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PREREGISTRATION AND RESERVATION FORM .............. A-562
The six hundred ninety-fifth meeting of the American Mathematical Society will be held at the University of Washington in Seattle, Washington, on Saturday, June 17, 1972. The Mathematical Association of America and the Society for Industrial and Applied Mathematics will hold Northwest Sectional Meetings in conjunction with this meeting of the Society. The Association will have sessions on Friday and Saturday, June 16 and 17; the Saturday sessions will be concerned primarily with community college problems. The SIAM sessions will be held on Friday, June 16.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, there will be two invited addresses. Professor Donald E. Sarason of the University of California, Berkeley, will lecture at 11:00 a.m. on Saturday on "Algebras of functions on the unit circle." Professor Peter Crawley of the California Institute of Technology and Brigham Young University will address the Society at 2:00 p.m. on Saturday. The title of his lecture is "Extensions of Ulm's theorem for abelian p-groups." These addresses will be given in room 134 of the General Engineering Building. There will be sessions for contributed papers on Saturday morning. Late papers will be accepted for presentation at the meeting, but late papers will not be listed in the printed program of the meeting. Sessions for contributed papers will be held in the General Engineering Building.

The registration desk will be located in room 111 of the General Engineering Building, and will be open for the duration of the meetings.

Dormitory accommodations will be available on Thursday, Friday, and Saturday nights. These accommodations are not recommended for families with children under twelve. The rates are $3.50 per person per night on a double occupancy basis, and $5.50 per person per night in a single room. No maid service is provided. Reservations should be sent, before June 2, 1972, to Professor Gomer Thomas, Department of Mathematics, University of Washington, Seattle, Washington 98195, and should be accompanied by a check payable to the University of Washington and in the amount covering the length of time desired. Room requests should include the expected time of arrival and the type of accommodations desired.

The following motels and hotels are located within walking distance of the campus:

UNIVERSITY TOWER HOTEL
4507 Brooklyn Avenue, N.E., Zip Code 98105
Phone (206) 634-2000
| Single  | $13.00 up |
| Double  | 15.50 up  |
| Twin    | 18.00 up  |

UNIVERSITY INN
4140 Roosevelt Way, N.E., Zip Code 98105
Phone (206) 632-5055
| Single  | $10.50 up |
| Double  | 12.50 up  |
| Twin    | 15.00 up  |
| Double-double | 19.00 up |

UNIVERSITY MOTEL
4731 12th Avenue, N.E., Zip Code 98105
Phone (206) 522-4724
| Single  | $10.00 up |
| Double  | 12.00 up  |
| Twin    | 12.00 up  |
| Double-double | 16.00 up |

UNIVERSITY TRAVELODGE
4725 25th Avenue, N.E., Zip Code 98105
Phone (206) 525-4612
| Single  | $10.00 up |
| Double  | 13.00 up  |
| Twin    | 14.00 up  |

Reservations should be made directly with
the desired motel or hotel. Breakfast and lunch will be available on Friday in the Union Building on campus. There are numerous restaurants in Seattle, and several of them are located in the area adjacent to the campus.

Seattle is served by major airlines and bus companies. The University Tower Hotel provides transportation for its patrons from the Seattle-Tacoma International Airport. Other air travellers can take the airport limousine to the Olympic Hotel in downtown Seattle and then take a taxi to the university district. Persons driving to the meeting on Interstate 5 should take the 45th N.E. or 50th N.E. exit, drive east to 17th Avenue, N.E., turn right onto 17th Avenue, N.E., and proceed onto the campus. Campus officers on duty at the entrance will give further directions. Special parking permits may be purchased at the registration desk, but will not be needed for the initial arrival on campus.

**PROGRAM OF THE SESSIONS**

The time limit for each contributed paper is ten minutes. To maintain this schedule, the time limit will be strictly enforced.

**SATURDAY, 9:00 A.M.**

**Session on Analysis, Room 222, General Engineering Building**

9:00-9:10

(1) On Wong's solution of the Rice problem. Preliminary report
   Professor William M. Stone, Oregon State University (695-F1)

9:15-9:25

(2) An upper bound on the product of eigenvalues of an optimal linear system
   Dr. Clyde F. Martin, NASA, Ames Research Center, Moffett Field, California (695-C1)

9:30-9:40

(3) A theorem on Cesàro summability for fractional orders of integral
   Mr. Santiranjan Mukhoti, Council of Scientific and Industrial Research, Rafi Marg, New Delhi, India (695-B4)

9:45-9:55

(4) Value distribution of potentials in three real variables
   Professor Peter A. McCoy, Washington State University (695-B2)

10:00-10:10

(5) Flat Banach spaces and smoothness. Preliminary report
   Professor John H. M. Whitfield, University of Washington (695-B3)

10:15-10:25

(6) Generalized Banach algebras. Preliminary report
   Professor Ralph E. DeMarr, University of New Mexico (695-B1)

10:30-10:40

(7) The topological structure of the Bohr compactification of the group of integers. Preliminary report
   Mr. James A. Draper, University of Washington (695-G4)

**SATURDAY, 9:00 A.M.**

**Session on Topology, Room 223, General Engineering Building**

9:00-9:10

(8) Numerical homotopy invariants and epimorphisms in the homotopy category
   Professor Allan L. Edelson, University of California, Davis (695-G7)

9:15-9:25

(9) Fixed point sets of conjugations
   Professor Allan L. Edelson, University of California, Davis (695-G7)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.*
9:30-9:40  On the countable union of cellular decompositions of n-manifolds. Preliminary report
Professor William L. Voxman, University of Idaho (695-G2)

9:45-9:55  A new characterization of tame 2-spheres in \( E^3 \)
Dr. Lawrence R. Weill, Hughes Aircraft Company, Culver City, California (695-G1)

10:00-10:10  Open and proper maps between convergence spaces
Professor Gary D. Richardson, East Carolina University, and
Professor Darrell C. Kent*, Washington State University (695-G5)

10:15-10:25  The topological complementation theorem \( \alpha \) la Zorn
Professor Paul S. Schnare, University of Florida (695-G6)

10:30-10:40  Nonstandard continuities and the closure monad. Preliminary report
Mr. Gary L. Bender, Hughes Aircraft Company, Fullerton, California (695-G3)
(Introduced by Professor Jan Mycielski)

SATURDAY, 9:30 A. M.

Session on Algebra, Room 224, General Engineering Building

9:30-9:40  Reflexive subsemigroups and identity-reflexive congruences on semigroups.
Preliminary report
Professor Takayuki Tamura, University of California, Davis (695-A5)

9:45-9:55  The study of left cancellative semigroups
Professor Donald G. Burnell*, Washington State University, and
Professor Takayuki Tamura, University of California, Davis (695-A4)

10:00-10:10  Dense subrings of \( \text{Hom}_R(M,M) \), for \( R \) QF, and \( R M \) a faithful projective-injective.
Preliminary report
Dr. Stuart A. Seligson, Redwood City, California (695-A2)

10:15-10:25  Relations in categories
Dr. Jeanne Meisen, University of British Columbia (695-A3)
(Introduced by Professor Armin Frei)

10:30-10:40  Primes within unit distances
Professor Earl F. Ecklund, Jr., Northern Illinois University, and
Professor James H. Jordan*, Washington State University (695-A1)

SATURDAY, 11:00 A. M.

Invited Address, Room 134, General Engineering Building
Algebras of functions on the unit circle
Professor Donald E. Sarason, University of California, Berkeley

SATURDAY, 2:00 P. M.

Invited Address, Room 134, General Engineering Building
Extensions of Ulm's theorem for abelian p-groups
Professor Peter Crawley, California Institute of Technology and
Brigham Young University

Eugene, Oregon
Kenneth A. Ross
Associate Secretary
The seventy-seventh summer meeting of the American Mathematical Society will be held at Dartmouth College, Hanover, New Hampshire, from Tuesday, August 29, through Friday, September 1, 1972. All sessions of the meeting will take place on the campus of the college. The times listed for events of the meeting are EASTERN DAYLIGHT SAVING TIME throughout.

There will be two sets of Colloquium Lectures. Professor Stephen Smale of the University of California, Berkeley, will present four lectures entitled "Applications of global analysis to biology, economics, electrical circuits, and celestial mechanics." These addresses will be given on Tuesday, August 29, at 1:30 p.m. and on Wednesday, Thursday, and Friday at 9:00 a.m. The other Colloquium Lecturer will be Professor John T. Tate of Harvard University. His topic will be "The arithmetic of elliptic curves." Professor Tate's four lectures will be given on Tuesday, August 29, at 2:45 p.m. and on Wednesday, Thursday, and Friday at 10:15 a.m. The remaining Colloquium Lectures will be presented in the Center Theater which is also located in Hopkins Center. The AMS Committee on Employment and Educational Policy will hold a panel discussion on Tuesday, August 29, at 4:00 p.m. in Spaulding Auditorium.

By invitation of the Committee to Select Hour Speakers for Annual and Summer Meetings, there will be nine invited hour addresses: Professor George E. Andrews of The Pennsylvania State University, "A general theory of identities of the Rogers-Ramanujan type," 1:30 p.m. on Friday, September 1; Professor R. C. Bose of Colorado State University, "Representation of non-Desarguesian projective planes in projective hyperspaces," 2:45 p.m. on Friday, September 1; Professor Glen E. Bredon of Rutgers University, "Locally smooth actions on manifolds," 1:30 p.m. on Thursday, August 31; Professor Chuan C. Hsiung of Lehigh University, "Submanifolds of Riemannian manifolds," 1:30 p.m. on Friday, September 1; Professor Joachim Lambek of McGill University, "Noncommutative localization," 2:45 p.m. on Friday, September 1; Professor Seymour Sherman of Indiana University, "Monotonicity and magnetism," 2:45 p.m. on Thursday, August 31; Professor Michael Shub of the University of California, Santa Cruz, "Dynamical systems, filtrations, and entropy," 4:00 p.m. on Friday, September 1; Professor Charles C. Sims of Rutgers University, "The construction of large permutation groups," 1:30 p.m. on Thursday, August 31; and Professor Herbert S. Wilf of the University of Pennsylvania, "Bounds for the chromatic number," 2:45 p.m. on Thursday, August 31.

Two special sessions of selected twenty-minute papers will be scheduled. Professor Joseph B. Keller of the Courant Institute of Mathematical Sciences, New York University, is arranging a session on asymptotic and perturbation methods in fluid mechanics and wave propagation from 1:30 p.m. to 5:00 p.m. on Friday, September 1. Professor Gerald E. Sacks of the Massachusetts Institute of Technology is organizing a session on recursion theory from 1:30 p.m. to 3:30 p.m. on Thursday, August 31, and from 10:00 a.m. to noon on Friday, September 1.

Sessions for contributed ten-minute papers will be held during the morning on Wednesday, Thursday, and Friday. Abstracts of contributed papers should be sent to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904; the deadline for receipt of abstracts is July 5, 1972. There is no limit on the number of papers that will be accepted for presentation. No provisions will be made for late papers.

This meeting will be held in conjunction with meetings of the Institute of Math-
ematical Statistics, the Mathematical Association of America, and Pi Mu Epsilon. The Institute of Mathematical Statistics will meet from Monday, August 28, through Thursday, August 31, Professor Peter Huber, Eidgenössische Technische Hochschule, Zürich, will present the Wald Lectures on the subject of robustness. The Mathematical Association of America will meet from Monday through Wednesday. The Earle Raymond Hedrick Lectures, sponsored by the Association, will be given by Professor Peter Lax of the Courant Institute of Mathematical Sciences, New York University; the title of the lectures will be "Scattering theory." Pi Mu Epsilon will meet concurrently with the Association and the Society. Dr. John G. Kemeny will address the fraternity on Tuesday evening, August 29, at 8:00 p.m.; the title of his lecture will be "Mathematical models and the computer."

COUNCIL AND BUSINESS MEETING

The Council of the Society will meet at 5:15 p.m. on Tuesday, August 29, in the Drake Room of Hopkins Center. The Business Meeting of the Society will be held in the Spaulding Auditorium at 4:00 p.m. on Thursday, August 31. The Steele Prizes will be awarded prior to the Business Meeting.

An amendment to the bylaws, requested by the Trustees, has been approved by the Council. It provides for an associate treasurer, who is an ex officio member of the Council with one vote and an ex officio member of the Board of Trustees. The amendment will be offered to the Business Meeting for approval in accord with Article XIII of the bylaws.

PREREGISTRATION AND REGISTRATION

Dartmouth College will operate a Housing Bureau as a special service to those attending the meetings. Since Hanover and the surrounding country is a popular tourist area, and hotels and motels are generally filled during the latter part of August, it is urged that arrangements for accommodations be made early. The College has made arrangements with several nearby motels to hold rooms for participants, in addition to the dormitory rooms. The use of the College Housing Bureau requires preregistration for the meeting. In addition to aid in finding preferred accommodations, those who preregister will pay a lower registration fee than those who register at the meeting, as indicated in the schedule below. Preregistrants will be able to pick up their badges and programs when they arrive at the meeting.

The registration fees for the meeting are as follows:

<table>
<thead>
<tr>
<th></th>
<th>Preregistration</th>
<th>At meeting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Member</td>
<td>$ 5</td>
<td>$ 7</td>
</tr>
<tr>
<td>Student or unemployed member</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Nonmember</td>
<td>10</td>
<td>14</td>
</tr>
</tbody>
</table>

There will be no extra charge for members of the families of registered participants.

The unemployed status refers to any member currently unemployed and actively seeking employment. It is not intended to include members who have voluntarily resigned or retired from their latest position.

Students are considered to be only those currently working toward a degree who do not receive an annual compensation totalling more than $7,000 from employment, fellowships, and scholarships.

Checks for the preregistration fee (plus $15 room deposit for motel rooms only) should be mailed not later than August 1. Participants need not utilize the services of the Housing Bureau to make reservations but it is essential to complete the preregistration section of the form found on the last page of these Notices to take advantage of the lower registration fees.

The Registration Desk will be located at the "Top of the Hop," on the second floor of the Hopkins Center. It will be open on Sunday from 2:00 p.m. to 8:00 p.m.; on Monday from 8:00 a.m. to 5:00 p.m.; on Tuesday, Wednesday, and Thursday from 8:30 a.m. to 4:30 p.m.; and on Friday from 8:30 a.m. to 1:00 p.m.

EMPLOYMENT REGISTER

The Mathematical Sciences Employment Register will be maintained from 9:00 a.m. to 4:00 p.m. on Tuesday, August 29, and from 9:00 a.m. to 5:40 p.m. on Wednesday and Thursday, August 30–31,
### SUMMARY OF ACTIVITIES

**INSTITUTE OF MATHEMATICAL STATISTICS**

**MATHEMATICAL ASSOCIATION OF AMERICA**

#### SUNDAY, August 27

<table>
<thead>
<tr>
<th>Time</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>9:00 a.m.</td>
<td>MAA Board of Governors</td>
</tr>
<tr>
<td>5:30 p.m.</td>
<td>Buffet Supper - Alumni Hall and Hanover Inn</td>
</tr>
<tr>
<td>8:00 p.m.</td>
<td>Feature Length Movie - 28 Silsby</td>
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</table>

#### MONDAY, August 28

<table>
<thead>
<tr>
<th>Time</th>
<th>Activity</th>
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</thead>
<tbody>
<tr>
<td>9:00 a.m.</td>
<td>Welcome on behalf of the College</td>
</tr>
<tr>
<td>9:00 a.m.</td>
<td>Computer Course: BASIC and Dartmouth Time</td>
</tr>
<tr>
<td>9:15 a.m.</td>
<td>THE EARLE RAYMOND HEDRICK LECTURES:</td>
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<tr>
<td></td>
<td>Scattering Theory, Lecture I</td>
</tr>
<tr>
<td></td>
<td>Peter D. Lax</td>
</tr>
<tr>
<td>10:00 a.m.</td>
<td>Ledyard Canoe Club - canoe and kayak lessons</td>
</tr>
<tr>
<td>10:30 a.m.</td>
<td>JOINT SESSION -- THE PRESIDENT'S COMMISSION</td>
</tr>
<tr>
<td></td>
<td>ON FEDERAL STATISTICS, AND BEYOND</td>
</tr>
<tr>
<td>1:00 p.m.</td>
<td>INVITED PAPERS I</td>
</tr>
<tr>
<td>1:30 p.m.</td>
<td>THE EARLE RAYMOND HEDRICK LECTURES:</td>
</tr>
<tr>
<td></td>
<td>Lecture II</td>
</tr>
<tr>
<td></td>
<td>Peter D. Lax</td>
</tr>
<tr>
<td>2:40 p.m.</td>
<td>PANEL DISCUSSION: Student Self-Paced</td>
</tr>
<tr>
<td></td>
<td>Calculus</td>
</tr>
<tr>
<td>3:00 p.m.</td>
<td>WALD LECTURES:</td>
</tr>
<tr>
<td></td>
<td>Lecture I - Peter Huber</td>
</tr>
<tr>
<td>3:00 p.m.</td>
<td>Organized Softball - all ages - College Green</td>
</tr>
<tr>
<td></td>
<td>Ledyard Canoe Club - canoe and kayak lessons</td>
</tr>
<tr>
<td></td>
<td>Boathouse</td>
</tr>
<tr>
<td>4:15 p.m.</td>
<td>INVITED PAPERS II</td>
</tr>
<tr>
<td>4:45 p.m.</td>
<td>Challenge Cup Softball - College Green</td>
</tr>
<tr>
<td>5:00 p.m.</td>
<td>Rathskeller: draught beer by the pitcher</td>
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<tr>
<td></td>
<td>Tavern Room, Hanover Inn</td>
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<tr>
<td>7:00 p.m.</td>
<td>IMS Council Meeting</td>
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<tr>
<td>7:30 p.m.</td>
<td>Feature Length Movie - 28 Silsby</td>
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<tr>
<td>8:00 p.m.</td>
<td>Concert: Guarnieri String Quartet - Spaulding</td>
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<tr>
<td></td>
<td>Auditorium</td>
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#### TUESDAY, August 29

<table>
<thead>
<tr>
<th>Time</th>
<th>Activity</th>
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<tbody>
<tr>
<td>8:00 a.m.</td>
<td>Dartmouth Outing Club - all day climb - Mt. Moosilauke</td>
</tr>
<tr>
<td>8:30 a.m.</td>
<td>CONTRIBUTED PAPERS I</td>
</tr>
<tr>
<td>9:00 a.m.</td>
<td>THE EARLE RAYMOND HEDRICK LECTURES:</td>
</tr>
<tr>
<td></td>
<td>Lecture III</td>
</tr>
<tr>
<td></td>
<td>Peter D. Lax</td>
</tr>
<tr>
<td>9:00 a.m.</td>
<td>Computer Course: BASIC and Dartmouth Time</td>
</tr>
<tr>
<td></td>
<td>Sharing - Filene Auditorium</td>
</tr>
<tr>
<td>10:00 a.m.</td>
<td>Ledyard Canoe Club - canoe and kayak lessons</td>
</tr>
<tr>
<td>10:00 a.m.</td>
<td>MAA Business Meeting</td>
</tr>
<tr>
<td>11:00 a.m.</td>
<td>SPECIAL INVITED PAPERS I</td>
</tr>
<tr>
<td>11:10 a.m.</td>
<td>INVITED ADDRESS: Is mathematics relevant</td>
</tr>
<tr>
<td></td>
<td>and, if so, to what?</td>
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<tr>
<td></td>
<td>Felix Browder</td>
</tr>
<tr>
<td>12:15 p.m.</td>
<td>PI MU EPSILON - Council Luncheon</td>
</tr>
<tr>
<td>1:00 p.m.</td>
<td>CONTRIBUTED PAPERS II</td>
</tr>
<tr>
<td>Time</td>
<td>Event</td>
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<tr>
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<td>----------------------------------------------------------------------</td>
</tr>
<tr>
<td>1:30 p.m.</td>
<td><strong>TUESDAY, August 29</strong></td>
</tr>
</tbody>
</table>
|          | **COLLOQUIUM LECTURES:** Applications of global analysis to biology, economics, electrical circuits, and celestial mechanics, Lecture I  
Stephen Smale |
| 2:45 p.m. | **COLLOQUIUM LECTURES:** The arithmetic of elliptic curves, Lecture I  
John T. Tate |
| 3:00 p.m. | Organized Softball - all ages - College Green  
Ledyard Canoe Club - canoe and kayak lessons - Boathouse |
| 3:15 p.m. | **EMPLOYMENT PANEL**                                                |
| 3:45 p.m. | **SPECIAL INVITED PAPERS II**                                       |
| 4:00 p.m. | **COLLOQUIUM LECTURES II**                                          
Stephen Smale |
| 4:15 p.m. | **INVITED PAPERS III**                                             |
| 5:45 p.m. | **INVITED PAPERS IV**                                               |
| 6:30 p.m. | **COLLOQUIUM LECTURES II**                                          
Peter Huber |
| 7:00 p.m. | **COLLOQUIUM LECTURES II**                                          
Peter Huber |
| 7:30 p.m. | **COLLOQUIUM LECTURES II**                                          
Peter Huber |
| 8:00 p.m. | **COLLOQUIUM LECTURES II**                                          
Peter Huber |
| 8:30 a.m. | **PI MU EPSILON - Dutch Treat Breakfast**                           |
| 9:00 a.m. | **COLLOQUIUM LECTURES II**                                          
John T. Tate |
| 9:00 a.m. | **SESSIONS OF TEN MINUTE PAPERS**                                   |
| 9:00 a.m. | **SESSIONS OF TEN MINUTE PAPERS**                                   |
| 10:00 a.m. | Computer Course: BASIC and Dartmouth Time Sharing - Filene Auditorium |
| 10:15 a.m. | **INVITED PAPERS III**                                             |
| 10:40 a.m. | **PI MU EPSILON - CONTRIBUTED PAPERS**                              |
| 1:00 p.m. | **INVITED PAPERS IV**                                               |
| 2:00 p.m. | **COLLOQUIUM LECTURES II**                                          
Peter Huber |
| 3:00 p.m. | Organized Softball - all ages - College Green  
Ledyard Canoe Club - canoe and kayak lessons - Boathouse |
| 4:00 p.m. | **PI MU EPSILON - CONTRIBUTED PAPERS**                              |
| 5:30 p.m. | **INVITED ADDRESS:**                                               
What every college president should know about mathematics  
John G. Kemeny |
| 7:00 p.m. | **PI MU EPSILON - CONTRIBUTED PAPERS**                              |
| 7:30 p.m. | **COLLOQUIUM LECTURES II**                                          
Peter Huber |
| 9:00 p.m. | **COLLOQUIUM LECTURES II**                                          
Peter Huber |
| midnight | **COLLOQUIUM LECTURES II**                                          
Peter Huber |
<table>
<thead>
<tr>
<th>Time</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>7:30 a.m.</td>
<td>Shelburne Museum Trip</td>
</tr>
<tr>
<td>8:00 a.m.</td>
<td>Dartmouth Outing Club - Mt. Lafayette</td>
</tr>
<tr>
<td>8:30 a.m.</td>
<td>CONTRIBUTED PAPERS IV</td>
</tr>
<tr>
<td>9:00 a.m.</td>
<td>COLLOQUIUM LECTURES III</td>
</tr>
<tr>
<td></td>
<td>Stephen Smale</td>
</tr>
<tr>
<td>9:00 a.m.</td>
<td>SESSIONS OF TEN MINUTE PAPERS</td>
</tr>
<tr>
<td>9:00 a.m.</td>
<td>Computer Course - BASIC and Dartmouth Time Sharing - Filene Auditorium</td>
</tr>
<tr>
<td>10:00 a.m.</td>
<td>Ledyard Canoe Club - canoe and kayak lessons - Boathouse</td>
</tr>
<tr>
<td>10:15 a.m.</td>
<td>INVITED PAPERS V</td>
</tr>
<tr>
<td></td>
<td>COLLOQUIUM LECTURES III</td>
</tr>
<tr>
<td></td>
<td>John T. Tate</td>
</tr>
<tr>
<td>10:30 a.m.</td>
<td>INVITED PAPERS VI</td>
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<tr>
<td></td>
<td>SESSIONS OF TEN MINUTE PAPERS</td>
</tr>
<tr>
<td>1:00 p.m.</td>
<td>SPECIAL INVITED PAPERS III</td>
</tr>
<tr>
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<td>INVITED ADDRESS: Glen E. Bredon</td>
</tr>
<tr>
<td></td>
<td>Locally smooth actions on manifolds</td>
</tr>
<tr>
<td></td>
<td>INVITED ADDRESS: Charles C. Sims</td>
</tr>
<tr>
<td></td>
<td>The construction of large permutation groups</td>
</tr>
<tr>
<td>1:30 p.m.</td>
<td>SPECIAL SESSION OF TWENTY MINUTE PAPERS: Recursion Theory</td>
</tr>
<tr>
<td>2:45 p.m.</td>
<td>INVITED ADDRESS: Seymour Sherman</td>
</tr>
<tr>
<td></td>
<td>Monotonicity and magnetism</td>
</tr>
<tr>
<td>2:45 p.m.</td>
<td>INVITED ADDRESS: Herbert S. Wilf</td>
</tr>
<tr>
<td></td>
<td>Bounds for the chromatic number</td>
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<tr>
<td>3:00 p.m.</td>
<td>SPECIAL INVITED PAPERS III</td>
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<td>Organized Softball - all ages - College Green</td>
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<td>Ledyard Canoe Club - canoe and kayak lessons - Boathouse</td>
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<tr>
<td>4:00 p.m.</td>
<td>AMS Business Meeting</td>
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<tr>
<td>8:00 p.m.</td>
<td>Concert - Spaulding Auditorium</td>
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<tr>
<td>9:00 a.m.</td>
<td>COLLOQUIUM LECTURES IV</td>
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<td>Stephen Smale</td>
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<tr>
<td>9:00 a.m.</td>
<td>SESSIONS OF TEN MINUTE PAPERS</td>
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<tr>
<td>10:00 a.m.</td>
<td>SPECIAL SESSION OF TWENTY MINUTE PAPERS: Recursion Theory</td>
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<tr>
<td>10:15 a.m.</td>
<td>INVITED PAPERS VI</td>
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<td>COLLOQUIUM LECTURES IV</td>
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<td>John T. Tate</td>
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<tr>
<td>10:30 a.m.</td>
<td>INVITED PAPERS VI</td>
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<td>SESSIONS OF TEN MINUTE PAPERS</td>
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<tr>
<td>1:30 p.m.</td>
<td>INVITED ADDRESS: George E. Andrews</td>
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<td>A general theory of identities of the Rogers-Ramanujan type</td>
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<td>1:30 p.m.</td>
<td>INVITED ADDRESS: Chuan C. Hsiung</td>
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<td>Submanifolds of Riemannian manifolds</td>
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<td>1:30 p.m.</td>
<td>SPECIAL SESSION OF TWENTY MINUTE PAPERS: Asymptotic and perturbation</td>
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<td>methods in fluid mechanics and wave propagation</td>
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<tr>
<td>2:45 p.m.</td>
<td>INVITED ADDRESS: Joachim Lambek</td>
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<td>Noncommutative localization</td>
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<td>2:45 p.m.</td>
<td>INVITED ADDRESS: R. C. Bose</td>
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<td>Representation of non-Desarguesian projective planes in projective</td>
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<td>hyperspaces</td>
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<td>4:00 p.m.</td>
<td>INVITED ADDRESS: Michael Shub</td>
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<tr>
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<td>Dynamical systems, filtrations, and entropy</td>
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in the Alumni Hall of the Hopkins Center. Alumni Hall is located on the second floor of the Center adjacent to the registration area.

EXHIBITS

Book exhibits and exhibits of educational media will be displayed in the Art Studios, located on the first floor of the Hopkins Center across from the Snack Bar, on Monday, August 28, from 1:00 p.m. to 5:00 p.m.; on Tuesday and Wednesday, August 29 and 30, from 9:00 a.m. to 5:00 p.m.; and on Thursday, August 31, from 9:00 a.m. to 1:00 p.m. All participants are encouraged to visit the exhibits sometime during the meeting.

BOOK SALE

Books published by the Society will be sold for cash prices below the usual prices when the same books are sold by mail.

RESIDENCE HALL HOUSING

College facilities have been set aside for the exclusive use of the joint Mathematics Meetings participants. All dormitories are within a five-minute walk of the dining hall, auditoria, and lecture halls to be used during the meeting. A variety of College dormitory rooms including single rooms, two and three room suites, suites with private half baths, and rooms and suites with full baths, are available. Most dormitories have coin-operated washers and dryers as well as ironing facilities.

College rooms can be occupied from 1:00 p.m., Saturday, August 26, to 1:00 p.m., Saturday, September 2. Dormitory clerks will be available from 8:00 a.m. until 9:00 p.m. Room registration for all College rooms will take place in Hopkins Center next to the Registration Desk and will be open from 1:00 p.m. to 9:00 p.m. on Saturday, August 26, and continuously thereafter from 8:30 a.m. on Sunday, August 27, to 10:00 a.m. Friday, September 1.

The daily rate per person is as follows:

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<td>Suites or singles without bath</td>
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<td>Suites or singles with full bath*</td>
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* N.B. There are only a limited number of rooms and suites with full baths and preference will be given to families making reservations prior to July 18.

The rate for children given above assumes occupancy in suite with parents; the first child in a separate suite will be subject to the full adult rate. Since the College cannot provide cribs and there is no rental agency in town, parents are advised to bring their own if needed. There is no charge for children in cribs sharing a room with their parents.

The rate for children given above assumes occupancy in suite with parents; the first child in a separate suite will be subject to the full adult rate. Since the College cannot provide cribs and there is no rental agency in town, parents are advised to bring their own if needed. There is no charge for children in cribs sharing a room with their parents.

FOOD SERVICES

Thayer Hall, the College dining hall, will be open for breakfast on Monday, August 28, and will serve all meals through luncheon on Friday, September 1, except for dinner Wednesday, August 30. Hours of service and prices for individual meals are:

- Breakfast 7:15 a.m.-8:45 a.m. $1.25
- Luncheon 11:30 a.m.-1:00 p.m. 1.75
- Dinner 5:00 p.m.-6:30 p.m. 3.00

On Sunday evening, August 27, from 5:30 p.m. to 8:30 p.m. the Hanover Inn will provide a special Conference Buffet for $2.75 per person.

A special contract meal ticket may be purchased at registration for $20 (children under six for $10). It includes all thirteen meals from breakfast on Monday, August 28, through luncheon on Friday, September 1, with the exception of the clambake on Wednesday, August 30.

Available also are a number of local restaurants and nearby inns. There is a Snack Bar at the Hopkins Center; the hours of operation will be posted in the registration area.

HOTELS AND MOTELS

The College has reserved several blocks of rooms at nearby motels which cannot be held after August 1. Those desiring accommodations in motels are encouraged to preregister prior to August 1.
1. Hopkins Center
2. Hanover Inn
3. Thornton Hall
4. Dartmouth Hall
5. Wentworth Hall
6. Thayer Dining Hall

DARTMOUTH
COLLEGE
HANOVER · NEW HAMPSHIRE
A $15 deposit is required for motel reservations. Area motels are listed below with the number of rooms reserved, telephone numbers, and their distance from the Dartmouth campus. Participants wishing motel accommodations should make their reservations, with deposit, prior to August 1 through the College Conference Bureau, using the form provided on the last page of these Notice). The price and assignment will be confirmed by the motel upon receipt of the reservation deposit from the College Conference Bureau. All prices are subject to change without notice. The motels and hotels are listed below with the following coded information: FP - Free Parking; SP - Swimming Pool; AC - Air Conditioned; TV - Television; CL - Cocktail Lounge; RT - Restaurant.

HANOVER INN
(603) 643-4300
Hanover, N. H. 03755 - 100 rooms
Singles $14-$24
Doubles 17-30
Code: FP-TV-partial AC-CL-RT
Next to Conference Center

HANOVER INN MOTOR LODGE
(603) 643-4400
Hanover, N. H. 03755 - 30 rooms
Singles $13-$15
Doubles 18-22
Code: FP-TV-AC
Next to Conference Center

NORWICH INN
(802) 649-1143
Norwich, Vt. 05055 - 15 rooms
Doubles $16-$20
Code: FP-TV-CL-RT
1 mile from campus

CHIEFTAIN MOTEL
(603) 643-2550
Hanover, N. H. 03755 - 15 rooms
Singles $14
Doubles 18
(all double rooms have two double beds)
Code: FP-TV-Continental Breakfast
2 miles from campus

SUNSET MOTEL
(603) 298-2721
West Lebanon, N.H. 03784 - 13 rooms
Singles $15
Doubles 19
Code: FP-TV
3 miles from campus

HOWARD JOHNSON'S MOTOR INN
(802) 295-3015
White River Jct., Vt. 05001 - 75 rooms
Singles $15
Doubles 21
Code: FP-SP-AC-TV-CL-RT-Sauna
5 miles from campus

HOLIDAY INN
(802) 295-7537
White River Jct., Vt. 05001 - 50 rooms
Singles $15
Doubles 22
Code: FP-AC-TV-CL-RT-SP-Sauna
5 miles from campus

COACH AND FOUR
(802) 295-2210
White River Jct., Vt. 05001 - 20 rooms
Singles $16-$17
Doubles 18-20
Code: FP-TV-AC
5 miles from campus

MAPLE LEAF MOTEL
(802) 295-2817
White River Jct., Vt. 05001 - 18 rooms
Singles $12-$15
Doubles 16-18
Code: FP-AC-TV-Continental Breakfast
7 miles from campus

PARKING
No permits will be required for on-campus parking. Maps showing the location of the various college parking lots open to participants will be available at the Registration Desk. No on-street overnight parking is allowed in Hanover.

CAMPING
Campsites are available at the Storrs Pond Recreation area, a Hanover-owned park, two miles from the campus. Tents or trailers can be accommodated but no electrical or water hookups are furnished. Hot water showers and grills are available. The charge is $3 per night, and reservations should be made directly with the Hanover Improvement Society, Nugget Building, Hanover, N.H. 03755.

BOOKSTORE
There is no bookstore on campus. There are several good bookstores in the town, however, which are open daily during normal working hours.

LIBRARIES
Baker Library, Dartmouth's undergraduate library, will be open Monday through Friday from 8:00 a.m. to 5:00 p.m. It is located at the center of the campus and contains many interesting
collections as well as the Orozco murals, the College's most famous work of art. The Mathematics Library is located on the second floor of Bradley Hall and will be open from 9:00 a.m. to 10:00 p.m. daily.

MEDICAL SERVICES

The 400 bed Mary Hitchcock Memorial Hospital and Clinic is two blocks north of the Conference Center. The Campus Police may be called (646-2234) for transportation to the hospital at any time day or night.

