Notices

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Notices
OF THE
AMERICAN MATHEMATICAL SOCIETY

Edited by Everett Pitcher and Gordon L. Walker

CONTENTS

The Seventy-Seventh Summer Meeting
Dartmouth College
Hanover, New Hampshire
August 29-September 1, 1972

The seventy-seventh summer meeting of the American Mathematical Society will be held at Dartmouth College, Hanover, New Hampshire, from Tuesday, August 29, through Friday, September 1, 1972. All sessions of the meeting will take place on the campus of the college. The times listed for events of the meeting are EASTERN DAYLIGHT SAVING TIME throughout.

There will be two sets of Colloquium Lectures. Professor Stephen Smale of the University of California, Berkeley, will present four lectures entitled "Applications of global analysis to biology, economics, electrical circuits, and celestial mechanics." These addresses will be given on Tuesday, August 29, at 1:30 p.m. and on Wednesday, Thursday, and Friday at 9:00 a.m. The other Colloquium Lecturer will be Professor John T. Tate of Harvard University. His topic will be "The arithmetic of elliptic curves." Professor Tate's four lectures will be given on Tuesday, August 29, at 2:45 p.m. and on Wednesday, Thursday, and Friday at 10:15 a.m. The first address of each series will be given in Spaulding Auditorium, Hopkins Center; the remaining Colloquium Lectures will be presented in the Center Theater which is also located in Hopkins Center. The AMS Committee on Employment and Educational Policy will hold a panel discussion on Tuesday, August 29, at 4:00 p.m. in Spaulding Auditorium.

By invitation of the Committee to Select Hour Speakers for Annual and Summer Meetings, there will be nine invited hour addresses: Professor George E. Andrews of the Pennsylvania State University, "A general theory of identities of the Rogers-Ramanujan type," 1:30 p.m. on Friday, September 1; Professor R. C. Bose of Colorado State University, "Representation of non-Desarguesian projective planes in projective hyperspace," 2:45 p.m. on Friday, September 1; Professor Glen E. Bredon of Rutgers University, "Locally smooth actions on manifolds," 1:30 p.m. on Thursday, August 31; Professor Chuan C. Hsiung of Lehigh University, "Submanifolds of Riemannian manifolds," 1:30 p.m. on Friday, September 1; Professor Joachim Lambek of McGill University, "Noncommutative localization," 2:45 p.m. on Friday, September 1; Professor Seymour Sherman of Indiana University, "Monotonicity and magnetism," 2:45 p.m. on Thursday, August 31; Professor Michael Shub of the University of California, Santa Cruz, "Dynamical systems, filtrations, and entropy," 4:00 p.m. on Friday, September 1; Professor Charles C. Sims of Rutgers University, "The construction of large permutation groups," 1:30 p.m. on Thursday, August 31; and Professor Herbert S. Wilf of the University of Pennsylvania, "Bounds for the chromatic number," 2:45 p.m. on Thursday, August 31.

Two special sessions of selected twenty-minute papers will be scheduled. Professor Joseph B. Keller of the Courant Institute of Mathematical Sciences, New York University, is arranging a session on Asymptotic and Perturbation Methods in Fluid Mechanics and Wave Propagation from 1:30 p.m. to 5:30 p.m. on Friday, September 1, in the Hartman Rehearsal Hall, Hopkins Center. The speakers will be Mark Ablowitz, Andrew J. Callegari, W. Stephen Childress, Pao-Liu Chow, Frank Hoppensteadt, Martin D. Kruskal, Reese T. Prosser, and M. C. Shen.

Professor Gerald E. Sacks of the Massachusetts Institute of Technology is organizing a session on Recursion Theory from 1:30 p.m. to 3:30 p.m. on Thursday, August 31, and from 10:00 a.m. to noon on Friday, September 1; both sessions will be held in the Faulkner Recital Hall, Hopkins Center. The speakers will be Harvey Friedman, Eugene M. Kleinberg, Manuel Lerman, Anil Nerode, Richard Platek, Robert W. Robinson, Hartley Rogers, Jr., and Robert I. Soare.

There will be regular sessions for contributed ten-minute papers during the morning on Wednesday, Thursday, and Friday.

This meeting will be held in conjunction with meetings of the Institute of Mathematical Statistics, the Mathematical Association of America, and Pi Mu Epsilon. The Institute of Mathematical Statistics will meet from Monday, August 28, through Thursday, August 31. Professor Peter Huber, Eidgenössische Technische Hochschule, Zürich, will present the Wald Lectures on the subject of Robustness. Professor A. Dvoretzky, Hebrew University, Jerusalem, will speak at a memorial session for Paul Lévy. The Mathematical Association of America will meet from Monday through Wednesday. The Earle Raymond Hedrick Lectures, sponsored by the Association, will be given by Professor Peter Lax of the Courant Institute of Mathematical Sciences, New York University; the title of the lectures will be "Scattering theory." Pi Mu Epsilon will meet concurrently with the Association and the Society. Dr. John G. Kemeny will address the fraternity on Tuesday evening, August 29, at 8:00 p.m.; the title of his lecture will be "Mathematical models and the computer."

COUNCIL AND BUSINESS MEETING

The Council of the Society will meet at 5:15 p.m. on Tuesday, August 29, in the Drake Room of Hopkins Center. The Business Meeting of the

Society will be held in the Spaulding Auditorium at 4:00 p.m. on Thursday, August 31. The Steele Prizes will be awarded prior to the Business Meeting.

An amendment to the bylaws, requested by the Trustees, has been approved by the Council. It provides for an associate treasurer, who is an ex officio member of the Council with one vote and an ex officio member of the Board of Trustees. The amendment will be offered to the Business Meeting for approval in accord with Article XIII of the bylaws.

REGISTRATION

The Registration Desk will be located at the "Top of the Hop," on the second floor of the Hopkins Center. It will be open on Sunday from 2:00 p.m. to 8:00 p.m.; on Monday from 8:00 a.m. to 5:00 p.m.; on Tuesday, Wednesday, and Thursday from 8:30 a.m. to 4:30 p.m.; and on Friday from 8:30 a.m. to 1:00 p.m.

The registration fees for the meeting are as follows:

Member	$ 7
Student or unemployed member	1
Nonmember	14

There will be no extra charge for members of the families of registered participants.

The unemployed status refers to any member currently unemployed and actively seeking employment. It is not intended to include members who have voluntarily resigned or retired from their latest position.

Students are considered to be only those currently working toward a degree who do not receive an annual compensation totalling more than $7,000 from employment, fellowships, and scholarships.

EMPLOYMENT REGISTER

The Mathematical Sciences Employment Register will be maintained from 9:00 a.m. to 4:00 p.m. on Tuesday, August 29, and from 9:00 a.m. to 5:40 p.m. on Wednesday and Thursday, August 30–31, in the Alumni Hall of the Hopkins Center. Alumni Hall is located on the second floor of the Center adjacent to the registration area.

EXHIBITS

Book exhibits and exhibits of educational media will be displayed in the Art Studios, located on the first floor of the Hopkins Center across from the Snack Bar, on Monday, August 28, from 1:00 p.m. to 5:00 p.m.; on Tuesday and Wednesday, August 29 and 30, from 9:00 a.m. to 5:00 p.m.; and on Thursday, August 31, from 9:00 a.m. to 1:00 p.m. All participants are encouraged to visit the exhibits sometime during the meeting.

BOOK SALE

Books published by the Society will be sold for cash prices below the usual prices when the same books are sold by mail.

RESIDENCE HALL HOUSING

College facilities have been set aside for the exclusive use of the joint Mathematics Meetings participants. All dormitories are within a five-minute walk of the dining hall, auditoria, and lecture halls to be used during the meeting. A variety of College dormitory rooms including single rooms, two and three room suites, suites with private half baths, and rooms and suites with full baths, are available. Most dormitories have coin-operated washers and dryers as well as ironing facilities.

College rooms can be occupied from 1:00 p.m., Saturday, August 26, to 1:00 p.m., Saturday, September 2. Dormitory clerks will be available from 8:00 a.m. until 9:00 p.m. Room registration for all College rooms will take place in Hopkins Center next to the Registration Desk and will be open from 1:00 p.m. to 9:00 p.m. on Saturday, August 26, and continuously thereafter from 8:30 a.m. on Sunday, August 27, to 10:00 a.m. Friday, September 1.

The daily rate per person is as follows:

	Registrant	Spouse	Children
Suites or singles <u>without</u> bath	$6	$3	$1.50
Suites or singles <u>with</u> full bath	8	4	2

The rate for children given above assumes occupancy in suite with parents; the first child in a separate suite will be subject to the full adult rate. Since the College cannot provide cribs and there is no rental agency in town, parents are advised to bring their own if needed. There is no charge for children in cribs sharing a room with their parents.

Guests must register in advance to be assured of residence hall housing. Please use the form provided on page A-562 of the June issue of these *Notices*. Residence hall reservation requests will be acknowledged by the Conference Center. It is probable that rooms may be available for those not registering in advance, but this cannot be guaranteed.

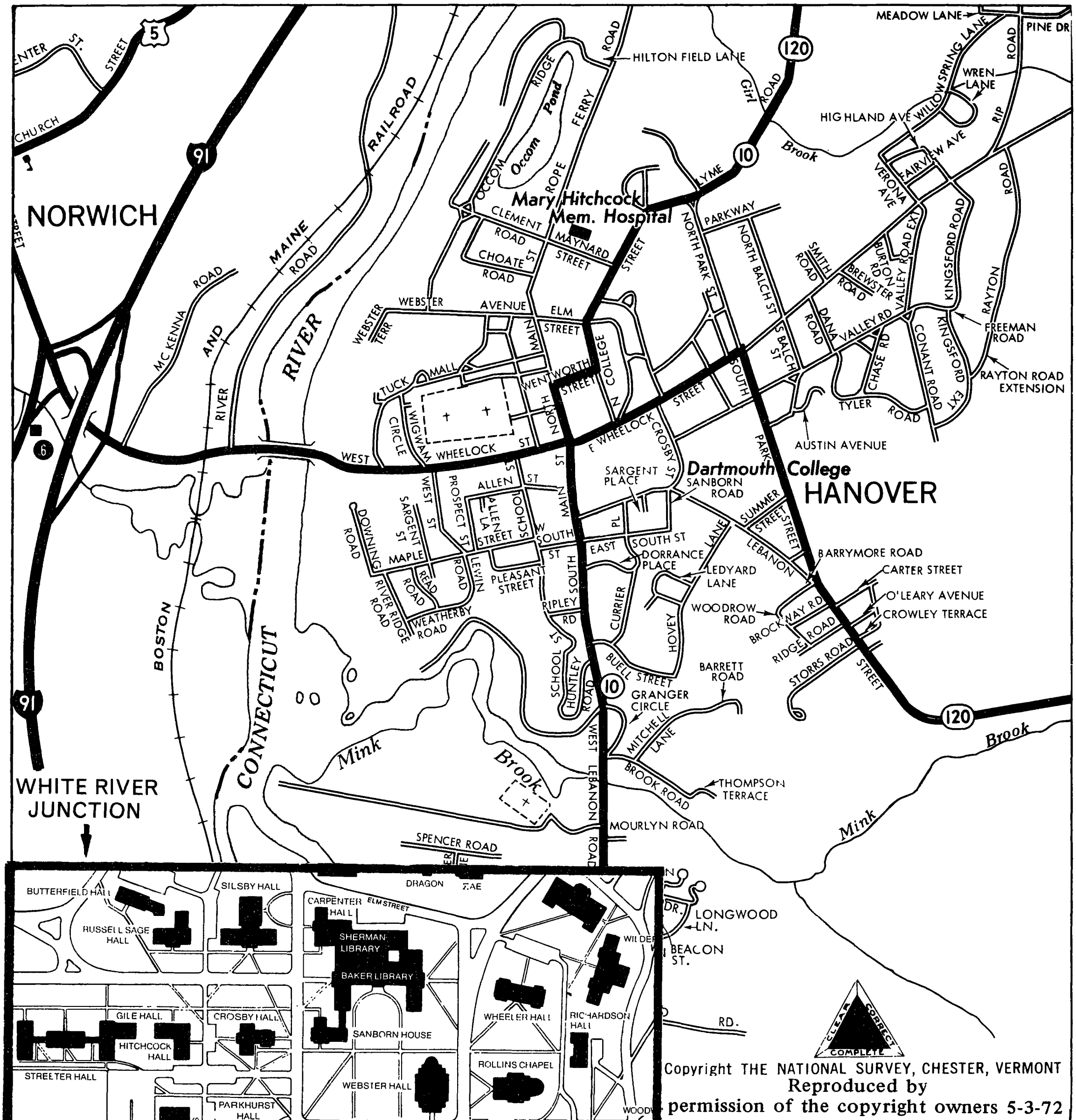
FOOD SERVICES

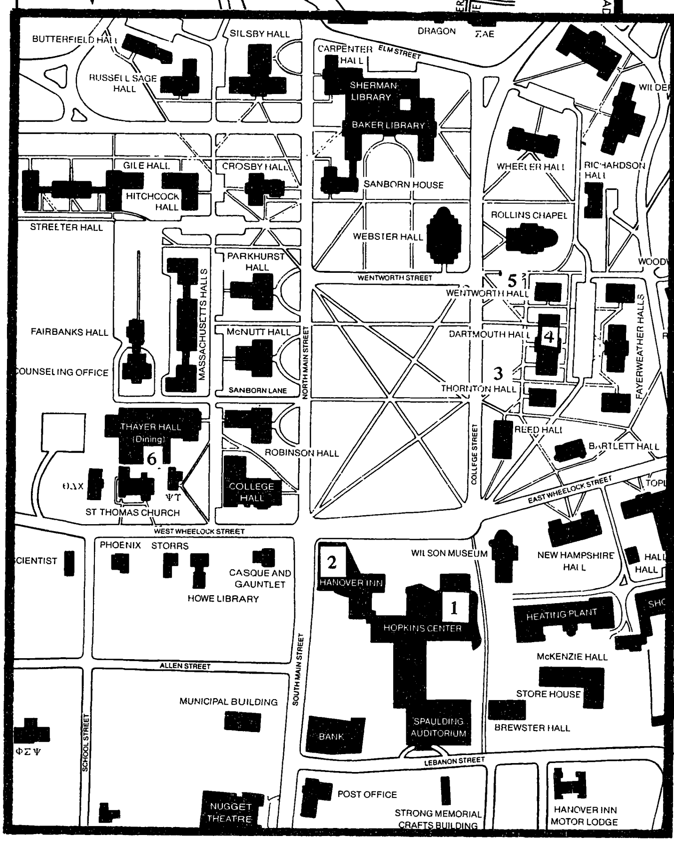
Thayer Hall, the College dining hall, will be open for breakfast on Monday, August 28, and will serve all meals through luncheon on Friday, September 1, except for dinner Wednesday, August 30. Hours of service and prices for individual meals are:

Breakfast	7:15 a.m -8:45 a.m.	$1.25
Luncheon	11:30 a.m. -1:00 p.m.	1.75
Dinner	5:00 p.m. -6:30 p.m.	3.00

On Sunday evening, August 27, from 5:30 p.m. to 8:30 p.m. the Hanover Inn will provide a special Conference Buffet for $2.75 per person.

A special contract meal ticket may be purchased at registration for $20 (children under six for $10). It includes all thirteen meals from breakfast on Monday, August 28, through lun-





1. Hopkins Center
2. Hanover Inn
3. Thornton Hall
4. Dartmouth Hall
5. Wentworth Hall
6. Thayer Dining Hall

DARTMOUTH COLLEGE

HANOVER · NEW HAMPSHIRE

cheon on Friday, September 1, with the exception of the clambake on Wednesday, August 30.

Available also are a number of local restaurants and nearby inns. There is a Snack Bar at the Hopkins Center; the hours of operation will be posted in the registration area.

HOTELS AND MOTELS

Area motels are listed below with the number of rooms reserved, telephone numbers, and their distance from the Dartmouth campus. Participants should make their own reservations. The College Conference Bureau cannot assist with motel or hotel reservations after August 1. All prices are subject to change without notice. The motels and hotels are listed below with the following coded information: FP - Free Parking; SP - Swimming Pool; AC - Air Conditioned; TV - Television; CL - Cocktail Lounge; RT - Restaurant.

HANOVER INN (603) 643-4300
Hanover, N. H. 03755 - 100 rooms
Singles $14-$24
Doubles 17- 30
Code: FP-TV-partial AC-CL-RT
Next to Conference Center

HANOVER INN MOTOR LODGE (603) 643-4400
Hanover, N. H. 03755 - 30 rooms
Singles $13-$15
Doubles 18- 22
Code: FP-TV-AC
Next to Conference Center

NORWICH INN (802) 649-1143
Norwich, Vt. 05055 - 15 rooms
Doubles $16-$20
Code: FP-TV-CL-RT
1 mile from campus

CHIEFTAIN MOTEL (603) 643-2550
Hanover, N. H. 03755 - 15 rooms
Singles $14
Doubles 18
(all double rooms have two double beds)
Code: FP-TV-Continental Breakfast
2 miles from campus

SUNSET MOTEL (603) 298-2721
West Lebanon, N. H. 03784 - 13 rooms
Singles $15
Doubles 19
Code: FP-TV
3 miles from campus

HOWARD JOHNSON'S MOTOR INN (802) 295-3015
White River Jct., Vt. 05001 - 75 rooms
Singles $15
Doubles 21
Code: FP-SP-AC-TV-CL-RT-Sauna
5 miles from campus

HOLIDAY INN (802) 295-7537
White River Jct., Vt. 05001 - 50 rooms
Singles $15
Doubles 22
Code: FP-AC-TV-CL-RT-SP-Sauna
5 miles from campus

COACH AND FOUR (802) 295-2210
White River Jct., Vt. 05001 - 20 rooms
Singles $16-$17
Doubles 18- 20
Code: FP-TV-AC
5 miles from campus

MAPLE LEAF MOTEL (802) 295-2817
White River Jct., Vt. 05001 - 18 rooms
Singles $12-$15
Doubles 16- 18
Code: FP-AC-TV-Continental Breakfast
7 miles from campus

PARKING

No permits will be required for on-campus parking. Maps showing the location of the various college parking lots open to participants will be available at the Registration Desk. No on-street overnight parking is allowed in Hanover.

CAMPING

Campsites are available at the Storrs Pond Recreation area, a Hanover-owned park, two miles from the campus. Tents or trailers can be accommodated but no electrical or water hookups are furnished. Hot water showers and grills are available. The charge is $3 per night, and reservations should be made directly with the Hanover Improvement Society, Nugget Building, Hanover, N. H. 03755.

BOOKSTORE

There is no bookstore on campus. There are several good bookstores in the town, however, which are open daily during normal working hours.

LIBRARIES

Baker Library, Dartmouth's undergraduate library, will be open Monday through Friday from 8:00 a.m. to 5:00 p.m. It is located at the center of the campus and contains many interesting collections as well as the Orozco murals, the College's most famous work of art.

The Mathematics Library is located on the second floor of Bradley Hall and will be open from 9:00 a.m. to 10:00 p.m. daily.

MEDICAL SERVICES

The 400 bed Mary Hitchcock Memorial Hospital and Clinic is two blocks north of the Conference Center. The Campus Police may be called (646-2234) for transportation to the hospital at any time day or night.

ENTERTAINMENT
AND FAMILY EVENTS

Dartmouth College has planned a program for mathematicians and their families to take advantage of the informality and beauty of the North Country of New England. The program of recreational and educational events is for families as well as for participants, and families are encouraged to attend.

Hopkins Center will offer two formal concerts during the meeting. The Dartmouth College Department of Mathematics will arrange a third concert of Handel's "The Messiah" for anyone interested in singing (or listening). Dartmouth

College will supply music. The Guarnieri String Quartet will present the first formal concert on Monday evening, August 28, at 8:00 p.m. in Spaulding Auditorium. The second formal concert, probably a folk singer, is tentatively scheduled for Thursday evening.

On Wednesday, August 30, Thayer Hall will be closed for dinner, and a New England Clambake will be offered, weather permitting, at the College Park on the banks of the Connecticut River. Those wishing to attend should indicate the number of tickets desired on the preregistration form. Only a limited number of tickets will be available at the conference based on the preliminary estimates determined from the preregistration form. The cost will be $6.50 for the complete bake of clams, lobster, and chicken, or $3 (children under six, $2) for the chicken portion only. In case of rain, the clambake will be served at Thayer Hall.

If sufficient interest is indicated on the reservation form, a bus tour will go to the Shelburne Museum on Thursday, August 31. One of the foremost attractions of New England, the 45-acre museum is located on the shores of Lake Champlain south of Burlington, Vermont. Its extensive "collection of collections" depicts the early life of New England. The day-long trip will cost $11 per person and will include chartered bus transportation, Museum entrance fee, and lunch.

A four-hour computer course introducing the Dartmouth Time-Sharing system and the computer language BASIC will be provided for families. A fee of $15 per family will include extensive computer time as well as lectures. The course will be held on Monday through Thursday at 9:00 a.m. in Filene Auditorium, Bradley Hall.

The Dartmouth Outing Club will lead two one-day climbs for anyone over the age of 12. The Ledyard Canoe Club will provide (in addition to canoe rentals) a series of four one-hour canoeing and kayak lessons at a minimal charge for participants and families. The ability to swim is required.

An organized program of softball will take place each afternoon on the Hanover Green. On Monday at 4:45 p.m. the Department of Mathematics softball team will play any challenge team for the MAA/IMS/IIME/AMS Challenge Cup to be awarded by the Dartmouth College Office of Summer Programs.

The College's athletic facilities will be available throughout the meeting during hours posted at the Information Center, adjacent to the Registration Desk at the "Top of the Hop," Hopkins Center. Tennis, indoor swimming, volley ball, and canoeing are located on the campus; the golf course and hiking trails are less than a mile from the campus. Storrs Pond, Hanover's town recreation area, located two miles from the campus, provides an outdoor Olympic-sized pool, a pond, picnic areas, trail hiking, and campsites for a nominal fee. Shuttle transportation will be provided.

Free movies, selected with children in mind, will be shown Sunday through Thursday nights. Other activities and items of interest include the Dartmouth College Museum, art exhibits, campus tours, the Daniel Webster College, and numerous nearby lakes and scenic drives.

A nursery play-school for children ages one to six will be available Monday through Friday from 9:00 a.m. to 1:00 p.m. The charge will be $2 per day per child. Group babysitters will be available in the dormitories during the evening hours. The charge for this service will be $1 per child per evening. A list of individual babysitters will be available at the Registration Desk.

The Tavern Room at the Hanover Inn will be open exclusively for registrants and their guests from 5:00 p.m. to midnight daily. Draught beer will be served in this pub-like location.

TRAVEL

Bus and air transportation serve Hanover from Boston and New York. Executive Airlines from Boston and Northeast Airlines from New York use the Lebanon Regional Airport located five miles from Hanover. Bus service is through White River Junction, Vermont, located approximately seven miles away. Taxis to Hanover are available at both locations. The two main auto routes are Interstate 91 from Connecticut to Exit 13 in Norwich, Vermont, or Interstate 93 from Boston, connecting with Interstate 89 at Concord, New Hampshire, to the Hanover Exit 18 which is four miles from campus.

Those planning to fly to Hanover should be warned that early morning flights are cancelled with some frequency in August because of ground fog. For those interested in renting a car at Logan Airport in Boston and driving to Hanover, the driving time is two and one-half hours. The Hanover Avis office is one block from Hopkins Center.

A travel desk will be maintained daily in the registration area by a local travel agency to assist conferees in making or changing travel plans.

WEATHER

The average maximum temperature during this week is 67.2°. The temperature may range from 55° to 80° in any one day with the evenings somewhat cool. Rainfall in August averages 3.07".

MAIL AND MESSAGE CENTER

Individuals may be addressed at Mathematics Meeting, Hopkins Conference Center, Dartmouth College, Hanover, New Hampshire 03755. The telephone number of the Message Center will be (603) 646-3218. Messages may be left for registrants at any time.

COMMITTEE

H. L. Alder (ex officio), Mrs. Sandi J. Garland, Kenneth I. Gross (chairman), Walter H. Gottschalk (ex officio), Reese T. Prosser, William E. Slesnick, G. L. Walker (ex officio), and H. D. Weed.

TIMETABLE

DH - Dartmouth Hall
HC - Hopkins Center
HI - Hanover Inn

(Eastern Daylight Saving Time)

Sunday August 27	Institute of Mathematical Statistics	Mathematical Association of America
9:00 a.m.		Board of Governors Drake Room, H. C.
2:00 p.m. - 8:00 p.m.	REGISTRATION - "Top of the Hop" - H. C.	
4:00 p.m.	Council Meeting, Room 112, H.I.	
5:30 p.m.	Buffet Supper - Alumni Hall and Hanover Inn	
		Center Theater, H.C.
7:00 p.m.		Films
7:00 p.m. - 7:07 p.m.		An Allendoerfer arithmetic film: Equivalent sets (in color)
7:10 p.m. - 7:35 p.m.		The Gauss-Bonnet theorem - a lecture by Carl B. Allendoerfer (in color)
7:45 p.m.		Films of the college geometry project (in color)
7:45 p.m. - 7:57 p.m.		Inversion - Dan Pedoe
7:58 p.m. - 8:10 p.m.		Equidecomposable polygons - J. D. E. Konhauser
8:00 p.m.	Feature Length Movie - 28 Silsby	
8:11 p.m. - 8:25 p.m.		Symmetries of the cube - H. S. M. Coxeter and W. O. J. Moser
8:30 p.m. - 8:56 p.m.		Isometries - W. O. J. Moser and S. Schuster
9:00 p.m. - 9:16 p.m.		Projective generation of conics - S. Schuster

Monday August 28	Institute of Mathematical Statistics	Mathematical Association of America
8:00 a.m. - 5:00 p.m.	REGISTRATION - "Top of the Hop" - H.C.	
9:00 a.m. - 9:15 a.m.		Welcome on behalf of the College Spaulding Auditorium, H.C.
9:00 a.m.	Computer Course: BASIC and Dartmouth Time Sharing - Filene Auditorium	
9:15 a.m. - 10:15 a.m.		THE EARLE RAYMOND HEDRICK LECTURES: Scattering theory, Lecture I Peter D. Lax Spaulding Auditorium, H.C.
10:00 a.m.	Ledyard Canoe Club - canoe and kayak lessons - Boathouse	
10:30 a.m. - 11:30 a.m.	JOINT SESSION -- FEDERAL STATISTICS, THE PRESIDENT'S COMMISSION AND BEYOND Joseph B. Kadane William H. Kruskal David L. Wallace Spaulding Auditorium, H.C.	
11:30 a.m. - 12:00 noon	General discussion by the panel and audience	
1:00 p.m. - 5:00 p.m.	EXHIBITS - Art Studios - H.C.	
1:00 p.m. - 2:45 p.m.	Invited Papers, I Room 105, D.H.	
1:30 p.m. - 2:30 p.m.		THE EARLE RAYMOND HEDRICK LECTURES: Lecture II Peter D. Lax Spaulding Auditorium, H.C.
2:40 p.m. - 3:40 p.m.		Panel discussion: Student self-paced calculus Arthur H. Copeland, Jr. David W. Henderson M. Evans Munroe (moderator) J. Roger Teller Spaulding Auditorium, H.C.

DH - Dartmouth Hall
HC - Hopkins Center
HI - Hanover Inn

Monday August 28	Institute of Mathematical Statistics	Mathematical Association of America
3:00 p.m. - 4:00 p.m.	WALD LECTURES: Robustness, Lecture I Peter J. Huber Room 105, D.H.	
3:00 p.m.	Organized Softball - all ages - College Green Ledyard Canoe Club - canoe and kayak lessons - Boathouse	
3:40 p.m. - 4:15 p.m.		General discussion by the panel and the audience
4:15 p.m. - 6:15 p.m.	Invited Papers, II Room 105, D.H.	
4:45 p.m.	Challenge Cup Softball - College Green	
5:00 p.m. - midnight	Rathskeller - draught beer by the pitcher Tavern Room, Hanover Inn	
		Center Theater, H.C.
7:00 p.m.		Films A film of the Encyclopaedia Britannica Educational Corporation (in color)
7:00 p.m. - 7:19 p.m.		What is a computer?
7:25 p.m.		Films of Charles Eames, distributed by the Encyclopaedia Britannica Educational Corporation (in color)
7:25 p.m. - 7:39 p.m.		House of science
7:30 p.m.	Feature Length Movie - 28 Silsby	
7:30 p.m.	Business Meeting Hartman Rehearsal Hall, H.C.	
7:40 p.m. - 7:52 p.m.		Introduction to feedback
7:53 p.m. - 8:16 p.m.		Communications primer
8:00 p.m.	Concert: Guarnieri String Quartet - Spaulding Auditorium	
8:30 p.m. - 8:41 p.m.		Computer glossary
8:42 p.m. - 8:53 p.m.		The information machine
8:54 p.m. - 9:06 p.m.		Mathematics peep shows
9:07 p.m. - 9:21 p.m.		View from the people wall

Tuesday August 29	Institute of Mathematical Statistics	Mathematical Association of America
8:00 a.m.	Dartmouth Outing Club - all day climb - Mt. Moosilauke	
8:30 a.m. - 4:30 p.m.	REGISTRATION - "Top of the Hop" - H.C.	
8:30 a.m. - 10:00 a.m.	Panel discussion: Social responsibility of scientists Cuthbert Daniel Richard M. Dudley Joseph Harris Joseph W. Lamperti (chairman) Room 105, D.H.	
9:00 a.m. - 9:50 a.m.		THE EARLE RAYMOND HEDRICK LECTURES: Lecture III Peter D. Lax Spaulding Auditorium, H.C.
9:00 a.m.	Computer Course: BASIC and Dartmouth Time Sharing - Filene Auditorium	
9:00 a.m. - 4:00 p.m.	EMPLOYMENT REGISTER - Alumni Hall - H.C.	
9:00 a.m. - 5:00 p.m.	EXHIBITS - Art Studios - H.C.	
10:00 a.m.	Ledyard Canoe Club - canoe and kayak lessons - Boathouse	

DH - Dartmouth Hall
HC - Hopkins Center
HI - Hanover Inn

Tuesday August 29	Institute of Mathematical Statistics	Mathematical Association of America
10:00 a.m. - 11:00 a.m.		Business Meeting: Presentation of Lester R. Ford awards Spaulding Auditorium, H.C.
11:00 a.m. - 12:00 noon	Special Invited Paper I: The complexity of classification algorithms in pattern recognition Thomas M. Cover Room 105, D.H.	
11:10 a.m. - 12:15 p.m.		Invited address: Is mathematics relevant and, if so, to what? Felix E. Browder Spaulding Auditorium, H.C.
12:15 p.m.	PI MU EPSILON - Council Luncheon - Thayer Dining Hall	

Tuesday August 29	Institute of Mathematical Statistics	American Mathematical Society
1:00 p.m. - 2:30 p.m.	Contributed Papers I Room 105, D.H.	
1:30 p.m. - 2:30 p.m.		Colloquium Lectures: Applications of global analysis to biology, economics, electrical circuits, and celestial mechanics, Lecture I Stephen Smale Spaulding Auditorium, H.C.
2:45 p.m. - 3:45 p.m.	WALD LECTURES II: Peter J. Huber Room 105, D.H.	Colloquium Lectures: The arithmetic of elliptic curves, Lecture I John T. Tate Spaulding Auditorium, H.C.
3:00 p.m.	Organized Softball - all ages - College Green Ledyard Canoe Club - canoe and kayak lessons - Boathouse	
3:15 p.m. - 5:15 p.m.	PI MU EPSILON - Contributed Papers - Room 206, D.H.	
4:00 p.m.		Employment Panel W. L. Duren, Jr. (chairman) Spaulding Auditorium, H.C.
4:00 p.m. - 5:00 p.m.	Special Invited Paper II: Bayesian nonparametric statistics Thomas S. Ferguson Room 105, D.H.	
5:00 p.m. - 6:15 p.m.	Contributed Papers II Room 105, D.H.	
5:00 p.m. - midnight	Rathskeller - draught beer by the pitcher Tavern Room, Hanover Inn	
5:15 p.m.		Council Meeting Drake Room, H.C.
6:30 p.m.	PI MU EPSILON - Banquet - Thayer Dining Hall	
7:00 p.m.	Council Meeting Room 122, H.I.	

Tuesday August 29	Institute of Mathematical Statistics	Mathematical Association of America
		Center Theater, H.C.
7:00 p.m.		Films Films of the MAA Individual Lectures Film Project (ILFP) (in color)
7:00 p.m. - 7:25 p.m.		Shapes of the future I - some unsolved problems in geometry - two dimensions with Victor Klee
7:30 p.m.	Feature Length Movie - 28 Silsby	
7:35 p.m. - 8:15 p.m.		Shapes of the future II - some unsolved problems in geometry - three dimensions with Victor Klee

DH - Dartmouth Hall
HC - Hopkins Center
HI - Hanover Inn

Tuesday August 29	Institute of Mathematical Statistics	Mathematical Association of America
8:00 p.m.	PI MU EPSILON - Invited Speaker - John G. Kemeny Mathematical models and the computer - Sanborn House	
8:00 p.m.	Participatory Concert - Sight reading of Handel's "Messiah" Spaulding Auditorium, H. C.	
8:30 p.m.		Films of the NCTM Series: Elementary Mathematics for Teachers and Students (in color)
8:30 p.m. - 8:39 p.m.		Graphing inequalities (marvelous marshes)
8:40 p.m. - 8:50 p.m.		Reciprocals - multiplicative inverses (sunglasses)
8:51 p.m. - 9:03 p.m.		Dividing with fractions: missing factor method (Lola and Arthur)
9:04 p.m. - 9:13 p.m.		Subtraction with mixed numerals (stones)

Wednesday August 30	Institute of Mathematical Statistics	American Mathematical Society
8:00 a.m.	PI MU EPSILON - Dutch Treat Breakfast - Thayer Dining Hall	
8:30 a.m. - 4:30 p.m.	REGISTRATION - "Top of the Hop" - H. C.	
8:30 a.m. - 10:00 a.m.	Contributed Papers III Room 105, D.H.	
9:00 a.m. - 10:00 a.m.		Colloquium Lectures II Stephen Smale Center Theater, H. C.
9:00 a.m.	Computer Course: BASIC and Dartmouth Time Sharing - Filene Auditorium	
9:00 a.m. - 5:00 p.m.	EXHIBITS - Art Studios - H. C.	
9:00 a.m. - 5:40 p.m.	EMPLOYMENT REGISTER - Alumni Hall - H.C.	
		SESSIONS FOR CONTRIBUTED PAPERS
9:00 a.m. - 10:25 a.m.		Algebra I, Room 104, Reed Hall
9:00 a.m. - 10:25 a.m.		Analysis, Room 108, Reed Hall
9:00 a.m. - 10:25 a.m.		General Session I, Room 105, Thornton Hall
9:00 a.m. - 10:25 a.m.		Number Theory, Room 204, Wentworth Hall
9:00 a.m. - 10:25 a.m.		Probability and Statistics, Room 307, Wentworth Hall
10:00 a.m.	Ledyard Canoe Club - canoe and kayak lessons - Boathouse	
10:15 a.m. - 11:15 a.m.		Colloquium Lectures II John T. Tate Center Theater, H.C.
10:15 a.m. - 12:00 noon	Invited Papers III Room 105, D.H.	
		SESSIONS FOR CONTRIBUTED PAPERS
10:30 a.m. - 11:55 a.m.		Complex Analysis I, Room 108, Reed Hall
10:30 a.m. - 11:55 a.m.		Functional Analysis, Room 104, Reed Hall
10:30 a.m. - 11:55 a.m.		Geometry, Room 105, Thornton Hall
10:30 a.m. - 11:55 a.m.		Graph Theory, Room 204, Wentworth Hall
10:30 a.m. - 11:55 a.m.		Topology I, Room 307, Wentworth Hall
10:40 a.m. - 12:40 p.m.	PI MU EPSILON - Contributed Papers - Room 206, D.H.	

DH - Dartmouth Hall
HC - Hopkins Center
HI - Hanover Inn

Wednesday August 30	Institute of Mathematical Statistics	Mathematical Association of America
1:30 p.m. - 2:30 p.m.	Memorial Session: In honor of Paul Lévy A. Dvoretsky Hartman Rehearsal Hall, H.C.	
2:00 p.m. - 3:30 p.m.		Panel discussion: Mathematics and the social sciences, its place in the university William F. Lucas Robert Z. Norman (moderator) F. S. Roberts Herbert E. Scarf Spaulding Auditorium, H.C.
2:45 p.m. - 3:45 p.m.	WALD LECTURES III Peter J. Huber Room 105, D.H.	
3:00 p.m.	Organized Softball - all ages - College Green Ledyard Canoe Club - canoe and kayak lessons - Boathouse	
3:30 p.m. - 3:50 p.m.		General discussion by the panel and the audience Spaulding Auditorium, H.C.
4:00 p.m. - 5:00 p.m.		Invited Address: What every college president should know about mathematics John G. Kemeny Spaulding Auditorium, H.C.
5:30 p.m.	Clambake - College Park	
7:30 p.m.	Feature Length Movie - 28 Silsby	
9:00 p.m. - midnight	Draught Beer - Live Entertainment Tavern Room, Hanover Inn	

Thursday August 31	American Mathematical Society	Institute of Mathematical Statistics
7:30 a.m.	Shelburne Museum Trip	
8:00 a.m.	Dartmouth Outing Club - Mt. Lafayette	
8:30 a.m. - 10:00 a.m.		Contributed Papers IV Room 105, D.H.
8:30 a.m. - 4:30 p.m.	REGISTRATION - "Top of the Hop" - H.C.	
9:00 a.m.	Computer Course: BASIC and Dartmouth Time Sharing - Filene Auditorium	
9:00 a.m. - 10:00 a.m.	Colloquium Lectures III Stephen Smale Center Theater, H.C.	
	SESSIONS FOR CONTRIBUTED PAPERS	
9:00 a.m. - 10:25 a.m.	Algebra II, Room 104, Reed Hall	
9:00 a.m. - 10:25 a.m.	General Session II, Room 105, Thornton Hall	
9:00 a.m. - 10:25 a.m.	Group Theory I, Room 204, Wentworth Hall	
9:00 a.m. - 10:10 a.m.	Numerical Analysis, Room 108, Reed Hall	
9:00 a.m. - 10:10 a.m.	Real Analysis, Room 307, Wentworth Hall	
9:00 a.m. - 1:00 p.m.	EXHIBITS - Art Studios - H.C.	
9:00 a.m. - 5:40 p.m.	EMPLOYMENT REGISTER - Alumni Hall - H.C.	
10:00 a.m.	Ledyard Canoe Club - canoe and kayak lessons - Boathouse	
10:15 a.m. - 11:15 a.m.	Colloquium Lectures III John T. Tate Center Theater, H.C.	
10:15 a.m. - 12:00 noon		Invited Papers IV Room 105, D.H.

DH - Dartmouth Hall
HC - Hopkins Center
HI - Hanover Inn

Thursday August 31	American Mathematical Society	Institute of Mathematical Statistics
	SESSIONS FOR CONTRIBUTED PAPERS	
10:30 a.m. - 11:40 a.m.	Applied Mathematics, Room 105, Thornton Hall	
10:30 a.m. - 11:40 a.m.	Complex Analysis II, Room 108, Reed Hall	
10:30 a.m. - 11:55 a.m.	Functional Analysis II, Room 104, Reed Hall	
10:30 a.m. - 11:40 a.m.	Matrix Theory, Room 204, Went-worth Hall	
10:30 a.m. - 11:55 a.m.	Topology II, Room 307, Wentworth Hall	
10:30 a.m.	ASSOCIATION FOR WOMEN IN MATHEMATICS Business Meeting - Hartman Rehearsal Hall - H.C.	
1:00 p.m. - 2:45 p.m.		Invited Papers V Room 105, D.H.
1:30 p.m. - 2:30 p.m.	INVITED ADDRESS: Glen E. Bredon Locally smooth actions on manifolds Spaulding Auditorium, H.C.	
1:30 p.m. - 2:30 p.m.	INVITED ADDRESS: Charles C. Sims The construction of large permutation groups Center Theater, H.C.	
1:30 p.m. - 3:30 p.m.	SPECIAL SESSION Recursion Theory I Faulkner Recital Hall, H.C.	
2:45 p.m. - 3:45 p.m.	INVITED ADDRESS: Seymour Sherman Monotonicity and magnetism Spaulding Auditorium, H.C.	
2:45 p.m. - 3:45 p.m.	INVITED ADDRESS: Herbert S. Wilf Bounds for the chromatic number Center Theater, H.C.	
3:00 p.m. - 4:00 p.m.		Special Invited Paper III Asymptotic expansions in nonparametric statistics Peter J. Bickel Room 105, D.H.
3:00 p.m.	Organized Softball - all ages - College Green Ledyard Canoe Club - canoe and kayak lessons - Boathouse	
4:00 p.m.	Business Meeting Presentation of L. P. Steele Prizes Spaulding Auditorium, H.C.	
4:00 p.m. - 5:30 p.m.		Contributed Papers V Room 206, D.H.
8:00 p.m.	Concert - Spaulding Auditorium	

Friday September 1	American Mathematical Society	
8:30 a.m. - 1:00 p.m.	REGISTRATION - "Top of the Hop" - H.C.	
9:00 a.m. - 10:00 a.m.	Colloquium Lectures IV Stephen Smale Center Theater, H.C.	

DH - Dartmouth Hall
HC - Hopkins Center
HI - Hanover Inn

Friday September 1	American Mathematical Society	
	SESSIONS FOR CONTRIBUTED PAPERS	
9:00 a.m. - 10:25 a.m.	Algebra III, Room 104, Reed Hall	
9:00 a.m. - 10:10 a.m.	Approximation Theory, Room 105, Thornton Hall	
9:00 a.m. - 10:25 a.m.	Functional Analysis III, Room 108, Reed Hall	
9:00 a.m. - 10:25 a.m.	Group Theory II, Room 204, Wentworth Hall	
9:00 a.m. - 10:25 a.m.	Topology III, Room 307, Wentworth Hall	
10:00 a.m. - 12:00 noon	SPECIAL SESSION Recursion Theory II Faulkner Recital Hall, H.C.	
10:15 a.m. - 11:15 a.m.	Colloquium Lectures IV John T. Tate Center Theater, H.C.	
	SESSIONS FOR CONTRIBUTED PAPERS	
10:30 a.m. - 11:10 a.m.	Combinatorics, Room 105, Thornton Hall	
10:30 a.m. - 11:40 a.m.	Complex Analysis III, Room 108, Reed Hall	
10:30 a.m. - 11:55 a.m.	Differential Equations, Room 104, Reed Hall	
10:30 a.m. - 11:25 a.m.	Operator Theory, Room 204, Wentworth Hall	
10:30 a.m. - 11:55 a.m.	Topology IV, Room 307, Wentworth Hall	
1:30 p.m. - 2:30 p.m.	INVITED ADDRESS: George E. Andrews A general theory of identities of the Rogers-Ramanujan type Center Theater, H.C.	
1:30 p.m. - 2:30 p.m.	INVITED ADDRESS: Chuan C. Hsiung Submanifolds of Riemannian manifolds Spaulding Auditorium, H.C.	
1:30 p.m. - 5:30 p.m.	SPECIAL SESSION Asymptotic and perturbation methods in fluid mechanics and wave propagation Hartman Rehearsal Hall, H.C.	
2:45 p.m. - 3:45 p.m.	INVITED ADDRESS: Joachim Lambek Noncommutative localization Spaulding Auditorium, H.C.	
2:45 p.m. - 3:45 p.m.	INVITED ADDRESS: R. C. Bose Representation of non-Desarguesian projective planes in projective hyperspace Center Theater, H.C.	
4:00 p.m. - 5:00 p.m.	INVITED ADDRESS: Michael Shub Dynamical systems, filtrations, and entropy Center Theater, H.C.	

PRESENTERS OF PAPERS

Following each name is the number corresponding to the speaker's position on the program

Abdelaziz, N. H. #101
Ablowitz, M. #190
Abrahamse, A. F. #71
Agarwal, A. K. #162
• Andrews, G. E. #182
Arkin, J. #160
Arsove, M. G. #7
Asadulla, S. #24
Aull, C. E. #58
Baartmans, A. H. #75
Bakshi, J. S. #89
Barker, G. P. #110
Bartelt, M. W. #133
Bateman, J. M. #146
Berman, A. #106
Beyl, F. R. #76
Billera, L. J. #45
Blanchard, A. A. #130
Bolstein, R. #173
Bonar, D. D. #33
• Bose, R. C. #193
Bottorff, G. A. #21
• Bredon, G. E. #117
Brown, J. B. #87
Brunner, H. #82
Brylawski, T. H. #159
Buck, R. J. #88
Byrnes, J. S. #22
Callegari, A. #186
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Childress, W. S. #188
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Dickerson, C. E. #59
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Foulis, D. J. #29
Frame, J. S. #78
Friedman, H. #122
Garbanati, D. A. #147
Gardner, R. B. #125
Geller, D. P. #50
Giellis, G. R. #138
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Gowrisankaran, K. N. #149
Greenberg, J. M. #168
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Haddad, H. M. #113
Hahn, K. T. #165
Hales, R. S. #53
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Hebert, D. J. #175
Hechler, S. H. #67
Heisey, R. E. #48
Henney, D. R. #37
Herda, H.-H. #46
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• Lambek, J. #192
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Pitcher, T. S. #25
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Poreda, S. J. #132
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• Sims, C. C. #118
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• Wilf, H. S. #123
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Young, J. D. #83

• Invited one-hour lectures

PROGRAM OF THE SESSIONS

The time limit for each contributed paper in the general sessions is ten minutes and in the special sessions is twenty minutes. To maintain this schedule, the time limits will be strictly enforced.

TUESDAY, 1:30 P. M.

Colloquium Lectures: Lecture I, Spaulding Auditorium, H. C.
Applications of global analysis to biology, economics, electrical circuits, and celestial mechanics. Professor STEPHEN SMALE, University of California, Berkeley

TUESDAY, 2:45 P. M.

Colloquium Lectures: Lecture I, Spaulding Auditorium, H. C.
The arithmetic of elliptic curves. Professor JOHN T. TATE, Harvard University

TUESDAY, 4:00 P. M.

Panel Discussion on Employment and Educational Policy, Spaulding Auditorium, H. C.
Professor W. L. DUREN, JR. (chairman), University of Virginia

WEDNESDAY, 9:00 A. M.

Colloquium Lectures: Lecture II, Center Theater, H. C.
Applications of global analysis to biology, economics, electrical circuits, and celestial mechanics. Professor STEPHEN SMALE, University of California, Berkeley

WEDNESDAY, 9:00 A. M.

Session on Algebra I, Room 104 - Reed Hall

9:00- 9:10 (1) Some special polynomials over a finite field. I. Dr. ROBERT G. VAN METER, State University of New York, College at Oneonta (696-12-1)

9:15- 9:25 (2) Polynomial subfields of k(x). Mr. ALAN McCONNELL, Howard University (696-12-2)

9:30- 9:40 (3) An inseparable Galois theory. Professor NICHOLAS HEEREMA and Mr. JAMES K. DEVENEY*, Florida State University (696-12-3)

9:45- 9:55 (4) Units of algebraic number fields. Preliminary report. Dr. CHARLES J. PARRY, Virginia Polytechnic Institute and State University (696-12-4)

10:00-10:10 (5) Antidifferentiation of differential polynomials. Professor MARTIN D. KRUSKAL, Princeton University, and Professor ROBERT M. MIURA*, Vanderbilt University (696-12-5)

10:15-10:25 (6) Totally archimedian semigroups. Professor NAOKI KIMURA, University of Arkansas (696-20-12)

WEDNESDAY, 9:00 A. M.

Session on Analysis, Room 108 - Reed Hall

9:00- 9:10 (7) Local behavior of subharmonic functions. Professor MAYNARD G. ARSOVE*, University of Washington, and Professor ALFRED O. HUBER, Eidgenössische Technische Hochschule, Zürich, Switzerland (696-31-1)

9:15- 9:25 (8) Metrizability of order intervals in locally convex spaces ordered by biorthogonal systems. Professor HARRY F. JOINER II* and Professor THURLOW A. COOK, University of Massachusetts (696-40-1)

9:30- 9:40 (9) Convergence and regularity of meromorphic Ritt series. Professor GUTTALU R. VISWANATH, Howard University (696-40-2)

9:45- 9:55 (10) Absolute Nörlund summability factors of a Fourier series. Preliminary report. Mr. D. S. GOEL and Dr. BADRI N. SAHNEY*, University of Calgary (696-42-1)

10:00-10:10 (11) The almost periodic part of an ergodic function. Professor GORDON WOODWARD, University of Nebraska (696-42-2)

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*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.

10:15-10:25 (12) Sampling expansion with derivatives for finite Hankel and other transforms. Professor ABDUL JABBAR JERRI, American University in Cairo, United Arab Republic, and Clarkson College of Technology, and Professor DALE W. KREISLER*, Castleton State College (696-44-1)

WEDNESDAY, 9:00 A. M.

General Session I, Room 105 - Thornton Hall

9:00- 9:10 (13) WQO's and hierarchies of recursive functions. Professor D. H. J. de JONGH, State University of New York at Buffalo, and Professor ROHIT J. PARIKH*, State University of New York at Buffalo and Boston University (696-02-2)

9:15- 9:25 (14) Minimal and conservative extensions of arithmetic. Preliminary report. Professor ROBERT G. PHILLIPS, University of South Carolina (696-02-3)

9:30- 9:40 (15) Nonrecursive tilings of the plane. Preliminary report. Professor WILLIAM P. HANF, University of Hawaii (696-02-6)

9:45- 9:55 (16) Almost huge cardinals and Vopenka's principle. Dr. WILLIAM C. POWELL, State University of New York at Buffalo (696-02-8)

10:00-10:10 (17) The tensor interpretation of Grassmann's Ausdehnungslehre. II: Combinatorial products. Dr. VIVIAN EBERLE SPENCER, Bureau of the Census, Washington, D. C., and Professor DOMINA EBERLE SPENCER*, University of Connecticut (696-01-1)

10:15-10:25 (18) Graphing counterexamples in multivariate analysis. Preliminary report. Sister M. CORDIA EHRMANN, Villanova University (696-98-1)

WEDNESDAY, 9:00 A. M.

Session on Number Theory, Room 204 - Wentworth Hall

9:00- 9:10 (19) A combinatorial method in number theory. II. Dr. EMANUEL VEGH, Naval Research Laboratory, Washington, D. C. (696-10-1)

9:15- 9:25 (20) The intersection of polygonal numbers of different rank. Professor GREGORY WULCZYN, Bucknell University (696-10-2)

9:30- 9:40 (21) Proof of the "Littlewood Conjecture" for infinitely many pairs. Dr. GERALD A. BOTTORFF, Pennsylvania State University, Mont Alto Campus (696-10-3)

9:45- 9:55 (22) On a partition problem of Frobenius. Dr. JAMES S. BYRNES, University of Massachusetts, Boston Campus (696-10-4)

10:00-10:10 (23) Odd perfect numbers are divisible by at least 7 distinct primes. Preliminary report. Dr. CARL POMERANCE, Harvard University (696-10-5) (Introduced by Professor John T. Tate)

10:15-10:25 (24) A note on Fermat numbers. Dr. SYED ASADULLA, St. Francis Xavier University (696-10-6)

WEDNESDAY, 9:00 A. M.

Session on Probability and Statistics, Room 307 - Wentworth Hall

9:00- 9:10 (25) Convergence behavior of random T-fractions. Professor TOM S. PITCHER, University of Hawaii (696-60-1)

9:15- 9:25 (26) Conditional probabilities, determinate pairs of sets and indices. Preliminary report. Professor ROHIT J. PARIKH, Boston University, and Professor MILTON N. PARNES*, State University of New York at Buffalo (696-60-2)

9:30- 9:40 (27) Another ridge regression. Professor JAMES M. LOWERRE, Clarkson College of Technology (696-62-1) (Introduced by Professor Victor Lovass-Nagy)

9:45- 9:55 (28) Sparse and crowded cells in a homogeneous multinomial distribution. Preliminary report. Professor MILTON SOBEL, University of Minnesota, and Dr. VENKATA R. R. UPPULURI*, Oak Ridge National Laboratory (696-62-2)

10:00-10:10 (29) Stability of pure weights under conditioning. Professor DAVID J. FOULIS, University of Massachusetts (696-60-3)

10:15-10:25 (30) Bayesian inference on nonclassical stochastic models. Preliminary report. Professor CHARLES H. RANDALL, University of Massachusetts (696-62-3)

WEDNESDAY, 10:15 A. M.

Colloquium Lectures: Lecture II, Center Theater, H. C.

The arithmetic of elliptic curves. Professor JOHN T. TATE, Harvard University

WEDNESDAY, 10:30 A. M.

Session on Complex Analysis I, Room 108 - Reed Hall

10:30-10:40 (31) Bazilevič functions and close-to-convex p-valent functions. Professor PETRU MOCANU, Babeş-Bolyai University, Cluj, Romania, Professor ELIGIUSZ ZŁOTKIEWICZ, Universitatis Mariae Curie-Skłodowska, Lublin, Poland, and Professor MAXWELL O. READE*, University of Michigan (696-30-1)

10:45-10:55 (32) Annular functions form a residual set. Preliminary report. Dr. DANIEL D. BONAR, Denison University, and Dr. FRANCIS W. CARROLL*, Ohio State University (696-30-2)

11:00-11:10 (33) Not every annular function is strongly annular. Preliminary report. Dr. DANIEL D. BONAR*, Denison University, and Dr. FRANCIS W. CARROLL, Ohio State University (696-30-8)

11:15-11:25 (34) Typically real functions of order α. Preliminary report. Professor ROBERT W. REDDING, Clark University and Worcester State College (696-30-3)

11:30-11:40 (35) Successive remainders of the Newton series. Preliminary report. Professor J. K. SHAW* and Professor GEORGE W. CROFTS, Virginia Polytechnic Institute and State University (696-30-4)

11:45-11:55 (36) Coefficient bounds for k-fold symmetric univalent functions of bounded rotation. Preliminary report. Professor PREM K. KULSHRESTHA, Louisiana State University in New Orleans (696-30-7)

WEDNESDAY, 10:30 A. M.

Session on Functional Analysis, Room 104 - Reed Hall

10:30-10:40 (37) Properties of the space of convex compact sets. Professor DAGMAR R. HENNEY, George Washington University (696-46-1)

10:45-10:55 (38) B-convexity and reflexivity. Dr. SRINIVASA SWAMINATHAN, Dalhousie University (696-46-5)

11:00-11:10 (39) Projections in locally convex spaces. Mr. ROBERT H. LOHMAN, Kent State University (696-46-4)

11:15-11:25 (40) A generalization of echelon spaces. Preliminary report. Professor GEORGE W. CROFTS, Virginia Polytechnic Institute and State University (696-46-14)

11:30-11:40 (41) Complete topologies on spaces of Baire measures. Professor RONALD B. KIRK, Southern Illinois University (696-46-3)

11:45-11:55 (42) Vector valued measures and summable functions. Preliminary report. Professor ERIK G. F. THOMAS, Yale University (696-46-15)

WEDNESDAY, 10:30 A. M.

Session on Geometry, Room 105 - Thornton Hall

10:30-10:40 (43) Three infinite families of tetrahedral space-fillers. Preliminary report. Mr. MICHAEL GOLDBERG, Washington, D. C. (696-50-1)

10:45-10:55 (44) On the intersection of maximal m-convex subsets. Dr. JAMES J. TATTERSALL, Providence College (696-52-1)

11:00-11:10 (45) A characterization of Pareto surfaces. Preliminary report. Professor LOUIS J. BILLERA*, Cornell University, and Professor ROBERT E. BIXBY, University of Kentucky (696-52-2)

11:15-11:25 (46) A new characterization of spheres. Preliminary report. Dr. HANS-HEINRICH HERDA, Boston State College (696-52-3)

11:30-11:40 (47) An analytical criterion for the completeness of Riemannian manifolds. Preliminary report. Dr. WILLIAM B. GORDON, Naval Research Laboratory, Washington, D. C. (696-53-1)

11:45-11:55 (48) A closed embedding theorem. Preliminary report. Mr. RICHARD E. HEISEY, Cornell University (696-58-1)

WEDNESDAY, 10:30 A. M.

Session on Graph Theory, Room 204 - Wentworth Hall

10:30-10:40 (49) Matroid basis graphs. I. Preliminary report. Mr. STEPHEN B. MAURER, Princeton University (696-05-1) (Introduced by Professor A. W. Tucker)

10:45-10:55 (50) Classes of mappings between digraphs. Professor DENNIS P. GELLER, State University of New York at Binghamton (696-05-2)

11:00-11:10 (51) Product graphs for given subgroups of the wreath product of two groups. Dr. JAMES W. UEBELACKER, Syracuse University (696-05-3)

11:15-11:25 (52) A generalized measure of independence and its relation to a cartesian product of graphs. Professor BRUCE H. BARNES and Ms. KAREN E. MACKEY*, Pennsylvania State University (696-05-7)

11:30-11:40 (53) Numerical invariants and graph products. Professor R. STANTON HALES, Pomona College (696-05-9)

11:45-11:55 (54) A stability theorem for minimum edge graphs with given abstract group. Professor DONALD J. McCARTHY*, St. John's University, and Professor LOUIS V. QUINTAS, Pace College (696-05-10)

WEDNESDAY, 10:30 A. M.

Session on Topology I, Room 307 - Wentworth Hall

10:30-10:40 (55) A zero-set condition for $\beta X \times Y = \beta X \times \beta Y$. Preliminary report. Professor FRANK C. KOST, State University of New York, College at Oneonta (696-54-2) (Introduced by Professor Guy T. Hogan)

10:45-10:55 (56) C_λ-compact spaces. Preliminary report. Dr. KOK-KEONG TAN* and Mr. TECK-CHEONG LIM, Dalhousie University (696-54-3)

11:00-11:10 (57) Product spaces with compactness-like properties. Professor JERRY E. VAUGHAN, University of North Carolina at Chapel Hill (696-54-11)

11:15-11:25 (58) Initial and final properties. Preliminary report. Professor CHARLES E. AULL, Virginia Polytechnic Institute and State University (696-54-8)

11:30-11:40 (59) An algebraic characterization of compact zero-dimensional spaces. Mr. CHARLES E. DICKERSON, Purdue University (696-54-7)

11:45-11:55 (60) Some epi-reflective subcategories and their relationship with covering properties. Preliminary report. Mr. GEORGE D. REYNOLDS* and Mr. MICHAEL D. RICE, Wesleyan University (696-54-9)

THURSDAY, 9:00 A. M.

Colloquium Lectures: Lecture III, Center Theater, H. C.

Applications of global analysis to biology, economics, electrical circuits, and celestial mechanics. Professor STEPHEN SMALE, University of California, Berkeley

THURSDAY, 9:00 A. M.

Session on Algebra II, Room 104 - Reed Hall

9:00- 9:10 (61) Normal pairs. Professor EDWARD D. DAVIS, State University of New York at Albany (696-13-1)

9:15- 9:25 (62) F-rings. Preliminary report. Miss MARGARET W. TAFT, Clark University (696-16-1)

9:30- 9:40 (63) Finite subgroups of radical rings. Dr. B. ARTHUR MILLER, Mount Allison University (696-16-2)

9:45- 9:55 (64) A condition equivalent to finite Goldie dimension. Preliminary report. Dr. PATRICK J. FLEURY, State University of New York, College at Plattsburgh (696-16-3)

10:00-10:10 (65) Automorphisms of commutative algebras and separable subalgebras. Professor HERBERT F. KREIMER, Florida State University (696-13-2)

10:15-10:25 (66) Truncated fields. Professor WILLIAM E. JENNER, University of North Carolina at Chapel Hill (696-17-1)

THURSDAY, 9:00 A. M.

General Session II, Room 105 - Thornton Hall

9:00- 9:10 (67) On the structure of open sets in $\beta N - N$. Professor STEPHEN H. HECHLER, Case Western Reserve University (696-04-1)

9:15- 9:25 (68) Cycle indices of certain classes of types of quasiorders or topologies. Preliminary report. Dr. JOHN A. WRIGHT, University of Prince Edward Island (696-06-1)

9:30- 9:40 (69) Archimedean-like classes of lattice-ordered groups. Dr. JORGE MARTINEZ, University of Florida (696-06-2)

9:45- 9:55 (70) Prime ideals in the lattice structure of an f-ring. Preliminary report. Professor HARIHARAIER SUBRAMANIAN, State University of New York at Buffalo (696-06-3)

10:00-10:10 (71) Uniform integrability of derivatives on σ-lattices. Professor ALLAN F. ABRAHAMSE, University of Southern California (696-28-2)

10:15-10:25 (72) On the existence of sum and product integrals. Professor FRED M. WRIGHT* and Mr. DEAN R. KENNEBECK, Iowa State University (696-28-4)

THURSDAY, 9:00 A. M.

Session on Group Theory I, Room 204 - Wentworth Hall

9:00- 9:10 (73) Minimal non-M-groups. Preliminary report. Dr. V. C. VARADACHARI, University of Wisconsin-River Falls (696-20-1) (Introduced by Professor Alfred Aeppli)

9:15- 9:25 (74) A further extension of Frobenius's theorem. Preliminary report. Professor JAMES W. RICHARDS, Kent State University (696-20-3)

9:30- 9:40 (75) Groups with characteristic cyclic series. Preliminary report. Professor ALPHONSE H. BAARTMANS* and Professor JAMES J. WOEPPEL, Southern Illinois University (696-20-8)

9:45- 9:55 (76) The classification of metacyclic p-groups. Preliminary report. Professor F. RUDOLF BEYL, Cornell University (696-20-9)

10:00-10:10 (77) On the number of generators of an H_p group. Preliminary report. Professor GUY T. HOGAN, State University of New York, College at Oneonta (696-20-10)

10:15-10:25 (78) The projective characters of the Rudvalis simple group. Preliminary report. Professor J. SUTHERLAND FRAME, Michigan State University (696-10-8)

THURSDAY, 9:00 A. M.

Session on Numerical Analysis, Room 108 - Reed Hall

9:00- 9:10 (79) A general N-station solution of the optical triangulation problem. Dr. MARK M. LOTKIN, General Electric Company, Philadelphia, Pennsylvania (696-65-1)

9:15- 9:25 (80) Error estimates for numerical differentiation. Dr. R. D. RIESS, Virginia Polytechnic Institute and State University (696-65-2)

9:30- 9:40 (81) Computable error bounds for inner product evaluation. Dr. NAI-KUAN TSAO, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio (696-65-3) (Introduced by Dr. Paul J. Nikolai)

9:45- 9:55 (82) The numerical solution of the generalized Abel integral equation by piecewise polynomials. Preliminary report. Dr. HERMANN BRUNNER, Dalhousie University (696-65-4)

10:00-10:10 (83) Cubic spline formulation for matrix method for second order ordinary differential eigenvalues. Preliminary report. Dr. JONATHAN D. YOUNG* and Professor PAUL LUGER, University of California, Lawrence Berkeley Laboratory (696-65-5)

THURSDAY, 9:00 A. M.

Session on Real Analysis, Room 307 - Wentworth Hall

9:00- 9:10 (84) On separating capacity in a class of c-dense cartesian products. Professor HENRYK FAST, Wayne State University (696-26-1)

9:15- 9:25 (85) On a result of Besicovitch. Preliminary report. Dr. SIOBHÁN VERNON, University College, Cork, Ireland (696-26-2) (Introduced by Professor David Rosen)

9:30- 9:40 (86) Functions with a concave modulus of continuity. Professor HENRY E. WHITE, JR., Ohio University (696-26-3)

9:45- 9:55 (87) Concerning Ceder's differentiable restrictions of arbitrary real functions. Professor JACK B. BROWN, Auburn University (696-26-4)

10:00-10:10 (88) Comparison theorems for generalized Besicovitch dimensions of compact planar sets. Preliminary report. Professor ROBERT J. BUCK, University of California, Davis (696-28-1)

THURSDAY, 10:15 A. M.

Colloquium Lectures: Lecture III, Center Theater, H. C.

The arithmetic of elliptic curves. Professor JOHN T. TATE, Harvard University

THURSDAY, 10:30 A. M.

Session on Applied Mathematics, Room 105 - Thornton Hall

10:30-10:40 (89) Exact and approximate solution for normal modes of vibrations. Preliminary report. Mr. J. S. BAKSHI, Federal City College (696-73-1) (Introduced by Dr. Victor J. Katz)

10:45-10:55 (90) Correlation between new theory and numerical results in plate vibrations. Professor WILLIE R. CALLAHAN, St. John's University (696-73-2)

11:00-11:10 (91) Finite difference solution of the incompressible, time dependent Navier-Stokes equations in polar coordinates. Dr. SUHRIT K. DEY, Eastern Illinois University (696-76-1) (Introduced by Dr. Nicholas Petridis)

11:15-11:25 (92) Extension of Griffiths inequalities to Gaussian spin configuration models. II. Preliminary report. Dr. VENKATA R. R. UPPULURI and Dr. JAMES M. DOLAN*, Oak Ridge National Laboratory (696-82-1)

11:30-11:40 (93) Generalized multicomponent systems. Professor MURRAY HOCHBERG, City University of New York, Brooklyn College (696-94-1)

(94) WITHDRAWN

THURSDAY, 10:30 A. M.

Session on Complex Analysis II, Room 108 - Reed Hall

10:30-10:40 (95) Value distribution of linear combinations of axisymmetric harmonic polynomials and their derivatives. Professor PETER A. McCOY, U. S. Naval Academy (696-30-5)

10:45-10:55 (96) Properties of the generalized Koebe function. Preliminary report. Dr. DOUGLAS M. CAMPBELL*, Brigham Young University, and Dr. JOHN A. PFALTZGRAFF, University of North Carolina at Chapel Hill (696-30-6)

11:00-11:10 (97) On polynomial approximation in $A_q(D)$. Professor THOMAS A. METZGER, Texas A & M University (696-30-10)

11:15-11:25 (98) Functions of bounded variation and topological indices. Professor FRED M. WRIGHT and Mr. J. N. LING*, Iowa State University (696-30-11)

11:30-11:40 (99) A radial analog of Carathéodory's corner mapping theorem. Professor KARL F. BARTH, Syracuse University, and Professor WALTER J. SCHNEIDER*, Carleton University (696-30-12)

THURSDAY, 10:30 A. M.

Session on Functional Analysis II, Room 104 - Reed Hall

10:30-10:40 (100) Theory of sets of constant width in B-spaces. Professor DAGMAR R. HENNEY and Mr. WALTER LINCOLN*, George Washington University (696-46-2)

10:45-10:55 (101) Spectral properties of some positive operators in a Banach space with the decomposition property. Dr. NAZAR H. ABDELAZIZ, University of Maryland (696-46-6)

11:00-11:10 (102) Bounded linear operators into vector valued Banach function spaces. Professor NEIL E. GRETSKY*, University of California, Riverside, and Professor J. JERRY UHL, JR., University of Illinois (696-46-8)

11:15-11:25 (103) Classification of biorthogonal sequences. Professor WILLIAM H. RUCKLE, Clemson University (696-46-11)

11:30-11:40 (104) A common fixed point theorem for commuting nonexpansive mappings. Preliminary report. Professor MOON WUKI KIM, Seton Hall University (696-46-12)

11:45-11:55 (105) Some results on nonlinear contractions. Dr. SANKATHA P. SINGH, Memorial University of Newfoundland (696-54-10)

THURSDAY, 10:30 A. M.

Session on Matrix Theory, Room 204 - Wentworth Hall

10:30-10:40 (106) Cones and iterative methods for singular systems. Professor ABRAHAM BERMAN*, Université de Montréal, and Professor ROBERT J. PLEMMONS, University of Tennessee (696-15-2)

10:45-10:55 (107) A pivot theory of skew matrices with applications. Preliminary report. Miss MARJORIE L. STEIN, Princeton University (696-15-3)

11:00-11:10 (108) On the best least-squares solution of an incompatible system arising from the 9-point approximation to the Neumann problem. Professor JOHN F. DALPHIN*, Indiana and Purdue Universities at Indianapolis and Clarkson College of Technology, and Professor VICTOR LOVASS-NAGY, Clarkson College of Technology (696-15-4)

11:15-11:25 (109) A noniterative method for computing the Moore-Penrose generalized inverse of an arbitrary matrix. Professor VICTOR LOVASS-NAGY* and Professor DAVID L. POWERS, Clarkson College of Technology (696-15-5)

11:30-11:40 (110) Stochastic matrices over cones. Professor GEORGE P. BARKER, University of Missouri-Kansas City (696-15-6)

THURSDAY, 10:30 A. M.

Session on Topology II, Room 307 - Wentworth Hall

10:30-10:40 (111) The theorem of Mulliken-van Est for unicoherent Peano spaces. Professor EDWARD D. TYMCHATYN* and Professor JOHN H. V. HUNT, University of Saskatchewan (696-54-1)

10:45-10:55 (112) Closed sets closed under continuous maps. Preliminary report. Dr. KOK-KEONG TAN and Mr. TECK-CHEONG LIM*, Dalhousie University (696-54-4)

11:00-11:10 (113) On a class of contraction mappings. Professor HADI M. HADDAD* and Professor MILAN KUBR, University of Libya, Tripoli, Libya (696-54-5)

11:15-11:25 (114) Between the closed and the pseudo-open mappings. Dr. PAUL L. STRONG, Bucknell University (696-54-12)

11:30-11:40 (115) A note on 0-regular maps. Preliminary report. Mr. CARL P. PIXLEY, State University of New York at Binghamton (696-54-15)

11:45-11:55 (116) On spaces whose diagonal is a set of interior condensation. Preliminary report. Professor HOWARD H. WICKE, Ohio University (696-54-16)

THURSDAY, 1:30 P. M.

Invited address, Spaulding Auditorium, H. C.

(117) Locally smooth actions on manifolds. Professor GLEN E. BREDON, Rutgers University (696-57-5)

THURSDAY, 1:30 P. M.

Invited address, Center Theater, H. C.

(118) The construction of large permutation groups. Professor CHARLES C. SIMS, Rutgers University (696-20-11)

THURSDAY, 1:30 P. M.

Special Session on Recursion Theory, Faulkner Recital Hall, H. C.

1:30- 1:50 (119) Recursion and metarecursion. Professor HARTLEY ROGERS, JR., Massachusetts Institute of Technology

2:00- 2:20 (120) Automorphisms of the lattice of recursively enumerable sets. II: Complete sets. Professor ROBERT I. SOARE, University of Illinois at Chicago Circle (696-02-5)

2:30- 2:50 (121) Combinatorial functions on algebraic structures and Dedekind types. Preliminary report. Professor ANIL NERODE*, Cornell University, and Professor JOHN CROSSLEY, Monash University, Melbourne, Australia (696-02-7)

3:00- 3:20 (122) Borel sets and hyperdegrees. Preliminary report. Professor HARVEY FRIEDMAN, Stanford University (696-02-10)

THURSDAY, 2:45 P. M.

Invited Address, Center Theater, H. C.

(123) Bounds for the chromatic number. Professor HERBERT S. WILF, University of Pennsylvania (696-05-4)

THURSDAY, 2:45 P. M.

Invited Address, Spaulding Auditorium, H. C.

(124) Monotonicity and magnetism. Professor SEYMOUR SHERMAN, Indiana University

THURSDAY, 4:00 P. M.

Business Meeting, Spaulding Auditorium, H. C.
Presentation of the Steele Prizes

FRIDAY, 9:00 A. M.

Colloquium Lectures: Lecture IV, Center Theater, H. C.
Applications of global analysis to biology, economics, electrical circuits, and celestial mechanics. Professor STEPHEN SMALE, University of California, Berkeley

FRIDAY, 9:00 A. M.

Session on Algebra III, Room 104 - Reed Hall

9:00- 9:10 (125) Some properties of the exterior rank of modules. Preliminary report. Professor ROBERT B. GARDNER, University of North Carolina at Chapel Hill (696-15-1)

9:15- 9:25 (126) The lattice $\mathcal{L}_m$ of equational classes of m-semigroups. Dr. W. F. PAGE* and Professor ALTON T. BUTSON, University of Miami (696-08-1)

9:30- 9:40 (127) The lattice of varieties of semirings. Preliminary report. Professor GILBERT F. ORR* and Professor ALTON T. BUTSON, University of Miami (696-08-2)

9:45- 9:55 (128) On homomorphic relations and weak homomorphisms of algebras. Professor MOHAMMAD ISHAQ, Laval University (696-10-7)

10:00-10:10 (129) A general theory of compact Hausdorff objects. Professor ERNEST G. MANES, University of Massachusetts (696-18-1)

10:15-10:25 (130) Structure species and forgetful functors. Dr. ANDREW A. BLANCHARD, U. S. Naval Academy (696-18-2) (Introduced by Professor G. Ralph Strohl, Jr.)

FRIDAY, 9:00 A. M.

Session on Approximation Theory, Room 105 - Thornton Hall

9:00- 9:10 (131) The constant error curve problem for varisolvant families. Preliminary report. Professor WILLIAM H. LING, Rensselaer Polytechnic Institute, and Professor J. EDWARD TORNGA*, Union College (696-41-1)

9:15- 9:25 (132) Best approximation by meromorphic functions with free poles. Preliminary report. Professor STANLEY J. POREDA, Clark University (696-41-2)

9:30- 9:40 (133) Strongly unique best approximates to a function on a set and a finite subset thereof. Professor MARTIN W. BARTELT, Rensselaer Polytechnic Institute (696-41-3)

9:45- 9:55 (134) Upper bounds for Dirichlet kernels and for Tychebycheff polynomials of the second kind. Dr. BENJAMIN LEPSON, Naval Research Laboratory, Washington, D. C. (696-41-4)

10:00-10:10 (135) On the universal existence of generalized interpolating spline functions. Preliminary report. Professor JOSEPH W. JEROME, Northwestern University (696-41-5)

FRIDAY, 9:00 A. M.

Session on Functional Analysis III, Room 108 - Reed Hall

9:00- 9:10 (136) The structure of Hilbert modules. Professor JAMES F. SMITH, Le Moyne College (696-46-7)

9:15- 9:25 (137) Semigroups with positive definite structure. Professor PARFENY P. SAWOROTNOW, Catholic University of America (696-46-9)

9:30- 9:40 (138) On a class of vector inner product modules. Preliminary report. Professor GEORGE R. GIELLIS, U.S. Naval Academy (696-46-16)

9:45- 9:55 (139) Extreme functionals on an upper semicontinuous function space. Professor FREDERIC CUNNINGHAM, JR., Bryn Mawr College, and Professor NINA M. ROY*, Rosemont College (696-46-10)

10:00-10:10 (140) Finitely generated submodules of differentiable functions. II. Professor BEN G. ROTH, University of Wyoming (696-46-13)

10:15-10:25 (141) Köthe families in vector lattices. II. Preliminary report. Professor CHARLES G. DENLINGER, Millersville State College (696-46-17)

FRIDAY, 9:00 A. M.

