essex SS1 2JY, England.

PI MU EPSILON

The Pi Mu Epsilon Fraternity will hold a banquet for members and guests on Tuesday, August 31, 1971, at 6:30 p.m. in Dining Room B of the Hetzel Union Building on the campus of The Pennsylvania State University. Professor John W. Randolph will present a lecture entitled "Reflections on the worth of mathematics." A Dutch Treat Breakfast meeting for members and guests will be held on Wednesday, September 1, at 8:00 a.m. in the Waring Hall Dining Room. Participants will go through the cafeteria line before convening for this meeting. The Governing Council will meet on Tuesday at 12:15 p.m. in the Waring Hall Dining Room. Sessions for contributed papers will be held on Tuesday at 3:15 p.m. and on Wednesday at 10:40 a.m. in Rooms 312-313-314 of the Conference Center.

INTERNATIONAL CONFERENCE ON HARMONIC ANALYSIS

The Department of Mathematics at the University of Maryland is planning an International Conference on Harmonic Analysis, scheduled for November 8–12, 1971, to highlight its Special Year in Functional Analysis. The conference will focus on two areas of abstract harmonic analysis—spectral synthesis and group representations—and the program will include a number of invited talks by noted speakers, with ample time for informal discussions. For further information, please write to Professor Denny Gulick, Department of Mathematics, University of Maryland, College Park, Maryland 20742.

PERSONAL ITEMS

GRAHAM D. ALLEN of the University of Wisconsin has been appointed to an assistant professorship at Texas A & M University.

JOAN S. BIRMAN of the Stevens Institute of Technology has been appointed to a visiting assistant professorship at Princeton University for the fall term 1971-1972.

LINCOLN E. BRAGG of the University of Kentucky has been appointed to an assistant professorship at the Florida Institute of Technology.

RONALD P. BROWN of Simon Fraser University, British Columbia, has been appointed to an assistant professorship at the University of Hawaii.

ARNOLD B. CALICA of the U. S. Naval Academy has been appointed to an assistant professorship at the University of Hawaii.

HASKELL B. CURRY has been appointed Visiting Andrew Mellon Professor at the University of Pittsburgh for the fall and winter terms of 1971-1972.

THADDEUS G. DANKEL of Duke University has been appointed to an associate professorship at the University of North Carolina at Wilmington.

DAVID FRASER of Brown University has been appointed to an assistant professorship at Worcester Polytechnic Institute.

JOHN W. GREINER of North American Rockwell Corporation has been appointed to the staff of Long Beach City College.

JAMES C. HALSEY of North Carolina State University has been appointed to an assistant professorship at the University of North Carolina at Wilmington.

ARTHUR M. HOBBS of the University of Waterloo has been appointed to an assistant professorship at Texas A & M University.

RAYMOND T. HOOBLER of the Graduate Center, CUNY, has been appointed to a visiting assistant professorship at Rice University.

HOWARD J. JACOBOWITZ of the Institute for Advanced Study has been appointed to an assistant professorship at Rice University.

LAMBERT H. KOOPMANS of the University of New Mexico will be on sabbatical leave at the University of California, Santa Cruz, until July 1972.

DAVID G. KOSTKA of Northwestern University has been appointed to an assistant professorship at Texas A & M University.

JOHN S. LANCASTER of Indiana University has been appointed to an assistant professorship at the University of Hawaii, Hilo College.

WILLIAM J. LEVEQUE of the University of Michigan has been appointed to a professorship at Claremont Graduate School.

SUE-CHIN LIN of the Institute for Advanced Study has been appointed to an associate professorship at the University of Illinois at Chicago Circle.

ADOLF MADER of the University of Hawaii is on sabbatical leave during the academic year 1971-1972. He will spend his leave at the Universität Tübingen, Federal Republic of Germany.

JOHN C. MARTIN of Rice University has been appointed to an assistant professorship at the University of Hawaii.

THOMAS A. METZGER of Purdue University has been appointed to an assistant professorship at Texas A & M University.

ITREL MONROE of Dartmouth College has been appointed to an assistant professorship at the University of Hawaii.

DALE MYERS of the University of California, Berkeley, has been appointed to an assistant professorship at the University of Hawaii.

NAGENDRA PANDEY of Oregon State University has been appointed to an assistant professorship at Montana College of Mineral Science and Technology.

WILLIAM L. PERRY of the University of Illinois has been appointed to an assistant professorship at Texas A & M University.

MYRON F. ROSSKOPF was named

as Clifford Brewster Upton Professor of mathematical education at the Teachers College, Columbia University.

MARTIN RUTSCH of the Universität Saarbrücken, Federal Republic of Germany, has been appointed to a professorship at the Universität Karlsruhe, Federal Republic of Germany.

GABRIEL SABBAGH of Yale University has been appointed Attaché de Recherches at the Centre National de la Recherche Scientifique, Antony, France.

JESSE SHAPIRO of Augsburg College has been appointed to an associate professorship at the University of the Negev, Beer Sheva, Israel.

PHILIP B. SHELDON of Wisconsin State University, Whitewater, has been appointed to an assistant professorship at Virginia Polytechnic Institute and State University.

ROBERT R. STOLL of Oberlin College has been appointed to a professorship and to the chairmanship of the Department of Mathematics at Cleveland State University.

PAUL L. STRONG of the University of Illinois has been appointed to an assistant professorship at Bucknell University.

ARTHUR H. STROUD of SUNY at Buffalo has been appointed to a professorship at Texas A & M University.

JEFFREY L. TOLLEFSON of Iowa State University has been appointed to an assistant professorship at Texas A & M University.

FAUSTO A. TORANZOS of the Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires, has been appointed to a professorship in the Facultad de Farmacia y Bioquimica at that university.

JOSEPH F. TRAUB of Bell Telephone Laboratories and the University of Washington has been appointed to a professorship and to head of the Department of Computer Science at Carnegie-Mellon University.

HISAHARU UMEGAKI of the Department of Mathematics, Tokyo Institute of Technology, has been appointed to a professorship in the Department of Information Sciences at that university.

BENJAMIN B. WELLS, Jr., of Tecnica del Estado, Santiago, Chile, has been appointed to an associate professorship at the University of Hawaii.

CALVIN H. WILCOX of the University of Denver, currently visiting at the Institute de Physique Théorique, Geneva, has been appointed to a professorship at the University of Utah.

PROMOTIONS

To Dean of Academic Advising. Worcester Polytechnic Institute: JOHN P. VAN ALSTYNE.

To Chairman, Department of Mathematics. Massachusetts Institute of Technology: KENNETH M. HOFFMAN; Wartburg College: WILLIAM L. WALTMANN.

To Professor. University of Calgary: RANJIT S. DHALIWAL; University of Hawaii: NOBUO NOBUSAWA; Indiana University: PETER A. FILLMORE; Ohio University: S. K. JAIN.

To Associate Professor. Bucknell University: EUGENE M. LUKS; University of Hawaii: JACK WILLIAMSON; Indiana University: DAVID L. COLTON, DANIEL P. MAKI; Indiana & Purdue Universities at Indianapolis: JOSEPH E. KUCZKOWSKI; Lehigh University: DAVID TRUTT; Rice University: F. REESE HARVEY; St. Francis Xavier University, Nova Scotia: SYED ASADULLA; Worcester Polytechnic Institute: BRUCE C. McQUARRIE.

DEATHS

Dr. SAMUEL BEATTY of the University of Toronto died on July 3, 1970, at the age of 89. He was a member of the Society for 56 years.

Professor L. G. BUTLER of San Pedro, California, died on January 26, 1971, at the age of 75. He was a member of the Society for 46 years.

Professor H. PETER DEMBOWSKI of the Universität Tübingen, Federal Republic of Germany, died in January 1971 at the age of 42. He was a member of the Society for 15 years.

Professor HENRY GERHARDT of Waynesville, North Carolina, died on May 19, 1971, at the age of 87. He was a member of the Society for 37 years.

Professor Emeritus HEINZ HOPF of the Eigdenössiche Technische Hochschule, Zürich, died on June 3, 1971, at the age of 77. He was a member of the Society for 23 years.

Mrs. ROBIN W. ROBINSON of Urbana, Illinois, died on July 6, 1970, at the age of 25. She was a member of the Society for 5 years.

Professor JORDAN T. ROSENBAUM of the University of Pittsburgh died on June 22, 1971, at the age of 35. He was a member of the Society for 6 years.

Professor HARRY SILLER of Hofstra University died on April 15, 1971, at the age of 60. He was a member of the Society for 34 years.

Professor DAVID M. TOPPING of Tulane University died on October 21, 1970, at the age of 37. He was a member of the Society for 11 years.

NEWS ITEMS AND ANNOUNCEMENTS

CUPM-MSSB PROJECT

The Committee on the Undergraduate Program in Mathematics and the Mathematical Social Science Board are seeking interesting problems, or illustrative examples, from each of the social sciences, whose solutions and study make use of ideas and techniques from the following topics in undergraduate mathematics: sets and relations, differential and integral calculus, matrices and linear algebra, and probability. It is proposed that such examples will be collected into a book to be used mainly by mathematics teachers and students as a source (1) of current social science applications of mathematics, and (2) of material for textbook and classroom exercises to illustrate how topics in collegiate mathematics arise in a social science context. Included also will be annotated bibliographies of articles and books involving applications of mathematics to the various social sciences. Preferred contributions will be expositions giving (a) the social science problem and its background; (b) the reduction of the problem to mathematical form; (c) the mathematical analysis, perhaps with associated numerical results obtained on a computer; and (d) the meaning and insights provided by the mathematical analysis when related back to the original social science problem. Less desirable, but still welcome, will be reprints including material from which such an exposition could be extracted. References to the literature will be helpful. The CUPM-MSSB Project Committee is composed of Donald W. Bushaw, Samuel Goldberg, Harold Kuhn, R. Duncan Luce, and Henry Pollak. Please send contributions to the CUPM-MSSB Project, P.O. Box 1024, Berkeley, California 94701.

ESSAY COMPETITION

The editors of Philosophia Mathematica have announced an essay competition on The Nature of Modern Mathematics. The essay should be less than 6,000 words (in English), and the prize will be $200. Explicit emphasis should be placed on philosophical aspects of modern mathematics in the process of creation rather than expositional or logical analysis, namely from the standpoint of working mathematicians. The editors have stated that the entrants should have backgrounds at no less than the graduate level. Essays should be double spaced with wide margins, and should be free from any complex symbols unnecessary in a paper of a philosophical nature. Footnotes, numbered consecutively, should also be double spaced and on separate sheets. Essays (together with a carbon copy) may be sent at any time before May 31, 1972, to Professor J. Fang, 7543 Calmbach, Federal Republic of Germany. The selection committee will consist of the editor and associate editors of Philosophia Mathematica, and the winning essay will appear in either volume 9 (1972) or volume 10 (1973) of Philosophia Mathematica.

NEW AMS PUBLICATIONS

MEMOIRS OF THE AMERICAN MATHEMATICAL SOCIETY

Memoir 112

ON THE MIXED PROBLEM FOR A HYPERBOLIC EQUATION
By Tadeusz Bałaban

119 pages; List Price $2.20; Member Price $1.65

The aim of this Memoir is to generalize to a problem with variable coefficients Agmon's result on a mixed problem for hyperbolic equations with constant coefficients, announced at the Paris conference in June 1962. By the locality properties of this problem, it is sufficient to consider the equation with boundary conditions, that is, $Pu = f$ in Ω, $Q_j u = g_j$, $j = 1, \ldots, \varkappa$, where Ω is a domain on one side of the smooth surface S, P is a strongly hyperbolic operator of order m. The operators Q_j satisfy a condition which is a generalization of the well-known "coercivity condition" in elliptic boundary problems. The characteristic polynomial of the operator P has no double real zeros on the surface S in the direction of the normal to S. Under these conditions "the fundamental inequality," that is

$$\int_\Omega |D^{m-1+k}u|^2\,dx + \int_S |D^{m-1+k}u|^2\,dS$$
$$\leq C\Big(\int_\Omega |D^k Pu|^2 dx$$
$$+ \sum_{j=1}^{\kappa} \int_S |D^{m-1+k-m_j} Q_j u|^2 dS\Big),$$
$$u \in C^\infty_{(0)}(\Omega),$$

is proved. The "dual inequality," that is the inequality for the adjoint problem, is also obtained. From these inequalities, the existence and uniqueness of solutions of the mixed problem is obtained.

Memoir 113

INVARIANT DIFFERENTIAL OPERATORS AND COHOMOLOGY OF LIE ALGEBRA SHEAVES
By Franz W. Kamber and Philippe Tondeur

127 pages; List Price $2.20; Member Price $1.65

For a Lie algebra sheaf $\underline{L}$ of derivations of a sheaf of rings $\underline{O}$ on a space X global cohomology groups and local cohomology sheaves are introduced and analyzed. Global and local splitting obstructions for extensions of modules over a Lie algebra sheaf are studied.

In the applications considered, $\underline{L}$ is a Lie algebra sheaf of vectorfields on a manifold M, $\underline{O}$ the structure sheaf or M. For vectorbundles E, F on M on which $\underline{L}$ acts, the existence of invariant differential operators $D: \underline{E} \to \underline{F}$ whose symbols are preassigned equivariant maps is discussed in terms of these splitting obstructions. Lie algebra sheaves defined by Lie group actions are considered.

This theory is applied in particular to the case of a transitive $\underline{L}$. The splitting obstructions for extensions of modules over a transitive Lie algebra sheaf are analyzed in detail. The results are then applied to the problem of the existence of invariant connections on locally homogeneous spaces. The obstruction is computed in some examples.

Number 114

MIXING SEQUENCES OF RANDOM VARIABLES AND PROBABILISTIC NUMBER THEORY
By Walter Philipp

104 pages; List Price $2.00; Member Price $1.50

This memoir presents the results of a partially successful attempt to unify probabilistic number theory by means of mixing sequences of random variables. The first chapter deals with the central limit problem and the uniform law of the iterated logarithm for such sequences. In the subsequent chapters these results are applied to number theoretic questions and we obtain limit theorems for additive functions, Diophantine approximation, the discrepancies of certain sequences uniformly distributed modulo 1, continued fractions, and related algorithms.

PROCEEDINGS OF THE STEKLOV INSTITUTE

Number 108 (1968)

INFINITE-DIMENSIONAL GAUSSIAN DISTRIBUTIONS
By Ju. A. Rozanov

165 pages; List Price $12.60; Member Price $9.45

In this volume, problems are considered which in one way or another are concerned with equivalence of Gaussian probability distributions in various infinite-dimensional spaces (infinite-dimensional Gaussian distributions). In the four successive chapters, the author discusses some results pertaining to the definition of Gaussian probability measures in various linear spaces; results concerning equivalence of Gaussian probability distributions and densities of such distributions; the applications of some of these results to problems of mathematical statistics, in particular to problems of constructing the best unbiased estimates for an unknown mean value and the correlation function of an arbitrary Gaussian probability distribution; and finally, problems are considered concerning the structure of linear measurable functionals and transformations in an arbitrary linear space with a Gaussian measure. The investigations carried out reveal an harmonious connection between these problems posed and certain well-known problems in the theory of probability and in mathematical statistics.

TRANSLATIONS OF MATHEMATICAL MONOGRAPHS

Volume 27

THE STEFAN PROBLEM
By L. I. Rubenšteĭn

427 pages; List Price $26.70; Member Price $20.13

In recent years there has been an intensive development of the Stefan problem with which the author has long been concerned. In this monograph, the author has limited himself to brief descriptions of finite difference algorithms which have been proposed for the solution of the Stefan problem; as a rule no justification of the algorithm has been given, but the reader is referred to the original literature. The author is, on the one hand, mainly interested in the physical formulation of a problem which reduces to some form of the Stefan problem and, on the other hand, in the examination of general theoretical problems, i.e. problems of a qualitative character. The volume includes three supplements; the first two supplements present certain known results which are used in the book, and the third supplement presents illustrative examples of numerical solutions. Included also is a bibliography of the problem.

Volume 32

TRIANGULAR AND JORDAN REPRESENTATIONS OF LINEAR OPERATORS
By M. S. Brodskiĭ

254 pages; List Price $14.30; Member Price $10.73

Since the first results of 1954 for nonselfadjoint operators, many mathematicians have been extending the classes of operators for which infinite-dimensional analogs can be obtained for the finite-dimensional theorems that any square matrix is reducible to triangular form by a unitary transformation, and that a matrix reduced to Jordan form consists of only one Jordan cell (i.e., is "unicellular") if and only if, given any two of its invariant subspaces, one is contained in the other. The present book brings these investigations up to date, with natural emphasis on Volterra operators.

PROCEEDINGS OF SYMPOSIA IN PURE MATHEMATICS

Volume XIX

COMBINATORICS
Edited by Theodore S. Motzkin

263 pages; List Price $19.70; Member Price $14.78

This volume constitutes the proceedings of a symposium on combinatorics held in the spring of 1968 at the University of California, Los Angeles. Emphasis at the symposium was placed on the theory of simple general or homogeneous structures. Of the twenty-four papers in the present volume, eight treat general structures; nine treat designs (homogeneous structures); six treat applications of the first two topics to sets of integers, algebra, and complex analysis; and one is a survey article mainly on general structures and partly on sets of integers. (Asymptotic results occur in seven of the thirteen papers on general structures or applications thereof; computers were used in three papers.) With its numerous and varied open questions and new methods and results, extending from the solution of a century-old problem on designs to algebra-geometric and number and function theoretic studies, this volume reflects work done and in progress and should contribute to growth and change in combinatorics.

RECENT REPRINTS

MATHEMATICAL SURVEYS
Number 7

THE ALGEBRAIC THEORY OF SEMIGROUPS, Volume II
By A. H. Clifford and G. B. Preston

368 pages; List Price $14.20; Member Price $10.65

"The algebraic theory of semigroups, Volume II," (Number 7 in the Mathematical Surveys series) by A. H. Clifford and G. B. Preston has been extensively revised by the authors and has been reprinted. The volume was published in 1967 and was reprinted in 1971. Most of the subject matter in this volume is taken from papers published prior to the drawing up of the original plan for both volumes. The greater part of the book deals not with the deeper development of the topics initiated in Volume I, but with additional branches of the theory to which there was at most passing reference in Volume I. The topics dealt with in Volume II include what the authors judge to be the more important developments to date.

MEMOIR 41

ISOCLINIC n-PLANES IN EUCLIDEAN 2n-SPACE, CLIFFORD PARALLELS IN ELLIPTIC (2n - 1)-SPACE, AND THE HURWITZ MATRIX EQUATIONS
By Yung-Chow Wong

118 pages; List Price $3.40; Member Price $2.55

Since its publication in 1961, this Memoir has led to developments in three different directions. First, J. A. Wolf has generalized the main results in this Memoir from the field of real numbers to the field of complex numbers and the field of real quaternions, obtaining a complete analysis, in all three cases, of the sets of mutually Clifford parallel subspaces in a projective space provided with the usual elliptic. Independent of and different from Wolf's work, J. G. Semple and J. A. Tyrrell have given an alternative development of generalized Clifford parallelism in the context of complex projective geometry, revealing much insight into the construction and properties of the sets of mutually Clifford parallel subspaces. Lastly, the author himself has developed a unified treatment of the geometry in Euclidean and pseudo-Euclidean spaces and the differential geometry of Grassmann manifolds and Cartan domains, over the field of real numbers, the field of complex numbers, and the field of real quaternions. The subject of this Memoir is an interesting one, both historically and mathematically. It gives the first generalization to higher spaces of the parallelism in elliptic 3-space discovered by the British mathematician William K. Clifford in 1873, and it is related in some unexpected ways to several important problems in other branches of mathematics. The recent developments mentioned above have shown that this subject is worthy of further study and that the ideas and methods first conceived and developed in this Memoir have been quite useful.

NEWS ITEMS AND ANNOUNCEMENTS

NSF COMPREHENSIVE TEACHER EDUCATION PROGRAM

The National Science Foundation seeks proposals for its new Comprehensive Teacher Education Program and has published a "Guide for Preparation of Proposals." This program will support an interrelated set of activities for improving pre-college teacher education, and combines programs that in the past have been funded separately by the Foundation. Projects should be designed to meet demonstrated science education needs in a given geographical area, in a particular subject, and/or concerning other factors deemed appropriate by the proposing institution. Projects are expected to produce changes in the teacher education activities of the host institutions and in the way they cooperate with schools. There is no specific deadline for the submission of proposals; processing of a proposal requires approximately six months. Institutions eligible to apply for grants under the Comprehensive Teacher Education Program are appropriate nonprofit organizations and colleges and universities which grant at least the baccalaureate level degree in science and mathematics. For additional information and a copy of the "Guide for Preparation of Proposals, Comprehensive Grants for Teacher Education" (NSF E-71-4), please write to Program Director for Academic Year Study, Division of Pre-College Education in Science, National Science Foundation, Washington, D. C. 20550. Telephone: 202-632-5970.

COURANT INSTITUTE POSTDOCTORAL VISITING MEMBERSHIPS

The Courant Institute of Mathematical Sciences of New York University offers postdoctoral Visiting Memberships to mathematicians, scientists, and engineers who are interested in its program of training and research in a broad range of pure and applied mathematics. Applications for the academic year 1972-1973 must be submitted before January 1, 1972. Inquiries should be addressed to the Visiting Membership Committee of the Courant Institute, 251 Mercer Street, New York, New York 10012.

CANADIAN MATHEMATICAL CONGRESS

The members of the Canadian Mathematical Congress will be listed in the 1971-1972 issue of the Combined Membership List to be published in the fall of 1971. The members of the Congress will not receive free copies of the 1971-1972 edition, but they will receive free copies of the 1972-1973 edition. This will bring the distribution of complimentary copies to the Congress in phase with the distribution of complimentary copies to the members of the Society, who receive such copies in even-numbered years. Free copies are available to members of the Mathematical Association of America in odd-numbered years.

BACKLOG OF MATHEMATICS RESEARCH JOURNALS

Information on the backlog of papers for research journals is published in the February and August issues of these *Notices* with the cooperation of the respective editorial boards. Since all columns in the table are not self-explanatory, we include further details on their meaning.

Column 3. This is an estimate of the number of printed pages which have been accepted but are not necessary to maintain copy editing and printing schedules.

Column 5. The first (Q_1) and third (Q_3) quartiles are presented to give a measure of normal dispersion. They do not include misleading extremes, the result of unusual circumstances arising in part from the refereeing system.

The observations are made from the latest issue of each journal received at the Headquarters Offices before the deadline for the appropriate issue of these *Notices*. Waiting times are measured in months from receipt of manuscript in final form to receipt of final publication at the Headquarters Offices. When a paper is revised, the waiting time between an editor's receipt of the final revision and its publication may be much shorter than is the case otherwise, so these figures are low to that extent.

	1	2	3		4	5		
JOURNAL	No. issues per year	Approx. No. pages per year	BACKLOG 6/30/71	12/31/70	Est. time for paper submitted currently to be published (in months)	Observed waiting time in latest published issue (in months) Q_1	Med.	Q_3
Acta Informatica	4	370	0	---	4-6			
American J. of Math.	4	1100	472	0	12	9	11	14
Annals of Math. Stat.	6	2200	0	0	9	8	11	17
Annals of Math.	6	1200	NR*	NR*	12	13	14	18
Arch. Rational Mech. Anal.	25	2000	0	---	6	6	###	9
Canad. J. of Math.	6	1200	100	680	6-9	6	7	9
Comm. Math. Physics	17	1428	0	---	6			
Duke Math. J. **	4	192	340	900	24	24	25	26
Illinois J. of Math.	4	700	1700	1700	24	33	33	34
Indiana Univ. Math. J.	12	1200	500	500	4#	8	7	6
Inventiones Math.	14	1176	0	---	6	7	8	10
J. Amer. Stat. Assoc.	4	976	99	NR*	15		--	
J. Assoc. for Comp. Mach.	4	NR*	NR*	0	NR*	6	7	10
J. Diff. Geometry	4	830	600	300	9-12	18	19	22
J. Math. Physics	12	3000	400	NR*	9 1/2	9	10	12
J. Symbolic Logic	4	800	0	0	10	15	20	23
Linear Algebra and Appl.	4	420	0	80	NR*	15	20	28
Math. Biosciences	6	1400	120	100	NR*		--	
Math. Systems Theory	4	384	0	---	4-6	13	16	21
Math. of Comp.	4	1000	0	0	8	8	10	13
Math. Annalen	19	1596	0	---	9	9	10	11
Math. Zeitschrift	22	1980	180	---	9	6	7	14
Michigan Math. J.	4	400	150	150	12	13	14	16
Numerische Math.	10	900	94	---	4-8	12	14	20
Operations Research	6	1650	400	700	13	17	21	27
Pacific J. of Math.	12	3200	NR*	NR*	14	13	15	18
Proceedings of AMS	12	3250	300	50	10	10	12	14
Proc. Nat'l Acad. Sci.	12	300##	0	0	2	3	###	3
Quarterly of Appl. Math.	4	650	650	600	12-15	14	15	17
SIAM J. of Appl. Math.	8	1700	0	0	8-10	10	11	14
SIAM J. on Computing	4	700	0	---	8-10		***	
SIAM J. on Control	4	650	0	0	8-10	9	11	13
SIAM J. on Math. Anal.	4	650	0	0	8-10	9	10	11
SIAM J. on Numer. Anal.	4	700	0	0	8-10	12	14	17
SIAM Review	4	650	0	0	8-10	12	13	16
Transactions of AMS	12	5500	200	50	11	12	16	22
Z. Wahrscheinlichkeits theorie	16	1408	260	---	15	16	17	18

*NR means that no response was received to a request for information.
**The Duke Mathematical Journal is presently not accepting articles for publication because of the extensive backlog.
***New journal–first issue has not yet appeared.
#After acceptance.
##In mathematics.
###The latest issue of this journal consisted of only two articles.
--Date of receipt of manuscript not indicated in this journal.
---This journal is new to this compilation. Figures regarding backlog as of 12/31/70 are not available.

VISITING MATHEMATICIANS

The list of visiting mathematicians includes both foreign mathematicians visiting in the United States and Canada, and Americans visiting abroad. Note that there are two separate lists.

American and Canadian Mathematicians Visiting Abroad

Name and Home Country	Host Institution	Field of Special Interest	Period of Visit
Aczél, J. (Canada)	Istituto Nazionale di Alta Matematica	Geometry, Finite Differences	9/71 - 12/71
	Monash University, Clayton, Australia		2/72 - 6/72
	Math. Forschungsinstitut, Oberwolfach, Fed. Rep. of Germany		7/72 - 8/72
Baragar, A. F. (Canada)	International Centre for Theoretical Physics, Trieste	Quantum Field Theory	7/71 - 6/72
Bell, Howard E. (Canada)	University of Leeds, England	Rings	8/71 - 7/72
Bennett, Ralph B. (U. S. A.)	Polish Academy of Sciences	Topology	1/72 - 6/72
Bleistein, Norman (U. S. A.)	University of Dundee, Scotland	Asymptotic Methods for Partial Differential Equations	1/72 - 6/72
Chang, Shao-Chien (Canada)	Taing Hua University, Taiwan	Summability	8/71 - 7/72
Chen, Yuh-Ching (U. S. A.)	Université de Paris VII, France	Algebraic Topology	9/71 - 6/72
Chover, Joshua (U. S. A.)	Weizmann Institute of Science, Israel	Probability, Functional Analysis	7/71 - 6/72
Christian, Robert R. (U. S. A.)	Government of Sierra Leone	Mathematics Education	7/71 - 6/72
Coven, Ethan M. (U. S. A.)	Universität Erlangen-Nürnberg, Fed. Rep. of Germany	Topological Dynamics	1/71 - 6/72
Dhaliwal, Ranjit S. (Canada)	City University, London, England	Elasticity	9/71 - 6/72
Dickey, R. W. (U. S. A.)	University of Newcastle upon Tyne, England	Applied Mathematics	9/71 - 9/72
Dubisch, Roy (U. S. A.)	Ministry of Education, Nairobi, Kenya	Mathematical Education	9/71 - 6/72
Dwass, Myer (U. S. A.)	Hebrew University of Jerusalem, Israel	Statistics	7/71 - 7/72
Eames, W. (Canada)	Sir John Cass College, London, England	Analysis	9/71 - 9/72
Earle, Clifford (U. S. A.)	Mittag-Leffler Institute, Sweden	Complex Variable	1/72 - 6/72
Ebin, David (U. S. A.)	Université de Paris, France	Global Analysis	9/71 - 1/72
Epstein, Bernard (U. S. A.)	Technion—Israel Institute of Technology	Partial Differential Equations	7/71 - 8/72
Fischer, Frederic E. (U. S. A.)	University of Stuttgart, Fed. Rep. of Germany	Statistics: Computer Applications to Education	9/71 - 9/72
Fossum, Robert M. (U. S. A.)	Mathematics Institute, University of Aarhus, Denmark	Algebra	9/71 - 8/72
Fugate, Brauch (U. S. A.)	University of Warwick, England	Topology	9/71 - 6/72
Garg, K. (Canada)	Institut Henri Poincaré, France	Real Analysis	6/71 - 5/72
Gil De Lamadrid, Jésus (U. S. A.)	Université de Rennes, France	Functional Analysis	9/71 - 6/72
Glauberman, George (U. S. A.)	Oxford University, England	Group Theory	9/71 - 8/72
Goldberg, Samuel I. (U. S. A.)	Hebrew University of Jerusalem, Israel	Differential Geometry	2/72 - 8/72
Greville, Thomas N. E. (U. S. A.)	Federal University of Pernambuco, Brazil	Spline Functions	10/71 - 12/71

Name and Home Country	Host Institution	Field of Special Interest	Period of Visit
Groh, H. (Canada)	Technical University of Aachen, Fed. Rep. of Germany	Combinatorial Geometry	5/71 - 9/72
Hamilton, Richard (U. S. A.)	University of Warwick, England	Global Analysis	9/71 - 5/72
Hano, Jun-Ichi (U. S. A.)	Université de Grenoble, France	Lie Theory	9/71 - 8/72
Horwitz, Lawrence (U. S. A.)	University of Geneva, Switzerland	Mathematical Physics, Algebra	9/71 - 6/72
Husseini, Sufian Y. (U. S. A.)	Mathematical Institute, Oxford University, England	Topology	9/71 - 6/72
Klemola, Tapio (Canada)	Eidgenössische Technische Hochschule, Zürich	Analysis and Algebra	9/71 - 5/72
Koschorke, Ulrich (U. S. A.)	Bonn University, Fed. Rep. of Germany	Global Analysis	9/71 - 6/72
Lappan, Peter A. (U. S. A.)	Imperial College of Science and Technology, England	Analysis	9/71 - 8/72
Latta, Gordon E. (U. S. A.)	University of Edinburgh, Scotland	Applied Mathematics	9/71 - 6/72
Levin, Frank (U. S. A.)	Imperial College, University of London, England Erlangen University, Fed. Rep. of Germany	Group Theory	9/71 - 1/72
Levitt, Norman J. (U. S. A.)	University of Aarhus, Denmark	Differential Topology	2/72 - 6/72
Lumer, Gunter (U. S. A.)	Université de Paris, France		9/71 - 6/72
Maag, Urs (Canada)	Eidgenössische Technische Hochschule, Zürich	Statistics	6/71 - 8/71
Mack, John (U. S. A.)	University of Warwick, England	Topology	1/72 - 8/72
Matlis, Eben (U. S. A.)	University of London, England	Algebra	6/71 - 9/72
May, J. Peter (U. S. A.)	Cambridge University, England	Topology	8/71 - 6/72
McHugh, Richard B. (U. S. A.)	University of London, England	Biometry	9/71 - 9/72
Meyer, Richard E. (U. S. A.)	University of Essex, England	Applied Mathematics	9/71 - 4/72
Milner, E. C. (Canada)	Cambridge University, England	Set Theory, Combinatorics	9/71 - 6/72
Myhre, Janet M. (U. S. A.)	Eidgenössische Technische Hochschule, Zürich	Mathematical Statistics	8/71 - 8/72
Mysak, Lawrence A. (Canada)	Cambridge University, England	Geophysical Fluid Dynamics	8/71 - 7/72
Ney, Peter E. (U. S. A.)	Technion—Israel Institute of Technology Weizmann Institute of Science, Israel	Probability	9/71 - 12/71 12/71 - 6/72
Noble, Ben (U. S. A.)	Hertford College, Oxford, England	Integral Equations, Variational Methods	9/71 - 9/73
Nohel, John A. (U. S. A.)	École Polytechnique Federale Lausanne, Switzerland	Differential Equations	8/71 - 8/72
O'Malley, Robert E., Jr. (U. S. A.)	University of Edinburgh, Scotland	Singular Perturbation Theory	9/71 - 6/72
Orlik, Peter (U. S. A.)	University of Oslo, Norway	Algebraic Topology	6/71 - 8/72
Parlett, Beresford N. (U. S. A.)	Université de Paris, Quai St. Bernard, France	Numerical Analysis	9/71 - 4/72
Pfeffer, Washek F. (U. S. A.)	University of Ghana	Topology	7/71 - 6/73
Petro, John W. (U. S. A.)	University of Heidelburg, Fed. Rep. of Germany	Algebra	8/71 - 8/72
Rallis, Steven (U. S. A.)	University of Strasbourg, France	Group Representations	9/71 - 1/72
Reich, Edgar (U. S. A.)	Eidgenössische Technische Hochschule, Zürich	Complex Analysis	9/71 - 9/72
Robbin, Joel (U. S. A.)	Institut des Hautes Études Scientifique, France	Global Analysis, Dynamical Systems	9/71 - 8/72
Rothman, Neal J. (U. S. A.)	Hebrew University of Jerusalem, Israel	Harmonic Analysis	9/71 - 8/72

Name and Home Country	Host Institution	Field of Special Interest	Period of Visit
Salkauskas, K. (Canada)	University of Southampton, England	Numerical Analysis	9/71 - 6/72
Sampson, Allan Robert (U.S.A.)	University of Tel Aviv, Israel	Multivariate Analysis, Mathematical Statistics	9/71 - 8/72
Schlomiuk, Dana (Canada)	University of Rome, Italy	Logic	9/71 - 5/72
Schlomiuk, Norbert (Canada)	University of Rome, Italy	Algebraic Topology	9/71 - 5/72
Seebach, J. Arthur, Jr. (U.S.A.)	Eidgenössische Technische Hochschule, Zürich	Category Theory	9/71 - 8/72
Segal, Irving (U.S.A.)	University of Lund, Sweden	Quantum Mechanics	8/71 - 5/72
Sell, George (U.S.A.)	University of Florence, Italy	Ordinary Differential Equations	9/71 - 6/72
Shoemaker, Edward Milton (U.S.A.)	University of New South Wales	Plasticity Theory	5/71 - 12/71
Simmonds, James G. (U.S.A.)	Technical University of Denmark University of Delft, The Netherlands	Elasticity	8/71 - 6/72
Solomon, Louis (U.S.A.)	Queen Mary College, England	Algebra	6/71 - 6/72
Stone, William M. (U.S.A.)	University of Innsbruck, Austria	Information Theory	9/71 - 6/72
Sullivan, Francis (U.S.A.)	Gothenburg University, Sweden	Computer Science, Formal Languages	9/71 - 9/72
Weichsel, Paul M. (U.S.A.)	Hebrew University of Jerusalem, Israel	Algebra	9/70 - 8/72
Westbrook, D. R. (Canada)	University of Newcastle, England	Perturbation Theory in Partial Differential Equations, Bifurcation Theory, Theory of Shells and Plates	9/71 - 6/72
Wilcox, Theodore W. (U.S.A.)	Technische Universität, München, Fed. Rep. of Germany	Abstract Harmonic Analysis	9/71 - 8/72
Williams, Gareth (U.S.A.)	Imperial College, University of Wales, Bangor	Relativity Theory, Differential Geometry	9/71 - 8/72
Wong, James S.W. (U.S.A.)	University of Dundee, Scotland University of Hong Kong	Ordinary Differential Equations, Topology	9/71 - 2/72 2/72 - 9/72
Zidek, James V. (Canada)	University College, London, England	Statistics	7/71 - 6/72

Foreign Mathematicians Visiting in the United States and Canada

Abubakar, Iya (Nigeria)	City University of New York, Hunter College	Partial Differential Equations	9/71 - 6/72
Aharonov, Dov (Israel)	University of Michigan	Analysis	9/71 - 12/71
Amann, Herbert (Fed. Rep. of Germany)	University of Kentucky	Partial Differential Equations	9/71 - 6/72
Anderssen, R. S. (Australia)	Princeton University	Numerical Analysis	2/72 - 6/72
Andersson, Karl G. (Sweden)	Rutgers University	Partial Differential Equations	9/71 - 6/72
Anvari, Morteza (Iran)	Michigan State University	Analysis	9/71 - 8/72
Atzmon, Aharon (Israel)	University of California, Los Angeles	Harmonic Analysis	7/71 - 6/72
Axelsson, Owe (Sweden)	University of Pittsburgh	Computer Science (Data Structures)	9/71 - 9/72
Baas, Nils A. (Norway)	University of Virginia	Topology	9/71 - 6/72
Baker, C. (United Kingdom)	University of Toronto	Numerical Solution of Integral Equations	9/71 - 12/71
Barner, M. (Fed. Rep. of Germany)	University of Waterloo	Differential Geometry, Higher Geometry	9/71 - 12/71
Barnes, Frank W. (United Kingdom)	University of Michigan	Number Theory	9/71 - 6/72

Name and Home Country	Host Institution	Field of Special Interest	Period of Visit
Bhatnagar, P. L. (India)	University of Waterloo	Summability of Fourier and Applied Series, Astronomy and Astrophysics, Fluid Mechanics, Magneto-hydrodynamics and Plasma Physics	9/71 - 8/72
Billard, L. (Australia)	University of Waterloo	Stochastic Processes, Sequential Analysis, Epidemiology, Multivariate Analysis, Nonparametric Methods	9/71 - 8/72
Borosh, Itshak (Israel)	University of Illinois	Number Theory	9/70 - 6/72
Bowtell, Andrew J. (United Kingdom)	Texas A & M University	Algebra	9/71 - 7/72
Brelot, Marcel E. (France)	University of Michigan	Analysis	10/71
Burridge, Robert (United Kingdom)	New York University, Courant Institute	Elastic Wave Propagation and Applications in Geophysics	3/71 -
Cac, Nguyen P. (Vietnam)	University of British Columbia	Functional Analysis	9/71 - 8/72
Callaert, Herman (Belgium)	University of Rochester	Stochastic Processes and Applications	9/71 - 6/72
Choquet, Gustave (France)	University of Maryland	Analysis and Topology	9/71 - 1/72
Conde, Antonio (Brazil)	Institute for Advanced Study	Algebraic Topology	11/71 - 5/72
Day, Brian (Australia)	University of Chicago	Category Theory	10/70 - 9/72
Dellacherie, Claude (France)	Institute for Advanced Study	Markov Processes and Potential Theory	9/71 - 4/72
Dieudonné, Jean (France)	University of Maryland	Algebraic Geometry	9/71 - 6/72
Donnelly, John D. P. (United Kingdom)	University of Wisconsin, Madison	Partial Differential Equations	9/71 - 8/72
Dugué, Daniel (France)	Catholic University of America	Probability and Statistics	8/71 - 9/71
Eckhoff, Knut S. (Norway)	New York University, Courant Institute	Partial Differential Equations and Applications	9/71 - 8/72
Ejike, Uwadiegwu (Nigeria)	University of Illinois	Applied Mathematics	9/71 - 6/72
Erdos, John (United Kingdom)	University of British Columbia	Functional Analysis	7/71 - 6/72
Ewens, Warren J. (Australia)	University of Wisconsin, Madison	Mathematical Genetics	7/71 - 12/71
Florentin, J. J. (United Kingdom)	University of Waterloo	Theory of Computation, Software Development	1/71 - 12/71
Freud, Geza (Hungary)	University of Alberta	Analysis, Approximation Theory	12/71 - 5/72
Fryer, John Graeme (United Kingdom)	University of North Carolina, Chapel Hill	Statistics	8/71 - 7/72
Geoghegan, Ross (Ireland)	Institute for Advanced Study	Topology	9/71 - 4/72
Gheorghita, Vitalii (Rumania)	New York University, Courant Institute	Elasticity	
Gossez, J.-P. (Belgium)	University of Chicago	Analysis	9/71 - 6/72
Goulaouic, C. (France)	Purdue University	Partial Differential Equations	9/71 - 6/72
Graddon, Christopher J. (United Kingdom)	University of Illinois	Group Theory	9/71 - 6/72
Gruenberg, Karl W. (United Kingdom)	University of Illinois	Group Theory	2/72 - 6/72
Gutiérrez, Mauricio A. (Mexico)	Institute for Advanced Study	Knot Theory, Differential Topology	9/71 - 4/72
de Guzmán, Miguel (Spain)	Washington University	Harmonic Analysis	9/71 - 2/72
de Haan, Laurens (The Netherlands)	Stanford University	Probability and Mathematical Statistics	9/71 - 6/72
Halin, Rudolf (Fed. Rep. of Germany)	Western Michigan University	Graph Theory	8/71 - 4/72
Hannabuss, Keith C. (United Kingdom)	Massachusetts Institute of Technology	Representation Theory	9/71 - 6/72

Name and Home Country	Host Institution	Field of Special Interest	Period of Visit
Hedberg, Torbjorn (Sweden)	SUNY at Albany	Functional Analysis	5/71 - 9/71
Herrero, Domingo (Argentina)	SUNY at Albany	Functional Analysis	9/71 - 9/72
Hildebrandt, Stefan (Fed. Rep. of Germany)	New York University, Courant Institute	Calculus of Variations	9/71 - 1/72
	California Institute of Technology	Functional Analysis	1/72 - 6/72
Hitchin, Nigel (United Kingdom)	Institute for Advanced Study	Topology	9/71 - 4/72
Hotta, Ryoshi (Japan)	Institute for Advanced Study	Differential Geometry, Group Representations	9/71 - 4/72
Huber, Peter J. (Switzerland)	Yale University	Statistics	9/71 - 1/72
Iitaka, Shigeru (Japan)	Institute for Advanced Study	Classification of Algebraic Varieties	9/71 - 4/72
Iwahori, Nagayoshi (Japan)	University of Oregon	Lie Algebras and Algebraic Groups	5/71 - 8/71
Kahane, Jean-Pierre (France)	University of Maryland	Fourier Analysis	9/71 - 1/72
Kasteleyn, Pieter Williem (The Netherlands)	University of Waterloo	Matroid Theory	2/72 - 4/72
Katznelson, Yitzhak (Israel)	University of Maryland	Fourier Analysis	9/71 - 1/72
Kawakubo, Katsuo (Japan)	SUNY at Albany	Differential Topology	9/71 - 6/72
Kimura, Tosihusa (Japan)	University of Minnesota	Ordinary Differential Equations	9/71 - 6/72
King, Clarence A. (West Indies)	Michigan State University	Group Theory—Algebra	9/71 - 8/72
Kjellberg, Bo (Sweden)	York University	Complex Analysis	7/71 - 6/72
Koethe, Gottfried M. (Fed. Rep. of Germany)	University of Maryland	Functional Analysis	9/71 - 6/72
Kolomy, Joseph (Czechoslovakia)	Carnegie-Mellon University	Nonlinear Analysis	9/71 - 6/72
Kozma, Ilan (Israel)	University of Chicago	Topology	10/71 - 9/72
Kulkarni, Ravindra S. (India)	Institute for Advanced Study	Differential Geometry	9/71 - 4/72
Lance, Edward C. (United Kingdom)	University of Pennsylvania	Functional Analysis	7/71 - 6/72
Laursen, Kjeld B. (Denmark)	University of California, Los Angeles	Functional Analysis	7/71 - 6/72
Leeb, Klaus (Austria)	University of Minnesota	Combinatorics	9/71 - 6/72
Leopoldt, Heinrich W. (Fed. Rep. of Germany)	University of Maryland	Number Theory	9/71 - 6/72
Levy, Azriel (Israel)	Yale University	Set Theory	9/71 - 6/72
Loh, Hooi-Tong (Malaysia)	University of Waterloo	Graph Theory	10/71 - 4/72
Makan, Amirali (Tanzania)	University of Alberta	Algebra	7/71 - 6/72
Mangeron, Demetre (Romania)	University of Alberta	Control Theory	10/71 - 10/72
Martin, Phillippe (Switzerland)	University of Denver	Mathematical Physics, Functional Analysis	9/71 - 6/72
Maus, Eckart (Fed. Rep. of Germany)	University of British Columbia	Algebraic Number Theory	7/71 - 6/72
McMullen, John R. (Australia)	Institute for Advanced Study	Harmonic Analysis on Locally Compact Groups	9/71 - 4/72
Meinardus, Gunter (Fed. Rep. of Germany)	Michigan State University	Numerical Analysis	9/71 - 8/72
Meyberg, Kurt (Fed. Rep. of Germany)	University of Virginia	Mathematics Algebra	9/71 - 6/72
Mitra, Sujit K. (India)	Indiana University	Statistics	6/71 - 6/72
Mochizuki, K. (Japan)	University of Utah	Partial Differential Equations	1/72 - 6/72
Moh, Tzuong-Tsieng (Republic of China)	Institute for Advanced Study	Algebraic Geometry	9/71 - 4/72
Moran, William (United Kingdom)	Indiana University	Harmonic Analysis	9/71 - 6/72

Name and Home Country	Host Institution	Field of Special Interest	Period of Visit
Motoo, Minoru (Japan)	New York University, Courant Institute	Probability	9/71 - 8/72
Mukhopadhyay, Salya N. (India)	University of British Columbia	Properties of Derivatives	7/71 - 6/72
Naimpally, S.A. (India)	Lakehead University	Topology and Analysis	9/71 - 6/72
Nakai, Yoshikazu (Japan)	Northern Illinois University	Algebraic Geometry	9/71 - 6/72
Nath, Prem (India)	McMaster University	Ergodic Theory	5/70 - 6/71
Nathan, Gad (Israel)	University of North Carolina, Chapel Hill	Statistics	8/71 - 7/72
Natterer, Frank (Fed. Rep. of Germany)	Indiana University	Numerical Analysis	9/71 - 6/72
Neumann, Walter D. (Fed. Rep. of Germany)	Institute for Advanced Study	Differentiable Transformation Groups	9/71 - 4/72
Nicholls, Peter (United Kingdom)	Northern Illinois University	Complex Analysis	9/71 - 6/72
Niederreiter, Harry (Austria)	University of Illinois	Number Theory	9/71 - 6/72
Nishida, Takaaki (Japan)	New York University, Courant Institute	Partial Differential Equations	9/71 - 8/72
Ockendon, Hilary (United Kingdom)	New York University, Courant Institute	Applied Mathematics, Fluid Mechanics, Singular Perturbation	7/71 - 1/72
Ockendon, John (United Kingdom)	New York University, Courant Institute	Applied Mathematics, Fluid Mechanics, Singular Perturbation	7/71 - 1/72
Okumura, Masafumi (Japan)	Michigan State University	Geometry	9/71 - 8/72
Osborne, M.R. (Australia)	Stanford University	Numerical Analysis	9/71 - 12/71
Ostrowski, A. (Switzerland)	SUNY at Buffalo	Analysis, Numerical Analysis	10/71 - 11/71
Ozawa, Mitsuru (Japan)	Washington University	Theory of Functions, Complex Variables	9/71 - 4/72
Pareigis, Bodo (Fed. Rep. of Germany)	SUNY at Albany	Homological Algebra	9/71 - 6/72
Parthasarathy, Rajagopalan (India)	Massachusetts Institute of Technology	Lie Groups	9/71 - 6/72
Passi, I.B.S. (India)	University of Alberta	Algebra	1/71 - 3/72
Patodi, Vijay K. (India)	Institute for Advanced Study	Partial Differential Operators	9/71 - 4/72
Peregrine, Dennis H. (United Kingdom)	University of Wisconsin, Madison	Applied Mathematics	9/71 - 9/72
van der Put, Marius (The Netherlands)	Purdue University	Algebraic Geometry	2/72 - 6/72
Rado, F. (Romania)	University of Waterloo	Functional Equations, Geometry, Numerical Analysis	9/71 - 8/72
Rado, R. (United Kingdom)	University of Waterloo	Combinatorics, Algebra, Geometry, Number Theory, Classical Analysis	9/71 - 8/72
Ringel, Claus Michael (Fed. Rep. of Germany)	Carleton University	Ring Theory and Categories	7/71 - 4/72
Rivaud, Juan José (Mexico)	Institute for Advanced Study	Topology	9/71 - 4/72
Robson, J.C. (United Kingdom)	University of Utah	Ring Theory	9/71 - 6/72
Sabbagh, Gabriel A. (France)	University of Wisconsin, Madison	Logic, Algebra, Combinatorics	9/71 - 9/72
Saffari, B. (France)	University of Ottawa	Analytic and Probabilistic Number Theory	7/71 - 6/72
Sagher, Yoram (Israel)	University of Minnesota	Trigonometric Series	9/71 - 3/72
Salinas, Norberto (Argentina)	Institute for Advanced Study	Operator Theory	9/71 - 4/72
Sanderson, Brian (United Kingdom)	SUNY at Stony Brook	Differential Topology	9/71 - 5/72

Name and Home Country	Host Institution	Field of Special Interest	Period of Visit
Shibaoka, Yasumitsu (Japan)	University of Oregon New York University, Courant Institute	Algebra	4/71 - 8/71 9/71 - 6/72
Shintani, Takuro (Japan)	Institute for Advanced Study	Unitary Representations and Dirichlet Series	9/71 - 4/72
Skowronski, J. M. (Poland)	University of Ottawa	Nonlinear Differential Equations, Control Theory, Differential Games	11/71 - 3/72
Smythe, Neville F. (Australia)	Dartmouth College	Topology	9/70 - 7/72
Sneddon, Ian N. (United Kingdom)	North Carolina State University	Fracture Mechanics and Transform Methods	3/72
Spies, Guenther (Fed. Rep. of Germany	New York University, Courant Institute	Statistical Mechanics, Theoretical Plasma Physics	7/71 - 6/72
Srebro, Uri (Israel)	University of Minnesota	Quasiregular Functions	9/71 - 3/72
Stuhler, Ulrich (Fed. Rep. of Germany)	Massachusetts Institute of Technology	Algebraic Geometry	9/71 - 6/72
Suwa, Tatsuo (Japan)	University of Michigan	Topology	9/71 - 6/72
Szucs, Joseph (Hungary)	Indiana University	Functional Analysis	9/71 - 6/72
Tanaka, Hisao (Japan)	University of Illinois, Urbana	Group Theory	9/70 - 6/72
Taylor, S. J. (United Kingdom)	University of Minnesota	Probability	9/71 - 6/72
Thomas, Charles B. (United Kingdom)	Yale University	Topology	9/71 - 6/72
Thomée, Vidar (Sweden)	University of Wisconsin, Madison	Partial Differential Equations	8/71 - 7/72
Thompson, Mark (United Kingdom)	New York University, Courant Institute	Scattering Theory	9/70 - 9/72
Tits, Jacques C. (Fed. Rep. of Germany)	University of Michigan Institute for Advanced Study	Algebra Group Theory	9/71 10/71 - 4/72
Tsuchiya, Akihiro (Japan)	Institute for Advanced Study	Algebraic Topology	9/71 - 4/72
Turnbull, Bruce W. (United Kingdom)	Stanford University	Applied Probability	9/71 - 6/72
Vaisman, Izu (Romania)	University of Illinois	Differential Geometry	9/71 - 6/72
Vidossich, Giovanni (Italy)	University of Chicago	Nonlinear Analysis	11/71 - 6/72
Vorel, Zdenek (Czechoslovakia)	University of Southern California	Applied Mathematics	9/71 - 6/72
Wanby, Goren (Sweden)	University of Kentucky	Partial Differential Equations	7/71 - 6/72
Ward, Martin (Australia)	York University	Group Theory	8/71 - 5/72
Wattel, Everb (The Netherlands)	Carnegie-Mellon University	General Topology	1/72 - 9/72
Weisspfenning, Volker (Fed. Rep. of Germany)	Yale University	Model Theory	10/71 - 9/72
Winkelnkemper, H. Elmar (Fed. Rep. of Germany)	Institute for Advanced Study	Differential Topology	9/71 - 4/72
Wong, P.K. (Republic of China)	McMaster University	Banach Algebra	9/70 - 8/71
Wright, K. (United Kingdom)	University of Toronto	Numerical Analysis	7/71 - 6/72
Yau, Shing-Tung (Republic of China)	Institute for Advanced Study	Differential Geometry	9/71 - 4/72
Yokoi, Hideo (Japan)	University of Maryland	Number Theory	9/71 - 6/72
Zamir, Shmuel (Israel)	University of California, Los Angeles	Game Theory	7/71 - 6/72
Zarantonello, Eduardo H. (Argentina)	University of Wisconsin, Madison	Hydrodynamics and Functional Analysis	1/72 - 6/72
Zehnder, Eduard (Switzerland)	New York University, Courant Institute	Stability of Orbits	9/71 - 8/72
Zwas, Gideon Z. (Israel)	New York University, Courant Institute	Numerical Analysis, Partial Differential Equations	9/71 - 8/72

ABSTRACTS OF CONTRIBUTED PAPERS

Preprints are available from the author in cases where the abstract number is starred

The June Meeting in Corvallis
June 19, 1971

686-B11. CHARLES F. AMELIN, California State Polytechnic College, Pomona, California 91766. A stability of index theorem for operators on Frechet spaces. Preliminary report.