ENTERTAINMENT AND FAMILY EVENTS

Dartmouth College has planned a program for mathematicians and their families to take advantage of the informality and beauty of the North Country of New England. The program of recreational and educational events is for families as well as for participants, and families are encouraged to attend.

Hopkins Center will offer two formal concerts during the meeting. The Dartmouth College Department of Mathematics will arrange a third concert of Handel's "The Messiah" for anyone interested in singing (or listening). Dartmouth College will supply music. The Guarnieri String Quartet will present the first formal concert on Monday evening, August 28, at 8:00 p.m. in Spaulding Auditorium. The second formal concert, probably a folk singer, is tentatively scheduled for Thursday evening.

On Wednesday, August 30, Thayer Hall will be closed for dinner, and a New England Clambake will be offered, weather permitting, at the College Park on the banks of the Connecticut River. Those wishing to attend should indicate the number of tickets desired on the preregistration form. Only a limited number of tickets will be available at the conference based on the preliminary estimates determined from the preregistration form. The cost will be $6.50 for the complete bake of clams, lobster, and chicken, or $3 (children under six, $2) for the chicken portion only. In case of rain, the clam bake will be served at Thayer Hall.

If sufficient interest is indicated on the reservation form, a bus tour will go to the Shelburne Museum on Thursday, August 31. One of the foremost attractions of New England, the 45-acre museum is located on the shores of Lake Champlain south of Burlington, Vermont. Its extensive "collection of collections" depicts the early life of New England. The day-long trip will cost $11 per person and will include chartered bus transportation, Museum entrance fee, and lunch.

A four-hour computer course introducing the Dartmouth Time-Sharing system and the computer language BASIC will be provided for families. A fee of $15 per family will include extensive computer time as well as lectures. The course will be held on Monday through Thursday at 9:00 a.m. in Filene Auditorium, Bradley Hall.

The Dartmouth Outing Club will lead two one-day climbs for anyone over the age of 12. The Ledyard Canoe Club will provide (in addition to canoe rentals) a series of four one-hour canoeing and kayaking lessons at a minimal charge for participants and families. The ability to swim is required.

An organized program of softball will take place each afternoon on the Hanover Green. On Monday at 4:45 p.m. the Department of Mathematics softball team will play any challenge team for the MAA/IMS/PME/AMS Challenge Cup to be awarded by the Dartmouth College Office of Summer Programs.

The College's athletic facilities will be available throughout the meeting during hours posted at the Information Center, adjacent to the Registration Desk at the "Top of the Hop," Hopkins Center. Tennis, indoor swimming, volleyball, and canoeing are located on the campus; the golf course and hiking trails are less than a mile from the campus. Storrs Pond, Hanover's town recreation area, located two miles from the campus, provides an outdoor Olympic-sized pool, a pond, picnic areas, trail hiking, and campsites for a nominal fee. Shuttle transportation will be provided.

Free movies, selected with children in mind, will be shown Sunday through Thursday nights. Other activities and items of interest include the Dartmouth College Museum, art exhibits, campus tours, the Daniel Webster Cottage, and numerous
nearby lakes and scenic drives.

A nursery play-school for children ages one to six will be available Monday through Friday from 9:00 a.m. to 1:00 p.m. The charge will be $2 per day per child. Group babysitters will be available in the dormitories during the evening hours. The charge for this service will be $1 per child per evening. A list of individual babysitters will be available at the Registration Desk.

The Tavern Room at the Hanover Inn will be open exclusively for registrants and their guests from 5:00 p.m. to midnight daily. Draught beer will be served in this pub-like location.

**TRAVEL**

Bus and air transportation serve Hanover from Boston and New York. Executive Airlines from Boston and Northeast Airlines from New York use the Lebanon Regional Airport located five miles from Hanover. Bus service is through White River Junction, Vermont, located approximately seven miles away. Taxis to Hanover are available at both locations. The two main auto routes are Interstate 91 from Connecticut to Exit 13 in Norwich, Vermont, or Interstate 93 from Boston, connecting with Interstate 89 at Concord, New Hampshire, to the Hanover Exit 18 which is four miles from campus.

Those planning to fly to Hanover should be warned that early morning flights are cancelled with some frequency in August because of ground fog. For those interested in renting a car at Logan Airport in Boston and driving to Hanover, the driving time is two and one-half hours. The Hanover Avis office is one block from Hopkins Center.

A travel desk will be maintained daily in the registration area by a local travel agency to assist conferees in making or changing travel plans.

**WEATHER**

The average maximum temperature during this week is 67.2°. The temperature may range from 55° to 80° in any one day with the evenings somewhat cool. Rainfall in August averages 3.07”.

**MAIL AND MESSAGE CENTER**

Individuals may be addressed at Mathematics Meeting, Hopkins Conference Center, Dartmouth College, Hanover, New Hampshire 03755. The telephone number of the Message Center will be (603) 646-3218. Messages may be left for registrants at any time.

**COMMITTEE**

H. L. Alder (ex officio), Mrs. Sandi J. Garland, Kenneth I. Gross (chairman), Walter H. Gottschalk (ex officio), Reese T. Prosser, William E. Slesnick, G. L. Walker (ex officio), and H. D. Weed.

Walter H. Gottschalk
Associate Secretary
Middletown, Connecticut

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**The Six Hundred Ninety-Seventh Meeting**
**Brown University**
**Providence, Rhode Island**
**October 28, 1972**

The six hundred ninety-seventh meeting of the American Mathematical Society will be held at Brown University, Providence, Rhode Island, on Saturday, October 28, 1972.

It is expected that there will be two one-hour addresses. The names of the lecturers and the titles of their addresses will appear in the August issue of these Notices.

There will be sessions for contributed papers both morning and afternoon. Abstracts for contributed papers should be sent to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904, so as to arrive prior to the deadline of September 7, 1972.

Walter H. Gottschalk
Associate Secretary
Middletown, Connecticut
MEMORANDA TO MEMBERS

INFORMAL SESSIONS AT SUMMER AND ANNUAL MEETINGS

The Council has approved a recommendation of the Committee to Monitor Problems in Communication that the Society experiment with arranging informal sessions at the summer and annual meetings. These sessions would be similar to, but not necessarily exactly like, those that Saunders Mac Lane has been organizing very successfully in category theory for the past few years. Paul Halmos arranged for an equally successful informal session in noncommutative approximation theory at the annual meeting in January of this year.

The basic idea for these sessions is to make it easy for a small group of mathematicians who have specialized interests in common to get together for an afternoon or evening and discuss their work, present short papers, and generally meet and talk to other mathematicians in a particular field. Organizers of these sessions might wish to arrange for a series of ten- or twenty-minute talks, or merely set aside time for unorganized discussions, or instead limit the time to one or two invited speakers. In any event, organizers are invited to experiment freely.

Members who are interested in arranging informal sessions for future annual and summer meetings should write directly to the associate secretary in charge of the meeting. The associate secretary will arrange for meeting space and have announcements carried in the Notices.

CHANGES IN ABSTRACTS

The Board of Trustees has approved a new policy on changes made in abstracts after the deadline. A fee of ten dollars will be charged for any changes made in abstracts after the deadline; no charge will be made for the withdrawal of abstracts, however. The deadlines appear on the inside front cover of each issue of the Notices; for abstracts of papers to be read at a meeting, the deadline is shown in the fourth column of the Calendar, and the deadline for abstracts of papers not being read at a meeting (by title) appears as a footnote. This policy will become effective for abstracts of contributed papers that will be presented at the 1972 summer meeting at Dartmouth (deadline, July 5, 1972), and for abstracts scheduled to appear in the August 1972 Notices but not to be presented in person at the meeting (June 28, 1972).

NEWS ITEMS AND ANNOUNCEMENTS

THEODORE VON KÁRMÁN PRIZE IN APPLIED MATHEMATICS

The first Theodore von Kármán Prize in Applied Mathematics, established by the Society for Industrial and Applied Mathematics, will be awarded to Sir Geoffrey I. Taylor, internationally-acclaimed British authority in applied mathematics and mechanics. The award will be made by Professor C. C. Lin on Tuesday, June 13, 1972, at the twentieth anniversary meeting of SIAM in Philadelphia, Pennsylvania.

SALEM PRIZE

The Salem Prize for 1971 has been awarded to Charles Fefferman of the University of Chicago for his work in harmonic analysis. This prize, created in 1968, is given every year to a young mathematician for an outstanding paper on Fourier series and related questions. Previous winners were Nicholas Varopoulos in 1968, Richard Hunt in 1969, and Yves Meyer in 1970. The jury consists of A. Zygmund, C. Pisot, and J.-P. Kahan.
NOMINATIONS BY PETITION

In accord with Council action of March 31, 1972, names of candidates for the position of member-at-large of the Council may be placed on the ballot by suitable petition.

There are several rules and operational considerations of which the members must be aware.

1. To be considered, petitions must be addressed to Everett Pitcher, Secretary, Box 6248, Providence, R. I. 02904, and must arrive by August 1, 1972.

2. The name of the candidate must be given as it appears in the Combined Membership List. If the name does not appear in the list, as in the case of a new member or by error, it must be as it appears in the mailing lists, for example on the mailing label of the *Notice*.

3. The petition for a single candidate may consist of several sheets each bearing the petition statement and signatures. The name of the candidate must be exactly the same on all sheets.

4. On the facing page is a sample form for petitions. Copies may be obtained from the Secretary. However, petitioners may make and use photocopies or reasonable facsimiles.

5. A signature is valid when it is clearly that of the member whose name and address is given in the left-hand column. At least fifty valid signatures are required for a petition to be considered further.

6. The signature may be in the style chosen by the signee. However, the printed name and address will be checked against the Combined Membership List and the mailing lists. No attempt will be made to match variants of names with the form of name in the CML. A name not in the CML or on the mailing lists is not that of a member. (Example: The name Everett Pitcher is that of a member. The name E. Pitcher appears not to be. Note that the current mailing label of the *Notice* can be peeled off and affixed to the petition as a convenient way of presenting the printed name correctly.)

7. When a petition meeting these various requirements appears, the Secretary will ask the candidate whether he is willing to have his name on the ballot. His assent is the only other condition of placing it there. Petitioners can facilitate the procedure by accompanying the petitions with a signed statement from the candidate giving his consent.

Members should know that the current authorization for the petition procedure covers the elections of 1972 and 1973 only. The situation is to be evaluated in January 1974. In 1973, the slate of candidates for member-at-large to be proposed by the Nominating Committee will be known in time to be of service to petitioners, whereas that is not true of 1972. One should remember in this regard that the slate proposed by the Nominating Committee does not necessarily agree with the names finally appearing on the ballot, both because proposed candidates sometimes decline and because candidates proposed by the Nominating Committee are not always accepted by the Council.
The undersigned members of the American Mathematical Society propose the name of ___________________ as a candidate for the position of member-at-large of the Council of the American Mathematical Society for a three year term beginning January 1, 1973.

Printed or typed name and address or (Notices) mailing label

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The purpose of this center is to maintain a file on prospective symposia, colloquia, institutes, seminars, special years, meetings of other associations, and to notify the organizers if conflicts in subject matter, dates or geographical area become apparent. A first announcement will be published in the *Notices* if it contains a call for papers, place, date, and subject, where applicable; a second announcement must contain reasonably complete details of the meeting in order for it to be published. Information on the pre-preliminary planning will be stored in the files, and will be available to anyone desiring information on prospective conferences. All communications on special meetings should be sent to the Special Meetings Information Center of the American Mathematical Society.

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**June 5–9, 1972**  
**MATHEMATICAL PROBLEMS IN THE BIOLOGICAL SCIENCES**  
Michigan State University, East Lansing, Michigan  
*Program*: Sol I. Rubinow, principal speaker  
*Sponsors*: CBMS and NSF  
*Information*: Professor Charles J. Martin, Department of Mathematics, Michigan State University, East Lansing, Michigan 48823

**June 12–16, 1972**  
**REGIONAL CONFERENCE ON BANACH ALGEBRA TECHNIQUES IN THE THEORY OF TOEPLITZ OPERATORS**  
University of Georgia, Athens, Georgia  
*Program*: Ten lectures by Ronald G. Douglas; contributed papers  
*Support*: NSF  
*Information*: Professor Bernard B. Morrel, Department of Mathematics, Graduate Studies Research Center, University of Georgia, Athens, Georgia 30601

**June 15–19, 1972**  
**REGIONAL CONFERENCE ON APPROXIMATION THEORY**  
University of California, Riverside, California  
*Program*: Ten lectures by George G. Lorentz  
*Support*: NSF (pending)  
*Participants*: Support for limited number  
*Information and applications*: Professor Bruce L. Chalmers, Department of Mathematics, University of California, Riverside, California 92502

**June 19–23, 1972**  
**CBMS REGIONAL CONFERENCE**  
Carleton College, Northfield, Minnesota  
*Program*: Lectures on "Decidable and undecidable problems in mathematics and set theory" by Paul J. Cohen  
*Support*: NSF (pending)  
*Participants*: Contingent upon grant, support will be available for twenty-five participants

**June 25 – July 2, 1972**  
**SIXTH INTERNATIONAL CONGRESS ON THE APPLICATION OF MATHEMATICS IN ENGINEERING**  
Weimar, German Democratic Republic  
*Information*: Professor Dr. Habil H. Matzke, Tagungleitung I.K.M., Karl Marx Platz, 253 Weimar, German Democratic Republic

**June 26–30, 1972**  
**REGIONAL CONFERENCE ON APPROXIMATION OF EIGENVALUES OF DIFFERENTIAL OPERATORS**  
Vanderbilt University, Nashville, Tennessee  
*Program*: Ten lectures by Hans Weinberger; contributed papers  
*Support*: CBMS and NSF  
*Participants*: Travel and subsistence for twenty-five participants  
*Information*: Professor Philip Crooke, Box 6205, Department of Mathematics, Vanderbilt University, Nashville, Tennessee 37235

**June 28–30, 1972**  
**CONFERENCE ON GROUP THEORY**  
Wingspread Conference Center, Racine, Wisconsin  
*Program*: Invited speakers: W. W. Boone, K. Gruenberg, J. E. Roseblade, J. R. Stallings; contributed papers  
*Support*: University of Wisconsin  
*Participants*: Support for twenty-five participants  
*Information*: Professors R. W. Gatterdam and K. W. Weston, University of Wisconsin–Parkside, Kenosha, Wisconsin 53140
July 3-10, 1972
THIRD INTERNATIONAL CONFERENCE ON NUMERICAL METHODS IN FLUID DYNAMICS
Orsay, France
Information: Secréariat de la 3e Conférence Internationale sur les Méthodes Numériques en Dynamique des Fluides, Université de Paris-Sud (Paris XI), Batiment 425, 90 Orsay, France

July 3-22, 1972
THIRTEENTH SESSION OF THE SUMMER SCHOOL ON NUMERICAL ANALYSIS
Bréa-sans-Nappe (near Rambouillet), France
Topic: Singular perturbations
Information: M. Steinberg, 17 avenue de la Libération, 92 Clamart, France

July 3-28, 1972
SEMINAR ON FOLIATED VARIETIES AND GLOBAL DIFFERENTIAL GEOMETRY
University of Montreal, Montreal, Quebec, Canada
Program: Invited speakers include Lipman Bers, Raoul Bott, Herbert B. Lawson, Ngo Van Que, Georges H. Reeb, Harold Rosenberg
Sponsors: National Research Council of Canada, Department of Education of the Government of Quebec
Participants: May apply for support
Information and applications: Séminaire de Mathématiques Supérieures, Université de Montréal, C. P. 6128, Montréal 101, P. Q., Canada

July 5 — August 25, 1972
SUMMER INSTITUTE IN APPLIED MATHEMATICS
Dartmouth College, Hanover, New Hampshire
Topic: Asymptotic and perturbation methods in fluid dynamics and wave propagation
Support: Office of Naval Research
Information: Professor E. L. Reiss, Courant Institute of Mathematical Sciences, New York University, New York, New York 10012

July 14-20, 1972
INTERNATIONAL CONFERENCE
Rethymno (Isle of Crete), Greece
Topic: Problems in the teaching of mathematics
Sponsors: L'Union Balkanique des Mathématiciens (UBM)
Information: Academicien N. Teodorescu, Secrétair General UBM, Str. Academiei no. 14, Bucharest, Romania

July 17-28, 1972
SEMINAR ON STOCHASTIC DIFFERENTIAL EQUATIONS
University of Alberta, Edmonton, Alberta, Canada
Program: Lectures by Mark Kac, Wendell Fleming, Frank Spitzer, Richard Griego
Sponsor: Rocky Mountain Mathematics Consortium
Information: Professor Reuben Hersh, Department of Mathematics and Statistics, The University of New Mexico, Albuquerque, New Mexico 87106

August 7-12, 1972
SYMPOSIUM ON VECTOR AND OPERATOR VALUED MEASURES AND APPLICATIONS
Snowbird, Utah
Sponsors: NSF, University of Utah, University of Florida, University of Pittsburgh
Participants: Limited support
Information: Professor D. H. Tucker, Department of Mathematics, University of Utah, Salt Lake City, Utah 84112

August 9-21, 1972
NATO ADVANCED STUDY INSTITUTE ON DYNAMICAL ASTRONOMY
Cortina d'Ampezzo, Italy
Information: Professor Victor Szebehely, The University of Texas at Austin, Austin, Texas 78715

August 14-16, 1972
ASSOCIATION FOR COMPUTING MACHINERY MEETING
Prudential Center, Boston, Massachusetts
Program: Three sessions on use of symbolic and algebraic manipulation systems
Information: Dr. James H. Griesmer, IBM Thomas J. Watson Research Center, P.O. Box 218, Yorktown Heights, New York 10598

August 14-18, 1972
AUSTRALIAN AND NEW ZEALAND ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE, SECTION 8 (Mathematics)
University of New South Wales, Australia
Information: Mr. B. V. Hamon, P. O. Box 21, Cronulla, New South Wales 2308, Australia

August 14-18, 1972
CONFERENCE IN COMBINATORIAL THEORY
Memphis State University, Memphis, Tennessee
Lecturers: Marshall Hall, Jr., California Institute of Technology (principal lecturer); Trevor Evans, Emory University; Charles Lindner, Auburn University; M. D. Plummer, Vanderbilt University; C. C. Rousseau, Memphis State University
Information: Professor R. J. Faudree or Professor R. H. Schelp, Department of Mathematics, Memphis State University, Memphis, Tennessee 38111

August 24 — September 2, 1972
CONFERENCE ON GEOMETRIC THEORY OF MEASURE AND MINIMAL SURFACES
Varena, Italy
Information: Professor E. Bombieri, Istituto Matematico, via Derna 1, 56100 Pisa, Italy
August 25-31, 1972
INTERNATIONAL SYMPOSIUM ON
TOPOLOGY AND ITS APPLICATIONS
Budva (near Dubrovnick), Yugoslavia
Information: Topološki Simpozij, P. P. 791,
Belgrade, Yugoslavia

September 26-28, 1972
CONFERENCE ON THE MATHEMATICAL
THEORY OF THE DYNAMICS OF BIOLOGICAL
POPULATIONS
University of Oxford
Program: Lectures by S. Karlin, W. Bodmer,
M. Williamson, M. S. Bartlett, R. M. Cormack,
N. Bailey, J. Skellam, D. Chapman, R. Hiorns,
X. Pielou
Information and applications: Secretary and
Registrar, The Institute of Mathematics and Its
Applications, 2565 The Mall, University of Hawaii,
Honolulu, Hawaii 96822

Topics: Information sciences, computer sci­
ences, communication theory, control theory,
and system theory
Abstracts: Deadline September 1, 1972
Information: Conference Chairman, Professor
Bharat Kinariwala, Information Sciences Pro­
gram, 2565 The Mall, University of Hawaii,
Honolulu, Hawaii 96822

March 29-31, 1973
TENTH SYMPOSIUM ON BIOMATHEMATICS
AND COMPUTER SCIENCE IN THE LIFE
SCIENCES
Houston, Texas
Topics: Mathematical, statistical, and computing
applications to biology and medicine
Abstracts: Dr. Stuart O. Zimmerman, The Uni­
versity of Texas, M. D. Anderson Hospital and
Tumor Institute, 6723 Bertner, Houston 77025,
prior to December 15, 1972
Information: Office of the Dean, The University
of Texas Graduate School of Biomedical Sciences
at Houston, Division of Continuing Education,
P. O. Box 20367, Houston, Texas 77025

September 10-15, 1973
SEVENTH INTERNATIONAL CONGRESS OF
CYBERNETICS
Namur, Belgium
Abstracts: Deadline January 1, 1973
Information: Secretariat of the International
Association for Cybernetics, Palais des Expo­
sitions, Place André Rijckmans, Namur, Belgium

January 9-11, 1973
CONFERENCE ON REGIONAL AND URBAN
SYSTEMS (SIXTH HAWAII INTERNATIONAL
CONFERENCE ON SYSTEM SCIENCES)
University of Hawaii, Honolulu, Hawaii

NEWS ITEMS AND ANNOUNCEMENTS

CATALOG OF LECTURE NOTES
A short catalog of lecture notes is
published regularly in the CONTENTS
OF CONTEMPORARY MATHEMATICAL
JOURNALS AND NEW PUBLICATIONS.
Unfortunately, the list is not sufficiently
complete, and the Society is making an
effort to acquire information on more
notes. This information is very impor­
tant, as is evidenced by the number of re­
quests that are received annually from
individuals and libraries. The importance
of expanding the list of lecture notes can­
not be over-emphasized. The specific in­
formation needed is title, author, date,
price and postage (with indication of
whether prepayment is necessary), num­
ber of pages, and address from which
notes may be obtained. The list is re­
stricted to lecture notes, published and
unpublished, and does not include pre­
prints or theses. There is no charge for
this service. Department chairmen and
individual mathematicians are urged to
send information on lecture notes to

CCMJ & NP, American Mathematical
Society, P. O. Box 6248, Providence,
Rhode Island 02904.

MATHEMATICS IN ISRAEL AND ITALY
Articles on mathematics in Israel
and Italy by Professor Robert H. Owens
of the Department of Applied Mathematics
and Computer Science of the University
of Virginia have been published by the
Office of Naval Research. "Mathematical
Science at Selected Israeli Institutions"
appeared in ONR London Report R-31-71
(20 August 1971), and "Applied Mathemat­
ics in Italy" in ONR London Report R-41-
71 (15 October 1971). A limited number
of copies are available. Copies of the
former may be obtained by writing to
Miss Sandra D'Allesandro, American
Mathematical Society, P.O. Box 6248,
Providence, Rhode Island 02904. Copies
of the latter may be obtained by writing
to Commanding Officer, Office of Naval
Research, Branch Office, Box 39, FPO
New York 09510.
LETTERS TO THE EDITOR

Editor, the Notices

Professor J. A. Zilber begins his recent letter, these Notices 19 (1972), 153-154, as follows:

"The words 'negative' and 'minus' are not synonymous. In fact, I can't think of any instance in which a literate mathematician would regard them as interchangeable."

It seems to me that the first of these sentences responds adequately to the challenge set forth in the second.

W. Wistar Comfort

Editor, the Notices

In the February 19, 1972, issue of The New Yorker, there appeared an article purporting to describe mathematics and mathematicians today. This article includes many dubious claims of which the following are examples: "The insanity and suicide levels among mathematicians are probably the highest in any of the professions." "Their non-mathematical efforts are, on the whole, pitifully inept." "The financial adventures of mathematicians consist of wildly speculative stock market excursions—always exciting but usually unsuccessful—rationalized by elaborate but irrelevant formulas and systems." "There is no such thing as a man who does not create mathematics and yet is a fine mathematics teacher."

Since the author of the article is listed in directories of mathematicians and in the author index of Mathematical Reviews, one might expect these statements to be based on knowledge. However, mathematicians who find them alarming or annoying may be interested to know that a letter to the author elicited no reference to data but merely the reply that these claims "are items that statistics will not really enhance."

Available evidence suggests that more comprehensive information would refute these groundless claims. Indeed the undersigned knows of no data to support them and of many counterexamples to them. For example, two well-known cases of successful mathematical specula-

tors are C. F. Gauss and G. A. Miller. The former built up a large estate in the last few years of his life, The latter willed a large fortune to the University of Illinois.

There is one claim in the article that is supported by both theoretical considerations and data, namely "the fact, amusing at first and then puzzling, that nearly all mathematicians are eldest sons." Now the known distribution of the number of siblings in families (see William Petersen, Population, NY (Macmillan), 1961, page 227) coupled with a simple probability calculation indicates that about 75% of males in the general population are eldest sons. For professional families, in which the distribution of family size is skewed more heavily toward small families than in the population as a whole, this probability is still higher. In the population of college students, from which virtually all mathematicians come, the proportion of first-born children is higher than among children of professionals as a whole. (For data and a good bibliography, see Stanley Schachter, "Birth Order, Eminence and Higher Education," American Sociological Review (1963), vol. 28, no. 5, pp. 757-768.) Accordingly, the eldest son phenomenon is neither surprising nor peculiar to mathematics.

Kenneth O. May

Editor, the Notices

I think the readers will be interested in some of the reactions to my article, "Can Mathematics be Saved?", these Notices, Vol. 16, No. 6, October 1969. I received over twenty letters of strong endorsement and encouragement. The article was republished in the Ontario Secondary School Mathematics Bulletin, December 1969; the Darmstader Blatter, wir lesen für Sie, January 1970, in German; the Suri-Kagaku, Mathematical Sciences, May 1970, in Japanese; and the Bulletin of the Institute of Mathematics and its Applications, Essex, England, February 1971. It was also honored by an award of 180 DM by the Bibliographisches Institut, February 1972.

W. G. Spohn, Jr.
CHUNG-MING AN of Johns Hopkins University has been appointed to an assistant professorship at Seton Hall University.

DENNIS ANSON of Western Kentucky University has been appointed to an assistant professorship at Alfred University, Alfred, New York.

HERMANN G. BURCHARD of Indiana University has been appointed to an associate professorship at Oklahoma State University.

HARVEY COHN of the University of Arizona has accepted the Emil Post Professorship at the City College (CUNY).

ALEJANDRO D. De ACOSTA of the Massachusetts Institute of Technology has been appointed to a professor titular adjunto at the Universidad Nacional de La Plata, Buenos Aires, Argentina.

DAVID D. DIX of the Harvard University Computing Center has been appointed vice president of Systems and Operations at Ticketron, Inc., New York, New York.

BENNY D. EVANS of the Institute for Advanced Study has been appointed to an assistant professorship at Oklahoma State University.

RONALD R. FLAMBOE of the TRW Systems Group has been appointed a mathematician at the University of California, Riverside.

ESTHER E. GUERIN of the University of Wyoming has been appointed to an assistant professorship at Seton Hall University.

HEINI HALBERSTRAM of the University of Nottingham has been appointed vice president of The Institute of Mathematics and Its Applications in Essex, England.

DAGMAR R. HENNEY of George Washington University will participate this summer in an international conference on Mathematische Methoden der Unternehmungsforschung held in Oberwolfach, Federal Republic of Germany, and a seminar on Eigenwertaufgaben und Ihre Numerische Behandlung at the Universität Freiburg. As an advisory member of the President's Committee for the advancement of women scientists she will visit a number of European universities.

DHANDAPANI KANNAN of New York University has been appointed to an assistant professorship at the University of Georgia.

PETER KOHN of the University of Chicago has been awarded the Vaclav Hlavaty research assistant professorship at Indiana University.

JOSEPH LEHNER of the University of Maryland will retire at the close of the 1971–1972 academic year. He has been appointed to a Mellon professorship at the University of Pittsburgh, effective September 1972.

ODELAR L. LINHARES of the University of Campinas has been appointed head of the Department of Computing Sciences and Statistics at the University of Sao Paulo, Brazil.

JAMES P. LONG of the College of Steubenville has been appointed president of Saint Francis College, Loretto, Pennsylvania.

JOEL M. McKEAN of the U.S. Air Force Academy has been transferred to Headquarters, Strategic Air Command, Omaha, Nebraska.

YAICHI SHINOHARA of the University of Toronto has been appointed to a visiting assistant professorship at the University of Georgia.

CARL R. SPITZNAGEL of the University of Kentucky has been appointed to an assistant professorship at John Carroll University, Cleveland, Ohio.

JAN E. WYNN of Colorado State University has been appointed to an assistant professorship at Brigham Young University.

PROMOTIONS

To Distinguished Professor, University of Dayton: KENNETH C. SCHRAUT.

To Chairman, Department of Mathematics, Mellon Institute of Science: RICHARD A. MOORE; Rensselaer Polytechnic Institute: RICHARD C. DI PRIMA.
NEWS ITEMS AND ANNOUNCEMENTS

STATEMENT BY THE AMS COUNCIL ON TEACHING ASSISTANTSHIPS AND JUNIOR FACULTY POSITIONS

With the current and prospective unfavorable employment prospects for Ph.D.'s, the Council notes, with favor, reports that many graduate mathematics departments are cutting down on the number of their teaching assistantships and are using the released money to increase the number of junior faculty positions. The Council strongly recommends that all graduate mathematics departments having significant numbers of teaching assistants make every effort to effect suitable changes in their teaching staffs, both to provide more positions for young Ph.D.'s and to effect a significant reduction in the production of future Ph.D.'s. An overall reduction of twenty percent in the number of teaching assistantships over the next two years with corresponding increases in junior faculty positions should reduce the number of teaching assistantships nationally by one thousand and increase the number of junior faculty positions by about three hundred.

The Council makes the preceding statement with the full knowledge that in those universities which cannot provide sufficient additional money, it may well be necessary to increase some teaching loads or to increase some class sizes, or both, in effecting the proposed change in faculty-teaching assistant distribution.

COLLECTED PAPERS OF NORBERT WIENER

The MIT Press is in the process of publishing the Collected Papers of Norbert Wiener. The papers will be reproduced by photo-offset as they appeared in the original publication. Photography is done from one reprint of each of Wiener's papers (bound volumes are generally of little use). Few such reprints are at present available to the MIT Press. Mathematicians are being requested to lend the MIT Press any reprints of Wiener's papers they may have; they will be returned promptly. Please send them to Professor Gian-Carlo Rota, Department of Mathematics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139.
STATEMENT ON COOPERATION OF SCIENTISTS

On March 7, 1972, the Council of The Federation of American Scientists issued the following statement:

"Mankind is faced with threats from many sides: nuclear annihilation; resource depletion; environmental degradation; and the debasement of traditional human values. Science and scholarship have helped to produce some of these threats. But they will be resolved, or forestalled, only with the application of educated thought. All nations must make intellectual contributions to solving these global problems. Scientists must unite in cooperating among themselves to facilitate solutions. And nations must make this cooperation possible. Indeed, nations who make international cooperation difficult for their scientists can only retard their own national development. A nation can help mankind go forward, or hold itself back. This is the only choice.

"We, therefore, urge all nations to facilitate intellectual exchanges between scientists and scholars. No limitations should be placed on the rights of scientists to attend conferences of their peers or to invite their foreign colleagues to their own country. No censorship should prevent their correspondence on matters of scientific and scholarly interest.

"We call on all scientists and scholars to assist each other in requiring that nations fulfill these obligations."

NATIONAL ACADEMY OF SCIENCES

On April 25, 1972, the National Academy of Sciences elected seventy-five new members to the academy "in recognition of their distinguished and continuing achievements in original research." Total membership is now 950. Among the new members are four members of the Society: Richard J. Duffin, Carnegie-Mellon University; Ralph E. Gomory, T. J. Watson Research Center; Samuel Karlin, Stanford University; and George W. Whithed, Massachusetts Institute of Technology.

NAS-NRC COMMITTEE ON NATIONAL STATISTICS

The National Academy of Sciences-National Research Council has announced the appointment of a Committee on National Statistics (CONS), under the chairmanship of William H. Kruskal of the University of Chicago. The committee is charged "with providing policy guidance for the application of statistics to problems of national interest." The first meeting of this committee was held on January 29, and it is assessing priorities for its attention. Support is provided from the Program Initiation and Development Fund of NAS-NRC.

BOWLING GREEN STATE UNIVERSITY PH.D. PROGRAM

The Department of Mathematics of Bowling Green State University has announced that it will begin a Ph.D. program in the fall of 1972. The program embodies three main features: (1) a research and dissertation requirement; (2) a program of seminars in teaching college and university-level mathematics; and (3) an independent study program in cognate areas of mathematical application. For further details on the program, please write to the Department of Mathematics, Bowling Green State University, Bowling Green, Ohio 43403.

INTERNATIONAL EDUCATIONAL EXCHANGE PROGRAM

The International Educational Exchange Program, authorized by the Fulbright-Hays Act, has recently announced the grants that will be available in 1973-1974 in engineering, mathematics, and physics. The grants are exclusively for university lecturing and advanced research abroad. Details on the terms of awards for a particular country, available information on specific openings, and application forms may be obtained from the Committee on International Exchange of Persons, 2101 Constitution Avenue, Washington, D. C. 20418.
ABSTRACTS PRESENTED TO THE SOCIETY

Preprints are available from the author in cases where the abstract number is starred.

The papers printed below were accepted by the American Mathematical Society for presentation by title. The abstracts are grouped according to subjects chosen by the author from categories listed on the abstract form. The miscellaneous group includes all abstracts for which the authors did not indicate a category.

An individual may present only one abstract by title in any one issue of the Notices but joint authors are treated as a separate category. Thus, in addition to abstracts from two individual authors, one joint abstract by them may also be accepted for an issue.

Algebra & Theory of Numbers

*72T-A114. R. M. RAPHAEL, Sir George Williams University, Montreal 107, Quebec, Canada.

Strongly regular rings.

A ring R is strongly regular if given x ∈ R there exists y ∈ R such that x = x²y (cf. R. F. Arens and I. Kaplansky, "Topological representation of algebras," Trans. Amer. Math. Soc. 1(1950), 165-171). Strongly regular rings are presented as a subcategory of the category of regular rings. Theorem. R is strongly regular iff given x ∈ R there exists a y ∈ R and a positive integer n such that x = xyy and (xy - yx)ⁿ = 0.

Theorem. Strong regularity implies the following condition and is equivalent to it if 1 + 1 is not a zero-divisor: given x ∈ R, there exists y ∈ R such that x = xyy and x(xy - yx) = (xy - yx)x. Theorem. The category of strongly regular rings is equationally definable. (Received February 10, 1972.)

*72T-A115. HENRY W. GOULD, West Virginia University, Morgantown, West Virginia 26506. Note on functions studied by LeVan, Chawla, and Euler.