Session on Group Theory II, Room 204 - Wentworth Hall

9:00- 9:10 (142) Mappings of degree n from groups to abelian groups. Mr. BRIAN KIRKWOOD SCHMIDT, Princeton University (696-20-2) (Introduced by Professor John R. Stallings)

9:15- 9:25 (143) The order of the automorphism group of certain semidirect products. Preliminary report. Dr. STANLEY L. STEPHENS, Anderson College (696-20-4) (Introduced by Professor Albert D. Otto)

9:30- 9:40 (144) Intersection theorem for groupnets. Preliminary report. Professor RICHARD H. CROWELL*, Dartmouth College, and Professor NEVILLE F. SMYTHE, Dartmouth College and Australian National University, Canberra, Australia (696-20-5)

9:45- 9:55 (145) A note on torsion free abelian groups of finite rank. Professor W. J. WICKLESS* and Professor CHARLES I. VINSONHALER, University of Connecticut (696-20-6)

10:00-10:10 (146) The fundamental ideal functor and its adjoint. Preliminary report. Professor JAMES M. BATEMAN, Michigan State University (696-20-7)

10:15-10:25 (147) Classes of nonsingular abelian group matrices over fields. Dr. DENNIS A. GARBANATI, University of California, Santa Barbara (696-10-9)

FRIDAY, 9:00 A. M.

Session on Topology III, Room 307 - Wentworth Hall

9:00- 9:10 (148) Topological group extensions. Preliminary report. Professor ERIC C. NUMMELA, University of Florida (696-22-1)

9:15- 9:25 (149) Suslin and Lusin topologies on a set. Professor KOHUR N. GOWRISANKARAN, McGill University (696-28-3)

9:30- 9:40 (150) On isotopics of homeomorphisms of 3-manifolds that are branched cyclic coverings. Professor JOAN S. BIRMAN, Stevens Institute of Technology, and Professor HUGH M. HILDEN*, University of Hawaii (696-54-13)

9:45- 9:55 (151) The R_0 separation axiom in closure spaces. Professor WOLFGANG J. THRON, University of Colorado, and Dr. R. H. WARREN*, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio (696-54-6)

10:00-10:10 (152) Locally compact convergence spaces. Professor DARRELL C. KENT, Washington State University, and Mr. GARY D. RICHARDSON*, East Carolina University (696-54-14)

10:15-10:25 (153) Covering properties of uniform spaces. Preliminary report. Mr. MICHAEL D. RICE* and Mr. GEORGE D. REYNOLDS, Wesleyan University (72T-G93)

FRIDAY, 10:00 A. M.

Special Session on Recursion Theory, Faulkner Recital Hall, H. C.

10:00-10:20 (154) Effective reduction of finite types. Professor RICHARD PLATEK, Cornell University

10:30-10:50 (155) Degrees below $\underset{\sim}{0}'$. Professor ROBERT W. ROBINSON, University of California, Berkeley (696-02-4)

11:00-11:20 (156) Maximal sets and admissible ordinals. Preliminary report. Professor MANUEL LERMAN, Yale University (696-02-1)

11:30-11:50 (157) On large ordinals in recursion theory. Mr. EUGENE M. KLEINBERG, Massachusetts Institute of Technology (696-02-9)

FRIDAY, 10:15 A. M.

Colloquium Lectures: Lecture IV, Center Theater, H. C.

The arithmetic of elliptic curves. Professor JOHN T. TATE, Harvard University

FRIDAY, 10:30 A. M.

Session on Combinatorics, Room 105 - Thornton Hall

10:30-10:40 (158) Construction of nonisomorphic reverse Steiner quasigroups. Professor CHARLES C. LINDNER, Auburn University (696-05-5)

10:45-10:55 (159) Modular flats and the critical problem of combinatorial geometries. Professor THOMAS H. BRYLAWSKI, University of North Carolina at Chapel Hill (696-05-6)

11:00-11:10 (160) The Arkin-Hoggatt game and the solution of a classical problem. Dr. JOSEPH ARKIN*, Spring Valley, New York, and Professor VERNER E. HOGGATT, JR., San Jose State College (696-05-8)

FRIDAY, 10:30 A. M.

Session on Complex Analysis III, Room 108 - Reed Hall

10:30-10:40 (161) The fundamental group of the space of moduli. Preliminary report. Professor DAVID PATTERSON, Stevens Institute of Technology (696-30-9)

10:45-10:55 (162) A note on the geometric means of entire functions of two complex variables. Dr. ARUN KUMAR AGARWAL, Grambling College (696-32-1)

11:00-11:10 (163) The bounded extension problem. Discs in polydiscs. Professor EDGAR LEE STOUT, University of Washington (696-32-2)

11:15-11:25 (164) Branched projective structures and flat vector bundles on Riemann surfaces. Preliminary report. Dr. RICHARD MANDELBAUM, University of Massachusetts (696-32-3)

11:30-11:40 (165) Generalization of the Bloch theorem for meromorphic mappings on the hyperball. Preliminary report. Professor KYONG T. HAHN, Pennsylvania State University (696-32-4)

FRIDAY, 10:30 A. M.

Session on Differential Equations, Room 104 - Reed Hall

10:30-10:40 (166) Existence theorems for two-point boundary value problems for second order non-linear differential systems. Preliminary report. Professor STEPHEN R. BERNFELD, University of Missouri-Columbia, and Professor GANGARAMS S. LADDE* and Professor V. LAKSHMIKANTHAM, University of Rhode Island (696-34-1)

10:45-10:55 (167) On bounded perturbations of controllable systems. Professor JERALD P. DAUER, University of Nebraska (696-49-1)

11:00-11:10 (168) The Cauchy problem for the quasi linear wave equation. Dr. JAMES M. GREENBERG, State University of New York at Buffalo (696-35-1) (Introduced by Professor Nicholas D. Kazarinoff)

11:15-11:25 (169) A representation theorem for solutions of parabolic equations with discontinuous coefficients. Preliminary report. Dr. NEIL EKLUND, Vanderbilt University (696-35-2)

11:30-11:40 (170) On the nonpropagation of zero sets of solutions of homogeneous linear partial differential equations across noncharacteristic hyperplanes. Professor DAVID K. COHOON, University of Minnesota (696-35-4)

11:45-11:55 (171) Existence and uniqueness results for some possibly noncoercive evolution problems with regular data. Professor TAPAS MAZUMDAR, Wright State University (696-35-3)

FRIDAY, 10:30 A. M.

Session on Operator Theory, Room 204 - Wentworth Hall

10:30-10:40 (172) Wave and scattering operators for some second order elliptic operators in R^m, $m \geqq 3$. Preliminary report. Dr. GOSULA NARAYANA REDDY, Universidade Federal da Bahia, Salvador, Bahia, Brasil (696-47-1)

10:45-10:55 (173) Subnormal operators in strictly cyclic algebras. Dr. RICHARD BOLSTEIN* and Dr. WARREN R. WOGEN, University of North Carolina at Chapel Hill (696-47-2)

11:00-11:10 (174) Abstract Volterra operators. Preliminary report. Mr. KAI-JAUNG PEI, University of Illinois at Chicago Circle (696-47-3)

11:15-11:25 (175) Linear approximation of nonlinear semigroups. Preliminary report. Professor D. J. HEBERT, University of Pittsburgh (696-47-4)

FRIDAY, 10:30 A. M.

Session on Topology IV, Room 307 - Wentworth Hall

10:30-10:40 (176) Spanning surfaces of companioned knots. Professor WILBUR WHITTEN, Institute for Advanced Study (696-55-1)

10:45-10:55 (177) On branched coverings of knots and links. Professor S. KINOSHITA, Florida State University (696-55-2)

11:00-11:10 (178) On the homotopy type of irregular sets. Professor PAUL F. DUVALL, JR., Oklahoma State University, and Professor LAWRENCE S. HUSCH*, University of Tennessee (696-57-1)

11:15-11:25 (179) Concordances and isotopies. Professor KENNETH C. MILLETT, University of California, Santa Barbara (696-57-2)

11:30-11:40 (180) The missing boundary theorem for 3-manifolds. Mr. THOMAS W. TUCKER, Princeton University (696-57-3)

11:45-11:55 (181) Isotopy classification of homeomorphisms of a 2-sphere with n holes. Preliminary report. Dr. DAVID J. SPROWS, Villanova University (696-57-4) (Introduced by Professor August A. Sardinas)

FRIDAY, 1:30 P. M.

Invited Address, Center Theater, H. C.

(182) A general theory of identities of the Rogers-Ramanujan type. Professor GEORGE E. ANDREWS, Pennsylvania State University (696-10-10)

FRIDAY, 1:30 P. M.

Invited Address, Spaulding Auditorium, H. C.

(183) Submanifolds of Riemannian manifolds. Professor CHUAN C. HSIUNG, Lehigh University (696-53-2)

FRIDAY, 1:30 P. M.

Special Session on Asymptotic and Perturbation Methods in Fluid Mechanics and Wave Propagation, Hartman Rehearsal Hall, H. C.

1:30- 1:50 (184) Representations of solutions as sums of poles. Professor MARTIN D. KRUSKAL, Princeton University (696-76-4)

2:00- 2:20 (185) Analysis of some problems having matched asymptotic expansion solutions. Professor FRANK HOPPENSTEADT, Courant Institute, New York University (696-76-2)

2:30- 2:50 (186) A stability problem for the Sine-Gordon equation. Professor ANDREW J. CALLEGARI* and Professor EDWARD L. REISS, Courant Institute, New York University (696-35-5)

3:00- 3:20 (187) Inverse scattering problems. Professor REESE T. PROSSER, Dartmouth College (696-81-1)

3:30- 3:50 (188) Multiple-scale theory of hydromagnetic dynamos. Professor W. STEPHEN CHILDRESS, Courant Institute, New York University (696-86-1)

4:00- 4:20 (189) Propagation of a laser beam through a strongly fluctuating random medium. Professor PAO-LIU CHOW, Courant Institute, New York University (696-78-1)

4:30- 4:50 (190) Semiresonant interactions and frequency dividers. Professor MARK ABLOWITZ, Clarkson College of Technology (696-94-2)

5:00- 5:20 (191) Nonlinear propagation of surface waves. Professor M. C. SHEN, University of Wisconsin (696-76-3)

FRIDAY, 2:45 P. M.

Invited Address, Spaulding Auditorium, H. C.

(192) Noncommutative localization. Professor JOACHIM LAMBEK, McGill University (696-16-4)

FRIDAY, 2:45 P. M.

Invited Address, Center Theater, H. C.

(193) Representation of non-Desarguesian projective planes in projective hyperspace. Professor R. C. BOSE, Colorado State University

FRIDAY, 4:00 P. M.

Invited Address, Center Theater, H. C.

(194) Dynamical systems, filtrations, and entropy. Professor MICHAEL SHUB, University of California, Santa Cruz (696-58-2)

Walter H. Gottschalk
Associate Secretary

Middletown, Connecticut

NEWS ITEMS AND ANNOUNCEMENTS

ALBERT EINSTEIN AWARD

The Albert Einstein Award, presented by the Lewis and Rosa Strauss Memorial Fund, was awarded this year to Eugene P. Wigner, professor emeritus of mathematical physics at Princeton University, for his contributions to the natural sciences. He was an instigator of the World War II Manhattan Project and a principal mover in the application of physics to atomic energy.

MATHEMATICAL SCIENCES AT THE JOHNS HOPKINS UNIVERSITY

The Johns Hopkins University announces the creation of a Department of Mathematical Sciences. This new department will conduct graduate and undergraduate programs in the broad area of modern applied mathematics including such fields as statistics, probability, discrete and continuous optimization, control theory, game theory, and numerical analysis. The graduate program will emphasize solid training in mathematics and exposure to several of the above areas before the beginning of specialized work and thesis research.

WORLD DIRECTORY OF HISTORIANS OF MATHEMATICS

The World Directory of Historians of Mathematics is now available from Historia Mathematica, Department of Mathematics, University of Toronto, Toronto 181, Canada. The Directory contains seven hundred names and addresses of mathematicians indexed by forty countries and three hundred research specialties. The price is \$3.00 when payment accompanies the order and \$4.00 otherwise.

PRELIMINARY ANNOUNCEMENTS OF MEETINGS

The Six Hundred Ninety-Seventh Meeting
Brown University
Providence, Rhode Island
October 28, 1972

The six hundred ninety-seventh meeting of the American Mathematical Society will be held at Brown University, Providence, Rhode Island, on Saturday, October 28, 1972. The sessions of the meeting will be held in Barus-Holley Science Building which is located near the corner of George and Hope Streets.

It is expected that there will be two one-hour addresses. The names of the lecturers and the titles of their addresses will appear in the October issue of these *Notices*.

There will be sessions for contributed papers both morning and afternoon. Abstracts for contributed papers should be sent to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904, so as to arrive prior to the deadline of September 7, 1972. No provision will be made for late papers.

Registration will be held on the first floor of Barus-Holley. The registration desk will be open from 9:00 a.m. to 12:00 noon and from 1:30 p.m. to 3:30 p.m. Parking space will be available in the George Street parking lot located at the corner of George and Brook Streets.

There are several hotels and motels in Providence, some of which are listed below:

BILTMORE HOTEL (center of city, 1/2 mile from campus)
Dorrance Street
(401) 421-9200

Singles	\$13-\$16.50
Doubles	18- 21.50

COLONIAL HILTON INN (5 miles from campus)
Route 1A, Cranston
(401) 467-8800

Singles	\$16-\$20
Doubles	21- 25

HOLIDAY INN (Downtown) (adjacent to Interstate 95, 3/4 mile from campus)
21 Atwells Avenue
(401) 831-3900

Singles	\$16
Doubles	22

WAYLAND MANOR (east side of city, 1/2 mile from campus)
500 Angell Street
(401) 751-7700

Singles	\$12-\$18
Doubles	18- 25

Participants should write or telephone hotels or motels if reservations are desired.

Luncheon will be served in the Ivy Room located in the Sharpe Refectory at the corner of George and Thayer Streets. The entrance to the Refectory is on George Street. A list of local restaurants will be available at the registration desk.

Providence is served by Allegheny Airlines, American Airlines, Eastern Airlines, National Airlines, and United Airlines.

Limousine service is available from the airport to the downtown area with stops at the Holiday Inn (Downtown) and the Biltmore Hotel. The charge for this service is about \$1.75.

Regular passenger rail service is available from Boston and New York; bus service is available with some frequency from most major cities in New England as well as from New York. Both the bus station and the railroad station are located in the center of the city approximately one block from the Biltmore Hotel.

Those coming by car should leave Interstate 95 at the Interstate 195, East, exit. They should then take the Downtown Providence exit and proceed north on Dyer Street to the traffic light; continue through the traffic light across to South Main Street and turn left. Turn right off South Main onto College Street, right onto Benefit Street, left onto George Street, and continue up George to the corner of Hope Street.

The campus may be reached from the downtown area by bus, namely, buslines No. 41, "Elmgrove-Tunnel," No. 38B, "Rumford-Tunnel," or No. 40, "Butler-Tunnel." Each bus runs every half hour from the corner of Dorrance and Westminster Streets (center of the city and two blocks from the Biltmore Hotel); a detailed schedule may be obtained by telephone from the Rhode Island Transit Authority (781-9400). The best stop for access to Barus-Holley is at the corner of Hope and Waterman Streets. Barus-Holley is located two blocks south of the corner (turn right after getting off bus).

The headquarters of the American Mathematical Society, located at 321 South Main Street, will be open on Saturday from 10:15 a.m. to 11:45 a.m. and from 1:30 p.m. to 3:00 p.m. Participants are invited to tour the Society's office at their convenience; directions to the building will be posted in the lobby of Barus-Holley Science Building.

Walter H. Gottschalk
Associate Secretary

Middletown, Connecticut

The Six Hundred Ninety-Eighth Meeting
University of California, San Diego
La Jolla, California
November 18, 1972

The six hundred ninety-eighth meeting of the American Mathematical Society will be held at the University of California, San Diego, in La Jolla, California, on Saturday, November 18, 1972.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, there will be two invited addresses. They will be given by Professors Marc A. Rieffel and Ichiro Satake, both of the University of California, Berkeley. The titles of their addresses will appear in the October issue of these *Notices*.

There will be sessions for contributed papers. Abstracts for contributed papers should be sent to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904, so as to arrive prior to the deadline of October 5, 1972. Late papers will be accepted for presentation at the meeting, but late papers will not be listed in the printed program of the meeting.

Kenneth A. Ross
Associate Secretary

Eugene, Oregon

The Six Hundred Ninety-Ninth Meeting
University of North Carolina
Chapel Hill, North Carolina
November 24-25, 1972

The six hundred ninety-ninth meeting of the American Mathematical Society will be held at the University of North Carolina at Chapel Hill, Chapel Hill, North Carolina, on Friday and Saturday, November 24-25, 1972.

By invitation of the Committee to Select Hour Speakers for Southeastern Sectional Meetings, there will be two one-hour addresses, both of which will be held in the Peabody Hall Auditorium. Professor James K. Brooks of the University of Florida will give an address entitled "Measure and integration theory in Banach spaces." An address entitled "Some results on polynomial rings over a commutative ring" will be given by Professor Robert Gilmer of Florida State University.

There will be sessions for contributed papers both Friday afternoon and Saturday morning. Abstracts should be sent to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904, so as to arrive prior to the deadline of October 5, 1972.

There will be two special sessions held in addition to the regular sessions. Professor R. D. Anderson of Louisiana State University is arranging a session on Infinite Dimensional Topology. The speakers will include T.A. Chapman, Douglas Curtis, Ross Geoghegan, William K. Mason, R.M. Schori, R. Richard Summerhill, James E. West, Raymond Y.-T. Wong. Professor Christopher Hunter of Florida State University is arranging a session on New Areas for Applied Mathematics (i.e., areas other than those relating to Physics and Engineering). The slate of speakers will be announced later. In the past a few contributed papers have been chosen by the organizing chairmen of special sessions to be added to the program. Anyone desiring to have a paper so considered should have his abstract in the Providence office by September 14. The abstract should be plainly marked for consideration for twenty-minute papers.

The registration desk will be located in Phillips Hall. Registration hours will be 10:00 a.m. to 5:00 p.m. on Friday, November 24, and 9:00 a.m. to 12:00 noon on Saturday, November 25. The sessions will be held in Phillips Hall and Peabody Hall.

Chapel Hill is located on U.S. 15-105 and is also accessible from Interstate 85 via N.C. 86. It is 17 miles from the Raleigh-Durham Airport, which is served by Eastern, United, Delta, and Piedmont airlines. Limousine service is available from the airport to the Carolina Inn. The limousine fare is \$3.25 and the trip takes approximately 40 minutes. Taxis are also available from the Tarheel Cab Company at \$8.00 one way for one person, or \$4.00 each for two or more persons.

Meals are available only at commercial establishments. The cafeteria in the Carolina Inn will be available for lunch and dinner, and the dining room will be open for all three meals.

A block of rooms is being reserved in the Carolina Inn with November 10 as deadline. Requests for reservations should be made directly to the hotel.

CAROLINA INN
West Cameron Avenue
(one block from Phillips Hall)
Phone: (919) 933-2001

Single (one person per room)	\$ 9.00 - \$15.00
One Double Bed (two persons per room)	11.00 - 14.00

Two Beds 12.00 - 19.00
(two persons per room)

HOLIDAY INN
15-105 Bypass
(three miles from campus)
Phone: (919) 929-2171

Single (one person per room)	$13.00
One Double Bed (two persons per room)	17.00
Two Beds (two persons per room)	19.00

UNIVERSITY MOTEL
Raleigh Road
P.O. Box 2118
(one mile from campus)
Phone: (919) 942-4132

Single (one person per room)	$10.50 - $14.00
One Double Bed (two persons per room)	14.00
Two Beds (two persons per room)	14.00 - 18.00

All reservations should be made directly with the hotel and motels as soon as possible.

Messages may be left for delivery by phoning (919) 933-2028.

O. G. Harrold
Associate Secretary

Tallahassee, Florida

The Seven Hundreth Meeting
Case Western Reserve University
Cleveland, Ohio
November 25, 1972

The seven hundredth meeting of the American Mathematical Society will be held at Case Western Reserve University, Cleveland, Ohio, on Saturday, November 25, 1972.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, there will be two one-hour addresses. Professor Calvin R. Putnam of Purdue University will address the Society at 11:00 a.m. Professor Mary-Elizabeth Hamstrom of the University of Illinois will speak at 1:45 p.m. The titles of both lectures will be given in the October issue of these *Notices*.

There will be sessions for the presentation of contributed ten-minute papers both morning and afternoon. Those having time preference for the presentation of papers should so indicate on their abstracts. Abstracts should be submitted to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904, so as to arrive prior to the deadline of October 5, 1972. There will be a session for late papers if one is needed, but late papers will not be listed in the printed program of the meeting.

There will be two special sessions of selected twenty-minute papers. Professor Lamberto Cesari of the University of Michigan is arranging one such session on the subject of Optimization Theory and Optimal Control; the list of speakers will include Lamberto Cesari, Henry G. Hermes, Edward J. McShane, and Lucien W Neustadt. The other special session is being arranged by Professor Alan C. Woods of the Ohio State University, on the subject of the Geometry of Numbers. Most of the papers presented at these two sessions will be by invitation. Anyone contributing an abstract for the meeting, however, who feels that his paper would be particularly appropriate for one of these special sessions should indicate this emphatically on his abstract and submit it three weeks earlier than the above deadline, namely by September 14, 1972, in order to allow time for the additional handling necessary.

On Friday, November 24, 1972, the day before the meeting itself, Case Western Reserve University will sponsor a brief symposium on the subject of Optimization Theory and Optimal Control. The special session on Optimization Theory and Optimal Control mentioned earlier will be an extension of this symposium.

One-hour invited addresses will be given by Professors Wendell Fleming (Brown University), R. E. Kalman (University of Florida) and R. V. Gamkrelidze (Steklov Institute, Moscow).

Detailed information about travel and accommodations will appear in the October issue of these *Notices*. There is a Howard Johnson's Motor Inn adjacent to the campus of Case Western Reserve University, and the Sheraton-Cleveland Hotel is ten minutes away by public transit. A beer party is planned on Friday evening.

Paul T. Bateman
Associate Secretary

Urbana, Illinois

Session of Contributed Papers in Biomathematics
Washington, D. C.
December 27, 1972

A one-day session of contributed papers in Biomathematics will be held on Wednesday, December 27, 1972, at a site in Washington, D. C. to be announced. This session is cosponsored by the American Mathematical Society and the Society for Industrial and Applied Mathematics, and is being held in cooperation with Section A (Mathematics) of the American Association for the Advancement of Science.

Contributed papers will be presented at both morning and afternoon sessions. Those persons desiring to present a paper should submit an abstract, on a standard AMS abstract form, to Professor Jack D. Cowan, Chairman, Department of Theoretical Biology, The University of Chicago, 939 East 57th Street, Chicago, Illinois 60637. Abstracts should be mailed so as to arrive prior to the deadline of September 7, 1972. Those persons having time preferences for the presentation of their papers should so indicate on their abstracts.

The program will be arranged by a committee selected by the presidents of the American Mathematical Society and the Society for Industrial and Applied Mathematics. A complete program of the sessions will be included in the November issue of these *Notices*.

Jack D. Cowan, Chairman
AMS-SIAM Committee on
Mathematics in the Life Sciences
Chicago, Illinois

Special News Item

Wilkes College lost its entire library, including 7,000 mathematics books and 95 journals, in the flood accompanying Hurricane Agnes. Wilkes is a liberal arts college with a masters program in mathematics; they cover pure mathematics, applied mathematics, and computer science. At present, the department has 120 mathematics majors and between 30 and 35 graduate students. Books at the junior, senior, and first-year graduate levels are desperately needed. Books should be sent to Prof. Bing Kuen Wong, Chairman, Dept. of Mathematics, Wilkes College, Wilkes-Barre, Pa. 18703.

MEMORANDA TO MEMBERS

CONTRIBUTING MEMBERS

The Society acknowledges with gratitude the support rendered by the contributing members, who pay a minimum of $30 per year in dues. The extra dues paid by these members provide vital support to the work of the Society. In addition to those on the following list, six contributing members asked that their names remain anonymous.

Abbott, James H.
Adams, J. Frank
Amir-Moez, Ali R.
Anderson, Richard D.
Andrews, Donald H.
Andrews, George E.
Apostol, Tom M.
Arenstorf, Richard F.
Aroian, Leo A.
Ayer, Miriam C.
Babcock, William W.
Ballou, Donald H.
Bauer, Frances B.
Baumslag, Gilbert
Beals, Richard W.
Beck, William A.
Beckenbach, Edwin F.
Beesley, E. Maurice
Bennett, Ralph B.
Bennewitz, William C.
Berg, Kenneth R.
Bernstein, Leon
Bharucha-Reid, A. T.
Bing, R. H.
Borisewich, John
Botts, Truman A.
Brauer, George U.
Brickman, Louis
Bristol, Edgar H.
Brobeck, Barbara C.
Brunswick, Natascha A.
Bryan, William S.
Bryant, Jack D.
Burington, Richard S.
Burke, James E.
Burt, Howard H.
Carson, Albert B.
Carter, Joan Cooley
Caywood, Thomas E.
Chellis, Thomas W.
Clark, Harry E.
Clemens, Charles H.
Clifford, Alfred H.
Cohen, Henry B.
Cohen, Teresa
Coleman, A. John
Collins, Heron S.
Colson, Henry D.
Cook, E. Allen, Jr.
Cooke, Roger Lee
Cornelius, Eugene F., Jr.
Cowan, John C., III
Coxeter, H. S. MacDonald
Cunkle, Charles H.
Danskin, John M.
Daus, Paul H.
Davis, Robert D.
DeFacio, Brian
DeFrancesco, Henry F.
De Marr, Ralph E.
Devault, John L.
Dinneen, Gerald P.
Donoghue, William F., Jr.
Durst, Lincoln K.
Eachus, J. J.
Earle, Clifford J., Jr.
Eisele, Carolyn
Ellis, James W.
Embree, Earl O.
Epstein, Irving J.
Evans, George W., II
Fair, Wyman G.
Farrell, Roger H.
Fass, Arnold L.
Feustel, Charles D.
Findley, George B.
Fine, Nathan J.
Francis, Eugene A.
Fuller, Leonard E.
Galant, David
Garrison, George N.
Gillman, Leonard
Gilmer, Robert
Gordon, Hugh
Gottschalk, Walter H.
Gould, Henry W.
Grace, Edward E.
Graves, Robert L.
Green, John W.
Greene, Peter H.
Greif, Stanley J.
Guggenbuhl, Laura
Hacker, Sidney G.
Halpern, James D.
Hamilton, Norman T.
Hardy, F. Lane
Harris, Charles D.
Hart, William L.
Hashisaki, Joseph
Heard, Melvin L.
Hendrickson, Morris S.
Herwitz, Paul S.
Hilt, Arthur L.
Hochstadt, Harry
Hodges, John H.
Howell, James L.
Huff, Melvyn E.
Hufford, George A.
Hukle, George W.
Humphreys, M. Gweneth
Hunt, Burrowes
Hutchinson, George A.
Ingraham, Mark H.
Isbell, John R.
Jackson, Stanley B.
James, R. D.
Jarnagin, Milton P., Jr.
Jeffers, Lionel N.
Katzin, Martin
Kauffman, Robert M.
Keisler, James E.
Kelly, John B.
Kiernan, Bryce M.
Kirk, Joe E., Jr.
Kist, Joseph E.
Kohls, Carl W.
Koss, Walter E.
Kossack, C. R.
Kunen, Kenneth
Lanczos, Cornelius
Laning, J. H.
Laush, George
Lawrence, Sidney H.
Lee, Philip F.
Lemay, William H.
Levinson, Norman
Lewis, Hugh L.
Ling, Donald P.
Lipman, Joseph
Lubben, R. G.
Macy, Josiah, Jr.
Madi-Raj, Hagzl-Rao V.
Mamelak, Joseph S.
Mansfield, Maynard J.
Mansfield, Ralph
Marchand, Margaret O.
Maskit, Bernard
Mattson, H. F., Jr.
Mayor, John R.
McBrien, Vincent O.
McIntosh, William D.
McLeod, Robert M.
McNaughton, Robert
McNeill, Robert B.
Meder, Albert E., Jr.
Miller, Harlan C.
Miller, W. F.
Mitchell, Alfred K.
Mizel, Victor J.
Moore, Hal G.
Morrey, Charles B., Jr.
Morse, Anthony P.
Moursund, Andrew F.
Muller, David E.
Mullikin, Thomas W.

Nelson, Eric J.
Newhouse, Sheldon E.
Nohel, John A.
Norman, Edward
Norris, Donald O.
Norris, Eugene M.
Norton, Karl K.
Offenbacker, Robert E.
Orloff, Leo N.
Outcalt, David L.
Paige, Eugene C.,Jr.
Paige, Lowell J.
Palais, Richard S.
Pate, Robert S.
Peabody, Mary K.
Pearson, Robert W.
Pell, William H.
Persinger, Carl A.
Pflaum, C. W.
Plivka, Andrew D.
Poe, Robert L.
Potter, Meredith W.
Purzitsky, Norman
Quade, Edward S.
Raudenbush, H. W.
Reardon, Philip C.
Redheffer, Raymond M.
Rees, Carl J.
Rees, Mina S.
Reid, James D.
Rich, Ellis J.
Rinehart, Robert F.
Riney, John S.
Riordan, John
Roberts, J. H.
Rochon, Lloyd J.
Rose, Donald C.
Rosenblum, Marvin
Roth, Emile B.
Rovnyak, James L.
Russak, Ira Bert
Russo, Bernard
Sampson, Charles H.
Santosuosso, Giulio
Sawyer, Stanley A.
Saxon, Stephen A.
Schoenberg, Isaac J.
Schurrer, Augusta L.
Scott, Dana S.
Scott, Walter T.
Seligman, George B.
Sexauer, Norman E.
Shanks, Merrill E.
Sheffer, Isador M.
Shiffman, Max
Sinke, Carl J.
Sinn, Frederick W.,Jr.
Sloan, Thomas D.
Smith, Duane B.
Smith, P. A.
Sorgenfrey, Robert H.
Sternberg, David
Stolberg, Harold J.
Stone, Lawrence D.
Sudler, Culbreth, Jr.
Sweigart, James R.
Tellefsen, Carl R.
Thompson, Claude C.
Thompson, James M.
Thompson, Layton O.
Tsao, Sherman
Tucker, Albert W.
Turquette, Atwell R.
Vallarta, M. S.
Walker, James R.
Wallach, Sylvan
Walsh, Joseph L.
Weiss, Paul
Wendroff, Burton
Weyl, F. Joachim
Wheeler, Lyle Mary
White, George N.,Jr.
Whitmore, William F.
Whitney, D. Ransom
Whitney, Hassler
Whittaker, James V.
Wilf, Herbert S.
Wilkins, J. Ernest W.,Jr.
Winkel, Brian J.
Woeppel, James J.
Wolman, Eric
Wonenburger, Maria J.
Zemel, Jacqueline L.
Zink, Robert E.

TO THE MEMBERS OF THE AMERICAN MATHEMATICAL SOCIETY

This letter is to inform the members of the American Mathematical Society of the serious financial crisis faced by the Mathematical Reviews which may threaten the very existence of this journal—at least in its present form. The Trustees of the Society have budgeted a deficit for MR of approximately $90,000 for 1972 to be met out of general funds. The preliminary budget for 1973 contained a projected deficit of approximately $290,000. The large increase in the cost of publishing MR has two main sources: increased unit cost of production, particularly for printing, and the very large increase in the number of reviews which must be produced to keep abreast of the literature. For 1972 approximately 16,500 reviews will be printed and it is estimated that this will have to be increased to about 25,000 for 1973 (including in part a carryover of a backlog from 1972). This figure is based on current policies for covering the mathematical literature which are recognized to be inadequate in coverage of applied mathematics. Should the recommendations of a committee on coverage of applied mathematics, headed by Professor Ralph Boas, be followed in 1973 the number of reviews would have to be increased to 40,000!

The President, on the recommendation of the Treasurer, has recently appointed a Crisis Committee for Mathematical Reviews. The Chairman of this committee is Professor Boas. While it has not yet had the time to produce a long range program which it is hoped will insure the continuance of MR and perhaps increased coverage of applied mathematics, the committee has made some preliminary recommendations which were considered at a recent joint meeting of the Executive Committee and the Board of Trustees of the Society. Further studies and recommendations will be made by the Crisis Committee acting jointly with the Editorial Board for MR. Of the recommendations made thus far the only one which seemed likely to offer short term relief for MR's financial imbalance was an increase in subscription rates. This recommendation was adopted by the Board of Trustees. The new rates are eminently fair in view of the growth of the journal. In fact, a comparison of individual members' subscription rates and number of reviews indicates clearly that MR is a much better bargain now than it was in 1940, since the number of reviews which will be printed in 1972 is 8.2 times that of 1940, whereas members' cost in 1972 is 6.2 times the 1940 figure. These two ratios for 1973 vs 1940 will be twelve and 9.2 respectively. Moreover, this does not take into account the inflation which has occurred. Nevertheless, no one can predict what will be the outcome of the increase in subscription rates since there is a real possibility that tight library budgets may result in some cancellation of subscriptions.

MR was founded in 1940 at a time when the Hitlerian oppression was threatening the scientific life of Germany and had begun to have an adverse effect on its mathematical journals. It was felt at the time by many that it was necessary to have a new review journal published outside of Germany. MR has had a remarkably distinguished career since its inception and it has become the best review journal for mathematics and the most important journal published by AMS. Mathematicians throughout the world would regard it a major calamity if financial problems were to force this journal out of existence.

I am now appealing to the membership for help to alleviate the financial crisis of MR. One way members (and other users of MR) can help is by direct contributions to a Mathematical Reviews Fund. These could be for immediate use as MR income. Moreover, since it is clear that the problem will persist for a number of years, pledges for contributions for a number of years will be extremely useful. Also new individual subscriptions are strongly urged. Another important way mathematicians can help is to impress librarians of their institutions with the importance of MR in order to insure continuance of existing subscriptions to this journal. The libraries should be urged to give the highest priority to MR and to economize by cancelling subscriptions to some primary journals if this should become absolutely necessary.

It is our intention to keep the membership informed on future developments in this crisis. Suggestions on ways of solving this problem will be welcomed by the Crisis Committee.

N. Jacobson
President

LETTERS TO THE EDITOR

Editor, the *Notices*

B. E. Rhoades, in the American Mathematical Monthly, April, 1972, criticizes the lack of qualitative information in letters of recommendation we write for our students. This lack of information is particularly noticeable in files from placement offices. Job seekers should be warned against reliance on these files, and we should supply information for them reluctantly, if at all. A letter dated 1957 can be of little use to anyone in assessing a candidate's current capabilities. A professor who checks the box indicating that he knew a student slightly and then says, "I would give his expectation of success a high rating and recommend him to any possible employer," does harm to himself and the student.

H. E. Reinhardt

Editor, the *Notices*

I am attempting to make an up-to-date report on the status of the 26 problems in the book KNOT GROUPS (Annals of Mathematics Studies, Study 56, Princeton University Press, 1965). I would appreciate it if anyone having any information whatsoever on results pertinent to these problems would communicate them to me at 100 Prospect Avenue, Princeton, New Jersey 08540.

Lee P. Neuwirth

SPECIAL MEETINGS INFORMATION CENTER

The purpose of this center is to maintain a file on prospective symposia, colloquia, institutes, seminars, special years, meetings of other associations, and to notify the organizers if conflicts in subject matter, dates or geographical area become apparent. A first announcement will be published in the *Notices* if it contains a call for papers, place, date, and subject, where applicable; a second announcement must contain reasonably complete details of the meeting in order for it to be published. Information on the pre-preliminary planning will be stored in the files, and will be available to anyone desiring information on prospective conferences. All communications on special meetings should be sent to the Special Meetings Information Center of the American Mathematical Society.

August 14–18, 1972
CONFERENCE ON NUMBER THEORY
University of Colorado, Boulder, Colorado
Program: 20 invited half-hour lectures and a limited number of contributed papers
Support: NSF (for limited number of invited speakers)
Information: Professors John H. Hodges and Wolfgang M. Schmidt, University of Colorado, Boulder, Colorado 80302

August 15-24, 1972
COLLOQUIUM ON ANALYSIS
Federal University of Rio de Janeiro, Rio de Janeiro, Brazil
Program: Lectures by K. G. Andersson, Lund Universitets; K. Bierstedt, Universität Mainz; J. Blatter, Universität Bonn; H. Brézis, Université de Paris; R. Carroll, University of Illinois; G. Coeuré, Université de Nancy; B. Coleman, Carnegie-Mellon University; M. De Wilde, Université de Liège; M. Dostal, Stevens Institute of Technology; H. G. Garnir, Université de Liège; J. A. Goldstine, Tulane University; C. P. Gupta, Northern Illinois University; L. Harris, University of Kentucky; N. D. Kazarinoff, SUNY at Buffalo; P. Lelong, Université de Paris; J. L. Lions, Université de Paris; L. Narici, St. John's University; E. Peschl, Universität Bonn; U. Richard, Università di Padova; C. E. Rickart, Yale University; M. Schottenloher, Universität Munchen; S. Simmons, University of California, Santa Barbara; W. H. Summers, University of Arkansas; F. Trèves, Rutgers University. Sessions for research announcements.
Sponsors: Universidade Federal do Rio de Janeiro, Conselho Nacional de Pesquisas, CAPES, Secretaria de Ciência e Tecnologia do Estado da Guanabara, COPPE/UFRA
Information: Professor Mario C. De Matos, Instituto de Matemática, Universidade Federal do Rio de Janeiro, Caixa Postal 1835, ZC-00, 20000-Rio de Janeiro, GB, Brazil

August 29–30, 1972
PI MU EPSILON SUMMER MATHEMATICS MEETING
Dartmouth College, Hanover, New Hampshire
Program: Two sessions of contributed papers. John Kemeny will address the members on "Mathematical models and the computer." Traditional banquet and Dutch-treat breakfast. Complete program of these sessions is included in the timetable on page 217 of this issue of these *Notices*.
Information: Professor Richard Andree, Department of Mathematics, University of Oklahoma, Norman, Oklahoma 73069

August 28–30, 1972
MATHEMATICAL ASSOCIATION OF AMERICA 53rd SUMMER MEETING
Dartmouth College, Hanover, New Hampshire
Program: Twenty-first series of Earl Raymond Hedrick Lectures to be delivered by Professor Peter Lax of the Courant Institute of Mathematical Sciences, New York University on "Scattering theory." Panel discussions on the following subjects: Student Self-Paced Calculus, The President's Commission on Federal Statistics, and Beyond (joint with IMS), and Mathematics and the Social Sciences, Its Place in the University. Invited addresses presented by Professor Felix Browder and Dr. John G. Kemeny. Complete program of the meeting is included in the timetable on page 217 in this issue of these *Notices*.
Information: The Mathematical Association of America, 1225 Connecticut Avenue, N.W., Washington, D.C. 20036

August 28–31, 1972
INSTITUTE OF MATHEMATICAL STATISTICS 35th ANNUAL MEETING
Dartmouth College, Hanover, New Hampshire
Program: Wald Lectures to be delivered by Professor Peter J. Huber of ETH, Zürich on "Robustness." Special invited papers by Professors Peter J. Bickel (University of California, Berkeley), Thomas M. Cover (Massachusetts Institute of Technology and Harvard University), and Thomas S. Ferguson (University of California, Los Angeles). Professor A. Dvoretzky, Hebrew University, Jerusalem, will speak at a memorial session for Paul Lévy. A panel discussion entitled "The President's Commission on Federal Statistics, and Beyond" will be sponsored jointly with MAA. Five sessions for contributed papers. Complete program of the meeting is included in the timetable on page 217 in this issue of these *Notices*.
Information: Professor Robert Wijsman, Program Chairman, Department of Mathematics, University of Illinois, Urbana, Illinois 61801

September 5–8, 1972
CONGRÈS EUROPÉEN DE LA SOCIÉTÉ D'ECONOMÉTRIE
Budapest, Hungary
Information: Mme. Jalat, Econométrie, Tour 46, Université de Paris VI, 11 quai Saint-Bernard, 75-Paris, France

September 11–13, 1972
ADVANCED SEMINAR ON MATHEMATICAL PROGRAMMING
Mathematics Research Center, University of Wisconsin, Madison, Wisconsin
Program: Ten invited addresses. Speakers and session chairmen: M. L. Balinsky, City University of New York; G. B. Dantzig, Stanford University; E. V. Denardo, Yale University; D. R. Fulkerson, Cornell University; R. F. Garfinkel, University of Rochester; R. E. Gomory, IBM Research Center; E. A. Johnson, IBM Research Center; R. M. Karp, University of California, Berkeley; C. E. Lemke, Rensselaer Polytechnic Institute; W. F. Lucas, Cornell University; G. L. Nemhauser, Cornell University; L. S. Shapley, The RAND Corporation; A. W. Tucker, Princeton University; P. Wolfe, IBM Research Center; R. D. Young, Rice University
Information: Professor T. C. Hu or Dr. S. M. Robinson, c/o Mrs. Gladys G. Moran, Mathematics Research Center, University of Wisconsin, 610 Walnut Street, Madison, Wisconsin 53706

October 2–3, 1972
COLLOQUE: MODÈLE D'ENSEIGNEMENT, ÉVALUATION ET CALCULATEUR
Paris, France
Information: Institut de Recherche d'Informatique et d'Automatique, Domaine de Voluceau, 78-Rocquencourt, France

October 3–5, 1972
USA—JAPAN COMPUTER CONFERENCE
Tokyo, Japan
Sponsors: American Federation of Information Processing Societies, Inc., and Information Processing Society of Japan
Information: USA—Japan Computer Conference, c/o AFIPS Headquarters, 210 Summit Avenue, Montvale, New Jersey 07645

October 16–17, 1972
COMPUTER SCIENCE AND STATISTICS: SIXTH ANNUAL SYMPOSIUM ON THE INTERFACE
University of California, Berkeley, California
Program: Keynote speaker: J. W. Tukey. Six workshops
Sponsors: Division of Measurement Sciences and University of California, Berkeley, Extension
Information: Dr. Michael E. Tarter, Interface Chairman, Division of Measurement Sciences, School of Public Health, University of California, Berkeley, California 94720

November 20–23, 1972
CONFERENCE ON ABELIAN GROUPS
Rome, Italy
Information: Professor Giuseppe Scorza, Istituto Nazionale di Alta Matematica, Città Universitaria, Roma, Italy

December 12–15, 1972
CONFERENCE ON THE THEORY OF GROUP REPRESENTATIONS
Rome, Italy
Information: Professor Giuseppe Scorza, Istituto Nazionale di Alta Matematica, Città Universitaria, Roma, Italy

January 1–7, 1973
INTERNATIONAL CONFERENCE ON FUNCTIONAL ANALYSIS AND ITS APPLICATIONS
Madras, India
Program: Sessions on Topological Vector Spaces, Approximation Theory, Harmonic Analysis, Partial Differential Equations. One-hour invited lectures, half-hour lectures, and ten-minute communications
Sponsor: MATSCIENCE—The Institute of Mathematical Sciences, India
Information: Professor K. R. Unni, MATSCIENCE, Madras-20, India

January 1–7, 1973
SECOND ANNUAL CASAMCU RESEARCH SYMPOSIUM
University College of Science and Technology, Calcutta, India
Program: Theories of plasma and assemblies of charged particles
Contributed papers: Deadline for papers and their abstracts: November 21, 1972
Sponsors: Centre of Advanced Study in Applied Mathematics of Calcutta University in collaboration with the Calcutta Mathematical Society, Indian Physical Society, and Indian Chemical Society
Information: Dr. M. Dutta, Centre of Advanced Study in Applied Mathematics, Calcutta University, 92, Acharya Prafulla Chandra Road, Calcutta-9, India

January 2–6, 1973
REGIONAL CONFERENCE ON MULTIPLE TIME SERIES ANALYSIS AND SYSTEM IDENTIFICATION
Department of Statistics, University of North Carolina, Chapel Hill, North Carolina
Program: Principal speaker: Emanuel Parzen. Open discussion. Contributed papers
Support: NSF (pending)
Participation: Research workers, younger faculty, and senior students. Support for travel and allowance for accommodations for 28 participants
Information: Organizing committee, Professor S. Cambanis, E. J. Wegman, or M. R. Leadbetter, Department of Statistics, Phillips Hall, University of North Carolina, Chapel Hill, North Carolina 27514

January 3-5, 1973
CONFERENCE ON TECHNIQUES OF ASYMPTOTIC EXPANSION
University of Surrey, Guildford, England
Program: Introductory talk by A. Erdélyi, Mathematical Institute, Edinburgh. Three sessions: Integrals, lectures by F. J. Ursell, University of Manchester; Differential Equations, lectures by J. Heading, University College of Wales, Aberystwyth; Singular Perturbations, lectures by L. E. Fraenkel, University of Cambridge
Information: Secretary and Registrar, The Institute of Mathematics and Its Applications, Maitland House, Warrior Square, Southend on Sea, Essex SS1 2JY, England

January 18–23, 1973
CONFERENCE ON SYMPLECTIC GEOMETRY AND MATHEMATICAL PHYSICS
Rome, Italy
Information: Professor Giuseppe Scorza, Istituto Nazionale di Alta Matematica, Città Universitaria, Roma, Italy

January 22–24, 1973
SYMPOSIUM ON APPROXIMATION THEORY
University of Texas, Austin, Texas
Program: Invited lectures and sessions for contributed papers
Information: Professor G. G. Lorentz, Department of Mathematics, University of Texas, Austin, Texas 78712

February 6–9, 1973
CONFERENCE ON INFORMATION THEORY
Rome, Italy
Information: Professor Giuseppe Scorza, Istituto Nazionale di Alta Matematica, Città Universitaria, Roma, Italy

March 8–13, 1973
CONFERENCE ON POINT-SET TOPOLOGY AND GENERAL TOPOLOGY
Information: Professor Giuseppe Scorza, Istituto Nazionale di Alta Matematica, Città Universitaria, Roma, Italy

April 5–10, 1973
CONFERENCE ON STRUCTURES IN ALGEBRAIC FIELDS
Rome, Italy
Information: Professor Giuseppe Scorza, Istituto Nazionale di Alta Matematica, Città Universitaria, Roma, Italy

May 22–25, 1973
CONFERENCE ON GEOMETRIC THEORY OF INTEGRATION AND MINIMAL VARIETIES
Rome, Italy
Information: Professor Giuseppe Scorza, Istituto Nazionale di Alta Matematica, Città Universitaria, Roma, Italy

NEWS ITEMS AND ANNOUNCEMENTS

FRANCO-AMERICAN COMMISSION FOR EDUCATIONAL EXCHANGE

The Franco-American Commission for Educational Exchange, the official Fulbright commission in France, has announced that it is organizing a clearing house service to facilitate direct teaching exchange between French and American universities. In order to provide an incentive for the project, the Franco-American commission will set aside each year a number of round-trip travel allowances for individual French and American exchange lecturers. An American professor will be paid the salary normally provided to a French staff member of corresponding rank, and the French lecturer will expect to be paid the salary corresponding to his university rank at the American university. No limitation has been made with regard to academic disciplines or duration of appointments, although one academic year or at least one semester is preferable. American professors applying for the program must indicate their ability to use the French language in teaching. Inquiries and prospective applications should be sent to Madame Claude Taudin, Clearing House Service, Franco-American Commission for Educational Exchange, 9, rue Chardin, 75-Paris 16e, France.

CENTER OF ADVANCED STUDY IN APPLIED MATHEMATICS

The Center of Advanced Study in Applied Mathematics at Calcutta University, India, has recently been reorganized. Areas of interest include basic mathematics for application, mathematical methods, statistical methods and information theory, mathematical physics, mathematics of technology and social sciences, computer sciences, and biomathematics. In addition to regularly scheduled research and instruction seminars, colloquium lectures, invited lectures, and memoir lectures are being organized. Annual symposia on different branches of applied mathematics are part of the program. Quarterly reports of the academic activities, including abstracts and lecture notes, are published. Besides the academic staff (professors, readers, research associates, and research fellows), there are provisions for inviting distinguished scientists and scholars as visiting professors and fellows. Further details on the Center can be obtained by writing to Dr. P. K. Bose, Pro-Vice Chancellor of Academic Affairs, Calcutta University, 92, Acharya Prafulla Chandra Road, Calcutta-9, India.

BACKLOG OF MATHEMATICS RESEARCH JOURNALS

Information on the backlog of papers for research journals is published in the February and August issues of these *Notices* with the cooperation of the respective editorial boards. Since all columns in the table are not self-explanatory, we include further details on their meaning.

Column 3. This is an estimate of the number of printed pages which have been accepted but are not necessary to maintain copy editing and printing schedules.

Column 5. The first (Q_1) and third (Q_3) quartiles are presented to give a measure of normal dispersion. They do not include misleading extremes, the result of unusual circumstances arising in part from the refereeing system.

The observations are made from the latest issue of each journal received at the Headquarters Offices before the deadline for the appropriate issue of these *Notices*. Waiting times are measured in months from receipt of manuscript in final forms to receipt of final publication at the Headquarters Offices. When a paper is revised, the waiting time between an editor's receipt of the final revision and its publication may be much shorter than is the case otherwise, so these figures are low to that extent.

	1	2	3		4	5		
JOURNAL	No. issues per year	Approx. no. pages per year	BACKLOG 6/30/72	12/31/71	Est. time for paper submitted currently to be published (in months)	Observed waiting time in latest published issue (in months) Q_1	Med.	Q_3
Acta Informatica	2	192	0	200	5		**	
American J. of Math.	4	1500	NR*	1150	NR*	8	10	13
Annals of Math. Stat.	6	2100	0	0	15	8	9	13
Annals of Math.	6	1200	NR*	NR*	12	10	11	12
Arch. Rational Mech.	25	2000	0	164	6-8	7	7	9
Canad. J. of Math.	6	1200	700	800	15	6	8	13
Comm. Math. Physics	19	1593	0	250	5	5	6	6
Duke Math. J.	4	800	0	0	9	5	6	7
Illinois J. of Math.	4	700	1000	1000	18	29	31	33
Indiana Univ. Math. J.	12	1200	400	600	8	9	10	10
Inventiones Math.	12	1032	0	254	6-7	9	12	12
J. Amer. Stat. Assoc.	4	1040	70	NR*	15	8	10	12
J. Assoc. for Comp. Mach.	4	700	NR*	300	NR*	8	9	11
J. of Comp. and Sys. Scis.	6	600	200	---	6-9	10	11	14
J. Diff. Geometry	4	860	780	800	11-13	9	12	13
J. Math. Physics	12	2545	0	0	5	6	8	16
J. Symbolic Logic	7	800	0	0	10	15	18	22
Linear Algebra and Appl.	4	450	120	NR*	6-8	20	21	23
Math. Systems Theory	NR*	NR*	NR*	NR*	NR*	11	12	14
Math. of Comp.	4	1000	0	0	10	10	13	14
Math. Annalen	16	1376	NR*	255	10	9	11	14
Math. Zeitschrift	20	1850	0	168	9	8	8	9
Michigan Math. J.	4	400	150	150	14	9	12	15
Numerische Math.	10	940	0	NR*	12	10	13	18
Operations Research	6	1200	0	200	13	18	24	31
Pacific J. of Math.	12	3200	NR*	NR*	20	13	17	20
Proceedings of AMS	12	3900	150	50	7-11	10	12	14
Proc. Nat'l Acad. Sci.	12	3500-4000	0	0	2	3	3	3
Quarterly of Appl. Math.	4	600	600	520	12	10	12	13
Semigroup Forum	8	736	0	---	7	6	7	9
SIAM J. of Appl. Math.	8	1500	0	0	8-10	10	10	14
SIAM J. on Computing	4	500	0	0	6-8	5	6	9
SIAM J. on Control	4	650	0	0	8-10	8	9	10
SIAM J. on Math. Anal.	4	650	0	0	8-10	10	12	15
SIAM J. on Numer. Anal.	4	800	0	0	8-10	13	14	15
SIAM Review	4	600	0	0	8-10	12	13	16
Transactions of AMS	12	6000	400	100	13-16	12	14	16
Z. Wahrscheinlichkeitstheorie	13	1112	0	NR*	11	9	12	13

*NR means that no response was received to a request for information.
**Not computable for this journal.
---This journal is new to this compilation. Figures re backlog as of 12/31/71 are not available.

VISITING MATHEMATICIANS

The list of visiting mathematicians includes both foreign mathematicians visiting in the United States and Canada, and Americans visiting abroad. Note that there are two separate lists.

American and Canadian Mathematicians Visiting Abroad

Name and Home Country	Host Institution	Field of Special Interest	Period of Visit
Akemann, Charles A. (U.S.A.)	University of Copenhagen, Denmark	C*-algebras	7/72 - 12/72
Alperin, Jonathan L. (U.S.A.)	University of Warwick, England	Group Theory	1/73 - 6/73
Anderson, Glen (U.S.A.)	University of Helsinki, Finland	Analysis	9/72 - 8/73
Arens, Richard (U.S.A.)	CNRS, Marseille, France	Functional Analysis	10/72 - 12/72
Bisshopp, Fredric E. (U.S.A.)	University of Paris, France	Mathematical Physics	9/72 - 8/73
Bragg, Louis (U.S.A.)	Mittag-Leffler Institute, Djursholm, Sweden	Partial Differential Equations	8/72 - 4/73
Chaney, Robin (U.S.A.)	University of Goteborg, Sweden	Optimization, Harmonic Analysis, Measure Theory	9/72 - 6/73
Cohen, Haskell (U.S.A.)	University of Amsterdam, Netherlands	Topology	9/72 - 8/73
Cornell, J. A. (U.S.A.)	Birkbeck College, University of London	Design of Experiments	9/72 - 6/73
Dubisch, Roy (U.S.A.)	East Africa Regional Mathematics Program, Addis Ababa, Ethiopia	Teacher Training: Elementary and Secondary	9/72 - 6/73
Eagon, John (U.S.A.)	University of Sheffield, England	Algebra	9/72 - 6/73
Elder, Ernest (U.S.A.)	National Research Institute for Mathematical Sciences, Council for Scientific and Industrial Research, Pretoria, South Africa		6/72 - 9/72
Fadell, Edward (U.S.A.)	University of Heidelberg, Fed. Rep. of Germany	Topology	8/72 - 6/73
Fattorini, Hector (U.S.A.)	University of Buenos Aires	Differential Equations	7/72 - 6/73
Fefferman, Solomon (U.S.A.)	Oxford University, England University of Paris, France	Logic	9/72 - 12/72 1/73 - 6/73
Galvin, Fred (U.S.A.)	Hungarian Academy of Sciences	Logic	9/72 - 6/73
Halmos, Paul R. (U.S.A.)	University of Edinburgh, Scotland	Functional Analysis	1/73 - 6/73
Hartnett, William E. (U.S.A.)	Romania	Modern Applied Mathematics	4/73 - 5/73
Hewitt, Edwin (U.S.A.)	Steklov Institute, U.S.S.R.	Harmonic Analysis	4/73 - 9/73
Infante, Ettore F. (U.S.A.)	University of Paris, VI, France	Stability Theory	9/72 - 8/73
Ingraham, Edward (U.S.A.)	University of London, England	Algebra	9/72 - 8/73
Jenkins, Terry L. (U.S.A.)	University of Groningen, Netherlands	Algebra	9/72 - 8/73
Kambayashi, T. (U.S.A.)	Kyoto University, Japan	Algebra	7/72 - 6/73
Kemperman, J.H.B. (U.S.A.)	University of Amsterdam, Netherlands	Probability, Analysis	8/72
Kinney, John (U.S.A.)	University of Salzburg, Austria	Analysis	9/72 - 8/73

Name and Home Country	Host Institution	Field of Special Interest	Period of Visit
Levine, Howard (U. S. A.)	Battelle Institute, Frankfurt, Fed. Rep. of Germany	Partial Differential Equations	9/72 - 12/72
Lorch, L. (Canada)	Royal Institute of Technology, Stockholm, Sweden	Differential Equations	5/72 - 8/73
Lumer, Gunter (U. S. A.)	University of Paris, VI, France	Function Algebras, Functional Analysis	9/72 - 6/73
Lumer, Linda (U. S. A.)	University of Nancy, France	Potential Theory, Applications	9/72 - 6/73
Mac Lane, Saunders (U. S. A.)	University of Cambridge, England	Category Theory	10/72 - 12/72
Maddox, B.H. (U. S. A.)	University of Cambridge, England	Algebra, Undergraduate Mathematics	9/72 - 1/73
McCarthy, Charles (U. S. A.)	University of Göteborg, Sweden	Functional Analysis	9/72 - 6/73
McQuillan, Donald (U. S. A.)	University of Bordeaux, France	Algebraic Number Theory	9/72 - 8/73
Megibben, Charles K. (U. S. A.)	Oxford University, England	Algebra	9/72 - 11/72
Muldoon, M. E. (Canada)	University of Glasgow, Scotland	Differential Equations	7/72 - 8/73
Nakai, Mitsuru (U. S. A.)	Nagoya University, Japan	Complex Analysis	7/72 - 6/73
Narasimhan, Raghavan (U. S. A.)	University of Geneva, Switzerland	Several Complex Variables	10/72 - 9/73
Nymann, James E. (U. S. A.)	University of Liberia	Algebra, Number Theory	9/72 - 6/73
Rabinowitz, Paul (U. S. A.)	University of Aarhus, Denmark University of Pisa, Italy University of Paris, France	Functional Analysis	6/72 - 8/73
Rhoades, B. E. (U. S. A.)	University of Stuttgart, Fed. Rep. of Germany University of Tel-Aviv, Israel	Summability	9/72 - 1/73 2/73 - 4/73
Robertson, James B. (U. S. A.)	University of Warwick, England	Ergodic Theory and Nonlinear Prediction Theory	9/72 - 6/73
Rothaus, Oscar S. (U. S. A.)	University of Tel-Aviv, Israel	Complex Variables	9/72 - 6/73
Schiffer, Max (U. S. A.)	Federal Institute of Technology, Zürich, Switzerland Weizmann Institute of Sciences, Israel	Partial Differential Equations, Complex Analysis	
Schober, Glenn E. (U. S. A.)	Forschungsinstitut für Mathematik, Zürich, Switzerland	Complex Analysis	8/72 - 8/73
Segal, Sanford L. (U. S. A.)	University of Nottingham, England	Number Theory, Complex Analysis	9/72 - 7/73
Shimrat, M. (Canada)	University of Jerusalem, Israel	Topology, Set Theory and Logic	7/72 - 7/73
Smythe, Robert T. (U. S. A.)	University of Strasbourg, Fed. Rep. of Germany University of Paris, France	Probability Theory	9/72 - 1/73 2/73 - 8/73
Spitzer, Frank L. (U. S. A.)	Mittag-Leffler Institute, Djursholm, Sweden	Probability Theory	9/72 - 6/73
Steinberg, Robert (U. S. A.)	Tata Institute, Bombay, India University of Paris, France	Algebra	9/72 - 6/73
Stout, Edgar Lee (U. S. A.)	University of Leeds, England	Function Theory, Function Algebras	9/72 - 6/73

Name and Home Country	Host Institution	Field of Special Interest	Period of Visit
Thompson, A. C. (Canada)	University of Edinburgh, Scotland	Functional Analysis	8/72 - 8/73
Turner, R. E. L. (U. S. A.)	Mathematical Institute, Oxford, England		8/72 - 8/73
Wang Hsien-Chung (U. S. A.)	University of Warwick, England	Lie Groups, Differential Geometry	9/72 - 6/73
Wilansky, Albert (U. S. A.)	University of Reading, England	Functional Analysis, Topology	
Williams, James P. (U. S. A.)	University of Newcastle, England	Functional Analysis	9/72 - 6/73
Zukowski, Lucille P. (U. S. A.)	Iranzamin, Teheran, Iran	General Undergraduate Program	9/72 - 8/73

Foreign Mathematicians Visiting in the United States

Adams, J. F. (United Kingdom)	University of Chicago	Topology	4/73 - 6/73
Akaike, Hirotugu (Japan)	University of Hawaii	Time Series Analysis	1/72 - 12/72
Balslev, Erik (Denmark)	University of California, Los Angeles	Operator Theory	7/72 - 6/73
Blight, B. J. N. (United Kingdom)	University of Florida	Time Series	9/72 - 6/73
Borosh, Itshak (Israel)	Texas A & M University	Number Theory, Diophantine Approximations	9/72 - 7/73
Brezis, Haim (France)	University of Chicago	Analysis	1/73 - 3/73
Broman, Arne (Sweden)	Western Washington State College	Harmonic Analysis	9/72 - 6/73
Bruggeman, Reolof W. (Netherlands)	Yale University	Automorphic Forms, Algebraic Groups	9/72 - 6/73
Chang Chao-ping (Australia)	University of Chicago	Trigonometric Series	1/73 - 8/73
Crossley, John N. (Australia)	Cornell University	Logic	7/72 - 12/72
Dalenius, Tore E. (Sweden)	Brown University	Applied Statistics	7/72 - 8/73
Dieudonné, Jean (France)	California State University, San Diego	Topology, Analysis, Algebraic Geometry	12/72 - 3/73
Essén, Matt S. (Sweden)	University of Minnesota	Mathematical Analysis	9/72 - 12/72
Field, Michael (United Kingdom)	University of Minnesota	Complex Manifold Theory	9/72 - 6/73
Filippi, P. (France)	Stanford University	Analysis of Partial Differential Equations	9/72 - 6/73
Foguel, Shaul (Israel)	University of Minnesota	Functional Analysis, Ergodic Theory	3/73 - 6/73
Friberg, Jöran (Sweden)	University of Minnesota	Partial Differential Equations	9/72 - 6/73
Furstenberg, Harry (Israel)	Yale University	Stochastic Processes	9/72 - 10/72
Glasner, Samuel (Israel)	University of Minnesota	Topological Dynamics	9/72 - 6/73
Goethals, Jean-Marie (Belgium)	Syracuse University	Combinatorics	8/72 - 12/72
Göktepe, Ismail (Turkey)	Cornell University	Partial Differential Equations	9/72 - 6/73
Hida, Takeyuki (Japan)	University of Minnesota	Stochastic Processes	9/72 - 3/73
Jaeger, Willi (Fed. Rep. of Germany)	University of Minnesota	Partial Differential Equations	2/73 - 6/73
Kart, Cevat (Turkey)	Brown University	Partial Differential Equations	8/72 - 7/73
Kearton, Cherry (United Kingdom)	Princeton University	Topology	9/72 - 6/73
Kozma, Ilan (Israel)	University of Chicago	Topology	10/71 - 9/73
Lau Ka-sing (Hong Kong)	University of Chicago	Functional Analysis	10/72 - 9/73
Lee Shing-meng (Republic of China)	Cornell University	Analysis	6/72 - 6/73
Lovász, László (Hungary)	Vanderbilt University	Graph Theory, Universal Algebra	9/72 - 6/73
Martensson, Sven Krister (Sweden)	Brown University	Control Theory	8/72 - 2/73

Name and Home Country	Host Institution	Field of Special Interest	Period of Visit
Massari, Umberto (Italy)	University of Minnesota	Partial Differential Equations	9/72 - 6/73
Matsumoto, Hideya (France)	Yale University	Arithmetic Subgroups	4/73 - 5/73
Okamoto, Kiyosato (Japan)	University of Chicago	Group Representations	1/73 - 6/73
Ooms, Alfons (Belgium)	University of Massachusetts	Lie Algebra	9/72 - 8/73
Park, David M. R. (United Kingdom)	Syracuse University	Theory of Computation	1/73 - 4/73
Rao, C. R. (India)	Indiana University	Statistics	8/72 - 2/73
Rapoport, Michael (Fed. Rep. of Germany)	Princeton University	Number Theory	9/72 - 6/73
Regev, Amitai (Israel)	University of California, Los Angeles	Algebra	7/72 - 6/73
Rodriguez-Rosell, Juan (Colombia)	Brown University	Computer Science	9/72 - 8/73
Sjölin, Per (Sweden)	University of Chicago	Analysis	10/72 - 12/72
Trudinger, Neil (Australia)	Stanford University	Partial Differential Equations	4/73 - 6/73
Vervaat, Wim (Netherlands)	University of Washington	Probability Theory	9/72 - 9/73
Wahlbin, Lars B. (Sweden)	University of Chicago	Numerical Analysis	10/72 - 9/73
Ward, James (Australia)	University of Chicago	Group Theory	6/72 - 3/73
Weisspfenning, Volker (Fed. Rep. of Germany)	Yale University	Model Theory	9/72 - 8/73
Wienholtz, Ernst (Fed. Rep. of Germany)	University of Minnesota	Partial Differential Equations	3/73 - 6/73
Yoshizawa, Taro (Japan)	Michigan State University	Analysis	9/72 - 8/73

NEWS ITEMS AND ANNOUNCEMENTS

ACADEMY OF SCIENCES HONORS MATHEMATICIAN

The Academy of Sciences at its 109th annual meeting honored eleven scientists for their contributions to science and mankind. Kurt Otto Friedrichs of New York University received the National Academy of Sciences Award in Applied Mathematics and Numerical Analysis for distinguished achievements in fundamental research relevent to other fields of science.

COURANT INSTITUTE POSTDOCTORAL VISITING MEMBERSHIPS

The Courant Institute of Mathematical Sciences of New York University offers postdoctoral visiting memberships to mathematicians, scientists, and engineers who are interested in its program of training and research in a broad range of pure and applied mathematics. Applications for the academic year 1973–1974 must be submitted before January 1, 1973. Inquiries should be addressed to the Visiting Membership Committee of the Courant Institute of Mathematical Sciences, 251 Mercer Street, New York, New York 10012.

NAS ELECTS NEW MEMBERS

The National Academy of Sciences has elected 75 new members. The five AMS members honored were Richard J. Duffin of Carnegie-Mellon University, Ralph E. Gomory of the T. J. Watson Research Center of IBM and Cornell University, Samuel Karlin of Stanford University and the Weizmann Institute, and Willem V. R. Malkus and George W. Whitehead of the Massachusetts Institute of Technology. Twelve individuals were honored as NAS Foreign Associates. Among those was AMS member Henri Cartan of the Institut Henri Poincaré, Paris, France.

NEW AMS PUBLICATIONS

SIAM-AMS PROCEEDINGS

MATHEMATICAL ASPECTS OF STATISTICAL MECHANICS, edited by James C. T. Pool (Volume V)

90 + x pages; list price $8.00; member price $6.00

Utilizing the properties of individual constituents of matter, for example atoms or molecules, statistical mechanics seeks to predict the results of experiments involving macroscopic quantities of matter. Statistical mechanics provides an adequate description of many phenomena; indeed, in many cases the quantitative agreement with experimental observations is remarkable. Thus, in marked contrast to many areas of physics, for example high energy physics, the physical fundamentals of statistical mechanics are generally accepted. Despite the general acceptance of the physical fundamentals, the utilization of these fundamentals to predict the behavior of macroscopic matter involves significant, unresolved difficulties. There is an increasing belief that many difficulties can be resolved by more penetrating mathematical analysis of existing theories. The seven papers presented in this volume survey the state-of-the-art in various mathematical aspects of statistical mechanics, and involve a spectrum of inquiry ranging from physical intuition to penetrating mathematical analysis, a characteristic feature of the history of statistical mechanics. They also exhibit opportunities for exploration of mathematical problems originating in a field of physics which historically has contributed to and benefited from numerous areas of mathematical research.

TRANSLATIONS OF MATHEMATICAL MONOGRAPHS

THEORY OF CONVEX PROGRAMMING by E. G. Gol'steĭn (Volume 36)

57 + v pages; list price $9.60; member price $7.20

This volume is based on the lectures on convex programming delivered by the author in the summer of 1968 to the students of the All-Union School of Mathematical Programming in the city of Alma-Ata. Besides the basic facts of the theory of convex programming, the book includes a number of results on the marginal values of convex programming problems. Although the presentation is carried out for the case of finite-dimensional problems, many of the theorems remain in force for more general infinite-dimensional problems. Moreover, the transition to functional spaces does not, as a rule, involve any change in the structure of the corresponding arguments.

MEMOIRS OF THE AMERICAN MATHEMATICAL SOCIETY

THE CAUCHY-GOURSAT PROBLEM by Paul DuChateau (Number 118)

60 + ii pages; list price $2.00; member price $1.50

This article begins by exploiting techniques developed by Treves involving the Ovcyannikov theorem to obtain abstract theorems of existence, uniqueness, and regularity of the solutions of differential equations whose solutions take their values in scales of Banach spaces. It is then shown that when the ingredients of the abstract theorems are appropriately chosen, theorems of a classical nature are immediately produced. These classical theorems then bear on the existence, uniqueness, and regularity of the solutions of that generalization of the Cauchy problem in partial differential equations known as the Goursat problem. Besides providing a certain degree of unity of approach and perhaps additional insight to this area of partial differential equations, the abstract methods have the additional advantage that the results obtained remain true when the partial differential operators involved are replaced by more general operators, e.g. convolution operators, pseudo-differential operators.

ORDERED STRUCTURES AND PARTITIONS by Richard Stanley (Number 119)

104 + vi pages; list price $2.00; member price $1.50

In this Memoir, a general theory is developed for the enumeration of order-reversing maps of partially ordered sets P (usually finite) into chains. This theory comprehends many apparently disparate topics in combinatorial theory, including (1) ordinary partitions, (2) ordered partitions (compositions), (3) plane and multidimensional partitions, with applications to Young tableaux, (4) the Eulerian numbers and their refinements, (5) the tangent and secant numbers (or Euler numbers) and their refinements, (6) the indices of permutations, (7) trees, (8) stacks, and (9) protruded partitions, with applications to the Fibonacci numbers. The main tool used is that of generating functions. In particular, the influence of the structure of P on the form of the generating functions under consideration is studied. As an application, new combinatorial relationships between a finite partially ordered set P and its distributive lattice of order ideals are derived.

GROTHENDIECK SPACES IN APPROXIMATION THEORY by Jörg Blatter (Number 120)

121 + ii pages; list price \$2.40; member price \$1.80

The theory of approximation of the functions of a given set by means of elements of a given subset, which began with the Weierstrass theorem on approximation of continuous real functions by polynomials and was extended by theorems of Stone, Kakutani, and Grothendieck, is here developed for closed vector subspaces of the set $C_0(T, X)$ of continuous functions from T to X, where T is a locally compact Hausdorff topological space, and X is a real Banach space. These vector subspaces are analogous to Grothendieck's set of closed vector subspaces of the set $C_0(T, \mathbb{R})$, where $\mathbb{R}$ is the real field, and to two subsets—Kakutani-Stone and Weierstrass-Stone subspaces—analogous to the sets of closed vector sublattices and closed subalgebras of $C_0(T, \mathbb{R})$.

TRELLIS THEORY by Helen Skala (Number 121)

41 + iv pages; list price \$2.30; member price \$1.73

Trellis theory is a generalization of the theory of lattices. Though the trellis operations corresponding to join and meet are nonassociative, and the order in a trellis is nontransitive, nearly all the basic theorems of lattice theory have analogs in trellis theory. The assumptions of transitivity and associativity, previously regarded as essential in the proofs of many lattice theorems, are now seen to be in fact superfluous. Thus, one gains new insight into the nature of lattices.

A GENERAL CHARACTER THEORY FOR PARTIALLY ORDERED SETS AND LATTICES by K. Keimel and K. H. Hofmann (Number 122)

124 + iv pages; list price \$2.80; member price \$2.10

Various classical and not-so-called duality theories between certain categories of partially ordered sets, semilattices, lattices, and suitable categories of topological spaces receive a unified treatment through co-representable cofunctors. A good deal of not merely functorial analysis is given in numerous individual cases. Among other categories, the authors discuss the categories of distributive, pseudocomplemented, Boolean, Stonean lattices; and the topics from general topology covered in the process include spectral spaces (i.e. spaces arising as prime ideal spaces of lattices and rings), compactifications, and projective covers of spaces.

EXPOSED POINTS OF CONVEX SETS AND WEAK SEQUENTIAL CONVERGENCE by Edmond E. Granirer (Number 123)

80 + iv pages; list price \$2.50; member price \$1.88

Let K be a convex set in a locally convex space and A a convex subset in the closure of K. It is shown that, in certain cases, the exposed points of A belong to the sequential closure of K. Applications are given to invariant means on locally compact groups and discrete semigroups. Among other results, the author shows that, for any locally compact group G, W(G) = UC(G) if and only if G is compact. Other applications are to existence of invariant measures for Markov operators on an L_1 space, to existence of invariant probability measures on the Baire sets of an arbitrary topological space, and to existence of ultraweakly continuous invariant states on a W*-algebra.

SINGULAR TORSION AND THE SPLITTING PROPERTIES by K. R. Goodearl (Number 124)

90 + vi pages; list price \$2.50; member price \$1.88

This Memoir may be divided into three parts, the first of which is an expository development of the singular torsion theory. The second part develops characterization and structure theorems for rings with all nonsingular right modules projective, and for rings with all singular right modules injective. The final part is concerned with studying necessary and sufficient conditions on a ring such that for certain classes of modules, the singular submodule of any member of the class is a direct summand of the module. The classes of interest are the class of all right modules, the class of all finitely generated right modules, and the class of those right modules whose singular submodules have bounded order.

PROCEEDINGS OF THE STEKLOV INSTITUTE

BOUNDARY VALUE PROBLEMS OF MATHEMATICAL PHYSICS. VI, edited by O. A. Ladyženskaja (Number 110 (1970))

210 + vi pages; list price \$19.20; member price \$14.36

The articles in this volume fall into two groups. Those in the first group are concerned with problems of quantum mechanics, and in the second with boundary problems for linear and quasilinear equations and systems of elliptic and parabolic type. The first group includes the papers by V. S. Buslaev, "Quantization and the WKB-method"; V. S. Buslaev and S. P. Merkur'ev, "On the third group integral in quantum-mechanical statistics"; and O. A. Jakubovskiĭ, "Structure of the resolvent of the Schrödinger operator for a system of $\underline{n}$ particles with decreasing pair interaction." The remaining papers belong to the second group: A. V. Ivanov, "Local estimates for the maximum modulus of the first derivatives of solutions of quasilinear nonuniformly elliptic and nonuniformly parabolic equations and systems of general type"; N. M. Ivočkina and A. P. Oskolkov, "Nonlocal estimates for the first derivatives of solutions of the first boundary problem for certain classes of nonuniformly elliptic and nonuniformly parabolic equations and systems"; A. P. Oskolkov, "Interior estimates for the first order derivatives for a class of quasilinear elliptic systems"; and V. A. Solonnikov, "On Green's matrices for elliptic boundary value problems."

CBMS REGIONAL CONFERENCE SERIES IN MATHEMATICS

ARRANGEMENTS AND SPREADS by Branko Grünbaum (Number 10)

114 + vi pages; list price \$4.50; individual price \$3.38

This survey deals mostly with rather elementary mathematics, so elementary in fact that most of its results and problems are (or at least should be) understandable to undergraduates. It was written out of the conviction that many neglected aspects of elementary geometry deserve a wider dissemination because of their inherent beauty and interest, and for the inspiration and understanding they can impart to students and mathematicians. Throughout the survey, many conjectures are explicitly stated, and many additional problems are hinted at. The subject obviously offers extremely varied opportunities for research. Due to the elementary nature of the topic, the solutions of many of the open problems will probably require more inspiration than erudition, but that is a hallmark of beauty in mathematics.

NEWS ITEMS AND ANNOUNCEMENTS

COOPERATIVE SCIENCE PROGRAMS

The National Science Foundation is offering a new program to foster and support scientific and technological cooperation between the United States and Bulgaria, Czechoslovakia, Hungary, and Romania. The program will promote collaboration and exchange of information between scientists, engineers, scholars, and institutions of research and higher learning of the United States and the cooperating countries. Under this program, cooperative activities may be conducted in any branch of science and technology, including basic and applied aspects of natural sciences and mathematics, the engineering sciences, and the social sciences. American institutions eligible to participate in this program include universities and colleges, professional societies, academies of sciences, and other nonprofit scientific organizations. Support is available for American scientists who are U.S. citizens or have at least five years of professional employment beyond the doctorate in U.S. institutions and who are currently affiliated with an eligible U.S. institution. The types of activities for which financial support will be given are cooperative research projects, seminars, and scientific visits.