(For notation cf. Abstract 677-47-11, these *Notices* 17(1970), 789.) Let X (Y) be a complete T_2 lcs with saturated calibration Γ_X (Γ_Y). Then the cont. lin. op. $T: X \to Y$ ($T \in L(X,Y)$) is connection strictly singular (CSS) iff there exists a connection $\varphi: \Gamma_X \to \Gamma_Y$ such that $T \in \mathfrak{Z}_\varphi(X,Y)$ and, for any inf. dim. $M \subset X$ and $\epsilon > 0$, $\exists$ an inf. dim. $N \subset M$ such that $p(Tx) \leq \epsilon\varphi p(x)$ $\forall$ $x \in N$, $\forall p \in \Gamma_Y$. $T \in \varphi SS(X,Y)$ iff $T \in CSS(X,Y)$ for φ. Lemma. If $T \in \varphi SS(X,Y)$ then $T_{\varphi f}: X_{\varphi f} \to Y_f$ is strictly singular in the classical Banach space sense. Definition. $\delta_\varphi(T,S) = \sup((\sup(\inf(\varphi p(x_1 - x_2)^2 + p(Tx_1 - Sx_2)^2)^{\frac{1}{2}}: (x_2, Sx_2) \in G(S)) \Phi p(x_1)^2 + p(Tx_1)^2 = 1)\ p \in \Gamma_Y)$. $\hat{\delta}_\varphi(T,S) = \max(\delta_\varphi(T,S), \delta_\varphi(S,T))$ extends the $\|\cdot\|_\varphi$ topology of $\mathfrak{Z}_\varphi(X,Y)$ to $L(X,Y)$. Lemma. $\varphi SS(X,Y)$ is closed in the $\hat{\delta}_\varphi$-topology. Then nuclear operators are contained in $CSS(X,Y)$. $CSS(X,X)$ is a left ideal in $L(X,X)$. Theorem. Let X or Y be metrizable and let $T \in L(X,Y)$ be Fredholm. If $S \in CSS(X,Y)$ for the transformation connection φ_T, then $\text{index}(T) = \text{index}(T+S)$. (Received May 6, 1971.)

686-C5. HOWARD TASHJIAN, National Aeronautics and Space Administration, Ames Research Center, Moffett Field, California 94035. Computation of subsonic flow patterns of Bergman type. Preliminary report.

In S. Bergman, "Application of the kernel function for the computation of flows of compressible fluids", Quart. Appl. Math. 26(1968), 301-310, an explicit formula for the potential function of a compressible fluid flow in a channel C has been given. Here C is a channel bounded by segments of straight lines and segments of free boundaries. In Bergman's formula the fundamental solution of a linear differential equation for the potential function of compressible fluids in the pseudo-logarithmic plane appears. The solution involves iterated double integrals. See formula (2.6 b) and (3.1 b) of Bergman's paper. Their evaluation is a computational problem in itself. The author shows how using a special computational system, IBM 360 Formac, these iterated double integrals can be computed. (Received May 26, 1971.) (Author introduced by Professor Stefan Bergman.)

THE AUGUST MEETING IN PENNSYLVANIA AUGUST 31-SEPTEMBER 3, 1971

02 Logic and Foundations

*687-02-1. JULIA F. KNIGHT, Pennsylvania State University, University Park, Pennsylvania 16802. U-extensions of countable structures.

The main result here is the following Theorem. There exists a countable structure M, of countable type, with a unary relation U^M, such that M has a proper U-extension N (i.e., an elementary extension N such that $U^N = U^M$), but M has no U-extension with more than one new element. This result answers an unpublished question of Baumgartner. It also solves a problem of Keisler, stated on p. 37 of his paper, "Logic with the quantifier 'there exist uncountably many' " (Ann. Math. Logic 1(1970), 1-93). The result is reminiscent of one in John Gregory's Abstract 70T-E81 (these *Notices* 17(1970), 967), although it is not clear how to get either result from the other. The theorem has variations involving (a) a finite chain of proper U-extensions $M_0 < M_1 < \ldots < M_n$ such that M_0 has no U-extension with more than n new elements, (b) an infinite chain of proper U-extensions $M_0 < M_1 < M_2 < \ldots$ such that M_0 has no uncountable U-extension. In the proof of the theorem, M and N are constructed by a purely model-theoretic use of Cohen's forcing technique. The same kind of construction yields results on weak second order extensions, similar to the ones on U-extensions. (Received January 14, 1971.)

*687-02-2. ROBERT F. JOLLY, R. F. D. Rte. #1, Box 1120, Davis, California 95616. A psychological view of the foundations of mathematics. Preliminary report.

Recognizing the human brain as a bio-computer, we discover that ordinary cultural environment introduces inconsistent programs which inhibit within the mechanism their own discovery. In the process of locating these points of blockage by means of subtle new psychological methods, we uncover several hidden assumptions within the interface between mathematics and the physical universe. The exposure of these flaws in the mirroring of reality suggests several dramatic changes in the area of applied mathematics. (Received March 25, 1971.)

687-02-3. R. G. JEROSLOW, University of Minnesota, Minneapolis, Minnesota 55455. Canonical representations of formal systems. Preliminary report.

We develop the notion of a K-description, which is essentially due to Kreisel (and which he calls a "canonical representation" in "Lectures in modern math," vol. 3, Wiley, New York, 1965, p. 154). A finitely axiomatized system is exhibited for which the following holds for any extension theory T: T has a provably infinite K-description iff T has the schema of induction: T has a provably infinite K-description iff T has all K-descriptions of all formal systems (in the sense of E. Post). We then apply K-descriptions to obtain a generalization of Theorem 5.6 of S. Feferman's "Arithmetization of metamathematics in a general setting," (Fund. Math. 49(1960/61), 66), and show that the formula alpha-star of Feferman's Theorem 5.9 is not a K-description of the r.e. theory it describes. (Received June 28, 1971.)

687-02-4. E. WILLIAM CHAPIN, JR., University of Notre Dame, Notre Dame, Indiana 46556. An axiomatization of the set theory of Zadeh.

Previous study of the "fuzzy" set theory of L. A. Zadeh (e.g., Information and Control 8(1965), 338-353 and Information and Control 18(1971), 32-39) has depended on constructions within or additions to some (unspecified) "real" set theory. A Zermelo-Fraenkel-like axiomatization of Zadeh set theory is given. It is shown that this theory has for models all of the models of usual Z - F set theory, models of the Zadeh set theory, and models of the various generalizations of the Zadeh theory. The inclusion relation $\in$ ("is an element of") in this theory is ternary, x,y, and z being so related (roughly) if it is true that x is an element of y to the extent z. (Received July 7, 1971.)

05 Combinatorics

*687-05-1. HENRY W. GOULD, West Virginia University, Morgantown, West Virginia 26506. Two remarkable extensions of the Leibniz differentiation formula.

The Leibniz formula $D^n(uv) = \sum\binom{n}{k}D^k u \cdot D^{n-k}v$ may be extended in at least two ways by the introduction of a third function as a parameter. One of these formulas, due to Cauchy, says that $\sum\binom{n}{k}D^{k-1}(f^k Du)D^{n-k-1}(f^{n-k}Dv) = D^{n-1}(f^n D(uv))$ and another, due to J. J. Walker, says that $\sum\binom{n}{k}D^{k-1}(f^k Du)D^{n-k}(f^{n-k}v) = D^n(f^n uv)$, where, in both sums, $0 \leq k \leq n$, and $D^{-1}(Du) = u$. Such formulas are not very widely known. We give rapid proofs of them by infinite series techniques and show that to every such formula there exists a special binomial identity so that new light is shed on the nature of certain combinatorial identities. Proofs of the formulas by induction are not exactly trivial. Some conjectures based on the analogy with combinatorial identities are given. An inductive proof of Walker's formula, in particular, suggests the existence of a nontrivial proof of a nontrivial combinatorial identity. The formulas also have application to certain problems of special functions. (Received February 1, 1971.)

*687-05-2. MURRAY HOCHBERG, Brooklyn College, City University of New York, Brooklyn, New York 11210. A characterization of s-systems with restrictions. Preliminary report.

A map f from the first r natural numbers to the power set of a set Ω of cardinality n is called an "r-decomposition" if $f(i) \cap f(j) = \emptyset$, $i \neq j$, and $\bigcup f(i) = \Omega$. A set σ of r-decompositions is an "s-system of order r" if, for each index i, no two sets in the set $\{f(i) : f \in \sigma\}$ satisfy an inclusion relation. Meshalkin (Teor. Verojatnost. i Primenen 8(1963), 219-220) proved that the cardinality of σ cannot exceed the maximum multinomial coefficient. This generalizes an earlier result of Sperner (Math. Z. 27(1928), 544-548), who had studied the case r = 2. The main theorem of Hochberg and Hirsch (Ann. N. Y. Acad. Sci. 175(1970), 224-237) states that there is an essentially unique maximal s-system and characterizes this system. In this report, the author studies s-systems with two types of restrictions. In both cases, he derives an upper bound for the cardinality of the maximal s-system which can be achieved subject to these restrictions and he investigates the uniqueness of the maximal s-system. (Received April 2, 1971.)

*687-05-3. NICHOLAS D. KAZARINOFF and ROGER WEITZENKAMP, University of Michigan, Ann Arbor, Michigan 48104. On existence of compound perfect squared squares.

The authors describe a collection of planar graphs that may be recursively constructed and may be used to generate all compound (and simple) perfect squared rectangles. They use this theory to prove nonexistence of compound perfect squared squares composed of 21 or fewer squares. This means that no perfect squared square composed of less than twenty subsquares exists. The authors confirm many of A. J. W. Duijvestijn's computations, and they present examples of: simple perfect squared rectangles with side-ratios 3/5 and 5/7, a perfect squared rectangle with largest subsquare in its interior, and a perfect squared rectangle with a perfect squared subrectangle in its interior. Statistics on simple squared rectangles of various kinds and other examples are given. (Received June 28, 1971.)

687-05-4. GERHARD RINGEL, University of California, Santa Cruz, California 95060. The maximum genus of the complete graph.

An embedding of a graph G into an orientable surface S without crossovers of arcs is called cellular if the components of $S-G$ are open 2-cells. The maximum genus of an orientable surfact in which a cellular embedding of G exists is defined as the maximum genus of G. An embedding of the complete graph K_n into an orientable surface S such that $S-K_n$ has only one component (one 2-cell) if $n \equiv 1, 2 \pmod 4$ and has two components if $n \equiv 0, 3 \pmod 4$ will be constructed. Using this embedding and Euler's formula one can determine the maximum genus of K_n. (Received July 1, 1971.)

*687-05-5. VERA S. PLESS, Air Force Cambridge Research Laboratories, L. G. Hanscom Field, Bedford, Massachusetts 01730. Self-orthogonal codes of half dimension over GF(2).

For $2 \leqq n \leqq 18$, the self-orthogonal codes of dimension n/2 of F_2^n are completely classified. We call two codes equivalent if one is the image of the other under a permutation of the coordinate indices. For each n in the range described, the number of subspaces equivalent to a canonical subspace are given, and the entire group of coordinate permutations leaving the subspace invariant is determined. The weight of a vector in F_2^n is the number of its nonzero components. The weight distribution of each self-orthogonal subspace described above is also given. (Received June 30, 1971.)

08 General Mathematical Systems

*687-08-1. ROBERT WILLIS QUACKENBUSH, University of Manitoba, Winnipeg, Manitoba, Canada. A finite non-Boolean algebra.

A. H. Diamond and J. C. C. McKinsey constructed an infinite non-Boolean algebra $\langle A; \wedge, \vee, ', 0, 1\rangle$ such that every 2-generated subalgebra is Boolean ("Algebras and their subalgebras", Bull. Amer. Math. Soc. 53(1947), 959-962). In this paper a finite non-Boolean algebra such that every 2-generated subalgebra is Boolean is constructed. This solves Problem 18 of (G. Grätzer, "Lattice theory, first concepts and distributive lattices," W. H. Freeman and Co., San Francisco, 1971). The algebra $\mathfrak{A}$ has 16 elements:

$A = \{0,1,a,a',b,b',c,c',d,d',e,e',f,f',g,g'\}$. The operations on $\mathfrak{A}$ are defined by the 7 8-element Boolean sublattices of $\mathfrak{A}$. The atoms of these 7 sublattices are: $\{a,b',d\}$, $\{b,c',e\}$, $\{a',c,f\}$, $\{d,e,f\}$, $\{a,e,g\}$, $\{b,f,g\}$, $\{c,d,g\}$. Since $a \vee b = b$, $b \vee c = c$, and $c \vee a = a$, $\vee$ is not associative. Hence $\mathfrak{A}$ is not a Boolean algebra. (Received June 17, 1971.)

10 Number Theory

*687-10-1. GERALD E. BERGUM, South Dakota State University, Brookings, South Dakota 57006. Complete residue systems in the quadratic domain $Z(e^{2\pi i/3})$.

Let $\omega = e^{2\pi i/3}$ and $Z(\omega) = \{a+b\omega \mid a, b \text{ are integers}\}$. In this paper, we illustrate several types of complete residue systems modulo γ for $\gamma \in Z(\omega)$. In particular, it is shown that if T_1 is the set of points interior to the hexagon whose vertices are $(\gamma/3)(1-\omega)e^{\pi ki/3}$ where $1 \leqq k \leqq 6$ and T_2 is the set of points on the line segments $(-(\gamma/3)(1-\omega), (\gamma/3)(1-\omega)e^{4\pi i/3})]$, $[(\gamma/3)(1-\omega)e^{4\pi i/3}), (\gamma/3)(1-\omega)e^{5\pi i/3})]$ and $[(\gamma/3)(1-\omega)e^{5\pi i/3}), (\gamma/3)(1-\omega))$ then $T = T_1 \cup T_2$ is the absolute minimal representation of a complete residue system modulo γ. It is also shown that the cardinality of a complete residue system modulo $\gamma = a + b\omega$ is $a^2 - ab + b^2$. (Received February 22, 1971.)

*687-10-2. GREGORY WULCZYN, Bucknell University, Lewisburg, Pennsylvania 17837. On representations of generalized polygonal numbers as the sum of two such numbers in m ways.

If (r,s,t) is a primitive Pythagorean triple, $r = 2mn$, $s = m^2 - n^2$, $t = m^2 + n^2$, there exist generalized polygonal numbers $P(n) = (n/2)[un + 2v - u]$ such that $P(kr+a) + P(ks+b) = P(kt+c)$. Hence there exist an infinite number of generalized polygonal numbers each of which can be represented in m ways as the sum of two such generalized polygonal numbers. (Received June 11, 1971.)

*687-10-3. WILLIAM A. WEBB, Washington State University, Pullman, Washington 99163. A number theoretic sum involving divisors of polynomial values.

Let $f(n)$ denote an integer valued polynomial and let $\sigma_s(k)$ denote the sum of the sth powers of the divisors of k. Using Bombieri's theorem on primes in arithmetic progression, an asymptotic formula for $\sum_{p\leq x} \sigma_s(f(p))$ is obtained for $s > 0$. (Received June 21, 1971.)

*687-10-4. BRUCE C. BERNDT, University of Illinois, Urbana, Illinois 61801. The evaluation of character series by contour integration.

A classical and well-known application of the calculus of residues occurs in the evaluation of series of the form, $\sum_{n=-\infty}^{\infty} f(n)$ or $\sum_{n=-\infty}^{\infty} (-1)^n f(n)$, where f is a suitable meromorphic function. We extend this theory by showing how to evaluate by contour integration character series of the form, $\sum_{n=-\infty}^{\infty} \chi(n)f(n)$ or $\sum_{n=-\infty}^{\infty} (-1)^n \chi(n)f(n)$, where χ is a primitive character. In particular, we apply our results to the evaluation of Dirichlet L-functions at positive integral arguments. (Received July 6, 1971.)

687-10-5. JOSEPH B. MUSKAT, Bar-Ilan University, Ramat-Gan, Israel and University of Pittsburgh, Pittsburgh, Pennsylvania 15213. On the representations of primes by different binary quadratic forms.

If $p \equiv 1 \pmod 8$, the biquadratic character of 2 is given by $\frac{1}{2}$ ind $2 \equiv B \equiv C \pmod 2$, where $p = A^2 + 16B^2 = (4C+1)^2 + 8D^2$. Dirichlet proved one criterion and derived the other by showing that $B \equiv C \pmod 2$ [H. J. S. Smith, "Report on the theory of numbers," Chelsea, New York, 1965, p. 313]. The method of Dirichlet, which involves nothing deeper than quadratic residues, can be applied to several other instances where a prime p is represented by different binary quadratic forms. For example, if $(-7|p) = +1$, $p = L^2 + 7M^2$. If, furthermore, $(2|p) = +1$, either $p = S^2 + 14T^2$ or $p = 7X^2 + 2Y^2$; which of the latter decompositions holds can be determined in terms of L, M, and p (mod 8). (Received July 7, 1971.)

*687-10-6. INDA LEPSON, University of Maryland, College Park, Maryland 20742. The reduced set of residues mod Πp_n.

Let $R(\Pi p_n)$ be the reduced set of residues mod Πp_n, the product of the first n primes. Let $D_n = \{x\}$, $0 < x < \Pi p_n$, $(x, p_i) = 1$, $i = 1,2,3,\ldots,n-1$, and $p_n | x$. Let D_n' be the complement of D_n relative to $R(\Pi p_{n-1}) \cup \{b_{n-1}\Pi p_{n-1} + a_i\}$, $b_{n-1} = 1,2,3,\ldots,p_n - 1$ and $a_i \in R(\Pi p_{n-1})$. It is shown that $R(\Pi p_n) = D_n'$ and an explicit construction for it is given in terms of $R(\Pi p_{n-1})$. Use is made of the Πp_n place value system in which a positive integer C, symbolized by $c_n c_{n-1} c_{n-2} \cdots c_2 c_1 c_0$, is equal to $\Sigma_{k=1}^{n} (c_k \Pi p_k) + c_0$, where $c_k = 0,1,2,\ldots,(p_{k+1} - 1)$, $c_n \neq 0$ and $c_0 = 0,1$. An algorithm for a sieve for the prime numbers belonging to $R(\Pi p_n)$ and an algorithm for calculating $\pi(\Pi p_n)$ are constructed. (Received July 7, 1971.)

*687-10-7. RICHARD B. LAKEIN, State University of New York, Buffalo, New York 14226. Bounds for consecutive kth power residues in the Eisenstein integers.

This paper extends to the Eisenstein integers $a + b\rho$ $(a, b \in Z, \rho^2 + \rho + 1 = 0)$ the problem of the existence of a bound on the size of a sequence of m consecutive kth power residues of p, for all but a finite number of primes p and independent of p. The least such bound is denoted by $\Lambda_E(k,m)$. It is shown that $\Lambda_E(k,2)$ is finite for $k = 2,3,4$ or $6n \pm 1$. On the other hand, for every k, $\Lambda_E(2k,3) = \Lambda_E(3k,4) = \Lambda_E(k,6) = \infty$. Similar results are obtained for the related bound for m consecutives all in the same coset modulo the subgroup of kth power residues. (Received July 7, 1971.)

12 Algebraic Number Theory, Field Theory and Polynomials

*687-12-1. TOSHIHIKO YAMADA, Queen's University, Kingston, Ontario, Canada. Simple components of group algebras Q[G] which are central over totally real fields.

Let ℓ be a prime number and c be a positive integer. Set $k_{\ell,c} = Q(\zeta_{\ell^c} + \zeta_{\ell^c}^{-1})$. Theorem. A central simple algebra A over $k_{\ell,c}$ appears in some Q[G] (up to similarity) if and only if A has Hasse invariant 0 or $\frac{1}{2}$ at every prime $\mathfrak{P}$ of $k_{\ell,c}$, and for any $\mathfrak{P}$ and $\mathfrak{P}'$ of $k_{\ell,c}$ dividing a rational prime p, the Hasse invariants of A at $\mathfrak{P}$ and $\mathfrak{P}'$ are the same. Method of Proof. The following groups G yield simple algebras central over

$k_{\ell,c}$ with desired Hasse invariants. (1) $\ell \neq 2$, $p \neq 2$: $G = \langle u,v,w,x,y\rangle$, $u^{\ell^c} = 1$, $v^p = 1$, $w^2 = 1$, $\langle u,v,w\rangle =$ abelian, $x^{p-1} = u^{(p-1)/2}$ or $x^{p-1} = wu^{(p-1)/2}$, $y^2 = 1$, $xy = wuyx$, $xu = ux$, $xv = v^r x$ (r is primitive mod p), $xw = wx$, $yu = u^{-1}y$, $yv = vy$, $yw = wy$. (2) $\ell \neq 2$, $p = 2$: $G = \langle u,v,x,y\rangle$, $u^{\ell^c} = 1$, $v^4 = 1$, $x^2 = v$, $y^2 = v^2$ or $y^2 = 1$, $xy = vyx$, $xu = u^{-1}x$, $yu = uy$, $xv = vx$, $yv = v^{-1}y$. (3) $\ell = 2$, $p \not\equiv \pm 1 \pmod{2^c}$: $G = \langle u,v,x,y\rangle$, $u^{2^c} = 1$, $v^p = 1$, $x^{p-1} = u^{(p-1)/2}$, $y^2 = 1$, $xy = u^{1+2^{c-1}}yx$, $xu = ux$, $yu = u^{-1}y$, $xv = v^r x$ (r is primitive mod p), $yv = vy$. (4) $\ell = 2$, $p \equiv \pm 1 \pmod{2^c}$: $G = \langle u,v,x,y\rangle$, $u^{2^c} = 1$, $v^p = 1$, $x^{p-1} = u^{2^{c-1}}$, $y^2 = 1$, $xy = yx$, $xu = ux$, $xv = u^r x$, $yu = u^{-1}y$, $yv = vy$. (5) For any ℓ: $G = \langle u,x\rangle$, $u^{2\ell^c} = 1$, $x^2 = u^{\ell^c}$, $xu = u^{-1}x$. The Hasse invariants of corresponding simple algebras are computed by class field theory. Remark. The statement of Theorem does not hold if $k_{\ell,c}$ is replaced with any real quadratic field $Q(\sqrt{m})$, $m \equiv 3 \pmod 4$. (Received June 16, 1971.)

*687-12-2. ROBERT G. VAN METER, State University College of New York, Oneonta, New York 13820.
Generalized k-linear equations over a finite field.

Let K be a finite field with q elements. Let Z and Z^+ respectively be the set of integers and the set of positive integers. Restrict k and n to Z^+. Let $[1,n]$ be $\{z \in Z : 1 \leq z \leq n\}$. Let χ be the function from K into Z defined by $\chi(0) = q - 1$ and $\chi(a) = -1$ for all $a \in K - \{0\}$. A. D. Porter [Math. Nachr. 32(1966), 277-279] has determined the number of solutions ($\in K^{nk}$) of the k-linear equation $\sum_{i=1}^n a_i \prod_{j=1}^k x_{ij} = a$, where $a \in K$ and $a_i \in K - \{0\}$ for all $i \in [1,n]$. In this paper, the following more general result is proved: If $a \in K$ and, for all $i \in [1,n]$, $a_i \in K - \{0\}$, $k_i \in Z^+$ and $\gcd\{e_{ij} \in Z^+ : j \in [1,k_i]\} = 1$, then the number of solutions of the equation $\sum_{i=1}^n a_i \prod_{j=1}^{k_i} x_{ij}^{e_{ij}} = a$ is $q^{(\sum k_i)-1} + q^{n-1}\{\prod_{i=1}^n [q^{k_i-1} - (q-1)^{k_i-1}]\} \cdot \chi(a)$. (Received June 15, 1971.)

*687-12-3. WILLIAM M. BOYCE, Bell Telephone Laboratories, Inc., Murray Hill, New Jersey 07974.
On polynomials which commute with a given polynomial.

By extending the proof of Jacobsthal's Theorem 19 ("Über vertauschbare Polynome," Math. Z. 63(1955), 243-276), we obtain: Theorem I. Let $g(x) = \sum_{i=0}^n b_i x^{n-1}$, $\beta = b_1/nb_0$, $n > 1$, and $m > 0$. Then there is a unique monic polynomial h of degree m such that if $\alpha^{n-1} = (b_0)^{m-1}$, then $f(x) = \alpha h(x) - \beta$ is the unique polynomial of degree m with leading coefficient α such that $f(g(x)) = g(f(x)) + o(x^{(n-1)m})$. Theorem II. Let $g_\beta(x) = g(x-\beta) + \beta = \sum_{i=0}^n d_i x^{n-i}$ and $J(g) = \mathrm{GCD}\{i - 1 \mid d_{n-i} \neq 0\}$. Then if f is an mth degree polynomial which commutes with g, then the mth degree polynomials which commute with g are exactly those of the form $uf(x) + (u-1)\beta$, where u is a J(g)th root of unity. Thus for each $m > 0$, there are either J(g) or zero distinct polynomials of degree m which commute with g. (Received July 6, 1971.)

13 Commutative Rings and Algebras

*687-13-1. MAX D. LARSEN and DAVID E. BROWN, University of Nebraska, Lincoln, Nebraska 68508. Separation of nonassociates by valuations.

In many classical integral domains, given two nonassociates it is possible to find a valuation on the quotient field of the domain which is nonnegative on the domain and for which the nonassociates have different values. Recent work by Griffin, Harrison, and Manis has extended valuation theory to commutative rings with identity which contain zero divisors. In this paper we investigate the separation of nonassociates by valuations for the extended valuation theory. Our main result states that if R is a ring with a von Neumann regular total quotient ring, then nonassociates can be separated by valuations if and only if there is no unit in the integral closure of R which is not a unit in R. In addition, examples are given to show that this result can be improved. (Received June 17, 1971.)

*687-13-2. R. DOUGLAS WILLIAMS, 713 East Capitol Street, Washington, D. C. 20003. Primary ideals in rings of analytic functions.

Let A be the ring of all analytic functions on a connected, noncompact Riemann surface. In this paper the valuation theory of the ring A is used to analyze the structure of the primary ideals of A. The structure of the primary ideals of A is found to be strikingly similar to the structure of the primary ideals of the ring C of all real valued continuous functions on a completely regular topological space. (Received July 6, 1971.)

*687-13-3. CHIN-PI LU, University of Colorado, Denver, Colorado 80202. Contracted ideals in power series rings.

A commutative ring R with unity is said to have property C with respect to (w.r.t.) an overring S, if each ideal of R is the contraction of an ideal of S. Also we say that R has property ξ w.r.t. an overring S if, for every integer $n \geqq 1$, each system of n linear equations over R which is solvable over S is also solvable over R. In this article, we will prove that property ξ of certain rings R w.r.t. S implies property C of power series rings $R[[x_\lambda]]_{\lambda\in\Lambda}$ w.r.t. $S[[x_\lambda]]_{\lambda\in\Lambda}$ for any index set Λ. Lemma. Let A be a topological ring with a basis Q of neighborhoods of 0 consisting of ideals, such that every finitely generated ideal of the completion $\hat{A}$ of A is closed. If A has property C w.r.t. an overring B, then so does $\hat{A}$ w.r.t. the completion $\hat{B}$ of B for the topology induced by the basis $Q^e = \{q^e = qS ; q \in Q\}$ of neighborhoods of 0. Theorem 1. If a noetherian ring R has property ξ w.r.t. an overring S, then the power series ring $\bigcup_{F\subseteq\Lambda} R[[x_\lambda]]_{\lambda\in F}$ has property C w.r.t. $\bigcup_{F\subseteq\Lambda} S[[x_\lambda]]_{\lambda\in F}$, where F runs through all finite subsets of Λ. The same conclusion holds for the ring $\bigcup_{F\subseteq\Lambda} R\{x_\lambda\}_{\lambda\in F}$ of restricted formal power series w.r.t. $\bigcup_{F\subseteq\Lambda} S\{x_\lambda\}_{\lambda\in F}$ provided that R is equipped with an ideal-adic topology. The first part of Theorem 1 was proved by Gilmer and Mott (Duke Math J. 37(1970), 751-761) in a different manner. Theorem 2. If R is a linearly compact ring having property ξ w.r.t. an overring S, then the power series ring $R[[x_\lambda]]_{\lambda\in\Lambda}$ has property C w.r.t. $S[[x_\lambda]]_{\lambda\in\Lambda}$. (Received July 7, 1971.)

14 Algebraic Geometry

687-14-1. ANTHONY A. IARROBINO, JR., University of Texas, Austin, Texas 78712. The type of an ideal in the ring of power series.

The type T or characteristic function of an ideal I is a power series ring A over a field k is the sequence of integers $(t_0, \ldots, t_j, \ldots)$ describing of growth of the ideal: t_j = codimension of the vector space of degree-j initial forms of elements of I, in the space of all degree-j homogeneous forms of A. We characterize the sequences that actually occur as types. We describe a partial order on the types, such that a family I_t of ideals of type T can have as a limit an ideal I_0 of type T' only if $T \leqq T'$. In the case $A = k[[x, y]]$, k algebraically closed and $\Sigma t_j = n > \text{char}(k)$ the ideals (graded ideals) of type T are parametrized by a variety Z_T (G_T) that is locally an affine space, is irreducible, nonsingular, and whose dimension we calculate. Also, G_T is complete and Z_T is a locally trivial bundle over G_T (but not in general a vector bundle!). We describe the G_T's in some special cases. These results do not extend to higher dimensions. They depend heavily upon putting an ideal of $k[[x,y]]$ into a normal form. (Received June 4, 1971.)

15 Linear and Multilinear Algebra, Matrix Theory (Finite and Infinite)

*687-15-1. LINCOLN E. BRAGG, Florida Institute of Technology, Melbourne, Florida 32901. Selfadjointness with indefinite quadratic forms.

Let V be a finite dimensional vector space over a field F (characteristic $\neq 2$), and let G be a nonsingular symmetric bilinear form on V. Let L be an operator such that $(Lu) \cdot Gv = u \cdot G(Lv)$. Then there exists a direct sum decomposition of V into mutually orthogonal subspaces U which are "elementary" invariant subspaces under L. Lemma. If σ is a prime polynomial, $T = \sigma(L)$ is singular, and T^{k+1} and T^k have the same nullspace N, then there exists $x_0 \in N$ such that $x_0 \cdot G(T^{k-1}x_0) \neq 0$. Each U has a σ, k, and x_0 associated, and has a linear function f_0 on the space P of polynomials of degree less than σ's. The matrix of G on a basis of U is expressed in terms of the functions $(\xi, \eta) \in P \times P \to f_0(\alpha)$ and $f_0(\beta)$, where $\xi\eta = \alpha\sigma + \beta$. For the real field and small Sylvester index, the list of possible matrices is short; insight is provided, besides this list. The extension to the case when G is hermitian with respect to an automorphism of F involves some of the U's not being elementary, but being direct sums of two isotropic invariant subspaces. (Received June 7, 1971.)

687-15-2. GERNOT MICHAEL ENGEL, University of Wisconsin, Madison, Wisconsin 53706. The behavior of the determinant and permanent on sets of equimodular matrices.

Let $M_{n \times p}$ denote the set of matrices with complex coefficients. For $A \in M_{n \times p}$ let Ω_A denote the set of all $B \in M_{n \times p}$ such that $|b_{ij}| = |a_{ij}|$ and A(j) denote the submatrix of A with the jth column deleted. For $A \in M_{n \times n}$ it can be shown that $\min_{B \in \Omega_A} |\det B| \leqq \min_{B \in \Omega_A} |\text{Per } B| \leqq \max_{B \in \Omega_A} |\det B|$. It is known that if $\min_{B\in\Omega_A} |\det B| \neq 0$ there is a permutation matrix P and $B \in \Omega_A$ such that $PB = (\ell_{ij})$ where L is a nonsingular M-matrix. In this case it can be shown that $\min_{B\in\Omega_A} |\det B| = \det L$. Let Q be a column and row linear matrix function on $M_{n\times n}$ such that if $P \in M_{n\times n}$ is a permutation matrix and I is the identity matrix then $|Q(P)| \leq |Q(I)|$. The following are equivalent: (1) $\forall B \in M_{n\times n}$, B diagonally dominant implies $Q(B) \neq 0$ and (2) $\forall A \in M_{n\times n+1}$, A diagonally dominant implies $|Q(A(n+1))| > |Q(A(n))|$. For $A \in M_{n\times n+1}$ and Q the determinant function the following are equivalent: (3) $\forall B \in \Omega_A$, $|\det B(n+1)| > |\det B(n)|$ and (4) $\exists x > 0$, $x = (x_1, x_2, \ldots, x_n)$ and a permutation $\sigma \in S_n$ such that $|a_{i\sigma(i)}| > \sum_{1\leq j\leq n;\, j\neq i} |a_{i\sigma(j)}| x_j/x_i + |a_{in+1}| x_n/x_i$ for $1 \leq i \leq n$. (Received June 28, 1971.)

*687-15-3. PETER M. GIBSON, University of Alabama, Huntsville, Alabama 35807. Localization of the zeros of the permanent of a characteristic matrix.

Let $A = (a_{ij})$ be an n-square nonnegative matrix, and let $\Psi(A)$ be the set of all n-square complex matrices $B = (b_{ij})$ for which $|b_{ij}| = a_{ij}$ for $i, j = 1, \ldots, n$. Using results of J. L. Brenner [Duke Math. J. 26(1959), 563-567] and of P. Camion and A. J. Hoffman [Pacific J. Math. 17(1966), 211-214], we prove the following: Theorem. If $\det B \neq 0$ for every $B \in \Psi(A)$, then $\operatorname{per} B \neq 0$ for every $B \in \Psi(A)$. It follows from this theorem that many bounds for the eigenvalues of a complex matrix C are also bounds for the zeros of the permanent of the characteristic matrix of C. (Received June 18, 1971.)

687-15-4. ANDREAS Z. ZACHARIOU, Johns Hopkins University, Baltimore, Maryland 21218. The Gram-Schmidt process without iteration: applications. Preliminary report.

The following theorem and its converse are true: Theorem. Let $(V, \langle\ ,\ \rangle)$ be an inner-product space over R and $B = \{x_0, x_1, \ldots, x_n, \ldots\}$ a basis of V. For each $n \geq 0$, let V_n be the subspace of V with basis $B_n = \{x_0, x_1, \ldots, x_n\}$. Let $f_n : V_n \to R$, $n = 1, 2, \ldots$, be a sequence of linear transformations such that: $f_n(x_k) = 0$ for all $k = 0, 1, \ldots, n-1$ and $f_n(x_n) \neq 0$. Let a_n be the unique element of V_n such that: $f_n(x) = \langle a_n, x\rangle$ for all x in V_n, $n = 1, 2, \ldots$. Finally, suppose that: there exists a subset $C = \{y_0, y_1, \ldots, y_n, \ldots\}$ of V, with y_n in V_n, for each $n = 0, 1, 2, \ldots$, having the following properties: (i) $y_0 = x_0/\|x_0\|$. Furthermore, for each $n = 1, 2, \ldots$, $f_n(y_k) = 0$ for all $k = 0, 1, \ldots, n-1$ and $f_n(y_n) \neq 0$. (ii) $f_n(x_n)\, f_n(y_n) > 0$ for all $n = 1, 2, \ldots$. (iii) $a_n = f_n(y_n) y_n$ for all $n = 1, 2, \ldots$. Then C is an orthonormal basis of V. Furthermore, C is the orthonormal basis of V obtained from B by applying the Gram-Schmidt process. Applications. Explicit orthonormal bases are obtained for $V = R[x]$ with $0 \leq x \leq 1$ or $-1 \leq x \leq 1$ (Legendre Polynomials) by applying the above theorem with $B = \{1, x, x^2, \ldots, x^n, \ldots\}$. Other applications are: the inversion of matrices and the solution of systems of linear equations. (Received June 24, 1971.)

*687-15-5. RICHARD D. SINKHORN, University of Houston, Houston, Texas 77004. Continuous dependence on A in the D_1AD_2 theorems.

The author and Paul Knopp ["Concerning nonnegative matrices and doubly stochastic matrices", Pacific J. Math. 21(1967), 343-348] have shown that if A is a nonnegative nonzero square matrix, then there exists a unique doubly stochastic matrix of the form D_1AD_2 where D_1 and D_2 are diagonal matrices with positive main diagonals if and only if every positive entry of A lies on at least one positive diagonal. It was also shown that the limit of the iteration of alternately scaling the rows and column sums of A exists and is doubly stochastic if and only if A contains at least one positive diagonal. In case D_1AD_2 exists, it is the limit of this iteration. In this paper the following is established. Theorem. Let A be a nonnegative square matrix with at least one positive diagonal. Let $\bar{A}$ be such that $\bar{a}_{ij} = a_{ij}$ whenever a_{ij} lies on at least one positive diagonal and $\bar{a}_{ij} = 0$ otherwise. Let D_1 and D_2 be diagonal matrices with positive main diagonals such that $D_1\bar{A}D_2$ is doubly stochastic. Then the limit of the iteration of alternately scaling the row and column sums of A is equal to $D_1\bar{A}D_2$. Corollary. Let A be such that the doubly stochastic form D_1AD_2 exists. Then the map $A \rightarrow D_1AD_2$ is a continuous function of A. Likewise the limit of the iteration of alternately scaling the row and column sums of A (when the limit exists) depends continuously upon A. (Received June 30, 1971.)

687-15-6. WITHDRAWN

16 Associative Rings and Algebras

*687-16-1. IRVING REINER and STEPHEN V. ULLOM, University of Illinois, Urbana, Illinois 61801. Classgroups of p-groups.

Let G be a finite group and Λ a maximal order of QG containing ZG. Theorem. If G is a p-group, the kernel D(ZG) of the (surjective) homomorphism of classgroups $C(ZG) \rightarrow C(\Lambda)$ is a p-group. The proof uses Jacobinski's description of classgroups. We calculate D(ZG) in some examples. (Received April 13, 1971.)

687-16-2. DAVID JACOBSON, Rutgers University, New Brunswick, New Jersey 08903. The isomorphism problem for polynomial rings. Preliminary report.

Let R and S be rings with identity such that $\varphi : R[x] \rightarrow S[x]$ is an isomorphism. Is R necessarily isomorphic to S? D. B. Coleman and E. E. Enochs ["Isomorphic polynomial rings," Proc. Amer. Math. Soc. 27(1971), 247-252] have called R strongly invariant if $S[x] = S[\varphi(x)]$. For R strongly invariant, R is isomorphic to S. Utilizing methods due to Coleman and Enochs, along with commutative techniques, we obtain the following: If R is (von Neumann) regular, then R is strongly invariant. If R is a commutative ring of Krull dimension zero, then R is strongly invariant. If the Jacobson radical of a ring R is a maximal ideal, then R is strongly invariant. A direct sum of local rings is strongly invariant. If R is a commutative strongly invariant ring, then the ring of all endomorphisms of any free R-module is strongly invariant. In line with the question of S. Abhyankar and W. Heinzer (Abstract 683-A16, these *Notices* 18(1971), 359) we have the following: Theorem. If R is a commutative (von Neumann) regular ring and $\varphi : R[x_1, \ldots, x_n] \to S[y_1, \ldots, y_n]$ is an isomorphism, then R and S are isomorphic and $\varphi(R) = S$. (Received May 3, 1971.)

*687-16-3. KONRAD JOHN HEUVERS, Michigan Technological University, Houghton, Michigan 49931.
A generalization of the Clifford algebra to Hermitian world spaces and their related Euclidian world spaces.

An attempt is made to extend the notion of a Clifford algebra to an n-dimensional Hermitian world space H. Through the introduction of a fixed conjugation operator Π a related Euclidian world space E is uniquely determined. The investigation leads to the discovery of two families of associative algebras which are generated by E. The Clifford algebra C is the chief representative of the first family. The other family is generated by E subject to the defining relations $x \cdot y\Pi\Phi + y \cdot x\Pi\Phi = 2(x|y)$ and $\lambda \cdot x = x \cdot \lambda^*$ where λ^* denotes the complex conjugate, where $(x|y)$ is the inner product in E, and where Φ is an involutory isometry in E which also satisfies $\Phi = \Pi\Phi\Pi$. In both families of algebras, formulas are found for the isometries of E. In the second family the algebra generated by $\Phi = I$ is found to play a role similar to that of the Clifford algebra C in the first family. However the relationship between the algebra and the dimension n of the world space is reversed. (Received June 7, 1971.)

*687-16-4. SHEILA O. COLLINS, Tulane University, New Orleans, Louisiana 70118. A general definition of injectivity.

The following definition of injectivity for unital (left) modules generalizes most of the previously studied types. For an arbitrary, but fixed class Γ of R-monomorphisms, an R-module M is called Γ-injective if for each $\alpha : A \to B \in \Gamma$ and for each $f : A \to M$, there exists an $\bar{f} : B \to M$ such that $\bar{f} \circ \alpha = f$. The following results hold: Theorem 1. Epimorphic images of injective modules are Γ-injective if and only if for each exact sequence $E : 0 \to A \overset{\alpha}{\to} B \to C \to 0$ with $\alpha \in \Gamma$ and for each epimorphism $\psi : F \to A$ with F projective, E lies in the image of the connecting group-homomorphism $\psi_* : \mathrm{Ext}^1_R(C, F) \to \mathrm{Ext}^1_R(C, A)$. Corollary. Assume that for each $\alpha : A \to B \in \Gamma$, B is projective; then Γ-injectivity is closed under epimorphic images if and only if for each $\alpha : A \to B \in \Gamma$, A is likewise projective. Theorem 2. Every module has a unique maximal Γ-injective submodule if and only if Γ-injectivity is closed under direct sums and epimorphic images. (Received June 21, 1971.)

17 Nonassociative Rings and Algebras

*687-17-1. GEORGE F. LEGER, Tufts University, Medford, Massachusetts 02155 and EUGENE M. LUKS, Bucknell University, Lewisburg, Pennsylvania 17837. Cohomology theorems for Borel-like solvable Lie algebras.

The cohomology groups $H^i(B, B)$ are studied for solvable Lie algebras B having structural properties like those of Borel subalgebras of complex semisimple Lie algebras. In particular, these algebras are semidirect sums of nilpotent ideals and abelian subalgebras which act on the ideals in a semisimple fashion. Techniques for computing $H^1(B, B)$, the outer derivation algebra, are given. For a large class of such algebras, which exist in any characteristic $\neq 2$, $H^i(B, B) = 0$ for $i \leq 2$. For the aforementioned Borel subalgebras themselves, $H^i(B, B) = 0$ for all i. (Received July 7, 1971.)

687-17-2. VOLODYMYR BOHUN-CHUDYNIV, Morgan State College, Baltimore, Maryland 21212 and BORIS BOHUN-CHUDYNIV, Seton Hall University, South Orange, New Jersey 07079. On loops satisfying Cayley relations.

The authors determine the existence of 2 nonisotopic types of loop of order 2^k $(k \geqq 4)$ satisfying A. Cayley's relation $x(y \cdot xz) = (xy \cdot x)z$, where x, y, z are any 3 elements of a loop (A. Cayley, "Note on a system of imaginaries," Philos. Mag. 30(1847), p. 257). The first type consists of loops with the binary operation defined by Kirkman triple systems (KTS) of $2^k - 1$ order and relation $\alpha^2 = 0$ (0 is the identity element), denoted $KSL_{(2^k)}$. The number of nonisomorphic types of $KSL_{(2^k)}$ equals the number of nonisomorphic types of KTS of the same order. The second type consists of double loops with binary operation defined by B double triple-systems of 2^k $(k \geqq 4)$ order constructed by the senior author, and appearing here for the first time, denoted $BSL_{(2^k)}$. The total number of BSL_{16} equals 240, all of which are isomorphic and isotopic, i.e. BSL_{16} are all G-loops. Fundamental theorems are proved, and numerous illustrative examples are given. (Received July 7, 1971.)

687-17-3. VOLODYMYR BOHUN-CHUDYNIV and WALTER R. TALBOT, Morgan State College, Baltimore, Maryland 21212. On algorithms for constructing distributive idempotent quasi-groups of order $2(3q+r)$ $(r=1,2)$.

An idempotent quasi-group is called distributive if for any of its 3 elements the following relations hold: $a(bc) = ab \cdot ac$; $(ab)c = ac \cdot bc$. In a paper, "On distributive idempotent quasi-groups defined by n-tuple systems" presented at a MAA meeting on April 29, 1971, at Loyola College, Baltimore, the first author introduced the 4 nonisotopic types of idempotent quasi-groups and algorithms for constructing them. In this paper the authors introduce distributive idempotent quasi-groups of order $2(3q+r)$ $(r=1,2)$, and algorithms for constructing them. Each algorithm consists of two parts. Part one is a triple-system of $3q + r$ $(r = 1, 2)$ order. The second part consists of two operators constructed from the triplets of the system. For triple-systems of different orders there are 24 different numbers of pairs of operators. Therefore the total number of distributive idempotent quasi-groups equals $30 \times 24 = 720$ distributive idempotent quasi-groups of the 8th order. 30 is the number of triple-systems of the 7th order. Illustrative examples are given and theorems are proved. (Received July 7, 1971.)

18 Category Theory, Homological Algebra

*687-18-1. STEPHEN BARON, Clark University, Worcester, Massachusetts 01610. Group sequential topological groups. Preliminary report.

