Notices

OF THE AMERICAN MATHEMATICAL SOCIETY

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June 1973
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Calendar

This Calendar lists all of the meetings which have been approved by the Council up to the date this issue of the *Notices* was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change; this is particularly true of meetings to which no numbers have yet been assigned.

Abstracts should be submitted on special forms which are available in most departments of mathematics; forms can also be obtained by writing to the headquarters of the Society. Abstracts to be presented at the meeting *in person* must be received at the headquarters of the Society in Providence, Rhode Island, on or before the deadline for the meeting.

Meeting Number	Date	Place	Deadline for Abstracts* and News Items
706	August 20–24, 1973 (78th Summer Meeting)	Missoula, Montana	June 28, 1973
707	October 27, 1973	Cambridge, Massachusetts	Sept. 6, 1973
708	November 3, 1973	Minneapolis, Minnesota	Sept. 6, 1973
709	November 16–17, 1973	Atlanta, Georgia	Oct. 1, 1973
710	November 24, 1973	Tucson, Arizona	Oct. 1, 1973
711	January 15–19, 1974 (80th Annual Meeting)	San Francisco, California	Oct. 31, 1973
712	March 7–8, 1974	Gainesville, Florida	
713	April 10–13, 1974	New York, New York	
714	April 27, 1974	Santa Barbara, California	
	August 1974	No summer meeting; International Congress (see below)	
	January 23–27, 1975 (81st Annual Meeting)	Washington, D. C.	
	January 22–26, 1976 (82nd Annual Meeting)	San Antonio, Texas	

*Deadline for abstracts not presented at a meeting (by title).

August 1973 issue: June 21
October 1973 issue: August 30
November 1973 issue: September 24

OTHER EVENTS

August 13–16, 1973	Conference on the Influence of Computing on Mathematical Research and Education Missoula, Montana
August 17–18, 1973	Preceptorial Introduction to Computer Science for Mathematicians Missoula, Montana
September 3–15, 1973	International Meeting on Combinatorial Theory Rome, Italy
August 21–29, 1974	International Congress of Mathematicians Vancouver, B.C., Canada

Please affix the peel-off label on these *Notices* to correspondence with the Society concerning fiscal matters, changes of address, promotions, or when placing orders for books and journals.

The *Notices* of the American Mathematical Society is published by the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904, in January, February, April, June, August, October, November, and December. Price per annual volume is $10. Price per copy $3. Special price for copies sold at registration desks of meetings of the Society, $1 per copy. Subscriptions, orders for back numbers (back issues of the last two years only are available), and inquiries should be addressed to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904. Second class postage paid at Providence, Rhode Island, and additional mailing offices.

Notices
OF THE
AMERICAN MATHEMATICAL SOCIETY

Everett Pitcher and Gordon L. Walker, Editors
Wendell H. Fleming, Associate Editor

CONTENTS

The Seven Hundred Fifth Meeting
Western Washington State College
Bellingham, Washington
June 16, 1973

The seven hundred fifth meeting of the American Mathematical Society will be held at Western Washington State College in Bellingham, Washington, on Saturday, June 16, 1973. The Mathematical Association of America and the Society for Industrial and Applied Mathematics will hold Northwest Sectional Meetings in conjunction with this meeting of the Society. The Association will have sessions on Friday and Saturday, June 15 and 16. Aspects of the program will emphasize numerical analysis and its role in the mathematics curriculum. Professor George Polya, Stanford University, will lecture to members of all of the organizations in the Ridgeway Commons at 7:00 p.m. on Friday, June 15, 1973. He will lecture on "Galileo: His life and contributions to the scientific method."

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, there will be two invited addresses. Professor Burton Rodin of the University of California, San Diego, will lecture at 11:00 a.m. on Saturday. The title of his talk is "The method of extremal length." Professor Haskell P. Rosenthal of the University of California, Berkeley, will lecture at 2:00 p.m. on Saturday on "The Banach spaces $C(K)$ and $L^p(\mu)$." These addresses will be given in Room 163 of Miller Hall. There will be sessions for contributed papers on Saturday morning. These sessions will be held in Bond Hall. Overhead projectors will be available for use by the speakers. Persons planning to use one should check at the registration desk in advance. Late papers will be accepted for presentation at the meeting, but will not appear in the printed program of the meeting.

The registration desk will be located in Room 104 of Bond Hall, and will be open for the duration of the meetings.

Dormitory rooms are available on campus for the nights of June 14, 15, and 16. The rates are $4 per person per night on a double occupancy basis, and $6 per person per night in a single room. Towels, sheets, and bedding are supplied, but there is no maid service. Families are welcome at the above rates. Reservations for dormitory rooms should have been sent prior to June 4, 1973. The dormitory assignments will be available at the registration desk.

The following motels are located in Bellingham (zip code 98225), but none of them is within easy walking distance of the campus.

BELL MOTEL
208 North Samish Way
Phone: (206) 733-2520

Single	$ 7.95 up
Double	9.95 up
Twin	10.95 up

KEY MOTEL
212 North Samish Way
Phone: (206) 733-4060

Single	$10.00 up
Double	13.00 up
Twin	15.00 up

LEOPOLD INN
1224 Cornwall Avenue
Phone: (206) 733-3500

Single	$ 8.00 - $12.50
Double	12.00 - 20.00
Twin	13.00 - 20.00

MOTEL SIX
3701 Byron Street
Phone: (206) 734-6940

Single	$ 6.60
Double	8.80

ROYAL MOTOR INN
215 North Samish Way
Phone: (206) 734-8830

Single	$10.00 - $15.00
Double	12.00 - 17.00
Twin	14.00

TRAVELODGE MOTEL
East Holly and Railroad Avenue
Phone: (206) 734-1900

Single	$11.00 up
Double	13.00 up

Reservations should be made directly with the desired motel.

Lunch service will be available on Friday and Saturday at a college dining hall for $1.75. There will be a salmon barbecue on Friday evening, June 15. Reservations should have been sent to Mathematics Meetings, Department of Mathematics and Computer Science, Western Washington State College, Bellingham, Washington 98225, prior to June 4.

Bellingham is served by Harbor Airlines with daily flights arriving from Seattle at 9:15 a.m., 1:45 p.m., 6:16 p.m., and 10:15 p.m. There is frequent Greyhound bus service from Seattle and Vancouver, British Columbia. Burlington Northern (Amtrak) train service leaves Seattle at 5:50 p.m. daily for Bellingham, and leaves Bellingham at 8:40 a.m. daily. Taxi service is available from the plane, bus, and train terminals.

Persons driving to the meeting on Interstate 5 should take the W.W.S.C. freeway exit, follow signs to the campus along College Parkway (1 mile), turn right on College Drive into the campus, and turn right again on Campus Drive to parking lot number 17A. Parking on campus is free to registrants, but a pass (obtainable at the registration desk) must be displayed inside the car.

Emergency messages may be delivered to participants at any hour by telephoning W.W.S.C. Campus Security (206) 676-3556. During the day, participants can be reached by calling the Department of Mathematics and Computer Science (206) 676-3785.

PROGRAM OF THE SESSIONS

The time limit for each contributed paper is ten minutes.
To maintain this schedule, the time limit will be strictly enforced.

SATURDAY, 9:00 A. M.

General Session, Room 106, Bond Hall

9:00- 9:10	(1)	On the decision problem for equational theories of quasi-groups. Preliminary report. Professor DON L. PIGOZZI, Iowa State University (705-E2)
9:15- 9:25	(2)	A representation theorem for quasi-varieties categorical in power. Mr. STEVEN R. GIVANT, University of California, Berkeley (705-E1)
9:30- 9:40	(3)	The nonexistence of certain rank 5 permutation groups. Preliminary report. Dr. J. STEPHEN MONTAGUE, University of Tennessee (705-A2)
9:45- 9:55	(4)	Vertices missed by longest paths or circuits. Professor BRANKO GRÜNBAUM, University of Washington (705-A1)
10:00-10:10	(5)	On Wiener's shortest-line conjecture. Mr. RICHARD R. JOSS, University of Washington (705-D1). (Introduced by Professor Branko Grünbaum)
10:15-10:25	(6)	On normal subgroups of differentiable homeomorphisms. Professor JAMES V. WHITTAKER, University of British Columbia (705-G1)
10:30-10:40	(7)	Two-plane fields and bordism. Dr. ULRICH KOSCHORKE, Rutgers University (705-G2)

SATURDAY, 9:00 A. M.

Session on Analysis, Room 112, Bond Hall

9:00- 9:10	(8)	Duals of Fréchet spaces and a generalization of the Banach-Dieudonné theorem. Preliminary report. Professor KALMAN G. BRAUNER, JR., University of Maryland (705-B3)
9:15- 9:25	(9)	Finite dimensionality and duality of B*-algebras. Professor BANSHI D. MALVIYA, North Texas State University (705-B2)
9:30- 9:40	(10)	Two types of hyperinvariant subspaces. Professor ROBERT M. KAUFFMAN, Western Washington State College (705-B4)
9:45- 9:55	(11)	The deficiency index of an ordinary differential operator. Professor RICHARD C. GILBERT, California State University, Fullerton (705-B1)
10:00-10:10	(12)	On a particular solution of certain differential equations via linear mapping of finite dimensional vector spaces. Preliminary report. Professor SADANAND VERMA, University of Nevada, Las Vegas (705-B7). (Introduced by Mr. Louis J. Simonoff)
10:15-10:25	(13)	Majorization in multivariate distributions. Professor ALBERT W. MARSHALL*, University of Rochester, and Professor INGRAM OLKIN, Stanford University (705-F1)
10:30-10:40	(14)	Classes of sentences recognizable in (exponentially) less space than their length, viewed algebraically as abstract families of languages (AFLs). Preliminary report. Professor FREDERICK N. SPRINGSTEEL, University of Montana (705-C1). (Introduced by Professor Merle Manis)
10:45-10:55	(15)	On the degree of approximation of continuous functions. Preliminary report. Dr. BADRI N. SAHNEY* and Dr. D. S. GOEL, University of Calgary (705-B8)

SATURDAY, 11:00 A. M.

Invited Address, Room 163, Miller Hall

(16) The method of extremal length. Professor BURTON RODIN, University of California, San Diego (705-B6)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.

SATURDAY, 2:00 P. M.

Invited Address, Room 163, Miller Hall

(17) The Banach spaces C(K) and $L^p(\mu)$. Professor HASKELL P. ROSENTHAL, University of California, Berkeley (705-B5)

Kenneth A. Ross
Associate Secretary

Eugene, Oregon

Symposium on Some Mathematical Questions in Biology
México, D. F.
June 30, 1973

The seventh annual symposium on Some Mathematical Questions in Biology will be held on June 30, 1973, at the Unidad de Congresos del Centro Médico Nacional, Avenida Cuauhtemoc 330, México, D. F. This symposium will be co-sponsored by the American Mathematical Society and the Society for Industrial and Applied Mathematics and is being held in conjunction with meetings of the American Association for the Advancement of Science and the Consejo Nacional de Ciencia y Technologia. The symposium will be supported by a grant from the National Science Foundation. Registration and hotel arrangements were announced in the May 11 issue of Science; a provisional program appears in the May 18 issue of the same publication.

The program has been arranged by Hans J. Bremermann, Hirsh G. Cohen, Jack D. Cowan, and Murray Gerstenhaber, all of whom are members of the AMS-SIAM Committee on Mathematics in the Life Sciences, with the assistance of José Negrete Martínez, Instituto de Investigaciones Biomédicas, Universidad Nacional Autónoma de México.

PROGRAM

June 30, 9:00 a. m.

Chairman: Jack D. Cowan, University of Chicago

Positional information and the development of pattern. LEWIS WOLPERT, Middlesex Hospital Medical School, London

The control of development. ANTHONY D. J. ROBERTSON, University of Chicago

A mathematical model in cellular biology. STEPHEN SMALE, University of California, Berkeley

Principles of vertebrate embryology. RENE THOM, Institut des Hautes Etudes Scientifiques, Paris

June 30, 3:00 p. m.

Chairman: José Negrete Martínez, Instituto de Investigaciones Biomédicas, Universidad Nacional Autónoma de México

The competition exclusion principle: How similar can coexisting species be? ROBERT M. MAY, Princeton University

A theory for cooperative activity in nervous tissue. JACK D. COWAN, University of Chicago

Computational languages for biological theory. S. PAPERT, Massachusetts Institute of Technology

Lecture (title to be announced). JOSÉ NEGRETE MARTÍNEZ, Instituto de Investigaciones Biomédicas, Universidad Nacional Autónoma de México

June 30, 8:30 p. m.

Panel Discussion: Mathematical education for life scientists. JOSÉ NEGRETE MARTÍNEZ (moderator), Instituto de Investigaciones Biomédicas, Universidad Nacional Autónoma de México

Jack D. Cowan
Chairman

Chicago, Illinois

PRELIMINARY ANNOUNCEMENTS OF MEETINGS

The Seventy-Eighth Summer Meeting, Conference on the Influence of Computing on Mathematical Research and Education, and Preceptorial Introduction to Computer Science for Mathematicians University of Montana Missoula, Montana August 13– 24, 1973

CONFERENCE ON THE INFLUENCE OF COMPUTING ON MATHEMATICAL RESEARCH AND EDUCATION

With the support of the National Science Foundation, the American Mathematical Society and the Mathematical Association of America will sponsor a four-day Conference on the Influence of Computing on Mathematical Research and Education. The conference will be held on the campus of the University of Montana, Missoula, Montana, from Monday, August 13, through Thursday, August 16, 1973.

This conference is open to all who wish to participate. Attendance is limited only by availability of dormitory and motel accommodations. Those wishing to attend should write to Dr. Gordon L. Walker, American Mathematical Society, P.O. Box 6887, Providence, Rhode Island 02904. Prospective participants are advised to preregister on the form which appears on the last page of these *Notices* in order that accommodations be held for them. Requests for participation will be acknowledged.

The topic of the conference was selected by the AMS-SIAM Committee on Applied Mathematics (Donald G.M. Anderson, Hirsh G. Cohen, Joaquin B. Diaz, Harold Grad, Stanislaw M. Ulam, and Richard S. Varga, chairman) and the officers of the American Mathematical Society and the Mathematical Association of America. The Organizing Committee includes William S. Dorn (University of Denver), Stephen J. Garland (Dartmouth College), Thomas E. Hull (University of Toronto), Donald E. Knuth (Stanford University), and Joseph P. LaSalle (Brown University), chairman.

The program will consist of nine invited one-hour lectures, five panel discussions, each of ninety minutes duration, and contributed papers selected and refereed in advance. Sessions for thirty-minute contributed papers are scheduled each afternoon. Abstracts for contributed papers should be submitted on the standard AMS abstract form to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904. Abstracts should be sent to arrive no later than June 21, 1973, marked clearly "For Computer Conference." There may be some financial support available for participants selected to present a paper at the conference. Since there are a limited number of time slots available for contributed papers, a specific notation should be made if the author would like to have the paper reconsidered for presentation at the regular AMS meeting (August 21–24). Also under consideration is an evening informal session devoted to innovative ideas and projects in this field.

The major emphasis in the lectures will be on the influence of the computer on mathematical research and on the applications of mathematics; secondarily, the lectures will consider what this means for the education of mathematicians and the teaching of mathematics. The speakers and titles are as follows: Thomas Cheatham (Harvard University), "Unexpected impact of the computer on science and mathematics"; Edward A. Feigenbaum (Stanford University), "The computer and thought: What's new"; Ulf Grenander (Brown University), "Mathematical statistics and probability"; Peter Lax (Courant Institute of Mathematical Sciences, New York University), "Applied mathematics–applications to the physical sciences"; Derrick H. Lehmer (University of California, Berkeley), "Number theory"; Charles C. Sims (Rutgers University), "Algebra"; Joseph F. Traub (Carnegie-Mellon University), "Numerical mathematics"; Stanislaw M. Ulam (University of Colorado), "Mathematics and the computer: What lies ahead"; Philip Wolfe (International Business Machines), "Optimization and operations research."

The panel discussions will emphasize the influence of the computer on the education of mathematicians and the teaching of mathematics. The titles of the panels, and the moderator for each, are: "The computer and calculus," Stephen Garland (Dartmouth College); "The computer and its relation to less traditional courses," Richard A. Karp (University of California, Berkeley); "Logistics of computer usage in education," William B. Kehl (University of California, Los Angeles); "The computer and its relation to other traditional courses," Cleve B. Moler (University of New Mexico); and "CUPM Report on the impact of the computer on the teaching of mathematics," Alex Rosenberg (Cornell University).

PRECEPTORIAL INTRODUCTION TO COMPUTER SCIENCE FOR MATHEMATICIANS

The American Mathematical Society will sponsor a two-day preceptorial introduction to computer science for mathematicians on the campus of the University of Montana, Missoula, Montana, on Friday, August 17, and Saturday, August 18, 1973.

This short course is open to all who wish to participate. Attendance is limited only by availability of dormitory and motel accommodations. Prospective participants are advised to preregister as soon as possible on the form which appears on the last page of these *Notices* in order to be sure that accommodations will be held for them.

The program is under the direction of Professor Jacob T. Schwartz, Courant Institute of Mathematical Sciences, New York University. This short course was recommended by the AMS Committee on Employment and Educational Policy whose members are Richard D. Anderson (chairman), Michael Artin, John W. Jewett, Calvin C. Moore, Richard S. Palais, and Martha Kathleen Smith.

The program will consist of six to eight lectures on various aspects of computer science intended to provide a concentrated introduction to the field, thus making it possible for the participants to judge if computer science is a subject which they would be interested in pursuing further. Lecturers will include Richard A. Karp (University of California, Berkeley), "Lower and upper bounds on the computational complexity of combinatorial problems"; Albert Meyer (Massachusetts Institute of Technology), "Discrete computation: Theory and open problems"; Jacob T. Schwartz, "Programming: The technique of algorithm description"; and possibly one additional lecturer whose name and topic will be announced in the August issue of these *Notices*.

SEVENTY-EIGHTH SUMMER MEETING

The seventy-eighth summer meeting of the American Mathematical Society will be held at the University of Montana, Missoula, Montana, from Tuesday, August 21, through Friday, August 24, 1973. All sessions of the meeting will take place on the campus of the university.

Two sets of Colloquium Lectures are scheduled. Professor Felix E. Browder of the University of Chicago will lecture on "Nonlinear functional analysis, and its applications to nonlinear partial differential and integral equations." The other set of lectures will be given by Professor Errett A. Bishop of the University of California, San Diego; the title of these lectures will be "Schizophrenia in contemporary mathematics: The problem and the cure." Both series of lectures will be given in the Ballroom of the University Center.

There will be twelve invited one-hour addresses at the meeting. The names of the lecturers, the titles of their addresses, and the dates and times for each lecture are listed in the Summary of Activities which follows this announcement. Sessions for contributed ten-minute papers will be held during the morning on Wednesday, Thursday, and Friday. Abstracts of contributed papers should be sent to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904; the deadline for receipt of abstracts is June 28, 1973. There is no limit on the number of papers that will be accepted for presentation. No provisions will be made for late papers.

The AMS Committee on Employment and Educational Policy is planning two panel discussions. The first, which is cosponsored by the MAA, is scheduled for 4:30 p.m. on Monday, August 20, 1973. Professor John W. Jewett of Oklahoma State University will serve as moderator; the topic to be discussed is "The role of the Ph.D. in two-year college teaching." The second panel discussion is scheduled for 7:30 p.m. on Thursday, August 23, 1973, and will be devoted to an analysis of the state of the job market based on the employment data which will have been collected earlier in the summer as part of the Seventeenth Annual AMS Survey. The panel members will be Professor Richard D. Anderson, Louisiana State University, and Professor Martha Kathleen Smith, Washington University (St. Louis), both members of the AMS Committee on Employment and Educational Policy.

This meeting will be held in conjunction with meetings of the Mathematical Association of America and Pi Mu Epsilon. The Mathematical Association of America will meet from Monday, August 20, through Wednesday, August 22. The Earle Raymond Hedrick Lectures, sponsored by the Association, will be given by Dr. Henry O. Pollak of the Bell Telephone Laboratories. The series of lectures, entitled "Some recent applications of mathematics," will be presented at 9:00 a.m. on Monday ("An application of geometry"), at 1:30 p.m. on Monday ("An application of algebra"), and at 9:00 a.m. on Tuesday ("An application of analysis"). The Business Meeting of the Association, presentation of the Lester R. Ford Awards and Two Special Awards will take place at 10:00 a.m. on Tuesday. All sessions of the Association's meeting will be held in the Ballroom of the University Center. Pi Mu Epsilon will meet concurrently with the Association and the Society. Professor Victor Klee, University of Washington, will address the fraternity on Tuesday evening, August 21, at 8:00 p.m.; the title of his lecture will be "Some unsolved problems from intuitive geometry."

COUNCIL AND BUSINESS MEETINGS

The Council of the Society will meet at 5:00 p.m. on Tuesday, August 21, in the McLeod Room of the Florence Motor Inn. The Business Meeting of the Society will be held in the Ballroom of the University Center at 4:00 p.m. on Thursday, August 23.

MEETING REGISTRATION

The registration desk for the meetings will be located in the University Center on the first floor mall. Following are the hours that the desk will be open:

	Computer Conference
August 12	2:00 p.m. - 7:00 p.m.
August 13	8:00 a.m. - 4:00 p.m.
August 14–16	8:30 a.m. - 4:00 p.m.
	Computer Short Course
August 17	8:00 a.m. - 4:00 p.m.
August 18	8:00 a.m. - 3:00 p.m.

AMS–MAA Summer Meeting	
August 19	2:00 p.m. - 8:00 p.m.
August 20	8:00 a.m. - 5:00 p.m.
August 21–23	8:30 a.m. - 4:30 p.m.
August 24	8:30 a.m. - 1:30 p.m.

Participants who wish to preregister for the meetings should complete the Meeting Preregistration Form on the last page of these *Notices*. Those who preregister will pay a lower registration fee than those who register at the meeting, as indicated in the schedule below. Preregistrants will be able to pick up their badges and programs when they arrive at the meeting. Complete instructions on procedure for making hotel, motel, or dormitory reservations is given in the sections entitled RESIDENCE HALL HOUSING and HOTELS AND MOTELS.

Please note that separate registration is required for each of the three meetings. Registration fees for the meetings are as follows:

	Computer Conference	
	Preregistration (deadline July 20)	At meeting
All Participants	$10	$12

	Computer Short Course	
	Preregistration (deadline July 20)	At meeting
All Participants	5	7

	AMS–MAA Summer Meeting	
	Preregistration (deadline July 27)	At meeting
Member	7	10
Student or unemployed member	1	1
Nonmember	12	15

There will be no extra charge for members of the families of registered participants.

The unemployed status refers to any member currently unemployed and actively seeking employment. It is not intended to include members who have voluntarily resigned or retired from their latest position. Students are considered to be only those currently working toward a degree who do not receive an annual compensation totaling more than $7,000 from employment, fellowships, and scholarships.

> Checks for the preregistration fee(s) should be mailed to arrive not later than July 27, 1973, for the AMS–MAA Summer Meeting, and not later than July 20, 1973, for the computer conference and computer short course. Participants may make their own reservations directly with any hotel or motel in the area if they wish. It is essential, however, to complete the Meeting Preregistration Form on the last page of these *Notices* to take advantage of the lower meeting registration fee(s).

A fifty percent refund of preregistration fee will be reimbursed for all cancellations received prior to August 13. There will be no refunds granted for cancellations received after that date or to persons who do not attend the meetings.

EMPLOYMENT REGISTER

The Mathematical Sciences Employment Register will not be in operation at this meeting.

EXHIBITS

The book and educational media exhibits will be displayed on the first level of the University Center at the following times: August 15 (Wednesday), noon to 5:00 p.m.; August 16–17 (Thursday, Friday), 9:00 a.m. to 5:00 p.m.; August 18 (Saturday), 9:00 a.m. to 2:00 p.m.; August 20–22 (Monday-Wednesday), 9:00 a.m. to 5:00 p.m.; August 23 (Thursday), 9:00 a.m. to noon. All participants are encouraged to visit the exhibits sometime during the meeting.

RESIDENCE HALL HOUSING

College facilities have been set aside for the exclusive use of the Mathematics Meetings participants and for participants of the Computer Conference and Computer Short Course. All dormitories are within a five-minute walk of the University Center and the Liberal Arts Building which will be used during the meetings.

Accommodations will be in a traditional residence hall. The rooms are clean, comfortable, and adequate. Single beds, chests or dressers, study tables, chairs, and wastebaskets are provided. Bedding provided includes mattress pad, pillow, one woolen blanket, sheets, pillow cases, and towels. Closet space is ample, and lavatory facilities are traditional dormitory facilities. Maid service is available at an extra charge of $2 per person per day. Keys, curtains, glasses, and soap are also provided. Each dormitory has a fully equipped laundry room with coin-operated washers and dryers. Ironing facilities are also available. A limited number of irons (3) will be available at each hall desk.

Residence hall rooms can be occupied from 1:00 p.m. on Sunday, August 12, to 1:00 p.m. on Thursday, August 16, for participants in the Computer Conference; from 1:00 p.m. on Thursday, August 16, to 1:00 p.m. on Sunday, August 19, for participants in the Computer Short Course; and from 1:00 p.m. on Sunday, August 19, to 1:00 p.m. on Saturday, August 25, for those attending the AMS–MAA Summer Meeting. Residence hall clerks will be available from 8:00 a.m. to midnight daily in the individual halls. Housing assignments may be picked up in the University Center adjacent to the meeting registration desk. The room registration clerk will be on duty from 8:00 a.m. to midnight daily, August 18–22. Full payment must be made at time of check-in.

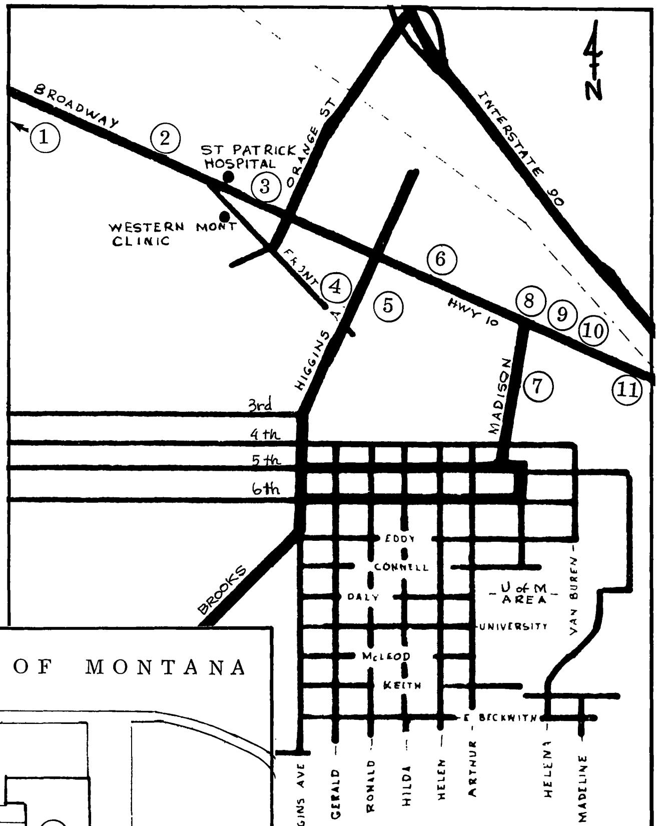
The daily rate per person is as follows:

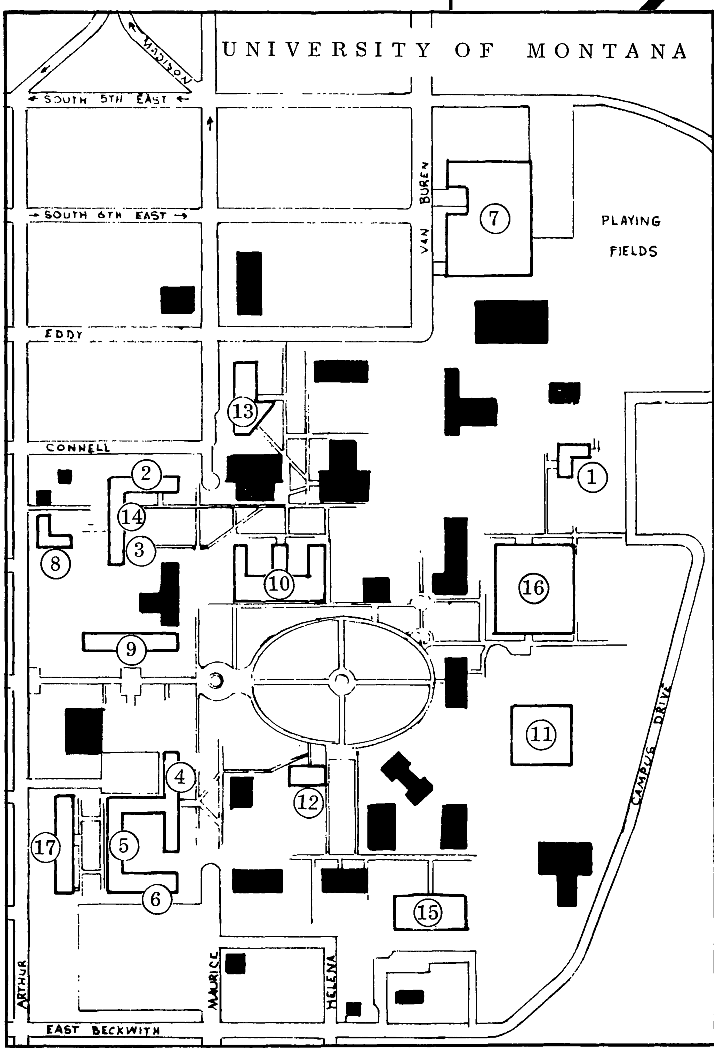
Singles	$4 per person per day
Doubles	$3 per person per day

Children will be housed at the regular rates in rooms adjacent to their parents. Cribs and cots are not available in the residence halls.

Since only a limited number of dormitory rooms will be available, guests should register in advance to be assured of residence hall housing. Please use the Room Reservation Form provided on the last page of these *Notices*. Residence hall reservation requests will be acknowledged by the university.

1. Holiday Inn
2. Red Lion Motor Inn
3. Travelodge
4. Florence Motor Inn
5. Executive Motor Inn
6. Bel Aire Motel
7. Village Motor Inn
8. The Lodge Motel
9. Trade Winds Motel
10. Ponderosa Lodge
11. Thunderbird Motel





1. Aber Hall
2. Brantly Hall
3. Corbin Hall
4. Craig Hall
5. Duniway Hall
6. Elrod Hall
7. Field House
8. Jesse Hall
9. Knowles Hall
10. Liberal Arts
11. Library
12. Mathematics
13. Music
14. North Corbin Hall
15. Science Complex
16. University Center
17. Miller Hall

FOOD SERVICES

Meals will be available on an a la carte, pay-as-you-go basis in the Gold Oak Room at the University Center. Hours of service and representative prices are as follows:

Breakfast	7:00 a.m. -	10:00 a.m.	$1.25
Lunch	10:30 a.m. -	2:30 p.m.	1.60
Dinner	5:00 p.m. -	6:30 p.m.	2.35

For children under 12, meal prices in the Gold Oak Room are two-thirds of those for adults. Light snacks and beverages will be available in the Gold Oak Room between the hours of 6:30 a.m. and 10:00 p.m.

MOTELS

The university has reserved several blocks of rooms at nearby motels. Participants desiring accommodations in motels are encouraged to preregister, and to complete the form, or reasonable facsimile thereof, which will be found on the last page of these *Notices*. Forms should be returned to Mathematics Meetings Housing Bureau, P.O. Box 6887, Providence, Rhode Island 02904. The Housing Bureau will forward the room reservation to Missoula for processing of accommodation requests. Reservations will be made in accordance with preferences indicated on the reservation form, insofar as this is possible, and all reservations will be confirmed. Participants will be informed of any deposit requirements at the time of confirmation. Please note that only those motels designated by an asterisk have reserved a block of rooms during the week of the Computer Conference and Computer Short Course. Space is limited and early reservations are suggested. The special post office box number, given above, will expedite processing of preregistrations and reservations. All participants are urged to use this special number.

Deadline for receipt of reservation requests in Providence is July 20, 1973, for participants in the Computer Conference and Computer Short Course, and July 27, 1973, for participants in the AMS-MAA Summer Meeting.

Motels are listed below with the following coded information: SP, Swimming Pool; KF, Kitchen Facilities; TV, Television; RT, Restaurant; CL, Cocktail Lounge; T, Telephone; AC, Air Conditioning; DB, Double Bed; QB, Queen-Size Bed; KB, King-Size Bed. All prices are subject to change without notice.

The bracketed numbers appearing next to each category of rooms in the listing below refer to the number of available rooms in that price range.

*BEL AIRE MOTEL (406) 549-5134
300 East Broadway - 26 rooms

[15]	Singles, DB	$14.50
[11]	Doubles, DB	14.50
	Doubles, two DB	20.50

Code: SP-TV-RT-T-AC
1 mile from campus

*EXECUTIVE MOTOR INN (406) 543-7221
201 East Main Street - 30 rooms

[30]	Singles	$20.00
	Doubles	20.00
	Triples	22.00
	Quadruples	24.00
	Rollaway	2.00

Code: SP-TV-limited RT-T-AC
1 mile from campus

FLORENCE MOTOR INN (406) 543-6631
111 North Higgins - 100 rooms

[8]	Studios	$ 9.50
[10]	Singles	11.50
[45]	Twins	14.50
[35]	Doubles	14.50
[2]	Suites	32.50
	Rollaway	2.00

Code: TV-RT-CL-T-AC
1 mile from campus

HOLIDAY INN (406) 543-7231
U.S. 10W & Mullen - 30 rooms

[30]	Singles	$14.00
	Doubles	19.00

Code: SP-TV-CL-T-AC
5 miles from campus

THE LODGE MOTEL (406) 549-2387
630 East Broadway - 10 rooms

[8]	Doubles	$18.00
	Triples	20.00
	Quadruples	22.00
[2]	Suites	28.00 and $30.00

Code: SP-TV-T-AC
1/2 mile from campus

*RED LION MOTOR INN (406) 728-3300
700 West Broadway - 32 rooms

[20]	Singles, QB	$14.00-$18.00
[12]	Doubles, two QB	20.00- 24.00
	Triples, two QB	23.00- 27.00
	Quadruples, two QB	26.00- 30.00
	Rollaway	3.00

Code: SP-TV-RT-CL-T
3 miles from campus

PONDEROSA LODGE (406) 543-3102
800 East Broadway - 11 rooms

[2]	Singles, QB	$16.00
[6]	Triples, two QB	22.00
	Quadruples, two QB	24.00
[3]	Suites, 2 rooms, QB	26.00

Code: TV-AC
1/2 mile from campus

*THUNDERBIRD MOTEL (406) 543-7251
1009 East Broadway - 8 rooms

[2]	Singles	$17.50
[6]	Doubles, two DB	24.00
	Rollaway	2.00

Code: SP-TV-T-AC
1/2 mile from campus

TRADE WINDS MOTEL (406) 549-5134
744 East Broadway - 5 rooms

[5]	Doubles, two DB	$24.00

Code: SP-KF-TV-T-AC
1/2 mile from campus

TRAVELODGE (406) 728-4500
420 West Broadway - 40 rooms

[12]	Singles	$13.00
	Doubles	16.50
[8]	Singles, KB	14.00
	Doubles, KB	18.00
[20]	Doubles, two DB	20.50
	Triples, two DB	22.50
	Quadruples, two DB	24.00
	Rollaway	3.00

Code: TV-RT-T-AC
1 1/2 miles from campus

VILLAGE MOTOR INN (406) 728-3100
100 Madison - 50 rooms

[10]	Singles	$14.00
[10]	Doubles, two DB	19.00
[30]	Quadruples	23.50

Code: SP-KF-TV-RT-CL-T-AC
6 blocks brom campus

PARKING

No permits will be required for parking on campus. Maps showing the location of the various college parking lots will be available at the registration desk.

CAMPING

There are several National Forest Campgrounds within easy commuting distance of Missoula. A partial listing by highway and distance from Missoula includes the following: U.S. 93 South, Charles Waters Memorial, 27 miles, 10 campsites; U.S. 12 West, Lewis and Clark, 27 miles, 22 campsites; Lee Creek, 36 miles, 15 campsites; I 90 East (Rock Creek Road, 24 miles east of Missoula). Five separate campgrounds with a total of 51 campsites are located on the Rock Creek Road; the distances from Missoula range from 38 to 51 miles. The area is noted for its excellent fishing. Private camping, camper, and trailer facilities are available at El Mar Trailer Village and KOA Kampground (406) 549-0881, Route 9, Missoula, Montana 59801, 5 miles from campus, 2 people $3.00 + $0.25 for each additional person in the same party; Mobil City Trailer Park (406) 549-2218, 1509 River Road, Missoula, Montana 59801, 2 1/2 miles from campus, 2 people $3.00 + $0.50 for each additional person in the same party. Requests for reservations can be made directly by mail or telephone.

BOOKSTORE

A bookstore is located in the University Center. Its hours of operation are 9:00 a.m. to 5:00 p.m., Monday through Friday.

LIBRARIES

The University Library will be moving to its new building between summer session and autumn quarter, hence probably will not be open.

Current mathematical journals are housed in Room 205 of the Mathematics Building.

Missoula City—County Library
Schedule: 10:00 a.m. - 9:00 p.m. Weekdays
10:00 a.m. - 6:00 p.m. Saturdays
Closed Sundays and legal holidays

MEDICAL SERVICES

Missoula is the medical center of Western Montana. The Western Montana Clinic, comprising 30 physicians and surgeons, is at 501 West Broadway; many physicians have offices in Professional Village. St. Patrick Hospital is at 500 West Broadway, and Community Hospital is on West South Avenue. Aid in case of emergency may be had by calling the university telephone exchange (dial "0").

ENTERTAINMENT

The University of Montana is planning a program of recreation and entertainment for mathematicians and their families. Participants and their families are encouraged to take advantage of the natural beauty of the area.

A buffet will be served in the University Center on Sunday, August 19, from 6:30 p.m. to 8:30 p.m. The cost will be $1.74 for adults and half-price for children under 12 years.

On Wednesday, August 22, there will be a western barbecue picnic at Pattee Canyon Picnic Area. The menu will feature a choice of barbecued spare ribs, barbecued beef, and chopped steak. The cost will be $4 per adult, $1.75 per child (under 12 years). Bus transportation from the University Center to the picnic area will be provided. Service will begin at 5:30 p.m.

A beer party is being arranged for Wednesday, August 22, beginning at 8:00 p.m. at one of the local establishments at a nominal cost.

Fun-do courses will be offered in the University Center on Monday through Thursday from 10:00 a.m. to noon. The topics will be selected from among the following areas: painting and drawing, improvisational theater, folk music, dance, and yoga, organized with children in mind. A nominal fee may be charged. Also with children in mind, free movies will be shown at 8:00 p.m. on Monday, Tuesday, and Thursday in the University Center. Further entertainment is being planned for Monday and Tuesday evenings, August 20 and 21. Details will be available at the registration desk.

An organized program of volleyball and/or softball will take place each afternoon at 3:00 p.m. Up to three hikes of varying grades of difficulty are being scheduled for Tuesday, Wednesday, and Thursday.

If sufficient interest is indicated at registration time, the following tours will be held: (1) Tuesday, August 21, a half-day trip to the National Bison Range which is a unique wildlife reserve with deer, elk, antelope, and buffalo. This refuge is 36 miles from Missoula, and the trip through the reserve covers about 19 miles. There will be a charge for transportation. (2) Wednesday, August 22, a half-day tour of Missoula, including such areas as Fort Missoula (one of the first military posts in Montana) and the Smokejumpers Center. (3) Thursday, August 23, an all-day tour of ghost towns in western Montana. Leave Missoula 8:30 a.m.; return 10:00 p.m. There is a $10 fee for transportation, or drive your own car. Bring sack lunch, pay for own dinner. (4) Thursday, August 23, a half-day river-float trip. A $10 fee per person includes raft rental and transportation. Bring sack lunch and fishing gear. This trip is limited to 25.

The following pre- and post-meeting trips will be scheduled if sufficient interest is indicated on responses received by July 15. (1) Leave Missoula, Friday, August 17, 7:00 a.m. and return Sunday, August 19, 6:00 p.m. A horseback pack trip to either Anaconda-Pintlar Wilderness (fee $75) or Bob Marshall Wilderness (fee $75). The fee includes all expenses. Bring only sleeping bag, personal gear, and fishing equipment. Each of the above trips is limited to 15. (2) Leave Missoula, Saturday, August 25, 10:00 a.m. and return Monday, August 27, 9:00 p.m. A backpack hike in Glacier National Park. A $15 fee per person covers guide service and transportation. Furnish own food, sleeping bag, and personal gear. This trip is limited to 12.

If you are interested in any of these trips, please send name, address, and number of persons in party to Professor Gloria Hewitt, University of Montana, not later than July 15 with a clear indication of first, second, and third choice of trips [(1) horseback trip to Anaconda; (2) horseback trip to Bob Marshall; (3) backpack in Glacier].

The Rocky Mountaineers Club will have hikes (day or overnight) scheduled on the weekends of August 18–19 and August 25–26. Interested persons may obtain details by writing Professor W. R. Ballard, University of Montana.

Most university recreational facilities will be open. Swimming, tennis, bowling, table tennis, and billiards will be available. There are numerous picnic areas and fishing opportunities (licenses required) in the area. Hiking possibilities are essentially unlimited. The surrounding countryside offers much to the inquisitive traveler.

TRAVEL

During the summer, Missoula is on MOUNTAIN DAYLIGHT SAVING TIME. Its situation on two major highways, Interstate 90 (Old U.S. 10) and U.S. 93 (the Pan-American Highway, with connections to the Alcan Highway) makes Missoula the "Crossroads of Northwestern North America." Missoula is also on U.S. 12 (the Lewis and Clark Trail) and scenic Montana 200. Information regarding highway travel in Montana is available from the Montana Department of Highways, Helena, Montana 59601. The Greyhound Bus Line provides service to and from points west (via Spokane) and east. Frontier Airlines serves Missoula with connections from Billings, Denver, and Salt Lake City, and Northwest Airlines has flights with connections through Billings and Spokane. Participants are advised to check airline schedules carefully to obtain the best connecting flights to and from Missoula. Early reservations are also suggested. Rail service to Missoula is by Amtrak which currently has three trains per week from both the east and west. Rental cars available are limited so that prior reservations would be advisable through Avis, Budget, Ford, Hertz, and National Car Rentals. Rental agencies located at the airport terminal are Avis, Hertz, and National.

Missoula provides no regular transportation to and from the air terminal, and cab service is extremely limited. During the meetings, shuttle bus service will be provided to and from the air terminal, tickets for which may be purchased through the Mathematics Meetings Housing Bureau along with preregistration at a charge of $1 per person one way (children under six free). Tickets will be $2 per person one way if purchased at the terminal. The schedule of service will be determined on the basis of responses on the preregistration form, and tickets will be mailed.

MAIL AND MESSAGE CENTER

All mail and telegrams for persons attending the meetings should be addressed to the specific meeting in care of University Center Mall, Registration Area, University of Montana, Missoula, Montana 59801. The telephone number will be (406) 243-5665. Messages may be left for registrants during hours given in section entitled MEETING PREREGISTRATION AND REGISTRATION.

LOCAL ARRANGEMENTS COMMITTEE

H. L. Alder (ex officio), William R. Ballard (co-chairman), Charles A. Bryan, Rodney T. Hansen, Gloria C. Hewitt, Don O. Loftsgaarden, D. George McRae, W. M. Myers, Jr. (co-chairman), Kenneth A. Ross (ex officio), F. N. Springsteel, Robert R. Stevens, Gordon L. Walker (ex officio), and I. Keith Yale.

WEATHER

The normal daytime high temperature during this period is $79^{\circ}F$ to $82^{\circ}F$. Normal nighttime low is $45^{\circ}F$ to $47^{\circ}F$. Rainfall in August averages 0.72 inches; precipitation may accordingly be expected to be light and scattered. If there is precipitation, it may fall as snow on nearby mountains. Humidity normally ranges from an afternoon low of about 20 percent to a nighttime high of 70 percent. Record high and low temperatures for August are $105^{\circ}F$ and $32^{\circ}F$ respectively.

SUMMARY OF ACTIVITIES

The AMS Committee to Monitor Problems in Communication has recommended that a Summary of Activities appear in the issue of the *Notices* which contains a reservation form for either an annual or a summer meeting. The purpose of this summary is to provide assistance to registrants in the selection of arrival and departure dates. The program, as outlined below, is based on the information available at press time.