Inquiries and requests for further information should be addressed to East Europe Cooperative Science Program, Office of International Programs, National Science Foundation, Washington, D.C. 20550.

SALEM PRIZE

The Salem Prize for 1972 has been awarded to Thomas Körner of the University of Cambridge, England, for his work on perfect sets and trigonometric series. The prize, established in 1968, is given every year to a young mathematician who is judged to have done an outstanding paper on Fourier series and related topics. Previous recipients were Nicholas Varopoulos in 1968, Richard Hunt in 1969, Yves Meyer in 1970, and Charles Fefferman in 1971. The jury consists of A. Zygmund, C. Pisot, and J.-P. Kahane.

BROUWER MEMORIAL LECTURE

The second Brouwer Memorial Lecture will be held the last week of April 1973 in Leiden, Netherlands, on the subject of Foundations. Professor A. Robinson of Yale University will give the lecture entitled "Standard and nonstandard number systems." The Memorial Lecture was established in 1969 by the Dutch Mathematical Society, with financial support from the Netherlands government, in memory of L.E.J. Brouwer. The first Brouwer lecture was held in Amsterdam in 1970 on the subject of Topology and was delivered by R. Thom of Institut des Hautes Études Scientifiques.

PERSONAL ITEMS

KENDALL E. ATKINSON of Indiana University has been appointed to an associate professorship at the University of Iowa.

J. F. G. AUCHMUTY of SUNY at Stony Brook has been appointed to a visiting assistant professorship at Indiana University.

JOHN A. BEACHY of Northern Illinois University has been appointed to a visiting assistant professorship at Indiana University.

ABRAHAM BERMAN of the University of Montreal has been appointed a senior lecturer at the Technion-Israel Institute of Technology, Haifa, Israel.

ANDREW D. BOOTH of the University of Saskatchewan, Saskatoon, has been appointed president and vice-chancellor of Lakehead University, Thunder Bay, Canada.

ALFRED T. BRAUER was awarded the honorary degree Doctor of Laws from the University of North Carolina at Chapel Hill.

MELVYN CIMENT of the University of Michigan has been appointed a mathematician in the Mathematical Analysis Division of the Naval Ordnance Laboratory, Silver Spring, Maryland.

ROBERT A. FONTENOT of Louisiana State University has been appointed to an assistant professorship at Oakland University.

D. RAY FULKERSON of Cornell University was honored with an award for professional achievement from the Southern Illinois University at Carbondale, his alma mater.

MICHAEL C. GEMIGNANI of Smith College has been appointed to a professorship and to the chairmanship of the Department of Mathematics at Indiana University-Purdue University, Indianapolis.

SUDHANSHU K. GHOSHAL of Jadavpur University, Calcutta, has been appointed to a professorship and to the chairmanship of the Department of Mathematics at Regional Engineering College, Durgapur, India.

ROBERT T. GLASSEY of Brown University has been appointed to an assistant professorship at Indiana University.

VICTOR GOODMAN of the University of New Mexico has been appointed to an assistant professorship at Indiana University.

GEORGE A. GRÄTZER of the University of Manitoba has won the 1971 Steacie Prize in the Natural Sciences. He is the first mathematician to win this prize.

EDWIN HEWITT of the University of Washington has been appointed to a professorship at the University of Texas at Austin.

NATHAN JACOBSON of Yale University was awarded the honorary degree Doctor of Science from the University of Chicago.

NICHOLAS D. KAZARINOFF of the State University of New York at Buffalo has been appointed Martin Professor of Mathematics at that university.

DEAN K. KUKRAL of Indiana University has been appointed to an assistant professorship at Wichita State University.

PETER D. LAX of New York University has been appointed director of NYU's Courant Institute of Mathematical Sciences and head of the all-University department of mathematics.

JAMES I. LEPOWSKY of Brandeis University has been appointed to an assistant professorship at Yale University.

KENNETH O. MAY of the University of Toronto will be on leave for the academic year 1972-1973. He will do research on information retrieval in mathematics with support from a Killam Award of the Canada Council.

GARY H. MEISTERS of the University of Colorado has been appointed to a professorship at the University of Nebraska, Lincoln.

CLAUDE L. SCHOCHET of the Hebrew University of Jerusalem, Israel, has been appointed to an assistant professorship at Indiana University.

DANA S. SCOTT of Princeton University has been appointed a professor of Mathematical Logic at the Mathematical Institute, Oxford, and a fellow at Merton College, University of Oxford, England.

HAROLD S. SHAPIRO of the University of Michigan has been appointed to a professorship at the Royal Institute of Technology, Stockholm, Sweden.

ROBERT C. SHARPLEY III of the University of Texas at Austin has been appointed to an assistant professorship at Oakland University.

ARON SIMIS of Queen's University has been appointed a postdoctoral fellow at Brandeis University.

JOHN JULIUS SMITH of NASA Langley Research Center has been appointed an environmental protection specialist with the Department of Environmental Resources of the State of Pennsylvania.

PHILIP W. SMITH of Purdue University has been appointed to an assistant professorship at Texas A & M University.

GERALD D. TAYLOR of Michigan State University will be on sabbatical leave for the year 1972-1973. He will spend the year at the Department of Computer Science, Stanford University.

ELLEN M. TORRANCE of Gordon-Conwell Theological Seminary has been appointed to an associate professorship and to acting head of the Department of Mathematics at Sterling College.

STANLEY J. WERTHEIMER of Georgia Institute of Technology has been appointed to an assistant professorship at Connecticut College.

WILLIAM H. WHEELER of Yale University has been appointed to an assistant professorship at Indiana University.

PROMOTIONS

To Chairman, Department of Mathematics. Oakland University: GEORGE F. FEEMAN.

To Acting Head, Department of Mathematics and Statistics. University of Massachusetts: MURRAY EISENBERG.

To Professor. Boston University: ROHIT J. PARIKH; Lehigh University: GREGORY T. McALLISTER; Long Island University, Brooklyn Center: NATHANIEL R. STANLEY, ISRAEL ZUCKERMAN; Western Michigan University: DON R. LICK, JACK R. MEAGHER.

To Associate Professor. Long Island University, Brooklyn Center: RICHARD C. RETH; New York City Community College (CUNY): WARREN PAGE; Stevens Institute of Technology: JOAN S. BIRMAN, LAWRENCE E. LEVINE; Western Michigan University: S. F. KAPOOR.

DEATHS

Professor A. ADRIAN ALBERT of the University of Chicago died on June 6, 1972, at the age of 66. He was a member of the Society for 44 years.

Professor GEORGE W. EVANS II of the University of Santa Clara died on June 7, 1972, at the age of 51. He was a member of the Society for 22 years.

Professor TAH-KAI HU of Western Washington State College died on May 26, 1972, at the age of 31. He was a member of the Society for 5 years.

Professor Emeritus AUBREY E. LANDRY of the Catholic University of America died on May 3, 1972, at the age of 91. He was a member of the Society for 58 years.

Professor Emeritus WILLIAM M. WHYBURN of the University of North Carolina at Chapel Hill died on May 6, 1972, at the age of 70. He was a member of the Society for 44 years.

NEWS ITEMS AND ANNOUNCEMENTS

ARTHUR B. COBLE MEMORIAL LECTURES

The Department of Mathematics of the University of Illinois, Urbana-Champaign, has announced that the third annual series of Arthur B. Coble Memorial Lectures will be delivered by Professor Derrick H. Lehmer of the University of California, Berkeley, on September 26, 27, and 28, 1972. These lectures are supported by the University of Illinois Foundation through a fund established by the late Professor Coble's family. The topics of the individual lectures will be announced later, but will reflect Professor Lehmer's long-term interest in number theoretic and computational matters.

NATO SENIOR FELLOWSHIPS IN SCIENCE

The National Science Foundation and the Department of State have announced that forty American scientists have been awarded NATO Senior Fellowships in Science. The fellowships enable universities and nonprofit scientific research institutions in the United States to send senior staff members to research and educational institutions in other NATO countries. Only one fellowship in mathematics was announced. It was awarded to William F. Ames of the University of Iowa; his host institution will be Universität Karlsruhe, Federal Republic of Germany.

NSF AWARDS GRANTS TO IMPROVE CLASSROOM TEACHING

Awards designed to improve classroom instruction received by students in science and mathematics have been announced by the National Science Foundation. The grants support comprehensive year-round study opportunities for 2,500 elementary and high school teachers for full- and part-time training during the summer and academic year. The awards were made to six colleges and universities. The three grants in mathematics went to William R. Orton of the University of Arkansas, Abraham Goetz of the University of Notre Dame, and Robert Pruitt of San Jose State College.

ABSTRACTS PRESENTED TO THE SOCIETY

Preprints are available from the author in cases where the abstract number is starred.

The papers printed below were accepted by the American Mathematical Society for presentation by title. The abstracts are grouped according to subjects chosen by the author from categories listed on the abstract form. The miscellaneous group includes all abstracts for which the authors did not indicate a category.

An individual may present only one abstract by title in any one issue of the *Notices* but joint authors are treated as a separate category. Thus, in addition to abstracts from two individual authors, one joint abstract by them may also be accepted for an issue.

Algebra & Theory of Numbers

72T-A159. HYO CHUL MYUNG, University of Northern Iowa, Cedar Falls, Iowa 50613. A characterization of the Jacobson radical in ternary rings.

Let T be a τ-ring defined by W. G. Lister (Trans. Amer. Math. Soc. 154(1971), 37-55), and let R(T) be the Jacobson radical of T. For an element $a \in T$ we form a ring T_a by setting $x \cdot y = xay$ on the group (T,+). An element a in T is called properly quasi-invertible (p.q.i.) if a is quasi-invertible in all T_x. Theorem. If $t \to 2t$ is an additive automorphism of T, then R(T) is equal to each of the following: (1) the set of all $x \in T$ such that the principal right ideal xTT is quasi-regular in T, (2) $\bigcap_{x\in T} R(T_x)$, (3) the set of all p.q.i. elements in T. Setting $P(u)x = uxu$ for x, u in T, the additive group T together with the quadratic mapping P becomes a Jordan triple system introduced by K. Meyberg (Abstract, Conference on Lie algebras and related topics, Ohio State Univ., October, 1971). In view of (2) or (3), R(T) coincides with the Jacobson radical of T regarded as a Jordan triple system. (Received April 6, 1972.)

72T-A160. KIM KI-HANG BUTLER, Pembroke State University, Pembroke, North Carolina 28372 and GEORGE MARKOWSKY, Harvard University, Cambridge, Massachusetts 02138. Partially ordered and quasi-ordered sets.

By $\pi(n)$ we mean the set of all unordered partitions of n. If $k \in \pi(n)$, by k(i) we mean the number of times i appears as a part of k. Let P(n), $P_c(n)$, Q(n), and $Q_c(n)$ denote the number of partial orders, connected partial orders, quasi-orders, and connected quasi-orders on n points. Let $P^*(n)$, $P_c^*(n)$, $Q^*(n)$, and $Q_c^*(n)$ denote the number of isomorphism classes in P(n), $P_c(n)$, Q(n), and $Q_c(n)$. Let $\Psi_i = \binom{P_c^*(i)+k(i)-1}{k(i)}$ and $\Phi_i = \binom{Q_c^*(i)+k(i)-1}{k(i)}$. In general notation follows from Riordan ["Introduction to combinatorial analysis," Wiley, 1958]. Theorem 1. (i) $P(n) = \sum_{m=1}^{n} s(n,m)\,Q(m)$; (ii) $Q(n) = \sum_{m=1}^{n} S(n,m)\,P(m)$. Theorem 2. (i) $P(n) = Y_n(P_c(1),\ldots,P_c(n))$; (ii) $P_c(n) = Y_n(fP(1),\ldots,fP(n))$ $(f^k \equiv f_k = (-1)^{k-1}(k-1)!)$. Theorem 3. (i) $Q(n) = Y_n(Q_c(1),\ldots,Q_c(n))$; (ii) $Q_c(n) = Y_n(fQ(1),\ldots,fQ(n))$ $(f^k \equiv f_k = (-1)^{k-1}(k-1)!)$. Theorem 4. (i) $P^*(n) = \sum_{\pi_n}(\prod_{i=1}^{n}\Psi_i)$; (ii) $Q^*(n) = \sum_{\pi_n}(\prod_{i=1}^{n}\Phi_i)$. (Received June 15, 1972.)

72T-A161. EARL S. KRAMER and DALE M. MESNER, University of Nebraska, Lincoln, Nebraska 68508. Intersections among Steiner systems. Preliminary report.

A Steiner system $S(\ell,m,n)$ is a system of subsets of size m (called blocks) from an n-set S, such that each ℓ-subset from S is contained in precisely one block. Two Steiner systems have intersection k if they share exactly k blocks. The possible intersections among S(5,6,12)'s, among S(4,5,11)'s, among S(3,4,10)'s, and among S(2,3,9)'s are determined, together with associated orbits under the action of the automorphism group of an initial Steiner system. The following are results: (i) the maximal number of mutually disjoint S(5,6,12)'s is two and any two such pairs are isomorphic; (ii) the maximal number of mutually disjoint $S(4,5,11)$'s is two and any two such pairs are isomorphic; (iii) the maximal number of mutually disjoint $S(3,4,10)$'s is five and any two such sets of five are isomorphic; (iv) there are exactly two nonisomorphic ways to partition all 3-subsets of a 9-set into seven mutually disjoint $S(2,3,9)$'s. (Received April 28, 1972.)

72T-A162. J. L. BRENNER, 10 Phillips Road, Palo Alto, California 94303 and University of Arizona, Tucson, Arizona 85721. Zolotarev's theorem on the Legendre symbol.

In 1872 (Nouvelles Annales 2nd ser) Zolotarev proved that the sign of the permutation $i \pmod p \to ia \pmod p$ is equal to the Legendre symbol (a/p). He used this to give a new proof of the law of quadratic reciprocity. In the present paper, a matrix-theoretic proof of Zolotarev's lemma is given, based on the ideas of the paper "Roots and canonical forms of circulant matrices," by C. M. Ablow and J. L. Brenner, Trans. Amer. Math. Soc. 107(1963), 360-376; MR 27 #5775. This article will appear in Pacific J. Math. (Received May 1, 1972.)

*72T-A163. PAUL JEAN CAHEN, Queen's University, Kingston, Ontario, Canada. Torsion theory and associated primes. Preliminary report.

A torsion theory partitions the spectrum of the base ring into two sets. Over a Noetherian ring, every suitable partition of the spectrum gives rise to one and only one torsion theory. It is possible to know whether a module is torsion or torsion-free by looking at its associated primes. The example of the polynomial torsion theory, introduced by the author and J. L. Chabert ("Coéfficients et valeurs d'un polynôme," Bull. Sci. Math. 95(1971), 295-304), is developed. (Received May 1, 1972.)

*72T-A164. FLORINDA KATSUME MIYAOKA, Instituto de Matemática, Universidade Federal do Paraná, Curitiba, Brasil. About 3-rings and their lattice structures.

It is shown that the result in (Abstract 71T-A149, these *Notices* 18(1971), 795) is not correct. If R is a 3-ring with unity, then the most general expressions for the lower bound and the upper bound that are possible to be defined in R are: $x \wedge y = (p+2q)x^2y^2 + (2p^2+q^2)(x^2y+xy^2) + pq^2(x^2+y^2) + (pq^2+2p+q)xy + (2pq+q^2)(x+y)$, and $x \vee y = (2p+q)x^2y^2 + (p^2+2q^2)(x^2y+xy^2) + 2pq^2(x^2+y^2) + (2pq^2+p+2q)xy + (2pq+p^2)(x+y)$, where p and q are two elements of R, such that $pq^2 + p^2q = 2p^2 + 2q^2 + 2pq + 1 = 0$, which respectively become under the operations $\wedge$ and $\vee$, the greatest and the least elements of the lattice. (Received April 19, 1972.) (Author introduced by Professor Haroldo C. Affonso da Costa.)

*72T-A165. JONATHAN S. GOLAN, McGill University, Montreal, Quebec, Canada and MARK L. TEPLY, University of Florida, Gainesville, Florida 32601. Torsion-free covers.

This paper studies the properties of torsion-free covers with respect to a hereditary torsion theory $(\mathcal{T},\mathfrak{F})$ of left R-modules such that $R \in \mathfrak{F}$. See Teply [Pacific J. Math. 28(1969), 441-453] for definitions. If $\mu : F \to M$ is a torsion-free cover, then a necessary and sufficient condition for M to be $\mathcal{T}$-injective [injective] is that F be $\mathcal{T}$-injective [injective]. If the injective hull of M has a torsion-free cover, so does the $\mathcal{T}$-injective hull of M. The direct sum of the torsion-free covers of a finite number of modules is the torsion-free cover of their direct sum. A definition of relative neatness is introduced in order to prove a generalization of the main theorem of Enochs [Arch. Math. 22(1971), 37-52] on liftings of homomorphisms. $(\mathcal{T},\mathfrak{F})$ is called universally covering if every R-module has a torsion-free cover relative to $(\mathcal{T},\mathfrak{F})$. If the inclusion map of R into the quotient ring $Q_{\mathcal{T}}$ or the theory is a left localization, then the study of when $(\mathcal{T},\mathfrak{F})$ is universally covering can be reduced to the case where $R = Q_{\mathcal{T}}$. Applications of this are made to perfect torsion theories, torsion theories with noetherian quotient rings, and torsion theories generated by one simple module. (Received May 5, 1972.)

*72T-A166. AMASSA C. FAUNTLEROY, University of Illinois, Urbana, Illinois 61801. Rational points of commutator subgroups of solvable algebraic groups. Preliminary report.

Let G be a connected algebraic group defined over a field k. Denote by $G(k)$ the group of k-rational points of G. Suppose that A and B are closed subgroups of G defined over k. Then $[A, B](k)$ is not equal to $[A(k), B(k)]$ in general. Here $[A, B]$ denotes the group generated by commutators $aba^{-1}b^{-1}$, $a \in A$, $b \in B$. We say that a field k of characteristic p is p-closed if given any additive polynomial $f(x)$ in $k[x]$ and any element c in k, there exists an element α in k such that $f(\alpha) = c$. Theorem 1. Let G be a connected solvable algebraic group defined over the p-closed field k. Let A and B be closed connected subgroups of G, which are also defined over k, and suppose A normalizes B. Then $[A, B](k) = [A(k), B(k)]$. Theorem 2. If G, A and B are as above and k is only assumed to be perfect then there exists a finite extension k_0 of k such that if K is the maximal p-extension of k_0, then $[A, B](K) = [A(K), B(K)]$. (Received May 8, 1972.)

72T-A167. FRED CLARE, University of Colorado, Boulder, Colorado 80302. Embeddings and elementary equivalence for infinite symmetric groups. Preliminary report.

For notation see Scott, "Group theory." For a group G, a set I, and a filter $\mathfrak{F}$ on I, let G^I denote the direct power, $G^{0,I}$ the external direct sum, and $G^I/\mathfrak{F}$ the reduced power of G. Let m, n, p, q, r denote infinite cardinals; let $A(m,n,p)$ stand for: there is a family $\mathfrak{F}$ contained in the power set of m such that $|\mathfrak{F}| = n$, $\bigcup\mathfrak{F}=m$, $|X| = p\ \forall\, X \in \mathfrak{F}$, and $\forall X, Y \in \mathfrak{F}(X \neq Y \Rightarrow |X \cap Y| < p)$. Theorem 1. If $A(m,n,p)$ and $n > m$, then $\mathrm{Sym}(n, \mathrm{cf}(p)) \mathrel{\overset{\subset}{\sim}} \mathrm{Sym}(m, p^+)/\mathrm{Sym}(m, p)$. Corollary 1. If $p, q > 1$ and p is the least cardinal such that $q^p > m$, then (i) $\mathrm{Sym}(q^p, \mathrm{cf}(p)) \mathrel{\overset{\subset}{\sim}} \mathrm{Sym}(m, p^+)/\mathrm{Sym}(m, p)$, (ii) $\mathrm{Sym}(m, p^+)/\mathrm{Sym}(m, p) \not\mathrel{\overset{\subset}{\sim}} \mathrm{Sym}(m)^I$ for any set I, and (iii) $\mathrm{Sym}(m, p^+)$ does not split over $\mathrm{Sym}(m, p)$. Corollary 2. For all m, $\mathrm{Sym}(m^+, \aleph_0) \mathrel{\overset{\subset}{\sim}} \mathrm{Sym}(m)/\mathrm{Sym}(m, m)$. Corollary 3. If p is regular, then for any n there is an $m > n \cup p$ such that $m^+ \geqq r > p \Rightarrow \mathrm{Sym}(m, r)$ does not split over $\mathrm{Sym}(m, p)$. Corollaries 1 and 3 give a partial solution to a problem in Scott, "Group theory," p. 317. Theorem 2. For any m, $\mathrm{Sym}(\aleph_0)/\mathrm{Sym}(\aleph_0, \aleph_0) \not\mathrel{\overset{\subset}{\sim}} \mathrm{Sym}(m, 2^{\aleph_0})$. Theorem 3. If $\mathfrak{F}$ is any filter such that $\forall A \subseteq m(|A| < m \Rightarrow m \sim A \in \mathfrak{F})$, then $(\mathrm{Sym}(m)^m/\mathfrak{F})^{0,m^+} \mathrel{\overset{\subset}{\sim}} \mathrm{Sym}(m)^m/\mathfrak{F}$. Corollary 1. The same hypotheses give $\mathrm{Sym}(m)^m/\mathfrak{F} \not\mathrel{\overset{\subset}{\sim}} \mathrm{Sym}(m)$. Theorem 4. $\mathrm{Sym}(\aleph_0)$ is not elementarily equivalent to $\mathrm{Sym}(\aleph_0, \aleph_0)$. Theorem 5. If $m^+ \geqq n > p$, then $\mathrm{Sym}(m)$ is not elementarily equivalent to $\mathrm{Sym}(m, n)/\mathrm{Sym}(m, p)$. (Received May 8, 1972.) (Author introduced by Professor J. Donald Monk.)

72T-A168. ROBERTO L. O. CIGNOLI, Instituto de Matemâtica, Universidad Nacional del Sur, Bahía Blanca, Argentina. Stone filters and ideals in distributive lattices.

Let L be a distributive lattice with zero 0 and unit 1, and let $B = B(L)$ be the Boolean algebra of all complemented elements of L. A Stone ideal is an ideal of L generated by an ideal of B. A Stone ultraideal is a proper maximal Stone ideal. Stone filters and Stone ultrafilters are defined dually. The main result of this paper is that the following conditions are equivalent: (1) Any Stone ultraideal is a prime ideal of L; (2) any Stone ultrafilter is contained in a unique ultrafilter of L; and (3) the minimal prime ideals of L are just the Stone ultraideals. As an application, a necessary and sufficient condition in order that L be a Stone algebra is given. (Received May 8, 1972.)

72T-A169. DUNCAN SUTTLES, University of British Columbia, Vancouver 8, British Columbia, Canada. A counterexample to a conjecture of Albert. Preliminary report.

The following is an example of a power associative commutative nil algebra A of nil-index four which is solvable but not nilpotent. A has a basis (over an arbitrary field) a, b, c, d, e, where $ab = c$, $ac = d$, $ae = (-c)$, $bc = e$, $bd = c$ and all other products are zero. (Received May 9, 1972.)

72T-A170. JENNIFER WALLIS, University of Newcastle, New South Wales, 2308, Australia and ALBERT LEON WHITEMAN, University of Southern California, Los Angeles, California 90007. Some classes of Hadamard matrices with constant diagonal.

The concepts of circulant and back-circulant matrices are generalized in the case of additive abelian groups. These results are then used to show the existence of skew-Hadamard matrices of order $8(4f+1)$ when f is odd and $8f+1$ is a prime power. This shows the existence of skew-Hadamard matrices of orders 296, 592, 1184, 1640, 2280, 2368 which were previously unknown. A construction is given for regular symmetric Hadamard matrices with constant diagonal of order $4(2m+1)^2$ when a symmetric conference matrix of order $4m+2$ exists and there are Szekeres difference sets, X and Y, of size m satisfying $x \in X \Rightarrow -x \notin X$, $y \in Y \Rightarrow -y \notin Y$. (Received May 11, 1972.)

*72T-A171. PHILIP G. BUCKHIESTER, Clemson University, Clemson, South Carolina 29631. Extending an $s \times t$ matrix of rank r to an $n \times m$ matrix of rank $r+k$.

Let $GF(q)$ denote a finite field of order q. Let A be an $s \times t$ matrix of rank r over $GF(q)$. The number of $n \times m$ matrices $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ of rank $r+k$ over $GF(q)$ is found. The procedure used in finding this number yields an algorithm which can be used to extend a given $s \times t$ matrix of rank r over any field, finite or infinite, to an $n \times m$ matrix of rank $r+k$. (Received May 12, 1972.)

*72T-A172. MICHAEL DOOB, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada. On graph products and association schemes.

A metrically regular graph of diameter m is the natural extension of the concept of a strongly regular graph, i.e., there are parameters p^i_{jk}, $i, j, k = 1, \ldots, m$ such that if the distance between two vertices is i, there are exactly p^i_{jk} vertices at distance j from the first and at distance k from the second. Theorem. For any integer m, there exist at least $[(m+2)/2]$ nonisomorphic metrically regular graphs of diameter m with the same parameters. Theorem. The above graphs determine PBIB's which are nonisomorphic with the same parameters. Necessary and sufficient conditions for the Cartesian product of metrically regular graphs to be metrically regular are given. (Received May 15, 1972.)

*72T-A173. LEO J. ALEX, State University College of New York, Oneonta, New York 13820. Simple groups of order $2^a 3^b 5^c 7^d p$.

Let $PSL(n, q)$ denote the projective special linear group of degree n over $GF(q)$, the field with q elements. The following theorem is proved. Theorem. Let G be a simple group of order $2^a 3^b 5^c 7^d p$, $a > 0$, p an odd prime. If the index of a Sylow p-subgroup of G in its normalizer is two, then G is isomorphic to one of the groups $PSL(2,5)$, $PSL(2,7)$, $PSL(2,9)$, $PSL(2,8)$, $PSL(2,16)$, $PSL(2,25)$, $PSL(2,27)$, $PSL(2,81)$, and $PSL(3,4)$. (Received May 15, 1972.)

*72T-A174. KENNETH K. HICKIN, Michigan State University, East Lansing, Michigan 48823. Countable type local theorems in algebra.

Techniques are developed for proving the following type of theorem: If enough countable subalgebras of a (general) algebra A belong to a class Σ, then $A \in \Sigma$. A local cover $\mathcal{L}$ of a set S is a directed subset-cover of S. $\mathcal{L}$ is countably complete if $\mathcal{L}$ contains the union of each of its countable chains. A W-system of S is a collection of subsets of S having some member in common with each countably complete local cover of S of countable subsets. A class Σ is W-local if for every algebra A possessing a W-system of Σ subalgebras we have $A \in \Sigma$. Sample results: (A) Let Σ = class of centerless groups with Max-N, and $DS\Sigma$ = class of (restricted) direct products of Σ groups; $DS\Sigma$ is W-local. (B) The class of algebras A satisfying $|\mathrm{Aut}(A)| \leq |A|$ is W-local. Some of these concepts have also been studied by D. W. Kueker, Bull. Amer. Math. Soc. 78 (1972), 211-215. (Received May 15, 1972.) (Author introduced by Professor Richard E. Phillips.)

*72T-A175. CHARLES R. WALL and THOMAS S. HSU, East Texas State University, Commerce, Texas 75428. A characterization of totients.

An arithmetic function f is a totient if f can be written as the Dirichlet product of a completely multiplicative function and the inverse of a completely multiplicative function; any totient is multiplicative. Theorem. A multiplicative function f is a totient if and only if, for each prime p, $f(p)$, $f(p^2)$, $f(p^3), \ldots$ is a geometric progression. (Received May 8, 1972.)

*72T-A176. YECHEZKEL ZALCSTEIN, Datalogy Institute, University of Copenhagen, Copenhagen, Denmark. A finiteness condition for matrix semigroups.

A classical theorem of Schur states that a finitely generated periodic group of $n \times n$ complex matrices is finite. A semigroup is periodic if for all x in S there are positive integers r, m such that $x^{r+m} = x^r$. Schur's theorem generalizes as follows. Theorem. A finitely generated regular periodic semigroup of $n \times n$ complex matrices is finite. (Received May 18, 1972.)

72T-A177. WITHDRAWN.

*72T-A178. MOURAD EL-HOUSSIENY ISMAIL, University of Alberta, Edmonton, Alberta, Canada. Two new congruences.

In this paper we study the congruences (1) $n\sigma^*(n) \equiv 2 \pmod{\varphi^*(n)}$, and (2) $\varphi^*(n)\tau^*(n) + 2 \equiv 0 \pmod{n}$, where $\varphi^*(n) = \Pi_1^r (p_i^{a_i}-1)$, $\sigma^*(n) = \Pi_1^r (p_i^{a_i}+1)$ and $\tau^*(n) = 2^r$; and $\Pi_{i=1}^r p_i^{a_i}$ is the canonical representation of n. We conjecture that the only solutions of (1) ((2)) are powers of primes, 6 and 22 (powers of primes). These are indeed solutions. To support these conjectures we show that any other solution of (1) ((2)) is $> 3 \times 2^{91}$ (has at least four distinct odd prime factors). (Received May 15, 1972.)

72T-A179. PHILIP KELENSON, City University of New York, John Jay College of Criminal Justice, New York, New York 10010. Normal Schrier varieties of universal algebras.

A variety K of universal algebras is called a normal Schrier variety if each subalgebra of a K-free algebra is K-freely generated by any minimal set of generators. Normal Schrier varieties are characterized. The variety of groups is a typical example. This generalizes results of the author, Abstract 72T-A110, these Notices 19(1972), A-435. (Received May 22, 1972.)

*72T-A180. ALBERT A. MULLIN, Army Research Office, Arlington, Virginia 22204. On interpolation lemmas for the Hardy-Ramanujan theorem.

Definitions. Let $f(n)$ be the number of distinct prime factors of n, $F(n)$ be the total number of prime factors of n counting repetitions, $f^*(n)$ be the number of distinct primes in the mosaic of n, and $F^*(n)$ be the total number of primes in the mosaic of n counting repetitions. Lemma 1. $f(n) \leqq f^*(n) \leqq F^*(n) \leqq F(n)$ for every natural number n with all equalities holding iff n is square-free. Lemma 2. The average value of $(F^*(n) - f^*(n)) < \sum_p 1/p(p-1) < 1$, since $0 \leqq (F^*(n) - f^*(n)) \leqq (F(n) - f(n))$ for every n with all inequalities holding on a recursive set of positive density. Lemma 3. The average order and the normal order (in the senses of Hardy and Ramanujan) of both $F^*(n)$ and $f^*(n)$ is $\log \log n$. Lemma 4. Let real $G(\cdot)$ be unbounded for large arguments. Then, for almost all n, $(F^*(n) - f^*(n)) < G(n)$. Lemma 5. Let real $H(\cdot)$ satisfy $H(x)/(\log \log x)^{1/2} \to \infty$ as $x \to \infty$. Then almost all numbers not exceeding x have between $\log \log x \pm H(x)$ primes in their mosaics regardless of whether one counts distinct primes or repetitions. (Received May 23, 1972.)

*72T-A181. WALTER DEUBER, Mathematisches Institut, Technische Universität, Hannover, Federal Republic of Germany. A conjecture of R. Rado on regular sets.

In Math. Z. 36(1933), 424-480, R. Rado defines a system $\sum_1^n a_{i,k}x_k = 0$ ($1 \leqq i \leqq m$, $a_{i,k}$ rational) to be regular iff for all partitions of N into finitely many classes at least one of the classes contains a solution of the system. A subset M of N is regular iff every regular system of linear homogeneous equations can be solved in M. Theorem. For all partitions of any regular set into finitely many classes one of the classes is regular. This was conjectured by R. Rado in the above paper. (Received May 22, 1972.) (Author introduced by Professor E. Specker.)

72T-A182. THOMAS J. SHARP, West Georgia College, Carrollton, Georgia 30117. A note on projection-invariant subgroups. Preliminary report.

Suppose G is an abelian group. A subgroup H of G is called projection-invariant if every projection of G onto a direct summand maps H into itself [L. Fuchs, "Infinite abelian groups," Vol. I, Academic Press, 1970]. It is clear that projection-invariance is a transitive property. Lemma. If $G = A \oplus B$ and if K is projection-invariant in G, then $G/K = (A\pi) \oplus (B\pi)$, where π denotes the natural homomorphism. Theorem. If K is projection-invariant in G and if H/K is projection-invariant in G/K, then H is projection-invariant in G. Several other results are presented. (Received May 22, 1972.)

72T-A183. HANSRAJ GUPTA, 402 Mumfordganj, Allahabad 2, India and Panjab University, Chandigarh, India and G. BAIKUNTH NATH, University of Queensland, St. Lucia, Australia. Enumeration of stochastic cubes. Preliminary report.

A cube of side n is divided into n^3 unit cubes called cells. Each cell is filled up with a nonnegative integer so that every line sum in the direction of an edge is r. In analogy with matrices having this property, such a cube will be called a stochastic cube of weight r. In this paper, it is shown that for $n = 3$ the number of stochastic cubes of weight r is $H(r) = \{20736\binom{r+5}{8} + 9504\binom{r+4}{6} + 1368\binom{r+3}{4} + 74\binom{r+2}{2} + (-1)^r 27(r+d)\}/128$, where $d = 1$ or 2 according as r is odd or even. It is surprising that the relation $H(r) = H(-r-3)$ which had been seen to hold in the case of matrices (Duke Math. J. 33(1966), 757-770) still holds good. It looks reasonable to conjecture that $H(r) = (-1)^{n-1}H(-r-n)$ will hold for cubes of side n. The theorem of Marshall Hall, Jr. concerning the representation of stochastic matrices as sums of permutation matrices (stochastic matrices of weight 1) cannot be extended to stochastic cubes. (Received May 23, 1972.)

72T-A184. E. M. WRIGHT, University of Aberdeen, Aberdeen, United Kingdom. Unlabelled graphs with many nodes and edges. IV. Prêliminary report.

An (n, q) graph has n unlabelled nodes and q undirected edges, each pair of different nodes being not joined or joined by just one edge. We write $T = T(n,q)$ for the number of different (n,q) graphs, $t = t(n,q)$ for the number of these which are connected and $\beta = \beta(n,q) = t/T$ for the probability that an (n,q) graph is connected. I have shown (Proc. Amer. Math. Soc. (to appear); Abstract 71T-A258, these *Notices* 18(1971), 1094) that, contrary to what one might expect, β does not increase steadily with q (even in a nonstrict sense). We write $N = n(n-1)/2$, ψ any positive number such that $\psi \to \infty$ as $n \to \infty$, $\chi = (1-x^2)^{1/2} - (x/2)\log\{(1-x)/(1+x)\}$ and c_k for the coefficient of x^{2k} in the expansion of $\chi^{k-1}/(2k-1)$. We write also $q_0 = N - (n/2)$ $\cdot\{\log n + 1 - \sum_{k=1}^{\infty} c_k (\log n)^{1-2k}\}$. Then we prove that, for large enough n, (i) $\beta(n,q) < \beta(n,q+1)$ when $(n/2)$ $\cdot(\psi + \log n) < q < q_0 - An^{1-C}$ and (ii) $\beta(n,q) > \beta(n,q+1)$ when $q_0 + An^{1-C} < q \leqq N - n$, where A, C are positive numbers independent of n and q. Of course, $\beta = 1$ for $N - n + 2 \leqq q \leqq N$. (Received May 24, 1972.)

*72T-A185. ABRAHAM BERMAN, Centre de recherches mathématiques, Montréal 101, Québec, Canada. Incidence matrices of Boolean functions and (0,1) programming.

Incidence matrices of Boolean functions are defined and used to check the consistency of Boolean expressions. This, in turn, is applied to check the feasibility and find the optimal solutions of (0,1) programs. (Received May 24, 1972.)

72T-A186. ARUN K. SRIVASTAVA, Indian Institute of Technology, Hauz Khas, New Delhi-29, India. Stable and local adjunctions.

D. Harris in his "Wallman compactification as a functor" (unpublished) introduces stably coreflective subcategories. Stable coreflections, besides others, include a host of projective covers in topology (see B. Banaschewski, "General topology and its relations to modern analysis and algebra", Proceedings of the Kanpur Topological Conference, 1968). Locally adjunctable functors appear in J. J. Kapur (Illinois J. Math. 16 (1972), 86-94). It is not hard to define stably adjunctable functors, stable colimits and local colimits. Main results. (i) If $\underline{A}$ has coequalizers, then $T : \underline{A} \to \underline{B}$ is right adjunctable if and only if it is stably right adjunctable and preserves coequalizers. (ii) A faithful functor $T : \underline{A} \to \underline{B}$ is right adjunctable and stably right adjunctable. Thus, (iii) a subcategory is coreflective if and only if it is both locally and stably coreflective. (iv) A category is cocomplete if and only if it is both stably and locally cocomplete. (Received May 26, 1972.) (Author introduced by Dr. S. P. Franklin.)

*72T-A187. BJARNI JONSSON, Vanderbilt University, Nashville, Tennessee 37235. Multiplicity types of algebras and their congruence lattices.

The multiplicity type of an algebra A is the sequence of cardinals $\mu = \langle \mu_0, \mu_1, \mu_2, \ldots \rangle$ where each μ_k is the number of basic operations of A that have rank k. If μ and μ' are two such sequences, then $\mu \leqq_c \mu'$ means that for each algebra A having the multiplicity type μ there exists an algebra A' having the multiplicity type μ' such that A and A' have the same universe and have exactly the same congruence relations. Theorem. $\mu \leqq_c \mu'$ iff $\mu_n + \mu_{n+1} + \ldots \leqq \mu'_n + \mu'_{n+1} + \ldots$ for $n = 1, 2, \ldots$. For the special case when $\mu_1 + \mu_2 + \ldots$ and μ'_1 $\mu'_2 + \ldots$ are finite this was proved by Thomas P. Whaley. (Cf. Abstract 71T-A224, these *Notices* 18(1971), 941.) Our construction and proof make extensive use of his ideas. (Received May 30, 1972.)

*72T-A188. IVAN RIVAL, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada. Maximal sublattices of finite distributive lattices.

H. Sharp ("Cardinality of finite topologies," J. Combinatorial Theory 5(1968), 82-86) and D. Steven ("Topology on finite sets," Amer. Math. Monthly 75(1968), 739-741) have proven that if L is a finite Boolean lattice with $|L| \geqq 4$ and M is a maximal proper sublattice of L then (i) $|M| = (3/4)|L|$ and (ii) $\ell(M) = \ell(L)$, where $\ell(L)$ denotes the length of L. The purpose of this note is to provide a simple proof of this result, as well as to establish an analogous result for finite distributive lattices. Theorem. If L is a finite distributive lattice with $|L| \geqq 3$ and M is a maximal proper sublattice of L then (i) $|M| \geqq (2/3)|L|$ and (ii) $\ell(M) \geqq \ell(L) - 1$. (Received May 30, 1972.) (Author introduced by Professor George A. Grätzer.)

72T-A189. JOEL HARKOW, State University of New York, Stony Brook, New York 11790. On planar Cayley diagrams. Preliminary report.

Under the assumption of this theorem, the word problem for G is reduced to finding the order γ of $(x_1x_2\ldots x_n)$. Let Γ be an embedding of the Cayley diagram of the group $G = F/N$ on the generators $a_1, a_2, \ldots, a_n$ in the plane. Γ is locally finite if every finite region of the plane contains a finite number of points of Γ. Let O_v be the clockwise ordering of the edges about the vertex $v \in \Gamma$; $-O_v$ be the counterclockwise ordering of the edges about v. If w is any word in F, let $\overline{w}$ denote the path in Γ determined by w. Theorem. Let $G = \langle a_1, a_2, \ldots, a_n; a_1^{\beta_1}, a_2^{\beta_2}, \ldots, a_n^{\beta_n}, \ldots \rangle$, $3 \leqq \beta_i < \infty$, $n \geqq 2$. Assume G has a locally finite planar Cayley diagram Γ such that $O_{v_\delta} = \pm O_{v_\mu}$, all $v \in \Gamma$. Assume that D is a finite connected component of the complement of Γ in the plane, and that D has boundary $\overline{w}$, a Jordan curve. Then either (1) $w = a_i^{\beta_i}$, $i \in (1, 2, \ldots, n)$, or (2) (a) $w = (x_1x_2\ldots x_n)^\gamma$ where $x_j = a_i^{\epsilon_i}$, i, $j \in (1, 2, \ldots, n)$, $\epsilon_i = \pm 1$, $1 \leqq \gamma < \infty$, $x_j \neq x_k^{\pm 1}$, (b) every path $\overline{(x_1x_2\ldots x_n)^\gamma}$ is a Jordan curve and determines a finite connected component of the complement of Γ in the plane, and (c) $O_{v_\delta} = +O_v$, all $v \in \Gamma$. The proof is combinatorial. (Received May 30, 1972.)

*72T-A190. JEFFREY DAWSON and DAVID E. DOBBS, Rutgers University, New Brunswick, New Jersey 08903. On going down in polynomial rings.

An extension $A \subset B$ of domains is mated if every prime ideal P of A such that $PB \neq B$ is unibranched in B. Let $R \subset T$ be domains. Necessary and sufficient conditions are given that the extension of polynomial rings $R[x] \subset T[x]$ be mated (resp., satisfy going down). If T is contained in the quotient field of R and $R[x] \subset T[x]$ satisfies going down, then $R[x] \subset T[x]$ is mated. If R is pseudo-Bezout and $T = R[u]$ for some u in the quotient field of R, then: $R[x] \subset T[x]$ satisfies going down $\Leftrightarrow$ $R[x] \subset T[x]$ is mated $\Leftrightarrow$ $R \subset T$ satisfies going down $\Leftrightarrow$ $R \subset T$ is mated $\Leftrightarrow$ T is a flat R-module $\Leftrightarrow$ T is a localization of R. Thus, R is Bezout $\Leftrightarrow$ R is pseudo-Bezout and $R \subset R[u]$ satisfies going down for all u in the quotient field of R. (Received May 30, 1972.)

72T-A191. BERNHARD AMBERG, Mathematisches Institut, Universität Mainz, Mainz, Federal Republik of Germany. Groups which are the product of two abelian subgroups. Preliminary report.

The following theorem generalizes a result which was first stated by E. Schenkman in Proc. Amer. Math. Soc. 21(1969), 202-204, where an error was found in its proof; see W. R. Scott, Abstract 667-137, these *Notices* 16(1969), 796. Theorem 1. Let the group $G = AB$ be the product of two abelian subgroups A and B, one of which is noetherian. (a) If $G \neq 1$, there exists a nontrivial normal subgroup of G contained in A or B. (b) If $A \neq G \neq B$, there exists a proper normal subgroup of G containing A or B. Theorem 2. If the group $G = AB$ is the product of two noetherian abelian subgroups A and B, then G is polycyclic and $h(G) = h(A) + h(B) -$

$h(A \cap B)$, where $h(X)$ is the Hirsch-number of X; furthermore $F(G) = (A \cap F(G))(B \cap F(G))$, where $F(G)$ is the fitting subgroup of G. (Received May 31, 1972.)

72T-A192. GERNOT MICHAEL ENGEL and HANS SCHNEIDER, University of Wisconsin, Madison, Wisconsin 53706. Cyclic and diagonal products. I.

Let D_n be the set of $n \times n$ matrices with elements in an integral domain D. A diagonal product for A is $\Pi_{i=1}^{n} a_{i\sigma(i)}$ where σ is a permutation. A cyclic product for A is $\Pi_{k=1}^{r-1} a_{i_k i_{k+1}}$ where $(i_1, \ldots, i_{r-1})$ are distinct integers and $i_1 = i_r$. For $A \in D_n$, define $A^S = B$, a totally supported matrix, by $b_{ij} = a_{ij}$ if a_{ij} is one of the factors of a nonzero diagonal product, and $b_{ij} = 0$ otherwise. Define $A^C = F$, a completely reducible matrix, by $f_{ij} = a_{ij}$ if a_{ij} is one of the factors of a nonzero cyclic product, and $f_{ij} = 0$ otherwise. Partially order D_n by defining $A \overset{0}{\leqq} B$ if either $b_{ij} = a_{ij}$ or $b_{ij} = 0$. Suprema are taken with respect to this partial order. Theorem 1. Let $1 \leqq k \leqq n$ and $m = \max\{k, n+1-k\}$. Let $A \in D_n$ with $a_{ij} = 0$ if $1 \leqq i \leqq k$, $k \leqq j \leqq n$. Then there exist permutation matrices $P_1, \ldots, P_m$ such that $A^S = \sup\{P_i^{-1}(P_iA)^C : i = 1, \ldots, m\}$. Theorem 2. Let $a_{ij} = 0$ if either $1 \leqq i \leqq k$, $k \leqq j \leqq n$ or $k+1 \leqq i \leqq n$, $1 \leqq j \leqq k-1$, and let $a_{ij} = 1$ otherwise. If $A^S = \sup\{P_i^{-1}(P_iA)^C : i = 1, \ldots, r\}$ then $r \geqq m = \max\{k, n+1-k\}$. Theorem 3. Let $A \in D_n$. (i) If $A^S \neq 0$, then there exists a permutation matrix P such that $A^S = P^{-1}(PA)^C$. (ii) If $A^S = 0$, then there exist permutation matrices $P_1, \ldots, P_n$ such that $A^S = \{P_i^{-1}(P_iA)^C : i = 1, \ldots, n\}$. (Received June 2, 1972.)

*72T-A193. ROBERT W. MILLER and DARRELL R. TURNIDGE, Kent State University, Kent, Ohio 44242. A non-Artinian Noetherian injective module.

Question. Does there exist a Noetherian injective module which is not Artinian? This question arises quite naturally when investigating chain conditions on injective modules. Conversations with other mathematicians have not revealed the existence of such an example. J. W. Fisher recently posed the same question in the Proceedings of the Utah Conference on Ring Theory (March, 1971). The following answers the question in the affirmative. Let C be the ring of countably infinite square upper triangular matrices over a field A. Let e be the idempotent with the identity of A in the (1,1) position and zeros elsewhere. Then, as a right C-module, eC is Noetherian injective but is not Artinian. (Received June 5, 1972.)

72T-A194. HERBERT S. GASKILL, Simon Fraser University, Burnaby 2, British Columbia, Canada. On the relationship of a distributive lattice to its lattice of ideals.

Let $\mathcal{L}$ be a finite distributive lattice and $\mathcal{J}(\mathcal{L}')$ denote the lattice of ideals of $\mathcal{L}'$. Theorem 1. If $\mathcal{L}'$ is a distributive lattice and φ embeds $\mathcal{L}$ in $\mathcal{J}(\mathcal{L}')$ then there is an embedding ψ of $\mathcal{L}$ in $\mathcal{L}'$ such that for all x and $y \in L$, $x\psi \in y\varphi$ if and only if $x \leqq y$. Corollary 1. A distributive lattice and its lattice of ideals have exactly the same set of finite substructures. Theorem 2. Let $\mathcal{L}'$ be any lattice (not necessarily distributive) and $\mathcal{L}$ satisfy the added condition that no point is both join and meet reducible. If φ embeds $\mathcal{L}$ in $\mathcal{J}(\mathcal{L}')$ then there is a ψ embedding $\mathcal{L}$ in $\mathcal{L}'$ such that for all x and $y \in \mathcal{L}$, $x\psi \in y\varphi$ if and only if $x \leqq y$. (Received June 6, 1972.)

*72T-A195. TORRENCE D. PARSONS, Pennsylvania State University, University Park, Pennsylvania 16802. Extremal chromatic graphs. Preliminary report.

Given graphs A, B, the Ramsey number $f(A,B)$ is the least integer N such that, for every graph G with N vertices, either G contains A or the complement of G contains B as a subgraph. Various numbers $f(A,B)$, and the corresponding extremal graphs, are determined in this paper. In particular, if K_n, P_n, C_n denote respectively the complete graph on n points, the path of length n, and the cycle of length n, then $f(K_m, P_n)$, $f(C_k, P_n)$ and $f(P_n, P_k)$ are determined for all m, n and for certain values of k. (Received June 7, 1972.)

72T-A196. ALAN A. BISHOP, Western Michigan University, Kalamazoo, Michigan 49001. Completions of lattices with semicomplementation. Preliminary report.

A lattice L is a generalized section semicomplemented (or gSSC) lattice if for every $x \in L$ there exists an $e \in L$ such that $e \leqq x$ and such that $e \leqq b < a$ implies the existence of a $c \in L$ satisfying $e < c \leqq a$ and $e = b \wedge c$. Generalized dual section semicomplemented (gDSSC) lattices are defined dually. The completion B(L) of a lattice L is the set of all complete ideals of L which are bounded above. The set B(L), partially ordered by set inclusion, is a conditionally complete lattice and is a sublattice of the complete lattice K(L) of all complete ideals. The mapping $x \mapsto J_x$ regularly embeds L into B(L). Theorem. If L is a conditionally upper continuous modular (resp. distributive) gSSC lattice, then B(L) is a conditionally upper continuous relatively complemented modular (resp. distributive) lattice. Under these same conditions B(L) is the conditional completion by cuts $\tilde{L}$ of L, L is a gDSSC lattice, and the completion by cuts $\overline{L}$ of L is modular (resp. distributive). (Received June 12, 1972.)

*72T-A197. J. ARTHUR GERHARD, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada. Injectives in equational classes of idempotent semigroups.

In this paper an investigation of injectives in equational classes of idempotent semigroups is begun. It is shown that subclasses of the class defined by the equation $(xyzx = xzyx)$ all have enough injectives while the remaining classes do not. In fact, in the classes which are not subclasses of the class defined by $(xyzx = xzyx)$, the only injectives are the injective semilattices. In the classes which are subclasses of $(xyzx = xzyx)$ the injectives are characterized as retracts of products of powers of maximal subdirectly irreducibles. An internal description of the injectives is either trivial or in the literature for each of these classes except (essentially) the class defined by the equation $(xyz = xzy)$. In this class we give necessary conditions that a semigroup be injective which are shown to be sufficient in the finite case. (Received June 12, 1972.)

72T-A198. MOTUPALLI SATYANARAYANA, Bowling Green State University, Bowling Green, Ohio 43403. On left cancellative semigroups. Preliminary report.

Let S be an l.c. semigroup. The existence of idempotents of S is shown to be related to the existence of maximal right ideals. The following are some of the results obtained about ideal structure of S and the idempotents. The first following result in a slightly different form was announced by Tamura recently. (1) If S has idempotents, then S is a right group or contains a unique maximal right ideal M, which is also the unique maximal ideal such that every right ideal is contained in M. (2) S has idempotents if S has a nontrivial maximal right ideal which is two-sided. (3) If S has no idempotents and if S has no proper maximal ideals, then $S \neq S^2$ and S^2 is the intersection of maximal right ideals. (4) If S is a left uniform semigroup (intersection of any two left ideals is nonempty) and if S has idempotents, then S has a unique idempotent, which is the identity of S. Some ideal theoretic properties of l.c. simple semigroups are obtained. (Received June 14, 1972.)

*72T-A199. MATTHEW I. GOULD, Vanderbilt University, Nashville, Tennessee 37235. Automorphism and subalgebra structure in algebras of finite type.

For a universal algebra $\mathfrak{A}$, $G(\mathfrak{A})$ denotes its set of automorphisms and $S(\mathfrak{A})$ its family of subuniverses.

Consider a nonvoid set A, a group $\mathscr{G}$ of permutations of A, and an intersection-closed family $\mathscr{S}$ of subsets of A. For $X \subseteq A$ let $[X]$ denote the intersection of all members of $\mathscr{S}$ which contain X and set $X^* = \{a | a \in A$ and for all $\alpha, \beta \in \mathscr{G}$, $\alpha|_X = \beta|_X$ implies $a\alpha = a\beta\}$. Consider these four conditions: (I) $\mathscr{G} = G(\mathfrak{A})$ and $\mathscr{S} = S(\mathfrak{A})$ for some algebra $\mathfrak{A}$ having finitely many operations. (II) Same as (I) for $\mathfrak{A}$ having at most two operations. (III) Same as (I) for $\mathfrak{A}$ having precisely one operation. (IV) There exist positive integers n and k such that: (IV. i) A permutation of A belongs to $\mathscr{G}$ if on every n-element subset of A it agrees with a member of $\mathscr{G}$; (IV. ii) for all $B \subseteq A$, $B \in \mathscr{S}$ if $B \supseteq [X]$ whenever $X \subseteq B$ and $|X| \leqq k$; (IV. iii) for all $X \subseteq A$ and $\alpha \in \mathscr{G}$, $|X| \leqq k$ implies $|[X]| \leqq \aleph_0$ and $[X] \subseteq X^*$ and $[X]\alpha \in \mathscr{S}$. Theorem. (I), (II), and (IV) are equivalent. If $\emptyset \in \mathscr{S}$ then all four conditions are equivalent. Remarks. (1) Two corollaries give further concrete and abstract characterizations of the pair $\langle G(\mathfrak{A}), S(\mathfrak{A})\rangle$ under suitable conditions. (2) Using different methods M. G. Stone (Abstract 70T-A111, these *Notices* 17(1970), 647) gave a concrete characterization of $\langle G(\mathfrak{A}), S(\mathfrak{A})\rangle$ with no requirement concerning the cardinality of the set of operations. (Received June 14, 1972.)

72T-A200. ARNOLD L. ROSENBERG, IBM, T. J. Watson Research Center, Yorktown Heights, New York 10598. An algebraic approach to addressability in data graphs.

A monoid system (MS) is a pair $\langle M, S\rangle$ where M is a finitely generated monoid and S is a submonoid of M. $\langle M, S\rangle$ is a transitive MS (TMS) if $(\forall s, t \in S)(\exists m \in M)[sm = t]$. Let f be the function associating with each data graph $\Gamma = (C, \Lambda)$ the MS $\Gamma f = \langle \Lambda^T, \mathrm{Tot}\rangle$, where Tot is the submonoid of Λ^T comprising all and only total functions. Theorem. If Γ is addressable, then Γf is a TMS. Let $\langle M, S\rangle$ be an MS, M having $g = \{g_1, \ldots, g_n\}$ as a set of generators. Let h_G be the function associating with $\langle M, S\rangle$ the system $(S, \{\gamma_1, \ldots, \gamma_n\})$, where each γ_i is the partial transformation of S defined by $s\gamma_i = sg_i$ if $sg_i \in S$, for all $s \in S$. Theorem. For each TMS $\langle M, S\rangle$, M having generators G, the system $\langle M, S\rangle h_G$ is an addressable data graph. Theorem. A data graph Γ is addressable if and only if $\Gamma f h_\Lambda$ is isomorphic to Γ. Definitions and notation are from A. L. Rosenberg, "Data graphs and addressing schemes," J. Comput. System Sci. 5(1971), 193-238. (Received June 14, 1972.)

72T-A201. JOHN ZOLNOWSKY, Computer Science Department, Stanford University, Stanford, California 94305. A direct combinatorial proof of the Jacobi identity.

An alternate form of the Jacobi identity is equivalent to the assertion that the number of partitions of a Gaussian integer r + si into an odd number of distinct nonzero Gaussian integers p + qi such that $|p - q| <= 1$, $p >= 0$, $q >= 0$ is equal to the number of partitions into an even number of such integers, except when r and s are consecutive triangular numbers. A proof of this assertion is given, based on a dot diagram analogous to that used in Fabian Franklin's proof of Euler's theorem relating to the number of partitions of a natural integer into an odd and an even number of distinct parts. (Received June 15, 1972.)

72T-A202. FAWZI M. YAQUB, American University of Beirut, Beirut, Republic of Lebanon. α-complete extensions of distributive lattices.

Let $\mathcal{K}$ be a class of α-complete distributive lattices and let L be a distributive lattice. The free $\mathcal{K}$-extension of L is a lattice $L_\alpha(\mathcal{K}) \in \mathcal{K}$ such that $L_\alpha(\mathcal{K})$ has a sublattice L_0 isomorphic to L and every homomorphism of L_0 into $L' \in \mathcal{K}$ can be extended uniquely to an α-homomorphism of $L_\alpha(\mathcal{K})$ into L'. Let $\mathcal{K}_\alpha$ be the class of all α-complete distributive lattices and $\mathcal{K}_\alpha^*$ the class of all α-complete lattices satisfying the infinite distributive laws $x \wedge (\bigvee_{t\in T} y_t) = \bigvee_{t\in T}(x \wedge y_t)$ and $x \vee (\bigwedge_{t\in T} y_t) = \bigwedge_{t\in T}(x \vee y_t)$, where $|T| \leqq \alpha$. The following are proved: (1) For every L, $L_\alpha(\mathcal{K}_\alpha)$ and $L_\alpha(\mathcal{K}^*_\alpha)$ exist. (2) For every L, $L_\sigma(\mathcal{K}^*_\sigma)$ is isomorphic to a σ-ring of sets. (3) A sufficient condition is given in order that $L_\alpha(\mathcal{K}^*_\alpha)$ be isomorphic to an α-ring of sets. Moreover, the above results are used to investigate regular α-extensions of distributive lattices. (Received June 16, 1972.)

*72T-A203. DAVID E. DOBBS, Rutgers University, New Brunswick, New Jersey 08903. Amitsur cohomology of cubic extensions of algebraic integers.

Let R be a domain with quotient field K and let S be the integral closure of R in a cubic (i.e., three-dimensional) Galois field extension L of K. Theorem. Assume that R is integrally closed, S is a flat R-module and the unit group U(R) is torsion with no element of order 3. Then $H^1(S/R, UK/U) = 0$. Corollary. Assume that K is either the rational field Q or a complex quadratic number field other than $Q(\sqrt{-3})$. Then $H^2(S/R, U) = 0$. Moreover, inflation and class number tables give information about cohomology arising from certain noncyclic cubics. The proofs depend upon the results of Canad. J. Math. 24(1972), 239-260. (Received June 16, 1972.)

*72T-A204. SEYMOUR LIPSCHUTZ, Temple University, Philadelphia, Pennsylvania 19122. Roots in eighth-groups. Preliminary report.

Theorem. Let G be a torsion-free eighth-group. Then G has unique roots. The proof uses the fact that, for any $W \neq 1$ in G, we have: (i) W is not conjugate to W^{-1}, and (ii) $W, W^2, W^3, \ldots$ are in different conjugate classes. (Received June 16, 1972.)

*72T-A205. R. M. RAPHAEL, Sir George Williams University, Montreal, Quebec, Canada. On the unit problem for rings.

On page 167 of "Complemented modular lattices and regular rings," L. A. Skornyakov asks the question: "Can every element of a regular ring with unit element be represented as a sum of elements having inverses?" Here regularity means regularity in the sense of von Neumann. Although the answer to the question is no (Utumi, "On continuous regular rings and self-injective rings," Canad. J. Math. 12(1960), 597-605), one is able to show that it is true for many regular rings provided that 2 is a unit. These include all commutative regular rings, full matrix rings which are regular, and left or right self-injective ones. The latter case is an extension of Utumi's results in the above paper. Crucial to the arguments is the result that any idempotent is a sum of two units in a ring in which 2 is a unit. Rings which are generated by their units are discussed more generally. (Received June 16, 1972.)

72T-A206. GEORGE S. SACERDOTE, Mathematical Institute, Oxford University, Oxford, England. SQ-universality of 1-relation groups. Preliminary report.

A countable group G is SQ-universal if for each countable group H, there is a quotient Q of G into which H can be embedded. A group is a 1-relation group if it has a presentation by generators and relations with one relation. Theorem. If G is a countable 1-relation group, then G is either SQ-universal or else G is one of the following: (i) cyclic, or (ii) metabelian and isomorphic to $\langle a, b;\ a^{-1}b^m a = b^n\rangle$ for $|m| = 1$ or $|n| = 1$. The theorem is proved by using the Moldovanskii construction to show that any noncyclic 1-relation group is an HNN extension of another 1-relation group. A careful analysis of this subgroup and the way it is embedded into the given noncyclic 1-relation group allows one to conclude that either the given group is one of the above metabelian groups or else it is SQ-universal as a consequence of a theorem of the author and P. E. Schupp. (The theorem just cited gives sufficient conditions to guarantee the SQ-universality of HNN groups.) (Received June 19, 1972.)

*72T-A207. FELIPE RUIZ, Public Health Department, State of Michigan, Lansing, Michigan 48933 and ESTHER SEIDEN, Michigan State University, East Lansing, Michigan 48823. Some results on construction of orthogonal Latin squares by the method of sum composition.

A method of sum composition of orthogonal Latin squares was introduced by A. Hedayat and E. Seiden [Proc. Conf. Combinatorial Geometry and its Applications, Perugia, Italy, 1970, pp. 239-256]. The method is applied to prove the following theorems. Theorem 1. For any prime p and an even integer $n \neq 6$ one can construct a pair of orthogonal Latin squares of size $p^{\alpha} + n$ provided that $p \geqq 2n$, α an integer. Effective method of construction was obtained using two distinct patterns. Theorem 2. For p of the form $4n + 1$ or $p \equiv 1, 2, 4 \pmod 7$ a method of construction of two orthogonal squares of size $p^{\alpha} + 4$ was obtained. The construction does not preclude obtaining three mutually orthogonal Latin squares. However no more than two could actually be produced. Theorem 3. It is shown that the restriction $xy = 1$ used by Hedayat and Seiden for construction of a pair of orthogonal Latin squares of size $p^{\alpha} + 3$ is necessary in all cases in which the two projection procedures (horizontal and vertical) recover mixtures of transversals belonging to the sets S and T. (Received June 21, 1972.)

*72T-A208. EMIL GROSSWALD, Technion, Haifa, Israel. On the number of solutions of Euler's equation.

Let $N(n)$ denote the number of solutions of the equation $\varphi(x) = n$, where $\varphi(x)$ stands for the Euler function. Exact formulae are obtained for $N(n)$, in case $n = 2^m$ and also if $n \equiv a \pmod{2^m \cdot 3}$, $a \not\equiv 2^m \pmod{3 \cdot 2^m}$, for $m \leqq 4$. In all these instances one obtains as an easy corollary of the formula for $N(n)$ that $N(n) \neq 1$. This is not surprising, since it follows from a recent result of Donelly that if $N(n) = 1$, then $n \equiv 0 \pmod{2^m}$ for some $m \geqq 4$. In case $n = 2^m$, the result is essentially a sharpening of a theorem due to Carmichael. Sample theorem. If $n \equiv 2 \pmod{12}$, then $N(n) = 0$, unless $n = p^{2m-1}(p-1)$ with p a prime, $p \equiv -1 \pmod{12}$, when $N(n) = 2$, or $n = 2$, when $N(2) = 3$. Somewhat similar results were recently announced by Mourad El-Houssieny Ismail (Abstract 72T-A121, these *Notices* 19(1972), A-501) for the equation $\varphi^*(x) = n$, where $\varphi^*(x)$ is the unitary analogue of $\varphi(x)$.
(Received June 22, 1972.)

72T-A209. THOMAS W. CUSICK, State University of New York at Buffalo, Amherst, New York 14226. Evaluation of a Diophantine approximation constant. Preliminary report.

For each pair of real numbers α, β define the constant $c(\alpha, \beta)$ to be the infimum of those $c > 0$ such that the inequality $\max(|x|(\alpha x - y)^2, |x|(\beta x - z)^2) < c$ has infinitely many solutions in integers x, y, z with $x \neq 0$. It is proved that $\sup c(\alpha, \beta) = 2/7$, where the supremum is taken over all α, β such that $1, \alpha, \beta$ is a basis for a real cubic number field. This proves a conjecture of W. W. Adams (Pacific J. Math. 30(1969), 2). The proof is based on an extension of the results in the author's paper "Formulas for some Diophantine approximation constants" (to appear in Math. Ann.). (Received June 23, 1972.)

72T-A210. JUDITH D. SALLY and WOLMER V. VASCONCELOS, Rutgers University, New Brunswick, New Jersey 08903. Stable rings and a conjecture of Bass. Preliminary report.

In [Math. Z. 82(1963), 8-28], Bass proved the following Theorem. Let A be a 1-dimensional Noetherian ring with finite integral closure. If every ideal of A is generated by 2 elements, then every ideal is projective in its endomorphism ring. The authors give two different proofs of the converse to this theorem which was conjectured by Bass. (Received June 23, 1972.)

*72T-A211. ROBERT GILMER and BRIAN J. WESSELINK, Florida State University, Tallahassee, Florida 32306. On a purely inseparable extension of a normal extension of a field.

Let F be a field of characteristic $p \neq 0$ and let K be a normal field extension of F. Theorem 1. Let $L = K(\alpha_1, \ldots, \alpha_r)$ be a purely inseparable field extension of K. Then L/F is normal if and only if $[L:K] = [K(\{a_{1j}^{1/p^{s_1}}\}_0^{t_1}, \ldots, \{a_{rj}^{1/p^{s_r}}\}_0^{t_r}):K]$, where f_i, the minimal polynomial of α_i over F, is a polynomial in $X^{p^{s_i}}$ but not in $X^{p^{s_i+1}}$ and $\{a_{ij}\}_{j=0}^{t_i}$ is the set of coefficients of f_i. Corollary. In the notation of Theorem 1, if $L = K(\alpha)$ is a simple purely inseparable extension of K, and if K/F is separable, then $K(\alpha)/F$ is normal if and only if $[F(\{a_j^{1/p^s}\}_0^t):F] = [K(\alpha):K]$. Theorem 2. If $F^{1/p} = F(\theta^{1/p})$ is a simple extension of F, then for each pair of fields L and K_0 with K_0/F normal and L/K_0 purely inseparable, L/F is normal. If F is not separably algebraically closed, and if $F^{1/p}$ is not a simple extension of F, then there exist fields L and K_0 such that K_0/F is normal, L/K_0 is purely inseparable, and L/F is not normal. Theorem 3. Let Σ be the purely inseparable part of K/F. If $K \cdot \Sigma^{1/p}$ is a simple extension of K, then each purely inseparable extension L of K has the property that L/F is normal. If $\Sigma \neq K$ and if $K \cdot \Sigma^{1/p}$ is not a simple extension of K, then there exists a purely inseparable extension L of K such that L/F is not normal. (Received June 26, 1972.)

*72T-A212. ROBERT WILLIS QUACKENBUSH, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada. The descending varietal chain of a variety.

Let V be a variety (equational class) of algebras and let V^n be the variety of all algebras which satisfy all n-variable identities of V. Equivalently V^n is the variety of all algebras $\mathfrak{A}$ such that every n-generated subalgebra of $\mathfrak{A}$ is in V. Clearly $V^1 \supseteq V^2 \supseteq \ldots \supseteq V$ and $V = \bigcap_{n=1}^{\infty} V^n$; the chain $V^1 \supseteq V^2 \supseteq \ldots$ is called the descending varietal chain of V. G. Grätzer and N. Gupta have raised the problem of which patterns of equality and inequality are possible in this chain. Theorem. Let I be a subset of the integers ≥ 2. There is a variety V with countably many binary operations and one nullary operation such that $V^{n-1} \neq V^n$ if and only if $n \in I$. (Received June 26, 1972.)

72T-A213. ARMIN THEDY, Matematisk Institut, Aarhus Universitet, DK 8000 Aarhus C., Denmark. On right alternative rings. Preliminary report.

Let R be a 2-torsion free right alternative algebra (over a unital associative and commutative ring k). Then $N_\beta := \{v \in R \mid x(xv) = x^2 v\}$ is a subalgebra. If R is simple, then $R = N_\beta$ or N_β is a commutative and associative subalgebra containing no right ideal $\neq 0$ of R. Put $(a,b,c) := (ab)c - a(bc)$ and $[a,b] := ab - ba$ for $a, b, c \in R$. If R is simple and not alternative, then each element of R is a linear combination of elements of type (a,a,b) or $([a,b],b,a)$ (such elements are nilpotent by results of Micheev, resp. Kleinfeld). If $[R,R] \subset N_\beta$ and R is simple, then R is alternative. If R is simple and if $x \in R$ satisfies $xy + yx = 0$ for all $y \in R$, then $x = 0$. If k is a field and R is a simple k-algebra of finite dimension over k, then R is alternative (in case characteristic 2 there are no strongly right alternative nil algebras of finite dimension). If R is simple and 6-torsion free, then $R = [R,R] + [R,R]R$. If, moreover, $[[R,R],R] \subset N_\beta$ or $(a,a,b) \in N_\beta$ for all $a, b \in R$, then R is alternative. If R is simple and 6-torsion free and if $x \in R$ satisfies $[[x,R],R] = 0$, then $[x,R] = 0$. The above results generalize earlier results of Albert, Kleinfeld and others. The proofs do not involve Peirce decompositions. (Received June 26, 1972.)

*72T-A214. JORGE MARTINEZ, University of Florida, Gainesville, Florida 32601. Some pathology in connection with groups of divisibility. Preliminary report.

In a partially ordered abelian group G, two elements a, $b > 0$ are pseudo-disjoint if each 0-ideal which is maximal with respect to not containing a contains b, and vice versa. G is a pseudo-lattice group if every $0 \| g \in G$ can be written as the difference of a pair of pseudo-disjoint elements. We prove the following Theorem. Suppose G is an abelian pseudo-lattice group; if there is an $x > 0$ and a finite set of pairwise pseudo-disjoint elements $x_1, x_2, \ldots, x_k$ all of which exceed x, and in addition this set is maximal with respect to the above properties, then G is not a group of divisibility. The main consequence of this result is that every so-called "v-group" $V(\Lambda, R_\lambda)$ for a given partially ordered set Λ, and where R_λ is a subgroup of the additive reals in their usual order, is a group of divisibility only if Λ is a root system, and hence $V(\Lambda, R_\lambda)$ is a lattice-ordered group.
(Received June 27, 1972.)

*72T-A215. GUSTAVE A. EFROYMSON, Department of Mathematics and Statistics, University of New Mexico, Albuquerque, New Mexico 87106. Local reality for algebraic varieties.

A ring A is real if $\sum_{i=0}^{m} \lambda_i^2 = 0$ implies all $\lambda_i = 0$. An ideal $\mathfrak{a} \subset A$ is real if $A/\mathfrak{a}$ is real. Next let k be real closed and let $\mathfrak{a}$ be an ideal in $k[X_1, \ldots, X_n]$. Define $V_k(\mathfrak{a}) = \{a \in k^{(n)} | f(a) = 0 \ \forall\ f \in k[X_1, \ldots, X_n]\}$. Then Dubois' nullstellensatz states $\mathfrak{a}$ is real iff $\mathfrak{a} = I(V_k(\mathfrak{a}))$. A short proof of this result is given using Tarski-Seidenberg. For the local theory assume $\mathfrak{a}$ is a real prime ideal in $k[X_1, \ldots, X_n]$ and $P = (0, \ldots, 0) \in V_k(\mathfrak{a})$. Then $\mathfrak{a}k[[X_1, \ldots, X_n]]$ has irredundant prime decomposition $\mathcal{P}_1 \cap \ldots \cap \mathcal{P}_r$. Theorem. The following are equivalent: (1) P is a limit point of nonsingular points of $V_k(\mathfrak{a})$ meaning that for all $\epsilon > 0$, ϵ in k, we have... . (2) Some $\mathcal{P}_i$ is real. (3) On a desingularization of some branch of $V(\mathfrak{a})$ thru P, there is a real point above P. The proof uses Tarski-Seidenberg extensively. Also a "real Rückert nullstellensatz" is proved using ultraproducts.
(Received June 26, 1972.)

72T-A216. JOHN FEGREUS, Clark University, Worcester, Massachusetts 01610. Calculation of conjugacy classes in the subgroup U of a finite Chevalley group. Preliminary report.

CONJCLAS is a SNOBOL4 program which aids in determing conjugacy class representatives for the subgroup U of a finite Chevalley group, where U is the subgroup generated by the groups $\{\mathcal{X}_\alpha(t) | t \in Q,$ Q a finite field$\}$ for each positive root α. The commutator relations are calculated using a matrix representation; or instead, may be input when known. An input element of U is then conjugated by a typical element i.e. $\Pi \mathcal{X}_\alpha(t_\alpha)$. An attempt is then made to calculate the order of the centralizer in U of the input element by determining if any of the t_α must either be zero or linearly dependent upon other of the t_α's. These results are output before another element is examined; hence, a judicious ordering of the input data will lower the execution time. In determining the class representatives for U of $G_2(Q)$ where the characteristic of Q was not equal to 2 or 3, approximately 1000 seconds were used on an RCA 70-46. (Received June 28, 1972.) (Author introduced by Professor John F. Kennison.)

72T-A217. OLGA TAUSSKY TODD, California Institute of Technology, Pasadena, California 91109. The factorization of an integral matrix into two integral symmetric matrices. II.

It is shown that any 2×2 integral matrix $A = (a_{ik})$ can be factorized as above if the binary quadratic form $-\lambda^2 a_{12}^2/\delta^2 + \lambda\mu[(a_{22} - a_{11}) - 2a_{12}y_0\delta] + \mu^2(z_0\delta - y_0^2)$, where $\delta = \text{g.c.d.}\ [(a_{22} - a_{11}), a_{12}]$ and $(a_{22} - a_{11})y_0$

$+ a_{21}\delta - a_{12}z_0 = 0$, is congruent to a form whose middle term has coefficient $\neq$ trace A. It follows that for every A there is an integer m such that $A + mI$ factorizes as above. (Received June 28, 1972.)

Analysis

72T-B184. WILLIAM D. L. APPLING, North Texas State University, Denton, Texas 76203. Two convergence theorems.

U, F, p_B, p_{AB}, p_A^+, A_m for m in p_A^+ and the notion of integral are as in previous abstracts of the author. Suppose a is in p_B, m is in p_A^+, $\{h_i\}_{i=1}^\infty$ is a sequence of elements in p_{AB}, and g is a function from F into $\mathbb{R}$ such that if V is in F, then $h_n(V) \to g(V)$, $n \to \infty$. Theorem 1. If $\{h_i\}_{i=1}^\infty$ is uniformly absolutely continuous with respect to m, $\int_U am$ exists and $\infty > \sup\{\int_U |h_n| : n = 1, 2, \ldots\}$, then g is in A_m and $\int_U ah_n \to \int_U ag$, $n \to \infty$. Theorem 2. If, for each n, $m - \int |h_n|$ is in p_A^+ and $\int_U ah_n$ exists, then g is in p_{AB}, $\int_U ag$ exists and $\int_U ah_n \to \int_U ag$, $n \to \infty$. (Received March 7, 1972.)

72T-B185. RAM SINGH, Punjabi University, Patiala, India. On Bazilevič functions. Preliminary report.

Let α be a fixed positive real number. Let $B(\alpha)$ be the class of functions $f(z) = z + \sum_{n=2}^\infty a_n z^n$ which are regular in $E = \{z \mid |z| < 1\}$ and satisfy the condition: $\mathrm{Re}[z f'(z) \{f(z)\}^{\alpha-1}/\{g(z)\}^\alpha] > 0$, $z \in E$, where $g(z)$ is some normalised starlike function in E. Let $B_1(\alpha)$ be the subclass of $B(\alpha)$ which corresponds to $g(z) \equiv z$. $B(\alpha)$ is a subclass of the so-called class of Bazilevič univalent functions in E. The following theorems have been proved: Theorem 1. The set of all points $\log[z f'(z)/\{f(z)\}^{1-\alpha}]$, for a fixed $z \in E$ and $f(z)$ ranging over the class $B(\alpha)$ is convex. Theorem 2. If $f(z) \in B(\alpha)$, where α is an integer, then the function $F(z)$, defined by $F^\alpha(z) = ((1+\alpha)/z) \int_0^z [f(t)]^\alpha dt$ also belongs to $B(\alpha)$. Theorem 3. If $F(z) \in B(\alpha)$, α an integer, and $f(z)$ is defined by $F^\alpha(z) = ((1+\alpha)/z) \int_0^z [f(t)]^\alpha dt$, then $f(z)$ is Bazilevič in $|z| < [\alpha + 1 - \sqrt{2(\alpha+1)}]/(\alpha - 1)$. Also, for $f(z) \in B_1(\alpha)$ sharp bounds on $|a_2|$, $|a_3|$, $|a_4|$ and $|a_3 - \lambda a_2^2|$, where λ is any complex number, have been established. (Received March 7, 1972.) (Author introduced by Professor V. Singh.)