We say that a topological group is group sequential if any finer topology that will make the group a topological group will result in strictly fewer convergent sequences. It is shown that the category of group sequential topological groups is a coreflective subcategory of the category of topological groups. An adjoint $\overline{F}$ is found for the underlying functor from topological groups to topological spaces. Continuous homomorphisms from $\overline{F}(\omega+1)$ to a topological group correspond to convergent sequences in the given group. ($\omega+1$ is the set of ordinals up to and including the first infinite ordinal with the order topology.) It is shown that any group sequential topological group is an extremal quotient object of a direct sum of copies of $\overline{F}(\omega+1)$. (Received June 1, 1971.)

*687-18-2. DONOVAN H. VAN OSDOL, University of New Hampshire, Durham, New Hampshire 03824. Coalgebras, sheaves, and cohomology.

The category of sheaves on the topological space X with values in the algebraic category A is shown to be cotripleable under the stalk category $A^{|X|}$. If K is a field then the category of K-coalgebras is shown to be cotripleable under the category of K-vector spaces. This makes possible the interpretation of the first group of the associated triple cohomology complex, as in Beck's thesis (Columbia University, 1967). In particular, our H^1 is isomorphic to Jonah's H^2 (D. W. Jonah, "Cohomology of coalgebras," Memoirs of the American Mathematical Society, No. 82, American Mathematical Society, Providence, Rhode Island, 1968). (Received June 29, 1971.)

687-18-3. V. SANKRITI KRISHNAN, Temple University, Philadelphia, Pennsylvania 19122. Dual connections and Galois connections. Preliminary report.

Given a pair of categories C, C' a dual connection (T, T', n, n') between them is made up of two contravariant functors $T: C \to C'$, $T': C' \to C$, and two natural equivalences $n: 1_C \to T'T$, $n': 1_{C'} \to TT'$. If we replace the requirement that n, n' be natural equivalences by only requiring that they be natural transformations, we get the definition of a Galois connection (T, T', n, n'). More generally, if C, C' have (grounding) covariant functors $b: C \to B$, $b': C' \to B'$, we define a weak dual connection (or a weak Galois connection) to be (T, T', m, m') with T, T' contravariant functors as before, but m, m' being natural equivalences (or natural transformations for the Galois connection) $m: b \to bT'T$, $m': b' \to b'TT'$. The main results are: starting from a weak Galois connection (T, T', m, m') we can find a pair of full subcategories C^*, C'^* of C, C' such that there is a dual connection (T, T', n, n') between them (with T, T' restrictions of the given T, T'), with $b \cdot n = m$, $b' \cdot n' = m'$ and $Tn = (n'T)^{-1}$; and this pair of full subcategories is the largest such pair. C^* is closed for J-products and C'^* is closed for J-coproducts if T, B are J-product preserving, while T', b' are J-coproduct preserving. Examples are given. (Received July 6, 1971.)

20 Group Theory and Generalizations

687-20-1. RICHARD A. SANERIB, JR., University of Colorado, Boulder, Colorado 80302. Automorphism groups of ultrafilters. Preliminary report.

Let I be an infinite set, $\mathfrak{F}_1$ and $\mathfrak{F}_2$ nonprincipal ultrafilters on I, Sym(I) the symmetric group on I, Sym(I,$\mathfrak{m}$) the set of permutations of I which move less than $\mathfrak{m}$ elements, Alt(I) the set of even permutations of Sym(I, $\aleph_0$) and Aut*($\mathfrak{F}_1$) = $\{\pi \in \mathrm{Sym(I)} : \pi(\mathfrak{F}_1) = \mathfrak{F}_1\}$. When $\mathfrak{F}_1$ is considered as a lattice, there is a natural isomorphism between Aut*($\mathfrak{F}_1$) and the automorphism group Aut($\mathfrak{F}_1$). Theorem 1. The only nontrivial normal subgroups of Aut*($\mathfrak{F}_1$) are Sym(I,$\mathfrak{m}$) $\cap$ Aut*($\mathfrak{F}_1$) and Alt(I) where $\mathfrak{m} \leq |I|$. Theorem 2. $\mathfrak{F}_1$ is isomorphic to $\mathfrak{F}_2$ if and only if Aut($\mathfrak{F}_1$) $\cong$ Aut($\mathfrak{F}_2$). Theorem 3. If H is a normal subgroup of Aut*($\mathfrak{F}_1$), then Sym(I) is isomorphic with a subalgebra of Aut*($\mathfrak{F}_1$)/H. Theorem 4. [Sym(I) : Aut*($\mathfrak{F}_1$)] $\geq \mathfrak{m}^+$ if $\mathfrak{F}_1$ is uniform and $|I| = \mathfrak{m}$, [Sym(I) : Aut*($\mathfrak{F}_1$)] $\leq \mathfrak{m}^{\mathfrak{n}}$ where $\mathfrak{n}$ is the minimum cardinality of sets in $\mathfrak{F}_1$, [Sym(I) : Aut*($\mathfrak{F}_1$)] = $2^{\mathfrak{m}}$ if $\mathfrak{m}^{\mathfrak{m}} = \mathfrak{m}$ and $\mathfrak{F}_1$ is uniform, [Sym(I) : Aut*($\mathfrak{F}_1$)] = $\mathfrak{m}$ if $\mathfrak{m}^{\mathfrak{n}} = \mathfrak{m}$. Theorem 5. $\mathfrak{F}_1$ is isomorphic to $\mathfrak{F}_2$ if and only if Aut*($\mathfrak{F}_1$)$\cdot$ Aut*($\mathfrak{F}_2$) $\neq$ Sym(I). Theorem 6. $\mathfrak{F}_1$ is uniform if and only if Aut*($\mathfrak{F}_1$)/Sym(I, $|I|$) $\cap$ Aut*($\mathfrak{F}_1$) $\not\cong$ Sym(I)/Sym(I, $|I|$). (Received May 6, 1971.)

687-20-2. HIDEGORO NAKANO, Wayne State University, Detroit, Michigan 48202. On characterization of automorphism groups. Preliminary report.

We characterized automoration groups (Abstract 682-20-4, these *Notices* 18(1971), 127). However we cannot characterize automorphism groups. We can easily prove that any group is isomorphic to some inner automorphism group. (Received May 6, 1971.)

*687-20-3. KIM KI-HANG BUTLER, Pembroke State University, Pembroke, North Carolina 28372. A note on semigroups 0.

Recently, R. Korfhage ["Solutions of $X^2 = I$ for matrices over finite ring with unity," Amer. Math. Monthly 75(1968), 634-636 (Theorem 2)] asserted that if B is a Boolean algebra of 2^m elements, then the number of n $\times$ n matrices over B satisfying $X^2 = I$ is $T_n = (g_n \sum_{t=0}^{[n/2]} (2^{-t(2n-3t)}/(g_t g_{n-2t})))^m$, where I is the n $\times$ n identity matrix, $g_0 = 1$, and $g_t = \sum_{k=0}^{t-1} (2^t - 2^k)$ ($k \geq 1$). The purpose of this note is to point out errors in this assertion. This note investigation is restricted to m = 1. Let B_n denote the set of all n $\times$ n matrices over the Boolean algebra B = $\{0,1\}$ of order 2 (i.e., $0+0=0$, $0+1=1+0=1+1=1$, $0\cdot 0 = 1\cdot 0 = 0 \cdot 1 = 0$, $1 \cdot 1 = 1$). Theorem 1. If π_n denotes the number of n $\times$ n involutory matrices in B_n, then π_n equals the sum of all symmetric permutation matrices in B_n. Hence $\pi_n = \sum_{k=0}^{[n/2]} (n!/k!\,2^k(n-2k)!)$. Corollary 2. $\pi_n = (n-1)\pi_{n-2} + \pi_{n-1}$ ($\pi_1 = 1$, $\pi_2 = 2$). (Received June 7, 1971.)

687-20-4. LUIS RIBES, Carleton University, Ottawa, Ontario, Canada. Amalgamated products of profinite groups.

Let $\underline{C}$ be a class of finite groups. Let A_1 and A_2 be pro-$\underline{C}$ groups with a common closed subgroup H. The pro - $\underline{C}$ amalgamated product of A_1 and A_2 with amalgamated subgroup H is defined to be the push-out A of A_1 and A_2 over H in the category of pro - $\underline{C}$ groups if the canonical homomorphisms of A_1 and A_2 into A are monomorphisms. If A_1 and A_2 are profinite groups it is proved that the profinite amalgamated product of A_1 and A_2 over a common closed subgroup H exists if one of the following conditions holds: (1) H is finite, (2) H is in the center of either A_1 or A_2, (3) H is normal in both A_1 and A_2 and finitely generated as a topological group. If A_1 and A_2 are pro - p groups satisfying the first axiom of countability, necessary and sufficient conditions are given for the existence of the pro - p amalgamated product of A_1 and A_2 over a common closed subgroup. It is shown that the pro - p amalgamated product of two pro - p groups over a common closed pro-cyclic subgroup always exists. Examples are given to show that pro - $\underline{C}$ amalgamated products do not always exist. (Received June 28, 1971.)

687-20-5. RICHARD H. CROWELL and NEVILLE F. SMYTHE, Dartmouth College, Hanover, New Hampshire 03755. A monomorphism theorem for groupoids. Preliminary report.

The following construction of a mapping cylinder and monomorphism theorem for (Brandt) groupoids subsumes the theory of free products with amalgamated subgroups and the HNN-construction for groups. The techniques also yield intersection theorems such as R. H. Fox's: If $A_1 \leftarrow A_0 \rightarrow A_2$ is a diagram of groups and if the group P with morphisms $g_i : A_i \rightarrow P$ is the pushout, then $g_1(A_1) \cap g_2(A_2) = g_0(A_0)$. Given a mapping diagram A, i.e., a family of groupoids and morphisms indexed by a directed graph, a groupoid C called the mapping cylinder of A is defined analogously to the topological construction. For each groupoid A_i of A, there is a morphism $g_i : A_i \rightarrow C$, and, for each morphism $f: A_i \rightarrow A_j$, there is a morphism $h: A_i \times I \rightarrow C$ which is a homotopy between g_i and $g_j f$. The cylinder C and morphisms into it satisfy the universal property, and I is the 4-element edge-path groupoid of the unit interval. Monomorphism Theorem. If every morphism f of A is a monomorphism, then so is every morphism g_i and homotopy h into C. If the graph of A is acyclic and its groupoids connected, then C has the homotopy type of the pushout of A. (Received June 28, 1971.)

687-20-6. JOHN SHAFER, University of Massachusetts, Amherst, Massachusetts 01002. Homomorphisms between free contents.

Let F_n be the free semigroup generated by the set $\{a_1, a_2, \ldots, a_n\}$. The free content, C_n, is the set of all words in F_n each of which uses every a_i, $i = 1, 2, \ldots, n$, at least once. C_n is a subsemigroup of F_n, but it is not finitely generated. Let F_n^1 be the semigroup F_n with the "empty word" adjoined as an identity. Theorem. Any homomorphism $\varphi : C_m \rightarrow C_n$ can be extended to a unique homomorphism $\psi : F_m^1 \rightarrow F_n^1$. If φ is one-to-one or onto, then so is ψ. Corollary. $C_m(F_m)$ is isomorphic onto $C_n(F_n)$ if and only if $m = n$; $C_m(F_m)$ is homomorphic onto $C_n(F_n)$ if an only if $m \geqq n$. The automorphism groups of C_m and F_m are both isomorphic to the symmetric group on m letters. The theorem determines a functor which embeds the category of free contents with semigroup homomorphisms into the category of free semigroups with adjoined identity. (Received July 6, 1971.)

*687-20-7. PHILLIP E. McNEIL, University of Cincinnati, Cincinnati, Ohio 45221. Finite commutative subdirectly irreducible semigroups.

This paper is devoted to completing the solution to the problem of constructing all finite commutative subdirectly irreducible semigroups. Those semigroups of this type which were formerly unknown are realized as certain permutation group extensions of nilpotent semigroups. The results in this paper extend the efforts in this area by G. Thierrin and B. M. Schein. (Received June 17, 1971.)

*687-20-8. TAKAYUKI TAMURA, University of California, Davis, California 95616. Irreducible $\mathfrak{N}$-semigroups. Preliminary report.

By an $\mathfrak{N}$-semigroup we mean a commutative archimedean cancellative semigroup without idempotent. An $\mathfrak{N}$-semigroup is called irreducible if S has no proper $\mathfrak{N}$-congruence on S except the equality relation. Theorem 1. An irreducible $\mathfrak{N}$-semigroup is isomorphic to a positive real semigroup with addition. An irreducible archimedean positive real semigroup is called an IAPR-semigroup. Theorem 2. Let Q be an additive group of real numbers and Q^+ the positive cone of Q. Let $\mathfrak{P}$ be the system of all nonempty additive semigroups of positive integers. Let ψ be a mapping of Q^+ into $\mathfrak{P}$ satisfying (1) $\psi(a) \cap \psi(b) \subseteq \psi(a+b)$ for all $a, b \in Q^+$. (2) $((1/n)) \cdot \psi(a) \subseteq \psi(n \cdot a)$ for all $a \in Q^+$, all positive integers n where $((1/n)) \cdot \psi(a) = \{x/\text{g.c.d.}(n,x) : x \in \psi(a)\}$. Let $S = \{a \in Q^+ : \psi(a) = \{1, 2, 3, \ldots\}\}$. Then S is an IAPR-semigroup and Q is the quotient group of S. Every IAPR-semigroup can be obtained in this manner. The correspondence $\psi \to S$ is one to one. It is noticed that every $\mathfrak{N}$-semigroup is homomorphic onto an irreducible $\mathfrak{N}$-semigroup. (Received July 6, 1971.)

687-20-9. WITHDRAWN.

687-20-10. J. SUTHERLAND FRAME and ARUNAS RUDVALIS, Michigan State University, East Lansing, Michigan 48823. A curious pairing of characters of $Sp_{2n}(2)$ and of its orthogonal subgroups. Preliminary report.

A striking correspondence, called pairing, is found in a study of the 81 irreducible characters of the symplectic group $Sp_8(2)$, of order $2^{16}3^5 5^2 7 \cdot 17 = 47{,}377{,}612{,}800$, which appears to generalize for the family of groups $Sp_{2n}(2)$ of order $2^{n^2} \Pi a_{ni} b_{ni}$, where $a_{ni} = 2^{n+1-i} + (-1)^{n+1-i}$ and $b_{ni} = 2^{n+1-i} - (-1)^{n+1-i}$. Character degrees have the form $2^{e_k} f_{nk}/d_k$, where f_{nk} is a product of an even number of factors a_{ni} and b_{ni}, $1 \leq i \leq n+1$, and d_k divides f_{nk} for all n. Pairing maps characters of the subgroup $O^-_{2n}(2)$ of index $2^{n-1} b_{n1}$ onto characters of the subgroup $O^+_{2n}(2)$ of index $2^{n-1} a_{n1}$ while interchanging factors a_{ni} and b_{ni} in their degrees, so that the induce-restrict tables for $Sp_{2n}(2)$ over these two subgroups are interchanged by interchanging paired characters of $Sp_{2n}(2)$. The two permutation characters of degrees $2^{n-1} b_{n1}$ and $2^{n-1} a_{n1}$ are a reducible pair whose values have sums that are powers of 2 and differences that are $\pm 2^s$ or 0. The group $Sp_8(2)$ has 16 self-paired characters, 20 sets of paired characters, and 25 characters with degree factor $a_{n+1} = 2$, which are unpaired because their pairs vanish, since $b_{n+1} = 0$. These unpaired characters introduce an asymmetry into many relationships, including the induce-restrict tables. (Received July 7, 1971.)

*687-20-11. R. B. HORA and NAOKI KIMURA, University of Arkansas, Fayetteville, Arkansas 72701. Left translations of the direct sum of semigroups.

Let $\Phi = \{S_i : i \in I\}$ be a family of semigroups. Let S be the direct sum of Φ. Then the semigroup $\mathcal{L}(S)$ of all left translations of S is canonically isomorphic with the semigroup S with the identity element adjoined, whenever the cardinality of I is greater than 2. The detail structure of $\mathcal{L}(S_1 + S_2)$ will be established, in which two congruence relations in S_i will play a decisive role. (Received July 7, 1971.)

*687-20-12. ALPHONSE H. BAARTMANS, Southern Illinois University, Carbondale, Illinois 62901. Solvable groups with abelian Carter subgroups. Preliminary report.

In this paper we obtain some results on A-groups and generalize some known results on A-groups to solvable groups with abelian Carter subgroup. Theorem 1. Let G be an A-group, and let D and K denote a system normalizer and Carter subgroup of G. Then (i) $K = D\cdot(G'\cap K)$ and $D \cap (G' \cap K) = 1$, (ii) $C_G(D) \cap C_G(G'\cap K) = K$, (iii) if $C_G(G'\cap K) = K$, then $G' \cap K = (1)$ and $K = D$, (iv) if $K \neq C_G(G'\cap K)$, then K is a system normalizer of $C_G(G'\cap K)$. Theorem 2. Let G be a solvable group with abelian Carter subgroup K, and let H be a subgroup of K. We set $H_0 = H$, $N_0 = N_G(H)$, and define groups H_i and N_i for $i \geqq 1$ recursively by the following conditions: H_i is the system normalizer of N_{i-1} and $N_i = N_G(H_i)$. Then the following holds: $H_0 \leqq H_1 \leqq H_2 \ldots \leqq H_k = N_k \leqq N_{k-1} \leqq \ldots \leqq N_0$ and $H_k = N_k$ is a Carter subgroup of G. This generalizes a result due to Carter (Proc. London Math. Soc. 5(1962), 535-562). (Received July 7, 1971.)

22 Topological Groups, Lie Groups

*687-22-1. ALBERT R. STRALKA, University of California, Riverside, California 92502. Extending congruences on compact semigroups.

We define φ to be the canonical homomorphism associated with the congruence $[\varphi]$. A semigroup S is medial if it satisfies the condition abcd = acbd for all $a,b,c,d \in S$. For a semigroup S, Reg S is the set of those elements $s \in S$ for which there exist elements $t \in S$ such that sts = s and tst = t. Theorem. Let S be a compact medial topological semigroup and let A be a closed subsemigroup of Reg S. If $[\varphi]$ is a closed congruence on A such that $\dim \varphi(A)/\mathcal{H} = 0$ then $[\varphi]$ can be extended to a closed congruence on S, i.e., there is a closed congruence $[\Phi]$ on S such that $[\Phi] \cap A \times A = [\varphi]$. Theorem. Let X be the usual compact semilattice on the product of a countable family of copies of $\{0,1\}$. Then X is dimensionally stable. Corollary. There is a closed chain C in X and a closed congurence $[\varphi]$ on C such that $[\varphi]$ cannot be extended to X. (Received June 7, 1971.)

*687-22-2. KENNETH I. GROSS, Dartmouth College, Hanover, New Hampshire 03755. A degenerate principal series of representations of $Sp(n+1, \mathbb{C})$.

Let $\Sigma = Sp(n+1, \mathbb{C})$ and G a maximal parabolic subgroup which leaves fixed a one-dimensional subspace of $\mathbb{C}^{2(n+1)}$. G is the semidirect product of a normal nilpotent (Heisenberg) group Z and a subgroup C which is the direct product of $Sp(n, \mathbb{C})$ with the multiplication group A of complex numbers. The degenerate principal series of representations $T(\cdot, \pi)$ of Σ induced from the unitary characters of G is indexed by $\pi \in A^*$, the dual group of A. Theorem. $T(\cdot, \pi)$ is irreducible if and only if π is not the identity character. This appears to be the first example in the literature of a reducible representation of a complex semisimple Lie group that lies in a (degenerate) principal series. The proof of the theorem hinges upon analyzing the restriction to the subgroup transpose to G of the representations $T(\cdot, \pi)$, and the techniques center upon nonabelian Fourier analysis on the group Z. (Received June 25, 1971.)

*687-22-3. ARTHUR LIEBERMAN, University of South Florida, Tampa, Florida 33620. The structure of certain unitary representations of infinite symmetric groups.

Let S be an infinite set and d be an infinite cardinal number. Let G be the group of those permutations g of S such that the support of g has cardinal number less than d. Give G the topology of pointwise convergence on S; G is a topological group but is not locally compact. Any weakly continuous unitary representation of G is the direct sum of irreducible representations of G. The irreducible weakly continuous representations of G can be explicitly described. There is a canonical one-to-one correspondence between the set of equivalence classes of irreducible weakly continuous unitary representations of G and the set of equivalence classes of irreducible representations of the symmetric group on n symbols, as n ranges over the nonnegative integers. (Received June 30, 1971.)

28 Measure and Integration

687-28-1. CHARLES A. HAYES, JR., University of California, Davis, California 95616. Derivation in Orlicz spaces.

$(R, \mathfrak{M}, \mu)$ is a measure space, μ is σ-finite, $\mathcal{B}$ is a derivation basis with domain $E \subset R$ that derives all μ-integrals with bounded integrands. Given any μ-finite μ-integral ν, a criterion that $\mathcal{B}$ derives ν to its integrand a.e. is established. L_Φ and L_Ψ are dual Orlicz spaces with norms $\| \ \|_\Phi$ and $\| \ \|_\Psi$, respectively (notation of A. C. Zaanen, "Linear analysis"). $\mathcal{B}$ is L_Φ-strong iff for each $X \subset E$ with $\mu(\overline{X}) < +\infty$, each $\mathcal{B}$-fine covering $\mathcal{V}$ of X, and each $\epsilon > 0$, there exists a finite family $\mathcal{K} \subset \mathcal{V}$ with $\mu(\overline{X} - \overline{X} \cup (\bigcup \mathcal{K})) < \epsilon$, $\mu(\bigcup \mathcal{K} - \overline{X} \cap (\bigcup \mathcal{K})) < \epsilon$, and $\|e_{\mathcal{K}}\|_\Phi < \epsilon$ where $e_{\mathcal{K}}$ measures the overlap of $\mathcal{K}$. Then $\mathcal{B}$ derives the integrals of functions in L_Ψ. Two other conditions on $\mathcal{B}$ are introduced and shown to imply L_Φ-strength. Under certain restrictions, these conditions are shown to be equivalent to L_Φ-strength. (Received June 25, 1971.)

687-28-2. FRED M. WRIGHT and DEAN R. KENNEBECK, Iowa State University, Ames, Iowa 50010. On the definition and existence of an (LIR)-refinement integral.

Let f_1, f_2, f_3, g, h, k be real-valued functions on a closed interval $[a, b]$ of the real axis. For a partition $\Delta = \{a = x_0 < x_1 < x_2 < \ldots < x_m = b\}$ of $[a, b]$, choose a point ζ_i of the open interval (x_{i-1}, x_i) for each integer $i = 1, 2, \ldots, m$, and form the sum $S(\Delta; \zeta_1, \zeta_2, \ldots, \zeta_m) =$
$\sum_{i=1}^m \{f_1(x_{i-1}) \cdot [g(x_i) - g(x_{i-1})] + f_2(\zeta_i) \cdot [h(x_i) - h(x_{i-1})] + f_3(x_i) \cdot [k(x_i) - k(x_{i-1})]\}$. When these sums

$S(\Delta;\zeta_1, \zeta_2, \ldots, \zeta_m)$ have a finite refinement limit, this limit will be denoted by (LIR) $\int_a^b (f_1dg + f_2dh + f_3dk)$. Here existence theorems for the (LIR)-refinement integral are discussed which generalize the existence theorems for the (LR)-refinement integral presented by the authors (Abstract 677-28-8, these *Notices* 17(1970), 770). If there is a point p of $[a,b)$ such that h is not continuous from the right at p and such that $f_2(p^+)$ does not exist, then (LIR) $\int_a^b (f_1dg + f_2dh + f_3dk)$ does not exist. Similarly, (LIR) $\int_a^b (f_1dg + f_2dh + f_3dk)$ does not exist if there is a point p of $(a,b]$ such that h is not continuous from the left at p and such that $f_2(p^-)$ does not exist. If h is continuous and of bounded variation on $[a,b]$ and if the Riemann-Stieltjes norm integral $\int_a^b f_2dh$ does not exist, then (LIR) $\int_a^b (f_1dg + f_2dh + f_3dk)$ does not exist. Special attention is paid to the important case when g, h, and k are of bounded variation on $[a,b]$. (Received July 7, 1971.)

30 Functions of a Complex Variable

*687-30-1. MAXWELL O. READE, University of Michigan, Ann Arbor, Michigan 48104 and ELIGUISZ ZŁOTKIEWICZ, M. Curie-Skłodowska University, Lublin, Poland. On the values omitted by univalent functions with two preassigned values. Preliminary report.

Let $\mathfrak{M}$ denote the set of all functions f(z) each of which is analytic and univalent in the unit disc Δ and satisfies $f(0) = 0$ and $f(z_0) = z_0$. Here z_0 is a fixed point in Δ, $0 < r_0 = z_0 < 1$. Let $m(R,f)$ denote the one-dimensional Lebesgue measure on the circle $|w| = R$ of the complement of the intersection of $f(\Delta)$ and the circle $|w| = R$. Here R must satisfy $\rho = (1-r_0^2)/4 \leqq R \leqq 1$ [Lewandowski, Ann. Univ. Mariae Curie-Skłodowska Sect. A 13(1959), 115-126]. The main result contained in the present note is the following one. Theorem. $m(R) \equiv \sup[m(R,f) \mid f \in \mathfrak{M}] = 2R \arccos(1-2a^2)$, where $a = [(2(1-\rho)/(1+\rho)^2)(\sqrt{R} - \rho/\sqrt{R}) + 8\rho/(1+\rho)^2 - 1]$, where $\rho \leqq R \leqq 1$. This result complements an earlier one due to Jenkins, who considered the same problem, but for the class S of univalent functions f(z) normalized by the conditions $f(0) = 0$ and $f'(0) = 1$ [Amer. J. Math. 75(1953), 406-408]. (Received January 20, 1971.)

687-30-2. ADOLPH W. GOODMAN, University of South Florida, Tampa, Florida 33620. On close-to-convex functions of higher order.

A function $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ that is regular in the unit disk E is said to be close-to-convex of order $\beta > 0$ if there is a $C \neq 0$ and a normalized convex function $\varphi(z)$ such that $|\arg Cf'(z)/\varphi'(z)| \leqq \beta\pi/2$ in E. The case $\beta \leqq 1$ has been studied earlier. The case of arbitrary real β is considered in the present work. The family $K(\beta)$ of all close-to-convex functions of order β is a linear-invariant family of order $\alpha = \beta + 1$. Various inequalities are obtained for functions in the class $K(\beta)$. In particular if β is a positive integer, then we obtain the sharp upper bound for $|a_n|$, but if β is not an integer, the correct bound is still open. (Received March 11, 1971.)

*687-30-3. JOSEPH A. CIMA, University of North Carolina, Chapel Hill, North Carolina 27514. On the dual of a space of locally schlicht functions.

Let X be the set of holomorphic functions in the unit disk with $f'(z) \neq 0$, $f(0) = 0$, $f'(0) = 1$. Define linear operations in X as $f \oplus g(\zeta) = \int_0^\zeta f'(t) \cdot g'(t)dt$ and $\alpha \otimes f(\zeta) = \int_0^\zeta (f'(t))^\alpha dt$ and a norm as follows: $\|f\| = \sup(1-|z|)|f''(z)/f'(z)|$, $|z| < 1$. X is a Banach space. We determine a subspace Y* of the topological dual of X* which is weak-star dense in X*. (Received April 15, 1971.)

687-30-4. RICHARD MANDELBAUM, University of Massachusetts, Amherst, Massachusetts 01002. Branched structures and affine and projective bundles on Riemann surfaces.

Suppose M is a Riemann surface of genus g. Let A(M) and P(M) denote the set of branched affine and projective structures on M respectively. For any nonnegative integer k, let $A_k(M) \subset A(M)$ and $P_k(M) \subset P(M)$ be the subset of structures of total branching order k. Then it can be shown: Theorem. There exist maps $j_1 : A(M) \to H^1(M, GA(1, C))$ and $j_2 : P(M) \to H^1(M, PL(1, C))$, such that $j_1|A_{2g-2(\varkappa)}$ is injective and $j_2|\bigcup_{i=0}^{i=g-1} P_{2i}(M)$ is injective. In the noninjective cases, conditions are determined under which two structures are mapped into the same bundle and these conditions are related to the distribution of Weierstrass points on the surface. (Received July 1, 1971.)

687-30-5. CARL P. McCARTY, LaSalle College, Philadelphia, Pennsylvania 19141 and DAVID E. TEPPER, Delaware State College, Dover, Delaware 19901. A note on the 2/3 conjecture for starlike functions.

Let $w = f(z) = z + a_2z^2 + \ldots$ be regular and univalent for $|z| < 1$ and map $|z| < 1$ onto a region which is starlike with respect to $w = 0$. If r_0 denotes the radius of convexity of $w = f(z)$, $d_0 = \min|f(z)|$ for $|z| = r_0$, and $d^* = \inf|\beta|$ for $f(z) \neq \beta$, then it has been conjectured that $d_0/d^* \geqq 2/3$. It is shown here that $d_0/d^* \geqq 0.380\ldots$ which improves both the original estimate $d_0/d^* \geqq 0.268\ldots$ [Schild, Proc. Amer. Math. Soc. 4(1953), 43-51] and the more recent estimate $d_0/d^* \geqq 0.343\ldots$ (unpublished). In addition, an upper bound for d^* which depends on $|a_2|$ is given. (Received July 7, 1971.)

*687-30-6. JAMES L. FRANK and JOHN K. SHAW, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. Abel-Gončarov polynomial expansions.

Let $\{d_n\}$ denote a nondecreasing sequence of positive numbers such that $\{d_{n+1}/d_n\}$ is nonincreasing and has limit 1, and let $\{z_n\}$ be a sequence of complex numbers. In terms of these sequences we define a "generalized derivative" operator $\mathcal{D}$ and a sequence of polynomials $\{Q_n\}$. If $d_n = n$, then $\mathcal{D}$ reduces to the ordinary derivative operator D and Q_n is the nth Gončarov polynomial G_n. If $|z_n| \leqq 1$ and f is an entire function of exponential type less than the Whittaker constant W, then the series $\sum f^k(z_k)G_k(z)$ is known to converge to the function f. In the present paper, we investigate the convergence of the series $\sum \mathcal{D}^k f(z_k)Q_k(z)$, where $\mathcal{D}^k$ is the kth successive iterate of $\mathcal{D}$. If $R = \{R_n\}$ is a nondecreasing sequence of positive numbers such that $\{R_{n+1}/R_n\}$ has limit 1, then the R-type of a function $f(z) = \sum a_k z^k$ is defined to be $\limsup |a_n R_1 R_2 \ldots R_n|^{1/n}$. We prove the following. Theorem A. Let $\{d_n\}$ and $\{R_n\}$ be given. There is a constant $W(\mathcal{D})$, depending only on the operator

$\mathcal{D}$, such that if f has R-type less than 1 and $|z_n| \leqq W(\mathcal{D})R_n/d_n$, then $f(z) = \sum \mathcal{D}^k f(z_k)Q_k(z)$, with uniform convergence on compact subsets of the domain of f. We also show that $W(\mathcal{D})$ cannot be replaced by a larger number in Theorem A. (Received July 6, 1971.)

687-30-7. JAMES D. BUCKHOLTZ, University of Kentucky, Lexington, Kentucky 40508 and JAMES L. FRANK, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. Whittaker constants. II.

Let $\{d_n\}_{n=1}^{\infty}$ and $\{R_n\}_{n=1}^{\infty}$ be two nondecreasing sequences of positive numbers such that $\{d_{n+1}/d_n\}_{n=1}^{\infty}$ is nonincreasing with limit 1 and $\{R_{n+1}/R_n\}_{n=1}^{\infty}$ has limit 1. Denote by $e_0 = 1$, $e_n = (d_1 \cdots d_n)^{-1}$, $n = 1, 2, \ldots$. For a function analytic in a neighborhood of 0, we define the operator $\mathcal{D}$ which transforms the function $f(z) = \sum_0^{\infty} a_n z^n$ into $\mathcal{D}^k f(z) = \sum_{n=k}^{\infty} (e_{n-k}/e_n) a_n z^{n-k}$. We define the R-type of the function f to be the number $\tau_R(f) = \limsup_{n\to\infty} |a_n R_1 \cdots R_n|^{1/n}$. Let $r_n(f)$ denote the minimum distance from 0 to a zero if $\mathcal{D}^n f$. Theorem. There is an absolute constant $W(\mathcal{D})$ such that if f is an analytic function of R-type 1 or less, and f is not a polynomial, then $\limsup_{n\to\infty}(d_n r_n(f)/R_n) \geqq W(\mathcal{D})$. Moreover, there is an analytic function F with R-type $\tau_R(F) = 1$ such that equality holds in the previous inequality. (Received July 6, 1971.)

31 Potential Theory

687-31-1. STUART P. LLOYD, Bell Telephone Laboratories, Inc., Murray Hill, New Jersey 07974. Extension of axiomatic potential theory to the parabolic case. Preliminary report.

Let $\mathcal{H}$ be a real function sheaf containing local constants on connected locally compact R, and for open $U \subset R$ with $\bar{U}$ compact put $H(\bar{U}) = \{f \in C(\bar{U}) : f|U \in H(U) \in \mathcal{H}\}$. The heat equation suggests the Definition. U is regular provided $H(\bar{U})$ separates $\bar{U}$ and the Choquet boundary δU of $H(\bar{U}) \subset C(\bar{U})$ is a closed subset of $\bar{U} - U$ (so that the restriction mapping $H(\bar{U})\,|\,\delta U$ is a nonnegative 1-preserving isomorphism onto $C(\delta U)$). The second axiom of Brelot (regular U are a base) is retained, with the modified definition of regularity. A one sided Harnack inequality replaces the third axiom. (Received June 21, 1971.)

32 Several Complex Variables and Analytic Spaces

*687-32-1. ARUN KUMAR AGARWAL, Grambling College, Grambling, Louisiana 71245. On the maximum modulus and the mean value of an entire function of two complex variables.

Let $f(z_1, z_2) = \sum_{k_1 k_2=0}^{\infty} a_{k_1 k_2} z_1^{k_1} z_2^{k_2}$ be a transcendental entire function of two complex variables z_1 and z_2. In this paper the well-known results due to Brinkmier and Valiron have been extended to the case of the function of two complex variables. The results obtained are the inequalities connecting the maximum modulus, mean value, maximum term and the order of the function $f(z_1, z_2)$. (Received June 25, 1971.)

687-32-2. EDGAR LEE STOUT, University of Washington, Seattle, Washington 98105. The multiplicative Cousin problem with bounded data. Preliminary report.

We understand by B_N the unit ball in $\mathbb{C}^N$, i.e., the set $\{(z_1,\ldots,z_N) \in \mathbb{C}^N : |z_1|^2+\ldots+|z_N|^2 < 1\}$. Theorem. The multiplicative Cousin problem with bounded data is solvable on B_N. Explicitly, if $\mathcal{V} = \{V_\alpha\}_{\alpha \in I}$ is an open cover of $\bar{B}_N$, if for each α, f_α is a function holomorphic and bounded on $B_N \cap V_\alpha$, and if for all $\alpha, \beta \in I$, f_α/f_β is bounded on $B_N \cap V_\alpha \cap V_\beta$, then there is a function F bounded and holomorphic on B_N such that, for each α, F/f_α is bounded and bounded away from zero on $B_N \cap V_\alpha$. The theorem holds for a somewhat more general class of smoothly bounded, convex domains. (Received July 6, 1971.)

687-32-3. HUGH M. COLLINS, Tulane University, New Orleans, Louisiana 70118. Coherence of ${}_\Lambda\theta$ and $\mathcal{J}_V$ when Λ infinite. Preliminary report.

Let Λ be infinite. Let D be an open subset of C^Λ. (Let ${}_\Lambda\theta$ be the sheaf of germs of holomorphic functions on D.) Let V be a subvariety of D and $\mathcal{J}_V$ be the ideal sheaf of V. Theorem. ${}_\Lambda\theta$ is a coherent sheaf of ${}_\Lambda\theta$-modules. Theorem. $\mathcal{J}_V$ is a coherent sheaf of ${}_\Lambda\theta$-modules. Theorem. ${}_\Lambda\theta/\mathcal{J}_V$ is a coherent sheaf of ${}_\Lambda\theta$-modules. Theorem. Let Λ be countable. Let $D_1, D_2, \ldots$ be disks in C and assume all but a finite number of the D_i are relatively compact in C. Let $D = D_1 \times D_2 \times \ldots$. Then $H^p(D, {}_\Lambda\theta) = 0$, $\forall\, p \geqq 1$. (Received July 6, 1971.)

34 Ordinary Differential Equations

*687-34-1. G. Di ANTONIO, Pennsylvania State University, Middletown, Pennsylvania 17057. Closed orbit solutions to $x'' + g(x)(x')^2 + f(x) = 0$.

Theorem. Let f(x) be continuous, and g(x) Riemann integrable on an interval [a,b]. Then the differential equation has a closed orbit solution on the interval if f(x) changes sign twice on (a,b). The alternate plane equation is given by $c(x) = 2\int f(x) \exp[2\int g(x)dx]\,dx$. Hence there results $c'(x) = 2f(x)\exp[2\int g(x)dx]$. Since this is continuous the result follows from the theorem stated in (Abstract 673-17, these *Notices* 17(1970), 396). Example. The perihelion shift equation $x'' = c - x + dx^2$ has a closed orbit solution if $cd < 0$, for in this case the parabola $y = c - x + dx^2$ changes sign twice on the x-axis. (Received November 16, 1970.)

*687-34-2. THOMAS A. BRONIKOWSKI, Marquette University, Milwaukee, Wisconsin 53233 and JAMES E. HALL and JOHN A. NOHEL, University of Wisconsin, Madison, Wisconsin 53706. Quantitative estimates for a nonlinear system of integrodifferential equations.

Consider the real nonlinear system $u'(t) = -\int_0^c \alpha(x)T(x,t)dx$, $T_t(x,t) = (b(x)T_x(x,t))_x - q(x)T(x,t) + \eta(x)\sigma(u(t))$, for $0 < x < c$, $0 < t < \infty$, with initial and boundary conditions: $u(0) = u_0$, $T(x,0) = f(x)$, $d_1T(0,t) + d_2T_x(0,t) = 0$, $d_3T(c,t) + d_4T_x(c,t) = 0$. The authors are concerned with the asymptotic behavior of the solution u(t), T(x,t) as $t \to \infty$. Precise quantitative estimates are obtained in a variety of cases. If zero is not an eigenvalue for the associated Sturm-Liouville problem: $(b(x)y')' + (\lambda - q(x))y = 0$, $d_1y(0) = d_2y'(0) = 0$, $d_3y(c) + d_4y'(c) = 0$, then $u(t)$, $T(x,t) = O(e^{-\omega t})$ for some $\omega > 0$. If, however, zero is an eigenvalue the function $T(x,t)$ may approach a nonzero limit exponentially or may be unstable. The technique used combines the Galerkin approximation with suitable Liapunov functions for related systems of ordinary differential equations. Extensions to more general problems are also considered. (Received May 21, 1971.)

*687-34-3. CRAIG COMSTOCK, Naval Postgraduate School, Monterey, California 93940. The Poincaré-Lighthill perturbation technique and its generalizations.

The known generalizations of the Poincaré-Lighthill perturbation technique of strained coordinates are investigated and compared. Some new limitations are shown and some new conditions for the method's applicability are conjectured. (Received June 28, 1971.)

687-34-4. ABOLGHASSEM GHAFFARI, NASA Goddard Space Flight Center, Greenbelt, Maryland 20771. Criteria for the solvability of a nonlinear second order differential equation.

H. Knothe ("Satellites and Riemannian geometry," Celestial Mech. 1(1969/70), 36-45) obtained the differential equation (1) $y'' = (1+y'^2)(\psi_y - y'\psi_x)$, where the prime denotes derivative with respect to x. The integration of (1) can be carried out either by quadratures or reduced to a first order differential equation according to the form and structure of the function $\psi(x,y)$. Theorem 1. Let the expression $(\psi_y dx - \psi_x dy)$ be a perfect differential of a function $U(x,y)$, i.e., (2) $U_y = -\psi_x$, $U_x = +\psi_y$, and suppose $\psi \in C^2$, $U \in C^2$ in R^2. Then the solutions of (1) can be given in the form (3) $U(x,y) + C_2 = \omega$, where $U(x,y) = \int_{x_0}^{n} \psi_y(x,y)dx - \int_{y_0}^{y} \psi_x(x_0,y)dy + C_1$, and the angle ω has been defined by Knothe (ref. cit.) such that the tangent vector of the trajectory $x(t)$ and $y(t)$ has the components $\cos\omega$ and $\sin\omega$ respectively, and C_1, C_2 are integration constants. It can be deduced from the Cauchy-Riemann conditions (2) that the expression $(\nabla z)^2$ is an invariant on either surfaces $z = U(x,y)$, $z = \psi(x,y)$. Theorem 2. If $\psi(x,y)$ is an homogeneous function of degree zero, then equation (1) will be reduced to the first order differential equation in u and t: $t(dt/du + 1) = f'(u)[1 + u(t+u)][1 + (t+u)^2]$, where $y = xu$ and $xu' = t$. The details will appear in Celestial Mechanics, Vol. 4, No. 2. (Received June 29, 1971.)

687-34-5. GARY D. JONES, Murray State University, Murray, Kentucky 42071. Oscillation properties of third order differential equations.

Oscillation properties of elements of possible bases for the solution space of the differential equation (1) $y''' + p(x)y' + q(x)y = 0$ are studied. Theorem 1. If (1) is Class I, and if some solution oscillates, then the solution space of (1) has bases with 2 or 3 oscillatory elements and every basis is of one of these types. Theorem 2. If (1) is Class II, and if some solution oscillates, then the solution space of (1) has bases with 0, 1, 2 oscillatory elements and every basis is of one of these types. (Received July 6, 1971.)

*687-34-6. GARRET J. ETGEN and C. D. SHIH, University of Houston, Houston, Texas 77004. Disconjugacy of third order differential equations with nonnegative coefficients.

In [J. Math. Anal. Appl. 24(1968), 212-224], J. H. Barrett studied the third order linear differential equation (1) $y''' + P(x)y' + Q(x)y = 0$, where $P(x)$ and $Q(x)$ are continuous, nonnegative functions on $[a, \infty)$ such that $P(x) + Q(x) \not\equiv 0$ on any subinterval of $[a, \infty)$. The objective of his paper is to establish conditions which will imply that (1) is not disconjugate on $[a, \infty)$. The purpose of our paper is to indicate two arithmetical mistakes in the arguments leading to Barrett's key result: Theorem. If (1) is disconjugate on $[a, \infty)$ and $u(x)$ is the solution of (1) satisfying $u(a) = u'(a) = 0$, $u''(a) = 1$, then $u''(x) > 0$ on $[a, \infty)$. We show that if the coefficients of (1) satisfy $Q(x) > 0$, $P(x)/Q(x)$ nondecreasing on $[a, \infty)$, in addition to the conditions stated above, then the Theorem holds. With the strengthened hypotheses on $P(x)$ and $Q(x)$, the remaining results of Barrett's paper are valid. (Received July 7, 1971.)

35 Partial Differential Equations

*687-35-1. RICHARD KRAFT, National Bureau of Standards, Washington, D. C. 20234. Riemann function type solutions of boundary initial problems.

Auxilliary functions (called boundary Riemann functions) are defined. These functions permit the solutions of boundary initial problems for linear hyperbolic systems in two independent variables to be expressed as quadratures of the boundary and initial data. The definitions result from a generalization of the idea that was used to define Riemann functions for pure initial problems in [R. Kraft, J. Math. Anal. Appl. 25(1969)]. (Received June 11, 1971.)

*687-35-2. THOMAS J. LANGAN, Hydrodynamics Branch 552, Naval Ship Research and Development Center, Carderock, Maryland 20034 and University of Maryland, College Park, Maryland 20742. On the existence of not necessarily unique solutions of semilinear hyperbolic systems in two independent variables.

Existence and uniqueness of solutions to the initial value problem for semilinear hyperbolic systems are proved under more general conditions than in the past. In this paper a solution means a solution to a system of integral equations along the characteristics, and it may or may not have continuous derivatives. Previously, the least restrictive regularity assumption for local existence of solutions of the Cauchy problem for hyperbolic systems assured at the same time uniqueness. A study of the convergence of sequences of functions, which satisfy approximating integral equations, leads to the concept of quasi-equicontinuous operators. It is shown that, if a hyperbolic system is associated with a quasi-equicontinuous operator, then there exists at least one solution to the initial value problem. Perron-type uniqueness conditions are shown to apply to these solutions; moreover, if the solution is unique, it is proven that the entire sequence of approximate solutions must converge to the solution. This latter result is used to obtain an estimate to the difference between the approximate solution and the solution. (Received June 18, 1971.)

40 Sequences, Series, Summability

*687-40-1. REINHART HITZ, Old Dominion University, Norfolk, Virginia 23517. On certain polynomials occuring in continued fractions.

The denominators B_n of the nth approximant of continued fractions of the form $1/b_1 + 1/b_2 + 1/b_3 + \ldots$ are functions of complex variables $b_1, b_2, b_3, \ldots$, represented as sums of certain homogeneous polynomials $P^n_{k,i}$ and $Q^n_{k,i}$; these polynomials are defined by recurrence formulas as sums of products of i distinct factors from the sequence $\{b_p\}_{p=1}^{\infty}$. For Theorems 1–3, suppose the series $\sum|b_{2p}|$ is convergent, the sequence of partial sums of the series $\sum b_{2p-1}$ is bounded, and n is a nonnegative integer. Then Theorem 1. (a) Each of $\{Q^n_{2k,2i-1}\}_{k=1}^{\infty}$ and $\{P^n_{2k,2i}\}_{k=1}^{\infty}$, $i = 1, 2, 3, \ldots$, is absolutely convergent; (b) each of $\{Q^n_{2k,2i}\}_{k=1}^{\infty}$ and $\{P^n_{2k-1,2i-1}\}_{k=1}^{\infty}$, $i = 1, 2, 3, \ldots$, is bounded. Theorem 2. If $\epsilon > 0$, there is a positive integer k such that if $r > k$, $|P^n_{2(r+s)+1,\,2i+1} - P^n_{2r+1,\,2i+1}| < \epsilon$ and $|Q^n_{2(r+s),\,2i} - Q^n_{2r,\,2i}| < \epsilon$, $i = 1, 2, 3, \ldots$; $s = 1, 2, 3, \ldots$. Theorem 3. (1) Each of $\{\sum_{q=1}^{k}\sum_{p=q}^{k} Q^n_{2p,\,2q-2}b_{2(n+p)}\}_{k=1}^{\infty}$ and $\{\sum_{q=1}^{k}\sum_{p=q}^{k} P^n_{2p-1,\,2q-1}b_{2(n+p)}\}_{k=1}^{\infty}$ is absolutely convergent; (2) each of $\{\sum_{q=1}^{k}\sum_{p=q}^{k} Q^n_{2p,\,2q-1}b_{2(n+p)+1}\}_{k=1}^{\infty}$ and $\{\sum_{q=1}^{k}\sum_{p=q}^{k} P^n_{2p-2,\,2q-2}b_{2(n+p)-1}\}_{k=1}^{\infty}$ is bounded; (3) if $\epsilon > 0$, there is a positive integer k such that if $r > k$, $s = 1, 2, 3, \ldots$, $|\sum_{q=2}^{s} P^n_{2(r+s)-1,\,2q-1} - \sum_{q=2}^{r} P^n_{2r-1,\,2q-1}| < \epsilon$ and $|\sum_{q=1}^{s} Q^n_{2(r+s),\,2q} - \sum_{q=1}^{r} Q^n_{2r,\,2q}| < \epsilon$. Theorem 4. Suppose $\sum|P_{2p-1,1}b_{2p}|$ is convergent. If n is a nonnegative integer, then the sequence $\{\sum_{q=1}^{k}\sum_{p=q}^{k} P^n_{2p-1,\,2q-1}b_{2(n+p)}\}_{k=1}^{\infty}$ is convergent. (Received June 28, 1971.)

41 Approximations and Expansions

687-41-1. BRUCE L. CHALMERS, University of California, Riverside, California 92502. A unified approach to uniform real approximation by polynomials with linear restrictions.

Problems concerning approximation of real-valued continuous functions of a real variable by polynomials of degree smaller than n with various linear restrictions have been studied by several authors. This paper is an attempt to provide a unified approach to these problems. In particular, the notion of restricted derivatives approximation is seen to fit into the theory and includes as special cases the notions of monotone approximation and restricted range approximation. Also bounded coefficients approximation, ϵ-interpolator approximation, and polynomial approximation with interpolation fit into our scheme. (Received May 11, 1971.)

*687-41-2. A. KLIMAS and GUIDO SANDRI, Aeronautical Research Associates of Princeton, Inc., 50 Washington Road, Princeton, New Jersey 08540. Comparison of PLK and time scale uniformizations.

A class of differential equations for three-dimensional (real) vectors with a small parameter is studied. The equations occur in a wide variety of physical situations. The direct perturbation expansion in the small parameter is nonuniform in the independent variable (time). We show that with linear time scales (A. Klimas, R. V. Ramnath and G. Sandri, J. Math. Anal. Appl. 32(1970), 482) we can uniformize the perturbation expansion while the Poincaré-Lighthill-Kuo technique fails except for very special cases. The class of equations is of relevance to recent work by P. Usher (Quart. Appl. Math. 28(1971), 463). (Received June 28, 1971.)

