NOTICES
OF THE
AMERICAN MATHEMATICAL SOCIETY

Edited by Everett Pitcher and Gordon L. Walker

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RESERVATION FORM . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 992
The six hundred eighty-eighth meeting of the American Mathematical Society will be held at the Massachusetts Institute of Technology, Cambridge, Massachusetts, on Saturday, October 30, 1971.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, Professor Barry Mazur of Harvard University will lecture on "Frobenius and the Hodge filtration" at 11:00 a.m., and Professor Gerald E. Sacks, Massachusetts Institute of Technology, will present an address entitled "The differential closure of a differential field" at 2:00 p.m. Both lectures will be presented in the Compton Auditorium, Room 26-100. There will be sessions for contributed papers at 9:30 a.m. and at 3:15 p.m. in Rooms 2-190 and 2-390.

Registration will be on the first floor of Building 2. The registration desk will be open from 9:00 a.m. to 12:00 noon and from 1:30 p.m. to 3:30 p.m. Parking space will be available in the East Parking Garage on the MIT grounds, at the corner of Main and Vassar Streets.

MIT is an eight-minute walk from the Kendall Square station of the Cambridge–Dorchester subway; the convenient entrance is then at the northwest corner of the Hayden Memorial Library. Those coming by taxi or bus will find it convenient to use the main entrance, 77 Massachusetts Avenue. Taxi fare from Logan Airport is approximately $4; limousine service is available from the airport to downtown Boston for about $1.50.

Lunch will be served in the MIT Student Center cafeteria. A list of nearby restaurants in Boston and Cambridge will be available at the registration desk.

PROGRAM OF THE SESSIONS

The time limit for each contributed paper is ten minutes. The contributed papers are scheduled at fifteen minute intervals. To maintain this schedule, the time limit will be strictly enforced.

SATURDAY, 9:30 A.M.

Session on Algebra, Room 2-190

9:30-9:40
(1) Knot and linkage projections. Preliminary report
Professor Henry W. Levinson, Rutgers University (688-A5)

9:45-9:55
(2) Short exact sequences which look split
Mr. David Eisenbud* and Mr. R. M. Hamsher, Brandeis University (688-A7)

10:00-10:10
(3) Local Jordan algebras
Dr. Marvin E. Camburn, Mercyhurst College (688-A6)

10:15-10:25
(4) Central simple algebras over real quadratic fields which appear in some \( \mathbb{Q}[G] \)
Professor Toshihiko Yamada, Queen's University at Kingston (688-A2)

10:30-10:40
(5) Primes in progressions modulo \( p^r \)
Professor Patrick X. Gallagher, Barnard College, Columbia University (688-A8)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.
SATURDAY, 9:30 A.M.

Session on Analysis, Room 2-390

9:30-9:40

(6) Perturbing uniform ultimate bounded differential systems
    Professor Stephen R. Bernfeld, University of Missouri-Columbia and University of Rhode Island (688-B2)

9:45-9:55

(7) Iterated fine limits and iterated nontangential limits
    Professor Kohur N. Gowrisankaran, McGill University (688-B4)

10:00-10:10

(8) A class of potentialtheoretic operators
    Professor Maynard G. Arsove*, University of Washington, and Professor Heinz Leutwiler, University of Indiana (688-B5)

10:15-10:25

(9) Uniformization of asymptotic expansions with the methods of Poincaré and Lighthill, "Multiple time scales," "Extension"
    Mr. F. X. Murphy, Jr., City University of New York, and Dr. Guido Sandri*. Aeronautical Research Associates of Princeton, Inc., Princeton, New Jersey (688-B6)

SATURDAY, 11:00 A.M.

Invited Address, Compton Auditorium, Room 26-100

Deformation of principal bundles
    Professor Barry Mazur, Harvard University

SATURDAY, 2:00 P.M.

Invited Address, Compton Auditorium, Room 26-100

The differential closure of a differential field
    Professor Gerald E. Sacks, Massachusetts Institute of Technology

SATURDAY, 3:15 P.M.

General Session I, Room 2-190

3:15-3:25

(10) The Ramsey number N(3,3,3,3;2)
    Dr. Earl Glen Whitehead, Jr., Courant Institute, New York University (688-A1)

3:30-3:40

(11) A combinatorial invariant for graphs. Preliminary report
    Professor Paul Kainen, Case Western Reserve University (688-A3)

3:45-3:55

(12) Formulae for well formed formulae (w.f.f.)
    Professor Dow Tamari, State University of New York at Buffalo (688-E2)

4:00-4:10

(13) Combinatorially triangulating 4-manifolds. Preliminary report
    Mr. Ralph Jones, University of Massachusetts (688-G2)

4:15-4:25

(14) Self-universal crumpled cubes and a dogbone space
    Professor Edmund H. Anderson, Mississippi State University (688-G3)
SATURDAY, 3:15 P. M.

General Session II, Room 2-390

3:15-3:25
(15) Integrals of automorphic forms corresponding to holomorphic sections of elliptic surfaces
Professor William L. Hoyt, Rutgers University (688-A4)

3:30-3:40
(16) Regular ultrafilters and long ultrapowers. Preliminary report
Mr. Murray A. Jorgensen, University of British Columbia and Waterloo Lutheran University (688-E1)

3:45-3:55
(17) The norm of the $L^p$-Fourier transform on unimodular groups. Preliminary report
Professor Bernard Russo, University of California, Irvine (688-G1)

4:00-4:10
(18) The $p$-classes of an $H^*$-algebra
Professor James F. Smith, Le Moyne College (688-B3)

4:15-4:25
(19) On the Hahn-Banach theorem. Preliminary report
Professor Hidegoro Nakano, Wayne State University (688-B1)

NEWS ITEMS AND ANNOUNCEMENTS

ARTHUR B. COBLE MEMORIAL LECTURES

The Department of Mathematics of the University of Illinois, Urbana-Champaign, has announced that the second annual series of Arthur B. Coble Memorial Lectures will be delivered by Professor Samuel Eilenberg of Columbia University on October 11, 12, and 13, 1971. The topics of the three lectures will be "Theory of automata," "Topology and automata," and "Categories and automata." These lectures are supported by the University of Illinois Foundation through a fund established by the late Professor Coble's family. This year they are supported, in addition, by a generous contribution from the International Business Machines Corporation.

FELLOWSHIP AND RESEARCH OPPORTUNITIES IN THE MATHEMATICAL SCIENCES

In its annual brochure on Fellowship and Research Opportunities in the Mathematical Sciences, the Division of Mathematical Sciences of the National Research Council calls attention to a number of fellowships and other kinds of support for research in the mathematical sciences at both the predoctoral and postdoctoral levels to be awarded during the year 1971-1972. Copies of this brochure are available from the Division of Mathematical Sciences, National Research Council, 2101 Constitution Avenue, N. W., Washington, D. C. 20418.

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The six hundred eighty-ninth meeting of the American Mathematical Society will be held at Auburn University in Auburn, Alabama, on Friday and Saturday, November 19-20, 1971.

By invitation of the Committee to Select Hour Speakers for Southeastern Sectional Meetings, there will be three one-hour addresses, all of which will be presented in Room 307 of the Commons Building. Professor J.C. Cantrell of the University of Georgia will give an address entitled "Locally flat embeddings of manifolds." An address entitled "Developments in the theory of Schauder bases" will be given by Professor Charles W. McArthur of Florida State University, and Professor John Neuberger of Emory University will give an address entitled "Quasi-analyticity and semigroups."

There will be sessions for contributed papers both Friday afternoon and Saturday morning. Abstracts for contributed papers should be sent to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904, so as to arrive prior to the deadline of October 4, 1971.

The registration desk will be located in the entrance hall of the Commons Building, Physical Science Center, where all sessions will be held. Registration hours will be 10:00 a.m. to 5:00 p.m. on Friday, November 19, and 9:00 a.m. to 12:00 noon on Saturday, November 20.

Auburn is located on Interstate 85 approximately one-third of the way from Montgomery, Alabama, to Atlanta, Georgia. Auburn is also accessible via U.S. 280, U.S. 29, and U.S. 80. The nearest commercial airline terminals are Atlanta, Georgia (two hours driving time); Columbus, Georgia, and Montgomery, Alabama (one hour driving time, each). It is likely that several pick-ups can be arranged at these airports for persons flying in and for whom car rental is impractical. Persons requiring such service should write to Professor L. P. Burton, Head, Department of Mathematics, by November 15.

Meals, except for snacks, must be taken at commercial establishments. Coffee and doughnuts will be served each morning in Room 244, Commons Building. A dutch-treat beer party is planned for Friday evening.

The two motels which are within walking distance of the meetings and which are holding blocks of rooms for reservations (deadline November 10) are:

University Motor Lodge (25-room block)
125 North College Street
P. O. Box 831
Phone: 887-6583

<table>
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<tr>
<th>Room Type</th>
<th>Rate</th>
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</thead>
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<tr>
<td>Single</td>
<td>$9.36</td>
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<tr>
<td>One Double Bed</td>
<td>$12.48</td>
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<tr>
<td>Two Double Beds</td>
<td>$14.56</td>
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Heart of Auburn Motel (80-room block)
333 South College Street
P. O. Box 632
Phone: 887-3462

<table>
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<td>$12.48</td>
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<tr>
<td>Two Double Beds</td>
<td>$15.60</td>
</tr>
</tbody>
</table>

This meeting is conducted under the auspices of the American Mathematical Society, a non-profit educational organization.
Additional accommodations are as follows:

Holiday Inn
Birmingham Highway 280
P. O. Box 551
Phone: 887-7065

Single $9.45
(one person per room)
One Double Bed $11.55
(two persons per room)
Two Double Beds $16.80
(two persons per room)

(five miles north of campus on Alabama 147, cocktail lounge)

Holiday Inn
Junction I-85 and U.S. 280
P. O. Box 391
Opelika, Alabama 36801
Phone: 745-6331

One Double Bed $11.55
(one person per room)

Stoker's Motel
1208 Opelika Highway
Auburn, Alabama 36830
Phone: 887-3481

Single $8.40
(one person per room)
One Double Bed $10.50
(two persons per room)
Two Double Beds $12.60
(two persons per room)

(four miles east of campus)

All reservations should be made directly with the motels as early as practicable.

O. G. Harrold
Associate Secretary
Tallahassee, Florida
The Six Hundred Ninetieth Meeting  
University of Wisconsin—Milwaukee  
Milwaukee, Wisconsin  
November 27, 1971

The six hundred ninetieth meeting of the American Mathematical Society will be held at the University of Wisconsin-Milwaukee, Milwaukee, Wisconsin, on Saturday, November 27, 1971. The sessions of the meeting will be held in the Science Complex, which houses the Mathematics Department, and in the adjoining Physics and Engineering Building. These buildings are located near the corner of Cramer Street and Kenwood Boulevard in northeast Milwaukee.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, there will be two one-hour addresses. Professor Raghavan Narasimhan of the University of Chicago will address the Society at 11:00 a.m.; his subject will be "Deformations of principal bundles." Professor Mary-Ellen Rudin of the University of Wisconsin, Madison, will speak at 1:45 p.m. on the topic "Set theory and general topology." By invitation of the same committee there will be three special sessions of selected twenty-minute papers. Professor Morris Marden of the University of Wisconsin-Milwaukee is arranging one such session on the subject of Function Theory; the list of speakers will include Stephen B. Agard, Patrick R. Ahern, Herbert J. Alexander, Albert Baernstein II, Peter L. Duren, Stephen D. Fisher, Michael B. Freeman, Simon Hellerstein, Albert Marden, Joseph B. Miles, Ricardo Nirenberg, Walter J. Schneider, and Daniel F. Shea. Another special session is being arranged by Professor Frank A. Raymond of the University of Michigan on the subject of Transformation Groups; the list of speakers will include Gary C. Hamrick, John J. Hinrichsen, Ronnie Lee, Reinhard E. Schultz, and Philip D. Wagner. The third special session is being arranged by Professor Thomas G. McLaughlin of the University of Illinois on the subject of Recursive Functions; the speakers will be Louise Hay, Carl G. Jockusch, Jr., Alistair H. Lachlan, Manuel Lerman, Michael Machtey, Yiannis N. Moschovakis, James C. Owings, Jr., Marian B. Pour-El, Robert W. Robinson, Gerald E. Sacks, Robert I. Soare, and Hisao Tanaka.

There will be sessions for the presentation of contributed ten-minute papers both morning and afternoon. Those having time preferences for the presentation of their papers should so indicate on their abstracts. Abstracts should be submitted to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904, so as to arrive prior to the deadline of October 4, 1971. There will be a session for late papers if one is needed, but late papers will not be listed in the printed program of the meeting.

On Friday afternoon, November 26, 1971, the day before the meeting itself, the University of Wisconsin-Milwaukee will sponsor a brief symposium on the subject "Function Theory in the 1970's." This symposium will be a part of the dedication ceremonies of the Science Complex. The speakers will be Professor Frederick W. Gehring of the University of Michigan, Professor Robert C. Gunning of Princeton University, and Professor Walter Rudin of the University of Wisconsin, Madison. In addition, the special session on Function Theory mentioned earlier will be an extension of this symposium.

REGISTRATION

The registration desk will be located in the ground floor lobby of the Science Complex. The desk will be open from 1:00 p.m. to 5:00 p.m. on Friday, November 26, and from 8:00 a.m. to 4:00 p.m. on Saturday, November 27.
ACCOMMODATIONS

Dormitory accommodations in the Carl Sandburg Dormitory (3400 North Maryland) have been reserved by the Department of Mathematics for at least one hundred participants at this meeting. A single room, in room suites with bath, is $8. There is ample parking available in the building for a fee. In addition, the following two hotels are recommended. Both are located near downtown Milwaukee about two miles from the campus. Both offer free indoor parking. The rates listed are special ones for those attending the meeting.

THE PFISTER HOTEL AND TOWER
424 East Wisconsin Avenue
(at Jefferson Street)
Singles $12.50
Doubles 17.50

THE MILWAUKEE INN
916 East State Street
Singles $13.00
Doubles 18.00

Reservations for the dormitory or for either of the hotels listed above should be made through the department at least two weeks in advance. The reservation form on the last page of these Notices should be completed and mailed to arrive not later than November 12, 1971.

The YMCA and YWCA are located at 915 West Wisconsin Avenue and 626 North Jackson Street, respectively. Both are approximately 3 1/2 miles from the campus. Reservations should be made directly.

FOOD SERVICE

A dining room located in Carl Sandburg dormitory will be serving meals at a cafeteria-style snack bar. The hours of operation will be posted at the registration desk. Some well-known Milwaukee restaurants are Dutch's Sukiyaki House, the John Ernst Cafe, Frenchy's Restaurant, Mader's German Restaurant, and Karl Ratzsch's Restaurant.

TRAVEL AND LOCAL INFORMATION

Milwaukee is served by Air Michigan, Eastern Airlines, North Central Airlines, Northwest Orient Airlines, Ozark Airlines, and United Airlines.

Limousine service is available from the airport to the downtown area with stops at the Pfister Hotel and Milwaukee Inn. The charge for this service is about $1.40. If enough prior commitments are received, limousine service may be arranged directly to the Sandburg Dormitory. Registrants interested in the direct service from the airport to the dormitory should so indicate on the reservation form included on the last page of these Notices. Direct service will be arranged only if it is economically feasible.

Milwaukee is serviced from Minneapolis and Chicago by the National Railroad Passenger Corporation. Those coming by car should leave the North-South Freeway (Route 141) at the Locust Street exit (about 2 1/2 miles north of the intersection of the North-South Freeway with the East-West Freeway). They should then proceed about 1 1/2 miles eastward on Locust Street and then turn north on either Oakland or Mary­land Avenue.

The campus may be reached from the downtown area by bus, namely, busline No. 30, "Prospect-Maryland" (or "Jackson-Downer" if one doesn't mind an additional four block walk west); exact fare of forty cents is necessary. From the rail terminal, No. 57 connects with No. 30 at Wisconsin Avenue and 4th Street.

ENTERTAINMENT

There will be a complimentary beer and cheese party between 5:00 p.m. and 8:00 p.m. on Friday, November 26, in the fourth floor conference room and lounge of the new Science Complex Building.

Paul T. Bateman
Associate Secretary

Urbana, Illinois
The Seventy-Eighth Annual Meeting
Sahara Hotel
Las Vegas, Nevada
January 17-20, 1972

The seventy-eighth annual meeting of the American Mathematical Society will be held at the Sahara Hotel in Las Vegas, Nevada, January 17-20, 1972. The meeting is being held in conjunction with the annual meeting of the Mathematical Association of America (January 19-21). The Conference Board of the Mathematical Sciences will present a panel discussion on Performance Contracting and Mathematics in the Schools on Wednesday, January 19, at 4:00 p.m. The AMS Committee on Employment and Educational Policy will conduct a panel discussion on Wednesday evening, January 19, at 8:30 p.m.

On the recommendation of the Council of the Society, the invited addresses will be scheduled without regard to the scheduling of sessions for ten-minute contributed papers. See page 485 of the April 1971 issue of these Notices for a more detailed explanation of this new policy. By invitation of the Committee to Select Hour Speakers for the Summer and Annual Meetings, there will be eight invited addresses. They are scheduled as follows:

Blaine Lawson, University of California, Berkeley
Monday, January 17, 9:00 a.m.

Richard S. Palais, Brandeis University
Tuesday, January 18, 1:30 p.m.

Charles C. Pugh, University of California, Berkeley
Thursday, January 20, 2:45 p.m.

Hugo Rossi, Brandeis University
Wednesday, January 19, 4:00 p.m.

Jacob T. Schwartz, Courant Institute
Thursday, January 20, 1:30 p.m.

Robert M. Solovay, University of California, Berkeley
Monday, January 17, 10:30 a.m.

Myles Tierney, Rutgers University
Tuesday, January 18, 9:00 a.m.

Robert F. Williams, Northwestern University
Tuesday, January 18, 10:30 a.m.

The Josiah Willard Gibbs Lecture will be given by Professor Freeman J. Dyson of the Institute for Advanced Study at 8:30 p.m. on Monday, January 17, in Rooms 2 and 3 of the Sahara Hotel Convention Center. The title of Professor Dyson's lecture is "Missed opportunities."

There will be no limit on the number of ten-minute papers presented at the meeting. The deadline for abstracts to be received in the Providence office is November 4, 1971. The Providence office will not be able to accept changes in abstracts. Authors are requested to notify the Providence office of papers to be withdrawn.

Two special sessions of selected twenty-minute papers are scheduled. Professor Louis Auslander of the City University of New York is arranging a session on Harmonic Analysis on Solvable and Nilpotent Lie Groups; this session will begin at 1:30 p.m. on Monday, January 17. Professor Morris W. Hirsch of the University of California, Berkeley, is organizing a session on Foliations and Stable Manifolds; this session is scheduled for 2:45 p.m. on Tuesday, January 18. Lists of speakers will appear in the November issue of the Notices. At least one informal session will be scheduled. Professor Paul R. Halmos of Indiana University is arranging such a session; it will begin at 2:45 p.m. on Tuesday, January 18. Further details on the topics for this session and information on other possible sessions will be given in the November issue of these Notices.

The registration desk will be in the lobby of the Sahara Hotel Convention Center, located on the second floor of the hotel. The desk will be open from
2:00 p.m. to 8:00 p.m. on Sunday, January 16; from 8:00 a.m. to 5:00 p.m. on Monday, January 17; from 8:30 a.m. to 4:30 p.m. on Tuesday through Thursday, January 18-20; and from 8:30 a.m. to 2:30 p.m. on Friday, January 21.

The registration fees for the meetings are as follows:

- Member: $5.00
- Student and Unemployed: $1.00
- Nonmember: $10.00
- Families of registered participants: no charge

The Council of the Society will meet at 2:00 p.m. on Sunday, January 16, in Rooms 11 and 12 of the Sahara Hotel. These rooms are located in the "North Hall" section of the hotel on the second floor. The Business Meeting of the Society will be held on Wednesday, January 19, at 2:30 p.m. in Rooms 2 and 3 of the main convention hall in the Sahara Hotel.

The Mathematical Sciences Employment Register will be maintained from 9:00 a.m. to 4:00 p.m. on Tuesday, January 18; and from 9:00 a.m. to 5:40 p.m. on Wednesday and Thursday, January 19-20, in the International Room of the Stardust Hotel. The International Room is located on the second floor of the hotel above the casino.

Further information and a reservation form will be incorporated into the November issue of these Notice.

Kenneth A. Ross
Associate Secretary
Eugene, Oregon
The sixth annual symposium on Some Mathematical Questions in Biology will be held on December 26–27, 1971, in the Viennese Room of the Bellevue Stratford Hotel, Philadelphia, Pennsylvania. The symposium is cosponsored by the American Mathematical Society and the Society for Industrial and Applied Mathematics, and is being held in cooperation with Section A (Mathematics) of the American Association for the Advancement of Science. It is anticipated that the symposium will be supported by the Institute for Defense Analyses and the National Science Foundation. Registration and hotel arrangements will be announced in SCIENCE.

This is the sixth in a series of annual symposia whose purpose is to stimulate direct communication between mathematicians and biologists with some mathematical background. The main theme of the symposium will be biological organization at various levels including chemical, developmental, and neuronal. Among the speakers will be René Thom of the Institut des Hautes Études Scientifiques.

The program, consisting of six lectures, was arranged by the AMS-SIAM Joint Committee on Mathematics in the Life Sciences assisted by Hirsh Cohen of the Society for Industrial and Applied Mathematics. The members of the committee are Hans J. Bremermann, Jack D. Cowan (chairman), Murray Gerstenhaber, Alston S. Householder, Richard C. Lewontin, and Robert MacArthur. A complete program of the sessions will be included in the November issue of these (Notices).

Jack D. Cowan
Chicago, Illinois
Chairman
MATHEMATICAL TITLE SERVICE

In 1968, the American Mathematical Society began operation of the MATHEMATICAL OFFPRINT SERVICE (MOS). This service, which provided subscribers with numerous options and alternatives, was designed to assist mathematicians in keeping abreast of the current research in their particular fields of interest. Despite the strong support of subscribers, the deficit was such that the Society could not foresee the time when MOS might become widely enough used to warrant full support by the Society. Upon the recommendation of the AMS Committee to Monitor Problems in Communication and with the approval of the Board of Trustees, the system has been revised and simplified so that it may become more widely usable.

The new service will be called the MATHEMATICAL TITLE SERVICE (MTS). Only one product will be offered to subscribers. MTS will process current research articles concurrently with their publication. Each subscriber will receive monthly a list of articles selected especially for him on the basis of the definition of his interests which he will have supplied to MTS. Every listing will provide complete publication information on an article: author(s), title, source document, volume, issue, and page numbers. In addition, the complete set of subject classification numbers assigned to the article from the AMS (MOS) Subject Classification Scheme (1970) will be printed with each article listing. Listings are priced at $0.10 each.

Package subscriptions will also be available. Each package is based on an MTS profile form designed to identify the current research results in the field of the package. A subscriber need only purchase a subscription to one of these packages (filling out no individual profile form), and he will receive monthly listings selected for that particular package. These subscriptions are available in the following fields: ring theory, combinatorics, diophantine approximations and related geometry of numbers, algebraic topology, linear initial value problems, linear boundary value problems, and nonlinear boundary value problems. Sample monthly listings for each package are available upon request.

For more information on the MATHEMATICAL TITLE SERVICE or package subscriptions, please write to MATHEMATICAL TITLE SERVICE, American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904.

VISITING LECTURER PROGRAM IN STATISTICS

A Visiting Lecturer Program in Statistics has been organized for the ninth successive year. The program is sponsored jointly by the American Statistical Association, The Biometric Society, and the Institute of Mathematical Statistics. The National Science Foundation provides partial financial support. Forty-three leading teachers and research workers in statistics— from universities, industry, and government—have agreed to participate as lecturers. Lecture topics include subjects in experimental and theoretical statistics, as well as in such related areas as probability theory, information theory, and stochastic models in the physical, biological, and social sciences. The purpose of the program is to provide information to students and college faculty about the nature and scope of modern statistics, and to provide advice about careers, graduate study, and college curricula in statistics. The organizing committee consists of H. T. David, S. W. Greenhouse, S. S. Gupta, W. J. Hall, L. Katz, L. H. Koopmans, I. Olkin, D. B. Owen (chairman), and G. J. Resnikoff. Further information about this program may be obtained by writing to Visiting Lecturer Program in Statistics, Department of Statistics, Southern Methodist University, Dallas, Texas 75222.
PANEL OF VOLUNTEERS
FOR CAREER INFORMATION

The American Mathematical Society and the many students who ask for career information from the Society are indebted to the volunteers listed below, who have, with impressive care and thoughtfulness, encouraged students in mathematics by answering their letters. An average of approximately seventy requests for career information are received every month. Most of them are routine in nature and are answered with a form letter and pamphlets available for free distribution by the Society. Some of the letters, however, show a real interest in mathematics; these are sent to the panel of volunteers to be answered. During the past year, eighty-one letters were turned over to these volunteers. The present volunteers are Richard A. Alo (Carnegie-Mellon University), Richard V. Andree (University of Oklahoma), William F. Atchison (University of Maryland), Prem N. Bajaj (Wichita State University), Thomas L. Bartlow ( Villanova University), Barnard H. Bissinger (Pennsylvania State University), Wray G. Brady (Slippery Rock State College), R. C. Carson (University of Akron), S. Charmonman (University of Missouri at Columbia), Daniel Clock (Northern Michigan College), Romae J. Cormier (Northern Illinois University), Charles H. Cunkle (Slippery Rock State College), John M. Danskin (University of California, Santa Cruz), Richard C. DiPrima (Rensselaer Polytechnic Institute), Underwood Dudley (DePauw University), Joseph H. Engel (The Franklin Institute), Harry A. Gehman (State University of New York at Buffalo), Herbert A. Gindler (San Diego, California), A. K. Gupta (University of Arizona), Deborah T. Haimo (University of Missouri at St. Louis), Franklin Haimo (Washington University), R. G. Helsel (Ohio University), Hubert L. Hunzeker (Michigan Technological University), Ladis D. Kovach (Salinas, California), David M. Krabill (Bowling Green State University), George R. Kuhn (Northwestern Michigan College), John B. Lane (Edinboro State College), William J. LeVeque (Claremont Graduate School), William F. Lucas (Ithaca, New York), Eugene Lukacs (Washington, D. C.), Kenneth O. May (University of Toronto), Bernard McGovern (Cherry Hill, New Jersey), Robert A. Melter (Southampton College, L. I. University), Paul D. Minton (Southern Methodist University), Richard C. Morgan (St. John's University), Abraham Nemeth (Detroit, Michigan), Sam Newman (Atlantic City, New Jersey), Malcolm W. Oliphant (Hawaii Loa College), Otway Pardee (Syracuse University), George Pirianan (University of Michigan), Lyle E. Pursell (University of Missouri at Rolla), Gordon Raisbeck (Arthur D. Little, Inc.), Stewart M. Robinson (Cleveland Heights, Ohio), Ervin Y. Rodin (Washington University), Alex Rosenberg (Cornell University), Paul Rotter (The Mutual Life Insurance Company), Jules P. Russell (Polytechnic Institute of Brooklyn), Albert J. V. Sade (Pertuis, Vaucluse, France), I. Richard Savage (Stanford, California), Thomas H. Southard (California State College at Hayward), Raymond A. Spong (General Dynamics/Electric Boat), Nancy Tapper (Ithaca, New York), Charles J. Thorne (U. S. Naval Missile Center), C. Ionescu Tulcea (University of Illinois), Daniel H. Wagner (Daniel H. Wagner Associates), H. Westcott Vayo (University of Toledo), Myron E. White (Teanake, New Jersey), W. Thurmon Whitney (Marshall University), John W. Young, Jr. (The National Cash Register Company).

Anyone who would be willing to be a part of this service is invited to send his name, address, and field of interest to Career Information, American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904.

EXCHANGE PROGRAMS

The International Research and Exchanges Board (IREX) has recently published a new pamphlet entitled "Exchange Programs with Eastern Europe and the Soviet Union for 1972-1973." This pamphlet gives details on the many programs administered by IREX which include exchanges for a semester or an academic year, short term travel grants, and awards for support of collaborative projects. Copies may be obtained by writing to IREX, 110 East Fifty-Ninth Street, New York, New York 10022.
MEMORANDA TO MEMBERS

FINAL REPORT ON 1971 SUMMER RESEARCH INSTITUTE ON PARTIAL DIFFERENTIAL EQUATIONS

The Society held its eighteenth annual summer research institute at the University of California, Berkeley, from August 9 to August 27, 1971. The topic for the institute was selected by the Committee on Summer Institutes which at the time was composed of Lynn H. Loomis (chairman), Joseph L. Doob, Gilbert A. Hunt, Louis Nirenberg, and Edwin Spanier. The Organizing Committee consisted of Alberto P. Calderón, Lars V. Hörmander, Charles B. Morrey, Jr., Louis Nirenberg (chairman), James B. Serrin, Isadore M. Singer, and Donald C. Spencer. The program for the institute was divided into several parts: (1) six lecture series on recent developments in various topics of partial differential equations; (2) five seminars with invited one-hour talks on subjects of a more technical nature than those presented in the lecture series; (3) five introductory expository lectures presented in an evening program organized by students and young faculty members. A total of 256 mathematicians attended the institute; this number included 40 participants from foreign countries. The National Science Foundation provided financial support to the institute. The proceedings will be published by the Society during the coming year as one of the series of Proceeding of Symposia in Pure Mathematics.

DELIVERY OF THE Notices

The Notices are shipped to members via second-class mail and readers do not always receive the final announcement of a meeting in time to make plans to attend. Subscribers whose mailing address is in the United States or Canada may arrange for first-class mail delivery by paying an additional $6 per year. Requests for this service should be sent in writing to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904.

COLLOQUIUM LECTURES

A limited number of lecture notes of the two series of Colloquium Lectures presented at the summer meeting in University Park, Pennsylvania, in August 1971 are still available. These lectures were "Uniformization, moduli and Kleinian groups" by Professor Lipman Bers of Columbia University, and "Algebraic groups and arithmetic groups" by Professor Armand Borel of the Institute for Advanced Study. Requests for notes should be accompanied by a check for $1 for each set, made payable to the American Mathematical Society, to cover the cost of handling; and requests should be mailed to the Society, P.O. Box 6248, Providence, Rhode Island 02904.

RETIRED MATHEMATICIANS

The List of Retired Mathematicians Available for Employment will once again be published in January 1972 and will be distributed to subscribers to the Employment Register lists when the January issue is mailed. Besides being available to subscribers, the list is available on request from the Employment Register office, American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904. Copies will also be available at the annual meeting in Las Vegas, Nevada, January 17-21, 1972.

Retired mathematicians who are interested in being included in the list may either request a form from the Employment Register office or send the following information: name, date of birth, highest degree earned and where it was obtained, most recent employment, present address, date available, references, preference for academic or industrial employment, and geographic location preferred. The deadline for receipt of either the completed form or the preceding information is December 1, 1971.
The International Room of the Stardust Hotel in Las Vegas, Nevada, will be the location of the Mathematical Sciences Employment Register during the annual meeting. The Employment Register will be open for three days, January 18 through January 20, 1972. Hours of operation will be from 9:00 a.m. to 4:00 p.m. on Tuesday and from 9:00 a.m. to 5:40 p.m. on Wednesday and Thursday. If necessary, evening interviews will be scheduled on Wednesday and Thursday.

Registration for the Employment Register is separate and apart from meeting registration; it is, therefore, most important that both applicants and employers sign in at the Employment Register desk as early as they can on Tuesday morning, January 18.

The new system of operation introduced in January 1971 will again be in effect for the open Register in Las Vegas, with a few administrative changes. The most significant change is that a registration fee of $10 for each employer in the open Register has been established. Also, applicants MUST BE REGISTERED for the general Mathematics Meeting before registering for the open Register; there is no open Register fee for applicants participating in interview schedules. Registration fees, location of the registration area, and hours of operation for the registration of participants for the Mathematics Meeting are listed in the preliminary announcement of the meeting included in this issue of the Notices.

Applicants and employers should secure an instruction sheet to acquaint themselves with the rules and operating regulations. These instruction sheets will be available on request in the International Room registration area at 9:00 a.m. on Tuesday. There will be no interviews scheduled for the first day. Please keep in mind that the registration for the open Register is separate and apart from both meeting registration and from the published listings, and it is imperative that both applicants and employers who wish to participate in the open Register sign in at the Employment Register desk as early as they can on Tuesday morning. Appointments will be scheduled only for those people who have actually signed in at the Register and obtained a code number. Requests for appointments can be submitted on Tuesday and Wednesday only and these interviews will be scheduled on Wednesday and Thursday respectively.

The January published lists will be mailed on approximately December 22. Applicants and employers who wish to be listed in the published list should write to the Mathematical Sciences Employment Register, P.O. Box 6248, Providence, Rhode Island 02904, for applicant qualification forms or position description forms. These forms must be completed and returned to the Employment Register not later than December 1, 1971, in order to be included in the January lists. There is no charge for listing in the published lists except when the late listing charge of $5 is applicable. Provision will be made for anonymity of applicants upon payment of $5 to defray the cost involved in handling such a listing.

A subscription to the lists, which includes three issues (January, May, and August) of both the applicants list and the positions list, is available for $30 a year; the individual issues of both lists may be purchased in January, May, or August for $15. A subscription to the applicants list alone or single copies of that list is not available. Copies of the positions list only may be purchased for $5. A subscription to the list of positions, which also includes three issues (January, May, August) is available for $12 a year. Employers who wish to display literature pertinent to available positions may do so. The charge for this service is $15. Checks should be made payable to the American Mathematical Society and sent to the address given above.

It should also be carefully noted that lists are mailed Book Rate (which means average delivery time from Providence to most locations is approximately 14 to 21 days) unless the purchaser either indicates a willingness in advance to pay the First Class or Air Mail charges or includes the fee for this service when prepayment is made. The applicable postage charges, determined by the location of the purchaser, will be furnished on request to those persons who would like to take
advantage of this service. A limited number of the published lists will also be available at the meeting on a first-come-first-served basis.

The Employment Register is sponsored by the American Mathematical Society, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics for the purpose of establishing communication between mathematical scientists available for employment and employers with positions to fill.

NEWS ITEMS AND ANNOUNCEMENTS

MATHEMATICIANS HONORED

Professor Saunders Mac Lane was awarded the degree of LL.D. from the University of Glasgow, Scotland, on June 23, 1971.

A symposium on Symmetric Functions in Statistics was held at the University of Windsor, Windsor, Canada, on March 13, 1971, in honor of Professor Paul Sumner Dwyer who is retiring this year from the University of Michigan. Invited addresses were given by Professors Dwyer, Florence N. David, Douglas S. Robson, and Edward J. Carney. A number of contributed papers was also presented. The dinner speaker was Professor Cecil C. Craig. Professor Derrick S. Tracy, on behalf of the University of Windsor, spoke of the guidance and inspiration their graduate program in statistics has received from Professor Dwyer. The University of Windsor, in recognition of Professor Dwyer's services and achievements, will award him a Doctorate honoris causa at its fall Commencement on October 23, 1971. Professor Dwyer will be the Commencement speaker at this occasion, which also marks the opening of the new Mathematics Building of the university.

Dedication ceremonies were held on August 14, 1971, for Evans Hall, the new mathematical sciences building on the Berkeley campus of the University of California. The dedication honored Professor Griffith C. Evans, eighty-four. Professor Evans, who was president of the Society in 1939-1940, is credited with building the mathematical sciences faculty and program at Berkeley to its present stature. Among the mathematicians who took part in the ceremony were Professors Charles B. Morrey, Jr., George Evans, Michael Rabin, John M. Hammersley, and John Milnor.

ITALIAN NATIONAL RESEARCH COUNCIL FELLOWSHIPS

The Italian National Research Council will award fifteen fellowships to citizens of other countries who intend to do research in mathematics at Italian universities during the academic year 1972-1973. Each fellowship will carry a monthly stipend of 180,000 Italian lire (approximately $290) for a maximum of twelve months. In addition, recipients will be given a reimbursement for their travel expenses. Applications, written in English, French, or Italian, should be addressed to Consiglio Nazionale delle Ricerche, Servizio Affari Scientifici e Tecnologici, Ufficio Attivita di Ricerca, Sezione Borse di Studio, Piazzale delle Scienze 7, 00100 Roma, Italy. Application should include date and place of birth, citizenship, and residence; nature of the proposed research; names of Italian mathematicians with whom the applicant would like to associate and collaborate; knowledge of Italian, if any, or other foreign languages; address; brief curriculum vitae; two letters of reference; and, if possible, a letter of invitation from an Italian mathematician. Application with all supporting documents must reach the offices of the Consiglio Nazionale delle Ricerche before February 15, 1972. Further information may be obtained by writing to Alessandro Figà-Talamanca, Istituto Matematico, Università di Genova, via L. B. Alberti 4, 16132 Genova, Italy.
EMPLOYMENT IN MATHEMATICAL SCIENCES

Summer 1971

At the summer meeting at Pennsylvania State University, the AMS Committee on Employment and Educational Policy held a panel discussion in which preliminary data available from the salary survey were presented, and estimates of the state of the current job market were discussed. The members of the committee are Richard D. Anderson, William L. Duren, Jr. (chairman), John W. Jewett, and Gail S. Young. This report summarizes some of the data and inferences that served as the basis for that discussion. The salary survey data are presented in detail on pp. 868 and 869 of this issue of Notices. Information on the employment of new recipients of doctorates was obtained by Anderson from a study of the dissertation forms and the starting salary forms. His estimate of the state of the job situation as of the end of August 1971 is illustrated in the left half of the flow chart: of the approximately 1,350 people known at that time to have received doctorates from U.S. and Canadian universities between July 1970 and June 1971, nearly 200 took positions outside of the U.S.; some 130 took positions in business, industry, or government; about 80 were unemployed; no information was available on about 70; and the remainder took academic positions in the U.S., some 20 of them in junior colleges.

The other half of the flow chart was constructed with information obtained from the salary survey forms, which suggest that the national mathematics faculty of four-year colleges and universities (approximately 17,000 last year) has grown by only about 200 positions this year. The annual number of deaths and retirements is currently about 200; this year roughly 500 holders of master's degrees were replaced, and of these, CBMS estimates indicate that about 300 are included among those who received doctorates this year; and there appears to have been an excess of about 50 doctorates entering from other countries or other fields over the number displaced from academic positions. No information is available on the number of doctorates leaving academic positions this year; some estimates suggest that it might be as much as one or two hundred.

CBMS preliminary figures for a year ago indicate a net increase in the national mathematics faculty then of over 800; 440 masters were replaced, 343 of whom received Ph.D.'s; and 169 Ph.D.'s left academia. The number of doctorates entering from other sources, however, was only 70 a year ago.

Thus it appears that not only has the rate of increase of the national mathematics faculty been greatly reduced, but the rate of replacement of masters by doctors has nearly doubled. One result is that...
there has been a significant reduction in the number of instructorships and assistant professorships held by people who have not earned doctoral degrees.

The salary tables (pp. 868-869) indicate that, at least for those departments reporting, an overall reduction in staff was anticipated for universities in Groups I and II, but some expansion was expected for Groups III and IV. Similar patterns appear in the reports on graduate student support (page 871): departments in Groups I and II expected an overall decrease in the number of graduate students, while those in Groups III and IV anticipated a modest expansion.

Estimates on the number of first-year graduate students in 1971-1972 compared to 1970-1971 indicate an overall increase of 5.4%, broken down as follows: Group I, -2.3%; Group II, +1.1%; Group III, +5.8%; Group IV, +14%; master's granting, +12%. Restrictions on departments of "mathematics" only, from among all departments of "mathematical sciences," yields an increase of only 3.7%, with the following breakdown: Group I, -3.6%; Group II, -3.7%; Group III, +1.9%; Group IV, +14%; master's granting, +10%.

L. K. Durst

NEWS ITEMS AND ANNOUNCEMENTS

CONFERENCES IN THE MATHEMATICAL SCIENCES

The National Science Foundation is seeking proposals for five-day regional conferences on subjects of current research interest in the mathematical sciences. The conferences are to be held during the summer or fall of 1972. The objective of the regional conference project is to stimulate and broaden mathematical research activity, particularly in regions of the country where such activity needs further development. The organization of the conferences, evaluation of proposals, and arrangements for publication of conference-related expository papers are to be carried out by the Conference Board of the Mathematical Sciences under contract with the NSF. About a dozen conferences per year are projected, each to take place at a host institution during a summer week, or possibly within a recess of the succeeding fall term. Topics may be concerned with one or more of the various disciplines of the mathematical sciences, including pure mathematics, applied mathematics, statistics, computer science, operations research, and management science.

Each conference should plan for a single principal guest lecturer and about twenty-five other participants, the latter to be active research mathematicians from the broad geographic region around the host institution. It is expected that the lecturer would give two lectures per day during the five days of the conference, with the remainder of the time available for study, informal discussion, and exchange of ideas.

Participants in a conference receive allowances for travel and subsistence; the principal lecturer receives, in addition, a fee for delivering his lectures and for organizing these into a substantial expository paper. The Conference Board arranges for the editing and publication of these expository papers. Inquiries regarding details of proposals for these regional conferences may be addressed to the Conference Board of the Mathematical Sciences, 834 Joseph Henry Building, 2100 Pennsylvania Avenue, N. W., Washington, D. C. 20037. Proposals by prospective host institutions should be sent directly to the Mathematical Sciences Section, National Science Foundation, 1800 G Street, N. W., Washington, D. C. 20550, and should be received by December 1, 1971. Proposals will be evaluated by a panel of the Conference Board and awards of grants will be made by the National Science Foundation with the advice of the panel.
ANNUAL SALARY SURVEY

As has been the practice for several years, questionnaires were sent again this year to departments in the mathematical sciences asking for information on salaries. The two-year colleges were sent a new form this year which allowed them to report salaries without regard to academic rank, because many of the junior colleges do not rank their teaching staffs. These results have been included in a listing under "two-year colleges." In past years the nondegree granting institutions were mainly two-year colleges so the new category covers practically the same institutions. The starting salary survey results have been given separately for men and women in this year's compilation. Except for the starting salary survey, the institutions are divided into groups according to the highest degree offered in the mathematical sciences.

Ph.D. Granting Institutions

Group I and Group II include the universities with leading departments of mathematics according to the findings of a survey made by the American Council of Education in 1969* in which departments were ranked according to the quality of their graduate faculty. Group I is composed of the twenty-seven institutions in which the departments of mathematics were ranked highest, and Group II is made of the thirty-eight institutions which were also listed in that report as having leading departments. It should be noted that most of these institutions have several departments in the mathematical sciences (see figures below) but that the rating was on the strength of the department of mathematics. It is to be noted also that the institutions are different from those shown last year, because this present report is based on the latest survey of the American Council of Education.

Group III includes those universities that have granted three or more doctorates during the last three years according to the list of doctorates which is published in these Notices.

Group IV consists of institutions that have granted two or less doctorates during the past three years.

All Canadian universities maintaining Ph.D. granting programs are included in Groups III and IV.

Master's Degree Granting Institutions.

Bachelor's Degree Granting Institutions.

Two-year Colleges.

The total number of institutions and of departments in the mathematical sciences in the various groups are as follows.

<table>
<thead>
<tr>
<th>Type of institution</th>
<th>Number of Institutions</th>
<th>Number of Departments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ph.D. granting</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group I</td>
<td>27</td>
<td>81</td>
</tr>
<tr>
<td>Group II</td>
<td>38</td>
<td>89</td>
</tr>
<tr>
<td>Group III</td>
<td>74</td>
<td>140</td>
</tr>
<tr>
<td>Group IV</td>
<td>36</td>
<td>49</td>
</tr>
<tr>
<td>Master's granting</td>
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<td>281</td>
</tr>
<tr>
<td>Bachelor's granting</td>
<td>838</td>
<td>868</td>
</tr>
<tr>
<td>Two-year colleges</td>
<td>575</td>
<td>575</td>
</tr>
<tr>
<td></td>
<td>1834</td>
<td>2083</td>
</tr>
</tbody>
</table>

Faculty Salaries

This survey is the fifteenth in an annual series begun in 1957 by the Society's Committee on the Economic Status of Teachers. The 1971 survey is based on returns from 816 departments in the mathematical sciences. The institutions include 63 two-year colleges which do not classify staff according to academic rank, a group not included in previous surveys. The number of returns was smaller than it might have been because no returns were included which were returned after August 15. It should, therefore, be kept in mind that none of this information reflects changes due to the wage freeze.

The 816 departments in the survey cover 11,443 academic positions held in 1970-1971 and 11,592 positions in 1971-1972. Institutions submitted a minimum, median, and maximum salary figure for each of four academic ranks, both for staff members with master’s degrees and for those with doctorates, creating forty-eight categories of salary figures. In some instances, relatively few universities or colleges reported, and inasmuch as there were no significant figures available, salaries could not be listed.

In the following two pages the data in the parentheses give the range of the middle fifty percent of salaries reported. The figures outside the parentheses represent the minimum and maximum salary listed by any reporting institution. Salaries are given in "hundreds of dollars."

All salaries refer to an academic year of nine or ten months. Grants and contracts are included but sabbatical payments and other part-time salaries are not.

*The findings were published in "A Rating of Graduate Programs" by Kenneth D. Roose and Charles J. Anderson, American Council of Education, Washington, D.C., 1969, 115 pp. The information on mathematics was reprinted by the Society and can be found on pages 338-340 of the February 1971 issue of these Notices.