Let the prime factorization of n be p₁...pᵣ. Define g(n) = 1 + ∑ᵣᵢ=₁ aᵢ(pᵢ⁻¹). This was first given by Euler (1739) in a treatise on music, and he used g to measure euphonic value of chords. It is Euler's gradus suavitatis function, g(n) - 1 is completely additive, and because of this we can easily obtain \( \sum_{d|n} g(d) = \frac{\tau(n) (1 + g(n))}{2} \). Now define Q(n) as the number of solutions of the equation g(x) = n. Then Q(n) = coefficient of \( \frac{t^{n-1}}{n!} \) in \( \Pi_{p \leq n} (1 - \frac{t^{p-1}}{p^{p-1}})^{-1} \), and moreover we show that Q satisfies the recurrence relation \( Q(n) = -\sum_{d|n} P(n, d) \mu(d) Q(n - g(d)) \), n ≥ 2, where P(n) = product of all primes ≤ n. We show that Q(2n - 1) = Q(2n), confirming a table by Euler and also prove that x₀ ∈ \( \{ x | g(x) = 2n - 1 \} \) if and only if x₀ ∈ \( \{ x | g(x) = 2n \} \). The functions g and Q are developed in parallel with functions f and q studied by L. M. Chawla and M. O. Levan. Tables and other properties are given. (Received February 10, 1972.)


Ford (Trans. Amer. Math. Soc. 27(1925), 146-154) proved the following generalization of Hurwitz'theorem. If x is any complex irrational number (x ∉ Q(i)), then there exist infinitely many p, q ∈ Z[i] such that (*) \( |x - p/q| < 1/3^{1/2} |q|^2 \). The first proof using continued fractions is given here. Using the algorithm of
"nearest Gaussian integer" due to Hurwitz (Acta Math, 11(1888), 187-200), a complex number x is expanded in a regular continued fraction (CF) \( x = \langle a_0, a_1, \ldots, a_n, x_{n+1} \rangle \), \( a_n \in \mathbb{Z}[i] \), \( x_n = a_n + 1/x_{n+1} \). We prove the following Theorem 1. Let \( x \) be any complex irrational number (\( x \not\in \mathbb{Q}(i) \)), and let \( p_n/q_n \) (\( p_n, q_n \in \mathbb{Z}[i] \)) be the \( n \)th convergent of the CF expansion of \( x \). Then there are infinitely many indices \( n \) for which \( |x - p_n/q_n| < 1/3^{1/2} \). That is, infinitely many solutions to (*) may be found among the convergents to \( x \). In addition we show that Theorem 1 cannot be sharpened in the expected way ("for one of any 3 successive convergents...`).

For \( n \geq 1 \), denote \( M_n(x) = |x|/|q_n| \), \( q_n = x - p_n \) so that \( p_n/q_n \) is a solution to (*) iff \( M_n(x) > 3^{1/2} \). Theorem 2. Given any positive integers \( K, N \), there exist complex \( y \) and \( n \geq N \) such that \( M_n(y) < 3^{1/2} \) for \( j = 0, 1, \ldots, K \). Finally we prove Theorem 3, For a slightly modified CF algorithm, Theorem 1 is false: there exist \( z \) such that \( M_n(z) < 3^{1/2} \) for all \( n \geq 2 \). (Received March 3, 1972.)

*72T-A117. JAMES W. PETTICREW, Indiana State University, Terre Haute, Indiana 47809. Cartesian product semigroups.

Definition. A semigroup with zero is called annihilator ordered if two elements are equal when they have the same left and right annihilators and it satisfies \( abc = 0 \) implies \( ab = 0 \) or \( ba = 0 \). Theorem. If \( S \) is an annihilator ordered semigroup and \( a \leq b \) is defined by \( b \in A_a \subset A_a \) and \( A_b \subset A_a \), then \( (S, \leq) \) is a partially ordered semigroup. Lemma. If \( ab \neq 0 \), then \( ab = a \cdot a = A_a \) and \( A_{ab} = A_b \). Let \( A_{L(R)} \) denote all left (right) annihilators and let \( T = A_{L(R)} \times A_{R(L)} \) and define the product \( (a \cdot A_{ab}) \cdot (c \cdot A_{cd}) \) to be \( (a \cdot A_{ab}) \cdot (c \cdot A_{cd}) \) if \( b \neq 0 \) and to be \( (S, \leq) \) if \( b = 0 \); then \( (T, \leq) \) is a semigroup with \( (S, \leq) \) as zero. Let \( A_{L(R)} = A_{L(R)} \times \{ [S] \} \) and let \( T' = A_{L(R)} \times A_{C(L)} \); then \( T' \) is a subsemigroup of \( T \) if and only if the only element with left or right annihilator \( S \) is \( 0 \). Theorem. Define \( \varphi = (a \cdot A_{ab}) \cdot (c \cdot A_{cd}) \); then if \( S \) is annihilator ordered, \( \varphi \) is a monomorphism. Further, if \( S \) is \( 0 \)-simple, then \( \varphi \) is an isomorphism between \( S \) and \( T' \). Definition. A semigroup with involution * is called *-regular if \( aa**a = a \) for all \( a \in S \). Further, define \( a_{\lambda} = aa^* \) and \( a_{\rho} = a^*a \). Definition. The cartesian product semigroup on \( A \) is the set of all \( B \times C \); \( B, C \subset A \), together with the operation of composition. Theorem. \( S \) is isomorphic to a cartesian product semigroup if and only if \( S \) is an annihilator ordered, \( 0 \)-simple, *-regular semigroup and \( A_{L(R)} \) is a complete atomic boolean algebra in which \( a_{\lambda} \wedge b_{\lambda} = 0 \) implies \( a_{\lambda} b_{\lambda} = 0 \). (Received March 6, 1972.)

*72T-A118. GARY CHARTRAND, Western Michigan University, Kalamazoo, Michigan 49001 and SEYMOUR SCHUSTER, Carleton College, Northfield, Minnesota 55057. A note on cycle Ramsey numbers.

Definition. For \( m, n \geq 3 \), the cycle Ramsey number \( c(m, n) \) is the least integer \( p \) such that for any graph \( G \) of order \( p \), either \( G \) has an \( m \)-cycle or its complement \( \overline{G} \) has an \( n \)-cycle. Lower bounds for \( c(m, n) \) are announced in the following theorems. Theorem 1. If \( 3 \leq m \leq n \), where \( m \) is odd, then \( c(m, n) \leq 2n - 1 \). Theorem 2. If \( 3 < m \leq n \) and \( m \) is even, then \( c(m, n) \leq n + m/2 - 1 \) if \( n \geq (3/2)m \), \( c(m, n) \leq n + m/2 - 1 \) if \( n \) is even and \( n \leq (3/2)m \), \( c(m, n) \leq 2m - 1 \) if \( n \) is odd and \( n \leq (3/2)m \). The proofs rest on exhibiting specific graphs that establish the lower bounds. Since Bondy has proved \( c(m, n) \leq 2m - 1 \), Theorem 1 implies that \( c(m, n) = 2m - 1 \) for \( m \) odd. Further, we offer the conjecture that the lower bounds stated in Theorems 1 and 2 are greatest lower bounds except for the two early cases \( m = n = 3 \) and \( m = n = 4 \). (Received March 8, 1972.)

Let $S$ be a category with finite inverse limits and $U : G \to S$ a tripleable functor. Under these limit assumptions, the classifying complex construction $\overline{W}(K) \to W(K)$ of Mac Lane is defined as well as the augmented $k$-coskeleton triple $\text{Cosk}^k$ for augmented complexes in $G$ and $S$. In particular, for any abelian group object $\pi$ in $G$, the Eilenberg–Mac Lane complex $K(\pi, n)$ exists. Let $SS\mathcal{Z}[K_1, K_2]$ be the set of homotopy classes of simplicial maps of $K_1$ into $K_2$ and $G'(X)$ be the standard cotriple resolution of an object $X$ in $G$ so that the triple cohomology groups $H^n_U(X; \pi)$ are defined. Theorem 1 (homotopy representability). $H^n_U(X; \pi) \simeq SS\mathcal{Z}[G'(X), K(\pi, n)]$.

Definition. A simplicial fiber space $F \to E \to K(\pi, n)$ in $G$ will be called a $K(\pi, n)$-torsor above $X$ (relative to $U$) provided it satisfies the following conditions: (a) $E$ is augmented above $X$ and is $U$-contractible. (b) $U(E) \simeq [\text{Cosk}^{n-2}(U(E))] \times \overline{W}(K(U(\pi), n-1))$. If $\text{TOR}^n_U(X; \pi)$ is the group of connected component classes of $K(\pi, n)$-torsors, then we have Theorem 2 (classification). $H^n_U(X; \pi) \simeq \text{TOR}^n_U(X; \pi)$.

The proof is based on the construction of a "standard" $K(\pi, n)$-torsor in each class defined by a given $n$-cocycle $\xi$ using a theorem of Beck and the following Lemma (applied for $k = n - 1$). There exists a canonical homomorphism $\Sigma : H^n_U(X; \pi) \to H^{n-k}(\text{Cosk}^{k-1}(G'(X)); \pi), n > k$. (Received April 3, 1972.)

#72T-A120. PAUL HELLE, Université de Montréal, Montréal, Québec, Canada and JAROSLAV NEŠETŘIL, Charles University, Prague, Czechoslovakia. Groups and monoids of regular graphs (and of graphs with bounded degrees).

It is shown that groups of large orders cannot be represented as groups of automorphisms (or monoids of endomorphisms) of graphs with bounded degrees. Every group of at most countable order is represented as the monoid of endomorphisms of a 3-regular graph (and for a finite group the graph can be chosen finite), thus both strengthening and extending a result of R. Frucht (1949). Moreover, this is best possible, as for each uncountable $\gamma$ there exists a group of order $\gamma$ which cannot be represented as the monoid of endomorphisms (group of automorphisms) of any 3-regular graph. Given a cardinal $\alpha$, the following cardinals are characterised: all cardinals $\beta$ such that every group of order $\beta$ can be represented as the group of automorphisms of a suitable graph with $\alpha$-bounded degrees; all cardinals $\beta$ such that every group of order $\beta$ can be represented as the monoid of endomorphisms of a graph with $\alpha$-bounded degrees; all cardinals $\beta$ such that there exists an asymmetric (rigid) graph with $\alpha$-bounded degrees with cardinality of its vertex-set equal to $\beta$; all cardinals $\beta$ such that there exists an asymmetric (rigid) connected graph with $\alpha$-bounded degrees with cardinality of its vertex-set equal to $\beta$. (Received March 10, 1972.) (Authors introduced by Professor Gert O. Sabidussi.)

#72T-A121. MOURAD EL-HOUSSIENI ISMAIL, University of Alberta, Edmonton, Alberta, Canada. On the equation $\phi^*(x) = n$. Preliminary report.

Let $\phi(n)$ be the Euler totient function and $\phi^*(n)$ its unitary analogue. A classical conjecture of Carmichael, still unsolved, says that, for a given positive integer $m$, the equation $\phi^*(x) = m$ has either no solution or more than one solution in $x$. We here examine the solutions of the equation $\phi^*(x) = m$. The analogue of Carmichael's conjecture can be shown to fail here. The case where $m$ is odd is easily disposed of. Where
m = 2^k, the solutions are explicitly given for all \( k < 32 \). It is shown here that for \( 32 \leq k < 2^{17} \), there are no solutions. For \( k \geq 2^{17} \) the problem is tied up with the unsolved problem concerning the existence of Fermat primes \( \equiv F_{17} \). The case when \( m = 2^r \) \((r \text{ odd})\) is also discussed in detail. (Received March 13, 1972.)

72T-A122. KIM KI-HANG BUTLER, Pembroke State University, Pembroke, North Carolina 28372.

A Moore-Penrose inverse for Boolean relation matrices.

Let \( B_n \) be the set of all \( n \times n \) Boolean relation matrices over the Boolean algebra \( \{0,1\} \) of order 2. Consider the matrix equations (i) \( A = AXA \), (ii) \( X = XAX \). A unique solution to the systems (i) and (ii) is called a Moore-Penrose inverse of \( A \). We shall call a matrix \( A \) of \( B_n \) invertible if \( A \) has a Moore-Penrose inverse. We say that a matrix \( A \) of \( B_n \) is nonsingular if \( A \) is regular \((i.e., A \in B_n)\) and row rank of \( A = n = \) column rank of \( A \). Let \( H_n^i \) \((i=1,\ldots,m)\) denote the nonisomorphic nonsingular \( \kappa \)-classes of \( B_n \).

**Theorem.** If \( V(B_n) \) denotes the set of all invertible matrices of \( B_n \), then \( |V(B_n)| = \sum_{i=1}^{m_i} (n!)^2/|H_n^i| \). (Received March 13, 1972.)

*72T-A123. PHILIP GEORGE BUCKHIESTER, Department of Mathematical Sciences, Clemson University, Clemson, South Carolina 29631. The number of \( n \times n \) matrices of rank \( r \) and trace \( \alpha \) over a finite field.

Let \( GF(q) \) denote a finite field of order \( q \). Let \( N(n,q,r,\alpha) \) denote the number of \( n \times n \) matrices of rank \( r \) and trace \( \alpha \) over \( GF(q) \), where \( 0 \leq r \leq n \) and \( \alpha \) is in \( GF(q) \). The number \( N(n,q,r,\alpha) \) is found by obtaining and solving a difference equation in \( N(n,q,r,\alpha) \). (Received March 16, 1972.)


For unexplained terminology see A. F. Pixley, "The ternary discriminator function in universal algebra," Math. Ann. 191(1971), 167-180. Let \( \mathcal{B} \subset \Pi \{\mathcal{B}_i \mid i \in I\} \). For \( i \in I \), let \( \sigma_i \) be the \( i \)-th projection of \( \mathcal{B} \) and let \( \theta_i \) be the congruence relation on \( \mathcal{B} \) induced by \( \sigma_i \). \( \mathcal{B} \) is called regular if for every finite \( J \subset I \), where \( \theta_i \equiv \theta_j \) iff \( i = j \) for all \( i, j \in J \), \( \sigma_j(\mathcal{B}) = \Pi(\{\mathcal{B}_i \mid i \in J\}) \). **Theorem 1.** Let \( \mathcal{B} \) be nontrivial. Then \( \mathcal{B} \) is locally quasi primal iff for every \( I \neq \emptyset \) and every \( \mathcal{B} \subset \mathcal{B}_I \), \( \mathcal{B} \) is a regular subalgebra of \( \Pi(\sigma_i(\mathcal{B}) \mid i \in I) \). (Received March 17, 1972.)

72T-A125. VANCE FABER, University of Colorado, Denver, Colorado 80202. An upper bound for the minimum valency of a graph embedded in a surface of Euler characteristic \( X \). Preliminary report.

**Theorem.** Let \( G \) be a connected graph (we allow loops and multiple edges) with no vertices of valency smaller than \( \nu \geq 3 \). (We count each loop only once in the valency.) Suppose \( G \) is embedded in a compact 2-manifold \( M \) of Euler characteristic \( \chi \) such that \( M \setminus G \) is the disjoint union of open 2-cells and such that no face has a boundary with fewer than \( \nu^* \geq 3 \) distinct edges. Let \( M = \max(\nu,\nu^*) \) and \( m = \min(\nu,\nu^*) \). If \( \chi \geq 6 \), then (i) \( m \leq X + 2 \), (ii) \( M \leq [m(X - 1)/(m - 2)] \) when \( m \) is odd and \( m \leq \frac{1}{2} + \frac{1}{2}\sqrt{1 + 8X} \). (iii) \( M \leq [X(m + 1)/(m - 1)] \)

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when $m$ is odd and $m \equiv \frac{1}{2} + \sqrt{1 + 8X}$, (iv) $M \equiv [Xm/(m - 2)]$ when $m$ is even and (v) all bounds are sharp. The author also has complete information about the case $0 \leq X \leq 5$. (Received March 17, 1972.)

72T-A126. HYO CHUL MYUNG, University of Northern Iowa, Cedar Falls, Iowa 50613. von Neumann regularity in ternary rings. Preliminary report.

Let $T$ be a ternary ring ($\tau$-ring) defined by W. G. Lister (Trans. Amer. Math. Soc. 154(1971), 37-55). An element $a \in T$ is called regular (in the sense of von Neumann) if there exists an element $x \in T$ with $a = axa$. $T$ is called regular if every element of $T$ is regular. Among other results, the unique maximal regular ideal of $T$ is shown to be the set of all $x \in T$ such that the principal ideal $(x)$ in $T$ generated by $x$ is a regular ideal.

Let $x\mu(a) = axa$ for $x, a \in T$. An additive subgroup $S$ of $T$ is called a quadratic ideal of $T$ if $x\mu(S) \subseteq S$.

Theorem 1. A $\tau$-ring $T$ is regular if and only if we have $M\mu(S) = M \cap S$ for all medial ideals $M$ of $T$ and quadratic ideals $S$ of $T$.

Theorem 2. Let $A = T \oplus T^2$ be a direct enveloping ring of $T$ where $x \rightarrow 2x$ is an additive automorphism of $A$, and let $x\mu_a = axa$ for $x, a$ in the Jordan ring $A^+$. Then $T$ is regular if and only if we have $M \cap Q \subseteq M\mu(Q)$ for all medial ideals $M$ of $T$ and quadratic ideals $Q$ of $A^+$.

Theorem 3. Let $T$ be a $\tau$-ring with the d.c.c. on right ideals of $T$. Then $T$ is regular if and only if it is semisimple (the Jacobson radical of $T$ is 0). (Received March 20, 1972.)


Notation. Let $\mathcal{F}$ be the category of pointed compact simple semigroups $(S, e)$, where $e \in E(S)$, and homomorphisms, $\mathcal{G}$ be the category of inverse limit preserving functors from compact Lawson semilattices into $\mathcal{F}$ and natural transformations, and $\mathcal{H}$ be the category of compact normal bands of groups possessing a Lawson semilattice cross-section of its D-classes and homomorphisms. Further, let $\Sigma$ denote the category of inverse limit preserving functors from compact Lawson semilattices into the category $\mathcal{J}$ of compact groups and homomorphisms, and $\mathcal{C}$ be the category of compact Clifford semigroups over compact Lawson semilattices and homomorphisms. The following result is established, Theorem. The categories $\mathcal{F}$ and $\mathcal{H}$ are equivalent.

Corollary (Bowman). The categories $\mathcal{G}$ and $\mathcal{C}$ are isomorphic. (Received March 20, 1972.)

*72T-A128. JEAN-MARIE De KONINCK, Temple University, Philadelphia, Pennsylvania 19122. On

$$\sum_{2 \leq n \leq x} (\omega(n))^{-1}$$. 

Let $\omega(n)$ be the number of distinct prime divisors of $n$, R. L. Duncan (Proc. Amer. Math. Soc. 25 (1970), 191-192) proves that $\sum_{2 \leq n \leq x} (\omega(n))^{-1} = O(x(\log \log x)^{-1})$. We prove the following: Given a positive integer $\alpha$, then $\sum_{2 \leq n \leq x} (\omega(n))^{-1} = x\sum_{1 \leq a \leq A_1} (\log \log x)^{-1} + O(x(\log \log x)^{-\alpha-1})$ where the $A_1$ are constants ($A_1 = 1$). The proof uses a result of A. Selberg (J. Indian Math. Soc. 18(1954), 83-87). We also get similar estimates for $\sum_{2 \leq n \leq x} (\Omega(n))^{-1}$ and $\sum_{2 \leq n \leq x} \mu_2^2(n)/\omega(n)$, where $\Omega(n)$ denotes the total number of prime factors of $n$ and $\mu$ is the Möbius function. (Received March 20, 1972.)

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K. D. Magill, Jr. first proved that two Boolean rings are isomorphic if and only if their respective endomorphism semigroups are isomorphic as semigroups. His proof, however, relied on topological techniques. More recently C. J. Maxson [Semigroup Forum 4(1972), 78-82] proved the same result using purely algebraic techniques. In this paper the authors extend this result to arbitrary $p^k$-rings in the sense of A. L. Foster. An example is included to show that the result cannot be generalized to arbitrary rings which satisfy the identity $x^n = x$. Only algebraic techniques are used throughout the proofs. (Received March 20, 1972.)

(-1,1) rings can arise from at least two sources: (1) associative rings, and (2) strongly (-1,1) rings, i.e. right alternative rings satisfying $[[x,y],z] = 0$. We show these sources are essentially the only ones. We show simple (-1,1) rings are associative. $[[x,y],[z,w,v]]$ is an alternating function of its five arguments. We characterize all (-1,1) rings with an idempotent. We show how they can be constructed from associative rings. When $R$ is generated by two elements, then $(R,R,R) \subseteq$ center of $R$. Thus $(R,R,R)^2 = (R,R,R)(R,R) = (R,R,(R,R,R)) = 0$. We construct the free strongly (-1,1) ring on two generators and an example of a strongly (-1,1) ring of real valued functions of two variables. In a strongly (-1,1) ring, the set of all nilpotent elements is an ideal and the quotient ring is associative and commutative. This means all finitely dimensional strongly (-1,1) algebras over algebraically closed fields are a direct sum $\bigoplus_{i=1}^{n} R_i$ where $R_i$ is a nilpotent ring or a nilpotent ring with an identity adjoined. (Received March 24, 1972.)

A left duo-ring is one in which every left ideal is two-sided. In this paper we prove the analogues of certain well-known theorems in commutative rings to the class of left duo-rings. Theorem 1. In a left duo-ring with 1, if every prime ideal is a principal ideal, then every ideal is principal. Theorem 2. In a left duo-ring with 1, if every prime ideal is finitely generated, then every ideal is finitely generated. It is well known that a commutative ring $R$ with 1 is $\pi$-regular iff every prime ideal in $R$ is maximal. Here we prove Theorem 3. A duo-ring $R$ with 1 is $\pi$-regular iff every prime ideal in $R$ is maximal. Also we give examples to show that the condition that every prime ideal in a noncommutative ring $R$ is maximal is neither a necessary nor a sufficient condition for that ring $R$ to be $\pi$-regular. (Received March 24, 1972.)
Let $R$ be a ring with a unity element and $(\mathcal{P}_L, \mathcal{P}_R)$ left and right idempotent filters respectively. Then one can define a "two-sided ring of quotients," $Q_2(R) = Q_{\mathcal{P}_L, \mathcal{P}_R}(R)$ (W. Schelter, McGill Univ., preprint, 1971) with respect to $(\mathcal{P}_L, \mathcal{P}_R)$. **Main Theorem.** Let $R$ be an $f$-ring and $(\mathcal{P}_L, \mathcal{P}_R)$ left and right idempotent filters with $D \in \mathcal{P}_L$ ($D \in \mathcal{P}_R$) implying $RD^+ \in \mathcal{P}_L$ ($D^+R \in \mathcal{P}_R$); then $Q_2(R)$ is an $f$-ring. If $R$ is also totally ordered then so is $Q_2(R)$. In particular the maximal two-sided ring of quotients, as defined by Utumi, of an $f$-ring $R$ is a special case and is always an $f$-ring. This is not the case with the maximal right (left) ring of quotients (see W. Anderson, Canad. J. Math. 17(1965), 434-448). Included is also a discussion of some of the order properties of the $f$-ring $R$ which are inherited by the "two-sided ring of quotients" of $R$. The methods used are in general those introduced by Anderson (see above). (Received March 23, 1972.)

A local ring $R$ with the radical $W$ is said to be an $(s, 1)$-ring if $Q = R/W$ is commutative, $W^2 = 0$, $W$ has length $s$ and $W_R$ is simple. Thus, in particular, $(2, 1)$-rings are exceptional (see Abstract 691-16-13, these Notices 19(1972)). **Theorem 1.** Let $R$ be an $(s, 1)$-ring with infinite field $Q = R/W$. If $s \geq 4$, then there is an infinite number of nonisomorphic local left and an infinite number of colocal right $R$-modules of length $s - 1$. Consequently, there are indecomposable left and right $R$-modules of arbitrarily large lengths. **Theorem 2.** Let $R$ be a $(3, 1)$-ring. Let $u, v, w$ be a basis of the vector space $Q^W$. Then there are just five types of indecomposable left $R$-modules, viz. the simple type, $L_1 \cong R/R_J$, the injective type, $L_2 \cong R/(Ru \oplus Rv)$, the local type of length $3$, $L_3 \cong R/\langle u \rangle$, the projective type, $L_4 \cong R/\langle u, v \rangle$, and the indecomposable type of length 5, $X_5 \cong R/R_J$. Moreover, every left $R$-module is a direct sum of modules of types $L_1$, $L_2$, $L_3$, $L_4$ and $X_5$. Dually, there are just five types of indecomposable right $R$-modules, viz. the simple type, $C_1 \cong R/W$, the projective type, $C_2 \cong R$, the colocal type of length $3$, $C_3 \cong (R \oplus R)/R(u, v)$, the injective type, $C_4 \cong (R \oplus R \oplus R)/R(u, v, 0) + R(u, 0, w) R(u, v, w)$, and the indecomposable type of length 5, $Y_5 \cong (R \oplus R \oplus R)/R(u, v, w)$; and, every right $R$-module is a direct sum of modules of types $C_1$, $C_2$, $C_3$, $C_4$ and $Y_5$. (Received March 27, 1972.)

Let $\Gamma$ be an embedding of the Cayley diagram of the group $G \neq 1$. Call $\Gamma$ locally finite if every finite region of the plane contains only a finite number of vertices. Assume $\Gamma$ locally finite. **Theorem 1.** If a generator $x$ of $G$ has finite order $\infty$, then every polygon determined by the relation $x^\infty = 1$ bounds a component of the complement of $\Gamma$ in the plane. Let $O_{v_1}$ be the clockwise ordering of the edges about the vertex $v_1 \subset \Gamma$ so that $-O_{v_1}$ is the counterclockwise ordering about $v_1$. **Theorem 2.** If $G = (a, b; a^n, b^m, R) \neq (a, b; a^n, b^m)$, then

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n,m \geq 3, \Gamma \text{ locally finite and } O_{V_i} = \pm O_{V_j} \text{ for every vertex}, \text{ then } (ab)^q = 1, \epsilon = \pm 1, q > 1. \text{ Moreover, the closed path determined by } (ab)^q \text{ bounds a component of the complement of } \Gamma \text{ in the plane. The proof is combinatorial. (Received March 27, 1972.) (Author introduced by Professor Elvira Strasser Rapaport.)}

72T-A135. DAVID KELLY, Lehrstuhl für Mathematik I, Universität Mannheim, 68 Mannheim (Schloss), L 5/4 West Germany. A characterization of atomic compact structures.

A structure \( \mathfrak{A} \), the cardinality of whose type does not exceed \( |\mathfrak{A}| \), is atomic compact if and only if \( \mathfrak{A} \) is a retract of every elementary extension whose cardinality is \( |\mathfrak{A}| \). The standard example (see G. Grätzer, "Universal algebra," Van Nostrand, Princeton, N. J., 1968, p. 248) of a countable structure with no proper countable elementary extensions shows the above restriction on the type of \( \mathfrak{A} \) to be necessary. (Received March 31, 1972.) (Author introduced by Dr. G. H. Wenzel.)


J. Mycielski ("Some compactifications of general algebras," Colloq. Math. 13(1964), 1-9) asked whether every equationally compact algebra is an algebraic retract of a compact topological algebra (Problem 484). Various equational classes are known to have the property that every equationally compact member is such a retract, but W. Taylor showed ("Atomic compactness and graph theory," Fund. Math. 65(1969), 139-145) using an algebra with two unary operations that the question in general has a negative answer. For some time interest has been shown in Mycielski's question for the class of semilattices. A positive answer to this question is given here. Let \( \gamma = \langle S; \vee \rangle \) be an equationally compact semilattice. Then \( \gamma \) (as any semilattice) can be embedded with preservation of all suprema into the semilattice \( \langle 2^S; \cup \rangle \), which in its order topology is a compact topological semilattice. Denote the embedding by \( e \), let \( R = e(S) \), let \( \overline{R} \) denote the closure of \( R \) in the above topology, and let \( \overline{\gamma} \) be the compact topological semilattice \( \langle \overline{R}; \cup \rangle \). Then \( \overline{\gamma} \), it is shown, retracts onto \( \gamma \). The proof depends on the observation that if \( A, B \subseteq S \) have infima in \( \gamma \), then \( \bigcap (e(a)|a \in A) = \bigcap (e(b)|b \in B) \) implies \( \inf_{\gamma} A = \inf_{\gamma} B \). (Received March 31, 1972.) (Author introduced by Dr. G. H. Wenzel.)

72T-A137. HANSRAJ GUPTA, 402 Mumfordganj, Allahabad 2, India and Panjab University, Chandigarh 14, India. A problem in permutations and Stirling's numbers.

Stirling's numbers of the second kind arise in several different contexts. One such situation is provided by the Theorem. Among the \( r^n \) meaningful or meaningless \( n \)-letter words in a language with \( r \) letters of alphabet, there are exactly \( C(n, r, m) = r^n - \binom{m}{1} (r-1)^n + \binom{m}{2} (r-2)^n - \ldots + (-1)^m \binom{m}{m} (r-m)^n \) in which the first \( m \) letters of alphabet occur each at least once. The proof is based on the principle of inclusion and exclusion. Evidently, \( C(n, r, m) = 0 \) for \( 0 \leq n < m; C(n, r, m) = m! \) for \( n = m \). Finally for \( m = r - 1 \), \( C(n, r, m) \) is \( (r-1)! \) times a Stirling's number of the second kind. A particular case of the theorem has been given by Kim Ki-Hang Butler (Abstract 71T-A68, these Notices 18(1971), 549). (Received April 3, 1972.)
Let $\mathcal{F}$ be the set of all real $n \times n$ matrices such that (a) $m_{ii} > 0$ and $m_{ij} \leq 0$. Let $\mathcal{M}$ be the set of all matrices satisfying (a) and (b) $m_{ii} = m_{1i}$, where $m_{1i} = \sum_{j \neq i} |m_{ij}|$. If $M \in \mathcal{F}$, then $A = |M|$ is the matrix given by $a_{ij} = |m_{ij}|$. If $\beta$ is any subset of $\{1, \ldots, n\}$, then $M[\beta]$ is the principal minor lying in the rows and columns of $\beta$ (in their natural order), and $M[\beta']$ is the complementary principal minor. A matrix $M$ will be called triangulable if $P^{-1}A$ is triangular, for some permutation matrix $P$. Theorem 1. Let $M \in \mathcal{M}$. Then $\det M \geq \prod_{i=1}^{n} m_{ii}$, with equality if and only if (1) $M(i|i)$ is triangulable for all $i$, $1 \leq i \leq n$, such that $m_{ii} > m_{1i}$. Theorem 2. Let $M \in \mathcal{F}$, and let $A = |M|$. Then $\det M + \text{per } A = \prod_{i=1}^{n} m_{ii}$, with equality if and only if (2) for all proper subsets $\beta$ of $\{1, \ldots, n\}$ either $M[\beta]$ or $M[\beta']$ is triangulable. Theorem 3. If $M \in \mathcal{M}$ and $A = |M|$ then $\text{per } A = \prod_{i=1}^{n} m_{ii}$, with equality if and only if (1) and (2) hold. Corollary. If $A$ is stochastic and $a_{ii} \geq 1/2$, $1 = 1, \ldots, n$, then $\text{per } A \geq (1/2)^{n-1}$. Observe that for $n \geq 3$, $(1/2)^{n-1} > n! / n^n$. (Received April 6, 1972.)

Let $R$ be a commutative ring. A commutative $R$-algebra $S$ is a quasi-separable cover of $R$ if over each connected component of $\text{spec}(R)$ the restriction of $S$ is a direct limit of separable, projective subalgebras. The purpose of the paper is to prove that there is a topological groupoid (called the fundamental groupoid of $R$) such that the category of quasi-separable covers of $R$ is anti-equivalent to the category of profinite sets on which the groupoid acts. The groupoid is the Galois groupoid of the separable closure (in a suitable sense) of $R$. (Received April 7, 1972.)

It is well known that a divisor $d > 0$ of the positive integer $n$ is called unitary if $d|n$ and $(d, \delta) = 1$. For integers $a, b$ not both zero, let the symbol $(a, b)^{**}$ denote the greatest unitary divisor of both $a$ and $b$. A divisor $d > 0$ of the positive integer $n$ is called bi-unitary if $d|n$ and $(d, \delta)^{**} = 1$. Let $\tau^{**}(n)$ denote the number of bi-unitary divisors of $n$. Recently, the first author ("The theory of arithmetic functions," Lecture Notes in Math., vol. 251, Springer-Verlag, Berlin and New York, 1972, pp. 273-282) established an asymptotic formula for $\sum_{n \leq x} \tau^{**}(n)$ with an error term $O(x^{1/2} \log x)$. However, there is a mistake in the statement of the main term of the asymptotic formula. In this paper, we not only correct this mistake but also improve the order of the error term $E(x)$ in the asymptotic formula for $\sum_{n \leq x} \tau^{**}(n)$ to $E(x) = O(x^{1/2} \cdot \exp(-A \log^{3/5} x \log \log x^{-1/5}))$, $A$ being a positive constant. Further, on the assumption of the Riemann hypothesis, we prove that $E(x) = O(x^{(2-\alpha)/(5-4\alpha)}) \exp\{A \log x (\log \log x)^{-1}\}$, $A$ being a positive constant, where $\alpha$ is the number which appears in the Dirichlet divisor problem. It is known that $1/4 < \alpha < 1/3$. The best known result till now is $\alpha = 12/37 + \epsilon$ for every $\epsilon > 0$. (Received March 24, 1972.) (Authors introduced by Dr. J. G. Krishna.)

Let \( \Phi(\mathcal{L}) \) be the Frattini subalgebra of a Stone algebra \( \mathcal{L} = \langle L; \lor, \land, \ast, 0, 1 \rangle \). Let \( D(L) = \{ a/a^* = 0 \} \), \( S(L) = \{ a^*a; a \in L \} \), \( P(L) \), and \( Q(L) \) be respectively the dense set, skeleton, posets of prime ideals, and prime dual ideals of \( L \). For each \( a \in S(L) \), set \( F_a = \{ x/x^{**} = a \} \). A pair \( (P, Q) \) in \( P(L) \times Q(L) \) is said to be minimal if (1) \( P \cap Q \neq \emptyset \), (2) if \( (P^*, Q^*) \in P(L) \times Q(L) \), \( (P^*, Q^*) < (P, Q) \), then \( P^* \cap Q^* = \emptyset \). Theorem 1.