*687-41-3. MARTIN W. BARTELT and HARRY W. McLAUGHLIN, Rensselaer Polytechnic Institute, Troy, New York 12181. Strong unicity in approximation theory.

M denotes a subspace of the real normed linear space W, $f \in W$, $\|f\| = 1 = \inf_{m \in M} \|f-m\|$. The element $0 \in M$ is a strongly unique best approximate to f if there exists $r > 0$ such that $\|f-m\| \geqq 1 + r\|m\|$, $\forall m \in M$. The linear span of M and f is denoted by $\langle M,f\rangle$ and its dual by $\langle M,f\rangle^*$. $\mathcal{L} = \{L \in \langle M,f\rangle^* : Lf = \|L\| = 1\}$ and $K = \{z \in \langle M,f\rangle : Lz \leqq 1, \forall L \in \mathcal{L}\}$. The element $L_0 \in \mathcal{L}$ is defined by $L_0(m+af) = a$, $\forall m \in M$ and for all real a. Theorem 1 (D. Wulbert, Bull. Amer. Math. Soc. 77(1971), 88-91). The real number r satisfies $\|f-m\| \geqq 1 + r\|m\|$, $\forall m \in M$, if and only if $\sup_{L \in \mathcal{L}} L(m) \geqq r\|m\|$, $\forall m \in M$. Theorem 2. That 0 is a strongly unique best approximate to f is equivalent to both (1) $K \cap M$ is bounded and (2) (when M is finite dimensional) $\{z \in \langle M,f\rangle : L_0 z = 1\} \cap K = \{f\}$. Let $W = C(X)$ (continuous real functions on a compact Hausdorff space) and M = finite dimensional subspace with basis $m_1, \ldots, m_n$. Theorem 3. That 0 is a strongly unique best approximate to f is equivalent to both (1) $\max_{x \in A} f(x)\, m(x) > 0$, $\forall m\ (\neq 0) \in M$, where $A = \{x \in X : |f(x)| = \|f(x)\|\}$, and (2) 0 lies in the interior of the convex hull of $\{(f(x)m_1(x), \ldots, f(x)m_n(x)) : x \in A\}$. The corresponding results for a complex normed linear space are obtained. (Received July 6, 1971.)

*687-41-4. ANNETTE SINCLAIR, Purdue University, Lafayette, Indiana 47907. Determination of extremal functions in H^p by a Fortran program.

A method for determining extremal functions presented by the author at the Maryland Conference on Approximation Theory in October, 1970, has been programmed in Fortran and sixty extremal functions determined. Given z_j and w_j, $|w_j| < 1$, $|z_j| < 1$, $j = 1, \ldots, m$, the object was to determine parameters for writing f^* such that $\|f^*\|_p \leqq \|f\|_p$, $f \in H^p$, and $f(z_j) = w_j$, $j = 1, \ldots, m$, where $\|f\|_p = \int_{|z|=1} |f(z)|^p |dz|$. It is known [Rogosinski, Shapiro, Macintyre] that $f^*(z)$ is of the form $C[\Pi_{i=1}^{K}(z-c_i)/(1-\bar{c}_i z)] \cdot [\Pi_{i=1}^{m-1}(1-\bar{c}_i z)]^{2/p}[\Pi_{j=1}^{m}(1-\bar{z}_j z)]^{-2/p}$, $|c_i| \leqq 1$, $K \leqq m-1$, and that a function of this form satisfying the given interpolation conditions is just $f^*(z)$. For $p = 2$, $f^*(z) = p(z)[d(z)]^{-2/p}$, where $P(z)$ is the polynomial of degree $m-1$ such that $P(z_j) = w_j[d(z_j)]^{2/p}$, and $d(z) = \Pi_{j=1}^{m}(1-\bar{z}_j z)$. For arbitrary p and for $f_1(z) = P_{m-1}(z)[\Pi_{i=1}^{m-1}(1-\bar{c}_i z)]^{(2/p)-1}/[d(z)]^{2/p}$, $|c_i| \leqq 1$, then $f_1(z)$ can be written in the form of $f^*(z)$, but $f_1(z_j) = w_j[\Pi_{i=1}^{m-1}(1-\bar{c}_i z_j)]^{(2/p)-1}$. In the Fortran program the $w_j/[(1-\bar{c}_i z_j)]^{2/p-1}$ are next substituted for the original w_j. The procedure is repeated until $|[f(z_j)-w_j]/w_j| < .01e$, where "e" is a preassigned percentage error allowed at the z_j. The method has worked for all data and p-values tested, where p has ranged from 9/8 to 41. Any $p > 1$, and data (z_j, w_j), $j = 1, \ldots, m$, with $m \leqq 49$, can be read into the program. (Received July 7, 1971.)

43 Abstract Harmonic Analysis

687-43-1. PETER R. MUELLER-ROEMER, East Carolina University, Greenville, North Carolina 27834. A noncommutative, noncompact group with tauberian group algebra.

Let G be the multiplicative group of real matrices $\binom{a\ b}{0\ 1}$ with positive a, and let H denote the closed normal subgroup of all matrices $\binom{1\ b}{0\ 1}$. For f in $L^1(G)$ define $T_H f$ in $L^1(G/H)$ by $(T_H f)(xH) = \int_H f(xh)dh$. It is known that T_H is surjective and that it maps closed ideals into closed ideals, since H is amenable. Using methods of generalized L^1-algebras as developed by Horst Leptin [Invent. Math. 5(1968), 192-215] the author proves the Theorem. T_H maps proper closed ideals of $L^1(G)$ to proper closed ideals in $L^1(G/H)$. Corollary. The structure spaces of maximal modular two-sided ideals of the two algebras are topologically isomorphic. $L^1(G)$ is thus a completely regular tauberian algebra in the sense of Charles Rickart ["Banach algebras," van Nostrand, Princeton, N.J., 1970]. Remark. G operates on $H \cong (\mathbb{R},+)$ by real multiplication; the proof of the theorem depends essentially on a property of real multiplication for which there is no analogue in general locally compact groups. (Received July 6, 1971.)

44 Integral Transforms, Operational Calculus

*687-44-1. KUSUM K. SONI, University of Tennessee, Knoxville, Tennessee 37916. Riesz means and Hankel transform.

Recently Nasim has given some results relating the behavior of the Riesz means of the Hankel transform of a function in $L(0,\infty)$ with the behavior of the function near zero [Canad. J. Math. 21(1969)]. Let F(x) be the transform of a function f(x) with respect to the kernel k(x), $0 < x < \infty$. We prove that if $f(x) \sim x^{-\alpha}$, $x \to 0$, then under certain conditions, $F(x) \sim M(k, 1-\alpha)x^{\alpha-1}$, $x \to \infty$, where M(k,s) is the Mellin transform of k(x). If k*(x) is some regular mean of k(x) and F*(x), the corresponding mean of F(x), then under similar conditions, $F^*(x) \sim M(k^*, 1-\alpha)x^{\alpha-1}$. Nasim's results are special cases of these. (Received July 6, 1971.)

46 Functional Analysis

687-46-1. DANIEL JOHN RANDTKE, University of Georgia, Athens, Georgia 30601. A representation theorem for precompact maps. Preliminary report.

A linear map T from one locally convex space E into another F is PRECOMPACT if T maps a neighborhood of 0 in E into a precompact subset of F. A general "representation" theorem for precompact linear maps from one locally convex space into another has been proved. An application of this theorem implies that every precompact linear map T from a locally convex space E into a normed space has a representation of the form $Tx = \sum \lambda_n < x, a_n > y_n$ where λ is a 0-convergent sequence of scalars, $\{a_n\}$ is an equicontinuous sequence in the topological dual E' of E, and $\{y_n\}$ is an unconditionally summable sequence in a suitable Banach space. (Received May 10, 1971.)

687-46-2. ROBERT C. SHARPLEY, University of Texas, Austin, Texas 78712. Interpolation theorems for operators of weak type.

We give a general form of a theorem of Calderón and its application to mappings of Lorentz spaces. Let X_i, Y_i, X, Y $(i=1,2)$ be rearrangement invariant Banach function spaces on $[0,1]$. We call an operator from X into the measurable functions of weak type (X,Y) if $(Tf)^*(t)\varphi_Y(t) \leqq C \cdot \int_0^1 f^*(t)d\varphi_X(t)$, $f \in X$, $0 < t \leqq 1$, where $\varphi_X(t) = \|\chi_{(0,t)}\|_X$ and g^* denotes the decreasing rearrangement of $|g|$. We call (X,Y) a weak intermediate pair for the pairs (X_i, Y_i), $i = 1, 2$, if each operator of weak types (X_i, Y_i) is a bounded operator from X to Y. Let $\Lambda(X)$ and $M(X)$ be the Lorentz Λ and M spaces corresponding to $\varphi_X(t)$ and $t/\varphi_X(t)$, respectively. Let $\Psi(s,t) = \min_{i=1,2} \{\varphi_{X_i}(s)/\varphi_{Y_i}(t)\}$ and $F(s,t) = \Psi(s,t) \cdot \varphi_Y(t)/\varphi_X(s)$. Define $S(\sigma) f(t) = \int_0^1 f(s)d\Psi(s,t)$. Assume either $\varphi_{X_1}(0+)$ or $\varphi_{X_2}(0+)$ is zero. Theorem 1. The pair (X,Y) is weak intermediate for the pairs (X_i, Y_i) iff for each measurable g and each $f \in X$ the relation $g^*(t) \leqq S(\sigma) f^*(t)$, a.e. implies $g \in Y$. Theorem 2. The pair $(\Lambda(X), \Lambda(Y))$ is weak intermediate for the pairs (X_i, Y_i) iff $\int_0^1 F(s,t)/\varphi_Y(t)d\varphi_Y(t) \leqq C$, $0 < s \leqq 1$. Theorem 3. If the index $\bar{\gamma}_X < 1$, then $(M(X), M(Y))$ is a weak intermediate pair for the pairs (X_i, Y_i) iff (i) $\int_0^1 F(s,t)/\varphi_X(s)d\varphi_X(s) \leqq C_1$, $0 < t \leqq 1$, and (ii) $F(s,t) \leqq C_2$, $0 < s, t \leqq 1$. We hope to apply this to other interpolation theorems. (Received June 16, 1971.)

687-46-3. GARRETT O. VAN METER II, University of Maryland, College Park, Maryland 20742. The 2n-dimensional disc algebra. Preliminary report.

Let $C(T^n)$ be the collection of all continuous complex-valued functions on T^n, the n-dimensional torus in $\mathbb{C}^n$. Also the 2n-dimensional disc algebra, $A(T^n)$, is the sup-norm closure in $C(T^n)$ of the polynomials in the complex variables $z_1, \ldots, z_n$. Theorem. Let $(B_k)_{k \geqq 1}$ be a sequence of Banach subalgebras of $C(T^n)$ with the property that $A(T^n) \subseteq B_k \subseteq B_{k+1}$ for $k \geqq 1$. Then $\bigcup_{k=1}^{\infty} B_k$ is dense in $C(T^n)$ iff there is a k_0 for which $B_{k_0} = C(T^n)$. Theorem. Assume that $f_1, \ldots, f_k$ are in $C(T^n)$ and the algebra generated by $A(T^n)$ and $f_1, \ldots, f_k$ is dense in $C(T^n)$. Then there is a number $\epsilon > 0$ such that for each collection of functions $h_1, \ldots, h_k$ in $C(T^n)$ with $\|h_j - f_j\| < \epsilon$ for $j = 1, \ldots, k$, the algebra generated by $A(T^n)$ and $h_1, \ldots, h_k$ is dense in $C(T^n)$. (Received June 18, 1971.)

687-46-4. CHARLES G. DENLINGER, Millersville State College, Millersville, Pennsylvania 17551. Order summable families in vector lattices. Preliminary report.

Let E be a vector lattice. A family of elements x_i of E, indexed by a set I, is denoted $[x_i, I]$. Such a family is order summable if the net of sums $\sum_{i \in J} x_i$ (taken over finite $J \subset I$) order converges in E. Under coordinatewise operations, the space of all $[x_i, I]$ for which the sums $\sum_{i \in J} |x_i|$ are order bounded in E forms a vector lattice $\ell_I^1(E)$. Theorem 1. If E is Archimedean, every order summable family belongs to $\ell_I^1(E)$; if E is Dedekind complete, every family in $\ell_I^1(E)$ is order summable. Suppose E is Archimedean, and $\hat{E}$ denotes its Dedekind completion. Then each $[x_i, I]$ in $\ell_I^1(E)$ has a "sum" $\hat{x}$ in $\hat{E}$. Theorem 2. If the net $[x_i^{(\alpha)}, I]$ order converges in $\ell_I^1(E)$ to $[y_i, I]$, then the sums $\hat{x}^{(\alpha)}$ order converge in $\hat{E}$ to $\hat{y}$. Relative to a normed vector lattice $(E, \leq, \|\cdot\|)$, an ℓ-norm on $\ell_I^1(E)$ is any monotone norm $\|\cdot\|_\ell$ such that (a) for finite families $[x_i, I]$, $\|[x_i, I]\|_\ell = \|\Sigma_{i\in I} |x_i|\|$, and (b) if $\Sigma_{i\in J}|x_i| \leq y$ for all J, then $\|[x_i, I]\|_\ell \leq \|y\|$. In Theorem 3 minimum and maximum ℓ-norms $\|\cdot\|_1$ and $\|\cdot\|_1$ are obtained. Theorem 4. $\|\cdot\|$ is additive on $E^+ \Leftrightarrow \|\cdot\|_1$ is additive on $\ell_I^1(E)^+$. Theorem 5. If $\|\cdot\|$ is semicontinuous on E, then $\|\cdot\|_1$ is the only semicontinuous ℓ-norm on $\ell_I^1(E)$. Theorem 6. If $\|\cdot\|$ is continuous on E, then $\|\cdot\|$ is the only ℓ-norm on $\ell_I^1(E)$. Further related results are obtained. (Received June 28, 1971.)

687-46-5. PHILIP LAIRD, University of Calgary, Calgary, 44, Alberta, Canada. Some properties of continuous mean periodic functions. Preliminary report.

For the mean periodic continuous complex-valued functions of a real variable introduced by L. Schwartz ("Théorie générale des fonctions moyenne périodiques," Ann. of Math. (2) 48(1947), 857-929); the truncated convolution product of two such functions is mean periodic whereas their pointwise product need not be mean periodic. Conditions are given for when ordinary linear differential equations with periodic coefficients and linear differential-difference equations admit mean periodic solutions. (Received July 6, 1971.)

687-46-6. GEORGE W. CROFTS, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. Some properties of a special class of perfect Banach sequence spaces. Preliminary report.

A sequence space λ is said to be a step if λ is a perfect sequence space with $\ell^1 \subset \lambda \subset \ell^\infty$ and λ is a Banach space in its strong topology from λ^x. If ν is a sequence space, let ν^λ denote the diagonal maps of ν into λ. In Ruckle's terminology ν is λ-perfect if $\nu^{\lambda\lambda} = \nu$. For $1 \leq p < \infty$, let $\mu = \lambda^{1/p} = \{x|x$ is a complex sequence with $x^p = (x_i^p) \in \lambda\}$. Under the norm $\|x\| = \|x^p\|_\lambda^{1/p}$, μ is a step and μ is λ-perfect. From results of Dubinsky and Ramanujan this implies that each absolutely λ-summing operator is absolutely μ-summing. If λ is reflexive, then each diagonal map of μ into λ is compact and by a result of Holub the space of diagonal nuclear maps of λ into μ is $(\mu\cdot\lambda^x)^{xx}$. (Received July 6, 1971.)

*687-46-7. GEORGE R. GIELLIS, U. S. Naval Academy, Annapolis, Maryland 21403. A characterization of Hilbert modules.

A Hilbert module is a right module H over a proper H*-algebra A, with a vector inner product [,] mapping $H \times H$ into the trace class $\tau(A)$, where [,] has certain properties analogous to a scalar valued inner product on a Hilbert space. P. P. Saworotnow [Duke Math. J. 35(1968), 191-197] showed that a linear structure can be defined on H so that H becomes a Hilbert space. Theorem. Let K be a right module over A which is a Hilbert space with norm $\|f\| = (f,f)^{1/2}$. Then K is a Hilbert module over A iff $f(\lambda a) = (\lambda f)a = \lambda(fa)$, $f(ga) = (fa^*, g)$ and $fA = 0$ implies $f = 0$, for all complex λ, $a \in A$ and $f, g \in K$. (Received July 6, 1971.)

*687-46-8. HERBERT HALPERN, University of Cincinnati, Cincinnati, Ohio 45221. Unitary implementation of groups of automorphisms of a von Neumann algebra.

Let A be a semifinite von Neumann algebra in standard form on the Hilbert space H and let $g \to \varphi(g)$ be a homomorphism of a topological group into the group of *-automorphisms of A which leave the center of A invariant. Then there is a homomorphism $g \to u(g)$ of G into the unitary group of H such that $\varphi(g)(x) = u(g) \times u(g^{-1})$ for every x in A and g in G. If $g \to (\varphi(g)(x)\xi, \zeta)$ is continuous for every ζ, ξ in H, then $g \to u(g)$ is a strongly continuous unitary representation of G. This contains a result of R. Kallman [Trans. Amer. Math. Soc. 156(1971)] for G and H separable. The same line of reasoning yields results concerning unitary implementation of groups of center-fixing automorphisms which commute with the modular automorphisms attached to a normal state of any von Neumann algebra. This differs from the usual procedure in which the automorphisms are assumed to leave the state invariant. (Received July 6, 1971.)

687-46-9. CHARLES J. A. HALBERG, JR., University of California, Riverside, California 92502. On the spectra of mixed operators. Preliminary report.

Let A and B be bounded linear operators on the sequence space $\ell_p, 1 \leqq p \leqq \infty$, with infinite matrix representations (a_{ij}) and (b_{ij}) respectively. Let S and U be the operators defined by the infinite matrices (s_{ij}) and (u_{ij}) respectively, where s_{ij} is 1 if i is odd and $j = (i+1)/2$ and s_{ij} is zero otherwise, and where $u_{ij} = u_{ji} = 1$ if i is odd and $|i-j| = 1$ and $u_{ij} = 0$ otherwise. Let T be the shift operator defined by the matrix (t_{ij}) where $t_{ij} = 1$ if $i - j = 1$ and $t_{ij} = 0$ otherwise. If P is an operator defined by the matrix (p_{ij}), let P^t be the operator defined by the transpose of the matrix (p_{ij}). We consider the mixing operators $M(A,B)$ and $M^+(A,B)$ where $M(A,B) = SAS^t + TSB(TS)^t$ and $M^+(A,B) = M(A,B)U$. In a paper to appear in Math. Scand. the author and Åke Samuelsson determined the fine spectral structure of $M(A,B)$ in terms of the spectra of A and B. Here we determine the spectra of $M^+(A,B)$ and various relationships among $M(A,B)$, $M^+(A,B)$, A and B and their spectra. Some of this theory is applied to determine the spectral structure of certain special operators. (Received July 7, 1971.)

47 Operator Theory

687-47-1. MICHAEL B. DOLLINGER, Louisiana State University, Baton Rouge, Louisiana 70803 and KIRTI K. OBERAI, Queen's University, Kingston, Ontario, Canada. Maximal functional calculi.

Let T be a bounded linear operator on a complex Banach space X and let $\sigma(T)$ be the spectrum of T. Let $H(\sigma(T))$ be the set of germs of analytic functions on $\sigma(T)$ equipped with the usual inductive limit topology. (In $H(\sigma(T))$ we identify the functions which agree on a neighbourhood of $\sigma(T)$.) Let $\psi_0 : H(\sigma(T)) \to L(X)$ be defined by $\psi_0(f) = f(T)$. Definition. A functional calculus for T is a pair (A, ψ) such that A is a locally convex algebra of equivalence classes of complex valued functions with a continuous algebra homomorphism $\eta : H(\sigma(T)) \to A$ and ψ is a continuous algebra isomorphism of A into L(X) such that $\psi \circ \eta = \psi_0$. We define a partial ordering on the set of functional calculi for T by $(A, \psi) \leqq (B, \varphi)$ if there is a continuous algebra isomorphism $\mu : A \to B$ such that

$\varphi \circ \mu = \psi$. The main result of the paper is to prove the existence of maximal elements in this partial ordering. Examples show that the maximal element need not be unique. Of course ψ_0 is the minimal element in this partial ordering. (Received June 28, 1971.)

50 Geometry

687-50-1. J. M. THOMAS, 60 Slocum Street, Philadelphia, Pennsylvania 19119. The four color theorem.

If map M_{n+1} arises from M_n by bonding unequal faces F, H across G by side xy, erasure of xy converts a root of map system X_{n+1} into a root of $X_n - F - G - H + (F^2 - 1) + (H^2 - 1)$. Bonding F to itself across G by a side marked +- converts a root of X_n into a root of X_{n+1}. Let R be the set of faces joining G. System S_n is had from $X_n - G$ by replacing each equation F, where face F is in R, by $F^2 - f$ with mark f an indeterminate to be 0 or 1. The consistency condition of S_n is a polynomial t_n whose use gives for the theorem a new, simple, direct proof depending on only one reduction, erasure. (Received July 6, 1971.)

52 Convex Sets and Geometric Inequalities

*687-52-1. MURRAY S. KLAMKIN, Ford Motor Company, P. O. Box 2053, Dearborn, Michigan 48121. Duality in triangle inequalities.

If for $\triangle ABC$, we let $x = s - a$, $y = s - b$, $z = s - c$, then $x, y, z \geqq 0$. Conversely, for any $x, y, z \geqq 0$, there exists a triangle with sides $a = y + z$, $b = z + x$, $c = x + y$. This gives a duality between all triangle inequalities and all inequalities for three nonnegative numbers. We list, side by side, a large number of dual inequalities in the above variables as well as R, r, s; A, B, C and the elementary symmetric functions T_1, T_2, T_3 of x, y, z. Each representation has its pros and cons. To simplify the proofs via T_1, T_2, T_3, we list a number of best symmetric homogeneous inequalities of degrees 2 - 6 (there still being some open ones of degrees 5, 6), e.g., $T_1^3 + 9T_3 \geqq 4T_1T_3$ is the best cubic containing T_1^3. One advantage of the latter variables is that it is very easy to write down valid inequalities, e.g., $\sum(x^3 - y^3)(x - y) \geqq 0$, $\sum x(x - y)(x - z) \geqq 0$. Also, it is very easy to make transformations preserving inequality, e.g., if $x' = 1/x$, etc., then $T_1' = T_2/T_3$, $T_2' = T_1/T_3$, $T_3' = 1/T_3$. The latter is self-dual and transforms best inequalities into best inequalities. The duality also applies to spherical triangles enabling us to give an elementary proof of the isoperimetric theorem and that $\tan R \geqq 2 \tan r$. Finally, we give some representation results as sums of squares. (Received July 6, 1971.)

*687-54-1. ROBERT W. HEATH and DAVID J. LUTZER, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. A note on monotone normality.

P. Zenor defined that a space X is monotonically normal if to each pair (A, B) of disjoint closed subsets of X one can assign an open set $G(A, B)$ in such a way that $A \subset G(A, B) \subset G(A, B)^- \subset X \backslash B$ and if (A', B') is another pair of disjoint closed sets having $A \subset A'$ and $B \supset B'$, then $G(A, B) \subset G(A', B')$. In this paper we establish three principle results : (i) every linearly ordered topological space is (completely) monotonically normal ; (ii) every F_σ-subspace of a monotonically normal space is monotonically normal ; (iii) a monotonically normal p-space with a G_δ-diagonal is metrizable. The first two results are used to construct examples which illustrate the relation between monotonic normality and certain more familiar topological properties--for example, there is a compact Hausdorff space which is not monotonically normal. (Received May 3, 1971.)

*687-54-2. ERIC J. BRAUDE, Seton Hall University, South Orange, New Jersey 07079. The projection of a compact zeroset is a zeroset. Preliminary report.

Example. There exist normal Hausdorff spaces P and Q, as well as a continuous, open and closed map of P onto Q which does not map every compact zeroset of P onto a zeroset of Q. Theorem. Let X and Y be topological spaces and let π be the projection map of $X \times Y$ onto X. If K is a compact zeroset of $X \times Y$, then $\pi(K)$ is a zeroset of X. (Received May 3, 1971.)

*687-54-3. EDYTHE P. WOODRUFF, State University of New York, Binghamton, New York 13901. Disks in E^3/G.

Suppose G is a point-like decomposition of E^3 and a distance $\epsilon > 0$ is given. If D is a disk in E^3/G, does there exist a disk D' in E^3 such that $P[D']$ is a disk that is ϵ-homeomorphic to D? The analogous 2-sphere question was posed by Steve Armentrout in [Ann. of Math. Studies, No. 60, Princeton Univ. Press, Princeton, N. J., 1966, pp. 1-25]. Counterexamples for both the 2-sphere and disk problems exist in the (2,1) toroidal decomposition of E^3. (For this terminology, see Sher in [Fund. Math. 61(1967/68), 225-241].) The disk problem with the added hypothesis that the given disk be the image of a 2-complex in E^3 also is answered in the negative by a counterexample. For this counterexample, a construction called a knit Cantor set of nondegenerate elements is defined. The original disk question has an affirmative answer (1) if the number of nondegenerate elements is countable and E^3/G is homeomorphic to E^3, or (2) if the decomposition is definable by 3-cells. (Received June 3, 1971.)

*687-54-4. ROBERT A. McCOY, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. A Baire space extension.

A Baire space is a space such that every nonempty open subset is of second category in the space. Theorem 1. X is a Baire space iff every countable point finite open cover of X is locally finite at a dense set of points. Let F denote the set of all free open ultrafilters on X. If X is not a Baire space, then $F \neq \emptyset$. Let X_F be the disjoint union of X and F. For each open U in X, let $U^* = U \cup \{\mathfrak{F} \in F \mid U \in \mathfrak{F}\}$. Let X_F have the topology generated by the base $\{U^* \mid U \text{ is open in } X\}$, so that X is a dense subspace of X_F. Theorem 2. X_F is a Baire space. Corollary. Every space is a dense subspace of some compact Baire space. Theorem 3. If f is a continuous function from X into a T_3-space Y such that $\overline{f(X)} = Y$, then there exists a subspace Z of X_F containing X and a continuous function g from Z onto Y such that $g|_X = f$ and $g|_{Z/X}$ is a homeomorphism from Z/X onto $Y \backslash f(X)$. Furthermore, if f is a homeomorphism, then g is a homeomorphism. Corollary. If Y is Hausdorff, then it is a compactification of X iff it can be embedded as a compact subspace of X_F containing X. (Received June 21, 1971.)

*687-54-5. EUGENE M. NORRIS, West Virginia University, Morgantown, West Virginia 26506. The Hausdorffication functor.

According to a theorem of J. F. Kennison [Trans. Amer. Math. Soc. 118(1965), 303-315], the full subcategory of Hausdorff spaces is reflective in the category of all topological spaces and continuous functions. We give a simple construction of the Hausdorffication (i.e., reflection) of an arbitrary space. Proposition. The Hausdorffication of a space X is the pair (X/F, p), where F is the meet of all those equivalence relations E on X for which X/E is Hausdorff and $p: X \to X/F$ is the canonical quotient map. The result follows from a well-known induced function theorem (e.g., VI, 3.2 of Dugundji's "Topology"). (Received May 13, 1971.)

687-54-6. JOHN D. BAILDON, Pennsylvania State University, Dunmore, Pennsylvania 18512. Euler characteristics of 2-manifolds and light open maps. Preliminary report.

In his extensive study of light open maps between 2-manifolds, G. T. Whyburn showed they are finite-to-one and established a relationship between the degree (order) of the map, the Euler characteristics of the spaces involved, and the cardinalities of the image of the singular set and the inverse of this ("Analytic topology," Amer. Math. Soc. Colloq. Publ. No. 28, p. 202). In this paper, Whyburn's result is extended to a light open map from a space X which is the union of m oriented 2-manifolds without boundary, A_i, intersecting pairwise in a finite union of disjoint arcs. Theorem. Let f be a light open map from X onto an oriented 2-manifold without boundary. Then $k\chi(B) - \sum_{i=1}^{m} \chi(A_i) = kr - n - \sum_{x \in Q} d(x)$ where k is the degree of f; r is the cardinality of the set Y of all points in B whose inverses contain a point where f/A_i is not a local homeomorphism for some i; n is the cardinality of $f^{-1}(Y)$; Q is the set of points common to $f^{-1}(Y)$ and the intersecting arcs; and $d(x) = -1 + \#\{A_i : x \text{ is in } A_i\}$. Use is made of a theorem by Titus and Young (Trans. Amer. Math. Soc. 103(1962), 330) to show that f/A_i is open for each i. Whyburn's result is then applied to each of these and the terms are combined to produce the formula of the theorem. (Received June 28, 1971.)

*687-54-7. MARY E. POWDERLY, William Paterson College of New Jersey, Wayne, New Jersey 07470. Some theorems on topological expansions.

Theorem. Let τ be any infinite cardinal number. There exist topological spaces which are τ-maximal with respect to the properties of being a T_4 and a T_5. A special case of this theorem follows from a result of Katětov obtained many years ago, i.e., there exist topological spaces which are $\aleph_0$-maximal with respect to the property of being a T_4. This also strengthens a result of B. Pospisil, which asserts the existence of T_5 spaces with any dispersion character. The theorem given here can also be used to provide a generalization of previous results obtained by the author which answer a problem of Hewitt (Duke Math. J. 10(1943), 332) concerning the existence of normal spaces which cannot be expanded to a completely normal space without change in dispersion character (cf. Abstract 71T-G88, these *Notices* 18(1971), 670). (Received July 1, 1971.)

*687-54-8. LOUIS F. McAULEY, State University of New York, Binghamton, New York 13901 and DAVID F. ADDIS, Texas Christian University, Fort Worth, Texas 76129. Sections and selections.

A function $\Phi : X \to 2^Y$ (where 2^Y is the set of nonempty closed subsets of Y) is called a carrier. For $S \subset Y$, let $L(S) = \{x \in X \mid \Phi(x) \cap S \neq \emptyset\}$. The carrier Φ is point-compact iff $L(\{y\})$ is compact for all $y \in Y$ and is small if, for each $y \in Y$, and open set $U \supset L(\{y\})$, there exists a neighborhood V of y with $L(\{y\}) \subset L(V) \subset U$. An at most n selection for Φ is a carrier $\sigma : X \to 2^Y$ such that, for every $x \in X$, $\sigma(x) \subset \Phi(x)$, $\sigma(x)$ consists of at most n points, and $\{y \in Y \mid (\exists x) y \in \sigma(x)\}$ is closed in $\{y \in Y \mid (\exists x) y \in \Phi(x)\}$. The following theorem generalizes a result of Roberts and Civin. Theorem. Suppose that X and Y are metric spaces with $\dim X \leqq n$ and that $\Phi : X \to 2^Y$ is a lower semicontinuous, small, and point-compact carrier such that $\Phi(x)$ is a complete subset of Y for every $x \in X$. Then if $B \subset X$ is a closed (perhaps empty) subset and $g : B \to Y$ is a map such that $g(x) \in \Phi(x)$ for all $x \in X$ then there exists an at most $n + 1$-selection σ for Φ so that $\Phi(x) = \{g(x)\}$ for $x \in B$. It is shown that usc carriers on compact X are small and point-compact. Furthermore the theorem yields at most n-sections for certain open maps. (Received July 6, 1971.)

*687-54-9. WAYNE M. LAWTON, Wesleyan University, Middletown, Connecticut 06457. No infinite dimensional compact group admits an expansive automorphism.

The result stated in the above title has been proved using character theory for compact abelian groups and the theory of symbolic shift dynamical systems. (Received July 6, 1971.)

687-54-10. HOWARD H. WICKE, Ohio University, Athens, Ohio 45701 and JOHN M. WORRELL, JR., Sandia Laboratories, Division 1721, P. O. Box 5800, Albuquerque, New Mexico 87115. Local properties and a p-space analogue of a theorem of Smirnov. Preliminary report.

For conditions β_b, β_c, λ_b, λ_c, cf. the article by Wicke in General Topology and its Applications 1 (1971). For conditions β_d and λ_d cf. Wicke's article in the Proceedings of the 1971 Houston Conference on Point Set Topology. Theorem. Suppose α is one of the above-mentioned conditions. If X is a regular space and each point of X is in an open subspace of X which satisfies α, then X satisfies α. If $(A_n)_{n \in N}$ is a sequence of subspaces of X which satisfy α, then $\bigcap \{A_n : n \in N\}$ satisfies α. This theorem has numerous consequences. As one example an analogue of Smirnov's theorem concerning local metrizability may be obtained: Suppose X is a Tychonoff θ-refinable space covered by a collection of open subspaces which are p-spaces. Then X is a p-space (equivalently, an M-space). It may be noted that a stronger result holds: The first occurrence of "p-space" in the preceding theorem may be replaced by "β_d-space." (Received July 6, 1971.)

687-54-11. ROBERT W. HEATH, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. A note on quasi-metric spaces. Preliminary report.

In "On quasi-metric spaces", Duke Math. J. 36(1969), 65-71, Ronald A. Stoltenberg asks if every Moore space is quasi-metrizable. In this paper it is shown that the "Niemytski plane" (or "tangent disc" space; see V. Niemytski, "On the third axiom of metric spaces",Trans. Amer. Math. Soc. 29(1927), 507-513) is not quasi-metrizable. It is also proved that a linearly ordered space with a point-countable base is quasi-metrizable. Hence, if a Souslin space exists, there is one which is quasi-metrizable. (Received July 7, 1971.)

687-54-12. CHARLES C. ALEXANDER, University of Mississippi, University, Mississippi 38677. A metrization theorem.

A topological space X is said to be semidevelopable if there is a sequence of (not necessarily open) covers of X, $\mu = \{\mu_n\}_1^\infty$, such that for each $x \in X$, $\{St(x, \mu_n)\}_1^\infty$ is a neighborhood base at x. In this case, μ is called a semidevelopment. If μ and α are collections of subsets of X, then we say that μ is cushioned in α if one can assign to each M in μ an A(M) in α such that, for every $\mu' \subset \mu$, $\mathrm{Cl}(\bigcup\{M : M \in \mu'\}) \subset \bigcup\{A(M) : M \in \mu'\}$. By a cushioned pair-semidevelopment for X we shall mean a pair of semidevelopments (μ, α) such that μ_n is cushioned in α_n, for each n. Theorem. A space is metrizable if and only if it is T_0 and has a cushioned pair-semidevelopment. This theorem may be regarded as a metrization theorem for semistratifiable spaces, since they are precisely the spaces which have a σ-cushioned pair-network. Also it may be applied to semimetric spaces, since they are the T_0 semidevelopable spaces. The theorem has as immediate corollaries metrization theorems of Morita, Alexandroff, Stone, and Arhangel'skii. (Received July 7, 1971.)

*687-54-13. JERRY E. VAUGHAN, University of North Carolina, Chapel Hill, North Carolina 27514. Linearly stratifiable spaces and a definition given by H. Tamano.

H. Tamano has defined the class of perfectly paracompact spaces, which he intended to be a subclass of the class of paracompact spaces (see Abstracts 68T-125 and 654-31, these *Notices* 15(1968), 229, 345). His definition, however, cannot be what he wanted because a well-known nonparacompact space of V. Niemytzki is perfectly paracompact. Tamano's definition is in terms of a special "pair of bases" for the topology of the space, and resembles the following definition: a pair-base $\mathcal{P}$ is called linearly cushioned provided there is a linear order $\leqq$ on $\mathcal{P}$ such that for every subset $\mathcal{P}' \subset \mathcal{P}$ which has an upper bound, we have $(\bigcup[P_1 : P = (P_1, P_2) \in \mathcal{P}'])^-$ is a subset of $\bigcup[P_2 : P = (P_1, P_2) \in \mathcal{P}']$. Linearly stratifiable spaces, which are paracompact, were defined in Abstract 682-54-19 these *Notices* 18(1971), 210. Theorem. A T_1-space is inearly stratifiable if and only if it has a linearly cushioned pair-base. (Received July 7, 1971.)

687-54-14. LEE H. MINOR, Western Carolina University, Cullowhee, North Carolina 28723. Pseudo-expansive maps and transformation groups.

Let (X,d) be a metric space. A map $f: X \to X$ is pseudo-expansive on (X,d) if there exists $c > 0$ such that $f^n \neq f^m$ implies $d(f^n(x), f^m(x)) > c$ for some $x \in X$. This definition was introduced by Gerald Jungck ("Periodicity via equicontinuity," Amer. Math. Monthly 75(1968), 265-267), who proved that a continuous map of a compact metric space onto itself is periodic if and only if the map is pseudo-expansive and its iterates form an equicontinuous family. In this paper we extend the definition of pseudo-expansiveness to arbitrary families of maps and transformation groups and give an example which shows that Jungck's result does not generalize to arbitrary transformation groups. (Received July 7, 1971.)

55 Algebraic Topology

687-55-1. DAVID L. SMALLEN, University of Rochester, Rochester, New York 14620. The group Eq(M), of self equivalences of certain manifolds. Preliminary report.

Let M be a closed n-manifold with $\pi_1(M)$ finite of order > 2 and $\pi_i(M) = 0$ for $1 < i < n$. (This includes all closed three-manifolds with finite fundamental group, of order > 2.) We study the group Eq(M) of homotopy equivalences of M modulo homotopy. The following is proved: Theorem. Eq(M) is a normal subgroup of $\mathrm{Aut}(\pi_1(M))$ and consists of precisely those automorphisms φ such that $\varphi^*(k) = \pm k$ where k is the first Postnikov invariant of the Postnikov system for M $(k \in H^{n+1}(\pi_1(M), Z))$. The method of proof is to apply results of Olum ("Mappings of manifolds and the notion of degree," Ann. of Math. (2) 58(1953), 458-480) and to use the correspondence between Eq(M) and Eq(E), where E is the total space of the first stage of the Postnikov system for M. Corollaries which follow are: Corollary 1. If $G \times H$ acts on S^n without fixed points and the order of $G \times H > 2$ then $\mathrm{Eq}(S^n/G\times H) \approx \mathrm{Eq}(S^n/G) \times \mathrm{Eq}(S^n/H)$. (This situation frequently arises for three manifolds (see Milnor "Groups which act on S^n without fixed points").) Corollary 2. $\mathrm{Eq}(S^3/Q) \approx S_4$ (the symmetric group), where Q denotes the 8 element quaternionic group. $\mathrm{Eq}(S^{2n+1}/Z_m) \approx Z_k$ where k is the number of integers t with $1 \leq t \leq m$ satisfying $t^{n+1} \equiv \pm 1 \pmod{m}$. A result originally due to Olum in the above paper. Other calculations follow using Corollary 1 along with known results. (Received May 25, 1971.)

687-55-2. CASPER R. CURJEL, University of Washington, Seattle, Washington 98105 and ROY R. DOUGLAS, University of British Columbia, Vancouver 8, British Columbia, Canada. On Stasheff's fifth problem.

Stasheff's fifth problem (Conference on H-spaces at Neuchatel, Switzerland held in August 1970, Lecture Notes in Math., no. 196, Springer-Verlag, Berlin and New York, 1971, p. 123) should read: Let A be a candididate for $H^*(X; Z)$, i.e. an associative graded simply-connected algebra over the integers, Z. Let T(A) be the set of homotopy types of simply-connected finite complexes having integral cohomology ring isomorphic to

A. Question. For which A is T(A) a finite set? We conjectured then and proved recently the following: Theorem. If $A \otimes Q$ is an exterior algebra on odd dimensional generators (over the rationals, Q), then T(A) is a finite set. Remark. There are, of course, other types of Z-algebras A, for which T(A) is finite and nonempty. (Received July 6, 1971.)

*687-55-3. WOLFGANG H. HEIL, Florida State University, Tallahassee, Florida 32306. On Kneser's conjecture for bounded 3-manifolds. Preliminary report.

Kneser's conjecture (proved by J. Stallings for closed 3-manifolds) is generalised as follows. Theorem. Let M be a 3-manifold with incompressible boundary. If the fundamental group of M is a free product $G_1 * G_2$, then M is the connected sum of two 3-manifolds M_1, M_2 such that M_1, M_2 have fundamental groups G_1, G_2, respectively. The proof is similar to that of Stallings and uses the unique decompositions (rel. to free products) of finitely presented groups. (The conjecture is false for manifolds with compressible boundary.) It follows that knot groups are not (nontrivial) free products. Furthermore, Waldhausen's theorem ("On irreducible 3-manifolds which are sufficiently large," Ann. of Math. (2) 87(1968), 56-88, Corollary 6.5) can be generalised to boundary irreducible 3-manifolds which are connected sums of sufficiently large 3-manifolds and handles. (Received July 7, 1971.)

57 Manifolds and Cell Complexes

*687-57-1. JOHN P. NEUZIL, Kent State University, Kent, Ohio 44242. Embedding the dunce hat in E^4.

It has been conjectured by E. G. Zeeman that the complement of a tame dunce hat in E^4 is contractible. Here we give an embedding which has a nonsimply connected complement, giving a negative answer to the conjecture. (Received July 6, 1971.)

58 Global Analysis, Analysis on Manifolds

687-58-1. JOEL C. GIBBONS, 238 S. Summit Avenue, Villa Park, Illinois 60181 and St. Procopius College, Lisle, Illinois 60532. One-dimensional basic sets in the three-sphere.

This paper is a continuation of Williams' classification of one-dimensional attracting sets of a diffeomorphism on a compact manifold (Topology 6(1967)). After defining the knot presentation of a solenoid in S^3 and some knot-theoretic preliminaries we prove Theorem. If Σ_1, h_1 and Σ_2, h_2 are shift classes of oriented solenoids admitting elementary presentations K, g_1 and K, g_2, resp., where $g_{1*} = (g_{2*})^t$: $H_1(K) \to H_1(K)$, there is an Anosov-Smale diffeomorphism f of S^3 such that $\Omega(f)$ consists of a source Λ^- and a sink Λ^+ for which Λ^+, f/Λ^+ and Λ^-, f^{-1}/Λ^- are conjugate, resp., to Σ_1, h_1 and Σ_2, h_2. (The author has proved (Proceedings, to appear) that if f is an Anosov-Smale map of S^3, $\Omega(f)$ has dimension one, and contains no hyperbolic sets, then f has the above structure.) We also prove Theorem. There is a nonempty C^1-open set F_2 in the class of such diffeomorphisms for which $K = S^1$ and $g_1 = g_2$ is the double covering such that each f in F_2 defines a loop t in S^3, stable up to C^1 perturbations, for which at every x in t the generalized stable and unstable manifolds through x are tangent at x. (Received July 6, 1971.)

60 Probability Theory and Stochastic Processes

*687-60-1. CLARK H. KIMBERLING, University of Evansville, Evansville, Indiana 47701. A probabilistic interpretation of complete monotonicity.

A sequence (a_n) of nonnegative real numbers with $a_0 = 1$ is completely monotonic if and only if there exists a probability space with events A_n satisfying $P(A_{n_1} \dots A_{n_m}) = a_n$ for all $n_1 < \dots < n_m$ and $m = 1, 2, \dots$. For such a sequence, $a_{m+n} \geqq a_m a_n$ and $a^n_{m+1} \leqq a_{m+n} a^{n-1}_m$ for $m = 0, 1, \dots$ and $n = 1, 2, \dots$. A function f from $[0, \infty)$ to $(0, 1]$ is completely monotonic if and only if there exist random variables with continuous distribution functions F_n and joint distribution functions given by $F_{n_1, \dots, n_m}(x_1, \dots, x_m) = f[\sum_{j=1}^m f^{-1}(F_{n_j}(x_j))]$. In this case, $F_{n_1, \dots, n_m} \geqq F_{n_1} \cdots F_{n_m}$. If events $[\xi_n \leqq x_n]$ are so distributed, then their complements are distributed by such an f only in the case of independence. (Received May 10, 1971.)

*687-60-2. PETER C.-C. WANG and GLENN A. STOOPS, Naval Postgraduate School, Monterey, California 93940. Overlapping recurrent patterns.

In binomial experiments, one traditionally studies the occurrence of fixed patterns (e.g., success runs of length k) by the classical theory of recurrent events. Direct application of the theory requires that the patterns be nonoverlapping, a somewhat artificial constraint. In particular, one would like to observe only the "recent" past, rather than the entire history, to see if a pattern has occurred. In this paper the nonoverlapping condition is removed, and it is seen that the general theory of (delayed) recurrent events is still applicable. Generating functions, and some closed form expressions, are derived both for arrival and interarrival times, and for the number of patterns occurring in a fixed sequence of n trials. (Received June 21, 1971.)

*687-60-3. LAWRENCE A. SHEPP, Bell Telephone Laboratories, Inc., Murray Hill, New Jersey 07974 Covering the circle with random arcs.

Arcs of length ℓ_n, $0 < \ell_{n+1} \leqq \ell_n < 1$, $n = 1, 2, \dots$, are thrown independently and uniformly on a circumference C of unit length. The union of the arcs covers C with probability one if and only if at least one of the following holds: (1) $\limsup_{n \to \infty} (1/n) \exp(\ell_1 + \dots + \ell_n) = \infty$; (2) $\sum_{n=1}^{\infty} (1/n^2) \exp(\ell_1 + \dots + \ell_n - n\ell_n) = \infty$. (Received June 28, 1971.)

62 Statistics

687-62-1. BENNET P. LIENTZ, University of Southern California, Los Angeles, California 90007. Procedures for nonparametric modal intervals. Preliminary report.

For a random variable X with distribution function F and a point x define the function $S(x,\beta;F)$ for $\beta \in [0,1]$ as the smallest nonnegative quantity satisfying $F(x+S(x,\beta;F)) - F(x-S(x,\beta;F)) \geqq \beta$. Properties such as differentiability and continuity are explored. Estimation procedures are developed with application to reliability. It is shown that S is continuous for any F allowing discontinuities of the first kind. The paper extends results in (Lientz, "Results on nonparametric modal intervals," SIAM J. Appl. Math. 19(1970), 356-366). (Received December 21, 1970.)

65 Numerical Analysis

*687-65-1. CHARLES S. DURIS and MICHAEL TEMPLE, Drexel University, Philadelphia, Pennsylvania 19104. An exchange algorithm for determining the "strict" Chebyshev solution to overdetermined linear equations.

The purpose of this paper is to investigate Chebyshev solutions of the system of linear equations $Ax = b$, where A is an $m \times n$ real matrix of rank $n \leqq m-1$ and b is a real m-vector. In particular, a method for determining the unique "strict" solution is given. Defining x_p^* to be the unique real n-vector such that $\|b - Ax_p^*\|_p = \min_x \|b - Ax\|_p$ then the "strict" solution, x^*, has the following interesting property $x^* = \lim_{p\to\infty} x_p^*$. A complete algorithm is given which incorporates a one for one exchange algorithm to permit a maximal ascent over $(n+1) \times n$ subsystems. In addition, a stable procedure for updating the pseudoinverse of an $(n+1) \times n$ matrix is presented. (Received July 1, 1971.)

687-65-2. R. D. RIESS, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. Error estimates for Clenshaw-Curtis quadrature. Preliminary report.

The Clenshaw-Curtis quadrature formula, $Q_N(f)$, can be analyzed as a closed interpolatory scheme on the roots of $U_{N-1}(x)$, the $(N-1)$st degree Chebyshev polynomial of the second kind, and the interval endpoints, +1 and -1. Similar quadrature formulas can be derived by deleting the use of the endpoints, and/or using the roots of $T_{N-1}(x)$, the $(N-1)$st degree Chebyshev polynomial of the first kind. Several types of error estimates are derived for these quadrature formulas operating on different classes of functions, such as functions of low-order continuity and functions which are analytic and square-integrable in an elliptic disk. The basis for most of the error bounds is the relative ease with which one can compute $Q_N(T_k)$ and $Q_N(U_k)$ for $k \geqq N$. (Received July 6, 1971.)

*687-65-3. DIRAN SARAFYAN, Louisiana State University, New Orleans, Louisiana 70122. Determination of an optimal step-size for Runge-Kutta processes.

Let $y(x)$ be the solution of the initial-value problem $y' = f(x,y)$, $y(x_0) = y_0$. Furthermore, let $\tilde{y}_n(x_0+h)$ represent an nth order Runge-Kutta approximation to $y(x_0+h)$, where h is the chosen step-size. If e is the given tolerance, that is the largest absolute error which is considered as negligible, then the step-size H_n for which the absolute local truncation error in $\tilde{y}_n(x_0+H_n)$ is equal to e will be called the theoretical optimal step-size. For practical use, a reliable approximation for the theoretical optimal step-size will be given. This practical optimal step-size, designated by h_n, is neither too large nor unnecessarily small. Usually, $\frac{1}{2}h_n \leqq H_n \leqq 2h_n$. (Received July 6, 1971.)

73 Mechanics of Solids

687-73-1. K. L. ARORA, Punjab Engineering College, Punjab University, Chandigarh, India. Flexure of an initially curved cuboid dielectric.

Eringen's constitutive equations for a homogeneous, hyperelastic dieletric are applied to the problem of flexural deformation of an initially curved cuboid. Electrostatic field and stress tensors are determined. New effects have been observed which are not present in an unpolarized initially curved elastic cuboid. In the absence of the charge, the results reduce to those of the elasticity solution. (Received April 6, 1971.)