72T-B186. BILLY E. RHOADES, Indiana University, Bloomington, Indiana 47401. The Chebyshev summability method. Preliminary report.

The regular Chebyshev summability method introduced by G. G. Bilodeau [J. Math. and Phys. 40(1961), 289-299] is of interest in its own right. For example, it is translative, is stronger than all of the Cesaro methods, but is not stronger than the Euler methods (E, q) for $q \geqq (\sqrt{8} - 1)/4$. Also the set of limit points of the Chebyshev transform of each bounded sequence is connected. (Received April 3, 1972.)

*72-B187. JERROLD E. MARSDEN, Institut des Hautes Études Scientifiques and University of California, Berkeley, California 94720. Smoothness of nonlinear semigroups. Preliminary report.

Let $\mathbb{E}$ be a Banach space (or manifold) and let $X : D \subset \mathbb{E} \to \mathbb{E}$ have a C^0 semigroup $F_t : \mathbb{E} \to \mathbb{E}$, $t \geqq 0$ [as in Chernoff-Marsden, Bull. Amer. Math. Soc. 76(1970), 1044]. We suppose D has a linear [or manifold] structure, inclusion is continuous [or smooth], dense and X is of class C^1. F_t may only be locally defined. Theorem. Suppose the tangent operator $TX : D \times D \to \mathbb{E} \times \mathbb{E}$, $(x, u) \mapsto (x, DX(x) \cdot u)$ is the generator of a C^0 semigroup on $\mathbb{E} \times \mathbb{E}$. Assume $DX(x)$ is a linear generator whose semigroup leaves D invariant, D is a core for $DX(x)$ for each $x \in D$ and $DX(x)$ is resolvent continuous on D. Then for each $t \geqq 0$, $F_t : D \to D$ is of class C^1. Examples. This may be applied to quasi-linear, first order symmetric hyperbolic systems, or to the strictly hyperbolic systems of Leray (the latter was already observed by Y. Choquet-Bruhat, C.R. Acad. Sci. Paris 274(1972), 843). Proofs of this result and similar ones for a semigroup depending on a parameter will appear shortly (A. Fischer and J. Marsden, Comm. Math. Phys., to appear). (Received April 10, 1972.)

72T-B188. A. R. REDDY, University of Missouri, St. Louis, Missouri 63121. Rational approximation to e^x. Preliminary report.

Recently D. J. Newman has studied the problem of obtaining uniform approximation to e^{-x} by general rational functions on the whole positive axis. In this direction, we present a result for e^x. Theorem. Let $P(x)$ and $Q(x)$ be any polynomials of degree at most $(n-1)$. There must exist a point somewhere in $[0, 3n]$ where $\|e^x - P(x)/Q(x)\| > (320)^{-n}$. (Received April 24, 1972.)

*72T-B189. WILLIAM E. DIETRICH, JR., University of Texas, Austin, Texas 78712. Ideals in function algebras.

Function algebras A on completely regular spaces X are introduced and under certain restrictions on X, an abundance of prime ideals in A is obtained by observing that C(K) is a homomorphic image of A for an appropriate thin compact subset K of the (generalized) Shilov boundary (Γ_A) of A. In particular, a uniform algebra which is, say, Dirichlet or logmodular on an infinite compact metric space always has infinite krull dimension. As an application, we observe that the hull of a countably generated, closed ideal of A always meets Γ_A in an open-closed set. The results serve to confirm the surprisingly pivotal role which the Shilov boundary plays in determining algebraic structure. (Received May 5, 1972.)

*72T-B190. RICHARD C. BROWN, Pennsylvania State University, University Park, Pennsylvania 16802. The adjoint and fredholm index of a linear system with general boundary conditions.

Let L_p be the system $L_p y = y' + Py$, $\int_0^1 y\, d\nu = 0$. P is an $n \times n$ continuous matrix valued function on $[0,1]$ and ν is an $m \times n$ matrix valued b.v. measure with lebesgue decomposition $\nu_c << \mu$, $\nu_s \perp \mu$. $\mathcal{D}(L_p) = \{y : y \text{ is a.c. in } L_n^p(\mu; 0,1) \text{ and } L_p(y) \text{ exists a.e. in } L_n^p(\mu; 0,1)\}$. Theorem 1. L_p is closed and densely defined, $1 \leqq p < \infty$, if the operators $f_t(v) = \int_0^t d\nu_s^*$ separates points in R^m. Theorem 2. Given the hypothesis of Theorem 1, $\mathcal{D}(L_p^*) = \{z : z(t) + \int_0^t d\nu_s^* \varphi \text{ is a.c. in } L_n^q(\mu; 0,1);\ 1/q + 1/p = 1\}$ and $L_p^* = -z' + P^*z + (d\nu_c^*/d\mu)\varphi$ where φ is a well defined parameter in R^m. Theorem 3. L_p, L_p^* are fredholm with indices $\pm m - n$ and mutually orthogonal ranges and kernels. L_p^* is similar to the preconjugate defined by Bryan, "A linear differential system with general boundary conditions" (J. Differential Equations 5(1969), 38-48) in the case $p = \infty$. These results also extend those of Krall and Brown ("Adjoints of multipoint-integral boundary value problems," Proc. Amer Math. Soc., to appear). (Received April 28, 1972.)

*72T-B191. RICHARD KRAFT, Section 205.01, National Bureau of Standards, Washington, D.C. 20234. A priori inequalities for families of linear hyperbolic systems in two independent variables.

A priori inequalities are derived for the solutions to the family of semidiscrete equations, $(\mu_i^N \partial_x + \partial_t) U^{Ni}(x,t) - \sum_{j=1}^N B_{ij}^N U^{Nj} = f^{Ni}$, $i = 1, \dots, N$, in x, $t \geqq 0$, with either absorbing or reflective boundary conditions. The derivation uses the technique in [Richard Kraft, "Convergence of semidiscrete approximations of linear transport equations", J. Math. Anal. Appl. 37(1972), 412-431] together with some bounds obtained by using monotonicity properties of a system of auxiliary Volterra integral equations. (Received May 1, 1972.)

*72T-B192. JAMES R. McLAUGHLIN, Pennsylvania State University, University Park, Pennsylvania 16802. The Denjoy-Luzin theorem on trigonometric series.

Let $\{g_n\}$ be a sequence of transformations from the real line to the real line. Throughout this paper assume that f is periodic with period $p = b - a$ (> 0), $f \in L^1[a,b]$, and set $f_n(x) = f(g_n(x))$. The sequence $\{g_n\}$ will be called an A-sequence with respect to f if the absolute convergence of $\sum c_n f_n(x)$ in a set of positive measure implies that $\sum |c_n|$ converges. The classical Denjoy-Luzin theorem on trigonometric series states that $g_n(x) = nx + B_n$ is an A-sequence with respect to $f(x) = \cos x$, where $\{B_n\}$ is any sequence of numbers. The present author generalizes the result by considering the infimum of the averages of a function over sets of constant measure. Inequalities are obtained relating these infimum for the functions $f(x)$ and $f(g(x))$ when $g(x)$ is a strictly monotonic absolutely continuous function. The following result is a simple corollary. The sequence $g_n(x) = A_n x + B_n$ is an A-sequence for every periodic f that is essentially nonzero (i.e., nonzero almost everywhere) if and only if $\liminf |A_n| > 0$. (Received May 1, 1972.)

*72T-B193. DOUGLAS MOREMAN, Auburn University, Auburn, Alabama 36830. A fixed point theorem involving a new metric convexity. Preliminary report.

In the context of a metric space S, d: Definitions. The statement that r is a radius of the point set M relative to the point set H means that r is the greatest lower bound of the set of all numbers x such that some spherical domain centered at a point of H and of radius x contains M. P is a center of M relative to H provided that P is a point of H, M has a radius r relative to H, and if $x > r$ then the spherical domain with center P and radius x contains M. (For other definitions, see Abstract 689-G6, these *Notices* 18(1971), 1065.) Theorems. (1) If H is a closed and spherically convex point set and K is the set of all centers relative to H of a point set M then K is closed, bounded, and spherically convex. (2) If the point set H is closed, spherically convex, and spherically compact, and the point set M is bounded then M has a center relative to H. (3) If M is a closed, bounded, spherically convex, and spherically perfectly compact point set such that if W is a nondegenerate subset of M then some point of $\mathscr{I}(W)$, the spherical convex hull of W, is not a center of $\mathscr{I}(W)$ relative to $\mathscr{I}(W)$, and if T is a transformation from M onto a subset of M such that if X and Y are points of M then $d(T(X), T(Y)) \leqq d(X,Y)$ then T leaves some point fixed. (Received May 2, 1972.)

72T-B194. AVRAHAM UNGAR, Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario Canada. A relation between derivatives of an implicitly defined function. Preliminary report.

Let (*) $t = \sum_{i=0}^n x_i f_i(\lambda)$, $\partial t/\partial\lambda \neq 0$, where $x_0 \equiv 1$, x_j $(j = 1, \dots, n)$ are real variables and $f_k(\lambda)$ $(k = 0, 1, \dots, n)$ are analytic functions of the complex variable λ. For an index exponent $p = (p_1, p_2, \dots, p_n)$, whose components are integers, $|p| = p_1 + p_2 + \dots + p_n$, let $\partial^p = \partial^{|p|}/\partial x_1^{p_1} \partial x_2^{p_2} \dots \partial x_n^{p_n}$, $f^p(\lambda) = f_1^{p_1}(\lambda) f_2^{p_2}(\lambda) \dots f_n^{p_n}(\lambda)$, $\dot\lambda = \partial\lambda/\partial t$ and $\partial_t^{|p|} = \partial^{|p|}/\partial t^{|p|}$. Theorem 1. Let $\lambda(x_1, x_2, \dots, x_n, t)$ be defined implicitly by (*) and let F be an arbitrary analytic function of a complex variable. Then $\partial^p\{F(\lambda)\dot\lambda\} = (-1)^{|p|}\partial_t^{|p|}\{f^p(\lambda)F(\lambda)\dot\lambda\}$. Example. Let λ be defined implicitly by $t = x \sin\alpha \cosh\lambda - iy \sin\alpha \sinh\lambda + z \cos\alpha$, $0 < \alpha < \pi$. Then λ is given explicitly by $\lambda = \cosh^{-1}((t - z\cos\alpha)/r \sin\alpha) + i\theta$ $(x = r\cos\theta,\ y = r\sin\theta)$ and $\dot\lambda = ((t - z\cos\alpha)^2 - r^2\sin^2\alpha)^{-1/2}$. By Theorem 1, $F(\lambda)\dot\lambda$ satisfies $\Delta\varphi = \partial^2\varphi/\partial t^2$ and derivatives of $F(\lambda)\dot\lambda$ with respect to the (space) variables x, y, z can be replaced by derivatives with respect to the single (time) parameter t. $\mathrm{Re}(\dot\lambda)$ describes a supersonic wave field due to a moving point source. Other definitions for λ can be found such that $\dot\lambda$ describes a subsonic wave field due to a moving point source or a wave field due to the impulsive point source $H(t - R)/R$. With the aid of Theorem 1, the problems of acoustic, elastic and electromagnetic wave propagation in horizontally layered media emitted by a moving or impulsive point source have been solved algebraically. (Received May 2, 1972.)

72T-B195. SA-GE LEE, University of California, Santa Barbara, California 93106. Invariance of the relative spectrum under C-algebra isomorphisms. Preliminary report.

Lemma. Let A be a bounded operator on a Hilbert space H. Let $\sigma(A^*A)$ be the spectrum of A^*A. Then the following two conditions are equivalent. (i) Range(A) is closed. (ii) Either $0 \notin \sigma(A^*A)$, or $0 \in \sigma(A^*A)$ and 0 is an isolated point of $\sigma(A^*A)$. Notation. Let A be a bounded operator on a Hilbert space H. Denote by $\sigma_{\mathrm{rel}}(A)$ the set of all complex numbers λ such that Range$(A-\lambda)$ is not closed. M. Embry [preprint, "Operators with closed range" (1972)] identified this set with the relative spectrum of A, introduced by E. Asplund [Ark. Mat. 3(1958), 425-427]. Proposition. Let $C^*(A)$ be the C*-algebra generated by A and the identity operator I on a Hilbert space H. Suppose there is a faithful *-representation π of $C^*(A)$ into the algebra of all bounded operators on a second Hilbert space K such that $\pi(I)$ is the identity operator on K. Then, $\sigma_{\mathrm{rel}}(A) = \sigma_{\mathrm{rel}}(\pi(A))$. (Received May 2, 1972.)

*72T-B196. JACK S. SHAPIRO, City University of New York, Bernard M. Baruch College, New York, New York 10010 and MORRIS SNOW, City University of New York, Queens College, Flushing, New York 11367. The Fredholm spectrum of the sum and product of two operators. Preliminary report.

Let X be a Banach space, $L(X)$ the set of bounded linear operators on X, $C(X)$ the set of closed operators on X with dense domain, $\mathcal{K}(X)$ the set of compact operators on X, and $\Phi(X)$ the set of Fredholm operators on X. Let $\sigma_\Phi(A) = \{\lambda : (\lambda - A) \notin \Phi(X)\}$. Theorem 1. Let $B \in L(X)$, $A \in C(X)$. Suppose $\exists\, n > 0$ and $K \in \mathcal{K}(X)$ such that $B : D(A^n) \to D(A)$ and $BAx = ABx + Kx \ \forall x \in D(A^n)$. Then $\sigma_\Phi(A+B) \subseteq \sigma_\Phi(A) + \sigma_\Phi(B)$. If $\sigma_\Phi(A)$ is empty, we interpret $\sigma_\Phi(A) + \sigma_\Phi(B)$ to be the empty set. Theorem 2. Let A and B satisfy the same conditions as in Theorem 1. Assume $0 \notin \sigma(B)$. Then BA is a closed linear operator, and $\sigma_\Phi(BA) \subseteq \sigma_\Phi(B)\, \sigma_\Phi(A)$. (Received May 5, 1972.)

*72T-B197. LOUIS BRICKMAN, DAVID J. HALLENBECK, THOMAS H. MacGREGOR and DONALD R. WILKEN, State University of New York, Albany, New York 12222. Convex hulls and extreme points of families of starlike and convex mappings. Preliminary report.

Let $St(\alpha)$ denote the family of starlike functions of order α; that is, $f \in St(\alpha)$ if f is analytic for $|z| < 1$, $f(0) = 0$, $f'(0) = 1$ and $\mathrm{Re}\{z f'(z)/f(z)\} > \alpha$ $(\alpha < 1)$. The closed convex hull of $St(\alpha)$ is proved to be the set of all functions represented by $f(z) = \int_X (z/(1-xz)^{2-2\alpha})\, d\mu(x)$ where μ varies over the probability measures on the unit circle X. A similar representation is obtained for the closed convex hull of the k-fold symmetric functions in $St(\alpha)$ and for the convex functions of order α, denoted $K(\alpha)$. The extreme points of these families are also determined. The arguments depend on ideas introduced by three of the authors in [Trans. Amer. Math. Soc. 156(1971), 91-107] and a generalization of a result in that paper. Several extremal problems are solved using the knowledge of these extreme points. In particular, the upper bounds are found for the coefficients of a function subordinate to or majorized by some function in $St(\alpha)$. Also, the problem $\min_{f\in K(\alpha)} \min_{|z|=r} \mathrm{Re}(f(z)/z)$ is solved for each r, $0 < r < 1$, and for $\alpha \geqq 0$. The result implies that if $f \in K(\alpha)$ and $\alpha \geqq 0$ then $\mathrm{Re}(f(z)/z) > (1 - 2^{2\alpha-1})/(1-2\alpha)$ for $|z| < 1$. This generalizes the well-known result corresponding to $\alpha = 0$, namely that $\mathrm{Re}(f(z)/z) > 1/2$ $(|z| < 1)$ for convex, univalent mappings. (A written communication from W. Kirwan asserts that D. Brannan, J. Clunie and Kirwan had earlier found the extreme points of the hull of $St(\alpha)$ for $\alpha \geqq 1/2$.) (Received May 5, 1972.)

72T-B198. MYRON S. HENRY, Montana State University, Bozeman, Montana 59715 and STANLEY E. WEINSTEIN, University of Utah, Salt Lake City, Utah 84112. Best Chebyshev composite approximations. Preliminary report.

Let $f(x,y)$ be continuous on $D = I \times J$, and let $F(A,x)$, $A \in P \subseteq E^n$ be a varisolvent function on I. Define $f_y(x) = f(x,y)$, $y \in J$, and suppose that (*) $\rho(y) = \inf_{A\in P} \sup_I |F(A,x) - f_y(x)|$ has the solution $A(y^*)$ at $y^* \in J$. If for all $A \in P$, $m(A) \leqq n$, and if $mA(y^*) = n$, then (*) has a solution $A(y)$ for all y in an appropriate neighborhood $N(y^*)$, and $A(y)$ is a continuous function at each $y \in N(y^*)$. If $A(y)$ is a continuous function on J, then the best composite approximation to $f(x,y)$ on D is $F[\overline{H}(\overline{B},y),x]$, $\overline{H}(\overline{B},y) = (H(B_1,y),\ldots,H(B_n,y))$ where $H(B,y)$, $B \in Q \subseteq E^m$, is an appropriate approximating function, where $A(y) = (a_1(y), a_2(y),\ldots,a_n(y))$, and where $\|H(B_i,y) - a_i(y)\|_J = \inf_{B\in Q} \sup_J |H(B,y) - a_i(y)|$. Examples are given and errors are considered in the cases that $F(A,x)$ is a rational function and $H(B,y)$ is a rational function or an appropriate linear combination of a Chebyshev set. Also an example is given to demonstrate that if $mA(y) = n$ for all $y \in J - \{y^*\}$, but $mA(y^*) < n$, then $A(y)$ may be badly discontinuous at y^*, i.e., limits from the left and right of y^* do not exist. (Received May 12, 1972.)

72T-B199. ABRAR A. KHAN, College of Engineering and Technology, Aligarh Muslim University, Aligarh, India. $|N, p_n|_k$ summability factors of infinite series.

Let $\sum a_n$ be a given infinite series with s_n as its nth partial sum and let $\{p_n\}$ be a sequence of constants, real or complex, such that $P_n = \sum_{i=0}^n p_i$. The series $\sum a_n$ is said to be summable $|N,p_n|_k$, $k \geqq 1$, if $\sum n^{k-1}|t_n - t_{n-1}|^k < \infty$, where $t_n = (1/P_n) \sum_{v=0}^n p_{n-v} s_v$, $P_n \neq 0$. If $p_n = 1$ summability $|N,p_n|_k$ reduces to the summability $|C, 1|_k$. In this note the following theorem has been proved which generalizes certain results due to T. Singh (Proc. Indian Sci. Congress, part III, 1964–65, p. 13), Nandkishore (Riv. Mat. Univ. Parma (2) 6(1965), 129-134) and R. K. Jain (Colloq. Math. 23(1971), 157-164). Theorem. If $\{p_n\}$ is a nonnegative and nonincreasing sequence such that $\sum_{n=2v+1}^{\infty} (n^{k-1}(\log n)^{k-1}(p_{n-v} - p_n)^k/P_{n-1}^k) = O(1)$, $v \geqq 1$ and if $\sum a_n$ is summable $|C,1|_k$, $k \geqq 1$, then $\sum_1^{\infty} (a_n P_n/n)$ is summable $|N,p_n|_k$. (Received May 16, 1972.) (Author introduced by Dr. S. M. Mazhar.)

*72T-B200. DAVID J. HALLENBECK and THOMAS H. MacGREGOR, State University of New York, Albany, New York 12222. Subordination and extreme-point theory. Preliminary report.

Let $\Delta = \{z : |z| < 1\}$ and for two functions f and g analytic in Δ let $f < g$ denote that f is subordinate to g in Δ. If F is any family of functions analytic in Δ then $F^* = \{f : f < g$ for some g in $F\}$. With respect to the topology given by uniform convergence on compact subsets of Δ, we study the question of finding the closed convex hull of F^* and the extreme points of this hull for various families F. For example, let St denote the set of analytic functions that are univalent in Δ so that $f(0) = 0$, $f'(0) = 1$ and $f(\Delta)$ is starlike with respect to 0. Then the hull of St* consists of the functions represented by $f(z) = \int_X (xz/(1-yz)^2)\, d\mu(x,y)$ where X is the product of the unit circle with itself and μ varies over the probability measures on X. The set of extreme points of this hull is $\{f : f(z) = xz/(1-yz)^2,\ |x| = |y| = 1\}$. Similar results are obtained for the convex and close-to-convex mappings and for other families F. A second consideration asks similar questions for the family of functions subordinate to a given function F in Δ. In particular, we show that the functions $F(xz)$, $|x| = 1$, are always extreme points. Also, if $F \in H'$ (the Hardy class) and if φ is an inner function with $\varphi(0) = 0$, then $F(\varphi)$ is an extreme point. Several examples are discussed and some applications are presented concerning the solution to related extremal problems. (Received May 16, 1972.)

*72T-B201. PATRICK WILLIAM DARKO, Cornell University, Ithaca, New York 14850. Sections of coherent analytic sheaves with growth on complex spaces. Preliminary report.

Let X be a complex space; $Y \subset\subset X, Y' \subset X$, $\Omega \subset \mathbb{C}^n$, $\Omega' \subset \mathbb{C}^n$ open sets with $\bar{\Omega} \subset \Omega'$, $\bar{Y} \subset Y'$, η a closed imbedding of Y' onto Ω' and let there be a system of neighborhoods of $\bar{\Omega}$ which are domains of holomorphy, a plurisubharmonic function φ on Ω and $K_1 > 0$, $K_2, K_3 \geqq 1$, $K_4 > 0$ such that $z \in \Omega$ and $|z - \xi| \leqq \exp(-K_1\varphi(z)-K_2)$ imply $\xi \in \Omega$ and $\varphi(\xi) \leqq K_3\varphi(z) + K_4$. Let $\mathcal{O}$ be the structure sheaf of $\mathbb{C}^n$ and $\mathfrak{F}$ a coherent analytic sheaf on a neighborhood of $\bar{\Omega}$, $H_\varphi(\Omega, \mathfrak{F}) = \bigcup_{\rho \geqq 0}\{f \in \Gamma(\Omega, \mathfrak{F}) : \exists\, h \in \Gamma(\Omega, \mathcal{O}^q),\ q \geqq 1,\ \sup_{z\in\Omega}|h(z)|\exp(-\rho\varphi(z)) < \infty$ and for some open $U \supset \bar{\Omega}$, $\exists\, \alpha \in \Gamma(U, \mathrm{Hom}_{\mathcal{O}}(\mathcal{O}^q, \mathfrak{F}))$, $\alpha(\mathcal{O}^q) = \mathfrak{F}$ and $\alpha(h) = f\}$. For $\mathcal{S}$ a coherent analytic sheaf on Y', $\eta_*(\mathcal{S})$ the 0th direct image on Ω' we set $H_\varphi(Y, \mathcal{S}) = \{f \in \Gamma(Y, \mathcal{S}) : \eta_*(f) \in H_\varphi(\Omega, \eta_*(\mathcal{S}))\}$. Theorem. Let $\mathcal{S}_1, \mathcal{S}_2$ be coherent analytic sheaves on Y', $\lambda \in \Gamma(Y', \mathrm{Hom}_{\mathcal{O}}(\mathcal{S}_1, \mathcal{S}_2))$ surjective. Then $H_\varphi(Y, \mathcal{S}_2) \subset \lambda(H_\varphi(Y, \mathcal{S}_1))$. The methods are similar to those in R. Narasimhan, Lecture Notes in Math., no. 155, Springer-Verlag, 1970, pp. 141-150, and Y. T. Siu, Duke Math J. (1970), 77-84. (Received May 4, 1972.)

72T-B202. THOMAS R. CAPLINGER, Memphis State University, Memphis, Tennessee 38111 and University of Mississippi, University, Mississippi 38677. On certain classes of analytic functions. Preliminary report.

Let A denote the class of functions $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ which are analytic in the open unit disk. Also let $D(\alpha)$ be the subclass of A of functions $f(z)$ satisfying $|(f'(z)-1)/(f'(z)+1)| < \alpha$, $0 < \alpha \leqq 1$. Theorem 1. If $f(z)$ is in $D(\alpha)$, then $|a_n| \leqq 2\alpha/n$. The result is sharp. The author also determines sharp distortion theorems and sharp estimates on the area of f $(|z| < r)$ and the arc length of f $(|z| = r)$, $0 < r < 1$. Theorem 2. If $F(z)$, $g(z)$ and $h(z)$ are in A and $F(z) = (2/z)\int_0^z f(t)\,[g(t)/h(t)]^\beta dt$, where $F(z)$ is starlike, both $g(z)$ and $h(z)$ are convex and $\beta \geqq 1/2$, then $f(z)$ is univalent and starlike for $|z| < (2\beta + 3 - \sqrt{4\beta^2 + 12\beta + 1})/4$. The result is sharp and generalizes a theorem of E. Calys [Compositio Math. 23(1971), 467-470]. (Received May 19, 1972.) (Author introduced by Professor W. M. Causey.)

*72T-B203. REUBEN HERSH, University of New Mexico, Albuquerque, New Mexico 87106. Maxwell's coefficients are conditional probabilities.

Given a disjoint system of closed, bounded conductors $\Gamma_1, \ldots, \Gamma_n$, there are defined Maxwell's "capacitance" or "induction coefficients" c_{ij}, which are familiar in both electrostatics and conformal mapping. Theorem. $-c_{ij}/c_{ii}$ is the conditional probability that a Brownian particle which is initially near Γ_i first meets the boundary at Γ_j. An application is a simple new proof that each c_{ij} is monotonic as a domain functional. (Received May 23, 1972.)

72T-B204. PETER HESS, University of California, Berkeley, California 94720. A strongly nonlinear parabolic equation.

Let $\Omega \subset R^n$ be open, bounded, and $Q = \Omega \times]0, T[$, T finite. The following problem is considered: To find a function $u = u(x,t)$ $((x,t) \in Q)$ such that (1) $\partial u/\partial t + \mathcal{Q}u + p(u) = f$ in Q; (2) $u(x,0) = u_0(x)$ $(x \in \Omega)$; (3) $u(x,t) = 0$ $((x,t) \in \partial\Omega \times]0, T[)$. Here $\mathcal{Q} = \sum_{|\alpha|,|\beta| \leqq 1}(-1)^{|\alpha|}D^\alpha a_{\alpha\beta}D^\beta$; $p: R \to R$ is a continuous function, and f, u_0 are given. If p is monotone increasing, strong solutions of (1)—(3) are implied by the theory of nonlinear contraction semigroups in Hilbert space. Our interest is in nonmonotone p. Assume (I) $a_{\alpha\beta}(x,t) \in L^\infty(Q)$ $(|\alpha|, |\beta| \leqq 1)$. Let $V = L^2(0, T; H_0^1(\Omega))$ and a be the bilinear form on $V \times V$ associated with $\mathcal{Q}$. Suppose further (II) $a(u,u) \geqq c_0\|u\|_V^2$ $(c_0 > 0)$ for $u \in V$; (III) $p(s)s \geqq 0$ $(s \in R)$; (IV) $|p(s)| \leqq c_1 + c_2|s|^\gamma$ $(s \in R)$ for some $\gamma > 0$. Let $q = 1 + \gamma$ and $W = L^q(Q)$

$\cap$ V. Let V*, W* denote the spaces conjugate to V, W, and (f, u) the duality pairing, either between $f \in V^*$, $u \in V$, or $f \in W^*$, $u \in W$. Theorem. Assume conditions (I)—(IV) are satisfied. Then for given $f \in V^*$ and $u_0 \in L^2(\Omega)$ problem (1)—(3) has a weak solution: there exists $u \in V$ with $\partial u/\partial t \in W^*$ such that $(\partial u/\partial t, w) + a(u, w) + \int_Q p(u) w \, dx \, dt = (f, w)$ for $w \in W$; $u(0) = u_0$. In case $\gamma \leq 1$, the theorem follows by a result of J. L. Lions (Bull. Soc. Math. France 93(1965), 155-175). The case $\gamma > 1$ is reduced to that by truncating p. (Received May 22, 1972.)

*72T-B205. HASKELL P. ROSENTHAL, University of California, Berkeley, California 94720. On factors of C([0, 1]) with nonseparable dual.

Let C denote the Banach space of scalar-valued continuous functions defined on the closed unit interval. It is proved that if X is a Banach space and $T : C \to X$ is a bounded linear operator with T^*X^* nonseparable, then there is a subspace Y of C, isometric to C, so that $T|Y$ is an isomorphism. An immediate consequence of this and a result of A. Pelczyński is that every complemented subspace of C with nonseparable dual is isomorphic (linearly homeomorphic) to C. (Received May 26, 1972.)

*72T-B206. STEPHEN GROSSBERG, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. Limit distributions of some reverberating neural networks. Preliminary report.

Consider the system $\dot{x}_i = -(\alpha + I_i)x_i + (\beta - x_i)E_i$, $i = 1, 2, \ldots, n$, where $E_i = f(x_i)$, $I_i = \sum_{k \neq i} f(x_k)$, and $0 \leq x_i(0) \leq \beta$. This system describes the interaction of n neural populations interconnected by an on-center off-surround recurrent anatomy, with $E_i(I_i)$ the total excitatory (inhibitory) input, and $x_i(\beta - x_i)$ the number of (un)excited states in the ith population. Let $y_i = x_i x^{-1}$, where $x = \sum_k x_k$, and let $P_i = \lim_{t \to \infty} y_i(t)$. Consider the existence and distribution of the probabilities P_i for various choices of f(w). If for some $\epsilon > 0$ and all $t \geq 0$, $x(t) \geq \epsilon$, the reverberation is persistent. If $P_i = y_i(0)$, it is fair; if for some i, $P_i = 1$, it is 0-1; if for all i, $P_i = 1/n$, it is uniform. Theorem. Suppose that the reverberation is persistent. If $f(w) = \gamma w$, $\gamma > 0$, it is fair. If $f(w) = \gamma w(\delta + w)^{-1}$, γ, $\delta > 0$, it is uniform. If $f(w) = \gamma w [\delta + \exp(-\epsilon w)]^{-1}$, γ, δ, $\epsilon > 0$, and there exists a unique i such that $x_i(0) > x_k(0)$, $k \neq i$, it is 0-1. If $f(w) = \gamma w^2 (\delta + w^2)^{-1}$, $2\delta > \beta$, and there exists a unique i such that $x_i(0) > x_k(0)$, $k \neq i$, it is 0-1. By contrast, for sufficiently large $\beta\delta^{-1}$ and initial x(0) (precise estimates exist), it is uniform. Conditions that guarantee persistence have been derived. The theorem can be used to show that networks can store different features (such as the boundary) of an input pattern by choosing different f(w)'s. (Received May 26, 1972.)

*72T-B207. BERNARD N. HARVEY, California State University, Long Beach, California 90840. The complex moment problem. Preliminary report.

Let S(k) be the set of all sequences $(i_1 j_1 i_2 j_2 \cdots i_k j_k)$ of nonnegative integers. If W_i and Y_i are in $\mathbb{C}$ $(i = 1, 2, \ldots, k)$, let, for ω in S(k), $(W, Y)^\omega = W_1^{i_1} Y_1^{j_1} W_2^{i_2} Y_2^{j_2} \cdots W_k^{i_k} Y_k^{j_k}$. Let $(W, Y) = W_1 Y_1 W_2 Y_2 \cdots W_k Y_k$. If $\omega = (i_1 j_1 i_2 j_2 \cdots i_k j_k)$ let $\omega' = (j_1 i_1 j_2 i_2 \cdots j_k i_k)$. Theorem. Let f_ω be an infinite sequence of complex numbers (ω in S(k)) with $\overline{f}_\omega = f_{\omega'}$ and let $\overline{Q} = Q \subseteq \mathbb{C}^k$. If $p = \sum (a, \overline{b})(z, \overline{z})^\omega$, let $f(p) = \sum (a, \overline{b}) f_\omega$. Then there is a real Borel measure F in $\mathbb{C}^k$ supported in Q such that $f_\omega = \int (z, \overline{z})^\omega dF$ iff $f(p) \geq 0$ whenever $p \geq 0$ in Q. To prove this one considers $m_v = \sum \alpha_\omega f_\omega$ $(v, \omega \in S(k))$ where $(x, y)^v = \sum \alpha_\omega (z, \overline{z})^\omega$. The formula $f_\omega = \sum \beta_v m_v$ is shown, where $(z, \overline{z})^\omega = \sum \beta_v (x, y)^v$. The m_v are proven real by the condition $\overline{f}_\omega = f_{\omega'}$. If $q = \sum (c, d)(x, y)^v$ let $M(q) = \sum (c, d) m_v$. By the well-known real moment theorem $\exists$ a positive Borel measure F supported in Q such that $m_v = \int (x, y)^v dF$ iff $M(q) \geq 0$ whenever $q \geq 0$ on Q. Here we identify $\mathbb{R}^{2k}$ and $\mathbb{C}^k$. Since every $q = \sum (c, d)(x, y)^v$ can be written

$p = \sum(a,\bar{b})(z,\bar{z})^{\omega}$, and conversely, the conditions "$M(q) \geqq 0$ whenever $q \geqq 0$ on Q", and "$f(p) \geqq 0$ whenever $p \geqq 0$ on Q" are equivalent. Lastly $\sum \beta_v m_v = \int \sum \beta_v (x,y)^v dF$, $f_\omega = \int (z,\bar{z})^{\omega} dF$, and the Theorem follows.
(Received May 24, 1972.)

*72T-B208. JAMES H. OLSEN, North Dakota State University, Fargo, North Dakota 58102. An ergodic theorem for convex combinations of isometries induced by point transformations of the unit interval.

Isometries of $L_p(0,1)$, $1 < p < \infty$, induced by point transformations $\tau : (0,1) \to (0,1)$ of the form $\tau x = x^k$, $k > 0$, are considered. The author has previously proved an ergodic theorem for the convex combination of two such isometries. This result is combined with the proof of S. A. McGrath of the same result for the convex combination of any two positive invertible isometries to give the Theorem. Let T be a convex combination of three isometries of $L_p(0,1)$, $1 < p < \infty$, induced by point transformations of the form $\tau x = x^k$, $k > 0$. Then the sequence $\{T^n f\}_{n=0}^{\infty}$ converges cesàro to an L_p function a.e. for every f in $L_p(0,1)$. (Received May 26, 1972.)

*72T-B209. JOSEPH LEHNER, University of Maryland, College Park, Maryland 20742. Bounded and integrable automorphic forms.

Let Γ be a finitely-generated Fuchsian group of the second kind acting on $U : |z| < 1$, let $A_q(\Gamma)$ be the set of automorphic forms f of weight q (degree $-2q$) $\geqq 1$ and multiplier system v on Γ that are integrable ($\iint_R M(z)\,(1-|z|^2)^{-2} dx\,dy < \infty$, $M(z) = (1-|z|^2)^q \cdot |f(z)|$, R = fundamental polygon) and let $B_q(\Gamma)$ be the set of bounded automorphic forms ($M(z) < B$ for $|z| < 1$). The author gives a simple, elementary proof of the known Theorem. $A_q(\Gamma) \subset B_q(\Gamma)$, $q \geqq 1$ (D. Drasin—C. J. Earle, Proc. Amer. Math. Soc. 19(1968), 1039-1042; T. A. Metzger-K. V. R. Rao, Proc. Amer. Math. Soc. 28(1971), 562-566; M. I. Knopp, to be published). Namely, it is easily shown, using Cauchy's integral formula, that $M(z_0) < (2\pi)^{-1} \iint_\Delta M(z)\,(1-|z|^2)^{-2} dx\,dy$, $z_0 \in U$, $\Delta = \{|z-z_0| < (1-|z_0|)/2\}$. Also there exists $0 < r_0 < 1$ such that for $|z_0| > r_0$, z_0 lying in the "funnel" between the two conjugate arcs of a hyperbolic element of Γ, Δ meets at most 2 fundamental polygons. Hence $M(z_0) < m_1$. In $|z| \leqq r_0$ obviously $M(z_0) \leqq m_2$. The remainder of $R \cap \{|z| > r_0\}$ consists of finitely many parabolic "cusp sectors" ending at p_i, and it is known (and easily proved) that $f \to 0$ as $z \to p_i$ within R provided $q \geqq 1$. Q. E. D.
(Received May 30, 1972.)

72T-B210. WITHDRAWN.

72T-B211. G. P. KAPOOR, Indian Institute of Technology, Kanpur-16, India. On extreme rates of growth of functions analytic in a finite disc. Preliminary report.

Let f(z) be analytic in $D_R \equiv \{z : |z| < R \leqq \infty\}$. Growth of f(z) is measured by its maximum modulus $M(r,f) = \max_{|z|=r} |f(z)|$, $0 < r < R \leqq \infty$. For $R = \infty$, Lepson [J. Math. Anal. Appl. 36(1971)] proved existence of an entire function whose upper rate of growth is arbitrarily fast and simultaneously whose lower rate of growth is arbitrarily slow. Using Lepson's result the following is proved. Theorem. Let $\lambda(r)$ and $\mu(r)$ be positive functions of r for $0 < r < R < \infty$ such that $\log \lambda(r) \neq O(-\log(R-r))$ as $r \to R$. Then there exists a function f(z) analytic in D_R with $0 < R < \infty$, and two sequences $\{s_n\}$ and $\{t_n\}$ of positive numbers, tending monotonically to R, such that for every positive integer n, $M(s_n, f) > \mu(s_n)$ and $M(t_n, f) < \lambda(t_n)$. Further, it is also shown that there exists a function analytic in D_R with $0 < R < \infty$ which has two arbitrarily prescribed rates of growth on different, although unspecified, sequences tending to R. (Received May 30, 1972.) (Author introduced by Professor Prabha Gaiha.)

72T-B212. WITHDRAWN.

72T-B213. PRABHA JAIN, Department of Mathematics and Statistics, Aligarh Muslim University, Aligarh, India. An integrability theorem for power series.

The following theorem has been proved which generalizes a theorem of Heywood (J. London Math. Soc. 32(1957), 22-27) and extends a result of Khan (Acta Sci. Math. 30(1969), 255-257). Theorem. Let $F(x) = \sum_0^\infty c_n x^n$, $0 \leq x < 1$, and $\gamma < 1$. Suppose $c_n > -K n^{(1-\gamma)/p-1-\epsilon}$, $0 < p \leq \infty$, K a positive constant, for all sufficiently large values of n. Then $(1-x)^{-\gamma}(F(x))^p \in L(0,1)$ iff $\sum n^{\gamma-2}(\sum_{k=1}^n |c_k|)^p < \infty$. (Received May 31, 1972.)

72T-B214. EDMOND E. GRANIRER, University of British Columbia, Vancouver, British Columbia, Canada. On amenable locally compact groups. Preliminary report

Let G be a locally compact σ-compact group, (TLIM) LIM denote the set of (topological) left invariant means on $L^\infty(G)$. Theorem. Let G be amenable as a discrete group. Then LIM = TLIM if and only if G is discrete. In particular, if G is compact and amenable as a discrete group, then LIM = $\{\lambda\}$ (λ denoting Haar measure) if and only if G is finite. This generalises a result of F. Greenleaf (written communication). (Received June 1, 1972.)

72T-B215. DAVID F. DAWSON, North Texas State University, Denton, Texas 76203. Some strict inclusions for matrix summability methods.

R. P. Agnew (Bull. Amer. Math. Soc. 52(1946), 128-132) obtained a simple sufficient condition for a regular matrix method to be stronger than convergence. In this note, Agnew's result is extended to Cesàro summability of any nonnegative integral order (Theorem 1), and a related result of the author (Abstract 70T-B173, these *Notices* 17(1970), 822) is extended (Theorem 2). Difference notation is as follows: $\Delta_2^1 a_{pq} = a_{pq} - a_{p,q+1}$, $\Delta_2^2 a_{pq} = \Delta_2^1 a_{pq} - \Delta_2^1 a_{p,q+1}$, etc. Theorem 1. If $A = (a_{pq})$ is a regular complex matrix which sums every (C,k)-summable sequence, $q^k a_{pq} \to 0$ as $q \to \infty$, $p = 1, 2, 3, \ldots$, and $q^k \Delta_2^k a_{pq} \to 0$ as $p, q \to \infty$, then A is stronger than (C,k). Theorem 2. If A is any complex matrix such that $q^k a_{pq} \to 0$ as $p + q \to \infty$, then A sums to 0 some sequence which is not (C,k)-summable. (Received June 1, 1972.)

*72T-B216. MANFRED STOLL, University of South Carolina, Columbia, South Carolina 29208. Properties of the space $\tilde{h}^p$ $(0 < p \leq 1)$ of harmonic functions on the unit disc. Preliminary report.

Definition. A harmonic function u on $|z| < 1$ belongs to the space $\tilde{h}^p$ $(0 < p < \infty)$ if there exists a harmonic function F such that $|u(z)|^p \leq F(z)$ for all z, $|z| < 1$. For $u \in \tilde{h}^p$ we define $N_p(u) = \inf[F(0) \mid F$ is harmonic, $F(z) \geq |u(z)|^p]$. The space of harmonic functions u for which $\|u\|_p = \sup_{0<r<1}[(1/2\pi)\int_0^{2\pi}|u(re^{i\theta})|^p d\theta]^{1/p} < \infty$ is denoted by h^p. Theorem 1. If u is a nonzero harmonic function in $|z| < 1$, then $u \in \tilde{h}^p$ (h^p) for all $p \leq 1$ with $N_p(u) = \|u\|_p^p = |u(0)|^p$. Theorem 2. If $u \in \tilde{h}^p$, $0 < p \leq 1$, then the harmonic conjugate v of u is in $\tilde{h}^q$ and h^q for all $q < p$. As a consequence one obtains that $\lim_{r\to 1} u(re^{i\theta})$ exists almost everywhere. Theorem 3. Every uniformly bounded family $\mathfrak{F}$ in $\tilde{h}^p$, $0 < p < \infty$, has compact closure in $\tilde{h}^p$. Theorem 4. If $u \in \tilde{h}^p$, then $N_p(u_r)$, $u_r(z) = u(rz)$, is a nondecreasing function of $r \in [0,1)$ with $\lim_{r\to 1} N_p(u_r) = N_p(u)$. (Received June 2, 1972.)

*72T-B217. PREM SINGH and SUSHEEL CHANDRA, Indian Institute of Technology, Kanpur, India. On starlike and convex functions of order α.

Denote by $S_p(\alpha)$ the class of functions $f(z) = z + \sum_{n=1}^{\infty} a_{np+1} z^{np+1}$ which are regular and starlike of order α in $D = \{z : |z| < 1\}$. And $C_p(\alpha)$ is the class of functions $f(z) = z + \sum_{n=1}^{\infty} a_{np+1} z^{np+1}$ which are regular univalent and convex of order α in D. Bernard Pinchuk [Duke Math. J. 35(1968), 721-734] solved some extremal problems for the classes of starlike univalent and convex univalent functions of order α in D. We solved the similar extremal problems for the classes $S_p(\alpha)$ and $C_p(\alpha)$. For the special case $p = 1$, we get the same results of Bernard Pinchuk. (Received June 2, 1972.) (Author introduced by Professor Phabha Gaiha.)

*72T-B218. ROBERT E. FENNELL and THOMAS G. PROCTOR, Clemson University, Clemson, South Carolina 29631. Perturbations of nonlinear differential equations.

Scalar and vector comparison techniques are used to study the asymptotic behavior of the systems (1) $x' = f(t,x)$ and (2) $y' = f(t,y) + g(t,y)$, where for a convex region Ω in $R^n f$, f_x and g are in $C(R^+ \times \Omega, R^n)$. Let $G(t,z) = [(\partial x/\partial\gamma)(t,z)]^{-1} g(t,x(t,z))$ where $x(t,\gamma)$ is the solution of (1), $x(0) = \gamma$. Conditions are given which give bounds for the solutions of (2) assuming a knowledge of $x(t,\gamma)$, $t \geqq 0$, and which guarantee the generalized asymptotic equivalence of (1) and (2). Assume $|G(t,z)| \leqq \omega(t,|z|)$, where $\omega(t,r)$ is in $C(R^+ \times R^+, R^+)$ and is nondecreasing in r for fixed t, and there is an $n \times n$ matrix $D(t)$ such that $|D(t)\varphi(t,z)| \leqq 1$ for $t \geqq 0$. Let $r(t,\delta)$ be the maximal solution of $r' = \omega(t,r)$, $r(0) = \delta$. Theorem. If $r(t,\delta)$ exists on $[0,T]$ and $\{z : |z-\gamma| \leqq r(T,\delta) - \delta\} \subset \Omega$ for some $|\gamma| \leqq \delta$, there is a solution y of (2) with $y(0) = \gamma$ and $|D(t)[y(t) - x(t,\gamma)]| \leqq r(t,\delta) - \delta$ for t in $[0,T]$. If $r(t,\delta)$ is bounded on R^+ and $\{z : |z-\gamma| \leqq r(\infty,\delta) - \delta\} \subset \Omega$ for some $|\gamma| \leqq \delta$, there is a solution y of (2) with $y(0) = \gamma$ and a solution x of (1) such that $\lim_{x\to\infty} D(t)[y(t) - x(t)] = 0$. Further, if $|\xi| < r(\infty,\delta)$, there is a solution y of (2) for large t such that $\lim_{t\to\infty} D(t)[y(t) - x(t,\xi)] = 0$. Similar results are obtained by assuming bounds on the components of G. (Received June 5, 1972.)

*72T-B219. VICTOR J. PELLEGRINI, University of Cincinnati, Cincinnati, Ohio 45221. General Wiener-Hopf operators and the numerical range of an operator. Preliminary report.

Let H be a Hilbert space and A a bounded linear operator on H. For a selfadjoint projection P on H we consider operators of the form $T_P(A) = PA_{R(P)}$. We show that $T_P(A)$ is invertible for all P if and only if 0 is not in the closure of the numerical range of A. We also show that if A is a 1-1, normal operator whose numerical range is contained in a cone of the form $\{z = x+iy : |y| \leqq cx\}$ where $c > 0$, then for each projection P there exist bounded operators A_- and A_+ such that $A = A_-A_+$, $\overline{R(A_+P)} = R(P)$ and $R(A_-Q) = R(Q)$ (where $Q = I - P$). (Received June 5, 1972.) (Author introduced by Professor Edward P. Merkes.)

72T-B220. WILLIAM J. DAVIS and W. B. JOHNSON, Ohio State University, Columbus, Ohio 43210, DOUGLAS F. HALE, Oklahoma State University, Stillwater, Oklahoma 74074 and JOSEPH DIESTEL, Kent State University, Kent, Ohio 44242. Remarks on Grothendieck spaces. Preliminary report.

Theorem. Let X be a Banach space. TFAE: (1) X is a Grothendieck space; (2) every linear continuous (l.c.) u: X → Y (Y weakly compactly generated) has a weak-star-to-weak continuous adjoint; (3) every l.c. u: X → Y (Y weakly compactly generated) is weakly compact; (4) given weakly compact operators u_n: X → Z (Z any Banach space) $(n = 1,2,3,\ldots)$ such that weak $\lim_n u_n x$ exists for each $x \in X$ then $u_0 x \equiv$ w-$\lim_n u_n x$ is weakly compact; (5) same as (4) except norm $\lim_n u_n x$ is supposed to exist for each $x \in X$. Corollary 1. Every weakly compactly generated quotient of a Grothendieck space is reflexive. Corollary 2 (Kalton). Every bounded additive map defined on a sigma-algebra of sets with values in a weakly compactly generated Banach space is strongly bounded (i.e., $\Sigma_n \mu(A_n)$ converges unconditionally whenever (A_n) are pairwise disjoint). Corollary 3. If $\mu_n : \Sigma$ (σ-algebra) $\to X$ (Banach space) are such that weak $\lim_n \mu_n(A) = \mu_0(A)$ exists for each $A \in \Sigma$ then μ_0 is strongly bounded provided each μ_n is strongly bounded. (Received June 9, 1972.)

*72T-B221. J. DAVID LOGAN, University of Arizona, Tucson, Arizona 85721. A canonical formalism for systems governed by certain difference equations.

In this paper a canonical method for reducing ν second order difference equations to a system of 2ν first order equations is developed. The basic assumption is that the ν governing equations can be obtained as a necessary condition for a finite vector sequence $r_n = (r_n^1, \ldots, r_n^\nu), n \in \Omega = \{M, \ldots, N, N+1\}$, to provide an extreme value to the summation $J\{r_n\} = \Sigma_{n=M}^N F(n, r_{n+1}, \Delta r_n)$ where Δ is the forward difference operator and $F(n,y,z)$ is given real-valued function defined on $\Omega \times R^{2\nu}$ which is twice differentiable in $y = (y_1, \ldots, y_\nu)$ and $z = (z_1, \ldots, z_\nu)$. The necessary condition on the optimal sequence is that it must satisfy the system of second order difference equations $F_{y_\alpha}(n, r_{n+1}, \Delta r_n) - \Delta F_{z_\alpha}(n, r_{n+1}, \Delta r_n) = 0$, $\alpha = 1, \ldots, \nu$. These ν equations are the discrete Euler equations for the system. If $p_n^\alpha = F_{z_\alpha}(n, r_{n+1}, \Delta r_n)$ and $H(n, r_{n+1}, p_n) = -F(n, r_{n+1}, \Delta r_n) + \Sigma_\alpha p_n^\alpha \Delta r_n^\alpha$, then the system of 2ν first order equations $p_{n+1}^\alpha - p_n^\alpha = -H_{y_\alpha}(n, r_{n+1}, p_n)$, $r_{n+1}^\alpha - r_n^\alpha = H_{z_\alpha}(n, r_{n+1}, p_n)$ is equivalent to the discrete Euler equations. In the course of proving this equivalence, a discrete version of the Hamilton-Jacobi equation, which turns out to be a first order differential-difference equation of the retarded type, is derived. Conservation laws are also obtained and the concepts are applied to an optimization problem. (Received June 9, 1972.)

72T-B222. R. A. BELL, Partridge Courts, Columbia, Maryland and SWARUPCHAND M. SHAH, University of Kentucky, Lexington, Kentucky 40506. Oscillating generalized polynomials in approximation theory. I.

Let $(g_\alpha)_{\alpha=0}^\infty$ be a sequence of functions, real-valued, nonnegative, and continuous on $[0,1]$ and analytic on $(0,1]$. Suppose that g_α is not a constant function if $\alpha \geqq 1$ and that $g_\alpha(0) = 0$ unless g_α is constant. Then $(g_\alpha)_{\alpha=0}^\infty$ is said to have property $\mathcal{P}$ if and only if the following hold: For every set of nonzero real numbers $\{c_0, c_1, \ldots, c_n\}$ and for every choice of integers $\{\alpha_0, \alpha_1, \ldots, \alpha_n\}$ with $0 \leqq \alpha_0 < \alpha_1 < \ldots < \alpha_n$, the number of zeros (counted with due regard to multiplicity) of the generalized polynomial $\Sigma_{k=0}^n c_k g_{\alpha_k}$, and of its derivative $\Sigma_{k=0}^n c_k g'_{\alpha_k}$, is at most equal to the number of variations of sign in the sequence $[c_0, c_1, \ldots, c_n]$. If $\{\alpha_0, \ldots, \alpha_n\}$ is a set of strictly increasing nonnegative integers and $\{A_0, \ldots, A_n\}$ is a set of nonzero real numbers, then $p(x) = \Sigma_{k=0}^n A_k g_{\alpha_k}(x)$ is said to be an oscillating generalized polynomial (O.G.P.) in $[0,1]$ if and only if $\max_{0 \leq x \leq 1} |p(x)|$ is attained for at least $n+1$ values of x in $[0,1]$. Properties of these polynomials will be studied in Part II of this abstract. (Received June 12, 1972.)

72T-B223. ERWIN O. KREYSZIG, University of Karlsruhe, Karlsruhe, Germany. Surface representations and Bergman operators.

Let $(x, y, u(x,y))$ and $(x, y, v(x,y))$ be surfaces related by grad $u = A$ grad v, $A = (a_{jk}(x,y))$ and suppose that the identity mapping of the one surface onto the other is equiareal. Then A is orthogonal. Suppose that A depends only on x. Then for the linear partial differential equation of the second order satisfied by v there exists a Bergman operator for which the nonlinear partial differential equations for the coefficients of the generating function $g(z, z^*, t) = \sum_{m=0}^{\infty} q_{2m}(z, z^*)t^{2m}$ can be transformed into linear form; here $z = x + iy$ and $z^* = x - iy$. (Received June 12, 1972.)

*72T-B224. ATHANASSIOS G. KARTSATOS, University of South Florida, Tampa, Florida 33620. Existence of periodic solutions of nonlinear systems. Preliminary report.

One of the results proved is the following Theorem. Assume that (i) $A(t)$ is a continuous $n \times n$ matrix on $(-\infty, +\infty)$, and such that the fundamental matrix $X(t)$ $(X(0) = I)$ of solutions of the system (*) $x' = A(t)x$ is periodic of period 2ω and satisfies $X(-t) = X(t)$, $t \in (-\infty, +\infty)$; (ii) $F(t,x)$ is defined and continuous on $(-\infty, +\infty) \times \mathbb{R}^n$ with values in $\mathbb{R}^n$, periodic in t of period 2ω, and $F(-t, u(t)) = -F(t, u(t))$, $t \in (-\infty, +\infty)$, for every continuous periodic function $u(t)$, of period 2ω, which satisfies $u(-t) = u(t)$, $t \in (-\infty, +\infty)$; (iii) $\|F(t,x)\| \leq r_2$ for every $t \in (-\infty, +\infty)$ and any $x \in \mathbb{R}^n$ with $\|x\| \leq r_1$, where r_1, r_2 are positive constants. Then if $K[\|\xi\| + 2\omega L r_2] \leq r_1$ $(K = \sup_{t \in [-\omega, +\omega]} \|X(t)\|$, $L = \sup_{t \in [-\omega, +\omega]} \|X^{-1}(t)\|)$, there exists at least one solution of (**) $x' = A(t)x + F(t,x)$, which satisfies $x(t + 2\omega) = x(t)$, $x(-t) = x(t)$, $t \in (-\infty, +\infty)$, and $x(0) = \xi$. The results are related to those of Medvedev [Differencial'nye Uravnenija 4(1968), 1258-1264] and Lasota and Opial [Ann. Polon. Math. 16(1964), 69-94]. (Received June 15, 1972.)

*72T-B225. GARY H. MEISTERS, University of Colorado, Boulder, Colorado 80302. Guichard theorems on connected monothetic groups.

Let G denote a compact connected Abelian group with dual group Γ. Choose $0 < \epsilon < 1$. Let $A_\epsilon(G)$ denote the linear space of all continuous functions f on G such that $\sum_{\xi \in \Gamma} |\hat{f}(\xi)|^\epsilon < \infty$, where $\hat{f}(\xi) = \int_G f(x)\overline{\xi(x)}\,dx$. We prove the following theorem for functions in $A_\epsilon(G)$, analogous to a theorem proved for entire functions by Claude Guichard [Ann. Sci. École Norm. Sup. 4(1887), 361-380]. Theorem. For each f in $A_\epsilon(G)$ with $\hat{f}(1) = 0$ there is a set G_f of Haar measure 1 contained in G, such that for each a in G_f there is a function g in $A_\epsilon(G)$ satisfying $f(x) = g(x) - g(x-a)$ for all x in G. One application. Using a little Pontryagin-van Kampen duality theory, the above theorem immediately yields Theorem III of Halmos and Samelson [Proc. Nat. Acad. Sci. U.S.A. 28(1942), 254-258] which states that if G is second countable then almost every element of G generates a dense cyclic subgroup of G. Another application. If Φ is any translation-invariant linear form on $A_\epsilon(G)$, then there exists a constant c such that $\Phi(f) = c \cdot \int_G f(x)\,dx$ for all f in $A_\epsilon(G)$. (Received June 19, 1972.)

*72T-B226. S. MANICKAM, Western Carolina University, Cullowhee, North Carolina 28723 and JAGDISH C. AGRAWAL, California State College, California, Pennsylvania 15419. Delta sequences and an intermediate value theorem.

A modified intermediate value theorem, with a specific expression as to the location of the point in question, is obtained using delta sequences. (Received June 19, 1972.)

72T-B227. CORNELIS W. ONNEWEER, University of New Mexico, Albuquerque, New Mexico 87106. Absolute convergence of Fourier series of convolution functions. Preliminary report.

For $\alpha > 0$, let $A(\alpha)$ be the set of all functions f on $[0, 2\pi)$ whose Fourier coefficients $\hat{f}(n)$ satisfy $\sum_{n=-\infty}^{\infty} |\hat{f}(n)|^\alpha < \infty$. Theorem 1. If $g, h \in L_p$, $1 < p \leq 2$, then $g * h \in A(p/(2p-2))$. For $1 < p \leq 2$ and β with $0 < \beta < p/(2p-2)$ there exist functions $g, h \in L_p$ such that $g * h \notin A(\beta)$. Theorem 2. If $g \in L_p$, $1 < p \leq 2$, and if $h \in \mathrm{Lip}\,\gamma$, $0 < \gamma \leq 1$, then $g * h \in A(\beta)$ for all $\beta > 2p/(2\gamma p + 3p - 2)$. Corollary. If $h \in \mathrm{Lip}\,\gamma$, $0 < \gamma \leq \frac{1}{2}$, and if $g \in L_p$ for some $p > 2/(2\gamma + 1)$, then $g * h \in A(1)$. For $1 < p < 2$ and $\gamma < (2-p)/2p$ we have constructed examples of functions $g \in L_p$ and $h \in \mathrm{Lip}\,\gamma$ such that $g * h \notin A(1)$. This extends a result of M. and S. Izumi, J. Approximation Theory 1(1968), 103-109, who proved for each p, $1 < p < 2$, and all $s > 2$, the existence of functions $g \in L_p$ and $h \in L_s$ such that $g * h \notin A(1)$. (Received June 19, 1972.)

72T-B228. JOHN E. COURY, University of British Columbia, Vancouver 8, British Columbia, Canada. Some results on lacunary Walsh series.

It is known that if a lacunary trigonometric series converges to 0 on a set of positive measure then the series vanishes identically. In the present paper, it is shown that a similar result does not hold for lacunary Walsh series. The following analogue for the Walsh system is proved. Theorem. A lacunary Walsh series which converges to 0 almost everywhere is identically zero. In particular, let $S(x)$ be a lacunary Walsh series with degree of lacunarity $q > 1$. Theorem. If $S(x)$ converges to 0 on a set of positive measure, or on a set of the second category having the property of Baire, then the series is a finite sum. Also, if $S(x)$ converges to 0 on a set of sufficiently large measure (the measure depending only on the degree of lacunarity q), then $S(x)$ is identically zero. Finally, sufficient conditions are given for a set to be a set of uniqueness for lacunary Walsh series. This paper extends the results of the author reported in Abstract 72T-B50, these *Notices* 19(1972), A-309. (Received June 26, 1972.)

*72T-B229. FRANK N. HUGGINS, University of Texas, Arlington, Texas 76010. Bounded slope variation.

Theorems which establish the properties of functions which have bounded slope variation with respect to an increasing function over an interval are given. (For a definition of bounded slope variation, see Pacific J. Math. 39(1971), 695.) Also given are theorems which establish sufficient conditions for a function to have bounded slope variation with respect to an increasing function over an interval. Theorem. If m is a continuous increasing function on $[a,b]$ and f is a function such that $D_m f(x)$, the derivative of f with respect to m at $(x, f(x))$, exists for each x in $[a,b]$, then in order that f have bounded slope variation with respect to m over $[a,b]$, it is necessary and sufficient that the function $D_m f$ be of bounded variation on $[a,b]$. Moreover, $V_a^b(df/dm) = V_a^b(D_m f)$. Theorem. If m is a continuous increasing function on $[a,b]$, f is a function such that $D_m^2 f(x)$, the second derivative of f with respect to m at $(x, f(x))$, exists for each x in $[a,b]$, and the Stieltjes integral $\int_a^b |D_m^2 f|\, dm$ exists, then f has bounded slope variation with respect to m over $[a,b]$ and $V_a^b(df/dm) = \int_a^b |D_m^2 f|\, dm$. (Received June 19, 1972.)

72T-B230. DONALD C. BENSON, University of California, Davis, California 95616. Underdamping of second order nonlinear systems. Preliminary report.

Consider the generalized vector Liénard equation $\ddot{\underset{\sim}{x}} + \underset{\sim}{f}(\underset{\sim}{x}, \dot{\underset{\sim}{x}}) + \underset{\sim}{h}(\underset{\sim}{x}) = \underset{\sim}{0}$ with $\underset{\sim}{f}$ and $\underset{\sim}{h}$ continuous. We say that a solution is overdamped if (i) $\underset{\sim}{x}(t) \to 0$ and $\dot{\underset{\sim}{x}}(t) \to 0$ as $t \to \infty$, and (ii) there exist real numbers t_0 and ϵ $(\epsilon > 0)$ and $\underset{\sim}{a}$ in E_n $(|\underset{\sim}{a}| = 1)$ such that $\underset{\sim}{x}(t) \cdot \underset{\sim}{a}/|\dot{\underset{\sim}{x}}(t)| < -\epsilon$ for all $t > t_0$ satisfying $\dot{\underset{\sim}{x}}(t) \neq 0$. If (i) is satisfied and (ii) is not, $\underset{\sim}{x}(t)$ is said to be underdamped. Theorem. All solutions $\underset{\sim}{x}(t)$ are underdamped provided that the following hypotheses hold: (A) The equation is globally asymptotically stable. (B) There exists a continuously differentiable nonnegative function $H(\underset{\sim}{x})$ on E_n such that $\underset{\sim}{h}(\underset{\sim}{x}) = \underset{\sim}{\mathrm{grad}}\, H(\underset{\sim}{x})$ and $H(\underset{\sim}{0}) = 0$. (C) There exists a positive real number α and a continuous real nonnegative function $g(\underset{\sim}{x})$ on E_n such that for all $\underset{\sim}{x}$ and $\underset{\sim}{y}$ in E_n we have $0 \leq |\underset{\sim}{f}(\underset{\sim}{x}, \underset{\sim}{y})| \leq g(\underset{\sim}{x})|\underset{\sim}{y}|^{\alpha}$. (D) For all $\underset{\sim}{x}$ and $\underset{\sim}{y}$ in E_n we have $\underset{\sim}{y} \cdot \underset{\sim}{f}(\underset{\sim}{x}, \underset{\sim}{y}) \geq 0$, with equality only in case $\underset{\sim}{y} = \underset{\sim}{0}$. (E) Either $\alpha \geq 2$ (in (C), above); or $0 < \alpha < 2$ and for each $\underset{\sim}{a} \neq \underset{\sim}{0}$ in E_n and each $\epsilon > 0$ we have $\liminf_{B \to 0^+} \limsup_{A \to 0^+} \int_A^B F(u, \alpha, \underset{\sim}{a}, \epsilon)\, du < 0$. The function $F(u, \alpha, \underset{\sim}{a}, \epsilon)$ is defined as follows. Put $h(u) =$

$\inf\{H(\underset{\sim}{x}) : \underset{\sim}{x} \cdot \underset{\sim}{a} = u, |\underset{\sim}{x}| \leq u/\epsilon\}$, $k(u) = \sup\{(1/\epsilon)g(\underset{\sim}{x}) : \underset{\sim}{x} \cdot \underset{\sim}{a} = u, |\underset{\sim}{x}| \leq u/\epsilon\}$, $K(u) = \int_0^u k(u_1)\,du_1$, $\beta = 2/(2-\alpha)$. Then we put $F(u, \alpha, \underset{\sim}{a}, \epsilon) = \beta^{-\beta}\alpha^{\beta-1}K(u)^{-1}k(u) - \beta h(u)K(u)^{-\beta-1}k(u)$. The theorem generalizes the main result of the author in Proc. Amer. Math. Soc. 33(1972), 101-106. (Received June 19, 1972.)

*72T-B231. PER ENFLO, University of California, Berkeley, California 94720. A counterexample to the approximation problem. Preliminary report.

The following result is proved. There exists a reflexive Banach space B with the following property: there exists a sequence (M_n) of finite-dimensional subspaces of B with $\dim(M_n) \to \infty$ such that if T is a finite rank operator on B and $\|T - I\| < 0{,}1$ on (M_n) then $\|T\| > [\log(\dim(M_n))]^{0,1}$. In particular B does not have the approximation property and B does not have a Schauder basis. (Received June 20, 1972.) (Author introduced by Professor Haskell P. Rosenthal.)

*72T-B232. THOMAS ERBER and ABE SKLAR, Illinois Institute of Technology, Chicago, Illinois 60616 and BERTHOLD SCHWEIZER, University of Massachusetts, Amherst, Massachusetts 01002. Mixing transformations on metric spaces. Preliminary report.

The homogenizing effects of mixing transformations are well known. The following theorems make the equally important dispersive effects explicit: Theorem 1. Let T be mixing with respect to a nonsingular probability measure P on a separable metric space (Ω, d). For any $S \subseteq \Omega$ and any $\chi > 0$, let $\chi_{(S,T,X)}$ denote the characteristic function of the set of positive integers m such that $\operatorname{diam}(T^m(S)) < X$. Then there exists an $X_0 > 0$ such that $\lim_{n\to\infty}(1/n)\sum_{m=1}^{n}\chi_{(S,T,X_0)}(m) = 0$. If $P(Q) > 0$ for any open subset Q of Ω, then X_0 can be chosen to be any number $< \operatorname{diam}(\Omega)$. Theorem 2. If, in addition to the above hypotheses, T is invertible then there is an $X_0 > 0$ such that for any P-measurable set S with $P(S) > 0$ there exists an integer $n(S)$ such that $\operatorname{diam}(T^m(S)) > X_0$, for all $m \geq n(S)$. Again, if open sets have positive measure, then X_0 can be chosen to be any number $< \operatorname{diam}(\Omega)$. (Received June 26, 1972.)

*72T-B233. WILLIAM P. ZIEMER, Indiana University, Bloomington, Indiana 47401. Behavior at the boundary of solutions to quasilinear equations.

We consider quasilinear equations of the form $\operatorname{div} A(x, u, u_x) = B(x, u, u_x)$ where A and B are subjected to certain standard structural inequalities. Let Ω be a bounded open set in E^n and let $u \in W_p^1(\Omega)$, $p > 1$, be a Sobolev function which is a weak solution to this equation. Assume also that u vanishes on the boundary of Ω in the sense that u is the limit in $W_p^1(\Omega)$ of $C^\infty(\Omega)$ function whose supports are contained in Ω. Whenever $A \subset E^n$ define the capacity $\Gamma_p(A)$ as the infimum of all numbers $\int_{E^n}|v_x|^p$ corresponding to those functions $v \in W_p^1$ with compact support for which A lies in the interior $\{x : v(x) \geq 1\}$. Theorem. For Γ_p quasi every point $x_0 \in \operatorname{bdry}\Omega$, $u(x) \to 0$ as $x \to x_0$, $x \in \Omega$. That is, Γ_p quasi every $x_0 \in \operatorname{bdry}\Omega$ is regular. Moreover, if for any $q < p$, $\lim_{r\to 0}\Gamma_q[B_{x_0}(r) - \Omega] \cdot r^{q-n} > 0$ where $B_{x_0}(r)$ denotes the ball of radius r with center at x_0, then x_0 is regular. (Received June 23, 1972.)

72T-B234. SUDHANSHU K. GHOSAL and MADAN CHATTERJEE, Regional Engineering College, Durgapur, West Bengal, India. Some results on common fixed point theorems in metric space. Preliminary report.

The following results on common fixed points in metric space have been deduced. Theorem 1. Let (x, d) be a complete metric space, and let there be two mappings $T_1 : X \to X$ and $T_2 : X \to X$; then if $d(T_1^p x, T_2^q y) \leq \lambda d(x, y)$ for $x, y \in X$ where $0 < \lambda < 1$ and p and q are positive integers, then T_1 and T_2 will have a unique common fixed point. Theorem 2. Let T_1 and T_2 be two operators mapping the complete metric space X into itself and if

$d(T_1^p x, T_2^q y) \leqq \lambda\{d(x, T_2^q y) + d(y, T_1^p x)\}$ where $0 < \lambda < 1/4$, T_1 and T_2 will have a unique common fixed point. Theorem 3. Let X be a complete metric space and let T_1 and T_2 be two operators mapping X into itself. If there exists a mapping K of X into itself which has a right inverse (i.e. $KK^{-1} = I$ = identity mapping), such that $d(K^{-1}T_1Kx, K^{-1}T_2Ky) \leqq \alpha d(x, K^{-1}T_1Kx) + \beta d(y, K^{-1}T_2Ky)$ where $\alpha + \beta < 1$, $\alpha > 0$, $\beta > 0$, then T_1 and T_2 will have a unique fixed point. Theorem 4. Let T_1 and T_2 be two mappings of X into itself; one of the transformations T_1^p, T_2^q is continuous, say T_2^q. If there exists an everywheredense subset E of X, such that for any two points $x, y \in E$, we have $d(T_1^p x, T_2^q y) \leqq \lambda d(x,y)$ where $0 < \lambda < 1$ then T_1 and T_2 have a unique common fixed point in X. (Received June 23, 1972.)

72T-B235. ALEXANDER WEINSTEIN, Georgetown University, Washington, D.C. 20007. On some inequalities of Lidskiĭ.

Let $H^{(0)}$ and H be two selfadjoint operators bounded below in a Hilbert space. Let $H - H^{(0)} \geqq 0$. Put $H_n v = -\Sigma_{k=1}^n (v, H^{(0)}p_k)p_k$, $n = 1, 2, \ldots$. Let $H^{(n)} = H^{(0)} + H_n$. In his preface to the Russian translation (1970) of the book by S. H. Gould ("Variational methods for eigenvalues," 2nd ed., 1966), V. B. Lidskiĭ formulates the following inequalities: (1) $H^{(n)} - H^{(0)} \geqq 0$; (2) $H - H^{(n)} \geqq 0$; (3) $H_{n+1} - H_n \geqq 0$. All these inequalities are incorrect. For instance, by (1) one would have $H_n = H^{(n)} - H^{(0)} \geqq 0$. Therefore the nonnegative operator H_n would be necessarily symmetric. However H_n is obviously nonsymmetric. Similar proofs show that (2) and (3) are also incorrect. Lidskiĭ adds several footnotes reproducing some statements of M. G. Kreĭn. However, in the proofs of these statements nonsymmetric operators are again tacitly assumed to be symmetric. (Received June 27, 1972.)

72T-B236. GEORGE YU-HUA CHI, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. The Radon-Nikodym theorem for vector measures with values in the duals of nuclear barreled spaces.

Let (Ω, Σ, μ) be a positive measure space, where Σ is a σ-field. Let E be a nuclear barreled space, and let F be a nuclear (F)-space, or a complete (DF)-space, or the sequential projective limit of nuclear (F)-spaces. Let E' and F' be the respective topological duals. Let $\vec{m}: \Sigma \to E'$ and $\vec{n}: \Sigma \to F$ be vector measures with respect to any locally convex topologies on E' and F consistent with the respective dualities $\langle E, E'\rangle$ and $\langle F, F'\rangle$. Theorem 1. There exists a $\sigma(E', E)$-integrable function $\vec{f}: \Omega \to E' \ni \vec{m}(A) = \int_A \vec{f}(\omega)d\mu(\omega)$, $\forall A \in \Sigma$ iff $\vec{m} << \mu$ (i.e. whenever $\mu(A) = 0$, $\vec{m}(a) = 0$). Theorem 2. There exists a $\sigma(F, F')$-integrable function $\vec{g}: \Omega \to F \ni \vec{n}(A) = \int_A \vec{g}(\omega)d\mu(\omega)$, $\forall A \in \Sigma$ iff $\vec{n} << \mu$. Corollary. Every weakly integrable random Schwartz distribution has a unique (up to equivalence) conditional expectation relative to a given sub σ-field of Σ. Martingale convergence theorem follows. (Received June 28, 1972.)

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72T-C37. NABIL A. KHABBAZ, Department of Mathematical Sciences, University of Iowa, Iowa City, Iowa 52240. Extent and parsing within the hierarchy. Preliminary report.

Definition. Let L be the collection of labeled linear grammars, L_0 the collection of context free languages. For $i > 0$ define $L_i = L/L_{i-1} = \{L(\mathcal{G}) \mid \mathcal{G} = (G, A),\ G \in L,\ A \in L_{i-1}\}$ and where $L(\mathcal{G}) = \{x \in \Sigma^* \mid \sigma \underset{\pi}{\overset{*}{\Rightarrow}} x,\ \pi \in A\}$, Σ being the alphabet of G. Then $\{L_i\}$ forms a proper hierarchy within the context sensitive languages. Theorem. $\bigcup_{i=0}^{\infty} L_i = \mathfrak{M}/L_0 = \bigcup_{i=0}^{\infty} \mathfrak{M}/L_i$ where $\mathfrak{M}$ is the collection of meta linear labeled grammars whose productions are of the form $1: \sigma \to \sigma_1\sigma_2 \ldots \sigma_n$, $1: \alpha \to a\beta$, $1: \alpha \to \beta a$, $1: \alpha \to \epsilon$, and such that

$D(\sigma_i)$ are mutually disjoint for each σ_i appearing on the right hand side of a production starting with σ. $D(\sigma_i) = \{\alpha \in V | \sigma_i \stackrel{*}{\Rightarrow} x\alpha y\}$, V being the variables of the meta linear grammar. The hierarchy is in general suitable for parsing. By defining a controlled language $L(\mathcal{S}) \in L_i$ to be a precedence language if the sequence of grammars defining $\mathcal{S}$ forms a sequence of precedence grammars, the algorithm of N. Wirth and H. Weber generalizes to yield a parsing algorithm within the hierarchy. (Received March 13, 1972.) (Author introduced by Professor Arthur C. Fleck.)

*72T-C38. MURLI M. GUPTA, University of Western Australia, Nedlands, 6009, Australia and RAM P. MANOHAR, University of Saskatchewan, Saskatoon, Saskatchewan, Canada. Numerical solution of second biharmonic boundary value problems.

The second boundary value problem for the biharmonic equation is equivalent to the Dirichlet problems for two Poisson equations. Two finite difference approximations are defined for the solution of these Dirichlet problems in a general not necessarily rectangular domain D. The discretization error is proved to be of order h^2, as $h \to 0$, where h is the mesh size. The relation between local truncation errors and the overall discretization error is also discussed. It is shown that in order to obtain a discretization error bound of order h^2, it is sufficient to approximate the biharmonic operator by its thirteen point discrete analogue in the interior of the domain D; whereas near the boundary, the biharmonic operator can be replaced by difference analogues with a truncation error of order as low as h^{-2}. It is also shown that splitting of the biharmonic equation helps in reducing the roundoff errors and also produces a numerically efficient procedure. (Received April 20, 1972.)