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### PH. D. GRANTING INSTITUTIONS. Group I

<table>
<thead>
<tr>
<th>DOCTORAL DEGREE</th>
<th>Total Number of Staff Members</th>
<th>1970-1971</th>
<th>1971-1972</th>
<th>Minimum</th>
<th>Median</th>
<th>Maximum</th>
<th>Minimum</th>
<th>Median</th>
<th>Maximum</th>
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</thead>
<tbody>
<tr>
<td>Instructor</td>
<td>88</td>
<td>73</td>
<td>70(85-105)</td>
<td>(97-110)</td>
<td>(100-110)</td>
<td>70(97-105)</td>
<td>(100-110)</td>
<td>70(97-105)</td>
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</tr>
<tr>
<td>Asst. Professor</td>
<td>353</td>
<td>308</td>
<td>90(105-115)</td>
<td>(114-118)</td>
<td>(120-135)</td>
<td>74(105-120)</td>
<td>(114-125)</td>
<td>74(105-120)</td>
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<tr>
<td>Assoc. Professor</td>
<td>238</td>
<td>261</td>
<td>118(128-145)</td>
<td>(135-154)</td>
<td>(150-172)</td>
<td>121(131-145)</td>
<td>(139-160)</td>
<td>121(131-145)</td>
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<tr>
<td>Professor</td>
<td>548</td>
<td>545</td>
<td>140(161-188)</td>
<td>(210-240)</td>
<td>(258-315)</td>
<td>140(169-189)</td>
<td>(215-240)</td>
<td>140(169-189)</td>
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### PH. D. GRANTING INSTITUTIONS. Group II

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<thead>
<tr>
<th>MASTER'S DEGREE</th>
<th>Instructor</th>
<th>93</th>
<th>63</th>
<th>52(78-96)</th>
<th>(84-99)</th>
<th>(85-110)</th>
<th>130(78-100)</th>
<th>(85-107)</th>
<th>(86-116)</th>
<th>138</th>
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</table>

### PH. D. GRANTING INSTITUTIONS. Group III

<table>
<thead>
<tr>
<th>MASTER'S DEGREE</th>
<th>Instructor</th>
<th>222</th>
<th>188</th>
<th>49(75-95)</th>
<th>(80-102)</th>
<th>(83-102)</th>
<th>54(75-95)</th>
<th>(80-101)</th>
<th>(83-106)</th>
<th>173</th>
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### PH. D. GRANTING INSTITUTIONS. Group IV

<table>
<thead>
<tr>
<th>MASTER'S DEGREE</th>
<th>Instructor</th>
<th>52</th>
<th>43</th>
<th>34(80-88)</th>
<th>(82-89)</th>
<th>(84-90)</th>
<th>70(80-88)</th>
<th>(84-93)</th>
<th>(90-93)</th>
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### DOCTORAL DEGREE

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<td>Asst. Professor</td>
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<td>200</td>
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<td>(116-123)</td>
<td>(130-140)</td>
<td>156</td>
<td>100(110-115)</td>
<td>(120-127)</td>
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<tr>
<td>Assoc. Professor</td>
<td>104</td>
<td>115</td>
<td>103(127-141)</td>
<td>(137-153)</td>
<td>(144-170)</td>
<td>215</td>
<td>103(131-146)</td>
<td>(140-167)</td>
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<td>403</td>
<td>424</td>
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<td>100(110-115)</td>
<td>(120-127)</td>
<td>(132-140)</td>
<td>165</td>
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### MASTER'S DEGREE GRANTING INSTITUTIONS

#### Number of Usable Returns: 146

### MASTER'S DEGREE

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<th>60(78-90)</th>
<th>(82-95)</th>
<th>(86-103)</th>
<th>164</th>
<th>65(80-95)</th>
<th>(84-100)</th>
<th>(88-107)</th>
<th>188</th>
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<td>481</td>
<td>69(92-105)</td>
<td>(99-113)</td>
<td>(106-124)</td>
<td>153</td>
<td>78(94-113)</td>
<td>(103-117)</td>
<td>(110-127)</td>
<td>159</td>
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<tr>
<td>Assoc. Professor</td>
<td>234</td>
<td>241</td>
<td>78(113-138)</td>
<td>(120-143)</td>
<td>(123-148)</td>
<td>182</td>
<td>93(115-138)</td>
<td>(125-146)</td>
<td>(126-152)</td>
<td>184</td>
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<tr>
<td>Professor</td>
<td>59</td>
<td>62</td>
<td>108(132-163)</td>
<td>(137-167)</td>
<td>(141-183)</td>
<td>298</td>
<td>108(137-166)</td>
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<td></td>
<td>1164</td>
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</tbody>
</table>

### BACHELOR'S DEGREE GRANTING INSTITUTIONS

#### Number of Usable Returns: 310

### BACHELOR'S DEGREE

<table>
<thead>
<tr>
<th>Instructor</th>
<th>290</th>
<th>247</th>
<th>60(78-92)</th>
<th>(81-93)</th>
<th>(83-96)</th>
<th>164</th>
<th>60(81-95)</th>
<th>(84-98)</th>
<th>(85-100)</th>
<th>171</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asst. Professor</td>
<td>527</td>
<td>474</td>
<td>50(89-104)</td>
<td>(93-108)</td>
<td>(95-114)</td>
<td>153</td>
<td>50(91-108)</td>
<td>(95-112)</td>
<td>(99-118)</td>
<td>160</td>
</tr>
<tr>
<td>Professor</td>
<td>81</td>
<td>80</td>
<td>90(120-156)</td>
<td>(120-163)</td>
<td>(120-167)</td>
<td>266</td>
<td>90(125-163)</td>
<td>(125-166)</td>
<td>(125-172)</td>
<td>266</td>
</tr>
<tr>
<td></td>
<td>1402</td>
<td>1563</td>
<td>310</td>
<td></td>
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<td>310</td>
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<td></td>
</tr>
</tbody>
</table>

### TWO-YEAR COLLEGES

#### Number of Usable Returns: 168

### MASTER'S DEGREE

<table>
<thead>
<tr>
<th>Instructor</th>
<th>876</th>
<th>897</th>
<th>36(81-101)</th>
<th>(93-124)</th>
<th>(100-148)</th>
<th>212</th>
<th>40(85-108)</th>
<th>(96-130)</th>
<th>(105-156)</th>
<th>235</th>
</tr>
</thead>
</table>

### DOCTORAL DEGREE

| Instructor       | 42  | 54  | 80(108-160) | (110-160) | (120-162) | 222 | 85(110-163) | (119-172) | (121-173) | 235 |

### TWO-YEAR COLLEGES WHOSE STAFF IS CLASSIFIED BY ACADEMIC RANK

(105 returns; INCLUDED in the figures above)

### MASTER'S DEGREE

<table>
<thead>
<tr>
<th>Instructor</th>
<th>230</th>
<th>217</th>
<th>60(84-99)</th>
<th>(88-108)</th>
<th>(92-117)</th>
<th>170</th>
<th>62(86-104)</th>
<th>(90-114)</th>
<th>(95-128)</th>
<th>170</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asst. Professor</td>
<td>181</td>
<td>207</td>
<td>75(94-110)</td>
<td>(99-118)</td>
<td>(104-130)</td>
<td>173</td>
<td>79(98-115)</td>
<td>(104-125)</td>
<td>(107-137)</td>
<td>193</td>
</tr>
<tr>
<td>Assoc. Professor</td>
<td>99</td>
<td>107</td>
<td>86(108-140)</td>
<td>(117-153)</td>
<td>(130-154)</td>
<td>212</td>
<td>86(110-142)</td>
<td>(120-150)</td>
<td>(121-163)</td>
<td>198</td>
</tr>
<tr>
<td>Professor</td>
<td>50</td>
<td>56</td>
<td>36(109-168)</td>
<td>(125-176)</td>
<td>(125-180)</td>
<td>190</td>
<td>41(125-186)</td>
<td>(137-189)</td>
<td>(137-189)</td>
<td>235</td>
</tr>
</tbody>
</table>
Starting Salary Survey for New Recipients of the Doctorate

The latest figures in this Survey were compiled from questionnaires sent to individuals who received a doctorate in the mathematical sciences during the academic year 1970–1971 from universities in the United States and Canada. Of 1,362 distributed, there were 605 returns considered usable, i.e., compatible with the Survey categories, 576 representing men and 29 representing women. Among the unusable returns, 4 representing males and 6 representing females reported part-time jobs only and were for that reason not used in the compilation of salary information.

In parentheses after each category below are listed the number of returns that represent men and women respectively. Men and women are also treated separately, as well as together, in the 1971 figures, 1971M referring to male replies and 1971F to female.

Most Ph.D.'s, 85%, have accepted academic positions. Of the remainder, 7% are in industry, 3% in research institutions, and 4% in government. The 29 women respondents, reported 25 academic, 2 industrial, and 2 governmental positions.

Geographically, the Survey indicates that 89% have accepted jobs in the United States, 85% men and 4% women; 7%, all men, in Canada; and 4%, again all men, in other locations. There were 25 returns, considered unusable as far as compatibility of salary was concerned, that indicated positions accepted in foreign countries. They include four in England, three each in Australia and Mexico, two each in Ireland and New Zealand, and one each in Brazil, Colombia, Denmark, East Pakistan, Federal Republic of Germany, Hong Kong, India, Iran, Israel, Korea, and Taiwan.

It is interesting to note that 56% of all returns were in the category of teaching, nine-month salary, and that, of all the academic positions, 68% were held in master's or Ph.D. granting institutions, 30% in institutions granting the bachelor's as its highest degree, and 2% in junior colleges. Quartiles are indicated for teaching, nine-month salary.

Salaries are listed in hundreds of dollars. Dashes indicate that not enough returns were received to warrant including the figures here.

<table>
<thead>
<tr>
<th>Year*</th>
<th>Min.</th>
<th>Median</th>
<th>Q3</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1967</td>
<td>65</td>
<td>96</td>
<td>140</td>
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<tr>
<td>1968</td>
<td>72</td>
<td>102</td>
<td>170</td>
<td></td>
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<tr>
<td>1969</td>
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<td>1971</td>
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<td>1971F</td>
<td>70</td>
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<td>110</td>
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<tr>
<td>1971F</td>
<td>70</td>
<td>95</td>
<td>105</td>
</tr>
</tbody>
</table>

*Figures cover degrees conferred in the academic year ending in June of the stated year. 1967 figures cover only degrees conferred during the first six months of 1967.
Graduate Student Support

The figures below represent the major sources of support (excluding tuition) of full-time graduate students enrolled for advanced degrees in 1970-1971 and estimates for 1971-1972. The figures have been converted into percentages by type of support, with the total number of students represented by the sample being given in the last column.

<table>
<thead>
<tr>
<th>SUPPORT SOURCES</th>
<th>Total Number of Students in Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>STUDENTS PRIMARILY SUPPORTED BY</td>
<td>Fellowships and Traineeships</td>
</tr>
<tr>
<td>Ph.D. GRANTING INSTITUTIONS</td>
<td></td>
</tr>
<tr>
<td>Group I (44 returns)</td>
<td></td>
</tr>
<tr>
<td>1970-1971</td>
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</tr>
<tr>
<td>United States Government</td>
<td>22%</td>
</tr>
<tr>
<td>Other Sources</td>
<td>9%</td>
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<tr>
<td>1971-1972</td>
<td>17.5%</td>
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<tr>
<td>United States Government</td>
<td>10%</td>
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<td>Other Sources</td>
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<tr>
<td>Ph.D. GRANTING INSTITUTIONS</td>
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<tr>
<td>Group II (55 returns)</td>
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<tr>
<td>1970-1971</td>
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<tr>
<td>United States Government</td>
<td>14%</td>
</tr>
<tr>
<td>Other Sources</td>
<td>4%</td>
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<tr>
<td>1971-1972</td>
<td>13%</td>
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<tr>
<td>United States Government</td>
<td>4.5%</td>
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<tr>
<td>Other Sources</td>
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<tr>
<td>Ph.D. GRANTING INSTITUTIONS</td>
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<tr>
<td>Group III (90 returns)</td>
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<td>1970-1971</td>
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<tr>
<td>United States Government</td>
<td>14%</td>
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<tr>
<td>Other Sources</td>
<td>8%</td>
</tr>
<tr>
<td>1971-1972</td>
<td>10%</td>
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<tr>
<td>United States Government</td>
<td>8%</td>
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<tr>
<td>Group IV (25 returns)</td>
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<td>1970-1971</td>
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<td>United States Government</td>
<td>7%</td>
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<td>Other Sources</td>
<td>7%</td>
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<tr>
<td>1971-1972</td>
<td>6%</td>
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<td>United States Government</td>
<td>9%</td>
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<tr>
<td>MASTER'S DEGREE GRANTING</td>
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<td>INSTITUTIONS (128 returns)</td>
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<td>1970-1971</td>
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<td>4%</td>
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<tr>
<td>Other Sources</td>
<td>3%</td>
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<tr>
<td>1971-1972</td>
<td>3.5%</td>
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<tr>
<td>United States Government</td>
<td>4.5%</td>
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<tr>
<td>Other Sources</td>
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</tr>
</tbody>
</table>

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SPECIAL MEETINGS INFORMATION CENTER

The purpose of this center is to maintain a file on prospective symposia, colloquia, institutes, seminars, special years, meetings of other associations, and to notify the organizers if conflicts in subject matter, dates or geographical area become apparent. A first announcement will be published in the Notices if it contains a call for papers, place, date, and subject, where applicable; a second announcement must contain reasonably complete details of the meeting in order for it to be published. Information on the pre-preliminary planning will be stored in the files, and will be available to anyone desiring information on prospective conferences. All communications on special meetings should be sent to the Special Meetings Information Center of the American Mathematical Society.

SYMPOSIUM ON THEORY OF COMPUTING

The Fourth Annual ACM Symposium on Theory of Computing, sponsored by SIGACT and the Department of Computer Science of the University of Colorado, will be held in Denver during three days of the first week of May 1972. Papers describing original research in the subject area are being sought. Research areas covered include automata and switching theory, computability and computational complexity theory, theory of formal languages, theoretical aspects of algorithm specification and analysis, theoretical aspects of computer organization, and theoretical aspects of programming and programming languages. An abstract of approximately three to five typewritten pages is suggested, though no length limit is imposed; it must provide sufficient detail to permit technical evaluation, and appropriate references and comparison with related work should be included. Send abstracts by December 1, 1971, to Dr. Arnold L. Rosenberg, IBM, T.J. Watson Research Center, P.O. Box 218, Yorktown Heights, New York 10598. Authors will be notified of acceptance or rejection of their papers by the middle of January 1972. Copies of accepted papers, typed on special forms, will be due at the above address by March 1, 1972. A limited amount of funds have been set aside for the partial support of travel for student authors. Highest priority will be given to the support of students who are to present a paper at the conference and have no nonstudent co-authors.

CONFERENCE ON STATISTICAL METHODS FOR THE EVALUATION OF COMPUTER SYSTEMS PERFORMANCE

A conference on Statistical Methods for the Evaluation of Computer Systems Performance will be held at Brown University on November 22–23, 1971. This conference will be devoted to a discussion of the relation between modern statistical methodology and computer systems performance evaluation, and will bring together statisticians with interests in system evaluation, and practicing systems experts with interests in quantitative methods. Support is being furnished by the Office of Naval Research. Attendance is by invitation only, and requests for invitations should be sent to any member of the Organizing Committee: Professor Walter Freiberger (chairman), Division of Applied Mathematics, Brown University, Providence, Rhode Island 02912; Professor Ulf Grenander, Brown University; Professor Barry Margolin, Department of Statistics, Yale University, New Haven, Connecticut 06520; Dr Rhett Tsao, Computer Science Department, Thomas J. Watson Research Center, IBM, Yorktown Heights, New York 10598.

CONFERENCE ON GENERAL TOPOLOGY

The University of Oklahoma has scheduled a conference on General Topology on March 23–25, 1972. This is a continuation of conferences held in the past at Arizona State University (1967),
Houston University (1968, 1971), Auburn University (1969), and Emory University (1970). The program will consist of several invited speakers and a number of contributed papers. Further information may be obtained by writing to Professor Li Pi Su, Department of Mathematics, The University of Oklahoma, Norman, Oklahoma 73069.

CONFERENCE ON LIE ALGEBRAS AND RELATED TOPICS

The Department of Mathematics of The Ohio State University will host a conference on Lie Algebras and Related Topics to be held on October 29–November 2, 1971. There will be approximately fifteen invited speakers including Professors A. Adrian Albert and Nathan Jacobson. Half of each day will be devoted to formal talks and half to informal discussions. Further information may be obtained by writing to Dr. Robert B. Brown, Department of Mathematics, The Ohio State University, Columbus, Ohio 43210.

CONFERENCE ON INFORMATION SCIENCES AND SYSTEMS

The Sixth Annual Princeton Conference on Information Sciences and Systems will be held at Princeton University on March 23–24, 1972. Authors are invited to submit papers describing new advances, applications, and ideas in the fields of computer science, communication theory, system and circuit theory, including contributions which explore the application of these and related disciplines to current societal issues. Two kinds of contributed papers are solicited. The first consists of papers requiring approximately thirty minutes for presentation; these will be reproduced in full (up to ten pages) in the conference proceedings. The second consists of short papers suitable for presentation in about fifteen minutes; one page summaries of these will be published in the conference proceedings. Title, fifty-word abstract (for thirty-minute papers only), and summary are to be submitted by January 10, 1972. Summaries should be of sufficient detail and length to permit careful reviewing. Authors will be notified of acceptance by February 1, 1972. Instructions for the preparation of accepted papers for the proceedings will be sent to each author. Manuscripts are to be submitted to Professor Murray Edelberg, Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08540.

MATHEMATICS RESEARCH INSTITUTE OF OBERWOLFACH

The Mathematics Research Institute of Oberwolfach (Mathematisches Forschungsinstitut Oberwolfach) has recently announced the program that is planned for 1972.

January 2–6
Arbeitstagung
Chairman: H. Salzmann, Tübingen

January 9–15
Arbeitstagung über Modelltheorie
Chairman: G.H. Müller, Heidelberg

January 16–22
Wahrscheinlichkeitsmassen auf Gruppen
Chairmen: H. Heyer, Tübingen
L. Schmetterer, Wien

February 6–12
Spezielle Funktionen
Chairmen: C. Meyer, Köln
F.W. Schäfke, Berlin

February 13–19
Funktionentheorie
Chairmen: C. Pommerenke, Berlin
J. Winkler, Berlin
H. Wittich, Karlsruhe

February 20–26
Medizinische Statistik
Chairmen: K. Überla, Ulm
E. Walter, Freiburg

February 27–March 4
Mathematische Statistik
Chairman: F. Eicker, Dortmund

March 5–11
Wahrscheinlichkeitsstatistik
Chairman: U. Krengel, Göttingen
March 12-18  
Finite Geometries  
Chairmen: D.R. Hughes, London  
H. Lüneburg, Kaiserslautern

March 19-25  
Gewöhnliche Differentialgleichungen  
Chairmen: H.-W. Knobloch, Würzburg  
R. Reissig, Bochum

March 26-April 1  
Regelungstheorie  
Chairman: P. Sagirow, Stuttgart

April 3-8  
Distributionen und partielle Differentialgleichungen  
Chairman: J. Wloka, Kiel

April 3-8  
Logic Group  
Chairman: G.H. Müller, Heidelberg

April 9-15  
Arbeitstagung  
Chairmen: M. Kneser, Göttingen  
P. Roquette, Heidelberg

April 16-22  
Mathematische Logik  
Chairmen: H. Hermes, Freiburg  
K. Schütte, München

April 23-29  
Methoden und Verfahren der mathematischen Physik  
Chairmen: B. Brosowski, Göttingen  
E. Martensen, Darmstadt

April 30-May 6  
Ringe, Moduln und homologische Methoden  
Chairmen: F. Kasch, München  
A. Rosenberg, Ithaca

May 7-13  
Gruppentheorie  
Chairmen: W. Gaschütz, Kiel  
K.W. Gruenberg, London

May 14-20  
Gruppen und Geometrien  
Chairmen: D. Higman, Ann Arbor  
H. Salzmann, Tübingen

May 21-27  
Grundlagen der Geometrie  
Chairmen: F. Bachmann, Kiel  
A. Barlotti, Perugia  
H. Freudenthal, Utrecht  
E. Sperner, Hamburg

May 28-June 3  
Höhere Geometrie  
Chairmen: R. Artzy, Philadelphia  
W. Benz, Waterloo  
F. Rado, Cluj

June 4-10  
Numerische Behandlung von Differentialgleichungen  
Chairmen: L. Collatz, Hamburg  
Not named

June 18-24  
Variationsrechnung  
Chairmen: E. Heinz, Göttingen  
S. Hildebrandt, Bonn  
W. Jager, Münster

June 25-30  
Graphentheorie  
Chairmen: G. Ringel, Berlin  
K. Wagner, Köln

July 2-8  
Konvexe Körper. Geometrische Ordnungen  
Chairmen: D. Derry, Vancouver  
G. Ewald, Bochum  
O. Haupt, Erlangen  
P. Scherk, Toronto

July 9-15  
Zahlentheorie  
Chairman: Th. Schneider, Freiburg

July 16-22  
Differentialgeometrie im Grossen  
Chairmen: M. Barner, Freiburg  
W. Klingenberg, Bonn  
S.S. Chern, Berkeley

July 23-29  
Kategorien  
Chairmen: J.W. Gray, Urbana  
H. Schubert, Düsseldorf

July 30-August 5  
Funktionalgleichungen  
Chairmen: J. Aczel, Waterloo  
O. Haupt, Erlangen  
A.M. Ostrowski, Basel
August 6-12
Systemtheoretische Probleme der Mechanik
Chairman: K. Magnus, München

August 13-19
Mathematische Methoden der Unternehmensforschung
Chairmen: R. Henn, Karlsruhe
H.P. Künzi, Zürich
H. Schubert, Düsseldorf

August 20-26
Endliche Gruppen und Permutationsgruppen
Chairman: B. Huppert, Mainz

August 27-September 2
Himmelsmechanik
Chairman: E. Stiefel, Zürich

September 3-9
Fastperiodische Funktionen
Chairman: K. Jacobs, Erlangen

September 3-16
Topologie
Chairmen: E. Brieskorn, Göttingen
T. tom Dieck, Saarbrücken
K. Jänich, Regensburg

September 17-23
Geometrie
Chairmen: P. Dombrowski, Köln
K. Leichtweiss, Stuttgart

October 1-7
Funktionalanalysis
Chairmen: H. König, Saarbrücken
G. Köthe, Frankfurt
H. Schaefer, Tübingen
H.G. Tillmann, Mainz

October 8-14
Arbeitstagung
Chairmen: M. Kneser, Göttingen
P. Roquette, Heidelberg

October 15-21
Angewandte Kombinatorik
Chairmen: H. Dinges, Frankfurt
K. Jacobs, Erlangen
D. Morgenstern, Hannover

October 22-28
Problemgeschichte der Mathematik
Chairmen: J.E. Hofmann, Ichenhausen
C.J. Scriba, Berlin

October 29-November 4
Automatentheorie und formale Sprachen
Chairmen: G. Hotz, Saarbrücken
H. Langmaack, Saarbrücken

November 5-11
Fortbildungslehrgang für Studienräte
Chairman: H. Salzmann, Tübingen

November 12-18
Fragen des Mathematikunterrichts an allgemeinbildenden Schulen
Chairman: Not named

November 19-25
Numerische Behandlung von Eigenwertaufgaben
Chairmen: L. Collatz, Hamburg
Not named

November 26-December 2
Algorithmen und Komplexitätstheorie
Chairmen: C.P. Schnorr, Frankfurt
A. Schönhage, Konstanz

December 3-9
Numerische, insbesondere approximationstheoretische Behandlung von Funktionalgleichungen
Chairmen: R. Ansorge, Hamburg
W. Törnig, Jülich

December 10-16
Fragen des Mathematikunterrichts an allgemeinbildenden Schulen
Chairman: Not named

Attendance at the sessions is by invitation only. Those wishing to attend should write directly to the chairmen of individual sessions requesting an invitation.
ALGEBRA SYMPOSIUM

An Algebra Symposium will be held at Carleton University, Ottawa, on March 14-17, 1972. The symposium, sponsored by the National Research Council of Canada and Carleton University, will consist of lectures, three main addresses, research seminars, and contributed papers primarily in the areas of the structure of rings, the structure of groups, and category theory. The conference is designed to bring together Canadian algebraists but will feature four principal speakers from outside Canada. The following mathematicians are among those who have been invited to participate: R. Baer, P.M. Cohn, S. Mac Lane, H. Tachikawa, B. Banaschewski, N. Divinsky, J. Lambek, N.S. Mendelsohn, B.M. Mitchell, P. Ribenboim, D. Solitar, and S. Takahashi. On Saturday, March 18, 1972, immediately following the symposium, Carleton University will host the 23rd Ontario Mathematical Meeting. Persons wishing to receive the second notice of the symposium, and who desire further information should write to Dr. John Poland, Department of Mathematics, Carleton University, Ottawa, Canada.

CONFERENCE ON GRAPH THEORY AND APPLICATIONS

A conference on Graph Theory and Applications will be held in the Department of Mathematics of Western Michigan University on May 11-13, 1972. The principal speakers will include R.L. Graham, R.K. Guy, R. Halin, V. Klee, E.A. Nordhaus, and G. Ringel. A number of invited thirty-minute papers will also be presented. In addition, time will be available for a limited number of contributed papers. Plans call for the publication of the proceedings of the conference. Further information may be obtained from Directors, Conference on Graph Theory and Applications, Department of Mathematics, Western Michigan University, Kalamazoo, Michigan 49001.

MEETING AT UNIVERSITY OF BRITISH COLUMBIA

A regional mathematics meeting will be held at the University of British Columbia in Vancouver on November 12-13, 1971. Three invited addresses will be presented by Professor E.A. Bishop of the University of California, San Diego; Dr. Paul C. Gilmore of the IBM, T.J. Watson Research Center, currently visiting at the University of British Columbia; and Professor Richard Rado of the University of Reading, currently visiting at Waterloo University. Arrangements are being made for sessions for contributed papers. A program of interest to mathematicians from the regional colleges and open discussions on topics such as the preparing of graduate students for teaching are planned. Abstracts should be submitted to Professor Afton H. Cayford, University of British Columbia, Vancouver 8, Canada.

SYMPOSIUM ON APPROXIMATION THEORY AND ITS APPLICATIONS

A symposium on Approximation Theory and its Applications will be held at Michigan State University on March 22-24, 1972. The symposium is being organized by the Department of Mathematics and the Center for Applied Mathematics at Michigan State University. Further information may be obtained by writing to Professor Gerald D. Taylor, Department of Mathematics, Michigan State University, East Lansing, Michigan 48823.

SEMINAR ON ABELIAN GROUPS

A seminar on Abelian Groups will be held at The University of Arizona during the spring semester of 1972. Professor L. Fuchs of Tulane University will be in residence during the entire semester giving advanced lectures. The following visitors will participate in the seminar for periods of approximately one week each: R.A. Beaumont, J.M. Irwin, C. Megibben, R.J. Nunke, E.A. Walker, and R.B. Warfield. Anyone wishing to attend part or all of the semester is invited to do so. Further information may be obtained by writing to Professor Elias Toubassi, Department of Mathematics, The University of Arizona, Tucson, Arizona 85721.
A symposium on Computational Complexity will be held by the Computer Science Department of the Courant Institute of Mathematical Sciences on October 25-26, 1971. This symposium is designed to facilitate the exchange of ideas among those actively engaged in this research, as well as to acquaint other computer scientists who possess a high level of competence in some area related to the subject. It is anticipated that the proceedings will be published. Support is being provided by the Mathematics Program of the Office of Naval Research. Participation is on a non-fee, limited enrollment basis. There are a limited number of subsidies covering partial expenses; however, prospective enrollees who are able to support their attendance out of personal, institutional, or contract funds are urged to do so. A partial list of speakers and participants includes Manual Blum, University of California, Berkeley; Ron Book, Harvard University; Robert Constable, Cornell University; Steve Cook, University of Toronto; Patrick Fischer, University of Waterloo; Don Loveland, Carnegie-Mellon University; Albert Meyer, University of California, Berkeley; Arnold Rosenberg, IBM, T. J. Watson Research Center; Jacob T. Schwartz, New York University; Philip Spira, University of California, Berkeley; Paul Young, Purdue University. Further information may be obtained by writing to Randall Rustin, Courant Institute, 251 Mercer Street, New York, New York 10012.

SYMPOSIUM IN SEQUENTIAL ANALYSIS AND OPTIMAL DESIGN

For the ninth consecutive year, New Mexico State University invites interested mathematicians to participate in a Holiday Symposium, December 27-31, 1971. A series of lectures will be given by Professor Herman Chernoff of Stanford University highlighting recent research in sequential analysis and optimal design; sessions for contributed papers and discussion are also scheduled. Some support for local living expenses will be available. The National Science Foundation is providing partial support for the symposium, and twenty-five guest participants will be invited as part of the Regional Conference Program of the Conference Board of the Mathematical Sciences. Further information may be obtained from Professor G. S. Rogers, Department of Mathematical Sciences, New Mexico State University, Las Cruces, New Mexico 88001.
LETTERS TO THE EDITOR

Editor, the Notices)

The quiet analysis of Gail S. Young (Notices, August 1971) of the sharp contrast between former employment predictions and present realities constitute a notable service to our mathematical community. Of course, he clearly warns us to be much more circumspect about whose advice we follow.

There is no time or proper place for public polemics or recriminations, so I simply propose that Professor Young’s suggestion, and example of quiet diplomacy, be followed by the formation of serious working committees to discuss what went wrong and thence to propose what remedies might now be taken. Of course, in doing so we must not once again listen only to those who formerly misled us. There are those who realized in the early sixties that many public policies, both announced and unannounced ones, were very wrong and who tried very hard throughout the mid-sixties to alter a course they knew would carry us to disaster. The massive support among mathematicians of the programs in the mid-sixties prevented opponents of it from being heard. This is the most important lesson of all, not to allow ourselves to be led by a herding instinct into blind ventures.

In spite of some lip service, we have yet to see any substantial movement to answer the call for genuine relevance in mathematical training or to accept the efforts of established mathematicians to convert portions of other fields into mathematical disciplines.

H. Melvin Lieberstein
Benton, Kansas

Editor, the Notices)

I would like to inquire whether the Society has considered the possibility of using re-cycled paper in its various publications. A number of paper companies claim to produce, at competitive prices, quality paper containing a high proportion of re-cycled material (Business Week, July 17, 1971, p. 86). Various business concerns, including major banks and publishers have used such paper to their satisfaction, and it seems to me that the Society should do the same.

Walter Meyer
Adelphi University

EDITOR’S NOTE

The Society has already started to use re-cycled paper. For example, the ballots for the 1972 election of officers and council members were prepared on Ecology Bond. Investigations are underway for using re-cycled paper for books and journals, but for this particular use we must be concerned with whether the quality is adequate for long-term library use. It appears that this may be the case, although re-cycled paper is not as strong as newly processed paper. Also, for several months the waste paper from the administrative office in Providence has been gathered and delivered to a collection depot each week. This has amounted to over two tons a month. The Society receives $20 a ton which covers about one-half of the costs for collection and delivery.
PERSONAL ITEMS

DONALD H. ADAMS of Monash University, Clayton, Australia, has been appointed to an assistant professorship at the University of Massachusetts.

DAVID C. BUCHTHAL of Purdue University has been appointed to an assistant professorship at the University of Akron.

JOSEPH T. BUCKLEY of the University of Massachusetts has been appointed to an associate professorship at Western Michigan University.

MICHAEL F. CAPOBIANCO of St. John's University has been appointed to the chairmanship of the Division of Natural Sciences at Notre Dame College of St. John's University, Staten Island.

SARVADAMAN CHOWLA of Pennsylvania State University has been appointed to a research professorship at SUNY at Buffalo. His appointment becomes effective in January 1972.

JOHN W. DETTMAN of Oakland University has returned from a leave of absence during which he was in residence at the University of Glasgow, Scotland.

RONALD A. DeVORE of Oakland University, Michigan, has been appointed to a visiting associate professorship at the University of Alberta, Edmonton, for the academic year 1971-1972.

RAYMOND A. DIBRELL, JR., of the American Research Corporation has been appointed vice president of Data Dynamics, Incorporated, Los Angeles, California.

VERENA HUBER DYSON of the University of Illinois at Chicago Circle has been appointed to a visiting associate professorship at the University of Alberta, for the academic year 1971-1972.

MICHAE L EDELSTEIN of Dalhousie University, Nova Scotia, will be on sabbatical leave for the year 1971-1972. He will spend much of the year at the University of California, Berkeley.

DELBERT RAY FULKERSON of the RAND Corporation has been appointed to the Maxwell M. Upson professorship in engineering, and to a professorship in operations research and applied mathematics at Cornell University.

BERNARD R. GELBAUM of the University of California, Irvine, has been appointed to a professorship and to the vice presidency of Academic Affairs at SUNY at Buffalo.

GEORGE E. HABORAK of the U.S. Naval Academy has been appointed to an associate professorship and to the chairmanship of the Department of Mathematics at the College of Charleston.

JAMES F. HURLEY of the University of California, Riverside, has been appointed to an associate professorship at the University of Connecticut.

LAWRENCE S. HUSCH of the Virginia Polytechnic Institute and State University has been appointed to an associate professorship at the University of Tennessee.

BURTON W. JONES of the University of Colorado has been appointed to a visiting professorship at the Colegio Universitario de Cayey, Puerto Rico, for the academic year 1971-1972.

G. SAMUEL JORDAN of the University of Wisconsin has been appointed to an associate professorship at the University of Tennessee.

NICHOLAS D. KAZARINOFF of the University of Michigan has been appointed to a professorship and to the chairmanship of the Department of Mathematics at SUNY at Buffalo.

JAMES R. KRABILL of the University of North Carolina at Greensboro has been appointed to an associate professorship at Pembroke State University, North Carolina.

H.F. MATTSON, JR., of Sylvania Electric Products, Incorporated, and Frazier Research Company has been appointed to a professorship in the Systems and Information Science Program at Syracuse University.

EDWARD J. MAYLAND, JR., of the University of Wisconsin has been appointed to an assistant professorship at York University, Toronto.

JAMES M. ORTEGA of the University of Maryland has been appointed to a visiting professorship at the University of California, San Diego.
ROHIT J. PARikh of Boston University has been appointed to a visiting associate professorship at SUNY at Buffalo.

JEFFREY B. RAUCH of the Courant Institute, New York University, has been appointed to an assistant professorship at the University of Michigan.

GEORGE W. REDDIEN, JR., of the Georgia Institute of Technology has been appointed to an assistant professorship at Vanderbilt University.

GEORGE M. REED of Auburn University has been appointed to an assistant professorship at Ohio University.

PETER SEIBERT of the Instituto Politécnico Nacional, México, has been appointed to a professorship at the Universidad Católica de Chile.

ALAN H. SHUCHAT of the University of Toledo has been appointed to an assistant professorship at Mount Holyoke College.

DURaiswamy Sundararaman of York College, CUNY, has been appointed to a visiting assistant professorship at SUNY at Buffalo.

ALDEN H. WRIGHT of the University of Utah has been appointed to an assistant professorship at Western Michigan University.

Toshihiko Yamada of McGill University, Montreal, has been appointed to a visiting associate professorship at Queen's University, Kingston, Ontario, for the academic year 1971-1972.

Mordecay Zippin of the University of California, Berkeley, has been appointed a senior lecturer at the Hebrew University of Jerusalem, Israel.

PROMOTIONS

To Associate Provost and Dean for Academic Planning, Vanderbilt University: Glen F. Clanton.

To Acting Associate Dean, College of Arts and Sciences, University of Delaware: Ronald H. Wenger.

To Director, Center for Applied Mathematics, Cornell University: William F. Lucas.

To Chairman, Department of Mathematics, University of Michigan: James M. Kister; St. John's University, New York: Edward J. Miranda; Spring Hill College, Alabama: Walter L. Furman.

To Acting Chairman, Department of Mathematics, Oakland University, Michigan: George F. Freeman.

To Associate Chairmen, Department of Mathematics, University of Michigan: Roger C. Lyndon, Maxwell O. Reade.

To Professor, University of Michigan: Wilfred M. Kincaid; SUNY at Buffalo: Robert Rosen; University of Tennessee: Henry Frandsen; Western Michigan University: John W. Petro.

To Associate Professor, University of Michigan: Chung-Tuo Shih; University of Minnesota: Harvey B. Keynes; Oakland University, Michigan: Jon Froemke; SUNY at Buffalo: Thomas W. Cusick; University of Tennessee: Robert J. Daverman, John W. Heidel.

INSTRUCTORSHIPS

Otero Junior College, Colorado: Gladwin E. Bartel; University of Rochester: Melvyn B. Nathanson; SUNY at Buffalo: Jussi A. Ketonen, Dallas E. Webster.

DEATHS

Professor Frank S. Beale of Perry, Maine, died on December 29, 1970, at the age of 72. He was a member of the Society for 46 years.

Mr. William J. Cahill of Satellite Beach, Florida, died on December 13, 1970, at the age of 69. He was a member of the Society for 11 years.

Professor Nathaniel Coburn of the University of Michigan died on June 22, 1971, at the age of 61. He was a member of the Society for 32 years.

Professor Emeritus Julian D. Mancill of the University of Alabama died on July 30, 1971, at the age of 67. He was a member of the Society for 38 years.

Professor Emeritus A. Boyd Mewborn of the Naval Postgraduate School
died on April 24, 1971, at the age of 67. He was a member of the Society for 31 years.

Mr. GERARD J. MITCHELL of the Institute for Defense Analyses, Princeton, New Jersey, died on May 11, 1971, at the age of 41. He was a member of the Society for 11 years.

Professor DONALD L. WEBB of the University of Arizona died on June 1, 1971, at the age of 63. He was a member of the Society for 35 years.

**NEWS ITEMS AND ANNOUNCEMENTS**

**THE NEW YORK PUBLIC LIBRARY**

The New York Public Library has just announced that effective January 1, 1972, services to the public in the Science and Technology Division of the Library will be terminated because of insufficient funds. This Division has not only served several hundred readers and researchers daily, but the staff has answered thousands of telephone calls and letters from around the world each year. The termination of these services must be considered a loss to worldwide scientific and technological communities and related industrial, commercial, and educational interests. Acquisition and processing of scientific and technological materials will continue as before, however, so that the Division may function fully when public service is restored. The largest single source of income for The Research Libraries comes from endowment funds, with the State of New York being the second largest contributor. The City of New York and gifts combine to make the remainder of the operating funds. The New York Public Library cannot charge admission or dues for services because it is chartered as a free public service. It is also the conviction of the trustees that educational materials should be made available to all citizens of the city, state, and nation, regardless of their ability to pay fees. The Library has requested that the members of professional organizations be informed of this situation, and has further requested that those who are concerned about this write to The Research Libraries, Room 210, New York Public Library, Fifth Avenue and 42nd Street, New York, New York 10018.

**CORPORATE MEMBERS AND INSTITUTIONAL ASSOCIATES**

The Society acknowledges with gratitude the support rendered by the following corporations who held either Corporate Memberships or Institutional Associateships in the Society as of June 1, 1971.

**Corporate Members**
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- General Motors Corporation
- International Business Machines Corporation
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- Shell Development Company
- Springer-Verlag
- Union Oil Company of California

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NEW AMS PUBLICATIONS

PROCEEDINGS OF SYMPOSIA IN PURE MATHEMATICS

Volume XXI

REPRESENTATION THEORY OF FINITE GROUPS AND RELATED TOPICS
Edited by Irving Reiner

184 pages; List Price $11.70; Member Price $8.78

This volume constitutes the proceedings of a symposium on Representation Theory of Finite Groups and Related Topics which was held in Madison, Wisconsin, April 14-16, 1970, in conjunction with a sectional meeting of the American Mathematical Society. The symposium was held in honor of Professor Richard Brauer whose fundamental work in representation theory lies at the heart of most of the further developments in this topic. These proceedings contain articles by the participants, based on their symposium lectures. The articles range from brief surveys of results to detailed outlines of proofs, and are intended to indicate the scope of current research in representation theory. Professor Irving Reiner of the University of Illinois, Urbana, was chairman of the Organizing Committee and editor of these proceedings.

Volume XXII

ALGEBRAIC TOPOLOGY
Edited by Arunas Liulevicius

290 pages; List Price $17.80; Member Price $13.35

This volume constitutes the proceedings of the seventeenth summer research institute of the American Mathematical Society, held at the University of Wisconsin, Madison, from June 29 to July 17, 1970. The program of the institute was divided into four parts: (1) survey talks on recent developments in the field of algebraic topology; (2) invited one-hour talks on important recent work; (3) sessions on problems; and (4) seminars organized by the participants. Appearing in this volume are the lecture notes of the survey talks presented by J. Frank Adams, Edgar H. Brown, Jr., Samuel Gitler, Richard K. Lashof, Franklin P. Peterson, Larry Smith, and James D. Stasheff. The remainder of the volume consists of the texts of the invited one-hour talks and a list of research problems. The list of research problems was edited by R. James Milgram.

CBMS REGIONAL CONFERENCE SERIES IN MATHEMATICS

Number 9

NOTES FROM A RING THEORY CONFERENCE
By I. N. Herstein

50 pages; List Price $2.60; Member Price $1.95

These notes represent the material presented in the first eight of ten lectures at a CBMS Regional Conference which was held in Norman, Oklahoma, in June 1970. The author chose those results which seemed to him representative of their areas both as to technique and flavor. The main topics are affine rings, rings of quotients, group algebras, W*-algebras, polynomial identities and their generalizations. In most of the areas touched upon, much work remains to be done; the results developed are merely beginnings of large areas of research that could, or should, be carried out.

MEMOIRS OF THE AMERICAN MATHEMATICAL SOCIETY

Number 95

EQUIVARIANT MAPS OF SPHERES INTO THE CLASSICAL GROUPS
By Jon Folkman

46 pages; List Price $2.40; Member Price $1.80

In this Princeton doctoral dissertation, necessary and sufficient conditions
are given for the existence of an equivariant mapping from the k-dimensional sphere to certain of the classical linear groups G, where k is small enough so that the kth homotopy group of G is stable. The results are applied to estimates for homotopy groups and to the study of vector bundles over lens-spaces, with a remarkable theorem on isometry of k-dimensional lens-spaces.

TRANSACTIONS OF THE MOSCOW MATHEMATICAL SOCIETY

Volume 20

288 pages; List Price $29.30; Member Price $21.98

This volume of translations contains eight papers published by the Moscow Mathematical Society in 1969. The papers include "τ-smooth functionals" by E.V. Maikov; "Tensor products and functors in categories of Banach spaces defined by KB-lineals" by V.L. Levin; "Tensors with a given invariance group and spherical functions on homogeneous spaces" by P.K. Raševskiǐ; "Some properties of singular semisimple algebraic groups over nonclosed fields" by B.Ju. Veĭsfeĭler; "On weak solutions of the Dirichlet and Neumann problems" by V.G. Maz'ja; "On the Poincaré boundary value problem" by M.B. Maljutov; "Mixed problems for a certain class of partial differential equations" by Ju.A. Dubinskiǐ; "Hypoelliptic pseudodifferential operators in the theory of functional spaces" by L.R. Volevič and V.M. Kagan.

Volume 21

320 pages; List Price $24.90; Member Price $18.68

This volume of translations contains seven papers published by the Moscow Mathematical Society in 1970. The papers include "Integral representations of holomorphic functions of several complex variables" by L.A. Aizenberg; "On some topological invariants of algebraic functions" by V.I. Arnol'd; "The generalized power moment problem" by Ju.M. Berezanskii; "Periodic solutions of quasilinear parabolic equations of second order" by Ju.S. Kolesov; "Limit functions in the restricted sense of trigonometric and orthogonal series" by D.E. Men'shov; "Extensions of modules" by B.M. Rudyk; "The conjugacy problem for equations of principal type with two independent variables" by G.I. Eskin.
### VISITING MATHEMATICIANS

#### Supplementary List

The following is a continuation of the list of visiting mathematicians which was given in the August 1971 issue of these Notices. Note that there are two lists.

#### American Mathematicians Visiting Abroad

<table>
<thead>
<tr>
<th>Name and Home Country</th>
<th>Host Institution</th>
<th>Field of Special Interest</th>
<th>Period of Visit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ahlors, Lars (U.S.A.)</td>
<td>Mittag-Leffler Institut, Sweden</td>
<td>Complex Analysis</td>
<td>9/71 - 1/72</td>
</tr>
<tr>
<td>Gehring, Frederick W. (U.S.A.)</td>
<td>Mittag-Leffler Institut, Sweden</td>
<td>Complex Analysis</td>
<td>1/72 - 6/72</td>
</tr>
<tr>
<td>Gustafson, Karl E. (U.S.A.)</td>
<td>Institut de Physique Théorique, Geneva, Switzerland Ecole Polytechnique Fédérale, Lausanne, Switzerland</td>
<td>Operator Theory</td>
<td>8/71 - 7/72</td>
</tr>
<tr>
<td>Hinman, Peter G. (U.S.A.)</td>
<td>University of Oslo, Norway</td>
<td>Logic</td>
<td>9/71 - 6/72</td>
</tr>
<tr>
<td>McBrien, Vincent O. (U.S.A.)</td>
<td>Trinity College, Dublin, Ireland</td>
<td>Algebraic Geometry</td>
<td>1971-72 academic year</td>
</tr>
<tr>
<td>Payton, Robert G. (U.S.A.)</td>
<td>University of Strathclyde, Glasgow, Scotland</td>
<td>Elasticity</td>
<td>1971-72 academic year</td>
</tr>
</tbody>
</table>

#### Foreign Mathematicians Visiting in the United States

<table>
<thead>
<tr>
<th>Name and Home Country</th>
<th>Host Institution</th>
<th>Field of Special Interest</th>
<th>Period of Visit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aarts, Jan (Netherlands)</td>
<td>University of Pittsburgh</td>
<td>General Topology</td>
<td>4/72 - 8/72</td>
</tr>
<tr>
<td>Atkin, A. O. L. (United Kingdom)</td>
<td>Brown University</td>
<td>Modular Forms</td>
<td>7/71 - 6/72</td>
</tr>
<tr>
<td>Boutet de Monvel, Louis (France)</td>
<td>Rutgers University</td>
<td>Partial Differential Equations</td>
<td>1972-73 year</td>
</tr>
<tr>
<td>Cartwright, Mary L. (United Kingdom)</td>
<td>Case Western Reserve University</td>
<td>Differential Equations and Complex Analysis</td>
<td>2/72 - 5/72</td>
</tr>
<tr>
<td>Eichhorn, B. (Israel)</td>
<td>Case Western Reserve University</td>
<td>Statistics</td>
<td>8/71 - 8/72</td>
</tr>
<tr>
<td>Gamkrelidze, R. (U.S.S.R.)</td>
<td>Case Western Reserve University</td>
<td>Control Theory</td>
<td>2/72 - 5/72</td>
</tr>
<tr>
<td>Jackson, C. J. (Jamaica)</td>
<td>University of Michigan</td>
<td>Actuarial Science</td>
<td>9/71 - 5/72</td>
</tr>
<tr>
<td>Khartishvili, G. L. (U.S.S.R.)</td>
<td>Case Western Reserve University</td>
<td>Control Theory</td>
<td>2/72 - 5/72</td>
</tr>
<tr>
<td>Lehto, Olli (Finland)</td>
<td>Case Western Reserve University</td>
<td>Complex Analysis</td>
<td>8/71 - 8/72</td>
</tr>
<tr>
<td>Watt, James M. (United Kingdom)</td>
<td>Texas A&amp;M University</td>
<td>Numerical Analysis</td>
<td>9/71 - 7/72</td>
</tr>
</tbody>
</table>
DOCTORATES CONFERRED IN 1970–1971

The following are among those who received doctorates in the mathematical sciences and related subjects from universities in the United States and Canada during 1970–1971. The numbers appearing in parentheses after each university indicate the following: the first number is the total number of degrees listed for that institution; the next seven numbers are the number of degrees in the categories of 1. Pure Mathematics, 2. Applied Mathematics, 3. Computer Science, 4. Statistics, 5. Operations Research, 6. Mathematical Education, 7. Other. Each entry contains the dissertation title. 137 universities are listed with a total of 1,362 individual names.