A subset \( M \) of a Stone algebra \( \mathcal{L} \) is a maximal subalgebra iff either (i) \( M \cap S(L) \) is a maximal subalgebra of the Boolean algebra \( S(L) \) and \( M = \bigcup (F_x/ x \in M \cap S(L)) \), or (ii) \( M = L - P \cap Q \), where \( (P, Q) \) is a minimal pair in \( P(L) \times Q(L) \) with \( (P \cap Q) \cap S(L) = \emptyset \). Theorem 2. \( \Phi(\mathcal{L}) = \{ 0 \} \cup (D(L) - E) \), where \( E = \bigcup (P \cap Q)/(P, Q) \), minimal pair in \( P(L) \times Q(L) \), \( P \cap Q \cap S(L) = \emptyset \). Theorem 3. Let \( \mathcal{L} \) be a finite Stone algebra, \( L = \prod_{i=1}^{n} K_i \), \( K_i \) is a finite distributive dense lattice. Let \( \Phi(K_i) \) be the Frattini sublattice of \( K_i \). Then \( \Phi(\mathcal{L}) = (\prod_{i=1}^{n} \Phi(K_i) \cup \{ 1 \}) \cup \{ 0 \} \). Corollary 1. Let \( \mathcal{L} \) be a finite Stone algebra. Then \( \Phi(\mathcal{L}) = \{ 0, 1 \} \) iff \( L \) is a direct product of finite chains. Corollary 2. Let \( K \) be a finite Stone algebra. Then \( K \cong \Phi(\mathcal{L}) \) for some finite Stone algebra \( \mathcal{L} \) iff (1) \( K \) is dense, (2) \( K - \{ 0 \} = \prod_{i=1}^{n} K_i \) where either \( |K_i| = 1 \) or \( K_i \cong \Phi(L_i) \) for some finite distributive lattice \( L_i \) such that \( L_i \) is \( \lor \)-reducible in \( L_i \). (Received April 10, 1972.)

*72T-A142. R. PADMANABHAN, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada. On M-symmetric lattices. Every \( \iota \)-symmetric relatively semi-orthocomplemented lattice is M-symmetric. This answers Problem 1 of F. Maeda and S. Maeda ("Theory of symmetric lattices," Springer-Verlag, Berlin and New York, 1970, terminology as in this book) in the affirmative. (Received April 10, 1972.)

*72T-A143. N. S. MENDELSOHN, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada. A construction related to the van der Waerden theorem.

A famous theorem of van der Waerden states that given integers \( k \) and \( m \), then there exists an integer \( N(k, n) \) such that if the integers from 1 to \( N \) be partitioned into \( k \) subsets, then at least one of the subsets contains an arithmetic progression of \( m \) terms. In the present paper the following is proved. Given an integer \( k \) the positive integers can be partitioned into \( k \) subsets \( S_1, S_2, \ldots, S_k \) such that none of the sets contains an infinite arithmetic progression but each of the sets contains arithmetic progressions of arbitrarily large size. The construction can be carried out in many ways. (Received April 10, 1972.)

*72T-A144. EDUARDO R. BASTIDA and ROBERT GILMER, Florida State University, Tallahassee, Florida 32306. Overrings and divisorial ideals of rings of the form \( D + M \).

Let \( V \) be a valuation ring of the form \( K + M \), where \( K \) is a field and \( M \) is the maximal ideal of \( V \). Let \( D \) be a subring of \( K \) and let \( D_1 = D + M \). If \( \{ D_{\lambda} \} \) is the family of subrings of \( K \) containing \( D \) and if \( \{ W_{\alpha} \} \) is the family of overrings of \( V \), then \( \{ D_{\lambda} + M \} \cup \{ W_{\alpha} \} \) is the family of overrings of \( D_1 \). This leads to numerous results about overrings of domains that have been considered in the literature. These include, for...
example, GQR-domains [W. Heinzer, Mathematika 15(1968), 164-170] and QQR-domains [R. Gilmer and W. Heinzer, J. Math. Kyoto Univ. 7(1967), 133-150]. The ideals of $D_1$ compare with $M$ under containment, and if $A_1$ is an ideal of $D_1$ which contains $M$, then $A_1 = A + M$ for some ideal $A$ and $D$. **Theorem.** If $A_1 = A + M$, $A \neq (0)$, is an ideal of $D_1$ containing $M$, then $A_1$ is divisorial if and only if $A$ is divisorial. Moreover, $M$ is a divisorial ideal of $D_1$. A similar result is proved for ideals of $D_1$ properly contained in $M$. The domain $D_1$ has the property that each nonzero ideal is divisorial if and only if each $D$-submodule of $K$ is a divisorial fractional ideal of $D$. We conclude by obtaining a formula for the dimension of $D_1[X_1, \ldots, X_m]$; specifically, $\dim D_1[X_1, X_2, \ldots, X_m] = \dim V + \dim D[X_1, \ldots, X_m] + \inf \{m, d\}$, where $d$ is the transcendence degree of $K$ over the quotient field of $D$. (Received April 10, 1972.)

*72T-A145. CHARLES H. BRASE, University of Hawaii, Honolulu, Hawaii 96822, A note on intersections of valuation ideals.*

Let $R \subseteq S$ be integral domains with unity such that $S$ is integrally dependent on $R$. Let $\overline{R}$ be the integral closure of $R$ in its quotient field. Let $\mathcal{V}$ be the set of all valuation ideals of $R$. Let $\mathcal{J}$ be the set of all ideals of $R$ which can be written as an intersection of valuation ideals of $R$. Let $C(S)$ be the set of all ideals of $R$ which are contracted from ideals of $S$. It is known that $\mathcal{V} = C(S)$ ($\mathcal{J} \subseteq C(S)$ has been proved by R. Gilmer). The following question is natural: What are necessary and sufficient conditions that $\mathcal{V} = C(S)$? **Theorem.** Let $R$ be a domain. Let $S$ be a domain integral over $R$ such that $R \subseteq S$. The following are equivalent: (1) $\mathcal{V} = C(S)$, (2) $\mathcal{J} = C(S)$, (3) $\overline{R}$ is a valuation ring. (4) The completion of each principal ideal of $R$ is a valuation ideal of $R$. (5) An ideal of $R$ is a valuation ideal of $R$ if and only if it is complete. (Received April 10, 1972.)

*72T-A146. H. LAKSER, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada. Simple sublattices of free products of lattices.*

**Theorem 1.** Each nontrivial sublattice of a free lattice has a homomorphism onto the two-element chain. Let $(L_i | i \in I)$ be a family of lattices and let $L$ be their free product. **Theorem 2.** If $M$ is a simple nondistributive sublattice of $L$ then there is an $i \in I$ such that $L_i$ contains an isomorphic copy of $M$. **Theorem 3.** If $M$ is a modular simple sublattice of $L$ of finite length > 2 then $M \cap L_i \neq \emptyset$ for the $i$ of **Theorem 2.** **Theorem 4.** If $M$ is isomorphic to either the subspace lattice of a projective geometry of dimension > 1 or the partition lattice of a finite set with more than three elements and $M$ is a sublattice of $L$ then $M$ is a sublattice of $L_i$ for some $i \in I$. **Theorem 5.** If $M$ is a simple lattice of length 2 then $L$ contains an isomorphic copy of $M$ which is a subset of no $L_i$ if and only if there is an $i \in I$ such that $L_i$ contains an isomorphic copy of either $M \times 2$ or $M'$, the lattice obtained by replacing an atom of $M$ by the two-element chain. (Received April 10, 1972.) (Author Introduced by Professor George A. Grätzer.)
1-factors in point determining graphs. Preliminary report.

Following the terminology in Abstract 691-05-22, these NO. 691-05-22, A-36, a point determining graph is one in which distinct nonadjacent vertices have distinct neighborhoods. For a point determining graph \( G \), \( G^0 \) represents the subgraph of \( G \) induced by the set of those vertices whose removal again results in a point determining graph. For an arbitrary graph \( G \), \( \pi(G) \) denotes the point determining graph obtained from \( G \) by identifying two nonadjacent vertices whenever they have the same neighborhood. Let \( G \) be a point determining graph and \( a \notin G^0 \). Theorem 1. \( [\pi(G-\{a\})]^0 \) is isomorphic to an induced subgraph of \( G^0 \) and if \( G^0 \) is complete, then \( [\pi(G-\{a\})]^0 = G^0 \). Theorem 2. If \( G^0 \) is complete, then \( |G| \equiv |G^0| \pmod{2} \). Theorem 3. If \( G^0 \) is complete minus one edge, then \( |G| \equiv (|G^0|+1) \pmod{2} \). Theorem 4. If \( G^0 \) is complete of even order or a nontrivial complete graph of odd order minus one edge or is one of \( K_{1,3}, K_2 \cup K_2, P_4 \), then \( G \) has a 1-factor. Several questions are also raised. (Received April 24, 1972.)

Rings in which every right ideal is quasi-injective.

A ring \( R \) with unity \( 1 \neq 0 \) is said to be a right (left) q-ring if each of its right (left) ideals is quasi-injective. Following are the main results established: (I) Every homomorphic image of a right artinian ring \( R \) is a right q-ring if and only if it is a uniserial right q-ring. (II) A ring \( R \) is uniserial if and only if it is a direct sum of matrix rings over those right artinian rings all of whose homomorphic images are right q-rings. (III) Let \( R \) be an indecomposable right artinian ring which is not uniserial. Then \( R \) is a generalized uniserial right q-ring if and only if there exists a division ring \( K \) and an integer \( n > 1 \), such that \( R \) is isomorphic to the ring \( S \) of all \( n \times n \)-matrices over \( K \) of the form \( \sum e_{ii}^0 + \sum_{i < k} e_{ii}^0 e_{ii+1}^0 + e_{nn}^0 e_{nn+1}^0 + e_{nn}^0 e_{nn+1}^0 \) in which the addition composition is the usual matrix addition and the multiplication is such that besides satisfying the usual rules of matrix multiplication, the matrix units \( e_{ij}^0 \) satisfy the condition \( e_{ij}^0 e_{ik}^0 = 0 \) whenever \( i \neq j \neq k \). (IV) Let \( R \) be an indecomposable artinian right q-ring. Then either \( R \) is simple or \( R \) is a d.u.o. self-injective ring or \( R \) is generalized uniserial. (Received April 14, 1972.) (Author introduced by Professor S. K. Jain.)

Equivalent conditions for every sheaf of \( R \)-modules to be quasi-coherent. Preliminary report.

Let \( R \) be a commutative ring with identity, and let \( \mathcal{R} \) be the structure sheaf of \( R \). Theorem. The following are equivalent: (1) Every sheaf of \( \mathcal{R} \)-modules is quasi-coherent; hence the category of \( R \)-modules is equivalent to the category of sheaves of \( \mathcal{R} \)-modules, (2) \( \text{Spec}(R) \) is Hausdorff, (3) \( R/N(R) \) is von Neumann regular where \( N(R) \) is the nil radical of \( R \). The equivalence of (1) and (2) is proved with the aid of C. J. Mulvey's criterion for \( \mathcal{R} \) to be a generator in its category of modules (J. Algebra 15(1970), 312-313). (Received April 17, 1972.)
The complement $\overline{G}$ of a graph $G$ is the graph whose vertex set is that of $G$ and such that two vertices are adjacent in $\overline{G}$ if and only if they are not adjacent in $G$. Let $\delta(G)$ and $\Delta(G)$ denote the minimum and maximum degrees of a graph $G$ respectively. Only finite unoriented graphs without loops and multiple edges are considered. It was shown by Alavi and Mitchum ["Recent trends in graph theory," Lecture Notes in Math., no. 186, Springer-Verlag, Berlin and New York, 1971] that for any graph $G$ of order $p \geq 2$, (i) $1 \leq \delta(G) + \delta(\overline{G}) \leq p - 1$, (ii) $0 \leq \delta(G) \cdot \delta(\overline{G}) \leq M(p)$, where $M(p) = \left[\frac{(p-1)/2}{(p-1)/2}\right]$ if $p \equiv 0, 1, 2 \pmod{4}$, and $= \left[\frac{(p-3)/2}{(p-3)/2}\right]$ if $p \equiv 3 \pmod{4}$.

It is shown here that for any graph $G$ of order $p \geq 2$, (i) $p - 1 \leq \Delta(G) + \Delta(\overline{G}) \leq 2p - 3$, (ii) $0 \leq \Delta(G) \cdot \Delta(\overline{G}) \leq M(p)$, where $M(p)$ is the same as above, and these bounds are best possible.

(Received April 17, 1972, Author introduced by Professor Remu Laskar.)

For a lattice $K$ let $K^b = K \cup \{0_b, 1_b\}$ denote the bounded lattice containing $K$, $0_b, 1_b \notin K$. If $L_i, i \in I$, are lattices and $L$ is a free product of the $L_i, i \in I$, then for every $a \in L$ there is a largest (resp. smallest) element $a^{(i)}$ (resp. $a^{(i)}$) of $L_i^b$ contained in (resp. containing) $a$. The free lattice is known to have the following properties: (F1) $x \land y \leq u \lor v$ implies that $x \leq u \lor v$ or $y \leq u \lor v$ or $x \land y \leq u \lor v$; (F2) $u = x \lor y = x \lor z$ implies that $u = x \lor (y \land z)$; (F3) is the dual of (F2). Let $A_i$ be a sublattice of $L_i^b, i \in I$. Theorem. Let $A$ be a sublattice of $L$ such that $a^{(i)}$ and $a^{(i)} \in A_i$ for all $a \in A$ and $i \in I$. If, for all $i \in I, A_i$ satisfies (F1) (resp. (F2) and (F3)), then $A$ satisfies (F1) (resp. (F2) and (F3)). The proof uses the results of G. Grätzer, H. Lakser, and C. R. Platt, Fund. Math. 69(1970), 233-240. (Received April 17, 1972.)

Theorem. Let $A$ be a sublattice of $L$ such that $a^{(i)}$ and $a^{(i)} \in A_i$ for all $a \in A$ and $i \in I$. If, for all $i \in I, A_i$ satisfies (F1) (resp. (F2) and (F3)), then $A$ satisfies (F1) (resp. (F2) and (F3)). The proof uses the results of G. Grätzer, H. Lakser, and C. R. Platt, Fund. Math. 69(1970), 233-240. (Received April 17, 1972.)

As in (Abstract 72T-ASS, these Notices 19(1972)) an algebra $A$ over a field $F$ is called scalar dependent if there is a map $g : A \times A \times A \rightarrow F$ satisfying $(xy)z = g(x, y, z)x(yz)$ for all $x, y, z$ in $A$. We prove that if $A$ is a scalar dependent algebra with an idempotent $e \neq 0$ then $A$ must automatically be associative. There do exist, however, examples of nil scalar dependent algebras which are not associative. (Received April 24, 1972.)

*72T-A151. GEORGE A. GRÄTZER and H. LAKSER, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada. Free lattice-like sublattices of free products of lattices.

(Received April 17, 1972, Author introduced by Professor Remu Laskar.)

*72T-A152. DAVID W. BALLEW and RONALD C. WEGE, South Dakota School of Mines and Technology, Rapid City, South Dakota 57701. Triangular numbers with all digits equal.

In 1905, E. B. Escott proved that 1, 3, 6, 55, 66, and 666 are the only triangular numbers of less than 30 digits that consist of a single repeated digit (Dickson, "History of the theory of numbers, VI, IP"). We have completed the proof of this result and shown that these are the only such numbers. The method reduced the problem to a finite number of cases and each of these was tested on the computer. (Received April 24, 1972.)

*72T-A153. RAYMOND F. COUGHLIN and MICHAEL RICH, Temple University, Philadelphia, Pennsylvania 19122. Scalar dependent algebras. II.

(Received April 24, 1972.)

Consider the 28 unordered pairs of distinct numbers taken from the numbers 0, 1, 2, 3, 4, 5, 6, 7. If one distributes these pairs into the squares of a 7 x 7 checkerboard in such a way that every square contains at most one pair, every number from 0 to 7 appears in some pair in each row, and every number from 0 to 7 appears in some pair in each column, then the resulting design is called a room design of side 7. Two room designs of side 7 are said to be equivalent iff one can be obtained from the other by permuting the rows, permuting the columns, permuting the numbers 0 to 7 which appear in the pairs, by taking the transpose of the checkerboard, considered as a 7 x 7 matrix, or by any combination of these operations. Theorem. There are exactly 6 equivalence classes of room designs of side 7. (Received April 24, 1972.)

The number of solutions of $A^2 = \emptyset$.

In this paper we obtain the enumeration of all binary relations defined on a finite set of n elements which satisfy $A^2 = \emptyset$. Let $\mathcal{R}_n$ denote the set of all binary relations on a set of n elements. Let $N(\mathcal{R}_n) = \{A \in \mathcal{R}_n : A = \emptyset\}$. Then the elements of $\mathcal{R}_n$ can be readily represented by $n \times n$ matrices over the two element Boolean algebra $\{0, 1\}$ or digraphs defined on a set of n elements which have no paths of length 2. Theorem, $|N(\mathcal{R}_n)| = \sum_{i=0}^{n} \binom{n}{i} (2^i - 1)^{n-i}$. (Received April 24, 1972.)

A HOM-functor for lattice-ordered groups. Preliminary report.

By defining a quasi-order on the $\Lambda$-homomorphisms of one lattice-ordered group to another (all groups abelian) one can set up a co-compatible system of partially ordered groups. Their colimit $L(A, B)$, where $A$ and $B$ are the $\Lambda$-groups in question, is a directed, semiclosed partially ordered group. If $A$ is a totally ordered group then $L(A, B)$ is simply the subgroup in $\text{Hom}(A, B)$ generated by the $\sigma$-homomorphisms. On the other hand if $B = R$, the additive group of reals with the usual order, then $L(A, B)$ is a cardinal sum of copies of $R$, one for each maximal $\Lambda$-ideal of $A$. In general the co-compatible system mentioned above is far from being directed. $L(\ldots, B)$ is a contravariant functor; not much happens functorially in the second variable. $L(\ldots, B)$ is additive, whenever the sum of two $\Lambda$-homomorphisms is defined. It transforms $\Lambda$-epimorphisms (onto maps) into $\sigma$-embeddings. The functor also preserves finite cardinal sums. If the sequence $A \rightarrow B \rightarrow C$ is exact, i.e. $C \cong B/A$, then $L(C, X) \rightarrow L(B, X) \rightarrow L(A, X)$ is exact at the first two places, for any totally ordered group $X$, provided $B \rightarrow C$ is a retraction. In particular this is so when $C$ is projective (relative to onto maps), or $B$ is divisible and $A$ is prime in $B$, or $B$ is a lexicographic, direct extension of $A$ by $C$. (Received April 27, 1972.)
A domain $D$ is said to have Property 1 if the complete integral closure of $D$ is not completely integrally closed, and it has Property 2 if it is completely integrally closed but is not an intersection of rank-one valuation overrings. Examples of domains with infinite Krull dimension satisfying these properties already exist, and Heinzer (J. Austral. Math. Soc. 9(1969), 310-314) has asked whether Prüfer domains with either of these properties were necessarily infinite-dimensional. In this paper, both of Heinzer's questions are answered in the negative. Two examples of Bezout domains are constructed, each with Krull dimension two, and satisfying Properties 1 and 2 respectively. The method of construction in each case consists of first constructing an appropriate lattice-ordered abelian group $G$, and then citing the existence of a Bezout domain $D$ with $G$ as its divisibility group. In showing that $D$ has the required properties, the following results are very useful: Theorem. In a Bezout domain $D$ there is a one-one, inclusion-preserving correspondence between proper prime ideals of $D$ and proper prime filters in the positive cone of the divisibility group of $D$. Corollary. The Krull dimension of a Bezout domain $D$ is determined by its divisibility group. (Received April 27, 1972.)

Unlabelled graphs with many nodes and edges. III. Preliminary report.

$T = T(n, q)$ is the number of graphs with $n$ unlabelled nodes and $q$ undirected edges, $N = n(n-1)/2$ and $\gamma = 2(q/n) - \log n$. Since $T(n, N-q) = T(n, q)$, we suppose $q \leq N/2$. The positive number $C$ is independent of $n$ and $q$, while $\eta = O(q^{-C})$; $C$ and $\eta$ are not the same at every occurrence. We improve the results in these $\mathbb{N}$otices A-3, by replacing the factor $1 + o(1)$ implicit in each asymptotic formula for $T$ by a factor $1 + \eta$. Again, we close the gap in the results and find asymptotic formulae for $T$ for all large $q$. From these we deduce that $T(n, q+1)/T(n, q) = (n^2/2q)(1+\eta)$ if $\gamma \geq 0$, and that $T(n, q+1)/T(n, q) = (\gamma^2/2q)(1+\eta)$ if $\gamma \leq 0$, where $v$ is the greatest integer such that $v \log v \leq 2q$. From these and the results of Acta Math. 126(1971), 1-9, we deduce that $T(n, q+1) > T(n, q)$ for $1 \leq q \leq (N/2) - 1$, and all $n >$ some $n_0$. From tables, this is false for $n = 5$ and true for $6 \leq n \leq 18$. A direct proof that this is true for all $n \geq 6$ would be interesting. (Received April 27, 1972.)
Analysis

72T-B135. CARLOS AUGUSTO SHOLL ISNARD, Instituto de Matemática Pura e Aplicada, Rio de Janeiro, Brasil. Orientation and degree in infinite dimensions.

Let \( E \) be a real Banach space, \( \Phi_0(E,E) = \{ \text{Fredholm operators } E \to E \text{ of index 0} \} \), \( \Phi_\ast (\mathbb{R}) \) is an open subset of \( L(E,E) \) with homotopy type of \( GL(\mathbb{R}) \), hence with two path-components that define orientations. Let \( X \) and \( Y \) be oriented \( C^1 \)-manifolds modeled after \( E \); then oriented degree is defined for all proper \( C^1 \)-\( \Phi_\ast \)-maps \( f: X \to Y \) (\( C^1 \)-\( \Phi_\ast \)-map: derivatives of compositions with charts in \( \Phi_\ast (\mathbb{R}) \)). If \( f \) is transversal to an oriented submanifold \( M \) of \( Y \), then \( f^{-1}(M) \) is an oriented manifold and \( \Gamma: f^{-1}(M) \to M \) is a \( C^1 \)-\( \Phi_\ast \)-map with the same oriented degree at points of \( M \) as \( f: X \to Y \). For every \( C^1 \)-\( \Phi_0 \)-map \( f: X \to Y \), some \( \gamma_f \in H^1(X, \mathbb{Z}^2) \) is defined, and it is shown that \( \gamma_f = 0 \) if and only if the restriction of \( f \) to \( \gamma_f \) is a \( C^1 \)-\( \Phi_\ast \)-map. So oriented degree can be defined for \( f \) when \( \gamma_f = 0 \) and \( f \) is proper. (Received February 3, 1972.)

72T-B136. A. R. REDDY, University of Missouri, St. Louis, Missouri 63121. Chebyshev rational approximation to certain entire functions in \([0, \infty)\).

Let \( \lambda_{m,n} = \min_{r_{m,n}} \max_{p \in \Pi_{m,n}} r_{m,n}(\xi) = \frac{\max_{p \in \Pi_{m,n}} |p(\xi) - f(\xi)|}{|p(\xi)|} \), where \( r_{m,n} = \frac{p_m(\xi)}{q_n(\xi)} \), \( p_m(\xi) \in \Pi_m \cdot \Pi_n \) denotes the class of all ordinary polynomials of degree at most \( m \), \( \Pi_{m,n} \) denotes the class of real rational functions of the form \( r_{m,n}(\xi) \). An entire function \( f(\xi) \) is of perfectly regular growth (\( \rho, \gamma \)) if and only if there exist two (finite) positive constants \( \rho \) and \( \gamma \) such that \( \rho M(\xi) / r^\rho \). Recently, Meinardus and Varga proved the following Theorem A. Let \( f(\xi) \) be an entire function of perfectly regular growth (\( \rho, \gamma \)) with the nonnegative coefficients, then \( \frac{1}{2} + 1/\rho \leq \lim_{n \to \infty} \rho M(\xi) / r^\rho \). In this we prove the following: Theorem. Let \( f(\xi) = \sum_{k=0}^{\infty} a_k \xi^k \) (\( a_k \geq 0 \) for \( k \geq 0 \)) be an entire function of perfectly regular growth (\( \rho, \gamma \)). Then \( \frac{1}{2} + 1/\rho \leq \lim_{n \to \infty} \rho M(\xi) / r^\rho \). (Received February 11, 1972.)

72T-B137. PHILIP J. BOLAND, University College of Dublin, Belfield, Dublin 4, Ireland. Entire functions of exponential type on a Banach space. Preliminary report.

For two complex normed linear spaces \( M \) and \( N \), let \( H(N, M) \) be the space of holomorphic functions from \( N \) to \( M \), and \( Exp(N, M) \) the subspace of functions of exponential type. Let \( N_1, N_2 \), and \( E \) be three complex normed linear spaces where \( E \) is Banach and \( N_1 \) is nontrivial. Proposition 1. If \( f \in H(E, N_2) \), then \( f \in Exp(E, N_2) \) iff \( \varphi \circ f \in Exp(E, C) \) for all \( \varphi \in E' \). Proposition 2. If \( f \in H(E, N_2) \), then \( f \in Exp(E, N_2) \) iff \( f \circ T \in Exp(N_1, E) \) for all continuous linear transformations \( T \) from \( N_1 \) to \( E \). Proposition 3. Let \( f \in H(E, N_2) \). Then \( f \in Exp(E, N_2) \) iff the restriction of \( f \) to every complex line in \( E \) belongs to \( Exp(C, N_2) \). These results hold for other types of growth in addition to exponential growth. (Received February 21, 1972.)
U, F, R_AB and the notion of integral are as in previous abstracts of the author. L and G denote, respectively, the sum supremum and sum infimum functional. **Theorem.** Suppose T is a function from R_AB into R. The following two statements are equivalent: (1) T is in the dual of R_AB (where \( \|h\| = \int_U |h(t)| \) for all h in R_AB), and (2) there is a set S with partial ordering P, with respect to which S is directed, and a bounded function w from S x F into R such that if h is in R_AB and Q is L or G, then T(h) = \( \lim_P \int_U Q(w(x, \cdot)h) \).

(Received February 28, 1972.)

**72T-B139. ERNEST E. BURNISTON and C. E. SIEWERT, North Carolina State University, Raleigh, North Carolina 27607.** Exact analytical solutions of the transcendental equation \( \alpha \sin \zeta = \zeta \).

Complex analysis is used to derive exact solutions of the transcendental equation \( \alpha \sin \zeta = \zeta \), where \( \alpha \) is an arbitrarily assigned complex number. The method involves the use of canonical solutions to suitably posed Riemann problems, and the explicit results are expressed in terms of elementary quadratures. (Received February 28, 1972.)


Let U, V be two arbitrary nonempty sets in a normed linear space X. The points \( \tilde{u} \in U, \tilde{v} \in V \) are called the proximal points of the sets U, V if \( \|\tilde{u} - \tilde{v}\| = \inf_{u \in U, v \in V} \|u - v\| \). Here we are interested in studying some characterizations of proximal points and also to obtain lower bounds on \( d(U, V) \), the distance between the two sets. This is done by introducing certain families of pairs of half spaces called the separating families, supporting families and strong supporting families. These notions enable us to obtain a duality theorem for \( d(U, V) \) and also to obtain sufficient conditions for proximal points. The applications of these results are discussed for the spaces \( C(T) \) (\( T \) compact Hausdorff). Incidentally, some results of our earlier paper concerning the proximal points for the case when both the sets U, V are convex are noted as special cases of the results of this paper. (See also Abstract 71T-B205, these Notices 18(1971); Theorem 1 of this abstract is now proven for arbitrary normed linear spaces.) (Received March 3, 1972.) (Author introduced by Professor P. C. Jain.)

**72T-B141. CHARLES F. DUNKL and DONALD E. RAMIREZ, University of Virginia, Charlottesville, Virginia 22903.** Weakly sequentially complete function algebras.

**Theorem.** If a function algebra A (possibly without a unit) is weakly sequentially complete, then it is finite dimensional. **Proof.** We may assume A is separable, and so the Shilov boundary \( \partial A \) is metrizable. By the Lebesgue dominated convergence theorem and the weak sequential completeness, the characteristic function of any peak point is in A. Similarly, the characteristic function of any denumerable set of peak points is also in A. Thus the set of peak points is finite and so equals \( \partial A \). (This result is based on a method of R. E. Edwards,
Remark. Convolution measure algebras are examples of weakly sequentially complete algebras. (Received March 6, 1972.)


Let \( A \) be a regular weighted mean matrix generated by nonnegative \( p_n \) with \( p_n / P_n \to 0 \), and \( f \) a continuous map of \([0,1]\) into itself. Then the iterative scheme \( x_1 = x_1, x_{n+1} = f(x_n), \text{ and } x_n = \sum_{k=1}^{n} a_k x_k, n = 1, 2, \ldots, \) converges to a fixed point of \( f \) on \([0,1]\). This result generalizes the theorem of R, L. Franks and R, P. Marzec [Proc, Amer, Math, Soc, 30(1971), 324-326] wherein \( A \) is taken as the \((C,1)\) method. The author of this abstract conjectures that the above result is true for any regular triangular matrix with nonnegative entries with each row sum not exceeding 1, and satisfying the property that the set of limit points of the \( A\)-transform of each bounded sequence is a connected set (see, \( \text{e.g.} \) Barone [Duke Math, J, 5(1939); MR 1, 218]). (Received March 6, 1972.)

*72T-B143. H, M. SRIVASTAVA, University of Victoria, Victoria, British Columbia, Canada. A reduction formula for generalized hypergeometric functions.

In the present paper the author obtains a generalization of a transformation formula, given recently by R. Rösler [Z, Angew, Math, Mech, 43(1963), 433], which expresses the generalized hypergeometric function \( _3F_2 \) as a finite sum of the Gaussian hypergeometric functions \( _2F_1 \). An application of this general reduction formula is shown to yield an interesting expansion associated with the product of two Bessel polynomials, which arise as solutions of the classical wave equation in spherical coordinates. This paper is scheduled to appear in Z, Angew, Math, Mech. (Received March 6, 1972.)

72T-B144. MICHAEL G. COWLING, School of Mathematical Sciences, Flinders University of South Australia, Bedford Park, South Australia 5042. Functions which are restrictions of \( L^p \)-multipliers.

Raouf Doss has given a sufficient condition for a measurable function \( \phi \) on a measurable subset \( \Lambda \) of an LCA group \( \Gamma \) to be the restriction (l.a.e.) to \( \Lambda \) of the Fourier transform of a bounded measure, i.e., a Fourier multiplier of type \((1,1)\). We generalise Doss' theorem, and prove that if the measurable function \( \phi \) on \( \Lambda \) is approximable on finite subsets of \( \Lambda \) by trigonometric polynomials which are \( L^p \) Fourier multipliers on \( \Gamma \) of norms no greater than \( C \), then \( \phi \) is the restriction (l.a.e.) to \( \Lambda \) of a Fourier multiplier of type \((p,p)\) and norm no greater than \( C \). (Received March 7, 1972.) (Author introduced by Dr, G, I, Gaudry.)

*72T-B145. JACK S. SHAPIRO, Baruch College, City University of New York, New York, New York 10010 and MARTIN SCHECHTER, Belfer Graduate School, Yeshiva University, New York, New York 10033. A generalized operational calculus developed from Fredholm operator theory.

Let \( \Lambda \) be a closed operator on the Banach space \( X \). We construct an operator, \( R_\Lambda(A) \), depending on the parameter, \( \lambda \), and having the following properties: \((\lambda - A) R_\Lambda(A) = I + F_1, R_\Lambda(A) (\lambda - A) = I + F_2, \) where \( F_1 \) and \( F_2 \) are
$F_2$ are bounded finite rank operators. $R'_\lambda (A)$ is defined and analytic in $\lambda$ for all $\lambda \in \Phi_A$ except for at most a countable set containing no accumulation point in $\Phi_A$. Let $\sigma_\Phi (A)$ be the complement of $\Phi_A$, and let $f \in \mathcal{S}'(\infty)$, where $\mathcal{S}'(\infty)$ denotes the set of complex valued functions which are analytic on $\sigma_\Phi (A)$ and at $\infty$. We then use the operator, $R'_\lambda (A)$, to construct an operational calculus for $A$, $f(A)$ is defined up to addition by a compact operator. We prove for our operational calculus analogues of the theorems for the classical operational calculus, including the Spectral Mapping Theorem. We then extend a theorem of Kato by using the operator, $R'_\lambda (A)$, to construct an analytic basis for $N(A-\lambda)$.

(Received March 8, 1972.)


Let $G$ be a bounded strongly pseudoconvex domain in $\mathbb{C}^n (n \geq 2)$, with a $C^5$ boundary. There is a positive constant $K$ with the following property: For $0 \leq q \leq n - 1$, $1 \leq p \leq \infty$, if $\alpha$ is an $(0,q+1)$-form with coefficients in $L^p(G)$ such that $\bar{\partial} \alpha = 0$ in the sense of distributions, then an $(0,q)$-form $\beta$ on $G$ with coefficients in $L^p(G)$ can be found such that $\bar{\partial} \beta = \alpha$ in the sense of distribution theory and $\|\beta\| L^p(0,q;G) \leq K\|\alpha\| L^p(0,q+1;G)$.

In addition, if the coefficients of $\alpha$ are continuous so are those of $\beta$, if the coefficients of $\alpha$ are smooth so are those of $\beta$. The methods used have a lot in common with those appearing in I. Grauert and I. Lieb ("Das Ramirezsche Integral und die gleichung $\bar{\partial} f = \alpha$ in Bereich der beschränkten Formen", Rice University Studies, 1970), I. Lieb ("Die Cauchy-Riemannschen Differentialgleichungen auf streng pseudokonvexen Gebieten", Math. Ann. 190(1971)) and the proofs hinge upon estimates of one of the Ramirez-Chenkin kernels. (Received March 7, 1972.) (Author introduced by Professor Wolfgang Fuchs.)

*72T-B147. ALBERT WILANSKY, Lehigh University, Bethlehem, Pennsylvania 18015. A two-norm characterization of the distinguished subspace $W$.

Let $E^\infty$ be the set of finite complex sequences. An FK space $E \supset E^\infty$ is called semi-conservative if $\{a^n\}$ is weakly Cauchy, where $a^n = (l,1,\ldots,1,0,0,\ldots)$ with $n$ ones. $W$ is the subspace of those $x$ for which $x^n \rightharpoonup x$ weakly, $x$ being the $n$th section $(x_1,x_2,\ldots,x_n,0,0,\ldots)$. Given some norm $p$ defined on $E^\infty$ and $S \subset E^\infty$, let $2_S(p)$, called the two-norm sequential closure of $S$, be the set of all $x \in E$ such that $x$ is the limit (in $E$) of a $p$-bounded sequence in $S$.

**Theorem 1.** $W \cap bv = 2 bv$, where $bv$ is the norm $\|x\|_{bv} = \sum |x_n - x_{n+1}|$.

**Theorem 2.** Let $E \supset c_0$. Then $W \cap m = 2_\infty E^\infty$ where $2_\infty$ refers to $\|x\|_\infty = \sup |x_n|$. A corollary is A. K. Snyder's characterization of comull spaces by $1 \in 2_\infty E^\infty$ since $E$ is comull iff $1 \in W$. However the above generalization does not assume $1 \in E$ and the theory can be carried forward without this assumption. (Received March 16, 1972.)