SUNDAY, August 12	American Mathematical Society	Mathematical Association of America
2:00 p.m. - 7:00 p.m.	REGISTRATION	
MONDAY, August 13	CONFERENCE ON THE INFLUENCE OF COMPUTING ON MATHEMATICAL RESEARCH AND EDUCATION	
8:00 a.m. - 4:00 p.m.	REGISTRATION	
9:30 a.m. - 11:30 a.m.	Panel discussion: The computer and calculus Stephen J. Garland, moderator	
1:00 p.m. - 2:00 p.m.	Invited address: Number theory Derrick H. Lehmer	
2:00 p.m. - 3:00 p.m.	Invited address: Algebra Charles C. Sims	
3:15 p.m. - 4:15 p.m.	Contributed Papers I	
TUESDAY, August 14		
8:30 a.m. - 4:00 p.m.	REGISTRATION	
9:00 a.m. - 10:00 a.m.	Invited address: Mathematical statistics and probability Ulf Grenander	
10:30 a.m. - 11:30 a.m.	Invited address: Numerical mathematics J. F. Traub	
1:00 p.m. - 3:00 p.m.	Panel discussion: Logistics of computer usage in education Glen R. Ingram William B. Kehl, moderator Thomas E. Kurtz	
3:00 p.m. - 4:30 p.m	Contributed Papers II	
WEDNESDAY, August 15		
8:30 a.m. - 4:00 p.m.	REGISTRATION	
9:00 a.m. - 10:00 a.m.	Invited address: Applied mathematics (applications to physical sciences) Peter Lax	
10:30 a.m. - 12:30 p.m.	Panel discussion: The computer and its relation to less traditional courses Richard A. Karp, moderator	
1:30 p.m. - 2:30 p.m.	Invited address: Unexpected impact of the computer on science and mathematics Thomas Cheatham	
2:30 p.m. - 3:30 p.m.	Invited address: The computer and thought: What's new Edward A. Feigenbaum	
3:30 p.m. - 5:00 p.m.	Contributed Papers III	
THURSDAY, August 16		
8:30 a.m. - 4:00 p.m.	REGISTRATION	
9:00 a.m. - 5:00 p.m.	EXHIBITS	
9:00 a.m. - 10:00 a.m.	Invited address: Mathematics and the computer: What lies ahead Stanislaw M. Ulam	
10:30 a.m. - 12:30 p.m.	Panel discussion: CUPM Report on the impact of the computer on the teaching of mathematics Alex Rosenberg, moderator	

THURSDAY, August 16	CONFERENCE ON THE INFLUENCE OF COMPUTING ON MATHEMATICAL RESEARCH AND EDUCATION
1:30 p.m. - 2:30 p.m.	Invited address: Optimization and operations research Philip Wolfe
2:30 p.m. - 4:30 p.m.	Panel discussion: The computer and its relation to other traditional courses Cleve B. Moler, moderator
4:30 p.m. - 5:30 p.m.	Contributed Papers IV

FRIDAY, August 17	PRECEPTORIAL INTRODUCTION TO COMPUTER SCIENCE FOR MATHEMATICIANS	
8:00 a.m. - 4:00 p.m.	REGISTRATION	
9:00 a.m. - 5:00 p.m.	EXHIBITS	
SATURDAY, August 18		
8:00 a.m. - 3:00 p.m.	REGISTRATION	
9:00 a.m. - 2:00 p.m.	EXHIBITS	

AMS — MAA SUMMER MEETINGS

SUNDAY, August 19	American Mathematical Society	Mathematical Association of America
9:00 a.m. - 4:00 p.m.		Board of Governors
2:00 p.m. - 8:00 p.m.	REGISTRATION-University Center-First Floor Mall	
6:30 p.m. - 8:30 p.m.	Buffet - University Center	
7:00 p.m.		MAA - MOTION PICTURES Films of the Topology Films Project (in color and with sound narration)
7:00 p.m. - 7:30 p.m.		Space filling curves
7:40 p.m. - 7:54 p.m.		Regular homotopies in the plane, Part I
7:55 p.m. - 8:14 p.m.		Regular homotopies in the plane, Part II
8:25 p.m. - 9:45 p.m.		A film of the MAA Individual Lectures Film Project (in black and white) The marriage theorem (two parts), with Gian-Carlo Rota

MONDAY, August 20	AMS	MAA
8:00 a.m. - 5:00 p.m.	REGISTRATION-University Center-First Floor Mall	
9:00 a.m. - 10:00 a.m.		THE EARLE RAYMOND HEDRICK LECTURES: Some recent applications of mathematics Lecture I: An application of geometry Henry O. Pollak
9:00 a.m. - 5:00 p.m.	EXHIBITS - University Center - First Level	
10:10 a.m. - 11:30 a.m.		Panel discussion: Beyond academe: The mathematician in the real world W. M. Boyce R. C. DiPrima, moderator R. E. Gaskell A. J. Goldman Shmuel Winograd
11:30 a.m. - 12:00 noon		General discussion by the panel and the audience
10:00 a.m. - 12:00 noon	Fun-Do Courses - University Center	
1:30 p.m. - 2:20 p.m.		THE EARL RAYMOND HEDRICK LECTURES: Lecture II: An application of algebra Henry O. Pollak
2:30 p.m. - 3:20 p.m.		Invited address: Subsets of finite sets H. J. Ryser

MONDAY, August 20	American Mathematical Society	Mathematical Association of America
3:00 p.m.	Softball - Volleyball	
3:30 p.m. - 4:20 p.m.		Invited address: Some machinations of Leonhard Euler C.T. Long
4:30 p.m. - 6:00 p.m.	AMS Committee on Employment and Educational Policy - Panel discussion: The role of the Ph. D. in two-year college teaching John W. Jewett, moderator	
7:00 p.m.		MAA - MOTION PICTURES Films of the college geometry project (in color)
7:00 p.m. - 7:10 p.m.		Caroms, with Chandler Davis
7:11 p.m. - 7:22 p.m.		Equidecomposable polygons, with J. D. E. Konhauser
7:23 p.m. - 7:33 p.m.		Central similarities, with Daniel Pedoe and H. S. M. Coxeter
7:34 p.m. - 7:47 p.m.		Dihedral kaleidoscopes, with H. S. M. Coxeter
8:00 p.m. - 9:20 p.m.		A film of the MAA Individual Lectures Film Project (in black and white): It's how you count that counts (Parts I and II), with H. N. Shapiro
8:00 p.m.	Family Movies - University Center	

TUESDAY, August 21	AMS	MAA
a.m.	National Bison Range Tour	
8:30 a.m. - 4:30 p.m.	REGISTRATION - University Center - First Floor Mall	
9:00 a.m. - 5:00 p.m.	EXHIBITS - University Center - First Level	
9:00 a.m. - 9:50 a.m.		THE EARLE RAYMOND HEDRICK LECTURES: Lecture III: An application of analysis Henry O. Pollak
10:00 a.m. - 10:50 a.m.		Business Meeting: Presentation of Lester R. Ford Awards and two special awards
10:00 a.m. - 12:00 noon	Fun-Do Courses - University Center	
11:00 a.m. - 11:50 a.m.		Invited address: Guessing and proving George Polya
12:15 p.m.	PI MU EPSILON - Council Luncheon	
1:30 p.m. - 2:30 p.m.	Colloquium Lectures: Nonlinear functional analysis, and its applications to nonlinear partial differential and integral equations, Lecture I Felix E. Browder	
2:45 p.m. - 3:45 p.m.	Colloquium Lectures: Schizophrenia in contemporary mathematics: The problem and the cure, Lecture I Errett A. Bishop	
3:00 p.m.	Softball - Volleyball	
3:15 p.m. - 5:15 p.m.	PI MU EPSILON - Contributed Papers	
5:00 p.m.	Council Meeting	
6:30 p.m.	PI MU EPSILON - Banquet	
7:00 p.m.		MAA - MOTION PICTURES
7:00 p.m. - 7:08 p.m.		A film produced by Bruce and Katharine Cornwell (in color): Newton's equal areas
7:10 p.m.		Films of the MAA mathematics today series (in color)
7:10 p.m. - 8:11 p.m.		Let us teach guessing, a demonstration with George Polya
8:20 p.m. - 9:20 p.m.		Fixed points, with Solomon Lefschetz

TUESDAY, August 21	American Mathematical Society	Mathematical Association of America
8:00 p.m.	Family Movies - University Center	
8:00 p.m.	PI MU EPSILON - Lecture - Victor Klee - Some unsolved problems from intuitive geometry	

WEDNESDAY, August 22	AMS	MAA
a.m.	Missoula Tour	
a.m.	Hikes	
8:00 a.m.	PI MU EPSILON - Dutch Treat Breakfast	
8:30 a.m. - 4:30 p.m.	REGISTRATION - University Center - First Floor Mall	
8:30 a.m. - 9:30 a.m.	INVITED ADDRESS: The Hilbert modular group F. E. P. Hirzebruch	
9:00 a.m. - 5:00 p.m.	EXHIBITS - University Center - First Level	
9:00 a.m. - 12:00 noon	Contributed Papers	
9:45 a.m. - 10:45 a.m.	Colloquium Lectures II Felix E. Browder	
10:00 a.m. - 12:00 noon	Fun-Do Courses - University Center	
10:40 a.m. - 12:40 p.m.	PI MU EPSILON - Contributed Papers	
11:00 a.m. - 12:00 noon	Colloquium Lectures II Errett A. Bishop	
1:30 p.m. - 2:20 p.m.		Invited address: A survey of lattice point problems T.M. Apostol
2:30 p.m. - 4:00 p.m.		Panel discussion: New directions in training undergraduate mathematics teachers John Hilzman J.M. Jobe D.E. Kibbey Joseph Lipson Robert McKelvey, moderator
3:00 p.m.	Softball - Volleyball	
4:00 p.m. - 4:30 p.m.		General discussion by the panel and the audience
5:30 p.m.	Western Barbecue Picnic - Pattee Canyon Picnic Area	
8:00 p.m.	Beer Party	
8:30 p.m.	Association for Women in Mathematics - Panel Discussion	

THURSDAY, August 23	AMS	MAA
a.m.	Hikes	
8:30 a.m. - 10:00 p.m.	Western Montana Ghost Town Tour	
8:30 a.m. - 4:30 p.m.	REGISTRATION - University Center - First Floor Mall	
8:30 a.m. - 9:30 a.m.	INVITED ADDRESS: The analytic theory of algebraic numbers Harold M. Stark	
9:00 a.m. - 12:00 noon	EXHIBITS - University Center - First Level	
9:00 a.m. - 12:00 noon	Contributed Papers	
9:00 a.m. - 4:00 p.m.	C B M S Council Meeting	
9:45 a.m. - 10:45 a.m.	Colloquium Lectures III Felix E. Browder	
10:00 a.m. - 12:00 noon	Fun-Do Courses - University Center	
11:00 a.m. - 12:00 noon	Colloquium Lectures III Errett A. Bishop	
1:30 p.m. - 2:30 p.m.	INVITED ADDRESS: Binomial probabilities Richard M. Dudley	
2:45 p.m. - 3:45 p.m.	INVITED ADDRESS: Piecewise linear methods in Riemannian geometry in the large Herman R. Gluck	

THURSDAY, August 23	American Mathematical Society	Mathematical Association of America
2:45 p.m. - 3:45 p.m.	INVITED ADDRESS: On the incompatibility of two conjectures concerning primes; a discussion of the use of computers in attacking a theoretical problem J. Ian Richards	
3:00 p.m.	Softball - Volleyball	
4:00 p.m.	Business Meeting	
7:30 p.m.	AMS Committee on Employment and Educational Policy - Panel discussion: An analysis of the state of the job market Richard D. Anderson Martha Kathleen Smith	
8:00 p.m.	Family Movies - University Center	

FRIDAY, August 24	AMS	MAA
8:30 a.m. - 9:30 a.m.	INVITED ADDRESS: On the approximation-solvability of equations involving A-proper and pseudo-A-proper mappings W.V. Petryshyn	
8:30 a.m. - 1:30 p.m.	REGISTRATION - University Center - First Floor Mall	
9:00 a.m. - 12:00 noon	Contributed Papers	
9:45 a.m. - 10:45 a.m.	Colloquium Lectures IV Felix E. Browder	
11:00 a.m. - 12:00 noon	Colloquium Lectures IV Errett A. Bishop	
1:30 p.m. - 2:30 p.m.	INVITED ADDRESS: Projective varieties of small codimension Robin Hartshorne	
1:30 p.m. - 2:30 p.m.	INVITED ADDRESS: Diffusion processes and migration models in population genetics theory Wendell H. Fleming	
2:45 p.m. - 3:45 p.m.	INVITED ADDRESS: Some invariants of flat bundles James Simons	
2:45 p.m. - 3:45 p.m.	INVITED ADDRESS: Approaches to Goldbach's problem Patrick X. Gallagher	
4:00 p.m. - 5:00 p.m.	INVITED ADDRESS: Toward a mathematical theory of memory Jan Mycielski	
4:00 p.m. - 5:00 p.m.	INVITED ADDRESS: Some structure theorems in several complex variables F. Reese Harvey	

Kenneth A. Ross
Associate Secretary

Eugene, Oregon

The Seven Hundred Eighth Meeting
University of Minnesota
Minneapolis, Minnesota
November 3, 1973

The seven hundred eighth meeting of the American Mathematical Society will be held at the University of Minnesota, Minneapolis, Minnesota, on Saturday, November 3, 1973.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, there will be hour addresses by Professor Charles L. Fefferman of the University of Chicago and by Professor Frank A. Raymond of the University of Michigan. The titles of their lectures will be announced later.

Sessions for contributed ten-minute papers will be held both morning and afternoon. Abstracts should be submitted to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904, so as to arrive prior to the deadline of September 6, 1973. Those having time preferences for the presentation of their papers should indicate them on their abstracts. There will be a session for late papers if one is needed, but late papers will not be listed in the printed program of the meeting.

There will probably be several special sessions of selected twenty-minute papers. The subjects of these special sessions and the names of the mathematicians arranging them will be announced in the August *Notices*. Some informal sessions may also materialize.

Detailed information about travel and accommodations will appear in the August issue of these *Notices*, and the final program of the meeting will appear in the October *Notices*.

Paul T. Bateman
Associate Secretary

Urbana, Illinois

HOUR SPEAKERS AT AMS MEETINGS

This section of these *Notices* lists regularly the individuals who agreed to address the Society at the times and places noted below.

<u>Bellingham, Washington, June 1973</u>

Burton Rodin	Haskell Rosenthal

<u>Missoula, Montana, August 1973</u>

Richard M. Dudley	F. E. P. Hirzebruch
Wendell Fleming	Jan Mycielski
Patrick X. Gallagher	Walter Petryshyn
Herman Gluck	J. Ian Richards
Robert C. Hartshorne	James Simons
F. Reese Harvey	Harold M. Stark

<u>Atlanta, Georgia, November 1973</u>

Ben Fitzpatrick	M. Z. Nashed
Paul Hill	

<u>Tucson, Arizona, November 1973</u>

Andrew P. Ogg	Robert R. Phelps

NOMINATIONS FOR MEMBER-AT-LARGE

Five positions of member-at-large of the Council are to be filled in the election of October 1973. The Council has nominated eight candidates for these positions, namely

Raymond O. Wells, Jr.
Joshua A. Leslie
Robion C. Kirby
Herbert B. Keller
Wolfgang M. Schmidt
Phillip A. Griffiths
Charles W. Curtis

It is intended that the Council will name one additional candidate for the position of member-at-large.

The names are published now to assist those who wish to participate in making additional nominations by petition. This is a procedure authorized on March 31, 1972, for a two-year trial in the elections of 1972 and 1973. The situation is to be evaluated in January 1974.

The name of a candidate for the position of member-at-large of the Council may be placed on the ballot by a petition that conforms to several rules and operational considerations, as follows:

1. To be considered, petitions must be addressed to Everett Pitcher, Secretary, Box 6248, Providence, Rhode Island 02904, and must arrive by August 1, 1973.

2. The name of the candidate must be given as it appears in the Combined Membership List. If the name does not appear in the list, as in the case of a new member or by error, it must be as it appears in the mailing lists, for example on the mailing label of the *Notices*.

3. The petition for a single candidate may consist of several sheets each bearing the statement of the petition and signatures. The name of the candidate must be exactly the same on all sheets.

4. On the facing page is a sample form for petitions. Copies may be obtained from the Secretary; however, petitioners may make and use photocopies or reasonable facsimiles.

5. A signature is valid when it is clearly that of the member whose name and address is given in the left-hand column. At least fifty valid signatures are required for a petition to be considered further.

6. The signature may be in the style chosen by the signer. However, the printed name and address will be checked against the Combined Membership List and the mailing lists. No attempt will be made to match variants of names with the form of name in the CML. A name not in the CML or on the mailing lists is not that of a member. (Example: The name Everett Pitcher is that of a member. The name E. Pitcher appears not to be. Note that the current mailing label of the *Notices* can be peeled off and affixed to the petition as a convenient way of presenting the printed name correctly.)

7. When a petition meeting these various requirements appears, the Secretary will ask the candidate whether he is willing to have his name on the ballot. His assent is the only other condition of placing it there. Petitioners can facilitate the procedure by accompanying the petitions with a signed statement from the candidate giving his consent.

NOMINATION PETITION FOR 1973 ELECTION

The undersigned members of the American Mathematical Society propose the name of ______________________________ as a candidate for the position of member-at-large of the Council of the American Mathematical Society for a three-year term beginning January 1, 1974.

Printed or typed name and address or *Notices* mailing label	Signature

A POSSIBLE PROGRAM FOR THE AMERICAN MATHEMATICAL SOCIETY
by Saunders Mac Lane

For many years the American Mathematical Society has continued to carry on its accustomed activities, modified in detail by the needs of the time, the 1971 Committee to Review Society Activities, and the ingenious ideas of Comm-Comm (the Committee to Monitor Problems in Communication). It now seems appropriate to again reconsider the overall objectives of the Society. This essay will be the author's own formulation of these objectives, set down here to initiate discussion.

A. PUBLICATION

Publication of mathematical research is a major portion of the activity of the Society. Such publication is fundamental; it provides the opportunity for members of the Society (and others) to present their new results to the world mathematical community. Currently, Society publications are extensively supplemented by other publication activities sponsored by universities, regional groups, or commercial publishers. But there is no present likelihood that the Society's publication activities could be replaced by such others.

Present publication activities may be grouped as follows:

1. Primary research journals. The Society maintains three general journals: Transactions, Proceedings, and Bulletin. They have clearly separated functions; namely, they publish longer research papers, shorter research papers, and finally research announcements plus review articles. In addition, research monographs are published in the various numbers of the Memoirs. These publications, taken together, provide for a broad scale presentation of research. What is missing is a smaller journal publication at a level comparable with the Annals of Mathematics for the very best and most impressive papers.

The Society also publishes one specialized journal, Mathematics of Computation. Many other specialized mathematical journals in other fields exist, chiefly under commercial auspices. It might be appropriate that the Society consider the total array of such journals with a view to discovering bad gaps or duplication in the list of specialties.

2. Secondary literature. Mathematical Reviews, perhaps the single most important publication of the Society, faces both financial and intellectual difficulties. It is important that the Society continue this publication by clearly meeting these difficulties. We need to provide for greater simplicity and at the same time greater coverage of the applications. The recently established Index of Mathematical Papers is another AMS secondary journal. It is probably too early to judge its success though there is a clear need to increasing its coverage to match that of MR. Our secondary literature also includes the various Translations; I observe that their success is more sensitive to economic considerations. Finally, the Society has several sets of Proceedings of symposia, containing papers of a survey or expository nature.

There are very serious gaps in the secondary publication program of the Society. The recent growth of mathematical publication means that evaluative and analytical summaries of the state of mathematical research and knowledge will become increasingly important. We have long needed more expository articles; now we also need more thorough going essays on the organization of knowledge. I suggest that the Society should take a lead in stimulating such development.

3. Books. The AMS has the Colloquium and Mathematical Survey series, active in the past, currently dormant as of new volumes, and worth continuing as a resource. Book publication, even for very specialized topics, is today almost wholly done commercially. There are problems of high prices; as of text books, there may be many shoddy books which should never have been published. The mathematical community has a clear obligation to provide a clear and quick reviewing service on textbooks; this is indeed provided by the Mathematical Association of America in the Monthly, supplemented by some reviews in the Bulletin.

B. MEETINGS

The second most important activity in the Society is the organization of meetings. The recent development of our science suggest that the increasing speed of communication and of meeting has indeed considerably accelerated the development of research. However, the external pattern of the meetings of the Society has not been changed for forty years, except for a one-month shift in the date of the annual meeting and a smaller shift in the date of the summer meeting.

1. The annual meeting has become top-heavy (attendance 4,256 in New Orleans 1969; 3,656 in San Antonio; 3,409 in Atlantic City; 2,851 at Las Vegas in 1972; and 3,200 at Dallas in January 1973). This big meeting is effective in getting "everybody" together, in joining with our sister societies, and in providing time and place for necessary committee meetings. The meeting provides scientific communication in variety: Colloquium Lectures to tie it all together, a good variety of hour lectures, and a vast array of ten-minute papers (at Dallas, 737 individuals were scheduled to speak at AMS sessions). The sessions of ten-minute papers give new members a chance to appear; they also seem to individualize the big meeting by bringing together specialists with common interests. The new special sessions of twenty-minute papers

can provide a somewhat sharper focus of interest; they have succeeded well. The time of the January meeting fits the job market, so many of our younger members come in search of interviews—which, given current conditions, can be a dismal discouragement. All these sizable activities mean, too, that the January meeting has heavy costs. The visible costs for travel are just the top of the iceberg: There are also the costs of time and energy spent in attending such a big assembly, the losses because of the things which are not accomplished, and the necessity of holding such a big meeting in a correspondingly big hotel or convention center, which leads to added travel costs. It becomes relevant to ask: What functions can be moved away from the January meeting? Are the benefits worth the costs?

2. The Summer Meeting had once a great attraction: a more relaxed atmosphere which allowed a leisurely presentation of mathematics at a comfortable college campus which was fun as a family enterprise. It no longer works so well; it has slipped in size and attractiveness; it is very hard to get colleges willing or able to take on the meeting.

3. Regional Meetings continue and seem attractive, but it may be that we adhere too much to tradition. For example, some regional meetings might be replaced by specialist meetings: one on a broad field (analysis, algebra, ...), the field to vary from year to year. There are possible topics such as "Teaching and research in topology".

4. Symposia (special meeting, conferences, ets.). These we hold and enjoy. Presumably more would be better, and that means financial support by grants or by registration fees. Recommendation: That the Society establish a committee to reexamine from first principles the idea and arrangements of meetings. Such a committee could get reports from the membership (say—questionnaire to those at a meeting: Was it good or why was it bad; what should be done differently?); it would propose substantial changes or trials and experiments; it must cooperate with (committees from?) our sister societies.

C. ENCOURAGEMENT OF RESEARCH

In addition to publications and meetings for the dissemination of research, the Society has traditionally encouraged high quality research. In particular, the Society's prizes provide such encouragement. With the recent institution (jointly with SIAM) of the Birkhoff and Wiener Prizes in Applied Mathematics, it is my judgment that the Society has a well-balanced program of prizes.

The recognition of high quality research also appears in the invited hour lectures and more particularly in the Colloquium and Gibbs Lectures. It is my observation that they are doing excellently: lectures good to splendid, subjects well chosen, attendance good.

Despite these positive words, there are still other respects in which the Society might act in encouragement of high quality research. For example, none of our actions so far are explicit, positive encouragement of high quality research by women. For my own part, the fact that I studied with Emmy Noether in Göttingen gives me a lively appreciation of what ought to be possible, and what we should help to foster.

With encouragement of research I count also the difficult problem of judging priorities in fields of research. The American mathematical community has now grown to a size where the distribution of mathematicians among fields may no longer be left to happy chance. Fields which are initially promising may seem out of promise or problems. Young mathematicians (and others) may seek help or guidance in picking the right field or in shifting as occasion suggests to new fields.

I recommend that the Society tackle this problem head on by establishing a committee to advise on priorities in research. This committee should in particular take note of challenging areas of research for the future. The part of this examination bearing on the applications would naturally be done in collaboration with our sister societies, notably SIAM.

D. THE ECONOMIC INTERESTS OF MATHEMATICS

The economic interests and concerns of our members should also be the concern of the American Mathematical Society. The times require it, but the good mechanism will not be easy to find. There are the problems of discrimination in hiring, mentioned in the resolutions to be presented for the San Francisco meeting. There are many other related problems. Specifically, there can be discrimination not only against women or minority groups, there can also be discrimination against men or majority groups, and the Society must be even handed in its support of protest against discrimination of any one of these sorts. We must strive more than we have to assist our young mathematicians in finding their first jobs. Moreover, finding the second job has now become a very touchy question. Economic pressures and previous careless actions mean that many young mathematicians holding positions as assistant professor may not be promoted to a permanent position with tenure. These problems have grown with great rapidity. There are cases of individual injustices and it is not even clear that the general mechanism, dependent so much on letters of reference, is in good shape. If the Society acts to support its members, it must in particular support those who have lost their first jobs.

Even at the most prestigious universities the current enthusiasm for applications and "social relevance" may sometimes result in a lack of understanding of how mathematics works. We may have much to do, even at the top, in encouraging the support of the best mathematicians.

Recommendation: That we establish a committee to examine the ways in which the Society could begin to support the economic interests of its members.

This action would be in keeping with the resolution adopted at the January 1972 Business Meeting of the Society in Las Vegas.

E. NEW POSITIONS

The above recommendations—like the present Employment Register and the newly established Employment Information for Mathematicians—may help to distribute better those mathematical jobs which already exist. How to really make new jobs in mathematics is a deeper problem, to which I do not have the answer. Serious suggestions are in order, beginning with more Ph. D.'s in junior colleges.

One suggestion I have heard calls for an assessment of the achievements of mathematics: a report examining outstanding cases in which mathematical research has contributed directly or indirectly to societal ends, and of cases in which public support of mathematical research has produced fundamental knowledge or understanding. Such an assessment is hard to prepare—especially if it looks to the future as well. Nevertheless, such an assessment might provide a basic argument for the public support of mathematics and hence of more mathematicians. The COSRIMS report was such an assessment, but times have changed, and a new try may be in order.

Such an assessment would be one part of the large problem of the public representations of mathematics—a problem which this essay does not adequately cover.

Also, when we look hard at getting new positions we will be in condition to look at the complementary problem of too many Ph. D.'s: has graduate work in mathematics expanded too rapidly; if so, what steps should we take?

F. TEACHING

Fifty years ago or so it seemed that problems of teaching could be left to the Mathematical Association of America while the Society concentrated just on research. This distinction is no longer so clear. Promotion often depends now on both teaching and research. Research mathematicians and teachers in these days will all deplore undue increases in teaching loads and excessive sizes of classes. However, it will do no good at all to simply "deplore" and "view with alarm". To anyone outside the profession, such statements will be discounted (or ignored) as partisan proclamations of self-interest.

The point is that teaching matters, and because it matters, we would do well to find out what things about it can be done better. On the one hand we should know what the facts are as to teaching load (a notable difficult quantity to measure) and as to class size. Harder yet we should try to discover the real effects on the students of big classes. Equally hard, we should pay attention to the question of how young mathematicians learn to teach. This is in fact a part of standard graduate study and would better be a conscious part. For example, E. E. Moise has suggested that each four or five graduate teaching assistants get together (in class and offices) to observe how they teach it and why. When and as the profession takes steps to know what it is doing and to improve what it is doing, we will also be able to deplore the wrong things with more effectiveness.

What appears to be needed is a joint committee with MAA on the training of graduate students to teach, and on the relative effect on the students of variables such as class size.

G. SUMMARY OF RECOMMENDATIONS.

a. Examination of the present array of specialized journals, with especial attention to gaps in fields covered and to applied fields.

b. A request that Comm-Comm consider the problems of encouraging evaluative and analytical summaries of the state of mathematical research.

c. A basic study of the style and arrangement of Society meetings, with special attention to possible meetings devoted to subfields of mathematics, more symposia, etc.

d. A committee to report on priorities of research, say in some selected field, with a view to advising young mathematicians.

e. A joint committee with SIAM on priorities in (some field of) applied mathematics.

f. Real examination of the way in which the Society could support the economic interests of its members.

g. Examination (committee or panel discussion) of the problems of mathematicians not promoted to tenure.

h. A joint committee with MAA on the training of graduate students to teach.

i. Information on the state of teaching loads and class size.

j. An assessment of the achievements of mathematics.

NEWS ITEMS AND ANNOUNCEMENTS

TANJAY COLLEGE, THE PHILIPPINES

Word has been received that Tanjay College is in dire need of books at the secondary school and college level. Tanjay College is located in the backwoods in the heart of the Philippines and was founded twenty-one years ago. The college is now trying to obtain books and educational materials that will assist them in their plans to obtain university accreditation. Individuals and organizations wishing to participate by collecting and shipping books to the college should observe the following shipping instructions. Arrangements have been made with Project Handclasp, 11th Naval District, San Diego, California, for free overseas shipment (dependent upon availability of space), provided that books are sent to the navy warehouse in San Diego. To save mailing charges, packages should be sent to San Diego by "book rate." Project Handclasp should be notified of a possible shipment, and packages should be clearly marked for Tanjay College, Tanjay, Negros Oriental, Philippines, c/o Dr. D. P. Villaflores, President.

TEACHING UNDERGRADUATE MATHEMATICS

A study that was conducted in 1965 showed that forty percent or more of the freshman-sophomore teaching was assigned to teaching assistants in more than half of the universities that reported [1]. Departments of mathematics and their faculties are becoming increasingly concerned with giving more guidance to graduate students who are assuming such a high percentage of the teaching load. In January 1972, the Committee on the Undergraduate Program in Mathematics published a booklet entitled Suggestions on the Teaching of College Mathematics which can be obtained free of charge from CUPM, P. O. Box 1024, Berkeley, California 94701. In the foreword, Professor Peter J. Hilton stresses the importance of good undergraduate teaching, not only to mathematics majors, but to the users of mathematics (biologists, economists, ecologists); and in addition, he discusses the importance of mathematics as a cultural heritage, making it important to "strive to awaken in as many people as possible, irrespective of their chosen vocation, an awareness of the nature of our science, and its significance for our civilization, material and spiritual."

The ideas offered in this booklet have been culled from materials prepared by a number of universities, and from various publications dealing with the teaching of mathematics. One particular publication was widely used, Some Helpful Hints to Good Teaching (1966), written by two graduate students at the University of Wisconsin, Leroy J. Dickey and Kenneth M. Hunter. Suggestions are given on conducting a course from the general class preparation by the instructor through ways to present a lecture, blackboard techniques, the use of textbooks and visual aids, assignments, tests, correcting of papers, and grading. The evaluation of instructors by students and senior faculty is discussed from the standpoint of the written evaluation by students at the end of a term and consultations with faculty. The final section deals with reading and seminars related to teaching, and provides an excellent list of reference material in the form of journals, periodicals, and books.

The CUPM Newsletter of February 1972 entitled New Methods for Teaching Elementary Courses and for the Orientation of Teaching Assistants, also available without charge from CUPM, contains seven articles that deal with various approaches to undergraduate teaching: "Orientation of teaching assistants" by Richard C. DiPrima, Rensselaer Polytechnic Institute; "An improved orientation program" by Siegfried K. Grosser, University of Minnesota; "Large-group instruction at the University of Maryland" by David Schneider, University of Maryland; "An exam-tutorial program in calculus at MIT" by Arthur P. Mattuck, Massachusetts Institute of Technology; "A new approach to mass instruction" by John W. Riner, The Ohio State University; "The calculus experiment at UCSD" by Burton Rodin, University of California, San Diego; and "Teaching elementary mathematics to large groups of non-mathematics majors" by A. Bruce Clark, Western Michigan University.

The University of Minnesota and Rensselaer Polytechnic Institute have devised orientation programs which, though different in scope, aim toward improving undergraduate teaching and assist graduate student teachers in the classroom situation. The teaching of large groups of students, many of whom are not mathematics majors nor even interested in mathematics, poses problems for many universities, and the programs developed by the University of Maryland, The Ohio State University, and Western Michigan University are well worth studying. Two quite different calculus programs have been tried by the Massachusetts Institute of Technology and the University of California, San Diego, with varying degrees of success.

Chairmen of departments and others interested in improving the teaching methods of teaching assistants should read the article by Professor R. L. Wilder entitled "The beginning teacher of college mathematics." [2] Professor Wilder, emeritus professor of mathematics, University of Michigan, is well-known for his excellent teaching, and his paper provides excellent guidelines for the beginning instructor. He also discusses the cultural position of mathematics and lists a bibliography that includes some publications not on the CUPM list.

It is self evident that good teaching is of vital importance to members of the mathematical community, most of whom will spend some or all of their lives teaching in some form or other. Good teaching provides its own rewards, to both student and instructor. The two CUPM publications and the article by Professor Wilder contain suggestions and programs that should prove of inestimable value to universities and to teaching assistants.

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[1] Report of the Survey Committee, Conference Board of the Mathematical Sciences, vol. I (1967), p. 36.

[2] American Council on Education, Effective College Teaching (1970), $3.50; reprinted, CUPM Newsletter, December 1970.

REPORT OF CARNEGIE COMMISSION ON HIGHER EDUCATION ON GRADUATES AND JOBS

In the recent report of the Carnegie Commission on Higher Education (Graduates and Jobs: Adjusting to a New Labor Market Situation, to be published by McGraw-Hill), conclusions were reached and recommendations made that are of particular importance to the mathematical community. The predictions on the job market for Ph. D.'s are reasonably close to those made by the AMS Committee on Employment and Educational Policy. The report says that the market will probably be "increasingly unfavorable" during this decade with a "surplus that will reach sizable proportions by 1980." The most serious problem, according to the report, is that of the white, male Ph. D. in view of current efforts of institutions to implement their affirmative action plans by hiring more women and members of minority groups. The report states that this majority "constitute a special potential crisis situation that will result in massive disappointments in the later years of the 1970's and the early 1980's. This is the most serious single problem area we see ahead." Because the vast majority of mathematicians are employed by academic institutions, this particular problem is a grave one.

The report, which concerns job opportunities for bachelor-, master-, and doctoral-level graduates, contains thirty-five recommendations. The first of these states: "Institutions of higher education and governments at all levels should not restrict undergraduate opportunities to enroll in college or to receive student aid because of less favorable trends in the job market for college graduates than have prevailed in the recent past." This echoes the statement made by Gail Young in his article, "The problems of employment in mathematical sciences," which appeared in the August 1971 *Notices*.

Two of the recommendations were particularly concerned with data gathering on the job market: "Associations of professional schools and professional societies should undertake the responsibility for careful studies of manpower supply and demand for graduates in their respective fields"; and "Agencies and individuals that have been conducting studies of future supply and demand for Ph. D.'s should continue to review and update their work. We are impressed by the differences in outlook among fields and believe that the time has come for increased emphasis on projections relating to individual fields or groups of fields and less reliance on broad aggregative studies." The AMS committee has been particularly active in studying trends and making reports that appear in these *Notices*.

The report also recommended government support for investigative data gathering. "The federal government should give high priority to the development of more adequate, sophisticated, and coordinated programs of data gathering and analysis relating to highly educated manpower. Because professional associations can be particularly helpful in these efforts, we also believe that federal government agencies should develop programs designed to elicit and support the efforts of these associations."

This article has been prepared on behalf of the Committee on Employment and Educational Policy whose members are Richard D. Anderson (chairman), Michael Artin, John W. Jewett, Calvin C. Moore, Richard S. Palais, and Martha Kathleen Smith.

NEWS ITEMS AND ANNOUNCEMENTS

SEVENTEENTH ANNUAL AMS SURVEY

Questionnaires for the Seventeenth Annual AMS Survey, 1973, have now been mailed to chairmen of departments in the mathematical sciences. The quality of the survey and the usefulness of the results to the mathematical community are dependent upon the cooperation of chairmen. The Committee on Employment and Educational Policy prepared the questionnaires, and the AMS staff will compile the data. The members of the committee and Professor Mac Lane, president of the Society, urge chairmen to assist in this project by completing the forms promptly so that the information gathered will be accurate and up to date.

THREE MATHEMATICIANS NAMED TO NATIONAL ACADEMY OF SCIENCES

The National Academy of Sciences has announced the election of ninety-five new members, three of whom are mathematicians. The three mathematicians chosen for membership are Professor Felix E. Browder, University of Chicago; Professor Joseph B. Keller, New York University; and Professor Irving E. Segal, Massachusetts Institute of Technology. All are members of the Society.

QUERIES

Edited by Wendell H. Fleming

The QUERIES column is published in each issue of these *Notices*. This column welcomes questions from AMS members regarding mathematical matters such as details of, or references to, vaguely remembered theorems, sources of exposition of folk theorems, or the state of current knowledge concerning published conjectures. When appropriate, replies from readers will be edited into a definitive composite answer and published in a subsequent column. All answers received to QUERIES will ultimately be forwarded to the questioner. Consequently, all items submitted for consideration for possible publication in this column should include the name and complete mailing address of the person who is to receive the replies. The queries themselves, and responses to such queries, should be typewritten if at all possible and sent to Professor Wendell H. Fleming, American Mathematical Society, Post Office Box 6248, Providence, Rhode Island 02904.

18. Irving Adler (North Bennington, Vermont 05257). Are there any published solutions on a cylindrical surface of the diffusion equation with a finite number of sources? References, please.

19. Herbert E. Salzer (941 Washington Ave., Brooklyn, New York 11225). In J. V. Uspensky and M. A. Heaslet, Elementary Number Theory, McGraw-Hill, New York, 1939 (MR 1, 38), on page 462, top line, there occurs the equation $\omega_0(n) = 0$. The writer is unable to deduce it from the preceding material, all of which has been verified as correct. Can anyone prove (or disprove) that $\omega_0(n) = 0$?

20. Stuart P. Lloyd (Bell Laboratories, Murray Hill, New Jersey 07974). Does a given non-negative $f(x)$, $0 \leqq x < \infty$, with the properties $f(0+) = f(0) = 0$ and $\limsup_{x\to\infty} f(x)/x < \infty$ have a least subadditive majorant u ($u \geqq f$ and $u(x+y) \leqq u(x) + u(y)$, $0 \leqq x, y < \infty$)?

21. David M. Bloom (Brooklyn College (CUNY), Brooklyn, New York 11210). Let affine n-space A be imbedded in projective n-space S (over a field F); let $M = (a_{ij})$ be a symmetric $(n+1) \times (n+1)$ matrix of rank r, and let Q be the graph of the equation

$$\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}x_ix_j + 2\sum_{j=1}^{n} a_{n+1,j}x_j + a_{n+1,n+1} = 0$$

in A. By a singular point of Q we mean a point $V \in S$ such that, for every point $P \neq V$ in Q, all A-points on the (projective) line through P and V lie on Q. Proposition 1: If $M \neq 0$ and Q is non-empty, then the set of all singular points of Q is a projective (n-r)-space. Proposition 1 has been proved when F is the reals. Query: Is it true over a wider class of fields? (Proposition 1 is the affine analog of a well-known projective result which holds over all fields of characteristic $\neq 2$.)

22. Robert Gilmer (Department of Mathematics, Florida State University, Tallahassee, Florida 32306). Is a good survey article available on progress that has been made concerning a determination of groups G that can be realized as Galois groups over the rationals? I am familiar with the material in Gründzuge der Galois'schen Theorie, by Tschebotaröw and Schwerdtfeger, and with the paper Polynomials with certain prescribed conditions on the Galois group, by Rowlinson and Schwerdtfeger.

RESPONSES TO QUERIES

Several responses have been received to queries published in recent issues of these *Notices*. The editor wishes to thank all those who have responded. The following summarizes information given therein, arranged according to query number and name of the questioner.

5. (Shields, Nov. 1972) The following new references should be added to those listed previously (Jan. 1973) in this column:

g. A. V. Pogorelov, A complete solution of Hilbert's fourth problem, Dokl. Akad. Nauk SSSR 208 (1973), 48–51 = Soviet Math. Dokl.

14(1973), 46–49.

h. V. A. Rohlin, Congruences modulo 16 in Hilbert's sixteenth problem, Funkcional. Anal. i Priložen. 6(1972), no. 4, 58–64.

i. V. M. Harlamov, The maximal number of components of a 4th degree surface in RP^3, Funkcional. Anal. i Priložen. 6(1972), no. 4, 101. (contribution to Hilbert's sixteenth problem)

j. Ken Hirose and Shigeaki Iida, A proof of a negative answer to Hilbert's tenth problem, Proc. Japan Acad. 49(1973), 10–12.

7. (Wood, Jan. 1973) The following references have been suggested:

a. F. G. Friedlander and A. E. Heins, On a singular boundary value problem for the Euler-Darboux equation, J. Differential Equations 4(1968), 460–491.

b. _____, On the representation theorems of Poisson, Riemann, and Volterra for the Euler-Poisson-Darboux equation, Arch. Rational Mech. Anal. 33(1969), 219–230.

c. Peter Henrici, A survey of I. N. Vekua's theory of elliptic partial differential equations with analytic coefficients, Z. Angew. Math. Phys. 8(1957), 169–203. MR 19, 38.

d. _____, Complete systems of solutions for a class of singular elliptic partial differential equations, Boundary Problems in Differential Equations, edited by Rudolph Langer, University of Wisconsin Press, 1960, 19–34. MR 22 #2785.

e. Johanna Stenzel, The Cauchy problem for a singular partial differential equation, Ph.D. dissertation, University of Michigan, 1968.

f. G.S.S. Ludford, Two topics in one-dimensional gas dynamics, Studies in Mathematics and Mechanics, Academic Press, 1954, p. 184. MR 16, 418.

8. (Goldberg, Jan. 1973) The following reference has been suggested:

a. S. T. Kuroda, On a generalization of Weinstein-Aronszajn formula and the infinite determinant, Scientific Papers of the College of General Education, University of Tokyo 11(1961), 1–12. MR 25 #1456.

9. (Huggins, Jan. 1973) According to several respondants, the irrationality of Euler's constant γ is still an open question. However, there are partial results and an incomplete proof in the literature. The relevant references may be obtained from Mathematical Reviews: MR 25 #3011, MR 29 #2222, MR 29 #3431, MR 32 #5599, MR 33 #5565, MR 36 #118, MR 36 #1395, MR 36 #1397, MR 38 #104, MR 39 #3181.

12. (Wilansky, Feb. 1973) In partial reply, for X a separable Banach space, X' contains a sequence of elements of norm 1 that converge w* to 0. See W. B. Johnson and H. P. Rosenthal, On w*-basic sequences and their applications to the study of Banach spaces, Studia Math. 43(1972), 85.

14. (Lee, Feb. 1973) The answer if "yes"–every open, orientable 3-manifold is parallelizable; in fact, such a manifold can be immersed in R^3. See J.H.C. Whitehead, The immersion of an open 3-manifold in Euclidean 3-space, Proc. London Math. Soc. 11(1961), 81–90.

15. (Lee, Feb. 1973) This can be proved by obstruction theory as outlined, for example, in part III of Steenrod's book, The topology of fibre bundles, Princeton Univ. Press, 1951 (MR 12, 522). The obstructions to a "parallelization" of 4-manifold, M^4, lie in the cohomology groups $H^i(M^4, \pi_{i-1}(SO(4)))$, and these cohomology groups are 0, except for $i=2$. It is readily seen that the 2-dimensional obstruction is exactly the Stiefel-Whitney class, w_2.

PROBLEM LISTS

Good lists of conjectures have played a useful role in stimulating mathematical advances. The 23 problems of Hilbert furnish a classical example; for up-to-date references about the status of the Hilbert problems see the responses to Shields' Query #5 in the January 1973 *Notices* and in the present column above. More recently, a number of thoughtfully prepared lists of conjectures have resulted from symposia, sessions of AMS meetings, summer conferences, and the like.

Problems in Ring Theory

Problems from the Ring Theory Problem Session at the Dallas Meeting of the American

Mathematical Society, edited by E. G. Evans, Jr.

1. Let R be a commutative noetherian domain. Following Nagata's book if P is a prime ideal of R then depth (P) is the Krull dimension of R and altitude (P) is the Krull dimension of R_P. We say that a domain R is catenary if for every localization at a maximal ideal m of R and every prime P_m of R_m, depth (P_m) + altitude (P_m)= Krull dimension R_m. The following are conjectured to be true:

(a) The integral closure of R in its quotient field is catenary.

(b) If P is a prime ideal of R with altitude $P \geqq 2$, then there is a prime ideal p in R such that $p \subset P$ and depth p = depth P + 1.

(c) If R is a domain such that every prime ideal of altitude 1 in R has depth equal to dim R - 1, then R is catenary.

(d) If R is catenary, then the integral closure of R in its quotient field is catenary.

2. The Serre Problem. Let K be a field and $R = K[X_1, \dots, X_n]$. Then is every finitely generated projective R module free? The simplest unknown case is M = 3 and P a rank 2 projective. This problem has been modified by Bass and Murthy to R = S[X] a noetherian ring of dim = $d < \infty$. Let P be a finitely generated projective of rank = d. Then is $P \simeq R \oplus P'$?

3. Let R be a complete local noetherian ring of dimension d. Let M be a finitely generated R module and depth (M) be the length of a longest R sequence on M. Does there exist a finitely generated R module M such that depth (M) = d? An affirmative answer to this would have several interesting consequences. On the other hand some assumption like completeness is necessary. Another way of viewing this problem is let R be a regular local ring and S a commutative R algebra which is finitely generated as an R module. Is there a homomorphism of S into n by n matrices over R which is compatible with the R algebra structure of both?

4. Let R and S be noetherian domains with R[X] isomorphic to S[X]. Is Krull dimension of R equal to the Krull dimension of S? Is the quotient field of R isomorphic to the quotient field of S? Same problems for power series.

5. Let D be a division ring which is finite dimensional over its center, F. When is it a crossed product? Say $\dim_F D = n^2$. D is a crossed product if D has a maximal commutative subfield L of dimension n over F where L is Galois over F.

This is false if char(F) = 0 and n is divisible by 8 or p^2 where P is an odd prime by a theorem of Amitsur. It is false if char(F) = P, (n, p) = 1, and n is divisible by the cube of a prime by a Theorem of Schacher and Small.

What if n is a prime? If n = 3, D is a crossed product. n = 5 is open. This is equivalent to the following question: Does there exist a polynomial $P(X_1, \dots, X_n)$ in noncommuting variables such that when you specialize the variables $P(A_1, \dots, A_n)$ is not always central when the A_i are 5 by 5 matrices but $P(A_1, \dots, A_n)^5$ is central for all A_i?