ALABAMA

AUBURN UNIVERSITY (10;10,0,0,0,0,0,0)

Ball, Eugene Spencer
Weak normality and related properties

Beck, Olian Oscar
Inverse limit groups

Bell, James Milton
A class of sequence spaces

Propes, John Carmichael
Complements of maximal ideals of semigroups

Reed, George Michael
Concerning dense subspaces of certain first countable spaces

Salzmann, Frank Louis, III
Cooperative matrices and positive operators on cones

Sidbury, James Richard
Oscillatory solutions in the solution space of third order differential equations

Spikes, Paul Winton
Oscillatory properties of the equation $y + qyP = r$

Van Cleave, John Truitt
Concerning spaces with a disconnected quasi-component and $Q$-compactness

Watford, Luck James, Jr.
M-matrices with respect to a cone

UNIVERSITY OF ALABAMA, TUSCALOOSA (10;8,1,0,0,1,0,0)

Abercrombie, Jerald D.
Biocomplex quaternionic function theory

Butler, Charles Morgan
On a combinatorial property of a renewal sequence with an alternating renewal sequence

Gibbs, Richard Morris
On generalized Dirichlet series

Gibson, Richard Gerald
Concerning Darboux functions

Hall, Japheth, Jr.
On the theory of structures in sets

Kellum, Kenneth Richard
Almost continuous real functions

Kiser, Lola Frances
Non-analytic solutions of differential equations

Klose, Karl Richard
On the uniform Jordanization of a family of matrices with non-distinct eigenvalues

Nelms, Dorothy
Concerning connected real functions

Rant, William Howard
A study of Steinitz rings and related rings

ARIZONA

ARIZONA STATE UNIVERSITY (4;3,0,0,0,0,1,0)

Annavedder, Edwin K.
Determination of the finite groups having eight conjugate classes

Barr, Dennis Neal
Existence and uniqueness of solutions of three point boundary value problems

Schlais, Harold Eugene
Non-aposyndesis and indecomposability

Williams, Billy Richard
Critical thinking ability as affected by a unit on symbolic logic

UNIVERSITY OF ARIZONA (5;2,2,0,1,0,0,0)

Brandler, Jacob Alfred
Residuacity properties of real quadratic units

Smith, Robert Paul
Contributions in sequential analysis

Suitt, Clifton B.
Existence and uniqueness of a two dimensional free streamline gravity flow for a liquid issuing from a container

Thomas, Gary Marshall
Prescalar elements in Banach algebras

Zachmann, David Walter
Existence of secondary solutions to some generalized Taylor problems

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Summers, Marguerite King
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Taiburt, John Randolph
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Leon, Jeffrey Samuel
Simple groups of order \(2^a b^r 2\)

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Applications of model theory to complex analysis

Department of Applied Mathematics
Delaney, Michael Ernest
1. Singular perturbation problems involving singular points and turning points
2. On the averaged Lagrangian technique for nonlinear dispersive waves

Ellison, James Auby
Existence, uniqueness and stability of solutions of a class of nonlinear partial differential equations

Everett, William Warren
On the determination of the properties of a medium from its reflection coefficient

Langford, William Finlay
Bifurcation theory of nonlinear boundary value problems

Department of Operations Research
Adler, Ilan
Abstract polytopes

Blackburn, Joseph Daniel, Jr.
Optimal control of queuing systems with intermittent service

Corcoran, Timothy Michael
A reliability reformulation with application to complex systems

Ferland, Jacques A.
Quasi-convex and pseudo-convex functions on solid complex sets

Harrison, John Michael
Queueing models for assembly-like systems

Jaquette, Stratton Christopher
Markov decision processes with a new optimality criterion

Kennedy, Douglas Peter
Some weak convergence theorems and the continuity of queues

McCallum, Charles John, Jr.
The linear complementarity problem in complex space

Notzon, Edmund Matthew, III
Minimax inventory and queuing models

Pinsky, Paul Douglas
Mathematical models concerning the measurement of educational achievement levels

Rosenthal, Robert Wernick
Stability analysis of cooperative games in effectiveness form

Sloss, Judith Shirlamith Lieberman
Stable points of directional preference relations

Department of Statistics
Arnold, Steven Ferris
Products of problems and patterned covariance matrices that arise from interchangeable random variables

Borglum, Dale Gutzon
Some estimators of parameters from a selected population

Brill, Edward Abe
Stochastic models of vehicular traffic congestion

Clevenson, Marshall Lawrence
Asymptotically efficient estimates of the parameters of a moving average time series

Cooke, Peter John
Sequential coverage in geometrical probability

Reading, James Cardon
A multiple comparison procedure for classifying all pairs out of \(k\) means as close or distant

Rogers, Warren Francis
Exact null distributions and asymptotic expansions for rank

Sampson, Allan Robert
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Withers, Christopher Stroude
Power and efficiency of a class of goodness-of-fit tests

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Allday, Christopher James
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Baumgartner, James Earl
Results and independence proofs in combinatorial set theory

Bendersky, Martin
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Camacho Manco, Cesar Leopoldo
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Chichilnisky, Graciela
Group actions on spin manifolds and a generalization of Bargmann's theorem

DeJean, Robert Willis
Zeros of the real part of the zeta function

Delver, Robert
Variational problems within the class of solutions of a partial differential equation

Goldstine, Jonathan
Abstract families of languages generated by bounded languages

Hill, Walter Lewis
Formal groups and zeta-functions of elliptic curves

Jacewitz, Chester Alan
Invariant subspaces on the torus

Jordan, Steven Lee
Some invariants for complex manifolds

Krener, Arthur James
A generalization of the Pontryagin maximal principle and the bang-bang principle

Kupka, Joseph Gregory
A new class of Banach spaces associated with a disintegrable measure

Lai Hon-fei
Characteristic classes of real manifolds immersed in complex manifolds

Leung, Dominic Shui-pui
Deformations of integrals of exterior differential systems

Levine, Ronald Dorian
The compact euclidean space forms of dimension four

Lindberg, Karl John
Contractive projections in Orlicz sequence spaces and continuous function spaces

Massey, Frank Jones
Applications of the theory of evolution equations to symmetric hyperbolic systems of partial differential equations

Minovitch, Michael Andrew
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Nash, David Henry George
HP theorems on a Riemann surface

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Potter, Thomas Franklin
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Discontinuous subgroups of extensions of semi-simple Lie groups

Shahshahani, Mehrdad Mirshams
Discontinuous subgroups of extensions of semi-simple Lie groups

Unsain, Ignacio
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Williams, James Garner
Local uniform spaces

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Anbar, Dan
On optimal estimation methods using stochastic approximation procedures

Drogin, Richard
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Finkelstein, Helen Fein
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Flynn, James O'Donnell
Linear production systems with incentives: A dynamic programming approach

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Yengoyan, Marie-Louise Agop
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Some complexities of simplicity: Concerning the grades of simplicity of recursively enumerable sets
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Bemiller, Stephen Gruber
The infinite topology on closed subsets

Fildes, Robert Alan
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Leezer, Roger William
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Shafer, John
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Travis, Curtis Clyde
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Kerlin, John E.
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Lancaster, Glenn A.
A generalization of spherical functions on complex semisimple Lie groups

Mucci, Anthony G.
Differential equations related to optimal selection problems

Walter, Martin E.
$W^*$algebras and non-Abelian harmonic analysis

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Aucoin, Paschal Joseph, Jr.
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Bachar, John Martin, Jr.
Composition mappings between function spaces

Beer, Gerald Alan
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Chang Chi-lung
Metabelian p-groups of maximal class

Chen Chang-shing
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Dell, Roger Marcus
Lower bounds for solutions to elliptic partial differential inequalities

Dubman, Morton Raymond
Estimates of the renewal function when the second moment is infinite

Dubrovsky, Diana Lydia
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Fraser, Grant Adam
Tensor products of distributive lattices and semilattices

Freedman, Benedict
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Galyean, Paul Hewitt
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Glassman, Bernard Alan
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Kasdan, John Martin
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Kawai, Kazumitsu
Evans-type P-harmonic functions on a Riemann surface

Krakowski, Don
Profinite groups and the Galois group of fields

Lay, Steven Ramar
Combinatorial separation theorems and convexity

Martinez, Daniel Gonzales
Applications of principal functions to fluid dynamics, thermodynamics and electro- and magneto-statics

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Applications of model theory to topology

Öksendal, Bernt Karsten
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Some results on invariant differential operators on a real semi-simple Lie algebra

Pierce, Sam
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Pugh, Edward Lawrence
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Rader, Jon Anthony
Biharmonic functions on Riemannian spaces with applications to elasticity

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Range, Michael Rolf
Bounded holomorphic functions on strictly pseudo-convex domains

Schiff, Joel Linn
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Scholz, Uwe Kurt
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Gordh, George Rudolph, Jr.
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Griesel, Martha Ann
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Lesley, Frank David
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Minda, Carl David
Extremal length, bounds for harmonic functions, and analytic arcs

Munoz, Diego
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Piele, Donald T.
Approximation of harmonic functions by potentials of unit mass

Vinge, Vernor Steffen
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Zavodnik, Raymond J.
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Ali, Shafqat Mohammad
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Cengiz, Bahattin
Isomorphic properties of spaces of continuous functions

Joiner, Charles Madison, Jr.
Some results on free topological groups

Loustau, John Arthur
Associative and commutative and structure theorems in certain classes of rings

Featherstone, John De Forest
Spaces with non-symmetric distance and compactness

Glassco, Donald Edward, II
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Jensen, Don Cook
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Lessner, Lawrence
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A general approach to one-step methods with application to eigenvalue problems

McCornock, Maxwell Duncan
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Passi, Ranjit Mohan
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Field, David Anthony  
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Fraker, Ross McKee  
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On the lattice of topologies
Lee, Robert Alex  
A quantitative solution to a problem of Schnizel
Maynard, Hugh Bardeen  
A Radon-Nikodym theorem for vector-valued measures
Shonkwiler, Ronald Wesley  
Spectral triples, dilations and reproducing kernels
Sikonia, William George  
Essential, singular, and absolutely continuous spectra
Zimmerman, Susan Jane  
Order-induced topological properties

Allen, Ernest Edgar  
Selected characteristics of junior high level mathematics teachers in Colorado
Almes, Philip C.  
Existence and stability of infinitesimal and finite periodic orbits in the restricted problem of three bodies
Carlson, Stanley L.  
Differences in aptitude, previous achievement, and nonintellectual traits (personality, values, interest, and attitude toward mathematics) of freshmen mathematics majors and transfers from the mathematics major at the University of Northern Colorado
Dilley, Larry Sylvester  
Nonassociative number theory
Lacher, Robert Joseph  
$\gamma$-dimension: A generalization of small and large inductive dimensions
Marsh, Dean A.  
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Meyerholtz, Roy Alden  
A comprehensive study of the development of the simplex algorithm for solving linear programming problems
Miller, Gordon L.  
Quine's new foundations: A modern appraisal
Milner, Billy Eugene  
Examples and counterexamples in elementary group theory
Nash, Gale B.  
Commutativity theorems for rings
Nelson, Norman N.  
A survey of the status of probability and statistics in the public secondary schools of Colorado
Richard, Tommy Harold  
The development and appraisal of a unit on Diophantine equations for prospective elementary school teachers
Werremeyer, Frederic Negley, Jr.  
A generalization of the derivative: The Dini derivates

Gregory, Michael B.  
p-Helson sets, $1 < p < 2$

Bauer, David Francis  
Contributions to nonparametric hypothesis testing and estimation
DeLisser, Oswald George  
Classification: The linear discriminant function with samples from mixed normal distributions
Lavoie, Richard Hector  
An ANOVA-type procedure for the analysis of rankings

Brown, Ian D.  
Irreducible representations of lattice subgroups of nilpotent Lie groups
Cahn, Robert Stern  
Some lattice point problems
Cherlin, Gregory Lyle  
A new approach to the theory of infinitely general structures
Cooper, Benjamin Grosvenor  
Co-products in the mod p cohomology of stable two-stage Postnikov systems
Coven, Carol Wood  
Forcing for infinitary languages
Hemperly, John Cecil  
The parabolic contribution to the number of linearly independent automorphic forms on a certain bounded domain
Hurley, Donal J.  
Structure of geodesic flows on compact manifolds
Johnson, David Randolph, Jr.  
The construction of elementary extensions and the Stone-Čech compactification
Pizer, Arnold Koster  
Type numbers of Eichler orders of arithmetically definite quaternion algebras
Ponomarev, Paul  
Class numbers of positive definite quadratic forms in four variables
Porter, Richard Dawson, Jr.
Defining systems and the functor D tor

Racine, Michel Louis
The arithmetics of quadratic Jordan algebras

Ralston, Elizabeth Wall
Fixed point free automorphisms of order pq

Department of Statistics
DuMouchel, William H.
Stable distributions in statistical inference

Ling, Robert F.
Cluster analysis

DELAWARE
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Albright, Richard Andrew
An integration theory for topological spaces

Moulis, Edward Jean
A generalization of univalent functions with bounded boundary rotation

Reidlinger, Roy Francis
Mappings on rings with involution

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AMERICAN UNIVERSITY (1;0,0,0,1,0,0,0)

Department of Mathematics and Statistics
Webster, David B.
An analysis of time series associated with a class of network flow problems

CATHOLIC UNIVERSITY OF AMERICA (5;2,0,0,3,0,0,0)
Archambault, Claire
A generalization of ideal theory to lattice-ordered residuated semigroups

Haborak, George
A generalized vectorial Bartle-integral

Statistical Laboratory
Gilroy, Edward J.
Linear least squares prediction for multivariate time series with missing observations

O'Neill, Robert T.
Some problems of sequential and two stage estimation in sampling from multivariate normal populations

Wiener, Howard L.
A test of significance for periodogram analysis

GEORGETOWN UNIVERSITY (2;2,0,0,0,0,0,0)
Kauflin, John Edward
Existence and regularity of solutions of nonlinear differential equations of parabolic type

Velez, Carmelo E.
On the existence of families of periodic solutions of systems admitting first integrals

GEORGE WASHINGTON UNIVERSITY (11;2,1,0,8,0,0,0)

Butler, Kim Ki-hang
On (0,1)-matrix semigroups

Jones, John, Jr.
Properties of strong solutions of first and second order differential equations and inequalities in Hilbert space

Junghenn, Hugo Dietrich
Almost periodic compactifications and applications to one-parameter semigroups

Department of Statistics
Brown, Charles Clark
Tests of hypotheses and estimations for mixtures of two normal distributions

Coffey, Jerry Lee
On estimates obtained by minimizing certain goodness-of-fit statistics

Fox, Mary Wilkinson
Testing for a change at an unknown point in time in the mean of normally distributed variables

Frishman, Fred
On ratios of random variables and an application to a nonlinear regression problem

Levy, Bert
A tutorial set of problems in mathematical statistics

Pettigrew, Hugh McClelland
On the mathematical theory of epidemics with two or more types of infectives

Saikia, Aroona
Asymptotic methods for estimating the mean of N stationary time series

Weiner, Sidney
Queueing models for passing delays on two and three lane freeways

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FLORIDA STATE UNIVERSITY (8;2,0,0,6,0,0,0)

Fields, David E.
Some ideal theoretic properties of power series rings

Levitz, Kathleen Bulger
Finiteness properties of certain rings

Department of Statistics
Blot, William James
Sequential experimental design procedures

Fairweather, David Warren
Generalized branching process theorems for the class of positively regular first moment matrices

Moore, Bobbie Iris
Relevance of asymptotic theory in small sample estimation
Patel, Kantilal Mangaldas
Randomization tests for the multivariate two-sample and paired-sample problems and the problem of independence

Sielken, Robert Lewis, Jr.
Sequentially determined bounded length confidence intervals for stochastic approximation procedures of the Robbins-Monro type

Taylor, Robert Lee
Some weak laws of large numbers for random elements in normed linear spaces and their applications

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Brooks, Burrow P.
Corings in the category of rings

Chae, Younki
Topological multigroups

Crummer, Arthur Allen
Topological means

Fay, Temple Harold
Relation theory in category

Maxwell, Stephen J.
Certain well-factored categories

Robbie, Desmond Alexander
Some theorems on binary topological algebras

Wright, Reverdy Edmond
Theorems for finite automata

Department of Statistics
McClave, James T.
On some problems of estimation and prediction for non-stationary time series

UNIVERSITY OF MIAMI (1;1,0,0,0,0,0,0)

Snider, Robert Leonard
Radicals of lattices

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EMORY UNIVERSITY (4; 4,0,0,0,0,0,0,0)

Carlisle, W. Homer, III
Some problems in the theory of semigroup varieties

Kropa, James Combs
Topological semigroups with identity metrizability and embeddability

May, Everett Lee, Jr.
Nonlinear spectral theory

Merryman, Emory Hughes
Applications of category theory to normed linear spaces

GEORGIA INSTITUTE OF TECHNOLOGY (8;2,3,3,0,0,0,0,0)

Buckley, James J.
Weak extensions of operators with emphasis on integrals

Lee, Phillip Francis
Periodic orbits of a dynamical system

Lovelady, David Lowell
Behavior of solutions of Stieltjes integral equations

Martens, Walter Fred, Jr.
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Reddien, George William
Projective methods for nonlinear boundary value problems

School of Information and Computer Science
Hoffman, John Marion
A pre-scheduler and management model for a class of computer-user systems

Poore, Jesse Hubbard, Jr.
Toward an algebra of computation

Siegmann, Robert Martin
Optimal simultaneous flow in single path communications networks

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Childress, Joseph Atkins, Jr.
Taming and unknotting problems in piecewise linear manifolds

Maxwell, James Walton
Embedding piecewise linear n-manifolds in euclidean (2n-1)-space

Penney, Carol Wilson
Piecewise linear embeddings of non-compact manifolds

Pfeiffer, George William
Asymptotic solutions of the equation $y'' + qy' + ry = 0$

Reese, Richard Harold
The local contractibility of the space of homeomorphisms of 2-complexes and certain decompositions of manifolds

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Carter, Michael
A Bayesian analysis of concomitant information

Helm, Russell F.
Non-standard techniques for estimating distribution parameters which are functionally related to the range of the random variable

Penn, Lucious Wright
An on-line statistical computer system for lay usage

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Ferguson, David John
Nilalgebras

Haworth, Bryan Anthony
Purely atomic measures
Florey, Francis
A generalization of noncommutative Jordan algebras

Jensen, Lawrence Paul
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Kimberling, Clark
Associative functions and an application to exchangeable stochastic processes

Maneki, Alfred
Multiplier group algebras

Meshboum, Stuart Joseph
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Sadd, Martin
Small deformations on large deformations in viscoelastic liquids

Sanders, William A.
Continued fractions associated with Jacobi series and an extension of Van Vleck's theorem

Bronn, Stephen D.
π-functors and cotorsion theories

Conde, Antonio
B-genus and non-imbeddings

Connell, Fred Jopling
Non-immersions of spin and almost spin manifolds

Fattahi, Abiabdollah
On certain classes of finite solvable groups

Filloy, Eugenio
Pointwise characterization of ideals of differentiable functions

Finkel, David
Some results on local and global structure of finite groups

Heitsch, James Lawrence
The cohomologies of classifying spaces for foliations

Jager, Thomas
Steinberg characters and Gallagher characters

Kochman, Stanley Oscar
The homology of the classical group over the Dyer-Lashof algebra, a Hirsch formula in homology and applications

Lyons, Richard Neil
Characterizations of some finite simple groups with small 2-rank

Machado, Hilton V.
Fixed point theorems for nonexpansive mappings in metric spaces

Pichorides, Stylianas
On the best values of the constants in the theorems of M. Riesz, Zygmund and Koomorov

Raupp, Marco Antonio
Galerkin methods for two-dimensional un-
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Sally, Judith D.
Regular overrings of regular local rings
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Group algebras
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Department of Theoretical Biology
Butz, Edward
Signal transmission in nerve cell dendrites
Nazarea, Apolinario
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Strobeck, Curtis
Population genetics: Evolution of genetic systems

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Barlow, William Jean
Coefficient properties of random variable sequences
Beck, Walter Edward
Efficient quotient representations of meromorphic functions in the disk
Behan, Donald F.
On some problems concerning composition semi-groups of analytic functions
Bertness, Charles Harwick
Corresponding residue systems in non-normal extensions of prime degree
Conatser, Charles Wiley
Contractions and deformations of Lie algebras
Cook, Jean Hall
Decompositions of finite rank, torsion-free Abelian groups
Cope, Douglas Anthony
Integral representations of the confluent hypergeometric function of the second kind
Fendrich, John William
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Gardner, Melvin Frank
Spectral decomposition of stochastic processes with parameter in a Hilbert space
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Gustafson, William Howard
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Hadlock, Charles Robert
Singular perturbations of a class of two point boundary value problems arising in optimal control
Henschen, Lawrence Joseph
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Kaufman, Edwin Henry, Jr.
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Krukenberg, Claire Emil
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Department of Computer Science
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Chase, Stephen Martin
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Ghatge, Pratibha
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Kelly, James Charles
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Quigley, John Brendan
An attempt to prove a Hurewicz theorem in a slightly different context from the theory of shape

Rayar, Mary
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Shrikhande, Mohan S.
Hereditary and cohereditary objects

Varughese, Thottathil V.
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Winkle, Brian J.
Quotient rings and a generalized Ore condition

Bucuzzo, Joseph J.
A cosheaf theory on Grothendieck topologies with application to the relative torsion functors

Chang Chan-nan
Local orthogonal groups over 2-adic fields

Cheng Chao-kun
H-spaces and higher order structures of two-stage Postnikov systems

Gaigalas, Michael Andrew
On the classification of metabelian Lie algebras
Hahn, Alexander John  
On the isomorphisms of the classical groups over infinite fields

Hee, Christopher E.  
A basis for the Wu relations

Martin, Kenneth E.  
Orthogonal group over $R(\pi)$

Meck, Michael Ray  
Cohomology of the Borel subgroups of a Chevalley group

Miglani, Ramesh K.  
Representation-theoretic study of maximal class p-groups

Ostlie, Randolph John  
Pure injective modules

Poss, Richard L.  
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Description and error analysis of a rapidly converging norm reduction method for eigenvectors

Eltze, Ervin Marvin  
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Fischer, Daryl Robert  
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Girolo, Jack Emilie  
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Oakland, David Oliver  
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Peters, Galen Roger  
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Saegrove, Marcus John  
On bitopological spaces

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Oleson, Gary K.  
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Conjugacy problems for \( \mathbb{R}^k \) actions

COLUMBIA UNIVERSITY (38;15,0,5,4,0,0,14)

Bloch, Spencer J.
Algebraic cohomology classes on algebraic varieties

Braude, Eric
Descriptive Baire and descriptive \( \mathbb{Z} \)-analytic sets

Edwards, David
The index program for \( K \)-theory with local coefficients

Gilman, Jane Piore
Relative modular groups in Teichmüller spaces

Jackson, Gregory T.
Submanifolds of euclidean and spherical spaces

Jayne, John E.
Spaces of Baire functions, Baire classes and Suslin sets

Kempf, George
The singularity of certain varieties in the Jacobian of a curve

Kiremidjian, Garo
Complex structures on Riemann surfaces

Linch, Michele R.
On metrics in Teichmüller spaces

Namba, Makoto
A study of complex families of compact complex manifolds

Olsen, Bruce A.
On higher order Weierstrass points

Singley, Donald
Pairs of metrics on Riemannian manifolds

Sundararaman, Duraiswamy
Normal filtration of \( H^1(M,\theta) \)

Williams, Eddie R.
The Poincaré lemma with estimates

Wu Tsu-chih
On elliptic boundary problems

Department of Electrical Engineering and Computer Science

Armanazi, Amr
State-space analysis of piecewise linear systems and networks

Arnold, Hamilton Webster
First order envelope statistics of a noise modulated FM wave with linear pre-detection filtering

Ashkinazy, Aron
Fault detection in asynchronous sequential machines

Bertrand, John
Resistive N-port networks

Cheng, George Tsung-hsien
A study of the synthesis and decomposition of threshold gate networks

Chervin, Robert Michael
Lyapunov stability and feedback control of two-stream plasma systems

Courtin, Patrick
Performance index sensitivity of optimal control systems

DiPerna, Richard Arthur
Computational methods for digital simulation of continuous systems

Eskin, Howard David
Sequencing for discrete event simulation

French, Larry Jack
(Title not available)

Jacusiel, Pedro
System optimization with specified controller structure

905
Kang Young-kook  
Sampling rates in digital realization of continuous dynamic systems

Lupatkin, William Lee  
Nonlocal effects in magnetoplasma waves in Bismuth

Sakanaka, Paulo Hiroshi  
Numerical studies of non-acoustic collisionless shocks and solitary waves

Schatz, Joseph George  
The optimal control of merging aircraft

Singhal, Kishore  
A class lumped and distributed R.C. multiports with multiloop feedback

Stern, William Abraham  
The rubidium-85 maser

Tang, Henry Yue-sun  
Parametric frequency conversion in optically pumped Rb$^{87}$ vapor

Tsai Jiann-dek  
Image formation through a random medium with partially coherent light

Department of Mathematical Statistics

Henry, Neil W.  
Problems in the statistical analysis of Markov chains

Khan, Rasul Ahmad  
On sequential distinguishability

Miller, Joan E.  
Transmission of analog signals over a Gaussian channel by permutation modulation coding

Tobias, Paul Arnold  
Some results concerning mixing rates

Center for Applied Mathematics

Crooke, Philip Schuyler  
On the Saffman model for the flow of a dusty gas and some related eigenvalue inequalities

Falk, Richard Steven  
Variational inequalities with order of convergence estimates

King, John Thomas  
Least squares methods for parabolic initial-boundary value problems

Leibovich, Lewis  
Exact and piecewise constant approximate solutions of quasilinear hyperbolic systems of equations with and without weak convexity conditions

Noble, David Franklin  
Circuit properties of dispersive coupled transmission lines and wave guides

Department of Computer Science

Adelson, Leonard Edward  
Singular perturbations in a class of improperly posed problems

Bewtra, Manju Mather  
Double centraliser theorems for algebras

Brownawell, Woodrow Dale  
Some transcendency results for the exponential function

Chang I-lok  
I. Value distribution of lacunary power series and lacunary Fourier series  
II. Analytic continuation of a class of functions

Iglarsh, Harvey Jerome  
Regularity for the initial-boundary value problem for the heat equation in infinite dimensions

Kerr, Sandria Neidus  
Infinite dimensional manifolds modelled on abstract Wiener spaces

Kime, Linda Almgren  
On purely inseparable field extensions: The diagram of an extension and modular extensions of unbounded exponent

Kozma, Ilan  
Relations between semi-groups of polyhedra

Millman, Richard Steven  
Complex structures on real product bundles with applications to differential geometry

Selby, Michael Allen  
Necessary and sufficient conditions to insure that a certain intrinsic metric is complete

Sternstein, Martin  
Necessary and sufficient conditions for homotopy classification by cohomology and homotopy homomorphisms

Sullivan, John Brendan  
The classification of Hopf algebras with integrals

Wagstaff, Samuel Standfield, Jr.  
On infinite matroids

Department of Operations Research

Garagorry, Fernando L.  
Foundations of deterministic optimization theory

Little, James G., Jr.  
Waiting line processes at signalized traffic intersections

Rom, Walter O.  
On classification theory in mathematical programming and applications

906
Department of Plant Breeding and Biometry

Male, Larry M.

A multiple comparison procedure for binomial random variables

Department of Statistics

Kambhampati, Chandrasekhararao

A chi-square statistic for goodness-of-fit tests

NEW YORK UNIVERSITY (27;17,8,1,0,0,0)

Chen Li-chieh

The buckling of a pre-stressed elastic circular plate

Ching Chin-hung

An existence and uniqueness theorem for the reduced wave equation

Czerniakiewicz, Anastasia J.

Endomorphisms of a free associative algebra of rank 2

Drillick, Albert

The Picard group

Fenichel, Neil

Invariant manifolds for flows: Persistence and smoothness

Fredericks, Albert A.

Iterative methods for minimizing functionals on a Hilbert space with application to optimal control problems

Friedman, Neal

Computation of the equilibrium of a plasma with helical symmetry

Graff, Samuel M.

On the conservation of hyperbolic invariant tori for Hamiltonian systems

Grossman, Edward

On the prime ideal divisors of $\gamma^n - \beta^n$

Harband, Joel J.

Three dimensional flow over a submerged object

Isaac, Donald F.

Simultaneous similarity for sets of matrices

Jacobowitz, Howard

Implicit function theorems and isometric embeddings

Jennings, Gray

Asymptotic behavior in the presence of shocks of difference schemes for nonlinear hyperbolic partial differential equations

Levinson, Henry William

Decomposable braids and linkages

Marlow, Norman A.

Asymptotic tail probabilities for a class of functionals of a Gaussian process

Murdock, James Alan

A generalized e-invariant in stable homotopy of CW-complexes

Quinn, Barbara Keyfitz

Time-decreasing functionals of nonlinear conservation laws

Rauch, Jeffrey

Energy inequalities for hyperbolic initial boundary value problems

Shenkin, Peter

On loss-convergence of estimators

Snider, Arthur David

Numerical solution of nonlinear boundary value problems using reflection

Spellman, Dennis

On the respective terms of the lower central series of pairs of free groups

Stork, Daniel Franklin

Structure and applications of Schreier coset graphs

Strauss, Monty J.

Uniqueness and norm convexity for the Cauchy problem

Swartz, Blair K.

$O(h^k - \omega(Dk^h))$ bounds on some spline interpolation errors

Venugopalkrishna, Ubaradakmittur

Some Toeplitz operators associated with strongly pseudoconvex domains in $\mathbb{C}^n$

Walker, Homer

Elliptic systems of partial differential equations in $\mathbb{R}^n$

Department of Computer Science

Firestone, Roger M.

Parallel programming: Operational model and detection of parallelism

RENSSELAER POLYTECHNIC INSTITUTE (8;0,3,4,1,0,0,0)

Davis, Paul W.

Generalization of the phase-shift-averaging method for oscillations in piecewise linear systems

Gelb, Jack P.

The computer solution of English probability problems

Hutchison, John S.

Finite difference solutions to delay differential equations

Peronilla, Alberto H.

Free-format input for integer programming: A machine code for a modified Gomory algorithm with applications to Weingartner's problem

Steinmetz, William J.

Existence, uniqueness, and asymptotic representation for large bearing numbers of the solution of the nonlinear Reynolds equation of gas bearing theory

Warfield, J. Thomas

Acoustic ray propagation in channels with a horizontal sound speed gradient

Williams, Thomas A.

Scheduling algorithms for a demand-response bus system

Department of Operations Research and Statistics

Hahn, Gerald J.

Prediction intervals for a normal distribution

907
UNIVERSITY OF ROCHELLE (8;7,1,0,0,0,0,0)

Aron, Richard Martin
Topological properties of the space of holomorphic mappings

Blanchard, Hardwin
Growth patterns of sumset sequences \(|CP = C + \ldots + C|\) in abelian groups: Holes, multiplicities, and rooted sets

Conover, Robert Allyn
Normality and products of linearly ordered spaces

Garner, Thomas B.
Vector fields on manifolds with boundary

Kim O-hoe
Properties of a positive linear mapping

Kitchen, Andrew Thornton
Algebraic Alexander-Spanier cohomology

Marathe, Kishore B.
Structure of relativistic spaces
deMatos, Mario Carvalho
Holomorphic mappings and domains of holomorphy

STATE UNIVERSITY OF NEW YORK AT BINGHAMTON (4;4,0,0,0,0,0,0)

Baildon, John David
Open maps and maps onto two-manifolds

Haver, William Emery
Cellular mappings on manifolds

Reed, Myra Jean
Decomposition spaces and separation properties

Woodruff, Edythe Parker
Concerning the condition that a disk in \(E^3/G\) be the image of a disk in \(E^3\)

STATE UNIVERSITY OF NEW YORK AT BUFFALO (8;4,1,2,1,0,0,0)

Georgantas, George T.
Inseparable ring extensions of exponent one

Hofer, Robert D.
Algebraic structures for families of continuous functions

Jackson, William D.
Tennenbaum density

Puttaswamy, Tumku K.
The solution in the large of a certain third order ordinary differential equation of W.B. Ford's type

Reddy, Gosula N.
Conditions for existence and completeness of the wave operators for Schrödinger operators with radial potential

Department of Computer Science

Eckhouse, Richard Henry
A high-level microprogramming language (MPL)

Shriver, Bruce
Interpolation formulas of Gauss type for approximate solution of the n-dimensional heat equation

Department of Statistics

Kryscio, Richard
Computation and estimation procedures in multidimensional right-shift processes with applications to epidemic theory

STATE UNIVERSITY OF NEW YORK AT STONY BROOK (5;0,5,0,0,0,0,0)

Balantrhan, V. K.
Certain spaces of vector-valued distributions and an application to the causality of convolution operators

Ean, Walter K.
Scattering of plane longitudinal elastic waves by a larger convex rigid object with a statistically corrugated surface

Joshi, V. N.
Contribution to the theory of the system of equations

Lee Yee-koo
On the spaces of type \(H\) and their transformations

Surendran, Kamayasamy
Control of quasistatic distributed parameter systems with application to reactor theory

SYRACUSE UNIVERSITY (1;1,0,0,0,0,0,0)

Geramita, Joan
The nature of coproduct of two compact semigroups with identity

YESHIVA UNIVERSITY (8;8,0,0,0,0,0,0)

Bak, Joseph
Muntz-Jackson theorems in \(L^p[0,1]\) and \(C[0,1]\)

Jacobson, David Norman
Mean value problems for specialized measures

Levy, Saul Yermie
Computational equivalence

Shapiro, Jack Sol
A generalized calculus developed from Fredholm operator theory

Snow, Morris
Unbounded linear operators on tensor products

Westreich, David
Bifurcation theory in a Banach space

Wilamowsky, David
Perturbations of unbounded Fredholm operators

Wohlgelernter, Devora Kasachkoff
Weighted \(L^2\) approximation of entire functions and related topics
Dorsey, Frank Colston  
A generalization of the Box-Draper result and some consequences

Krabill, James Richard  
Generalized Dedekind-Rademacher sums

Kramer, Thomas Rollin  
Product spaces and countably subparacompact spaces

Lane, Malcolm Graham  
The numerical solution of improperly posed problems

Shatoff, Larry David  
Multiples and extensions of combinatorial geometries

Webb, Geoffrey Roderick  
The functional central limit theorem for non-stationary mixing sequences of random variables

Williams, Henry Gordon, Jr.  
An extension of the Simon Newcomb problem

Halsey, James Clinton  
Finite automata admitting inverses with some application to cryptography

Hanafy, Lawrence Michael  
A transformation approach to optimal control problems with bounded state variables

Marin, Gerald Arnold  
The multitype branching process with random environment

Park, Stephen Kent  
On the equivalence of optimal control problems and the transformation of optimal control problems with compact control regions into Lagrange problems

Pian Chao-kuang  
On simple rings with uniform right ideals

Redeker, Lewis Timothy  
On the extension of the Davidon-Broyden class of rank one

Quasi-Newton minimization methods to an infinite dimensional Hilbert space with applications to optimal control problems

Taylor, William Wallace  
A theory of contractive mappings in uniform spaces with applications in linear topological spaces

Campbell, Douglas M.  
Generalizing Koch's estimates of variance components

Redeker, Lewis Timothy  
A study of ultraproducts and productive relations

Truitt, Samuel Augusta, Jr.  
Some locally convex topologies on the space of bounded holomorphic functions

Forthofer, Ronald N.  
Alternate family planning programs for India: A simulation experiment
Williams, O. Dale  
Analysis of categorical data with more than one response variable by linear models

Department of Computer Science  
Buttelmann, Henry William, III  
Syntax-semantics systems as structure manipulation systems: Phrase structure grammars and generalized finite automata

Department of Statistics  
Gerig, Thomas Michael  
Non-parametric estimation and testing procedures in multivariate two way layouts

OHIO

CASE WESTERN RESERVE UNIVERSITY  
(21;7,2,5,0,0,0,0)

Department of Mathematics and Statistics  
Belanger, David G.  
Functional characterizations of recurrence
Bien, Darl Dean  
Optimum redundancy for a series k-out-of-n system
Burden, Richard Lee  
Convergence of difference schemes for the heat equation
Coppotelli, Fred  
Elementary functorial syntax
Delfour, Michel Claude  
Hereditary differential equations, defined on a compact interval
Hammood, Abdul Wahab  
Some aspects of the classification problem in multivariate analysis
Klingel, Arthul Raymond  
The effect of parameters of error of observation in both variables on regression coefficients
Meek, Gary Edwin  
DF confidence intervals for coefficients in a linear model
Mida, Chaim  
Boundary behaviour of meromorphic functions
Rajlich, Vaclav  
Absolutely parallel languages and 2-way deterministic finite-state transducers
Reynolds, Albert Coburn, Jr.  
Convergent finite difference schemes for nonlinear parabolic equations
Satinder, Pushpa Kumari  
Completeness and cut-elimination in constructive infinitary arithmetics
Schneider, Leo Joseph  
Oscillatory properties of a fourth order differential equation
Sweet, Leonard  
A generalized approach to symmetric implementation designs
Weiner, Ede Ruth  
Permutations of finite abelian groups: Some counting theorems

Westerbeck, Kent Edward  
The equicontinuous structure relation for ergodic abelian flows

Department of Operations Research  
Brown, Melvin  
Performance evaluation via a systems approach: Some applications to baseball
Fricks, Robert  
Non-convex linear programming
Larson, Roy  
The optimal choice of corporate growth plans under risk
Lev, Benjamin  
Non-iteration algorithm for solving special types of transportation problems
Pill, Juri  
Technical management and control of large-scale urban studies: A comparative analysis of two cases

OHIO STATE UNIVERSITY  
(16;11,0,5,0,0,0,0)

Datta, Biswa Tosh  
On the non-existence of tangential 2 blocks
Engle, Jessie Ann  
Haar measure on left-continuous group and a related uniqueness theorem
Falk, Daniel Allen  
Orders in separable algebras
Johnson, Charles Edward  
a theory of group extensions
Karamanoukian, Zaven Albert  
On the covering problems for the Gaussian and Eisenstein fields
Logan, John David  
Noether's theorems and the calculus of variations
Pujara, Lakhpat Rai  
$L_1$ spaces and decompositions in Banach spaces
St. Andre, Richard Joseph  
Topics in semi-uniform spaces
Sehnert, James Ellis  
Minkowski's conjecture in three dimensions over the fields $Q(1)$ and $Q(e^{2n/3})$
Terrell, Thomas Richard  
Local ergodic theorems for N-parameter semigroups of contraction operators
Yanosko, Kenneth Paul  
A characterization of the family of finite simple groups $Sp(q)$ q even

Department of Statistics  
Heydorn, Richard Paul  
Non-parametric classification
Mayer, Lawrence S.  
Utilizing initial estimates in estimating the coefficients in a general linear model
Oprian, Charles Allen  
On the exact power functions of some rank tests for the trend alternative
Stewart, J. Richard  
Principal component analysis of time series
Sullivan, James Aubrey
On minimizing an expectation with constraints

UNIVERSITY OF CINCINNATI (3;2,1,0,0,0,0,0)

Hill, William Mack, II
Frattini extensions of finite groups
Park Soon-dal
On properties of preorders on the set of options induced by guiding rules
Peele, Charles VanBuren
(Title not given)

UNIVERSITY OF TOLEDO (1;1,0,0,0,0,0,0)

Cano, Julio V.
On fixed points, common fixed points and periodic points of continuous functions on a metric space

OKLAHOMA

OKLAHOMA STATE UNIVERSITY (8;6,0,0,1,1,0,0,0)

Department of Mathematics and Statistics
Broughton, John, III
Introduction to convex programming
Crawford, Albert Lee
Hereditarily indecomposable continua
Flusser, Peter R.
Some characterization problems of random variables with values in a locally compact abelian group
Knapp, Robert Charles
Homological methods and abelian groups
Knight, Ronald Allen
Characterizations of certain classes of planar dynamical systems
Lindstrom, Ralph Raymond
Estimable effects and interactions in an n-way cross classification with missing cells
Owens, Glenda Kay Hastings
The nonexistence of a convex curve with two equichordal points
Shult, Donald Frank
Uniformizable spaces

UNIVERSITY OF OKLAHOMA (7;6,0,1,0,0,0,0)

Bennett, Larry Frank
Integration theory in a Banach space and applications to generalized differential equations
Ganske, Gary Layton
Finite local rings
Jainchell, Richard Anthony
On the representation of integers on binary quartic forms
Kim, Churl S.
Spectral theory for singular linear systems of Hamiltonian differential equations

Tattersall, James Joseph
A generalization of convexity
Teska, John Thomas
A binary representation for finite Boolean functions

Department of Biostatistics and Epidemiology
Kalbfleisch, John H.
The algebraic partitioning of factorial arrangements

OREGON

OREGON STATE UNIVERSITY (9;6,1,0,1,1,0,0)

Higdem, Roger Leon
Sums of subsets of certain algebraic systems
Knoshaug, Ronald Norman
Flows of anisotropic fluids induced by rotating disks
Margolis, William Edward
Topological vector spaces and their invariant measures
Okajima, Isamu
Study of the topological structure of the Royden compactification of the unit disk
Olson, Arthur Eugene, Jr.
Density space theory: Discrete spaces
Swanson, Leonard George
Equivalence of finite measures
Vander Beek, John Wayne
Isoconjunctivity of Hermitian matrices

Department of Statistics
Bogdanoff, David A.
Bayesian inference for the Weibull distribution
Seymore, George Edward
Interval integer programming

UNIVERSITY OF OREGON (12;10,0,0,2,0,0,0)

Book, Stephen Alan
Large deviations for order statistics and weighted sums
Browder, David Sheldon
Derived algebras in $L_1$ of a compact group
Burke, Michael Edmund
Discrete non-linear and simultaneous approximation
Grisel, Ronald David
Singularities and singular differential operators on Riemannian manifolds
Hill, David Arneson
Restricted homological properties of modules
Horn, Peter Joseph
A generalization of symmetric algebras
Johnson, Bruce Rae
Asymptotic behavior of Bayes tests and Bayes risk
Lamet, Daniel Godlieb
Homotopy sets and cohomology operations
Meyer, Norman Christian
The ring of holomorphic functions on an open Riemann surface

Pond, Ralph Gregory
An algebraic and function-algebraic approach to finite-dimensional differential topology and geometry: Global theory

Ryden, Roy Warren, III
The multiplicative set of an arithmetic functions

Watson, William Braidon
Manifolds maps which commute with the Laplacian on p-forms

PENNSYLVANIA

CARNEGIE-MELLON UNIVERSITY

(13;3,2,5,3,0,0,0)

Benthien, George Womack
Semidiscrete approximations to linear boundary-initial-value problems

Cheney, Charles A.
Sets of operator-valued expectations

Daley, Robert Paul
Pseudo-recursiveness and pseudo-randomness within minimal program complexity hierarchies

Nyikos, Peter Joseph
N-compact spaces

Thurston, Louis Harold
The equation of forced vibrations in nonlinear mechanics

Withers, Nathaniel R.
A qualitative approach to the study of ordinary differential equations

Department of Computer Science

Lindstrom, Gary E.
Variability in language processors

Mitchell, James George
The design and construction of flexible and efficient interactive programming systems

Moore, James Aston, Jr.
The design and evaluation of a knowledge net for MERLIN

Pfefferkorn, Charles Elton
Computer design of equipment layouts using the design problem solver (DPS)

Department of Statistics

Blau, Roger Allen
On some optimization problems under uncertainty

Goel, Prem Kumar
Some estimation problems in the study of tampered random variables

Radhakrishnan, Ramaswamy
Some estimation problems in multivariate analysis

LEHIGH UNIVERSITY

(10;6,2,1,1,0,0,0)

Alpert, Stephen Ray
Applications of category theory to model theory

Call, Richard Lawrence
An investigation of generalized quantifiers

Cozzolino, Gerard Eugene
Uniform asymptotic solution of second order differential equations with a single simple turning variety

Michaels, Steven Jeffrey
Probabilistic investigation of boundary layer phenomena in singular perturbation problems

Mittra, Sitansu Sekhar
Isometries of compact hypersurfaces with boundary in a Riemannian space

Ostling, Edward Gregory
Polar topologies and KS spaces

Rohde, Steve Mark
Solutions of the simplest variational problem in n-dimensions in non-parametric form in a class of functions having simple discontinuities

Sabotka, Edward Francis
Some estimates for the discrete Green's function and its difference quotients

Simms, Nathan Frank, Jr.
Stable homotopy in Frobenius categories

Toubassi, Elias Hanna
The splitting of abelian groups and the group Ext(Q,T)

PENNSYLVANIA STATE UNIVERSITY

(25;13,0,7,5,0,0,0)

Bottorff, Gerald A.
On quaternary quadratic forms

Dangello, Frank
Maximally almost periodic semidirect products

Gould, William A.
Solution-preserving operators for a three-dimensional second-order partial differential equation

Green, Gary Brian
Linear differential systems with infinitely many boundary points

Hartzler, Jefferson
On unique factorization of ideals in power series rings

Hurd, Carl
Concerning ideals in $\mathbb{Z}[x]$ and $\mathbb{Z}_p[x]$

Jones, John P.
Local finiteness of periodic groups and algebraic algebras

Malbrock, Jane
Chebychev subspaces in spaces of bounded linear operators
Pirnot, Thomas  
Certain lattices of congruences on completely regular semigroups

Selman, Alan  
Arithmetical reducibilities and sets of formulas valid in finite structures

Shubert, Ronald  
On the torsion groups of units of torsion-free rings

Yu Chie-yunn  
Group rings over modular fields

Zitarelli, David  
Subdirectly irreducible finite inverse semigroups

Department of Computer Science

Ault, David Arthur  
Characterization theorems and algorithms for best approximation

Chaffee, Norman Frederick  
The strategic search component of the SOLID system

Elshoff, James Lester  
The binary floating point digital differential analyzer

Fitzgerald, John M.  
Numerical methods using integrals over subregions

Hughes, Charles Edward  
Representation of many-one degrees by Markov algorithms, semi-Thue systems, and tag systems

Springer, Gordon Kent  
AGISAR: A system for classifying digitized pictorial data

Vander Mey, James Edwin  
An investigation of communications methods for mini-computer systems with multiple low speed terminals

Department of Statistics

Atzinger, Erwin  
Variance components analysis

Hoetzl, George  
Confidence intervals for cross-product ratios and the $2 \times 2 \times 2$ contingency table

Kileen, Timothy  
Bivariate tests for location and their Bahadur efficiencies

Stiteler, William  
Measurement of spatial patterns in ecology

Tryon, Peter  
Nonparametric tests of homogeneity against restricted alternatives in a one-way classification

TEMPLE UNIVERSITY (6;6,0,0,0,0,0,0)

Gowdy, Spenser Oliver  
On Greendlinger's eighth-groups

Hofmann, Charles E., III  
Specializations of Pascal's theorem on an oval

Lord, Graham Frank  
A convolution operation in the theory of partitions

Mc Carty, Carl Patrick  
On the radius of univalence of certain analytic functions

Mc Clow, Jerold John  
On a class of small cancellation groups

Wiley, Samuel John  
On the extensions of left uniformly continuous functions on a topological semigroup

UNIVERSITY OF PENNSYLVANIA (14;7,0,0,2,5,0,0)

Adelman, Murray  
Free abelian categories

Glaser, Marlene  
Asymptotic abelianness of infinite factors

Hopenwasser, Alan Lawrence  
Completely isometric maps and triangular operator algebras

Kean, Orville E.  
(Title not given)

Lyczak, Alexander J.  
Some results on crossing numbers

Shar, Alan O.  
The homotopy groups of spaces whose rational cohomology is a truncated polynomial algebra

Sherwin, Margaret Weinblatt  
A categorical setting for the third cohomology group of an object with coefficients in module

Department of Statistics and Operations Research

Barron, Francis Hutton  
The potential of a doctor's decision role: A constrained optimization model of decision making under risk and uncertainty

Bhoj, Dinesh J.  
On multivariate tests of hypotheses on mean vectors with missing observations

Finnie, William C.  
Towards a systems analysis of racial inequality

Krishnan, K. S.  
On procedures for admitting university students

Simkowitz, Howard  
Common carrier vehicle scheduling over multi-link networks using optimal trip path estimation

Sinha, Bani K.  
Operations research in controlled acquisition and weeding of library collections

Siskin, Bernard  
Reserve decision discrimination

UNIVERSITY OF PITTSBURGH (11;4,0,0,3,4,0,0)

Buoni, John Joseph  
Intrinsic functions of some special Banach algebras
Lee Hua-tsun
On the equiconvergence between two Noerlund transformations

Reiser, Eugene Jacob
Stability of the solutions of elliptic partial differential equations with general boundary conditions

Rishel, Thomas Walter
Some aspects of Morita’s M-spaces

Department of Biostatistics
DeCoufle, Pierre
Mortality patterns of a group of retired asbestos workers

Fukushima, Kazuko
On the problem of bias caused by withdrawal in longitudinal surveys: An analysis of the adult health study in ABCC

Hankey, Benjamin F.
Application of a multivariate risk function in considering the association between food consumption and risk of stomach cancer

Department of Systems Management Engineering and Operations Research
Albosta, Chester A.
A program for the goal-efficient allocation of funds to R and D projects

Borbash, Steven R.
Design of a document retrieval system using pattern recognition and mathematical programming techniques

Hartsock, James H.
A stochastic inventory model for scheduling development drilling

Saatcioglu, Omer
Highway and transit capacity additions to urban transportation networks

RHODE ISLAND
BROWN UNIVERSITY (12;6,6,0,0,0,0,0)

Ewing, John Harwood
On the types of associative H-spaces

Fraser, David Fleming
Rates of convergence for a probabilistic solution to the Dirichlet problem

Hardt, Robert Miller
Slicing and intersection theory for chains associated with real varieties

Preskenis, Kenneth John
Approximation on disks

Stodghill, Jack Richard
On the Adams operators and exterior powers of a simple Lie algebra

Warfield, Virginia McShane
A stochastic maximum principle

Division of Applied Mathematics
Chong Tjee-hung
Linear and nonlinear effects in two and three dimensional supersonic dissipative gasdynamics

Everstine, Gordon Carl
Stress concentrations and boundary layers in fiber-reinforced materials

Kent, George Alan
Optimal control of functional differential equations of neutral type

Levine, Alan
Stability of the parallel flow of a mixture

Melvin, William Richard
A class of neutral functional differential equations

Weis, Dennis George
The asymptotic behavior of some Volterra integral equations arising from heat radiation problems

SOUTH CAROLINA
CLEMSON UNIVERSITY (3;3,0,0,0,0,0,0)

Breen, Marilyn Janet
A determination of the combinatorial type of a polytope by Radon partitions

Holmes, Billy Joe
Convergence theorems for series of Bessel functions

Protomastro, Gerard Philip
The separable projection property

UNIVERSITY OF SOUTH CAROLINA
(2;2,0,0,0,0,0,0)

Cicero, Joseph Edward
Pseudo-Boolean valued rings: Algebraic and geometric theory

Zahn, George D.
On the additive structure of non-standard models of arithmetic

TENNESSEE
GEORGE PEABODY COLLEGE
(10;0,0,1,0,0,9,0)

Allan, Richard Earl
The role of digital computers in the development of geographical methods for data analysis and presentation

Cromer, Fred Eugene
Structural models for predicting the difficulty of multiplication problems

Curry, Richard Dean
Arithmetic achievement as a function of concrete, semiconcrete and abstract teaching methods

Evans, Joe S.
An experimental study of the readability of textbooks as a factor in achievement in college algebra