72T-B148. ANDREAS Z. ZACHARIOS, Johns Hopkins University, Baltimore, Maryland 21218. Two representation theorems. Preliminary report.

**Theorem 1** (Generalized Riesz representation theorem). Let $(V, \langle \cdot, \cdot \rangle)$ be an inner-product space over $R$ and $f: V \rightarrow R$ a linear functional. Suppose that: (Ker $f$)$^\perp \neq 0$. Then: (i) $V = \text{Ker } f \oplus (\text{Ker } f)^\perp$, (ii) dim $\text{Ker } f^\perp = 1$, (iii) there exists a unique vector $a$ in $(\text{Ker } f)^\perp$ such that: $f(x) = \langle a, x \rangle$ for all $x$ in $V$. A-517
Application (Fundamental lemma of the calculus of variations). Let $a, b$ be real numbers with $a < b$. Let $x_0$ be a fixed point of $[a, b]$. Let $g: [a, b] \to \mathbb{R}$ be a continuous function. Suppose that $\int_a^b g(x) f(x) \, dx = 0$ for all continuous functions $f: [a, b] \to \mathbb{R}$ with $f(x_0) = 0$. Then $g = 0$.

Theorem 2 (Representation of linear functionals as determinants). Let $f: \mathbb{R}^n \to \mathbb{R}$ be any linear functional. Then there exist vectors $a_1, \ldots, a_{n-1}$ in $\mathbb{R}^n$ such that:
$$f(x) = \det(a_1, \ldots, a_{n-1}, x)$$
for all $x$ in $\mathbb{R}^n$.

We define a subclass of $p$-absolutely summing operators called $p$-extending. We prove that $T: A(K) \to X$ is $p$-extending iff it extends to an isometry $U: A^P(K, \mu) \to X$, where $A(K)$ is a closed logmodular subspace of $C(K)$, $X$ a Banach space, $\mu$ a probability measure on $K$ and $A^P(K, \mu)$ the closed subspace of $L^P(K, \mu)$ generated by $A(K)$. We use this result to give necessary and sufficient conditions under which scalar and subscalar operators are isometrically equivalent to multiplication by $z$ on $L^P(K, \mu)$ and $H^P(K, \mu)$ respectively. (Received March 16, 1972.)

A remark on the Polya-Schoenberg conjecture. Let $S$ be the class of normalized $(f(0) = 0, f'(0) = 1)$ schlicht functions in the unit disk, and $C$ and $K$ the close-to-convex and convex subsets of $S$, respectively. A well-known conjecture of Polya and Schoenberg [Pacific J. Math, 8 (1958), 295-334] asserts for $f(z) = \sum_{n=1}^{\infty} a_n z^n$, $g(z) = \sum_{n=1}^{\infty} b_n z^n \in K$ that $(f \ast g)(z) = \sum_{n=1}^{\infty} a_n b_n z^n \in K$. In this direction Suffridge [J. Math, Mech, 15 (1964), 795-804] proved that $f \ast g$ belongs to $C$ and is therefore schlicht. Recently Brickman, MacGregor, Wilkin [Trans. Amer. Math, Soc, 156 (1971), 91-107] observed that $\overline{\text{co}} K$, the closed convex hull of $K$ in the usual topology, is characterized by (1) $\text{Re} f/z > \frac{1}{2}$. Much earlier, Schur [J. Reine Angew. Math, 148 (1918), 122-145, Satz XIV] and others noted a result which implies that if $f$ and $g$ satisfy (1) then $f \ast g$ also satisfies (1). That is, if $f, g \in \overline{\text{co}} K$, then $f \ast g \in \overline{\text{co}} K$.

Therefore if $f, g \in K$, we remark that $f \ast g$ is not only close-to-convex but also in the closed convex hull of $K$. (Received March 17, 1972.)

Transformations of Walsh-Fourier coefficients.


In the present paper we have examined the analogous problem for Walsh-Fourier coefficients [for Walsh-Fourier series see Fine, Trans, Amer, Math, Soc, 65 (1949), 372-414]. Let $W$ denote the set of Walsh polynomials and $W_\infty$ denote the set of Walsh-series. The complementary space $E^*$ of space $E \subset W_\infty$ is the space of all functions $\hat{f} = \sum_{k=0}^{\infty} a_k \psi_k(x)$ such that for every function $\hat{g} = \sum_{k=0}^{\infty} b_k \psi_k \in E$ the sum $\langle \hat{f}, \hat{g} \rangle = \sum_{k=0}^{\infty} a_k b_k$ exists. The space $E_n$ is the subset of $E \subset W_\infty$ in which $\lim_{n \to \infty} \|s_n(\hat{f}) - \hat{f}\| = 0$, for $\hat{f} \in E$. Let $A = (a_{kj})$ be an infinite matrix.
matrix, \( a_j \) be Walsh-Fourier coefficients of functions belonging to \( E \) and \( a_k = \sum_{j=0}^{\infty} a_{kj} \) be Walsh-Fourier coefficients of functions belonging to \( E_1 \); then we say that \( A \) generates the transformation \( T \) and we write \( T \in (E, E_1) \). The following are the some typical results of this paper. **Theorem A.** \((L_p,)_N \subset L_p^k (1 \leq p \leq \infty), 1/p + 1/p' = 1, \)**

**Theorem B.** Let \( W \subset E \subset W_0^\infty \) be a Banach space and \( E_1 = L_q (1 < q \leq \infty) \); then \( T \in (E, E_1) \) if and only if \( \sum_{k=1}^{\infty} a_{kj}(t) \in E^* (k=1, 2, 3, \ldots) \) and \( \|T_n\|_{EE_1} = O(1) \) (\( n \rightarrow \infty \)), where \( T_n = \sum_{k=1}^{n} a_{kj}(t) \). **Theorem C.** If \( T \in (L_p, L_p^k), 1 < p < \infty, \) then \( T^* \in (L_p', L_p') \) where \( 1/p + 1/p' = 1 \) and \( T \) is generated by transpose of matrix \( A \). (Received March 17, 1972.)

*72T-B152. DOROTHY BROWNE SHAFFER, Courant Institute, New York University, New York, New York 10012. Distortion theorems for a special class of analytic functions.

Estimates are derived for the class of analytic functions \( P_{\alpha,n} \) with expansion \( p(z) = 1 + a_n z^n + \ldots \), \( n \geq 1, |z| < 1 \) which assume values in the disc \( |p(z) - 1/2 \alpha| \leq 1/2 \alpha \) where \( 0 \leq \alpha < 1 \). **Theorem 1.** Let \( p(z) \in P_{\alpha,n} \); then \( |p'(z)/p(z)| \leq (1+\alpha)\alpha^n \alpha^2 - (1-\alpha)\alpha^2 \) for \( |z| < 1 \) where \( c = 1 - 2\alpha \). The same estimate holds for \( |h'(z)/h(z)| \) where \( h(z) = 1/p(z) \) and \( h(z) \) satisfies \( \Re \{h(z)\} > \alpha \). **Corollary.** Let \( p(z) > 0 \), then \( |p'(z)/p(z)| \leq 2n|z|^{n-1} - (1-|z|^{2n}) \). **Theorem 2.** Let \( p(z) \in P_{\alpha,1} \) with \( 0 \leq \alpha \leq \frac{1}{2} \); then \( |p'(z)| \leq (1+\alpha)(1-\alpha)|z|^2 \) for \( |z| \leq r \), and \( |p'(z)| \leq (1+\alpha)(1+|z|^2)^2/4 - (1-|z|^2)(1+\alpha)|z|^2 \) for \( |z| > r \), \( r \) largest root of \( cx^3 + x^2(1-2c) + x(2-c) - 1 = 0 \). **Theorem 3.** Let \( f(z) \) be analytic in \( |z| < 1 \), \( f(0) = 0, f'(0) = 1 \), \( f(z) \in P_{\alpha,1} \); then \( f(z) \) maps the disc \( |z| \leq 1/(n+1) + (n-1)c \) onto a convex domain; \( f(z) \) assumes values in the annulus \( 1/c - (1+1)c \) onto \( c^2 \) for \( |z| < -1/c - (1+c) \log(1-c)/c \). All estimates are sharp. (Received March 20, 1972.)

*72T-B153. WITHDRAWN


For definitions see W. B. Johnson, "Markuschevich bases and duality theory," Trans. Amer. Math. Soc., 149(1970), 171–177. **Theorem.** A locally convex space \( E \) with a countable Markuschevich basis is semireflexive if each countable M-basis for \( E \) is shrinking or if each countable M-basis for \( E \) is boundedly complete. The proof is based on the existence theorem (Theorem III.6) of Johnson's article. The following corollaries result from two theorems of N. J. Kalton, "Schauder bases and reflexivity," Studia Math. 38(1970), 255–266. **Corollary 1.** Let \( E \) be a sequentially complete locally convex space with a Schauder basis. The following are equivalent: (1) \( E \) is semireflexive; (2) Each countable M-basis for \( E \) is shrinking; (3) Each Schauder basic sequence in \( E \) is shrinking; (4) Each countable M-basis for \( E \) is boundedly complete; (5) Each Schauder basic sequence in \( E \) is boundedly complete. **Corollary 2.** Let \( E \) be a complete barrelled space with a normalized Schauder basis. The following are equivalent: (1) \( E \) is reflexive; (2) Each countable M-basis for \( E \) is shrinking; (3) Each normalized Schauder basis for \( E \) is shrinking; (4) Each countable M-basis for \( E \) is boundedly complete; (5) Each normalized Schauder basis for \( E \) is boundedly complete. (Received March 21, 1972.)

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An abundance of prime ideals in many convolution subalgebras $A$ of $M(G)$ is obtained by observing that $C(K)$ is a homomorphic image of $A$ for an appropriate Helson set $K$. Normed ideals and subalgebras in $M(G)$ are introduced and their countably generated, closed ideals $J$ are discussed. Necessarily $Z(J)$ is open-closed; if $G$ is compactly generated, and $A$ contains such a nonzero $J$, $G$ must be $\mathbb{Z}^n \times C$, $C$ a compact group, and $J$ must consist of those $L^1$ functions whose Fourier transforms vanish on $T^n \times E$, where $E$ is a cofinite subset of the dual of $C$. In particular, a Segal algebra on $G$ (satisfying mild restrictions) can have a countably generated regular maximal ideal if and only if $G$ is finite. (Received March 23, 1972.)

On nonstandard topological vector spaces and invariant subspaces of compact operators. Preliminary report.

Let $H$ be a Hausdorff topological vector space for topology $\tau$ on $H$. Let $T$ be a linear operator mapping $H$ into $H$ such that $T[V]$ is $\tau$-compact for some $V$, a $\tau$-neighborhood of the zero vector in $H$. This paper gives two sufficient conditions for $(H, \tau)$ such that $T$ has a nontrivial, $\tau$-closed invariant linear subspace of $H$. The proofs and conditions are stated within the framework of nonstandard analysis. The sequence spaces $l(p_v)$ defined by S. Simons ("The sequence spaces $l(p_v)$ and $m(p_v)$," Proc. London Math. Soc. (3) 15(1965), 422-436) are examples of spaces that satisfy the two conditions stated in this paper. (Received March 24, 1972.)

Hyperplane and reflexivity. Preliminary report.

Let $X$ be a Banach space and $Q : X \to X^{**}$ be the canonical embedding. **Theorem.** $X$ is reflexive if and only if for any closed hyperplane $H$ of $X^*$, $\bigcap_{x^* \in H} x^{*-1}(0) \neq (0)$ if and only if for any closed hyperplane $K$ of $X^{**}$, $\bigcap_{x^{**} \in K} x^{**-1}(0) \neq (0)$. **Corollary.** For any $x^{**} \in X^{**}$, $x^{**} \notin QX$ if and only if $\bigcap_{x^* \in X^{**-1}(0)} x^{*-1}(0) = (0)$. (Received March 24, 1972.)

Distortion properties of $p$-fold symmetric alpha-starlike functions.

Starlike functions $f$ which are of Mocanu type $\alpha$ and have power series of the form $f(z) = z + a_{p+1}z^{p+1} + a_{2p+1}z^{2p+1} + \ldots$, where $p = 2, 3, 4, \ldots$, are shown to satisfy the relation $f(z) = [g(z^p)]^{1/p}$ where $g$ is of Mocanu type $\alpha$ with power series $g(z) = z + b_2z^2 + b_3z^3 + \ldots$. Distortion results dealing with the $\text{h}^{1/2}$-theorem and bounds on $f(z)$ are obtained. (Received April 11, 1972.)
Let $L^p_\otimes_X$ be the space obtained by taking the weak tensor product of $L^p = L^p(S, \Sigma, \mu)$, $1 \leq p < \infty$, and the Banach space $X$ ($\otimes$ denotes the inductive topology). We obtain a representation of the dual space $(L^p_\otimes X)^*$ by regarding $L^p_\otimes X$ as the subspace $L^p_\otimes(X)$ of the Pettis space $L^p_0(X)$ formed by the closure of simple functions. Suppose $X^*$ has the Radon-Nikodym property. Then for every $\tau \in L^p_0(X)^*$ there exists a unique function $g:S \to X^*$ satisfying: (i) $g \in L^q(X^*)$ (Bochner), $1/p + 1/q = 1$; (ii) $gf \in L^1$ for every $f \in L^p_0(X)$; $\tau(f) = \int gfd\mu$. Slightly weaker results hold if property R-N is not assumed. Also representations for operators $T:L^p_\otimes X \to Y$ are obtained. Precise statements and sketches of the proofs will appear in "Sur la représentation de l'espace dual de l'espace des fonctions intégrables au sens de Pettis", C. R. Acad. Sci. Paris. (Received March 27, 1972.)


The initial value problem for the quasilinear hyperbolic equation (1) $u_t + b(u)x = 0$ is studied by means of the theory of nonlinear semigroups. It is shown that a recent theorem of M. G. Crandall and T. M. Liggett (Amer. J. Math. 93(1971), 265-298) applies to the operator $A:f \mapsto (d/dx)b(f)$, domain of $A \subset L^1(R)$, which means that $A$ generates a semigroup $\{S_t\}$ of (nonlinear) contraction operators on $L^1(R)$. The function $t \mapsto S_t f$ ($f \in L^1(R)$) may thus be interpreted as a generalized solution of equation (1), with initial value $u(x,0) = f(x)$, and it is proved that for $f \in L^1 \cap L^\infty$, this function is a weak solution in the usual sense. Finally, semigroup methods are employed to derive the "ordering principle" for solutions of (1): if $u(x,0) \preceq v(x,0)$, then $u(x,t) \preceq v(x,t)$. (Received March 27, 1972.)

*72T-B161. MARCEL ADRIEN LABBE, University of Pittsburgh, Pittsburgh, Pennsylvania 15212 and JOHN E. WOLFE, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15212. Isomorphic classes of the spaces $C_\sigma(S)$.

Jerison introduced the Banach spaces $C_\sigma(S)$ of continuous real or complex-valued odd functions with respect to an involutory homeomorphism $\sigma$ on a compact Hausdorff space $S$. It has been conjectured that any Banach space of the type $C_\sigma(S)$ is isomorphic to a Banach space of all continuous functions on some compact Hausdorff space. This conjecture is shown to be true if either (1) $S$ is a Cartesian product of compact metric spaces or (2) $S$ is a linearly ordered compact Hausdorff space and $\sigma$ has at most one fixed point. (Received March 28, 1972.)

*72T-B162. GRAHAME BENNETT, Indiana University, Bloomington, Indiana 47401. Inclusion mappings between $L^p$ spaces. Preliminary report.

It is shown that the inclusion mapping from $L^p$ to $L^q$ is $(2pq/(pq - 2p + 2q), 1)$-absolutely summing if $1 \leq p \leq q \leq 2$, and $(p, 1)$-absolutely summing if $1 \leq p \leq 2 \leq q < \infty$ or $2 \leq p \leq q \leq \infty$. Moreover, these results
are best possible. As a corollary it follows that a \((p,1)\)-absolutely summing operator, \(p > 1\), need not have the Dunford–Petits property, this observation answering a question raised by Pelczynski. The above results are also used to study matrix transformations on \(l^p\) spaces. (Received March 24, 1972.)


Let \(U(z)\) be an analytic function in the upper half-plane whose values are uniformly bounded linear operators in a complex separable Hilbert space \(K\) and such that \(U(x) = \text{strong lim } U(x + iy) (y \to 0)\) (these limits are well-defined a.e.) are unitary operators \((u, o)\) in \(K\), for a.e, \(x \in \mathbb{R}\) (\(U\) is an inner function-operator in the sense of H. Helson: "Lectures on invariant subspaces", Academic Press, New York, 1964). If \(U(z)\) is also analytic on \(R\) and at \(\infty\), then \(\{U(x) : -\infty \leq x \leq \infty\}\) can be considered as a smooth curve in the space of \((u, o)\) in \(K\). Lemma. If \(A, B \in (u, o, (K), \sigma(A) \subset \{e^{\pi i} : -a \leq t \leq a\}\) and \(\sigma(B) \subset \{e^{\pi i} : -b \leq t \leq b\}\), then \(\sigma(AB) \subset \{e^{\pi i} : -a - b \leq t \leq a + b\}\). Theorem. Let \(U\) be a nonconstant inner function-operator, analytic on \(R\) and at \(\infty\), then the smooth curve \(\{U(x)\}\) (described above) has diameter two, and therefore its length is \(2\pi\). If \(U'(x) = iM(x) U(x)\) is the differential equation of \(U\), the inequality means that \(\int_{-\infty}^{\infty} |M(x)| dx \geq 2\pi\); moreover, the lower bound is attained iff \(U(z) = [(I-P) + (z-\alpha)/(z+\alpha)P]X\), where \(P\) is a nonzero projection in \(\mathbb{K}\), \(X \in (u, o, (K)\) and \(\text{Im } \alpha > 0\). There exist two constants, \(C \geq C > 0\), such that \(c/(1+x^2) \leq |M(x)| \leq C/(1+x^2)\). Theorem. Let \(U\) be an inner function-operator, analytic on the real segment \([a, b]\); let \(U(0) = I\) and assume that \(M(x)\) is a trace class operator for \(x \in [a, b]\), then \((\text{det } U(x))' = i(\text{trace } M(x)) \text{det } U(x)\), \(a < x < b\). (Received March 23, 1972.) (Author introduced by Dr. Marta B. Herrero.)


Let \(g\) be continuous on \([0,1]\), \(f(x,t) \geq 0\) be continuous on \([0,1] \times [0,\infty)\), and \(L = -D(pD) + q\) be the general Sturm-Liouville operator, with \(p > 0\) in \(C^1[0,1]\) and \(q \geq 0\) in \(C[0,1]\). Theorem 1. If \(\varphi \geq 0\) is in \(C[-a^2,0]\), \(\varphi(0) = 0\), and \(g(x) \in [-a^2,1]\), then the functional differential equation \(Lu(x) = \lambda f(x, \bar{u}(g(x)))\), \(u(0) = u(1) = 0\), where \(\bar{u}\) coincides with \(\varphi\) on \([-a^2,0]\) and with \(u\) on \([0,1]\), has a continuum of positive solutions \((\lambda, u)\) connecting \((0,0)\) to \(\infty\) in \(\mathbb{R}^+ \times C[0,1]\). Moreover, if \(f(x, \varphi(g(x))) \neq 0\), for \(g(x) \equiv 0\), the solutions are nontrivial. Theorem 2. If \(g \neq 0\), \(1\) and \(f\) satisfies the condition \(f(x,t) \geq \alpha \min (t^\beta, t')\), \(\alpha, \beta > 0\), \(\beta \geq \gamma \geq 0\), then the equation \(Lu = \lambda f(x, u(g(x)))\), \(u(0) = u(1) = 0\), has a continuum of nontrivial positive solutions as well as existence of solutions with internal nodes. The functions \(p\) and \(q\) may also be allowed to depend continuously on \(\lambda\). (Received April 3, 1972.)

*72T-B165. A. T. DASH, Indiana University, Bloomington, Indiana 47401. Tensor products and joint numerical range.

Let \(K_1, K_2, \ldots, K_n\) be complex Hilbert spaces. Further, let \(I_j\) be the identity operator and \(A_j\) an

\[ A-522 \]
arbitrary operator on \( V_j, 1 \leq j \leq n \). We consider the tensor product of operators \( T_j (1 \leq j \leq n) \) acting on the tensor product space \( V_1 \otimes \cdots \otimes V_n \) defined by \( T_1 = A_1 \otimes I_2 \otimes \cdots \otimes I_n \), \( T_2 = I_1 \otimes A_2 \otimes \cdots \otimes I_n \) and \( T_j = I_1 \otimes \cdots \otimes I_{j-1} \otimes A_j \otimes I_{j+1} \otimes \cdots \otimes I_n \) in general. Clearly, the operators \( T_j \) commute, it is shown that the joint numerical range of the operators \( T_j (1 \leq j \leq n) \) is the cartesian product of their numerical ranges; that is, \( W(T_1, \ldots, T_n) = \prod_{j=1}^n W(T_j) = \prod_{j=1}^n W(A_j) \). Thus it is convex. It is also shown that the joint spectrum of the operators \( T_j \) is contained in the closure of their joint numerical range. It should be pointed out that it is not in general known whether the joint numerical range of an \( n \)-tuple of commuting operators is convex; and also it is not known whether the joint spectrum of commuting \( n \)-tuples of operators is contained in the closure of their joint numerical range. For further details about joint numerical range and joint spectra the reader is referred to [A. T. Dash, Ph.D. Thesis, University of Toronto, 1969]. (Received April 3, 1972.)

*72T-B166. KENNETH A. ROSS, University of Oregon, Eugene, Oregon 97403. Fatou-Zygmund sets.

Let \( G \) be a compact Abelian group with character group \( X \). Let \( W \subset G \) be measurable so that \( \emptyset \neq W \subset (\text{int} \, W)^{-} \). A theorem of Edwards, Hewitt and Ross, "Lacunarity for compact groups. III," Studia Math. (1972), asserts that a symmetric subset \( P \) of \( X \) is an \( FZ(W) \)-set if and only if to every bounded Hermitian function \( \beta \) on \( P \) there corresponds a nonnegative measure \( \mu \) supported by \( (W)^{-1} \) and satisfying \( \lim \sup_{\beta} |\mu(x) - \beta(x)| < 1 \). If \( P \) is an \( FZ(W) \)-set for all \( W \), \( P \) is called a full \( FZ \)-set. We give a characterization of full \( FZ \)-sets as \( FZ(G) \)-sets satisfying a certain algebraic condition. In particular, if \( G \) is connected, then \( P \) is an \( FZ(G) \)-set if and only if it is a full \( FZ \)-set. Some of the techniques are adaptations of those of Déchamps-Gondim, C. R. Acad. Sci. Paris 271(1970), 590-592. In contrast to \( FZ(G) \)-sets and Sidon sets, the union of two full \( FZ \)-sets is not always another full \( FZ \)-set. If \( D \) is a countable dissociate set, then \( D \cup D^{-1} \) is a full \( FZ \)-set. (Received April 6, 1972.)

72T-B167. KOK-KEONG TAN, Dalhousie University, Halifax, Nova Scotia, Canada and TSU-TEH HSIEH, Carleton University, Ottawa, Ontario, Canada. Remark on almost weakly periodic isometries. Preliminary report.

This is a generalization of the Theorem of the Abstract 72T-B64, these Notices 19(1972), A-313. Let \( X \) be a Hausdorff locally convex space whose topology is generated by a family \( \theta \) of seminorms on \( X \).

Definition. \( f: X \to X \) is (i) nonexpansive iff for each \( p \in \theta \), \( p(f(x) - f(y)) \leq p(x - y) \), for all \( x, y \in X \); (ii) isometric iff for each \( p \in \theta \), \( p(f(x) - f(y)) = p(x - y) \), for all \( x, y \in X \); and (iii) almost weakly periodic iff for each \( x \in X \), \( x \in \overline{\text{co}}\{f(x), f^2(x), \ldots \} \). Theorem. Suppose \( X \) has normal structure. If \( f \) is an almost weakly periodic nonexpansive map on \( X \) which maps some nonempty bounded closed convex symmetric subset of \( X \) into itself, then \( f(0) = 0 \). Corollary. If \( X \) is over the real field and \( f \) is an almost weakly periodic isometry from \( X \) onto \( X \) which maps some nonempty bounded closed convex symmetric subset of \( X \) into itself, then \( f \) must be linear. (Received April 6, 1972.)

*72T-B168. STEPHEN A. McGRATH, Bowling Green State University, Bowling Green, Ohio 43403. Dominated estimates of certain \( L^p \) contractions. Preliminary report.

Let \( (X, \mathcal{F}, \mu) \) be a \( \sigma \)-finite measure space and \( T \) a linear operator of \( L^p(X, \mathcal{F}, \mu) \), \( p \) fixed, \( 1 < p < \infty \).
If there exists a constant $c$ such that, for each $f \in L^p(X, \mathcal{F}, \mu)$, 

$$
\int \sup_n |f, (f+Tf)/2, \ldots, (f+Tf+\ldots+T^{n-1}f)/n|^p d\mu \leq c^p \int |f|^p d\mu,
$$

then we say that $T$ admits of a dominated estimate with constant $c$. We say that an operator $T$ is a positive contraction if $\|T\| \leq 1$ and $f \geq 0$ implies $Tf \geq 0$. We obtain dominated estimates for compact contractions and normal contractions by applying a result in a paper by the author and R. V. Chacon [Pacific J. Math. 30(1969), 609-620].

**Theorem.** If $T$ is a positive compact contraction of $L^p(X, \mathcal{F}, \mu)$ then $T$ admits of a dominated estimate (the estimate obtained depends on $T$). **Theorem.** If $T$ is a positive normal contraction of $L^2(X, \mathcal{F}, \mu)$ then $T$ admits of a dominated estimate with constant 2. (Received April 7, 1972.)

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Consider the quasi-linear system (*) $x' = A(t,x)x + B(t,x)$, $t \in [0, +\infty)$. Conditions are given, for the matrix $A(t,x)$ and the vector $B(t,x)$, which guarantee the existence of convergent solutions of the system (*), which are defined on the whole interval $[0, +\infty)$. The results are then related to the problem of the asymptotic equivalence (with respect to a continuous nonsingular matrix) between (*) and its associated unperturbed system (**) $y' = A(t,y)y$. As in a previous paper of the first of the authors ["On the relationship between a nonlinear system and its nonlinear perturbation," J. Differential Equations, to appear], the matrix $A(t,x)$ is not supposed to be differentiable w. r. t. $x$. (Received April 7, 1972.)

*72T-B170. W. JOHN WILBUR, Pacific Union College, Angwin, California 94508. **Reflective and coreflective hulls in the category of locally convex spaces.**

This paper is the completion of work reported in Abstract 70T-G126, these Notices 17(1970), 840; Abstract 70T-G165, these Notices 17(1970), 972; Abstract 682-22-7, these Notices 18(1971), 137. A new method of characterizing epi-reflective and mono-coreflective hulls in certain subcategories of the category HAUS of Hausdorff spaces and continuous maps is given. The method is based on the "dual" characterization of continuous maps in terms of filters and open sets. The details are worked out for HAUS and for the category LCS of Hausdorff locally convex topological vector spaces and continuous linear transformations. The results for LCS are then used to study the distribution of reflective and coreflective subcategories in LCS and the hulls of certain classes of spaces. Subcategories of LCS that are both reflective and coreflective are also investigated and some consequences of results are the representation theorems that every space is a quotient of a complete space; a closed subspace of a barreled space; and, assuming the nonexistence of a strongly inaccessible cardinal, a closed subspace of a bornological space. (Received April 17, 1972.)

*72T-B171. J. E. KERLIN and ALAN L. LAMBERT, University of Kentucky, Lexington, Kentucky 40506. **Strictly cyclic shifts on $l_p$.**

Let $1 < p < \infty$ and let $\{e_0, e_1, \ldots\}$ be the standard Schauder basis for $l_p$. For $\alpha = \{\alpha_1, \alpha_2, \ldots\}$ a bounded sequence of nonzero complex numbers, let $S_\alpha$ be the weighted shift on $l_p$ with weight sequence $\alpha$. Let $\mathcal{A}(S_\alpha)$ be the weakly closed algebra generated by $S_\alpha$. Then $S_\alpha$ is strictly cyclic if $\mathcal{A}(S_\alpha) e_0 = l_p$. Let $\beta_0 = 1,$
Theorem. Let $1/p + 1/q = 1$. If $\sum_{m=0}^n |\beta_n^m / \beta_n|^{q} < \infty$ then $S_\alpha$ is strictly cyclic. The converse is valid if $|\alpha_{n+1}| \leq |\alpha_n|$ for all $n$. Corollary 1. Suppose $|\alpha_{n+1}| \leq |\alpha_n|$ for all $n$. Then $S_\alpha$ is strictly cyclic on $L^p$ if either $\sum |\alpha_n|^r < \infty$ for some $r > 0$ or $\sum |\beta_n^m / \beta_n|^q < \infty$. Corollary 2. Suppose there is a sequence $\{a_n\}$ in $L^q$ such that $|\beta_n^m / \beta_n| \leq |a_n| + |a_m|$ for all $n$ and $m$. Then $S_\alpha$ is strictly cyclic on $L^p$. (The case $p = 1$ was completely settled by Mary Embry in an article to appear in Pacific J. Math.) Shifts on $L^\infty$ are also considered and analogous results are obtained for strict cyclicity of the commutant of $S_\alpha$ on $L^p$. (Received April 10, 1972.)


For notations, see D. Ebin and J. Marsden, Ann. of Math. (2) 92(1970), 102-163. Theorem. Let $H_t$ denote the flow on $H^s (=H^s$ vectorfields on a compact $n$-manifold with boundary), $s > n/2 + 1$, defined by the Euler equations. Then for each fixed $t$, $H_t$ is a $C^\infty$ mapping. Indeed, if we pass to the diffeomorphism group $\mathcal{F}_\mu$, we get a $C^\infty$ flow $F_t: \mathcal{F}_\mu \rightarrow \mathcal{F}_\mu$ (locally defined). Then it is not hard to prove that the derivative of $H_t$ is given by the right pull back of the derivative of $F_t$ for all fixed $t$. Hence $H_t$ is $C^\infty$ for fixed $t$. Similar results hold for the Navier Stokes equations (the latter was proved by another method in two dimensions by Foias-Prodi, Rend. Sem. Mat. Univ. Padova 39(1967), 1-34). (Received April 10, 1972.)


Let $\mathfrak{A}$ be a von Neumann algebra, $\mathfrak{G}$ the center of $\mathfrak{A}$, and $\mathfrak{J}$ a uniformly closed weakly dense ideal of $\mathfrak{A}$. An operator $A \in \mathfrak{A}$ is called thin relative to $\mathfrak{J}$ if $A = Z + K$ for some $Z \in \mathfrak{G}$, $K \in \mathfrak{J}$. The lattice $\mathcal{D}$ of projections in $\mathfrak{G}$ is a directed set under the usual ordering; if $A \in \mathfrak{A}$, then $\{ \| AP - PA \| \}_{P \in \mathcal{D}}$ is a net of positive numbers. Theorem. An operator $A \in \mathfrak{A}$ is thin relative to $\mathfrak{J}$ if and only if $\lim_{P \in \mathcal{D}} \| AP - PA \| = 0$. This was proved by R. G. Douglas and C. Pearcy in the case where $\mathfrak{A}$ is all bounded linear operators on a Hilbert space and $\mathfrak{J}$ is the ideal of compact operators ("A characterization for thin operators," Acta. Sci. Math. (Szeged) 29(1968), 295-297). The theorem has also been previously shown by the author in the case where $\mathfrak{A}$ is a factor and in other special cases ("Thin operators in von Neumann algebras," to appear in Acta. Sci. Math. (Szeged)). The proof of the general case relies on the result in the factor case, and on the preliminaries used to obtain this result. (Received April 13, 1972.)

*72T-B174. ROBIN HARTE, University College, Cork, Ireland. The spectral mapping theorem for quasi-commuting systems.

If $a = (a_1, a_2, \ldots, a_n)$ is a quasi-commuting system [N. H. McCoy, Bull. Amer. Math. Soc. 36(1934), 327-340] of elements of a complex Banach algebra $A$, with identity, and if $f = (f_1, f_2, \ldots, f_m)$ is a system of polynomials in $n$ variables on $A$, then there is equality $\varphi_0(a) = \varphi(a)$. In particular $\sigma(a)$ is nonempty, where $\sigma(a)$ is the "joint spectrum" of a (Abstract 71T-B92, these Notices 18(1971), 558). (Received April 14, 1972.)
Let $X$ denote a real, locally convex, hausdorff topological vector space. We prove: Theorem 1. A necessary and sufficient condition that a weakly closed subset $A$ of $X$ is weakly compact is that $A \sim B$ for all weakly closed sets $B$ disjoint from $A$. Theorem 2. A quasi-complete space $X$ is semireflexive if and only if $A \sim B$, where $A$ is a bounded weakly closed set and $B$ a closed convex set disjoint from $A$. \(\delta\) is a certain proximity relation on $X.$ These results are generalization, to locally convex spaces, of the results of R. C. James, "Weak compactness and separation," Canad. J. Math. 16(1964), 204-206. (Received April 17, 1972.)

Let $T = (t_{nk})$ be a row-finite and reversible infinite matrix with $\lim_n t_{nk} = 1$ (\(k=1,2,\ldots\)) and with an inverse $T^{-1} = (s_{nk}).$ The $n$th $T$-section of a sequence $x$ is $t^n_x = (t_{n1}x_1, t_{n2}x_2, \ldots).$ An FK-space is a TB-space if $\{t^n_x\}_{n=1}^{\infty}$ is a bounded subset of $E$ for each $x$ in $E$. Let $E_{TB}^T = \{x \in E: \lim_n t^n_x = x \text{ in the topology of } E\}.$ Denote the summability field $[x: \lim_n \sum_{nk} x_{nk} \text{ exists}]$ by $c_T$ and the space of summability factors $[x: x \cdot y = (x_k y_k) \in c_T \text{ for every } y \in c_T]$ by $q_T.$ Suppose $c_T$ is a TB-space. Theorem 1. $q_T^1$ is a dense subset of $c_T$ and $c_T = (c_T)c_0^T.$ Theorem 2. $q_T$ is a BK-space and a TB-space, $(q_T)^{TB} = q_T \cap c_0,$ and $q_T = [x: \|x\| = \sum_{nk} x_{nk} + \lim_k |x_{nk}| < \infty].$ Theorem 3. Let $E$ be an FK-space. The following statements are equivalent: (a) $E$ is a TB-space; (b) $E = q_T \cdot E = (x,y) : x \in q_T, y \in E; \text{ (c) } E_{TB}^T = (q_T \cap c_0) \cdot E.$ Theorem 4. Let $E$ be an FK-space. Then $E = E_{TB}^T$ if and only if $E = (q_T \cap c_0) \cdot E.$ (Received April 19, 1972.)
**72T-B178.** SIMON COHEN, Polytechnic Institute of Brooklyn, Brooklyn, New York 11201. An inner product for a Banach algebra.  