6. Let R be a two sided noetherian ring. We say an R module M is a critical prime module if the Gabriel dimension of M defined and if $0 \neq N \subset M$ is a submodule, then dim M/N < dim M and ann N = ann M. If M is a critical prime and finitely generated R module and $f: M \to N \not\subseteq M$ and $N \neq 0$, then is dim M/N = dim M - 1?

7. Let K be a field of characteristic 0 and G a finite group. Then the group ring K[G] is a sum of matrix rings over division rings D_i. What kind of D_i can appear? They give a subgroup S(K), the Schur subgroup, of the Brauer group of K. If K = Q, the rationals, S(K) is the elements of order less than or equal to 2 in the Brauer group. What if K is any other field? If D occurs as $M_n(D)$ for some K[G], how small an n is possible for that K?

8. Does there exist a quasi-local integrally closed domain with a maximal ideal which is not finitely generated but such that some power of it is?

9. Let R be a Prüfer ring. Is every finitely generated ideal generated by two elements?

10. Let R be a simple noetherian ring with 1. Is R isomorphic to $M_n(D)$ where D is a simple noetherian domain? Is R Morita equivalent to a simple noetherian domain?

11. Let R be a commutitive ring such that every finitely generated module is a direct sum of cyclic modules.

(a) Does R have only finitely many minimal prime ideals?

(b) Does R have a noetherian maximal spectrum?

12. Let $R \subset S$ be rings such that each monic polynomial over R has a root in S, then does each monic polynomial over R split into linear factors in S? This is true if S is an integrally closed domain.

Problems in the Computation of Homeotopy Groups of 2-manifolds

L. V. Quintas, Solved and unsolved problems in the computation of homeotopy groups of 2-manifolds, Trans. New York Academy of Sciences, Ser. II, 30(1968), 919–938 (also available through Department of Mathematics, Pace College, New York, New York 10038).

Problems in Integration Theory

P. Y. Lee, Some problems in integration theory, Math. Chronicle Vol. 2, No. 1 (April 1973, Auckland, New Zealand). These problems concern nonabsolute integration.

NEWS ITEMS AND ANNOUNCEMENTS

MEMORANDUM TO WOMEN PH. D.'S IN MATHEMATICS

Our committee requested the American Mathematical Society to make a roster of women mathematicians with Ph. D.'s to improve the opportunities for women to participate in mathematical activities and to help in employment. The form for the roster appeared in the April 1973 issue of the *Notices* of the AMS on page 131 but there has been a very small response so far. We urge you now to fill in the questionnaire, or a photocopy of it, so that your name and qualifications will be on file with the American Mathematical Society. Knowing that some people object to being on file anyway, that some women object to being singled out from their male colleagues and that some people cannot be bothered to fill out still one more form we, nevertheless, urge you to cooperate in our present effort. Please send the form to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904, by June 29, 1973.

Cathleen S. Morawetz

LOST

The Committee on Women in Mathematics is trying to find the addresses of the following mathematicians. If anyone has information on their present whereabouts, please notify the Committee on Women in Mathematics, c/o American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904.

Jacqueline Anderson
Mira Bhargava
Marta C. Bynge
Natalie Calabro
Beverly Douglas Causey
Barbara J. Thomson Chicks
Virginia A. Leader Clark
Joan Ann Conn
Madeline Drufenbrock (Sister Sophie)
Karen Marie Atkins Duncan
Karen Norma Federick
Polly C. Bartholomew Feigl
Frances Adelia Frost
Nadia Makary Girgis
Florence Gordon
Elizabeth Mendel Grobe
Claudia In. Henschke
Cara Job Hughes
Carole Sloan Izen
Sister Mary Kenneth Keller
Gail S. Kinkle
Harriett Botta Kruse
Jean A. Larson
Laura Miller Lawson
Judith Ann Ormann Lewis
Joyce Cho-Hsin Ling
Erika Adrienne Mares
Irene Patricia Monahan
Halina Montvila
Ann Muzyka
Hanna I. Nassar
Norma A. Nelson
Jane Roberg
Esther Rodlitz
Hannah L. Wolfson Rosenbaum
Lalitha Sanathanan
Jitendriya Sarangi
Marlene Schick
Annemarie Schlette
Myra Schmeltzer
Ann Marie Singleterry
Nora Snoeck Smiriga
Diane Pirog Smith
Georgia A. Caldwell Smith
Leslie Sheila Smith
Ravipati Suhasini
Fidya Sagar Taneja
Diana Taylor
Evelyn Thornton
Patricia Anne Tucker
Patricia Ann Tulley
Kathleen Tunner
Harriet G. Wagman
Sandra Ann West
W. F. Whitmore (Elizabeth S. Arnold)
Helen Wittenberg
Elizabeth Yen (Yu-Yin Hsi)
Karen Kan Yuen
Rita Zemach

SPECIAL MEETINGS INFORMATION CENTER

The purpose of this center is to maintain a file on prospective symposia, colloquia, institutes, seminars, special years, meetings of other associations, and to notify the organizers if conflicts in subject matter, dates or geographical area become apparent. A first announcement will be published in the *Notices* if it contains a call for papers, place, date, and subject (when applicable); a second announcement must contain reasonably complete details of the meeting in order for it to be published. Information on the pre-preliminary planning will be stored in the files, and will be available to anyone desiring information on prospective conferences. All communications on special meetings should be sent to the Special Meetings Information Center of the American Mathematical Society.

June 18–22, 1973
REGIONAL CONFERENCE ON HOLOMORPHIC FUNCTIONS OF FINITE GROWTH
University of Wisconsin–Whitewater, Whitewater, Wisconsin
Speaker: Ten one-hour lectures devoted to the Nevanlinna theory of holomorphic and meromorphic functions of finite order in several complex variables by Wilhelm Stoll, University of Notre Dame
Support: CBMS and NSF (subsistence and limited travel allowances for twenty-five participants)
Information and applications: Dr. Rudolph M. Najar, Department of Mathematics, University of Wisconsin–Whitewater, Whitewater, Wisconsin 53190

July 2–3, 1973
JOURNEES D'ALGEBRE NON COMMUTATIVE
Lyon, France
Information: Université Claude Bernard, Département de Mathématique, 43 boul. 11 novembre 1918, 69621 Villeurbanne, France

July 9–12, 1973
FACHTAGUNG UBER AUTOMATENTHEORIE UND FORMALE SPRACHEN
University of Bonn, Federal Republic of Germany
Information: Professor Dr. K. H. Böhling, Institut für Angewandte Mathematik und Informatik, Universität Bonn, Wegelerstrasse 6, 53 Bonn, Federal Republic of Germany

July 15–21, 1973
LOGIC COLLOQUIUM
Bristol, England
Sponsor: Association for Symbolic Logic
Program: Invited addresses on philosophy of mathematics, category theory, proof theory, theory of computation, metamathematics of algebra; contributed papers (deadline was June 1)
Information: Dr. H. E. Rose, School of Mathematics, University of Bristol, University Walk, Bristol BS8 1TW, England

July 16–21, 1973
SUMMER SCHOOL ON GROUP THEORY AND COMPUTATION
University College, Galway, Eire
Information: Mr. M.P.J. Curran, Department of Mathematics, University College, Galway, Eire

July 25–26, 1973
CONFERENCE ON COMPUTATIONAL PROBLEMS IN STATISTICS
University of Essex, England
Speakers: G. A. Barnard (University of Essex), E.M.L. Beale (Scientific Control Systems Limited), C. Rogers and G. Wilkinson (Rothamsted Experimental Station), J. G. Hayes (National Physical Laboratory), J. M. Craddock (Meteorological Office, Bracknell), R. W. Hiorns (University of Oxford), J. H. Wilkinson and P. E. Gill (National Physical Laboratory), M.A.H. Dempster (University of Oxford), W. G. Gentlemen (University of Waterloo, Canada)
Information and applications: Secretary, The Institute of Mathematics and Its Applications, Maitland House, Warrior Square, Southend-on-Sea, Essex SS1 2JY, England

August 13–17, 1973
THIRD CONFERENCE ON STOCHASTIC PROCESSES AND THEIR APPLICATIONS
Sheffield, England
Information: Miss Catherine Colver, Department of Probability and Statistics, The University, Sheffield S3 7RH, England

August 24–28, 1973
REGIONAL CONFERENCE ON ALGEBRAIC K-THEORY
Colorado State University, Fort Collins, Colorado
Speaker: Hyman Bass, Columbia University
Support: NSF and CBMS
Information: Professor F. R. DeMeyer, Department of Mathematics, Colorado State University, Fort Collins, Colorado 80521

August 24 - September 2, 1973
CENTRO INTERNAZIONALE MATEMATICO ESTIVE (CIME)
Varenna, Italy
Subject: Spectral analysis
Speakers: Uy. M. Berezanskii (University of Kiev), L. Garding (University of Lund), Ch. Goulaouic (University of Paris), N. Schechter (Belfer Graduate School of Sciences, New York)
Applications: Submit, along with scientific curriculum and/or a letter of recommendation, a statement indicating the reason for particular interest in the course by June 20, 1973, to Professor R. Conti, Secretary, CIME, Istituto Matematico U. DINI, Viale Morgagni, 67/A, 50134 Firenze, Italy

September 3–8, 1973
INTERNATIONAL CONFERENCE ON NONLINEAR DIFFERENTIAL EQUATIONS
Brussels, Belgium
Speakers: N. T. Bhatia (USA), L. Cesari (USA), D. Graffi (Italy), A. Halanay (Romania), J. K. Hale (USA), V. M. Matrosov (USSR), Yu. A. Mitropolsky (USSR), M. Urabe (Japan), L. Salvadori (Italy), Th. Vogel (France)
Information: Professor Paul Janssens, Université Libre de Bruxelles, Faculté des Sciences Appliquées, avenue F. D. Roosevelt, 50, B-1050 Bruxelles, Belgium

September 4–7, 1973
ACM INTERNATIONAL COMPUTING SYMPOSIUM
Davos, Switzerland
Information: Dr. H. Lipps, Symposium Chairman, International Computing Symposium 1973, c/o CERN, CH-1211 Genf 23, Switzerland

September 10–12, 1973
CONFERENCE ON DIFFERENTIAL GEOMETRY AND ITS APPLICATIONS IN ENGINEERING
University of Southampton, England
Speakers: E. T. Davies (University of Waterloo, Canada), D. J. Dawe (Berkeley Nuclear Laboratory, Gloucestershire), J. Eells (Institute for Advanced Study), W. T. Koiter (Technische Hogeschool te Delft), L. Markus (University of Minnesota and University of Warwick), E. Tonti (Politecnico di Milano)
Contributed papers: Short abstract, not exceeding 200 words, to be submitted by June 15, 1973, to Professor J. W. Craggs, FIMA, Department of Mathematics, The University, Southampton, England
Information and applications: Secretary, The Institute of Mathematics and Its Applications, Maitland House, Warrior Square, Southend-on-Sea, Essex SS1 2JY, England

September 17–20, 1973
CONGRES FRANCAIS DE MECANIQUE
Poitiers, France
Information: Mlle. M.-C. Charpentier, Université Poitiers, Département de Mécanique, 40 avenue du Recteur Pineau, 86022 Poitiers, France

October 5–6, 1973
DANISH MATHEMATICAL SOCIETY 100th ANNIVERSARY
University of Copenhagen, Denmark
Speakers: Fr. Fabricius-Bjerre, W. Fenchel, S. Helgason, B. Jessen, L. Kristensen
Information: Danish Mathematical Society, H. C. Ørsted Institute, 2100 Copenhagen Ø, Denmark

October 8–10, 1973
JAHRESTAGUNG DER GESELLSCHAFT FÜR INFORMATIK
University of Hamburg, Hamburg, Federal Republic of Germany
Information: Programmausschuss GI 73, Institut für Informatik der Universität Hamburg, Schlüterstrasse 70, D-2000 Hamburg 13, Federal Republic of Germany

October 8–10, 1973
SEMINAR ON GENERALIZED INVERSES AND APPLICATIONS
Mathematics Research Center, University of Wisconsin-Madison
Program: Thirteen invited lectures dealing with the theory of generalized inverses of matrices and linear operators; computational aspects of least squares problems; and applications of generalized inverses in linear and nonlinear analysis, ill-posed problems, operator equations approximation, optimization and control theory, statistics, econometrics and operations research
Information: Professor M. Z. Nashed, Program Chairman, c/o Mrs. Gladys G. Moran, Mathematics Research Center, University of Wisconsin–Madison, 610 Walnut Street, Madison, Wisconsin 53706

October 17–21, 1973
COLLOQUE INTERNATIONAL SUR LES METHODES DE CALCUL SCIENTIFIQUE ET TECHNIQUE
Rocquencourt, France
Information: IRIA, Service Relations Exterieures, Rocquencourt, 78150 le Chesnay, France

October 22–23, 1973
SIXTEENTH ANNUAL CONGRESS AND ANNUAL GENERAL MEETING OF THE SOUTH AFRICAN MATHEMATICAL SOCIETY
University of South Africa, Pretoria, South Africa
Program: Invited survey lectures, contributed papers, symposium
Information: Professor J. H. van der Merwe, Department of Mathematics, University of South Africa, P.O. Box 392, Pretoria, Republic of South Africa

November 12–14, 1973
NATIONAL MEETING OF OPERATIONS RESEARCH SOCIETY OF AMERICA
San Diego, California
Program: Keynote speaker, three tutorial lectures, contributed papers
Information: J.V.J. Ravenis II, 44th National ORSA Meeting, P.O. Box 20673, San Diego, California 92120

November 20, 1973 - April 9, 1974
ISTITUTO NAZIONALE DI ALTA MATEMATICA
Rome, Italy
November 20–23, 1973
Anelli de funzioni continue
December 11–14, 1973
Gruppi infiniti
January 22–25, 1974
Gruppi topologici e gruppi de Lie
February 19–22, 1974
Teoria matematica dell'elettromagnetismo
March 12–15, 1974
Trasformazioni quasiconformi e questioni connesse
April 4–9, 1974
Programmazione matematica e sue applicazioni
Information: Professor Giuseppe Scorza, Istituto Nazionale di Alta Matematica, Roma, Italy

January 7–8, 1974
CONFERENCE ON RESEARCH IN MATHEMATICS
University of Reading, England
Speakers: J. M. Hammersley (University of Oxford), M. F. Atiyah (Mathematical Institute, Oxford), Hermann Boudi (Ministry of Defence), L. Fox (Oxford University Computing Laboratory), W. H. Cockcroft (University of Hull), H. Halberstam (University of Nottingham), C. A. Coulson (University of Oxford)
Information and application forms: Secretary, The Institute of Mathematics and Its Applications, Maitland House, Warrior Square, Southend-on-Sea, Essex SS1 2JY, England. (Note that numbers are limited and early registration is recommended.)

NOTES ON PREVIOUS ANNOUNCEMENTS

The Colloquium on Gyrodynamics, September 3–5, 1973, will be held in Louvain-la-Neuve, not Louvain, Belgium. Professor F. Buckens (for information) has sent a change of address; write to him at Bâtiment S. Stevin, Place du Levant, 2, B-1348 Louvain-la-Neuve, Belgium.

The Colloquium on the Exchanges at the Air/Sea Boundary, announced for September 1973 in the February 1973 *Notices*, will be held in September 1974.

The sponsor for the Second Conference on Societal Problems, June 29–July 2, 1973, is SIAM Institute for Mathematics and Society (SIMS).

Definite dates have been received for the Colloquium on the Dynamics of Machine Foundations, announced in the February 1973 *Notices* merely as September 1973. That colloquium is being held on October 29–31, 1973.

LETTERS TO THE EDITOR

Editor, the *Notices*

This letter is intended as a public appeal to the Editors of all mathematical periodicals to recognize and use their editorial responsibility to refuse to permit offensive remarks directed against groups of people to be included in articles published in their journals. We are sure that they would exercise that right automatically, without this reminder, in the case, say, of pornography.

The particular example which prompts this public appeal is the appearance in the current volumes of both the Canadian Mathematical Bulletin and Discrete Mathematics of the following statement of a well-known combinatorial problem:

> There are n ladies, and each of them knows some item of gossip not known to the others. They communicate by telephone and whenever one lady calls another, they tell each other all that they know at that time. How many calls are required before each gossip knows everything?

It is of course clear that this formulation is based on the customary derrogation of women as idle gossips. If the editors defend this formulation on the basis of realism, we suggest the following, as likely to be far more reflective of reality:

> There are n corporation presidents, and each of them knows some item of illegal price-fixing information not known to the others. They communicate by telephone and whenever one corporation president calls another they tell each other all that they know at that time. How many calls are required before each capitalist knows everything?

We have yet to see this formulation, or anything suggestive of the same underlying social concept, used in this problem. But we feel sure that many mathematicians would recognize at once that it contains not only a mathematical problem but also a definite social statement. We feel that the formulation in terms of ladies also contains a social statement, one which is both clear and obnoxious.

Paul T. Bateman	Anatole Beck
Judy Green	Ann Heard
John Kasdan	Jerome Dancis
James A. Donaldson	M. Solveig Espelie
Lee Lorch	Evelyn M. Silvia

EDITORIAL COMMENT:

The two papers are

Hajnal, A., Milner, E. C., and Szemerédi, E.: A cure for the telephone disease, Can. Math. Bull., 15, 447–450 (1972)

Baker, Brenda and Shoslak, Robert Gossips and telephones, Discrete Math., 2, 191–193 (1972)

One of the editors of the Canadian Mathematical Bulletin is Amram Meir.

Authors and editors were given the opportunity to reply, with the following two letters resulting.

Editor, the *Notices*

We do not deny the possibility that the ladies are corporation presidents or, indeed, mathematicians.

Brenda Baker and Robert Shostak

Editor, the *Notices*

Thanks for your letter of February 28. It is needless to say that it was without intention that a formulation of a problem which could be considered as offensive to a group of people (ladies), has appeared in one of the articles published in the Canadian Mathematical Bulletin.

I wish to express my sincere apologies to all ladies of the world (my mother, wife, daughter included).

I shall be grateful if all those interested in the "telephone problem" will replace the pertinent lines by the following:

Problem:

There are n Editors of Mathematical Periodicals; each of them knows the name of a person who submits trivial papers (not known to any of the other Editors). They communicate by telephone and whenever two Editors make a call they pass on to each other as many names as they know at the time.

How many calls are needed before all Editors know all the names?

A. Meir

Editor, the *Notices*

Having recently, in view of my approaching retirement, presented my resignation, I would greatly welcome the opportunity of a final word.

On the first line, I express to numerous members and officers of the Society my gratitude for many kindnesses in a variety of instances. I entered the profession in the depths of a great depression and it seems apparent that I leave it on the edge of another, at least so far as mathematical affairs are concerned. It has been my good fortune to participate in the luxury of being a mathematician in pleasant times, for which I am indeed happy.

It is a source of great pride to me to have once been a mathematician. Certainly not a great one, and perhaps not even a very good one, but it has meant much to have shared by reflection, if only from a distance, some of the intellectual glory of those who mastered some great fraction of the discipline.

On the second line, a few observations. It is my judgment that the most important functions of the Society are arranging meetings and publishing MR. A very close third is the publication of its journals. But by and large it seems that the Society ought to get out of the publishing business (including translations) to the extent that it can, and certainly where this activity is not supported by long term external funding. In this context I much regret the effort to enlarge MR to include "applied" mathematics, unless the entirety of this effort is profitably (to the Society) based upon extended support from outside. The point and purpose of mathematicians is to be mathematicians, and I take a decidedly dim view of this hurrying and scurrying around to save the world for science and technology. This is no doubt, a proper activity for the genius type, but of these we have too few to make the business useful.

With this probably unpopular remark, and with best regards and all good wishes for the future of the Society, I remain,

Alexander Doniphan Wallace

Editor, the *Notices*

I am writing this open letter about a minor personal disappointment, which I believe has important implications for the whole mathematical community.

I have been told that my research contract with the National Science Foundation has not been renewed. This is a common enough occurrence these days, and I understand and appreciate the need to create opportunities for young mathematicians. However, these contracts are competitions, with definite but strange rules. For example, I was told that my application failed because my work was not considered "exciting" by a group of mathematicians who were asked to comment on it. This raises an issue that transcends my own case.

For the last thirteen years, I have been working in the no-man's land between contemporary pure mathematics, particularly differential geometry and Lie group theory, and its potential scientific applications.

I have published eight books, and have finished two more which attempt the dual task of making ideas available to mathematically oriented scientists and of developing the mathematics which one needs. My work is not meant to be popular or expository, but to make a serious attempt at reintegration of mathematics with the scientific applications, which I believe is the ultimate foundation for genuine progress in mathematics.

Even at the beginning of this work, I saw clearly the current consequences of the isolation of the mathematical community from these applications. Unfortunately, there are too many people in position of leadership in the profession who take great pride in this isolation and who try to cut down those of us who are bold enough to attempt to combine a career transcending narrow disciplinary lines. Even in the prosperous years, when most marginally competent pure mathematicians were handsomely supported and enjoyed a satisfying academic career, several of my colleagues were quite cruelly and senselessly prevented from obtaining positions where their talent could serve the community.

The consequences of this foolish policy of over stimulating the growth of mathematics in one direction are now clear. Whoever makes such decisions has decided, quite wisely, that the last thing the world needs is more narrowly trained academic mathematicians who demand positions and research grants so that they can train replicas of themselves, who will then subdivide the available research problems into ever narrower categories, and so on.

The consequences of this blind urge to narrowness, purity, and specialization are tragic for the new generation of mathematicians whose careers will be seriously blighted by a lack of money. The best "applied mathematics" (in the traditional and literal sense) is now being done by young scientists in other disciplines, such as electrical engineering, computer science, biology and physics. I have a feeling that this is a loss, that there is a constructive role in the progress of science and technology for people trained in pure mathematics, but that seems to be irrelevant now, save for a few isolated cases. I suppose too that we will see our best students lost to those areas such as computer science and biology which are expanding and which present opportunities for using their mathematical talent. Of course, I must admit that the leaders of the profession (including those who label themselves as "applied mathematicians") are aware of these problems, but my experience has been that they are a complacent lot, better at high minded rhetoric and self-inflation that at the sort of realistic, global thinking that one expects from them in the current situation. Most seem content with their role in the ponderous academic bureaucracy that rules our careers.

Of course, I am being melodramatic in blowing up this small case concerning my own career into such a large issue. Perhaps this element of paranoia that many of us who have tried to do applications and still keep our ties with the mathematical community feel is our own personal problem. Certainly I have the highest respect for the integrity of the National Science Foundation, and have found that the people who work in the Mathematics Section are admirable, dedicated and hard working. I am sure that they feel far more agony over the present situation than those of us who still have comfortable positions and have ample, if reduced, opportunities for professional advancement. Certainly it is not their fault that the obvious tactic of meeting the problem in the short term by reducing the size of summer salaries to senior people was not chosen. (And why not?)

However, I must say that the policies used by the National Science Foundation over the years in awarding grants have contributed to the present crisis. I have no statistical evidence, but I am certain that they strongly favor precisely defined research proposals in a narrow technical area over the sort of broad, long-term interdisciplinary work that we need and that would tie in with applications. It is a classic bureaucracy, and it imposes its bureaucratic values on us. I suspect that they favor the type of work that is easiest to judge, classify, and rank in some linear ordering, and to justify to their superiors in the bureaucracy.

I suggest that some reordering of priorities, procedures and goals is badly needed, both by the mathematical community and the National Science Foundation.

Robert Hermann

Editor, the *Notices*

Such blatantly discriminatory advertisements as those occurring on pages A246 and A247 of the January 1973 issue (vol. 20, Number 1) should not be published in an official journal of the American Mathematical Society.

They are an insult to every member of the Society and pointedly show the real present day meaning of the phrase "equal opportunity". They also provide a sad commentary on the ever increasing governmental direction of all activities in this country.

A. Swimmer

EDITORIAL COMMENT:

It appears that the writer takes exception to the sentences "Women, blacks, Americans with Spanish surnames, as well as other minority ethnic groups, are urged to apply" and to the sentence "Montana State University is an equal opportunity employer, and solicits applications from members of minority groups."

Such statements are in current use as a result of federal regulations with the force of law that appear in the Federal Register. A pertinent sentence is the following

> The contractor should... inform all recruiting sources verbally and in writing of company policy, stipulating that these sources actively recruit and refer minorities and women for all positions listed.

The reference is to vol. 36, no. 234 - Saturday, December 4, 1971; more specifically to the Title 41, 360-2.20, item (b) (1), to be found on p. 23155.

When an editor referred the writer of the letter to the extract above, he in turn referred the editor to item (a) (1) of the same reference on p. 23154, which states that the contractor

> ...should recruit, hire, train, and promote personnel in all job classifications without regard to race, color, sex, or national origin...

The writer further states, in part, in his second letter

> [W]e all agree that discriminatory practices in such matters as hiring personnel for positions in public institutions should be abolished. However these practices cannot be abolished by following the directions of §60 of Title 41 since, besides being contradictory, they instruct contractors to replace one set of discriminatory practices by another one.

Editor, the *Notices*

The exhortation--"Write fewer papers!"--made by the distinguished members of the Mathematical Reviews Crisis Committee [1] reminded me of a story [2, Pt. III, p. 49] related by the eminent psychologist Baba Ram Dass (né Richard Alpert) who actually made and carried out the decision to write fewer papers.

> A woman once came to Mahatma Gandhi with her little boy. She asked, "Mahatma-ji, tell my little boy to stop eating sugar." "Come back in three days," said Gandhi. In three days the woman and the little boy returned and Mahatma Gandhi said to the boy, "Stop eating sugar." The woman asked, "Why was it necessary for us to return only after three days for you to tell my boy that?" The Mahatma replied: "Three days ago I had not stopped eating sugar."

1. Benney, D. J., Boas, R. P., Gehring, F.W., LeVeque, W. J., Palais, R. S., Schwartz, J.T., and Spencer, D. C., Report of the Mathematical Reviews Crisis Committee, Notices Amer. Math. Soc. 20 (1973), 121-122.
2. Ram Dass, B., Be here now, Lama Foundation,San Cristobal, New Mexico, 1971.

Robert B. Kelman

Editor, the *Notices*

I would like to make three suggestions for improving the procedures by which the AMS elections are conducted: (1) There should be a separate envelope to place the ballot in, which in turn would be placed in a second envelope to be signed and mailed. Thus there would be greater assurance of a true secret ballot. (2) All nominees should be identified with respect to their fields of mathematical interest. (3) Candidates for contested office should be required to submit a short statement of their intentions.

The great majority of the American Mathematical Society members are not familiar with many of the candidates, let alone their views. The latter two proposals would insure that our officers would more truly represent the membership's thinking with regard to both the future directions of the society as well as its day-to-day operations.

Albert Feuer

NEW AMS PUBLICATIONS

CBMS REGIONAL CONFERENCE SERIES IN MATHEMATICS

MEASURE ALGEBRAS by Joseph L. Taylor

Number 16
108 pages; list price \$4.70; individual price \$3.53; ISBN 0-8218-1666-7
To order, please specify CBMS/16

These notes were prepared in conjunction with the NSF regional conference on measure algebras held at the University of Montana during the week of June 19, 1972. The original objective in preparing these notes was to give a coherent, detailed, and simplified presentation of a body of material on measure algebras developed in a recent series of papers by the author. This material has two main thrusts: the first concerns an abstract characterization of Banach algebras which arise as algebras of measures under convolution (convolution measure algebras) and a semigroup representation of the spectrum (maximal ideal space) of such an algebra; the second deals with a characterization of the cohomology of the spectrum of a measure algebra and applications of this characterization to the study of idempotents, logarithms, and invertible elements. As the project progressed the original concept broadened. The final product is a more general treatment of measure algebras, although it is still heavily slanted in the direction of the author's own work.

LECTURES ON LINEAR PARTIAL DIFFERENTIAL EQUATIONS by Louis Nirenberg

Number 17
60 pages; list price \$4.00; individual price \$3.00; ISBN 0-8218-1667-5
To order, please specify CBMS/17

This volume is the outgrowth of a series of lectures presented at a CBMS Regional Conference held at Texas Tech University in May 1972. The theory of partial differential equations has seen a remarkable development in the last twenty years. New questions have been asked and new and powerful techniques developed, leading often to deeper understanding and resolution of old problems. In these lectures the author takes up several topics in the theory of linear partial differential equations, beginning with rather elementary, expository material, and going on to some of the current developments and techniques. The lectures are meant for the nonexpert, as an introduction to some of the current questions and ideas. Since the author wished to include some deep results, he has been technical on some occasions, but he has endeavored to describe the necessary background.

SELECTED TABLES IN MATHEMATICAL STATISTICS

Each volume of this series will contain several sets of extensive tables of interest to statisticians and users of statistical methods. The introductory material for each set discusses method of computation, accuracy, methods of interpolation (when required), and applications, and gives numerical examples of the use of the tables. This series is published by the Society for the Institute of Mathematical Statistics. Volume I, first published in 1970 as one of the Markham Series in Statistics, will be reprinted in 1974. This volume contains five sets of tables, four of which were freshly computed for this volume, while one is a reprint and extension of previously published tables. These tables deal with (1) the cumulative noncentral chi-square distribution, which permits a determination of the power function of the chi-square test, (2) two-sample Kolmogorov-Smirnov criterion, (3) signed rank test, (4) distribution of the product moment statistics for exponential, hald-Gamma and normal series, (5) orthogonal polynomials. The tentative plans are for Volume II to contain five new sets of tables. These will deal with (1) probability integral of the doubly noncentral t-distribution, (2) doubly noncentral F-distribution, (3) expected sample size for curtailed fixed sample size tests, (4) distribution of some product-moment statistics under a null permutation hypothesis, (5) zonal polynomials of order 1–12. Donald B. Owen is the chairman of the Committee on Tables, and James M. Davenport is the managing editor. Publication is expected in 1974. Standing orders for this series are now being accepted.

SELECTED TRANSLATIONS IN MATHEMATICAL STATISTICS AND PROBABILITY

Volume 11
292 pages; list price \$13.80; member price \$10.35; ISBN 0-8218-1461-3
To order, please specify STAPRO/11

The following authors are represented in this volume: A. Aleškjavičene [A. Aleškevičiene], I. I. Banis [J. Banys], A. Bikjalis [A. Bikelis], N. A. Bodin, N. G. Gamkrelidze, B. Grigelionis, S. V. Grigor'ev, Hoang Hyu N'y [Hoang Huu Nhu], G. Jasjunas [G. Jasiūnas], F. M. Kagan, V. F. Kolčin [V. Kolčinas], V. I. Ladohin, T. I. Mirskaja, A. V. Nagaev, A. S. Pąbedinskaĭte, V. Pipiras, J. Sapagovas, A. N. Šerstnev, V. A. Statuljavičjus [V. Statulevičius], P. Survila, A. A. Tempel'man, I. N. Volodin, and V. A. Zalgaller.

MEMOIRS OF THE AMERICAN MATHEMATICAL SOCIETY

WEB DERIVATIVES by Hewitt Kenyon and Anthony P. Morse

Number 132
178 pages; list price \$3.50; member price \$2.63; ISBN 0-8218-1832-5
To order, please specify MEMO/132

This Memoir presents a dimension- and metric-free setting for the differentiation of non-negative valued set functions. Conditions which guarantee the frequent existence of the derivative of one such function with respect to another are examined and then strengthened to insure the measurability of the derivative in case the denominator function is a measure. A Lebesgue decomposition of a function into a part which is the integral of its derivative and a remainder with derivative almost everywhere zero is obtained. In addition to unifying much of the existing theory, the Memoir makes applications to the differentiation of indefinite integrals, complex valued functions of bounded variation on the reals, and others.

ON SUMMABILITY METHODS FOR CONJUGATE FOURIER-STIELTJES INTEGRALS IN SEVERAL VARIABLES AND GENERALIZATIONS by T. Walsh

Number 131
108 pages; list price \$3.90; member price \$2.93; ISBN 0-8218-1831-7
To order, please specify MEMO/131

Abstract. Let K denote a Calderón-Zygmund singular integral kernel in R^n and $\hat{K}$ its principal valued Fourier transform.

Sufficient conditions are given for the summability, by general methods, of "conjugate Fourier integrals" of the form

$$(2\pi)^{-n} \int e^{ixy}\hat{K}(y)\hat{f}(y)\,dy$$

and their derivatives to the function f and its derivatives.

The results simplify and generalize known results of V. L. Shapiro and R. L. Wheeden and are proved by elementary methods.

TRANSLATIONS OF MATHEMATICAL MONOGRAPHS

STATISTICAL SEQUENTIAL ANALYSIS by A. N. Širjaev

Volume 38
174 pages; list price \$18.50; member price \$13.88; ISBN 0-8218-1588-1
To order, please specify MMONO/38

This book is based on lectures given by the author at the Mechanics-Mathematics Faculty of Moscow State University in 1966–1968 and (to a lesser extent) the Second All-Union School on Optimal Control in Shemakha in 1967. In these lectures, the author did not come close to covering all the problems of statistical sequential analysis, but restricted the exposition to the theory of optimal stopping rules and some of their applications. This is reflected particularly in the book's subtitle, Optimal stopping rules. In the remarks at the end of the book, the sources of the results are indicated, and also literature references are given to certain works related to the material presented.

VESTNIK OF THE LENINGRAD UNIVERSITY (MATHEMATICS)

List price \$50/volume; member price \$25/volume

This new journal will constitute the complete translation into English of the mathematics section of the Vestnik Leningradskogo Universiteta beginning with the Soviet publication of 1968. All fields of mathematics are covered in this journal. One volume (four issues) will be published each year, and the first issue should be ready for distribution in 1974.

THEORY OF PROBABILITY AND MATHEMATICAL STATISTICS

List price \$140/volume; member price \$70/volume

The Society is preparing for the publication of a cover-to-cover translation into English of the Teorija Verojatnosteĭ i Matematičeskaja Statistika published by Kiev University. This journal is being published for the Institute of Mathematical Statistics. One volume, beginning with the 1970 Soviet publication, will consist of four issues, and the first issue will be ready for distribution in 1974.

PERSONAL ITEMS

ANATOLE BECK of the University of Wisconsin–Madison has accepted the Chair of Mathematics at the London School of Economics, University of London.

KATHLEEN C. BERNARD of Clearwater, Florida, has been appointed a field service engineer with Honeywell Aerospace, St. Petersburg, Florida.

JOAN S. BIRMAN of Stevens Institute of Technology has been appointed to a professorship at Columbia University.

DAVID BIRNBAUM of the University of Illinois has been appointed to an assistant professorship at Amherst College.

LOKENATH DEBNATH of East Carolina University has been elected a fellow of the Institute of Mathematics and its Applications, England.

ROBERT P. GILBERT of Indiana University is spending the academic year 1972–1973 as the Visiting Unidel Chair Professor at the University of Delaware.

JONATHAN S. GOLAN of McGill University has been appointed to a lectureship at the University of Haifa, Israel.

DOUGLAS HALE of Oklahoma State University has been appointed to an assistant professorship at the University of Texas of the Permian Basin.

ROBERT L. KRUSE of Albuquerque, New Mexico, has been appointed to an associate professorship at Emory University.

WALTER LEIGHTON was named Defoe Distinguished Professor of Mathematics at the University of Missouri, Columbia.

G. T. McLOUGHLIN of Computer Devices of Canada has been appointed chief statistician and chief of the Mathematical Programming Division at the National Energy Board, Ottawa.

THOMAS E. OBERBECK has been appointed chief scientist at Headquarters Air Weather Service, United States Air Force, Scott AFB, Illinois.

RICHARD J. O'FARRELL of the University of Wisconsin, Milwaukee, has been appointed to an assistant professorship and to the chairmanship of the Department of Mathematics at Carroll College, Waukesha, Wisconsin.

DOUGLAS RAVENEL of the Massachusetts Institute of Technology has been appointed to an assistant professorship at Columbia University.

YOUNG H. RHIE of the University of Massachusetts has been appointed to an assistant professorship at Springfield College.

JAWAID H. RIZVI of the University of Western Ontario has been appointed to an associate professorship at the University of Karachi, Pakistan.

GERALD E. SACKS of the Massachusetts Institute of Technology has been appointed to a joint professorship at Harvard University and the Massachusetts Institute of Technology. He is only the third person in recent history to hold a joint professional appointment at the two universities.

PAUL A. SCHWEITZER of the Pontificia Universidade Catolica of Rio de Janeiro will be a visiting research fellow at Harvard University until August 1973.

LEE A. SEGEL of Rensselaer Polytechnic Institute is one of eight members of the faculty to be chosen Outstanding Educators of America for 1973.

JAU-SHYONG SHIUE of the National Taiwan Normal University has been appointed to an associate professorship and to the chairmanship of the Department of Mathematics at the National Chengchi University, Taipei, Taiwan.

IVAN E. STUX of Courant Institute of Mathematical Science has been appointed to an assistant professorship at Columbia University.

E. J. THIELE of the Freie Universität Berlin has been appointed to a professorship at the Technische Universität Berlin.

STEVEN K. THOMASON of Simon Fraser University, Burnaby, Canada, will be visiting the University of Canterbury, Christchurch, New Zealand, from May 1973 through April 1974. His field of special interest is Logic.

PROMOTIONS

To Chairman, Department of Applied Analysis and Computer Science. University of Waterloo: PATRICK C. FISCHER.

To Professor. Amherst College: DUANE W. BAILEY; Bucknell University: LAURENCE E. SIGLER; Hofstra University: AZELLE B. WALTCHER.

To Associate Professor. New Mexico State University: WARREN M. KRUEGER; University of Ottawa: W. D. BURGESS; University of Western Ontario: F. P. A. CASS; Wright State University: WON JOON PARK.

To Assistant Professor. Wright State University: PATRICK D. CASSADY.

DEATHS

Professor Emeritus HOBART C. CARTER of Mary Washington College died on February 24, 1973, at the age of 65. He was a member of the Society for 43 years.

Professor Emeritus PAUL H. DAUS of the University of California, Los Angeles, died on March 27, 1973, at the age of 79. He was a member of the Society for 51 years.

Professor Emeritus HILDA VON MISES GEIRINGER of the University of Berlin and Wheaton College and research fellow emeritus of Harvard University died on March 22, 1973, at the age of 79. She was a member of the Society for 33 years.

Dr. BERTRAM R. LEVY of New York University died on March 27, 1973, at the age of 40. He was a member of the Society for 9 years.

Dr. IVAN G. PETROVSKII of the Moscow State University died on January 15, 1973, at the age of 72. He was a member of the Society for 8 years.

Professor WILLIAM T. SHARP of the University of Toronto died in 1972 at the age of 45. He was a member of the Society for 21 years.

ERRATA

The following items correct or amplify items that appeared in this column in previous issues of the *Notices*.

YECHEZKEL ZALCSTEIN of the University of Copenhagen, Denmark, has been appointed to an assistant professorship in the Computer Science Department of SUNY at Stony Brook.

ALFRED PIETROWSKI of the University of Toronto has been appointed to a visiting assistant professorship at York University.

JOAN WICK PELLETIER of Sir George Williams University has been appointed to a visiting assistant professorship at York University.

MEMORANDA TO MEMBERS

RESOLUTIONS OF THE JANUARY 1974 BUSINESS MEETING

The following three resolutions will be on the agenda of the Business Meeting at the Annual Meeting of January 1974 in San Francisco, according to a procedural resolution from the Business Meeting of January 1973 in Dallas:

1. In cases of alleged discrimination against mathematicians in matters of hiring, promotion, or tenure, for reasons of racial, sex, or political bias, the AMS shall pay legal expenses for the complainant in those cases where the complainant, in the opinion of the Council, has made out a prima facie case of such discrimination.
2. The AMS declares itself in favor of the massive transfer of funds from the federal military budget to the support of education, including higher education, and calls upon the Council and staff to engage the Society in lobbying and publicity efforts directed to this end.
3. The AMS views with alarm the present practice of increasing class sizes and teaching loads, and other measures designed to save money by decreasing faculties at the expense of educational quality. The Council and staff of the Society are called upon to seek avenues for inducing colleges and universities to reverse this unhealthy trend.

Publication at this time is for information only and does not constitute the notification to the membership described in Article X, Section 1, of the bylaws.

Everett Pitcher
Secretary

RECIPROCITY AGREEMENT WITH VIJNANA PARISHAD OF INDIA

The Society has entered into a reciprocity agreement with the Vijnana Parishad of India. Below is the item on this society as it will appear in the next issue of the Report on Reciprocity Agreements.

Apply for membership to
The Secretary
Vijnana Parishad
D. V. Postgraduate College
(Kampur University)
Oral, U.P., India

Pay dues to
Vijnana Parishad

Amount of dues
$5, annual; $50, life

Privileges of membership
Jñānabha (Section A)
(one issue per year; back volumes available at twenty-five percent discount)

RECIPROCITY AGREEMENT WITH SOUTH AFRICAN MATHEMATICAL SOCIETY

The Society has recently concluded a reciprocity agreement with the South African Mathematical Society. This will enable AMS members to join the South African Mathematical Society for one-half the regular dues and enjoy other privileges of membership. The next Report on Reciprocity Agreements will list the Society as follows:

Apply for membership to
Dr. N.J.H. Heideman, Secretary
South African Mathematical Society
Rhodes University
Grahamstown, Republic of South Africa

Pay dues to
Dr. G.C.L. Brummer, Treasurer
South African Mathematical Society
University of Cape Town
Private Bag, Rondebosche,
Republic of South Africa

Amount of annual dues
$3.20

Privileges of membership
Notices of the S. A. Mathematical Society
(four issues per year containing information on mathematical activity in South Africa)
Proceedings of the Annual Congress (abstracts of papers presented)

NEWS ITEMS AND ANNOUNCEMENTS

ALSTON S. HOUSEHOLDER AWARD

In recognition of the outstanding services of Dr. A. S. Householder, former Director of the Mathematics Division of the Oak Ridge National Laboratory, to numerical analysis and linear algebra, it was decided at the Fourth Gatlinburg Symposium, April 1969, to establish a Householder Award. This award is $400 and was first presented in 1971 to Professor F. Robert of the University of Grenoble.

The prize is awarded to the author of the best thesis in numerical algebra. The term "numerical algebra" is intended to describe those parts of mathematical research which have both algebraic aspects and numerical content or implications; thus, the term covers, for example, linear algebra that has numerical applications, or the algebraic aspects of ordinary differential, partial differential, integral and nonlinear equations. The theses will be assessed by an international committee consisting of Peter Henrici (Zurich, Switzerland), James H. Wilkinson (Teddington, England), Hans Schneider, chairman (Madison, Wisconsin), and Richard S. Varga (Kent, Ohio).

To qualify, the thesis must be for a degree at the level of an American Ph. D. awarded in the calendar years 1971, 1972, or 1973. An equivalent piece of work will be acceptable from those countries where no formal thesis is normally written at that level. The candidate's sponsor (e. g. supervisor of his research) should submit an abstract of the thesis (or equivalent) together with his appraisal to A. S. Householder Prize, SIAM, 33 South 17th Street, Philadelphia, Pennsylvania 19103, by April 1, 1974. After a preliminary scrutiny of the abstract and the appraisal, the committee may request candidates to submit copies of the actual thesis. The award will be announced at the "Gatlinburg" meeting in Munich in 1975.

NSF GRADUATE FELLOWSHIPS

The National Science Foundation has announced the award of 457 Graduate Fellowships to graduate students of outstanding ability in the sciences, mathematics, and engineering. Sixty-two of the awards were made in mathematics. Each of the fellowships was awarded to first-year graduate students for three years of graduate study. Fellows may attend any appropriate non-profit U. S. or foreign institution of higher education, and fellowships may be used over a five-year period. In addition to the fellowships awarded, the Foundation accorded honorable mention to 1, 913 applicants.

NATIONAL ACADEMY OF ENGINEERING

The National Academy of Engineering recently announced the election of seventy new members including one member of the Society: Lotfi A. Zadeh, University of California, Berkeley.

HIRAM PALEY, MAYOR OF URBANA, ILLINOIS

Professor Hiram Paley of the University of Illinois at Urbana-Champaign was elected mayor of Urbana on April 3, 1973, and took office on May 1, 1973, for a four-year term. He is now on a partial leave of absence from his position in the Department of Mathematics of the University of Illinois. One of the features of his campaign was a fund-raising hot dog and beer party jocularly referred to as the Paley-Wiener Party.

SAUNDERS MAC LANE, VICE-PRESIDENT OF NATIONAL ACADEMY OF SCIENCES

The National Academy of Sciences has just announced the election of Professor Saunders Mac Lane as vice-president of the National Academy of Sciences, his four-year term to begin on July 1, 1973. Professor Mac Lane has been a member of the Academy since 1949, and was chairman of the Editorial Board of the Proceedings of the Academy from 1960 to 1968 and served on the Council from 1969 to 1972. He is Mason Distinguished Professor of Mathematics at the University of Chicago and assumed the presidency of the American Mathematical Society in January of this year.

POSITION OPEN FOR ASSOCIATE EDITOR OF MR

The Mathematical Reviews Editorial Committee wishes to announce the availability of a position as Associate Editor of Mathematical Reviews in Ann Arbor. The responsibilities of an Associate Editor include classification of articles from a wide range of mathematical areas and the assigning of these to appropriate reviewers. Fluency in reading French, German, and Russian is an essential requirement for the position. Persons interested in obtaining further details should get in touch with the Chairman of the Mathematical Reviews Editorial Committee, Professor Frederick W. Gehring, whose address is Department of Mathematics, University of Michigan, Ann Arbor, Michigan 48104.

WOMEN IN MATHEMATICS REPORT AVAILABLE

The report entitled Women in Mathematics, prepared by Cathleen S. Morawetz and published on pp. 131–132 of the current volume of these *Notices*, is a summary of a longer report of which a few copies are available on request from the Providence office of the Society.