Foisey, Heeter B.
Mathematics specialists in the elementary school

Forrest, Thomas Douglass
An experiment in teaching complex numbers in basic mathematics for college students
Hall, Wayne Hawkins
The effect on performance of number of exercises, feedback, and amount of detail
Hill, Warren Henry, Jr.
The effect of set theory instruction upon the ability of children to recognize valid and invalid inference patterns in sentential logic
McHenry, Hugh Lansden
An analysis of the relative effectiveness of two embodiments of rigor in a first course in mathematics for prospective elementary school teachers
Mealy, Edward Clark
An evaluation of the use of films as an aid to changing students' attitudes toward mathematics

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Anderson, Edwin Carey
Invariance and minimization of information in sequential tests
Chitwood, Howard
Generalized Green's matrices for linear differential systems
Worm, George H.
An explicit solution to a class of quadratic minimization problems

VANDERBILT UNIVERSITY (5;5,0,0,0,0,0)
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Generators for expansive properties and their application to liftings
Linnaestetter, Jerry Leroy
A generalized multistage problem of Bolza in the calculus of variations
Minor, Lee Hammack
Pseudo-expansive mappings
Wallace, Kyle David
On the classification and structure of abelian groups
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The convergence, periodicity, and rest-point behavior of orbits in nonlinear semigroups of contractions

TEXAS
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Becker, Joseph Abraham
Parameterizations of analytic varieties
Dendy, Joel Eugene, Jr.
Penalty Galerkin methods for partial differential equations
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A priori $L_2$ error estimates for Galerkin approximations to parabolic partial differential equations

TEXAS A&M UNIVERSITY (9;1,0,1,7,0,0)
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Endomorphism near rings

Department of Industrial Engineering
Anderson, Robert J.
A fire protection simulation model for evaluating management strategies

Institute of Statistics
Doyle, Jack E.
The estimation of parameters in certain statistical models utilizing optimization techniques
Early, Grady Gaston
Factor analysis as an aid to interpretation in the multivariate analysis of variance-MANOVA
Fawcett, Richard F.
Expected mean squares in variance components models based on unbalanced data
Oglesby, Jerry L.
Application of dynamic programming to a general class of discrete stochastic control problems
Rainosek, Alvin P.
The transportation problem with optimization of the origins
Shepard, Robert L.
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Vaughn, William K.
A technique for maximum likelihood estimation in mixed models

TEXAS CHRISTIAN UNIVERSITY (5;5,0,0,0,0,0)
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Fugate, James Kirk
Translation theory for semirings
Gray, James Michael
Green's functions and discrete semi-analytic kernels
Reynolds, Donald Paul
Preservation of topological properties under extensions of topologies
Salam, Dianne Joy
A semiring extension

TEXAS TECH UNIVERSITY (8;5,0,0,3,0,0)
Ahlers, Carl Wilkerson
Linear estimation with a continuum of data
Alston, Roy Dean
Fractional operators on generalized functions
Cooper, Alex Richard
Optimal designs for multi-linear regression
Momment, Charles Gardiner
Semi-invertible spaces on normoid spaces
Thrash, Joe Barham, Jr.
Non-linear transforms of improper integrals

Wald, Margaret Cowar
Time degenerate parabolic equations

Ward, James Franklin, Jr.
Weighted matrix pseudoinverses

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The numerical solution of degenerate parabolic equations

UNIVERSITY OF HOUSTON (7;7,0,0,0,0,0)

Barr, Betty J.
Monotone operators in Banach spaces

Hinman, Betty J. Freeman
Products of threads in topological semigroups

Kuykendall, Daniel P.
Irreducibility in inverse limits of continua

Lau Yiu-wa
Small semilattices and costability

Parks, James M.
Homotopy-systems, H-spaces and sheaf cohomology

Riecke, Carroll V.
The lattice of convergence structures

Scott, John B.
On the nature of the solution of the linear homogeneous fourth order differential equation

UNIVERSITY OF TEXAS AT AUSTIN (20;9,2,7,0,0,2,0)

Anderson, Robert Brockett
Some theoretical aspects of automatic theorem proving

Asar, Ali Osman
Conjugacy theorems in locally finite groups

Barton, Calvin Pascal
The rectilinear crossing number for complete simple graphs in $E_2$

Bean, William Clifton
On certain extensor structures in the calculus of variations

Bright, George Walter
The influence of opinions of professional reference groups on decisions of preservice teachers

Chance, Joseph Edward
Group exponential polynomials and enumeration of paths on a lattice

Dupsissey, Claude Vernon
Contractive projections in abstract Banach function spaces

Griffith, Hugh Wallace
Preliminary investigations using interval arithmetic in the numerical evaluation of polynomials

Hibbs, Edwin Bultan
On length and variation of simple graphs

Hitz, Reinhart Green
Concerning continued fractions

Kincaid, David Ronald
An analysis of a class of norms of iterative methods for systems of linear equations

McPherson, Ronald Victor
On some classes of Banach spaces whose duals are abstract L-spaces

Rhodes, Joe Bill
Modularity and decomposition of semilattices

Webb, Leland Frederick
Interaction effects between selected cognitive abilities and instructional treatment in algebra

Department of Computer Science

Baskett, Forest, III
Mathematical models of multiprogrammed computer systems

Everett, Gerald David
Data structure and algorithmic determinacy: A formal model for the comparison of data management systems

Guba, Ratan Kumar
Periodic decomposition of sequential machines and phrase structure grammars

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A computer assisted instruction course to teach the Fortran programming language

Howard, John Hayes, Jr.
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Schwetman, Herbert, Jr.
Experimental studies in computer system performance

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Harris, Robert Michael
Uniform approximation of functions: Approximation by partitioning

Heal, Earl Robert
Peak sets for uniform F-algebras

Lewis, Paul Weldon
An extension theorem and regularity conditions for vector measures with finite semi-variation

Olson, Keith Bennett
Products of simple groups

Price, Barbara Stewart
Lower bounds for P-module of linked curves

Sipinen, Fredrick
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Wixom, James Arthur
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Quinn, Frank Stringfellow, Jr.
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Wine, James Dennis
Paracompactifications

Abdeljaouad, M. Mohamed
On the automorphisms and derivations of algebras
Farmer, Frank Davis
Algebraic topology of reflexive relations
Ferguson, Helaman Rolfe Pratt
Some constants of Harish-Chandra
Goodarl, Kenneth Ralph
Torsion for modules
Lindley, Lawrence Merrill
A generalization of Frobenius extensions
Martin, Michael Stephen
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McLinden, Lynn
Minimax problems, saddle-functions and duality
Moore, Robert Barry
Pointwise convergence of generalized Walsh series
Murley, Charles Estep
Invariants for torsion-free abelian groups of finite rank, and the structure of certain classes of these groups
Rader, Cary B.
Spherical functions of a semi-simple Lie group
Silverman, Ruth
Decomposition of plane convex bodies

Menninga, Larry D.
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Hosmer, David Wylie, Jr.
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Wahl, Patricia Walker
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Kuzmanovich, James J.  
Localizations and completions of Dedekind prime rings  

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Mather, David Paul  
Commutativity properties of continuous operators on the space of entire functions  

Mayland, Edward J., Jr.  
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Montgomery, John Thomas  
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Nadel, Mark Edward  
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Overdeck, John Michael  
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Read, John Arthur  
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Rod, David L.  
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Scrimger, Edward B., Jr.  
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Sheldon, Philip B.  
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Shilpsky, Arnold C.  
Homogeneity and extension properties of embeddings of \( S^1 \) and \( E^3 \)  

Shilepsky, Carol  
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Soloway, Robin Naomi  
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Troccolo, Joseph Anthony  
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Naylor, William Clark  
Some studies in interactive machine imitation using character recognition  

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Roochvarg, Alan C.  
The combination lock version of a finite state machine  

Schultz, Hilbert K.  
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Shapiro, Stuart C.  
A data structure for semantic information processing  

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Zeidman, Edward A.  
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Newbold, Paul  
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Palit, Charles D.  
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Sredni, Jacobo  
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Tierney, David Edwin  
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Designs for regression in time series when the regression functions are not completely known, with applications to signal detection problem

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Krajewski, Lawrence L.
On expanding locally finite collections
Me Coy, Peter A.
Geometry of the zeros and singularities of axisymmetric potentials
Rothman, Stanley M.
Applications of the theory of two normed spaces to Banach spaces
Subramanian, Pudukkottai K.
Contributions to the theory and applications of two-norm spaces

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Dalla, Ronald H.
Matric equations and canonical forms over finite fields
Olson, Dwight Marius
A radical theory for semirings
Sutton, Warren Gordon
A Liapunov theory for boundary value problems
Varol, Yako Leon
Exit criteria for some numerical algorithms

CANADA
CARLETON UNIVERSITY (2;1,0,1,0,0,0)

Scott, Christine
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Shum, Daniel T. T.
Integral inequalities using Bensen's method

MCGILL UNIVERSITY (7;7,0,0,0,0,0,0)

Castonguay, Charles
Meaning and existence in mathematics: On the use and abuse of the theory of models in the philosophy of mathematics
Holton, Derek Allan
On Redfield enumeration methods - Application of group theory to combinatorics
Lim Chong-keang
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MacGibbon, Kathryn B.
K-analytic spaces and countable operations in topology
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Rideout, Donald Eric
On a generalization of a theorem of Stickelberger

Szabo, Manfred
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MCMASTER UNIVERSITY (8;8,0,0,0,0,0,0)

Bozel, Frank Paul
Bases and cones in locally convex spaces
Drake, James Stanley
Regular sets, scalar multiplications and abstractions of distance spaces
Eastman, Donald E.
Universal topological and uniform algebra
Girhiny, John
Generalized fibre spaces
Higgs, Denis
Matroids on complete Boolean algebras
Jansen, Arnold Victor
Some mapping and homological properties of g-functions
Ng Shu-bun
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Spoar, Gary Roy
Analysis of curves of minimal order as regards the type and number of singular points

QUEEN'S UNIVERSITY (7;4,1,0,2,0,0,0)

Burry, John H. W.
Measures and variations associated with functions of one or two variables
Chang Sau-tong
High order derivations and high order Lie-like elements
Feldman, William Alan
Topological spaces and their convergence function algebra
Hines, W. Gordon S.
A simple monitor for use with a time-varying communication channel
Horn, Ronald W.
Convex programming: The introduction of some numerical methods and a comparison of these methods with existing methods
Maxwell, George
Forms over a-rings
Wilmut, Michael J.
Signal detection in the presence of cochannel interference and noise

SIMON FRASER UNIVERSITY (1;1,0,0,0,0,0,0)

Baldwin, John Theodore
Countable theories categorical in uncountable power

UNIVERSITÉ DE MONTRÉAL (2;2,0,0,0,0,0,0)

Dubois, Jacques
Transformations affines extrémales agissant sur un simplexe en laissant un de ses points fixes
Leroux, Pierre
Extension a des catégories de morphismes
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Feng Yan-kwang
Some representation and distribution problems for generalized r-free integers
Hrycay, Rudolph
Weakly connected functions
Hursey, Robert J.
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Pareek, Chandra Mohan
A study of some generalizations of paracompact and metrizable spaces

UNIVERSITY OF BRITISH COLUMBIA (9;8,1,0,0,0,0,0)
Carson, Andrew Bruce
Sheaf methods applied to coherent rings
Chan Gin-hor
Minimal (k)-groups, their structure and relevance to (G,X)-spaces
Hsieh Tsu-teh
Existence of normal linear positive functionals on a von Neumann algebra invariant with respect to a semigroup of contractions
Kapoor, Jagmonan
Matrices which, under row permutations, give specified values of certain matrix functions
Ko Hwei-mei
Fixed point theorems for point-to-set mappings
Millington, Hugh G. R.
Cylinder measures over vector spaces
Qureshi, Hilal Ahmed
Constrained Hartree-Fock wave functions for atoms
Tam Ping-kwam
Inequivalence and equivalence of certain kinds of non-normal operators
Tan Kok-keong
Fixed point theorems for nonexpansive mappings on Hausdorff locally convex spaces

UNIVERSITY OF CALGARY (3;2,0,1,0,0,0,0,0)
Department of Mathematics, Statistics and Computing Science
Laywine, Charles F.
List processing by element numbers and its use in the analytical solution of ordinary differential equations
Niven, Scott
Ramsey type problems for complete bipartite graphs
Pethe, Sharadchandra
Some two-point expansions and related classes of functions

UNIVERSITY OF SASKATCHEWAN (1;0,1,0,0,0,0)
Koneru, Sarveswara Rao
The solution of non-steady and steady state boundary value problems

UNIVERSITY OF TORONTO (15;6,1,3,4,1,0,0)
Andersen, Kenneth Frank
Discrete Hilbert transformations on rearrangement invariant sequence spaces
Auer, Jan Willem
A spectral sequence for smooth fibre bundles and fibre integration
Banerjee, Pradeep Kumar
On sequential procedures in estimating a confidence interval of prescribed accuracy and the cost of not knowing values
Bruen, Aiden A.
Spreads, indicator sets and translation planes
Chin Seong-tah
Controls for nonlinear parabolic equations
Jeffries, Clark Debs
The theory of O-deformable (1,1)-tensor fields
MacKay, Ritchie Jock
Some applications of marginal likelihood to structural modules
Peskun, Peter Harold
The choice of transition matrix in Monte Carlo sampling methods using Markov chains
Walsh, Timothy Robert Stephen
Combinatorial enumeration of non-planar maps

Department of Computer Science
Jackson, Lorne Wayne
Automatic error analysis in the solution of ordinary differential equations
MacEwen, Glenn Hilton
Performance of disk storage devices in computer systems

Department of Industrial Engineering
Hoq, A. K. M. Sinajul
Contributions to life testing and distribution of T for samples from an exponential population
Horne, Garry J.
The optimal performance of linear dynamic systems by parameter specification
Salpe, Alan L.
Touring strategies on a stochastic network
Seeley, Douglas A. R.
A pattern discovery in sequences of symbols with application to neuro-electric data

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Feuer, Richard D.
Torsion-free subgroups of Fuchsian groups

Osler, Thomas J.
Leibniz rule, the chain rule, and Taylor's theorem for fractional derivatives

UNIVERSITY OF WISCONSIN (3;3,0,0,0,0,0,0)

Arnold, James E., Jr.
Local to global constructions in the theory of Hurewicz fibrations

Blumberg, Duane
Trivial spectral sequences in the theory of fibre spaces

Self, William M.
Homeomorphisms of some classical Banach algebras
ABSTRACTS OF CONTRIBUTED PAPERS

Preprints are available from the author in cases where the abstract number is starred.

THE AUGUST MEETING IN PENNSYLVANIA
AUGUST 31–SEPTEMBER 3, 1971


Abstraction yields elegant proofs for the results of Fred Supnick and Louis Quintas (“Extreme Hamiltonian paths, Odd case,” Proc. Amer. Math. Soc. 15 (1964), 454-456 and “Extreme Hamiltonian paths, Even case,” Proc. Amer. Math. Soc. 16 (1965), 1058-1061). Definitions. A polygon, \( (P_1, P_2, \ldots, P_n) \), is convex-like if \( i < j < k < l \) implies \( d(P_i, P_j) + d(P_k, P_l) \leq d(P_i, P_k) + d(P_j, P_l) \) and \( d(P_i, P_j) + d(P_k, P_l) \leq d(P_i, P_l) + d(P_j, P_k) \).

Two line segments, \( AB \) and \( CD \), pseudosect relative to a polygon if the points, \( A, B, C, \) and \( D \), are vertices of the polygon and appear in the order, \( A \cdots C \cdots B \cdots D \). The pseudosection number of a polygon relative to a fixed polygon is the number of pairs of pseudosecting edges it contains. A \( \beta \)-polygon is a polygon of maximal pseudosection number. Theorem. If \( P \) is a convex-like polygon and \( S \) is a polygon with the same vertices as \( P \), then there exist two sequences of polygons both starting at \( S \) and ending at \( P \) and a \( \beta \)-polygon of \( P \) respectively, such that (i) each polygon in the sequence is an arc inversion of its predecessor, (ii) the pseudosection numbers of the polygons in the sequence is strictly decreasing and increasing respectively, (iii) the lengths of the polygons are decreasing and increasing respectively, (iv) there are less than \( n \) polygons in each sequence. Sets of points which have convex-like polygons include (i) small perturbations of a convex, planar set of points, (ii) points lying on a nonplanar, convex curve of a surface in three space (distance is measured along geodesics). (Received July 8, 1971.)

687-20-13. FRANK M. MARKEL, University of Toronto, Toronto 5, Ontario, Canada. Groups all of whose irreducible representations are rational. Preliminary report.

Theorem. Let \( G \) be a finite group. Assume: (I) Every irreducible complex representation of \( G \) is equivalent to a rational representation. (II) \( G \) is supersolvable. Then if \( p \) is a prime number dividing the order of \( G \), \( p \) is either two or three. (Received July 29, 1971.) (Author introduced by Professor Peter G. Norton.)

687-31-2. PAUL A. NICKEL, North Carolina State University, Raleigh, North Carolina 27607. Linear topologies for continuity of Sario’s linear operator method.

Let \( \Phi : H(W) \rightarrow H(W') \) be the mapping sending each harmonic function \( p \) on an arbitrary Riemann surface \( W \) into its restriction \( p|_{W'} \), a singularity function defined on the regular boundary neighborhood \( W' \). Rodin
and Sario [Kōdai Math. Sem. Rep. 19(1967)] have shown that $\Phi : H(W)/K \rightarrow H(W')/LC(\alpha)$ is an isomorphism, where $K$ is the set of all constant functions of $H(W)$ and $LC(\alpha)$ is the set of all regular singularity functions on $W'$. **Theorem 1.** The mapping $\Phi$ is topological when each of the spaces $E$ (the domain of $\Phi$) and $F$ (the range of $\Phi$) is equipped with the quotient topology induced by compact convergence. The following theorem is of interest only when $H(W)$ distinguishes finite point sets of $W$. **Theorem 2.** The mapping $\Phi$ is topological for the weakened topologies $\sigma(E,E')$ and $\sigma(F,F')$. **Theorem 3.** If $E$ and $F$ are equipped with the quotient topology induced by simple convergence, then $\Phi$ fails to be topological, at least when $H(W)$ distinguishes finite point sets of $W$. (Received July 8, 1971.)

*687-46-10. CHARLES T. TUCKER, University of Houston, Houston, Texas 77004. Sequentially relatively uniformly complete Riesz spaces and Vulikh algebras.*

Suppose $V$ is an Archimedean Riesz space with a weak unit $e$ and a zero element $\theta$. The sequence $f_1, f_2, f_3, \ldots$ is called a relative uniform Cauchy sequence (with regulator $g$) if, for each $\epsilon > 0$, there is a number $N$ such that if $n$ and $m$ are positive integers and $n, m > N$, then $|f_n - f_m| < \epsilon g$. A subset $M$ of $V$ is said to be sequentially relatively uniformly complete, written $s.r.u.\text{-}complete$, whenever every relatively uniform Cauchy sequence of points of $M$ (with regulator in $V$) converges to a point of $M$. This property was defined by Luxemburg and Moore in ["Archimedean quotient Riesz spaces," Duke Math. J. 34(1967), 725-740].

It was shown that the property of being Archimedean and $s.r.u.\text{-}complete$ was intermediate to the properties of being $\sigma\text{-}complete$. The pair $(V,e)$ will be said to be a Vulikh algebra if a positive multiplication can be defined on $V$ which makes it an associative, commutative algebra with multiplicative unit $e$. Finally, a necessary and sufficient condition for $(V,e)$ to be a Vulikh algebra, under the assumption that $V$ is $s.r.u.\text{-}complete$, is given. (Received July 8, 1971.)

The October Meeting in Cambridge
October 30, 1971

**Algebra & Theory of Numbers**

*688-A1. EARL GLEN WHITEHEAD, JR., Courant Institute, New York University, New York, New York 10012. The Ramsey number $N(3,3,3,3;2)$.*

A partitioning of the nonzero elements of the finite abelian group $Z/7Z \times Z/7Z$ into four sum-free sets shows that $N(3,3,3,3;2) > 49$. Based on a matrix technique for analyzing the structure of the two nonisomorphic 16-point colorings nondegenerate with respect to $N(3,3,3;2)$, an involved argument proves that no 65-point coloring nondegenerate with respect to $N(3,3,3,3;2)$ exists. Thus $49 < N(3,3,3,3;2) \leq 65$. (Received June 28, 1971.)
**688-A2. TOSHIHIKO YAMADA, Queen's University, Kingston, Ontario, Canada.** Central simple algebras over real quadratic fields which appear in some \( \mathbb{Q}[G] \).

Let \( k \) be a prime number. **Theorem.** If \( k = \mathbb{Q}(\sqrt{I}), \ I \equiv 1 \pmod{4} \), or \( k = \mathbb{Q}(\sqrt{15}), \ I \equiv 13 \) or \( 17 \pmod{20} \), then a central simple algebra \( A \) over \( k \) appears in some \( \mathbb{Q}[G] \) (up to similarity) if and only if (i) \( A \) has Hasse invariant 0 or \( \frac{1}{2} \) at every prime \( \mathfrak{p} \) of \( k \) and (ii) for any rational prime \( p \), \( A \) has the same Hasse invariant at all the primes \( \mathfrak{p} \) of \( k \) dividing \( p \). **Remark 1.** The above statement also holds for \( k = \mathbb{Q}(\sqrt{-c}) \), where \( c \) is a positive integer (cf. 687-12-1, these Notices 18(1971), p. 756).

**Remark 2.** Let \( m \) be a positive integer. If \( k = \mathbb{Q}(\sqrt{-c}) \), where \( m \) is divisible by distinct two primes \( p \) and \( q \), or if \( k = \mathbb{Q}(\sqrt{m}) \) where \( m \) is divisible by a prime \( p \equiv 3 \pmod{4} \), then the above statement is not true.

(Received August 31, 1971.)

**688-A3. PAUL C. KAINEN, Case Western Reserve University, Cleveland, Ohio 44106.** A combinatorial invariant for graphs. Preliminary report.

Let \( G \) be a finite graph without loops or parallel edges, and suppose \( G \) is not a tree. Define \( \delta^g_G = m - (L/(L-2))(n - 2 + 2g) \) where \( m \) is the number of edges, \( n \) is the number of vertices, \( L \) is the girth (length of shortest cycle), and \( g \) is any nonnegative integer. Let \( \text{cr}_G \) be the crossing number of \( G \) on the \( g \)-handled sphere \( S^g \), and let \( \mu^G_G \) be the minimum number of edges whose removal allows \( G \) to be embedded in \( S^g \).**Theorem 1.** \( \mu^G_G \geq \delta^g_G \). **Corollary.** \( \text{cr}_G \geq \delta^g_G \). Let \( \mathcal{S} \) denote the class of all complete, bipartite complete, or cubical graphs. **Conjecture 1.** If \( G \in \mathcal{S} \), then \( \delta^g_G = \mu^G_G \) provided \( \delta^g_G \geq 0 \). The following two theorems are special cases of this conjecture. **Theorem 2.** \( \delta^0_{(p,q)} = \mu^0_{(p,q)} \). If \( p = \delta \), \( \text{cr} \), or \( \mu \), set \( \rho^g_k(G) = \rho^g_{\gamma(G)-k}(G) \), where \( \gamma(G) \) is the genus of \( G \) and \( 0 \leq k \equiv \gamma(G) \). **Theorem 3.** \( \delta^k_{(Q(d))} = \mu^k_{(Q(d))} \) provided \( k \leq 2d - 4 \). This theorem is proved by an inductive construction which also yields: **Theorem 4.** If \( k \leq 2d - 4 \), then \( 8k \equiv \text{cr}^k_{(Q(d))} \equiv 4k \). Let \( g(G) \) denote the largest integer \( g \) for which \( \delta^g_G \geq 0 \). **Conjecture 2.** Let \( G \in \mathcal{S} \). Then \( \text{cr}_G = \delta^g_G \) if \( g = g(G) \). Various special cases of this conjecture are known--for example, \( G = \mathbb{Q}(d), d \equiv 2 \), and \( G = K_n, 3 \equiv n \equiv 8 \). (Received September 1, 1971.)

**688-A4. WILLIAM L. HOYT, Rutgers University, New Brunswick, New Jersey 08903.** Integrals of automorphic forms corresponding to holomorphic sections of elliptic surfaces.

Let \( B_{\Gamma} \to \Delta_{\Gamma} \) be the elliptic modular surface attached to a suitable subgroup \( \Gamma \subset \text{SL}(2, \mathbb{Z}) \), as defined by T. Shioda [Proc. Japan Acad. 45(1969), 786-790 and 833-837]. **Theorem 1.** Each holomorphic section of \( B_{\Gamma} \to \Delta_{\Gamma} \) corresponds to an integral with integral form of weight 3 for \( \Gamma \), and conversely. More generally, let \( B \to \Delta \) be the basic member of a family \( J(G) \), as defined by K. Kodaira [Ann. of Math. (2) 77(1963), 563-626] in terms of a map \( \omega \) and a representation \( \beta \to (S_{\beta}) \) [loc. cit., 579-580]. Assume that \( \omega(\widetilde{u}) \) is an unramified cover. **Theorem 2.** There is a space \( T_3 \) of generalized automorphic forms such that each holomorphic section of \( B \to \Delta \) corresponds to an integral with integral periods of an element of \( T_3 \), and conversely. \( T_3 \) consists of holomorphic functions \( f \) on the universal cover \( U' \) of a subset \( \Delta' \subset \Delta \), with transformation law \( f(\beta(\widetilde{u})) = (c_{\beta} \omega(\widetilde{u}) + d_{\beta})^3 f(\widetilde{u}) \) involving \( \omega \) and \( \beta \to (S_{\beta}) \), and with behavior at cusps determined by the structure of \( B \to \Delta \) near corresponding points of \( \Delta - \Delta' \). (Received September 2, 1971.)
Necessary and sufficient conditions may be given for a finite presentation of a group to be the Wirtinger presentation of a knot group from one of its projections. An enumeration of all such presentations solves the enumeration problem for all knot projections. It also offers a solution of the knot enumeration problem independent of braids. Similar statements hold for linkages. (Received September 3, 1971.)

A unital quadratic Jordan algebra \( J \) is a local Jordan algebra, if (i) \( R(J) \), the Jacobson radical, is the unique maximal ideal of \( J \), (ii) \( \bigcap_{k=1}^{\infty} R(J)(k) = 0 \), where \( R(J)(1) = R(J) \) and \( R(J)(k+1) = \bigcup_{k} R(J)(k)(R(J)(k)) \) for \( k \geq 1 \), and (iii) \( J/R(J) \) satisfies the minimum condition. \( J \) is completely primary, if \( J/R(J) \) is a Jordan division algebra. Theorem 1. If \( J \) is a local Jordan algebra, then the completion of \( J \) as a module is a local Jordan algebra. Theorem 2. If \( J \) is a complete local Jordan algebra over a field of characteristic not 2, then \( J/R(J) \) has finite capacity \( n \), and (1) if \( n = 1 \), \( J \) is a complete, completely primary Jordan algebra, (2) if \( n = 2 \), \( J = J_1 \oplus J_2 \oplus S \), where \( J_1 \) is a completely primary Jordan algebra and \( S \) is a subspace of \( J \), (3) if \( n \geq 3 \), \( J \cong H(D_{n}, j) \), a Jordan matrix algebra, such that \( R(H) = R(D) \cap H \) and \( (D, j) \) is a unital algebra with involution satisfying: (i) \( (D, j) \) is a nonassociative alternative algebra such that \( R(D) \), the Smiley radical of \( D \), is the unique maximal ideal of \( (D, j) \) and \( D \) is complete in the topology induced by the \( \mathcal{A}(k) \), where \( R(H)(k) = A(k) \cap H \) for \( k \geq 1 \); or (ii) \( D \) is a complete semilocal associative algebra and \( R(D) \), the Jacobson radical of \( D \), is the unique maximal ideal of \( (D, j) \). Moreover, the algebras in (1) and (3) are complete local Jordan algebras. (Received September 9, 1971.)

Suppose that \( R \) is a commutative ring with 1, and \( E: 0 \to A \to M \to B \to 0 \) is a short exact sequence of \( R \)-modules. If \( M \cong A \oplus B \), need \( E \) split? The answer is no, even for abelian groups, as is easily seen. However, we do have: Theorem 1. If \( A \) and \( B \) are finitely presented \( R \)-modules and \( M \cong A \oplus B \), then \( E \) splits. // More generally, suppose that \( F: 0 \to A \to N \to B \to 0 \) is also a short exact sequence of \( R \)-modules. What can be said about the relationship of \( E \) to \( F \) (say as elements of the \( R \)-module \( \text{Ext}^1_R(B,A) \)) in case \( M \cong N \)? We will write \( \text{ann}_R E \) for the annihilator in \( R \) of \( E \in \text{Ext}^1_R(B,A) \), and if \( I \) is an ideal of \( R \), we define as usual \( \mathcal{A} = \{ x \in R | r^k x \in I \) for some \( k \} \). The following result is an easy consequence of Theorem 1.

Theorem 2. With the above notation, if \( A \) and \( B \) are finitely presented and \( M \cong N \), then \( \sqrt{\text{ann}_R E} = \sqrt{\text{ann}_R F} \). // Examples show that under the conditions of Theorem 2, we need not have \( \text{ann}_R E = \text{ann}_R F \), and that both theorems become false if we replace "finitely presented" by "finitely generated". (Received September 8, 1971.)
Theorem 1. Let \( q = p^T \) (\( p \) an odd prime). Then for each \( \gamma > 3/4 \), the Dirichlet \( L \)-functions to modulus \( q \) have no zeros in rectangles of the form: \( \left| t \right| \leq T, \sigma \geq 1 - c \cdot \log \gamma (qT) / (T^2) \). Here \( c \) depends on \( T \) and \( \gamma \).

The proof uses a formula due to A. G. Postnikov (Izv. Akad. Nauk SSSR 19(1955)) for \( x(1 + pu) \) and estimates due to I. M. Vinogradov (ibid. 14(1950); 15(1951)) for exponential sums. Using Montgomery estimates (Thesis, Cambridge, 1971) for the density of the zeros of \( L \)-functions, the following extension of the result of Barban, Linnik and Čudakov (Acta Arith. 9(1964)) is proved. Theorem 2. For \( q \) as above, the Prime Number Theorem holds in segments \([x, x+h]\) of progressions modulo \( q \) as \( x \to \infty \) provided \( h \ll q \cdot x^{3/5+\epsilon} \). (Received September 9, 1971.)

Analysis


In the Hahn-Banach theorem, the conditions of the functional \( P \) are replaced by (1) \( 0 \leq p(x) < +\infty \),

(2) \( \lim_{\xi \to 0} p(\xi x) = 0 \), and (3) \( p(x+y) \leq p(x) + p(y) \). (Received June 11, 1971.)

688-B2. STEPHEN R. BERNFELD, University of Missouri, Columbia, Missouri 65201 and University of Rhode Island, Kingston, Rhode Island 02881. Perturbing uniform ultimate bounded differential systems.

We obtain results on the eventual uniform boundedness and eventual uniform ultimate boundedness of solutions of the differential equation (P) \( \dot{x} = f(t, x) + g(t, x) \) given that solutions of the equation (E) \( \dot{x} = f(t, x) \) are uniform bounded and uniform ultimate bounded. Theorem 1. Assume solutions of \( \dot{x} = A(t)x \) are uniform bounded and uniform ultimate bounded. Consider the perturbed system (PL) \( \dot{x} = A(t)x + \gamma(t,x)g(t,x) \). Assume

\[
|\gamma(t,x)| \leq L(t)|x|, \quad \lim_{\tau \to \infty} \sup_{0 \leq t \leq \tau} t^{\epsilon+1} L(s)ds < Q \text{ for sufficiently small } Q \text{ and large } t_0. \quad \text{Assume}
\]

\[
|g(t,x)| \leq h(t)|x|, \quad \sup_{t \in [0,\infty)} t^{\epsilon+1} h(s)ds < \infty. \text{Then solutions of (PL) are eventually uniform bounded and eventually uniform ultimate bounded. Theorem 2. Assume solutions of (E) are uniform bounded and uniform ultimate bounded and that for each } \alpha > 0 \text{ there exists } L(\alpha) > 0 \text{ such that } f \text{ satisfies } |f(t,x) - f(t,y)| \leq \lambda(t)L(\alpha)|x - y|, \text{ for all } |x| \leq \alpha \text{ and } |y| \leq \alpha, \text{ and } \sup_{t \in [0,\infty)} t^{\epsilon+1} \lambda(s)ds < \infty. \text{If } g \text{ satisfies}
\]

\[
\sup_{0 \leq u \leq s} |g(t,x(s))ds| \to 0 \text{ as } t \to \infty \text{ for each bounded continuous function } x(s) \text{ then solutions of (P) are eventually uniform bounded and eventually uniform ultimate bounded. Examples show that the conditions on the perturbing term cannot be easily weakened in Theorems 1 and 2. (Received August 23, 1971.)
\]
Let $A$ be a proper $H^*$-algebra with norm $\|\cdot\|$. For $0 < p \leq \infty$ a nonnegative extended-real value $|a|_p$ is associated with each $a$ in $A$; then the $p$-class $A_p$ is defined to be $\{a \in A : |a|_p < \infty\}$. If $1 \leq p \leq \infty$, $A_p$ is then a two-sided $*$-ideal of $A$ (proper only if $p < 2$), and $(A_p, |\cdot|_p)$ is a normed $*$-algebra. $(A_2, |\cdot|_2)$ is $(A, \|\cdot\|)$; and for $1 \leq p \leq 2$, $(A_p, |\cdot|_p)$ is a Banach $*$-algebra, for which structure theorems are given. (Received August 27, 1971.)

Let $\Omega_k$, $k = 1$ to $n$, be harmonic spaces of Brelot and $u_k > 0$ harmonic functions on $\Omega_k$. For each $\omega$ in a class of multiply superharmonic functions it is shown that the iterated fine limits of $[w/u_1 \ldots u_n]$ exist up to a set of measure zero for the product of the canonical measures corresponding to $u_k$ and are independent of the order of iteration. This class contains all positive multiply harmonic functions on the product of $\Omega_k$'s. For a holomorphic function $f$ in the Nevanlinna class of the polydisc $U^n$, it is shown that the $n$th iterated fine limits exist and equal almost everywhere on $T^n$ the $n$th iterated nontangential limits of $f$, for any fixed order of iteration. It is then deduced that with the exception of a set of measure zero on $T^n$, the absolute values of the different iterated limits of $f$ are equal. It is also shown that the $n$th iterated nontangential limits are equal almost everywhere on $T^n$ for any $f$ in $N_1(U^n)$. (Received September 2, 1971.)

Let $u$ denote an arbitrary nonnegative superharmonic function on a region $\Omega$. A general theory is developed for the operators $Q_{\lambda}$ ($\lambda \geq 0$) defined by $Q_{\lambda}u = u - \hat{R}(u - \lambda)^+$, where $\hat{R}$ signifies the regularized reduced function. It turns out that $Q_{\lambda}u$ (after removal of removable singularities) is the largest superharmonic function $\varphi$ on $\Omega$ such that $\varphi \leq \lambda$ and $u = \varphi + \psi$ for $\psi$ superharmonic and nonnegative. Further $Q_{\lambda}$ satisfies the composition identity $Q_{\lambda}Q_{\mu}u = Q_{\lambda \mu}u$. The condition $\frac{dQ_{\lambda}u}{d\lambda}|_{\lambda=0} = u$ determines a class $\mathcal{J}$ of functions $u$ which can be characterized as (1) the class of all $u$ such that $Q_{\lambda}u = \lambda u$ for some (equivalently, all) $\lambda$ with $0 < \lambda < 1$ or, alternatively, as (2) the class of all $u \leq 1$ representable as $\lim_{\lambda \to 1 -} \hat{R}u_{[u \geq \lambda]}$. Moreover, $\mathcal{J}$ contains all capacitary potentials, and the harmonic functions in $\mathcal{J}$ are precisely the generalized harmonic measures of M. Heins. Theorem. The quasibounded part $Qu$ of any nonnegative superharmonic function $u$ can be represented as $Qu = \int_0^\infty \lambda u \, d\lambda$, where $\{v_{\lambda} \lambda \geq 0\}$ is a decreasing family of functions in $\mathcal{J}$. In the case when $u$ is harmonic, the functions $v_{\lambda}u$ are generalized harmonic measures on $\Omega$. (Received September 7, 1971.)
The technique of writing an asymptotic expansion (H. Schmidt, Math. Ann. 113(1936), 629) to approximate a function of one variable within a restricted domain can be generalized to encompass the approximation of a function which is a solution to a differential equation with small perturbation. The requirement that the expansion be uniform throughout some chosen domain introduces difficulties which have been attacked by several methods. One example is the Poincaré-Lighthill technique. An alternate method is the use of "multiple time scales." Both can be considered as examples of the introduction of an "extension" of the variables. The form of the asymptotic expansion is discussed for these cases and the methods are compared and contrasted. Summation of the expansion is discussed for several examples. (Received September 9, 1971.)

Logic and Foundations

*M688-E2. DOV TAMARI, State University of New York at Buffalo, Amherst, New York 14226. Formulae for well formed formulae (w.f.f.).

One considers strings $y_0 \ldots y_n y_{n+1}$ and cycles $(y_0 \ldots y_n y_0 \ldots y_n)$ of $n$ variables (nullary operators included) and $m+1$ operators $g_0, \ldots, g_m$ chosen from a set of fundamental operations $F = \{\ldots, f, \ldots\}$ of arities $a_f$ and multiplicities $m_f$ among the $g_j$; $\Sigma m_f = m + 1$; $\Sigma a_f m_f = a$ (total arity); a w.f.f. bent into a cycle is a w.f. cycle. Theorem 1. A string with $a = n + m$ admits one and only one cyclic permutation $y_0 \ldots y_n y_{n+1}$ which is a w.f.f.; in other words, $a = n + m$ is a n.a.s.c. for a cycle to be w.f.

Theorem 2. The number of w.f.f. $f(n) = \frac{m!}{a_0! \ldots a_m!} \frac{m + n + 1}{a_0 \ldots a_m}$ (The formula depends only on the multiplicities $m_f$, not on other peculiarities of the $g_j$). Example. $f(n) = 2^{n+1}$, $m_f = m + 1$, $a = n + m$ = $(2n-2)! / n! (n-1)!$. Theorem 1 has two proofs, both simple and significant; one based on building up w.f.f. from shorter ones; the other purely arithmetical or analytical, e.g. as implied by a Lemma. Let $r_0, \ldots, r_{n+m}$ be a sequence of real numbers $\neq 0$ with positive sum $s = r_{n+m} > 0$, $i - 1$ the smallest index with
Let \( G \) be a locally compact unimodular group and for \( 1 < p < 2 \) let \( \mathfrak{F}_p(G) \) be the \( L^p \)-Fourier transform on \( G \), i.e. the map \( f \in L^p(G) \rightarrow \mathfrak{F}_p(Lf) \), where \( \Gamma \) is the dual gauge space of \( G \), \( p' \) is the index conjugate to \( p \), and \( Lf \) is the operator on \( L^2(G) \) of left convolution by \( f \) (see R. A. Kunze, Trans. Amer. Math. Soc. 89 (1958), 519-540). The Hausdorff-Young theorem, proved by Kunze (loc. cit.), is the assertion \( \| \mathfrak{F}_p(G) \| \leq 1. \)

**Theorem 1.** There are nonzero vectors \( f \) in \( L^p(G) \) with \( \| Lf \|_p = \| f \|_p \) if and only if \( G \) has compact open subgroups. This was proved for Abelian groups by Hewitt and Hirschman, and for compact groups by Hewitt and Ross (Hewitt and Ross, "Abstract harmonic analysis, II," §43). Using the deep result of Babenko (Izv. Akad. Nauk SSSR (1961)) that \( \| \mathfrak{F}_p(R) \| < 1 \), it is easy to see that \( \| \mathfrak{F}_p(G) \| < 1 \) for every locally compact Abelian group \( G \) lacking compact open subgroups. **Theorem 2.** \( \| \mathfrak{F}_p(R \times H) \| < 1 \) for any separable unimodular Type I group \( H \) and all \( 1 < p < 2 \). **Theorem 3.** If \( M(2) \) is the group of rigid motions of the plane and \( p = 4/3 \) then \( \| \mathfrak{F}_p(M(2)) \| < 1 \). Theorems 2 and 3 are based on the direct integral decompositions of the regular representations of the groups involved. (Received September 7, 1971.)

**Combinatorially triangulating 4-manifolds.** Preliminary report.

There is a question whether manifolds which have triangulations can have combinatorial triangulations. Consider \( M \) to be a 4-manifold which is open (i.e. not compact and without boundary) and connected. **Theorem.** If an open 4-manifold \( M \) has a triangulation then it has a combinatorial triangulation. This theorem does not say that the given triangulation must be combinatorial. **Conjecture.** If \( M \) is a simply-connected, open 5-manifold with only one end, and if \( M \) has a triangulation, then \( M \) may have a combinatorial triangulation. (Received September 7, 1971.)

**Self-universal crumpled cubes and a dogbone space.**

Burgess and Cannon ("Embeddings of surfaces in \( E^3 \)," Rocky Mt. J. Math. (to appear)) have asked: Is every self-universal crumpled cube universal? Daverman and Bass ("A self-universal crumpled cube that is not universal," Bull. Amer. Math. Soc. 76(1970)) purport to answer this question but have failed to substantiate essential parts of their argument. The question is answered negatively here by presenting a certain dogbone space which is not topologically \( E^3 \) but which can be expressed as the result of sewing together two crumpled cubes, one of which is self-universal. (Received September 9, 1971.)
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Algebra & Theory of Numbers

*71T-A190. V. R. CHANDRAN, Madurai University, Madurai-2, India. On duo-rings. IV.

A duo-ring is one in which every one-sided ideal is two-sided. We prove the following theorems in this paper. **Theorem 1.** A right primitive right duo-ring is a division ring. **Corollary 1.** A semisimple (in the sense of Jacobson) right duo-ring is a dense subdirect sum of skew fields. **Corollary 2.** A duo-ring with more than one element is isomorphic to a direct sum of a finite number of skew fields if R has d.c.c. on ideals and has zero Jacobson radical. **Theorem 2.** For a right duo-ring R (not necessarily with 1), √I = γ(I) for every ideal I in R where √I = {x ∈ R|xn ∈ I} for some positive integer n and γ(I) denotes the McCoy radical of I.

**Theorem 3.** A right duo-ring R (not necessarily with 1) is regular iff every ideal in R coincides with its McCoy radical. **Theorem 4.** A right duo-ring R is regular iff every right ideal in R is the intersection of all maximal right ideals in R containing I. **Theorem 5.** Let R be a duo-ring. Suppose (1) Σ a commutative ring M with 1 such that R can be considered as an algebra over M and (2) for each element a in R, Σ an idempotent e (depending on a) such that ae = ea = a. Then R can be embedded in a duo-ring with 1. **Corollary.** A regular duo-ring can always be embedded in a regular duo-ring with 1. (Received April 23, 1971.) (Author introduced by Professor I. N. Herstein.)

71T-A191. ARON SIMIS, Queen's University, Kingston, Ontario, Canada. On the Krull-Schmidt theorem for orders over Hensel rings and valuation rings. Preliminary report.

Let A be a ring with identity and let C be a full subcategory of the category of all (left) A-modules which is closed under finite direct sums; C is said to have the Krull-Schmidt property if every object of C is a finite direct sum of indecomposable modules belonging to C and this decomposition is unique up to isomorphisms and order of the summands. **Theorem.** Let R be a commutative semilocal ring, A a finite R-algebra, C the category of all finite projective A-modules (resp. all A-modules which are R-noetherian). If (R, rad R) is a Henselian pair (i.e., if f(X) ∈ R[X] is monic and factorizes comaximally mod (rad R)R[X] then this factorization can be lifted to R[X]), then C has the Krull-Schmidt property. Let R be an integral domain, K its quotient field, A a finite K-algebra; a subring Λ of A containing R is called an R-order in A if Λ is integral over R and KA = A. A A-module M is called a A-lattice if M is R-torsion free and is contained in a finite R-submodule of KM. Let R be a normal local domain, K its quotient field, hR the Henselization of R, hK = K ⊗R hR; a

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finite-dimensional division ring $D$ over $K$ is called $R$-unibranch if $D \otimes_K hK$ is a division ring. Theorem. $R$ is a valuation ring, $K$ its quotient field, $A = \prod_k (D_k \otimes K)$ a finite semisimple $K$-algebra such that every $D_k$ is $R$-unibranch, $A$ a finite $R$-order in $A$. If $\mathcal{C}$ is the category of finite projective $A$-lattices (resp. $A$-lattices which are $R$-noetherian), then $\mathcal{C}$ has the Krull-Schmidt property. In the sequel, applications are made for grouprings of infinite rings of integers. (Received April 21, 1971.) (Author introduced by Professor Paulo Ribenboim.)

A note on semigroups. II.

Any notation and terminology should be referred to (see Abstract 71T-A143, these Notices 18(1971), 793) unless otherwise stated. If $A \in B_n$ and $\rho_r(A) = n = \rho_c(A)$, then we say that $A$ is reduced. Let $P_X$ denote the set of all posets on a finite set $X$. Let $P^*_X$ denote the set of all nonsomorphic posets on a finite set $X$. Let $R(D)$ denote the set of all reduced regular $\mathcal{D}$-classes in $B_n$. $E$ denotes the set of all reduced idempotents in $B_n$.

Theorem 1. $P^*_X \cong R(D)$. Corollary 2. $P_X \cong E$. Let $d(B_n)$ denote the set of all $n \times n$ dominant main diagonal matrices (i.e., $A = (a_{ij}) \in d(B_n) \iff a_{ii} = 1$ for every $i$) over $B$. Theorem 3. If $A \in d(B_n)$, then $\langle A \rangle$ is a cyclic subsemigroup of $d(B_n)$ such that $\langle A \rangle = \{A\}$ if $A^2 = A$, $\langle A \rangle = \{A, A^2, \ldots, A^{n-1}\}$ otherwise. (Received May 27, 1971.)

Finite solvable groups with Sylow normalizers maximal. Preliminary report.

Here, $G$ is a finite solvable group. The term "Sylow normalizer of $G" refers to a $p$-Sylow normalizer of $G$, $p \in \pi(G)$, which is a proper subgroup of $G$. If $H \not\leq G$, then $Core_G(H)$ is defined to be the largest normal subgroup of $G$ contained in $H$. An "A-group" is a solvable group all of whose Sylow subgroups are abelian.

Lemma. If every Sylow normalizer of $G$ is a maximal subgroup of $G$, then any other maximal subgroup of $G$ is normal in $G$. Theorem 1. If every Sylow normalizer of $G$ is maximal in $G$, then $G/\varphi(G)$ is isomorphic to a subgroup of $A \times B$, where $A$ is a direct product of cyclic groups of prime order and $B$ is the direct product over $\pi(G)$ of $Core_G(N(S_p))$, $S_p$ a Sylow $p$-subgroup of $G$. Theorem 2. If every Sylow normalizer of $G$ is a maximal subgroup of $G$, then the nilpotent length of $G$ is at most three and the $p$-length of $G$, for any $p \in \pi(G)$, is at most two. Theorem 3. Let $G$ be a nonabelian A-group whose Sylow normalizers are maximal subgroups of $G$. Then $G$ is metabelian. Theorem 4. Let $H$ be a finite group with the property that, for each $p \in \pi(H)$, $H$ has at most $p^2$ Sylow $p$-subgroups. Then $H$ is solvable and, if $H$ is nonnilpotent, either $H/\varphi(H)$ is metabelian or $H$ maps onto $\Sigma(4)$, the symmetric group on four letters. (Received April 29, 1971.)

Modules finite over endomorphism ring. II.

We show that a module $M$ has $\Sigma$-injective hull only if $R/\text{ann}_R M$ satisfies the a.c.c. on right ideals annihilated by subsets of $M$, and hence on right ideals generated by idempotents. (Corollary to a theorem in the author's 1966 Nagoya Math. J. paper.) Hence over a regular ring, this implies that $M$ and $R/\text{ann}_R M$ are semisimple, and that $M$ is a finendo injective module. Conversely, employing Fuchs's characterization of quasiinjective abelian groups, we show that the only finendo ones are the divisible groups containing a summand.
isomorphic to $\mathbb{Q}$, and the torsion groups of bounded order the $p$-components of which are direct sums of isomorphic cyclic groups. Over any ring, any finendo quasinjective module is injective over its biendomorphism ring. Portions of this paper, I and II, were presented to the Tulane Symposium on Ring Theory (1970–71), and the University of Arkansas Symposium, Jonesboro (1971). (Received June 14, 1971.)

71T-A195. RAVINDER KUMAR, Ramjas College, Delhi University, Delhi, India. Overrings of KE-domains. Preliminary report.