Let $X$ be a semisimple Banach algebra and let $\mathfrak{M}$ be the maximal ideal space of $X$. Let $m$ be a probability measure on $\mathfrak{M}$ which is positive on nonempty open sets. Then $(x,y) = \int X \, dm$, where $x$ and $y$ are the Gelfand functions associated with $x$ and $y$ resp., defines an inner product on $X$. The resultant inner product space, denoted by $X_2$, is shown to be a topological algebra (though not necessarily a normed algebra). Several rather strong conditions for completeness are established (e.g., the subalgebra of Gelfand functions of $C(\mathfrak{M})$ must be closed). $X_2$ is shown to be incomplete if $X$ is regular and $\mathfrak{M}$ is countably infinite. As a by-product, a new proof of the theorem that if $X$ is finite dimensional, then cardinal $\mathfrak{M} = \dim X$ is obtained. Several results are presented under the assumption that $X$ is an $A^*$ algebra. For example, if $X$ is an $A^*$ algebra, then $X_2$ is complete iff $X$ is finite dimensional. Furthermore, a maximal ideal $\mathfrak{M}$ will be $X_2$-closed if $M_1 \neq \{0\}$ and in this case we have $X = \mathfrak{M} \oplus M^2$. (Received April 20, 1972.)


We treat first-order algebraic differential equations whose coefficients belong to a field of functions, each analytic in a sectorial neighborhood of $\infty$ (which is a union of sectors) such that each element in the field except 0 is asymptotically equivalent ($\sim$) to a function of the form $cz^a$ (for $c \neq 0$ and real $a$) as $z \to \infty$ over a filter base $\mathcal{F}$ of sectorial neighborhoods of $\infty$. (This includes the case when the coefficients are rational functions. We use here the concepts of "< ~ " and "smaller rate of growth" $<<$ introduced by W. Strod [Mem. Amer. Math. Soc. No. 13(1967), p. 11].) We prove that if $h$ is any solution which is meromorphic in an element of $\mathcal{F}$ and which is "comparable" with all functions (*) $M = cz^a (\log z)^b$ (for $c \neq 0$ and real $a, b$) (i.e., for each $M$, either $h \ll M$, $M \ll h$, or $h \sim kM$ for some constant $k \neq 0$), then one of three possibilities must occur: (i) $z^a \ll h$ for all real $a$; (ii) $h \ll z^a$ for all real $a$; or (iii) for some $M$ of the form (*), $h \sim M$. By a previous result of the author [Trans. Amer. Math. Soc. 159(1971), 295], in (i) and (ii) (if $h \neq 0$), $h$ is then of the form $\exp \left[ cz^a (1 + o(1)) \right]$ over $\mathcal{F}$. When the coefficients are allowed to be $\sim$ to functions of the form (*), it is shown that the analogous result (i.e., where (iii) is $h \sim cz^a (\log z)^b (\log \log z)^d$) can fail to hold. (Received April 20, 1972.)

**72T-B180.** GORDON G. JOHNSON, University of Houston, Houston, Texas 77004. Inner products characterized by difference equations.  

A necessary and sufficient condition that a normed linear space $X$ be an inner product space is that there is an integer $k$ greater than 2 such that $\sum_{t=0}^{k} \left( (-1)^t \left\| a + tb \right\|^2 = 0 \right.$ if each of $a$ and $b$ is in $X$. (Received April 24, 1972.)
A study of the asymptotic solutions of a certain third order ordinary differential equation in a domain containing a multiple turning point.

This paper is concerned with the asymptotic solutions of the linear differential equation of third order

\( w''' + \lambda^2 z^{-2} (\alpha z^2 w'' + \beta zw' + \gamma w) = 0. \)

Here, the variable \( z \) is regarded complex, \( \lambda \) a complex parameter of large absolute value, \( k \) an integer \( \geq 2 \) and \( \alpha, \beta, \gamma \) arbitrary constants real or complex. In general, the point \( z = 0 \) is a multiple turning point of (1). It may be noted that for the special case with \( k = 2, \alpha = 0, \beta = 1 \), the point \( z = 0 \) is a turning point of first order and this case has been earlier studied by R. E. Langer (Duke Math. J. 22(1955), 525-542). (Received April 24, 1972.)


Let \( p > 1 \), \( S_n = \sum_{r=0}^{n} u_r \), \( S_\lambda (y) = (1+y)^{-\lambda-1} \sum_{n=0}^{\infty} (\lambda y)^n \), \( u_\lambda (y) = (1+y)^{-\lambda-1} \).

The sequence \( \{S_n\} \) is strongly \( \lambda \)-convergent with index \( p \) to \( \mathcal{L} \left( S_n \rightarrow \mathcal{L}[A\lambda]\right) \).

The sequence \( \{S_n\} \) is strongly \( \lambda \)-convergent with index \( p \) to \( \mathcal{L} \left( S_n \rightarrow \mathcal{L}[A\lambda]\right) \).

The main results are:

**Theorem 1.** For \( \lambda > 0 \), the necessary and sufficient conditions for the \( \mathcal{L}[A\lambda] \)-convergence of \( \{S_n\} \) to \( \mathcal{L}[A\lambda] \) are:

(i) \( S_n \rightarrow \mathcal{L}[A\lambda] \) and
(ii) \( \int_{0}^{1} [U_\lambda (t) - 1]^p dt = o(y) \) as \( y \rightarrow \infty \).

**Theorem 2.** For \( \lambda > 0 \), \( S_n \rightarrow \mathcal{L}[A\lambda] \) and

\( n \cdot u_n \rightarrow o[A\lambda-1] \).

**Theorem 3.** For \( \lambda > 0 \), \( S_n \rightarrow \mathcal{L}[A\lambda] \) if and only if \( S_n \rightarrow \mathcal{L}[A\lambda-1] \).

**Theorem 4.** If \( \lambda > -1 \), \( H_\lambda \) is a regular Hausdorff method and \( S_n \rightarrow \mathcal{L}[A\lambda] \) then \( S_n \rightarrow \mathcal{L}[A\lambda H_\lambda] \). (Received April 26, 1972.)

72T-B183. T. K. PUTTASWAMY, Ball State University, Muncie Indiana 47306 and R. PRABHAKARAN, University of Cincinnati, Cincinnati, Ohio 45221. The asymptotic solutions of a certain ordinary differential equation with reference to the Stokes' phenomenon about an irregular singular point. Preliminary report.

In this paper, the authors have solved in the large the linear homogeneous differential equation of order \( n \),

\( \sum_{j=0}^{n-1} (a_j + b_j z + c_j z^2) z^j \frac{d^j y}{dz^j} + (a_n + b_n z^n) z^n \frac{d^n y}{dz^n} = 0. \)

Here, the variable \( z \) is regarded as complex, as likewise the constants \( a_j, b_j, c_j \) \((j=0,1,2,\ldots, n, k=0,1,2,\ldots, n-1)\) with \( a_n \neq 0, b_n \neq 0 \) and \( c_{n-1} \neq 0 \). If \( \mu = -a_n/b_n \), (1) will have two finite regular points, namely \( z = 0 \) and \( z = \mu \) and an irregular singular point at \( z = \infty \).

The indicial equation corresponding to \( z = 0 \) is found to be (2)

\( \sum_{j=0}^{n-1} a_{n-1} \frac{n-1}{j} (n-j) = 0. \)

It is assumed that the roots \( h_i, i = 1, 2, \ldots, n, \) of (2) are such that no two of them differ by an integer. This paper is a sequel to an earlier paper by one of us (Puttaswamy, Abstract 72T-B134, these Notices 19(1972), A-448). (Received April 26, 1972.)

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A study is made of the unsteady hydromagnetic boundary layer flow induced by torsional oscillations of an infinite disk in an electrically conducting rotating fluid in the presence of a uniform magnetic field. Some quantitative and qualitative information is obtained about (i) the structure of the steady and the unsteady velocity field, (ii) the development of the associated hydrodynamic Rayleigh layer, the Ekman–Hartmann boundary layer and the Stokes–Ekman–Hartmann boundary layer with their physical implications, (iii) the total transient time required for the establishment of quasi-steady hydromagnetic boundary layers, (iv) the hydrodynamic and the hydromagnetic Ekman suction velocity and its importance and (v) the significant interaction of the electromagnetic and Coriolis forces. It is shown that the rapid attainment of the quasi-steady Ekman–Hartmann and the Stokes–Ekman–Hartmann boundary layers and the striking properties of the Ekman suction velocity appear to play a distinctive role on the hydromagnetic spin-up mechanism. Special attention is given to the physical interpretation of the mathematical results obtained. (Received February 16, 1972.)

A geometric hierarchy of languages. Preliminary report.

Theorem (A pumping lemma). Given a labeled metilinear (or nonterminal bounded) grammar $G = (V, \Sigma, \sigma, P, L)$ and a regular control set $A$, the language derived by $(G, A)$ is denoted by $L(G, A)$ and defined to be $L(G, A) = \{x \in \Sigma^* | \sigma \frac{S}{\pi} x, \pi \in A\}$. For $L(G, A)$ as above, $\exists$ natural numbers $p, q$ such that if $z \in L(G, A)$ and $|z| > p$ then $z = \prod_{i=1}^{r} u_i v_i w_i x_i y_i$ and $z_k = \prod_{i=1}^{r} u_i^{k_i} v_i^{k_i} w_i^{k_i} x_i^{k_i} y_i^{k_i} \in L(G, A)$ $\forall k = 0, 1, 2, \ldots$ where $\prod_{i=1}^{r} v_i x_i \neq \epsilon$ and $|\prod_{i=1}^{r} v_i x_i| < q$ and $r$ being bounded by the maximum number of variables that appear on the right hand of $\sigma$.

The symbol $\Pi$ denotes concatenation. Theorem. There is a hierarchy of languages $L_0 \subseteq L_1 \subseteq L_2 \subseteq \ldots \subseteq L_S$, where $L_0$ is the family of context free languages and $L_S$ is the family of context sensitive languages. Each language in every class satisfies a pumping lemma. The usual corollaries of the pumping lemma hold, e.g., decidability and regularity on one symbol alphabet. The hierarchy is defined by $L_0 = CF$, $L_1 = L_0 / L_{l-1}$ $\forall i > 0$, where $CF$ is the family of context free languages, $L$ is the family of labeled linear grammars, and $L / L_{l-1}$ is the family of languages obtained by controlling a grammar in $L$ by languages in $L_{l-1}$. (Received March 3, 1972.)

A concrete approach to abstract recursive definitions. Preliminary report.

A $\mu$clone of operations on a complete lattice is defined to be a clone of continuous functions which is also closed under a fixed-point operation. It is seen that $\mu$clones may be used for many of the same purposes as Wagner’s abstract recursive definitions [11th IEEE Conference on Switching and Automata Theory, 1971, pp. 192–201]. Theorem. The free $\mu$clone generated by a ranked set is isomorphic to the set of loop-representable flow diagrams (in the sense of Scott) with function symbols in the ranked set. This characterization allows great
simplification in the proofs of theorems about \( \mu \)-clones. It is shown that fixed points over simultaneous equations may be replaced by fixed points over single equations, analogous to the Kleene star. Several well-known theorems of language and automata theory are drawn as special cases of this result. (Received March 8, 1972.)

*72T-C30, KAILASH CHANDRA, Division of Physical Sciences, Institute of Advanced Studies, Meerut University, Meerut, U. P., India. Magneto-hydrodynamic stability of heterogeneous incompressible nondissipative conducting fluid with a radial gravitational force.

The stability of an incompressible, inviscid, ideally conducting fluid with density \( \rho (r) \), velocity \( \mathbf{u} = (0, V(r), W(r)) \), magnetic field \( \mathbf{H} = (0, H_\phi (r), H_z (r)) \), between two fixed coaxial cylinders, in the presence of a radial gravitational force \( g \) is discussed under linear theory by the normal dode technique for axisymmetric perturbations. If \( \left( \mu H_\phi^2 / \rho \right)^{1/2} = V \), then the swirl velocity \( V \) as well as the transverse magnetic field \( H_z \), has destabilizing or stabilizing effect according as \( \left( \mu H_\phi^2 / \rho \right)^{1/2} = W \neq 0 \) or \( H_z = W = 0 \). And thus the dual role of the swirl velocity as well as of the transverse magnetic field is manifested. Further, the case of the axial flow with the axial magnetic field (strong and weak) has been dealt with. (Received March 21, 1972.)

(Received March 8, 1972.)

(Received March 8, 1972.)

(Received March 21, 1972.)

(Received March 21, 1972.)

72T-C31, KAILASH CHANDRA and LAKSHMI NARAYAN, Division of Physical Sciences, Institute of Advanced Studies, Meerut University, Meerut, U. P., India. Thermal instability of a heterogeneous shear flow. Preliminary report.

The stability of an incompressible inviscid fluid layer in the presence of general streaming \( U(z) \) \( (z \) is the vertical coordinate) between two horizontal parallel planes \( (z = 0 \) and \( d \)) heated or cooled from below is discussed taking the resultant density stratification to be \( \rho = \rho_0 \left( e^{5z} - \alpha \beta z \right) \), where \( \rho_0 e^{5z} \) \( (\delta \) is real), \( \alpha \) and \( \beta \) are respectively the basic density of the fluid, coefficient of volume expansion and the temperature gradient maintained. (i) Squire's theorem is proved when \( \beta > 0 \) and \( d \rho / dz \leq 0 \) in \( (0,d) \). (ii) Sufficient conditions for the stability are obtained for adverse and nonadverse temperature gradients. (iii) The growth rate of an arbitrary unstable mode, \( \xi \), is bounded above by \( \left( e^{5|d|/2} \right) max |dU/dz| \) in case \( \beta > 0 \) and \( d \rho / dz \leq 0 \) in \( (0,d) \). (iv) Bounds for the phase velocity of an arbitrary stable, unstable or neutral mode are obtained. In case of small thermometric conductivity, (i) the phase velocity of an arbitrary unstable or neutral mode lies in the interval \( \left( U_{min}, U_{max} \right) \) if \( \beta > 0 \), (ii) the principle of exchange of stability is not valid for nonvanishing \( U \), (iii) \( C_1^2 < g(\delta - \alpha \beta e^{5d}) \) if \( U = \) constant and \( \beta > 0 \), and (iv) \( C_1^2 > - g \alpha \beta e^{-5d} \) if \( U = \) constant and \( d \rho / dz \leq 0 \) in \( (0,d) \). (Received March 21, 1972.) (Author introduced by Vice-Chancellor J. N. Kapur.)

*72T-C32, MURLI M. GUPTA, University of Western Australia, Nedlands 6009, Australia. Discretization error for Dirichlet problems. Preliminary report.

This paper deals with the Dirichlet problem for elliptic differential equations of order \( 2m \) \( (m \geq 1) \) in a bounded \( n \)-dimensional domain \( D \). A class of finite difference schemes is defined for its numerical solution. The finite difference operator \( L_h \) is assumed to be elliptic and consistent with the differential operator \( L \). The truncation error of the operator \( L_h \) is assumed to be of order \( h^k \) in the interior of domain \( D \) whereas near the
boundary it is assumed to be of order $h^k$ (\(k < \infty\), in general). Under certain conditions on the smoothness of the boundary, the discretization error is shown to be of order $h^q$, $q = \min(k, l + m + 1/2)$, where the exact solution belongs to $C^{2m+k}$. Some applications are given that include the Dirichlet problem for the biharmonic equation.

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*72T-C33. MURLI M. GUPTA, University of Western Australia, Nedlands, 6009, Australia and RAM P. MANOHAR, University of Saskatchewan, Saskatoon, Saskatchewan, Canada. On numerical solution of biharmonic equation by splitting. Preliminary report.

In order to obtain the numerical solution of a biharmonic boundary value problem, it is advantageous to split the biharmonic equation into two simultaneous Poisson equations and reduce this problem to the solution of two second order Dirichlet problems. However, in the case of the Dirichlet problem for the biharmonic equation, the boundary conditions for one of the Poisson equations remain undefined. This paper shows how these missing conditions can be approximated in order to obtain the maximum accuracy and to achieve the fastest rate of convergence. Several such approximations are studied and corresponding accuracy and the rates of convergence analyzed. These results are shown to have many practical advantages over the results of Smith (SIAM J. Numer. Anal. (1968)) and Ehrlich (ibid (1971)). (Received March 17, 1972.)

*72T-C34. HUI-HSIUNG KUO, University of Virginia, Charlottesville, Virginia 22903. On stochastic controllability in abstract Wiener space.

This paper deals with the controllability of linear (stochastic) dynamical systems in infinite dimensional spaces with infinite dimensional control. It has been shown recently by Kuperman and Repin (Dokl. Akad. Nauk SSSR 200(1971)) that the system $dx(t) = Ax(t) + \sum_{i=1}^{\infty} b_i \sigma_i(t)$ in an infinite dimensional Banach space is uncontrollable. In this paper it is shown that even if the control is infinite dimensional this system is still uncontrollable. However, it is $\epsilon$-controllable. As for the stochastic dynamical system $dX(t) = A(t)\,dW(t) + (DX(t) + u(t))\,dt$, $t_0 \leq t \leq t_1$, where $W(t)$ is a Wiener process in an abstract Wiener space $(I, H, B)$, it is proved to be stochastically controllable in the sense that for any $\epsilon > 0$ and any $x_0$ and $x_1$ in $B$ there exists a control $u$ which produces the response $X(t)$ guiding $x_0$ to the $\epsilon$-neighborhood of $x_1$ with probability larger than $1 - \epsilon$.

(Received April 11, 1972.)

*72T-C35. L. D. CHATTERJI, Applied Science Department, Madan Mohan Malviya Engineering College, Gorakhpur, U. P., India. Period radius relations of cepheid variable stars.

In this paper three polytropic and two stellar models with density variations as $\rho = \rho_c^\infty (1-r/R)$ and $\rho = \rho_c^\infty (1-r^2/R^2)$, where $R$ is the radius of the star and $r$ is the distance from the centre at any point have been studied. The mean densities of each model have been calculated and also their periods of pulsations. The period density relation $P \sqrt[\rho]$ gives the value $0.08 \pm 0.02$, nearly a constant, thus verifying one of the conclusions drawn from pulsation theory. The calculated values of masses show varying from 0.3 times to 11 times the mass of the sun, the periods derived are more or less in agreement with the period of cepheid $\delta$-cephei star. The radius of the classical cepheids in three different period ranges have been calculated by assuming a period radius relation

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of the form $R \propto P^n$, the five models are chosen to fit in and with a generalisation. The radii of five cepheid stars have been calculated in the three period ranges (i) $P < 8^d$, (ii) $8^d \leq P \leq 17^d$, and (iii) $P > 17^d$, and found to be in near agreement with the values obtained by Fernie (Astrophys. J. vol. 151, p. 197). (Received April 13, 1972.)


A design is proposed for a self reproducing machine which operates in an environment which consists of a surface on which a large number of parts walk at random. Bonds may be formed or broken between pairs of these parts according to locally defined rules. The machine itself is made up of linearly connected strings of parts. The machine is supposed to be able to compute any computable function or to construct other machines. The universe being nondeterministic, the machine is subject to malfunction; it is not yet determined how often the machine behaves as intended. (Received April 20, 1972.) (Author introduced by Dr. Michael Slater.)

Geometry


Bose-Chaudhuri-Hocquenghem codes [E. R. Berlekamp, "Algebraic coding theory", McGraw-Hill, New York, 1968, Chapter 7] and Justesen codes [J. Justesen, "Constructive, asymptotically-good, algebraic codes", IEEE Trans. Information Theory 18(1972), to appear] are used to pack equal spheres in $n$-dimensional Euclidean space with density $\Delta$ satisfying $\log_2 \Delta > -6n + o(n)$, for all $n$ of the form $m^2m$, where $m \geq 256$ is a power of 4. The construction uses Construction C of John Leech and the author [Canad. J. Math. 23(1971), 718-745]. These nonlattice packings appear to be the densest packings yet constructed in high dimensional space. (Received April 13, 1972.)

72T-D12. BANG-YEN CHEN, Michigan State University, East Lansing, Michigan 48823. Surfaces with parallel normal vector fields.

Let $M$ be a surface in a euclidean $m$-space $E^m$ with Gauss curvature $G$. A normal vector field is said to be parallel if it is parallel in the normal bundle. For a unit normal vector $e$, let $A(e)$ be the second fundamental form at $e$. The following theorems are proved: Theorem 1. If there exists a parallel unit normal vector field $e$ such that $\text{Trace } A(e)$ is constant and $G - \det (A(e))$ is nonzero almost everywhere, then $M$ is one of the following surfaces: (i) a surface in a hypersphere of $E^m$ with $e$ in the direction of radius vector field, (ii) a surface in a hyperplane of $E^m$ with $e$ as the hyperplane normal, or (iii) a surface in an affine 4-subspace of $E^m$. Theorem 2. Let $M$ be a surface in $E^4$ with constant mean curvature. If there exists a parallel unit normal vector field $e$ with Trace $A(e)$ being constant, then $M$ is contained in either a hyperplane of $E^4$ or a hypersphere of $E^4$. The conditions "$G - \det (A(e))$ is nonzero almost everywhere" in Theorem 1 and "$M$ with constant mean curvature" in Theorem 2 are essential. In the case the mean curvature vector $H$ is parallel,
Theorems 1 and 2 were obtained independently by S.-T. Yau, "Submanifolds with constant mean curvature. I", and the author, "Surfaces with parallel mean curvature vector" (to appear in Bull. Amer. Math. Soc. 78(1972)). (Received April 26, 1972.)

*72T-D13. DIMITRI KOUTROFIOTIS, University of California, Santa Barbara, California 93106. On a conjectured characterization of the sphere.

It is a standing conjecture that if on a $C^3$-ovaloid $S_i(k_1 - c)(k_2 - c) \leq 0$, $c =$ constant, then $S$ is a sphere. This conjecture is verified in the case that the ovaloid $S$ has a circular enveloping cylinder in some direction. The strictly convex compact surfaces of revolution are special cases (Mühlner, Comment Math. Helv. 41(1966), 58). (Received April 17, 1972.)


Let $M$ be an $n$-dimensional submanifold of a Riemannian manifold $N(c)$ of constant sectional curvature $c$, and let $S$ denote the square of the length of second fundamental form. The following theorems are proved:

**Theorem 1.** If the scalar curvature $R$ satisfies $R \geq (n-2)S + (n-2)(n-1)c$ (resp. $R > (n-2)S + (n-2)(n-1)c$), at a point $p$ in $M$, then the sectional curvature of $M$ at $p$ is nonnegative (resp. positive). As a consequence of Theorem 1 we have **Theorem 2.** Let $M$ be an $n$-dimensional compact submanifold of euclidean $(n+p)$-space $E^{n+p}$. Then the mean curvature vector $H$ is parallel in the normal bundle and we have $R > (n-2)S$ if and only if $M$ is a hypersphere of a linear $(n+1)$-subspace of $E^{n+p}$ when $n \geq 3$, and $M$ is a minimal surface of a hypersphere of $E^{2+p}$ with positive Gauss curvature when $n = 2$. (Remark. Theorem 1 is best possible.) (Received April 20, 1972.)


**Theorem.** Let $M$ be a compact submanifold of dimension $n$ immersed in a manifold $\overline{M}$ of nonnegative constant curvature of dimension $n+p$. Suppose that the connection of the normal bundle of $M$ in $\overline{M}$ is flat and that the mean curvature vector field of $M$ is parallel with respect to the connection of the normal bundle. If the second fundamental tensor satisfies the inequality (1) $\sum_{A=1}^p \text{trace } H_A^2 < (n-1)^{-1} \sum_{A=1}^p (\text{trace } H_A^2)^2$, then $M$ is totally umbilical. The inequality (1) can be replaced by (2) $K > n(n-1)c + (n-2) \sum_{A=1}^p (\text{trace } H_A^2)$, where $K$ is the scalar curvature. The proof of the theorem is based on the Lemma. If a given set of $n+1$ $(n \geq 2)$ real numbers $a_1, \ldots, a_n$ and $k$ satisfy the inequality $\sum_{i=1}^n a_i^2 + k^2 < (n-1)^{-1} (\sum_{i=1}^n a_i)^2$, then for any pair of distinct $i$ and $j = 1, \ldots, n$, we have $k^2 < 2a_ia_j$. A slight generalization of the theorem shall be announced by the author and B.-Y. Chen. (Received April 21, 1972.)
Logic and Foundations

72T-E51. G. P. MONRO, "Laputa", Seaview Avenue, Ferny Creek, Victoria 3786, Australia. Models of ZF with the same sets of sets of ordinals.

We exhibit an $\omega$-sequence of transitive models of ZF such that the kth and $(k+1)$st models have the same sets of sets of sets... (k times) of ordinals. The construction of the sequence of models proceeds by forcing, each model being a generic extension of its predecessor. (Received December 21, 1971.) (Author introduced by Professor John C. Shepherdson.)

72T-E52. FRANKLIN D. TALL, University of Toronto, Toronto 181, Ontario, Canada. Martin's axiom applied to complete spaces. Preliminary report.

A topological space is $\kappa$-complete if the intersection of fewer than $\kappa$ open dense sets is dense. A space is CCC if every collection of disjoint open sets is countable. One form of Martin's axiom states that every CCC compact Hausdorff space is $2^{\aleph_0}$-complete. Theorem. Martin's axiom implies that if $X$ is locally CCC, completely regular, and the intersection of fewer than $2^{\aleph_0}$-open sets in some compactification, then $X$ is $2^{\aleph_0}$-complete. Almost all of the consequences of Martin's axiom plus $2^{\aleph_0} > \aleph_1$ related to Souslin's hypothesis can be strengthened using this observation. For topologists it is pleasant to note that these consequences, including Souslin's hypothesis itself, can be deduced purely topologically from $2^{\aleph_0} > \aleph_1$ plus the topological version of Martin's axiom. (Received March 3, 1972.)

*72T-E53. GERARD P. HUET, Computer Science Department, Case Western Reserve University, Cleveland, Ohio 44106. The undecidability of the existence of a unifying substitution between two terms in the simple theory of types.

It is shown in this paper that it is recursively undecidable whether two terms in high order logic have a common instance. The proof, which consists in showing the equivalence of a special case of this problem with Post's correspondence problem over an alphabet with two letters, is valid for any logic of order $\geq 3$. Some implications of this result on the mechanization of high order theorem proving are discussed. (Received March 17, 1972.) (Author introduced by Professor William F. Ogden.)

*72T-E54. BARUCH GERSHUNI, Ibn Gvirol Street 43, Tel Aviv, Israel. The concepts "member" and "element" of a totality.

We adhere to the productive principle: There are no proper synonyms. If a language contains two words for one and the same concept, they mean in truth two related concepts. This principle need not be true in each case; its productivity justifies its use. The two mathematical names "member" and "element" are usually taken as meaning the same: the building stones of a totality. But the word "element" means building stones which are simple individuals, while the word "member" means also plurals. We call members (of the first order), members of members (members of the second order), a.s.o. generalized members of a totality. Let T denote a class of the type $\{a, b, c, \ldots\}$ or a set of the corresponding type $\{a, b, c, \ldots\}$, the generalized members of which are classes
and sets of the same types. Let \( \mathbb{R} \) be the reduction operator \( \bigcup \). Here the operator \( \bigcup \) transforms the class of sets \( \{a, b, c, \ldots\} \), \( \{a', b', c', \ldots\} \), \( \{a''', b''', c''', \ldots\} \) (the set \( \{a, b, c, \ldots\} \) in the class \( a, b, c, \ldots \)) and the class of classes \( a, b, c, \ldots \) in the same class. Let the operator \( \rho \) transform the class \( a, b, c, \ldots \) in the manifold \( a b c \ldots \). The operator \( \mathbb{R} \) applied a sufficiently large number of times on \( T \) and its transforms yields a manifold the members of which are mere simple individuals. They are the elements of \( T \). (Received March 3, 1972.)

*72T-E55. STEVEN K. THOMASON, Simon Fraser University, Burnaby 2, British Columbia, Canada. Noncompactness in propositional modal logic.

A formula \( A \) (of propositional modal logic without propositional quantifiers) is said to be a logical consequence of a set \( Y \) of formulas if \( A \) is valid in every frame \( (W, R) \) in which all members of \( Y \) are valid.

We construct a recursive set \( X \) of formulas (containing the axioms of \( T \) and contained in the theses of \( S4 \)), such that \( \Box p \not\rightarrow \Box \Box p \) is a logical consequence of \( X \) but not of any finite subset of \( X \). It follows that no set of finitary rules can be adequate for deriving, from an arbitrary recursive set \( Y \), exactly the logical consequences of \( Y \). It also follows that the relation "\( A \) is a logical consequence of \( Y \)" is not recursively enumerable. (Received March 27, 1972.)

72T-E56. GEORGE E. WEAVER, Department of Philosophy, Bryn Mawr College, Bryn Mawr, Pennsylvania 19010. A uniform interpolation lemma for first order logic with equality.

Let \( K \) be a set of nonlogical constants, excluding functional constants. Let \( L_K \) be a first order language over \( K \) where, for every formula \( A \) and every variable \( \alpha \) occurring in \( A \), \( \exists \alpha A \) is a formula if every occurrence of \( \alpha \) in \( A \) is free. For all \( B, A \) in \( L_K \) let \( r(A) \) (the rank of \( A \)) denote the number of distinct variables in \( A \); \( K(A) \) denote the members of \( K \) occurring in \( A \); and \( A \models B \) indicate that \( B \) is a logical consequence of \( A \). **Theorem.** For all \( A \) a sentence in \( L_K \) all \( K' \subset K \) and all \( n \models r(A) \) there is a sentence \( C \) in \( L_K \) such that \( A \models C \), \( r(C) = n \), \( K(C) = K' \) and for all \( B \) such that \( A \models B \), \( r(B) \leq n \) and \( K(A) \cap K(B) = K' \), \( C \models B \). (Received March 29, 1972.)


A formalization is given for a complete negationless first-order predicate calculus. While the completeness proof, which uses a tableau system, is classical, the concept of validity with respect to a model of the Kripke-type may be defined negationlessly, and falsity for the counterexamples may be considered in terms of constructible falsity of Nelson [J. Symbolic Logic 14(1949), 16-26]. The system is a sequent calculus which is an extension of the system \( P_\alpha \) of Nelson [J. Symbolic Logic 31(1966), 562-572], and also satisfies the realizability definition given for that system. (Received April 7, 1972.)
Models with cardinality \( \omega_1 \) for an infinitary language. Preliminary report.

Our theorem concerns the infinitary language which differs from the language of finitary logic only in that the predicate symbols are infinitary, i.e., they may be followed by an infinite sequence of variables. We show that if a set of formulas in this language has cardinality \( \omega_1 \) and has a model, then it must have a model with cardinality \( \omega_1 \) if it satisfies the following condition: For any two infinite sequences of variables appearing in the set of formulas, the set of variables which do not appear in exactly the same places in the two sequences is finite. We give an example to show that a set of formulas which does not have this property may have no models with cardinality less than the continuum. (Received April 12, 1972.) (Author introduced by Professor Martin D. Davis.)

Proper extensions of \( N \) which are minimal with respect to definable relations. Preliminary report.

Let \( *N \) be a proper elementary extension of \( N \) and let \( \lambda \) be any \( m \)-place \( *N \) definable relation. Let \( a_1, \ldots, a_n \) be the constants of \( *N \setminus N \) that occur in \( \lambda \). Then there is a formula of \( N \) with \( n + m \) free variables, \( \theta \), such that \( *N \models \lambda(b_1, \ldots, b_m) \) iff \( *N \models \theta(a_1, \ldots, a_n, b_1, \ldots, b_m) \). It is shown that for proper choice of \( *N \) (depending on \( \theta \)), the relation \( \lambda \) when restricted to \( N \) is definable in \( N \). Furthermore, there is an \( *N \) which works for every \( \theta \). For this \( *N \), every \( *N \) definable relation, when restricted to \( N \) is \( N \) definable. \( *N \) is constructed as an ultraproduct of \( N \) restricted to the set of definable functions from \( N \) to \( N \). (Received April 12, 1972.)

A decidable class of Krom formulae.

Let the class of Krom formulae \( Kr \) be the class of prenex formulae of the first order predicate calculus with a matrix in conjunctive normal form and binary disjunctions. The class of all closed Krom formulae with a prefix of the form \( AEA \ldots A \), with an arbitrary number of monadic or binary predicate symbols and without individual or function symbols has a recursively solvable decision problem with respect to satisfiability. A detailed proof consisting in a reduction to the case \( AEA \cap Kr \) shown decidable by S. Aanderaa ("On the decision problem for formulae in which all disjunctions are binary," Proc. Second Scand. Logico Sympos., 1971) will be published in the 1973 volume of the Z. Math. Logik Grundlagen Math. (Received April 14, 1972.)

On local proof restrictions for strong theories.

Here \( T \) is an r.e. axiomatized theory which interprets Peano arithmetic. A mapping \( F \) from sentences of \( T \) to r.e. subtheories of \( T \) is a proof restriction function provided \( F(\alpha) \vdash \alpha \) whenever \( T \vdash \alpha \). It has been shown that if \( T \) is consistent then there is no such recursive \( F \) where \( T \vdash \text{Con} \neg F(\alpha) \) for all \( \alpha \). (In many applications this prevents \( F(\alpha) \) from being finite, of bounded-depth, etc.) A more general result is
obtained where we drop \( T \vdash \text{Cons}_{\mathcal{F}(\alpha)} \) and ask rather that \( \mathcal{F}(\alpha) \) and \( \mathcal{F}(\neg \alpha) \) are explicitly distinguished by sentence \( \delta \) in case \( \delta \) is provable in exactly one of \( \mathcal{F}(\alpha), \mathcal{F}(\neg \alpha) \).

**Theorem.** Assume that \( T \) is consistent, that \( \mathcal{F} \) and \( \mathcal{D} \) are recursive functions such that \( \mathcal{F}(\alpha) \) and \( \mathcal{F}(\neg \alpha) \) are explicitly distinguished by \( \mathcal{D}(\alpha) \) for all \( \alpha \), and that \( T \) has a truth definition for all the \( \mathcal{D}(\alpha) \). Then \( \mathcal{F} \) is not a proof restriction function for \( T \). (Received April 21, 1972.)