90 Economics, Operations Research, Programming, Games

*687-90-1. BENJAMIN L. SCHWARTZ, American University, Washington, D. C. 20016. A single-label, maximum-flow algorithm.

Maximum-flow problems in capacitated networks have conventionally been solved by labelling procedures. Such a procedure defines a function L on a subset of the set of nodes N of the network to $N \times Sg \times R^+$, where Sg is the set $\{+,-\}$, and R^+ is the set of positive reals. Thus, $L(a)$ is an ordered triple. In certain special types of problem (e.g. networks with gains), labels may even be ordered higher dimensional n-tuples. In computational practice, the requirement to store each component of the n-tuple $L(a)$ is a limitation. This note describes an algorithm that retains all the simplicity and computational convenience of the most commonly used prior labelling algorithm, but is superior in requiring less computation and in having a label function L whose domain is N; i.e. $L(a)$ has only one component, and its value, rather than numerical, is a literal from a finite set. The concept has been reduced to practice and verified empirically as well as theoretically. This application involves relocation planning for scarce, specially trained personnel. (Received June 21, 1971.)

*687-94-1. WILLIAM N. ANDERSON, JR., University of Maryland, College Park, Maryland 20742, RICHARD J. DUFFIN, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213 and G. E. TRAPP, West Virginia University, Morgantown, West Virginia 26506. Incidence matrix concepts for the analysis of the interconnection of networks.

When electrical networks are interconnected, many properties of the connected networks are induced by the connection itself and not by the particular components connected; algebraic operations are thus induced on the parameters describing the networks. The operations induced by series, parallel and hybrid connections have been extensively analyzed (J. Math. Anal. Appl. 23(1969), 576-594; SIAM J. Appl. Math. 19(1966), 390-413). Motivated by these considerations, we study arbitrary connections between sets of terminals, as specified by incidence matrices or graphs. Necessary and sufficient conditions are given so that a connection will lead to an algebraic operation, and so that the operation will be associative. A generalization of associativity, called compatibility, is also considered. (Received July 1, 1971.)

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BARUCH GERSHUNI. The action of the $\cup$-operator upon plural classes, Abstract 71T-E13, Page 424.

Line 4 should read: "the operator, the sequence of plural classes $P_0, P_1 = \cup P_0$, $P_2 = \cup P_1 = \cup^2 P_0, \ldots$. The class P_1 is called the"

JACK T. GOODYKOONTZ, JR. Aposyndetic properties of hyperspaces. Preliminary report, Abstract 682-54-50, Page 219.

The second theorem should read: "If X is semi-aposyndetic, then K(X) is mutually aposyndetic."

JAMES L. HEITSCH. On the classifying space of Haefliger. II. Preliminary report, Abstract 71T-D3, Page 257.

The last line should read: "Theorem. $H_{4j-1}(B S\Gamma_q; Z)$ is infinitely generated for $4j > 2q$."

DAVID W. HENDERSON. Applications of infinite-dimensional manifolds to quotient spaces of metric ANR's, Abstract 682-54-49, Page 219.

To the hypotheses of Theorem 2 should be added: "Either $f: A \to B$ is a homeomorphism or A and B are compact".

ROBERT A. HERRMANN. Some results using the nonstandard theory of remoteness. I, Abstract 71T-G23, Page 431.

In line 5, replace the word "mapping" by the word "injection".

N. S. MENDELSOHN and STEPHEN H. Y. HUNG. On Steiner systems S(3, 4, 14) and S(4, 5, 15), Abstract 71T-A78, Page 552.

In line 3, in the phrase "... there are exactly three nonisomorphic systems..." the word "three" should be replaced by the word "four".

ROBERT S. SMITH. A note on distributive filter lattices. Preliminary report, Abstract 683-A11, Page 357.

On line 5: (3) should read: "If $x, y, z \in G$ and $x \subset yz$ but $x \not\subset y$ and $x \not\subset z \ldots$."

FRANKLIN D. TALL. A set-theoretic proposition implying the metrizability of normal Moore spaces, Abstract 71T-G13, Page 264.

The third script Y should be a script Z.
In the third definition, it should be "p splits" rather than "p split".

ABSTRACTS PRESENTED TO THE SOCIETY

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Algebra & Theory of Numbers

*71T-A142. ANDRÉ JOYAL, University of Montreal, Montreal, Quebec, Canada. Cohomology of spectral spaces. Preliminary report.

For notations and terminology see Abstract 71T-A18, these *Notices* 18(1971), 393. Theorem 1. For any sheaf $\mathfrak{F}$ of abelian groups over a spectral space X, the Čech cohomology $\check{H}^q(X, \mathfrak{F})$ are canonically isomorphic with the Grothendieck cohomology groups $H^q(X, \mathfrak{F})$. Theorem 2. For every integer $q \geqq 0$, the functor $\mathfrak{F} \mapsto H^q(X, \mathfrak{F})$ preserves direct limits. The combinatorial dimension of a spectral space X is defined as the least upper bound of the length of all finite chains of irreducible closed subsets of X. We have Theorem 3. If a spectral space X is of combinatorial dimension $\leqq n$, then X is of cohomological dimension $\leqq n$ (i.e. $H^q(X, \mathfrak{F}) = 0$ for any sheaf $\mathfrak{F}$ of abelian groups and any integer $q > n$). Remarks. Theorem 1, proved by elementary means, is a generalisation of a theorem of Grothendieck on the cohomology of quasi-compact schemes for the particular cases of quasi-coherent sheaves. Theorem 3 is also a generalisation of a theorem of Grothendieck on the cohomology of Noetherian spaces. (Received November 4, 1970.)

*71T-A143. KIM KI-HANG BUTLER, Pembroke State University, Pembroke, North Carolina 28372. A note on semigroups. I.

Let B be a Boolean algebra $\{0,1\}$ of order 2. Let B_n denote the set of all $n \times n$ matrices over B_n. Then B_n is a multiplicative semigroup with $n \times n$ identity matrix (having 1's along the main diagonal and zeros elsewhere) as multiplicative identity. Let $\rho_r(A)$ ($\rho_c(A)$) denote the row (column) rank of A in B_n. We remark that A in B_n need not have the same row and column rank. A matrix $T = (t_{ij}) \in B_n$ is called lower triangular if T has the following form: (i) $t_{ij} = 1$ if $i = j$, (ii) $t_{ij} = 0$ if $i < j$, (iii) $t_{ij} = 0$ or 1 if $i > j$. Theorem 1. (i) if $U \in B_n$ is regular, then $\rho_r(U) = \rho_c(U)$; (ii) a lower triangular matrix $T \in B_n$ with $\rho_r(T) = r$ is an idempotent if $r \leqq 2$ or if the following condition holds for $r \geqq 3$: $t_{ij} = 1 = t_{ki} \Rightarrow t_{kj} \neq 0$ $(1 \leqq j < i < k \leqq r)$; (iii) if D is a regular $\mathcal{D}$-class of B_n, then D contains at least one idempotent lower triangular matrix. If $A \in D$, then there exist U, $V \in D$ such that $A = UV = A^{(i),(j)}A_{(i),(j)}$ if $A^2 = A$, $= A^{(h),(k)}A_{(p),(q)}$ otherwise. Here $A^{(i),(j)}$ $(A_{(i),(j)})$ denote the interchanging of ith and jth column (row) of A. Theorem 2. If $U \in B_n$ is regular, then U has an inverse in D_U. Moreover, if $\rho_r(U) = n = \rho_c(U)$ then U has a unique inverse in D_U. (Received May 27, 1971.)

71T-A144. HELEN SKALA, 1927 South Racine, Chicago, Illinois 60608, and University of Massachusetts, Boston, Massachusetts 02116. Pseudo-ordered sets.

Any reflexive and antisymmetric binary relation $\leqq$ on a set A will be called a pseudo-order on A. We call a family $\mathcal{Q}$ of subsets of A consistent if it satisfies the condition: if R and S are any two distinct members of $\mathcal{Q}$, then there do not exist elements r_1 and r_2 of R, and s_1 and s_2 of S such that $r_1 \leqq s_1$ and $s_2 \leqq r_2$. On such a family $\mathcal{Q}$ the following pseudo-order is induced: $R \leqq S$ if and only if R and S contain elements r and s, respectively, such that $r \leqq s$. Any consistent family of subsets of A with the induced pseudo-order will be called a contraction set of A. Theorem. Each pseudo-ordered set is isomorphic with a contraction set of a partially ordered set. If any two elements of a pseudo-ordered set A have a l.u.b. and g.l.b. (defined similarly as for partially ordered sets), A is said to be a trellis. A is complete if any subset of A has a l.u.b. and a g.l.b. Theorem. Any complete trellis is isomorphic with a contraction trellis of some lattice. (Received March 10, 1971.)

*71T-A145. GEORGE A. GRÄTZER and HARRY LAKSER, University of Manitoba, Winnipeg 19, Manitoba, Canada. Identities for equational classes generated by tournaments.

A weakly-associative lattice $\langle L ; \wedge, \vee\rangle$ is an algebra satisfying the identities $a \wedge b = b \wedge a$, $(a \vee b) \wedge a = a$, $[(a \wedge c) \vee (b \wedge c)] \vee c = c$, and their duals (E. Fried and H. Skala, unpublished). If in a tournament we set $a \vee b = b$ and $a \wedge b = a$ if either $a = b$ or there is a directed edge from a to b then the tournament becomes a weakly-associative lattice. Let $\underset{\sim}{T}$ be the equational class of weakly-associative lattices generated by the class of all tournaments. For each integer $k \geqq 1$ let $r_{1k} = ((x \vee z_1) \wedge z_2) \vee \ldots z_k$, $s_{1k} = (((x \wedge y) \vee z_1) \wedge z_2) \vee \ldots z_k$, $r_{2k} = ((y \vee w_1) \wedge w_2) \vee \ldots w_k$, $s_{2k} = (((x \wedge y) \vee w_1) \wedge w_2) \vee \ldots w_k$, and let Σ_k be the identity $[[[u \vee (r_{1k} \wedge s_{1k})] \wedge (r_{1k} \vee s_{1k})] \vee (r_{2k} \wedge s_{2k})] \wedge (r_{2k} \vee s_{2k}) = [[[v \vee (r_{1k} \wedge s_{1k})] \wedge (r_{1k} \vee s_{1k})] \vee (r_{2k} \wedge s_{2k})] \wedge (r_{2k} \vee s_{2k})$. Theorem 1. The identites Σ_k for all k along with the identities for weakly-associative lattices given above define the class $\underset{\sim}{T}$. Theorem 2. An algorithm is given to derive a system (always infinite) of identities defining the equational class generated by any finite set of finite subdirectly irreducible tournaments. These results are based on proving that a method of K. Baker (Bull. Amer. Math. Soc. 77(1971), 97-102) which generates lattice identities generalizes to certain classes of weakly-associative lattices. (Received March 15, 1971.)

*71T-A146. V. R. CHANDRAN, Madurai University, Madurai-2, India. On duo-rings. II.

A right duo-ring R is one in which every right ideal is two-sided. In this paper, we prove many interesting theorems on duo-rings. Theorem 1. In a right duo-ring R with 1, the following are equivalent: (1) R is (von Neumann) regular. (2) Every simple right R-module is injective. We use Theorem 1 to prove H. Bass's conjecture in the class of left-duo-rings with 1, which is the following theorem. Theorem 2. A left duo-ring R with 1 is left perfect if and only if every left R-module has a maximal submodule and R has no infinite set of orthogonal idempotents. Recently, R. H. Cozzens (Bull. Amer. Math. Soc. 76(1970), 75-79) has given an example to show Bass's conjecture need not be valid in the general class of noncommutative rings. Theorem 3. In a right duo-ring R (not necessarily with 1) the intersection of two modular right ideals is again modular. Further an example is constructed to show the intersection of two modular right ideals need not be modular in a general noncommutative ring. Theorem 4. A right duo-ring which is also a prime ring is embeddable in a skew field of quotients. (Received March 15, 1971.) (Author introduced by Professor M. Rajagopalan.)

*71T-A147. SIN-MIN LEE, University of Manitoba, Winnipeg 19, Manitoba, Canada. Semigroups and rings satisfying $(xy)^n = xy = x^n y^n$.

For $n \geqq 2$, let I_n be the equational class of semigroups satisfying $x^n = x$ and $\mathcal{C}$ the equational class of all constant semigroups. Theorem 1. For a semigroup S, the following statements are equivalent: (1) $S \in I_n \vee \mathcal{C}$. (2) S satisfies $(xy)^n = xy = x^n y^n$. (3) S is an inflation of some semigroup in I_n. Theorem 2. The free semigroup $\mathfrak{F}_{I_n \vee \mathcal{C}}(X)$ in $I_n \vee \mathcal{C}$ is an inflation of $\mathfrak{F}_{I_n}(X)$ and $|\mathfrak{F}_{I_n \vee \mathcal{C}}(X)| = |X| + |\mathfrak{F}_{I_n}(X)|$. Corollary. The following are equivalent: (1) $I_n \vee \mathcal{C}$ is a locally finite equational class. (2) All groups of exponent $n - 1$ are locally finite. Theorem 3. The lattice $\mathcal{L}(I_n \vee \mathcal{C})$ of equational subclasses of $I_n \vee \mathcal{C}$ is isomorphic to the direct product of $\mathcal{L}(I_n)$ and a two-element chain. Theorem 4. Let $\mathcal{B}_n$ be the equational class of rings satisfying $x^n = x$, and let $\mathcal{C}_0$ be the equational class of all zero rings. The following statements are equivalent: (1) $R \in \mathcal{B}_n \vee \mathcal{C}_0$. (2) $R \cong B \times C$, $B \in \mathcal{B}_n$, $C \in \mathcal{C}_0$. (3) R satisfies the identities $(x+y)^n = x^n + y^n$, $(xy)^n = xy = x^n y^n$. Examples of rings satisfying $(x+y)^n = x^n + y^n$ or $(xy)^n = x^n y^n$ are given. These results generalize the case $n = 2$ which the author investigated before; see Abstract 71T-A83, these *Notices* 18(1971). (Received March 16, 1971.) (Author introduced by Professor George A. Grätzer.)

71T-A148. DAVID C. BUCHTHAL, Purdue University, Lafayette, Indiana 47907. Finite groups factorized by a single class of Sylow normalizers. Preliminary report.

In the following, S is a Sylow p-subgroup of a finite group G and $N = N_G(S)$ is its normalizer. Theorem 1. Suppose $G = NB$, with S contained in the normalizer of some nontrivial normal subgroup of B. Then G is nonsimple. Theorem 2. Suppose $G = NA$, with S cyclic and A abelian. Moreover, suppose some nonidentity element of S normalizes some nontrivial subgroup of A. Then G is nonsimple. Theorem 3. Suppose $G = NC$, with S and C cyclic. Suppose, moreover, that N is a nilpotent maximal subgroup of G. Then G is supersolvable. Theorem 4. If the solvable group $G = NC$, where S and C are cyclic and N is a supersolvable maximal subgroup of G, then G is supersolvable or G maps onto $\Sigma(4)$, the symmetric group on four letters. Theorem 5. Let $G = NC$, where G is solvable, S is cyclic, N is supersolvable, and C is a cyclic Hall-subgroup of G. Then G is supersolvable. (Received March 25, 1971.)

*71T-A149. JAYME M. CARDOSO, Universidade Estadual de Campinas, Caixa Postal 1170, Campinas, Brasil. On 3-rings and their lattice structures. II.

Let R be a 3-ring with unity 1. The most general expressions for $x \wedge y$ and $x \vee y$ that are possible to be defined in R are obtained in such a way that R with these operations is a lattice where $\wedge$ is the lower bound and $\vee$ the upper bound. $\wedge$ is defined by: $x \wedge y = Ax^2y^2 + B(x^2y + xy^2) + Cxy + D(x^2 + y^2) + E(x+y)$ where $A = (c-f)p^2q^2 - epq^2 + (1-h)qp^2 - gpq + dp^2 - cq^2 + p - q - d$, $B = -eq^2q^2 - fpq^2 - gp^2q - hpq + p^2$, $C = ap^2q^2 - p^2q + bp^2 + cq^2 - p + q + d$, $D = (c+a-f)p^2q^2 - hp^2q - epq^2 - gpq + (d+b)p^2$, $E = ep^2q^2 + fpq^2 + gp^2q + hpq - p^2 + 1$. Here, a, b, c, d, e, f, g and h are elements of R such that $a + c = c + d = b + d = e - h + 1 =$

$g - f + 1 = 0$ and p and q are two elements of R, arbitrarily chosen, which respectively become under the operations $\wedge$ and $\vee$, the greatest and the least elements of the lattice (see Abstract 70T-A224, these *Notices* 17(1970), 1066). (Received April 5, 1971.)

71T-A150. ARON SIMIS, Queen's University, Kingston, Ontario, Canada. A local version of a theorem by Passman on the nilradical of a group-ring. Preliminary report.

While trying to investigate the isomorphism problem of commutative group-rings via the study of their nilradical, the author stumbled over a theorem of Passman (Michigan Math. J. 9(1962), 375-384) and was then led to give a "local" version of Theorem II (Ibid.). What is here called the nilradical of a (not necessarily commutative) ring is the union of its two-sided nilideals. A commutative ring is called reduced if its nilradical is (0). Theorem. R is a reduced commutative ring, $\mathcal{P}$ is the set of rational primes, $P \subset \mathcal{P}$ is such that $\mathcal{P} \backslash P$ is infinite and the elements of P are units in R. If G is a group whose q-primary part is trivial for every $q \in \mathcal{P} \backslash P$, then the nilradical of RG is (0). Corollary. R is a reduced commutative ring, G a group. The nilradical of RG is (0) in the following cases: (i) G is torsion-free. (ii) G is finite and its order is a unit in R. Corollary. R is a reduced commutative $\mathbf{Q}$-algebra, G is a group with trivial q-primary part for an infinite set of primes q. Then the nilradical of RG is (0); in particular, if G is finite or finitely generated abelian, then RG has zero nilradical. Corollary. R is a commutative $\mathbf{Q}$-algebra, G and H are finitely generated abelian groups. If $RG \cong RH$ as R-algebras then G and H have the same free rank. (Received April 21, 1971.) (Author introduced by Professor Paulo Ribenboim.)

71T-A151. KHEE-MENG KOH, University of Manitoba, Winnipeg 19, Manitoba, Canada. Algebras representing $\langle 0, 0, 2, 10 \rangle$.

For notations and basic definitions, see Abstract 71T-A30, these *Notices* 18(1971), 397. Let K be the class of all algebras having two distinct symmetric essentially binary polynomials. In this note, it is proved that there is a four-element algebra $\mathfrak{A}(4)$ in K representing the sequence $\langle 0, 0, 2, 10 \rangle$. Conversely, if $\mathfrak{A}$ is an algebra in K which represents the sequence $\langle 0, 0, 2, 10 \rangle$, then $\mathfrak{A}$ can be represented as an algebra which contains $\mathfrak{A}(4)$ as a subalgebra. These results are applied to show that the sequence $\langle 0, 0, 2, 10 \rangle$ has the minimal extension property with respect to the class K. (Received April 22, 1971.)

*71T-A152. ESMOND E. DeVUN, Wichita State University, Wichita, Kansas 67208. Semigroups on the disk with thread boundaries. Preliminary report.

In an earlier work (Abstract 663-411, these *Notices* 16(1969), 205) commutative semigroups on the two-cell without zero divisors whose boundary consisted of two usual unit intervals were determined. In this note the semigroups with zero divisors are determined. Moreover, if S is a commutative semigroup on the two-cell whose boundary consists of two threads with E(S) (the set of idempotents of S) = $\{z, i\}$ where z is the zero for S and i is the identity for S, then a classification for S is obtained when the results here are combined with the earlier work listed above. The method involves constructing a class of closed congruence relations $\mathcal{R}$ on $(I, \cdot) \times (I, \cdot)$ (where $(I, \cdot)$ represents the unit interval $[0, 1]$ with the usual multiplication). Theorem. S is a commutative semigroup on the two-cell whose boundary consists of two threads with E(S) = $\{x, i\}$ if and only if there exist $R \in \mathcal{R}$ such that S is iseomorphic to S/R. (Received May 5, 1971.)

*71T-A153. GUENTER R. KRAUSE, University of Manitoba, Winnipeg 19, Manitoba, Canada. On fully left bounded left noetherian rings.

An associative ring R with identity is said to be left bounded if each nonzero essential left ideal of R contains a nonzero two-sided ideal. R is fully left bounded if R/P is left bounded for each prime ideal P of R. Theorem 1. The classical Krull-dimension and the left Krull-dimension of any fully left bounded, left noetherian ring are equal (for the definition of these dimensions see Abstract 677-16-4, these *Notices* 17(1970), 764). Theorem 2. The following properties of the left noetherian ring R with identity are equivalent: (1) The map β: E $\rightarrow$ Ass(E) induces a bijective map from the set of isomorphism classes of injective, indecomposable left R-modules onto the set of all prime ideals of R. (2) R is fully left bounded. (3) If E is an injective, indecomposable left R-module with Ass(E) = P, then there exists an element $e \neq 0$ in E such that the Krull-dimension of the module Re is equal to the left Krull-dimension of the ring R/P. (4) Each finitely generated tertiary left R-module is Goldman-primary. (Received May 10, 1971.)

*71T-A154. R. PADMANABHAN, University of Manitoba, Fort Garry 19, Manitoba, Canada. Characterization of a class of groupoids.

Let k and n be two positive integers. An equational class $\underset{\sim}{K}$ of algebras is said to have the property (k, n) if every algebra in $\underset{\sim}{K}$ generated by k distinct elements has exactly n elements. In this paper we show that an equational class $\underset{\sim}{K}$ of cancellative groupoids has the property (2, 5) iff $\underset{\sim}{K}$ has a binary polynomial symbol, say $x \cdot y$, satisfying the cancellative property and the identity $(xy)(y(z(zx))) = y$ and moreover, $\underset{\sim}{K}$ is unique up to definitional equivalence. A special case of this solves a problem of N. S. Mendelsohn (Problem 5793, American Math. Monthly 78(1971), 411). From the proofs we deduce the converse of a result of S. K. Stein (Pacific J. Math. 14(1964), 1091-1102), thereby characterizing the equational classes $\underset{\sim}{K}$ of cancellative groupoids with the property (2, 4). (Received May 10, 1971.)

*71T-A155. KWANGIL KOH and JIANG LUH, North Carolina State University, Raleigh, North Carolina 27607. Maximal regular right ideal space of a ring. Preliminary report.

Let R be a ring and let $\mathcal{L}(R)$ be the lattice of right ideals of R. Let X(R) be the set of maximal regular right ideals of R. For each $A \in \mathcal{L}(R)$, define $\operatorname{supp}(A) = \{I \in X(R) \mid A \not\subseteq I\}$. We give a topology to X(R) by declaring an open set of X(R) to be an arbitrary union of finite intersections of $\{\operatorname{supp}(A) \mid A \in \mathcal{L}(R)\}$. We say X(R) is irreducible if it is not a union of two proper closed sets. Otherwise it is reducible. Theorem 1. Let R be a (right) primitive ring. Then X(R) is reducible if and only if there is an idempotent $e \in R$ such that eRe is a finite field. Theorem 2. Let R be a (right) primitive ring. Then X(R) is a Hausdorff space if and only if either R is a dense subring of the ring of linear transformations of a vector space over a finite field such that R modulo the socle of R is a radical ring or R is a division ring. Corollary. If R is a primitive ring with 1, then X(R) is Hausdorff if and only if R is a finite ring, or R is a division ring. (Received May 10, 1971.)

*71T-A156. ANASTASIA CZERNIAKIEWICZ, Columbia University, New York, New York 10027. Automorphisms of a free associative algebra of rank 2. II.

Let R be a commutative domain with 1. $R\langle x, y\rangle$ stands for the free associative algebra of rank 2 on the free generators x and y over R; $R[\tilde{x}, \tilde{y}]$ denotes the polynomial ring in the two commuting indeterminates $\tilde{x}$ and $\tilde{y}$ over R. Theorem. The map $\mathrm{Aut}(R\langle x, y\rangle) \to \mathrm{Aut}(R[\tilde{x}, \tilde{y}])$ induced by the abelianization functor is a monomorphism. The proof is obtained by generalizing the techniques used in "Automorphisms of a free associative algebra of rank 2. I", A. Czerniakiewicz, Trans. Amer. Math. Soc. (to appear). As a Corollary. We obtain that if R is a Generalized Euclidean Ring then every automorphism of $R\langle x, y\rangle$ is tame (i.e. a product of elementary automorphisms). (Received May 10, 1971.)

71T-A157. STEPHEN M. GERSTEN, Rice University, Houston, Texas 77001. On the spectrum of algebraic K-theory. Preliminary report.

If A is a unital ring, then the work of Quillen and Segal shows that there is an Ω-spectrum whose 0th term has connected component $BGl(A)^+ \simeq z_\infty BGl(A)$. We describe the positive terms in the spectrum. If $\tilde{A} = \bigcup_n M_n(A)$, and CA and SA are the cone and suspension of A respectively (Karoubi and Villamayor, C. R. Acad. Sci. Paris Sér. A-B 269(1969), 416-419) then we prove Theorem. The space $\Omega BGl(SA)^+$ has the homotopy type of $K_0(A) \times BGl(A)^+$. Corollary. For $n \in Z$ we have $K_n(A) = K_{n+1}(SA)$. Here the functors K_n for $n \geqq 0$ are those defined by Bass, Milnor, and Quillen, while for $n < 0$ these are the functors defined by Bass (and denoted K^{-n} by Karoubi for $n<0$). An important step in the proof is Proposition. The space $BGl(CA)^+$ is contractible. We remark that the corollary above was known to Karoubi for $n < 2$. One interesting consequence of these results is Corollary. For any ring A and $n \in Z$ we have $K_n(A[t, t^{-1}]) = K_n(A) \oplus K_{n-1}(A) \oplus ?$. (Received May 13, 1971.)

*71T-A158. M. V. SUBBARAO and D. SURYANARAYANA, University of Alberta, Edmonton, Alberta, Canada. Exponentially perfect and unitary perfect numbers.

Let n be an integer > 1 and $n = \Pi_{i=1}^r p_i^{a_i}$ be its canonical representation. A divisor d of the form $d = \Pi_{i=1}^r p_i^{b_i}$ is called an exponential divisor (exponential unitary divisor) of n if $b_i | a_i$ (b_i unitarily divides a_i). We call an integer n to be exponentially perfect (exponentially unitary perfect) if $\sigma^{(e)}(n) = 2n$ ($\sigma^{(e)*}(n) = 2n$), where $\sigma^{(e)}(n)$ and $\sigma^{(e)*}(n)$ respectively denote the sum of the exponential divisors and the sum of the exponential unitary divisors of n. For example, 36 is both exponentially perfect and exponentially unitary perfect. Actually, there exist an infinity of exponentially perfect and an infinity of exponentially unitary perfect numbers. We have the following Unsolved Problem. Is there an odd exponentially perfect number. However, we can prove Theorem 1. There is no odd exponentially unitary perfect number. Among other results obtained in connection with odd exponentially perfect numbers, we mention the following: Theorem 2. Suppose $n = \Pi_{i=1}^r p_i^{a_i}$ is an odd exponentially perfect number. Then all the a_i's are squares except for one i, say a_1. If $p_1 \equiv 1 \pmod 4$, then a_1 is of the form $a_1 = q_1^{\alpha_1} b_1^2$, where q_1 is a prime, $(q_1, b_1) = 1$ and $\alpha_1 \equiv 1 \pmod 4$. If $p_1 \equiv 3 \pmod 4$ and a_1 is odd, then $a_1 = q_1^{\alpha_1} b_1^2$, where q_1 is an odd prime, b_1 is odd, $(q_1, b_1) = 1$ and

$\alpha_1 \equiv 1 \pmod 4$. If $p_1 \equiv 3 \pmod 4$ and a_1 is even, then $a_1 = q_1^{\alpha_1} b_1^2$, where q_1 is an odd prime, b_1 is even $(q_1, b_1) = 1$ and $\alpha_1 \equiv 1 \pmod 4$ or $a_1 = 2^{\beta_1} b_1^2$, where b_1 is odd and $\beta_1 \equiv 3 \pmod 4$. (Received May 21, 1971.)

*71T-A159. RICHARD R. HALLETT, University of British Columbia, Vancouver 8, British Columbia, Canada. On pseudo-injective modules. Preliminary report.

Let R be a ring with 1, and let all modules be left unital R-modules. A module M is pseudo-injective if every monomorphism from a submodule of M into M is the restriction of an endomorphism of M (S. Singh and S. K. Jain, "On pseudo injective modules and self pseudo injective rings", J. Math. Sci. 2(1967), 23-31). An example is given of a pseudo-injective module which is not quasi-injective. In fact most artinian nongeneralized uniserial rings have a module of this type. Every self pseudo-injective generalized uniserial ring is quasi-Frobenius. Every pseudo-injective module over a uniserial ring is quasi-injective. (Received May 14, 1971.) (Author introduced by Professor Tim Anderson.)

*71T-A160. CHARLES C. LINDNER, Auburn University, Auburn, Alabama 36830. Construction of orthogonal Steiner quasigroups.

A Steiner quasigroup is a quasigroup satisfying the identities $x^2 = x$, $x(xy) = y$, and $(yx)x = y$. A pair of Steiner quasigroups $(S, \circ)$ and $(S, \circledcirc)$ are said to be orthogonal provided that whenever x, y, z, and w are elements of S and $x \circ y = z \circ w$, $x \circledcirc y = z \circledcirc w$, then $\{x,y\} = \{z,w\}$. In this paper we give a recursive construction for pairs of orthogonal Steiner quasigroups. The construction uses the singular direct product for quasigroups along with the idea of a pair of conjugate orthogonal quasigroups. (Received June 28, 1971.)

71T-A161. TIMOTHY B. CARROLL and ANTHONY A. GIOIA, Western Michigan University, Kalamazoo, Michigan 49001. On extended linear functions. Preliminary report.

For any terminology not defined here, see Goldsmith (Pacific J. Math. 27(1968), 283-304). If $\mathcal{B}$ is any basic sequence, let $\mathcal{B}_1$ denote the basic sequence generated by all type I primitive pairs contained in $\mathcal{B}$, and let $\mathcal{B}_2$ denote the basic sequence generated by $\mathcal{B}_1$ together with all type II primitive pairs. Let p and q denote primes ; p and q are said to be severed in the integer n with respect to $\mathcal{B}_2$ if and only if there exist primes q_i $(i = 1, \dots, k)$ such that $q_i | n$, $p = q_1$, $q = q_k$, and $(q_i, q_{i+1}) \notin \mathcal{B}_2$ $(i = 1, \dots, k-1)$. The sets $S_0(\mathcal{B}_2; n) = \{p : p | n, (p,q) \in \mathcal{B}_2 \ \forall\, q | n\}$ and $S(\mathcal{B}_2; p, n) = \{q : p, q$ are severed in $n\}$ are called the severance classes of n relative to $\mathcal{B}_2$. It is shown that every integer $n > 1$ has an essentially unique severence canonical form $n = n_0 n_1 \dots n_j$, where the n_i are relatively prime in pairs, and p, q divide n_i $(i = 0, 1, \dots, j)$ if and only if p and q are in the same severance class of n. Using the above, results analogous to the classical results on linear functions are proved. For example, if f is an arithmetic function multiplicative relative to $\mathcal{B}_1$, necessary and sufficient conditions are determined for f to be multiplicative relative to $\mathcal{B}_2$. (Received May 24, 1971.)

*71T-A162. DIETRICH H. VOELKER, Clarkson College of Technology, Potsdam, New York 13676. On a class of polynomials. Preliminary report.

The complex Fourier coefficients $c_k = 1/(ik-\alpha)^{n+1}$ ($\alpha \neq ik$) ($n = 0,1,2,\dots$) belong--in the interval $(0,2\pi)$--to the real function $f(x) = e^{(x+2\pi)\alpha} \sum (2\pi)^{\nu+1} \Phi_{\nu-1}(e^{2\pi\alpha}) x^{n-\nu}/(1-e^{2\pi\alpha})^{\nu+1} \nu! (n-\nu)!$ where $\Phi_n(x) = x \sum \binom{n+1}{\nu} \Phi_{\nu-1}(x)(1-x)^{n-\nu}$ ($n = 0,1,2,\dots$) and $\Phi_{-1}(x) = 1/x$. (The sums are taken from $\nu = 0$ to $\nu = n$.) These $\Phi_n(x)$ are polynomials of degree n and satisfy the functional relation $\Phi_n(x) = (1+nx)\Phi_{n-1}(x) + (1-x)x\Phi'_{n-1}(x)$ from which their properties can be found. They are reciprocal, positive and monic. The zeros are not rational (except $x = -1$), they are simple and real negative and are separated by those of $\Phi_{n-1}(x)$. The coefficients a_{nk} are positive integers, symmetrical and increasing till the center. Their formula is $a_{nk} = \sum_{\nu=0}^{k} (-1)^{\nu} \binom{n+2}{\nu} (k+1-\nu)^{n+1}$. Recursion formula: $a_{n+1,k} = (k+1)a_{nk} + (n+2-k)a_{n,k-1}$. (Received May 24, 1971.)

*71T-A163. CHARLES R. WALL, East Texas State University, Commerce, Texas 75428. Bi-unitary perfect numbers.

A divisor d of an integer n is a unitary divisor if d and n/d are relatively prime; a divisor d of n is a bi-unitary divisor if the greatest common unitary divisor of d and n/d is 1. An integer is bi-unitary perfect if it equals the sum of its proper bi-unitary divisors. Theorem. The only bi-unitary perfect numbers are 6, 60 and 90. (Received May 21, 1971.)

71T-A164. ROBERTO L. O. CIGNOLI, Instituto de Matemática, Universidad Nacional del Sur, Bahía Blanca, Argentina. A Hahn-Banach theorem for distributive lattices.

Let L be a distributive lattice with 0 and 1, and B a complete Boolean algebra. A meet-homomorphism is a map $m: L \to B$ fulfilling $m(0) = 0$, $m(1) = 1$ and $m(x \wedge y) = m(x) \wedge m(y)$. A join-homomorphism j is defined dually. Theorem. Let S be a subset of L containing the elements 0 and 1. A map $f: S \to B$ can be extended to an homomorphism $h: L \to B$ such that $m(x) \leq h(x) \leq j(x)$ for any x in L if and only if the following properties hold: (1) $f(0) = 0$, $f(1) = 1$, and (2) if $x_1, \dots, x_n$, $y_1, \dots, y_m$ are in S and w, z are in L, then the relation $x_1 \wedge \dots \wedge x_n \wedge w \leq y_1 \vee \dots \vee y_m \vee z$ implies that $f(x_1) \wedge \dots \wedge f(x_n) \wedge m(w) \leq f(y_1) \vee \dots \vee f(y_m) \vee j(z)$. It is shown that some classical results of G. Birkhoff and M. H. Stone on ideals in distributive lattices, as well as a theorem of R. Balbes (Pacific J. Math. 21(1967), 405-420) asserting that complete Boolean algebras are injective objects in the category of distributive lattices, can be derived from the above theorem. When the theorem is applied to Boolean algebras, it yields a theorem of A. Monteiro (Bull. Sci. Math. 89(1965), 65-74), which is the analogue of the classical Hahn-Banach theorem for Boolean algebras. This paper will appear in Rev. Un. Mat. Argentina, 1971. (Received May 24, 1971.)

*71T-A165. ALBERT A. MULLIN, USARV, USAICCV (DDS), APO, San Francisco, California 96384. Strengthening four results of elementary additive number theory. Preliminary report.

For terminology see the author's research problem (Bull. Amer. Math. Soc. 76(1970), 974-975) and references loc. cit. The present note strengthens four well-known results. Lemma 1. Let $a \neq b$ be integers. Then there exist infinitely many positive integers n such that the mosaics of $(a+n)$ and $(b+n)$ have no prime in common. Lemma 2. Let a, b, c be distinct integers. Then there exist infinitely many positive integers n such that the mosaics of $(a+n)$, $(b+n)$, and $(c+n)$, pairwise, have no prime in common. Note. The analogous result with four distinct integers is false. Lemma 3. Every integer > 6 is a sum of two integers > 1 whose mosaics have no prime in common. Lemma 4. Every integer > 17 is a sum of three integers > 1 whose mosaics, pairwise, have no prime in common. Note. The "usual" proofs must be modified for these stronger results since, e.g., it is false that any two consecutive odd integers satisfy the condition that their mosaics have no prime in common (although they are relatively prime). (Received June 1, 1971.)

*71T-A166. CARLOS S. JOHNSON, JR. and FRED R. McMORRIS, Bowling Green State University, Bowling Green, Ohio 43403. Injective hulls of certain S-systems over a semilattice.

An S-system is a set M acted on by a semigroup S such that $m(s_1s_2) = (ms_1)s_2$ for all $m \in M$, $s_1, s_2 \in S$. Adapting the techniques of Bruns and Lakser (Canad. Math. Bull. 13(1970), 115-118), we construct, in the category of S-systems over a semilattice, the injective hulls of S-systems which are homomorphic images of S-subsystems of S. Among the corollaries, we obtain a characterization of injective cyclic S-systems over a semilattice and the result that a semilattice S is injective in the category of semilattices if and only if it is injective in the category of S-systems. (Received June 1, 1971.)

71T-A167. LARY SCHIEFELBUSCH, University of Chicago, Chicago, Illinois 60637. Sylow subgroups of simple groups. Preliminary report.

Suppose T is a 2-group which has a normal elementary abelian subgroup of order eight and has an element of order eight which generates its own centralizer in T. If T is a Sylow 2-subgroup of a simple group then T is isomorphic with a Sylow 2-subgroup of M_{12}, M_{22}, or A_{10}. (Received June 3, 1971.)

71T-A168. SEYMOUR LIPSCHUTZ, Temple University, Philadelphia, Pennsylvania 19122. On root decision problems in groups.

Consider a group G and an integer $r > 1$. We say the rth root problem is solvable for G if we can decide, for any $w \in G$, whether or not there exists $v \in G$ such that $w = v^r$. (That is, we can decide if w has an rth root.) Theorem 1. There exists a finitely presented group with all root problems unsolvable. Theorem 2. If r and s are positive integers such that neither r nor s divides the other, then the corresponding root problems are independent. Theorem 3. If $r_1, r_2, \ldots, r_k$ are positive integers such that, for each i, there exists a prime p_i dividing r_i but not dividing r_j for $j \neq i$, then the corresponding root problems are independent. (We say that the decision problems $D_1, D_2, \ldots, D_n$ are independent if for any subset $\{i_1, \ldots, i_m\}$ of $\{1, \ldots, n\}$ there exists a recursively presented group with $D_{i_1}, \ldots, D_{i_m}$ solvable, but the remaining D_i's unsolvable.) (Received June 7, 1971.)

71T-A169. E. FRIED and GEORGE A. GRÄTZER, University of Manitoba, Winnipeg 19, Manitoba, Canada. Free and partial weakly-associative lattices.

A weakly-associative lattice (WA-lattice) is an algebra $\langle A; \wedge, \vee \rangle$, where $\wedge$ and $\vee$ are idempotent and commutative, they satisfy the absorption identities, and the weak-associative identities: $((x \wedge z) \vee (y \wedge z)) \vee z = z$ and dually. This concept is due to E. Fried (Ann. Univ. Sci. Budapest. Eötvös Sect. Math. 13(1970), 151-164) and H. Skala (Algebra Universalis 1(1971)). Setting $x \leqq y$ for $x \vee y = y$, $x \wedge y = \inf\{x,y\}$ and $x \vee y = \sup\{x,y\}$. We characterize the partial WA-lattices and prove that a finite partial WA-lattice can be embedded into a finite WA-lattice. This implies that the word problem for WA-lattices is solvable. An effective solution is presented. A further application is an embedding theorem for directed graphs. These methods are then used to investigate the free WA-lattices. If F is a free WA-lattice and $X \subseteq F$, necessary and sufficient conditions on X are given for the subalgebra $[X]$ generated by X to be freely generated by X. This is applied to show that the free WA-lattice on two generators contains as a subalgebra the free WA-lattice of $\aleph_0$-generators. (Received June 7, 1971.)

*71T-A170. WILLIAM A. LAMPE, University of Manitoba, Winnipeg 19, Manitoba, Canada. On the independence of certain related structures of a universal algebra. I. Partial algebras with useless operations and other lemmas.

Let $\mathfrak{B}$ be any partial algebra and let H be a system (set) of congruences of $\mathfrak{B}$. Let $\mathfrak{F}(\mathfrak{B})$ be the algebra freely generated by $\mathfrak{B}$. Let $\mathfrak{F}(H)$ be the system of smallest extensions of members of H to congruences of $\mathfrak{F}(\mathfrak{B})$. This paper contains technical lemmas that provide sufficient conditions for $\mathfrak{F}(H)$ to have various properties that H has. These lemmas are the key to the constructions that will appear in the later papers in the series. The primary lemma provides sufficient conditions under which $\mathfrak{F}(H)$ is a closure system. A lemma of this sort was known for unary partial algebras, but its analogue for arbitrary partial algebras was false. (Received June 7, 1971.)

71T-A171. LINDSAY N. CHILDS, State University of New York, Albany, New York 12203, G. GARFINKEL, University of Illinois, Urbana, Illinois 61801 and M. ORZECH, Queen's University, Kingston, Ontario, Canada. The Brauer group of graded Azumaya algebras.

Let R be a commutative ring, trivially graded, G a finite abelian group, $\varphi: G \times G \to U(R)$ a fixed bilinear form. A G-graded R-algebra A is a graded Azumaya R-algebra if A is a separable R-algebra and R is both the left and the right graded center of A. Put an equivalence relation on graded Azumaya algebras as does M. A. Knus [Springer Lecture Notes in Math., No. 92, Springer-Verlag, New York and Berlin, 1969, pp. 117-133]; multiplication of equivalence classes induced by the φ-graded tensor product [Knus, op. cit.] yields a Brauer group B(R,G). Theorem 1. There is an exact sequence $1 \to B(R) \to B(R,G) \to C \to 1$ where C is a group of graded Galois extensions and B(R) is the usual Brauer group. Theorem 2. If G is cyclic of order $n = \Pi_{i=1}^{r} p_i^{e_i}$, there is an exact sequence $1 \to \mathrm{Gal}(R,G) \to C \xrightarrow{\beta} \mathrm{Cont}(\mathrm{Spec}(R), \Pi_{i=1}^{r} Z/2Z)$ where Gal(R,G) = the group of Galois extensions of R with group G and the image of $\beta = \{f \mid f(x) = \Pi_{i=1}^{r} f(x)_i$ where

$f(x)_i = 0$ if φ is degenerate on the p_i-Sylow subgroup of G, or if p_i is not a unit modulo $x\}$. These results extend those of H. Bass [Tata Notes #40] and C. Small [Trans. Amer. Math. Soc., to appear] for $G = Z/2Z$, and of Knus [op. cit.]. (Received June 8, 1971.)

*71T-A172. MURRAY M. SCHACHER, University of California, Los Angeles, California 90024 and LANCE W. SMALL, University of California at San Diego, La Jolla, California 92037. Noncrossed products.

Amitsur has proved recently that there are finite-dimensional division rings (over their centers) which are not crossed products. His examples are of characteristic 0. Using Amitsur's methods we prove: Theorem. Let p be a prime or 0 and n a positive integer; assume $(n,p) = 1$ if $p > 0$. Then there are n^2-dimensional division algebras of characteristic p which are not crossed products if n is divisible by the cube of a prime. (Received June 9, 1971.)

71T-A173. GERNOT MICHAEL ENGEL, University of Wisconsin, Madison, Wisconsin 53706. The Cassini oval bounds for permanental and determinantal roots are sharp. Preliminary report.

Let M_n denote the set of $n \times n$ matrices with complex coefficients. For $A \in M_n$ and $1 \leq i \leq n$ let $R_i(A) = \sum_{j \neq i} |a_{ij}|$. Let Ω_A^R denote the set of $B \in M_n$ such that $R_i(B) = R_i(A)$ and $a_{ii} = b_{ii}$. Let $S(\Omega_A^R)$ denote the set of eigenvalues of all matrices in Ω_A^R and $P(\Omega_A^R) = \{\lambda : \exists\, B \in \Omega_A^R, \text{ permanent } (B - \lambda I) = 0\}$. Let $C(A) = \bigcup_{i \neq j} \{\lambda : |a_{ii} - \lambda|\,|a_{jj} - \lambda| \leq R_i(A)\, R_j(A)\}$. For $n = 2$ it is easily verified that $P(\Omega_A^R) = S(\Omega_A^R) = \partial C(A)$. For $n > 2$ it can be shown that $P(\Omega_A^R) = S(\Omega_A^R) = C(A)$. (Received June 10, 1971.) (Author introduced by Professor Hans Schneider.)

*71T-A174. V. S. RAMAMURTHI, Madurai College, Madurai 11, Tamilnadu, India. On hereditary torsion theories. Preliminary report.

Let R be a ring and $\mathcal{J}$ a hereditary torsion class of right R-modules, $F(\mathcal{J})$ the associated topologising and idempotent filter of right ideals of R, and $K = \bigcap \{I ; I \in F(\mathcal{J})\}$. Theorem 1. $\mathcal{J}$ is a TTF class if and only if either $R \in \mathcal{J}$ or $\mathrm{Hom}_R(K, M) \neq 0$ for each $M \notin \mathcal{J}$ and $K^2 = K$. Corollary. Let $\mathcal{J}$ be the class of right R-modules each of whose homomorphic images has a simple submodule and let $\mathcal{J}$ be a TTF class. Then a right R-module $\in \mathcal{J} \Leftrightarrow$ its Jacobson radical $\in \mathcal{J}$. Theorem 2. If for each countable set $\{X_R\}$ of simple modules in $\mathcal{J}$, there is a countable set $\{Y_R\}$ of right R-modules such that $X_R \subseteq Y_R$ and $\mathrm{Ext}_R^1(R/I, \Sigma \oplus Y) = 0$ for each $I \in F(\mathcal{J})$, then $F(\mathcal{J})$ has ACC. Corollary. If direct sums of injective hulls of simple right R-modules are injective then R is right Noetherian [cf. R. P. Kurshan, Abstract 70T-A148, these *Notices* 17(1970), 807]. Theorem 3. If each semisimple module in $\mathcal{J}$ is injective, then for any two-sided ideal $A \in F(\mathcal{J})$, R/A is a finite direct sum of simple rings. Corollary. If R is commutative and all the semisimple R-modules are injective then R is Artin semisimple [cf. V. C. Cateforis and F. L. Sandomierski, Pacific J. Math. 31(1969), 289-292]. (Received June 10, 1971.) (Author introduced by Professor R. Padmanabhan.)

71T-A175. AWAD A. ISKANDER, Vanderbilt University, Nashville, Tennessee 37203 and FAWZI M. YAQUB, American University of Beirut, Beirut, Lebanon. Commutative rings with the congruence extension property.

A class K of rings is said to have the congruence extension property if for every $A \in K$ and every subring B of A, any ideal of B is the intersection of B with an ideal of A. A commutative ring A is strongly regular if for every $a \in A$, there is a polynomial $p_a(x)$ with integral coefficients such that $p_a(a) \cdot a^2 = a$. Theorem 1. Strongly regular rings have the congruence extension property. Theorem 2. Let A be a p-ring with 1, B a subring of A, and $1 \in B$. Then M is a maximal ideal of B iff M is the intersection of B with a maximal ideal of A. (Received June 14, 1971.)

71T-A176. CARL FAITH, Rutgers University, New Brunswick, New Jersey 08903. Modules finite over endomorphism ring. I.

For a right R-module M, let $B = \text{End } M_R$. Then M is finendo in case $B^n \to M \to 0$ is exact in B-mod, for an integer n. This implies (and when M is quasinjective is implied by) the exactitude of $0 \to R/\text{ann}_R M \to M^n$. (1) A finendo module is quasinjective iff injective modulo annihilator. (2) Same as (1) for f.g. modules over commutative rings. (3) R is right artinian iff every quasinjective right module is finendo. (4) R is right selfinjective iff every finendo faithful injective right R-module generates mod-R. (5) A selfinjective ring R is right PF iff every faithful (quasi) injective right module is finendo. (6) If R is right PF, and A is an ideal, then R/A is right PF iff the left annihilator of A is $Rz = zR$, for some $z \in R$. (7) Then, every factor ring of R is right PF only if R is artinian, hence uniserial. (8) A ring R is a right Goldie prime ring iff R has a faithful finendo indecomposable injective right R-module E with no nontrivial fully invariant submodules such that $D = \text{End } E_R$ is a field. In this case $Q = \text{End}_D E$ is the right quotient ring of R, and E is the (unique up to isomorphism) minimal right ideal of Q. (Received June 14, 1971.)

*71T-A177. CHESTER E. TSAI and DAVID M. FOSTER, Michigan State University, East Lansing, Michigan 48823. Primary ideal theory for quadratic Jordan algebras. II. Preliminary report.