ROCKY MOUNTAIN JOURNAL OF MATHEMATICS–RESEARCH PAPERS

The Rocky Mountain Journal of Mathematics has ended its moratorium on the acceptance of research papers. As in the past, the Journal will be divided approximately equally between expository papers and research papers.

ABSTRACTS PRESENTED TO THE SOCIETY

Preprints are available from the author in cases where the abstract number is starred.

The papers printed below were accepted by the American Mathematical Society for presentation by title. The abstracts are grouped according to subjects chosen by the author from categories listed on the abstract form. The miscellaneous group includes all abstracts for which the authors did not indicate a category.

An individual may present only one abstract by title in any one issue of the *Notices* but joint authors are treated as a separate category. Thus, in addition to abstracts from two individual authors, one joint abstract by them may also be accepted for an issue.

Algebra & Theory of Numbers

73T-A133. ANTHONY JOHN van WERKHOOVEN, Michigan State University, East Lansing, Michigan 48823. On a generalization of subnormality in infinite groups.

Following Richard E. Phillips ("Some generalizations of normal series in infinite groups", J. Austral. Math. Soc., to appear), it is said that the subgroup H of G is an f-subnormal subgroup (written $H \triangleleft f \triangleleft G$) if there exists a series $S: H = H_0 \subseteq H_1 \subseteq H_2 \subseteq \ldots \subseteq H_n = G$ of finite length such that either $H_i \triangleleft H_{i+1}$ or $|H_{i+1} : H_i| < \infty$. The following are typical of the results obtained. Theorem 1. If G is metabelian, finite-by-abelian, nilpotent-by-abelian, or FC-by-abelian, then the join of two finite f-subnormal subgroups of G is a finite f-subnormal subgroup. Theorem 2. There exists an abelian-by-finite group G which has two finite f-subnormal subgroups whose join is not an f-subnormal subgroup of G. Let $\mathfrak{M}_{\check{s}}$ ($\mathfrak{M}_{\hat{s}}$) denote the class of groups satisfying the minimal (maximal) condition for subnormal subgroups. Theorem 3. The join of finitely many f-subnormal solvable-by-finite-, $\mathfrak{M}_{\check{s}}$-, $\mathfrak{M}_{\hat{s}}$-subgroups is respectively a solvable-by-finite-, $\mathfrak{M}_{\check{s}}$-, $\mathfrak{M}_{\hat{s}}$-subgroup. Theorem 4. If $\mathfrak{X}$ is a class closed under the taking of subgroups and homomorphic images, then the following are equivalent: (1) The join of two finite f-subnormal subgroups of the $\mathfrak{X}$-group G is a finite f-subnormal subgroup of G. (2) The join of two f-subnormal $\mathfrak{M}_{\check{s}}$-subgroups of the $\mathfrak{X}$-group G is an f-subnormal $\mathfrak{M}_{\check{s}}$-subgroup of G. (Received January 15, 1973.)

73T-A134. RONSON J. WARNE, University of Alabama, Birmingham, Alabama 35233. Generalized ω-$\mathcal{L}$-unipotent bisimple semigroups.

Let S be a bisimple semigroup and let E(S) be the set of idempotents of S. If E(S) is an ω-chain of rectangular bands ($E_n : n \in N$, the nonnegative integers) and $\mathcal{L}$, Green's equivalence relation, is a left congruence on E(S), we term S a generalized ω-$\mathcal{L}$-unipotent bisimple semigroup. Let $(I, \circ)$ be an ω-chain of left zero semigroups $(I : k \in N)$; let $(n,r) \to \alpha_{(n,r)}$ be a homomorphism of C, the bicyclic semigroup, into $\mathrm{End}(I, \circ)$ (iteration); let $(j, *)$ be an ω-chain of right groups $(J_k : k \in N)$; let $(n,r) \to \beta_{(n,r)}$ be a homomorphism of C into $\mathrm{End}(J, *)$; let $j \to A_j$ be an upper antihomomorphism of $(J, *)$ into T_I, the full transformation semigroup on I; and let $I_k \cap J_k = (e_k)$, a single element for each $k \in N$ such that $g\beta_{(s,s)} = g * e_s$ for all $g \in J$; $I_r\alpha_{(n,k)} \subseteq I_{k+r-\min(n,r)}$ and $J_r\beta_{(n,k)} \subseteq J_{k+r-\min(n,r)}$; $I_rA_j \subseteq I_{\max(r,k)}$ if $j \in J_k$; $(r \circ s)A_x = rA_x \circ sA_{x*e_u}$ for $r, s \in I$ with $r \in I_u$; $\alpha_{(r,s)}A_j\alpha_{(s,r)}A_{j\beta_{(s,r)}}$ for all $j \in J$ and $r, s \in N$. We denote $((i,(n,k),j) : i \in I_n,\ j \in J_k, n, k \in N)$ under the multiplication $(i,(n,k),j)\,(u,(r,s),v) = (i \circ (uA_j\alpha_{(k,n)}),$ $(n+r-\min(k,r),\ k+s-\min(k,r)),\ j\beta_{(r,s)} * v)$ by (I, J, α, β, A). Theorem. S is a generalized ω-$\mathcal{L}$-unipotent bisimple semigroup if and only if $S \cong (I, J, \alpha, \beta, A)$ for some collection I, J, α, β, A. (Received February 1, 1973.)

73T-A135. TAO-CHENG YIT, Carleton University, Ottawa, Ontario K1S 5B6, Canada. On subdirect product of rings without symmetric divisors of zero. III. Preliminary report.

Unless otherwise stated, R will denote a semiprime and *-compressible ring. Let X be the set of all minimal *-prime ideals of R. Question. Do we have, as for rings without nilpotent elements, that X is a Hausdorff space under the topology admitting as open sets all the sets $\Gamma(A) = \{P \in X; A \not\subseteq P\}$ where A is any *-ideal of R? Proposition 1. In any semiprime ring R with involution the following conditions are equivalent: (1) R is *-compressible; (2) each minimal *-prime ideal of R is *-complete prime (i.e. if $sx \in P$ with $s = t + t^*$ or tt^*, then $s \in P$ or $x \in P$). Proposition 2. Let P be a minimal *-prime ideal of the ring R. For every norm or trace u in P, there is a norm v outside P such that $uv = 0$. Proposition 3. Let P be as in Proposition 2. Let $I = I^*$ be generated by a finite number of norms or traces. Then either $P \supseteq I$ or $P \supseteq \mathrm{ann}(I)$ $[= \underline{r}(I) = \underline{\ell}(I)]$, but not both. Theorem A. X is Hausdorff. Theorem B. If R is right noetherian, for any *-ideal I, ann(I) can be written in a unique way as an intersection of a finite number of minimal *-prime ideals. Conversely every minimal *-prime ideal is the annihilator of a symmetric ideal. Thus the symmetric annihilator ideals form a finite boolean lattice coinciding with the sublattice generated by the minimal *-prime ideals. (Received February 19, 1973.) (Author introduced by Professor Maurice Chacron.)

*73T-A136. RICHARD A. SANERIB, JR., University of Colorado, Boulder, Colorado 80302. Ultrafilters with no bases. Preliminary report.

A filter F in a Boolean algebra (BA) $\mathfrak{A}$ is said to have a basis (weakly independent set of generators) $\{a_i\}_{i\in I}$ if $F = \langle\{a_i\}_{i\in I}\rangle$ (i.e., $b \in F$ iff there exist $i_1,\ldots,i_n$ such that $a_{i_1} \cdot \ldots \cdot a_{i_n} \leq b$) and for $i_0,\ldots,i_{n+1} \in I$ distinct, $a_{i_0} \cdot \ldots \cdot a_{i_n} \not\leq a_{i_{n+1}}$. If G and F are filters in a BA $\mathfrak{A}$ with $G \supset F$, then $\{a_i\}_{i\in I} \subset G - F$ is a basis for G over F if $G = \langle F \cup \{a_i\}_{i\in I}\rangle$ and for $i_0,\ldots,i_{n+1}$ distinct, $-a_{i_0} + \ldots + -a_{i_n} + a_{i_{n+1}} \notin F$. Theorem. Let $\mathfrak{A}$ be a σ-complete BA, G an ultrafilter in $\mathfrak{A}$ and $F \subset G$ a filter in $\mathfrak{A}$. Then G has a basis over F iff there exists $b \in \mathfrak{A}$ such that $G = \langle F \cup \{b\}\rangle$. Corollary. Let $\mathfrak{A}$ be a σ-complete BA and $\mathfrak{B}$ a homomorphic image of $\mathfrak{A}$. Then no nonprincipal ultrafilter in $\mathfrak{B}$ has a basis. (Received February 21, 1973.)

*73T-A137. SHAFAAT AHMAD, Université Laval, Québec, Québec, Canada. On implicational completeness.

Let $\underline{A}$ be a (universal) algebra. $\underline{A}$ will be called singleton if $|A| = 1$, and atomic if $\underline{A}$ is embeddable in every nonsingleton subalgebra of $\underline{A}^2$. Theorem 1. Two element algebras and pseudo-primal algebras are atomic. A finite nonsingleton algebra $\underline{A}$ is defined to be pseudo-primal if for every set $B \subseteq A$ there exists a unary polynomial u(x) such that $B = \{u(a); a \in A\}$. If $\underline{A}$ generates a quasi-variety which has no proper sub-quasi-varieties then $\underline{A}$ is defined to be implicationally complete. Theorem 2. A finite minimal algebra is atomic if and only if it is implicationally complete. [$\underline{A}$ is minimal if $\underline{A}$ is the only nonsingleton subalgebra of $\underline{A}$.] Theorem 3. A quasi-variety of semigroups is implicationally complete if and only if it is generated by a two element semigroup or a group of prime order or the additive semigroup of positive integers. Theorem 4. There are uncountably many implicationally complete quasi-varieties of groupoids that are not varieties. (Received February 26, 1973.) (Author introduced by Professor Mohammad Ishaq.)

*73T-A138. FREDERIC W. SHULTZ, Wellesley College, Wellesley, Massachusetts 02181. A characterization of state spaces of orthomodular lattices.

It is shown that every rational polytope is affinely equivalent to the set of all states of a finite orthomodular lattice, and that every compact convex subset of a locally convex topological vector space is affinely homeomorphic to the set of all states of an orthomodular lattice. (Received February 28, 1973.)

*73T-A139. GORO AZUMAYA, Indiana University, Bloomington, Indiana 47401. A theorem on T-nilpotent ideals. Preliminary report.

Let I be a left ideal of a ring R. I is called left T-nilpotent if for any sequence $a_1, a_2, a_3, \ldots$ of elements in I there exists an n such that $a_1 a_2 \ldots a_n = 0$. Theorem. The following conditions are equivalent: (a) I is left T-nilpotent; (b) for any nonzero left R-module M, $IM \neq M$; (c) for any nonzero right R-module M, $\ell_M(I) \neq 0$, where $\ell_M(I)$ denotes the left annihilator of I in M. The theorem can be proved quite elementarily, but this makes it possible to provide simplified proofs to the implications (1) ⇒ (2) and (7) ⇔ (1) of Theorem P in (H. Bass, Trans. Amer. Math. Soc. 95(1960), 467) without using any transfinite method. (Received February 28, 1973.)

*73T-A140. ALBERT A. MULLIN, U.S. Army Research Office, Arlington, Virginia 22204. On extending a result by Rademacher and Toeplitz. Preliminary report.

This note extends the principal result of the last chapter in Rademacher's and Toeplitz' book "Von Zahlen und Figuren," Berlin, 1933. Recall they showed that 30 is the largest natural number satisfying the Condition A: all (sic) natural numbers less than it and relatively prime to it are prime; the sic reminds us, as the authors surely intended, that the natural number 1 is excluded from consideration. Definition. Consider only natural numbers $n > 1$. Then n enjoys the Property B provided all natural numbers less than n, whose mosaics have no prime number in common with the mosaic of n, are prime. Lemma. Every natural number satisfying Condition A enjoys Property B. However, 40, 42, 45, 60, and 75 are the only natural numbers enjoying Property B which do not satisfy Condition A. Corollary. There are precisely 13 natural numbers enjoying Property B and 75 is the largest such number. Scholium. The above ideas treat prime factors and the exponents of prime factors by the same uniform standard making it unnecessary to use a double standard for treating prime factors differently from the number of times those prime factors occur. (Received February 28, 1973.)

*73T-A141. VLASTIMIL DLAB, Carleton University, Ottawa, Ontario, Canada and CLAUS MICHA RINGEL, University of Tübingen, Tübingen, Federal Republic of Germany. Decomposition of filtered vector spaces.

Let F be a field containing a central field K such that F_K is finite dimensional. A finite ordered set S together with an order preserving mapping φ of S into the lattice of all subfields of F containing K is called a K-structure for F. For a subfield $K \subseteq G \subseteq F$ and a natural n, $I_n(G)$ denotes the K-structure defined by the chain $\{1 < 2 < \ldots < n\}$ such that $\varphi(1) = \varphi(2) = \ldots = \varphi(n) = G$, and N(G) denotes the K-structure given by the ordered set $\{i < j > k < \ell\}$ with $\varphi(i) = \varphi(j) = \varphi(k) = \varphi(\ell) = G$. Given a K-structure S for F, define $w(S) = \max_J \sum_{j \in J} \dim F_{F_j}$, where J are subsets of mutually unrelated elements of S. An S-space (W, W_i) is a right vector space W over F together with an F_i-subspace W_i for each $i \in S$, such that $i \leqq j$ implies $W_i \subseteq W_j$. For a given K-structure S, the S-spaces form an additive category in which the morphism $(W, W_i) \to (W', W_i')$ are F-linear mappings $\varphi : W \to W'$ satisfying $\varphi W_i \subseteq W_i'$, $i \in S$. A K-structure S is said to be of finite type if there is only a finite number of finite dimensional indecomposable S-spaces. In the case when $\varphi(i) = F$ for all $i \in S$, L. A. Nazarova and A. V. Roiter, and M. M. Kleiner have characterized the structures of finite type. Their results are extended in the following Theorem. Let S be a K-structure for F. Then S is of finite type if and only if $w(S) \leqq 3$ and S does not contain, as a full ordered subset, any of the following 7 structures: $I_2(F) \sqcup I_2(F) \sqcup I_2(F)$; $I_1(F) \sqcup I_3(F) \sqcup I_3(F)$; $I_1(F) \sqcup I_2(F) \sqcup I_5(F)$; $I_4(F) \sqcup N(F)$; $I_2(G) \sqcup I_2(F)$ with $[F:G] = 2$; $I_3(G) \sqcup I_1(F)$ with $[F:G] = 2$; and $I_2(G)$ with $[F:G] = 3$. (Received February 27, 1973.)

*73T-A142. PAVOL HELL, University of British Columbia, Vancouver, British Columbia, Canada. Absolute planar retracts and the four color conjecture.

We consider only planar graphs (finite, undirected, without loops and multiple edges). A subgraph G of the graph H is a retract of H if there is a homomorphism (adjacency preserving mapping) $r: H \rightarrow G$ such that $r(g) = g$ for all $g \in V(G)$. An absolute planar retract is a graph which is a retract of each graph of which is a subgraph. The four color conjecture is shown to be equivalent with the existence of absolute planar retracts, and also with some other conditions on the class of all planar graphs and all homomorphisms. As an application, a possible argument for the decidability of the 4CC is discussed and proved to be equivalent to the 4CC itself. Under the assumption that the 4CC is true, all absolute planar retracts are determined. (Received March 2, 1973.)

*73T-A143. JORGE MARTINEZ, University of Florida, Gainesville, Florida 32601. Purity and hyper-archimedean modules. Preliminary report.

Call an algebraic lattice hyper-archimedean if it is modular and every element is the meet of maximal (proper) elements. A module M over an associative ring with identity is hyper-archimedean if its lattice of submodules is hyper-archimedean. We show that a ring R has the property that every R-module is hyper-archimedean if and only if R is a V-ring (i.e. every left ideal is the intersection of maximal left ideals). If R is commutative then we are dealing with the von Neumann regular rings. If R is commutative, we prove that an R-module M is hyper-archimedean if and only if every submodule of M is pure. If R is also Noetherian each such module is a direct sum of simple submodules. Since the pure submodules are precisely those which are direct limits of summands, we are led to define an element of an algebraic lattice L to be pure if it is the join of a directed family of complemented elements of L. If L is distributive then each element of L is pure if and only if every prime element of L is maximal; in particular L is hyper-archimedean. (Received March 5, 1973.)

*73T-A144. URI LERON, University of Oregon, Eugene, Oregon 97403. Nil polynomials, power-central polynomials and a generalized Herstein's conjecture.

A polynomial $f(x_1, \ldots, x_k)$ in noncommuting variables is vanishing, nil or central in a ring R, if its value under every substitution from R is 0, nilpotent or central element of R, respectively. Theorem. If R has no nonvanishing multilinear nil polynomials (e.g. if R has no nonzero nilpotent elements) then the matrix ring R_n has neither. Theorem. Let R be a ring satisfying a polynomial identity modulo its nil radical, and let f be a multilinear polynomial. If f is nil in R then the ideal generated by all the values of f under substitutions from R is nil. Applied to the polynomial $xy - yx$, this establishes the validity of a conjecture of Herstein, in the presence of polynomial identity. Theorem. Let F be a field containing no mth roots of unity other than 1, and let f be a multilinear polynomial such that for some $n > 2$, f^m is central in F_n. Then f is central in F_n. The theorem applies, in particular, when $n = m = p$ is an odd prime and $F = Q$ or $F = Z_q(t)$, $q \leqq p$. This is closely related to the (non-) existence of noncrossed products among p^2-dimensional central division rings (Schacher and Small, "Central polynomials which are pth power", preprint). (Received March 26, 1973.)

*73T-A145. PAUL M. HARMS, Taylor University, Upland, Indiana 46989. Real function rings.

A definition of a real function ring and some consequences of this definition are given. A real function ring (X, A) is a ring of real valued functions defined on a nonempty set X such that: (i) A separates the points of X, and (ii) for each x in X the set $\{f(x) : f \in A\}$ is the set of real numbers. The structure of fixed maximal ideals in A is given. The following result is proved. Theorem. Let (X, A) be a real function ring. The space X can be imbedded in a space Y so that: (i) each function f in A has an extension f^{ν} defined on Y so that the mapping taking f to f^{ν} is an isomorphism of A onto the set of extensions, and (ii) a maximal ideal in the set of extensions is a fixed ideal at a point of Y if and only if it is a real ideal. (Received March 12, 1973.)

73T-A146. WITHDRAWN.

73T-A147. ANNE B. KOEHLER, Western College, Oxford, Ohio 45056. Rings with quasi-injective cyclic modules.

A ring is called a left (right) qc-ring if every cyclic left (right) R-module is quasi-injective (J. Ahsan, "Rings all of whose cyclic modules are quasi-injective", Proc. London Math. Soc., to appear). An internal characterization of qc-rings is given which extends a result of Klatt and Levy ("Pre-self injective rings", Trans. Amer. Math. Soc. 137(1969), 407-419) on commutative qc-rings. As a result of this characterization, a left qc-ring is a right qc-ring. It is also shown that a left qc-ring is a left q-ring. (Received March 12, 1973.)

73T-A148. CARLA WOLFE, University of Guelph, Guelph, Ontario, Canada. Endomorphism rings of quasi-projective modules over semiperfect and perfect rings. Preliminary report.

Let E be the endomorphism ring of a quasi-projective R-module M. Case A. R is semiperfect, M is finitely generated. Case B. R is perfect. In Case A: (1) E is semiperfect; (2) E is local iff M is indecomposable; (3) E is semiprimitive iff E is regular iff E is left (eq. right) hereditary iff E is left (eq. right) semihereditary iff E is semisimple; (4) E is simple iff E is left (eq. right) primitive iff E is semisimple and all indecomposable direct summands of M are isomorphic; (5) E is a division ring iff E is semisimple and M is indecomposable. In Case B: (1) E is F-semiperfect; (2) E is semilocal iff E is semiperfect iff E is left (eq. right) perfect iff E is semiprimary iff M is finitely generated; (3) E is local iff E is primary iff E is fully primary iff M is indecomposable; (4) E is semiprime iff E is semiprimitive iff E is regular iff E is left (eq. right) hereditary iff E is left (eq. right) semihereditary; (5) E is prime iff E is left (eq. right) primitive; (6) E is semisimple iff E is regular and M is finitely generated; (7) E is simple iff E is semisimple and all indecomposable direct summands of M are isomorphic; (8) E is a division ring iff E is semisimple and M is indecomposable. Further results hold if either R is commutative or the projective cover of M is a generator. (Received February 20, 1973.) (Author introduced by Dr. D. Fieldhouse and Dr. H. Pesotan.)

73T-A149. DAVID COHOON, University of Minnesota, Minneapolis, Minnesota 55455. A class of algebras which are not power associative. Preliminary report.

Theorem 1. Let G be a semigroup. Let F be a field. Let V denote a vector space of mappings from G into F. Let $\mathfrak{M}(V)$ denote the set of all mappings f from G into itself such that, if $g \to c(g)$ is in V, then the mapping $L_f(c)$ from G into F defined by $L_f(c)(g) = \sum_{f(g')=g} c(g')$, for $g \in f(G)$, and $L_f(c)(g) = 0$, for $g \notin f(G)$, is in V.

Then $\mathfrak{M}(V)$ is a monoid and $L_f: V \to V$ is a linear transformation for every f in $\mathfrak{M}(V)$. We define the semigroup ring multiplication in V by the rule $(c_1c_2)(g) = \sum c_1(g_1)c_2(g_2)$ $(g_1g_2=g,\ g_1 \in G,\ g_2 \in G)$. Then if f is in $\mathfrak{M}(V)$, it follows that if we define a multiplication on V by the rule $c_1 * c_2 = c_1L_f(c_2)$ (semigroup ring multiplication) then $(V,+,*)$ is an algebra and if there is a g in G such that $gf(g)f(g) \neq gf(gf(g))$ or such that $gf(g)f(gf(g)) \neq gf(gf(gf(g)))$ then $(V,+,*)$ is not power associative. (Received March 15, 1973.)

73T-A150. ALEXANDER ABIAN, Iowa State University, Ames, Iowa 50010. Conditional completion of rings without nilpotent elements.

Let R be an associative but not necessarily commutative ring without nonzero nilpotent elements. Then $(R,\leqq)$ is a partially ordered set where $x \leqq y$ if and only if $xy = x^2$ for every element x and y of R. For every element r of R define $I(r) = \{x | x \in R$ and $x \leqq r\}$ and consider the set C of all $\bigcap_{r\in S} I(r)$ where S is a nonempty subset of R. Then $(C,\subseteq)$ is a conditionally complete (i.e., every nonempty bounded above subset of C has a lub in $(C,\subseteq)$) partially ordered set. Addition $+$ and multiplication $\cdot$ are judiciously defined in C such that the mapping f from $(R,+,\cdot,\leqq)$ into $(C,+,\cdot,\subseteq)$ defined by $f(r) = I(r)$ is a ring as well as order isomorphism. For obvious reasons $(C,+,\cdot,\subseteq)$ is called the minimal conditional completion of $(R,+,\cdot,\leqq)$. Thus, every associative ring without nonzero nilpotent elements has a conditional completion with respect to the order $\leqq$ mentioned above. (Received March 15, 1973.)

73T-A151. CHRISTIAN HERRMANN, CLAUS MICHA RINGEL and RUDOLF WILLE, Technische Hochschule, Darmstadt, Federal Republic of Germany. On modular lattices with four generators. Preliminary report.

Theorem. A subdirectly irreducible sublattice of a finite dimensional complemented modular lattice is generated by four elements iff it belongs to the following list: M_4, $S(n,4)$, subspace lattices of linear spaces of dimension $n \geqq 3$ over prime fields, nondesargian planes with four generators. (Here the lattices $S(n,4)$ are those introduced in Day, Herrmann, Wille (Algebra Universalis (1973)).) Further on all subgroup lattices $S(\mathbb{Z}^n_{p^k})$ of abelian groups $\mathbb{Z}^n_{p^k}$ with $n \geqq 3$ and p prime are generated by four elements. The subgroup lattice of the nth power of the Prüfer group $\mathbb{Z}_{p^\infty}$ contains a subdirectly irreducible sublattice generated by four elements, into which all $S(\mathbb{Z}^n_{p^k})$ can be imbedded. The proof is based on the complete classification of indecomposable objects in the category of finite dimensional linear spaces with four subspaces by Gelfand and Ponomarev (Colloquia Math. Soc. J. Bolyai 5(1970)) and recent work of one of the authors with A. Huhn. (Received March 16, 1973.)

*73T-A152. JUDITH Q. LONGYEAR, Dartmouth College, Hanover, New Hampshire 03755. Small configurations and their arrays.

It is known that there is a bipartite $(n+1)$-regular graph $G(n)$ of girth 6 on $2(n^2+n+1)$ vertices iff there are $(n-1)$ mutually orthogonal latin squares of order n. When n is not a prime power, it is very hard to determine whether any $G(n)$ exist. None are known to exist, and many special theorems have shown that large classes of $G(n)$ do not exist. If $T(n,k)$ is used as a generic term for a bipartite $(n+1)$-regular graph of girth 6 on $2(n^2+n+1+k)$ vertices, then it is known that there are $T(n,k)$ for all n and all large k. Since $T(n,1)$ is so very nearly a $G(n)$, we mimic the construction of an affine plane from a projective plane, and then examine the set of square arrays which are associated with $T(n,1)$. Conditions on a set of arrays are determined which will produce a $T(n,1)$ for each set of $(n-1)$ arrays satisfying the conditions. The arrays are exhibited for $T(2,1)$ and $T(3,1)$. There is a list of several related questions. (Received March 16, 1973.)

73T-A153. SABINE KOPPELBERG, Universität Bonn, Bonn, Federal Republic of Germany. Free subalgebras of complete Boolean algebras.

The following theorems improve results of Ketonen, Solovay, and Vladimirov. Theorem 1. Every infinite complete Boolean algebra is completely generated by some free subalgebra. For every complete Boolean algebra B, let $\tau(B)$ be the least cardinal α s.t. B is completely generated by some subset of cardinality α and $t(B) = \sup\{\alpha | B$ has an antichain of power $\alpha\}$. B is said to be τ-homogeneous if, for every $a \in B$ s.t. $a > 0$, $\tau(\{x \in B | x \leq a\}) = \tau(B)$. Theorem 2. Let B be an infinite complete Boolean algebra s.t., for every τ-homogeneous factor C of B, t(C) is not weakly inaccessible. Then B has a free subalgebra whose cardinality is $\overline{\overline{B}}$. Corollary. Under the assumptions of Theorem 2, B has $2^{\overline{\overline{B}}}$ ultrafilters. Theorem 3 (GCH). Let B be an infinite complete Boolean algebra s.t. $\overline{\overline{B}}$ is not strongly inaccessible. Then B has a free subalgebra of cardinality $\overline{\overline{B}}$. B. J. Koppelberg proved that the conclusion of Theorem 3 also holds if $\overline{\overline{B}}$ is Π_1^1-indescribable. (Received March 19, 1973.) (Author introduced by Professor G. Hasenjaeger.)

73T-A154. WILLIAM F. KEIGHER, University of Illinois, Urbana, Illinois 61801. Differential categories are cotripleable. Preliminary report.

Let Rings (resp., $\mathcal{D}$) denote the category of commutative associative unitary rings (resp., ordinary differential rings). For any differential ring $(A, \underline{a})$, let $(A, \underline{a})$-Mod (resp., $(A, \underline{a})$-Alg) denote the category of differential modules (resp., commutative differential algebras) over $(A, \underline{a})$. Theorem. There exists an adjoint cotriple on Rings (resp., A-mod, resp., A-Alg) such that $\mathcal{D}$ (resp., $(A, \underline{a})$-Mod, resp., $(A, \underline{a})$-Alg) is cotripleable (and hence tripleable as well) over Rings (resp., A-Mod, resp., A-Alg). Theorem. $(A, \underline{a})$-Mod is a symmetric monoidal closed category. Theorem. There exists a natural ring homomorphism $\varphi_A : A[[t]] \to \omega A$ where $\Omega = (\omega, \delta, \epsilon)$ is the cotriple on Rings such that $\mathcal{D} \cong \text{Rings}_\Omega$, and if A contains the rationals Q, φ_A is an isomorphism. Theorem. For any $(A, \underline{a}) \in \mathcal{D}$ with $Q \subset A$, there is a natural open retraction $\rho_A : \text{Spec } A \to \text{Spec}_d A$ ($\text{Spec}_d A$ is the subspace of differential primes in A). Corollary. For any $(A, \underline{a}) \in \mathcal{D}$ with $Q \subset A$, $\text{Spec}_d A$ is quasi-compact and dense in Spec A. Additional results and applications will be announced in these *Notices* 20(1973), to appear. (Received March 19, 1973.)

*73T-A155. J. C. McCONNELL, University of Leeds, Leeds LS2 9JT, England. Representations of solvable Lie algebras and the Gelfand-Kirillov conjecture. Preliminary report.

The following theorem gives an affirmative answer to the conjecture of Gelfand and Kirillov, Publ. Math. Debrecen 31(1966), 509-523, for the case of solvable algebraic Lie algebras. Let k be a field of characteristic zero, B a commutative k-algebra, A a Lie algebra of derivations of B, $B \#_\sigma U(A)$ the corresponding ring of differential operators twisted by a 2-cocycle $\sigma \in Z^2(A, B)$. Theorem. Let g be a trigonalisable Lie algebra, P a prime ideal of U(g) and E the semicentre of U/P. Then (i) there exists $e \in E$ such that $(U/P)_e \cong B \#_\sigma U(A)$ where $B = R \otimes kS \otimes kG$, R a finitely generated commutative algebra, kS the symmetric algebra of a vector space S and kG the group algebra of a free abelian group G, A is an abelian Lie algebra of derivations of B and $\sigma \in Z^2(A,B)$. (ii) R is the centre of $(U/P)_e$ and there is a lattice isomorphism between the ideals of R and the ideals of $(U/P)_e$. (iii) If g is also algebraic then $(U/P)_e \cong R \otimes A_n \otimes A'_m$ for some integers $m, n \geq 0$ where A_n is a Weyl algebra and A'_m is the localisation of A_m with respect to the powers of the element $x_1 x_2 \ldots x_m$. (Received March 19, 1973.) (Author introduced by Professor Earl J. Taft.)

*73T-A156. CHARLES I. VINSONHALER and WILLIAM J. WICKLESS, University of Connecticut, Storrs, Connecticut 06268. Quasi-pure-injective torsion free abelian groups of rank two. Preliminary report.

In "Infinite abelian groups," Vol. I, Fuchs poses the problem of investigating the class of quasi-pure-injective abelian groups (i.e. groups G such that every homomorphism $P \to G$ with P a pure subgroup of G is induced by an endomorphism of G). Using the quasi-endomorphism algebra $Q \otimes_Z \mathcal{E}(G)$ the authors classify all quasi-pure-injective torsion free abelian groups of rank two. (Received March 22, 1973.)

*73T-A157. TORRENCE D. PARSONS, Pennsylvania State University, University Park, Pennsylvania 16802. Friendship sets and Ramsey graphs. Preliminary report.

Let $f(n)$ be the least positive integer N such that, for any graph G with N vertices, either G contains a simple cycle of length 4 or the complementary graph $\overline{G}$ has a vertex of valence n or more. Then $f(n) < n + \sqrt{n} + 2$. If $q \geqq 2$ is an integer such that $q \neq 5$ and q is not of the form $k^2 - 1$, k odd, then $f(q^2+1) \leqq q^2 + q + 2$. This bound arises from the study of eigenvalues of adjacency matrices. If $q = p^s$, p is a prime, then $q^2 + q + 1 < f(q^2+1)$ follows from a known construction based on the projective plane over $GF(q)$. This construction gives infinitely many examples of "nonhomogeneous friendship sets" recently defined by Skala. If $q = p^s$, p prime and $q \neq 5, 8$, then we have $f(q^2+1) = q^2 + q + 2$. (Received March 22, 1973.)

*73T-A158. SHARI L. LAWRENCE, Pennsylvania State University, University Park, Pennsylvania 16802. Cycle-star Ramsey numbers. Preliminary report.

Let $f(m,n)$ be the least positive integer N such that, for any graph G with N vertices, either G contains a cycle at length m or the complementary graph $\overline{G}$ contains a star of degree n. Then for m odd, $m \leqq 2n+1$, we have $f(m,n) = 2n+1$. For $m \geqq 2n+1$, $f(m,n) = m$. For m even, $m \leqq 2n$, it appears that $f(m,n)$ may have complicated behavior. (Received March 22, 1973.)

*73T-A159. STEVE BRAVY, University of Maryland, College Park, Maryland 20742. A characterization of semisimple rings as rings with acc and "enough" projectives.

This paper shows that for a Noetherian ring R the left global projective dimension is the sup of the projective dimensions of the R-modules containing no nonzero projective subobject. A corollary then shows that R is semisimple iff R has acc and every nonzero R-module contains a projective nonzero submodule. (Received March 29, 1973.) (Author introduced by Professor James C. Owings, Jr.)

73T-A160. S. K. JAIN, Ohio University, Athens, Ohio 45701. On a class of regular rings that are unit-regular.

The results of this paper are inspired by Henriksen's paper 'On a class of regular rings that are elementary divisor rings', Arch. Math. (Basel), to appear, and by Levitzki's results on the existence of matrix units in certain rings. All rings have identity and R^* will denote the group of units of a ring R. R is called regular (unit-regular) iff $\forall\ a \in R\ \exists\ x \in R(R^*)$ such that $axa = a$. Henriksen showed: 'If A is an $n \times n$ matrix over a unit-regular ring R then there exist nonsingular matrices P, Q such that PAQ is a diagonal matrix'. We assume that R is a regular ring with a polynomial constraint, such as polynomial identity, pivotal monomial, quasi-standard identity (for definitions see, for example, Belluce-Jain, J. Indian Math. Soc., 25(1968), 79-87). Some of the results proved are: (1) There exists a central idempotent $e \in R$ such that the ideal eR (as a ring) is unit-regular and, if A is any $n \times n$ matrix over R, then there exist matrices P, Q, P', Q' such that PAQ is a diagonal matrix with all entries as idempotents and $PP' = P'P = E = QQ' = Q'Q$, $E = \mathrm{diag}(e, e, \ldots, e)$. (2) If the center $Z(R^*)$ of R^* is of finite index then R itself is unit-regular. In particular, if R^* is a finite group of order n then R is a finite direct sum of matrix rings over the rings each of whose elements a satisfies $a^{n+1} = a$. (Received March 29, 1973.)

*73T-A161. MICHAEL N. BLEICHER, HANS SCHNEIDER and ROBERT L. WILSON, University of Wisconsin, Madison, Wisconsin 53706. Permanence of identities on algebras.

Any equational class closed under taking complex algebras can be defined by a set of identities using only words with no repeated symbols. In addition, we generalize this to partially ordered algebras. (Received March 30, 1973.)

*73T-A162. DAVID KELLY and IVAN RIVAL, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada. Fences, crowns and dismantlable lattices.

An element x of a lattice L is doubly irreducible in L if $x \neq y \vee z$ and $x \neq y \wedge z$ whenever y and z are elements of L distinct from x. A lattice L is dismantlable (see Abstract 72T-A220, these *Notices* 19(1972), A-677) if every sublattice S of L contains an element that is doubly irreducible in S. Sublattices and homomorphic images of dismantlable lattices are dismantlable. A (lower) fence is a partially ordered set $\{x_i | 0 \leqq i < n\}$ $(n \leqq \omega)$, for which the comparabilities that hold are precisely: (*) $x_i < x_{i+1}$ (i even), $x_i > x_{i+1}$ (i odd). For even $n \geqq 6$, a crown is a partially ordered set $\{x_i | 0 \leqq i < n\}$ for which $x_0 < x_{n-1}$ and (*) are precisely the comparabilities that hold. Theorem. Let L be a lattice which contains no infinite chains and no infinite fences. (a) L is dismantlable if and only if L contains no crowns. (b) If L is dismantlable and is not a chain, then L contains (at least) two incomparable doubly irreducible elements. Theorem. Let L be a modular lattice of finite length. L is dismantlable if and only if L contains no crown of order 6 (i.e., the breadth of L does not exceed 2). Corollary. A finite distributive lattice is dismantlable if and only if it is planar. (Received April 2, 1973.)

73T-A163. EDMUND H. FELLER, University of Wisconsin, Milwaukee, Wisconsin 53201 and MADHUKAR G. DESHPANDE, Marquette University, Milwaukee, Wisconsin 53233. The ring of endomorphisms of an essential extension of a Noetherian module.

Let I denote the injective hull of a Noetherian module N. If J is the Jacobson radical of $\mathrm{Hom}_R(I, I)$, then $J^n N = 0$, for some positive integer n. The endomorphism ring of the quasi-injective hull of N is semiprimary. (Received April 2, 1973.)

*73T-A164. JENNIFER R. GALOVICH, STEVE GALOVICH and SEYMOUR SCHUSTER, Carleton College, Northfield, Minnesota 55057. An elementary proof of the friendship theorem.

Theorem. If in a party of p people every two people have one common friend, then there is some person in the party who is a friend of everyone else. Several proofs of this theorem have been given since the original one by Erdős, Rényi and Sós ("On a problem of graph theory", Studia Math. Hungar. 1(1966), 215-235). The current paper presents a simple elementary proof resting only on the notions of a graph (vertices, edges and adjacency), degree of a vertex, and path of length n. (Received April 2, 1973.)

73T-A165. M. BHASKARAN, 140 Charles Street, West Perth, Western Australia 6005, Australia. A law of decomposition for nonabelian extensions. Preliminary report.

Let k denote a finite Galois extension of the rationals. For any rational prime ℓ, let e_ℓ, f_ℓ and g_ℓ denote the ramification index, residue class degree and the number of distinct conjugates respectively of a prime divisor in k lying above ℓ. Let p and q be rational primes and let $\mathrm{ord}_p q$ denote the smallest positive integer x such that $q^x \equiv 1 \bmod p$. Let $(a,b,c,\dots)$ denote the G.C.F. of rational integers $a,b,c,\dots$. Theorem. Let k be of degree n and e be a positive integer such that $(e, n/e) = 1$ and e divides $(g_q, e_p, p-1)/(n/g_q, g_q, e_p, p-1)$. Then, if q splits into principal divisors in k, e divides $c(p-1)/\mathrm{ord}_p q$ where $c = 1$ when e is odd or $p \equiv 1 \bmod 2e$ and $c = 2$ otherwise. (Received April 3, 1973.) (Author introduced by Mr. Desmond Fearnley-Sander.)

*73T-A166. O. A. VARGHESE, Mar Athanasius College, Kothamangalam, Kerala 686666, India. An easy method for numerical division. Preliminary report.

In this paper, we give certain conditions under which a number y is a factor of another number P, which help very much in arithmetic computations. Also we give a formula for the quotient in this case. A number P written in the form $a_1 a_2 \dots a_n$ means $P = 10^{n-1} a_1 + 10^{n-2} a_2 + \dots + 10 a_{n-1} + a_n$. Main Results. (1) Let P be the number $a_1 a_2 \dots a_n$. Then P is a multiple of y (not equal to zero) if and only if $\sum_{i=1}^{n} (a_i \times x^{n-i})$ is a multiple of y, where $x = 10 - y$. (2) Let the number P be $a_1 a_2 a_3 \dots a_n = Q \times y$ where $y \neq 10$; then Q is given by the formula $Q =$

$\Sigma_{i=1}^{n-1}((a_1a_2\dots a_i)\times x^{n-(i+1)}) + (1/y)\times\Sigma_{i=1}^{n}(a_i\times x^{n-i})$ where $x = 10 - y$. (Received April 4, 1973.) (Author introduced by Professor T. Thrivikraman.)

*73T-A167. JACQUES LEWIN, Syracuse University, Syracuse, New York 13210. A matrix representation and an application to PI algebras.

Let F be the free associative algebra on the set X over the field K. Let U, V be two ideals of F and $\{\partial(x), x\in X\}$ a basis for a free (F/U, F/V)-bimodule T. Theorem. The map $x \to \begin{pmatrix} x+V & 0 \\ \partial(x) & x+U\end{pmatrix}$ induces an injective homomorphism $F/UV \to \begin{pmatrix} F/V & 0 \\ T & F/U\end{pmatrix}$. Theorem. If F/U and F/V are embeddable in matrices (over a commutative algebra), so is F/UV. Theorem. A PI algebra with nilpotent radical satisfies all the identities of some full matrix algebra. (Received April 5, 1973.)

*73T-A168. B. GANTER, Technische Hochschule Darmstadt, 6100 Darmstadt, Federal Republic of Germany and HEINRICH WERNER, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada. Equational classes of Steiner systems.

Steiner triple systems can be coordinatized by Steiner quasi-groups. It is possible to give a similar construction for Steiner systems of type (2, q) where q is a prime power, and of type (3,4). In these cases (and in no others) equational classes of universal algebras exist such that the system of subalgebras of any algebra in these classes is a Steiner system of the given type. The classes can be chosen in such a way that each Steiner system of suitable type is coordinatized, and that any morphism between those Steiner systems can be represented by a homomorphism between coordinatizing algebras. The considered equational classes have interesting algebraic properties like strong amalgamation property, solvable word problem, permutable, normal and regular congruences and a unique factorization. In the case (2, q), the class can be chosen as a class of quasi-groups, the equations of which are given. (Received April 9, 1973.)

73T-A169. ROBERT J. BOND, Brown University, Providence, Rhode Island 02912. When does the Galois group determine the field? The function field case.

Let K and K' be two function fields in one variable with finite fields of constants k and k'. Assume that K and K' have a common separable closure L. Let G be the Galois group of L over K and G' the Galois group of L over K'. Assume that G and G' are isomorphic as profinite groups. What can be said about K and K'? Theorem 1. k and k' have the same number of elements. So one can assume that k = k'. Theorem 2. There is a one-to-one and onto set-theoretic map from the prime divisors of K to the prime divisors of K' which preserves the degree of a prime. Corollary. K and K' have the same zeta function and consequently the same genus and the same class number. In the genus zero case, K is a rational function field if and only if K' is. In the elliptic (genus one) case, the associated curves are isogenous. Theorem 3. Let J and J' denote the Jacobian varieties associated to K and K', respectively. Then, if E is a finite extension of k, the groups of E-rational points J(E) and J'(E) are isomorphic as g-modules where g is the Galois group of E over k. The proofs involve the cohomology of groups and class field theory. (Received April 5, 1973.)

73T-A170. JURGEN ROMPKE, McMaster University, Hamilton, Ontario, Canada. Regular, commutative, maximal semigroups of quotients. Preliminary report.

For each right reductive semigroup S one can construct its maximal semigroup of quotients Q(S), namely $Q(S) = \varinjlim \mathrm{Hom}_S(D, S)$, where the direct limit is taken over the downward directed family of dense right ideals. For S a commutative semigroup, Q(S) is regular if and only if S is separative and for each element $a\in S$ the ideal $\Gamma(a) = \{s\in S \mid$ there exists $b\in aS$ such that $st = bt$ for all $t\in aS\}$ is dense. (Received April 9, 1973.)

73T-A171. RONALD J. EVANS, University of Illinois, Urbana, Illinois 61801. Asymptotic formulae for extended Eulerian numbers. Preliminary report.

The extended Eulerian numbers $H(n,\lambda)$ are defined for $\lambda \neq 1$ by $(\lambda-1)/(\lambda-\zeta(s)) = \sum_{n=1}^{\infty} H(n,\lambda)n^{-s}$, where $\zeta(s)$ is the Riemann zeta function. Fix $\lambda > 1$ and let $n = p_1^{a_1}p_2^{a_2}\dots p_v^{a_v}$, where the p_i are distinct primes and v, $a_1,\dots,a_v$ are positive integers. Let c be the (unique) positive zero of the polynomial $\lambda x^v - \prod_{i=1}^{v}(x+a_i)$. Let $\Omega = \sum_{i=1}^{v} a_i$. We prove that for any constant $F < \frac{1}{2}$, $H(n,\lambda) = \alpha\beta\sqrt{\pi}\{1+O(\Omega^{-F})\}$, where $\alpha = \lambda^{-3/2}(\lambda-1)e^{-\Omega}\prod_{i=1}^{v}(c+a_i)^{a_i}/(a_i!)$ and $\beta = \{(1/2c)\sum_{i=1}^{v} a_i/(a_i+c)\}^{-1/2}$. This formula is obtained using the representation $H(n,\lambda) = ((\lambda-1)/\lambda)\sum_{k=0}^{\infty}\lambda^{-k}d_k(n)$, where $d_k(n)$ is the number of representations of n as a product of factors $\geqq 1$. Simpler asymptotic formulae are also given, subject, however, to certain restrictions on the growth of v and the a_i. These formulae are then used to refine some inequalities of Hille (Acta Arith. 2(1936), 134-144). (Received April 9, 1973.)

73T-A172. DAVID W. MATULA, University of Texas, Austin, Texas 78712 and Washington University, St. Louis, Missouri 63130. On minimum coverings of a graph by i-independent sets.