(KE)-domains were introduced by Singh ("Principal ideals and multiplication rings. II", J. London Math. Soc., to appear) and were characterized in various ways by Singh and Kumar ("(KE)-domains and their generalizations," Abstract 71T-A52, these Notas 18(1971), 403). Here we study rings each of whose overrings is a (KE)-domain. Following are the main results: (i) Each overring of (KE)-domain is a (KE)-domain. (ii) If $D$ is nonquasilocal with more than two noncomparable prime ideals then each proper overring of $D$ is a (KE)-domain iff $D$ is so. (iii) If $D$ has exactly two noncomparable prime ideals then, under the conditions of (ii) above, $D$ may not be a (KE)-domain. (iv) If $D$ is quasilocal then each proper overring of $D$ is a (KE)-domain iff $D$ is a rank one valuation ring. (v) Let $D$ be integrally closed and $J$ its integral closure in an algebraic extension $L$ of $K$, the quotient field of $D$. If $J$ is a (KE)-domain then $D$ is so. The converse is not true even if $(L:K) < \infty$. (Received July 1, 1971.) (Author introduced by Professor S. K. Jain.)

71T-A196. GEOFFREY LEWIS, University of New South Wales, Kensington, N.S.W., 2033, Australia. Coherence for closed functors.

Let $V = (V, \otimes, \otimes', I, a, b, c, d, e)$ and $V' = (V', \otimes', \otimes', I')$ be (symmetric monoidal) closed categories as in [Kelly-Mac Lane, "Coherence in closed categories," J. Pure and Appl. Algebra 1 (1970)]. Let $\varphi = (\varphi, \varphi', \varphi^0)$ be a closed functor $\varphi : V \to V'$. Define a category whose objects, called shapes, are formal functors, from repeated products of $V^{\otimes}$ and $V$, into $V'$, constructed from $I, \otimes, \otimes', I', \varphi$; and whose morphisms, called nats, are formal natural transformations constructed from the data $a, \ldots, e', \varphi, \varphi^0$ using the given functors and composition. Each such nat has a graph, showing how the variables are linked, and a $\varphi$-graph, needed to show how occurrences of $\varphi X$ are linked when $X$ is constant. Call a shape proper if it does not involve $[X, Y]$ or $[X, Y]'$ where $Y$ is constant and $X$ is not, or where $Y$ is independent of $\varphi$ and $X$ is not. Then two nats $f, g : S \to T$ between proper shapes coincide iff their graphs and $\varphi$-graphs coincide. (Received July 2, 1971.) (Author introduced by Professor Gregory M. Kelly.)

*71T-A197. ROBERT GILMER, Florida State University, Tallahassee, Florida 32306 and MATTHEW O'MALLEY, University of Houston, Houston, Texas 77004. Non-Noetherian rings whose proper subrings are Noetherian.

Theorem. In a ring $R$, the following conditions are equivalent: (a) $R$ is not Noetherian, but each proper subring of $R$ is Noetherian, (b) $R$ is not left Noetherian, but each proper left ideal of $R$ is left Noetherian, (c) $R$ is the zero ring on a $p$-quasicyclic group. The theorem is proved by first showing that the quasicyclic groups are the only nonfinitely generated abelian groups for which each proper subgroup is finitely

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HENRY HUNG, McGill University, Montreal 110, Quebec, Canada. The amalgamation property for G-metric spaces. Preliminary report.

Let G be a totally ordered abelian group. A G-metric space \((X, p)\) consists of a set \(X\) and a G-metric \(p: X \times X \to G\) (satisfying the usual axioms of a metric, with \(G\) replacing the set of real numbers). That the amalgamation property holds for the class of all metric spaces is attributed, by Morley and Vaught ("Homogeneous universal models," Math. Scand. 11 (1962), 41), to Sierpiński. The following theorem is proved. Theorem. The class of all G-metric spaces has the amalgamation property, if, and only if, \(G\) is order-complete (i.e., if, and only if, \(G\) is either the ordered group of the integers or the ordered group of the reals). (Received July 6, 1971.) (Author introduced by Professor S. Negrepontis.)

PRABHA GAIHA, National Institute for Bank Management, 85 Nepean Sea Road, Bombay-6, India. On quasiconvexity and pseudoconvexity of quadratic functions over an arbitrary cone. Preliminary report.

Recent works of Martos, Cottle and Ferland giving matrix theoretic criteria for the quasiconvexity and pseudoconvexity of quadratic functions over the nonnegative orthant are extended to giving necessary and sufficient conditions for a quadratic function to be quasiconvex and pseudoconvex over an arbitrary self-dual cone, with nonempty interior in an \(n\)-dimensional Euclidean space. For example, it is shown that it is necessary and sufficient for the quadratic form \(x^T C x\) to be quasiconvex w.r.t. a self-dual cone \(K\) with nonempty interior in \(\mathbb{R}^n\) if the negative of the spectral radius, \(-\rho(C)\), of the \(n \times n\) symmetric matrix \(C\) is its only negative eigenvalue. (Received July 6, 1971.) (Author introduced by Professor Z. R. Pop-Stojanovic.)

H. L. ABBOTT and D. SURYANARAYANA, University of Alberta, Edmonton, Alberta, Canada. Quasiperfect numbers.

It is well known that a positive integer \(n\) is called quasiperfect if \(\sigma(n) = 2n + 1\), where \(\sigma(n)\) is the sum of all the distinct divisors of \(n\). In 1951, P. Cattaneo proved that any quasiperfect number \(n\) is the square of an odd integer. He also proved that if \(n = p^{2a} m^2\) is quasiperfect and \(q \equiv 1 \pmod{p}\), then \((\frac{2}{p}) = 1\), where \((\frac{2}{q})\) is the Legendre symbol. It is mentioned in W. Sierpinski's book entitled "Theoria Liczk, Czesc. II" that A. Schinzel proved that any quasiperfect number \(n > 11,000\). In this paper we prove that any quasiperfect number \(n > 10^{20}\), using the following general results. (1) If \(n = p^{2a} q^{2b} m^2\) is quasiperfect and \(q \equiv 1 \pmod{p}\), then \(2\beta + 1 \neq 0 \pmod{p}\); (2) if \(n = p^{2a} q^{2b} m^2\) is quasiperfect and \(q\) belongs to the exponent \(d > 1\) modulo \(p\), then \(2\beta + 1 \neq 0 \pmod{d}\); and (3) if \(p\) is the smallest prime divisor of any odd integer \(n\), where \(3|n\), then \(\sigma(n)/n < ((2p + 4\omega(n) + 8)/(2p + 1))^{1/2}\), where \(\omega(n)\) is the number of distinct prime factors of \(n\). (Received July 7, 1971.)

CHIN-PI LU, University of Colorado, Denver, Colorado 80202. A generalization of Mori's theorem.

Let \((R, (q_n)_{n=0}^{\infty})\) be a filtered ring which is commutative with unity. \(R\) is called a Gelfand ring if its Jacobson radical is open, and \(R\) is said to be flat (flat hat) if the completion \(\hat{R}\) of \(R\) is a flat \(R\)-module. We say
that R is topologically artinian if R/q_n is an artinian ring for every n. Mori's theorem states that a Zariski ring R is a unique factorization domain (UFD) if its completion R̂ is a UFD. We consider a generalization of the theorem and prove Theorem 1. If R is a flat Gelfand ring, and it is a pure submodule of R̂, in particular R̂ is a faithfully flat R-module, then R is a UFD whenever R̂ is a UFD. Theorem 2. Let R be a flat Gelfand ring which is topologically artinian. If R̂ is a UFD, then R is also a UFD. Applying these results, we establish the following generalization of Nagata-Mori's theorem: Theorem 3. Let (R, q^n) be a flat ring which is either a noetherian ring, or a separated topologically artinian ring satisfying the ascending chain condition for principal ideals. If R̂ is a UFD and 1 + q_1 is generated by prime elements of R, then R is also a UFD. (Received July 7, 1971.)


That addition in a near field is abelian was first shown by B. H. Neumann (J. London Math. Soc. 15(1940), 203-208). In this paper the additive group of a near field is investigated further and shown to be characteristically simple. Theorem. A unitary near ring (R, +, \cdot) is a near field if and only if (R, +) is an abelian characteristically simple group and for all T \in A(R), the group of automorphisms of (R, +), (x(sT) = x(sT) for each x, r \in R and some s \in R. Corollary. Let (R, +, \cdot) be a near field. If R has characteristic 0, then (R, +) is the direct sum of groups each isomorphic to the group of rationals. If R has finite characteristic p, then (R, +) is a direct sum of groups of order p. Neumann (loc. cit.) worked with an algebraic system, which we call an N-system, that had different defining properties than a near field. It has been known that every finite N-system is a near field, but it was an open question whether this was true about infinite N-systems. An example is given that answers this question negatively. (Received July 8, 1971.)

*71T-A203. GEORGE D. POOLE, Texas Tech University, Lubbock, Texas 79409. Abelian groups which are quasi-injective as modules over their endomorphism rings. Preliminary report.

Suppose E is the endomorphism ring of the abelian group G. The divisible groups G which are quasi-injective over E are precisely those which are not mixed. All finite groups G and all torsion groups G for which each primary component G_p is homogeneous are quasi-injective over E. Certain mixed groups G may be quasi-injective over E. We also determine when quasi-injectivity of certain direct products and direct sums can be characterized in terms of their summands. For example, if G = \bigoplus G_i and each G_i is fully invariant in G, then G is quasi-injective over E if and only if each G_i is quasi-injective over its endomorphism ring E(G_i). (Received July 12, 1971.)


Exercise 8.14 of Harary's text ["Graph theory," Addison-Wesley, Reading, Mass., 1969] incorrectly asserts "A graph G is the line graph of some multigraph if and only if it has no induced subgraph of the form
In this paper we show that the correct condition involves four more forbidden induced subgraphs. A Krausz type characterization of the line graph of a multigraph is also given. (Received July 12, 1971.)


\((X, \ast)\) is a pre-order (transitive reflexive relation) and \(R\) is any ring, with identity. Any ring \(A\) is an \(R\)-ring iff \(A\) is an \(R\)-module. Any topology \((X, t)\) is an SOS topology iff every \(x\) in \(X\) has a smallest open set \(t(x)\) about it. \(I(X, \ast, R)\) is the class of functions from \(X^2\) into \(R\) subject to \(I(x, y) \neq 0\) only if \(x \ast y\). \textbf{Lemma.} \(I\) is an \(R\)-ring (G. C. Rota, "On the foundations of combinatorial theory. I," Z. Wahrscheinlichkeitstheorie und Verw. Gebiete 2(1964), 340-368). Define the SOS topology \(t\) on \(X\) as follows: \(t(x) = \{y \in X \mid x \ast y\}\). For the pre-order \((X_1, \ast_1)\) define similarly \(I_1\) and \(t_1\). \textbf{Lemma.} If \(g\) maps \(X\) to \(X_1\) then \(g: (X, t) \to (X_1, t_1)\) is continuous iff \(g: (X, t) \to (X_1, t_1)\) preserves the order. \textbf{Lemma.} If \(X\) is a finite set, T.F.A.E. \((a)\) \((X, t)\) and \((X_1, t_1)\) are homeomorphic. \((b)\) \((X, \ast)\) and \((X_1, \ast_1)\) are isomorphic. \((c)\) \(I(X, \ast, R)\) and \(I(X_1, \ast_1, R)\) are \(R\)-ring isomorphic for all \(R\). \((d)\) \(I(X, \ast, Z_2)\) and \(I(X_1, \ast_1, Z_2)\) are \(Z_2\)-ring isomorphic. (Received July 13, 1971.)

*71T-A206. WILLIAM A. LAMPE, University of Manitoba, Winnipeg 19, Manitoba, Canada. On the independence of certain related structures of a universal algebra. II. The congruence lattice and automorphism group are independent.

\textbf{Theorem.} If \(\mathcal{U}\) is a universal algebra with more than one element and if \(\mathcal{B}\) is any universal algebra, then there is a universal algebra \(\mathcal{E}\) whose congruence lattice is isomorphic to the congruence lattice of \(\mathcal{U}\) and whose automorphism group is isomorphic to the automorphism group of \(\mathcal{B}\). An incorrect proof of the above theorem is in E. T. Schmidt, Acta Math. Acad. Sci. Hungar. 15(1964), 37-45, as pointed out in Exercise 31 of Chapter 2 of "Universal algebra" by G. Grätzer (Van Nostrand, Princeton, N. J., 1968). The lemmas from Part I in this series (Abstract 71T-A170, these Notices 18(1971)) are used in proving the above Theorem. (Received July 14, 1971.)


All Moufang loops of order less than 26 are found. There are twelve such loops which are not groups (1 of order 12, 5 of order 16, 1 of order 20, and 5 of order 24). All of them are G-loops (a G-loop is a loop which is isomorphic to each of its loop-isotopes). Lagrange's theorem and Sylow's first theorem hold for all of them. All of them are solvable. The methods used were essentially combinatorial. (Received July 15, 1971.)

71T-A208. RÜDIGER GÖBEL, University of Texas, Austin, Texas 78712. A connection between cardinal numbers and chain conditions of groups.

If \(\Omega\) is an ordered set, we call a family \(\Theta\) of subgroups of a group \(G\) (ordered by inclusion) an \(\Omega\)-system of \(G\) if there is an order preserving map from \(\Omega\) onto \(\Theta\). The system \(\Theta\) is \(H\)-invariant for a subgroup \(H\) of\( G\)
iff all members of \( \Theta \) are invariant under inner automorphisms induced by \( H \). [We will assume that \( X \neq Y \) for all pairs \( X, Y \in \Theta \).] **Theorem.** Let \( \Omega \) be an ordered set and let \( M \) and \( N \) be subgroups of a group \( G \) such that 

\[ [G : M] < \aleph_0. \]

Then the following conditions are equivalent: (1) Every \( G \)-invariant \( \Omega \)-system of \( N \) has a cardinality less than \( \aleph_0 \). (2) Every \( M \)-invariant \( \Omega \)-system of \( N \) has a cardinality less than \( \aleph_0 \). For \( \aleph_0 = \aleph_0 \) and \( N = G \) the Theorem says, that the property to have only finite systems of normal subgroups of a given type \( \Omega \) is inherited by subgroups of finite index. If we restrict ourselves to \( \aleph_0 = \aleph_0 \) and \( \Omega \) to be ascendently or descendently ordered, the Theorem is due to J. L. Wilson [Math. Z. 114(1970), 19-21; MR 41 #3577]. (Received July 12, 1971.) (Author introduced by Professor Alfred Schild.)


A projective plane of Lenz-Barlotti type I-4 is coordinatized by a linear planar ternary ring which has associative multiplication and satisfies both distributive laws. Such a ternary ring is called a Planar Division Neo-ring (PDNR). D. R. Hughes [Trans. Amer. Math. Soc. 80(1955), 502-527] has shown that the additive loop of a finite PDNR is commutative and satisfies the inverse property. The main results of the present paper are:

(1) All finite PDNR's have commutative multiplication;
(2) if \( p \) is a divisor of \( n \neq 4 \) for which there exists an integer a such that \( p^a \) is congruent to -1 modulo \( n - 1 \), then there is no PDNR of order \( n \);
(3) if a divisor of \( n \neq 8 \) has exponent \( r \) modulo \( n - 1 \) such that (a) \( r \) is not divisible by 3, and (b) if \( r \) is odd, \( r > \left[ \frac{n-2}{6} \right] \), or if \( r \) is even, \( r > 2 \left[ \frac{n-2}{6} \right] \), then there is no PDNR of order \( n \) which has cyclic multiplication. A computable algorithm for construction of finite PDNR's with cyclic multiplication is also given. These results make use of several of Hughes' theorems. (Received July 12, 1971.) (Author introduced by Professor Reuben Sandler.)


The basic facts about free products of \( L \)-groups were discussed in the author's paper "Free products in varieties of lattice-ordered groups," still in preprint form. In this paper we restrict ourselves to the free product of two Archimedean \( o \)-groups in the category of abelian \( L \)-groups. One of our main theorems states that the free product \( G \) of two such \( o \)-groups \( A \) and \( B \) has no singular elements, and that every nonzero element has uncountably many values. We show further, that the following conditions for \( G \) are equivalent: (1) \( G \) is radical, (2) \( G \) is Archimedean, (3) \( G \) is a subdirect product of reals. If \( G \) satisfies one of these conditions there is a representation of \( G \) as an \( L \)-group of bounded, continuous functions on the positive real line, which are differentiable almost everywhere. We have some further partial results, and some theorems about free products of arbitrary \( L \)-groups. For example, if \( C \) and \( D \) are \( L \)-ideals of the \( L \)-groups \( A \) and \( B \) respectively, then the canonical map of \( C \square D \) into \( A \square B \) is an \( L \)-embedding; the image however is (except for the case when \( C = A \) and \( D = B \)) never an \( L \)-ideal of \( A \square B \). (Received July 16, 1971.)
GEORGE A. GRÄTZER and WILLIAM A. LAMPE, University of Manitoba, Winnipeg 19, Manitoba, Canada. Various representations of algebraic lattices as congruence lattices of universal algebras.

Congruence lattices of (universal) algebras were characterized as algebraic lattices by G. Grätzer and E. T. Schmidt (Acta Sci. Math. (Szeged) 24(1963), 34–59). Various proofs of this theorem (G. Grätzer, W. A. Lampe, E. T. Schmidt, and others) all rely on the same basic construction: the three-leaf extension. In this paper that construction is generalized. This generalization, combined with some of the results of "On the independence of certain related structures of a universal algebra. I", Abstract 71T-A170, these Notices 18(1971), makes it possible to obtain representations of algebraic lattices as congruence lattices of algebras with specific properties. A sample result follows. **Theorem.** Every modular algebraic lattice can be represented as the congruence lattice of a universal algebra having type 2 congruence lattice. (The congruence lattice of an algebra is of type 2 if for any pair of congruences \( \Theta \) and \( \Phi \) and elements \( a \) and \( b \), \( a = b(\Theta \lor \Phi) \) implies the existence of elements \( c \) and \( d \) satisfying \( a = c(\Theta) \), \( c = d(\Phi) \), and \( d = b(\Theta) \).) B. Jónsson proved that every modular lattice can be represented by partitions with type 2 joins (Math. Scand. 1(1953), 193–206). (Received July 16, 1971.)

PAUL JEAN CAHEN, 82 Toronto Street, Kingston, Ontario, Canada and Queen's University, Kingston, Ontario, Canada. Spectrum of the ring of integral valued polynomials. Preliminary report.

The ring \( R \) of integral valued polynomials, over an algebraic number field \( K \), is the ring of polynomials which map the ring \( A \) of algebraic integers into itself. This paper describes the prime ideals of \( R \). The fiber above the ideal \((0)\) of \( A \) is isomorphic to the ring of polynomials \( K[X] \), whereas the fiber above any maximal ideal \( m \) is shown to be dense in the ring of continuous functions from the \( m \)-adic completion of \( A \) into itself (using the Stone-Weierstrass theorem). The prime ideals of \( R \), which lie over \( m \), turn then out to be in one to one correspondence with the elements of this completion. The height of the prime ideals of \( R \) is discussed and similar results are developed for the ring of integral valued polynomials in several indeterminates, and for the ring of integral valued rational fractions, over a local ring. (Received July 21, 1971.)

MICHAEL DOOB, University of Manitoba, Winnipeg 19, Manitoba, Canada. On the spectral characterization of the line graph of a BIBD.

The spectrum of the line graph of a BIBD is computed and conditions are given under which the graph is characterized by its polynomial. The previously unpublished conditions which are shown include the **Theorem.** If \( \frac{1}{4}(r-k)^2 + r - \lambda \) is not a square, \( k \) is a prime, \( k \equiv 1 \mod 4 \), and \( r + k > 18 \), then the line graph of a BIBD with parameters \( (v, b, r, k, \lambda) \) is characterized by its polynomial. **Theorem.** If \( k \) is an odd prime, \( k = \lambda + 2 \), \( r + k > 18 \), then the line graph of a BIBD with parameters \( (v, b, r, k, \lambda) \) is characterized by its polynomial. **Corollary.** The line graph of a Steiner triple system is characterized by its polynomial if \( r > 15 \). (Received July 22, 1971.)
Let A be a simple power-associative algebra having a Peirce decomposition $A = A_{11} + A_{10} + A_{01} + A_{00}$ relative to any idempotent. It is shown that if the modules multiply as for associative algebras (i.e., $A_{ij} A_{k\ell} = \delta_{ij} A_{k\ell}$, $\delta$ being the Kronecker delta) and if A has an idempotent e which is not an identity element then either A is associative or is a noncommutative Jordan algebra of degree 2. Other results are obtained assuming that the modules have different multiplicative properties. (Received July 26, 1971.)

Lattice of equational subclasses of distributive semigroups.

Let $Sg(\Omega)$ denote the equational class of semigroups satisfying a set of identities $\Omega$ and let $\mathcal{L}(Sg(\Omega))$ denote the lattice of equational subclasses of $Sg(\Omega)$. We show that $S \in Sg(xyz = uvx)$ iff S is a strong inflation of the semigroup $[0]$, from which $\mathcal{L}(Sg(xyz = uvx))$ is completely described. The notion of "strong inflation" of semigroups was introduced by M. Petrich (Acad. Roy. Belg. Bull. Cl. Sci. 53(1967), 60-73). By showing that every semigroup $S$ which is a strong inflation of a semigroup $H$ is a subdirect product of the semigroup $H$ and a semigroup in $Sg(xyz = uvx)$, the following result follows immediately: $\mathcal{L}(Sg(xyz = xyyz = xzyz))$ is isomorphic to the direct product of $\mathcal{L}(Sg(xyz = uvx))$ and the eight element Boolean lattice. Semigroups satisfying $xyz = xyyz = xzyz$ are called distributive semigroups and were investigated by M. Petrich (C. R. Acad. Sci. Paris Ser. A-B 268(1969), A849-A852). (Received July 29, 1971.) (Author introduced by Professor George A. Grätzer.)

Fibonacci Catalan numbers: Arithmetic properties and a table of the first fifty numbers.

Let $\left[ \begin{array}{c} m \\ n \end{array} \right] = F_m F_{m-1} \cdots F_{m-n+1} / F_n F_{n-1} \cdots F_1$, $\left[ \begin{array}{c} 0 \\ 0 \end{array} \right] = 1$, be the Fibonomial coefficients, where $F_{n+1} = F_n + F_{n-1}$, $F_0 = 0$, $F_1 = 1$, are the Fibonacci numbers. The author has proved [Abstract 71T-A75, these Notices 18(1971), 551] that $K(n) = \left[ \begin{array}{c} 2n \\ n \end{array} \right] / 2^n$ is an integer for all integers $n \geq 0$. The numbers $K(n)$ are called Fibonacci Catalan numbers in analogy to $C(n) = \left( \begin{array}{c} 2n \\ n \end{array} \right) / (n+1)$, the ordinary Catalan numbers ($= 1, 1, 2, 5, 14, 42, 132, \ldots$ with many combinatorial interpretations). It is now proved that $K(n)$ is odd iff $n = 0, 1, 3 \cdot 2^k - 1$, a being any integer $\equiv 0$. This parallels a theorem of Silberger [Prace Mat. 10(1969), 91-96] for $C(n)$. Moreover, $5^F | K(n)$ iff $5^F | C(n)$. These and other divisibility properties parallel results of R. Alter and K. K. Kubota [J. Combinatorial Theory, to appear] for Catalan numbers. Thus, the set of K's not divisible by 5 occur in blocks of length 4. The situation for other odd primes is more involved. A table is given of $K(n)$ in factored form for $0 \leq n \leq 50$. The table was computed from tables of factors of $F_n$ using two methods as a check. Other arithmetical tables relating to $K(n)$ are given. $K(50)$ is square-free with 110 prime divisors. (Received July 30, 1971.)

In this dissertation, we begin a classification of simple totally antiflexible algebras (finite dimensional) over splitting fields of characteristic \( \neq 2, 3 \). For such an algebra \( A \) let \( P \) be the largest associative ideal in \( A^+ \) and let \( N \) be the radical of \( P \). We say that \( A \) is of type \((m,n)\) if \( N \) is nilpotent of class \( m \) with \( \dim A = n \).

Define \( N_i = N_{i-1} \cdot N \), \( N_1 = N \), then \( A \) is said to be of type \((m,n,d_1,d_2,\ldots,d_q)\) if \( A \) is of type \((m,n)\), \( \dim(N_i - N_{i-1}) = d_i \) for \( 1 \leq i \leq q \) and \( \dim(N_1 - N_{i+1}) = 1 \) for \( q < i < m \). We then determine all nodal simple totally antiflexible algebras of types \((n,n)\), \((n-k,n,k+1)\), \((n-2,n)\) (over fields of characteristic \( \neq 2, 3 \)) and of type \((3,6)\) (over the field of complex numbers). We also give preliminary results for nodal simple totally antiflexible algebras of type \((n-k,n,k,2)\) and of type \((m,n,d_1,\ldots,d_q)\) in general with \( m > 2 \) (the case \( m = 2 \) has been classified by D. J. Rodabaugh). (Received August 3, 1971.)

KHEE-MENG KOH, University of Manitoba, Winnipeg 19, Manitoba, Canada. On sublattices of a lattice. I.

After N. D. Filippov, a sublattice \( A \) of a lattice \( L \) is called completely isolated if \( L - A \) is a sublattice; homogeneous if for each \( a \in L - A \) either \( a\parallel x \) or \( a\nparallel x \) for all \( x \in A \). Let \( K \) be the \( L \)-sum of the family \( G \) of lattices (Alg. Univ. (1971), 108). Each lattice in \( G \) is called a component of the \( L \)-sum relative to \( K \).

Theorem 1. A lattice \( A \) of \( L \) is homogeneous, completely isolated iff there is a retraction \( \phi \) of \( L \) such that \( A = \bigcup_{i \in I} C_i \), where for each \( i \in I \), \( C_i \) is a component of \( L\phi \)-sum relative to \( L \) and \( \left[C_i \right]_{i \in I} \) forms a homogeneous chain in \( L\phi \). Corollary. A sublattice \( A \) of \( L \) is convex homogeneous completely isolated iff there is a retraction \( \phi \) of \( L \) such that \( A \) is a component of \( L\phi \)-sum relative to \( L \). Let \( S(L) \) be the lattice of sublattices of \( L \).

Theorem 2. The following are equivalent: (1) \( S(L) \) is modular, (2) \( S(L) \) is distributive, (3) \( S(L) \) is section complemented, (4) \( S(L) \) is relatively complemented, (5) \( S(L) \) is a Boolean lattice, (6) \( L \) is a chain.

Theorem 3. If \( L \) satisfies A.C.C. or D.C.C., then \( S(L) \) is complemented iff \( L \) is a chain. Let \( S^*(L) = S(L) - \{ \emptyset \} \). Then \( S^*(L) \) is a \( \vee \)-semilattice. Let \( \Theta \in C(L) \). Define a binary relation \( S^*(\Theta) \) on \( S^*(L) \) as follows:

\[ A = B(S^*(\Theta)) \text{ iff for each } a \in A \text{ there is } b \in B \text{ with } a = b(\Theta) \text{ and conversely.} \]

Theorem 4. \( S^*(\Theta) \subseteq C(S^*(L)) \) and \( S^*(L/\Theta) \cong S^*(L)/S^*(\Theta) \). (Received August 3, 1971.)

KATHLEEN B. LEVITZ, University of Kentucky, Lexington, Kentucky 40506. Another characterization of Dedekind domains. Preliminary report.

Let \( R \) be an integral domain with identity. A prime ideal of \( R \) is called a minimal prime ideal if it is of height 1. \( R \) has property \( \pi_n \), \( n \) a positive integer, if each ideal of \( R \) generated by \( n \) or fewer elements can be represented as a product of finitely many prime ideals. Lemma. If the domain \( R \) has property \( \pi_2 \), then \( R \) is a Krull domain in which each minimal prime ideal is invertible. Moreover, the minimal prime ideals of \( R \) are pairwise comaximal. Theorem. If \( R \) is an integral domain with identity, the following conditions are equivalent: (1) \( R \) has property \( \pi_2 \); (2) \( R \) has property \( \pi_n \), for any \( n \neq 2 \); (3) \( R \) is a Dedekind domain. (Received August 4, 1971.)
Denote by \( d(n) \) the number of positive divisors of the natural number \( n \); Erdős (Math. Student 36 (1968/69)) proved
\[
\sum_{1 \leq n \leq x} d(d(n)) = C_1 (1 + o(1)) x \log \log x \text{ with a certain constant } C_1 > 0.
\]
In this note, we study similar sums. Let \( v(n) = \sum_{p \mid n} 1 \), \( V(n) = \sum_{p \mid n} 1 \) (p prime). We show
\[
\sum_{1 \leq n \leq x} v(d(n)) = C_2 x + O(x^{1/2 + \epsilon})
\]
(\( \epsilon > 0 \), arbitrary) with a certain constant \( C_2 > 0 \). Furthermore,
\[
\sum_{1 \leq n \leq x} V(d(n)) = C_3 x \log \log x + C_4 x + O(x/\log x)
\]
and
\[
\sum_{1 \leq n \leq x} d(d(n)) = C_1 x \log \log x + C_5 x + O(x/\log x) \text{ with certain constants } C_3 > 0, C_4, C_5.
\]
(Received August 5, 1971.)

Let \( \varphi(n) \) be Euler's totient and \( \varphi^*(n) \) the unitary totient function, so that \( \varphi^*(n) = \prod (p^a - 1) \) when \( n \) has the prime-power decomposition \( n = \prod p^a \). The still unsolved celebrated problem of D. H. Lehmer (Bull. Amer. Math. Soc. 38(1932), 745-751) asks whether the divisibility of \( n - 1 \) by \( \varphi(n) \) implies that \( n \) is a prime. We here consider the analogous problem whether the divisibility of \( n - 1 \) by \( \varphi^*(n) \) implies that \( n \) is the power of a prime. This seems to be a deep problem and we here offer a partial solution. If \( n \) is a squarefree integer, our problem is the same as Lehmer's, in which case E. Lieuwens showed (Nieuw Arch. Wisk. 18(1970), 165-169) that \( n \) must have at least eleven distinct prime factors. In this paper we show that if \( n \) is not squarefree, and if \( \varphi^*(n) \) divides \( n - 1 \), then if \( n \) is not a prime-power, it must be odd and have at least 32 distinct prime factors. Some other results concerning such an integer \( n \) are also obtained. (Received August 5, 1971.)

Each \( T_0 \) topology on \( n \) points determines the finite semigroup \( S \) of continuous functions under composition. \( S \) is a subsemigroup of the full transformation group. The continuous functions, multiplication tables, Green's \( R \), \( L \), and \( J \) relations as well as the ideals are computed for the 15 nonhomeomorphic, nondiscrete \( T_0 \) topologies on four points. The APL/360 program used is also included. (Received August 6, 1971.)

Let \( A \) and \( B \) be abelian groups of exponents \( m \) and \( n \) respectively. The laws of the unrestricted wreath product \( A \mathcal{Wr} B \) have been known for some time in the cases \( (m, n) = 1 \) or \( m = p^k \) and \( n = p \). We demonstrate a basis for the laws of \( \mathcal{C}_p \mathcal{Wr} \mathcal{C}_2 \) and \( \mathcal{C}_p \mathcal{Wr} \mathcal{C}_p \), where \( \mathcal{C}_n \) is the cyclic group of order \( n \). (Received August 6, 1971.)
Let $n$ and $s$ be integers where $n \leq s \leq 2n - 1$. A square of side $s$ such that each cell is empty or contains an unordered pair of integers from amongst $1, 2, \ldots, 2n$ is called a Howell design of type $H(s, 2n)$ provided: (1) each integer from 1 to $2n$ appears exactly once in each row and in each column and (2) every unordered pair of integers appears at most once in a cell of the square. In this paper, one obtains designs of $H(s, 2n)$ for $n = 3$ to 15 and $s = n$ to $2n - 1$ for all values with four exceptions and also the following theorems.

**Theorem 1.** Designs of type $H(n, 2n)$ exist for all $n$ except $n = 2$ and possibly $n = 6$. We say a Howell design of type $H(s, 2n)$ satisfies a $*$-condition on a set of $2n - s$ integers if there are $2n - s$ integers amongst $1, 2, \ldots, 2n$ such that no pair of them occupy a cell. Many designs satisfy the $*$-condition. **Theorem 2.** If designs of type $H(s_1, 2n_1)$ and $H(s_2, 2n_2)$ exist and the first satisfies a $*$-condition and a pair of orthogonal latin squares of side $s_1$ exists then there is a design of type $H(s_1, s_2, (2n_2 - 1)s_1 + 2n_1 - s_1)$ which also satisfies a $*$-condition. **Theorem 3.** If a design of type $H(s, 2n)$ exists and there are orthogonal latin squares of order $k$ then $H(sk, (2n - 1)k + k)$ exists. **Theorem 4.** Let designs of type $H(2n - 1, 2n)$ and $H(s_1, 2n_1)$ exist and the latter contain a subdesign of type $H(s_2, 2n_2)$ where $s_1 - s_2 = 2n_1 - 2n_2$. With one other condition there exists a design of type $H((2n - 1)(s_1 - s_2) + s_2, (2n_1 - 2n_2)(2n - 1) + 2n_2)$. (Received August 12, 1971.)
KENNETH I. APPEL, University of Illinois, Urbana, Illinois 61801. The word problem for tame alternating knot groups is solvable.

The theorem of the title is proved using many of the techniques of small cancellation theory as employed by Lyndon and Schupp. The principal new tool is the dual of a small cancellation diagram which is a collection of closed curves in the plane of the knot projection. The presentation of the knot group is a minor modification of the Wirtinger projection. Basically the idea of the argument is to show that the small cancellation type conjugacy diagram which does not satisfy the conditions of Lyndon and Schupp can be replaced in an effective manner by a related diagram which retains the same information but has the appropriate graph theoretic properties to make possible arguments similar to those used by Schupp in his proof of the solvability of the conjugacy problem in small cancellation groups. (Received August 10, 1971.)

RAM AWTAR, Department of Mathematics and Statistics, Aligarh Muslim University, Aligarh, U. P., India. On the commutativity of nonassociative rings.

All the rings $R$ considered here are nonassociative with unity. We prove the following results: (I) Let $R$ be such that for a fixed $n \geq 1$, $(xy)^n = (yx)^n$, $x, y \in R$, and $(R,+)$ is $p$-torsion free for each prime $p \leq n$. Then $R$ is commutative. (II) Let $R$ be such that for a fixed $n \geq 1$, $(xy)^n = x^n y^n$, $x, y \in R$, and $(R,+)$ is $p$-torsion free for each prime $p \leq n$. Then $R$ is commutative. If $n$ is even, it is sufficient to take $p < n$. (III) Let $R$ be such that for a fixed $n \geq 1$, $(xy)^n = y^n x^n$, $x, y \in R$, and $(R,+)$ is $p$-torsion free for each prime $p \leq n + 1$. Then $R$ is commutative. If $n$ is odd, it is sufficient to take $p \leq n$. (IV) Let $R$ be such that for a fixed $n \geq 1$, $x^n y^n = y^n x^n$, $x, y \in R$, and $(R,+)$ is $p$-torsion free for each prime $p \leq n$. Then $R$ is commutative. (V) If $R$ is such that $(R,+)$ is torsion free and it satisfies any one of the following conditions: (i) $(xy)^n(x, y) = (yx)^n(x, y)$, $n(x, y) \geq 1$, $x, y \in R$, and $n(x, y) = n(x+1, y)$. (ii) $(xy)^n(x, y) = x^n y^n x^n y^n$, $x, y \in R$, $n(x, y) \geq 1$, and $n(x, y) = n(x+1, y)$. (iii) $(xy)^n(x, y) = y^n x^n y^n x^n y^n$, $x, y \in R$, $n(x, y) \geq 1$, and $n(x, y) = n(x+1, y)$. (iv) $x^n y^n = y^n x^n$, $x, y \in R$, $n(x, y) \geq 1$, and $n(x, y) = n(x+1, y)$. Then $R$ is commutative. (Received August 13, 1971.) (Author introduced by Professor S. K. Jain.)

SUBJEET SINGH, Department of Mathematics and Statistics, Aligarh Muslim University, Aligarh, U. P., India. (KE)-domains.

This paper is in continuation of our paper "(KE)-domains and their generalizations," Abstract 71T-A52, these Notices 18(1971), 403. All rings considered here are commutative with unity and all modules are unital. Following are the main results established: (I) If $A$ is a proper ideal of a domain $D$ such that the subring $A^*$ of $D$ generated by $A \cup \{1\}$ is Noetherian and every injective $D$-module is injective as an $A^*$-module, then $D = A^*$. (II) A domain $D$ is a (KE)-domain iff for every proper ideal $A$, $D = A^*$. (III) A domain $D$ is a (KE)-domain iff
for every proper ideal \( A \), \( D \) is a flat \( A^* \)-module. (IV) Let \( Z \) be the ring of integers and \( p \) be a prime number. If \( A \) is a (KE)-domain \( D \) (\( \neq \) its quotient field) is such that \( p \) is not invertible in \( D \), then \( D \) is embeddable in \( Z(p) \), the \( p \)-adic completion of \( Z(p) \). (V) A direct limit of a directed family of (KE)-domains associated with same quotient ring \( Z \) of \( Z \) is a (KE)-domain associated with \( Z \). (Received August 13, 1971.) (Author introduced by Professor S. K. Jain.)

71T-A230. DON R. LICK, Western Michigan University, Kalamazoo, Michigan 49001. A characterization of \( n \)-connected graphs.

The following result is proved. Let \( G \) be a graph with at least \( n + 1 \) points, \( n \geq 2 \). Then \( G \) is \( n \)-connected if and only if for each set \( S \) of \( n \) points of \( G \), any two points of \( S \) are contained in a cycle of \( G \) which avoids the other \( n - 2 \) points of \( S \). (Received August 13, 1971.)

71T-A231. OLGA TAUSSKY, California Institute of Technology, Pasadena, California 91109. A result concerning classes of matrices.

It is known that there is a 1-1 correspondence between the ideal classes in the order \( Z(\alpha) \) generated by an algebraic integer \( \alpha \) and the matrix classes \( S^{-1}A \), where \( A \) is a \( Z \)-matrix whose characteristic polynomial \( f(x) \) is the irreducible \( Z \)-polynomial for \( \alpha \), and \( S \) is a unimodular \( Z \)-matrix. A similar relation holds for the classes of \( Z \)-representations (see Zassenhaus, Latimer and McDuffee). Now an expression is obtained for the matrix class (or representation) corresponding to the product of two ideal classes (or representations) based on the concept of the ideal matrix (see McDuffee, Taussky, G. B. Wagner). Previous results for the transposed matrix class are examined in the light of the new result and lead to the following relation for the ideal matrix \( X' \) of the ideal \( \mathcal{I} \): \( p(C)S'X' = VX'U \), where \( p \) stands for polynomial, \( C \) for companion matrix of \( f(x) \), \( S \) for a certain unimodular matrix which transforms a companion matrix into its transpose, \( U, V \) are certain unimodular matrices and \( X' \) is the transpose. (Received August 16, 1971.)

71T-A232. ALEXANDER ABIAN, Iowa State University, Ames, Iowa 50010. Order in rings.

Generalizing the author's results (Proc. Amer. Math. Soc. 24(1970), 502-507), structures of rings are studied in connection with the order relation \( \preceq \) introduced in a ring \( R \) by: \( x \preceq y \) if and only if \( x^m(x) \cdot y^n(y) = x^{m(x)+n(y)} \) for every element \( x \) and \( y \) of \( R \), where \( m(x) \) and \( n(y) \) are integers depending on \( x \) and \( y \) respectively. Some generalizations for the case of noncommutative rings are also considered. (Received August 19, 1971.)

\*71T-A233. DONALD P. MINASSIAN, Butler University, Indianapolis, Indiana 46208. On the direct product of \( V \)-groups.

A \( V \)-group (resp. \( V^* \)-group) is a group in which every full (resp. partial) order for every subgroup extends to some full order for the group. A \( V \)AN-group is one in which every full order for every abelian normal subgroup extends to some full order for the group. Necessary and sufficient conditions that an abstract group be a \( V \)-group are known. These are used to establish: Lemma. Let \( G = \{ a, b \mid ba = ab^2 \} \) and \( H = \)


\[ c, d \text{ d c} = cd^3 \]. Then \( G \) and \( H \) are \( V \)-groups. **Theorem.** \( G \times H \) is not a \( B \)-group. **Corollary.** The direct product of \( V^* \)-groups is not a \( VAN \)-group. (Received August 25, 1971.)

*71T-A234. HARVEY E. WOLFF, University of Texas, Austin, Texas 78712. Monads and monoids on symmetric monoidal closed categories.

Let \( \mathcal{V} \) be a symmetric monoidal closed category. If \( T \) is a \( \mathcal{V} \)-functor on \( \mathcal{V} \) and \( M \) is an object of \( \mathcal{V} \), then it is well known that there exists a bijection between \( \mathcal{V} \)-natural transformation \( - \otimes M \rightarrow T \) and maps \( M \rightarrow T(I) \) (see Eilenberg and Kelly, "Closed categories," Proc. Conf. on Categorical Algebra, p. 503, Corollary 7.5). If \( T \) is the \( \mathcal{V} \)-functor part of a \( \mathcal{V} \)-monad \( T \) on \( \mathcal{V} \), then Kock ("Monads on symmetric monoidal closed categories," Arch. Math. 22(1970), 1-10) has shown that it carries two canonical structures as a monoidal functor. Using these structures \( T(I) \) can be made into a monoid in \( \mathcal{V} \) in two ways, and it turns out that one is the dual of the other. Note also that if \( M \) is a monoid in \( \mathcal{V} \), then there is an obvious way to make \( - \otimes M \) the \( \mathcal{V} \)-functor part of a \( \mathcal{V} \)-monad \( M \). We show that under the bijection mentioned above \( \mathcal{V} \)-monad maps \( M \rightarrow T(I) \) correspond to monoid maps \( M \rightarrow T(I) \). As a consequence we show that for every \( \mathcal{V} \)-monad on \( \mathcal{V} \) there is a monoid (namely \( T(I) \)) and a \( \mathcal{V} \)-monad map \( T : \mathcal{V} \rightarrow \mathcal{V} \) such that if \( M \) is a monoid and \( \alpha : M \rightarrow T(I) \) a \( \mathcal{V} \)-monad map, then there exists a unique \( \mathcal{V} \)-monad map \( \beta : M \rightarrow T(I) \) with \( \tau \beta = \alpha \). We also show that if \( T \) is a commutative monad (see Kock, op. cit., p. 7) then \( T(I) \) is a commutative monad. It should be noted that there are noncommutative monads \( T \) such that \( T(I) \) is a commutative monad. (Received August 25, 1971.)

*71T-A235. RUDOLF AHLSWEDE, Ohio State University, Columbus, Ohio 43210. Solution of Shannon's zero-error capacity problem for polygons. Preliminary report.

Shannon introduces in his paper, "The zero-error capacity of a noisy channel" (IRE Trans. Information Theory, IT-2(1956), 8-19), the zero-error capacity \( C_0 \) of a discrete memoryless channel and proposes as a problem to determine \( C_0 \) as an explicit function of the stochastic transmission matrix. The problem is equivalent to a graph theoretical problem, which we now formulate (c.f.C. Berge: "The theory of graphs and its applications," Chapter 4). Consider a finite graph \( H = (X, \Gamma) \), a set \( S \subseteq X \), is said to be internally stable if no two vertices of \( S \) are adjacent. Let \( \mathcal{S} \) be the family of internally stable sets of \( H \). \( \alpha(H) = \max_{S \in \mathcal{S}} |S| \) is called the coefficient of internal stability of the graph \( H \). Let now \( G^L \) be the graph obtained from a graph \( G \) after addition of loops, and let \( G = \prod_1^L G \) be the product graph with \( n \) factors. The capacity \( C(G) \) of the graph \( G \) is defined as \( C(G) = \lim_{n \to \infty} n^{-1} \log \alpha(G_n) \). Shannon's problem consists in finding \( C(G) \) explicitly for every finite graph \( G \). Shannon determines \( C(G) \) for all graphs with at most 5 vertices with the only exception of the pentagon graph \( P^5 \). In this case he shows that \( \frac{1}{2} \log 5 \leq C(P^5) \leq \log(5/2) \). We prove that, for \( t \geq 4 \), \( C(P^t) = \log(t/2) \), where \( P^t \) denotes the polygon graph with \( t \) vertices. As an immediate consequence of this result one obtains that for odd \( t \), \( t \geq 5 \), (\(*) \) \( \alpha((P^t)_{2k}) > (\alpha((P^t)_{2k}))^2 \) for infinitely many \( k \). It was undecided until now whether there exist graphs for which (\*) holds for infinitely many \( k \). (Received August 26, 1971.) (Author introduced by Professor D. K. Ray-Chaudhuri.)

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Let $Y$ be a connected prescheme. A quasicoherent $\mathcal{O}_Y$-algebra $\mathcal{A}$ is said to be locally finite etale if it is faithful and is the direct limit of its finite etale $\mathcal{O}_Y$-subalgebras. $\mathcal{A}$ is said to be locally Galois if it is connected, locally finite etale and $\mathcal{A}^G = \mathcal{O}_Y$ where $G = \text{Aut}_{\mathcal{O}_Y} \mathcal{A}$. Let $\mathcal{A}$ be a locally Galois $\mathcal{O}_Y$-algebra, we show that $G$ is a profinite group and the mappings $H \rightarrow \mathcal{A}^H$ and $\mathcal{A} \rightarrow \text{Aut}_{\mathcal{O}_Y} \mathcal{A}$ are inverse bijections between the set of closed subgroups of $G$ and the locally finite etale $\mathcal{O}_Y$-subalgebras of $\mathcal{A}$. This correspondence in affine case was given by Nagahara ["A note on Galois theory of commutative rings", Proc. Amer. Math. Soc. 18(1969)].

For integers $j, k, \ell, v$ such that $j \geq 1$ and $\ell \leq k \leq v$ we define $N_j(\ell, k, v)$ as the minimum number of $k$-sets of $V = \{1, 2, \ldots, v\}$ needed such that every $\ell$-set of $V$ occurs at least $j$ times. Fort and Hedlund in "Minimal coverings of pairs by triples" (Pacific J. Math. 8(1958), 709-719) determined $N_1(2, 3, v)$.

**Theorem 1.** $N_2(3, 2, v) = (n(n - 1))/3$ if $n \leq 0, 1 (3)$ and $N_2(3, 2, v) = \lfloor (n(n - 1))/3 \rfloor + 1$ if $n \geq 2 (3)$. Using the constructions of Fort and Hedlund and Theorem 1 various other $N_j(2, 3, v)$ are determined. (Received August 27, 1971.)

Let $(GF(q))_n$ denote the complete matrix ring of all $n \times n$ matrices over $GF(q)$ under normal matrix addition and multiplication. If a subring $M$ of $(GF(q))_n$ is a field, then $M$ is called a matrix field, and we say that $M$ is a subfield of $(GF(q))_n$. The author characterizes all subfields of $(GF(q))_n$ and in the special case that $F = GF(p)$ obtains the following **Theorem**. Let $M$ be a subring of $(F)_n$ having rank $r$. If $r = n$, then $M$ is a subfield of $(F)_n$ if and only if $M$ is similar over $F$ to the matrix field $k$-sum $(S_m(F)[C(f(x))])$ for some prime polynomial $f(x) \in F[x]$ of degree $m$, where $n = mk$ and $C(f(x))$ denotes the companion matrix of $f(x)$. If $1 \leq r < n$, then $M$ is a subfield of $(F)_n$ if and only if $M$ is similar over $F$ to the matrix field $1^n$-sum $(M';n-r,0)$, where $M'$ is a subfield of $(F)_r$ and has rank $r$. (Received August 30, 1971.)

Let $D$ be a Dedekind domain, $K$ its quotient field, $\Sigma$ a central simple $K$-algebra, $\Lambda$ a $D$-order in $\Sigma$. Several equivalent conditions for a two-sided ideal $I$ to be simultaneously left and right invertible are given; these conditions involve left and right projectivity, the left and right orders associated with the ideal and $\text{Hom}(I, \Lambda)$, where $I$ is considered as both a left $\Lambda$-module and a right $\Lambda$-module. From these conditions several known and new characterizations of maximal orders are obtained. For example, $\Gamma$ is a maximal $D$-order if and only if every two-sided ideal $I$ such that $K \otimes_D I \cong \Sigma$ is two-sided invertible. As a corollary, one obtains
The known result that if $\Gamma$ is maximal, the two-sided ideals $I$ such that $I \oplus D \cong K$ form an abelian group. (Received August 30, 1971.)