72T-C39. MICHAEL P. BENSON, 625 Skyview Place, Apt. 4, Madison, Wisconsin 53713 and University of Wisconsin, Madison, Wisconsin 53706. Asymptotic error expansions for certain product integration methods. Preliminary report.

Product integration methods for approximating $\int_0^1 f(s)\,g(s)\,ds$ are studied. For any positive integer n, define $h = n^{-1}$ and $s_\lambda = \lambda h$ for $0 \leq \lambda \leq n$. The product midpoint method is defined by replacing $g(s)$ with $g(s_{j-1/2})$ for $s_{j-1} \leq s < s_j$, $j = 1, 2, \ldots, n$. The product trapezoid method is defined by replacing $g(s)$ with its linear interpolant at the points s_j, $j = 0, 1, \ldots, n$. For even n, the product Simpson method is defined by replacing $g(s)$ on $s_{2j-2} \leq s \leq s_{2j}$ with its quadratic interpolant at $s_{2j-2}, s_{2j-1}, s_{2j}$ for $j = 1, 2, \ldots, \frac{1}{2}n$. Asymptotic expansions for the quadrature error in terms of h are derived for each of these methods for $g(s)$ smooth and $f(s) = s^\alpha \ln s\tilde{f}(s)$, $\alpha > -1$, $\tilde{f}(s)$ smooth or $f(s) = s^\alpha \tilde{f}(s)$, $\alpha > -1$, $\tilde{f}(s)$ smooth. Expansions are also obtained for $g(s)$ having the same type of singularity, not at $s = 0$. (Received May 4, 1972.) (Author introduced by Professor Seymour V. Parter.)

72T-C40. MARTIN SCHECHTER and ISAAC J. BULKA, Belfer Graduate School of Science, Yeshiva University, New York, New York 10033. Scattering for Schrödinger operators. Preliminary report.

Let $V(x)$ be a real valued function in E^n, and let Δ denote the Laplacian. Let H_0 and H be the Hamiltonians in $L^2(E^n)$ corresponding to $-\Delta$ and $-\Delta + V$, respectively. Assume that (a) $(1 + |x|)^\alpha V(x)$ is in $L^p(E^n)$ for $\alpha > 1 - [2n/(n+1)p]$. Theorem. If (a) holds, then the absolutely continuous parts of H_0 and H are unitarily equivalent. The wave operators exist and are complete. The theorem holds if (a) is replaced by (b) $\int |V(x)|\,(1 + |x-y|)^{(1-n)/2}\,dx \leq C_0$ for all $x \in E^n$. We employ methods of Kato and Kuroda. (Received May 19, 1972.)

*72T-C41. KAILASH CHANDRA, Division of Physical Sciences, Institute of Advanced Studies, Meerut Univer. Meerut, U. P., India. Hydromagnetic stability of a density stratified fluid column with a radial gravitational force. Preliminary report.

The stability of an incompressible inviscid ideally conducting fluid column with density $\rho(r)$ and magnetic field $\underline{H} = (0, H_\theta(r), H_z(r))$, between two fixed coaxial cylinders, in the presence of a radial gravitational force g towards the axis of the cylinders, is examined under linear theory by the normal mode technique against nonaxisymmetric perturbations. Four sufficient conditions for the stability are obtained as follows. The system will be stable if (i) $4ace - b^2e - ad^2 \geqq 0$, $e > 0$, and $a > 0$ everywhere in the fluid region; or if (ii) $2ac - b^2 \geqq 0$, $2ec - d^2 \geqq 0$, $c > 0$, and $a > 0$ everywhere in the fluid region; where a, b, c, d, and e are all taken from either of the sets. Set (I) $a = r\mu D(H_\theta^2/r^2) - g(D\rho)$, $b = 2r\mu D(H_\theta H_z/r)$, $c = -2r^2 g(D\rho) - 8\mu H_\theta^2$, $d = 2r^5 \mu D(H_\theta H_z/r^3)$, $e = -r\mu D(r^2 H_\theta^2) - r^4 g(D\rho)$. Set (II) $a = -g(D\rho) - \mu H_\theta^2/r^2$, $b = -2\mu H_\theta H_z/r$, $c = -2r^2 g(D\rho) - (\mu/r^4) D(r^5 H_\theta^2) - \mu D(rH_z^2)$, $d = -2r\mu H_\theta H_z$, $e = -r\mu D(r^2 H_\theta^2) - r^4 g(D\rho) - r^4 \mu D(H_z^2/r)$. Here D stands for d/dr and μ for the magnetic permeability of the vacuum. (Received May 23, 1972.) (Author introduced by Vice-Chancellor J. N. Kapur.)

*72T-C42. S. K. GUPTA and S. K. BHATT, Indian Institute of Technology, Kanpur, India. Linearization algorithm for pseudo-concave programs. Preliminary report.

An algorithm to solve the program (P) $\max_{x\in X} f(x)$, $X = \{x \in R^n | Ax = b\}$ is developed; f being a pseudo-concave function. The algorithm is based on a theorem of Kortanek and Evans, that if $x^* \in X$ solves the linear program (l.p.) $\max_{x\in X} x^T \nabla f(x^*)$, then x^* solves (P) and vice versa. The algorithm aims at finding such an x^*. The steps are: (I) Choose any $x^0 \in R^n$, $\nabla f(x^0) \neq 0$ and solve the l.p. $\max_{x\in X} x^T \nabla f(x^0)$. Let x^1 be any optimal basic feasible solution. (II) Set $i = 1$, solve the l.p. $(L_i): \max_{x\in X} x^T \nabla f(x^i)$ to get x^{i+1}. Let some p (properly chosen) vertices $x^1, \ldots, x^p$ be thus generated. (III) Solve $(\Lambda p): \max f(\lambda) = f(\sum_{i=1}^p \lambda_i x^i)$ subject to $\lambda \in \Lambda = \{\lambda \in R^p | \sum_{i=1}^p \lambda_i = 1, \lambda_i \geqq 0\}$ to get λ^1 say. Let $x^{\lambda^1} = \sum_{i=1}^p \lambda_i^1 x^i$. (IV) Solve the l.p. $\max_{x\in X} x^T \nabla f(x^{\lambda^1})$ to get $y^1 \in X$. Then if (i) $y^1 \in \{x^1, \ldots, x^p\}$ then x^{λ^1} solves (P) by Theorem 2. (ii) Otherwise set $x^{p+1} = y^1$, go to (III) with $x^1, \ldots, x^p, x^{p+1}$. The main feature of the linearization algorithm is that it solves (P) in finite steps provided (Λp) are solvable finitely for which a simplification of Zoutendijk's gradient method is proposed. (Received May 30, 1972.) (Authors introduced by Dr. Prabha Gaiha.)

*72T-C43. JAN MYCIELSKI, University of Colorado, Boulder, Colorado 80302. Monte Carlo interpolation of Boolean functions.

$\{0,1\}^m$ denotes the set of sequences of 0's and 1's of length m. A k-cylinder in $\{0,1\}^m$ is a set of the form $\{(x_1, \ldots, x_m) : (x_{i_1}, \ldots, x_{i_k}) = (c_1, \ldots, c_k)\}$, where $1 \leqq i_1 < \ldots < i_k \leqq m$. If $X \subseteq \{0,1\}^m$ and f is a function with domain X then f is called k-continuous if $f^{-1}\{r\}$ is a union of k-cylinders for every element r of the range of f. Let a sequence of n + 1 points $x_1, \ldots, x_n$, $x \in X$ be chosen at random, every element of X having probability 1/card(X). We assume that f is k-continuous. Lemma. With probability not less than $1 - \binom{m-k}{k}(1 - 4^{-k})^n$ the set $\{x_1, \ldots, x_n\}$ intersects every set $C \cap f^{-1}\{f(x)\}$, where C is a k-cylinder containing x. Theorem. If we choose by any method depending only on m, k, $x_1, \ldots, x_n$, $f(x_1), \ldots, f(x_n)$ and x a k-cylinder C such that $x \in C$, $C \cap \{x_1, \ldots, x_n\} \neq \emptyset$ and f restricted to $C \cap \{x_1, \ldots, x_n\}$ is a constant function with value v then $f(x) = v$ with probability not less than $1 - \binom{m-k}{k}(1 - 4^{-k})^n$. (Received May 30, 1972.)

72T-C44. MARTIN SCHECHTER, Belfer Graduate School of Science, Yeshiva University, New York, New York 10033. Estimates for singular potentials.

Let n be an integer > 2. For $s > 0$ and $\lambda > 0$, set $G_{s,\lambda}(x) = (4\pi)^{-n/2}\Gamma(s/2)^{-1} \cdot \int_0^\infty \exp\{-|x|^2/4t - \lambda t\}\, t^{(s-n)/2}\, dt/t$, and for $s < n$, $G_{s,0}(x) = \Gamma([n-s]/2)\, |x|^{s-n}/2^s \pi^{n/2}\Gamma(s/2)$. Theorem. Let $V(x) \geqq 0$ be a function locally integrable on E^n. Then $(Vu,u) \leqq C([\lambda - \Delta]^s u,u)$, $u \in C_0^\infty(E^n)$, iff $B_{s,\lambda}(V) \leqq C$, where Δ is the Laplacian and $B_{s,\lambda}(V) = \inf_{\psi \geqq 0} \sup_x \psi(x)^{-1} \int V(x)\, G_{2s,\lambda}(x-y)\, \psi(y)\, d^n y$. (Received June 7, 1972.)

*72T-C45. SANJO ZLOBEC and J.-J. NIQUET, McGill University, Montreal 110, Quebec, Canada and ADI BEN-ISRAEL, Technion, Haifa, Israel. Iterative computation of the Moore-Penrose inverse.

For any $m \times m$ matrix A the hyperpower iterative method for calculating the Moore-Penrose inverse A^+ is defined as follows: $T_k = I - X_k A$, $\tilde{X}_k = (I + T_k + \ldots + T_k^{p-2})\, X_k$, $X_{k+1} = X_k + T_k \tilde{X}_k$, $k = 0,1,\ldots,p \geqq 2$. The main result is the following theorem. Theorem. The sequence $\{X_k\}$ converges to A^+ if and only if both $\rho < 1$ and $X_0 \in R(A^*,A^*)$. (Here ρ is the spectral radius of $P_{R(A)} - AX_0$, where $P_{R(A)}$ is the orthogonal projection on the range of A, and $R(A^*,A^*) = \{X : X = A^*YA^* \text{ for some } Y\}$.) Various corollaries of this theorem are stated in terms of K_U-symmetric and K_U-p.d. matrices. (Received June 7, 1972.)

*72T-C46. MOSHE GOLDBERG, Department of Mathematical Sciences, Tel Aviv University, Ramat Aviv, Tel Aviv, Israel. Stability of locally perturbed Lax-Wendroff approximation. Preliminary report.

Consider the Cauchy problem $u_t = Au_x$, $-\infty < x < \infty$, $t \geqq 0$, where $A = A(x,t)$ is an $n \times n$ matrix and $u(x,t)$ a vector. Conditions for stability of the Lax-Wendroff approximation, perturbed locally by an arbitrary three-point scheme, are given. Also stability is proved for the perturbation due to a particular five-point scheme that results from iterating the Lax-Wendroff approximation. (Received June 22, 1972.) (Author introduced by Professor Harley Flanders.)

*72T-C47. M. RAY and JAGDISH C. AGRAWAL, California State College, California, Pennsylvania 15419. On hydromagnetic flow over conducting and nonconducting walls. Preliminary report.

A detailed numerical study of shear stress on the walls, displacement thickness, energy equation, heat transfer has been carried out. Investigation reveals contrasting results in the two cases, particularly in relation to the heat transfer and decay of magnetic field. (Received June 19, 1972.)

Geometry

72T-D16. WEI-LUNG TING, State University College of New York, Plattsburgh, New York 12901. On the Gauss-Bonnet theorem. Preliminary report.

For terminology see Levine (Proc. Liverpool Singularities Sympos. II, pp. 90-103) and White (J. Differential Geometry 4(1970), 207-223). We use a similar method as in (Wei-Lung Ting, "On differential invariant," (Abstract 72T-G114, these *Notices* 19(1972)) to give a new formulation of the Gauss-Bonnet theorem for a T-map in the sense of Levine. Let $f: M^n \to E^{p=n+k}$ be a T-map of an oriented compact manifold with smooth boundary B^{n-1}. Let v be the vector field which is tangent to M^n and normal to B^{n-1} and $\mathscr{S}(f)$ denote the set of singularities of f. Then the following is true for n odd: $-\frac{1}{2}\chi(B^{n-1}) = (-1)^n (I(g,q) - I(g',q))$, and for a even; $(1/O_n)\int_{M\setminus S(f)} K\, dV + (1/O_{n-1})\int_{B\setminus S(f)} K_v(N')\, dV_B = (-1)^{n-1}(I(g,q) - I(g',q))$. Here we adopt the notation of White's paper modifying it when necessary, by restricting the bundles to $M\setminus S(f)$. (Received April 20, 1972.)

72T-D17. PAUL EWING EHRLICH, State University of New York, Stony Brook, New York 11790. Deformation of Ricci curvature on Riemannian manifolds. Preliminary report.

Aubin showed (J. Differential Geometry 4(1970), 397) that a Riemannian manifold with a metric of nonnegative Ricci curvature and with positive Ricci curvature at a point admits a metric of positive Ricci curvature. However, Aubin did not claim to preserve completeness in changing the given metric. This is important in light of the result of Gromov that any noncompact manifold admits a metric such that the sectional curvatures take values in any prescribed open interval. Using a deformation $g(t) = e^{tf}g$ of the given metric g to construct a standard deformation on a convex disk of uniform radius, we prove Theorem A. Let (M, g) be a complete (respectively compact) Riemannian manifold of nonnegative Ricci curvature and positive Ricci curvature at a point. Then M admits a Riemannian metric $\hat{g}$ of positive Ricci curvature and $(M, \hat{g})$ is complete (respectively compact). Corollary B. Let (M, g) be a complete open Riemannian manifold of nonnegative Ricci curvature and positive Ricci curvature at a point. Then M is connected at infinity. Corollary B follows from Theorem A and Theorem 1 of Gromoll and Meyer (Ann. of Math. (2) 90(1969), 80). (Received May 9, 1972.)

*72T-D18. KENTARO YANO, Tokyo Institute of Technology, Tokyo, Japan and BANG-YEN CHEN, Michigan State University, East Lansing, Michigan 48823. Special conformally flat spaces and canal hypersurfaces.

Let V_n be an n-dimensional Riemannian manifold $(n \geqq 3)$ with metric tensor g_{ij} and Ricci tensor R_{ij}. Then we may define a tensor L_{ij} by $L_{ij} = (-R_{ij}/(n-2)) + (Rg_{ij}/2(n-1)(n-2))$, where R denotes the scalar curvature. Let C^h_{ijk} be the conformal curvature tensor. Then V_n is called a conformally flat space if $C^h_{ijk} = 0$ $(n > 3)$, $\nabla_i L_{jk} = \nabla_j L_{ik}$ $(n = 3)$. On a conformally flat space V_n, if there exist functions α and β such that $L_{ij} = -\frac{1}{2}\alpha^2 g_{jk} + \beta(\nabla_i\alpha)(\nabla_j\alpha), \alpha > 0$, then V_n is called a special conformally flat space. The following theorems are proved. Theorem 1. Every conformally flat hypersurface of euclidean space is special. Theorem 2. Every simply-connected special conformally flat space can be isometrically immersed in euclidean space as a hypersurface. Theorem 3. Every canal hypersurface of euclidean space is a special conformally flat space, where a canal hypersurface means the envelope of one-parameter family of hyperspheres. (Received May 15, 1972.)

72T-D19. GERALD L. ALEXANDERSON, University of Santa Clara, Santa Clara, California 95053 and JOHN E. WETZEL, University of Illinois, Urbana, Illinois 61801. Dissections of a simplex.

Let T^d be a simplex in E^d, the convex hull of $d + 1$ independent vertices $V_0, V_1, \ldots, V_d$, and denote by e_{ij} the edge of T^d that joins V_i and V_j. On each edge e_{ij} let $n_{ij} \geqq 0$ points (not vertices) be chosen. With the $d - 1$ opposite vertices of T^d, each such point determines a hyperplane that dissects T^d. What is the maximum number H_d of nonoverlapping $(d-1)$-polytopes into which the $\sum n_{ij}$ dissecting hyperplanes divide T^d? We find that $H_d = 1 + \sum_{k=1}^{d} \sigma_k^*$, where $\sigma_k^* = \sum^* n_{i_1j_1} n_{i_2j_2} \cdots n_{i_kj_k}$, the sum being extended over all k-sets of edges $e_{i_1j_1}, e_{i_2j_2}, \ldots, e_{i_kj_k}$ no subcollection of which forms a closed path on the simplex T^d. The proof is by induction on the dimension d. The case $d = 3$ appears in the authors' paper, "Dissections of a tetrahedron," J. Combinatorial Theory 11(1971), 58-66. (Received June 15, 1972.)

72T-D20. MICHAEL GOLDBERG, 5823 Potomac Ave., N. W., Washington, D. C. 20016. On the minimum track of a moved line segment.

The author's solution of part of a Ulam problem ("Problems of modern mathematics," 1964, p. 79) of transferring a line segment from one given position to another with the least motion of the ends of the line segment will appear in Math. Mag. in 1972. The present paper exhibits those solutions for which the track (or path) is minimized, and in which retracing of part of the track is permitted, either by one or both of the ends of the line segment. These solutions are obtained by the use of elementary statics in a mechanical analogy. (Received May 18, 1972.)

*72T-D21. ROBERT E. SEALL, Illinois Institute of Technology, Chicago, Illinois 60616. A characterization of Desargues and Pappus planes by the superassociativity of a single operation.

In a projective plane π let j be a line and 0, 1, ∞ three distinct points on j. Let U and V be two distinct points not on j and collinear with ∞. Consider each point of j as a 3-place function and define a substitution (rel. U, V) of any triple of disjoint points (B, C, D) ($C \neq \infty$) into any point A on j. For any two points P and Q on j (not both $=\infty$), denote by $P * Q$ the meet of the lines PV and QU. The join of $A * C$ and $0 * D$ intersects the join of $1 * C$ and $0 * B$ in a point P. The result of the substitution is the point A(B, C, D), defined as the meet of j with the join of P and U. According as $A = 0, 1, \infty$ the result is B, C, D. Theorem I. π is a Pappus plane if and only if the above substitution is superassociative in Menger's sense, i.e. if $(A(B,C,D))(E,F,G) = A(B(E,F,G), C(E,F,G), D(E,F,G))$. Theorem II. π is a Desargues plane if and only if the above formula holds at least in the two special cases where $B = E = 0$ and $C = F = 1$ and where $D = G = \infty$. Even in this case the result of the substitution is independent of the choice of U and V (collinear with ∞). Further special cases of the superassociativity characterize certain special Desargues planes. (Received June 26, 1972.) (Author introduced by Professor Abe Sklar.)

Logic and Foundations

*72T-E63. DON L. PIGOZZI, Iowa State University, Ames, Iowa 50010. On the decision problem for equational theories. I: Normal-form functions.

For notation see Tarski, "Equational logic and equational theories of algebras" in "Contributions to mathematical logic" (Colloquium, Hannover, 1966), North-Holland, 1968. Thus the terms "equational theory," "base of a theory," and "finitely based" are assumed understood. In the following θ, Φ represent arbitrary theories and σ, τ arbitrary terms. A function n from terms to terms is a normal form function (Nff) for θ if n and θ are (i) Turing reducible to one another, and, for all σ, τ, (ii) $(n\sigma = \sigma) \in \theta$, (iii) $(\sigma = \tau) \in \theta$ iff $n\sigma$ and $n\tau$ are identical, (iv) $n\sigma$ and σ are identical for every subterm σ of $n\tau$. For any set Γ of terms $Op(\Gamma)$ and $Va(\Gamma)$ are respectively the set of operation symbols and the set of variables occurring in at least one member of Γ. Theorem. For every θ there exists an Nff n satisfying the following conditions for every σ. (i) If $Op(\theta)$ contains a constant, then $Va(n\sigma) \subseteq Va(\sigma)$; otherwise, $Va(n\sigma) \subseteq Va(\sigma) \cup \{v\}$ for some variable not depending on σ. (ii) If there exists a base Γ for θ such that $Op(\Gamma)$ is finite (in particular if θ is finitely based), then length of $n\sigma \leqq$ length of σ. (Received April 28, 1972.)

72T-E64. WILLIAM D. JACKSON, Detroit Institute of Technology, Detroit, Michigan 48201. A remark on a result of S. Tennenbaum. Preliminary report.

For a set of nonnegative integers α and a function f, let $\alpha(n,f) = \text{card}\{x < n | f(x) \in \alpha\}$, and let $d(\alpha, f) = \liminf \alpha(n,f)/n$. S. Tennenbaum claimed (see Hartley Rogers, Jr., "Theory of recursive functions and effective computability," 1967, p. 156) the following: If $d(\alpha, f) > 0$ for a recursive function f and a hyperimmune set α, then there is an $x \in \alpha$ such that $d(\{x\}, f) > 0$. The following theorem shows that the above claim is unjustified. Theorem 1. Card $\alpha \geqq 2$ iff there is a recursive function f with $d(\alpha, f) > 0$ and $d(\{x\}, f) = 0$ for every $x \in \alpha$ (in fact, for every x). However, the following two theorems give results close to Tennenbaum's original claim. Theorem 2. If $d(\alpha, f) > 0$ for a hyperimmune set α and a recursive function f, then there is a finite set β with $d(\beta, f) > 0$. Theorem 3. If α' recursively enumerable is added to the hypothesis of Theorem 2, then $\beta \subseteq \alpha$ can be added to the conclusion. (Received May 3, 1972.)

72T-E65. KENNETH JON BARWISE, University of Wisconsin, Madison, Wisconsin 53706. Using Kueker's filter to approximate uncountable sets. Preliminary report.

Let $A = H(\varkappa)$ where $\varkappa$ is an uncountable cardinal or the class of all ordinals, and let $I = P_{\omega_1}(A)$. A set $X \subseteq I$ belongs to Kueker's countably complete filter D iff there is an $X \subseteq X$ such that (i) every $s \in I$ is $\subseteq s'$ for some $s' \in X'$ and (ii) X' is closed under unions of countable chains. Define, for any $a \in A$ and $s \in I$, the countable approximation $a_s = \{b_s : b \in a \cap s\}$. Theorem. Let $P(x, y)$ be a Σ_1 predicate of set theory. If $a, b \in A$ and $P(a, b)$ is true then $P(a_s, b_s)$ is true for almost all s. Corollary. If $P(x, y)$ is Δ_1 then for $a, b \in A$, $P(a, b)$ holds iff $P(a_s, b_s)$ holds for almost all s. Applications give Lévy's version of the Gödel collapsing lemma and generalization of Theorems 1 and 2 in Kueker [Bull. Amer. Math. Soc. 78(1972), 211-215]. There are analogous results for uncountable approximations. (Received May 4, 1972.)

*72T-E66. ALISTAIR H. LACHLAN, Simon Fraser University, Burnaby, British Columbia, Canada. Complete varieties of algebras. Preliminary report.

Let K be an equational class of algebras with a countable number of operations. Let K have a nontrivial finite member. Let T be the first order theory of the infinite members of K. Theorem 1. If T is complete then T is $\aleph_0$-categorical. Theorem 2. If T is $\aleph_0$-categorical but not $\aleph_1$-categorical then T is unstable but cannot have the strict order property. These results improve on earlier theorems of J. T. Baldwin. (Received May 23, 1972.)

*72T-E67. ALBERT R. MEYER, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. Weak S1S cannot be decided. Preliminary report.

WS1S is the weak monadic second order theory of successor. Büchi has shown that WS1S is recursive, and in fact primitive recursive. Let $t(0, n) = n$; $t(k+1, n) = 2^{t(k, n)}$. Theorem. There is an $\epsilon > 0$ such that any procedure (Turing machine) which decides the truth of sentences in the language of WS1S requires time and space exceeding $t(\epsilon \cdot \log_2 n, n)$ on some sentences of length n for all sufficiently large integers n. Corollary. WS1S is not elementary-recursive in the sense of Kalmar. The proof follows by exhibiting formulas of length n in the language of WS1S which define the predicate $\lambda y[y \geqq t(\epsilon \cdot \log_2 n, n)]$, and then using these formulas to obtain formulas of roughly length n which describe Turing machine computations of length $t(\epsilon \cdot \log_2 n, n)$. (Received May 10, 1972.)

*72T-E68. BARUCH GERSHUNI, Ibn Gvirol Street 43, Tel Aviv, Israel. The manifolds of members of an arbitrary totality.

A class is called a totality of the first kind; a set--a totality of the second kind. Moreover, the members of a class are called of the first kind; while those of a set--of the second kind. A class C which is a proper plural has two manifolds of members of the first kind. Suppose it is e.g. a so-called comma-class:

C = a, b, c, ... Then the first manifold of members of C consists of the usual members of C, namely a b c ... . They are written in the form of a manifold because each member appears in it as a separate entity. The second "manifold" is the comma-class C itself: a, b, c, ..., which is the unique member of this manifold. When the comma-class C is a singular, i.e. in this case a simple individual i, the two mentioned manifolds of members coincide. They form then one "manifold" consisting of one member, namely of i itself. A set {a, b, c, ...} has, as the simple individual, only one manifold of members, this time of the second kind, namely its usual members a b c ... . But the same set may also be grasped as a class, if we reckon the packing material (the braces) also as a member. Then this class is also representable by { } a, b, c, ... and is, it is sure, not a pure comma-class. It has, as any class, two manifolds of members, namely the manifold { } a b c ... and the class { } a, b, c, ... itself. (Received May 15, 1972.)

*72T-E69. STEPHEN G. SIMPSON, Yale University, New Haven, Connecticut 06520. Admissible selection operators.

Let M be an admissible set. An admissible selection operator for M is a function f with domain the ordinals of M and range M such that M is admissible relative to f. The known results for recursion theory on admissible ordinals, e.g. the Friedberg-Muchnik theorem (Sacks-Simpson, "The α-finite injury method," Ann. Math. Logic (to appear)) carry over straightforwardly to systems $\langle M, f\rangle$. It seems fair to say that there is a reasonable recursion theory with M as domain of individuals if and only if M has an admissible selection operator. Some results on the existence of admissible selection operators are presented. (Received May 31, 1972.)

72T-E70. CHI T. CHONG, Yale University, New Haven, Connecticut 06520. Some results on α-recursively enumerable degrees. Preliminary report.

For definitions see Sacks ("Post's problem, admissible ordinals, and regularity," Trans. Amer. Math. Soc. 124(1966), 1-23). Let α be an admissible ordinal. Sacks and Simpson ("The Friedberg-Muchnik theorem in α-recursion theory," Ann. Math. Logic (to appear)) proved that there exist two incomparable α-recursively enumerable (α-r.e.) degrees. Their argument is modified to prove the following generalization. Theorem 1. Let $\underline{a}$ be an α-r.e. non-α-recursive degree. Then there exist two incomparable α-r.e. degrees $\underline{b}, \underline{c}$ which are α-recursive in $\underline{a}$. Definition. An α-r.e. degree $\underline{a}$ is the greatest lower bound (glb) of two α-r.e. degrees $\underline{b}, \underline{c}$ iff for any α-r.e. $\underline{e}$, if $\underline{e}$ is α-recursive in $\underline{b}$ and $\underline{c}$, then $\underline{e}$ is α-recursive in $\underline{a}$. A similar technique proves Theorem 2. Let $\underline{a}$ be as in Theorem 1. Then there exists an α-r.e. $\underline{b}$, α-recursive in $\underline{a}$, which is not the glb of any other two α-r.e. degrees. The case $\alpha = \omega$ was proved by Lachlan ("Lower bounds for pairs of recursively enumerable degrees," Proc. London Math. Soc. 16(1966), 537-569). (Received May 26, 1972.)

72T-E71. PAUL J. CAMPBELL, St. Olaf College, Northfield, Minnesota 55057. Suslin logic.

An extension of the propositional part of the logical language $L_{\omega_1, \omega}$ is defined and investigated which is based on the Suslin operation A as an additional logical connective. (Concerning operation A, see W. Sierpinski, "Introduction to general topology," Univ. of Toronto, Toronto, 1952.) Logical completeness of the language is proved in a Boolean-algebraic manner heavily based on work of L. Rieger ("Concerning Suslin algebras and their representation," Czechoslovak Math. J. 5(1955), 99-142; (Russian)). The major innovation is the reconceptualization of a Suslin algebra as not merely a σ-algebra closed under operation A applied to Suslin systems of elements of the algebra, but as also satisfying an analogue of the "$A^2 = A$" property of analytic sets.

Various propositions are proved concerning distributive laws in Suslin algebras and their relation to "$A^2 = A$". The conditions under which a Suslin algebra is representable as a field of sets are explored. (Received June 1, 1972.)

72T-E72. DAVID W. KUEKER, University of Michigan, Ann Arbor, Michigan 48104. Some Löwenheim-Skolem theorems. Preliminary report.

For classes closed downward and related notions see the author's article in Bull. Amer. Math. Soc. 78(1972), 211-215. K_0 is the class of all models $\mathfrak{A}$ with finite $S \subseteq A$ such that every element of A is $L_{\infty\omega}$-definable in terms of the elements of S. P is a predicate of the language. Theorem. Let K be closed downward. Then (k) $\Leftrightarrow$ (k*) for $k = 1, 2, 3$. (1) Every countable model in K has $\leqq \omega$ automorphisms. (1*) $K \subseteq K_0$. (2) Some countable model in K has $\leqq \omega$ automorphisms. (2*) $K \cap K_0 \neq 0$. (3) There is a countable $\mathfrak{A}_0 \in K$ such that $\mathfrak{A}_0 \prec_{\infty\omega} \mathfrak{A}_1$ for some uncountable $\mathfrak{A}_1$ with $P^{\mathfrak{A}_0} = P^{\mathfrak{A}_1}$. (3*) There is some $\mathfrak{A}_1 \in K$ with a proper $L_{\infty\omega}$-elementary submodel $\mathfrak{A}_0$ with $P^{\mathfrak{A}_0} = P^{\mathfrak{A}_1}$. Remarks. (a) The theorem for $k = 1, 2$ generalizes Theorem 2.2 of the author's paper in "The syntax and semantics of infinitary languages," which is for $K = \mathrm{Mod}(\sigma)$ where σ is a complete sentence of $L_{\omega_1\omega}$. Analogous results hold generalizing Theorem 2.1 of that paper, and for similar properties. (b) The theorem for $k = 3$ is a two-cardinal result which implies, for example, Vaught's two-cardinal theorem for finitary theories. (c) The proofs for $k = 1, 2$ involve proving that K_0 is closed by showing $K_0 = \mathrm{Mod}(\sigma)$ for an appropriate $\sigma \in L^P(\omega)$. A consequence of this — that the class of all $\mathfrak{A}$ with $> |A|$ automorphisms is closed downward—was first shown by K. Hickin. The class of all $\mathfrak{A}_1$ with the property in (3*) is similarly shown to be closed downward. (Received June 1, 1972.)

72T-E73. JOHN W. DAWSON, JR., University of Michigan, Ann Arbor, Michigan 48104. Definability of ordinals in the rank hierarchy of set theory. Preliminary report.

Let Df(M) denote the class of sets first-order definable without parameters in the structure M. x is ordinal definable iff $x \in \mathrm{Df}(\langle R(\alpha), \epsilon\rangle)$ for some ordinal α; however, $x \in \mathrm{Df}(\langle R(\alpha), \epsilon\rangle)$ need not imply $x \in \mathrm{Df}(\langle R(\beta), \epsilon\rangle)$ for $\beta > \alpha$. Definition. $\alpha_\xi = \mu\alpha(\exists\gamma > \alpha)\ (\alpha \notin \mathrm{Df}(\langle R(\gamma), \epsilon, \alpha_\delta\rangle_{\delta<\xi}))$, $\gamma_\xi = \mu\gamma(\exists\alpha<\gamma)(\alpha \notin \mathrm{Df}(\langle R(\gamma), \epsilon, \alpha_\delta\rangle_{\delta<\xi}))$, each α_δ a constant of $R(\gamma)$. Theorem. $L(\alpha_\xi)$ and $L(\gamma_\xi)$ are models of ZFL minus power set, for all ξ. Moreover for ξ less than every fixed-point of the function $f(\xi) = \alpha_\xi$, $\alpha_\xi \notin \mathrm{Df}(\langle R(\gamma_\xi), \epsilon, \alpha_\delta\rangle_{\delta<\xi})$ and $L(\alpha_\xi)$ is an elementary substructure of $L(\gamma_\xi)$. As a consequence of a general definability theorem, γ_0 is always a cardinal of L. Thus $\omega_1^L \leqq \gamma_0 \leqq \omega_1$, and Theorem. Each of $\omega_1^L = \omega_1 = \gamma_0$, $\omega_1^L = \gamma_0 < \omega_1$, $\omega_1^L < \gamma_0 < \omega_1$, and $\omega_1^L < \gamma_0 = \omega_1$ is consistent relative to ZF + GCH and to ZFC + $\neg$GCH. The proofs involve forcing techniques of Cohen, Lévy, and McAloon; many carry over to the context of a measurable cardinal. Similarly, Theorem. For any countable standard model of V = L there is a Cohen extension preserving cardinals but not preserving α_0. Finally, Theorem. If $V = L_\mu$, μ a normal measure on a cardinal κ, then α_0 and γ_0 are indiscernibles in the sense of Silver. (Received June 5, 1972.)

*72T-E74. HASKELL B. CURRY, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. The consistency of a system of combinatory restricted generality.

In the forthcoming second volume of "Combinatory logic" (by the author, J. R. Hindley, and J. P. Seldin, North Holland Publishing Co., Amsterdam) there is proposed (without claim as to consistency) a system $\mathfrak{F}_{21}^*$, for restricted generality. This system has turned out to have the anomalous property that $\vdash$ YH, where Y is a fixed point (or paradoxical) combinator, and H represents the category of propositions. This is highly counterintuitive, and it raises serious doubt as to the consistency of the system. Here a new system, $\mathfrak{F}_{22}$, is proposed which is obtained by modifying the definition of canonicalness in $\mathfrak{F}_{21}$ so that $\mathbf{H}X$ is canonical just when X is, dropping the axiom $\vdash \mathbf{LH}$, and making other changes corresponding to this new conception. This has the effect that properties, such as certain axioms, expressed in $\mathfrak{F}^*_{21}$ by single formulas, can be expressed in $\mathfrak{F}_{22}$ only as rules. But the system is sufficient for the most important consequences of $\mathfrak{F}^*_{21}$, including the formulation of predicate calculus; and it can be shown that all provable formulas are canonical, whereas there are formulas which are not canonical. (Received June 8, 1972.)

72T-E75. ALEXANDER S. KECHRIS, University of California, Los Angeles, California 90024. Basis theorems for large analytical sets. Preliminary report.

It was proved by Sacks and (independently) Tanaka, that every Π^1_1 set of reals of positive measure contains a Δ^1_1 real. Similarly, Thomason and Hinman used methods of Feferman to prove the same result for nonmeager Π^1_1 sets. An extension to all odd levels of the analytical hierarchy is obtained here, using Projective Determinacy (PD). Theorem 1. PD $\Rightarrow$ every Π^1_{2n+1} set of positive measure or of the second category contains a Δ^1_{2n+1} real. In fact it contains reals of any Δ^1_{2n+1}-degree (this stronger version for $n = 0$ is due to Sacks and Tanaka independently). The following is a step in the proof and for $n = 1$ is due to Tanaka. Theorem 2. PD $\Rightarrow$ the measure of a Σ^1_n (Π^1_n, Δ^1_n) set is a Σ^1_n (Π^1_n, Δ^1_n) real number. (A real number is Σ^1_n (Π^1_n, Δ^1_n) if the set of rationals preceding it is Σ^1_n (Π^1_n, Δ^1_n).) One can use these results to extend the "measure-theoretic uniformity" results of Sacks to all appropriate levels of the analytical hierarchy. Other corollaries include the nonexistence of largest countable Σ^1_{2n+1} sets and a "Gandy-Kreisel-Tait" type characterization of the set of all Δ^1_{2n+1} reals. (Received June 12, 1972.)

72T-E76. THOMAS OTTMANN, Institut für Mathematische Logik und Grundlagenforschung, Westfälische Wilhelms-Universität, 44 Münster, West Germany. Arithmetical predicates on a class of finite automata.

If $I = I_1 \dot{\cup} I_r$ is the set of inputs and $O = O_1 \dot{\cup} O_r$ is the set of outputs of a finite, incomplete, initial, deterministic automaton X of Mealy-type and if $|I_1| = |O_r| = m$, $|I_r| = |O_1| = n$, X is called an automaton of type (m, n). $A(m,n)$ denotes the class of all automata of type (m,n). There is a natural way to concatenate automata of the same type. Concatenation is denoted by juxtaposition. Define $X \approx Y$ ($X \sim Y$, $X \leqq Y$) iff X is isomorphic to Y (initial state of X is equivalent to initial state of Y, X is isomorphically embeddable in Y). Lemma. $\forall m,n \geqq 2 : \exists M : (M \subseteq A(m,n),\ M \text{ finite}) : \forall X \in A(m,n) : \exists k : \exists X_1, \dots, X_k \in M : X \approx X_1 \dots X_k$. A set $M \subseteq A(m,n)$ with the properties of the Lemma is called a basis for $A(m,n)$. $\mathfrak{A}(m,n)$ is the smallest class of predicates on $A(m,n)$ with: (i) $\lambda XY \cdot X \approx Y$, $\lambda XY \cdot X \sim Y$, $\lambda XY \cdot X \leqq Y \in \mathfrak{A}(m,n)$, (ii) $\mathfrak{A}(m,n)$ is closed under substitution of the functions $\lambda XY \cdot XY$, $\lambda X_1 \dots X_n \cdot X_i$, C^X_0($X \in M$, M basis for $A(m,n)$), (iii) $\mathfrak{A}(m,n)$ is closed under first order logic. A mapping γ from $A(m,n)$ onto N is defined. If $\mathcal{P}$ and P are both k-ary predicates on $A(m,n)$ and N respectively, define: $\mathcal{P} \mathrel{\hat{=}_\gamma} P : \Leftrightarrow \forall X_1, \dots, X_k \in A(m,n) : (\mathcal{P}(X_1, \dots, X_k) \Leftrightarrow P(\gamma(X_1), \dots, \gamma(X_k)))$. Let $\mathfrak{A}_N$ denote the class of arithmetical predicates of natural numbers and define: $\mathfrak{A}_\gamma(m,n) := \{\mathcal{P}; \exists P \in \mathfrak{A}_N : \mathcal{P} \mathrel{\hat{=}_{\gamma_0}} P\}$. Theorem. $\forall m,n \geqq 2 : \mathfrak{A}(m,n) = \mathfrak{A}_\gamma(m,n)$. (Received June 16, 1972.) (Author introduced by Dr. Egon Börger.)

72T-E77. GREGORY J. CHAITIN, Mario Bravo 249, Buenos Aires, Argentina. Information-theoretic aspects of the Turing degrees. Preliminary report.

Use is made of the concept of the relative complexity of a finite binary string in one or more infinite binary strings. An infinite binary string is recursive in another iff $\exists c \forall n$ the relative complexity of its initial segment of length n is less than $c + \log_2 n$. With positive probability, an infinite binary string has the property that the complexity of its initial segment of length n relative to the rest of the string is asymptotic to n. One such string R recursive in φ' is defined. This infinite string R is separated into $\aleph_0$ independent infinite strings, i.e. the complexity of the initial segment of length n of any of these strings relative to all the rest of these strings is asymptotic to n. By joining these independent infinite strings one obtains Turing degrees greater than 0 and less than $0'$ with any denumerable partial order. From the fact that R is recursive in φ' it follows that there is a recursive predicate P such that asymptotically n bits of axioms are needed to determine which of the following n propositions are true and which are false: $\exists x \forall y\ P(x,y,a)$ $(a < n)$. (Received June 19, 1972.)

*72T-E78. CHARLES C. PINTER, University of California, Berkeley, California 94720. Simpler axioms for polyadic algebras.

A polyadic algebra is understood in the sense of Halmos (Fund. Math. 43(1956), 255-325), and a transformation algebra is understood in the sense of L. LeBlanc (Canad. J. Math. 13(1961), 602-613). If I is any set, a projection of I is any $\lambda \in I^I$ such that $\lambda\lambda = \lambda$; if λ is a projection, we write edm λ = domain $(\lambda - id_I)$ and ern λ = range $(\lambda - id_I)$. Let $\mathfrak{A} = \langle A, +, \cdot, -, 0, 1, S(\alpha), \exists(J)\rangle_{\alpha \in I^I, J \subseteq I}$ be an algebra such that $\langle A, +, \cdot, -, 0, 1, S(\alpha)\rangle_{\alpha \in I^I}$ is a transformation algebra and $\exists(J)$ are unary operations. Consider the following conditions on $\mathfrak{A}$: (QT) For all $K \subset I$ and all $x, y \in A$, (i) $\exists(K)(x+y) = \exists(K)x + \exists(K)y$, and (ii) $x \leqq \exists(K)x$. (PL) For all $K \subset I$ and every projection λ, if J = edm λ and L = ern λ, then (i) $S(\lambda)\exists(J) = \exists(J)$, (ii) $\exists(J)S(\lambda) = S(\lambda)$, (iii) $\exists(K)S(\lambda) = S(\lambda)\exists(K)$ if $K \cap (J \cup L) = \emptyset$, and (iv) $\exists(K)\exists(I-K) = \exists(I)$. Theorem. If $\langle A, +, \cdot, -, 0, 1, S(\alpha), \exists(J)\rangle_{\alpha \in I^I, J \subseteq I}$ is a polyadic algebra, then (QT) and (PL) hold. Conversely, if $\mathfrak{A} = \langle A, +, \cdot, -, 0, 1, S(\alpha), \exists(J)\rangle_{\alpha \in I^I, J \subseteq I}$ is an algebra such that $\langle A, +, \cdot, -, 0, 1, S(\alpha)\rangle_{\alpha \in I^I}$ is a transformation algebra and (QT) and (PL) hold, then $\mathfrak{A}$ is a polyadic algebra. (Received June 19, 1972.)

72T-E79. CARL G. JOCKUSCH, JR., University of Illinois, Urbana, Illinois 61801 and ROBERT I. SOARE, University of Illinois at Chicago Circle, Chicago, Illinois 60680. Post's problem and his hypersimple set.

A standard enumeration of the recursively enumerable (r.e.) sets is a 1:1 recursive function f with range $\{\langle x,y\rangle : x \in W_y\}$, where $\{W_y\}_{y \in N}$ is an acceptable numbering of the r.e. sets in the sense of Rogers, "Theory of recursive functions and effective computability," p. 41. In his quest for an incomplete r.e. set Post (Bull. Amer. Math. Soc. 50(1944), 284-316) constructed a hypersimple set H_f relative to a fixed but unspecified standard enumeration f. Although it was later shown that hypersimplicity does not guarantee incompleteness, the ironic possibility remained that Post's own particular hypersimple set might be incomplete. This seemed plausible in view of the fact that Post's hypersimple set construction is almost a priority argument (although requirements are never injured), and there is a great deal of "negative restraint" which keeps elements out of the set. We settle the question by proving that the H_f may be either complete or incomplete depending upon which standard enumeration f is used. In contrast, D. A. Martin has shown that Post's simple set S is complete for any standard enumeration. (Received June 19, 1972.)

72T-E80. C. R. MANN, Mathematical Institute, University of Oxford, St. Giles, Oxford, England. Two equivalence relations on proofs. Preliminary report.

J. Lambek and M. E. Szabo have developed a method for constructing free cartesian closed categories in which they set up a deductive system which closely resembles Gentzen's system LJ, and then defined an equivalence relation on the proofs of this system. D. Prawitz has proposed a definition of equivalence on the proofs of Gentzen's second system NJ. Theorem 1. Under the obvious translation from LJ to NJ, if two proofs are equivalent in the system LJ in the sense of Szabo, then the corresponding proofs in the system NJ are equivalent in the sense of Prawitz. Theorem 2. With the addition of explicit operations for the thinning, interchange and contraction of assumptions to the system NJ, the converse result holds also. This allows a simplification of Szabo's construction, and adds weight to Prawitz's conjecture. (Received June 21, 1972.) (Author introduced by Professor G. S. Sacerdote.)

72T-E81. J. L. HICKMAN, Institute of Advanced Studies, Australian National University, Canberra, ACT 2600, Australia. Some ordinal-theoretic properties.

We work in ZF set theory without AC: all sets referred to are assumed to be linearly ordered. A similarity $f: A \to B$ is an order-preserving injection; A, B are called "similar" if there exists a surjective similarity $f: A \cong B$. A map $f: A \cong A$ is called an "automorphism" if it is a surjective similarity. Given A, we consider the following statements. (I) A is well-ordered. (II) There is no similarity $f: \omega^* \to A$. (III) For any similarity $f: A \to A$, we have $x \leqq f(x)$ for all $x \in A$. (IV) A is not similar to any proper segment of itself. (IVa) A has only one automorphism, viz. the identity. It is routine to prove the implication chain (I) → (II) → (III) → ((IV) & (IVa)). These arrows cannot be reversed; moreover, (IV) and (IVa) are mutually independent. The demonstrations of (II) → (I), (III) → (II), and ((IV) & (IVa)) → (III) all require the construction of at least one "Cohen-type" ZF-model; the other demonstrations are straightforward. Let C(I) be the class of order types of sets satisfying (I), i.e. the ordinals, and define C(II), C(III), etc. similarly. The arithmetic of C(II) can be developed along the lines of ordinal arithmetic, but with some interesting contrasts. C(III) can be endowed with a fairly satisfactory additive structure, but it seems difficult to develop a multiplicative theory. (Received June 5, 1972.) (Author introduced by Professor J. N. Crossley.)

*72T-E82. STEVEN R. GIVANT, University of California, Berkeley, California 94720. Universal relational theories. Preliminary report.

Let Σ be a universal relational theory (i.e., axiomatizable by a set of universal sentences, with no operation symbols). $L(\Sigma)$ denotes the language of Σ, Σ_∞ denotes the theory obtained from Σ by adding the set of axioms which together assert that a model is infinite, and E denotes the theory of equality. If Σ_∞ is complete and consistent (in particular, if Σ is categorical in some infinite power) then it is essentially trivial. In fact, Theorem. If Σ_∞ is complete and consistent, it is an (open) definitional extension of E_∞ (i.e., with each (n-ary) relation symbol R in $L(\Sigma)$ is correlated an open formula φ_R in L(E) in such a way that $E_\infty \cup \{\forall v_0 \ldots \forall v_{n-1}(R(v_0, \ldots, v_{n-1}) \leftrightarrow \varphi_R) : R \text{ in } L(\Sigma)\}$ is an axiomatization of Σ_∞). The proof uses the methods of Ehrenfeucht-Mostowski (Fund. Math. 43(1956)). A class K of finite structures for $L(\Sigma)$ is the class of finite models of some universal theory Σ' with $\Sigma'_\infty = \Sigma_\infty$ if and only if (i) K contains the finite substructures of models of Σ_∞; (ii) K is closed under isomorphic images and substructures; (iii) for each (n-ary) R in $L(\Sigma)$, $\forall v_0 \ldots \forall v_{n-1}(R(v_0, \ldots, v_{n-1}) \leftrightarrow \varphi_R)$ holds in all sufficiently large members of K. (Received June 28, 1972.)

72T-E83. ALEXANDER ABIAN, Iowa State University, Ames, Iowa 50010. Set-theoretical models and Boolean rings.

It is well known that ZF-set theory (or for that matter ZF $\pm$ AC $\pm$ CH) viewed as a first order theory has a denumerable model. Putting aside all the formal paraphernalia, a denumerable model for ZF (or ZF $\pm$ AC $\pm$ CH) is a denumerable list of distinct symbols $a_0, a_1, a_2, a_3, \dots$ (each called a set) together with a truth "1" or falsehood "0" assignment to the atomic formulas $a_i \in a_j$ such that all the axioms of ZF (or ZF $\pm$ AC $\pm$ CH) are valid. In short, a denumerable model for ZF (or ZF $\pm$ AC $\pm$ CH) is a table whose uppermost row is $\ni | a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, \dots$ and whose other rows are, say, $a_0 | 0, 1, 1, 0, 1, 0, 0, 0, 0, 1, \dots$; $a_1 | 1, 0, 0, 1, 1, 1, 0, 1, 0, 0, \dots$; $a_2 | 0, 1, 0, 0, 1, 1, 0, 1, 0, 1, \dots$; etc., where the elementhood symbol $\in$ is inverted deliberately to facilitate in determining the elements of a_i. For example, the elements of a_2 are $a_1, a_4, a_5, a_7, a_9, \dots$. It can readily be verified that in view of the axioms of ZF the rows of the above table form a Boolean ring under coordinatewise addition and multiplication Mod 2. In fact, the entire model is represented by a Boolean ring. The validity or invalidity of set-theoretical axioms in the model imply the validity or invalidity of certain properties in the corresponding Boolean ring, and vice versa. In this paper various set-theoretical models are obtained as a result of specifying the corresponding Boolean rings, and vice versa. (Received June 28, 1972.)

Statistics and Probability

72T-F8. PHILIP TODD LAVIN, Division of Applied Mathematics, Brown University, Providence, Rhode Island 02912. Stochastic feature selection.

A methodology for looking at problems in pattern recognition is viewed through the development and analysis of three related mathematical models. These probabilistic models are viewed as deformation mechanisms and methods of mathematical statistics are used to extract data features from the following models: (I) Maximum likelihood is used to analyze a class of parameter dependent images deformed by white noise. The selection of these parameters is variance minimizing. (II) Karhunen-Loève expansions are utilized to decompose classes of nonstationary stochastic processes defined on the unit interval. Under certain restrictions, the best stochastic approximation of these processes is obtained using trigonometric polynomials. (III) Large sample radius estimation techniques are employed to estimate the radius of a circle with an unknown center when random sample of N points is drawn from a uniform distribution on the circle. Estimates are found that have variance proportional to $N^{-4/3}$. These models constitute an effort to attack feature selection problems using a mathematical framework. (Received April 28, 1972.)

*72T-F9. TZE-CHIEN SUN, Wayne State University, Detroit, Michigan 48202 and ARUNAVA MUKHERJEA and NICOLAS A. TSERPES, University of South Florida, Tampa, Florida 33620. On recurrent random walks on semigroups.

Let S be a locally compact semigroup, Σ its Borel subsets and μ a probability measure on Σ. Let X_n be the usual coordinate sequence of ind. ident. distri. r.v.'s on the probability space $(\Pi_{i=1}^{\infty} S_i, P)$ $(S_i = S \ \forall\ i)$, induced by μ. Let $Z_n = X_1X_2 \dots X_n$. Let $D = \overline{\bigcup_{n=1}^{\infty} F^n}$, F being the support of μ. Let $R' = \{x \in D | P(Z_n \in N(x)$ i.o.$) = 1$ for every nbd. $N(x)$ of $x\}$ and $R = \{x \in D | P_x(Z_n \in N(x)$ i.o.$) = 1$ for every nbd. $N(x)$ of $x\}$. Let S be completely simple $(= E \times G \times F$, usual representation) in the Theorems 1, 2, 3, 4 and 5. Theorem 1. $R' = \emptyset$ or $R' = D =$ a topological right group in S. Theorem 2. $R = \emptyset$ or $R = D =$ completely simple. Theorem 3. $R' \neq \emptyset$

iff $\Sigma_{n=1}^{\infty} P(Z_n \in N(x)) = \infty$ for all nbds. $N(x)$ of some x in D. Theorem 4. $R = D$ = completely simple if G is compact. Theorem 5. S can support a random walk iff G can support a random walk. Theorem 6. $R' = R = K$ = the kernel of S, if S is compact abelian. [These results extend previously known results of Chung and Fuchs and Loynes.] (Received May 22, 1972.)

Topology

72T-G116. WITHDRAWN.

72T-G117. CHANDRA MOHAN PAREEK, University of Saskatchewan, Regina, Saskatchewan, Canada. M_i-spaces $(i = 1, 2, 3)$. Preliminary report.

In this paper we prove: (a) A space X has a σ-closure preserving closed network (σ-closure preserving closed K-network) if and only if every point x of X has a sequence $\{U_n(x)\}_{n=1}^{\infty}$ of open neighborhoods satisfying (i) if $y \in U_n(x)$, then $U_n(y) \subset U_n(x)$, (ii) if $x \notin F$ ($K \cap F = \emptyset$) for any point x and a closed set F of X (for any compact set K and closed set F of X), then there is n for which $x \notin (U_n(y) | y \in F)$ $(K \cap U(U_n(y) | y \in F) = \emptyset)$. (The second condition characterizes spaces with σ-cushioned pair network.) (The second condition characterizes spaces with σ-cushioned pair K-network.) (b) A regular space is F_σ-screenable if and only if it is a continuous open σ-locally finite image of a zero dimensional paracompact Hausdorff space. (c) A T_1-space with σ-cushioned pair network is a one to one continuous σ-cushioned image of an M_3-space. (d) A T_3-space with σ-closure preserving network is a one to one continuous σ-closure preserving image of an M_1-space. (e) A first countable T_1-space is an M_3-space if and only if it has σ-cushioned pair K-network. Various other results in this direction have been obtained and several questions have been raised. (Received February 29, 1972.)

72T-G118. HAROLD BENNETT, Texas Tech University, Lubbock, Texas 79409. Real valued functions on semimetric spaces. Preliminary report.

Theorem 1. If f is a real valued function on a regular semimetrizable Baire space, then there is a dense subset Y of X such that f restricted to Y is continuous. Theorem 2. If X is a regular semimetric space such that $X \times X$ is a Baire space, then $X \times X$ has a dense Moore subspace. Corollary 3. If X is a regular, weakly complete semimetric space, then X has a dense metrizable subspace. The methods of proof generalize methods used by H. Blumberg (Trans. Amer. Math. Soc. 24(1922)). (Received March 9, 1972.) (Author introduced by Professor Charles N. Kellogg.)

72T-G119. THOMAS E. HAYS, University of Tennessee, Knoxville, Tennessee 37919. A characterization of some Clifford semigroups. Preliminary report.

Notation. Let $\mathcal{C}$ be the category of compact simple semigroups and homomorphisms, $\mathcal{RG}$, $\mathcal{G}$, and $\mathcal{RB}$ the subcategories of rectangular groups, groups, and rectangular bands, respectively. Let Φ be the category of inverse limit preserving functors whose domains are compact Lawson semilattices and codomains are $\mathcal{C}$, where the morphisms are defined in a natural manner. Let Σ, Γ, and Δ be those subcategories of Φ whose codomains are $\mathcal{RG}$, $\mathcal{G}$, and $\mathcal{RB}$, respectively. Let $\mathcal{J}$ be the category of compact semigroups S which are normal bands of groups and $S/\mathcal{D}$ is a Lawson semilattice and homomorphisms, and let $\mathcal{O}$, $\mathcal{A}$, and η be those subcategories whose objects are semigroups S with $E(S)^2 \subseteq E(S)$, $S/\mathcal{H}$ a semilattice, and $E(S) = S$, respectively. The following result is obtained. Theorem. The categories Φ and $\mathcal{J}$, Σ and $\mathcal{P}$, Γ and $\mathcal{O}$, and Δ and η, respectively are isomorphic. (Received April 3, 1972.)

72T-G120. LESLIE C. WILSON, University of Illinois, Urbana, Illinois 61801. Some properties of stratifications. Preliminary report.

Theorem 1. Let M, N be finite-dimensional C^{∞}-manifolds, S a closed subset of N, $\mathcal{P}$ a prestratification (in the sense of Mather) of S satisfying Whitney's condition (a). Then, in the fine C^{∞} topology, the set of all $f \in C^{\infty}(M, N)$ such that f is transversal to each of the strata of $\mathcal{P}$ is open. The proof is similar to the proof, for openness of transversality, in Abraham and Robbin ("Transversal mappings and flows," p. 47). Theorem 2. If $\mathcal{P}$ is a Whitney prestratification of a subset S of a finite-dimensional C^1-manifold, then there exists a canonical Whitney prestratification with only finitely many strata, equivalent to $\mathcal{P}$. The constructive proof is based on grouping in a certain way the strata of $\mathcal{P}$ of each fixed dimension. (Received May 5, 1972.) (Author introduced by Professor Felix R. Albrecht.)

*72T-G121. R. GRANT WOODS, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada. Maps that characterize weak normality properties and pseudocompactness. Preliminary report.

All topological spaces are assumed to be Tychonoff. A "map" is a continuous function. Definition. A map k from a space X onto a space Y is called a WN-map if $\mathrm{cl}_{\beta X} k^{\leftarrow}[Z] = (k^{\beta})^{\leftarrow}[\mathrm{cl}_{\beta Y} Z]$ for each zero-set Z of Y. (k^{β} denotes the Stone extension of k.) In [Trans. Amer. Math. Soc. 148(1970), 265-272], John Mack defines a space X to be δ-normally separated (weakly δ-normally separated) if each zero-set of X is completely separated from each closed (regular closed) subset of X disjoint from it. Theorem 1. The following conditions on a space X are equivalent: (i) X is δ-normally separated. (ii) Each closed map onto X is a WN-map. (iii) Each Z-mapping onto X is a WN-map. (A Z-mapping from Y onto X takes zero-sets of Y onto closed sets of X.) Theorem 2. Let E(X) denote the projective cover (absolute) of a space X and k the perfect irreducible map from E(X) onto X. Then X is weakly δ-normally separated iff k is a WN-map. Corollary. Let I denote the closed unit interval, and let E(X) and k be as in Theorem 2. If E(X) is realcompact and $k \times 1_I : E(X) \times I \to X \times I$ is a WN-map, then X is realcompact. Theorem 3. A space X is pseudocompact iff each map from X onto any Tychonoff space is a WN-map. (Received May 15, 1972.)

*72T-G122. MARLON C. RAYBURN, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada. Hard sets.

All spaces used are Tychonov, βX is the Stone-Čech compactification and υX is the Hewitt realcompactification. A set in X is hard if it is closed in $X \cup K$, where $K = \mathrm{cl}_{\beta X}(\upsilon X - X)$. Let $\delta X = \beta X - (K - X)$. A hard set is the restriction to X of a compact subset of δX. In X, every compact set is hard, and conversely iff $X = \delta X$; every hard set is closed and realcompact, and every closed set is hard iff X is realcompact. A complicated characterization of hard sets in terms of X is known. Closed subsets, arbitrary intersections, and finite unions of hard sets are hard. The union of a hard set and a realcompact set is realcompact. Perfect maps pull hard sets back to hard sets and homeomorphisms (onto) preserve hard sets. Hard sets of X are not necessarily hard in subspaces or superspaces of X. If f is a quotient map from realcompact X onto Y, then Y is realcompact iff the closure in Y of the set of points with multiple preimages is hard. (Received May 18, 1972.)

72T-G123. PEI LIU, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. Compatible topologies on homeomorphism groups.

Let (X, τ) be a topological space and let $H(X)$ be the homeomorphism group of X. Then (X, τ) is said to be a weak Galois space provided that for each $U \in \tau$, there exist $h \in H(X)$ and $x \in U$ such that $h(x) \neq x$ and $h|X - U = i|X - U$. Theorem 1. Let $(X, \mathcal{U})$ be a quasi-uniform space such that every $h \in H(X)$ is $\mathcal{U} \vee \mathcal{U}^{-1}$ uniformly continuous. Then $H(X)$ is a topological group under the topology of uniform convergence with respect to $\mathcal{U} \vee \mathcal{U}^{-1}$. Theorem 2. Let (X, τ) be a T_0 weak Galois (second countable) space. Then $H(X)$ admits a topology such that $H(X)$ is a nondiscrete Hausdorff (metrizable) topological group. Theorem 3. Let (X, τ) be a Tychonoff space which contains a nonempty open weak Galois subspace. Then $H(X)$ is a nondiscrete Hausdorff topological group under the topology of uniform convergence with respect to every compatible uniformity.

(Received June 5, 1972.)

*72T-G124. THOMAS J. SANDERS, University of Oklahoma, Norman, Oklahoma 73069. Shape groups and products.

If X is a compact Hausdorff space, $x_0 \in X$ and $\underline{X} = \{(X_a, x_a), p_{aa'}, A\}$ is an ANR-system associated with (X, x_0) then the nth shape group is defined by $\underline{\pi}_n(X, x_0) = \varprojlim \{\pi_n(X_a, x_a), (p_{aa'})_*, A\}$ where $\pi_n(X_a, x_a)$ is the nth homotopy group and $(p_{aa'})_*$ is the induced map. If X is a compactum then the nth shape group is isomorphic to the nth fundamental group defined by Borsuk [Fund. Math. 62(1968)]. If X_0 is the component of X containing x_0 then $\underline{\pi}_n(X, x_0) = \underline{\pi}_n(X_0, x_0)$. If x_0 and x_1 are in the same path component of X then $\underline{\pi}_n(X, x_0) = \underline{\pi}_n(X, x_1)$. A product of a family of inverse systems is defined such that if $\underline{X}^{\omega}$ is an ANR-system associated with X^{ω}, $\omega \in \Omega$, then $\Pi_{\omega \in \Omega} \underline{X}^{\omega}$ is an ANR-system associated with $\Pi_{\omega \in \Omega} X^{\omega}$. The product of shapes can then be defined: $\Pi_{\omega \in \Omega} \mathrm{Sh}(X^{\omega}) = \mathrm{Sh}(\Pi_{\omega \in \Omega} X^{\omega})$. Results similar to (12.3) and (13.1) of [Borsuk, Fund. Math. 64(1969)] are obtained for ASR and ANSR-sets. If $(X, x_0) = \Pi_{\omega \in \Omega}(X^{\omega}, x_0^{\omega})$ then $\underline{\pi}_n(X, x_0) = \Pi_{\omega \in \Omega}(\underline{\pi}_n(X^{\omega}, x_0^{\omega}))$. (Received May 26, 1972.)

*72T-G125. PETER W. HARLEY III, University of South Carolina, Columbia, South Carolina 29208. Metrization of closed images of metric spaces.

Let X be a metric space, Z any nondiscrete Hausdorff Fréchet (that is, each point in the closure of a subset A of X is the limit of a sequence in A) space and $f: X \to Y$ a closed, continuous, onto mapping. Here it is shown for separable, locally compact X, that Y is metrizable if $Y \times Z$ is Fréchet. This result is also established for metrizable X if $Y = X/A$ where A is a closed subset of X. (Received May 30, 1972.)

*72T-G126. MICHAEL OLINICK, Middlebury College, Middlebury, Vermont 05753. A sufficient condition for compactness of a monotone map of E^n.

Theorem. Let f be a monotone map of E^n onto itself. Let S be an $(n-1)$-sphere in E^n with complementary domains C and D. Suppose that f acts like the identity on S, $f(C) = C$ and $f(D) = D$. Then f is compact. The proof utilizes the fact that f can be written as the composition of two reflexive compact maps of E^n onto E^n, each strongly one-to-one on an open set; see "Factoring monotone maps of E^n," in "Topology of manifolds," Markham, Chicago, 1970, pp. 185-189. (Received May 30, 1972.)

*72T-G127. FRANKLIN D. TALL, University of Toronto, Toronto 181, Ontario, Canada. A counterexample in the theories of compactness and of metrization.

A completely regular space is Čech complete if it is a G_δ in some compactification of itself. A collection of sets is centered if every finite subcollection of it has nonempty intersection. Van der Slot ("Some properties related to compactness", Math. Centre Tract 19, Math. Centrum, Amsterdam, 1968) defines a space to be basis-compact if it has a base $\mathcal{B}$ such that if $\mathfrak{F} \subset \mathcal{B}$ is centered, then $\bigcap\{\overline{F} : F \in \mathfrak{F}\} \neq \emptyset$. He proves that basis-compactness coincides with completeness (and hence with Čech completeness) in metric spaces. Theorem. There is a basis-compact space which is not Čech complete; there is a Čech complete space which is not basis-compact. The latter example is also a metacompact nonmetrizable Moore space, which can, consistently with the axioms of set theory, be assumed to be normal. (Received May 30, 1972.)

*72T-G128. EUGENE M. NORRIS, West Virginia University, Morgantown, West Virginia 26506. Relationally induced semigroups.

Properties of difunctional relations allow a generalization of the induced semigroup theorems of A. D. Wallace [Jber. Deutsch. Math.-Verein., Bd. 73, Heft 1, 1971 and references therein] and of a previously announced result of the author [Abstract 689-A15, these *Notices* 18(1971)]. Theorem. If (T,X) is a compact or discrete recursion and $R \subseteq T \times X$ is a closed difunctional relation satisfying: (1) $TR = X$ and $X = TR$. (2) The set $Z = \{z \in T \mid \forall t, t' \in T,\ tR = t'R \text{ implies } t(zR) = t'(zR)\}$ is nonvoid. (3) If $tR = t'(t''R)$ then for all $z \in Z$, $t(zR) = t'(t''(zR))$. (4) For each $t', t'' \in T$ there is some $t \in T$ such that $t'(t''R) = tR$; then the quotient $ZR/R \circ R^{-1}$ is a compact topological semigroup with multiplication $*$ satisfying $zR * z'R = z(z'R)$. If, furthermore, $\exists z \in Z$ such that $\forall t \in T$, $t(zR) = tR$ and $z(tR) = tR$, then the semigroup is a monoid iseomorphic with a certain monoid end, where f end if $f : X/R \circ R^{-1} \to X/R \circ R^{-1}$ is continuous and satisfies the identity $tf(x) = f(tx)$. (Received May 30, 1972.)

72T-G129. MASSOOD SEYEDIN, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. Concerning topological homeomorphism groups. Preliminary report.

For terminology see "Quasi-uniform topological spaces," Noordhoff. Theorem. Let $(X, \mathcal{T})$ be an R_0 space, let $\mathcal{U}$ be a weakly locally symmetric quasi-uniformity for $(X, \mathcal{T})$ and let G be a subgroup of $H(X)$ such that G is quasi-equicontinuous relative to $\mathcal{U}$. Then G is a topological group under the topology of pointwise convergence. Theorem. Let $\{f_n\}_{n=1}^{\infty}$ be a sequence of functions from a topological space $(X, \mathcal{T})$ into a locally symmetric quasi-uniform space $(Y, \mathcal{U})$ such that $\{f_n\}_{n=1}^{\infty}$ converges quasi-uniformly to a function $g : X \to Y$. Then g is a continuous function. (Received May 31, 1972.)

*72T-G130. VICTOR A. NICHOLSON and JOHN P. NEUZIL, Kent State University, Kent, Ohio 44242. Nonfolding maps and the singular regular neighborhood theorem.

We consider some basic properties of nonfolding maps and their role in tame embeddings in 3-manifolds. Suppose f is a map of an n-manifold M into an n-manifold N, $p \in N$ and $f^{-1}(p)$ is cellular in M. Choose n-cells B and C in M and N respectively such that $f^{-1}(p) \subset \operatorname{Int} B$ and $f(B) \subset \operatorname{Int} C$. We say f folds (does not fold) at p if and only if $f \mid \operatorname{Bd} B$ is homotopically trivial (nontrivial) in $C - p$. The definition is independent of the choice of n-cells B and C. Suppose K is a subset of an n-manifold N and f is a map of an n-manifold M into N such that $K \subset f(M)$. We let G_K denote the decomposition of M whose elements are the inverse images of points in K and the singletons of $M - f^{-1}(K)$. Theorem. Suppose K is a connected topological complex which is a closed subset of a 3-manifold N, $f: M \to N$ is a map of a 3-manifold M into N such that $K \subset f(M)$, $f^{-1}(K)$ is a subcomplex of M and G_K is an upper-semicontinuous decomposition of M such that M/G_K is a 3-manifold. Then K is tame if (a) K contains a 3-cell, or (b) each element of G_K is cellular in M and f does not fold at some point of K, or (c) f maps M onto N and is the projection map of a monotone upper semicontinuous decomposition of M into compact sets. We give an example of a map (which folds) of E^3 onto itself that is a homeomorphism on a tame arc K, $f^{-1}f(K) = K$ and f(K) is wild. (Received June 1, 1972.)

*72T-G131. JEONG SHENG YANG, University of South Carolina, Columbia, South Carolina 29208. A note on the topological group C(X, G).

For a topological space X and a topological group G, let C(X, G) be the topological group of continuous functions from X into G endowed with the compact-open topology and with the pointwise multiplication. For p in X, let $C_p(X, G)$ denote the subgroup of C(X, G) composed of the functions f such that f(p) = e. The following theorems are observed: Theorem 1. If G is abelian, C(X, G) is isomorphic to $G \times C_p(X, G)$. Theorem 2. If X is a compact connected metrizable space and G is a locally compact group with equal left and right uniformities then $C_p(X, G)$ is homeomorphic to $1_2 \times C_p(X, K)$ and C(X, G) is homeomorphic to $G \times 1_2 \times C_p(X, K)$ for some compact group K. Theorem 3. If G is a locally compact group such that G/Z is compact, where Z is the center of G, then (1) there is an inverse system $\{(L_\alpha, f_{\beta\alpha}): \alpha, \beta \in A\}$ of Lie groups such that $C_p(X, G)$ is the projective limit of $C_p(X, L_\alpha)$ for any space X, (2) there is a vector group V and a normal subgroup H of G containing a compact open normal subgroup such that $C_p(X, G)$ is isomorphic to $C_p(X, V) \times C_p(X, H)$ and C(X, G) is isomorphic to $C(X, V) \times C(X, H)$ for any space X. The metrizability of the group C(X, G) is also considered. (Received June 2, 1972.)

*72T-G132. JOHANNES M. AARTS, Delft Institute of Technology, Delft, The Netherlands and DAVID J. LUTZER, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. Pseudocompleteness and the product of Baire spaces. Preliminary report.

Oxtoby [Fund. Math. 49(1961), 157-166; MR 25 #4054] introduced pseudocomplete spaces and proved that (a) any pseudocomplete space is a Baire space (= a countable intersection of dense open sets is dense); (b) Cech-complete spaces are pseudocomplete; (c) any product of pseudocomplete spaces is pseudocomplete. In our paper we prove that (a) for metrizable spaces or Moore spaces, pseudocompleteness is equivalent to the existence of a dense completely metrizable subspace; (b) the product of a quasi-regular [Oxtoby, loc. cit.] Baire space and a pseudocomplete space is a Baire space. We also give several examples which relate pseudocompleteness to other types of completeness. (Received June 5, 1972.)

*72T-G133. PETER FLETCHER, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061 and WILLIAM F. LINDGREN, Slippery Rock State College, Slippery Rock, Pennsylvania 16057. Some generalizations of compactness. Preliminary report.

A topological space X is of 2nd category if and only if every point finite open cover of X is locally finite somewhere. This result answers a question posed in [Arch. Math 22(1971), 528-533]. It is interesting to note that if X is regular, then every open cover of X has an open refinement which is locally finite somewhere; whereas a space is compact if and only if every open cover has a subcover which is point finite somewhere. A space is (countably) strongly orthocompact provided that every open cover has a subcover $\mathcal{R}$ with the property that, for each $x \in X$, $\bigcap\{R \in \mathcal{R} : x \in R\}$ is open. Some results concerning (countable) strong orthocompactness are established. (Received June 5, 1972.)

*72T-G134. IVAN L. REILLY, University of Auckland, Auckland, New Zealand. A generalized Banach contraction principle.

The classic Banach contraction mapping principle is extended to arbitrary Hausdorff spaces. The machinery required is the concept of a quasi-gauge structure for a topological space [Abstract 70T-G174, these *Notices* 17(1970), 975], and a suitable definition of Cauchy sequence in this setting. (Received June 6, 1972.)

*72T-G135. STANISLAW G. MROWKA, State University of New York at Buffalo, Amherst, New York 14226. All powers of the Sorgenfrey line are strongly 0-dimensional.

We prove that for every cardinal m the topological power S^m of the Sorgenfrey line S is strongly 0-dimensional (i.e., βS^m is 0-dimensional). (Received June 8, 1972.)

*72T-G136. PAUL R. MEYER, City University of New York, Lehman College, Bronx, New York 10468. On total orderings in topology.

The imposition of order hypotheses on a T_1 topological space X simplifies the behavior of the following cardinal functions: local neighborhood character χ, tightness τ, sequential or net character σ, and pseudocharacter ψ. Definitions. τX = least m with the property: $A \subset X$ and $x \in A^-$ implies $x \in B^-$ for some $B \subset A$ with $|B| \leq m$. ψX = least m such that each point of X is the intersection of at most m open sets. For definition of σ (and related earlier results) see these *Notices* 17(1970), 116. Theorem. If X satisfies one of the following conditions, then $\tau X = \sigma X = \chi X = \psi X$. (1) X is a generalized orderable space (GO space). (2) X is locally orderable. (3) Each point of X has a nested neighborhood base. (4) $X = \Pi\{X_i : i \in I\}$ with $|X_i| \geq 2$ and $|I| \geq \sup \chi X_i$. Theorem. The properties $\tau = \chi$ and $\psi = \chi$ are productive. Theorem (extension of (1) - (3)). If X is a strong chain net space, then $\tau X = \sigma X$. (Note that in spaces for which $\tau = \sigma$ the notions of c-space and sequential space coincide.) A total of nine order conditions are ranked according to generality; examples are given to show that most of the results cannot be improved in this context. (Received June 9, 1972.)

*72T-G137. B. V. S. THOMAS, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213. Epimorphisms of Hausdorff groups have dense range. Preliminary report.

Theorem 1. The epimorphisms in the category of Hausdorff topological groups are the continuous group homomorphisms with dense range. While this result is exactly what one would expect, its proof is nontrivial and makes use in an essential way of the existence of a free topological group over any topological space. This theorem in conjunction with other properties of the category of Hausdorff topological groups and continuous group homomorphisms gives Theorem 2. There is an epireflective compactification functor from the category of Hausdorff topological groups to the category of compact-Hausdorff topological groups. On locally compact abelian groups this functor yields the Bohr compactification. (Received June 9, 1972.)

72T-G138. KUNIO MURASUGI, Princeton University, Princeton, New Jersey 08540. On closed 3 braids. Preliminary report.

Let B_3 be the group of 3 braids generated by σ_1, σ_2. For any β in B_3, denote by $\hat{\beta}$ the closed braid obtained from β. Then the following theorem is proven: Theorem. Let β be a 3 braid. (i) $\hat{\beta}$ is splittable iff β is conjugate to σ_1^m. (ii) $\hat{\beta}$ is the torus knot of type (m, 2) iff β is conjugate to $\sigma_1^m \sigma_2$ or $\sigma_1^m \sigma_2^{-1}$. (iii) $\hat{\beta}$ is a nontrivial product knot or link iff β is conjugate to a split braid $\sigma_1^p \sigma_2^q$, p, q being nonzero integers. Theorem (iii) may be considered as a generalization of a theorem due to Magnus-Pelluso (Comm. Pure Appl. Math. 20(1967), 749-770). (Received June 12, 1972.)

72T-G139. JIMMIE A. ROBERTSON, Florida State University, Tallahassee, Florida 32306. Closed fake surfaces which are spines of 3-balls. Preliminary report.