The edge-connectivity, $\lambda(G)$, is the minimum number of edges of the graph G whose removal disconnects G. The strength, $\sigma(G)$, of the graph G is the maximum of the edge-connectivities of the subgraphs of G. A vertex set A of the graph G is an i-independent set of G if the induced subgraph $\langle A\rangle$ satisfies $\sigma(\langle A\rangle) \leqq i$. Thus 0-independent sets (ordinary independent sets) induce totally disconnected subgraphs, and 1-independent sets induce acyclic subgraphs. The X_i-covering number of the graph G is the minimum number of i-independent sets that cover the vertices of G. $X_0(G)$ is the chromatic number of the graph G and $X_1(G)$ is the point-arboricity of the graph G. Theorem 1. For $i \geqq 0$ and any graph G, $X_i(G) \leqq 1 + [\sigma(G)/(i+1)]$. Theorem 2. For $i \geqq 0$ and any connected but not complete graph G with maximum degree $\Delta(G) \geqq i+2$, where also G is not a cycle for $i=0$, $X_i(G) \leqq [(\Delta(G)+i)/(i+1)]$. Theorem 2 is generally weaker than Theorem 1, but is one unit sharper for r-regular graphs with $r = \Delta(G) = \lambda(G) = k(i+1)$. Most of the previously known upper bounds on the chromatic number and the point-arboricity follow from these theorems. (Received April 11, 1973.)

73T-A173. ARUN K. SRIVASTAVA, Indian Institute of Technology, Hauz Khas, New Delhi 110029, India. Every locally right adjunctable functor $F:\underline{A} \to \underline{C}$ gives rise to a monad in $\underline{A}$.

James J. Kaput [Illinois J. Math. 16(1972), 86-94] has generalized the notion of adjoint functors by calling a functor $F:\underline{A} \to \underline{C}$ locally right adjunctable if for each $f: FA \to X$, $A \in \underline{A}$ and $X \in \underline{C}$, there exist an object $fX \in \underline{A}$ and morphisms $f_1: A \to fX$ and $f_0: FfX \to X$ such that $f = f_0 \cdot Ff_1$. Moreover whenever $f = g \cdot Fh$ where $g: FA' \to X$, $A' \in \underline{A}$ and $h: A \to A'$, then there exists a unique $e: A' \to fX$ such that $e \cdot h = f_1$ and $f_0 \cdot Fe = g$. It is shown that every locally right adjunctable functor $F:\underline{A} \to \underline{C}$ gives rise to a monad in $\underline{A}$. (Received April 11, 1973.) (Author introduced by Professor S. Franklin.)

73T-A174. DAVID ZEITLIN, 1650 Vincent Avenue North, Minneapolis, Minnesota 55411. Identities for integer sequences involving the greatest integer function. II. Preliminary report.

Let M be a positive integer and W_n an integer sequence defined by $W_{n+2} = MW_{n+1} + W_n$, where W_0 and W_1 are integers; if $W_0 = 0$, $W_1 = 1$, then $W_n \equiv U_n$, and if $W_0 = 2$, $W_1 = M$, then $W_n \equiv V_n$. Let $P > 0$ and $N < 0$ be the roots of $x^2 - Mx - 1 = 0$; set $F = M/(M+1)$, $C = (PW_0 - W_1)/(P-N)$, and $W_{-n} = (-1)^n(W_0V_n - W_n)$, $n \geqq 1$. Six results, (R1),...,(R6), were given in my Abstract 73T-A90, these *Notices* 20(1973), A-263. In (R6), $n \geqq 1$, and not $n = 0$. Two additional related results are: (R7) If $F - 1 < \{C(1+P^{-2})\} \leqq F$, then $[P^{2n+1}W_{2n+1} + F] = W_{4n+2}$, $n \geqq 0$. (R8) If $-F \leqq C < 1 - F$, then for $n \geqq 0$, $[P^{2n}W_{2n} + F] = W_{4n}$. Since $x = P, N$ satisfy $x^k = xU_k + U_{k-1}$ and $W_{n+k} = U_{k-1}W_n + U_kW_{n+1}$, P^k and N^k can be removed from (R1),...,(R8) to obtain equivalent statements. Thus, in (R1), $[PU_kW_n + F] = U_kW_{n+1}$; in (R2), $[NU_kW_{-n} + F] = U_kW_{-n+1}$, and in (R8), $[PU_{2n}W_{2n} + F] = U_{2n}W_{2n+1}$. For

$M = 1$, $F = \frac{1}{2}$, $W_n \equiv H_n$, $U_n \equiv F_n$, and $V_n \equiv L_n$, the Fibonacci sequences. For $W_n \equiv L_n$, $C = 1$. Thus, from (R5), for $j = 0$, $[P^{2n+1}L_{2n+1} + \frac{1}{2}] = L_{4n+2} - 1$, $n \geqq 0$; and from (R6), for $j = 1$, $[P^{2n}L_{2n} + \frac{1}{2}] = L_{4n} + 1$, $n \geqq 1$. (R9) For $M = 1$, $|C| \leqq 1$, $[P^k H_n + \frac{1}{2}] = H_{n+k}$ for (a) $n \geqq k+2$, $k \geqq 2$, (b) $n \geqq 4$, $k = 1$. For $C = 1$, $H_n \equiv L_n$, and (R9) gives the result of L. Carlitz, Fibonacci Quart. 10(1972), 526. For $C = -1$, $H_n \equiv -L_n$, and so $[-P^k L_n + \frac{1}{2}] = -L_{n+k}$. (R10) For $W_n \equiv V_n$, $C = 1$. Thus, from (R5) with $j = 0$, we have $[P^{2n+1}V_{2n+1} + F] = V_{4n+2} - 1$, $n \geqq 0$. (Received April 12, 1973.)

*73T-A175. CLIFFORD S. QUEEN, Lehigh University, Bethlehem, Pennsylvania 18015. Euclidean subrings of function fields.

Let F be a function field of one variable over a finite field k, where k is algebraically closed in F. Let P range over all prime divisors of F; then if S is a nonempty finite set of prime divisors of F we get a ring $O_S = \bigcap_{P \notin S} O_P$, where O_P is the valuation ring associated to a prime divisor P. Theorem. If S is a finite nonempty set of prime divisors of F containing at least two elements, then O_S is a principle ideal domain if and only if it is a euclidean domain. Further if O_S is a principle ideal domain and S contains just one element, then O_S is one of four possible rings which are not euclidean. (Received April 12, 1973.)

*73T-A176. MICHAEL O. ALBERTSON, Swarthmore College, Swarthmore, Pennsylvania 19081. The independence number of a planar graph.

The independence number $\mu(G)$ of a graph G on N vertices is the maximum number of vertices of G no pair of which are adjacent. The Erdos-Vizing problem is to show for planar graphs that $\mu(G)/N \geqq 1/4$. The five color theorem implies $\mu(G)/N \geqq 1/5$. Theorem. G planar implies $\mu(G)/N \geqq 2/9$. (Received April 13, 1973.)

73T-A177. WITHDRAWN.

*73T-A178. JOHN K. KARLOF, University of Colorado, Boulder, Colorado 80302. The subclass algebra associated with a finite group and subgroup.

Let G be a finite group and let H be a subgroup of G. If $g \in G$, then the set $E_g = \{hgh^{-1} | h \in H\}$ is the subclass of G containing g and $\sum_{x \in E_g}$ is the subclass sum containing g. The algebra over the field of complex numbers K generated by these subclass sums is called the subclass algebra, denoted by S, associated with G and H. Let $\{M_1 \ldots M_s\}$ be the irreducible KG modules and let $\{N_1 \ldots N_t\}$ be the irreducible KH modules. Suppose $\{e_i\}_{i=1}^t$ is a set of orthogonal primitive idempotents of KH. Define the nonnegative integers $\{c_{ij}\}$ by $M_j|_H = \sum_{i=1}^t c_{ij} N_i$. An algebraic proof of E. P. Wigner's result, $\sum_{i,j} c_{ij}^2$ = the number of subclasses, is established. The irreducible modules of the subclass algebra are shown to be $\{e_i M_j\}$ with $\dim(e_i M_j) = c_{ij}$. Also, results about Schur algebras are utilized to develop formulas relating the irreducible characters of S with the irreducible characters of G and H. (Received April 16, 1973.)

73T-A179. BRUCE A. ANDERSON, Arizona State University, Tempe, Arizona 85281. A method of finding perfect 1-factorizations. Preliminary report.

A 1-factorization F of K_{2n}, the complete graph on 2n points, will be called semiregular iff for any i, j, k, m such that $i \neq j$, $k \neq m$, if $F_i, F_j, F_k, F_m \in F$, then $F_i \cup F_j$ and $F_k \cup F_m$ have identical cycle structures. F is perfect iff $i \neq j$ implies $F_i \cup F_j$ is a Hamiltonian circuit. A technique is considered for constructing 1-factorizations on certain K_{2n} that turns out to yield a small number of semiregular 1-factorizations which are then tested for perfectness. In most of the cases tested so far, the method gives some perfect ones. It is not yet clear why the idea sometimes works and sometimes fails. The following positive application of the procedure is apparently new. Theorem. There is a perfect 1-factorization of K_{28}. (See Abstract 73T-A126, these Notices 20(1973), A-322.

In reference to this abstract, some minor qualifying phrases were inadvertently omitted in the statement of the first theorem and its corollary. Insert the underlined phrases: GK_{p+1}, $\underline{p > 5}$; GA_{2p}, $\underline{p > 3}$; $p + 1 = 2q$, $\underline{p > 5}$.) (Received April 23, 1973.)

Analysis

73T-B137. WILLIAM D. L. APPLING, North Texas State University, Denton, Texas 76203. Homogeneity and continuity properties of upper and lower integral difference functionals.

U, F, p_B, p_{AB}, p_A^+ and the notions of integral, sum infimum and sum supremum functional are as in previous abstracts of the author. Suppose B is in p_B, and for each h in p_{AB} and V in F, $Z(h)(V) = \int_V [L(Bh)(I) - G(Bh)(I)]$ and $A(h)$ denotes the element of $\{k : k \text{ in } p_{AB}, \int_U B(I)k(I) \text{ exists}\}$ closest to h (variation norm). Theorem 1. If H is in p_B, w is in p_{AB} and $\int_U H(I)w(I)$ exists, then $Z(\int Hw)(V) = \int_V |H(I)| Z(w)(I)$ for all V in F. Theorem 2. If m is in p_A^+ and $0 < c$, then there is $d > 0$ such that if h is in p_{AB}, $m - \int |h|$ is in p_A^+ and $Z(h)(U) < d$, then $\int_U |h(I) - A(h)(I)| < c$. (Received January 23, 1973.)

*73T-B138. DOMINGO A. HERRERO, Universidade Estadual de Campinas, Campinas, S. P., Brasil. Burnside's theorem for the Fréchet space ω. Preliminary report.

Let ω denote the Fréchet space of all complex sequences $\{\xi_n\}_{n=1}^{\infty}$ under the topology given by the family of seminorms $\|\{\xi_n\}\|_p = \max\{|\xi_n| : 1 \leqq n \leqq p\}$, $p = 1, 2, \ldots$, and let $\mathcal{L}(\omega)$ be the algebra of all continuous linear maps from ω into itself. $\mathcal{L}(\omega)$ is a topological algebra under the strong operator topology (i.e., the topology of the pointwise convergence). The author generalizes a well-known density theorem for operator algebras in a Banach space (W. B. Arveson, "A density theorem for operator algebras", Duke Math. J. 34(1967), 635-647) to operator algebras in an arbitrary locally convex space. By using this generalization of Arveson's result and the results of K.-H. Körber about continuous endomorphisms of ω (Math. Ann. 181(1969), 8-34 and 182(1969), 95-103) it is shown that the only strongly closed subalgebra of $\mathcal{L}(\omega)$ without nontrivial closed invariant subspaces (i.e., different from (0) and ω) is $\mathcal{L}(\omega)$. A trivial example is given to show that, for a suitable normed space E, $\mathcal{L}(E)$ can contain a nontrivial subalgebra without closed invariant subspaces. (Received February 5, 1973.)

*73T-B139. DAVID J. HALLENBECK, University of Delaware, Newark, Delaware 19711. Convex hulls and extreme points of some families of univalent functions. Preliminary report.

Let C_K denote the K-fold symmetric close-to-convex functions. We determine the closed convex hull of C_K denoted by HC_K for $K = 2, 3, \ldots$. We prove that if $f(z) = \sum_{n=1}^{\infty} a_n z^n$ is subordinate to F a close-to-convex odd function, then $|a_n| < \sqrt{2}$ for $n = 1, 2, \ldots$. Let $F_p = \{\int_X ((1 - xz)^p (1 - \bar{x}z)^p)^{-1} d\mu(x) : \mu \in P\}$ where P is the set of probability measures on $X = \{x : \operatorname{Im} x \geqq 0 \text{ and } |x| = 1\}$. We prove that $F_p \cdot F_q \subset F_{p+q}$. Let $St_R(\alpha, K)$ denote those functions which are starlike of order α, K-fold symmetric, and are real on $(-1, 1)$. We determine that $H\,St_R(\alpha, K) = \{\int_X z((1 - xz^K)((1-\alpha)/K)(1 - xz^K)(1-\alpha)/K)^{-1} d\mu(x) : \mu \in P\}$ where X and P are as in the statement above about F_p. Let $St^*(\alpha) = \{F : f < g \text{ and } g \in St(\alpha)\}$ where $St(\alpha)$ denotes the class of functions starlike of order α. We determine that the closed convex hull of $St^*(\alpha)$ for $\alpha \leqq 0$ consists of the functions represented by $f(z) = \int_{X \times Y} (yz/(1-xz)^{2-2\alpha}) d\mu(x, y)$ where μ is a probability measure on $X \times Y$, the product of two unit circles. The extreme points of $H\,St^*(\alpha)$ consists of the functions $\{yz/(1 - xz)^{2-2\alpha} : |x| = |y| = 1\}$. We also determine the closed convex hull and extreme points of the class of typically real odd functions. (Received February 15, 1973.)

*73T-B140. HONNAPPA SIDDALINGAIAH, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. A constructive method of solving linear integral equations of the second kind. Preliminary report.

D. Enskog gives a series solution, convergent only in the L_2 norm but not uniformly, of a symmetric Fredholm integral equation $f = \varphi - \lambda K\varphi \equiv J\varphi$, where J is positive definite. Now a uniformly and absolutely convergent series solution of the above equation, along with an estimation of its truncation error, is obtained and it is also shown that the new solution is applicable to nonsymmetric Fredholm and weakly singular integral equations and thus the method is applicable to Fredholm-Poincaré integral equations of potential theory. Also a bilinear expansion of the resolvent H of the kernel K is realized. (Received February 21, 1973.)

*73T-B141. HAROLD S. SHAPIRO, Royal Institute of Technology, S-100 44 Stockholm, Sweden. Counterexamples to a majorant principle for trigonometric series with positive coefficients.

Let $(TS)_+$ denote the set of formal trigonometric series $\sum c_n e^{int}$ with $c_n \geqq 0$. N. Wiener proved that if f in $L^1(-\pi, \pi)$ has a Fourier series in $(TS)_+$ and belongs to $L^2(-\delta, \delta)$ for some $\delta > 0$ then $f \in L^2(-\pi, \pi)$. (This is also valid if f is assumed a priori to be merely a tempered distribution, not necessarily in $L^1(-\pi, \pi)$.) The analogous proposition with 2 replaced by p is easily demonstrated if $p = 4, 6, 8, \ldots$ or ∞, but Wainger (Proc. Amer. Math. Soc. 20(1969), 16-18) showed it is false for $p < 2$ and raised the question concerning $p > 2$. That the answer is generally negative is shown by Theorem. If $p > 2$ is not an even integer, then given $\epsilon > 0$ there exists a function f in $L^1(-\pi, \pi)$ with positive Fourier coefficients such that $f \in L^p(-\pi + \epsilon, \pi - \epsilon)$ but $\int_{-\infty}^{\infty} |f(t)|^p \, dt = \infty$. The proof is based on another "majorant" counterexample relating to a problem of Hardy and Littlewood and reported on in a forthcoming paper by G. Bachelis in Quart. J. Math. Oxford Ser. (Received February 23, 1973.)

*73T-B142. J. MARSHALL ASH and LAWRENCE GLUCK, DePaul University, Chicago, Illinois 60614. Counterexamples to Carleson's and Plessner's theorems for multiple Fourier series. Preliminary report.

C. Fefferman presented a spectacular counterexample to Carleson's theorem with a continuous f on T^2 with everywhere restrictedly divergent Fourier series $S(f)$. The authors can show: Theorem 1. There is a continuous H on T^n, $n > 1$, $\ni S(H)$ is of power series type, has uniformly bounded partial sums, and is everywhere restrictedly divergent. H is used to construct multiple Fourier series counterexamples to the classical Plessner theorem: $S(f)$ convergent on $E \Rightarrow S(\tilde{f})$ convergent a.e. in E. Theorem 2. There is a continuous F on T^n, $n > 2$, $\ni F$ and all its conjugates F_α are continuous, $S(F)$ is unrestrictedly convergent, while the $S(F_\alpha)$ are unrestrictedly bounded a.e. on T^n but divergent by any 2-parameter rectangular convergence method. Replacing H by a Kolmogorov-Marcinkiewicz divergent series yields Theorem 3. There is a G on $T^2 \ni G$ and all its conjugates G_α are integrable, $S(G)$ is unrestrictedly convergent, while the $S(G_\alpha)$ are unrestrictedly bounded but square divergent a.e. on T^2. In (Abstract 701-42-5, these *Notices* 20(1973), A-134) the authors announced a Plessner type theorem for $L_p(T^2)$, $p > 1$. Theorems 2 and 3 show that this result is the best possible. (Received March 1, 1973.)

*73T-B143. JUSSI KETONEN, University of California, Berkeley, California 94720. On Banach spaces of large cardinality. Preliminary report.

Let B be a Banach space of the cardinality of a Ramsey cardinal m. Theorem 1. B contains an unconditional basis-set of cardinality m. In particular, if B does not contain any subspace isomorphic to $\ell^1(m)$, we can select the basis-constant to be 1. Theorem 2. If T is a continuous linear operator on B so that the image of B under T has power m, then there is a closed subspace M of B of cardinality m so that T is a one-to-one map on M with a closed range. (Received February 26, 1973.)

*73T-B144. SUSHEEL CHANDRA and PREM SINGH, Indian Institute of Technology, Kanpur 16, India. Coefficient bounds of the functions of bounded boundary rotation. Preliminary report.

For $k \geqq 2$ denote by Λ_k the class of functions $f(z) = 1/z + a_0 + a_1 z + \ldots + a_n z^n + \ldots$ that are regular in $0 < |z| < 1$ which have boundary rotation at most $k\pi$ [cf. B. Pinchuk, J. Analyse Math. 24(1971), 101-150]. For fixed $n \leqq \min[(k+10)/4, k/2 + 1]$ we determine the maximum of the set of values of $|a_n|$. (Received March 1, 1973.) (Authors introduced by Professor Prabha Gaiha.)

*73T-B145. H. M. SRIVASTAVA, University of Victoria, Victoria, British Columbia, Canada and ROBERT G. BUSCHMAN, University of Victoria, Victoria, British Columbia, Canada and University of Wyoming, Laramie, Wyoming 82070. Composition of fractional integral operators involving Fox's H-function.

In the present paper the authors obtain an elegant expression for the composition of the generalized operators of fractional integration involving the H-function of C. Fox [Trans. Amer. Math. Soc. 98(1961), 395-429]. The various examples of special cases displayed in this paper are associated, for instance, with Appell's function F_1, Humbert's function Φ_1 and the Gaussian hypergeometric function ${}_2F_1$. (Received March 5, 1973.)

*73T-B146. H. M. SRIVASTAVA, University of Victoria, Victoria, British Columbia, Canada and REKHA PANDA, University of Victoria, Victoria, British Columbia, Canada and Ravenshaw College, Cuttack 3, India. Some recursion formulas associated with multiple hypergeometric functions. Preliminary report.

Let $F^{(3)}[x,y,z]$ denote a general triple hypergeometric function which was introduced several years ago by H. M. Srivastava [Proc. Cambridge Philos. Soc. 63(1967), 425-429; see p. 428]. The present paper discusses various classes of recurrence relations involving the function $F^{(3)}[x,y,z]$ as well as its extensions in three and more variables. It is shown, among other things, how some of the results proven in this paper by using simple analysis would unify and extend a large number of summation formulas scattered in the literature, e.g. those given recently by M. A. Pathan [Riv. Mat. Univ. Parma (2) 11(1970), 159-164] for certain of the fourteen hypergeometric functions of three variables belonging to Lauricella's set [G. Lauricella, Rend. Circ. Mat. Palermo 7(1893), 111-158; see p. 114]. (Received March 5, 1973.)

*73T-B147. JON C. HELTON, Arizona State University, Tempe, Arizona 85281. Equivalence of integrals in commutative Banach algebras.

All definitions and conventions are as in the following paper by the author: "Product integrals and exponentials in commutative Banach algebras" (Proc. Amer. Math. Soc., to appear). Theorem 1. Suppose X is commutative. If $\beta > 0$ and F and G are functions from $R \times R$ to X such that $|F| < 1 - \beta$ on $[a,b]$, $|G| < 1 - \beta$ on $[a,b]$, F and G are in OL° on $[a,b]$ and F^2 and G^2 are in OA° and OB° on $[a,b]$, then the following statements are equivalent: (1) $\int_a^b F$ exists, $\int_a^b G$ exists and $\exp\int_a^b F = \exp\int_a^b G$, and (2) F and G are in OC° on $[a,b]$ and ${}_a\Pi^b(1+F) = \{{}_a\Pi^b(1+G)\}\{\exp\int_a^b \sum_{n=2}^{\infty}(-1)^{n-1}(F^n - G^n)/n\}$. Theorem 2. Suppose X is commutative. If $\beta > 0$ and F and G are functions from $R \times R$ to X such that $|F| < 1 - \beta$ on $[a,b]$, $|G| < 1 - \beta$ on $[a,b]$, F and G are in OL° on $[a,b]$ and F^2 and G^2 are in OA° and OB° on $[a,b]$, then the following statements are equivalent: (1) $\int_a^b F$ exists, $\int_a^b G$ exists and $\exp\int_a^b F = \{\exp\int_a^b G\}\{\exp\int_a^b \sum_{n=2}^{\infty}(-1)^{n-1}(G^n - F^n)/n\}$, and (2) F and G are in OC° on $[a,b]$ and ${}_a\Pi^b(1+F) = {}_a\Pi^b(1+G)$. (Received March 5, 1973.)

*73T-B148. NIGEL J. KALTON, University College of Swansea, Swansea, Wales and WILLIAM H. RUCKLE, Clemson University, Clemson, South Carolina 29631. A series characterization of $L_p(\mu)$ spaces.

We derive the following improvement of the Dvoretzky-Rogers theorem: Theorem. A normed space E is not isomorphic to a subspace of some $L_p(\mu)$ space $(1 \leqq p \leqq \infty)$ if and only if there exists a series in E which does not converge absolutely but such that every continuous linear image of this series in ℓ_p converges absolutely. (Received February 26, 1973.)

*73T-B149. DEVENDRA S. GOEL, ANTHONY S. B. HOLLAND, CYRIL NASIM and BADRI N. SAHNEY, University of Calgary, Calgary, Alberta T2N 1N4, Canada. On best simultaneous approximation in normed linear spaces. Preliminary report.

Let X be a normal linear space and K be a subset of X. Given any two elements $x_1, x_2 \in X$ we define $d(x_1, x_2; K) = \inf_{k\in K}\max(\|x_1-k\|, \|x_2-k\|)$. An element $k^* \in K$ is said to be a best simultaneous approximation to x_1 and x_2 if $d(x_1, x_2; K) = \max(\|x_1-k^*\|, \|x_2-k^*\|)$. If K is a finite dimensional subspace and X is strictly convex, then the best simultaneous approximation exists and is unique. If K is a closed and uniformly convex subset of a Banach space X, then the best simultaneous approximation is unique, although K is not necessarily of finite dimension. In an inner product space, if g_1 is the best approximation to x_1 and g_2 is the best approximation to x_2 then the unique best simultaneous approximation is a convex combination of g_1 and g_2. (Received March 12, 1973.)

*73T-B150. J. M. BOWNDS and JIM M. CUSHING, University of Arizona, Tucson, Arizona 85721. On preserving stability of Volterra integral equations under a general class of perturbations.

Consider the system (P) $u(t) = \varphi(t) + \int_a^t K(t,s)u\,ds + \int_a^t p(t,s,u)\,ds$, $t \geqq a \geqq t_0$, where $p(t,s,0) \equiv 0$, φ is continuous and K, p are such that u exists locally, is unique, and is continuable. Several new and refined definitions of stability with respect to φ and a on a normed space are stated and then characterized for the associated linear system (L) for certain special spaces in terms of its fundamental matrix and/or its resolvent matrix. Using these characterizations we study the preservation of stability on a given normed space from (L) to (P) under a general class of perturbations p. Specifically, $p \equiv f + g + h$ where $f = f(t,s,z)$ is "higher order" in z and g, h are appropriate generalizations of typical perturbations found in the theory of ordinary differential equations: $|g| \leqq \gamma z, \int^{+\infty}\gamma\,dt < +\infty$ and $|h| \leqq \eta|z|$, $|\eta| \to 0$ as $t \to +\infty$. Results are derived on the basis of three different representation formulas and the continuability property of solutions u. The results include as corollaries many well-known theorems for differential equations as well as recent theorems for integral equations (where $g \equiv h \equiv 0$ and p has a factor of K(t, s)). This work greatly extends that announced in Abstract 698-B12, these *Notices* 19(1972), A-777. (Received March 12, 1973.)

*73T-B151. ATHANASSIOS G. KARTSATOS, University of South Florida, Tampa, Florida 33620. A boundary value problem on an infinite interval.

Consider the generalized boundary value problem: (I) $x' = A(t)x + F(t,x)$, (II) $Tx = r$, where A(t) is a continuous $n \times n$ matrix on $R_+ = [0,\infty)$, F is a continuous n-vector on $R_+ \times S$ (S = a suitable subset of R^n), T is a bounded linear operator defined on (or a subspace of) $C[R_+, R^n]$, the space of all bounded and continuous R^n-valued functions on R_+, and r is a fixed vector in R^n. Such problems have many applications in boundary layer theory. A solution to the problem (I), (II) is shown to exist, under the assumption that the homogeneous problem has only the zero solution. Special cases of (II) include $Mx(0) - Nx(\infty) = r$, where M,N are $n \times n$ constant matrices, and $x(\infty) = \lim_{t\to\infty} x(t)$. The method is based upon reduction of (I), (II) to an integral equation and an application of a fixed point theorem. An approximation technique is also considered. (Received March 15, 1973.)

73T-B152. JEFFREY B. RAUCH, University of Michigan, Ann Arbor, Michigan 48104. Necessary conditions for L_2-well-posedness of hyperbolic mixed problems with variable coefficients. Preliminary report.

Agemi and Shirota (J. Fac. Sci. Hokkaido Univ. Ser. I 22(1972), 146 ff.) have shown that if a hyperbolic mixed problem with variable coefficients is L_2-well-posed in a halfspace then each "frozen" constant coefficient problem is L_2-well-posed. In this note we show that, with the additional hypothesis that there is finite speed of propagation, the same result can be obtained for regions with curved boundary. In addition we can weaken the hypothesis of noncharacteristic boundary. (Received March 15, 1973.)

73T-B153. MARY S. RANADE, Illinois Institute of Technology, Chicago, Illinois 60616. Certain polynomials are isomorphic to Bernoulli shifts.

Let F be the set of nonnormality of the iterates of an nth degree polynomial $P_n(z)$. Then if F is a rectifiable Jordan curve, $P_n(z)$ is isomorphic on F, with respect to the equilibrium measure μ introduced by H. Brolin [Ark. Mat. 6(1965), 103-144], to the Bernoulli shift (1/n, 1/n,..., 1/n). Thus $P_n(z)$ has μ-measure-theoretic entropy log n. For example, the Tchebycheff polynomial, $T_n(z) = \cos n \arccos z$, with F the real interval [-2, 2] and $\mu = Cdx/\sqrt{4-x^2}$, and the power function, $p_n(z) = z^n$, with F the unit circle and μ Lebesgue measure, are each isomorphic to the Bernoulli shift (1/n,..., 1/n), and have μ-entropy log n. (Received March 19, 1973.) (Author introduced by Professor Abe Sklar.)

73T-B154. WITHDRAWN.

73T-B155. S. K. BAJPAI, Clark University, Worcester, Massachusetts 01610. On the univalence of some analytic functions.

Let $f(z) = z + \sum_{k=n+1}^{\infty} a_k z^k$ and $g(z) = z + \sum_{k=n+1}^{\infty} b_k z^k$ be analytic and satisfy $|(1-\delta)^{-1} [f(z)/(\lambda f(z) + (1-\lambda)g(z)) - \delta] - M| < M$, $0 \leqq \lambda \leqq 1$, $M \geqq 1$ if $\delta \neq 0$, $M > 1/2$ if $\delta = 0$. Then we determine the values of R for which f(z) is univalent and starlike for $|z| < R$ under the assumptions (a) $\mathrm{Re}\{g(z)/z\} > \eta$ or (b) $\mathrm{Re}\{zg'(z)/g(z)\} > \alpha$; $0 \leqq \eta, \alpha < 1$ and $1 - m\delta - \delta \geqq 0$, $m = (M-1)/M$. These results sharpen and generalize the results of G. M. Shah [see Pacific J. Math. 43(1972), 239-250]. (Received March 14, 1972.)

*73T-B156. DONALD D. ROGERS, Indiana University, Bloomington, Indiana 47401 and University of Edinburgh, Edinburgh, Scotland. Normal approximants of operators. Preliminary report.

For an arbitrary Hilbert space operator A define $\delta(A) = \inf\{\|A - N\| : N \text{ normal}\}$ and call a normal operator N_0 a normal approximant of A in case $\|A - N_0\| = \delta(A)$. It is shown that the one-way weighted shift with weights 1, 1/2, 1/3,..., 1/n,... does not have a normal approximant. More generally, the following theorem is proved. Call a nonzero vector f a maximal vector for an operator A in case $\|Af\| = \|A\| \cdot \|f\|$. Theorem. If A is an operator with a dense range such that $\delta(A) \leqq \frac{1}{2}\|A\|$ and such that the kernel of A contains a maximal vector for A^*, then A does not have a normal approximant. (This theorem applies to the adjoint of the operator above.) The proof uses elementary properties of normal operators and maximal vectors. (Received March 20, 1973.)

*73T-B157. REBECCA R. KITTO, University of Washington, Seattle, Washington 98195. Fourier self-transforms on self-dual groups. Preliminary report.

For the self-dual locally compact Abelian group of real numbers R the function $t \to e^{-t^2/2}$ is known to be carried to itself by the Fourier transformation (a self-transform) and may be used to generate a complete orthonormal set of self-transforms in $L_2(R)$ known as the Hermite functions. In 1933 Norbert Wiener proved this fact and in so doing obtained a very simple proof of Plancherel's theorem for R, since the Fourier transformation plainly carries an orthonormal basis of L_2 onto an orthonormal basis of L_2. Thus one is led to study a similar question, the construction of a complete orthonormal set of self-transforms in L_2, for some other self-dual groups. This paper proves the existence of a complete analogue of the Hermite functions for a self-dual group containing a compact open subgroup isomorphic to its own annihilator. In this case the characteristic function of the [self-annihilating] subgroup is a self-transform, and translates of this function can be used to generate an orthonormal basis. The theorem is applicable to many self-dual groups: p-adic numbers, the product of a LCA group with its dual group in certain cases, some finite groups. A basis of self-transforms is easily described for a finite direct product of self-dual groups, each of which has a set analogous to the Hermite functions, solving the problem for R^n and for the product of an arbitrary locally compact Abelian group with its dual group. (Received March 21, 1973.)

*73T-B158. EDWARD B. SAFF, University of South Florida, Tampa, Florida 33620 and T. SHEIL-SMALL, The University, Heslington, York, England. Coefficient and integral mean estimates for polynomials with restricted zeros. Preliminary report.

Let $T(\theta)$ be a trigonometric polynomial all of whose zeros are real, and let $M = \max_\theta |T(\theta)|$. We prove that $\int_0^{2\pi} |T(\theta)|\, d\theta \leqq 4M$, which settles a long-standing conjecture of P. Erdős. Furthermore, if $P_n(z) = \sum_0^n a_k z^k$ is a polynomial of degree n having all its zeros on $|z| = 1$ and $M = \max[|P_n(z)|; |z| = 1]$, then it is shown that $|a_k| \leqq M/2$, for all $k \neq n/2$. This settles a conjecture due to W. K. Hayman except in the case of a polynomial of even degree. For the case $n = 2m$, it is shown that $|a_m| \leqq M/\sqrt{3}$. Also when $n = 4$ the best possible estimate $|a_2| \leqq M/2$ is established. Additional results are included which concern coefficient estimates for "large" n as well as an $n/2$-theorem for the derivative of a self-inversive polynomial. (Received March 22, 1973.)

*73T-B159. JOHN WERMER, Brown University, Providence, Rhode Island 02912. Smooth generators for the ball algebra. Preliminary report.

B is the closed ball in $\mathbb{C}^n$, $A(B)$ the algebra of all f analytic on int B, continuous on B. Fix $f_1, \dots, f_k$ in $A(B)$ separating points on B and smooth up to ∂B. Put $\chi(z) = (f_1(z), \dots, f_k(z))$. A point z is "regular" if the matrix $((\partial f_i/\partial z_j))$ has rank n at z. Theorem. Assume each z in B is regular. Then $\exists$ a smoothly bounded strictly pseudoconvex domain W in C^k with $\chi(B) \subseteq \overline{W}$ and functions α_i, $i = 1, \dots, n$, smooth in $\overline{W}$, analytic in W, with $\alpha_i(\chi(z)) = z_i$, all i, z in B. The theorem is proved for $n = 1$, k arbitrary and n arbitrary, $k = n + 1$. Corollary. Assume (1) $\chi(B)$ is polynomially convex, and (2) each z in B is regular. Then $f_1, \dots, f_k$ generate $A(B)$. Note. Polynomial convexity of $\chi(B)$ plus regularity of each z in int B are necessary but not sufficient for $f_1, \dots, f_k$ to generate $A(B)$. The above answers questions raised by N. Sibony. (Received March 22, 1973.)

73T-B160. JORGE M. LOPEZ, University of Oregon, Eugene, Oregon 97403. Some results on Fatou-Zygmund sets. Preliminary report.

Let G be a compact abelian group with character group X. Let $W \subset G$ be a measurable set so that $W \subset (\text{int } W)^-$ and let $P \subset X$ be a nonvoid symmetric set. Theorem (for notation and definitions, see Edwards, Hewitt and Ross, "Lacunarity for compact groups. III," Studia Math. 44(1972), 429-476). The following conditions are equivalent and if P is countable they are equivalent to the FZ(W)-property: (i) There exist a finite symmetric set $P_0 \subset P$ and a constant $\varkappa \geqq 1$ so that $\|f\|_A \leqq \varkappa \|f^+ \xi_{W^-}\|_\infty$ for all $f \in \mathcal{T}_{P\backslash P_0, r}(G)$. (ii) There exist a finite symmetric set $P_0 \subset P$ and a constant $\varkappa \geqq 1$ so that if $f \in L^\infty_{P\backslash P_0, r}(G)$ then $\hat{f} \in \ell^1(X)$ and $\|\hat{f}\|_1 \leqq \varkappa \|f^+ \xi_W\|_\infty$. (iii) There exists a finite symmetric set $P_0 \subset P$ so that for each $\varphi \in B_h(P\backslash P_0)$ there exists some $\mu \in M_+(-W^-)$ so that $\varphi = \hat{\mu}|_{P\backslash P_0}$. (iv) There exists a finite symmetric set $P_0 \subset P$ so that for each $\varphi \in B_h(P)$ there exists some μ in $M_+(-W^-)$ and $g \in \mathcal{T}_{P_0, r}(G)$ so that $\varphi = (g + \mu)\hat{}\,|_P$. (v) For each $\varphi \in B_h(P)$ there exists some $\mu \in M_+(-W^-)$ and $g \in L^\infty_r(G)$ so that $\varphi = (g + \mu)\hat{}\,|_P$. (Received March 22, 1973.)

*73T-B161. JUDITH REEVES McKINNEY, University of Missouri, Columbia, Missouri 65201. Kernels of measures on completely regular spaces. Preliminary report.

A kernel is a weak* continuous and bounded function $\lambda : T \to M_x(S)$ from a topological space T into a space of measures $M_x(S)$. An operator A is defined by a kernel $A : C(S) \to C(T)$ by $Af(t) = \int f(s)\, \lambda(t, ds)$. With various spaces of measures under consideration, the adjoint $A^* : M_x(T) \to M_x(S)$ and is represented by $A^*v(E) = \int_T \lambda(t, E)\, dv$. A corollary states that in $M_\sigma(S)$ and $M_\tau(S)$ the bipolar of a weak* compact set is weak* compact, thus showing that the Mackey and strong Mackey topologies agree for the dual pairs $(C(S), M_\sigma(S))$ and $(C(S), M_\tau(S))$. (Received March 23, 1973.)

73T-B162. JAWAID H. RIZVI, University of Western Ontario, London, Ontario, Canada and University of Karachi, Karachi, Pakistan. Generalized absolute summability.

Let $s_n = \sum_{r=0}^{n} u_r$, $s_\lambda(y) = (1+y)^{-\lambda-1}\sum_{n=0}^{\infty}\binom{n+\lambda}{n} s_n (y/(1+y))^n$, $u_\lambda(y) = (1+y)^{-\lambda-1}\sum_{n=0}^{\infty}\binom{n+\lambda}{n} u_n (y/(1+y))^n$; $U_\lambda(y) = \lambda\int_0^y u_\lambda(t)\,dt$, and $p > 1$. Then $s_n \to \ell(A_\lambda)$ if $s_\lambda(y) \to \ell$ as $y \to \infty$ and $s_n \to \ell(A'_\lambda)$ if $U_\lambda(y) \to \ell$ as $y \to \infty$. Also $s_n \to \ell|A'_\lambda|_p$ if $s_n \to \ell(A_\lambda)$ and $\int_0^\infty y^{p-1}|(d/dy)s_\lambda(y)|^p\,dy < \infty$ and $s_n \to \ell|A'_\lambda|_p$ if $s_n \to \ell(A'_\lambda)$ and $\int_0^\infty y^{p-1}|(d/dy)U_\lambda(y)|^p\,dy < \infty$. Theorem 1. If $\lambda > \mu > 0$ and $s_n \to \ell|A'_\lambda|_p$, then $s_n \to \ell|A'_\mu|_p$. Theorem 2. If $\lambda > 0$ and $\int_0^\infty y^{-1}|U_{\lambda+1}(y) - \ell|^p\,dy < \infty$, then $s_n \to \ell|A'_\lambda|_p$. Theorem 3. For $\lambda > 0$, $s_n \to \ell|A_\lambda|_p$ if and only if $s_n \to \ell|A'_\lambda|_p$ and $nu_n \to o|A_{\lambda-1}|_p$. Theorem 4. For $\lambda > 0$, $s_n \to \ell|A'_\lambda|_p$ if and only if $s_n \to \ell|A_{\lambda-1}|_p$. Theorem 5. If $\lambda > -1$, H is a regular Hausdorff method and $s_n \to \ell|A_\lambda|_p$, then $s_n \to \ell|A_\lambda H|_p$. These results extend the earlier results of the author and Borwein in J. Reine Angew. Math. 247(1971), 139-145. (Received March 23, 1973.)

73T-B163. DAVID BORWEIN, University of Western Ontario, London, Ontario, Canada and JAWAID H. RIZVI, University of Western Ontario, London, Ontario, Canada and University of Karachi, Karachi, Pakistan. Tauberian theorems for product methods.

Let H be a regular Hausdorff method and A be the Abel method, and $\Delta s_n = s_n - s_{n-1}$. Theorem 1. If $n\Delta s_n = O(1)$ is a Tauberian condition for H, then it is also a Tauberian condition for the product method AH. Immediate consequences are the following Tauberian theorems for the scales of Abel-type summability methods A_λ $(\lambda > -1)$ and $(A,\alpha) \equiv A(C,\alpha)$ when (C,α) is the Cesàro method of order α $(\alpha > -1)$. Corollary 1. $n\Delta s_n = O(1)$ is a Tauberian condition for the method (A,α). Corollary 2. $n\Delta s_n = O(1)$ is a Tauberian condition for the method A_λ. Let L denote the logarithmic method of summability and let $(L,\alpha) = L(C,\alpha-1)$ for $\alpha > 0$. Theorem 2. $n\Delta s_n \log n = o(1)$ is a Tauberian condition for the method (L,α). This extends a result of Ishiguro. (Received March 23, 1973.)

*73T-B164. GRAHAME BENNETT, Indiana University, Bloomington, Indiana 47401. An extension of the Dvoretsky-Rogers theorem.

It is shown that the identity mapping on an infinite-dimensional normed space is not (p,q)-absolutely summing whenever $1 \leqq q \leqq p < 4q/(2-q+|2-q|)$. This result is best possible and generalizes Pietsch's generalization [Studia Math. 28(1967), 333-353] of the celebrated Dvoretsky-Rogers theorem. The proof is based on the original paper of A. Dvoretsky and C. A. Rogers [Proc. Nat. Acad. Sci. U.S.A. 36(1950), 192-197]. (Received March 26, 1973.)

73T-B165. JAMES R. McLAUGHLIN, Pennsylvania State University, University Park, Pennsylvania 16802. Absolute convergence of series of Fourier coefficients.

This paper is concerned with sufficiency conditions on functions f which imply $\sum_{k=1}^{\infty}|a_k|^\beta k^\gamma$ converges, where $\{a_k\}$ denotes the Fourier coefficients of the function f with respect to some orthonormal system in question. This article unifies and generalizes practically all known sufficiency results that are given in terms of the integrated modulus of continuity, best approximation, or bounded path variation. This is done for the trigonometric, Walsh, Haar, Franklin, and related systems as well as general orthonormal systems. Two of the numerous results in this paper are as follows. Theorem 1. The exact analogue of A. Konyushkov's result on the trigonometric system [Mat. Sb. 44(86) (1958), 74, Theorem 11(b) for $1 \leqq p \leqq 2$, $0 < \beta \leqq p'$] is valid for the Walsh and generalized Walsh system (as defined by J. J. Price [Canad. J. Math. 9(1957), 413] for $\{n_i\}$ bounded). Theorem 2. The exact analogue of Z. Ciesielski and J. Musielak's result on the Haar system [Colloq. Math. 7(1959), 63, Theorem 2] is valid for the Franklin and generalized Haar system (as defined by B. I. Golubov [Sibirsk. Mat. Ž. 9(1968), 297] for $\{p_n\}$ bounded). This article will appear in Trans. Amer. Math. Soc. (Received March 28, 1973.)

*73T-B166. RONALD J. LEACH, Howard University, Washington, D.C. 20001. Coefficients of symmetric functions of bounded boundary rotation. Preliminary report.

Suppose $f(z) = z + \sum_{p=1}^{\infty} a_{mp+1} z^{mp+1}$ is an m-fold symmetric function with boundary rotation at most $k\pi$. Sharp estimates on $|a_{mp+1}|$ are obtained: for all p if $k \leq 2m + 2$, for $p = 1$ and $p = 2$ for $2 \leq k \leq 2m + 2$, and for p sufficiently large if $2m - 2 < k$. Sharp bounds are also obtained for the valency of f. (Received March 28, 1973.)

*73T-B167. THOMAS H. MacGREGOR, State University of New York, Albany, New York 12202. Convex and starlike functions of order α. Preliminary report.

The family $K(\alpha)$ of convex functions of order α consists of the functions f analytic in $\Delta = \{z : |z| < 1\}$ such that $f(0) = 0$, $f'(0) = 1$ and $\mathrm{Re}\{zf''(z)/f'(z) + 1\} > \alpha$. The family $St(\alpha)$ of starlike functions of order α are the functions f analytic in Δ such that $f(0) = 0$, $f'(0) = 1$ and $\mathrm{Re}\{zf'(z)/f(z)\} > \alpha$. These families were introduced by M. S. Robertson (Ann. of Math. 37(1936), 374-408). The function, $F(z) = (1 - (1-z)^{2\alpha-1})/(2\alpha - 1)$ if $\alpha \neq \frac{1}{2}$, $F(z) = -\log(1-z)$ if $\alpha = \frac{1}{2}$, belongs to $K(\alpha)$ and is extremal for many problems in $K(\alpha)$. We show that if $f \in K(\alpha)$ and $0 \leq \alpha < 1$, then $zf'(z)/f(z)$ is subordinate to $zF'(z)/F(z)$ in Δ. In particular, this subordination affords the determination of the largest number $\beta = \beta(\alpha)$ so that each function in $K(\alpha)$ also belongs to $St(\beta)$, if $0 \leq \alpha < 1$. The problem of finding β was posed by I. S. Jack in (J. London Math. Soc. (2) 3(1971), 469-474), where upper and lower bounds were found for β. The case $\alpha = 0$ corresponds to the fact that each normalized univalent function for which $f(\Delta)$ is convex satisfies $\mathrm{Re}\{zf'(z)/f(z)\} > \frac{1}{2}$, as shown by A. Marx (Math. Ann. 107(1932/33), 40-67) and E. Strohhäcker (Math. Z. 37(1933), 356-380). (Received March 30, 1973.)

73T-B168. HYMAN J. ZIMMERBERG, Rutgers University, New Brunswick, New Jersey 08903. Reduction of symmetrizable problems with integral boundary conditions.

The study of symmetrizable linear vector differential problems with integral boundary conditions $y' - A(x)y = \lambda B(x)y$, $My(a) + Ny(b) + \int_a^b F(x)y\,dx = 0$, initiated by Jones [J. Differential Equations 3(1967), 191-202] is continued for the case when the lower right-hand submatrix of the symmetrizing matrix is singular, wherein the symmetrizing matrix has been partitioned into four equal size blocks. Such symmetrizable problems are shown to be equivalent to lower-dimensional nonhomogeneous boundary problems of the same type but involving a vector parameter and also, in general, equivalent to linear homogeneous integro-differential-boundary problems of the type discussed by Vejvoda and Tvrdý [Ann. Mat. Pura Appl. (4) 89(1971), 169-216] and of dimension equal to the rank of the lower right-hand submatrix of the symmetrizing matrix. (Received March 30, 1973.)