*71T-A240. KEITH R. PIERCE, University of Missouri, Columbia, Missouri 65201. Amalgamations of lattice ordered groups.

The author investigates the amalgamation property in various classes of $\ell$-groups. Theorem 1. The classes of abelian $\alpha$-groups and abelian $\ell$-groups have the amalgamation property, but the class of $\ell$-groups does not. Theorem 2. An $\ell$-subgroup $G$ of the $\ell$-groups $H$ and $K$ can be amalgamated in the class of $\ell$-groups if either (a) $G$ is an archimedean $\alpha$-group, or (b) $G$ is a direct product of archimedean $\alpha$-groups and $H$ and $K$ are representable. Corollary. Every $\ell$-group is embeddable in a divisible $\ell$-group. This is a result of Charles Holland, but Theorem 2 yields a proof more along the lines of B. H. Neumann's proof of the analogous result for groups. Corollary. Every $\ell$-group is embeddable in an $\ell$-group in which any two positive elements are conjugate. (Received August 30, 1971.)

*71T-A241. M. V. SUBBARAO and D. SURYANARAYANA, University of Alberta, Edmonton, Alberta, Canada. Arithmetical functions associated with the bi-unitary divisors of an integer.

It is well known that a divisor $d$ of $n$ is called unitary, if $d\delta = n$ and $(d, \delta) = 1$. For integers, $a, b$, not both zero, we define $(a, b)^{**}$ to be the greatest unitary divisor of both $a$ and $b$. We say that $a$ is bi-unitarily prime to $b$, if $(a, b)^{**} = 1$. We define a divisor $d$ of $n$ to be bi-unitary, if $d\delta = n$ and $(d, \delta)^{**} = 1$. One of the authors recently obtained asymptotic formula for the average order of $\tau^{**}(n)$, the number of bi-unitary divisors of $n$. In this paper, we establish estimates for the average orders of $\sigma^{**}(n)$ and $\varphi^{**}(n)$ defined respectively to be the sum of all the bi-unitary divisors of $n$ and the number of positive integers $\equiv n$ and bi-unitarily prime to $n$. Also, we establish the maximum order of $\tau^{**}(n)$. We note that $\varphi^{**}(n)$ is not multiplicative, whereas $\tau^{**}(n)$ and $\sigma^{**}(n)$ are multiplicative. We prove the following results. Theorem 1. $\sum_{n \leq x} \sigma^{**}(n) = \zeta(3) \pi^2 x^2 / 12 + O(x \log^3 x)$, where $\alpha = \prod_p (1 - 2/p^3 + 1/p^4 + 1/p^5 - 1/p^6)$. Theorem 2. $\sum_{n \leq x} \varphi^{**}(n) = \pi^2 x^2 / 30 + O(x \log^2 x)$, where $\beta = \prod_p (1 + 2/p^3 - 1/p^4)$. Theorem 3. $\lim (\log \tau^{**}(n) \log \log n / \log n) = \lim (\log \tau^{**}(n^2) \log \log n / \log n) = \log 2$. Finally by defining $(a, b)^{**}_k$ to be the greatest $k$th power unitary divisor of both $a$ and $b$ ($k$ being any fixed positive integer) we obtain analogous results for the corresponding generalized arithmetical functions. (Received September 1, 1971.)

71T-A242. HARRY F. SMITH, University of Iowa, Iowa City, Iowa 52240. On generalized alternative rings I and II. Preliminary report.

In (J. Algebra 18(1971), 304–325) Kleinfeld defines a generalized alternative ring $I$ to be a nonassociative ring satisfying the following identities: (i) $(wx, y, z) + (w, x, (y, z)) - w(x, y, z) - (w, y, z)x = 0$, (ii) $(w, x)(y, z) + (w, x, yz) - y(w, x, z) - (w, x, y)z = 0$, (iii) $(x, x, x) = 0$. Consider $R (A)$ to be a generalized alternative ring (algebra) $I$. Theorem 1. Assume the characteristic of $R$ prime to 2, 3. If $R$ contains an idempotent $e \neq 0, 1$, then $R$ semiprime implies $R$ has a Peirce decomposition relative to $e$. Theorem 2. Let $R$ be prime with characteristic prime to 2, 3. If $R$ contains an idempotent $e \neq 0, 1$, then $R$ is alternative.
Theorem 3. If \( A \) is finite-dimensional over a field \( F \) of characteristic \( \neq 2, 3 \), then \( A \) a nilalgebra implies \( A \) nilpotent. In addition, Kleinfeld has defined a generalized alternative ring \( II \) to be a nonassociative ring satisfying (i) (same as (i) above) and (ii) \( (x, y, x) = 0 \) (J. Algebra 18(1971), 326-339). Assuming characteristic of \( F \) only \( \neq 2 \), Theorem 3 above has been proven for this class of algebras as well. (Received September 1, 1971.)

*71T-A243. K. DEMYS, 844 San Ysidro Lane, Santa Barbara, California 93108. The accepted rule for matrix addition is in general false.

When terms or elements of two matrices are noncommutative with respect to each other, the accepted rule for matrix addition is in general not valid, although the law for matrix multiplication is universally valid.

One example (and there exist many) suffices to prove the general invalidity of the addition rule. Given \( M^2 = \frac{1}{2}E \), \( E = (1 \ 0 \ 1 \ 0) \), \( E \) is a nonassociative ring satisfying \( (x, y, x) = 0 \), then \( M^2 = \frac{1}{2}E \) is erroneous. The correct result is obtained by multiplying first, thus:

\[
M^2 = \frac{1}{2}(A^2 + B^2 + C^2 + D^2 + 2AC + 2BC) = \frac{1}{2}(A + B + C + D)^2,
\]

where \( A = \frac{1}{2}E \) and \( E = (1 \ 0 \ 0 \ 1) \). Thus \( -M = i \). The invalidity of the addition rule arises in this instance because of the structure of \( D \) and the fact that \( iE = -Ei \).

(Note that if \( D \) had been chosen as \( iE \), the general failure of the addition rule would not have been observable.) Since matrices in the form \( (i \ E) \) are used fundamentally in modern quantum physics, in connection with the creation and annihilation operators, this result is of practical importance. (Received September 2, 1971.)

*71T-A244. ALBERT J. KARAM, Universidad de Los Andes, Merida, Venezuela. Strong Lie ideals.

Let \( R \) be an associative semiprime 2-torsion free ring. If \( U \) is a Lie ideal of \( R \), \( U \) is \( R \)-strong iff \( aua \in U \) for all \( a \in R \), \( u \in U \). Let \( K \) be the skew-symmetric elements of a ring with involution. Similarly \( K \)-strong Lie ideals are defined. Theorem 1. If \( U \) is an \( R \)-strong Lie ideal, \( U \) contains a nonzero two-sided ideal. Theorem 2. If \( R \) is simple and \( U \) is a nonzero \( K \)-strong Lie ideal, \( K = U \). Topological annihilator rings are considered and results paralleling those obtained by Baxter [Pacific J. Math. 27(1968), 1-12] for Jordan ideals are obtained for \( K \)-strong Lie ideals. Theorem 3. If \( R \) is a topological annihilator ring and \( U \) is a nonzero closed \( K \)-strong Lie ideal, \( U = C \cap K \) where \( C \) is a closed two-sided ideal. Let \( R \) be a simple ring with involution, \( \star \), not of characteristic 2 with center, \( Z \), equal to zero or \( [R : Z] > 4 \). Let \( \phi \) be a nonzero additive mapping of \( R \) into a ring \( A \) such that the subring \( A \) generated by the image of \( R \) is a 2-torsion free noncommutative prime ring. Assume that \( \phi(xy - yx\star) = \phi(x)\phi(y) - \phi(y)\phi(x) \) for all \( x, y \in R \). This is the Lie analog of mappings considered by Small [Ph.D. Dissertation, Yale University, New Haven, Conn., 1968]. It follows that \( \phi \) is an associative isomorphism. (Received July 29, 1971.) (Author introduced by Professor Willard E. Baxter.)


Rho-set theory (Abstract 71T-E58, these Notices 18(1971), 666) sanctions operation augment, @, trivialized by classical set theory: \( \{p_1, \ldots, p_m\} \oplus \{q_1, \ldots, q_n\} \sim_{\rho} \{p_1, \ldots, p_m, q_1, \ldots, q_n\} \) (irredundantly).
Theorem 1. \( \odot \) is commutative, associative, nonidempotent, cancellation, closure operation; induces "power"; behaves on rho-set like product on factors of natural number \( n \), i.e. if \( F(n) \) is factor set of \( n \), \( F(j \cdot k) \odot F(l) \odot F(k) \). Lemma 1. In (distributive) lattice of numbers, \( L_n \), ordered by divisibility, an element is join-irreducible iff of form \( p^r \) \((r = 0, 1, \ldots)\), \( p \) prime. Label atoms of \( L_n \) as improper join-irreducibles, nonatomic ones as proper (isomorphic to pure augments of atoms or singletons); denote model of 2-chain as \( C_p \), \( p \) prime.

Lemma 2 (Birkhoff). Any nondegenerate \( L_n \) can be written uniquely as subdirect product of 2-chains.

Definition. Direct augment, \( I_n \), is the composition: (1) cartesian product (rendered associative) of lattice, \( \times L_i \); (2) augment (or product) of elements in each \( n \)-tuple; (3) map of this rho-set as alpha-set. Theorem 2. Every \( L_n \) can be written uniquely as direct augment of powers of 2-chains: if \( P_1 \ldots P_k = n \), \( L_n = C_{P_1}^{m_1} \ldots C_{P_k}^{m_k} \).

(Received September 2, 1971.)

**Analysis**

*71T-B194. H. M. SRIVASTAVA, University of Victoria, Victoria, British Columbia, Canada and J. P. SINGHAL, University of Jodhpur, Jodhpur, India. Some formulas involving the products of several Jacobi or Laguerre polynomials. Preliminary report.

By using the principle of mathematical induction and certain operational techniques, the authors prove a number of multiple series relations associated with the products of several Jacobi or Laguerre polynomials. The various results obtained in this paper are analogous to the recent extensions of the well-known Mehler formula for Hermite polynomials, given by L. Carlitz and the authors [Proc. Amer. Math. Soc. 31(1972), to appear; Rocky Mt. J. Math. 3(1972), in press; see also Abstracts 71 T-B47 and 71 T-B128, these *Notices* 18(1971), 412 and 645]. (Received June 21, 1971.)

*71T-B195. H. M. SRIVASTAVA, University of Victoria, Victoria, British Columbia, Canada. On a generating function for the Jacobi polynomial. II. Preliminary report.

This paper is a continuation of the earlier work of the present author [J. Math. Sci. 4(1969), 61-68; see also Yokohama Math. J. 17(1969), 65-71] and it discusses the possibilities of extending some generating relations involving Jacobi and related polynomials to hold for various classes of generalized hypergeometric polynomials. Several particular cases of the main results are considered briefly. (Received June 21, 1971.)


Let \( S \) be a compact Hausdorff space, and \( \mu \) a Borel measure on \( S \) such that \( \mu S = 1 \). Then, for some cardinal \( m \), the product measure space \( S \times I^m \) is point isomorphic, in a measure-preserving way, to \( I^m \). [Here \( I^m \) denotes the product of \( m \) copies of the unit interval, with product measure.] (Received June 25, 1971.)
Charles Byrne and Francis E. Sullivan, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. Representation of isometric involutions in an $L^p$ space.

Isometric involutions are linear isometries, $U$, such that $U^2 = I$. An isometric involution is called a reflection, and is said to be reduced if it leaves no range of a characteristic projection point-wise invariant.

The notion of an independent pair of complementary contractive projections is defined, using the idea of independence of subalgebras. The following theorem is then proved: Theorem. The following are equivalent:

1. $L^p$ admits a reduced reflection.
2. There exists an independent pair of complementary contractive projections on $L^p$.
3. There is a second measure, mutually absolutely continuous with the first, with respect to which the original measure algebra is a product of measure algebras, one of which has exactly four elements.

It is also shown that every $L^p$ isometry generates a reflection in a natural way. (Received July 1, 1971.) (Authors introduced by Professor H. B. Cohen.)

DAN MAULDIN, University of Florida, Gainesville, Florida 32601. An integral representation of functionals on $ca(S, \Sigma)$. Preliminary report.

Let $\Sigma$ be a $\sigma$-algebra of subsets of $S$ and $ca(S, \Sigma)$ be the Banach space of all real-valued countably additive set functions defined on $\Sigma$ under the variation norm. Suppose the cardinality of the space $ca(S, \Sigma)$ is $2^\aleph_0$ and the continuum hypothesis is true. Theorem. If $T$ is a bounded linear functional on $ca(S, \Sigma)$, then there is a bounded real-valued set function $\psi$ defined on $\Sigma$ such that $T(\mu) = \int_S \psi d\mu$ for every $\mu$ in $ca(S, \Sigma)$.

The integral is the refinement type limit of the approximating sums over the directed set of all sub divisions $D$ of the space $S$. If $D$ is a subdivision (a finite collection of mutually exclusive sets in $\Sigma$ filling up $S$), then the approximating sum over $D$ is: $\sum \psi(B)\mu(B)$, where the sum is taken over all $B$ in $D$. (Received July 2, 1971.)


We refer to the set $C$ of a previous paper of the author (J. London Math. Soc. 44(1969), 385-396).

Theorem. Each pair of elements of $C$ commute. (Received July 6, 1971.)

Claude W. Bardos, University of Paris, Paris 5E, France and Jeffery M. Cooper, University of Maryland, College Park, Maryland 20742. A nonlinear wave equation in a time dependent domain.

Let $B$ be the strip in $\mathbb{R}^{n+1}$ defined as $b = \mathbb{R}^n \times (0, T)$. Let $Q$ be an open set in $B$ with $\Omega(t_0) = Q \cap \{(x, t): t = t_0\}$. We say that $Q$ is monotone increasing if $\Omega(t)$ grows with $t$. A smooth mapping $\varphi = (\varphi_1, \ldots, \varphi_n)$ of $B$ onto $B$ is defined to be hyperbolic if the $n \times n$ matrix $\left(\frac{\partial \varphi_i}{\partial x_j}\right)$ is positive definite and $\left(\frac{\partial \varphi_i}{\partial t}\right)^2 - \sum_{i=1}^n \left(\frac{\partial \varphi_i}{\partial x_i}\right)^2 > c_0 > 0$. Let $f \in L^2(Q)$, $\mu_0 \in L^p(\Omega(0)) \cap H^1_0(\Omega(0))$, and $\mu_1 \in L^2(\Omega(0))$ be given. Suppose that there is a hyperbolic mapping $\varphi: B \to B$ such that $Q^* = \varphi(Q)$ is monotone increasing. Then there exists a weak solution of $\varphi_{tt} - \Delta u + |u|^{p-2}u = f$ such that $u \in L^p(Q) \cap L^\infty(0, T; H^1_0(\Omega(0)))$, $u_t \in L^2(Q)$, $u(x, 0) = u_0(x)$ and $u_t(x, 0) = u_1(x)$. If $Q$ can also be mapped hyperbolically onto $Q^*$, monotone decreasing, then the solution is unique for $p - 2 = 2/(n - 2)$, $n = 3$. Note that if
Q can be mapped hyperbolically onto a cylinder, in which case one has existence and uniqueness, then the lateral boundary of Q is time-like. The results are obtained by the penalty method and extend those of J. L. Lions (Revue Roumaine Math. Pure Appl. 9(1964), 11). (Received July 7, 1971.)


In a previous paper (Abstract 70T-B228, these Notices 17(1970), 959) we obtained transformation formulae for Kampe' de Feriet hypergeometric function. In this paper we have obtained a transformation for \( F_{1,2}^{1,3} \) corresponding to \( _4 F_3 \) (Cambridge Tract No. 32, p. 56, 7. 2(1)) from which a number of known and unknown transformations and sums, as particular cases are obtained. An important particular case is a Watson type sum of \( F_{1,1}^{1,1-a-2m-2n; c,c'; 2c,2c'} \). In the end Cayley Orr type results for the double hypergeometric function have been obtained from the above mentioned transformation (to appear in Rocky Mt. J. Math.). (Received July 1971.) (Author introduced by Dr. Brij M. Nayar.)

*71T-B202. STERLING K. BERBERIAN, University of Texas, Austin, Texas 78712. Equivalence of projections.

Theorem. An AW*-algebra is the ring generated by its projections if and only if it has no abelian summand. Corollary. Every equivalence in an AW*-algebra may be implemented by a partial isometry in the ring generated by the projections of the algebra. The corollary is extended to certain finite Baer *-rings. (Received July 8, 1971.)

*71T-B203. EDWARD R. ROZEMA, Purdue University, Lafayette, Indiana 47907. Almost Cebysev subspaces of finite dimension in \( L_1^E \).

Let \((\Omega,\mu)\) be a positive nonatomic space, E an arbitrary Banach space, and \( L_1^E \) the B-space of measurable functions \( f \) from \( \Omega \) into \( E \) such that \( \int \Omega ||f(s)||_E d\mu(s) \) is finite. Then no finite dimensional subspace of \( L_1^E \) is Cebysev, but in the terminology of A. L. Garkavi, every such subspace \( M \) is almost Cebysev, i.e. the set of elements in \( L_1^E \) which have unique best approximations out of \( M \) is dense and of the second category. (Received July 9, 1971.)


Let \( K \) be a separable, infinite dimensional Hilbert space, and let \( \mathcal{L}(K) \) denote the algebra of bounded linear operators on \( K \). It has sometimes been possible to prove that an operator \( T \) in \( \mathcal{L}(K) \) has a nontrivial invariant subspace by attaching some sort of "compactness" hypothesis to \( T \). One nice result in this direction is the theorem of Arveson and Feldman that appeared in their paper "A note on invariant subspaces", Michigan Math. J. 15(1968), 61-64. We have now obtained the following generalization of the Arveson-Feldman result.

Theorem. Let \( T \) be a quasitriangular operator, and suppose that there exist a sequence of rational functions \( \{ r_k \} \) and a nonzero compact operator K such that \( \| r_k(T) - K \| = 0 \). Then there exists a subspace \( \mathcal{M} \) different from
[0] and \( \mathcal{N} \) that is invariant under every rational function of \( T \). We show, moreover, that this theorem is not vacuous in the sense that there exists a quasitriangular operator \( T \) with the following properties: (1) the spectrum of \( T \) is connected, (2) there exists no nonzero compact operator in the uniformly closed algebra generated by the polynomials in \( T \), and (3) there exists a quasinilpotent compact operator in the uniformly closed algebra generated by the rational functions in \( T \). (Received July 12, 1971.)


Let \( K_1 \) and \( K_2 \) be two convex sets in a normed linear space \( X \). The points \( k_1^* \in K_1 \) and \( k_2^* \in K_2 \) are called the proximal points of the sets \( K_1 \) and \( K_2 \) if and only if \( \|k_1^* - k_2^*\| \leq \|k_1 - k_2\| \) for all points \( k_1 \in K_1 \) and \( k_2 \in K_2 \). The main results of the paper are the characterizations of the proximal points given by the following theorems: Theorem 1. Let \( X \) be a normed linear space whose dual space \( X^* \) is strictly convex and \( K_1, K_2 \) be two convex sets in \( X \) such that \( d(K_1, K_2) > 0 \). Then \( k_1^* \in K_1, k_2^* \in K_2 \) are proximal points if and only if there exists a linear functional \( L \in X^* \) such that: (1) \( \|L\| = 1 \); (2) \( \text{Re} \ L(k_1^*) = \inf_{k_1 \in K_1} \text{Re} \ L(k_1) \); (3) \( \text{Re} \ L(k^*_2) = \sup_{k_2 \in K_2} \text{Re} \ L(k_2) \); (4) \( \text{Re} \ L(k_1^* - k_2^*) = \|k_1^* - k_2^*\| \). Theorem 2. Let \( X, K_1 \) and \( K_2 \) be as in the previous theorem. Then \( k_1^* \in K_1, k_2^* \in K_2 \) are proximal points if and only if \( k_1^* \) is the nearest point of \( k_1^* \) in \( K_1 \) and \( k_2^* \) is the nearest point of \( k_2^* \) in \( K_2 \) i.e., \( \|k_1^* - k_1\| \leq \|k_1 - k_1^*\| \) for all \( k_1 \in K_1 \) and \( \|k_2^* - k_2\| \leq \|k_2^* - k_2\| \) for all \( k_2 \in K_2 \). (Received July 13, 1971.) (Author introduced by Professor P. C. Jain.)

*71TB206. A. G. BRANDSTEIN, University of Connecticut, Storrs, Connecticut 06268. On the closure in \( L^2 \) of \( R(X) \).

Let \( X \) be a compact set of the form \( \mathbb{C} - \bigcup_{i=0}^{\infty} X_i \) where \( X_0 = \{z : |z| > 1 \} \), \( X_i \) open and simply connected for each \( i > 0 \), and \( \text{cl}(X_i) \cap \text{cl}(X_{i+1}) = \emptyset, i \neq j \). Let \( \nu \) be any finite positive (Borel) measure supported on \( \bigcup_{i=0}^{\infty} \partial X_i = \Gamma, \nu \neq \text{point mass} \). Denote by \( R(X) \) the set of all functions in \( C(X) \), the continuous functions on \( X \), which can be approximated by rational functions whose poles lie off \( X \). Denote by \( R^2(\nu) \) the closure of \( R(X) \) in \( L^2(\nu) \). Theorem. There exists a \( \varphi \) in \( R^2(\nu), \varphi \neq 0 \) constant in \( R^2(\nu) \), with \( |\varphi| = 1 \) almost everywhere-\( \nu \) on \( X \). This result extends Lemma 2.13 in Brennan (J. Functional Analysis 7(1971), 285-310). Corollary. \( R^2(\nu) \) has a nontrivial invariant subspace. (Received July 12, 1971.) (Author introduced by Professor Eugene Spiegel.)


Let \( S_n \) be the nth partial sum of the Fourier-Bessel series of order \( \nu \) for \( f(x) \), i.e., the expansion in terms of the o.n. set in \( (0,1), J_\nu(s_0^x)/\|J_\nu(s_j^x)\|_2 \). \( J_\nu(s_j^x) \) = 0. Theorem 1. For \( f \in L^2(\varphi^c dx), -1 < \nu \) and \( (\nu + 1/2) \wedge 0 + 3/2 - \beta > 1/p > 1/2 - \beta - (\nu + 1/2) \wedge 0 \) it holds \( \int_0^1 |S_n(x) - f(x)| \varphi^c dx \) tends to zero for \( n \to \infty \). Theorem 2. The same result holds for Dini series. (Received July 12, 1971.)
The Fredholm spectra on tensor products.

active operators on the Banach space $x_1$ (resp. $x_2$) and on the completion of $x_1 \otimes x_2$ with respect to some crossnorm, $T_1 = A_1 \otimes I$ and $T_2 = I \otimes A_2$. If $\sigma_\Phi$ represents the (joint) Fredholm spectrum, then (1) $\sigma_\Phi(\sigma(A_1) \times \sigma(A_2)) \supseteq [\sigma_\Phi(A_1) \times \sigma_\Phi(A_2)]$ and if $f(z_1, z_2)$ is a function analytic in a neighborhood of $\sigma(A_1) \times \sigma(A_2)$, then (2) $\sigma_\Phi(f(\sigma(T_1, T_2)) \supseteq [f(\sigma_\Phi(A_1), \sigma_\Phi(A_2))] \cdot [f(\sigma_\Phi(A_1), \sigma_\Phi(A_2))]$. If $\sigma(A_1)$ and $\sigma(A_2)$ contain no Riesz pts., then equality holds in (2) and in addition $\sigma_\Phi(f(\sigma(T_1, T_2)) = \sigma(f(\sigma(T_1, T_2)))$. If for all $\lambda_1 \in \sigma(A_1)/\sigma_\Phi(A_1)$, $\lambda_1$ is a Riesz pt. of $A_1$ (e.g. normal or Riesz operators) then equality holds in (1) and (2).

(Received July 13, 1971.)

Nonsquare spaces with bases.

Let $B$ be a Banach space that is isomorphic to a uniformly nonsquare space. If $0 < 2\varphi < \varepsilon \leq 1 < \Phi$, then there are numbers $r$ and $s$ for which $1 < r < \infty$, $1 < s < \infty$ and, if $[a_i]$ is any normalized basis sequence in $B$ with characteristic not less than $\varepsilon$, then $\varphi[\sum |a_i|^s/\varepsilon] \leq \|\sum a_i e_i\| \leq \Phi[\sum |a_i|^s]/\varepsilon$, for all numbers $a_i$ such that $\sum a_i e_i$ is convergent. This also is true for any uniformly convex Banach space and for any unconditional basic subset in any nonseparable Banach space that is isomorphic to a space which is uniformly convex or uniformly nonsquare. Gurarii and Gurarii recently established the existence of $\Phi$ and $r$ for uniformly smooth spaces, and the existence of $\varphi$ and $s$ for uniformly convex spaces [Izv. Akad. Nauk SSSR Ser. Mat. 35(1971), 210-215].

(Received July 14, 1971.)

A geometric form for the linearized equations of a Hamiltonian system. Preliminary report.

The linearized equations about a solution $x(t)$ of the Hamiltonian system of differential equations $\dot{x} = A_3 H_x$ with two degrees of freedom ($A_3 = (0 1, 0 1)$, $I = (1 0)$) are obtained in natural coordinates by using the global unit tangent frame field $X_i$, $1 \leq i \leq 3$, to an energy manifold $H(x) = h$. Here $X_i = A_i N$ where $N = H_x/|H_x|$ is the unit normal and the $A_i$ are skew-symmetric orthogonal constant matrices corresponding to the quaternions $j, k$. The two linearized equations describing the behavior in the tangent subspace orthogonal to $x(t)$ separate from the other equations and can be put in the form $\dot{\alpha} = -\langle 0 1 \rangle (S_1(t) + S_2(t)) \alpha$ for $\alpha = [\alpha_1^1, \alpha_2^2]$, where $\det S_1(t) = \langle R(X_1, X_2) X_2, X_1 \rangle$ is the sectional curvature of $X_1, X_2$ in $H(x) = h$. Letting $D$ be the standard connexion in Euclidean 4-space and $LX = D_N(N)$ the Weingarten operator, then $S_2(t) = \langle LX_3, X_3 \rangle I$ and $\det(S_1(t) + S_2(t)) = \det S_1(t) + \langle LX_3, X_3 \rangle (\text{trace } L)$ where $\langle LX_3, X_3 \rangle$ is the normal curvature of $x(t)$ and trace $L$ is the mean curvature of $H(x) = h$ along $x(t)$. (Received July 14, 1971.)
Every separable Banach space $E$ can be equipped with a multiplication under which it becomes a commutative semisimple Banach algebra. One can use this algebra structure to study $E$ and its bounded linear operators. For instance, if $E$ has a Schauder basis, it has a bounded linear operator whose lattice of invariant subspaces is isomorphic to the lattice of subsets of the positive integers. The multiplication is constructed by considering a biorthogonal sequence $\{b_n, \beta_n\}$ whose sequence of linear functionals $\{\beta_n\}$ is a total subset of the dual space of $E$. It is then possible to find a sequence of nonzero scalars $\{d_n\}$ such that the bilinear function $x \cdot y$ defined on $E$ by $\beta_n(x \cdot y) = d_n \beta_n(x) \beta_n(y)$ turns $E$ into a commutative semisimple Banach algebra. The above construction depends only on the total biorthogonal sequence $\{b_n, \beta_n\}$ and hence can also be carried out in certain nonseparable spaces. Under various stronger assumptions on the biorthogonal sequence, it is possible to completely describe, respectively, the closed maximal ideals, the closed ideals, or the closed subalgebras of $E$. (Received July 19, 1971.)

We treat the problem of reaching the origin in least time within controllable control systems $\dot{x} = Ax + e_n u$, $|u(t)| \leq 1$, $e_n = (0, \ldots, 0, 1)$. The boundary of sets $R(\theta)$ reachable at time $\theta$ is $|x: T(x) = \theta|$, where $T$ is the minimal-time function (O. Hájek, "Geometric theory of time-optimal control," SIAM J. Control, to appear). Thus whenever the $T(x)$ exists, it is an exterior normal to $R(\theta)$ at $x$; and hence it describes, in a sense, the shape of $R(\theta)$. It is paradoxical that all coordinates of the gradient of $T(x)$ have the same sign: Theorem. There exists $\theta > 0$ and an open connected set $M$ such that all coordinates of the gradient of $T(x)$ are strictly positive in $M$, strictly negative in $(-1)M$; the remainder of $R(\theta)$ is nowhere dense. (Received July 19, 1971.)

We study the degenerate parabolic operator $Lu = \sum_{i,j=1}^n a^{ij}(x,t)u_{x^i x^j} + \sum_{i=1}^n b^i(x,t)u_{x^i} - cu + du$ where $a^{ij}, b^i, c, d$ are bounded, real-valued functions defined on a domain $D = \Omega \times (0,T) \subset R^{n+1}$. Assume that $L$ is parabolic. Classically, $c(x,t) \equiv 1$ or, equivalently, $c(x,t) \equiv 0 > 0$ for all $(x,t) \in \bar{D}$. We assume only that $c$ is nonnegative. We prove the standard version of the weak maximum principle. Harnack inequalities are likewise derived. Main Theorem. Assume that $a^{ij}$ is constant, the coefficients of $L$ and $f$ and their derivatives with respect to time are uniformly H"{o}lder continuous (exponent $\alpha$) in $\bar{D}$, $\bar{D}$ has sufficiently nice boundary, $c > 0$ on the normal boundary of $D$, $\psi \in \mathbb{C}^{2,\alpha}_{\bar{D}}$ and $L\psi = f$ on $\partial D = \partial (\Omega \times [0,T])$. Then there exists a unique solution $u$ of the first initial-boundary value problem $Lu = f$, $u = \psi$ on $\bar{D} + (\partial D \times [0,T])$; and, furthermore, $u \in \mathbb{C}^{2,\alpha}(\bar{D})$. All results require proofs that differ substantially from the classical ones, which depend heavily on the minimum of $c$ in $\bar{D}$. This work is contained in the author's dissertation written at Texas Tech University under the direction of Wayne T. Ford. (Received July 6, 1971.)
Let $S^*_\alpha$ be the class of regular univalent functions in $E = \{z : |z| < 1\}$ having expansion of the form $f(z) = z + a_2 z^2 + \ldots + a_n z^n + \ldots$, and satisfying $|zf'(z)/f(z)| - \alpha \leq a$, $\alpha \geq 1$. For $\alpha = \infty$, $S^*_\alpha$ coincides with the well-known class of univalent starlike functions. It is shown that the functions of the class $S^*_\alpha$ and their derivatives are uniformly bounded in $\overline{E}$, the closure of $E$. With $\alpha = \frac{1}{2}$, the following theorems have been established:

**Theorem 1.** If $f \in S^*_\alpha$ then (i) $|a_2| \leq \beta$, (ii) for $\frac{1}{2} \leq \alpha \leq 1$, $|a_n| \leq \beta/(\alpha - 1)$, $n \geq 3$, and (iii) for $n \geq 3$, $\alpha > (\alpha - 1)/2$, $|a_n| \leq n^{\beta}(2 - \beta(\alpha - 1))/n - 1$.

**Theorem 2.** If $f \in S^*_\alpha$ and $a_2, a_3$ and $a_4$ are real, then (A) $|a_4| \leq \beta/3$ for $1 \leq \alpha \leq 7/6$, (B) $|a_4| \leq (104\beta - 344(7\beta - 9)^{3/2})/324$ for $7/6 \leq \alpha \leq 95/78$ and (C) $|a_4| \leq \beta(2\beta - 1)(3\beta - 2)/6$ for $95/78 \leq \alpha \leq 3/2$. These inequalities are sharp. (A) is true even when the coefficients are complex but it has not been possible to establish (B) and (C) for this case. Some inequalities involving $a_3$ and $a_2$ have also been established. (Received July 21, 1971.)

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**71T-B215.** JAMES GUYKER, State University College of New York, Buffalo, New York 14222. Lifting normal operators commuting with a contraction.

Let $T$ be a contraction on a Hilbert space $H$ and let $X$ be a bounded operator on $H$ which commutes with $T$. It is known (B. Sz.-Nagy and C. Foiaş, "Harmonic analysis of operators on Hilbert space", American Elsevier, New York, 1970, p. 11) that there exists a unique minimal coisometry $V$ acting on a Hilbert space $K$ containing $H$ such that $T$ is the restriction of $V$ to $H$. By the lifting theorem of Sz.-Nagy and Foiaş ("Dilatation des commutants d'opérateurs", C. R. Acad. Sci. Paris 266(1968), 493-495), $X$ is the restriction of an operator $Y$ on $K$ which commutes with $V$ such that $\|Y\| = \|X\|$. In this paper we show that if $X$ is normal, then there is a unique choice of $Y$ such that $Y$ is normal and $Y$ has the same spectrum as $X$. The proof does not use the lifting theorem of Sz.-Nagy and Foiaş, but rather depends primarily on the spectral theorem for normal operators. (Received July 28, 1971.)

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**71T-B216.** PAUL WILLIG, Stevens Institute of Technology, Hoboken, New Jersey 07030. On hyperfinite W-* algebra.

A W-* algebra $R$ on separable Hilbert space $H$ is hyperfinite if there is an increasing sequence of finite dimensional *-subalgebras $R_n$ of $R$ such that $R$ is the strong closure of the union of the $R_n$. $R$ is strongly hyperfinite if the $R_n$ can be chosen to be factors of type $I_2^n$. Let $R = \int_{\Lambda} R(t) \mu(dt)$ be the direct integral decomposition of $R$ into factors. **Theorem 1.** $H = \{t \in R(t) = \{1\} \mu(A - H) = 0$. (Received July 28, 1971.)

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**71T-B217.** JOHN E. CONNETT, Northern Illinois University, DeKalb, Illinois 60115. On covering the unit ball in a Banach space.

For $E$ a normed linear space, $x \in E$, and $r \geq 0$, let $B(x, r) = \{y \in E : \|x - y\| \leq r\}$. We show that if $E$ is an infinite-dimensional Banach space, there is no covering of the unit ball $B(\overline{0}, 1)$ in $E$ of the form $\bigcup_{i=1}^{\infty} B(x_i, r_i)$, where $|r_i|$ is a sequence of positive numbers such that (a) $r < 1$, $1 \leq i < \infty$, and (b) $|r_i|$ converges to 0. (Received August 2, 1971.)
71T-B218. BRUCE R. HENRY, Syracuse University, Syracuse, New York 13210. Escape from the unit interval under the transformation $x \mapsto \lambda x(1-x)$. Preliminary report.

P. R. Stein and S. Ulam in "Nonlinear transformation studies on electronic computers," Rozprawy Mat. 39(1964), and others have studied iterations on the unit interval of the quadratic transformation $x \mapsto x' = \lambda x(1-x)$, with $3 < \lambda \leq 4$. In a population model the author has used the same map with $\lambda > 4$ in which case the unit interval no longer goes into itself. It is proved that on an $x$-set of measure 1 the iteration $x \mapsto x' \mapsto x'' \mapsto \ldots$ ultimately leaves the interval. The proof uses the parabolic structure of the function. A counterexample is given to motivate the ineffectiveness of topological arguments. (Received August 3, 1971.)


Let $(\Omega_\alpha, \Sigma_\alpha, \mu_\alpha, h_{\alpha \beta})$ be a projective system of measure spaces and let the projective limit measure $\mu = \lim_\alpha \mu_\alpha$ be countably additive. If each $T_\alpha : \Omega_\alpha \rightarrow \Omega_\alpha$ is a measure preserving transformation satisfying the consistency condition: $h_{\alpha \beta} T_\beta = T_\alpha h_{\alpha \beta}$ for $\alpha \neq \beta$, then the projective limit $T = \lim_\alpha T_\alpha$ can be defined on $\Omega = \lim_\alpha \Omega_\alpha$, and is measure preserving. If each $T_\alpha$ has the properties of ergodicity, mixing, weakly mixing, then so does $T$. To a projective system of measure preserving transformations $(\Omega_\alpha, \Sigma_\alpha, \mu_\alpha, T_\alpha, h_{\alpha \beta})$, we can associate an inductive system of algebraic systems $(\Gamma_\alpha, U_\alpha, \varphi_\alpha, h_{\alpha \beta})$, and the projective limit $T = \lim_\alpha T_\alpha$ has the inductive limit $(T, U, \varphi) = \lim_\alpha h_{\alpha \beta}(\Gamma_\alpha, U_\alpha, \varphi_\alpha)$ as a model. Conversely, every inductive system of algebraic ergodic systems $(\Gamma_\alpha, U_\alpha, \varphi_\alpha, h_{\alpha \beta})$ is an algebraic model for a certain projective system of measure preserving transformations. If each $T_\alpha$ has discrete model, then it is not known in general whether $T$ also has discrete model. However, if each $T_\alpha$ is the superposition of a continuous endomorphism and a rotation, or ergodic with discrete spectrum, or if each $T_\alpha$ has all iterates ergodic and with quasi-discrete spectrum, then so does $T$. (Received August 4, 1971.)


This paper may be regarded as an addition to papers by W. P. Coleman (Abstract 71T-B191, these Notices 18(1971), 817) and by J. R. Calder (Abstract 71T-B190, these Notices 18(1971), 817). In the context of a normed linear space $S$: Definition. The statement that the infinite point set $M$ is quasi-symmetric about the point $P$ means that if $\epsilon > 0$ then there exists a reversible transformation $T$ such that the initial set of $T$ and the final set of $T$ each consist of all but finitely many points of $M$ and if $X$ is in the initial set of $T$ then the distance from $X$ to $T(X)$ differs from twice the distance from $X$ to $P$ by less than $\epsilon$. Example. There exists a two-dimensional normed linear space containing a point set $M$, a point $P$, and a point $Q$ such that $P$ is a center of $M$, $M$ is quasi-symmetric about $Q$, $M$ is not quasi-symmetric about $P$, and $M$ is not almost symmetric about $Q$. Theorems. If $M$ is almost symmetric about $P$ then $M$ is quasi-symmetric about $P$. The following statements are equivalent: (1) no infinite point set has two centers, (2) no infinite and weakly closed point set has
two centers, (3) if $M$ is an infinite point set that is quasi-symmetric about a point $P$ then $M$ has no center distinct from $P$. (Received August 6, 1971.) (Author introduced by Professor Ben Fitzpatrick, Jr.)


**Theorem.** Let $m: R \rightarrow X$ be a finitely additive measure on a ring $R$, with values in a Banach space $X$. Then $m$ has a local control measure if and only if $m$ is locally $s$-bounded. If $m$ is countably additive then the control measure can be chosen to be countably additive. $m$ is said to be locally $s$-bounded if $m(A_n) \rightarrow 0$ for every sequence of disjoint sets $A_n \in R$ contained in a common set $A \in R$. A positive measure $\nu$ on $R$ is said to be a local control measure for $m$ if: (a) for every set $E \in R$ we have $\lim \nu(A) - 0 m(A) = 0, A \subseteq E$; (b) every $m$-negligible set is $\nu$-negligible. (Received August 10, 1971.)


Let $X$ be a real Banach space and $f$ be a $k$-set contraction ($k < 1$) which maps $X$ into $X$. Suppose that $f(\theta) = \theta$ and $f'(\theta)$ exists. Let $G$ be the closure of the set $\{ (\lambda, x) \in R \times X \mid x \neq \theta \text{ and } x = \lambda f(x) \}$. **Theorem.** Suppose that $\mu_0$ is an eigenvalue of odd multiplicity of $f'(\theta)$. Then $G$ possesses a component $C$ which contains $(\mu_0, \theta)$. This component is either unbounded (in $R \times X$) or contains $(\mu_1, \theta)$ where $\mu_1 \neq \mu_0$ is also an eigenvalue of $f'(\theta)$. This theorem is a generalization of a theorem due to P. Rabinowitz (Proc. Sympos. Nonlinear Functional Analysis, Madison, Wisconsin, 1971) and an extension of a theorem due to the author (a $k$-set contraction form of the well-known bifurcation theorem of Krasnosel’skiǐ, “Topological methods in the theory of nonlinear integral equations,” p. 196). (Received August 10, 1971.)

*71T-B223. LEROY J. DICKEY, University of Waterloo, Waterloo, Ontario, Canada, HANS-HEINRICH KAIRIES, Institut B für Mathematik, T. U. Braunschweig, 33 Braunschweig, West Germany and HERBERT S. SHANK, Department of Combinatorics and Optimization, University of Waterloo, Waterloo, Ontario, Canada. On the functional equation $f(x) = f(x/2) + f((x+1)/2)$ over $GF(p)$.

Let $p$ be a prime. In the multiplicative group of $GF(p)$, let $G$ be the subgroup generated by 2. There is a solution $f_1$ to the functional equation $f(x) = f(x/2) + f((x+1)/2)$, $f: GF(p) \rightarrow GF(p)$, which is defined by the equations $f_1(1/2) = 0$, $f_1(x+1) - f_1(x) = 1$ if $x \in C_1$ and $f_1(x+1) - f_1(x) = 0$ if $x \notin C_1$ for each coset $C_1$ of $G$. The general solution is a linear combination of these (independent) solutions. (Received August 3, 1971.)

*71T-B224. STEFAN BERGMAN, Stanford University, Stanford, California 94305. On pseudoconformal mappings of circular domains.

Let $B$ be a pseudoconformal image of a circular domain $C$, i.e., $B = \overline{f}(C)$, and let the invariant $J_B(z, \overline{z})$ not be constant inside $B$ and assume the boundary values $2/9 \pi^2$. See Bergman (2nd ed., Math. Surveys, no. 5, 1970, Chapter XI, in particular p. 182 ff). The first problem to be considered is the determination of $t =$
If \( J_C \) has an isolated critical point at \( O \), then \( t = \mathcal{T}(O) \) is a point where \( J_C \) has an isolated critical point. The center \( O \) cannot lie on a critical line of \( J_C \). Let \( s^n \) be an \((n\text{-dimensional})\) critical segment of \( J_C(z, \bar{z}) \), \( O \in s^n \). Here \( n = 2 \) or \( 3 \) and let \( N^4(s^n) \) be a neighborhood of \( s^n \) in \( B \). Suppose \( B = \mathcal{T}(C) \). If \( N^4(s^n) \) is connected and \( N^4(s^n) - t \) is disconnected, then \( t = \mathcal{T}(O) \), i.e., \( t \) is the image in \( B \) of the center \( O \). Suppose that \( B = \mathcal{T}(C) \) and \( t = \mathcal{T}(O) \). Then the representative \( R(B, t) \) is a circular domain. The pair \((v_{B}^{10}, v_{B}^{01})\) given in terms of \( K_B \) and its derivatives maps \( B \) onto the representative \( R(B, t) \), which in this case is a circular domain. Concerning \( R(B, t) \) and \((v_{B}^{10}, v_{B}^{01})\) see Bergman, p. 105, 187 ff.

(Received August 13, 1971.)

*71T-B226. JOHN J. F. FOURNIER, University of Virginia, Charlottesville, Virginia 22903. Fourier coefficients after gaps.

Let \( n_k \) be an increasing sequence of nonnegative integers and let \( \mu \) be a regular Borel measure on the unit circle \( T \). Suppose that its Fourier transform, \( \hat{\mu}(n) = \int_T e^{-in\theta} d\mu \), vanishes in a gap of length \( L_k \) just before each \( n_k \). That is, \( \hat{\mu}(n) = 0 \) whenever \( n_k - L_k \leq n < n_k \) for some \( k \). Let \( n_{k+1} - n_k \geq 1 \).

Theorem. (i) If, for some \( \delta > 0 \), \( L_{k+1} \geq \delta n_k \) for all \( k \), then \( \sum_{k=0}^{\infty} |\hat{\mu}(n_k)|^2 < \infty \). (ii) If \( L_k \to \infty \) as \( k \to \infty \), then \( \hat{\mu}(m_k) \to 0 \) and \( \hat{\mu}(n_k) \to 0 \) as \( k \to \infty \). As usual there are dual statements about interpolation by Fourier coefficients of continuous functions. Statement (i) is new whenever the sequence \( |n_{k+1} - n_k| \) is not bounded above. The proof uses the Hilbert space lemma of the author's paper on random Fourier-Stieltjes transforms (Proc. Cambridge Philos. Soc. 67(1970), 295-306). It is not known whether it also follows in (i) that \( \sum_{k=1}^{\infty} |\hat{\mu}(m_k)|^2 < \infty \). Statement (ii) follows from the argument given by Rudin (J. London Math. Soc. 32(1957), 307-311, see Theorem 4) for the case when \( \hat{\mu}(n) = 0 \) for all positive \( n \) except the \( |n_k| \). (Received August 13, 1971.)


For certain Lie groups \( G \) and compact subgroups \( K \) it is shown how the Darboux equation on \( G/K \) can be embedded in a "canonical" sequence of hyperbolic equations analogous to an EPD sequence. Canonical operations in the representation theory of \( G \) lead to canonical recurrence relations which yield formulas connecting solutions with different parameter values. This gives group theoretic meaning to some of the operations of EPD theory developed by Weinstein and others and yields new classes of equations where a parallel theory can be developed. For example if \( T = \mathbb{R}^2 \) with additive structure and \( K = SO(2) \) then \( G = T \times \gamma K \) (natural
semidirect product) gives rise to an EPD sequence. The case \( G = \text{SL}(2, \mathbb{R}) \) and \( K = \text{SO}(2) \) yields a sequence for \( m \equiv 0 \), \( w^m_{tt} + (2m+1)\coth(t)w^m_t + m(m+1)w^m = \Delta w^m \) with \( w^m(0,p) = f(p) \) and \( w^m_t(0,p) = 0 \) where \( p \in G/K \) and \( \Delta \) is the Laplace-Beltrami operator. One proves that if the function \( (\Delta - m(m+1))f \equiv 0 \) with \( f \in C^\infty \) then \( w^m(t,p) \) is monotone nondecreasing in \( t \) and convex in a function \( \rho(t) \) (cf. A. Weinstein, Ann. Mat. Pura Appl. 43 (1957), 325-340 and 60(1962), 87-91; R. Carroll, Ann. Mat. Pura Appl. 56(1961), 1-31; and B. Fusaro, J. Math. Mech. 18(1969), 603-606). (Received August 16, 1971.)


Let \( x \) denote a point in \( n \equiv 2 \) dimensional Euclidean space with rectangular coordinates \((x_i)^{1 \leq i \leq n}\). Let \( \theta_1 \) be the angle between the \( x_1 \) axis and \( x \). Given \( \alpha \), \( 0 < \alpha < \pi \), put \( K_\alpha = \{ x : 0 \leq \theta_1 < \alpha \} \). Let \( \rho \) denote the order of a positive harmonic function in \( K_\alpha \) with boundary values zero on \( \partial K_\alpha \). Let \( u \) be subharmonic in \( K_\alpha \) and put \( M(r) = \sup_{|x|=r} u(x) \), \( 0 < r < \infty \). For given \( \lambda \), \( 0 < \lambda < 1 \), the authors consider subharmonic functions \( u \) in \( K_\alpha \) which satisfy the boundary condition (i) \( \lim \sup_{x \to w} u(x) \leq C_\alpha M(|w|) < \infty \), when \( w \in \partial K_\alpha \). Under the assumption (i), they show that one of the following statements is true: (a) \( u \equiv 0 \), (b) \( \lim_{r \to 0} r^{-\lambda \rho} M(r) = \infty \), (c) \( |x|^{-\lambda \rho} u(x) \) has a limit as \( |x| \to \infty \) in the sense of Ahlfors-Heins (Ann. of Math. (2) 50(1949), 341-346) and Hayman (J. Math. Pures Appl.(9) 35(1956), 115-126). This theorem is an \( n \)-dimensional analogue of theorems by the authors in the plane (Essen, Trans. Amer. Math. Soc. 142(1969), 331-344, and Lewis, to appear in Trans. Amer. Math. Soc.). Several estimates of Azarin (Amer. Math. Soc. Transl. (2) 80(1969), 119-138) are also used in the proof. (Received August 16, 1971.)