In Part I (see Abstract 71T-A58, these *Notices* 18(1971), 406), the concept of a primary ideal was introduced for a unital quadratic Jordan algebra $J = (J, U, 1)$. Using the notation of Part I, we say an ideal Q of J is fully primary if, whenever A and B are ideals of J with $AU_B \subseteq Q$, then either $A \subseteq Q$ or $B \subseteq r(Q)$ and either $B \subseteq Q$ or $A \subseteq r(Q)$. J has the full Artin-Rees property if whenever A and B are ideals of J there exist integers p and q such that $A \cap D^p(B) \subseteq AU_B$ and $B \cap D^q(A) \subseteq AU_B$. An ideal A of J has an nfp-representation if A is the intersection of a finite number of fully primary ideals with distinct radicals. Lemma. The radical of a fully primary ideal of J is prime. Theorem. Suppose J is noetherian and $A \lhd J$. (1) A can be represented as a finite intersection of fully primary ideals iff J has the full Artin-Rees property. (2) If A has a representation as a finite intersection of fully primary ideals, then (a) A has an nfp-representation, (b) any two such representations has the same number of components and the radicals coincide in some order, (c) the primary components below the 'isolated' primes are uniquely determined, (d) r(A) is equal to the intersection of the 'isolated' primes. (Received June 14, 1971.)

71T-A178. WOLMER V. VASCONCELOS, Rutgers University, New Brunswick, New Jersey 08903 and ARON SIMIS, Queen's University, Kingston, Ontario, Canada. Projective modules over $R[X]$, R valuation ring, are free. Preliminary report.

A pseudo-bezoutian ring is an integral domain whose group of nonzero principal fractional ideals is lattice-ordered (with the usual order defined in terms of divisibility). Lemma. If R is pseudo-bezoutian, then $\mathrm{Pic}(R) = 0$, where $\mathrm{Pic}(R)$ stands for the group of isomorphism classes of rank 1 projective R-modules. Theorem. Let R be a valuation ring of finite rank and let A be a finite $R[X]$-algebra. Then any finitely presented A-module can be written in the form $A^r \oplus L$ $(r \geqq 0)$ where L is an A-module such that $f\text{-}\mathrm{rk}_A(L) \leqq 1$ (cf. H. Bass, "Algebraic K-theory," Benjamin, New York, 1968, for the terminology employed); in particular, finitely generated projective $R[X]$-modules are free. Using a standard procedure, the first author realized that the above result could be extended to yield: Theorem. Let R be an arbitrary valuation ring; then finitely generated projective $R[X]$-modules are free. (Received June 14, 1971.)

*71T-A179. RICHARD M. WILSON, Ohio State University, Columbus, Ohio 43210. Concerning the number of mutually orthogonal Latin squares.

Let $N(n)$ denote the maximum number of mutually orthogonal Latin squares of order n. It is shown that for large n, $N(n) \geqq n^{1/17} - 2$. In addition to a known number-theoretic result, the proof uses a new combinatori construction which also allows a quick derivation of the existence of a pair of orthogonal squares of all orders $n > 14$. In addition, it is proven that $N(n) \geqq 6$ whenever $n > 90$. (Received June 14, 1971.)

71T-A180. RICHARD M. WILSON and DWIJENDRA K. RAY-CHAUDHURI, Ohio State University, Columbus, Ohio 43210. Generalization of Fisher's inequality to t-designs. Preliminary report.

An $S_\lambda(t,k,v)$ consists of a set of v points together with a distinguished family of k-subsets (called blocks) such that every t-subset of points is contained in precisely λ of the blocks. Fisher's well-known inequality states that in an $S_\lambda(2,k,v)$ with $v \geqq k+1$, the number b of blocks is $\geqq v$. Moreover, if the blocks of the $S_\lambda(2,k,v)$ can be partitioned into a number r of $S_{\lambda'}(1,k,v)$'s for some r, λ', then $b \geqq v + r - 1$. In 1968, A. Ya. Petrenjuk [Mat. Zametki 4(1968), 417-425] proved that in an $S_\lambda(4,k,v)$ with $v \geqq k+2$, the number b of blocks is $\geqq \binom{v}{2}$ and he conjectured that in an $S_\lambda(t,k,v)$ with $v \geqq k + [t/2]$, $b \geqq \binom{v}{[t/2]}$. This conjecture has been established. Moreover, if the $S_\lambda(t,k,v)$ admits a partition into r $S_{\lambda'}([t/2],k,v)$'s, then $b \geqq \binom{v}{[t/2]} + r - 1$. (Received June 14, 1971.)

71T-A181. PHILIP DWINGER, University of Illinois, Chicago, Illinois 60680. Subdirect products of infinite chains.

Let C_∞ be the class of all lattices (lattice always means: distributive lattice) which are subdirect products of copies of infinite chains without extreme elements. Note that if $L \in C_\infty$ then L has no relatively complemented elements (the converse is not true!). Anderson and Blair (Math. Ann. 143(1961), 187-211) have given necessary and sufficient conditions for a lattice to belong to C_∞ and we use their results to investigate

C_∞ and to exhibit some important subclasses of C_∞. Typical result. Let C_L be the class of lattices which admit a nontrivial group structure; let C_H be the class of lattices L, $|L| \geqq 2$ which are homogeneous in the following sense: for $\forall\, a, b \in L$, $\exists\, f \in \mathfrak{A}(L)$ with $f(a) = b$ and if $a < b$ ($a \nless b$ and $a \ngtr b$) then $x < f(x)$ ($x \nless f(x)$ and $x \ngtr f(x)$) for $\forall\, x \in L$; let C_M be the class of lattices L, $|L| \geqq 2$ such that every proper prime ideal of L is neither maximal nor minimal. Then $C_L \subsetneq C_H \subset C_M \subset C_\infty$. Example of a lattice $L \in C_\infty \sim C_M$: every infinite free lattice. More generally: every infinite coproduct of lattices belongs to $C_\infty \sim C_M$.

(Received June 15, 1971.)

*71T-A182. ROBERT WILLIS QUACKENBUSH, University of Manitoba, Winnipeg, Manitoba, Canada. Models for Mal'cev conditions: n-permutability.

Let $\mathcal{K}$ be an equational class of algebras; for $\mathfrak{A} \in \mathcal{K}$, $\mathcal{C}(\mathfrak{A})$ is the congruence lattice of $\mathfrak{A}$. If θ, $\varphi = \mathcal{C}(\mathfrak{A})$ then they are n-permutable if $\theta \vee \varphi = \theta \circ \varphi \circ \theta \circ \ldots$ where $\circ$ appears $n-1$ times. If every pair of congruences in $\mathcal{C}(\mathfrak{A})$ are n-permutable then $\mathfrak{A}$ has n-permutable congruences; if every $\mathfrak{A} \in \mathcal{K}$ has n-permutable congruences then $\mathcal{K}$ has n-permutable congruences. For $n = 2$ we use the term permutable. A. Mal'cev has shown that $\mathcal{K}$ has permutable congruences iff $\mathcal{K}$ has a ternary polynomial $p(x,y,z)$ satisfying $p(x,z,z) = x = p(z,z,x)$. Similarly $\mathcal{K}$ has n-permutable congruences iff $\mathcal{K}$ has polynomials satisfying certain identities Σ_n. It is trivial that n-permutability implies (n+1)-permutability. Conversely, it is obvious that n-permutability does not imply (n - 1)-permutability. The purpose of this paper is to show that the obvious is indeed true; equational classes $\mathcal{K}_n$ are constructed which are n-permutable but not (n - 1)-permutable. An n-element algebra $\mathfrak{A}_n$ is constructed such that $\mathfrak{A}_n$ has n-permutable but not (n - 1)-permutable congruences and such that every function on $\mathfrak{A}_n$ which preserves $\mathcal{C}(\mathfrak{A}_n)$ is an operation of $\mathfrak{A}_n$. It is then shown that some of these operations satisfy Σ_n. $\mathcal{K}_n$ is then the equational class generated by $\mathfrak{A}_n$. (Received June 17, 1971.)

*71T-A183. JOHN HAYS, University of Maine, Orono, Maine 04473. Generalized theory of combinations. Preliminary report.

Elementary combination theory, as algebra, has analogues in Boolean algebra, and in classical set and statement theory. A rho-set analogue of Brouwer algebra and intuitionist logic (Abstract 71T-E58, these *Notices* 18(1971), 666) motivates a generalized algebra of combinations: elementary theory is quotient algebra. General Occupancy Problem. Assign $f \leqq i$ indistinguishables to c cells (maximal occupancy o_j in cell c_j, $j \leqq c$). General Combination Problem. Select $e \leqq d$ elements of t types (multiplicity m_k of type t_k, $k \leqq t$). Set $i = d$, $c = t$, $j = k$, $f = e$. Metalemma. The General Occupancy and Combination Problems are dual. Theorem. The generator for generalized combinations is $\Pi_1^t \Sigma_0^{m_j} t^j$; its enumerator is $\Pi \Sigma u^i$, for indeterminate u. Definition ("en sequential"). $, n = 0, 1, \ldots, n$; $,j(u) = 1, 1 + u, \ldots, 1 + u + \ldots + u^j$; $U[,j;r]$, tabulated by row $,j$, column r, enumerates combination densities, with unumerator $U[,j;r;u]$. Theorem. $U[,j;r] = \Sigma_0^i U[,(j-1);r-k]$, $U[,0;0] = 1$, $U[,j;r<u] = 0$; $U[,j;r;u] = \Sigma_0^j u^i \cdot \ldots \cdot \Sigma_0^1 u^i$. Algorithms are demonstrated for computing, from these functions, all generalized densities and distributions, for tables of densities. Extending an operation of rho-theory, a fundamental decomposition theorem is stated. The generalized theory is shown to determine a variety in universal algebra. (Received June 17, 1971.)

*71T-A184. CHUNG-HARNG SU, University of Wyoming, Laramie, Wyoming 82070. An algorithm for putting a matrix into a Jordan form. Preliminary report.

Let A be an $n \times n$ matrix over the complex field C, representing the linear mapping T. Let λ be an eigenvalue of T and denote $T - \lambda I$ by T_λ. A nonzero vector v such that $T_\lambda^m v = 0$ is called a generalized eigenvector (g.e.v.) of degree m. If $T_\lambda^m v = 0$ and $T_\lambda^{m-1} v \neq 0$, then v is called a g.e.v. of essential degree m, and the set $C(T_\lambda, v) = \{v, \cdots, T_\lambda^{m-1} v\}$ is called the cycle of v w.r.t. T_λ. The following results are used in the construction of the algorithm. Theorem 1. Let λ be an eigenvalue of T, α the ascent of T_λ. Then there exist $E_i = \{v_{ij} : j = 1, \cdots, f_i\}$, $i = 1, \cdots, \alpha$, such that (i) every v_{ij} is a g.e.v. of essential degree i for $j = 1, \cdots, f_i$; (ii) $f_i = \dim(N(T_\lambda^i)) - \dim(N(T_\lambda^{i-1}))$; (iii) $E_1 \cup \cdots \cup E_\alpha$ is linearly independent; (iv) $N(T_\lambda^b) = [E_1 \cup \cdots \cup E_b]$ for $b = 1, \cdots, \alpha$. Theorem 2. With the same notations of Theorem 1, let $d \leqq c \leqq \cdots \leqq \alpha$, $w \in E_d$, $x \in E_c, \cdots, z \in E_\alpha$. If $C(T_\lambda, x) \cup \cdots \cup C(T_\lambda, z)$ is linearly independent and $C(T_\lambda, w) \cup C(T_\lambda, x) \cup \cdots \cup C(T_\lambda, z)$ is linearly dependent, then (*) $T_\lambda^{d-1} w = \xi_{c-1} T_\lambda^{c-1} x + \cdots + \zeta_{\alpha-1} T_\lambda^{\alpha-1} z$. Theorem 3. With the same notations of Theorems 1 and 2, if (*) holds, then $[E_1 \cup \cdots \cup E_\alpha] = [E_1 \cup \cdots \cup E_\alpha \backslash \{w\} \cup C(T_\lambda, x) \cup \cdots \cup C(T_\lambda, z)]$. (Received June 22, 1971.) (Author introduced by Professor V. M. Sehgal.)

71T-A185. VANCE FABER, University of Colorado, Boulder, Colorado 80302. Reconstruction of graphs from indexed p - 2 point subgraphs. Preliminary report.

We prove the following theorem related to Ulam's conjecture (S. M. Ulam, "A collection of mathematical problems," p. 29): Theorem. Suppose G and H are graphs with $n \geqq 4$ points. If there are labelings of G and H such that for every $i \neq j$ the induced subgraphs G_{ij} and H_{ij} ($G_{ij} = G \backslash \{v_i, v_j\}$ where $G = \{v_1, \ldots, v_n\}$) have the same number of edges, then there is a label preserving isomorphism between G and H. We also prove generalizations of this theorem showing that any number of points from 2 up to $n - 2$ may be removed with the same results; and that similar versions hold for many other kinds of relational structures including k-uniform set systems, digraphs and arbitrary square matrices. The proofs are all based on the proof of the fact that the map $A = \{a_{ij}\} \to T(A)$, where $(T(A))_{ij} = \Sigma\{a_{\ell m} \mid \ell \neq i, m \neq j\}$, is a nonsingular linear transformation on the space of $n \times n$ matrices over a field of characteristic zero. (Received June 25, 1971.)

*71T-A186. BENNY EVANS and JAMES BOLER, Rice University, Houston, Texas 77001. The free product of residually finite groups amalgamated along retracts is residually finite. Preliminary report.

Let $\{G_i : i \in I\}$ be a collection of residually finite groups with distinguished subgroups $K_i < G_i$, such that K_i is isomorphic with K_j for each $i, j \in I$. If K_i is a retract of G_i for each i, then the generalized free product of the groups G_i amalgamated along the subgroups K_i is again a residually finite group. As a corollary, it is shown that a Knot space (the complement of a tamely embedded simple closed curve in S^3) has residually finite fundamental group if and only if each of its prime components has residually finite fundamental group. (Received June 18, 1971.) (Author introduced by Professor John P. Hempel.)

71T-A187. EDMUND H. FELLER and RICHARD L. GANTOS, University of Wisconsin, Milwaukee, Wisconsin 53201. Hereditary semigroups.

A S-system F is free provided there exists a subset $X \subset F$ such that each element of F has a unique representation of the form xs, $x \in X$, $s \in S$. A semigroup S is hereditary provided each right ideal is projective. Theorem. Every subsystem of a free S-system is the disjoint union of subsystems each isomorphic to a right ideal of S. Theorem. A semigroup is hereditary iff every subsystem of a projective system is projective. Theorem. If S is a cancellative principal ideal semigroup, then subsystem of free systems are free. (Received June 28, 1971.)

71T-A188. GEORGE S. MONK, University of Washington, Seattle, Washington 98115. A characterization of exchange rings. Preliminary report.

In [R. B. Warfield, "Exchange rings and decompositions of modules," Math. Ann., to appear], R. Warfield introduces the class of exchange rings and shows that among other interesting properties they possess, they are the endomorphism rings of modules with the finite exchange property. He then goes on to give a necessary condition for a ring to be an exchange ring. We can now prove: Theorem. The ring R (with identity) is an exchange ring if and only if given an element a in R, there is an idempotent f in R and elements c and d in R such that $caf = f$, $fc = c$, and $(1-f)d(1-a)(1-ca)(1-f) = 1-f$. It can also be shown that this condition on the ring R is strictly weaker than that given by Warfield. (Received June 28, 1971.)

*71T-A189. DENNIS SPELLMAN, Courant Institute, New York University, New York, New York 10012. On the respective terms of the lower central series of a free group and a normal subgroup.

Let F be a free group and S a normal subgroup of finite index. Let F_{m+1} and S_{m+1} be the $(m+1)$st terms in the lower central series of F and S respectively. Then S_{m+1} is a normal subgroup of F_{m+1}. The problem considered is the investigation of the structure of the quotient group F_{m+1}/S_{m+1}. Let $1 < \text{rank}(F) = n < \infty$ and $1 < |F:S| = j < \infty$. The principal results are the following: (1) There is an exact sequence $1 \to F_{m+1}/S_{m+1} \cap S/S_{m+1} \to F_{m+1}/S_{m+1} \to (F/S)_{m+1} \to 1$. (2) If $\max(j,n) > 2$, then $F_{m+1}/S_{m+1} \cap S/S_{m+1}$ is a finitely generated torsion free nilpotent group of class exactly m. Observe that if F/S is abelian, then $F_{m+1}/S_{m+1} \cap S/S_{m+1} = F_{m+1}/S_{m+1}$ for $m = 1,2,3,\ldots$. (3) If $j = n = 2$, then F_3/S_3 is free abelian of rank 3. (Received June 25, 1971.)

Analysis

71T-B161. SAMUEL ZAIDMAN, Université de Montréal, Montréal, Québec, Canada. A generator of a strongly continuous semigroup.

Let $\mathfrak{X} = C_0[0,1]$ -- the Banach space of continuous functions $\varphi(t)$ such that $\varphi(0) = \varphi(1) = 0$, with $\|\varphi\| = \max_{0\leq t\leq 1}|\varphi(t)|$. Let A be the differential operator $= d^2/dt^2$, where $D(A) = \{\varphi \in C^2(0,1) \cap C_0[0,1]$, such that $\varphi''(t) \in C_0[0,1]\}$. Then A is the infinitesimal generator of a strongly continuous contraction semigroup of linear continuous operators of $\mathfrak{X}$ into itself. (Received December 4, 1970.)

*71T-B162. ANDRE de KORVIN and LAURENCE E. KUNES, Indiana State University, Terre Haute, Indiana 47809. Some nonweak integrals defined by linear functionals.

For notations and basic definitions the reader is referred to Abstract 71T-B28, these *Notices* 18(1971), 406. Let H denote a normal space, and $C_{tb}(H, E)$ denote all continuous functions from H into E whose range is totally bounded (E a Banach space). $C_b(H, E)$ will denote functions which are continuous and bounded. Σ will denote the smallest field containing closed sets. Let T be a continuous linear operator from $C_b(H, E)$ into F (F a Banach space). The following concepts on T are introduced: T has the weak c.a. property, T has the strong c.a. property and T is of bounded variation. Theorem. Let T be a bounded linear operator from $C_b(H, E)$ into F. Assume T is weakly compact and has the weak c.a. property. Then there exists a finitely additive set function m from Σ into $L(E, F)$ such that $\tilde{m}_\Sigma(H) < \infty$, every φ in $C_{tb}(H, E)$ is m-integrable and $T(\varphi) = \int \varphi\, dm$, m_{x,y^*} is regular, $T^*(y^*)$ may be identified with m_{y^*}. If T is of bounded variation then $\bar{m}_\Sigma(H) < \infty$. If T is of bounded variation and has the strong c.a. property then m can be extended to a Borel measure. (Received March 11, 1971.)

*71T-B163. SWARUPCHAND M. SHAH, University of Kentucky, Lexington, Kentucky 40506 and SHANTILAL N. SHAH, Syracuse University, Syracuse, New York 13210. Entire functions of strongly bounded index. II.

This paper is in continuation of our earlier paper with the same title. Theorem. Let $a_n \neq 0$ $(n \geq 1)$, $\{a_n\}_{n=n_0}^\infty$ be a positive strictly increasing sequence such that $a_{n+1} - a_n \geq b_n$ $(n \geq n_0)$, where $\{b_n\}_{n=n_0}^\infty$ is positive, nondecreasing and $\sum(1/nb_n) < +\infty$. Let α, β be any two complex numbers. Then $f(z) = e^{\alpha z+\beta}\prod_{n=1}^\infty(1 + z/a_n)$ is an entire function of strongly bounded index. (Received March 31, 1971.)

71T-B164. H. M. SRIVASTAVA, University of Victoria, Victoria, British Columbia, Canada. Certain formulas associated with generalized Rice polynomials. II. Preliminary report.

This paper is a continuation of the earlier work of the present author [Ann. Polon. Math. 23(1970), 109-115; see also Abstract 69T-B27, these *Notices* 16(1969), 411] and it discusses the possibilities of extending certain linear and bilinear relations involving generalized Rice polynomials to hold for various classes of generalized hypergeometric polynomials. Among other observations, simple and direct proofs are attributed to a number of results involving generalized hypergeometric polynomials as well as to their generalizations presented in this paper. (Received April 2, 1971.)

71T-B165. RICHARD F. BASENER, Brown University, Providence, Rhode Island 02912. On certain rationally convex hulls in C^2. Preliminary report.

Let S be a compact connected subset of the closed unit disk Δ such that $\partial\Delta \subseteq S$. Let $X = X(S) = \{(z,w) \in C^2 \mid |z| = 1$ and $w \in S$, or $|w| = 1$ and $z \in S\}$. Wermer constructed a specific S such that the rationally convex hull of X, $h_r(X)$, does not contain analytic structure even though $h_r(X)\backslash X$ is nonempty ("On an example of Stolzenberg", Symposium on Several Complex Variables, Park City, Utah, 1970, Lecture Notes in Math., no. 184, Springer-Verlag, Berlin, 1971, pp. 78-84). Let S be arbitrary and choose a nested sequence $\{S_n\}$ of smoothly bounded compact connected subsets of Δ such that $S = \bigcap_{n=1}^{\infty} S_n$. Let $u_n \in C(S_n)$ be harmonic on int S_n, $u_n = 0$ on $\partial\Delta$ and $u_n = 1$ on $\partial S_n \backslash \partial\Delta$. For $z \in S$ let $u(z) = \lim_{n\to\infty} u_n(z)$. Theorem 1. $h_r(X) = \{(z,w) \in S \times S \mid u(z) + u(w) \leq 1\}$. Theorem 2. If $(z,w) \in h_r(X)\backslash X$ and $u(z) + u(w) < 1$, then (z,w) belongs to a Gleason part of $R(X)$ which has positive 4-dimensional measure. (Received April 29, 1971.)

71T-B166. ROBERT W. REDDING and HERBERT SILVERMAN, Clark University, Worcester, Massachusetts 01610. The Koebe constant for α-spiral-like functions. Preliminary report.

Definition. For $|\alpha| \leq \pi/2$, and for $f(z)$ analytic in $|z|<1$, $f(z)$ is α-spiral-like provided $\mathrm{Re}\{e^{i\alpha} zf'(z)/f(z)\} \geq 0$. Using the Herglotz representation theorem and variational techniques, the following is shown. Theorem I. For a given entire function $\varphi(w)$ and fixed z, $|z| < 1$, $\mathrm{Max}\,|\varphi(\log(f(z)/z))|$ in the class S_α is attained only by $f(z) = z/(1-e^{-it}z)^{2\cos(\alpha)\exp(-i\alpha)}$. Corollary. For $f(z) \in S_\alpha$ and $|z| < 1$, $|f(z)| \geq 1/4^{\cos^2(\alpha)}$ with equality only for $f(z) = z/(1-e^{-it}z)^{2\cos(\alpha)\exp(-i\alpha)}$. (Received May 4, 1971.)

71T-B167. PAUL WILLIG, Stevens Institute of Technology, Hoboken, New Jersey 07030. Type II W-* algebras are not normal.

A W-* algebra R with center Z is called normal if for every W-* subalgebra S of R containing Z we have $S = C(S,R)$, where $C(S,R) = (S' \cap R)' \cap R$. Fuglede and Kadison proved that no type II factor is normal (Proc. Nat. Acad. Sci. U.S.A. 37(1951), 420-425). Using direct integral decompositions, we extend this result as follows. Theorem. Let R be a W-* algebra of pure type II on separable Hilbert space H. Then R contains a W-* subalgebra S whose center is Z and for which $C(S,R) \neq S$. (Received May 4, 1971.)

71T-B168. SIDNEY A. MORRIS, University of Florida, Gainesville, Florida 32601. A topological group characterization of those locally convex spaces having their weak topology.

Theorem. Let E be a locally convex Hausdorff real (or complex) topological vector space. Then E has its weak topology if and only if every discrete subgroup (of the additive group structure) of E is finitely generated. (Received May 5, 1971.)

*71T-B169. JAMES W. NOONAN, 5600 54th Avenue, Apartment 106, Riverdale, Maryland 20840. Close to convex functions of order β. Preliminary report.

For $\beta \geqq 0$ denote by $K(\beta)$ the class of normalized functions f analytic in the unit disc such that for some starlike function s, $|\arg zf'(z)/s(z)| \leqq \beta\pi/2$. We use the following notation: $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$; $F_\beta(z) = (2(\beta+1))^{-1}\{((1+z)/(1-z))^{\beta+1} - 1\} = z + \sum_{n=2}^{\infty} A_n(\beta)z^n$; $M(r,f') = \max_{|z|=r}|f'(z)|$; $2\pi I_\lambda(r,f) = \int_0^{2\pi}|f(re^{i\theta})|^\lambda d\theta$. Theorem 1. Let $f \in K(\beta)$. Then $\omega = \lim(1-r)^{\beta+2}M(r,f')$ exists and is finite. Also, $\lim_{n\to\infty}|a_n|/n^\beta = \omega/\Gamma(\beta+2)$. Corollary. Let $f \in K(\beta)$. Then there exists an integer n(f) depending on f such that $|a_n| \leqq A_n(\beta)$ for $n \geqq n(f)$. Theorem 2. Let $f \in K(\beta)$ and $2\lambda > 1$. Then $\lim_{r\to 1}(1-r)^{\lambda(\beta+2)-1}I_\lambda(r,f') = \omega^\lambda C(\lambda,\beta)$ where $C(\lambda,\beta)$ depends only on λ and β. Also, for $\lambda \geqq 1$, $\lim\sup_{r\to 1}(1-r)^{\lambda(\beta+1)-1}I_\lambda(r,f) = 0$ if and only if $\lim\inf_{r\to 1}(1-r)^{\lambda(\beta+1)-1}I_\lambda(r,f) = 0$. Theorem 3. Let $\beta \geqq 0$, and m be a positive integer. Then $f \in K(\beta)$ if and only if $f'(z^m) = g'(z)^m$ where $g \in K(\beta/m)$ and is m-fold symmetric. For $\alpha > 0$, let $B(\alpha)$ be the class of α-Bazilevič functions. Theorem 4. If $f \in K(1/\beta)$, then $g \in B(\beta)$ where $g(z) = \{\beta\int_0^z t^{\beta-1}f'(t)^\beta dt\}^{1/\beta}$. If $g \in B(1/\alpha)$, then $f \in K(\alpha)$ where $f(z) = \int_0^z (g(t)/t)^{1-\alpha}(g'(t))^\alpha dt$. Corollary. $g \in B(\alpha)$ if and only if, for each fixed r, $0 < r < 1$, the tangent to the curve $C(r) = \{g(re^{i\theta})^\alpha : 0 < \theta \leqq 2\pi\}$ does not turn back on itself more than π radians. (Received May 14, 1971.)

71T-B170. GEORGE J. KERTZ, University of Toledo, Toledo, Ohio 43606 and FRANK H. LUEBBERT, 2407 Albert Lee Avenue, Sedalia, Missouri 65301 and St. Louis University, St. Louis, Missouri 63103. On a series associated with a Kennedy series.

In 1931, E. S. Kennedy (Amer. J. Math. 58(1941), 443-459) introduced a series which he called an exponential analogue of the Lambert series and designated as the "ordinary R-series." In this paper the derived series of the ordinary R-series is investigated. The regions of convergence, absolute convergence, and uniform convergence are determined; and sufficient conditions for the imaginary axis to be a natural boundary for the function defined by this series are obtained. (Received May 17, 1971.)

*71T-B171. DAVID H. CARLSON, University of Missouri, Columbia, Missouri 65201. Compact elliptic sets do not exist in E^n. Preliminary report.

Let π be a dynamical system on a locally compact T_2 space X. A compact set C contained in X is said to be elliptic if there is a neighborhood U of C such that for every point in U the α and ω limit points are contained in C. Theorem. A necessary condition for C to be elliptic is that C be contained in an open and compact invariant set. This answers Nemytskii's conjecture that elliptic points do not exist in E^n (V. V. Nemytskii, "Topological classification of singular points and generalized Lyapunov functions," Differential Equations 3(1967) (translation 1971)). Corollary. Let U be a compact neighborhood of a simple rest point. If the points elliptic to the rest point are closed in U then it is a saddle rest point with at least one parabolic trajectory. (Received May 17, 1971.)

*71T-B172. C. J. EVERETT and EDMOND D. CASHWELL, Los Alamos Scientific Laboratory, Los Alamos, New Mexico 87544. Functionals additive on a symmetric set.

A Galois correspondance $S \to T^*S$, $T \to S^*T$ is set up between a lattice S^* of symmetric sets S of pairs (X, Y) of vectors of a Hilbert space, and a lattice T^* of subspaces T of continuous functionals t on H, T^*S being the space of all t additive on S, and S^*T the additivity domain of the space T. Perhaps the most interesting of the problems discussed from this viewpoint is the characterization of those momentum distributions consistent with the relativistic elastic collision relation. Some positive results are (1) the only pairs on which all functionals "through the origin" are additive satisfy $\|X\| \|Y\| = 0$; (2) the only pairs on which all homogeneous functionals are additive are collinear; (3) if t is additive on all orthogonal pairs, and on all unidirectional pairs, or on all oppositely directed pairs, then t is linear. (Received May 17, 1971.)

*71T-B173. GEORGE M. MULLER, Stanford Research Institute, Menlo Park, California 94025. Linear iteration and summability. III.

An earlier result concerning the iterative solution of linear operator equations (Abstract 68T-313, these *Notices* 15(1968), 392) has been extended with the use of a summability method of A. Meir (Israel J. Math. 1(1963), 224) and the Dunford-Taylor representation of a bounded linear operator. Let $q(z)$ be a complex polynomial (of degree $k \geq 1$) such that $q(1) = 1$, S the set $\{z : \operatorname{Re} q(z) < 1\}$ and $\Lambda q(z)$ the polynomial $(q(z)-1)/(z-1)$. Let $\{d_n\}$ be a sequence of complex numbers for which the $[F, d_n]$ transformation sums the geometric series in $\operatorname{Re} z < 1$, uniformly on compact subsets--e.g., $d_n = n - 1$. Construct sequences $\{\alpha_n\}$, $\{\bar{\alpha}_n\}$ as follows: for $n = 1, 2, 3, \ldots$, set $\alpha_n = (1 + d_n)^{-1}$ and determine the k numbers $\beta_{n,i}$ (uniquely except for order) by $\alpha_n q(z) + 1 - \alpha_n \equiv \Pi_{i=0}^{k-1}(\beta_{n,i} z + 1 - \beta_{n,i})$; set $\bar{\alpha}_n = \beta_{m,i}$ where $m = [n/k]$, $i = n - mk$. In a complex Banach space with norm $\|\cdot\|$, let L be a bounded linear operator with spectrum in S, I the identity operator, v_0, f arbitrary vectors. Determine sequences $\{v_n\}$, $\{w_n\}$ by $v_n = (\alpha_n q(L) + 1 - \alpha_n)v_{n-1} + \alpha_n \Lambda q(L) f$; $w_0 = v_0$, $w_n = (\bar{\alpha}_n L + 1 - \bar{\alpha}_n) w_{n-1} + \bar{\alpha}_n f$. Then $v_n = w_{kn}$, and $\|v_n - (I - L)^{-1} f\|$, $\|w_n - (I - L)^{-1} f\| \to 0$, i.e., both sequences converge strongly to the (unique) solution of $y = Ly + f$. (Received May 19, 1971.)

71T-B174. R. M. GOEL and VIKRAMADITYA SINGH, Punjabi University, Patiala, India. Coefficient estimates for a class of star-like functions. Preliminary report.

Let $S^*(\alpha)$ denote the class of regular univalent functions $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ in the unit disc $|z| < 1$ satisfying the condition $|\arg(zf'(z)/f(z))| < \pi\alpha/2$, $0 < \alpha \leq 1$. In a recent paper [Canad. J. Math. 22(1970), 476-485] due to D. A. Brannan, J. Clunie and W. E. Kirwan sharp upper bounds for $|a_2|$ and $|a_3|$ have been obtained using the Herglotz formula for analytic functions whose real part is positive in $|z| < 1$. In this paper, using the method of Nehari and Netanyahu [Proc. Amer. Math. Soc. 8(1957), 15-23] the following theorem is proved. Theorem. Let $f(z) \in S^*(\alpha)$, then $|a_4| \leq 2\alpha/3$ for $0 < \alpha \leq \sqrt{2/17}$, $\leq 2\alpha(2\alpha^2 + 1)/9$ for $\sqrt{2/17} \leq \alpha \leq 1$, $|a_5| \leq \alpha/2$ for $0 < \alpha \leq \alpha_1$, $\leq \alpha^2(38\alpha^2 + 7)/9$ for $3/7 \leq \alpha \leq 1$, where α_1 is the smallest positive root of the equation $232\alpha^5 + 680\alpha^4 + 116\alpha^3 - 329\alpha^2 - 33\alpha + 36 = 0$. The bounds are sharp. Some other estimates on certain combinations of a_2 and a_3 have also been obtained. (Received June 2, 1971.)

*71T-B175. JOHN W. NEUBERGER, Emory University, Atlanta, Georgia 30322. Quasi-analyticity and semigroups of bounded linear transformations. Preliminary report.

Suppose H is a real Banach space and T is a strongly continuous semigroup of bounded linear transformations from H to H. Theorem. If $\lim\inf_{x\to 0}|T(x) - I| < 2$ then the set of all functionals of trajectories of T form a quasi-analytic collection and T(x) is invertible for all $x > 0$ (although $(T(x))^{-1}$ may be unbounded). Lemma. Suppose f is a real-valued continuous function with domain $[0,1]$ so that $f(x) = 0$ if $0 \leq x \leq 1/2$ and, if $y > 1/2$, then there is x in $(1/2, y)$ so that $f(x) \neq 0$. Suppose furthermore that $\{\delta_q\}_{q=1}^{\infty}$ is a sequence of positive numbers converging to 0 and Q is an open interval containing 1. If q is a positive integer for which there is a positive integer n so that $n\delta_q \in Q \cap [0, 1]$, then denote by z(q) the set of all such n, denote by n(q) the largest element of z(q) and denote $\sup\{|\sum_{r=0}^{n}\binom{n}{r}(-1)^{n-r} f(r\delta_q)| : n \in z(q)\}$ by F(q). Then $\lim_{q\to\infty} F(q)^{1/n(q)} = 2$. (Received June 7, 1971.)

*71T-B176. IGOR KLUVANEK, Flinders University of South Australia, Bedford Park, South Australia. The range of vector measure.

There is a set P, a σ-algebra S of its subsets and a vector measure $\mu : S \to X$ with values in a quasi-complete locally convex topological vector space without atoms (i.e. every set $E \in S$ with $\mu(E) \neq 0$ contains $F \in S$ with $0 \neq \mu(F) \neq \mu(E)$) such that the weak closure of its range $\{\mu(E) : E \in S\}$ is not convex. (Received June 7, 1971.) (Author introduced by Professor J. Jerry Uhl, Jr.)

*71T-B177. JOHN DAVID LOGAN, University of Dayton, Dayton, Ohio 45409. First integrals in the discrete variational calculus.

In the discrete calculus of variations, a necessary condition for the sum $J = \sum_M^N F(n, r_n, r_{n-1})$ to have an extremum for a given sequence r_n, $n = M, \ldots, N - 1$, is that r_n satisfy the second order difference equation (the discrete Euler equation) $F_y(n, r_n, r_{n-1}) + F_z(n+1, r_{n+1}, r_n) = 0$ where F_y and F_z denote partial derivatives of the continuously differentiable function $F(x, y, z)$. It is shown that if J is invariant with respect to an infinitesimal transformation of r_n given by $\bar{r}_n = r_n + \epsilon s_n$, then a first integral of the discrete Euler equation can be determined explicitly. This is the discrete analog of the classical Noether theorem. Also, for the higher order problem $J = \sum_M^N F(n, r_n, r_{n-1}, \ldots, r_{n-p})$, the discrete Euler equation is derived and first integrals are calculated using invariance procedures. (Received June 11, 1971.)

*71T-B178. RICHARD M. ARON, University of Kentucky, Lexington, Kentucky 40506. Holomorphy types for open subsets of a Banach space.

In "Holomorphic functions on a Banach space", Bull. Amer. Math. Soc. 76(1970), S. Dineen defines and studies the space of entire functions of α-holomorphy type θ from one complex Banach space E to another complex Banach space F, and the corresponding topology T_θ. We extend many of Dineen's results to the case $(H_\theta(U; F), T_\theta)$, where U is an arbitrary open subset of E. For example, we characterize the topology T_θ by exhibiting a generating family of seminorms, and use this to show that $(H_\theta(U; F), T_\theta)$ is complete. When θ is the current holomorphy type, $H_\theta(U;F) = H(U;F)$. A characterization of bounded and relatively compact subsets of $(H_\theta(U;F), T_\theta)$ is given, which extends a similar result in "Topology on spaces of holomorphic mappings" by L. Nachbin, Springer-Verlag, Berlin and New York, 1968. (Received June 14, 1971.)

71T-B179. MAX SHIFFMAN, 16,913 Meekland Avenue, Hayward, California 94541 and California State College, Hayward, California 94542. A notion of strong sub- or super-additivity of set functions.

Considering the exterior measure $m_e(S)$ of a set S is it shown to have a property which is here called strong subadditivity. Likewise for interior measure $m_i(S)$ and strong superadditivity. It is shown that average measure $m_a(S)$, while subadditive, is not strongly subadditive. Linear combinations $\alpha m_i(S) + \beta m_e(S)$ are discussed, considering additional properties such as monotonicity and nonnegativity. General set functions of the form $\varphi(m_i(S), m_e(S))$ are discussed, and an example is given in which the function is subadditive, and monotone increasing, and homogeneous of degree 1, and equal to m(S) for measurable sets, and which is not linear. However, it is not strongly subadditive. Some particular nonnegative set functions are here called the measure span, which is strongly subadditive, and the nonmeasurable ratio, which is subadditive; both are countably subadditive. It is shown that if $\varphi(m_i(S), m_e(S))$ is strongly subadditive, and monotone increasing, then it must be a convex function of $m_e(S)$ alone; if it is in addition equal to m(S) for measurable sets, or is homogeneous of degree one, then it must be $m_e(S)$. Similarly in connection with strong superadditivity and $m_i(S)$. (Received June 14, 1971.)

*71T-B180. PREM K. KULSHRESTHA, Louisiana State University, New Orleans, Louisiana 70122. Generalized convexity in conformal mappings. Preliminary report.

Let P denote the class of functions p(z) which are regular in the unit disk $E : \{z : |z| < 1\}$ and are such that $p(0) = 1$, $\operatorname{Re} p(z) > 0$. A class of regular univalent functions f(z) in E, expressible in terms of $p(z) \in P$, and consisting of starlike functions f(z) of order α, $0 \leqq \alpha < 1$, and normalized by the conditions $f(0) = 0$, $f'(0) = 1$ with the properties that $f(z)f'(z)/z \neq 0$ and $\operatorname{Re}\{(1-k)zf'(z)/f(z) + k[1 + zf''(z)/f'(z)]\} \geqq 0$ in E, where k is a real constant, $0 \leqq k \leqq 1$, is investigated and some mapping properties of functions of this class, specially those related to their extremum, distortion and rotation theorems are proved, which reduce to some well-known results for particular values of the parameters. (Received June 15, 1971.)

71T-B181. MICHAEL KEISLER, North Texas State University, Denton, Texas 76203. An elementary proof of a form of the Lipschitz portion of the Bochner-Radon-Nikodym theorem.

Suppose U is a set, F is a field of subsets of U, p is the set of real valued functions on F, $\int_v \varphi$, for $v \in F$ and $\varphi \in p$, is the refinement-wise limit of appropriate sums over finite subdivisions of v. Theorem. Suppose $\xi, \mu \in p$, each additive and bounded, such that for some $k \geqq 0$, $|\xi(I)| \leqq k|\mu(I)|$, for $I \in F$. If $c > 0$, then there exists $\beta \in p$ such that $\int_U \int_I |\xi - \beta(I)\mu| < c/2$ (and hence a subdivision D such that $\sum_D \int_I |\xi - \beta(I)\mu| < c$). Proof. Let $T = \int_U |\mu| + 1$; $\alpha_n = -k + nc/2T$, $n = 0, 1, \ldots, N$, where $\alpha_N \geqq k$; and $\beta(I) = \min\{\alpha_m | \int_I |\xi - \alpha_m \mu| = \min\{\int_I |\xi - \alpha_n \mu|\}_{n=0}^N\}$, for $I \in F$. There exists, for $I \in F$, $w(I) \in \{m | \xi(I) \in [\alpha_m \mu(I), \alpha_{m+1}\mu(I)]\}$ so that $\min\{|\xi(I) - \alpha_n \mu(I)|\}_{n=0}^N \leqq |\xi(I) - \alpha_{w(I)}\mu(I)| \leqq (\alpha_{w(I)+1} - \alpha_{w(I)})|\mu(I)| \leqq c|\mu(I)|/2T$. Since $\int_U \min\{\int_I |\xi - \alpha_n \mu|\}_{n=0}^N$

exists and $= \int_U \min\{|\xi(I) - \alpha_n \mu(I)|\}_{n=0}^{N}$, which is $\leqq \int_U c|\mu|/2T < c/2$, then by definition of β, $\int_U \int_I |\xi - \beta(I)|$ exists and is $< c/2$. (Received June 4, 1971.)

*71T-B182. PETRU MOCANU, Babĕs-Bolyai University, Cluj, Romania and MAXWELL O. READE, University of Michigan, Ann Arbor, Michigan 48104. The order of starlikeness of certain univalent functions.

In an earlier note [Abstract 683-B12, these *Notices* 18(1971), 365] the authors proved that every α-convex function f(z) is starlike for $0 \leqq \alpha \leqq 1$. The authors now show that every α-convex function f(z) is convex (and univalent) for $\alpha \geqq 1$. Moreover, for $\alpha \geqq 0$, the authors show that Re $zf'(z)/f(z) \geqq \beta(\alpha) \equiv \alpha/2^{2/\alpha} \int_0^1 x^{(1-\alpha)/\alpha} dx/(1+x)^{2/\alpha}$ holds; that is, for $\alpha \geqq 0$, each α-convex f(z) is also starlike of order $\beta(\alpha)$. Since $\beta(1) = 1/2$, the present result may be considered as a generalization of a classic result due to Marx [Math. Ann. 107(1932), 40-67] and Strohäcker [Math. Z. 37(1933), 356-380]. (Received June 21, 1971.)

*71T-B183. H. M. SRIVASTAVA, University of Victoria, Victoria, British Columbia, Canada and J. P. SINGHAL, University of Jodhpur, Jodhpur, India. Some extensions of the Mehler formula. Preliminary report. II.

In a recent paper [H. M. Srivastava and J. P. Singhal, "Some extensions of the Mehler formula", Proc. Amer. Math. Soc. 31(1972), to appear; see also Abstract 71T-B47, these *Notices* 18(1971), 412], the authors applied some operational techniques to prove an elegant unification of several interesting extensions of the well-known Mehler formula for Hermite polynomials given earlier by L. Carlitz [Collect. Math. 21(1970), 117-130; see also Boll. Un. Mat. Ital. Ser. IV 3(1970), 43-46]. This paper is a continuation of the earlier work of the present authors. It incorporates a number of further generalizations of the Mehler formula and it shows how these results may be applied to sum certain symmetric series involving the products of several Hermite polynomials. (Received June 21, 1971.)

71T-B184. C. J. EVERETT and NICHOLAS C. METROPOLIS, Los Alamos Scientific Laboratory, Los Alamos, New Mexico 87544. A generalization of the Gauss limit for iterated means.

It is shown that a recursive process, involving iteration of the means of orders $t_1, \ldots, t_N$, applied to any positive initial point of N-space, produces a sequence of points converging to a point $(a, a, \ldots, a)$ "on the diagonal." The special case $N = 2$, $t_1 = 0$, $t_2 = 1$, with both weights $\frac{1}{2}$, is the geometric-arithmetic-mean algorithm of Gauss. (Received June 21, 1971.)

*71T-B185. JAMES R. McLAUGHLIN, Pennsylvania State University, University Park, Pennsylvania 16802. Integrated orthonormal series.

Let $F_\alpha(t) = \sum_{m=1}^{\infty} |\int_a^t \varphi_m(x)dx|^\alpha$ where $0 < \alpha \leqq 2$, $a \leqq t \leqq b$, and $\{\varphi_m\}$ is a sequence in $L^1[a,b]$. If $\{\varphi_m\}$ is orthonormal, then Parseval's equality says that $F_2(t) = t - a$, $a \leqq t \leqq b$, if and only if $\{\varphi_m\}$ is complete. In this paper, the author studies $F_\alpha(t)$ with $0 < \alpha \leqq 2$ for the Haar, Walsh, trigonometric, and general orthonormal sequences. An application is then given regarding the absolute convergence of Walsh-Fourier series. For instance, it is proved that for the Haar sequence $F_\alpha(t)$ satisfies a Lipschitz condition of order $\alpha/2$ in $[0,1]$ and that this result is best possible for any complete orthonormal sequence. (Received June 23, 1971.)

*71T-B186. ANDERS LUNDBERG, University of Waterloo, Waterloo, Ontario, Canada. On generalized distributivity for real functions.

The generalized distributivity equation (1) $F(G(x,y),z) = H(K(x,z), L(y,z))$ for c.s.i. functions (i.e. continuous, strictly increasing) is easily reduced to (2) $fG(x,y) = G(g(x), h(y))$. We here suppose G to be c.s.i. from R^2 onto R and f, g, and h to be o-iseotopisms of T. Theorem 1. Let G be a fixed function. If there exist any solution (f,g,h) of (2), such that f, g, and h have no fixpoints, then every fixpoint-free solution is of the form $(f^\alpha, g^\alpha, h^\alpha)$, where $\{f^\alpha\}$, $\{g^\alpha\}$, and $\{h^\alpha\}$ are either continuous iteration groups or groups of natural iterates of some functions f^1, g^1, and h^1. Consider the continuous case, and suppose $f^1, g^1, h^1 > e$, where $e(x) \equiv x$. If we transform $(f^\alpha, g^\alpha, h^\alpha)$ into $(e+\alpha, e+\alpha, e+\alpha)$, then G is transformed into $(x,y) \to x+\Psi(y-x)$, where Ψ is c.s.i. Theorem 2. If f, g, and h are c.s.i. and satisfy the equation $f(x+\Psi(y-x)) = g(x)+\Psi(h(y)-g(x))$, then f, g, and h are all linear functions. Similar results can be obtained even under weaker conditions. (Received June 24, 1971.) (Author introduced by Professor John A. Baker.)

71T-B187. SWARUPCHAND M. SHAH, University of Kentucky, Lexington, Kentucky 40506. Absolute Norlund summability of Fourier series.

This paper supplements the results of M. Izumi and S. Izumi (Pacific J. Math. 26(1968), 289-301) and S. N. Lal (ibid. 35(1970), 661-667). Let (i) $p_0 > 0$, (ii) $\{p_n\}$ nonnegative and nonincreasing, (iii) $\{p_n - p_{n+1}\}$ nonincreasing, and (iv) $p_n = o(1)$. Let $f(x)$ be a 2π-periodic, integrable function. If $\sum p_n^p n^{p-2} < \infty$ $(1 < p \leq 2)$ and $(*)$ $\sum \omega(n^{-1}) P_n^{-1} n^{-1/q} < \infty$ $(1/p + 1/q = 1$: $P_n = \sum_0^n p_k$; ω modulus of continuity of f) then Lal has shown that the Fourier series is summable $|N, p_n|$. The series $(*)$ will fail to converge if $P_n/\omega(n^{-1})$ does not increase rapidly (e.g. $P_n \sim \{\omega(n^{-1})\}^{-1} \sim \log^2 n$). Theorem. Let $\{p_n\}$ satisfy conditions (i) - (iv) and suppose that $c_1 > 0$, $c_2 > 0$, $0 \leq \gamma < \frac{1}{2}$ and $c_1 n^\gamma \psi(n) \leq P_n \leq c_2 n^\gamma \psi(n)$ $(n \geq 1)$ where $\psi(n)$ is any slowly oscillating and nondecreasing function for $n \geq 1$. If $f(x)$ is a 2π-periodic function of bounded variation over $[0, 2\pi]$ and if the series $\sum(1/n)\omega(1/n)$ and $\sum(1/nP_n)\,\omega^{1/2}(1/n)$ are both convergent then the Fourier Series of f is summable $|N, p_n|$. (Received June 24, 1971.)

71T-B188. DOROTHY MAHARAM, University of Rochester, Rochester, New York 14627. Set homomorphisms and measurable transformations. Preliminary report.

Let (S_1, m_1) and (S_2, m_2) be σ-finite measure spaces, and E_1 and E_2 their respective measure algebras. A "set homomorphism" of S_1 onto S_2 is a σ-isomorphism θ of E_1 onto a subalgebra of E_2. It has been shown by C. Ionescu-Tulcea (see for instance A. and C. Tulcea, "Topics in the theory of lifting," Ergebnisse der Math. 48, Berlin, 1969, p.160) that if S_1 and S_2 are compact and the μ_i are Borel measures, then any such θ arises from a measurable transformation T of S_2 into S_1. In general $T(S_2)$ cannot be measurable (though it must be of full outer measure). It is shown here that if S_1 and S_2 are arbitrary products of unit intervals (with product measures) then T can be taken so that $T(S_2) = S_1$. (Received June 25, 1971.)

71T-B189. RONALD B. KIRK, Southern Illinois University, Carbondale, Illinois 62901. Sets which interpolate families of measurable sets.