*73T-B169. ETHELBERT N. CHUKWU, Cleveland State University, Cleveland, Ohio 44115. On the boundedness of solutions of third order differential equations. Preliminary report.

This research investigates the boundedness, as $t \to \infty$, of solutions of the real third order differential equation $x''' + f(x,x')x'' + g(x,x') + h(x) = p(t,x,x',x'')$. The method of approach is that initiated by Yoshizawa which is here reformulated for the general third order differential system $x' = \ell(x,y,z,t)$, $y' = m(x,y,z,t)$, $z' = n(x,y,z,t)$, where ℓ, m, n are continuous functions of the variables x, y, z and t. It rests entirely on the existence of appropriate Yoshizawa functions whose properties are comparable to those of Lyapunov functions. Indeed Lyapunov functions corresponding to certain differential systems and equations can be appropriated as complete or well-behaved Yoshizawa functions. The main object of this paper is to draw special attention to a method in current usage for supplementing these incomplete Yoshizawa functions appropriate signum functions to yield complete Yoshizawa functions. The discussion is in the context of the particular equation under study, and in the case of a certain explicitly given incomplete Yoshizawa function, the supplementation actually yields a complete Yoshizawa function with which ultimate boundedness is proved for all solutions of the equation under certain quite mild restraints on f, g, h and p. (Received March 30, 1973.)

*73T-B170. MARIA WELLEDA BALDONI, University of Genoa, 16132 Genova, Italy. An example of a lacunary set for a noncommutative group.

Let $G = \Pi U(d_n)$ where $U(d_n)$ is the group of unitary $n \times n$ matrices. Let $D_n : G \to U(d_n)$ be the canonical irreducible representations of G which map an element $x \in G$ into its nth coordinate. Let $E = \{D_n \otimes D_m : n \neq m\}$. Then E is a subset of Γ, the space of irreducible unitary representations of G. Theorem. E is a $\Lambda(p)$ set for every $p < \infty$, but E is not a Sidon set. (For a definition of $\Lambda(p)$ set and of Sidon set see E. Hewitt and K. Ross, "Abstract harmonic analysis", vol. II.) To the author's knowledge the above is the first example of a $\Lambda(p)$ set for every p which is not a Sidon set and with the property that the degrees of the representations it contains are not uniformly bounded. (Received April 2, 1973.) (Author introduced by Professor Alessandro Figà-Talamanca.)

73T-B171. WALTER HENGARTNER, Laval University, Quebec, Quebec, Canada and GLENN E. SCHOBER, Indiana University, Bloomington, Indiana 47401. Extremal functions and extreme points.

Let $\mathfrak{F}$ be the set of univalent functions in $H(\Omega)$ normalized by two linear functionals $\ell_1(f) = P$, $\ell_2(f) = Q$ with $D = \ell_1(Q) - \ell_2(P) \neq 0$. Set $U(w;f) = \ell((f-w)^{-1})/D$ where $\ell = \ell_2(1)\ell_1 - \ell_1(1)\ell_2$. Theorem 1. If $f \in \mathfrak{F}$ maximizes $\mathrm{Re}_{\mathfrak{F}} L$ for some $L \in H^*(\Omega)$, then each component of $\mathbb{C} - f(\Omega)$ is either a point or consists of finitely many analytic arcs whose tangents make an angle of at most $\pi/4$ with respect to the vector field $\mathrm{grad}[\mathrm{Re}\int\sqrt{U}\,dw]$. Only one component of $\mathbb{C} - f(\Omega)$ is unbounded and the only possible branching or nonanalytic points are the zeros of U. Theorem 2. Extreme points of $\mathfrak{F}$ in the sense of convexity have the weaker property that $\mathbb{C} - f(\Omega)$ consists of points and arcs that are monotone relative to the orthogonal trajectories, $\mathrm{Re}\int\sqrt{U}\,dw = \text{constant}$. Example. If $\mathfrak{F}$ is normalized at two points (i.e., $\ell_1(f) = f(p)$, $\ell_2(f) = f(q)$), these trajectories are the ellipses with foci P and Q. In this case $U \neq 0$ so that no branching can occur. (Received April 9, 1973.)

*73T-B172. HENDRIK J. KUIPER, Arizona State University, Tempe, Arizona 85281. Eigenvalue problems for noncontinuous operators associated with quasi-linear elliptic equations. Preliminary report.

For $u \in W_2^1(\Omega)$, where Ω is a bounded region in R^n with sufficiently smooth boundary, let ∇u denote $(D_1u, D_2u, \ldots, D_nu)$. For each $\lambda \geqq 0$ define the partial (or ordinary if n = 1) differential operator $L_\lambda(u) \equiv -D_i a_{ij}(x, u(x), \lambda)D_j + b_i(x, u(x), \nabla u(x), \lambda)D_i + c(x, u(x), \nabla u(x), \lambda)$ with domain = closure in $W_2^1(\Omega)$ of the C^∞ function of compact support in Ω. Consider the nonlinear eigenvalue problem $L_\lambda(u)u = \lambda F(x, u(x), \nabla u(x), \lambda)$ with $F(x, u, \nabla u, \lambda) \geqq k(x) > 0$ a.e. It is shown that this problem has a continuum of solutions (λ, u), $\lambda \geqq 0$, $u(x) \geqq 0$, joining $(0,0)$ and ∞ in $R^1 \times S$ where S is some real Banach space of functions satisfying $W_q^2(\Omega) \subset S$, a compact injection and $S \subset W_p^1(\Omega)$ a continuous injection, $p \geqq q > n$, $q \geqq 2$. Restrictions placed on the coefficients are such as to insure the existence of an inverse, $L_\lambda(u)^{-1}$ from $W_q^1(\Omega)$ to $W_q^2(\Omega) \cap \mathring{W}_2^1(\Omega)$. The restrictions on F are quite weak. For example, F is allowed to have jump discontinuities with respect to u. This, of course, means that the operator $L_\lambda(\cdot)^{-1}F(x, \cdot, \nabla\cdot, \lambda)$ from S to itself is allowed to be noncontinuous. (Received April 5, 1973.)

73T-B173. PAUL WILLIG, Stevens Institute of Technology, Hoboken, New Jersey 07030. Continuous W- algebras are nonnormal.

Let R be a W-* algebra with center Z acting on separable Hilbert space H. R is normal if for every W-* subalgebra S of R containing Z we have $S = S^{cc}$, where $S^{cc} = (S' \cap R') \cap R$. The author has shown that if R is type II then R is not normal (Abstract 71T-B167, these *Notices* 18(1971), 810; Proc. Amer. Math. Soc., to appear). A. Connes has shown (C. R. Acad. Sci. Paris 275(1972), 523-525) that every type III factor R contains a proper W-* subalgebra S such that $S^{cc} = R$, whence R is not normal. Using this fact and direct integral theory, we can show that if R is a W-* algebra of type III then R is not normal. Since if R is continuous $R = R_{II} + R_{III}$, R_{II} of type II and R_{III} of type III, we have the following result: Theorem. Every continuous W-* algebra on separable Hilbert space is nonnormal. (Received April 6, 1973.)

*73T-B174. C. J. EVERETT and P. R. STEIN, Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico 87544. Fermat functions and the general Wallis product.

A two-parameter family $F(a,p)$ of functions, $S(\theta)$, $C(\theta)$, is introduced, generalizing the functions $\sin\theta$, $\cos\theta$, and exploited to obtain a general "Wallis inequality" $(1+a+(n-1)p)/(a+np) < W_n^* < 1$, where $p \geqq 2$ is integral, $a > 0$ is real, $W_n^* = (n-1+(1+a)p^{-1})(a(a+p)\ldots(a+(n-1)p)\Gamma(a/p)/(1+a)\ldots(1+a+(n-1)p)\Gamma((a+1)/p))^p$. For $a = 1$, $p = 2$, one has the classical result. A general factorial $a(a+p)\ldots(a+(n-1)p)$, a, $p > 0$, is also defined and shown to have the asymptotic form $(2\pi n)^{1/2}(n/e)^n p^n n^{(a/p)-1}/\Gamma(a/p)$. For $a=p=1$, this is the Stirling formula. (Received April 6, 1973.)

*73T-B175. REKHA PANDA, University of Victoria, Victoria, British Columbia, Canada and Ravenshaw College, Cuttack 3, India. Some integrals associated with the generalized Lauricella functions.

In the present note the author evaluates eight definite integrals, four finite and four infinite, that involve certain products containing the generalized Lauricella function of several complex variables, introduced earlier by H. M. Srivastava and M. C. Daoust [Nederl. Akad. Wetensch. Proc. Ser. A 72 = Indag. Math. 31 (1969), 449-457; see also Math. Nachr. 53(1972), 151-159]. These integrals are shown to extend several recent results associated with the Kampé de Fériet function or its generalization in two variables [cf., e.g., Publ. Inst. Math. (Beograd) 9 (23) (1969), 199-202]. The paper concludes with a brief remark about several trivial variations or obvious particular forms of these known results that are still appearing in the literature. (Received April 9, 1973.)

*73T-B176. WILLIAM C. CONNETT and ALAN L. SCHWARTZ, University of Missouri, St. Louis, Missouri 63121. A multiplier theorem for Jacobi expansions.

If $f \sim \sum a_n h_n P_n^{\alpha\beta}(x)/P_n^{\alpha\beta}(1)$ is a Jacobi series, where h_n is the appropriate normalizing weight, and if $Mf \sim \sum m_n a_n h_n P_n^{\alpha\beta}(x)/P_n^{\alpha\beta}(1)$ is a multiplier transformation, we have the following result which generalizes and is sharper than our earlier result (Abstract 701-42-2, these *Notices* 20(1973), A-133). We use $[x]$ and $\langle x\rangle$ to denote the greatest integer in and fractional part of x respectively. Theorem. If $m_n = O(1)$ and $\sum_M^{2M} |\Delta^M m_n|^2 h_n = O(M^{-2\gamma})$ where $m = [\alpha+1]+1$ and $\gamma = 1-\langle\alpha+1\rangle$ then M is of strong type p-p for $1 < p < \infty$ and weak type 1-1. (Received April 9, 1973.)

73T-B177. KENNETH O. LELAND, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio 45433. Computer generated winding numbers and integer valued analogues of the log function. Preliminary report.

Let $A: z_0, z_1, \ldots, z_n = z_0$ be a sequence in C and let P be the closed polygonal path determined by A. Then there exists a unique integer valued map M, defined on C − P, such that for most $z \in P$, the change in M at z as one travels along a line cutting P at z is determined by the number and direction of the line segments of P passing through z. For $z \in C - P$, M(z) can be interpreted as the number of times P winds about z. The proof is by induction. For a triangular path $B: z_2, z_0, z_1, z_2$ the existence of M is trivial. B is "amalgamated" with $z_0, z_2, z_3, \ldots, z_n$ to obtain A, etc. A computer generation of M which works by "destroying" A is given. A generalization to E_n involving maps from ∂S^n, where S^n is an n-simplex, follows readily. If f is a map from a compact set H into $C - \{0\}$, then an integer valued map u on H is said to lie in Log(f), if for all $x \in H$, u is continuous at x if $x \in (-\infty, 0)$, and u + h is continuous at x if $x \notin (-\infty, 0)$, where h is the characteristic function of the lower half plane. These maps u are related to the maps M and to branches of the analytic log function ln. Applications are made to topological analysis (a lá G. T. Whyburn) and to the Eilenberg-Janiszewski proof of the Jordan curve theorem (which uses ln). (Received April 9, 1973.)

*73T-B178. CARL L. DeVITO, University of Arizona, Tucson, Arizona 85721. Characterizations of those ideals in L_1 which can be synthesized.

Let L_1 denote the Banach algebra of all complex-valued Lebesgue integrable functions on the real line with convolution as multiplication. It is well known that spectral synthesis fails in this algebra [see, for example, E. Hewitt and K. A. Ross, "Abstract harmonic analysis," vol. II, Springer-Verlag, New York, 1970]. If a given ideal is the intersection of the family of all regular maximal ideals in which it is contained, we shall say that the given ideal can by synthesized. For any fixed θ in L_∞ the transformation which takes each b in L_1 onto $\theta * b$ is a continuous, linear map from L_1 into L_∞. The null space of this map, call it $N(\theta)$, is a closed ideal in L_1. A transformation defined in this way will be called an L_∞-induced map. The map from L_1 into L_∞ induced by a (continuous) almost periodic function will be called a Bohr induced map. Theorem. For an ideal I in L_1 the three following conditions are equivalent: (i) I is the null space of a compact L_∞-induced map; (ii) I is the null space of a Bohr induced map; (iii) I can be synthesized. The proof of this theorem can be modified to yield the following result: A bounded, continuous, complex-valued function on the real line is an almost periodic function iff it induces a compact mapping from L_1 into L_∞. (Received April 11, 1973.)

73T-B179. A. A. JAFARIAN, University of Toronto, Toronto, Ontario M5S 1A1, Canada. Weak and quasi-decomposable operators. Preliminary report.

Let T be a bounded linear operator on a complex Banach space X. T is called weak-decomposable if for every finite open covering $\{G_i\}_{i=1}^n$ of $\sigma(T)$ there exists a system $\{Y_i\}_{i=1}^n$ of spectral maximal subspaces of T such that (i) $\sigma(T|Y_i) \subset G_i$, and (ii) $X = \bigvee_{i=1}^n Y_i$. The class of quasi-decomposable operators is also defined. This class contains the class of weak decomposable operators and, in fact, it is shown that the quasi-decomposable operators are those weak decomposable operators for which the linear manifolds $X_T(F)$ are closed for closed sets $F \subset C$. Many of the known results on decomposable operators are shown to have analogs for weak and quasi-decomposable operators. In particular the results on the single-valued extension property, similarity, quasi-similarity and restrictions, for both weak and quasi-decomposable operators; and the results on direct sum, the perturbation topology, and analytic functions for quasi-decomposable operators are derived. It is also shown that if the direct sum of a finite number of operators is a decomposable (quasi-decomposable) operator, then each of the summands is decomposable (quasi-decomposable). (Received April 16, 1973.) (Author introduced by Professor Peter M. Rosenthal.)

73T-B180. ARNOLD M. DUNN, Clarkson College of Technology, Potsdam, New York 13676 and State University College of New York, Potsdam, New York 13676. Schauder decompositions of quasi-reflexive Banach spaces. Preliminary report.

Let (R_n) be a countably infinite Schauder decomposition of a Banach space X. $[R_n]$ is the norm closed linear span of the R_n's. $B(X) = \{(r_n) : r_n \in R_n, \sup_n \|\Sigma r_n\| < \infty, \text{ for all } n\}$. Definitions. (1) (R_n) is k-shrinking iff $\dim(X^*/[R_n^*]) = k$. (2) (R_n) is k-boundedly complete iff (i) every $(k + 1)$ dim. l.s. of $B(X)$ has an element with convergent partial sums, and (ii) $\exists$ a $(k + 1)$ dim. l.s. of $B(X)$ for which the above element is unique up to a homothety. Note that (1) and (2) are extensions for bases as defined by I. Singer, and for Schauder decompositions as defined by Ruckle and Cook. (3) (R_n) is dual** k-boundedly complete iff $\dim(X^{**}/([R_n^{**}] \oplus [R_n^*]^\perp)) = k$. (4) $\mathrm{Ord}(X) = \dim(X^{**}/J(X))$ where $J(X)$ is the canonical embedding of X into X^{**}. Results. (1) $\Sigma\, \mathrm{Ord}(R_n) = \dim([R_n^{**}]/J(X))$. (2) If $\Sigma\, \mathrm{Ord}(R_n)$ is finite then Definitions (2) and (3) are equivalent. (3) $\mathrm{Ord}(X) = n$ iff $n = r + s + t$ where $r = \Sigma\, \mathrm{Ord}(R_n)$, and (R_n) is s-shrinking and t-boundedly complete. (4) Quasi-reflexive spaces are constructed with $n \geqq 0$ and $s = t = 0$. (5) A non-quasi-reflexive space is constructed with $s = t = 0$ which is also 0-dual** boundedly complete. (Received April 13, 1973.) (Author introduced by Professor Peter G. Jessup.)

*73T-B181. SERGE GAUTIER and PAUL PENOT, Faculte des Sciences Exactes, University of Pau, 64000 Pau, France. Contingent differential equations in Banach spaces.

Let F be a Banach space and E its dual endowed with the bounded weak topology σ_b^*. Let Ω be an open subset of $\mathbb{R} \times E$ and f a (multivalued) mapping from Ω into the set of nonempty closed bounded convex subsets of E. A solution of the differential equation (1) $x'(t) \in f(t, x(t))$, $x(t_0) = x_0$ is an absolutely continuous function $u: [t_0, t_0 + a] \to E$ such that $u(t_0) = x_0$, $(t, u(t)) \in \Omega$, $Du(t) \subset f(t, u(t))$ for every $t \in [t_0, t_0 + a]$ where $Du(t)$ denotes the set of σ_b^*-limit points of the ratio $h^{-1}[u(t+h) - u(t)]$ as $h \to 0$. Theorem. If f is upper semicontinuous, for every $(t_0, x_0) \in \Omega$ there exists a solution of (1). Corollary. If f is a one-point valued continuous mapping from Ω to E, for every $(t_0, x_0) \in \Omega$ there exists a solution u of the differential equation $x'(t) = f(t, x(t))$. Moreover u is everywhere σ_b^*-differentiable. Corollary. The same result holds if f is $\sigma(E, F)$-continuous. These statements generalize results of S.-N. Chow and J.D. Schuur [Abstract 691-34-41, these *Notices* 19(1972), A-137, and Bull. Amer. Math. Soc. 77(1971), 1018-1020]. (Received April 13, 1973.)

*73T-B182. ANDRE de KORVIN and CHARLES ROBERTS, Indiana State University, Terre Haute, Indiana 47809. A reduction theorem for the representation of averaging operators. Preliminary report.

For notations see [Abstract 73T-B127, these *Notices* 20(1973), A-330]. Theorem. Assume $L_\rho = M_\rho$ and $\rho[X_S] < \infty$. There exists an extremally disconnected compact space $\hat{S}$ such that $L_\rho(S, \Sigma, \mu)$ is isomorphic isometric with $L_\rho(\hat{S}, \hat{\mathcal{B}}, \hat{r})$ where $\hat{\mathcal{B}}$ is the field of clopen sets of $\hat{S}$ and $\hat{r}$ is a regular Borel measure on $\hat{\mathcal{B}}$. If $f \to \hat{f}$ denotes this isomorphism and if U is averaging for φ, for every $t \in T$ there exists a regular Borel measure H_t such that $\int f\, dG_t = \int \hat{f}\, dH_t$ where $G_t \in \mathfrak{A}_{\rho'}[\mu]$ with the property that $U(f)(t) = \int f\, dG_t$. Theorem. If U is averaging for φ and (1) $M_\rho = L_\rho$, (2) $\rho(X_S) < \infty$, (3) t is plural (essentially this means that $\hat{\varphi}^{-1}(t)$ is not an atom for H_t), (4) φ is irreducible (if $\mu(A) > 0$, for some t, $\varphi^{-1}(t) \subset A$ with $\varphi^{-1}(t) \neq \varphi$) then $|G_t|(S) < \rho(X_S)\|U\| - 1$. Under the conditions of the above theorem an inequality involving the norms of an averaging operator U of φ and U_1 of φ_1, where φ_1 is a restriction of φ, is obtained. (Received April 16, 1973.)

*73T-B183. RICHARD A. ALO, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213 and CHARLES ROBERTS and ANDRE de KORVIN, Indiana State University, Terre Haute, Indiana 47809. Relations between averaging operators for φ and its restriction in a Banach function space setting. Preliminary report.

For notations see [Abstract 73T-B127, these *Notices* 20(1973), A-330]. Let J be a Borel set of T, let $J_1 = \varphi^{-1}(J)$. Let Σ_1 be the restriction of Σ to J_1 and μ_1 the restriction of μ to Σ_1. If f denotes a function defined on J_1, $\overline{f} = f$ on J_1 and $\overline{f} = 0$ on $S - J_1$. Let $\rho_1(f) = \rho(\overline{f})$. Let $\varphi_1: J_1 \to J$ be the restriction of φ. Let U be a bounded linear operator from $L_\rho(S, \Sigma, \mu)$ into $C(T)$; U satisfies (1) if for $g \in C(J)$, $U[\varphi_1^0(g)]$ is an extension of g to a function in $C(T)$; φ satisfies (2) if for every $f \in L_\rho(S, \Sigma, \mu)$ with the property that, restricted to J_1, f is of the form $\varphi_1^0(g)$ for $g \in C(J)$ one can find $g' \in C(T)$ such that $f = \varphi^0(g')$. Theorem. Assume that the range of φ^0 is a subset of M_ρ and that φ admits an averaging operator U satisfying (1); then φ_1 admits an averaging operator U_1: $L_{\rho_1}(J_1, \Sigma_1, \mu_1) \to C(J_1)$. Conversely if φ_1 has an averaging operator U_1, if φ^0 is 1:1 and if φ satisfies (2) then φ admits an averaging operator. If $P_\varphi \subset J$, (2) may be replaced by (2'). Let $g \in C(J)$, $f \in L_\rho(S, \Sigma, \mu)$ with the property that f restricted to J_1 is $\varphi_1^0(g)$. Define g' on T by $g' = g$ on J and $g'(t) = f\varphi^{-1}(t)$ off J. Then one must have $g' \in C(T)$. (Received April 16, 1973.)

*73T-B184. K. L. SINGH, 267 Havelock Street, Toronto 173, Ontario, Canada and Rutgers University, New Brunswick, New Jersey 08903. Common fixed points for semi-quasi-nonexpansive maps.

Theorem 1. Let X be a uniformly convex Banach space and C a closed, bounded, convex subset of X. Let T_1 and T_2 be continuous mappings of C into itself which are semi-quasi-nonexpansive, i.e. $\|T_1x - T_2y\| \leq$

$1/3[\|x - T_1x\| + \|x - y\| + \|y - T_2y\|]$, for all x,y in C. Then there exists a common fixed point of T_1 and T_2. (Received April 16, 1973.) (Author introduced by Professor S. P. Singh.)

*73T-B185. PHILIP J. BOLAND and SEAN DINEEN, University College, Belfield, Dublin 4, Ireland. Convolution operators on G-holomorphic functions in infinite dimensions. Preliminary report.

For a complex vector space E, let $H_G(E)$ denote the space of G- (Gateaux-) holomorphic functions on E. ($f: E \to C$ is G-holomorphic if the restriction of f to every finite dimensional subspace of E is holomorphic in the usual sense.) The most natural topology on $H_G(E)$ is that of uniform convergence on finite dimensional compact subsets of E. A convolution operator A on $H_G(E)$ is a continuous linear mapping $A: H_G(E) \to H_G(E)$ such that A commutes with translations. The concept of a convolution operator generalizes that of a differential operator with constant coefficients. Theorem 1. If A is a convolution operator on $H_G(E)$, then the kernel of A is the closed linear span of the exponential polynomials contained in the kernel. Theorem 2. Any nonzero convolution operator on $H_G(E)$ is a surjective mapping. It is known that for $E = \sum_N C$ endowed with the locally convex inductive limit topology, the holomorphic functions $(H(\sum_N C))$ and the G-holomorphic functions coincide. Hence Theorems 1 and 2 also apply to $H(\sum_N C)$. (Received April 16, 1973.)

73T-B186. MEHDI RADJABALIPOUR, University of Toronto, Toronto 181, Canada. On decomposable operators. Preliminary report.

Let T be a bounded linear operator in a Banach space X whose spectrum lies on a rectifiable Jordan curve J. Theorem. Assume for any closed subarc F of J the manifold $X_T(F)$ is closed and $\sigma(\hat{T}) \subseteq \overline{J - F}$, where $\hat{T}$ is the operator induced on the quotient $X/X_T(F)$ by T. Then T is strongly decomposable. (For terminology see Colojoara and Foias, "Theory of generalized spectral operators".) Corollary. Assume J is a smooth Jordan curve with no singular point. Let $(z - T)^{-1}$, the resolvent of T, satisfy the growth condition $\|(z - T)^{-1}\| \leqq \exp\{\exp([\mathrm{dist}(z,J)]^{-p})\}$ for $z \notin J$ and some $p \in (0,1)$. Then T is strongly decomposable. In this corollary if we decrease the size of the majorant of the resolvent of T we can relax more restrictions on J. (Received April 17, 1973).

*73T-B187. IH-CHING HSU and ROBERT G. KULLER, Northern Illinois University, DeKalb, Illinois 60115. Convexity of vector-valued functions.

Let $(\mathcal{B}, <<)$ be a Banach lattice. A vector-valued function $F: (a,b) \to \mathcal{B}$ is defined to be weakly convex if there exists a positive continuous and nondecreasing function $G: (a,b) \to \mathcal{B}$ such that $p[F(s)] + tp[G(s)] \leqq p[F(s+t)]$, whenever s and s + t are in (a,b), for each positive linear functional p on $\mathcal{B}$. The following is proved: Theorem. If F is weakly convex on (a,b) and is bounded on an interval contained in (a,b), then $(B)\int_{a+\epsilon}^{x} G(s)\,dm = F(x) - F(a+\epsilon)$, where $(B)\int_{a+\epsilon}^{x} G(s)\,dm$ is the Bochner integral of G on $[a+\epsilon, x]$ with $0 < \epsilon$ and $a + \epsilon < x < b$. A function $F: (a,b) \to \mathcal{B}$ is called strongly convex if there exists a positive continuous and nondecreasing function $G: (a,b) \to \mathcal{B}$ such that $F(s) + tG(s) << F(s+t)$ whenever s and s + t are in (a,b). An open question is raised concerning the relation between weak and strong convexity. (Received April 17, 1973.)

73T-B188. SWARUPCHAND M. SHAH, University of Kentucky, Lexington, Kentucky 40506 and SELDEN Y. TRIMBLE, University of Missouri, Rolla, Missouri 65401. The order of an entire function with some derivatives univalent.

Let $\{n_p\}_{p=1}^{\infty}$ be a strictly increasing sequence of nonnegative integers. Theorem 1. Let f be an entire function of order Λ and suppose that each $f^{(n_p)}$ is univalent in the unit disk D. If $\log n_{p+1} \sim \log n_p$ then $\Lambda \leqq \Lambda^* \equiv 1/\{1 - \lim\sup_{p\to\infty} \log(n_p - n_{p-1})/\log n_p\}$. Corollary 1. If $n_p - n_{p-1} \leqq \mu$ for all large p, then f is of exponential type no bigger than $(.384)(\mu+1)^{7/2}$. Corollary 2. Let f be regular in D. Suppose φ is slowly oscillating and $1 \leqq \varphi(p) \leqq$

$n_p - n_{p-1} \leqq \theta(p)$ $(p > 1)$. Let $\xi = 1/\{1 - \lim\sup_{p\to\infty} \log\theta(p)/\log p\varphi(p)\}$. If either (a) $\theta(p) = o(p\varphi(p))$ or (b) $0 \leqq \xi < \infty$ then f is entire. In either case $\Lambda \leqq \Lambda^* \leqq \xi$. Theorem 2. Let $1 < \Lambda < \infty$. Then there exists a sequence $\{n_p\}_{p=1}^{\infty}$ and an entire function f such that (a) $n_{p+1} \sim n_p$, (b) $1 - 1/\Lambda = \lim\sup_{p\to\infty} \log(n_p - n_{p-1})/\log n_p$, (c) $f^{(n)}$ is univalent in D if and only if $n = n_p$ for some p and (d) f is of order Λ. Theorem 3. Let f be regular in D. If $\Lambda^* < \infty$, then f is an entire function of order no greater than Λ. If $\Lambda^* = \infty$, then f need not be entire, and if it is entire it may be of any order. (Received April 16, 1973.)

73T-B189. MORRIS MARDEN, University of Wisconsin, Milwaukee, Wisconsin 53201. Harmonic interpolation polynomials in R^3.

Corresponding to a given function $F(x,\rho)$ which is axisymmetric harmonic in an axisymmetric region $\Omega \subset R^3$ and to a set of $n+1$ circles C_n in an axisymmetric subregion $A \subset \Omega$, an axisymmetric harmonic polynomial $\Lambda_n(x, \rho; C_n)$ is found which on the C_n interpolates to $F(x, \rho)$ or to its partial derivatives with respect to x. An axisymmetric subregion $B \subset \Omega$ is found such that $\Lambda_n(x, \rho; C_n)$ converges uniformly to $F(x, \rho)$ on the closure of B. Also a $\Lambda_n(x, \rho; x_0, \rho_0)$ is determined, which, together with its first n partial derivatives with respect to x, coincides with $F(x, \rho)$ on a single circle (x_0, ρ_0) in Ω and converges uniformly to $F(x, \rho)$, is a torus with (x_0, ρ_0) as central circle. In derivation of these results, use is made of the Whitaker-Bergman relation $F(x, \rho) = (1/2\pi)\int_0^{\pi} f(x + i\rho\cos t)\,dt$ in which $f(\sigma)$ is holomorphic in the meridian section of Ω (complex plane). (Received April 23, 1973.)

*73T-B190. CARL L. DeVITO, KWANG-SHANG WANG and EDWARD C. WAYMIRE, University of Arizona, Tucson, Arizona 85721. On compact mappings between L_1-modules and almost periodic functions.

Let R denote the real numbers and for each p, $1 \leqq p \leqq \infty$, let L_p denote $L_p(R)$. For f in L_1 and h in L_p let $f * h(x) = \int f(x-y)h(y)\,dy$, where the integral is taken over R. Linear maps T from L_1 into L_p such that $T(f*g) = f * T(g)$ for all f, g in L_1 have proved useful in studying the ideals in L_1. We say that such a map is L_1-linear. Theorem 1. A linear map T from L_1 into L_∞ is L_1-linear and compact iff there is a (continuous) almost periodic function φ on R such that $Tf = f * \varphi$ for all f in L_1. Corollary. Any compact, L_1-linear map from L_1 into L_∞ is the limit, for the operator norm, of a sequence of L_1-linear maps each having finite dimensional range. We say h in L_p, $p < \infty$, is a Bohr function if the map which takes t in R to the translate of h by t is an almost periodic function on R with values in L_p. Theorem 2. A linear map T from L_1 into L_p is L_1-linear and compact iff there is a Bohr function h such that $Tf = f * h$ for all f in L_1. Corollary. If T is a compact, L_1-linear map from L_1 into itself, then there is a closed ideal which is invariant under T. (Received April 25, 1973.)

73T-B191. FRANK N. HUGGINS, University of Texas, Arlington, Texas 76010. Bounded slope variation and a uniform Lipschitz condition. Preliminary report.

A study is made of the relationship between the concept of bounded slope variation and that of a uniform Lipschitz condition of order p. Definition. A function f is said to satisfy a uniform Lipschitz condition of order $p > 0$ with respect to an increasing function m on [a,b] if there exists a nonnegative number K such that if x_1 and x_2 are numbers in [a,b], then $|f(x_1) - f(x_2)| \leqq K \cdot |m(x_1) - m(x_2)|^p$. (For a definition of bounded slope variation, see Pacific J. Math. 39(1971), 695.) Some representative theorems are: Theorem 1. If f satisfies a uniform Lipschitz condition of order $p \geqq 2$ with respect to m on [a,b], then f has bounded slope variation with respect to m over [a,b], but not conversely. Theorem 2. If f has bounded slope variation with respect to m over [a,b], then f satisfies a uniform Lipschitz condition of order 1 with respect to m on [a,b], but not conversely. (Received April 26, 1973.)

*73T-B192. CHAMAN L. SABHARWAL, St. Louis University, St. Louis, Missouri 63103 and R. L. WILKE, Forest Park Community College, St. Louis, Missouri 63128. S-distribution theory. II. Preliminary report.

For terminology we refer to Abstract 691-46-28, these *Notices* 19(1972), A-166. An irregularity order, I_f, of f in B' is defined. Theorem 1. For f & g in B', (i) $I_f(n) = I_f$ for all positive integers, (ii) $I_{f+g} < \max(I_f, I_g)$, (iii) $I_f > I_g$ implies $I_{f+g} = I_f$, (iv) if S(f) & S(g) are bounded on the left then (a) $I_{fg} \leqq I_f + I_g$, (b) if g is nonzero regular with $|g| \leqq M$ then $I_{fg} \leqq I_f \leqq I_{fg} + \ln(1+M)$. Various examples concerning equality and inequality are given. S-distributions of slow growth are defined. Theorem 2 (boundedness property). Let $f \in S'$ & $b \in S$; then there exists a constant C_f such that $|(f,b)| \leqq C_f \max_x (1+|x|^2)|b(x)|$. (Received April 26, 1973.)

Applied Mathematics

*73T-C31. LUDVIK JANOS, University of Newcastle, New South Wales 2308, Australia. Error estimate characterization of contractions. Preliminary report.

Let (X, ρ) be a metric space and $f: X \to X$ a continuous mapping of X into itself. Assuming that f has a unique fixed point $a \in X$ and that for each $x \in X$ the iteration process $f^n(x) \to a$ converges to it, our interest is drawn to estimation of the error $\rho(f^n(x), a)$. If there exists a numerical sequence $c_n \to 0$ converging to zero such that for each $x \in X$ holds $\rho(f^n(x), a) \leqq c_n$ $(n = 1, 2, \ldots)$ we say that the iteration process has a uniform error estimate. It turns out that in the compact case this property characterizes topological contractions in the following sense: Theorem. If (X, ρ) is compact then the above iteration process $f^n(x) \to a$ has a uniform error estimate iff there exists a metric ρ^* on X topologically equivalent to ρ such that f is a contraction with respect to ρ^*. (Received February 14, 1973.)

*73T-C32. NEIL E. BERGER, University of Illinois at Chicago Circle, Chicago, Illinois 60680. Estimates for the derivatives of the velocity and pressure in shallow water flow and approximate shallow water equations.

Estimates are derived on the time and space derivatives of the velocity and pressure of a shallow fluid flow. These estimates depend upon the velocity, pressure, and derivatives of the free surface, being bounded and the free surface having a long wave length compared to the depth of the fluid. The technique used is to derive L_2 estimates on the derivatives of the velocity and pressure and then to convert these to pointwise estimates. As a consequence of these results, the horizontal velocity is shown to be independent of the depth and the pressure hydrostatic to the first approximation. Higher order estimates lead to second order approximate equations which under additional physically motivated assumptions correspond to the Boussinesq and Korteweg-de Vries equations. (Received February 26, 1973.)

*73T-C33. MEEMPAT GOPINATH, Farook College, Calicut University, Calicut, Kerala, India and LOKENATH DEBNATH, East Carolina University, Greenville, North Carolina 27834. On the growth of unsteady boundary layers on porous flat plates.

An investigation is carried out on the unsteady boundary layer induced in an incompressible, homogeneous, viscous fluid bounded by (i) an infinite horizontal porous plate at $y = 0$ or (ii) two parallel horizontal rigid porous plates at $y = 0$ and $y = d$. The unsteady motion is generated in the above fluid configurations by moving the plate(s) impulsively in its (their) own plane with a prescribed time dependent velocity. Solutions for the unsteady velocity field are exactly solved by the Laplace transform treatment combined with the theory of residues. The structures of the associated boundary layers are determined. The effects of suction on the solutions and the boundary layers are investigated in detail. The limiting behaviour of the unsteady solution at time $t \to \infty$ is examined. Physical interpretations of the mathematical results are given. Finally, the frictional stresses on the plates are stated without detailed calculation. (Received March 12, 1973.)

*73T-C34. ANTONIO RESTIVO, Laboratorio di Cibernetica del C. N. R., Arco Felice, Napoli, Italia. On a question of McNaughton and Papert. Preliminary report.

McNaughton and Papert in their book "Counter-free automata" (M.I.T. Press, Cambridge, 1971) asked the question to find conditions on a code A in order that the submonoid A^* generated by A be "locally-testable" or "noncounting". The answer to this question and a characterization of these codes have been given in this paper by using algebraic tools developed by Schützenberger. Definitions. A prefix code is a subset A of a free monoid X^* such that $A \cap AXX^* = \emptyset$. An element f in a monoid is primitive iff any relation $f = g^p$ implies $f = g$. Every word f of a free monoid is in a unique matter a power of a primitive element which we designate by $\sqrt{f}$. A prefix code A is pure iff $f \in A^*$ implies $\sqrt{f} \in A^*$, and is very pure iff $\forall h, h' \in X^*$, $hh', h'h \in A^*$ implies $h, h' \in A^*$. Theorem 1. Let A be a finite prefix code. A^* is "noncounting" if and only if A is pure. Theorem 2. Let A be a finite prefix code. The following three conditions are equivalent: (a) A is very pure. (b) A has a finite synchronization delay. (c) A^* is "locally testable". It is to be pointed out the essential role played by the finiteness of A. The previous theorems do not hold without it. (Received March 19, 1973.) (Author introduced by Professor Aldo De Luca.)

*73T-C35. BISWANATH DATTA, University of Ottawa, Ottawa, Ontario, Canada. Matrix equation and relative primeness of two polynomials.

Let $a(\lambda)$ and $b(\lambda)$ be two monic polynomials of degree n. Then it has been shown by Barnett ["Linear algebra," vol. 3, 1970, pp. 7–9] and independently by Vogt and Bose [IEEE Trans. Automatic Control, AC-15(1970), 379-386] that these two polynomials are relatively prime if and only if a(B) is nonsingular, where B is the companion matrix of $a(\lambda)$. An alternative proof of this via matrix equation is presented in this note, thus establishing a "link between the matrix equation and companion matrix approaches for polynomials" for the problem of determining if two polynomials are relatively prime. Similar connections between these two approaches that currently exist in the literature for solving the classical Routh-Hurwitz and Schur-Cohn problems have been recently established by the author (Ph.D. Thesis, University of Ottawa, Ottawa, 1972). (Received March 28, 1973.) (Author introduced by Professor Remi Vaillancourt.)

*73T-C36. LI KUNG SHAW, P.O. Box 59 (Suc. 24), Buenos Aires, Argentina. A mathematical model of life and living. II.

The fundamental equation of life and its derived functions, discovered in part I, were more systematically presented and precisely formulated, on the broad basis of all higher animals, including man. The human mortality curve was refined by taking into account the intelligence factor. The Gompertz function was shown as a logical consequence of the present model. A general theory of mortality was formulated on the basis of the evolutional, ecological and accidental forces on the life functions. Analyzing the exchange times among mother, father and foetus/newborn during fertilization and reproduction, the physiology of human copulation, gestation, lactation, menses and menopauses were correctly related to the natural life span, in close conformity with observed data. The general model was applied to 27 primates and 8 other higher mammals of single birth. It shows the gestation period is proportional to the natural life span of each species. A geriatric and a precocial model were presented, to illustrate the flexibility of the general model and its possible applications to practical problems of geriatrics and sociology. (Received April 9, 1973.)

*73T-C37. R. G. JEROSLOW, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213. Problems unsolvable by branch-and-bound.

For each n, consider the zero-one integer program P(n), consisting of the maximization of $(-x_{n+1})$, subject to the single linear constraint $2x_1 + \ldots + 2x_n = n - x_{n+1}$. Theorem. For n odd, any branch-and-bound scheme, using any heuristics whatever, will have to solve at least $\exp(\langle n/2 \rangle)$ linear programs before determining the maximum in P(n). (Exp(x) denotes two to the power x.) Remark. Evidently problems of the type P(n) occur with very low probability in practice, because branch-and-bound successfully solves most problems in up to 150 variables. However, given present computer capabilities, P(149) requires at least 400 million years for its solution by branch-and-bound. (Received April 9, 1973.)

*73T-C38. EDGAR RECHTSCHAFFEN, Departamente de Engenharia Eletrica, Pontificia Universidade Catolica, Rio de Janeiro ZC 20, Brasil. Another equivalence between differential games and optimal control. Preliminary report.

We treat the differential game $\dot{x}(t) = Ax(t) - p(t) + q(t)$, $p(\cdot) \in P \subset R^n$, $q(\cdot) \in Q \subset R^n$. Data: initial position x(0); A real n-square matrix; P a convex body: Q compact and $0 \in Q$. Constraints: (a) the mappings $s \to p(s)$ and $s \to q(s)$ are piecewise constant (p and q are the pursuer and quarry controls); (b) the pursuer only has instantaneous information of quarry control; (c) the data are known to pursuer and quarry. The objective is: Given the above data and a subspace $L \subset R^n$ find a $T \geqq 0$ such that there exists a pursuer strategy $f: R^n \times R^1 \to R^n$ (if q is piecewise constant on [0, T] then so is $s \to f(q(s), s)$) such that for all admissible $q: [0, T] \to Q$, the choice $p(s) = f(q(s), s)$, $s \in [0, T]$, yields $x(T) \in L$. In case $T < \infty$ we say that the game can be completed. With our game we associate the system $\dot{x}(t) = Ax(t) - u(t)$, $u(\cdot) \in U$; $s \to u(s)$ is piecewise constant and $U = P \cap [(P + L_A) \underline{*} Q]$, where L_A is the largest subspace of L invariant under A (and $z \in V \underline{*} W$ if, and only if, $z + W \subset V$). This linear system is called the dual of the game. Theorem. Given x(0), the game can be completed in time $T \geqq 0$ if and only if in the dual system x(0) can be steered to L in time T. (Received April 9, 1973.) (Author introduced by Professor Otomar Hajek.)

*73T-C39. OTOMAR HAJEK, Case Western Reserve University, Cleveland, Ohio 44106. Quadratic targets in pursuit games. Preliminary report.

In the game $\dot{x} = Ax - p + q$ in n-space, with nonvoid, compact and convex constraint sets P, Q in R^n for the player control values, let forcing $x_0 \in R^n$ to target $T \subset R^n$ in time $t \geqq 0$ mean: for every quarry control $q: [0, t] \to Q$ there is a pursuer control $p: [0, t] \to P$, both measurable, and each p(s) depends only on s and q(s), such that $x(t) \in T$ for the solution $x(\cdot)$ of the equation with so chosen $p(\cdot), q(\cdot)$, and $x(0) = x_0$. Given square symmetric matrices M_k, let T_2 consist of all points x with $x'M_k x = 0$ for each k. Theorem. Forcing x_0 to the quadratic target T_2 in time t is equivalent to forcing x_0 in time t to a linear target T_1; furthermore, T_1 is one of the linear subspaces which satisfy $x'M_k y = 0$ for all x, y in T_1, all k. Problem. Which T_1? (Received April 9, 1973.)

*73T-C40. JOHN de PILLIS, University of California, Riverside, California 92502. On a result of Strassen. Preliminary report.

Let A and B be $m \times q$ and $q \times n$ matrices over field F. Let P(m, q, n) be the number of scalar products required, whose linear combinations over F, produce the mn entries of the product matrix AB. (The usual definition has P(m, q, n) = mqn.) It was Strassen who originally showed [Numer. Math. 13(1969), 354] that a scheme exists such that P(2, 2, 2) = 7. Gastinel [Numer. Math. 17(1971), 222] showed a scheme whereby P(3, 3, 3) = 25. Theorem 1. Schemes exist, using P(m, q, n) scalar products whose linear combinations (using only 0, 1, -1 if desired) generate the mn entries of AB, where P(3, 2, 3) = 15, P(3, 3, 3) = 24, P(4, 2, 4) = 26, P(8, 2, 8) = 103. Since our schemes do not use commutativity of field F, we have, relative to our schemes, Theorem 2. For all k = 1, 2, ..., $P(m^k, q^k, n^k) = [P(m, q, n)]^k$. (Received April 16, 1973.)

Geometry

73T-D11. JAMES J. TATTERSALL, Providence College, Providence, Rhode Island 02918. The Helly order for the family of 3-convex sets. II.

It has been called to my attention by Professor Branko Grunbaum that I actually show that the family of closed, connected, 3-convex sets in E^2 has infinite Helly number (Abstract 697-D2, these *Notices* 19(1972), A-731). The distinction between Helly order and Helly number appears in Proc. Sympos. Pure Math. 7(1963), 124. (Received March 13, 1973.)

73T-D12. ERIC MENDELSOHN, University of Toronto, Toronto, Ontario M5S 1A1, Canada. On subgroups of the collineation group of a projective plane obtainable as collineation groups of affine planes.

Theorem. Let G be a group; then there exists a projective plane P, such that the collineation group of P is G, and if $N \lhd G$ then there is a line $\ell \in P$ such that the collineation group of the affine plane $\ell = \infty$ is N. (Received March 19, 1973.)

*73T-D13. RICHARD S. MILLMAN, Southern Illinois University, Carbondale, Illinois 62901. Integrable bi-invariant f-structures on Lie groups. Preliminary report.

An f-structure on a manifold M^n is a nonzero tensor, f, of type (1,1) with constant rank r such that $f^3(X) + f(X) = 0$ for all tangent vectors X of M. An f-structure on a Lie group is bi-invariant if $dL_g f(X) = f dL_g(X)$ and $dR_g f(X) = f dR_g(X)$ for all tangent vectors X of G. (Here, R_g is right multiplication by $g \in G$ and L_g is left multiplication by $g \in G$.) Theorem. If G^n is a simply connected Lie group with integrable bi-invariant f-structure then there is a complex manifold G_1 of real dimension r and a real manifold G_2 of dimension $n - r$ such that G is isomorphic (as a Lie group) to $G_1 \times G_2$. Furthermore, if J is the complex structure of G_1 then $f = J \times 0$. (Received April 9, 1973.)