*72T-E62, MAHMOUD M. HAIFAWI, Middle East Technical University, Ankara, Turkey. A note on Novak’s paradoxical theorem. Preliminary report.*

J. Novak proved a paradoxical theorem [Fund. Math. 37(1950), 77-83] for a collection \( \{A_\xi, \xi < \omega_1\} \) of sets each of which has cardinality \( \aleph_0 \). For the extent of generalization we have, **Theorem.** There exists no ordinal valued function \( \varphi(\xi) \) defined for ordinal numbers of the second kind \( \xi < \omega_{\omega+1} \) having the properties \( \varphi(\omega_1) < \omega_1 \) for \( \xi < \omega_{\alpha+1} \) and \( \varphi(\omega_1) \neq \varphi(\omega_2) \) for \( \xi_1 < \xi_2 < \omega_{\alpha+1} \). **Theorem.** Let \( \{A_\xi, \xi < \omega_{\alpha+1}\} \) be a set of nonempty mutually exclusive sets such that \( |A_\xi| = \aleph_0 \) for all \( \xi < \omega_{\alpha+1} \). From \( A_0 \) we take away one element \( b_1 \) and add a new set \( A_1 \) to the remainder, proceeding in this fashion so that from \( \bigcup_{0 < \lambda < \beta} A_\lambda \) - \( \bigcup_{0 < \lambda < \beta} \{b_\lambda\} \) if not empty, we take away one element \( b_\beta \) and add to the remainder a new set \( A_\beta \). Then there exists an ordinal number \( \xi < \omega_{\alpha+1} \) such that \( \bigcup_{\lambda < \xi} A_\lambda = \bigcup_{\lambda < \xi} \{b_\lambda\} \). A counterexample is given for the case of an initial number with limit index. (Received April 17, 1972.) (Author introduced by Dr. Leif Arkeryd.)

**Statistics and Probability**

72T-F6, THOMAS M. LIGGETT, University of California, Los Angeles, California 90024.

A characterization of the invariant measures for an infinite particle system with interactions. Preliminary report.

Let \( S \) be a countable set, and \( p(x,y) \) be the transition function for a symmetric, irreducible, transient Markov chain on \( S \). Let \( \{\eta_\xi\} \) be the Markov process on \( X = \{0,1\}^S \) which describes the motion of infinitely many particles on \( S \) which move according to Spitzer’s simple exclusion interaction model (see § 3b of “Interaction of Markov processes", Advances in Math. 5(1970), 246-290). Let \( I \) be the set of probability measures on \( X \) which are invariant for \( \mathcal{I}_e \), and \( I_e \) be its set of extreme points. Put \( H = \{\alpha(\cdot) \mid 0 \leq \alpha \leq 1 \text{ and } \sum_y p(x,y) \alpha(y) = \alpha(x) \text{ for all } x \in S\} \). **Theorem.** There is a one-to-one correspondence between \( H \) and \( I_e \) which is given for \( \alpha \in H \) and \( \mu_\alpha \in I_e \) by (i) \( \alpha(\cdot) = \mu_\alpha \{\eta : \eta(x) = 1\} \) and (ii) \( \mu_\alpha \{\eta : \eta(x_n) = 1, \ldots, \eta(x_1) = 1\} = \lim_{t \to \infty} p_\alpha(\eta : \eta(x_1) = 1, \ldots, \eta(x_n) = 1) \nu_\alpha(d\eta) \) for \( x_1, \ldots, x_n \in S \), where \( \nu_\alpha \) is the product measure on \( X \) defined by \( \nu_\alpha \{\eta : \eta(x_1) = 1, \ldots, \eta(x_n) = 1\} = \alpha(x_1) \ldots \alpha(x_n) \). Furthermore, \( \mu_\alpha = \nu_\alpha \) if and only if \( \alpha \) is constant. In particular, if \( S \) is an abelian group and \( p(x,y) = p(0,y-x) \), then \( I_e \) consists only of the product measures \( \mu_\rho \) with constant density \( \rho \) for \( 0 \leq \rho \leq 1 \). On the other hand, there are many choices of \( p(x,y) \) for which \( H \) contains nonconstant functions. In those cases, there are measures in \( I_e \) which are not product measures.

(Received April 10, 1972.)
Let $b(s)$ be the Skorokhod-Breiman representation of a mean-zero random variable $X$. Here $b(s)$ is Brownian motion and $b(T) \equiv X$. Assume $X \equiv -M$ and (1) $P(|X| \leq t) \sim C t^\delta \exp(-ct^\delta)$. Let $\delta = e/(2+e)$. Then, if $0 < \delta \leq 1$, (2) $P(T \geq s) \sim C_1 s^\delta \exp(-hs^\delta)$. If $0 < e \leq 2$, an additional factor of $\int_0^M \exp(\gamma_1|u|^{2/3}) F(du)$ is introduced in (2), where $c = (3\delta - 1)/2$ and $F(du) = P(X \in du)$. For larger values of $e$, the correction factor becomes more complicated. Similarly, if $X \equiv -M$ and $P(X \leq t) \sim C/P^\alpha$, then $P(T \geq s) \sim C_2 s^{1/2}$. The condition (3) $X \equiv -M$ can probably not be removed. For example, if $T$ is the (univariate) Skorokhod representation of $X$, then (2) still holds for $0 < e \leq 1$ given (1) and (3), but if (3) is replaced by $X$ being symmetrically distributed, then (2) holds with a smaller value of $b$. For a collateral reference, see the author's "A remark on the Skorokhod representation", to appear in Z. Wahrscheinlichkeitstheorie und Verw. Gebiete. (Received April 27, 1972.)

**Topology**

72T-G84. CHANDRA MOHAN PAREEK, University of Saskatchewan, Regina, Saskatchewan, Canada. Some generalizations of Lindelöf and paracompact spaces. Preliminary report.

Let $m$ be any infinite cardinal number. In this note notions of $m$-Lindelöf, meta-$m$-Lindelöf, para-$m$-Lindelöf and closure-$m$-preserving property are defined and the following are the main results: (a) If a topological space $X$ has a point-$m$ open cover such that it has no subcover of cardinality $\leq m$, then $X$ has a discrete subset $S$ such that $|S| > m$. (b) In a regular $m$-Lindelöf space a subset $Y$ is $m$-Lindelöf if and only if $Y$ is a generalized $F_m$-set. (c) A topological space $X$ is $m$-Lindelöf if and only if it is meta-$m$-Lindelöf and it is $m$-Lindelöf in the sense of complete accumulation point. (d) A regular topological space $X$ is paracompact if and only if it is para-$m$-Lindelöf and it has the $m$-closure preserving property. (e) If $X$ is $m$-Lindelöf and $Y$ is $m$-separable then $X \times Y$ is para-$m$-Lindelöf if and only if $X \times Y$ is $m$-Lindelöf. (Received February 4, 1972.)


A b.t.s. $(X, \tau_1, \tau_2)$ is called minimal pairwise-$G$ iff $(X, \tau_3, \tau_4)$ is a pairwise-$G$ b.t.s. and $\tau_3 \subset \tau_1$ and $\tau_4 \subset \tau_2$ implies $\tau_3 = \tau_1$ and $\tau_4 = \tau_2$. We obtain characterisations of minimal pairwise-$G$ spaces for $G =$ Hausdorff, Urysohn, regular, completely regular by using various types of filter bases in the b.t.s. $(X, \tau_1, \tau_2)$. (Received February 22, 1972.) (Author introduced by Professor M. Rajagopalan.)

72T-G86. JAMES L. CORNETTE, Iowa State University, Ames, Iowa 50010. "Image of a Hausdorff arc" is extensible and reducible.

We assume a cyclic element theory for Hausdorff continua similar to that developed by G. T. Whyburn for metric continua. An arc is a Hausdorff continuum with only two noncut points. It is shown that a Hausdorff continuum $S$ is the continuous image of an arc (respectively arcwise connected) if and only if each cyclic element...
of $S$ is the continuous image of an arc (respectively arcwise connected). A corollary is that each Hausdorff dendron is the continuous image of an arc. (Received March 3, 1972.)

72T-G87. DAVID K. COHOON, University of Minnesota, Minneapolis, Minnesota 55455. The MSS question for the category of connected metric spaces and continuous maps.

The author shows the existence of a connected metric space $X$, a connected subspace $W$ of $X$, and a continuous map $f$ of $X$ onto itself such that $f(W)$ is a proper subset of itself. Furthermore, the triple $(X, W, f)$ has the property that if $E$ is a connected subset of $X$ containing $W$ such that $f(E) = E$, then there is a proper subset $F$ of $E$ containing $W$ such that $f(F) = F$. In other words there is no minimal connected metric space $E$ containing $W$ on which $f$ is surjective. (Received March 6, 1972.)

72T-G88. DAVID K. COHOON, University of Minnesota, Minneapolis, Minnesota 55455. The MSS question for the category of connected metric spaces and continuous maps.

The author shows the existence of a connected metric space $X$, a connected subspace $W$ of $X$, and a continuous map $f$ of $X$ onto itself such that $f(W)$ is a proper subset of itself. Furthermore, the triple $(X, W, f)$ has the property that if $E$ is a connected subset of $X$ containing $W$ such that $f(E) = E$, then there is a proper subset $F$ of $E$ containing $W$ such that $f(F) = F$. In other words there is no minimal connected metric space $E$ containing $W$ on which $f$ is surjective. (Received March 6, 1972.)

72T-G89. HAROLD BENNETT, Texas Tech University, Lubbock, Texas 79409. Developability of quasi-developable spaces. Preliminary report.

Theorem. A regular $T_1$ $\theta$-refinable quasi-developable space is a Moore space if and only if it is one of the following (i) a $\beta$-space, (ii) a $p$-space, (iii) a w$\Delta$-space, or (iv) an M-space. (Received March 6, 1972.)


A bordism mapping torus construction is introduced and used to compute $\pi_0(Diff: RP^{2^k-1})$ for $k > 8$, $k \equiv 6 \pmod 8$. Let $f: BSO \rightarrow BSO$ classify $(2^{\omega(\xi)} - 2)\xi$ where $\xi$ is the canonical line bundle over $RP^{2^k}$. Let the fibration $g: X \rightarrow BSO$ be the $k$th stage in the Moore-Postnikov decomposition of $f$. Let $\Omega_{2^k}(g)$ be the 2kth Lashof cobordism group of the fibration $g$. Theorem. If $k \equiv 6 \pmod 8$ and $k > 8$, then there is an exact sequence of groups $1 \rightarrow \Omega_{2^k}(g) \rightarrow \pi_0(Diff: RP^{2^k-1}) \rightarrow Z_2 + Z_2$. One of the factors $Z_2$ splits back into $\pi_0(Diff: RP^{2^k-1})$. Corollary. Modulo 2-torsion, $\Omega_{2^k}(g) \cong \pi_0(Diff: RP^{2^k-1})$. (Received March 7, 1972.)

*72T-G91. ROBERT A. DOOLEY, Oklahoma State University, Stillwater, Oklahoma 74074. Further extending a complete convex metric.

The following theorem strengthens a result of the author that was announced in Abstract 71T-G198 (these Notices 18(1971), 983) and is to be published in full ("Extending a complete convex metric," Proc. Amer. Math. Soc., to appear). Theorem. Let $M_1$ be a nonempty space with complete convex metric $D_1$ and let $M_2$ be
a nonempty locally connected generalized continuum. In order for there to exist a complete convex metric for $M_1 \cup M_2$ that extends $D_1$, it is necessary and sufficient that $M_1 \cap M_2$ be a nonempty subspace of $M_1$ and $M_2$ which is closed in $M_2$, and that the $M_2$ boundary of $M_1 \cap M_2$ be closed in $M_1$. Remark. The space $M_1 \cup M_2$ has the largest topology for which $M_1$ and $M_2$ are subspaces. (Received March 20, 1972.) (Author introduced by Dr. John M. Jobe.)


The equality $\nu(X \times Y) = \nu_X \times \nu_Y$ is studied for the case when one of the factors is a linearly ordered topological space (LOTS). Among the results obtained are the following: (1) If $X$ is any separable realcompact space and $Y$ is any LOTS of nonmeasurable cardinal, then $\nu(X \times Y) = \nu_X \times \nu_Y$. (2) If $X$ is a nonparacompact LOTS, then there is a paracompact LOTS $Y$ such that $\nu(X \times Y) \neq \nu_X \times \nu_Y$. (3) For any pair $X, Y$ of well-ordered spaces, $\nu(X \times Y) = \nu_X \times \nu_Y$. (Received March 9, 1972.)


This paper clarifies the relationship between certain covering properties of uniform spaces and their connection with the functors $\pi$, $e$, and $c$. (For a uniform space $uX$, $\pi uX$ is the completion of $uX$, $e uX$ has the basis of countable $u$-covers, and $c uX$ is generated by the $u$-uniformly continuous real valued functions.) The following theorems are proved: (0) $\pi$ and $c$ commute on uniform spaces with bases of star-finite uniform covers; (1) $\pi$ and $e$ commute on uniform spaces with bases of point-finite uniform covers; (2) $e uX$ is complete if and only if the uniformity generated by the point-finite $u$-covers is complete if and only if the uniformity generated by the star-countable $u$-covers is complete; (3) $c uX$ is complete if and only if the uniformity generated by the star-finite $u$-covers is complete; (4) Every star-countable uniform cover is a $\sigma$-uniformly discrete uniform cover; (5) Every uniform space with a basis of $\sigma$-uniformly discrete uniform covers has a basis of point-finite uniform covers. The last theorem shows that (1) is a generalization of the Ginsburg-Isbell theorem that $\pi$ and $e$ commute under the assumption of a $\sigma$-uniformly discrete base. In addition, theorems are presented which relate the covering properties of $uX$ to the covering properties of the various cardinal reflections of $uX$. (Received March 20, 1972.)

*72T-G94. JEONG SHENG YANG, University of South Carolina, Columbia, South Carolina 29208. A note on topological groups and homotopy groups.

Let $X$ be a topological space, $G$ a topological group, and let $C(X,G)$ be the topological group of all continuous functions from $X$ into $G$ with the compact-open topology and with pointwise multiplication. If $G$ and $H$ are topological groups, let $C^\bullet(G,H)$ denote the closed normal subgroup of $C(G,H)$ which consists of identity-preserving elements. The following results are obtained. Theorem 1. $G$ has equal uniformities if and only if $C(X,G)$ has equal uniformities. Theorem 2. If $G$ is locally compact and contractible to its identity element, then, for any $H$, $C^\bullet(G,H)$ is contractible to its identity element and is $n$-simple for every positive integer $n$.
Theorem 3. If $G$ is locally compact and locally contractible, then $C^*(G, H)$ is locally contractible for any $H$.

Theorem 4. If $G$ is a topological group, there are closed normal subgroups $H$ and $K$ of a contractible, locally contractible topological group $L$ such that (a) $\pi_1(G, e) = H/K$, (b) $\pi_k(G, e) = \pi_k(H, e_L)$ for each positive integer $k$, (c) $L$ may be assumed to have equal uniformities if $G$ has equal uniformities, and (d) $L$ may be taken to be a metrizable group if $G$ is metrizable. (Received March 9, 1972.)

72T-G95. WITHDRAWN.

72T-G96. RICHARD A. SNAY, Indiana University, Bloomington, Indiana 47401. Decompositions of $E^3$ into countably many dendrites. Preliminary report.

If $G$ is a decomposition of $E^3$, the letter $H$ will denote the collection of nondegenerate elements of $G$.

Theorem. If $G$ is an upper semicontinuous decomposition of $E^3$ such that $H$ consists of countably many tame dendrites, then $E^3/G \approx E^3$.

Definition. A space $X$ has the map separation property iff the following condition holds: If $D$ is a disk and $f_1, \ldots, f_n$ are maps of $D$ into $X$ such that (1) for each $i$, on some neighborhood of $f_i(Bd D)$, $f_i^{-1}$ is a function, (2) if $i \neq j$, $f_i(Bd D) \cdot f_j(D) = \emptyset$, and (3) $U$ is a neighborhood of $\sum_{i=1}^n f_i(D)$, then there exist maps $g_1, \ldots, g_n$ of $D$ into $X$ such that (1) for each $i$, $g_i|Bd D = f_i|Bd D$, (2) $\sum_{i=1}^n g_i(D) \subset U$, and (3) if $i \neq j$, $g_i(D) \cdot g_j(D) = \emptyset$.

Theorem. Let $G$ be an upper semicontinuous decomposition of $E^3$ such that $H$ consists of countably many cellular dendrites and such that if $h \in H$, then the boundary of $h$ as a subset of $E^3$ is 0-dimensional. Then $E^3/G \approx E^3$ iff $E^3/G$ has the map separation property. In particular, if $G$ is a cellular upper semicontinuous decomposition of $E^3$ such that $H$ is contained in a set homeomorphic to a 1-dimensional simplicial complex, then $E^3/G \approx E^3$ iff $E^3/G$ has the map separation property. (Received March 13, 1972.)


The following general theorem is proved. Theorem. Let $\{X_j, T_j\}_j$ be a sequence of $T_1$ spaces, let $\langle f_n \rangle$ be an infinite sequence in $\Pi_{j=1}^\infty X_j$. Suppose that for each $j$ in $\mathbb{N}$ and each subsequence $\langle f_{k_n} \rangle$ of $\langle f_n \rangle$ with $n_k \geq j$, there is a subsequence $\langle f_{k_{n_k}} \rangle$ of $\langle f_{k_n} \rangle$ such that $\langle f_{k_{n_k}} \rangle$ is compact. Then $\langle f_n \rangle$ has a subsequence whose closure is compact. Several results can be obtained from this theorem, of which the following is representative: Let $\{X_j, T_j\}_j$ be a sequence of weakly-k (Thomas W. Rishel, "On a subclass of $M$-spaces," Proc. Japan Acad. 45(1969), 544-546), $w^\infty$-spaces. Then $\Pi_{j=1}^\infty X_j$ is a weakly-k, $w^\Delta$-space. Burke and Stoltenberg ("A note on p-spaces and Moore spaces," Pacific J. Math. 30(1969), 601-608) have shown that for a Tychonoff space $X$, $X$ is a strict p-space if and only if $X$ has a sequence $\mathcal{S}_1, \mathcal{S}_2, \ldots$ of open covers satisfying: (1) $C(p) = \bigcap_{n=1}^\infty S(p, \mathcal{S}_n) = \text{compact}$ for each $p$ in $X$, (2) $\{S(p, \mathcal{S}_n) : n = 1, 2, \ldots\}$ is a base for $C(p)$. A decreasing sequence $\mathcal{S}_1, \mathcal{S}_2, \ldots$ of open covers satisfying (1) and (2) is called a strong $w^\Delta$-sequence for $X$, and is called a strong $w^\Delta$-space. Theorem. A countable product of strong $w^\Delta$-spaces (strict p-spaces) is a strong $w^\Delta$-space (strict p-space). This theorem can be used to give results concerning covering properties of product spaces, of which the following is representative: A countable product of metacompact, strong $w^\Delta$-spaces is metacompact. (Received March 13, 1972.)
For any cardinal \( \kappa \) let \( \mathcal{N}^\kappa \) be the product of \( \kappa \) copies of the countable discrete space with the usual product topology, let \( \mathcal{Q} \) be the space of the rational numbers with the usual topology, and, following S. Mrówka ("Further results on \( \mathcal{E} \)-compact spaces. I," Acta Math. Acad. Sci. Hungar. 130(1968), 163-165), let \( \text{Exp}(\mathcal{Q}) \) be the least cardinal \( \lambda \) such that \( \mathcal{Q} \) can be embedded as a closed subset of \( \mathcal{N}^\lambda \). Mrówka notes in the above that \( \text{Exp}(\mathcal{Q}) \) is uncountable but no larger than \( 2^{\aleph_0} \). We prove: Theorem 1. \( \text{Exp}(\mathcal{Q}) \) is equal to the cardinality of the smallest family of closed subsets of the irrationals whose union is the set of all irrationals.

Theorem 2. It is consistent with ZFC that \( \text{Exp}(\mathcal{Q}) \) be any uncountable regular cardinal no greater than \( 2^{\aleph_0} \).

Mrówka ("Some comments on the class \( \mathcal{M} \) of cardinals," Bull. Acad. Polon. Sci. 17(1969), 411-414) has asked about the extent of the class \( \mathcal{M} \) of those cardinals \( \kappa \) for which \( \mathcal{N}^\kappa \) contains a closed discrete subset of cardinality \( \kappa \). Theorem 3. If there exists a maximal almost-disjoint family (Abstract 70T-E26, these Notices 17(1970), 576-577) of cardinality \( \kappa \), then \( \kappa \in \mathcal{M} \). Theorem 4. Inaccessible cardinals below \( 2^{\aleph_0} \) may consistently belong to \( \mathcal{M} \). This extends Mycielski's ("a-compactness of \( \mathcal{N}^\kappa \)," Bull. Acad. Polon. Sci. 12(1964), 437-438) result that \( \mathcal{M} \) contains all cardinals below the first inaccessible. The proofs of Theorems 1 and 3 are combinatorial in nature while those of 2 and 4 follow from earlier independence results of the author.

(Received March 20, 1972.)

*72T-G98. STEPHEN H. HECHLER, Department of Mathematics and Statistics, Case Western Reserve University, Cleveland, Ohio 44106. Exponents of some \( \mathcal{N} \)-compact spaces. Preliminary report.

Embedding circle-like continua in \( \mathbb{E}^3 \).

If \( X \) is a locally planar space embedded as a closed subset of \( \mathbb{E}^3 \), we will say that \( X \) is \textit{locally tame} if each point of \( X \) has a neighborhood in \( X \) which lies on a tame disk in \( \mathbb{E}^3 \), and that \( X \) is \textit{strongly locally tame} if each cell-like subset of \( X \) has such a neighborhood. Our principal result is that every circularly chainable continuum has a strongly locally tame embedding in \( \mathbb{E}^3 \). As an application, it is shown that the hyperspace of subcontinua of a pseudosolenoid (a circularly chainable, hereditarily indecomposable continuum which is not chainable) has a particularly nice embedding in \( \mathbb{E}^4 \), and that this hyperspace is not embeddable in \( \mathbb{E}^3 \) unless the pseudosolenoid is embeddable in \( \mathbb{E}^2 \). (Received March 28, 1972.)

72T-G100. BARBARA LEHMAN, Iowa State University, Ames, Iowa 50010. Products of arcwise connected spaces. Preliminary report.

An arc is a Hausdorff continuum \( A \) which is either a single point or has only two noncut points called the end points of \( A \). A topological space \( X \) is arcwise connected if each two points of \( X \) are the end points of an arc in \( X \). Lemma. Let \( \{X_\alpha: \alpha \in (\mathcal{A}, \preceq)\} \) be a collection of nondegenerate arcs indexed by a well-ordered set \( (\mathcal{A}, \preceq) \). For each \( \alpha \) in \( \mathcal{A} \), let \( a_\alpha, b_\alpha \) be the end points of \( X_\alpha \). Let \( f, g \) be the points of the product space \( \prod_{\alpha \in \mathcal{A}} X_\alpha \) such that for each \( \alpha \) in \( \mathcal{A} \), \( f(\alpha) = a_\alpha \), \( g(\alpha) = b_\alpha \). For each \( \alpha \) in \( \mathcal{A} \), let \( A_\alpha = \{ h \in \prod_{\alpha \in \mathcal{A}} X_\alpha : h(\beta) = b_\beta, \beta < \alpha; h(\beta) = a_\beta, \alpha < \beta \} \). Then \( (\bigcup_{\alpha \in \mathcal{A}} A_\alpha) \cup \{ g \} \) is an arc in \( \prod_{\alpha \in \mathcal{A}} X_\alpha \) with end points \( f \) and \( g \).

Theorem. If \( \{Y_\alpha: \alpha \in \mathcal{A}\} \) is a collection of arcwise connected spaces, then \( \prod_{\alpha \in \mathcal{A}} Y_\alpha \) is arcwise connected, (Received March 30, 1972.) (Author introduced by Professor James L. Cornette.)

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Let $\alpha$ and $\beta$ be elements of finite order in $\prod_{m=1}^{n} S^n$ with mapping cones $C_{\alpha}$, $C_{\beta}$. We shall say $C_{\alpha}$ and $C_{\beta}$ are of the same genus if their localized spaces are homotopically equivalent for every prime $p$. Let $X$ and $Y$ denote the "wedge" or "one-point union" of $X$ and $Y$. Then the following are equivalent: (i) $C_{\alpha}$ and $C_{\beta}$ are of the same genus; (ii) $C_{\alpha} \vee T \cong C_{\beta} \vee T$ for $T$ a wedge of spheres; (iii) $C_{\alpha} \vee S^n \vee S^m \cong C_{\beta} \vee S^n \vee S^m$. If we restrict to the case of $n$ odd, $\alpha$ and $\beta$ of odd order, then also equivalent to (i) are (iv) $C_{\alpha} \vee S^n \cong C_{\beta} \vee S^k$; (v) $C_{\alpha} \vee S^m \cong C_{\beta} \vee S^m$; and (vi) $\alpha$ and $\beta$ generate the same subgroup of $\prod_{m=1}^{n} S^n$. Examples have been found of the following: (a) for $k = n, m$, $C_{\alpha}$ and $C_{\beta}$ are of the same genus but $C_{\alpha} \vee S^n \cong C_{\beta} \vee S^m$; (b) for $\alpha$ of infinite order, $C_{\alpha}$ and $C_{\beta}$ are of the same genus but $C_{\alpha} \vee S^n \vee S^m \neq C_{\beta} \vee S^n \vee S^m$.

*72T-G102, EDWARD T. ORDMAN, University of Kentucky, Lexington, Kentucky 40506. Free products of topological groups. Preliminary report.

If $\{G_{\alpha} : \alpha \in A\}$ is a collection of Hausdorff topological groups, Theorem 2.2 of S. A. Morris [Bull. Austral. Math. Soc. 4(1971), 17-29] states that the topological free product is Hausdorff. The proof given there is in error, in that the pseudometric topology $\tau_1$ constructed on $p$, 19 does not make $F$ a topological group (the map $t \rightarrow gtg^{-1}$ is discontinuous for $g \neq e$). In this paper the result is proven for the free product of locally invariant Hausdorff groups, i.e., Hausdorff groups whose topology is determined by their invariant pseudometrics. Our topology corresponding to $\tau_1$ involves $\rho(g, e) = \inf \sum_{\alpha} \rho_{\alpha(i)}(g_{i}, e_{i})$ where $g = g_1 g_2 \ldots g_n$ ranges over words representing $g$, and $e = e_1 e_2 \ldots e_n$ ranges over a certain class of words representing the identity, in the free product. The result is also extended to a class of free products with amalgamated subgroup.

*72T-G103, CHANDAN S. VORA, University of Genoa, Genoa, Italy and Indiana University, Bloomington, Indiana 47401. Fixed point theorems for certain symmetric product mappings of $A$-$ANR$ and a manifold. Preliminary report.

Let $X$ be a topological space, $G$ a group of permutations of the letters $[1, \ldots, n]$. Then $G$ can be considered as a group of homeomorphisms on $X^n$ by definition, for $g \in G$ and $(x_1, \ldots, x_n) \in X^n$, $g(x_1, \ldots, x_n) = (x_{g(1)}, x_{g(2)}, \ldots, x_{g(n)})$. The orbit space under this action with identification topology will be denoted by $X^n/G$. An element $x \in X$ is a fixed point of a continuous map $f: X \rightarrow X^n/G$, if $x$ belongs to the orbit of $f(x)$. If $X$ is a compact $A$-$ANR$, then the concept of Lefschetz number of $f$, $\Lambda(f)$, is defined and the following theorem is proved, "If $X$ is a compact $A$-$ANR$ and $f: X \rightarrow X^n/G$ is such that $\Lambda(f) \neq 0$, then $f$ has a fixed point." The concept of Lefschetz number of a map $f: M \rightarrow M^n/G$, $M$ a topological manifold, is defined and the following theorem is proved: "Let $M$ be an $m$ manifold (with or without boundary); let $X$ be an $(m-2)$-connected $ANR$ imbedded as a closed subset of $M$ and let $U$ be a component of $M - X$ whose closure is not compact. Let $f: (M-U, X) \rightarrow (M^n/G, (M-U)^n/G)$ be a compact map and let $f^*: X \rightarrow (M-U)^n/G$ denote the map defined by the restriction of $f$. Then there exist $C$-Lefschetz maps $u: M - U \rightarrow (M-U)^n/G$ and $v: M \rightarrow M^n/G$ such that $u$ is an
extension of \( f \), \( v \) an extension of \( f \), \( \Lambda(v) = \Lambda(u) \) and \( F_v \subset F_u \cap F \) (\( F_v \) denotes fixed point set of \( v \)). In particular, if \( \Lambda(v) \neq 0 \), then \( f \) has a fixed point." (Received March 29, 1972.)

*72T-G104. MELVIN C. THORNTON, University of Nebraska, Lincoln, Nebraska 68508. Lattice characterization of lower semicontinuous functions.

Lower semicontinuous real-valued functions on a space \( X \) form a conditionally complete distributive lattice \( L(X) \). Those lattices which can be represented as \( L(X) \) for some \( X \) are characterized algebraically. All spaces producing isomorphic lattices are determined. Necessary and sufficient conditions are found for a lattice to have a representation \( L(X) \) unique up to homeomorphism of \( X \). (Received April 3, 1972.)

*72T-G105. ANTHONY W. HAGER, GEORGE D. REYNOLDS and MICHAEL D. RICE, Wesleyan University, Middletown, Connecticut 06457. Uniform completeness and nonmeasurability.

The cardinal \( m \) is measurable if when \( |X| = m \), there is a free \( X \)-ultrafilter with the "\(< m \)-intersection property". \( ( \aleph_0 \) is measurable, and the first uncountable measurable cardinal is the first Ulam-measurable cardinal \( \kappa \).) Theorem. Let \( m \) be measurable, and let \( (X, \mu) \) be a complete uniform space such that; \( \mu \) has a basis of covers of power \( < m \); each point of \( X \) is the intersection of \( < m \) \( \mu \)-open sets. Then \( |X| < m \). Proof. If \( \mathcal{F} \) is an ultrafilter with \( < m \)-ip, then the \( \mu \)-open members of \( \mathcal{F} \) form a \( \mu \)-Cauchy filter, \( \mathcal{F}_0 \), and \( \mathcal{F}_0 \to x \). Then \( \{x\} = \bigcap \mathcal{F} \), where \( |\mathcal{F}| < m \) and the members are \( \mu \)-open. \( \mathcal{F} \subset \mathcal{F}_0 \subset \mathcal{F} \), so \( \bigcap \mathcal{F} \in \mathcal{F} \) and \( \mathcal{F} \) is fixed. One obtains immediately: (1) a complete precompact (i.e. compact) discrete uniform space is finite; (2) (Juhasz, Bull. Acad. Polon. Sci. 17(1969), 219–223) a realcompact topological space in which each point is the intersection of \( < \kappa \) open sets has power \( < \kappa \); (3) (the authors, Fund. Math., to appear) a Borel-complete topological space has power \( < \kappa \). This paper will not be published formally. (Received April 6, 1972.)

*72T-G106. DENNIS K. BURKE, Miami University, Oxford, Ohio 45056. On the nonpreservation of a \( G_\delta \)-diagonal under a perfect map.

An example is constructed of a locally compact Hausdorff space with a \( G_\delta \)-diagonal and a perfect map from this space onto a space without a \( G_\delta \)-diagonal. This answers negatively the question asked by R. W. Heath (in "General topology and its applications," Vol. 1, no. 1) about the preservation of \( G_\delta \)-diagonals under perfect maps. This example is a modification of an example constructed previously (see Abstract 71T-G150, these Notices 18(1971), 838) of a nondevelopable locally compact Hausdorff space with a \( G_\delta \)-diagonal. (Received April 7, 1972.)

*72T-G107. OFELIA T. ALAS, Caixa Postal 20570 (Iguatemi) S\'o Paulo, S\'o Paulo, Brazil. Closed continuous images of some paracompact spaces.

Let \( m \) be an infinite regular cardinal number, \( X \) be a Hausdorff paracompact space such that the intersection of less than \( m \) open subsets is an open set, \( Y \) be a Hausdorff space and \( f \) be a closed continuous function of \( X \) onto \( Y \). Definition. A subset \( A \) of \( X \) is compact-m if every open covering of \( A \) has a
subcovering of cardinality less than \( m \). **Theorem.** Suppose that every point of \( X \) has a fundamental system of neighborhoods compact-m. Then there is a closed subset \( F \) of \( X \) such that \( f(F) \) is a discrete subspace of \( Y \) and \( Y - f(F) \) is locally compact-m. This result is a generalization of Edwin Duda's theorem for locally compact metric spaces. (Received April 11, 1972.)

72T-G108. LOWELL E. JONES, University of California, Berkeley, California 94720. **Combinatorial symmetries of the m-disc.** I. Preliminary report.

In Abstract 70T-G110, these \( \mathcal{N} \)otices 17(1970), 688, the author announced the proof of the converse to the fixed point theorem of P. A. Smith, provided the potential fixed point set is a PL manifold. For more general polyhedra the converse is not true. Let \( p \) be an odd prime of the form \( p = 4q + 1 \) with \( q = \text{odd} \). Let \( K \) be a \( \mathbb{Z}_p \)-homology manifold; \( K \) will denote both the polyhedron and its dimension. **Theorem 1.** There is a characteristic class \( \beta^p_{4k+1} \in H_{4k+1}(K/\partial K, \mathbb{Z}_2) \), which must vanish if \( K \) is the fixed point set of a combinatorial symmetry \( \mathbb{Z}_p \times N \to N \) defined on some PL manifold \( N \). **Theorem 2.** \( \exists K \) which is a \( \mathbb{Z}_p \)-homology disc, but for which \( \beta^p_{4k+1} \neq 0 \). (Received April 3, 1972.) (Author introduced by Professor F. Thomas Farrell.)


A full subcategory \( \mathcal{G} \) of a category \( \mathcal{L} \) of topological spaces is said to be universal in \( \mathcal{L} \) provided that \( \mathcal{G} \) is epireflective in \( \mathcal{L} \) and each reflection map is an embedding. Van der Slot (ZW 1966-011 Math Centrum Amsterdam) has characterized the universal subcategories of the category of Tychonoff spaces as precisely those epireflective subcategories which contain all compact Hausdorff spaces. A result of Herrlich and Strecker (Math. Ann. 177(1968)) implies that the category of Hausdorff spaces has no proper universal subcategory. We show that there exists a category \( \mathcal{G} \) of completely Hausdorff spaces containing the Tychonoff spaces as a proper subcategory which contains a proper universal subcategory \( \mathcal{S} \) which is minimal in the sense that it is contained in any universal subcategory of \( \mathcal{G} \). We further show that the category of completely Hausdorff spaces contains no proper universal subcategory and that no category between the category of regular completely Hausdorff spaces and the category of Hausdorff spaces is simple. (Received March 24, 1972.)