A 2-polyhedron P is called a closed fake surface if each point of P has a neighborhood homeomorphic to one of three simple types described by Ikeda in [Topology 10(1971), 9-36]. $S_i(P)$ is defined to be the closure of all points in P which have a neighborhood homeomorphic to type i. Define $E(s,t)$ to be the set of all acyclic (integer coefficients) closed fake surfaces P which embed in 3-manifolds and such that $S_2(P)$ and $S_3(P)$ have s and t components respectively. Define $B(s,t) = \{P \in E(s,t) \mid \text{a regular neighborhood of } P \text{ is a ball}\}$. Ikeda has shown that $B(s,t) = E(s,t)$ if $t \leqq 2$. The classical dodecahedral space of Poincaré shows that $B(1,5) \neq E(1,5)$. Lemma. If $B(s,t) = E(s,t)$ for all s, t such that $t \leqq t_0$ then $B(s,t_0+1) = E(s,t_0+1)$ for all $s \geqq 2$. Theorem. $B(s,3) = E(s,3)$ for all s. By the lemma and the results of Ikeda, it suffices to show that $B(1,3) = E(1,3)$. This is done by proving several results about the presentation of the fundamental group of any possible counterexample. The computation of several first homology groups then shows that no counterexample exists. (Received June 13, 1972.) (Author introduced by Professor T. Perrin Wright.)

*72T-G140. EARL PERRY, West Georgia College, Carrollton, Georgia 30117. A note on unions of duods.

In a recent paper (Abstract 72T-G112, these Notices 19(1972), A-546), the author defined the terms duod, hereditarily duodic, and unod. Example 1. Duods A and B whose intersection is a continuum, but whose union is not a duod. Example 2 (due to Professor Ralph Bennett). Hereditarily duodic continua A and B whose union is a continuum which has no cut point. Example 3. Hereditarily duodic continua A and B such that $A \cap B$ is a continuum, the only cut point of A lies in $A \cap B$, the only cut point of B lies in $A \cap B$, $A' = A - (A \cap B)$ is disconnected, $B' = B - (A \cap B)$ is disconnected, and $A \cup B$ is a duod. Theorem 1. Suppose each of A and B is a hereditarily duodic continuum, $A \cap B$ is a continuum, each cut point of A lies in $A \cap B$, each cut point of B lies in $A \cap B$, neither of $A' = A - (A \cap B)$ and $B' = B - (A \cap B)$ is connected, but $\beta_{A'}$ is connected. If $\beta_{A'}$ is degenerate, then $A \cup B$ is a duod; if $\beta_{A'}$ is nondegenerate, then $\overline{A'}$ is a continuum and each of its cut points lies in $\beta_{A'}$. Theorem 2. Suppose each of A and B is a hereditarily duodic continuum, $A \cap B$ is a continuum, and B is locally connected. Then $A \cup B$ is hereditarily duodic. (Received June 16, 1972.)

72T-G141. RICHARD E. HEISEY, Cornell University, Ithaca, New York 14850. Manifolds with the bounded weak-* topology. Preliminary report.

Let B be a separable infinite-dimensional Banach space. By $B^*(b^*)$ denote the conjugate, B^*, of B with its bounded weak-* topology. Let M and N denote paracompact, connected $B^*(b^*)$-manifolds. Theorem 1. $B^*(b^*)$ is homeomorphic to $Q^\infty = \varinjlim Q^n$ where Q is the Hilbert cube. In particular, $B^*(b^*)$ is homeomorphic to ℓ_2 with its bounded weak topology. Theorem 2. Let $C(B^*(b^*))$ be the space of maps from $B^*(b^*)$ to itself with c-o topology. There is a contraction of $C(B^*(b^*))$ to the identity map which simultaneously contracts (i) all homeomorphisms, (ii) all embeddings, (iii) all closed maps, and (iv) all open maps. Theorem 3. Any microbundle or locally trivial bundle with base M and fiber $B^*(b^*)$ is trivial. Theorem 4. If $f: M \to N$ is a closed split embedding, then f(M) is a neighborhood retract of N and there is an open embedding $g: M \times B^*(b^*) \to N \times B^*(b^*)$ such that $g(m,0) = (f(m),0)$ for every $m \in M$. Theorem 5. If $B \oplus B$ is linearly homeomorphic to B and $f, g: M \to N \times \{0\}$ are homotopic, closed embeddings, then f and g are ambient invertibly isotopic in $N \times B^*(b^*)$. Remark. There are analogues of Theorems 2, 3, 4, and 5 for more general TVS's; in particular, for $R^\infty = \varinjlim R^n$. (Received June 19, 1972.)

*72T-G142. CHARLES L. HAGOPIAN, California State University, Sacramento, California 95819. Planar λ connected continua.

A continuum M is said to be λ connected if any two points of M can be joined by a hereditarily decomposable continuum in M. Theorem 1. If a plane continuum M is the union of countably many λ connected continua, then M is λ connected. Theorem 2. The topological product of countably many plane continua is λ connected if and only if each factor is λ connected. Theorem 3. If a λ connected continuum M and a disc D lie in a plane, then each nondegenerate component of $M \cap D$ is λ connected. (Received June 19, 1972.)

*72T-G143. ANTHONY W. HAGER, Wesleyan University, Middletown, Connecticut 06457. Perfect maps and epi-reflective hulls.

In either of the categories of Tychonoff spaces or Uniform spaces, let $\mathcal{A}$ be a class of spaces, let $O(\mathcal{A})$ (respectively, $\mathcal{R}(\mathcal{A})$) be all (resp., all closed) subspaces of products of members of $\mathcal{A}$, and let o and r be the associated epi-reflectors. Let $\ell f\mathcal{A}$ be the left-fitting hull of $\mathcal{A}$, i.e., all spaces which admit a perfect map into a member of $\mathcal{A}$. (In Unif, a map is perfect if it is topologically perfect and uniformly continuous.) The paper mainly concerns the classes $\ell f\mathcal{R}(\mathcal{A})$; these are epi-reflective; call the functor p. Let $\mathcal{K}$ be compact spaces, and k the associated epi-reflector (Stone-Čech or Samuel compactification). Then $\mathcal{R}(\mathcal{A},\mathcal{K}) \subset \mathcal{R}(\ell f\mathcal{A}) \subset \ell f\mathcal{R}(\mathcal{A})$; equality holds in Tych, but not in Unif. One description of pX is as the pullback of the $\mathcal{K}$-reflection of rX and the image under k of the $\mathcal{R}(\mathcal{A})$-reflection of X. The main theorems are that (1) $O(\mathcal{A}) \cap \ell f\mathcal{R}(\mathcal{A}) = \mathcal{R}(\mathcal{A})$; (2) op = r. (2) gives a factorization of an arbitrary epi-reflector r into a product of one (o) for which all reflection maps are onto, and one (p) for which all reflection maps are homeomorphisms (but not embeddings in Unif). (Received June 19, 1972.)

72T-G144. D. M. GIARRUSSO, Fordham University, Bronx, New York 10458 and Yale University, New Haven, Connecticut 06520. On the topology of certain homogeneous spaces of linear analytic groups. Preliminary report.

Using a recent rigidity theorem of G. D. Mostow, concerning discrete co-compact subgroups of semisimple analytic groups with no compact factors, the author obtains the following result: Theorem. Let G_i be a linear analytic group with simply connected radical R_i such that G_i/R_i has no compact factors $(i=1,2)$. Let $\tilde{\Gamma}_i$ be a discrete subgroup of G_i with $G_i/\tilde{\Gamma}_i$ compact $(i=1,2)$, and suppose $\theta: \tilde{\Gamma}_1 \to \tilde{\Gamma}_2$ is an isomorphism. Then there exist subgroups of finite index Γ_1, Γ_2 in $\tilde{\Gamma}_1$, $\tilde{\Gamma}_2$ respectively, with $\Gamma_2 = \theta(\Gamma_1)$, such that the homogeneous spaces G_1/Γ_1 and G_2/Γ_2 are diffeomorphic. (Received June 21, 1972.)

*72T-G145. JOSEPH F. KENT, University of Florida, Gainesville, Florida 32601. Locally connected almost periodic minimal sets.

Let X be an almost periodic minimal set in R^n or C^n under the flow $\varphi: X \times R \to X$. For $x \in X$, φx is a (vector-valued) almost periodic function on R where $(\varphi x)(t) = \varphi(x,t)$. If $S(\varphi x)$ denotes the spectrum of φx, we note that for $x, y \in X$, $S(\varphi x) = S(\varphi y)$. Hence we associate with X a spectrum S(X). As X is a compact solenoidal topological group, we may associate in addition a character group X*. The character group is shown to be algebraically isomorphic to the subgroup of the reals generated by S(X). Using this result we show that quasiperiodic functions are the almost periodic functions whose images have locally connected closures. An alternate proof of a result of M. L. Cartwright equating the dimension of an almost periodic orbit closure and the rank of its spectrum is proven. The main result of the paper shows that locally connected almost periodic minimal sets in R^n or C^n are flow isomorphic to irrational flows on tori. (Received June 22, 1972.)

*72T-G146. DAVID B. GAULD, University of Auckland, Auckland, New Zealand. Tubular neighbourhoods for submersions of topological manifolds.

Let $\varphi: M^m \to N^n$ be a submersion from a metrizable manifold to any (topological) manifold, let $B \subset M$ be compact, $y \in N$ and $C \subset \varphi^{-1}(y)$ be a compact neighbourhood (in $\varphi^{-1}(y)$) of $B \cap \varphi^{-1}(y)$. Theorem. There is a neighbourhood U of y in N and an embedding $\epsilon: U \times C \to M$ such that $\varphi\epsilon$ is projection on the first factor, $\epsilon(y,x) = x$ for each $x \in C$ and $B \cap \varphi^{-1}(U) \subset \epsilon(U \times C)$. The proof involves a straightening of overlapping coordinate patches using the isotopy extension theorem of Edwards and Kirby [Ann. of Math (2) 93(1971), 63-88]. Corollary 1. If φ is surjective and has strongly p-connected fibres then φ is p-connected. Corollary 2. If C is a compact regular leaf of a foliation F on M and U is any neighbourhood of C then there is a saturated neighbourhood V of C contained in U which is the union of compact regular leaves of F. (Received June 23, 1972.)

72T-G147. ROBERT A. HERRMANN, U. S. Naval Academy, Annapolis, Maryland 21402. Modified monads. Preliminary report.

In this paper we study the modified monads of nonstandard topology. In particular, if (X,T) is a topological space, then a c-monad $\mu(\bar{p}) = \text{Nuc}\ \{\bar{G} \mid G \in T \text{ and } p \in G\} = \text{Nuc}$ (filter generated on X by the set of all closed neighborhoods of $p \in X$), an r-monad $\mu_C(p) = \text{Nuc}\ M_C(p)$, where $M_C(p)$ is the filter on X generated by $G_C(p) = \{G \cap C \mid G \in T, p \in G, C \subset X\}$, the set-monads and others. We have established numerous theorems including: Theorem 1. A space X is Urysohn iff $\mu(\bar{p}) \cap \mu(\bar{q}) = \emptyset$ for each pair of distinct points $p, q \in X$. Theorem 2. Let X and Y be regular. Then $f: X \to Y$ continuously iff $g[\mu(\bar{p})] \subset \mu(\overline{f(p)})$, where g is the unique extension of f on $\hat{X}$ into $\hat{Y}$. Theorem 3. Given $S \subset X$. Then S contains the closure of an open set iff $\mu(\bar{p})$ for some $p \in S$. Theorem 4. X is T_2 iff for each $p, q \in X$ whenever $q \in \mu(\bar{p})$, then $p = q$. Theorem 5. Let Z be a theorem characterizing a topological concept for a space X in terms of monads in $\hat{X}$ and assume that C is a nonempty subspace in X. If we substitute throughout Z the space C for X and r-monads for monads, then the new theorem holds for the subspace C. (Received June 22, 1972.)

72T-G148. JOHN G. MICHAELS, State University College of New York, Brockport, New York 14420. A generalization of a theorem of Lusin for nonseparable spaces. Preliminary report.

A classical result of N. Lusin states: if f is a continuous map defined on a complete separable metric space X such that $f^{-1}(y)$ is countable for all y in the range, then $X = H_1 \cup H_2 \cup \dots$ where each H_i is Borel and the restriction of f to each set H_i is 1-1. If k is an infinite cardinal, the collection of k-Borel sets of a space X is the smallest family of subsets of X containing the open sets and closed under complementation and intersections of k or fewer sets. A map f (between two absolute Borel metric spaces) is bimeasurable if f and f^{-1} preserve absolute Borel sets. The following is obtained: if f is a continuous, bimeasurable map defined on a complete space X of (infinite) weight k such that $f^{-1}(y)$ is σ-discrete for all points y of the range, then there exist k k-Borel sets H_λ, $\lambda \in \Lambda$, such that $X = \bigcup\{H_\lambda \mid \lambda \in \Lambda\}$ and the restriction of f to each set H_λ is 1-1. In the presence of separability, that is when $k = \aleph_0$, Lusin's theorem is obtained. (Received June 28, 1972.)

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01 History and Biography

696-01-1. VIVIAN EBERLE SPENCER, Bureau of the Census, Department of Commerce, Washington, D.C. 20233 and DOMINA EBERLE SPENCER, University of Connecticut, Storrs, Connecticut 06268. The tensor interpretation of Grassmann's Ausdehnungslehre. II: Combinatorial products.

The paper continues the study of Grassmann's Ausdehnungslehre from a modern tensor viewpoint. The formulation of products is considered in a very general fashion. The combinatorial product of Grassmann is defined and its properties are discussed. A method is given for elimination of unknowns from algebraic equations by combinatorial multiplication. (Received July 5, 1972.)

02 Logic and Foundations

696-02-1. MANUEL LERMAN, Yale University, New Haven, Connecticut 06520. Maximal sets and admissible ordinals. Preliminary report.

Several definitions of maximal α-r.e. sets were introduced by Kreisel and Sacks ["Metarecursive sets," J. Symbolic Logic 30(1965), 318-338]. A new definition is introduced which seems to be a more natural generalization of the ordinary recursion theoretic definition of a maximal r.e. set. Let α be an admissible ordinal, and let α^* denote the projectum of α. For any set X, let $\overline{X}$ denote the relative complement of X in α. Call a subset of α L-finite if it is α-finite and has order-type less than α^*. An α-r.e. set M is maximal if $\overline{M}$ is not L-finite and, for every α-r.e. set B, either $B \cap \overline{M}$ or $\overline{B} \cap \overline{M}$ is L-finite. Theorem. If α is an uncountable admissible ordinal, then there are no maximal α-r.e. sets. This theorem also holds for all the Kreisel-Sacks definitions of maximal α-r.e. sets. (Received May 5, 1972.)

696-02-2. D. H. J. de JONGH, State University of New York at Buffalo, Amherst, New York 14226 and ROHIT J. PARIKH, State University of New York at Buffalo, Amherst, New York 14226 and Boston University, Charles River Campus, Boston, Massachusetts 02215. WQO's and hierarchies of recursive functions.

It is a common practice in recursive function theory to construct a hierarchy as follows. One selects a class I of "initial" functions and a set $\{F_1, \ldots, F_n\}$ of "closure operations". If A is a class of recursive functions then $A \subseteq F_i(A)$. Let $C = \{F_1, \ldots, F_n\}^*$ be the set of all strings on $F_1, \ldots, F_n$. For $x = G_1 \ldots G_m$ in C we define A_x to be $G_1(G_2(\ldots G_m(I)\ldots))$. The classes A_x are partially ordered by inclusion and some suitable family of these A_x is our hierarchy. Hierarchies of Grzegorczyk, Cleave, Parsons, etc., fit in this pattern. What seems to have escaped notice is that this method will invariably give a hierarchy. Definition. For $x, y \in C$, $x \leq y$ if x can be embedded in y preserving order; e.g., $F_1F_2 \leq F_1F_3F_2$ but $F_1F_2 \nleq F_2F_1$. Remark. If $x \leq y$ then $A_x \subseteq A_y$. Theorem. Let $\leq^+$ be a linear ordering extending $\leq$. Then $\leq^+$ is a well ordering of ordinal $\alpha < \omega^{\omega^\omega}$. (Thus $\leq$ is a well quasi-ordering.) Corollary. Let $D \subseteq C$ such that $H = \{A_x : x \in D\}$ is linearly ordered. Then H is well ordered of ordinal $\alpha < \omega^{\omega^\omega}$. (Received June 14, 1972.)

*696-02-3. ROBERT G. PHILLIPS, University of South Carolina, Columbia, South Carolina 29208. Minimal and conservative extensions of arithmetic. Preliminary report.

Let P denote Peano's axioms with induction formulated in a countable first-order language L. We have shown that each model of P has a proper elementary extension which conserves the arithmetical relations expressible in L. Gaifman has shown that each model of P has a minimal proper elementary extension. By an alternation of the two constructions used for the aforementioned results we show that each model of P has a proper elementary extension which is both minimal and conservative. (Received June 29, 1972.)

696-02-4. ROBERT W. ROBINSON, University of California, Berkeley, California 94720. Degrees below $\mathbf{0}'$.

Interpolation and join properties are studied. Theorem 1. If $\mathbf{a}$, $\mathbf{b}$ are $\neq \mathbf{0}$ degrees $\leq \mathbf{0}'$ then there is $\mathbf{c}$ with $\mathbf{a} \cup \mathbf{c} = \mathbf{0}'$ and $\mathbf{b} \nleq \mathbf{c}$. Theorem 1 generalizes in some respects the unpublished theorem of C. E. M. Yates that if $\mathbf{a}$, $\mathbf{d}$ are r.e. and $\mathbf{0} < \mathbf{a} \leq \mathbf{d}$ then $\mathbf{a} \cup \mathbf{c} = \mathbf{d}$ for some $\mathbf{c} < \mathbf{d}$. This suggests replacing $\mathbf{0}'$ by an arbitrary r.e. degree $\geq \mathbf{a}$ in Theorem 1, but that remains an open problem. The proof of Theorem 1 uses a lemma of S. B. Cooper which sometimes helps in replacing a $\neq \mathbf{0}$ r.e. degree by a $\neq \mathbf{0}$ degree below $\mathbf{0}'$. Improvements of Theorem 1 obtained by modifying the basic construction include replacing $\mathbf{b}$ by $\{\mathbf{b}_i\}_{i \in N}$ where $\mathbf{b}_i \neq 0$ for all i and $\{\mathbf{b}_i\}_{i \in N} \leq \mathbf{0}'$, replacing $\mathbf{a}$ by a finite independent set of degrees below $\mathbf{0}'$, and getting $\mathbf{c}' = \mathbf{0}'$. Various interpolation theorems known for the r.e. degrees can be modified to apply to degrees $\leq \mathbf{0}'$. For example Theorem 2. If $\mathbf{a} \leq \mathbf{b}$, $\mathbf{b}$ r.e., $\mathbf{f} \nleq \mathbf{a}$, $\mathbf{a}' \leq \mathbf{u}$, and $\mathbf{u}$ is r.e. in $\mathbf{b}$ then $\mathbf{a} \leq \mathbf{c} \leq \mathbf{b}$, $\mathbf{f} \nleq \mathbf{c}$, and $\mathbf{c}' = \mathbf{u}$ for some $\mathbf{c}$. With the help of Theorem 2 it is not hard to see that in Theorem 1 one can require $\mathbf{c}' = \mathbf{u}$ for any complete degree $\mathbf{u}$ which is r.e. in $\mathbf{0}'$. Like Theorem 1, Theorem 2 is capable of numerous improvements by variations on the basic construction. (Received July 5, 1972.)

696-02-5. ROBERT I. SOARE, University of Illinois at Chicago Circle, Chicago, Illinois 60680. Automorphisms of the lattice of recursively enumerable sets. II: Complete sets.

Post's program (as opposed to Post's problem) was to find a simple property P of complements of recursively enumerable (r.e.) sets which guarantees that a set satisfying P has degree strictly between $\mathbf{0}$ and $\mathbf{0}'$. Although Post's problem was solved by Friedberg and Muchnik, his program remains open as has recently been pointed out by G. E. Sacks in "Degrees of unsolvability," and by C. E. M. Yates in his review (J. Symbolic Logic, March 1971) of Hartley Rogers' book, "Theory of recursive functions and effective computability." We give a partial answer to the question by proving that no lattice-invariant property guarantees incompleteness. Theorem. Given any nonrecursive r.e. set A there is an automorphism Φ of the lattice of r.e. sets $\mathcal{E}$ such that $\Phi(A)$ has degree $\mathbf{0}'$. Corollary (Yates). There is a complete maximal set. (In our earlier "Automorphisms of the lattice of recursively enumerable sets. I: Maximal sets," Abstract 72T-E50, these *Notices* 19(1972), A-458, we should, of course, have specified in Corollary 1 that the maximal sets $\{A_i\}_{i \leq k}$ and $\{B_i\}_{i \leq k}$ have pairwise disjoint complements, since we stated the result for automorphisms of $\mathcal{E}$ rather than of $\mathcal{E}/\mathcal{F}$.) (Received July 5, 1972.)

*696-02-6. WILLIAM P. HANF, University of Hawaii, Honolulu, Hawaii 96814. Nonrecursive tilings of the plane. Preliminary report.

A finite set of unit squares with colored edges is said to tile the plane if there exists an arrangement of translated (but not rotated) copies of the squares which fill the plane in such a way that abutting edges of the squares have the same color. The problem of whether there exists a finite set of tiles which can be used to tile the plane but not in any periodic fashion was proposed by Hao Wang and solved by Robert Berger. Economical solutions to this and related problems are described in Raphael Robinson's paper in Invent. Math. 12(1971), 177-209. Dale Myers asked whether there exists a finite set of tiles which can tile the plane but not in any recursive fashion. If we make an additional restriction (called the origin constraint) that a given tile must be used at least once, then the answer is positive. The construction uses the method of §4 of Robinson's paper together with a Turing machine that will halt unless the set represented by a sequence of zeros and ones on the initial tape separates a given pair of recursively inseparable, recursively enumerable sets. (Received July 5, 1972.)

696-02-7. ANIL NERODE, Cornell University, Ithaca, New York 14850 and JOHN CROSSLEY, Monash University, Melbourne, Australia. Combinatorial functions on algebraic structures and Dedekind types. Preliminary report.

Some existence and uniqueness theorems in the authors' theory of combinatorial functors and functions are given. The detailed presentation will appear in the authors' forthcoming monograph. (Received July 5, 1972.)

*696-02-8. WILLIAM C. POWELL, State University of New York at Buffalo, Amherst, New York 14226. Almost huge cardinals and Vopenka's principle.

Call a cardinal α almost huge if there exists an elementary embedding $j: V_\beta \to M$ such that α is the first ordinal moved, $j(\alpha) = \beta$ and for $\gamma < \beta$, ${}^\gamma M \subseteq M$. (We implicitly assume that M is some transitive class with the standard membership relation understood.) We can show α is almost huge iff there exists an elementary embedding $j: V \to M$ such that α is the first ordinal moved and, for all $\gamma < j(\alpha)$, ${}^\gamma M \subseteq M$. K. Kunen defined α to be huge if there exists an elementary embedding $j: V \to M$ such that α is the first ordinal moved and ${}^{j(\alpha)}M \subseteq M$. Clearly every huge cardinal α is almost huge and bigger than α almost huge cardinals. Vopenka's principle asserts that every proper class of relational structures of the same type has two structures, one of which can be elementarily embedded into the other. Theorem. If α is almost huge, then $V_{\alpha+1}$ satisfies Vopenka's principle. (Received July 5, 1972.)

*696-02-9. EUGENE M. KLEINBERG, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. On large ordinals in recursion theory.

There exist relatively simple questions in recursion theory whose solutions seem to depend upon the existence of large ordinals from set theory. As an example we shall consider a collection A of Π^1_1 sets of sets of natural numbers with the following properties: (1) Each set in A is nonempty. (2) Under the assumption of a large cardinal axiom (such as the existence of a measurable cardinal) each set in A contains a member which is retraceable. (3) Without the use of enough set theory to at least prove the existence of uncountably many infinite cardinals each set in A cannot be shown to contain a member which is retraceable. (Received July 5, 1972.)

*696-02-10. HARVEY FRIEDMAN, Stanford University, Stanford, California 94305. Borel sets and hyperdegrees. Preliminary report.

We present results about the hyperdegrees of elements of Borel subsets of ω^ω. The results are most easily stated for parameterless classes, the relativizations being straightforward. Gandy has shown that every nonempty parameterless analytic set contains an element of strictly lower hyperdegree than Kleene's 0, and Feferman has shown that for some uncountable parameterless projective sets, any two elements are of incomparable hyperdegree. We show that a much wider basis theorem than Gandy's is true for parameterless Borel sets, at the same time that Feferman's result is false for parameterless Borel sets. Specifically, we show that every uncountable parameterless Borel set contains an element of any hyperdegree at least as great as Kleene's

0. Virtually all the work is involved in showing this true for closed sets; that is, sets of infinite paths through recursive trees. (Received July 5, 1972.)

04 Set Theory

*696-04-1. STEPHEN H. HECHLER, Department of Mathematics and Statistics, Case Western Reserve University, Cleveland, Ohio 44106. On the structure of open sets in $\beta N - N$.

We classify the open sets in $\beta N - N$ in terms of their construction from clopen sets. Thus the degree of an open set U is defined to be the smallest cardinal m such that U can be expressed as a union of m clopen sets. Similarly, U is called scattered iff it is a disjoint union of clopen sets and monic if it is an increasing union. It is easily seen that Theorem 1. An open set is both scattered and monic iff it is of degree at most $\aleph_0$. It would be useful to know which sets are regular, but we have Theorem 2. Martin's axiom implies that all open sets of degree less than $2^{\aleph_0}$ are regular; however it is consistent with ZFC + $2^{\aleph_0} \neq \aleph_1$ that there exist open sets of degree $\aleph_1$ which are not regular. The simplest nonclopen open set is the set of degree $\aleph_0$, and it is easily seen that this set is unique up to permutation of N. Thus we should be able to say something about its exterior which we denote by E. (The exterior of a set is the interior of its complement.) However, Theorem 3. It is consistent with ZFC that the degree of E be any cardinal m such that $\aleph_0 < \mathrm{cf}(m) \leq m \leq 2^{\aleph_0}$, and for each regular such cardinal greater than $\aleph_1$ it is consistent both that E be monic and that E not be monic. We also use these criteria to establish various necessary conditions for two disjoint open sets to have nondisjoint closures. (Received July 3, 1972.)

05 Combinatorics

696-05-1. STEPHEN B. MAURER, Princeton University, Princeton, New Jersey 08540. Matroid basis graphs. I. Preliminary report.

A matroid is a finite set whose subsets satisfy certain abstract properties of linear independence. The basis graph of a matroid has a vertex for each basis (maximal independent subset) and an edge for each pair of bases which differ by the exchange of two elements. A common neighbor subgraph (CNS) is any induced subgraph on two vertices distance 2 apart and all vertices adjacent to both. A leveling from vertex v is a partition of the vertices of a graph according to their distance from v. Theorem. G is a basis graph iff (1) it is connected; (2) each CNS is a square, square pyramid or octahedron; (3) in every leveling each CNS lies in or across levels in one of a few prescribed ways; and (4) in some leveling, level 1 is the line graph of a bipartite graph. This result can be strengthened. Also, another characterization in terms of embeddings in certain "complete" basis graphs is given. (Received May 25, 1972.)

696-05-2. DENNIS P. GELLER, School of Advanced Technology, State University of New York, Binghamton, New York 13901. Classes of mappings between digraphs.

A mapping $\varphi : D \to D'$ between digraphs is admissible, if, given that $\varphi(u) = \varphi(v)$ and that uw is an arc of D, then there is a point $\overline{w}$ in D such that $\varphi(\overline{w}) = \varphi(w)$ and $v\overline{w}$ is an arc of D. Results relating the admissibility of a composition $\varphi = \varphi_1\varphi_2$ of mappings with the admissibility of the components φ_1 and φ_2 are discussed. (Received June 16, 1972.)

*696-05-3. JAMES W. UEBELACKER, Syracuse University, Syracuse, New York 13210. Product graphs for given subgroups of the wreath product of two groups.

For any nontrivial finite groups F and G and any proper normal subgroup K of G, if F wr G denotes the wreath product of G by F, then $(F \times G) \cdot (1 \text{ wr } K)$ is a subgroup of F wr G containing $F \times G$ (embedded in F wr G). It is shown that there are graphs W and X with automorphism groups F and G, respectively, such that there is a graph "product" of W and X whose automorphism group is $(F \times G) \cdot (1 \text{ wr } K)$. When F is the symmetric group on two letters, S_2, it is shown that for all subgroups L of S_2 wr G containing $1 \times G$ (embedded in S_2 wr G), there is a graph X with automorphism group G, such that there is a graph "product" of K_2 (the complete 2-graph) and X whose automorphism group is L. A concept of graph product which is a generalization of Izbicki's definition (see W. Imrich, "Associative Produkt von Graphen," Österreich. Akad. Wiss. Math.-Natur. Kl. S.-B. II (to appear)) is employed. (Received June 23, 1972.)

696-05-4. HERBERT S. WILF, University of Pennsylvania, Philadelphia, Pennsylvania 19104. Bounds for the chromatic number.

A review of known bounds for the chromatic number of a graph will first be given. These include bounds which involve maximum valence, largest eigenvalue, minimum valence in a subgraph, edge connectivities of subgraphs, path lengths and so forth. The results have been found by Brooks, Szekeres, Folkman, Wilf, and others, with refinements by H. Sachs as well as new proofs by several authors. Following this review, new work of M. Albertson and Wilf will be discussed. We consider the list of all possible colorings of the boundary of a planar graph G which can be extended to colorings of the whole graph. A variety of theorems will be proved about the size and other properties of this list of extendable colorings and some unsolved problems will be mentioned. Bounds for the chromatic polynomial will be deduced as a corollary. (Received June 23, 1972.)

*696-05-5. CHARLES C. LINDNER, Auburn University, Auburn, Alabama 36830. Construction of nonisomorphic reverse Steiner quasigroups.

A Steiner quasigroup is a quasigroup satisfying the identities $x(xy) = y$, $(yx)x = y$, and $x^2 = x$. It is well known that the spectrum for Steiner quasigroups is the set of all positive integers $n \equiv 1$ or 3 (mod 6). A Steiner quasigroup is said to be reverse provided that its automorphism group contains an involution fixing exactly one element. Such an automorphism is called a reverse automorphism. Recently, A. Rosa ("On reverse Steiner triple systems," Discrete Math. 1(1972), 61-71) has shown that a necessary condition for the existence of a reverse Steiner quasigroup of order n is $n \equiv 1, 3, 9$, or 19 (mod 24). In this same paper Rosa gives a construction for reverse Steiner quasigroups for all $n \equiv 1, 3$, or 9 (mod 24), except possibly $n = 25$. Subsequently Jean Doyen ("A note on reverse Steiner triple systems," Discrete Math. 1(1972), 315-319) constructed a reverse Steiner quasigroup of order 25, for every $n = 24(4k-1) + 19$ where k is any positive integer, and for every sufficiently large $n \equiv 19$ (mod 24). This paper gives a different construction for reverse Steiner quasigroups which is then used to construct large numbers of nonisomorphic reverse Steiner quasigroups of a given order; for example, 10^{500} nonisomorphic reverse Steiner quasigroups of order 171. (Received July 3, 1972.)

*696-05-6. THOMAS H. BRYLAWSKI, University of North Carolina, Chapel Hill, North Carolina 27514. Modular flats and the critical problem of combinatorial geometries.

R. Stanley made an investigation of modular flats in geometries (Algebra Universalis, 1-2 (1971), 214-217) and proved that such flats produced a factorization of $\chi(G)$, the characteristic polynomial of a geometry

G. We characterize modular flats by extensive lattice properties as well as by strong versions of the circuit elimination axiom and the MacLane-Steinitz exchange axiom. Modular flats are shown to have many of the useful properties of distributive flats (separators) in addition to being much more prevalent. Theorem. Given geometries G and H and x a modular flat of G as well as a subgeometry of H, a geometry $P = P_x(G, H)$ is constructed which is a pushout in the category of injective strong maps and $\chi(P) = \chi(G)\chi(H)/\chi(x)$. The closed set structure, rank function, and independent sets of P are given and applications to chain groups, unimodular geometries, transversal geometries and especially graphs are given showing that this theorem is the generalization to geometries of the theorem relating the chromatic polynomials of two graphs and their vertex join across a common clique. (Received July 3, 1972.)

696-05-7. BRUCE H. BARNES and KAREN E. MACKEY, Computer Science Department, Pennsylvania State University, University Park, Pennsylvania 16802. A generalized measure of independence and its relation to a cartesian product of graphs.

Let $1, 2, \ldots, n$ denote the vertices of an undirected finite graph G and let $\{C_j \mid j = 1, \ldots, m\}$ be the set of all cliques of G. To obtain the generalized measure of independence $\beta_k(G)$, assign a nonnegative weight x_i to each vertex i such that for all j: (1) no more than k vertices are weighted nonzero in C_j, and (2) $\sum_{i \in C_j} x_i \leqq 1$. The kth independence number $\beta_k(G)$ is then defined to be the maximum $\sum_{i \in G} x_i$ for all such weightings $\{x_i \mid i = 1, \ldots, n\}$. $\beta_1(G)$ is identical to the independence number of G. It is shown that $\beta_k(G \times H)$, where the product $\times$ is defined in the sense of Berge, "The theory of graphs," p. 38, can be bounded above by $N\beta_N(H)$ with N equal to $\beta_k(G)$. Rosenfeld in Proc. Amer. Math Soc. 18(1967), 315-319, showed that given G, $\beta_1(G \times H) = \beta_1(G)\beta_1(H)$ for all H if and only if $\beta_1(G) = \max_k\{\beta_k(G)\}$. The sufficiency of this condition can be derived from the above bound. Also a bound can be obtained for the capacity θ of G, as defined by Shannon in (Institute of Radio Engineers, Transactions on Information Theory IT-2(1956), 8-19) where $\theta(G)$ is equal to $\lim_{n \to \infty} \sqrt[n]{\beta_1(G^n)}$. (Received July 3, 1972.)

*696-05-8. JOSEPH ARKIN, 197 Old Nyack Turnpike, Spring Valley, New York 10977 and VERNER E. HOGGATT, Jr., San Jose State College, San Jose, California 95114. The Arkin-Hoggatt game and the solution of a classical problem.

In this paper Arkin and Hoggatt proved for the first time in the history of the orthogonal 10×10 square problem that diagonals in a 10×10 orthogonal square can be found such that each has a sum equal to any row or column sum. The solution was found without the use of any computing machine. In fact the new method they introduce requires only the use of a few pieces of paper. Of the many solutions found by the authors we list the diagonal which was used in the proof that a 10×10 can be orthogonally magic in the diagonals also: 63 11 28 05 37 51 98 58 64 80. (Received July 3, 1972.)

*696-05-9. R. STANTON HALES, Pomona College, Claremont, California 91711. Numerical invariants and graph products.

This paper completes work initially reported in Abstract 658-11, these *Notices* 15(1968), 722, and Abstract 69T-H41, these *Notices* 16(1969), 855, and suggests the following problem. Let $\otimes$ be a binary graph product and φ a numerical graph invariant. Then φ, with respect to $\otimes$, is: (I) natural if always $\varphi(G \otimes H) = \varphi(G)\varphi(H)$; (II) weak if not natural and always $\varphi(G \otimes H) \leqq \varphi(G)\varphi(H)$; and (III) strong if not natural and always $\varphi(G \otimes H) \geqq \varphi(G)\varphi(H)$. For given $\otimes$, determine its natural, weak, and strong invariants, their relationships, and conditions of the various inequalities. Example. For φ = gamma product, the invariants β = independence number, ρ = Rosenfeld number, and σ = clique-covering number are weak, natural, and strong, respectively. Some conditions for inequalities are established. Other known examples are listed and first results of further classification are reported. (Received July 5, 1972.)

696-05-10. DONALD J. McCARTHY, St. John's University, Jamaica, New York 11432 and LOUIS V. QUINTAS, Pace College, New York, New York 10038. A stability theorem for minimum edge graphs with given abstract group.

Throughout, G denotes a finite group; S_m, D_m, C_m denote the symmetric, dihedral and cyclic groups respectively. All graphs are finite and undirected, without loops or multiple edges. A graph is called a G-graph if its automorphism group is (abstractly) isomorphic to G. For arbitrary G, there exist G-graphs on n vertices whenever n is sufficiently large. The following problem has received some attention: Given G, for each positive integer n decide if there exists a G-graph on n vertices, and if so determine the minimum number e(G, n) of edges possible. To date, this minimum edge problem has been completely solved only for a limited class of groups; namely, for S_m and D_m (for $m \geqq 1$) and for C_3. In all cases there exists a graph such that for n sufficiently large e(G, n) is attained by adding to this fixed graph M a standard asymmetric forest. Such a result holds in general. More precisely, suppose the direct product decomposition of G involves precisely r nontrivial symmetric groups as direct factors; a G-graph which has at most r nonisomorphic asymmetric components is called semireduced. Among those semireduced G-graphs for which the cycle rank minus the number of components is minimized, select M as a graph having the smallest number of vertices. (Received July 5, 1972.)

06 Order, Lattices, Ordered Algebraic Structures

696-06-1. JOHN A. WRIGHT, University of Prince Edward Island, Charlottetown, Prince Edward Island, Canada. Cycle indices of certain classes of types of quasiorders or topologies. Preliminary report.

Let K be any class of isomorphism types of quasiorders on $x_1, \ldots, x_n$. Define a polynomial $Z(K)(s_1, s_2, \ldots)$ as the sum of the cycle indices of automorphism groups of arbitrary members of the types in K. It is shown that $k = |K|$ is the sum of the coefficients and $k' = |\bigcup K|$ is n! times the coefficient of s_1^n. Let Q_n be the class of types of n-point quasiorders (equivalently, topologies). Let P_n be the types of partial orders (T_0 topologies). Let QC_n and C_n be the types of quasiorders (respectively, partial orders) connected by comparability; equivalently, connected topologies. We define three other classes $\subset P_n$, of which one called S_n is contained in the others, and show how the polynomials for the rest of the classes mentioned can be derived from those of S_m, $m \leqq n$. We construct S_m for $m \leqq 7$, derive the polynomials, and obtain among other results the following: For n = 6: c = 238, qc = 512, p = 318, q = 718; c' = 101642, qc' = 158175, p' = 130023, q' = 209527. For n = 7, the corresponding values are 1650, 3485, 2045, 4535; 5106612, 7724333, 6129859, 9535241. Evans, Harary and Lynn (Comm. ACM (1967)) obtained the same values of p' and q' by computer construction. (Received February 22, 1972.)

*696-06-2. JORGE MARTINEZ, University of Florida, Gainesville, Florida 32601. Archimedean-like classes of lattice-ordered groups.

Suppose $\mathcal{C}$ denotes a class of totally ordered groups closed under taking subgroups and quotients by 0-homomorphisms. We study the following classes: Res($\mathcal{C}$), all ℓ-groups which are subdirect products of groups in $\mathcal{C}$; Hyp($\mathcal{C}$), all ℓ-groups in Res($\mathcal{C}$) having all their ℓ-homomorphic images in Res($\mathcal{C}$); Para($\mathcal{C}$), all

ℓ-groups having their principal convex ℓ-subgroups in Res($\mathcal{C}$). If $\mathcal{C}$ is the class of archimedean totally ordered groups, Res($\mathcal{C}$) is the class of all subdirect products of reals, Para($\mathcal{C}$) the class of all archimedean ℓ-groups, and Hyp($\mathcal{C}$) the class of all hyper-archimedean ℓ-groups. Under a very mild condition on such a class $\mathcal{C}$, we show that any representable ℓ-group has a unique largest convex ℓ-subgroup in Hyp($\mathcal{C}$); it is a characteristic subgroup. We consider several examples of this "hyper-$\mathcal{C}$-kernel," and investigate some of its properties. For any class $\mathcal{C}$ as above we show that the free lattice-ordered group on a set X in the variety generated by $\mathcal{C}$ is always in Res($\mathcal{C}$). We also prove that Res($\mathcal{C}$) has free products. (Received April 27, 1972.)

696-06-3. HARIHARAIER SUBRAMANIAN, State University of New York at Buffalo, Amherst, New York 14226. Prime ideals in the lattice structure of an f-ring. Preliminary report.

Let R be an ℓ-semisimple commutative f-ring with unity. Theorem. For any $f \in R$ and any prime ideal P in the lattice structure of R, either $P \subseteq f + P$ or $f + P \subseteq P$, where $f + P$ denotes the translate $\{f + g \in R | g \in P\}$ of P by f. (Received June 23, 1972.)

08 General Mathematical Systems

*696-08-1. W. F. PAGE and ALTON T. BUTSON, University of Miami, Coral Gables, Florida 33124. The lattice $\mathcal{L}_m$ of equational classes of m-semigroups.

An m-semigroup is an algebra $\langle A ; f \rangle$ with one m-ary operation f satisfying $[(x_1 x_2 \ldots x_m) f x_{m+1} \ldots x_{2m-1}]f = [x_1 x_2 \ldots x_i (x_{i+1} \ldots x_{m+i+1}) f x_{m+i+2} \ldots x_{2m-1}] f$ for all $x_1, x_2, \ldots, x_{2m-1}$ in A and $i = 1, 2, \ldots, m-1$. It is called commutative if, furthermore, $(x_1 x_2 \ldots x_m) f = (x_{\sigma(1)} x_{\sigma(2)} \ldots x_{\sigma(m)}) f$ for every permutation σ of $\{1, 2, \ldots, m\}$ and for all $x_1, x_2, \ldots, x_m$ in A. It is idempotent if $(xx\ldots x) f = x$ for all x in A. In this paper the study of $\mathcal{L}_m$ for $m > 2$ is begun. It is shown that the sublattice $\mathcal{C}_m$ of equational classes of commutative m-semigroups has the finite basis property, is countable, satisfies no special lattice laws, and has a large distributive sublattice. All the atoms of $\mathcal{C}_m$ are determined. The lattice $\mathcal{L}_m$ is shown to have uncountably many elements. All the atoms of $\mathcal{L}_3$ are determined. For $m > 3$, those atoms of $\mathcal{L}_m$ which are nonidempotent classes of m-semigroups are also found. Consequently, the only atoms yet to be determined are those classes which are idempotent and noncommutative. (Received June 30, 1972.)

696-08-2. GILBERT F. ORR and ALTON T. BUTSON, University of Miami, Coral Gables, Florida 33124. The lattice of varieties of semirings. Preliminary report.

A semiring is an algebra $\langle S, +, \cdot \rangle$ such that: (1) $(S, +)$ is a semigroup, (2) $(S, \cdot)$ is a semigroup, and (3) $a(b+c) = ab + ac$; $(a+b)c = ac + bc$ for all a, b, c in S. Those semiring varieties which are atoms in the lattice of varieties of semirings are determined. It is shown the lattice of varieties of semirings contains a sublattice, lattice isomorphic to the lattice of varieties of semigroups. (Received June 30, 1972.)

10 Number Theory

*696-10-1. EMANUEL VEGH, Mathematics Research Center (Code 7840), Naval Research Laboratory, Washington, D. C. 20390. A combinatorial method in number theory. II.

Using combinatorial methods the following results are shown: Theorem 1. Let s be a natural number and n a natural number with prime decomposition $p_1^{\alpha_1} p_2^{\alpha_2} \ldots p_t^{\alpha_t}$. There is a natural number N depending only on s such that if $\alpha_i > \log_{p_i} N$ $(i = 1, 2, \ldots, t)$ then there is an arithmetic progression of integers (incongruent modulo n), of length s, each of which is a λ-primitive root of n. This result generalizes (and improves) the results stated in Abstract 653-176, these *Notices* 15(1968), 130. For the definition of λ-primitive roots see, for example, LeVeque ("Topics in number theory", vol. 1, Reading, Mass., 1956). Theorem 2. If $p = 2^{\alpha}q^{\beta} + 1 > 7$ and q are primes, then there is at least one pair of consecutive primitive roots modulo p. This result is not covered by the theorems given by the author in ("A note on the distribution of the primitive roots of a prime", J. Number Theory 3(1971), 13-18). (Received June 1, 1972.)

696-10-2. GREGORY WULCZYN, Bucknell University, Lewisburg, Pennsylvania 17837. The intersection of polygonal numbers of different rank.

There are an infinite number of polygonal numbers $\overset{u}{P}_n = (n/2)\,[(u-2)n-(u-4)]$ identical to $\overset{v}{P}_m = (m/2)\,[(v-2)m-(v-4)]$ unless: (1) $u = 2r+1$, $v = 2s+1$, $(2r-1)(2s-1)$ is an integer square; (2) $u = 2r$, $v = 2s$, $(r-1)(s-1)$ is an integer square; (3) $u = 2r+1$, $v = 2s$, $2(2r-1)(s-1)$ is an integer square. (Received May 30, 1972.)

*696-10-3. GERALD A. BOTTORFF, Pennsylvania State University, Mont Alto, Pennsylvania 17237. Proof of the "Littlewood Conjecture" for infinitely many pairs.

The "Littlewood Conjecture" is the following: Given two irrational numbers u_1, u_2, there exists an infinite sequence of triples of integers p, q, r, which make $|q|\,|u_1q-p|\,|u_2q-r|$ arbitrarily small. Call u_1, u_2 a P. V. J. P. pair if the Jacobi-Perron algorithm is purely periodic, positive, convergent and the characteristic equation has one real root which is a P. V. number. The author proves the following result. Theorem. If u_1, u_2 is P. V. J. P., then there exists an infinite sequence of triples p, q, r, satisfying: $|q|\,|u_1q-p|\,|u_2q-r| << \ln^{-1}(q)$. There are infinitely many P. V. J. P. pairs. An example is given by the pair $x, x^2 - x$ where x is the real root of $x^3 - x^2 - x - 1 = 0$. The algorithm is purely periodic length one. (Received June 15, 1972.)

*696-10-4. JAMES S. BYRNES, University of Massachusetts, Boston, Massachusetts 02116. On a partition problem of Frobenius.

We consider the following problem, which was raised by Frobenius: Given n relatively prime positive integers $a_1, a_2, \ldots, a_n$, what is the largest integer $M(a_1, a_2, \ldots, a_n)$ omitted by the linear form $\sum_{i=1}^{n} a_i x_i$, where x_i are variable nonnegative integers? Sylvester ["Mathematical questions" Educational Times 41(1884), 21] showed that $M(a_1, a_2) = a_1a_2 - (a_1 + a_2)$, but the problem has remained essentially unsolved for n = 3. We solve the following special case for n = 3: Theorem. Let $1 < a_1 < a_2 < a_3$, g.c.d. $(a_1, a_2, a_3) = 1$, and $a_2 \equiv 1 \pmod{a_1}$. Then $M(a_1, a_2, a_3) = a_1a_2 - (a_1 + a_2)$ if $a_3 \geqq ja_2$; $M(a_1, a_2, a_3) = (a_1 - m - j)a_3/j + (m-1)a_2 - a_1$ if $(j-m)a_2 < a_3 < ja_2$; $M(a_1, a_2, a_3) = (a_1 - m - j)a_3/j + (j-1)a_2 - a_1$ if $a_3 < (j-m)a_2$. Here, j and m are defined by: $a_3 \equiv j \pmod{a_1}$, $0 \leqq j\ a_1$, and (if $j \neq 0$) $a_1 \equiv m(j)$, $1 \leqq m \leqq j$. (Received June 23, 1972.)

*696-10-5. CARL POMERANCE, Harvard University, Cambridge, Massachusetts 02138. Odd perfect numbers are divisible by at least 7 distinct primes. Preliminary report.

No proof that odd perfect numbers are nonexistent is known, but it is known that no odd number divisible by less than 6 distinct primes is perfect. In the present paper the result is extended to show that odd numbers divisible by exactly 6 distinct primes are not perfect. The following lemma, due to Birkhoff and Vandiver, was invaluable (here F_n denotes the nth cyclotomic polynomial): If n and a are integers $\geqq 3$, then $F_n(a)$ is divisible by a prime which does not divide any $F_m(a)$ for all $m < n$. A useful lemma proved in the paper is that

if p is a prime $\geqq 7$, then $F_p(3)$ is divisible either by a prime $q > 1000$ with $q \not\equiv 17$ (36) or by two primes $r > s > 1000$ with $r \equiv s \equiv 17$ (36). A similar lemma is proved about $F_p(5)$ where "17 (36)" is replaced by "49 (100)". The proofs involve some calculations and make use of quadratic reciprocity. It is known that if r is an odd perfect number, then among the primes dividing r, exactly one prime p has odd exponent c. It is proved here that if $5^b | r$, $5^{b+1} \nmid r$, and $5^b \nmid \sigma(p^c)$, then r is divisible by two primes $\equiv 1$ (5) one of which is $\geqq 1381$. The proof of this was accomplished by constructing a table of the prime factors of $F_5(p)$ for p a prime $\equiv 1$ (5) and < 1381.
(Received June 23, 1972.)

696-10-6. SYED ASADULLA, St. Francis Xavier University, Antigonish, Nova Scotia, Canada. <u>A note on Fermat numbers</u>.

The nth Fermat number is defined as $F_n = 2^{2^n} + 1$. <u>Theorem</u> 1. The digital root of F_n is 5 or 8 according as n is odd or even. <u>Corollary</u>. The digital root of $F_0 F_1 \ldots F_{n-1}$ is 3 or 6 according as n is odd or even. <u>Theorem</u> 2. (i) $\frac{1}{2}(F_{n+1} - F_n) = \Delta(F_n - 2)$, an even triangular number. (ii) $F_n - 2$ is a triangular number iff $8F_n - 15$ is a perfect square. (iii) $\Delta(F_n - 2)$ is a perfect number iff $n = 1$. <u>Theorem</u> 3. (i) $\frac{1}{2}(F_{n+k} - F_n) = \sum_{i=n}^{n+k-1} \Delta(F_i - 2)$, the sum of k even triangular numbers. (ii) $\Delta(F_n - 2) = 2\Delta(F_{n-1} - 2)\Delta(F_{n-1} - 1)$. (iii) $\Delta(F_{n+k} - 2) = 2^k \Delta(F_n - 2) \Pi_{i=n}^{n+k-1} \Delta(F_i - 1)$. (iv) $\Delta(F_n - 2)$ divides $\frac{1}{2}(F_{n+k} - F_n)$. (Received June 22, 1972.)

696-10-7. MOHAMMAD ISHAQ, Laval University, Québec 10, Québec, Canada. <u>On homomorphic relations and weak homomorphisms of algebras</u>.

Let $[A, \Omega]$ and $[B, \Lambda]$ be two algebras, and (ρ, σ) a homomorphic relation (see M. Ishaq, "On homomorphic relations in algebras", Abstract 691-16-25, these *Notices* 19(1972), A-78) from $[A, \Omega]$ to $[B, \Lambda]$. Let θ be a mapping from A into B and ρ_θ a relation from Ω to Λ defined as follows: $(\omega, \lambda) \in \rho_\theta$ iff $\omega \circ \theta = \theta^n \circ \lambda$, $\omega \in \Omega, \lambda \in \Lambda$, where ω is of rank n, and θ^n denotes the functorial extension of cartesian products to mappings. The mapping θ is called a <u>weak homomorphism</u> of $[A, \Omega]$ into $[B, \Lambda]$ if for each $\omega \in \Omega$, there exists a $\lambda \in \Lambda$ such that $(\omega, \lambda) \in \rho_\theta$, and if for each $\lambda' \in \Lambda$, there exists an $\omega' \in \Omega$ such that $(\omega', \lambda') \in \rho_\theta$. A number of results involving a homomorphic relation, a weak homomorphism and a B-congruence have been obtained.
(Received June 15, 1972.)

696-10-8. J. SUTHERLAND FRAME, Michigan State University, East Lansing, Michigan 48823. <u>The projective characters of the Rudvalis simple group</u>. Preliminary report.

On April 23, 1972, Arunas Rudvalis conjectured the existence of a new simple group R of order $2^{14}3^3 5^3 \cdot 7 \cdot 13 \cdot 29$, having a rank 3 permutation representation of degree 4060, with subdegrees 1, 1755, 2304, on the cosets of the subgroup $F = {}^2F_4(2)$. Starting with the characters of the simple Tits group F' of index 2 in F, we constructed the character tables of F and then of R. Characters of R induced from linear characters of F split as $1_i + 783_i + 3276_i$ and $406_i + 3654_i$. Noting that almost all values of $1_i + 783_i$ were squares, this author conjectured the existence of a projective character 28_i of degree 28, which should be a complex character of a covering group $\hat{R}$ with center $Z = \{I, -I\}$, such that $R \cong \hat{R}/Z$. Its even Kronecker powers provided characters of R. This conjecture was supported by completing the character table of $\hat{R}$. Projective characters of R, which are faithful for $\hat{R}$, include nine complex pairs of degrees 28, 1248, 3276, 4032, 7280, 7308, 10556, 38976, 98280, and seven symplectic characters of degrees 8192, 8192, 34944, 48256, 87696, 221184, 250560. The existence of the Rudvalis simple group was established by Conway, using the 28-dimensional representation. (Received June 29, 1972.)

*696-10-9. DENNIS A. GARBANATI, University of California, Santa Barbara, California 93106. Classes of nonsingular abelian group matrices over fields.

Let G be a finite abelian group of order n. Let F be a field. Let P be the regular representation of G so that P(g) is an n-square permutation matrix and P(gh) = P(g)P(h) for all g, h $\in$ G. A matrix of the form $\sum_{g \in G} a_g P(g)$ where $a_g \in F$ is called a group matrix for G over F. This paper establishes a theorem of Hasse-Minkowski type for abelian group matrices which reads as follows. Let F be a global field whose characteristic does not divide 2n. Let Ω denote the set of all nontrivial spots on F. Let ^{T} denote transposition. Let M and L be symmetric nonsingular group matrices for G over F. There exists a group matrix A for G over F such that $M = A^T LA$ if and only if for each $\mathfrak{p} \in \Omega$ there exists a group matrix $A_\mathfrak{p}$ for G over $F_\mathfrak{p}$ such that $M = A_\mathfrak{p}{}^T L A_\mathfrak{p}$ where $F_\mathfrak{p}$ is a completion of F at $\mathfrak{p}$. The existence of the group matrices $A_\mathfrak{p}$ can be determined by making a finite number of Hilbert symbol computations.
(Received July 3, 1972.)

*696-10-10. GEORGE E. ANDREWS, Pennsylvania State University, University Park, Pennsylvania 16802. A general theory of identities of the Rogers-Ramanujan type.

Isolated partition identities have appeared in mathematical literature since the time of Euler. Euler noted the infinite product identity $\prod_{n=1}^{\infty}(1+q^n) = \prod_{n=1}^{\infty}(1-q^{2n-1})^{-1}$, and from this he deduced that for each positive integer n the partitions of n into distinct parts are equinumerous with the partitions of n into odd parts. In 1894, L. J. Rogers proved that, for b = 1 or 2, $1 + \sum_{n=1}^{\infty} q^{n^2 + (b-1)n}/(1-q)(1-q^2)\ldots(1-q^n) = \prod_{n=0}^{\infty}(1-q^{5n+b})^{-1}(1-q^{5n+5-b})^{-1}$. These identities have become known as the Rogers-Ramanujan identities since Ramanujan rediscovered them around 1913. P. A. MacMahon observed that the Rogers-Ramanujan identities imply a partition identity: Namely, for each positive integer n the number of partitions of n in which each pair of summands differ by at least 2 and each summand is $\geqq$ b equals the number of partitions of n into parts congruent to $\pm$ b modulo 5. Prior to 1961, there existed a mere handful of partition identities, and the entire subject appeared to be an interesting but limited subtopic in the theory of basic hypergeometric series. In the last ten years, however, a large number of infinite families of partition identities have been found. This talk will begin with a description of a lattice-theoretic setting for partition identities. As well as unifying many seemingly unrelated results, the lattice-theoretic approach allows us to consider the fundamental classification questions for partition identities. The last portion of the talk will describe current methods of treating partition identities. The technique that has been used most extensively is that of q-difference equations; recently, however, certain sieve techniques have provided new families of identities which contain the Rogers-Ramanujan identities.
(Received July 5, 1972.)

12 Algebraic Number Theory, Field Theory and Polynomials

*696-12-1. ROBERT G. VAN METER, State University College of New York, Oneonta, New York 13820. Some special polynomials over a finite field. I.

The main theorem gives the number of solutions of the equation $\sum_{i=1}^{n} a_i f_i(x_{i1}, \ldots, x_{ir_i}) = a$ over a finite field K with q elements, where $f_i \in K[X_{i1}, \ldots, X_{ir_i}]$ and the number of solutions of the equation $f_i(x_{i1}, \ldots, x_{ir_i}) = a$ is a linear combination over Z of one or more of $\underline{1}$(a,), $\underline{\varkappa}$(a) and $\underline{\psi}$(a), where $\underline{1}$, $\underline{\varkappa}$ and $\underline{\psi}$ are K to Z functions defined as follows: $\underline{1}(a) = 1$ for all $a \in K$; $\underline{\varkappa}(0) = q - 1$, $\underline{\varkappa}(a) = -1$ for all $a \in K - \{0\}$; $\underline{\psi}(0) = 0$, $\underline{\psi}(a) = 1$ for all sqares $a \in K - \{0\}$ and $\underline{\psi}(a) = -1$ for all nonsquares $a \in K$. This result and corollaries are too lengthy for quotation here. (Received June 21, 1972.)

*696-12-2. ALAN McCONNELL, Howard University, Washington, D.C. 20001. Polynomial subfields of $k(x)$.

Let k be a field of characteristic 0; call L, $K \subset L \subset k(x)$, a polynomial subfield if $L = k(f)$, $f(x) \in k[x]$. (x is an indeterminate.) Let $k(f)$, $k(g)$ be two polynomial subfields of $k(x)$, with $\deg f = [k(x) : k(f)] = m$, $\deg g = [k(x) : k(g)] = n$. Theorem. $k(f) \cap k(g)$ is a polynomial subfield of $k(x)$. Furthermore, either $k(f) \cap k(g) = k$ or $[k(x) : k(f) \cap k(g)] = \mathrm{l.c.m.}(m, n)$. Corollary. If $m|n$, then either $k(g) \subset k(f)$ or $k(g) \cap k(f) = k$. The proof of the theorem uses a technique of Fried and MacRae [Illinois J. Math. 13(1969), 165-171]. One can easily derive theorems on pushouts in the category of nonsingular complete algebraic curves of genus zero, or — for $k = \mathbb{C}$ — in the category of 2-spheres and ramified covering maps. The theorem fails if char $k > 0$. (Received June 26, 1972.)

*696-12-3. NICHOLAS HEEREMA and JAMES K. DEVENEY, Florida State University, Tallahassee, Florida 32306. An inseparable Galois theory.

Let K be a field having characteristic $p \neq 0$ and let $H^t(K)$ be the group of rank t higher derivations on K. If k is the field of constants of an abelian set $S = \{d^{(1)}, \ldots, d^{(n)}\}$ of iterative higher derivations on $H^t(K)$ having first nonzero maps $\{d^{(1)}_{i_1}, \ldots, d^{(n)}_{i_n}\}$ linearly independent over K, then $K = k(x_1) \otimes \ldots \otimes k(x_n)$ where $[k(x_j) : k] = p^{n_j}$, $n_j = \min\{q+1 | p^q \cdot i_j \geqq t\}$. If d is iterative of index r (d_r is the first nonzero map) then $V(d) = h \in H^t(K)$ is given by $h_{(r+1)j} = d_{rj}$ and $h_i = 0$ if $(r+1) \nmid i$. S is normal if the subscript i_j of the first nonzero map is the least integer $\geqq t + 1/p^r$ for some $r > 0$. Let $\overline{S} = \{V^i(d) | \, i \geqq 0, d \in S\}$. Theorem. A subgroup G of $H^t(K)$ is Galois if and only if G is generated over K by a set $\overline{S}$ where S is a finite abelian independent set of normal iterative higher derivations. This theorem is related to but distinct from a similar characterization of Galois groups of higher derivations due to Gerstenhaber-Zaromp. It has the advantage of an explicit description of minimal generating sets S and simple proofs. (Received June 29, 1972.)

*696-12-4. CHARLES J. PARRY, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. Units of algebraic number fields. Preliminary report.

Suppose that k is a proper subfield of an algebraic number field K and that k and K have the same number of fundamental units. Such fields k and K are characterized and relations between the units of k and K are discussed. Generalizations and new proofs are obtained for some results occurring in a paper by Peter Dénes [Monatsh. Math. 55(1951), 161-163] and in a recent article by C. R. MacCluer and the present author. (Received July 3, 1972.)

696-12-5. MARTIN D. KRUSKAL, Program in Applied Mathematics, Princeton University, Princeton, New Jersey 08540 and ROBERT M. MIURA, Vanderbilt University, Nashville, Tennessee 37023. Antidifferentiation of differential polynomials.

A differential polynomial is a polynomial in variables $u_0, u_1, u_2, \ldots$ representing a generic function u_0 and its first, second,... (ordinary) derivatives. In the linear space of differential polynomials, those that are derivatives form a subspace. Operators that annihilate only the subspace can serve to test whether a polynomial is a derivative. Relations among such operators are discussed and many operators are exhibited explicitly, some as infinite products. Other operators corresponding to them are exhibited explicitly which, applied to a polynomial, produce its primitive if it has one. (Received June 19, 1972.)

13 Commutative Rings and Algebras

*696-13-1. EDWARD D. DAVIS, State University of New York, Albany, New York 12222. Normal pairs.

A pair of domains (A, B) is called normal if A is a subring of B with all intermediate rings normal in B; a domain A is trivial if there is no $B \neq A$ with (A, B) normal. The localization of a domain with respect to a set of prime ideals means the intersection of the localizations at the primes of the set. Theorem. The pair (A, B) is normal iff B is a localization of A with respect to a set of primes including all P with A_P trivial. Call a domain A Krull-like if the radical of each principal ideal $\neq$ A, 0 is the intersection of a finite number of height 1 primes; call such an A einbettungsfrei if its localization with respect to its set of height 1 primes is A itself. Theorem. For Krull-like A, the pair (A, B) is normal iff B is a localization of A with respect to a set of maximal ideals excluding only such M with A_M a valuation ring; the excluded set is unique for A einbettungsfrei. Corollary (conjectured by Kaplansky). For a Noetherian domain A, the set of overrings B with (A, B) normal is bijective with the set of subsets of the set of invertible maximal ideals of A; the ring corresponding to a given set of invertible maximal ideals is the localization of A with respect to the complementary set of maximal ideals. (Received June 8, 1972.)

*696-13-2. HERBERT F. KREIMER, Florida State University, Tallahassee, Florida 32306. Automorphisms of commutative algebras and separable subalgebras.

If B is a commutative ring with 1, G is a finite group of automorphisms of B, and A is the subring of G-invariant elements of B; then there exists a maximal separable A-subalgebra of B, which is necessarily unique and G-stable. The assertion is proved by use of Zorn's lemma and the following two results. (1) For any separable A-subalgebra A' of B and any prime ideal p of A, A'_p is a free A_p-module of rank not exceeding the order of G. (2) A commutative A-algebra A' is separable if and only if A'_p is a separable A_p-algebra for every prime ideal p of A. (Received June 16, 1972.)

15 Linear and Multilinear Algebra, Matrix Theory (Finite and Infinite)

696-15-1. ROBERT B. GARDNER, University of North Carolina, Chapel Hill, North Carolina 27514. Some properties of the exterior rank of modules. Preliminary report.

In his paper "On free exterior powers" (Trans. Amer. Math. Soc. 145 (1969), 357-367) H. Flanders asks, if M is a module over a commutative ring and if $\wedge^p M$ is generated by q elements does $\wedge^{p+q} M = 0$. He further remarks that it is doubtful that it is true. In this report we will establish that the implication is correct. In addition, we will generalize and simplify a result of C. Jensen, "A remark on flat and projective modules" (Canad. J. Math. 18 (1966), 943-949) by establishing the following result. Theorem. Let R be an integral domain with quotient field Q and M a finitely generated R-module; then ext rank $M = \dim_Q Q \otimes M$ implies M is projective and coherent. (Received June 15, 1972.)

*696-15-2. ABRAHAM BERMAN, Centre de recherches mathématiques, Université de Montréal, Montréal 101, Quebec, Canada and ROBERT J. PLEMMONS, University of Tennessee, Knoxville, Tennessee 37916. Cones and iterative methods for singular systems.

The concept of a regular splitting of a real nonsingular matrix, introduced by R. Varga, is used in iterative methods for solving linear systems. This concept is extended to rectangular linear systems (*) Ax = b, in two directions: firstly, replacing A^{-1} by A^+, the Moore-Penrose inverse of A and, secondly, by considering

... $= M - N$ is called proper if $R(A) = R(M)$ and ... nonsingular. Consider the iteration (**) $x^{i+1} = M^+ N x^i + M^+ b$. It is shown that for a proper splitting, $\rho(M^+N)$ (the spectral radius of M^+N) is less than 1, if and only if the iteration (**) converges to A^+b, the best least square approximate solution of the system (*). Necessary and sufficient conditions for $\rho(M^+N) < 1$ are given, extending results of Varga, Ortega and Rheinboldt, Mangasarian and Vandergraft. This approach has the advantage of avoiding the normal system $A^TAx = A^Tb$ in solving (*). Results of Collatz and Schröder on monotone iteration in the nonsingular case are also extended. (Received June 19, 1972.)

696-15-3. MARJORIE L. STEIN, Princeton University, Princeton, New Jersey 08540. A pivot theory of skew matrices with applications. Preliminary report.

Given a skew matrix A over an ordered field, consider the finite class of skew matrices $\bar{A}$ such that $XA = Y$ and $\bar{X}\bar{A} = \bar{Y}$ have the same solution sets, where $\bar{x}_i = y_i$, $\bar{y}_i = x_i$ for some i (perhaps none) and $\bar{x}_i = x_i$, $\bar{y}_i = y_i$ for the remaining i. Theorem. For each i there exists an $\bar{A}$ such that $\bar{a}_{ij} \geqq 0$ for all j. A skewsymmetrization technique permits an extension to arbitrary rectangular matrices. Other applications include new proofs for "complementary slackness" theorems, von Neumann's minimax theorem, results in linear programming theory and classical theorems on linear inequalities. Elementary determinant theory and "pivot algebra" combine to yield simple proofs for fundamental facts about skew matrices. (Received June 28, 1972.)

*696-15-4. JOHN F. DALPHIN, Indiana and Purdue Universities, Indianapolis, Indiana 46205 and Clarkson College of Technology, Potsdam, New York 13676 and VICTOR LOVASS-NAGY, Clarkson College of Technology, Potsdam, New York 13676. On the best least-squares solution of an incompatible system arising from the 9-point approximation to the Neumann problem.

Consider the Neumann problem: $\nabla^2 u = f$ in a rectangle R and $\partial u/\partial n = g$ on the boundary of R. The 9-point finite-difference approximation to this problem yields a matrix equation $Mz = h$, where $M = A \otimes I_m + (1/6)\, A \otimes B + I_n \otimes B$. Here A and B are singular tridiagonal matrices of order n and m respectively, I_m and I_n are identity matrices and $\otimes$ denotes the tensor product. Even if the original problem satisfies the compatibility conditions, the discretized problem may not. In such a case, the "best least-squares solution" $z_b = M^+h$ (where M^+ denotes the Moore-Penrose generalized inverse of M) may be useful. In this paper, a noniterative method will be developed to obtain z_b in terms of the eigenvalues and eigenvectors of A and B. (Received June 30, 1972.)

*696-15-5. VICTOR LOVASS-NAGY and DAVID L. POWERS, Clarkson College of Technology, Potsdam, New York 13676. A noniterative method for computing the Moore-Penrose generalized inverse of an arbitrary matrix.

Suppose A is an $m \times n$ matrix of rank r. Let A^+ denote the Moore-Penrose generalized inverse of A, and let A^c be any conditional inverse of A (i.e. $AA^cA = A$). Then A^+ may be represented in terms of A^c thus: $A^+ = (A^+A)A^c(AA^+)$. The left factor may be constructed from solutions of $Au = 0$ as follows. The equation $AA^+A = A$ implies $A(I - A^+A) = 0$. Also $I - A^+A$ is hermitian and idempotent, and of rank $n - r$. Hence $I - A^+A = UU^+$, where U is $n \times (n-r)$, of rank r and $AU = 0$. It can be verified in a similar way that the right factor may be constructed from solutions of $A^*v = 0$. A conditional inverse of A may be found by Jordan elimination. It can also be shown that $A^+A = I - XX^*$ and $AA^+ = I - YY^*$, where the columns of X and Y are the normalized eigenvectors belonging to the zero eigenvalues of A^*A and AA^*, respectively. (Received July 3, 1972.)

696-15-6. GEORGE P. BARKER, Univ[illegible] over cones.

The definition of a stochastic matrix is extended to [illegible] [illegible]ant. The usual theorems dealing with the convergence of $1/n \sum_{k=1}^{n} A^k$ are obtained in this setting. Finally, compact groups of matrices which leave the cone invariant are discussed. (Received July 3, 1972.)

16 Associative Rings and Algebras

696-16-1. MARGARET W. TAFT, Clark University, Worcester, Massachusetts 01610. F-rings. Preliminary report.

R is an F-ring if every element of R can be written in the form $r + n$ where n is nilpotent, $rn = nr = 0$, and r is strongly regular—that is, there is an $s \in R$ such that $rs = sr$, $rsr = r$, $srs = s$. Such decompositions are unique. An ideal or the center of an F-ring is an F-ring. All finite rings are F-rings; all rings with unit satisfying the dcc are F-rings; but not all F-rings satisfy the dcc. Not all rings with unit satisfying the acc are F-rings. (Received July 3, 1972.)

*696-16-2. B. ARTHUR MILLER, Mount Allison University, Sackville, New Brunswick, Canada. Finite subgroups of radical rings.

A ring R is (Jacobson) radical if given $x \in R$, there exists $y \in R$, with $x \circ y = x + y - xy = 0$. To each radical ring R there is associated a group R°, the adjoint group of R. It is well known that if R is a finite radical ring, R° is a nilpotent group. Theorem 1. Let R be a radical algebra over a field. If G is a finite subgroup of R°, G is a nilpotent group. Theorem 2. If R is a radical ring, finite subgroups of R° are nilpotent if: (i) R^+ is a torsion group, (ii) R^+ is torsion free, (iii) R^+ is divisible, (iv) R^+ is reduced, or (v) R is a semiprime ring. These results are used to prove Theorem 3. Let R be a radical ring. If G is a finite subgroup of R°, G is an extension of an abelian group by a nilpotent group. In particular, G is solvable. (Received July 5, 1972.)

696-16-3. PATRICK J. FLEURY, State University College of New York, Plattsburgh, New York 12901. A condition equivalent to finite Goldie dimension. Preliminary report.

Theorem. The following are equivalent: (1) M has finite Goldie dimension. (2) If $M_1 \subset M_2 \subset \ldots$ is a sequence of submodules of M, there is an N and M_i is essential in M_{i+1} for $i \geq N$. Theorem. The following are equivalent: (1) M has finite Goldie dimension. (2) Every submodule of M has a finitely generated essential submodule. Theorem. If a ring R has finite Goldie dimension, so does R[X] where X is any indeterminate. (Received July 5, 1972,)

*696-16-4. JOACHIM LAMBEK, McGill University, Montreal, Quebec, Canada. Noncommutative localization.

Given any injective right R-module I, one obtains for each R-module A its torsion-submodule $T(A) = \{a \in A \mid \mathrm{Hom}_R(A, I) = 0\}$ and its divisible hull $D(A)$ defined by $D(A)/A = T(I(A)/A)$, where $I(B)$ denotes the injective hull of B. One then obtains a functor Q of Mod R into itself, called localization, defined by $Q(A) = D(A/T(A))$. $Q(A)$ is the module of quotients of A, $Q(R)$ the ring of quotients of R with respect to I. This concept agrees with that introduced by Gabriel and others. It frequently happens that Q is exact. Then $Q(A)$ is a dense submodule of $S(A) = \mathrm{Hom}_E(\mathrm{Hom}_R(A, I), I)$, where E is the ring of endomorphisms of I, in the finite (product) topology of $S(A)$. In particular, $Q(R)$ is a dense subring of the bicommutator of I. One may also

associate with I a topology on each module A, the I-adic topology: a fundamental system of open neighborhoods of 0 consists of all kernels of homomorphisms $A \to I^n$, n being some positive integer. Then S(A) may be veiwed as the I-adic completion of Q(A). Of special interest is the case where I = I(R/P), R being a right Noetherian ring and P a prime ideal. Michler and the author have investigated a number of equivalent conditions which assure the exactness of Q. They have also obtained conditions for the I-adic topology on any finitely generated Q(R)-module to coincide with the classical PQ(R)-adic topology, which generalize the usual Artin-Rees Lemma.
(Received July 5, 1972.)

17 Nonassociative Rings and Algebras

*696-17-1. WILLIAM E. JENNER, University of North Carolina, Chapel Hill, North Carolina 27514. Truncated fields.

Let K be any field and $L = K(\theta)$ a simple algebraic extension of degree $n \geqq 4$. Let $B = K + K\theta + \ldots + K\theta^{n-s}$ and $H = K\theta^{n-s+1} + \ldots + K\theta^{n-1}$ so that L = B + H, vector space direct sum. This determines a structure of truncation algebra on B by defining the product in B to be the projection on the subspace B of the ordinary product in L (cf. J. Elisha Mitchell Sci. Soc. 86(1970), 196-197). The resulting algebra is isotopically simple for $2 \leqq s \leqq n/2$. (Received June 9, 1972.)

18 Category Theory, Homological Algebra

*696-18-1. ERNEST G. MANES, University of Massachusetts, Amherst, Massachusetts 01002. A general theory of compact Hausdorff objects.

A topological space X is compact if and only if each projection $X \times Y \to Y$ is closed, and is Hausdorff if and only if the diagonal map $X \to X \times X$ is closed. The suggested general definition is posed, yielding familiar results such as "a continuous map from a compact to a Hausdorff is closed." If T is an algebraic theory, the category R(T) of relational T-models comes equipped with a closure operator whose compact Hausdorff objects are the T-algebras; the Tychonoff theorem holds in R(T) and each object in R(T) has an algebra reflection. Appropriate choice of T produces a common construction of the Stone-Čech compactification of a space and the completion of a partial algebra (P. Burmeister and J. Schmidt, "On the completion of partial algebras," Colloq. Math. 17(1967), 235-245). (Received May 25, 1972.)

*696-18-2. ANDREW A. BLANCHARD, U.S. Naval Academy, Annapolis, Maryland 21402. Structure species and forgetful functors.

This paper reformulates the classical theory of structures of Bourbaki with the language of category theory. For a category X, we define the notion of a structure species on X, and also define the category of structure species on X. To each structure species is associated a forgetful [faithful, good, and transportable] functor on X, and more importantly, we prove that to each forgetful functor on X is associated a structure species on X. We use this result to show that the category of structure species on X is equivalent to the category of forgetful functors on X. (Received June 12, 1972.)

20 Group Theory and Generalizations

696-20-1. V. C. VARADACHARI, University of Wisconsin, River Falls, Wisconsin 55402. Minimal non-M-groups. Preliminary report.

A finite group G is an M-group (monomial group) if every irreducible complex character of G is induced from a linear character of a subgroup of G. What is the structure of a non-M-group G, all of whose proper subgroups and proper factor groups are M-groups? First, let G be of odd order and the Fitting group of G a Hall subgroup of G. In this case, G is an extension of an extraspecial p-group P of exponent p of order p^{2n+1}, by a group A where $(p, |A|) = 1$ and A is trivial on Z(P) and irreducible on P/Z(P). If furthermore $(2n, |A|) = 1$, then A is of order a prime $q \neq p$. In the case of solvable groups of even order the structure of G may be different from above. For example, q cannot be equal to 2, for then $P \cdot Q$ is a supersolvable group and so an M-group. Also there is an example of a group G which is an extension of an extraspecial p-group by the quaternion group of order 2^3 such that all proper subgroups and proper factor groups of G are M-groups but G itself is not an M-group. Next let G be simple and all proper subgroups of G M-groups. In view of Thompson's classification theorem, G is isomorphic to one of the following groups: projective special linear groups $L_2(2^p)$, p any prime, $L_2(3^p)$, p any odd prime, $L_2(p)$, p any prime > 3 such that $p^2 + 1 \equiv 0 \pmod 5$; the Suzuki groups $SZ(2^p)$, p any prime. (Received June 19, 1972.)