Let (X, Σ, m) be a measure space and let Γ be a subset of Σ consisting of sets of positive measure. Then a set $A \in \Sigma$ is said to interpolate Γ if $0 < m(A \cap B) < m(B)$ for all $B \in \Gamma$. Theorem. Assume that m is an atomless, σ-finite measure. If Γ is a countable subset of Σ consisting of sets of positive measure, then there is a set $A \in \Sigma$ which interpolates Γ. As an application of this theorem, the following is proved. Corollary. Let X be a separable metric space and let m be an atomless, σ-finite regular Borel measure on X. Then there is a Borel set A such that $0 < m(A \cap B) < m(B)$ for every open set B of positive measure. (Received June 25, 1971.)

*71T-B190. JAMES R. CALDER, Auburn University, Auburn, Alabama 36830. Concerning almost symmetric point sets in normed linear spaces.

Suppose S is a normed linear space, M is an infinite bounded point set, P is in S, $w > 0$, and $B(P,w)$ is the open ball with center P. "R(M) is the radius of M" means R(M) is the largest number, k, such that if Q is in S, $r > 0$, and if $M - M \cap B(Q,r)$ is finite, then $r \geq k$. "M is almost symmetric about P" means that if $r > 0$ there is a subset, R, of M such that $M - R$ is finite and if x is in R, then $2P - x$ is in R or $B(2P-x, r) \cap R$ is infinite. Theorem. Each two of the following five statements are equivalent: (1) S is u.c.e.d. [see James and Swaminathan, Abstract 71T-B157, these *Notices* 18(1971)]; (2) no infinite bounded point set has two centers; (3) if K is an infinite bounded point set almost symmetric about Q, then Q is the only center of K; (4) ((5)) if K is the intersection of two unit balls, then K has only one center $(R(K) < 1)$. (Received June 25, 1971.)

*71T-B191. WILLIAM P. COLEMAN, Auburn University, Auburn, Alabama 36830. Banach spaces with property H.

Suppose S is a Banach space with open unit ball U. "S has property H" means that if $h > 0$ there exists a $d \in (0,1)$ such that if K is the intersection of an open $(1-d)$-ball with U then there is an open $(1-d)$-ball $B(Q, 1-d)$ covering K such that $\| Q \| < h$. Theorem. If S has property H, every infinite bounded point set has a center. Theorem. Each of c_0, c, and m (usual norms) has property H. Theorem. If S* contains a total point sequence then: (1) S can be renormed so as to be u.c.e.d. [see James and Swaminathan, Abstract 71T-B157, these *Notices* 18(1971)]; (2) if S is separable, S can be renormed so as to be u.c.e.d. and locally uniformly convex. (Received June 25, 1971.)

71T-B192. WILLIAM D. L. APPLING, North Texas State University, Denton, Texas 76203. A further remark on the Bochner-Radon-Nikodym theorem.

U, F, R, R_{AB}, R_A^+ and the notion of integral are as in previous abstracts of the author. Theorem. Suppose m is in R_A^+, f is in R_{AB} and b is in R. The following two statements are equivalent: (1) $0 = \int_U [\int_V |f(I) - b(V)m(I)|]$; (2) f is absolutely continuous with respect to m and $0 = \int_U |f(I) - b(I)m(I)|$. (Received June 28, 1971.)

71T-B193. CHARLES G. DENLINGER, Millersville State College, Millersville, Pennsylvania 17551. Banach order limits in Archimedean vector lattices. Preliminary report.

In this paper, methods of S. Simons ("Banach limits, infinite matrices and sublinear functionals," J. Math. Anal. Appl. 26(1969), 640-655) for real sequences are extended to vector lattices. Let E denote an Archimedean vector lattice, $\hat{E}$ denote its Dedekind completion, and $m(E)$ denote the vector lattice of order bounded sequences in E. Let $\sigma : m(E) \to m(E)$ denote the "shift operator," $L : m(E) \to \hat{E}$ denote the sublinear map $L(\{x_n\}) = \overline{\lim}\ x_n$ and $S : m(E) \to \hat{E}$ denote the map $S(\{x_n\}) = \sup x_n$. A Banach order limit is a linear map $g : m(E) \to \hat{E}$ such that $f \circ \sigma \leq g \leq S$. It is shown that a linear $g : m(E) \to \hat{E}$ is a Banach order limit if and only if $g \circ \sigma = g$, $0 \leq g \leq L$, and $g(\{x_n\}) = \hat{x}$ whenever $\{x_n\}$ order converges to $\hat{x}$ in $\hat{E}$. Simons' methods are extended to prove the existence of Banach order limits on $m(E)$ for arbitrary Archimedean E. "Almost order convergent" sequences are defined and considered. Banach order limits are not necessarily order continuous. However, if $\{x_n\}^{(k)}$ is relatively uniformly convergent to 0 in $m(E)$, as $k \to \infty$, and g is a Banach order limit, then $g(\{x_n\}^{(k)})$ is relatively uniformly convergent to 0 in $\hat{E}$. (Received June 28, 1971.)

Applied Mathematics

71T-C18. STEPHEN GROSSBERG, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. A neural theory of punishment and avoidance. Preliminary report.

Neural networks are derived from psychological postulates about punishment and avoidance. The networks are embedding fields (Proc. Nat. Acad. Sci. U.S.A. 68(1971), 828), or nonlinear functional-differential, cross-correlated flows describing nonstationary pattern prediction, learning, and discrimination. Positive incentive-motivational on-cells $\mathcal{A}_f^+$ and negative-incentive motivational off-cells $\mathcal{A}_f^-$ interact via rebound from $\mathcal{A}_f^+$ to $\mathcal{A}_f^-$ when shock terminates. Rebound is due to asymmetric transmitter accumulation in $\mathcal{A}_f^+$ and $\mathcal{A}_f^-$ driven by asymmetric distribution of aversive phasic inputs and internal tonic inputs. Classical conditioning from sensory cells $\mathcal{S}$ to arousal cells $\mathcal{A}_f = (\mathcal{A}_f^+, \mathcal{A}_f^-)$ can occur. Positive net feedback from $\mathcal{A}_f$ to $\mathcal{S}$ releases sampling by $\mathcal{S}$ of motor output controls. Various difficulties in two-factor theories are avoided. Estes' stimulus sampling theory is neurally interpreted. An analogy with adrenergic and cholinergic interactions at lateral and ventromedial hypothalamic sites is noted. Recent data and concepts are analyzed, such as relaxation theory; effective reinforcement; instrumental properties of a CS+, CS-, and contingent feedback stimuli; self-stimulation under drugs; inverted U in learning; emotional depression; analgesia. (Received May 11, 1971.)

Geometry

71T-D14. GERALD A. BEER, University of California, Los Angeles, California 90024. The index of convexity and pseudo-starshaped sets. Preliminary report.

The visibility function, v, assigns to each point x of a fixed measurable set E in a Euclidean space the Lebesgue outer measure of $\{y : rx+(1-r)y \subset E\ \forall r \text{ in } [0,1]\}$. When $m(E) < \infty$ and v is measurable, let I(E) denote $\int_E (v/m(E)^2)\,dm$ if $m(E) > 0$. If $m(E) = 0$, let $I(E) = 1$. Typical results for compact sets are the following: Theorem. If $I(E) = \alpha$ and $I(F) = \beta$, $I(E \times F) = \alpha \times \beta$. Theorem. I(E) is upper semicontinuous on $\langle C, \bar{d}\rangle$, the space of compact sets in E_n with the metric $\bar{d}(E,F) = \sup\{d(E,F), m(E \triangle F)\}$ where d denotes the Hausdorff metric. Theorem. Let $P\,\mathrm{Ker}(E) = \{x : v(x) = m(E)\}$. If $m(E) > 0$ and $P\,\mathrm{Ker}(E) \neq \emptyset$, E is the union of a compact starshaped set whose convex kernel is P Ker(E) and a set of measure zero. Corollary. If $\{E_\alpha\}$ is a collection of compact sets in E_n, $I(E_\alpha) = 1\ \forall\alpha$, such that every $n+1$ intersect in a set of positive measure, then $\bigcap E_\alpha \neq \emptyset$. Corollary. $\{E : P\,\mathrm{Ker}(E) \neq \emptyset\}$ is a closed subspace of $\langle C, \bar{d}\rangle$. (Received April 7, 1971.)

71T-D15. JOHN DeCICCO, Illinois Institute of Technology, Chicago, Illinois 60616 and ROBERT V. ANDERSON, Université du Québec à Montréal, Montréal 110, Québec, Canada. On nets of special harmonic character.

In a set of isothermal parameters (x,y) let the linear element of a surface Σ be given by $ds^2 = [u(x,y)]^{-2}(dx^2+dy^2)$, where $u(x,y) = d\sigma/ds > 0$. The class of surfaces is studied, on each of which there exists at least one orthogonal isothermal net of $2\infty^1$ curves C_1 such that $e^{-2\sigma}u^{-2}dK_1/ds_1$ is a finite real harmonic function. Such a net is said to be of special harmonic character. Such nets are extensions of isothermal nets. On a surface Σ of constant Gaussian curvature, every orthogonal isothermal net is of special harmonic character. There is at least one orthogonal isothermal net of $2\infty^1$ Minding circles on a surface Σ if and only if there exists a set of isothermal parameters (x,y) such that $ds^2 = [\alpha(x)+\beta(y)]^{-2}(dx^2+dy^2)$. If the Gaussian curvature K is a real positive function of a nonconstant real harmonic function $t = t(x,y)$, then there is at least one orthogonal isothermal net of $2\infty^1$ curves C with the special harmonic character if and only if the surface Σ is a conformal image of a surface $\bar{\Sigma}$ which is applicable to a surface of revolution and whose linear element is of the form $d\bar{s}^2 = e^{2\mu(r)}[(r_x)^2+(r_y)^2](dx^2+dy^2)$ where $\mu = \mu(r)$ is a real function of a real nonconstant harmonic function r(x,y). (Received May 7, 1971.)

71T-D16. SHERMAN K. STEIN, University of California, Davis, California 95616. A nonlattice tiling by a star body. Preliminary report.

Theorem. There is a symmetric star body that tiles R^{10}, ten-dimensional Euclidean space, but cannot tile R^{10} in a lattice manner. The existence part of the proof, using a theorem of Zaremba on finite abelian groups, provides a tiling whose centers constitute a finite family of cosets of a group. The nonexistence of a lattice tiling depends on the impossibility of expressing the nonzero elements of certain finite abelian groups as the disjoint union of sections of cyclic groups of a fixed prescribed order. (Received May 7, 1971.)

*71T-D17. HELEN R. MEINES and DAVID W. BALLEW, South Dakota School of Mines and Technology, Rapid City, South Dakota 57701. Some models of a bounded geometry. Preliminary report.

This paper considers a model for a geometry which is defined as consisting of all the points of the disk determined by a circle of radius a in the Euclidean plane, together with the agreement to identify all the points on the circumference of the circle as one point. The points of the model are all the Euclidean points of the disk and the lines are all the chords of the circle. The distance between any pair of points is defined to be the minimum of the two Euclidean distances on a chord through the two points. Angle measure is defined to be the Euclidean angle measure. A system of axioms is developed for the model in which each axiom is the same as or similar to an axiom of absolute geometry and hence contains it as a special case, except for the triangle axiom which fails completely. The major differences between this model and absolute geometry are the following: the lengths of the lines vary; convexity depends only on a connected line segment and does not require any condition on shortest distance; some triangles have a configuration impossible in absolute geometry; the triangle congruence axiom mentioned above fails; and the shortest distance between two points may not always occur on a line. (Received May 17, 1971.)

71T-D18. KINETSU ABE, Johns Hopkins University, Baltimore, Maryland 21218. On Kählerian submanifolds in $\mathbb{C}^N$. Preliminary report.

Let f be a holomorphic and isometric immersion of a Kählerian manifold of complex dimension n into N-dimensional complex Euclidean space $\mathbb{C}^N$. Let ν be the index of relative nullity of the immersion, where ν is the complex dimension. Theorem. If M^n is complete and if there exists a point x in M where the tangent space TM_x to M^n contains a complex $(n-\nu)$-dimensional subspace such that for any nonzero vector X in the subspace, the holomorphic sectional curvature $K(X)$ is not zero, then M^n is ν-cylindrical, i.e., M^n is a Kahlerian product of ν-dimensional complex space $\mathbb{C}^\nu$ and $(n-\nu)$-dimensional Kahlerian manifold $M_1^{n-\nu}$. This result generalizes a theorem by Rosenthal, "Kähler manifolds of constant nullity," Michigan Math. J. 15(1968), 433-440, and the result by the author, Abstract 70T-D10, these *Notices* 17(1970), 669. (Received May 24, 1971.)

*71T-D19. ROBERT MALTZ, The Hebrew University, Jerusalem, Israel. Isometric immersions of Riemannian products in Euclidean space. Preliminary report.

Theorem. Suppose M is the Riemannian product $M_1 \times M_2 \times \ldots \times M_k \times \mathbb{R}^n$, where dimension $M_i = n_i \geqq 2$ and the M_i are assumed complete. Then if $f: M \to \mathbb{R}^N$ is an isometric immersion of codimension k, f must be the product $(f_1, \ldots, f_k, \mathrm{Id})$ where $f_i: M_i \to \mathbb{R}^{n_i+1}$ are hypersurface immersions and $\mathrm{Id}: \mathbb{R}^n \to \mathbb{R}^n$ is the identity immersion. This result sharpens a cylindricity theorem of S. B. Alexander, and removes all compactness assumptions in previous theorems of this type due to S. B. Alexander and J. D. Moore. (Received June 7, 1971.)

*71T-D20. BANG-YEN CHEN, Michigan State University, East Lansing, Michigan 48823. Surfaces of Gauss curvature $\geqq 0$ in euclidean 4-space. Preliminary report.

Let M be a surface immersed in E^4. Let H be the mean curvature vector, h the second fundamental form of M in E^4. If there exists a function f on M such that $h(X,Y) \cdot H = fX \cdot Y$, for all tangent vectors X, Y on M, then the surface M is called a pseudo-umbilical surface in E^4. Theorem 1. A closed pseudo-umbilical surface of Gauss curvature zero in E^4 is a Clifford torus. Theorem 2. Let M be a closed pseudo-umbilical surface of Gauss curvature $\geqq 0$ in E^4. If the mean curvature vector H is nowhere zero, then M is either a Clifford torus or a sphere in a hyperplane E^3. Theorem 3. Let M be a flat torus in E^4. Then the mean curvature vector H satisfies $\int_M H \cdot H \, dV \geqq 2\pi^2$. The equality sign holds when and only when M is a Clifford torus, where "$\cdot$" denotes the scalar product of E^4 and dV the volume element of M. The proofs of Theorems 1 and 2 are based on the Laplacian of the second fundamental form of pseudo-umbilical surfaces in E^4 [(2.30) of the previous paper entitled "Some results of Chern-do Carmo-Kobayashi type and the length of second fundamental form" to appear in Indiana Univ. Math. J. 20(1971)]. The proof of Theorem 3 is based on Theorem 1. (Received June 21, 1971.)

Logic and Foundations

71T-E65. ALBERT J. SADE, 364 Cours de la République, Pertuis Vaucluse, France 84. Fonctions implicationnelles Recensement des thèses.

Pour qu'une fonction soit accessible il faut et il suffit qu'elle couvre un atome. Soit M une matrice couvrant z ; P_0 l'ensemble des points nuls de M; H celui des points $M = 1$, $z = 0$; $a_1, \ldots, a_j$ les formules qui ont respectivement pour point nul unique chacun des éléments de H; alors $M = f = Ca_jC \cdots a_3Ca_2Ca_1z$. Soient $b_1, \ldots, b_i$ les formules qui ont resp. pour point nul unique chacun des éléments de P_0; alors $M = f = CC \cdots Cb_1Cb_2zzCb_3zzC \cdots zzCb_izz$. Le groupoide $\mathscr{G}$ de toutes les fonctions accessibles sur μa pour cardinal $\sum(-)^{u+1}\binom{c}{u}2ex2^{c-u}$, $u = 1, \ldots, c$. Pour $c = 2, 3, 4$ on trouve Card $\mathscr{G} = 6$, 38, 942. La relation d'équivalence $ab = cd$ définit sur $\mathscr{G} \times \mathscr{G}$ une partition dont le groupoide quotient est isomorphe à $\mathscr{G}$. L'ensemble des f qui couvrent a est $\mathscr{G}a$. Toute thèse est une implication ab où b couvre a. Le nombre des thèses ab où a est donné est Card $\mathscr{B}a_0$, où a_0 est l'ensemble des points $a = 0$. Pour $c = 3$ il y a 144 thèses non banales (autres que aa ou a1) dont 33 non isomorphes. Toute thèse connue dérive d'une de ces 33 thèses. Une large prospection montre que presque la moitié de ces 33 modèles n'apparaît pas dans la littérature. (Received February 25, 1971.

71T-E66. RICHARD E. LADNER, Simon Fraser University, Burnaby 2, British Columbia, Canada. A r.e. set is autoreducible if and only if it is mitotic.

A recursively enumerable (r.e.) set is mitotic if it is the disjoint union of two r.e. sets both of the same degree. B. A. Trahtenbrot defined the notion of autoreducible (Soviet Math. Dokl. 11(1970)). A set is autoreducible if there exists a number e suchthat, for all n, $c_A(n) = \{e\}^{A \cup \{n\}}(n) = \{e\}^{A - \{n\}}(n)$. Theorem. A r.e. set is autoreducible if and only if it is mitotic. (Received April 16, 1971.)

*71T-E67. KEITH J. DEVLIN, University of Bristol, Bristol, England. Note on a theorem of J. Baumgartner. Preliminary report.

In Abstract 70T-E79, these *Notices* 17(1970), 967, J. Baumgartner announced the following result. If $V = L[A]$ where $A \subseteq \omega_1$, there is an Aronzajn tree order-embeddable in the reals but not in the rationals. A stronger result is possible. Theorem. Assume Jensen's axiom $\Diamond$. Then there are $2^{\aleph_1}$ nonisomorphic such trees. The proof uses ideas of Herbacek and Jech. (Received April 21, 1971.) (Author introduced by Professor John C. Shepherdson.)

*71T-E68. W. WISTAR COMFORT, Wesleyan University, Middletown, Connecticut 06457 and STELIOS NEGREPONTIS, McGill University, Montréal, Québec, Canada. On families of large oscillation.

Definitions. $\mathfrak{F} \subset \gamma^{\alpha}$ has $\varkappa$-large oscillation provided that if $(f_{\xi})_{\xi<\lambda}$ are (distinct) in $\mathfrak{F}$ and $(\eta_{\xi})_{\xi<\lambda}$ are in γ (with $\lambda < \varkappa$) then there is σ with $f_{\xi}(\sigma) = \eta_{\xi}$; $S(X)$ and $d(X)$ are respectively the Souslin number and density character of the space X; $(\Pi_i X_i)_{\varkappa}$ is the product space $\Pi_i X_i$ with the (less than) $\varkappa$-box topology; $\Omega_{\varkappa}(\alpha)$ is the set of all $\varkappa$-complete ultrafilters on α; the Rudin-Keisler partial order $<$ on $\underset{\sim}{\beta}(\alpha)$ is given by: $p < q$ if $f_*(q) = p$ for some $f \in \alpha^{\alpha}$. Theorem 1. For $\omega \leq \varkappa \leq \alpha$, these are equivalent: (a) $\alpha = \alpha^{\underline{\varkappa}}$; (b) there are, among any α^+ sets with fewer than $\varkappa$ elements, α^+ sets whose pairwise intersections coincide; (c) $S((\Pi_i X_i)_{\varkappa}) \leq \alpha^+$ if each $d(X_i) \leq \alpha$; (d) there exists $\mathfrak{F} \subset \alpha^{\alpha}$ with $\varkappa$-large oscillation and $|\mathfrak{F}| = 2^{\alpha}$; (e) there exists $\mathfrak{F} \subset \alpha^{\alpha}$ with $\varkappa$-large oscillation and $|\mathfrak{F}| = \alpha^+$; (f) $d((\alpha^{(2^{\alpha})})_{\varkappa}) = \alpha$; (g) $d((\alpha^{(\alpha^+)})_{\varkappa}) = \alpha$. [(a) $\Leftrightarrow$ (b) follows also from Erdös-Rado, J. London Math. Soc. 44(1969), 467-479.] Theorem 2. If $\omega \leq \varkappa \leq \alpha = \alpha^{\underline{\varkappa}}$, $\varkappa$ is regular, each $\varkappa$-complete filter on α extends to an element of $\Omega_{\varkappa}(\alpha)$, then each $S \subset \Omega_{\varkappa}(\alpha)$ with $|S| \leq 2^{\alpha}$ is bounded above in $\Omega_{\varkappa}(\alpha)$. Corollary. If $\alpha \geq \omega$, $(2^{\alpha})^+ = 2^{2^{\alpha}}$, and $\Omega_{\omega^+}(\alpha) = \alpha$, then $\underset{\sim}{\beta}(\alpha)$ contains a well-ordered, cofinal subset of $2^{2\alpha}$ α^+-good ultrafilters. (Received May 21, 1971.)

71T-E69. S. B. COOPER, University of Leeds, Leeds 2, England. The jumps of hyperhyperimmune sets below $\underline{0}'$. Preliminary report.

A set of numbers, A, is hyperhyperimmune if there is no recursively enumerable mutually disjoint collection of finite recursively enumerable sets all of whose members intersect with A. A is strongly hyperhyperimmune if the finiteness requirement is removed. This work continues the author's investigations into the relationships between jumps of degrees, structure of the degrees, and sparseness of (or tortuousness of approximation to) representatives of degrees. It is proved: (1) that below $\underline{0}'$ hyperhyperimmunity implies strong hyperhyperimmunity, (2) that every hyperhyperimmune set recursive in $\underline{0}'$ has jump $\underline{0}''$ (C. G. Jockusch has proved every set below $\underline{0}'$ with jump $\underline{0}''$ to be Turing equivalent to a strongly hyperhyperimmune set). The proof of both requires approximation methods although that of (1) is easy. (Received May 3, 1971.) (Author introduced by Dr. F. R. Drake.)

71T-E70. GONZALO E. REYES, Université de Montréal, Montréal 101, Québec, Canada. $L_{\omega_1\omega}$ is enough.

We consider theories in infinitary languages with finite strings of quantifiers and finitary predicate symbols (cf. Karp's book: "Languages with expressions of infinite length"). We assume that μ is a nonmeasurable cardinal (in the sense of a σ-measure). Theorem. For every theory T in a language $L_{\mu\omega}$ there is a theory T' in $L_{\omega_1\omega}$ which is equivalent to T in the following sense: the predicate symbols of the language of T can be mapped into formulas of the language of T' and vice versa in such a way that every model of one theory determines (via these maps) a model of the other having the same universe. Refinements of this result (including cardinality bounds for T') are discussed. The proof uses a theorem of Sikorski on ω-homomorphisms of μ-complete boolean algebras. (Received June 25, 1971.)

71T-E71. WITHDRAWN.

71T-E72. SAHARON SHELAH, University of California, Los Angeles, California 90024. On problems from the list "Unsolved problems in set theory" of Erdős and Hajnal.

We deal with three of their problems. See "Axiomatic set theory," Proc. Sympos. Pure Math, Vol. 13, Amer. Math. Soc., Providence, R. I., 1971. On problem 36 (Hajnal). (It follows from $\omega_1 \to (\alpha)^2_n$ of Hajnal and Baumgartner, but here is a generalization.) Theorem 1. Let f be a set mapping defined on $\aleph_{\alpha+1}$, $\mu < \lambda \Rightarrow \aleph^{\mu}_{\alpha} = \aleph_{\alpha}$, $x \neq y \Rightarrow |f(x) \cap f(y)| < \aleph_{\alpha}$, ζ an ordinal $< \lambda^+$. Then there is an independent set $\subset \aleph_{\alpha+1}$ of type ζ. On problem 74 (Erdős, Rado). Let K_λ be the class of graphs with λ vertices, each of valence $< \lambda$. Theorem 2. If $\lambda = \beth_\delta$ ($\delta = \bigcup\delta$) then there is in K_λ a universal graph G_0. That is, any other graph in K_λ is isomorphic to a subgraph of G_0, spanned by a set of vertices of G_0. (The problem was on $\lambda = \beth_\omega$, with G.C.H.) On problem 75 (Erdős, Milner). Definition. An ultrafilter D over I is λ-decomposable if there are disjoint I_k, $k < \lambda$, $I = \bigcup_{k<\lambda} I_k$ such that for every $A \subset \lambda$, $|A| < \lambda$; $[\bigcup_{k\in A} I_k] \notin D$. Theorem 3. Let $\beth_{\delta+1} = \beth^+_\delta$, $\mathrm{cf}(\delta) = \lambda < \beth_\delta$, and assume there is a uniform ultrafilter over $\beth_\delta$ which is μ-indecomposable for every $\lambda < \mu < \beth_\delta$. Then there is a family K of subsets of $\beth_\delta$, $K \subset [\beth_\delta]^\lambda$, $|K| > \beth_\delta$, such that: $\beth_\delta = A \cup B$, $|A \cap B| < \beth_\delta$ implies $|\{x \in K | x \subset A\}| \leq \beth_\delta$ or $|\{x \in K | x \subset B\}| \leq \beth_\delta$. (On consistency results concerning the assumption see Prikry, Ph.D. Thesis, University of California, Berkeley, Calif., 1968). (Received May 10, 1971.) (Author introduced by Professor Chen Chung Chang.)

*71T-E73. GERALD E. SACKS, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. The density number of a quasi-totally transcendental theory. Preliminary report.

T is a countable, consistent, complete theory that admits elimination of quantifiers. $\mathcal{B}$ is any substructure of any model of T. $S\mathcal{B}$ is the Stone space whose points correspond to the isomorphism types (over $\mathcal{B}$) of the simple extensions of $\mathcal{B}$. L. Blum calls T quasi-totally transcendental (q.t.t.) if for every $\mathcal{B}$, the Morley ranked points of $S\mathcal{B}$ are dense in $S\mathcal{B}$. (T is totally transcendental if for every $\mathcal{B}$, every point of $S\mathcal{B}$ has a Morley rank.) She defines the density number, d_T, of a q.t.t. T to be the least ordinal γ such that for all $\mathcal{B}$, the points of $S\mathcal{B}$ of Morley rank less than γ are dense in $S\mathcal{B}$. Theorem. If T is quasi-totally transcendental, then d_T is countable. (Received May 17, 1971.)

*71T-E74. LEONARD P. SASSO, University of California, Berkeley, California 94720. Degrees of unsolvability of partial functions.

This continues work announced in Abstract 666-39, "A minimal partial degree of unsolvability," these *Notices* 16(1969), 649. Let $\mathcal{P}$, $\mathcal{T}$, and $\mathcal{S}$ denote the sets of degrees of arbitrary partial functions, total functions, and single-valued partial functions respectively and let $\leq$ be the partial order of $\mathcal{P}$ naturally induced by $\leq_T$. The following results are obtained in addition to those announced in the above abstract. $(\mathcal{P}, \leqq)$ is an upper-semilattice but not a lattice and $\mathcal{T}$ and $\mathcal{S}$ are upper-semilattices in $\mathcal{P}$. For any $\underset{\sim}{a}, \underset{\sim}{b} \in \mathcal{P}$ and $\underset{\sim}{c} \in \mathcal{S}$, if $\underset{\sim}{a} \leqq \underset{\sim}{b} \leqq \underset{\sim}{a} \cup \underset{\sim}{c}$ then $\underset{\sim}{b} = \underset{\sim}{a} \cup \underset{\sim}{d}$ for some $\underset{\sim}{d} \in \mathcal{S}$ and if $\underset{\sim}{b} \in \mathcal{T}$ and $\underset{\sim}{b} \leqq \underset{\sim}{a} \cup \underset{\sim}{c}$ then $\underset{\sim}{b} \leqq \underset{\sim}{a}$. Any countable (possibly finite) ascending sequence $\underset{\sim}{d}_0 < \underset{\sim}{d}_1 < \ldots$ of degrees in $\mathcal{P}$ has an upper bound $\underset{\sim}{d}$ which is not the join of any $\underset{\sim}{d}_i$ with a degree in $\mathcal{S}$ and such that any degree in $\mathcal{T}$ below $\underset{\sim}{d}$ is already below some $\underset{\sim}{d}_i$. Hence every countable ideal in $\mathcal{T}$ is the intersection of $\mathcal{T}$ with a principal ideal in $\mathcal{P}$. The intersection of $\mathcal{S}$ with any principal ideal in $\mathcal{P}$ is a principal ideal in $\mathcal{S}$ and thus naturally induced embedding of $\mathcal{T}$ into $\mathcal{S}$ is properly into and not join preserving. The jump operator for $\mathcal{T}$ generalizes naturally to $\mathcal{P}$ but the notion of "recursively enumerable in" does not. (Received May 17, 1971.)

*71T-E75. DALE W. MYERS, University of California, Berkeley, California 94720. Separation properties for first-order languages.

For any similarity type σ let σ^* be the class of structures of type σ; for any $K \subseteq \sigma^*$ and $n \in \omega$ let $\forall_n^0(K)$ be the collection of subclasses of K definable by a prenex sentence whose prefix has n homogeneous blocks of quantifiers, the first being universal. By Shoenfield [see Addison, "Some problems in hierarchy theory," Proc. Sympos. Pure Math., vol. 5, Amer. Math. Soc., Providence, R. I., 1962, pp. 123-130], $\forall_n^0(\sigma^*) \in \mathrm{Sep}_I$ (i.e., has the first separation property) for any type σ and any $n \geqq 2$. From the theorems below, derived by Ehrenfeucht-Fraissé game techniques, it follows that, for all but the simplest σ, $\exists_n^0(\sigma^*) \notin \mathrm{Sep}_I$ if $n \geqq 2$, and $\exists_n^0(\sigma^*)$, $\forall_n^0(\sigma^*) \notin \mathrm{Sep}_{II}$ (i.e., fail to have the second separation property) if $n \geqq 1$. The only such questions involving the σ^*-classes which remain unsettled reduce to those of determining whether or not $\exists_2^0(\sigma^*) \in \mathrm{Sep}_I$ when σ consists of exactly two or more unary operations. We conjecture that it does.
Theorem. If K is the class of linear orders and $n \geqq 2$, then $\exists_n^0(K) \notin \mathrm{Sep}_I$. Theorem. If σ consists of exactly one unary operation and $n \geqq 1$, then $\exists_n^0(\sigma^*) \in \mathrm{Sep}_I$ and $\exists_n^0(\sigma^*)$, $\forall_n^0(\sigma^*) \notin \mathrm{Sep}_{II}$. See also Krom, "Separation principles in the hierarchy theory of pure first-order logic," J. Symbolic Logic 28(1963), 222-236. (Received May 24, 1971.)

*71T-E76. STELIOS NEGREPONTIS, McGill University, Montréal, Québec, Canada. The growth of subuniform ultrafilters.

Let α be an infinite regular cardinal. $U(\alpha^+)$ denotes the space of uniform ultrafilters on α^+, $\Omega(\alpha^+)$ denotes the space of subuniform ultrafilters on α^+, i.e., those ultrafilters on α^+ which are not uniform, but which have only elements of cardinality at least α. The Stone-Čech compactification of S is denoted by βS
Theorem. If $\alpha = \alpha^{\underset{\sim}{\alpha}}$ then $\beta(\Omega(\alpha^+)) \backslash \Omega(\alpha^+)$ (= the growth of $\Omega(\alpha^+)$) has the following property: for every nonempty closed subset F of $\beta(\Omega(\alpha^+)) \backslash \Omega(\alpha^+)$ which is equal to the intersection of at most α open-and-closed

sets and every family $\{V_\eta, \eta<\alpha\}$ of nonempty pairwise disjoint open-and-closed subsets of $\beta(\Omega(\alpha^+))\setminus\Omega(\alpha^+)$, there are p in F and an open-and-closed set N, with p in N and such that $|\{\eta<\alpha: V_\eta \cap N \neq \emptyset\}| < \alpha$. Corollary. If $\alpha = \alpha^{\aleph}$ then $U(\alpha^+)$ is not homeomorphic to $\beta(\Omega(\alpha^+))\setminus\Omega(\alpha^+)$. In particular $\beta(\Omega(\alpha^+))\setminus\Omega(\alpha^+)$ is not C*-embedded in $\beta(\alpha^+)$. (For $\alpha = \omega$ this last statement has been proved by Mrs. N. M. Warren (Doctoral Dissertation, University of Wisconsin, 1970).) Corollary (E. Specker). If $\alpha = \alpha^{\aleph}$ then α^+ has property Q in the sense of Erdős and Tarski ("On some problems involving inaccessible cardinals," Essays on the Foundations of Mathematics, Magnes Press, 1966, pp. 50-82). (Received May 24, 1971.)

*71T-E77. PETER H. KRAUSS, State University of New York, New Paltz, New York 12561. Universally complete theories of algebras. II. Preliminary report.

For unexplained notation see Abstract 71T-E19, these *Notices* 18(1971), 426. Theorem 3. Suppose Σ is a universally complete set of universal Horn sentences which has a nontrivial finite model. Then there exists $\mathfrak{A} \in \mathrm{Mod}\ \Sigma$ such that for every $\mathfrak{B} \in \mathrm{Mod}\ \Sigma$, (i) $\mathfrak{A} \in \mathcal{S}\mathcal{P}\mathfrak{B}$ and (ii) $\mathfrak{B}$ is isomorphic to a subdirect power of $\mathfrak{A}$. A set Σ of sentences is called Ribeiro complete if (i) Σ is consistent and (ii) for every universal sentence σ, either $\Sigma \models \sigma$ or there exists $n < \omega$ such that $\Sigma \models \sigma \leftrightarrow \exists^{\leq n}$. Theorem 4. Suppose Σ is a set of universal Horn sentences which has a nontrivial finite model. Then Σ is Ribeiro complete iff (i) there exists $\mathfrak{A} \in \mathrm{Mod}\ \Sigma$ such that every $\mathfrak{B} \in \mathrm{Mod}\ \Sigma$ is isomorphic to a subdirect power of $\mathfrak{A}$ and (ii) for all $\mathfrak{A}, \mathfrak{B} \in \mathrm{Mod}\ \Sigma$, if $\overline{\overline{\mathfrak{A}}} \leq \overline{\overline{\mathfrak{B}}} < \omega$ then $\mathfrak{A} \in \mathcal{S}\mathcal{P}\mathfrak{B}$. (Received May 25, 1971.)

*71T-E78. JOHANN A. MAKOWSKY, Swiss Federal Institute of Technology, 8006 Zürich, Switzerland. On a problem of Morley and Shelah. Preliminary report.

In ["On strongly minimal sets," J. Symbolic Logic, to appear] Baldwin and Lachlan studied strongly minimal sets. Almost strongly minimal theories, as defined and proved in [Baldwin, Abstract 70T-E59, these *Notices* 17(1970), 834], are $\aleph_1$-categorical. Morley and Shelah [Shelah, "Stability, the f.c.p. ...," to appear] asked the question, whether there exists a finitely axiomatizable complete $\aleph_1$-categorical theory. Theorem. No complete, almost strongly minimal, $\aleph_0$-categorical theory is finitely axiomatizable. The proof uses, besides the theory of strongly minimal sets of Baldwin and Lachlan, a criterion of finite axiomatizability derived from Ehrenfeucht's game theoretic definition of elementary equivalence and some lemmata on automorphisms of models of $\aleph_0$- and $\aleph_1$-categorical theories. (Received June 1, 1971.) (Author introduced by Professor Erwin Engeler.)

71T-E79. JONATAN STAVI, The Hebrew University, Jerusalem, Israel. On the free σ-complete and $<\kappa$-complete Boolean algebras on $\aleph_0$ generators. Preliminary report.

Given a cardinal $\kappa > \aleph_0$ and a $<\kappa$-complete Boolean algebra (B.a.), B freely $<\kappa$-generated by a sequence $(a_n)_{n\in\omega}$, we denote: C = smallest σ-subalgebra of B that includes $\{a_n | n \in \omega\}$, A = set of atoms of C = set of atoms of B. Given moreover a fixed sequence $(c_n)_{n\in\omega}$ from C, let B_1 = smallest $<\kappa$-complete subalgebra of B that includes $\{c_n | n \in \omega\}$, c_1 = smallest σ-subalgebra of C that includes $\{c_n | n\in\omega\}$, $A_1 = \{\bigcap^C_{n\in\omega} x_n | \forall_n \in \omega\ (x_n = c_n \text{ or } x_n = \sim c_n)\}$. Thus $A_1 - \{0\}$ = set of atoms of C_1 = set of atoms of B_1. Theorem 1. $B_1 = B$ iff $C_1 = C$ iff $A_1 - \{0\} \subseteq A$. Theorem 2. B_1 is freely $<\kappa$-generated by $(c_n)_{n\in\omega}$ iff C_1 is freely σ-generated by $(c_n)_{n\in\omega}$

iff $0 \notin A_1$. Now suppose $b \in C$. Put $H_b = \{x \in A | x \leq b\}$, and $I_b = \{x \in b | x \leq b\}$ with the B.a. structure induced by B. Theorem 3. (1) If $|H_b| \leq \aleph_0$ then the B.a. I_b is atomic and isomorphic to the B.a. of all subsets of H_b. (2) If $|H_b| > \aleph_0$ then I_b is isomorphic to B (hence, by the Gaifman-Hales theorem $|I_b| \geq \varkappa$). Note. The theorems hold, suitably interpreted, for "$\varkappa = \infty$". (Received June 15, 1971.) (Author introduced by Professor Haim Gaifman.)

*71T-E80. KENNETH A. BOWEN, Syracuse University, Syracuse, New York 13210. Forcing and natural models of set theory. Preliminary report.

Let $\mu < \lambda$ be infinite cardinals, μ regular. Let K be a class of structures of fixed similarity type which is closed under increasing unions of length $< \lambda$. Let L be a usual infinitary language $L_{\alpha\beta}$ or one of Barwise's $\mathcal{L}_A$ such that each sentence of L is of length $< \mu$ and for any $\mathcal{O} \in K$, $L(\mathcal{O})$ has $< \lambda$ sentences. For each $\mathcal{O} \in K$ and sentence $A \in L(\mathcal{O})$ a notion of forcing $\mathcal{O} \# A$ is defined. $\mathcal{O} \in K$ is complete if for every sentence $A \in L(\mathcal{O})$, $\mathcal{O} \# A$ or $\mathcal{O} \# \neg A$. Theorem. For all $\mathcal{O} \in K$, there is a complete $\mathcal{B} \in K$, $\mathcal{O} \subseteq \mathcal{B}$. Theorem. If $\mathcal{O} \in K$ is complete and $A \in L(\mathcal{O})$, then $\mathcal{O} \# A$ iff $\mathcal{O} \models A$. Theorem. If $\mathcal{O}, \mathcal{B} \in K$ are complete and $\mathcal{O} \subseteq \mathcal{B}$, then $\mathcal{B}$ is an L-elementary extension of $\mathcal{O}$. Corollary. The complete structures of K for an L-model complete cofinal subclass of K. Corollary. If λ is inaccessible, there are λ ordinals $\alpha < \lambda$ such that $\langle R(\alpha), \epsilon \rangle$ is an L-elementary substructure of $\langle R(\lambda), \epsilon \rangle$. (Received June 18, 1971.)

71T-E81. MURRAY A. JORGENSEN, University of British Columbia, Vancouver 8, British Columbia, Canada and Waterloo Lutheran University, Waterloo, Ontario, Canada. Images of ultrafilters and cardinality of ultrapowers. Preliminary report.

Let D be an ultrafilter on I and α a cardinal no greater than $|I|$. An image of D on α is a uniform ultrafilter E on α such that for some $f: I \to \alpha$ we have $X \in E$ iff $f^{-1}(X) \in D$. Assume the GCH. Theorem. For regular cardinals α the following are equivalent statements: (1) D is α-descendingly incomplete. (2) D has an image on α. (3) $|\alpha^I/D| > \alpha$. (4) $\mathrm{cf}(|\alpha^I/D|) > \alpha$. Remark. The equivalence of (1), (2) and (3) needs only the LCH for proof; and that of (1) and (2) needs no special hypothesis. Corollary (LCH). If α is regular then $|(\alpha^+)^I/D| > \alpha^+$ implies $|\alpha^I/D| > \alpha$. A better result is the following, which is a consequence of some work of A. Adler and needs no form of GCH. Theorem. For any cardinal α, $|(2^\alpha)^I/D| > 2^\alpha$ implies $|\alpha^I/D| > \alpha$. (Received June 21, 1971.)

71T-E82. LEO A. HARRINGTON, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. Structures with recursive presentations. Preliminary report.

A countable structure, $\mathcal{O}$, has a recursive presentation if there is a listing, $c_0, c_1, \ldots, c_n, \ldots, n < \omega$, of the members of $|\mathcal{O}|$, such that $T(\langle \mathcal{O}, c_0, c_1, \ldots \rangle)$ is recursive, DCF_0 is the model completion of the theory of differential fields of characteristic 0. Lenore Blum has shown that DCF_0 has a prime model. Theorem. The prime model of DCF_0 has a recursive presentation. John Baldwin has shown that an $\aleph_1$-categorical theory has finite transcendence rank. A consequence of his proof is: Theorem. Every countable model of a recursive $\aleph_1$-categorical theory has a recursive presentation. (Received June 21, 1971.)

71T-E83. ALEXANDER ABIAN, Iowa State University, Ames, Iowa 50010. Mitotic ordinals and their applications.

Definition. A nonzero ordinal w is called mitotic if and only if it can be partitioned into $\overline{\overline{w}}$ pairwise disjoint subsets each of type w. Theorem 1. Every mitotic ordinal is a limit ordinal. Moreover, an infinite ordinal is mitotic if and only if it is equipollent to the last term of its normal expansion. Theorem 2. Let w be a mitotic ordinal and $(c_i)_{i<w}$ a nondecreasing sequence of type w of nonzero cardinals c_i. Then $\Pi_{i<w} c_i = (\Pi_{i<w} c_i)^{\overline{\overline{w}}} = (\sup_{i<w} c_i)^{\overline{\overline{w}}}$. Corollary. Let $(c_i)_{i<v}$ be an infinite nondecreasing sequence of type v of cardinals c_i. Then $\Pi_{i<v} c_i = \Pi_{j<v} (\Pi_{i\leq j} c_i)$. (Received June 28, 1971.)

*71T-E84. STEPHEN L. BLOOM, Stevens Institute of Technology, Hoboken, New Jersey 07030 and DONALD J. BROWN, Yale University, New Haven, Connecticut 06520. Classical abstract logics. I.

All "algebras" $A, B, \ldots$ have at least a unary function $-a$ and a binary function $a \vee b$. A morphism from A to B is a function preserving at least $-$ and $\vee$. A closure operation Cn on A is negative if $a \in Cn(X) \Leftrightarrow Cn(X,-a) = A$; disjunctive if $Cn(a,X) \cap Cn(b,X) = Cn(X, a \vee b)$, all $X \subseteq A$, $a, b \in A$. Cn is classical if Cn is finite (= algebraic) negative and disjunctive. A (classical) abstract logic $\langle A, Cn\rangle$ consists of an algebra A and (classical) closure operation on A. If $L_i = \langle A_i, Cn_i\rangle$, $i = 1, 2$, are classical, a logical morphism h from L_1 to L_2 is a morphism from A_1 to A_2 s.t. $Cn_1(h^{-1}X) = h^{-1}X$, for all $X \subseteq A_2$ such that $Cn_2(X) = X$. Let $\mathbb{L}$ be the category of classical abstract logics and logical morphisms. Let $\mathbb{B}$ = category of Boolean spaces and continuous maps. Theorem. There are contravariant functors $F : \mathbb{L} \to \mathbb{B}$, $G : \mathbb{B} \to \mathbb{L}$ such that (1) $GF : \mathbb{B} \to \mathbb{B}$ is the identity and (ii) there is a natural transformation from the identity on $\mathbb{L}$ to $FG : \mathbb{L} \to \mathbb{L}$. The proof uses the concept of a dual space of an abstract logic. Corollary. A closure system C is classical iff C is isomorphic (as a lattice) to the closure system of all filters of a Boolean algebra. Let $a \in TAUT(A)$ if $h(a) = 1$, all morphisms $h : A \to \underline{2}$, the two element Boolean algebra. Theorem. $\langle A, Cn\rangle$ is classical iff $TAUT(A) \subseteq Cn(\emptyset)$, Cn is finite and Cn satisfies the "deduction property": $a \in Cn(X,b) \Leftrightarrow -b \vee a \in Cn(X)$. (Received June 28, 1971.)

*71T-E85. EUGENE M. KLEINBERG, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. Rowbottom cardinals and Jonsson cardinals are almost the same. Preliminary report.

All models here are of countable length. A cardinal κ is said to be a Jonsson cardinal if every model of power κ has a proper elementary submodel of power κ. A model $\langle D; U, \ldots\rangle$ where $U \subseteq D$ is said to be of type $\langle \overline{\overline{D}}, \overline{\overline{U}}\rangle$. (If x is any set, $\overline{\overline{x}}$ denotes the cardinality of x.) If $\kappa \geq \lambda \geq \delta$ are cardinals, the notation $\langle\kappa, \lambda\rangle \to \langle\kappa, \delta\rangle$ denotes the assertion "every model of type $\langle\kappa, \lambda\rangle$ has an elementary submodel of type $\langle\kappa, \delta\rangle$". Theorem. Let κ be any Jonsson cardinal. Then there is a δ less than κ such that for infinitely many λ, $\kappa > \lambda > \delta$, $\langle\kappa, \lambda\rangle \to \langle\kappa, \delta\rangle$. Theorem. Let κ be the least Jonsson cardinal. Then there is a δ less than κ such that for every λ, $\kappa > \lambda > \delta$, $\langle\kappa, \lambda\rangle \to \langle\kappa, \delta\rangle$. (A cardinal κ is said to be a Rowbottom cardinal if for every λ, $\kappa > \lambda > \omega$, $\langle\kappa, \lambda\rangle \to \langle\kappa, \omega\rangle$.) The proofs of these two theorems are entirely different from one another. (Received June 30, 1971.)

Statistics and Probability

71T-F5. WANG CHUNG LEE, University of Southern California, Los Angeles, California 90007. Convergence of the probability measure of an infinite particle system. Preliminary report.

Consider the motion of a system of particles on a d-dimensional lattice Z_d. Assume that (a) no point of Z_d can be occupied by more than one particle and (b) a particle at time t can jump to an unoccupied neighboring site with probability $dt + o(dt)$ in the time interval $(t, t+dt)$. Let $\{\xi_t\}$ be the resulting Markov process on $\Xi = \{\xi \mid \xi : Z_d \to \{0,1\}\}$ and μ_t the state (i.e., probability measure) of the system at time $t \geq 0$. This is the case of "simple exclusion" discussed by Spitzer and by Holley (Advances in Math. 5(1970)). For positive integers n and M, define V_{nM} to be the set $\{A \subset Z_d \mid |A| = n \text{ and } 0 < \|x-y\| \leq M \text{ for some } x, y \in A\}$, where $| \ |$ denotes cardinality of a set and $\| \ \|$ the maximum norm in Z_d. Suppose that (1) μ_0 is translational invariant, (2) $0 < E\xi_t(x) = \rho < 1$, (3) there exists a double sequence of positive numbers δ_{nM} such that for each n, $\delta_{nM} \to 0$ as $M \to \infty$ and $\rho^n - \delta_{nM} \leq \mu_0\{\xi(x) = 1, x \in A\} \leq \rho^n + \delta_{nM}$ for every $A \subset Z_d$, $|A| = n$ and $A \notin V_{nM}$. Then μ_t converges weakly to the probability measure μ defined on Ξ by $\mu\{\xi(x) = 1, x \in A\} = \rho^{|A|}$ for each $A \subset Z_d$, $|A| < \infty$. (Received May 17, 1971.)

71T-F6. KENNETH S. MILLER and M. ROCHWARGER, Riverside Research Institute, New York, New York 10023. A covariance approach to spectral moment estimation.

Let $\{x(t) \mid t \in T\}$ (the signal) and $\{\nu(t) \mid t \in T\}$ (the noise) be independent, mean zero, stationary, discrete parameter, complex Gaussian processes. Using vector samples $Z = \{z(t_1), \ldots, z(t_m)\}$ where $z(t) = x(t) + \nu(t)$, estimators for determining the spectral moments of the signal process may be constructed. The approach is to use the fact that the derivatives of the covariance function evaluated at zero are the spectral moments, and to base the spectral moment estimators on estimates of the covariance function. In particular, if $m = 2$, the estimators are the maximum likelihood solutions. Using these solutions, asymptotic (with sample size) formulas for the means and variances of the normalized first spectral moment and normalized standard deviation are derived. It is shown that the leading term in the variance computations is identical with the Cramér-Rao lower bound calculated using the Fisher information matrix. Also considered is the case where T is an interval and n samples of continuous data each of finite duration are measured. Asymptotic (with n) formulas are also derived in this case for the means and variances of the spectral first moment and standard deviation. (Received June 11, 1971.)

*71T-F7. MARK A. PINSKY, Northwestern University, Evanston, Illinois 60201. Stochastic integral representations of multiplicative operator functionals of a Wiener process.