73T-D14. LUCIO L. RODRIGUEZ, Universidade Federal de Pernambuco, Recife, Brazil. The two-piece property.

For an immersion $f: M \to R^m$ of a manifold, with or without boundary, Banchoff introduced the concept of the two-piece property (T.P.P.): We say that $f: M \to R^m$ has the T.P.P. if for every hyperplace $H \subset R^m$, $f^{-1}(R^m - H)$ has at most two components. Theorem 1. If M is topologically a 2-dimensional sphere with a finite number of disjoint discs removed, then $f|\partial M$ consists of plane convex curves, and f is an embedding into the boundary of the convex hull of f(M) contained in some three-dimensional affine subspace of R^m. We give an example of a torus with a disc removed that has the T.P.P., but whose boundary curve is not convex. However, using a condition studied by Nirenberg, we have Theorem 2. If $M \subset N$ where N is a closed orientable surface in R^m, both having the T.P.P., and $\text{grad}(K) \neq 0$ on the boundary of the region of negative Gaussian curvature k, then ∂M consists of convex curves in the boundary of the convex hull of M. (Received April 10, 1973.)

*73T-D15. ARTHUR A. SAGLE and JOSEPH R. SCHUMI, University of Minnesota, Minneapolis, Minnesota 55455. Multiplications on homogeneous spaces, nonassociative algebras and connections.

In this paper we show how nonassociative algebras over the real numbers arise from multiplications on reductive homogeneous spaces, that is, an analytic function $\mu: M \times M \to M$. Then these algebras are used to obtain an invariant connection ∇ on the homogeneous space and we give some applications of nonassociative algebras to these topics. Conversely every finite dimensional nonassociative algebra over the real numbers arises from an invariant connection and a local multiplication on a homogeneous space. Thus, analogous to the theory of Lie groups and Lie algebras, much of the basic theory of nonassociative algebras can be formulated in terms of multiplications and connections and conversely. (Received April 23, 1973.)

73T-D16. S. M. KARNIK, University of Alberta, Edmonton, Alberta, Canada. Abstract geometries. Preliminary report.

We define a pair $(X,\mathcal{S})$ to be an 'abstract geometry' if X is a nonempty set and $\mathcal{S}$ is a collection of subsets of X (ordered by set-inclusion) satisfying: (i) For every two points a, b of X, the subcollection $\mathcal{S}_{ab}$ consisting of all those members of $\mathcal{S}$ which contain both a and b has minimal members. (These minimal members may be called 'joins' or 'segments' and are denoted by M_{ab}, M'_{ab}, etc.) (ii) If a, b, c are points of X such that $b \in M_{ac}$ for some M_{ac}, then there exist M_{ab} and M_{bc} such that $M_{ab} \cup M_{bc} = M_{ac}$. (iii) $\varphi \in \mathcal{S}$, $X \in \mathcal{S}$. The usual Euclidean geometry, partially ordered sets, vector spaces over partially ordered fields, topological continua are some of the nontrivial examples of abstract geometries. We have a method for 'segmentizing' the metric spaces in general as well as a necessary and sufficient condition for linearly ordering the straight lines in abstract geometries in a 'compatible' way. A characterization of the real line up to a suitable mapping is obtained. We have also characterized the Euclidean geometry. A link with the graph theory is forged. Moreover, some results about a natural convexity are obtained. (Received April 23, 1973.)

Logic and Foundations

*73T-E42. R. STEVE NEWBERRY, 1415 Bellevue Avenue, Burlingame, California 94010. Finite quantification theory of second-order is stronger than you think.

Let $\mathcal{P}$ denote pure second-order logic (Henkin 1950, 1953) and $\mathcal{B}$ denote what is left when quantifiers are restricted to finite ground-domains. By using .Ax[Sx ≠ 0]. as a local condition instead of an axiom, I can simulate in $\mathcal{B}$ the zero, unit and successor functions definable in $\mathcal{P}$. The simulation is good enough to get Gödel-sentences in $\mathcal{B}$, which proves that the intended-interpretation of Gödel-sentences is only sound on infinite ground-domains. Moral. For automated-logic people willing to specialize in problem-classes defined on discrete, finite grounds, $\mathcal{B}$ is as good as $\mathcal{P}$ and is equivalent to truth-functional logic, and hence decidable. (Received February 15, 1973.)

*73T-E43. M. MAKKAI, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada. Generalized Vaught sentences.

We have gone through the fairly straightforward work of generalizing part of R. Vaught's theory ["Descriptive set theory in $L_{\omega_1\omega}$", Proc. 1971 Summer School in Cambridge, U.K., to appear] for $L_{K\omega}$ and L_{KK} for strong limit K with cofinality ω. The theory becomes interesting (i) from a technical point of view because it involves what is essentially Carol Karp's notion of truth in an ascending chain of structures ["Infinite-quantifier languages and ω-chains of models", Tarski Sympos., to appear] and (ii) because it gives results sharper than existing ones for ordinary model theory as follows. Theorem (covering). For every $\varphi \in L_{K\omega}(\vec{R})$ there are $\delta_\alpha \in L_{KK}$ $(\alpha < K^+)$ such that for any $\psi \in L_{K\omega}(\vec{S})$, $\vec{S}$ not in $L(\vec{R})$, if $\varphi \models \psi$, then for some δ_α, $\varphi \models \delta_\alpha \models \psi$. This strengthens a theorem of C. C. Chang [Symposia Math., vol. V (INDAM, Rome, 1969/70), Academic Press, London, 1971, pp. 5-19]. Corollary(!). Every structure of power $\leq K$ can be characterized among $\leq K$-power structures by a sentence $\bigwedge_{\alpha<K^+} \delta_\alpha$ with $\delta_\alpha \in L_{KK}$. This strengthens a result of C. C. Chang [Lecture Notes in Math., vol. 72, 1968, pp. 36-63]. Further results involve homomorphic images and positive sentences and the like. (Received February 16, 1973.)

73T-E44. CHARLES FONTAINE MARTIN, General Electric Space Division, 450 Persian Drive, Sunnyvale, California 94086. Doner-Tarski operations restricted to limit ordinals. Preliminary report.

In Doner-Tarski, Fund. Math. 65(1969), a recursively defined sequence of binary operations O_γ on Ω (the class of ordinals) is studied. O_0, O_1, and O_2 are essentially addition, multiplication, and exponentiation. Beyond O_2, the odd-indexed operations are essentially "dummies" used to construct the next even-indexed operation. The author's Abstract 702-E1, these *Notices* 20(1973), A-354, shows that the operations beyond O_ω exhibit similar behavior. The author found stronger similarities of the operations beyond O_3 when the operations are restricted to Λ, the class of infinite limit ordinals. For any $\delta \in \Omega$ and $f: \delta \to \Omega$ we consider the algebraic structure $\mathfrak{B}_f = \langle \Lambda, O_{2 \cdot f\gamma}, O_{2 \cdot f\gamma+1} \rangle_{\gamma \in \delta}$. Theorem 1. If g, h are strictly increasing functions such that $g0 = h0 = 0$ and $g1 = h1 = 1$, then the same universal sentences hold in $\mathfrak{B}_g$ and $\mathfrak{B}_h$. Theorem 2. Assume that, in addition to the hypotheses of Theorem 1, the following condition holds: for all $\gamma < \delta$, $g(\gamma + 1) = g\gamma + 1$ iff $h(\gamma + 1) = h\gamma + 1$. Then the same weak second order sentences hold in $\mathfrak{B}_g$ and $\mathfrak{B}_h$. The proofs use the Fraissé-Ehrenfeucht method. (Received February 22, 1973.)

*73T-E45. RUDOLF von BITTER RUCKER, State University College of New York, Geneseo, New York 14454. Truth and infinity. Preliminary report.

We are concerned here with the possibility, suggested in Dr. Gödel's 1946 talk at Princeton, that every sentence of set theory may be decidable from ZF plus some true axioms of infinity. Definition. σ is an R-sentence iff σ has the form $(Q_1 R_{\beta_1}) \ldots (Q_n R_{\beta_n}) \psi [R_{\beta_1}, \ldots, R_{\beta_n}]$, where each Q_i is an $\forall$ or an $\exists$, each R_{β_1} ranges over the class of partial universes, and ψ has no unbound quantifiers. We let R be the set of Gödel numbers of R-sentences. For any structure M, Th(M) is the theory of M (our metatheory is ZF or MK). V = OD is the ZF-sentence saying "every set is ordinally definable". We identify a theory with the set of Gödel numbers of sentences in the theory. Theorem. (i) If M is a model of ZF, then $\mathrm{Th}(M) \cap R \cup ZF$ is a complete theory. (ii) If M is a model of ZF + V = OD, then $\mathrm{Th}(M) \cap R$ is a complete theory. The first proof is somewhat more syntactic than the second. (Received February 15, 1973.)

*73T-E46. SAHARON SHELAH, Hebrew University of Jerusalem, Jerusalem, Israel. Differentially closed fields.

L. Blum proved that the theory of differential field of characteristic zero has a model completion T^o_{dc} which is totally transcendental (that is $\aleph_0$-stable). Hence, by results of Morley, over every differential field there is a prime one, which is unique by a result of the author (see Sacks, "Saturated model theory," Benjamin, 1972). L. Blum asked whether the prime models are minimal. Theorem 1. Even the prime model of T^o_{dc} is not minimal. Theorem 2. In every $\lambda > \aleph_0$ there are 2^λ nonisomorphic models of T^o_{dc} of power λ. C. Wood proved that the theory of radical differential fields of characteristic p (radical means that $\sqrt[p]{\ }$ is in the language) has a model completion T^p_{rdc}, and it is not $\aleph_0$-stable. Theorem 3. T^p_{rdc} is stable but not superstable. Theorem 4. Over every radical differential field there is a prime model (of T^p_{rdc}) among the $\aleph_0$-saturated ones. (Received March 2, 1973.)

73T-E47. FRED J. CLARE, University of Colorado, Boulder, Colorado 80302. Some results on elementary classes of groups. Preliminary report.

If K is a class of groups, let HK denote the class of all homomorphic images of members of K, and let PK denote the class of all direct products of members of K. For other notation and definitions see Bell and Slomson, "Models and ultraproducts." Theorem 1. If K is a class of divisible abelian groups, then $K \in EC_\Delta \Rightarrow HK \in EC_\Delta$. Theorem 2. If K is a class of abelian groups each having finite exponent, then $K \in EC_\Delta \Rightarrow HK \in EC_\Delta$. Theorem 3. There is a class of groups $K \in EC_\Delta$ such that $HK \notin EC_\Delta$. This solves a problem posed by A. Tarski which appears

as Problem 1.68 in the Kourovskaya Tetrad, 3rd edition, 1969. Theorem 4. There is a class of abelian groups $K \in EC_\Delta$ such that $PK \notin EC_\Delta$. Theorem 5. There is a class $K \in EC_\Delta$ such that K contains only simple groups.
(Received March 7, 1973.)

73T-E48. PAUL C. EKLOF, Stanford University, Stanford, California 94305. Another finitely axiomatizable decidable theory with no finitely axiomatizable complete extensions.

Ehrenfeucht (Abstract 73T-E21, these *Notices* 20(1973), A-284) has given an example of such a theory. Another example can be obtained from abelian group theory. Let p be a fixed prime and let T be the theory of abelian groups plus the sentences: (i) $\forall x(px = 0 \to x = 0)$; and (ii) $\exists x \forall y(py \neq x)$. T is decidable (Smzielew, Fund. Math. 41 (1955), 203-271). T has no finite models and therefore by a theorem of Smzielew (ibid., Theorem 5.4) T has no models whose theory is finitely axiomatizable. (Received March 8, 1973.)

73T-E49. STEVEN R. GIVANT, University of California, Berkeley, California 94720. A union decomposition theorem for unary algebras and an application to certain theories categorical in power.

For unary algebras $\mathfrak{A}$ introduce the notation $\mathfrak{A} = \sum_{\mathfrak{C}} \{\mathfrak{B} : \mathfrak{B} \in K\}$ iff $\mathfrak{A} = \bigcup K$, $\mathfrak{C} \subset \mathfrak{B}$ for $\mathfrak{B} \in K$, and $\mathfrak{B} \cap \mathfrak{B}' = \mathfrak{C}$ for distinct $\mathfrak{B}, \mathfrak{B}' \in K$. $\mathfrak{A}$ is $\mathfrak{C}$-irreducible if its only such decomposition is $K = \{\mathfrak{A}\}$. Theorem 1. $\mathfrak{A} \supset \mathfrak{C}$ implies $\mathfrak{A} = \sum_{\mathfrak{C}} \{\mathfrak{B} : \mathfrak{B} \in K\}$ for a unique $K \subseteq \{\mathfrak{B} : \mathfrak{C} \subset \mathfrak{B} \subseteq \mathfrak{A}$ and $\mathfrak{B}$ is $\mathfrak{C}$-irreducible$\}$. Theorem 1 and its generalizations are derivable from a theorem of Jónsson ("Ordinal algebras," Tarski Sympos., p. 112,120, to appear). $\mathfrak{A} = \sum_{\mathfrak{C}} \{\mathfrak{B} : \mathfrak{B} \in K\}$ is a multiple of $\mathfrak{B}$ over $\mathfrak{C}$ if $\mathfrak{B} \in K$ and $\langle \mathfrak{B}', c\rangle_{c \in C} \cong \langle \mathfrak{B}, c\rangle_{c \in C}$ for $\mathfrak{B}' \in K$. $\mathfrak{C}^{(\mathfrak{A})}$ is the subalgebra of $\mathfrak{A}$ whose universe is the union of the finite sets first-order definable in $\mathfrak{A}$. Theorem 2. Suppose M, a UC_Δ class of unary algebras with λ operations, is κ-categorical where $\kappa > \lambda + \omega$ or $\kappa = \lambda + \omega \leqq \omega_1$ and let $\mathfrak{A} \in M$ be infinite. Then there is a $\mathfrak{B} \supset \mathfrak{C}^{(\mathfrak{A})}$ such that the subalgebras of infinite members of M are, up to isomorphisms, exactly the multiples of $\mathfrak{B}$ over $\mathfrak{C}^{(\mathfrak{A})}$ together with the subalgebras of $\mathfrak{C}^{(\mathfrak{A})}$; any $x \in B \sim C^{(\mathfrak{A})}$ generates $\mathfrak{B}$ and proper subalgebras of $\mathfrak{C}^{(\mathfrak{A})}$ are finite. The conclusion holds also when $\kappa = \lambda > \omega_1$ assuming there is a saturated member of power κ. Theorem 2 improves the author's Theorem 3, Abstract 72T-E98, these *Notices* 19(1972), A-717. It solves, for this special case, several open problems concerning theories categorical in power, e.g. for M as in Theorem 2 it gives the number of infinite $\mathfrak{A} \in M$ of power $< \kappa$; if M is ω-categorical, it is ω_1-categorical. (Received March 14, 1973.)

*73T-E50. CAROL WOOD, Yale University, New Haven, Connecticut 06520. Prime model extensions of differential fields, characteristic $p \neq 0$.

The terminology is that of Sacks, "Saturated model theory," Benjamin, 1972. The author has shown [Wood, "The model theory of differential fields of characteristic $p \neq 0$," Proc. Amer. Math. Soc., to appear] that the theory DPF_p of differentially perfect fields of characteristic p has a model completion DCF_p, which is moreover not ω-stable. Theorem. For any $\mathfrak{F} \models DPF_p$, the isolated points form a dense subset of the Stone space of $\mathfrak{F}$. Corollary. Every differentially perfect field has a prime model extension. The proof of the theorem uses the concept of a set of constrained elements over a differential field (cf. Kolchin, "Differential algebra and algebraic groups," Academic Press, 1973). A correspondence exists between atomic types over a model of DPF_p and sets of constrained elements over the model. This procedure also provides new axioms for DCF_p. (Received March 14, 1973.)

*73T-E51. JOHN W. BERRY, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada. N-ary almost recursive functions.

In "Closure properties of regressive functions," Proc. London Math. Soc. (3) 15(1965), 226-238, Dekker generalized the notion of regressiveness to functions of n variables. We carry out a similar programme for almost recursive functions and verify that theorems analogous to Dekker's hold also in this case. $(N^n, \leq)$ is the direct product of n copies of the nonnegative integers N with the usual ordering. A subset A of N^n is initial if the set of predecessors of every element of A is included in A. A function $f: N^n \to A$ is almost recursive if (i) δf is initial, (ii) f is injective, and (iii) each of the n mappings $f(x_1, \ldots, x_n) \to x_i$ has a partial recursive extension. A set X is in ARE(n) (respectively AR(n)) if X is the range of an n-ary almost recursive (respectively strictly increasing almost recursive) function. We show that ARE(n) = ARE(1) but that $AR(n) \subsetneq AR(n+1)$. (Received March 22, 1973.)

73T-E52. DANIEL SARACINO, Yale University, New Haven, Connecticut 06520. On existentially complete solvable groups. Preliminary report.

In 1970, Eklof and Sabbagh proved that the theory T_1 of abelian groups has a model companion but that the theory T of groups does not. Shortly afterward, A. Macintyre strengthened the negative result for groups by showing that the class of existentially complete groups is distinct from the class of infinitely generic ones. Here we consider the corresponding questions for an increasing sequence of (universal) elementary classes of groups intermediate between the extremes given by T_1 and T. Namely, for any $n \geqq 1$ let T_n denote the theory of groups solvable of length n. (Thus T_1 gives abelian groups, T_2 gives metabelian groups, etc.) Theorem 1. For any $n \geqq 2$, T_n has no model companion, i.e. the class of existentially complete models of T_n is not elementary in the wider sense. For the case n = 2 we prove a stronger result: Theorem 2. There is an $\exists\forall\exists$ sentence which holds in every infinitely generic metabelian group and fails in every finitely generic one. Consequently the class of existentially complete metabelian groups is distinct from the class of infinitely generic ones, and the finite and infinite forcing companions of T_2 are different. Our principal tool in obtaining these results is the standard unrestricted wreath product. (Received March 26, 1973.) (Author introduced by Professor Abraham Robinson.)

*73T-E53. JEKERI OKEE, Makerere University, Kampala, P.O. Box 7062, Kampala, Uganda. Completeness of the algebra of species.

The algebra of species is complete in the sense that a formula $\mathfrak{h}$ of the algebra of species is provable in the algebra of species if and only if it is valid in every algebra of species of all subspecies of any infinite species. There is a one-to-one mapping φ of the formulae of the intuitionistic propositional calculus onto the formulae of the algebra of species such that a formula H of the propositional calculus is provable if and only if $\varphi(H)$ is valid in every algebra of species of all subspecies of any infinite species. (See Abstract 72T-E95, these *Notices* 19(1972), A-716.) φ maps the axioms of the propositional calculus onto the independent axioms of the algebra of species. (See Jekeri Okee, "Untersuchungen über den einstelligen intuitionistischen prädikatenkalkül der ersten Stufe", Z. Math. Logik Grundlagen Math., Band 18(1972), 37-48.) If a formula H is provable by the rules of detachment and/or substitution then $\varphi(H)$ is provable by the rules of species-detachment and/or substitution. Since the algebra of all subspecies of any infinite species is an adequate matrix for the system of provable formulae of the intuitionistic propositional calculus it follows that the algebra of species is complete in the above sense. (Received March 28, 1973.)

73T-E54. MIRIAM A. LIPSCHUTZ-YEVICK, Princeton University, Princeton, New Jesery 08540 and Rutgers University, New Brunswick, New Jersey 08903. A formalism of holographic operations.

Let A(x,y), $H_{O,R}(x,y)$ and $T_{A,B}(x,y)$, etc., represent the complex amplitude transmittance at the point (x,y) of physical transparencies (objects), denoted respectively by A, H(O, R), T(A, B), etc. If H(x,y) denotes a complex amplitude in the plane of A, the product $H \cdot A$ represents the complex amplitude transmitted through A. Let $H_A(u,v)$, $T_{A,B}(x,y)$, $C_{A,B}(x,y)$ now represent the Fourier transform, the cross-correlation and the convolution of the corresponding functions and let $\mathfrak{H}(A)$, $\mathfrak{T}(A,B)$ and $\mathfrak{C}(A,B)$ denote the physical operations which realize these transformations. All holographic operations can be expressed in terms of the following two primitive operations from objects to objects. Recording the hologram $\mathfrak{H}(O,R): H_O(x,y) \cdot H_R^*(x,y) + H_O^*(x,y)\, H_R(x,y) = H_{O,R}(x,y)$. Filtering $\mathfrak{F}(A, H_{O,R}) = \mathfrak{H}(H_A \cdot H_{O,R}) = \mathfrak{X}(R^*, C_{A,O}) + \mathfrak{C}(T_{A,O}, R) = \mathfrak{X}(O^*, C_{R,A}) + \mathfrak{C}(O, T_{R,A})$. From which follow: realizing the cross-correlation of a known object A with an arbitrary object O: $\mathfrak{F}(A, H_{O,R})$, reconstructing the object O: $\mathfrak{F}(R, H_{O,R})$, reconstruction of the object from a part of the hologram $\mathfrak{F}(R, H'_{O,R})$ and, its dual, from a part of the reference: $\mathfrak{F}(R', H_{O,R})$. The first degrades the resolution, the second diminishes the intensity. (Received March 28, 1973.)

*73T-E55. BARUCH GERSHUNI, Bloch Street 38, Tel Aviv, Israel. An axiom-system for the totality of all the natural numbers 1,2,3,... plus the nonnatural number 0.

The number 0 is genuinely not a natural number. But it is sometimes useful to consider the natural numbers together with the number 0, e.g. in order to have a coordinate-initial point for them. The number 0 plays an exceptional role: it is the unique number represented by a point; the natural numbers are represented by intervals: 1 by 0 ÷ 1, 2 by 0 ÷ 2 a.s.o. We formulate here an axiom-system for the totality of the natural numbers plus the number 0, which has 8 axioms, one axiom more than the modified axiom-system of Peano for the natural numbers. We adhere namely to the principle that any axiom should be an indivisible assertion and we consider also the fact that the axiom-system of Peano is short of one axiom, viz. that the successor of any number is unique. The so modified and completed axiom-system contains 7 axioms. Here is our axiom-system: (1) 0 is a number; (2) 0 is not a natural number; (3) any natural number is a number; (4) any number has a successor; (5) the successor of any number is a natural number; (6) the successor x' of any number x is unique; (7) if 0 has an arbitrary property E and if the successor of any number having the property E has the same property E, then any number has the property E. From (2) and (5) there follows that 0 is not the successor of any number. (Received March 30, 1973.)

73T-E56. STÅL AANDERAA, IBM T.J. Watson Research Center, Yorktown Heights, New York 10598 and WARREN D. GOLDFARB, Department of Philosophy, Harvard University, Cambridge, Massachusetts 02138. Finite controllability of the Maslov case.

A prenex formula of pure quantification theory is Krom if its matrix is a conjunction of binary disjunctions of signed atomic formulas. Theorem. Let F be any Krom formula with prefix $\exists\ldots\exists\forall\ldots\forall\exists\ldots\exists$; if F has a model then F has a finite model. The theorem strengthens Maslov's result that the class of such F is decidable for satisfiability [Soviet Math. Dokl. 5(1964), 1420-1424]. The proof is a generalization and modification of Gödel's argument [Monatsh. Math. Phys. 40(1933), 433-443] for the class of prenex formulas with prefix $\forall\forall\exists\ldots\exists$. In particular, Gödel's combinatorial Lemmas 1 and 2 are generalized to handle n rather than just 2 universal quantifiers. (Received April 2, 1973.)

73T-E57. WITHDRAWN.

*73T-E58. STEPHEN G. SIMPSON, University of California, Berkeley, California 94720. Pointwise definable models of arithmetic and set theory.

Theorem. Let $M = \langle |M|, +, \cdot \rangle$ (resp. $M = \langle |M|, \epsilon^M \rangle$) be a countable model of Peano arithmetic (resp. ZFC). Then there is a set $U \subseteq |M|$ such that (i) $\langle M, U \rangle$ satisfies the schema of first order induction (resp. replacement) for formulas involving an extra predicate symbol $\underline{U}x$; and (ii) every element of $|M|$ is first order definable in $\langle M, U \rangle$. (Received April 5, 1973.)

*73T-E59. J. D. MAITLAND WRIGHT, The University, Reading, England. All operators on a Hilbert space are bounded.

Let DC be the dependent choice axiom; this axiom implies that given a countable collection of nonempty sets then there exists a choice function. Let DC be adjoined to Zermelo-Fraenkel set theory and denote the enlarged system by ZF + DC. It is proved that when the proposition 'All linear operators on a Hilbert space are bounded' is interpreted in Solovay's model for ZF + DC then this proposition becomes a true statement. Hence this proposition cannot be disproved in ZF + DC. (Received April 6, 1973.)

73T-E60. WILFRID A. HODGES, Bedford College, Regent's Park, London NW1, England. Infinitary preservation results for universal maps. Preliminary report.

Let A, B be varieties of algebras, of similarity types Ω, Σ respectively; let E be the set of defining equations of A, and let F be a forgetful functor from A to B. Let L be a language of form $M \cap L^{\alpha}_{\infty\lambda}$, where λ is a regular cardinal, α is ∞ or an ordinal $\geqq 2$, α bounds the quantifier ranks of formulae, and M is a transitive $\mathrm{Prim}(\omega, \Omega, \Sigma, E, \mathscr{P}_{<\lambda})$-closed set. (For most interesting A, B, this includes the languages $L_{\varkappa\lambda}$ as well as the admissible languages of Barwise.) Theorem. The left adjoint of F is $\mathrm{Prim}(\omega, \Omega, \Sigma, E)$ and preserves L-elementary equivalence and L-elementary embeddings. If Ω, Σ are allowed to have $(<\lambda)$-ary operations, the theorem still holds, but with F $\mathrm{Prim}(\omega, \Omega, \Sigma, E, \mathscr{P}_{<\lambda})$. This is a common extension of results of Feferman and Gaifman. (Received April 6, 1973.)

73T-E61. GONZALO E. REYES, Université de Montréal, Montréal 101, Québec, Canada. Modelling theories in topoi.

If B is any boolean algebra, we let Sh($\mathbb{B}$) be the category of sheaves over $\mathbb{B}$ for the Grothendieck topology of arbitrary coverings (i.e., $\{a_i : i \in I\}$ covers a iff $\bigvee\langle a_i : i \in I\rangle = a$). Using Karp's terminology (cf. C. Karp's "Languages with expressions of infinite length") we have Theorem. Let $\varkappa$ be a regular cardinal and let $\mathbb{B}$ be the regular open algebra of the product space $(K^*)^{\omega}$, where $K^* = \Sigma\langle \varkappa^{\mu} : \mu \in \varkappa\rangle$ is given the discrete topology. Every theory in a $L_{\varkappa\varkappa}$ language having at most $\varkappa$ primitive symbols and consistent in the calculus $B_{\varkappa\varkappa}(\Sigma)$, where Σ contains the μ law of dependent choices for all $\mu < \varkappa$ only, has a model in Sh($\mathbb{B}$). Furthermore, the model is faithful (i.e., nonequivalent formulas are mapped into distinct sheaves). As a corollary, countable first order theories have faithful models in the subcategory of sheaves for the double negation topology of the category of sheaves over the irrationals. The proof uses Kripke's theorem on continuous embeddings of boolean algebras. Mansfield's completeness theorem (J. Symbolic Logic 37(1972), 31-34) is seen in this context as the existence of "nonstandard" models of infinitary theories in Sh($\mathbb{B}$). (Received April 9, 1973.)

73T-E62. JAN MYCIELSKI, University of Colorado, Boulder, Colorado 80302. A decidable class of formulas.

Let A denote the class of atomic or negations of atomic formulas of first order logic which may involve arbitrary terms with function symbols. Theorem 1. There exists a decision procedure for checking satisfiability of universal formulas of the form $(\alpha_1 \vee \beta_1) \wedge \ldots \wedge (\alpha_n \vee \beta_n)$, where $\alpha_i, \beta_i \in A$ and $n = 1, 2, \ldots$. Theorem 2. The class of satisfiable formulas of the form $(\alpha_1 \vee \beta_1 \vee \gamma_1) \wedge \ldots \wedge (\alpha_n \vee \beta_n \vee \gamma_n)$, where $\alpha_i, \beta_i, \gamma_i \in A$ and $n = 1, 2, \ldots$, constitutes a reduction class for satisfiability for first order logic. (Received April 9, 1973.)

73T-E63. GEORGE FRANK McNULTY, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada. Structural diversity in the lattice of equational theories of algebras. II. Preliminary report.

Fix a similarity type of algebras. An equation ϵ is simple if ϵ is a tautology or, in the case that the similarity type has no operation symbols of rank more than one, no variables occur in ϵ. Let α be the cardinality of

the set of nonsimple equations. For any set β the lattice of equivalence relations on β is denoted by π_β. A term is trivial if the sum of the ranks of the distinct operation symbols occurring in it is less than two. Theorem 1. Let T be an equational theory and τ be a nontrivial term such that some variable different from x occurs in τ and $\tau = x \in T$. (i) π_α is isomorphic to an interval in the lattice of subtheories of T and (ii) for any nonsimple equation ϵ there is a set F of subtheories of T so that (a) $|F| = 2^\alpha$, (b) each member of F has the same finite models as T, (c) the join of any two distinct elements of F is T, (d) if T is finitely based then T covers each element of F, and (e) if $\Delta \in F$ then the theory based on $\Delta \cup \{\epsilon\}$ includes T. Remark. Provided (e) is deleted and α is replaced everywhere by ω, the condition that a variable different from x occurs in τ can be omitted. Theorem 2. If the similarity type has a binary operation symbol then π_α is isomorphic to an interval of the lattice of theories extending the commutative law.
(Received April 13, 1973.)

73T-E64. GREGORY L. CHERLIN, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. On ω_1-categorical theories of commutative rings. Preliminary report.

Let $\underline{R}$ be an infinite commutative noetherian ring with identity. Call a ring $\underline{A}$ a special ring iff $\underline{A}$ is a finite type extension of a completely primary commutative ring $\underline{B}$ whose residue field $\underline{B}/\mathrm{rad}(\underline{B})$ (here $\mathrm{rad}(\underline{B})$ is the nilpotent radical of $\underline{B}$) is algebraically closed and such that $\mathrm{rad}(\underline{B}) = \underline{pB}$ where $\underline{p}$ = the characteristic of the residue field $\underline{B}/\mathrm{rad}(\underline{B})$. Theorem. (a) $\mathrm{Th}(\underline{R})$ is ω-stable iff $\underline{R}$ is the direct product of a finite ring and finitely many special rings. (b) $\mathrm{Th}(\underline{R})$ is ω_1-categorical iff $\underline{R}$ is the direct product of a finite ring and one special ring. This extends results found indepently by J. Reineke, announced in Abstract 73T-E39, these *Notices* 20(1973), A-340. As by-products of this work, we have information concerning nonnoetherian ω-stable or ω_1-categorical commutative rings. (Received April 16, 1973.)

*73T-E65. FRANKLIN D. TALL, University of Toronto, Toronto 181, Ontario, Canada. Hausdorff's gaps and limits and a new iterated extension axiom.

Consider the quasi-order E on subsets of ω defined by $a \leqq b$ if $a - b$ is finite. Say $a < b$ if $a \leqq b$ and not $b \leqq a$. E has an (Ω, ω^*) gap if there exist $\{a_\alpha\}_{\alpha<\omega_1}$ and $\{b_n\}_{n<\omega}$ in E such that $a_0 < a_1 < \ldots < a_\alpha < \ldots \ldots < b_\alpha < \ldots < b_1 < b_0$ and there is no $d \in E$ such that $a_\alpha < d < b_n$ for every $\alpha < \omega_1$ and every $n < \omega$. E has an Ω-limit if there exists $\{a_\alpha\}_{\alpha \leqq \omega_1}$ in E such that $a_0 < a_1 < \ldots < a_\alpha < \ldots < a_{\omega_1}$, and there is no $d \in E$ such that $a_\alpha < d$ for all $\alpha < \omega_1$, and $d < a_{\omega_1}$. F. Rothberger (Fund. Math. 35(1948), 29-46) proved Theorem 1. (a) If there are no Ω-limits, there are no (Ω, ω^*)-gaps. (b) If there are no (Ω, ω^*)-gaps, $2^{\aleph_0} > \aleph_1$. (c) If there are no Ω-limits, $2^{\aleph_0} = 2^{\aleph_1}$. In this paper it is shown that none of these implications can be reversed. (b) and (c) are settled by adjoining $\aleph_2$ Cohen reals to L. Let 'W' stand for Martin's axiom restricted to countable chain condition partial orders constructible from a real. Theorem 2. (a) W plus $2^{\aleph_0} > \aleph_1$ implies Q. (b) W is consistent with $\aleph_1 < 2^{\aleph_0} < 2^{\aleph_1}$. Thus 1(a) cannot be reversed. (Received April 17, 1973.)

*73T-E66. MARTIN SEBASTIAN GERSON, Simon Fraser University, Burnaby 2, British Columbia, Canada. A neighbourhood frame for T with no equivalent relational frame.

A neighbourhood or Scott-Montague frame for modal propositional calculi is presented and shown to model a set Γ of formulae but not to model another formula, (10). It is then shown (relying heavily on a construction of S. K. Thomason's) that any relational frame which models Γ also models (10). Since it is already known that for every relational frame there is an equivalent neighbourhood frame, the result shows that, even when restricted to models of T, the neighbourhood semantics is stronger, or has more depth, than the relational semantics for modal logics. (Received April 19, 1973.)

73T-E67. HERBERT B. ENDERTON, University of California, Los Angeles, California 90024. Completeness in Chang's modal model theory. Preliminary report.

A language related to R. Montague's "pragmatics" has been introduced by C. C. Chang, "Modal model theory," Proc. Summer School in Logic at Cambridge, Springer-Verlag, to appear. In this language symbols for modal operators are included but can be arbitrarily interpreted. Thus for a sentence to be valid, it must be true under all interpretations of (inter alia) the modal operators. We construct a deductive calculus appropriate for Chang's semantics. It is (roughly) obtained from an ordinary deductive calculus for first-order logic by addition of such axioms as $\forall x[\varphi(x) \leftrightarrow \psi(x)] \rightarrow [Ny\varphi(y) \leftrightarrow Ny\psi(y)]$ and $x = y \rightarrow [Nx\varphi(x) \leftrightarrow Ny\varphi(y)]$. Theorem. This deductive calculus is sound and complete; any consistent set of sentences has a model. Corollary. The set of valid sentences (under a recursive numbering of the language) is recursively enumerable. Another corollary is the compactness theorem, but this was already established by Chang through use of ultraproducts. (Received April 23, 1973.)

73T-E68. FRANCIS LOWENTHAL, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. Some results on measure and category in α-recursion.

Let α be a countable admissible ordinal, $\alpha > \omega$, let μ be the probability measure on 2^{α} and τ the topology defined by α-finite neighborhood conditions. Theorem 1. Let $A \subseteq \alpha$ be such that A is not α-recursive. Then $\{f \in 2^{\alpha} | A \leq_{\alpha} f\}$ has measure 0 and is of 1st category (the second part was proved independently by R. Jhu, Abstract 73T-E35, these *Notices* 20(1973), A-339). Theorem 2. The set of all α-degrees $\underline{d}$ such that $\underline{d}$ is the union of 2 incomparable degrees less than $\underline{d}$ has measure 1. Theorem 3. Let $A \subseteq \alpha$ be such that A is not α-calculable. Then $\{Y \subseteq \alpha | A \leq_{c} y\}$ has measure 0 and is of 1st category. Theorem 4. The set $\{X \subseteq \alpha | X$ is regular and hyperregular$\}$ is of 2nd category and has measure 0, but for the usual product topology, this set is of 1st category. (Received April 24, 1973.)

73T-E69. DAVID W. KUEKER, University of Maryland, College Park, Maryland 20742. Automorphisms of models with cardinality cofinal with ω.

Let κ be a strong limit cardinal cofinal with ω, and let $\mathfrak{A}$ be a model of cardinality κ. Theorem. The following two conditions are equivalent: (1) There is an $S \subseteq A$ with $|S| < \kappa$ such that every element of A is $L_{\infty\kappa}$-definable in $\mathfrak{A}$ in terms of the elements of S. (2) There is a $U_0 \subseteq A$ with $|U_0| < \kappa$ such that $(\mathfrak{A}, U)$ has $\leq \kappa$ automorphisms whenever $U_0 \subseteq U \subseteq A$, $|U| < \kappa$. Corollary 1. $\mathfrak{A}$ has $< \kappa$ automorphisms if and only if (1) holds where S is $L_{\infty\kappa}$-definable in $\mathfrak{A}$. Corollary 2. If every $S \subseteq A$ with $|S| < \kappa$ is contained in some set of cardinality $< \kappa$ which is $L_{\infty\kappa}$-definable in terms of finitely many elements of A, then (1) holds if and only if $\mathfrak{A}$ has $\leq \kappa$ automorphisms. Examples show that in general the conditions in the thoerem do not imply that $\mathfrak{A}$ has $\leq \kappa$ automorphisms, if $\kappa \neq \omega$. In condition (2) the number of automorphisms could equivalently be changed to either $< \kappa$ or $< 2^{\kappa}$. Similar results hold concerning the $L_{\infty\kappa}$-definability of subsets of A in terms of $< \kappa$ parameters. These theorems improve results mentioned in our paper in "The syntax and semantics of infinitary languages". (Received April 25, 1973.)

Statistics and Probability

73T-F10. THOMAS H. SAVITS, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. Space-time harmonic functions applied to age-dependent processes. Preliminary report.

Let X be an age-dependent process with lifetime distribution G and age-dependent generating function $f(y, s) = \sum_{k=0}^{\infty} p_k(y) s^k$. We assume that G is right-continuous and $G(0+) = G(0) = 0$. The base state space S is $[0, T)$ where $T = \inf\{t : G(t) = 1\}$. Define $p_k = \int_{(0,T]} p_k(y)\, dG(y)$ and $m = \sum_{k=1}^{\infty} kp_k$. Theorem 1. Extinction occurs with probability one iff $m \leq 1$. In the case where $m > 1$, define λ to be the unique (positive) root of $\int_{(0,T]} m(y) e^{-\lambda y}\, dG(y)$

$= 1$, where $m(y) = \sum_{k=1}^{\infty} kp_k(y)$, and set $\varphi(x) = [1 - G(x)]^{-1} e^{\lambda x} \int_{(x,T]} m(y) e^{-\lambda y}\, dG(y)$ on S. Theorem 2. $h(x,t) = e^{-\lambda t}\varphi(x)$ is a V-space-time harmonic function of the process X. Theorem 3. The nonnegative martingale $W_t = e^{-\lambda t}\varphi(X_t)$ converges w.p.1. to a random variable W; moreover W is nondegenerate iff $\sum_{k=2}^{\infty}(k \log k)p_k < \infty$. Let Z_t denote the number of particles at time t. Theorem 4. If $0 < \alpha \leq \varphi \leq \beta < \infty$, for some constants α, β, then $\beta^{-1}W \leq \underline{\lim}_{t\to\infty} e^{-\lambda t} Z_t \leq \overline{\lim}_{t\to\infty} e^{-\lambda t} Z_t \leq \alpha^{-1}W$ w.p.1. (Received March 30, 1973.)

73T-F11. THEODORE E. HARRIS, University of Southern California, Los Angeles, California 90007. Interactions on a lattice. Preliminary report.

Let Z_d be the d-dimensional integers, Ξ the set of mappings $\xi : Z_d \to \{0,1\}$, identifying ξ with the set $\{x : \xi(x) = 1\}$. Write $y \sim x$ if y is one of the 2d neighbors of $x \in Z_d$. Let $N = N(\xi, x) = \sum_{y:y\sim x} \xi(y)$. Let $\mu, \lambda_0, \lambda_1, \ldots, \lambda_{2d}$ be positive numbers. If $\xi_t = \xi$, the intensity for a change $0 \to 1$ of $\xi_t(x)$ is λ_N, and for $1 \to 0$ is μ. Let $\lambda_0 = 0$ until further notice. Let $m_t(\xi) = \mathcal{E}\{\xi_t(Z_d) | \xi_0 = \xi\}$, $p_t(\xi) = P\{\xi_t(Z_d) \neq 0 | \xi_0 = \xi\}$. Theorem. If $\lambda_k\uparrow$ in k and $\lambda_k/k\downarrow$ in k, then m_t and p_t are, for each t, subadditive set functions of ξ. If $\lambda_k\uparrow$ and $(\lambda_{k+n} - \lambda_k)\downarrow$ for fixed n, m_t and p_t are strongly subadditive. If only $\lambda_k\uparrow$, m_t and p_t are increasing in ξ. The theorem is used to bound parameter regions where $\int_0^{\infty} m_t(\xi)\, dt < \infty$ if $\xi(Z_d) < \infty$. Moreover, if $\lambda_0 > 0$, there is a related process $\{\xi_t^*\}$ with $\lambda_0^* = 0$, such that if $\int_0^{\infty} m_t^*(\xi)\, dt < \infty$ when $\xi(Z_d) < \infty$ then $\{\xi_t\}$ is ergodic. (Received April 16, 1973.)

Topology

73T-G70. JOSEF K.H. DORTMANN and FLORINDA K. MIYAÒKA, Federal University of Parana, Curitiba, Brazil. Semi-open sets in topological spaces. II. Preliminary report.

In a recent paper (Abstract 73T-G62, these *Notices* 20(1973), A-344), the authors defined strictly semi-open (strictly semiclosed) sets, s-sets and φ-sets in a topological space (X, T). We define the kernel and the subkernel of a topological space (X, T) as follows: the former as $K = \bigcap A_j$, where (A_j) is the class of open sets which are strictly semi-open and whose closure is X, the latter as $K_0 = \bigcap B_i$, where (B_i) is the class of strictly semi-open sets. Distinction is made between the proper kernel, when K is open, and the improper kernel when K is not open. It is shown that in connected topological spaces the kernel always exists and is unique. It is also shown that in connected spaces the proper kernel K is always a disconnected subspace of (X, T). (Received January 29, 1973.)

*73T-G71. LOUIE M. MAHONY, State University of New York, Binghamton, New York 13901. Integral representation theory on manifolds. Preliminary report.

M(G, R) denotes the matrix class semigroup of R-equivalent finite dimensional R-matrix representations of a finite group G, where R is the integers Z or rationals Q. If $p : M(G,Z) \to M(G,Q)$ is the inclusion, then $p^{-1}(x)$ is finite and nonempty whose elements will be called the Z-concordance classes of x. By equivariant surgery, the following can be proved. Theorem. There exist a $k - 1$ connected compact π-manifold N^{2k} ($\partial N = \emptyset$), a finite group G and two free actions ρ_1, ρ_2 of G on N^{2k} as orientation preserving diffeomorphisms such that: (1) the Q-representation of G on $H^k(N, Q)$ afforded by ρ_1, ρ_2 determine the same element x in M(G, Q); (2) the Z-representation $[\rho_1], [\rho_2]$ of G on $H^k(N, Z)$ are not integrally equivalent and consequently give two distinct Z-concordance classes of x. (Received March 1, 1973.)

*73T-G72. R. GRANT WOODS, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada. A Tychonoff almost realcompactification.

Let X be any Tychonoff space and let βX be its Stone-Čech compactification. An ultrafilter $\mathcal{U}$ on the Boolean algebra $\mathcal{R}(X)$ of regular closed subsets of X converges to a point p of βX if $\{p\} = \{\mathrm{cl}_{\beta X} A : A \in \mathcal{U}\}$. Let

$a_1X = \{p \in \beta X$: there exists an ultrafilter $\mathcal{U}$ on $\mathcal{R}(X)$ with the countable intersection property that converges to $p\}$. If n is a positive integer > 1, let $a_nX = a_1(a_{n-1}X)$, and put $aX = \bigcup\{a_nX : n \in N\}$. Theorem. aX is an almost realcompact Tychonoff space containing X as a dense subspace. If Y is any almost realcompact Tychonoff space and if $f: X \to Y$ is continuous, then f extends continuously to $f^a : aX \to Y$. Theorem. Let $E(X)$ denote the projective cover of X (note that $E(\beta X)$ can be identified with $\beta E(X)$). Then aX is the smallest space T between X and βX such that $E(T)$ is realcompact. Theorem. Let uX denote the minimal c-realcompactification of X. The following are equivalent: (i) $uX = aX$. (ii) $\upsilon E(X) = E(T)$ for some T such that $X \subseteq T \subseteq \beta X$. (iii) $\upsilon E(X) = E(aX)$. Theorem. Let X and Y be two Tychonoff spaces. If $\upsilon(X \times Y) = \upsilon X \times \upsilon Y$, then $a(X \times Y) = aX \times aY$. (Received March 2, 1973.)

73T-G73. DAVID J. LUTZER and HAROLD MARTIN, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. A note on the Dugundji extension theorem. Preliminary report.

Suppose that A is a closed, metrizable G_δ-subset of a collectionwise normal space X and let $C(A)$ and $C(X)$ denote the vector spaces of all continuous, real-valued functions on A and X respectively. Then there is a linear transformation $U: C(A) \to C(X)$ with the property that for each $g \in C(A)$, $u(g)$ extends g and the range of $u(g)$ is contained in the convex hull of the range of g. An analogous theorem holds for continuous functions with values in a locally convex topological vector space. An example due to Heath and Lutzer shows that the hypothesis that A is a G_δ-subset of X cannot be removed, contrary to a remark appearing on p. 805 of ["Some extension theorems for continuous functions," Pacific J. Math. 3(1953), 789-806]. (Received March 2, 1973.)

73T-G74. FRED D. CRARY, Mathematics Research Center, University of Wisconsin, Madison, Wisconsin 53706. Improved radial engulfing. Preliminary report.