*72T-G110. ROSS GEOGHEGAN and R. RICHARD SUMMERHILL, Institute for Advanced Study, Princeton, New Jersey 08540. **Pseudo-boundaries and pseudo-interiors in euclidean spaces and topological manifolds.**

The partition of the Hilbert cube into pseudo-boundary and pseudo-interior is now understood to be a special case of a general procedure; that procedure is here applied to Euclidean n-space \( E^n \) \( (n \neq 4) \). For each nonnegative integer \( k < n \) we construct two kinds of dense \( k \)-dimensional pseudo-boundaries, one built out of Menger universal \( k \)-dimensional compacta, the other out of \( k \)-dimensional polyhedra. The corresponding pseudo-interiors are dense \( G_\delta \)-subsets of \( E^n \) of dimension \( n - k - 1 \). The uniqueness and negligibility...
theorems of infinite-dimensional topology apply. Typical Theorem. $\mathbb{E}^{2k+1}$ is the union of two dense disjoint $k$-dimensional subsets $B$ and $P$ such that (1) any $\sigma$-compact subset of $P$ is strongly negligible in $P$; (2) any compact subset of $B$ is strongly negligible in $B$; and (3) any separable metric space of dimension $\leq k$ can be embedded in $P$ or $B$. Such pseudo-boundaries can also be constructed on topological manifolds. (Received April 26, 1972.)

*72T-G111. BRIAN D. WARRACK, University of Alberta, Edmonton, Alberta, Canada. Uniformly continuous pairs of uniform subspaces. Preliminary report.

J. Wilker [Fund. Math., 71(1971), 127-130] has defined a pair $(A, B)$ of subspaces of a metric space $X$ to be a Lipschitz pair if every function Lipschitz on each of $A$ and $B$ is necessarily Lipschitz on their union. We define uniformly continuous pairs and proximally continuous pairs analogously for subspaces of uniform and proximity spaces respectively. Both uniformly continuous and proximally continuous pairs are characterized, and it is shown that every uniformly continuous pair of subspaces of a uniform space $(X, \mu)$ is a proximally continuous pair with respect to the induced proximity $\delta(\mu)$. (Received April 19, 1972.)

*72T-G112. EARL PERRY, West Georgia College, Carrollton, Georgia 30117. On duods and hereditarily duodic continua.

A continuum will be a bicompact connected set and all spaces will be considered to be metric. A continuum $M$ is defined to be a duod or to be duodic if it is the union of two nondegenerate subcontinua $A$ and $B$ whose intersection is degenerate. The point in $A \cap B$ is called a cut point of $M$, and $A$ and $B$ are said to be a pair of unods of $M$ determined by the cut point which is in $A \cap B$. If each nondegenerate subcontinuum of a nondegenerate continuum $M$ is duodic, then $M$ is said to be hereditarily duodic. It is immediately apparent that any nondegenerate dendrite is hereditarily duodic. In this paper there is exhibited an example of a hereditarily duodic continuum which contains uncountably many hereditarily duodic continua, no one of which is a dendrite, and no two of which are homeomorphic. Following an example of a hereditarily duodic continuum which has a unique cut point is a proof of this Theorem. There exists no hereditarily duodic continuum with the property that each subcontinuum has a unique cut point. (Received April 13, 1972.)


Theorem. There exists an equationally compact semigroup which is not a retract of any compact topological semigroup. This theorem settles Problem 4.8 of the author [Fund. Math. 65(1969), 139-145]. Refer to this article or to page 424 of the author's article [Ann. of Math. Logic 3(1971), 395-438] for definitions and background. The proof is to appear in Semigroup Forum. (Received April 20, 1972.)


For terminology see Levine (Proc. Liverpool Singularities Sympos. II, pp. 90-103) and White (J.
Let $M$ be an $n$-dimensional oriented compact manifold. Let $\sigma : A \to B$ be a $T$-homomorphism of an $(n+1)$-bundle into a $p$-plane trivial bundle over $M$, $n < p$. Let $L$ be a line subbundle of $A$, such that $A = A \oplus L$. Let $A$, $B$, $L$ be the restriction of these bundles to $M \setminus S(\sigma)$. Define $N = \text{coker}(\sigma : A \to B)$. Let $i : N \to B$ be an inclusion; we define $\tilde{N}$ to be $i(N) \oplus L$ and include in it for each k-plane the oriented $(k-1)$-sphere of points at infinity, where $k = p - n$. Let $N \subset \tilde{N}$ be the fibre space over $M \setminus S(\sigma)$ such that at each point $m$, the fibre is the half k-plane spanned by $N_{\infty}$ and the positive half-line of $L_{\infty}$. The boundary $\partial N$ of $N$ is $N' \cup \{q\}$. Let $q \in E$ such that $g(\partial N) \cap \{q\} = \emptyset$.

Define $(-1)^{n-1} I(g, q) = -(1/O_{-1}) \int_{\partial N} N dO_{-1} - (1/O_{-1}) \int_{\partial N} W_N dO_{-1}$. Theorem 1. $I(g, q)$ is well defined.

Theorem 2. For $n$ odd, $(-1)^{n-1} I(g, q) = -(1/O_{-1}) \int_{\partial N} N dO_{-1} - (1/O_{-1}) \int_{M \setminus S(\sigma)} L dV$, and for $n$ even, $(-1)^{n-1} I(g, q) = -(1/O_{-1}) \int_{\partial N} N dO_{-1} + \frac{\chi}{(\tilde{A})}$. If we let $g = h$ we have; Theorem 3. For $n$ odd $(-1)^{n-1} I(h, q) = 0$, and for $n$ even $(-1)^{n-1} I(h, q) = \chi(\tilde{A})$. (Received April 20, 1972.)


Let $M$ be a connected 2-manifold. In [Proc. Amer. Math. Soc. 28(1971), 581-583] it was shown that any open 2-manifold $M$ has at most countably infinite domain rank. The complete solution of the problem concerning the domain rank of connected 2-manifolds is given by the following: Theorem. If $M$ is a connected 2-manifold, then the domain rank of $M$ is countably infinite. Furthermore, $M$ has finite domain rank if and only if $M$ is homeomorphic to a domain of a compact 2-manifold. (Received April 24, 1972.)

Miscellaneous Fields

72T-H2. ROBERT H. COWEN, Queens College, City University of New York, Flushing, New York 11367, HENRY FRISZ, Bronx Community College, City University of New York, Bronx, New York 14068 and ALAN GRENADEIR, 221-22 Manor Road, Queens Village, New York 11427. Some new axiomatizations in group theory. Preliminary report.

We give axiomatizations of group theory and abelian group theory in terms of the binary group operation $f(a, b) = ab^{-1}$. Suppose $G$ is a first order theory with equality with a binary operation symbol $f$, whose nonlogical axioms are: (G1) $f(x, x) = f(y, y)$, (G2) $f(x, f(y, y)) = x$, (G3) $f(f(x, z), f(y, z)) = f(x, y)$. Then $G$ is an axiomatization of group theory ($1$ is defined as the (unique) element $f(x, x)$ and $xy$ as $f(x, f(1, y)))$. For abelian groups, G1, G2, G3 are replaced by: (A1) $f(x, f(y, z)) = f(z, f(y, x))$, (A2) $f(x, f(x, y)) = y$, or, by the single axiom (A), $f(f(x, y), f(f(x, z), y)) = z$. Some other single axioms for abelian groups are (A') $f(f(x, f(y, z))), f(x, y) = z$ and (A'') $f(x, f(y, f(z, f(x, y)))) = z$. (Received March 20, 1972.)
The March Meeting in St. Louis, Missouri
March 27-April 1, 1972

693-A40. DERRICK H. LEHMER, University of California, Berkeley, California 94720. Reciprocally weighted partitions.

Let $S$ be the set of all positive integers or any subset thereof. Let $P$ be any partition of $n$ into parts taken from $S$ and let $w(P)$ be the reciprocal of the product of the parts. We define $W_n(S)$ to be the sum of $w(P)$ taken over all the partitions of $n$. A method of calculating $W_n(S)$ recursively is presented that avoids any of the actual partitions of $n$. Theorems are proved that give asymptotic values of $W_n(S)$ as $n \to \infty$. For example if $S$ is the set of all integers $> 1$ then $\lim W_n(S) = \exp(-\gamma)$ where $\gamma$ is Euler's constant. The sets $S$ considered include every arithmetic progression. (Received April 7, 1972.)

693-A41. ARTHUR L. STONE, Simon Fraser University, Burnaby 2, British Columbia, Canada. Localization, colocalization, (the flying machine) and limit tripleable categories. Preliminary report.

Given a functor $F: C \to A$ with $C$ small and $A$ complete and bi-well powered, the density triple and codensity cotriple are the duals of the familiar density cotriple and codensity triple. The flying machine is the resulting set of four adjoint towers. Theorem. $A$ is limit tripleable for the density triple if $\text{Im}(F)$ is generating ("generating" in the stronger sense). Consequently $A$ is limit tripleable on a left adjoint into $A$ iff the image is generating. The density triple tower stops in one step if $\text{Im}(F)$ is projective--giving a tripleability criterion based on the left (rather than right) adjoint. The density triple tower stops at $\sigma$ if the objects of $\text{Im}(F)$ are $\sigma$-presentable in the sense of Gabriel-Ulmer. Other stopping conditions are given. (Received April 10, 1972.)


A nonlinear equation of Hammerstein type is an equation of the form (1) $u + ANu = v$, where $A$ is a linear mapping from a Banach space $X$ into its dual $X^*$ and $N$ is a nonlinear mapping of $X^*$ into $X$. In recent years there has been an extensive work on these equations under various types of monotonicity hypotheses on the mappings $A$ and $N$ (see F. E. Browder, "Contributions to nonlinear functional analysis," by E. H. Zarantonello, Academic Press, New York, New York, 1971, pp. 425-500, and the bibliography therein). In this paper we establish new existence theorems for equation (1) when $A$ is not necessarily a monotone mapping. (Received April 10, 1972.)
The June Meeting in Seattle, Washington
June 17, 1972

Algebra & Theory of Numbers


In the real integers only 2 and 3 (and -2 and -3) are so related that both are primes and the distance between the two is one only. In the Gaussian integers only 1 + i and 2 + i (their associate and conjugates) are so related. In Z(\sqrt{2}) only the triple -1 + \sqrt{2}, \sqrt{2}i, 1 + \sqrt{2}i (and their conjugates) form a string of three with the desired property. In the Eisenstein integers, Z(1/2 + \sqrt{-3}/2), many examples can be found. Examples are given for the other imaginary quadratic extensions whose integers form a unique factorization domain, i.e., Z(-1/2 + \sqrt{-p}/2) for p = 7, 11, 19, 43, 67 and 163. A connection exists between these related primes and some well-known prime generating functions. (Received April 27, 1972.)

695-A2. STUART A. SELIGSON, 1752 Edgewood Road, Redwood City, California 94062. Dense subrings of Hom\(_R(M, M)\), for R QF, and \(_R M\) a faithful projective-injective. Preliminary report.

We show the following: If R is a QF ring, \(_R M\) a faithful projective-injective, and B = Hom\(_R(M, M)\), then \(\forall\) subrings \(A \subseteq B\), dense in the finite topology of \(M\) on \(B\), \(M_A\) is (1) quasi-injective, and \(\forall N_A \subseteq M, \lceil N \rceil = N\), (2) quasi-projective (\(\forall\) homo, of \(M\) into a quotient module of \(M\) can be factored), and \(\forall N_A \subseteq M, \# N = N\), where \#\(N\) = \(\{r \in R : rM \subseteq N, \forall T_R \subseteq R, \#T = TM\},\ (3) artinian and noetherian, and (4) \(\forall m \in M, m\# = 0 \Rightarrow m = 0\). Conversely, if \(M_A\) satisfies (1)-(4), \(R = Hom_A(M, M)\) is QF, \(_R M\) a faithful projective-injective, and if \(M_A\) is faithful \(A\) is iso, to a dense subring of \(B, Hom_R(M, M)\). These results are developed using a generalization of Jacobson's "closure" theorem ("Structure of rings", p. 31) giving: if (1) holds for \(M_A\), faithful, and \(B\) is the second centralizer of \(A\), then \(\forall T_A \subseteq B, Cl(T) = \lceil T \rceil^\perp\). We also prove a dual theorem giving: if (2) holds, \(B\) is as above, and \(*M = Hom_B(M, M), \forall T_A \subseteq B, \lceil \text{In}(T) \rceil = T \cap *MM = \lceil T \rceil M = \lceil T \rceil\), where \#\(T = \{m \in M : mMM \subseteq T\}\). Then, \(A\) is dense in \(B = (4)\), when (1) holds, and \#\(A = 0 = A \cap *MM = 0, \) when (2) holds. Thus, results by Jacobson for primitive rings and completely reducible rings of endomorphisms generalize. These considerations are developed in somewhat greater generality, with emphasis on the duality involved, and various further results are derived. (Received May 4, 1972.)

*695-A3. JEANNE MEISEN, University of British Columbia, Vancouver 8, British Columbia, Canada. Relations in categories.

Relations are studied in categories which have finite products, pullbacks, and \((E, M)\) factorization systems. Relations are the morphisms of a Benabou's bicategory, provided that pullbacks preserve \(E\)-class morphisms. More generally, when there is assigned a full reflexive subcategory \(Hom'(A, B)\) to each hom category, \(Hom(A, B)\), of a given bicategory, a sufficient condition is obtained for \(Hom'(A, B)\) to be the hom category in another bicategory. This is also applied to obtain a bicategory whose morphisms are the pullback spans. Some properties of a relation \(R\) and its converse \(\overline{R}\) are investigated. All pullback relations are

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von Neumann regular (i.e., $R = R \cdot R \cdot R$). The converse of this statement is studied. Applications are made to Barr's exact categories. Furthermore, some results known in algebraic categories are extended to exact categories. (Received May 4, 1972.)

695-A4. DONALD G. BURNELL, Washington State University, Pullman, Washington 99163 and TAKAYUKI TAMURA, University of California, Davis, California 95616. The study of left cancellative semigroups.

(1) A left cancellative semigroup with idempotent is either a right group or the disjoint union of a right group and a maximal proper ideal without idempotent. (2) The following question is raised. If $T$ is a left cancellative semigroup without idempotent, what conditions must exist on a group $G$ such that there exists a left cancellative right reductive semigroup $S = T \cup G$? This question is answered in the cases where $T$ is an $\mathbb{R}$-semigroup, an $\mathbb{R}^*$-semigroup and a finitely generated free semigroup. (3) A left cancellative semigroup can be faithfully represented by the product function in terms of a class of one-to-one functions defined over disjoint sets indexed by a left cancellative right reductive semigroup with identity. (4) Let $S$ be a left cancellative semigroup and $I$ be a right ideal in $S$. If $I$ is commutative then so is $S$. (Received May 4, 1972.)

695-A5. TAKAYUKI TAMURA, University of California, Davis, California 95616. Reflexive subsemigroups and identity-reflexive congruences on semigroups. Preliminary report.

Let $H$ be a subsemigroup of a semigroup $S$. $H$ is called reflexive if $xy \in H$ implies $yx \in H$; unitary if $ax \in H$ and $a \in H$ implies $x \in H$, and if $xa \in H$ and $a \in H$ implies $x \in H$. If $T$ is a subsemigroup of $S$, $\rho_T$ denotes the congruence generated by $\{(x, xa) : x \in S, a \in T\}$. Let $D$ be a semigroup with identity $e$. $D$ is called identity-reflexive if $\{e\}$ is reflexive. Theorem 1. Let $H$ be a subsemigroup of $S$ and let $\rho_H$ be an identity-reflexive congruence on $S$. Then the following are equivalent: (1) $H$ is reflexive and unitary in $S$. (2) $H$ is peable in $S$, i.e., $x \in S$, $ab \in H$, $axb \in H$ implies $x \in H$. (3) $H = \text{Ker } \rho_H$. $H$ is called cofinal in $S$ if $ax \in H$ and $a \in H$ implies $x \in H$, and if $xa \in H$ and $a \in H$ implies $x \in H$. If $H$ is cofinal reflexive unitary in $S$, $\rho_H$ is a group-congruence on $S$, and if $H$ is reflexive, unitary but not cofinal, then $\rho_H$ is a group-with-zero congruence on $S$. Theorem 2. All reflexive subsemigroups of $S$ are cofinal in $S$ if $S$ is semilattice indecomposable. (Received May 4, 1972.)

Analysis


A pola (denoted by $A$) is a real linear associative algebra which is partially ordered so that it is a directed partially ordered linear space and $0 \leq xy$ whenever $x, y \in A$, $0 \leq x, 0 \leq y$. We also assume that $A$ has a multiplicative identity $1 \geq 0$. A Dedekind $\sigma$-complete pola (dsc-pola) $A$ is one having the property: If $x_n \in A$, $0 \leq \ldots \leq x_2 \leq x_1$, then $\text{inf} \{x_n\}$ exists. We now assume that $A$ is a dsc-pola and that there exists $u \in A$ such that $u^2 = u$, $ux = xu$ for all $x \in A$ and $1 \leq 2u$. Next define $e = 1 - u$, $N = \{x : ux = x\}$ and $B = \{y : ye = y\}$. The real linear algebra $B$ is called a generalized Banach algebra (GBA) and $N$ is the real linear
algebra of estimating (i.e., "norming") elements. For each \( y \in \mathcal{B} \) there exists \( x \in \mathcal{N} \) such that \( -x \preceq y \preceq x \).

The abstract "norms" may be real-valued functions, matrices, etc. Norms of functional type are characterized by the assumption: if \( x \geq u \), then there exists \( z \geq 0 \) such that \( xz = u \). (Received April 17, 1972.)


The value distribution of several classes of non-axisymmetric potentials in \( \mathbb{R}^3 \) is studied by means of Bergman's integral operator method. For example, let \( H \) be a potential generated by Bergman's operator \( B_3 (f, \zeta) \) [see Stefan Bergman, "Integral operators in the theory of linear partial differential equations," 2nd rev. printing, Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 23, Springer-Verlag, New York, 1969, p. 43] where \( f = f(\tau, \zeta) \) is an analytic function of \( \tau = x + \frac{3}{2}(iy + z) \zeta + \frac{3}{2}(iy - z) \zeta^{-1} \) for \( \tau \) over a region in the complex plane and a continuous function of \( \zeta \) for \( \zeta \) over \( \mathcal{L} \). The contour \( \mathcal{L} \) is chosen from a class of contours satisfying a specified smoothness condition. For a given complex constant \( \gamma \), a region \( \Omega \) in \( \mathbb{R}^3 \) is constructed in which \( H(x, y, z) \neq \gamma n(\zeta, 0) \) where \( n(\zeta, 0) \) is the winding number of \( \zeta \) with respect to the origin. Furthermore, subregions \( \Omega_\delta \) of \( \Omega \) and open sectors \( S_\delta \), functions of \( \Omega_\delta \), which are contained in the complex plane are determined. In these \( H(x, y, z) \neq (\gamma + k) n(\zeta, 0) \) for all \( k \) in \( S_\delta \). (Received April 26, 1972.)


The notion of a flat Banach space was introduced by R. Harrell and L. Karlovitz ["Girths and flat Banach spaces," Bull. Amer. Math. Soc. 76(1970), 1288-1291]. If \( X \) is a flat Banach space, then \( g: [0, 2] \rightarrow X \) such that \( g([0, 2]) \subset S = \{ x: \| x \| = 1 \} \), \( g(0) = -g(2) \) and \( g \) is Lipschitz continuous with constant 1, is called a girth curve of \( X \). Some results concerning flatness and smoothness are: (1) If \( X \) is flat, then (a) \( X \) cannot be renormed with an equivalent Frechet differentiable norm; (b) the norm on \( X \) is not Frechet differentiable at any point on \( S \) through which a girth curve passes. (2) If \( X^{**} \) is smooth, then \( X \) is not flat. (3) If \( X \) is flat and smooth, then \( X^* \) is not rotund. (Received May 3, 1972.)


We suppose that \( A(u) \) is of bounded variation in every interval \( 1 \leq t \leq U \) and that \( A(1) = 0 \), and we consider the Cesàro summability, for fractional orders, of the Stieltjes integral \( \int_1^U \phi(u) A(u) \) and establish an equivalence theorem, Theorem (the case \( k \) fractional). Suppose that \( \phi(u) \) is positive and \( \phi[k](t) \) is absolutely continuous for \( t \geq 1 \), and (i) \( \Phi_k(x) = o(x^k) \) as \( x \rightarrow \infty \), where \( \Phi_k(x) = (T(k) -1)^{-1} \int_1^x (x - u)^{-k} \phi(u) \) \( du \); (ii) \( \int_1^x \int_1^w (1-w)^{-k} \phi[k](3/2w) \) \( W_k(x, w) \) \( dw \leq K \int_1^x W_k(x, w) \) \( dw \); (iii) \( \int_1^x w^{-k} (3/2w) \phi[k] W_k(x, w) \) \( dw = o(x^k) \), where \( W_k(x, w) = \theta^k(t) \Phi_k(x) \) \( (x - u)^{-k} \phi(u) \) \( du \). Then necessary and sufficient conditions for (i) \( \int_1^U \phi(u) A(u) \) to be summable \( (C, k) \) to \( B \) are that (II) \( -\int_1^U \phi[t] A(u) \) \( du \) should be summable \( (C, k) \) to \( B \) and (III) \( A(x)\phi(x) = o(1) \) \( (C, k) \). (Received February 23, 1972.)
Applied Mathematics


Let \( L: \dot{x} = Fx + Gu \) be a controllable linear system with \( x \) an \( n \)-vector, \( u \) an \( m \)-vector and \( F \) and \( G \) compatible matrices. \( J(u) = J;:\int_0^\infty x'Qx + u'Ru \) is the performance criteria with \( Q \) and \( R \) positive definite matrices. Assume that \( L \) is in control canonical form [D. G. Luenberger, IEEE Trans. Automatic Control AC-12(1967), 290]. The optimal state feedback is given by \( u(t) = R^{-1}GPx(t) \) where \( F'P + PF + PGR^{-1}G'P = Q \). Let \( K = F + GR^{-1}G'P \). Let \( F_1, \ldots, F_m \) be the \( m \)-columns of \( F \) which contain at most \( m \) nonzero elements and assume that the canonical zeros are repressed so that \( F_i \) is an \( m \)-vector. Similarly for \( K \). \( (x,y)_R = x'y. \) Theorem 1. \( \langle K_i, K_j \rangle_R = q_{ij} + \langle F_i, F_j \rangle_R. \) Lemma 1. Let \( \det(x_1, \ldots, x_m) \) be considered as a multilinear mapping from \( \mathbb{R}^m \times \cdots \times \mathbb{R}^m \to \mathbb{R} \). Then supremum \( \det(x_1, \ldots, x_m) \), subject to the \( m \) constraints \( x_i'Ri = a_i \), is attained by any set \( \{x_1, \ldots, x_m\} \) where \( x_i'Ri = 0 \) and \( x_i'Ri = a_i^2 \) for some ordering of the indices. By expanding \( \det K \) about the rows in which a canonical 1 appears, it follows that \( \det K = \det(K_1, \ldots, K_m). \) Theorem 2. The absolute value of the product of the eigenvalues of \( K \) is bounded by \( \prod_{i=1}^m (q_{ii} + \langle F_i, F_i \rangle_R) / (\det R)^{1/2} \) Furthermore if the \( q_{ii} \) are sufficiently large the bound is the best possible. The proof follows from Lemma 1 with constraints as in Theorem 1. (Received April 27, 1972.)

Statistics and Probability


E. Wong (SIAM J. Appl. Math. 14(1966), 1246-1254) seems to have made the first real progress with the S. O. Rice (Bell System Tech. J. 23(1944), 282-332) problem of the distribution of the time intervals between the zeros of a continuous time Gaussian process. To achieve closed forms of density and distribution Wong imposed severe restrictions on the autocorrelation of the process. A compound pseudo-filter technique, devised by K. J. Hammerle of the Boeing Company, serves to put this restriction in realistic form and allows study of the effect of small variations. (Received April 28, 1972.)

Topology

*695-G1. LAWRENCE R. WEILL, Hughes Aircraft Company, Culver City, California 90230. A new characterization of tame 2-spheres in \( \mathbb{E}^3 \).

It is shown in Theorem 1 that a 2-sphere \( S \) in \( \mathbb{E}^3 \) is tame from \( A = \text{Int} S \) if and only if for each compact set \( F \subset A \) there exists a 2-sphere \( S' \) tamely embedded in \( \mathbb{E}^3 \) with complementary domains \( A' = \text{Int} S', B' = \text{Ext} S' \) such that \( F \subset A' \subset A \) and for each \( x \in S' \) there exists a path \( \overline{xy} \) in \( \overline{B} \) having initial point \( x \in S' \), terminal point \( y \in S \), and whose diameter is less than \( \rho(F, S) \). Furthermore, the theorem also holds when \( A \) is replaced by \( B, A' \) by \( B' \), \( B' \) by \( A' \), and Int by Ext. The characterization given by Theorem 1 is compared with Bing's original characterization which states that a 2-sphere \( S \) in \( \mathbb{E}^3 \) is tame in the
complementary domain \( C \) if for each \( \varepsilon > 0 \) there is an \( \varepsilon \)-homeomorphism of \( S \) into \( C \). Two applications of Theorem 1 are presented. Theorem 2 states that a 2-sphere \( S \) is tame from the complementary domain \( C \) if for arbitrarily small \( \varepsilon > 0 \), \( S \) has a metric \( \varepsilon \)-envelope in \( C \) which is a tame 2-sphere. Theorem 3 answers affirmatively the following question: Is a 2-sphere \( S \subset E^3 \) tame in \( E^3 \) if there exists an \( \varepsilon > 0 \) such that if \( a, b \in S \) satisfy \( \rho(a, b) < \varepsilon \), then there exists a path in \( S \) of diameter \( \leq \rho(a, b) \) which connects \( a \) and \( b \)?

(Received December 13, 1971.)


Let \( M \) be a metric space and suppose \( K \) is a collection of mutually disjoint subsets of \( M \). If \( g \in K \), then \( K \) is said to be continuous at \( g \) if for each \( \varepsilon > 0 \), there exists an open subset \( V \) of \( M \) containing \( g \) such that if \( g' \in K \) and \( g' \not\in V \), then \( g \not\in S_\varepsilon(g') \). \textbf{Theorem.} Suppose that \( G_1, G_2, \ldots \) is a countable collection of cellular decompositions of an \( n \)-manifold with boundary \( M (n \neq 4) \) such that (1) if \( g \in H_{G_1} \) and \( g \cap H_{G_j} \neq \emptyset \), then \( g \in H_{G_j} \) for each \( k = 1, 2, \ldots, \), if \( g \in H_{G_k} \), then \( [H_{G_1} : i \neq k] \cup \{g\} \) is continuous at \( g \) (3) for \( i = 1, 2, \ldots, \), \( M/G_i \) is homeomorphic to \( M \); and (4) the decomposition \( G = \{h : h \in \bigcup H_{G_1} \text{ or } h \text{ is a point of } M - \bigcup H_{G_1}\} \) is upper semicontinuous. Then \( M/G \) is homeomorphic to \( M \).

(Received January 31, 1972.)


A new concept of an infinitesimal neighborhood of a point \( p \) in a topological space \( (X, T) \) is defined as \( \mathcal{V}(p) = \bigcap_{p \in \mathcal{V}} \mathcal{V}_p \), where \( \mathcal{V} \) is the closure of the set \( V \), and \( *V \) is the enlargement as defined by A. Robinson ('"Nonstandard analysis," North-Holland, Amsterdam, 1966). \textbf{Theorem 1.} A topological space \( (X, T) \) is regular iff \( \mu(p) = \nu(p) \) for every \( p \in X \) where \( \mu(p) \) is the monad of \( p \) (ibid.). \textbf{Definition.} A standard mapping \( f \) is continuous\( i \) \((i = 1, 2, 3, 4)\) at a standard point \( p \) iff (for \( i = 1 \)) \( f(\mu(p)) \subset \mu(f(p)) \); (for \( i = 2 \)) \( f(\nu(p)) \subset \nu(f(p)) \); (for \( i = 3 \)) \( f(\mathcal{V}(p)) \subset \mathcal{V}(f(p)) \); (for \( i = 4 \)) \( f(\mathcal{V}(p)) \subset \mathcal{V}(f(p)) \). "\( i \rightarrow j \)" is written for "continuity\( i \) at a standard point \( p \) implies continuity\( j \) at \( p' \). \textbf{Theorem 2.} \( 3 \rightarrow 1 \) for \( i = 1, 2, 4 \); \( 1 \rightarrow 2 \) for \( i = 1, 3, 4 \) and these are the only positive implications among these continuities. \textbf{Theorem 3.} A standard function \( f \) is continuous\( i \) at the standard point \( p \) iff for every open set \( V \) containing \( f(p) \) there exists an open set \( U \) containing \( p \) such that \( U = f^{-1}(V) \) for \( i = 1, U \subset f^{-1}(V) \) for \( i = 2, \bar{U} \subset f^{-1}(V) \) for \( i = 3 \), and \( \bar{U} \subset f^{-1}(V) \) for \( i = 4 \). (Received March 22, 1972.)


Assume all spaces have the homotopy type of a CW-complex; write \( \dim Y \leq r \) if the homotopy type of \( Y \) contains an \( r \)-dimensional CW-complex and \( \text{conn } Y = s \) if \( Y \) is \( s \)-connected. Let \( \text{cat } \) denote the Lusternik-Schnirelmann category and let \( \mathbb{N} \) denote any of the related numerical homotopy invariants: weak category, conilpotency, cup-length. If \( f : X \to Y \) has a right homotopy inverse, a classical theorem asserts that \( \text{cat } Y \leq \text{cat } X \).
cat X and it is well known that \( N(Y) \leq N(X) \). This paper treats the more general case in which \( f \) is only required to be epic in the homotopy category, i.e. \( gf \cong hf \) implies \( g \cong h \). Theorem 1. If \( f \) is epic, \( N(Y) \leq N(X) \). Theorem 2. If \( f \) is epic, \( \text{cat} X \geq 1 \) and \( \text{conn} Y \geq 1 \), then \( \dim Y \leq 2(k+1) (\text{conn} Y+1)-3 \) implies \( \text{cat} Y \leq \text{Max}(k, \text{cat} X) \). Corollary. If \( f \) is epic, \( X \) is a co-H-space and \( \dim Y \leq 4 (\text{conn} Y)+1 \), \( Y \) is a co-H-space. Theorem 1 is a special case of theorems on right homotopy structures in the sense of Peterson. Theorem 2 follows from a recent result of Y. Nomura (Osaka J. Math. 7(1970), 353-373) on epimorphisms applied to T. Ganea's structure for \( L,-S \) category. (Received May 1, 1972.)


Open and proper (perfect) maps extend in a natural way to the realm of convergence spaces. Let \( f: (S, q) \to (T, p) \) be a map (continuous and onto) between convergence spaces. If \( f \) is open, then \( f \) induces an open translation of the decomposition series of \( (S, q) \) onto that of \( (T, p) \). If \( (S, q) \) is topological (pretopological), then so is \( (T, p) \). In the case when \( f \) is proper, the induced translation of the decomposition series need not be proper, but each component map in this translation is closure-preserving and has the property that inverse images of singletons are compact, and the induced map between the topological modifications is indeed proper. The image of a topological space under a proper map is almost topological (coincides with a topology relative to ultrafilter convergence), but, surprisingly, the analogous result does not hold for pretopological spaces. If a pair of spaces is topologically coherent, then their images under either proper or open maps are also topologically coherent; for open maps, this result extends to pretopological and total coherence as well. (Received May 3, 1972.)


A. K. Steiner ["The lattice of topologies; structure and complementation," Trans. Amer. Math. Soc. 122(1966), 379-397] proved that the lattice of topologies on a fixed set \( X \), denoted \( \Sigma \) or \( \Sigma(X) \), is complemented. In fact, she showed that each \( t \in \Sigma \) has a complement in \( \Pi = \Pi(X) \), the sublattice of principal topologies. (A topology \( t \in \Sigma \) is principal iff each point \( x \in X \) has a smallest \( t \)-neighborhood: \( B_t(x) \).) Her proof was quite complicated; van Rooij [Canad. J. Math. 20(1968), 805-807] gave a simpler proof which in addition to Zorn’s lemma, however, required two applications of transfinite induction. The purpose of this note is to give a short proof of Steiner's theorem via a standard Zornification utilizing the simple trick of suitably adjoining a new point \( p \) to \( X \) and subsequently discarding it. (Received May 3, 1972.)

*695-G7. ALLAN L. EDELSON, University of California, Davis, California 95616. Fixed point sets of conjugations.

It is known that the fixed point set of a conjugation on an almost complex manifold \( \mathbb{M}^{2n} \) is an \( n \)-dimensional submanifold (Conner and Floyd, "Differentiable periodic maps", Ergebnisse Der Mathematik und
Ihrer Grenzgebiete, Band 33). We study the question of realizing an $n$-manifold as such a fixed point set. An examination of the generators shows that every oriented cobordism class contains the fixed point set of a conjugation. It is shown that under certain conditions a surgery on the fixed point set $F^n$ extends to an equivariant surgery on $M^{2n}$, preserving the almost complex structure. Hence any manifold $\chi$-equivalent to $F^n$ will also be the fixed point set of a conjugation. Applications to 3-manifolds are given. (Received May 4, 1972.)


This work shows that the Bohr compactification of the group of integers is a bundle with the circle group as its base space, a compact subgroup of the bundle as its fiber, and open projection. (Received May 4, 1972.)

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MATHEMATICIAN, TEACHING AND RESEARCH. Ph.D. 1958. Age 37. Several published articles and a textbook. Fourteen years (13 years in U.S.) of experience in teaching various courses at undergraduate and graduate level and in curriculum development. References and vita available upon request. 1382 DeWitt Drive, Akron, Ohio 44313.


MATHEMATICIAN. Ph.D. 1966, University of Illinois. Topology. Seven research publications, author of graduate level textbook, editor, reviewer; administrative experience; university graduate level teaching experience, small liberal arts college teaching experience. Employed. Age 32. Desire responsible constructive position. Salary and location secondary. SW9


ERRATA

Volume 18

LAWRENCE J. KRAJEWSKI. The product of a totally normal countably paracompact space and [0, 1] need not be totally normal. Abstract 71T-G197, Page 983.

Instead of "Bing's Example G", it should read "Bing's Example F is an example...".
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