Let M be a multiplicative operator functional of (X, L) where X is a d-dimensional Wiener process and L is a separable Hilbert space (for the definitions and elementary properties see the author's announcement, Bull. Amer. Math. Soc. 77(1971), 377-380). Sufficient conditions are given in order that M be equivalent to a solution of the linear Itô equation $dM = \sum_{j=1}^{d} MB_j dx_j + MB_0 dt$, where $B_0, \ldots, B_d$ are bounded operator functions on R^d. The conditions require that the equation $T(t)f = E[M(t)f(x(t))]$ define a semigroup on $L^2(R^d)$ whose infinitesimal generator has a domain which contains all linear functions of the coordinates $(x_1, \ldots, x_d)$. The proof of this result depends upon an a priori representation of the semigroup T(t) in terms of the Wiener semigroup and a first order matrix differential operator. A second result characterizes solutions of the above Itô equation with $B_0 = 0$. A sufficient condition that M belong to this class is that $E_x[M(t)]$ be the identity operator on L and that M(t) be invertible for each $t > 0$. The proof of this result uses the martingale stochastic integral introduced by H. Kunita and S. Watanabe. (Received June 14, 1971.)

*71T-F8. NARESH C. JAIN and WILLIAM E. PRUITT, University of Minnesota, Minneapolis, Minnesota 55455. Further limit theorems for the range of random walk.

Let $\{X_n, n \geqq 1\}$ be a sequence of independent identically distributed random variables with values in the d-dimensional integer lattice E_d, and let $S_0 = 0$, $S_n = X_1 + \ldots + X_n$. R_n, the range of the random walk $\{S_n, n \geqq 0\}$ up to time n, is the cardinality of the set $\{S_0, \ldots, S_n\}$. Let $p = \text{Prob } \{S_1 \neq 0, S_2 \neq 0, \ldots\}$. It is shown that for $d = 3$ the variance of R_n behaves asymptotically as $n\Psi(n)$, where $\Psi(n)$ is a slowly varying, nondecreasing function and that $(R_n - np)\,(n\Psi(n))^{-1/2}$ is asymptotically normal with mean 0, variance 1, except in the trivial case $p = 1$ (then $R_n = n+1$). This extends our previous result (J. Analyse Math. 24(1971), 369-393) which was proved under the assumptions that $EX_1 = 0$ and $E|X_1|^2 < \infty$, in which case $\Psi(n) = c \log n$ for some $c > 0$. An invariance principle is also established which applies to R_n when $d \geqq 3$ and in some cases when $d = 1$ or 2. This then gives further limit theorems for functionals of R_n. (Received June 23, 1971.)

*71T-G123. T. G. RAGHAVAN, Thiagarajar College of Engineering, Madurai 15, India. Quasi ordered bitopological spaces.

A b.t.s. equipped with a quasi order of graph G is under discussion. $H_i^l(A)$ $(H_i^m(A))$ denotes the smallest decreasing (increasing) τ_i-closed set containing A. A set A is called τ_i - G_δ^l (τ_i - G_δ^m) iff A is the countable intersection of decreasing (increasing) τ_i-open sets. A b.t.s. is called (a) G-p-completely normal iff $(A \cap H_1^l(B)) \cup (H_2^m(A) \cup B) = \emptyset$ implies there exist an increasing τ_1-open set $U \supset A$ and a decreasing τ_2-open set $V \supset B$ such that $U \cap V = \emptyset$, (b) G-p-perfectly normal iff it is G-p-normal (T. G. Raghavan, "Quasi ordered bitopological spaces. I, II," Math. Student (submitted)) and every decreasing τ_1-closed set is τ_2-G_δ^l and every increasing τ_2-closed set is τ_1-G_δ^m. It is proved (i) G-p-regular and second countability $\Rightarrow$ G-p-perfect normality $\Rightarrow$ G-p-complete normality $\Rightarrow$ G-p-normality. (2) G-p-perfect normality $\Leftrightarrow$ for each nonempty decreasing τ_1-closed set A and point $b \notin A$, there is a real valued monotone function f on X such that $f^{-1}(0) = A$ and $f(b) = 1$, f is τ_2-u.s.c. and τ_1-l.s.c., $f(x) \subset [0,1]$ and for each nonempty increasing τ_2-closed set B and a point $a \notin B$ there is a real valued antitone function g on X such that $g^{-1}(0) = B$ and $g(a) = 1$. g is τ_1-u.s.c. and τ_2-l.s.c., $g(X) \subset [0,1]$. (Received February 22, 1971.)
(Author introduced by Professor M. Rajagopalan.)

71T-G124. MYRA JEAN REED, State University of New York, Binghamton, New York 13901. Hausdorff-like separation properties. Preliminary report.

Properties similar to the Hausdorff condition are compared. While accessibility and Hausdorff are equivalent in first-countable spaces, they are independent, in general, even in compact spaces. Other conditions weaker than first-countability are investigated in conjunction with these and related properties. Typical theorems. An almost-accessibility space is first-countable if and only if it is weakly first-countable in the sense of Arhangel'skii. An accessibility space in which points are G_δ has compact subsets closed. (Received March 5, 1971.)

*71T-G125. JEONG SHENG YANG, University of South Carolina, Columbia, South Carolina 29208. On pointwise periodic transformation groups.

Recently, S. K. Kaul ["On pointwise periodic transformation groups", Proc. Amer. Math. Soc. 27 (1971), 391-394] has generalized a result of D. Montgomery ["Pointwise periodic homeomorphisms", Amer. J. Math. 59(1937), 118-120] by showing that if X is a connected metrizable manifold without boundary, and if (X, T) is a transformation group such that T is countable and pointwise periodic, then T is periodic. It is shown that Kaul's result remains true without assuming that X is metrizable. (Received March 18, 1971.)

71T-G126. LUDVIK JANOS, University of Florida, Gainesville, Florida 32601. Some results on a conjecture of de Groot in dimension theory.

Let X be a metrizable topological space and M(X) the set of all metrics on X compatible with the topology of X. If $\rho \in M(X)$ we say that two subsets $A, B \subset X$ are ρ-congruent if there is a one to one mapping of A onto B preserving ρ. We introduce the function i(X) by: $i(X) = -1$ iff X is empty; $i(X) \leq n$ iff there is a metric $\rho \in M(X)$ such that no two different subsets of cardinality $n + 2$ are ρ-congruent, and finally $i(X) = n$ iff $i(X) \leq n$ and $i(X) \nleq n - 1$. The question was raided by de Groot whether $i(X) = \dim(X)$ for X separable. Denoting by ind(X) the weak inductive dimension we have the following results: Theorem 1. $i(X) \leq n$ implies $\operatorname{ind}(X) \leq n$. Theorem 2. If X is separable then $i(X) = 0 \Leftrightarrow \operatorname{ind}(X) = 0$ and $i(X) = 1 \Rightarrow \operatorname{ind}(X) = 1$. Theorem 3. For the real line R, $i(R) = 1$ holds and the corresponding metric can be obtained by identifying R with the graph $\{(x,x^2) | x \in (0,1)\}$. (Received March 25, 1971.)

*71T-G127. LI PI SU, University of Oklahoma, Norman, Oklahoma 73069. A theorem on pairwise-normality of bitopological spaces.

A bitopoligical space $(X, \mathcal{T}, \mathcal{T}')$ is pairwise Lindelöf iff every pairwise open covering of $(X, \mathcal{T}, \mathcal{T}')$ has a countable subcovering. Lemma. If $(X, \mathcal{T}, \mathcal{T}')$ is pairwise Lindelöf and F is a $\mathcal{T}$-closed subset of X, then F is $\mathcal{T}'$-Lindelöf. Theorem. Every pairwise Lindelöf, regular space $(X, \mathcal{T}, \mathcal{T}')$ is pairwise-normal. (Received April 7, 1971.)

71T-G128. JAMES M. PARKS, University of Houston, Houston, Texas 77004. Locally constant sheaves, mapping theorems and continuity.

Base spaces are Hausdorff and sheaves are in the category $\underline{\mathfrak{A}}_X$ ($\underrightarrow{\mathfrak{A}}_X$) of locally constant sheaves (limits of locally constant sheaves) on X. If $f, g: X \to Y$ such that $f \simeq g$ and X is compact, then it may be shown that $f^*\mathcal{A} \approx g^*\mathcal{A}$ for $\mathcal{A} \in \underline{\mathfrak{A}}_Y$ (and thus for $\mathcal{A} \in \underrightarrow{\mathfrak{A}}_Y$). Therefore not all sheaves are limits of locally constant sheaves. If $\mathcal{A} \in \underline{\mathfrak{A}}_Y$ with respect to the cover G of Y, $f: X \to Y$ is a homotopy equivalence and $F = f^{-1}(G)$, then we say f is an (F,G)-homotopy equivalence iff $fg \simeq 1_Y$ by a G-homotopy and $gf \simeq 1_X$ by an F-homotopy. A typical mapping theorem obtained is the following. Theorem. If $\mathcal{A} \in \underline{\mathfrak{A}}_Y$ with respect to a cover G of Y, $f: X \to Y$ is an (F,G)-homotopy equivalence, X and Y are compact, then there exists a sheaf $\mathcal{B} \in \underline{\mathfrak{A}}_X$ such that $H^*(X, \mathcal{B}) \approx H^*(Y, \mathcal{A})$. The result extends to sheaves in $\underrightarrow{\mathfrak{A}}_Y$. Restrict the base spaces to be locally path connected and compact. By relaxing the matching condition on an inverse system of base spaces to allow matching up-to-F_α-homotopies (where the covers F_α are determined by the respective sheaves in $\underline{\mathfrak{A}}_{X_\alpha}$) a continuity theorem is obtained. If the sheaves are constant one has the corresponding result for Alexander cohomology with the more general homotopy-inverse systems (matching condition satisfied up-to-homotopy). (Received April 26, 1971.)

71T-G129. GIOVANNI A. VIGLINO, Wesleyan University, Middletown, Connecticut 06457. Extensions of functions and spaces.

Let φ be a (continuous) map from a space X to a space Y (denoted by $[X,\varphi,Y]$). $[E,\Phi,Y]$ is an extension of $[X,\varphi,Y]$ if E extends X and Φ extends φ to E. We assume $\varphi(X)$ is dense in Y and all spaces are Hausdorff. X is φ-absolutely closed if every φ-convergent filter has nonempty adherent set. Theorems. (1) $[X,\varphi,Y]$ has no proper extensions if and only if X is φ-absolutely closed. (2) If Y is compact then X is φ-absolutely closed if and only if X is absolutely closed. (3) If X is semiregular and φ is injective then X is φ-absolutely closed if and only if φ is a homeomorphism. (4) If X is regular then X is φ-absolutely closed if and only if φ is a perfect map. (5) Let $[X,\varphi,Y]$ and $[Y,\gamma,Z]$ be given. If X is $\gamma\varphi$-absolutely closed then X is φ-absolutely closed and Y is γ-absolutely closed. (6) Let $[X,\varphi,Y]$ be given. Then there exists an extension $[E,\Phi,Y]$ with $\Phi(E) = Y$. In the case that Y is regular, the extension may be chosen so that $E\backslash X$ is homeomorphic to $Y\backslash\varphi(X)$. (7) Let $\{[X_\alpha,\varphi_\alpha,Y_\alpha]\}_{\alpha\in A}$ be given. Then $\Pi_{\alpha\in A}X_\alpha$ is $\Pi_{\alpha\in A}\varphi_\alpha$-absolutely closed if and only if each X_α is φ_α-absolutely closed. (8) Let $[X,\varphi,\Pi_{\alpha\in A}Y_\alpha]$ be given. If X is $\pi_{\alpha_0}\varphi$-absolutely closed for some $\alpha_0 \in A$, then X is φ-absolutely closed. (9) There exists a maximal extension for $[X,\varphi,Y]$. (Received April 29, 1971.)

*71T-G130. V. KANNAN, Madurai University, Madurai-2, India. Notes in categorical topology.

In this paper we deal with topological reflections and coreflections. The first section gives an internal description of the Hausdorff reflection of a topological space, thereby solving a problem posed by H. Herrlich ("Categorical topology", Pittsburgh Conference in Topology, June, 1970). The second section shows that the epireflection of a topological space in an arbitrary epireflexive subcategory is built out of extremal epireflections and simple reflections. This is then used to describe the epireflection. In the third section we study the behaviour of some cardinal invariants which arise on the consideration of coreflexive subcategories. The fourth section describes a method of constructing comparable topologies with the same class of continuous self-maps, thereby improving some results of a paper by Yu Lee Lee (Pacific J. Math. 20(1967)). In the fifth section we show that several nice topological categories have the property that they admit no reflexive cum coreflexive subcategory. Lastly we show that a reflexive subcategory preserves a coreflexive subcategory if and only if the viceversa holds. (Received May 3, 1971.) (Author introduced by Mr. V. R. Chandran.)

*71T-G131. PETER L. RENZ, Wellesley College, Wellesley, Massachusetts 02181. Equivalent flows and partitions of unity.

By considering the vector field $\eta(x) = x$ defined on the open unit ball U of a separable Banach space B we show that if the flow α associated with this vector field is C^{p-1} equivalent (in the sense of P. L. Renz, Indiana Univ. Math. J. 20(1971)) to a vector field having flow domain then B admits C^{p-1} partitions of unity subordinate to any open cover. This result was suggested by Theorem 2 of David H. Carlson's paper (Abstract 71T-B85, these *Notices* 18(1971), 556) but is independent of his work. This proof follows from the Indiana Univ. Math. J. paper cited above, a crucial comment of R. Bonic and results of Bonic and Frampton ("Smooth functions on Banach manifolds," J. Math Mech. 15(1966), 877-897). (Received May 5, 1971.)

*71T-G132. P. L. SHARMA and S. A. NAIMPALLY, Indian Institute of Technology, Kanpur-16, India. Compactification numbers.

Let $\varkappa$ be a function from the set Σ of all completely regular Hausdorff spaces into the set of cardinal numbers, where $\varkappa(X)$ for $X \in \Sigma$ is the number of distinct Hausdorff compactifications of X. Similarly u(X) is the number of admissible uniformities on X. Theorem. For a positive integer m there exists $X \in \Sigma$ with $\varkappa(X) = m$ iff m is the number of all possible equivalence relations on some finite set. Theorem. For each infinite cardinal m, there exists $X \in \Sigma$ with $\varkappa(X) = 2^m$. Theorem. For any nonpseudocompact member X of Σ, $\varkappa(X) \geqq 2^c$. Theorem. If X is realcompact noncompact then $\varkappa(X) \geqq 2^c$. Theorem. For the real line R, $\varkappa(X) = u(X) = 2^c$. Theorem. Let $m \leqq c$. Then $\varkappa(X) = m$ iff $u(X) = m$. Theorem. If X is separable or discrete then $\varkappa(X) = u(X)$. The real line R has no n-point compactification for any countable $n \geqq 3$. Several other interesting results are given. (Received May 6, 1971.)

*71T-G133. P. L. SHARMA, Indian Institute of Technology, Kanpur-16, India. Lattice invariance in topological spaces.

In this paper it is proved that pseudocompactness, paracompactness, para-Lindelöf, complete regularity, extremal disconnectedness, basic disconnectedness, realcompactness and several other topological properties are lattice invariant. A topological space is CP-compact iff for any decreasing sequence of nonempty open sets, the intersection of their closures is nonempty. It is shown that CP-compactness lies strictly between countable compactness and pseudocompactness and that it is lattice invariant. If two topological spaces are lattice equivalent then a lattice isomorphism between the collections of their closed sets yields a lattice isomorphism between the collections of their zero sets. Thus the sets of Z-filters of such spaces are lattice isomorphic. Two separated uniform spaces may be lattice equivalent without being uniformly isomorphic or even homeomorphic. Total-boundedness is not a lattice invariant property. Several other interesting results are given. (Received May 6, 1971.) (Author introduced by Professor S. A. Naimpally.)

*71T-G134. JONATHAN K. SIMON, University of Iowa, Iowa City, Iowa 52240. An algebraic classification of knots in S^3.

To each polygonal knot K in the 3-sphere S^3 is associated a finitely presented group in such a way that two knots are of the same type iff their classifying groups are isomorphic. Specifically, if K is a knot, $J(1,\pm 2;K)$ the $(1,\pm 2)$ cable knots about K, $CG_{\pm}(K) = \pi_1(S^3 - J(1,\pm 2;K))$, $CG_I(K)$ the free product $CG_+(K) * CG_-(K)$, R a prime, amphicheiral knot with Property P (R has Property P iff it is impossible to obtain a simply connected manifold by removing a regular neighborhood of R from S^3 and sewing it back differently) other than a cable knot (e.g. R = Figure-eight knot), K # R the composition ("sum") of K and R, and $CG_R(K) = CG_I(K \# R)$, then we have the following: Theorem 1. If K_2 is a composite knot and K_1 is a composite knot or any other knot with Property P that is not a cable knot, then $CG_I(K_1) \cong CG_I(K_2)$ if $\exists$ homeomorphism $h:(S^3,K_1) \to (S^3,K_2)$. Theorem 2. For any knots K_1, K_2, $CG_R(K_1) \cong CG_R(K_2)$ iff K_1, K_2 are of the same knot type. (Received May 10, 1971.)

*71T-G135. VICTOR A. NICHOLSON, Kent State University, Kent, Ohio 44242. Tame and nice are equivalent in 3-manifolds.

Let K be a topological complex in a 3-manifold M^3 and $p \in K$. The subdivision K_p of K is defined by: $K_p = K$ if p is a vertex of K, and if p is not a vertex of K then K_p has exactly one more vertex than K, namely p. We say K is nice at p if for each sufficiently small open connected set U containing p there is an open set V such that $p \in V \subset U$ and if W is any open connected set such that $p \in W \subset V$ and $W \cap \Delta$ is connected for each simplex Δ of K_p then for each nonempty component W' of W - K the image of the inclusion homomorphism $i_*: \pi(W') \to \pi(U')$ is a free group on m - 1 generators where U' is the component of U - K containing W' and m is the number of components of st(p) - {p} that meet Cl(W'). We say K is nice if K is nice at each of its points. Proposition 1. Suppose p is in the topological boundary of K and the link of p in K_p is connected. Then K is nice at p iff $M^3 - K$ is 1-LC at p. Proposition 2. Suppose K is a finite graph and $p \in K$. Then K is nice at p iff $M^3 - K$ has 1-FLG at p. Theorem 1. Suppose K is a topological complex which is a closed subset of a 3-manifold M^3. Then K is tame iff K is nice. (Received May 14, 1971.)

*71T-G136. ANTOINE DERIGHETTI, Institut de Mathématiques, Université de Lausanne, Lausanne 1015, Switzerland. On weak containment. II.

For notations see [1] Abstract 70T-B224, these *Notices* 17(1970), 958. Let π be an arbitrary continuous unitary representation of a locally compact group G. We show that $d(\pi) \geqq 1$ if and only if π does not weakly contain i_G. This result improves [1] and [2] (A. Derighetti "On the property p_1 of locally compact groups," Comment. Math. Helv., to appear). We could sketch the main argument as follows: Under the assumption $d(\pi) < 1$, we prove at first $2|\int_G f(x)dx| \leqq \|\pi(f)\| + \|\pi(\bar{f})\|$ where $\pi(f)$ is the bounded operator $\int_G f(x)\pi(x)dx$ [2, Proposition 12]. Let $\mathcal{A}$ be the convex hull of $\{A_x | x \in G\}$ where A_x is the linear map of $L^1(G)$ into itself defined by $A_x f = f_x \Delta_G(x)$. The inequality $\inf\{\|\pi(A\bar{f})\| \mid A \in \mathcal{A}\} \leqq |\int_G f(x)dx|$ [2, Proposition 15] together with $\|\pi(Af)\| \leqq \|\pi(f)\|$ for every $A \in \mathcal{A}$ gives us $|\int_G f(x)dx| \leqq \|\pi(f)\|$. (Received May 14, 1971.)

*71T-G137. ARTHUR L. STONE, Simon Fraser University, Burnaby 2, British Columbia, Canada. Topological cauchy spaces.

Let a topological cauchy structure on a set be an equivalence class of uniformities on the set, where uniformities are equivalent if they make the same filters cauchy. Cauchy maps are those functions which carry cauchy filters to cauchy filters. (So uniformly continuous implies cauchy, and cauchy implies continuous.) Where A and B are cauchy spaces let [A, , B] denote the set of cauchy maps $A \to B$ with the double-cauchy structure: $\mathbb{F}$ is cauchy if $\{\{f(a) | a \in E, f \in F\} | E \in \mathbb{E}, F \in \mathbb{F}\}$ is cauchy on B for every $\mathbb{E}$ cauchy on A. The category of cauchy spaces is cartesian closed, and the forgetful functor to sets has both adjoint and coadjoint. [A, , B] frequently has the compact-open topology. Nonstandard Characterization. Where A* is an adequate nonstandard model of A, a cauchy structure on A is a partial equivalence relation on A* over A which is G_δ in the $\beta A \times \beta A$ topology. (Recall: a uniformity is an equivalence relation over A which is closed in the $\beta(A \times A)$ topology.) (Received May 17, 1971.)

*71T-G138. NEAL R. WAGNER, University of Texas, El Paso, Texas 79968. The space of retractions of a 2-manifold. Preliminary report.

For any second-countable 2-manifold M^2, let Λ be the embedding of M^2 into its space of retractions which maps each point to the constant retraction to that point. Denote by $L(M^2)$ the component containing the image of Λ. The embedding Λ, with range restricted to $L(M^2)$, is shown to be a weak homotopy equivalence in three cases: (1) if M^2 is compact and the compact-open (= sup-metric) topology is used, (2) if M^2 is complete and the sup-metric topology is used, or (3) if M^2 is homeomorphic to a compact 2-manifold with a closed subset of its boundary omitted and the compact-open topology is used. The proofs are similar to those in the author's article "The space of retractions of the 2-sphere and the annulus," to appear in Trans. Amer. Math. Soc. (Received May 17, 1971.)

71T-G139. JAMES C. CANTRELL, University of Georgia, Athens, Georgia 30601. A local flatness criterion for triangulated manifolds. Preliminary report.

Theorem. Let M be a triangulated $(n-1)$-manifold in which each open simplex of M is locally flat in M, and let f be an embedding of M into the interior of an n-manifold N such that the image of each closed simplex is locally flat in N. Then f(M) is locally flat in N. Glaser has recently shown that the first condition will be satisfied if M is a simplicial homotopy manifold. Cantrell has shown (Abstract 70T-G162, these *Notices* 17(1970), 971) that, if $n \neq 4$, the second condition holds if the image of each open $(n-1)$-simplex and each closed $(n-2)$-simplex is locally flat in N. (Received May 19, 1971.)

71T-G140. GERHARD X. RITTER, University of Wisconsin, Madison, Wisconsin 53706. Free Z_8 actions on S^3.

Theorem. Any free action of Z_8 on S^3 is topologically equivalent to one of the two standard actions of Z_8 on S^3. (Received May 24, 1971.)

71T-G141. WITHDRAWN.

71T-G142. C. J. M. RAO and S. A. NAIMPALLY, Indian Institute of Technology, Kanpur-16, India. A realcompactification for limit spaces.

A T_2 limit space (X,q) [H. R. Fischer, Math. Ann. 137(1959), 269-303] is realcompact iff every real ultrafilter (i.e. an ultrafilter closed under countable intersections) q-converges. Set $X_1 = \{\mathscr{U} : \mathscr{U}$ a real ultrafilter on $X\}$. For $A \subset X$, set $\hat{A} = \{\mathscr{U} \in X_1 : A \in \mathscr{U}\}$ and for a filter $\mathfrak{F}$ on X, set $\hat{\mathfrak{F}}$ to be the filter on X_1 with base $\{\hat{F} : F \in \mathfrak{F}\}$. Define a limit structure q_1 on X_1 as follows: for $\varphi \in F[X_1]$, $\varphi \in q_1(\mathfrak{F})$ iff (a) $\varphi \geq \hat{\mathscr{G}}$ for some $\mathscr{G} \in q(x)$, if $\mathfrak{F} = \dot{x}$ for some $x \in X$, (b) $\varphi \geq \hat{\mathfrak{F}}$ otherwise. (I) (X_1, q_1) [denoted by νX] is a T_2 realcompactification of (X,q). (II) If $f: (X,q) \to (Y,q')$ is a continuous function, then f has a continuous extension $f': \nu X \to \nu Y$. (III) If βX denotes the Stone-Čech compactification of a T_2 limit space (X,q) [G. D. Richardson, Proc. Amer. Math. Soc. 25(1970), 403-404], then (a) $X \subset \nu X \subset \beta X$, (b) $\beta[\nu X] = \beta X$. (Received May 19, 1971.)

71T-G143. RICHARD N. CAIN, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213. A spectral sequence that relates homology of a polyhedron to the homology presheaf. Preliminary report.

We find that there is a filtered space whose spectral sequence relates the general homology of a polyhedron to the homology presheaf. In detail: let $U = \{U^i | i \in I\}$ be an open covering of a cw complex X, with indexing set I finite and each $X - U^i$ a subcomplex. Let ΔI be a simplex whose vertex set is I. For each subset $s \subset I$ let ∇s be the face of ΔI whose vertex set is $I - s$. The filtered space: $\ldots X \times (\Delta I)^n \cup K \supset X \times (\Delta I)^{n+1} \cup K \supset \ldots$, where $(\Delta I)^n = \bigcup \{\nabla s | \dim \Delta s \geqq n\}$, $K = \bigcup \{\nabla s \times (X - U^s) | s \subset I\}$, and $U^s = \bigcap_{i \in s} U^i$. The spectral sequence: if h is a (twisted) general homology theory, the h spectral sequence of the filtered space has $E_1^{n,-j}(U) =$ alternating cochains on Nerve(U) with coefficients in the presheaf $h_j(X, X - \mathcal{O})$ ($\mathcal{O}$ open $\subset$ X), while E_∞ is the graded group associated with a certain filtration of h(X). The differential $d_1^{n,-j}$ agrees with the usual cochain cohomology operator to within sign. (Received May 20, 1971.)

*71T-G144. STEWART S. CAIRNS, University of Illinois, Urbana, Illinois 61801. Isotopies of complexes.

Given a finite linear simplicial complex K in euclidean n-space E^n, let $\delta = \frac{1}{2} \min \{d(\sigma, \tau) | \sigma, \tau \in K$ and $\sigma \cap \tau = \emptyset\}$. Theorem. If $p_1, \ldots, p_\alpha$ are the vertices of K and if q_i is a point at distance $< \delta$ from p_i $(i = 1, \ldots, \alpha)$, then $p_i \to q_i$ induces an isomorphism of K onto a complex L with vertices $q_1, \ldots, q_\alpha$. The theorem becomes false if δ is replaced by any $\epsilon > \delta$. The number δ is continuous in $p_1, \ldots, p_\alpha$. A space $\varkappa(K, E^n)$ of complexes L isomorphic to K is defined as a subspace of $E^{\alpha n}$. The above theorem shows that $\varkappa(K, E^n)$ is open in $E^{\alpha n}$. The theorem may also be useful in simplifying the proofs of certain results pertaining to isotopies of complexes. (Received May 25, 1971.)

*71T-G145. PETER FLETCHER, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061 and WILLIAM F. LINDGREN, Southern Illinois University, Carbondale, Illinois 62901. Precompact transitive quasi-uniformities.

A topological space is compact if every compatible transitive quasi-uniformity is precompact. This result lends support to the conjecture that every topological space has a compatible complete transitive quasi-uniformity. (Received June 1, 1971.)

71T-G146. JOSEPH B. FUGATE, University of Kentucky, Lexington, Kentucky 40506 and LEE K. MOHLER, University of Saskatchewan, Saskatoon, Canada. Confluent images of irreducible continua. Preliminary report.

A continuum is a compact connected metric space. A continuum X is irreducible provided there are points p and q in X such that no proper subcontinuum of X contains both p and q. A mapping $f: X \to Y$ is confluent provided if K is a subcontinuum of Y and A is a component of $f^{-1}[K]$, then $f[A] = K$. Theorem. Suppose that X is an irreducible continuum, Y is a continuum, $f: X \to Y$ is confluent and for each $y \in Y$, $f^{-1}(y)$ has finitely many components. Then Y is irreducible. Corollary. If X is an irreducible continuum, Y is a continuum and f is a local homeomorphism of X onto Y, then Y is irreducible. (This extends a result of Mohler.) Theorem. Suppose that X is an irreducible continuum, Y is a continuum and $f: X \to Y$ is a quasi-monotone mapping. Then Y is irreducible. (Received June 1, 1971.)

71T-G147. CHIEN WENJEN, California State College, Long Beach, California 90801. On a theorem of K. Fan and N. Gottesman.

The results presented in this paper are mainly deductions from a theorem of Fan and Gottesman which states that a regular space T with a normal base can be embedded in a compact Hausdorff space T* [Nederl. Akad. Wetensch. Proc. Sec. A 55(1952), 504-510]. (1) A regular space T is completely regular if and only if it is pseudo-normal. (T is called pseudo-normal if for any two open subsets A, B of T with disjoint closures $\bar{A}$, $\bar{B}$, there are disjoint open sets containing $\bar{A}$, $\bar{B}$, respectively.) (2) For a regular space the Fan-Gottesman normal base is equivalent to Frink normal base for closed sets [Amer. J. Math. 86(1964), 602-607], that is, the existence of one implies the existence of the other. (3) The Fan-Gottesman compactification T* with respect to the normal base formed by all open sets of a completely regular space T coincides with Stone-Čech compactification βT and provides an internal construction for βT.
(4) (Generalized Tietze extension theorem) Let $\bar{A}$ be the closure of a nonvoid open subset A of a completely regular space T. Then any continuous function on $\bar{A}$ to $[-1,1]$ has a continuous extension over T to $[-1,1]$. (Received June 4, 1971.)

*71T-G148. B. J. BALL, University of Georgia, Athens, Georgia 30601 and JO W. FORD, Auburn University, Auburn, Alabama 36830. Spaces of ANR's.

Let 2^X_h denote the space of ANR's lying in the compactum X, with the "homotopy metric" defined by K. Borsuk [Fund. Math. 37(1950), 137-160]. It was shown by Borsuk that 2^X_h is separable and complete and that, for finite dimensional X, many topological and most homotopy properties of an ANR in X are shared by all ANR's sufficiently close to it in 2^X_h. The present paper is concerned with topological properties of the space 2^X_h, particularly for $X = S^2$. If X is a polyhedron with dim $X \geqq 2$ at each point, then 2^X_h is almost everywhere infinite dimensional and almost nowhere locally compact, and for any such X, the set $\mathcal{P}$ of proper subpolyhedra of X is of the first (Baire) category in 2^X_h. If $X = S^2$, then $\mathcal{P}$ is dense in 2^X_h, and the set $\hat{\mathcal{P}}$ of topological polyhedra in X is a dense G_δ in 2^X_h. For any X, in order that elements A, B of 2^X_h be connected by an arc in 2^X_h it is sufficient that A be isotopically deformable onto B in X, and for $X = S^2$ and A, B connected, it is necessary and sufficient that A and B be homotopically equivalent. Several additional results are given, and a number of problems are posed. (Received June 7, 1971.)

71T-G149. PEI LIU, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. On $\mathcal{a}$-stable spaces. Preliminary report.

Let $\mathcal{a}$ be a collection of open covers of a topological space $(X, \mathcal{T})$. Then $(X, \mathcal{T})$ is $\mathcal{a}$-stable provided that if $\mathcal{C} \in \mathcal{a}$ there is $h \in H(X) - \{i\}$ such that $h(A) \subset A$ for each $A \in \mathcal{C}$; and $\mathcal{a}$ has property L provided that if $\mathcal{C} \in \mathcal{a}$ there is $x \in X$ and $U \in \mathcal{T}$ such that $x \in U$ and U intersects only finitely many members of

$\mathcal{C}$. A space $(X,\mathcal{T})$ is a weak Galois space provided that if C is a proper closed set, then there is $h \in H^*(X) - \{i\}$ such that $h|C = i|C$. Theorem. If $(X,\mathcal{T})$ is a weak Galois space and $\mathcal{Q}$ is a collection of covers having property L, then $(X,\mathcal{T})$ is $\mathcal{Q}$-stable. A Q-cover of space $(X,\mathcal{T})$ is an open cover $\mathcal{C}$ such that for each $x \in X$, $\bigcap \{C \in \mathcal{C}, x \in C\} \in \mathcal{T}$ [M. Sion and R. C. Willmott, "Hausdorff measures on abstract spaces," Trans. Amer. Math. Soc. 123(1966), 275-309]. Theorem. Let $(X,\mathcal{T})$ be a locally compact metric weak Galois space and let $\mathcal{Q}$ be the collection of all Q-covers. Then $(X,\mathcal{T})$ is $\mathcal{Q}$-stable. There exists a weak Galois homogeneous Tychonoff space which is not a Galois space. (Received June 7, 1971.)

*71T-G150. DENNIS K. BURKE, Miami University, Oxford, Ohio 45056. A nondevelopable locally compact Hausdorff space with a G_δ diagonal. Preliminary report.

An example is constructed of a T_2 locally compact space which has a G_δ diagonal but is not developable. This answers negatively the question of whether every p-space with a G_δ diagonal is developable. (Received June 14, 1971.)

*71T-G151. JUN-ITI NAGATA, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. Characterizations of some generalized metric spaces.

Theorem 1. A T_1-space X is Σ (E. Michael, "On Nagami's Σ-spaces and some related matters," Proc. Washington State University Conference on General Topology, 1970, 13-19) if and only if every point x of X has a sequence $\{U_n(x) | n = 1, 2, \dots\}$ of open nbds (= neighborhoods) satisfying (i) if $y \in U_n(x)$, then $U_n(y) \subset U_n(x)$, (ii) if $x \in U_n(x_n)$, $n = 1, 2, \dots$, for a fixed point x of X, then $\{x_n\}$ has a cluster point. (A space with open nbds satisfying (ii) is called a β-space by R. Hodel, "Moore spaces and wΔ-spaces," Pacific J. Math. (to appear)). Theorem 2. A regular space X is M_2 (J. Ceder, "Some generalizations of metric spaces," Pacific J. Math. 11(1961), 105-125) if and only if every point x of X has a sequence $\{U_n(x) | n = 1, 2, \dots\}$ of open nbds satisfying (i) if $y \in U_n(x)$, then $U_n(y) \subset U_n(x)$, (ii) if $x \notin F$ for a point x and a closed set F of X, then there is n for which $x \notin [\bigcup \{U_n(y) | y \in F\}]^-$. (The second condition characterizes stratifiable spaces as proved by R. Heath.) (Received June 14, 1971.)

*71T-G152. ROBERT H. OVERTON, University of Washington, Seattle, Washington 98103. Čech homology for movable compacta.

A pair of metric compacta (X,A) is movable if it is the inverse limit of an ANR-pair sequence $((X_n, A_n), p_{n,k} : (X_k, A_k) \to (X_n, A_n))$ with the property that for any n there exists $n' \geqq n$ such that if $i \geqq n'$, then there is a map $s_{i,n'} : (X_{n'}, A_{n'}) \to (X_i, A_i)$ with $p_{n,i}\, s_{i,n'}$ homotopic to $p_{n,n'}$. This property is a shape invariant. Theorem 1. The Čech homology of a movable pair is exact. Theorem 2. The Čech homology of a pair of movable compacta is not necessarily exact. These theorems answer a question of Borsuk (Fund. Math. 66(1969), 137-146). (Received June 15, 1971.)

*71T-G153. MARTHA KATHLEEN SMITH, Rice University, Houston, Texas 77001. Regular representations of unimodular groups.

Suppose G is a locally compact, unimodular group. Let W(G) denote the W* algebra generated by the image of the left regular representation of G. Theorem 1. W(G) has a finite part if and only if G has a compact, invariant neighborhood of the identity. Let Δ denote the subgroup of G consisting of all elements whose conjugacy classes have compact closure. Theorem 2. If W(G) has a type I_n part, then $[G:\Delta] \leq n^2$. (Received June 16, 1971.)

*71T-G154. FRANK SIWIEC, St. John's University, Jamaica, New York 11432. Quotient mappings with restricted domain and range.

In a forthcoming paper the author has introduced the concept of a sequence-covering mapping and has proved that every almost-open mapping with first countable domain is sequence-covering. As a partial answer to a problem in that paper, we have: every sequence-covering s-mapping onto a first countable space is almost-open. With the domain and range restricted further, it is to be expected that other classes of mappings of interest in general topology are equivalent. The relations that exist under certain assumptions upon the domain and range are surveyed by means of diagrams. The case in which the domain and range are the set of real numbers is also considered. (Received June 17, 1971.)

71T-G155. WITHDRAWN.

*71T-G156. ROBERT E. ATALLA, Ohio University, Athens, Ohio 45701. P-sets and F-spaces.

Let X be compact T^2. A closed set in X is called a P-set if it is interior to any zero set which contains it. (See these *Notices* 17(1970), 931.) Theorem 1. X is an F-space iff the closure of any cozero set is a P-set. Theorem 2. If X is an F-space, then (a) the intersection of two P-sets is a P-set, (b) the closure of a countable union of P-sets is a P-set, (c) if Ω is the first uncountable ordinal, and $\{K_\alpha : \alpha < \Omega\}$ are monotonically decreasing P-sets, then their intersection is a P-set. Theorem 3. An F-space with countable chain condition is extremally disconnected. (Proved by K. Hoffman and A. Ramsay for disconnected F-spaces.) Theorem 4. Let X be basically disconnected and satisfy an $\aleph_1$ chain condition. Then X contains no nowhere dense P-set iff it satisfies the countable chain condition. Theorem 5. A P-set in a basically disconnected space is basically disconnected in the subspace topology. (Received June 23, 1971.)

71T-G157. ROBERT A HERRMANN, U. S. Naval Academy, Annapolis, Maryland 21402. A collection of nonacceptable topologies for 2^X.

Let (X,t) be a topological space and 2^X the set of all nonempty closed subsets in X. Let $\underline{G} \subset t$ and $\{G_i\}$ be a finite subset of $\underline{G}$. For each $G \in t$, let $N(G) = \{F \in 2^X \mid F \cap G \neq \emptyset\}$ and for each $\{G_i\} \subset \underline{G}$, let $N(XG_i) = \{F \in 2^X \mid F \cap G_i \neq \emptyset$ for all $G_i \in \{G_i\}\}$. Definition. The T-topology for 2^X is the topology generated by the subbase $\{N(G) \mid G \in t\} \cup \{N(XG_i) \mid \{G_i\} \subset \underline{G}\}$. It is clear that the strongest T-topology is the finite topology and the weakest the upper semifinite. Theorem 1. If (X,t) is T_1 and there exists a nonempty $G_1 \in t$ such that $G_1 \not\subset \bigcup \{G \mid G \in \underline{G}\}$ and $G_1 \cup (\bigcup \{G \mid G \in \underline{G}\}) \neq X$, then T is a nonacceptable and non-T_1 topology for 2^X. Among the various properties any T-topology shares with the finite topology on 2^X, we have the following where X is T_1. Theorem 2. If $f: 2^X \to Y$ is continuous with respect to the T-topology, then the function $g(x) = f(\{x\})$ is continuous on X into Y. Theorem 3. If $F(X) = \{F \in 2^X \mid F \text{ is finite}\}$, then $F(X)$ is dense in $(2^X, T)$. Theorem 4. (X,t) is separable iff $(2^X, T)$ is separable. Theorem 5. (X,t) is second countable iff $C(X) = \{F \in 2^X \mid F \text{ is compact}\}$ is second countable with respect to the topology induced by T. Theorem 6. If $F_n(X) = \{F \in 2^X \mid F \text{ has at most } n \text{ elements}\}$, then the surjection $f: X^n \to F_n(X)$ defined by $f((x_1, \ldots, x_n)) = \{x_1, \ldots, x_n\}$ is continuous with respect to the product topology on X^n and that induced by T on $F_n(X)$. Theorem 7. Assume $F(X) \subset A \subset 2^X$. If X is connected, then $F_n(X)$ and A are connected in $(2^X, T)$. (Received June 24, 1971.)

*71T-G158. CARLOS R. BORGES, University of California, Davis, California 95616. Elastic spaces are monotonically normal. Preliminary report.

Recently, Zenor, Lutzer and Heath have announced studies of the class of monotonically normal spaces (which was introduced by Zenor). When we first studied stratifiable spaces (Pacific J. Math. 17(1966), 1-16), we were led by Lemma 4.2 to study the class (which we called central spaces) of spaces X which satisfy the following property: For each $x \in X$ and open neighborhood $U \subset X$ of x there exists neighborhood U_x of x such that $U_x \cap V_y \neq \emptyset$ implies $x \in V$ or $y \in V$. It turns out that this class of spaces is exactly the class of monotonically normal spaces of Zenor. This enables us to answer some questions of Heath, Lutzer and Zenor and to prove that these spaces are preserved under closed continuous images. It also turns out that the elastic spaces of Tamano and Vaughan (Proc. Amer. Math. Soc. 28(1971), 297-303) are monotonically normal. (Received June 21, 1971.)

71T-G159. JOHN D. BAILDON, Pennsylvania State University, Dunmore, Pennsylvania 18512. Open simple maps and periodic homeomorphisms. Preliminary report.

A map f is said to be of order $\leq$ k if, for any y, $f^{-1}(y)$ contains at most k points. A map of order ≤ 2 is called a simple map if some point inverse contains two points. Sieklucki has shown (Fund. Math. 48(1960), 217-228) that every continuous function of finite order on a compact, finite dimensional space is a composition of a finite number of simple maps. Since $w = z^3$ on the unit sphere is of order 3, the corresponding theorem does not hold for factoring open maps into open simple maps due to the following Theorem. If the open map f between 2-manifolds without boundary is the composition of n open simple maps, then f is of order 2^n. A theorem by M. H. A. Newman on homeomorphisms of period two is extended to homeomorphisms of period 2^k on manifolds with or without boundary. Theorem. If a homeomorphism of period 2^k on an n-manifold M with or without boundary is the identity on a domain, then it is the identity on M. Theorem. Let h be a periodic homeomorphism of period $p_1 p_2 \ldots p_k$ defined on a compact space X. Then the orbit map (decomposition map) associated with h can be written as the composition of k open maps of orders $p_1, p_2, \ldots, p_k$. In particular, if h is of period 2^k, then the orbit map is the composition of k open simple maps. (Received June 28, 1971.) (Author introduced by Professor Louis F. McAuley.)

71T-G160. ROBERT J. DAVERMAN, University of Tennessee, Knoxville, Tennessee 37916. On the scarcity of tame disks in wild cells. Preliminary report.

Let $B^k = \{(x_1, \ldots, x_k) \in E^k | x_1^2 + \ldots + x_k^2 \leq 1\}$, $B^2 = \{(x_1, x_2, 0, \ldots, 0) \in B^k\}$, and $B^1 = \{(x_1, 0, \ldots, 0) \in B^k\}$. Let $\pi : B^k \to B^1$ denote the map sending $(x_1, \ldots, x_k)$ to $(x_1, \ldots, 0)$. Definition. A k-cell K in E^n can be squeezed to an arc iff there exist a map f of E^n onto itself, a homeomorphism h of B^k onto K, and a homeomorphism g of B^1 onto f(K) such that f takes E^n - K homeomorphically onto E^n - f(K) and $fh = g\pi$. Theorem 1. Suppose $3 \leq k \leq n$ and $n \geq 4$. There exists an embedding e of B^k in E^n such that $e(B^k)$ is locally tame modulo a Cantor set but, for some $\delta > 0$, if f is any homeomorphism of B^k onto itself moving no point of B^2 as much as δ, then $ef(B^2)$ is wild. Theorem 2. In case $3 \leq k \leq n - 2$, the cell $K = e(B^k)$ promised by Theorem 1 cannot be squeezed to an arc. (Received June 28, 1971.)

*71T-G161. DAVID J. LUTZER, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. Ordinals and paracompactness in linearly ordered spaces.

Let ω_1 be the first uncountable ordinal and let $X = [0, \omega_1[$ have the usual open-interval topology. In [Amer. Math. Monthly 64(1957), 351] Mary Ellen Rudin proved that there is a subset $A \subset X$ such that neither A nor $X \backslash A$ contains an uncountable closed subset of X. Using this set A we construct an example of a nonparacompact first-countable linearly ordered topological space Y in which every countably compact set is compact. In particular, Y does not contain a topological copy of X even though Y (being nonparacompact) contains uncountable strictly increasing nets. (Received June 28, 1971.)

71T-G162. PAUL L. STRONG, University of Illinois, Urbana, Illinois 61801. Peripheral separability in metric spaces. Preliminary report.

A set Y in a space is peripherally separable (F. B. Jones) if Bd Y is separable. We say a space has condition A if whenever $\{F_\lambda \mid \lambda \in \Lambda\}$ is a disjoint collection of nonempty closed sets in X, then there exists an $x_\lambda \in F_\lambda$ for each $\lambda \in \Lambda$ such that $\mathrm{cl} \bigcup \{F_\lambda \mid \lambda \in \Lambda\} - \bigcup \{F_\lambda \mid \lambda \in \Lambda\} \subset \mathrm{cl}\{x_\lambda \mid \lambda \in \Lambda\}$. Theorem. For a metric space X, the following are equivalent. (a) X has condition A. (b) Every open subset is peripherally separable. (c) X can be expressed as $D \cup S$ where D is an open discrete subspace and S is a separable subspace. (d) Every subset is peripherally separable. (e) cl Y - Y is separable for every $Y \subset X$. Conditions (b) - (e) are also equivalent for a stratifiable space (Borges, "On stratifiable spaces," Pacific J. Math. 17(1966), 1-16), and are implied by condition A in such a space. (Received June 28, 1971.)

Miscellaneous Fields

71T-H4. RICHARD LAVER, University of Bristol, Bristol, England. On well-quasi-ordering sequences.

Let $A = \bigcup_{n<\omega} ({}^n n)$. Quasi-order A by the rule $f \leq_a g \leftrightarrow \exists$ 1-1 increasing $H : \mathrm{dom}\, f \to \mathrm{dom}\, g$ such that $\forall x \in \mathrm{dom}\, f\ H(f(x)) \leq g(H(x))$. Let $A' = \{f \in A : \forall x \in \mathrm{dom}\, f\ fx \leq x\}$. Theorem. A' is well-quasi-ordered, but A is not, under $\leq_a$. Let $P = \{f : \exists n < \omega$ f is a permutation of $n\}$. Quasi-order P by $f \leq_b g \leftrightarrow \exists$ 1-1 increasing I: $\mathrm{dom}\, f \to \mathrm{dom}\, g$ such that $\forall x, y \in \mathrm{dom}\, f\ f(x) < f(y) \leftrightarrow g(I(x)) < g(I(y))$. Let $P' = \{f \in P : f$ is cyclic$\}$ where f is cyclic $\leftrightarrow \exists m \subset n < \omega$ $(m,n) = 1$ dom $f = n$ and $f(i) = mi \pmod n$. Theorem. P' is well-quasi-ordered, but P is not, under $\leqq_b$. A corollary is that the generalized Peterson graphs are ωq^0 under homeomorphic embeddability. If Q is a quasi-ordered set, quasi-order $Q^{<\omega}$ by the rule $f \leqq_c g \leftrightarrow \exists$ 1-1 increasing $J: \mathrm{dom}\, f \to \mathrm{dom}\, g$ such that $\forall x \in \mathrm{dom}\, f$ $f(x) \leq g(J(x))$ and if $x+1 \in \mathrm{dom}\, f$ and $J(x) \leq z < J(x+1)$ then rank $f(x) \leq$ rank $g(z)$. A proposition used in the proofs is that if $\leq$ is an appropriate notion of embeddability, ωq^0 on $Q^{<\omega}$, and if $\langle S_n : n < \omega \rangle$ is a sequence from $Q^{<\omega}$, then there exists $i_1 < i_2 < \ldots$, $K_1, K_2, \ldots$ such that $S_{i_n} \leqq_{K_n} S_{i_{n+1}}$ and such that the direct limit of the K_n's is well ordered. (Received May 28, 1971.)

71T-H5. ROBERT L. CONSTABLE, Computer Science Department, Cornell University, Ithaca, New York 14850. SR-schemata.

In "Loop schemata" (Proc. of Third Annual ACM Symposium on Theory of Computing, Shaker Heights, Ohio, 1971, pp. 24-39) the author gave an algorithm to decide the functional equivalence of monadic loop schemata. There is also an algorithm for multiple argument schemata, but for SR-schemata (adding downward go to's and conditionals to loop schemata) equivalence is undecidable. (Received June 18, 1971.)

ERRATA
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FRANKLIN D. TALL. On the existence of metacompact normal Moore spaces (perfectly normal spaces with σ-point finite-bases) which are not metrizable, Abstract 71T-G89, Page 670.

(1) of the Theorem should be omitted.

WILLIAM F. TRENCH. Strong minimum variance estimation of polynomial plus random noise, Abstract 682-65-13, Page 237.

Replace Equation (2) by: "(2) $E(u) = L(f)$".

DAVID WESTREICH. Bifurcation of a nonlinear operator equation in a real Banach space. Preliminary report, Abstract 682-46-38, Page 186.

The equation in line 1 should read "$x = \lambda Lx + T(\lambda, x)$" instead of "$x = Lx + T(\lambda, x)$". The inequality in line 4 should read "$0 < |\varphi| < c$" instead of "$0 < \varphi < c$".

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