An algebra \( \mathcal{A} \) of bounded linear operators in a complex \( B \)-space \( X \) has finite strict multiplicity if (by definition) there exist \( x_1, \ldots, x_n \in X \) such that \( X = \text{lin span} \{ Ax_j : A \in \mathcal{A}, \ j = 1, \ldots, n \} \). Theorem. Let \( \mathcal{A} \) be a strongly closed algebra of finite strict multiplicity containing the identity; let \( \mathcal{B} \) be a transitive algebra (i.e., \( \mathcal{B} \) has no nontrivial closed invariant subspaces) containing \( \mathcal{A} \) and let \( T \) be a densely defined linear map commuting with \( \mathcal{A} \). Then: (1) The only dense linear manifold invariant under \( \mathcal{A} \) is \( X \); (2) \( \mathcal{B} \) is equal to the algebra of all bounded linear operators in \( X \); (3) \( T \) is everywhere defined and bounded; (4) if range(\( T \)) is dense, then range(\( T \)) = \( X \); (5) If \( \lambda \) is any point of the spectrum of \( T \) and \( T - \lambda \neq 0 \), then either kernel(\( T - \lambda \)) or closure range(\( T - \lambda \)) is a nontrivial hyperinvariant subspace of \( T \). Complete proofs of these results will appear in Rev. Un. Mat. Argentina. (Received August 17, 1971.)

71T-B230. WITHDRAWN.
A closed symmetric operator $A$ in a Hilbert space is said to be simple if there is no nontrivial reducing subspace in which $A$ is selfadjoint. It is shown that if $A$ has no eigenvalues and has a selfadjoint extension with a discrete spectrum, then $A$ is simple. It follows that the minimal closed symmetric operator $T_0$ in $L^2(a, b)$ determined by a formally selfadjoint linear ordinary differential operator $L$ on $(a, b)$ is simple if $L$ is regular at both ends. If $E(\lambda)$ denotes the set of solutions of $A^*u = \lambda u$, M.G. Kreín has shown that $A$ is simple if and only if $f$ perpendicular to $E(\lambda)$ for all nonreal $\lambda$ implies $f = 0$. Using this characterization, it is shown that $T_0$ is simple if $L$ is regular or quasiregular at one end. The proof uses a generalized resolvent and spectral function of $T_0$ and the Stieltjes inversion formula. Finally it is shown that if $L$ has order $n = 1$ or if $L$ has real coefficients and order $n = 2$, then $T_0$ is simple whatever the behavior of $L$ at the ends of $(a, b)$. (Received August 18, 1971.)

Duality of functors in the category of Banach spaces.

The duality of functors is defined in the category $B$ of all Banach spaces and linear transformations of norm not exceeding one as in Mityajin and Svarc (Russian Math. Surveys 19(1964)): $D$ is a contravariant endofunctor on the category $B^B$ of functors from $B$ to $B$; for $F \in B^B$, $X \in B$, $DFX = [F, \Sigma_X]$, where $\Sigma_X$ is the tensoring functor, $\Sigma_XY = X \otimes Y$, and where the set of natural transformations from $F$ to $\Sigma_X$ is viewed as an element of $B$ in a natural way. $F$ is said to be reflexive if $F = D^2F$ (isometric isomorphism); $F$ is said to be finite dimensional if $F = \Sigma_A$ (isomorphism), where $A$ is a finite dimensional Banach space. The question, "Are all finite dimensional functors reflexive?" is posed in Mityajin and Svarc. We prove Theorem. A finite dimensional functor $F$ is reflexive if and only if $F(i_X)$ is an extremal monomorphism for every $X \in B$, where $i_X: X \to X^{**}$ is the canonical embedding. Corollaries and remarks follow. (Received August 19, 1971.)

Simply invariant subspaces on finite bordered Riemann surfaces.

Let $R$ be a finite open Riemann surface with boundary $\partial R$ consisting of a finite nonzero number of disjoint analytic Jordan curves. Let $A$ be the algebra of functions continuous on $\partial R$ and possessing continuous extensions to $\bar{R} = R \cup \partial R$ which are holomorphic on $R$. Fix a point $a \in R$ and let $e_a$ denote evaluation on $A$ at $a$. If $m$ is a representing measure for $e_a$ and $X \subset (\text{complex}) L^p(m)$, let $[X]_p$ denote the $L^p$ closure of $X$. For $1 \leq p < \infty$ define $H^p(m) = [A]_p$ and let $A_0 = \{f \in A | f(a) = 0\}$. A (closed) subspace $M$ of $L^p(m)$ is called simply invariant if $[A_0M]_p \subset M$. By adapting a method of Sarason, we obtain a fairly simple proof of the following Classification Theorem. Any simply invariant subspace of $L^p(m)$, $1 \leq p < \infty$, is of the form $uH^p(m)$, where $u \in \{f \in L^\infty(m) | 1/f \in L^\infty(m)\}$. We then use an invariant subspace approach to present an elementary proof of Forelli's theorem that if $f$ is an extreme point of the unit ball of $H^1(\mu)$, where $\mu$ is harmonic measure for $a$, then the codimension of $[A\mu]_1$ in $H^1(\mu)$ does not exceed one-half of the first Betti number of $R$. (Received August 2, 1971.)
Let $\alpha$ and $\beta$ be positive real constants, and let $g(n)$ be a real-valued function over the nonnegative integers. Consider the new function $M(n)$ defined as follows: $M(0) = g(0)$ and $M(n+1) = g(n+1) + \min_{0 \leq k < n} \alpha M(k) + \beta M(n-k))$. Asymptotic formulas are derived for $M(n)$ in the cases where $g(n) = \delta_n^\alpha g(n) = n$, and $g(n) = n^2$. The case $g(n) = n$ is of particular interest and the remainder of this notice is restricted to consideration of this case. Given $\alpha$ and $\beta$ with $\alpha > 1$, $\beta > 1$, there exist positive constants $c_1$ and $c_2$ such that $c_1 n^{(1+1/\gamma)} < M(n) < c_2 n^{(1+1/\gamma)}$ where $\gamma$ is the positive constant satisfying $(1/\alpha)^\gamma + (1/\beta)^\gamma = 1$. If in addition, $\log \alpha/\log \beta$ is irrational, then $M(n) \sim cn^{(1+1/\gamma)}$ for some positive $c$. When $\log \alpha/\log \beta$ is rational, in some cases $\lim(M(n)/n(1+1/\gamma))$ exists, and in other cases it has been shown not to exist. The behavior of $M(n)$ is intimately related to that of a number-theoretic function $H(z)$ defined as follows: Let $S$ be the smallest multiset satisfying the following: $S = \{1\} \cup \{\alpha s + 1, \beta s + 1 : s \in \mathbb{S}\}$. Thus when $\alpha = 2, \beta = 3, S = \{1, 3, 4, 7, 9, 10, 13, 15, 19, 21, 22, 27, 28, 31, 33, \ldots\}$. $H(z) = \text{card}(s \in S$ and $s \equiv z)$. For integers $\alpha, \beta, \alpha > 1, \beta > 1$, we know that when $\log \alpha/\log \beta$ is irrational, $\lim H(z)/z^\gamma = c > 0$, and when $\log \alpha/\log \beta$ is rational, $\lim H(z)/z^\gamma$ does not exist. (Received August 20, 1971.)


We show $L(1, \infty) \neq \{0\}$. We also show that for $1 < p < \infty$, unless the underlying measure space consists of finitely many atoms, $L(p, \infty)$ is strictly larger than $L(p', \infty)$, where $1/p + 1/p' = 1$. For the definitions of these spaces, and for related results, see R. A. Hunt, “On $L(p,q)$ spaces,” L’Enseignement Math. 12(1966), 249–276. (Received August 20, 1971.)


Let $f(x) \in \text{Lip } \alpha$ for $0 < \alpha < 1$ in the range $(-\pi, \pi)$ and periodic with period $2\pi$, outside this range. Also let (*) $f(x) \sim \sum_{\nu=0}^\infty \lambda_\nu(x)$ be the Fourier series associated with the function $f(x)$. If we define a generating sequence $[p_n]$ such that it is nonnegative, nonincreasing and $P(n) = p_0 + p_1 + \ldots + p_n \rightarrow \infty$ as $n \rightarrow \infty$, then we can define the trigonometric polynomial $T_n(x)$ by $T_n(x) = \sum_{k=0}^n (P_k S_{n-k}(x)/P(n))$ where $S_n(x)$ is the partial sum of the series (*). We consider the norm as the maximum modulus (for the space $C^*$) and let the degree of approximation be given by $E_n^\alpha(f) = \min_{T_n} \|f - T_n\| = \min_{T_n} \max_{x} |f(x) - T_n(x)|$. Some of the recent related results are due to G. Alexits and L. Lienard (Acta Math. Acad. Sci. Hungar. 16(1965), 27–32) and G. Alexits and D. Karlit (Acta Math. Acad. Sci. Hungar. 16(1965), 43–49). The following theorem is proved. Theorem. If $f(x)$ is periodic and belongs to the class $\text{Lip } \alpha$, $0 < \alpha \leq 1$, and if $\int_1^\infty (P(y)/y^{1+\alpha}) dy = O(P(n)/n^\alpha)$, where the sequence $P(n)$ is defined above, then $E_n^\alpha(f) = \min_{T_n} \|f - T_n\| = O(1/n^\alpha)$. (Received August 24, 1971.)
I = [a, b] a closed interval of reals. \((I, \Sigma, \mu)\) a positive measure space with \(\sigma\) field \(\Sigma\) containing Borel sets in \(I\). \(\nu = \mu \times \mu\) a product measure on \(I \times I\). Define \(S = \{(x, y) : x, y \in I, x \neq y\} \subset I \times I\) and for \(f \in L^\infty = L^\infty(I, \Sigma, \mu)\), the set of \(\mu\) essentially bounded real functions on \(I\), let \(S_f = \{(x, y) \in S : f(x) > f(y)\} \subset I \times I\). Define \(S = \{(x, y) : x, y \in I, x \leq y\} \subset I \times I\). Let \(M \subset L^\infty\) the set of almost monotonic (nondecreasing) functions i.e. all \(f \in L^\infty\) for which \(\nu(S_f) = 0\). Problem. For \(f \in L^\infty\) find \(g \in M\) such that \(\|f - g\|_w = \inf_{h \in M} \|f - h\|_w\). Here for \(f \in L^\infty\), \(\|f\|_w = \sup_{x \in I} |w(x)||f(x)|\) where \(w \in L^\infty\), \(w \geq 0\) \(\mu\) a.e. is given. Theorem. \(f \in L^\infty - M\). Then \(0 < \theta \equiv \nu\) ess sup \((x, y) \in S\{(w(x)w(y))/(w(x) + w(y))\}\) \((f(x) - f(y)) = \min_{h \in M} \|f - h\|_w\). Let \(g(x) = \mu\) ess sup \(f(x) = \mu\) ess sup \((w(x)w(y))/(w(x) + w(y))\) \((f(x) - f(y))\) then \(G \in M, G \equiv g\mu\) a.e. & \(\|f - g\|_w = \|f - \overline{g}\|_w = \min_{h \in M} \|f - h\|_w\). Further, for \(g \in M, \|f - g\|_w = \min_{h \in M} \|f - h\|_w\) holds if & only if \(g \equiv g\equiv g\mu\) a.e. Theorem. When \(w = 1\) there exists an operator \(T : L^\infty - M\) such that \(\|f - Tf\|_w \leq \|f - h\|_w\) all \(h \in M\) & \(\|Tf - Tk\|_w \leq \|f - k\|_w\) where \(f, k \in L^\infty\). When \(\mu = \mu_0\) where \(\mu_0\) is a measure for which \(\mu_0(x) = 1\) all \(x \in I\), \(L^\infty(I, \Sigma, \mu_0)\) becomes the set of bounded functions on \(I\) & \(M\) the set of nondecreasing functions on \(I\). Also \(\|f\|_w = \sup_{x \in I} |w(x)||f(x)|\). For this case we have the Theorem. \(f, w\) are continuous & \(f \notin M\). Then \(g, g\) are continuous & there exists an infinitely differentiable function \(g \in M\) such that \(\|f - g\|_w = \min_{h \in M} \|f - h\|_w\). (The paper discusses several other results.) (Received August 25, 1971.)

71T-B238. CLIFFORD V. COMISKY, Naval Electronic Laboratory Center, San Diego, California 92152. Weakly compact multipliers.

Let \(A\) be a Banach algebra with bounded approximate unit, and let \(W^*\) be both a dual space and a Banach \(A\)-module. Let \(T\) be a multiplier (module homomorphism) from \(A\) to \(W^*\). Theorem. If \(T\) is weakly compact, it is of the form \(a \otimes w^* a\) for some \(w^*\) in \(W^*\). On the other hand, an operator of that form is not, in general, weakly compact. (Received August 27, 1971.)

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71T-B239. MICHAEL COWLING, Australian National University, Canberra, A.C.T., 2601, Australia. Extension of multipliers by periodicity.

The following theorems, which generalise results of de Leeuw, Lohoue, and Saeki about multipliers of type \((p, p)\), are proved. Let \(\Gamma_0\) be a closed subgroup of the LCA group \(\Gamma\), and \(\pi\) be the canonical mapping of \(\Gamma\) onto \(\Gamma/\Gamma_0\). The first two theorems state that if \(1 \leq p < q \leq \infty\) and \(\varphi\) is a Fourier multiplier of type \((p, q)\) on \(\Gamma/\Gamma_0\), then (i) if \(\Gamma_0\) is compact, \(\varphi \circ \pi\) is a Fourier multiplier of type \((p, q)\) on \(\Gamma\), and (ii) if \(\Gamma_0\) is not compact, then \(\varphi \circ \pi\) is a Fourier multiplier of type \((p, q)\) on \(\Gamma\) only if \(\varphi\) is null. The third theorem states that if \(\Gamma\) is discrete, \(1 \leq q < p \leq \infty\), and \(\varphi\) is a Fourier multiplier of type \((p, q)\) on \(\Gamma/\Gamma_0\), then \(\varphi \circ \pi\) is a Fourier multiplier of type \((p, q)\) on \(\Gamma\). (Received August 27, 1971.) (Author introduced by Dr. Garth I. Gaudry.)
Let $u = (u_1, \ldots, u_n)$ be a real vector-valued function defined on $\Omega$, a bounded subset of $\mathbb{R}^n$. Denote $D_j = \partial/\partial x_j$ and let $p = n/(n-1)$. It is frequently necessary in the theory of elasticity to estimate the norm of $u$ (in some sense) in terms of norms of linear combinations of the $D_j u_j$. Two versions of an inequality related to Korn's inequality and a Sobolev inequality are proved. Theorem 1. For all $u$ in $H^1_0(\Omega)$, \( \|u\|_{L^p(\Omega)} \leq C \sum_{i,j=1}^n \|D_i u_j + D_j u_i\|_{L^1(\Omega)}. \) Theorem 2. Let $\Omega$ be a convex subset of $\mathbb{R}^2$ with piecewise $C^1$ boundary. Then for all $u$ in $H^{1,2}(\Omega)$ such that $\int_{\Omega} u_1 = \int_{\Omega} u_2 = 0$, there exists a function $g(x)$ equal to $\frac{1}{2} \int_{\partial K} u$ at each point $x$ in $\Omega$ such that \( \|u\|_{L^2(\Omega)} \leq C \left( \|D_1 u_1\|_{L^1(\Omega)} + \|D_2 u_2\|_{L^1(\Omega)} + \|D_1 u_1 + g D_2 u_2\|_{L^1(\Omega)} \right). \) (Received August 30, 1971.)

Equivalence of integrals.

Suppose $S$ is a linearly ordered set, $R$ is the set of numbers, $N$ is the set of positive numbers, $G$ and $F$ are functions from $S \times S$ to $R$ and $R \times R$ to $N$, respectively. Definitions of most of the symbols and notations used may be found in "Integral equations and product integrals", Pacific J. Math. 16(1966), by B. W. Helton.

Theorem 1. The following are equivalent: (a) $\int_S \alpha^b G$ exists and $G$ is of controlled variation on $[a,b]$ and (b) $\int_a^b G$ exists and $G$ is almost additive on $[a,b]$.

Theorem 2. If $G$ is a function from $S \times S$ to $R$ such that $\int_a^b (G)$ exists and for each function $f$ from $S$ to $R$ of bounded variation on $[a,b]$, $\int_a^b \alpha^b f G$ exists, then $\int_a^b \alpha^b f G = 0$. Theorem 3. If $\alpha = (\alpha_1, \ldots, \alpha_n)$ with $\sum_{j=1}^n \alpha_j \leq r$, then $\int_X K \in H(K)$.

Theorem 4. If $H$ is a function from $R \times R$ to $N$, $H$ is in $R \times R$ and $[\alpha, \beta]$ is in $R \times R$ then the following are equivalent: (1) $\int_a^b \alpha^b F = 0$ and (2) if for each $[x,y]$ of $[a,b]$, $\int_x^y F$ exists or $\Pi^y(1+F)$ exists, then the other exists and $\int_x^y F = \log \Pi^y(1+F)$. (Received September 1, 1971.)
Suppose we embed a real k-dimensional $C^0$ manifold $M^k$ in $\mathbb{C}^n$. If $M^k$ is locally generic at $p \in M^k$ and the Levi form at $p$ is nonzero, we know that we can extend all holomorphic functions on $M^k$ to holomorphic functions on a manifold of real dimension $k + 1$.

**Theorem.** If $M^k$ is locally C-R at $p \in M^k$ and the Levi form at $p$ is nonzero, then $M^k$ is extendible to a set containing a manifold of real dimension $k + 1$.

If $M^k$ is locally C-R at $p \in M^k$ and the Levi form at $p$ is nonzero, then $M^k$ is not locally holomorphically convex at $p$.

**Theorem.** If $M^k$ is locally C-R at $p \in M^k$ and the Levi form at $p$ is nonzero, then $M^k$ is not locally holomorphic at $p$.

The manifold $M^k$ is locally holomorphic at $p$ if and only if $M^k$ is locally C-R. (Received September 2, 1971.)

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**71T-B244.** JON C. HELTON, Arizona State University, Tempe, Arizona 85281. Some interdependencies of sum and product integrals. Preliminary report.

In the following all functions are from $\mathbb{R} \times \mathbb{R}$ to $\mathbb{R}$, where $\mathbb{R}$ denotes the real numbers, and notation and definitions are the same as used by B. W. Helton [Pacific J. Math. 16(1966), 297-322, and Proc. Amer. Math. Soc. 23(1969), 493-500].

**Theorem.** If $|G| < 1 - \beta$ on $[a, b]$ for some $\beta > 0$ and $\int_a^b G^2$ exists, then $\int_a^b G$ exists iff $\pi^b_a (1 + G)$ exists and is not zero.

**Theorem.** If $\pi^b_x (1 + G) = \exp \int_x^b G$ for $a \leq x < y \leq b$, then $\int_a^b G^2 = 0$.

**Theorem.** If $|G| < 1 - \beta$ on $[a, b]$ for some $\beta > 0$, $\int_a^b G$ exists and $\int_a^b G^2$ exists, then $G \in \mathcal{C}^0[a, b]$.

**Theorem.** If $\pi^b_a (1 + G)$ exists and is not zero and $\int_a^b G$ exists, then $G \in \mathcal{C}^0[a, b]$. The first theorem is an extension of a result of Davis and Chatfield [Proc. Amer. Math. Soc. 25(1970), 743-747]. (Received September 2, 1971.)

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Under consideration are various dual pairs in which one component is a space of measures, e.g., $M(G)$ where $G$ is a l.c.t.a.g., or $M(X)$ where $X$ is a l.c.h.s. or a normal space. In each case, the basic space is assumed to be $\sigma$-compact. Under investigation are various topologies associated with these dual pairs, mainly $\sigma, \beta,$ and $\tau$. The following set of theorems is only a sample.

**Theorem 1.** $(M(G), \sigma(M(G), [\Gamma]))$ is metrizable $\sigma = \Gamma$ is countable.

**Theorem 2.** Consider any dual pair, $(X, Y)$, $\beta X$ and $Y$ are n.v.s. and $\gamma$ elements of $X \times Y$, $(x, y) : |\langle x, y \rangle| \leq \|x\| \|y\|$. Then, $(X, \sigma(X, Y))$ is barrelled $\sigma \dim X$ is finite.

**Theorem 3.** Consider the case: $G$ is compact and metrizable. Then, $(M(G), \sigma(M(G), [\Gamma]))$ is complete $\sigma \dim M(G)$ is finite. **Theorem 4.** Consider any n.v.s., $X$. Then, $(X, \sigma(X, \widehat{X}))$ is semi-Montel $\sigma X$ is reflexive.

**Theorem 5.** Consider the case: $X$ is normal and the field of scalars is $C$. Then $(M(X), \sigma(M(X), \overline{M(X)}))$ is semi-Montel $\sigma \dim M(X)$ is finite.

**Theorem 6.** Consider the case: $X$ is a l.c.h.s. Then, $M(X)$ is $\sigma(M(X), C(X))$-closed. Questions on relative compactness have also been considered. (Received September 2, 1971.)
Here is a network whose output cells fire in response to arbitrary learnable classes of spatial patterns, which bias network response when presetting cells are active. Terminology is as in Studies in Appl. Math. 49(1970), 135-166 and J. Theoret. Biol. 27(1970), 291. The set \( v_1 = \{v_j\} \) contains \( n \) outstar source cells that send axons to the set \( v_2 = \{v_j\} \) of \( m \) cells, \( i = 1, \ldots, n, j = n + 1, \ldots, n + m \). The sets \( v_3 = \{v_{j+m}\} \) and \( v_4 = \{v_{j+2m}\} \) also contain \( m \) cells. The axons \( v_{j+m} - v_j \) and \( v_{j+m} - v_{j+2m} \) are excitatory. The axons \( v_j - v_{j+2m} \) are inhibitory. The \( v_{j+m} - v_{j+2m} \) excitatory signal has larger absolute value than the net \( v_{j+m} - v_j - v_{j+2m} \) inhibitory signal. \( v_4 \) is a high-band filter for the output cell \( v_5 = v_{n+3m+1} \). Spiking thresholds of \( v_2 - v_4 \) and \( v_4 - v_5 \) axons are zero. \( v_5 \) is an outstar source for motor control cells \( v_6 \). The spatial pattern \( \theta_1 \) learned from \( v_2 \) by \( v_1 \) presets the network if only \( v_1 \) in \( v_1 \) is active so that only \( \theta_1 \) emitted by \( v_3 \) fires \( v_5 \). A cerebellar analogy is possible; e.g., \( v_1 - v_2 \) axons = parallel fibers, \( v_2 \) = Purkinje cells, \( v_3 - v_2 \) axons = climbing fibers; \( v_4 \) = nuclear cells. (Received May 24, 1971.)

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Every wave function \( W(\lambda+i\theta) \) (see Abstract 71T-B149, these Notices 18(1971), 651 for definitions) defines an e-m field by means of the Hertz vector \( H = (0, 0, W) \). For example, the e-m field defined by \( W = \text{Re}(\lambda+i\theta) = \lambda \) is the field of a moving point charge with uniform velocity \( \sigma = c/\cosh \alpha < c \) if \( \lambda = \lambda_\beta \) or \( \sigma = c/\cos \alpha > c \) if \( \lambda = \lambda_c \). Let \( u(\lambda+i\theta) \) and \( v(\lambda+i\theta) \) be two wave functions. Then \( f(u, v) \) is also a wave function for an arbitrary (differentiable) function \( f \). The e-m field defined by \( f(u, v) \) will be called an e-m structure. If the structure \( f[u(\lambda+i\theta), v(\lambda+i\theta)] \) is disturbed so that \( v(\lambda+i\theta) \) becomes \( v(\lambda+i\theta') \), \( \theta' \neq \theta \), which physically means that the wave \( v \) is rotating in angle of \( \theta' - \theta \) about the \( z \)-axis, then--generally--\( f[u(\lambda+i\theta), v(\lambda+i\theta')] \) is no more a wave function. But there are appropriate parameters \( \beta \) and \( \gamma \) so that if \( \alpha \) which appears in the expression of the two \( \lambda \)'s is replaced by \( \beta \) in the first \( \lambda \) and by \( \gamma \) in the second \( \lambda \), then \( f[u(\lambda_\beta+i\theta), v(\lambda_{\gamma+i\theta'})] \) is a wave function which defines an e-m structure. The two components \( u \) and \( v \) of this disturbed structure have no common velocity and we say that the disturbed structure is disintegrated. (Received July 19, 1971.)

Implicit Runge Kutta methods for second kind Volterra integral equations.

Implicit Runge Kutta methods for ordinary differential equations are generalized to Volterra integral equations of the second kind, \( y(t) = g(t) + \int_0^t K(t, s, y(s))ds, 0 \leq s \leq t \leq T \). Two classes of methods, one of which requires values of \( K(t, s, y(s)) \) outside the region \( 0 \leq s \leq t \leq T \) are derived. Every method in each class corresponds to an interpolatory quadrature formula. Theorem 1. The implicit Runge Kutta methods are convergent of order \( n \), where \( n \) is the degree of precision of the associated quadrature formula.

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*71T-C20. AVRAHAM UNGAR, Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario, Canada. Fission of electromagnetic structures. Preliminary report.

Theorem 2. The implicit Runge Kutta methods are strongly stable. (Received August 10, 1971.) (Authors introduced by Dr. Garth I. Gaudry.)

*71T-C22. OSCAR H. IBARRA, Department of Computer Science, University of Minnesota, Minneapolis, Minnesota 55455. A note concerning nondeterministic tape complexities.

Let $L(n)$ be a monotonically increasing computable function from positive integers into positive integers (called a tape function), and let $\mathcal{L}(L(n))$ be the class of all sets accepted by $L(n)$-tape bounded nondeterministic Turing machines. $L(n)$ is said to be constructable (respectively, fully constructable) if there is a $L(n)$-tape bounded deterministic Turing machine $U$ which halts for all inputs and which has the property that for each $n$, $U$ uses exactly $L(n)$ storage cells in its computation for at least one input (respectively, for all inputs) of length $n$. The following result is proved: Theorem. Let $L_1(n)$ and $L_2(n)$ be tape functions with $L_1(n)$ fully constructable and $L_2(n) \equiv \lceil \log_2 n \rceil \land (x)$ is the least integer $\leq x$. Then $\mathcal{L}(L_1(n))$ contains a set not in $\mathcal{L}(L_2(n))$ if the following conditions are satisfied: (1) There exist $k \geq 2$ and tape functions $f_i(n) \equiv n (1 \leq i \leq k)$ such that $L_2(f_i(n))$ is fully constructable, (2) $\mathcal{L}(L_1(f_i(n)))$ contains $\mathcal{L}(L_2(f_{i+1}(n)))$ for $1 \leq i < k - 1$, and (3) $L_1(f_k(n)) \equiv \lceil \log_2 n \rceil$ is constructable and $\inf_{n=\omega} (L_2(f_{i+1}(n)))^2 / L_1(f_k(n)) = 0$. Interesting corollaries arise. For example, it follows that $\mathcal{L}(n^{k+1})$ properly contains $\mathcal{L}(n^{k} \lceil \log_2 n \rceil)$. (Received August 13, 1971.) (Author introduced by Professor J. B. Rosen.)

**Geometry**

71T-D21. JOHN DeCICCO, Illinois Institute of Technology, Chicago, Illinois 60616 and ROBERT V. ANDERSON, Université du Québec à Montréal, Montréal, Québec, Canada. Concerning the Ptolemy stereographic projection of a surface applicable to a sphere.

There exists a Ptolemy stereographic projection $T$, which is conformal, upon a Euclidean plane $\Pi$, of a surface $\Sigma$ which is applicable to a sphere, such that its system of $\omega^2$ geodesics $C$ is represented on $\Pi$ by the system of $\omega^2$ circles $C_1$ and the proper pencil of $\omega^1$ straight lines $L$, with proper vertex at the origin $0$, which depicts the $\omega^1$ meridians of $\Sigma$, with the property that every circle $C_1$ of the system and every straight line of the proper pencil is invariant under the Moebius transformation $R$ defined by the complex equation: $Z = -1/Z$.

Every simple family of $\omega^1$ curves $C$ which is both parallel and isothermal on $\Sigma$ is pictured in $\Pi$, by $T$, either by the pencil of $\omega^1$ concentric circles $x^2 + y^2 = \text{constant}$, which represents its $\omega^1$ circles of latitude, or else, by an elliptic pencil of $\omega^1$ circles $C$ whose two distinct foci $F$ and $F'$ correspond by the Moebius transformation $R$. Consequently, on a surface $\Sigma$ which is applicable to a sphere there is an aggregate of $\omega^2$ simple families of $\omega^1$ curves $C$, each of which is both parallel and isothermal. (Received May 14, 1971.)

71T-D22. JAYME MACHADO CARDOSO, Universidade de Campinas, Campinas, São Paulo, Brazil. Ellipse as homologous of the circle. $\Pi$.

Let $\Delta$ be an ellipse whose center is at the point $O$; $a$ and $b$ the lengths of its semi-axes. Let $\Gamma$ be a circle which has its center in the point $C$, different from $O$, and that is carried into $\Delta$ by an affinity (planar
homology whose center is the point at infinity on the line OC). Let $\Gamma'$ be a circle whose center is at the point O and whose radius is $r = OC$. Then it is shown that $\Delta$ can be transformed, by affinity, in every circle $\Gamma_i$ whose center is on $\Gamma'$ and is tangent to an ellipse $\Delta'$ with center at the point O. The semi-axis major (minor) of $\Delta'$ lies along the semi-axis minor (major) of $\Delta$ and its length is $r + a(r + b)$. Moreover if there are nonparallel common tangents to $\Gamma_i$ and $\Delta$ then they meet in points of an ellipse $\Delta''$ such that $\Delta$ and $\Delta''$ are confocal conics. (Received August 10, 1971.)


It was shown in two previous papers ("Notes on inequalities involving triangles or tetrahedrons," Publ. Electrotehn. Fak. Ser. Mat. Fiz. Univ. Beograd No. 330-337(1970), 1-15 and "Asymmetric triangle inequalities," Ford Motor Company Preprint, April 26, 1971; Publ. Electrotehn. Fak. Ser. Mat. Fiz. Univ. Beograd (to appear), that a large number of triangle inequalities could be obtained as special cases of a nonnegative form which in symmetric form is given by $x^2 + y^2 + z^2 \equiv (-1)^n + 1[2yz \cos nA + 2zx \cos nB + 2xy \cos nC]$, where $x, y, z$ are real numbers, $n$ an integer and $A, B, C$ are angles of a triangle. If $|x|, |y|, |z|$ do not form a triangle, there is strict inequality. Here, sharp bounds are obtained for these cases for the form on the r.h.s. as well as other inequalities related to the above form for certain nonintegral $n$. Additionally, other nonnegative forms and their associated triangle inequalities are considered. Finally, elementary solutions are given for the plane case of two problems of extremum simplexes recently treated by Slepian and Ali. These lead to two triangle inequalities relating the elements of two triangles. (Received June 7, 1971.)

*71T-D24. A BRAHAM BERMAN, Centre de Recherches Mathématiques, Université de Montréal, Montréal 101, Québec, Canada. Consistency of linear inequalities over sets.

Sufficient and necessary and sufficient conditions for the equivalence of the statements (I) the system, $b - Ax \in T, x \in S,$ is consistent, (II) $y \in T^*, A H y \in S^* \equiv Re(b, y) \geq 0$ are given in terms of the sets $S$ and $T$ and the matrix $A$. (Received July 8, 1971.)


Let $M^n$ be an $n$-dimensional manifold immersed in an $(n+p)$-dimensional Riemannian manifold $M^{n+p}$ and let $(h^\alpha_{ij})$ (i,j=1,...,n; $\sigma = n+1,...,n+p$) be the second fundamental form of the immersion. The normal curvature is defined to be $\sum_{\alpha, \beta, i, j, k} (h^\alpha_{ik} h^\beta_{jk} - h^\alpha_{jk} h^\beta_{ik})^2$. The following theorems are obtained. Theorem 1. The complex projective line and the complex quadratic in $CP^2$ are the only compact Riemann surfaces in complex projective space with constant normal curvature. Theorem 2. Let $M$ be a complex submanifold of a $K$ähler manifold. The normal curvature vanishes at $p \in M$ if and only if the second fundamental form vanishes at $p$. (Received July 26, 1971.)
A variational problem for submanifolds of Euclidean space.

Let $M^n$ be a compact differentiable manifold and $R^{n+k}$ Euclidean space. An immersion $\psi : M^n \to R^{n+k}$ induces a Riemannian metric on $M^n ; M^n$ with this Riemannian metric is denoted by $M^n_\psi$. Let $\overline{x}$ denote the positions vector in $R^{n+k}$ and $\overline{x}_c$ the center of mass of $M^n_\psi$ in $R^{n+k}$. i.e., $\overline{x}_c = V^{-1} \int_{M^n_\psi} \overline{x} dv$ where $V = \int_{M^n_\psi} dv$ and $dv$ is the volume element of $M^n_\psi$. A necessary and sufficient condition is given for an immersion $\phi : M^n \to R^{n+k}$ to be a stationary immersion for $J = \int_{M^n_\psi} (\overline{x} - \overline{x}_c, \overline{x} - \overline{x}_c) dv$ subject to the side condition $V = \int_{M^n_\psi} dv = a$ fixed constant. In particular minimal submanifolds of spheres satisfy this condition. (Received July 26, 1971.)

Reducibility of isometric immersions. Preliminary report.

An isometric immersion $f$ from a riemannian product $M_1 \times M_2$ into $N$-dimensional euclidean space $E^N$ is said to be reducible if there is an orthogonal product decomposition $E^N = E_1^N \times E_2^N$ together with isometric immersions $f_1 : M_1 \to E_1^N$ and $f_2 : M_2 \to E_2^N$ so that $f = f_1 \times f_2$. We will say that a riemannian manifold $M$ satisfies condition $A(p)$ at a point $m$ if there are vectors $u, v$ in the tangent space $V$ to $M$ at $m$ so that $R(u, v)$ has rank at least $2p$, where $R : V \times V \to V$ is the Riemann-Christoffel curvature operator.

Reducibility Theorem. For $i = 1, 2$, let $M_i$ be a connected riemannian manifold which satisfies condition $A(p_i)$ almost everywhere. Then any isometric immersion of the riemannian product $M_1 \times M_2$ into euclidean space of dimension $p_1 + p_2 + \text{dim}(M_1 \times M_2)$ is reducible. An induction, based on this theorem, yields a generalization of Theorem 1 in the author's paper, "Isometric immersions of riemannian products," J. Differential Geometry 5(1971), 159-168. (Received August 10, 1971.)

Logic and Foundations

Functors which preserve elementary operations. Preliminary report.

The functors $F$ from the category $S$ of sets into itself preserving elementary operations of first order logic (projection, complement, etc.) are easily seen to be those which preserve finite left limits and finite (disjoint) sums. These functors form a category denoted by $E$. Let $\mathcal{U}$ be the category whose objects are pairs $(I, \mathcal{F})$ where $\mathcal{F}$ is an ultrafilter over the set $I$ and whose morphisms are mappings preserving ultrafilters. Let $\mathcal{U}P$ be the category of ultrapoints which is the quotient category of $\mathcal{U}$ by the following equivalence relation between the morphisms from $(I, \mathcal{F})$ to $(J, \mathcal{G})$: $f \equiv g$ iff $EF \subseteq \mathcal{G}$ such that $f[F] = g[F]$. Theorem. The dual category of $E$ is equivalent to the category of pro-objects of the category $\mathcal{U}P$ of ultrapoints. Remarks. The straightforward proof of this theorem can be adapted to prove a theorem of Keisler on elementary extensions of complete structures. We have similar theorems for the category $S/Y$ of sets with base the set $Y$ (via ultraproducts instead of ultrapowers).
Construct Boolean terms (B.t.'s) from variables \( (x^1)^{\alpha<\delta} \) by the operations \(-, \wedge, \vee \) where \( \wedge, \vee \) may be applied to any set of terms. The depth \( D(f) \) of a B.t. \( f \) is defined by induction: \( D(x^1) = 0 \), \( D(g) = D(g) + 1 \), and \( D(\wedge X) = D(\vee X) = \sup \{ D(g) + 1 | g \in X \} \). Define the regular cardinal \( K(\alpha) = \min [\alpha : \alpha \text{ is a regular cardinal and } \text{ in the formation of } f, \wedge, \vee \text{ are applied only to sets of } <\alpha \text{ terms}] \). The value \( f(b ; B) \) of the B.t. \( f \) in the complete B.a. \( B \) under the assignment \( (b^1)^{\delta<\delta} \) (where \( \delta = B \)) is defined by induction on \( f \) in the well-known way. If always \( f(b ; B) = g(b ; B) \), write \( f = g \).

**Theorem.** Let \( f_1, f_2 \) be B.t.'s, \( f_1 = f_2 \). Then there is a B.t. \( f_3 \) such that \( f_1 = f_2 = f_3 \) and \( K(f_3) \leq K(f_1) \) and \( D(f_3) \leq D(f_2) \). This theorem answers for regular cardinals \( \alpha \) the question of Gaifman, Fund. Math. 54(1964), 232, about \( B^\alpha \) and \( B^\beta \). (Received June 15, 1971.) (Author introduced by Professor Haim Gaifman.)

*71T-ES8. STEPHEN L. BLOOM, Stevens Institute of Technology, Hoboken, New Jersey 07030 and DONALD J. BROWN, Yale University, New Haven, Connecticut 06520. Classical abstract logics. II

This continues Abstract 71T-ES4, these Notices 18(1971). Here we consider an abstract logic as \( \langle A, C \rangle \) where \( C \) is a closure system on the algebra \( A \). (There is a natural bijection between closure systems and closure operations.)

**Definition.** Let \( \langle A, C \rangle \) and \( \langle B_i, C_i \rangle \) \( (i \in I) \) be abstract logics, and let \( H_i \) be a subset of morphisms from \( A \) to \( B_i \). \( C \) is projectively generated by \( H_i \) and \( C_i \) \( (i \in I) \) if \( C \) is the coarsest closure system on \( A \) such that every morphism in \( \bigcup H_i \) is logical. A Boolean logic \( \langle B, C \rangle \) consists of a Boolean algebra \( B \) and the closure system \( C \) of all filters on \( B \).

**Theorem.** \( \langle A, C \rangle \) is classical iff there is a Boolean logic \( \langle B, C_B \rangle \) and a surjective morphism \( h:A \rightarrow B \) such that \( C \) is projectively generated by \( \{ h \} \) and \( C_B \). **Remark.** Classical logics are not closed under projective generation. There are classical logics \( \langle B_n, C_n \rangle, n = 1, 2, \ldots \), and morphisms \( h_n:A \rightarrow B_n \) such that the closure system projectively generated on \( A \) by \( \{ h_n \} \) and \( C_n \) is not classical. (Received June 28, 1971.)

71T-E89. WITHDRAWN.

71T-E90. JUDITH GREEN, University of Maryland, College Park, Maryland 20742. Beth's Theorem and Craig's Theorem for finite quantifier infinitary languages. Preliminary report.

Let \( \kappa \) be a regular cardinal or a singular cardinal of cofinality \( \omega \). We can show that if a relation is implicitly definable in \( L_{\kappa}, \omega \) then it can be explicitly defined in \( L_{(2^{\kappa})^+, \kappa} \), by a sentence whose only infinite quantifier is prenex and universal. If in addition \( \kappa \) is strongly inaccessible or a strong limit cardinal of cofinality \( \omega \), we can find a sentence of this prenex form in \( L_{\kappa, \kappa} \). These results follow from a refinement of Craig's Theorem for \( L_{\kappa, \omega} \) which we prove using a generalization of Keisler's consistency property for \( L_{\omega_1, \omega} \), and in which the interpolation sentence is of the form mentioned above. In all these cases we can also find existential prenex infinite quantifier formulae in addition to the universal ones. For Craig's Theorem, an example in Malitz's Thesis (Berkeley, 1966) shows that if \( \kappa \) is a limit cardinal no smaller quantifier will suffice. In the case of Beth's Theorem, John Gregory has shown a finite quantifier will not suffice. (Received July 14, 1971.)
Some valid identities in Doner-Tarski's extended arithmetic of ordinal numbers. Preliminary report.

For notation, see Doner-Tarski, Fund. Math. 65 (1969), 95-127. A transfinite sequence of recursively defined binary operations \( O_\gamma \) on and to ordinals is studied there; \( O_0, O_1 \), and \( O_2 \) are essentially addition, multiplication, and exponentiation. The authors raise the problem whether there is an identity involving operations \( O_0, \ldots, O_\gamma \) with \( \gamma < \omega \) which is not a logical consequence of identities involving only \( O_0, \ldots, O_3 \). The solution of this problem is affirmative. Theorem 1. The equation \( a + \{3 + \{3 + a \} \} \) holds for all \( \alpha \) and \( \beta \). If, however, we restrict ourselves to equations not involving operations \( O_0, \ldots, O_3 \), the solution becomes negative. Theorem 2. Let \( E \) be an equation containing any number of operation symbols \( O_\gamma \) with \( 4 \in \gamma < \omega \) (and no other operation symbols or constants). If \( E \) is valid for all ordinals, or even just for finite ordinals, then it is a tautology. As regards operations \( O_\gamma \) with \( \gamma \equiv \omega \), an example is given op. cit. of a nontautological identity without constants involving only operations \( O_0 \) and \( O_\gamma \) where \( \gamma \neq 0 \) is a limit ordinal.

An improvement of this is: Theorem 3. For any \( \gamma \equiv \omega \) there is a nontautological identity without constants whose only operation is \( O_\gamma \). (Received July 26, 1971.) (Author introduced by Professor Alfred Tarski.)

Complete types and \( L_{\omega_1 \omega} \)-sentences. Preliminary report.

Theorem. Let \( \eta = (\omega, +, *, <, n \in \omega) \), and let \( \sigma \) be an \( L_{\omega_1 \omega} \)-sentence such that for all countable structures \( \mathfrak{A} \) of the proper type, \( \mathfrak{A} \models \eta \) iff \( \mathfrak{A} \models \eta \). Let \( \Sigma \) be a type (i.e., a set of formulas with just the variable \( \nu \) free), and let \( T \) be a theory, in a countable elementary first order language with the relation and function symbols of \( \sigma \) and perhaps some new symbols as well. Suppose that \( \mathfrak{A} \in \text{Mod}(\sigma) \) iff \( \mathfrak{A} \) can be expanded to a model of \( T \) which omits \( \Sigma \) (i.e., such that no element satisfies all of the formulas of \( \Sigma \)). Then \( \Sigma \) is not complete with respect to \( T \); i.e., the theory generated by the set of sentences \( T \cup \Sigma(e) \) is not complete, where \( \Sigma(e) \) is the result of replacing \( \nu \) by the new constant \( e \) in all of the formulas of \( \Sigma \). This theorem answers a question asked by Malitz ("The Hanf number for complete \( L_{\omega_1 \omega} \)-sentences," in "The syntax and semantics of infinitary languages," Lecture Notes in Math., vol. 72, Springer-Verlag, Berlin and New York, 1968, pp. 166-181). The proof uses the \( \omega \)-Completeness Theorem and the fact that if \( \alpha \) is an infinite subset of \( \omega \) and \( \mathfrak{A} \) is an expansion of \( \eta \) in which \( \alpha \) is definable by a formula \( \varphi(\nu) \), then the type \( \Gamma = \{ \nu \neq n : n \in \omega \} \cup \{ \varphi(\nu) \} \) is not complete with respect to \( \text{Th}(\mathfrak{A}) \). (Here \( \mathfrak{A} \) is \( \equiv \mathfrak{M} \) for \( n \in \omega \).) (Received August 2, 1971.)

Linear \( \mathbf{E} \)-resolution. Preliminary report.

For notation see R. Anderson, "Completeness results for \( \mathbf{E} \)-resolution," Proc. Spr. Joint Comp. Conf., 1970, pp. 653-656. Let \( S \) be a fully factored set of clauses. \( C_1, C_2, \ldots, C_n \) is a linear \( \mathbf{E} \)-resolution deduction of \( C_n \) from \( C_1 \) within \( S \) iff (1) \( C_1 \in S \); (2) \( C_{i+1} \) follows from \( C_i \) unless \( C_i \) is a resolvent composed of all positive equality literals whereupon \( C_{i+1} \in S \) or \( C_{i+1} = C_j \) for some \( j < i \); (3) if \( C_{i+1} \) follows from \( C_i \), then \( C_{i+1} \) is a factor of \( C_i \), a resolvent of \( C_i \) and \( B_j \), or a paramodulation of \( B_j \) into \( C_i \) where \( B_j \in S \) or \( B_i = C_j \) for some \( j < i \). Theorem. If \( S \) is a minimal, fully factored, functionally reflexive set of clauses with no normal model and
clause $C \in S$ has no positive equality literals, then there exists a linear $E$-resolution refutation of $S$ with first clause $C$. Both the definition and theorem can be strengthened. (Received August 10, 1971.)

*71T-E94. DONALD ALVIN ALTON and EUGENE W. MADISON, University of Iowa, Iowa City, Iowa 52240. Computability of Boolean algebras and their extensions.

A Boolean algebra is computable if there is a one-to-one enumeration $(O_n)_{n \in N}$ of its domain which associates recursive functions with sup, inf, and complement. A computable Boolean algebra $\mathfrak{B}$ with enumeration $(U_n)$ is a constructive extension of its computable subalgebra $\mathfrak{A}$ with enumeration $(O_n)$ if there is a recursive function $h$ such that $O_n = U_n h(n)$. Let $\mathfrak{A}$ be a computable Boolean algebra with enumeration $(O_n)$ whose elements are the clopen sets in some Boolean space $\mathcal{S}$. A subset $U$ of $\mathcal{S}$ is recursive open (respectively, recursively regular open) iff there are recursive functions $f$ and $g$ such that $U = \bigcup_{n \in N} O_f(n)$ and $U^{-1} = \bigcup_{n \in N} O_g(n)$ (respectively, $U = \bigcup_{n \in N} O_f^{-1}(n) = \bigcap_{n \in N} O_g^{-1}(n)$). The recursively regular open sets, a recursive analog of the completion of $\mathfrak{A}$, form a subalgebra of the minimal completion consisting of regular open sets. A simple extension of $\mathfrak{A}$ obtained by adjoining a regular open set $U$ can be given the structure of a constructive extension of $\mathfrak{A}$ iff $U$ is recursive open. Henceforth assume $\mathcal{S}$ is the Cantor space, so that $\mathfrak{A}$ is atomless. The recursive open sets which are regular do not form a Boolean algebra. $\mathfrak{A}$ possesses computable extensions which are constructive and others which are not constructive. (Received August 10, 1971.)

*71T-E95. JMRE SIMON, Department of Applied Analysis and Computer Science, University of Waterloo, Waterloo, Ontario, Canada and Universidade de São Paulo, São Paulo, Brazil. On the time required by the Davis-Putnam tautology recognition algorithm.

M. Davis and H. Putnam (J. Assoc. Comput. Mach. 7(1960), 201-215) proposed an algorithm to decide whether a formula in conjunctive normal form is a contradiction. A dual algorithm (DP) decides whether a formula in disjunctive normal form (DNF) is a tautology. Definition. Let $A$ be a formula in DNF. The length $l(A)$, is the number of occurrences of atomic formulae in $A$. Theorem. For any polynomial $Q(t)$ in $t$, there is a formula $A$ in DNF, such that DP, applied to $A$, yields an intermediate formula longer than $Q(l(A))$, no matter in which order the atomic formulae in $A$ are eliminated by Rule III of DP. Corollary. If $T$ is a deterministic Turing machine simulating DP then $T$ does not operate within polynomial time. This solves an open problem by S. Cook and provides further evidence for his conjecture (Third Annual ACM Symp. on Theory of Computing, 1971, 151-158). (Received August 10, 1971.) (Author introduced by Professor Patrick C. Fischer.)

71T-E96. ALBERT J. SADE, 364 Cours de la République, Pertuis Vaucluse, France 84. Univers des axiomes implicationnels.

Dans Abstract 71T-E41 (ces Notes) 18(1971), 661 on a vu que l'ensemble des formules implicationnelles d'indice de vérité indéterminé sur un ensemble d'atomes est un groupoïde $G$, au moyen duquel toutes les thèses sur cet ensemble peuvent être construites. Chacune de ces thèses, considérée comme axiome, définit l'implication mais peut aussi définir un ou plusieurs autres foncteurs parmi les 16 opérations

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On donne les conditions générales pour chaque foncteur. Sur un ensemble de 3 atomes, on obtient 32 modèles de thèses qui, regardées comme axiomes, définissent C, et V. Six d'entre elles définissent en outre E, neuf définissent B et deux D. Seize seulement, en écartant la solution banale V, peuvent entrer dans la composition d'un système d'axiomes pour définir C. (Received August 25, 1971.)

7IT-E97. BARUCH GERSHUNI, Ibn Gvirol Street 43, Tel Aviv, Israel. The antinomy of Russell for arbitrary kinds of totalities.