696-20-2. BRIAN KIRKWOOD SCHMIDT, Princeton University, Princeton, New Jersey 08540. Mappings of degree n from groups to abelian groups.

The basic problem is to characterize all mappings of degree n or less with domain a given group. The problem is approached by means of a universal mapping problem which gives rise to a covariant functor Θ_n from groups to abelian groups. If $\mathcal{I}$ is the fundamental ideal of the integral group-ring of a group G, $\Theta_n(G) \cong \mathcal{I}/\mathcal{I}^{n+1}$. Given a free group F on a finite set of generators, $\Theta_n(F)$ is computed, and the mappings of degree n on F are described using the free differential calculus. Methods are developed for computing Θ_n of a finitely presented group or a product of groups. And it is shown that over the field of rational numbers every mapping of degree n is the sum of mappings which satisfy the added condition of homogeneity $(f(x^r) = r^m f(x))$. (Received June 26, 1972.)

*696-20-3. JAMES W. RICHARDS, Kent State University, Kent, Ohio 44242. A further extension of Frobenius's theorem. Preliminary report.

The author extablishes the following result. Theorem. Let H be a subgroup of a finite group G with the property that $H \cap H^g$ is a cyclic p-group, p a fixed prime, for all g not in H. If H is not a cyclic p-group, then H has a normal complement in G when one of the following conditions holds: (i) if p = 2, then H contains a Sylow 2-subgroup of G; (ii) H is not a Frobenius group where the Frobenius complements are cyclic p-groups. This result is the best possible in the sense of (i) and (ii). One only has to look at the normalizers of the Sylow 2-subgroups of Sz(8) and also at the normalizers of the Sylow 3-subgroups of A_6. (Received June 26, 1972.)

696-20-4. STANLEY L. STEPHENS, Anderson College, Anderson, Indiana 46011. The order of the automorphism group of certain semidirect products. Preliminary report.

A group G is called a semidirect product of H by K if G is generated by H and K where H is a normal subgroup of G and K is a subgroup of G having trivial intersection with H. Let G be the semidirect product of H by K where $H = \bigoplus_{i=1}^{t} \langle h_i \rangle$, $K = \langle k \rangle$, H and K are p groups for some odd prime p, and $k^{-1}h_i k \in \langle h_i \rangle$ for each i, $i = 1, \ldots, t$. In this case the order of G divides the order of the automorphism group of G. (Received April 21, 1972.)

696-20-5. RICHARD H. CROWELL, Dartmouth College, Hanover, New Hampshire 03755 and NEVILLE F. SMYTHE, Dartmouth College, Hanover, New Hampshire 03755 and Australian National University, Canberra (ACT 2600), Australia. Intersection theorem for groupnets. Preliminary report.

Instead of the term groupoid (as employed by Brandt) the word groupnet will be used. A morphism of groupnets is defined to be monotone if the inverse image of every identity is connected. Consider the mapping diagram $A_1 \leftarrow A_0 \rightarrow A_2$ of groupnets formed by morphisms $f_k : A_0 \rightarrow A_k$ for $k = 1, 2$. The following is proved: Intersection Theorem. If the morphisms f_1 and f_2 are monotone and if the groupnet A and morphisms $g_k : A_k \rightarrow A$, for $k = 0, 1, 2$, constitute the pushout of the diagram, then $g_1(A_1) \cap g_2(A_2) = g_0(A_0)$. The statement is false without the requirement that f_1 and f_2 are monotone. For groups, where every morphism is monotone, the theorem has been proved by R. H. Fox. The necessity of the monotonicity condition for the more general theorem helps explain the nontriviality of the theorem for groups. For an earlier reference, see Abstract 687-20-5, these *Notices* 18(1971), 766. (Received June 15, 1972.)

*696-20-6. W. J. WICKLESS and CHARLES I. VINSONHALER, University of Connecticut, Storrs, Connecticut 06268. A note on torsion free abelian groups of finite rank.

Let G be a torsion free abelian group of rank n and $X = \{x_1, \ldots, x_n\}$ a set of rationally independent elements in G. If $1 \leqq i \leqq n$ and $i \notin J \subset \{1, \ldots, n\}$, let $A^i_J(G,X) = \{\alpha \in Q \mid \alpha_1 x_1 + \ldots + \alpha_i x_i + \ldots + \alpha_n x_n \in G$ for some $\alpha_1 \ldots \alpha_{i-1}, \alpha_{i+1} \ldots \alpha_n$ in Q with $\alpha_j = 0$ whenever $j \in J\}$. There exist natural isomorphisms $A^i_J / A^i_{\{j\} \cup J} \overset{\theta}{\cong} A^j_J / A^j_{\{i\} \cup J}$ for $j \notin J$. Let $S(G,X)$ be this collection of rank one group isomorphisms. It is well known if rank G = 2, then G may be recovered from $S(G,X)$. This result does not extend to torsion free abelian groups of rank greater than two. However, the following result is obtained. Let G, G' be torsion free abelian groups of finite rank with $S(G,X) = S(G',X')$ for suitable X, X'. Let F, F' be the free subgroups of G, G' generated by X, X'. Then $G/F \cong G'/F'$. (Received June 30, 1972.)

696-20-7. JAMES M. BATEMAN, Michigan State University, East Lansing, Michigan 48823. The fundamental ideal functor and its adjoint. Preliminary report.

Let $\Delta(G)$ be the fundamental ideal of group ring $R[G]$ with $1 \in R$. Let $^{\circ}S$ ($\upsilon(S)$) be the group of quasiregular elements (units) of R-algebra S (with 1). Extend these mappings naturally to functors. Group G is embedded naturally in $^{\circ}\Delta(G)$ with its nonidentity elements an R-basis for $\Delta(G)$. Theorem. $^{\circ}$ is the adjoint of Δ. Let E (F) be the natural adjunction of a 1 (the functor forgetting 1) for R-algebras (with 1). Theorem. $^{\circ}F$ ($E\Delta / \Delta / {}^{\circ}$) is naturally equivalent υ ($R[\cdot]$ / a subfunctor of $FR[\cdot]$ / a subfunctor of υE). Theorem. Every unit of $R[G]$ is trivial $\Leftrightarrow$ $^{\circ}\Delta$ fixes G. The nth dimension subgroup of H is $\Delta(H)^n \cap H$. The natural epimorphism $\Delta^{\circ}(\Delta(G)) \rightarrow \Delta(G)$ is used to prove Theorem. The nth dimension subgroup of $^{\circ}\Delta(G)$ is $^{\circ}(\Delta(G)^n) = \Delta(G)^n \cap {}^{\circ}\Delta(G)$. (Received June 30, 1972.)

696-20-8. ALPHONSE H. BAARTMANS and JAMES J. WOEPPEL, Southern Illinois University, Carbondale, Illinois 62901. Groups with characteristic cyclic series. Preliminary report.

Let G be a finite group, let Aut G denote the automorphism group of G. By a cyclic characteristic series of a group we mean a finite series $1 = G_0 \subseteq G_1 \subseteq G_2 \subseteq \ldots \subseteq G_n = G$ such that each G_i is characteristic in

G and each G_{i+1}/G_i is cyclic. In their paper "Groups with characteristic cyclic series," J. Algebra 18(1971), 453-460, J. R. Durbin and M. McDonald make the following Conjecture. If G is a p-group and Aut G is supersolvable then G has a cyclic characteristic series. Theorem 1. Let G be a p-group with a supersolvable automorphism group; then G has a characteristic cyclic series. Theorem 2. Let G be a supersolvable group such that Aut (G) is supersolvable. If $Z(G) = 1$, then G has a cyclic characteristic series. (Received July 3, 1972.)

*696-20-9. F. RUDOLF BEYL, Cornell University, Ithaca, New York 14850. The classification of metacyclic p-groups. Preliminary report.

A metacyclic group (MC-group) is a finite group which is an extension of a cyclic group by a cyclic group. Typically an MC-group is supported by different metacyclic extensions. All metacyclic p-groups are enumerated, indeed for each such group a supporting extension is distinguished. This is achieved mainly by (a) an easily manageable formula for the second cohomology group of a cyclic group in a cyclic group, (b) an enumeration of all weak congruence classes of metacyclic p-extensions, and (c) the determination of a maximal kernel extension for each metacyclic p-group. Two extensions are called weakly congruent if there is an equivalence $e_0 \rightarrow e$ in the category of extensions. A maximal kernel of the metacyclic group G is a subgroup containing G' which has maximal order among all such subgroups. Theorem. For an odd prime p, every nonabelian metacyclic p-group determines a unique weak congruence class of metacyclic maximal kernel extensions. For $p = 2$, the same statement is true, except for the groups $(a, b: a^M = 1,\ bab^{-1} = a^{-1},\ b^N = a^{M/2})$ with $8|M$ and $4|N$. For odd p and for $p = 2$, a nonabelian metacyclic p-group is supported by at most one split metacyclic extension and split metacyclic extensions are alwavs maximal kernel extensions. (Received July 3, 1972.)

*696-20-10. GUY T. HOGAN, State University College of New York, Oneonta, New York 13820. On the number of generators of an H_p group. Preliminary report.

If G is a finite p-group of exponent greater than p admitting an automorphism σ, such that $g^{1+\sigma+\sigma^2+\cdots+\sigma^{p-1}} = 1$ for all $g \in G$, we say that G is an H_p group. Theorem. If G is an H_p group, and $G^p \not\subseteq G_2$, then G requires at least $p - 1$ generators. (If $p=2$ or 3, then the restriction on G^p is not necessary.) The proof is accomplished by showing that any abelian H_p group always contains a σ-invariant subgroup which is an H_p group of rank $p - 1$. (Received July 5, 1972.)

696-20-11. CHARLES C. SIMS, Rutgers University, New Brunswick, New Jersey 08903. The construction of large permutation groups.

• The proofs of existence of several of the recently discovered sporadic finite simple groups, those not part of a known infinite family of groups with similar structure, have depended on being able to determine the order and structure of the group generated by a given set of permutations on thousands or even millions of points. This talk presents a survey of the techniques available and illustrates them with an outline of the construction as a permutation group of degree 8835156 of the simple group predicted by Lyons. (Received July 5, 1972.)

*696-20-12. NAOKI KIMURA, University of Arkansas, Fayetteville, Arkansas 72701. Totally archimedian semigroups.

A semigroup S is called totally archimedian if every subsemigroup of S is archimedian. A semigroup is called power-joined if any two nonempty subsemigroups intersect. Main Theorem. A semigroup is totally archimedian if and only if it is power-joined. Also several equivalent conditions to the totally archimedian property will be given, one of which contains Levin's theorem (Pacific J. Math. 27(1968), 371) as a special case. (Received July 5, 1972.)

22 Topological Groups, Lie Groups

*696-22-1. ERIC C. NUMMELA, University of Florida, Gainesville, Florida 32601. Topological group extensions. Preliminary report.

Let X be a uniformizable topological monoid. A topological ring R is free over X if (1) there is a continuous monoid homomorphism $i: X \to R$, and (2) given any topological ring S together with a continuous monoid homomorphism $f: X \to S$, there is a unique continuous ring homomorphism $\bar{f}: R \to S$ such that $\bar{f}i = f$. Given any such X, there is a unique (up to isomorphism) topological ring RX free over X. In case X is the colimit of an expanding sequence of compact spaces, then RX is the Markov free abelian topological group over X, made into a ring in the natural way. In this case the natural concepts of topological X-module and RX-module coincide. (RX-modules are always X-modules.) In the general case, given any RX-module A, we define the cohomology of X with coefficients in A to be $H^*(X,A) = \mathrm{Ext}^*_{RX}(Z,A)$, where Z is the discrete group of integers considered as a trivial RX-module, and Ext* is as defined in Abstract 689-A45, these *Notices* 18(1971), 1051. Thus $H^*(X,A)$ theoretically may be computed via RX-projective resolutions of Z. Moreover, $H^1(X,A)$ can be realized as continuous derivations modulo continuous inner derivations; and if X is a group, $H^2(X,A)$ classifies those topological extensions of A by X which admit continuous sections. (Received June 19, 1972.)

26 Functions of Real Variables

*696-26-1. HENRYK FAST, Wayne State University, Detroit, Michigan 48202. On separating capacity in a class of c-dense cartesian products.

Let $\mathcal{C}$ denote the class of subsets of $[0,1]^2$ of the form $X \times Y$ where X and Y are c-dense in [0,1] (i.e. of cardinality continuum in any subinterval of [0,1]). Let $P = P(x,y)$ be a polynomial with rational coefficients. For this given P a subclass $\mathcal{S}$ of $\mathcal{C}$ of cardinality continuum consisting of disjoint sets is constructed with the additional property that the sets are P-separated or, what is the same, that each of the level-lines $P^{-1}(z)$ meets each of the sets from $\mathcal{S}$ at most once. This gives some idea as to the "separating capacity" of P in this class or, if preferred, the "packing capacity" of the sets from $\mathcal{S}$ with respect to P such that it still would permit for a "free flow" of the level-lines of P through $[0,1]^2$ with minimal meeting of the sets. Denoting by $\mathcal{P}^+$ the class of such polynomials with the additional requirement of nonnegativity of coefficients, we construct another class $\mathcal{S}$ having the same property with respect to the whole class $\mathcal{P}^+$, i.e. to all the level-lines of all the $P \in \mathcal{P}^+$ at once. Corollary. A class $\mathcal{S}$ with similar properties exists for a function of the form $h(P(g_0,g_1))$ as well as for the whole class of functions $\{h(P(g_0,g_1)) : P \in \mathcal{P}^+\}$ where $g_i : [0,1] \xrightarrow{\text{onto}} [0,1]$ $(i=0,1)$, $h: [0,1] \to [0,1]$ are arbitrary. (Received June 13, 1972.)

696-26-2. SIOBHÁN VERNON, University College, Cork, Ireland. On a result of Besicovitch. Preliminary report.

Let $\mathfrak{P}$ denote the class of power series with nonnegative coefficients defined on the interval [0,1). Besicovitch (J. London Math. Soc. 38(1963), 223-225) proved the following result: There is a function f positive, continuous and increasing on $[0,1)$ such that (i) $\int_0^1 f(x)\,dx = \infty$, and (ii) for all $p \in \mathfrak{P}$, $p(x) \leqq f(x) \Rightarrow \int_0^1 p(x)\,dx < \infty$. The function f is constructed as follows: Let $g(x) = (1-x)^{-1/2}$ and let $x_r = 1 - 2^{-2^r}$; then $f = f(g, \{x_r\})$ is defined by the equation $f(x) = g(x)$ at the points $0, x_1, x_2, \ldots,$ and by linear interpolation elsewhere on $[0,1)$. The author generalises this result and shows that for any function g positive, continuous, and strictly increasing to infinity on $[0,1)$, and satisfying $\int_0^1 g(x)\,dx < \infty$, there corresponds a sequence $\{x_r\}$, $0 < x_r < x_{r+1}$ (all r), $x_r \to 1$ $(r \to \infty)$, such that the function $f = f(g, \{x_r\})$, defined as above, satisfies (i) and (ii). (Received June 14, 1972.)

*696-26-3. HENRY E. WHITE, JR., Ohio University, Athens, Ohio 45701. Functions with a concave modulus of continuity.

A modulus of continuity is a function $\sigma: [0,1] \to [0,\infty)$ such that, if $0 < x \leqq y \leqq 1$, then $0 < \sigma(x) \leqq \sigma(y)$, and $\lim_{t \to 0+} \sigma(t) = \sigma(0) = 0$. If σ is a modulus of continuity, $C(\sigma)$ denotes the set of all f in $C[0,1]$ such that $|f(y) - f(x)| \leqq \sigma(|y-x|)$ for all x, y in $[0,1]$. Here $C[0,1]$ is the set of all continuous, real valued functions defined on $[0,1]$. A modulus of continuity σ is concave if, whenever $0 \leqq x < y \leqq 1$, $\sigma(t) \geqq L(t)$ for all t in (x,y), where L is the linear function such that $L(x) = f(x)$ and $L(y) = f(y)$. Lebesgue measure is denoted by m. Theorem. If σ is a concave modulus of continuity and $0 < M < 1/24$, then the set of all f in $C(\sigma)$ such that $m(\{x : f(x) = g(x)\}) = 0$ for all g in $C(M\sigma)$ is residual in $C(\sigma)$. Corollary. Suppose $0 < \alpha < 1$. Then there is a function f which satisfies on $[0,1]$ a Holder condition of exponent α (i.e. there is $M > 0$ such that $|f(y) - f(x)| \leqq M|y - x|^{\alpha}$ for all x, y in $[0,1]$) such that $m(\{x : f(x) = g(x)\}) = 0$ for all functions g satisfying on $[0,1]$ a Holder condition of exponent $> \alpha$. (Received June 27, 1972.)

*696-26-4. JACK B. BROWN, Auburn University, Auburn, Alabama 36830. Concerning Ceder's differentiable restrictions of arbitrary real functions.

J. G. Ceder proved (Fund. Math. 65(1969), 351-358) a theorem from which it follows that if A is an uncountable subset of the reals R, then for every $f : A \to R$, there exists a bilaterally dense in itself set $B \subset A$ such that $f|B$ is differentiable (where infinite derivatives are allowed). Uncountability of A is necessary, and B cannot be made to be uncountable. The main purpose here is to characterize those sets $A \subset R$ for which it is true that for every $f : A \to R$, there exists a bilaterally c-dense (c denotes the cardinality of R) in itself set $W \subset A$ and a dense in W set B such that $f|W$ is differentiable on B. The characterization is as follows: A cannot be expressed as a countable union $L_1 \cup L_2 \cup \ldots$, where each L_i has the property that if $N \subset L_i$ is nowhere dense in L_i, then N has cardinality less then c. Perfect sets A, for example, have this property, while Lusin sets are uncountable sets which do not. (Received July 3, 1972.)

28 Measure and Integration

*696-28-1. ROBERT J. BUCK, University of California, Davis, California 95616. Comparison theorems for generalized Besocivitch dimensions of compact planar sets. Preliminary report.

Let τ be a translation invariant outer measure on the convex subsets of R^2, such that $\tau(A)$ tends to zero with the diameter $\delta(A)$. A collection, K, of rectangles with bounded diameters having edges parallel to the coordinate axes is called a covering class if K is closed under translations and contains rectangles of arbitrarily small diameter. For $0 < \beta$, let $K^{\beta}(E) = \inf\{\sum \tau(A_i)^{\beta} : \cup A_i \geqq E,\ A_i \in K\}$ for $E \subseteq R^2$. The generalized Besicovitch dimension of E relative to K and τ is $K_{\tau}(E) = \sup\{\beta : K_{\tau}^{\beta}(E) > 0\}$. For each finite, positive measure of compact support, let $K_{\tau}(\mu) = \liminf (\log \bigvee_x \mu(R+x)/\log \tau(R))$ as $\tau(R) \to 0$, $R \in K$.

Theorem 1. For all K and τ, $K_\tau(E) = \sup\{K_\tau(\mu) : \mu \in M(E)\}$, where M(E) is the class of all finite, positive measures supported in the compact set E. Corollary. $M_\tau(E) \leqq K_\tau(E)$ for all compact E provided there is a map $\varphi : K \to M$ such that (i) $\lim \log \tau(\varphi(R))/\log \tau(R) = 1$, and (ii) $\lim \log\{\delta(\pi_j R)/\delta(\pi_j\varphi(R))\}/\log \tau(R) = 0$ $(j=1,2)$ as $\tau(R) \to 0$. Theorem 2. If m denotes Lebesgue measure, $2K_m(E) = K_\delta(E)$ for all compact E in R^2, if and only if $K_m(E) = S_m(E)$ for all such E, where S consists of squares. The results extend intact to R^n. (Received April 13, 1972.)

*696-28-2. ALLAN F. ABRAHAMSE, University of Southern California, Los Angeles, California 90007. Uniform integrability of derivatives on σ-lattices.

Let P be a probability measure, and let φ be a finite signed measure on some measurable space. Let $\mathfrak{M}$ be a σ-lattice of measurable sets. Definition. φ is absolutely continuous with respect to P on $\mathfrak{M}$ if whenever $\Lambda \in \mathfrak{M}$ and $P(\Lambda) = 0$, then $\varphi(\Lambda) \leqq 0$, and whenever $\Lambda^c \in \mathfrak{M}$ and $P(\Lambda) = 0$, then $\varphi(\Lambda) \geqq 0$. Now let $\mathcal{D}$ represent the class of Radon-Nikodym derivatives of φ with respect to P, taken over all the sub-σ-lattices of $\mathfrak{M}$ [Johansen, Pacific J. Math. 21(1967)]. Theorem. $\mathcal{D}$ is a uniformly integrable class if and only if φ is absolutely continuous with respect to P on $\mathfrak{M}$. (Received June 7, 1972.)

*696-28-3. KOHUR N. GOWRISANKARAN, McGill University, Montreal 110, Quebec, Canada. Suslin and Lusin topologies on a set.

Let T_1 and T_2 be two Suslin topologies on a set X. It is shown that the Borel sets for the two topologies are identical if and only if X provided with the $\sup(T_1, T_2)$ topology is Suslin. In the case of Lusin topologies necessary and sufficient conditions for the above to happen are given in terms of subdivisions corresponding to the two topologies. With similar conditions on the subdivisions in the case of Suslin topologies it is shown that X with $\sup(T_1, T_2)$ topology is a Radon space. (Received July 5, 1972.)

*696-28-4. FRED M. WRIGHT and DEAN R. KENNEBECK, Iowa State University, Ames, Iowa 50010. On the existence of sum and product integrals.

Let [a, b] be a closed interval of the real axis, and let G be a real-valued function on the set of all ordered pairs (x, y) of points of [a, b] such that $x < y$. The first theorem generalizes a result of W. P. Davis and J. A. Chatfield (Proc. Amer. Math. Soc. 25(1970), 743-747). The proof in this paper is shorter and simpler than the one given by Davis and Chatfield. Theorem 1. Let m be a positive integer greater than 1. Suppose that $\int_a^b G^j$ $(j=2, 3, \ldots, m-1)$ exists and that $\int_a^b |G|^m$ exists and is 0. Then, $\int_a^b G$ exists iff $\Pi_a^b(1+G)$ exists, and in this case $\Pi_a^b(1+G) \neq 0$ and $\ln \Pi_a^b(1+G) = \int_a^b G + \sum_{j=2}^{m-1}((-1)^{j+1}/j) \cdot \int_a^b G^j$. The next theorem concerns the question of when the existence of $\Pi_a^b(1+G)$ implies the existence of $\Pi_\alpha^\beta(1+G)$ for every closed subinterval $[\alpha, \beta]$ of [a, b]. Theorem 2 shows that this is so when $G \geqq 0$. Theorem 2. For each closed subinterval $[\alpha, \beta]$ of [a, b], suppose there are a positive real number $M([\alpha, \beta])$ and a partition $\Delta([\alpha, \beta])$ of $[\alpha, \beta]$ such that $|\Pi_{i=1}^n [1 + G(x_{i-1}, x_i)]| \geqq M([\alpha, \beta])$ for every partition $\Delta = \{\alpha = x_0 < x_1 < x_2 < \ldots < x_n = \beta\}$ of $[\alpha, \beta]$ which is a refinement of $\Delta([\alpha, \beta])$. Suppose $\Pi_a^b(1+G)$ exists. Then, $\Pi_\alpha^\beta(1+G)$ exists for every closed subinterval $[\alpha, \beta]$ of [a, b]. (Received July 5, 1972.)

30 Functions of a Complex Variable

*696-30-1. PETRU MOCANU, Babēs-Bolyai University, Cluj, Romania, ELIGIUSZ ZŁOTKIEWICZ, Universitatis Mariae Curie-Skłodowska, Lublin, Poland and MAXWELL O. READE, University of Michigan, Ann Arbor, Michigan 48104. Bazilevič functions and close-to-convex p-valent functions.

Let $B(\beta)$, β a positive constant, denote the set of all functions $f(z) = z + a_2z^2 + \ldots$ which are analytic in the unit disc Δ and for which there exists a starlike univalent function $\sigma(z) = z + b_2z^2 + \ldots$ such that $\mathrm{Re}[zf'(z)/f^{(1-\beta)}(z)\sigma^{(\beta)}(z)] \geqq 0$ holds in Δ. If β is a rational number, $\beta = m|p$, then it is shown that $[f(z^m)]^{p/m}$ is a p-valent close-to-convex function [Umezawa, Proc. Amer. Math. Soc. 8(1957), 869-874]. This leads to various distortion and coefficient theorems for the Bazilevič functions $B(\beta)$, $\beta > 0$. The possibility of univalent members of $B(\beta)$, for β negative, is also investigated. (Received March 15, 1972.)

*696-30-2. DANIEL D. BONAR, Denison University, Granville, Ohio 43023 and FRANCIS W. CARROLL, Ohio State University, Columbus, Ohio 43210. Annular functions form a residual set. Preliminary report.

Let D be the open unit disk and define $H(D) = \{f : f$ is analytic in $D\}$ and let H(D) be provided with the (complete metric) topology of almost uniform convergence. [That is, a base of open neighborhoods being $f + V(K,\epsilon)$ where, for K a compact subset of D and $\epsilon > 0$, $V(K,\epsilon) = \{f \in H(D) : |f(z)| < \epsilon$ for all $z \in K\}$.] Definitions. $f \in H(D)$ is annular if there exists a sequence of closed Jordan curves J_n in D converging outward to the boundary C of D such that the minimum of $|f|$ on J_n converges to ∞. (If the J_n are circles concentric with D then f is strongly annular.) Let A, S and W be respectively the set of all annular functions, all strongly annular functions, and all $f \in H(D)$ for which $Z'(f,0) = C$ where $Z'(f,0)$ is the derived set of $Z(f,0) = \{z : f(z) = 0\}$. Letting $S_n = \{f \in H(D)$: there exists $r = r(f)$ with $(1-1/n) < r < 1$ such that $\min|f(z)| > n$ for $|z| = r\}$ and $W_n = \{f \in H(D) : (Z(f,0) - \{0\}) \cap \{z : ((k-1)2\pi/n) < \arg z < (k2\pi)/n\} \neq \emptyset,\ k = 1,2,\ldots,n\}$, it is shown that S_n and W_n are open and dense in H(D) and hence the following theorem results. Theorem. A, S and W are residual sets in H(D). (Received June 12, 1972.)

696-30-3. ROBERT W. REDDING, Clark University, Worcester, Massachusetts 01610 and Worcester State College, Worcester, Massachusetts 01610. Typically real functions of order α. Preliminary report.

Definition 1. $f(z) \in TR(\alpha)$, $\alpha < 1$, provided $f(z)$ is analytic in $|z| < 1$, $f(0) = 0$, $f'(0) = 1$, and $\mathrm{Re}\{1 - z^2/z\, f(z)\} \geqq \alpha$ in $|z| < 1$. For $\alpha = 0$, this is the well-known class of typically real functions. Theorem 1. $f(z) \in TR(\alpha)$ iff $1/1 - \alpha\, \{f(z) - \alpha z/1 - z^2\} \in TR(0)$. Using known results for $TR(0)$, the following theorems are proved. Theorem 2. If $f(z) \in TR(\alpha)$, then $\mathrm{Im}\, f(z) \geqq \mathrm{Im}\, \alpha z/1 - z^2$ if $\mathrm{Im}\, z > 0$, and $\mathrm{Im}\, f(z) \leqq \mathrm{Im}\, \alpha z/1 - z^2$ if $\mathrm{Im}\, z < 0$. Theorem 3. If $f(z) = z + \sum_{n=2}^{\infty} a_n z^n \in TR(\alpha)$, then $|a_n| \leqq (1-\alpha)n$ for n even and $|a_n| \leqq n - \alpha(n-1)$ for n odd. These are sharp. An integral representation is found and the following result is shown. Theorem 4. Let $|c| < 1$ and let $F(w_0, w_1, \ldots, w_{m+1})$ be analytic on $U(f(c), f'(c), \ldots, f^{(m)}(c), c)$, $f \in TR(\alpha)$. Then $\max (\min) \mathrm{Re}\{F(f(c), f'(c), \ldots, f^{(m)}(c), c)\}$ is attained by a function of the form $f(z) = \alpha z/1 - z^2 + (1-\alpha) \cdot \sum_{k=0}^{p} t_k z/1 - 2s_k z + z^2$ where $p \leqq m + 1$, $-1 \leqq s_k \leqq 1$, $t_k > 0$, $\sum_{k=0}^{p} t_k = 1$. (Received June 19, 1972.)

*696-30-4. J. K. SHAW and GEORGE W. CROFTS, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. Successive remainders of the Newton series. Preliminary report.

If f is analytic in the open unit disc D and λ is a sequence of points in D converging to 0, then f admits the Newton series expansion $f(z) = f(\lambda_1) + \sum_{n=1}^{\infty} \Delta_\lambda^n f(\lambda_{n+1})(z-\lambda_1)(z-\lambda_2)\ldots(z-\lambda_n)$, where $\Delta_\lambda^n f(z)$ is the nth divided difference of f with respect to the sequence λ. The Newton series reduces to the Maclaurin series in case $\lambda_n \equiv 0$. The present paper investigates relationships between the behavior of zeros of the normalized remainders $\Delta_\lambda^k f(z) = f(\lambda_{k+1}) + \sum_{n=k+1}^\infty \Delta_\lambda^n f(\lambda_{n+1})(z-\lambda_{k+1})\dots(z-\lambda_n)$ of the Newton series and zeros of the normalized remainders $\sum_{n=k}^\infty a_n z^{n-k}$ of the Maclaurin series for f. Let C_λ be the supremum of numbers $c > 0$ such that if f is analytic in D and each of $\Delta_\lambda^k f(z)$, $0 \leqq k < \infty$, has a zero in $|z| \leqq c$, then $f \equiv 0$. The corresponding constant for the Maclaurin series (C_λ, where $\lambda_n \equiv 0$) is called the Whittaker constant for remainders and is denoted by W. We prove that $C_\lambda \geqq W$, for all λ, and, moreover, $C_\lambda = W$ if $\lambda \in \ell_1$. In obtaining this result, we prove that functions f analytic in D have expansions of the form $f(z) = \sum_{n=1}^\infty \Delta_\lambda^n f(z_n) C_n(z)$, where $|z_n| \leqq W$, for all n, and $C_n(z)$ is a polynomial of degree n determined by the conditions $\Delta_\lambda^j C_k(z_j) = \delta_{jk}$. (Received June 26, 1972.)

696-30-5. PETER A. McCOY, U.S. Naval Academy, Annapolis, Maryland 21402. Value distribution of linear combinations of axisymmetric harmonic polynomials and their derivatives.

We obtain results on the value distribution of linear combinations of axisymmetric harmonic polynomials (AHP) and their derivatives which are analogous to results from the classical theory of polynomials of one complex variable. For example, let H be an AHP of degree n, $D_x H$ the axial derivative of H and let α be a point in the complex plane $\mathbb{C}$. Consider the operator $\mathcal{L}(\underline{\nu})H = (1-\nu_1 D_x)\dots(1-\nu_p D_x)H$ with $\underline{\nu} = \{\nu_1,\dots,\nu_p\} \subset \mathbb{C}$. Open sectors $S \subset \mathbb{C}$ and open cones $K \subset E^3$, functions of H, α, and n, are determined. In K, $\mathcal{L}(\underline{\nu})H \neq \alpha$ for all $\underline{\nu} \in S$ and all p with $1 \leqq p \leqq n$. Similar results are considered for other operators on H along with extensions to generalized axisymmetric harmonic polynomials. (Received July 3, 1972.)

696-30-6. DOUGLAS M. CAMPBELL, Brigham Young University, Provo, Utah 84601 and JOHN A. PFALTZGRAFF, University of North Carolina, Chapel Hill, North Carolina 27514. Properties of the generalized Koebe function. Preliminary report.

The function $k_\alpha(z) = (1/2\alpha)([(1+z)/(1-z)]^\alpha - 1)$, $\alpha \in \mathbb{C} - \{0\}$, is known as the generalized Koebe function and is an extremal function in U_α for many of the same problems for which the Koebe function ($\alpha = 2$) is 'the' extremal in S. Theorem. The order of $k_\alpha(z)$, $\alpha = a + ib$, is $\{\frac{1}{2}(|c|^2 + 1 + ((1-|c|^2)^2 + 4b^2)^{1/2})\}^{1/2}$. If order $f = \beta$ and $\mathfrak{C}(e^{i\theta}, f)$ [Pommerenke, 1964] consists of a single function $g(z)$, then $g(z) = k_\alpha(z)$, $\alpha \in \mathbb{C}$, where α must lie in the ellipse $(\beta^2 - 1)a^2 + \beta^2 b^2 \leqq \beta^2(\beta^2 - 1)$. Several recent papers [Duren; Duren and McLaughlin] have pointed out particular properties that $F(z) = \log k_2'(z)$ possesses. Theorem. Let $F_\alpha(z) = \log k_\alpha'(z)$, $\alpha \in \mathbb{C}$. Then $F_\alpha'(z)$ is close-to-convex univalent iff $|\alpha| \geqq 1$; convex univalent iff $|\alpha| = 1$; starlike if $\alpha^2 \geqq 1$. Furthermore, if $\alpha^2 \geqq 1$, $F_\alpha(z)$ has radius of convexity R_α, where $R_\alpha^2 = s_\alpha$ is the unique root in $(0,1]$ of $\alpha^2 s^5 + (32 - 25\alpha^2) \cdot s^4 + (27\alpha^4 - 17\alpha^2)s^3 + (-27\alpha^4 + 9\alpha^2)s^2 - 27\alpha^4 s + 27\alpha^4 = 0$. (Received July 5, 1972.)

*696-30-7. PREM K. KULSHRESTHA, Louisiana State University, New Orleans, Louisiana 70122. Coefficient bounds for k-fold symmetric univalent functions of bounded rotation. Preliminary report.

Let (*) $f(z) = z + \sum_{n=1}^\infty a_{nk+1}^{(k)} z^{nk+1}$ be a regular univalent function in the open unit disk K, where k is a natural number. We say that $f \in BR(\alpha, \zeta, M)$ if and only if f is defined by (*) and there are real numbers α, ζ and M, $|\alpha| < \pi/2$, $\zeta \in [0,1)$ and $M \geqq 1$, such that $|[e^{i\alpha} f'(z) - i \sin\alpha - \cos\alpha]/[(1-\zeta)\cos\alpha] - M| < M$ for $z \in K$. Coefficients bounds for $|a_{nk+1}^{(k)}|$ are obtained when $f \in BR(\alpha, \zeta, M)$, and the results reduced to those given by T. H. MacGregor (Proc. Amer. Math. Soc. 15(1964), 311-317) for $k = 1$, $M = 1$, $\alpha = \zeta = 0$. (Received July 5, 1972.)

*696-30-8. DANIEL D. BONAR, Denison University, Granville, Ohio 43023 and FRANCIS W. CARROLL, Ohio State University, Columbus, Ohio 43210. Not every annular function is strongly annular. Preliminary report.

A function f analytic on the open unit disk D is annular if there exists a sequence of closed Jordan curves J_n in D converging outward to the unit circle K such that the minimum of $|f|$ on J_n converges to ∞ as $n \to \infty$. If the J_n are circles concentric with D then f is strongly annular. Notation. For $n = 1, 2, \ldots, \beta_n = 1 - 1/(2n-1)$, $\beta'_n = 1 - 1/2n = \alpha_n$, $\alpha'_n = 1 - 1/(2n+1) = \beta_{n+1}$, $S_n = [\alpha_n, \alpha'_n] \cup [i\beta_n, i\beta'_n]$, $a_n = \alpha'_n + (1/3)(\alpha_{n+1} - \alpha'_n)$, $a'_n = \alpha'_n + (2/3)(\alpha_{n+1} - \alpha'_n)$, $b_n = \beta'_n + (1/3)(\beta_{n+1} - \beta'_n)$, $b'_n = \beta'_n + (2/3)(\beta_{n+1} - \beta'_n)$, J_n is the circle of radius $r = (a_n^2 + b_n^2)/2a_n$ with real center $c = (a_n^2 - b_n^2)/2a_n$, J'_n similarly defined using a'_n and b'_n, $D_n = \mathrm{Cl}(\mathrm{int}\, J'_n)$, and finally $E_n(z) = (z-c)/r$. Construct, inductively, a sequence of functions $\{f_n\}$ given by $f_n(z) = g_{n-1}(z) + [E_n(z)]^{k_n}$, where g_{n-1} is an entire function obtained using Mergelyan's theorem in approximating g with $g(z) = f_{n-1}(z)$ on D_{n-1} and $g(z) = 0$ on S_n, and k_n is a large enough integer to ensure that (1) $|f_n(z)| < 1/2^n$, $z \in S_n$, (2) $|f_n(z) - f_{n-1}(z)| < 1/2^n$, $z \in D_{n-1}$, and (3) $|f_n(z)| > 10^n$, $z \in J'_n$. The sequence f_n converges almost uniformly on D to a function which is annular but is not strongly annular. (Received July 5, 1972.)

696-30-9. DAVID PATTERSON, Stevens Institute of Technology, Hoboken, New Jersey 07030. The fundamental group of the space of moduli. Preliminary report.

Let $S(g,n)$ be a compact Riemann surface of genus g with n punctures and $X(g,n)$ the space of moduli, i.e. the space of conformal equivalence classes of surfaces homeomorphic to $S(g,n)$. Recently Maclachlan [Proc. Amer. Math. Soc. 29(1971), 85-86] has shown that $X(g,0)$ is simply-connected for all g. This result can be improved as follows. Theorem. $X(g,n)$ is simply-connected if any of the following hold: (1) $n = 0$; (2) g is even; (3) $g = 1$; (4) $2g + 2$ divides n. In any event, it can be shown that the fundamental group of $X(g,n)$ is cyclic of order one or two. It is reasonable to conjecture that $X(g,n)$ is simply-connected in all cases. The problem can be reduced, as Maclachlan did, to showing that the mapping class group (the Teichmuller modular group) of $S(g,n)$ is generated by elements of finite order. (Received July 5, 1972.)

*696-30-10. THOMAS A. METZGER, Texas A & M University, College Station, Texas 77843. On polynomial approximation in $A_q(D)$.

Let D be a bounded Jordan domain with rectifiable boundary. The Bers space, $A_q(D)$ $(1 < q < \infty)$, is defined to be the space of functions holomorphic on D such that $\iint_D |f| \gamma_D^{2-q}\, dx\, dy < \infty$, where $\gamma_D(z)$ is the Poincaré metric for D. Using elementary methods and known facts on areal approximation and schlicht functions, it is shown that the polynomials are dense in $A_q(D)$ for $q > 3/2$. (Received July 5, 1972.)

*696-30-11. FRED M. WRIGHT and J. N. LING, Iowa State University, Ames, Iowa 50010. Functions of bounded variation and topological indices.

The first theorem concerns the composition of an analytic function and a continuous complex-valued function of bounded variation on a closed interval of the real axis. Theorem A. Let g be an analytic complex-valued function in an open connected subset S of the complex plane. Let $\vec{\varphi}$ be a continuous complex-valued function with domain a closed interval $[a,b]$ of the real axis and with range in S such that $V_a^b \vec{\varphi}$ is finite. Let $\vec{\nu}$ be a continuous complex-valued function with domain $[a,b]$. Let $\vec{\omega}$ be the composite complex-valued function with domain $[a,b]$ such that $\vec{\omega}(t) = g(\vec{\varphi}(t))$ for all t in $[a,b]$. Then, $\vec{\omega}$ is of bounded variation on $[a,b]$. Moreover, $\int_a^b \vec{\nu}(t)\, d\vec{\omega}(t) = \int_a^b \vec{\nu}(t)\, g'(\vec{\varphi}(t))\, d\vec{\varphi}(t)$. Theorem A may be used in establishing the following result which is related to the concept of a topological index discussed by Whyburn ("Topological analysis," Princeton Univ. Press, 1958).

Theorem B. Let $\vec{\varphi}$ be a continuous complex-valued function with domain a closed interval $[a,b]$ of the real axis such that $V_a^b\vec{\varphi}$ is finite. Let p be a complex number not in the range of $\vec{\varphi}$. Let $\vec{u}$ be a continuous complex-valued function with domain $[a,b]$ such that $e^{\vec{u}(t)} = \vec{\varphi}(t) - p$ for all t in $[a,b]$. Then $V_a^b\vec{u}$ is finite. An alternate direct proof of Theorem B is also indicated. (Received July 5, 1972.)

696-30-12. KARL F. BARTH, Syracuse University, Syracuse, New York 13210 and WALTER J. SCHNEIDER, Carleton University, Ottawa 1, Ontario, Canada. A radial analog of Caratheodory's corner mapping theorem.

The following theorem represents a radial analog of the well-known theorem of Caratheodory on the angular behavior of a conformal map in the neighborhood of a corner point. Theorem. Let $c(z)$ map the upper half plane conformally onto a Jordan domain bounded by the curve $w(t)$ $(0 \leq t < 2\pi)$. Also suppose $w(0) = 0$, $\lim_{t\to 0} \arg w(t) = 0$, $\lim_{t\to 2\pi} \arg w(t) = \alpha\pi$ and $c(0) = 0$. Then if $r_\epsilon = \{z \mid |z| = r, \epsilon < \arg z < 2\pi - \epsilon\}$ and $M(r_\epsilon)$ $(m(r_\epsilon))$ = maximum (minimum) distance away from the origin of $c(r_\epsilon)$ then $\lim_{r\to 0} M(r_\epsilon)/r^\alpha = \lim_{r\to 0} m(r_\epsilon)/r^\alpha = 1$. It is possible to show by easy examples that the above theorem does not hold for $\epsilon = 0$; however if additional smoothness conditions are put on the curve $w(t)$ or if r is required to go to zero avoiding certain exemptional sets then an $\epsilon = 0$ type theorem can be proved. Also it is possible to give analogs of the second author's extension of Caratheodory's theorem (Abstract 632-54, these *Notices* 13(1966), 342). (Received July 5, 1972.)

31 Potential Theory

*696-31-1. MAYNARD G. ARSOVE, University of Washington, Seattle, Washington 98195 and ALFRED O. HUBER, Eidgen Tech Hochschule, Zurich, Switzerland. Local behavior of subharmonic functions.

Let u be a subharmonic function on the unit disc and M the concentrated mass at 0 associated with u. It is shown that $u(re^{i\theta})/\log r$ tends to M as $r \to 0$ provided either (i) θ omits values in some set A for which the points $e^{i\theta}$ on the unit circle form a set of logarithmic capacity zero or (ii) r omits values in some set B for which $\int_B (1/r^2)\, dr < +\infty$. After inversion, the problem reduces to the study of entire subharmonic functions of potential type with regard to their behavior at infinity. The following Wiener-type criterion for a Borel set E to be thin at infinity is derived and forms the principal tool in the proof. For E to be thin at infinity it is necessary and sufficient that $\sum_{n=1}^{\infty} n/\log(1/\gamma_n) < +\infty$, where γ_n is the logarithmic capacity of the part of E lying in the annulus $R^n < |z| \leq R^{n+1}$. (Received July 3, 1972.)

32 Several Complex Variables and Analytic Spaces

*696-32-1. ARUN KUMAR AGARWAL, Grambling College, Grambling, Louisiana 71245. A note on the geometric means of entire functions of two complex variables.

Let $f(z_1, z_2) = \sum_{\varkappa_1,\varkappa_2=0}^{\infty} a_{\varkappa_1\varkappa_2} z_1^{\varkappa_1} z_2^{\varkappa_2}$ be an entire function of two complex variables z_1 and z_2, holomorphic for $|z_t| \leq r_t$, $t = 1, 2$. In this note a few properties of the geometric means of the function $|f(z_1, z_2)|$ have been obtained. The results are given in the form of theorems. (Received June 29, 1972.)

*696-32-2. EDGAR LEE STOUT, University of Washington, Seattle, Washington 98195. The bounded extension problem. Discs in polydiscs.

We say that a subvariety E of an open set W in $\mathbb{C}^n$ has the bounded extension property if every bounded holomorphic function on E is the restriction to E of a bounded holomorphic function on W. Theorem. Let $\varphi : U \to U^2$ be a proper holomorphic map of the unit disc into the unit bidisc. If φ extends holomorphically to a neighborhood Ω of $\overline{U}$, then the variety $V = \varphi(U)$ in U^2 has the bounded extension property if and only if every point of $\varphi(\Omega) \cap \partial U^2 \backslash T^2$ is a regular point of $\varphi(\Omega)$ and at every point of $T^2 \cap \varphi(\Omega)$ distinct local branches of $\varphi(\Omega)$ have distinct tangents. There is an analogous but slightly more involved result in the case that U^2 is replaced by the unit n-dimensional polydisc U^n. (Received July 3, 1972.)

*696-32-3. RICHARD MANDELBAUM, University of Massachusetts, Amherst, Massachusetts 01002. Branched projective structures and flat vector bundles on Riemann surfaces. Preliminary report.

Let M be a Riemann surface of genus g. For any nonnegative integer k, let ${}_pV(2k)$ be the space of projective branched structures of total branching order 2k on M. Then ${}_pV(2k)$ is a disjoint union of complex analytic varieties, and there exists a map $j: {}_pV(2k) \to H^1(M, PL(1, \mathbb{C}))$, holomorphic on each such variety and injective for $0 \leq k < g$. Letting $u: H^1(M, SL(2, \mathbb{C})) \to H^1(M, PL(1, \mathbb{C}))$ be the canonical map we get the following results. Theorem. Let k be an integer with $-g \leq k < g$. Then: (1) If $T \in H^1(M, SL(2, \mathbb{C}))$ is an irreducible flat vector bundle with divisor order k then u(T) is an element of $j_{g-1-k}({}_pV(2g-2-2k))$. (2) Conversely if $\Phi \in j_{g-1-k}({}_pV(2g-2-2k))$ then $\Phi = u(T)$ for some $T \in H^1(M, SL(2, \mathbb{C}))$ and $\mathrm{div}(T) \geq k$. Furthermore if $k \geq 0$ then $\mathrm{div}(T) = k$. Corollary. Every irreducible unstable flat projective line bundle on M is the image of a unique branched projective structure on M of total branching order $\leq 2g-2$. (Received July 3, 1972.)

696-32-4. KYONG T. HAHN, Pennsylvania State University, University Park, Pennsylvania 16802. Generalization of the Bloch theorem for meromorphic mappings on the hyperball. Preliminary report.

Let B be the hyperball in the space C^n with the standard Kaehler metric ds_B^2 and let $P_n(C)$ be the complex projective space of dimension n with the Fubini-Study metric $d\sigma^2$. For a holomorphic mapping $f: B \to P_n(C)$, we define on B the following nonnegative functions: $q_f(z) = \inf_X (f^*(d\sigma^2)/ds_B^2)^{1/2}(z, X)$ and $Q_f(z) = \sup_X (f^*(d\sigma^2)/ds_B^2)^{1/2}(z, X)$, where inf and sup are taken for X in the tangent space at $z \in B$, and $f^*(d\sigma^2)$ denotes the pull-back form of $d\sigma^2$ by f. Then q_f and Q_f are invariant under any holomorphic automorphism of B. Using this fact and the generalization of the Landau theorem (Abstract 694-B10, these *Notices* 19(1972), A-476), the author generalizes the Bloch theorem to a family of meromorphic mappings on the hyperball in C^n with an explicit lower bound for the Bloch constant. Theorem. Let $f: B \to P_n(C)$ be a holomorphic mapping of B into $P_n(C)$ such that $q_f(0) \geq a > 0$ and $\sup_{z \in \overline{B}} Q_f(z) \leq N$ for some constants a and N. Then there is a ball of radius $\beta > 0$ (measured in $d\sigma$) in $P_n(C)$ which is the one-to-one image of an open subset of B under f. Furthermore, $\beta \geq \mathrm{Tan}^{-1}(3^{1/2}a^2/18N)$. (Received July 5, 1972.)

34 Ordinary Differential Equations

*696-34-1. STEPHEN R. BERNFELD, University of Missouri, Columbia, Missouri 65201 and GANGARAMS S. LADDE and V. LAKSHMIKANTHAM, University of Rhode Island, Kingston, Rhode Island 02881. Existence theorems for two-point boundary value problems for second order nonlinear differential systems. Preliminary report.

In this paper, we employ the Lyapunov functions to obtain the existence of solutions of the boundary value problem: $x'' = f(t, x, x')$, $x(0) - A_0 x'(0) = 0$, $x(1) + A_1 x'(1) = 0$, where $f: [[0,1] \times R^n \times R^n, R^n]$ is continuous. We use two techniques: (1) Leray-Schauder's theory and (2) modification function approach. In the special case, our results reduce to those of Lasota and Yorke ["Existence of solutions of two-point boundary value problems for nonlinear systems," J. Differential Equations 11(1972), 509-518]. (Received July 5, 1972.)

*696-35-1. JAMES M. GREENBERG, State University of New York at Buffalo, Amherst, New York 14226. The Cauchy problem for the quasi-linear wave equation.

We consider the Cauchy problem for the hyperbolic system (E) $\partial u/\partial t - \partial v/\partial x = 0$ and $\partial v/\partial t - \partial\sigma(u)/\partial x = 0$, $-\infty < x < \infty$ and $t > 0$. We assume that σ is $C^2(L,\infty)$ and satisfies $\sigma'(u_1) > 0$, $\sigma''(u_1) < 0$, $\int_{u_1}^{\infty}\sqrt{\sigma'(s)}\,ds = +\infty$, and $\sqrt{\sigma'(u_1)}\sqrt{\sigma'(u_2)} \geqq (\sigma(u_2) - \sigma(u_1))/(u_2 - u_1)$ for every $L < u_1 < u_2 < \infty$. The principal result of this investigation is Theorem 1. If the initial data $u(x,0)$ and $v(x,0)$ is bounded and measurable and of locally bounded variation and if $q_0 \overset{\text{def}}{=} \frac{1}{2}[\inf_x[v(x,0) + \int_{u_1}^{u(x,0)}\sqrt{\sigma'(s)}\,ds] - \sup_x[v(x,0) - \int_{u_1}^{u(x,0)}\sqrt{\sigma'(s)}\,ds]]$ satisfies $\int_{u_1}^{L^+}\sqrt{\sigma'(s)}\,ds < q_0$, then the Cauchy problem for (E) has a "generalized" solution. □ This theorem generalizes Nishida's results. He considered the situation where $\sigma = -1/u$, $u > 0$. There one has $\sqrt{\sigma'(u_1)}\sqrt{\sigma'(u_2)} = (\sigma(u_2) - \sigma(u_1))/(u_2 - u_1)$, $0 < u_1 < u_2$. Our results cover conservation laws when $\sigma = \alpha u^p - \beta/u^q$ and α and β are positive and $0 < p$ and $q < 1$. (Received June 19, 1972.)

*696-35-2. NEIL EKLUND, Vanderbilt University, Nashville, Tennessee 37203. A representation theorem for solutions of parabolic equations with discontinuous coefficients. Preliminary report.

Let Ω be a bounded domain in E^n with compact boundary $\partial\Omega$. Let $T > 0$ be fixed. Set $Q = \Omega x(0,T]$; Q is called a cylinder. Let $a_{ij}(x,t)$ be bounded measurable functions on Q for $i,j = 1,2,\ldots,n$ and assume the matrix $(a_{ij}(x,t))$ is uniformly elliptic a.e. on Q. Let L denote the parabolic operator $Lu = u_t - \Sigma_{i,j=1}^n\{a_{ij}(x,t)u_{x_i}\}_{x_j}$. The function $u(x,t) \equiv u(x_1,\ldots,x_n,t)$ is in $L^2_{loc}(Q)$ if for each compact cylinder $K \subset Q$, $u \in L^2(K)$. We then say u is a weak solution of $Lu = 0$ in Q if: (i) $(u_{x_1},\ldots,u_{x_n}) \equiv u_x$ exists, (ii) $u, u_x \in L^2_{loc}(Q)$, and (iii) $\int\int[\Sigma_{i,j=1}^n a_{ij}(x,t)u_{x_i}\varphi_{x_j} - u\varphi_t]\,dx\,dt = 0$ for $\varphi \in C^1(Q)$ vanishing in a neighborhood of the boundary of Q. The following theorems are obtained. Theorem 1. Let f be a given continuous function on $\partial_p Q \equiv \{\Omega x(0)\} \cup \{\partial\Omega x[0,T]\}$. Then there is a unique weak solution of the boundary value problem (*) $Lu = 0$ in Q, $u(x,t) = f(x,t)$ on $\partial_p Q$. Theorem 2. Let L and Q be as defined above. For each $(x,t) \in Q$ there is a unique measure $\mu_{(x,t)}$ defined on a σ-algebra of subsets of $\partial_p Q$ containing the Borel sets such that for each $f \in C(\partial_p Q)$ the solution u of the boundary value problem (*) can be represented in the form $u(x,t) = \int f\,d\mu_{(x,t)}$. (Received June 22, 1972.)

*696-35-3. TAPAS MAZUMDAR, Wright State University, Dayton, Ohio 45431. Existence and uniqueness results for some possibly noncoercive evolution problems with regular data.

Let $(H,(\ ,\ ))$ be a separable Hilbert space and $\forall t \in [0,T]$, $T < \infty$, $(V_t,((\ ,\ ))_t) \subset H$ be a family of Hilbert spaces dense in H with continuous injections. Let S_t be the standard operator with domain V_t such that $((x,y))_t = (S_t x, S_t y)$. Assume S_t^{-2} is weakly C^1 with $(S_t^{-2}h,k)' = (\dot{S}_t^{-2}h,k)$. Define $W = \{u \in L^2(H) \mid Su \in L^2(H)\}$. Let $a(t;\cdot,\cdot)$ be a measurable family of sesquilinear forms on $V_t \times V_t$ with $|a(t;x,y)| \leqq c\|x\|_t\|y\|_t$. Let $f \in W$, $u_0 \in V_0$ and set $F = \{v \in W \mid v' \in L^2(H),\ v(T) = 0\}$. Assume that (*) $\mathrm{Re}\int_0^T a(t;S_t^{-2}v,v)\,dt + \frac{1}{2}\int_0^T(\dot{S}_t^{-2}v,v)\,dt + \lambda(1-\delta)\cdot\int_0^T|S_t^{-1}v|^2\,dt \geqq 0$ $\forall v \in F$ for some constants $\lambda > 0$, $\delta > 0$. Then $\exists u \in W$ such that $-\int_0^T(u,v')\,dt + \int_0^T a(t;u,v)\,dt = \int_0^T(f,v)\,dt + (u_0,v(0))$ $\forall v \in F$. If $G: H \to H$ is bounded, 1-1, selfadjoint and positive, with $G(V_t) \subset V_t$, and if $\mathrm{Re}\int_0^T a(t;Gv,v)\,dt > 0$ $\forall v \in F$, $v \neq 0$, then this solution u is unique iff $G(F)$ is dense in W. Proofs utilize generalized projection theorems. For existence in the constant domain version, the assumption (*) can be replaced by $\mathrm{Re}\int_0^T a(t;\varphi,S^2\varphi)\,dt \geqq Q$ $\forall\varphi \in S^{-2}(F)$, with $Q \equiv 0$. This is an improvement upon the known result, $Q = \alpha\|\varphi\|_V^2$, obtained by putting $n = 2$, $\beta = 1$ in Abstract 682-35-1 these *Notices* 18(1971), 164 (cf. also Carroll and Mazumdar, "Solutions of some possibly noncoercive evolution problems with regular data," J. Applicable Anal. 1(1972), 381-395). Highly noncoercive examples exist where Re $a(t;u,u)$ takes up positive as well as negative values. (Received June 23, 1972.)

*696-35-4. DAVID K. COHOON, University of Minnesota, Minneapolis, Minnesota 55455. On the nonpropagation of zero sets of solutions of homogeneous linear partial differential equations across noncharacteristic hyperplanes.

The author introduces the spaces $\gamma^M(\Omega)$ for every map M of the set of n-tuples of nonnegative integers into the positive cone in $\mathbb{R}^n$ for every open subset Ω of $\mathbb{R}^n$. He notes that if $M_i(x)^{1/x}/x$ is an increasing function of x for $x \geqq 1$ for $i = 1, 2$, then if f and g are real valued functions in $\gamma^{M_1}(\mathbb{R})$ and $\gamma^{M_2}(\mathbb{R})$, respectively, then $f(g)$ and $g(f)$ are in $\gamma^{M_1M_2}(\mathbb{R})$. The space $\gamma^M(\Omega)$ is defined to be the set of f in $C^\infty(\Omega)$ such that for every $\epsilon > 0$ and for every compact subset K of Ω there is a $C > 0$ such that $|D^\alpha f(x)|\epsilon^{-|\alpha|} \cdot [\Pi_{k=1}^n M_k(\alpha)]^{-1} \leqq C$ for all x in K and all n-tuples of nonnegative integers α. The author proves the nonpropagation of zero sets of solutions of homogeneous linear partial differential equations across a noncharacteristic hyperplane, even if the solution vanishes identically on one side of the hyperplane, if the coefficients lie in a special class of spaces $\gamma^M(\mathbb{R}^2)$. Related results are obtained for solutions defined on manifolds. (Received July 5, 1972.)

696-35-5. ANDREW J. CALLEGARI and EDWARD L. REISS, Courant Institute, New York University, New York New York 10012. A stability problem for the Sine-Gordon equation.

A stability problem for the nonlinear Sine-Gordon equation is studied. The stability of a class of time independent solutions is analyzed using the linear dynamic stability theory. The two time method is used to construct an asymptotic expansion of the solution of the nonlinear problem for the transient response. A proof of the asymptotic validity of the expansion is given. Applications are given to analyzing the stability of the inverted, continuous pendulum and to the Josephson tunnel junction. (Received July 5, 1972.)

40 Sequences, Series, Summability

*696-40-1. HARRY F. JOINER II and THURLOW A. COOK, University of Massachusetts, Amherst, Massachusetts 01002. Metrizability of order intervals in locally convex spaces ordered by biorthogonal systems.

Let E be a locally convex space and $(x_\alpha; f_\alpha)$, $\alpha \in A$, be a (not necessarily countable) biorthogonal system on E. Then $(x_\alpha; f_\alpha)$, $\alpha \in A$, is a summability basis for E provided the net of all finite sums of the family $(f_\alpha(x) x_\alpha)$ converges to x for each x in E. If E is partially ordered and $a \leqq b$, then let $[a,b]$ denote the order interval from a to b. Theorem. Let E be a locally convex Hausdorff space ordered by the positive cone of a summability basis $(x_\alpha; f_\alpha)$, $\alpha \in A$. If $x \in E$ and $x \geqq 0$, then $[0,x]$ is a compact metric space iff $f_\alpha(x) \neq 0$ for at most countably many $x \in A$. R. E. Fullerton ["Geometric structure of absolute basis system in a linear topological space," Pacific J. Math. 12(1962), 137-147] proved the sufficiency of the second condition by embedding $[0,x]$ in the Hilbert cube. Our proof avoids use of the Hilbert cube and uses standard duality techniques. (Received June 12, 1972.)

696-40-2. GUTTALU R. VISWANATH, Howard University, Washington, D.C. 20001. Convergence and regularity of meromorphic Ritt series.

Let B be any class of entire functions of bounded index which contains all polynomials and is such that if $f_1, f_2, \ldots, f_n$ are elements of B and $P_1, P_2, \ldots, P_n$ are any polynomials then $P_1f_1 + P_2f_2 + \ldots + P_nf_n$ is of bounded index. Series of the form $\sum_{n=1}^{\infty}\{f_n(z)/g_n(z)\}\exp(-\lambda_n^z)$, where for each n, $f_n(z)$ is an entire function of class B and $g_n(z)$ is a polynomial of degree ν_n in the complex variable z, which have no common zeros, and where $\{\lambda_n\}$ is an increasing sequence of positive real numbers tending to infinity, are called meromorphic Ritt series. Under specified conditions it is shown that the regions of convergence and absolute convergence of the meromorphic Ritt series are both punctured half planes (which may be the punctured plane or the empty set). Furthermore the series is uniformly convergent in every bounded subset of the complex plane whose closure is included in the region of convergence, and it is M-convergent in every such subset included in the region of absolute convergence. A Landau-Pringsheim type theorem has been established for the singularities of the function represented by a meromorphic Ritt series. (Received June 29, 1972.)

41 Approximations and Expansions

*696-41-1. WILLIAM H. LING, Rensselaer Polytechnic Institute, Troy, New York 12181 and J. EDWARD TORNGA, Union College, Schenectady, New York 12308. The constant error curve problem for varisolvent families. Preliminary report.

Charles B. Dunham first noted the problem of a constant error curve for unisolvent and varisolvent families in 1968. Since that time a number of approaches have been taken to eliminate this possibility. We have proved the following Theorem. Let V be a varisolvent family, on an interval [a, b], which is dense compact. Then V does not permit a best approximation to $f \in C[a, b]$ with a constant error curve. The theorem is applied to simultaneous approximation to show a constant error may not arise there. Examples are given. (Received May 9, 1972.)

696-41-2. STANLEY J. POREDA, Clark University, Worcester, Massachusetts 01610. Best approximation by meromorphic functions with free poles. Preliminary report.

Let $D = \{|z| < 1\}$ and $U = \{|z| = 1\}$. Theorem. Let $f(z) = \sigma(z)/\prod_{k=1}^{n}(z - a_k)$ where $a_k \in D$ for k = 1, 2, ..., n and $\sigma(z)$ is analytic in D and continuous in $\bar{D}$. There then exists a unique function r(z), meromorphic in D, continuous in $\bar{D}$, having n - 1 or fewer poles in D, of best uniform approximation to f on U. Furthermore, the function r(z) is given by: $r(z) = f(z) - R(z)\prod_{k=1}^{n}[(1 - \bar{a}_k z)/(z - a_k)]$ where $R(z) = \lambda\prod_{j=1}^{K}[(1 - \bar{c}_j z)/(z - c_j)]$, and where λ and the c_j's can be explicitly determined in terms of f, and $K \leqq n - 1$. (Received June 19, 1972.)

*696-41-3. MARTIN W. BARTELT, Rensselaer Polytechnic Institute, Troy, New York 12181. Strongly unique best approximates to a function on a set and a finite subset thereof.

Let X be a compact Hausdorff space and let C(X) denote the space of continuous real valued functions on X endowed with the Chebyshev norm. Let M denote a finite dimensional subspace of C(X) and let $f \in C(X)$, $\pi \in M$ and $E = \{x : |f(x) - \pi(x)| = \|f - \pi\|\}$. It is well known that, using the Kolmogorov criterion characterizing best Chebyshev approximates, one can show that π is a best approximate to f on X if and only if π is a best approximate to f on a finite subset of E. Using characterizations of strong unicity related to the Kolmogorov criterion, it is shown that if π is a strongly unique best approximate to f on X, then π is a strongly unique best approximate to f on a finite subset A of E. The converse holds under the additional hypothesis that $m/A = 0$ implies $m \equiv 0$ on X. It is also shown that strong unicity does not occur (nontrivially) in any normed linear space W with a strictly convex dual, e.g. L^p, $1 < p < \infty$. (Received July 3, 1972.)

*696-41-4. BENJAMIN LEPSON, Naval Research Laboratory, Code 7104, Washington, D. C. 20390. Upper bounds for Dirichlet kernels and for Tchebycheff polynomials of the second kind.

This investigation concerns the behavior of $\sin n\theta$ and $(\sin n\theta)/\sin\theta$ for positive integral values of n. We start from the inequality $|\sin n\theta| < (\sin\theta)(2\cos\theta)^{-n-1}$ for $\pi/3 < \theta < \pi/2$ and $n \geqq 1$ of Byrnes and Newman [Trans. Amer. Math. Soc. 138(1969)]. We derive several systems of such inequalities, for example: Theorem. Let p and $n > 1$ be positive integers, $\alpha = p\pi/(2p+1) < \theta < \pi/2$, and $R = (n-1)\cdot\cot^2\alpha$. Then $|\sin n\theta| < (\sin\theta)[(\cos\alpha)/(\cos\theta)]^R$. This result is best possible for all p and $n \equiv 1 \pmod{2p+1}$. The case $p = 1$ therefore gives the best possible inequality of the above type. Corollary. Let $\pi/3 < \theta < \pi/2$ and $r = (n-1)/3$. Then $|\sin n\theta| < (\sin\theta)(2\cos\theta)^{-r}$. These results may be expressed in terms of Tchebycheff polynomials. Using Lanczos' definition, we write $S_n(x) = (\sin(n+1)\theta)/\sin\theta$, where $x = 2\cos\theta$, and have, for example, an equivalent form of the corollary as $|S_n(x)| < x^{-n/3}$ for $0 < x < 1$. A direct proof of the last result only, using the known recurrence relations, has been communicated to the author by J. S. Frame. Finally, upper bounds for the Dirichlet kernels follow from the above. The corollary, in particular, implies that $|D_n(x)| < 2^{-1}(2+2\cos x)^{-n/3}$ for $2\pi/3 < x < \pi$. (Received July 3, 1972.)

696-41-5. JOSEPH W. JEROME, Northwestern University, Evanston, Illinois 60201. On the universal existence of generalized interpolating spline functions. Preliminary report.

Let (a,b) be a finite interval in R and, for $m \geqq 1$, let $W^{m,2}(a,b)$ be the real Sobolev space of functions f with $D^{m-1}f$ absolutely continuous on (a,b) and $D^m f \in L^2(a,b)$ with the inner product $(u,v)_{m,2} = \Sigma_{j=0}^m (D^j u, D^j v)_2$. Let $B(u,v) = \Sigma_{i,j=0}^m (a_{ij} D^i u, D^j v)_2$ be a prescribed bilinear form on $W^{m,2}(a,b)$ with $a_{mm} = 1$ and $a_{ij} \in L^\infty(a,b)$. Finally, let V be a bounded linear operator from $W^{m,2}(a,b)$ onto a Hilbert space X such that the kernel K of V is dense in $L^2(a,b)$. Theorem 1. For each $f \in W^{m,2}(a,b)$ there exists $u \in K$ satisfying (1) $B(u-f,v) = 0$ for all $v \in K$. Lemma. There exists a closed linear operator Λ in $L^2(a,b)$ with domain $D(\Lambda)$ (the latter a Hilbert space in the graph norm) and a bounded linear operator U from $D(\Lambda)$ into X satisfying $B(u,v) = (\Lambda u, v)_2 + (Uu, Vv)_X$ for all $u \in D(\Lambda)$, $v \in W^{m,2}(a,b)$. Theorem 2. Any solution of (1) is characterized by the three conditions: (i) $\Lambda s = 0$; (ii) $Vs = Vf$; (iii) $Us = 0$. (Received July 5, 1972.)

42 Fourier Analysis

*696-42-1. D. S. GOEL and BADRI N. SAHNEY, Department of Mathematics, Statistics and Computing Science, University of Calgary, Calgary, Alberta T2N-1N4, Canada. Absolute Nörlund summability factors of a Fourier series. Preliminary report.

An infinite series $\Sigma\alpha_n$ is said to be summable $|N,p_n|$ if the sequence $t_n = (1/P_n)\Sigma P_{n-k}\alpha_k$ is of bounded variation, where $P_n = p_0 + \ldots + p_n$ is a sequence of positive constants. Let f be a periodic function with period 2π and integrable L. At a given point x we define a function $\varphi(t)$ by $\varphi(t) = \frac{1}{2}[f(x+t) + f(x-t) - 2f(x)]$, and write $\varphi_\alpha(t) = (1/\sqrt{\alpha})\int_0^t (t-u)^\alpha \varphi(u)\,du$, $\alpha > 0$. Let $\Sigma_{n=0}^\infty A_n$ be the Fourier series associated with the function f. We prove the following theorem for the $|N,p_n|$ summability of the factored series $\Sigma_0^\infty A_n\lambda_n$ at the point x, where λ_n is a sequence of nonincreasing positive constants. Theorem. If, for $0 \leqq \alpha \leqq 1$, $\varphi_\alpha(t)$ is of bounded variation, then the series $\Sigma A_n\lambda_n$ is summable $|N,p_n|$ at the point x, where p_n is a sequence of positive constants satisfying the following conditions: (1) $(n+1)p_n/P_n$ is of bounded variation, (2) $((n+1)/P_n)\Sigma_{k=1}^n |\Delta p_k|$ is bounded and (3) $\mathrm{Sup}_n\,(P_n/n^\alpha)\Sigma_{k=n}^\infty k^{\alpha}\lambda_k/P_k(k+1)$ is bounded. This includes many results proved in this direction. (Received May 18, 1972.)

*696-42-2. GORDON WOODWARD, University of Nebraska, Lincoln, Nebraska 68508. The almost periodic part of an ergodic function.

A function $\varphi \in \ell^{\infty}(Z)$ is ergodic if $\mathfrak{F}(\varphi)(t) \equiv M(\varphi e^{-it(\cdot)})$ is independent of the choice of the positive, translation-invariant, linear functional M on $\ell^{\infty}(Z)$ with norm 1, for each $t \in [0, 2\pi)$. Let $\overline{Z}$ be the Bohr compactification of Z. For an ergodic $\varphi \in \ell^{\infty}(Z)$, define $F(\varphi) \in L^{\infty}(\overline{Z})$ by $F(\varphi)^{\wedge} = \mathfrak{F}(\varphi)$. Let $\mathcal{E}(\overline{Z})$ be the set of all such $F(\varphi)$. Theorem. There exists $\psi \in \mathcal{E}(\overline{Z})$ such that $\psi^2 \notin \mathcal{E}(\overline{Z})$. In particular the "almost periodic part" of the general ergodic function does not correspond to a Weyl-almost periodic function. (For further reference see Abstract 684-B30, these *Notices* 18(1971), 519, and "Une classe d'ensembles épars," C. R. Acad. Sci. Paris 274(1972), 221-223.) (Received July 5, 1972.)

44 Integral Transforms, Operational Calculus

*696-44-1. ABDUL JABBAR JERRI, Clarkson College of Technology, Potsdam, New York 13676 and American University in Cairo, Cairo, United Arab Republic and DALE W. KREISLER, Castleton State College, Castleton, Vermont 05735. Sampling expansion with derivatives for finite Hankel and other transforms.

A method for obtaining sampling expansions involving a function, represented by a finite integral transform, and its derivatives is outlined. This includes the sampling expansions with the derivatives for the finite Hankel and Legendre transforms. It is shown that in parallel to the known special case of the finite Fourier transform the advantage of sampling with N-derivatives is to increase by $(N+1)$-fold the asymptotic spacing between the sampling points. The application of such results in the analysis of time or spacial variant systems is indicated. (Received July 3, 1972.)

46 Functional Analysis

*696-46-1. DAGMAR R. HENNEY, George Washington University, Washington, D. C. 20006. Properties of the space of convex compact sets.

Let Y be a bounded compact set in Euclidean n-space. Let $C(Y)$ denote the collection of all sets Y. One defines two operations on $C(Y)$, the operation of addition of sets and scalar multiplication: for $A, B \in C(Y)$, let $\delta A + B = \{\delta a + b : a \in A, b \in B\}$. Note that $\delta A + B \in C(Y)$. Then $C(Y)$ forms an abelian semigroup since the addition is associative and commutative. Furthermore, the cancellation law holds. Also it is easily verified that $\delta(A_1 + A_2) = \delta A_1 + \delta A_2$, $\delta A + \mu A = (\delta + \mu)A$ for $\delta\mu \geq 0$, $\delta(\mu A) = (\delta\mu)A$ and $1A = A$ so that $C(Y)$ forms an abelian semigroup with scalar operators. (For further terminology see Dagmar Henney, Abstract 630-199, these *Notices* 13(1966), 118.) The authors show how, by considering equivalence classes in $C(Y)$, more sophisticated structure can be introduced. (Received May 19, 1972.)

696-46-2. DAGMAR R. HENNEY and WALTER LINCOLN, George Washington University, Washington, D. C. 20006. Theory of sets of constant width in B-spaces.

Let R^n denote Euclidean n-space. A convex compact set $A \subset R^n$ is of constant width if the distance between any two parallel support hyperplanes is constant. In R^n, H. G. Eggleston has shown that a set of constant width is not a proper subset of any set of equal diameter. The converse is also true in R^n but not generally in any arbitrary Banach space. The authors examine finite dimensional Banach for which the converse can be established. (Received May 19, 1972.)

*696-46-3. RONALD B. KIRK, Southern Illinois University, Carbondale, Illinois 62901. Complete topologies on spaces of Baire measures.

Let X be a completely regular Hausdorff space, let L be the linear space of all finite linear combinations of the point measures on X and let M_σ denote the space of Baire measures on X. The following is proved: If M_σ is endowed with the topology of uniform convergence on the uniformly bounded, equicontinuous subsets of C^b, then M_σ is a complete locally convex space in which L is dense and whose dual is C^b (provided there are no measurable cardinals). Let M_c be the subspace of M_σ consisting of those measures which have compact support in the realcompactification of X. If M_c is endowed with the topology of uniform convergence on the pointwise bounded and equicontinuous subsets of C, then M_c is a complete locally convex space in which L is dense and whose dual is C (provided there are no measurable cardinals). Let M denote the Banach dual of C^b. If M is endowed with the topology of uniform convergence on the norm compact subsets of C^b, then M is a complete locally convex space in which L is dense. The situation in the presence of measurable cardinals is also described completely. Finally, the following is proved: If M_c has the Mackey topology for the pair (M_c, C), then M_c is complete and L is dense in M_c. (Received June 7, 1972.)

*696-46-4. ROBERT H. LOHMAN, Kent State University, Kent, Ohio 44242. Projections in locally convex spaces.

A result of Diestel, Morris and Saxon ("Varieties of linear topological spaces," Trans. Amer. Math. Soc., to appear) on product spaces is used to give simple proofs of the following theorems. Theorem 1. Let E be a separable LCS and let F be a subspace of E such that F is topologically isomorphic to c_0. Then there exists a continuous projection of E onto F. Theorem 2. Let E be an LCS and let F be a subspace of E such that F is a Banach space which is complemented in every Banach space containing F. Then there exists a continuous projection of E onto F. Theorem 3. Let E be an LCS and let F, G be subspaces of E such that F and G are Banach spaces. If F and G are totally incomparable, then $F + G$ is a Banach space. Theorem 1 shows that Sobczyk's famous result on c_0 extends to separable locally convex spaces. Theorem 3 shows that a theorem of H. P. Rosenthal ("On totally incomparable Banach spaces," J. Functional Analysis 4(1969), 167-175) holds when the underlying space is a locally convex space. (Received June 9, 1972.)

*696-46-5. SRINIVASA SWAMINATHAN, Dalhousie University, Halifax, Nova Scotia, Canada. B-convexity and reflexivity.

A Banach space X is B-convex if there exists an integer $k > 0$ and a real number $\epsilon > 0$ such that for any choice of elements $a_1, a_2, \ldots, a_k$ in the unit ball of X, there exists a combination of the + and - signs such that $\|\pm a_1 \pm a_2 \pm \cdots \pm a_k\| < k(1-\epsilon)$. Using general, regular summability methods in the characterisation of Anatole Beck [Symposium on Ergodic Theory, Tulane University, 1961] of such spaces in terms of a strong law of large numbers for X-valued random variables, we obtain a form of the law for reflexive Banach spaces. This leads to the determination of a class of reflexive B-convex spaces. (Received June 14, 1972.)