We prove that a codimension 4 radial engulfing theorem of Bing is true in codimension 3 and improve his theorem in codimensions 4 and greater. Theorem. Suppose that U is an open subset of a manifold M^n without boundary, P is a closed polyhedron in M^n with $\dim P \leq n - 3$, Q is a subpolyhedron of P with $Q \subset U$, $R^r = Cl(P - Q)$ is compact, and $\{A_\alpha\}$ is a collection of sets in M^n such that finite r-complexes in M^n can be pulled into U along $\{A_\alpha\}$ [as defined by Bing in "Radial engulfing," Conf. on Topology of Manifolds]. Then for each $\epsilon > 0$, there is an engulfing isotopy $H: M^n \times [0,1] \to M^n$ such that $H_0 = \mathrm{id}$, $H_t = \mathrm{id}$ on Q, $H_1(U) \supset P$ and, for each point $x \in M^n$ that is moved by h, $H(x \times [0,1])$ lies in the ϵ-neighborhood of the union of s elements of $\{A_\alpha\}$, where $s = 1$ if $r = 0$, $s = r$ if $0 < r < n - 3$, and $s = r + 1$ if $r = n - 3$. (See Theorem A in Bing's paper.) For $r < n - 3$, the proof is the same as Bing's proof of Theorem A except that the induction begins with 1 instead of 0. For $r = n - 3$, careful use of Zeeman's piping lemma can be used to obtain the extra dimension. The estimates of track size in the engulfing lemma of Edwards and Glaser ["A method of shrinking decompositions of certain manifolds"] can be reduced to these estimates. This theorem for $r = n - 3$ was previously claimed by Perrin Wright, but was later withdrawn. (Received March 5, 1973.)

*73T-G75. GARY F. GRUENHAGE, University of California, Davis, California 95616. A separable continuously normal space is metrizable. Preliminary report.

The author has given an example of a continuously perfectly normal space which is not first countable (Abstract 73T-G37, these Notices 20(1973), A-291). The following results have now been obtained: Theorem. A separable continuously completely regular space (for definition, see the Abstract 701-54-52 of Phillip Zenor, these Notices 20(1973), A-182) is metrizable if and only if it is a Frechet space. Theorem. A continuously normal space X is Frechet at each point which is G_δ in X. Theorem. A separable continuously normal space is metrizable. (Received March 8, 1973.)

*73T-G76. ROBERT F. BROWN, University of California, Los Angeles, California 90024. Fixed points of endomorphisms of compact groups.

For a function $f: X \to X$, let $\Phi(f) = \{x \in X \mid f(x) = x\}$. If X is a group (Lie group, vector space) and h is a homomorphism, then $\Phi(f)$ is a group (Lie group, vector space). Let $h: G \to G$ be an endomorphism of a topological group, let $H^*(G)$ denote real Čech cohomology, h^* the induced endomorphism of $H^*(G)$, and $h^{*,1}$ its restriction to $H^1(G)$. Use the symbol $\Phi_0(h)$ for the identity component of $\Phi(h)$. Theorem. If G is a compact, connected abelian topological group and h is an endomorphism of G, then the dimension of the topological group $\Phi_0(h)$ is equal to the dimension of the vector space $\Phi(h^{*,1})$. Now assume that G is a compact Lie group and let $\underset{\sim}{H}^*(G)$ denote the subspace of $H^*(G)$ spanned by a set of algebra generators for $H^*(G)$. Let h be an automorphism of G and let $\underset{\sim}{h}^*$ be the restriction of h^* to $\underset{\sim}{H}^*(G)$. Theorem. If G is a compact, connected Lie group and h is an automorphism of G, then the dimension of a maximal torus of the Lie group $\Phi_0(h)$ is equal to the dimension of the vector space $\Phi(\underset{\sim}{h}^*)$. (Received March 12, 1973.)

*73T-G77. DALE P. ROLFSEN, University of Wyoming, Laramie, Wyoming 82070 and University of British Columbia, Vancouver 8, British Columbia, Canada. Localized Alexander invariants and isotopy of links.

The Alexander invariant of a link of two n-spheres in R^{n+2} is the homology $H_*(\tilde{X})$ of the universal abelian cover of the complement, considered as a module over the ring Λ of finite Laurent polynomials in two variables with integer coefficients. Let $\Sigma = \{f(x)g(y)\}$ be the set of nonzero members of Λ which factor into terms involving only one variable. Form the localization Λ/Σ and the corresponding localization $H_*(\tilde{X})/\Sigma$ of the Alexander invariant. Theorem (PL category). If two links are isotopic, then their localized Alexander invariants are isomorphic as Λ/Σ-modules. Here isotopy refers to nonambient isotopy (= continuous family of embeddings), a relation under which, for instance, PL knot theory in all dimensions becomes trivial. The localized invariant settles the corresponding question for link theory, hitherto known only for $n = 1$. Corollary. For each $n \geqq 1$, there are infinitely many PL isotopy classes of links of two n-spheres in R^{n+2}. (Received March 5, 1973.) (Author introduced by Professor Joseph M. Martin.)

*73T-G78. ELIZABETH B. CHANG, Hood College, Frederick, Maryland 21701. Characterizations of some zero-dimensional spaces.

A Hausdorff space is collectionwise ultranormal if for every locally finite collection $\{A_i : i \in I\}$ of mutually disjoint closed subsets there is a collection $\{C_i : i \in I\}$ of pairwise disjoint clopen sets such that $A_i \subseteq C_i$ for each i in I. It is shown that collectionwise spaces are characterized by the existence of extensions for continuous functions defined on closed subsets with values in complete metric spaces. They are also characterized by the existence of continuous selections for certain types of set-valued functions (specifically, l.s.c. carriers with values which are generally compact subsets of complete metric spaces). Analogues of these results are presented for some further classes of zero-dimensional spaces. (Received March 26, 1973.)

*73T-G79. KENNETH A. PERKO, JR., One Chase Manhattan Plaza, New York, New York 10005. On the classification of knots.

The Tait-Little tables of 166 presumably prime, 10-crossing knots were recently rechecked for completeness and republished along with algebraic invariants which distinguished all but 31 pairs. [See J. H. Conway's paper in "Computational problems in abstract algebra" (Pergamon, Oxford, 1970) but beware of false "beliefs".] Linking numbers between the branch curves of appropriate noncyclic covering spaces of these examples newly distinguish all but the pair 10_{5II} 10_{6VI} which turns out to be a duplication in Little's table that Conway overlooked; thus these tables contain precisely 165 distinct, new knot types. Such linking numbers also solve the amphicheirality problem for the 6 remaining examples in Reidemeister's table, proving that the only amphicheirals up to nine crossings are those identified as such by Tait when this table first appeared. [Compare "Knotentheorie," vol. III, p. 15, and the results reported in Canad. J. Math. 22(1970), 200-201.] (Received February 26, 1973.)

73T-G80. SUKHJIT SINGH, Pennsylvania State University, University Park, Pennsylvania 16802. A 3-dimensional compact absolute retract which contains no 2-dimensional compact absolute retract.

By an AR we understand a compact absolute retract for the category of metric spaces. Let E^3 denote the 3-dimensional Euclidean space and B^3 denote the closed ball of unit radius in E^3. Let n and k be positive integers with $2 \leqq k \leqq n$. An n-dimensional AR X will be called $(\text{irreducible})^k$ if and only if X does not contain any proper AR of dimension $k, (k-1), \ldots, 2$. Theorem. There exists an upper semicontinuous decomposition G of B^3 whose nondegenerate elements form a countable null family of arcs such that the decomposition space B^3/G is a 3-dimensional AR which is $(\text{irreducible})^3$. This solves a problem posed by Bing and Borsuk (Fund. Math. 54(1964), 159-175). Whether there exists an n-dimensional AR X with $n > 3$ such that X is $(\text{irreducible})^k$, with $2 \leqq k \leqq n$, is an open problem. Corollary. There is an upper semicontinuous decomposition G of E^3 whose nondegenerate elements form a countable null collection of arcs such that the decomposition space E^3/G does not contain any 2-dimensional AR. (Received March 30, 1973.)

*73T-G81. SIDNEY A. MORRIS and E. T. ORDMAN, University of New South Wales, Kensington, New South Wales 2033, Australia. Remarks on free products of topological groups. Preliminary report.

We are able to prove the following, which settle several previously announced questions: Theorem 1. If the abstract group $G * H$ has a topology making it a Hausdorff, nondiscrete topological group, such that the natural maps $G * H \to G$ and $G * H \to H$ are continuous, then $G * H$ is not locally compact. Corollary. If the free product of two or more topological groups is locally compact, all factors and the product are discrete. Theorem 2. If G and H are Hausdorff topological groups and k_w-spaces, the subgroup of their free product $G * H$ generated by $[G,H] = \{g^{-1}h^{-1}gh \mid g \in G,\ h \in H\}$ is the (Graev) free topological group on $[G,H]$. Theorem 3. If G and H are Hausdorff topological groups admitting continuous monomorphisms into locally invariant (SIN) groups, then $G * H$ also admits such a continuous monomorphism. Example. The free product of two groups which are k-spaces need not be a k-space. Let $G = S * S$, the free product of two circle groups, and let H = the rationals. Then G, H, $G \times G$, $H \times H$ are k-spaces but $G \times H$ and $G * H$ are not k-spaces. (Received March 30, 1973.)

*73T-G82. RONALD H. ROSEN, University of Michigan, Ann Arbor, Michigan 48104. An annulus theorem for suspension spheres. Preliminary report.

A suspension sphere of dimension $n - 1$ is a space X whose suspension, $S^\circ * X$, is homeomorphic to S^n. From Kirby's proof that orientation preserving homeomorphisms of S^n are stable for $n \geqq 5$ (Ann. of Math. (2) 89(1969), 575-582) we now know that the ordinary annulus conjecture is true if $n \neq 4$. The author uses Kirby's work to prove the following annulus type theorem. Theorem. Let X be a suspension sphere of dimension $n - 1$ and let $n \geqq 5$. Let $X_i = f_i(X)$, $i = 1,2$, so that $f_i : X \to S^n$ is an embedding and X_i is bicollared in S^n. If X_1 and X_2 are disjoint and U is the region between them then the closure of U is homeomorphic to $X \times I$. The proof is not completely straightforward since if $X \neq S^{n-1}$ there is no single canonic family of embeddings of X in S^n (similar to the round $(n-1)$-spheres in S^n). (Received April 2, 1973.)

*73T-G83. JOHN M. ATKINS, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. A note on metacompact developable spaces.

We assume all spaces are at least T_1. Theorem 1. X is a metacompact developable space if and only if

X is a β-space with a σ-point finite open base. Theorem 2. Let f be a closed continuous map from X onto a metacompact developable space Y with the fibers of f metacompact developable and the boundaries of the fibers of f compact. Then the following are equivalent: (1) X is a metacompact developable space; (2) X has a G_δ-diagonal; (3) X is a $\sigma^{\#}$-space. (Received April 5, 1973.)

73T-G84. MARTIN G. SCHARLEMANN, University of California, Berkeley, California 94720. A fake homotopy structure on $S^3 \times S^1 \# S^2 \times S^2$.

A homotopy equivalence f is defined between a manifold M and the connected sum $S^3 \times S^1 \# S^2 \times S^2$. f is normally cobordant but not s-cobordant to the identity. Let K be the Poincare homology 3-sphere. M is obtained by doing surgery on a circle representing a generator of the finite part of the fundamental group of $K \times S^1$. There are locally flatly imbedded 2-spheres S_1 and S_2 in M representing the homotopy classes of both $S^2 \times$ (point) and (point) $\times S^2$ in $S^3 \times S^1 \# S^2 \times S^2$. However, the linking circle to each S_i in M is not null-homotopic in $M - S_i$, so the existence of a fake homotopy structure on $S^3 \times S^1$ remains an open question. There is a locally flat non-PL imbedding of $K \times S^1$ in $S^4 \times S^1$ inducing an epimorphism on fundamental groups. From this are derived (i) a structure theorem for closed orientable non-PL 5-manifolds, (ii) information on the homotopy type of the complement of a non-PL 3-knot in S^5, and (iii) a very weak form of the product structure theorem for four-manifolds. (Received April 9, 1973.)

*73T-G85. DAVID A. EDWARDS and ROSS GEOGHEGAN, State University of New York, Binghamton, New York 13901. Compacta weak shape equivalent to CW complexes.

If C is a category, there is a category called pro-C whose objects are inverse systems of C: the set of morphisms from $\{X_\alpha\}$ to $\{Y_\beta\}$ is $\varprojlim_\beta \varinjlim_\alpha C(X_\alpha, Y_\beta)$ [see Artin-Mazur's "Étale homotopy," Lecture Notes in Math., vol. 100, Springer-Verlag, Berlin and New York, 1969]. $\{X_\alpha\}$ in pro-C is movable if $\forall \alpha \exists \beta \geqq \alpha$ such that $\forall \gamma \geqq \alpha$, $\exists r_{\beta\gamma}: X_\beta \to X_\gamma$ making $p_{\gamma\alpha} r_{\beta\gamma} = p_{\beta\alpha}$, where the p's bond. Let H_0 be the homotopy category of pointed connected CW complexes. The Čech construction defines a functor $\check{\mathcal{C}}$: (pointed connected spaces) $\to$ pro-H_0. For X metric, (X, x_0) is essentially the Borsuk-Fox shape of (X, x_0). The nth homotopy pro-group [resp. shape group] of (X, x_0) is the induced inverse system [resp. limit] of the nth homotopy groups. $f: \check{\mathcal{C}}(Y, y_0) \to \check{\mathcal{C}}(X, x_0)$ is a weak equivalence [resp. very weak equivalence] if f induces an isomorphism on homotopy pro-groups [resp. shape groups]. (X, x_0) is movable if $\check{\mathcal{C}}(X, x_0)$ is movable in pro-H_0: this agrees with Borsuk's definition when X is compact metric. If each homotopy group is given the discrete topology, the shape groups acquire an interesting inverse limit topology. Theorem. Let (X, x_0) be a connected pointed movable compactum. There is a pointed CW complex (Q, q_0) and a very weak equivalence $f: \check{\mathcal{C}}(Q, q_0) \to \check{\mathcal{C}}(X, x_0)$. Moreover f can be chosen to be a weak equivalence if and only if the shape groups of (X, x_0) are discrete in the inverse limit topology. (Examples are given.) (Received April 13, 1973.)

*73T-G86. TEODOR PRZYMUSIŃSKI, Institute of Mathematical Sciences, Śniadeckich 8, Warsaw, Poland and FRANKLIN D. TALL, University of Toronto, Toronto 181, Ontario, Canada. The undecidability of the existence of a nonseparable normal Moore space satisfying the countable chain condition.

Theorem. Martin's axiom plus $2^{\aleph_0} > \aleph_1$ implies the existence of a nonseparable normal metacompact Moore space satisfying the countable chain condition. Corollary. The assertion that normal Moore spaces satisfying the countable chain condition are separable is consistent with and independent of the usual axioms of set theory, including the axiom of choice. (Received April 17, 1973.)

*73T-G87. NANCY M. WARREN, Metropolitan State College, Denver, Colorado 80204. Normality problems under perfect mappings. Preliminary report.

The following results are proven in this paper. There is a normal space X whose associated extremally disconnected space E_X is not normal. There exists a nonnormal space X whose image Y under a perfect mapping is normal. This result extends to the following result on products. Let I be the unit interval; then there exist spaces X and Y, where Y is the image of X under a perfect mapping, $I \times Y$ is normal, but $I \times X$ is not normal. (Received April 20, 1973.)

*73T-G88. IVAN L. REILLY and BRUCE HUTTON, University of Auckland, Auckland, New Zealand. Compactness and finiteness in topological spaces.

This paper considers a class of topological spaces, the icn (infinite complement neighbourhood) spaces, defined as follows: $(X, \mathcal{T})$ is icn if for each point p in X and for each infinite subset A of X there is an open set G containing p such that A - G is infinite. In particular, the relationship between icn spaces and the cf spaces (in which compactness and finiteness are equivalent) of Levine is discussed. Theorem 1. Every cf space is icn. Theorem 2. In the first countable space $(X, \mathcal{T})$ the following are equivalent: (a) $(X, \mathcal{T})$ is cf. (b) $(X, \mathcal{T})$ is icn. (c) Every subset of X is free of ω accumulation points. (d) X is free of ω accumulation points. (e) Each point of X is contained in some finite open set. Corollary. Any first countable cf space is a quasi-ordered space. (Received April 23, 1973.)

*73T-G89. LOUIE M. MAHONY, P. ROY and JOHN J. WALSH, State University of New York, Binghamton, New York 13901. Plumbing 3-manifolds along trees. Preliminary report.

The quadratic form given by $q(x) = 8x_1^2 + 6x_1x_2 + \sum_{i=2}^{8} 2x_i^2 + \sum_{i=2}^{7} 2x_ix_{i+1}$ has the properties that it is of type II, its rank and index are both 8, the determinant of the associated matrix is 1, and therefore, is integrally equivalent to the matrix of Milnor. The tree associated with q(x) is however, a straight line tree, whereas the matrix of Milnor has a branch. Proposition 1. If M^3 is boundary of the 4-dimensional manifold obtained by plumbing according to the tree of q(x), then the fundamental group of M^3 is a perfect infinite group on two generators x, y with relations $x^{29} = (x^7y)^3 = y^{11}$. Remark. There are only three other trees with eight nodes which can satisfy the conditions above. (Received April 23, 1973.)

73T-G90. WITHDRAWN.

*73T-G91. ROBERT A. HERRMANN, U.S. Naval Academy, Annapolis, Maryland 21401. Nonstandard topological extensions. II.

We continue our study of nonstandard extensions. (See Abstract 73T-G41, these *Notices* 20(1973), A-292.)

In what follows, (X,T) will be a noncompact completely regular (not necessarily T_1) space. Theorem 1. Each normal base B for (X,T) determines nonstandard normal compactifications $H(X,B)$ and $w(X,B)$ such that $X \subset w(X,B) \subset H(X,B) \subset \hat{X}$. Theorem 2. If (X,T) is T_1 and B a normal base, then the nonstandard extension $w(X,B)$ is homeomorphic to the T_2 Wallman compactification of X in the sense of Frink. Theorem 3. If L is a subspace where $w(X,B) \subset L \subset H(X,B)$ and $w(X,B) \neq L$, then L is a nonstandard non-T_1 normal compactification of X. Theorem 4. Assume (X,T) is T_1 and B is a normal base. Then B determines infinitely many distinct homeomorphic T_2 Wallman compactifications of X (i.e. $\{w(X,B)_h \mid h \in S(B)\}$), as well as infinitely many distinct non-T_1 normal compactifications. Theorem 5. Assume (X,T) is T_1 and that B, C are normal bases where $B \subset C$. Then $w(X,B) \cong w(X,C)$ iff for each $h \in S(C)$ we have that $w(X,B)_h = w(X,C)_h$ as spaces. (Received April 23, 1973.)

73T-G92. JOE A. GUTHRIE, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. Ascoli theorems and the pseudocharacter of mapping spaces. Preliminary report.

Spaces are Hausdorff, maps are continuous, and α and β represent infinite cardinals. Let $\psi(X)$ represent the pseudocharacter of the space X and $\delta(X)$ the density. Let $\mathcal{C}_\alpha(X,Y)$ represent the space of maps from X to Y with a set-open topology finer than the topology of pointwise convergence, in which for each subbasic open set (A,U), $\delta(A) \leq \alpha$. Let $\mathcal{C}(X,Y)$ denote the space of maps with the compact-open topology. Theorem. $\psi(Y) \leq \psi(\mathcal{C}_\alpha(X,Y)) \leq \psi(Y)\,\delta(X)$. Theorem. Let X be completely regular and let Y contain a nondegenerate path. Then (1) if $\psi(\mathcal{C}_\alpha(X,Y)) \leq \beta$, then $\delta(X) \leq \alpha\beta$, and (2) $\psi(\mathcal{C}_\alpha(X,Y)) \leq \alpha\psi(Y)$ if and only if $\delta(X) \leq \alpha\psi(Y)$. Theorem. Let X be a separable sequential space and Y a regular space (uniform space) such that $\psi(Y) = \aleph_0$. Then $\mathfrak{F} \subset \mathcal{C}(X,Y)$ is compact if and only if (a) $\mathfrak{F}$ is closed, (b) $\mathrm{cl}\{f(x) \mid f \in \mathfrak{F}\}$ is compact for every $x \in X$, and (c) $\mathfrak{F}$ is evenly continuous (equicontinuous) on each convergent sequence in X. (Received April 23, 1973.)

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*704-B36. CHAO-PING CHANG, Davis Hall, 1164 East 58th Street, Chicago, Illinois 60637 and University of Auckland, Auckland, Private Bag, New Zealand. A stronger form of the fundamental theorem of calculus for complex line integrals.

In the present paper, the author proves a stronger form of the fundamental theorem of calculus for complex line integrals. Let $\gamma : [a,b] \to \mathbb{C}$ be a rectifiable curve on the complex plane $\mathbb{C}$ and let $|\gamma|$ be the range of γ, i.e., the set of all points on $\mathbb{C}$ traced by γ. The author proves that if f is a complex-valued function defined and continuous on $|\gamma|$, and if there exists a complex-valued function F defined on the set $|\gamma|$, such that for every point $z_0 \in |\gamma|$, $\lim(F(z) - F(z_0))/(z - z_0) = f(z_0)$ as $z \to z_0$ through values of z confined to $|\gamma|$, then $\int_\gamma f(z)\,dz = F(\gamma(b)) - F(\gamma(a))$. This is stronger than the existing version of the theorem, in which one has to assume that F is defined in some open set D containing the set $|\gamma|$, F has continuous complex derivative at all points of D and $F'(z_0) = f(z_0)$ for all points $z_0 \in |\gamma|$. The author also proves a stronger version of the theorem of change of variables for complex line integrals. The method of proof by the author is by way of complex Riemann-Stieltjes integrals. (Received March 6, 1973.)

704-C2. EDWARD W. NG, Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California 91103. Recent progress in the computerization of closed form integration. Preliminary report.

In recent years mathematical results by Risch [Trans. Amer. Math. Soc. 13(1969), 167-189, Bull. Amer. Math. Soc. 76(1970, 605-608] and computational implementation by Moses [Comm. ACM 8(1971), 548-560] have opened new vistas toward automation of analytic integration. In this presentation we first give a brief description of Risch's algorithm for the integration in terms of elementary functions. We then examine Moses' proposed ideas of field-extension operators [Comm. ACM 15(1972), 550-556] and describe some of our current attempts in the application of such algorithmic approaches to include special functions. (Received May 4, 1973.) (Author introduced by Professor Richard A. Askey.)

The June Meeting in Bellingham, Washington
June 16, 1973

Algebra & Theory of Numbers

Invited addresses are indicated by •

*705-A1. BRANKO GRÜNBAUM, University of Washington, Seattle, Washington 98195. Vertices missed by longest paths or circuits.

Let G denote a graph with v(G) vertices, and let c(G) and p(G) denote the maximal numbers of vertices contained in any simple circuit or path in G. We denote by c(j,k) [p(j,k)] the greatest lower bounds of v(G) where G is a k-connected graph such that for every choice of j vertices of G there exists in G a simple circuit [path] with c(G) [p(G)] vertices that misses the j chosen vertices. Analogously defined numbers with G restricted to planar graphs are denoted by $c_0(j,k)$ and $p_0(j,k)$. Extending various known results (see, for example, T. Zamfirescu, J. Combinatorial Theory 13(1972), 116-121) we prove: $c_0(1,3) \leqq 124$, $p_0(1,3) \leqq 484$, $c(2,3) \leqq 90$ and $p(2,3) \leqq 324$. (Received April 27, 1973.)

705-A2. J. STEPHEN MONTAGUE, University of Tennessee, Knoxville, Tennessee 37916. The nonexistence of certain rank 5 permutation groups. Preliminary report.

Several of the sporadic simple groups discovered in the past several years have arisen as transitive extensions of small rank of known groups. In Math. Z. 86(1964), 145-156, D. G. Higman has developed the techniques for investigating possible rank 3 extensions. In J. Algebra 14(1970), 506-522 and the present report, the author develops similar techniques for higher rank extensions and applies them to rank 5 groups having two doubly transitive constituents of different degrees. Theorem 1. Let G be a rank 5 group with subdegrees $d_0 = 1$, d_1, d_2, d_3, d_4, with d_1 and $d_2 > d_1$ corresponding to doubly transitive constituents. Then by a classical theorem of W. A. Manning either (i) $d_3 = d_1(d_1 - 1)/\ell_1$ and $d_4 = d_2(d_2 - 1)/\ell_2$, or (ii) $d_3 = d_1(d_1 - 1)/\ell_1 = d_2(d_2 - 1)/\ell_2$, where in either case ℓ_i is an integer less than $d_i - 1$. In case (i), either $d_2 = d_1\ell_2 + 1$ and $1 + \ell_1\ell_2$ divides $d_1 - 1$ or $d_3 + d_4 = d_1d_2$. Similar conditions can be obtained for case (ii), but they are much less compact. Theorem 2. (a) There do not exist groups G as in Theorem 1 with d_1 and d_2 being respectively 5 and 6, 7 and 8, 7 and 15, 8 and 15, or 22 and 56. (b) If rank 5 extensions of $Sp_{2n}(2)$ with $d_1 = 2^{n-1}(2^n - 1)$ and $d_2 = 2^{n-1}(2^n + 1)$ exist, they must come from case (ii) of Theorem 1. (Received May 3, 1973.)

Analysis

*705-B1. RICHARD C. GILBERT, California State University, Fullerton, California 92634. The deficiency index of an ordinary differential operator.

Let L be a formally selfadjoint linear ordinary differential operator defined on $[a,\infty)$. The dimensions n_i and n_{-i} of the spaces of solutions of $Ly = \pm iy$ in $L^2[a,\infty)$ are called the deficiency numbers of L, and the pair (n_i, n_{-i}) is called the deficiency index. The deficiency index is studied for the cases that the coefficients of L are constant, arbitrary, bounded, real, or pure imaginary. For a first order operator it is shown that the deficiency index is (1,1), (0,1) or (1,0). For the case $Ly = -iy^{(3)} + qy$, where $|q(x)| \to \infty$, it is shown that the deficiency index is (1,2) or (2,2). In this last case it is shown that the equation $Ly = \pm i\tau y$, $\tau \neq 0$, has 3 linearly independent solutions of the form $y_s = \rho^{-1}[1 + o(1)] \exp \int_{x_0}^{x} w_s(\xi)\, d\xi$, $s = 1,2,3$, where $\rho(x) = q^{1/3}(x)[1 - i\tau/q(x)]^{1/3}$, $w_s(x) = \omega_s \rho(x)$, $\omega_1, \omega_2, \omega_3$ are the cube roots of $-i$. The square integrability of these solutions is then studied. (Received April 5, 1973.)

*705-B2. BANSHI D. MALVIYA, North Texas State University, Denton, Texas 76203. Finite dimensionality and duality of B^*-algebras.

For the notations and terminology used in this abstract, a reference is made to Rickart's book on Banach algebras. We prove the following theorems: Theorem 1. A weakly completely continuous B^*-algebra with an identity is finite dimensional. Corollary 2. A reflexive weakly completely continuous B^*-algebra is finite dimensional. Theorem 3. Let A be a B^*-algebra. Then the following statements are equivalent: (1) A is a dual algebra; (2) for every maximal modular left ideal M of A there exists a right identity modulo M that is weakly completely continuous. Theorems 1 and 3 were obtained earlier by B. J. Tomiuk [Glasgow Math. J. 13(1972), 56-60] using complicated methods. We give the proof of these theorems based on well-known results and techniques. (Received April 20, 1973.)

*705-B3. KALMAN G. BRAUNER, JR., University of Maryland, College Park, Maryland 20742. Duals of Fréchet spaces and a generalization of the Banach-Dieudonné theorem. Preliminary report.

The purpose of this paper is to explore a class of locally convex spaces that can be characterized by the fact that they are duals of Fréchet spaces with the topology of precompact convergence. I call these spaces dF spaces, since the category of such spaces is equivalent to the dual category of the category of Fréchet spaces. In §1, dF spaces are shown to be k-spaces, Krein-Šmulian spaces, and semi-Montel spaces; and they are shown to remain stable under the taking of closed subspaces, separated quotients, and countable direct sums. In §2, spaces of continuous linear maps from Fréchet spaces to dF spaces and from dF spaces to Fréchet spaces play the central role. With the topology of precompact convergence, it is shown that a set of continuous linear maps is precompact if the operators in the set are "uniformly compact". In addition, a generalization of the Banach-Dieudonné theorem is proved where, rather than the scalars, the codomain space is allowed to be any dF space. Thus if E is metrizable and F is dF, then the topology of precompact convergence is the finest topology that gives to equicontinuous, pointwise precompact subsets of L(E, F) the same relative topology as the topology of pointwise convergence. (Received April 27, 1973.)

*705-B4. ROBERT M. KAUFFMAN, Western Washington State College, Bellingham, Washington 98225. Two types of hyperinvariant subspaces.

Let A be a bounded operator in a Banach space B. Suppose that A has the single valued extension property. Given a closed set F in the complexes, define $\sigma_A(F)$ to be the set of all x in B such that there is an analytic function $x(\lambda)$ from the complement of F to B with $(A - \lambda I)x(\lambda) = x$. A is said to have property Q if $\sigma_A(F)$ is a closed subset of B for every F. Let A be, again, a bounded operator in a Banach space B. Given a real number b, define $S_A(b)$ to be the set of all x in B such that exp(-ct)exp(At)x is a bounded function from the nonnegative reals to B for all $c > b$. A is said to have property P if $S_A(b)$ is a closed subspace of B for all b. These two properties are discussed in this paper. (Received April 30, 1973.)

*705-B5. HASKELL P. ROSENTHAL, University of California, Berkeley, California 94720. The Banach spaces $C(K)$ and $L^p(\mu)$.

• Besides their natural intrinsic interest, $C(K)$- and $L^p(\mu)$-spaces play a central role in the theory of general Banach spaces. For example, one of the earliest results is that every separable Banach space is isometric both to a subspace of $C[0,1]$ and a quotient space of $L^1[0,1]$. On the other hand, a striking result of Grothendieck asserts that if a Banach space is both isomorphic (linearly homeomorphic) to a quotient space of $C[0,1]$ and a subspace of $L^1[0,1]$, then this space is isomorphic to a Hilbert space. It is moreover an open question if every infinite-dimensional Banach space contains a subspace isomorphic to c_0 or to ℓ^p for some $1 \leq p < \infty$. Recent results suggest that the answer to this question may be affirmative. This talk will deal mainly with isomorphism problems concerning the separable $C(K)$- and L^p-spaces. The central problems are to characterize the complemented subspaces of these spaces, and to determine properties of their subspaces and quotient spaces. Although complete solutions to these problems may be unattainable, remarkable progress has recently been made concerning them. Some of this progress, as well as the main research techniques involved in it, will be surveyed. The rich interconnections between this field and others, such as probability theory and harmonic analysis, will also be discussed. Particular attention will be devoted to applications of stable random variables and recent discoveries connecting the theory of Λ_p sets with the theory of reflexive subspaces of $L^1[0,1]$. (Received May 2, 1973.)

705-B6. BURTON RODIN, University of California, San Diego, La Jolla, California 92037. The method of extremal length.

• The concept of extremal length is an outgrowth of the length-area estimates used in classical complex analysis. In its modern form it provides a convenient tool which has proved useful in a wide variety of areas. This talk will survey the fundamental results and some recent applications. (Received May 3, 1973.)

*705-B7. SADANAND VERMA, University of Nevada, Las Vegas, Nevada 89154. On a particular solution of certain differential equations via linear mapping of finite dimensional vector spaces. Preliminary report.

A commonly used method in obtaining a particular solution y_p of a class of ordinary nonhomogeneous linear differential equations $L(y) = f(x)$ is "the annihilator method," also known as the method of undetermined coefficients or the method of judicious guessing. All books on the subject illustrate the applicability of the method by working out exercises rather than giving a general proof of the fact that the method must work out if $f(x)$ belongs to a certain class of functions. The purpose of this paper is twofold: (1) To give a general proof of "the annihilator method" by making use of the theory of linear mappings of finite dimensional vector spaces after suitably restricting the domain V and carefully selecting the range W of the linear operator L associated with the given differential equation. (2) To simplify the method of construction of a particular solution y_p. In fact, it turns out that y_p is the product of the inverse of the matrix associated with the linear transformation L cut down to V and the vector $f(x)$, where $f(x)$ is regarded as a column vector in the range of L/D with regard to a suitable basis. (Received May 3, 1973.)

*705-B8. BADRI N. SAHNEY and D. S. GOEL, University of Calgary, Calgary, Alberta T2N 1N4, Canada. On the degree of approximation of continuous functions. Preliminary report.

The degree of approximation of a periodic function f with period 2π and belonging to the class $\mathrm{Lip}\,\alpha$ is given by (*) $\max_{0 \leq x \leq 2\pi} |f(x) - T_n(x)| = O((1/P_n)\sum_{k=1}^{n} P_k/k^{1+\alpha})$ where $T_n(x)$ are (N, p_n) means of its Fourier series, and the sequence $\{p_n\}$ is positive and nonincreasing, such that $P_n = p_0 + p_1 + \dots + p_n \to \infty$ as $n \to \infty$. Corollary 1. If $f(x)$ is the same as above, then the degree of approximation of (C, δ) means of its Fourier series for $0 < \alpha < \delta \leq 1$ is of the order $n^{-\delta}$. Corollary 2. For $0 < \alpha \leq \delta \leq 1$, the degree is of the order $n^{-\alpha} \log n$. (Received May 3, 1973.)

Applied Mathematics

*705-C1. FREDERICK N. SPRINGSTEEL, University of Montana, Missoula, Montana 59801. Classes of sentences recognizable in (exponentially) less space than their length, viewed algebraically as abstract families of languages (AFLs). Preliminary report.

The languages (sets of sentences) mentioned above have been characterized by Chomsky's formal models (called context-sensitive grammars) for natural and programming languages. As such they share many common algebraic properties which have come to be accepted as the definition of an AFL (see "Studies in abstract families of languages," Mem. Amer. Math. Soc., no. 87, 1969). Here several of these classes of languages—especially those recognizable in exponentially less space than sentence length (denoted by n)—are shown to have interesting properties in addition to those of AFLs and pre-AFLs. Focus is on the open problem of determinism vs. nondeterminism and especially on whether $[\log n]^2$ is required for recognition of (a simpler model) context-free (or ALGOL-like) (CF) languages. Some typical theorems, where one identifies the special class mentioned above as the $[\log n]$ tape complexity class, are: Theorem. The $[\log n]$ class enjoys all the defining properties of a "pre-AFL" but is not a full AFL (closed under arbitrary homomorphisms). Theorem. If the $[\log n]$ class is an AFL, then it contains all CF languages. (Notice that if the hypothesis here is true, the conjecture that $[\log n]^2$ is required is false.) (Received May 3, 1973.)

Geometry

*705-D1. RICHARD R. JOSS, University of Washington, Seattle, Washington 98195. On Wiener's shortest-line conjecture.

Norbert Wiener proved that the curve of least length dividing an area in a given ratio will consist of one or more arcs of circles (Proc. Cambridge Philos. Soc. 18(1914), 56-58). The present paper proves Wiener's final conjecture: "It is almost self-evident that the shortest line to divide a convex area in a given ratio is a single arc of a circle, but this I have not been able to prove." (Received April 30, 1973.)

Logic and Foundations

705-E1. STEVEN R. GIVANT, University of California, Berkeley, California 94720. A representation theorem for quasi-varieties categorical in power.

For q.v. (quasi-variety) or v. (variety) K, $\mathfrak{F}_\omega K$ denotes the K-free algebra on ω generators, K_∞ the class of infinite $\mathfrak{A} \in K$, and Th(K) the first-order theory of K. Given a set A, define c_n^A, a unary, and d_n^A an n-ary operation on ${}^nA$: $c_n^A(\langle x_1,\ldots,x_n\rangle) = \langle x_2,\ldots,x_n,x_1\rangle$; $d_n^A(\langle x_{1,1},\ldots,x_{1,n}\rangle,\ldots,\langle x_{n,1},\ldots,x_{n,n}\rangle) = \langle x_{1,1},\ldots,x_{n,n}\rangle$. Set $K_n = \{\mathfrak{B} : \exists_{\mathfrak{A}}(\mathfrak{A} \in K \wedge \mathfrak{B} \cong \langle {}^n\mathfrak{A}, d_n^A, c_n^A\rangle)\}$. Theorem 1. If a q.v./v. K is categorical in some power $\geqq |\mathfrak{F}_\omega K|$ and $\mathrm{Th}(K_\infty)$ has a strongly minimal formula, then there is a q.v./v. L, all of whose nontrivial members are L-free, and an integer $n \geqq 1$ so that K is polynomially equivalent to L_n. Q.v./v. with all nontrivial members free have been described by the author, Abstract 72T-E112, these *Notices* 19(1972), A-767. K' trivially extends K if Th(K') extends Th(K) in a language with at most one new individual constant, and every $\mathfrak{A} \in K$ is the reduct of a $\mathfrak{B} \in K'$. Theorem 2. If q.v./v. K is categorical in some power $\kappa \geqq |\mathfrak{F}_\omega K|$ there is a κ-categorical q.v./v. K' trivially extending K and $\mathrm{Th}(K'_\infty)$ has a strongly minimal formula. If Th(K) has a constant term, we may take K' = K. Corollary. For K as in Theorem 2 with $|\mathfrak{F}_\omega K| > \omega$, K_∞ has a prime model and every $\mathfrak{A} \in K_\infty$ has a minimal prime extension. Theorem 3. If q.v. K with $|\mathfrak{F}_\omega K| = \omega$ is not ω-categorical, but $\mathfrak{F}_\omega K$ is the only countable member not finitely generated, then K is ω_1-categorical. (Received April 23, 1973.)

705-E2. DON L. PIGOZZI, Iowa State University, Ames, Iowa 50010. On the decision problem for equational theories of quasi-groups. Preliminary report.

For notation and terminology see [Abstract 72T-E108, these *Notices* 19 (1972), A-766]. Thus the notions of an (equational) theory and of an equation generating a theory are assumed understood. By a theory of quasi-groups we mean any theory with binary operation symbols $\cdot$, $/$, $\backslash$ which contains the following equations: $x\cdot(x\backslash y)=y$, $x\backslash(x\cdot y)=y$, $(x/y)\cdot y=x$, $(x\cdot y)/y=x$. Theorem 1. For any Turing degree δ there exists an equation $\sigma=\tau$ with operation symbols $\cdot$, $/$, $\backslash$ such that $\sigma=\tau$ generates a theory of quasi-groups of T-degree δ. Theorem 2. Let E be the set of all equations in the operation symbols $\cdot$, $/$, $\backslash$ which individually generate a decidable theory of quasi-groups. Then E is a maximal Σ_3-set in the Kleene-Mostowski hierarchy. The proofs are based in part on a proof that there exists a uniform way of interpreting an arbitrary equational theory θ in a theory θ' of quasi-groups of the same T-degree. The results of both Theorems 1 and 2 can be extended to various nonassociative theories. (Received May 3, 1973.)

Statistics and Probability

*705-F1. ALBERT W. MARSHALL, University of Rochester, Rochester, New York 14627 and INGRAM OLKIN, Stanford University, Stanford, California 94305. Majorization in multivariate distributions.

A real function φ of n real variables is called Schur-concave if $\varphi(x) \geqq \varphi(y)$ whenever $x \prec y$ in the partial ordering that Hardy, Littlewood and Pólya call "majorization." It is shown that if φ and f are Schur-concave, then under appropriate integrability conditions, $\int\varphi(x-\theta)f(x)\,dx$ is a Schur-concave function of θ. As consequences, various inequalities for multivariate distributions are obtained. (Received May 1, 1973.)

Topology

*705-G1. JAMES V. WHITTAKER, University of British Columbia, Vancouver 8, British Columbia, Canada. On normal subgroups of differentiable homeomorphisms.

Let X be the line and $E_n(X)$ the family of all homeomorphisms from X onto X that have at each point of X a local polynomial approximation of degree n. Then $E_n(X)$ is a group under composition, and its normal subgroups are studied in this paper. The minimal normal subgroup of $E_n(X)$ is shown to be the family $SE_n(X)$ of all those members with compact support, and $SE_n(X)$ turns out to be a simple group. The commutator subgroup of $E_n(X)$ is shown to be the family $E_n^+(X)$ of all monotone increasing members. The results are just like those in the topological case. (Received April 17, 1973.)

*705-G2. ULRICH KOSCHORKE, Rutgers University, New Brunswick, New Jersey 08903. Two-plane fields and bordism.

Let $\Omega_n(2)$ (resp. $\Omega_n^{or}(2)$) be the bordism group of oriented, smooth n-manifolds with arbitrary (resp. oriented) 2-plane fields, and let $\mathfrak{N}_n(2)$ and $\mathfrak{N}_n^{or}(2)$ be the corresponding bordism groups based on unoriented manifolds. The natural forgetful maps f from these groups into the corresponding bordism groups of BO(2) (resp. BSO(2)) fit into long exact sequences with third term 0, $\mathbf{Z}_2$ or $\mathbf{Z}_2 \oplus \mathbf{Z}_2$, depending on n and the orientation conditions. Here a bordism class $[M,\xi]$ of $B(S)\circ(2)$ (where ξ is a 2-plane-bundle over the n-manifold M) gets mapped into a linear combination of the Whitney numbers $w(M)w(\xi)^{-1}[M]$ and $(w_1(M)+w_1(\xi))w(M)w(\xi)^{-1}[M]$. This leads to the determination of the groups $\Omega_n(2)$, $\Omega_n^{or}(2)$, etc., e.g., f is injective in all four cases except that the kernel of f in $\mathfrak{N}_n^{or}(2)$ is $\mathbf{Z}_2$ for even n. All this can be applied to foliations, e.g., every element in $\mathfrak{N}_n^{or}(2)$ can be represented by a plane field which is transversal to a foliation. Also, every oriented manifold M^n $(n\neq 5)$ is orientedly bordant to one with a 2-codimensional foliation $\mathfrak{F}$. The normal bundle of $\mathfrak{F}$ can be orientable iff M^n has even Euler number. (Received May 3, 1973.)

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MATHEMATICS PROFESSOR, TEACHING AND RESEARCH. Age 31. Ph.D. 1972. Several published articles. Interest: Numerical Analysis, Statistics and Operations Research. Several years experience in teaching; one year in a Canadian University. Available Sept. 1973. B. Datta, Mathematics Department, University of Ottawa, Ottawa, Canada.

MATHEMATICS INSTRUCTOR. M.A. 1970. Major: Mathematics, Minor: Statistics. Age 27. Married. Four years college and university teaching experience. Computer programming experience. References and résumé available upon request. Available June 1973. Paul W. Guy, 740 Santa Barbara, Pasadena, California 91106, 213-795-1651.

MATHEMATICS TEACHING OR ADMINISTRATION. Ph.D. 1962. Age 36. Differential Equations. Eleven years teaching experience, three as department chairman. Primary interest: teaching in small, liberal innovative college where individuals have chance to develop. Available Fall 1973. Vita and references on request. John G. Moore, 1116 Cherokee St., Orangeburg, South Carolina 29115.

MATHEMATICS PROFESSOR, TEACHING AND RESEARCH. University of London Ph.D. 1965. Seven published papers, two submitted. Many years of college and university teaching, including three years in America. Wide range of undergraduate and graduate courses, including Elementary Statistics and Operations Research. S. Mukhoti, F-1919-10, University Drive, Cedar Falls, Iowa 50613.

MATHEMATICS PROFESSOR. Age 28. Ph.D. 1972. University of Georgia under J. G. Horne. Topological Algebra and Semigroups. One article accepted for publication and one submitted. One year teaching experience. References and résumé on request. Available August 1973. W. Sheffield Owen, Department of Mathematics, Auburn University, Auburn, Alabama 36830.

MATHEMATICS INSTRUCTOR. Age 28. Ph.D. Yale 1970. Instructor at Yale, 1 yr., MIT 2 yrs. Published research, representation of Lie groups. Co-author of 2 freshmen texts. Dedicated teacher. Résumé and references on request. Leonard Richardson, 17 Short St., Brookline, Massachusetts 02146.

MATHEMATICS PROFESSOR. Ph.D. 1968. Age 30. Speciality: Analysis. Five years teaching experience as an assistant professor in graduate and undergraduate courses. Three publications. Available summer 1973. Résumé and references on request. Leonard Sternbach, Department of Mathematics, University of South Carolina, Columbia, South Carolina 29208.

MATHEMATICIAN, TEACHING AND RESEARCH. Purdue Ph.D. 1965. Age 32. Specialties: Real Analysis, General Topology. One article published; in past year, two accepted for publication, two more submitted. Seven years teaching experience. Any teaching rank with opportunity to continue research. Available immediately. Reference and résumé upon request. H. E. White, Jr., 251 N. Blackburn Road, Athens, Ohio 45701.

ANONYMOUS

RESEARCH MATHEMATICIAN. Ph.D. 1962. Age 36. Speciality: Algebra, Number Theory. Four published articles. Over ten years experience in teaching and academic research in Britain, Germany and U.S. Any region. Available September 1973. SW17

MATHEMATICS PROFESSOR, TEACHING OR TEACHING AND RESEARCH, Ph.D. 1963. Age 39. Specialties: Algebraic Topology, General Topology, Analysis, Applied Mathematics. Publications. Post-doctoral research, graduate and undergraduate teaching experience in U.S., Canada and abroad. Qualified to teach core courses in pure, applied and engineering mathematics. Employed. U.S. preferred. Vita and references upon request. SW18

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