In the simplified theory of totalities every totality contains its elements either directly or, when allround packed up, by means of the packing material. In the graphical representation of the totalities packed up, the packing material is symbolized by braces. We call the first-named totalities, e.g., abc...; a b c...; a,b,c,... classes and the second-named |abc...|, etc. sets. The sets are all singulars because of the packing material. The von Neumann condition that the elements of classes and sets can be only sets means that these elements can be only synthetic singulars. We enlarge now the range of the admitted elements by permitting every singular, also the individuals, to be elements. Furthermore we state the Axiom. The elements of classes may be singulars and plurals; the elements of sets may be only singulars. The first part of this axiom is stated in order not to hamper the mathematical analysis too much. According to this axiom any class contains itself as its unique, it is sure, plural element. In this manner the totality of Russell R (x ∈ R = x ∈ x) for classes become the null class, i.e. nothing. Thus the Russell antinomy for classes is solved. For sets again, the Russell totality does not lead by the usual considerations to an antinomy. (Received August 30, 1971.)

Statistics and Probability

*7IT-F9. NAresh C. JAIN and William E. PruitT, University of Minnesota, Minneapolis, Minnesota 55455. The law of the iterated logarithm for the range of random walk.

Let \( \{X_n, n \geq 1\} \) be a sequence of independent identically distributed random variables with values in the d-dimensional integer lattice \( E_d \), and let \( S_0 = 0, S_n = X_1 + \ldots + X_n \). \( R_n \), the range of the random walk \( \{S_n, n \geq 0\} \) up to time \( n \), is the cardinality of the set \( \{S_0, \ldots, S_n\} \). Let \( p = P \{S_1 \neq 0, S_2 \neq 0, \ldots\} \). Then we prove: Theorem. If \( p < 1 \) and either \( d \geq 4 \) or the random walk is strongly transient, then there exists a constant \( c > 0 \), which depends on the distribution of the random walk, such that \( \lim \sup_n (R_n - np)(cn \log \log n)^{-1/2} = 1 \), \( \lim \inf_n (R_n - np)(cn \log \log n)^{-1/2} = -1 \) with probability one. (Received July 9, 1971.)

*7IT-F10. PHILIP S. MARcus, Indiana University, South Bend, Indiana 46624. Connections between stochastic processes and probabilistic metric spaces.

The theory of probabilistic metric spaces was created by K. Menger and has been applied by B. Schweizer and A. Sklar to the study of quantized spaces. In this theory, the ordinary triangle inequality involving distances and addition is generalized to a triangle inequality involving distribution functions and two-place functions called \( t \)-norms. Consider any stationary Markov chain. Given two states \( a, b \) define a transition probability \( P_{ab} \) as the probability that at a given time \( t \), the process will have reached state \( b \) at some time between 0 and \( t \). given
that the process is in state \( a \) at time 0. Then the total set of states together with the transition probabilities satisfy the Menger triangle inequality for the \( t \)-norm \( \text{Prod} \). Although the transition probabilities are not necessarily symmetric the formation of commutators produces a Menger space under the \( t \)-norm \( T_m \). This result can be generalized to stationary continuous processes. The argument uses Renyi's axiomatic treatment of conditional probability. (Received July 12, 1971.)

71T-F11. JOSEPH HOROWITZ and DONALD GEMAN, Department of Mathematics and Statistics, University of Massachusetts, Amherst, Massachusetts 01002. Palm probabilities and additive functionals. I. Preliminary report.

Given a stationary flow \( \theta_t, t \in \mathbb{R} \), on a probability space \((\Omega, \mathcal{F}, P)\), and an additive functional \( \alpha = (\alpha_t) \) with \( E \alpha_1 = 1 \), the "Palm" probability \( \mathcal{P}_\alpha \) is defined as \( \mathcal{P}_\alpha(A) = E \int_0^1 I_A \theta_t \alpha_t \), following Maruyama, Totoki, and others. It is shown that \( \mathcal{P}_\alpha \) is the analogue of the Palm probabilities given by Ryll-Nardzewski (Proc. Fourth Berk. Symp. on Math. Stat. and Probability, vol. 2, 1961, pp. 455-466) for stationary streams of "calls," and many of his results carry over. Some other results are: (1) For a fixed \( \alpha \), there is a unique correspondence \( P \rightarrow \mathcal{P}_\alpha \); an inversion formula is obtained. (2) If \( x(t) \) is stationary Gaussian with finite second spectral moment, and \( \alpha_t \) the stream of zeros of \( x(t) \), then \( \mathcal{P}_\alpha \) is the horizontal-window probability \( P(\cdot | x(0) = 0 \text{ h.w.}) \), which is equivalent to the usual \( P(\cdot | x(0) = 0) \); the Radon-Nikodym derivative is identified. (3) For particular \( \alpha_t \), e.g. local times, \( \mathcal{P}_\alpha \) is an ordinary (Radon-Nikodym) conditional probability. Further questions on Palm probabilities will be treated in a subsequent paper. (Received August 2, 1971.)


A family of random variables \( \{\xi_{ij}\}_{i,j=1}^\infty \) is said to be strictly stationary (s.s.), if for each integer \( n \geq 1 \) and each set \( \{ (k_i, k_j) : k_1, \ldots, n \} \) of pairs of integers with \( k_i, j_k \geq 1 \), the joint distribution of \( \{\xi_{ik_i, j_k} \}_{i,j=1}^n \) is the same as the joint distribution of \( \{\xi_{ik_i+s, j_k+t} \}_{i,j=1}^n \) for every \( (s,t) \), where \( s \) and \( t \) are integers and \( i_k, j_k \geq 1 \) and \( k_1 \leq s \leq 1, k_1 \leq t \leq 1, k_1 \leq 1, \ldots, n \). The following is an extension of a theorem of Billingsley ("Convergence of probability measures", Wiley, New York, 1968, pp. 174-177). Theorem. Suppose a family of random variables \( \{\xi_{ij}\}_{i,j=1}^\infty \) with mean zero and variance one is given, which is s.s. and \( \varphi \)-mixing (i.e., the rows and columns are \( \varphi \)-mixing). Let \( X_n(s,t) = (S_{-in\lceil n\rceil}, n\lceil n\rceil, n\lceil n\rceil, n\lceil n\rceil) \)/\( n \), \( 0 \leq s, t \leq 1 \), where \( S_{r,h} = \sum_{s=t}^1 \sum_{t=h}^1 \xi_{st} \). Then if \( \sum \alpha_n^{1/2} < \infty \) and \( \sigma_n^2 = E(\xi_{11}^2) + 2 \sum_{k,r=2}^\infty (E(\xi_{11}^2) + E(\xi_{11}^2)) + 2 \sum_{r=2}^\infty (E(\xi_{11}^2) + E(\xi_{11}^2)) > 0 \), then \( X_n \) converges in distribution to the two-parameter Yeh-Wiener process. (Received August 30, 1971.)

71T-F13. THOMAS M. COVER, Stanford University, Stanford, California 94305. Determining the rationality of the bias of a coin.

Consider a sequence \( X_1, X_2, \ldots \) of independent identically distributed coin tosses with unknown bias \( p = \Pr[X_1 = 1] \). Let \( Q = \{r_{ij}\}_{i,j=1}^\infty \) denote the rationals. Consider the countable set of hypotheses \( H_i : p = r_i \), \( i = 1, 2, \ldots \), together with the null hypothesis \( H_0 : p \) is irrational. We have evolved a test which makes a decision after each new coin flip and makes only a finite number of mistakes with probability one for every
where \( N_0 \) is a set of irrationals of Lebesgue measure zero. A modified test will decide whether or not the sequence \( X_1, X_2, \ldots \) of i.i.d. random variables with finite but unknown second moment has a mean \( \mu \) lying in some countable subset \( Q \) of the real line. Again, this test will make only a finite number of mistakes with probability one for any mean \( \mu \notin N_0 \), where \( N_0 \) is a subset of \( \mathbb{R} - Q \) of Lebesgue measure zero. Thus it is theoretically possible to determine whether empirically determined physical constants belong to certain sets of special numbers. (Received August 31, 1971.)

**Topology**


Recently Sam B. Nadler found an interesting example of a noncomplete metric space in which every contraction has a fixed point. This suggests the following Definition. A metric space \((X, \rho)\) is said to be pseudocomplete, if each contraction \( f: X \rightarrow X \) in it has a fixed point. Theorem. Let \((X, \rho)\) be a bounded connected metric space and \( X^\ast \) its completion. If there exists an \( \epsilon > 0 \) such that every connected subset \( A \subseteq X \) with \( \delta(A) < \epsilon \) has a positive distance from \( X^\ast - X \), then \((X, \rho)\) is pseudocomplete. (Received June 7, 1971.)

71T-G164. SIDNEY A. MORRIS, University of Florida, Gainesville, Florida 32601. Quotient groups of topological groups with no small subgroups.

It is well known that a quotient group of a Lie group is a Lie group, or equivalently that a quotient of a locally compact group with no small subgroups is a locally compact group with no small subgroups. Irving Kaplansky asked: if \( G \) is a topological group with no small subgroups and \( H \) is a closed normal subgroup of \( G \), is \( G/H \) (necessarily) a group with no small subgroups? We answer the question in the negative. (Received June 11, 1971.)


The structure that we shall describe is analogous to the Godement cohomology spectral sequence of a covering but is not complete enough to earn the name "spectral sequence". Yet, it does provide enough information so that one can deduce the existence of a functorial spectral sequence that relates general homology of a space to the general homology presheaf. (Cf. another announcement of this author (Abstract 71T-G143, these Notices 18(1971)) concerning a different construction of perhaps the same general homology spectral sequence but which is not known to be functorial.) (Received June 21, 1971.)

71T-G166. J. J. UCCI, Syracuse University, Syracuse, New York 13210. Symmetric maps of spheres.

A based map \( f: X^m \rightarrow X \) is a G-map if \( f(\sigma \cdot x) = f(x) \), all \( x \in X \), \( \sigma \in G \) (\( G \) a given subgroup of the symmetric group \( S(m) \)). If \( X = S^n \) the James number \( J(f) \) of a G-map \( f \) is the degree of the composite \( f \circ i \), \( i \) the axial embedding \( i(x) = (x, e, \ldots, e) \). Let \( f: (S^{2t+1})^m \rightarrow S^{2t+1} \) be a G-map. A known result: Theorem 1. For
Let \( p \) be an odd prime, and let \( X \) be a 1-connected mod \( p \) H-space (i.e. a space with continuous product \( \mu : X \times X \to X \) such that if \( \phi_j : X \to X \times X \), \( j = 1, 2 \), are the natural inclusions, then \( (\mu * \phi_j)^* = 1_{H^*(X; \mathbb{Z}_p)} \)). Suppose \( X \) has no \( p \)-torsion and that \( H^*(X; \mathbb{Z}_p) = H^*(S^{n_1} \times \ldots \times S^{n_l}; \mathbb{Z}_p) \), \( n_1 \leq \ldots \leq n_l \), \( n_l \) odd, \( i = 1, \ldots, l \).

**Theorem.** If \( p \geq (n_1 - n_1 + 4)/2 \), then there exists \( f : S^{n_1} \times \ldots \times S^{n_l} \to X \) such that \( f^* \) is an isomorphism in \( \mathbb{Z}_p \)-cohomology. This result coincides with a result of Serre when \( X \) is a compact Lie group. It improves on a result of the author for \( X = G/K \), a compact symmetric space, and is best possible for the irreducible symmetric spaces. (Received July 6, 1971.)

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**71T-G167. PAUL G. KUMPEL, JR., State University of New York, Stony Brook, New York 11790.**

\( p \)-equivalences of mod \( p \) H-spaces.

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**71T-G168.** FRANK SWIEC, St. John's University, Jamaica, New York 11432. A note on spaces in which every compact subspace is metrizable. Preliminary report.

For Hausdorff spaces we say that a space is KM if every compact subspace is metrizable. KM is a hereditary and countably productive property implied by conditions such as the existence of a point countable base or a \( G_\delta \)-diagonal. A compact-covering image of a KM-space is KM. A space is KM iff it is a compact-covering image of a metric space. TFAE for a space \( Y \): \( Y \) is sequential + KM, \( Y \) is \( k + \) KM, \( Y \) is a quotient + compact-covering image of a metric space. A KM-space of point countable type is first countable. A locally compact space is KM iff it is locally metrizable. These and several related results have trivial proofs. (Received July 6, 1971.)

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**71T-G169. LEE K. MOHLER, State University of New York at Buffalo, Amherst, New York 14226.**

A characterization of hereditarily decomposable snake-like continua. Preliminary report.

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**71T-G170. KENNETH HARDY, Carleton University, Ottawa, Ontario, Canada and RUSSELL GRANT WOODS, University of Manitoba, Winnipeg 19, Manitoba, Canada.**

On c-realcompact spaces and locally bounded normal functions. Preliminary report.

The concept of a c-realcompact space was introduced for the class of completely regular Hausdorff.
spaces by Nancy Dykes (Pacific J. Math. 33(1970)). Let $\text{LN}(X)$ denote the lattice-ordered ring of locally bounded normal upper semicontinuous functions on $X$. The $c$-realcompact spaces between $X$ and $\beta X$ can be identified as intersections of families of locally compact, $c$-realcompact spaces indexed by functions in $\text{LN}(X)$ (analogous to the Gillman-Jerison description of the realcompact spaces between $X$ and $\beta X$). Thus, one has an external construction for the $c$-realcompactification of $X$. If $E(X)$ denotes the absolute of $X$, then several necessary and sufficient conditions are given for the equality $E(\kappa X) = \nu E(X)$ to hold, where $\nu X$ is the Hewitt realcompactification of $X$. Various related results and applications have been obtained. (Received July 12, 1971.)


The following are generalizations of the concept of strict $p$ space and of $\tilde{G}_0$-diagonal: (1) $X \in \tilde{p}(\sigma)$ iff for some bicompactification $bX$ of $X$, there exists a sequence $\{P_i\}$ of sets of open sets of $bX$ such that:

(a) $\bigcup\{P_i : i \in \mathbb{N}\}$ covers $X$, (b) $\bigcap\{P_i(x) : P_i(x) \neq \emptyset \} \subseteq X$, and (c) $x \in X$ and $m, n \in \mathbb{N}$ such that $P_n(x) \neq \emptyset$ and $P_m(x) \neq \emptyset$ implies there $k \in \mathbb{N}$ such that $x \in P_k(x) \subseteq P_n(x) \cap P_m(x)$. (2) $X \in \tilde{G}(\sigma)$ iff there exists a sequence $\{G_i\}$ of sets of open sets such that: (a) $\bigcap\{G_i(x) : G_i(x) \neq \emptyset \} = \emptyset$; and (b) $x \in X$ and $m, n \in \mathbb{N}$ such that $G_n(x) \neq \emptyset$ and $G_m(x) \neq \emptyset$ implies there exists $k \in \mathbb{N}$ such that $x \in G_k(x) \subseteq G_n(x) \cap G_m(x)$. Some of the results obtained are:

Theorem. $X$ is a completely regular quasi-developable space iff $X \in \tilde{G}(\sigma)$ and $X \in \tilde{p}(\sigma)$. Theorem. $X \in \tilde{G}(\sigma)$, $X \in \tilde{p}(\sigma)$ and $X$ perfect iff $X$ is completely regular and developable iff $X \in \tilde{p}(\sigma)$ and $X$ symmetrizable. Theorem. $X \in \tilde{p}(\sigma)$ iff there exists a sequence $\{H_i\}$ of sets of open sets of $X$ such that: (a) $\bigcup\{H_i : i \in \mathbb{N}\}$ is a cover of $X$, (b) $H_x \cap \bigcap\{H_i(x) : x \in X\}$ is bicom pact, and (c) $\bigcap\{H_i(x) : i \in \mathbb{N}\}$ is a base for $H_x$. (Received July 14, 1971.)

*71T-G172. CHARLES L. HAGOPIAN, Sacramento State College, Sacramento, California 95819. $\lambda$ connected plane continua.

A continuum $M$ is said to be $\lambda$ connected if any two distinct points of $M$ can be joined by a hereditarily decomposable continuum in $M$. In this paper it is proved that every bounded semi-aposyndetic plane continuum is $\lambda$ connected. It is also proved that if $M$ is a $\lambda$ connected compact metric continuum and $f$ is a continuous function on $M$ into the plane, then $f(M)$ is $\lambda$ connected. This result follows from a theorem which indicates that if a continuous image of a compact hereditarily decomposable metric continuum lies in the plane, then the boundary of each of its complementary domains must be hereditarily decomposable. (Received July 14, 1971.)

*71T-G173. ROBIN B. S. BROOKS, Bowdoin College, Brunswick, Maine 04011. On the sharpness of the $\Delta_1$ and $\Delta_2$ Nielsen numbers.

Theorem 1. Suppose $X$ and $Y$ are connected, locally path connected, semilocally simply connected Hausdorff spaces, $X$ is a homotopy retract of an $n$-dimensional polyhedron for some $n \geq 3$, and $Y_0 \subseteq Y$ is such that $\pi_m(Y_0 - Y) = 0$ for $m < n$. Then for any map $f : X \to Y$ there is a map $f'$ homotopic to $f$ with exactly $N(f, y_0, \Delta_2)$ coincidence classes. Theorem 2. Suppose $X$ and $Y$ are connected, locally path connected, semilocally simply connected Hausdorff spaces, $X$ is a homotopy retract of an $n$-dimensional polyhedron for some $n \geq 3$, and $\pi_m(Y \times Y, Y \times Y - d(Y)) = 0$ for $m < n$. Then for any two maps $f, g : X \to Y$ there are maps $f'$
and \( g' \) homotopic to \( f \) and \( g \) respectively such that \( f' \) and \( g' \) have exactly \( N(f, g, A_1) \) root classes. Here \( d(Y) = \{(y, y')|y \in Y\} \), \( N(f, y_0', \Delta_2) \) and \( N(f, g, \Delta_1) \) are the \( \Delta_2 \) and \( \Delta_1 \) Nielsen numbers as defined in Brooks and Brown, "A lower bound for the \( \Delta \)-Nielsen numbers," Trans. Amer. Math. Soc. 143(1969), 555-564, and \( X \) is said to be a homotopy retract of \( P \) when there are maps \( \varphi: X \rightarrow P \) and \( \psi: P \rightarrow X \) such that \( \varphi \circ \psi \) is homotopic to the identity on \( X \). An example is given to show that much stronger hypotheses are needed on \( X \) (e.g., \( X \) is a manifold) in order to reduce the root (coincidence) classes to singletons. (Received July 15, 1971.)


In the bitopological space \((X, \tau_1, \tau_2)\), \( \tau_1 \) is said to be locally compact with respect to \( \tau_2 \) if for each point \( x \in X \) there is a \( \tau_1 \) open neighbourhood of \( x \) whose \( \tau_2 \) closure is pairwise compact. \((X, \tau_1, \tau_2)\) is pairwise locally compact if \( \tau_1 \) is locally compact with respect to \( \tau_2 \) and \( \tau_2 \) is locally compact with respect to \( \tau_1 \). Such a bitopological space is not just a pair of locally compact topologies. \((X, \tau_1, \tau_2)\) can be pairwise Hausdorff and pairwise locally compact without \( \tau_1 = \tau_2 \). Various simple properties are examined and results obtained; for example, if \((X, \tau_1, \tau_2)\) is pairwise Hausdorff and pairwise locally compact then it is pairwise regular. A bitopological analogue of the Alexandroff one point compactification is defined, and its properties are investigated. (Received July 19, 1971.)

71T-G175. CHIEN WENJEN, California State College, Long Beach, California 90801. Pseudo retracts. Preliminary report.

A nonvoid closed subset of topological space is called canonical if it is the closure of its interior. A Tychonoff space \( X \) is said to be a pseudoabsolute retract (PAR*) [PANR*] whenever a topological image of \( X \) as a canonical subset \( X_1 \) of an arbitrary Tychonoff space \( Y \) is necessarily a retract [a neighborhood retract] of \( Y \).

1. (Borsuk extension theorem.) In order that a Tychonoff space \( X \) be a PAR* [PANR*], it is necessary and sufficient that for any canonical closed subset \( S_0 \) of an arbitrary Tychonoff space \( S \), every mapping \( f: S_0 \rightarrow X \) can be extended over \( S \) [some open subset \( U \supset S_0 \) of \( S \)] with respect to \( X \).

2. (Borsuk homotopy extension theorem.) If a mapping \( h: S_0 \times [0, 1] \rightarrow X \) for a canonical closed subset \( S_0 \) of a Tychonoff space \( X \) into a PAR* [PANR*] \( X \) can be extended over \( S \times [0, a] \) for some \( 0 < a < 1 \), then \( h \) can be extended over \( S \times [0, 1] \).

3. (Borsuk uniform homotopy extension theorem.) Let \( X \) be a compact AR* [ANR*] and \( S_0, S \) be the same as in 2. If \( f_0 \) and \( f_1 \) are uniformly homotopic mappings of \( S_0 \) into \( X \) and \( f_0 \) can be extended to a mapping \( F_0 \) over \( S \), then \( f_1 \) can be similarly extended to \( F_1 \) with \( F_0 \) and \( F_1 \) homotopic. (Received July 19, 1971.)


Let \( f: M^m \rightarrow N^n \) be a surjective map between closed, \( K \)-orientable manifolds (\( K \) a field). Define \( A^q(f) \) to be the set of points \( y \in N \) for which \( f^{-1}(y) \) is not acyclic in dimension \( q \) (using \( \acute{C}ech \) cohomology with coefficients in \( K \)). Let \( p \) be an integer, \( p < n \). Theorem. If \( \dim A^q(f) \equiv 0 \) for \( q < p \) then \( A^q(f) \) is finite for \( q < p \) and \( q \equiv m - p \). Therefore, if in addition \( 2p \equiv m \), \( A^q(f) \) is finite for all \( q \). The proof is based on an improved version.
of Theorem 1 of [E. G. Skljarenko, "Almost acyclic mappings", Mat. Sb. 4(1968), 267-272] and a finiteness theorem due to the author and D. R. McMillan, Jr. (to appear). (Received July 19, 1971.)


Let $G^n_q$ be the space of oriented $q$ planes in $R^n$. It is known, that except when $n$ is even and $q$ is odd, $K^0(G^n_q) = 0$ and $K^0(G^n_q)$ is free. In this paper we calculate the ring structure of $K^*(G^n_q)$ for the case $n$ even and $q$ odd. There is a canonical $n-q$ plane bundle over $G^n_q$. A main tool is to describe its Thom space as $G^{n+1}_q/G^{n+1}_q$. Theorem. Let $q = 2r + 1$ and $n-q = 2m+1$. There are elements $A_0 = 1, A_1, \ldots, A_r$, $B_0 = 1, B_1, \ldots, B_m$ lying in $K^0(G^n_q)$ and $\nu$ lying in $K^1(G^n_q)$ such that $K^0(G^n_q) = Z[A_1, \ldots, A_r, B_1, \ldots, B_m]$ modulo relations

$$\sum_{j=0}^{r} A_j B_{k-j} = 0$$

for each $1 \leq k \leq r+m, A_j = 0$ for $j > r, B_j = 0$ for $j > m$. $K^1(G^n_q) = K^0(G^n_q)(\nu)$ with $\nu^2 = 0$.

(Received July 2, 1971.)

*71T-G178. DOUGLAS W. CURTIS, Louisiana State University, Baton Rouge, Louisiana 70803. Total and absolute paracompactness.

A paracompact space $X$ is totally paracompact if every open base for $X$ contains a locally finite cover. If for every closed imbedding $h : X \to Y$ into a paracompact space $Y$, every open base for $Y$ contains a locally finite (in $Y$) cover of $h(X)$, then $X$ is absolutely paracompact. It is shown that absolute paracompactness is weakly hereditary, and that if every nonempty closed subspace of a paracompact space $X$ is locally absolutely paracompact at some point, then $X$ is absolutely paracompact. A countable sum theorem is obtained, from which it follows that $F_\sigma$-subsets of totally paracompact spaces, and paracompact countable sums of closed absolutely paracompact spaces, are totally paracompact. $\mathfrak{m}$-absolute paracompactness is the restriction of absolute paracompactness to the metric category. This property is particularly useful since it is obtained for all $\sigma$-locally separable totally paracompact metric spaces, and behaves analogously to absolute paracompactness. It is shown that products of $\sigma$-locally separable totally paracompact metric spaces with $\sigma$-locally compact metric spaces are totally paracompact. Every Hurewicz space is totally paracompact; this generalizes in one direction a theorem of Lelek [Fund. Math. 64(1969), 209-218] that a metric space is Hurewicz if and only if it is separable and totally paracompact. (Received July 30, 1971.)


It is known that neither closed maps nor compact-open maps preserve the existence of a $G_\delta$-diagonal and in [General Topology and its Applications, Vol. 1, no. 1] R. W. Heath asked whether perfect maps preserve $G_\delta$-diagonals. In this note we obtain a partial solution to Heath's problem. We show, for example, that if $X$ is perfectly paracompact (=paracompact and closed sets are $G_\delta$'s) and has a $G_\delta$-diagonal, then any perfect image of $X$ has the same properties. Also, if $X$ is a Hausdorff space with a $G_\delta$-diagonal and if $f : X \to Y$ is a perfect map with at most finitely many nondegenerate fibres, then $Y$ has a $G_\delta$-diagonal. (Received August 4, 1971.)

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Let \( f: S^m \to S^{m+k} \) be a generic metastable immersion and \( D(f) - \Delta(f) \) its double point cover in the sense of Haefliger. Then \( f \) determines a unique isotopy class \( F(f) \) of stable bundle maps \( \nu(\Delta(f)) \to k \xi \), with \( \xi \) the canonical line bundle over \( P_\xi \). Thus \( (\Delta(f), F(f)) \) is a \( k \xi \)-manifold (Wells, "Cobordism groups of immersions," Topology 5(1966)). Theorem. Every \( k \xi \)-surgery of \( (\Delta(f), F(f)) \) is realized by a generic regular homotopy of \( f \).

This theorem refines theorems of Haefliger and Connolly in the case \( S^m \to S^{m+k} \). Let \( N = \tilde{N} \) be a double cover of closed manifolds with \( \tilde{N} \) homotopy equivalent to \( S^k \times S^k \) and \( N \) orientable and \( k \equiv 4 \). Theorem. If \( k \equiv 4, 6 \mod 8 \), then there is \( k \in \mathbb{Z} \) such that there is a generic metastable immersion \( f: S^{m-1} \to S^{m+k} \) with \( D(f) - \Delta(f) = \tilde{N} \). Theorem. For \( k \equiv 0 \mod 8 \) there is \( \tilde{N} - N \) as above which is not a component of \( D(f) - \Delta(f) \) for any generic immersion \( M^{2k+1} \to S^{2k+2} \).

For metastable generic immersions \( f: \tilde{M} \to \tilde{S}^{m+k} \) with \( D(f) - \Delta(f) = \tilde{N} \), \( \nu(\Delta(f): \tilde{M}) \) trivial. (Received August 4, 1971.)

71T-G181. MASAMI WAKAE, Soka University, Hachioji, Tokyo 192, Japan. The generalized Smith indices. III. In [M. Wakae and O. Hamara, "The generalized Smith indices. I," Math. Z. 122(1971), 71-81] the Smith classes in the case of a finite abelian group \( G \) acting properly discontinuously on a space \( X \) is defined in a classical way. In this note a functorial construction of Smith classes is given. Notation and terminology are the same as those of Wakae and Hamara. If \( C \) is a cyclic group of order \( n \) which is divisible by an odd prime number \( p \), then \( H^*(C; J_p) = \Lambda(a) \otimes J_p(b) \) (dim \( a = 1 \), dim \( b = 2 \)) where \( \Lambda(a) \) and \( J_p(b) \) refer to the exterior and the polynomial algebra over \( J_p \), respectively. Therefore using induction on \( s(1) \) and the Künneth formula, we have \( H^*(B_G; J_p) = H^*(G; J_p) = \Lambda(a_1, \ldots, a_{s(1)}) \otimes J_p(b_1, \ldots, b_{s(1)}) \) where \( B_G \) is the classifying space of \( G \). We define \( A^{2m}(B_G) = (a_1 + \ldots + b_{s(1)})^m \) and \( A^{2m+1}(B_G) = (a_1 + \ldots + b_{s(1)})^m (b_1 + \ldots + b_{s(1)}) \). Thus we may define the Smith classes \( A^i(X) \) as the image of \( A^i(B_G) \) under the characteristic map \( \sigma^*: H^*(B_G; J_p) \to H^*(X_G; J_p) \). Then it may be shown that \( \rho(1)A^i(X, g_\xi) = A^i(X) \) (up to an automorphism in the cohomology algebra \( H^*(X_G; J_p) \)). (Received August 10, 1971.)

*71T-G182. JOHN E. MACK and MARLON C. RAYBURN, University of Kentucky, Lexington, Kentucky 40506 and RUSSELL GRANT WOODS, University of Manitoba, Winnipeg 19, Manitoba, Canada. Lattices of topological extensions for locally-\( P \) spaces.

All spaces considered are completely regular and Hausdorff. These results generalize those of K. D. Magill, Jr., "The lattice of compactifications of a locally compact space" (Proc. London Math. Soc. 18(1968), 231-244). An axiomatic description of property \( P \) is given sufficient to insure the existence of the \( P \)-reflection \( \gamma X \) in \( \beta X \) and the largest one-point \( P \)-extension \( \ast X \) for space \( X \); examples of \( P \) include (a) compactness, (b) realcompactness, (c) almost-realcompactness in the sense of Frolik, (d) \( m \)-boundedness. The complete lattice of minimal \( P \)-ifications of \( X \) above \( \ast X \) is called \( P^* \), and \( D^* \) consists of those \( T \) in \( P^* \) for which the
quotient map (fixing X pointwise) \( f: \beta X \rightarrow \beta T \) takes \( \gamma X \) onto \( T \). It is found that if \( P^*(X) \) is lattice-isomorphic to \( P^*(Y) \), then \( \gamma X - X \) is homeomorphic to \( \gamma Y - Y \). A partial converse is obtained which is sufficient to yield the Corollary. If \( \gamma X - X \) and \( \gamma Y - Y \) are \( C^* \)-embedded in \( \gamma X \) and \( \gamma Y \) and \( D^* = P^* \) for both, then \( P^*(X) \) is isomorphic to \( P^*(Y) \) if and only if \( \gamma X - X \) is homeomorphic to \( \gamma Y - Y \). (Received August 10, 1971.)

71T-G183. RICHARD C. DETMER, University of Wisconsin, Madison, Wisconsin 53706. Taming subsets of complexes in \( E^3 \). Preliminary report.

The definition of 1-FLG used by McMillan ("Local properties of the embedding of a graph in a three-manifold," Canad. J. Math. 18(1966), 517-528) may be generalized to apply at any point of a 3-manifold in \( E^3 \).

Theorem. If \( K \) is a 3-complex topologically embedded in \( E^3 \), then \( K \) is tame iff \( E^3 - K \) has 1-FLG at each point of \( K \). Definition. Suppose \( K \) is a 3-complex topologically embedded in \( E^3 \) and \( X \) is a compact subset of \( K \); \( X \) is said to be tame in \( K \) iff given \( r > 0 \), there is an embedding \( g: K \rightarrow E^3 \) which fixes \( X \) and moves no point further than \( r \) and such that \( g(K) \) is tame in the usual sense. An arc may be wild in a sphere yet tame in a 2-complex. An arc may be wild on a Cantor set yet tame in a 2-complex. Theorem. If \( A \) is an arc in \( E^3 \) and \( X \) is a compact subset of \( A \), then \( X \) is tame in \( A \) iff the following condition holds: for every \( \epsilon > 0 \) there is a \( \delta > 0 \) such that loops in \( E^3 - X \) which bound on \( \delta \)-sets in \( E^3 - X \) also shrink on \( \epsilon \)-sets in \( E^3 - X \).

Theorem. Suppose \( K \) is a complex embedded in \( E^3 \) which is 2-dimensional at each point; suppose \( X \) is a compact subset of \( K \) which has no degenerate components and contains no vertex of \( K \); and suppose \( E^3 - K \) is 1-ULC in \( E^3 - X \); then \( X \) is tame in \( K \). (Received August 10, 1971.)

71T-G184. JIMMIE A. ROBERTSON, Florida State University, Tallahassee, Florida 32306. On the collapsibility of \( K \times I \). Preliminary report.

A method is given to collapse \( K \times I \) when \( K \) is the wedge of two circles \( a \) and \( b \) with two 2-cells \( D \) and \( E \) attached by the words \( a^n b^r \) and \( a^r b^n \) respectively where \( n \) and \( r \) are any positive integers. This generalizes several special cases which were indicated by Zeeman to be true although no explicit proof was given. (Received August 10, 1971.)


Suppose \( X \) is a nonempty set, \( \Sigma(X) \) is the lattice of all topologies on \( X \) and \( w*(\Sigma(X)) \) is the largest integer \( m \) such that \( \Sigma(X) \) has a family of \( m \) mutually complementary elements. Theorem. If \( p \) is an odd prime and \( |X| = p, p + 1, 2p - 1 \) or \( 2p \), then \( w*(\Sigma(X)) \) is respectively \( p, p, 2p - 1, 2p - 1 \). The case \( |X| = p \) is due to J. B. Kelly. The problem of determining \( w*(\Sigma(X)) \) is shown to be related to the existence of a certain type of 1-factorization of the complete graph on an even number of points. In this form, the above theorem allows one to characterize those graphs on \( p \) or \( 2p - 1 \) (\( p + 1 \) or \( 2p \)) points which have Hamiltonian paths (circuits). For an infinite analogue of the above theorem, see "Families of mutually complementary topologies," Proc. Amer. Math. Soc. 29(1971), 362-368. (Received August 10, 1971.)
Let $M$ be a smooth $(C^\infty)$ 3-manifold and let $|ht|_{t \in \mathbb{R}}$ be a smooth flow on $M$. A closed subset $A$ of $M$ is invariant if $ht(A) = A$ for all $t$ and minimal if it is invariant and has no proper closed invariant subsets.

**Theorem 1.** If $A$ is a compact, 1-dimensional minimal set lying in the interior of $M$ then $A$ is (topologically) a solenoid (i.e., $A$ is homeomorphic to the inverse limit of a sequence of simple closed curves with onto bonding maps). Following Conley and Easton ("Locating invariant sets," Proc. Sympos. Pure Math., vol. 14, Amer. Math. Soc., Providence, R.I., 1970, p. 55), we say that an invariant set $A$ is isolated if there is an open neighborhood $U$ of $A$ such that $A$ is the maximal invariant set in the closure of $U$. **Theorem 2.** If, in addition to the hypotheses of Theorem 1, $A$ is isolated, then $A$ is a periodic orbit. The proof of Theorem 1 is essentially topological. Smoothness is used in proving Theorem 2, where the Alexander-Spanier cohomology of the set $A$ is computed well enough to see that the only possible solenoid is the circle itself. (Received August 13, 1971.)

**Heegaard splittings of homology 3-spheres.** Preliminary report.

Let $\{w_1, \ldots, w_n\}$ be a Heegaard diagram of a closed 3-manifold $M$. In ["Topology of manifolds" (Proc. Univ. of Georgia Inst.), Markham, Chicago, 1970, pp. 140-160] Haken determines a necessary and sufficient condition that there exist an embedding $h(H) \subset S^3$ with each $h(w_i) \sim 0$ in $S^3 - h(H)$. Since it is known that there is such an embedding if $M$ is simply-connected, it might be expected that this condition would provide at least a partial algorithm for simple-connectivity. However, we have: **Theorem 1.** There is an embedding $h(H) \subset S^3$ with each $h(w_i) \sim 0$ in $S^3 - h(H)$ if and only if $H_1(M) = (0)$. Even stronger embedding properties fail to distinguish simple-connectivity: we construct, for each positive integer $k$, a non-simply-connected homology 3-sphere $M_k$, which has a Heegaard diagram admitting an embedding $h(H) \subset S^3$ with the images of the Heegaard curves in the $k$th term of the lower central series of $\pi_1(S^3 - h(H))$. A similar property, previously known for Heegaard diagrams of simply-connected manifolds [Papakyriakopoulos, Ann. of Math. (2) 77(1963), 250-305], is generalized to homology 3-spheres in: **Theorem 2.** If $H_1(M) = (0)$, then there are canonical longitudes $1_1, \ldots, 1_n$ for $H$, with each $1_i \sim w_i$ on $Bd(H)$. (Received August 16, 1971.) (Author introduced by Professor Joseph M. Martin.)

**1-ULC properties of embedded curves and surfaces in $E^3$.

We develop properties of curves and surfaces in $E^3$ (some well known but proved elsewhere by other means) which depend on locally homotopy and homology properties and can be proved without Bing's Side Approximation Theorem: (1) If $S$ is an $(n-1)$-sphere in $S^n$ and $X$ is a closed subset of $S$ that lies on a flat $(n-1)$-sphere in $S^n$, then $S^n - S$ is 1-ULC in $S^n - X$. (2) **Corollary.** If $S$ is a 2-sphere in $E^3$, then there is a 0-dimensional $F$ set $F$ in $S$ such that $F \cup \text{Int } S$ and $F \cup \text{Ext } S$ are 1-ULC. (3) **Corollary.** If $F$ and $S$ are as in (2) and $G$ is a finite graph in $S - F$, then $G$ has a singular regular neighborhood in $E^3$. (4) If a finite graph

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G in $E^3$ has a singular regular neighborhood in $E^3$, then G is tame. (5) **Corollary.** Each disk in $E^3$ contains many tame graphs. (6) **Corollary.** A graph G in $E^3$ is tame if $E^3 - G$ has 1-FLG at each point of G.

(7) **Corollary.** A simple closed curve in $E^3$ is tame if it is deformation-free. (8) An arc in $E^3$ is tame if it satisfies singular analogues of the lpu and lu conditions. (9) **Corollary.** A simple closed curve that is homogeneous by isotopies of $E^3$ and pierces a singular disk is tame. (10) Locally spherical and locally capped $(N - 1)$-spheres in $E^n$ have 1-ULC complements. (11) If C is a crumpled cube in $E^3$, there is an embedding of C in $E^3$ which has 1-ULC complement. We suppress definitions and references for lack of space. (Received August 16, 1971.)

71T-G189. WITHDRAWN.

*71T-G190. STANISLAW G. MROWKA, State University of New York at Buffalo, Amherst, New York 14226. The Sorgenfrey plane $S^2$ is strongly 0-dimensional.

We prove the result in the title ($X$ is strongly 0-dimensional means that $\partial X$ is 0-dimensional). We do not know if $S^3$ is strongly 0-dimensional. We also prove that every continuous real function on $S^n$, $n$ an arbitrary cardinal, is of the 1st Baire class with respect to the Euclidean topology. (Received August 17, 1971.)

*71T-G191. KAPIL D. JOSHI, Indiana University, Bloomington, Indiana 47401. A characterization of plane ANR's by existence of local means.

A local mean (more precisely a local 2-mean) on a topological space $X$ is defined as a pair $(U, m)$ where $U$ is a symmetric neighbourhood of the diagonal $DX$ in $X \times X$ and $m$ is a map from $U$ to $X$ satisfying (i) $m(x, y) = m(y, x)$ for each $(x, y)$ in $U$ and (ii) $m(x, x) = x$ for each $x$ in $X$. If $U = X \times X$, then $m$ becomes a 2-mean as studied by Bacon (Fund. Math. 67(1970), 11-13). **Theorem.** A plane Peano continuum is an absolute neighbourhood retract if and only if it admits a local mean. This characterization is used to give an example of a contractible Peano continuum which admits no 2-mean, thus answering a question raised by Bacon. After these results were obtained another paper by Bacon (Colloq. Math. 21(1970), 211-215) appeared in which he also considers the example, although by an entirely different argument. (Received August 20, 1971.)
In 1943, J. Nielson proved that every element of finite order \( s \) in the mapping class group of a compact, orientable 2-manifold can be realized by a topological mapping of order \( s \) \cite{Acta Math. 75(1943), 23-115}. Using an argument based on results in Teichmüller theory, we extend Nielson's theorem to prove that if an element (of finite or infinite order) in the mapping class group belongs to the normalizer of an element of finite order, then there exists a topological mapping which belongs to the normalizer of the corresponding topological mapping of finite order. As a result, it follows that every element in the normalizer of an element of finite order in the mapping class group can be represented by a topological mapping which is the lift of another topological mapping from an appropriate quotient space. The quotient space will in general have lower genus than the original 2-manifold, and both the original mapping and its projection will have fixed points which depend on the fixed points of the element of finite order and its powers. (Received August 23, 1971.)

Let \( X \) be a compact polyhedron and \( F: X \to X \) a continuous set valued map. It has been shown by B. O'Neill that if, for some fixed \( n \), \( F(x) \) consists of 1 or \( n \) acyclic components for each \( x \), then \( F \) induces a homomorphism on the Čech homology groups which satisfies the Lefschetz Fixed Point Theorem. In general the pair \( (1, n) \) is maximal. However, it is possible that under certain conditions \( F(x) \) may have 1, 2, \ldots, \( n \) acyclic components and still induce a nontrivial homomorphism. \textbf{Theorem.} Let \( F: X \to X \) be a continuous set valued function. Define \( K_j = \{ x: x \in X \text{ and } F(x) \text{ consists of } 1 \text{ or } j \text{ acyclic components} \} \). If \( K_1, K_2, \ldots, K_n \) are polyhedra and if \( X = K_1 \cup K_2 \cup \ldots \cup K_n \), then \( F \) induces a nontrivial homomorphism that satisfies the Lefschetz Fixed Point Theorem. The proof follows by several applications of the Mayer–Vietoris sequence. (Received August 24, 1971.)
corresponding collections of uniformly continuous weak compactifications are isomorphic as lattices. (Received August 26, 1971.)


Consider the following properties that might be possessed by a topological space: (i) $X$ is pseudocompact. (ii) $X$ is $T_{3\frac{1}{2}}$. (iii) $X$ is countably compact. (iv) The diagonal in $X \times X$ is a $G_\delta$-set. (v) The diagonal in $X \times X$ is a countable intersection of closed neighborhoods of it. **Theorem.** Properties (i), (ii), and (v) together imply that $X$ is metrizable. **Remarks.** It is well known (Proizvolov (1964), Anderson (1967)) that (iii) and (v) together imply that $X$ is metrizable. Also well known are examples showing that neither of the combinations ((i), (ii), and (iv)) or ((i) and (v)) are sufficient for metrizability [Gillman and Jerison]. (Received August 31, 1971.)

*71T-G196. ROBION C. KIRBY, University of California, Berkeley, California 94720 and LAURENCE C. SIEBENMANN, University of Paris, Orsay, France. Codimension two locally flat imbeddings.

**Theorem.** A locally flat imbedding $f: M^m \to Q^{m+2}$ of a TOP $m$-manifold without boundary into a TOP $(m+2)$-manifold has up to isotopy a unique normal bundle if $m \neq 2$. (The expected relative version of this holds.) Bjorn Friberg found a gap in the first author's proof of this theorem (with no dimensional restriction) which appears in "Topology of manifolds," edited by Cantrell and Edwards, Markham, 1970, pp. 416-423. Consequently we will publish an old proof built on an announcement in Abstract 69T-640, these Notices 16(1969), 582. The proof involves a relative handle straightening problem which can be solved without surgery. (Received September 2, 1971.)

*71T-G197. LAWRENCE L. KRAJEWSKI, Viterbo College, La Crosse, Wisconsin 54601. The product of a totally normal countably paracompact space and $[0, 1]$ need not be totally normal.

It is shown that Bing's Example G as modified by Michael (Canad. J. Math. 7(1955), 275-279) is an example of a totally normal countably paracompact space whose product with $[0, 1]$ is not totally normal (see Theorem 2' in V. J. Mancuso, "A note on Morita's P-spaces," Proc. Japan Acad. 46(1970), 290-294). The result follows from the following **Theorem.** If $X$ has a point $p$ which is not a $G_\delta$, then $(X \times [0, 1]) \setminus \{(p, 0)\}$ is not countably paracompact. (Received September 2, 1971.)

*71T-G198. ROBERT A. DOOLEY, Oklahoma State University, Stillwater, Oklahoma 74074. Extending a complete convex metric.

A metric $D$ is convex if for every two points $x$ and $z$ there is a third point $y$ such that $D(x, y) + D(y, z) = D(x, z)$. A generalized continuum is a connected, locally compact, separable metric space. It is shown that if $M_1$ is a space with a complete convex metric $D_1$ and $M_2$ is a locally connected generalized continuum whose intersection with $M_1$ is nonempty and compact, there is a complete convex metric for $M_1 \cup M_2$ that extends $D_1$. This result generalizes a theorem of Bing [Bull. Amer. Math. Soc. 55(1949), 812-819] by relaxing the
compactness of the spaces \( M_1 \) and \( M_2 \). By use of this result, several classes of locally connected generalized continua, including spaces that are homeomorphic to an interval of the real line, can be characterized by the type of complete convex metric they admit. (Received August 20, 1971.) (Author introduced by Dr. John M. Jobe.)

\[ \mathbb{T}_\nu \text{-closed spaces, } \nu = 2, 3. \]

\( \mathbb{T}_\nu \) -closure, \( \mathbb{T}_\nu \)-minimality, \( \nu = 2, 3 \), open filters and regular filters are defined as usual, see "\( \mathbb{T}_\nu \)-Abgeschlossenheit und \( \mathbb{T}_\nu \)-Minimalität" by H. Herrlich, Math. Z. 88 (1965), 285–294 and "Some topological properties weaker than compactness" by G. Goss and G. Viglino, Pacific J. Math. 35 (1970). For \( \nu = 2, 3 \), a \( \mathbb{T}_\nu \) space \( X \) is \( \mathbb{T}_\nu \)-c-closed if every continuous function on \( X \) into a \( \mathbb{T}_\nu \) space is closed. A closed subset \( C \) of a topological space \( X \) is said to be \( \mathbb{T}_2 \)-closed (resp. \( \mathbb{T}_3 \)-closed) if for every point \( x \notin C \), there exist two open sets \( O_1, O_2 \) such that \( x \in O_1 \), \( C \subseteq O_2 \) and \( O_1 \cap O_2 = \emptyset \) (resp. if for every closed subset \( D \) disjoint from \( C \), there exist two disjoint open sets \( O_1, O_2 \) such that \( C \subseteq O_1 \) and \( D \subseteq O_2 \)). A filter \( \mathcal{F} \) is said to converge to a set \( A \) if for every nbd \( N \) of \( A \), there is an \( F \in \mathcal{F} \) such that \( F \subseteq N \). Theorem 1. A \( \mathbb{T}_\nu \) space is \( \mathbb{T}_\nu \)-c-closed iff every \( \nu \)-filter (=open filter for \( \nu = 2 \); =regular filter for \( \nu = 3 \)) with \( \mathbb{T}_\nu \)-closed adherence converges to its adherence.

Theorem 2. A \( \mathbb{T}_2 \) space is \( \mathbb{T}_2 \)-c-closed iff it is \( \mathbb{T}_2 \)-closed and weakly seminormal (for definition, see Abstract 682–54–33, these \( \mathcal{N} \)otica 18 (1971), 214). A theorem for Urysohn space similar to Theorem 1 is also obtained. (Received August 23, 1971.) (Author introduced by Professor A. C. Thompson.)

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Replace statement 2" by the following one: "2" \( \langle A, \equiv \rangle \) has totally ordered chains of the order type \( \lambda \) (of the real number system); this follows from Theorem 1 of R. Montague, J. Symbolic Logic 27 (1962), 195–211;"

**Errata. Volume 18**

K. DEMYS. Meta–Cayley hypercomplex algebras must include at least one hypercomplex square root of positive unity, Abstract 683–A5, Page 355.

Instead of \( i^4 i^8 = \epsilon_1 \), read: \( i^4 i^8 = i^8 i^4 = -\epsilon_1 \). The simplest form of the 8 basic matrices is as follows, all governed by the law \( \epsilon_n i = i \epsilon_n = i_0 \), where \( n = 1, 2 \) or 3; also \( \epsilon_2 = \epsilon_3 = 1 \), where \( n = 0, 1, 2 \) or 3. Then \( \epsilon_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \); \( i_0 = \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix} \), where \( i = i_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \); \( i_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \); \( i_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \); \( \epsilon_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \).\( \epsilon_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \); \( \epsilon_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \).

GEORGE M. MULLER. Linear iteration and summability, Abstract 71T-B173, Page 812.

In line 8, replace "\( m = [n/k] \), \( i = n - mk \)" by "\( m = \lfloor (n-1)/k \rfloor + 1 \), \( i = n - 1 \) - (\( m-1 \)k)."
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