*696-46-6. NAZAR H. ABDELAZIZ, University of Maryland, College Park, Maryland 20742. Spectral properties of some positive operators in a Banach space with the decomposition property.

Let E be an ordered Banach space with the decomposition property and a closed normal generating cone K. A positive operator T of E is strongly quasi-interior if for each $0 \neq x \in K$ there exists a natural number $n \equiv n(x)$ such that $T^n x$ is quasi-interior to K. Here we prove first Theorem 1. If E is an ordered Banach space with a closed cone K then E has the decomposition property and K is normal and generating if and only if the Banach dual space of E is a locally convex lattice. This result is then used to obtain Theorem 2. Let T be a positive operator of E whose spectral radius r is a pole of the resolvent $R(\lambda, T)$, with residue P: (i) if PE is of finite dimension and K has a nonempty interior then the peripheral spectrum of T consists only of poles of $R(\lambda, T)$; (ii) if T is quasi-interior (H. H. Schaefer, Pacific J. Math. 10(1960)) then the peripheral spectrum of T is of the form rH, H is the union of a finite number of cyclic subgroups of the unit circle; each $\alpha \in rH$ is a simple pole and a simple eigenvalue; (iii) if T is strongly quasi-interior then r is the only element in the peripheral spectrum of T; the eigenspace of T associated to r is a one dimensional subspace passing through a quasi-interior point and that of T' passes through a strictly positive linear functional. (Received June 19, 1972.)

*696-46-7. JAMES F. SMITH, Le Moyne College, Syracuse, New York 13214. The structure of Hilbert modules.

Let H be a right Hilbert A-module (where A is a proper H^*-algebra), as defined by Saworotnow [Duke Math. J. 35(1968), 191-198]. Let I be a closed two-sided ideal of A, and let $M = \{f \in H : fI^{\perp} = (0)\}$; then the closed submodule M is in fact a Hilbert I-module, called the annihilator submodule with associated ideal I. Theorem 1. H is the orthogonal direct sum of its minimal annihilator submodules M_λ, with associated ideals I_λ, where each I_λ is a minimal closed two-sided ideal of A, and therefore a topologically simple H^*-algebra. Now let A be a topologically simple H^*-algebra, let Ω be the index set of a primitive projection base E in A, and let Γ be the index set of a maximal family of nonzero elements $f_\gamma e_\gamma \in H$ such that each $e_\gamma \in E$ and $(f_{\gamma_1} e_{\gamma_1}, f_{\gamma_2} e_{\gamma_2}) = 0$ if $\gamma_1 \neq \gamma_2$, $(\cdot, \cdot)$ denoting the vector inner product in H. Theorem 2. H is isomorphic to the orthogonal direct sum $\sum_{\gamma \in \Gamma} (l_2(\Omega))_\gamma$, considered as a right Hilbert module over the Schmidt class σc on $l_2(\Omega)$. If $\sum \xi_\gamma$ and $\sum \eta_\gamma$ correspond to f and $g \in H$, respectively, then the element of the trace class τc corresponding to (f,g) is $\sum \eta_\gamma \otimes \bar{\xi}_\gamma$ (convergence in the τc norm). Applications are made to the p-classes of H introduced by the author (paper to appear in Proc. Amer. Math. Soc.; see Abstract 692-B33, these *Notices* 19(1972), A-365). (Received June 23, 1972.)

*696-46-8. NEIL E. GRETSKY, University of California, Riverside, California 92502 and J. JERRY UHL, JR., University of Illinois, Urbana, Illinois 61801. Bounded linear operators into vector valued Banach function spaces.

Vector measure representations for bounded linear operators from an arbitrary Banach space to a Banach space of vector valued functions are obtained. Included is a representation for bounded linear operators into a space of indefinite integrals. A theorem deciding when these latter operators actually have range in the function space is also given. (Received June 23, 1972.)

696-46-9. PARFENY P. SAWOROTNOW, Catholic University of America, Washington, D.C. 20017. Semigroups with positive definite structure.

Let G be a semigroup with an identity and an involution $x \mapsto x^*$ ($(xy)^* = y^*x^*$ and $x^{**} = x$ holds for all $x, y \in G$). A complex valued function q on G is said to be positive definite if $\sum \lambda_i \lambda_j q(x_i^* x_j) \geq 0$ for all complex numbers $\lambda_1, \lambda_2, \ldots, \lambda_n$ and $x_1, x_2, \ldots, x_n \in G$. In a similar fashion one can define an H^*-algebra valued positive definite function. Assume that for each $x \in G$ there exists a positive number K_x such that $q(x^*x) \leq K_x q(1)$ for every complex positive definite function q on G. If it is shown that each H^*-algebra valued positive definite function on G is of the form $p(x) = (f, T_x f)$ for some representation $x \mapsto T_x$ of G on a Hilbert module H and $f_0 \in H$. Also there is an analogue of Bochner theorem for G. (Received June 26, 1972.)

696-46-10. FREDERIC CUNNINGHAM, JR., Bryn Mawr College, Bryn Mawr, Pennsylvania 19010 and NINA M. ROY, Rosemont College, Rosemont, Pennsylvania 19010. Extreme functionals on an upper semicontinuous function space.

Let Ω be compact Hausdorff. An upper semicontinuous function space on Ω is a linear space X of functions on Ω, whose values at $t \in \Omega$ are in a normed linear space X_t depending on t, satisfying: (1) for each $x \in X$ the norm function $t \mapsto \|x(t)\|_{X_t}$ is upper semicontinuous on Ω, and (2) for $f \in C(\Omega)$ the operator $x \mapsto fx$, where $fx(t) = f(t)x(t)$, takes X into X. X is given the norm $\|x\| = \sup_t \|x(t)\|_{X_t}$. For convenience assume $X_t = \{x(t) : x \in X\}$ for each $t \in \Omega$. Theorem. The extreme points of the unit ball in X^* are the "evaluation functionals" e_{t,p_t} defined for $t \in \Omega$ with $X_t \neq \{0\}$ and p_t an extreme point of the unit ball in X_t^* by $e_{t,p_t}(x) = p_t(x(t))$. This includes a result announced by W. J. Strøbele (Abstract 72T-B119, these *Notices* 19(1972), A-443). The proof is a generalization of a known proof of the Arens-Kelley Theorem, which is the case $X = C(\Omega)$. (Received June 26, 1972.)

*696-46-11. WILLIAM H. RUCKLE, Clemson University, Clemson, South Carolina 29631. Classification of biorthogonal sequences.

Various equivalences are established for a complete biorthogonal sequence $\{e_i, E_i\}$ in a Banach space X. Examples. (A) The following are equivalent: (1) For each K-element subset of X the identity mapping I from X into X lies in the closure of $[E_i \otimes e_i]$ with respect to the (non-Hausdorff) topology of operators given by pointwise convergence on that set. (2) For each finite dimensional continuous linear mapping T from X into X of rank $\leqq K$, $E_i(Te_i) = 0$ for each i implies T has zero trace. (3) For $\{x_1', \ldots, x_K'\} \subset X^*$ there is a BK-space $\mathcal{L}$ and a continuous linear functional E on $\mathcal{L}$ such that $(x_n'(e_i)E_i(x)) \in \mathcal{L}$ for $n = 1, 2, \ldots, K$ and each $x \in X$ and $E(x_n'(e_i)E_i(x)) = x_n'(x)$. (4) For each K-dimensional subspace F of X there is a sequence of finite dimensional diagonal linear mappings $\{T_n^{(F)}\}$ such that $\lim_n T_n^{(F)} y = y$ for $y \in F$. (B) The following are equivalent: (1) X has the approximation property and if T is a nuclear mapping from X into X for which $E_i(Te_i) \geqq 0$ for each i, the trace of T is $\geqq 0$. (2) There is a BK-space $\mathcal{L}$ containing all sequences of the form $(x'(e_i)E_i(x))$ with x in X and x' in X* and a positive continuous linear functional E on $\mathcal{L}$ for which $E(x'(e_i)E_i(x)) = x'(x)$. (3) X has the approximation property and I is in the weak* closure of the cone determined by $\{E_i \otimes e_i\}$. (Received July 3, 1972.)

*696-46-12. MOON WUKI KIM, Seton Hall University, South Orange, New Jersey 07079. A common fixed point theorem for commuting nonexpansive mappings. Preliminary report.

The following generalization of Kakutani-Markov theorem in the Banach space context is proved: Theorem. Let X be a Banach space and let C be a nonempty compact star domain of X. If F is a nonempty commutative family of nonexpansive self mappings of C, then there exists a common fixed point for F in C. (Received July 3, 1972.)

*696-46-13. BEN G. ROTH, University of Wyoming, Laramie, Wyoming 82070. Finitely generated submodules of differentiable functions. II.

Let $\mathcal{E}(\Omega)$ be the space of real-valued infinitely differentiable functions on an open set Ω in R^n equipped with the topology of uniform convergence of all derivatives on all compact subsets of Ω. Let $[\mathcal{E}(\Omega)]^p$ denote the Cartesian product of $\mathcal{E}(\Omega)$ with itself p-times equipped with the product topology. $[\mathcal{E}(\Omega)]^p$ is an $\mathcal{E}(\Omega)$-module. In the present paper, the finitely generated submodules of $[\mathcal{E}(\Omega)]^p$ which are closed in $[\mathcal{E}(\Omega)]^p$ are characterized for $\Omega \subset R^1$. Partial results are obtained for $\Omega \subset R^n$ and applications are made to systems of variable coefficient linear differential equations. (Received July 3, 1972.)

*696-46-14. GEORGE W. CROFTS, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. A generalization of echelon spaces. Preliminary report.

Let E be a locally convex space. For $1 \leq r < \infty$, $\ell^r\{E\}$ denotes the space of all sequences $x = (x_n)$ in E for which $P(x) = (\sum_{n=1}^{\infty}(p(x_n))^r)^{1/r} < \infty$, for each continuous seminorm p on E. $\ell^r\{E\}$ is an l.c.s. under coordinate operations and the seminorms P. Let $\omega(E)$ denote the space of all sequences in E. We consider sequences of operators, $\{T^k\}$, of $\omega(E)$ into itself where $T^k(x)$ is defined coordinatewise as $(T_n^k(x_n))_n$ and each T_n^k is a linear homeomorphism on E. If $\mathcal{N}(E)$ is a zero neighborhood base of barrels for E such that $T_n^{k+1}(U) \subset T_n^k(U)$, for each $U \in \mathcal{N}(E)$ and each n and k, then $S(E) = \bigcap_k T^k(\ell^r\{E\})$ is a subspace of $\omega(E)$. When S(E) is given the projective topology defined by the maps $\{(T^k)^{-1}\}$, we call S(E) an echelon space over E. For U and V elements of $\mathcal{N}(E)$, we denote by $\tilde{T}_n^j$ the map from $\hat{E}_V$ into $\hat{E}_{T_n^j(V)}$ induced by T_n^j, and by $\widetilde{(T_n^k)^{-1}}$ the map from $\hat{E}_{T_n^k(U)}$ into $\hat{E}_U$ induced by $(T_n^k)^{-1}$. Theorem. S(E) is a nuclear space iff for each k and each $U \in \mathcal{N}(E)$, there is a j and a $V \in \mathcal{N}(E)$, with $\mathrm{E}_{n=1}^{\infty}\nu(\widetilde{(T_n^k)^{-1}} \circ \widetilde{(T_n^j)}) < \infty$, where ν is the nuclear norm for maps from $\hat{E}_V$ into $\hat{E}_U$. A similar characterization of Schwartz echelon spaces over E may be given. (Received July 3, 1972.)

*696-46-15. ERIK G. F. THOMAS, Yale University, New Haven, Connecticut 06520. Vector valued measures and summable functions. Preliminary report.

Theorem 1. Let E be a separable or weakly sequentially complete Banach (or Frechet) space and let H be a subset of the dual, separating the points of E. Let $m : \mathcal{A} \to E$ be a set function, defined on a σ-algebra $\mathcal{A}$, such that x^*m is countably additive for all $x^* \in H$. Then m is countably additive. The use of the theorem of Banach-Krein-Smulian enables one to reduce this to the case $H = E^*$, i.e. to the Orlicz-Pettis theorem.
Theorem 2. Assume that E, with the above properties, is included in a locally convex space F, with a continuous linear injection $E \hookrightarrow F$. Let S be a compact Hausdorff space with a positive Radon measure μ, and let $\vec{f} : S \to F$ be a μ-measurable (Lusin) and Pettis integrable function such that $m(A) = \int_A \vec{f}\, d\mu \in E$ for all Borel sets A. Then m is a μ-continuous E-valued vector measure. If E is separable or a reflexive Banach space, m has μ-measurable E-valued Pettis integrable density with respect to μ, if and only if $\vec{f}(s) \in E$ μ-almost everywhere. Here the first assertion follows from Theorem 1; the second uses the theorems of Grothendieck-Phillips and P. A. Meyer respectively to assert that $\vec{f}$, when taking values in E, is automatically an E-valued μ-measurable function [cf. L. Schwartz, "Radon measures in arbitrary topological spaces," Tata Institute, Bombay]. (Received July 5, 1972.)

*696-46-16. GEORGE R. GIELLIS, U.S. Naval Academy, Annapolis, Maryland 21402. On a class of vector inner product modules. Preliminary report.

Let A be a proper H*-algebra. A multiplier module is a right module over A which has a vector inner product and a certain Banach space structure. It is shown that a multiplier module generates a Hilbert module over A, and multiplier modules are characterized in terms of their association with Hilbert modules. Also, the p-classes of a Hilbert module H over A developed by J. F. Smith (see Abstract 692-B33, these *Notices* 19(1972), A-365) can be embedded as submodules of a multiplier module M over A. For $p > 2$, the completion of the p-class is also a submodule of M. (Received July 5, 1972.)

*696-46-17. CHARLES G. DENLINGER, Millersville State College, Millersville, Pennsylvania 17551. Köthe families in vector lattices. II. Preliminary report.

Let E be an Archimedean vector lattice, $\hat{E}$ its Dedekind completion and $E^{\#}$ its universal completion. Given a weak order unit 1 in $E^{\#}$, there is a unique product defined on $E^{\#}$ for which 1 is the identity. In Abstract

691-46-58, these *Notices* 19(1972), 176, the author considered vector lattices $\ell_I^p(E,1)$ $(1 \leq p \leq \infty)$ defined in terms of this product; these are ideals in the vector lattice $\omega_I(E)$ of all families $[x_i, I]$ of elements of E indexed by a set I. If $\lambda \subseteq \omega_I(E)$ we define its Köthe χ-dual, relative to 1, to be the set $\lambda^\times = \{[y_i, I] \in \omega_I(E) : \forall [x_i, I] \in \lambda, [x_i y_i, I] \in \ell_I^1(\hat{E})\}$, which is seen to be an ideal in $\omega_I(E)$. The notion of χ-perfect subsets of $\omega_I(E)$ is defined. Conditions are examined under which Köthe's theory of α-dual sequence spaces generalizes to this context. In particular, if $1 \in \hat{E}$ and $\hat{E}$ is closed under the multiplication, then $[\ell_I^\infty(E)]^\times = \ell_I^1(E)$. If 1 is a strong order unit for $\hat{E}$, and if $0 < p + q < \infty$, then $[\ell_I^p(E,1)]^\times = \ell_I^q(E,1)$; and the only self χ-dual subset of $\omega_I(E)$ is $\ell_I^2(E,1)$. Given a vector sublattice λ of $\omega_I(E)$, each $y \in \lambda$ determines a natural map $y^* : \lambda \to E$, by $y^*(x) = \sum x_i y_i$. This map is order-bounded, positive if y is positive, and order-continuous if λ is an ideal. (Received July 5, 1972.)

47 Operator Theory

*696-47-1. GOSULA NARAYANA REDDY, Universidade Federal da Bahia, Salvador, Bahia, Brasil. Wave and scattering operators for some second order elliptic operators in R^m, $m \geq 3$. Preliminary report.

Let $\tilde{L}_0$ be the selfadjoint operator in $L^2(R^m)$, $m \geq 3$, associated with the differential expression $-\sum_{i=1}^m (\partial/x_j)(p(|x|))\partial/x_j$ and $L_0 = \tilde{L}_0$ with $p \equiv 1$. Let L_{0k} and $\tilde{L}_{0k}$ be operators in $L^2(0,\infty)$ unitarily equivalent to the restrictions respectively of L_0 and $\tilde{L}_0$ to the subspace $H_{k\ell} = \{Y_{k\ell}\} \otimes L^2(0,\infty; r^{m-1})$ where $Y_{k\ell}$ is a spherical harmonic of order m. Let Q be the multiplication operator associated with q(r). In this paper the existence and completeness of the wave operators $W_\pm(L_0 + Q, L_0)$ and $W_\pm(\tilde{L}_0 + Q, \tilde{L}_0)$ are proved under the following conditions on the real valued function $q(|x|)$: (i) $\int_0^1 |q(r)|^2 r^2 \psi(r)\,dr < \infty$ where $\psi(r)$ equals $1 \cdot r\,|\ln r|$ or r according as m is 3, 4 or more, (ii) $\sup_{r \geq 1} \int_r^{r+1} |q(t)|^2 dt < \infty$, and (iii) $\int_1^\infty q(r)\,dr$ and certain conditions on the real-valued function $p(|x|)$ in case of $W(\tilde{L}_0 + Q, \tilde{L}_0)$. Under the above conditions $Q^{1/2}(L_{0k} - 2i)^{-1}$ and $Q^{1/2}(\tilde{L}_{0k} - 2i)^{-1}$ are shown to be Hilbert-Schmidt operators and a theorem of Kuroda is applied to prove the existence and completeness of $W_\pm(L_{0k} + Q, L_{0k})$ and $W_\pm(\tilde{L}_{0k} + Q, \tilde{L}_{0k})$ and hence of $W_\pm(L_0 + Q, L_0)$ and $W_\pm(\tilde{L}_0 + Q, \tilde{L}_0)$. This implies the existence of the scattering operators $W_+^*(L_0 + Q, L_0) W_-(L_0 + Q, L_0)$ and $W_+^*(\tilde{L}_0 + Q, \tilde{L}_0) W_-(L_0 + Q, L_0)$. Absolute continuity of $\tilde{L}_{0k}$ and $\tilde{L}_0$ is proved under an additional condition on the function p. (Received May 17, 1972.)

*696-47-2. RICHARD BOLSTEIN and WARREN R. WOGEN, University of North Carolina, Chapel Hill, North Carolina 27514. Subnormal operators in strictly cyclic algebras.

A set $\mathcal{A}$ of bounded linear operators on a complex Hilbert space $\mathcal{H}$ is said to be strictly cyclic if there is a vector x such that $\mathcal{A}x = \{Ax : A \in \mathcal{A}\} = \mathcal{H}$. In this case, x is called a strictly cyclic vector for $\mathcal{A}$. A vector x such that no two operators in $\mathcal{A}$ agree at x is called a separating vector for $\mathcal{A}$. Theorem. Let $\mathcal{A}$ be a uniformly closed algebra of operators which has a separating vector x such that $\mathcal{A}x$ is a closed subspace of $\mathcal{H}$ (this is the case for example if x is strictly cyclic). If $T \in \mathcal{A}$ is subnormal, then T is normal and has finite spectrum. Corollary. A subnormal operator with a strictly cyclic commutant is normal and has finite spectrum. Let μ be a finite positive Borel measure in the plane with compact support, let $H^2(\mu)$ be the closure of the polynomials in $L^2(\mu)$, and let $H^\infty = H^2 \cap L^\infty$. It is shown that $H^\infty = H^2$ only if μ has finite support. This fact is used in the proof of the theorem. (Received June 19, 1972.)

696-47-3. KAI-JAUNG PEI, University of Illinois at Chicago Circle, Chicago, Illinois 60680. Abstract Volterra operators. Preliminary report.

This thesis, written under the supervision of Professor Shmuel Kantorovitz, is an extension of his theory reviewed in MR 43 #6764. By applying the model mentioned there to $L_p(0,\infty)$, one is led to the following setting. Let $M(t)$ be a strongly continuous group of operators (infinitesimal generator iM with domain $D(M)$); a bounded operator N is said to be M-Volterra with respect to A if $ND(M) \subset D(M)$ and $[N, M] \subset AN^2$ with $A \neq 0$ bounded and commuting with both M and N. Theorem 1. The following statements are equivalent: (a) N is M-Volterra with respect to A; (b) $[N, R(\lambda;M)] = AR(\lambda;M)N^2R(\lambda;M)$, $\lambda \in \rho(M)$; (c) $[N, M(t)] = itANM(t)N$. Let $\{N(\zeta); \operatorname{Re}\zeta > 0\}$ be a "regular" holomorphic semigroup of type ν with $N(\zeta)D(M) \subset D(M)$ and $N = N(1)$ M-Volterra with respect to A. Denote $T_\zeta = M + \zeta AN$ and let $T_\zeta(t)$ be the strongly continuous group with infinitesimal generator iT_ζ. Theorem 2. T_ζ is similar to T_α if $\operatorname{Re}\zeta = \operatorname{Re}\alpha$. Theorem 3. Given $\|M(t)\| \leqq B$, T_ζ is not similar to T_α if $|\operatorname{Re}\zeta| \neq |\operatorname{Re}\alpha|$. Theorem 4. Given $\|M(t)\| \leqq B$, T_ζ is not spectral if $\operatorname{Re}\zeta \notin Z^+ \cup \{0\}$. The inequality $C(\xi)e^{-2\nu|\eta|} \leqq (1+\|AN\|\,|t|)^{-|\xi|}\|T_\zeta(t)\| \leqq He^{2\nu|\eta|}$, where $C(\xi)$ is strictly positive and H is a constant, is the main tool for proving Theorem 3 and Theorem 4. (Received June 30, 1972.)

*696-47-4. D. J. HEBERT, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. Linear approximation of nonlinear semigroups. Preliminary report.

Let A be a nonlinear operator whose domain is a dense subspace D of a Banach space B. Suppose that A is Gateaux differentiable and K-accretive, i.e. there is a positive real valued function K on D such that for each u in D and positive r, $\|u - v + v(Au - Av)\| \geqq (1 - K(u))\|u - v\|$. Suppose further that the derivative $dA(u)$ at each point u in D is the infinitesimal generator of a linear semigroup $S_t(u)$. Conditions are found under which the semigroups $S_t(u)$ can be used to approximate a solution to the initial value problem $\partial u/\partial t + Au = 0$, $u(0) = u_0$. An example is given by an elliptic operator A with coefficients dependent upon u in the Banach space $C_0(\mathbb{R}^n)$ of continuous functions with compact support. Here this method of approximations leads to connections with measure theory similar to those found in the study of markov processes and diffusion equations. (Received July 5, 1972.)

49 Calculus of Variations and Optimal Control

696-49-1. JERALD P. DAUER, University of Nebraska, Lincoln, Nebraska 68508. On bounded perturbations of controllable systems.

Suppose the system $\dot{x} = A(t)x + k(t,u)$ is completely controllable on $I = [t_0, t_1]$, where A and k are continuous and $x \in E^n$, $u \in E^m$. Let $g(t,x,u)$ be bounded on $I \times E^n \times E^m$. Theorem 1. Assume that for sufficiently large ρ the set $k(t, S_\rho(0)) + g(t,x,S_\rho(0))$ is convex, then the perturbed system $\dot{x} = A(t)x + k(t,u) + g(t,x,u)$ is completely controllable. Theorem 2. If there is an integrable function $\omega(t)$ satisfying $|g(t,x,u) - g(t,y,u)| \leqq \omega(t)|x-y|$, then the perturbed system is approximately controllable. (Received May 15, 1972.)

50 Geometry

696-50-1. MICHAEL GOLDBERG, 5823 Potomac Avenue, N. W., Washington, D.C. 20016. Three infinite families of tetrahedral space-fillers. Preliminary report.

Hilbert's eighteenth problem asked for the search for polyhedra which do not necessarily appear as fundamental regions of groups of motions, but which could still fill all of three-dimensional space by replication and suitable orientations. As a partial answer, five distinct space-filling tetrahedra have been described in the literature (Sommerville, Proc. Edinburgh Math. Soc. 41(1923), 49-57; Baumgartner, Math. Nachr. 48(1971), 213-224). In this paper, the author describes the construction of three infinite families of suitable tetrahedra of which four tetrahedra in the literature are only special cases. (Received June 26, 1972.)

52 Convex Sets and Geometric Inequalities

*696-52-1. JAMES J. TATTERSALL, Providence College, Providence, Rhode Island 02918. On the intersection of maximal m-convex subsets.

A set S, in a linear topological space E, is said to be m-convex if for each m distinct points of S at least one of the $C_{m,2}$ segments between these m points is contained in S. Let the segment between two points x and y in E be denoted by xy. For any point x in $S \subset E$, let $S_x = \{y : xy \subset S\}$. The kernel of a set S is then defined as $\ker S = \{x \in S : S_x = S\}$. It is shown that the kernel of a set S is always a subset of the intersection of all the maximal m-convex subsets of S. A sufficient condition is given for the intersection of all the maximal m-convex subsets of a set S to be the kernel of S. (Received June 19, 1972.)

696-52-2. LOUIS J. BILLERA, Cornell University, Ithaca, New York 14850 and ROBERT E. BIXBY, University of Kentucky, Lexington, Kentucky 40506. A characterization of Pareto surfaces. Preliminary report.

Let u_i be continuous concave functions on the unit m-cube I^m, for $i = 1,\dots,n$. The set $V = \{x \in R^n | x_i \leq u_i(y^i), y^i \in I^m, \sum_{i=1}^n y^i = e\}$ where $e = (1,\dots,1) \in I^m$, is called the attainable set for $u_1,\dots,u_n$. The Pareto surface of V is the set of all maximal points of V (with respect to the normal partial order on R^n). Pareto surfaces are of interest in economics. Theorem. $V \subseteq R^n$ is the attainable set for some concave continuous $u_1,\dots,u_n$ on some unit m-cube if and only if $V = C - R^n_+$ where $C \subseteq R^n$ is compact and convex. For any attainable set in R^n, a representation can be given with $m \leq n(n-1)$. This result is applied to obtain some partial results on characterizing so-called market games without side payments. (Received July 3, 1972.)

696-52-3. HANS-HEINRICH HERDA, Boston State College, Boston, Massachusetts 02115. A new characterization of spheres. Preliminary report.

Let K be a compact convex subset of E^3 which has interior points. Denote the (finite) surface area of K by s. In each vector direction $\vec{v}$ consider that plane $p(\vec{v})$ which cuts K into two pieçes each having surface area s/2. Denote the area of $p(\vec{v})$ cut off by K by $a(\vec{v})$. Let the inf of $a(\vec{v})$ over all vector directions be a. Then $s \geqq 4a$, and equality is attained only for spheres. Generalization of this problem to more general compact sets will be discussed. (Received July 5, 1972.)

53 Differential Geometry

*696-53-1. WILLIAM B. GORDON, Mathematics Research Center, Naval Research Laboratory, Washington, D. C. 20390. An analytical criterion for the completeness of Riemannian manifolds. Preliminary report.

If M is a (not necessarily complete) Riemannian manifold with metric tensor g_{ij}, and f is any proper real valued function on M, then M is necessarily complete with respect to the metric $\tilde{g}_{ij} = g_{ij} + (\partial f/\partial x^i)(\partial f/\partial x^j)$. Using this construction one can easily prove that a Riemannian manifold is complete if and only if it supports a proper function whose gradient is bounded in modulus. (Received June 13, 1972.)

696-53-2. CHUAN C. HSIUNG, Lehigh University, Bethlehem, Pennsylvania 18015. Submanifolds of Riemannian manifolds.

• The purpose of this paper is to give a collection of global results of the following types concerning compact submanifolds of a Riemannian manifold: (1) Minimal compact submanifolds. (2) Congruence of compact hypersurfaces. (3) Isometries of compact submanifolds without boundary as well as compact hypersurfaces with boundary. (4) Umbilical property of a compact submanifold relative to a fixed normal vector field. (Received July 3, 1972.)

54 General Topology

*696-54-1. EDWARD D. TYMCHATYN and JOHN H. V. HUNT, University of Saskatchewan, Saskatoon, Saskatchewan S7N 0W0, Canada. The theorem of Mulliken-van Est for unicoherent Peano spaces.

In 1923, A. Mulliken proved the following Theorem. If $M_1, M_2, \ldots$ is a sequence of disjoint closed sets in the plane no one of which disconnects the plane, then, $M_1 \cup M_2 \cup \ldots$ does not disconnect the plane. van Est extended the theorem to Euclidean spaces in 1952. J. H. V. Hunt proved in 1971 that the above theorem can not be extended to the class of all unicoherent Peano spaces (i.e. locally compact, connected locally connected metric spaces). The Mulliken-van Est theorem can be extended as follows: Theorem. Let X be a Peano space having a covering by a sequence $U_1 \subset \overline{U}_1 \subset U_2 \subset \overline{U}_2 \subset \ldots$ of unicoherent regions with compact closures. If M_1, $M_2, \ldots$ is a sequence of disjoint closed subsets of X no one of which disconnects X then $M_1 \cup M_2 \cup \ldots$ does not disconnect X. In particular, the theorem holds for compact unicoherent Peano spaces. (Received May 30, 1972.)

696-54-2. FRANK C. KOST, State University College of New York, Oneonta, New York 13820. A zero-set condition for $\beta X \times Y = \beta X \times \beta Y$. Preliminary report.

I. Glicksberg (Trans. Amer. Math. Soc. 90(1959), 369-382) and Z. Frolik (Czechoslovak Math. J. 10(1960), 339-349) showed that $\beta X \times Y = \beta X \times \beta Y$ if and only if $X \times Y$ is pseudocompact. We offer a necessary and sufficient condition for this equality in terms of the zero-sets of $X \times Y$. Definition. A family of zero sets Z separates another family of zero sets Z' if for $F_1, F_2 \in Z$ and $F_1 \cap F_2 = \emptyset$ there is $H_1, H_2 \in Z'$ such that $F_1 \subset H_1$ and $F_2 \subset H_2$ and $H_1 \cap H_2 = \emptyset$. Let Z_1 be the family of zero sets of X and Z_2 those from Y. Set $Z_1 \times Z_2 = \{F \times H : F \in Z_1, H \in Z_2\}$ and denote the family of finite unions of members of $Z_1 \times Z_2$ by $Z_1 \times Z_{2_\Sigma}$. Theorem. $\beta X \times Y = \beta X \times \beta Y$ if and only if $Z_1 \times Z_{2_\Sigma}$ separates the zero sets of $X \times Y$, e.g. $\beta R \times R \neq \beta R \times \beta R$ since $Z_1 \times Z_{2_\Sigma}$ fails to separate the y-axis and $\{(x, 1/x) : x \neq 0\}$. Here R is the real numbers. (Received June 16, 1972.)

*696-54-3. KOK-KEONG TAN and TECK-CHEONG LIM, Dalhousie University, Halifax, Nova Scotia, Canada. C_λ-compact spaces. Preliminary report.

Definition. A topological space (X, τ) is C_λ-compact $(0 \leqq \lambda \leqq \omega)$ iff given any τ-closed subset A of X and any τ-open cover $\mathcal{G}$ of A, there exist finite subfamilies $\mathcal{G}'$ and $\mathcal{G}''$ of $\mathcal{G}$ such that (i) $A \subseteq \bigcup\{G' : G' \in \mathcal{G}'\} \cup \bigcup\{\overline{G''} : G'' \in \mathcal{G}''\}$ and (ii) Card $\mathcal{G}'' \leqq \lambda$. The notion of C_λ-compactness is a topological property lying strictly between C-compactness and compactness. A sequentially compact, C_1-compact, noncompact Hausdorff space is constructed whose product with $[0, \omega]$ is C-compact but not C_λ-compact for all $\lambda = 1, 2, \ldots$, while its product with $[0, \Omega]$ is not functionally compact. Moreover, a general method for constructing such an example is indicated. (Received June 19, 1972.)

696-54-4. KOK-KEONG TAN and TECK-CHEONG LIM, Dalhousie University, Halifax, Nova Scotia, Canada. Closed sets closed under continuous maps. Preliminary report.

A closed subset C of a Hausdorff space X is said to satisfy property (P) iff for every continuous function f from X into a Hausdorff space Y, f(C) is closed in Y. It is well known that if C is compact, then C satisfies (P). An open set O is called r-open iff every point in O has a closed neighborhood contained in O. Theorem 1. A closed subset of a Hausdorff space satisfies (P) iff every open cover $\mathscr{G}$ of C satisfying (i) each member of $\mathscr{G}$ is regular open, and (ii) $\bigcup\{G : G \in \mathscr{G}\}$ is r-open, contains a finite subfamily $\{G_1, \ldots, G_n\}$ such that $C \subseteq \bigcup_{i=1}^{n} \bar{G}_i$. Corollary 1. A closed subset C of a semiregular Hausdorff space satisfies (P) iff every open cover $\mathscr{G}$ of C such that $\bigcup\{G : G \in \mathscr{G}\}$ is r-open, contains a finite subfamily $\{G_1, \ldots, G_n\}$ such that $C \subseteq \bigcup_{i=1}^{n} \bar{G}_i$. Corollary 2. A closed subset C of a regular Hausdorff space is compact iff C satisfies (P). Theorem 2. A Hausdorff space is compact iff every proper closed subset with the relative topology is H-closed. (Received June 21, 1972.)

*696-54-5. HADI M. HADDAD and MILAN KUBR, University of Libya, Tripoli, Libya. On a class of contraction mappings.

Let X be a complete metric space. In this paper we study a class of mappings from X to X which satisfy the following conditions. There exists $0 \leqq \lambda \leqq 1$ such that: (a) $d(T^2x, Tx) \leqq \lambda d(Tx, x)$, for all x in X. (b) $d(Tx, Ty) \leqq \lambda[d(Tx, x) + d(Ty, y)]$, for all x, y in X. The main result of this paper is that every mapping of this type has a unique fixed point. It is shown that this condition is independent of continuity. Sequences of mappings of this type are also considered and the behavior of the sequences of their fixed points are investigated. (Received June 23, 1972.)

696-54-6. WOLFGANG J. THRON, University of Colorado, Boulder, Colorado 80302 and R. H. WARREN, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio 45431. The R_0 separation axiom in closure spaces.

A closure space (in the sense of Čech) is a set X with an operator c mapping the power set of X into the power set of X such that $c(\emptyset) = \emptyset$. $c(A \cup B) = c(A) \cup c(B)$ for all $A, B \subset X$ and $A \subset c(A)$ for all $A \subset X$. A closure space (X, c) is called an R_0-space iff $x \in c([y])$ implies $y \in c([x])$ for all x, y in X. Five characterizations of R_0-closure spaces are given. The most interesting is: every R_0-closure space is a subspace of a product of a certain number of copies of a fixed R_0-closure space. Three characterizations of R_0-topological spaces are shown to be false for R_0-closure spaces. (Received June 27, 1972.)

*696-54-7. CHARLES E. DICKERSON, Purdue University, Lafayette, Indiana 47907. An algebraic characterization of compact zero-dimensional spaces.

Let X be a compact Hausdorff space, and let C(X) be the ring of real-valued continuous functions on X. C(X) will be called a B-ring if f, g ∈ C(X) with 1 ∈ (f, g) implies there exists t ∈ C(X) such that 1 ∈ (f + tg). Compare with Theorem 5.7 of Gillman and Henriksen's "Rings of continuous functions" [Trans. Amer. Math. Soc. 82(1956), 366-390]. Studying a zero-dimensional space X is done by studying a dense subring of C(X) which is a B-ring (where C(X) is given the topology of uniform convergence); let A(X) be the subring of all continuous functions with finite range. Theorem. C(X) is a B-ring if and only if A(X) is dense in C(X). It is well known that A(X) is dense in C(X) if and only if X is zero-dimensional. Hence, X is zero-dimensional if and only if C(X) is a B-ring. This result generalizes Gillman and Henriksen's Theorem 5.5. Further theorems are deduced by studying Hermite B-rings. (Received June 30, 1972.)

*696-54-8. CHARLES E. AULL, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. Initial and final properties. Preliminary report.

A topological space X is initially P (w.r.t. property Q) if every continuous image (satisfying Q) of X satisfies property P. A topological space X is finally P (w.r.t. property Q) if every finer topology (satisfying Q) satisfies property P. It is proved that: if P is a property such that (a) regular Lindelöf $\rightarrow$ P, (b) P + $\aleph_1$-compactness + normality $\rightarrow$ Lindelöf, (c) P is hereditary w.r.t. closed sets, then a T_4 space is initially P w.r.t. T_4 spaces iff it is Lindelöf. A T_2 paracompact space (T_3 countably paracompact space) is initially paracompact (initially countably paracompact) w.r.t. T_2 iff it is compact (countably compact). A regular or normal space is finally regular or normal iff every dense set is open. (Received June 30, 1972.)

*696-54-9. GEORGE D. REYNOLDS and MICHAEL D. RICE, Wesleyan University, Middletown, Connecticut 06457. Some epi-reflective subcategories and their relationship with covering properties. Preliminary report.

Given a "covering condition" on a topological space, what "completeness conditions" must that space satisfy? This paper is partially concerned with some specific cases of this general problem. The main completeness condition discussed is weak-Borel completeness. Every Borel ultrafilter with the countable intersection property converges. (A space is Borel complete if every Borel ultrafilter with C. I. P. is fixed; see Hager, Reynolds, Rice: "Borel complete topological spaces," Fund. Math. 73(1972), and Simon: "A note on Rudin's example of a Dowker space," Comment. Math. Univ. Carolinae 12(1971).) The following are sample results: (1) every almost realcompact space is weak-Borel complete; (2) subparacompact and weak paracompact spaces are weak-Borel complete; and (3) weak-Borel complete spaces are closed complete. Weak-Borel complete and almost realcompact spaces are epi-reflective subcategories of regular T_1-spaces, and examples of weak paracompact, resp. subparacompact, Borel complete spaces which are not almost realcompact are given. (In Simon's paper, Rudin's example is shown to be closed complete but not weak-Borel complete.) We prove that a space is closed complete if and only if $\{X - F : F \in \mathfrak{F}\}$ has a σ-disjoint open refinement for every free ultrafilter of closed sets $\mathfrak{F}$. Finally, various topological conditions are characterized (1) in terms of images of certain filters under continuous maps to metric spaces, and (2) in terms of uniform structures. (Received June 29, 1972.)

696-54-10. SANKATHA P. SINGH, Memorial University of Newfoundland, St. John's, Newfoundland, Canada. Some results on nonlinear contractions.

(1) Let X be a Banach space and C be a closed, bounded convex subset of X. Let $T : C \rightarrow C$ be a densifying mapping and let $T_\lambda = \lambda I + (1-\lambda)T$ such that $\|T_\lambda x - y\| < \|x - y\|$, where $x \in C - F(T_\lambda)$ and $y \in F(T_\lambda)$. Then the sequence of iterates $\{T_\lambda^n x_0\}$ converges strongly to a fixed point of T. (2) Let H be a Hilbert space and $T : H \rightarrow H$ be a coercive mapping such that $1 - T$ is 1-set contraction with the following condition: If $\{x_n\}$ is a sequence in H such that $x_n - Tx_n$ converges to y as $n \rightarrow \infty$ then there exists an x_0 in H such that $x_0 - Tx_0 = y$. Then T is onto, i.e. given any $y \in H$ there exists x in H such that $Tx = y$. (Received July 3, 1972.)

*696-54-11. JERRY E. VAUGHAN, University of North Carolina, Chapel Hill, North Carolina 27514. Product spaces with compactness-like properties.

This paper gives sufficient conditions on topological spaces such that certain infinite products will be countably compact, Lindelöf, and in general, $[a, b]$-compact in the sense of open covers. Definition. Let m be an infinite cardinal number. A space X is said to have property $(1)_m$ if for every filter base F of cardinality $\leq$

m on X, there exists a compact set $K \subset X$ and a filter base G on X of cardinality $\leq$ m such that G is finer than both F and the filter base of all open neighborhoods of K. Theorem. Every product of at most m^+ spaces, each of which satisfies condition $(1)_m$, is initially m-compact (i.e., every open cover of cardinality $\leq$ m has a finite subcover). Corollary 1 (V. Saks and R. M. Stephenson, Jr.). Every product of at most $\aleph_1$ spaces, each of which has the property that every sequence has a subsequence with compact closure, is countably compact. Corollary 2 (C. T. Scarborough and A. H. Stone). Every product of at most $\aleph_1$ sequentially compact spaces is countably compact. Corollary 3 (N. Noble). Every product of at most m^+ spaces, each of which is initially m-compact and of character $\leq$ m, is initially m-compact. Some results concerning other [a, b]-compact products are given. (Received July 3, 1972.)

*696-54-12. PAUL L. STRONG, Bucknell University, Lewisburg, Pennsylvania 17837. Between the closed and the pseudo-open mappings.

Several classes of mappings which lie between the open mappings and the pseudo-open mappings have been studied by Michael, Arhangel'skii, McDougle, and others. Several analogous classes of mappings lying between the closed and the pseudo-open mappings are defined here. For example, a mapping $f: X \to Y$ is almost closed if for each subset S of Y, there exists a subset T of $f^{-1}(S)$ containing exactly one point of each $f^{-1}(s)$, such that $f(\mathrm{Cl}\, T) = \mathrm{Cl}\, S$. The elementary properties of these new mappings are given, some results are proved which relate their behavior to conditions on their domains, and a collection of examples is provided which distinguishes between the newly defined classes and which answers questions about the relative strengths of these new classes and other known classes of mappings. (Received July 3, 1972.)

696-54-13. JOAN S. BIRMAN, Stevens Institute of Technology, Hoboken, New Jersey 07030 and HUGH M. HILDEN University of Hawaii, Honolulu, Hawaii 96822. On isotopics of homeomorphisms of 3-manifolds that are branched cyclic coverings.

Let $p: X \to \underline{X}$ be a branched cyclic covering of closed oriented 3-manifolds. Let G be a homeomorphism of X that preserves fibres, and let $\underline{G}$ be the homeomorphism of $\underline{X}$ induced by $\underline{G}$. In this report, we investigate the question: "When does G isotopic to Id imply $\underline{G}$ isotopic to the identity?" We prove the following Theorem. If $\pi_2(X) = 0$, the branching set is a link in X, and G is isotopic to the identity then G leaves each component of the link fixed. (Received July 5, 1972.)

*696-54-14. DARRELL C. KENT, Washington State University, Pullman, Washington 99163 and GARY D. RICHARDSON, East Carolina University, Greenville, North Carolina 27834. Locally compact convergence spaces.

A convergence space is called locally compact if each convergent filter contains a compact subset. It is shown that the k-spaces are precisely the topological modifications of the locally compact convergence spaces. Also, the product of a pair of k-spaces is a k-space iff their locally compact modifications are topologically coherent. A convergence space (S, q) is defined to be τ-regular if $\Gamma_{\lambda q}\mathfrak{F}$ converges to x whenever $\mathfrak{F}$ converges to x, where λq denotes the topological modification of q. Products of minimal τ-regular and τ-regular-closed convergence spaces are investigated, and an example is given to show that "τ-regular-closed" is not a productive property. (Received July 5, 1972.)

696-54-15. CARL P. PIXLEY, State University of New York, Binghamton, New York 13901. A note on 0-regular maps. Preliminary report.

A proper open mapping f from metric space Y onto metric space X is a 0-regular mapping if for each $x \in X$ and $y \in f^{-1}(x)$ and $\epsilon > 0$ there is a $\delta > 0$ such that any two points in $f^{-1}(x') \cap N_\delta(y)$ can be joined by a path in $f^{-1}(x) \cap N_\epsilon(y)$ for each $x' \in X$. Theorem. If X and Y are separable metric spaces and f is a proper open 0-regular mapping from Y onto X such that $f^{-1}(x)$ is an arc for each $x \in X$, then f is locally equivalent to a product, i.e. for each $x \in X$ there is a neighborhood U of x and a homeomorphism h from $U \times [0,1]$ onto $f^{-1}(U)$ such that $\pi_1 = f \circ h$. This theorem is apparently due to H. Whitney ["Regular families of curves," Ann. of Math. 34(1933), 244-270]. Our proof is an application of Theorem 3.1" of E. Michael ["Continuous selections. I," Ann. of Math. (2) 63(1956), 361-382] to a certain space of functions. The technique is similar to that of Hamstrom and Dyer ["Completely regular mappings," Fund. Math. 45(1957), 103-118]. (Received July 5, 1972.)

696-54-16. HOWARD H. WICKE, Ohio University, Athens, Ohio 45701. On spaces whose diagonal is a set of interior condensation. Preliminary report.

For a definition of set of interior condensation see Pacific J. Math. 37(1971), 270, or Proc. Houston Conference on Point Set Topology, 1971, p. 83. For β_c see General Topology and Appl. 1(1971), 89, or the Houston Proceedings, p. 88. The class of β_c spaces properly includes the classes of p-spaces and $w\Delta$ spaces (Houston Proceedings, p. 89). Theorem. A regular T_0-space X has a base of countable order if and only if it is a β_c space and the diagonal in $X \times X$ is a set of interior condensation. Corollary (Bennett and Burney, Abstract 72T-G47, these *Notices* 19(1972), A-346). A T_3-space with G_δ diagonal has a base of countable order if it is either a p-space or a $w\Delta$ space. Further results on the class of spaces of the title have been obtained including some mapping invariances and an intrinsic characterization. Some related work on characterizing spaces having bases of countable order using sets of interior condensation is reported on by Worrell in Abstract 70T-G81, these *Notices* 17(1970), 976-977. (Received July 5, 1972.)

55 Algebraic Topology

*696-55-1. WILBUR WHITTEN, Institute for Advanced Study, Princeton, New Jersey 08540. Spanning surfaces of companioned knots.

There is a knot with nonisotopic minimal spanning surfaces [W. R. Alford, Ann. of Math. (2) 91(1970), 419-424]. Along with all fibered knots, however, there are nonfibered knots without this property. Begin with a link $L = K_1 \cup K_2$, where K_1 is trivial and K_2 is the trefoil. K_2 bounds an orientable surface that is the union of a two-cell and two bands, and K_1 links one of the bands simply. T-surgery on L yields the examples [the author, Bull. Amer. Math. Soc. 78(1972), 99-103; Proc. Amer. Math. Soc. (to appear); Amer. J. Math. (to appear)]. The proof involves general position arguments with applications of results from [H. Schubert, Acta Math. 90(1953), 131-286] and [L. P. Neuwirth, "Knot groups," Ann of Math. Studies, no. 56, Princeton Univ. Press, Princeton, N.J., 1965]. (Received June 19, 1972.)

*696-55-2. S. KINOSHITA, Florida State University, Tallahassee, Florida 32306. On branched coverings of knots and links.

A compact closed orientable 3-manifold is a branched covering of a 3-sphere S^3, with branching over a tame link and the index of branching is at most 2 (J. W. Alexander, Bull. Amer. Math. Soc. 26(1919), 370-372). The method of a proof of this theorem is applied to determine branched coverings of some knots and links in S^3. Especially, the branched covering of a knot in the Alexander-Briggs table with a monodromy map onto the symmetric group S_3 is either S^3, $S^2 \times S^1$ or a lens space, if it exists, and that of 9_{34} is $L(5,2)$. The branched covering of a doubled knot k over a trivial knot with a monodromy map onto S_3 is S^3, if it exists, and the link over k of this branched covering is also determined. (Received June 26, 1972.)

57 Manifolds and Cell Complexes

*696-57-1. PAUL F. DUVALL, JR., Oklahoma State University, Stillwater, Oklahoma 74074 and LAWRENCE S. HUSCH, University of Tennessee, Knoxville, Tennessee 37916. On the homotopy type of irregular sets.

If (X,d) is a metric space and h is a homeomorphism of X onto itself, then h is regular (positively regular) at $x \in X$ if, for each $\epsilon > 0$, there exists $\delta > 0$ such that if $d(x,y) < \delta$, then $d(h^i(x), h^i(y)) < \epsilon$ for all i $(i > 0)$. The main result of this paper is that if X is an open connected manifold and h is a homeomorphism of X onto itself such that h is positively regular on all of X and the set of points, $\mathrm{Irr}(h)$, at which h fails to be regular is a nonseparating compactum, then $\mathrm{Irr}(h)$ is a strong deformation retract of X. (Received June 12, 1972.)

*696-57-2. KENNETH C. MILLETT, University of California, Santa Barbara, California 93106. Concordances and isotopies.

For codimensions greater than two, J. F. P. Hudson has shown that concordant proper piecewise linear embeddings of one compact manifold into another are isotopic. Indeed, the concordance is isotopic to an isotopy. This may be interpreted as proving the triviality of the relative π_0 for an appropriate pair of (semi)-simplicial complexes, the result announced here concerns the higher homotopy groups of this pair. Theorem. Let $f: \Sigma^{m-1} \to \partial N$ be a p.l. embedding. If $m \geqq 1$ and $n - m \geqq 4$ then $\pi_s(C(D^m, N; f), I(D^m, N; f)) = 0$, where $C(D^m, N; f)$ and $I(D^m, N; f)$ denote the complexes of concordances and isotopies of embeddings of D^m into N which restrict to f on Σ^{m-1}, respectively. Corollary. $\pi_s(E(D^m, D^m \times \Sigma^n; j)) \cong \pi_{s+m}(\Sigma^n) \oplus \pi_{s+m}(F_n, G_n)$ if $n \geqq 4$ and $m \geqq 1$, where $E(D^m, D^m \times \Sigma^n; j)$ denotes the complex of proper embeddings of D^m into $D^m \times \Sigma^n$ which restrict to j on Σ^{m-1} with $j(x) = (x, (1, 0, \ldots, 0, 1))$. (Received June 12, 1972.)

*696-57-3. THOMAS W. TUCKER, Princeton University, Princeton, New Jersey 08540. The missing boundary theorem for 3-manifolds.

In this paper, we prove the following theorem partially conjectured by L. S. Husch ("Finding a boundary for a 3-manifold," Proc. Amer. Math. Soc. 21(1969), 64-68). Theorem. Let M be a P^2-irreducible connected 3-manifold. Suppose for every compact 3-submanifold C of M, each component of $M - \mathrm{int}\ C$ has finitely generated fundamental group. Then there is an embedding $h: M \to N$ where N is compact and $h(\mathrm{int}\ M) = \mathrm{int}\ N$ (i.e. M is a compact manifold with some closed set removed from the boundary). (Received June 26, 1972.)

696-57-4. DAVID J. SPROWS, Villanova University, Villanova, Pennsylvania 19085. Isotopy classification of homeomorphisms of a 2-sphere with n holes. Preliminary report.

Let S_n be a 2-sphere from which has been removed the interiors of n disjoint closed discs. Let $H(S_n, \partial S_n)$ denote the group of isotopy classes (rel ∂S_n) of homeomorphisms of S_n onto itself which keep the boundary pointwise fixed. Using twist homeomorphisms and homeomorphisms which spin the boundary components of S_n, a presentation is obtained for $H(S_n, \partial S_n)$. In particular, it is shown that, for $n \geqq 4$, $H(S_n, \partial S_n)$ made abelian is the free abelian group on $(n-1)(n-2)/2 - 1 + n$ generators. (Received July 3, 1972.)

696-57-5. GLEN E. BREDON, Rutgers University, New Brunswick, New Jersey 08903. Locally smooth actions on manifolds.

• Locally smooth actions of a compact Lie group on a manifold form a class of actions lying between that of continuous actions and that of differentiable actions. It is a natural class of actions to study and it avoids some of the pathology associated with both of the latter categories. We take the point of view that the orbit space of the actions to be studied is known as a stratified space, and we prove a basic lemma concerning the structure in the neighborhood of a stratum. Thus, we are concerned with a type of equivariant tubular neighborhood theorem. The analog of this theorem can also be formulated for smooth actions, but it is much deeper than the smooth equivariant tubular neighborhood theorem and its validity is unknown in general. Some remarks will be made about the cases of actions with two or three orbit types, where some classification theorems are known.

(Received July 5, 1972.)

58 Global Analysis, Analysis on Manifolds

696-58-1. RICHARD E. HEISEY, Cornell University, Ithaca, New York 14850. A closed embedding theorem. Preliminary report.

Definitions. Let B be an infinite-dimensional Banach space with conjugate B*. B* carries both its metric topology and its bounded weak-* (= b*) topology. A B*-manifold M is a (C^p, b^*)-manifold, $0 \leqq p \leqq \infty$, if M has an atlas $\{(U_\alpha, \varphi_\alpha)\}$ whose transition maps $\varphi_\beta \varphi_\alpha^{-1}$ are C^p-diffeomorphisms in the metric topology and homeomorphisms in the b* topology onto b*-open sets of B*. A (C^{-1}, b^*)-manifold is a topological manifold modelled on B* with b* topology. Define (C^p, b^*)-morphisms and (C^p, b^*)-embeddings in the natural way, $-1 \leqq p \leqq \infty$. In the following theorems M is a (C^p, b^*)-manifold as above and M(b*) denotes M with its induced b* topology. Theorem 1. If M(b*) is paracompact, then M admits (C^p, b^*)-partitions of unity w.r.t. b*-open covers, $-1 \leqq p \leqq \infty$. Theorem 2. If M(b*) is regular and Lindelöf and if there is a closed linear split embedding $B \oplus B \to B$, then there is a closed split (C^p, b^*)-embedding $f: M \to B^*$, $-1 \leqq p \leqq \infty$. Remark. If B is separable, any (C^{-1}, b^*)-manifold modelled on B* may be regarded as a manifold modelled on ℓ_2 with its bounded weak topology. (See Abstract 72T-G141, these *Notices* 19(1972).) (Received June 19, 1972.)

696-58-2. MICHAEL SHUB, Division of Natural Sciences II, University of California, Santa Cruz, California 95060. Dynamical systems, filtrations, and entropy.

• The role of filtrations for discrete dynamical systems will be considered. Filtrations play a role in a new genericity conjecture, i.e. almost all discrete dynamical systems have a fine sequence of filtrations, and in constructing the simplest model of a diffeomorphism in an isotopy class. The simplest diffeomorphisms are described via structural stability and the asymptotic number of orbits, which is to say topological entropy. Lower bounds for the entropy of an arbitrary diffeomorphism and certain structurally stable diffeomorphisms will be given in terms of the absolute values of the eigenvalues of the induced maps on the homology and cohomology groups of the manifolds. (Received July 5, 1972.)

60 Probability Theory and Stochastic Processes

*696-60-1. TOM S. PITCHER, University of Hawaii, Honolulu, Hawaii 96822. Convergence behavior of random T-fractions.

In the T-fraction $1 + d_0 z + \frac{z}{1+d_1 z+} \ \frac{z}{1+d_2 z+}$ let the d_i be independent identically distributed complex random variables. It has been shown that under certain conditions on z and the distribution of d the T-fraction converges with probability one. We can prove, however, that if the distribution of d is equivalent to Lebesgue measure then the T-fractions do not converge uniformly in any open set. (Received July 5, 1972.)

*696-60-2. ROHIT J. PARIKH, Boston University, Boston, Massachusetts 02215 and MILTON N. PARNES, State University of New York at Buffalo, Amherst, New York 14226. Conditional probabilities, determinate pairs of sets and indices. Preliminary report.

We continue our investigations of "Relative probability and uniform sets," Abstract 72T-B129, these *Notices* 19(1972), A-446. Our remark at the end of Theorem 1 is incorrect and there does exist a ρ satisfying conditions (a)—(g) as well as: (i) If $\overline{\overline{A}} < \overline{\overline{B}}$ and B is infinite, then $\rho(A,B) = 0$. (j) For the real line, if $A_n = A \cap [-n,n]$, $B_n = B \cap [-n,n]$ and $\lim_{n\to\infty} \rho(A_n, B_n) = \alpha$ exists, then $\rho(A,B) = \alpha$. Definition 1. We call a pair A,B of sets determinate if the value $\rho(A,B)$ is the same for all ρ satisfying conditions (a)—(j). Remark. If A is the usual example of a nonmeasurable subset of $[0,1]$ (congruence modulo the rationals etc.), then the pair A, $[0,1]$ is determinate and $\rho(A, [0,1]) = 0$. Definition 2. We define $i(A,B)$, the index of A relative to B, to be $\rho(A, A\cup B)/\rho(B, A\cup B)$. $i(A,B)$ takes values in $[0,\infty]$, both end values permitted. Using the notions of Definitions 1 and 2 above we are able to perform calculations quite similar to those of Ellentuck and Bumby in Fund. Math. 65(1969), 33-42. (Received July 5, 1972.)

*696-60-3. DAVID J. FOULIS, University of Massachusetts, Amherst, Massachusetts 01002. Stability of pure weights under conditioning.

Complete stochastic models for the empirical universe of discourse represented by a coherent collection of compound physical operations synthesized from a basic collection of primitive physical operations are represented mathematically by so-called weight functions. These weight functions form a convex set, the extreme points of which are the pure weights. We prove that these pure weights are mapped back into themselves under a certain natural class of conditionings. (Received July 5, 1972.)

62 Statistics

*696-62-1. JAMES M. LOWERRE, Clarkson College of Technology, Potsdam, New York 13676. Another ridge regression.

The usual ridge regression estimate of β in $y = x\beta + e$ is the solution to $(kI + X'X)^{-1}X'Y$. In this paper a different modification of the normal equations is developed, its relationship with the usual method discussed, and an example from experimental design shown. (Received July 3, 1972.)

696-62-2. MILTON SOBEL, Department of Statistics, University of Minnesota, Minneapolis, Minnesota 55455 and VENKATA R. R. UPPULURI, Mathematics Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830. Sparse and crowded cells in a homogeneous multinomial distribution. Preliminary report.

Suppose n balls are dropped independently into c cells. A cell is said to be u-sparse if the number of balls in it is less than or equal to u. Let b ($\leq c$) of the cells be colored blue, and let p be the probability that a ball may fall in a blue cell. We obtain the probability distribution of the number of u-sparse blue cells in terms of incomplete Dirichlet integrals. This distribution is used in the sparse cell test which is an extension of the empty cell test (see F. N. David, "Two combinatorial tests of whether a sample has come from a given population", Biometrika 37(1950), 97-110). As a dual concept, a cell is said to be t-crowded if it has at least t balls in it. We obtain the distribution of t-crowded blue cells, the joint distribution of t-crowded and u-sparse blue cells, and study some properties of the moments. (Received July 5, 1972.)

696-62-3. CHARLES H. RANDALL, University of Massachusetts, Amherst, Massachusetts 01002. Bayesian inference on nonclassical stochastic models. Preliminary report.

The Bayesian rule of inference can be utilized to modify, in the face of new experimental data, complete stochastic models, even when they are defined for nonclassical empirical universes of discourse such as found in quantum mechanics. The resulting procedure then does satisfy a Bayes type theorem. (Received July 5, 1972.)

65 Numerical Analysis

*696-65-1. MARK M. LOTKIN, General Electric Company, Philadelphia, Pennsylvania 19101. A general N-station solution of the optical triangulation problem.

The problem of accurately determining the position in space of a moving object that is being tracked simultaneously by a number of fixed optical sensors is solved here by a maximum likelihood approach in which the weighted sum of the squares of the distances between all pairs of lines of sight from the optical sensors to the object are minimized. The method provides also an estimate of the propagated position covariance matrix. Application of this "REDOP" method to a number of simulated trajectories has shown that positions may be calculated to within 5-8 feet, and that measurement errors such as biases and standard deviations may be recovered to within 80% of their exact values. (Received June 13, 1972.)

*696-65-2. R. D. RIESS, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. Error estimates for numerical differentiation.

A special type of Richardson extrapolation for numerical evaluation of the derivative of a function at a point is considered. Convergence of the procedure is established for functions whose derivatives are "smooth" [Zygmund, Duke Math. J. 12(1945), 47-76]. Error estimates are also given for functions of different orders of continuity. (Received June 16, 1972.)

*696-65-3. NAI-KUAN TSAO, Applied Mathematics Research Laboratory, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio 45433. Computable error bounds for inner product evaluation.

In floating-point computations, the accurate evaluation of the inner product $s_n^0 = \sum_{i=1}^n a_i b_i$ is very important in solving linear algebraic problems. Due to round-off errors in actual computation, the computed s_n satisfies $s_n = \sum_{i=1}^n a_i b_i + \epsilon$ where ϵ is correction necessary to make the equation hold exactly. In this paper we give an a posteriori bound for ϵ which is simply the absolute sum of all intermediate computed products and sums. This bound is sharp compared with the one obtained using backward error analysis [J. H. Wilkinson, "Rounding errors in algebraic processes," Prentice Hall, Englewood Cliffs, N. J., 1963, pp. 18-19]. It can further be sharpened if chopped arithmetic is used to evaluate s_n^0. Two numerical examples are included. (Received June 19, 1972.)

*696-65-4. HERMANN BRUNNER, Dalhousie University, Halifax, Nova Scotia, Canada. The numerical solution of the generalized Abel integral equation by piecewise polynomials. Preliminary report.

Piecewise polynomials of degree $m \geqq 1$ and of continuity class $C[0,a]$ are used to obtain (global) approximations to the solution of a given generalized Abel integral equation on $[0,a]$. For each interval given by two successive knots the unknown coefficients in the approximate solution $s(x)$ are determined by requiring that $s(x)$ satisfy the integral equation at m distinct points chosen suitably between these two knots. The existence of a unique set of coefficients for each such interval is based on the Haar property of the basis of $s(x)$. It is shown that this approach yields, under certain conditions on the kernel of the integral equation, convergent methods of order m. The method will be compared with the so-called "product integration methods" which were introduced recently by several authors. (Received June 23, 1972.)

*696-65-5. JONATHAN D. YOUNG and PAUL LUGER, University of California, Lawrence Berkeley Laboratory, Berkeley, California 94720. Cubic spline formulation for matrix method for second order ordinary differential eigenvalues. Preliminary report.

This report describes a cubic spline formulation for the matrix method in solving for second order ordinary differential eigenvalues. The matrix method as described in Reference 1 formulates the matrices in terms of finite difference approximations. Rows of the matrices corresponding to boundary points relate to boundary conditions only and not to the differential equation. The cubic spline formulation constructs the "boundary point" rows in such a way that both boundary condition and differential equation are satisfied. For each eigenvalue so approximated, a corresponding eigenfunction is computed. An integral ratio (modified Rayleigh quotient) process is applied to this function to improve the eigenvalue approximation. Numerical examples are given to illustrate the method and compare it with the finite difference formulation. (Received June 15, 1972.)

73 Mechanics of Solids

696-73-1. J. S. BAKSHI, Federal City College, Washington, D. C. 20001. Exact and approximate solution for normal modes of vibrations. Preliminary report.

When rotary inertia and transverse shear are included the partial differential equations governing the motion of plates $\partial^2 U_i/\partial x^2 + \partial^2 U_i/\partial y^2 + d_i^2 U_i = 0$, $i = 1, 2, 3$, for a case of simply supported circular ring become $\partial^2 U_i/\partial r^2 + (1/r)\partial U_i/\partial r + (1/r)\partial^2 U_i/\partial\theta^2 + d_i^2 \cdot U_i = 0$ where U_1 and U_2 are displacement components and U_3 a potential function for twist: d_1, d_2 and d_3 express relationship between rotary inertia, coefficient of shear, thickness of plate, angular frequency, and material constants. For free vibrations the line integral gives W the plate displacement, M_r the bending moment, $M_{r\theta}$ the twisting moment which expressed in terms of U_i and other plate parameters must be equal to zero on the boundary. A solution of type $U_i = \sum_{m=0}^{\infty} [A_{im} J_m(r, d_i) + B_{im} Y_m(r, d_i)] \cdot K_i$ where $K_i = \mathrm{Cos}\, M\theta + \mathrm{Sin}\, m\theta$ for $i = 1, 2$ and $K_i = \mathrm{Cos}\, m\theta - \mathrm{Sin}\, m\theta$ for $i = 3$ yields a solution for normal modes in a closed form. An asymptotic solution for this case for the first four modes was worked out. The results were within 8%. This asymptotic approximation is being extended to various other shapes of the plates where the exact solutions are not available. (Received May 12, 1972.)

696-73-2. WILLIE R. CALLAHAN, St. John's University, Jamaica, New York 11432. Correlation between new theory and numerical results in plate vibrations.

Numerical results are given which substantiate the theory advanced by E. Reissner, R. Mindlin, and Uflyand of the Soviet Union for a plate bounded by elliptical and hyperbolic cylinders. The theory which takes transverse shear and rotatory inertia into account is compared with the classical theory of plates and algorithms are given illustrating the methods used in finding the higher modes of vibrations. (Received July 5, 1972.)

76 Fluid Mechanics

*696-76-1. SUHRIT K. DEY, Eastern Illinois University, Charleston, Illinois 61920. Finite difference solution of the incompressible, time dependent Navier-Stokes equations in polar coordinates.

The unsteady, laminar, viscous flow past a circular cylinder has been studied numerically in this work. In the equations of motion, written in polar coordinates, the dependent variables are vorticity ζ and stream function ψ. For the sake of simplifications in the numerical calculations, a transformation $r = e^{(\varphi)}$ was used. The method adopted for solution is implicit in both space and time, so that the two difference equations must be solved simultaneously sweeping the field iteratively at each time-step. Accelerated Gauss-Seidel type iteration was employed and optimized relaxation factor was used to accelerate the rate of convergence. The convergence properties of the iterative solution were studied by matrix analysis. By computer experimentation it was verified that in the present problem, the accelerated Gauss-Seidel algorithm is a well-posed computation. While the previous researchers have been restricted by the time-step; cell-size and Reynolds number, such restrictions were greatly overcome by the optimized convergence acceleration in the present effort. Some close agreements were obtained with experimental results as regards to the production of the vortices, the time-lag for separation, the locations of the points of separation, and the shedding frequency of the vortices. The maximum free stream Reynolds number used was 2×10^3 and the maximum time-step was 0.01 sec. (Received May 8, 1972.)

696-76-2. FRANK HOPPENSTEADT, Courant Institute, New York University, New York, New York 10012. Analysis of some problems having matched asymptotic expansion solutions.

Many problems of mathematical physics, in particular problems of fluid mechanics, can be formulated as nonlinear evolution equations of the form $du/dt = F(u, \lambda)$, where u is an element of some abstract space, F is some operator acting in that space and λ is some parameter occurring in the problem. If a steady solution u_0 is known for some parameter value λ_0 (i.e. $F(u_0, \lambda_0) = 0$), then the method of matched asymptotic expansions may often be used to study behavior of solutions of the full problem beginning near $u = u_0$ with λ near λ_0. For various classes of problems, this approach can be justified. This classification is described in terms of the linear operator $F_u(u_0, \lambda_0)$ and the form of the nonlinearity in $F(u, \lambda)$. (Received July 5, 1972.)

696-76-3. M. C. SHEN, University of Wisconsin, Madison, Wisconsin 53706. Nonlinear propagation of surface waves.

An asymptotic theory describing the nonlinear propagation of small amplitude waves on the surface of a rotating fluid of variable depth is presented. The wavelength is long compared to the depth and short compared to the horizontal length scale. The result is that the amplitude propagates according to the Korteweg-deVries equation along the rays of linear theory. (Received July 5, 1972.)

696-76-4. MARTIN D. KRUSKAL, Princeton University, Princeton, New Jersey 08540. Representations of solutions as sums of poles.

The special solutions of the Korteweg-deVries equation $u_t + uu_x + u_{xxx} = 0$ which consist of nothing but a finite number of solutions (solitary waves) can be represented as sums of appropriately drifting poles in the complex plane. (Received July 5, 1972.)

78 Optics, Electromagnetic Theory

696-78-1. PAO-LIU CHOW, Courant Institute, New York University, New York, New York 10012. Propagation of a laser beam through a strongly fluctuating random medium.

The propagation of a focused laser beam through a turbulent medium is studied using our asymptotic results for high frequency obtained previously [Pao-Liu Chow, "Application of function space integrals to problems in wave propagation in random media," J. Math. and Phys. (1972)]. The mean, covariance and the fourth moment of the transmitted field are obtained. Two special cases are treated in detail and the results are compared with others wherever possible. (Received July 5, 1972.)

81 Quantum Mechanics

696-81-1. REESE T. PROSSER, Dartmouth College, Hanover, New Hampshire 03755. Inverse scattering problems.

A variety of special results for inverse scattering problems for potential, refraction and boundary scattering are briefly reviewed, and then a general framework for the analysis of such problems is proposed. This framework is based on the inversion of appropriate perturbation expansions for the associated direct problems. It is shown that most of the special results discussed can be obtained within this framework. (Received July 5, 1972.)

82 Statistical Physics, Structure of Matter

696-82-1. VENKATA R. R. UPPULURI and JAMES M. DOLAN, Mathematics Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830. Extension of Griffiths inequalities to Gaussian spin configuration models. II. Preliminary report.

In a previous paper (Abstract 691-82-1, these *Notices* 19(1972), A-243), we proved the validity of Griffiths inequalities when the Hamiltonian is given by $H = -J_0 \sum_{1 \leqq u < v \leqq n} \sigma_u \sigma_v$, and the random vector $(\sigma_1, \sigma_2, \ldots, \sigma_n)$ has the equicorrelated multivariate Gaussian distribution $N(0, A)$, with $A = (\lambda\, a_{ij})$, $\lambda > 0$, $a_{ii} = 1$, $a_{ij} = \rho$ for $i \neq j$. In this paper we consider a more general Hamiltonian given by $H = -\sum_{1 \leqq u < v \leqq n} J_{uv} \sigma_u \sigma_v$ and prove the following Theorem. In the case of an equicorrelated Gaussian "spin" configuration model, with the conditions (i) $-1/(2n-3) < \rho < 1$, (ii) $-\rho/\{(1-\rho)[1+(n-1)\rho]\} < \lambda\beta J_{k\ell}$, for $k, \ell = 1, 2, \ldots, n$ and (iii) for $k \neq \ell$, $\lambda\beta \sum_{\ell=1}^{n} J_{k\ell} < \{(1-\rho) + (n-1)(\rho - |\rho|)\}/\{(1-\rho)[1+(n-1)\rho]\}$ for $k = 1, 2, \ldots, n$ we have Griffiths inequalities (a) $\langle \sigma_k \sigma_\ell \rangle \geqq 0$ and $\langle \sigma_k \sigma_\ell \sigma_u \sigma_v \rangle - \langle \sigma_k \sigma_\ell \rangle \langle \sigma_u \sigma_v \rangle \geqq 0$ for $k, \ell, u, v = 1, 2, \ldots, n$. (Received July 3, 1972.)

86 Geophysics

696-86-1. W. STEPHEN CHILDRESS, Courant Institute, New York University, New York, New York 10012. Multiple-scale theory of hydromagnetic dynamos.

The multiple-scale perturbation methods utilized in some recent models of the earth's magnetic field are outlined, and examples of the resulting class of hydromagnetic dynamos are discussed. (Received July 5, 1972.)

94 Information and Communication, Circuits, Automata

*696-94-1. MURRAY HOCHBERG, City University of New York, Brooklyn College, Brooklyn, New York 11210. Generalized multicomponent systems.

Using the theory of Hirsch, Meisner and Boll, we study the consequences of interchanging parts within a generalized coherent structure. This procedure has been termed "cannibalization". We extend the theory of cannibalization to the case where each component can operate at several levels of partial performance and we permit the structure to take on several possible values of performance. The main result is a representation theorem, which expresses the state of a system as a function of the number of working parts at each level. We then study the stochastic theory of these systems and derive a formula for the probability distribution of the cannibalized structure function. (Received April 19, 1972.)

696-94-2. MARK ABLOWITZ, Clarkson College of Technology, Potsdam, New York 13676. Semiresonant interactions and frequency dividers.

The question of evolution of a system through an internal resonance configuration is studied. In particular the time dependent Duffing equation is considered and the surprising result is found that forward and reverse transitions are possible without the presence of dissipation. (Received July 5, 1972.)

98 Mathematical Education, Collegiate

*696-98-1. SISTER M. CORDIA EHRMANN, Villanova University, Villanova, Pennsylvania 19085. Graphing counterexamples in multivariate analysis. Preliminary report.

This paper constitutes a progress report on what is conceived to be a long-ranged undertaking. The project involves the construction of functional (versus merely decorative) models to illustrate various concepts and counterexamples in multivariate calculus, real variables, topology, and calculus of variations. The set of models to be displayed in conjunction with the presentation of the paper includes Steiner's (Roman) surface. This is the graph of a quartic equation which has the topological property of being a closed one-sided figure. Also shown will be an array of level curves of Steiner's surface executed on a Calcomp Plotter. It is now planned that an offshoot of this research will be the subsequent development of a system of sensory aids for blind students of college mathematics. (Received June 8, 1972.)

SITUATIONS WANTED

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ERRATA
Volume 18

K. DEMYS. The accepted rule for matrix addition is in general false, Abstract 71T-A243, Page 947.

Add: Note, however, that AD = DA does in fact hold since Ei = - iE, and thus the matrix addition rule remains sustained, but the problem of finding $\begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}^{1/3}$ remains, and we conjecture that there are matrices whose square roots are not matrices.

ERRATA
Volume 19

JOHN T. BALDWIN. Equational classes categorical in uncountable power, Abstract 72T-A97, Page A-431.

Replace "no finite members" by "a finite member".

DAVID BORWEIN and S. J. H. RIZVI. Generalized strong summability, Abstract 72T-A182, Page A-528.

Theorem 2 should read as follows: Theorem 2. For $\lambda > 0$, $S_n \to \ell[A_\lambda]_p$ if and only if $S_n \to \ell[A'_\lambda]_p$ and $n \cdot u_n \to O[A_{\lambda-1}]_p$.

GARY CHARTRAND and SEYMOUR SCHUSTER. A note on cycle Ramsey numbers, Abstract 72T-A118, Page A-500.

The abstract, as printed, reads: "Since Bondy has proved $c(m,n) \leqq 2m-1$, Theorem 1 implies that $c(m,n) = 2m-1$ for m odd." It should read: "Since Bondy has proved $c(m,n) \leqq 2m-1$, Theorem 1 implies that $c(m,m) = 2m-1$ for m odd."

BYOUNG-SONG CHWE and WILLIAM H. RANT. A characterization of the minimum condition for a local ring and certain perfect rings, Abstract 72T-A25, Page A-8.

In the last sentence of the abstract "ascending" should be replaced by "descending".

HENRY E. FETTIS. A hyperelliptic integral, Abstract 691-33-1, Page A-125.

First integral should read $\int_0^x (1-t^8)^{1/2} dt$.

BARUCH GERSHUNI. Some achievements of the simplified Elementary Theory of Totalities (ETT). I, Abstract 72T-E21, Page A-332.

Line 12 should read "Not any class may be transformed to a set".

MOURAD EL-HOUSSIENY ISMAIL. On the equation $\varphi^*(x) = n$. Preliminary report, Abstract 72T-A121, Page A-501.

On line 4, the word "where" should read "when".

JOHN DAVID LOGAN. Higher dimensional problems in the calculus of variations, Abstract 72T-B120, Page A-443.

The title should read "Higher dimensional problems in the discrete calculus of variations".

ELWOOD G. PARKER. Concerning some generalizations of separability. Preliminary report, Abstract 691-54-26, Page A-202.

The phrase "for $m = \aleph_0$" should precede "(4) implies (3) in locally compact T_2 spaces" in the last line.

WILLIAM R. WADE. Uniqueness of Haar series which are summable to Denjoy integrable functions, Abstract 691-42-1, Page A-150.

There is a third hypothesis to Theorem 1 which was omitted: (iii) $\lim_{n\to\infty} \sum_{k=0}^{n} (1 + k/(n+1)) |\alpha_k \chi_k(\rho)| < \infty$ for every dyadic rational $\rho \in (0,1)$. The author has counterexamples to show that Theorem 1 is false if (iii) fails at even one dyadic rational ρ.

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